GENERALIZATION OF RESTRICTED PLANAR LOCATION PROBLEMS: UNIFIED META-HEURISTICS FOR SINGLE FACILITY CASE

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ABSTRACT

GENERALIZATION OF RESTRICTED PLANAR LOCATION PROBLEMS:
UNIFIED META-HEURISTICS FOR SINGLE FACILITY CASE

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A planar single facility location problem, also known as the Fermat–Weber problem, is to find a facility location such that the total weighted distance to a set of given demand points is minimized. A variation to this problem is obtained if there is a restriction coming from congested regions. In this study, congested regions are considered as arbitrary shaped polygonal areas on the plane where location of a facility is forbidden and traveling is charged with an additional fixed cost. The traveling fixed cost or penalty can be thought of the cost of risks taken when passing through the region or the cost of purchasing license or special equipment in order to be able to pass through the region. In this study we show that the restricted planar location problem with congested regions having fixed traveling costs maintains generality over two most studied related location problems in the literature, namely restricted planar facility location problems with forbidden regions and barriers. It is shown that this problem is non-convex and nonlinear under Euclidean distance metric; hence using heuristic approaches is reasonable. We propose three meta-heuristic algorithms, namely simulated annealing, evolutionary algorithm, and particle swarm optimization based on variable neighborhood search to solve the problem. The proposed algorithms are applied on test instances taken from the literature and the favorable computational results are presented.

Keywords: Single facility location, Restricted location, Congested regions, Meta-heuristic

ÖZ

KISITLI DÜZLEMSEL YER SEÇİMİ PROBLEMLERİNİN GENELLEŞTİRİLMESİ: TEK TESİS ÖRNEĞİ İÇİN BİRLEŞTİRİLMİŞ META-SEZGİSELLER

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Fermat–Weber problemi olarak bilinen düzlemsel tek tesis yerleştirme problemi, verilen talep noktalarına ağırlıklandırılmış mesafelerin toplamını en aza indirecek bir tesisin yerini seçmek-tir. Eğer kalabalık bölgelerden gelen bir kısıt olursa bu problemin bir varyasyonu elde edilir. Bu çalışmada, kalabalık bölgelere bir tesisin yerleştirilmesi yasaktır ve bu bölgelerden geçmek için ek sabit bir maliyet tahsil edilmektedir. Bölgeler rastgele şekillendirilmiş poligonal alanlara olarak kabul edilmektedir. Sabit seyahat maliyeti veya cezası, bölgeyi geçebilmek için gereken lisans yada özel ekipman satın almanın maliyeti, yani geçişte alınan risklerin maliyeti olarak düşünülebilir. Bu çalışmada, kalabalık bölgelerdeki sabit yolculuk maliyetini içeren sınırlı düzlemsel tek tesis yerleştirme probleminin, literatürde en çok çalışılan ilgili iki yer seçimi probleminin, yani yasak bölgeleri ve bariyerleri içeren kısıtlı düzlemsel tesis yerleşim problemini genelleştirildiği gösterilmiştir. Bu problemin Öklid mesafe metriği altında, dış-bükey olmadığı ve nonlineer olduğu gösterilmiştir; dolayısıyla çözüm için sezgisel yaklaşım-ların kullanılması doğaldır. Üç meta-sezgisel yöntem önerilmiştir. Bunlar tavlama benzetimi, evrimsel algoritma ve değişken komşuluk arama esaslı parçacık sürüsü eniyileme yöntemleridir. Önerilen yöntemler literatürden alınan örnek test problemleri üzerinde uygulan-mış ve olumlu hesaplama sonuçları elde edilmiştir.

Anahtar Kelimeler: Tek tesis yerleşimi, Yasaklı yerleşim, Sıkışık bölgeler, Meta-sezgisel yöntemler

To my family
for their love and unconditional support.

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CHAPTER 1

INTRODUCTION

The classical planar single facility location problem, also known as the *Fermat–Weber problem* or simply the Weber problem, is about placing a single facility on the plane to serve a finite number of demand points having different demand levels. Its aim is to find a facility location to minimize the sum of weighted distances from the facility to the demand points. While the Weber problem is well-studied in the literature, it fails to model many real life location problems. In the real life situations, there might be restrictions on the location of the facility or on the origin-destination travel paths. These limitations mostly come from environmental and geographical factors. For example, consider the existence of a lake, rivers or an urban area on the plane in which the facility is going to be located. If there are some regions that restrict the facility placement on the plane, the problem is called the *restricted (constrained) facility location problem*. Some of the restricted regions might also limit the facility-demand routes such that passing through them becomes costly, risky, or impossible. Examples of such regions are national parks, military zones, and mountains. Traveling through a national park may require paying a certain penalty whereas passing over a mountain is prohibitively expensive.

All restricted planar location problems have two common properties.

1. The facilities cannot be located within certain restricted regions on the plane.
2. The minimum travel distance between any two points in the plane may get longer by the presence of the restricted regions.

Having these properties, restricted facility location problems can be classified according to their constraint type. If the location of the facility is prohibited in some regions but traveling through those regions is free, the problem is called restricted facility location with *forbidden regions*. An example of forbidden regions is protected areas where facility construction is not allowed because of the recognized natural, ecological, and/or cultural values of those areas.

Another class of restricted facility location problems is the problems with *barriers*. Barriers are referred to those regions inside which facility location is infeasible and through which traveling is impossible. Examples of barrier regions are lakes and mountains that are obstacles to travel.

The other class of such problems is the restricted facility location problem with *congested regions*. In the limited literature of restricted location problems, congested regions are defined as the regions that forbid placing of a facility but allow traveling at an extra cost. For example, traffics on the roads of a city considerably increase traveling cost. These problems are studied under the rectilinear distance metric while the traveling costs within or through congested regions are assumed to be equal to a certain cost per-unit distance traveled.

When the restrictions are included – whether they are imposed by forbidden regions, barriers, or congested regions – the underlying problem becomes a non-convex optimization problem on a non-convex feasible set that results in computational difficulties. Providing solution approaches becomes more difficult under Euclidean distance measure where the objective function of the problem becomes non-linear.

In this study, we consider the restricted planar single facility location problem under Euclidean distance metric where the restrictions are caused by the presence of congested regions on the plane where passing through them is penalized by certain *fixed* costs. Our problem is different than the related location problems with congested regions in three ways. Firstly, the available studies consider the problem under rectilinear distance measure, while our problem uses Euclidean distance norm. Secondly, the traveling costs of congested regions addressed in this study are fixed. Other studies, however, consider a certain cost per unit distance traveled in a congested region. Finally, like most restricted planar location problems, they assume each congested region is a convex polygon. On the other hand, congested regions in this study can have both convex and non-convex polygonal shapes. We show that the problem studied in this thesis, i.e. planar single facility location problem restricted by congested regions with fixed traveling costs, is a general case of the two most studied related problems in the literature, namely restricted planar location problems with forbidden regions and barriers. Moreover, lower and upper bounds for the problem are introduced. The structure of our problem allows us to have different constraint types instead of having exactly one of the solid forbidden region or barrier restrictions.

The presented solution approaches for the problem is based on well-known meta-heuristic approaches. The implemented meta-heuristics are *simulated annealing*, *evolutionary algorithm*, and *particle swarm optimization*. Besides, the techniques of *variable neighborhood search* are used in the search procedure of the proposed algorithms. We believe that this study is the first that provides extended consideration over a general case of restricted planar single facility location problem and presents meta-heuristic solution approaches.

To illustrate the performance of the meta-heuristics, all available related problem instances in the literature are solved. Additional test problem instances are also generated in a standard scheme using the available instances. Furthermore, to assess the performance of our heuristics on large problem instances we run our heuristics on the test instances taken from the traveling salesman problem and the vehicle routing problem literature.

In this thesis, we review the literature of the restricted planar location problems in Chapter 2. The literature of meta-heuristics applications on the location problems is also reviewed in the same chapter. Next, formulation and applications of the restricted planar single facility location problem with congested regions having fixed costs are given in Chapter 3. The features of the meta-heuristic algorithms and the details about the proposed solution approaches are presented in Chapter 4. In Chapter 5, we first provide information about the test problem instances. Then, preliminary experiments on the test problems and parameter adjustment for each heuristics are explained. Afterwards, analysis of the computational experiments and the performance of meta-heuristics are given. Finally, Chapter 6 states conclusive remarks, extensions to the problem and future works.

CHAPTER 2

LITERATURE SURVEY

The well-known Weber problem is a single facility location problem on the plane with the objective of minimizing total facility-demand weighted distances. The application of this problem is studied in early 20th century by Weber (see Francis et al., 1992). To deal with this problem, Weiszfeld proposed an algorithm in 1937 that starts with an initial solution and iteratively updates the solution based on the weights and distances until it converges. The *Weiszfeld's algorithm* benefits from the fact that this problem is an unconstrained optimization problem of a convex function, however, its performance is highly sensitive to the initial solution. More details on the Weiszfeld's algorithm, other facility location models, solution approaches, and applications can be found in Drezner and Hamacher (2002). Besides, classification for various location problems is provided by Hamacher and Nickel (1998).

When dealing with real life planar facility location problems, we often face environmental restrictions that limit our location decisions. To illustrate, placing facilities on the lakes or in national parks cannot be possible. When such restrictions are imposed on the problem, the classical available solution methods do not work anymore. Despite their numerous real life applications, *restricted planar facility location problems* have not attained much attentions compared to the unrestricted problems in the location literature. Although the problem is shown to be non-convex which makes it hard to solve (see Katz and Cooper, 1981), there are a few studies that provide an optimal solution approach.

In this chapter an overview of the literature on the restricted planar location problems is given. Moreover, since we are using meta-heuristic approaches for the solution of the problem, the literature of meta-heuristic approaches for facility location problems is also considered. In Section 2.1 we provide a literature survey for restricted planar facility location problems. A review on meta-heuristic approaches for related facility location problems is given in Section 2.2.

2.1 Restricted Planar Facility Location Problems

The Weber problem with restrictions on facility location and/or traveling has been considered widely in recent years (see Butt and Cavalier, 1997, Hamacher and Nickel, 1995, Klamroth, 2002). Depending on the type of restrictions, such problems are divided into three categories. The category considers planar facility location problems in which restrictions come from existence of *forbidden regions*. Forbidden regions refer to prohibition of facility placement but allowance of free traveling. In the second category, restrictions are imposed by *barriers*. Barrier are defined as regions where neither locating a facility on nor passing through is allowed. The last category considers restricted planar location problems with *congested regions*. Congested regions are bounded areas in the plane that forbid facility location however passing through their interior is possible at some extra traveling cost. In the following sections, the details of these categories are reviewed.

2.1.1 Restricted Problems with Forbidden Regions

Hamacher and Nickel (1995) provided a broad overview on facility location problems with forbidden regions. They consider both center and median problems as well as their applications and provided some solution approaches to these problems.

Batta et al. (1989) proposed a solution method for the planar p-median problems with both arbitrary convex forbidden regions and arbitrary shaped barriers. While employing rectilinear distance metric for the problems, the authors showed that the search for an optimal solution can be limited to a finite set of points. Their solution procedure was dividing the plane into cells in which the objective function is convex. The cell formation technique (also called grid construction method) is useful when coping with rectilinear distances. This method was introduced by Larson and Sadiq (1983) for the planar location problems with barriers.

The planar location problems with forbidden regions and Euclidean distance metric was further studied by Aneja and Parlar (1994). They claimed that the optimal solution for the unconstrained problem (the Weber problem) is a lower bound for the location problem restricted by forbidden regions. For a special case, they showed that if the optimal solution to the unrestricted problem is feasible in the restricted one, it also satisfies optimality in the restricted problem with forbidden regions. Next, they proved that if the solution to the constrained problem becomes infeasible and falls inside a region when constraints are considered, then the optimal solution to the constrained problem will fall on the boundary of that region. Based on this idea, they proposed an algorithm for convex polygonal forbidden regions that starts with the solution of unconstrained problem and in the case of infeasibility, it searches edges of the corresponding forbidden region for the optimal solution to the constrained problem. They also provide an efficient method to find the optimal solution when forbidden regions are non-convex polygons. The proposed solution procedures are then used to find the exact solution to a problem instance which considers locating a facility on the plain with a non-convex polygonal forbidden region.

The facility location problem with forbidden regions was studied further by Muñoz-Pérez and Saameño-Rodríguez (1999) who considered the problem of locating an undesirable facility in a bounded polygonal region with polygonal forbidden regions, using Euclidean distances. They considered the problem with an objective function that generalizes the maximin and maxisum criteria, and includes other criteria such as the linear combinations of them. When the objective is maximin (maxisum), the undesirable facility is located such that its minimum (total) distance to the set of given existing points is maximized while satisfying forbidden region constraints. The authors identified a finite set of dominating solutions for this problem and indicated that an optimum solution could be found in polynomial time in the number of vertices of the regions and the number of demand points.

Location problems constrained by forbidden regions are also studied in the area of minimax objective functions and Euclidean distance metric. Hamacher and Schöbel (1997) provide a polynomial time algorithm to find the optimal solution when forbidden regions are convex polygons. The algorithm is based on *level curves* and *level sets* of the objective function. The procedure starts with the solution to the unconstrained problem and if that solution is infeasible the algorithm search on the edges of the restricting forbidden region for candidate solutions. Woeginger (1998) proposed a faster algorithm for this problem by applying standard techniques from computational geometry.

A connection between the location problem with forbidden regions and congested regions with fixed cost is established when all fixed costs of congested regions are set to zero. In chapter 3 the detailed discussion will be provided.

2.1.2 Restricted Problems with Barriers

A special case for facility locations with forbidden regions is obtained when the traveling through regions also becomes restricted. Regions that forbid both facility location and traveling are called barriers – they are also called forbidden regions sometimes (see Butt and Cavalier, 1996), but to make a distinction between the forbidden region concept given in this study, we use the term of barriers. A comprehensive overview about the continuous location problems incorporating barriers is provided by Klamroth (2002).

Presenting barriers as a restriction on planar location problems was first introduced by Katz and Cooper (1981). They studied The Weber problem in the presence of one circular barrier while considering Euclidean distance measure. The authors showed that the objective function of this problem is non-convex and discontinuous. They also proposed a heuristic solution approach based on a sequential unconstrained minimization technique for nonlinear problems. The authors provide some problem instances where restrictions come from circular barriers. Klamroth (2004) analyzed algebraic properties of the same problem and introduced a solution procedure based on dividing the feasible region into some convex regions in which the objective function of the Weber problem is convex. In this procedure, the number of convex regions depends polynomially on the number of demand points. As the set of demand points becomes larger, construction of such convex regions becomes harder and, thus, not preferable in practice. The studies provided in Katz and Cooper (1981) and Klamroth (2004) are different from ours as we assume polygonal regions as a restriction on the problem. Nevertheless, Butt and Cavalier (1996) and Bischoff and Klamroth (2007) worked on the problem instances given in Katz and Cooper (1981) by approximating the circular regions by regular convex polygons (e.g. hexagons). In this way they were able to solve the problem instance when the assumption of convex polygonal regions holds.

The planar location problems with barriers was further studied by Aneja and Parlar (1994). The authors studied the problem under general l_p–metric distances and convex or non-convex polygonal barriers assumptions. They proposed a solution procedure that consists of simulated annealing meta-heuristic for generating candidate locations for facility under Euclidean distance measure. For each generated solution, they consider the problem as a network problem by using *visibility graph* concept. Visibility graph is a graph of inter-visible locations, generally for a set of points and obstacles in the plane. Each node in the graph represents a point location (demand point, facility, or region vertex), and each edge represents a visible connection between them. That is, if the direct line segment connecting two locations does not pass through any obstacle, an edge is drawn between them in the graph (see Klamroth, 2001a, 2002). The visibility graph is constructed in order to find the *shortest path* between any candidate location and demand points. To find such a path they used *Dijkstra's algorithm* which runs in polynomial time. The authors also present some problem instance in which there exist both convex and non-convex polygonal barriers. To deal with such problem, they used simulated annealing meta-heuristic approach and reported the solutions to those problems. Some other variants to the same problem instance is given and solved in Aneja and Parlar (1994) by changing the number of regions in the original problem. Although the reported solutions are found using a heuristic method, they are later verified by Butt and Cavalier (1996) and Bischoff and Klamroth (2007) as the best solutions known for those problems.

Butt and Cavalier (1996) considered the restricted problem with convex polygonal barriers and Euclidean distance metric. They developed an iterative algorithm to find some local optima to the problem. The authors used the same visibility graph concept presented in Aneja and Parlar (1994). The solution procedure consists of partitioning the feasible region into subregions in which the shortest barrier distance between two points remain constant throughout the region, i.e., the shortest path between two points passes through same points. By solving the unconstrained problem in each of such regions they obtain a local optimum to the original problem. The main disadvantage of this approach is that the boundaries of subregions are generally nonlinear and not easily determined. To avoid finding boundaries of subregions, they developed a heuristic search algorithm that finds a local optimum to the problem. They also gave a problem instance with two convex polygonal barriers to illustrate the proposed heuristic algorithm. With their methodology, the authors also verified the best found solution for one of the problem instances given in Aneja and Parlar (1994).

A different decomposition method for both center and median restricted location problems with barriers was introduced by Klamroth (2001a). The author suggested *visibility grid* approach where the feasible region is divided into cells. Constructing visibility grid is also based on the visibility graph which converts the problem into network. The cells formed by this method has linear boundaries, making this method more efficient than the decomposition method given in Butt and Cavalier (1996). The author proved that in each cell of the constructed grid, the objective function is convex. Therefore, the exact solution of the non-convex problem can be obtained by reducing it to a finite number of convex subproblems and solving these underling problems. Still, the number of subproblems depends on the number of demand points and barrier vertices that make inefficient to generate and to solve all subproblems. An algorithm was provided which finds the set of global minima by searching all cells and boundaries in the constructed grid. Because this algorithm is computationally expensive, the author provided a second algorithm that can find a heuristic solution to the problem. However, a high quality solution requires a large number of iteration in this algorithm and thus decreases its efficiency.

To overcome the difficulties arising from subproblem generation, Bischoff and Klamroth (2007) introduced a genetic algorithm to find a heuristic solution to the problem. In the proposed algorithm subproblems are selected in an iterative manner to find the candidate solutions of the global problem. The visibility concept are also used in the study to reduce the number of subproblems that need to be considered. The authors also considered appropriate assignment of points in the facility-demand shortest paths. The proposed solution approach is only valid when barriers are convex polygons. In their study, Bischoff and Klamroth tried to solve some problem instances which are either from the literature or generated arbitrarily. The problems from the literature consist of those given in Katz and Cooper (1981) and those given in Aneja and Parlar (1994). Since the problem instances provided by Katz and Cooper (1981) consist circular barriers, Bischoff and Klamroth replaced circles with regular convex polygons. Then, they generated variants to those problems by changing some features of the polygons and reported the heuristic solutions obtained by their approach. Some other problem instances they considered are those given in Aneja and Parlar (1994). Since their solution approach is valid for convex polygonal barriers, they modified those problems by replacing non-convex polygonal barriers by the convex hull of them. This is based on the fact that the solution and objective function value of the modified problem is identical to the original problem unless there is at least one demand point in the non-convex region of a barrier (see Butt and Cavalier, 1996).

A different solution approach is used by McGarvey and Cavalier (2003). They used *Big Square Small Square* method for the Euclidean distanced Weber problem with polygonal barriers. The big square small square method is a branch and bound technique that divides the continuous feasible region into discrete square subregions. This iterative algorithm begins by determining the smallest square that encloses all the points (demand points and barrier vertices) in the instance problem. This square is then divided into four equal sub-squares. A lower bound is calculated for each sub-square, and sub-squares are pruned based on the computed bounds. At the start of the next iteration, every current sub-square is divided into four new sub-squares and the process repeats. The solution method terminates when the current sub-squares have sides of length less than a small positive number. The solutions will be represented as the center of constructed squares. They used visibility concept in calculating distances to each square. This method produces a solution within a very small tolerance of the optimal solution. Their procedure can applied on the problem with convex polygonal barriers, and in the case of non-convexity, the barrier is replaced by its convex hull. The largest instance they experimented contains 100 demand locations and 7 barriers. However, no information about the location of demand points and barriers in the problem instances is given. It is concluded that the computational times depend on the number of demand points and barrier vertices.

The Restricted planar location problem with barriers was also considered in the case of rectilinear distance l_1. For the first time, Larson and Sadiq (1983) considered the p-median problem with rectilinear distances and polygonal barriers. Based on a network determined by the problem, they defined a special structured grid of nodes and edges, They discovered that the set of nodes provides a finite *dominating set* of solution points for the problem. A dominating set consists of candidate solutions for optimality. The grid is made of horizontal and vertical lines passing through location points in the problem and their intersection points. In each cell of that grid, the objective function is shown to be convex. Afterwards, the authors proved that the optimal solutions fall on either one of the corners of grid cells or the intersection of cell edges and barrier edges. Based on this idea a polynomial time algorithm was introduced to search for the optimal solution among the candidate points.

The work of Larson and Sadiq (1983) motivated Batta et al. (1989) to extend the work by considering both convex forbidden regions and arbitrary shaped barriers while the metric is l_1. They defined a new grid structure for arbitrary shaped barriers, yet the set of optimal solution still consists of intersection points of the grid lines. Furthermore, Dearing and Segars (2002a) considered the problem under rectilinear distance measure and with any convex, nondecreasing function of distance of l_1 norm. They introduced a modification method to change barrier shapes and proved that the objective function values of the original problem and the modified problem are identical. Their method is based on rectilinear distance properties and allows some non-convex barrier shapes to be equivalent to convex ones if there are no demand points in the non-convex regions. The authors were able to reduce the feasible region with their modification as well as partitioning the feasible region into rectangular cells in which the problem is convex. As a sequel discussion, Dearing and Segars (2002b) showed that an optimal solution is not restricted to nodes of the network. Besides, they provided bounds for the objective function value in each cell generated using this method.

Dearing et al. (2002) provided similar results based on partitioning the feasible region into cells for the rectilinear center problems. Considering polygonal barriers and l_1 distances, they developed an algorithm that finds the optimal solution to the problem. The procedure searches for a dominating set and identifies the best solution. They extended these results by considering *block norm* distances instead of rectilinear distances (see Dearing et al., 2005). Block norm distance is defined in the plane with respect to a symmetric polytope as its unit ball. The polytope is assumed to have $2p$ distinct extreme points, for an integer $p \geq 2$. The authors also provided a similar method of Dearing and Segars (2002a) to modify barriers. Block norms in the Weber problem with barriers was first discussed by Hamacher and Klamroth (2000) who established a discretization result based the grid construction method. The grid defined in this paper is constructed using the existing facilities and the fundamental directions of the polyhedral distances. They showed that the barrier problem can be solved with a polynomial algorithm with the presented method.

Along with the existing literature on the Weber problems with polygonal or circular barriers, a line barrier case was introduced by Klamroth (2001b). The problem is considering general l_p norm and existence of a line shaped barrier with a given number of passages. In this case, the barrier divides the plane into two sub-planes. Traveling from one sub-plane to the other is only possible through one of the given passages. The problem becomes a combinatorial problem when there are more than one passages. An algorithm was developed for solving the problem when the number of passages is 2. Complexity of the problem increases exponentially with the number of passages but remains polynomial when the number of passages is fixed. Klamroth and Wiecek (2002) worked on the multi objective median problems with line barriers and different measures of distance. Based on the special structure of the problems, they proposed a polynomial algorithm for bi-criteria problems to find the set of efficient solutions. The restricted planar location problem with a line barrier is also the subject of interest in Canbolat and Wesolowsky (2010). They considered the problem with rectilinear distances where the position of the barrier is not deterministic. The presence of a line barrier in their problem occurs randomly on a given horizontal route on the plane. Some properties of such probabilistic problem are reported and a solution algorithm is provided in that paper.

From a different point of view, Frieß et al. (2003) conducted a simulation study and suggested a solution strategy to the restricted center location problem with Euclidean distances. They implemented a theoretical approach as a physical experiment using water tanks in a lab environment and developed a computer simulation based on the propagation of circular wavefronts. Considering the behavior of water waves in their approach makes it also valid for convex congested regions. In the experiment, barriers are taken as islands in the water tank and congested regions can be made by changing the depth of water in those regions. Another experimental based study on the Weber problem in the presence of convex barriers is recently done by Canbolat and Wesolowsky (2012). They used Varignon frame, a mechanical system of strings, weights, and a board with holes that has been used to identify an optimal location for the classical Weber problem. They showed through analytical results that the same approach can also be used for some of the Weber problems in the presence of barriers. This method provides rapid solutions, allows for flexibility, and enables one to visualize the problem. However, conducting experiments like the ones given in Frieß et al. (2003) and Canbolat and Wesolowsky (2012) require time and effort. Not every problem can be simulated in these ways and computational errors regarding physical experiments are not negligible either.

The literature of the restricted location problems with barriers also contains the multi-facility decision problems. Bischoff et al. (2009) was the first study that considered this problem. They developed alternate location and allocation procedures for the resulting multi-dimensional mixed-integer optimization problem that works by iteratively decomposing the problem into single-facility subproblems.

There is also a relation between restricted planar location problems with barriers and the problem considered in this study. For any congested region in our problem, if the fixed cost is set to infinity, the region becomes a barrier.

2.1.3 Restricted Problems with Congested Regions

The literature of planar facility location problems restricted by congested regions is more limited. Butt and Cavalier (1997) was the first study that considered existence of congested regions in the median planar location problems. The authors considered the p-median case with the rectilinear distance measure and convex polygonal congested regions. Each congested region in their study is characterized by a *congested factor*. Congested factor is defined as a nonnegative number representing a per-unit distance cost which is an additional cost faced when a traveling occurs in the congested region. The authors introduced the *least cost paths* concept and conclude that a rectilinear least cost path between two points in this problem may not necessarily be the path of shortest length. They provided a linear program to find least cost paths. Based on their finding, the authors proposed an extension of the same grid construction procedure as in Larson and Sadiq (1983) and claimed that at least one least cost path would always coincide with the segments of constructed grid. Then, the problem was transformed to an unconstrained p-median problem on a network where an optimal set of new facility locations is chosen from a finite dominating set of points. Hence, the problem was reduced to a combinatorial search where an optimal set of facilities locations is chosen from a finite set of candidate points. They also described the connection of their problem with location problems with forbidden regions and barriers.

Later, Sarkar et al. (2004) demonstrated that the proposed grid line method given in Butt and Cavalier (1997) to find the rectilinear least cost paths is not correct under certain conditions. The authors prove their claim by giving a contradictory example. They also provide a mixed integer linear programming formulation to determine the least cost path for that example. They claim that the difficulties arising from least cost rectilinear path calculations are much more than those mentioned in Butt and Cavalier (1997). Years after, they considered the problem of finding the least cost paths for rectilinear location problem with congested regions (see Sarkar et al., 2009). They established that the state-space for the problem of finding least cost path could be exponential. Moreover, they gave an upper bound for the number of entry/exit points of a rectilinear path between two points and based on this a memory-based algorithm is proposed. However, the computation of least cost path becomes prohibitively expensive when the underlying problem becomes large.

The constrained location problems with congested regions are studied under the assumption of per-unit traveling costs in congested regions. As a result, finding the least cost path becomes challenging as it depends on determining proper entry and exit points for each traveling path in congested regions. Note that our problem differs with the mentioned location problems with congested regions in three terms. Firstly, instead of the rectilinear distance metric in those problems, the distance metric in our problem is Euclidean. In this case, the traveler is not limited to move on vertical/horizontal paths. With euclidean distances the properties of the problem becomes completely different than those with rectilinear distances. Therefore, the solution approaches available in the literature is not valid for our case. Secondly, we considered fixed costs for passing trough congested regions rather than variable costs given in the literature. This also changes the properties of the problem as traveling through regions matters. Lastly, ignoring the assumption of congested region convexity in our problem makes it more realistic.

Despite the fact that the Euclidean planar 1-median problem with congested regions maintains generality over other restricted planar problems and has many real-life applications, as we mention in Chapter 3, it has not attained any attention in the literature so far. Moreover, All solution procedures in the literature that consider restricted planar location problems are given for either forbidden region or barrier restrictions. If a problem contains both restriction types (there exist some barriers and some forbidden regions as well), the available solution procedures in the literature are not valid. Besides, when considering the restricted problem with barrier case and Euclidean distances, the solution procedures in the literature are only applicable under the assumption of convex polygonal regions. If there is a non-convex region with some demand points located inside its non-convex part, the available methods in the literature cannot be used. The solution approach presented in this thesis is flexible in such a way that it is not limited to the type of constraining regions in the problem or to the location of demand points. Hence, the problems containing restrictive regions with different fixed costs and shapes can be solved using our proposed solution approaches as long as they meet the assumptions noted in Chapter 3.

2.2 Meta-heuristic Approaches for Single Facility Location Problems

Since we use meta-heuristic approaches in finding solutions to our restricted problem, it is important to review the literature of meta-heuristic applications for location problems. We concentrate more on the literature of using simulated annealing, evolutionary algorithm, particle swarm optimization, and variable neighborhood search for location problems. More details on meta-heuristic algorithms can be found in Blum and Roli (2003).

The application of meta-heuristics for the p-median problem is well studied in the literature. Mladenović et al. (2007) provides a survey about using well-known meta-heuristics such as simulated annealing, tabu search, variable neighborhood search, and evolutionary algorithms for the p-median problems.

Simulated annealing is a probabilistic search algorithm based on the annealing procedure of a heated metal. It was introduced by Kirkpatrick et al. (1983) and then widely used for the traveling salesman problem (TSP) and other combinatorial problems. A comprehensive review of simulated annealing as a tool for both single and multi-objective optimization and its applications is presented in Suman and Kumar (2006). More examples on the use of simulated annealing for the p-median problems are Al-khedhairi (2008) and the references therein.

Genetic algorithm, which is a class of the evolutionary algorithms, is also used to the p-median location problems. Alp et al. (2003) used genetic algorithm for the p-median problems and showed that their algorithm had a relatively high performance. Chaudhry et al. (2003) applied genetic search on the p-median problem with a maximum distance constraint.

Another implemented meta-heuristic in this area is particle swarm optimization. Particle swarm optimization is a population based meta-heuristic inspired by social behavior of bird flocking or fish schooling. It was first presented by Kennedy and Eberhart (1995) and attained many attentions in optimization problems afterwards. An extensive review on particle swarm optimization and its structure is given by Poli et al. (2007). An example of using particle swarm optimization for the p-median problems is Sevkli et al. (2012) where the search method is designed for the discrete p-median problems. Brito (2007) proposed a particle swarm optimization modified with a local search method to solve the continuous p-median problem.

Variable neighborhood search, introduced by Mladenović and Hansen (1997), is a technique of changing neighborhood to search for better solutions in a systematic manner. It is also used as a meta-heuristic approach to deal with a wide range of optimization problems. In the p-median location problems, for example, variable neighborhood search is used by Hansen and Mladenović (1997). Hansen et al. (2010) provided a review on the literature of variable neighborhood search as well as its different methods and applications. The special structure of variable neighborhood search enables it to be combined with other heuristics in order to improve the overall performance. In this thesis we refer to a basic principle of variable neighbor hood search given in Hansen et al. (2010) and use it as an advanced search process in other proposed meta-heuristics.

Meta-heuristic algorithms are also used in other types of location problems. For the planar multi-facility problem, Abdullah et al. (2008) applied simulated annealing on the uncapacitated planar multi-facility location problem where there is a fixed cost associated with opening a given facility in different zones on the plane. Aras et al. (2007) used simulated annealing for the capacitated multi-source Weber problem under different distance measures.

Houck et al. (1996) developed a genetic algorithm for the multi-source Weber problem and gave a comparison between the proposed genetic algorithm and traditional search algorithms. Brimberg et al. (2000) compared also various heuristics such as tabu search, genetic search, and different versions of variable neighborhood search for the uncapacitated multi-source Weber problem.

Güner and Sevkli (2008) implemented particle swarm optimization on the uncapacitated facility location problem. Parsopoulos and Vrahatis (2002) investigated particle swarm optimization in the constrained problems and compared its performance with evolutionary algorithms.

When the restricted planar facility location problems are considered, not many intensive studies can be found in the literature. The most relevant work and the first one in this area was done by Aneja and Parlar (1994). They used simulated annealing approach to find a heuristic solution to the restricted planar single facility location problem with polygonal barriers. However, no information about the exploited simulated annealing and its structure is provided in Aneja and Parlar (1994). Another relevant use of meta-heuristics for this types of problems is given by Bischoff and Klamroth (2007). Even so, the way that they used this algorithm is different from our procedure. They solved the problem based on the decomposition of the main problem to subproblems with mixed-integer programming (see Klamroth, 2001a) and used genetic algorithm to make a selection among subproblems that are going to be solved. On the contrary, in our study, the solution to the main problem is produced directly by proposed evolutionary algorithm. Besides, even though good results are obtained in our study from particle swarm optimization and variable neighborhood search coping with the problem, no more relevant study is found in the literature that uses particle swarm optimization or variable neighborhood search for the restricted planar location problems.

In Chapter 4 the application of three well-known meta-heuristics modified with variable neighborhood search, namely simulated annealing, evolutionary algorithm and particle swarm optimization, on the restricted planar single facility location problems with fixed cost congested regions is introduced. We believe that this study is the first that extensively considers application of meta-heuristics on a general restricted planar location problem with Euclidean distances.

CHAPTER 3

RESTRICTED SINGLE FACILITY LOCATION PROBLEM

Restricted facility location problems often refer to the problems where there are limitations on the facility location. Location problems imposing restrictions on locating facilities in and/or traveling through specific regions are typically referred as constrained or restricted problems (see Sarkar et al., 2004). Such problems have two topographical properties:

(a) The new facilities cannot be located within certain restricted areas in the plane.
(b) The minimum travel time between any two points in the plane may be made longer due to the presence of the restricted regions.

The restricted location problems in the literature are considering *forbidden regions*, *barriers*, or *congested regions* with variable costs. In this section we show that congested regions with fixed cost is another restriction which can be generalized into two types of restrictions, namely as forbidden regions and barriers. In this study, congested region is defined as a polygonal region where facility location is not possible while traveling through is permitted at a fixed cost.

There are many applications for the problems with congested regions. In real life situations, it is possible that traveling through forbidden regions is not completely free and requires facing some risks, like passing through nuclear plants. Another situation is that existence of a large barrier on our way may lengthen the route so much that we prefer undergoing some cost to be able to pass over the barrier, like purchasing aircraft to pass over mountains.

In this chapter, we first formulate our problem and discuss its relation with the restricted location problems studied in the literature. Next, the mathematical formulation of the problem is given followed by defining an upper and lower bound on its objective function value.

3.1 Problem Formulation

The classical planar single facility location problem is a well-known optimization problem where a set of demand points are served by a facility. Each demand point is located at $X_m = (x_m, y_m)$ with a weight w_m, $m = 1, ..., M$ while the facility location is denoted by $X_f = (x_f, y_f)$. The objective is to find X_f such that the total weighted distances between the facility and demand points is minimized. In other words, $\sum_{m=1}^{M} w_m l_p(X_f, X_m)$ is minimized over a p-norm distance measure, where,

$$l_p\left(X_f, X_m\right) = \left(\left|x_f - x_m\right|^p + \left|y_f - y_m\right|^p\right)^{1/p}, \qquad 1 \leq p < \infty \tag{3.1}$$

This problem is called the Weber problem if it is defined on the plane with Euclidean distance measure, l_2. For simplicity, we call this kind of unconstrained problems as *LocP*.

Yet, most of the real world location problems are not as simple as LocP. There may be some restrictions over the facility location which prevent the decision maker from locating the facility in certain areas on the plane. Moreover, additional restrictions may be imposed on traveling through these regions such that direct access between the facility and demand points is obstructed. If there are some regions which limit location of a facility and/or traveling from the facility to demand points, the problem is defined as the restricted planar single facility location problem or the restricted Weber problem. For simplicity, we refer to such problems as *RLocP*.

Some examples of such regions are mountains, lakes, national parks, and highways where locating the facility on or inside them is forbidden while passing through them is free, costly, or even impossible. Note that the existence of such restrictions can make the solution set non-convex. Therefore, along with non-convexity, discontinuity, and nonlinearity of the objective function under Euclidean distance norm, the problem becomes more difficult to solve than the (unrestricted) Weber problem.

In this study, we consider a general case where restrictions on the facility location are imposed by a finite number of arbitrary shaped polygonal regions on the plane. The set of regions is denoted by R where each region $r \in R$ has a bounded interior set $B_r \subset \mathbb{R}^2$. Locating a facility inside any B_r is prohibited, however, passing through some B_r is penalized by a fixed cost (or risk) denoted by c_r, $r \in R$. This cost can be any value from zero to infinity. Zero and infinity costs, respectively, imply that traveling is free and forbidden.

If there is a region on any facility-demand pathway, the decision maker should either make her way around that region or pay the fixed cost to be able to pass directly through that region and reach her destination.

As an example, consider the problem of locating a chemicals factory so that the total delivery time from the facility to some depots is minimized. Suppose that depots are distributed around but not inside an urban zone. Construction of the factory inside the zone is prohibited by environmental regulations. The zone offers shorter ways to depots but it has heavier traffic on its roads which increases delivery times significantly. Therefore, we should decide to pass either through the zone and face the traffic or round the zone and settle for the longer way.

The costs associated with congested regions can be:

- The cost of obtaining a certificate or paying for some special services provided in that region, like military and touristic zones;
- The cost of providing a vehicle for special use, like renting a ship to pass through a lake, or the money spent to modify vehicles regarding tight transportation codes;
- The risk of passing through dangerous and unsafe regions, like nuclear plants or war zones.

In any case, these fixed costs should be considered in the objective function to make it possible to decide whether the decision maker should face some extra cost for passing through a region or she should make the way longer and round that region. So, if there is at least one region with nonzero fixed cost on a facility-demand way, that facility-demand cost is not simply the facility-demand Euclidean distance in LocP any more, but a higher cost. For simplicity, we name RLocP in the presence of congested region with fixed cost as *RLocP-CR*.

In the following section we explain how regions are treated based on the associated fixed costs and what the special cases of our problem and their relations are.

3.2 Relation with Existing Restricted Location Problems

Here, assuming that all restrictive regions are polygonal, we introduce two types of RLocP that are highly connected to RLocP-CR. The aim is to make a distinction between these two types and explain their relation with our problem.

The first type is RLocP with forbidden regions, indicated by *RLocP-FR*. Forbidden regions are defined as those regions in which facility location is forbidden but through which traveling is freely possible. Butt and Cavalier (1997) argued that forbidden region is equivalent to congested region if the congestion factor of the region is zero. Their claim is also valid for other distance metrics and if the cost of regions are fixed instead of variable. Therefore, RLocP-FR is a special case of our problem if $c_r = 0$ for all $r \in R$.

The next type is RLocP with barriers where neither locating a facility in nor traveling through is possible. This type of problems can be considered as the special case of RLocP's with forbidden regions where additional restrictions are imposed on traveling through regions. Butt and Cavalier (1997) also claimed that if the congestion factor of any congested region is set to an infinity large number, that region behaves like an obstacle to travel. This is also true for the case of having fixed costs. Thus, RLocP with barriers, referred as *RLocP-BR*, can be converted to our problem when all c_r's are set to infinity. The reason is that with penalties $c_r = \infty$, $\forall r \in R$, no facility-demand way which falls inside a region will minimize the objective function.

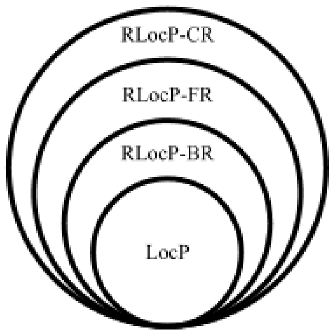


Figure 3.1: Relation of LocP and RLocP's

According to the discussion in this section, one can think of RLocP-CR as the general case of the well-known two problems, namely RLocP-FR and RLocP-BR. Figure 3.1 shows the relation of RLocP's we mentioned so far with each other and their relation with our problem.

Although RLocP with congested regions is said to be more general case than the others, in Section 3.5 we show how consideration of these two kinds of problems can be useful in finding lower and upper bounds for our problem, RLocP-CR.

3.3 Mathematical Formulation

Mathematical formulation of RLocP-CR is introduced in this section. Assumptions and parameters of the problem are as follows.

Assumptions:

- The problem is defined on the continuous, two dimensional space with Euclidean distance measure, l_2.
- There is only a single facility to locate.
- There is not any cost related to locating the facility.
- The size of the facility is negligible and it can be considered as a point on the plane.
- There is a finite number of congested regions with nonnegative fixed traveling costs.
- All congested regions are either line segments or closed arbitrary shaped polygons with bounded interior sets $B_r \subset \mathbb{R}^2$, $r \in R$.
- Each congested region $r \in R$ has a finite set of vertices.
- Locating a facility in the interior of any congested region is prohibited.
- There are a finite number of demand points with nonnegative weights to be served by the facility.
- No demand point is located in the interior of congested regions.

Although our problem structure and solution procedure are also valid when there are fixed costs related to location of the facility or if there exist some demand points inside a region, which are more general cases, we keep our assumption, without loss of generality, to be more connected with the literature.

Parameters:

R = set of all congested regions

U_r = set of vertices in region r, $\forall r \in R$

c_r = fixed traveling cost of region r, $r \in R$

$\mathcal{V}$ = set of all region vertices,
$\mathcal{V} = \{X_v \in \bigcup_{r\in R} U_r, v \in V\}$, $V = \{1, ..., \sum_{r\in R} |U_r|\}$

$\mathcal{M}$ = set of all demand points,
$\mathcal{M} = \{X_m, m \in M\}$, $M = \{|\mathcal{V}| + 1, ..., |\mathcal{V}| + |\mathcal{M}|\}$

w_m = wight of the m^{th} demand point, $m \in M$

$\mathcal{N}$ = set of all region vertices and demand points,
$\mathcal{N} = \{X_i \in \mathcal{V} \cup \mathcal{M}, i \in N\}$, $N = \{1, ..., |\mathcal{V}| + |\mathcal{M}|\}$

$$p_{ij}^r = \begin{cases} 1, & \text{if the direct way between } X_i \text{ and } X_j \text{ passes through region } r, \forall i, j \in N, r \in R \\ 0, & \text{otherwise} \end{cases}$$

Decision Variable:

X_f = location of the facility on the plane with coordinates x_f and y_f

Formulation: When there are no congested regions, the least cost path between the facility and the m^{th} demand point is simply equal to the length of the direct facility-demand way, i.e. $l_2(X_f, X_m)$. However, there may be some congested regions in the way of facility toward some demand point X_m which makes traveling so costly that we prefer the path rounding the region to the direct way.

Let $N' = N \cup \{f\}$, where $f = |N| + 1$ is the index referring the facility location X_f. Let $\bar{l}_2(X_f, X_m)$ be the least cost pathway from X_f to X_m. Let $E_{ij}^{(u)}$ be the least cost pathway from X_i to X_j for which all intermediate points are in the set $\{X_1, ..., X_u\} \subset \mathcal{V}$ and $i, j \in N'$. Moreover, Define the feasible set, F, be $\mathbb{R}^2 \setminus \bigcup_{r\in R} B_r$. Then, the problem is formulated as

$$\min Z\left(X_f\right) = \sum_{m\in M} w_m \bar{l}_2\left(X_f, X_m\right) \tag{3.2}$$

Subject to:

$$X_f \in F \tag{3.3}$$

Where,

$$\bar{l}_2\left(X_f, X_m\right) = E_{fm}^{(u)} = \begin{cases} d_{fm}, & u = 0 \\ \min\left\{E_{fm}^{(u-1)}, E_{fi}^{(u-1)} + E_{im}^{(u-1)}\right\}, & 1 \le u \le |V| \end{cases} \tag{3.4}$$

$$d_{ij} = l_2\left(X_i, X_j\right) + \sum_{r\in R} p_{ij}^r c_r, \quad \forall i, j \in N' \tag{3.5}$$

The objective is to minimize the sum of weighted least-cost facility-demand pathways where $Z(X_f)$ is a non-linear and non-convex function of X_f over the feasible set F which is a non-convex set in many situations. Calculating $E_{fm}^{(v)}$ is the same as finding the least cost way from the facility location to the m^{th} demand point. In the next section we provide algorithmic approaches to find such pathways.

3.4 Insights into Solution Procedure

Due to the nature of our problem, i.e. having a nonlinear, non-convex and discontinuous objective function and a non-convex feasible set, minimizing the objective function is not as simple as solving the unrestricted problem. To illustrate, obtaining the exact solution for RLocP with barriers requires decomposition of the original problem into sub-problems and separately solving those sub-problems and compare their solution to find the optimal one (see Butt and Cavalier, 1996, Klamroth, 2001a). However, decomposition procedure and determining sub-problems is itself a complex procedure. Besides, the number of sub-problems increases exponentially when the number of regions increases which decreases the efficiency of this method. After all, there is no optimization method in the literature, so far, that deals with non-convex regions. Therefore, using heuristics to deal with our problem is justifiable.

In this section, we first explain the evaluation of any solution X_f. For any facility location X_f, the objective is to find all weighted least-cost ways to all demand locations. Later, the concepts used to calculate $Z(X_f)$ for a given X_f are given and at the end we provide an upper limit for congested regions fixed costs, instead of infinity, that makes sure no route falls inside that region. Chapter 4 describes our search algorithms finding a good location for the facility, X_f.

3.4.1 Direct Access Cost

The direct access cost of point j from point i, denoted by d_{ij} in Equation 3.5, is the cost of going directly from point i to point j without lengthening the way to reach another point (called intermediate point). To clarify, let $i, j \in N'$, then, the cost associated with $d_{ij} = d_{ji}$ is $l_2(X_i, X_j) + \sum_{r=1}^{R} p_{ij}^r c_r$. That is, for every direct pathway of i to j passing through a region r, we add the traveling penalty cost c_r to the Euclidean distance between the points i and j. Note that, in this study, we consider each unit-distance of traveling as a unit-cost.

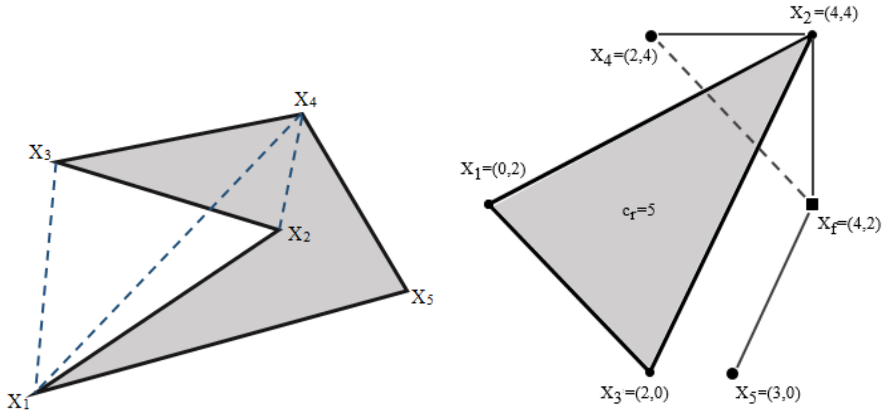
A simple approach given in Algorithm 3.1 is used to find the d_{ij} values. If the whole way between any two vertices of a region r falls inside r, their direct access route is charged with c_r. In Line 3, we check whether an arbitrary point on the way of two vertices falls inside the region. If so, the penalty is considered, otherwise, another check is performed in Line 7.

Algorithm 3.1 Direct access cost

Input: two points X_i and X_j, $i, j \in N'$
Output: d_{ij}, the direct access cost of X_i,X_j
1: $d_{ij} \leftarrow l_2(X_i, X_j)$ /* initialization */
2: **for each** region $r \in R$ **do**
3:   **if** $X_i, X_j \in U_r$ **and** the midpoint of line segment X_i, X_j is in the interior of r **then**
4:     $d_{ij} \leftarrow d_{ij} + c_r$ /* region r is on the way! */
5:     **continue for**
6:   **end if**
7:   **for each** edge e in r **do**
8:     **if** line segment X_i, X_j intersects edge e **then**
9:       $d_{ij} \leftarrow d_{ij} + c_r$ /* region r is on the way! */
10:       **exit for**
11:     **end if**
12:   **end for**
13: **end for**

An example is shown in Figure 3.2(a) where a non-convex region is considered. None of the direct ways X_1 to X_2 or X_1 to X_3 are charged with fixed cost since all the ways (including the midpoints) fall outside the interior of the region. However, the entire way of X_2 to X_4 falls inside the region (including the midpoint) so the fixed cost is considered in their direct access. X_1 to X_4 pathway falls partially in the region. Although the midpoint of this way is not inside the region, the edge X_2,X_3 crosses the X_1, X_4 direct way. Therefore, the direct X_1,X_4 access way is penalized with the fixed cost.

Figure 3.2(b) shows the direct access route (dashed line) between X_f and X_4. Note that since region r in on this way, $p_{f4}^r = 1$ and $d_{f4} = l_2(X_f, X_4) + c_r = 2\sqrt{2} + 5$. However, the direct access X_f to X_2, shown by a solid line, is not passing through a region, so its cost is simply the euclidean distance between them, i.e. $d_{f2} = 2$.



(a) Direct access routes in a non-convex region (b) Facility-demand least cost pathways

Figure 3.2: Direct access routes and least-cost ways

3.4.2 Least Cost Pathway

Alternative path finding approaches to the shortest path method given in Aneja and Parlar (1994) is used by Bischoff and Klamroth (2007), Dan (2009) and Lee and Preparata (1984). In this study we use another approach. If there is no region on the way of X_f to X_m for some $m \in M$, or all the regions on this way have zero fixed cost, then $l_2(X_f, X_m)$ is the least cost pathway from X_f to X_m. However, if there is a region on their way, we are not sure that the direct way of going from X_f to X_m is the least-cost possible way. Figure 3.2(b) shows an example of a situation where the direct way X_f to X_1 happens to be not the least cost way. In this case, going to an intermediate point and reaching the demand point from there is more preferable than directly reaching the demand point from the facility.

Generally, if there exists a vertex of a region, X_v, $v \in V$, for which $d_{iv} + \bar{l}_2(X_v, X_j) < d_{ij}$ then X_v is definitely visited in the X_i, X_j route, $i, j \in N'$. Thus, not only the direct facility-demand pathways should be considered, but also all possible combinations of region vertices as intermediate points in those ways should be considered in order to find the least cost facility-demand ways. In order to calculate the least cost X_i, X_j way for any $i, j \in N'$, i.e. $\bar{l}_2(X_i, X_j)$, we use Algorithm 3.2. This algorithm is similar to *Floyd–Warshall algorithm* (see Cormen et al., 2009) except one has not to restrict herself to the graph in order to find least cost ways. The Floyd–Warshall algorithm is a polynomial time algorithm that finds all-pairs shortest paths on a graph. The advantage of using Algorithm 3.2 is that once the it is implemented, the information about least cost pathways between all pairs of points becomes available which can be used several times without the need for recalculating the least cost paths. We will focus on this issue later in Chapter 4.

The complexity of Algorithm 3.2 is $O(|V| \times |N|^2)$ excluding the initialization step. But since demand-to-demand least cost ways are not needed the algorithm is reduced and can run in $O(|V|^2 \times |N|)$.

Algorithm 3.2 Calculation of the least cost pathways

Input: All locations of X_i's, $\forall i \in N'$
Output: all least cost path ways, $\bar{l}_2(X_i, X_j), \forall i, j \in N'$
  for all i, j pairs in N **do**
    $\bar{l}_2(X_i, X_j) \leftarrow d_{ij}$ /* initializing */
  end for
  for all v in V **do**
    for all i, j pairs in N **do**
      $\bar{l}_2(X_i, X_j) = \min\left\{\bar{l}_2(X_i, X_j), \bar{l}_2(X_i, X_v) + \bar{l}_2(X_v, X_j)\right\}$
    end for
  end for

3.4.3 An Upper Limit for Fixed Costs

When c_r is set to ∞ for any $r \in R$ no route falls inside region r and that region behaves like a barrier to travel. But, is there a finite $c_r < \infty$ that also prevents traveling through region r? The following proposition answers this question.

Proposition 3.1 *If $c_r \geq pm_r/2$ for any region r, where pm_r is the perimeter of region r, no traveling occurs in that region.*

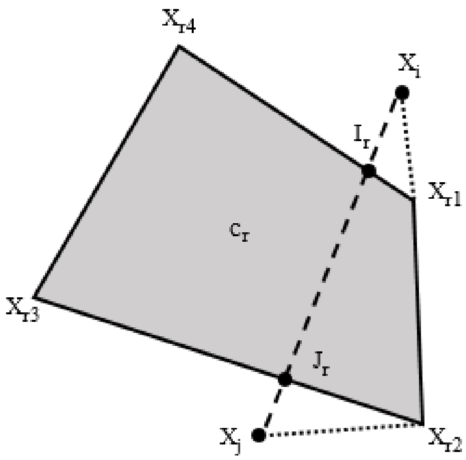


Figure 3.3: Upper limit for fixed costs. $W_r = \{X_{r1}, X_{r2}\}$ and $W'_r = \{X_{r3}, X_{r4}\}$. The dashed line shows the way passing through the region and the dotted line is the least-cost route when $c_r = \infty$. For values $c_r \geq \frac{1}{2}$(perimeter of region r), no traveling occurs in region r.

Proof. Let X_i and X_j be two points on the plain. Let region r defined by set of vertices U_r be a convex polygonal region on the plane. If $p^r_{ij} = 0$ then for any cost, no X_i, X_j route passes through region r. Assume $p^r_{ij} = 1$ and let I_r and J_r the intersection points of line segment X_i, X_j with region r which are respectively closest to X_i and X_j (see Figure 3.3). Let $W_r = \{X_{rk}, ..., X_{rl}\}$ be set of verices of region r that are intermediate points in $\bar{l}_r(X_i, X_j)$ when $c_r = \infty$ and ordered in the direction of traveling from X_i to X_j. Let $W'_r = \{X_{rk'}, ..., X_{rl'}\}$ be the set of remaining vertices in the same order. Let W_l be the total length of edges corresponding to the verices in W_r. Let W_l' be the total length of edges corresponding to the vertices in W_r'. Then, we prove Proposition 3.1 by contradiction as follows.

If $c_r = pm_r/2$ and the least-cost X_i, X_j pathway passes through region r then we get

$$\bar{l}_2(X_i, X_j) = l_2(X_i, I_r) + l_2(I_r, J_r) + l_2(J_r, X_j) + \frac{pm_r}{2} \tag{3.6}$$

On the other hand, from triangular inequality we have

$$l_2(I_r, J_r) + \frac{pm_r}{2} < l_2(I_r, X_{rk}) + W_l + l_2(X_{rl}, J_r) \tag{3.7}$$

It is obvious that

$$l_2(I_r, X_{rk}) + W_l + l_2(X_{rl}, J_r) < l_2(I_r, X_{rk'}) + W_l' + l_2(X_{rl'}, J_r) \tag{3.8}$$

Otherwise, the least cost route would be the other way around when $c_r = \infty$. However, since $l_2(I_r, X_{rk}) + W_l + l_2(X_{rl}, J_r) + l_2(I_r, X_{rk'}) + W_l' + l_2(X_{rl'}, J_r) = pm_r$, from Equation 3.8 we get

$$l_2(I_r, X_{rk}) + W_l + l_2(X_{rl}, J_r) < \frac{pm_r}{2} \tag{3.9}$$

Now, from Equation 3.7 we obtain $l_2(I_r, J_r) + pm_r/2 < pm_r/2$ or $l_2(I_r, J_r) < 0$ which is a contradiction. ■

3.5 Lower and Upper Bounds

Klamroth (2002) provided upper and lower bounds for RLocP with barriers. The author showed that the optimal objective function value of the unrestricted problem is always less than or equal to that of restricted problem with forbidden regions, i.e.,

$$Z\left(X_f^U\right) \leq Z\left(X_f^{FR}\right) \tag{3.10}$$

where, X_f^U is the optimal solution for unconstrained LocP and X_f^{FR} is the optimal solution to RLocP-FR. Klamroth (2002) also proved that the optimal objective function value of RLocP with barriers cannot be less than that of RLocP with forbidden regions. Or,

$$Z\left(X_f^{FR}\right) \leq Z\left(X_f^{BR}\right) \tag{3.11}$$

Where X_f^{BR} is the optimal solution for RLocP-BR.

Here, we give lower and upper bounds for our problem in the following theorem.

Theorem 3.1 *Let X_f^* be the optimal solution for the RLocP with congested regions. Let X_f^U be the optimal solution to the unrestricted problem LocP. Let X_f^{FR} be the optimal solution for RLocP-FR where $c_r = 0$, $\forall r$. Let X_f^{BR} be the optimal solution for RLocP-BR where $c_r = \infty$, $\forall r$. Then,*

$$Z\left(X_f^U\right) \leq Z\left(X_f^{FR}\right) \leq Z\left(X_f^*\right) \leq Z\left(X_f^{BR}\right) \tag{3.12}$$

Proof. For the first inequality relation, we have $Z(X_f^U) \leq Z(X_f^{FR})$ from Klamroth (2002). For the last two inequality relations, $Z(X_f^{FR}) \leq Z(X_f^*) \leq Z(X_f^{BR})$, consider the optimal solution to RLocP with congested regions. If no X_f^* to demand path passes through any region while all paths are direct, or all region traveling costs are zero then X_f^* and X_f^{FR} are identical, i.e. $Z(X_f^{FR}) = Z(X_f^*)$ is hold. On the other hand, suppose that there exists at least one demand point m, and one region r such that the path f, m passes through region r in the forbidden region problem, i.e. $p_{fm}^r = 1$. Thus, $d_{fm}^{FR} = l_2(X_f^{FR}, X_m) + \sum_{r=1}^R p_{fm}^r c_r = l_2(X_f^{FR}, X_m)$. Now, suppose that $c_r > 0$ while all other regions have zero traveling costs. If c_r is small enough, i.e. $c_r \leq \bar{l}_2(X_f^{FR}, X_m) - l_2(X_f^{FR}, X_m)$, then X_f^* and X_f^{FR} are the same but $d_{fm} = l_2(X_f^{FR}, X_m) + c_r > d_{fm}^{FR}$ resulting $Z(X_f^{FR}) < Z(X_f^*)$. Otherwise, if $c_r > \bar{l}_2(X_f^{FR}, X_m) - l_2(X_f^{FR}, X_m)$ then passing through region r is not preferred and the path f, m passes around region r. In this case, $d_{fm} = \bar{l}_2(X_f^*, X_m)$ which is greater than or equal to $l_2(X_f^{FR}, X_m)$ and again $d_{fm} > d_{fm}^{FR}$. Therefore, $Z(X_f^{FR}) \leq Z(X_f^*)$.
Likewise, if all region traveling costs are ∞ then no X_f^* to demand path passes through any region. Hence, X_f^* and X_f^{BR} are identical and $Z(X_f^{BR}) = Z(X_f^*)$ is hold. On the other hand, suppose that there exists at least one demand m, and one region r such that the path f, m passes through region r in the congested region problem, i.e. $p_{fm}^r = 1$. Thus, $d_{fm} = l_2(X_f^*, X_m) + \sum_{r=1}^R p_{fm}^r$. It means that for this case, $c_r \leq \bar{l}_2(X_f^*, X_m) - l_2(X_f^*, X_m) < \infty$. Therefore, $d_{fm} = l_2(X_f^*, X_m) + c_r$ which is less than or equal to $d_{fm}^{BR} = \bar{l}_2(X_f^{BR}, X_m)$ resulting $Z(X_f^*) \leq Z(X_f^{BR})$. ■

Based on the results of Theorem 3.1, we use an algorithm based on the idea given in Aneja and Parlar (1994) which finds the optimal solution of the single facility location problems with forbidden regions. In chapter 4 we introduce such an algorithm. The solutions for RLocP-FR can be used as a lower bound for our problem. For the barrier case, we rely on the solutions obtained by heuristic approaches presented in this study for the upper bounds.

CHAPTER 4

PROPOSED META-HEURISTICS

The term *meta-heuristic* was introduced to define heuristic methods that can be applied to a wide set of problems. In other words, a meta-heuristic designates a general algorithmic framework or computational method which can be adapted to a specific problem with few modifications and applied with a few assumptions about the problem being optimized. The aim is to guide the search procedure for finding a (near) optimal solution (see Blum and Roli, 2003).

This chapter explains how the search of the problem's good solution in the continuous solution space is done in order to obtain the best possible objective function value. All meta-heuristic algorithms given in this study start with an initial solution (or an initial set of solutions) and iteratively try to improve that solution (or solutions) with regard to their quality. We introduce three different meta-heuristics namely evolutionary algorithm (EA), particle swarm optimization (PSO), and simulated annealing (SA) based on variable neighborhood search (VNS) technique. SA is used for both p-median location problems (Mladenović et al., 2007) and RLocP with barriers (Aneja and Parlar, 1994). However, no information about how SA algorithm is given in the latter work. EA's are also used for both p-median location problems (Mladenović et al., 2007) and RLocP with barriers (Bischoff and Klamroth, 2007). Bischoff and Klamroth (2007) proposed an EA based meta-heuristic (genetic algorithm) which is used in selection of subproblems of the original problem where they used the decomposition approach given in Klamroth (2001a). Hence, it is completely different than EA introduced here. PSO is introduced in this study as another population based algorithm but with a behavior different from EA.

Firstly, we explain how the solutions generated by heuristic algorithms are evaluated and then infeasible and outlying solutions are clarified. Next, the variable neighborhood search concept is introduced and the proposed meta-heuristics along with their basic ideas and structures are given.

4.1 Solution Evaluation

Meta-heuristic algorithms generate new solutions based on the information available from previously generated solutions. In this study, quality of a solution is indicated by the corresponding objective function value, i.e. the lower the objective function value of a solution, the better that solution is.

In the proposed algorithms, the chance that more solutions are generated around the good solutions is increased as time passes. Therefore, we need to evaluate the quality of generated solutions. However, since the number of generated solutions by meta-heuristics is usually high, it is computationally expensive to calculate objective function for each solution as described in Section 3.4. In the following sections, we give an efficient way to compute objective function values, then we define infeasible and outlying solutions and the way we treat them.

4.1.1 Preprocessing Procedures

Here, we address two important procedures that are performed before our meta-heuristic is initialized. Firstly, it is explained how the problem space can be reduced so that we deal with less points in the problem. Next, another strategy is given to eliminate running least cost pathway approach for any generated solutions. Both given strategies help us to eliminate some steps in the computation of objective function values and, consequently, reduce the computational times.

4.1.1.1 Problem Space Reduction

Aneja and Parlar (1994) showed that in the facility location problem with barriers ($c_r = \infty, \forall r \in R$) any barrier that totally falls outside the convex hull of the demand points will not be effective in objective function value. Their claim is also true for the congested region case, thus based on this idea, we can eliminate the regions that are outside the convex hull of the demand points using *region elimination* algorithm given in Algorithm 4.1.

Algorithm 4.1 Region elimination algorithm

```
for each region r in R do
    if all vertices of r fall outside the convex hull of demand points then
        eliminate region r
    end if
end for
```

4.1.1.2 Least Cost Demand-Vertex Pathways

The least cost pathway from a facility location X_f to a demand point location X_m for $m \in M$ is either their direct access, d_{fm} or a pathway that has at least one intermediate point, i.e. $\exists v \in V : \bar{l}_2(X_f, X_m) = d_{fv} + \bar{l}_2(X_v, X_m)$. Therefore, once we have $\bar{l}_2(X_v, X_m)$ values for all $v \in V$, we only need to find an intermediate point X_v, $v \in V$ that minimizes $d_{fv} + \bar{l}_2(X_v, X_m)$. Note that $\bar{l}_2(X_v, X_m)$ values, $\forall v \in V$, are independent of X_f. Therefore, instead of running least cost pathway procedure for each generated solution X_f, we only need to find $\bar{l}_2(X_v, X_m)$ values, $\forall v \in V$ by running that procedure once and later, for each generated solution we only need to find $\bar{l}_2(X_f, X_m) = \min\left\{d_{fm}, \min_{v \in V}\left[d_{fv} + \bar{l}_2(X_v, X_m)\right]\right\}$ for all $m \in M$. Algorithm 4.2 shows how the objective function is calculated. Not that the $\bar{l}_2(X_v, X_m)$ values in Line 5 are available from Algorithm 3.2.

Algorithm 4.2 Objective function calculation

Input: a solution point X_f

Output: $Z(X_f)$

```
Z(X_f) ← 0    /* initialization */
for each demand point X_m, m ∈ M do
    for each region vertex X_v, v ∈ V do
        calculate d_fm and d_fv using Algorithm 3.1
        l̄_2(X_f, X_m) ← min{d_fm, d_fv + l̄_2(X_v, X_m)}
    end for
    Z(X_f) ← Z(X_f) + w_m l̄_2(X_f, X_m)
end for
```

4.1.2 Infeasible and Outlying Solutions

Infeasible solutions are those who fall inside a congested region, i.e. if $\exists r : X \in B_r$ then X is called infeasible solution. Aneja and Parlar (1994) showed that the solution of RLocP with barriers always fall inside the convex hull of instance points (demand points and barrier vertices). It is also true for RLocP with congested regions. However, due to random factors, it is possible that our heuristic algorithms generate a solution outside the convex hull of the instance points. We call these solutions outliers. For simplicity, instead of actual convex hull of the instance points, the smallest enclosing horizontal rectangle of the problem instance points called *rectangular convex hull* is considered. Therefore, solutions falling outside the rectangular convex hull are considered as outliers.

Repairing is one of the most important factors in our meta-heuristic algorithms. An efficient repairing procedure can increase the performance of the algorithm significantly. One way to deal with infeasible solutions and outliers is penalizing them with a very large objective function value, i.e. setting $Z(X) = \infty$ for any infeasible or outlier solution X. In this case any infeasible or outlier solution is excluded without providing useful information. Another way is to repair them so that they become feasible and more qualified. The advantage of repairing is that by moving infeasible or outlier solutions to feasible regions, we get some information about the objective function value in their close neighborhood. Whenever a solution falls inside a congested region we repair it by projecting it to the nearest edge of that region. Likewise, whenever a generated solution falls outside the rectangular convex hull we repair it by moving it to the nearest edge of the hull. The `Check(X)` procedure (Algorithm 4.3) shows how a solution X is checked for repairment and the fast repairing procedure if required. Checking the solution is done as soon as it is generated in any procedure.

Algorithm 4.3 Check procedure

Requires: a solution point X
Ensures: X is feasible
1: **if** X is outside the rectangular convex hull of the instance points **then**
2:     /* X needs to be repaired */
3:     $X \leftarrow$ the nearest projection of X onto the rectangular convex hull
4:     **go to** 13
5: **end if**
6: **for each** region r **do**
7:     **if** X is inside region r **then**
8:         /* X needs to be repaired */
9:         $X \leftarrow$ the nearest projection of X onto region r's edges
10:         **go to** 13
11:     **end if**
12: **end for**
13: /* X is repaired. Algorithm terminates */

4.1.3 Solutions to the Unrestricted and Restricted Problems with Forbidden Regions

If the problem is considered without any constraint it is simply LocP in which the objective function is convex. In this study, solutions to these problems are found by using Weiszfeld algorithm given in Francis et al. (1992). The iterative Weiszfeld's algorithm used in our study terminates when the Euclidean distance between two consecutive solutions is less than or equal to a small positive number, ε.

Aneja and Parlar (1994) proved that for any RLocP-FR, if the solution to the unconstrained problem, X_f^U, satisfies the constraint set in the restricted problem it is also optimal to RLocP-FR. On the contrary, if that solution does not satisfy any constraint in RLocP-FR, the optimal solution to the original problem is on the boundary of the particular forbidden region which actually contains X_f^U. Based on this claim, we use the Algorithm 4.4 to find the solution to RLocP-FR.

Algorithm 4.4 Algorithm to find RLocP-FR's solution

Output: Optimal solution X_f^{FR}
1: find X^U the optimal solution to the unrestricted problem by removing all the regions in RLocP-FR
2: **if** X^U is feasible in the original RLocP-FR **then**
3:     **return** X^U /* X^U is also optimal to RLocP-FR */
4: **else**
5:     find r, the forbidden region inside which X falls
6:     **for each** edge e of r **do**
7:         perform a line search on e for a candidate solution X_e
8:     **end for**
9:     **return** the best of X_e's
10: **end if**

The objective function is convex on each edge of the region r in which the solution falls (see Aneja and Parlar, 1994). Therefore, a line search procedure can be used to find a candidate solution on each edge. The line search method in Line 7 of Algorithm 4.4 is *Golden Section* search technique given in Kiefer (1953).

4.2 Variable Neighborhood Search (VNS)

VNS, first proposed by Mladenović and Hansen (1997), is a meta-heuristic or a framework based on the idea of systematically changing the neighborhood in order to search for better solutions. In this study, instead of using VNS as a whole single heuristic, we use its basic concepts and embed a simple one-step VNS approach, similar to Reduced VNS in Hansen et al. (2010), in our main meta-heuristics. Since the VNS framework in our study is used to enhance the search procedure in other meta-heuristics, the simplicity and efficiency of the structure matters. The `VNS( )` function used in meta-heuristic algorithms is given in Algorithm 4.5. Note that `Check( )` procedure is inserted inside VNS algorithm to prevent generating infeasible or outlier solution in this step.

Algorithm 4.5 A basic VNS procedure

Input: a solution point X, and a neighborhood size s
Output: a (better) solution point in the neighborhood of X
1: generate a random solution Y in $Nbr_s(X)$
2: `Check(`Y`)` /* check for neccessary repair procedures */
3: **if** $Z(Y) < Z(X)$ **then**
4:     **return** Y /* a better solution is found around X */
5: **else**
6:     **return** X /* failed to find a better solution */
7: **end if**

$Nbr_s(X)$ in line 1 is a rectangular area centered at location X with width and height that are at most sW and sH respectively. Here, W and H are the width and the height of the rectangular convex hull, respectively. In all meta-heuristic algorithms, s is initially set to 1 and decreases afterwards. To prevent generation of an outlier solution around X we bound its neighborhood by the boundaries of the rectangular convex hull. Figure 4.1 shows the rectangular convex hull and the neighborhood of two generated solutions, shown by filled squares, when $s = 0.25$ on an instance given in Butt and Cavalier (1996). Note that the neighborhood of X_{f2} is bounded by the rectangular convex hull.

4.3 Simulated Annealing (SA)

SA, first developed by Kirkpatrick et al. (1983), is a trajectory based meta-heuristic inspired from annealing process in metallurgy. It uses an analogy between the way in which a heated metal cools down into a minimum energy crystalline structure and the search for a global optimum in a more general system. SA forms a generic probabilistic search approach for finding a good approximation of the optimum solution as it benefits from uphill and non-improving moves to escape local traps. Its applications contain a vast area of optimization problems specially for combinatorial and highly nonlinear problems. In facility location problems it is applied on both the p-median problem (Mladenović et al., 2007) and RLocP with barriers (Aneja and Parlar, 1994).

SA generates only one solution at a time which makes it different from EA and PSO where a number of solutions in the population interact with each other. In this section we propose a modified SA algorithm using a variable neighborhood search and show how it is easily tuned to be implemented for our problem. Following sections address general components of SA and its main algorithm.

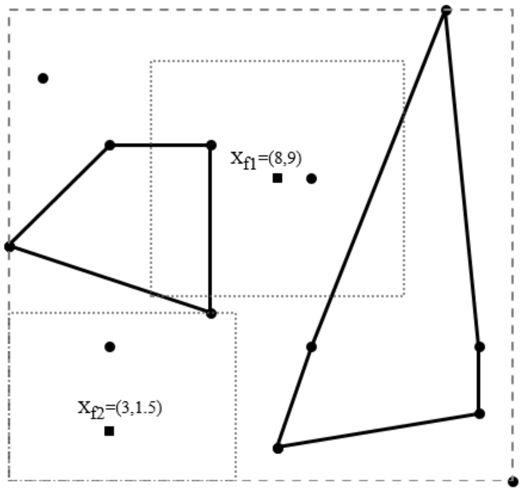


Figure 4.1: Rectangular convex hull (Dashed rectangle) and neighborhoods of two generated solutions X_{f1} and X_{f2} (dotted rectangles)

4.3.1 Main Components

Here, the main elements of SA considered in this study are presented. In the next section we see how these elements are used in the structure of SA.

Solution Representation and Evaluation:
Solutions in SA are represented as they are, i.e. a vector of two elements of x and y coordinates in the plain. Solutions are evaluated based on the corresponding objective function values. In its search procedure, SA tries to find a solution X_f which gives the minimum objective function value.

Initialization:
SA starts with an initial solution located randomly on one of the instance points. Besides, the system should be heated to a high temperature which brings a high thermodynamic free energy enabling the system to explore the search space more. The initial temperature, indicated by T_0, is a parameter to the algorithm.

Annealing Schedule:
The temperature of the system cools down through time. At any time, the temperature of the system, denoted by T, identifies the *state* of the system. The annealing schedule designates how the system cools down from initially high temperature T_0 to a freezing and stable state. Among several cooling schedules available in the literature, we use the simple *constant rate* cooling method by defining a cooling rate, α. With $0 < \alpha < 1$, the system cools down from the current state with temperature T to the next state with temperature αT. The value of α is given as a parameter.

Stopping Criteria:
Cooling down the system continues until the system freezes. With the annealing schedule described before, the temperature of the system never drops to zero. For this reason, we define a temperature threshold, ϵ as the freezing state of the system. Therefore, the algorithm terminates whenever the current temperature of the system, reaches or drops below ϵ, i.e. $T \leq \epsilon$.

Number of Iterations:
Defining constant cooling rate enables us to calculate the total number of iterations the algorithm runs. Let $Iter_T$ be the total number of remaining iterations when the temperature is T. Then, it is trivial to show

$$Iter_T = \left\lceil \log_\alpha \frac{\epsilon}{T} \right\rceil \tag{4.1}$$

Therefore, the total number of iterations is given by $Iter_{T_0} = \lceil \log_\alpha \epsilon/T_0 \rceil$, where, $\lceil a \rceil$ for any $a \in \mathbb{R}$ is the smallest integer greater than or equal to a.

Inner Repetitions:
In each state, SA can generate several solutions. The number of trials for generating a new solutions in each state is called inner repetition. Implementation of inner repetitions in SA gives more chance to the algorithm to find better solutions. The number of inner repetitions can be constant or dynamic. In this study we use linear dynamic repetition method by defining average number of inner repetitions, $AvgRep$, as follows. The algorithm starts with 0.5 of $AvgRep$ and ends with 1.5 of $AvgRep$. Using this idea, we let algorithm to exploit more around good solutions in later iterations. Let Rep_T be the number of inner repetitions at state T. Then,

$$\begin{aligned} Rep_T &= AvgRep \times \left(0.5 + \frac{Iter_{T_0} - Iter_T}{Iter_{T_0}}\right) \\ &= AvgRep \times \left(0.5 + \frac{\log_\alpha T/T_0}{\log_\alpha \epsilon/T_0}\right) \\ &= AvgRep \times \left(0.5 + \log_{\epsilon/T_0} \frac{T}{T_0}\right) \end{aligned} \tag{4.2}$$

Neighborhood:
Here we use the same definition for the neighborhood for solutions as given in Section 4.2. The neighborhood of solution X with size s, denoted by $Nbr_s(X)$, is a rectangular area centered at X with dimensions equal to s times dimensions of the rectangular convex hull of the instance points. In each state, SA generates the next solution, in the neighborhood of the incumbent solution. It is observed that decreasing the neighborhood size through time, has a significant effect on the convergence of the algorithm. Therefore, we define a state dependent neighborhood size as follows. Initial we set $s = 1$ and then, in each state, if an improving solution is found we multiply s by α using the same cooling rate. Decreasing the neighborhood size helps the algorithm to exploit better in the final iterations which yields to better convergence to a good solution.

Non-improving Moves:
Performing non-improving or uphill moves is essential for SA as it provides the opportunity of escaping from local optima while conserving the exploration factor. When a better solution than the incumbent solution is found in inner repetition steps, the algorithm immediately updates the incumbent solution to the newly generated better solution. It is possible that SA changes the current solution to a worse solution at a certain probability. Let Δ be the difference between objective function value of the newly generated solution Y and that of current solution X, i.e $\Delta = Z(Y) - Z(X)$. Then the probability of accepting a non-improving solution at state T, denoted as Pr_T, is given as

$$Pr_T = \begin{cases} 1 & \Delta < 0 \\ e^{-\Delta/T} & \Delta \geq 0 \end{cases} \tag{4.3}$$

Note that as the state of the system becomes more stable, the probability of accepting a non-improving solution decreases. The acceptance of a worse solution also becomes more unlikely as its objective function gets higher values. If any solution fails to improve or to perform an uphill move, we replace it by the best solution found so far.

Repairing:
Repairing infeasible and outlying solutions is essential to our meta-heuristic algorithms. In SA, the generated solutions are checked whether they are infeasible or outliers. If so, they are repaired by calling the `Check( )` procedure given in Algorithm 4.3.

4.3.2 The Algorithm

Following parameters are inputs to SA algorithm. In Chapter 5 we explain how proper parameter values for each meta-heuristic are chosen.

Parameters:

- Initial temperature (T_0)
- Cooling rate (α)
- Average inner repetitions ($AvgRep$)
- Temperature Threshold (ϵ)

Algorithm 4.6 SA algorithm

Input: the parameters T_0, α, $AvgRep$, and ϵ

```
T ← T_0 and s ← 1
initialize a solution, X
X_f ← X
while T > ε do
  update Rep_T using Equation 4.2
  for i = 1 to Rep_T do
    generate a new solution Y ∈ Nbr_s(X)
    Check(Y)   /* inspection for any required repairing */
    Y ←VNS(Y, s)
    Δ ← Z(Y) − Z(X)
    if Δ < 0 then
      X ← Y    /* improvement! update the current solution */
      if Z(X) < Z(X_f) then
        X_f ← X    /* update the best solution */
      end if
    else
      generate a standard uniform random number, rnd
      if rnd < e^{−Δ/T} then
        X ← Y    /* a non-improving move is accepted! */
      else
        X ← X_f    /* improve the current solution by moving it to the best location */
      end if
    end if
  end for
  if any improvement has been achieved then
    s ← αs
  end if
  T ← αT
end while
```

Output: the best solution generated, X_f

4.4 Evolutionary Algorithm (EA)

EA's are heuristic search methods which take their inspiration from biological evolution. EA is a generic population based meta-heuristic optimization algorithm that is widely used in solving some combinatorial optimization problems like the p-median problem (see Mladenović et al., 2007).

Through generations, EA follows the strategy of *survival of the fittest* in the population. The solutions with high fitness (or quality) are selected based on a selection method and recombined with other solutions using a reproduction procedure. Individuals (solutions) are also mutated by making a small change to their elements. The new solutions are more likely to be produced around the good solutions which have already been seen. After producing new solutions a replacement strategy is followed to keep fittest individuals for the next generation.

Before giving the main algorithm, let us explain general components of EA in details.

4.4.1 Main Components

In this section, general components of EA, namely coding scheme, fitness function, initialization, selection strategy, reproduction operators, replacement strategy, stopping criteria, and repairing are presented. In the next section, the algorithm structure is introduced.

Coding Scheme:
We define our solutions as a chromosome of two genes. The first gene represents the x-coordinate of the solution and the second gene represents its y-coordinate. With this representation, each gene can hold any real value.

Fitness Function:
Fitness function is used to measure the adaption of individuals to their environment. Here, the fitness of an individual is inversely proportional to its objective function value.

Initialization:
The number of individuals evolving in each generation of EA is denoted as Pop which is a parameter of the algorithm. One simple way to initialize these solutions is to generate them with random location in the plane. In this case there is a possibility that a generated solution becomes infeasible. Checking and repairing randomly generated solutions or generating only feasible solutions is time consuming. Another way, is selecting random locations from instance points (without replacement) and placing initial solutions on them. In this case, we ensure that initial solutions are feasible.

It is observed that different values for Pop changes the performance of EA. If the population size is too low, the algorithm cannot find a good solution and if it is too high, the CPU time increases dramatically when the problem size is large. Moreover, selecting large Pop for small instances is not so beneficial since a solution with almost the same quality can be obtained in less computational time using small Pop. Therefore, selecting a proper value for population size (Pop) is related to the problem size. For this reason, we define a population limit, *PopLimit*, and we set Pop as the minimum of *PopLimit* and half of the number of instance points, i.e.

$$Pop = \min\left\{PopLimit, \left\lceil \frac{|N|}{2} \right\rceil\right\} \tag{4.4}$$

Selection Strategy:
EA works by selecting one or more solutions called *parents* from a population of solutions and producing one or more new solutions from them. The produced solutions are called *offspring* who carry some characteristics of their parents. EA favors good solutions in the population by giving them more chance to reproduce.

Among several selection schemes presented in Bäck and Hoffmeister (1991), we used the *linear ranking* selection method. In this method all individuals are ranked based on their fitness such that the individual with the lowest objective function value has rank $i = 1$ and the individual with the highest objective function value has rank $i = Pop$. Then, the probability of selecting individual i as a parent is assigned as

$$Pr_i = \frac{1}{Pop}\left[1 + \pi - 2\pi\left(\frac{i-1}{Pop-1}\right)\right], \qquad i = 1, ..., Pop \tag{4.5}$$

where, $\pi \in [0, 1]$ is called selection pressure. In this study we use low selection pressure ($\pi = 0.5$) and high selection pressure ($\pi = 1$). The other strategy we used is random parent selection where parents are selected randomly without replacement. Note that setting $\pi = 0$ in Equation 4.5 implies random selection.

Reproduction Operations:
Reproduction methods in EA consists of two main operators: *crossover* and *mutation* operators which are applied on the selected individual(s) with a certain probability. The crossover operator generates offspring from selected parents by combining them. Let X and Y be two individuals selected as parents. Then two offspring O_X and O_Y are produced using the following equations.

$$O_X = X + \Gamma \otimes (Y - X) \tag{4.6}$$

$$O_Y = Y + \Gamma \otimes (X - Y) \tag{4.7}$$

Where, Γ is a vector of random numbers generated uniformly between -1 and 1, and $\otimes$ is component-wise multiplication. For instance, $(\gamma_1, \gamma_2) \otimes (x, y) = (\gamma_1 x, \gamma_2 y)$, for any vector (γ_1, γ_2) and (x, y). The random element, Γ, enables the algorithm to perform better exploration in the solution space.

The mutation operation is often applied on the offspring and changes it to a new solution by altering its gene. This operator plays an important role in EA's hill-climbing as well as preserving randomness and variety in the population to prevent fast convergence to local optima. Let X be an individual subjected to mutate. Then, the mutated individual is

$$X \leftarrow \texttt{VNS}(X, rnd) \tag{4.8}$$

where *rnd* is a standard uniform random number and $\texttt{VNS}(X, rnd)$ is a solution in the neighborhood of X with size *rnd* produced by using variable neighborhood concept in Section 4.2.

Replacement Strategy:
After producing offspring from selected parent, EA should decide which solutions to keep for the next generation and which solutions to discard. Two methods exist for replacement. In the first method, named generational approach, among all individuals and produced offspring, bests of them are survived in the next generation and the rest is discarded. The second method is steady-state method that makes replacements as soon as offspring are produced.

Süral et al. (2010) performed experiments on TSP and TSP with back-hauls using two EA algorithms. The first algorithm is based on generational strategy while the second one uses steady-state method. The authors concluded that the results obtained from the second algorithm are further better than the first one. Therefore, we use only the steady-state strategy which operates as follows. Among two parents and their offspring, the best offspring replaces the worst parent unconditionally and the other offspring replaces the remaining parent only if it is better that that parent.

Stopping Criteria:
The stopping condition determines the time through which the algorithm runs. It can be the total number of generations to evolve, denoted by *NGen*, or a factor of the population convergence. For the latter one, a number δ can be set for the upper limit fo the percentage deviation of objective function value of the worst individual from that of the best individual in a generation. For small values of δ, it can be said that the population is converged. Our EA terminates whenever the population deviation is less than or equal to a given number δ, or, the total number of generations is reached.

Repairing:
Repairing is another component of EA. Whenever a new solution is generated, it is inspected to make sure that this solution is useful, i.e. it is not infeasible or outlier. Repairing operation is done using `Check( )` procedure as soon as an offspring is produced.

4.4.2 The Algorithm

The following parameters are inputs to EA.

Parameters:

- Population size limit (*PopLimit*)
- Selection pressure (π)
- Crossove probability (P_c)
- Mutation probability (P_m)
- Maximum population deviation (δ) and maximum number of generations (*NGen*)

Algorithm 4.7 Evolutionary algorithm

Input: the parameters *PopLimit*, π, P_c and P_m, δ and *NGen*

1: set *Pop* and initialize the population
2: $X_f \leftarrow$ the best individual in the population
3: $gen \leftarrow 0$ /* set the generaton counter to zero */
4: **repeat**
5: **for** $i = 1$ to $Pop/2$ **do**
6: generate *rnd*, a standard uniform random number
7: **if** $rnd < P_c$ **then**
8: randomly select two parents, X and Y, from the population
9: produce two offspring O_X and O_Y /* perform crossover operation */
10: `Check(`O_X`)` and `Check(`O_Y`)` /* inspection for any required repairing */
11: $s \leftarrow 1 - \frac{gen}{NGen}$ /* decrease the neighborhood size */
12: $O_X \leftarrow$`VNS(`O_X, s`)` and $O_Y \leftarrow$`VNS(`O_Y, s`)`
13: generate *rnd*, rnd_1, and rnd_2 three standard uniform random numbers
14: **if** $rnd_1 < P_m$ **then**
15: $O_X \leftarrow$`VNS(`O_X, rnd`)` /* mutate the first offspring */
16: **end if**
17: **if** $rnd_2 < P_m$ **then**
18: $O_Y \leftarrow$`VNS(`O_Y, rnd`)` /* mutate the second offspring */
19: **end if**
20: **if** $Z(O_X) < Z(X_f)$ **then**
21: $X_f \leftarrow O_X$ /* update the best solution */
22: **end if**
23: **if** $Z(O_Y) < Z(X_f)$ **then**
24: $X_f \leftarrow O_Y$ /* update the best solution */
25: **end if**
26: follow the replacement strategy for selected parents and their offspring
27: **end if**
28: **end for**
29: $gen \leftarrow gen + 1$
30: **until** deviation of the worst solution's quality from $Z(X_f)$ is $\leq \delta$ **or** $gen = NGen$

Output: the best solution generated, X_f

Note that in Line 11 we set the size of the neighborhood in which `VNS( )` is going to generate a solution. As generations passes, this size decreases, allowing the algorithm to explore more in earlier generations and exploit better in later generations.

4.5 Particle Swarm Optimization (PSO)

PSO, developed by Kennedy and Eberhart (1995), is a population based meta-heuristic which regards the interaction between the particles in the population. In PSO, a number of simple entities (particles) are placed across the search space of a problem or function representing a solution to the problem. Each particle evaluates the objective function at its position and, then, decides to move in search space to find a better position. A particle's movement is determined by its current location and the best position visited by itself combined with those of one or more particles in the swarm and some random perturbations. In every single iteration, all particles in the swarm are moved and their attributes are updated accordingly. Throughout several iterations, the swarm of particles as a whole is likely to move close to the best value of the function.

Here we present a continuous PSO algorithm to be applied in our problem and show that how the idea of directional swarm intelligence can be easily justified and implemented for our problem. In the following sections we introduce main components and notations of PSO and we then we continue by giving the main algorithm.

4.5.1 Main Components

In this section, some general components of PSO, namely particle representation, initialization, update procedure, stopping criteria, and repairing are explained.

Particle Representation:
From a mathematical point of view, each particle's position X_i, is a 2-dimensional vector, where the first element of the vector represents x-coordinate and the second element represents y-coordinate of X_i's location. Each particle has a velocity, V_i, that has the same dimension as X_i. V_i is usually kept in the range $[-V_{max}, V_{max}]$ where V_{max}, a parameter to PSO, is a vector representing the maximum value that both elements of V_i can get. In addition to velocity, we define another attribute for each particle called *Age*. Each particle i ages whenever it fails to improve itself in its movement. Particles are allowed to get old until a given *AgeLimit*. If a particle's age reaches *AgeLimit*, the particle is immediately replaced by the best particle in the population. We observed that when aging concept with a proper *AgeLimit* value is added to PSO, its performance improves significantly.

Initialization:
The initialization in PSO is the same as that of EA explained in Section 4.4.1. That is, the number of particles in the system, Pop, is set by using Equation 4.4. All initial Pop particles are positioned at locations of Pop instance points selected randomly. Furthermore, all particle velocity vectors, V_i for $i = 1, ..., Pop$ are initialized such that each element of V_i is randomly generated by uniform distribution between 0 and V_{max}. Initial random velocity is a necessary element of PSO when exploring the solution space and escaping from local optima matter. Here, instead of setting a constant number for V_{max}, we set V_{max} as a fraction of far, where far is the distance between the farthermost points in the instance. Finally, all particles are initiated with zero ages, $Age_i = 0$ for $i = 1, ..., Pop$.

Update Procedure (Particle Movement):
New location for a particle X_i is obtained when it moves from its previous location at velocity V_i. Besides, previous best position of particle i, defined as vector P_i, can provide useful information about where better solutions might exist. Therefore, the direction from X_i towards P_i can also be included to find a new location with a random perturbation:

$$V_i \leftarrow V_i + \Phi_1 \otimes (P_i - X_i) \tag{4.9}$$

Where Φ_1 is a 2-dimensional vector in which both entries are equal to a random number uniformly generated between 0 and a constant φ_1.

However, a particle itself has almost no power to solve any problem and real progress occurs when particles interact which is the cornerstone of PSO. Not only do particles consider the history of their movements, but they also keep an eye on progress of other particles too. As a result of communication between particles and a *social network* topology, they become able to choose better directions that lead to better solutions. A comprehensive review on topologies is introduced in Poli et al. (2007). The implemented topology in this study is called *global best* where all particles are influenced by the best particle in the entire population. Therefore, V_i can be updated with respect to the direction towards the global best position, X_f, as following:

$$V_i \leftarrow V_i + \Phi_1 \otimes (P_i - X_i) + \Phi_2 \otimes (X_f - X_i) \tag{4.10}$$

Where Φ_2 is a random 2-dimensional vector in which both entries are equal to a random number uniformly generated between 0 and a constant φ_2.

φ_1 and φ_2 determine the magnitude of the random forces in the direction of personal best P_i and global best X_f. These are often called acceleration coefficients. The behavior of a PSO changes radically with the value of φ_1 and φ_2. Clerc and Kennedy (2002) noted that the acceleration of particles highly depends on V_{max} and acceleration coefficients. Therefore, they proposed a constriction coefficient, χ, that is multiplied to all additional vectors added to X_i in order to ensure convergence and eliminate the unwanted effect of V_{max}:

$$V_i \leftarrow \chi(V_i + \Phi_1 \otimes (P_i - X_i) + \Phi_2 \otimes (X_f - X_i)) \tag{4.11}$$

Where

$$\chi = \frac{2}{\varphi - 2\sqrt{\varphi^2 - 4\varphi}} \tag{4.12}$$

for $\varphi = \varphi_1 + \varphi_2$. The best setting that introduced by authors is $\chi = 0.7298$ with $\varphi_1 = \varphi_2 = 2.05$. Experiments showed that this setting also works well for our problem. Therefore, we set both acceleration factors, φ_1 and φ_2 to the same number, 2.05.
Finally the location of particle i is updated as:

$$X_i \leftarrow X_i + V_i \tag{4.13}$$

Stopping Criteria:
The stopping condition is the same as the one explained in Section 4.4.1. That is, PSO terminates when the deviation of objective function value of the worst individual from the objective function value of the best individual in a generation is less than or equal to a given number δ, or, the maximum number of iterations, $NIter$, is reached.

Repairing:
Every generated solution should be checked for repairing. Inspection and required repairing procedure is done whenever a particle's location is updated.

4.5.2 The Algorithm

Parameters:

- Population size limit ($PopLimit$)
- Aging limit ($AgeLimit$)
- Maximum population deviation (δ) and the maximum number of iterations ($NIter$)

Algorithm 4.8 PSO algorithm

Input: the parameters *PopLimit*, *AgeLimiti*, δ and *NGen*

1: set *Pop* and initialize the population
2: $X_f \leftarrow$ the best individual in the population
3: $iter \leftarrow 0$ /* set the iteration counter to zero */
4: **repeat**
5: **for each** particle X_i in the population **do**
6: update the particle's velocity, V_i, using Equation 4.11
7: update the particle's location, X_i, using Equation 4.13
8: `Check(`X_i`)` /* inspection for any required repairing */
9: $s \leftarrow 1 - \frac{iter}{NIter}$ /* decrease the neighborhood size */
10: $X_i \leftarrow$`VNS(`X_i, s`)`
11: **if** $Z(X_i) < Z(P_i)$ **then**
12: $P_i \leftarrow X_i$ /* update particle i's personal best */
13: $Age_i \leftarrow 0$ /* particle is improved */
14: **else**
15: $Age_i \leftarrow Age_i + 1$ /* the particle gets old */
16: **end if**
17: **if** $Age_i \geq AgeLimit$ **then**
18: $X_i \leftarrow X_f$ /* replace the particle with the best solution */
19: **end if**
20: **if** $z(X_i) < z(X_f)$ **then**
21: $X_f \leftarrow X_i$ /* update the best solution */
22: **end if**
23: **end for**
24: $iter \leftarrow iter + 1$
25: **until** deviation of the worst solution's quality from $Z(X_f)$ is $\leq \delta$ **or** $iter = NIter$

Output: the best solution generated, X_f

Again, the neighborhood size is reduced in Line 9 to explore around the good solutions more in final iterations.

Chapter 5 provides the experimental results obtained by running the three meta-heuristics presented in this chapter on different problem instances.

CHAPTER 5

COMPUTATIONAL EXPERIMENTS

After introducing meta-heuristic algorithms it is necessary to examine their performance. Performance of a search procedure is mainly characterized by its ability to find a high quality solution as well as the time it takes to find such a solution. The better the final solution and the lower the required processing time, the higher performance the algorithm has.

In this chapter we explain the software package which is coded to deal with the problem. Then, information about problem instances is provided and the preliminary experiments that yield to best parameter settings of heuristic algorithms is given. Finally, the experimental results on all problem instances are presented. The chapter also contains concluding remarks from experiments.

5.1 Software Development

To run the algorithms and to work on problem instances, a software package is developed in Microsoft Visual Studio 2010 environment using Visual Basic .Net language. The software has the following features:

- An adequate graphical user interface (GUI) that enables users to open and visually see problem instances.
- Enabling the user to change the problem instance. For example, changing the location or the weight of demand points, adding or removing demand points, modifying the location or the shape of regions, and changing region fixed costs are easily applicable.
- Enabling the user to choose the meta-heuristic algorithm she wants to implement.
- Enabling the user to manage the settings for each meta-heuristic algorithm.
- Making it possible to enable or disable VNS and/or repairing procedure in meta-heuristics algorithms.
- Visualizing the solution generating procedure in algorithms so that the user can see the behavior of meta-heuristics by visualizing the location of generated solution. This feature enables the user to observe how solutions are generating, moving, or converging to certain locations on the plain.
- Showing the convex hull of instance point, convex hull of regions, convex hull of demand points and rectilinear convex hull. The information about instance points (like minimum, maximum and range of x and y coordinate values) and the geometry of these convex hulls is also given so that the user can find out the percentage of instance convex hull that is occupied by the regions. The eliminated regions in the preprocessing procedure can also be seen.
- Showing the best solution when running an algorithm is done. Location of the facility on the

plain is visualized and the least cost pathways to all demand points are shown by lines. The user can notice the intermediate points on all ways and the cost of that way. All penalized pathways are shown with a dashed line. Therefore, when the final solution is shown, the user can have an idea about the facility location and all traveling routes by a single look.

- Providing statistical data of each generated solution, the initial solution(s) and the final solution as well as CPU time. This data can be imported in statistical or spreadsheet softwares for further analysis.

Figure 5.1 illustrates the problem instance BC13 given in Butt and Cavalier (1996). A facility location with barriers and the traveling routes are shown. The facility is located at $(6.857, 6.143)$ having objective function value of 29.838055 which is believed to be the optimal solution (see Butt and Cavalier, 1996). The facility-demand pathways are also shown by lines. Note that in this figure, since the problem considers barrier regions, no path passes through the regions.

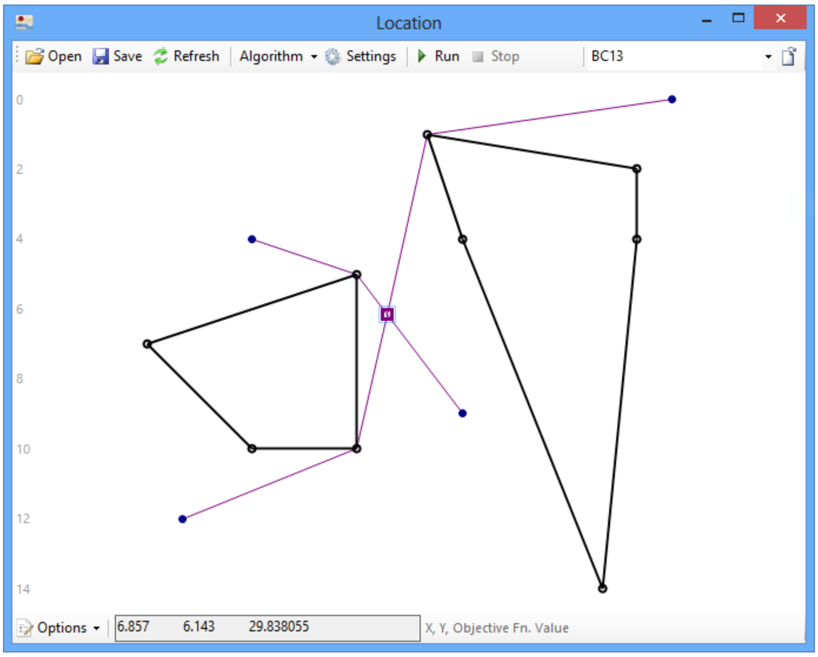


Figure 5.1: A snapshot of the application from BC13 instance. The best location for the facility is shown by a square

In Appendix D, more details and instructions about using the program is given.

All experiments are done using a PC with 2.99 GHz CPU and 3.49 GB RAM running Microsoft Windows XP operation system.

5.2 Problem Instances

Since our problem is a general restricted facility location problem, any LocP or RLocP which meets our assumptions can be solved using the proposed meta-heuristics. Hence, we tried to collect all RLocP problem instances available in the literature. Unfortunately, there is no extensive experimental studies available in the literature, possibly, due to the difficulties arising from the nature of Euclidean

RLocP's (i.e. nonlinearity and non-convexity of the objective function value). There are only a few numerical examples available for which the best known solution is reported. Table 5.1 shows the available instances in the literature, the problem type, and the type of their regions. The number of demand points ($|M|$), number of regions ($|R|$), total number of region vertices ($|V|$) and total number of points in that instance ($|N|$) is also provided in Table 5.1. To be able to refer to these instances easily, we name them as given in the same table.

Table 5.1: Available problem instances in the literature

#	Name	Source	Problem	Regions	$\|M\|$	$\|R\|$	$\|V\|$	$\|N\|$
1	AP25	Aneja and Parlar (1994)	RLocP-FR	non-convex polygon	4	1	21	25
2	AP70	Aneja and Parlar (1994)	RLocP-BR	non-convex polygons	18	12	52	70
3	BC13	Butt and Cavalier (1996)	RLocP-BR	convex polygons	4	2	9	13
4	D26	Dan (2009)	RLocP-BR	convex polygons	6	5	20	26
5	KC5	Katz and Cooper (1981)	RLocP-BR	circular	5	1	–	5
6	KC10	Katz and Cooper (1981)	RLocP-BR	circular	10	1	–	10

The name of instances contains the initials of authors' name and the number of points (demand points + region vertices) in the problem. Since KC5 and KC10 contain circular regions, corresponding number of vertices is not applicable for them.

Aneja and Parlar (1994) provided variants of their example, AP70, by removing some of the regions and reported their solutions up to 2 digits. The variants is denoted by AP70R# where the symbol # can be 10, 8, 6, 4, 2 or 0 showing the number of regions ($|R|$) in the problem (out of 12 regions). AP70R0 refers to the problem where all barriers are removed, the unrestricted problem. Since the regions that are given in KC5 and KC10 are circular (see Katz and Cooper, 1981), we replace them by equilateral polygons. The polygons are regular convex polygons that can approximate the circles from inside or outside. The same approach was also used by Bischoff and Klamroth (2007). They reported best solutions for KC5 and KC10 instances in Katz and Cooper (1981) by approximating the circular regions with regular polygons both from outside and inside. The polygons have different number of edges. The instances used in Bischoff and Klamroth (2007) with the information of the number of edges in the approximative polygon and whether it is approximated from outside (circumscribed) or inside (inscribed) are reported in Table 5.2. Number of sides in the polygons is equal to the number of vertices or $|V|$. Note that the number of demand points and regions are the same as the source problems. Because of our assumption for considering polygonal regions, the problem instances given in Table 5.2 is used instead of KC5 and KC10.

The name of instances in Table 5.2 contains the name given in Table 5.1, followed by a *c* or *i*, indicating that the approximative polygon is circumscribed or inscribed respectively, and the number of edges ($|V|$) in that polygon.

Bischoff and Klamroth (2007) also reported the solutions for the variants of AP70 problem given in Aneja and Parlar (1994) up to 4 digits. The reported best solutions and corresponding objective function values for all instances available in the literature are given in Table 5.3. From now on, we refer to the best known solution or the best found solution by *BSol* notation.

Table 5.2: Modified problem instances of KC5 and KC10 in Katz and Cooper (1981)

#	Name	Polygon Type	$\|V\|$	$\|N\|$
1	KC5c16	circumscribed	16	21
2	KC5c32	circumscribed	32	37
3	KC5c64	circumscribed	64	69
4	KC5c128	circumscribed	128	133
5	KC5c256	circumscribed	256	261
6	KC5c512	circumscribed	512	517
7	KC5i16	inscribed	16	21
8	KC5i32	inscribed	32	37
9	KC5i64	inscribed	64	69
10	KC5i128	inscribed	128	133
11	KC5i256	inscribed	256	261
12	KC5i512	inscribed	512	517
13	KC10c16	circumscribed	16	26
14	KC10c128	circumscribed	128	138
15	KC10i16	inscribed	16	26
16	KC10i128	inscribed	128	138

Table 5.3: Best solutions reported for the problem instances in the literature

#	Name	X_f^*	$Z(X_f^*)$
1	AP70	$(8.7667, 4.9797)$	119.1387
2	AP70R10	$(8.7667, 4.9797)$	119.1047
3	AP70R8	$(9.1873, 5.4860)$	116.3976
4	AP70R6	$(9.2658, 6.2527)$	114.5610
5	AP70R4	$(9.2173, 6.1528)$	113.7656
6	AP70R2	$(9.0372, 6.1150)$	111.6889
7	AP70R0	$(8.9127, 6.3554)$	110.0068
8	AP25	$(5.50, 0)$	48.50
9	BC13	$(6.857, 6.143)$	29.838
10	D26	$(31, 26)$	29.10
11	KC5c16	$(-1.201580, 2.077647)$	48.281797
12	KC5c32	$(-1.190873, 2.067660)$	48.261460
13	KC5c64	$(-1.185968, 2.062756)$	48.256464
14	KC5c128	$(-1.186446, 2.060556)$	48.255225
15	KC5c256	$(-1.186174, 2.060530)$	48.254917
16	KC5c512	$(-1.186063, 2.060519)$	48.254840
17	KC5i16	$(-1.181308, 2.057875)$	48.241865
18	KC5i32	$(-1.181308, 2.057875)$	48.251504
19	KC5i64	$(-1.186927, 2.058351)$	48.253988
20	KC5i128	$(-1.185897, 2.060503)$	48.254609
21	KC5i256	$(-1.185953, 2.060508)$	48.254764
22	KC5i512	$(-1.186050, 2.060516)$	48.254802
23	KC10c16	$(3.324784, -0.085586)$	88.468917
24	KC10c128	$(3.307095, -0.067167)$	88.325077
25	KC10i16	$(3.303454, -0.062217)$	88.249042
26	KC10i128	$(3.305932, -0.067746)$	88.321938

Lack of any intensive instance library for RLocP's and limited number of available problem instance in the literature encouraged us to design and generate more problems to perform computational experiments. In the following section the way of generating more problem instances is explained.

5.2.1 Generating Problem Instances

For generating more problem instances, we tried to use available problems in the literature as the original (seed) problems and create other instances by keeping some features in the original ones as they are. The term *pattern* is used for this purpose.

5.2.1.1 Instance Patterns

Patterns are introduced to enable us generating more problem instances from original ones. In generating instances we focus on the fixed cost and the size of regions. A pattern for a problem instance is given by two positions as

$$Pos1\text{-}Pos2$$

Pos1 indicates the level of congested region fixed costs that can be either high (h) or low (l). Following the discussion in Section 3.4, low variant assigns a fixed cost $c_r = rnd(0, 0.4) \times pm_r/2$ and high variants assigns $c_r = rnd(0.6, 1) \times pm_r/2$ to each region $r \in R$, where, $rnd(a, b)$ is a random number distributed uniformly between a and b and pm_r is the perimeter of any region $r \in R$.

To justify our results using upper and lower bounds given in Section 3.5, Pos1 can also take FR and BR values for which c_r are set to 0 and $pm_r/2$ respectively for all $r \in R$.

Pos2 gives information about size of the regions in the problem. It is given as a percentage: 50 (when the scale of all regions are one half of original scale) or 25 (where all regions are scaled as 25% of the original scale). If Pos2 is set to "*o*" it refers to the original sizes.

When the instance pattern is U, it means that all regions in the original problem is removed and the problem is considred as unconstrained problem. Moreover, the pattern O is assigned to an instance it means that the instance is the original problem given in Table 5.3. To illustrate, consider the problem AP70. If pattern O is assigned, the problem is the original problem. AP70 with pattern h-50 corresponds to the AP70 problem in which all barriers are turned into 50% smaller congested regions with high fixed cost levels. Finally, the problem AP70 with pattern U is the unrestricted AP70 problem (which is, in this case, AP70R0 in Table 5.3).

By using this scheme we are able to have 12 different variants for each 25 instances listed in Table 5.3 that has restriction regions. Furthermore, 6 problems in Table 5.1 are considered without restriction (with pattern U). Thus, we have 306 instances in total originated from the RLocP literature.

5.2.1.2 Large Problem Instances

In addition to the problem instances given in Section 5.2.1.1, we also include large problem instances from TSP and VRP online libraries. Our aim is to analyze the performance of proposed heuristics when the problem sizes become large. The list of instances and the corresponding number of demand points are given in Table 5.4.

Table 5.4: Large TSP and VRP Instances

#	Name	Source	$\|M\|$
1	C600	VRP [a]	601
2	R800	VRP	801
3	RC800	VRP	801
4	R1000	VRP	1001
5	RC1000	VRP	1001
6	u2319	TSP [b]	2319
7	fnl4461	TSP	4461
8	pla7397	TSP	7397
9	usa13509	TSP	13509
10	pla33810	TSP	33810

[a] From Homberger's instance collection in Vehicle Routing Problem (2012): `http://neo.lcc.uma.es/vrp/vrp-instances`

[b] From TSPLIB (2008): `http://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95`

The demand points in the VPR instances are clustered (shown by C), uniformly distributed (shown by R), or distributed as a mix of C and R (shown by RC). Problem instances given in Table 5.4 contain no regions and the sizes represents only the number of customers. Thus, we generate more instances by imposing congested region restrictions in the original problems. Two types of regions are considered in this case, lines and polygons. The shapes and location of the regions are completely arbitrary, however, to stick with our assumptions and preserving the number of costumers, we project demand points that fall inside the located regions to the nearest edge of corresponding regions. If the variant of the original instance contains line-shaped regions, its pattern is indicated by *LR* and if it contains polygonal regions, the pattern is indicated by *PR*. The fixed costs are set to $rnd(0, 1) \times pm_r/2$ for any located region $r \in R$. The original problem with no restriction is shown with pattern *U*. In the problems with patterns U and LR, distribution of demand points remain as original. Table 5.5 shows more details.

Thus, for each problem instance in Table 5.4 we have two different patterns providing 20 instances (listed in Table 5.5) in addition to 10 unconstrained ones. Consequently, we have totally 336 instances in our experiments.

Table 5.5: Modified large TSP and VRP problem instances in the literature

#	Name	Pattern	$|R|$	$|V|$	$|N|$
1	C600	LR	3	6	607
2	C600	PR	2	8	609
3	R800	LR	3	6	807
4	R800	PR	3	10	811
5	RC800	LR	2	4	805
6	RC800	PR	2	10	811
7	R1000	LR	4	8	1009
8	R1000	PR	3	12	1013
9	RC1000	LR	3	6	1007
10	RC1000	PR	2	7	1008
11	u2319	LR	2	4	2323
12	u2319	PR	1	5	2324
13	fnl4461	LR	2	4	4465
14	fnl4461	PR	2	6	4467
15	pla7397	LR	2	4	7401
16	pla7397	PR	3	11	7408
17	usa13509	LR	3	6	13515
18	usa13509	PR	3	9	13518
19	pla33810	LR	2	4	33814
20	pla33810	PR	2	7	33817

5.3 Parameter Settings

The performance of heuristic algorithms highly depends on the parameter setting they work with. Among 336 instances, 10 of them are arbitrary selected to perform our preliminary experiments. The aim is to determine the best parameter setting for each meta-heuristic in order to run for all instances. Parameter settings of each heuristic is given in Table 5.6.

Each combination of parameter levels of each heuristic is replicated 10 times on our 10 test instances. Two way interaction analysis are used to select the best settings for each algorithm. The performance of the algorithms (determined by the best objective function value and CPU time) was considered the main response to the parameter factors. In Appendix C, examples of interaction plots are given. Table 5.7 shows the resulting best levels for each heuristic parameters.

Table 5.6: Parameter settings of each meta-heuristics

(a) SA's parameters

Parameters	Levels 1	2	3
T_0	500	1000	–
α	0.82	0.92	–
$AvgRep$	5	10	15
ϵ	0.005	–	–

(b) EA's parameters

Parameters	Levels 1	2	3
$PopLimit$	10	20	30
π	0.0	0.5	1.0
P_c	0.85	0.95	–
P_m	0.2	–	–
δ	0.05	–	–
$NGen$	90	–	–

(c) PSO's parameters

Parameters	Levels 1	2	3
$PopLimit$	10	20	30
V_{max}	0.3	0.6	–
$AgeLimit$	3	6	12
φ_1	2.05	–	–
φ_2	2.05	–	–
δ	0.05	–	–
$NIter$	90	–	–

Table 5.7: Best parameter adjustments for each meta-heuristics

(a) SA's best settings

Parameters	Value
T_0	1000
α	0.92
$AvgRep$	10
ϵ	0.005

(b) EA's best settings

Parameters	Value
$PopLimit$	20
π	0.75
P_c	0.85
P_m	0.2
δ	0.05
$NGen$	90

(c) PSO's best settings

Parameters	Value
$PopLimit$	20
V_{max}	0.3
$AgeLimit$	12
φ_1	2.05
φ_2	2.05
δ	0.05
$NIter$	90

The parameter π in EA often results in better objective function values if it is set to 1, however, the CPU time increases, significantly. For this reason, we performed other experiments on the test instances by setting the best parameter values obtained from previous experiments and $\pi = 0.75$ (see Table 5.7(b)).

It is observed that the algorithm performed better in this case by producing good quality solutions in less time compared to $\pi = 1$ case (some examples are given in Appendix C). Therefore, $\pi = 0.75$ is set for further experiments.

Moreover, we doubled the population limit for EA and PSO to solve large problem instances given in Table 5.4. In this way we have more fair and accurate computations since the number of points in these instances are much more than that in other instances.

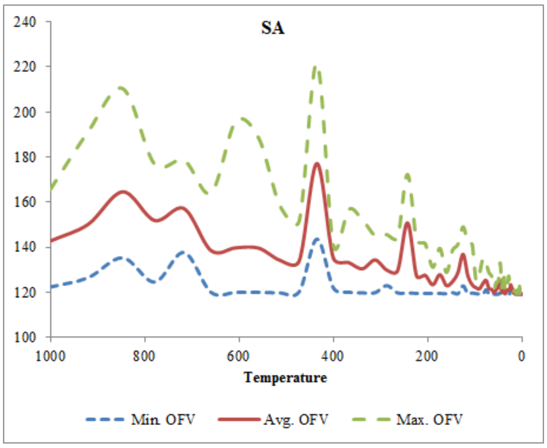
5.4 Convergence

In this section we give some illustrations to show the behavior of meta-heuristic algorithms around the best solution they found. Furthermore, we graphically show the convergence of the solutions to the solution of unrestricted problem when the fixed costs decreases to zero.

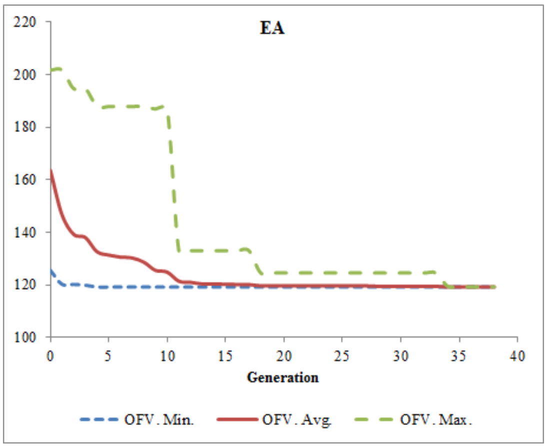
5.4.1 Converging to the Best Solution

Exploration and exploitation are two important processes that the algorithms undergo in solving a problem. At first, since a limited information is available, it is better to explore the solution space for a chance of finding a good solution. As time passes, the algorithms try to exploit more around the good solution in order to improve that solution. Figure 5.2 shows the change of objective function values corresponding to the generated solution over time. The information comes from one replication of each algorithm on AP70 problem instance. See Appendix C for more plots of convergence of the algorithms.

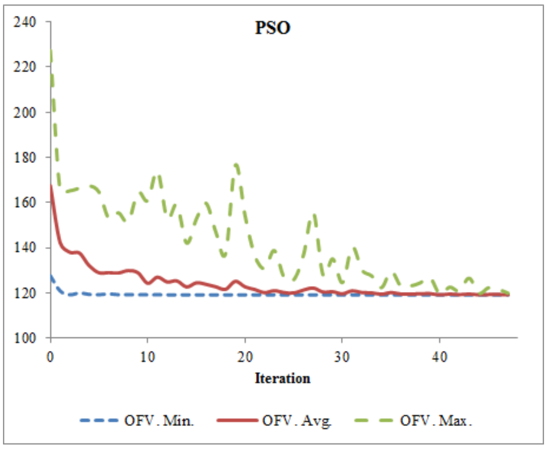
In Figure 5.2(a) the maximum, average and minimum objective function values (OFV's) are plotted with respect to the temperature. In any temperature, SA generates several solutions in the inner repetition steps. In Figure 5.2(a), 'Min. OFV', 'AVg. OFV' and 'Max. OFV' respectively correspond to the best, average and the worst of objective function values of generated solutions at a specific temperature. As the system cools down, all tree values converge to the best objective function value. The uphill moves and generation of non-improving solutions can be observed from this graph. EA updates a population of solutions in a generation. Figure 5.2(b) shows the objective function values of the best solution ('Min. OFV' curve) and the worst solution ('Max. OFV' curve) as well as the average objective function value of all solutions of a population ('AVg. OFV' curve) through generations. Since EA generates a population, it is more likely to have a good solution even in the first generation. It can be seen that EA has a faster convergence than SA. Similar behavior of particles' objective function values over iterations in PSO is also shown in Figure 5.2(c). 'Min. OFV', 'AVg. OFV' and 'Max. OFV' in Figure 5.2(c), respectively shows the best objective function value, average objective function value and the best objective function value of the generated solutions trough iterations.



(a) SA's convergence



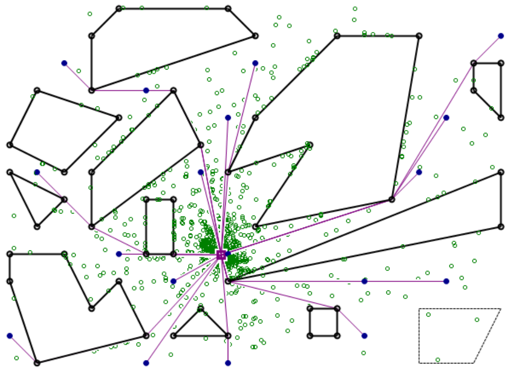
(b) EA's convergence



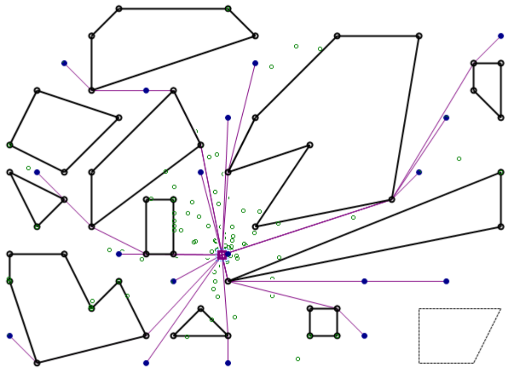
(c) PSO's convergence

Figure 5.2: Convergence of three meta-heuristics to the BSol in AP70 problem instance: Objective function values.

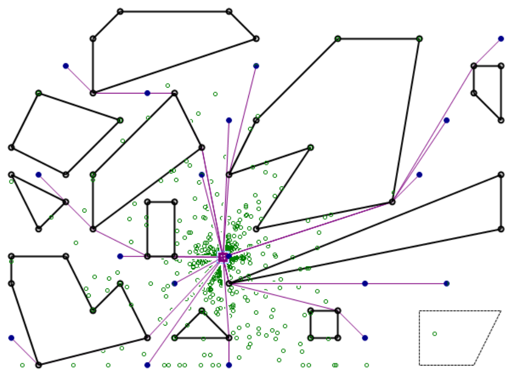
A special characteristic of planar location problems considered in this study is that they are defined on a plane. Therefore, it is easy to picture problem instances, their solutions and facility-demand routes. Figure 5.3 illustrates the behavior of different algorithms when solving AP70. All generated solutions are displayed by small circles and the final solution (facility location) is shown by a square. The facility-demand pathways are also shown by lines. Region boundaries are shown by black lines and demand points are filled circles.



(a) SA's convergence



(b) EA's convergence



(c) PSO's convergence

Figure 5.3: Convergence of three meta-heuristics to the BSol in AP70 problem instance: Generated solutions.

It can be seen from Figure 5.3 that:

1. The bottom-right most region (shown by dashed edges) is eliminated since it is totally outside the convex hull of demand points.

2. No solution is generated inside the regions since all infeasible ones are repaired and moved to the edges of corresponding infeasible region.
3. More solutions are generated around the final solution. Most of these solutions are generated in final iterations when the algorithms exploit the BSol they found.

It can also be noticed that SA explores the solution space more than EA and PSO (see Figure 5.3(a) in which more solutions are spread in the plain compared to EA (Figure 5.3(a)) and PSO (Figure 5.3(c))). This verifies the results given in Figure 5.2 where convergence of EA and PSO is faster than SA.

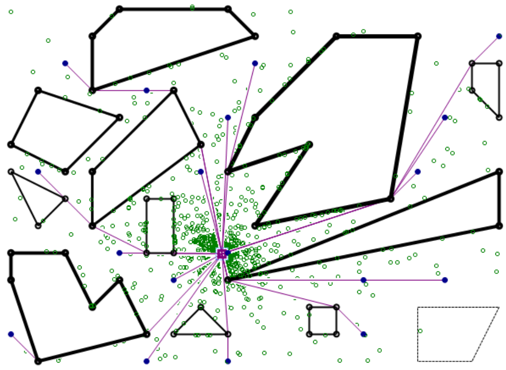
5.4.2 Dependency on Initial Solutions

One important characteristic of the proposed meta-heuristics is that their are independent to the initial solution(s). Regardless of the initial locations of solutions, the algorithms finally converge to the BSol. Small deviations from BSol for a particular instance, discussed in Section 5.5, demonstrates almost the same convergence for all replications. An illustration is given in Figure 5.4. Figure 5.4 shows the convergence of the generated solutions to the BSol of AP70 problem when all solutions in all meta-heuristics are initialized at $(19, 13)$. $(19, 13)$ is the location of the upper-right most demand point in the instance. Again, all algorithms were able to converge to the BSol located at $(8.7667, 4.9797)$.

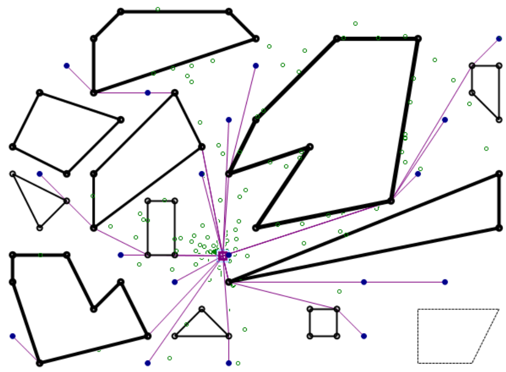
5.4.3 Converging to the Unrestricted Problem Solution

Figure 5.5 shows the fnl4461 instance with pattern LR where two line regions are placed in the plane among many demand points. Initially the traveling costs of both regions are set to BR level, meaning that no facility-demand way will pass through the lines. We solved this problem using PSO (under its best settings) and record the solution as the solution for the highest fixed cost level. Each time the fixed costs of both regions are decreased by $\%10$ and the solution is recorded. This procedure is continued until the cost of both regions became zero, i.e. FR cost level. The solution of the unconstrained problem is also found using Weiszfeld's algorithm. Each time we decrease the region costs, the location of the final solution became closer to the solution of the unrestricted problem. Figure 5.5 shows how solutions of different fix cost levels approach to X^U.

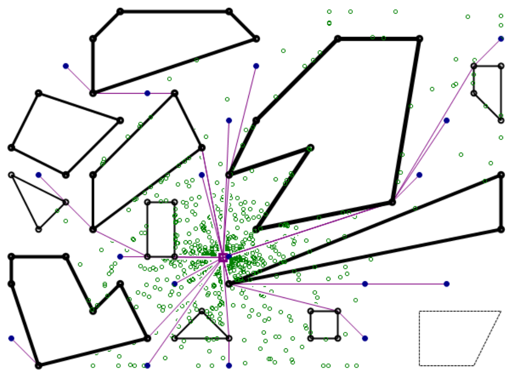
Convergence to the solution of unconstrained problem is investigated more in the next section where it is concluded that as the cost level of congested regions increases the gap between the objective function values of the restricted problem and unrestricted problem increases too.



(a) SA's convergence



(b) EA's convergence



(c) PSO's convergence

Figure 5.4: Convergence of three meta-heuristics to the BSol in AP70 problem instance: Generated solutions under a different initialization.

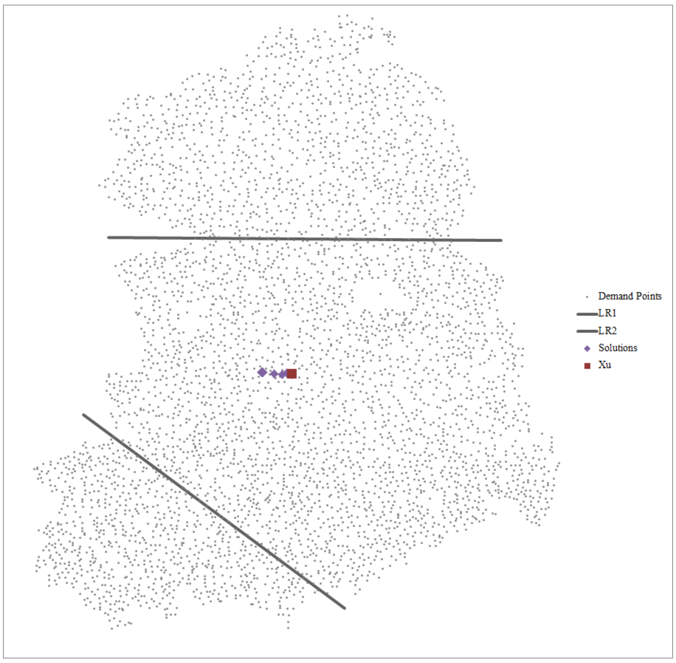


Figure 5.5: Convergence to the solution of the unrestricted problem. Solutions (shown by diamonds) approach the unrestricted problem's solution (shown by a square) as fixed cost levels decreases

5.5 Computational Results

All problem instances are solved by using proposed meta-heuristic algorithms and the results are given in Appendix A and B. All heuristics are replicated 10 times on each problem instance and the calculations are done using single decimal precision (up to 6 digits). The BSol for all unrestricted problems, i.e. problems with pattern U, and restricted problems with forbidden regions (problems with pattern FR-#) are found using the approaches discussed in Chapter 4. The obtained BSols and the reported BSols in the literature are used to examine the performance of our heuristics. In the following sections we focus on the solution results as well as the performance of the proposed meta-heuristic algorithms.

5.5.1 Solution Results

Tables in Appendix A show the information about the objective function values and the best solutions generated by meta-heuristic algorithms. Detailed information for different instances are given in separate tables. For every implemented meta-heuristic, these information contain:

- The instance name and its pattern in the first and second columns.
- The applied meta-heuristic algorithm in the 'Alg.' column.
- The *x* and *y* coordinates of the best solution out of 10 replications.
- The best objective function value found in 10 replications, indicated by 'Min. OFV'.
- 'Avg. OFV', the average of objective function values in 10 replications.
- 'Max. OFV' showing the maximum objective function value of 10 replications.
- 'Avg. %Gap U', the percentage gap between the average objective function value of 10 replica-

tions and the objective function value of the unconstrained problem.

To see the effect of imposing different restrictions on the objective function values, we compare the objective function value of each problem with that of unrestricted problem. 'Avg. %Gap U' is calculated as.

$$\text{Avg. \%Gap U} = 100 \times \frac{\text{Avg. OFV} - Z(X^U)}{Z(X^U)}. \tag{5.1}$$

Tables A.1 to A.25 in Appendix A show that for all problem instances in Table 5.3, at least one of meta-heuristic algorithms could find the reported solution. Thus we have the same result as Table 5.3 for original problems. One exception occurs for KC10c128 instance for which we were able to find a slightly better solution than the reported one in Bischoff and Klamroth (2007). The difference between two objective function values is about 2.6×10^{-6} percent. It is observed that in KC5 and KC10 instances, as the number of edges of the polygons increases, (the shape of polygons becomes closer to the circular regions), the objective function improves (see Table 5.3 for variants of KC instances). Therefore, approximating non-polygonal regions is important when using the proposed solution approach. In Section 5.5.2 we explain the trade off between good approximation of non-polygonal regions and the required time to solve the problem.

Table 5.8 shows the effect of problem instance patterns on the BSol found by each meta-heuristic. This analysis is done on the problem instances from the literature given in Table 5.3 and their variants. Considering all 306 instances, this table shows the mean of 'Avg. %Gap U' in Tables A.1 to A.25.

Table 5.8: Effect of instance patterns on the average %GAP U

Pattern		Algorithm		
		SA	EA	PSO
Cost Level	U	0.00	0.20	0.01
	FR	0.22	0.22	0.22
	l	1.28	1.28	1.28
	h	1.42	1.42	1.42
	BR	1.42	1.42	1.42
Region Size	25	0.04	0.04	0.04
	50	0.37	0.37	0.37
	o	2.85	2.85	2.85

It can be concluded from Table 5.8 that:

1. When considering cost levels, the lower and upper bounds introduced in Chapter 3 are justified. That is, the BSol's objective function value of the restricted problems is not less than that of the unrestricted problem since all gaps for cost levels *FR*, *l*, *h*, and *BR* are positive. Moreover, the objective function value of the restricted problems with forbidden regions (i.e. FR-#) are always less than or equal to that of restricted problems with higher cost levels for congested regions. Finally, the restricted problems with barriers (i.e. BR-#) always result in the highest objective function values compared to the restricted problem with lower fixed cost levels of congested regions and unrestricted problem. It can be justified since the gap values for *BR* is highest among other cost levels. In short, the computational results given in Appendix A hold the conditions in Theorem 3.1 in Chapter 3, as expected.
   Besides, the chance of improvement in objective function value increases if the fixed costs of the regions decrease. For example in AP70 problem, when the fixed cost are decreased from level *h*

to level l, improvements in objective function values are observed. Even less objective function values is achieved if the costs are further decrease to zero (see Table A.2 for more details). The reason for this trend is that, as expected, the fall in fixed costs of congested regions may result in less facility-demand direct access costs so that passing through regions becomes more favorable than going the way around them.

2. The objective function value improves when the size of the regions decreases, as expected. The reason is that smaller restriction regions provide larger feasible space. In some cases the regions are so small that they become redundant and yield identical solutions to unrestricted problems (as an example, when the regions in AP25 are squeezed by 75%, the solution becomes the same as that of unrestricted AP25. See AP25 problems with #-25 patterns in Table A.1).

However, changes in objective function values with respect to changes in patterns highly depend on the distribution of demand points. The value of the objective function may remain unaffected when the regions become smaller or passing costs of congested regions become higher. Even so, one can not have better objective function if the congested regions become larger or more expensive to pass. An example to this is AP25 problem where objective function values remained the same for *BR* and l level fixed costs. In addition, decreasing the size of the forbidden region from 50 to 25 does not lower the objective function value (see Table A.1).

Table 5.9 shows the effect of patterns in large problem instances on the average %GAP U. From this table wee see that with a polygonal congested region, the objective function values differ more than linear congested regions.

Table 5.9: Effect of large instance patterns on the average %GAP U

	Algorithm		
Pattern	SA	EA	PSO
U	0.03	0.00	0.00
LR	8.84	8.74	8.71
PR	9.60	9.57	9.57

5.5.2 Performance Measure

Appendix B shows the performance of our algorithm in different problem instances. Each table presents detailed information for each instance and all three algorithms. The performance information contains:

- The instance name and its pattern in the first and second column.
- The applied meta-heuristic algorithm in the 'Alg.' column.
- '%Imp.' that shows how the meta-heuristic algorithms were able to improve their initial solutions. To be more specific, we focus on three values indication different improvements, considering 10 replications of each meta-heuristic:
  - 'Min. %Imp.' showing that on average how much the algorithms could improve (in percent) the best individual of initial population before termination.
  - 'Avg. %Imp.' denoting how much on average the algorithm could improve the initial population to the final population.
  - 'Max. %Imp.' which shows the average percent improvement on the worst solution in the initial population.

- ‘%DV’ that indicates the percent deviation of final solutions from the BSol:
  - ‘Min. %DV’ shows the percent deviation of the best objective function found in 10 replications from BSol.
  - ‘Avg. %DV.’ is the average percent deviation of 10 final solutions from BSol.
- ‘BSol Hits’ showing the number of times the algorithm generated final solution with the same objective function value as BSol running in 10 replications.
- ‘CT’ which refers to CPU time. CPU time is the computational time the algorithm required to return a solution. The values regarding average computational time of 10 runs is given in ‘CT’ column.

Note that the final population is the generated solutions just before the algorithm terminates. The final population usually converges to the best individual in the population. Since SA produces a single solution at a time and does not work with a population of solutions, we give the percent improvement on the single initial solution as ’Min. %Imp.’ while ’Avg. %Imp.’ and ’Max. %Imp.’ is not applicable for SA.

The BSol of each problem is obtained from given approaches in Chapter 4 if the problem is unrestricted (has pattern U) or RLocP-FR (with pattern FR-#). For problems given in Table 5.3, the BSol corresponds the reported solutions in the literature. For other type of problems, BSol is the best solution found by any of the meta-heuristic algorithms for that problem (when each heuristic runs 10 times, BSol is the best solution found in all 30 replications). For example, the BSol of AP25 with pattern BR-o is the solution with minimum ’Min. OFV’ value in its SA, EA and PSO rows (see the first three rows of Table A.1). Once having BSol, we can calculate ‘%DV’ values from Equations 5.2 and 5.2.

$$\text{Min. \%DV} = 100 \times \frac{Min.OFV - Z(BSol)}{Z(BSol)} \tag{5.2}$$

$$\text{Avg. \%DV} = 100 \times \frac{Avg.OFV - Z(BSol)}{Z(BSol)} \tag{5.3}$$

Table 5.10 shows a performance summary of three meta-heuristics over all 26 instances in Table 5.3.

‘Average %Imp.’ shows the percentage improvement meta-heuristics achieved for the initial solution or the initial population average. ‘Average %DV’ is the average of percent deviation from the BSol. ‘BSol Hits’ is the number of BSols generated by each algorithm in 10 replications. Since we have 26 instances and 10 replications, ’Tot. BSol Hits’ is out of $26 \times 10 = 260$. Average computational times are also provided in Table 5.10.

Table 5.10: Overall meta-heuristics performances on problem instances in Table 5.3

Algorithm	Average %Imp.	Average %DV	Tot. BSol Hits	Avg. CT
SA	10.13	0.00	66 (%25.4)	88.93
EA	11.62	0.03	194 (%74.6)	41.73
PSO	11.60	0.00	187 (%71.9)	59.98

From Table 5.10 we can see that the percentage deviation values given by ‘Average %DV’ is negligible for all meta-heuristics and all of them were able to improve the initial solution/population average by about 11%. What is more, SA required the most time, on average, to solve the problems while EA took the least time. Although SA has the lowest rate of BSol hits, it produced good quality solutions with almost zero deviations, on average.

Similarly, Table 5.11 shows the algorithm performances on the large problems in Table 5.4 and their variants (30 instances in total).

Table 5.11: Overall meta-heuristics performances on large problem instances in Table 5.4

Algorithm	Average %Imp.	Average %DV	Tot. BSol Hits	Avg. CT
SA	23.03	0.16	0 (0.0%)	76.71
EA	25.41	0.11	163 (54.3%)	68.83
PSO	25.14	0.10	44 (14.7%)	101.65

Table 5.11 shows that all algorithms improved the initial solution(s) approximately by 25%. SA has the most average percent deviation by 0.17%. It also failed to find the BSol to any of 30 problems. In this case, EA did the best by finding the BSol 54.3% of the time. It also required less time than SA or PSO. Finally, PSO has the longest CPU time, but the lowest Average %DV.

At last, Table 5.12 shows the meta-heuristic performances on all of our problems.

Table 5.12: Overall meta-heuristics performances on all problem instances

Algorithm	Average %Imp.	Average %DV	Tot. BSol Hits	Avg. CT
SA	10.63	0.01	856 (25.5%)	90.24
EA	11.84	0.01	2459 (73.2%)	37.53
PSO	11.81	0.01	2081 (61.9%)	59.77

'Tot. BSol Hits' is out of 336 × 10 = 3360. It can be observed from Table 5.12 that on average the meta-heuristic algorithms have 0.01% average deviation from BSol's. It also took less time for EA to solve the problems compared to SA and PSO.

The effect of various instance patterns on CPU time is shown in Table 5.13. The instances considered here are the ones given in Table 5.3. As it can be seen, patterns have no significant effect on the computational times, except for the unrestricted problem where no least cost way is calculated.

Table 5.13: Effect of instance patterns on the computational time

		Algorithm		
Pattern		SA	EA	PSO
Cost Level	U	0.02	0.00	0.00
	FR	87.87	27.90	49.00
	l	94.60	36.84	58.78
	h	95.32	38.27	58.78
	BR	95.78	37.59	60.57
Region Size	25	96.39	32.72	56.15
	50	93.57	34.46	55.94
	o	90.22	38.27	58.25

Table 5.14 provides information about effects of the large instance patterns on average computational times.

Table 5.14: Effect of large instance patterns on average computational times

	Algorithm		
Pattern	SA	EA	PSO
U	9.09	9.07	12.43
LR	86.25	79.76	110.24
PR	134.81	117.65	182.27

Table 5.14 shows that with polygonal regions, which implies larger number of vertices and more infeasible area, the computational times increase.

Furthermore, Tables B.1 to B.23 in Appendix B show that the average CPU time increases as the number of region vertices in the problem increases. This is reasonable since the calculation of the least cost path depends directly on the number of region vertices in the problem instance. To illustrate, Table 5.15 summarizes the average computational time of all meta-heuristics over variants of KC5 and KC10 instances given in Table 5.2. It is obvious that as the number of edges in the polygons increases (the non-polygonal region is approximated more accurately) the computational time increases.

Table 5.15: Effect of number of region vertices in the problem on CPU time: KC5 and KC10 instances

		Algorithm		
Instance	$\|V\|$	SA	EA	PSO
KC5	0	0.02	0.00	0.00
KC5	16	1.22	0.26	0.37
KC5	32	3.99	1.47	2.31
KC5	64	14.20	5.78	8.51
KC5	128	53.76	21.01	32.09
KC5	256	206.54	77.81	123.95
KC5	512	809.40	298.94	492.48
KC10	0	0.02	0.00	0.00
KC10	16	1.43	0.38	0.56
KC10	128	56.09	25.13	37.81

The performance of the proposed meta-heuristic algorithms, namely SA, EA, and PSO, are close to each other. One of their significant differences is CPU time. The computational time, other than the problem itself, depends on the nature of the algorithm and since the meta-heuristics proposed in this study have dissimilar nature and behavior, such variety of computational times is reasonable. The other significant difference is the rate at which the heuristics were able to find BSol. But, even when the number of BSol hits is small, the deviations are close to zero. Considering the continuous solution space of the problem, as long as deviations are negligible, the performances are acceptable.

CHAPTER 6

CONCLUSION

This chapter provides an overview of the work done in this study, introduces some extensions to the problem, and addresses the future studies of this research.

In this research, a general type of planar single facility location problems is studied. The problem can be restricted by congested regions. Congested regions are referred to the regions on the plane inside which locating a facility is infeasible but through which traveling is possible at a certain fixed cost. Congested regions can have different fixed costs at different levels which make our problem more general than the most studied restricted problems in the literature, namely restricted planar single facility location problems with forbidden regions and barriers. The problem is formulated under weak assumptions that enables us to model numerous real-life problems.

Three different solution approaches are provided for solution of the problem. Solution methods are based on well-known meta-heuristics, specifically simulated annealing, evolutionary algorithm, and particle swarm optimization. In addition, variable neighborhood search technique is integrated with each meta-heuristic to improve the search procedure. The structure of all heuristics is explained in detail and the procedures needed to calculate objective function values are provided. This study is the first that designed and implemented evolutionary algorithm and particle swarm optimization to deal with restricted planar single facility location problems.

A software package, coded in visual basic .net language, with a graphical user interface is developed. The software enables us to visually see and manage problem instances as well as to implement the proposed algorithms on them.

The performance of the proposed meta-heuristics are investigated using available restricted planar problem instances in the literature. Large TSP and VRP examples are also solved to illustrate the execution of meta-heuristics on large problems. Besides, more problem instances in a structured scheme are generated. The parameters of the meta-heuristics are adjusted by performing preliminary experiments on a small number of test instances.

Computational results showed that all three heuristic algorithms performed well in solving problem instances. We were able to justify our meta-heuristics by finding all the best known solutions of the problems reported in the literature. In unrestricted problems, the insignificant deviation of heuristic solutions from the Weiszfeld's solution and small computational times support this claim (computational times are about 10 seconds for an instance having 7397 demand points).

Provided solution approaches in this study are flexible in a way that distance measures other than Euclidean norm can also be used by small adjustments in the distance calculation procedure. Furthermore, minimax objective function can also be adopted by modest changes in the objective function calculation procedure. In this case, one is able to solve restricted planar 1-center problems, as well.

The solution methods are also valid for the problems where some or all demand points are located inside congested regions. This can even generalize the problem more. For instance, suppose that a distribution center is to be located for delivering equipment to some military units in several war zones. Moreover, it is unsafe to build the distribution center inside the war zones. Making deliveries to the units in the war is essential but dangerous. Such a problem can be solved with the proposed solution method if the fixed costs are properly set to express the risks.

Moreover, each congested region may have different contour levels. If different fixed costs can assigned to each contour level of a congested region, the problem becomes more realistic. Examples are nuclear plants, mountains, and war zones. Usually, the boundaries of such regions cannot be specified, or the risk is not uniformly the same across the region. However, it is known that the closer to the center of the region we travel, the higher the risk, or cost, we encounter. The region with the highest risk (or cost) level can be expanded to regions with lower risk levels around it. Traveling can occur in the contour regions with lowest levels of risks but the drawback is the longer traveling distance. In Figure 6.1, there exists a region with different levels of fixed costs. As the region becomes darker the fixed cost becomes larger. The alternative traveling pathways from X_f to X_m are shown. The path shown by a dashed line is the most costly but it is the shortest way. If the solid route is used, less fixed cost is faced but a longer way is traveled. Doted line shows the longest path without any risk. Having such congested regions is equivalent to having multiple regions with different fixed cost levels. The least cost way between two points can be found by enumerating over all contours of all congested regions.

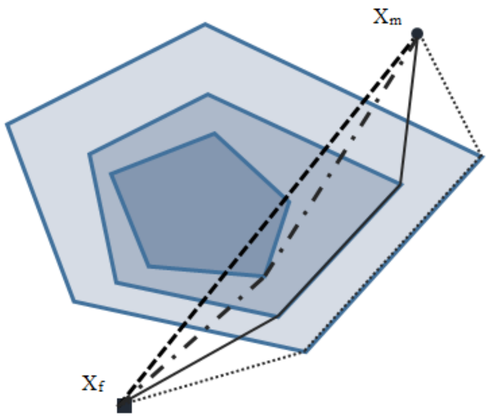


Figure 6.1: A congested regions with contours of fixed costs

Congested regions defined in this study restrict the facility location. However, there may exist zones on the plane that allow location of a facility but at a determined fixed cost (a location problem with zone-dependent fixed cost is studied by Brimberg and Salhi (2005)). The traveling through the zones may be free or costly. For example, consider location a terminal in a rural area. To construct the terminal we should pay for the land. The cost of land may vary in different zones of the area. For such problems, new definition of congested regions is suggested: the regions where location of the facility is costly and traveling is charged with a fixed cost. Note that if the location cost of such regions is set to infinity, they become identical to the congested regions of our problem. Therefore, we can assign another cost factor to the congested regions regarding the fixed location cost. In this case, solutions generated inside congested regions with a finite location cost are not infeasible anymore. Small changes in repairing procedure and the objective function can handle this problem. The new term added to the objective function is $L(X_f)$ indicating the location cost at (x_f, y_f). Note that in places outside the congested regions, $L(X_f) = 0$. Therefore, similar solution approaches is valid in this case. Following the last discussion, contours for location costs of congested regions can be defined, as well.

Future studies of this work may contain solving the restricted planar multi-facility (p-median) problems with congested regions having (different levels of) fixed traveling costs (and location costs). Even though the multi-facility problems have different characteristics than the single facility case, the flexibility and adaptability of meta-heuristics enables us to provide similar solution approaches for those problems.

Furthermore, planar facility location problems restricted by congested regions with *variable* traveling costs is in the area of this research. With variable costs, restricted regions allow traveling at certain per-unit traveling costs. The problem is more generalized with variable traveling costs where finding entry and exit points to congested regions matters. Besides, finding one best passage trough a congested region (with a fixed or variable traveling cost) to reach demand points behind the region is an interesting problem to be considered. If several passages is unified into one, the decision maker will not to face a fixed cost of passage for every demand point. But, finding the number of passages and their location is an additional decision in the least cost path finding problem. Congested regions can have predefined gates. In this case, the best gate-to-gate passage can be found. For example, in Figure 6.2 a facility is located on a plane where there exists a triangular congested region (at the top) and a long linear region (at the bottom). Demand points are shown by dots and facility location is shown by a square. Note that to reach the demand points on the other side of the line, one passage is used, instead of two, if the fixed cost of the linear region is high. By this aggregation, high amount of money could be saved, although traveling distances are longer. Dashed lines are for passages and the path from facility to the passages are shown by a solid line. When the regions are passed, the way to demand points follows the dotted line. To serve demand points falling behind the triangular regions, two penalty costs are paid instead of different costs for each.

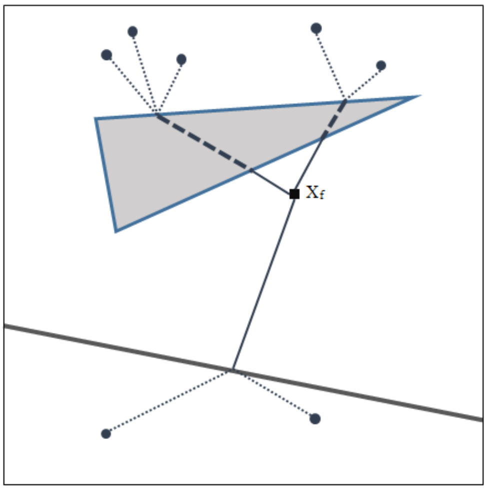


Figure 6.2: Unified passages through congested regions

The restricted obnoxious facility location problem in the presence of congested regions is another future research problem. In this problem, the facility(facilities) are to be located in order to serve the customers. However, the facility should be far from some unfavorable points or undesirable regions. For instance, building an airport too close to a swampland is unfavorable. In this case, congested region can have another cost factor. With this factor, the closer the facility location to the undesirable region, the more cost is encountered.

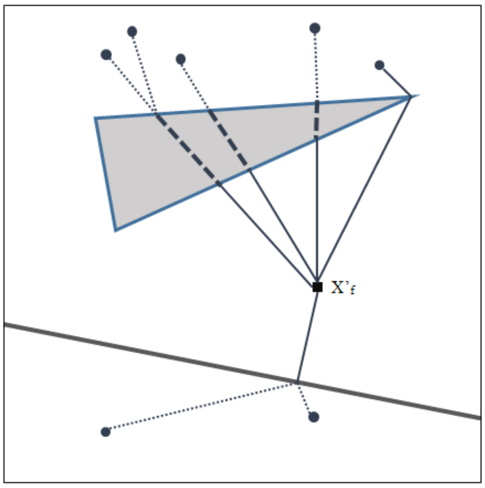


Figure 6.3: Facility location in the presence of undesirable congested region

For example, suppose that the triangular region in Figure 6.2 is undesirable, i.e. locating a facility near that region is risky or unfavorable. Then the new facility location can be the one shown in Figure 6.3 which is now farther from that region. The additional decision in these problems should be made about locating the facility near an undesirable region to reach demand points better or far from the undesirable regions to face less risk but having longer traveling ways.

Finally, stochastic models of restricted planar facility locations are also in the area of interest. In stochastic models, one or all of the following terms can be nondeterministic: location of the demand points, location of the congested regions, and/or the fixed traveling costs of the congested regions.

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APPENDIX A

TABLES OF SOLUTIONS OF ALL PROBLEM INSTANCES

In this appendix, information about the solutions and corresponding objective function values is provided for all meta-heuristics and all instances and their patterns. The information given in Tables A.1 to A.35 contains:

- The instance name and its pattern in the first and second columns.
- The applied meta-heuristic algorithm in the 'Alg.' column.
- The *x* and *y* coordinates of the best solution out of 10 replications.
- The best objective function value found in 10 replications, indicated by 'Min. OFV'.
- 'Avg. OFV', the average of objective function values in 10 replications.
- 'Max. OFV' showing the maximum objective function value of 10 replications.
- 'Avg. %Gap U', the percentage gap between the average objective function value of 10 replications and the objective function value of the unconstrained problem.

Table A.1: Solution results for AP25

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
AP25	BR-o	SA	5.5011	-3.2774	50.213366	50.213367	50.213369	3.58
AP25	BR-o	EA	5.5000	-3.2778	50.213366	50.213367	50.213374	3.58
AP25	BR-o	PSO	5.4997	-3.2773	50.213366	50.213370	50.213385	3.58
AP25	BR-50	SA	5.5001	-0.5712	48.601305	48.601307	48.601309	0.25
AP25	BR-50	EA	5.5000	-0.5714	48.601305	48.601306	48.601306	0.25
AP25	BR-50	PSO	5.5000	-0.5716	48.601305	48.601312	48.601343	0.25
AP25	BR-25	SA	5.5013	0.7999	48.479275	48.479276	48.479278	0.00
AP25	BR-25	EA	5.5000	0.8000	48.479275	48.479275	48.479278	0.00
AP25	BR-25	PSO	5.5004	0.8008	48.479275	48.479278	48.479296	0.00
AP25	h-o	SA	5.4997	-3.2768	50.213366	50.213368	50.213370	3.58
AP25	h-o	EA	5.5004	-3.2768	50.213366	50.213367	50.213369	3.58
AP25	h-o	PSO	5.5009	-3.2772	50.213366	50.213368	50.213383	3.58
AP25	h-50	SA	5.5004	-0.5736	48.601306	48.601307	48.601310	0.25
AP25	h-50	EA	5.5000	-0.5714	48.601305	48.601306	48.601312	0.25
AP25	h-50	PSO	5.4999	-0.5714	48.601305	48.601306	48.601309	0.25
AP25	h-25	SA	5.4990	0.7998	48.479275	48.479275	48.479277	0.00
AP25	h-25	EA	5.5000	0.8000	48.479275	48.479277	48.479294	0.00
AP25	h-25	PSO	5.5016	0.8015	48.479275	48.479276	48.479281	0.00
AP25	l-o	SA	5.5009	-3.2773	50.213366	50.213367	50.213369	3.58
AP25	l-o	EA	5.4992	-3.2778	50.213366	50.213367	50.213370	3.58
AP25	l-o	PSO	5.5001	-3.2776	50.213366	50.213369	50.213375	3.58
AP25	l-50	SA	5.4996	-0.5692	48.601306	48.601306	48.601307	0.25
AP25	l-50	EA	5.5001	-0.5711	48.601305	48.601322	48.601465	0.25
AP25	l-50	PSO	5.4995	-0.5702	48.601306	48.601308	48.601319	0.25
AP25	l-25	SA	5.5001	0.8006	48.479275	48.479276	48.479278	0.00
AP25	l-25	EA	5.5000	0.7984	48.479275	48.479373	48.480202	0.00
AP25	l-25	PSO	5.5002	0.7966	48.479275	48.479278	48.479294	0.00
AP25	O	SA	5.5003	0.0000	48.501095	48.501095	48.501095	0.05
AP25	O	EA	5.5000	0.0000	48.501095	48.501095	48.501095	0.05
AP25	O	PSO	5.4999	0.0000	48.501095	48.501095	48.501095	0.05
AP25	FR-50	SA	5.4996	0.7966	48.479275	48.479275	48.479275	0.00
AP25	FR-50	EA	5.5004	0.7993	48.479275	48.479276	48.479277	0.00
AP25	FR-50	PSO	5.5016	0.8032	48.479275	48.479277	48.479283	0.00
AP25	FR-25	SA	5.5007	0.7997	48.479275	48.479275	48.479277	0.00
AP25	FR-25	EA	5.5000	0.8000	48.479275	48.479300	48.479516	0.00
AP25	FR-25	PSO	5.4998	0.8002	48.479275	48.479280	48.479290	0.00
AP25	U	SA	5.4999	0.7970	48.479275	48.479276	48.479279	0.00
AP25	U	EA	3.8259	0.7982	48.846559	48.880350	48.959196	0.83
AP25	U	PSO	5.3694	1.1494	48.485660	48.487938	48.493253	0.02

Table A.2: Solution results for AP70

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
AP70	O	SA	8.7662	4.9795	119.138730	119.138742	119.138784	8.30
AP70	O	EA	8.7667	4.9797	119.138730	119.138730	119.138730	8.30
AP70	O	PSO	8.7667	4.9798	119.138730	119.138730	119.138731	8.30
AP70	BR-50	SA	9.0360	6.1557	110.565933	110.565936	110.565942	0.51
AP70	BR-50	EA	9.0364	6.1561	110.565933	110.565933	110.565937	0.51
AP70	BR-50	PSO	9.0360	6.1558	110.565933	110.565934	110.565940	0.51
AP70	BR-25	SA	8.9360	6.3091	110.054606	110.054610	110.054624	0.04
AP70	BR-25	EA	8.9361	6.3088	110.054606	110.054606	110.054606	0.04
AP70	BR-25	PSO	8.9358	6.3086	110.054606	110.054607	110.054610	0.04
AP70	h-o	SA	8.7673	4.9797	119.138730	119.138744	119.138775	8.30
AP70	h-o	EA	8.7667	4.9797	119.138730	119.138730	119.138730	8.30
AP70	h-o	PSO	8.7667	4.9797	119.138730	119.138730	119.138733	8.30
AP70	h-50	SA	9.0357	6.1554	110.565934	110.565938	110.565958	0.51
AP70	h-50	EA	9.0364	6.1557	110.565933	110.565934	110.565936	0.51
AP70	h-50	PSO	9.0362	6.1559	110.565933	110.565933	110.565935	0.51
AP70	h-25	SA	8.9360	6.3085	110.054606	110.054608	110.054610	0.04
AP70	h-25	EA	8.9358	6.3086	110.054606	110.054607	110.054612	0.04
AP70	h-25	PSO	8.9354	6.3090	110.054606	110.054606	110.054606	0.04
AP70	l-o	SA	8.5520	5.0226	117.514750	117.514757	117.514770	6.82
AP70	l-o	EA	8.5523	5.0223	117.514750	117.514750	117.514750	6.82
AP70	l-o	PSO	8.5521	5.0221	117.514750	117.514751	117.514760	6.82
AP70	l-50	SA	9.0360	6.1564	110.565933	110.565937	110.565945	0.51
AP70	l-50	EA	9.0362	6.1559	110.565933	110.565934	110.565947	0.51
AP70	l-50	PSO	9.0364	6.1562	110.565933	110.565935	110.565940	0.51
AP70	l-25	SA	8.9351	6.3076	110.054607	110.054611	110.054616	0.04
AP70	l-25	EA	8.9358	6.3086	110.054606	110.054606	110.054607	0.04
AP70	l-25	PSO	8.9356	6.3082	110.054606	110.054607	110.054610	0.04
AP70	FR-o	SA	8.9129	6.3565	110.006837	110.006839	110.006844	0.00
AP70	FR-o	EA	8.9124	6.3554	110.006837	110.006837	110.006837	0.00
AP70	FR-o	PSO	8.9125	6.3554	110.006837	110.006840	110.006855	0.00
AP70	FR-50	SA	8.9121	6.3562	110.006837	110.006846	110.006885	0.00
AP70	FR-50	EA	8.9127	6.3554	110.006837	110.006837	110.006837	0.00
AP70	FR-50	PSO	8.9126	6.3554	110.006837	110.006839	110.006846	0.00
AP70	FR-25	SA	8.9128	6.3556	110.006837	110.006840	110.006846	0.00
AP70	FR-25	EA	8.9126	6.3553	110.006837	110.006837	110.006837	0.00
AP70	FR-25	PSO	8.9129	6.3551	110.006837	110.006839	110.006847	0.00
AP70	U	SA	8.9121	6.3552	110.006837	110.006842	110.006853	0.00
AP70	U	EA	8.9126	6.3554	110.006837	110.006909	110.007030	0.00
AP70	U	PSO	8.9129	6.3548	110.006837	110.006889	110.007025	0.00

Table A.3: Solution results for AP70R10

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
AP70R10	O	SA	8.7670	4.9794	119.104667	119.104681	119.104721	8.27
AP70R10	O	EA	8.7667	4.9797	119.104667	119.104667	119.104667	8.27
AP70R10	O	PSO	8.7667	4.9797	119.104667	119.104667	119.104671	8.27
AP70R10	BR-50	SA	9.0359	6.1554	110.565933	110.565935	110.565938	0.51
AP70R10	BR-50	EA	9.0361	6.1559	110.565933	110.565934	110.565946	0.51
AP70R10	BR-50	PSO	9.0362	6.1558	110.565933	110.565934	110.565938	0.51
AP70R10	BR-25	SA	8.9352	6.3088	110.054606	110.054612	110.054653	0.04
AP70R10	BR-25	EA	8.9358	6.3086	110.054606	110.054606	110.054606	0.04
AP70R10	BR-25	PSO	8.9361	6.3088	110.054606	110.054607	110.054614	0.04
AP70R10	h-o	SA	8.7666	4.9794	119.104667	119.104672	119.104684	8.27
AP70R10	h-o	EA	8.7667	4.9797	119.104667	119.104667	119.104667	8.27
AP70R10	h-o	PSO	8.7667	4.9798	119.104667	119.104667	119.104667	8.27
AP70R10	h-50	SA	9.0362	6.1561	110.565933	110.565938	110.565947	0.51
AP70R10	h-50	EA	9.0362	6.1559	110.565933	110.565935	110.565948	0.51
AP70R10	h-50	PSO	9.0364	6.1558	110.565933	110.565936	110.565944	0.51
AP70R10	h-25	SA	8.9368	6.3086	110.054607	110.054610	110.054619	0.04
AP70R10	h-25	EA	8.9358	6.3086	110.054606	110.054606	110.054606	0.04
AP70R10	h-25	PSO	8.9357	6.3085	110.054606	110.054606	110.054607	0.04
AP70R10	l-o	SA	8.7052	5.0483	118.158977	118.158986	118.159002	7.41
AP70R10	l-o	EA	8.7051	5.0481	118.158977	118.158977	118.158978	7.41
AP70R10	l-o	PSO	8.7049	5.0483	118.158977	118.158977	118.158978	7.41
AP70R10	l-50	SA	9.0363	6.1557	110.565933	110.565935	110.565940	0.51
AP70R10	l-50	EA	9.0361	6.1558	110.565933	110.565933	110.565934	0.51
AP70R10	l-50	PSO	9.0362	6.1558	110.565933	110.565934	110.565938	0.51
AP70R10	l-25	SA	8.9361	6.3087	110.054606	110.054614	110.054630	0.04
AP70R10	l-25	EA	8.9358	6.3086	110.054606	110.054606	110.054608	0.04
AP70R10	l-25	PSO	8.9358	6.3083	110.054606	110.054607	110.054612	0.04
AP70R10	FR-o	SA	8.9127	6.3556	110.006837	110.006840	110.006850	0.00
AP70R10	FR-o	EA	8.9127	6.3554	110.006837	110.006838	110.006846	0.00
AP70R10	FR-o	PSO	8.9128	6.3557	110.006837	110.006837	110.006841	0.00
AP70R10	FR-50	SA	8.9137	6.3541	110.006839	110.006842	110.006851	0.00
AP70R10	FR-50	EA	8.9127	6.3553	110.006837	110.006837	110.006837	0.00
AP70R10	FR-50	PSO	8.9128	6.3554	110.006837	110.006838	110.006840	0.00
AP70R10	FR-25	SA	8.9122	6.3551	110.006837	110.006841	110.006854	0.00
AP70R10	FR-25	EA	8.9127	6.3554	110.006837	110.006837	110.006841	0.00
AP70R10	FR-25	PSO	8.9123	6.3553	110.006837	110.006838	110.006847	0.00

Table A.4: Solution results for AP70R8

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
AP70R8	O	SA	9.1873	5.4861	116.397638	116.397646	116.397669	5.81
AP70R8	O	EA	9.1874	5.4860	116.397638	116.397638	116.397639	5.81
AP70R8	O	PSO	9.1873	5.4860	116.397638	116.397638	116.397639	5.81
AP70R8	BR-50	SA	9.0451	6.1920	110.548562	110.548565	110.548574	0.49
AP70R8	BR-50	EA	9.0450	6.1915	110.548561	110.548561	110.548562	0.49
AP70R8	BR-50	PSO	9.0451	6.1916	110.548561	110.548566	110.548607	0.49
AP70R8	BR-25	SA	8.9356	6.3092	110.054606	110.054610	110.054621	0.04
AP70R8	BR-25	EA	8.9358	6.3085	110.054606	110.054606	110.054606	0.04
AP70R8	BR-25	PSO	8.9357	6.3087	110.054606	110.054607	110.054611	0.04
AP70R8	h-o	SA	9.1874	5.4861	116.397638	116.397642	116.397649	5.81
AP70R8	h-o	EA	9.1873	5.4860	116.397638	116.397638	116.397639	5.81
AP70R8	h-o	PSO	9.1874	5.4860	116.397638	116.397639	116.397640	5.81
AP70R8	h-50	SA	9.0450	6.1921	110.548562	110.548564	110.548569	0.49
AP70R8	h-50	EA	9.0448	6.1914	110.548561	110.548561	110.548561	0.49
AP70R8	h-50	PSO	9.0450	6.1914	110.548561	110.548562	110.548563	0.49
AP70R8	h-25	SA	8.9350	6.3093	110.054607	110.054609	110.054612	0.04
AP70R8	h-25	EA	8.9358	6.3086	110.054606	110.054606	110.054606	0.04
AP70R8	h-25	PSO	8.9357	6.3085	110.054606	110.054607	110.054611	0.04
AP70R8	l-o	SA	8.9839	5.8194	113.526344	113.526348	113.526356	3.20
AP70R8	l-o	EA	8.9838	5.8189	113.526344	113.526347	113.526375	3.20
AP70R8	l-o	PSO	8.9838	5.8192	113.526344	113.526345	113.526353	3.20
AP70R8	l-50	SA	9.0461	6.1908	110.548563	110.548566	110.548575	0.49
AP70R8	l-50	EA	9.0450	6.1914	110.548561	110.548561	110.548562	0.49
AP70R8	l-50	PSO	9.0449	6.1913	110.548561	110.548562	110.548562	0.49
AP70R8	l-25	SA	8.9361	6.3078	110.054606	110.054607	110.054610	0.04
AP70R8	l-25	EA	8.9358	6.3086	110.054606	110.054606	110.054607	0.04
AP70R8	l-25	PSO	8.9358	6.3086	110.054606	110.054606	110.054607	0.04
AP70R8	FR-o	SA	8.9125	6.3561	110.006837	110.006841	110.006848	0.00
AP70R8	FR-o	EA	8.9127	6.3554	110.006837	110.006838	110.006843	0.00
AP70R8	FR-o	PSO	8.9127	6.3555	110.006837	110.006837	110.006838	0.00
AP70R8	FR-50	SA	8.9131	6.3545	110.006837	110.006841	110.006857	0.00
AP70R8	FR-50	EA	8.9123	6.3555	110.006837	110.006839	110.006857	0.00
AP70R8	FR-50	PSO	8.9126	6.3558	110.006837	110.006837	110.006837	0.00
AP70R8	FR-25	SA	8.9126	6.3562	110.006837	110.006841	110.006856	0.00
AP70R8	FR-25	EA	8.9127	6.3554	110.006837	110.006839	110.006847	0.00
AP70R8	FR-25	PSO	8.9127	6.3551	110.006837	110.006837	110.006839	0.00

Table A.5: Solution results for AP70R6

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
AP70R6	O	SA	9.2655	6.2527	114.561027	114.561034	114.561046	4.14
AP70R6	O	EA	9.2658	6.2527	114.561027	114.561027	114.561027	4.14
AP70R6	O	PSO	9.2658	6.2529	114.561027	114.561027	114.561030	4.14
AP70R6	BR-50	SA	9.0064	6.2923	110.399570	110.399574	110.399583	0.36
AP70R6	BR-50	EA	9.0067	6.2926	110.399570	110.399570	110.399570	0.36
AP70R6	BR-50	PSO	9.0063	6.2928	110.399570	110.399570	110.399571	0.36
AP70R6	BR-25	SA	8.9322	6.3186	110.053444	110.053453	110.053482	0.04
AP70R6	BR-25	EA	8.9317	6.3179	110.053443	110.053443	110.053445	0.04
AP70R6	BR-25	PSO	8.9317	6.3177	110.053443	110.053445	110.053450	0.04
AP70R6	h-o	SA	9.2653	6.2531	114.561027	114.561032	114.561050	4.14
AP70R6	h-o	EA	9.2658	6.2527	114.561027	114.561028	114.561037	4.14
AP70R6	h-o	PSO	9.2661	6.2524	114.561027	114.561028	114.561032	4.14
AP70R6	h-50	SA	9.0067	6.2926	110.399570	110.399576	110.399586	0.36
AP70R6	h-50	EA	9.0067	6.2926	110.399570	110.399619	110.399848	0.36
AP70R6	h-50	PSO	9.0069	6.2921	110.399570	110.399570	110.399571	0.36
AP70R6	h-25	SA	8.9319	6.3176	110.053444	110.053446	110.053450	0.04
AP70R6	h-25	EA	8.9318	6.3179	110.053443	110.053444	110.053445	0.04
AP70R6	h-25	PSO	8.9316	6.3180	110.053443	110.053444	110.053447	0.04
AP70R6	l-o	SA	8.9869	6.2312	112.068568	112.068572	112.068586	1.87
AP70R6	l-o	EA	8.9867	6.2309	112.068567	112.068567	112.068569	1.87
AP70R6	l-o	PSO	8.9867	6.2308	112.068567	112.068568	112.068570	1.87
AP70R6	l-50	SA	9.0065	6.2923	110.399570	110.399572	110.399574	0.36
AP70R6	l-50	EA	9.0068	6.2921	110.399570	110.399570	110.399570	0.36
AP70R6	l-50	PSO	9.0066	6.2925	110.399570	110.399570	110.399571	0.36
AP70R6	l-25	SA	8.9315	6.3185	110.053444	110.053448	110.053460	0.04
AP70R6	l-25	EA	8.9317	6.3179	110.053443	110.053443	110.053447	0.04
AP70R6	l-25	PSO	8.9316	6.3180	110.053443	110.053445	110.053455	0.04
AP70R6	FR-o	SA	8.9128	6.3563	110.006837	110.006840	110.006850	0.00
AP70R6	FR-o	EA	8.9127	6.3554	110.006837	110.006837	110.006838	0.00
AP70R6	FR-o	PSO	8.9127	6.3550	110.006837	110.006837	110.006839	0.00
AP70R6	FR-50	SA	8.9135	6.3555	110.006837	110.006840	110.006848	0.00
AP70R6	FR-50	EA	8.9126	6.3554	110.006837	110.006837	110.006837	0.00
AP70R6	FR-50	PSO	8.9129	6.3558	110.006837	110.006838	110.006842	0.00
AP70R6	FR-25	SA	8.9128	6.3562	110.006837	110.006845	110.006872	0.00
AP70R6	FR-25	EA	8.9124	6.3563	110.006837	110.006837	110.006838	0.00
AP70R6	FR-25	PSO	8.9131	6.3553	110.006837	110.006837	110.006839	0.00

Table A.6: Solution results for AP70R4

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
AP70R4	O	SA	9.2172	6.1526	113.765606	113.765611	113.765621	3.42
AP70R4	O	EA	9.2173	6.1528	113.765606	113.765606	113.765607	3.42
AP70R4	O	PSO	9.2176	6.1526	113.765606	113.765607	113.765612	3.42
AP70R4	BR-50	SA	9.0028	6.3075	110.397543	110.397547	110.397563	0.36
AP70R4	BR-50	EA	9.0030	6.3075	110.397543	110.397546	110.397564	0.36
AP70R4	BR-50	PSO	9.0030	6.3078	110.397543	110.397544	110.397546	0.36
AP70R4	BR-25	SA	8.9309	6.3179	110.053444	110.053446	110.053453	0.04
AP70R4	BR-25	EA	8.9317	6.3179	110.053443	110.053443	110.053444	0.04
AP70R4	BR-25	PSO	8.9318	6.3180	110.053443	110.053444	110.053445	0.04
AP70R4	h-o	SA	9.2168	6.1530	113.765606	113.765610	113.765621	3.42
AP70R4	h-o	EA	9.2175	6.1529	113.765606	113.765606	113.765608	3.42
AP70R4	h-o	PSO	9.2177	6.1530	113.765606	113.765608	113.765617	3.42
AP70R4	h-50	SA	9.0032	6.3085	110.397544	110.397548	110.397559	0.36
AP70R4	h-50	EA	9.0030	6.3078	110.397543	110.397547	110.397577	0.36
AP70R4	h-50	PSO	9.0031	6.3077	110.397543	110.397544	110.397547	0.36
AP70R4	h-25	SA	8.9318	6.3179	110.053443	110.053446	110.053455	0.04
AP70R4	h-25	EA	8.9317	6.3179	110.053443	110.053443	110.053444	0.04
AP70R4	h-25	PSO	8.9318	6.3180	110.053443	110.053446	110.053453	0.04
AP70R4	l-o	SA	8.9841	6.2176	112.985891	112.985896	112.985907	2.71
AP70R4	l-o	EA	8.9841	6.2182	112.985891	112.985891	112.985891	2.71
AP70R4	l-o	PSO	8.9840	6.2181	112.985891	112.985891	112.985891	2.71
AP70R4	l-50	SA	9.0038	6.3080	110.397544	110.397547	110.397566	0.36
AP70R4	l-50	EA	9.0030	6.3078	110.397543	110.397543	110.397544	0.36
AP70R4	l-50	PSO	9.0029	6.3075	110.397543	110.397547	110.397579	0.36
AP70R4	l-25	SA	8.9317	6.3190	110.053444	110.053447	110.053461	0.04
AP70R4	l-25	EA	8.9317	6.3179	110.053443	110.053443	110.053445	0.04
AP70R4	l-25	PSO	8.9316	6.3179	110.053443	110.053444	110.053446	0.04
AP70R4	FR-o	SA	8.9132	6.3561	110.006837	110.006840	110.006845	0.00
AP70R4	FR-o	EA	8.9127	6.3554	110.006837	110.006837	110.006837	0.00
AP70R4	FR-o	PSO	8.9129	6.3554	110.006837	110.006839	110.006845	0.00
AP70R4	FR-50	SA	8.9132	6.3553	110.006837	110.006842	110.006863	0.00
AP70R4	FR-50	EA	8.9128	6.3554	110.006837	110.006837	110.006837	0.00
AP70R4	FR-50	PSO	8.9129	6.3551	110.006837	110.006837	110.006837	0.00
AP70R4	FR-25	SA	8.9128	6.3564	110.006837	110.006844	110.006857	0.00
AP70R4	FR-25	EA	8.9127	6.3554	110.006837	110.006838	110.006840	0.00
AP70R4	FR-25	PSO	8.9123	6.3554	110.006837	110.006838	110.006840	0.00

Table A.7: Solution results for AP70R2

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
AP70R2	O	SA	9.0373	6.1138	111.688863	111.688867	111.688880	1.53
AP70R2	O	EA	9.0372	6.1150	111.688862	111.688866	111.688901	1.53
AP70R2	O	PSO	9.0370	6.1147	111.688862	111.688868	111.688912	1.53
AP70R2	BR-50	SA	8.9584	6.2468	110.276942	110.276945	110.276950	0.25
AP70R2	BR-50	EA	8.9585	6.2465	110.276942	110.276945	110.276970	0.25
AP70R2	BR-50	PSO	8.9587	6.2465	110.276942	110.276958	110.277067	0.25
AP70R2	BR-25	SA	8.9284	6.3125	110.052979	110.052982	110.052989	0.04
AP70R2	BR-25	EA	8.9289	6.3132	110.052978	110.052978	110.052980	0.04
AP70R2	BR-25	PSO	8.9291	6.3131	110.052978	110.052981	110.052995	0.04
AP70R2	h-o	SA	9.0374	6.1144	111.688863	111.688867	111.688871	1.53
AP70R2	h-o	EA	9.0373	6.1148	111.688862	111.688862	111.688864	1.53
AP70R2	h-o	PSO	9.0372	6.1151	111.688862	111.688876	111.688976	1.53
AP70R2	h-50	SA	8.9584	6.2470	110.276942	110.276946	110.276967	0.25
AP70R2	h-50	EA	8.9584	6.2465	110.276942	110.277076	110.278257	0.25
AP70R2	h-50	PSO	8.9584	6.2465	110.276942	110.276942	110.276944	0.25
AP70R2	h-25	SA	8.9289	6.3142	110.052979	110.052984	110.053008	0.04
AP70R2	h-25	EA	8.9290	6.3130	110.052978	110.052979	110.052981	0.04
AP70R2	h-25	PSO	8.9289	6.3131	110.052978	110.052986	110.053023	0.04
AP70R2	l-o	SA	9.0370	6.1146	111.688863	111.688868	111.688878	1.53
AP70R2	l-o	EA	9.0372	6.1150	111.688862	111.688865	111.688884	1.53
AP70R2	l-o	PSO	9.0373	6.1147	111.688862	111.688863	111.688865	1.53
AP70R2	l-50	SA	8.9588	6.2467	110.276942	110.276945	110.276948	0.25
AP70R2	l-50	EA	8.9585	6.2466	110.276942	110.276943	110.276946	0.25
AP70R2	l-50	PSO	8.9584	6.2464	110.276942	110.276951	110.277008	0.25
AP70R2	l-25	SA	8.9290	6.3136	110.052979	110.052984	110.052997	0.04
AP70R2	l-25	EA	8.9290	6.3130	110.052978	110.052981	110.053004	0.04
AP70R2	l-25	PSO	8.9291	6.3130	110.052978	110.052981	110.052996	0.04
AP70R2	FR-o	SA	8.9130	6.3554	110.006837	110.006840	110.006860	0.00
AP70R2	FR-o	EA	8.9127	6.3554	110.006837	110.006837	110.006838	0.00
AP70R2	FR-o	PSO	8.9127	6.3555	110.006837	110.006838	110.006840	0.00
AP70R2	FR-50	SA	8.9133	6.3549	110.006837	110.006848	110.006924	0.00
AP70R2	FR-50	EA	8.9127	6.3553	110.006837	110.006838	110.006844	0.00
AP70R2	FR-50	PSO	8.9128	6.3555	110.006837	110.006845	110.006882	0.00
AP70R2	FR-25	SA	8.9126	6.3555	110.006837	110.006843	110.006863	0.00
AP70R2	FR-25	EA	8.9125	6.3558	110.006837	110.006839	110.006850	0.00
AP70R2	FR-25	PSO	8.9129	6.3553	110.006837	110.006839	110.006856	0.00

Table A.8: Solution results for BC13

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
BC13	O	SA	6.8562	6.1434	29.838055	29.838057	29.838063	13.67
BC13	O	EA	6.8582	6.1442	29.838055	29.839717	29.849380	13.68
BC13	O	PSO	6.8568	6.1433	29.838055	29.838095	29.838312	13.67
BC13	BR-50	SA	6.5494	6.9584	26.476606	26.476607	26.476614	0.87
BC13	BR-50	EA	6.5504	6.9589	26.476606	26.476802	26.478317	0.87
BC13	BR-50	PSO	6.5515	6.9585	26.476606	26.476629	26.476715	0.87
BC13	BR-25	SA	6.7194	7.0986	26.249339	26.249340	26.249344	0.00
BC13	BR-25	EA	6.7186	7.0988	26.249339	26.249824	26.252510	0.00
BC13	BR-25	PSO	6.7183	7.0977	26.249339	26.249349	26.249377	0.00
BC13	h-o	SA	6.8556	6.1414	29.838055	29.838056	29.838058	13.67
BC13	h-o	EA	6.8571	6.1416	29.838055	29.838097	29.838429	13.67
BC13	h-o	PSO	6.8576	6.1448	29.838055	29.838077	29.838154	13.67
BC13	h-50	SA	6.5520	6.9579	26.476606	26.476606	26.476610	0.87
BC13	h-50	EA	6.5506	6.9588	26.476606	26.476618	26.476649	0.87
BC13	h-50	PSO	6.5514	6.9615	26.476608	26.476653	26.476877	0.87
BC13	h-25	SA	6.7202	7.0988	26.249339	26.249339	26.249340	0.00
BC13	h-25	EA	6.7184	7.0986	26.249339	26.249507	26.250521	0.00
BC13	h-25	PSO	6.7182	7.0967	26.249339	26.249354	26.249382	0.00
BC13	l-o	SA	6.8578	6.1441	29.838055	29.838057	29.838064	13.67
BC13	l-o	EA	6.8565	6.1402	29.838056	29.838888	29.843626	13.67
BC13	l-o	PSO	6.8570	6.1445	29.838055	29.838068	29.838116	13.67
BC13	l-50	SA	6.5520	6.9585	26.476606	26.476607	26.476609	0.87
BC13	l-50	EA	6.5517	6.9634	26.476612	26.476930	26.477862	0.87
BC13	l-50	PSO	6.5508	6.9580	26.476606	26.476616	26.476668	0.87
BC13	l-25	SA	6.7167	7.0984	26.249339	26.249340	26.249342	0.00
BC13	l-25	EA	6.7192	7.1000	26.249339	26.249734	26.252192	0.00
BC13	l-25	PSO	6.7204	7.0996	26.249339	26.249368	26.249522	0.00
BC13	FR-o	SA	6.7188	7.0988	26.249339	26.249339	26.249341	0.00
BC13	FR-o	EA	6.7183	7.0986	26.249339	26.249541	26.250829	0.00
BC13	FR-o	PSO	6.7200	7.0984	26.249339	26.249353	26.249429	0.00
BC13	FR-50	SA	6.7183	7.0995	26.249339	26.249339	26.249341	0.00
BC13	FR-50	EA	6.7177	7.0984	26.249339	26.251620	26.267642	0.01
BC13	FR-50	PSO	6.7165	7.0966	26.249339	26.249362	26.249479	0.00
BC13	FR-25	SA	6.7192	7.0988	26.249339	26.249339	26.249339	0.00
BC13	FR-25	EA	6.7183	7.0986	26.249339	26.249419	26.249928	0.00
BC13	FR-25	PSO	6.7166	7.0987	26.249339	26.249341	26.249352	0.00
BC13	U	SA	6.7166	7.0984	26.249339	26.249339	26.249342	0.00
BC13	U	EA	6.5527	7.5336	26.308552	26.330397	26.335858	0.31
BC13	U	PSO	6.7159	7.0918	26.249347	26.252119	26.253307	0.01

Table A.9: Solution results for D26

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
D26	O	SA	30.9262	25.9467	29.102174	29.102175	29.102177	1.05
D26	O	EA	30.9270	25.9468	29.102174	29.102175	29.102188	1.05
D26	O	PSO	30.9320	25.9459	29.102174	29.102175	29.102177	1.05
D26	BR-50	SA	32.0137	26.0330	28.830820	28.830821	28.830822	0.11
D26	BR-50	EA	32.0114	26.0337	28.830820	28.830848	28.831097	0.11
D26	BR-50	PSO	32.0098	26.0325	28.830820	28.830820	28.830822	0.11
D26	BR-25	SA	32.1565	25.8831	28.803781	28.803782	28.803784	0.01
D26	BR-25	EA	32.1634	25.8889	28.803781	28.803790	28.803866	0.01
D26	BR-25	PSO	32.1628	25.8839	28.803781	28.803782	28.803785	0.01
D26	h-o	SA	30.9312	25.9425	29.102174	29.102175	29.102180	1.05
D26	h-o	EA	30.9270	25.9467	29.102174	29.102175	29.102180	1.05
D26	h-o	PSO	30.9279	25.9483	29.102174	29.102177	29.102200	1.05
D26	h-50	SA	32.0108	26.0358	28.830820	28.830820	28.830821	0.11
D26	h-50	EA	32.0101	26.0339	28.830820	28.830820	28.830822	0.11
D26	h-50	PSO	32.0060	26.0367	28.830820	28.830822	28.830827	0.11
D26	h-25	SA	32.1597	25.8871	28.803781	28.803782	28.803784	0.01
D26	h-25	EA	32.1600	25.8848	28.803781	28.803785	28.803808	0.01
D26	h-25	PSO	32.1588	25.8857	28.803781	28.803783	28.803786	0.01
D26	l-o	SA	31.4376	25.9885	29.065096	29.065098	29.065107	0.92
D26	l-o	EA	31.4351	25.9867	29.065096	29.065097	29.065102	0.92
D26	l-o	PSO	31.4366	25.9904	29.065096	29.065100	29.065130	0.92
D26	l-50	SA	32.0120	26.0334	28.830820	28.830821	28.830825	0.11
D26	l-50	EA	32.0089	26.0323	28.830820	28.830821	28.830831	0.11
D26	l-50	PSO	32.0184	26.0308	28.830820	28.830821	28.830823	0.11
D26	l-25	SA	32.1624	25.8869	28.803781	28.803781	28.803782	0.01
D26	l-25	EA	32.1600	25.8841	28.803781	28.803783	28.803798	0.01
D26	l-25	PSO	32.1601	25.8850	28.803781	28.803787	28.803820	0.01
D26	FR-o	SA	32.0712	25.7754	28.800501	28.800503	28.800513	0.00
D26	FR-o	EA	32.0739	25.7737	28.800501	28.800503	28.800515	0.00
D26	FR-o	PSO	32.0792	25.7687	28.800501	28.800504	28.800522	0.00
D26	FR-50	SA	32.0741	25.7776	28.800501	28.800501	28.800502	0.00
D26	FR-50	EA	32.0747	25.7738	28.800501	28.800501	28.800502	0.00
D26	FR-50	PSO	32.0740	25.7811	28.800501	28.800501	28.800504	0.00
D26	FR-25	SA	32.0750	25.7716	28.800501	28.800502	28.800504	0.00
D26	FR-25	EA	32.0745	25.7738	28.800501	28.800502	28.800505	0.00
D26	FR-25	PSO	32.0727	25.7771	28.800501	28.800505	28.800528	0.00
D26	U	SA	32.0732	25.7758	28.800501	28.800501	28.800503	0.00
D26	U	EA	31.9060	25.9517	28.801154	28.804356	28.809158	0.01
D26	U	PSO	32.1248	26.0313	28.801072	28.801632	28.801907	0.00

Table A.10: Solution results for KC5c16 and KC5U

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5c16	O	SA	-1.2017	2.0770	48.281797	48.281798	48.281802	1.93
KC5c16	O	EA	-1.2016	2.0777	48.281797	48.281800	48.281812	1.93
KC5c16	O	PSO	-1.2016	2.0775	48.281797	48.281799	48.281806	1.93
KC5c16	BR-50	SA	-0.7087	1.0949	47.533282	47.533286	47.533300	0.35
KC5c16	BR-50	EA	-0.7097	1.0940	47.533281	47.533330	47.533575	0.35
KC5c16	BR-50	PSO	-0.7098	1.0941	47.533281	47.533282	47.533285	0.35
KC5c16	BR-25	SA	-0.3951	0.5717	47.387318	47.387321	47.387323	0.04
KC5c16	BR-25	EA	-0.3949	0.5721	47.387318	47.387425	47.388022	0.04
KC5c16	BR-25	PSO	-0.3952	0.5714	47.387318	47.387323	47.387358	0.04
KC5c16	h-o	SA	-1.2023	2.0785	48.281797	48.281798	48.281802	1.93
KC5c16	h-o	EA	-1.2016	2.0786	48.281797	48.281857	48.282107	1.93
KC5c16	h-o	PSO	-1.2018	2.0773	48.281797	48.281798	48.281804	1.93
KC5c16	h-50	SA	-0.7096	1.0940	47.533281	47.533285	47.533299	0.35
KC5c16	h-50	EA	-0.7097	1.0940	47.533281	47.533282	47.533285	0.35
KC5c16	h-50	PSO	-0.7095	1.0937	47.533281	47.533284	47.533305	0.35
KC5c16	h-25	SA	-0.3953	0.5720	47.387318	47.387322	47.387332	0.04
KC5c16	h-25	EA	-0.3949	0.5721	47.387318	47.387360	47.387724	0.04
KC5c16	h-25	PSO	-0.3951	0.5720	47.387318	47.387325	47.387347	0.04
KC5c16	l-o	SA	-1.2011	2.0775	48.281797	48.281798	48.281804	1.93
KC5c16	l-o	EA	-1.2003	2.0781	48.281797	48.281800	48.281822	1.93
KC5c16	l-o	PSO	-1.2011	2.0786	48.281797	48.281799	48.281803	1.93
KC5c16	l-50	SA	-0.7096	1.0933	47.533282	47.533284	47.533294	0.35
KC5c16	l-50	EA	-0.7097	1.0940	47.533281	47.533285	47.533310	0.35
KC5c16	l-50	PSO	-0.7093	1.0942	47.533281	47.533282	47.533286	0.35
KC5c16	l-25	SA	-0.3954	0.5714	47.387318	47.387320	47.387323	0.04
KC5c16	l-25	EA	-0.3953	0.5720	47.387318	47.387329	47.387393	0.04
KC5c16	l-25	PSO	-0.3949	0.5722	47.387318	47.387384	47.387822	0.04
KC5c16	FR-o	SA	0.0936	2.0000	47.609492	47.609492	47.609492	0.51
KC5c16	FR-o	EA	0.0923	2.0000	47.609492	47.609492	47.609496	0.51
KC5c16	FR-o	PSO	0.0942	2.0000	47.609492	47.609492	47.609493	0.51
KC5c16	FR-50	SA	-0.0193	1.0000	47.391583	47.391583	47.391583	0.05
KC5c16	FR-50	EA	-0.0190	1.0000	47.391583	47.391716	47.392910	0.05
KC5c16	FR-50	PSO	-0.0190	1.0000	47.391583	47.391584	47.391592	0.05
KC5c16	FR-25	SA	-0.0916	0.5411	47.367374	47.367375	47.367380	0.00
KC5c16	FR-25	EA	-0.0923	0.5409	47.367374	47.367933	47.372801	0.00
KC5c16	FR-25	PSO	-0.0921	0.5402	47.367374	47.367418	47.367544	0.00
KC5	U	SA	-0.0915	0.5409	47.367374	47.367375	47.367376	0.00
KC5	U	EA	-0.0857	0.5384	47.367383	47.382740	47.392978	0.03
KC5	U	PSO	-0.0836	0.5435	47.367387	47.370764	47.371991	0.01

Table A.11: Solution results for KC5c32

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5c32	O	SA	-1.1908	2.0674	48.261460	48.261462	48.261466	1.89
KC5c32	O	EA	-1.1909	2.0677	48.261460	48.261460	48.261461	1.89
KC5c32	O	PSO	-1.1909	2.0679	48.261460	48.261460	48.261461	1.89
KC5c32	BR-50	SA	-0.7059	1.0840	47.527769	47.527771	47.527773	0.34
KC5c32	BR-50	EA	-0.7059	1.0838	47.527769	47.527769	47.527770	0.34
KC5c32	BR-50	PSO	-0.7059	1.0839	47.527769	47.527770	47.527777	0.34
KC5c32	BR-25	SA	-0.3884	0.5640	47.386346	47.386353	47.386382	0.04
KC5c32	BR-25	EA	-0.3883	0.5641	47.386346	47.386346	47.386347	0.04
KC5c32	BR-25	PSO	-0.3885	0.5641	47.386346	47.386348	47.386365	0.04
KC5c32	h-o	SA	-1.1913	2.0671	48.261460	48.261463	48.261468	1.89
KC5c32	h-o	EA	-1.1907	2.0678	48.261460	48.261462	48.261480	1.89
KC5c32	h-o	PSO	-1.1909	2.0677	48.261460	48.261460	48.261462	1.89
KC5c32	h-50	SA	-0.7064	1.0834	47.527769	47.527776	47.527806	0.34
KC5c32	h-50	EA	-0.7061	1.0839	47.527769	47.527769	47.527771	0.34
KC5c32	h-50	PSO	-0.7054	1.0836	47.527769	47.527769	47.527771	0.34
KC5c32	h-25	SA	-0.3883	0.5648	47.386347	47.386366	47.386448	0.04
KC5c32	h-25	EA	-0.3886	0.5642	47.386346	47.386356	47.386414	0.04
KC5c32	h-25	PSO	-0.3884	0.5647	47.386346	47.386346	47.386348	0.04
KC5c32	l-o	SA	-1.1911	2.0677	48.261460	48.261462	48.261470	1.89
KC5c32	l-o	EA	-1.1909	2.0677	48.261460	48.261462	48.261482	1.89
KC5c32	l-o	PSO	-1.1904	2.0683	48.261460	48.261460	48.261463	1.89
KC5c32	l-50	SA	-0.7063	1.0826	47.527770	47.527776	47.527795	0.34
KC5c32	l-50	EA	-0.7059	1.0838	47.527769	47.527769	47.527769	0.34
KC5c32	l-50	PSO	-0.7060	1.0839	47.527769	47.527769	47.527771	0.34
KC5c32	l-25	SA	-0.3886	0.5641	47.386346	47.386349	47.386362	0.04
KC5c32	l-25	EA	-0.3886	0.5638	47.386346	47.386349	47.386367	0.04
KC5c32	l-25	PSO	-0.3886	0.5643	47.386346	47.386347	47.386351	0.04
KC5c32	FR-o	SA	0.0931	2.0000	47.609492	47.609492	47.609492	0.51
KC5c32	FR-o	EA	0.0934	2.0000	47.609492	47.609492	47.609492	0.51
KC5c32	FR-o	PSO	0.0937	2.0000	47.609492	47.609492	47.609492	0.51
KC5c32	FR-50	SA	-0.0190	1.0000	47.391583	47.391583	47.391583	0.05
KC5c32	FR-50	EA	-0.0190	1.0000	47.391583	47.391583	47.391583	0.05
KC5c32	FR-50	PSO	-0.0190	1.0000	47.391583	47.391583	47.391583	0.05
KC5c32	FR-25	SA	-0.0920	0.5409	47.367374	47.367375	47.367376	0.00
KC5c32	FR-25	EA	-0.0923	0.5408	47.367374	47.367382	47.367451	0.00
KC5c32	FR-25	PSO	-0.0929	0.5408	47.367374	47.367376	47.367381	0.00

Table A.12: Solution results for KC5c64

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5c64	O	SA	-1.1863	2.0634	48.256464	48.256466	48.256473	1.88
KC5c64	O	EA	-1.1866	2.0630	48.256464	48.256466	48.256488	1.88
KC5c64	O	PSO	-1.1860	2.0628	48.256464	48.256464	48.256465	1.88
KC5c64	BR-50	SA	-0.7029	1.0793	47.526747	47.526751	47.526769	0.34
KC5c64	BR-50	EA	-0.7038	1.0793	47.526747	47.526747	47.526747	0.34
KC5c64	BR-50	PSO	-0.7037	1.0786	47.526747	47.526747	47.526748	0.34
KC5c64	BR-25	SA	-0.3887	0.5627	47.386218	47.386220	47.386224	0.04
KC5c64	BR-25	EA	-0.3884	0.5630	47.386218	47.386219	47.386222	0.04
KC5c64	BR-25	PSO	-0.3885	0.5628	47.386218	47.386218	47.386219	0.04
KC5c64	h-o	SA	-1.1853	2.0637	48.256464	48.256467	48.256471	1.88
KC5c64	h-o	EA	-1.1860	2.0627	48.256464	48.256464	48.256464	1.88
KC5c64	h-o	PSO	-1.1860	2.0628	48.256464	48.256464	48.256469	1.88
KC5c64	h-50	SA	-0.7039	1.0798	47.526747	47.526750	47.526756	0.34
KC5c64	h-50	EA	-0.7039	1.0795	47.526747	47.526749	47.526769	0.34
KC5c64	h-50	PSO	-0.7038	1.0793	47.526747	47.526747	47.526749	0.34
KC5c64	h-25	SA	-0.3881	0.5641	47.386220	47.386226	47.386234	0.04
KC5c64	h-25	EA	-0.3884	0.5630	47.386218	47.386219	47.386229	0.04
KC5c64	h-25	PSO	-0.3884	0.5627	47.386218	47.386218	47.386219	0.04
KC5c64	l-o	SA	-1.1869	2.0621	48.256464	48.256467	48.256472	1.88
KC5c64	l-o	EA	-1.1860	2.0626	48.256464	48.256464	48.256466	1.88
KC5c64	l-o	PSO	-1.1860	2.0628	48.256464	48.256464	48.256464	1.88
KC5c64	l-50	SA	-0.7047	1.0781	47.526748	47.526750	47.526754	0.34
KC5c64	l-50	EA	-0.7038	1.0793	47.526747	47.526747	47.526747	0.34
KC5c64	l-50	PSO	-0.7038	1.0793	47.526747	47.526747	47.526747	0.34
KC5c64	l-25	SA	-0.3882	0.5633	47.386218	47.386223	47.386233	0.04
KC5c64	l-25	EA	-0.3884	0.5630	47.386218	47.386218	47.386218	0.04
KC5c64	l-25	PSO	-0.3884	0.5630	47.386218	47.386218	47.386219	0.04
KC5c64	FR-o	SA	0.1860	1.9914	47.608134	47.608135	47.608136	0.51
KC5c64	FR-o	EA	0.1873	1.9912	47.608134	47.608134	47.608134	0.51
KC5c64	FR-o	PSO	0.1870	1.9913	47.608134	47.608134	47.608134	0.51
KC5c64	FR-50	SA	-0.0187	1.0000	47.391583	47.391583	47.391583	0.05
KC5c64	FR-50	EA	-0.0190	1.0000	47.391583	47.391583	47.391583	0.05
KC5c64	FR-50	PSO	-0.0190	1.0000	47.391583	47.391583	47.391584	0.05
KC5c64	FR-25	SA	-0.0913	0.5406	47.367374	47.367375	47.367380	0.00
KC5c64	FR-25	EA	-0.0923	0.5409	47.367374	47.367374	47.367375	0.00
KC5c64	FR-25	PSO	-0.0921	0.5404	47.367374	47.367374	47.367375	0.00

Table A.13: Solution results for KC5c128

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5c128	O	SA	-1.1868	2.0603	48.255225	48.255228	48.255236	1.87
KC5c128	O	EA	-1.1864	2.0606	48.255225	48.255226	48.255235	1.87
KC5c128	O	PSO	-1.1863	2.0610	48.255225	48.255225	48.255226	1.87
KC5c128	BR-50	SA	-0.7023	1.0794	47.526496	47.526503	47.526528	0.34
KC5c128	BR-50	EA	-0.7033	1.0791	47.526495	47.526496	47.526505	0.34
KC5c128	BR-50	PSO	-0.7034	1.0791	47.526495	47.526495	47.526497	0.34
KC5c128	BR-25	SA	-0.3882	0.5622	47.386183	47.386192	47.386218	0.04
KC5c128	BR-25	EA	-0.3883	0.5624	47.386183	47.386197	47.386310	0.04
KC5c128	BR-25	PSO	-0.3879	0.5626	47.386183	47.386184	47.386189	0.04
KC5c128	h-o	SA	-1.1861	2.0606	48.255225	48.255227	48.255229	1.87
KC5c128	h-o	EA	-1.1863	2.0605	48.255225	48.255225	48.255227	1.87
KC5c128	h-o	PSO	-1.1868	2.0606	48.255225	48.255225	48.255225	1.87
KC5c128	h-50	SA	-0.7033	1.0804	47.526496	47.526499	47.526506	0.34
KC5c128	h-50	EA	-0.7035	1.0791	47.526495	47.526495	47.526496	0.34
KC5c128	h-50	PSO	-0.7035	1.0791	47.526495	47.526495	47.526496	0.34
KC5c128	h-25	SA	-0.3890	0.5635	47.386185	47.386191	47.386206	0.04
KC5c128	h-25	EA	-0.3881	0.5622	47.386183	47.386183	47.386186	0.04
KC5c128	h-25	PSO	-0.3882	0.5623	47.386183	47.386183	47.386183	0.04
KC5c128	l-o	SA	-1.1864	2.0615	48.255225	48.255228	48.255232	1.87
KC5c128	l-o	EA	-1.1865	2.0606	48.255225	48.255226	48.255234	1.87
KC5c128	l-o	PSO	-1.1864	2.0606	48.255225	48.255225	48.255226	1.87
KC5c128	l-50	SA	-0.7040	1.0785	47.526496	47.526498	47.526500	0.34
KC5c128	l-50	EA	-0.7035	1.0791	47.526495	47.526496	47.526503	0.34
KC5c128	l-50	PSO	-0.7031	1.0790	47.526495	47.526495	47.526496	0.34
KC5c128	l-25	SA	-0.3889	0.5625	47.386183	47.386188	47.386195	0.04
KC5c128	l-25	EA	-0.3885	0.5623	47.386183	47.386183	47.386183	0.04
KC5c128	l-25	PSO	-0.3882	0.5623	47.386183	47.386183	47.386184	0.04
KC5c128	FR-o	SA	0.1867	1.9913	47.608134	47.608135	47.608135	0.51
KC5c128	FR-o	EA	0.1865	1.9913	47.608134	47.608134	47.608134	0.51
KC5c128	FR-o	PSO	0.1868	1.9913	47.608134	47.608134	47.608135	0.51
KC5c128	FR-50	SA	-0.0342	0.9995	47.391572	47.391572	47.391572	0.05
KC5c128	FR-50	EA	-0.0342	0.9995	47.391572	47.391572	47.391572	0.05
KC5c128	FR-50	PSO	-0.0343	0.9995	47.391572	47.391572	47.391572	0.05
KC5c128	FR-25	SA	-0.0920	0.5410	47.367374	47.367375	47.367378	0.00
KC5c128	FR-25	EA	-0.0923	0.5408	47.367374	47.367375	47.367380	0.00
KC5c128	FR-25	PSO	-0.0925	0.5400	47.367374	47.367377	47.367405	0.00

Table A.14: Solution results for KC5c256

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5c256	O	SA	-1.1851	2.0607	48.254917	48.254920	48.254927	1.87
KC5c256	O	EA	-1.1862	2.0605	48.254917	48.254917	48.254918	1.87
KC5c256	O	PSO	-1.1862	2.0604	48.254917	48.254918	48.254924	1.87
KC5c256	BR-50	SA	-0.7037	1.0801	47.526428	47.526431	47.526435	0.34
KC5c256	BR-50	EA	-0.7033	1.0791	47.526427	47.526427	47.526427	0.34
KC5c256	BR-50	PSO	-0.7033	1.0794	47.526427	47.526427	47.526427	0.34
KC5c256	BR-25	SA	-0.3883	0.5628	47.386175	47.386190	47.386237	0.04
KC5c256	BR-25	EA	-0.3882	0.5624	47.386175	47.386175	47.386179	0.04
KC5c256	BR-25	PSO	-0.3882	0.5624	47.386175	47.386176	47.386180	0.04
KC5c256	h-o	SA	-1.1864	2.0601	48.254917	48.254919	48.254932	1.87
KC5c256	h-o	EA	-1.1862	2.0605	48.254917	48.254917	48.254918	1.87
KC5c256	h-o	PSO	-1.1853	2.0615	48.254917	48.254917	48.254917	1.87
KC5c256	h-50	SA	-0.7023	1.0793	47.526427	47.526430	47.526440	0.34
KC5c256	h-50	EA	-0.7034	1.0790	47.526427	47.526427	47.526428	0.34
KC5c256	h-50	PSO	-0.7033	1.0791	47.526427	47.526428	47.526432	0.34
KC5c256	h-25	SA	-0.3877	0.5622	47.386175	47.386182	47.386202	0.04
KC5c256	h-25	EA	-0.3882	0.5624	47.386175	47.386175	47.386176	0.04
KC5c256	h-25	PSO	-0.3881	0.5619	47.386175	47.386175	47.386176	0.04
KC5c256	l-o	SA	-1.1872	2.0602	48.254917	48.254921	48.254942	1.87
KC5c256	l-o	EA	-1.1857	2.0599	48.254917	48.254917	48.254917	1.87
KC5c256	l-o	PSO	-1.1861	2.0606	48.254917	48.254917	48.254918	1.87
KC5c256	l-50	SA	-0.7033	1.0781	47.526427	47.526431	47.526454	0.34
KC5c256	l-50	EA	-0.7033	1.0791	47.526427	47.526427	47.526428	0.34
KC5c256	l-50	PSO	-0.7034	1.0788	47.526427	47.526427	47.526429	0.34
KC5c256	l-25	SA	-0.3879	0.5626	47.386175	47.386181	47.386224	0.04
KC5c256	l-25	EA	-0.3877	0.5624	47.386175	47.386175	47.386178	0.04
KC5c256	l-25	PSO	-0.3882	0.5623	47.386175	47.386175	47.386175	0.04
KC5c256	FR-o	SA	0.1871	1.9913	47.608134	47.608135	47.608136	0.51
KC5c256	FR-o	EA	0.1867	1.9913	47.608134	47.608134	47.608134	0.51
KC5c256	FR-o	PSO	0.1869	1.9913	47.608134	47.608134	47.608134	0.51
KC5c256	FR-50	SA	-0.0269	0.9996	47.391556	47.391556	47.391556	0.05
KC5c256	FR-50	EA	-0.0265	0.9997	47.391556	47.391556	47.391556	0.05
KC5c256	FR-50	PSO	-0.0266	0.9997	47.391556	47.391556	47.391556	0.05
KC5c256	FR-25	SA	-0.0923	0.5405	47.367374	47.367376	47.367382	0.00
KC5c256	FR-25	EA	-0.0932	0.5406	47.367374	47.367374	47.367376	0.00
KC5c256	FR-25	PSO	-0.0921	0.5412	47.367374	47.367374	47.367375	0.00

Table A.15: Solution results for KC5c512

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5c512	O	SA	-1.1862	2.0603	48.254840	48.254842	48.254845	1.87
KC5c512	O	EA	-1.1861	2.0605	48.254840	48.254841	48.254848	1.87
KC5c512	O	PSO	-1.1860	2.0604	48.254840	48.254840	48.254840	1.87
KC5c512	BR-50	SA	-0.7037	1.0786	47.526405	47.526409	47.526419	0.34
KC5c512	BR-50	EA	-0.7033	1.0790	47.526405	47.526407	47.526419	0.34
KC5c512	BR-50	PSO	-0.7033	1.0790	47.526405	47.526405	47.526406	0.34
KC5c512	BR-25	SA	-0.3882	0.5621	47.386173	47.386180	47.386199	0.04
KC5c512	BR-25	EA	-0.3881	0.5624	47.386173	47.386307	47.387515	0.04
KC5c512	BR-25	PSO	-0.3876	0.5629	47.386173	47.386173	47.386175	0.04
KC5c512	h-o	SA	-1.1853	2.0617	48.254841	48.254845	48.254864	1.87
KC5c512	h-o	EA	-1.1857	2.0604	48.254840	48.254840	48.254843	1.87
KC5c512	h-o	PSO	-1.1861	2.0605	48.254840	48.254840	48.254842	1.87
KC5c512	h-50	SA	-0.7032	1.0790	47.526405	47.526409	47.526417	0.34
KC5c512	h-50	EA	-0.7034	1.0789	47.526405	47.526405	47.526406	0.34
KC5c512	h-50	PSO	-0.7038	1.0790	47.526405	47.526406	47.526408	0.34
KC5c512	h-25	SA	-0.3888	0.5623	47.386173	47.386178	47.386195	0.04
KC5c512	h-25	EA	-0.3882	0.5624	47.386173	47.386189	47.386327	0.04
KC5c512	h-25	PSO	-0.3883	0.5619	47.386173	47.386173	47.386176	0.04
KC5c512	l-o	SA	-0.3143	2.1216	47.987851	47.987854	47.987864	1.31
KC5c512	l-o	EA	-0.3145	2.1217	47.987851	47.987852	47.987860	1.31
KC5c512	l-o	PSO	-0.3145	2.1216	47.987851	47.987851	47.987851	1.31
KC5c512	l-50	SA	-0.7030	1.0793	47.526405	47.526407	47.526412	0.34
KC5c512	l-50	EA	-0.7033	1.0790	47.526405	47.526407	47.526416	0.34
KC5c512	l-50	PSO	-0.7035	1.0789	47.526405	47.526405	47.526406	0.34
KC5c512	l-25	SA	-0.3887	0.5624	47.386173	47.386178	47.386191	0.04
KC5c512	l-25	EA	-0.3874	0.5620	47.386173	47.386174	47.386183	0.04
KC5c512	l-25	PSO	-0.3880	0.5624	47.386173	47.386173	47.386174	0.04
KC5c512	FR-o	SA	0.1753	1.9923	47.608123	47.608123	47.608124	0.51
KC5c512	FR-o	EA	0.0389	1.9542	47.595094	47.606820	47.608124	0.51
KC5c512	FR-o	PSO	0.1753	1.9923	47.608123	47.608124	47.608128	0.51
KC5c512	FR-50	SA	-0.0265	0.9996	47.391551	47.391551	47.391551	0.05
KC5c512	FR-50	EA	-0.0265	0.9996	47.391550	47.391550	47.391551	0.05
KC5c512	FR-50	PSO	-0.0266	0.9996	47.391550	47.391551	47.391551	0.05
KC5c512	FR-25	SA	-0.0915	0.5404	47.367374	47.367375	47.367379	0.00
KC5c512	FR-25	EA	-0.0923	0.5408	47.367374	47.367378	47.367398	0.00
KC5c512	FR-25	PSO	-0.0925	0.5402	47.367374	47.367375	47.367381	0.00

Table A.16: Solution results for KC5i16

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5i16	O	SA	-1.1813	2.0577	48.241865	48.241866	48.241869	1.85
KC5i16	O	EA	-1.1814	2.0576	48.241865	48.241913	48.242140	1.85
KC5i16	O	PSO	-1.1813	2.0586	48.241865	48.241865	48.241866	1.85
KC5i16	BR-50	SA	-0.7020	1.0740	47.522416	47.522421	47.522444	0.33
KC5i16	BR-50	EA	-0.7020	1.0742	47.522416	47.522418	47.522425	0.33
KC5i16	BR-50	PSO	-0.7022	1.0734	47.522416	47.522417	47.522424	0.33
KC5i16	BR-25	SA	-0.3829	0.5568	47.385427	47.385432	47.385450	0.04
KC5i16	BR-25	EA	-0.3822	0.5563	47.385426	47.385456	47.385644	0.04
KC5i16	BR-25	PSO	-0.3823	0.5564	47.385426	47.385442	47.385474	0.04
KC5i16	h-o	SA	-1.1808	2.0583	48.241865	48.241866	48.241867	1.85
KC5i16	h-o	EA	-1.1813	2.0579	48.241865	48.241872	48.241901	1.85
KC5i16	h-o	PSO	-1.1811	2.0590	48.241865	48.241867	48.241881	1.85
KC5i16	h-50	SA	-0.7031	1.0730	47.522416	47.522426	47.522475	0.33
KC5i16	h-50	EA	-0.7020	1.0737	47.522416	47.522421	47.522440	0.33
KC5i16	h-50	PSO	-0.7017	1.0739	47.522416	47.522418	47.522425	0.33
KC5i16	h-25	SA	-0.3823	0.5571	47.385427	47.385433	47.385443	0.04
KC5i16	h-25	EA	-0.3821	0.5564	47.385426	47.385566	47.386564	0.04
KC5i16	h-25	PSO	-0.3821	0.5563	47.385426	47.385436	47.385476	0.04
KC5i16	l-o	SA	-1.1820	2.0579	48.241865	48.241868	48.241874	1.85
KC5i16	l-o	EA	-1.1814	2.0578	48.241865	48.241893	48.242142	1.85
KC5i16	l-o	PSO	-1.1814	2.0578	48.241865	48.241866	48.241874	1.85
KC5i16	l-50	SA	-0.7029	1.0735	47.522416	47.522422	47.522432	0.33
KC5i16	l-50	EA	-0.7024	1.0734	47.522416	47.522433	47.522575	0.33
KC5i16	l-50	PSO	-0.7019	1.0734	47.522416	47.522424	47.522467	0.33
KC5i16	l-25	SA	-0.3821	0.5566	47.385427	47.385432	47.385443	0.04
KC5i16	l-25	EA	-0.3822	0.5563	47.385426	47.385496	47.386091	0.04
KC5i16	l-25	PSO	-0.3825	0.5562	47.385427	47.385432	47.385456	0.04
KC5i16	FR-o	SA	0.2724	1.9458	47.597710	47.597711	47.597711	0.49
KC5i16	FR-o	EA	0.2717	1.9460	47.597710	47.597712	47.597716	0.49
KC5i16	FR-o	PSO	0.2720	1.9459	47.597710	47.597710	47.597711	0.49
KC5i16	FR-50	SA	-0.0823	0.9836	47.390527	47.390527	47.390528	0.05
KC5i16	FR-50	EA	-0.0818	0.9837	47.390527	47.390528	47.390536	0.05
KC5i16	FR-50	PSO	-0.0821	0.9837	47.390527	47.390528	47.390530	0.05
KC5i16	FR-25	SA	-0.0924	0.5407	47.367374	47.367376	47.367378	0.00
KC5i16	FR-25	EA	-0.0933	0.5401	47.367374	47.367376	47.367382	0.00
KC5i16	FR-25	PSO	-0.0929	0.5417	47.367374	47.367378	47.367401	0.00

Table A.17: Solution results for KC5i32

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5i32	O	SA	-1.1808	2.0579	48.251504	48.251506	48.251513	1.87
KC5i32	O	EA	-1.1810	2.0580	48.251504	48.251504	48.251505	1.87
KC5i32	O	PSO	-1.1812	2.0579	48.251504	48.251504	48.251506	1.87
KC5i32	BR-50	SA	-0.7018	1.0753	47.525732	47.525735	47.525743	0.33
KC5i32	BR-50	EA	-0.7016	1.0748	47.525732	47.525732	47.525732	0.33
KC5i32	BR-50	PSO	-0.7016	1.0748	47.525732	47.525732	47.525734	0.33
KC5i32	BR-25	SA	-0.3877	0.5614	47.386093	47.386096	47.386102	0.04
KC5i32	BR-25	EA	-0.3878	0.5617	47.386092	47.386097	47.386116	0.04
KC5i32	BR-25	PSO	-0.3881	0.5618	47.386092	47.386093	47.386096	0.04
KC5i32	h-o	SA	-1.1810	2.0575	48.251504	48.251507	48.251516	1.87
KC5i32	h-o	EA	-1.1813	2.0579	48.251504	48.251505	48.251509	1.87
KC5i32	h-o	PSO	-1.1815	2.0582	48.251504	48.251504	48.251505	1.87
KC5i32	h-50	SA	-0.7023	1.0735	47.525733	47.525741	47.525788	0.33
KC5i32	h-50	EA	-0.7016	1.0748	47.525732	47.525737	47.525772	0.33
KC5i32	h-50	PSO	-0.7018	1.0751	47.525732	47.525732	47.525732	0.33
KC5i32	h-25	SA	-0.3884	0.5613	47.386092	47.386098	47.386120	0.04
KC5i32	h-25	EA	-0.3882	0.5617	47.386092	47.386094	47.386112	0.04
KC5i32	h-25	PSO	-0.3883	0.5621	47.386092	47.386092	47.386094	0.04
KC5i32	l-o	SA	-1.1810	2.0575	48.251504	48.251506	48.251513	1.87
KC5i32	l-o	EA	-1.1813	2.0579	48.251504	48.251505	48.251509	1.87
KC5i32	l-o	PSO	-1.1811	2.0578	48.251504	48.251504	48.251505	1.87
KC5i32	l-50	SA	-0.7019	1.0743	47.525732	47.525734	47.525737	0.33
KC5i32	l-50	EA	-0.7016	1.0748	47.525732	47.525734	47.525747	0.33
KC5i32	l-50	PSO	-0.7016	1.0749	47.525732	47.525732	47.525732	0.33
KC5i32	l-25	SA	-0.3880	0.5619	47.386092	47.386098	47.386106	0.04
KC5i32	l-25	EA	-0.3882	0.5617	47.386092	47.386093	47.386096	0.04
KC5i32	l-25	PSO	-0.3881	0.5621	47.386092	47.386092	47.386093	0.04
KC5i32	FR-o	SA	0.1860	1.9817	47.605008	47.605008	47.605008	0.50
KC5i32	FR-o	EA	0.1855	1.9817	47.605008	47.605008	47.605008	0.50
KC5i32	FR-o	PSO	0.1853	1.9818	47.605008	47.605008	47.605008	0.50
KC5i32	FR-50	SA	-0.0503	0.9950	47.391225	47.391225	47.391225	0.05
KC5i32	FR-50	EA	-0.0499	0.9951	47.391225	47.391225	47.391225	0.05
KC5i32	FR-50	PSO	-0.0501	0.9951	47.391225	47.391225	47.391225	0.05
KC5i32	FR-25	SA	-0.0920	0.5397	47.367374	47.367375	47.367379	0.00
KC5i32	FR-25	EA	-0.0925	0.5409	47.367374	47.367374	47.367376	0.00
KC5i32	FR-25	PSO	-0.0924	0.5410	47.367374	47.367375	47.367377	0.00

Table A.18: Solution results for KC5i64

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5i64	O	SA	-1.1853	2.0814	48.254236	48.265124	48.291410	1.90
KC5i64	O	EA	-1.1869	2.0584	48.253988	48.253988	48.253990	1.87
KC5i64	O	PSO	-1.1869	2.0583	48.253988	48.253988	48.253989	1.87
KC5i64	BR-50	SA	-0.7032	1.0790	47.526240	47.526244	47.526254	0.34
KC5i64	BR-50	EA	-0.7031	1.0790	47.526240	47.526240	47.526240	0.34
KC5i64	BR-50	PSO	-0.7026	1.0791	47.526240	47.526240	47.526241	0.34
KC5i64	BR-25	SA	-0.3886	0.5617	47.386147	47.386154	47.386167	0.04
KC5i64	BR-25	EA	-0.3882	0.5618	47.386147	47.386147	47.386149	0.04
KC5i64	BR-25	PSO	-0.3881	0.5618	47.386147	47.386148	47.386150	0.04
KC5i64	h-o	SA	-1.1868	2.0591	48.253988	48.253989	48.253993	1.87
KC5i64	h-o	EA	-1.1869	2.0584	48.253988	48.253988	48.253992	1.87
KC5i64	h-o	PSO	-1.1863	2.0593	48.253988	48.253988	48.253989	1.87
KC5i64	h-50	SA	-0.8389	0.5433	47.475875	47.521205	47.526245	0.32
KC5i64	h-50	EA	-0.7032	1.0789	47.526240	47.526240	47.526240	0.34
KC5i64	h-50	PSO	-0.7032	1.0789	47.526240	47.526240	47.526240	0.34
KC5i64	h-25	SA	-0.3880	0.5620	47.386147	47.386151	47.386160	0.04
KC5i64	h-25	EA	-0.3882	0.5618	47.386147	47.386148	47.386155	0.04
KC5i64	h-25	PSO	-0.3877	0.5620	47.386147	47.386147	47.386149	0.04
KC5i64	l-o	SA	-0.3149	2.1207	48.137397	48.137399	48.137404	1.63
KC5i64	l-o	EA	-0.3144	2.1209	48.137397	48.137400	48.137424	1.63
KC5i64	l-o	PSO	-0.3146	2.1209	48.137397	48.137397	48.137399	1.63
KC5i64	l-50	SA	-0.7028	1.0785	47.526240	47.526244	47.526252	0.34
KC5i64	l-50	EA	-0.7031	1.0790	47.526240	47.526240	47.526240	0.34
KC5i64	l-50	PSO	-0.7031	1.0790	47.526240	47.526241	47.526249	0.34
KC5i64	l-25	SA	-0.3889	0.5610	47.386148	47.386155	47.386174	0.04
KC5i64	l-25	EA	-0.3883	0.5617	47.386147	47.386147	47.386149	0.04
KC5i64	l-25	PSO	-0.3876	0.5622	47.386147	47.386147	47.386147	0.04
KC5i64	FR-o	SA	0.1399	1.9931	47.607613	47.607613	47.607613	0.51
KC5i64	FR-o	EA	0.1401	1.9931	47.607613	47.607613	47.607614	0.51
KC5i64	FR-o	PSO	0.1401	1.9931	47.607613	47.607613	47.607613	0.51
KC5i64	FR-50	SA	-0.0349	0.9983	47.391445	47.391445	47.391446	0.05
KC5i64	FR-50	EA	-0.0344	0.9983	47.391445	47.391445	47.391445	0.05
KC5i64	FR-50	PSO	-0.0343	0.9983	47.391445	47.391445	47.391445	0.05
KC5i64	FR-25	SA	-0.0926	0.5409	47.367374	47.367375	47.367376	0.00
KC5i64	FR-25	EA	-0.0923	0.5408	47.367374	47.367374	47.367377	0.00
KC5i64	FR-25	PSO	-0.0917	0.5408	47.367374	47.367374	47.367376	0.00

Table A.19: Solution results for KC5i128

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5i128	O	SA	-1.1869	2.0599	48.254609	48.254610	48.254613	1.87
KC5i128	O	EA	-1.1859	2.0605	48.254609	48.254609	48.254609	1.87
KC5i128	O	PSO	-1.1857	2.0608	48.254609	48.254610	48.254618	1.87
KC5i128	BR-50	SA	-0.7039	1.0779	47.526359	47.526363	47.526383	0.34
KC5i128	BR-50	EA	-0.7030	1.0791	47.526359	47.526360	47.526363	0.34
KC5i128	BR-50	PSO	-0.7031	1.0790	47.526359	47.526359	47.526359	0.34
KC5i128	BR-25	SA	-0.3879	0.5617	47.386167	47.386178	47.386219	0.04
KC5i128	BR-25	EA	-0.3881	0.5623	47.386166	47.386168	47.386187	0.04
KC5i128	BR-25	PSO	-0.3884	0.5624	47.386166	47.386166	47.386167	0.04
KC5i128	h-o	SA	-1.1659	2.0452	48.255003	48.267001	48.288779	1.90
KC5i128	h-o	EA	-1.1859	2.0605	48.254609	48.254609	48.254610	1.87
KC5i128	h-o	PSO	-1.1855	2.0613	48.254609	48.254609	48.254609	1.87
KC5i128	h-50	SA	-0.7035	1.0797	47.526359	47.526361	47.526363	0.34
KC5i128	h-50	EA	-0.7031	1.0790	47.526359	47.526359	47.526361	0.34
KC5i128	h-50	PSO	-0.7031	1.0790	47.526359	47.526359	47.526360	0.34
KC5i128	h-25	SA	-0.3887	0.5625	47.386167	47.386169	47.386174	0.04
KC5i128	h-25	EA	-0.3881	0.5623	47.386166	47.386166	47.386166	0.04
KC5i128	h-25	PSO	-0.3879	0.5621	47.386166	47.386167	47.386170	0.04
KC5i128	l-o	SA	-1.1860	2.0600	48.254609	48.254611	48.254623	1.87
KC5i128	l-o	EA	-1.1859	2.0605	48.254609	48.254609	48.254612	1.87
KC5i128	l-o	PSO	-1.1859	2.0607	48.254609	48.254609	48.254610	1.87
KC5i128	l-50	SA	-0.7020	1.0797	47.526359	47.526360	47.526361	0.34
KC5i128	l-50	EA	-0.7031	1.0790	47.526359	47.526359	47.526360	0.34
KC5i128	l-50	PSO	-0.7030	1.0786	47.526359	47.526359	47.526359	0.34
KC5i128	l-25	SA	-0.3878	0.5621	47.386166	47.386170	47.386176	0.04
KC5i128	l-25	EA	-0.3881	0.5623	47.386166	47.386166	47.386167	0.04
KC5i128	l-25	PSO	-0.3881	0.5623	47.386166	47.386167	47.386174	0.04
KC5i128	FR-o	SA	0.1635	1.9928	47.607967	47.607968	47.607968	0.51
KC5i128	FR-o	EA	0.1636	1.9928	47.607967	47.607967	47.607968	0.51
KC5i128	FR-o	PSO	0.1635	1.9928	47.607967	47.607967	47.607967	0.51
KC5i128	FR-50	SA	-0.0272	0.9993	47.391524	47.391524	47.391524	0.05
KC5i128	FR-50	EA	-0.0266	0.9993	47.391524	47.391524	47.391524	0.05
KC5i128	FR-50	PSO	-0.0266	0.9994	47.391524	47.391524	47.391524	0.05
KC5i128	FR-25	SA	-0.0928	0.5396	47.367374	47.367375	47.367377	0.00
KC5i128	FR-25	EA	-0.0923	0.5408	47.367374	47.367385	47.367478	0.00
KC5i128	FR-25	PSO	-0.0921	0.5422	47.367374	47.367374	47.367375	0.00

Table A.20: Solution results for KC5i256

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5i256	O	SA	-1.1855	2.0607	48.254764	48.254765	48.254766	1.87
KC5i256	O	EA	-1.1869	2.0597	48.254764	48.254764	48.254764	1.87
KC5i256	O	PSO	-1.1860	2.0604	48.254764	48.254764	48.254764	1.87
KC5i256	BR-50	SA	-0.7026	1.0796	47.526394	47.526398	47.526412	0.34
KC5i256	BR-50	EA	-0.7032	1.0790	47.526394	47.526396	47.526413	0.34
KC5i256	BR-50	PSO	-0.7034	1.0790	47.526394	47.526394	47.526395	0.34
KC5i256	BR-25	SA	-0.3884	0.5616	47.386173	47.386180	47.386198	0.04
KC5i256	BR-25	EA	-0.3886	0.5624	47.386173	47.386185	47.386294	0.04
KC5i256	BR-25	PSO	-0.3886	0.5622	47.386173	47.386175	47.386183	0.04
KC5i256	h-o	SA	-1.1866	2.0603	48.254764	48.254766	48.254767	1.87
KC5i256	h-o	EA	-1.1859	2.0605	48.254764	48.254764	48.254764	1.87
KC5i256	h-o	PSO	-1.1859	2.0605	48.254764	48.254764	48.254764	1.87
KC5i256	h-50	SA	-0.7030	1.0783	47.526394	47.526397	47.526400	0.34
KC5i256	h-50	EA	-0.7032	1.0790	47.526394	47.526394	47.526394	0.34
KC5i256	h-50	PSO	-0.7035	1.0779	47.526394	47.526394	47.526394	0.34
KC5i256	h-25	SA	-0.3876	0.5624	47.386173	47.386180	47.386196	0.04
KC5i256	h-25	EA	-0.3882	0.5623	47.386173	47.386175	47.386184	0.04
KC5i256	h-25	PSO	-0.3882	0.5623	47.386173	47.386173	47.386173	0.04
KC5i256	l-o	SA	-1.1851	2.0614	48.254764	48.254769	48.254795	1.87
KC5i256	l-o	EA	-1.1859	2.0605	48.254764	48.254764	48.254764	1.87
KC5i256	l-o	PSO	-1.1860	2.0605	48.254764	48.254764	48.254764	1.87
KC5i256	l-50	SA	-0.7043	1.0789	47.526394	47.526399	47.526406	0.34
KC5i256	l-50	EA	-0.7033	1.0790	47.526394	47.526394	47.526396	0.34
KC5i256	l-50	PSO	-0.7034	1.0788	47.526394	47.526394	47.526394	0.34
KC5i256	l-25	SA	-0.3883	0.5626	47.386173	47.386180	47.386194	0.04
KC5i256	l-25	EA	-0.3882	0.5623	47.386173	47.386175	47.386195	0.04
KC5i256	l-25	PSO	-0.3882	0.5623	47.386173	47.386173	47.386173	0.04
KC5i256	FR-o	SA	0.1744	1.9922	47.608074	47.608074	47.608075	0.51
KC5i256	FR-o	EA	0.1751	1.9922	47.608074	47.608074	47.608075	0.51
KC5i256	FR-o	PSO	0.1755	1.9921	47.608074	47.608074	47.608074	0.51
KC5i256	FR-50	SA	-0.0308	0.9995	47.391551	47.391551	47.391551	0.05
KC5i256	FR-50	EA	-0.0303	0.9995	47.391551	47.391551	47.391551	0.05
KC5i256	FR-50	PSO	-0.0304	0.9995	47.391551	47.391551	47.391551	0.05
KC5i256	FR-25	SA	-0.0923	0.5401	47.367374	47.367375	47.367376	0.00
KC5i256	FR-25	EA	-0.0924	0.5408	47.367374	47.367375	47.367376	0.00
KC5i256	FR-25	PSO	-0.0922	0.5407	47.367374	47.367375	47.367380	0.00

Table A.21: Solution results for KC5i512

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC5i512	O	SA	-1.1859	2.0604	48.254802	48.254805	48.254812	1.87
KC5i512	O	EA	-1.1860	2.0605	48.254802	48.254802	48.254803	1.87
KC5i512	O	PSO	-1.1866	2.0600	48.254802	48.254802	48.254802	1.87
KC5i512	BR-50	SA	-0.7041	1.0789	47.526399	47.526400	47.526405	0.34
KC5i512	BR-50	EA	-0.7032	1.0790	47.526399	47.526402	47.526410	0.34
KC5i512	BR-50	PSO	-0.7037	1.0793	47.526399	47.526400	47.526411	0.34
KC5i512	BR-25	SA	-0.3881	0.5630	47.386174	47.386183	47.386208	0.04
KC5i512	BR-25	EA	-0.3880	0.5624	47.386174	47.386176	47.386194	0.04
KC5i512	BR-25	PSO	-0.3880	0.5623	47.386174	47.386174	47.386175	0.04
KC5i512	h-o	SA	-1.1851	2.0612	48.254802	48.254803	48.254807	1.87
KC5i512	h-o	EA	-1.1860	2.0605	48.254802	48.254802	48.254802	1.87
KC5i512	h-o	PSO	-1.1861	2.0605	48.254802	48.254802	48.254803	1.87
KC5i512	h-50	SA	-0.7041	1.0789	47.526399	47.526402	47.526408	0.34
KC5i512	h-50	EA	-0.7032	1.0790	47.526399	47.526399	47.526399	0.34
KC5i512	h-50	PSO	-0.7033	1.0790	47.526399	47.526400	47.526408	0.34
KC5i512	h-25	SA	-0.3881	0.5629	47.386174	47.386182	47.386197	0.04
KC5i512	h-25	EA	-0.3882	0.5623	47.386174	47.386174	47.386174	0.04
KC5i512	h-25	PSO	-0.3885	0.5619	47.386174	47.386174	47.386176	0.04
KC5i512	l-o	SA	-1.1850	2.0612	48.254802	48.254804	48.254808	1.87
KC5i512	l-o	EA	-1.1860	2.0605	48.254802	48.254802	48.254804	1.87
KC5i512	l-o	PSO	-1.1862	2.0604	48.254802	48.254802	48.254804	1.87
KC5i512	l-50	SA	-0.7034	1.0792	47.526399	47.526403	47.526422	0.34
KC5i512	l-50	EA	-0.7033	1.0783	47.526399	47.526399	47.526399	0.34
KC5i512	l-50	PSO	-0.7033	1.0791	47.526399	47.526399	47.526399	0.34
KC5i512	l-25	SA	-0.3873	0.5623	47.386175	47.386186	47.386208	0.04
KC5i512	l-25	EA	-0.3882	0.5624	47.386174	47.386175	47.386188	0.04
KC5i512	l-25	PSO	-0.3881	0.5621	47.386174	47.386175	47.386179	0.04
KC5i512	FR-o	SA	0.1817	1.9917	47.608110	47.608110	47.608111	0.51
KC5i512	FR-o	EA	0.1806	1.9918	47.608110	47.608110	47.608110	0.51
KC5i512	FR-o	PSO	0.1811	1.9917	47.608110	47.608110	47.608110	0.51
KC5i512	FR-50	SA	-0.0291	0.9995	47.391553	47.391553	47.391553	0.05
KC5i512	FR-50	EA	-0.0292	0.9995	47.391553	47.391553	47.391553	0.05
KC5i512	FR-50	PSO	-0.0292	0.9995	47.391553	47.391553	47.391554	0.05
KC5i512	FR-25	SA	-0.0919	0.5411	47.367374	47.367375	47.367379	0.00
KC5i512	FR-25	EA	-0.0923	0.5408	47.367374	47.367374	47.367375	0.00
KC5i512	FR-25	PSO	-0.0922	0.5412	47.367374	47.367374	47.367376	0.00

Table A.22: Solution results for KC10c16 and KC10U

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC10c16	O	SA	3.3247	-0.0855	88.468917	88.468921	88.468927	5.50
KC10c16	O	EA	3.3246	-0.0856	88.468917	88.468921	88.468946	5.50
KC10c16	O	PSO	3.3247	-0.0852	88.468917	88.468920	88.468931	5.50
KC10c16	BR-50	SA	1.8635	-0.0327	84.691801	84.691805	84.691819	1.00
KC10c16	BR-50	EA	1.8636	-0.0328	84.691801	84.691802	84.691807	1.00
KC10c16	BR-50	PSO	1.8636	-0.0326	84.691801	84.691805	84.691818	1.00
KC10c16	BR-25	SA	0.9940	-0.0109	83.952812	83.952824	83.952870	0.11
KC10c16	BR-25	EA	0.9938	-0.0107	83.952812	83.952823	83.952904	0.11
KC10c16	BR-25	PSO	0.9939	-0.0107	83.952812	83.952816	83.952848	0.11
KC10c16	h-o	SA	3.3246	-0.0842	88.468918	88.468921	88.468925	5.50
KC10c16	h-o	EA	3.3247	-0.0856	88.468918	88.468924	88.468955	5.50
KC10c16	h-o	PSO	3.3248	-0.0857	88.468918	88.468919	88.468925	5.50
KC10c16	h-50	SA	1.8628	-0.0336	84.691802	84.691810	84.691824	1.00
KC10c16	h-50	EA	1.8635	-0.0328	84.691801	84.691801	84.691803	1.00
KC10c16	h-50	PSO	1.8631	-0.0331	84.691801	84.691811	84.691857	1.00
KC10c16	h-25	SA	0.9938	-0.0110	83.952812	83.952818	83.952827	0.11
KC10c16	h-25	EA	0.9938	-0.0108	83.952812	83.952817	83.952854	0.11
KC10c16	h-25	PSO	0.9936	-0.0106	83.952812	83.952813	83.952819	0.11
KC10c16	l-o	SA	3.2894	-0.1020	88.470573	88.480271	88.501641	5.51
KC10c16	l-o	EA	3.3248	-0.0862	88.468918	88.468918	88.468919	5.50
KC10c16	l-o	PSO	3.3247	-0.0849	88.468918	88.468918	88.468920	5.50
KC10c16	l-50	SA	1.8634	-0.0324	84.691801	84.691810	84.691836	1.00
KC10c16	l-50	EA	1.8635	-0.0328	84.691801	84.691802	84.691805	1.00
KC10c16	l-50	PSO	1.8638	-0.0329	84.691801	84.691801	84.691803	1.00
KC10c16	l-25	SA	0.9938	-0.0111	83.952812	83.952816	83.952822	0.11
KC10c16	l-25	EA	0.9938	-0.0108	83.952812	83.952814	83.952830	0.11
KC10c16	l-25	PSO	0.9936	-0.0111	83.952812	83.952818	83.952857	0.11
KC10c16	FR-o	SA	2.3029	1.9397	85.617491	85.617492	85.617493	2.10
KC10c16	FR-o	EA	2.3032	1.9394	85.617491	85.617495	85.617522	2.10
KC10c16	FR-o	PSO	2.3031	1.9396	85.617490	85.617491	85.617492	2.10
KC10c16	FR-50	SA	1.3913	0.5608	84.145983	84.145983	84.145983	0.34
KC10c16	FR-50	EA	1.3912	0.5611	84.145982	84.145982	84.145983	0.34
KC10c16	FR-50	PSO	1.3912	0.5609	84.145982	84.145982	84.145983	0.34
KC10c16	FR-25	SA	0.7500	0.0630	83.873630	83.873631	83.873631	0.02
KC10c16	FR-25	EA	0.7500	0.0632	83.873630	83.873664	83.873972	0.02
KC10c16	FR-25	PSO	0.7500	0.0633	83.873630	83.873630	83.873632	0.02
KC10	U	SA	0.5260	0.0197	83.856913	83.856915	83.856924	0.00
KC10	U	EA	0.5159	0.0105	83.856962	83.866423	83.914454	0.01
KC10	U	PSO	0.5278	0.0182	83.856915	83.857273	83.858100	0.00

Table A.23: Solution results for KC10c128

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC10c128	O	SA	3.2674	-0.0576	88.328089	88.341839	88.393078	5.35
KC10c128	O	EA	3.3071	-0.0672	88.325075	88.325075	88.325075	5.33
KC10c128	O	PSO	3.3074	-0.0674	88.325075	88.325075	88.325075	5.33
KC10c128	BR-50	SA	1.8336	-0.0335	84.666962	84.666965	84.666973	0.97
KC10c128	BR-50	EA	1.8329	-0.0334	84.666961	84.666961	84.666961	0.97
KC10c128	BR-50	PSO	1.8329	-0.0328	84.666961	84.666961	84.666962	0.97
KC10c128	BR-25	SA	0.9829	-0.0112	83.950485	83.950488	83.950493	0.11
KC10c128	BR-25	EA	0.9829	-0.0112	83.950485	83.950486	83.950494	0.11
KC10c128	BR-25	PSO	0.9829	-0.0112	83.950485	83.950485	83.950486	0.11
KC10c128	h-o	SA	3.3068	-0.0673	88.325075	88.325078	88.325084	5.33
KC10c128	h-o	EA	3.3071	-0.0672	88.325075	88.325076	88.325084	5.33
KC10c128	h-o	PSO	3.3072	-0.0670	88.325075	88.325076	88.325081	5.33
KC10c128	h-50	SA	1.8321	-0.0346	84.666963	84.666966	84.666977	0.97
KC10c128	h-50	EA	1.8331	-0.0335	84.666961	84.666961	84.666961	0.97
KC10c128	h-50	PSO	1.8329	-0.0330	84.666961	84.666961	84.666964	0.97
KC10c128	h-25	SA	0.9833	-0.0106	83.950485	83.950489	83.950494	0.11
KC10c128	h-25	EA	0.9829	-0.0112	83.950485	83.950487	83.950501	0.11
KC10c128	h-25	PSO	0.9829	-0.0112	83.950485	83.950485	83.950486	0.11
KC10c128	l-o	SA	3.3066	-0.0666	88.325075	88.325081	88.325090	5.33
KC10c128	l-o	EA	3.3071	-0.0672	88.325075	88.325075	88.325078	5.33
KC10c128	l-o	PSO	3.3071	-0.0672	88.325075	88.325075	88.325077	5.33
KC10c128	l-50	SA	1.8326	-0.0328	84.666961	84.666965	84.666978	0.97
KC10c128	l-50	EA	1.8329	-0.0330	84.666961	84.666961	84.666961	0.97
KC10c128	l-50	PSO	1.8329	-0.0330	84.666961	84.666961	84.666961	0.97
KC10c128	l-25	SA	0.9828	-0.0112	83.950485	83.950490	83.950506	0.11
KC10c128	l-25	EA	0.9829	-0.0112	83.950485	83.950485	83.950485	0.11
KC10c128	l-25	PSO	0.9829	-0.0109	83.950485	83.950485	83.950487	0.11
KC10c128	FR-o	SA	2.4845	1.6815	85.586560	85.586561	85.586562	2.06
KC10c128	FR-o	EA	2.4849	1.6810	85.586559	85.586560	85.586560	2.06
KC10c128	FR-o	PSO	2.4857	1.6797	85.586553	85.586559	85.586560	2.06
KC10c128	FR-50	SA	1.3913	0.5608	84.145974	84.145975	84.145977	0.34
KC10c128	FR-50	EA	1.3911	0.5613	84.145974	84.145981	84.146036	0.34
KC10c128	FR-50	PSO	1.3909	0.5617	84.145974	84.145974	84.145974	0.34
KC10c128	FR-25	SA	0.7437	0.0976	83.873080	83.873080	83.873081	0.02
KC10c128	FR-25	EA	0.7437	0.0977	83.873080	83.873080	83.873082	0.02
KC10c128	FR-25	PSO	0.7437	0.0977	83.873080	83.873080	83.873081	0.02

Table A.24: Solution results for KC10i16

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC10i16	O	SA	3.3033	-0.0630	88.249042	88.249045	88.249055	5.24
KC10i16	O	EA	3.3035	-0.0623	88.249042	88.249277	88.251136	5.24
KC10i16	O	PSO	3.3034	-0.0615	88.249042	88.249044	88.249057	5.24
KC10i16	BR-50	SA	1.8099	-0.0390	84.657198	84.657204	84.657224	0.95
KC10i16	BR-50	EA	1.8091	-0.0391	84.657197	84.657197	84.657198	0.95
KC10i16	BR-50	PSO	1.8092	-0.0394	84.657197	84.657198	84.657200	0.95
KC10i16	BR-25	SA	0.9800	-0.0117	83.950113	83.950118	83.950127	0.11
KC10i16	BR-25	EA	0.9799	-0.0117	83.950113	83.950145	83.950406	0.11
KC10i16	BR-25	PSO	0.9799	-0.0116	83.950113	83.950126	83.950174	0.11
KC10i16	h-o	SA	3.3035	-0.0620	88.249042	88.249044	88.249048	5.24
KC10i16	h-o	EA	3.3036	-0.0622	88.249042	88.249048	88.249084	5.24
KC10i16	h-o	PSO	3.3035	-0.0622	88.249042	88.249043	88.249048	5.24
KC10i16	h-50	SA	1.8095	-0.0384	84.657198	84.657199	84.657202	0.95
KC10i16	h-50	EA	1.8091	-0.0391	84.657197	84.657199	84.657209	0.95
KC10i16	h-50	PSO	1.8088	-0.0389	84.657197	84.657213	84.657278	0.95
KC10i16	h-25	SA	0.9795	-0.0126	83.950114	83.950120	83.950128	0.11
KC10i16	h-25	EA	0.9801	-0.0116	83.950113	83.950117	83.950145	0.11
KC10i16	h-25	PSO	0.9799	-0.0117	83.950113	83.950117	83.950135	0.11
KC10i16	l-o	SA	3.3032	-0.0623	88.249042	88.249046	88.249053	5.24
KC10i16	l-o	EA	3.3034	-0.0622	88.249042	88.249052	88.249096	5.24
KC10i16	l-o	PSO	3.3033	-0.0623	88.249042	88.249043	88.249044	5.24
KC10i16	l-50	SA	1.8094	-0.0395	84.657198	84.657203	84.657225	0.95
KC10i16	l-50	EA	1.8090	-0.0391	84.657197	84.657211	84.657337	0.95
KC10i16	l-50	PSO	1.8090	-0.0390	84.657197	84.657199	84.657209	0.95
KC10i16	l-25	SA	0.9806	-0.0108	83.950114	83.950121	83.950143	0.11
KC10i16	l-25	EA	0.9800	-0.0117	83.950113	83.950113	83.950114	0.11
KC10i16	l-25	PSO	0.9800	-0.0117	83.950113	83.950114	83.950119	0.11
KC10i16	FR-o	SA	2.4404	1.6438	85.510639	85.510640	85.510641	1.97
KC10i16	FR-o	EA	2.4404	1.6437	85.510638	85.510639	85.510639	1.97
KC10i16	FR-o	PSO	2.4404	1.6438	85.510638	85.510639	85.510639	1.97
KC10i16	FR-50	SA	1.4228	0.3878	84.134557	84.134557	84.134558	0.33
KC10i16	FR-50	EA	1.4229	0.3873	84.134556	84.134557	84.134561	0.33
KC10i16	FR-50	PSO	1.4229	0.3875	84.134556	84.134557	84.134557	0.33
KC10i16	FR-25	SA	0.7293	0.1042	83.871273	83.871273	83.871274	0.02
KC10i16	FR-25	EA	0.7294	0.1036	83.871273	83.871274	83.871279	0.02
KC10i16	FR-25	PSO	0.7292	0.1046	83.871273	83.871273	83.871274	0.02

Table A.25: Solution results for KC10i128

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
KC10i128	O	SA	3.2938	-0.1017	88.322511	88.335796	88.349756	5.34
KC10i128	O	EA	3.3060	-0.0677	88.321938	88.321939	88.321941	5.32
KC10i128	O	PSO	3.3059	-0.0678	88.321938	88.321938	88.321939	5.32
KC10i128	BR-50	SA	1.8326	-0.0348	84.666280	84.666285	84.666298	0.97
KC10i128	BR-50	EA	1.8327	-0.0329	84.666277	84.666277	84.666279	0.97
KC10i128	BR-50	PSO	1.8327	-0.0330	84.666277	84.666277	84.666279	0.97
KC10i128	BR-25	SA	0.9824	-0.0106	83.950414	83.950417	83.950425	0.11
KC10i128	BR-25	EA	0.9826	-0.0110	83.950414	83.950415	83.950419	0.11
KC10i128	BR-25	PSO	0.9828	-0.0108	83.950414	83.950415	83.950417	0.11
KC10i128	h-o	SA	3.3358	-0.0328	88.323573	88.333176	88.343879	5.34
KC10i128	h-o	EA	3.3059	-0.0677	88.321938	88.321938	88.321939	5.32
KC10i128	h-o	PSO	3.3060	-0.0682	88.321938	88.321938	88.321938	5.32
KC10i128	h-50	SA	1.8329	-0.0332	84.666277	84.666282	84.666288	0.97
KC10i128	h-50	EA	1.8327	-0.0329	84.666277	84.666277	84.666277	0.97
KC10i128	h-50	PSO	1.8327	-0.0326	84.666277	84.666278	84.666279	0.97
KC10i128	h-25	SA	0.9828	-0.0118	83.950415	83.950419	83.950433	0.11
KC10i128	h-25	EA	0.9826	-0.0110	83.950414	83.950414	83.950414	0.11
KC10i128	h-25	PSO	0.9826	-0.0110	83.950414	83.950414	83.950415	0.11
KC10i128	l-o	SA	2.3395	1.8777	87.037288	87.037984	87.038785	3.79
KC10i128	l-o	EA	2.3282	1.8914	87.037127	87.037196	87.037512	3.79
KC10i128	l-o	PSO	2.3288	1.8907	87.037131	87.037138	87.037145	3.79
KC10i128	l-50	SA	1.8327	-0.0331	84.666277	84.666280	84.666291	0.97
KC10i128	l-50	EA	1.8327	-0.0329	84.666277	84.666277	84.666278	0.97
KC10i128	l-50	PSO	1.8326	-0.0327	84.666277	84.666277	84.666280	0.97
KC10i128	l-25	SA	0.9827	-0.0116	83.950415	83.950421	83.950437	0.11
KC10i128	l-25	EA	0.9826	-0.0110	83.950414	83.950414	83.950414	0.11
KC10i128	l-25	PSO	0.9830	-0.0107	83.950414	83.950414	83.950416	0.11
KC10i128	FR-o	SA	2.4613	1.7137	85.585339	85.585340	85.585342	2.06
KC10i128	FR-o	EA	2.4614	1.7135	85.585336	85.585339	85.585339	2.06
KC10i128	FR-o	PSO	2.4609	1.7142	85.585314	85.585335	85.585339	2.06
KC10i128	FR-50	SA	1.3985	0.5411	84.145626	84.145626	84.145627	0.34
KC10i128	FR-50	EA	1.3984	0.5414	84.145626	84.145626	84.145626	0.34
KC10i128	FR-50	PSO	1.3984	0.5414	84.145625	84.145626	84.145626	0.34
KC10i128	FR-25	SA	0.7442	0.0914	83.873027	83.873027	83.873027	0.02
KC10i128	FR-25	EA	0.7441	0.0920	83.873027	83.873027	83.873029	0.02
KC10i128	FR-25	PSO	0.7441	0.0921	83.873027	83.873027	83.873027	0.02

Table A.26: Solution results for C600

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
C600	PR	SA	159.3546	150.8110	68447.395586	68467.293764	68486.611355	8.39
C600	PR	EA	159.0532	150.9916	68447.120020	68447.120021	68447.120024	8.36
C600	PR	PSO	159.0530	150.9917	68447.120020	68447.120022	68447.120027	8.36
C600	LR	SA	149.9124	141.2108	66209.058878	66221.993020	66256.751125	4.84
C600	LR	EA	149.8277	140.0098	66206.276295	66206.276298	66206.276320	4.81
C600	LR	PSO	149.8276	140.0096	66206.276295	66206.276296	66206.276299	4.81
C600	U	SA	147.1092	137.4711	63167.955303	63183.741020	63213.171621	0.03
C600	U	EA	146.2022	138.3442	63165.196719	63165.196721	63165.196731	0.00
C600	U	PSO	146.2022	138.3443	63165.196719	63165.196721	63165.196724	0.00

Table A.27: Solution results for R800

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
R800	PR	SA	204.6092	221.8558	143599.422942	143628.855257	143680.758402	15.69
R800	PR	EA	203.2698	221.2940	143596.148950	143596.148954	143596.148970	15.67
R800	PR	PSO	203.2697	221.2940	143596.148950	143596.148967	143596.149031	15.67
R800	LR	SA	150.0992	198.7629	129745.000868	130061.722669	130464.976894	4.76
R800	LR	EA	172.5945	200.0000	126345.170674	129048.867862	129665.329561	3.95
R800	LR	PSO	200.0000	200.0000	124757.257460	128701.370479	129665.325797	3.67
R800	U	SA	202.5701	190.9705	124148.723017	124165.913930	124204.640343	0.02
R800	U	EA	203.1676	191.7661	124146.985175	124146.985181	124146.985237	0.00
R800	U	PSO	203.1677	191.7664	124146.985175	124146.985182	124146.985216	0.00

Table A.28: Solution results for RC800

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
RC800	PR	SA	210.8576	216.2635	125528.500741	125571.852243	125624.482724	3.34
RC800	PR	EA	212.7191	214.9645	125521.684081	125521.684081	125521.684082	3.29
RC800	PR	PSO	212.7189	214.9645	125521.684081	125521.684095	125521.684147	3.29
RC800	LR	SA	203.0977	210.9984	131813.287388	131837.205524	131888.019708	8.49
RC800	LR	EA	202.1087	211.5063	131812.250702	131812.250891	131812.252561	8.47
RC800	LR	PSO	202.1081	211.5065	131812.250703	131812.250748	131812.251022	8.47
RC800	U	SA	213.2664	211.7665	121518.849072	121544.994086	121582.708161	0.02
RC800	U	EA	212.7780	211.5238	121518.369525	121518.369525	121518.369526	0.00
RC800	U	PSO	212.7781	211.5237	121518.369525	121518.369534	121518.369568	0.00

Table A.29: Solution results for R1000

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
R1000	PR	SA	246.0102	241.8248	197750.217133	197783.877526	197858.155498	2.82
R1000	PR	EA	245.0167	242.6992	197746.767258	197746.767283	197746.767505	2.80
R1000	PR	PSO	245.0167	242.6992	197746.767258	197746.767437	197746.769025	2.80
R1000	LR	SA	248.8401	244.0818	197514.862499	197530.387251	197588.535537	2.69
R1000	LR	EA	250.1162	244.5198	197512.424796	197512.424799	197512.424814	2.68
R1000	LR	PSO	250.1161	244.5199	197512.424797	197512.424810	197512.424839	2.68
R1000	U	SA	248.3364	248.8844	192369.777626	192389.794606	192440.489227	0.01
R1000	U	EA	247.4621	247.4064	192364.285938	192364.285950	192364.286007	0.00
R1000	U	PSO	247.4623	247.4067	192364.285938	192364.285954	192364.286038	0.00

Table A.30: Solution results for RC1000

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
RC1000	PR	SA	337.0140	190.9632	225597.555074	225659.307091	225823.069607	14.45
RC1000	PR	EA	338.2353	192.6926	225592.144136	225592.144145	225592.144226	14.42
RC1000	PR	PSO	338.2354	192.6930	225592.144136	225592.144157	225592.144221	14.42
RC1000	LR	SA	273.3606	235.4182	207341.729687	207384.903640	207494.410223	5.18
RC1000	LR	EA	272.8082	235.8678	207340.765485	207340.765516	207340.765786	5.16
RC1000	LR	PSO	272.8079	235.8678	207340.765485	207340.765493	207340.765533	5.16
RC1000	U	SA	267.8947	242.6387	197167.616542	197227.248665	197322.787867	0.03
RC1000	U	EA	268.0093	241.8427	197166.314828	197166.314830	197166.314837	0.00
RC1000	U	PSO	268.0094	241.8427	197166.314828	197166.314859	197166.315028	0.00

Table A.31: Solution results for u2319

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
u2319	PR	SA	6621.4048	4293.6270	5039431.975201	5040871.065195	5043393.817839	14.57
u2319	PR	EA	6616.6572	4307.2119	5039373.022379	5039373.025059	5039373.048520	14.53
u2319	PR	PSO	6616.6558	4307.2090	5039373.022386	5039373.023690	5039373.032034	14.53
u2319	LR	SA	5781.1431	4281.2759	4594089.958745	4594708.257487	4595925.573494	4.43
u2319	LR	EA	5790.4399	4282.9072	4594048.482312	4594048.482356	4594048.482730	4.41
u2319	LR	PSO	5790.4399	4282.9092	4594048.482314	4594048.482581	4594048.483338	4.41
u2319	U	SA	5942.8789	4408.8320	4400240.904198	4401272.299239	4403091.360358	0.03
u2319	U	EA	5954.6729	4381.2471	4399857.061474	4399857.061511	4399857.061831	0.00
u2319	U	PSO	5954.6738	4381.2490	4399857.061476	4399857.061644	4399857.062915	0.00

Table A.32: Solution results for fnl4461

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
fnl4461	PR	SA	7108.054	7439.100	6972635.3952	6974076.3058	6976375.0031	13.99
fnl4461	PR	EA	7103.300	7434.924	6972442.4372	6972442.4382	6972442.4469	13.96
fnl4461	PR	PSO	7103.300	7434.923	6972442.4372	6972442.4463	6972442.4859	13.96
fnl4461	LR	SA	7362.708	7668.590	7042386.7917	7043671.6135	7046458.7310	15.13
fnl4461	LR	EA	7360.406	7665.550	7042372.2581	7042372.2588	7042372.2640	15.11
fnl4461	LR	PSO	7360.406	7665.550	7042372.2580	7042372.2583	7042372.2595	15.11
fnl4461	U	SA	7375.443	7657.929	6118235.2705	6119888.6883	6122392.3573	0.03
fnl4461	U	EA	7381.071	7658.548	6118196.3481	6118196.3482	6118196.3490	0.00
fnl4461	U	PSO	7381.0701	7658.548	6118196.3481	6118196.3508	6118196.3718	0.00

Table A.33: Solution results for pla7397

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
pla7397	PR	SA	209914.797	279007.313	2402159312.4905	2402460471.4661	2402851997.8214	7.70
pla7397	PR	EA	211135.594	278710.094	2402153411.0584	2402153411.1236	2402153411.5210	7.68
pla7397	PR	PSO	211136.406	278710.406	2402153411.0628	2402153411.6552	2402153413.4557	7.68
pla7397	LR	SA	262762.313	276539.906	2491340823.4622	2491530913.0554	2491759211.4848	11.69
pla7397	LR	EA	262784.906	278156.906	2491316004.7726	2491316010.0631	2491316057.3263	11.68
pla7397	LR	PSO	262783.688	278156.406	2491316004.7792	2491316005.1669	2491316005.8747	11.68
pla7397	U	SA	270926.000	278983.500	2230777473.3308	2230988239.9437	2231482174.5602	0.01
pla7397	U	EA	272463.688	278059.594	2230761223.4886	2230761223.5795	2230761224.2159	0.00
pla7397	U	PSO	272464.000	278059.594	2230761223.4888	2230761225.3240	2230761235.9436	0.00

Table A.34: Solution results for usa13509

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
usa13509	PR	SA	404704.316	879876.875	1633641736.6751	1634456545.8128	1636581830.2728	8.38
usa13509	PR	EA	404127.688	879543.375	1633609539.2599	1633609539.2602	1633609539.2625	8.33
usa13509	PR	PSO	404127.688	879543.375	1633609539.2599	1633609539.4479	1633609541.0152	8.33
usa13509	LR	SA	402383.188	875497.625	1808547271.5773	1809337329.7138	1811579116.0556	19.98
usa13509	LR	EA	402110.813	875633.813	1808542200.9146	1808542200.9341	1808542201.0110	19.93
usa13509	LR	PSO	402111.000	875633.688	1808542200.9156	1808542201.0435	1808542201.3938	19.93
usa13509	U	SA	388922.406	877223.875	1508084318.6475	1509071369.8034	1512810720.5106	0.07
usa13509	U	EA	388922.406	877223.875	1508040777.8456	1508040777.8561	1508040777.9080	0.00
usa13509	U	PSO	388922.500	877224.125	1508040777.8468	1508040777.9556	1508040778.5498	0.00

Table A.35: Solution results for pla33810

Inst.	Pattern	Alg.	x	y	Min. OFV	Avg. OFV	Max. OFV	Avg. %Gap U
pla33810	PR	SA	335262.906	292148.688	7345821677.8468	7347739401.2874	7349455574.3430	6.65
pla33810	PR	EA	336061.594	291743.313	7345795236.9079	7345795237.6735	7345795243.1184	6.62
pla33810	PR	PSO	336061.594	291743.313	7345795236.9079	7345795237.3489	7345795239.1268	6.62
pla33810	LR	SA	337941.406	310350.500	7661962405.5517	7664321066.0807	7669805236.8895	11.25
pla33810	LR	EA	339548.500	308993.188	7661708978.6298	7661708978.6344	7661708978.6573	11.21
pla33810	LR	PSO	339548.500	308993.188	7661708978.6301	7661708981.1509	7661708992.1780	11.21
pla33810	U	SA	337381.188	307836.188	6889595345.8628	6890496084.1618	6893945518.1842	0.01
pla33810	U	EA	337478.906	308525.406	6889564892.4200	6889564892.4491	6889564892.6278	0.00
pla33810	U	PSO	337478.813	308525.406	6889564892.4200	6889564892.7574	6889564893.5646	0.00

APPENDIX B

TABLES OF META-HEURISTICS PERFORMANCES FOR ALL PROBLEM INSTANCES

In this appendix, information about the performance of the algorithms on all instances and their patterns is provided. The information given in Tables B.1 to B.35 contains:

- The instance name and its pattern in the first and second column.
- The applied meta-heuristic algorithm in the 'Alg.' column.
- '%Imp.' that shows how the meta-heuristic algorithms were able to improve their initial solutions. To be more specific, we focus on three values indication different improvements, considering 10 replications of each meta-heuristic:
  – 'Min. %Imp.' showing that on average how much the algorithms could improve (in percent) the best individual of initial population before termination.
  – 'Avg. %Imp.' denoting how much on average the algorithm could improve the initial population to the final population.
  – 'Max. %Imp.' which shows the average percent improvement on the worst solution in the initial population.
- '%DV' that indicates the percent deviation of final solutions from the BSol:
  – 'Min. %DV' shows the percent deviation of the best objective function found in 10 replications from BSol.
  – 'Avg. %DV.' is the average percent deviation of 10 final solutions from BSol.
- 'BSol Hits' showing the number of times the algorithm generated final solution with the same objective function value as BSol running in 10 replications.
- 'CT' which refers to CPU time. CPU time is the computational time the algorithm required to return a solution. The values regarding average computational time of 10 runs is given in 'CT' column.

Table B.1: Performance results for AP25

			%Imp.			%DV		BSol	
Inst.	Pattern	Alg.	Min.	Avg.	Max.	Min.	Avg.	Hits	CT
AP25	BR-o	SA	9.98	N/A	N/A	0.00	0.00	3	1.87
AP25	BR-o	EA	3.96	8.62	15.85	0.00	0.00	6	0.48
AP25	BR-o	PSO	3.87	8.82	16.16	0.00	0.00	3	0.64
AP25	BR-50	SA	6.20	N/A	N/A	0.00	0.00	2	1.90
AP25	BR-50	EA	2.36	6.76	14.57	0.00	0.00	5	0.52
AP25	BR-50	PSO	2.26	7.18	14.56	0.00	0.00	2	0.60
AP25	BR-25	SA	6.67	N/A	N/A	0.00	0.00	7	1.97
AP25	BR-25	EA	1.11	5.13	14.69	0.00	0.00	9	0.46
AP25	BR-25	PSO	1.10	3.97	11.81	0.00	0.00	7	0.57
AP25	h-o	SA	9.53	N/A	N/A	0.00	0.00	4	1.88
AP25	h-o	EA	4.00	8.99	15.54	0.00	0.00	7	0.43
AP25	h-o	PSO	3.99	9.28	16.15	0.00	0.00	6	0.58
AP25	h-50	SA	6.91	N/A	N/A	0.00	0.00	0	1.89
AP25	h-50	EA	2.16	6.47	14.56	0.00	0.00	5	0.49
AP25	h-50	PSO	2.26	6.85	14.56	0.00	0.00	4	0.73
AP25	h-25	SA	3.48	N/A	N/A	0.00	0.00	9	1.96
AP25	h-25	EA	1.14	4.63	14.69	0.00	0.00	8	0.43
AP25	h-25	PSO	1.17	4.44	12.76	0.00	0.00	8	0.67
AP25	l-o	SA	6.40	N/A	N/A	0.00	0.00	6	1.87
AP25	l-o	EA	1.46	5.61	11.66	0.00	0.00	8	0.52
AP25	l-o	PSO	1.67	5.73	12.96	0.00	0.00	3	0.58
AP25	l-50	SA	5.49	N/A	N/A	0.00	0.00	0	1.89
AP25	l-50	EA	2.17	5.89	14.57	0.00	0.00	6	0.47
AP25	l-50	PSO	2.30	5.82	14.56	0.00	0.00	0	0.55
AP25	l-25	SA	7.41	N/A	N/A	0.00	0.00	6	1.96
AP25	l-25	EA	1.14	4.94	14.69	0.00	0.00	4	0.34
AP25	l-25	PSO	1.10	4.42	13.73	0.00	0.00	5	0.56
AP25	O	SA	4.38	N/A	N/A	0.00	0.00	10	1.86
AP25	O	EA	0.17	5.59	14.66	0.00	0.00	10	0.52
AP25	O	PSO	0.29	5.14	14.65	0.00	0.00	10	0.69
AP25	FR-50	SA	4.30	N/A	N/A	0.00	0.00	10	1.87
AP25	FR-50	EA	0.24	3.84	14.68	0.00	0.00	7	0.36
AP25	FR-50	PSO	0.27	3.40	13.47	0.00	0.00	5	0.56
AP25	FR-25	SA	0.67	N/A	N/A	0.00	0.00	8	1.97
AP25	FR-25	EA	0.36	3.60	13.37	0.00	0.00	6	0.38
AP25	FR-25	PSO	0.36	3.03	14.69	0.00	0.00	1	0.48
AP25	U	SA	14.71	N/A	N/A	0.00	0.00	8	0.02
AP25	U	EA	13.99	14.00	14.00	0.76	0.83	0	0.00
AP25	U	PSO	14.66	14.68	14.69	0.01	0.02	0	0.00

Table B.2: Performance results for AP70

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
AP70	O	SA	26.62	N/A	N/A	0.00	0.00	1	12.55
AP70	O	EA	3.61	29.42	45.99	0.00	0.00	10	5.20
AP70	O	PSO	2.22	29.12	46.51	0.00	0.00	9	8.16
AP70	BR-50	SA	23.12	N/A	N/A	0.00	0.00	4	12.60
AP70	BR-50	EA	3.79	27.77	44.51	0.00	0.00	9	5.04
AP70	BR-50	PSO	5.43	28.36	46.92	0.00	0.00	6	7.59
AP70	BR-25	SA	21.68	N/A	N/A	0.00	0.00	2	10.40
AP70	BR-25	EA	5.77	26.82	46.48	0.00	0.00	10	5.15
AP70	BR-25	PSO	3.02	25.20	46.10	0.00	0.00	5	6.02
AP70	h-o	SA	26.05	N/A	N/A	0.00	0.00	1	12.57
AP70	h-o	EA	3.60	29.92	47.01	0.00	0.00	10	6.55
AP70	h-o	PSO	3.67	28.58	45.96	0.00	0.00	9	8.40
AP70	h-50	SA	22.85	N/A	N/A	0.00	0.00	0	12.57
AP70	h-50	EA	5.97	27.41	46.42	0.00	0.00	7	4.82
AP70	h-50	PSO	3.95	27.09	46.31	0.00	0.00	7	7.39
AP70	h-25	SA	24.82	N/A	N/A	0.00	0.00	2	10.41
AP70	h-25	EA	3.83	24.49	42.66	0.00	0.00	9	4.07
AP70	h-25	PSO	3.55	24.76	46.18	0.00	0.00	10	6.24
AP70	l-o	SA	26.77	N/A	N/A	0.00	0.00	1	12.53
AP70	l-o	EA	2.02	28.63	45.72	0.00	0.00	10	5.68
AP70	l-o	PSO	4.54	29.89	46.02	0.00	0.00	5	8.51
AP70	l-50	SA	27.21	N/A	N/A	0.00	0.00	1	12.63
AP70	l-50	EA	3.72	25.42	42.64	0.00	0.00	9	5.65
AP70	l-50	PSO	3.59	25.80	45.40	0.00	0.00	6	6.79
AP70	l-25	SA	27.48	N/A	N/A	0.00	0.00	0	10.40
AP70	l-25	EA	4.27	25.85	44.77	0.00	0.00	9	4.10
AP70	l-25	PSO	5.62	25.73	43.86	0.00	0.00	8	6.24
AP70	FR-o	SA	27.23	N/A	N/A	0.00	0.00	3	12.63
AP70	FR-o	EA	1.91	26.32	44.84	0.00	0.00	10	5.73
AP70	FR-o	PSO	2.53	27.00	47.64	0.00	0.00	6	6.75
AP70	FR-50	SA	22.29	N/A	N/A	0.00	0.00	4	12.64
AP70	FR-50	EA	3.70	25.69	44.90	0.00	0.00	10	5.78
AP70	FR-50	PSO	3.37	26.60	44.85	0.00	0.00	7	7.28
AP70	FR-25	SA	27.80	N/A	N/A	0.00	0.00	4	10.42
AP70	FR-25	EA	3.04	25.24	46.79	0.00	0.00	10	4.65
AP70	FR-25	PSO	3.45	25.01	45.25	0.00	0.00	6	6.11
AP70	U	SA	23.17	N/A	N/A	0.00	0.00	1	0.04
AP70	U	EA	3.84	26.65	43.11	0.00	0.00	2	0.01
AP70	U	PSO	4.45	26.03	42.40	0.00	0.00	1	0.01

Table B.3: Performance results for AP70R10

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
AP70R10	O	SA	23.49	N/A	N/A	0.00	0.00	2	10.74
AP70R10	O	EA	2.47	27.53	46.52	0.00	0.00	10	4.72
AP70R10	O	PSO	3.12	28.09	47.27	0.00	0.00	9	7.30
AP70R10	BR-50	SA	18.77	N/A	N/A	0.00	0.00	2	10.82
AP70R10	BR-50	EA	4.96	25.78	41.20	0.00	0.00	8	4.63
AP70R10	BR-50	PSO	4.33	25.52	43.73	0.00	0.00	5	6.66
AP70R10	BR-25	SA	22.26	N/A	N/A	0.00	0.00	1	8.78
AP70R10	BR-25	EA	2.78	23.68	42.85	0.00	0.00	10	4.01
AP70R10	BR-25	PSO	4.40	23.83	43.37	0.00	0.00	7	4.67
AP70R10	h-o	SA	30.95	N/A	N/A	0.00	0.00	2	10.74
AP70R10	h-o	EA	2.75	27.90	45.51	0.00	0.00	10	6.26
AP70R10	h-o	PSO	3.39	27.56	45.88	0.00	0.00	10	7.00
AP70R10	h-50	SA	25.61	N/A	N/A	0.00	0.00	1	10.80
AP70R10	h-50	EA	3.17	25.75	46.66	0.00	0.00	7	4.89
AP70R10	h-50	PSO	4.94	25.48	44.45	0.00	0.00	4	5.86
AP70R10	h-25	SA	22.08	N/A	N/A	0.00	0.00	0	8.80
AP70R10	h-25	EA	4.09	23.59	43.20	0.00	0.00	10	4.07
AP70R10	h-25	PSO	4.45	23.93	42.77	0.00	0.00	8	5.48
AP70R10	l-o	SA	23.24	N/A	N/A	0.00	0.00	1	10.72
AP70R10	l-o	EA	3.03	27.98	47.56	0.00	0.00	9	4.85
AP70R10	l-o	PSO	3.36	26.25	44.15	0.00	0.00	9	7.18
AP70R10	l-50	SA	21.36	N/A	N/A	0.00	0.00	2	10.80
AP70R10	l-50	EA	3.88	25.12	41.61	0.00	0.00	9	4.90
AP70R10	l-50	PSO	4.73	25.45	39.61	0.00	0.00	7	6.78
AP70R10	l-25	SA	17.68	N/A	N/A	0.00	0.00	1	8.82
AP70R10	l-25	EA	4.74	24.00	40.32	0.00	0.00	7	3.23
AP70R10	l-25	PSO	3.80	23.55	44.62	0.00	0.00	9	5.54
AP70R10	FR-o	SA	14.70	N/A	N/A	0.00	0.00	5	10.81
AP70R10	FR-o	EA	1.59	25.09	43.63	0.00	0.00	9	4.96
AP70R10	FR-o	PSO	2.88	27.18	42.98	0.00	0.00	9	6.96
AP70R10	FR-50	SA	19.11	N/A	N/A	0.00	0.00	0	10.82
AP70R10	FR-50	EA	4.39	25.82	44.89	0.00	0.00	10	5.05
AP70R10	FR-50	PSO	4.47	23.93	38.03	0.00	0.00	6	6.08
AP70R10	FR-25	SA	23.63	N/A	N/A	0.00	0.00	4	8.77
AP70R10	FR-25	EA	4.61	22.77	39.29	0.00	0.00	9	3.93
AP70R10	FR-25	PSO	3.56	23.83	42.42	0.00	0.00	8	4.97

Table B.4: Performance results for AP70R8

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
AP70R8	O	SA	23.28	N/A	N/A	0.00	0.00	1	7.83
AP70R8	O	EA	3.12	26.97	46.32	0.00	0.00	8	3.75
AP70R8	O	PSO	2.53	27.70	45.91	0.00	0.00	7	5.10
AP70R8	BR-50	SA	22.77	N/A	N/A	0.00	0.00	0	7.91
AP70R8	BR-50	EA	2.99	25.26	41.11	0.00	0.00	8	3.32
AP70R8	BR-50	PSO	4.66	26.06	42.22	0.00	0.00	2	4.29
AP70R8	BR-25	SA	21.01	N/A	N/A	0.00	0.00	2	6.17
AP70R8	BR-25	EA	3.36	23.41	41.78	0.00	0.00	10	2.52
AP70R8	BR-25	PSO	3.07	23.51	43.45	0.00	0.00	9	3.66
AP70R8	h-o	SA	19.02	N/A	N/A	0.00	0.00	1	7.85
AP70R8	h-o	EA	3.72	27.93	45.66	0.00	0.00	8	3.57
AP70R8	h-o	PSO	2.59	25.59	43.55	0.00	0.00	3	5.18
AP70R8	h-50	SA	25.93	N/A	N/A	0.00	0.00	0	7.89
AP70R8	h-50	EA	3.34	24.91	43.79	0.00	0.00	10	3.68
AP70R8	h-50	PSO	2.97	24.41	41.03	0.00	0.00	4	4.92
AP70R8	h-25	SA	16.44	N/A	N/A	0.00	0.00	0	6.16
AP70R8	h-25	EA	3.89	24.14	43.81	0.00	0.00	10	3.20
AP70R8	h-25	PSO	3.45	23.46	41.18	0.00	0.00	7	3.68
AP70R8	l-o	SA	23.35	N/A	N/A	0.00	0.00	1	7.86
AP70R8	l-o	EA	3.93	28.42	46.41	0.00	0.00	9	3.49
AP70R8	l-o	PSO	2.27	27.61	45.42	0.00	0.00	8	5.21
AP70R8	l-50	SA	22.83	N/A	N/A	0.00	0.00	0	7.89
AP70R8	l-50	EA	2.47	25.58	41.86	0.00	0.00	9	3.32
AP70R8	l-50	PSO	4.57	25.03	42.44	0.00	0.00	5	4.84
AP70R8	l-25	SA	25.73	N/A	N/A	0.00	0.00	3	6.17
AP70R8	l-25	EA	4.23	24.35	44.03	0.00	0.00	9	3.01
AP70R8	l-25	PSO	4.55	22.97	41.88	0.00	0.00	9	3.82
AP70R8	FR-o	SA	25.04	N/A	N/A	0.00	0.00	2	7.91
AP70R8	FR-o	EA	2.15	26.18	43.78	0.00	0.00	9	3.26
AP70R8	FR-o	PSO	1.77	25.05	45.20	0.00	0.00	9	4.39
AP70R8	FR-50	SA	22.35	N/A	N/A	0.00	0.00	2	7.91
AP70R8	FR-50	EA	2.89	24.93	45.42	0.00	0.00	9	3.33
AP70R8	FR-50	PSO	3.70	25.10	45.53	0.00	0.00	10	4.57
AP70R8	FR-25	SA	16.51	N/A	N/A	0.00	0.00	1	6.18
AP70R8	FR-25	EA	3.21	23.79	45.60	0.00	0.00	6	2.29
AP70R8	FR-25	PSO	2.15	22.91	41.76	0.00	0.00	9	3.93

Table B.5: Performance results for AP70R6

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
AP70R6	O	SA	30.98	N/A	N/A	0.00	0.00	1	4.62
AP70R6	O	EA	3.47	27.81	45.85	0.00	0.00	10	2.21
AP70R6	O	PSO	4.05	28.47	47.28	0.00	0.00	8	2.65
AP70R6	BR-50	SA	28.55	N/A	N/A	0.00	0.00	2	4.60
AP70R6	BR-50	EA	3.00	26.23	44.45	0.00	0.00	10	2.11
AP70R6	BR-50	PSO	3.43	26.21	45.01	0.00	0.00	9	2.89
AP70R6	BR-25	SA	22.64	N/A	N/A	0.00	0.00	0	3.30
AP70R6	BR-25	EA	2.26	25.22	43.90	0.00	0.00	7	1.57
AP70R6	BR-25	PSO	2.72	25.46	43.47	0.00	0.00	1	1.72
AP70R6	h-o	SA	20.21	N/A	N/A	0.00	0.00	2	4.61
AP70R6	h-o	EA	1.01	27.33	44.83	0.00	0.00	7	1.88
AP70R6	h-o	PSO	2.82	28.13	46.65	0.00	0.00	5	2.55
AP70R6	h-50	SA	28.55	N/A	N/A	0.00	0.00	1	4.61
AP70R6	h-50	EA	3.61	26.90	45.44	0.00	0.00	6	1.61
AP70R6	h-50	PSO	3.69	26.45	43.81	0.00	0.00	7	2.88
AP70R6	h-25	SA	18.66	N/A	N/A	0.00	0.00	0	3.29
AP70R6	h-25	EA	2.95	25.50	45.85	0.00	0.00	7	1.46
AP70R6	h-25	PSO	3.16	25.49	41.42	0.00	0.00	4	2.02
AP70R6	l-o	SA	24.34	N/A	N/A	0.00	0.00	0	4.62
AP70R6	l-o	EA	2.46	27.03	45.33	0.00	0.00	9	2.06
AP70R6	l-o	PSO	2.30	27.78	46.17	0.00	0.00	5	2.81
AP70R6	l-50	SA	24.16	N/A	N/A	0.00	0.00	1	4.61
AP70R6	l-50	EA	2.42	26.47	43.73	0.00	0.00	10	1.95
AP70R6	l-50	PSO	3.02	26.67	45.90	0.00	0.00	9	2.74
AP70R6	l-25	SA	19.15	N/A	N/A	0.00	0.00	0	3.30
AP70R6	l-25	EA	2.40	26.00	46.31	0.00	0.00	9	1.58
AP70R6	l-25	PSO	2.30	25.21	43.48	0.00	0.00	4	1.99
AP70R6	FR-o	SA	25.86	N/A	N/A	0.00	0.00	3	4.61
AP70R6	FR-o	EA	2.38	26.97	44.77	0.00	0.00	9	2.08
AP70R6	FR-o	PSO	2.91	27.25	45.42	0.00	0.00	9	2.87
AP70R6	FR-50	SA	26.02	N/A	N/A	0.00	0.00	3	4.59
AP70R6	FR-50	EA	3.22	25.85	43.29	0.00	0.00	10	1.85
AP70R6	FR-50	PSO	2.61	26.21	44.88	0.00	0.00	7	2.62
AP70R6	FR-25	SA	17.03	N/A	N/A	0.00	0.00	4	3.29
AP70R6	FR-25	EA	2.34	24.32	42.30	0.00	0.00	9	1.47
AP70R6	FR-25	PSO	3.42	26.53	46.83	0.00	0.00	9	1.99

Table B.6: Performance results for AP70R4

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
AP70R4	O	SA	22.65	N/A	N/A	0.00	0.00	1	2.40
AP70R4	O	EA	1.31	27.52	46.58	0.00	0.00	9	1.01
AP70R4	O	PSO	2.71	26.74	43.15	0.00	0.00	8	1.22
AP70R4	BR-50	SA	26.61	N/A	N/A	0.00	0.00	2	2.38
AP70R4	BR-50	EA	3.72	27.63	44.82	0.00	0.00	6	0.76
AP70R4	BR-50	PSO	3.61	26.04	43.32	0.00	0.00	3	1.25
AP70R4	BR-25	SA	27.96	N/A	N/A	0.00	0.00	0	2.37
AP70R4	BR-25	EA	4.06	26.75	45.44	0.00	0.00	6	0.96
AP70R4	BR-25	PSO	3.23	26.22	43.73	0.00	0.00	3	1.27
AP70R4	h-o	SA	23.67	N/A	N/A	0.00	0.00	2	2.39
AP70R4	h-o	EA	2.23	27.69	46.62	0.00	0.00	9	0.98
AP70R4	h-o	PSO	4.23	29.12	47.26	0.00	0.00	5	1.24
AP70R4	h-50	SA	22.45	N/A	N/A	0.00	0.00	0	2.37
AP70R4	h-50	EA	2.97	26.56	43.65	0.00	0.00	6	0.82
AP70R4	h-50	PSO	3.26	27.63	45.56	0.00	0.00	3	1.25
AP70R4	h-25	SA	26.83	N/A	N/A	0.00	0.00	1	2.37
AP70R4	h-25	EA	2.32	26.17	40.63	0.00	0.00	8	1.00
AP70R4	h-25	PSO	2.42	26.19	41.36	0.00	0.00	4	1.20
AP70R4	l-o	SA	24.27	N/A	N/A	0.00	0.00	3	2.40
AP70R4	l-o	EA	3.46	27.06	43.92	0.00	0.00	10	0.95
AP70R4	l-o	PSO	2.00	27.23	43.73	0.00	0.00	10	1.40
AP70R4	l-50	SA	24.68	N/A	N/A	0.00	0.00	0	2.36
AP70R4	l-50	EA	1.58	26.60	43.08	0.00	0.00	8	1.13
AP70R4	l-50	PSO	3.72	26.86	43.50	0.00	0.00	7	1.30
AP70R4	l-25	SA	27.93	N/A	N/A	0.00	0.00	0	2.37
AP70R4	l-25	EA	3.84	27.73	46.84	0.00	0.00	8	1.11
AP70R4	l-25	PSO	4.21	26.84	43.28	0.00	0.00	4	1.16
AP70R4	FR-o	SA	33.98	N/A	N/A	0.00	0.00	2	2.40
AP70R4	FR-o	EA	3.50	27.34	46.32	0.00	0.00	10	0.97
AP70R4	FR-o	PSO	3.25	28.03	46.47	0.00	0.00	6	1.22
AP70R4	FR-50	SA	20.52	N/A	N/A	0.00	0.00	4	2.37
AP70R4	FR-50	EA	3.00	25.96	44.58	0.00	0.00	10	0.82
AP70R4	FR-50	PSO	3.67	28.23	48.25	0.00	0.00	10	1.32
AP70R4	FR-25	SA	24.97	N/A	N/A	0.00	0.00	1	2.38
AP70R4	FR-25	EA	2.39	26.83	45.95	0.00	0.00	6	0.81
AP70R4	FR-25	PSO	4.49	27.12	44.17	0.00	0.00	7	1.21

Table B.7: Performance results for AP70R2

			%Imp.			%DV		BSol	
Inst.	Pattern	Alg.	Min.	Avg.	Max.	Min.	Avg.	Hits	CT
AP70R2	O	SA	26.15	N/A	N/A	0.00	0.00	0	1.00
AP70R2	O	EA	5.39	29.78	46.26	0.00	0.00	7	0.29
AP70R2	O	PSO	4.27	28.68	46.08	0.00	0.00	3	0.38
AP70R2	BR-50	SA	30.84	N/A	N/A	0.00	0.00	2	0.98
AP70R2	BR-50	EA	4.10	28.97	46.21	0.00	0.00	9	0.29
AP70R2	BR-50	PSO	1.72	26.72	41.80	0.00	0.00	3	0.36
AP70R2	BR-25	SA	21.09	N/A	N/A	0.00	0.00	0	0.98
AP70R2	BR-25	EA	5.99	28.38	46.55	0.00	0.00	7	0.29
AP70R2	BR-25	PSO	6.71	28.27	44.89	0.00	0.00	2	0.42
AP70R2	h-o	SA	27.41	N/A	N/A	0.00	0.00	0	1.01
AP70R2	h-o	EA	5.62	29.75	47.88	0.00	0.00	7	0.29
AP70R2	h-o	PSO	4.76	29.63	47.27	0.00	0.00	3	0.39
AP70R2	h-50	SA	25.48	N/A	N/A	0.00	0.00	1	0.98
AP70R2	h-50	EA	5.50	28.96	42.96	0.00	0.00	6	0.30
AP70R2	h-50	PSO	4.66	28.17	44.27	0.00	0.00	7	0.41
AP70R2	h-25	SA	31.25	N/A	N/A	0.00	0.00	0	0.98
AP70R2	h-25	EA	1.55	26.30	42.48	0.00	0.00	6	0.29
AP70R2	h-25	PSO	4.43	28.54	46.84	0.00	0.00	3	0.41
AP70R2	l-o	SA	36.14	N/A	N/A	0.00	0.00	0	1.01
AP70R2	l-o	EA	1.67	27.61	45.53	0.00	0.00	6	0.26
AP70R2	l-o	PSO	4.82	28.91	47.22	0.00	0.00	3	0.38
AP70R2	l-50	SA	28.78	N/A	N/A	0.00	0.00	2	0.98
AP70R2	l-50	EA	8.13	29.53	46.28	0.00	0.00	6	0.25
AP70R2	l-50	PSO	3.82	27.84	44.78	0.00	0.00	3	0.35
AP70R2	l-25	SA	20.66	N/A	N/A	0.00	0.00	0	0.99
AP70R2	l-25	EA	1.44	26.17	40.38	0.00	0.00	6	0.28
AP70R2	l-25	PSO	2.24	28.00	45.09	0.00	0.00	1	0.40
AP70R2	FR-o	SA	23.85	N/A	N/A	0.00	0.00	6	1.01
AP70R2	FR-o	EA	2.50	29.05	45.76	0.00	0.00	9	0.27
AP70R2	FR-o	PSO	6.92	29.34	44.80	0.00	0.00	5	0.38
AP70R2	FR-50	SA	21.81	N/A	N/A	0.00	0.00	2	0.98
AP70R2	FR-50	EA	3.43	27.93	45.53	0.00	0.00	9	0.32
AP70R2	FR-50	PSO	5.52	27.92	44.76	0.00	0.00	3	0.36
AP70R2	FR-25	SA	24.24	N/A	N/A	0.00	0.00	2	0.99
AP70R2	FR-25	EA	2.39	27.98	44.89	0.00	0.00	8	0.26
AP70R2	FR-25	PSO	4.09	27.23	42.05	0.00	0.00	8	0.46

Table B.8: Performance results for BC13

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
BC13	O	SA	21.80	N/A	N/A	0.00	0.00	5	0.53
BC13	O	EA	2.36	22.74	43.26	0.00	0.01	2	0.06
BC13	O	PSO	2.89	24.52	44.94	0.00	0.00	1	0.08
BC13	BR-50	SA	17.91	N/A	N/A	0.00	0.00	8	0.53
BC13	BR-50	EA	4.39	18.56	32.87	0.00	0.00	1	0.07
BC13	BR-50	PSO	4.84	20.81	36.86	0.00	0.00	1	0.08
BC13	BR-25	SA	11.97	N/A	N/A	0.00	0.00	5	0.54
BC13	BR-25	EA	4.69	17.37	33.06	0.00	0.00	2	0.06
BC13	BR-25	PSO	5.55	17.25	30.59	0.00	0.00	2	0.09
BC13	h-o	SA	20.99	N/A	N/A	0.00	0.00	5	0.54
BC13	h-o	EA	2.03	22.28	42.54	0.00	0.00	2	0.07
BC13	h-o	PSO	4.52	25.04	44.14	0.00	0.00	3	0.09
BC13	h-50	SA	15.71	N/A	N/A	0.00	0.00	8	0.53
BC13	h-50	EA	4.74	18.38	33.11	0.00	0.00	3	0.07
BC13	h-50	PSO	4.73	18.75	35.65	0.00	0.00	0	0.08
BC13	h-25	SA	17.36	N/A	N/A	0.00	0.00	7	0.55
BC13	h-25	EA	5.09	17.86	33.23	0.00	0.00	3	0.07
BC13	h-25	PSO	4.85	18.38	34.37	0.00	0.00	4	0.09
BC13	l-o	SA	22.48	N/A	N/A	0.00	0.00	4	0.53
BC13	l-o	EA	2.09	22.16	39.08	0.00	0.00	0	0.07
BC13	l-o	PSO	2.31	22.45	38.69	0.00	0.00	2	0.09
BC13	l-50	SA	18.36	N/A	N/A	0.00	0.00	6	0.53
BC13	l-50	EA	3.69	19.92	36.15	0.00	0.00	0	0.05
BC13	l-50	PSO	4.49	19.66	35.50	0.00	0.00	3	0.09
BC13	l-25	SA	21.94	N/A	N/A	0.00	0.00	7	0.54
BC13	l-25	EA	4.68	17.09	34.30	0.00	0.00	2	0.07
BC13	l-25	PSO	4.69	16.04	30.42	0.00	0.00	3	0.08
BC13	FR-o	SA	22.57	N/A	N/A	0.00	0.00	8	0.53
BC13	FR-o	EA	4.23	22.39	38.78	0.00	0.00	2	0.07
BC13	FR-o	PSO	4.48	22.21	40.18	0.00	0.00	2	0.09
BC13	FR-50	SA	14.87	N/A	N/A	0.00	0.00	6	0.53
BC13	FR-50	EA	2.64	15.64	29.25	0.00	0.01	1	0.06
BC13	FR-50	PSO	2.83	16.78	31.61	0.00	0.00	3	0.09
BC13	FR-25	SA	14.15	N/A	N/A	0.00	0.00	10	0.54
BC13	FR-25	EA	3.88	16.05	29.52	0.00	0.00	3	0.06
BC13	FR-25	PSO	3.94	15.48	28.33	0.00	0.00	6	0.10
BC13	U	SA	22.43	N/A	N/A	0.00	0.00	7	0.02
BC13	U	EA	3.09	5.75	8.26	0.23	0.31	0	0.00
BC13	U	PSO	8.55	17.87	25.47	0.00	0.01	0	0.00

Table B.9: Performance results for D26

			%Imp.			%DV		BSol	
Inst.	Pattern	Alg.	Min.	Avg.	Max.	Min.	Avg.	Hits	CT
D26	O	SA	14.79	N/A	N/A	0.00	0.00	5	2.25
D26	O	EA	7.13	14.18	23.95	0.00	0.00	8	0.59
D26	O	PSO	6.81	13.56	23.43	0.00	0.00	7	0.70
D26	BR-50	SA	13.52	N/A	N/A	0.00	0.00	5	2.27
D26	BR-50	EA	8.08	14.41	22.97	0.00	0.00	8	0.57
D26	BR-50	PSO	7.95	14.11	26.05	0.00	0.00	8	0.72
D26	BR-25	SA	17.41	N/A	N/A	0.00	0.00	6	2.28
D26	BR-25	EA	8.89	14.09	22.94	0.00	0.00	7	0.54
D26	BR-25	PSO	8.82	14.28	26.22	0.00	0.00	4	0.74
D26	h-o	SA	12.79	N/A	N/A	0.00	0.00	7	2.26
D26	h-o	EA	6.53	13.83	23.85	0.00	0.00	8	0.60
D26	h-o	PSO	6.17	14.33	26.06	0.00	0.00	7	0.71
D26	h-50	SA	16.30	N/A	N/A	0.00	0.00	9	2.26
D26	h-50	EA	8.20	14.70	25.89	0.00	0.00	7	0.50
D26	h-50	PSO	7.98	14.49	25.79	0.00	0.00	6	0.69
D26	h-25	SA	14.95	N/A	N/A	0.00	0.00	2	2.26
D26	h-25	EA	8.51	13.68	22.62	0.00	0.00	5	0.50
D26	h-25	PSO	8.44	14.19	24.99	0.00	0.00	3	0.74
D26	l-o	SA	14.53	N/A	N/A	0.00	0.00	4	2.26
D26	l-o	EA	6.69	13.91	23.31	0.00	0.00	9	0.58
D26	l-o	PSO	6.14	13.92	22.96	0.00	0.00	7	0.81
D26	l-50	SA	15.86	N/A	N/A	0.00	0.00	8	2.27
D26	l-50	EA	7.65	14.16	24.38	0.00	0.00	7	0.52
D26	l-50	PSO	8.16	14.09	23.76	0.00	0.00	6	0.81
D26	l-25	SA	14.59	N/A	N/A	0.00	0.00	6	2.27
D26	l-25	EA	8.74	14.15	23.67	0.00	0.00	8	0.53
D26	l-25	PSO	8.44	13.92	24.67	0.00	0.00	3	0.72
D26	FR-o	SA	13.24	N/A	N/A	0.00	0.00	6	2.26
D26	FR-o	EA	7.77	14.67	25.75	0.00	0.00	8	0.58
D26	FR-o	PSO	6.96	14.67	25.30	0.00	0.00	6	0.73
D26	FR-50	SA	14.66	N/A	N/A	0.00	0.00	9	2.26
D26	FR-50	EA	8.21	14.01	25.10	0.00	0.00	9	0.61
D26	FR-50	PSO	8.59	14.62	25.23	0.00	0.00	9	0.76
D26	FR-25	SA	14.00	N/A	N/A	0.00	0.00	7	2.26
D26	FR-25	EA	8.67	13.95	24.35	0.00	0.00	8	0.50
D26	FR-25	PSO	8.60	14.07	25.33	0.00	0.00	6	0.69
D26	U	SA	16.65	N/A	N/A	0.00	0.00	9	0.02
D26	U	EA	11.59	19.18	26.54	0.00	0.01	0	0.00
D26	U	PSO	13.37	20.46	27.21	0.00	0.00	0	0.00

Table B.10: Performance results for KC5c16 and KC5U

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5c16	O	SA	4.41	N/A	N/A	0.00	0.00	5	1.20
KC5c16	O	EA	0.78	6.21	25.60	0.00	0.00	5	0.28
KC5c16	O	PSO	0.53	5.50	22.61	0.00	0.00	5	0.36
KC5c16	BR-50	SA	8.17	N/A	N/A	0.00	0.00	0	1.23
KC5c16	BR-50	EA	0.37	7.30	29.07	0.00	0.00	2	0.21
KC5c16	BR-50	PSO	0.38	6.22	26.42	0.00	0.00	4	0.42
KC5c16	BR-25	SA	6.99	N/A	N/A	0.00	0.00	1	1.28
KC5c16	BR-25	EA	0.21	6.31	28.50	0.00	0.00	6	0.28
KC5c16	BR-25	PSO	0.20	5.82	25.51	0.00	0.00	6	0.41
KC5c16	h-o	SA	3.09	N/A	N/A	0.00	0.00	8	1.19
KC5c16	h-o	EA	0.68	5.86	23.61	0.00	0.00	3	0.24
KC5c16	h-o	PSO	0.65	5.66	22.42	0.00	0.00	8	0.39
KC5c16	h-50	SA	4.37	N/A	N/A	0.00	0.00	1	1.24
KC5c16	h-50	EA	0.40	7.86	29.49	0.00	0.00	3	0.23
KC5c16	h-50	PSO	0.35	5.82	23.36	0.00	0.00	5	0.39
KC5c16	h-25	SA	5.39	N/A	N/A	0.00	0.00	1	1.26
KC5c16	h-25	EA	0.21	5.81	25.03	0.00	0.00	7	0.30
KC5c16	h-25	PSO	0.29	6.81	27.07	0.00	0.00	3	0.39
KC5c16	l-o	SA	4.57	N/A	N/A	0.00	0.00	3	1.20
KC5c16	l-o	EA	0.45	5.73	22.75	0.00	0.00	8	0.31
KC5c16	l-o	PSO	0.50	6.58	25.19	0.00	0.00	4	0.37
KC5c16	l-50	SA	5.46	N/A	N/A	0.00	0.00	0	1.24
KC5c16	l-50	EA	0.32	6.03	23.36	0.00	0.00	6	0.26
KC5c16	l-50	PSO	0.26	5.45	23.66	0.00	0.00	4	0.38
KC5c16	l-25	SA	1.16	N/A	N/A	0.00	0.00	2	1.27
KC5c16	l-25	EA	0.26	6.23	26.01	0.00	0.00	6	0.32
KC5c16	l-25	PSO	0.21	6.16	26.22	0.00	0.00	3	0.37
KC5c16	FR-o	SA	4.46	N/A	N/A	0.00	0.00	10	1.13
KC5c16	FR-o	EA	0.07	6.42	26.31	0.00	0.00	9	0.22
KC5c16	FR-o	PSO	0.08	7.46	28.58	0.00	0.00	9	0.33
KC5c16	FR-50	SA	0.84	N/A	N/A	0.00	0.00	10	1.17
KC5c16	FR-50	EA	0.03	5.24	27.73	0.00	0.00	9	0.23
KC5c16	FR-50	PSO	0.02	6.34	28.13	0.00	0.00	7	0.33
KC5c16	FR-25	SA	3.26	N/A	N/A	0.00	0.00	3	1.22
KC5c16	FR-25	EA	0.01	7.05	28.18	0.00	0.00	6	0.22
KC5c16	FR-25	PSO	0.01	6.16	27.75	0.00	0.00	2	0.31
KC5	U	SA	17.55	N/A	N/A	0.00	0.00	4	0.02
KC5	U	EA	11.77	22.22	31.27	0.00	0.03	0	0.00
KC5	U	PSO	8.04	19.43	31.29	0.00	0.01	0	0.00

Table B.11: Performance results for KC5c32

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5c32	O	SA	2.23	N/A	N/A	0.00	0.00	4	3.93
KC5c32	O	EA	0.39	4.74	25.95	0.00	0.00	9	1.47
KC5c32	O	PSO	0.36	4.73	25.32	0.00	0.00	9	2.39
KC5c32	BR-50	SA	0.83	N/A	N/A	0.00	0.00	1	4.04
KC5c32	BR-50	EA	0.28	3.55	22.12	0.00	0.00	9	1.44
KC5c32	BR-50	PSO	0.33	3.95	25.65	0.00	0.00	8	2.33
KC5c32	BR-25	SA	3.68	N/A	N/A	0.00	0.00	3	4.14
KC5c32	BR-25	EA	0.17	2.99	22.24	0.00	0.00	9	1.45
KC5c32	BR-25	PSO	0.18	3.24	25.44	0.00	0.00	8	2.41
KC5c32	h-o	SA	3.64	N/A	N/A	0.00	0.00	1	3.93
KC5c32	h-o	EA	0.41	3.51	20.97	0.00	0.00	8	1.43
KC5c32	h-o	PSO	0.46	4.08	22.43	0.00	0.00	9	2.57
KC5c32	h-50	SA	2.06	N/A	N/A	0.00	0.00	1	4.05
KC5c32	h-50	EA	0.26	3.96	26.31	0.00	0.00	6	1.39
KC5c32	h-50	PSO	0.25	4.07	26.43	0.00	0.00	8	2.40
KC5c32	h-25	SA	3.43	N/A	N/A	0.00	0.00	0	4.18
KC5c32	h-25	EA	0.16	3.62	25.23	0.00	0.00	8	1.42
KC5c32	h-25	PSO	0.20	3.83	27.42	0.00	0.00	8	2.50
KC5c32	l-o	SA	5.77	N/A	N/A	0.00	0.00	5	3.92
KC5c32	l-o	EA	0.42	4.17	22.23	0.00	0.00	7	1.75
KC5c32	l-o	PSO	0.33	3.60	20.84	0.00	0.00	7	2.21
KC5c32	l-50	SA	4.55	N/A	N/A	0.00	0.00	0	4.05
KC5c32	l-50	EA	0.25	3.30	23.56	0.00	0.00	10	1.78
KC5c32	l-50	PSO	0.24	4.40	26.57	0.00	0.00	7	2.35
KC5c32	l-25	SA	0.59	N/A	N/A	0.00	0.00	2	4.17
KC5c32	l-25	EA	0.19	3.83	26.30	0.00	0.00	8	1.57
KC5c32	l-25	PSO	0.19	3.00	22.57	0.00	0.00	7	2.51
KC5c32	FR-o	SA	2.39	N/A	N/A	0.00	0.00	10	3.66
KC5c32	FR-o	EA	0.02	4.13	25.47	0.00	0.00	10	1.70
KC5c32	FR-o	PSO	0.01	3.45	25.70	0.00	0.00	10	2.05
KC5c32	FR-50	SA	0.33	N/A	N/A	0.00	0.00	10	3.77
KC5c32	FR-50	EA	0.01	4.11	27.37	0.00	0.00	10	1.37
KC5c32	FR-50	PSO	0.01	2.82	23.11	0.00	0.00	10	2.10
KC5c32	FR-25	SA	1.42	N/A	N/A	0.00	0.00	5	3.98
KC5c32	FR-25	EA	0.00	2.30	21.84	0.00	0.00	8	1.14
KC5c32	FR-25	PSO	0.00	3.78	28.18	0.00	0.00	5	1.80

Table B.12: Performance results for KC5c64

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5c64	O	SA	4.16	N/A	N/A	0.00	0.00	3	14.00
KC5c64	O	EA	0.34	3.34	19.50	0.00	0.00	8	4.92
KC5c64	O	PSO	0.34	3.60	19.55	0.00	0.00	9	8.77
KC5c64	BR-50	SA	1.12	N/A	N/A	0.00	0.00	2	14.38
KC5c64	BR-50	EA	0.28	2.02	14.97	0.00	0.00	10	6.42
KC5c64	BR-50	PSO	0.27	2.24	19.10	0.00	0.00	8	9.11
KC5c64	BR-25	SA	0.59	N/A	N/A	0.00	0.00	2	14.71
KC5c64	BR-25	EA	0.16	1.80	15.07	0.00	0.00	6	6.58
KC5c64	BR-25	PSO	0.18	2.54	20.36	0.00	0.00	9	9.18
KC5c64	h-o	SA	2.23	N/A	N/A	0.00	0.00	2	13.97
KC5c64	h-o	EA	0.39	3.19	20.41	0.00	0.00	10	6.29
KC5c64	h-o	PSO	0.40	2.95	16.48	0.00	0.00	9	9.18
KC5c64	h-50	SA	1.01	N/A	N/A	0.00	0.00	2	14.58
KC5c64	h-50	EA	0.30	2.92	22.46	0.00	0.00	9	5.71
KC5c64	h-50	PSO	0.30	2.60	17.17	0.00	0.00	9	8.84
KC5c64	h-25	SA	1.20	N/A	N/A	0.00	0.00	0	14.90
KC5c64	h-25	EA	0.22	2.15	19.19	0.00	0.00	9	5.85
KC5c64	h-25	PSO	0.16	2.05	19.03	0.00	0.00	9	8.80
KC5c64	l-o	SA	1.63	N/A	N/A	0.00	0.00	3	13.94
KC5c64	l-o	EA	0.43	3.06	17.63	0.00	0.00	7	5.42
KC5c64	l-o	PSO	0.44	3.10	17.86	0.00	0.00	10	9.16
KC5c64	l-50	SA	1.08	N/A	N/A	0.00	0.00	0	14.51
KC5c64	l-50	EA	0.36	2.38	16.14	0.00	0.00	10	6.46
KC5c64	l-50	PSO	0.26	2.06	14.39	0.00	0.00	10	8.81
KC5c64	l-25	SA	3.64	N/A	N/A	0.00	0.00	1	14.85
KC5c64	l-25	EA	0.16	1.44	12.62	0.00	0.00	10	6.65
KC5c64	l-25	PSO	0.17	3.31	26.65	0.00	0.00	6	8.84
KC5c64	FR-o	SA	6.31	N/A	N/A	0.00	0.00	2	12.97
KC5c64	FR-o	EA	0.02	2.00	15.62	0.00	0.00	10	5.19
KC5c64	FR-o	PSO	0.02	3.12	22.76	0.00	0.00	10	7.55
KC5c64	FR-50	SA	0.50	N/A	N/A	0.00	0.00	10	13.45
KC5c64	FR-50	EA	0.02	2.03	20.12	0.00	0.00	10	5.00
KC5c64	FR-50	PSO	0.01	1.72	19.69	0.00	0.00	9	7.30
KC5c64	FR-25	SA	0.85	N/A	N/A	0.00	0.00	2	14.12
KC5c64	FR-25	EA	0.00	2.16	22.62	0.00	0.00	8	5.60
KC5c64	FR-25	PSO	0.01	1.57	18.08	0.00	0.00	6	7.21

Table B.13: Performance results for KC5c128

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5c128	O	SA	1.73	N/A	N/A	0.00	0.00	1	52.85
KC5c128	O	EA	0.44	2.48	10.80	0.00	0.00	8	22.01
KC5c128	O	PSO	0.46	2.67	14.58	0.00	0.00	8	32.83
KC5c128	BR-50	SA	0.92	N/A	N/A	0.00	0.00	0	54.77
KC5c128	BR-50	EA	0.27	2.19	14.67	0.00	0.00	9	23.82
KC5c128	BR-50	PSO	0.26	1.87	12.48	0.00	0.00	9	32.85
KC5c128	BR-25	SA	0.59	N/A	N/A	0.00	0.00	1	55.81
KC5c128	BR-25	EA	0.21	1.17	9.62	0.00	0.00	5	15.62
KC5c128	BR-25	PSO	0.19	1.17	9.19	0.00	0.00	8	31.11
KC5c128	h-o	SA	5.47	N/A	N/A	0.00	0.00	1	52.79
KC5c128	h-o	EA	0.33	2.90	18.30	0.00	0.00	9	23.25
KC5c128	h-o	PSO	0.37	2.59	13.33	0.00	0.00	10	34.15
KC5c128	h-50	SA	2.90	N/A	N/A	0.00	0.00	0	54.21
KC5c128	h-50	EA	0.26	1.82	10.67	0.00	0.00	8	22.87
KC5c128	h-50	PSO	0.26	1.50	9.38	0.00	0.00	9	38.13
KC5c128	h-25	SA	1.81	N/A	N/A	0.00	0.00	0	55.59
KC5c128	h-25	EA	0.15	1.61	16.32	0.00	0.00	9	22.38
KC5c128	h-25	PSO	0.16	1.51	15.27	0.00	0.00	10	33.42
KC5c128	l-o	SA	1.78	N/A	N/A	0.00	0.00	1	52.80
KC5c128	l-o	EA	0.45	2.87	15.15	0.00	0.00	9	26.39
KC5c128	l-o	PSO	0.34	2.56	12.56	0.00	0.00	9	33.32
KC5c128	l-50	SA	0.84	N/A	N/A	0.00	0.00	0	54.25
KC5c128	l-50	EA	0.29	1.82	11.61	0.00	0.00	7	20.02
KC5c128	l-50	PSO	0.31	1.46	7.70	0.00	0.00	7	33.59
KC5c128	l-25	SA	0.49	N/A	N/A	0.00	0.00	1	55.47
KC5c128	l-25	EA	0.19	1.97	19.01	0.00	0.00	10	23.75
KC5c128	l-25	PSO	0.16	1.62	15.50	0.00	0.00	9	33.63
KC5c128	FR-o	SA	0.97	N/A	N/A	0.00	0.00	2	48.12
KC5c128	FR-o	EA	0.01	1.98	18.40	0.00	0.00	10	18.95
KC5c128	FR-o	PSO	0.02	1.54	11.75	0.00	0.00	9	27.68
KC5c128	FR-50	SA	0.37	N/A	N/A	0.00	0.00	10	50.39
KC5c128	FR-50	EA	0.00	0.88	8.28	0.00	0.00	10	15.04
KC5c128	FR-50	PSO	0.01	0.54	4.43	0.00	0.00	10	26.27
KC5c128	FR-25	SA	1.40	N/A	N/A	0.00	0.00	4	53.10
KC5c128	FR-25	EA	0.01	1.01	12.72	0.00	0.00	8	16.25
KC5c128	FR-25	PSO	0.01	1.84	19.83	0.00	0.00	7	27.86

Table B.14: Performance results for KC5c256

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5c256	O	SA	1.82	N/A	N/A	0.00	0.00	2	203.04
KC5c256	O	EA	0.37	1.83	5.61	0.00	0.00	9	90.33
KC5c256	O	PSO	0.33	2.13	9.08	0.00	0.00	8	119.74
KC5c256	BR-50	SA	0.99	N/A	N/A	0.00	0.00	0	210.86
KC5c256	BR-50	EA	0.33	1.16	4.27	0.00	0.00	10	89.03
KC5c256	BR-50	PSO	0.26	1.61	1N/A	0.00	0.00	10	144.59
KC5c256	BR-25	SA	0.57	N/A	N/A	0.00	0.00	1	216.53
KC5c256	BR-25	EA	0.17	1.29	10.19	0.00	0.00	9	90.86
KC5c256	BR-25	PSO	0.19	1.12	10.19	0.00	0.00	8	128.74
KC5c256	h-o	SA	4.06	N/A	N/A	0.00	0.00	3	204.31
KC5c256	h-o	EA	0.46	2.10	9.41	0.00	0.00	9	92.64
KC5c256	h-o	PSO	0.38	2.41	11.38	0.00	0.00	10	124.75
KC5c256	h-50	SA	1.27	N/A	N/A	0.00	0.00	5	208.86
KC5c256	h-50	EA	0.25	1.47	8.18	0.00	0.00	9	72.99
KC5c256	h-50	PSO	0.31	1.17	3.76	0.00	0.00	8	118.50
KC5c256	h-25	SA	0.48	N/A	N/A	0.00	0.00	2	214.43
KC5c256	h-25	EA	0.19	0.93	6.74	0.00	0.00	9	82.74
KC5c256	h-25	PSO	0.17	1.20	8.74	0.00	0.00	8	120.18
KC5c256	l-o	SA	1.42	N/A	N/A	0.00	0.00	2	208.37
KC5c256	l-o	EA	0.44	1.66	5.72	0.00	0.00	10	93.40
KC5c256	l-o	PSO	0.39	2.10	12.18	0.00	0.00	9	133.83
KC5c256	l-50	SA	0.99	N/A	N/A	0.00	0.00	6	209.08
KC5c256	l-50	EA	0.25	1.38	7.24	0.00	0.00	9	67.52
KC5c256	l-50	PSO	0.30	2.08	13.15	0.00	0.00	9	126.01
KC5c256	l-25	SA	0.57	N/A	N/A	0.00	0.00	3	214.96
KC5c256	l-25	EA	0.17	1.27	11.62	0.00	0.00	9	78.88
KC5c256	l-25	PSO	0.18	0.94	6.75	0.00	0.00	10	139.72
KC5c256	FR-o	SA	0.90	N/A	N/A	0.00	0.00	4	183.75
KC5c256	FR-o	EA	0.02	1.16	5.82	0.00	0.00	10	66.74
KC5c256	FR-o	PSO	0.01	1.26	7.78	0.00	0.00	10	105.96
KC5c256	FR-50	SA	0.48	N/A	N/A	0.00	0.00	10	195.33
KC5c256	FR-50	EA	0.01	0.89	6.84	0.00	0.00	10	70.28
KC5c256	FR-50	PSO	0.01	0.83	7.45	0.00	0.00	10	116.89
KC5c256	FR-25	SA	0.19	N/A	N/A	0.00	0.00	4	206.80
KC5c256	FR-25	EA	0.00	0.85	8.22	0.00	0.00	6	56.88
KC5c256	FR-25	PSO	0.00	0.36	3.47	0.00	0.00	8	103.90

Table B.15: Performance results for KC5c512

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5c512	O	SA	1.45	N/A	N/A	0.00	0.00	2	801.96
KC5c512	O	EA	0.43	2.20	8.74	0.00	0.00	9	368.05
KC5c512	O	PSO	0.42	1.82	4.34	0.00	0.00	10	580.15
KC5c512	BR-50	SA	1.06	N/A	N/A	0.00	0.00	3	847.22
KC5c512	BR-50	EA	0.26	0.99	1.62	0.00	0.00	7	319.76
KC5c512	BR-50	PSO	0.25	1.00	2.09	0.00	0.00	7	515.19
KC5c512	BR-25	SA	0.59	N/A	N/A	0.00	0.00	2	862.75
KC5c512	BR-25	EA	0.19	1.22	10.61	0.00	0.00	8	258.19
KC5c512	BR-25	PSO	0.16	0.92	6.53	0.00	0.00	9	513.13
KC5c512	h-o	SA	1.56	N/A	N/A	0.00	0.00	0	811.07
KC5c512	h-o	EA	0.45	1.90	5.52	0.00	0.00	9	316.93
KC5c512	h-o	PSO	0.40	2.00	6.68	0.00	0.00	9	531.77
KC5c512	h-50	SA	1.30	N/A	N/A	0.00	0.00	1	821.35
KC5c512	h-50	EA	0.27	1.12	3.43	0.00	0.00	9	357.94
KC5c512	h-50	PSO	0.25	1.13	3.79	0.00	0.00	7	515.28
KC5c512	h-25	SA	0.64	N/A	N/A	0.00	0.00	3	842.34
KC5c512	h-25	EA	0.20	0.78	3.88	0.00	0.00	8	288.42
KC5c512	h-25	PSO	0.17	1.02	7.22	0.00	0.00	8	487.22
KC5c512	l-o	SA	1.24	N/A	N/A	0.00	0.00	1	767.62
KC5c512	l-o	EA	0.24	1.37	6.79	0.00	0.00	8	359.93
KC5c512	l-o	PSO	0.15	1.24	6.00	0.00	0.00	10	493.07
KC5c512	l-50	SA	0.67	N/A	N/A	0.00	0.00	2	816.75
KC5c512	l-50	EA	0.25	0.69	1.17	0.00	0.00	7	269.46
KC5c512	l-50	PSO	0.27	0.79	2.35	0.00	0.00	7	457.34
KC5c512	l-25	SA	0.45	N/A	N/A	0.00	0.00	1	843.97
KC5c512	l-25	EA	0.18	0.58	1.43	0.00	0.00	9	285.27
KC5c512	l-25	PSO	0.16	0.54	0.84	0.00	0.00	9	501.72
KC5c512	FR-o	SA	0.91	N/A	N/A	0.03	0.03	0	720.48
KC5c512	FR-o	EA	0.08	0.92	2.94	0.00	0.02	1	237.95
KC5c512	FR-o	PSO	0.01	0.91	2.89	0.03	0.03	0	411.49
KC5c512	FR-50	SA	0.41	N/A	N/A	0.00	0.00	0	762.15
KC5c512	FR-50	EA	0.01	0.52	3.01	0.00	0.00	8	254.68
KC5c512	FR-50	PSO	0.01	0.52	3.58	0.00	0.00	4	399.58
KC5c512	FR-25	SA	0.13	N/A	N/A	0.00	0.00	5	807.46
KC5c512	FR-25	EA	0.00	0.15	0.32	0.00	0.00	6	187.75
KC5c512	FR-25	PSO	0.01	0.30	2.63	0.00	0.00	6	408.54

Table B.16: Performance results for KC5i16

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5i16	O	SA	2.65	N/A	N/A	0.00	0.00	4	1.21
KC5i16	O	EA	0.50	6.66	27.64	0.00	0.00	7	0.30
KC5i16	O	PSO	0.44	6.90	24.97	0.00	0.00	8	0.38
KC5i16	BR-50	SA	3.25	N/A	N/A	0.00	0.00	1	1.23
KC5i16	BR-50	EA	0.36	6.33	26.32	0.00	0.00	7	0.28
KC5i16	BR-50	PSO	0.32	5.78	25.66	0.00	0.00	6	0.38
KC5i16	BR-25	SA	4.86	N/A	N/A	0.00	0.00	0	1.26
KC5i16	BR-25	EA	0.22	7.17	29.28	0.00	0.00	4	0.26
KC5i16	BR-25	PSO	0.26	7.06	29.70	0.00	0.00	1	0.38
KC5i16	h-o	SA	4.22	N/A	N/A	0.00	0.00	6	1.19
KC5i16	h-o	EA	0.72	6.09	24.57	0.00	0.00	7	0.28
KC5i16	h-o	PSO	0.46	5.89	24.06	0.00	0.00	7	0.35
KC5i16	h-50	SA	9.15	N/A	N/A	0.00	0.00	3	1.23
KC5i16	h-50	EA	0.31	6.27	24.17	0.00	0.00	6	0.27
KC5i16	h-50	PSO	0.36	6.93	26.08	0.00	0.00	8	0.42
KC5i16	h-25	SA	5.89	N/A	N/A	0.00	0.00	0	1.27
KC5i16	h-25	EA	0.26	5.66	24.85	0.00	0.00	5	0.29
KC5i16	h-25	PSO	0.19	6.01	27.72	0.00	0.00	1	0.38
KC5i16	l-o	SA	6.17	N/A	N/A	0.00	0.00	2	1.19
KC5i16	l-o	EA	0.44	6.62	26.55	0.00	0.00	9	0.25
KC5i16	l-o	PSO	0.61	6.93	26.05	0.00	0.00	8	0.42
KC5i16	l-50	SA	3.18	N/A	N/A	0.00	0.00	2	1.24
KC5i16	l-50	EA	0.39	5.95	27.09	0.00	0.00	6	0.24
KC5i16	l-50	PSO	0.35	6.75	29.07	0.00	0.00	4	0.37
KC5i16	l-25	SA	3.06	N/A	N/A	0.00	0.00	0	1.27
KC5i16	l-25	EA	0.17	6.20	28.15	0.00	0.00	3	0.22
KC5i16	l-25	PSO	0.25	5.63	24.25	0.00	0.00	0	0.36
KC5i16	FR-o	SA	4.95	N/A	N/A	0.00	0.00	4	1.14
KC5i16	FR-o	EA	0.12	5.80	24.94	0.00	0.00	4	0.20
KC5i16	FR-o	PSO	0.16	4.78	21.87	0.00	0.00	7	0.31
KC5i16	FR-50	SA	3.99	N/A	N/A	0.00	0.00	9	1.17
KC5i16	FR-50	EA	0.01	6.24	26.87	0.00	0.00	8	0.21
KC5i16	FR-50	PSO	0.02	5.27	24.53	0.00	0.00	6	0.31
KC5i16	FR-25	SA	2.17	N/A	N/A	0.00	0.00	1	1.22
KC5i16	FR-25	EA	0.02	6.58	27.34	0.00	0.00	7	0.24
KC5i16	FR-25	PSO	0.01	4.61	24.70	0.00	0.00	4	0.35

Table B.17: Performance results for KC5i32

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5i32	O	SA	1.27	N/A	N/A	0.00	0.00	3	3.97
KC5i32	O	EA	0.42	3.91	19.95	0.00	0.00	9	1.53
KC5i32	O	PSO	0.33	4.01	22.27	0.00	0.00	9	2.39
KC5i32	BR-50	SA	3.24	N/A	N/A	0.00	0.00	2	4.04
KC5i32	BR-50	EA	0.25	3.29	23.15	0.00	0.00	10	1.59
KC5i32	BR-50	PSO	0.30	3.70	23.36	0.00	0.00	9	2.21
KC5i32	BR-25	SA	7.32	N/A	N/A	0.00	0.00	0	4.14
KC5i32	BR-25	EA	0.16	3.55	27.72	0.00	0.00	7	1.54
KC5i32	BR-25	PSO	0.15	3.17	23.76	0.00	0.00	6	2.30
KC5i32	h-o	SA	1.96	N/A	N/A	0.00	0.00	2	3.92
KC5i32	h-o	EA	0.38	4.12	22.90	0.00	0.00	8	1.73
KC5i32	h-o	PSO	0.43	4.14	24.85	0.00	0.00	9	2.66
KC5i32	h-50	SA	1.72	N/A	N/A	0.00	0.00	0	4.07
KC5i32	h-50	EA	0.28	3.32	25.06	0.00	0.00	6	1.24
KC5i32	h-50	PSO	0.26	3.62	24.81	0.00	0.00	10	2.37
KC5i32	h-25	SA	0.49	N/A	N/A	0.00	0.00	1	4.18
KC5i32	h-25	EA	0.18	2.84	23.00	0.00	0.00	7	1.55
KC5i32	h-25	PSO	0.17	2.73	2N/A	0.00	0.00	7	2.42
KC5i32	l-o	SA	1.37	N/A	N/A	0.00	0.00	2	3.92
KC5i32	l-o	EA	0.29	3.33	20.97	0.00	0.00	8	1.52
KC5i32	l-o	PSO	0.28	3.99	27.23	0.00	0.00	9	2.26
KC5i32	l-50	SA	1.08	N/A	N/A	0.00	0.00	3	4.07
KC5i32	l-50	EA	0.31	3.63	24.34	0.00	0.00	9	1.43
KC5i32	l-50	PSO	0.25	3.48	24.39	0.00	0.00	10	2.51
KC5i32	l-25	SA	2.80	N/A	N/A	0.00	0.00	1	4.19
KC5i32	l-25	EA	0.20	3.95	28.15	0.00	0.00	7	1.63
KC5i32	l-25	PSO	0.17	3.30	23.60	0.00	0.00	9	2.65
KC5i32	FR-o	SA	3.02	N/A	N/A	0.00	0.00	10	3.67
KC5i32	FR-o	EA	0.06	3.56	20.96	0.00	0.00	10	1.35
KC5i32	FR-o	PSO	0.02	3.39	22.64	0.00	0.00	10	1.85
KC5i32	FR-50	SA	0.36	N/A	N/A	0.00	0.00	10	3.79
KC5i32	FR-50	EA	0.00	3.14	26.10	0.00	0.00	10	1.29
KC5i32	FR-50	PSO	0.01	2.98	21.60	0.00	0.00	10	2.16
KC5i32	FR-25	SA	0.23	N/A	N/A	0.00	0.00	3	3.98
KC5i32	FR-25	EA	0.00	3.00	23.46	0.00	0.00	8	0.94
KC5i32	FR-25	PSO	0.00	2.74	24.05	0.00	0.00	6	1.94

Table B.18: Performance results for KC5i64

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5i64	O	SA	1.34	N/A	N/A	0.00	0.02	0	14.01
KC5i64	O	EA	0.33	3.03	20.21	0.00	0.00	9	5.04
KC5i64	O	PSO	0.39	2.76	15.57	0.00	0.00	9	10.03
KC5i64	BR-50	SA	2.90	N/A	N/A	0.00	0.00	3	14.52
KC5i64	BR-50	EA	0.29	2.95	21.96	0.00	0.00	10	5.82
KC5i64	BR-50	PSO	0.30	2.36	16.57	0.00	0.00	9	8.82
KC5i64	BR-25	SA	1.26	N/A	N/A	0.00	0.00	1	14.96
KC5i64	BR-25	EA	0.17	1.97	16.48	0.00	0.00	7	6.85
KC5i64	BR-25	PSO	0.17	2.14	16.93	0.00	0.00	7	8.66
KC5i64	h-o	SA	5.11	N/A	N/A	0.00	0.00	7	14.46
KC5i64	h-o	EA	0.39	3.36	20.63	0.00	0.00	9	5.98
KC5i64	h-o	PSO	0.44	3.35	18.99	0.00	0.00	9	8.16
KC5i64	h-50	SA	3.08	N/A	N/A	0.00	0.10	1	14.40
KC5i64	h-50	EA	0.26	2.87	21.27	0.11	0.11	0	6.12
KC5i64	h-50	PSO	0.25	2.97	22.66	0.11	0.11	0	8.94
KC5i64	h-25	SA	0.55	N/A	N/A	0.00	0.00	1	14.81
KC5i64	h-25	EA	0.18	2.26	19.80	0.00	0.00	8	5.80
KC5i64	h-25	PSO	0.17	2.41	18.54	0.00	0.00	7	8.17
KC5i64	l-o	SA	4.02	N/A	N/A	0.00	0.00	1	13.44
KC5i64	l-o	EA	0.09	2.16	17.65	0.00	0.00	8	5.47
KC5i64	l-o	PSO	0.15	2.70	18.98	0.00	0.00	7	7.99
KC5i64	l-50	SA	2.16	N/A	N/A	0.00	0.00	2	14.48
KC5i64	l-50	EA	0.34	2.58	18.54	0.00	0.00	10	7.24
KC5i64	l-50	PSO	0.34	2.77	20.82	0.00	0.00	7	9.41
KC5i64	l-25	SA	3.62	N/A	N/A	0.00	0.00	0	14.87
KC5i64	l-25	EA	0.19	2.67	22.19	0.00	0.00	8	5.66
KC5i64	l-25	PSO	0.17	2.22	19.82	0.00	0.00	10	8.63
KC5i64	FR-o	SA	0.60	N/A	N/A	0.00	0.00	10	12.81
KC5i64	FR-o	EA	0.03	3.37	25.88	0.00	0.00	9	5.83
KC5i64	FR-o	PSO	0.01	2.71	22.20	0.00	0.00	10	7.70
KC5i64	FR-50	SA	0.48	N/A	N/A	0.00	0.00	9	13.48
KC5i64	FR-50	EA	0.01	0.94	7.87	0.00	0.00	10	4.68
KC5i64	FR-50	PSO	0.00	2.25	20.23	0.00	0.00	10	7.28
KC5i64	FR-25	SA	2.49	N/A	N/A	0.00	0.00	5	14.24
KC5i64	FR-25	EA	0.00	2.38	21.48	0.00	0.00	8	4.13
KC5i64	FR-25	PSO	0.01	1.53	18.72	0.00	0.00	8	7.66

Table B.19: Performance results for KC5i128

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5i128	O	SA	1.37	N/A	N/A	0.00	0.00	5	53.10
KC5i128	O	EA	0.45	2.90	16.83	0.00	0.00	10	20.58
KC5i128	O	PSO	0.42	2.76	14.80	0.00	0.00	7	31.35
KC5i128	BR-50	SA	0.96	N/A	N/A	0.00	0.00	2	54.70
KC5i128	BR-50	EA	0.34	1.33	4.91	0.00	0.00	8	19.26
KC5i128	BR-50	PSO	0.25	2.23	17.52	0.00	0.00	10	37.70
KC5i128	BR-25	SA	0.58	N/A	N/A	0.00	0.00	0	56.10
KC5i128	BR-25	EA	0.19	1.71	14.92	0.00	0.00	8	20.95
KC5i128	BR-25	PSO	0.21	1.78	15.30	0.00	0.00	9	35.72
KC5i128	h-o	SA	2.00	N/A	N/A	0.00	0.03	0	59.40
KC5i128	h-o	EA	0.38	2.48	15.01	0.00	0.00	9	27.65
KC5i128	h-o	PSO	0.49	2.56	12.96	0.00	0.00	10	34.47
KC5i128	h-50	SA	1.15	N/A	N/A	0.00	0.00	2	54.86
KC5i128	h-50	EA	0.28	2.11	15.84	0.00	0.00	8	21.54
KC5i128	h-50	PSO	0.33	1.95	12.32	0.00	0.00	8	32.43
KC5i128	h-25	SA	0.50	N/A	N/A	0.00	0.00	0	56.08
KC5i128	h-25	EA	0.17	1.42	11.64	0.00	0.00	10	24.01
KC5i128	h-25	PSO	0.17	1.25	10.27	0.00	0.00	7	31.95
KC5i128	l-o	SA	1.03	N/A	N/A	0.00	0.00	5	52.51
KC5i128	l-o	EA	0.29	2.15	14.70	0.00	0.00	8	20.08
KC5i128	l-o	PSO	0.29	1.81	10.41	0.00	0.00	9	30.42
KC5i128	l-50	SA	0.94	N/A	N/A	0.00	0.00	4	54.72
KC5i128	l-50	EA	0.28	1.81	12.13	0.00	0.00	9	22.29
KC5i128	l-50	PSO	0.30	1.63	10.92	0.00	0.00	10	33.58
KC5i128	l-25	SA	5.50	N/A	N/A	0.00	0.00	1	56.15
KC5i128	l-25	EA	0.16	1.38	11.83	0.00	0.00	8	21.68
KC5i128	l-25	PSO	0.16	2.03	18.97	0.00	0.00	8	35.16
KC5i128	FR-o	SA	0.79	N/A	N/A	0.00	0.00	5	48.23
KC5i128	FR-o	EA	0.08	1.82	12.85	0.00	0.00	8	19.43
KC5i128	FR-o	PSO	0.01	1.50	9.05	0.00	0.00	10	27.91
KC5i128	FR-50	SA	1.03	N/A	N/A	0.00	0.00	10	50.61
KC5i128	FR-50	EA	0.01	1.64	15.69	0.00	0.00	10	20.55
KC5i128	FR-50	PSO	0.00	1.36	13.84	0.00	0.00	10	26.57
KC5i128	FR-25	SA	0.21	N/A	N/A	0.00	0.00	2	53.70
KC5i128	FR-25	EA	0.01	1.30	15.54	0.00	0.00	7	15.81
KC5i128	FR-25	PSO	0.00	1.39	15.57	0.00	0.00	8	27.99

Table B.20: Performance results for KC5i256

Inst.	Pattern	Alg.	%Imp. Min.	Avg.	Max.	%DV Min.	Avg.	BSol Hits	CT
KC5i256	O	SA	1.58	N/A	N/A	0.00	0.00	7	204.22
KC5i256	O	EA	0.50	2.19	9.40	0.00	0.00	10	83.57
KC5i256	O	PSO	0.43	1.91	5.00	0.00	0.00	10	127.94
KC5i256	BR-50	SA	2.88	N/A	N/A	0.00	0.00	2	210.86
KC5i256	BR-50	EA	0.32	1.06	2.11	0.00	0.00	9	77.25
KC5i256	BR-50	PSO	0.29	1.15	4.25	0.00	0.00	9	130.47
KC5i256	BR-25	SA	0.52	N/A	N/A	0.00	0.00	3	216.80
KC5i256	BR-25	EA	0.20	0.83	5.11	0.00	0.00	8	75.09
KC5i256	BR-25	PSO	0.16	1.07	8.11	0.00	0.00	8	128.82
KC5i256	h-o	SA	2.31	N/A	N/A	0.00	0.00	2	202.98
KC5i256	h-o	EA	0.41	1.76	4.43	0.00	0.00	10	99.72
KC5i256	h-o	PSO	0.39	2.26	9.32	0.00	0.00	10	121.53
KC5i256	h-50	SA	3.93	N/A	N/A	0.00	0.00	2	211.16
KC5i256	h-50	EA	0.31	2.17	14.78	0.00	0.00	10	84.49
KC5i256	h-50	PSO	0.24	1.23	5.91	0.00	0.00	10	123.59
KC5i256	h-25	SA	0.60	N/A	N/A	0.00	0.00	2	215.74
KC5i256	h-25	EA	0.18	0.70	3.14	0.00	0.00	7	75.10
KC5i256	h-25	PSO	0.16	0.82	5.08	0.00	0.00	10	123.35
KC5i256	l-o	SA	1.91	N/A	N/A	0.00	0.00	4	202.98
KC5i256	l-o	EA	0.59	1.77	3.25	0.00	0.00	10	90.03
KC5i256	l-o	PSO	0.48	2.23	7.85	0.00	0.00	10	131.60
KC5i256	l-50	SA	1.18	N/A	N/A	0.00	0.00	2	211.52
KC5i256	l-50	EA	0.24	1.39	8.35	0.00	0.00	9	72.13
KC5i256	l-50	PSO	0.29	1.70	11.71	0.00	0.00	10	131.41
KC5i256	l-25	SA	0.57	N/A	N/A	0.00	0.00	1	216.32
KC5i256	l-25	EA	0.19	0.94	6.72	0.00	0.00	8	72.06
KC5i256	l-25	PSO	0.16	0.86	5.72	0.00	0.00	10	146.57
KC5i256	FR-o	SA	1.22	N/A	N/A	0.00	0.00	9	187.07
KC5i256	FR-o	EA	0.01	1.03	4.60	0.00	0.00	9	70.16
KC5i256	FR-o	PSO	0.03	1.06	4.59	0.00	0.00	10	114.43
KC5i256	FR-50	SA	0.38	N/A	N/A	0.00	0.00	10	194.96
KC5i256	FR-50	EA	0.02	0.69	6.06	0.00	0.00	10	66.08
KC5i256	FR-50	PSO	0.01	0.81	6.05	0.00	0.00	10	105.35
KC5i256	FR-25	SA	0.19	N/A	N/A	0.00	0.00	2	206.12
KC5i256	FR-25	EA	0.00	1.27	14.12	0.00	0.00	5	49.45
KC5i256	FR-25	PSO	0.00	0.45	4.66	0.00	0.00	5	107.04

Table B.21: Performance results for KC5i512

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC5i512	O	SA	1.93	N/A	N/A	0.00	0.00	2	798.88
KC5i512	O	EA	0.32	1.93	6.25	0.00	0.00	9	421.88
KC5i512	O	PSO	0.45	1.83	3.79	0.00	0.00	10	527.56
KC5i512	BR-50	SA	1.19	N/A	N/A	0.00	0.00	4	827.93
KC5i512	BR-50	EA	0.26	1.49	6.69	0.00	0.00	6	273.43
KC5i512	BR-50	PSO	0.26	1.14	4.25	0.00	0.00	9	504.24
KC5i512	BR-25	SA	0.58	N/A	N/A	0.00	0.00	1	852.94
KC5i512	BR-25	EA	0.18	0.70	3.15	0.00	0.00	9	294.86
KC5i512	BR-25	PSO	0.17	0.56	0.85	0.00	0.00	9	536.70
KC5i512	h-o	SA	1.62	N/A	N/A	0.00	0.00	5	802.57
KC5i512	h-o	EA	0.44	1.95	5.96	0.00	0.00	10	348.04
KC5i512	h-o	PSO	0.36	2.00	6.43	0.00	0.00	9	535.77
KC5i512	h-50	SA	1.04	N/A	N/A	0.00	0.00	2	826.95
KC5i512	h-50	EA	0.25	0.99	1.59	0.00	0.00	10	303.10
KC5i512	h-50	PSO	0.34	1.01	1.58	0.00	0.00	9	520.88
KC5i512	h-25	SA	0.54	N/A	N/A	0.00	0.00	3	847.16
KC5i512	h-25	EA	0.19	0.77	4.31	0.00	0.00	10	355.09
KC5i512	h-25	PSO	0.17	0.78	3.88	0.00	0.00	6	511.01
KC5i512	l-o	SA	1.42	N/A	N/A	0.00	0.00	1	800.26
KC5i512	l-o	EA	0.38	2.14	9.15	0.00	0.00	6	285.36
KC5i512	l-o	PSO	0.43	1.92	4.46	0.00	0.00	9	528.28
KC5i512	l-50	SA	0.96	N/A	N/A	0.00	0.00	2	826.82
KC5i512	l-50	EA	0.29	1.33	6.05	0.00	0.00	10	338.38
KC5i512	l-50	PSO	0.31	1.19	3.79	0.00	0.00	10	506.86
KC5i512	l-25	SA	0.54	N/A	N/A	0.00	0.00	0	850.77
KC5i512	l-25	EA	0.21	1.06	8.01	0.00	0.00	9	331.34
KC5i512	l-25	PSO	0.20	0.57	0.87	0.00	0.00	7	516.46
KC5i512	FR-o	SA	0.52	N/A	N/A	0.00	0.00	8	730.78
KC5i512	FR-o	EA	0.02	1.05	4.42	0.00	0.00	10	272.60
KC5i512	FR-o	PSO	0.03	0.85	2.31	0.00	0.00	10	454.37
KC5i512	FR-50	SA	0.31	N/A	N/A	0.00	0.00	10	757.20
KC5i512	FR-50	EA	0.03	0.44	1.94	0.00	0.00	10	233.82
KC5i512	FR-50	PSO	0.01	0.63	4.96	0.00	0.00	9	438.09
KC5i512	FR-25	SA	0.16	N/A	N/A	0.00	0.00	2	798.32
KC5i512	FR-25	EA	0.00	0.39	3.41	0.00	0.00	6	212.24
KC5i512	FR-25	PSO	0.01	0.33	2.86	0.00	0.00	8	424.72

Table B.22: Performance results for KC10c16 and KC10U

			%Imp.			%DV		BSol	
Inst.	Pattern	Alg.	Min.	Avg.	Max.	Min.	Avg.	Hits	CT
KC10c16	O	SA	9.39	N/A	N/A	0.00	0.00	3	1.53
KC10c16	O	EA	0.78	9.49	29.42	0.00	0.00	7	0.37
KC10c16	O	PSO	0.46	9.75	30.20	0.00	0.00	7	0.55
KC10c16	BR-50	SA	6.66	N/A	N/A	0.00	0.00	1	1.51
KC10c16	BR-50	EA	0.59	9.07	30.59	0.00	0.00	9	0.47
KC10c16	BR-50	PSO	0.57	9.77	30.43	0.00	0.00	4	0.53
KC10c16	BR-25	SA	6.51	N/A	N/A	0.00	0.00	1	1.52
KC10c16	BR-25	EA	0.30	10.27	32.80	0.00	0.00	5	0.43
KC10c16	BR-25	PSO	0.36	10.21	33.59	0.00	0.00	5	0.62
KC10c16	h-o	SA	10.28	N/A	N/A	0.00	0.00	2	1.52
KC10c16	h-o	EA	0.78	10.57	29.14	0.00	0.00	7	0.39
KC10c16	h-o	PSO	0.92	10.46	30.03	0.00	0.00	8	0.59
KC10c16	h-50	SA	9.05	N/A	N/A	0.00	0.00	0	1.51
KC10c16	h-50	EA	0.57	10.43	31.51	0.00	0.00	9	0.43
KC10c16	h-50	PSO	0.50	10.62	32.74	0.00	0.00	3	0.54
KC10c16	h-25	SA	15.95	N/A	N/A	0.00	0.00	1	1.53
KC10c16	h-25	EA	0.30	9.53	29.28	0.00	0.00	7	0.41
KC10c16	h-25	PSO	0.34	11.85	32.54	0.00	0.00	3	0.61
KC10c16	l-o	SA	10.83	N/A	N/A	0.00	0.01	0	1.49
KC10c16	l-o	EA	0.35	8.54	28.12	0.00	0.00	9	0.37
KC10c16	l-o	PSO	0.34	9.38	29.26	0.00	0.00	8	0.62
KC10c16	l-50	SA	17.05	N/A	N/A	0.00	0.00	1	1.51
KC10c16	l-50	EA	0.62	9.39	30.61	0.00	0.00	7	0.40
KC10c16	l-50	PSO	0.64	11.01	33.08	0.00	0.00	8	0.66
KC10c16	l-25	SA	5.70	N/A	N/A	0.00	0.00	3	1.52
KC10c16	l-25	EA	0.33	10.47	34.02	0.00	0.00	8	0.49
KC10c16	l-25	PSO	0.29	9.75	31.05	0.00	0.00	5	0.59
KC10c16	FR-o	SA	6.73	N/A	N/A	0.00	0.00	0	1.19
KC10c16	FR-o	EA	0.25	9.24	30.52	0.00	0.00	0	0.34
KC10c16	FR-o	PSO	0.29	8.58	30.43	0.00	0.00	1	0.49
KC10c16	FR-50	SA	5.08	N/A	N/A	0.00	0.00	0	1.27
KC10c16	FR-50	EA	0.04	10.08	33.49	0.00	0.00	6	0.32
KC10c16	FR-50	PSO	0.09	8.61	29.52	0.00	0.00	5	0.47
KC10c16	FR-25	SA	5.34	N/A	N/A	0.00	0.00	4	1.49
KC10c16	FR-25	EA	0.02	11.14	34.06	0.00	0.00	9	0.39
KC10c16	FR-25	PSO	0.02	8.98	32.82	0.00	0.00	9	0.54
KC10	U	SA	22.72	N/A	N/A	0.00	0.00	2	0.02
KC10	U	EA	9.83	23.55	33.71	0.00	0.01	0	0.00
KC10	U	PSO	10.56	21.52	33.88	0.00	0.00	0	0.00

Table B.23: Performance results for KC10c128

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC10c128	O	SA	3.75	N/A	N/A	0.00	0.02	0	52.26
KC10c128	O	EA	0.57	4.26	19.92	0.00	0.00	10	24.29
KC10c128	O	PSO	0.59	4.17	18.79	0.00	0.00	10	46.20
KC10c128	BR-50	SA	3.22	N/A	N/A	0.00	0.00	0	59.31
KC10c128	BR-50	EA	0.40	3.66	21.81	0.00	0.00	10	27.67
KC10c128	BR-50	PSO	0.45	2.64	12.25	0.00	0.00	9	38.31
KC10c128	BR-25	SA	0.75	N/A	N/A	0.00	0.00	4	60.00
KC10c128	BR-25	EA	0.35	2.35	17.85	0.00	0.00	8	28.61
KC10c128	BR-25	PSO	0.29	2.06	14.62	0.00	0.00	8	36.09
KC10c128	h-o	SA	2.24	N/A	N/A	0.00	0.00	2	59.18
KC10c128	h-o	EA	0.37	3.95	19.95	0.00	0.00	9	22.78
KC10c128	h-o	PSO	0.40	3.60	13.15	0.00	0.00	8	36.26
KC10c128	h-50	SA	2.33	N/A	N/A	0.00	0.00	0	59.00
KC10c128	h-50	EA	0.65	3.64	20.15	0.00	0.00	10	25.60
KC10c128	h-50	PSO	0.39	3.65	21.86	0.00	0.00	8	38.13
KC10c128	h-25	SA	0.93	N/A	N/A	0.00	0.00	1	59.50
KC10c128	h-25	EA	0.31	2.96	20.98	0.00	0.00	8	26.93
KC10c128	h-25	PSO	0.34	2.97	22.90	0.00	0.00	9	39.99
KC10c128	l-o	SA	2.85	N/A	N/A	0.00	0.00	3	58.72
KC10c128	l-o	EA	0.47	3.87	17.46	0.00	0.00	8	24.20
KC10c128	l-o	PSO	0.53	3.98	16.25	0.00	0.00	8	38.51
KC10c128	l-50	SA	1.41	N/A	N/A	0.00	0.00	3	58.65
KC10c128	l-50	EA	0.60	3.42	18.49	0.00	0.00	10	27.89
KC10c128	l-50	PSO	0.43	3.10	16.62	0.00	0.00	10	38.89
KC10c128	l-25	SA	0.99	N/A	N/A	0.00	0.00	1	59.45
KC10c128	l-25	EA	0.31	3.12	23.68	0.00	0.00	10	29.79
KC10c128	l-25	PSO	0.30	3.23	20.58	0.00	0.00	8	36.25
KC10c128	FR-o	SA	1.24	N/A	N/A	0.00	0.00	0	44.73
KC10c128	FR-o	EA	0.01	3.28	22.41	0.00	0.00	0	23.28
KC10c128	FR-o	PSO	0.07	3.48	22.41	0.00	0.00	1	33.67
KC10c128	FR-50	SA	1.44	N/A	N/A	0.00	0.00	2	47.81
KC10c128	FR-50	EA	0.01	2.75	23.48	0.00	0.00	8	15.42
KC10c128	FR-50	PSO	0.01	2.21	19.45	0.00	0.00	10	30.36
KC10c128	FR-25	SA	0.42	N/A	N/A	0.00	0.00	9	53.84
KC10c128	FR-25	EA	0.00	1.80	18.51	0.00	0.00	8	15.24
KC10c128	FR-25	PSO	0.00	1.47	13.59	0.00	0.00	9	31.24

Table B.24: Performance results for KC10i16

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC10i16	O	SA	7.27	N/A	N/A	0.00	0.00	1	1.41
KC10i16	O	EA	0.83	10.86	30.24	0.00	0.00	5	0.30
KC10i16	O	PSO	0.78	9.46	29.08	0.00	0.00	7	0.57
KC10i16	BR-50	SA	6.09	N/A	N/A	0.00	0.00	0	1.43
KC10i16	BR-50	EA	0.52	10.57	32.88	0.00	0.00	9	0.37
KC10i16	BR-50	PSO	0.61	9.04	30.79	0.00	0.00	5	0.61
KC10i16	BR-25	SA	12.21	N/A	N/A	0.00	0.00	2	1.46
KC10i16	BR-25	EA	0.42	10.52	33.03	0.00	0.00	4	0.34
KC10i16	BR-25	PSO	0.34	11.05	34.28	0.00	0.00	2	0.52
KC10i16	h-o	SA	7.36	N/A	N/A	0.00	0.00	3	1.40
KC10i16	h-o	EA	0.67	11.02	30.67	0.00	0.00	6	0.34
KC10i16	h-o	PSO	0.88	9.46	31.29	0.00	0.00	5	0.58
KC10i16	h-50	SA	5.05	N/A	N/A	0.00	0.00	0	1.44
KC10i16	h-50	EA	0.43	10.80	33.45	0.00	0.00	5	0.35
KC10i16	h-50	PSO	0.50	12.03	33.70	0.00	0.00	5	0.55
KC10i16	h-25	SA	12.05	N/A	N/A	0.00	0.00	0	1.47
KC10i16	h-25	EA	0.39	9.96	32.38	0.00	0.00	6	0.43
KC10i16	h-25	PSO	0.36	9.23	31.21	0.00	0.00	2	0.55
KC10i16	l-o	SA	8.45	N/A	N/A	0.00	0.00	1	1.40
KC10i16	l-o	EA	0.70	10.67	32.17	0.00	0.00	6	0.34
KC10i16	l-o	PSO	0.95	10.89	30.06	0.00	0.00	5	0.57
KC10i16	l-50	SA	8.13	N/A	N/A	0.00	0.00	0	1.44
KC10i16	l-50	EA	0.60	11.12	31.56	0.00	0.00	8	0.42
KC10i16	l-50	PSO	0.56	9.36	30.67	0.00	0.00	2	0.53
KC10i16	l-25	SA	8.78	N/A	N/A	0.00	0.00	0	1.47
KC10i16	l-25	EA	0.33	10.31	33.83	0.00	0.00	8	0.42
KC10i16	l-25	PSO	0.36	10.40	33.69	0.00	0.00	4	0.61
KC10i16	FR-o	SA	9.02	N/A	N/A	0.00	0.00	0	1.18
KC10i16	FR-o	EA	0.39	9.09	31.24	0.00	0.00	1	0.37
KC10i16	FR-o	PSO	0.20	8.90	28.95	0.00	0.00	1	0.61
KC10i16	FR-50	SA	3.15	N/A	N/A	0.00	0.00	0	1.25
KC10i16	FR-50	EA	0.08	10.18	32.84	0.00	0.00	2	0.38
KC10i16	FR-50	PSO	0.06	10.43	33.20	0.00	0.00	2	0.50
KC10i16	FR-25	SA	3.62	N/A	N/A	0.00	0.00	9	1.34
KC10i16	FR-25	EA	0.02	9.18	33.90	0.00	0.00	9	0.32
KC10i16	FR-25	PSO	0.02	8.21	29.38	0.00	0.00	9	0.50

Table B.25: Performance results for KC10i128

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
KC10i128	O	SA	2.48	N/A	N/A	0.00	0.02	0	60.75
KC10i128	O	EA	0.49	4.46	21.52	0.00	0.00	6	21.73
KC10i128	O	PSO	0.45	4.40	22.71	0.00	0.00	9	41.95
KC10i128	BR-50	SA	3.36	N/A	N/A	0.00	0.00	0	58.57
KC10i128	BR-50	EA	0.52	3.71	18.27	0.00	0.00	8	25.50
KC10i128	BR-50	PSO	0.52	3.74	19.36	0.00	0.00	9	40.61
KC10i128	BR-25	SA	1.10	N/A	N/A	0.00	0.00	2	59.47
KC10i128	BR-25	EA	0.30	2.69	21.82	0.00	0.00	8	27.23
KC10i128	BR-25	PSO	0.34	2.79	21.50	0.00	0.00	7	36.28
KC10i128	h-o	SA	3.86	N/A	N/A	0.00	0.01	0	60.67
KC10i128	h-o	EA	0.42	4.75	23.55	0.00	0.00	8	25.44
KC10i128	h-o	PSO	0.55	4.27	20.42	0.00	0.00	10	39.65
KC10i128	h-50	SA	3.98	N/A	N/A	0.00	0.00	1	58.16
KC10i128	h-50	EA	0.40	3.41	18.45	0.00	0.00	10	25.28
KC10i128	h-50	PSO	0.46	3.42	20.33	0.00	0.00	7	34.88
KC10i128	h-25	SA	6.43	N/A	N/A	0.00	0.00	0	58.75
KC10i128	h-25	EA	0.32	2.60	17.23	0.00	0.00	10	23.85
KC10i128	h-25	PSO	0.32	2.16	15.50	0.00	0.00	9	37.79
KC10i128	l-o	SA	2.62	N/A	N/A	0.00	0.00	0	54.46
KC10i128	l-o	EA	0.11	3.56	21.46	0.00	0.00	1	48.26
KC10i128	l-o	PSO	0.18	2.44	11.44	0.00	0.00	0	62.79
KC10i128	l-50	SA	3.67	N/A	N/A	0.00	0.00	3	57.95
KC10i128	l-50	EA	0.51	2.80	18.35	0.00	0.00	9	27.87
KC10i128	l-50	PSO	0.42	2.59	18.33	0.00	0.00	8	35.79
KC10i128	l-25	SA	0.99	N/A	N/A	0.00	0.00	0	58.94
KC10i128	l-25	EA	0.38	2.60	18.72	0.00	0.00	10	25.08
KC10i128	l-25	PSO	0.30	2.95	21.55	0.00	0.00	9	38.46
KC10i128	FR-o	SA	1.55	N/A	N/A	0.00	0.00	0	44.53
KC10i128	FR-o	EA	0.13	3.30	22.60	0.00	0.00	0	24.03
KC10i128	FR-o	PSO	0.12	2.63	17.59	0.00	0.00	1	33.76
KC10i128	FR-50	SA	0.53	N/A	N/A	0.00	0.00	0	47.60
KC10i128	FR-50	EA	0.05	4.11	28.23	0.00	0.00	0	19.58
KC10i128	FR-50	PSO	0.01	2.26	18.99	0.00	0.00	1	31.14
KC10i128	FR-25	SA	1.78	N/A	N/A	0.00	0.00	10	53.75
KC10i128	FR-25	EA	0.00	2.31	21.55	0.00	0.00	8	17.64
KC10i128	FR-25	PSO	0.01	1.76	14.27	0.00	0.00	10	30.55

Table B.26: Performance results for C600

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
C600	PR	SA	32.95	N/A	N/A	0.00	0.03	0	13.12
C600	PR	EA	2.62	25.80	45.73	0.00	0.00	8	11.12
C600	PR	PSO	1.66	25.81	45.13	0.00	0.00	3	17.06
C600	LR	SA	23.72	N/A	N/A	0.00	0.02	0	11.09
C600	LR	EA	2.29	27.48	46.21	0.00	0.00	9	10.38
C600	LR	PSO	2.20	25.96	46.11	0.00	0.00	7	14.95
C600	U	SA	21.21	N/A	N/A	0.00	0.03	0	0.85
C600	U	EA	1.78	25.84	44.90	0.00	0.00	6	0.84
C600	U	PSO	2.13	26.07	45.53	0.00	0.00	1	1.16

Table B.27: Performance results for R800

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
R800	PR	SA	19.58	N/A	N/A	0.00	0.02	0	18.33
R800	PR	EA	2.10	20.53	37.98	0.00	20.63	6	16.92
R800	PR	PSO	1.09	20.07	39.62	0.00	20.63	2	34.44
R800	LR	SA	24.80	N/A	N/A	4.00	4.25	0	14.93
R800	LR	EA	1.94	26.25	43.78	1.27	3.44	0	24.90
R800	LR	PSO	1.45	24.19	41.17	0.00	3.16	1	28.33
R800	U	SA	17.71	N/A	N/A	0.00	0.02	0	1.15
R800	U	EA	1.65	27.43	45.00	0.00	0.00	7	1.00
R800	U	PSO	2.39	26.95	44.57	0.00	0.00	2	1.50

Table B.28: Performance results for RC800

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
RC800	PR	SA	24.84	N/A	N/A	0.01	0.04	0	22.16
RC800	PR	EA	0.72	26.91	44.98	0.00	0.00	8	21.79
RC800	PR	PSO	1.77	26.55	44.89	0.00	0.00	2	25.67
RC800	LR	SA	28.47	N/A	N/A	0.00	0.02	0	9.82
RC800	LR	EA	2.34	24.86	44.68	0.00	0.00	6	8.91
RC800	LR	PSO	1.49	25.71	43.27	0.00	0.00	0	12.56
RC800	U	SA	27.30	N/A	N/A	0.00	0.02	0	1.14
RC800	U	EA	1.95	25.99	45.16	0.00	0.00	9	1.10
RC800	U	PSO	1.90	27.02	45.12	0.00	0.00	2	1.44

Table B.29: Performance results for R1000

Inst.	Pattern	Alg.	%Imp. Min.	Avg.	Max.	%DV Min.	Avg.	BSol Hits	CT
R1000	PR	SA	24.61	N/A	N/A	0.00	0.02	0	33.50
R1000	PR	EA	1.97	28.21	44.91	0.00	0.00	8	28.61
R1000	PR	PSO	1.64	27.49	45.17	0.00	0.00	4	46.50
R1000	LR	SA	23.13	N/A	N/A	0.00	0.01	0	24.62
R1000	LR	EA	0.81	25.70	44.98	0.00	0.00	4	20.17
R1000	LR	PSO	1.47	25.76	45.22	0.00	0.00	0	29.71
R1000	U	SA	23.95	N/A	N/A	0.00	0.01	0	1.42
R1000	U	EA	1.81	25.97	44.64	0.00	0.00	8	1.21
R1000	U	PSO	0.95	26.24	44.23	0.00	0.00	4	1.96

Table B.30: Performance results for RC1000

Inst.	Pattern	Alg.	%Imp. Min.	Avg.	Max.	%DV Min.	Avg.	BSol Hits	CT
RC1000	PR	SA	14.79	N/A	N/A	0.00	0.03	0	19.44
RC1000	PR	EA	1.40	21.08	38.96	0.00	0.00	7	22.22
RC1000	PR	PSO	1.24	20.17	39.41	0.00	0.00	5	31.25
RC1000	LR	SA	26.05	N/A	N/A	0.00	0.02	0	17.88
RC1000	LR	EA	0.94	26.81	44.16	0.00	0.00	8	15.34
RC1000	LR	PSO	1.06	25.75	45.55	0.00	0.00	5	22.66
RC1000	U	SA	24.76	N/A	N/A	0.00	0.02	0	1.42
RC1000	U	EA	2.01	27.25	45.42	0.00	0.00	7	1.36
RC1000	U	PSO	2.05	25.44	45.08	0.00	0.00	1	1.82

Table B.31: Performance results for u2319

Inst.	Pattern	Alg.	%Imp. Min.	Avg.	Max.	%DV Min.	Avg.	BSol Hits	CT
u2319	PR	SA	19.18	N/A	N/A	0.00	0.03	0	22.84
u2319	PR	EA	0.68	21.16	39.76	0.00	0.00	4	22.85
u2319	PR	PSO	0.70	20.39	38.92	0.00	0.00	0	41.53
u2319	LR	SA	27.20	N/A	N/A	0.00	0.01	0	27.53
u2319	LR	EA	1.94	26.93	46.90	0.00	0.00	4	24.26
u2319	LR	PSO	1.35	26.89	46.32	0.00	0.00	0	34.68
u2319	U	SA	26.90	N/A	N/A	0.01	0.03	0	3.22
u2319	U	EA	1.65	26.69	45.48	0.00	0.00	6	3.18
u2319	U	PSO	2.21	27.60	46.23	0.00	0.00	0	4.58

Table B.32: Performance results for fnl4461

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
fnl4461	PR	SA	24.99	N/A	N/A	0.00	0.02	0	71.60
fnl4461	PR	EA	1.44	21.68	43.33	0.00	0.00	7	71.57
fnl4461	PR	PSO	0.96	22.48	46.17	0.00	0.00	0	87.10
fnl4461	LR	SA	18.84	N/A	N/A	0.00	0.02	0	50.27
fnl4461	LR	EA	1.80	27.32	48.39	0.00	0.00	4	41.46
fnl4461	LR	PSO	1.61	26.17	49.34	0.00	0.00	2	66.37
fnl4461	U	SA	20.56	N/A	N/A	0.00	0.02	0	6.19
fnl4461	U	EA	2.05	26.12	48.70	0.00	0.00	7	5.99
fnl4461	U	PSO	1.73	25.28	46.68	0.00	0.00	1	8.04

Table B.33: Performance results for pla7397

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
pla7397	PR	SA	16.32	N/A	N/A	0.00	0.01	0	210.54
pla7397	PR	EA	3.20	17.55	29.53	0.00	0.00	3	188.24
pla7397	PR	PSO	2.96	16.95	29.72	0.00	0.00	0	247.21
pla7397	LR	SA	22.68	N/A	N/A	0.00	0.01	0	79.87
pla7397	LR	EA	3.20	18.75	30.23	0.00	0.00	3	68.08
pla7397	LR	PSO	3.41	18.45	30.21	0.00	0.00	0	87.09
pla7397	U	SA	16.17	N/A	N/A	0.00	0.01	0	10.23
pla7397	U	EA	3.89	19.74	33.11	0.00	0.00	4	10.54
pla7397	U	PSO	2.96	19.55	32.88	0.00	0.00	0	11.42

Table B.34: Performance results for usa13509

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
usa13509	PR	SA	20.31	N/A	N/A	0.00	0.05	0	317.18
usa13509	PR	EA	0.46	29.03	64.38	0.00	0.00	6	305.78
usa13509	PR	PSO	0.86	30.09	65.03	0.00	0.00	1	470.23
usa13509	LR	SA	30.77	N/A	N/A	0.00	0.04	0	221.64
usa13509	LR	EA	0.43	27.26	66.48	0.00	0.00	1	172.31
usa13509	LR	PSO	0.61	30.15	66.79	0.00	0.00	0	297.79
usa13509	U	SA	24.39	N/A	N/A	0.00	0.05	0	18.60
usa13509	U	EA	0.89	31.07	66.60	0.00	0.00	5	17.84
usa13509	U	PSO	0.91	29.88	66.71	0.00	0.00	0	27.04

Table B.35: Performance results for pla33810

Inst.	Pattern	Alg.	%Imp. Min.	%Imp. Avg.	%Imp. Max.	%DV Min.	%DV Avg.	BSol Hits	CT
pla33810	PR	SA	18.63	N/A	N/A	0.00	0.03	0	619.40
pla33810	PR	EA	2.00	26.35	46.01	0.00	0.00	0	487.44
pla33810	PR	PSO	1.19	26.35	46.23	0.00	0.00	1	821.69
pla33810	LR	SA	19.63	N/A	N/A	0.00	0.03	0	404.82
pla33810	LR	EA	1.41	28.25	54.63	0.00	0.00	4	411.82
pla33810	LR	PSO	1.17	28.63	53.97	0.00	0.00	0	508.29
pla33810	U	SA	26.37	N/A	N/A	0.00	0.01	0	46.66
pla33810	U	EA	1.54	28.28	47.70	0.00	0.00	4	47.60
pla33810	U	PSO	1.29	26.42	46.42	0.00	0.00	0	65.32

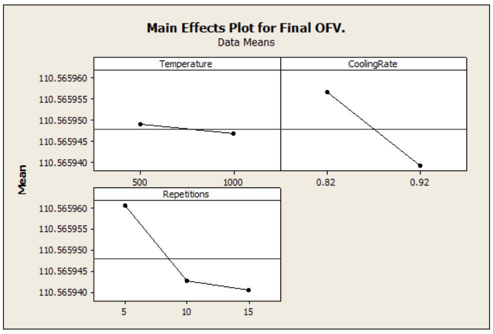
APPENDIX C

STOCHASTIC ANALYSIS OF TEST PROBLEM INSTANCES

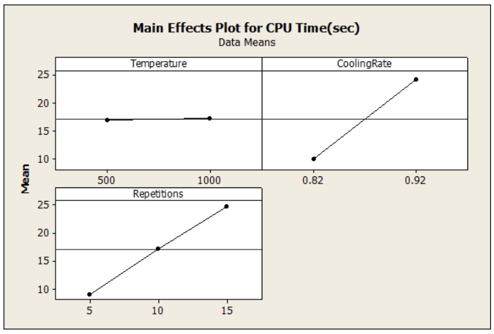
This appendix is about stochastic Analysis for two problem instances AP70R10 with pattern h-50 and KC5c16 with pattern BR-o as examples. The interaction plots of the algorithms parameter settings for both problems is given in Section C.1. The interaction plots are used to derive best parameter settings for each meta-heuristic algorithm. In Section C.2 we illustrate the convergence of all meta-heuristic algorithms under the best parameter settings for the same problems.

C.1 Interaction Plots

This section provides all three algorithms' parameters effect plots for two problem instances AP70R10 with pattern h-50 and KC5c16 with pattern BR-o.



(a) Effects on the objective function value

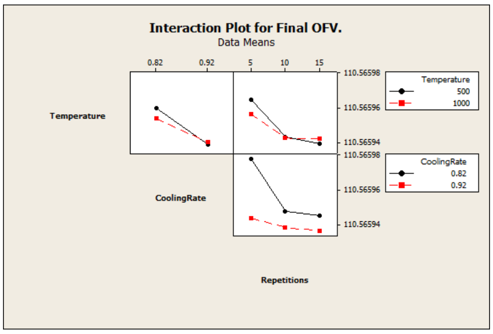


(b) Effects on the CPU time

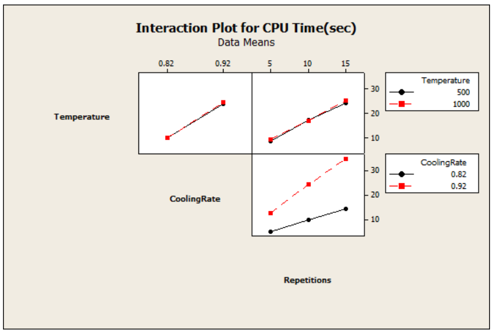
Figure C.1: Main effect plots of SA parameters for AP70R10. The effect of each parameter level on the objective function value and CPU time is shown in (a) and (b) respectively.

C.2 Algorithms Convergence

In this section we graphically show the how objective function values converge to the best objective function value through time. The information is given for all three meta-heuristics considering two problem instances AP70R10 with pattern h-50 and KC5c16 with pattern BR-o as examples.

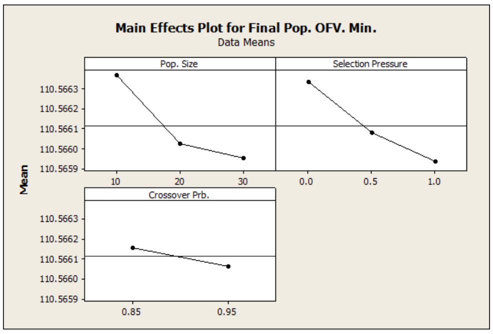


(a) Effects on the objective function value

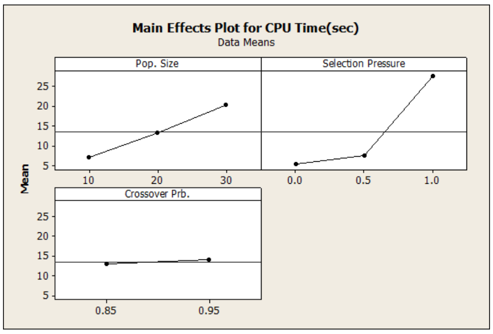


(b) Effects on the CPU time

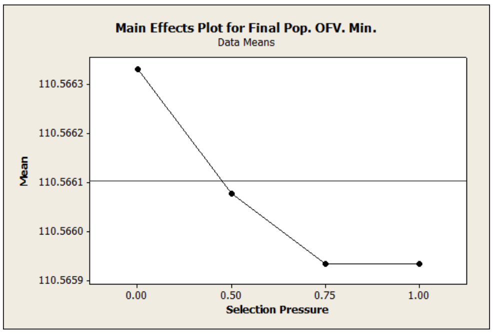
Figure C.2: Interaction plots of SA parameters for AP70R10. The effect of the parameters interactions on the objective function value and CPU time is shown in (a) and (b) respectively.



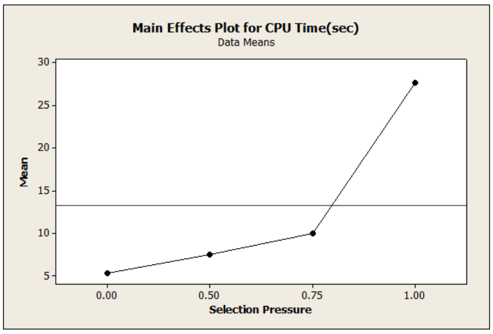
(a) Effects on the objective function value



(b) Effects on the CPU time

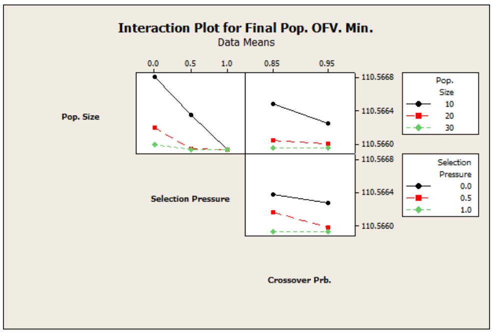


(c) Effects on the objective function value

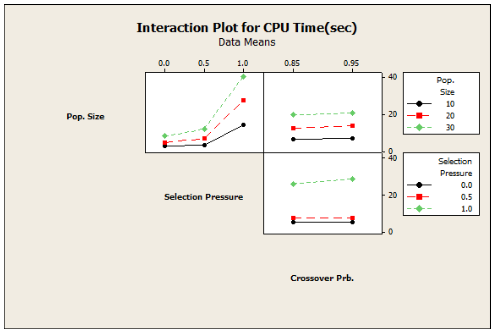


(d) Effects on the CPU time

Figure C.3: Main effect plots of EA parameters for AP70R10. The effect of each parameter level on the objective function value and CPU time is shown in (a) and (b) respectively. (c) and (d) show the effect of different selection pressure levels on the objective function value and CPU time respectively. when other parameters are set to their best values.

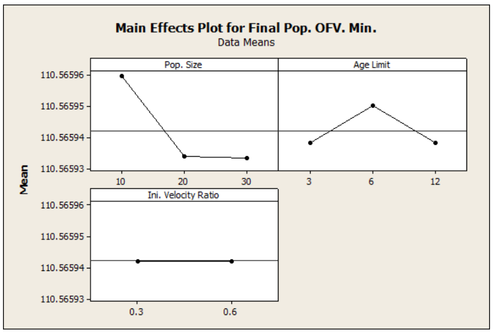


(a) Effects on the objective function value

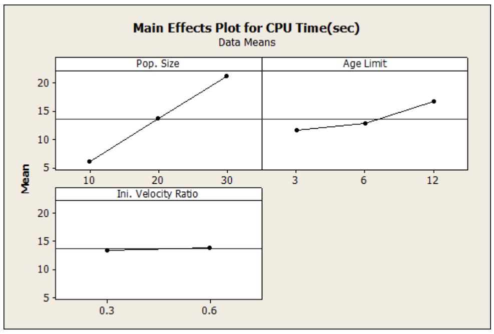


(b) Effects on the CPU time

Figure C.4: Interaction plots of EA parameters for AP70R10. The effect of the parameters interactions on the objective function value and CPU time is shown in (a) and (b) respectively.

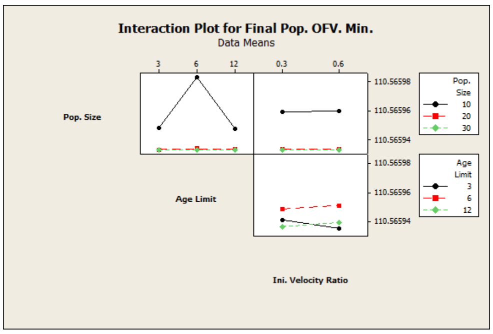


(a) Effects on the objective function value

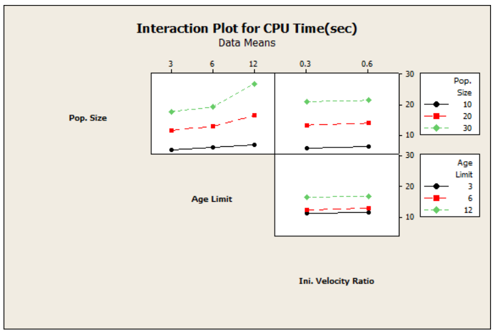


(b) Effects on the CPU time

Figure C.5: Main effect plots of PSO parameters for AP70R10. The effect of each parameter level on the objective function value and CPU time is shown in (a) and (b) respectively.

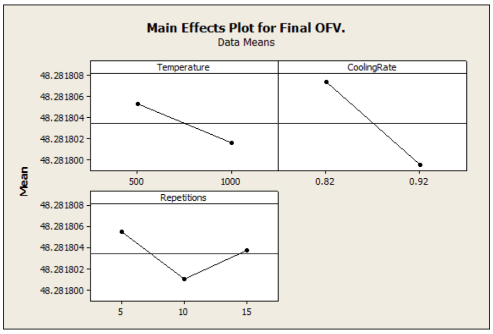


(a) Effects on the objective function value

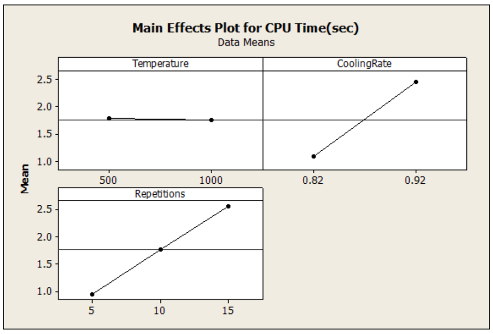


(b) Effects on the CPU time

Figure C.6: Interaction plots of PSO parameters for AP70R10. The effect of the parameters interactions on the objective function value and CPU time is shown in (a) and (b) respectively.

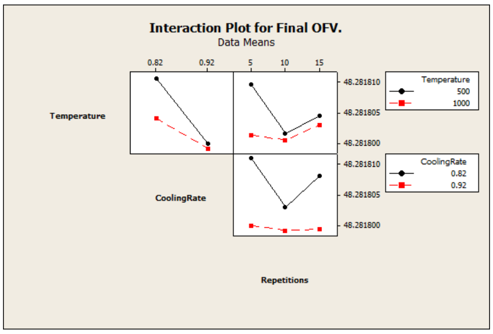


(a) Effects on the objective function value

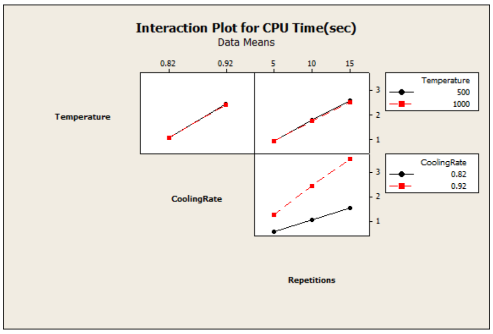


(b) Effects on the CPU time

Figure C.7: Main effect plots of SA parameters for KC5c16. The effect of each parameter level on the objective function value and CPU time is shown in (a) and (b) respectively.

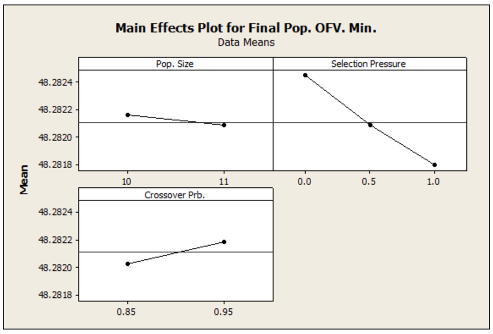


(a) Effects on the objective function value

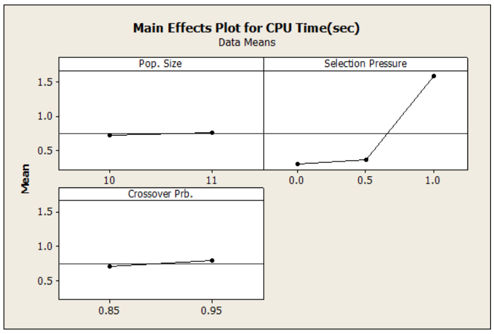


(b) Effects on the CPU time

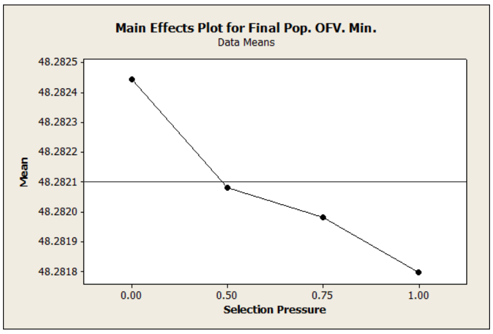
Figure C.8: Interaction plots of SA parameters for KC5c16. The effect of the parameters interactions on the objective function value and CPU time is shown in (a) and (b) respectively.



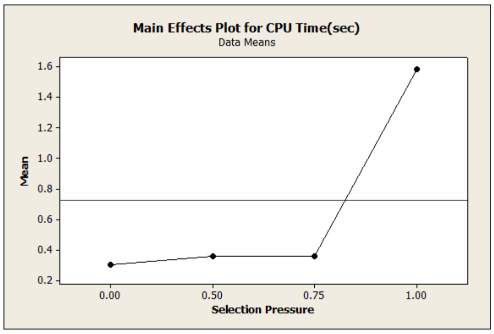
(a) Effects on the objective function value



(b) Effects on the CPU time

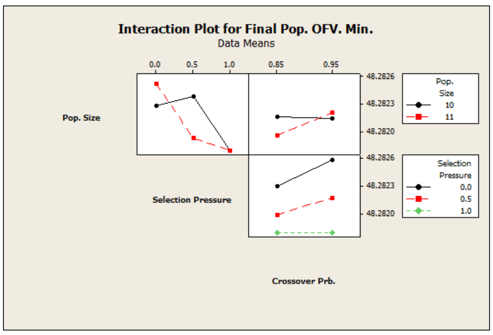


(c) Effects on the objective function value

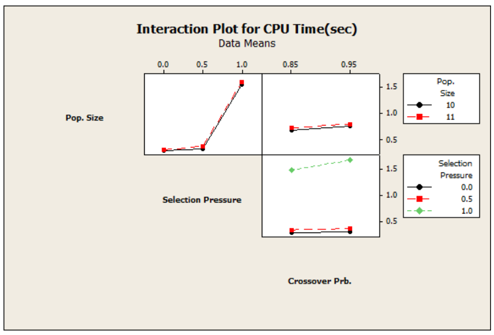


(d) Effects on the CPU time

Figure C.9: Main effect plots of EA parameters for KC5c16. The effect of each parameter level on the objective function value and CPU time is shown in (a) and (b) respectively. (c) and (d) show the effect of different selection pressure levels on the objective function value and CPU time respectively. when other parameters are set to their best values.

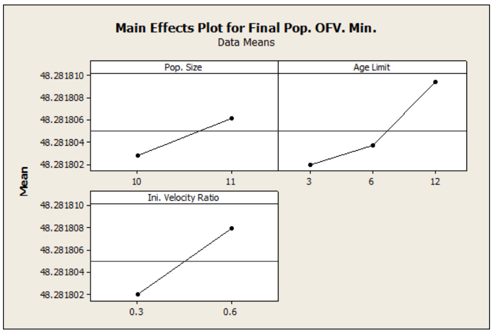


(a) Effects on the objective function value

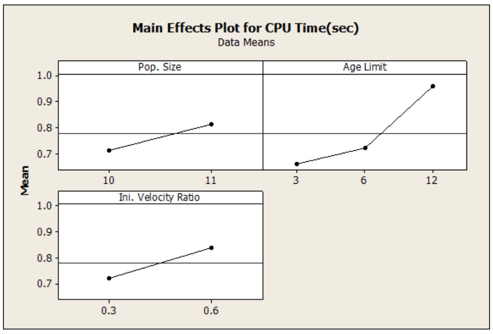


(b) Effects on the CPU time

Figure C.10: Interaction plots of EA parameters for KC5c16. The effect of the parameters interactions on the objective function value and CPU time is shown in (a) and (b) respectively.

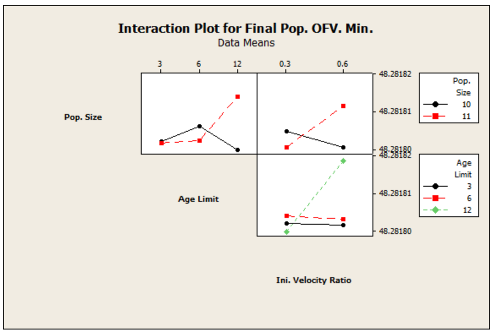


(a) Effects on the objective function value

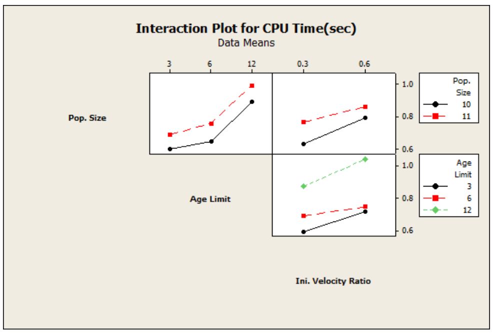


(b) Effects on the CPU time

Figure C.11: Main effect plots of PSO parameters for KC5c16. The effect of each parameter level on the objective function value and CPU time is shown in (a) and (b) respectively.

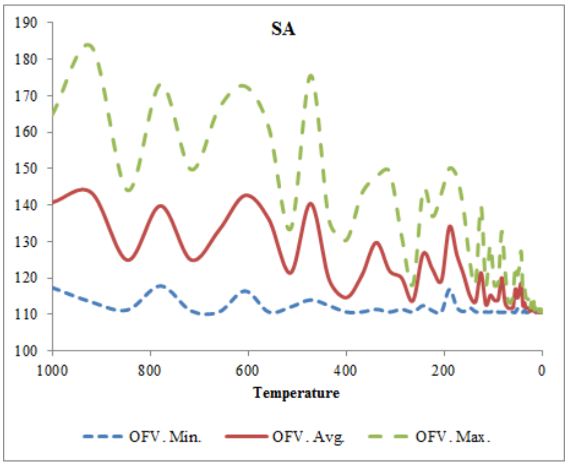


(a) Effects on the objective function value

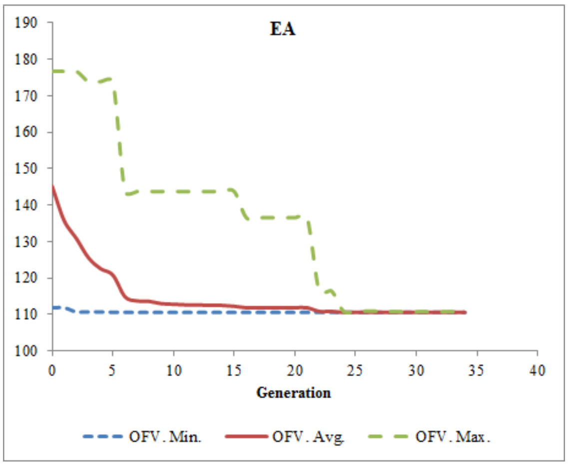


(b) Effects on the CPU time

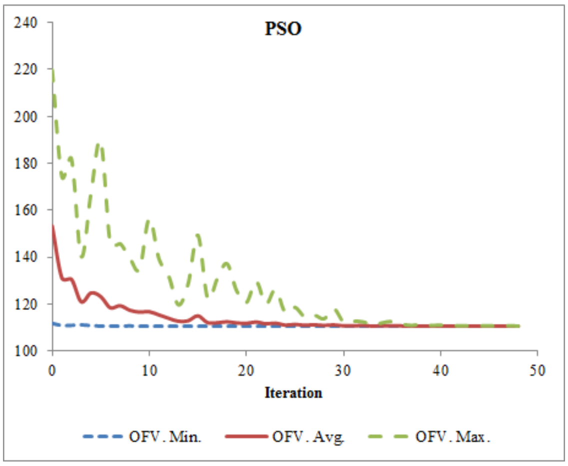
Figure C.12: Interaction plots of PSO parameters for KC5c16. The effect of the parameters interactions on the objective function value and CPU time is shown in (a) and (b) respectively.



(a) SA's convergence

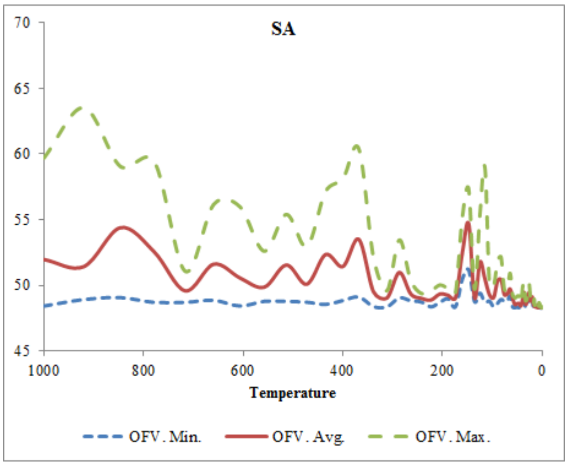


(b) EA's convergence

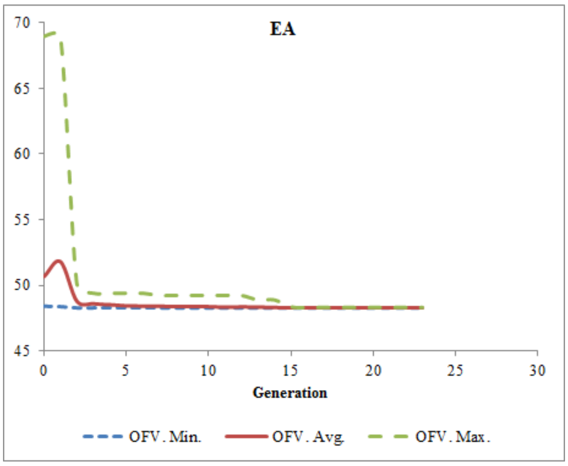


(c) PSO's convergence

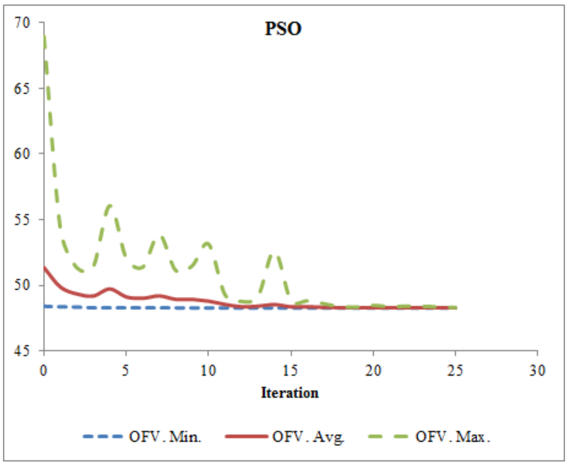
Figure C.13: Convergence of different meta-heuristics to the BSol for AP70R10 instance.



(a) SA's convergence



(b) EA's convergence



(c) PSO's convergence

Figure C.14: Convergence of different meta-heuristics to the BSol for KC5c16 instance.

APPENDIX D

USER MANUAL FOR THE SOFTWARE PACKAGE

In this appendix an instruction for using the developed software package is provided. The software is written in MS Visual Studio 2010 environment using visual basic language with .Net framework technology (VB.Net). The program is executable on MS Windows XP or later having .Net framework 4.0 (or later) installed.

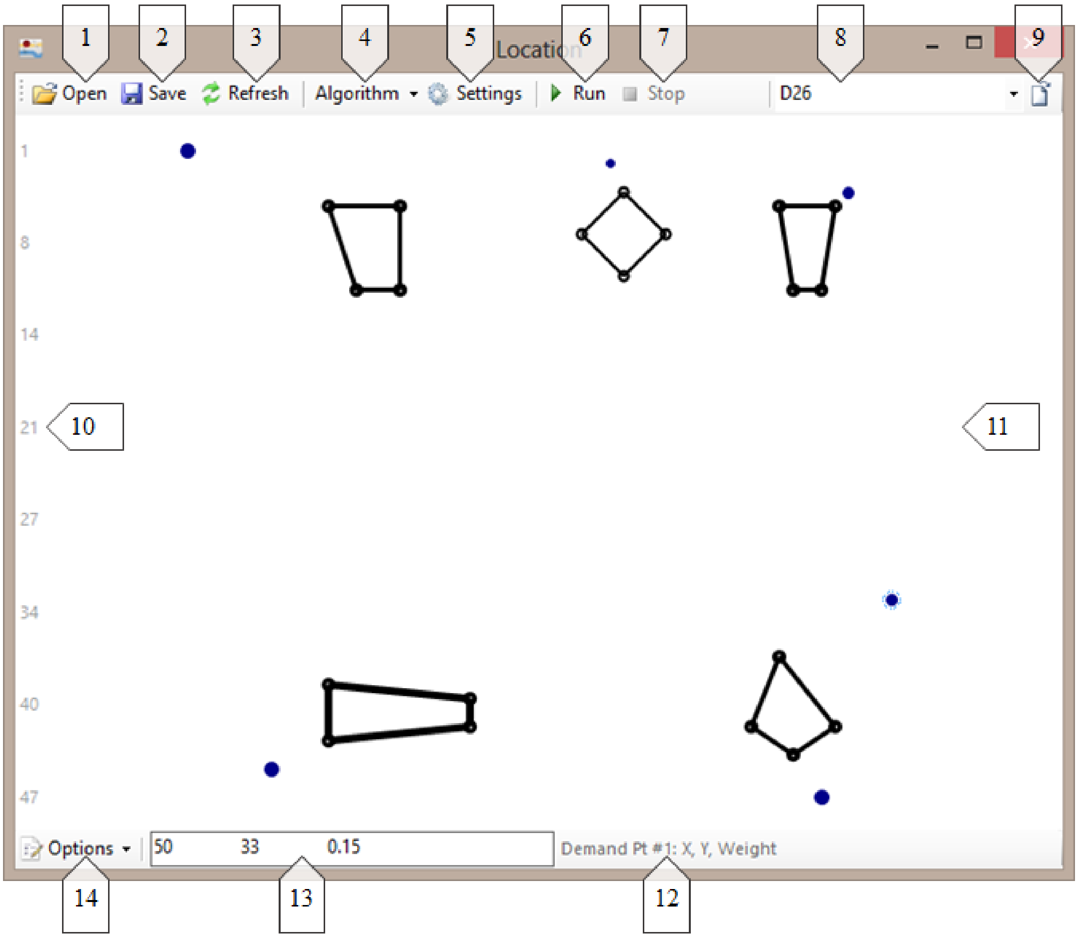


Figure D.1: A snapshot of the application's main window.

Figure D.1 displays the main window of the program.

The numbered objects in the window indicate:

1. Opening a problem instance file. If more than one file is selected, the programs considers them as a batch. Any text file can be opened as long as has the compatible format. Each line of the text corresponds to a point in the instance. All points are indexed starting from 0. Point data are separated by a 'tab' character.
   The data of a region vertex should be entered as follows. The region number \t the vertex number \t the *x*-coordinate \t the *y*-coordinate \t the fixed cost of the region. The vertex 0 is the first vertex followed by next adjacent vertex and so on. The last vertex is the other adjacent vertex of vertex 0. Where \t is the tab character. Providing information about number of the region and its cost in data line of the first vertex is necessary.
   For instance, the data first region of the D26 instance is entered in the instance file as follows (the header row provides information for the reader and it is not needed in the text file).

Region No.	Vertex No.	x	y	Fixed Cost
0	0	10	39	14
	1	20	40	
	2	20	42	
	3	10	43	

   The demand points are entered like: the demand number \t its *x*-coordinate \t its *y*-coordinate \t its weight. For example the first two demand points in D26 are entered as:

Demand No.	x	y	Weight
0	47	4	0.15
1	50	33	0.15

2. Save a problem instance. When changes on an instance is made, it can be saved as an instance file. The saved file contains data of the instance points and regions and information about instance at the end of the file. This in formation contains the minimum, maximum and range of *x* and *y*coordinates, the distance between two farthest points in the instance (called diameter), the scale of the instance, total number of vertices ($|V|$) and total number of points in the instance ($|N|$), the area of the convex hull of instance points, total area of the regions, and the percentage area of the instance occupied by the regions.
3. Refresh the panel. Clears the generated solutions on the panel, facility location and pathways. Any changes made on the instance can also be undone using this button.
4. Select an algorithm to run. The user can select some or all of three meta-heuristics to be implemented on the problem instance. Simulated annealing (SA), evolutionary algorithm (EA) and particle swarm optimization (PSO) are available heuristic algorithms to run.
5. Manage the settings of the runs and/or meta-heuristics. The user can adjust the settings for runs, like setting the number of replications, enabling or disabling variable neighborhood search (VNS) or repairing features, deciding whether or not to save log files regarding the generated solutions, setting the decimal precision for in calculations. Log files contain the details of generated solutions, parameter details of the applied meta-heuristic and computational times.
   Moreover, the user can access the parameter settings of the meta-heuristics and change the parameter values as needed. Meta-heuristics have different behaviors under different parameter adjustments. More than one level for each parameter can also be set. The algorithms run for all combinations of parameter levels (batch run).
6. Run the algorithm(s). The selected algorithm(s) can be applied if with this button.
7. Stop the currently running procedure.
8. The name of the problem instance is displayed in this box. Other available instances in the same

directory are also listed here.

9. Open the text file of the instance. The information of the instance points are available in that file, so the user can see or change the instance data.
10. A ruler for vertical axis. The minimum and maximum y coordinate values and the scale of the problem can be using the ruler.
11. Main Panel. Instance file is visualized in the panel. In Figure D.1, D26 instance is opened and shown in the panel. Demand points are shown by filled circles. The larger the circle, the more weight the demand has. Congested regions are the polygons with linear edges and circular vertices. Regions with thicker edges have higher fixed traveling costs.

    The user can select any object in this panel, i.e. a demand point, a vertex or an edge. The user can also selet a facility location after it is generated to see its coordinate and objective function value. For every selected object, the related information is displayed in the box at the bottom tool bar (numbered 13 in Figure D.1).

    Moreover, any object, like demand point or a region, can be deleted by selecting the object in the panel and pressing Delete button.
12. Shows which object is selected by user. The attributes of the selected object whose data is shown in the data box are also given. For example, in Figure D.1 it shows that demand point 1 is selected and the data box contains the information about its x coordinate, y coordinate, weight.
13. Data box. It shows the data of the selected object or generated final solution by the algorithms. If a demand point is selected this data contains its x coordinate t̂ y coordinate t̂ weight. If a region vertex is selected it shows x coordinate t̂ y coordinate t̂ the traveling cost of the region. If a region edge is seleted the region traveling cost is displayed. If a facility location is selected, its x coordinate t̂ y coordinate t̂ objective function value is give. When no object is selected data box shows x coordinate t̂ y coordinate of the cursor moving through the panel.

    The user can change any value shown in the data box. In Figure D.1 the data for demand point 1 is shown. If the user changes the x attribute of that point from 50 to, for example, 100 in the data box, the demand point 1 changes and moves from $(50, 33)$ to $(100, 33)$.
14. Options. Some options in this menu are graphical options like changing the display colors of the objects, or showing/hiding the generated solutions or facility locations and paths. Another options is Testing a location for a facility. When this option is selected, the programs waits for the user to click a position on the panel. Then, the program calculates the objective function value as if a facility is located at that position. Selecting a zone to start or restart algorithms is another options. With a rectangular zone, the user can limit the algorithms to generate solutions only in that zone. It is useful when the user wants to investigate a particular area for better solutions.

    This menu also contains options to change the instance. For instance, the user can remove all the demand points and distribute new demand points. In addition, the user can change the congested region traveling costs or sizes. There is also an option to replace non-convex regions by convex hull of them. The user can also change the scale of the instance by entering a number as the scale. All point location in the instance is then scaled between 0 and the entered number.

    Finally, in the Options menu, the type of the problem can be chosen as 1-median (minisum objective function) or 1-center (minimax objective function).

The software is developed for the context of this thesis. Although it can handle working on all the problems studied here, it is still under development for adding more features as well as considering the extensions to the problem discussed in Chapter 6.