A MIXED INTEGER SECOND ORDER CONE PROGRAMMING REFORMULATION FOR A CONGESTED LOCATION AND CAPACITY ALLOCATION PROBLEM ON A SUPPLY CHAIN NETWORK

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ABSTRACT

A MIXED INTEGER SECOND ORDER CONE PROGRAMMING REFORMULATION FOR A CONGESTED LOCATION AND CAPACITY ALLOCATION PROBLEM ON A SUPPLY CHAIN NETWORK

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Supply chain network design involves location decisions for production facilities and distribution centers. We consider a make-to-order supply chain environment where distribution centers serve as cross-docking terminals. Long waiting times may occur at a cross-docking terminal, unless sufficient handling capacity is installed. In this study, we deal with a facility location problem with congestion effects at distribution centers. Along with location decisions, we make capacity allocation (service rate) and demand allocation decisions so that the total cost, including facility opening, transportation and congestion costs, is minimized.

Response time to customer orders is a critical performance measure for a supply chain network. The decisions like where the plants and distribution centers are located affect the response time of the system. Response time is more sensitive to these decisions in a make-to-order business environment. In a distribution network where distribution centers function as cross-docking terminals, capacity or the service rate decisions also affect the response time performance.

This study is closely related to a recent work Vidyarthi et al. (2009) which models distribution centers as M/G/1 queuing systems. They use the average waiting time formula of M/G/1 queuing model. Thus, the average waiting time at a distribution center is a nonlinear function of the demand rate allocated to and the service rate available at the distribution center. The authors Vidyarthi et al. (2009) propose a linear approximation approach and a Lagrangian based heuristic for the problem.

Different than the solution approach proposed in Vidyarthi et al. (2009), we propose a closed form formulation for the problem. In particular, we show that the waiting time function derived from M/G/1 queuing model can be represented via second order conic inequalities. Then, the problem becomes a mixed integer second order cone programming problem which can be solved by using commercial branch-and-bound software such as IBM ILOG CPLEX. Our computational tests show that proposed reformulation can be solved in reasonable CPU times for practical size instances.

Keywords: Congestion, Cross docking, Second order conic programming

ÖZ

TEDARİK ZİNCİRİ AĞ TASARIMINDA BİR SIKIŞIK YER BELİRLEME VE KAPASİTE ATAMA PROBLEMİNİN KARIŞIK TAMSAYILI İKİNCİ DERECE KONİK PROGRAMLAMA İLE YENİDEN FORMÜLASYONU

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Tedarik zinciri ağ tasarım problemleri üretim ve dağıtım merkezlerinin yerlerinin belirlenmesi kararlarını içerir. Bu tezde siparişe üretim yapan bir tedarik zinciri ve çapraz yükleme işlevi gören dağıtım merkezlerinin bulunduğu bir problemi çözmeyi amaçlıyoruz. Yeterli yükleme indirme kapasitesi olmayan bir dağıtım merkezinde uzun bekleme süreleri oluşabilir. Bu çalışmada ele alınan yer belirleme problemi dağıtım merkezlerindeki bekleme sürelerini de dikkate alıyor. Çalışmada yer belirleme kararları kapasite ve talep atama kararları ile birlikte verirken tesis açma, taşıma ve dağıtım merkezlerindeki sıkışıklık maliyetlerinin toplamı minimize edilmeye çalışılıyor. Tedarik zinciri ağlarında siparişe yanıt süresi de önemli bir performans ölçüsüdür. Siparişe yanıt süresi üretim ve dağıtım tesisleri yer belirleme kararlarından etkilenir. Siperişe üretim yapan sistemlerde yanıt süresi bu kararlardan daha çok etkilenir. Ele alınan tipte dağıtım ağlarında dağıtım merkezinin kapasitesi ve işleme hızı da yanıt süresini etkiler. Bu tezde ele alınan problem Vidyarthi ve arkadaşları(2009) tarafından yapılan çalışmaya oldukça yakındır. O çalışmada dağıtım merkezleri M/G/1 kuyruk sistemleri olarak modellenmiştir. Yani, bir dağıtım merkezinde siparişlerin ortalama bekleme süreleri merkezin işleme hızı ve merkeze atanan talebin doğrusal olmayan bir fonksiyonu olarak modellenmektedir. Vidyarthi ve arkadaşları(2009) bu problemedoğrusal yaklaşıklama ve Lagrange temelli sezgisel algoritmalar önermişlerdir. Bu tezde Vidyarthi ve arkadaşlarından (2009) farklı olarak probleme kesin çözüm öneren bir formülasyon önerilmektedir. Dağıtım merkezlerinde M/G/1 kuyruk modelinin getirdiği toplam bekleme süresi fonksiyonunun ikinci derece konik programlama kısıtlarıyla ifade edilebildiği gösterilmiştir. Böylece çözülen problemin karışık tamsayılı ikinci derece konik programlama problemi olarak modellenebildiği ve IBM ILOG CPLEX gibi ticari dal-sınır yazılım paketleriyle çözülebilir olduğu gösterilmiştir. Yapılan hesaplamalı deneylerde gerçekçi boyutlarda problem örneklerinin makul sürelerde çözülebildiği gösterilmiştir.

Anahtar Kelimeler: Sıkışıklık, Çapraz yü kleme, İkinci derece konik programlama

This thesis is dedicated to my parents and who have supported me all the way since the beginning of my studies. Also, this thesis is dedicated to my Iranian friends who has been a great source of motivation and inspiration. Finally, this thesis is dedicated to all those who believe in the richness of learning.

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CHAPTER 1

INTRODUCTION

In this thesis, we consider a supply chain network design problem with congested distribution centers. We consider plant and distribution center location decisions along with capacity and demand allocations.
A supply chain includes all flows and transformations from the initial raw materials to the purchase of finished-items by the users. Each node of a supply chain network perform some activities such as manufacturing, product assembly or sales. These activities, however, necessitate logistical support, e.g., storage of intermediate or end goods, consolidation of orders for each consumer, and transportation.

Make-to-order (MTO) supply chain system is a business strategy which is applied in the cases of high product variety, variable customer demand, perishable products or obsolescence. Regarding to the strategic importance of response time in global business environment, MTO is a production process in which manufacturing resumes only after a customer's order is received. Due to extensive customization and competition, many firms adopt an MTO strategy to offer wide range of variety contrary to make-to-stock (MTS) supply chains in which the customer orders are met from stocks of finished products. The disadvantage associated with holding inventory of finished products may outweigh the advantage, particularly when we deal with products which become obsolete as technology advances or fashion changes. In addition, for many reasons, product and technology life cycles are getting shorter. Competitive market require more frequent product changes or innovation and consumers demand a greater variety of products than ever before.
Two critical decisions in supply chain network design are the location decisions for production plants and distribution centers. Transportation and inventory decisions are periodic and short term decisions which can be changed in response to external changes like demand variability. On the other hand, location decisions have long-term effects. Inefficient location choices made for plants and distribution centers can cause considerable surplus costs even the other factors like transportation or quantities of production are optimal. However, these long term decisions are subject to demand uncertainty at the time these decisions must be made. The collection of uncertainty in demand or capacity decisions and demand allocations, if not made carefully, may cause congestion in system or shortage in inventory which in turn make location decisions more critical.
The other challenge in supply chain management is the response time. Sule (2009) has performed a survey which supports the hypothesis that among the quality parameters of logistics the time is the most important one, even overtaking the price. A critical step to achieve targeted response time is the design of distribution network. The distribution network design addresses where to locate distribution centers, how much capacity to install and which demand points to be served from each distribution center. Distribution centers (DC) play major role in distribution networks. In fact, DC is a specific type of a warehouse. Frazelle (2002) refers DCs as distribution warehouses and defines them as facilities that accumulate and consolidate products from various manufacturing plants within a single firm, or from several firms, for combined shipment (economies of scale) to common customers. They

perform valuable functions which support the movement of materials. Storing goods (temporarily or longer), processing products, de-aggregating vehicle loads, creating SKU assortments, and assembling shipments are all activities commonly performed in these facilities. The main classification of DCs are: make-bulk/break-bulk consolidation terminal, a cross-docking center, a transshipment node, an assembly facility, a product fulfillment center or a returned goods depot.
Cross-docking (CD) is a strategy which appeared to cut the time items spend in the supply chain and reduce transportation cost. CD is a logistics technique applied in the retail and trucking industries to quickly consolidate shipments from separate sources and realize economies of scale in outbound transportation. There are three methods of CD (Burt, 2000):

- Manufacturing cross-docking: finished goods transfered from production line to a waiting truck or items produced are staged for later loading, are the categories.
- Distribution center cross-docking: consolidate inbound products from different suppliers which can be delivered when the last inbound shipment is received
- Terminal cross-docking: Products from DCs are dispatched to a break-bulk terminal for shipment of mixed loads to customers.

CD essentially eliminates the expensive inventory-holding costs of a warehouse, while still allowing short and temporary holding for consolidation and shipping functions. The idea is to transfer shipments directly from incoming (large scale) truck trailers to outgoing (small scale) truck trailers, without storage in between. Figure 1.1 shows schematic of CD centers (Yang et al. (2010)). With the process of moving shipments from the receiving dock (strip door) to the shipping dock (stack door), goods typically spend less than 24 hours in a cross-dock, sometimes even less than an hour.

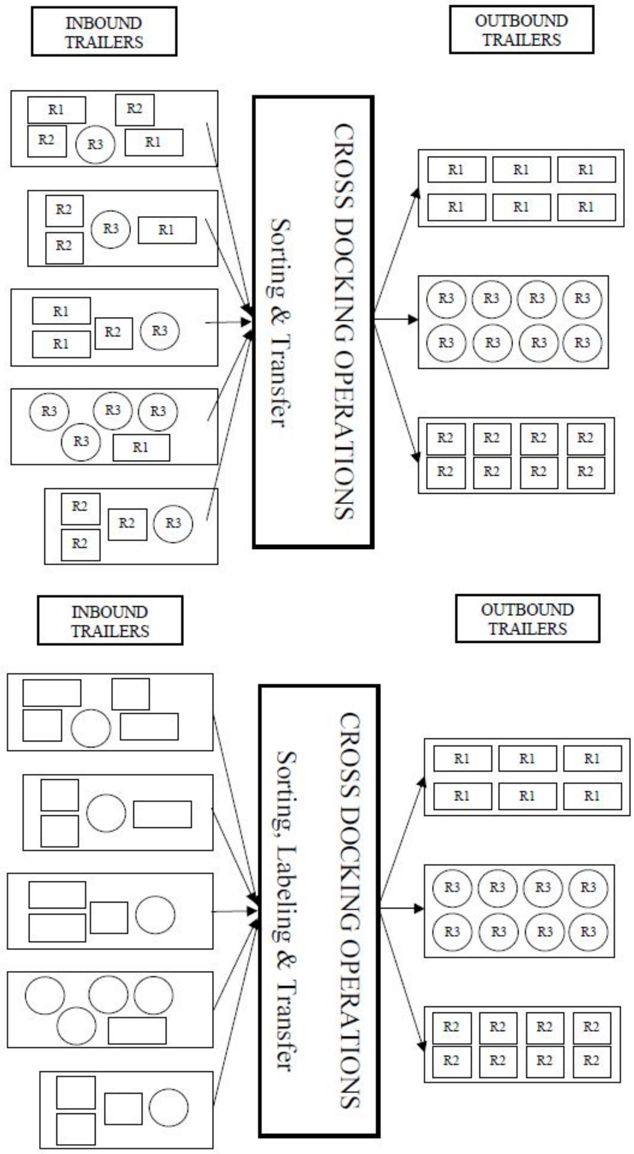

Figure 1.1: Cross docking center

The main advantages of utilizing CD are listed below:

- Elimination of activities associated with storage of products, such as incoming inspection, put-away, storage, pick-location replenishment, and order picking.
- Faster product flow and improved customer service.
- Reduced product handling.
- Cuts in inventory.
- Lower costs due to elimination of the above-mentioned activities.

But, in practice, both plant and CD centers face congestion for various reasons. Congestion in a distribution center may have several reasons. Below we list some of them which are observed especially in CD centers:

- **Interference among forklifts**: When a forklift makes a delivery to a stack door it must turn and maneuver its way in. Since loads are frequently bulky and hard to manipulate or carry, and there is usually freight sitting in the center of the dock, the forklift blocks each other trying to pass by that stack door. This phenomenon, usually referred as so-called interference, is most noticeable on docks that are operating close to capacity.
- **Dragline congestion:** a worker interacts with the dragline by pulling empty carts off the line and placing full carts on the line. Depending on the number of full and empty carts passing his door, he has to wait during either of these operations.
- **Congested floor space:** Sometimes workers, because of shipment consolidation cannot load a shipment directly into a stack door, but must park it temporarily on the floor nearby. Undoubtedly the existence of congestion in system increases response time which results higher inventory cost and lower customer attraction. Innumerous papers have been published in hierarchical congestion, hub congestion, emergency service congestion, distribution network congestion aiming to control congestion effect. In this thesis we consider the effect (cost) of distribution network congestion in MTO supply chain.

The congestion in a CD center can be controlled by either changing rate of service (capacity decision) or the amount of demand allocated. In fact, in congestible systems, there is the tradeoff between rate of service and demand allocation.

1.1 Contribution

In this thesis, we design a location and allocation model for supply chain network design. The decisions to be made are locations of plants, locations and capacities of CD centers and the assignment of customer demand to plants and CD centers. The objective is to minimize the total cost, which includes facility opening costs, transportation costs, and congestion costs. We assume that a customer's demand can be assigned to multiple plants and a single CD center. The CD centers are modeled as spatially distributed queues with Poisson arrivals. We assume the CD centers function as M/G/1 servers (to capture the dynamics of the response time). The model is a mixed integer nonlinear programming

problem and presence of the congestion function makes it hard to solve.
We show that the problem can be reformulated as a Mixed Integer Second Order Cone Programming problem (MISOCP). This, in contrary to previous studies which propose linear approximation and heuristic algorithms (Vidyarthi et al. (2009) and Huang et al. (2002)), allows us to solve the problem using a commercial software which can solve subproblems using Second Order Conic Programming (SOCP) algorithm. To the best of our knowledge, this is the first exact approach for the plants and distribution center location with congested distribution centers.

The paper is organized as follows. In chapter 2, we present literature review. In Chapter 3, we present the mathematical model for the problem and analyze M/G/1 waiting time function and its characteristics. Then we develop a SOCP reformulation for the congestion (total waiting time) function. Finally, in Chapter 4, we give the computational results.

CHAPTER 2

LITERATURE REVIEW

In this thesis, we study a supply chain network design problem with facilities in high level, CD centers in middle level and customers in downstream. The objective is to minimize the total cost including fixed cost, transportation cost and congestion cost by finding the best locations for facilities and CD centers with appropriate handling capacities and assigning customer demand to facilities and CD centers. In this section, we give literature review, we consider five major groups of work: supply chain network design, DC location problem, structure of CD center, congestion (waiting time) and SOCP.

2.1 Supply Chain Design and Hierarchical System Design

The supply chain design problems have received significant attention from different aspects. The range of literature in this area is widespread but because location decisions play a critical role in the strategic design of a supply chain network most of the studies pertain location and allocation decisions. Melo et al. (2009) provide literature review of facility location models associated with supply chain management.

Olivares Benitez et al. (2010) introduce a bi-objective optimization problem for two echelon supply chain system where cost and time related objectives are considered. The most important feature added to the problem is to consider transportation mode as a decision to be made. Each mode represents a specific type of service with different costs. In our problem, transportation mode is not a decision and moreover, we consider waiting time cost.

One important aspect of supply chain design problems is the existence of different layers or echelons in the system, In other words, there exists a hierarchy in the system therefore we can say every multi layer supply chain is a hierarchical system. In our study, we have two interacting layers, facilities and CD centers. Each has a specified role in the system. Sahin and Sural (2007) classify the hierarchical facility location problems according to the features of the systems studied. They group them according to the flow pattern considered, service availability at each level, and spatial configuration of services. Moreover they investigate the applications, MIP models, and solution methods presented for the problem. Jayaraman et al. (2003) propose an integer programming model that would solve a comprehensive hierarchical problem to locate service facilities. The objective of the model is maximizing demand coverage while number of facilities is given and allocates different levels of service to the open facilities and also discuss some of the contributions to the current state-of-the-art in design of distribution systems.

Similar to above models, we define a hierarchical system which consists of facilities as upstream level and CD centers as downstream level. We assign each customer to multiple plants, but all demand goes to customers through a single CD center. We, like most of the other studies, consider probabilistic demand in system which is more realistic. Demand is assumed to follow Poisson distribution.

2.2 Distribution center location problem

DC's are the foundation of a supply network. They, as middle layer of supply chain, has considerable effect on total logistic costs and timely service. Therefore DC related problems especially location problems have received special interest. A handful of publications address models and solution approaches for DC location problems. Higginson and Bookbinder (2005) explain the DC applications and different roles they can play in a supply chain.

Nozick and Turnquist (2001) integrated inventory and location decisions for DCs. They developed a model to find optimal location of DCs and to determine the minimum inventory level necessary to ensure a specified stockout probability for a given product. Their target is to retain service level by stocking optimal amount of safety stock in the best location.

Klose and Drexl (2005) illustrated different models for single stage capacitated and uncapacitated facility location problems (CFLP and UFLP) and then extended to two echelons cases (plant and depot) where fixed costs of both levels and transportation cost are considered. In addition, they discussed dynamic model of UFLP in which a given planning horizon is divided into two periods and all the fixed costs changed accordingly. Two echelons (plants and DC) case is the scenario that we consider in this thesis. The facilities have known capacity and CD centers have known service rates.

Our model deals with distribution network design problem for a two stage single product supply chain model similar to what Sourirajan et al. (2007) present. Their model seeks for DC locations such that lead time, including make up, replenishment and congestion times, is minimized and risk pooling benefits are maximized. They propose a Lagrangian heuristic. For congestion in DC, they assume M/M/1 queuing system whereas we consider M/G/1 system.

Aghezzaf (2005) presents two MIP models, one for the deterministic case and another for the robust optimization case. He considers strategic capacity planning and warehouse location problem. He integrates the issue of capacity expansion with distribution location. To incorporate demand uncertainty in the capacity expansion and warehouse location plans he utilized the concept of robust optimization then proposed a Lagrangian decomposition method. Similarly, our problem considers integration of capacity in CD centers locations.

In most of the aforementioned work, DCs behave as warehouse in which commodities are temporarily stored. The companies prefer to hold products as close as possible to customers to be able to satisfy demand faster and remain in competition. The DCs are also considered as assembly facilities to cover more customers taste. We consider DC for cross docking. In CD centers items are sorted and sent but not stored so there is no inventory cost considerations.

2.3 CD center Location Problem

A DC can be called a warehouse, a fulfillment center, a cross-dock facility, a bulk break center, and a package handling center. The name by which the distribution center is known is commonly based on the purpose of the operation. In this research area, existing work usually focus on two aspects: operational issues and the importance of the CD technique.

Galbreth et al. (2008) describe a multi-echelon supply chain in which both direct shipments and CD centers are available to move products from the manufacturer to customer locations. In their model, products can be shipped by truckload from a single supplier to a CD center. They propose a model for a single supplier and multiple CD centers supply chain. To show total costs saved by having the CD centers they compare the total costs of two supply chains, one with CD facilities and one without. The supply chain structure in this paper is similar to our design. Differently, we assume stochastic demand and in addition they consider fixed and transportation costs only whereas we also consider the congestion effect.

CD are sometimes called just in time distribution because most shipments spend less than 24 hours in CD centers. Napolitano (2000) and Bookbinder and Locke (1986), Shuib and Fatthi (2012), Vogt (2010) considered CD centers in supply chain management and propose some methods to design them optimally to improve operation. Yang et al. (2010) investigate the effects of various factors on the operation of CD centers. Computer simulation was done to monitor the travel and congestion time. The most comprehensive paper about congestion in CD center is Bartholdi and Gue (1999). They propose different congestion function for different parts of CD centers.

Despite the appealing affect of CD centers in supply chain network, there are few literature which consider CD centers in this area. These papers only cover CD centers location whereas our study consider location and capacity of them simultaneously and balance flows among them by adding congestion cost.

2.4 Waiting Time and Congestion

The waiting time is a result of congestion in a system. It has negative effect on customer's utility and hence, on a company's demand. There are different classes of literature which consider congestion in different ways. For instance, in demand capturing problems, customers are distance and time (congestion) sensitive. The relevant motivations are health and emergency services, banking or ticket selling centers. Usually, waiting time costs are calculated either by adding service level or queuing theory functions.

The studies in the literature, which consider service level, usually use Hillier and Liberman (1986) probability function of number of customers in system. They give different elasticity coefficients to distance and time to define specific levels of service. Marianov and Serra (2002) incorporate service level constraints to their model. The problem is to trade off between investment, operating cost and service quality. They chose heuristic concentration method (HC) for large instances. Sliva and Serra

(2007) consider a new version of demand capturing problem which not only takes into account the effect of traveling time but the waiting time on the market share. They propose metaheuristics which offer accurate results within acceptable computing time. They solved problem by ant colony optimization approach and because of time limitation of the methodology they used concentration algorithm for larger problems.

Marianov (2003) formulate a model for locating multiple server, congestible facilities. They control congestion in system by demand equilibrium constraint which is a nonlinear function of demand rate. They utilize traditional Lagrangean relaxation plus an iterative procedure. In Marianov et al. (2005), the goal is to maximize the number of people who travel to centers and stay in line until inoculated. They consider M/M/s/K queuing system and use the same demand equilibrium constraint as Marianov (2003). Obviously, the usage of Hillier and Liberman (1986) probability function of number in system increase computation times dramatically.

Marianov et al. (2008) present a model to maximize market share captured by the entering companies. They assume facilities function as M/M/m/K and customers are sensitive to distance and waiting time. To find shares in market they use logit function of cost which consists of convex combination of travel time and waiting time. Because of the probabilistic expression that model the number of customers captured (Hillier and Liberman, 1986), most of the constraints are nonlinear and complicated. They suggest ad hoc heuristics to solve the problem. All of these studies model congestion in facility location problem.

Aboolian et al. (2008) present the problem of locating facilities and allocation of servers on a congested network in order to reduce the costs of fixed installation, variable server, travel time and waiting time in the facilities act as M/M/K server. They proposed two heuristic approaches, descent approach and simulated annealing. In our model, allocation of service units is based on the service rate μ whereas they assume they can control the number of servers.

Berman and Drezner (2006) investigate the problem of locating a given number of facilities which can serve no more than a prespecified number of users at the same time. The goal is to maximize the number of customers captured. Unlike the other papers which use continuous variables to show proportion of arrival demand to each facility, they suggest following new form of waiting time function:

$$w(x) = \frac{1}{\mu - \sum_{i=1}^{n} \lambda \exp(-\alpha d(x,i))} \leq UB$$

which is convex. They reformulate problem as capacitated facility location problem without fixed charge and compare exact (Cplex), Ascent algorithm, Simulated annealing and Tabu search approaches.

Castillo et al. (2009) consider two capacity choice scenarios, choosing a service rate for the servers and choosing the number of servers for the optimal location of facilities (M/M/s) which influence both the travel time cost and the waiting time of customers. The policy which they followed is replacing congestion term in the objective function by a simpler one and then applying a Lagrangean heuristic. A unique feature of their model is to deal with the social optimum rather than a user equilibrium. In another scenario with multiple servers, they approximate the number of servers by a continuous variable.

Marianov and Serra (2011) propose a multi server model for fixed number of facilities. They use famous standard equations for an M/M/s/K queuing system (Hillier and Liberman, 1986). The objective is Min(Z(1) , Z(2)) where Z(1) and Z(2) are travel and congestion cost respectively. The presented model is a combinatorial, nonlinear optimization problem. They suggest a metaheuristic, the Max-Min Ant system, to obtain an initial solution. Then they use tabu search to improve the initial solutions.

In hierarchical systems, congestion may occur in different levels. Therefore, in some models, service constraints are incorporated to keep service level at a desirable level. The goal of these models is to find the minimum number of servers and their location which will cover a given region with distance or acceptable waiting time. Marianov and Serra (2001) control service level by adding probabilistic constraints whereby the probability of a customer standing in a line with b other customer is at most α, finally they apply a bi-level heuristic approach.

Another important class of network which may encounter congestion is hub-and-spoke network. Elhedhli and Hu (2005) proposed a model for uncapacitated single assignment p-hub location problem. In their model instead of using waiting time expression, a nonlinear power-law cost function was incorporated. Linearization and then Lagrangean heuristic were applied. Marianov and Serra (2003) analyze the queue formed by airplanes waiting for landing. To control congestion they insert probabilistic equations which bound the probability of the event that more than b airplanes on queue. They give a two phase heuristic approach, first greedy adding heuristic to find the initial set of p locations and second a one-opt exchange heuristic.

The capacity investment problem is another research area which take into account congestion cost. Rajagopalana and Yub (2001) address internal congestion in factories. Machines were modeled as nodes which act as M/G/1 queues. They include a constraint to guarantee that lead time satisfy target service level with prespecified probability, similar to service level constraint proposed in above papers. Although the research topic is different, the idea in this paper is close to our study in the sense that both consider tradeoff between fixed cost and congestion cost.

One similar paper to our model is Huang et al. (2005). They model a specific type of distribution network in which flows between origin and destination node must pass through connections which are congestible. Although the framework is similar to the one we use (three echelons system with congestion in the middle echelon), their solution approach is different. They consider M/G/1 server and use mean service time and second moment of service in congestion terms and set them as variable to find capacity in each connection. The resulting model is not convex or concave and they apply heuristics (Outer approximation and Lagrangean approaches). Differently, we reformulate the congestion terms and write them in convex form and then cast them as SOCP inequalities.

Model in Vidyarthi et al. (2009) is similar to our work. The objective of this paper is to model MTO and ATO supply chain with congestible DCs. Our work differ in two aspects. First one is the supply chain definition: they consider designing an MTO system, where manufacturing plants produce the wide range of disassembled items then ship them to DCs and after receiving orders the items are assembled according to customers expectations. In our model, orders are received in facilities and finished products are sent to customers through CD centers. Second one is the solution approach: in MTO case they suggest linearization method for M/G/1 term in the objective function. This is an approximation approach and may require a large number of linear inequalities to add. For ATO case they give a Lagrangian heuristic approach.

Despite the difference in basic definitions, our formulation is roughly the same. We show that the congestion function of M/G/1 model used by Vidyarthi et al. (2009), can be represented by SOCP inequalities. Hence, we show that the problem can be solved to optimum using MISOCP solvers instead of using approximation and heuristic methods. In their work, for MTO system the computational results reveal that the cutting plane algorithm provides optimal solution up to moderate instances in reasonable cpu time and for ATO system, the heuristic solution is on average within 6% of its optimal.

2.5 Second order cone programming

SOCP is a relatively new area in optimization and classified in convex optimization problem. An extensive review on SOCP is by Alizadeh and Goldfarb (2003).They present an overview of the SOCP problem and show SOCP form for LP, QP, quadratically constrained QP, and other classes of optimization problems. Lobo et al. (1998) recast different families of problem as SOCP and describe an efficient primal dual interior-point method for solving SOCP.

Gürel (2011) considered a multi-commodity network flow problem. The problem involves tradeoff between the total congestion and the capacity expansion costs on a given network. He estimated congestion on an arc by a convex increasing power function of the flow on it, then formulated problem as MISOCP problem and solved using Cplex. Atamtürk et al. (2012) studied joint facility location and multi-commodity inventory management problems with stochastic demand in uncapacitated and capacitated facilities case. They show how to formulate these problems as MISOCP. Valid inequalities were added to strengthen the model and improve the computational results.

Günlük and Linderoth (2008) describe the convex hull of mixed integer set. They show that for many classes of problems, the convex hull can be expressed via conic quadratic constraints, and can be solved via SOCP. They illustrated their approach on quadratic facility location and network design with congestion. They also show that congestion function of M/M/1 queuing model can be represented via SOCP inequalities.

In this thesis, we show that a congestion function based on M/G/1 model can be represented as SOCP inequalities and practical size problems can be solved in reasonable CPU time

2.6 Summary

As observed, the congestion phenomenon was studied in different contexts. Some papers consider congestion as part of service level, some present it as cost term in the objective function. Congestion is usually modeled as a nonlinear function and usually hard to deal with in mathematical programming models.
Studies in the literature often focus on how to model congestion. They apply heuristic algorithms to

solve the problem, in particular, for M/G/1 case to the our best knowledge no exact solution is proposed so far. Contrary to previous studies, we focus on reformulating the waiting time expression for an M/G/1 model and present a new formulation. Gürel (2011), Günlük and Linderoth (2008) and Atamtürk et al. (2012) consider SOCP reformulation in different area.

CHAPTER 3

PROBLEM DEFINITION

In this thesis, we study a plant and DC location problem in a MTO supply chain system. Along with location decisions for plants and DCs (CD centers), we consider capacity and demand allocation decisions to minimize the overall cost of the system. We consider a system where the demand location and demand rates are given. We need to find where to place plants and DCs and the capacity level to install at each DC. The objective to minimize is the sum of costs of opening plants, capacity of DCs, transportation and the congestion.
In our model, we suppose following supply processes: Plants receive orders and after the production, consolidate with other orders and transmit them, in large cargoes, to DCs for sorting and distribution.

We assume demand is concentrated at cities and we consider each city as a demand point. City k has a demand rate λ_k and the demand follows a Poisson distribution. There are two different sets of potential locations for plants and DCs. Furthermore, we assume a supply chain system which behaves like a referral hierarchy system in which a user cannot go to higher level server unless a low level server refers them to it (Narula, 1985), in other words, customers cannot receive their orders directly from plants.

While making location decisions we also consider capacity levels and demand allocation decisions. We decide which customer (demand point) will be served from which plants through which DC. We model the system in the following way: for the plants, requests of service are the union of all the requests for orders of all customers in its assignment set. Hence, demand in a plant can be viewed as a stochastic process, equal to the sum of several Poisson processes. By superposition property, this stochastic process known to be a Poisson process. Our problem involves the decision of which customer will be served by which plants and DCs. Therefore, we include a decision variable z_{ijk} denoting the proportion of customer k (λ_k) assigned to plant i and distributed from CD center j. Thereby, total customer demand assigned to plant i is:

$$\sum_j \sum_k \lambda_k z_{ijk}$$

In order to determine the input intensity of the DCs (second echelon) and also flow distribution between the first and the second echelon, we first recall the equivalence property for $M/M/1$ and $M/M/m$ queuing systems (Larson and Odoni, 1981). According to this property, if the system has an infinite (or large enough) queue capacity, and an arrival process of intensity of $\sum_j \sum_k \lambda_k z_{ijk}$, under steady state conditions, the departure process is also a Poisson process with the same intensity. Since, the input rates to DCs are sum of several Poisson departure rates from plants, according to Poisson superposition property (only some of the event being counted where this selection is made at random), we can conclude that the arrival rate to the DC j is also a Poisson process. Consequently, for DC j the arrival rate is:

$$\sum_i \sum_k \lambda_k z_{ijk}$$

The customers' orders arriving at the DCs are met on a First Come First Serve (FCFS) basis. We assume that each DC operates as a single flexible capacity server with infinite buffers to accommodate customer orders waiting for service. Under all these mentioned assumptions the DCs are modeled as a M/G/1 with service rates proportional to their capacity levels. We define discrete levels for capacity decisions with different costs.

3.1 Mathematical model

In this section, we give the mathematical formulation of the problem. We start with the notation used in the remaining part of the thesis. Initially, we write the nonlinear congestion function ($F(z, y)$) in the general form to make model more understandable, and later we present the explicit form of it.

Indices and parameters:

i	: index for plants $i = 1, 2, \ldots, I$
j	: index for DCs $j = 1, 2, \ldots, J$
k	: index for customers $k = 1, 2, \ldots, K$
F_i	: fixed cost of opening plant at location i
f_{js}	: fixed cost of opening DC j and acquiring capacity level s (\$/period)
c_{jk}	: unit transportation cost of serving customer k from DC j (\$/unit)
λ_k	: mean demand rate for the product from customer k (units/period)
C_{ij}	: unit cost of sending the product from plant i to DC j (\$/unit)
t	: mean response time cost per unit time per customer (\$/period/customer)
μ_s	: mean service rate, if capacity level s is allocated to a DC
μ^F	: mean service rate of plant
$G_j(\vec{Z}_j, \vec{Y}_j)$	: congestion function of DC j

Decision variables:

z_{ijk}	= fraction of demand k produced in plant i and distributed from DC j
y_{js}	= 1 if DC j is opened and capacity level s is acquired, 0 otherwise
x_i	= 1 if plant i is opened, 0 otherwise
w_{jk}	= 1 if customer k is assigned to DC j, 0 otherwise
$\vec{Z}_j$	: $\{z_{ijk} : \forall i, k\}$
$\vec{Y}_j$	: $\{y_{js} : \forall s\}$

$$\textbf{Min}: \sum_i F_i x_i + \sum_j \sum_s f_{js} y_{js} + \sum_j \sum_k c_{jk} \lambda_k w_{jk} + \sum_i \sum_j \sum_k C_{ij} \lambda_k z_{ijk} + t \sum_j G_j(\vec{Z}_j, \vec{Y}_j)$$

$$\text{s.t.} \quad \sum_i z_{ijk} = w_{jk} \qquad \forall j, k \tag{3.1}$$

$$\sum_i \sum_k \lambda_k z_{ijk} \leq \sum_s \mu_s y_{js} \qquad \forall j \tag{3.2}$$

$$\sum_s y_{js} \leq 1 \qquad \forall j \tag{3.3}$$

$$\sum_j w_{jk} = 1 \qquad \forall k \tag{3.4}$$

$$\sum_j \sum_k \lambda_k z_{ijk} \leq \mu^F x_i \qquad \forall i \tag{3.5}$$

$$w_{jk}, x_i, y_{js} \in \{0, 1\} \qquad z_{ijk} \in [0, 1]$$

The first and the second terms in objective function are the fixed costs of opening plants and DCs, respectively. The third term is the transportation cost from DCs to customers and the fourth term is the transportation costs from plants to DCs. The last term represents the congestion cost in the system. The t value can vary from customer to customer, but we assume that, it is the same across customers.

Constraint (3.1) ensures that demand of customer k is produced by the plants and transported through a selected DC. Constraint set (3.2) retains the steady state for DCs. In stochastic systems, when the system is in the steady state, then the probabilities that various states will be repeated will remain constant. Constraints (3.3) ensure that for a DC at most one capacity level is selected. Constraints (3.4) guarantee that every customer is assigned to a DC. Constraints (3.5) give the capacity limitation for each opened plant. To reduce the congestion and transportation costs we would open more plants which imposes more fixed cost. The model without $\sum_j G_j(\vec{Z}_j, \vec{Y}_j)$ is a simple hierarchical location problem which is a linear mixed integer programming model. We next discuss the congestion cost $G_j(\vec{Z}_j, \vec{Y}_j)$ that we use in our model.

3.2 The congestion function $G_j(\vec{Z}_j, \vec{Y}_j)$

$G_j(\vec{Z}_j, \vec{Y}_j)$ is a nonlinear function representing the expected total waiting time at DC j, assuming the service times at each DC follow a general distribution M/G/1. The following notations are used for each DC: $\mu_j = \sum_{s=1}^{S} \mu_s y_{js}$ is the mean service rate and $\sigma_j^2 = \sum_{s=1}^{S} \sigma_{js}^2 y_{js}$ is the variance of the service rate. Assume X is independent and identically distributed (i.i.d) random variables denoting service time of a customer, then $E[X] = \tau_j = 1/\mu_j$ represents mean service time. The $E[X^2]$ is the second moment of service time, ρ_j is utilization value ($\rho_j = \lambda_j/\mu_j$) and CV_j^2 is squared coefficient of variation of service time ($CV_j^2 = \sigma_j^2/\tau_j^2$).

Under steady state and FCFS conditions, the average waiting time (including the service time) at a DC j is given by:

$$E[w_j(M/G/1)] = \frac{\Lambda_j E[x^2]}{2(1 - \Lambda_j E[x])} + E[x] \tag{3.6}$$

Where $\Lambda_j = \sum_i \sum_k \lambda_k z_{ijk}$. One possible way to present waiting time in M/G/1 queue system is the above expression. In the following proposition we show the above formula is convex.

Proposition 1 *$E[w_j(M/G/1)]$ is convex in z if steady state condition in constraint (3.2) holds.*

Proof. The denominator is always positive if (3.2) holds, then the denominator is concave. Numerator is convex and from result in Bector (1968), we can conclude that function F is convex. ■
But in our model capacity level is a decision to be made, i.e, $E[X^2]$ and $E[X]$ are decision variables. As a result, the function is not convex anymore. To use the waiting time formula in (3.6), we utilize the Pollaczek - Khintchine (PK) formula:

$$E[w_j(M/G/1)] = \frac{\rho + \Lambda\mu \mathrm{Var}(S)}{2(\mu - \Lambda)} + \mu^{-1}$$

Now by using $CV_j^2 (= \sigma_j^2 \mu_j^2$ (Vidyarthi et al. (2009)), we come up with the following formula:

$$E[w_j(M/G/1)] = \left(\frac{1 + CV_j^2}{2}\right)\frac{\tau_j \rho_j}{1 - \rho_j} + \tau_j = \left(\frac{1 + CV_j^2}{2}\right)\frac{\Lambda_j}{\mu_j(\mu_j - \Lambda_j)} + \frac{1}{\mu_j}$$

To obtain the expected total waiting time in the entire system, $G_j(\vec{Z}_j, \vec{Y}_j)$, $E[w_j(M/G/1)]$ is multiplied by the total demand rate from plants arriving to DC j and all resulting terms are summed together:

$$\begin{aligned} G_j(\vec{Z}_j, \vec{Y}_j) = 1/2 \sum_j \Bigg[\left(1 + \sum_s CV_{js}^2 y_{js}\right) & \frac{\sum_i \sum_k \lambda_k z_{ijk}}{\sum_s \mu_s y_{js} - \sum_i \sum_k \lambda_k z_{ijk}} \\ + \left(1 - \sum_s CV_{js}^2 y_{js}\right) & \frac{\sum_i \sum_k \lambda_k z_{ijk}}{\sum_s \mu_s y_{js}} \Bigg] \end{aligned} \tag{3.7}$$

In (3.7), we assume, there are different squared coefficient of variation of service time, CV, for every pair of (j,s), i.e. CV_{js}. The $M/G/1$ expected waiting term is still not convex because of variable y in the denominator. We introduce the decision variable z_{ijks} which is the fraction of demand k satisfied by plant i and distributed through DC j with capacity s. The congestion function in (3.7) becomes:

$$\begin{aligned} F(\vec{Z}_j) = 1/2 \sum_j \sum_s \Bigg[(1 + CV_{js}^2) & \frac{\sum_i \sum_k \lambda_k z_{ijks}}{\mu_s - \sum_i \sum_k \lambda_k z_{ijks}} \\ + (1 - CV_{js}^2) & \frac{\sum_i \sum_k \lambda_k z_{ijks}}{\mu_s} \Bigg] \end{aligned} \tag{3.8}$$

In the next proposition we prove (3.8) is a convex function and then mathematical model will be presented to include the new decision variables and the new objective function.

Proposition 2 *Function $F(\vec{Z}_j)$ given as* (3.8)*, is a convex function when $\mu_s > \sum_i \sum_k \lambda_k z_{ijks}$.*

Proof. By introducing the new variable z_{ijks} in place of z_{ijk} and adding:

$$z_{ijks} \leq y_{js} \quad \forall i, j, k, s$$

to the constraint sets, we don't need variable y_{js} anymore because if DC j is not opened then, $y_{js} = 0$, according to (3.2), and no flow passes through that center i.e. $\sum_i \sum_k \lambda_k z_{ijks} = 0$. Thus average waiting time for that center will be 0. On the other hand, due to adding capacity index to variables z_{ijk}, each term in (3.8) represents congestion term in DC j with capacity setting s. As a result, nonconvex fractional term in (3.7) turn to general following form:

$$\frac{x}{c-x} \quad \text{(c is constant)}$$

Where $x = \sum_i \sum_k \lambda_k z_{ijks}$. When $c > x$, the second derivative is positive and the function is convex. ■
Now the model with new variables and constraints is as follows:

$$\textbf{Min}: \sum_i F_i x_i + \sum_j \sum_s f_{js} y_{js} + \sum_j \sum_k c_{jk} \lambda_k w_{jk} + \sum_i \sum_j \sum_k \sum_s C_{ij} \lambda_k z_{ijks}$$
$$+t/2 \sum_j \sum_s \left[(1 + CV_{js}^2) \frac{\sum_i \sum_k \lambda_k z_{ijks}}{\mu_s - \sum_i \sum_k \lambda_k z_{ijks}} + (1 - CV_{js}^2) \frac{\sum_i \sum_k \lambda_k z_{ijks}}{\mu_s} \right]$$

$$\text{s.t.} \quad \sum_i \sum_s z_{ijks} = w_{jk} \qquad \forall j,k \qquad (3.9)$$
$$\sum_s y_{js} \le 1 \qquad \forall j \qquad (3.10)$$
$$\sum_i \sum_k \sum_s \lambda_k z_{ijks} \le \sum_s \mu_s y_{js} \qquad \forall j \qquad (3.11)$$
$$z_{ijks} \le y_{js} \qquad \forall i,j,k,s \qquad (3.12)$$
$$\sum_j w_{jk} \le 1 \qquad \forall k \qquad (3.13)$$
$$\sum_j \sum_k \sum_s \lambda_k z_{ijks} \le \mu^F x_i \qquad \forall i \qquad (3.14)$$
$$w_{jk}, x_i, y_{js} \in \{0,1\} \qquad z_{ijks} \in [0,1]$$

3.3 SOCP representation of model

Recent developments in SOCP and the available commercial software allow us to solve a variety of convex problems by using SOCP inequalities.
In the most simple form, a SOCP is a convex optimization problem of the form:

$$\begin{aligned} &Min \quad f^T x \\ &s.t \quad \|A_i x + b_i\| \le c_i^T x + d_i \qquad i = 1,..,N \end{aligned}$$

Where $x \in \mathbb{R}^n$ is a variable of the problem, $f \in \mathbb{R}^n$ are scalars and $A_i \in \mathbb{R}^{(n_i-1)\times n}$, $b_i \in \mathbb{R}^{(n_i-1)}$, $c_i \in \mathbb{R}^n$ and $d_i \in \mathbb{R}$ are parameters. The norm in constraint is the standard Euclidean norm which is called SOCP constraint. Generally, the definition of standard second-order (convex) cone of dimension k is:

$$\xi_k = \left\{ \begin{bmatrix} u \\ t \end{bmatrix} | u \in R^{k-1}, t \in R, \|u\| \le t \right\}$$

A n-dimensional convex set C is SOCP representable if, possibly after introducing auxiliary variables, it can be represented by a number of SOCP constraints and also function f is SOCP representable if its epigraph$\{(x,t)|f(x) \le t\}$ has a SOCP representation. In other words, if in the problem:

$$\begin{aligned} &Min \quad f(x) \\ &s.t \quad x \in C \end{aligned}$$

f and C are SOC representable, then convex optimization can be reformulated as an SOCP and solved via algorithm available in most commercial optimization software (more information in Alizadeh and Goldfarb (2003), Tal and Nemirovski (2001) and Lobo et al. (1998)). In our model, we have linear constraints which are a special case of SOCP (Alizadeh and Goldfarb, 2003). But, $f(x)$ consists congestion cost terms.

In the next proposition, we explain how we can cast expected waiting time for M/G/1 system as SOCP.

Proposition 3 *Expected total waiting time of M/G/1 queue as given in* (3.8)*, is SOCP representable.*

Proof. (3.8) includes linear and nonlinear terms. To prove SOCP representability we will consider the nonlinear parts. To this end, we define an auxiliary variable S_{js} where:

$$S_{js} \geq \frac{\sum_i \sum_k \lambda_k z_{ijks}}{\mu_s - \sum_i \sum_k \lambda_k z_{ijks}} \qquad \forall j, s \tag{3.15}$$

then the nonlinear term in the objective function will be replaced by S_{js}. The new total expected waiting time for entire system will be:

$$1/2 \sum_j \sum_s \left[(1 + CV_{js}^2) S_{js} + (1 - CV_{js}^2) \frac{\sum_i \sum_k \lambda_k z_{ijks}}{\mu_s} \right]$$

For simplicity, define $R_{js} = \sum_i \sum_k \lambda_k z_{ijks}$. We will also drop indices of decision variables. We first multiply both sides in (3.15) with μ:

$$S\mu(\mu - R) \geq \mu R$$

We add R^2 to both side:

$$R^2 + S\mu(\mu - R) \geq R^2 + \mu R$$

$$R^2 \leq S\mu(\mu - R) - R(\mu - R)$$

$$R^2 \leq (S\mu - R)(\mu - R) \tag{3.16}$$

The constraint 3.16, is a **hyperbolic constraint**(hyperbolic constraints are the constraints which describe half a hyperboloid) of the form:

$$w^2 \leq xy, \quad x \geq 0, \quad y \geq 0, \Leftrightarrow \left\| \begin{bmatrix} 2w \\ x - y \end{bmatrix} \right\| \leq x + y \tag{3.17}$$

or in the matrix form :

$$w^T w \leq xy, \quad x \geq 0, \quad y \geq 0, \Leftrightarrow \left\| \begin{bmatrix} 2w \\ x - y \end{bmatrix} \right\| \leq x + y$$

The hyperbolic constraint, is a variety of conic SOCP sets (Alizadeh and Goldfarb (2003)). The SOCP will be accepted for solution by the optimizers if it can be transformed to the following convex SOC constraint:

$$-c_0 x_0^2 + \sum_i c_i x_i^2 \leq 0$$

Therefore we need to define new variables and replace hyperbolic constraints by them. the following procedure is done for all (j, s). Again for simplicity, we drop indices of variables:

$$P1 = S\mu - R - \mu + R$$

$$P2 = S\mu - R + \mu - R$$

$$P3 = 2R$$

$$P3^2 + P1^2 \leq P2^2$$

Hence, (3.8) is SOCP representable.1 ■

At the end, SOCP of our location and allocation model will be:

$$\textbf{Min} : \sum_i F x_i + \sum_j \sum_s f_{js} y_{js} + \sum_j \sum_k c_{jk} \lambda_k w_{jk} + \sum_i \sum_j \sum_k \sum_s C_{ij} \lambda_k z_{ijks}$$
$$+ t/2 \sum_j \sum_s \left[(1 + CV_{jk}^2) S_{js} + (1 - CV_{jk}^2) \frac{\sum_i \sum_k \lambda_k z_{ijks}}{\mu_s} \right]$$

s.t.

$$\left(\sum_i \sum_k \lambda_k z_{ijks} \right)^2 \leq \left(S_{js} \mu_s - \sum_i \sum_k \lambda_k z_{ijks} \right) \left(\mu_s - \sum_i \sum_k \lambda_k z_{ijks} \right)$$

$\forall j, s$

(and) constraints (3.9)-(3.14)

$$w_{jk}, x_{ic}, y_{js} \in \{0, 1\} \qquad z_{ijks} \in [0, 1] \quad S_{js} \quad Free$$

3.4 Summary

In this section we first presented a mathematical model for the problem. The model was a MINLP. We showed that it can represented as MISOCP.

What we did is, to represent each waiting time expression in the objective function by a set of new decision variables, and linear and SOCP inequalities. This help to solve the problem to exact optimum as the problem size permits via commercial MISOCP solvers such as Cplex. This allows to employ strong *B&B*, preprocessing and cut generation features of such solvers in solving our problem.

CHAPTER 4

COMPUTATIONAL STUDY

In this section, we report our computational results for the proposed MISOCP reformulation. We tested MISOCP formulation over several instances generated by using SGB128 data (City Distance Datasets, www.sc.fsu.edu) which describes 128 cities in North America. Appendix A.1 gives the map of the cities in the dataset. All instances were coded in MATLAB version R2012a to generate LP files and were solved by using ILOG CPLEX 12.5 on 8 GB RAM, 2.66 GHz computer.
We assume that the demand of each city is concentrated in the city center and hence consider each city as an individual customer. Demand rate (λ_k) at a city k is proportional to the population and obtained by dividing the population of each city by 1000 (The name and population of cities are given in Appendix A). We generate 50, 100 and 128 cities instances.
We also assume that plants and DCs are located close to city centers, so out of selected cities, We randomly choose two sets of candidate locations, for plants (N) and DCs (M). We assume the capacity of plants and DCs are both proportional to total demand,

$$\text{Total demand rate} = \sum_k \lambda_k$$

The capacity levels of possible plants are given, and we assume the fixed cost of opening a plant is a function of its capacity. The capacity and the fixed cost of plants are obtained by:

$$\mu^F = U(0.1, 0.5) \times \text{total demand rate} \tag{4.1}$$

$$F = \beta \times U(1000, 2000) \times \sqrt{\mu^F} \tag{4.2}$$

Where β is a constant multiplier and it is used to explore the sensitivity of solutions to different levels of fixed cost. In the model, we assume the candidate locations for DCs, are different from the candidate location of plants, and as mentioned earlier, we choose them randomly. The capacity of DCs is a decision variable (y_{js}), and we assume, the available capacities are discrete and proportional to total population. We define three levels for capacities, according to the number of demand point. The capacities of DCs and the corresponding fixed cost are generated by the formulas (4.3) and (4.4), respectively,

$$\mu_s = \alpha_s \times \text{total demand} \tag{4.3}$$

$$f = \beta \times 100 \times \sqrt{\mu_s} \tag{4.4}$$

α is dependent to number of customers where $\alpha_s = 0.15$, 0.2, 0.45 for $K = 50$; $\alpha_s = 0.1$, 0.2, 0.3 for $K = 100$ and $\alpha_s = 0.1$, 0.15, 0.2, 0.3, 0.45 for $K = 128$. To calculate transportation costs (c_{jk}, C_{ij}), we use the distance matrix in SGB128 dataset. We divide distances between plants and DCs, and between DCs and customers by .01 and .04 respectively, to differentiate the types of transportation in network. These types of difference between transportation mode is consistent to our motivation, where DCs act

CD operation in supply chain network (one of the main incentive to use CD centers in supply chain is to reduce the transportation costs by altering vehicles type). We use a multiplier in calculation of transportation cost, δ, to explore the model sensitivity to this cost. To show the effects of the fixed and transportation costs, we set them in two levels (low and high), and to explore the effect of congestion cost accurately, we set it in three levels (low, mid and high).

The response waiting cost can be expressed in different cost functions such as piecewise linear function or exponential cost function. However, in practice, determining the value of average response time is difficult. In this computational study, we generate values of t by using the following formula:

$$t = \theta \times \left(\frac{\sum_i \sum_j \lambda_k c_{ij}}{I \times J} \right) \tag{4.5}$$

where θ is the response time cost coefficient . The high θ can be interpreted to indicator of a situation which losing customers due to high expected waiting time is costly. The numerator of fraction of expression in (4.5) is total transportation cost in system. For M/G/1 expected waiting time, the coefficient of variation (CV) is set to 1.5.
All experimental factors and their levels are listed in Table 4.1.

Table 4.1: Experimental factors

Parameters	Levels
K	50,100,128
N	10,20
M	10,20
δ	1,10
β	1,10
θ	.01,5,10

We have $3\times2\times2\times2\times2\times3 = 144$ experimental settings and for each setting we have three replications. All results are given in Appendix B. We set two hours time limit for all runs.

Tables 4.2 - 4.4, show the summary of all experimental results. The column NP is the number of times which optimal solution was found for each instance in three replications. The column $Gap(\%)$ is the gap between the best integer objective and the objective of the best remaining node. The columns, $FC(\%)$, $TC(\%)$ and $CC(\%)$ represent the proportion of the fixed cost, transportation cost and congestion cost in total cost respectively.
When all costs are low, the total cost almost consists of fixed and transportation costs and congestion cost is negligible. As the θ increases, the congestion cost's proportion increases, and it is what we expect. The greatest value of congestion's proportion is when the congestion cost is mid or high and the other costs are low.
The results show that, the CPU time and gap are affected by the size of problem, the congestion cost and fixed cost. As the number of customers increase average CPU time and gap increase. Maximum number of non optimal solution is observed when fixed cost (β) and congestion cost (θ) are both high. One possible reason is, due to high congestion cost, we need to open more DC or increase the capacities to reduce total response time, but on the other hand, the fixed cost is also high. In most instances which the congestion cost is low, we obtain optimal solutions or very small gap, but when congestion cost is high, CPU is more likely to hit time limit with a significant gap.

When the congestion cost is low and fixed cost is high, less plants and DCs are opened because waiting cost is negligible, and conversely when fixed costs are low and the other is high, it is profitable to open more plants or DCs with higher capacity (Table 4.8).

Table 4.2: K = 50

N	M	θ	δ	β	NP	Gap(%)	CPU(s)	FC(%)	TC(%)	CC(%)
10	10	Low	Low	Low	3	0	70.67	64	35	0.6
				High	3	0	77.5	94	5	0.1
			High	Low	3	0	17.25	16	84	0.2
				High	3	0	43.84	64	36	0.1
		Mid	Low	Low	3	0	141.09	53	31	16.4
				High	3	0	557.16	90	5	4.6
			High	Low	3	0	16.74	15	78	7.2
				High	3	0	64.87	63	33	3.9
		High	Low	Low	3	0	191.35	44	27	29.3
				High	3	0	3868.08	88	5	7.1
			High	Low	3	0	27.02	17	71	11.1
				High	3	0	146.09	60	34	6.1
	20	Low	Low	Low	3	0	169.73	67	33	0.2
				High	3	0	80.5	95	5	0.1
			High	Low	3	0	26.13	17	83	0.1
				High	3	0	141.93	66	34	0.1
		Mid	Low	Low	3	0	459.98	55	28	16.4
				High	2	2	7204.3	90	6	4.5
			High	Low	3	0	24.52	16	78	5.8
				High	3	0	208.2	65	32	2.7
		High	Low	Low	3	0	358.54	48	26	26.2
				High	0	5	7204.97	89	5	5.9
			High	Low	3	0	47.97	19	68	12.4
				High	3	0	219.81	62	32	6.3
20	10	Low	Low	Low	3	0	180.87	64	36	0.4
				High	3	0	153.43	94	6	0.1
			High	Low	3	0	54.65	15	85	0.1
				High	3	0	236.22	64	36	0.1
		Mid	Low	Low	3	0	412.81	54	30	16.8
				High	0	0	5337.46	90	5	4.5
			High	Low	3	0	74.74	19	74	6.7
				High	3	0	1455.43	63	32	4.2
		High	Low	Low	3	0	2486.78	42	28	29.4
				High	3	3	7203.33	87	6	7.2
			High	Low	3	0	50.17	17	71	11.6
				High	3	0	597.52	59	34	6.8
	20	Low	Low	Low	3	0	676.78	70	30	0.3
				High	3	0	1927.38	96	4	0.1
			High	Low	3	0	44.66	19	81	0.1
				High	3	0	475.51	70	30	0.1
		Mid	Low	Low	3	0	2820.14	56	27	17.3
				High	0	3	7206.36	91	5	4.6
			High	Low	3	0	126.14	18	74	8.0
				High	3	0	2525.98	66	30	4.1
		High	Low	Low	3	6	7205.87	47	26	26.7
				High	0	8	7204.3	87	6	6.2
			High	Low	3	0	121.12	26	61	12.9
				High	3	0	2885.66	64	28	7.4

Table 4.3: K = 100

N	M	θ	δ	β	NP	Gap(%)	CPU(s)	FC(%)	TC(%)	CC(%)
10	10	Low	Low	Low	3	0	178.51	54	46	0.3
				High	3	0	5567.21	92	8	0.1
			High	Low	3	0	35.83	11	89	0.2
				High	3	0	147.95	54	46	0.1
		Mid	Low	Low	3	0	593.74	47	38	14.6
				High	0	0	4944.73	89	7	3.7
			High	Low	3	0	120.12	11	83	6.0
				High	3	0	351.14	55	42	3.3
		High	Low	Low	3	0	1625.61	39	36	25.6
				High	0	4	7200.85	86	8	5.7
			High	Low	3	0	105.29	16	77	7.1
				High	3	0	862.95	51	44	5.0
	20	Low	Low	Low	3	0	398.47	57	43	0.4
				High	3	0	940.11	93	7	0.1
			High	Low	3	0	58.08	16	84	0.1
				High	3	0	645.74	57	43	0.1
		Mid	Low	Low	3	0	1562.6	50	35	14.4
				High	0	10	7200.62	90	6	3.2
			High	Low	3	0	105.89	12	83	4.4
				High	3	0	360.96	57	40	2.3
		High	Low	Low	1	0	3834.46	44	35	21.7
				High	0	25	7203.83	84	9	7.0
			High	Low	3	0	193.38	18	74	7.9
				High	3	0	1295.7	54	41	5.0
20	10	Low	Low	Low	3	0	709.51	56	43	0.3
				High	3	0	1692.83	93	7	0.1
			High	Low	3	0	106	11	88	0.1
				High	3	0	554.27	56	44	0.1
		Mid	Low	Low	3	0	7202.35	47	36	16.3
				High	3	4	7200.2	88	7	5.0
			High	Low	3	0	540.86	15	80	5.1
				High	3	0	3867.23	56	40	3.8
		High	Low	Low	3	13	7203.41	38	36	26.2
				High	0	6	7202.41	85	8	6.5
			High	Low	3	0	465.73	13	77	9.5
				High	3	0	2126.68	52	42	6.6
	20	Low	Low	Low	3	0	2733.62	61	39	0.2
				High	3	0	2959.67	94	6	0.1
			High	Low	3	0	145.02	19	81	0.0
				High	3	0	4072.03	61	39	0.0
		Mid	Low	Low	1	7	7202.97	50	35	15.4
				High	0	5	7202.86	89	7	4.5
			High	Low	3	0	340.55	16	79	5.5
				High	3	5	7203.69	58	39	3.4
		High	Low	Low	0	16	7203.42	46	35	19.2
				High	0	8	7204.44	87	7	5.9
			High	Low	3	0	347.09	21	71	8.3
				High	3	2	7201.33	56	37	7.1

Table 4.4: K = 128

N	M	θ	δ	β	NP	Gap(%)	CPU(s)	FC(%)	TC(%)	CC(%)
10	10	Low	Low	Low	3	0	308.94	51	49	0.3
				High	1	0	231.16	91	9	0.1
			High	Low	3	0	73.21	13	87	0.1
				High	3	0	326.14	51	49	0.1
		Mid	Low	Low	0	0	655.47	50	42	8.3
				High	0	2	7204.14	90	8	2.2
			High	Low	3	0	124.61	11	87	1.9
				High	0	0	546.71	54	45	1.3
		High	Low	Low	0	0	913.87	40	41	19.0
				High	0	0	2186.99	86	9	5.8
			High	Low	3	0	60.92	12	83	5.5
				High	1	0	601.93	49	47	3.8
	20	Low	Low	Low	3	0	1014.6	55	44	0.1
				High	3	0	7127.78	92	8	0.1
			High	Low	3	0	159.03	15	85	0.0
				High	3	0	1078.65	55	45	0.0
		Mid	Low	Low	3	0	7100.18	52	40	7.3
				High	0	3	7202.46	89	8	2.7
			High	Low	3	0	247.35	12	86	1.7
				High	3	0	2273.22	55	43	1.5
		High	Low	Low	0	0	4423.13	47	38	14.9
				High	0	25	7202.69	85	9	6.1
			High	Low	3	0	412.93	14	81	4.3
				High	2	0	2967.23	54	43	2.8
20	10	Low	Low	Low	3	0	4065.93	54	46	0.2
				High	2	19	7201.97	92	8	0.1
			High	Low	3	0	427.99	11	89	0.1
				High	3	0	1893.62	54	46	0.0
		Mid	Low	Low	0	7	7203.44	50	40	9.8
				High	0	11	7202.99	87	9	4.2
			High	Low	3	0	1452.1	14	84	1.9
				High	0	4	7203.02	54	44	1.9
		High	Low	Low	0	17	7201.85	42	41	17.4
				High	0	47	7201.86	58	8	34.3
			High	Low	3	0	687.04	14	82	4.1
				High	1	6	7202.28	52	45	3.0
	20	Low	Low	Low	0	4	7202.43	58	42	0.3
				High	0	1	7203.41	93	7	0.1
			High	Low	3	0	319.71	21	79	0.0
				High	2	0	5219.28	58	42	0.1
		Mid	Low	Low	0	9	7213.88	51	39	9.1
				High	0	7	7201.29	87	10	3.4
			High	Low	3	0	885.12	15	83	2.2
				High	0	5	7207.81	56	43	1.4
		High	Low	Low	0	26	7202.41	40	39	20.7
				High	3	0	702.43	20	74	6.0
			High	Low	0	10	7202.08	55	40	5.3
				High	0	26	7200.68	86	9	4.7

The last row in Table 4.4 is result of a instance. We didn't get any integer solution within two hours for two other replications.
Table 4.5 shows the effect of number of candidate plant locations, DCs locations and customers on CPU time and gap. It gives statistic of the model's performance for all sets of plants, DCs and customers. The column *Opt* indicates how much percent of instances solved to optimum. The column *Gap* shows the average gap of non optimal solutions. As the size of problems get larger, as we expect, the number of optimal solutions decrease and CPU time increases. The model could solve around 87% of instances for $k = 50$, 73% for $k = 100$ and 64% for $k = 128$. Totally, 75% of instances were solved optimally. Regarding to two hours time limit, the results indicate that the model is able to solve the practical size instances of this problem.

Table 4.5: The model performance

N	M	K	Opt(%)	CPU(s)	Gap(%)
10	10	50	100	333.03	-
		100	83	936.86	3
		128	47	581.78	6
10	20	50	88	194.14	3
		100	88	1108.88	14
		128	72	2153.94	9
20	10	50	92	1019.65	2
		100	92	1259.80	7
		128	53	2034.85	10
20	20	50	83	1216.37	7
		100	69	1927.15	9
		128	30	1253.88	9

In the remaining parts of this section, we explore the effects of costs on solutions. To observe the sensitivity of solutions to transportation cost and fixed cost, we show the relation between this cost and the number of DCs to be opened in Table 4.6. The data are for $N = 10, M = 10$ and $\theta = .01$ instances from appendix B. The column *Op.DCs* indicates the average number of opened DCs for different number of customers. When the transportation cost is high ($\delta = 10$), on average, more DCs were opened to reduce transportation cost. When fixed cost is high ($\beta = 10$), on average, less DCs were opened. Next, we will explain the number of DCs are not just affected by these two parameters, but also it is affected by waiting time cost.

Table 4.6: Transportation cost and fixed cost versus CD centers

K	δ	β	Op. DCs
50	Low	Low	5
	Low	High	3
	High	Low	6
	High	High	5
100	Low	Low	6
	Low	High	4
	High	Low	7
	High	High	6
128	Low	Low	6

Continued on next page

Table 4.6: Transportation cost and fixed cost versus CD centers

K	δ	β	Op. DCs
	Low	High	3
	High	Low	7
	High	High	6

The waiting time cost or congestion cost actually is penalty of not fulfilling customer commitments in lead time and the value we assign to it. It can present customers delay sensitivity. In the case of high sensitivity, waiting time imposes too much cost and take grater percentage of total cost. In Table 4.7, we can observe different levels of congestion cost (different level of θ) and their waiting time and cost. The data are for $N = 10, M = 10, K = 100$. The CC column is the average congestion cost percentage in the total cost, and $E(W)$ column represents the average total expected waiting time in the system. Table 4.7 shows that, as the congestion cost increases, the proportion of congestion cost in total cost increases. To mitigate the congestion cost, we open more DCs or increase capacities, consequently the total waiting time ($E(W)$) decreases. In Table 4.7, when the congestion cost is very low ($\theta = .01$), the E(W) is very high. The reason is, because of low congestion rate, we can open less DCs to reduce fixed cost which in turn, increases the waiting time in system.

Table 4.7: Effect of waiting time cost on total expected waiting time

θ	CC	E(W)
.01	.1	183.25
5	3.8	17.31
10	7.2	14.76

Another observation in experimental results, is the relation between congestion cost and capacity decision in DCs. To show this relation accurately, we solved extra instances with more levels. We tabulate the solutions for $M = 10, N = 10, K = 50$ with different levels of congestion cost in Table 4.8. In this table, NOP is the number of opened plants, NODC is the number of opened DCs and DC capacity column is the capacities of the opened DCs. The numbers in parenthesis indicate the capacities, where 1 corresponds to lowest and 3 corresponds to highest capacity. As θ increases, the DCs capacity increases. In fact, we can say, because the cost of upgrading the capacity is lower than opening new facility, the number of opened DCs doesn't change considerably as the congestion cost increases, but the capacities increases.

Table 4.8: Effect of waiting time cost on chosen capacity levels

θ	NOP	NODC	DCs capacity
0.01	9	7	1(2)4(2)5(1)7(2)8(2)9(2)10(3)
0.1	9	6	1(2)4(2)5(2)6(3)8(3)9(3)
1	9	7	1(3)4(3)5(2)7(3)8(3)9(3)10(3)
10	9	8	1(3)2(3)4(3)5(3)7(3)8(3)9(3)10(3)

Note that, because transportation and fixed cost of plants are constant, the number of opened plants remained unchanged. The capacity decision in our model has direct and indirect effects on total cost. According to (4.2), the fixed cost of establishing CD centers depends on capacities. On the other hand,

the capacity decision is a tool to control congestion cost (3.8). In fact, we can say capacity increment is a cheap alternative for establishing a new DC and it is what we observed in 4.8, where by increasing congestion cost, only highest level capacities were opened.
In the model, we assumed capacity levels are proportional to entire demand. We set the different sets of discrete capacities in accordance with the number of demand points. The selection of the levels of capacities is very important. The experimental results reveal that, any change in the levels of capacities cause considerable changes in the total expected waiting time in the system. To realize the effect of capacity levels, we did extra runs for $M = 10, N = 10, K = 50$. We defined three sets of levels, tight, moderate, loose (all other setting parameters remain unchanged). Table 4.9 shows the results. The values in the $E(W)$ depends on the fixed cost of opening a DC and the cost of congestion. For instance, when fixed cost is low and the levels of capacities are tight, we can open more DCs to reduce $E(W)$.

Table 4.9: Effect of capacity in E(W)

	θ	E(W)
Tight capacities	0.01	184.92
	1	30.86
	200	13.36
Moderate capacities	0.01	131.11
	1	10.29
	200	5.89
Loose capacities	0.01	27.97
	1	8.24
	200	5.39

In the next chapter, we present the conclusion and bibliography.

CHAPTER 5

CONCLUSION AND FUTURE STUDY

In this thesis, we studied a congestible supply chain network design problem. We consider make-to-order supply chain which consists of plants, distribution centers and customers. Distribution centers act as Cross docking terminals. The resulting problem was MINLP and non convex. We showed the waiting time function derived from M/G/1 queuing model can be represented via second order conic inequalities. We proposed a closed form formulation for the problem. Then, the problem became a MISOCP which can be solved by using commercial branch-and-bound software such as IBM ILOG CPLEX.
The experiments showed that the model can solve practical size problems in reasonable CPU times. We implemented the model on 128 cities of U.S, and it could find optimal solution in 75% of instances. In the comparison to the approaches in the literature, this exact approach is easier to employ and there is no need to use any heuristics. The proposed SOCP formulation in this thesis, is not restricted to supply chain design problems, it is applicable in all M/G/1 queuing systems.

One possible extension of our model is considering congestion in both echelons, plants and DCs, simultaneously. It means that every customer may have to wait to get service in plants as may in DC. In fact, because establishing a new plant or even developing existing ones (to increase capacity) is very costly, possibility of congestion in higher level in supply chain increases.
In most studies, it is assumed that plants act as M/M/1 or M/M/s queuing systems. We can add plant capacity decision problem in our model in the cast of expected waiting time to control congestion in upstream level and then, similarly like DCs, utilize SOCP reformulation to solve new problems. But the key point is, now we have two sets of conic quadratic constraints, average waiting time function of M/M/1 and M/G/1 for plants and DCs respectively, and in this case Cplex or other softwares can solve at most moderate size problems in reasonable CPU time because as seen in computational results section the conic constraint have most effect in CPU time. The research is needed, first to model the problem as SOCP and second figure out whether we are able to solve large problems by adding some valid constraints or heuristics approach should also be applied.

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APPENDIX A

SGB128 DATA

Table A.1: Name and population of cities

Name	Population	Name	Population
Youngstown, OH	115436	Springfield, OH	72563
Yankton, SD	12011	Springfield, MO	133116
Yakima, WA	49826	Springfield, MA	152319
Worcester, MA	161799	Springfield, IL	100054
Wisconsin Dells, WI	2521	Spokane, WA	171300
Winston-Salem, NC	131885	South Bend, IN	109727
Winnipeg, MB	564473	Sioux Falls, SD	81343
Winchester, VA	20217	Sioux City, IA	82003
Wilmington, NC	139238	Shreveport, LA	205820
Wilmington, DE	70195	Sherman, TX	30413
Williston, ND	13336	Sheridan, WY	15146
Williamsport, PA	33401	Seminole, OK	8590
Williamson, WV	5219	Selma, AL	26684
Wichita Falls, TX	94201	Sedalia, MO	20927
Wichita, KS	279835	Seattle, WA	493846
Wheeling, WV	43070	Scranton, PA	88117
West Palm Beach, FL	63305	Scottsbluff, NB	14156
Wenatchee, WA	17257	Schenectady, NY	67972
Weed, CA	2879	Savannah, GA	141634
Waycross, GA	19371	Sault Sainte Marie, MI	14448
Wausau, WI	32426	Sarasota, FL	48868
Waukegan, IL	67653	Santa Rosa, CA	83320
Watertown, SD	15649	Santa Fe, NM	48953
Watertown, NY	27861	Santa Barbara, CA	74414
Waterloo, IA	75985	Santa Ana, CA	204023
Waterbury, CT	103266	San Jose, CA	629546
Washington, DC	638432	San Francisco, CA	678974
Warren, PA	12146	Sandusky, OH	31360
Walla Walla, WA	25618	San Diego, CA	875538
Waco, TX	101261	San Bernardino, CA	118794
Vincennes, IN	20857	San Antonio, TX	786023
Victoria, TX	50695	San Angelo, TX	73240
Vicksburg, MS	25434	Salt Lake City, UT	163697
Vancouver, BC	414281	Salisbury, MD	16429

Continued on next page

Table A.1: Name and population of cities (continued)

Name	Population	Name	Population
Valley City, ND	7774	Salinas, CA	80479
Valdosta, GA	37596	Salina, KS	41843
Utica, NY	75632	Salida, CO	44870
Uniontown, PA	14510	Salem, OR	89233
Tyler, TX	70508	Saint Paul, MN	270230
Twin Falls, ID	26209	Saint Louis, MO	453085
Tuscaloosa, AL	75211	Saint Joseph, MO	76691
Tupelo, MS	23905	Saint Joseph, MI	9622
Tulsa, OK	360919	Saint Johnsbury, VT	7150
Tucson, AZ	330537	Saint Cloud, MN	42566
Trinidad, CO	9663	Saint Augustine, FL	11985
Trenton, NJ	92124	Saginaw, MI	77508
Traverse City, MI	15516	Sacramento, CA	275741
Toronto, ON	599217	Rutland, VT	18436
Topeka, KS	115266	Roswell, NM	39676
Toledo, OH	354635	Rocky Mount, NC	41283
Texarkana, TX	31271	Rock Springs, WY	19458
Terre Haute, IN	61125	Rockford, IL	139712
Tampa, FL	271523	Rochester, NY	241741
Tallahassee, FL	81548	Rochester, MN	57890
Tacoma, WA	158501	Roanoke, VA	100220
Syracuse, NY	170105	Richmond, VA	219214
Swainsboro, GA	7602	Richmond, IN	41349
Sumter, SC	24890	Richfield, UT	5482
Stroudsburg, PA	5148	Rhinelander, WI	7873
Stockton, CA	149779	Reno, NV	100756
Stevens Point, WI	22970	Regina, SA	162613
Steubenville, OH	26400	Red Bluff, CA	9490
Sterling, CO	11385	Reading, PA	78686
Staunton, VA	21857	Ravenna, OH	11987

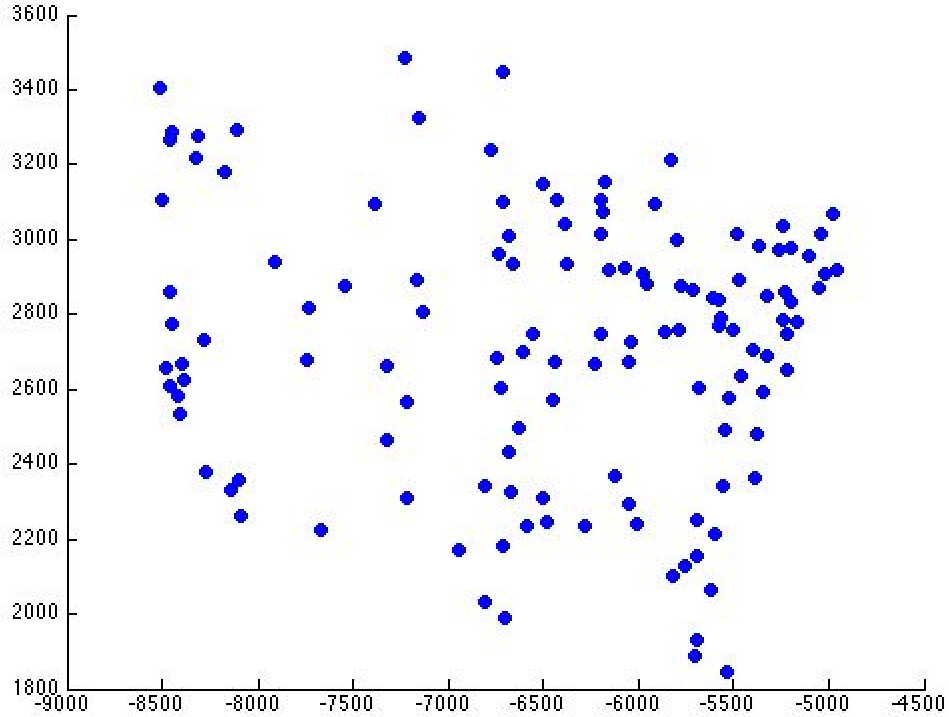

Figure A.1: Map of cities

APPENDIX B

COMPUTATIONAL RESULTS

The computational results of experiments are reported here. The meaning of abbreviations used in tables are as follows.

- **Rep** : Number of replications
- **n** : Number of candidate location for facilities
- **m** : Number of candidate location for CD centers
- **k** : Number of customers
- θ : Multiplier of waiting time cost
- δ : Multiplier of transportation cost
- β : Multiplier of fixed cost
- **FC** : Fixed Cost
- **CC** : Congestion Cost
- **E(W)** : Total waiting time cost
- **NOP** : Number of opened Plants
- **NODC** : Number of opened CD centers

Table B.1: Experimental results

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
1	10	10	50	0.01	1	1	0	120,317	70.67	76,931	42,679	707	94.74	3	4
2	10	10	50	0.01	1	1	0	108,414	66.52	71,998	35,921	495	77.43	3	5
3	10	10	50	0.01	1	1	0	115,642	47.67	75,292	39,875	475	64.33	3	5
1	10	10	50	0.01	1	10	0	808,642	77.50	763,413	44,312	917	122.24	3	3
2	10	10	50	0.01	1	10	0	747,010	65.19	709,032	37,098	880	135.33	3	3
3	10	10	50	0.01	1	10	0	783,368	195.74	740,205	42,161	1,001	133.49	3	3
1	10	10	50	0.01	10	1	0	496,763	17.25	78,244	417,523	995	134.33	3	6
2	10	10	50	0.01	10	1	0	426,618	16.40	72,546	353,245	827	128.79	3	6
3	10	10	50	0.01	10	1	0	469,785	19.70	75,735	392,970	1,080	145.45	3	6
1	10	10	50	0.01	10	10	0	1,196,805	43.84	769,314	426,784	707	94.74	3	4
2	10	10	50	0.01	10	10	0	1,077,378	54.94	717,684	358,320	1,374	213.82	3	5
3	10	10	50	0.01	10	10	0	1,148,802	57.30	753,674	394,142	986	133.72	3	6
1	10	10	50	5	1	1	0	157,667	141.09	83,264	48,471	25,932	9.48	4	8
2	10	10	50	5	1	1	0	143,375	157.16	77,328	41,478	24,569	9.53	4	8
3	10	10	50	5	1	1	0	151,509	218.93	80,724	45,559	25,226	9.45	4	8
1	10	10	50	5	1	10	0	888,558	557.16	799,499	48,045	41,014	13.71	4	5
2	10	10	50	5	1	10	0	822,369	819.01	742,549	41,672	38,148	13.58	4	5
3	10	10	50	5	1	10	0	860,200	1032.68	775,168	45,523	39,509	13.56	4	5
1	10	10	50	5	10	1	0	547,575	16.74	82,160	426,184	39,231	13.21	4	7
2	10	10	50	5	10	1	0	479,833	20.08	76,303	367,866	35,664	12.84	4	7
3	10	10	50	5	10	1	0	517,761	14.9	79,654	399,168	38,940	13.4	4	7
1	10	10	50	5	10	10	0	1,279,432	64.87	802,914	426,933	49,585	16.46	4	6
2	10	10	50	5	10	10	0	1,158,091	110.07	749,261	367,053	41,777	14.85	4	6
3	10	10	50	5	10	10	0	1,226,139	89.97	778,478	399,573	48,088	16.37	4	6
1	10	10	50	10	1	1	0	184,063	191.35	80,757	49,328	53,978	9.77	3	7
2	10	10	50	10	1	1	0	168,551	185.50	75,002	42,889	50,660	9.76	3	7

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
3	10	10	50	10	1	1	0	177,629	295.05	78,299	46,545	52,785	9.79	3	7
1	10	10	50	10	1	10	0	908,621	3,868.08	796,530	47,953	64,138	11.20	3	6
2	10	10	50	10	1	10	0	841,572	1,978.92	739,791	42,237	59,544	11.08	3	6
3	10	10	50	10	1	10	0	879,977	1,037.27	772,317	45,980	61,679	11.07	3	6
1	10	10	50	10	10	1	0	600,601	27.02	105,100	428,845	66,656	11.55	4	7
2	10	10	50	10	10	1	0	529,333	37.08	75,002	386,974	67,357	12.25	3	7
3	10	10	50	10	10	1	0	572,141	24.12	78,299	423,915	69,927	12.25	3	7
1	10	10	50	10	10	10	0	1,320,970	146.09	796,530	444,417	80,023	13.43	3	6
2	10	10	50	10	10	10	0	1,198,607	108.25	739,791	384,032	74,784	13.35	3	6
3	10	10	50	10	10	10	0	1,269,205	115.05	772,317	420,672	76,216	13.16	3	6
1	10	10	100	0.01	1	1	0	201,029	178.51	108,061	92,278	690	93.57	3	6
2	10	10	100	0.01	1	1	0	191,516	159.76	105,186	85,562	768	120.13	3	7
3	10	10	100	0.01	1	1	0	198,663	165.44	107,312	90,601	750	101.30	3	6
1	10	10	100	0.01	1	10	0	1,163,236	5,567.21	1,066,868	95,242	1,126	150.13	3	4
2	10	10	100	0.01	1	10	0	1,123,386	681.07	1,033,864	88,593	929	142.92	3	4
3	10	10	100	0.01	1	10	0	1,154,605	504.68	1,059,428	94,208	969	129.15	3	4
1	10	10	100	0.01	10	1	0	1,024,902	35.83	108,879	913,985	2,038	273.32	3	7
2	10	10	100	0.01	10	1	0	957,677	33.79	138,229	818,018	1,430	220.97	4	7
3	10	10	100	0.01	10	1	0	1,006,301	26.40	108,125	896,712	1,464	196.64	3	7
1	10	10	100	0.01	10	10	0	1,999,936	147.95	1,077,246	921,138	1,552	209.20	3	6
2	10	10	100	0.01	10	10	0	1,905,712	157.17	1,051,849	852,109	1,754	271.63	3	7
3	10	10	100	0.01	10	10	0	1,976,979	120.92	1,069,731	905,090	2,158	290.09	3	6
1	10	10	100	5	1	1	0	244,413	593.74	114,556	94,084	35,772	12.61	4	8
2	10	10	100	5	1	1	0	234,408	669.34	111,016	90,141	33,251	12.49	4	8
3	10	10	100	5	1	1	0	241,153	922.48	113,764	92,683	34,706	12.54	4	8
1	10	10	100	5	1	10	0	1,255,171	4944.73	1,117,236	90,998	46,937	15.74	4	6

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
2	10	10	100	5	1	10	0	1,213,625	6597.75	1,082,666	87,142	43,817	15.64	4	6
3	10	10	100	5	1	10	2	1,246,861	7203.03	1,109,434	91,201	46,226	15.86	4	6
1	10	10	100	5	10	1	0	1,012,873	120.12	113,139	838,533	61,201	19.74	4	7
2	10	10	100	5	10	1	0	967,961	81.88	109,642	801,532	56,788	19.51	4	7
3	10	10	100	5	10	1	0	997,311	68.45	112,356	825,678	59,277	19.61	4	7
1	10	10	100	5	10	10	0	2,025,065	351.14	1,114,635	843,733	66,698	21.38	4	6
2	10	10	100	5	10	10	0	1,948,195	230.24	1,080,146	805,193	62,856	21.41	4	6
3	10	10	100	5	10	10	0	2,003,770	830.3	1,106,851	832,386	64,533	21.22	4	6
1	10	10	100	10	1	1	0	286,675	1,625.61	111,196	102,188	73,291	12.85	3	7
2	10	10	100	10	1	1	0	272,670	1,355.54	107,763	95,870	69,037	12.86	3	7
3	10	10	100	10	1	1	0	282,191	1,533.29	110,428	99,625	72,138	12.94	3	7
1	10	10	100	10	1	10	4	1,290,574	7,200.85	1,111,991	104,712	73,871	12.93	3	7
2	10	10	100	10	1	10	5	1,255,731	7,202.46	1,075,069	103,077	77,585	14.25	3	7
3	10	10	100	10	1	10	3	1,280,133	7,203.69	1,090,161	102,801	87,172	15.10	3	6
1	10	10	100	10	10	1	0	1,150,113	105.29	180,131	887,752	82,230	14.11	5	8
2	10	10	100	10	10	1	0	1,080,567	100.31	174,567	828,306	77,694	14.15	5	8
3	10	10	100	10	10	1	0	1,133,904	100.07	143,952	900,414	89,538	15.44	4	7
1	10	10	100	10	10	10	0	2,165,527	862.95	1,106,001	950,526	109,000	18.01	3	7
2	10	10	100	10	10	10	0	2,067,186	424.25	1,075,069	895,529	96,588	17.04	3	7
3	10	10	100	10	10	10	0	2,136,046	683.46	1,098,287	935,650	102,109	17.38	3	7
1	10	10	128	0.01	1	1	0	238,975	308.94	121,598	116,563	814	109.35	3	6
2	10	10	128	0.01	1	1	0	228,107	219.15	118,151	109,195	761	117.87	3	6
3	10	10	128	0.01	1	1	0	234,114	298.82	120,133	113,173	808	108.72	3	6
1	10	10	128	0.01	1	10	0	1,315,323	231.16	1,192,194	121,865	1,264	168.50	3	3
2	10	10	128	0.01	1	10	0	1,275,004	487.19	1,158,426	115,573	1,005	154.68	3	3
3	10	10	128	0.01	1	10	0	1,299,815	372.20	1,177,881	120,727	1,207	160.87	3	3

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
1	10	10	128	0.01	10	1	0	1,277,165	73.21	162,053	1,113,882	1,230	165.09	4	8
2	10	10	128	0.01	10	1	0	1,197,243	68.33	156,187	1,039,647	1,409	217.54	4	7
3	10	10	128	0.01	10	1	0	1,242,718	70.90	121,631	1,118,951	2,136	285.61	3	7
1	10	10	128	0.01	10	10	0	2,379,284	326.14	1,215,956	1,161,108	2,220	297.12	3	6
2	10	10	128	0.01	10	10	0	2,271,180	193.61	1,179,503	1,089,788	1,889	291.49	3	6
3	10	10	128	0.01	10	10	0	2,330,396	304.01	1,201,358	1,125,786	3,252	434.58	3	6
1	10	10	128	5	1	1	0	256,738	655.47	127,962	107,368	21,408	7.72	4	6
2	10	10	128	5	1	1	0	246,520	793.47	124,334	102,090	20,097	7.71	4	6
3	10	10	128	5	1	1	0	251,308	1811.84	126,426	104,001	20,881	7.71	4	6
1	10	10	128	5	1	10	2	1,383,345	7204.14	1,240,528	112,663	30,155	10.17	4	4
2	10	10	128	5	1	10	9	1,342,321	7202.61	1,201,898	105,349	35,074	12.33	4	4
3	10	10	128	5	1	10	9	1,370,110	7202.77	1,225,644	114,941	29,525	10.2	4	4
1	10	10	128	5	10	1	0	1,212,673	124.61	129,558	1,060,597	22,518	8.07	4	7
2	10	10	128	5	10	1	0	1,156,293	105.63	125,885	1,009,126	21,282	8.1	4	7
3	10	10	128	5	10	1	0	1,175,053	144.63	128,357	1,024,484	22,212	8.1	4	7
1	10	10	128	5	10	10	0	2,358,408	546.71	1,262,775	1,065,737	29,896	10.48	4	6
2	10	10	128	5	10	10	0	2,266,861	752.83	1,216,351	1,022,171	28,339	10.37	4	5
3	10	10	128	5	10	10	0	2,305,154	480.14	1,238,520	1,038,119	28,515	10.07	4	5
1	10	10	128	10	1	1	0	301,702	913.87	121,037	123,321	57,344	10.25	3	7
2	10	10	128	10	1	1	0	286,284	703.13	117,610	115,047	53,627	10.20	3	7
3	10	10	128	10	1	1	0	295,004	764.11	119,582	120,125	55,297	10.15	3	7
1	10	10	128	10	1	10	0	1,373,008	2,186.99	1,175,896	117,290	79,822	13.40	3	5
2	10	10	128	10	1	10	0	1,326,206	2,858.31	1,142,601	108,570	75,035	13.39	3	5
3	10	10	128	10	1	10	0	1,353,059	1,653.05	1,161,781	113,474	77,804	13.38	3	5
1	10	10	128	10	10	1	0	1,323,410	60.92	156,765	1,093,359	73,286	12.53	4	7
2	10	10	128	10	10	1	0	1,232,004	59.89	152,327	1,009,234	70,443	12.74	4	7

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
3	10	10	128	10	10	1	0	1,282,376	55.91	154,881	1,052,070	75,425	13.08	4	7
1	10	10	128	10	10	10	0	2,411,425	601.93	1,175,896	1,143,434	92,095	15.13	3	5
2	10	10	128	10	10	10	0	2,287,186	661.54	1,142,601	1,058,454	86,131	15.05	3	5
3	10	10	128	10	10	10	0	2,354,800	310.01	1,161,781	1,101,022	91,998	15.42	3	5
1	10	20	50	0.01	1	1	0	116,432	169.73	77,445	38,752	235	31.32	3	4
2	10	20	50	0.01	1	1	0	105,914	130.60	71,927	33,798	189	29.10	3	4
3	10	20	50	0.01	1	1	0	113,131	199.71	74,720	37,966	445	59.86	3	4
1	10	20	50	0.01	1	10	0	805,891	80.50	763,413	41,542	936	124.74	3	3
2	10	20	50	0.01	1	10	0	744,817	42.70	709,032	34,994	791	121.72	3	3
3	10	20	50	0.01	1	10	0	780,933	193.60	740,205	39,813	915	121.99	3	3
1	10	20	50	0.01	10	1	0	460,114	26.13	78,625	381,206	283	38.19	3	6
2	10	20	50	0.01	10	1	0	407,095	25.12	72,475	334,403	218	33.65	3	5
3	10	20	50	0.01	10	1	0	450,959	29.55	75,661	375,002	295	39.50	3	5
1	10	20	50	0.01	10	10	0	1,161,329	141.93	770,640	389,959	730	98.16	3	4
2	10	20	50	0.01	10	10	0	1,055,477	150.82	715,744	338,941	792	122.47	3	4
3	10	20	50	0.01	10	10	0	1,127,006	127.42	747,212	378,543	1,251	167.35	3	4
1	10	20	50	5	1	1	0	150,884	459.98	83,264	42,930	24,689	9.13	4	8
2	10	20	50	5	1	1	0	137,507	290.22	77,328	37,063	23,116	9.1	4	8
3	10	20	50	5	1	1	0	145,717	210.91	80,724	40,926	24,067	9.12	4	8
1	10	20	50	5	1	10	2	888,482	7204.3	799,499	49,179	39,804	13.37	4	5
2	10	20	50	5	1	10	2	821,002	7206.17	752,802	36,716	31,484	11.59	4	6
3	10	20	50	5	1	10	1	857,570	7203.36	775,168	43,193	39,209	13.47	4	5
1	10	20	50	5	10	1	0	518,075	24.52	84,368	403,764	29,943	10.6	4	9
2	10	20	50	5	10	1	0	453,172	36.83	78,353	343,928	30,891	11.42	4	9
3	10	20	50	5	10	1	0	496,619	29.78	81,794	384,755	30,070	10.85	4	9
1	10	20	50	5	10	10	0	1,259,988	208.2	817,765	408,470	33,753	11.83	4	7

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
2	10	20	50	5	10	10	0	1,139,981	320.58	752,802	353,083	34,096	12.37	4	6
3	10	20	50	5	10	10	0	1,215,347	658.39	782,175	390,801	42,370	14.53	4	6
1	10	20	50	10	1	1	0	174,376	358.54	82,965	45,674	45,736	8.62	3	9
2	10	20	50	10	1	1	0	158,918	581.43	77,052	39,517	42,349	8.52	3	9
3	10	20	50	10	1	1	0	168,831	405.57	80,439	44,476	43,916	8.51	3	9
1	10	20	50	10	1	10	5	911,135	7,204.97	807,569	49,407	54,159	9.80	3	7
2	10	20	50	10	1	10	4	841,057	7,203.25	729,538	36,514	75,005	13.39	3	5
3	10	20	50	10	1	10	5	876,740	7,209.09	772,317	41,753	62,670	11.21	3	6
1	10	20	50	10	10	1	0	541,346	47.97	105,100	369,092	67,154	11.62	4	7
2	10	20	50	10	10	1	0	476,587	54.63	75,002	344,445	57,140	10.72	3	7
3	10	20	50	10	10	1	0	528,465	49.70	101,902	360,356	66,206	11.72	4	7
1	10	20	50	10	10	10	0	1,272,193	219.81	785,491	406,896	79,805	13.40	3	5
2	10	20	50	10	10	10	0	1,145,986	221.91	729,538	341,334	75,114	13.40	3	5
3	10	20	50	10	10	10	0	1,233,422	327.51	772,317	395,485	65,620	11.63	3	6
1	10	20	100	0.01	1	1	0	188,390	398.47	107,503	80,161	726	97.16	3	5
2	10	20	100	0.01	1	1	0	178,603	311.61	104,256	73,882	465	71.58	3	5
3	10	20	100	0.01	1	1	0	187,361	322.55	106,835	79,920	606	80.82	3	5
1	10	20	100	0.01	1	10	0	1,148,758	940.11	1,066,868	80,833	1,057	140.88	3	4
2	10	20	100	0.01	1	10	0	1,109,461	2,463.40	1,033,864	74,877	720	110.81	3	4
3	10	20	100	0.01	1	10	0	1,142,469	2,398.56	1,059,428	82,023	1,018	135.71	3	4
1	10	20	100	0.01	10	1	0	885,344	58.08	142,977	741,901	467	62.74	4	7
2	10	20	100	0.01	10	1	0	819,672	69.31	138,557	680,463	652	100.60	4	7
3	10	20	100	0.01	10	1	0	881,556	61.84	141,985	738,986	585	78.51	4	7
1	10	20	100	0.01	10	10	0	1,873,770	645.74	1,072,451	799,687	1,632	218.03	3	5
2	10	20	100	0.01	10	10	0	1,779,393	434.40	1,039,274	738,786	1,333	205.89	3	5
3	10	20	100	0.01	10	10	0	1,866,246	490.87	1,064,971	800,244	1,031	138.17	3	5

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
1	10	20	100	5	1	1	0	228,281	1562.6	114,556	80,835	32,890	11.8	4	8
2	10	20	100	5	1	1	0	217,637	1429.2	111,016	75,245	31,376	11.93	4	8
3	10	20	100	5	1	1	0	226,349	1775.9	113,764	80,487	32,098	11.79	4	8
1	10	20	100	5	1	10	10	1,252,047	7200.62	1,131,410	80,922	39,715	13.71	4	7
2	10	20	100	5	1	10	10	1,211,196	7203.39	1,093,881	78,327	38,987	14.34	4	7
3	10	20	100	5	1	10	10	1,252,161	7203.89	1,115,760	86,277	50,124	17.38	4	7
1	10	20	100	5	10	1	0	937,391	105.89	116,192	779,865	41,334	14.25	4	10
2	10	20	100	5	10	1	0	875,752	132.3	112,602	722,834	40,316	14.69	4	10
3	10	20	100	5	10	1	0	928,932	100.29	117,211	774,375	37,345	13.37	4	11
1	10	20	100	5	10	10	0	1,964,290	360.96	1,128,809	789,964	45,517	15.42	4	7
2	10	20	100	5	10	10	0	1,871,651	501.78	1,093,881	735,297	42,473	15.31	4	7
3	10	20	100	5	10	10	0	1,947,978	992.04	1,118,343	781,454	48,181	16.6	4	7
1	10	20	100	10	1	1	0	265,225	3,834.46	115,447	92,226	57,552	10.64	3	10
2	10	20	100	10	1	1	0	252,181	4,762.76	111,885	86,137	54,159	10.64	3	10
3	10	20	100	10	1	1	0	262,769	4,413.18	114,652	91,202	56,915	10.75	3	10
1	10	20	100	10	1	10	25	1,313,241	7,203.83	1,106,001	114,969	92,271	15.64	3	7
2	10	20	100	10	1	10	24	1,257,864	7,203.72	1,071,784	103,644	82,436	14.97	3	7
3	10	20	100	10	1	10	8	1,310,380	7,203.14	1,087,578	117,310	105,493	17.80	3	6
1	10	20	100	10	10	1	0	992,944	193.38	181,288	732,794	78,862	13.65	5	9
2	10	20	100	10	10	1	0	924,945	239.34	174,567	673,875	76,503	13.98	5	8
3	10	20	100	10	10	1	0	988,747	298.59	180,033	731,472	77,242	13.68	5	9
1	10	20	100	10	10	10	0	2,014,624	1,295.70	1,097,817	816,490	100,317	16.65	3	6
2	10	20	100	10	10	10	0	1,912,019	1,637.57	1,063,854	752,417	95,748	16.85	3	6
3	10	20	100	10	10	10	0	2,002,262	1,035.60	1,090,161	813,909	98,192	16.68	3	6
1	10	20	128	0.01	1	1	0	218,884	1,014.60	121,312	97,344	228	30.80	3	5
2	10	20	128	0.01	1	1	0	208,282	1,559.84	116,978	91,060	244	37.60	3	4

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
3	10	20	128	0.01	1	1	0	216,898	1,283.14	118,737	97,837	324	43.72	3	4
1	10	20	128	0.01	1	10	0	1,297,133	7,127.78	1,190,856	105,418	859	114.51	3	3
2	10	20	128	0.01	1	10	0	1,254,629	3,243.37	1,157,127	96,700	802	123.31	3	3
3	10	20	128	0.01	1	10	0	1,278,647	2,800.01	1,176,559	101,197	891	118.77	3	3
1	10	20	128	0.01	10	1	0	1,086,198	159.03	161,020	924,887	291	39.27	4	7
2	10	20	128	0.01	10	1	0	1,009,937	221.40	155,561	854,132	244	37.58	4	6
3	10	20	128	0.01	10	1	0	1,063,139	167.70	157,999	904,761	379	50.83	4	6
1	10	20	128	0.01	10	10	0	2,186,300	1,078.65	1,211,357	974,410	533	71.40	3	5
2	10	20	128	0.01	10	10	0	2,080,666	2,658.43	1,169,790	910,585	291	44.74	3	4
3	10	20	128	0.01	10	10	0	2,165,714	763.87	1,187,388	977,865	461	61.97	3	4
1	10	20	128	5	1	1	0	248,909	7100.18	129,917	100,765	18,226	6.83	4	7
2	10	20	128	5	1	1	0	237,199	2167.68	126,233	93,988	16,978	6.78	4	7
3	10	20	128	5	1	1	0	244,256	2838.34	128,357	98,346	17,552	6.76	4	7
1	10	20	128	5	1	10	3	1,397,710	7202.46	1,247,044	112,950	37,716	12.68	4	5
2	10	20	128	5	1	10	10	1,349,203	7204.03	1,201,898	111,178	36,128	12.59	4	4
3	10	20	128	5	1	10	4	1,382,799	7202.88	1,219,207	119,461	44,131	14.64	4	4
1	10	20	128	5	10	1	0	1,142,451	247.35	134,638	987,966	19,847	7.4	4	11
2	10	20	128	5	10	1	0	1,064,755	241.82	130,469	914,657	19,629	7.74	4	11
3	10	20	128	5	10	1	0	1,115,879	299.4	132,202	963,896	19,781	7.59	4	11
1	10	20	128	5	10	10	0	2,306,037	2273.22	1,270,474	1,001,801	33,763	11.93	4	7
2	10	20	128	5	10	10	0	2,195,005	3561.91	1,237,968	929,430	27,607	10.51	4	7
3	10	20	128	5	10	10	0	2,267,448	4500.58	1,260,173	977,539	29,737	10.85	4	7
1	10	20	128	10	1	1	0	268,043	4,423.13	125,778	102,426	39,840	7.30	3	6
2	10	20	128	10	1	1	7	256,323	7,202.57	122,211	96,081	38,031	7.38	3	6
3	10	20	128	10	1	1	6	269,491	7,202.38	127,417	102,512	39,562	7.49	3	8
1	10	20	128	10	1	10	25	1,445,193	7,202.69	1,224,524	132,308	88,361	14.67	3	5

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
2	10	20	128	10	1	10	8	1,400,705	7,204.77	1,188,785	97,519	114,402	18.99	3	5
3	10	20	128	10	1	10	11	1,453,845	7,202.03	1,221,524	120,100	112,221	15.23	3	5
1	10	20	128	10	10	1	0	1,143,379	412.93	165,097	929,583	48,699	8.58	4	7
2	10	20	128	10	10	1	0	1,066,214	603.74	160,416	857,883	47,915	8.89	4	7
3	10	20	128	10	10	1	0	1,123,363	340.85	163,110	910,258	49,995	8.93	4	7
1	10	20	128	10	10	10	0	2,278,032	2,967.23	1,231,698	982,595	63,739	10.76	3	5
2	10	20	128	10	10	10	0	2,169,275	2,949.04	1,184,150	920,810	64,315	11.30	3	4
3	10	20	128	10	10	10	0	2,255,796	3,305.04	1,219,804	978,836	57,156	10.01	3	5
1	20	10	50	0.01	1	1	0	115,780	180.87	73,815	41,496	469	63.00	3	4
2	20	10	50	0.01	1	1	0	104,058	324.42	68,552	35,082	424	66.04	3	4
3	20	10	50	0.01	1	1	0	110,901	264.33	71,567	38,907	427	57.48	3	4
1	20	10	50	0.01	1	10	0	776,206	153.43	730,908	44,371	927	123.55	3	3
2	20	10	50	0.01	1	10	0	717,073	287.51	678,831	37,391	851	130.99	3	3
3	20	10	50	0.01	1	10	0	751,212	104.91	708,666	41,632	914	121.80	3	3
1	20	10	50	0.01	10	1	0	487,373	54.65	74,196	412,697	480	64.05	3	4
2	20	10	50	0.01	10	1	0	417,253	53.90	69,454	347,263	536	82.56	3	5
3	20	10	50	0.01	10	1	0	458,082	56.16	72,508	384,886	688	92.21	3	5
1	20	10	50	0.01	10	10	0	1,151,904	236.22	738,135	412,675	1,094	146.35	3	4
2	20	10	50	0.01	10	10	0	1,035,470	359.77	685,543	349,206	721	111.76	3	4
3	20	10	50	0.01	10	10	0	1,102,491	252.55	715,673	385,306	1,512	202.27	3	4
1	20	10	50	5	1	1	0	155,299	412.81	83,264	45,926	26,108	9.53	4	8
2	20	10	50	5	1	1	0	141,376	1245.47	77,328	39,673	24,375	9.47	4	8
3	20	10	50	5	1	1	0	149,877	1101.43	80,724	43,679	25,474	9.52	4	8
1	20	10	50	5	1	10	0	883,732	5337.46	799,499	44,197	40,037	13.44	4	5
2	20	10	50	5	1	10	0	818,809	4714.72	742,549	38,840	37,420	13.36	4	5
3	20	10	50	5	1	10	0	855,863	4320.67	775,168	41,463	39,231	13.48	4	5

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
1	20	10	50	5	10	1	0	537,348	74.74	101,872	399,387	36,089	12.33	5	8
2	20	10	50	5	10	1	0	470,002	107.49	94,256	342,546	33,200	12.14	5	8
3	20	10	50	5	10	1	0	506,858	95.79	79,654	384,230	42,974	14.56	4	7
1	20	10	50	5	10	10	0	1,265,755	1455.43	801,588	411,323	52,844	17.34	4	6
2	20	10	50	5	10	10	0	1,144,641	2809.27	739,008	351,424	54,209	18.55	4	5
3	20	10	50	5	10	10	0	1,208,177	1084.43	771,471	379,212	57,494	18.87	4	5
1	20	10	50	10	1	1	0	182,913	2,486.78	77,508	51,680	53,725	9.74	3	7
2	20	10	50	10	1	1	0	166,579	2,660.11	71,981	44,231	50,367	9.71	3	7
3	20	10	50	10	1	1	0	175,868	1,393.23	75,146	48,470	52,252	9.71	3	7
1	20	10	50	10	1	10	3	875,943	7,203.33	764,025	48,930	62,988	11.04	3	6
2	20	10	50	10	1	10	3	810,281	7,200.88	709,590	42,011	58,680	10.95	3	6
3	20	10	50	10	1	10	1	847,095	7,200.13	740,778	45,412	60,905	10.95	3	6
1	20	10	50	10	10	1	0	577,295	50.17	99,664	410,469	67,162	11.63	4	6
2	20	10	50	10	10	1	0	505,245	72.81	92,558	349,493	63,194	11.63	4	6
3	20	10	50	10	10	1	0	548,440	59.73	96,628	388,404	63,407	11.31	4	6
1	20	10	50	10	10	10	0	1,267,191	597.52	752,986	428,575	85,630	14.22	3	5
2	20	10	50	10	10	10	0	1,142,061	758.62	699,337	364,394	78,330	13.88	3	5
3	20	10	50	10	10	10	0	1,213,746	481.03	730,074	401,618	82,054	14.00	3	5
1	20	10	100	0.01	1	1	0	183,211	709.51	102,994	79,645	572	76.55	3	5
2	20	10	100	0.01	1	1	0	174,480	912.42	99,810	74,218	452	69.92	3	5
3	20	10	100	0.01	1	1	0	181,410	2,305.21	102,279	78,444	687	92.15	3	5
1	20	10	100	0.01	1	10	0	1,102,907	1,692.83	1,021,787	80,241	879	117.24	3	4
2	20	10	100	0.01	1	10	0	1,065,953	7,210.32	990,163	75,040	749	115.26	3	4
3	20	10	100	0.01	1	10	0	1,095,169	7,201.27	1,014,650	79,477	1,042	138.94	3	4
1	20	10	100	0.01	10	1	0	898,318	106.00	102,994	794,068	1,256	167.77	3	5
2	20	10	100	0.01	10	1	0	840,731	137.75	99,810	739,988	933	143.86	3	5

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
3	20	10	100	0.01	10	1	0	886,189	121.34	102,279	782,832	1,078	144.31	3	5
1	20	10	100	0.01	10	10	0	1,823,786	554.27	1,021,787	799,866	2,133	284.34	3	4
2	20	10	100	0.01	10	10	0	1,738,940	797.46	998,093	739,773	1,074	165.48	3	5
3	20	10	100	0.01	10	10	0	1,805,430	686.92	1,019,410	784,166	1,854	248.72	3	5
1	20	10	100	5	1	1	0	239,328	7202.35	113,139	87,124	39,066	13.53	4	7
2	20	10	100	5	1	1	3	232,147	7202.6	109,642	86,357	36,147	13.35	4	7
3	20	10	100	5	1	1	4	237,671	7203.08	112,356	86,578	38,737	13.7	4	7
1	20	10	100	5	1	10	4	1,256,388	7200.2	1,103,062	90,148	63,177	20.3	4	5
2	20	10	100	5	1	10	4	1,219,535	7203.02	1,082,666	90,042	46,827	16.54	4	6
3	20	10	100	5	1	10	4	1,249,670	7200.2	1,103,485	88,891	57,294	19.22	4	6
1	20	10	100	5	10	1	0	967,929	540.86	140,361	778,132	49,436	16.44	5	8
2	20	10	100	5	10	1	0	920,230	354.95	133,274	732,053	54,903	18.95	5	6
3	20	10	100	5	10	1	0	950,676	417.07	136,573	757,373	56,729	18.88	5	6
1	20	10	100	5	10	10	0	1,976,739	3867.23	1,112,034	790,291	74,415	23.62	4	6
2	20	10	100	5	10	10	0	1,898,456	3108.6	1,077,626	751,112	69,719	23.54	4	6
3	20	10	100	5	10	10	0	1,950,728	3523	1,095,359	780,209	75,160	24.18	4	5
1	20	10	100	10	1	1	13	279,626	7,203.41	106,687	99,651	73,288	12.85	3	7
2	20	10	100	10	1	1	14	271,132	7,201.83	103,392	93,758	73,982	13.60	3	7
3	20	10	100	10	1	1	13	280,587	7,200.93	107,098	100,406	73,083	13.16	3	8
1	20	10	100	10	1	10	6	1,247,649	7,202.41	1,064,309	102,856	80,484	13.96	3	7
2	20	10	100	10	1	10	8	1,217,292	7,201.26	1,020,153	112,022	85,118	15.26	3	6
3	20	10	100	10	1	10	8	1,256,759	7,204.49	1,039,434	102,795	114,530	19.25	3	6
1	20	10	100	10	10	1	0	1,023,110	465.73	137,526	788,699	96,885	16.16	4	6
2	20	10	100	10	10	1	0	959,434	671.68	133,276	736,048	90,110	16.01	4	6
3	20	10	100	10	10	1	0	1,009,307	512.10	170,011	759,464	79,832	14.04	5	7
1	20	10	100	10	10	10	0	2,006,869	2,126.68	1,038,562	836,663	131,644	21.04	3	5

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
2	20	10	100	10	10	10	0	1,905,498	1,839.05	1,006,418	775,937	123,143	20.93	3	5
3	20	10	100	10	10	10	0	1,989,332	2,264.87	1,031,308	827,946	130,077	21.27	3	5
1	20	10	128	0.01	1	1	0	213,916	4,065.93	115,667	97,742	507	68.97	3	5
2	20	10	128	0.01	1	1	0	203,230	2,789.22	112,676	90,208	346	53.65	3	5
3	20	10	128	0.01	1	1	0	209,121	6,317.96	114,277	94,349	495	66.67	3	5
1	20	10	128	0.01	1	10	19	1,250,339	7,201.97	1,147,777	100,972	1,590	211.98	3	4
2	20	10	128	0.01	1	10	1	1,210,070	7,202.83	1,115,825	93,064	1,181	181.75	3	4
3	20	10	128	0.01	1	10	0	1,231,885	7,203.33	1,127,469	103,489	926	123.53	3	3
1	20	10	128	0.01	10	1	0	1,093,302	427.99	115,667	976,882	753	101.70	3	5
2	20	10	128	0.01	10	1	0	1,014,911	271.47	112,878	901,763	270	41.57	3	5
3	20	10	128	0.01	10	1	0	1,058,076	268.46	114,277	943,280	519	69.97	3	5
1	20	10	128	0.01	10	10	0	2,134,311	1,893.62	1,156,667	976,986	657	89.00	3	5
2	20	10	128	0.01	10	10	0	2,027,725	2,379.95	1,123,916	902,693	1,116	172.18	3	5
3	20	10	128	0.01	10	10	0	2,086,582	3,278.36	1,142,783	943,280	519	69.97	3	5
1	20	10	128	5	1	1	7	255,235	7203.44	127,244	103,093	24,898	8.8	4	6
2	20	10	128	5	1	1	3	241,664	7201.89	123,986	95,666	22,012	8.32	4	6
3	20	10	128	5	1	1	6	246,727	7202.86	126,426	98,473	21,828	7.99	4	6
1	20	10	128	5	1	10	11	1,403,628	7202.99	1,220,980	122,996	59,653	18.46	4	3
2	20	10	128	5	1	10	11	1,347,769	7200.41	1,186,389	98,839	62,541	20.37	4	3
3	20	10	128	5	1	10	10	1,364,702	7203.38	1,222,101	105,361	37,240	12.59	4	4
1	20	10	128	5	10	1	0	1,152,453	1452.1	159,745	970,575	22,133	7.99	5	8
2	20	10	128	5	10	1	0	1,089,651	738.6	152,221	911,099	26,331	9.6	5	6
3	20	10	128	5	10	1	0	1,109,666	899.52	125,429	949,468	34,769	11.79	4	6
1	20	10	128	5	10	10	4	2,296,756	7203.02	1,243,227	1,009,154	44,375	14.57	4	5
2	20	10	128	5	10	10	4	2,196,491	7202.96	1,221,819	939,139	35,532	12.7	4	6
3	20	10	128	5	10	10	4	2,227,774	7201.8	1,228,311	956,902	42,561	14.37	4	5

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
1	20	10	128	10	1	1	17	287,439	7,201.85	119,981	117,344	50,114	8.90	3	6
2	20	10	128	10	1	1	9	253,212	7,200.87	115,380	94,668	43,164	8.15	3	5
3	20	10	128	10	1	1	22	306,809	7,201.05	117,369	116,950	72,490	12.74	3	6
1	20	10	128	10	1	10	47	1,989,770	7,201.86	1,154,198	153,757	681,815	98.22	3	4
2	20	10	128	10	1	10	6	1,327,203	7,202.30	1,159,284	104,428	63,491	11.47	3	6
3	20	10	128	10	1	10	6	1,344,390	7,202.29	1,153,868	103,520	87,001	14.22	3	4
1	20	10	128	10	10	1	0	1,155,820	687.04	157,022	951,159	47,639	8.40	4	6
2	20	10	128	10	10	1	0	1,078,697	644.99	152,574	881,031	45,092	8.44	4	6
3	20	10	128	10	10	1	0	1,120,856	565.97	155,134	917,287	48,435	8.67	4	6
1	20	10	128	10	10	10	6	2,232,004	7,202.28	1,167,888	997,600	66,516	11.05	3	4
2	20	10	128	10	10	10	0	2,119,729	5,876.49	1,145,788	914,653	59,288	10.69	3	5
3	20	10	128	10	10	10	12	2,241,923	7,201.41	1,150,325	1,008,888	82,710	13.76	3	4
1	20	20	50	0.01	1	1	0	106,174	676.78	74,272	31,572	330	44.80	3	5
2	20	20	50	0.01	1	1	0	97,396	637.97	68,429	28,578	389	60.54	3	4
3	20	20	50	0.01	1	1	0	103,415	743.17	72,010	31,084	321	43.56	3	5
1	20	20	50	0.01	1	10	0	765,128	1,927.38	730,908	33,317	903	120.40	3	3
2	20	20	50	0.01	1	10	0	709,088	1,779.92	678,831	29,395	862	132.54	3	3
3	20	20	50	0.01	1	10	0	742,309	452.11	708,666	32,696	947	126.31	3	3
1	20	20	50	0.01	10	1	0	387,840	44.66	74,862	312,637	342	46.76	3	6
2	20	20	50	0.01	10	1	0	348,904	62.49	69,525	278,869	510	79.82	3	6
3	20	20	50	0.01	10	1	0	379,704	118.28	96,204	283,355	145	20.14	4	7
1	20	20	50	0.01	10	10	0	1,058,545	475.51	736,809	320,941	796	106.21	3	4
2	20	20	50	0.01	10	10	0	970,495	1,041.68	684,312	285,794	389	60.54	3	4
3	20	20	50	0.01	10	10	0	1,031,049	753.80	720,110	310,493	446	60.24	3	5
1	20	20	50	5	1	1	0	148,736	2820.14	83,264	39,739	25,733	9.42	4	8
2	20	20	50	5	1	1	0	135,599	3059.62	77,328	34,700	23,570	9.23	4	8

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
3	20	20	50	5	1	1	0	143,956	1262.47	80,724	38,516	24,716	9.31	4	8
1	20	20	50	5	1	10	3	881,798	7206.36	799,499	42,050	40,250	13.5	4	5
2	20	20	50	5	1	10	11	835,325	7203.83	742,549	39,776	52,999	18.01	4	5
3	20	20	50	5	1	10	8	858,460	7201.85	785,872	41,472	31,116	11.15	4	6
1	20	20	50	5	10	1	0	479,148	126.14	83,987	356,613	38,547	13.07	4	9
2	20	20	50	5	10	1	0	418,582	195.39	76,974	301,283	40,325	14.26	4	8
3	20	20	50	5	10	1	0	464,208	133.72	81,794	347,506	34,908	12.24	4	9
1	20	20	50	5	10	10	0	1,214,946	2525.98	805,400	359,921	49,625	16.28	4	6
2	20	20	50	5	10	10	0	1,098,523	3836.3	742,549	309,143	46,832	16.17	4	5
3	20	20	50	5	10	10	0	1,174,296	2031.59	780,890	348,454	44,952	15.27	4	6
1	20	20	50	10	1	1	6	168,251	7,205.87	79,716	43,582	44,953	8.51	3	9
2	20	20	50	10	1	1	8	153,966	7,205.30	74,031	37,589	42,346	8.52	3	9
3	20	20	50	10	1	1	10	162,996	7,202.55	78,356	42,693	41,947	8.23	3	10
1	20	20	50	10	1	10	8	887,241	7,204.30	775,064	57,580	54,597	9.86	3	7
2	20	20	50	10	1	10	8	827,772	7,203.78	709,590	40,477	77,705	13.79	3	6
3	20	20	50	10	1	10	7	859,565	7,204.94	747,785	50,372	61,409	11.13	3	7
1	20	20	50	10	10	1	0	473,448	121.12	125,132	287,413	60,903	10.75	5	8
2	20	20	50	10	10	1	0	425,217	157.53	116,210	253,623	55,385	10.46	5	8
3	20	20	50	10	10	1	0	453,697	171.84	121,320	274,216	58,161	10.56	5	8
1	20	20	50	10	10	10	0	1,188,485	2,885.66	764,025	336,780	87,680	14.51	3	6
2	20	20	50	10	10	10	0	1,086,758	3,584.33	709,590	302,617	74,551	13.32	3	6
3	20	20	50	10	10	10	4	1,160,514	7,203.86	737,081	335,598	87,834	14.98	3	6
1	20	20	100	0.01	1	1	0	167,613	2,733.62	102,363	64,924	327	43.93	3	4
2	20	20	100	0.01	1	1	0	159,255	3,246.74	99,019	59,812	424	65.90	3	4
3	20	20	100	0.01	1	1	0	165,713	3,088.90	101,648	63,766	299	40.32	3	4
1	20	20	100	0.01	1	10	0	1,081,201	2,959.67	1,013,619	66,633	949	126.56	3	3

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
2	20	20	100	0.01	1	10	19	1,045,187	7,201.69	982,251	62,155	781	120.12	3	3
3	20	20	100	0.01	1	10	0	1,073,043	2,429.26	1,006,542	65,518	983	131.03	3	3
1	20	20	100	0.01	10	1	0	730,447	145.02	136,784	593,486	177	24.32	4	6
2	20	20	100	0.01	10	1	0	686,746	223.91	132,834	553,562	350	54.62	4	7
3	20	20	100	0.01	10	1	0	720,972	256.00	136,643	584,164	165	22.94	4	7
1	20	20	100	0.01	10	10	0	1,672,060	4,072.03	1,021,803	649,736	521	70.11	3	4
2	20	20	100	0.01	10	10	0	1,587,979	3,333.66	990,181	597,148	650	100.57	3	4
3	20	20	100	0.01	10	10	0	1,654,313	3,212.93	1,014,668	638,842	803	107.88	3	4
1	20	20	100	5	1	1	7	228,763	7202.97	114,296	79,187	35,279	12.55	4	8
2	20	20	100	5	1	1	7	215,958	7202.97	111,016	72,988	31,954	12.1	4	8
3	20	20	100	5	1	1	4	227,782	7203.53	116,062	80,132	31,588	11.7	4	10
1	20	20	100	5	1	10	5	1,257,052	7202.86	1,117,236	83,724	56,092	18.31	4	6
2	20	20	100	5	1	10	7	1,243,234	7204.72	1,111,271	87,858	44,105	16.22	4	9
3	20	20	100	5	1	10	5	1,246,740	7203.99	1,123,509	85,954	37,277	13.28	4	7
1	20	20	100	5	10	1	0	887,757	340.55	140,919	697,655	49,183	16.43	5	9
2	20	20	100	5	10	1	0	832,140	443.93	136,562	651,592	43,986	15.76	5	9
3	20	20	100	5	10	1	0	884,799	416.27	141,351	699,592	43,856	15.24	5	10
1	20	20	100	5	10	10	5	1,935,378	7203.69	1,114,635	754,181	66,562	21.39	4	6
2	20	20	100	5	10	10	1	1,840,457	7200.46	1,091,361	690,168	58,928	20.4	4	7
3	20	20	100	5	10	10	4	1,913,842	7201.83	1,106,851	743,709	63,283	20.84	4	6
1	20	20	100	10	1	1	16	242,583	7,203.42	112,078	84,045	46,460	8.72	3	10
2	20	20	100	10	1	1	17	230,852	7,203.47	107,130	78,589	45,133	8.93	3	9
3	20	20	100	10	1	1	23	250,755	7,204.64	111,293	95,452	44,010	8.53	3	10
1	20	20	100	10	1	10	8	1,192,740	7,204.44	1,041,595	81,155	69,990	12.64	3	7
2	20	20	100	10	1	10	26	1,234,117	7,202.19	1,052,151	113,899	68,068	11.98	3	6
3	20	20	100	10	1	10	11	1,230,663	7,202.40	1,050,111	110,210	70,342	11.45	3	6

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
1	20	20	100	10	10	1	0	821,780	347.09	172,470	580,819	68,491	11.86	5	8
2	20	20	100	10	10	1	0	767,768	371.17	167,649	539,063	61,056	11.33	5	8
3	20	20	100	10	10	1	0	803,285	445.18	171,790	568,188	63,307	11.33	5	8
1	20	20	100	10	10	10	2	1,828,726	7,201.33	1,028,928	670,081	129,717	20.41	3	4
2	20	20	100	10	10	10	0	1,736,913	6,622.79	1,006,799	623,285	106,829	18.29	3	5
3	20	20	100	10	10	10	10	1,824,359	7,202.13	1,039,824	666,267	118,268	19.48	3	6
1	20	20	128	0.01	1	1	4	201,706	7,202.43	116,275	84,875	555	74.33	3	5
2	20	20	128	0.01	1	1	9	194,298	7,203.33	113,355	80,083	860	132.29	3	5
3	20	20	128	0.01	1	1	8	198,026	7,201.58	115,589	81,290	1,147	153.08	3	6
1	20	20	128	0.01	1	10	1	1,238,470	7,203.41	1,150,611	87,192	666	88.83	3	4
2	20	20	128	0.01	1	10	1	1,208,522	7,204.28	1,126,985	81,048	489	75.49	3	5
3	20	20	128	0.01	1	10	1	1,221,317	7,203.16	1,136,802	83,827	687	91.66	3	4
1	20	20	128	0.01	10	1	0	920,448	319.71	192,492	727,548	407	55.21	5	9
2	20	20	128	0.01	10	1	0	867,199	483.32	186,584	680,206	409	64.28	5	9
3	20	20	128	0.01	10	1	0	895,910	283.83	190,471	705,131	308	41.92	5	9
1	20	20	128	0.01	10	10	0	2,008,253	5,219.28	1,159,826	847,400	1,027	137.19	3	5
2	20	20	128	0.01	10	10	9	1,968,172	7,203.50	1,132,230	833,385	2,557	394.13	3	6
3	20	20	128	0.01	10	10	0	1,964,211	3,031.49	1,136,802	826,203	1,206	160.86	3	4
1	20	20	128	5	1	1	9	248,647	7213.88	127,962	98,101	22,585	8.05	4	6
2	20	20	128	5	1	1	6	234,735	7204.13	124,334	91,199	19,202	7.44	4	6
3	20	20	128	5	1	1	7	243,738	7209.84	128,003	94,504	21,231	7.85	4	7
1	20	20	128	5	1	10	7	1,446,347	7201.29	1,253,560	143,782	49,005	15.55	4	5
2	20	20	128	5	1	10	2	1,329,483	7201.27	1,205,384	95,050	29,049	10.38	4	4
3	20	20	128	5	1	10	10	1,475,885	7200.76	1,215,664	99,382	160,840	48.59	4	4
1	20	20	128	5	10	1	0	1,074,729	885.12	161,023	890,265	23,441	8.51	5	10
2	20	20	128	5	10	1	0	1,015,704	806.21	156,172	837,812	21,720	8.42	5	10

Table B.1: Experimental results (continued)

Rep	N	M	K	θ	δ	β	Gap(%)	obj	CPU(s)	FC	TC	CC	E(w)	NOP	NODC
3	20	20	128	5	10	1	0	1,059,139	670.74	159,853	877,118	22,168	8.23	5	10
1	20	20	128	5	10	10	5	2,251,762	7207.81	1,251,814	968,646	31,301	10.69	4	5
2	20	20	128	5	10	10	5	2,144,500	7204.58	1,223,515	889,440	31,545	11.61	4	6
3	20	20	128	5	10	10	6	2,218,970	7201.57	1,247,204	931,272	40,494	13.88	4	6
1	20	20	128	10	1	1	26	314,257	7,202.41	125,265	124,072	64,920	11.76	3	11
2	20	20	128	10	1	1	31	311,556	7,203.36	122,005	129,699	59,852	11.56	3	11
3	20	20	128	10	1	1	31	329,014	7,204.95	120,898	134,150	73,966	13.28	3	9
1	20	20	128	10	1	10	26	1360401.96	7200.68	1170320	125116	64965.96	11.99	3	7
2	20	20	128	10	1	10	-								
3	20	20	128	10	1	10	-								
1	20	20	128	10	10	1	0	988,586	702.43	196,536	733,046	59,004	10.51	5	9
2	20	20	128	10	10	1	0	933,964	652.80	190,972	686,468	56,524	10.65	5	9
3	20	20	128	10	10	1	0	964,046	737.78	193,586	708,658	61,802	11.16	5	9
1	20	20	128	10	10	10	10	2,153,060	7,202.08	1,186,301	853,181	113,578	18.46	3	7
2	20	20	128	10	10	10	3	2,035,663	7,200.55	1,153,568	791,368	90,726	15.84	3	6
3	20	20	128	10	10	10	12	2,104,800	7,203.39	1,176,154	823,046	105,601	17.69	3	7