PRE-SERVICE ELEMENTARY MATHEMATICS TEACHERS' KNOWLEDGE ABOUT DEFINITIONS OF INTEGERS AND THEIR KNOWLEDGE ABOUT ELEMENTARY STUDENTS' POSSIBLE MISCONCEPTIONS AND ERRORS IN DESCRIBING INTEGERS

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ABSTRACT

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The aim of the study is to examine the nature of pre-service elementary mathematics teachers' subject matter knowledge about definition of integers and the nature of pre-service elementary mathematics teachers' pedagogical content knowledge about definitions of integers. For this purpose, pre-service mathematics teachers' knowledge about definitions of integers, their interpretations of quoted definitions of integers, their knowledge about elementary students' possible misconceptions and errors regarding definition of integers, and possible reasons of elementary students' definitions of integers were examined. The thesis made use of two open-ended questions in order to gain information about pre-service teachers' definitions of integers, their knowledge of elementary students' possible misconceptions and errors about integers, and possible reasons of elementary students' errors and misconceptions. In addition, this study also made use of interview questions that were related to quoted integer definitions. In the beginning of the study, two open-ended questions were administered to 38 pre-service mathematics teachers who were taking teaching practice course. 4 voluntary pre-service mathematics teachers were interviewed.

The results of the study indicated that pre-service teachers preferred three categories for defining integers: "core concepts", "representation", and "other definitions". They suggested several mistakes which elementary students might make. The results of the study showed that misconceptions and errors suggested by the participants are parallel with the related literature. The reasons of the misconceptions and errors classified on the basis of the mistakes into three: negative transfer of former knowledge about number sets, students' general insufficiency, and teaching approach.

Key Words: Concept Definition, Pre-service Teachers, Teacher Knowledge, Integers

ÖZ

İLKÖĞRETİM MATEMATİK ÖĞRETMEN ADAYLARININ TAMSAYI TANIMI HAKKINDAKİ VE İLKÖĞRETİM ÖĞRENCİLERİNİN TAMSAYI TARİFLERİ HAKKINDAKİ OLASI KAVRAM YANILGISI VE HATALARINA İLİŞKİN BİLGİSİ

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Bu çalışmanın amacı, ilköğretim matematik öğretmen adaylarının tamsayıların tanımına ilişkin konu alan bilgisi ve pedagojik alan bilgisini incelemektir. Bunun için, öğretmen adaylarının tamsayı tanımına ilişkin bilgileri, öğretmen adaylarının alıntı tamsayı tanımlarına ilişkin bilgileri, aday öğretmenlerin ilköğretim öğrencilerinin tamsayı tarifleri hakkındaki olası kavram yanılgısı ve hatalarına ilişkin yorumları, öğretmen adaylarının ilköğretim öğrencilerinin tamsayı tarifleri hakkındaki olası kavram yanılgısı ve hatalarının nedenlerine ilişkin bilgileri incelenmiştir. Çalışmada bu amaçları gerçekleştirebilmek için iki araç kullanılmıştır: bunlardan birisi açık uçlu iki soru, diğeri ise görüşme sorularıdır. Çalışmanın başlangıcında açık uçlu bu iki soru, staj dersini almakta olan 38 aday öğretmene uygulanmştır. Bu öğrencilerin dördüyle ise gönüllülük esasına dayanılarak görüşme yapılmıştır. Çalışmanın sonucuna gore, öğretmen adayları, tamsayı kavramını tanımlarken 3 yol izlemişlerdir: "çekirdek kavramlar", "gösterim", ve "diğer tanımlar". Çalışmanın sonuçları, öğretmen adaylarının yaptıkları bazı tanımlamalarda eksiklik ve yanlışlık olduğunu göstermektedir.

Öğretmen adayları aynı zamanda ilköğretim öğrencilerinin kavram yanılgısı ve hatalarına ilişkin bir çok öneride bulunmuşlardır. Çalışmanın sonuçları, öğretmen adaylarının önerdikleri kavram yanılgısı ve hataların, önceki çalışmalarda değinilen sonuçlarla paralellik gösterdiğini ortaya koymuştur. Önerdikleri kavram yanılgıları ve hatalara dayanılarak, bu kavram yanılgısı ve hataların kaynakları üçe ayrılır: sayı kümeleriyle ilgili sahip olunan bilginin olumsuz transferi, öğrencilerin genel yetersizlikleri, ve öğretme yaklaşımları.

Anahtar Kelimeler: Kavram Tanımı, Aday öğretmen, Öğretmen bilgisi, Tam sayılar

To whom thinks the study is valuable…

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LIST OF ABBREVIATIONS

TTKB: Talim ve Terbiye Kurulu Başkanlığı

MoNE: Ministry of National Education

NCTM: National Council of Teachers of Mathematics

CHAPTER I

INTRODUCTION

Teachers are key people in the mathematics learning process of students. They may help students to clear the confusions that may exist in their minds regarding the content of mathematics or to gain positive attitudes or beliefs towards it. Considering students' long educational lives, it can be argued that students seek answers to questions about mathematics itself as well as the purposes in learning it. Students may think that one of the major goals of learning mathematics is to make computations, solve mathematical problems, or play with numbers. Also, they may see mathematics as the accumulation of disconnected rules (Lappan & Even, 1989). Elementary students' perception of mathematics in this way may continue throughout the following years of their educational life. Pre-service elementary mathematics teachers may also think in the same way as elementary students, or they may have ambiguous opinions in terms of the purposes in learning or teaching of elementary mathematics to their students. To establish a consistency between students' ideas regarding mathematics, and mathematics itself, teachers may need to rethink what mathematics is all about, why students need to study mathematics, and the role of teachers in the learning and teaching process.

The National Council of Teaching Mathematics (NCTM, 2000) standards in the USA and the Ministry of National Education (MoNE, 2009) curriculum documents in Turkey include widely accepted guidelines on these issues. The major aims of the elementary school curricula in Turkey include helping students to gain skills of critical thinking, creativity, communication, investigation, problem solving, using information technologies, and entrepreneurship (MoNE, 2009). In addition to these skills, the mathematics curriculum specifically focuses on problem solving, communication, reasoning, and connecting concepts (MoNE, 2009). These skills enable students to engage in more meaningful learning of mathematics (MoNE, 2009).

When students are doing mathematics, they learn problem solving, how to explain and defend their own ideas about mathematics to their friends, and to see how mathematics is in relation to other disciplines, and mathematical concepts (MoNE, 2009). Another important recommendation of mathematics curriculum in Turkey is to create learning environments which expects students to investigate, invent, solve problems, and share ideas and approaches (MoNE, 2009).

Considering all these goals and standards, the success of acquisition relies mostly on the teachers who are key people for the success of school curriculum (Çakıroğlu & Çakıroğlu, 2003). Thus, to effectively implement elementary school curriculum, it is necessary to have qualified teachers (Işıksal, Koç, & Bulut, 2007), and teachers should have in-depth knowledge and understanding of the mathematics they teach to students (Hill, Ball, & Schilling, 2008).

Ma (1999) suggests that "teachers' knowledge might directly affect mathematics teaching and learning" (p. xix). To be effective in teaching, teachers' knowledge of mathematics and teaching mathematics are critical factors (Işıksal et. al., 2007) and teachers' mathematical knowledge is so important that it is related to student achievement (Hill, Rowan, & Ball, 2005). "Teachers, who do not know subjects well are not likely to have the knowledge they need to help students learn this content" (Ball, Thames, Phelps, 2008, p.404). Kahan and Cooper (2003) agreed that "when policy makers and professional mathematicians consider the problems of school mathematics, they frequently conclude that students would learn more mathematics if their teachers knew more mathematics" (p. 223). In other words, students' learning of mathematics depends mostly on teachers' knowledge about mathematics. Students' difficulties and mistakes can be overcome with the teachers' help and teachers' help is directly related with teachers' knowledge (Bayazit & Aksoy, 2010).

Considering the goals of mathematics education in schools, an important consideration is how teachers should be equipped to be able to teach mathematics content. A response to this question relies on what really goes on in teachers' work in their mathematics classrooms instead focusing on the curriculum itself (Ball, Thames, & Phelps, 2008). According to Lappan and Even (1989), analyzing experiences in learning environments is one of the ways to examine what really goes on in teachers' work. Such experiences enable teachers to make sense of mathematics and to learn effective strategies in teaching mathematics. Lappan and

Even (1989) point out that teachers need to learn "doing mathematics" (p.3) to help students and themselves to create what mathematics is and what teachers need to make sense of mathematics and to learn effective teaching strategies. According to Ball et. al. (2008), the prevailing view is that teachers need to know everything that the curriculum covers and they need to study some years on college mathematics. Teachers also need to have in-depth insight in the curriculum together with some pedagogical content knowledge. As mathematics teachers teach mathematics, they should have a strong understanding of the subject they teach. If they do not know the subject well, they would probably not be able to help students to learn the subject. Even if they know the subject well, they may not be able to help students because of not knowing how to instruct (Ball, et. al., 2008). Thus, teachers should not only know their subjects well but also know how to design the subject of the teaching lesson as effectively as possible.

Teacher preparation programs help pre-service teachers to find out what they need to know when they enter classrooms. In preparation programs, pre-service teachers are educated to teach effectively. Effective teaching needs to be ensured by being competent in knowledge of mathematics and is shaped by conceptual understanding of that content (Kwoen, 2000). In teacher education programs, pre-service teachers improve themselves before they enter real school environments, and many of the programs are designed for pre-service teachers to practice the necessary knowledge and learn through content courses, method courses, or training sections (Nilsson, 2008; Ball et. al., 2008; Jegede & Taplin, 2000; Inoue, 2009; Zakaria & Zaini, 2009). Pre-service teachers learn how to teach their future students (Morris, Hiebert, & Spitzer, 2009). According to Morris et. al. (2009), the aim of teacher preparation programs is to ensure that pre-service teachers acquire the necessary skills to learn how to teach in a systematic way. Preparation programs provide two streams of work which address the knowledge and skills pre-service teachers have. The first stream focuses on the competencies needed to learn to teach effectively, and the other stream focuses on the development of mathematical knowledge for teaching.

It might be said that, in Turkey, teacher education programs focus on how pre-service teachers can be graduated as qualified teachers. To do this, one of the ways might be considered as being a proficient mathematics teacher in mathematical knowledge. It might be important that pre-service teacher need to know required knowledge in Turkish mathematics curriculum when they graduate from teacher education programs as qualified teachers. It might be said that Turkish mathematics curriculum helps pre-service teachers what to focus on or what need to be known for mathematical knowledge. In response to this, the elementary school mathematics curriculum in Turkey focus on mathematical concepts and their relationship to each other, the meanings of operations and acquisition of skills.

Having knowledge in mathematical concepts might mainly be related to knowing the definitions of the concepts. Knowledge in the definition of concepts is essential since it affects teachers' way of teaching, how they order the topics to be taught, and what set of theorems and proofs are to be covered. Consequently, one aspect of the content knowledge which teachers need to know is how to make definitions of mathematical concepts (Shield, 2004).

Defining mathematics concepts is important for elementary students that it enables elementary students to be aware of the relationships among mathematical concepts (Shield, 2004; Zaskis & Leikin, 2008). It is also important for pre-service teachers that definitions of mathematical concepts, structure of the definitions, and process of defining have an important part in teachers' subject matter knowledge regarding (Zaskis & Leikin, 2008). Defining is also important for their pedagogical content knowledge. Leikin & Zaskis (2010) describes the location of definition in teachers' pedagogical content knowledge:

> *This knowledge includes awareness of the cognitive, social and affective characteristics of a mathematics classroom in which a mathematical definition is an object of learning and teaching. It also includes understanding of students' cognitive development, of their common concept images and concept definitions of particular mathematical concepts, and of students' conceptions of the definition of meta-mathematical constructs per se. Teachers' pedagogical content knowledge determines their ability to match the teaching of definitions and defining with a particular classroom,*

> *and to attend to students' ability levels, affective needs and motivation* (p.454).

It shows that mathematical definitions improve students' understanding of mathematical concepts and constructions, and enable teachers to see what their students' conceptions about the issue. Knowledge of mathematical definitions in pedagogical perspective enables teachers to be aware of mathematical classrooms in the aspect of cognitive, social, and effective characteristics of the classrooms. What's more, it enables teachers to understand their students' cognitive development, their common concept images and concept definitions for mathematical concepts, and their definition of meta-mathematical constructs.

Although definition of mathematical concepts is important for students in mathematics departments, it is known that they do not know definitions they need to perform in proving theorems better (Edwards & Ward, 2008). According to Edwards and Ward (2008) making definitions are not an easy task for students in mathematics departments.

> *It seems to be common knowledge in mathematics departments that many students do not "know" the definitions they need to know in order to perform mathematical tasks such as proving theorems. Often, in an attempt to solve this problem students are asked to memorize the pertinent definitions in the course and sometimes they are given credit in examinations for repeating those definitions* (p.225).

To overcome the difficulty students have about defining in mathematics, defining activities which promote deeper conceptual understanding of mathematics involved, promote an understanding of the nature or the characteristics of mathematical definitions, and promote an understanding of the role of definitions in mathematics might be designed to use in mathematics courses (Edwards & Ward, 2008).

It might be important to examine students in undergraduate mathematics programs or mathematics teacher education programs for definition of mathematical concepts to deeply investigate what they know about the mathematical concepts or what they need to know.

It might be meaningful to study with students in mathematics teacher education programs since they have future students who will need to learn about mathematical concepts. In this way, it might be important to examine pre-service teachers' definitions of mathematical concepts since they will be responsible for the learning of their future students. Accordingly, their interpretations of any given definitions of mathematical concepts might be important since their subject matter knowledge and pedagogical content knowledge are fed from this type of knowledge. In this manner, one of mathematical concepts in which elementary students or teachers have difficulty to comprehend might be meaningful to examine.

Integers are one of the main mathematical concepts for elementary students and teachers in learning and teaching mathematics. So far, however, there has been little discussion about pre-service teachers' knowledge about definition of the concept of integers which is in close relationship with number systems in which the concept of integer is an extended number system which is not a trivial matter in elementary grades to teach (Levenson, 2012).

1.1 STATEMENT OF THE PROBLEM

Many researchers argue that one of the concepts that elementary students and pre-service mathematics teachers have problems in constructing conceptually is integers (Smith, 2002; Köroğlu & Yeşildere, 2004; Steiner, 2009; Spang, 2009; Ercan, 2010; İşgüden, 2008). Research studies show that elementary students have problems in operations of integers, especially in terms of addition and subtraction. Elementary students may get confused when they encounter integers with different signs, and most students just memorize the rule of changing the signs in subtraction problems (Steiner, 2009).

Another example of a problematic area regarding the concept of integers is zero. Elementary students have difficulty in operations that require distinguishing between "division of zero", and "division with zero" (Smith, 2002). What zero means also makes elementary students confused (Spang, 2009; Steiner, 2009; Smith, 2002; Van De Walle, Karp, Karp, & Bay-William, 2010). According to Spang (2009), the meanings of positive and negative integers, and the signs of "+" or "–" are another source of difficulty for elementary students. The evidence of the difficulty can be clearly seen in interpreting the problem itself, or creating a problem, which was designed using different meanings of integers. These studies above show that elementary students have problems with "integers" concepts.

The concept of integers might be thought as an extended set of whole numbers and cardinal numbers. "There are several junctions throughout the school years where the number system is extended. With each extension the teacher and the students must consider how the “new” numbers behave differently from previously recognized numbers as well as how known operations must be extended to include the new numbers. Extending the number system calls for definitions of new operations and terms as well as modifications of previously defined operations" (Levenson, 2012, p.211). To develop students’ understanding of mathematical concepts, it is important that teachers know and construct definitions since students encounter many definitions of mathematical concepts when they are learning (Shield, 2004). Teachers have problems in defining some mathematical concepts, such as circle, function, integral, sphere, slope (Leikin & Zaskis, 2010), fractions (Işıksal, 2006), and absolute value (Vinner, 1991). According to Leikin and Zaskis (2010), "teachers’ knowledge of mathematical definitions is a core element of teachers’ content knowledge. Teachers’ concept images and their personal definitions of mathematical concepts, as well as their understanding of definition as a construct guides the instructional design they create, the explanations they provide in the classroom, the way they guide their students in proving and solving procedures, and how they conduct mathematical discussions" (p.455). What’s more, teachers’ ways of teaching is influenced by their understanding of mathematical concepts (Leikin & Zaskis, 2010). Students may have problems with integers since their teachers may hold conceptions which cause misconceptions/mistakes or difficulties for elementary students.

It is important for pre-service teachers to be equipped with knowledge that does not cause misconceptions/mistakes or difficulties in students’ minds while teaching integers. How pre-service teachers conceptualize integers is a crucial point for future teachers and their students. How integers are defined by pre-service teachers may be a starting point to understand their conceptions of integers.

In this study, pre-service teachers' conceptions of integers were investigated based on the following research questions:

1. What is the nature of pre-service elementary mathematics teachers' subject matter knowledge regarding definition of integers?
   a. What are the types of integer definitions made by pre-service elementary mathematics teachers?
   b. How appropriate are the integer definitions made by pre-service elementary mathematics teachers?
   c. What are the interpretations made by pre-service teachers regarding quoted definitions of integers?
2. What is the nature of pre-service elementary mathematics teachers' pedagogical content knowledge in definitions of integers?
   a. What pre-service elementary mathematics teachers know about elementary students' possible misconceptions and errors regarding definition of integers?
   b. What do pre-service elementary mathematics teachers know about the possible reasons of misconceptions and mistakes of students regarding definition of integers?

1.2 SIGNIFICANCE OF THE STUDY

It is important for pre-service teachers to have content and pedagogical knowledge in the subject matter to teach mathematics effectively and overcome problematic situations that may arise in classrooms (Morris et. al., 2009; Stylianides et. al., 2010). It is considered that "definitions of mathematical concepts, the underlying structures of the definitions and the process of defining are some of the fundamental components of the subject matter knowledge of teachers of mathematics" (Zaskis &

Leikin, 2008, p. 133). Accordingly, this study aims to describe pre-service teachers' content knowledge regarding definition of integers. This study also aims to describe pre-service teachers' pedagogical content knowledge which "includes awareness of the cognitive, social and affective characteristics of a mathematics classroom in which a mathematical definition is an object of learning and teaching" (Leikin & Zaskis, 2009, p.454).

Examining pre-service teachers' knowledge of definition of integers is significant since they will have future students for whom relationship between number sets is an essential learning point to develop their understanding of mathematics. What's more, pre-service teachers guide their peers in cases where they have similar nature of conceptions in their minds. Then, pre-service teachers are given opportunity to correct their mistakes, or confusions before real classroom experiences.

According to Shield (2004), not only is creating definitions important for teachers in their development of understanding mathematical concepts, but also important to explore the accuracy of quoted definitions since these contribute significantly to students' understanding of the concept. In this study, pre-service teachers are posed questions regarding quoted definitions in order to understand their conceptions of integers deeply.

It is worthwhile to note that teachers and teacher educators need to investigate their students' logical aspects of mathematical activities to guide them in making educated pedagogical decisions (Van Dormolen & Zaslavsky, 2003). Teachers and teacher educators do not prefer to fill students' minds with piles of formal knowledge. However, they need to understand logical aspects of mathematical activities related to their teaching content. In this way, they need to understand the nature of the content to be taught before understanding the pedagogy of the content area (Van Dormolen & Zaslavsky, 2003). In this manner, definition of integers provided by pre-service teachers may help teacher educators to better understand pre-service teachers' logic of the concept of integers.

Teachers are one of the important components in students' learning process as their understanding of mathematics is central for effective teaching (Feueborn, Chinn, &

Morlan, 2009). This study helps to reveal pre-service teachers' conceptualization of the integer concept. Identifying difficulties or problems pre-service teachers have may help researchers to predict their performance in the future, in their real classrooms. In this way, teacher educators can design their content or method courses by taking into consideration the difficulties or problems that pre-service teachers have to enrich pre-service teachers' knowledge in that content.

Examining pre-service teachers' pedagogical content knowledge of integers regarding possible misconceptions and errors of elementary students helps to reveal the pre-service teachers' strengths and weaknesses about elementary students' thinking of integers. Their strengths and weaknesses show how pre-service teachers are able to combine their knowledge in mathematics and knowledge about students. To know the strengths and weaknesses of pre-service teachers regarding students' perceptions enable teacher educators to design tasks and to develop them in their method courses. In addition, by means of this study, examining pre-service teachers' knowledge of elementary students' possible sources of misconceptions and errors may help them to be prepared in overcoming problematic situations derived from the sources.

In order to make students learn meaningfully, it may be considered that teachers need to think deeply on student learning during teaching. To be more successful in mathematics, a comprehensive understanding of integers regarding misconceptions about integers which cause difficulties is needed (Vlassis, 2004; Akyüz, Stephan, & Dixon, 2012). Thus, teachers need to pay attention to students' mistakes, misunderstandings or misconceptions, and design their lessons taking these into consideration (Shulman, 1986). In this study, misconceptions and errors suggested by pre-service teachers enable teacher educators to gain insight into pre-service teachers' (in) experiences in knowledge of content and students. This helps to design method courses and to design practice teaching courses.

Several studies have produced insight in understanding the concept of integers. They are mostly related to algorithms of integers, role of negative sign, and concrete materials or models used to teach integers more effectively. However, far too little attention has been paid to definition of integers. It is hoped that this study may contribute to related literature on integers.

1.3 DEFINITION OF THE IMPORTANT TERMS

In this study, the important terms are intended to be used with the meanings below:

Pre-service elementary mathematics teachers: Pre-service elementary mathematics teachers are students in teacher education programs in their last years. They have been educated in mathematics to teach elementary grades from 6 to 8. They have completed all the required courses to graduate from the elementary mathematics teacher education program.

Integers: The whole numbers, 0, 1, 2, 3, 4… , together with the negatives of the whole numbers, - 1, -2, -3, -4,…. are called integers (Bennett & Nelson, 2001).

Definition: A form of words used to specify that concept (Tall & Vinner, 1981, p. 152).

Misconception: A student conception that produces a systematic pattern of errors (Smith, diSessa, & Roschelle, 1993, p.119).

Error: "An error is a mistake, slip, blunder, or inaccuracy, and a deviation from accuracy" (Luneta & Makonye, 2010, p.36).

In this study, misconception and error were not separately considered.

Subject matter knowledge: In this study, subject matter knowledge refers to "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p.9). For this study, I referred specifically to pre-service teachers' knowledge regarding definition of integers as their subject matter knowledge.

Pedagogical content knowledge: For this study, pedagogical content knowledge specifically referred to knowledge of pre-service teachers about elementary students' misconceptions and errors related to definition of integers.

CHAPTER II

LITERATURE REVIEW

In this chapter, theoretical framework about teachers' knowledge and knowledge of definition will be reviewed. Related literature regarding the concept of integers and making definitions will be reviewed as well.

2.1 FRAMEWORKS ABOUT TEACHERS' KNOWLEDGE

What teachers need to know was considered by many researchers and they have similar or extended ideas. Although there is no contradiction in whether teachers should learn mathematics conceptually, or teachers need to understand mathematics in order to teach it. However, it is not clear for many how much, or which content needs to be learned for effective learning of mathematics (Stacey, 2008). Researchers supported their claims with special components of mathematics learning and teachers' knowledge or they handle the components in frameworks (Shulman, 1986; Lappan & Even, 1989; Ball, 1990; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993; Ball, 2000; Chick, Pham, & Baker, & Cheng 2006; Escudero & Sanchez, 2007; Stacey, 2008; Ball et. al., 2008).

Frameworks on teachers' knowledge are critical to better understand how teachers need to be educated. One of the prominent concepts of teacher's knowledge was brought up by Shulman (1986). Besides Shulman, researchers came up with ideas about what teachers need to know. In this part, studies which categorize teachers' knowledge, including Shulman's framework, will be reviewed comparatively.

Shulman (1986) raised the questions of what teachers know, how they construct new knowledge on to the old one, and what the sources of teachers' knowledge are. In response to them, he suggested three domains of content knowledge that teachers need to know: subject matter, pedagogical content knowledge (PCK), and curricular knowledge.

Subject matter knowledge refers to "the amount and organization of knowledge per se in the mind of the teacher" (p.9). According to Shulman (1986), teachers must be competent in subjects in a domain with accepted rules, or truths, and be able to explain why the subjects are worth knowing and how they relate with other subjects in the domain, or out of the domain considering the theories underlying the subjects and how they reflect on practice. Thus, "teachers need not only understand *that* something is so; the teacher must further understand *why* it is so" (p.9). According to Leikin & Zaskis (2010, p.454):

> *Subject matter knowledge is associated with definitions and includes knowledge of the structure of mathematics, the place and the role of axioms, definitions and theorems within this structure, understanding of mathematical concepts (including personal definitions and concept images consistent with formal concept definitions) and understanding the meaning of defining and proving. Subject-matter knowledge also includes such meta-mathematical elements as understanding of what a definition is, how it is different from an axiom or a theorem, and what are its properties and logical structure.*

For instance, teachers need to know the definition of integers to understand mathematical concepts such as rational numbers, reel numbers, or to understand some theorems related with division of integers (Euclid’s Algorithm) or prime numbers (Herstein, 1996). For example, integers are crucial for the division theorem, which requires two properties: $0\leq r < a$ and $b=aq+r$. In this theorem r should be bigger than zero, and the theorem is not valid if a equals to zero or negative integers (Humphreys & Prest, 2004). According to Leikin and Zaskis (2010), teachers need to have content knowledge of integers to interpret problematic mathematical definitions of students and then to clear their misunderstandings. Interpreting definitions which are not actually formal definitions, but created ones gives teachers the opportunity to overcome students’ difficulties and gives students the opportunity to develop their understanding of definitions. The different ways give students the opportunity to recognize the hierarchical structure in the classification. Defining in different ways gives teachers the opportunity to perceive why some mathematical concepts are defined in a certain way and some are based on arbitrary rules or explanations (Even, 1990).

In addition to subject matter knowledge, teachers must be competent in content knowledge for teaching, which requires useful ways of representing those ideas, and powerful analogies, illustrations, useful examples to be comprehended by students (Shulman, 1986). Teachers need knowledge of students' preconceptions, or misconceptions or strategies to overcome difficulties of students to organize students' understandings (Shulman, 1986). Understanding what makes learning of a subject easy or difficult in the teaching process is easier if teachers have *pedagogical content knowledge (PCK)*. "Teachers' pedagogical content knowledge determines their ability to match the teaching of definitions and defining with a particular classroom, and to attend to students' ability levels, affective needs and motivation" (Leikin & Zaskis, 2010, p.454). For example, teachers need to adjust the definitions of integers to their students' possible definitions of integers for more effective teaching. In other words, teachers need to be familiar with or anticipate their students' problems in defining integers in order to implement effective teaching practices.

According to Shulman (1986), the last domain for teacher knowledge is *curricular knowledge (CK)*. This knowledge requires teachers to know "the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contra indications for the use of particular curriculum, or program materials in particular circumstances" (Shulman, 1986, p.10). In other words, which materials to use, what alternative ways to design instruction such as additional readings, computer based programs, or visual materials are curricular knowledge for a teacher. For example, "they need to have knowledge of the learning sequences in which various definitions of mathematical concepts are used, and the connections between various curricular topics in which different definitions of mathematical concepts may appear" (Leikin & Zaskis, 2010, p. 454). Specifically, when the mathematical concept is integers, teachers are expected to be aware of the location of the definition of integers in the elementary school curriculum, whether and how it is related to other mathematical concepts in the curriculum.

From Shulman’s (1986) perspective, it is seen that there are three main components for knowledge of teachers: subject matter knowledge, pedagogical content knowledge, and curricular knowledge. According to Ball (1990), "the goal of mathematics teaching is for students to develop mathematical understanding." (p.457). To develop mathematical understanding of students teachers need to have two critical knowledge components: "substantive knowledge of mathematics" and "knowledge about mathematics" (p.458).

Substantive knowledge of mathematics is about knowing particular concepts and procedures about a mathematical content. This knowledge has three dimensions which characterize what teachers need. One dimension is knowledge about concepts, facts, or procedures to be applied in mathematics content. In this type of knowledge, teachers are expected to apply rules without making mistakes. For example, calculation of multiplication of fractions, area of a rectangle, or graphing of a function should be correctly applied by teachers. Another dimension is that teachers need to understand "underlying principles and meanings of them" (p.458). That is, teachers should know why certain principles or rules are so (Shulman, 1986). For example, what division means, or how the result of $\frac{1}{2}:\frac{3}{4}$ is interpreted need to be comprehended by teachers. The last dimension is that "teachers must appreciate and understand the connections among mathematical ideas" (p.458). "How fractions are related to division, how place value figures in multiplication computation, and the connections and distinctions among measurements of distance, area, and volume (p.458)" are examples for this dimension.

The other component for what teachers need to know is *knowledge in mathematics* (p.458). Knowledge in mathematics requires "knowing about nature of mathematical knowledge, and of mathematics as a field" (p.458). In other words, teachers should have ideas about what doing mathematics means, when an answer of a mathematics question can be valid, what the distinction is between logical and unreasonable way to follow in problems, and what having a background of mathematics involves.

Similar to Ball’s (1990) detailed description of subject matter knowledge of Shulman (1986), Chick et. al. (2006) represented a detailed framework, multi faces, and specified elements for PCK. The categorization in Table 2.1 is quite detailed in giving ideas about which points are needed to be considered for teachers' PCK. Chick et. al. (2006) reported that the framework was organized based on particular aspects of PCK, which were shown in the literature (Shulman, 1986; Ball, 2000; Ma, 1999) and the framework provides details about teachers' PCK.

Table 2.1 shows that PCK was examined in terms of three main components: "clearly PCK", "content knowledge in a pedagogical context", and "pedagogical knowledge in a content context" which is shown below (p. 299):

PCK Category	Evident when the teacher
Clearly PCK	
Teaching Strategies	Discusses or uses strategies or approaches for teaching a mathematical concept
Student thinking	Discusses or addresses student ways of thinking about a concept or typical levels of understanding
Student thinking -misconceptions	Discusses or addresses student misconceptions about a concept
Explanations	Explains a topic, concept or procedure
Cognitive demands of Task	Identifies aspects of the task that affect its complexity
Appropriate and Detailed Representations of Concepts	Describes or demonstrates ways to model or illustrate a concept (can include materials or diagrams)
Knowledge of Resources	Discusses/uses resources available to support teaching
Curriculum Knowledge	Discusses how topics fit into the curriculum
Purpose of Content Knowledge	Discusses reasons for content being included in the curriculum or how it might be used
Content knowledge in a Pedagogical Context	
Profound understanding of fundamental Mathematics	Exhibits deep and through conceptual understanding of identified aspects of mathematics
Deconstructing content to key components	Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept
Mathematical structure and connections	Makes connections between concepts and topics, including interdependence of concepts
Procedural Knowledge	Displays skills for solving mathematical problems (conceptual understanding need not be evident)
Methods of Solution	Demonstrates a method for solving a math problem
Pedagogical Knowledge in a Content Context	
Goals for Learning	Describes a goal for student' learning (may or may not be related to specific mathematics content)
Getting and Maintaining Student Focus	Discusses strategies for engaging students
Classroom Techniques	Discusses generic classroom practices

Table 2.1: Framework for analyzing pedagogical content knowledge

(Chick, et. al., 2006 p. 299)

Clearly PCK is where pedagogical content knowledge of teachers is revealed in classroom environments. According to this PCK category, teachers need to specifically pay attention to using strategies in teaching, how students tend to think about a concept, their misconceptions about the concept, methods used for explanations of a topic or content, tasks which make students learning easier, representations or resources which enable students' conceptual understanding of a topic, topics, place of the topics, and their sequence in the curriculum. Another element is *content knowledge in a pedagogical context,* which refers to "the ability to deconstruct knowledge to its key components, awareness of mathematical structure and connections, and PUFM" (Profound Understanding for Mathematics) (p.298). In this category, teachers need to pay attention to make connections between concepts, to discuss mathematical components which are critical for a concept, and to guide students in utilizing methods to solve problems. The last element is *pedagogical knowledge in a content context* which "covers situations where teaching knowledge is applied to particular content area, and includes knowledge of strategies for getting and maintaining student focus and knowledge of classroom techniques" (p.298). It requires teachers to support specific achievements in students' learning and to help students focus on the lessons. It also requires that teachers use classroom techniques which are appropriate for specific teaching contents.

The framework seems to have detailed descriptions for PCK. There are subcomponents for each category. Compared with Shulman and Ball (1990), the framework seems to be more prominent in terms of analyzing PCK.

Another categorization of content knowledge for teachers was provided by Ball et. al. (2008). Different from her previous study, Ball (1990) and her colleagues categorized Shulman's (1986) both subject matter and pedagogical content knowledge dimensions into subcategories (Ball et. al., 2008). According to Ball et. al. (2008), the categorization of content knowledge comprises "common content knowledge", "horizon content knowledge", and "specialized content knowledge"; and pedagogical content knowledge, which was categorized as "knowledge of content and students", "knowledge of content and teaching", and "knowledge of content and curriculum" (p.403) can be seen in Figure 2.1:

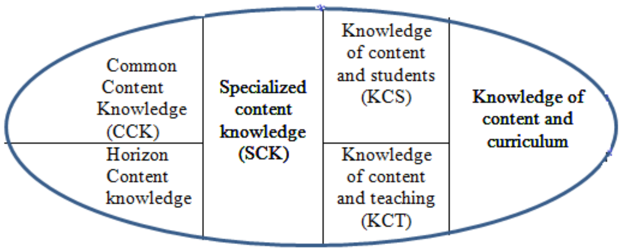


Figure 2.1: Domains of Mathematical Knowledge for Teaching
(Ball et. al., 2008, p. 403)

According to Ball et. al. (2008), teachers should have common content knowledge (CCK). She emphasized that *common* does not mean everyone has this knowledge. It means that this is knowledge of a kind used in a wide variety of settings, not specifically for teaching, but for the ability to answer mathematical questions that do not require specialized knowledge. For example, teachers, or anybody who knows mathematics generally know which numbers are integers. However, teachers also need to know specialized content knowledge (SCK) which is not known by everybody who knows some mathematics (Ball, et. al., 2008). For example, teachers need to know why integers are necessary and how the numbers are historically developed.

In addition to these, the last category within subject matter is *horizon content knowledge*, which requires an "awareness of how mathematics topics are related over the span of mathematics included in the curriculum" (p.403). For example, 9^{th} grade mathematics teacher may need to know 11^{th} grade mathematics content to see the relationship of content in different grade levels, and to make decisions about how to teach effectively the content providing sufficient and uncomplicated knowledge for students.

As teachers think about how students learn more effectively, it may be considered that they need to think deeply on student learning during teaching. Thus, teachers need to pay careful attention to students' mistakes, misunderstandings or misconceptions. They need to design their lessons considering the reflections of situations (Shulman, 1986). Thus, an interaction between students and teachers are inevitable during teaching. The interaction requires that teachers need *knowledge of content and students (KCS).* KCS is "knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing." (Ball et. al., 2008, p. 401). For example, teachers need to know the concept of division of fractions with elementary students' possible misconceptions about the concept.

According to Ball et. al. (2008), teachers need to know how their students act on content. This action includes close interaction between teachers and their students' misconceptions, questions, common errors, or any behavior related with the content. For example, teachers should know that their students have a common confusion of whether zero is an integer or not. However, although teachers know students' probable actions on content, it may be inefficient to implement solutions if the teachers do not know how to do it. To do this, teachers need to know the design of an instruction and present the solutions in the instruction. They also need to know how to represent a subject efficiently, know which examples can be suitable for a content to be used for teaching students effectively, or which connections should be covered to help the teacher think deeply in a content etc. Briefly, they should have *knowledge of content and teaching (KCT).* "KCT combines knowing about teaching and knowing about mathematics" (Ball et. al., 2008, p.401). Accordingly, teachers need to be capable of integrating the effective strategy to overcome the misconception that zero is not an integer.

Lastly, teachers need to know curriculum knowledge (Ball et. al., 2008). It means teachers need knowledge of content and curriculum. For example, teachers have curriculum knowledge when they are being aware of the location of the fraction concept in the curriculum.

In conclusion, Ball et. al. (2008) divided Shulman's subject matter and pedagogical content knowledge conceptions in categories. These categories help to consider relationships between teachers' content knowledge and their students' achievements, to study different approaches to teacher development with particular aspects of teachers' pedagogical content knowledge, and to consider content knowledge in teaching how to design a lesson with supplementary curriculum materials.

It is considered that the approaches or theories mentioned above about teachers' knowledge may provide a new perspective for what teachers are expected to know. What's more, they provide deeper understanding of teachers' knowledge.

In this study, subject matter knowledge refers to "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p.9). For this study, I referred specifically to pre-service teachers' knowledge regarding definition of integers as their subject matter knowledge. In this study, key elements related to pedagogical content knowledge were also considered. For this study, pedagogical content knowledge is related to knowledge of pre-service teachers about elementary students' misconceptions and errors related to definition of integers. In the next section, studies related to teachers' knowledge regarding integers will be reviewed.

2.2 STUDIES ON INTEGER CONCEPTS

The purpose of the study is to examine pre-service teachers' subject matter knowledge and pedagogical content knowledge regarding definitions of integers. To better understand the concept of integers and related frameworks for the study, related literature was examined based on elementary school students' and teachers' perspectives.

2.2.1 Studies on elementary school students

In this part of the related literature, studies on integers conducted on elementary school students are mentioned.

Köroğlu and Yeşildere (2004) studied how the unit of integers was handled with 7th grade elementary students. Research results show that elementary students' difficulties can be divided into four: difficulties in expressing set of integers, misconceptions about signs, difficulties in operations with two integers, and division with zero.

To explain the results briefly, elementary students were not able to determine the elements of integers, and tended to define zero as a non-integer, or included only negative numbers in the integers set. In addition, they have difficulty in deciding on the signs when making addition. Students cannot decide clearly whether the signs of integers (+, or -) indicate the operation of integers or the signs of the integers. Students are confused about how to make calculations/operations with integers with different signs. Also, students have problems taking the exponential of integers, and finding the result of the division of an integer by zero.

Another researcher found similar difficulties with the previously mentioned study. Melezoğlu (2005) found that elementary students' difficulties may be presented under three titles: (1) difficulty of making operations with negative integers, (2) ordering integers and (3) word problems.

The previous two studies have similar findings in the aspects of making operations with integers having different signs, and taking the exponential of numbers. The similarity between the studies describe that elementary students have problems uncovering them.

In addition to the problems, meanings of signs seem to make elementary students confused. Spang's (2009) study shows that elementary students were not able to distinguish between the subtraction sign and the sign of a negative number nor could they distiguish between the plus sign and the sign of a positive number. The study suggested using the "pebbles in a bag" active to enable students to understand the meaning of integers and the meaning of the signs of operations. By means of the "pebbles in a bag" activity, students gained deeper understanding of integers and their symbols.

Körükçü (2008) suggested that teaching with concrete models help elementary students comprehend meaning of integers. Prepared activities using visual manipulatives (counter, number line, and thermometer) helped 6^{th}-grade students in doing addition or subtraction of integers, and to compare integers with each other. Research results show that the "zero principle" also helped them to learn making operations with plus and minus signs. Results report that there was a positive effect of teaching integers with visual materials on elementary students' mathematics achievement in integers, together with operations of integers, and their level of recalling them.

In addition to these studies in which activities were conducted, technology supported mathematics lessons to teach integers were also looked into. Thompson and Dreyfus (1988) conducted a study with two elementary students of average competence in making operations to see the nature of their conceptions of integers with microworld program. Research results show that the microworld program gives students the opportunity to make operations with integers when both of the integers are positive or negative. However, while students used microworld program which adjusted operations of integers with different signs, some students were confused in comprehending the results of the operations.

According to Akyüz et. al. (2012), teachers' instructional activities have an efficient role supporting elementary students' understanding of integers. Their research result shows that students have three main images in which students argue for ideas about integers supported by an experienced mathematics teacher. The images are (1) 'situation specific imagery: assets, debts and north worth' which are the informal knowledge of students that need meanings related to integers to be constructed by teachers, (2) 'notational imagery: flexibly structuring space vertically' which supports students' notation imagery using a vertical number line on which assets, debts and north worth are shown; and (3) 'situation-specific to notational imagery: good and bad decisions' which require interpreting the notations of +(+), –(-), +(-), –(+) as good or bad situations. These images are revealed with the teacher's help. Teacher's role is so crucial that s/he designs learning trajectory, which is created by supportive contexts.

İşgüden (2008) studies the difficulties 7^{th} and 8^{th} grade students' experience in integers regarding writing a set of integers, placing positive and negative integers on a number line, absolute value of positive and negative integers, taking exponential of the integers, and addition and subtraction of integers. The study revealed that students experienced the following difficulties: whether zero is an integer or not, where the negative integers are on the number line, how negative integers are ordered, and what the meaning of absolute value is.

Ercan (2010) conducted a study with 628 randomly selected students of grade seven. Students were administered a test including numbers which are examples and non-examples of integers. Research results show that the sign of a number was an important factor in deciding whether a number was an integer or not and students were not sure when integers were written in a decimal or a rational number format. This study also required students to give reasons about their decisions of why some items in the test are the examples of integers and some are not. The research study revealed that students were undecided about the written form of integers through different number sets (e.g. 3.0 is an integer and a decimal number) and they articulated many different reasons to justify why some items are integers and the others are not. The reasons pointed out by these elementary students indicate that the concept of integer is perceived differently and thus they have different integer descriptions.

Research studies show that elementary students' conceptions about integers are varied, and classroom activities supported with models, manipulatives, or technology help elementary students learn integers. However, they may have difficulty in the integer set, placing integers on a number line, ordering integers especially ordering two negative integers, or relating integers with other number sets such as whole numbers, counting numbers, or decimal numbers. Meaning of integers, operations of them, especially the subtraction operation between two negative integers, are also difficult for elementary students.

Students' conceptions with integers may give an idea of teachers about which points teachers should be careful about when teaching the integers concept. For this reason, it might be important to determine how the same concept of integers is covered by in-service and pre-service teachers. In the next part, studies on pre-service teachers and in-service teachers will be mentioned.

2.2.2 Studies on pre-service teachers and in-service teachers

In this part, studies of integers conducted with pre-service and in-service teachers will be mentioned.

One of the ways for teaching integers to elementary school students might be using manipulatives or models. Smith (2002) prepared a guide for pre-service teachers to teach integers using manipulatives. He points out that counters can be used in integers operations to operate them, which enables students to learn the concept of integers effectively. The researcher emphasized that students need to understand "zero principle" before operating integers. Most elementary students just memorize rules about change in signs in operation of integers (Steiner, 2009). To make integers more comprehensible for elementary students, the "novel model" was created by the researcher and it was used with pre-service teachers. The model was used to construct meaning of integers and signs of integers in pre-service teachers' minds, and to clear confusions in their minds. To mention briefly, the novel model requires to design appropriate examples with integers operations using 'money own', and 'money debt' concepts instead of using positive and negative integers, and to show the examples on a number line model.

Steiner's (2009) research results show that the model revealed that pre-service teachers have problems with set of integers. They also have some misconceptions such as "negative integers have the same meaning with zero, there is nothing", and problems in deciding "whether decimals are integers or not", "which sets are included by integers", "whether a zero is positive or negative", "how to model addition of integers with different signs", or "how to model subtraction with two negative numbers". Although there are such problems mentioned, the novel model helped pre-service teachers to overcome them.

Research studies on pre-service or in-service teachers show that they need knowledge or practice in using manipulatives to teach integers, modeling integers, meaning of signs of integers, and what zero means.

As mentioned above, several studies have produced estimates of difficulties regarding concept of integers. Studies on literature focus on what elementary students or teachers' difficulties or confusions about making computations of integers, modeling integers, meanings of signs of integers so on. However, no research has been found that surveyed the definition of integers which helps to reveal elementary students or pre-service and in-service teachers' images of the concept of integers. There seems to be a gap in the concept of integers related available literature on studies regarding definition of integers. In other words, there have been no studies focusing on the meaning of an integer to reveal pre-service teachers' perceptions regarding the definition of integers. To understand the teachers' conceptions of definition of integers better, what features are looked for are required. In the following section, frameworks about definition and definition of integers in different resources will be reviewed.

2.3 FRAMEWORKS ABOUT DEFINITIONS

"Definition is an important language form in the register of mathematics" (Shield, 2004, p.28). Defining takes important part in teachers' subject matter knowledge regarding definitions of mathematical concepts, structure of the definitions, and process of defining (Zaskis & Leikin, 2008). Definitions of a mathematical concept is essential because it affects teachers' way of teaching, order of topic to be learned, and what set of theorems and proofs are to be covered. Definitions of mathematical concepts are also essential in preventing students from misunderstandings (Levenson, 2012) and to enable students to interpret them and build an understanding (Shield, 2004). It would be more helpful for students to build their understanding on explorations of a concept (Shield, 2004). It might be thought that students are allowed to explore the concept to be defined with teachers' help (Shield, 2004).

According to Shield (2004), there are no unique definitions for mathematical concepts; rather, several definitions may be used to define them. Using different ways to make definitions allows students to develop their awareness of relationships between ideas. What's more, the different ways give students the opportunity to recognize the hierarchical structure in the classification.

As students encounter many definitions of mathematical concepts when they are learning, they experience difficulty in developing their understanding about a definition when they use a single formal definition. Therefore, they need to understand the structure of a definition to construct their own definitions and to make sense of the definitions. With understanding the structure of a definition, "students can construct their own definitions as part of organizing their thoughts about the concepts they have explored" (Shield 2004, p.28). To know the structure of a definition enables students to understand further definitions and create their own definitions forming all their knowledge about a concept (Shield, 2004). According to Shield (2004), students' understanding of mathematical concepts is developed by two important aspects which enable students to construct their definition.

One of them is that, 'a definition is based on the least number of features requires to establish the item uniquely, that is, the necessary and sufficient conditions.' (p. 26). For example, when defining a triangle, it is necessary to say that any closed plane shape with three straight sides is sufficient to call it a triangle, without the necessity to add that there are three angles totaling 180 degrees. When students define a concept, they have difficulty in confirming to necessary and sufficient conditions. However, definitions of students tend to have more features than required (Shield, 2004). The other aspect is that students realize that a concept to be defined is a member of a class. The class helps to define the concept. For example, a triangle is a member of class of polygons. Therefore, a triangle can be regarded as a polygon: 'a triangle is a polygon with three sides' (p.26).

To sum up, according to the characteristics mentioned by Shield (2004), formal definitions have three parts: item, class and features. For example, the meaning of these parts can be seen in the following example: "A triangle is a closed plane shape with three straight sides". In this definition, triangle is the item which belongs to the class of closed plane shapes with features of three straight sides.

Vinner (1991) claims that there are five assumptions upon which definitions in many textbooks and classrooms are partly based:

1. Concepts are mainly acquired by means of their definitions.

2. Students will use definitions to solve problems and prove theorems when necessary from a mathematical point of view.

3. Definitions should be minimal. (By this we mean that definitions should not contain parts which can be mathematically inferred from other parts of the definitions. For instance, if one decides to define a rectangle in Euclidean geometry by means of its angles it is preferable to define it as a quadrilateral with 3 right angles and not as a quadrilateral having 4 right angles. This is because in Euclidean geometry, if a quadrilateral has 3 right angles one can prove that its fourth angle is also a right angle.)

4. It is desirable that definitions will be elegant. For instance, some mathematicians think that the definition of the absolute value as $|x| = \sqrt{x^2}$ *is more elegant than its definition as:*

$$|x| = \begin{cases} x; \text{ if } x \geq 0 \\ x; \text{ if } x \leq 0 \end{cases}$$

Also, some mathematicians believe that the definition of a prime number (in the domain of whole numbers) as a number having exactly two different divisors is more elegant than its definition as a number greater than 1 divisible only by 1 and itself.

5. Definitions are arbitrary. Definitions are "man-made". Defining in mathematics is giving a name. (For instance, when defining a trapezoid, one can define it as a quadrilateral having at least one pair of opposite sides which are parallel. On the other hand, he or she can define it, if they wish, as a quadrilateral having exactly one pair of opposite sides which are parallel. If you choose the first definition, a parallelogram is also a trapezoid. If you choose the second one, it is not. Now, if the idea that definitions are arbitrary is well understood the above fact will not cause a confusion, otherwise it might cause a great deal.) (p. 66).

Although Vinner (1991) put forward the five assumptions upon which a definition is based, he also emphasized that definitions "do not necessarily reflect all the aspects of definitions in higher mathematics" (p.66).

According to Van Dormolen and Zaslavsky (2003), there are seven criteria which are seen as fundamental components in a deductive system and which are logical necessities for a definition: (1) criterion of hierarchy, (2) criterion of existence, (3) criterion of equivalence, (4) criterion of axiomatization, (5) criterion of minimality, (6) criterion of elegance, and (7) criterion of degenerations.

Criterion of hierarchy

This criterion expresses that "any new concept must be described as a special case of a more general concept. One or more properties must be used to describe this special case." (p.94). For example, when the following definition is considered “A right angle is an angle of which the legs are perpendicular to each other” angle is the general concept and the general concept was supported with the property of “the legs are perpendicular to each other.” (p. 94).

Criterion of existence

According to Aristotle, whether there exists an instance of a such a newly defined concept within the current system must be proven that it really exists in the current system. For example, when a circlesquare is attempted to be defined within the Euclidean geometry context as "a circlesquare as a square for which all points have the same distance to a certain point", however, there is no such thing in Euclidean geometry" (p.94).

Criterion of equivalence

Another criterion is that definitions should be equivalent to each other. That is, when more than one definition is given for a concept, equivalency of the definitions must be proven. In this manner, one of the definitions is chosen as a definition of a concept, the other then becomes a theorem to be proved. For example, if the first definition of parallelogram given below am is chosen as a definition, the others should be proved that they are parallelogram as well.

"A parallelogram is a quadrilateral in which the opposite sides are parallel."
"A parallelogram is a quadrilateral in which the opposite sides are equal."
"A parallelogram is a quadrilateral in which the sides of one pair of opposite sides are both equal and parallel."
"A parallelogram is a quadrilateral that is symmetrical with respect to a point."

Criterion of axiomatization

The criterion is closely related criterion to criterion of hierarcy. Although mathematical concepts are defined with the help of general concepts, it is not possible to define some concepts according to the Aristotelian criterion of hierarchy. At that point, "Aristotle wrote that axioms or postulates implicitly define such concepts." (p.96). For example, when natural numbers are defined, there are several ways to do it. Peano axioms may be used or cardinality of finite sets helps us to define natural numbers. When Peano axioms are used, one of the axioms states that there is a first element which is called ‘one’. Therefore, the existence of the axiom which points out ‘one’ helps us to define natural numbers. When cardinality of finite sets is used, axioms of sets indicate that there exists an empty set.

Criterion of minimality

This criterion requires that "no more properties of the concept be mentioned than is required for its existence" (p.95). The following description would not be a good definition within Euclidean geometry: A rectangle is a quadrilateral with four right angles. However, it is proven that the sum of the four angles of a quadrilateral equals to 360 degrees. It is enough to define the rectangle concept as following: "A rectangle is a quadrilateral with three right angles." (p.95).

This criterion can be examined from two aspects in terms of acceptability. One side asserts that describing a concept with non-minimal concepts allows students to develop certain concepts or theories.

According to Van Dormolen and Zaslavksy (2003), describing a concept with not more properties than required allows students to develop certain concepts or theories. What's more, every definition may not give the opportunity to determine whether it is minimal or not. However, one has to investigate the definition with minimal descriptions so that s/he investigates the new definition.

Criterion of elegance

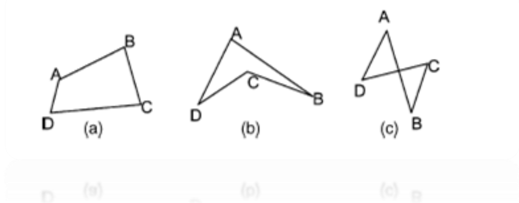
"Sometimes a textbook author, for example, has to choose between two definitions that are equivalent, but one looks nicer, needs fewer words or less symbols, or uses more general basic concepts from which the newly defined concept is derived." (p. 97).

In the following examples, according to Van Dormolen and Zaslavsky (2003), the first definition might be chosen by the author as it is more general and shorter.

> ***Definition I***. *The distance between two objects is the minimum length of a segment that has one endpoint on one object and the other endpoint on the other object.*
>
> ***Definition II.*** *Let two objects be given in a Cartesian coordinate system by the equations F(x, y, z) = 0 and G(x, y, z) = 0.*
>
> *Then the distance between the two objects is the minimum of F(x, y, z)= 0 and G(x, y, z)= 0* $\sqrt{(x_F - x_G)^2 + (y_F - y_G)^2 + (z_F - z_G)^2}$ *where* $F(x_{GF}, y_F, z_F) = 0$ *and* $G(x_G, y_G, z_G) = 0$

Criterion of degenerations

The criterion tells that "the consequence of a definition is sometimes that it allows instances that do not conform to our intuitive idea of the concept". The definition of the quadrilateral in the following is degenerated since the definition allows three kinds of quadrangles in which on one of them does not belong to quadrilateral (figure in c)even though it entails the features in the definition: "A quadrilateral is a set of four points A, B, C, D of which no three are collinear and four segments AB, BC, CD and DA." (p.99).



(Van Dormolen & Zaslavsky, 2003, p.99)

According to Van Dormolen and Zaslavsky (2003), there are the seven criteria to be verified for being a mathematical definition.

Similar to Shield’s (2004) characteristics, and Van Dormolen and Zaslavsky’s (2003) criteria, Zaskis and Leikin (2008) mentioned that well-known mathematicians consider that a definition should have the following features: a) the concept to be defined should be mentioned in the definition once, b) the definition should have necessary and sufficient conditions, c) the conditions should be minimal, d) only previously learned concepts can be used, and e) a definition is arbitrary.

A Framework for Analyzing Teacher-Generated Definitions

Leikin and Zaskis (2010) suggested criteria for the analysis of teacher-generated examples of definitions as shown in Table 2.2 below. According to the framework, definitions can be evaluated in terms of the following criteria: accessibility, correctness (appropriateness), richness, generality/concreteness.

Table 2.2

Criteria of a framework for the analysis of teacher-generated examples of definitions (Leikin & Zaskis, 2010, p.457)

Criteria	Focus of analysis	Inference of analysis	Setting
Accessibility	Ability to generate examples with and without prompts	Mental fluency associated with definitions	Oral
Correctness (appropriateness)	Necessary and sufficient conditions Minimality	Understanding of the notion of definition Understanding of a Mathematical concept	Oral and written
Richness	The number of different appropriate examples of definitions	Mental flexibility Understanding of equivalency of definitions	Oral and written
Generality/ concreteness	Ability to provide specific and precise examples of definitions	Understanding of a mathematical concept Understanding of the notion of definition	Oral

Accessibility: "Accessibility refers to the ability to generate examples with and without prompts" about a concept (p.456). This criterion can be determined through oral settings such as an interview. For example, when a student suggests definitions for rational numbers, whether s/he continues giving examples by his/her attempts or interviewer's attempts may be an action for the criterion.

Correctness: "Correctness refers to the properties of the examples generated" (p.456). It was examined as appropriate and inappropriate example statements.

Inappropriate statements: "Inappropriate examples of definitions of mathematical concepts are examples which are lacked either necessary or sufficient conditions, so that they represented mostly specific instances of the concepts" (p.459).

Appropriate statements: Appropriate examples of definitions of mathematical concepts are identified in two: "(1) appropriate rigorous examples of definitions are examples which include necessary and sufficient conditions of the defined concept as well as accurate mathematical terminology and symbols, and are usually minimal and (2) appropriate but not rigorous examples of definitions are examples which usually omit some constraint or use imprecise terminology because of a lack of attentiveness on the part of the PMT or a lack of rigor in the mathematical language in the usual mathematics classroom." (p.457)

Richness: Richness of examples is that examples of concepts vary in type and structure, and whether they are situated in a particular context or drawn from a variety of contexts are examined. For example, when examples of irrational numbers are examined, the number of different appropriate examples of definitions for rational numbers suggested by students may enable them to see richness. This category can be determined through oral and written settings.

Generality/concreteness: "Generality/concreteness refers to the ability to provide examples that are specific and precise rather than general descriptions" (p.457). "Were the examples specific or general?" (Zaskis & Leikin, 2008, p.20).

According to the framework of Leikin and Zaskis (2010), for the analysis of teacher-generated examples of definitions, there are four criteria (accessibility, correctness, richness, and generality/concreteness) to examine definitions of teachers.

From the features of and frameworks of definitions in this part, mathematical definitions need to have the following characteristics or criteria to be a mathematical definition: definitions are arbitrary, they are preferred to be minimal, they are preferred to be not degenerated, they should be based on previously learned concepts, and should have necessary and sufficient conditions about the concept to be defined. According to Van Dormolen and Zaslavsky (2003), minimality may not exist in mathematical definitions as a necessary condition. Moreover, criterion of elegance and criterion of degenerations are also subjective criteria to be discussed.

In this study, the stated features will partly guide the analysis and discussion of the responses of pre-service teachers.

2.4 STUDIES ON DEFINITION

There are some research studies on definitions of mathematical concepts. The studies will be reviewed in this section.

Leikin and Zaskis (2010) conducted a study on prospective teachers' definitions in different mathematical areas such as geometry, algebra, and calculus. Their knowledge in definitions of mathematical concepts was examined based on the accuracy and richness of the examples of their definitions. Results revealed that accuracy of the prospective teachers' definitions of the mathematical concepts show that there are appropriate and inappropriate examples of definitions suggested for mathematical concepts. Although prospective teachers suggested appropriate examples, some of their examples did not include constraints and have imprecise terminology. For example, when a second degree equation is defined as $ax^2+bx+c=0$, it is essential to say that *a* does not equal to *zero*. Prospective teachers also suggested inappropriate examples which did not have necessary and sufficient information for defining the concept. The examples showed that some of the mathematical concepts were misunderstood. Results of the study also reported that defining is a challenging task for prospective teachers. Richness of the participants' responses showed that they used similar numbers of topics for appropriate examples in algebra and calculus, and more number of topics in geometry. The study also revealed that prospective teachers have lack of knowledge about what a definition is. The similar result was revealed in the following study as well.

Levenson (2012) conducted a study on three junior high school teachers' knowledge about the nature of definition of integers through zero exponents. Results of the study reported that not all of the teachers were clear on which expressions were and were not definitions. For example, the teachers were not sure about whether a^n=a.a.a.a… n times is a theorem and a^0=1 is a definition. Results also revealed that the teachers have difficulty in explaining what a definition is in mathematics and deciding whether definitions or theorems are proven.

Martin, Oehrtman, Roh, Swinuard, and Hart-Weber (2011) conducted a research on two students who were taking a calculus course whose topics included sequences, series, and Taylor series. The authors had six teaching sessions with the students on reinvention of definition of series and pointwise convergence. Research results show that the students invented and unpacked the definitions of series and pointwise convergence in a short time. Although students had difficulty in this defining process, teachers' instructional design was helpful for them not only in defining the concepts, but also in using the definition in graph series. They also recognized similarities between definitions of series and pointwise convergence and interpreted components within the definitions.

Dede and Soybaş (2011) examined pre-service teachers' experiences about function and equation concepts. Research results related with definition of the concepts showed that most pre-service teachers defined equation concept accurately. However, they had difficulty in defining function as they perceived functions as a mechanism that makes transformations in algebraic or arithmetic operations.

The research studies report that defining mathematical concepts are not an easy task not only for students but also for teachers. They also report that not only pre-service teachers or students in universities but also high school teachers need to be capable of describing mathematical concepts. It is seen in the studies that what a definition is, differences between theorems, axioms and definitions need to be comprehended by the participants to understand the nature of mathematics (Vinner, 1998; Edwards & Ward, 2008; Mason, 2010; Levenson, 2012). These studies are also conducted with mathematical concepts in higher mathematics. However, as mentioned in the significant of the study and literature parts, there were few studies focusing on pre-service teachers' subject matter and pedagogical content knowledge regarding definition of mathematical concepts of elementary grades.

Research studies on definition of pre-service teachers are needed that "for a teacher, knowing a definition is not sufficient. If the teacher cannot explain why some concept or term is defined in a certain way and not in another, then students may get the feeling that mathematics is an arbitrary collection of rules and definitions" (Even, 1990, p.218).

In my review of literature, I could not reach any studies about pre-service teachers' definitions of integers. The concept of integers is a problematic content for elementary students and teachers. Definition of the concept of integers is an important mathematical concept for pre-service teachers. As the concept is related to many mathematical concepts of number sets, such as whole numbers, rational numbers, or decimal numbers, definition of the concept by pre-service teachers are needed to be capable of obtaining the related or unrelated ideas between them. Moreover, it is worthwhile for teacher educators to investigate in depth about definitions of integers of pre-service teachers to know how pre-service teachers perceive a mathematical concept.

In this research study, my aim is to investigate pre-service teachers' subject matter and pedagogical content knowledge in integers which is one of elementary topics in the curriculum.

In the following section, definitions of integers in different resources of which each of them offers a different perspective on the definition of integers will be mentioned.

Definition of Integers in Different Resources

"The definition of a concept, once determined in a curriculum, influences the approach to teaching mathematics, the learning sequence, the set of theorems and proofs" (Zazkis & Leikin, 2008, p.132). Concept of integers is seen in elementary, secondary, and university levels. Students encounter the concept of integers from elementary school to university in differentiated contexts. For example, in elementary school, integers are seen in measurement of temperature, credit and bills, sea level, or blood groups, while they are used to clear some theorems related with division of integers (Euclid's Algorithm) or prime numbers in university (Herstein, 1996).

Actually, students have conceptions of integers before their elementary education. They see integers in daily life before even going to school. Then students continue learning integers in elementary mathematics lessons and if selected, students use knowledge of integers to understand theorems and proofs in university level.

In the elementary mathematics curriculum, integers are explained as follows: "A set which comprises of positive integers, negative integers, and zero; and the set are shown with the notation of Z" (TTKB, 2005, p.132).

Integers are also seen in elementary or secondary school books. A similar definition with TTKB (2005) was made by O'Daffer, Charles, Cooney, Dossey, and Schielack (2008): "The set of integers, I, consists of the positive integers, the negative integers, and zero. I= {…,-4, -3, -2, -1, 0, 1, 2, 3, 4 …}" (p.249).

Integers are defined by Bennett and Nelson (2001) as "the whole numbers, 0, 1, 2, 3, 4… , together with the negatives of the whole numbers, - 1, -2, -3, -4,…. are called integers." (p.248). Musser, Burger, and Peterson (2003) give a more detailed account of positive and negative integers: "A set in which 1, 2, 3 … are called positive integers, and "the numbers -1, -2, -3… are called negative integers". In addition to these sets, zero does not have a sign and it is "neither a positive nor a negative integer" (p. 319). Hubbard and Robinson (1996) remind the necessity of negative numbers in daily life by exemplifying with temperature and bank account that is overdrawn. To do this, whole number set is expanded and this expanded set is called as set of integers: J= {…, -3, -2, -1, 0, 1, 2, 3 …}. Sonnabend (2004) similarly defined integers by elements of the set. Integers are "the union of the set of whole numbers and the set of negative integers, and denoted by I= {…-3, -2, -1, 0, 1, 2, 3 …}" (p.237).

The word 'integers' may be seen as "signed numbers". Dyke, Rogers, and Adams (2009) state that "positive numbers, zero and negative numbers are called signed numbers" (p.674). In this definition, signed numbers are not used as integers. In other words, all positive numbers, zero and negative numbers are not integers even though the definition says so. Therefore, teachers should explain integers and use integers rather than the signed numbers terminology. If not, students may have difficulty in the terminology.

These definitions in elementary or secondary school books mentioned above reported that integer is a set, and combination of elements in the set. Furthermore, the books also reported that integer is the expanded set of whole numbers. In addition to the elementary or secondary school books, definitions of integers are defined and described in mathematics dictionaries with little differences. There are different definitions of integers in mathematics dictionaries. These definitions differ from each other in terms of utilizing basic number set concepts. To understand integers better, these differences were reviewed.

The basic number set concepts are counting numbers, whole numbers, and rational numbers. Integer is "any positive or negative counting number or zero" (Parker, 1997, p.123). In this definition, counting number is the basic number set concept. In the following definitions, whole numbers is the basic number set concept: "Integer is one of the "whole" numbers… -3, -2, -1, 0, 1, 2, 3 … the set of all integers is often denoted by Z" (Clapham & Nicholson, 2005, p.231).

"Integer is a number that may be expressed as the sum or difference of two natural numbers; a member of the set {…,-3, -2, -1, 0, 1, 2, 3 …} usually denoted Z" (Borowski & Borwein, 1989, p.298). In this part of the definition, whole numbers is the basic concept of the definition similar to the previous one. However, the definition is different even if the basic concept is whole numbers. The difference is that the definition emphasizes addition property of set of integers. Specifically, two sub properties need to be known: (1) the set of integers is closed under addition, for any a; b ϵ Z, a + b ϵ Z. (2) For any a ϵ Z, there exists an additive inverse a ϵ Z satisfying a + (a) = 0 = (a) + a. Therefore, if someone uses the definition, it is necessary to explain the property as it is not acceptable for previously learned concepts, which are counting numbers and whole numbers, or to suggest the definition to who knows the property.

In the following definition, integers were defined based on the concept of rational numbers: "the integers are the closure of the natural numbers under subtraction, and are identified with the rational numbers with denominator 1" (Borowski & Borwein, 1989, p.298). In this definition, natural number was used as a previously learned concept; however, rational number is learned after the concept of integers. As regards Zaskis and Leikin (2008), the definition does not fit into previously learned concepts. However, as Vinner (1991) claims, definitions may not necessarily fit into all the assumptions in higher mathematics.

In the following definition, integers were defined based on elements of the elements of integers: Integer is "any of the numbers 1, 2, 3, etc. These are often spoken of as positive integers in contradistinction to the negative integers, -1, -2, -3, etc. The entire class of integers consists of 0, ±1, ±2 …" (James & James, 1959, p.298).

The literature of definitions of integers shows that definitions are formed based on number sets and elements of the set of integers. It might be better to evaluate the definitions based on elements of the set of integers as explanations of concept of integers as they do not fit into assumptions of being a definition (Zaskis & Leikin, 2008; Shield, 2004; Vinner, 1991).

Similar to elementary or secondary school books, dictionaries of mathematics defined integers related to whole numbers and elements of the set of integers. Different from the elementary or secondary school books, dictionaries of mathematics showed that integers are defined based on counting numbers and rational numbers.

When defining a concept, it is important to base a definition on prior concepts (Zaskis & Leikin, 2008; Shield, 2004). Although integers are defined based on counting numbers, rational numbers, and whole numbers, elementary students may not be expected to define integers using rational numbers as they are assumed to have knowledge in numbers involving only whole numbers, and fractions before starting 6^{th} grade (MoNE, 2009). In other words, they may be expected to define integers based on whole numbers and fractions. It may be considered that elementary students may define integers using the fraction concept with denominator 1. However, the definition of integers would be missing if it were not negative. As the fraction concept is always positive, definition of integers with fraction concept would be missing.

There are seven objectives which are considered as directly related with integers (TTKB, 2005). They are shown in Table 2.3:

Table 2.3

Objectives related with integers

	Students should be able to;
6^{th} grade	Explain integers. Explain meaning of absolute value. To order and to compare integers each other.
7^{th} grade	Compute operations of addition and subtraction of integers. Compute operations of multiplication and division of integers. Solve and create problems related with integers.
8^{th} grade	To determine integers’ negative exponents, and to represent as rational numbers.

According to Table 2.3, it might be said that objectives aim that students should be able to define integers, their relationship with other number sets, and to compute operations with them. Related with the study, students are expected to explain integers as "a set which comprises of positive integers, negative integers, and zero; and the set are shown with the notation of Z" (TTKB, 2005, p.132).

In 6^{th} grade, students are taught where and why integers are used, the meaning of distance of integers to zero, and how to order integers on the number line. After basic concepts about integers are given in 6^{th} grade, in 7^{th} grade creating appropriate problems for the expressions, making computation of operations with integers, and the importance of signs of numbers are focused on. Finally, elementary students are expected to relate decimal numbers when writing integers with negative exponents.

2.5 COMMON MISCONCEPTIONS OR ERRORS OF STUDENTS

When students face knowledge that is new or unusual for them, they may not totally learn what is intended to be taught, or may offer unintended ways of solution. In other words, students sometimes fail to understand what they are taught. According to Newton (2000), failing to understand derives from several reasons: lack of prior knowledge to construct mental representation of information, constructing only partially correct mental representation to link with the others, or being unable to notice or construct necessity relationships between prior and new information.

Contrary to popular belief, which views that students have no idea of conceptions they have not learned in schools, students have explanations created in their minds for the conceptions, even if the explanations are not an accepted form in mathematics (Smith, et.al., 1993). It can be said that students have ideas before learning from their teachers because of the fact that students create the concepts themselves (Küçük & Demir, 2009). Considering the pre-conceptions in students' mind, what the role of teachers is in teaching concepts may be wondered.

Existing knowledge in learners' mind may not be changed easily. Students are convinced to change concepts in their minds if knowledge which will be learned 1) are understood easily, 2) make students believe the knowledge is true, and 3) make students think that new knowledge is useful for them (Hewson, 1992).

According to Küçük and Demir (2009), the roles of teachers in this process are to help students in imaging, correcting, and recognizing the relationship between concepts in mathematics taught in a designed appropriate classroom environment. In other words, teachers design their lessons giving students accurate and perfect essential knowledge not only on the qualities of a concept, but also on what the concept is not (Küçük & Demir, 2009). If students are unwilling to make connections between pre-concepts and new attained knowledge, they tend to do the same with their prior knowledge (Barke, Hazari, & Yibarek, 2009). How a conception that is interpreted and shaped in students' mind by teachers' help transforms into a misconception or error in students' minds may be a source of curiosity for teachers.

Misconception

Misconception is "a student conception that produces a systematic pattern of errors" (Smith, diSessa, & Roschelle, 1993, p.119). According to Yağbasan and Gülçiçek (2003), misconception is a deviation among internalized form of a concept and scientific meaning of a concept. These definitions focus on the fact that it has lack of quality to be a scientific definition. Even if misconceptions are different from widely accepted truth in science (Smith et. al., 1993), misconceptions seem to be ordinary explanations for an asked question, and it is difficult to separate misconceptions from other explanations about a topic (Yağbasan & Gülçiçek, 2003).

Misconceptions are derived from many sources. The sources include having lack of correct information, understanding the information in a wrong way, not being careful while listening to new information, having distracted attention, not being capable of watching the learning process or creating new personal information (Newton, 2000). In addition, Tekkaya, Çapa, and Yılmaz (2000) conclude that misconceptions result from the following: teachers' lack of knowledge in certain topics, inefficient prior knowledge and having incorrect prejudgment, teaching methods based on rote learning and teacher-centered models, topics in curriculum which are disconnected to each other and to real life, incorrect information in textbooks and lack of revision of the books. Moreover, misconceptions can be hidden in correct answers of careless teachers (Luneta & Makonye, 2010; Smith, et. al., 1993).

In addition to making errors in mathematics, students may have misconceptions. According to Drews (2011), misconceptions are "misapplication of a rule, understanding of a concept over its own meaning rather thinking alternative, or generalization of a concept in incorrect situations" (p.15).

Misconceptions are clue for students' content knowledge about a topic (Smith et. al., 1993). It is said that misconceptions might enable that students' gaps in subject matter knowledge come out to the surface, and misconceptions can bridge these gaps.

Error

"An error is a mistake, slip, blunder, or inaccuracy, and a deviation from accuracy" (Luneta & Makonye, 2010, p.36).

Students make errors deriving from different sources. According to Drews (2007), errors could be derived from being careless or having lack of awareness in what is done in answers, misinterpretation and lack of knowledge about what a text or symbol expresses. Students may forget the requirements to answer questions or they may really not know how to answer (Ryan & William, 2007; Drews, 2011). Errors of students could also be derived from misconceptions (Ryan & William, 2007; Drews, 2011).

Errors can be recognized in students’ exam papers, or their discourses used in mathematics lessons (Luneta & Makonye, 2010; Smith, et. al., 1993).

Drews (2011) lists the reasons for making errors. According to the researcher, an error may be derived from "lack of knowledge about topics, knowledge related with misconceptions, being careless or not being awareness in giving and checking answers, or misinterpretation of symbols or text". Furthermore, the errors might be derived from "teachers’ selection of inappropriate tasks, or regarding that students understood teachers told as teachers wish" (p.14). Although errors seem to be easy to make because of many reasons mentioned above, they might be corrected carrying out the right application at the right time.

Misconception(s) & Error(s) in Integers

Students may have misconceptions or errors in numbers and integers as well. Even if the misconceptions or common errors seem to be held by students, they can sometimes be held by teachers as well. Teachers may have the experience to know common errors and misconceptions of students in mathematics topics to be able to block the errors and misconceptions before constructed in students’ mind. However, it might not be possible for every topic. If teachers are assumed to know the errors and misconceptions, students may construct knowledge base easier by not struggling to design their knowledge base again. Pre-service teachers may need to acquire knowledge of errors and misconceptions because of being inexperienced. To know the knowledge might save time for correction rather than trying to correct; and this saved time can be utilized to better design their teaching in their lessons.

One of the misconceptions and errors students make is related with notations. The notations include the + and – signs where negative integers are used for subtraction and positive integers and used for addition. To attribute meaning to the sign of operation and the sign of integers, notations of integers become important.

Although symbols for the operations of addition and subtraction are + and -, the symbols may appear in front of numbers such as +3, or $^{+}3$; and -3, or $^{-}3$; or may appear as in 3 – (–2), 3 – (+2). In these situations, students may become puzzled in terms of operations and symbols. Teachers are required to point out that signs in front of parentheses are not interpreted as negative, but opposite (e.g.: – (–2)) , and signs in front of the numbers is interpreted not as subtraction but as negative (Dyke et. al., 2009, p.676; MoNE, 2005).

Students may have problems with usage of parentheses in these examples not being able to decide whether the parentheses are separated from operations or not. Students need to be reminded that parentheses help to make the number sentence more easily than not separating the operations from number signs (Van de Walle et. al., 2010).

When computing integers, students seem to regard the integers separately independent of their signs, especially when computing two integers with different signs. This time, students tend to compute numbers regardless of their signs. After the computation of numbers with their absolute value form, students place the sign of the first number or any sign in front of the result number (e.g.: 4+ (-5) may be resulted as 9 or –9) (Ryan & William, 2007; Schuter & Anderson, 2005).

Another misconception and error is in relation to the sign of zero. Zero might be regarded as positive, +0, or negative integer, -0. However, zero does not have a sign. It is "neither positive, nor negative integer" (Musser et. al., 2003, p. 319).

Ercan's (2010) study also shows that elementary students have confusions regarding integers. For instance, students may not be sure about whether a zero is an integer or not, or they may have different views about set of integers.

According to Ryan and William (2007), numbers with the minus sign are smaller than those with the plus sign. As a result of considering the sign at the beginning, they tend to order the integers in terms of their magnitude as follows: "0, –1, –2, –3, +1, +2, +3"; or "–1, –2, –3, 0, +1, +2, +3". As a result of this, they may place these numbers on number lines incorrectly (p.24). When students make subtraction of integers, some students may think that "doubling the first integer will give the accurate result" (Schuter & Anderson, 2005, p.27). For example, –5 – (+5) = –10 can be written as –5 × 2; however, students need to be reminded that the situation is not always valid.

Misconceptions and errors in integers show that there are many misconceptions and errors committed by students. The results may enable one to see the concepts from students' perspectives, and they will help to guide teachers in teaching the concept of integers. In addition to the misconceptions and errors in integers mentioned above, this study may also help to expand the examples of errors and misconceptions.

Summary

As it is seen in available literature, integers may not be an easy content to learn for students who have misconceptions or errors (Thompson & Dreyfus, 1988; Melezoğlu, 2005; İşgüden, 2008; Spang, 2009; Ercan, 2010; Körükçü, 2008). At the same time, integers may not be easy to teach for pre-service teachers since they need to know the content deeply, how to design mathematics lessons with instructional activities, how to use effective teaching strategies with concrete materials, models, or with technology supported mathematics lessons (Thompson & Dreyfus, 1988; Smith, 2002; Steiner, 2009; Akyüz, 2012). It is said that there seems to be a gap in related available literature about pre-service teachers' content knowledge about integers regarding definition of integers and pre-service teachers' pedagogical content knowledge about integers regarding student thinking.

It is also said that the definition of integers is not seen in available literature of concept definitions. Literature about definitions of mathematical concepts reports that mathematical concepts are incompletely defined by pre-service teachers and the definitions have an important part in mathematics since they help pre-service teachers to understand the nature of mathematics (Leikin & Zaskis, 2010; Levenson, 2012; Martin et. al., 2011; Dede & Soybaş, 2011). It is said that there seems to be a gap in related available literature about concept definitions of students and teachers related to elementary levels. One of the concept definitions which is important in elementary level is the concept of integers.

There seems to be a gap in the concept of integers and definitions of mathematical concepts related literature on studies regarding concept definition of integers in pre-service teachers.

CHAPTER III

METHODOLOGY

In this study, the nature of pre-service elementary mathematics teachers' subject matter knowledge and their pedagogical content knowledge with respect to definitions of integers were examined. For this purpose, pre-service mathematics teachers' knowledge in definitions of integers, their interpretations of quoted definitions of integers, their knowledge about elementary students' possible misconceptions and errors regarding definition of integers, and possible reasons of elementary students about definition of integers were examined.

In this chapter, the research methodology used in this study is mentioned. Participants of the study, data collection procedures, data sources, and design of the study are explained.

3.1 GENERAL DESCRIPTION OF THE DESIGN OF THE STUDY

To describe pre-service elementary mathematics teachers' conceptions of definition of integers and knowledge about students' possible misconceptions and errors regarding definition of integers, a qualitative research design was used.

According to Merriam (1998), "qualitative research is an umbrella concept covering several forms of inquiry that helps to understand the meaning of social phenomena with as little disruption of the natural settings as possible." (p.5). In other words, researchers understand the meaning of a social phenomenon with the qualitative research design. The meaning of the phenomenon is supported by how participants constructed the meanings of the phenomenon in their mind. In this manner, the critical point to be considered in qualitative studies is the importance of participants' ideas, viewpoints, interpretations, or approaches about the phenomenon.

Qualitative research provides researchers with the ability to explore and to gain deeper understanding of a phenomenon, and to produce detailed information from small part of people or cases (Creswell, 2005; Frankel & Wallen, 2006). Products of qualitative research designs help to describe the phenomenon in natural settings conducted for a study (Merriam, 1998).

Qualitative researchers focus on understanding meanings participants construct in their mind, and give importance to how the participants understand the world with their experiences (Merriam, 1998). They also focus on the study of social phenomena and on giving voice to the feelings and perceptions of the participants under study (Lodico, Spaulding, & Voegtle, 2006). It is important for researchers to understand a phenomenon by looking into participants' way of thinking regardless of researchers' own perspective about it.

In this study, the investigator interested pre-service elementary mathematics teachers' conceptions of integers and aimed to have deep understanding of their ideas regarding that issue. Therefore, qualitative research design was used to describe pre-service elementary mathematics teachers' knowledge regarding definitions of integers and their knowledge of students' possible misconceptions and errors in the definition of integers.

Case Study

In this study, the case study design was used. "Case study research is a form of qualitative research that endeavors to discover meaning to investigate processes, and to gain insights into and in-depth understanding of an individual, group, or situation" (Lodico et. al., 2006, p. 269). Case studies are preferred to be used in questions which are searching for the reasons of why and how an event occurs under the least control of the researcher on that event (Yin, 2003). Case study designs make events possible to be characterized with whole and meaningful features in real life contexts (Yin, 2003). Also, the designs enable researchers to understand given situations deeply (Merriam, 1998).

In research studies, cases in which an individual, classroom, or school takes part might be central to a study. Also, an event, an activity or an ongoing process might be identified as cases (Fraenkel & Wallen, 2006). In this study, senior pre-service elementary mathematics teachers studying in the department of elementary mathematics teacher education program in Middle East Technical University are the case of the study. In order to examine pre-service elementary mathematics teachers' conceptions related to definition of integers and knowledge about students' possible misconceptions and errors regarding definition of integers, a single case embedded design was selected. The context was the elementary mathematics education program. Pre-service teachers' pedagogical content knowledge and subject matter knowledge were considered in that context.

In this study, not only pre-service teachers' subject matter knowledge was examined, but also their pedagogical content knowledge and relationship between them were given attention. The research design was created based on Yin's (2003) multiple embedded design in a single case. It is shown in Figure 3.1:

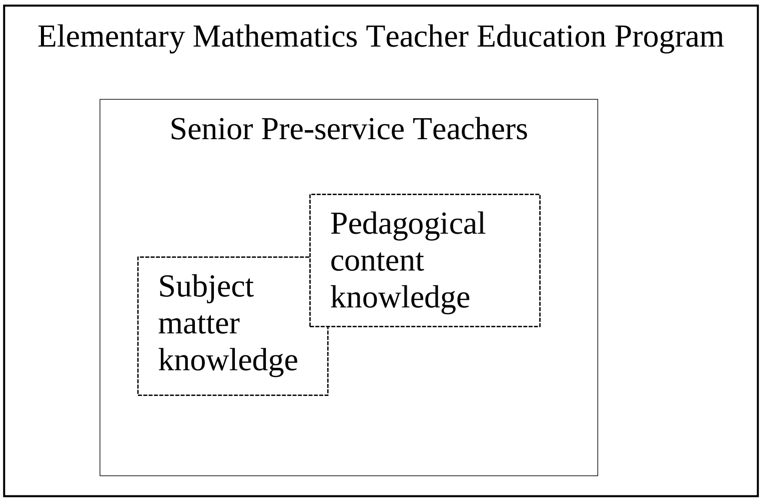


Figure 3.1: A single case design with multiple embedded unit of analysis (Yin, 2003)

3.1.1 Context of the study

The context of the study was elementary mathematics teacher education program. Elementary mathematics teacher education program is a four-year undergraduate program in METU. The program "aims to develop teachers with a sound understanding of how children learn mathematics; with confidence in using technology; with competence in problem-solving; with sensitivity to human rights, democracy, and ethics. The program emphasizes critical thinking, personal reflection, and professional development of pre-service math teachers" ("Department of Elementary Mathematics Education", 2012). By means of this program students become mathematics teachers for elementary grades from 1^{st} to 8^{th} grades (METU, 2009).

Pre-service teachers learn how to design the learning and teaching process of mathematics during their methodology courses and practice teaching courses. Methods of teaching mathematics courses are offered in their third year. Practice teaching courses are offered in their last year of education. Except for courses related to educational sciences, they are required to attend pure mathematics courses and courses of physics, history, or Turkish (METU, 2009). Courses of pre-service teachers in elementary mathematics department can be seen in Table 3.1.

Methods of teaching courses are based on objectives regarding how to teach mathematical concepts to elementary grade students. Pre-service teachers take the courses in their third year. In methods of teaching course which is taken in the second semester of the third year, number sets, specifically integers for elementary grade levels is one of the mathematical contents the course entails. In the course, pre-service teachers generally discuss meanings of mathematical concepts and the importance of the mathematical concepts; they suggest strategies for teaching the concepts effectively, and lastly they discuss each of their peers' activity sheets which are prepared after they discuss the critical points. Specifically, the following aspects of integers are discussed: how operations of integers are modeled, what elementary students' difficulties might be, and how to overcome the difficulties elementary students experience in their conceptual understanding.

Table 3.1

Courses in the elementary mathematics education program

1ST YEAR	MATH111	Fundamentals Of Mathematics	MATH112	Discrete Mathematıcs
	MATH115	Analytic Geometry	MATH116	Basıc Algebraic Structures
	MATH119	Calculus With Analytic Geometry	MATH120	Calculus For Functions Of Several Variables
	EDS200	Introduction To Education	CEIT100	Computer Applications In Education
	ENG101	English For Academic Purposes I	ENG102	Englısh For Academic Purposes II
	IS100	Introduction To Information Technologies And Applicatıons		
2ND YEAR	PHYS181	Basic Physics I	PHYS182	Basıc Physıcs II
	MATH219	Introduction To Differentıal Equations	MATH201	Elementary Geometry
	STAT201	Introduction To Probability &Stat. I	STAT202	Introduction To Probability &Stat.II
	ELE221	Instructional Principles And Methods	ELE225	Measurement And Assessment
	EDS220	Educational Psychology	ENG211	Academic Oral Presentation Skılls
	HIST2201	Prıncıples Of Kemal Atatürk I	HIST2202	Principles Of Kemal Atatürk II
	HIST2205	History Of The Turkish Revolution I	HIST2206	History Of The Turkish Revolution II
	MATH260	Basic Linear Algebra	ELE310	Community Service
3RD YEAR	ELE341	Methods Of Teaching Mathematics I	ELE329	Instructıonal Technology And Material Development
	TURK201	Elementary Turkish	ELE342	Methods Of Teaching Mathematics
	TURK305	Oral Communication	EDS304	Classroom Management
		Elective	TURK202	Intermediate Turkish
		Elective	TURK306	Written Expression
4TH YEAR	ELE301	Research Methods	ELE420	Practice Teaching In Elementary Education
	ELE435	School Experience	EDS416	Turkish Educational System And School Management
	ELE465	Nature Of Mathematical Knowledge For Teaching	EDS424	Guidance
		Restricted Electıve		Elective
		Elective		

Source: METU, 2009

3.1.2 Participants of the study

According to Merriam (1998), in qualitative studies, researchers spend most of their time in connection with participants. Furthermore, in qualitative studies, researchers want to get the necessary information from those who provide them the most. Not all the qualitative studies, but most of them use nonrandom, purposeful, and small samples instead of using random and larger samples used in quantitative studies. There are two basic types of sampling which are probability and non-probability sampling (Merriam, 2009). According to Merriam (2009), probability sampling enables researchers to make generalizations of results from the sample to the population. Since the purpose of qualitative studies is not to make generalizations, probability sampling is not preferred to be used. Rather than probability sampling, non-probability sampling is preferred. "The most common form of non-probability sampling is purposive sampling which based on the assumption that the investigator wants to discover, understand, and gain insight and therefore must select a sample from which the most can be learned" (Merriam, 2009, p.77). In other words, "the researcher will want to select participants so that they will be able to provide the key information essential for the study" (Lodico et. al., 2006, p. 266).

In this study, it was valuable for the researcher to study the pre-service elementary mathematics teachers at METU, since pre-service teachers are considered as qualified teachers after they graduate from the program. What’s more, participants of the study were studying in their last year in their education, and they had completed all their required courses to graduate from elementary mathematics teacher education program at METU. Pre-service elementary mathematics teachers were selected by means of purposeful sampling.

Convenience sampling was used as a type of purposive sampling. It enables the researcher to select sample based on time, money, energy, location, or accessibility of respondents, etc. (Merriam, 1998; Fraenkel & Wallen, 2006). In this study, pre-service elementary mathematics teachers at METU, who were studying in their last year in 2010 – 2011 spring semesters were selected as the potential of the study.

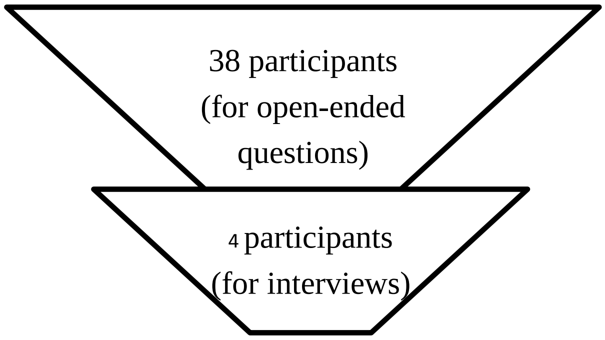


Figure 3.2: Sample of the main study

As shown in Figure 3.2, Thirty eight participants at METU voluntarily participated in two open-ended questions and 4 of these participants volunteered to participate in the interviews.

3.2 DATA COLLECTION

In case studies, only a single qualitative method may not be used. "Instead, multiple techniques including interviews, observations, and at times, the examination of documents and artifacts are employed" (Lodico et. al., 2006, p. 269).

One important form of data in qualitative research is "documents produced by key participants in the events being observed" (Slavin, 2007, p. 133). In this study, the documents consisted of written questions on integers, and an interview related to the same issues in the two open-ended questions was used as techniques for the case study. The researcher collected data through the two open-ended questions and interview questions. Written questions were one of the data sources for the study. Before the questions were administered to the participants, the participants were informed that they would be participating a study. And then, each participant was given the open-ended questions, and the researcher reminds the participants that participation in the study is not obligatory. At the same time, the researcher promised that nobody else would see the responses or written sheets except for the researcher.

Also, the researcher guaranteed that the results of the study would not affect their grades received from the course where the study is applied.

The two questions were applied to 38 pre-service elementary mathematics teachers at METU in the various sections of a mathematics teaching methods course methodology course which was taken in their last year, specifically during the 2011-2012 spring semesters. After the implementation of the questions, four participants voluntarily participated in the interview when the participants were available. Interviews were conducted face to face outside of the school area.

In this study, interviews of participants were videotaped by permission. The participants responded to the questions verbally.

3.2.1 Open-ended Questions

For this study, 2 open-ended questions, which were expected to provide answers to the research questions of this study, were used. Pre-service teachers were expected to give written answers to these two questions. These two open-ended questions were prepared to identify pre-service teachers' knowledge in definition of integers. They were prepared also to examine their knowledge about elementary students' possible misconceptions and errors about defining integers and to examine their knowledge of the possible reasons underlying the misconceptions and errors of elementary students about definition of integers. Two open-ended questions in which one of them has two sub questions, which were prepared based on the objectives of elementary mathematics curriculum (MoNE, 2009):

Open-ended questions

1. How can you define integers? Which numbers are included in the set of integers? ["Tam sayı"yı nasıl tanımlarsınız? Tam sayılar kümesi hangi elemanlardan oluşur?]
2. While explaining integers, teacher asked the students: “How can you define integers?"
   a) What might be misconceptions / errors of the definition students may have?
   b) What might be the reasons for the misconceptions / errors?

   [Öğretmen, tam sayılar konusunu anlatırken, öğrencilerine "tam sayıyı nasıl tanımlarsınız?" diye sordu.
   a) İlköğretim öğrencilerinin, bu soruyu cevaplarken düşebilecekleri yanılgı/lar ne/ler olabilir?
   b) Öğrencilerin bu yanılgısı/yanılgıları neden /nelerden kaynaklanıyor olabilir?]

The first question was prepared to understand pre-service teachers’ knowledge of key facts and concepts about definition of integers. It was inspired by one of the elementary mathematics curriculum objectives which state that "students are capable of explaining integers." (p.118). The second question focuses on pre-service teachers’ conception or knowledge of elementary students’ common misconceptions and errors in their view of what an integer is. The question was also inspired by the objective previously mentioned.

Pilot studies are used to improve data gathered from participants and to improve data collection procedures to be followed with necessary changes on data sources (Yin, 2003). The researcher conducted a pilot study during the spring semester of the 2010-2011 academic years.

The two open-ended questions were piloted on 19 senior students in Başkent University. The questions were also revised by 2 graduate students from elementary mathematics education, and 1 senior student from the same department to share their ideas about the open-ended questions to overcome problematic aspects of the content of the questions. What's more, 3 mathematics education professionals and one mathematician examined the questions and the objectives in the curriculum. With the help of their points of view, questions were redesigned until the researcher and the experts were in full agreement. Lastly, two Turkish language specialists examined the items for the appropriateness of the language used in the questions. Participants were given approximately 40-45 minutes to complete the questions. Their responses and participants' verbal reactions to items were examined to redesign the questions.

3.2.2 Interview Questions

Yin (2003) states that the interview is an essential data collection tool in case studies. The tool enables researchers to supply the needs of the line of inquiry with friendly and nonthreatening questions in open-ended interviews.

According to Yin (2003), interview types are categorized in three: open-ended nature, focused interview, and survey. In this study, the focused interview type was used. According to focused interview, respondents are interviewed with certain set of questions, and may be interviewed together with open-ended questions.

The purpose of the interview was to investigate participants' interpretations on quoted definitions of integers and to examine their responses to understand their conceptions of defining integers deeply. The interview included eight questions. For this study, responses of the first question were used since the interview question answers related research questions.

Quoted definitions used in the interviews

a..: It is a set which is combined of negative, positive, and zero numbers. [Negatif, pozitif ve sıfır sayısının birleşiminden oluşan kümedir.]

b.: Integers are numbers which are used for identification of plurality and are not written in a/b form. [Çoklukları tanımlamak amacıyla kullanılan ve a/b şeklinde yazılmayan, sayılardır.]

c.: Integers are numbers which are ascending and descending by one and represent a certain kind of plurality. [Birer birer ardışık olarak artan veya azalan ve bir çokluğu tam olarak ifade eden sayılara tamsayı denir.]

d.: Integers are rational numbers that do not have fractional part. [Kesir kısmı olmayan rasyonel sayılardır.]

e.: Integers are rational numbers of which the denominator is 1. [Paydası 1 olan bütün rasyonel sayılara tamsayı denir.]

Are the definitions correct? Why? [Bu tanımlar sizce doğru mudur? Neden?]

The interview question was asked to explore the accuracy of the quoted definitions as quoted definitions contribute much to understand students' understanding of the concept (Shield, 2004). There were appropriate and inappropriate definitions of integers in the interview. Table 3.2 presents a summary of the quoted definitions and their assessment based on the correctness criterion of Leikin and Zaskis (2010):

Table 3.2

Correctness of quoted definitions

	Inappropriate examples	*Appropriate rigorous examples of definitions*	*Appropriate but not rigorous examples of definitions*
a..: It is a set which is composed of negative, positive, and zero numbers.			X
b.: Integers are numbers which are used for identification of amount of objects and are not written in a/b form.	X		
c.: Integers are numbers which are ascending and descending by one and represent amount of objects.	X		
d.: Integers are rational numbers that do not have fractional part.			X
e.: Integers are rational numbers with denominator 1.		X	

All the definitions except e were prepared based on participants' responses in open-ended questions and a dictionary of mathematics used for definition e. Quoted definitions were prepared with the help of the quoted statements from several resources. To mention briefly, definition a was created based on responses of participants P3, P30, P32, and P33; definition b was created based on response of P20, definition c was created based on a response of P10, and definition d was created based on P1.

Interviews were conducted with four pre-service elementary mathematics teachers who accepted to participate in the study. For the purpose of participants’ knowledge about definition of integers, subjects were asked about quoted definition of integers.

Definition a and definition d were appropriate but not rigorous examples of definition of integers. They were asked to participants so as to participants recognize imprecise terminology usage in the definitions.

Definition b and definition c were inappropriate examples of definitions of integers so as to they have not necessary and sufficient information. Definition e is appropriate rigorous example of definition of integers since it has necessary and sufficient conditions.

A pilot study was conducted for the interview questions. One participant from METU voluntary participated in this pilot study. The participant and the researcher had a discussion on the interview questions and the open-ended questions to be able to determine the difficulties participants experienced in each data tool. Participants’ feedbacks to these data tools were helpful for the researcher to redesign them.

In conclusion, the items in open-ended questions and interview questions were developed and redesigned regarding grammar, content, and format by analyzing participants’ feedbacks and reactions to the open-ended items and interview questions. Based on the feedbacks, the order of some of the items in open-ended questions and interview questions were replaced with more appropriate and clearer ones, the scope of the questions were narrowed, and the format of the questions were arranged. Participants of the pilot study were summarized in Table 3.3 below:

Table 3.3

Participants for pilot studies

Universities pilot study conducted	Participants of pilot study for…	
	Interview questions	Open-ended questions
Başkent University	-	20 senior students and 2 graduate students
METU	1 senior student	-

3.3 PROCEDURES

The main study was conducted during the 2010-2011 spring semester. Participants of the main study were pre-service elementary mathematics teachers in METU. The conceptions of integers consist of two main components: (1) definition of integers, (2) misconceptions and errors about integers. The components were measured by two data tools: two open-ended questions and interview questions.

The total time to be allowed to complete the two questions was approximately 40 – 45 minutes. Participants were informed that the results of the questions would not be graded and not reflect onto their letter grades received from their courses. Interview sessions were arranged according to participants' own timetable. The total time spent on the interviews with participants was approximately 10-15 minutes for the interview questions.

Table 3.4 presents the time schedule for this study. It shows what procedures are followed. Each of the procedures will be mentioned in the following parts:

Table 3.4

Time schedule for the data collection process

Date of the time	Actions
February 2011–March 2011	Gathering questions, and design of the questions as an instrument
March 2011 – April 2011	Data collection from professionals, graduates and senior student, and redesign the questions
April 2011 – May 2011	Data collection – Implementation of questions
May 2011 – July 2011	Data collection – Interview
July 2011 – January 2012	Data analysis

3.4 DATA ANALYSIS

"Data analysis is the process of bringing order, structure, and meaning to the mass of collected data." (Marshall & Rossman, 1989, p. 112). According to Bogdan and Biklen (1998), "analysis involves working with data, organizing them, breaking them into manageable units, synthesizing them, searching for patterns, discovering what is important and what is to be learned, and deciding what you will tell others." (p.157).

In qualitative research, "researcher relies on the views of participants, asks broad, general questions, collects data consisting largely of words (or text) from participants, describes and analyzes these words for themes, and conducts the inquiry in a subjective, biased manner" (Creswell, 2005, p.46).

In this study, responses of participants were categorized as themes according to the similarities and the relationships of the responses. And then, they were coded based on the tendency of responses of the participants, literature, and the researchers' own experiences. If necessary, the categories were divided into subcategories.

According to Lodico et. al. (2006), coding is the process of identifying different segments of the data that describe related phenomena and labeling these parts using broad category names. Data are identified as major and minor themes in the coded data. "Themes are typically big ideas that combine several codes in a way that allows the researcher to examine the foreshadowed questions guiding the research" (p. 307). Coding data processes have three main steps: At first, all data are prepared and organized. And they are read, and the researcher has an idea of what data include and whether there are enough data collected. All of these done, data are coded into categories.

In this study, after codes and themes were prepared, they were reread, reexamined, and categorized by the researcher first and then by the second coders in order not to miss any detail. During the process, new codes and themes were added; some codes were replaced with more appropriate ones. After the necessary changes, three categories were created for participants' conceptions about definitions of integers: "core concepts", "representation", and "other definitions". Prior to explaining the results, it is meaningful to say that some categories might seem to be intertwined, although the definitions were evaluated on the category it is considered in agreement with the decision between coders.

Definitions under the categories were evaluated regarding their correctness. Correctness of the definitions were assessed based on Leikin and Zaskis' (2010) framework in which correctness is one of the criteria for the analysis of teacher-generated examples of the definitions. Some of the participants defined integers suggesting more than one definition. Therefore, number of responses is not equal to number of participants.

Furthermore, three categories were created for participants' reasons about students' possible misconceptions and errors: "negative transfer of former knowledge about number sets", "students' general insufficiency" and "teachers' approach". These categories were created based on responses of participants by the researcher.

3.5 ETHICS and LIMITATIONS OF THE STUDY

According to Fraenkel and Wallen (2006), there are three essential ethical issues which need to be addressed by every researcher: the protection of participants from harm, the ensuring of confidentiality of research data, and the question of deception of subjects.

Protecting participants from harm

"It is a fundamental responsibility of every researcher to do all in his or her power ro ensure that participants in a research study are protected from physical or psychological harm, discomfort, or danger that may arise due to research procedures." (Fraenkel & Wallen, 2006, p.56).

In this study, the researcher avoids hurting participants' feelings, opinions, or experiences. The participants of the study were informed about the purpose of the study and methods to be used. What's more, they were informed that their answers to the questions would not be graded in any course. Furthermore, they were informed that participation of the study was not obligatory, participation required being voluntary.

Ensuring confidentiality of research data

"Once the data in a study have been collected researchers should make sure that no one else (other than perhaps a few key research assistants) has access to the data" (Fraenkel & Wallen, 2006, p.56). In this study, the participants were informed that their responses would be held only by the researcher and would not be shared by making any links to their identities. Furthermore, fake names were used for the participants. No one else other than the researcher and the supervisor has access to the data.

Deception of subjects

"Sometimes it is better to deceive subjects than to cause them pain or trauma, as investigating a particular research question might require" (Fraenkel & Wallen, 2006, p.57). In this study, no problems are foreseen for the question of deception of subjects.

In addition to the ethical issues addressed, research studies might be discussed in terms of their strengths and limitations.

Merriam (1998) explains that case study designs ensure understanding phenomenon with its rich and holistic account. Accordingly, readers' perspectives towards insights and how meaning is derived from a phenomenon in the design expand with the designs. However, selection of the design comes with certain limitations mentioned below.

The first limitation is that "the amount of description, analysis, or summary material is up to the investigator" (p.42). Although the researcher wants to describe his/her case study design with rich and detailed description of data, it may not be possible because of his/her limited time, money, or energy to describe the data. In this study, the data related to definitions of integers were presented with rich and thick descriptions with respect to the limited time and energy of the researcher.

The second limitation is that "the researcher is the primary instrument for data collection and analysis." (p.42). This means that researchers do not have certain guidelines to collect and analyze their data. In other words, data collection and analysis of the data depend on the investigator’s own instincts and ability. In this study, the researcher was aware of biases which affect the final product of the study. To reduce the biases, the researcher underwent training in conducting interviews and applying open-ended items with pilot studies. In analyzing the collected data, the researcher received help from many graduate students and the thesis supervisor to reduce the researcher’s bias.

Reliability and validity are also limitations for the qualitative research studies. The limitations will be in the next title. Another limitation is related to generalizability of the study. "Generalizability is a standard aim in quantitative research and is normally achieved by statistical sampling procedures" (Silverman, & Marvasti, 2008, p.163). The sampling procedures are supported in two ways: (1) to be sure that your sample is represented by population, (2) the representativeness of sample of your study allow you to make general inferences. However, the sampling procedures are not available for qualitative research studies (Silverman & Marvasti, 2008). In this research study, data are derived from one case, pre-service elementary mathematics teachers. Pre-service elementary mathematics teachers at METU might not be representative of students who are studying in the same departments at other universities in Turkey. Thus, generalizability is not intended and may not be relevant for qualitative research designs and for this study.

The last limitation is about participants’ honesty. In this study, it was assumed that all the participants responded to the questions honestly and sincerely.

3.6 TRUSTWORTHINESS OF THE RESEARCH DESIGN

According to Merriam (2009), valid and reliable knowledge always makes sense in research studies. To trust research results, research studies need to be determined by validity and reliability of the design. "Regardless of the type of research, validity and reliability are concerns that can be approached through careful attention to a study’s conceptualization and the way in which the data are collected, analyzed, and interpreted, and the way in which the findings are presented" (Merriam, 2009). However, whether research results are trustworthy or not depends on some rigor in carrying out the study. As standards of rigor in research studies differ in quantitative and qualitative studies, credibility, transferability, dependability and confirmability are discussed in qualitative researches instead of the terminology of internal validity, external validity, reliability, and objectivity (Lincoln & Guba, 1985).

3.6.1 Internal validity or credibility

"Internal validity deals with the question of how research findings match with reality (Merriam, 1998). Another word, internal validity or credibility "…refers to whether the participants’ perceptions of the setting or events match up with the researcher’s portrayal of them in the research report" (Lodico et. al., 2006, p. 273).

To overcome probable problems regarding credibility, Merriam (2009) offers five strategies: triangulation, member checks, adequate engagement in data collection, researcher’s position, and peer examination. In this study, triangulation, researcher’s position, and peer examination were used to increase credibility of the study.

Triangulation

"The most well-known strategy to shore up the internal validity of a study is what is known as *triangulation*."(Merriam, 2009, p.215). Triangulation refers to studying with many investigators, many sources to gather and confirm data with each other, and many plans to match and agree on research findings (Slavin, 2007; Lodico et. al., 2006; Merriam, 1998; Stake, 1995).

According to Denzin (1978), there are four types of triangulation: the use of multiple methods, multiple sources of data, multiple investigators, and multiple theories. "Triangulation with multiple sources of *data* means comparing and cross-checking data collected through observations at different times or in different places, or interview data collected from people with different perspectives or from follow up interviews with the same people" (Merriam, 2009, p.216).

In this study, data were gathered from 38 pre-service elementary mathematics teachers using two open-ended questions. 4 of them also participated in the interview. Open-ended questions and interview were designed to evaluate the same knowledge, knowledge of defining integers. Data gathered from the sources were compared at two different times. Studying with the participants of the open-ended questions and interview, the researcher used many data sources to gather and confirm data from each of the participants, and have a chance to confirm the data collection tools with each other. In this way, credibility of the study was increased by data triangulation.

Researcher's position

One of the characteristics for all qualitative studies is that researchers are the primary instrument for gathering and analyzing data (Merriam, 1998). All observations and analyses are determined by researchers' worldview, values, and perspectives. The researchers can make arrangements be able to gather and produce meaningful information from their data. Even if researchers take into consideration for maximizing opportunities for collecting and producing meaningful information, they may overlook some particular situations, make mistakes, or biases stand out.

"Rather than trying to eliminate these biases or subjectivities, it is important to identify them and monitor them as to how they may be shaping the collection and interpretation of data" (Merriam, 2009, p.15). "Investigators need to explain their biases, dispositions, and assumptions regarding the research to be undertaken"

(Merriam 2009, p.219). Explanation of the biases and assumptions reveals researchers' expectations and values which influence research studies conducted (Fraenkel & Wallen, 2006). In this study, the researcher was inexperienced with pilot study and qualitative research design. To reduce these biases, the research needed to develop herself in these issues. To do this, the researcher conducted a pilot study for the open-ended questions and interview questions. In addition, she attended the 'qualitative research seminar' to gain knowledge about the design. What's more, she described in detail how the research setting was created and what the research findings were.

Peer examination

Peer examination means making critic on research findings together with colleagues (Merriam, 1998). "But such an examination or review can also be conducted by a colleague either familiar with the research or one new to the topic" (Merriam, 2009, p.220).

In this study, the researcher reexamined findings with the help of one graduate student from the mathematics education department and three graduate students from other departments. In addition, the researcher has also worked with her supervisor. The researcher studied with graduate students and the supervisor to scan the data and assess whether the findings were convenient with the data.

3.6.2 Reliability or Consistency/Dependability

Reliability is another concern for trustworthiness of the research (Merriam, 2009; Slavin, 2007; Fraenkel & Wallen, 2006). According to Merriam (1998), reliability refers to the extent to which research findings can be replicated. It also refers to getting consistent scores between measurements when measuring the same things (Slavin, 2007; Fraenkel & Wallen, 2006).

Although reliability is related to yielding the same results repeatedly, in social sciences, it is hard to get the same results (Merriam, 2009). Thus, Lincoln and Guba (1985) prefer to use the term of "dependability" and "consistency" of results instead of reliability. These terms refer to achieving consistency between data collected and the results obtained. The purpose is not to find the same or similar findings with the previous studies which measure the same things, rather the results need to be consistent with the data collected (Merriam, 1998).

According to Merriam (2009), peer examination, the investigator's position, triangulation, and audit trail are techniques to increase dependability of results. The first three techniques to increase internal validity of the study, which were discussed in the previous title, also increase the reliability of the study.

3.6.3 External validity or Transferability

External validity is related with the generalizability of research results to other situations (Merriam, 1998). According to Patton (1990), qualitative research requires providing viewpoints for research results instead of getting at the truth, to make local decisions by local decision makers' personal theories instead of making the decisions by help of theories which are universally accepted, and personal investigations examined in contexts instead of using the investigations to make generalizations. Thus, making generalizations are not the requirement for qualitative studies, rather qualitative researchers share context-bound investigations. In other words, making generalizability is not required because the major goal of qualitative research is to provide an in-depth understanding of a limited setting, group, or person (Lodico et. al., 2006).

"In qualitative research, a single case or small, nonrandom, purposeful sample is selected precisely because the researcher wishes to understand the particular in depth, not to find out what is generally true of the many" (Merriam, 2009, p.224). Although qualitative researchers do not expect their findings to be generalizable to all other settings, it is likely that the lessons learned in one setting might be useful to others (Lodico et. al., 2006). However, what to do to ensure external validity (transferability) in qualitative studies is enhanced by rich and thick description for research situation, and be able to transfer research findings to natural situations (Merriam, 1998). In other words, researchers need to describe whole detailed description about the sample, design, strategies used for preparation of data tools, etc. whatever covered in their research, and then research findings can be transferrable to settings which have similar or the same descriptions with the researchers' settings. In this study, research findings are transferred to cases which are similar or the same as the qualities of this study. Therefore, results of the study can be transferred to pre-service elementary mathematics teachers who have similar experiences with the participants of the study on integers.

CHAPTER IV

RESULTS

This chapter comprises two parts. The first part includes findings regarding pre-service teachers' subject matter knowledge on definition of integers. The first part was examined under two subcategories: participants' own definitions of integers and participants' interpretations of quoted definitions of integers. The second part covers pre-service teachers' pedagogical content knowledge on definition of integers. It was examined under two subcategories: participants' knowledge on elementary students' possible misconceptions and errors regarding definition of integers and their knowledge on possible reasons of elementary students' misconceptions and errors in defining integers.

Responses taken in each part were categorized based on the tendency of the responses of the participants, literature on dictionary of mathematics, and the researchers' own experiences. After the emerging categorization of the responses, they were titled under themes which are consistent with the categories. Findings from the open-ended questions and the interview questions were reported; data taken from the two sources were revised to create categories and themes.

4.1 THE NATURE OF PARTICIPANTS' SUBJECT MATTER KNOWLEDGE ON DEFINITION OF INTEGERS

As stated above, this study attempted to present the findings of subject matter knowledge on definition of integers. According to Shulman's framework of teachers' knowledge, defining integers can be considered as subject matter knowledge, which is key knowledge for teaching integers.

In this study, pre-service teachers' subject matter knowledge was determined by one of the questions in open-ended questions in which the definition of integers was asked. Participants' subject matter knowledge was also determined by interviews in which participants were asked about quoted definitions of integers. The findings were reported under two subcategories: (1) participants' own definition of integers and (2) participants' interpretations of quoted definitions of integers.

4.1.1 The nature of participants' own definitions of integers

In this section, participants' knowledge on definition of integers is presented. To do this, data from open-ended questions were used. Pre-service teachers were asked the following question related to subject matter knowledge on definition of integers: *How can you define integers? Which numbers are included in an integers set?*

The analysis of the data revealed that most pre-service elementary mathematics teachers' definition of integers could be categorized as "core concepts", "representation", and "other definitions."

Definitions by core concepts

Core concepts refer to definitions of integers that were described as and constructed by central concepts, or prior concepts which are "whole numbers", "counting numbers", and "rational numbers". In other words, core concepts are some other concepts that the participants attempted to use while trying to define integers.

Some of the definitions of the participants that attempted to use core concepts are explained below. The definitions are assessed whether they are appropriate or inappropriate based on the appropriateness criterion of Leikin and Zaskis (2010). The appropriateness of the definitions can be seen in tables at the end of the related parts.

Core Concept 1: Definitions Based on Whole Numbers

According to the results of data analysis, some of the participants defined integers based on whole numbers. In other words, the whole number concept helped the participants to create the definition. There were eleven participants who defined integers based on whole numbers.

Four of the participants attempted to define integers as "*If we combine whole numbers and their negatives, we have integers." [Tamsayı doğal sayılara negatiflerinin de eklenmiş halidir.]* (n=4). The definition has necessary and sufficient conditions to define integers. It is consistent with the definition of Bennett and Nelson (2000) which says that "the whole numbers, 0, 1, 2, 3, 4… , together with the negatives of the whole numbers, - 1, -2, -3, -4,…. are called integers." (p.248). The definition is categorized as an appropriate rigorous example of a definition of integers according to Leikin and Zaskin's (2010) framework.

Six of the participants preferred to emphasize that integers are a number set: "*If we add the set of negative integers to the whole number set, we have the set of integers." [Doğal sayılar kümesine negatif tamsayıları da eklersek tamsayılar kümesini elde ederiz.]* (n=6). This definition attempted to define the concept of integers with the concept of integers. In this definition, 'negatives of whole numbers' should have been used rather than 'negative integers'. Therefore, the term 'negative integers' is an imprecise terminology for the definition. According to Leikin and Zaskin's (2010) framework, this definition is appropriate but not a rigorous example of a definition of integers due to the fact that it has imprecise terminology.

Similar with the previous definition, one of the participants (P30) has also used imprecise terminology in his definition*: "A number set which includes whole numbers and negative numbers." [Doğal sayılar ve negatif sayılardan oluşan sayı kümesidir.]* The problem here is the use of the phrase "negative numbers" without further qualification. In this statement, this participant should have used "negatives of whole numbers" rather than "negative numbers" to describe or define integers. It is said that imprecise terminology was used for the definition. Therefore, the definition is appropriate but not a rigorous example of a definition of integers. Table 4.1 shows all examples of definitions of integers based on whole numbers:

Table 4.1

Correctness of the definitions based on whole numbers

Appropriate Definitions (N=11)	Frequency	Examples
Appropriate rigorous examples of definitions	4	*If we combine whole numbers and their negatives, we have integers. [Tamsayı doğal sayılara negatiflerinin de eklenmiş halidir.]*
Appropriate but not rigorous examples of definitions	6	*If we add the set of negative integers to the whole number set, we have set of integers. [Doğal sayılar kümesine negatif tamsayıları da eklersek tamsayılar kümesini elde ederiz.]*
	1	*A number set which includes whole numbers and negative numbers. [Doğal sayılar ve negatif sayılardan oluşan sayı kümesidir.]*

In summary, 11 of the participants attempted to define or describe integers based on the whole numbers concept. Four of the definitions were appropriate rigorous examples of definitions of integers. The other definitions were also appropriate but not rigorous examples of definitions of integers since the pre-service teachers used imprecise terminology in their definitions.

Core concept 2: Definitions Based on Counting Numbers

Another core concept that participants based their definitions on was the concept of counting numbers, which was defined as "one of the numbers 1, 2, 3, …" (James & James, 1959, p. 311). P19 stated, "*Integers are numbers which we use in daily life and they are (not only counting numbers) but also negatives of counting numbers." [Tamsayılar günlük hayatta kullandığımız sayma sayılarının negatiflerinin de düşünüldüğü sayılardır.]* As this example shows, P19 based her definition on counting numbers. However, this definition is incomplete for not mentioning the number zero. That makes the definition appropriate but not a rigorous example of a definition of integers according to Leikin and Zaskis's (2010) framework.

Two participants, P20 and P35, added "zero" to their definitions: *"I tell my students that integers are a set of numbers which include counting numbers, negatives of counting numbers, and zero". [Sayma sayıları kümesine negatiflerini ve 0 eklenerek* *oluşan küme diye anlatırım.](P20).* The definition has necessary and sufficient information for a definition of integers. The definition is consistent with the definition of Parker (1997) whose definition says that an integer is "any positive or negative counting number or zero" (p.123). They are appropriate rigorous examples of definitions of integers. Examples of definition regarding correctness of definitions based on counting numbers are shown in Table 4.2:

Table 4.2

Correctness of the definitions based on counting numbers

Appropriate Definitions (N=3)	**Frequency**	**Examples**
Appropriate rigorous examples of definitions	2	*Integers are numbers which we use in daily life and they include (not only counting numbers, but also) negatives of counting numbers. [Tamsayılar günlük hayatta kullandığımız sayma sayılarının negatiflerinin de düşünüldüğü sayılardır.]*
Appropriate but not rigorous examples of definitions	1	*I tell my students that integers are sets which include counting numbers, negatives of counting numbers, and zero. [Sayma sayıları kümesine negatiflerini ve 0 eklenerek oluşan küme diye anlatırım.]*

According to the research results, 3 participants attempted to define integers with the concept of counting numbers. One of them was an appropriate but not a rigorous example of a definition of integers since it omits zero. The other two definitions were appropriate rigorous examples of definitions of integers.

Core concept 3: Definitions Based on Rational numbers

The other core concept that was used by pre-service teachers in defining integers was rational numbers. A rational number is a different concept from the previous two core concepts because it is defined based on integers as "a number that can be written in the form a/b, where a and b are integers, with b≠0" (James & James, 1959, p. 387).

According to P1, an integer is "*a rational number which does not have a fractional part."[Kesir kısmı olmayan rasyonel sayılardır.].* Fractional part is a terminology usually seen in decimal numbers. When the term is used with rational numbers, it is necessary to convert rational numbers to decimal numbers to better understand the definition. Therefore, it may be said that the term 'rational number' in a participant's statement may be expanded to the concept of 'number'. The term 'rational number' was used imprecisely. Therefore, it is concluded that the definition is appropriate but not a rigorous example of a definition of integers.

In her definition, P5 used a more careful language regarding the concept of the fraction: "*The set of integers is composed of positive and negative numbers which have denominators equal to one." [Tamsayılar kümesi paydası bire eşit olan pozitif ve negatif sayılardan oluşur.]* In this definition, it seems that the participant intended to define integers based on rational numbers. However, she did not use the term rational numbers but instead said positive and negative numbers which have denominators equal to one. This definition is problematic in that all of the positive or negative numbers whose denominator is 1 may not be an integer. For example $\frac{\sqrt{2}}{1}$, $\frac{\sqrt[3]{-5}}{1}$ are not integers. The definition used imprecise terminology by using the term 'numbers' instead of 'rational numbers'.

Different from the previous definition, P8, P24, and P25 defined integers based on the concept of rational number without describing the concept. Instead of using descriptions of rational numbers, the participants preferred to describe integers focusing on the denominator of the rational numbers: *"All the rational numbers which have denominators equal to 1 are integers." [Paydası 1 olan bütün rasyonel sayılara tamsayı denir.](P25).* This definition has necessary and sufficient information to define integers. It is similar to the definition given by Borowski and Borwein (1989), whose definition is that " integers are the closure of the natural numbers under subtraction, and are identified with the rational numbers with denominator 1" (p.298).

Another participant P24 attempted to expand her definition by specifying boundary limits for integers: "*integers include rational numbers with the denominator equaling to one and they are from -∞ to +∞." [Tam sayılar -∞ dan +∞'a paydası 1 olan bütün rasyonel sayıları içerir.].* P24's response is not inaccurate. However, mentioning the boundaries is not necessary to define integers, if it is based on rational numbers. This information is not needed, but the participant preferred to state that integers are from -∞ to +∞. As extra information, it is said that the definition is not minimal. The definition is an appropriate rigorous example of a definition of integers.

Another participant, P33, used the knowledge that indicates that the denominator of an integer equals to 1: "*integers are numbers which are written in the form of a/b when b=1 and 'a' equals to positive or negative whole numbers." [a/b şeklinde yazılan, b=1 olmak şartı ile, a ise pozitif ya da negatif bir doğal sayı olmak şartı ile yazılabilen sayılara tamsayılar denir.]*. This definition includes the combination of two knowledge parts: (1) a/b form when b equals to 1 and (2) when a is a positive or negative whole number. The definition seems to be parallel with Borowski and Borwein (1989) in the sense that "the integers are identified with the rational numbers with denominator 1" (p.298). It is an appropriate rigorous example of a definition of integers.

P20 also used the knowledge that integers are written in a/b form. In addition to that knowledge, b needs to be exactly divided by a: "i*ntegers are numbers which can be written in a/b form when b is divisible by a." [a/b şeklinde yazılabilen rasyonel sayılar kümesinden paydanın payı tam olarak böldüğü sayıların oluşturduğu, rasyonel sayılar kümesinin bir alt kümesidir.]*. This definition seems to require that "a" needs to be multiple of b, and b is equal to the value 1. It is an appropriate rigorous example of definition of integers.

Correctness of the definitions based on rational numbers is shown in Table 4.3:

Table 4.3

Correctness of the definitions based on rational numbers

Appropriate Definitions (N=6)	Frequency	Examples
Appropriate rigorous examples of definitions	3	*All the rational numbers which have denominators equal to 1 are integers. [Paydası 1 olan bütün rasyonel sayılara tamsayı denir.]*
	1	*Integers are numbers which are written in a/b form when b=1 and 'a' equals to positive or negative whole numbers. [a/b şeklinde yazılan, b=1 olmak şartı ile, a ise pozitif ya da negatif bir doğal sayı olmak şartı ile yazılabilen sayılara tamsayılar denir.].*
	1	*Integers include rational numbers with the denominator equaling to one and they are from -∞ to +∞. [Tam sayılar -∞ dan +∞'a paydası 1 olan bütün rasyonel sayıları içerir.].*
	1	*Integers are numbers which are written as rational number in the form of a/b when b is divisible by a. [a/b şeklinde yazılabilen rasyonel sayılar kümesinden paydanın payı tam olarak böldüğü sayıların oluşturduğu, rasyonel sayılar kümesinin bir alt kümesidir.].*
Appropriate but not rigorous examples of definitions	1	*A rational number that does not have fractional part. [Kesir kısmı olmayan rasyonel sayılardır.]*
	1	*The set of integers is composed of positive and negative numbers which have denominators equal to one. [Tamsayılar kümesi paydası bire eşit olan pozitif ve negatif sayılardan oluşur.]*

To summarize the responses of the participants, there are 2 appropriate but not rigorous examples of definitions of integers and 6 appropriate rigorous examples of definitions of integers.

In summary, 11 of the definitions were based on the concept of whole numbers.

Definitions by Representations

Together with these core concepts, *representation* is another way to define integers. In this context, representation refers to written symbols which are the mathematical symbols or their forms which are expressed in words (Clement, 2004).

In the general sense, some of the participants in this study defined integers mainly by representing through symbols. Some of the participants attempted to define integers by using Z notation. In other words, integers were defined by using mathematical symbols of the integer's domain, Z, Z^{+}, Z^{-}. To define integers, four of the participants used this notation and stated the definition as "$Z=Z^{+} \cup \{0\} \cup Z^{-}$" (P3, P7, P18, and P32). In this definition, integers are attempted to be defined by the concept itself.

Other participants attempted to define integers by using numbers in the domain of integers. For example P28, P13, and P2 defined integers by writing this set: "{…-2, -1, 0, 1, 2, 3…}". In a similar way, P36 wrote a list of integers without using brackets and wrote in the open-ended questions "…-4,-3,-2,-1, 0, 1,2,3,4… "

P35, P9, and P6 used the letter Z to denote the set of integers and stated "Z={…-3,-2,-1, 0, 1,2,3,…}"

P12, P17, P21, P22, P27 had similar attempts to define integers. But they indicated that the numbers are between negative and positive infinity: "*{-∞…-2, -1, 0, 1, 2, 3…+∞}*." Negative and positive infinity signs are unnecessary information for the representation.

Correctness of the definitions based on representation of integers is seen in Table 4.4:

Table 4.4

Correctness of the definitions based on representation of integers

Representation of integers (N=16)	Frequency	Examples
	3	{…-2, -1, 0, 1, 2, 3…}
Appropriate rigorous examples of definitions	1	…-4,-3,-2,-1, 0, 1,2,3,4…
	3	Z={…-3,-2,-1, 0,1,2,3,…}
Inappropriate examples	5	{-∞…-2, -1, 0, 1, 2, 3…+∞}
	4	$Z=Z^{+} \cup \{0\} \cup Z^{-}$

The results show that 16 of the participants' responses attempted to define integers through the representations such as written symbols.

Other definitions

In addition to the research results, there are some responses which cannot be categorized under core concepts and representation titles. These responses were categorized as 'other definitions' as they are not in harmony with the previous categories. Participants used either appropriate or inappropriate examples of definition of integers. Their definitions were appropriate when they had necessary and sufficient conditions for a definition of integers. Although they had the conditions, imprecise terminology or omitting some constraints was usually seen as problems in appropriate definitions.

According to the research results, some participants attempted to define or describe integers with the concept of integers itself though the concept is not defined by the concept itself (Zaskis & Leikin, 2008; Shield, 2004). For example, P1 divided integers into its elements rather than describing what the elements include: *"Negative and positive integers, and zero."[Negatif ve pozitif tamsayılar ve sıfır.]* P19 used a similar way to define integers.

Although some participants divided the set of integers into its elements correctly, P3, P38, P30, and P32 did not use the precise language. In other words, they did not use the word 'numbers' in their definition appropriately. For example, P30 stated that "*integers composed of positive, negative numbers and zero." [Tamsayılar pozitif, negatif sayılar ve sıfırdan oluşur.].* P3, P38, and P32 used a similar way to define integers. The statement is obviously problematic. The participants stated that integers are any positive or negative numbers. But for instance, positive numbers such as $\sqrt{3}$ or 0.7 are not integers.

Although zero was emphasized as an integer by the participant, he did not need to emphasize zero separately from positive and negative integers: "*integer is a set which is composed of negative and positive integers." [Negatif ve pozitif tamsayıların birleşmesiyle oluşan kümedir.].* The definition has two problems. One of them is that he did not specify zero as an element of integers. The definition omits the number zero. The other is that the definition attempts to define integers by the concept itself.

While defining integers, some of the participants emphasized that these numbers are between -∞ to +∞. For instance, P13 stated that "*integers are numbers which are between -∞ and +∞." [Tamsayılar -∞ dan +∞ aralığındaki sayılardır.]* P14 and P27 suggested similar definitions. It is obvious that this definition is mathematically inappropriate because there are not necessary or sufficient conditions for defining integers. On the other hand, the participant needed to emphasize that there are infinitely many integers.

P28 used of "certain boundary" in her definition: "*integers are numbers which have certain boundaries and they have negative, positive, and zero values, with the denominator 1." [Negatif, pozitif ve sıfır değerlerine sahip belli aralıklarla sınırlanan paydası 1 olan sayılar].* This participant tried to use informal language to define integers. However, she made several mistakes in her definition, such as the use of the term ‘certain boundaries’, not being specific about which number the starting point is, and nominator of the number is also unclear. The definition omits some constraints by not specifying certain boundaries, the starting point and the nominator of the number with whole numbers and negatives of the whole numbers. Therefore, the definition is appropriate but not a rigorous example of a definition of integers.

P10 explains that the certain boundary needs to be 1 and the starting point needs to be zero: "*integers are numbers which are found by adding 1 to and subtracting 1 from numbers which start from zero, and indicate distance from zero on a number line." [0'dan başlanarak sayılara 1 eklenmesi ve çıkartılmasıyla elde edilen ve o sayıların sayı doğrusu üzerinde sıfıra olan uzaklıklarını belirten sayılardır.]* Definition of P10 determined that the range is 1 and the range starts from zero. This part of the definition seems to be clearer than the previous one. Also, it is parallel with Moskowitz's (2003) explanation which states that " the set has the property that 1 is in it and if anything in it, then so the next thing (if n$\in Z^+$, then also n+1$\in Z^+$)" (p.3). ""Adding 1 to a positive integer gives us another one" (p.4) in integers. This part of the statement is enough to define integers; therefore, this part of the statement is an appropriate rigorous example of definition of integers. However, the remaining part of the definition which says that "…they are numbers which indicate distance from zero on number line" seems to be unnecessary incorrect information. The unnecessary incorrect information reflects the lack of rigor in mathematical language related to distance from zero. Distance from zero on a number line is always expressed by positive integers. Therefore, the statement does not define integers. In this way, the definition is appropriate but not rigorous example of definition of integers.

Another participant, P22 described an integer as *"numbers express wholeness." [Bütünü ifade eden sayılar.]* This participant tried to imply that whole numbers are the ones that represent cardinality. In this sense, this set only includes natural numbers. Negative of natural numbers and zero are not concluded from the set. Therefore, it does not have the sufficient conditions to define integers. The definition is inappropriate for the insufficiency.

In summary, 13 definitions of integers were based on many mathematical concepts which are used in an intertwined way. Table 4.5 shows the correctness evaluation of these definitions.

Table 4.5

Correctness of other definitions

Definition (N=15)	**Frequency**	**Examples**
Appropriate but not rigorous examples of definitions	1	*Integers are numbers which have a certain boundary to each other, (they) are negative, positive, and zero, and (numbers) with denominator 1. [Negatif, pozitif ve sıfır değerlerine sahip belli aralıklarla sınırlanan paydası 1 olan sayılar].*
	1	*Integers are numbers which are found by adding 1 to and subtracting 1 from numbers which start from zero, and indicate distance from zero in number line. [0'dan başlanarak sayılara 1 eklenmesi ve çıkartılmasıyla elde edilen ve o sayıların sayı doğrusu üzerinde sıfıra olan uzaklıklarını belirten sayılardır.]*
	4	*Integers are composed of positive, negative numbers and zero. [Tamsayılar pozitif, negative sayılar ve sıfırdan oluşur.].*
Inappropriate examples	3	*Integers are numbers which are between -∞ and +∞. [Tamsayılar -∞ dan +∞ aralığındaki sayılardır.]*
	1	*Numbers which express wholeness. [Bütünü ifade eden sayılar.]*
	2	*Negative and positive integers, and zero. [Negatif ve pozitif tamsayılar ve sıfır.]*
	1	*Integer is a set which is composed of negative and positive integers. [Negatif ve pozitif tamsayıların birleşmesiyle oluşan kümedir.]*

In conclusion, participants used core concepts, representation, and other definitions to define integers. In addition, some participants did not provide any description or definition of integers. Frequency of participants' responses for definition of integers is shown in Table 4.6:

Table 4.6

Frequency of responses of participants for definition of integers

	Frequency
Core Concepts (CCs)	22
Representation	16
Other definitions	13
No answer	2

According to research results, 22 responses were categorized as core concepts for the definition of integers. This number is followed by "representation" which is seen in 16 responses of participants. 13 responses were categorized as "other definitions". Finally, 2 responses of participants did not suggest any definitions.

4.1.2 The nature of participants' ideas on quoted definitions of integers

As mentioned in earlier chapters, four of the participants were interviewed by an open-ended question. The purpose of the interview was to explore the correctness of the quoted definitions as quoted definitions contribute much to understand learners' understanding of the concept (Shield, 2004). It was aimed to investigate the nature of interpretations made by pre-service teachers regarding the quoted definitions of integers. In other words, how pre-service teachers evaluate different definitions was aimed at by asking the interview questions seen in Table 4.7:

Table 4.7

Interview questions

Interview question 1: a..: It is a set which is composed of negative, positive, and zero numbers. b.: Integers are numbers which are used for identification of amount of objects and are not written in a/b form. c.: Integers are numbers which are ascending and descending by one and represent amount of objects. d.: Integers are rational numbers that do not have a fractional part. e.: Integers are all rational numbers with the denominator 1. Are the definitions true? Why?

Five statements that supposedly try to define integers were given to the participants one by one. The nature of the participants' ideas on quoted definitions of integers was represented in the following.

Pre-service teachers' ideas about definition a: "It is a set which is composed of negative, positive numbers and zero"

Definition *a* says that a set of integers include negative numbers, positive numbers, and zero. In this definition, the interview participants were expected to recognize the inappropriate use of 'numbers'. In other words, they were expected to say that the word 'numbers' is a general concept which includes not only integers but also rational numbers or complex numbers.

PA thought that the definition is similar to the definition in her mind. After comparing the quoted definitions, PA seemed to accept the definition a as true: "*There are whole numbers, counting numbers, negative of counting numbers, and zero. That definition is close to the definition of integers." [Doğal sayılar, ve sayma sayıları olacak, sayma sayılarının eksileri olacak ve sıfır olacak gibi. Yakın.]*

However, PB realized that 'numbers' needs to be replaced with 'integers'. If it is not changed, the word may cause misunderstandings: "*(The students) should have added* *integers or…students may think (the word of the number) as a whole number. However, whole numbers are not negative. I mean, the 'number' should have characterized which type of number it is…" [Burada şeyi eklemesi gerekiyordu, tamsayı demesi gerekiyordu ya da… öğrenci bunu doğal sayı diye de düşünebilir, gerçi doğal sayılar negatif olamıyorda hani burada sayıyı nitelemesi gerekiyordu, hangi tür sayılar…]*

Similar to PB, PC corrected the definition by changing the word of 'numbers' to 'integers': "*It should have been said that (integers) are set of numbers that is composed of negative integers, positive integers, and zero." [Negatif tamsayılar, pozitif tamsayılar, ve sıfırdan oluşan kümedir deseydi olurdu...]*

PD corrected the definition for the inappropriately used word 'numbers': "*I would say that there are negative numbers which are not integers. Thus, I would say that the statement (definition b) is so general that it may not be true. There are integers which are not rational numbers. For example, if we say that (integers) consist of negative numbers, non-integer numbers are included in set of integers. We say the information as additional. I would give examples."* [*Şey derdim, negatif olup tamsayı olmayan sayılarda var. O yüzden bu genel bir bilgi olur, doğru olmayabilir derdim. Tamsayı olmayan rasyonel sayılar var, mesela negatif sayıların birleşiminden dersek tamsayı olmayan sayıları da içine dahil ederiz, fazladan söylemiş oluruz gibi. Örnek verirdim.]*

Research results show that three of the participants recognized the inappropriate wording of numbers, and the other participant did not recognize it. However, the participant probably considered the word of numbers as whole numbers and counting numbers, which make the definition as true.

Pre-service teachers' ideas about definition b: "Integers are numbers which are used for identification of amount of objects and are not written in a/b form."

The definition says that integers are used to show the amount of objects and they are not written in the form of a/b. In this definition, participants were expected to recognize that integers can be written in a/b form. In addition, they were expected to identify that this definition does not include negative integers.

PA explained that integers were written as a/b giving counter examples. According to the participant, there are examples to show the form: "*I write in a/b form, 2/1, or 2/2.*" [a*/b şeklinde yazarım, 2/1 diye yazarım, 2/2 diye yazarım.]*

PB followed a similar way with PA explaining that integers can be written in the a/b form through counter examples. PB also thinks that the written form of a/b is an unnecessary detail to mention in the definition: "*We write some numbers in a/b form. I would represent it as 5/1. We would say that there is no need for such an explanation."[a/b şeklinde yazabiliriz bazı sayıları. 5/1 gibi bunu sunardım. Bu tür bir açıklamaya gerek yok derdik.]*

Although PC also wrote integers in a/b form, PC focused on which number sets are not written in a/b form. Then, PC realized that integers are written in a/b form: "*I mean, it (integer) will not be written as a rational number, so it (the number) may be a whole number, counting number, integer but… we write them (integers) as such (a/b form) if the denominator of the number equals to 1. For example, 2/1, -8/1." [Yani rasyonel sayı şeklinde yazılmayacak, o zaman, doğal sayılarda olabilir, sayma sayıları da olabilir, tamsayılarda olabilir, ama… yazabiliriz aslında paydalarına 1 koyarsak, o şekilde yazabiliriz. Mesela, 2/1, -8/1.]*

Although PC accepted the written form, the explanations seem to reveale that the participant is not sure about which number sets are written in a/b form and which are not. According to the participant's first statement, whole numbers, cardinal numbers, or integers are not written in a/b form before thinking that the denominator might be equal to 1.

PD similarly focused on number sets which are not written in a/b form: "*There are numbers which are not written in a/b. For example, complex numbers." [a/b şeklinde yazılmayan başka sayılarda var. Mesela, karmaşık sayılar.]*

According to PD, the explanation about the written form of a/b is not unique for integers. In other words, PD thinks that not only integers but also complex numbers are not written in a/b form. In brief, the participant failed to explain that integers are written in a/b form when b=1.

According to the research results regarding this definition, two of the participants were aware of the fact that integers can be written in a/b form. However, the others were not sure about the numbers which are written in a/b form.

Pre-service teachers' ideas about definition c: "Integers are numbers which are ascending and descending by one and represent amount of objects."

This definition says that integers are numbers which have one unit distance to each other and they are used to represent the amount of objects. In this definition, participants were expected to recognize the missing information about the starting point. The starting point should be zero. The participants' responses are as follows:

PA agrees with the definition: "a*scending and descending by one… yes, they (integers) increase and decrease one by one, 0, -1, 1, okay." [Birer birer artan… evet, birer birer artar veya azalır, 0, 1, -1…tamam.].* When asked the participant about the following part of the definition, *'numbers which represent a certain kind of plurality',* the participant commented that the word 'certain' is related to rational numbers: *"It is as if they can include rational numbers too I mean, while it says 'which represent a certain kind of plurality', does it refer to all plurality? I think it does. I took the meaning that we can express all kinds of plurality with integers." [İçine rasyonel sayıları da alabilirmiş gibi, hani, bir çokluğu tam olarak derken, tamamını mı diyor ki, ben öyle yorumladım. Bu sayılarla her çokluğu ifade edebiliriz gibi bir anlam çıkardım ben.]*

PB agreed with the definition: "*The definition is in fact correct,… in fact, …it seems correct…" [Aslında tanım doğru ama…doğru gibi geldi aslında.]*

PC said that the participant preferred to use the definition *d* instead of using definition *a* when analyzing whether definition *a* is correct or not: "I would use the other definition, for example, ascending one by one, like zero, one, one, one, increasing exactly. I would say that the distance between the two numbers should be exact (exact number)." *[…onun yerine diğer tanımı (definition d) kullanabilirim* *mesela, birer birer artan sıfır, bir gibi, bir, bir, bir tam bir şekilde artan. Tam olması gerektiğini söylerdim.]* When the participant was asked whether the definition was sufficient as a definition of integer, PC thought that definition *d* seems to have sufficient information for an integer definition: *"It is probably enough, negative, positive, it's correct." [Yeterli heralde. 0, negatif, pozitif, doğru.]*

In contrast to the agreement of the participants above, PD realized that the reference point for ascending and descending was not mentioned; therefore, he disagreed with the definition: "*Now, it is possible that 1,1; 2,1 increase one by one. I would state that it mistakes may occur." [Şimdi bu durumda da şöyle bir şey olabilir, 1,1; 2,1 birer birer artıyor, yanlışlık olur diye belirtirim.]*

Results of this part of the research indicate that participants tend to accept the definition though the definition does not include the necessary information. In other words, participants were expected to recognize the missing information regarding the starting point, which is zero.

Pre-service teachers' ideas about definition d: "Integers are rational numbers that do not have fractional part."

This definition says that integers are a form of rational numbers in which the fractional part does not exist. Participants were expected to explain the written forms of rational numbers having a fractional component. This definition is problematic as long as the fractional part is not clear.

Fractional part means the nonnegative part of a number which comes after the decimal point For example, $\frac{5}{4}$ is a number which is written as1+ $\frac{1}{4}$ or 1.25. In this number, $\frac{1}{4}$ or 0.25 are fractional parts of the numbers. As seen from the examples, the fractional part of a number does not have to be a fraction. When an integer is written as $\frac{8}{2}$ or 4.0, the integer has a fractional part as well.

For example, PA does not agree with the definition:

No. They (Integers) have a fractional part. I can write as such (as if they have a fractional part). We say that rational numbers are things, like…the proportion of two integers, I can write in this way. Rational numbers are, well, they have a numerator and denominator, they are written as $\frac{a}{b}$*, … thus, when this part ("which do not have fractional part") is not considered, it is meaningless. That is, then its having a rational number feature disappears. When this part (b) is omitted…*

[Yoo.. Kesir kısmı var, yazabilirim yani. Zaten rasyonel sayıları da şey diyoruz ya, hani, iki tamsayının oranı, yazabilirim yani. Rasyonel sayı şey zaten, payı ve paydası olan, a/b şeklinde yazılan, yani burayı (b) atınca çok manasız bir… o zaman rasyonel sayı olma özelliği gidiyor yani. Burayı atınca…]

Another participant, PB, was confused about what 'fraction part' refers to in the definition. She stated:

So, is it intended to say that integers do not have denominators? It (Fractional part) might be numbers which have no remaining numbers. Generally, rational numbers are in $\frac{a}{b}$ *form. Instead of this (* $\frac{a}{b}$ *form), he/she (the student) probably intended to say integers; we mentioned 2/1 earlier. We can in fact write an integer in this way, but when written in this way as 2, the number does not have a fractional part. Since he/she (the student) did not think of this, he/she defined it (integer) in this way.*

[Yani paydası olmayan mı demek istiyor acaba. Küsurat kısmı olabilir. Normalde rasyonel sayılar a/b şeklinde. Yani bunun yerine, heralde tamsayı demek istemiş, hani biraz önce 2/1 demiştik, aslında tamsayıyı bu şekilde yazabiliyoruz ama, bu şekilde, 2, yazdığımızda kesir kısmı yok. Bunu düşünemediği için, bu şekilde bir tanım yapmış.]

Another participant, PC agreed with the definition. The participant considered that the definition might help to relate integers and rational numbers. However, ‘fractional part’ caused confusion in his mind:

Here (in this definition) it is like…there is a whole part, but not a fractional part. Rational numbers already include integers. It is correct, in fact. After teaching integers and rational numbers, it may be used for reasoning. However, while teaching rational numbers, it is unnecessary for such a purpose (to use for reasoning). It seems to be correct. But I do not understand what 'not fractional part' means. I can't exactly understand what it means. 'Not having a fractional part' means that they (integers) are rational numbers which are whole numbers). If that was intended, it is allright.

[Burda şey gibi geliyor, tam kısmı olacak ama kesir olmayacak. Rasyonel sayılar tamsayıları kapsıyor zaten. Doğru aslında. Tamsayıları öğrettikten sonra rasyonel sayıları öğrettikten sonra çıkarım yapmak için olabilir. Ama rasyonel sayıları öğretirken bu türlü bir amaçla gerekmez bence. Doğru gibi duruyor. Ama kesir kısmı olmayan derken onu anlamadım, tam olarak kestiremiyorum. Kesir kısmı olmayan, yani tam olan rasyonel sayılardır. Onu kastediyorsa tamam.]

PC seems to say that numbers which have no fractional part means that the numbers are not written in $\frac{a}{b}$ form.

PD agreed with the definition, and said: *"I would probably say that it is correct. For example, is 5/1 a fraction?" [Buna doğru derdim heralde. 5/1 kesir mi mesela.].* PD is confused whether 5/1 is a fraction or not.

In the following comment of PD, however, the participant was also confused about what 'fractional part' is. Moreover, the participant considers 'remaining part' in relation to 'fraction part': "*Not having a fractional component part in the first question. Yes I would ask that, what that means. Let me answer based on that part ." [İlk sorudaki küsuratı olmayan kısmı. Evet onu sorardım, ne demek istediğini, ona göre cevap vereyim.]* The participant also pays attention to what elementary students intend by saying *fraction part*.

Research results show that those participants are confused about what 'fractional part' means. However, they suggested several possibilities about what it means.

Pre-service teachers' ideas about definition e: "Integers are all rational numbers with denominator 1."

Definition *e* is the only clear definition in the quoted definitions. Participants were expected to interpret definition *e* as a correct definition of integers. Their responses are as follows:

PA agreed, though being unsure: "*It might be possible, but…*" [*Olabilir aslında da…]*

PB also agrees with the definition but has few concerns about zero: "*I think it is right, but zero, 0/1, 0, okay." [Doğru bence. ama sıfır, 0/1, 0, tamam.]*

PC also agrees with the definition: "*That's good. This is the correct form (of the definition). You write rational number in a/b form. If you write 1 for the value of b, you get it right." [Güzel. Bu haliyle doğru. Rasyonel sayıyı a/b olarak yazarsın. b yerine 1 yazarsak, doğru çıkıyor.]*

PD did not agree with the definition because integers are defined with the concept of integers with the definition:

> While *teaching that definition to students, if students ask me what the numerator is when the denominator equals to 1 and I give the response as an integer, If I would be adding a definition to the definition. For example, if I say 5.7/1, though an extreme example, a student would ask whether this ıs an integer also? While explaining that 5.7 is not an integer, I would try to define integers again. Thus, it is necessary to give a complete definition of integers…*
>
> *Şimdi bu tanımı verebilmek için paydası 1 olanın payı ne diye sorduğunda çocuk, tamsayı dersem tanıma yine tanım katmış olurum. Yani atıyorum, uç örnek olacak belki ama 5,7/1 desem mesela bu da mı tamsayı der? 5,7 tamsayı değil ki, derken, açıklamasını yaparken, yine bir tamsayı tanımı vermeye çalışırım… o yüzden, bunun yerine tam bir tamsayı tanımı….*

In other words, this definition is not the definition of integers either because the definition of rational numbers requires knowing integers. Similar to PD, some participants attempted to define integers with integers. According to P1 and P19, integers are: "*Negative, positive integers and zero." [Negatif ve pozitif tamsayılar ve 0.]*

Most of the participants agree with the definition; however, one of them does not agree. According to the participant, the definition is not acceptable since it is not consistent with learning consequences of number sets.

According to the participants' responses in the interview, their agreement statuses are seen in Table 4.8 below:

Table 4.8

Frequency of agreement statuses of participants

	Correctness of the definitions	**Agreement**	**Disagreement**	**Not sure**
Definition a	Appropriate but not rigorous example of integer definition	-	2	2
Definition b	Inappropriate example of integer definition	-	4	-
Definition c	Inappropriate example of integer definition	3	1	-
Definition d	Appropriate but not rigorous example of integer definition	2	1	1
Definition e	Appropriate rigorous example of integer definition	3	1	-

Except for definition *e*, the other definitions need some clarity. Table 4.8 shows that for definition *a*, half of the participants have disagreements and the other half are not sure about the accuracy of the definition. All of the participants disagreed with definition *b*. Three of the participants agreed that definition *c* is an accurate definition statement for integers. However, one of them disagreed with the definition. For definition *d*, two participants agreed that it is true, one participant disagreed, and one participant was not sure about the accuracy. For definition *e*, three participants agreed that it was correct, and one participant considered that it was not.

In conclusion, participants in this part had difficulty in identifying correctness of the definitions of integers when they were given quoted definitions of integers.

4.2 THE NATURE OF PARTICIPANTS' PEDAGOGICAL CONTENT KNOWLEDGE ABOUT ELEMENTARY STUDENTS' MISCONCEPTIONS and ERRORS

In this part, the nature of participants' views about elementary students' misconceptions and errors in defining integers were examined. The participants of the study suggested several reasons why elementary students have misconceptions and make errors in definition of integers.

4.2.1 The nature of participants' views on elementary students' misconceptions and errors in definition of integers

Participants were asked two open-ended questions which possible misconceptions and/or errors elementary students may have while defining integers. The following questions were asked to examine the participants' views on elementary students' misconceptions and errors in definition of integers: *While explaining integers, the teacher asked the students: "How can you define integers?" a) What might be misconceptions / errors of the definition students may have? b) What might be the reasons for the misconceptions / errors?*

Data revealed that pre-service teachers could identify several misconceptions and errors that elementary students might have in definition of integers. A list of these misconceptions and errors are presented in Table 4.9. In the following table, elementary students' possible misconceptions and errors are presented with some examples of participants' views.

Table 4.9

Misconceptions and errors in definition of integers

Misconceptions and errors		***Some examples of participants' views on misconceptions and errors***
zero is the smallest number	1	*It is thought that the smallest number is zero. Naturally, they have difficulty changing this schema. [En küçük sayının sıfır olduğu düşünülüyor. Bu şemayı değiştirmekte haliyle zorlanıyorlar.]*
zero means nothing	1	*If they think that zero is nullity, they may have an idea that there is nothing smaller than nullity. [Sıfırı yokluk olarak düşünüyorlarsa yok olan bir şeyden daha küçük ne olabilir düşüncesi oluşabilir.]*
zero is not an integer	7	*They may not include zero. [Sıfırı dahil etmeyebilirler.]*
zero is a positive integer	1	*While placing zero into the positive or negative category, they may place it in the positive category. [Sıfır negatif ya da pozitif olarak sınıflarken pozitife dahil edebilirler.]*
There is not an existing relationship between number sets	1	*Students may not realize that numbers are a single set. They may think that integers, rational numbers, natural numbers, etc. are separate sets. [Öğrenciler sayıların tek bir küme olduğunu fark etmeyebilir. Tamsayıları, rasyonel sayıları, doğal sayıları vs. ayrı ayrı kümeler olarak düşünebilirler.]*
	2	*They may not know numbers as separate sets. [Sayıları tam olarak küme küme bilmemelerinden.]*
Integers are composed of positive integers	8	*They may take into account only the positive integers. [Sadece pozitif tamsayıları göz önüne alabilirler.]*
	1	*They may define integers as a group of numbers consisting of only 0 and positive integer numbers. [Tamsayıyı sadece 0 ve pozitif sayılardan oluşan bir sayılar grubu olarak tanımlayabilirler.]*
	2	*They never take into consideration the negatives of integers. [Tamsayıların negatiflerini düşünmeyebilir.]*
Integers are whole numbers	1	*They may confuse (integers with) whole numbers [Doğal sayılarla karıştırıyor olabilir.]*
	1	*They may think of whole numbers when they say integers. [Tamsayılar deyince, doğal sayıları düşündükleri için olabilir.]*

Participants think that elementary students make errors and have problems in the definition of integers because of they need to have further subject matter knowledge on the concept of zero. According to them, elementary students may have misconceptions and errors in their minds due to the lack of knowledge regarding the zero. These misconceptions are as follows: *zero is the smallest number, zero means nothing, zero is not an integer, zero is a positive integer.* What's more, they may lack knowledge on number sets. This may cause the misconception and error which says *there is not an existing relationship between number sets.* Finally, elementary students may think that *integers are composed of positive integers,* or *integers are whole numbers.*

Misconception 1: Zero is the smallest number.

Participants suggested the idea that elementary students may think that there is no number below zero. Thus, they find it hard to construct negative integers in their minds. For example, one of the participants said, "it is thought that the smallest number is zero. Naturally, they find it hard to change the form of this schema."[En küçük sayının sıfır olduğu düşünülüyor. Bu şemayı değiştirmekte haliyle zorlanıyorlar.] (n=1)

Misconception 2: Zero means nothing.

Elementary students may avoid considering negative integers. They tend to think of zero and numbers greater than zero since for them there cannot be numbers less than zero; which means nothing. One participant suggested the misconception as follows: "If they think that zero is nullity, they may hold the idea that there is nothing smaller than nullity" [Sıfırı yokluk olarak düşünüyorlarsa yok olan bir şeyden daha küçük ne olabilir düşüncesi oluşabilir.] (n=1)

Misconception 3: Zero is not an integer.

According to some of the participants, zero seems to be a difficult concept for elementary students to deal with. It creates problems when zero is excluded from the integer set. Seven of the participants suggested the misconceptions stating that "they may not include zero." [Sıfırı dahil etmeyebilirler.]

Misconception 4: Zero is a positive integer.

Zero might be thought as a positive integer by elementary students. This misconception was suggested by one participant as follows: "While placing zero into the positive or negative category, zero is placed into the positive category." [Sıfır negatif ya da pozitif olarak sınıflarken pozitife dahil edebilirler.] (n=1)

These misconceptions and errors until this point reveal that participants suggest misconceptions and errors of students regarding the zero. According to the participants, elementary students need to be clarified in the misconceptions and errors concerning the zero.

The participants also consider that elementary students make errors and have misconceptions in definitions of integers because of the need to have further subject matter knowledge on positive integers and the relationship between number sets. These misconceptions and errors are illustrated in the following examples:

Misconception 5: There is not existing relationship among number sets

According to participants, elementary students would think that there is not an existing relationship among number sets. In other words, number sets are separate sets and there is no relationship among them. For example, one participant said that "students may not realize that numbers are a single set. They may think that integers, rational numbers, natural numbers, etc. are separate sets." [Öğrenciler sayıların tek bir küme olduğunu fark etmeyebilir. Tamsayıları, rasyonel sayıları, doğal sayıları vs. ayrı ayrı kümeler olarak düşünebilirler.] (n=1)

Some participants emphasized that although elementary students have knowledge about number sets, it may be confusing for students. The following examples of this point will better clarify this point: "*There are many number sets, they may be confusing for students." [Çok fazla sayı sistemi var, karıştırıyor olabilirler.] n=1*

Misconception 6: Integers are concluded from positive integers

According to participants, elementary students would think that integers consist of positive integers. In addition to this, participants predict that elementary students may think that negative numbers do not belong to the domain of integers. Eight of the participants expressed that "they may take into account only the positive integers." [Sadece pozitif tamsayıları göz önüne alabilirler.] (n=8)

Misconception 7: Integers are whole numbers

Participants specify integers to be whole number sets as follows: *"They mean whole numbers when they say integers." [Tamsayılar deyince, doğal sayıları düşündükleri için olabilir.] (n=1)*

As it is seen, participants suggested several misconceptions of elementary students regarding their definition of integers. In the following section, the reasons underlying elementary students’ misconceptions and errors will be described.

4.2.2 The nature of participants’ views on the reasons underlying elementary students’ misconceptions and errors in definition of integers

Participants were asked about the reasons of elementary students’ possible difficulties in defining integers. The data collected showed that pre-service teachers expressed many reasons for these misconceptions and errors and also there are many subreasons which underlay the causes of these reasons.

According to the participants of the study, elementary students may have misconceptions and errors in definition of integers. There are three reasons for this: "negative transfer of former knowledge on number sets", "students’ general insufficiency", and "the teaching approach".

Reason 1: Negative transfer of former knowledge on number sets

The term *negative transfer* has come to be used in this study to mean transferring former knowledge that entails gaps or incorrect knowledge to new knowledge. In other words, previously learned knowledge blocks learning new knowledge. Although former knowledge seems to take advantage of students to understand new knowledge to learn, it may not be practical when students transfer the knowledge with misconception(s) and error(s). Two subreasons were suggested by the participants to understand why negative transfer happens: (1) being un/familiar with number sets and (2) problematic prior knowledge.

Subreason 1: Being un/familiar with number sets

The term "being familiar with…" has been applied to situations where students tend to use their previously learned knowledge which they are familiar to gain new knowledge. The importance of the former knowledge is that elementary students have accurate former knowledge. According to some participants, however, they have difficulty in using the former knowledge for transferring new knowledge.

Specifically, the participants suggested the subreason that elementary students are familiar with positive integers because they spend much time on these numbers. In other words, elementary students tend to consider the number concept as limited by whole numbers. As a result of this, they have difficulties in negative integers. P32, P13, P18, and P5 made the following comment: "*students may find it difficult to use negative numbers because they have been familiar with whole numbers since their childhood." [Öğrenciler doğal sayılara küçük yaştan itibaren aşina oldukları için negatif sayıları kabullenmekte* zorlanabilirler*."]*

What's more, they tend to make operations with positive numbers; therefore, they may have resistance to accept negative numbers: "s*tudents find it difficult to accept negative numbers since they made operations with positive numbers until their 6th grade." ["6. sınıfa kadar hep pozitif sayılarla işlem yaptıkları için negatif sayıları kabullenmek onlar için zor olabilir."] (P14)*

In addition to these responses, P22 emphasized that elementary students are familiar with other number sets: *"(the reason is that) they only know whole numbers, fractions, and counting numbers." [Şimdiye kadar sadece doğal sayılar, kesirler ve sayma saylarını biliyor olmalarından."]*

However, P22 fails to consider that elementary students learn fractions before the concept of integers. According to TTKB (2009), before elementary students are taught integers, they already know counting numbers and whole numbers, not fractions.

Responses of the participants show that elementary students are familiar with positive integers. However, they are not familiar with negative numbers. The reasons why the familiarity happens may be divided into three groups: spending much time on positive numbers, considering the concept of number as limited by whole numbers or making operations with positive numbers for a long time.

The term "being unfamiliar with…" has been applied to situations where elementary students have a resistance to receive new knowledge on integers. In other words, elementary students may have difficulty in defining integers as they are unfamiliar with the knowledge required to learn related to the concept of integers.

Specifically, elementary students might not be familiar with integers. According to P6 and P30, "*they may be encountering the concept of integers for the first time." [Tamsayılar konusuyla ilk defa karşılaşıyor olabilir.]*

Or the unfamiliarity of students may be derived from being unfamiliar with negative numbers: "*they may consider negative numbers as a separate number set because they encounter the concept of integers for the first time in their elementary grades." [iköğretimde negatif sayıları yeni gördüklerinden onları tamsayılardan ayrı bir sınıf/küme gibi düşünebilirler.] (P15).*

Responses of the participants show that elementary students are not familiar with the concept of integers and negative numbers. The reasons why the unfamiliarity happens may be divided into two groups: needing acceptance of integers as a new number set and of negative numbers as integers.

Subreason 2: Problematic Prior Knowledge on Number Sets

The term "problematic prior knowledge" has been applied to situations where elementary students' prior knowledge which contains information gaps and disconnections create difficulties in transferring their old knowledge to new one.

Specifically, prior knowledge of elementary students which contains gaps or disconnections in number sets seems to block learning new knowledge on integers: "*the reason may be that prior concepts such as whole numbers, counting numbers* *may not be comprehended by students." [Daha önceki doğal sayılar, sayma sayıları gibi sayı kümelerinin tanımlarının tam olarak oturmamış olması olabilir.] (P30).*

In addition to number sets, participants think that inaccurate knowledge, rejected knowledge, or confusion with former knowledge on integers may create difficulties in the definition of integers: "*The reason may be that (students) have to accept knowledge which was not accepted before." [Önceden kabul edilmeyen bilgileri, kabul etmek zorunda kalmaları da olabilir.] (P33)*

"The reason may be that their previously learned knowledge may be incorrect" [Daha önceden öğrendikleri yanlış bilgiler olabilir.] (P37)

In conclusion, students' mathematics knowledge before learning integers is so important that it affects their knowledge on definition of integers as participants said: "*... and students' content knowledge is important."[…ve de çocukların matematik bilgisi önemlidir.] (P28)*

"Knowledge learned prior to integers may create obstacles for students." [Tamsayılara kadar öğrendikleri, tamsayılar konusunda engel oluşturabilir.] (P11)

"Prior knowledge may be the reason." [Geçmiş bilgilerinden kaynaklanıyor olabilir.] (P32)

The results reveal that according to participants, if elementary students transfer what they already know to knowledge they will learn incorrectly, it would be difficult for students to provide correct explanations for definition of integers.

P20 and P34 mentioned that the reason why elementary students cannot define integers correctly is lacking sufficient content knowledge: *"Errors may be due to lack of knowledge or due to teacher errors." [Bilgi eksikliğinden ya da öğretmen hatalarından kaynaklı olabilir.] (P20)*

"(It is because) they do not know; if we make them think (about their errors), they will easily learn." [Bilmediklerinden, eğer devamlı düşünmelerini sağlarsak kolaylıkla öğreneceklerdir.] (P34)

According to the participants, if students are un/familiar with number sets and if they have problematic prior knowledge on number sets, they may have difficulty in describing integers.

Reason 2: Students' general insufficiency

Participants of the study explain that what teachers' tell students might be misunderstood, misinterpreted, or forgotten by the students. Students may also have lack of conceptual understandings about what teachers' tell them. In other words, students may misunderstand or misinterpret what teachers tell them because of their own conducts. Examples of this point are: "*(students) may miss critical points related to their lessons." [Derslerde konuyla ilgili kritik bir kaç nokta kaçırmış olabilir.] (P20), "they may make up the topic by not listening to the topic (explained by the teacher).[Konuyu anlatırken iyi dinlemeyip kafalarından uydurmuş olabilirler.](P35), "They may interpret according to their own ideas." [Kafalarında o şekilde yorumlayabilir.] (P32), "It may derive from the fact that they may know it wrongly or they may not be thinking carefully." [Yanlış biliyor olmaktan ya da dikkatli düşünmemekten kaynaklanıyor olabilir.] (P24)*

In addition, because students have difficulty in remembering some knowledge regarding integers, integers may be considered to be a difficult concept to define. Participants think that forgetting some parts of subject matter knowledge on integers may result in making errors or facing problems in the definition of integers. Examples of this point are stated below:

"Zero may be forgotten." [0, (sıfırı) unutabilirler*.] (P6)*

"Negative integers may be forgotten." [Negatif tam sayılar unutulabilir.] (P4, P5, P6, P9, P13, P17, P30)

"They define (integers) as normal numbers and they may forget minus (negative) numbers." [Normal sayılar diye tanımlar ve eksi sayıları unutabilirler.] (P8)

It shows that elementary students might have misconceptions and errors because of their own conducts.

Reason 3: The Teaching Approach

The last reason suggested by the participants for the reasons of misconceptions and errors of defining integers is the "teaching approach". Here; it is referring to teachers' lack of subject matter knowledge or pedagogical content knowledge; which results in incorrect definitions students make. According to the pre-service teachers, the way the integer concept is taught to students may play a role in their misconceptions and errors. The reason was mentioned in general by P20 and P7:

"Teachers' errors may be one reason for (misconceptions or errors in definition of integers)" […öğretmen hatalarından kaynaklı olabilir] (P20) or "(students) may interpret their teachers' explanations in a wrong way." [Öğretmenin yanlış anlatımından / ifadesinden böyle bir sonuca varmış olabilirler.] (P7)

According to the participants, this reason has two subreasons: uncorrected explanations by teachers and teaching strategies.

Subreason 1: Uncorrected explanations

According to the participants, elementary students may encounter explanations of teachers which include inaccurate knowledge. The explanations may include inaccurate statements, formulas, or instruction on the concept of integers with many missing parts, or examples which cause students to misunderstand.

The participant predicts that some of uncorrected explanations might relate to zero: *"This misconception is derived from the explanation that the lowest number is zero and there are no numbers less than zero." [Bu yanılgı, öğrencilere bu yaşa gelene kadar en küçük sayı 0, sıfırdan küçük sayı olmaz (denilmesinden kaynaklanır)](P26)* or *"it is said that numbers start from zero. Numbers less than zero are mentioned late." [Sayıların hep 0'dan başlamasının söylenmesinden. Sıfırdan küçük sayıların varlığından geç bahsedilmesinden…] (P29)*

Some are related to teachers' instruction with many missing parts: "*(they derive from) definitions that include wrong or many missing parts." [Tanımların eksik ya da yanlış öğretilmesinden.] (P29)* or *(students) have difficultiy in giving meaning to* *negative numbers because of (teachers)' explanations such as you can not subtract 3 from 2." [2'den 3 çıkmaz gibi açıklamalar yapıldığı için öğrencilerin tamsayılarda negatif sayıları anlamlandırmaları zor oluyor olabilir.] (P27).*

According to P3, given examples may cause students to misunderstand: "*students may think that integers consist of negative integers. The reason is that they are given examples such as 'how could you express the height below the sea'." [Çünkü verilen örneklerde 'denizin altındaki yüksekliği nasıl belirtiriz?' şeklindeki sorular sorulduğu için, öğrenci tamsayının sadece negatif tamsayılar olduğunu düşünebilir.]* This subreason explains that teachers' explanations may cause misconceptions and errors of elementary students.

Subreason 2: Teaching Strategies

According to participants, strategies used in lessons to teach integers may result in misconceptions and errors in definition of integers. P28 touched upon the subreason generally saying that "*strategies in teaching are important." [Öğretmenin anlatım şekli önemlidir.]*

P21 believes that elementary students are taught procedurally rather than conceptually, saying "*making students memorize definitions may lead to misconceptions. They may have such misconceptions because they have not learned the concept (of integers) conceptually." [Tanımların ezberletilmesi de öğrencilerin kavram yanılgısına düşmesine sebep olur. Konuyu kavramsal olarak öğrenmedikleri için bu tür yanılgılara sahip olabilirler.]*

What's more, teachers do not make any effort to support teaching of knowledge with the help of visual aids: "*showing (number sets) on a number line can be more effective for (students). I mean, teachers teach (the concept of integers) like a formula. They do not use teaching methods or visual aids which help (students) to make sense of (what is taught)." [Sayı doğrusu kullanılarak bu sayılar bir arada gösterilse daha kalıcı olabilir. Yani öğretmenler sadece formül gibi öğretiyorlar. Görsel olarak, ya da anlamlandırmaya yardımcı olacak öğretim kullanmıyorlar.] (P25)*

In addition, teachers may not give opportunity to students to think critically when students are learning integers: "*while numbers are classified, students may not be made aware of of the differences between the numbers. The teacher may not have given the necessary importance to the classifications of definitions." [Sayıların sınıflandırılması yapılırken aralarındaki farklar, fark ettirilmemiş olabilir. Öğretmen, tanımları sınıflandırılmaları üzerinde gereken önemi vermemiş olabilir.] (P8)* or "*students learn only by definition. I wish (students are told) how the number sets (whole numbers, rational numbers, integers) intersect each other." [Öğrenciler sadece tanım olarak öğreniyorlar. Bunların (doğal sayıların, rasyonel, tamsayıların) küme olarak birbiriyle nasıl kesiştiği anlatılsa..](P25)*

One of participants also predicts that teachers repeat the same explanations over and over again and ask elementary students certain question types on the subject learned. Students may have misconceptions or make errors when encountered with different types of questions:

> *Different kinds of questions are not asked to (students). Because if the student solves the same kinds of questions, they may have a particular point of view on the issue, (and then) they never have a broader perspective on the issue. In order to overcome this situation, different questions which do not cause misunderstandings should be asked on the subject.*
>
> *[Farklı tür sorular sormuyor olunmasından. Çünkü öğrenci hep aynı tür sorular çözerse konuyla ilgili belirli bir bakış açısı edinip daha geniş bir bakış açısıyla düşünmeyebilir. Bu durumun üstesinden gelmek için, konuyla ilgili farklı yanlış anlaşılmaların oluşmamasını sağlayacak sorular sorulmalıdır.] (P10)*

These findings show that teaching strategies are so important for students as they might cause misconceptions and errors. According to the participants, teaching strategies may not be used effectively by teachers; therefore, elementary students may learn subjects with many confusions or gaps in their minds.

To sum up the responses, why teaching strategies are not effectively used by teachers may be grouped as follows:

- Preferring to teach procedurally instead of conceptually
- Not making effort to support teaching with visual aids
- Not providing opportunities for critical thinking
- Using only certain types of questions

In addition to these responses, two participants say that "*the definition may not be comprehended (by students)" [Tanımı kavrayamamış olabilir.] (P26)* or "*negative numbers may not be comprehended (by students)" [Negatif sayıları tam anlayamama.] (P23).* The situations above may be derived from students' work or their teachers may not have succeeded in conceptualizing the definition for them. Although the responses seem to be categorized as teachers, it is not so easy to say that. Therefore, these responses were not categorized since possible reasons were not determined.

Table 4.10

Possible reasons for not defining integers

Reasons (N=11)	Sub reasons	Frequency	Examples
Negative transfer of former knowledge on number sets	Being un/familiar with number sets	4	*Students find it difficult to accept negative numbers as they are familiar with whole numbers.*
		1	*They may consider negative numbers as a separate number set as they encountered the concept of integers for the first time in their elementary grades.*
	Problematic Prior Knowledge about number sets	1	*The reason may be that prior concepts such as whole numbers, counting numbers may not be comprehended by students.*
		1	*The reason may be that their previously learned information may be incorrect.*
Students' general insufficiency		1	*They may interpret according to their own ideas.*
		1	*Zero may be forgotten.*
Teaching approach	Uncorrected explanations	1	*The misconception is derived from the explanation that the lowest number is zero and there are no numbers less than zero.*
	Teaching strategies	1	*Students learn only by definition. I wish (students are told) how the number sets (whole numbers, rational numbers, integers) intersect each other.*

Briefly, there are three main reasons for misconceptions and errors of not defining integers. The results from the nature of participants' views on integers' definition show that pre-service elementary mathematics teachers' responses help to understand reasons underlying misconceptions or errors in defining integers.

CHAPTER V

CONCLUSION, DISCUSSION, AND IMPLICATION

The purpose of the study was to examine the nature of pre-service mathematics teachers' knowledge about definitions of integers and their knowledge about elementary students' misconceptions and errors on definition of integers. In this chapter, research findings, conclusions, and implications/recommendations for further studies will be addressed.

Research findings, and conclusions based on the research findings and the previous literature will be mentioned regarding research questions.

5.1 The nature of pre-service teachers' conceptions about definitions of integers

Defining integers might be critical during teaching process. Research has shown that not all pre-service teachers are able to make proper explanations when defining integers. As it is explicitly inferred from recent studies, the concept of integers is challenging to be described by the majority of math teachers and elementary students (Smith, 2002; Köroğlu & Yeşildere, 2004; Melezoğlu, 2005; Körükçü, 2008; Akyüz et. al., 2012; İşgüden, 2008; Spang, 2009; Ercan, 2010). Although the correctness and clarity of definitions have an important role in learning new mathematical concepts, the research findings revealed that most responses of pre-service teachers use imprecise terminology in defining integers.

A good example of this is the following definition: *"A number set which includes whole numbers and negative numbers."* [Doğal sayılar ve negatif sayılardan oluşan sayı kümesidir.] (P30). The definition is a typical example of imprecise terminology use made by pre-service teachers when defining the concept of integers. As it is seen in the definition, the pre-service teacher defines the concept of integers as a combination of whole numbers and negative numbers although the concept is supposed to be defined as the combination of whole numbers and negative whole numbers. This example explicitly shows that pre-service teachers are not careful enough to pick the appropriate wording when defining integers and they are supposed to put more effort into defining the concept of integers by using the precise terminology.

Mathematicians consider that a definition should have necessary and sufficient conditions, but they differ in the aspect of being necessary (Vinner, 1991; Zaskis & Leikin, 2008). Some of them claim that a mathematical definition has to be minimal while some others think that the definitions with unnecessary conditions might be accepted as appropriate definitions as well. Accordingly, the definition below might be both appropriate and not appropriate depending on the perspectives of mathematicians regarding the concept of minimality.

"*Integers include rational numbers of which denominator equals to one and they are from* -∞ *to* +∞*." [Tam sayılar* -∞ *dan* +∞*'a paydası 1 olan bütün rasyonel sayıları içerir.].*

For instance, boundary limits were used for integers, although indicating the limit was not necessary information. In this definition, "they are from -∞ to +∞" is unnecessary information so that rational numbers are numbers from -∞ to +∞. In other words, indicating the boundary limits are not necessary information so that the information can be inferred from being rational numbers. According to Van Dormolen and Zaslavsky (2003), additional information need to be removed from definitions or descriptions of integers. In other words, a definition needs to be economic. The result of this study indicates that pre-service teachers focus on defining integers regardless of the details they give are necessary or not. For example, in the following definition, the boundary limits of integers are not necessary: "*integers include rational numbers with denominator equals to one and they are from* -∞ *to* +∞*." [Tam sayılar* -∞ *dan* +∞*'a paydası 1 olan bütün rasyonel sayıları içerir.].* The criterion of being a definition economic or not should be used in method courses as a pedagogical advantage so that it provides understanding pre-service teachers' concept images (Van Dormolen & Zaslavsky, 2003).

There is a variety of definitions made for integers in dictionaries of mathematics and textbooks and what is seen in these resources is that the definitions of integers are based on one of the three concepts which are counting numbers, rational numbers, and whole numbers. Except from these three concepts, some participants constructed the concept of integers. For example, one of the pre-service teachers defined integers as following: *Integers are numbers which are found by adding 1 to and subtracting 1 from numbers which start from zero, and indicate distance from zero in number line.* It might be said that pre-service teachers created their own definitions and their definitions are man-made (Vinner, 1991).

Some of participants, however, support the idea that the concept of integers is defined by previously defined number sets which are whole numbers and counting numbers. For instance, *"if we combine whole numbers and their negatives, we have integers"(n=4)* or *"integers are sets which include counting numbers, negatives of counting numbers, and zero".*

Although some of participants defined the concept of integers based on previously learned concepts, some others defined the concept based on concept to be learned later. In this study, six participants defined the concept of integers by the concept of rational numbers which is taught following the concept of whole number and the concept of counting number in reference to TTKB (2005). In this way, it might be said that the following definition is not accepted definition so that integer is attempted to be defined by the concept of rational number which is learned after the concept of integer was taught (TTKB, 2005): *"All the rational numbers which have denominators equal to 1 are integers."* According to Zaskis and Leikin (2008), only previously learned concept can be used in definitions. However, the concept of rational numbers is latter concept in the elementary Turkish mathematics curriculum.

According to Zaskis and Leikin (2008), one of the main principles of defining mathematical concepts suggested by mathematicians is to make the definitions by using previously learned concepts, without including any unknown words or phrases for the students. Accordingly, the definition made by P1 and P19 which is *'negative and positive integers and zero'* cannot be accepted as a mathematical definition as they define the concept of integers by using the concept itself and elementary students might highly possibly have difficulty comprehending the concept of integers as they are not familiar with the concept from previous lessons.

There are several possible explanations for these results. Pre-service teachers may describe integers considering their all knowledge about number sets, they may be not careful about the consequences of the number sets in the curriculum, or they may not know that they need to be careful about the consequences. It is possible that pre-service teachers may need to know what a definition of a mathematical concept requires to be an appropriate description and they may need to know prerequisite knowledge about mathematical concepts which are related each other to define a mathematical concept.

16 responses of definitions of integers made by pre-service elementary mathematics teachers include definitions if integers based on representing. Most of the definitions are related to representing integers with set of numbers of integers: {-∞…-2, -1, 0, 1, 2, 3…+∞}. The definition was appropriate and rigorous examples of definition of integers. However, it is away from being minimality because of the usage of boundary limits which is not necessary. The following definition is inappropriate so as to the concept of integers was defined with the integer concept: Z=$Z^{+} \cup \{0\} \cup Z^{-}$. It might be said that most of the pre-service teachers pretend defining integer as a formal or informal statement.

We now turn to another dimension of the study where pre-service teachers' ideas on quoted definitions of integers were described.

5.2 The nature of participants' ideas on quoted definitions of integers

In this part, pre-service teachers' knowledge on quoted definitions is discussed. In other words, research findings regarding pre-service teachers' knowledge in terms of quoted definitions of integers will be discussed.

Findings of the study revealed that pre-service teachers have suggested many descriptions in defining integers. Furthermore, when pre-service teachers were asked to evaluate the correctness of quoted definitions, they have difficulty correcting them. Although pre-service teachers considered what problematic words are and how the words need to be corrected regarding their various possible meanings, and they mostly are aware of the imprecise terminology shown in the definitions, they pretend saying clearly that the quoted definitions are correct or not. A possible explanation for this might be that pre-service teachers are not sure about what the quoted definitions intend to say.

However, none of the participants preferred to examine the definitions with axiomatic structure (Van Dormolen & Zaslavsky, 2003). For instance, concept of rational numbers is structured in the curriculum (TTKB, 2005) after integers are taught to students. In this way, definition of integers based on concept of rational numbers is not appropriate for axiomatic structure of number sets. The reason for this is not clear but it may have something to do with the learning consequences of number sets. It can be suggested that pre-service teachers should pay attention to hierarchy of topics, specifically number sets in this case.

An interview question which includes quoted definitions of integers was prepared based on pre-service teachers' responses in open-ended questions and related literature. The way I followed to prepare the questions in the interview might be used to design method courses. List of students' responses or solutions may guide instructors about the way to be followed. The way not only provides the instructors to understand pre-service teachers' background knowledge, it also provides the teachers with the chance to discuss mathematical ideas and to make sense of examining mathematical language used. It is a common pedagogical strategy in which teachers ask students to respond to ideas of their classmates (Lampert, 1990; Zaskis & Leikin, 2008).

5.3 The nature of participants' views on misconceptions and errors of elementary students

In this part, pre-service mathematics teachers' knowledge about elementary students' misconceptions and errors regarding definition of integers will be discussed.

Research results show that pre-service teachers suggested several misconceptions and errors which elementary students may have: *zero is the smallest number*, *zero means nothing*, *zero is not an integer*, *zero is a positive integer*, *there is not an* *existing relationship between number sets, and integers are concluded from positive integers.*

The research findings revealed the fact that pre-service teachers have suggested similar misconceptions and errors in the literature and they are aware of elementary students' possible misconceptions and errors regarding defining integers. Accordingly, pre-service teachers have knowledge about students' misconceptions and errors regarding defining integers. Most of misconceptions and errors participants suggested are related to the concept of zero. Pre-service teachers supported the idea that zero may be misinterpreted by elementary students. For instance, "zero is the least number", "zero means nothing", "zero is a positive integer", and "zero is not an integer". The results of this research support the idea that elementary students have difficulty in zero (Köroğlu & Yeşildere, 2004; Steiner, 2009; Ercan, 2010; Ryan & William, 2007; Schuter & Anderson, 2005; Musser et. al., 2003).

The research findings revealed the fact that pre-service teachers pay more attention to zero as misconceptions and errors when elementary students are defining integers. Contrary to expectations, two participants did not emphasize zero separated from positive and negative integers: "*integer is a set which is composed of negative and positive integers." [Negatif ve pozitif tamsayıların birleşmesiyle oluşan kümedir.].* The reason for this is not clear but in my opinion, a possible explanation for this might be that zero is thought as a positive integer. It would be interesting to assess how the role of overcoming these misconceptions and errors about zero is related to elementary students' understanding of concept of integers.

One of the significant findings to emerge from this study is that elementary students may consider number sets such as whole numbers, rational numbers, or integers independent from each other. Therefore, pre-service teachers should pay attention to connections between number sets and between mathematical concepts as well when teaching.

The evidence from this study also suggests that most of the pre-service teachers stated that negative numbers might confuse elementary students. Minus sign is important for development of understanding and using negative numbers (Vlassis, 2004). An implication of this is the possibility that elementary students may have difficulty related to minus sign and that may reflect as difficulty in negative sign. Pre-service teachers and in-service teachers should be careful about minus sign as it is related to development of negative numbers.

The results of this study also indicates that, pre-service teachers have confusions about defining integers though they suggested several examples about elementary students' possible misconceptions and errors about describing integers. They have learned which misconceptions and errors students may have within the scope of their method courses. If not, method courses should be designed as pre-service teachers learn about students' thinking deeply.

The results of this study indicate that pre-service teachers are familiar with common conceptions or mistakes elementary students have or they may have difficulty in. If the common conceptions or mistakes are taken for each of mathematical content, pre-service teachers may guide their students better. It could be easily implied that pre-service teachers are given emphasis on environments in which they discuss the common conceptions or mistakes for every mathematical contents.

5.4 The nature of pre-service elementary mathematics teachers' conceptions about reasons of misconceptions and errors of students about definition of integers

This study has shown that elementary students' possible misconceptions and errors are derived from three reasons: negative transfer of former knowledge about number sets, students' general insufficiency, and teachers' approach.

Pre-service teachers stated that being un/familiar with number sets and problematic prior knowledge about number sets could be the sub-reasons for negative transfer of former knowledge about number sets. The categorization helps us to understand pre-service teachers' views about students' knowledge. In other words, with the categorization, they emphasized that elementary students' knowledge could be seen in two ways. One of them is that students may have prior knowledge for a content to learn. The other is that they may not have the prior knowledge. Students' prior knowledge or their problems about their prior knowledge are important concerns for teachers' knowledge and they are required to develop pre-service teachers (Van der Valk, 1999; Ball et. al., 2008).

Teachers are one of the important components for students learning so that their understanding of mathematics is essential for effective teaching (Feueborn, et. al., 2009). Although teachers know the subject to teach, they may still have barriers for effective teaching. The barriers may be grouped as lack of ability for instructional design, low content knowledge, or pedagogy concerns (Feueborn, et. al., 2009). In this study, participants seem to refer 'uncorrected explanations' to low content knowledge and 'teaching strategies' to instructional design. They identified as two sub-reasons for the misconceptions and errors based on teachers' approach.

Furthermore, elementary students' general insufficiency was suggested as another main reason for misconceptions and errors. What is surprising is that students seem to be responsible for their own faulty. A serious weakness with this argument, however, is that another factor, teachers, may cause for students' faulty. The following situation might happen in relation to teachers' design of their lesson: *"(students) may miss critical points related to their lessons." [Derslerde konuyla ilgili kritik bir kaç nokta kaçırmış olabilir.].* However, the situation might happen in relation to students' carelessness when their teachers were emphasizing the critical points.

An implication of these findings is that pre-service teachers in teacher education programs may be given problematic classroom scenarios and they want to examine the scenarios based on these reasons. Furthermore, they may be required to produce ways to overcome problematic in these scenarios.

5.5 Implications and Recommendations for Further Research Studies

The purpose of the current study was to determine pre-service teachers' subject matter knowledge and pedagogical content knowledge about describing integers. Considering the research findings, further research studies were suggested.

This study supports the fact that pre-service teachers have difficulty in defining concept of integers (Smith, 2002; Steiner, 2009). Although it only focused on describing the concept of integers, further work needs to be done to establish whether pre-service teachers have difficulties in concept of integers except from describing what an integer is.

Although the concept of integers were investigated in only one teacher education program in Turkey, findings were helpful to suggest that pre-service teachers have some trouble in defining integers. Future research should therefore concentrate on the investigation of defining integers. There should be more programs to have a whole picture of understanding pre-service teachers' conceptions about defining integers.

This study also supports the idea that pre-service teachers need to share their own ideas to improve their subject matter knowledge and pedagogical content knowledge (Inoue, 2009; Kinach, 2002). In this study, four interview participants have great contributions to understand their ideas about defining integers deeply. Thus, further research in this field regarding the role of sharing or discussing of knowledge face to face about a mathematical content would be of great help in changing pre-service teachers' perspectives about the content.

Pre-service teachers are inexperienced in students' cognitive process and they need to make practice in real classroom environments to be more experienced (Morris et. al., 2009; Stylianides & Stylianides, 2010; Inoue, 2009). However, it was a surprising result that pre-service teachers have several suggestions about elementary students' mistakes about defining integers. These finding provide the following insights for future research: what sources pre-service teachers are fed from to know elementary students' mistakes, what role of method courses for pre-services in understanding elementary students' mistakes, or how pre-service teachers gain experience.

5.6 Limitations of the study

There are three important limitations that need to be considered for the current study. Representativeness of pre-service teachers, researcher position, and selected content for the study are the limitations for the study.

First, representativeness of pre-service teachers is the main limitation of the study. In the current study, pre-service teachers from one teacher education program participated to the study. Data from open-ended questions were limited to 38 pre-service teachers and data from interview were limited to four pre-service teachers. Therefore, the participants and data gathered from the participants may not represent participants in other teacher education programs in Turkey. It is desirable to repeat the study for larger population of students.

Researcher position is the second limitation of the study. The limitation was mentioned in methodology part.

Lastly, in this study, concept of integers was selected to research. Defining of integers and mistakes of elementary students about defining integers were the main issues discussed in the study. Therefore, attitudes, beliefs, or values of teachers towards mathematics teaching were not considered in this study.

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APPENDIX

TEZ FOTOKOPİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü

Sosyal Bilimler Enstitüsü

Uygulamalı Matematik Enstitüsü

Enformatik Enstitüsü

Deniz Bilimleri Enstitüsü

YAZARIN

Soyadı : KUBAR
Adı : Ayşenur
Bölümü : İlköğretim Fen ve Matematik Eğitimi

TEZİN ADI (İngilizce) : Pre-Service Elementary Mathematics Teachers' Knowledge About Definitions Of Integers And Their Knowledge About Elementary Students' Possible Misconceptions And Errors In Describing Integers

TEZİN TÜRÜ : Yüksek Lisans Doktora

1. Tezimin tamamı dünya çapında erişime açılsın ve kaynak gösterilmek şartıyla tezimin bir kısmı veya tamamının fotokopisi alınsın.

2. Tezimin tamamı yalnızca Orta Doğu Teknik Üniversitesi kullancılarının erişimine açılsın. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.)

3. Tezim bir (1) yıl süreyle erişime kapalı olsun. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.)

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