A COMPARATIVE STUDY ON DIRECT ANALYSIS METHOD AND EFFECTIVE LENGTH METHOD IN ONE-STORY SEMI-RIGID FRAMES

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A COMPARATIVE STUDY ON DIRECT ANALYSIS METHOD AND EFFECTIVE LENGTH METHOD IN ONE-STORY SEMI-RIGID FRAMES

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ABSTRACT

A COMPARATIVE STUDY ON DIRECT ANALYSIS METHOD AND EFFECTIVE LENGTH METHOD IN ONE-STORY SEMI-RIGID FRAMES

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For steel structures, stability is a very important concept since many steel structures are governed by stability limit states. Therefore, stability of a structure should be assessed carefully considering all parameters that affect the stability of the structure. The most important of these parameters can be listed as geometric imperfections, member inelasticity and connection rigidity. Geometric imperfections and member inelasticity are taken into account with the stability method used in the design. At this point, the stability methods gain importance. The Direct Analysis Method, the default stability method in 2010 AISC Specification, is a new, more transparent and more straightforward method, which captures the real structure behavior better than Effective Length Method. In this thesis, a study has been conducted on the semi-rigid steel frames to compare Direct Analysis Method and Effective Length Method and to investigate the effect of flexible connections to stability. Four frames are designed for different connection rigidities with stability methods existing in the 2010 AISC Specification: Direct Analysis Method and Effective Length Method. At the end, conclusions are drawn about the comparison of these two stability methods and the effect of semi-rigid connections to stability.

Keywords: Direct Analysis Method, Effective Length Method, Semi-Rigid Frames, Frame Stability

ÖZ

DİREKT ANALİZ METODU İLE EFEKTİF UZUNLUK METODUNUN TEK KATLI YARI RİJİT BAĞLANTILI ÇERÇEVELERDE KARŞILAŞTIRILMASI

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Çelik yapılar için stabilite kavramı çok önemlidir çünkü çoğu çelik yapı stabilite limit durumlarına göre tasarlanmaktadır. Bundan dolayı, bir yapının stabilitesi onu etkileyebilecek bütün parametreler göz önünde bulundurularak itinayla değerlendirilmelidir. Geometrik kusurlar, elemanların inelastisitesi ve bağlantı rijitliği bu parametrelerin en önemlileridirler. Geometrik kusurlar ve eleman inelastisiteleri tasarım esnasında kullanılan stabilite metodu ile değerlendirilirler. Bu noktada stabilite metotları önem kazanmaktadır. 2010 AISC şartnamesinde geçerli stabilite metodu olan Direkt Analiz Metodu yeni, daha şeffaf ve daha dolambaçsız bir metot olup Efektif Uzunluk Metodu'na göre gerçek yapı davranışını daha iyi yansıtmaktadır. Bu tezde, Direk Analiz Metodu ile Efektif Uzunluk Metodunu karşılaştırmak ve esnek bağlantıların stabiliteye etkisini incelemek için yarı rijit çelik çerçeveler üzerine bir çalışma yapılmıştır. 2010 AISC şartnamesinde mevcut olan stabilite metotları (Direkt Analiz ve Efektif Uzunluk Metotları) kullanılarak değişik bağlantı rijitlikleri ile dört tane çelik çerçeve tasarlanacaktır. En sonda, bu iki

metodun kıyaslanması ve yarı rijit bağlantıların stabiliteye etkisi üzerine sonuçlar çıkartılacaktır.

Anahtar Kelimeler: Direkt Analiz Metodu, Efektif Uzunluk Metodu, Yarı Rijit Çerçeveler, Çerçevelerin Stabilitesi

To My Family

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LIST OF SYMBOLS / ABBREVATIONS

A	Cross-sectional area (m^2)
A_g	Gross cross-sectional area of member (m^2)
AISC	American Institute of Steel Construction
AISC 360-05	2005 AISC Specification for Structural Steel Buildings
AISC 360-10	2010 AISC Specification for Structural Steel Buildings
ASD	Allowable stress design
B_1	Multiplier to account for P-δ
B_2	Multiplier to account for P-Δ
C_b	Lateral-torsional buckling modification factor for non-uniform moment diagrams
C_m	Coefficient accounting for non-uniform moment
C_w	Warping constant (m^6)
D	Displacement matrix of the system
DAM	Direct analysis method
D/C	Demand/capacity ratio
E	Modulus of elasticity (MPa)
ELM	Effective length factor
F_e	Elastic buckling stress (MPa)
F_y	Minimum yield stress (MPa)
H	Story shear, in the direction of translation being considered, produced by the lateral forces used to compute Δ_H (N)
I	Moment of inertia in the plane of bending (m^4)
I_x	Moment of inertia in x-direction (m^4)
I_y	Moment of inertia in y-direction (m^4)
J	Torsional constant (m^4)
K	Effective length factor
K	Stiffness matrix of the system
K_x	Effective length factor for flexural buckling about x-axis

K_y	Effective length factor for flexural buckling about y-axis
K_1	Effective length factor in the plane of bending, calculated based on the assumption of no lateral translation at the member ends
L	Length of member (m)
LRFD	Load and resistance factor design
L_b	Length between points that are either braced against lateral displacement of compression flange or braced against twist of the cross-section (m)
L_p	Limiting laterally unbraced length for the limit state of yielding (m)
L_r	Limiting laterally unbraced length for the limit state of inelastic lateral-torsional buckling (m)
M_A	End moment at joint A (N·m)
M_B	End moment at joint B (N·m)
M_c	Available flexural strength
M_{lt}	First-order moment due to lateral translation of the structure only (N·m)
M_{nt}	First-order moment with the structure restrained against lateral translation (N·m)
M_1	Smaller moment at end of unbraced length (N·m)
M_2	Larger moment at end of unbraced length (N·m)
N_i	Notional load applied at i^{th} level (N)
P_c	Available axial strength (N)
P_{cr}	Critical buckling load (N)
P_e	Elastic critical buckling load (N)
P_{e1}	Elastic critical buckling strength of the member in the plane of bending (N)
P_{lt}	First-order axial force with the structure restrained against lateral translation (N)
P_{mf}	Total vertical load in columns in the story that are part of moment frames (N)

P_n	Nominal axial strength (N)
P_{nt}	First-order axial force due to lateral translation of the structure only (N)
P_r	Required axial compressive strength (N)
P_{estory}	Elastic critical buckling strength for the story in the direction of translation being considered (N)
P_{story}	Total vertical load supported by the story including loads in columns that are not part of the lateral force resisting system (N)
P_y	Axial yield strength (N)
Q	Force vector of the system
R_{kiA}	Initial stiffness of rotational spring at joint A (N·m)
R_{kiB}	Initial stiffness of rotational spring at joint B (N·m)
R_M	Coefficient to account for influence of P-δ and P-Δ
S_x	Section modulus about x-axis (m^3)
S_y	Section modulus about y-axis (m^3)
Y_i	Gravity load applied at i^{th} level (N)
Z_x	Plastic section modulus about x-axis (m^3)
Z_y	Plastic section modulus about y-axis (m^3)
b	Width of the flange (m)
d_b	Displacement matrix of the beam
d_c	Displacement matrix of the column
h	Total depth of the cross-section (m)
h_o	Distance between the flange centroids (m)
k_b	Stiffness matrix of the beam
k_c	Stiffness matrix of the column
r	Radius of gyration (m) [Eqn. 2.4]
r	End-fixity ratio [Eqn. 2.20]
r_x	Radius of gyration in x-direction (m)
r_y	Radius of gyration in y-direction (m)
s_{ii}^*	Stability function
s_{ij}^*	Stability function

s_{jj}^{*}	Stability function
s_{jj}^{*}	Stability function
t_f	Thickness of flange (m)
t_w	Thickness of web (m)
u	Lateral degree of freedom
v	Axial degree of freedom
α	ASD/LRFD force level adjustment factor
Δ_H	First-order interstory drift due to lateral forces (m)
λ	Slenderness parameter
λ_{pf}	Limiting slenderness parameter for compact flange
λ_{rf}	Limiting slenderness parameter for non-compact flange
$Ø_b$	Resistance factor for flexure
$Ø_c$	Resistance factor for compression
$Ø_c$	Stability stiffness function
$Ø_1$	Stability stiffness function
$Ø_2$	Stability stiffness function
$Ø_3$	Stability stiffness function
$Ø_4$	Stability stiffness function
θ	Rotational degree of freedom
θ_A	Rotation of joint A (rad)
θ_B	Rotation of joint B (rad)
θ_{rA}	Relative rotation between the joint A and the beam (rad)
θ_{rB}	Relative rotation between the joint B and the beam (rad)
τ_b	Stiffness reduction parameter

CHAPTER 1

INTRODUCTION

1.1. MOTIVATION

Stability is a very important concept for steel structures since most steel structures are governed by stability limit states. Local instability, such as compression flange buckling, and member instability, such as buckling of a column, may lead the structure to collapse. Therefore, stability provisions of steel design specifications are continuously improved to capture the real structure behavior and so to minimize the destabilizing effects.

In the Appendix 7 of 2005 AISC Specification for Structural Steel Buildings (AISC 360-05), Direct Analysis Method (DAM) was first introduced as an alternative to the Effective Length Method (ELM). Then in 2010 AISC Specification for Structural Steel Buildings (AISC 360-10) it became the default stability design method as it is given in Chapter C.

The need to develop a new method is the drawbacks of the ELM. These drawbacks can be listed as:

- ELM is based on many assumptions, which are hardly satisfied in a real structure. Inconsistencies between the assumptions and the real structure behavior lead to wrong estimation of internal forces and moments.
- ELM underestimates the internal forces and moments, due to this reason ELM cannot be used for structures having drift-ratio greater than 1.5 that means ELM is not applicable to all structures.

- Geometric imperfections and member inelasticity are not accounted for in the analysis instead they are accounted for in the resistance terms that causes misinterpretation of both analysis results and member strengths.

On the other hand, DAM is a more straightforward, transparent and accurate stability design method. It considers member inelasticity and geometric imperfections in the analysis and it calculates compressive strength of members with an effective length factor equals to 1.00. Therefore, the DAM captures the real structure behavior better than the ELM and it provides the designer a simpler and straightforward stability design procedure.

To obtain realistic analysis results, the stability method used in the analysis is important along with the realistic modeling of the structure. With the help of advanced commercial software, detailed 3-D modeling of structures is possible. However, there is still an important idealization in modeling that makes the structural model away from the real structure behavior: connections.

Steel frames are designed under the assumption that the beam-to-column connection is either fully rigid or ideally pinned. However in reality, any connection is neither fully rigid nor ideally pinned. Connection rigidity has an influence on the internal force distribution of the system and lateral drift of the structure. Therefore, connection rigidity should be modeled such that it reflects the connection behavior.

1.2. LITERATURE SURVEY

In this section, the researches conducted on comparison of ELM and DAM and the researches carried out with semi-rigid frames are discussed.

Ziemian et al [1] investigated eleven two-and-three-dimensional structural systems to evaluate and compare ELM and DAM. Also advanced-second order inelastic analyses were used to assess the adequacy of all design methods. They concluded that ELM and DAM provide similar results and for beam-columns subjected to minor-axis bending DAM is slightly unconservative.

In the study of Surovek et al [2], an 11-bay single-story frame was studied to discuss the three design approaches (Direct Analysis Method, Effective Length Method and Advanced Analysis) for the assessment of frame stability. The primary attribute of this frame was that it is sensitive to initial imperfection effects. To illustrate distinctive features of the design approaches, large gravity loads were applied to produce significant P-Δ effects. They concluded that axial forces are similar in each method but the internal moments differ substantially. ELM underestimates the internal moments since the moments in the frame are highly sensitive to out-of-plumbness of the structure which is not directly considered in the analysis with ELM, and DAM is conservative when calculating the internal moments since the columns are elastic at the factored load although the stiffnesses are reduced due to inelasticity.

In his study, Prajzner [3] dealt with the evaluation of case studies including a portal frame, a leaning column frame, a multi-story structure, and a multi-bay frame in order to assess the adequacy of ELM and DAM. To provide a reasonable representation of real frame behavior second-order plastic analysis approach was used as the third method and the results obtained with this method were treated as real results. In this study, ELM produced unsafe designs in structures where second-order effects are significant. On the other hand, DAM produced overly conservative designs for the same type of structures. He suggested a “sway” factor to quantify the second-order effects. The intent of the “sway” factor is to calibrate the results of ELM and DAM in structures where the second-order effects are significant.

Surovek et al [4] presented an approach that allows for the consideration of non-linear connection using commonly available elastic analysis software. The partially restrained frames were analyzed using Direct Analysis Method. The aim of the proposed connection approach was to simplify the consideration non-linear connection response in the analysis of partially restrained frames. By using Direct Analysis Method, they intended to simplify also the strength assessment of the structure by eliminating the calculation of effective length factor. In this study, the proposed method for handling the connection non-linearity along with the Direct Analysis Method have been shown to make it simpler to obtain realistic distribution of internal forces in partially restrained frames.

Kartal et al [5] developed a finite element program SEMIFEM in FORTRAN language to perform structural analysis that considers semi-rigid connections. The aim of their study was to investigate the effect of semi-rigid connections on the structure behavior. In their study, they adopted the formula suggested by Monforton and Wu [6] to define the connection stiffness in terms of the connection stiffness-beam stiffness ratio.

The formula suggested by Monforton and Wu [6] was also adopted by Xu [7] in his study on calculation of critical buckling loads of semi-rigid steel frames and by Patodi et al [8] in their study on first order analysis of plane frames with semi-rigid connections. They used this formula to define the stiffness of the connection in terms of the stiffness of the beam that the connection is attached to.

1.3. OBJECT AND SCOPE

Before development of DAM, ELM was the common stability method that has been used widely by many engineers. However ELM is based on some assumptions which are hardly satisfied in real structures and it has many drawbacks which make stability design very complex and challenging in some cases. DAM was developed and presented in the latest version of AISC Specification for Steel Structures to compensate the drawbacks of ELM and to make stability design easier and more straightforward for engineers. A number of studies were conducted to compare DAM and ELM in order to investigate whether DAM is a more straightforward method and superior to ELM.

In this study, DAM and ELM are compared in one-story semi-rigid frames. The objective of this study is to compare the two stability methods in semi-rigid frames and to investigate the influence of connection rigidity on the stability of the frame.

Four frames are used as case studies: first three ones are one-story one-bay frames consisting of two columns and one beam where the columns are oriented such that they are in major-axis bending. The only difference between these three frames is the loads. In the first frame, horizontal load is the highest but the axial load is the lowest and in the third frame, horizontal load is the lowest whereas the axial load is the highest. The aim in selecting these three frames is to investigate the load effects on the stability methods. The fourth frame is a one-story three-bay frame consisting of two columns in major-axis bending, two columns in minor-axis bending and three beams. The aim in selecting the fourth frame, where the some of the columns are in minor-axis bending, is to investigate the influence of column orientation on stability methods. The beams in all cases are connected to the columns with semi-rigid connections. The frames are analyzed with different stiffness values of semi-rigid connections and with both stability methods. At the end, critical columns in each case are designed according to the AISC 360-10 and demand/capacity ratios of columns are obtained for different connection stiffnesses and for both DAM and ELM. The conclusion part of the study deals with these demand/capacity ratios.

This study is composed of five chapters. In first chapter, an introduction part exists which gives a general information about the study, a brief background for the DAM along with semi-rigid frames and the aim of the study. In second chapter, the theory of methods, which are used in third chapter, are given and explained. These are Direct Analysis Method, Effective Length Method, Analysis of Semi-Rigid Frames and Approximate Second-Order Analysis. The analysis of frames and the design of columns are given in third chapter. The results of the analyses and designs in the third chapter and the discussions related to the results are given in fourth chapter. The last chapter, Chapter 5, contains conclusion of the study and future recommendations.

CHAPTER 2

THEORY

2.1. DIRECT ANALYSIS METHOD

Direct Analysis Method (DAM) was first introduced in 2005 version of the AISC Specification for Structural Steel Buildings as an alternative method to the Effective Length Method (ELM) and First-Order Analysis Method. Then in 2010 version of the specification, it became the standard stability design method as it is addressed in Chapter C. DAM has many advantages, such as; it obtains the analysis results more accurately and realistic, it is applicable to all type of structures and it eliminates the calculation of K factor.

ELM neglects initial imperfections and inelasticity during analysis and underestimates member demand. To compensate this underestimate, it requires the use of K factor to decrease the member capacity. Therefore, in ELM, the forces and capacities obtained do not reflect the real behavior of the structure. In DAM, initial imperfections and inelasticity are considered during the analysis and this eliminates the need for the K factor. Thus, DAM results in a design which is very close to the real structure behavior.

DAM is the most applicable method among all stability methods. It can be used for all types of steel structures such as braced frames, moment frames and combined systems without any limitation. The ratio of second-order drift to first-order drift shall be equal to or less than 1.5 in ELM however there is no such a limitation in DAM.

The biggest advantage that DAM provides is the elimination of K factor calculation. In ELM, the analysis is performed with neglecting the geometric imperfections and inelasticity and they are accounted for in member capacity calculations with increasing the K factor. In DAM, geometric imperfections and inelasticity are included in the analysis therefore the need to calculate the K factor is unnecessary and it can be taken as 1.0 for all members.

Since DAM has many advantages, it is expected that there are too many sophisticated requirements however the requirements of DAM are simple and easy to apply. There are three main requirements of DAM which are;

1. A rigorous second-order analysis including both P-Δ and P-δ effects should be conducted. Use of approximate methods is also permitted.
2. The effect of initial imperfections should be taken into account. The out-of-plumbness of columns can be directly modeled by displacing the points of intersection of members from their nominal locations or notional loads can be used.
3. Reduced stiffness of members should be used in the analysis. This reduction accounts for system reliability (uncertainty in stiffness and strength) and inelasticity.

Second-Order Analysis

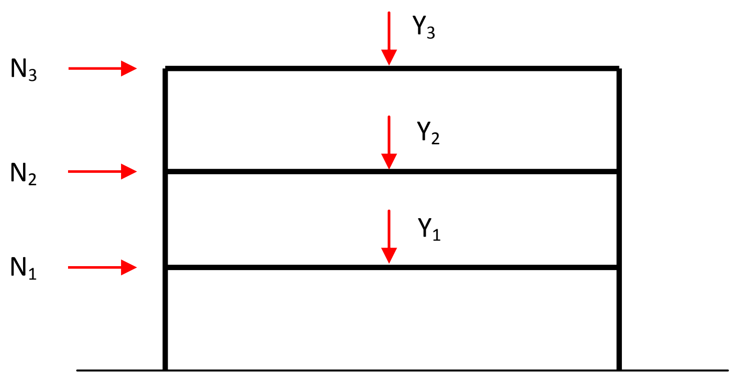
To reflect a real structure behavior, a rigorous second-order analysis considering both P-Δ and P-δ effects should be conducted. It is also acceptable to obtain second-order results by an approximate method given in Appendix 8 of the AISC 360-10. In this alternative and approximate method there are two multipliers; B_1 and B_2. By applying these multipliers to the results of a first-order analysis an approximate second-order solution may be obtained. This method will be explained in details in Section 2.4.

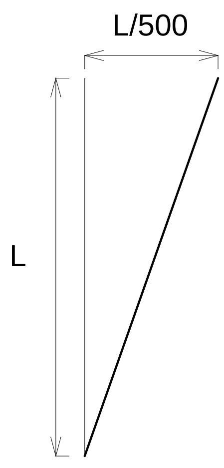
Initial Imperfections

The effect of out-of-plumbness of columns should be taken into account by considering the initial imperfections in the intersection of members. These imperfections can be directly modeled by displacing the intersection of members from their nominal locations or can be represented by notional loads. The notional loads at each level of the structure are calculated as;

$$N_i = 0.002 \cdot \alpha \cdot Y_i \tag{2.1}$$

where α = 1.0 for LRFD and 1.6 for ASD
N_i = notional load applied at i^{th} level (N)
Y_i = gravity load applied at i^{th} level (N)





The notional load coefficient 0.002 is based on a nominal initial out-of-plumbness ratio of L/500 which is the maximum tolerance on column plumbness specified in the AISC Code of Standard Practice.

If the second-order drift to first-order drift ratio is smaller than or equal to 1.7, it is not obligatory to use notional loads in a load combination which includes other lateral loads.

Reduced Stiffness

After rolling or welding process, the cross-section of the steel member begins to cool. First the extreme fibers of the section cool, then the remaining portions of the section cool. When the remaining portions cool, their contraction is prevented by extreme fibers that have already cooled. This results in development of tensile and compressive stresses in the cross-section. When a compressive force is applied to this cross-section, yielding will first occur in the portions of the section which are under compressive residual stress. Therefore, the spread of plasticity in the cross-section is affected by the presence of residual stress [9]. To account for geometric imperfections in the cross-section and the spread of plasticity due to residual stresses, stiffness reduction factor $0.8\tau_b$ is applied to the stiffness of members which are considered to contribute to the stability of the building.

A factor of 0.80 should be applied to the stiffnesses of all members whether or not they contribute to the stability of the building. The aim of the application of this reduction to stiffnesses of all members is to prevent an artificial distortion of the structure and unintended redistribution.

An additional τ_b factor is applied for the following conditions;

$$(i)\ When \quad \frac{\alpha P_r}{P_y} \leq 0.5 \qquad \tau_b = 1.0 \tag{2.2}$$

$$(ii)\ When \quad \frac{\alpha P_r}{P_y} \geq 0.5 \qquad \tau_b = 4 \cdot \left(\frac{\alpha P_r}{P_y}\right) \cdot \left(1 - \frac{\alpha P_r}{P_y}\right) \tag{2.3}$$

Where P_r = required axial compressive strength using LRFD or ASD load combinations (N)

P_y = axial yield strength ($=F_y \cdot A_g$)

A_g = gross cross-sectional area of the member (m^2)

When the $\alpha P_r/P_y$ ratio is higher than 0.5, the calculation and application of τ_b for each member can be painful therefore it is permissible to use τ_b = 1.0 for all members if a notional load of $0.001 \cdot \alpha \cdot Y_i$ is applied at all levels.

The Reasons for Use of 0.80 Factor

In AISC 360-10 Chapter E3, columns are separated into two groups for determination of compressive strength: slender columns and intermediate or stocky columns. For these two groups, stiffness reduction factor should be determined separately.

For slender columns, effective length method implies a safety factor, $\O P_n=0.9(0.877P_e)=0.79P_e$. Stiffness reduction factor for DAM should compensate this safety factor of 0.79 therefore stiffness reduction factor for slender columns is chosen as 0.80 [10].

Stiffness reduction factor for stocky columns should account for additional softening under combined axial compression and bending and the stiffness reduction factor for stocky columns is also 0.80 and this is a fortunate coincidence that for both groups the stiffness reduction factor is the same. The stiffness reduction factor $0.8\tau_b$ is valid for all columns regardless of their slenderness whereas the τ_b factor accounts for stiffness loss under high compressive loads [10].

2.2. EFFECTIVE LENGTH METHOD

2.2.1. Introduction

The effective length method has been used widely in column design for many years. It can be considered as mathematically reducing the evaluation of critical stress for columns to that of equivalent pinned-ended braced columns. In Eqn. 2.4, Euler buckling stress of a pinned-ended braced column is given and this can be used for all elastic column buckling problems by substituting the actual length of the column (L) with an effective length (KL). The effective length factor K can be obtained by performing a buckling analysis of the structure [11]. For idealized structures, K factor may be obtained from alignment charts (or nomographs) given in AISC 360-10 Appendix 7 however to be able to use these alignment charts, the assumptions that was considered during derivation of nomographs should not be violated. One of these assumptions is that "All joints are rigid". Therefore, for frames with semi-rigid connections, the alignment charts cannot be used. To obtain effective length factor K for semi-rigid frames, buckling analysis is needed.

$$F_e = \frac{\pi^2 \cdot E}{\left(\frac{L}{r}\right)^2} \tag{2.4}$$

Where

E	= Modulus of elasticity (MPa)
L	= Length of the column (m)
r	= Radius of gyration (m)

2.2.2. Buckling Analysis of Semi-Rigid Frames

To perform a buckling analysis for a semi-rigid frame, the modified stiffness matrices of columns and beams that constitute the frame should be considered. Beam matrices should include the connection flexibility, which will be discussed in Section

2.3.4 in details, whereas column matrices should include stability functions which include the effective length factor.

Stiffness matrix of a column is given in "Stability Design of Steel Frames" [12] and it is used in this thesis. One may refer to the reference for the details.

$$k = \frac{EI}{L}\begin{bmatrix} \frac{A}{I} & 0 & 0 & -\frac{A}{I} & 0 & 0 \\ 0 & \frac{12}{L^2}\emptyset_1 & \frac{6}{L}\emptyset_2 & 0 & -\frac{12}{L^2}\emptyset_1 & \frac{6}{L}\emptyset_2 \\ 0 & \frac{6}{L}\emptyset_2 & 4\emptyset_3 & 0 & -\frac{6}{L}\emptyset_2 & 2\emptyset_4 \\ -\frac{A}{I} & 0 & 0 & \frac{A}{I} & 0 & 0 \\ 0 & -\frac{12}{L^2}\emptyset_1 & -\frac{6}{L}\emptyset_2 & 0 & \frac{12}{L^2}\emptyset_1 & -\frac{6}{L}\emptyset_2 \\ 0 & \frac{6}{L}\emptyset_2 & 2\emptyset_4 & 0 & -\frac{6}{L}\emptyset_2 & 4\emptyset_3 \end{bmatrix} \tag{2.5}$$

In Eqn. 2.5, $\emptyset_1$, $\emptyset_2$, $\emptyset_3$ and $\emptyset_4$ are the stability stiffness functions and for the case when P_{cr} is a compressive axial load:

$$\emptyset_1 = \frac{(kL)^3 sinkL}{12\emptyset_c} \tag{2.6}$$

$$\emptyset_2 = \frac{(kL)^2(1 - coskL)}{6\emptyset_c} \tag{2.7}$$

$$\emptyset_3 = \frac{kL(sinkL - kLcoskL)}{4\emptyset_c} \tag{2.8}$$

$$\emptyset_4 = \frac{kL(kL - sinkL)}{2\emptyset_c} \tag{2.9}$$

In which

$$\emptyset_c = 2 - 2coskL - kLsinkL \tag{2.10}$$

$$k = \sqrt{\frac{P_{cr}}{EI}} \tag{2.11}$$

Once the stiffness matrix of the structure ($K_{structure}$) is constructed, the determinant of the $K_{structure}$ is set equal to zero (det $|K_{structure}|=0$) to obtain k. From k, we get the effective length factor K:

$$P_{cr} = F_e \cdot A = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \cdot A = \frac{\pi^2 EI}{(KL)^2} \tag{2.12}$$

$$k = \sqrt{\frac{P_{cr}}{EI}} = \sqrt{\frac{\frac{\pi^2 EI}{(KL)^2}}{EI}} = \sqrt{\frac{\pi^2}{(KL)^2}} = \frac{\pi}{KL} \quad \rightarrow \quad K = \frac{\pi}{kL} \tag{2.13}$$

Where

P_{cr} = Critical buckling load

F_e = Critical buckling stress (Eqn. 2.4)

A = Cross-sectional area of the column

2.2.3. AISC 360-10 Requirements

The requirements for ELM are given in Appendix 7 of the AISC 360-10. These requirements can be listed as below;

1. Maximum second-order drift to maximum first-order drift ratio shall be equal to or less than 1.5. If this requirement is not satisfied, the ELM cannot be used.
2. Nominal stiffnesses of members shall be used, no stiffness reduction is necessary.
3. Notional loads shall be applied in the analysis. The same rules as in the DAM are valid for the ELM.

2.3. ANALYSIS OF SEMI-RIGID FRAMES

In this section, types, behavior, modeling and analysis of semi-rigid connections are discussed. At the end of this section, stiffness matrix of a flexible-ended beam will be obtained. The related sections of "Stability Design of Steel Frames" [12] are discussed here, for the details one may refer to this book.

2.3.1. Introduction

In reality, all steel frames behave as semi-rigid however to simplify the analysis and design, they are idealized as fully rigid or perfectly pinned. In this idealization, rigid connections are assumed that they exhibit no deformation and pinned connections are assumed to have no moment capacity. However, in reality, rigid frame connections exhibit deformation and pinned connections have moment capacity even if it is small. For a more realistic and correct analysis, connection flexibility should be taken into account.

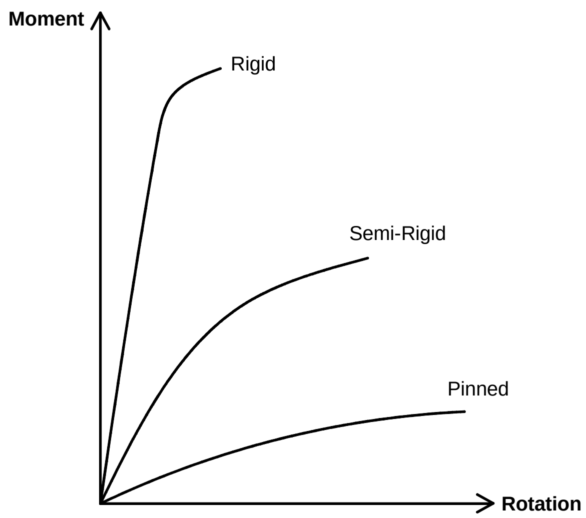


Figure 2.1 - Typical Moment-Rotation Curves for the Connection Types

2.3.2. Types of Semi-Rigid Connections

There exist many beam-to-column connection types but the most commonly used ones are briefly discussed here. These common types of beam-to-column connections can be listed as single web angle, single plate, double web angle, header plate, top and seat angle, top and seat angle with double web angle, extended end-plate, flush end-plate and t-stub connections. Each connection type has a different moment-rotation curve and these curves are given in Figure.2.2.

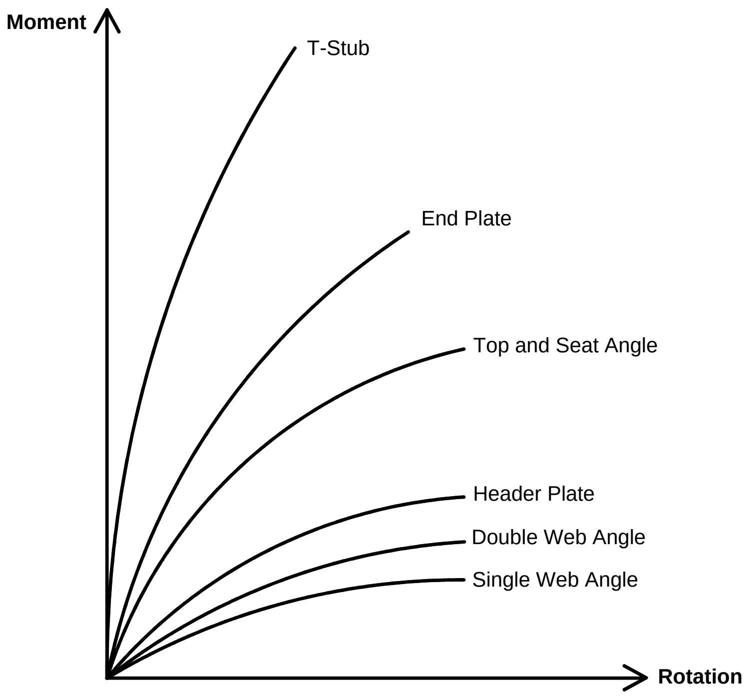


Figure 2.2 – Moment-Rotation Curves of Connections

There are many modeling types for semi-rigid connections to obtain moment-rotation curves. Key parameters such as initial connection stiffness or ultimate connection moment capacity are determined from these models. However the derivation of initial connection stiffness and determination of ultimate connection moment capacity are not in the aim of this study therefore only brief descriptions of the connections are given.

Single Web Angle

The beam is connected to the column through an angle member, welded or bolted both to the beam and column. This is a very flexible connection, it has very little moment capacity and it is considered as shear connection.

Single Plate

In this connection, a plate is used instead of an angle member. The plate member is welded or bolted to the beam and column. This connection type is also very flexible and considered as shear connection.

Double Web Angle

The beam is connected to column through two angle members on the both side of the beam web. The moment-rotation rigidity of this connection is higher than the rigidities of single web angle and single plate connections however this connection type is also considered as shear connection.

Header Plate Connections

Header plate connection consists of an end plate which is welded to the web of the beam and bolted to the flange of the column. In this connection type, the length of the plate is less than the depth of the beam. The moment-rotation rigidity of this connection is similar to that of double web angle connection. This connection is also considered as shear connection.

Top and Seat Angle Connections

This connection consists of two angle members welded or bolted to the beam, one is at the bottom (seat angle) and the other one is at the top. The angles are bolted to the column flange. The seat angle carries gravity loads but does not contribute significantly to the moment capacity of the connection. The top angle is for the lateral stability of the beam and does not carry any gravity loads. The experimental results show that this type of connections is capable of resisting some of the end moment of the beam.

Top and Seat Angle Connections with Double Web Angle

This connection is the combination of double web angle connection and top and seat angle connection. This type is considered as semi-rigid connection.

Extended & Flush End-Plate Connections

Flush end-plate connections consist of a plate welded to the beam end (along both top and bottom flanges and web) and bolted to the column. If the end-plate extends on tension side or both on tension and compression sides, this connection type is called extended end-plate connection. These two types of connections are considered as moment connections.

T-stub Connections

This connection type is one of the stiffest connections and consists of two T-stubs bolted to the beam at the top and bottom flange. The t-stubs are also bolted to the column. This connection gets stiffer when used with double web angles.

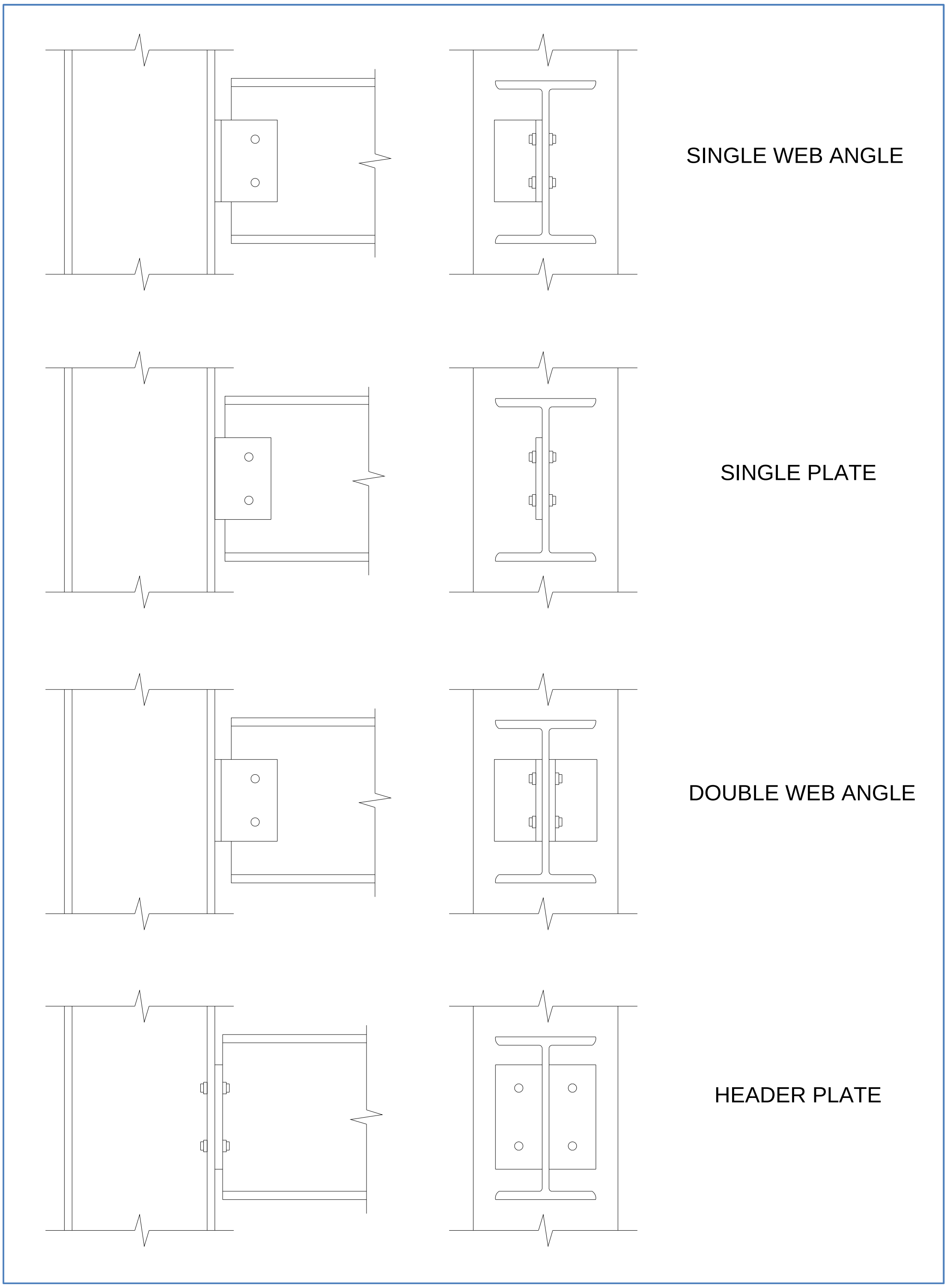


Figure 2.3 – Types of Semi-Rigid Connections

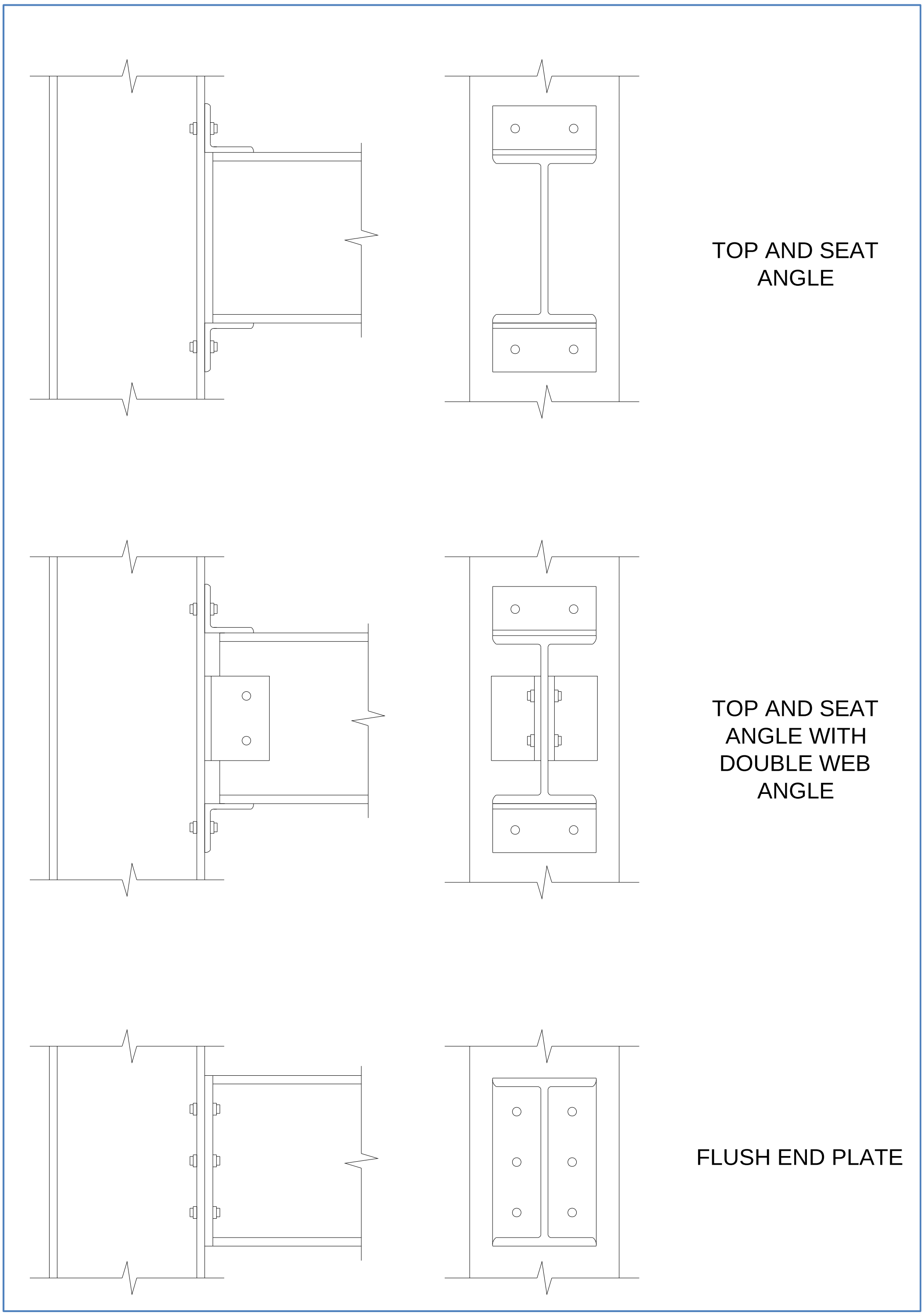


Figure 2.3 – Continued

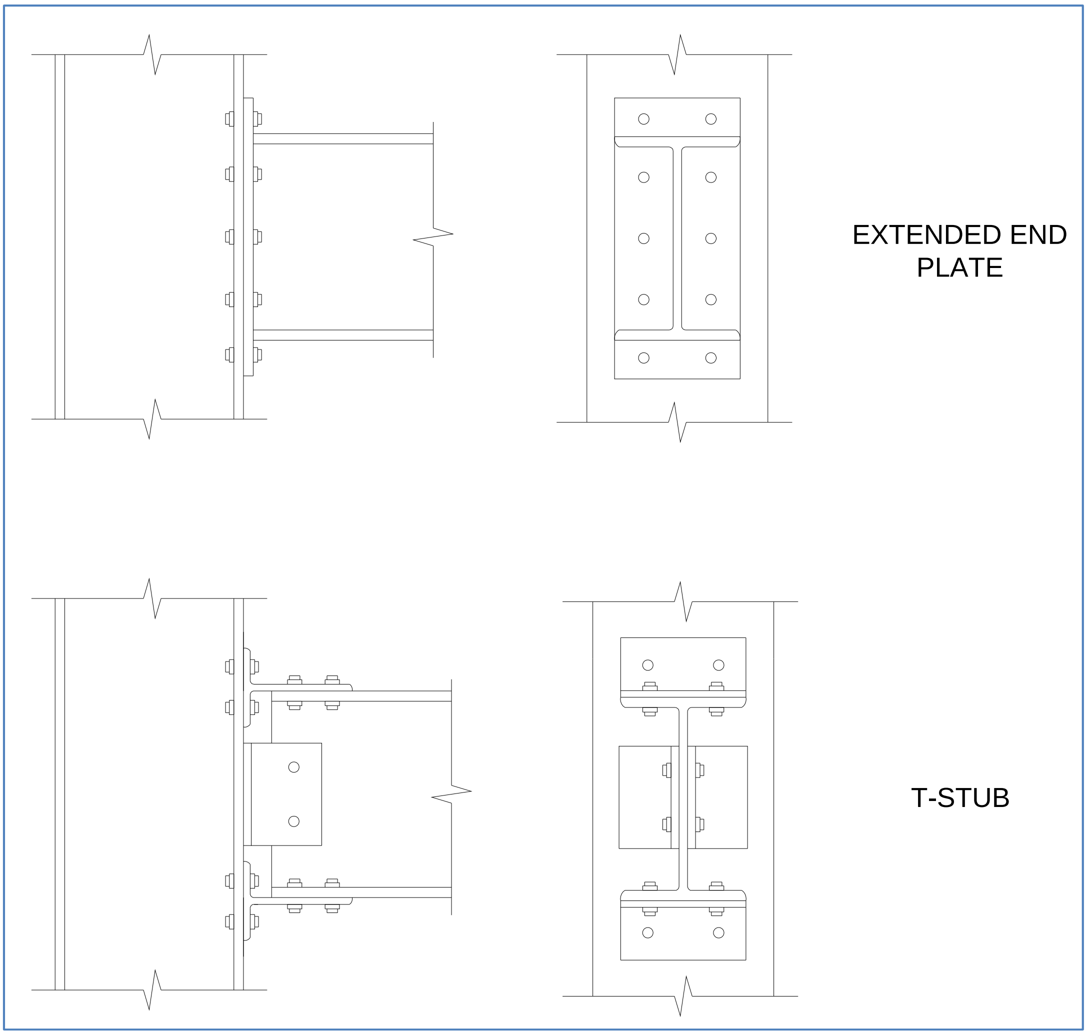


Figure 2.3 – Continued

2.3.3. Behavior and Modeling of Connections

The general behavior of a connection is that when a moment is applied to it, it exhibits rotational deformation. If the response of the connection to moment is plotted, the moment-rotation curve of the connection is obtained. The moment-rotation curves of different types of semi-rigid connections were given in Figure 2.2

and a typical moment-rotation behavior of a semi-rigid connection is given in Figure 2.4.

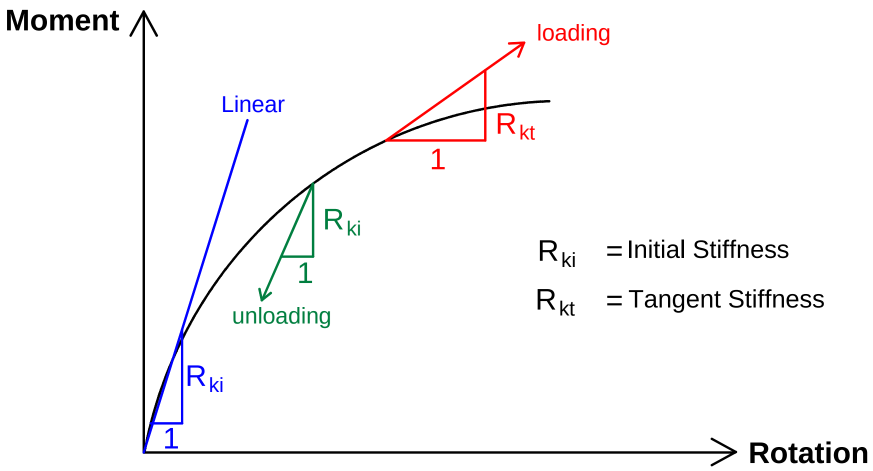


Figure 2.4 – Typical Moment-Rotation Behavior of a Semi-Rigid Connection

As it is seen in the Figure 2.4, the behavior of the connection is nonlinear. Factors such as bolt slip, stress concentration and local yielding lead connection to exhibit nonlinear response. Also the response of the connection to loading and unloading is different. To predict the actual behavior of the connection, nonlinearity and loading/unloading characteristic of the connection should be accounted for modeling the connection. There are many types of models such as linear models (linear, bi-linear, piecewise linear), polynomial model, b-spline model, power models and exponential models. Among these models, linear model is the weakest model to predict the actual behavior of the structure however it is the simplest one. Since the aim of this study is to compare effective length method and direct analysis method in semi-rigid frames and to investigate the effect of flexible connections to stability, it is sufficient to use the linear connection model. The linear model is shown in Figure 2.4. The only parameter in linear model is the initial stiffness, R_{ki}.

2.3.4. Analysis of Semi-Rigid Frames

To take into account the effect of flexible joints in the analysis, it is necessary to modify the stiffness matrix of the beam elements in the structure. To obtain the stiffness matrix of a beam element which considers flexible joints, the slope-deflection equation of the beam element need to be modified. In this section, the slope-deflection equation of the beam element is modified and the stiffness matrix considering flexible joint effect is obtained.

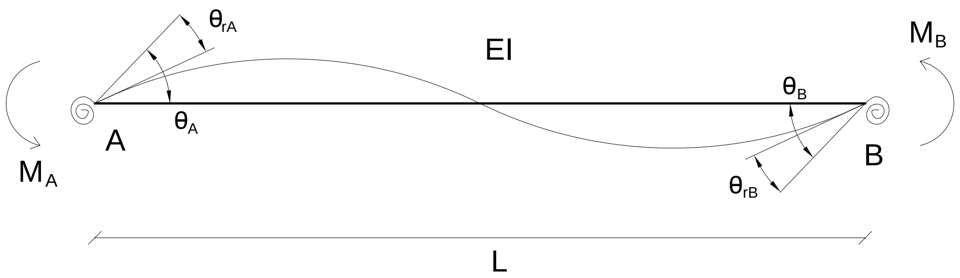


Figure 2.5 – Beam Element

Consider the beam element in Figure 2.5. It is subjected to end moments M_A and M_B. To represent the flexible connection effect, rotational springs are assigned at both ends of the beam. The initial stiffnesses of these springs are denoted as R_{kiA} and R_{kiB}. The relative rotations between the joint and the beam end due to the rotational springs are θ_{rA} and θ_{rB} at ends A and B, relatively. These relative rotations can be expressed as;

$$\theta_{rA} = \frac{M_A}{R_{kiA}} \qquad and \qquad \theta_{rB} = \frac{M_B}{R_{kiB}} \tag{2.14}$$

If the joint rotations at A and B are denoted as θ_A and θ_B, respectively, then the slope-deflection equations of the beam can be written as;

$$M_A = \frac{EI}{L}[4(\theta_A - \theta_{rA}) + 2(\theta_B - \theta_{rB})] \;=\; \frac{EI}{L}\left[4\left(\theta_A - \frac{M_A}{R_{kiA}}\right) + 2\left(\theta_B - \frac{M_B}{R_{kiB}}\right)\right] \quad (2.15a)$$

$$M_B = \frac{EI}{L}[2(\theta_A - \theta_{rA}) + 4(\theta_B - \theta_{rB})] = \frac{EI}{L}\left[2\left(\theta_A - \frac{M_A}{R_{kiA}}\right) + 4\left(\theta_B - \frac{M_B}{R_{kiB}}\right)\right] \quad (2.15b)$$

The equations 2.15a and 2.15b can be expressed as;

$$M_A = \frac{EI}{L}\left(s_{ii}^* \cdot \theta_A + s_{ij}^* \cdot \theta_B\right) \quad (2.16a)$$

$$M_B = \frac{EI}{L}\left(s_{ji}^* \cdot \theta_A + s_{jj}^* \cdot \theta_B\right) \quad (2.16b)$$

Where

$$s_{ii}^* = \frac{\left(4 + \frac{12EI}{LR_{kiB}}\right)}{R^*} \quad (2.17a)$$

$$s_{jj}^* = \frac{\left(4 + \frac{12EI}{LR_{kiA}}\right)}{R^*} \quad (2.17b)$$

$$s_{ij}^* = s_{ji}^* = \frac{2}{R^*} \quad (2.17c)$$

In which

$$R^* = \left(1 + \frac{4EI}{LR_{kiA}}\right) \cdot \left(1 + \frac{4EI}{LR_{kiB}}\right) - \left(\frac{EI}{L}\right)^2 \cdot \left(\frac{4}{R_{kiA} \cdot R_{kiB}}\right) \quad (2.18)$$

Figure 2.6 – Degrees of Freedom

For the degrees of freedom shown in the Figure 2.6, the stiffness matrix of the beam is obtained as;

$$k_{beam} = \frac{EI}{L}\begin{bmatrix} \frac{A}{I} & 0 & 0 & -\frac{A}{I} & 0 & 0 \\ 0 & \frac{\left(s_{ii}^{*}+2s_{ij}^{*}+s_{jj}^{*}\right)}{L^2} & \frac{\left(s_{ii}^{*}+s_{ij}^{*}\right)}{L} & 0 & -\frac{\left(s_{ii}^{*}+2s_{ij}^{*}+s_{jj}^{*}\right)}{L^2} & \frac{\left(s_{ij}^{*}+s_{jj}^{*}\right)}{L} \\ 0 & \frac{\left(s_{ii}^{*}+s_{ij}^{*}\right)}{L} & s_{ii}^{*} & 0 & -\frac{\left(s_{ii}^{*}+s_{ij}^{*}\right)}{L} & s_{ij}^{*} \\ -\frac{A}{I} & 0 & 0 & \frac{A}{I} & 0 & 0 \\ 0 & -\frac{\left(s_{ii}^{*}+2s_{ij}^{*}+s_{jj}^{*}\right)}{L^2} & -\frac{\left(s_{ii}^{*}+s_{ij}^{*}\right)}{L} & 0 & \frac{\left(s_{ii}^{*}+2s_{ij}^{*}+s_{jj}^{*}\right)}{L^2} & -\frac{\left(s_{ij}^{*}+s_{jj}^{*}\right)}{L} \\ 0 & \frac{\left(s_{ij}^{*}+s_{jj}^{*}\right)}{L} & s_{ij}^{*} & 0 & -\frac{\left(s_{ij}^{*}+s_{jj}^{*}\right)}{L} & s_{jj}^{*} \end{bmatrix} \tag{2.19}$$

End-Fixity Factor

In the analysis of semi-rigid frames, EI/L represents the stiffness of the beam member and R_{ki} represents the stiffness of connection. There should be a relation between these two such that it gives a physical interpretation of the rigidity available in the connection [7].

Monforton and Wu [6] define an end-fixity factor and suggest a formula;

$$r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} \tag{2.20}$$

End-fixity factor, r, is an indicator of the relation between the beam stiffness and the connection stiffness. It simplifies the analysis procedure and provides designers to compare the structural responses of a member with semi-rigid connections [7]. The upper and lower boundaries of end-fixity factor can be checked by setting connection stiffness, R_{ki}, to 1 N·m and 10^6 N·m;

$$R_{ki} = 1\,N \cdot m \qquad r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} = \frac{1}{1 + \frac{3000}{1}} = \frac{1}{3001} = 0.00033 \approx 0.00$$

$$R_{ki} = 10^6\,N \cdot m \qquad r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} = \frac{1}{1 + \frac{3000}{10^6}} = \frac{1}{1.003} = 0.99701 \approx 1.00$$

As the connection stiffness approaches to zero which is the case of a pin connection, the end-rigidity (r) approaches to zero and as the stiffness approaches to infinity which is a fully rigid connection case, r approaches to 1.

The Eqn. 2.20 suggested by Monforton and Wu [6] is also used in this thesis. The connection stiffnesses in the case studies are determined by this equation.

2.4. APPROXIMATE SECOND-ORDER ANALYSIS

In a structural analysis, if the original (or undeformed) geometry of the structure is considered when writing the equilibrium and kinematic relationships then the analysis is referred to as a first-order analysis. However, if the deformed geometry of the structure is considered when writing the equilibrium and kinematic relationships then the analysis is referred to as a second-order analysis. For the stability consideration of structures, second-order analysis is a must [12]. Both of the stability methods, Direct Analysis Method and Effective Length Method, require that a rigorous second-order analysis including both P-Δ and P-δ effects. As an alternative to a rigorous second-order analysis, the approximate method presented in Appendix 8 of the AISC 360-10 can be used. This method is based on the amplification of first-order analysis forces and moments by the multipliers, B_1 and B_2. The B_1 factor accounts for P-δ effects and the B_2 factor accounts for P-Δ effects. B_1 is a member parameter and applied to the moment due to gravity loads to account for the displacements between the two ends of the column member. B_2 is a story parameter and applied to the moment and axial force due to the lateral loads to account for the lateral displacement of the story.

The approximate second-order moment and axial force are determined as follows;

$$M_r = B_1 \cdot M_{nt} + B_2 \cdot M_{lt} \tag{2.21}$$

$$P_r = P_{nt} + B_2 \cdot P_{lt} \tag{2.22}$$

Where

M_{lt} = 1st order moment due to lateral translation of the structure only (N·m)

M_{nt} = 1st order moment with the structure restrained against lateral translation (N·m)

M_r = required 2nd order flexural strength (N·m)

P_{lt} = 1st order axial force due to lateral translation of the structure only (N)

P_{nt} = 1st order axial force with the structure restrained against lateral translation (N)

P_r = required 2nd order axial strength (N)

Calculation of B_1

B_1 is a member parameter and for each member it is calculated as follows;

$$B_1 = \frac{C_m}{1 - \alpha \frac{P_r}{P_{e1}}} \geq 1 \tag{2.23}$$

Where

C_m = coefficient assuming no lateral translation of the frame determined as follows:

a) For beam-columns not subjected to transverse loading between supports in the plane of bending

$$C_m = 0.6 - 0.4 \cdot (M_1/M_2) \tag{2.24}$$

Where M_1 and M_2 are the smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.

b) For beam-columns subject to transverse loading between supports, the value of C_m shall be determined either by analysis or conservatively taken as 1.0 for all cases.

P_{e1} = elastic critical buckling strength of the member in the plane of bending, calculated based on the assumption of no lateral translation at the member ends (N)

$$P_{e1} = \frac{\pi^2 \cdot EI^*}{(K_1 L)^2} \tag{2.25}$$

EI^* = flexural rigidity required to be used in the analysis ($=0.8\tau_b EI$ when used in DAM)

K_1 = effective length factor in the plane of bending, calculated based on the assumption of no lateral translation at the member ends, set equal to 1.0 unless analysis justifies a smaller value

Calculation of B_2

B_2 is a story parameter and calculated for each story and each direction of lateral translation as follows;

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} \geq 1.0 \tag{2.26}$$

Where

P_{story} = total vertical load supported by the story including loads in columns that are not part of the lateral force resisting system (N)

P_{estory} = elastic critical buckling strength for the story in the direction of translation being considered and calculated as;

$$P_{e\,story} = R_M \frac{HL}{\Delta_H} \tag{2.27}$$

Where

R_M	$= 1 - 0.15 \cdot (P_{mf}/P_{story})$
L	= story height (mm)
P_{mf}	= total vertical load in columns in the story that are part of moment frames, if any, in the direction of translation being considered (N)
Δ_H	= 1st order interstory dirft, in the direction of translation being considered, due to lateral forces computed using the stiffness required to be used in the analysis (mm).
H	= story shear, in the direction of translation being considered, produced by the lateral forces used to compute Δ_H.

CHAPTER 3

CASE STUDIES

3.1. GENERAL INFORMATION AND ASSUMPTIONS

To compare DAM and ELM in semi-rigid frames, four case studies are analyzed. In each case study, a frame is designed according to AISC 360-10 and stability design of these frames is conducted according to both DAM and ELM. 21 analyses are performed for different values of end-fixity factor (ranging from 0 to 1 with 0.05 increments) for each stability method. Total of 168 analyses are performed for the four cases. The analyses are performed with Microsoft Office – Excel software however for each case, one of the analyses is described in details for both ELM and DAM in Sections 3.2, 3.3, 3.4 and 3.5 to explain the procedure in Excel spreadsheets. For each case study and for each stability method, the analysis with end-fixity factor of 0.75 is selected to be performed in this chapter. The analyses are summarized in Figure 3.1.

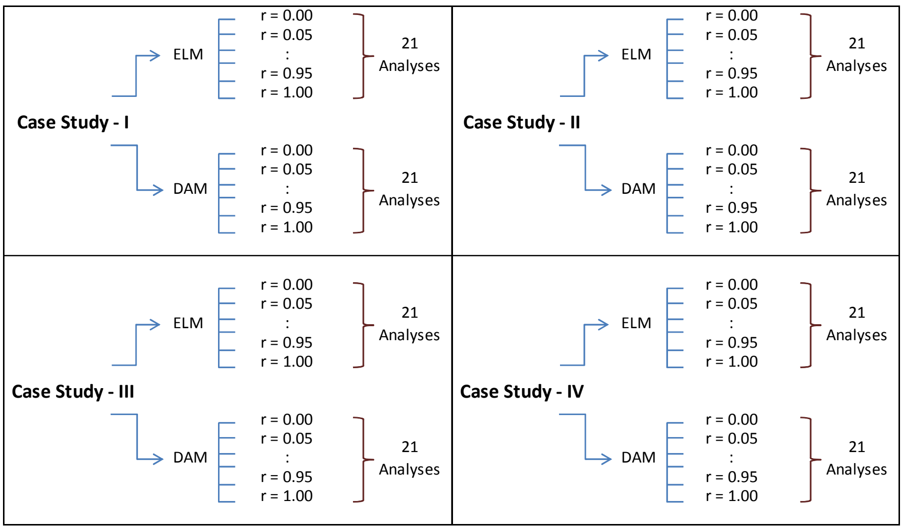


Figure 3.1 – Summary of Analyses

Each ELM analysis consists of structural analysis, buckling analysis and column design parts whereas DAM consists of structural analysis and column design parts. The flowcharts of both ELM and DAM are given in Figure 3.2.

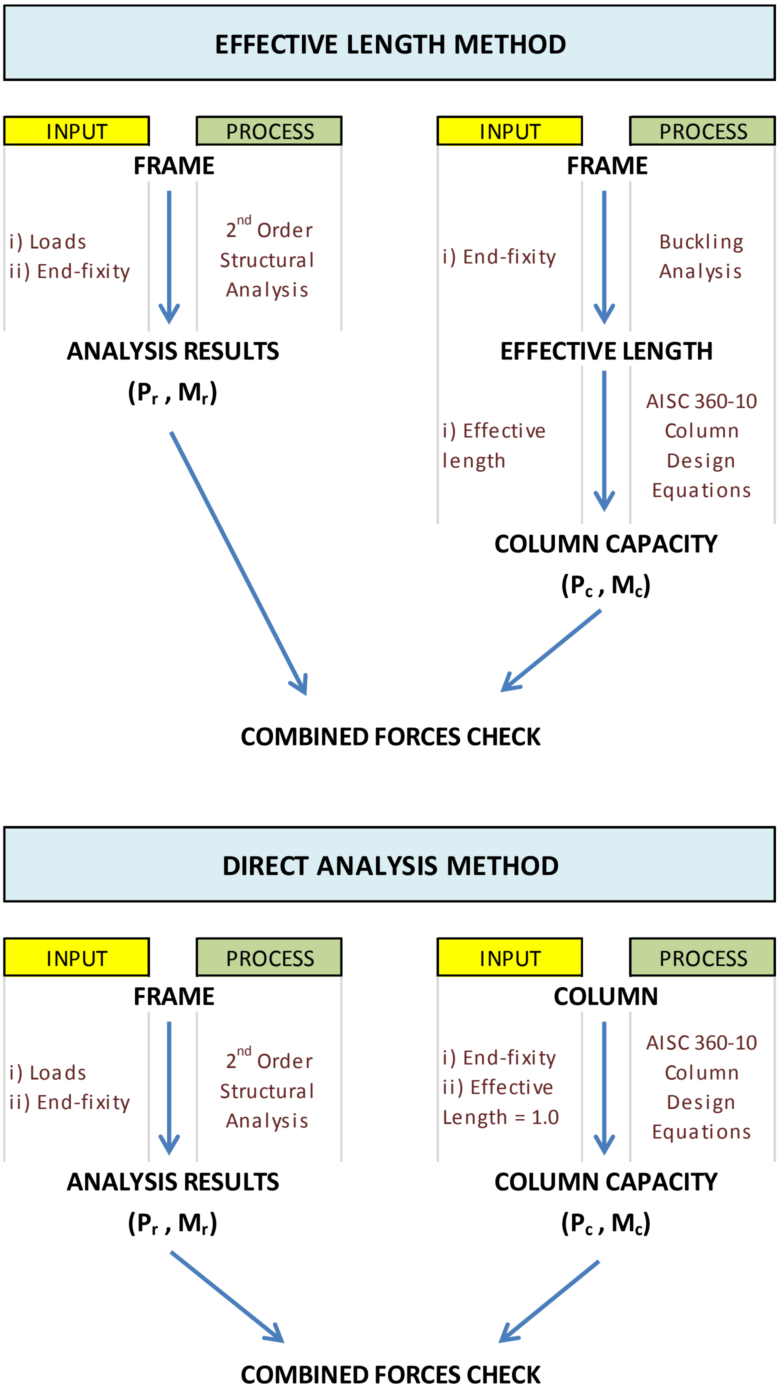
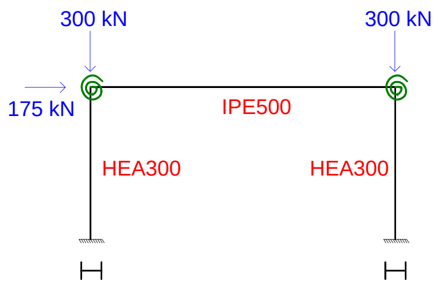
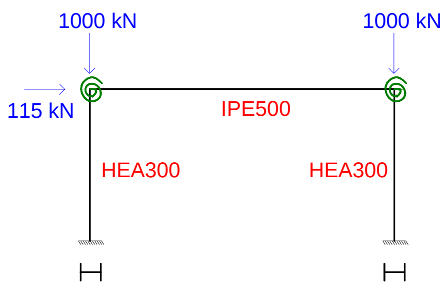


Figure 3.2 – Flowcharts of ELM and DAM

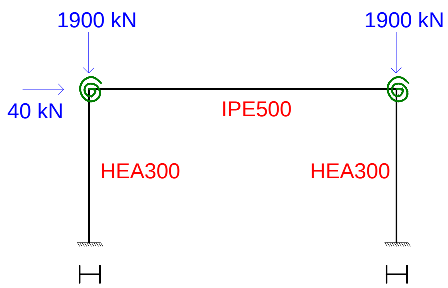
The general overview of the case studies is given in Figure 3.3.



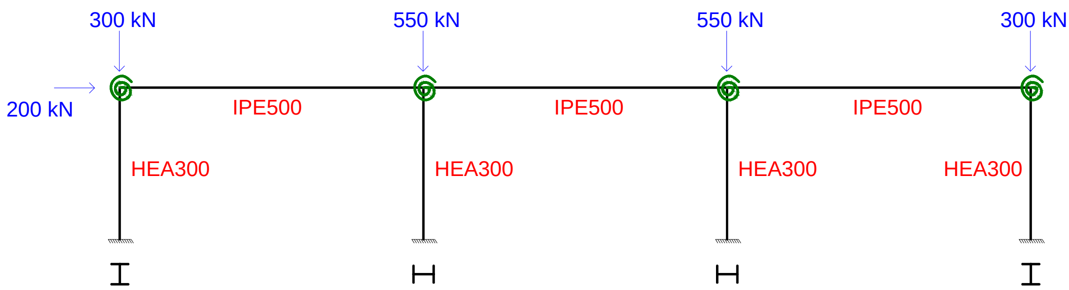
Case – I



Case – II



Case – III



Case – IV

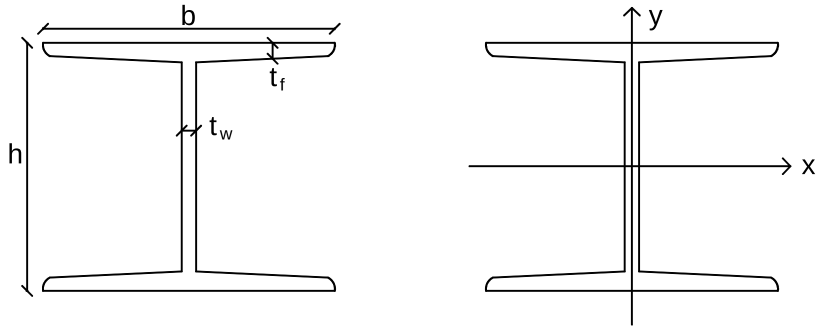
Figure 3.3 – The General Overview of Case Studies

Some general assumptions are made for the case studies. These can be listed as below;

1. All column-to-base connections are fixed whereas all beam-to-column connections are semi-rigid.
2. All springs (representing semi-rigid connections) in each case are identical, in other words they have the same stiffnesses.
3. In all cases, columns are HEA300 and beams are IPE500.
4. All members are made of the same material and they have the same yield strength, F_y = 345 MPa.
5. The beam members are assumed as axially rigid.
6. All columns, regardless of their orientation, are assumed as braced at their midpoint in out-of-plane direction. In other words, out-of-plane length of the columns is half of the in-plane length.
7. The applied loads are assumed as factored loads therefore no need to multiply these loads with load factors again.
8. Load and Resistance Factor Design (LRFD) is adopted for design calculations.

The physical properties of sections HEA300 and IPE500 are given in Table 3.1.

Table 3.1 – Physical Properties of Sections



	HEA300	IPE500
E : elastic modulus	200000 MPa	200000 MPa
F_y : yield strength	345 MPa	345 MPa
b : width of the flange	300 mm	200 mm
h : total depth	290 mm	500 mm
t_f : thickness of flange	14 mm	16 mm
t_w : thickness of web	8.5 mm	10.2 mm
I_x : moment of inertia in x-dir.	182600000 mm^4	482000000 mm^4
I_y : moment of inertia in y-dir.	63100000 mm^4	21420000 mm^4
A : cross-sectional area	11300 mm^2	11300 mm^2
r_x : radius of gyration in x-dir.	127.1 mm	203.8 mm
r_y : radius of gyration in y-dir.	74.7 mm	43.0 mm
J : torsional constant	878000 mm^4	891000 mm^4
S_x : section modulus about x-axis	1259310 mm^3	1928000 mm^3
S_y : section modulus about y-axis	420667 mm^3	214200 mm^3
Z_x : plastic section modulus about x-axis	1833000 mm^3	2194000 mm^3
Z_y : plastic section modulus about y-axis	641000 mm^3	336000 mm^3

3.2. CASE STUDY – I

One-bay, one-story portal frame is analyzed and designed in this case. The portal frame consists of two columns with a height of 4m and one beam having 8m length. The geometry of the frame, sections and labels of members are shown in Figure 3.4. The column sections are HEA300 and the beam section is IPE500. The columns are oriented such that their strong axes are in the plane of bending.

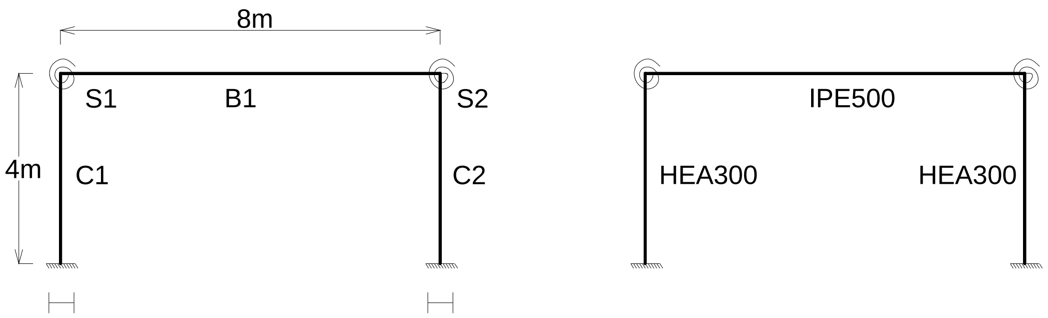


Figure 3.4 – Geometry of the Frame and Sections of Members in Case – I

The degrees of freedom and loads acting on the frame are shown in Figure 3.5. There are 5 degrees of freedom: u represents the lateral drift of the frame, v_1 and v_2 represent the axial deformation of columns and θ_1 and θ_2 represent the rotational deformations at each end of the beam. The horizontal load is 175 kN and the vertical loads acting on top of each column are 300 kN.

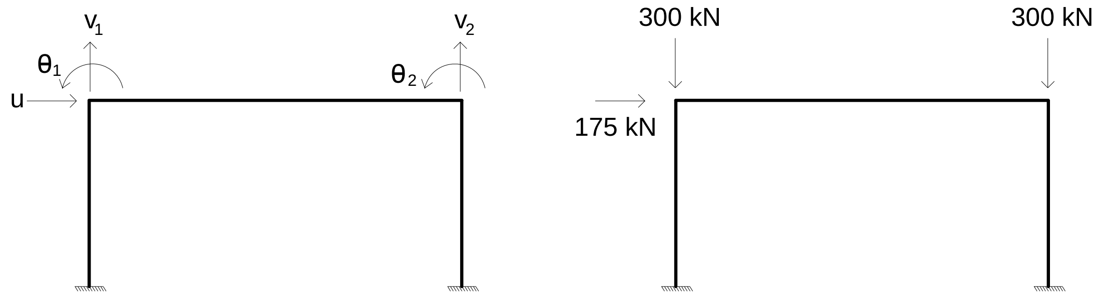


Figure 3.5 – Degrees of Freedom and Loads Acting on the Frame in Case - I

The portal frame is analyzed and designed with an end-fixity factor of 0.75 by using both stability methods, DAM and ELM.

3.2.1. Design with Effective Length Method

To design columns in the frame, first-order structural analysis should be conducted, and then the buckling length of the columns should be determined. At the end, the columns are designed according to AISC 360-10 Chapter H1.

As explained in Section 2.2, nominal stiffnesses of members are used during design with ELM. In addition, there is no need to use notional loads since there exists a horizontal load and the drift ratio is smaller than 1.5 (calculated in Section 3.2.1.1 as B_2).

3.2.1.1. Structural Analysis

Structural analysis of the frame is conducted by using stiffness method. First, the stiffness matrices of members are constructed then the system matrix is obtained. For the beam, stiffness matrix in Eqn. 2.19 in Section 2.3.4 is used. For the columns, the stiffness matrix in Eqn. 3.1 for the degrees of freedom given in Figure 3.6 is used.

Figure 3.6 – Degrees of Freedom for the Stiffness Matrix in Eqn. 3.1

$$k_{column} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad (3.1)$$

Since the columns are identical, their stiffness matrices are identical too. For the physical properties of HEA300 given in Table 3.1 (E, I and A) and column length of 4m (L), the stiffness matrix in Eqn. 3.1 becomes (units are in N and mm),

$$k_c = \begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix} \quad (3.2)$$

The displacement matrices of columns are,

$$d_{C1} = \begin{bmatrix} v_1 \\ u \\ \theta_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad d_{C2} = \begin{bmatrix} v_2 \\ u \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.3a \;\&\; 3.3b)$$

The stiffness matrix of the beam is constructed by using the matrix in Eqn. 2.19 but first connection stiffnesses should be determined. The end-fixity factor is chosen as 0.75 and accordingly the connection stiffnesses is determined by using Eqn. 2.20.

$$r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} \quad \rightarrow \quad 0.75 = \frac{1}{1 + \frac{3 \cdot (2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot R_{ki}}} \quad \rightarrow \quad R_{ki} = 1.085 \cdot 10^{11} N \cdot mm$$

Since the connections are assumed identical, R_{ki}, obtained above, is used in the below formulas for both R_{kiA} and R_{kiB}.

$$R^* = \left(1 + \frac{4EI}{LR_{kiA}}\right)\left(1 + \frac{4EI}{LR_{kiB}}\right) - \left(\frac{EI}{L}\right)^2 \left(\frac{4}{R_{kiA}R_{kiB}}\right)$$

$$R^* = \left(1 + \frac{4 \cdot (2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot 1.085 \cdot 10^{11}}\right)^2 - \left(\frac{2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000}\right)^2 \left(\frac{4}{(1.085 \cdot 10^{11})^2}\right)$$

$$R^* = 2.037$$

$$s_{ii}^* = \frac{\left(4 + \frac{12EI}{LR_{kiA}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{jj}^* = \frac{\left(4 + \frac{12EI}{LR_{kiB}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{ij}^* = s_{ji}^* = \frac{2}{R^*} = \frac{2}{2.037} = 0.98$$

The stiffness matrix in Eqn. 2.19 in Section 2.3.4 can be written as in Eqn. 3.4 (units are in N and mm) by substituting E, I, and A with the values given in Table 3.1, L with 8m, s_{ii}^*, s_{jj}^* and s_{ij}^* with the values found above. In addition, since the beam is assumed as axially rigid, the degree of freedom representing the axial displacement of the beam is removed.

$$k_{B1} = \begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix} \quad (3.4)$$

The displacement matrix of the beam member is,

$$d_{B1} = \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} \quad (3.5)$$

The stiffness matrix of the system, K, is constructed by combining the member stiffness matrices according to the system displacement matrix given in Eqn. 3.6.

$$D = \begin{bmatrix} u \\ v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} \quad (3.6)$$

$$K = \begin{bmatrix} 1.37x10^4 & 0 & 1.37x10^4 & 0 & 1.37x10^7 \\ 0 & 5.66x10^5 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 1.37x10^4 & 5.42x10^6 & 6.81x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ 0 & -1.36x10^3 & -5.42x10^6 & 5.66x10^5 & -5.42x10^6 \\ 1.37x10^4 & 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 6.81x10^{10} \end{bmatrix} \quad (3.7)$$

The force vector, Q, can be written as,

$$Q = \begin{bmatrix} 175000 \\ -300000 \\ 0 \\ -300000 \\ 0 \end{bmatrix} \begin{matrix} N \\ N \\ N \cdot m \\ N \\ N \cdot m \end{matrix} \quad (3.8)$$

The Eqn. 3.9 is solved to obtain displacement matrix, D.

$$Q = K\,D \quad \rightarrow \quad D = K^{-1}\,Q \tag{3.9}$$

The inverse of system stiffness matrix is taken and K^{-1} is obtained.

$$K^{-1} = \begin{bmatrix} 1.11x10^{-4} & 3.65x10^{-7} & -1.91x10^{-8} & -3.65x10^{-7} & -1.91x10^{-8} \\ 3.65x10^{-7} & 1.77x10^{-6} & -1.83x10^{-10} & 7.38x10^{-10} & -1.83x10^{-10} \\ -1.91x10^{-8} & -1.83x10^{-10} & 1.85x10^{-11} & 1.83x10^{-10} & 6.69x10^{-13} \\ -3.65x10^{-7} & 7.38x10^{-10} & 1.83x10^{-10} & 1.77x10^{-6} & 1.83x10^{-10} \\ -1.91x10^{-8} & -1.83x10^{-10} & 6.69x10^{-13} & 1.83x10^{-10} & 1.85x10^{-11} \end{bmatrix} \tag{3.10}$$

Multiplying K^{-1} by Q gives the displacement matrix D.

$$K^{-1} \qquad x \quad Q \quad = \quad D$$

$$\Uparrow \qquad \Uparrow \qquad \Uparrow$$

$$\begin{bmatrix} 1.11x10^{-4} & 3.65x10^{-7} & -1.91x10^{-8} & -3.65x10^{-7} & -1.91x10^{-8} \\ 3.65x10^{-7} & 1.77x10^{-6} & -1.83x10^{-10} & 7.38x10^{-10} & -1.83x10^{-10} \\ -1.91x10^{-8} & -1.83x10^{-10} & 1.85x10^{-11} & 1.83x10^{-10} & 6.69x10^{-13} \\ -3.65x10^{-7} & 7.38x10^{-10} & 1.83x10^{-10} & 1.77x10^{-6} & 1.83x10^{-10} \\ -1.91x10^{-8} & -1.83x10^{-10} & 6.69x10^{-13} & 1.83x10^{-10} & 1.85x10^{-11} \end{bmatrix} x \begin{bmatrix} 175000 \\ -300000 \\ 0 \\ -300000 \\ 0 \end{bmatrix} = \begin{bmatrix} 19.470 \\ -0.4671 \\ -0.0033 \\ -0.5949 \\ -0.0033 \end{bmatrix}$$

$$D = \begin{bmatrix} u \\ v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 19.470 \\ -0.4671 \\ -0.0033 \\ -0.5949 \\ -0.0033 \end{bmatrix} \begin{matrix} mm \\ mm \\ rad \\ mm \\ rad \end{matrix} \tag{3.11}$$

After the displacement matrix D is determined, it is multiplied with member stiffness matrices to obtain the member forces. For columns, C1 and C2, the stiffness matrix in Eqn. 3.2 and displacement matrices in Eqn. 3.3a and Eqn. 3.3b are used, respectively. For the beam, stiffness matrix in Eqn. 3.4 and displacement matrix in Eqn. 3.5 are used.

Member forces for C1 is (units are in kN and m),

$$\begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix} x \begin{bmatrix} -0.4671 \\ 19.470 \\ -0.0033 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -263.9 \\ 87.5 \\ 144.5 \\ 263.9 \\ -87.5 \\ 205.5 \end{bmatrix}$$

Member forces for C2 is (units are in kN and m),

$$\begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix} x \begin{bmatrix} -0.5949 \\ 19.470 \\ -0.0033 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -336.1 \\ 87.5 \\ 144.5 \\ 336.1 \\ -87.5 \\ 205.5 \end{bmatrix}$$

Member forces for B1 is (units are in kN and m),

$$\begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix} x \begin{bmatrix} -0.4671 \\ -0.0033 \\ -0.5949 \\ -0.0033 \end{bmatrix} = \begin{bmatrix} -36.1 \\ -144.5 \\ 36.1 \\ -144.5 \end{bmatrix}$$

These are the first order analysis results and can be shown on the system as in Figure 3.7.

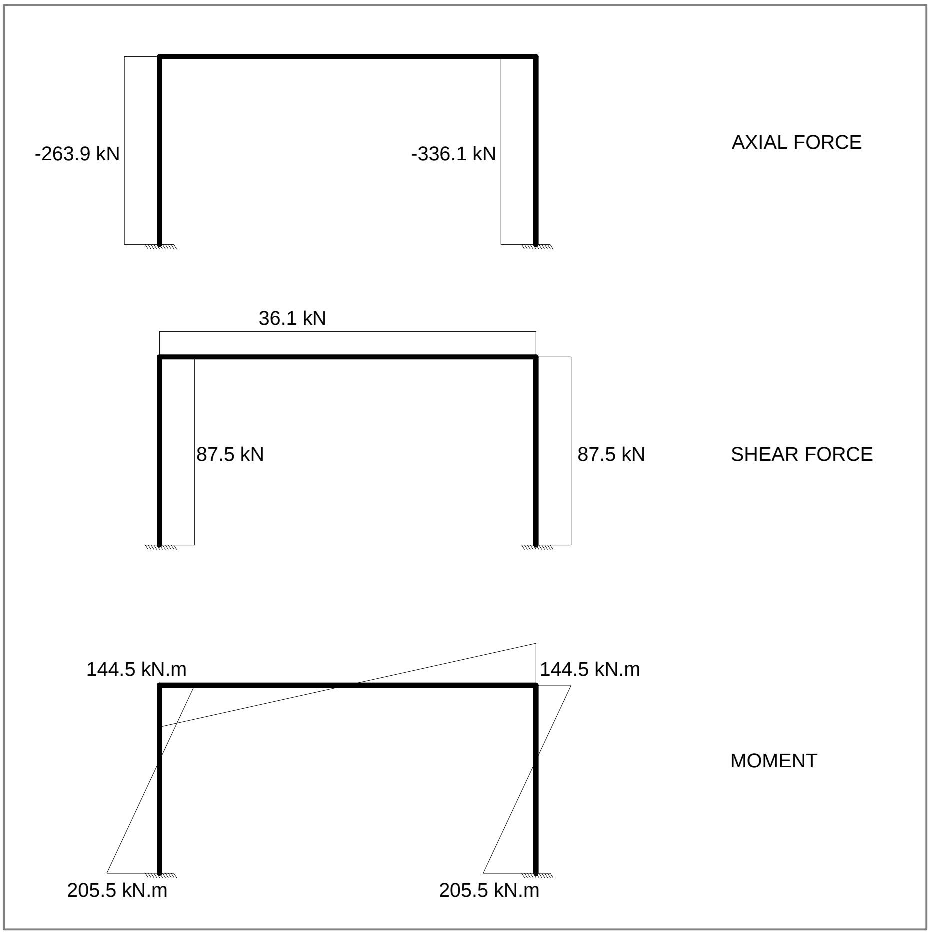


Figure 3.7 – First-Order Analysis Results of Case – I with ELM

As seen from the results, the critical column is C2 since the axial compressive force is higher on C2. C2 governs the column design therefore the first-order forces and moments of C2 are converted to second-order forces and moments with the approximate method described in Section 2.4. The second order design axial load P_r and design bending moment M_r are calculated as,

$$M_r = B_1 \cdot M_{nt} + B_2 \cdot M_{lt} = B_1 \cdot 0 + 1.0200 \cdot 205.5 = 209.7\ kN \cdot m$$

$$P_r = P_{nt} + B_2 \cdot P_{lt} = 300 + 1.02 \cdot 36.1 = 336.8\ kN \cdot m$$

$$M_{nt} = 0\ kN \cdot m \qquad \rightarrow \qquad No\ need\ to\ calculate\ B_1$$

$$M_{lt} = 205.5\ kN \cdot m$$

$$P_{nt} = 300\ kN$$

$$P_{lt} = 336.1 - 300 = 36.1\ kN$$

$$B_2 = \frac{1}{1 - \frac{\alpha \cdot P_{story}}{P_{e\ story}}} = \frac{1}{1 - \frac{1 \cdot 600}{30560}} = 1.02$$

$$P_{story} = 300 + 300 = 600\ kN$$

$$P_{e\ stroy} = R_M \frac{H \cdot L}{\Delta_H} = 0.85 \cdot \frac{175 \cdot 4000}{19.47} = 30560\ kN$$

$$R_M = 1 - 0.15 \cdot \frac{P_{mf}}{P_{story}} = 1 - 0.15 \cdot \frac{600}{600} = 0.85$$

$$\Delta_H = u = 19.47\ mm$$

As a summary, the second-order design forces and moments for the critical column C2 are,

$P_r = 336.8$ kN $\qquad$ $M_r = 209.7$ kN.m

3.2.1.2. Buckling Analysis

Buckling length of the frame is determined by setting the determinant of the stiffness matrix of the system equal to zero. However, during constructing the stiffness matrix, the column matrices shall be modified as described in Section 2.2 and so the stability functions including the term K (effective length factor) are included into the system. The smallest value of K, which makes the determinant equal to zero, is the buckling length of the frame.

The buckling analyses are performed with the help of Microsoft Office – Excel and in this section the calculation procedure in the spreadsheet is explained. The effective length factor is obtained by trial and error method in the spreadsheet therefore, for simplicity, only the calculation steps for the exact value of effective length are presented here.

For end-fixity factor of 0.75 the effective length factor K is determined as 1.193. Using Eqn. 2.13 k is obtained as,

$$K = \frac{\pi}{kL} \quad \rightarrow \quad k = \frac{\pi}{KL} = \frac{\pi}{1.193 \cdot 4000} = 6.58x10^{-4}$$

$$kL = 6.58x10^{-4} \cdot 4000 = 2.632$$

By using Eqns. 2.6, 2.7, 2.8, 2.9 and 2.10 the stability functions are obtained.

$$\emptyset_c = 2 - 2coskL - kL \cdot sinkL = 2 - 2 \cdot \cos(2.632) - 2.632 \cdot \sin(2.632) = 2.466$$

$$\emptyset_1 = \frac{(kL)^3 sinkL}{12\emptyset_c} = \frac{(2.632)^3 \cdot \sin(2.632)}{12 \cdot 2.466} = 0.300$$

$$\emptyset_2 = \frac{(kL)^2(1 - coskL)}{6\emptyset_c} = \frac{(2.632)^2\left(1 - cos(2.632)\right)}{6 \cdot 2.466} = 0.878$$

$$\emptyset_3 = \frac{kL(sinkL - kLcoskL)}{4\emptyset_c} = \frac{2.632\left(sin(2.632) - 2.632 \cdot cos(2.632)\right)}{4 \cdot 2.466} = 0.744$$

$$\emptyset_4 = \frac{kL(kL - sinkL)}{2\emptyset_c} = \frac{2.632\left(2.632 - sin(2.632)\right)}{2 \cdot 2.466} = 1.146$$

After determining stability functions, they are inserted into the column stiffness matrix as described in Eqn.2.5. The stiffness matrix of the columns C1 and C2 given in Eqn. 3.2 is modified as below,

$$k_c = \begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 2.06x10^3 & 1.20x10^7 & 0 & -2.06x10^3 & 1.20x10^7 \\ 0 & 1.20x10^7 & 2.72x10^{10} & 0 & -1.20x10^7 & 2.09x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -2.06x10^3 & -1.20x10^7 & 0 & 2.06x10^3 & -1.20x10^7 \\ 0 & 1.20x10^7 & 2.09x10^{10} & 0 & -1.20x10^7 & 2.72x10^{10} \end{bmatrix}$$

System stiffness matrix is formed with combining column matrices calculated above with the beam matrix obtained in Eqn. 3.4. In system stiffness matrix, elastic modulus (E) is the same for all members and each term of the matrix includes E therefore while taking the determinant of the matrix, E can be taken as a common multiple. To simplify the calculation, the elastic modulus E is assumed as 1. The system stiffness matrix with E=1 MPa is,

$$K = \begin{bmatrix} 0.02 & 0 & 60.14 & 0 & 60.14 \\ 0 & 2.83 & 27.11 & -0.01 & 27.11 \\ 60.14 & 27.11 & 293625 & -27.11 & 59155 \\ 0 & -0.01 & -27.11 & 2.83 & -27.11 \\ 60.14 & 27.11 & 59155 & -27.11 & 293625 \end{bmatrix}$$

Determinant of K equals to zero.

3.2.1.3. Column Design

In this part, the axial compressive strength and moment capacity of a HEA300 column having 4m length and in-plane effective length factor of 1.193 are determined and the column is checked under the combined effect of compression and flexure.

In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C1 and C2 is $M_c = 413.1$ kN.m. The calculation steps are given in Appendix A. The compressive strength of C1 and C2 is $P_c = 3164$ kN. The calculation steps are given in Appendix B.

The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand/capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{336.8}{3164.0} = 0.106 < 0.200$$

$$D/C = \frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{336.8}{2 \cdot 3164.0} + \frac{209.7}{413.1} = 0.561 < 1.000 \qquad OK!!$$

3.2.2. Design with Direct Analysis Method

In design according to DAM, the structural analysis is performed in the same way with ELM however in DAM reduced stiffnesses are used instead of nominal stiffnesses. The notional loads are not used in DAM too, with the same reason as in ELM. The stiffness reduction factor is determined according to Section 2.1 (Eqn. 2.2 & 2.3). It requires an iterative procedure to determine the reduction factor. First, a τ_b value is assumed and the structure is analyzed with this value then using the obtained forces, the assumed τ_b value is checked whether it is acceptable or not. If it is unacceptable, another value is assumed for τ_b and the same procedure is followed until τ_b satisfies the conditions. For the Case – I, in the first iteration step, τ_b is assumed as 1.0 and the structure is analyzed with the member stiffnesses reduced by 1.0x0.80 and the axial compressive load, P_r, is obtained as 337 kN (obtained in Section 3.2.2.1) for the critical column C2.

$$\frac{\alpha P_r}{P_y} = \frac{1 \cdot 337000}{11300 \cdot 345} = 0.09 < 0.50 \qquad \rightarrow \qquad \tau_b = 1.0$$

In the first iteration step τ_b is obtained. The stiffness reduction factor is 0.8 for both columns, C1 and C2 and for the beam. The P_r value above is calculated for the critical column C2, the axial load on C1 is smaller than the one on C2 therefore the ratio for C1 is also smaller than 0.50.

3.2.2.1. Structural Analysis

The structural analysis by using DAM is conducted by using the same procedure as described in ELM. Therefore, the calculation steps are skipped and only the resultant stiffness matrix of the system, displacement matrix and member forces are presented. The stiffness matrix of the system is,

$$K = \begin{bmatrix} 1.10x10^4 & 0 & 1.10x10^4 & 0 & 1.10x10^7 \\ 0 & 4.53x10^5 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 1.10x10^4 & 4.34x10^6 & 5.45x10^{10} & -4.34x10^6 & 9.46x10^{10} \\ 0 & -1.08x10^3 & -4.34x10^6 & 4.53x10^5 & -4.34x10^6 \\ 1.10x10^4 & 4.34x10^6 & 9.46x10^{10} & -4.34x10^6 & 5.45x10^{10} \end{bmatrix}$$

The displacement matrix of the system is,

$$D = \begin{bmatrix} u \\ v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 24.3380 \\ -0.5838 \\ -0.0042 \\ -0.7436 \\ -0.0042 \end{bmatrix} \begin{matrix} mm \\ mm \\ rad \\ mm \\ rad \end{matrix}$$

The first-order analysis results are as in Figure 3.8.

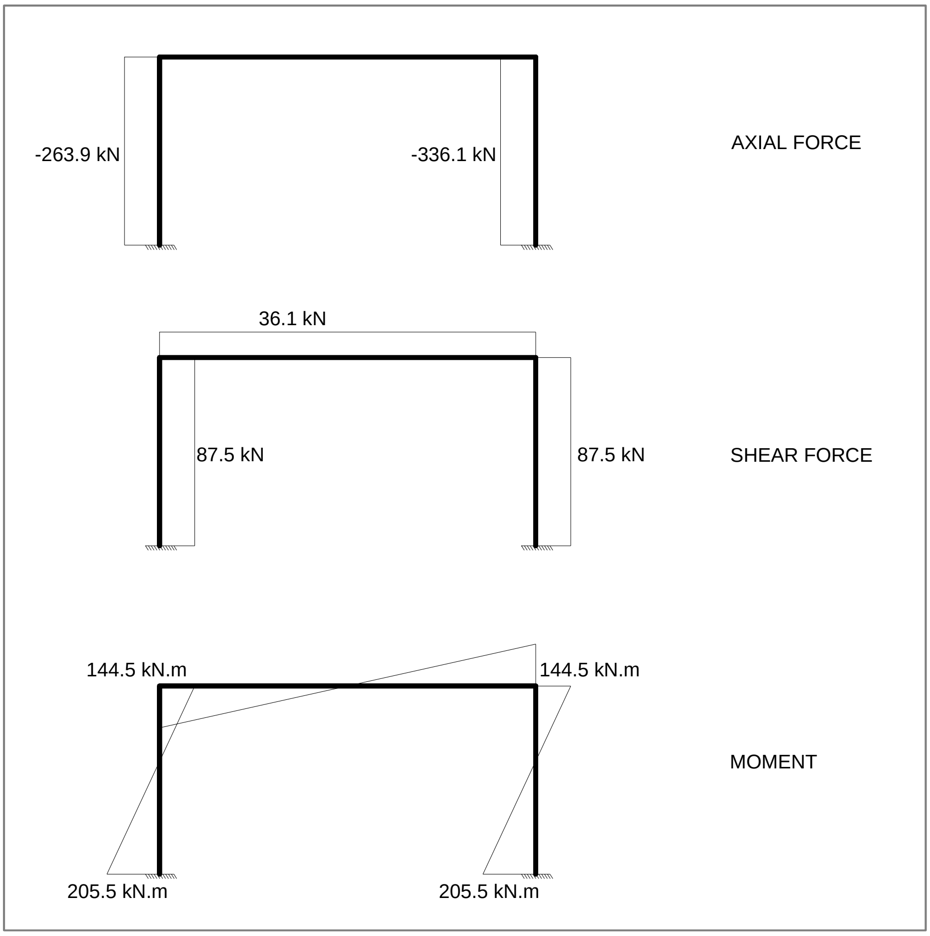


Figure 3.8 – First-Order Analysis Result of Case – I with DAM

The second-order analysis results are obtained as;

$$M_r = B_1 \cdot M_{nt} + B_2 \cdot M_{lt} = B_1 \cdot 0 + 1.0252 \cdot 205.5 = 210.7\ kN \cdot m$$

$$P_r = P_{nt} + B_2 \cdot P_{lt} = 300.0 + 1.0252 \cdot 36.1 = 337.0\ kN \cdot m$$

$$M_{nt} = 0\ kN \cdot m \qquad \rightarrow \qquad No\ need\ to\ calculate\ B_1$$

$$M_{lt} = 205.5\ kN \cdot m$$

$$P_{nt} = 300.0\ kN$$

$$P_{lt} = 336.1 - 300.0 = 36.1\ kN$$

$$B_2 = \frac{1}{1 - \frac{\alpha \cdot P_{story}}{P_{e\ story}}} = \frac{1}{1 - \frac{1 \cdot 600}{24447}} = 1.0252$$

$$P_{story} = 300 + 300 = 600\ kN$$

$$P_{e\ stroy} = R_M \frac{H \cdot L}{\Delta_H} = 0.85 \cdot \frac{175 \cdot 4000}{24.338} = 24447\ kN$$

$$R_M = 1 - 0.15 \cdot \frac{P_{mf}}{P_{story}} = 1 - 0.15 \cdot \frac{600}{600} = 0.85$$

$$\Delta_H = u = 24.338\ mm$$

As a summary, the second-order design forces and moments for the critical column C2 are,

P_r = 337.0 kN M_r = 210.7 kN.m

3.2.2.2. Column Design

In this part, the axial compressive strength and moment capacity of a HEA300 column with 4m length and in-plane effective length factor of 1.0 are determined and the column is checked under the combined effect of compression and flexure.

In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C_1 and C_2 is M_c = 413.1 kN.m. The calculation steps are given in Appendix A. The compressive strength of C1 and C2 is P_c = 3263 kN. The calculation steps are given in Appendix B.

The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{337.0}{3263.0} = 0.103 < 0.200$$

$$\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{337.0}{2 \cdot 3263.0} + \frac{210.7}{413.1} = 0.562 < 1.000 \qquad OK!!$$

3.3. CASE STUDY – II

The portal frame in Case – I is analyzed and designed with different loads in this case. The compressive loads are increased and the horizontal load is decreased. The geometry of the frame, sections and labels of members are shown in Figure 3.9. The degrees of freedom and loads acting on the frame are shown in Figure 3.10. The only difference between this frame and the frame in Case – I is the loads acting on it.

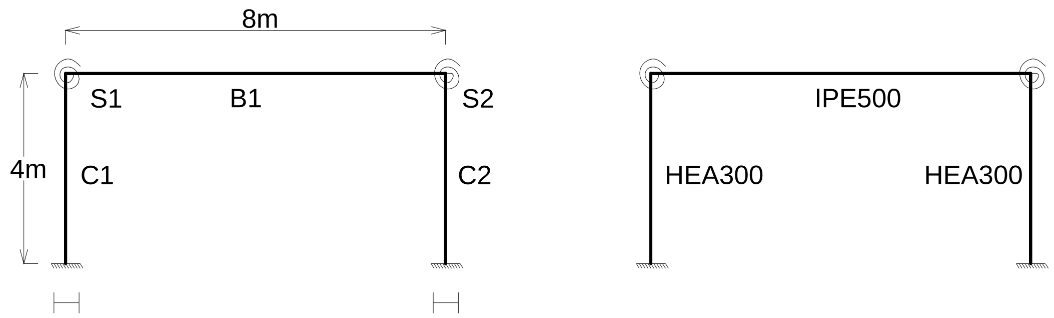


Figure 3.9 – Geometry of the Frame and Sections of Members in Case – II

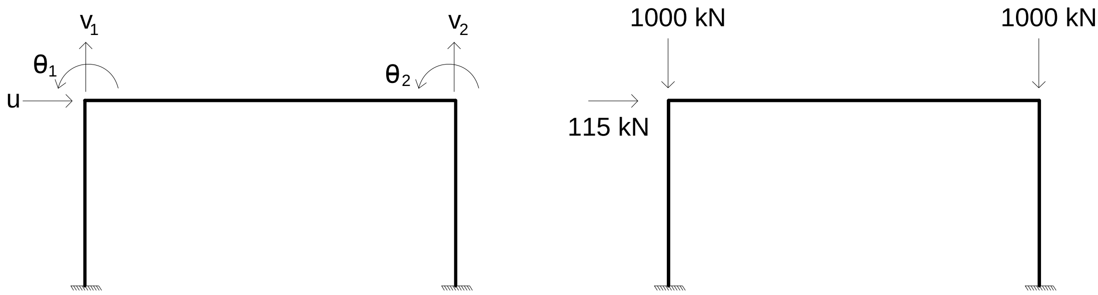


Figure 3.10 – Degrees of Freedom and Loads Acting on the Frame in Case - II

The portal frame is analyzed and designed with an end-fixity factor of 0.75 by using both stability methods, DAM and ELM.

3.3.1. Design with Effective Length Method

In Case – II, like in Case – I, first structural analysis is conducted, then buckling length of the columns is determined and at the end, the columns are designed. The procedures used in Case – I to obtain member forces and buckling length of the frame are followed in Case – II too, therefore in this part only the results are presented, the steps are given in Appendix C. In analysis with ELM, nominal stiffnesses of members are used and the notional loads are not used due to the presence of a horizontal load.

3.3.1.1. Structural Analysis

The stiffness matrix of the system, displacement matrix, force vector and member forces are presented here, for details of calculation please see Appendix C. The stiffness matrix of the system is,

$$K = \begin{bmatrix} 1.37x10^4 & 0 & 1.37x10^4 & 0 & 1.37x10^7 \\ 0 & 5.66x10^5 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 1.37x10^4 & 5.42x10^6 & 6.81x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ 0 & -1.36x10^3 & -5.42x10^6 & 5.66x10^5 & -5.42x10^6 \\ 1.37x10^4 & 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 6.81x10^{10} \end{bmatrix}$$

The displacement matrix D and force vector Q are,

$$D = \begin{bmatrix} u \\ v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 12.795 \\ -1.7279 \\ -0.0022 \\ -1.8119 \\ -0.0022 \end{bmatrix} \begin{matrix} mm \\ mm \\ rad \\ mm \\ rad \end{matrix} \qquad Q = \begin{bmatrix} 115000 \\ -1000000 \\ 0 \\ -1000000 \\ 0 \end{bmatrix} \begin{matrix} N \\ N \\ N \cdot m \\ N \\ N \cdot m \end{matrix}$$

The first-order analysis results are as in Figure 3.11.

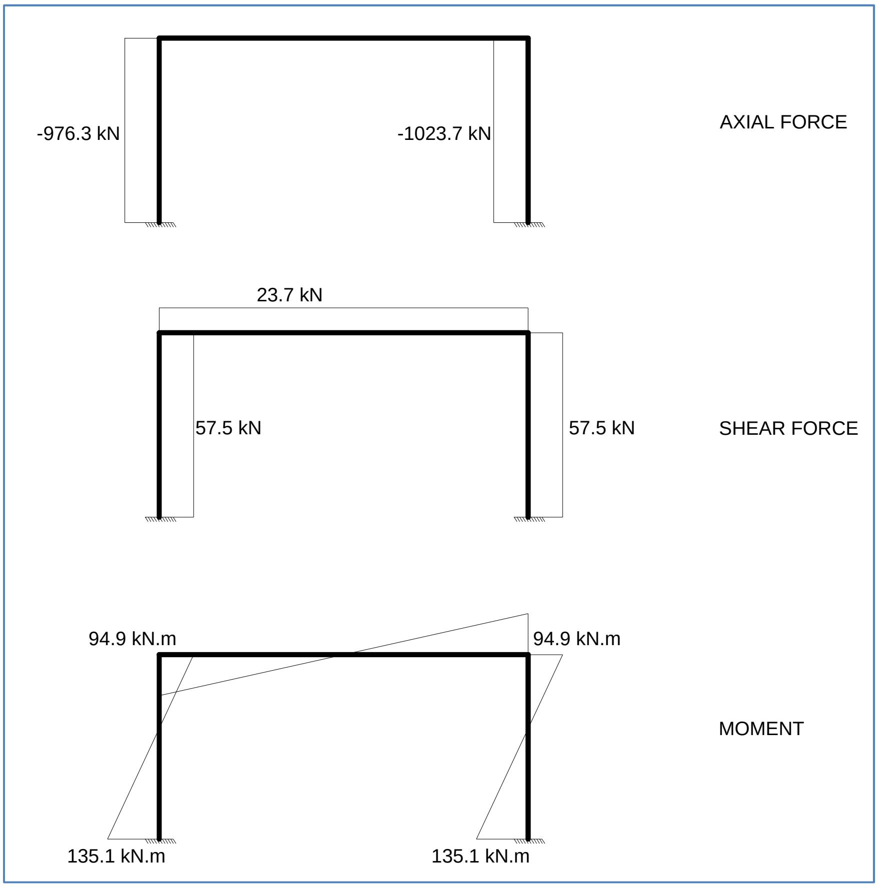


Figure 3.11 – First-Order Analysis Results of Case – II with ELM

The second-order design forces and moments for the critical column C2 are,

P_r = 1025.4 kN M_r = 144.5 kN.m

3.3.1.2. Buckling Analysis

The buckling analysis results for Case – II are the same with the results of Case – I since the frames are identical. The loads on the frame do not affect the buckling length of the frame. The buckling length of the frame for r = 0.75 is determined as 1.193.

3.3.1.3. Column Design

In this part, the axial compressive strength and moment capacity of a HEA300 column having 4m length and in-plane effective length factor of 1.193 are determined and the column is checked under the combined effect of compression and flexure.

In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C1 and C2 is $M_c = 413.1$ kN.m. The calculation steps are given in Appendix A. The compressive strength of C1 and C2 is $P_c = 3164$ kN. The calculation steps are given in Appendix B.

The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand/capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{1025.4}{3164.0} = 0.324 > 0.200$$

$$D/C = \frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{1025.4}{3164.0} + \frac{8}{9} \cdot \frac{144.5}{413.1} = 0.635 < 1.000 \qquad OK!!$$

3.3.2. Design with Direct Analysis Method

The columns are designed with effective length factor of 1.0 after determining the second-order forces from the structural analysis. During the analysis, reduced stiffnesses are used and the notional loads are ignored due to the presence of a horizontal load. The stiffness reduction factor is determined as described in Section 3.2.2. For the Case – II, the τ_b is obtained as 1.0 and the validity of it can be shown as;

$$\frac{\alpha P_r}{P_y} = \frac{1 \cdot 1025800}{11300 \cdot 345} = 0.26 < 0.50 \qquad \rightarrow \qquad \tau_b = 1.0$$

3.3.2.1. Structural Analysis

The stiffness matrix of the system, displacement matrix, force vector and member forces are presented here, for details of calculation please see Appendix C. The stiffness matrix of the system is,

$$K = \begin{bmatrix} 1.10x10^4 & 0 & 1.10x10^4 & 0 & 1.10x10^7 \\ 0 & 4.53x10^5 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 1.10x10^4 & 4.34x10^6 & 5.45x10^{10} & -4.34x10^6 & 9.46x10^{10} \\ 0 & -1.08x10^3 & -4.34x10^6 & 4.53x10^5 & -4.34x10^6 \\ 1.10x10^4 & 4.34x10^6 & 9.46x10^{10} & -4.34x10^6 & 5.45x10^{10} \end{bmatrix}$$

The displacement matrix D and force vector Q are,

$$D = \begin{bmatrix} u \\ v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 15.993 \\ -2.1599 \\ -0.0027 \\ -2.2649 \\ -0.0027 \end{bmatrix} \begin{matrix} mm \\ mm \\ rad \\ mm \\ rad \end{matrix} \qquad Q = \begin{bmatrix} 115000 \\ -1000000 \\ 0 \\ -1000000 \\ 0 \end{bmatrix} \begin{matrix} N \\ N \\ N \cdot m \\ N \\ N \cdot m \end{matrix}$$

The first-order analysis results are as in Figure 3.12.

The second-order design forces and moments for the critical column C2 are,

P_r = 1025.8 kN $\qquad$ M_r = 147.1 kN.m

3.3.2.2. Column Design

In this part, the axial compressive strength and moment capacity of a HEA300 column with 4m length and in-plane effective length factor of 1.0 are determined and the column is checked under the combined effect of compression and flexure.

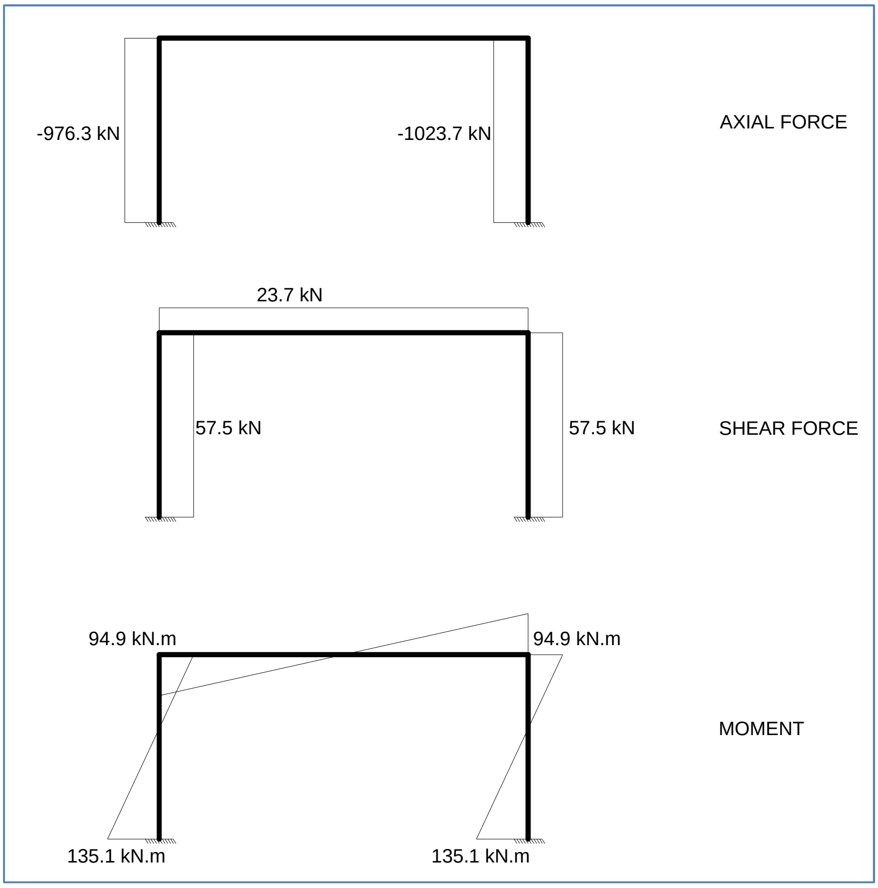


Figure 3.12 – First-Order Analysis Results of Case – II with DAM

In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C_1 and C_2 is $M_c = 413.1$ kN.m. The calculation steps are given in Appendix A. The compressive strength of C1 and C2 is $P_c = 3263$ kN. The calculation steps are given in Appendix B.

The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{1025.8}{3263.0} = 0.314 < 0.200$$

$$D/C = \frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{1025.8}{3263.0} + \frac{8}{9} \cdot \frac{147.1}{413.1} = 0.631 < 1.000 \qquad OK!!$$

3.4. CASE STUDY – III

The portal frame in Case – I and Case - II is analyzed and designed with different loads in this case. The compressive loads are increased and the horizontal load is decreased when compared with Case - II. The geometry of the frame, sections and labels of members are shown in Figure 3.13. The degrees of freedom and loads acting on the frame are shown in Figure 3.14. The only difference between this frame and the frames in Case – I and Case - II is the loads acting on it.

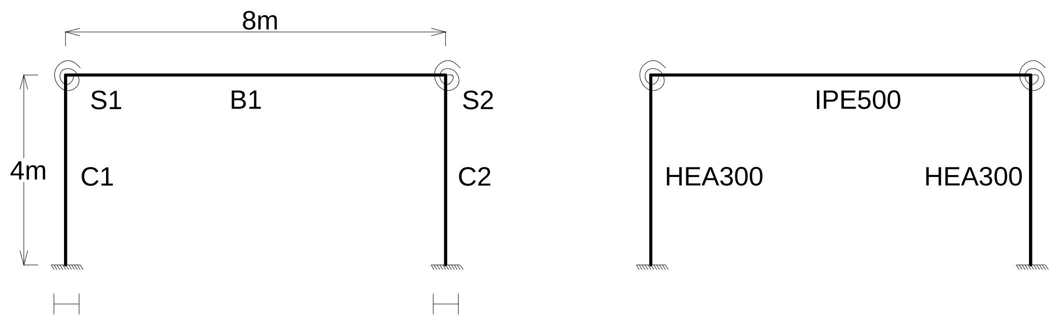


Figure 3.13 – Geometry of the Frame and Sections of Members in Case – III

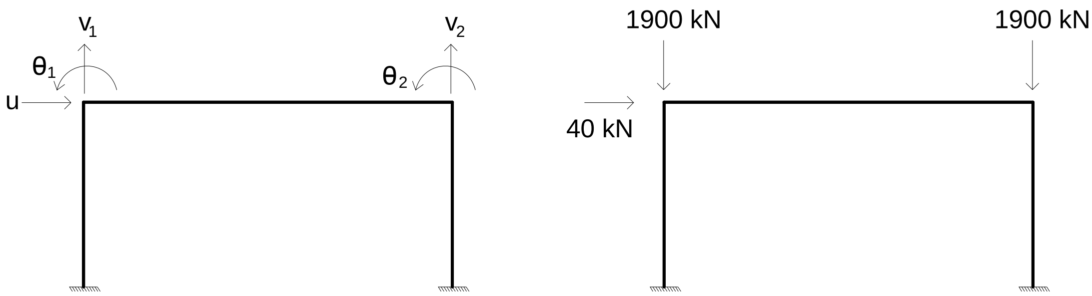


Figure 3.14 – Degrees of Freedom and Loads Acting on the Frame in Case - III

The portal frame is analyzed and designed with an end-fixity factor of 0.75 by using both stability methods, DAM and ELM.

3.4.1. Design with Effective Length Method

In Case – III, like in Case – I, first structural analysis is conducted, then buckling length of the columns is determined and at the end, the columns are designed. The procedures used in Case – I to obtain member forces and buckling length of the frame are followed in Case – III too, therefore in this part only the results are presented, the steps are given in Appendix C. In analysis with ELM, nominal stiffnesses of members are used and the notional loads are not used due to the presence of a horizontal load.

3.4.1.1. Structural Analysis

The stiffness matrix of the system, displacement matrix, force vector and member forces are presented here, for details of calculation please see Appendix C. The stiffness matrix of the system is,

$$K = \begin{bmatrix} 1.37x10^4 & 0 & 1.37x10^4 & 0 & 1.37x10^7 \\ 0 & 5.66x10^5 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 1.37x10^4 & 5.42x10^6 & 6.81x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ 0 & -1.36x10^3 & -5.42x10^6 & 5.66x10^5 & -5.42x10^6 \\ 1.37x10^4 & 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 6.81x10^{10} \end{bmatrix}$$

The displacement matrix D and force vector Q are,

$$D = \begin{bmatrix} u \\ v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 4.450 \\ -3.3482 \\ -0.0008 \\ -3.3774 \\ -0.0008 \end{bmatrix} \begin{matrix} mm \\ mm \\ rad \\ mm \\ rad \end{matrix} \qquad Q = \begin{bmatrix} 40000 \\ -1900000 \\ 0 \\ -1900000 \\ 0 \end{bmatrix} \begin{matrix} N \\ N \\ N\cdot m \\ N \\ N\cdot m \end{matrix}$$

The first-order analysis results are as in Figure 3.15.

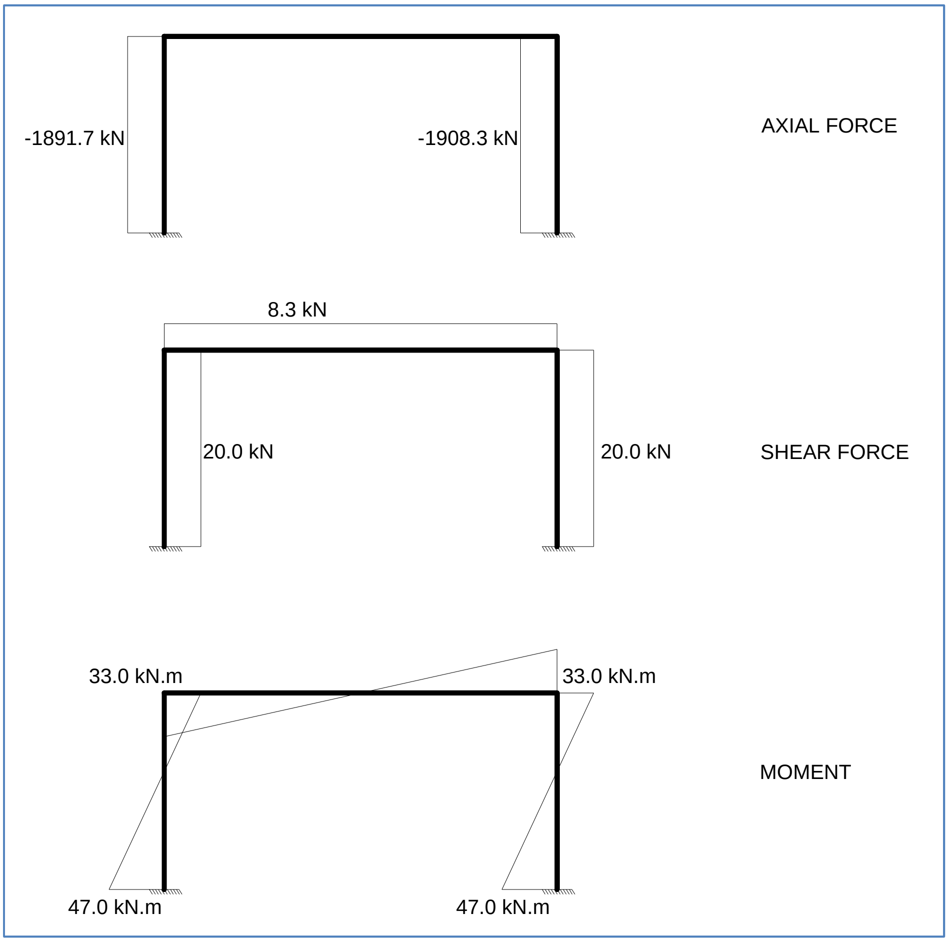


Figure 3.15 – First-Order Analysis Results of Case – III with ELM

The second-order design forces and moments for the critical column C2 are,

P_r = 1909.4 kN M_r = 53.7 kN.m

3.4.1.2. Buckling Analysis

The buckling analysis results for Case – III are the same with the results of Case – I and Case - II since the frames are identical. The loads on the frame do not affect the buckling length of the frame. The buckling length of the frame for r = 0.75 is determined as 1.193.

3.4.1.3. Column Design

In this part, the axial compressive strength and moment capacity of a HEA300 column having 4m length and in-plane effective length factor of 1.193 are determined and the column is checked under the combined effect of compression and flexure.

In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C1 and C2 is $M_c = 413.1$ kN.m. The calculation steps are given in Appendix A. The compressive strength of C1 and C2 is $P_c = 3164$ kN. The calculation steps are given in Appendix B.

The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand/capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{1909.4}{3164.0} = 0.603 > 0.200$$

$$D/C = \frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{1909.4}{3164.0} + \frac{8}{9} \cdot \frac{53.7}{413.1} = 0.719 < 1.000 \qquad OK!!$$

3.4.2. Design with Direct Analysis Method

The columns are designed with effective length factor of 1.0 after determining the second-order forces from the structural analysis. During the analysis, reduced stiffnesses are used and the notional loads are ignored due to the presence of a horizontal load. The stiffness reduction factor is determined as described in Section 3.2.2. For the Case – III, the τ_b is obtained as 1.0 and the validity of it can be shown as;

$$\frac{\alpha P_r}{P_y} = \frac{1 \cdot 1909800}{11300 \cdot 345} = 0.49 < 0.50 \qquad \rightarrow \qquad \tau_b = 1.0$$

3.4.2.1. Structural Analysis

The stiffness matrix of the system, displacement matrix, force vector and member forces are presented here, for details of calculation please see Appendix C. The stiffness matrix of the system is,

$$K = \begin{bmatrix} 1.10x10^4 & 0 & 1.10x10^4 & 0 & 1.10x10^7 \\ 0 & 4.53x10^5 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 1.10x10^4 & 4.34x10^6 & 5.45x10^{10} & -4.34x10^6 & 9.46x10^{10} \\ 0 & -1.08x10^3 & -4.34x10^6 & 4.53x10^5 & -4.34x10^6 \\ 1.10x10^4 & 4.34x10^6 & 9.46x10^{10} & -4.34x10^6 & 5.45x10^{10} \end{bmatrix}$$

The displacement matrix D and force vector Q are,

$$D = \begin{bmatrix} u \\ v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 5.563 \\ -4.1853 \\ -0.0010 \\ -4.2218 \\ -0.0010 \end{bmatrix} \begin{matrix} mm \\ mm \\ rad \\ mm \\ rad \end{matrix} \qquad Q = \begin{bmatrix} 40000 \\ -1900000 \\ 0 \\ -1900000 \\ 0 \end{bmatrix} \begin{matrix} N \\ N \\ N \cdot m \\ N \\ N \cdot m \end{matrix}$$

The first-order analysis results are as in Figure 3.16.

The second-order design forces and moments for the critical column C2 are,

P_r = 1909.8 kN $\qquad$ M_r = 55.6 kN.m

3.4.2.2. Column Design

In this part, the axial compressive strength and moment capacity of a HEA300 column with 4m length and in-plane effective length factor of 1.0 are determined and the column is checked under the combined effect of compression and flexure.

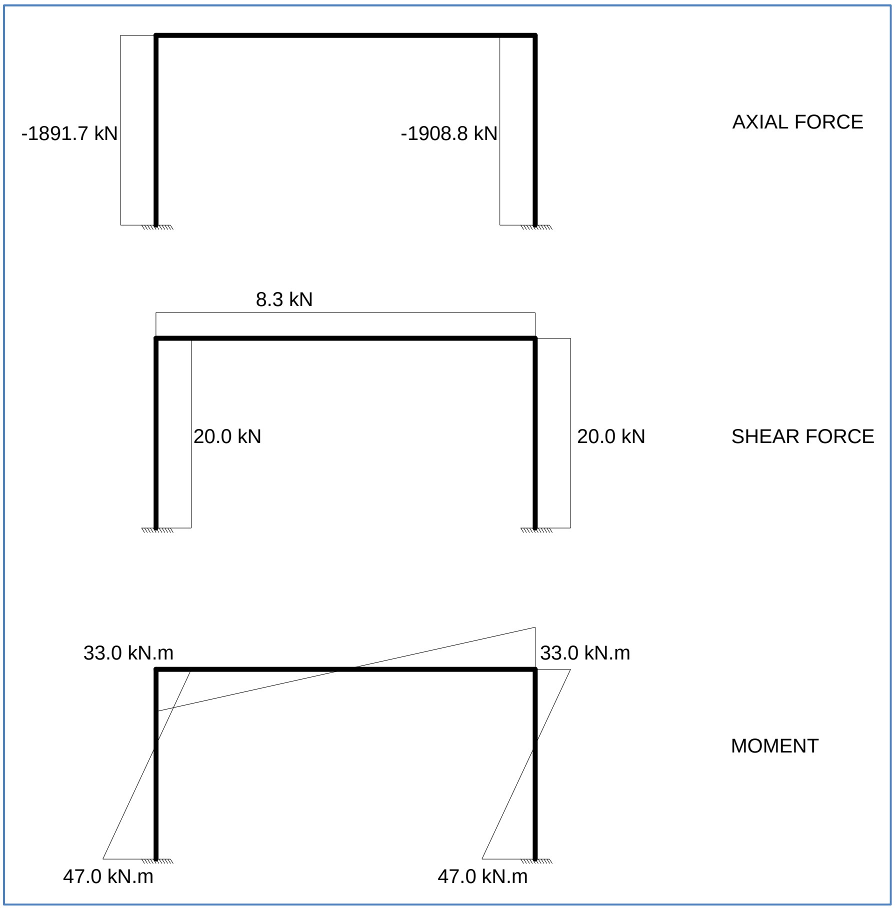


Figure 3.16 – First-Order Analysis Results of Case – III with DAM

In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C_1 and C_2 is $M_c = 413.1$ kN.m. The calculation steps are given in Appendix A. The compressive strength of C1 and C2 is $P_c = 3263$ kN. The calculation steps are given in Appendix B.

The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{1909.8}{3263.0} = 0.585 < 0.200$$

$$D/C = \frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{1909.8}{3263.0} + \frac{8}{9} \cdot \frac{55.6}{413.1} = 0.705 < 1.000 \qquad OK!!$$

3.5. CASE STUDY – IV

Three-bay, one-story portal frame is analyzed and designed in this case. The portal frame consists of four columns with a height of 4m and three beams having 8m length. The geometry of the frame, sections and labels of members are shown in Figure 3.17. The column sections are HEA300 and the beam sections are IPE500. The exterior columns (C1 & C4) are oriented such that their weak axes are in the plane of bending whereas interior columns (C2 & C3) are oriented such that their strong axes are in the plane of bending.

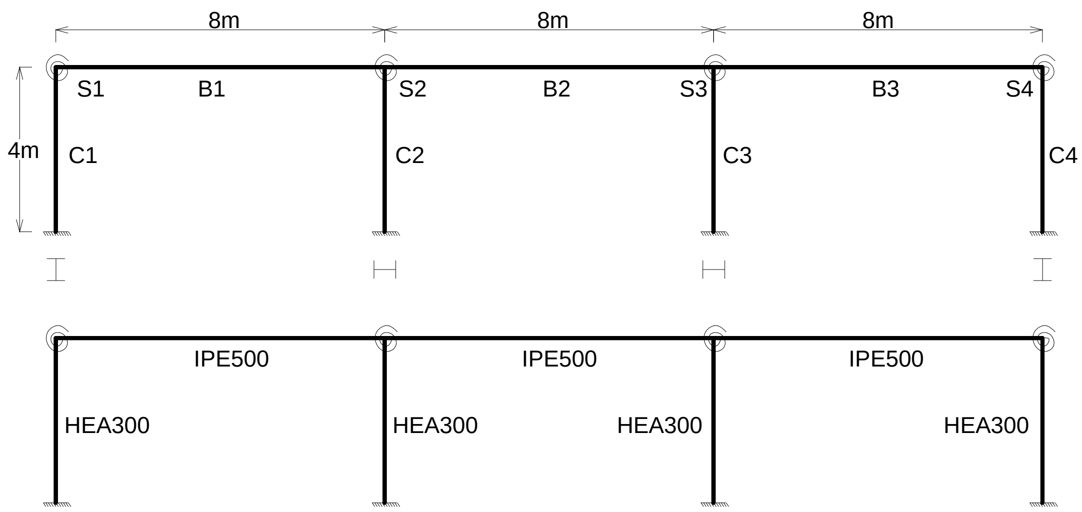


Figure 3.17 – Geometry of the Frame and Sections of Members in Case - IV

The degrees of freedom and loads acting on the frame are shown in Figure 3.18. There are total of 9 degrees of freedom: u represents the lateral drift of the frame, v_1, v_2, v_3 and v_4 represent the axial deformation of columns and θ_1, θ_2, θ_3 and θ_4 represent the rotational deformations at each end of the beam. The horizontal load is 200 kN and the vertical load acting on top of exterior columns is 300 kN and top of interior columns is 550 kN.

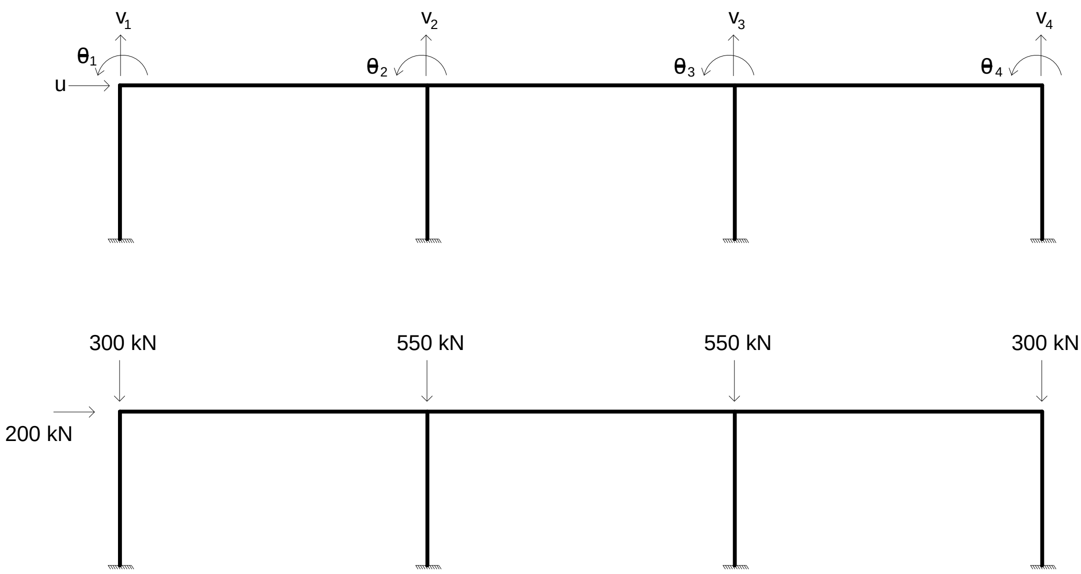


Figure 3.18 – Degrees of Freedom and Loads Acting on the Frame in Case – IV

The portal frame is analyzed and designed with an end-fixity factor of 0.75 by using both stability methods, DAM and ELM.

3.5.1. Design with Effective Length Method

In Case – IV, like in Case – I, first structural analysis is conducted, then buckling length of the columns is determined and at the end, the columns are designed. The procedures used in Case – I to obtain member forces and buckling length of the frame are followed in Case – IV too, therefore in this part only the results are presented, the steps are given in Appendix C. In analysis with ELM, nominal stiffnesses of members are used and the notional loads are not used due to the presence of a horizontal load.

3.5.1.1. Structural Analysis

The stiffness matrix of the system, displacement matrix, force vector and member forces are presented here, for details of calculation please see Appendix C. The stiffness matrix of the system is given in Figure 3.20. The displacement matrix D and force vector Q are,

$$D = \begin{bmatrix} u \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 13.739 \\ -0.0011 \\ -0.0016 \\ -0.0016 \\ -0.0010 \\ -0.5062 \\ -0.9680 \\ -0.9781 \\ -0.5566 \end{bmatrix} \begin{matrix} mm \\ rad \\ rad \\ rad \\ rad \\ mm \\ mm \\ mm \\ mm \end{matrix} \qquad Q = \begin{bmatrix} 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ -300 \\ -550 \\ -550 \\ -300 \end{bmatrix} \begin{matrix} kN \\ \\ \\ \\ \\ kN \\ kN \\ kN \\ kN \end{matrix}$$

The first order analysis results are as in Figure 3.19.

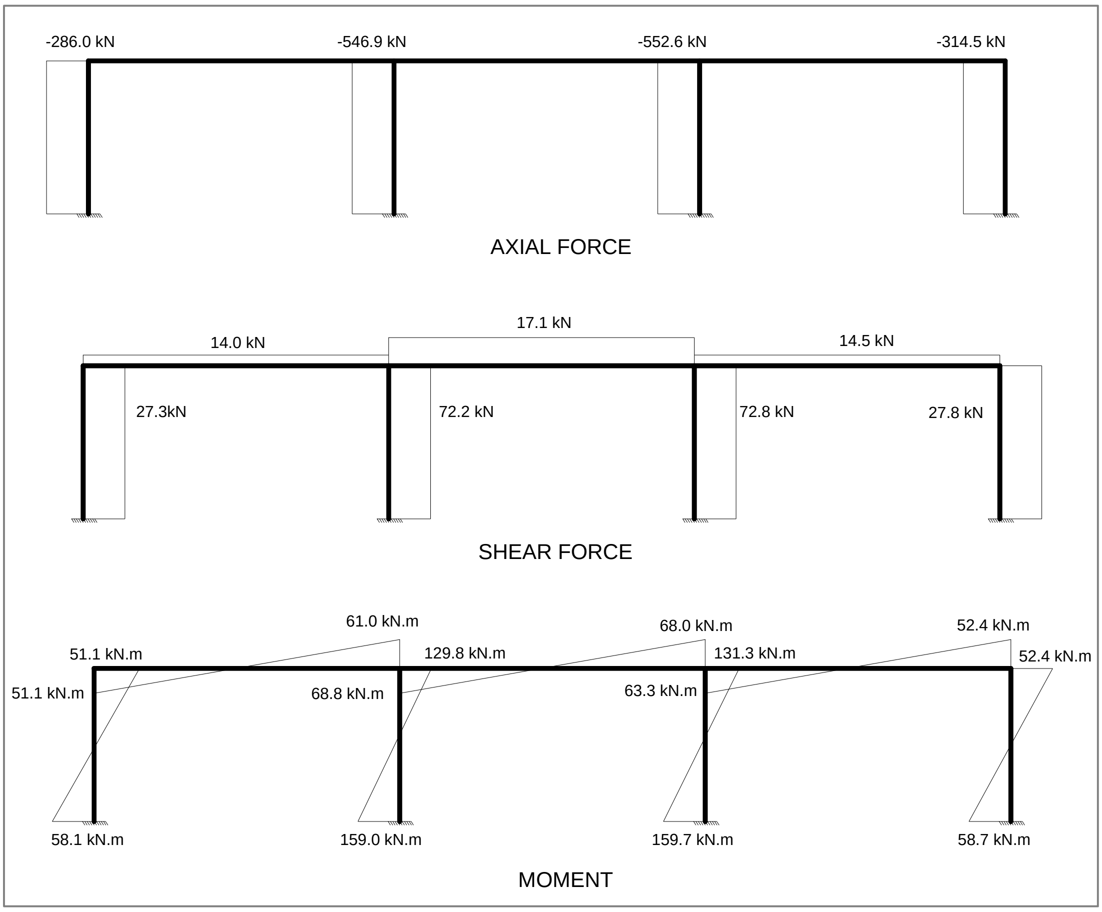


Figure 3.19 – First-Order Analysis Results of Case – IV with ELM

$$
K = \begin{bmatrix}
1.843x10^{4} & 4.733x10^{6} & 1.370x10^{7} & 1.370x10^{7} & 4.733x10^{6} & 0 & 0 & 0 & 0 \\
4.733x10^{6} & 4.417x10^{10} & 1.183x10^{10} & 0 & 0 & 5.423x10^{6} & -5.423x10^{6} & 0 & 0 \\
1.370x10^{7} & 1.183x10^{10} & 9.962x10^{10} & 1.183x10^{10} & 0 & 5.423x10^{6} & 0 & -5.423x10^{6} & 0 \\
1.370x10^{7} & 0 & 1.183x10^{10} & 9.962x10^{10} & 1.183x10^{10} & 0 & 5.423x10^{6} & 0 & -5.423x10^{6} \\
4.733x10^{6} & 0 & 0 & 1.183x10^{10} & 4.417x10^{10} & 0 & 0 & 5.423x10^{6} & -5.423x10^{6} \\
0 & 5.423x10^{6} & 5.423x10^{6} & 0 & 0 & 5.664x10^{5} & -1.356x10^{3} & 0 & 0 \\
0 & -5.423x10^{6} & 0 & 5.423x10^{6} & 0 & -1.356x10^{3} & 5.677x10^{5} & -1.356x10^{3} & 0 \\
0 & 0 & -5.423x10^{6} & 0 & 5.423x10^{6} & 0 & -1.356x10^{3} & 5.677x10^{5} & -1.356x10^{3} \\
0 & 0 & 0 & -5.423x10^{6} & -5.423x10^{6} & 0 & 0 & -1.356x10^{3} & 5.664x10^{5}
\end{bmatrix}
$$

Figure 3.20 – The Stiffness Matrix of the System in Case – IV with ELM

The second-order design forces and moments for the critical columns are;

Critical interior column	: C3	P_r = 552.7 kN	M_r = 165.4 kN·m
Critical exterior column	: C4	P_r = 315.0 kN	M_r = 60.8 kN·m

3.5.1.2. Buckling Analysis

The buckling analysis is performed with Microsoft Office - Excel and the buckling length of the frame is determined as 1.095. The resultant stiffness matrix of the system is presented in Figure 3.21. Like in the Case – I, elastic modulus E is assumed as 1 instead of 200000 to simplify the solution. The determinant of the matrix is equal to zero.

3.5.1.3. Column Design

3.5.1.3.1. Design of Interior Columns

The compressive and flexural strength of interior columns (C2 & C3) are determined and they are checked under the combined effect of compression and flexure. The in-plane effective length factor is calculated as 1.095. In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C2 and C3 is M_c = 413.1 kN.m. The calculation steps are given in Appendix A. The compressive strength of C2 and C3 is P_c = 3216 kN. The calculation steps are given in Appendix B. The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{552.7}{3216.3} = 0.172 < 0.200$$

$$D/C = \frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{552.7}{2 \cdot 3216.3} + \frac{165.4}{413.1} = 0.486 < 1.000 \qquad OK!!$$

$$K = \begin{bmatrix} 0.015 & 20.207 & 58.476 & 58.476 & 20.207 & 0 & 0 & 0 & 0 \\ 20.207 & 201266.896 & 59154.545 & 0 & 0 & 27.113 & -27.113 & 0 & 0 \\ 58.476 & 59154.545 & 441434.097 & 59154.545 & 0 & 27.113 & 0 & -27.113 & 0 \\ 58.476 & 0 & 59154.545 & 441434.097 & 59154.545 & 0 & 27.113 & 0 & -27.113 \\ 20.207 & 0 & 0 & 59154.545 & 201266.896 & 0 & 0 & 27.113 & -27.113 \\ 0 & 27.113 & 27.113 & 0 & 0 & 2.832 & -0.007 & 0 & 0 \\ 0 & -27.113 & 0 & 27.113 & 0 & -0.007 & 2.839 & -0.007 & 0 \\ 0 & 0 & -27.113 & 0 & 27.113 & 0 & -0.007 & 2.839 & -0.007 \\ 0 & 0 & 0 & -27.113 & 27.113 & 0 & 0 & -0.007 & 2.832 \end{bmatrix}$$

Figure 3.21 – The Stiffness Matrix of the System for Buckling Analysis in Case – IV

3.5.1.3.2. Design of Exterior Columns

The compressive and flexural strength of exterior columns (C1 & C4) are determined and they are checked under the combined effect of compression and flexure. The in-plane effective length factor is calculated as 1.095. In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C1 and C4 is M_c = 187.8 kN.m. The calculation steps are given in Appendix A. The compressive strength of C1 and C4 is P_c = 2732 kN. The calculation steps are given in Appendix B. The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{315.0}{2732.1} = 0.115 < 0.200$$

$$D/C = \frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{315.0}{2 \cdot 2732.1} + \frac{60.8}{187.8} = 0.381 < 1.000 \qquad OK!!$$

3.5.2. Design with Direct Analysis Method

The columns are designed with effective length factor of 1.0 after determining the second-order forces from the structural analysis. During the analysis, reduced stiffnesses are used and the notional loads are ignored due to the presence of a horizontal load. The stiffness reduction factor is determined as described in Section 3.2.2. For the Case – IV, the τ_b is obtained as 1.0 and the validity of it can be shown as;

$$\frac{\alpha P_r}{P_y} = \frac{1 \cdot 552800}{11300 \cdot 345} = 0.14 < 0.50 \qquad \rightarrow \qquad \tau_b = 1.0$$

3.5.2.1. Structural Analysis

The stiffness matrix of the system, displacement matrix, force vector and member forces are presented here, for details of calculation please see Appendix C. The stiffness matrix of the system is given in Figure 3.23. The displacement matrix D, force vector Q and first-order analysis results are as below;

$$D = \begin{bmatrix} u \\ \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 17.1741 \\ -0.0014 \\ -0.0020 \\ -0.0019 \\ -0.0013 \\ -0.6327 \\ -1.2100 \\ -1.2227 \\ -0.6957 \end{bmatrix} \begin{matrix} mm \\ rad \\ rad \\ rad \\ rad \\ mm \\ mm \\ mm \\ mm \end{matrix} \qquad Q = \begin{bmatrix} 200 \\ 0 \\ 0 \\ 0 \\ 0 \\ -300 \\ -550 \\ -550 \\ -300 \end{bmatrix} \begin{matrix} kN \\ \\ \\ \\ \\ kN \\ kN \\ kN \\ kN \end{matrix}$$

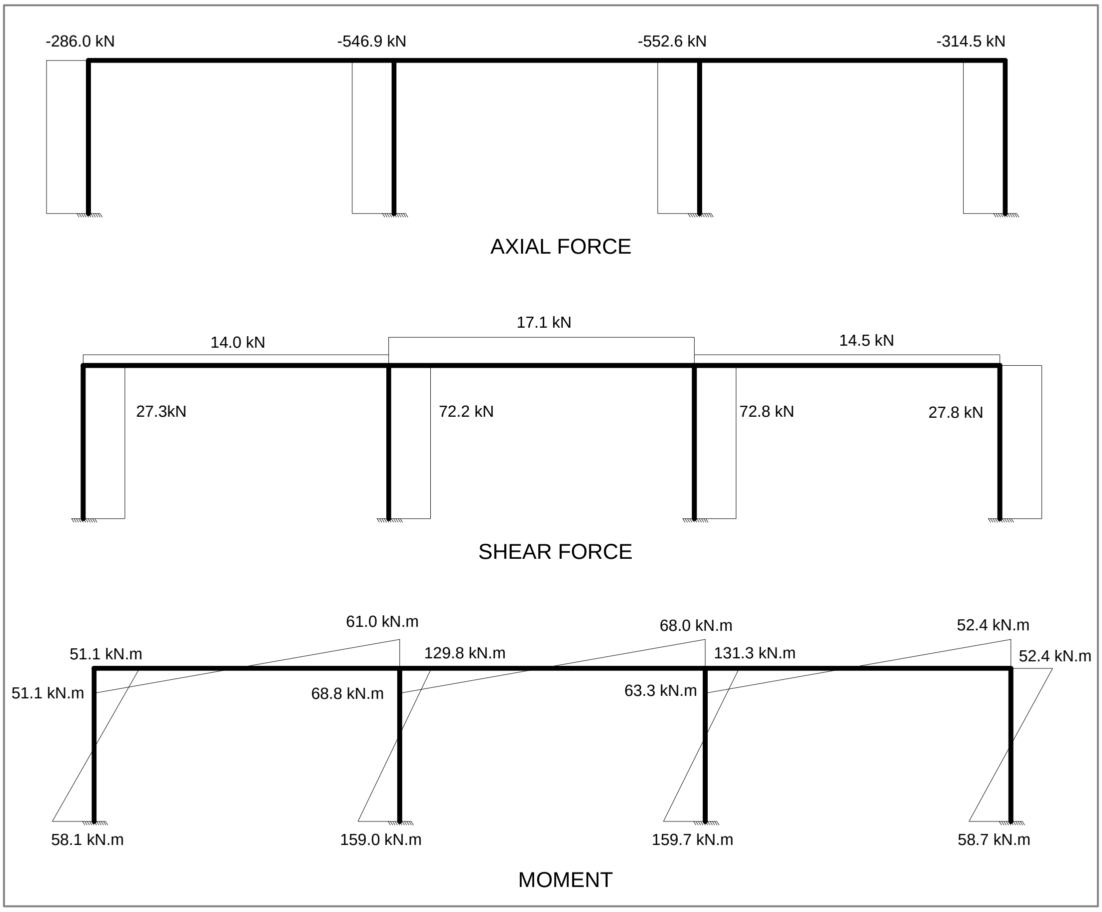


Figure 3.22 – First-Order Analysis Results of Case – IV with DAM

$$
K = \begin{bmatrix}
1.474x10^{4} & 3.786x10^{6} & 1.096x10^{7} & 1.096x10^{7} & 3.786x10^{6} & 0 & 0 & 0 & 0 \\
3.786x10^{6} & 3.534x10^{10} & 9.465x10^{9} & 0 & 0 & 4.338x10^{6} & -4.338x10^{6} & 0 & 0 \\
1.096x10^{7} & 9.465x10^{9} & 7.969x10^{10} & 9.465x10^{9} & 0 & 4.338x10^{6} & 0 & -4.338x10^{6} & 0 \\
1.096x10^{7} & 0 & 9.465x10^{9} & 7.969x10^{10} & 9.465x10^{9} & 0 & 4.338x10^{6} & 0 & -4.338x10^{6} \\
3.786x10^{6} & 0 & 0 & 9.465x10^{9} & 3.534x10^{10} & 0 & 0 & 4.338x10^{6} & -4.338x10^{6} \\
0 & 4.338x10^{6} & 4.338x10^{6} & 0 & 0 & 4.531x10^{5} & -1.085x10^{3} & 0 & 0 \\
0 & -4.338x10^{6} & 0 & 4.338x10^{6} & 0 & -1.085x10^{3} & 4.542x10^{5} & -1.085x10^{3} & 0 \\
0 & 0 & -4.338x10^{6} & 0 & 4.338x10^{6} & 0 & -1.085x10^{3} & 4.542x10^{5} & -1.085x10^{3} \\
0 & 0 & 0 & -4.338x10^{6} & -4.338x10^{6} & 0 & 0 & -1.085x10^{3} & 4.531x10^{5}
\end{bmatrix}
$$

Figure 3.23 – The Stiffness Matrix of the System in Case – IV with DAM

The second-order design forces and moments for the critical columns are;

Critical interior column	: C3	$P_r = 552.8$ kN	$M_r = 166.9$ kN·m
Critical exterior column	: C4	$P_r = 315.1$ kN	$M_r = 61.3$ kN·m

3.5.2.2. Column Design

3.5.2.2.1. Design of Interior Columns

The compressive and flexural strength of interior columns (C2 & C3) are determined and they are checked under the combined effect of compression and flexure. The in-plane effective length factor is 1.00. In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C2 and C3 is M_c = 413.1 kN.m. The calculation steps are given in Appendix A. The compressive strength of C2 and C3 is P_c = 3263 kN. The calculation steps are given in Appendix B. The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{552.8}{3263.1} = 0.169 < 0.200$$

$$D/C = \frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{552.8}{2 \cdot 3263.1} + \frac{166.9}{413.1} = 0.489 < 1.000 \qquad OK!!$$

3.5.2.2.2. Design of Exterior Columns

The compressive and flexural strength of exterior columns (C1 & C4) are determined and they are checked under the combined effect of compression and flexure. The in-plane effective length factor is calculated as 1.00. In-plane flexural strength of the column is determined according to Chapter F of the AISC 360-10. The in-plane flexural strength of C1 and C4 is M_c = 187.8 kN.m. The calculation steps are given in Appendix A. The compressive strength of C1 and C4 is P_c = 2848 kN. The calculation steps are given in Appendix B. The check of the column considering the interaction of flexure and compression is conducted according to Chapter H1 of AISC 360-10 and demand capacity ratio is obtained.

$$\frac{P_r}{P_c} = \frac{315.1}{2847.9} = 0.111 < 0.200$$

$$D/C = \frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{315.1}{2 \cdot 2847.9} + \frac{61.3}{187.8} = 0.382 < 1.000 \qquad OK!!$$

CHAPTER 4

RESULTS AND DISCUSSIONS OF RESULTS

4.1. RESULTS

In Section 3.1, it is explained that there are total of 168 analyses. In Sections 3.2, 3.3, 3.4 and 3.5, for each case one analysis is explained in details and the results of that analysis are presented at the end of those sections. In this section, all results for all analyses are presented.

The terms used in the tables are explained as below;

r : end-rigidity factor
P_r : second-order design compressive load (kN)
M_r : second-order design moment (kN·m)
P_c : compressive strength (kN)
M_c : flexural strength (kN·m)
K : effective length factor
B_2 : the ratio of second-order drift to first-order drift
D/C : demand/capacity ratio

In the tables given below, second-order forces and moments (P_r & M_r), member compressive and flexural strengths (P_c & M_c), effective length factors (K), drift ratios (B_2) and demand/capacity ratios for all end-rigidity factors (ranging from 0 to 1) and for both stability methods are presented. After each table, a figure showing the D/C ratios for both stability methods is presented.

4.1.1. Results of Case - I

Analysis and design results of the critical column C2 in Case – I are presented in Table 4.1.

Table 4.1 – Analysis and Design Results of C2 in Case – I

	Effective Length Method							*Direct Analysis Method*						
r	P_r	M_r	P_c	M_c	K	B_2	D/C	P_r	M_r	P_c	M_c	K	B_2	D/C
1.00	339.5	198.2	3203.4	413.1	1.120	1.018	0.533	339.7	199.1	3263.1	413.1	1.00	1.022	0.534
0.95	339.0	200.1	3196.6	413.1	1.133	1.018	0.537	339.2	201.0	3263.1	413.1	1.00	1.023	0.539
0.90	338.6	202.2	3189.7	413.1	1.146	1.018	0.543	338.7	203.1	3263.1	413.1	1.00	1.023	0.544
0.85	338.0	204.5	3182.2	413.1	1.160	1.019	0.548	338.2	205.4	3263.1	413.1	1.00	1.024	0.549
0.80	337.5	206.9	3174.2	413.1	1.175	1.019	0.554	337.6	208.0	3263.1	413.1	1.00	1.024	0.555
0.75	336.8	209.7	3164.9	413.1	1.192	1.020	0.561	337.0	210.7	3263.1	413.1	1.00	1.025	0.562
0.70	336.1	212.7	3154.4	413.1	1.211	1.021	0.568	336.3	213.8	3263.1	413.1	1.00	1.026	0.569
0.65	335.4	216.0	3143.3	413.1	1.231	1.021	0.576	335.6	217.2	3263.1	413.1	1.00	1.027	0.577
0.60	334.5	219.7	3130.3	413.1	1.254	1.022	0.585	334.7	220.9	3263.1	413.1	1.00	1.028	0.586
0.55	333.6	223.9	3115.9	413.1	1.279	1.023	0.595	333.7	225.2	3263.1	413.1	1.00	1.029	0.596
0.50	332.5	228.6	3099.6	413.1	1.307	1.024	0.607	332.7	229.9	3263.1	413.1	1.00	1.030	0.608
0.45	331.2	233.9	3080.6	413.1	1.339	1.025	0.620	331.4	235.4	3263.1	413.1	1.00	1.032	0.621
0.40	329.8	240.0	3058.8	413.1	1.375	1.027	0.635	330.0	241.6	3263.1	413.1	1.00	1.033	0.635
0.35	328.2	247.1	3033.5	413.1	1.416	1.028	0.652	328.4	248.9	3263.1	413.1	1.00	1.035	0.653
0.30	326.2	255.5	3003.9	413.1	1.463	1.030	0.673	326.4	257.4	3263.1	413.1	1.00	1.038	0.673
0.25	324.0	265.4	2969.0	413.1	1.517	1.032	0.697	324.1	267.5	3263.1	413.1	1.00	1.040	0.697
0.20	321.2	277.4	2926.6	413.1	1.581	1.035	0.726	321.4	279.8	3263.1	413.1	1.00	1.044	0.727
0.15	317.7	292.2	2875.5	413.1	1.656	1.038	0.763	317.9	295.0	3263.1	413.1	1.00	1.048	0.763
0.10	313.4	311.0	2811.6	413.1	1.747	1.042	0.809	313.5	314.3	3263.1	413.1	1.00	1.053	0.809
0.05	307.7	335.6	2730.4	413.1	1.859	1.047	0.869	307.8	339.6	3263.1	413.1	1.00	1.060	0.869
0.00	300.0	369.0	2624.7	413.1	2.000	1.054	0.950	300.0	374.1	3263.1	413.1	1.00	1.069	0.952

The demand/capacity ratios of C2 for ELM and DAM are drawn in Figure 4.1.

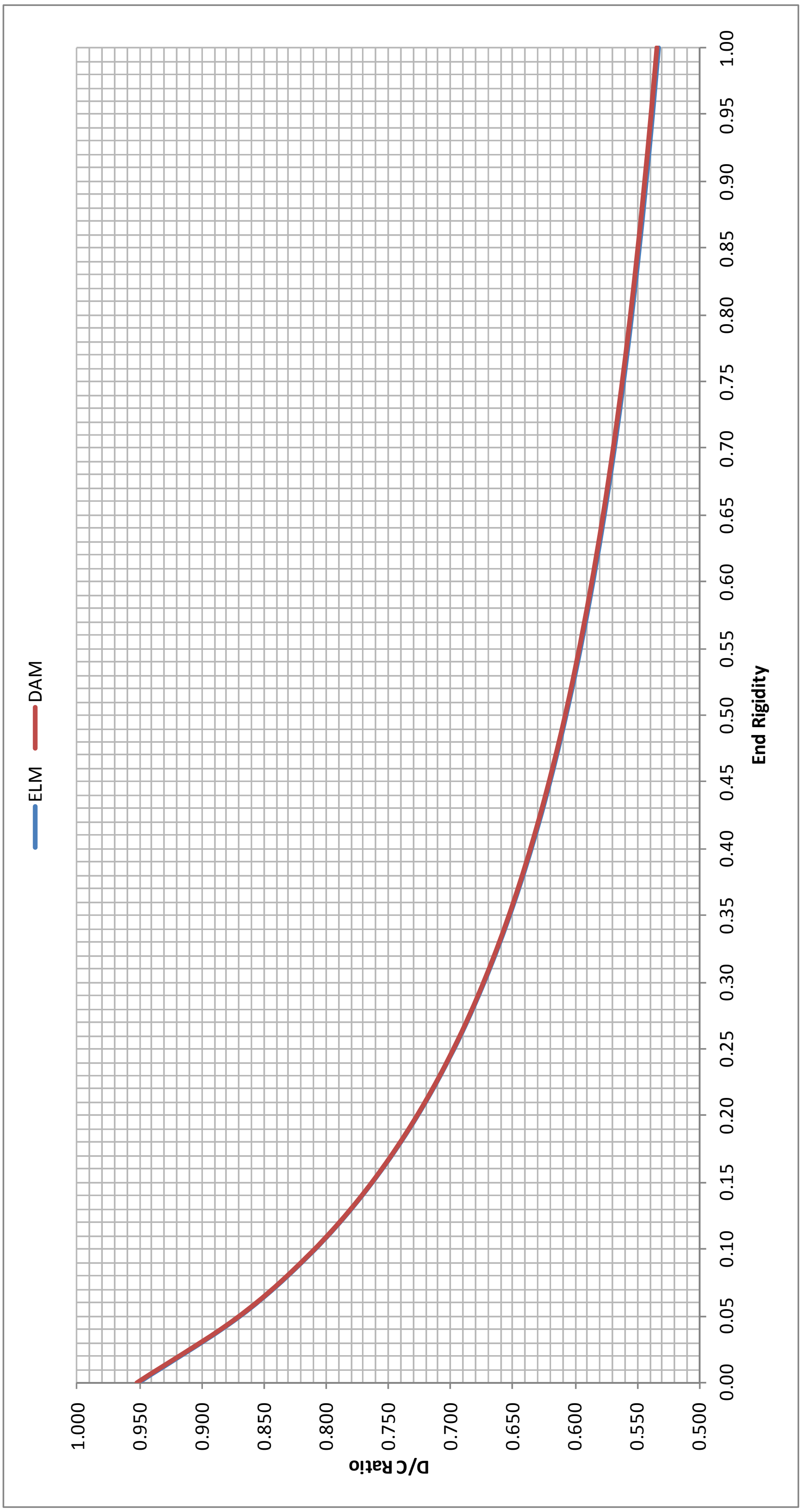


Figure 4.1 – Demand/Capacity vs. End-Rigidity for Case – I

4.1.2. Results of Case - II

Analysis and design results of the critical column C2 in Case – II are presented in Table 4.2.

Table 4.2 – Analysis and Design Results of C2 in Case – II

	Effective Length Method							*Direct Analysis Method*						
r	P_r	M_r	P_c	M_c	K	B_2	D/C	P_r	M_r	P_c	M_c	K	B_2	D/C
1.00	1027.1	135.8	3203.4	413.1	1.120	1.061	0.613	1027.5	137.9	3263.1	413.1	1.00	1.077	0.612
0.95	1026.8	137.2	3196.6	413.1	1.133	1.063	0.617	1027.2	139.4	3263.1	413.1	1.00	1.079	0.615
0.90	1026.5	138.8	3189.7	413.1	1.146	1.064	0.621	1026.9	141.1	3263.1	413.1	1.00	1.081	0.618
0.85	1026.2	140.6	3182.2	413.1	1.160	1.066	0.625	1026.6	142.9	3263.1	413.1	1.00	1.084	0.622
0.80	1025.8	142.5	3174.2	413.1	1.175	1.068	0.630	1026.2	144.9	3263.1	413.1	1.00	1.086	0.626
0.75	1025.4	144.5	3164.9	413.1	1.192	1.070	0.635	1025.8	147.1	3263.1	413.1	1.00	1.089	0.631
0.70	1025.0	146.8	3154.4	413.1	1.211	1.072	0.641	1025.4	149.5	3263.1	413.1	1.00	1.092	0.636
0.65	1024.5	149.4	3143.3	413.1	1.231	1.075	0.647	1024.9	152.3	3263.1	413.1	1.00	1.096	0.642
0.60	1023.9	152.3	3130.3	413.1	1.254	1.078	0.655	1024.4	155.3	3263.1	413.1	1.00	1.099	0.648
0.55	1023.3	155.5	3115.9	413.1	1.279	1.081	0.663	1023.8	158.7	3263.1	413.1	1.00	1.104	0.655
0.50	1022.6	159.1	3099.6	413.1	1.307	1.085	0.672	1023.1	162.6	3263.1	413.1	1.00	1.109	0.663
0.45	1021.8	163.3	3080.6	413.1	1.339	1.089	0.683	1022.3	167.1	3263.1	413.1	1.00	1.114	0.673
0.40	1020.9	168.1	3058.8	413.1	1.375	1.094	0.696	1021.4	172.2	3263.1	413.1	1.00	1.121	0.684
0.35	1019.8	173.8	3033.5	413.1	1.416	1.100	0.710	1020.3	178.3	3263.1	413.1	1.00	1.128	0.696
0.30	1018.5	180.5	3003.9	413.1	1.463	1.107	0.727	1019.1	185.4	3263.1	413.1	1.00	1.138	0.711
0.25	1017.0	188.5	2969.0	413.1	1.517	1.115	0.748	1017.5	194.1	3263.1	413.1	1.00	1.149	0.729
0.20	1015.1	198.3	2926.6	413.1	1.581	1.126	0.774	1015.6	204.7	3263.1	413.1	1.00	1.162	0.752
0.15	1012.8	210.6	2875.5	413.1	1.656	1.138	0.805	1013.2	218.2	3263.1	413.1	1.00	1.179	0.780
0.10	1009.8	226.5	2811.6	413.1	1.747	1.155	0.847	1010.2	235.6	3263.1	413.1	1.00	1.201	0.817
0.05	1005.7	247.8	2730.4	413.1	1.859	1.177	0.901	1006.0	259.2	3263.1	413.1	1.00	1.231	0.866
0.00	1000.0	277.7	2624.7	413.1	2.000	1.207	0.979	1000.0	292.9	3263.1	413.1	1.00	1.274	0.937

The demand/capacity ratios of C2 for ELM and DAM are drawn in Figure 4.2.

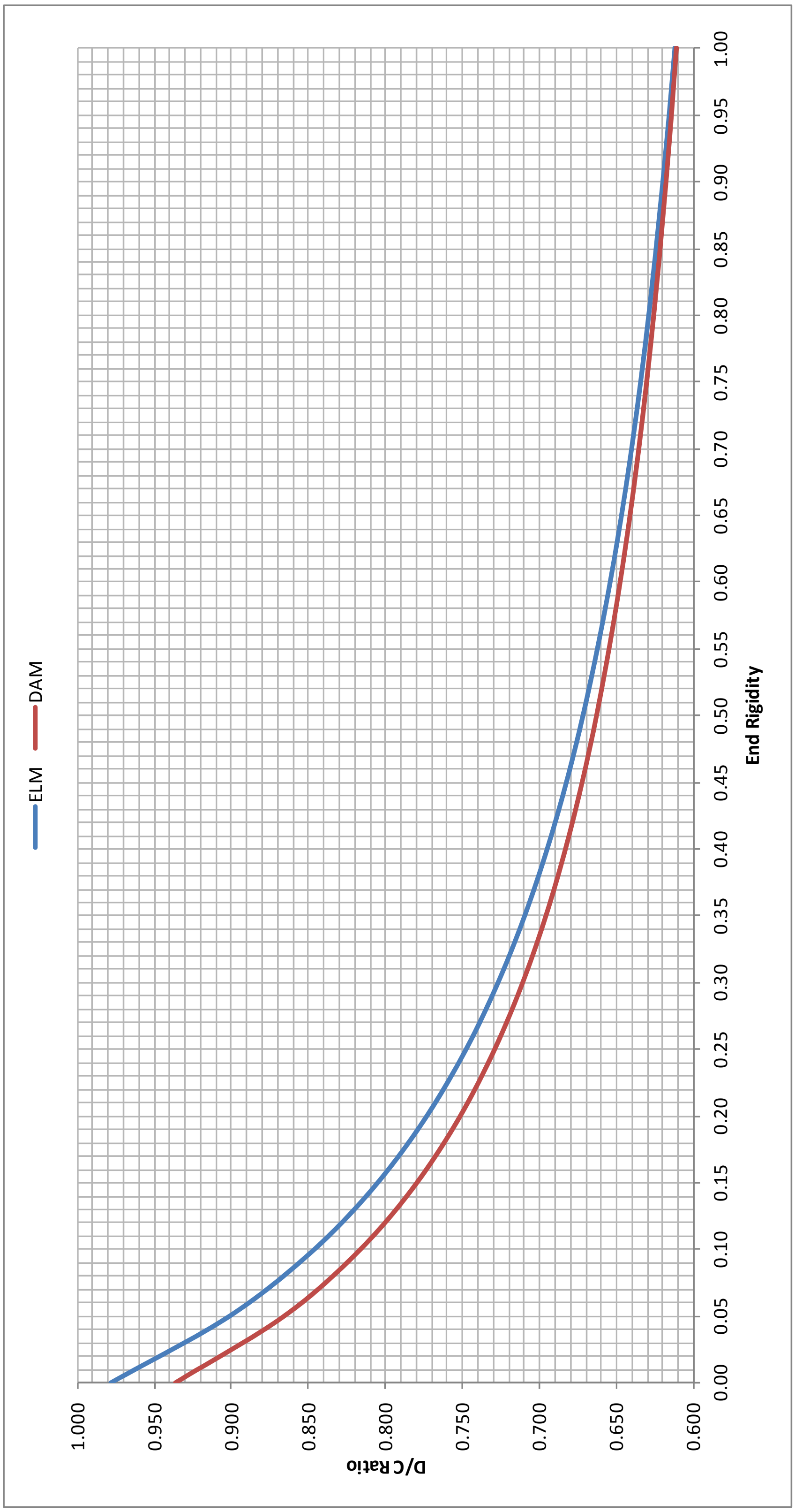


Figure 4.2 – Demand/Capacity vs. End-Rigidity for Case – II

4.1.3. Results of Case - III

Analysis and design results of the critical column C2 in Case – III are presented in Table 4.3.

Table 4.3 – Analysis and Design Results of C2 in Case – III

	Effective Length Method							*Direct Analysis Method*						
r	P_r	M_r	P_c	M_c	K	B_2	D/C	P_r	M_r	P_c	M_c	K	B_2	D/C
1.00	1910.0	50.0	3203.4	413.1	1.120	1.123	0.704	1910.3	51.6	3263.1	413.1	1.00	1.158	0.696
0.95	1909.9	50.6	3196.6	413.1	1.133	1.126	0.706	1910.2	52.2	3263.1	413.1	1.00	1.162	0.698
0.90	1909.8	51.2	3189.7	413.1	1.146	1.129	0.709	1910.1	53.0	3263.1	413.1	1.00	1.167	0.699
0.85	1909.7	52.0	3182.2	413.1	1.160	1.133	0.712	1910.0	53.8	3263.1	413.1	1.00	1.172	0.701
0.80	1909.6	52.8	3174.2	413.1	1.175	1.137	0.715	1909.9	54.6	3263.1	413.1	1.00	1.178	0.703
0.75	1909.4	53.7	3164.9	413.1	1.192	1.142	0.719	1909.8	55.6	3263.1	413.1	1.00	1.184	0.705
0.70	1909.3	54.6	3154.4	413.1	1.211	1.147	0.723	1909.6	56.7	3263.1	413.1	1.00	1.191	0.707
0.65	1909.1	55.7	3143.3	413.1	1.231	1.153	0.727	1909.5	57.9	3263.1	413.1	1.00	1.199	0.710
0.60	1908.9	57.0	3130.3	413.1	1.254	1.159	0.732	1909.3	59.3	3263.1	413.1	1.00	1.208	0.713
0.55	1908.7	58.4	3115.9	413.1	1.279	1.167	0.738	1909.1	60.9	3263.1	413.1	1.00	1.217	0.716
0.50	1908.5	59.9	3099.6	413.1	1.307	1.175	0.745	1908.9	62.7	3263.1	413.1	1.00	1.229	0.720
0.45	1908.3	61.8	3080.6	413.1	1.339	1.185	0.752	1908.7	64.8	3263.1	413.1	1.00	1.242	0.724
0.40	1907.9	63.9	3058.8	413.1	1.375	1.196	0.761	1908.4	67.2	3263.1	413.1	1.00	1.258	0.729
0.35	1907.6	66.4	3033.5	413.1	1.416	1.209	0.772	1908.0	70.1	3263.1	413.1	1.00	1.276	0.736
0.30	1907.1	69.5	3003.9	413.1	1.463	1.225	0.784	1907.6	73.6	3263.1	413.1	1.00	1.298	0.743
0.25	1906.6	73.2	2969.0	413.1	1.517	1.245	0.800	1907.0	77.9	3263.1	413.1	1.00	1.326	0.752
0.20	1905.9	77.8	2926.6	413.1	1.581	1.269	0.819	1906.4	83.4	3263.1	413.1	1.00	1.360	0.764
0.15	1905.1	83.7	2875.5	413.1	1.656	1.300	0.843	1905.5	90.5	3263.1	413.1	1.00	1.405	0.779
0.10	1903.9	91.5	2811.6	413.1	1.747	1.341	0.874	1904.3	100.1	3263.1	413.1	1.00	1.466	0.799
0.05	1902.4	102.5	2730.4	413.1	1.859	1.399	0.917	1902.6	113.8	3263.1	413.1	1.00	1.554	0.828
0.00	1900.0	118.8	2624.7	413.1	2.000	1.485	0.979	1900.0	135.1	3263.1	413.1	1.00	1.689	0.873

The demand/capacity ratios of C2 for ELM and DAM are drawn in Figure 4.3.

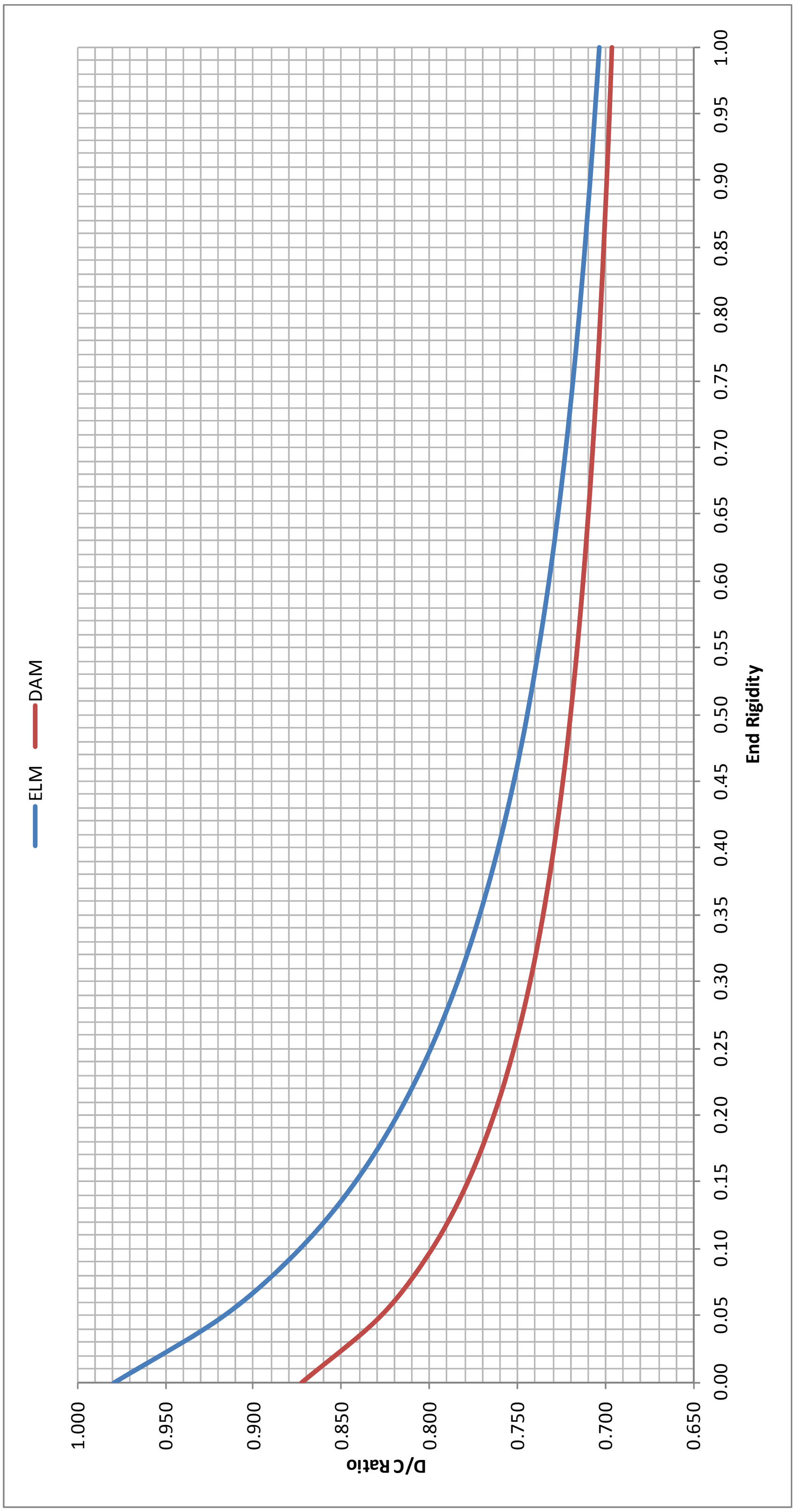


Figure 4.3 – Demand/Capacity vs. End-Rigidity for Case – III

4.1.4. Results of Case - IV

Analysis and design results of critical columns C3 (interior) and C4 (exterior) in Case – IV are presented in this part.

4.1.4.1. Results of C3 (Interior Column)

Table 4.4 - Analysis and Design Results of C3 (interior column) in Case – IV

	Effective Length Method							*Direct Analysis Method*						
r	P_r	M_r	P_c	M_c	K	B_2	D/C	P_r	M_r	P_c	M_c	K	B_2	D/C
1.00	553.8	160.4	3234.9	413.1	1.058	1.033	0.474	553.8	161.7	3263.1	413.1	1.00	1.041	0.476
0.95	553.5	161.2	3233.4	413.1	1.061	1.033	0.476	553.6	162.6	3263.1	413.1	1.00	1.042	0.478
0.90	553.3	162.1	3229.4	413.1	1.069	1.034	0.478	553.4	163.5	3263.1	413.1	1.00	1.042	0.480
0.85	553.1	163.1	3224.9	413.1	1.078	1.034	0.481	553.2	164.5	3263.1	413.1	1.00	1.043	0.483
0.80	552.9	164.2	3219.8	413.1	1.088	1.035	0.483	553.0	165.6	3263.1	413.1	1.00	1.044	0.486
0.75	552.7	165.4	3216.3	413.1	1.095	1.036	0.486	552.8	166.9	3263.1	413.1	1.00	1.045	0.489
0.70	552.6	166.8	3210.1	413.1	1.107	1.036	0.490	552.6	168.3	3263.1	413.1	1.00	1.046	0.492
0.65	552.4	168.4	3206.0	413.1	1.115	1.037	0.494	552.4	170.0	3263.1	413.1	1.00	1.047	0.496
0.60	552.2	170.2	3199.2	413.1	1.128	1.038	0.498	552.2	171.9	3263.1	413.1	1.00	1.048	0.501
0.55	552.0	172.3	3190.2	413.1	1.145	1.039	0.504	552.0	174.0	3263.1	413.1	1.00	1.050	0.506
0.50	551.8	174.8	3180.6	413.1	1.163	1.041	0.510	551.8	176.6	3263.1	413.1	1.00	1.051	0.512
0.45	551.6	177.7	3170.9	413.1	1.181	1.042	0.517	551.7	179.6	3263.1	413.1	1.00	1.054	0.519
0.40	551.5	181.3	3156.1	413.1	1.208	1.044	0.526	551.5	183.3	3263.1	413.1	1.00	1.056	0.528
0.35	551.3	185.6	3139.3	413.1	1.238	1.047	0.537	551.3	187.8	3263.1	413.1	1.00	1.059	0.539
0.30	551.1	191.0	3115.3	413.1	1.280	1.049	0.551	551.1	193.4	3263.1	413.1	1.00	1.063	0.553
0.25	550.9	198.0	3091.3	413.1	1.321	1.053	0.568	550.9	200.7	3263.1	413.1	1.00	1.067	0.570
0.20	550.7	207.3	3055.8	413.1	1.380	1.058	0.592	550.7	210.3	3263.1	413.1	1.00	1.074	0.594
0.15	550.5	220.3	3004.5	413.1	1.462	1.065	0.625	550.5	223.9	3263.1	413.1	1.00	1.082	0.626
0.10	550.3	239.7	2932.6	413.1	1.572	1.075	0.674	550.3	244.2	3263.1	413.1	1.00	1.095	0.676
0.05	550.1	271.6	2823.0	413.1	1.731	1.091	0.755	550.1	277.9	3263.1	413.1	1.00	1.116	0.757
0.00	550.0	333.5	2624.7	413.1	2.000	1.122	0.927	550.0	343.9	3263.1	413.1	1.00	1.157	0.917

The demand/capacity ratios of C3 for ELM and DAM are drawn in Figure 4.4.

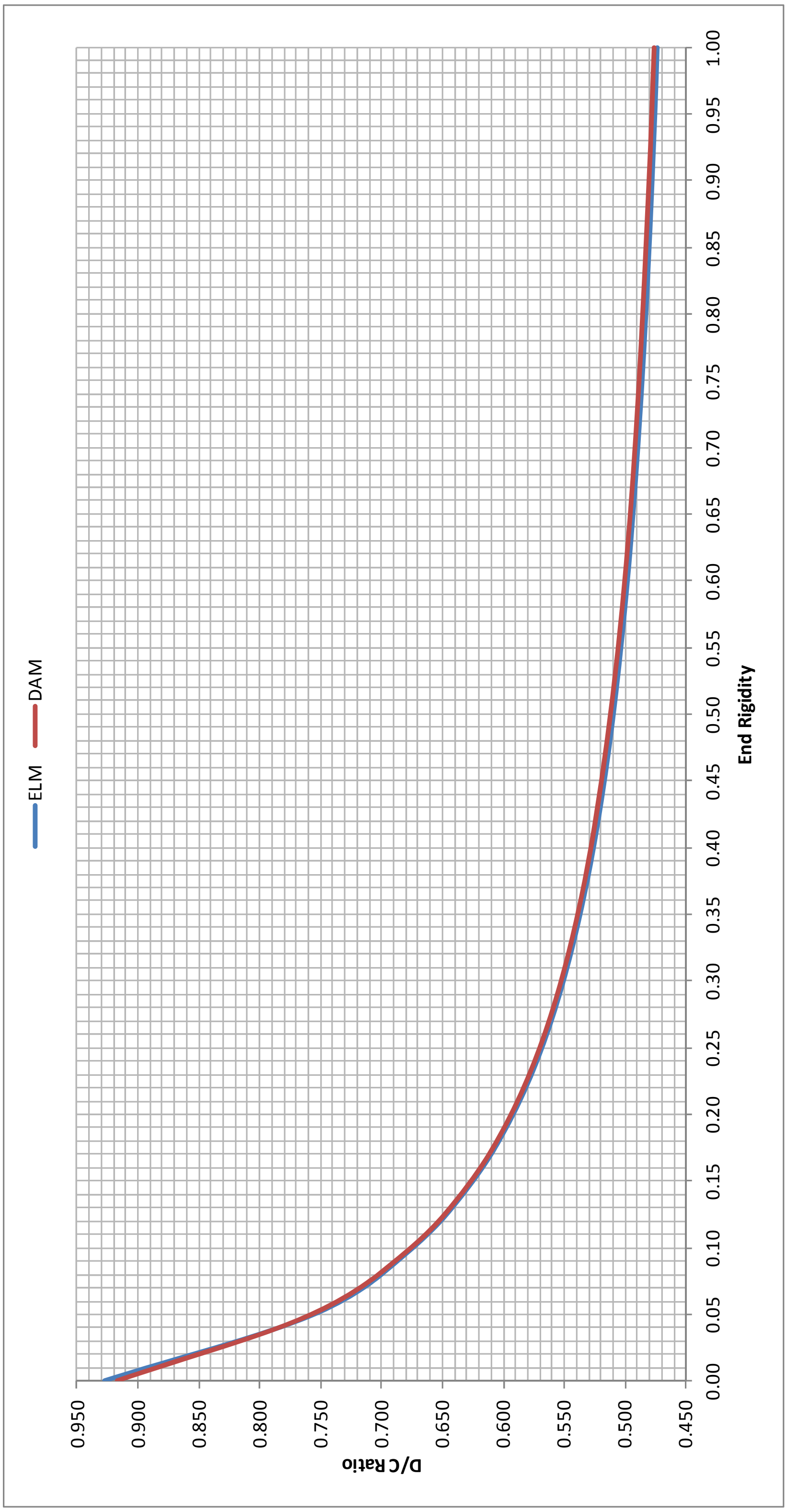


Figure 4.4 – Demand/Capacity vs. End-Rigidity for C3 in Case – IV

4.1.4.2. Results of C4 (Exterior Column)

Table 4.5 - Analysis and Design Results of C4 (exterior column) in Case – IV

	Effective Length Method							***Direct Analysis Method***						
r	**P_r**	**M_r**	**P_c**	**M_c**	**K**	**B_2**	**D/C**	**P_r**	**M_r**	**P_c**	**M_c**	**K**	**B_2**	**D/C**
1.00	315.2	58.4	2777.9	187.8	1.058	1.033	0.368	315.3	58.9	2847.9	187.8	1.00	1.041	0.369
0.95	315.2	58.8	2770.5	187.8	1.064	1.033	0.370	315.3	59.3	2847.9	187.8	1.00	1.042	0.371
0.90	315.1	59.3	2761.9	187.8	1.071	1.034	0.373	315.3	59.8	2847.9	187.8	1.00	1.042	0.374
0.85	315.1	59.7	2753.2	187.8	1.078	1.034	0.375	315.2	60.2	2847.9	187.8	1.00	1.043	0.376
0.80	315.0	60.2	2743.3	187.8	1.086	1.035	0.378	315.2	60.8	2847.9	187.8	1.00	1.044	0.379
0.75	315.0	60.8	2732.1	187.8	1.095	1.036	0.381	315.1	61.3	2847.9	187.8	1.00	1.045	0.382
0.70	314.9	61.4	2719.6	187.8	1.105	1.036	0.385	315.0	62.0	2847.9	187.8	1.00	1.046	0.385
0.65	314.8	62.1	2704.5	187.8	1.117	1.037	0.389	314.9	62.7	2847.9	187.8	1.00	1.047	0.389
0.60	314.6	62.8	2688.1	187.8	1.130	1.038	0.393	314.8	63.4	2847.9	187.8	1.00	1.048	0.393
0.55	314.5	63.7	2669.0	187.8	1.145	1.039	0.398	314.6	64.3	2847.9	187.8	1.00	1.050	0.398
0.50	314.3	64.7	2646.0	187.8	1.163	1.041	0.404	314.4	65.4	2847.9	187.8	1.00	1.051	0.403
0.45	314.1	65.9	2620.2	187.8	1.183	1.042	0.411	314.2	66.6	2847.9	187.8	1.00	1.054	0.410
0.40	313.8	67.2	2587.7	187.8	1.208	1.044	0.419	313.9	68.0	2847.9	187.8	1.00	1.056	0.417
0.35	313.4	68.8	2548.4	187.8	1.238	1.047	0.428	313.5	69.7	2847.9	187.8	1.00	1.059	0.426
0.30	312.9	70.8	2499.4	187.8	1.275	1.049	0.440	313.1	71.7	2847.9	187.8	1.00	1.063	0.437
0.25	312.3	73.3	2437.9	187.8	1.321	1.053	0.454	312.5	74.3	2847.9	187.8	1.00	1.067	0.451
0.20	311.5	76.5	2358.2	187.8	1.380	1.058	0.474	311.6	77.7	2847.9	187.8	1.00	1.074	0.468
0.15	310.3	80.9	2250.4	187.8	1.459	1.065	0.500	310.5	82.2	2847.9	187.8	1.00	1.082	0.492
0.10	308.5	87.1	2099.3	187.8	1.569	1.075	0.537	308.7	88.8	2847.9	187.8	1.00	1.095	0.527
0.05	305.6	97.0	1877.8	187.8	1.731	1.091	0.598	305.8	99.2	2847.9	187.8	1.00	1.116	0.582
0.00	300.0	115.2	1523.0	187.8	2.000	1.122	0.712	300.0	118.9	2847.9	187.8	1.00	1.157	0.686

The demand/capacity ratios of C4 for ELM and DAM are drawn in Figure 4.5.

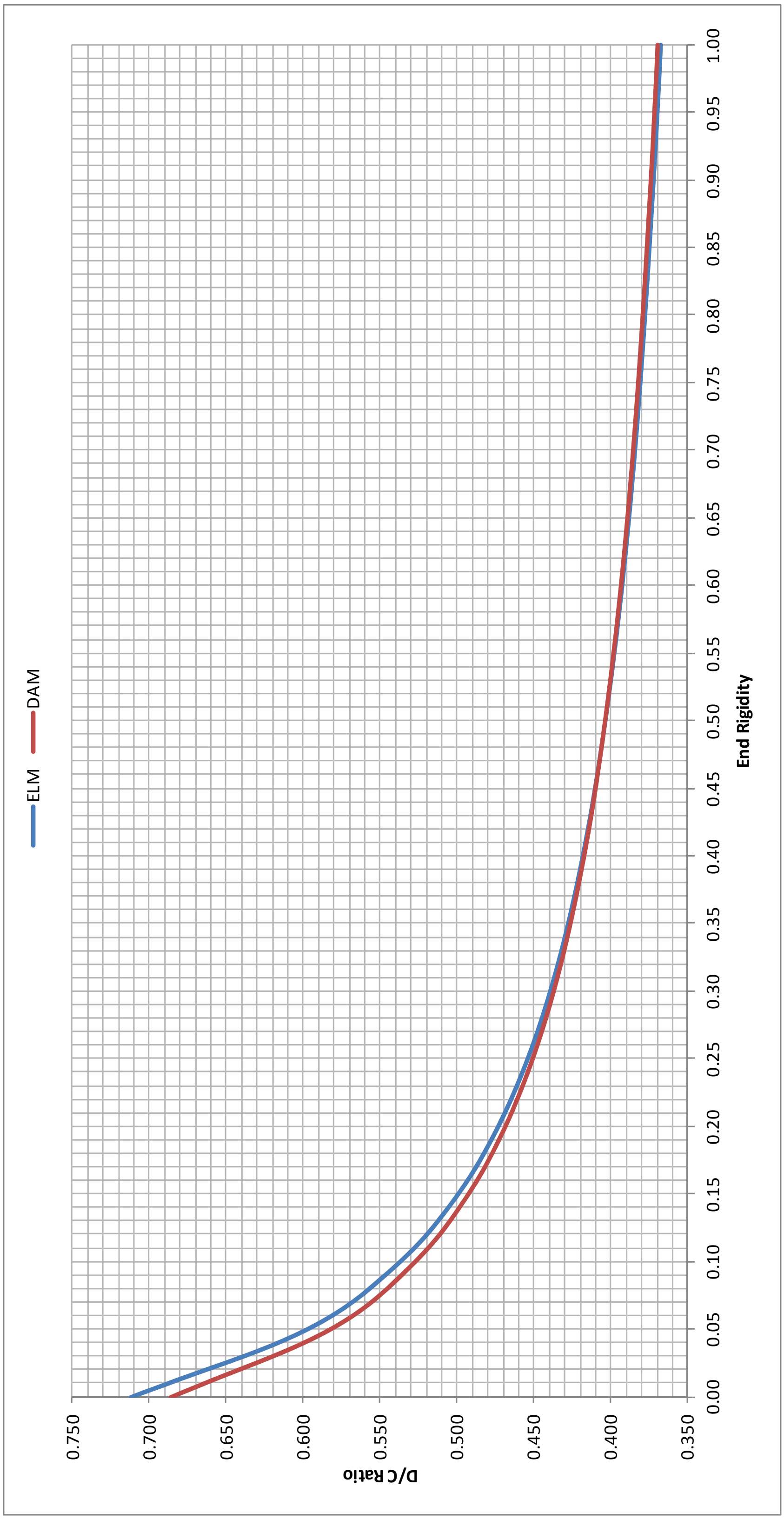


Figure 4.5 – Demand/Capacity vs. End-Rigidity for C4 in Case – IV

4.2. DISCUSSION OF RESULTS

In this section, each table and each figure given in Section 4.1 are discussed separately.

Table 4.1

- In Case – I, the contribution of compression to the D/C ratio is very low when compared with flexural bending. For example, for the end-rigidity ratio of 0.15, the contribution of compression is 5.52% in ELM and 4.87% in DAM whereas the contribution of flexural bending is 70.73% in ELM and 71.41% in DAM.

<table>
<tr><td colspan="4">r = 0.15 (selected randomly)</td></tr>
<tr><td colspan="2">Effective Length Method</td><td colspan="2">Direct Analysis Method</td></tr>
<tr><td colspan="2">$\frac{P_r}{2P_c} + \frac{M_r}{M_c}$</td><td colspan="2">$\frac{P_r}{2P_c} + \frac{M_r}{M_c}$</td></tr>
<tr><td colspan="2">$\frac{317.7}{2 \cdot 2875.5} + \frac{292.2}{413.1}$</td><td colspan="2">$\frac{317.9}{2 \cdot 3263.1} + \frac{295.0}{413.1}$</td></tr>
<tr><td>5.52%</td><td>70.73%</td><td>4.87%</td><td>71.41%</td></tr>
<tr><td colspan="2">76.25%</td><td colspan="2">76.28%</td></tr>
</table>

- Second-order forces and moments (P_r & M_r) obtained with DAM are greater than those obtained with ELM.

- D/C ratios of DAM and ELM are very close in Case – I.

- The ratio of second-order drift to first-order drift, B_2, is very low in Case – I.

Figure 4.1

- The effect of flexible connections is better observed in the figure than the table. In the pinned connected case (r = 0.00) the D/C ratio is about 0.95 whereas in rigidly connected case (r = 1.00) the D/C ratio is about 0.53 for both methods. Between end-rigidity values of 1.00 and 0.50 the increase in the D/C ratio is not significant, it rises from 0.53 to 0.61 for both methods. However, between end-rigidity values of 0.50 and 0.00 the increase in the D/C ratio is significant. It increases from 0.61 to 0.95 for both methods.

Table 4.2

- In Case – II, the contribution of compression to the D/C ratio is lower than the contribution of flexural bending but close to it. For example, for the end-rigidity ratio of 0.15, the contribution of compression is 35.22% in ELM and 31.05% in DAM whereas the contribution of flexural bending is 45.32% in ELM and 46.95% in DAM.

<table>
<tr><td colspan="4">r = 0.15 (selected randomly)</td></tr>
<tr><th colspan="2">Effective Length Method</th><th colspan="2">Direct Analysis Method</th></tr>
<tr><td colspan="2">$\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c}$</td><td colspan="2">$\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c}$</td></tr>
<tr><td colspan="2">$\frac{1012.8}{2875.5} + \frac{8}{9} \cdot \frac{210.6}{413.1}$</td><td colspan="2">$\frac{1013.2}{3263.1} + \frac{8}{9} \cdot \frac{218.2}{413.1}$</td></tr>
<tr><td>35.22%</td><td>45.32%</td><td>31.05%</td><td>46.95%</td></tr>
<tr><td colspan="2">80.54%</td><td colspan="2">78.00%</td></tr>
</table>

- Second-order forces and moments (P_r & M_r) obtained with DAM are greater than those obtained with ELM.

- D/C ratios of DAM and ELM are close to each other however as r factor approaches to 0.00, the D/C ratios become different in Case – II.

- The ratio of second-order drift to first-order drift, B_2, is average in Case – II.

Figure 4.2

- In pinned connected case (r = 0.00) the D/C ratio is 0.98 for ELM and 0.94 for DAM whereas in rigidly connected case (r = 1.00) the D/C ratio is about 0.61 for both methods. Between end-rigidity values of 1.00 and 0.50, the increase in the D/C ratio is not significant; it rises from 0.61 to 0.67 for ELM and to 0.66 for DAM. However, between end-rigidity values of 0.50 and 0.00 the increase in the D/C ratio is significant. It increases from 0.67 to 0.98 for ELM and it increases from 0.66 to 0.94 for DAM.

Table 4.3

- In Case – III, the contribution of compression to the D/C ratio is greater than the contribution of flexural bending. For example, for the end-rigidity ratio of 0.15, the contribution of compression is 66.25% in ELM and 58.40% in DAM whereas the contribution of flexural bending is 18.01% in ELM and 19.47% in DAM.

<table>
<tr><td colspan="4">r = 0.15 (selected randomly)</td></tr>
<tr><td colspan="2">Effective Length Method</td><td colspan="2">Direct Analysis Method</td></tr>
<tr><td colspan="2">$\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c}$</td><td colspan="2">$\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c}$</td></tr>
<tr><td colspan="2">$\frac{1905.1}{2875.5} + \frac{8}{9} \cdot \frac{83.7}{413.1}$</td><td colspan="2">$\frac{1905.5}{3263.1} + \frac{8}{9} \cdot \frac{90.5}{413.1}$</td></tr>
<tr><td>66.25%</td><td>18.01%</td><td>58.40%</td><td>19.47%</td></tr>
<tr><td colspan="2">84.26%</td><td colspan="2">77.87%</td></tr>
</table>

- Second-order forces and moments (P_r & M_r) obtained with DAM are greater than those obtained with ELM.

- As r approaches to 0.00 from 1.00, the difference between the D/C ratios of ELM and DAM increases. The D/C ratios of ELM are greater than the D/C ratios of DAM.

- The ratio of second-order drift to first-order drift, B_2, is high in Case – III.

Figure 4.3

- In pinned connected case (r = 0.00) the D/C ratio is 0.98 for ELM and 0.87 for DAM whereas in rigidly connected case (r = 1.00) the D/C ratio is about 0.70 for both methods. Between end-rigidity values of 1.00 and 0.50, the increase in the D/C ratio is not significant; it rises from 0.70 to 0.75 for ELM and to 0.72 for DAM. However, between end-rigidity values of 0.50 and 0.00 the increase in the D/C ratio is significant. It increases from 0.75 to 0.98 for ELM and it increases from 0.72 to 0.87 for DAM.

Table 4.4

- For the critical interior column C3 In Case – IV, the contribution of compression to the D/C ratio is low when compared with flexural bending. For example, for the end-rigidity ratio of 0.15, the contribution of compression is 9.16% in ELM and 8.44% in DAM whereas the contribution of flexural bending is 53.33% in ELM and 54.20% in DAM.

<table>
<tr><td colspan="4">r = 0.15 (selected randomly)</td></tr>
<tr><td colspan="2">Effective Length Method</td><td colspan="2">Direct Analysis Method</td></tr>
<tr><td colspan="2">$\frac{P_r}{2P_c} + \frac{M_r}{M_c}$</td><td colspan="2">$\frac{P_r}{2P_c} + \frac{M_r}{M_c}$</td></tr>
<tr><td colspan="2">$\frac{550.5}{2 \cdot 3004.5} + \frac{220.3}{413.1}$</td><td colspan="2">$\frac{550.5}{2 \cdot 3263.1} + \frac{223.9}{413.1}$</td></tr>
<tr><td>9.16%</td><td>53.33%</td><td>8.44%</td><td>54.20%</td></tr>
<tr><td colspan="2">62.49%</td><td colspan="2">62.64%</td></tr>
</table>

- Second-order forces and moments (P_r & M_r) obtained with DAM are greater than those obtained with ELM.

- D/C ratios of DAM and ELM are very close for the interior column C3 in Case – IV.

- The ratio of second-order drift to first-order drift, B_2, is low in Case – IV.

<u>Figure 4.4</u>

- In the pinned connected case (r = 0.00) the D/C ratio is about 0.92 whereas in rigidly connected case (r = 1.00) the D/C ratio is about 0.47 for both methods. Between end-rigidity values of 1.00 and 0.50 the increase in the D/C ratio is not significant, it rises from 0.47 to 0.51 for both methods. However, between end-rigidity values of 0.50 and 0.00 the increase in the D/C ratio is significant. It increases from 0.51 to 0.92 for both methods.

<u>Table 4.5</u>

- For the critical exterior column C4 In Case – IV, the contribution of compression to the D/C ratio is low when compared with flexural bending. For example, for the end-rigidity ratio of 0.15, the contribution of compression is 6.89% in ELM and 5.45% in DAM whereas the contribution of flexural bending is 43.08% in ELM and 43.77% in DAM.

<table>
<tr><td colspan="4">r = 0.15 (selected randomly)</td></tr>
<tr><td colspan="2">Effective Length Method</td><td colspan="2">Direct Analysis Method</td></tr>
<tr><td colspan="2">$\frac{P_r}{2P_c} + \frac{M_r}{M_c}$</td><td colspan="2">$\frac{P_r}{2P_c} + \frac{M_r}{M_c}$</td></tr>
<tr><td colspan="2">$\frac{310.3}{2 \cdot 2250.4} + \frac{80.9}{187.8}$</td><td colspan="2">$\frac{310.5}{2 \cdot 2847.9} + \frac{82.2}{187.8}$</td></tr>
<tr><td>6.89%</td><td>43.08%</td><td>5.45%</td><td>43.77%</td></tr>
<tr><td colspan="2">49.97%</td><td colspan="2">49.22%</td></tr>
</table>

- Second-order forces and moments (P_r & M_r) obtained with DAM are greater than those obtained with ELM.

- D/C ratios of DAM and ELM are close for the exterior column C4 in Case – IV.

- The ratio of second-order drift to first-order drift, B_2, is low in Case – IV.

<u>Figure 4.5</u>

- In the pinned connected case (r = 0.00) the D/C ratio is about 0.70 whereas in rigidly connected case (r = 1.00) the D/C ratio is about 0.37 for both methods. Between end-rigidity values of 1.00 and 0.50 the increase in the D/C ratio is not significant, it rises from 0.37 to 0.40 for both methods. However, between end-rigidity values of 0.50 and 0.00 the increase in the D/C ratio is significant. It increases from 0.40 to 0.70. Between r values of 0.00 and 0.25 (where the effective length factor begins to increase significantly) it is observed that DAM becomes slightly unconservative when compared with ELM. At r equals to 0.25 the D/C ratio is 0.45 for both methods. At r equals to 0.00 the D/C ratio for ELM is 0.71 and for DAM it is 0.69.

CHAPTER 5

CONCLUSIONS AND FUTURE RECOMMENDATIONS

5.1. CONCLUSIONS

To compare DAM and ELM in semi-rigid frames and to investigate the effect of semi-rigid joints to stability, four case studies were conducted and the results of these cases were obtained in terms of the demand/capacity ratio. The results were presented and discussed in Chapter 4. Based on these discussions, following conclusions were obtained.

- In Case – I, the contribution of compression to D/C ratio is very low and accordingly the results of ELM and DAM are very close. In Case – II, the contribution of compression increases and accordingly ELM becomes more conservative than DAM. In Case – III, the D/C ratios are governed by compression therefore the difference between ELM and DAM becomes large. When comparing Case – I, Case – II and Case – III, it is concluded that as the contribution of compression to D/C ratio increases, the D/C ratios obtained with ELM become greater than the D/C ratios obtained with DAM. This means that as the compressive force increases ELM gives conservative results when compared with DAM.

- ELM underestimates the internal forces and moments when compared with DAM since geometric imperfections and member inelasticity are not accounted for in the analysis whereas DAM considers these in the analysis. To compensate the underestimation of internal forces and moments, ELM decreases the compressive strength of members by using effective length factors. In cases where ELM and DAM give similar design results for the members, the internal forces obtained with ELM are lower than the forces obtained with DAM and this creates problem in connection design. The connections are designed with smaller forces and moments with ELM. Connection design is very important and critical for steel design since the fail of a connection may lead the fail of the entire structure therefore, design of connections with smaller forces than required is an undesired and unacceptable situation.

- From the results of the exterior column in Case – IV it is observed that in minor-axis bending, DAM becomes slightly unconservative when compared with ELM between the end-rigidity factor range of 0.00 and 0.25. This conclusion is also obtained in the study of Ziemian and Martinez-Garcia [1]. As the end-rigidity factor approaches from 0.25 to 0.00, the influence of semi-rigid connections becomes more significant and this is reflected by the effective length factors in ELM. Since the influence of effective length factor on the compressive strength is more pronounced in minor-axis bending in general, the compressive strength of the members decreases more than it decreases in major axis bending within this end-rigidity range.

- From the results of all cases, it is observed that between end-rigidity values of 0.00 and 0.50, the connection stiffness significantly affects the D/C ratios whereas between 0.50 and 1.00 it loses its influence. If the structure is designed assuming the joints are ideally pinned (end-rigidity values is zero), the structure will have more capacity than the calculated since the connections are not perfectly pinned. In other words, small moment capacity of the connections may increase the resistance of the structure significantly if the connections are designed as pinned. This means that economy can be provided by using semi-rigid connections instead of pinned connections. If the structure is designed assuming the joints are fully rigid, the actual capacity of the structure will be less than the calculated. In this case, an unsafe design comes out. However if the connections are designed as semi-rigid instead of fully rigid, safety of the design is guaranteed with a little bit increase in the cost of the structure. The increase in the cost is low because within the end-rigidity range of 0.50-1.00 the increase in the demand/capacity ratio is small.

5.2. FUTURE RECOMMENDATIONS

The followings are recommended for future studies:

- One may perform the analyses assuming column bases semi-rigid instead of fully rigid to determine the influence of column bases to the stability of the structure.
- Model the semi-rigid connections as non-linear instead of linear modeling to obtain more realistic results.
- Model a 3-D structure to investigate the effect of stability methods on major-axis or minor-axis bending of columns.

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APPENDIX A

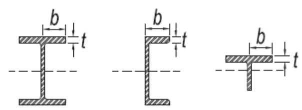
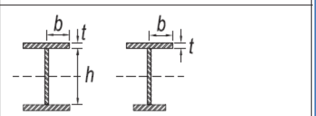
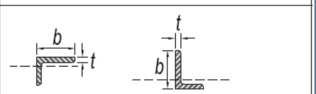
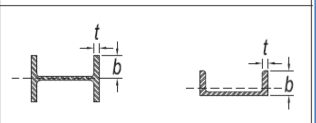
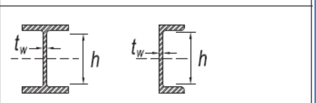
FLEXURAL STRENGTH OF COLUMNS

a) <u>Flexural Strength of Columns in Case – I, Case – II and Case – III</u>

In-plane flexural strength of a column is determined according to Chapter F of the AISC 360-10. First, the compactness of HEA300 should be checked. The limiting values are given in Table B4.1b of AISC 360-10 which is given in Table A1.

Table A1 – Compactness Limits

TABLE B4.1b
Width-to-Thickness Ratios: Compression Elements
Members Subject to Flexure

	Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio λ_p (compact/noncompact)	Limiting Width-to-Thickness Ratio λ_r (noncompact/slender)	Examples
Unstiffened Elements	10	Flanges of rolled I-shaped sections, channels, and tees	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
Unstiffened Elements	11	Flanges of doubly and singly symmetric I-shaped built-up sections	b/t	$0.38\sqrt{\frac{E}{F_y}}$	[a] [b] $0.95\sqrt{\frac{k_c E}{F_L}}$	
Unstiffened Elements	12	Legs of single angles	b/t	$0.54\sqrt{\frac{E}{F_y}}$	$0.91\sqrt{\frac{E}{F_y}}$	
Unstiffened Elements	13	Flanges of all I-shaped sections and channels in flexure about the weak axis	b/t	$0.38\sqrt{\frac{E}{F_y}}$	$1.0\sqrt{\frac{E}{F_y}}$	
Unstiffened Elements	14	Stems of tees	d/t	$0.84\sqrt{\frac{E}{F_y}}$	$1.03\sqrt{\frac{E}{F_y}}$	
	15	Webs of doubly-symmetric I-shaped sections and channels	h/t_w	$3.76\sqrt{\frac{E}{F_y}}$	$5.70\sqrt{\frac{E}{F_y}}$	

Compactness Check of Flanges

$$0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{200000}{345}} = 9.15 \qquad 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{200000}{345}} = 24.08$$

$$\frac{b}{t} = \frac{300/2}{14} = 10.71 \quad \rightarrow \quad 0.38\sqrt{\frac{E}{F_y}} < \frac{b}{t} < 1.0\sqrt{\frac{E}{F_y}} \quad \rightarrow \quad Non-compact\ flange$$

Compactness Check of Web

$$3.76\sqrt{\frac{E}{F_y}} = 3.76\sqrt{\frac{200000}{345}} = 90.53 \qquad 5.70\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{200000}{345}} = 137.2$$

$$\frac{h}{t_w} = \frac{290 - 14 \cdot 2}{8.5} = 30.8 \quad \rightarrow \quad \frac{h}{t_w} < 3.76\sqrt{\frac{E}{F_y}} < 5.7\sqrt{\frac{E}{F_y}} \quad \rightarrow \quad Compact\ web$$

Flexural Strength (According to Chapter F.3 of AISC 360-10)

i. Lateral Torsional Buckling

$$L_b = 4000\ mm$$

$$L_p = 1.76 \cdot r_y \cdot \sqrt{\frac{E}{F_y}} = 1.76 \cdot 74.7 \cdot \sqrt{\frac{200000}{345}} = 3165\ mm$$

$$L_r = 1.95 \cdot r_{ts} \cdot \frac{E}{0.7F_y}\sqrt{\frac{J \cdot c}{S_x \cdot h_o} + \sqrt{\left(\frac{J \cdot c}{S_x \cdot h_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} = 10868\ mm$$

Where

E = 200000 MPa

F_y = 345 MPa

J = 878000 mm^4

c = 1

S_x = 1259310 mm^3

I_y = 63100000 mm^4

$$h_o = h - t_f = 290 - 14 = 276\ mm$$

$$C_w = \frac{I_y \cdot h_o^2}{4} = \frac{631x10^5 \cdot 276^2}{4} = 1.202x10^{12}$$

$$r_{ts} = \sqrt{\frac{\sqrt{I_y \cdot C_w}}{S_x}} = \sqrt{\frac{\sqrt{631x10^5 \cdot 1.202x10^{12}}}{1259310}} = 83.15$$

$$L_p = 3165\ mm\ <\ L_b = 4000\ mm\ <\ L_r = 10868\ mm$$

$$M_p = F_y \cdot Z_x = 345 \cdot 1383000 = 477\ kN \cdot m$$

$$M_n = C_b \left[M_p - \left(M_p - 0.7F_yS_x\right)\left[\frac{L_b - L_p}{L_r - L_p}\right]\right] \le M_p$$

$$M_n = 2.213 \cdot \left[4.77x10^8 - (4.77x10^8 - 0.7 \cdot 345 \cdot 1259310)\left[\frac{4000 - 3165}{10868 - 3165}\right]\right]$$

$$\mathrm{M_n} = 1014\ kN \cdot m$$

$$M_n = 1014\ kN \cdot m > M_p = 477\ kN \cdot m \quad \rightarrow \quad M_n = 477\ kN \cdot m$$

Where C_b is calculated as;

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} = \frac{12.5 \cdot 209.7}{2.5 \cdot 209.7 + 3 \cdot 58.1 + 4 \cdot 31.2 + 3 \cdot 120.4}$$

$$\mathrm{C_b} = 2.213$$

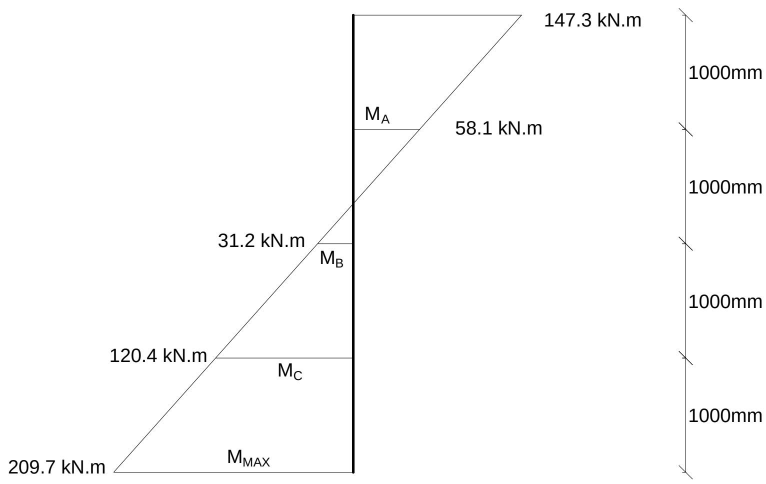


Figure A1 – Moment Values for C_b Calculation

In C_b calculation, the moment values of the critical column in Case – I are used. In this part it is shown that C_b does not affect the moment capacity of the columns therefore for the Cases II and III it is unnecessary to calculate C_b value.

ii. Compression Flange Buckling

$$\lambda = \frac{b}{2t_f} = \frac{300}{2 \cdot 14} = 10.71 \qquad \lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}} = 9.15 \qquad \lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}} = 24.08$$

$$M_n = M_p - \left(M_p - 0.7F_yS_x\right)\left[\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right] = 459\ kN \cdot m$$

The nominal flexural strength, M_n, shall be the smaller of the values determined according to the limit states of lateral-torsional buckling and compression flange buckling.

$$M_n = min(477\ kN \cdot m\,; 459\ kN \cdot m) = 459\ kN \cdot m$$

The design moment capacity of the column is;

$$M_c = \emptyset_b \cdot M_n = 0.9 \cdot 459 = 413.1\ kN \cdot m$$

b) Flexural Strength of Columns in Case – IV

There are two types of columns in Case IV: interior columns and exterior columns. For both types the columns are HEA300 however their orientations are different which causes their in-plane flexural capacities to differ.

i. Interior Columns

The flexural strength of interior columns is the same as the flexural strength of columns in Case - I since the all the properties are the same. The design moment capacity of interior columns is M_c = 413.1 kN.m.

ii. Exterior Columns

Lateral Torsional Buckling

Since the out-of-plane axis is stronger than in-plane axis, lateral torsional buckling does not apply. The flexural strength is determined according to compression flange buckling.

Compression Flange Buckling

$$M_p = F_y \cdot Z_x = 345 \cdot 641000 = 221.1\ kN \cdot m$$

Where Z_x refers to Z_y in Table 3.1 in Section 3.1.

$$\lambda = \frac{b}{2t_f} = \frac{300}{2 \cdot 14} = 10.71 \qquad \lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}} = 9.15 \qquad \lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}} = 24.08$$

$$M_n = M_p - \left(M_p - 0.7F_yS_x\right)\left[\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right] = 208.7\ kN \cdot m$$

The design moment capacity of the column is;

$M_c = \emptyset_b \cdot M_n = 0.9 \cdot 208.7 = 187.8\ kN \cdot m$

APPENDIX B

COMPRESSIVE STRENGTH OF COLUMNS

a) Compressive Strength of Columns in Case – I, Case – II and Case – III

i. Design with ELM

Compressive strength of the columns is determined according to Chapter E of AISC 360-10. The physical properties of HEA300 are given in Section 3.1 and the effective length factor of the column is calculated as 1.193 in Section 3.2.1.2. The calculated effective length factor is for the in-plane buckling, the out-of-plane effective length is assumed as 0.5 in assumption 6 in Section 3.1.

$$K_x = 1.193 \qquad r_x = 127.1\,mm \qquad \frac{K_x L}{r_x} = \frac{1.193 \cdot 4000}{127.1} = 37.5$$

$$K_y = 0.500 \qquad r_y = 74.7\,mm \qquad \frac{K_y L}{r_y} = \frac{0.5 \cdot 4000}{74.7} = 26.8$$

$$\frac{KL}{r} = 37.5 \quad (the\ critical\ one\ will\ be\ used)$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{200000}{345}} = 113.4 > 37.5 = \frac{KL}{r}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y = \left(0.658^{\frac{345}{1403}}\right)345 = 311.1\,MPa$$

$$where \quad F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 200000}{(37.5)^2} = 1403\ MPa$$

$$P_n = F_{cr} \cdot A_g = 311.1 \cdot 11300 = 3515\ kN \quad : nominal\ compressive\ strength$$

$$\emptyset_c \cdot P_n = 0.9 \cdot 3515 = 3164\ kN \quad : design\ compressive\ strength$$

Available axial strength of the column is P_c = 3164 kN.

ii. Design with DAM

The physical properties of HEA300 are given in Section 3.1, the in-plane effective length factor of the column is 1.0 and the out-of-plane effective length factor is 0.5 as specified in Section 3.1.

$$K_x = 1.0 \qquad r_x = 127.1\ mm \qquad \frac{K_x L}{r_x} = \frac{1.0 \cdot 4000}{127.1} = 31.5$$

$$K_y = 0.5 \qquad r_y = 74.7\ mm \qquad \frac{K_y L}{r_y} = \frac{0.5 \cdot 4000}{74.7} = 26.8$$

$$\frac{KL}{r} = 31.5 \quad (the\ critical\ one\ will\ be\ used)$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{200000}{345}} = 113.4 > 31.5 = \frac{KL}{r}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y = \left(0.658^{\frac{345}{1989}}\right) 345 = 320.8\ MPa$$

$$where \quad F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 200000}{(37.5)^2} = 1989\ MPa$$

$P_n = F_{cr} \cdot A_g = 320.8 \cdot 11300 = 3625\ kN \quad : nominal\ compressive\ strength$

$\emptyset_c \cdot P_n = 0.9 \cdot 3625 = 3263\ kN \quad : design\ compressive\ strength$

Available axial strength of the column is P_c = 3263 kN.

b) <u>Compressive Strength of Columns in Case – IV</u>

i. Interior Columns

<u>*Design with ELM*</u>

Compressive strength of the columns is determined according to Chapter E of AISC 360-10. The physical properties of HEA300 are given in Section 3.1 and the effective length factor of the column is calculated as 1.095 in Section 3.5.1.2. The calculated effective length factor is for the in-plane buckling, the out-of-plane effective length is assumed as 0.5 in assumption 6 in Section 3.1.

$$K_x = 1.095 \qquad r_x = 127.1\ mm \qquad \frac{K_x L}{r_x} = \frac{1.095 \cdot 4000}{127.1} = 34.5$$

$$K_y = 0.500 \qquad r_y = 74.7\ mm \qquad \frac{K_y L}{r_y} = \frac{0.5 \cdot 4000}{74.7} = 26.8$$

$$\frac{KL}{r} = 34.5 \quad (the\ critical\ one\ will\ be\ used)$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{200000}{345}} = 113.4 > 34.5 = \frac{KL}{r}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right) F_y = \left(0.658^{\frac{345}{1658}}\right) 345 = 316.2\ MPa$$

$$where \quad F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 200000}{(34.5)^2} = 1658\ MPa$$

$$P_n = F_{cr} \cdot A_g = 316.2 \cdot 11300 = 3573\ kN \quad : nominal\ compressive\ strength$$

$$\emptyset_c \cdot P_n = 0.9 \cdot 3573 = 3216\ kN \quad : design\ compressive\ strength$$

Available axial strength of the column is P_c = 3216 kN.

Design with DAM

The interior columns in Case - IV have the same properties with those of columns in Case I (both of them are HEA300, have 4m length and effective length factor of 1.00) therefore interior columns in Case - IV (C2 & C3) have the same compressive strength with the columns in Case - I.

Available axial strength of the interior columns is P_c = 3263 kN.

ii. Exterior Columns

Design with ELM

The only difference between the interior and exterior columns is the orientation. The flexural strength of exterior columns is calculated in the same way with the interior columns however the physical properties given for x-direction is accepted as they are for y-direction and the properties given for y are for x-direction.

$$K_x = 1.095 \qquad r_x = 74.7\ mm \qquad \frac{K_x L}{r_x} = \frac{1.095 \cdot 4000}{74.7} = 58.5$$

$K_y = 0.500 \qquad r_y = 127.1\ mm \qquad \frac{K_y L}{r_y} = \frac{0.5 \cdot 4000}{127.1} = 15.7$

$\frac{KL}{r} = 58.5 \quad (the\ critical\ one\ will\ be\ used)$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{200000}{345}} = 113.4 > 58.5 = \frac{KL}{r}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y = \left(0.658^{\frac{345}{577}}\right)345 = 268.6\ MPa$$

$$where \qquad F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 200000}{(58.5)^2} = 577\ MPa$$

$P_n = F_{cr} \cdot A_g = 268.6 \cdot 11300 = 3035\ kN \quad : nominal\ compressive\ strength$

$\emptyset_c \cdot P_n = 0.9 \cdot 3031 = 2732\ kN \quad : design\ compressive\ strength$

Available axial strength of the column is P_c = 2732 kN.

<u>*Design with DAM*</u>

The in-plane effective length factor of the exterior columns is 1.0 and the out-of-plane effective length factor is 0.5 as specified in Section 3.1. The properties given for x and y directions are replaced as described in *Design with ELM* for exterior columns in Case – IV.

$K_x = 1.00 \qquad r_x = 74.7\ mm \qquad \frac{K_x L}{r_x} = \frac{1.00 \cdot 4000}{74.7} = 53.5$

$K_y = 0.50 \qquad r_y = 127.1\ mm \qquad \frac{K_y L}{r_y} = \frac{0.5 \cdot 4000}{127.1} = 15.7$

$$\frac{KL}{r} = 53.5 \quad (the\ critical\ one\ will\ be\ used)$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{200000}{345}} = 113.4 > 53.5 = \frac{KL}{r}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y = \left(0.658^{\frac{345}{577}}\right)345 = 280.0\ MPa$$

$$where \quad F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 200000}{(53.5)^2} = 688\ MPa$$

$$P_n = F_{cr} \cdot A_g = 280.0 \cdot 11300 = 3164\ kN \quad : nominal\ compressive\ strength$$

$$\emptyset_c \cdot P_n = 0.9 \cdot 3164 = 2848\ kN \quad : design\ compressive\ strength$$

Available axial strength of the column is P_c = 2848 kN.

APPENDIX C

STRUCTURAL ANALYSIS STEPS OF CASES

1. Case - II

a) Design with ELM

Stiffness matrix of the columns C1 and C2,

$$k_c = \begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix}$$

Displacement matrices of columns C1 and C2,

$$d_{C1} = \begin{bmatrix} v_1 \\ u \\ \theta_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad d_{C2} = \begin{bmatrix} v_2 \\ u \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness matrix of the beam B1,

$$k_{B1} = \begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix}$$

Displacement matrix of beam B1,

$$d_{B1} = \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix}$$

The connection stiffness and stability functions are calculated as,

$$r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} \quad \rightarrow \quad 0.75 = \frac{1}{1 + \frac{3 \cdot (2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot R_{ki}}} \quad \rightarrow \quad R_{ki} = 1.085 \cdot 10^{11} N \cdot mm$$

$$R^* = \left(1 + \frac{4EI}{LR_{kiA}}\right)\left(1 + \frac{4EI}{LR_{kiB}}\right) - \left(\frac{EI}{L}\right)^2 \left(\frac{4}{R_{kiA}R_{kiB}}\right)$$

$$R^* = \left(1 + \frac{4 \cdot (2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot 1.085 \cdot 10^{11}}\right)^2 - \left(\frac{2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000}\right)^2 \left(\frac{4}{(1.085 \cdot 10^{11})^2}\right)$$

$$R^* = 2.037$$

$$s_{ii}^* = \frac{\left(4 + \frac{12EI}{LR_{kiA}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{jj}^* = \frac{\left(4 + \frac{12EI}{LR_{kiB}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{ij}^* = s_{ji}^* = \frac{2}{R^*} = \frac{2}{2.037} = 0.98$$

Member forces for C1 is (units are in kN and m),

$$\begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix} x \begin{bmatrix} -1.7279 \\ 12.7947 \\ -0.0022 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -976.3 \\ 57.5 \\ 94.9 \\ 976.3 \\ -57.5 \\ 135.1 \end{bmatrix}$$

Member forces for C2 is (units are in kN and m),

$$\begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix} x \begin{bmatrix} -1.8119 \\ 12.7947 \\ -0.0022 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1023.7 \\ 57.5 \\ 94.9 \\ 1023.7 \\ -57.5 \\ 135.1 \end{bmatrix}$$

Member forces for B1 is (units are in kN and m),

$$\begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix} x \begin{bmatrix} -1.7279 \\ -0.0022 \\ -1.8119 \\ -0.0022 \end{bmatrix} = \begin{bmatrix} -23.7 \\ -94.9 \\ 23.7 \\ -94.9 \end{bmatrix}$$

b) Design with DAM

Stiffness matrix of the columns C1 and C2,

$$k_c = \begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 5.48x10^3 & 1.10x10^7 & 0 & -5.48x10^3 & 1.10x10^7 \\ 0 & 1.10x10^7 & 2.92x10^{10} & 0 & -1.10x10^7 & 1.46x10^{10} \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -5.48x10^3 & -1.10x10^7 & 0 & 5.48x10^3 & -1.10x10^7 \\ 0 & 1.10x10^7 & 1.46x10^{10} & 0 & -1.10x10^7 & 2.92x10^{10} \end{bmatrix}$$

Displacement matrices of columns C1 and C2,

$$d_{C1} = \begin{bmatrix} v_1 \\ u \\ \theta_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad d_{C2} = \begin{bmatrix} v_2 \\ u \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness matrix of the beam B1,

$$k_{B1} = \begin{bmatrix} 1.08x10^3 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 4.34x10^6 & 2.52x10^{10} & -4.34x10^6 & 9.46x10^9 \\ -1.08x10^3 & -4.34x10^6 & 1.08x10^3 & -4.34x10^6 \\ 4.34x10^6 & 9.46x10^9 & -4.34x10^6 & 2.52x10^{10} \end{bmatrix}$$

Displacement matrix of beam B1,

$$d_{B1} = \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix}$$

The connection stiffness and stability functions are calculated as,

$$r = \frac{1}{1+\frac{3EI}{LR_{ki}}} \quad \rightarrow \quad 0.75 = \frac{1}{1+\frac{3\cdot(160000\cdot 482\cdot 10^6)}{8000\cdot R_{ki}}} \quad \rightarrow \quad R_{ki} = 8.676\cdot 10^{10} N\cdot mm$$

$$R^* = \left(1+\frac{4EI}{LR_{kiA}}\right)\left(1+\frac{4EI}{LR_{kiB}}\right) - \left(\frac{EI}{L}\right)^2\left(\frac{4}{R_{kiA}R_{kiB}}\right)$$

$$R^* = \left(1+\frac{4\cdot(160000\cdot 482\cdot 10^6)}{8000\cdot 8.676\cdot 10^{10}}\right)^2 - \left(\frac{160000\cdot 482\cdot 10^6}{8000}\right)^2\left(\frac{4}{(8.676\cdot 10^{10})^2}\right)$$

$$R^* = 2.037$$

$$s_{ii}^* = \frac{\left(4+\frac{12EI}{LR_{kiA}}\right)}{R^*} = \frac{\left(4+\frac{12\cdot 160000\cdot 482\cdot 10^6}{8000\cdot 8.676\cdot 10^{10}}\right)}{2.037} = 2.62$$

$$s_{jj}^* = \frac{\left(4+\frac{12EI}{LR_{kiB}}\right)}{R^*} = \frac{\left(4+\frac{12\cdot 160000\cdot 482\cdot 10^6}{8000\cdot 8.676\cdot 10^{10}}\right)}{2.037} = 2.62$$

$$s_{ij}^* = s_{ji}^* = \frac{2}{R^*} = \frac{2}{2.037} = 0.98$$

Member forces for C1 is (units are in kN and m),

$$\begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 5.48x10^3 & 1.10x10^7 & 0 & -5.48x10^3 & 1.10x10^7 \\ 0 & 1.10x10^7 & 2.92x10^{10} & 0 & -1.10x10^7 & 1.46x10^{10} \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -5.48x10^3 & -1.10x10^7 & 0 & 5.48x10^3 & -1.10x10^7 \\ 0 & 1.10x10^7 & 1.46x10^{10} & 0 & -1.10x10^7 & 2.92x10^{10} \end{bmatrix} x \begin{bmatrix} -2.1599 \\ 15.9934 \\ -0.0027 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -976.3 \\ 57.5 \\ 94.9 \\ 976.3 \\ -57.5 \\ 135.1 \end{bmatrix}$$

Member forces for C2 is (units are in kN and m),

$$\begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 5.48x10^3 & 1.10x10^7 & 0 & -5.48x10^3 & 1.10x10^7 \\ 0 & 1.10x10^7 & 2.92x10^{10} & 0 & -1.10x10^7 & 1.46x10^{10} \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -5.48x10^3 & -1.10x10^7 & 0 & 5.48x10^3 & -1.10x10^7 \\ 0 & 1.10x10^7 & 1.46x10^{10} & 0 & -1.10x10^7 & 2.92x10^{10} \end{bmatrix} x \begin{bmatrix} -2.2649 \\ 15.9934 \\ -0.0027 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1023.7 \\ 57.5 \\ 94.9 \\ 1023.7 \\ -57.5 \\ 135.1 \end{bmatrix}$$

Member forces for B1 is (units are in kN and m),

$$\begin{bmatrix} 1.08x10^3 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 4.34x10^6 & 2.52x10^{10} & -4.34x10^6 & 9.46x10^9 \\ -1.08x10^3 & -4.34x10^6 & 1.08x10^3 & -4.34x10^6 \\ 4.34x10^6 & 9.46x10^9 & -4.34x10^6 & 2.52x10^{10} \end{bmatrix} x \begin{bmatrix} -2.1599 \\ -0.0027 \\ -2.2649 \\ -0.0027 \end{bmatrix} = \begin{bmatrix} -23.7 \\ -94.9 \\ 23.7 \\ -94.9 \end{bmatrix}$$

2. Case - III

a) *Design with ELM*

Stiffness matrix of the columns C1 and C2,

$$k_c = \begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix}$$

Displacement matrices of columns C1 and C2,

$$d_{C1} = \begin{bmatrix} v_1 \\ u \\ \theta_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad d_{C2} = \begin{bmatrix} v_2 \\ u \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness matrix of the beam B1,

$$k_{B1} = \begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix}$$

Displacement matrix of beam B1,

$$d_{B1} = \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix}$$

The connection stiffness and stability functions are calculated as,

$$r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} \quad \rightarrow \quad 0.75 = \frac{1}{1 + \frac{3 \cdot (2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot R_{ki}}} \quad \rightarrow \quad R_{ki} = 1.085 \cdot 10^{11} N \cdot mm$$

$$R^* = \left(1 + \frac{4EI}{LR_{kiA}}\right)\left(1 + \frac{4EI}{LR_{kiB}}\right) - \left(\frac{EI}{L}\right)^2 \left(\frac{4}{R_{kiA} R_{kiB}}\right)$$

$$R^* = \left(1 + \frac{4 \cdot (2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot 1.085 \cdot 10^{11}}\right)^2 - \left(\frac{2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000}\right)^2 \left(\frac{4}{(1.085 \cdot 10^{11})^2}\right)$$

$$R^* = 2.037$$

$$s_{ii}^* = \frac{\left(4 + \frac{12EI}{LR_{kiA}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{jj}^* = \frac{\left(4 + \frac{12EI}{LR_{kiB}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{ij}^* = s_{ji}^* = \frac{2}{R^*} = \frac{2}{2.037} = 0.98$$

Member forces for C1 is (units are in kN and m),

$$\begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix} x \begin{bmatrix} -3.3482 \\ 4.4503 \\ -0.0008 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1891.7 \\ 20.0 \\ 33.0 \\ 1891.7 \\ -20.0 \\ 47.0 \end{bmatrix}$$

Member forces for C2 is (units are in kN and m),

$$\begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix} x \begin{bmatrix} -3.3774 \\ 4.4503 \\ -0.0008 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1908.3 \\ 20.0 \\ 33.0 \\ 1908.3 \\ -20.0 \\ 47.0 \end{bmatrix}$$

Member forces for B1 is (units are in kN and m),

$$\begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix} x \begin{bmatrix} -3.3482 \\ -0.0008 \\ -3.3774 \\ -0.0008 \end{bmatrix} = \begin{bmatrix} -8.3 \\ -33.0 \\ 8.3 \\ -33.0 \end{bmatrix}$$

b) Design with DAM

Stiffness matrix of the columns C1 and C2,

$$k_c = \begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 5.48x10^3 & 1.10x10^7 & 0 & -5.48x10^3 & 1.10x10^7 \\ 0 & 1.10x10^7 & 2.92x10^{10} & 0 & -1.10x10^7 & 1.46x10^{10} \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -5.48x10^3 & -1.10x10^7 & 0 & 5.48x10^3 & -1.10x10^7 \\ 0 & 1.10x10^7 & 1.46x10^{10} & 0 & -1.10x10^7 & 2.92x10^{10} \end{bmatrix}$$

Displacement matrices of columns C1 and C2,

$$d_{C1} = \begin{bmatrix} v_1 \\ u \\ \theta_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad d_{C2} = \begin{bmatrix} v_2 \\ u \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness matrix of the beam B1,

$$k_{B1} = \begin{bmatrix} 1.08x10^3 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 4.34x10^6 & 2.52x10^{10} & -4.34x10^6 & 9.46x10^9 \\ -1.08x10^3 & -4.34x10^6 & 1.08x10^3 & -4.34x10^6 \\ 4.34x10^6 & 9.46x10^9 & -4.34x10^6 & 2.52x10^{10} \end{bmatrix}$$

Displacement matrix of beam B1,

$$d_{B1} = \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix}$$

The connection stiffness and stability functions are calculated as,

$$r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} \quad \rightarrow \quad 0.75 = \frac{1}{1 + \frac{3 \cdot (160000 \cdot 482 \cdot 10^6)}{8000 \cdot R_{ki}}} \quad \rightarrow \quad R_{ki} = 8.676 \cdot 10^{10} N \cdot mm$$

$$R^* = \left(1 + \frac{4EI}{LR_{kiA}}\right)\left(1 + \frac{4EI}{LR_{kiB}}\right) - \left(\frac{EI}{L}\right)^2 \left(\frac{4}{R_{kiA} R_{kiB}}\right)$$

$$R^* = \left(1 + \frac{4 \cdot (160000 \cdot 482 \cdot 10^6)}{8000 \cdot 8.676 \cdot 10^{10}}\right)^2 - \left(\frac{160000 \cdot 482 \cdot 10^6}{8000}\right)^2 \left(\frac{4}{(8.676 \cdot 10^{10})^2}\right)$$

$$R^* = 2.037$$

$$s_{ii}^* = \frac{\left(4 + \frac{12EI}{LR_{kiA}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 160000 \cdot 482 \cdot 10^6}{8000 \cdot 8.676 \cdot 10^{10}}\right)}{2.037} = 2.62$$

$$s_{jj}^* = \frac{\left(4 + \frac{12EI}{LR_{kiB}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 160000 \cdot 482 \cdot 10^6}{8000 \cdot 8.676 \cdot 10^{10}}\right)}{2.037} = 2.62$$

$$s_{ij}^* = s_{ji}^* = \frac{2}{R^*} = \frac{2}{2.037} = 0.98$$

Member forces for C1 is (units are in kN and m),

$$\begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 5.48x10^3 & 1.10x10^7 & 0 & -5.48x10^3 & 1.10x10^7 \\ 0 & 1.10x10^7 & 2.92x10^{10} & 0 & -1.10x10^7 & 1.46x10^{10} \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -5.48x10^3 & -1.10x10^7 & 0 & 5.48x10^3 & -1.10x10^7 \\ 0 & 1.10x10^7 & 1.46x10^{10} & 0 & -1.10x10^7 & 2.92x10^{10} \end{bmatrix} x \begin{bmatrix} -4.1853 \\ 5.5629 \\ -0.0010 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1891.7 \\ 20.0 \\ 33.0 \\ 1891.7 \\ -20.0 \\ 47.0 \end{bmatrix}$$

Member forces for C2 is (units are in kN and m),

$$\begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 5.48x10^3 & 1.10x10^7 & 0 & -5.48x10^3 & 1.10x10^7 \\ 0 & 1.10x10^7 & 2.92x10^{10} & 0 & -1.10x10^7 & 1.46x10^{10} \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -5.48x10^3 & -1.10x10^7 & 0 & 5.48x10^3 & -1.10x10^7 \\ 0 & 1.10x10^7 & 1.46x10^{10} & 0 & -1.10x10^7 & 2.92x10^{10} \end{bmatrix} x \begin{bmatrix} -2.2649 \\ 15.9934 \\ -0.0027 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1908.3 \\ 20.0 \\ 33.0 \\ 1908.3 \\ -20.0 \\ 47.0 \end{bmatrix}$$

Member forces for B1 is (units are in kN and m),

$$\begin{bmatrix} 1.08x10^3 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 4.34x10^6 & 2.52x10^{10} & -4.34x10^6 & 9.46x10^9 \\ -1.08x10^3 & -4.34x10^6 & 1.08x10^3 & -4.34x10^6 \\ 4.34x10^6 & 9.46x10^9 & -4.34x10^6 & 2.52x10^{10} \end{bmatrix} x \begin{bmatrix} -2.1599 \\ -0.0027 \\ -2.2649 \\ -0.0027 \end{bmatrix} = \begin{bmatrix} -8.3 \\ -33.0 \\ 8.3 \\ -33.0 \end{bmatrix}$$

3. Case - IV

a) Design with ELM

Stiffness matrix of the columns C1 and C4,

$$k_{c1} = k_{c4} = \begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 2.37x10^3 & 4.73x10^6 & 0 & -2.37x10^3 & 4.73x10^6 \\ 0 & 4.73x10^6 & 1.26x10^{10} & 0 & -4.73x10^6 & 6.31x10^9 \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -2.37x10^3 & -4.73x10^6 & 0 & 2.37x10^3 & -4.73x10^6 \\ 0 & 4.73x10^6 & 6.31x10^9 & 0 & -4.73x10^6 & 1.26x10^{10} \end{bmatrix}$$

Stiffness matrix of the columns C2 and C3,

$$k_{c2} = k_{c3} = \begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 6.85x10^3 & 1.37x10^7 & 0 & -6.85x10^3 & 1.37x10^7 \\ 0 & 1.37x10^7 & 3.65x10^{10} & 0 & -1.37x10^7 & 1.83x10^{10} \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -6.85x10^3 & -1.37x10^7 & 0 & 6.85x10^3 & -1.37x10^7 \\ 0 & 1.37x10^7 & 1.83x10^{10} & 0 & -1.37x10^7 & 3.65x10^{10} \end{bmatrix}$$

Displacement matrices of columns C1, C2, C3 and C4,

$$d_{C1} = \begin{bmatrix} v_1 \\ u \\ \theta_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad d_{C2} = \begin{bmatrix} v_2 \\ u \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad d_{C3} = \begin{bmatrix} v_3 \\ u \\ \theta_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad d_{C4} = \begin{bmatrix} v_4 \\ u \\ \theta_4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness matrix of the beams B1, B2 and B3,

$$k_{B1} = k_{B2} = k_{B3} = \begin{bmatrix} 1.36x10^{3} & 5.42x10^{6} & -1.36x10^{3} & 5.42x10^{6} \\ 5.42x10^{6} & 3.15x10^{10} & -5.42x10^{6} & 1.18x10^{10} \\ -1.36x10^{3} & -5.42x10^{6} & 1.36x10^{3} & -5.42x10^{6} \\ 5.42x10^{6} & 1.18x10^{10} & -5.42x10^{6} & 3.15x10^{10} \end{bmatrix}$$

Displacement matrices of beams B1, B2 and B3:

$$d_{B1} = \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} \qquad d_{B2} = \begin{bmatrix} v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{bmatrix} \qquad d_{B3} = \begin{bmatrix} v_3 \\ \theta_3 \\ v_4 \\ \theta_4 \end{bmatrix}$$

The beams are identical therefore, stiffness matrices, connection stiffnesses and stability functions are identical too. The connection stiffness and stability functions are calculated as,

$$r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} \quad \rightarrow \quad 0.75 = \frac{1}{1 + \frac{3 \cdot (2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot R_{ki}}} \quad \rightarrow \quad R_{ki} = 1.085 \cdot 10^{11} N \cdot mm$$

$$R^* = \left(1 + \frac{4EI}{LR_{kiA}}\right)\left(1 + \frac{4EI}{LR_{kiB}}\right) - \left(\frac{EI}{L}\right)^2 \left(\frac{4}{R_{kiA} R_{kiB}}\right)$$

$$R^* = \left(1 + \frac{4 \cdot (2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot 1.085 \cdot 10^{11}}\right)^2 - \left(\frac{2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000}\right)^2 \left(\frac{4}{(1.085 \cdot 10^{11})^2}\right)$$

$$R^* = 2.037$$

$$s_{ii}^* = \frac{\left(4 + \frac{12EI}{LR_{kiA}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{jj}^{*} = \frac{\left(4 + \frac{12EI}{LR_{kiB}}\right)}{R^{*}} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^{5} \cdot 482 \cdot 10^{6}}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{ij}^{*} = s_{ji}^{*} = \frac{2}{R^{*}} = \frac{2}{2.037} = 0.98$$

Member forces for C1 (units are in kN and m);

$$\begin{bmatrix} 5.65x10^{5} & 0 & 0 & -5.65x10^{5} & 0 & 0 \\ 0 & 2.37x10^{3} & 4.73x10^{6} & 0 & -2.37x10^{3} & 4.73x10^{6} \\ 0 & 4.73x10^{6} & 1.26x10^{10} & 0 & -4.73x10^{6} & 6.31x10^{9} \\ -5.65x10^{5} & 0 & 0 & 5.65x10^{5} & 0 & 0 \\ 0 & -2.37x10^{3} & -4.73x10^{6} & 0 & 2.37x10^{3} & -4.73x10^{6} \\ 0 & 4.73x10^{6} & 6.31x10^{9} & 0 & -4.73x10^{6} & 1.26x10^{10} \end{bmatrix} x \begin{bmatrix} -0.5062 \\ 13.739 \\ -0.0011 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -286.0 \\ 27.3 \\ 51.1 \\ 286.0 \\ -27.3 \\ 58.1 \end{bmatrix}$$

Member forces for C2 (units are in kN and m);

$$\begin{bmatrix} 5.65x10^{5} & 0 & 0 & -5.65x10^{5} & 0 & 0 \\ 0 & 6.85x10^{3} & 1.37x10^{7} & 0 & -6.85x10^{3} & 1.37x10^{7} \\ 0 & 1.37x10^{7} & 3.65x10^{10} & 0 & -1.37x10^{7} & 1.83x10^{10} \\ -5.65x10^{5} & 0 & 0 & 5.65x10^{5} & 0 & 0 \\ 0 & -6.85x10^{3} & -1.37x10^{7} & 0 & 6.85x10^{3} & -1.37x10^{7} \\ 0 & 1.37x10^{7} & 1.83x10^{10} & 0 & -1.37x10^{7} & 3.65x10^{10} \end{bmatrix} x \begin{bmatrix} -0.9680 \\ 13.739 \\ -0.0016 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -546.9 \\ 72.2 \\ 129.8 \\ 546.9 \\ -72.2 \\ 159.0 \end{bmatrix}$$

Member forces for C3 (units are in kN and m);

$$\begin{bmatrix} 5.65x10^{5} & 0 & 0 & -5.65x10^{5} & 0 & 0 \\ 0 & 6.85x10^{3} & 1.37x10^{7} & 0 & -6.85x10^{3} & 1.37x10^{7} \\ 0 & 1.37x10^{7} & 3.65x10^{10} & 0 & -1.37x10^{7} & 1.83x10^{10} \\ -5.65x10^{5} & 0 & 0 & 5.65x10^{5} & 0 & 0 \\ 0 & -6.85x10^{3} & -1.37x10^{7} & 0 & 6.85x10^{3} & -1.37x10^{7} \\ 0 & 1.37x10^{7} & 1.83x10^{10} & 0 & -1.37x10^{7} & 3.65x10^{10} \end{bmatrix} x \begin{bmatrix} -0.9781 \\ 13.739 \\ -0.0016 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -552.6 \\ 72.8 \\ 131.3 \\ 552.6 \\ -72.8 \\ 159.7 \end{bmatrix}$$

Member forces for C4 (units are in kN and m);

$$\begin{bmatrix} 5.65x10^5 & 0 & 0 & -5.65x10^5 & 0 & 0 \\ 0 & 2.37x10^3 & 4.73x10^6 & 0 & -2.37x10^3 & 4.73x10^6 \\ 0 & 4.73x10^6 & 1.26x10^{10} & 0 & -4.73x10^6 & 6.31x10^9 \\ -5.65x10^5 & 0 & 0 & 5.65x10^5 & 0 & 0 \\ 0 & -2.37x10^3 & -4.73x10^6 & 0 & 2.37x10^3 & -4.73x10^6 \\ 0 & 4.73x10^6 & 6.31x10^9 & 0 & -4.73x10^6 & 1.26x10^{10} \end{bmatrix} x \begin{bmatrix} -0.5062 \\ 13.739 \\ -0.0011 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -314.5 \\ 27.8 \\ 52.4 \\ 314.5 \\ -27.8 \\ 58.7 \end{bmatrix}$$

Member forces for B1 (units are in kN and m);

$$\begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix} x \begin{bmatrix} -0.5062 \\ -0.0011 \\ -0.9680 \\ -0.0016 \end{bmatrix} = \begin{bmatrix} -14.0 \\ -51.1 \\ 14.0 \\ -61.0 \end{bmatrix}$$

Member forces for B2 (units are in kN and m);

$$\begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix} x \begin{bmatrix} -0.9680 \\ -0.0016 \\ -0.9781 \\ -0.0016 \end{bmatrix} = \begin{bmatrix} -17.1 \\ -68.8 \\ 17.1 \\ -68.0 \end{bmatrix}$$

Member forces for B3 (units are in kN and m);

$$\begin{bmatrix} 1.36x10^3 & 5.42x10^6 & -1.36x10^3 & 5.42x10^6 \\ 5.42x10^6 & 3.15x10^{10} & -5.42x10^6 & 1.18x10^{10} \\ -1.36x10^3 & -5.42x10^6 & 1.36x10^3 & -5.42x10^6 \\ 5.42x10^6 & 1.18x10^{10} & -5.42x10^6 & 3.15x10^{10} \end{bmatrix} x \begin{bmatrix} -0.9781 \\ -0.0016 \\ -0.5566 \\ -0.0010 \end{bmatrix} = \begin{bmatrix} -14.5 \\ -63.3 \\ 14.5 \\ -52.4 \end{bmatrix}$$

b) <u>Design with DAM</u>

Stiffness matrix of the columns C1 and C4:

$$k_{c1} = k_{c4} = \begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 1.89x10^3 & 3.79x10^6 & 0 & -1.89x10^3 & 3.79x10^6 \\ 0 & 3.79x10^6 & 1.01x10^{10} & 0 & -3.79x10^6 & 5.05x10^9 \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -1.89x10^3 & -3.79x10^6 & 0 & 1.89x10^3 & -3.79x10^6 \\ 0 & 3.79x10^6 & 5.05x10^9 & 0 & -3.79x10^6 & 1.01x10^{10} \end{bmatrix}$$

Stiffness matrix of the columns C2 and C3:

$$k_{c2} = k_{c3} = \begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 5.48x10^3 & 1.10x10^7 & 0 & -5.48x10^3 & 1.10x10^7 \\ 0 & 1.10x10^7 & 2.92x10^{10} & 0 & -1.10x10^7 & 1.46x10^{10} \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -5.48x10^3 & -1.10x10^7 & 0 & 5.48x10^3 & -1.10x10^7 \\ 0 & 1.10x10^7 & 1.46x10^{10} & 0 & -1.10x10^7 & 2.92x10^{10} \end{bmatrix}$$

Displacement matrices of columns C1, C2, C3 and C4:

$$d_{C1} = \begin{bmatrix} v_1 \\ u \\ \theta_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad d_{C2} = \begin{bmatrix} v_2 \\ u \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad d_{C3} = \begin{bmatrix} v_3 \\ u \\ \theta_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad d_{C4} = \begin{bmatrix} v_4 \\ u \\ \theta_4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Stiffness matrix of the beams B1, B2 and B3:

$$k_{B1} = k_{B2} = k_{B3} = \begin{bmatrix} 1.08x10^3 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 4.34x10^6 & 2.52x10^{10} & -4.34x10^6 & 9.46x10^9 \\ -1.08x10^3 & -4.34x10^6 & 1.08x10^3 & -4.34x10^6 \\ 4.34x10^6 & 9.46x10^9 & -4.34x10^6 & 2.52x10^{10} \end{bmatrix}$$

Displacement matrices of beams B1, B2 and B3:

$$d_{B1} = \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix} \qquad d_{B2} = \begin{bmatrix} v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{bmatrix} \qquad d_{B3} = \begin{bmatrix} v_3 \\ \theta_3 \\ v_4 \\ \theta_4 \end{bmatrix}$$

The beams are identical therefore, stiffness matrices, connection stiffnesses and stability functions are identical too. The connection stiffness and stability functions are calculated as:

$$r = \frac{1}{1 + \frac{3EI}{LR_{ki}}} \rightarrow 0.75 = \frac{1}{1 + \frac{3 \cdot (0.8 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot R_{ki}}} \rightarrow R_{ki} = 8.676 \cdot 10^{10} N \cdot mm$$

$$R^* = \left(1 + \frac{4EI}{LR_{kiA}}\right)\left(1 + \frac{4EI}{LR_{kiB}}\right) - \left(\frac{EI}{L}\right)^2 \left(\frac{4}{R_{kiA}R_{kiB}}\right)$$

$$R^* = \left(1 + \frac{4 \cdot (0.8 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6)}{8000 \cdot 8.676 \cdot 10^{10}}\right)^2 - \left(\frac{0.8 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000}\right)^2 \left(\frac{4}{(8.676 \cdot 10^{10})^2}\right)$$

$$R^* = 2.037$$

$$s_{ii}^* = \frac{\left(4 + \frac{12EI}{LR_{kiA}}\right)}{R^*} = \frac{\left(4 + \frac{12 \cdot 2 \cdot 10^5 \cdot 482 \cdot 10^6}{8000 \cdot 1.085 \cdot 10^{11}}\right)}{2.037} = 2.62$$

$$s_{jj}^{*}=\frac{\left(4+\frac{12EI}{LR_{kiB}}\right)}{R^{*}}=\frac{\left(4+\frac{12\cdot 2\cdot 10^{5}\cdot 482\cdot 10^{6}}{8000\cdot 1.085\cdot 10^{11}}\right)}{2.037}=2.62$$

$$s_{ij}^{*}=s_{ji}^{*}=\frac{2}{R^{*}}=\frac{2}{2.037}=0.98$$

Member forces for C1 (units are in kN and m);

$$\begin{bmatrix} 4.52x10^{5} & 0 & 0 & -4.52x10^{5} & 0 & 0 \\ 0 & 1.89x10^{3} & 3.79x10^{6} & 0 & -1.89x10^{3} & 3.79x10^{6} \\ 0 & 3.79x10^{6} & 1.01x10^{10} & 0 & -3.79x10^{6} & 5.05x10^{9} \\ -4.52x10^{5} & 0 & 0 & 4.52x10^{5} & 0 & 0 \\ 0 & -1.89x10^{3} & -3.79x10^{6} & 0 & 1.89x10^{3} & -3.79x10^{6} \\ 0 & 3.79x10^{6} & 5.05x10^{9} & 0 & -3.79x10^{6} & 1.01x10^{10} \end{bmatrix} x \begin{bmatrix} -0.6327 \\ 17.1741 \\ -0.0014 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -286.0 \\ 27.3 \\ 51.1 \\ 286.0 \\ -27.3 \\ 58.1 \end{bmatrix}$$

Member forces for C2 (units are in kN and m);

$$\begin{bmatrix} 4.52x10^{5} & 0 & 0 & -4.52x10^{5} & 0 & 0 \\ 0 & 5.48x10^{3} & 1.10x10^{7} & 0 & -5.48x10^{3} & 1.10x10^{7} \\ 0 & 1.10x10^{7} & 2.92x10^{10} & 0 & -1.10x10^{7} & 1.46x10^{10} \\ -4.52x10^{5} & 0 & 0 & 4.52x10^{5} & 0 & 0 \\ 0 & -5.48x10^{3} & -1.10x10^{7} & 0 & 5.48x10^{3} & -1.10x10^{7} \\ 0 & 1.10x10^{7} & 1.46x10^{10} & 0 & -1.10x10^{7} & 2.92x10^{10} \end{bmatrix} x \begin{bmatrix} -1.2100 \\ 17.1741 \\ -0.0020 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -546.9 \\ 72.2 \\ 129.8 \\ 546.9 \\ -72.2 \\ 159.0 \end{bmatrix}$$

Member forces for C3 (units are in kN and m);

$$\begin{bmatrix} 4.52x10^{5} & 0 & 0 & -4.52x10^{5} & 0 & 0 \\ 0 & 5.48x10^{3} & 1.10x10^{7} & 0 & -5.48x10^{3} & 1.10x10^{7} \\ 0 & 1.10x10^{7} & 2.92x10^{10} & 0 & -1.10x10^{7} & 1.46x10^{10} \\ -4.52x10^{5} & 0 & 0 & 4.52x10^{5} & 0 & 0 \\ 0 & -5.48x10^{3} & -1.10x10^{7} & 0 & 5.48x10^{3} & -1.10x10^{7} \\ 0 & 1.10x10^{7} & 1.46x10^{10} & 0 & -1.10x10^{7} & 2.92x10^{10} \end{bmatrix} x \begin{bmatrix} -1.2227 \\ 17.1741 \\ -0.0019 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -552.6 \\ 72.8 \\ 131.3 \\ 552.6 \\ -72.8 \\ 159.7 \end{bmatrix}$$

Member forces for C4 (units are in kN and m);

$$\begin{bmatrix} 4.52x10^5 & 0 & 0 & -4.52x10^5 & 0 & 0 \\ 0 & 1.89x10^3 & 3.79x10^6 & 0 & -1.89x10^3 & 3.79x10^6 \\ 0 & 3.79x10^6 & 1.01x10^{10} & 0 & -3.79x10^6 & 5.05x10^9 \\ -4.52x10^5 & 0 & 0 & 4.52x10^5 & 0 & 0 \\ 0 & -1.89x10^3 & -3.79x10^6 & 0 & 1.89x10^3 & -3.79x10^6 \\ 0 & 3.79x10^6 & 5.05x10^9 & 0 & -3.79x10^6 & 1.01x10^{10} \end{bmatrix} x \begin{bmatrix} -0.6957 \\ 17.1741 \\ -0.0013 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -286.0 \\ 27.3 \\ 51.1 \\ 286.0 \\ -27.3 \\ 58.1 \end{bmatrix}$$

Member forces for B1 (units are in kN and m);

$$\begin{bmatrix} 1.08x10^3 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 4.34x10^6 & 2.52x10^{10} & -4.34x10^6 & 9.46x10^9 \\ -1.08x10^3 & -4.34x10^6 & 1.08x10^3 & -4.34x10^6 \\ 4.34x10^6 & 9.46x10^9 & -4.34x10^6 & 2.52x10^{10} \end{bmatrix} x \begin{bmatrix} -0.6327 \\ -0.0014 \\ -1.2100 \\ -0.0020 \end{bmatrix} = \begin{bmatrix} -14.0 \\ -51.1 \\ 14.0 \\ -61.0 \end{bmatrix}$$

Member forces for B2 (units are in kN and m);

$$\begin{bmatrix} 1.08x10^3 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 4.34x10^6 & 2.52x10^{10} & -4.34x10^6 & 9.46x10^9 \\ -1.08x10^3 & -4.34x10^6 & 1.08x10^3 & -4.34x10^6 \\ 4.34x10^6 & 9.46x10^9 & -4.34x10^6 & 2.52x10^{10} \end{bmatrix} x \begin{bmatrix} -1.2100 \\ -0.0020 \\ -1.2227 \\ -0.0019 \end{bmatrix} = \begin{bmatrix} -17.1 \\ -68.8 \\ 17.1 \\ -68.0 \end{bmatrix}$$

Member forces for B3 (units are in kN and m);

$$\begin{bmatrix} 1.08x10^3 & 4.34x10^6 & -1.08x10^3 & 4.34x10^6 \\ 4.34x10^6 & 2.52x10^{10} & -4.34x10^6 & 9.46x10^9 \\ -1.08x10^3 & -4.34x10^6 & 1.08x10^3 & -4.34x10^6 \\ 4.34x10^6 & 9.46x10^9 & -4.34x10^6 & 2.52x10^{10} \end{bmatrix} x \begin{bmatrix} -1.2227 \\ -0.0019 \\ -0.6957 \\ -0.0013 \end{bmatrix} = \begin{bmatrix} -14.5 \\ -63.3 \\ 14.5 \\ -52.4 \end{bmatrix}$$

APPENDIX D

LATERAL DRIFT VALUES OF CASES

1. Case - I

Table D.1 – Lateral Drift Values for Case - I

	Effective Length Method		*Direct Analysis Method*	
r	**1st Order Drift (mm)**	**2nd Order Drift (mm)**	**1st Order Drift (mm)**	**2nd Order Drift (mm)**
1.00	17.11	17.41	21.38	21.86
0.95	17.50	17.82	21.88	22.37
0.90	17.93	18.26	22.42	22.93
0.85	18.40	18.75	23.00	23.55
0.80	18.91	19.28	23.64	24.22
0.75	19.47	19.86	24.34	24.95
0.70	20.09	20.50	25.11	25.76
0.65	20.77	21.21	25.96	26.66
0.60	21.53	22.00	26.91	27.66
0.55	22.38	22.89	27.97	28.78
0.50	23.33	23.89	29.16	30.05
0.45	24.42	25.03	30.52	31.49
0.40	25.66	26.34	32.08	33.15
0.35	27.10	27.86	33.87	35.07
0.30	28.78	29.64	35.98	37.33
0.25	30.78	31.76	38.47	40.02
0.20	33.18	34.33	41.47	43.28
0.15	36.13	37.50	45.16	47.32
0.10	39.84	41.51	49.80	52.43
0.05	44.64	46.75	55.81	59.13
0.00	51.11	53.89	63.89	68.29

2. Case - II

Table D.2 – Lateral Drift Values for Case - II

	Effective Length Method		*Direct Analysis Method*	
r	1st Order Drift (mm)	2nd Order Drift (mm)	1st Order Drift (mm)	2nd Order Drift (mm)
1.00	11.24	11.93	14.05	15.14
0.95	11.50	12.22	14.38	15.52
0.90	11.78	12.54	14.73	15.93
0.85	12.09	12.89	15.11	16.38
0.80	12.43	13.27	15.53	16.87
0.75	12.79	13.69	15.99	17.42
0.70	13.20	14.16	16.50	18.02
0.65	13.65	14.67	17.06	18.69
0.60	14.15	15.25	17.68	19.44
0.55	14.70	15.90	18.38	20.29
0.50	15.33	16.64	19.17	21.25
0.45	16.05	17.48	20.06	22.35
0.40	16.86	18.45	21.08	23.63
0.35	17.81	19.59	22.26	25.12
0.30	18.91	20.94	23.64	26.89
0.25	20.22	22.56	25.28	29.04
0.20	21.80	24.54	27.25	31.67
0.15	23.74	27.02	29.68	34.99
0.10	26.18	30.23	32.73	39.30
0.05	29.34	34.52	36.67	45.14
0.00	33.59	40.56	41.99	53.47

3. Case - III

Table D.3 – Lateral Drift Values for Case - III

	Effective Length Method		Direct Analysis Method	
r	1st Order Drift (mm)	2nd Order Drift (mm)	1st Order Drift (mm)	2nd Order Drift (mm)
1.00	3.91	4.39	4.89	5.66
0.95	4.00	4.50	5.00	5.81
0.90	4.10	4.63	5.12	5.98
0.85	4.21	4.77	5.26	6.16
0.80	4.32	4.92	5.40	6.36
0.75	4.45	5.08	5.56	6.59
0.70	4.59	5.27	5.74	6.83
0.65	4.75	5.47	5.93	7.11
0.60	4.92	5.70	6.15	7.43
0.55	5.11	5.97	6.39	7.78
0.50	5.33	6.27	6.67	8.19
0.45	5.58	6.61	6.98	8.67
0.40	5.87	7.01	7.33	9.22
0.35	6.19	7.49	7.74	9.88
0.30	6.58	8.06	8.22	10.68
0.25	7.03	8.76	8.79	11.66
0.20	7.58	9.62	9.48	12.90
0.15	8.26	10.74	10.32	14.51
0.10	9.11	12.21	11.38	16.69
0.05	10.20	14.27	12.76	19.82
0.00	11.68	17.35	14.60	24.67

4. Case - IV

Table D.4 – Lateral Drift Values for Case - IV

	Effective Length Method		*Direct Analysis Method*	
r	**1st Order Drift (mm)**	**2nd Order Drift (mm)**	**1st Order Drift (mm)**	**2nd Order Drift (mm)**
1.00	12.67	13.08	15.83	16.48
0.95	12.84	13.27	16.05	16.72
0.90	13.03	13.47	16.29	16.98
0.85	13.24	13.70	16.55	17.27
0.80	13.48	13.95	16.85	17.59
0.75	13.74	14.23	17.17	17.94
0.70	14.03	14.54	17.54	18.34
0.65	14.36	14.90	17.95	18.80
0.60	14.74	15.30	18.42	19.31
0.55	15.17	15.77	18.96	19.90
0.50	15.67	16.31	19.59	20.59
0.45	16.26	16.94	20.32	21.41
0.40	16.95	17.71	21.19	22.38
0.35	17.80	18.63	22.25	23.56
0.30	18.85	19.78	23.56	25.04
0.25	20.18	21.25	25.22	26.92
0.20	21.92	23.19	27.40	29.42
0.15	24.31	25.88	30.39	32.88
0.10	27.78	29.85	34.72	38.02
0.05	33.29	36.31	41.61	46.44
0.00	43.41	48.70	54.27	62.78