ORBIT TRANSFER OPTIMIZATION OF SPACECRAFT WITH IMPULSIVE THRUSTS USING GENETIC ALGORITHM

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ABSTRACT

ORBIT TRANSFER OPTIMIZATION OF A SPACECRAFT WITH IMPULSIVE THRUST USING GENETIC ALGORITHM

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This thesis addresses the orbit transfer optimization problem of a spacecraft. The optimal orbit transfer is the process of altering the orbit of a spacecraft with minimum propellant consumption. The spacecrafts are needed to realize orbit transfer to reach, change or keep its orbit. The spacecraft may be a satellite or the last stage of a launch vehicle that is operated at the exo-atmospheric region. In this study, a genetic algorithm based orbit transfer method has been developed. The applicability of genetic algorithm based orbit transfer method has been verified using orbit transfers which are optimal at specific cases. The solution to orbit transfer problem is also searched using steepest descent algorithm. While genetic algorithm can reach the optimal solution, steepest descent algorithm can reach optimal solution when a good initial prediction is provided. The effects of the initial orbital values on the orbit transfer solutions are also studied.

Keywords: Orbit transfer, optimal orbit transfer, minimum propellant consumption, impulsive maneuvers, genetic algorithms.

ÖZ

GENETİK ALOGRİTMA KULLANILARAK BİR UZAY ARACININ DARBESEL İTKİYLE YÖRÜNGE TRANSFER OPTİMİZASYONU

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Bu tez çalışması bir uzay aracının yörünge transfer eniyileme problemini içermektedir. Eniyilenmiş yörünge transferi asgari yakıt tüketimi ile uzay aracının yörüngesinin değiştirilmesidir. Uzay araçları yörüngeye erişmek, değiştirmek ve korumak amaçlı yörünge transferi gerçekleştirirler. Uzay aracı uydu ya da atmosfer dışı bölgede kullanılan uydu fırlatma aracı son kademesidir. Bu çalışmada bir genetik algoritma tabanlı yörünge transfer metodu geliştirilmiştir. Genetik algoritma tabanlı yörünge transfer yönteminin uygulanabilirliğinin belirli koşullarda eniyilenmiş yörünge transferleri kullanılarak doğrulanmıştır. Yörünge transfer problemine çözüm ayrıca 'steepest descent' algoritması ile de aranmıştır. Genetik algoritma eniyilenmiş sonuca ulaşabilirken, 'steepest descent' algoritması yanlızca iyi bir ilk tahmin sağlandığında eniyilenmiş sonuca ulaşabilmektedir. İlk yörünge değerlerinin yörünge transfer problem çözümüne etkileri de bu çalışmada incelenmiştir.

Anahtar Kelimeler: Yörünge transferi, eniyilenmiş yörünge transferi, asgari yakıt tüketimi, darbeli manevra, genetik algoritma.

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The work presented in this thesis could not have been accomplished without the support from many individuals. Finally, my thanks go to all those who are not specifically mentioned here.

Ankara, 5 August 2012

Ahmet Yılmaz

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LIST OF SYMBOLS

a	Semimajor axis
e	Eccentricity
i	Inclination
Ω	Right ascension of ascending node
ω	Argument of perigee
v	True anomaly
r_a	Apogee point
r_p	Perigee point
N1	Descending node
N2	Ascending node
h	Specific angular momentum vector magnitude
$\overline{\mathbf{x}}$	Orbital elements column matrix
b	Semiminor axis
$\overline{\mathbf{a}}$	Acceleration column matrix
c_a	Cost function coefficient of semimajor axis
c_e	Cost function coefficient of eccentricity
c_i	Cost function coefficient of inclination
c_Ω	Cost function coefficient of right ascension of ascending node
c_ω	Cost function coefficient of argument of perigee
c_v	Cost function coefficient of impulsive velocity vector magnitude
δ_a	Semimajor axis tolerance
δ_e	Eccentricity tolerance
δ_i	Inclination tolerance
δ_Ω	Right ascension of ascending node tolerance
δ_ω	Argument of perigee tolerance
J	Cost function

LIST OF ABBREVIATIONS

ECI	Earth Centered Inertial
SS	Sun Synchronous
LEO	Low Earth Orbit
MEO	Medium Earth Orbit
GEO	Geostationary Earth Orbit
GTO	Geostationary Transfer Orbit
OIC	Only Inclination Change
GA	Genetic Algorithm

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

A spacecraft is a vehicle that is designed for spaceflight. Spacecraft can be categorized in two main groups. These are satellites and launch vehicles.

Satellite is a natural or an artificial body that revolves around a celestial body. In this study only man-made (artificial) satellites are considered. Starting from this point the satellite is used instead of artificial satellite.

The idea of satellite was originated at the late 1800s. The modern satellite application concept was originated from a paper of Arthur C. Clarke published in Wireless World magazine in 1945. In this article he stated that "*... I would like to close by mentioning a possibility of the more remote future perhaps half a century ahead. An 'artificial satellite' at the correct distance from the earth would remain stationary above the same spot and would be within optical range of nearly half the earth's surface. Three repeater stations, 120 degrees apart in the correct orbit, could give television and microwave coverage to the entire planet*" (Clarke, 1945). The orbit that Clarke suggested is actually a geostationary orbit (35,786 km altitude, circular). Therefore, a geostationary orbit is also sometimes called a Clarke orbit.

Satellites are mainly used for communication (Figure 1), Earth observation (Figure 2), military and scientific purposes.

Figure 1 Geostationary satellite (Maini, Anil K.; Argawal, Varsha, 2011)

Figure 2 Earth observation satellite (Maini, Anil K.; Argawal, Varsha, 2011)

In 1955 the United States and Russia announced that their plans were to design and insert an artificial satellite to an orbit. On 4 October 1957, Russia inserted the first man-made satellite 'Sputnik' (Figure 3) into an elliptic orbit. It serviced for 92 days.

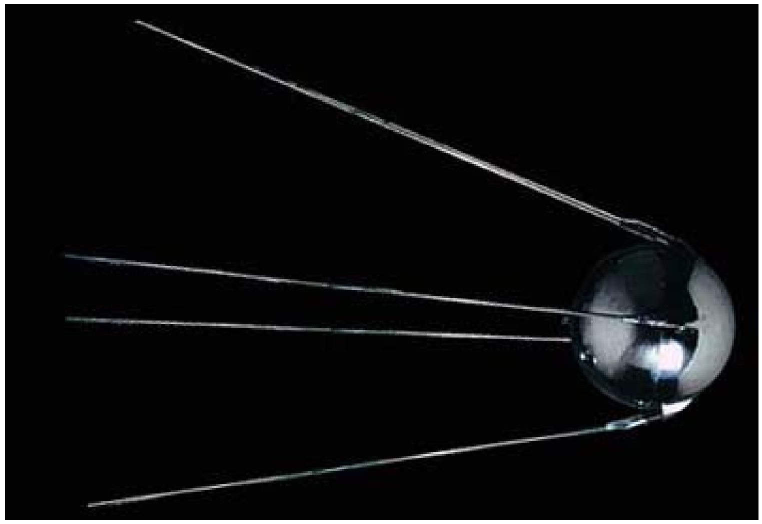

Figure 3 Sputnik-1 spacecraft (Krebs, 2012)

Russia inserted Sputnik-2 into the orbit also in 1957. Sputnik-2 carried a dog named "Laika" to space. In 1958 the United States inserted their first satellite, Explorer-1, into the orbit successfully on 31 January 1958.

Sputnik and Explorer series satellites provide very important knowledge about satellite and satellite launch technologies. Different types of satellites were inserted into different orbits to specify possible application areas of satellites from 1960 to 1965. The first satellites were communication satellites inspired from Arthur Clarke. Communication satellites followed by Earth observation, broadcasting and navigation purposed satellites.

Launch vehicles are a kind of multi-stage rocket system that carries satellites to orbit. Launch vehicles have also evolved in order to meet launch demands of different categories of satellites. (Maini, Anil K.; Argawal, Varsha, 2011) stated that both smaller launch vehicles capable of launching satellites into low Earth orbits and giant sized launch vehicles that can deploy multiple satellites into geostationary transfer orbit have shown improvements in their design over the last four decades of their history. In the earlier stages, the need to develop launch vehicles by countries like the United States and Russia (earlier Soviet Union) was targeted to acquire superiority in space technology. This led them to use the missile technology developed during the Second World War era to build launch vehicles. This was followed by their desire to have the capability to launch bigger satellites to different orbits.

Launch vehicles are expected to carry satellites to their desired orbits as close as possible. However, satellite is separated from the launch vehicle within a limited accuracy. This is also defined as injection/separation accuracy and generally defined in the datasheets or manuals of launch vehicles. For example in the Proton launch vehicle user manual, accuracy and performance values for both Low Earth Orbit and Geostationary Earth Orbit are given (Proton Mission Planner's Guide, 2009). The injection accuracy is mainly specified depending on the final stage guidance and the propulsion system performances.

It is obvious that, in the highly competitive environment of the launch service market, the future trend of spacecraft will be to reduce the cost and size of the spacecraft as well as increasing the performance of spacecraft payload. If orbit transfers which require less amount of energy can be obtained, the mass of onboard propellant of a spacecraft is reduced. Therefore orbit transfer is an important and contemporary subject.

1.2 MOTIVATION

Artificial satellites are positioned around Earth for mainly surveillance, reconnaissance, communications and scientific missions. A launch vehicle carries the spacecraft to the orbit. Based on the launch vehicle and the launch area, the launch vehicle can carry the spacecraft up to the desired region in the space with a restricted accuracy. This is also defined as the injection/separation accuracy and generally defined in the datasheets or the manuals of the launch vehicles.

After the separation of the spacecraft from the launch vehicle, the spacecraft is alone to perform all its orbit/attitude correction operations. During most of these operations the spacecraft consumes its propellant, which has to be spared as much as possible to increase the useful life of the spacecraft. These operations can be described briefly as follows:

1. Orbit transfer: This is the modification of the spacecraft’s initial orbit to the desired orbit. Orbit transfer may be carried out for many reasons. Mostly, after the separation from a launch vehicle, a satellite is generally placed closely but not exactly at its mission orbit. In that case, the satellite uses its own propulsion system to reach its mission orbit. Other reasons of orbit transfers are rendezvous requirements with another spacecraft or asteroid, escaping from a threat, etc.
2. Orbit maintenance: This is the effort of keeping the spacecraft in the desired mission orbit.
3. Attitude control: Actually, the attitude control is mainly realized by means of additional actuators such as reaction wheels, magneto torque bars, etc, which do not consume

propellant. Spacecraft propulsion system is only used as a secondary actuator at the attitude control tasks.

4. De-orbiting: The orbital maneuvers which cause leaving mission orbit at the end of the life of a satellite.

Orbit transfer can be described as the changing of one or more orbital elements using the thrust force. Most of the propellant of a spacecraft is consumed during orbit transfers. During attitude control tasks only a small amount of propellant is used.

The main critical parameter that specifies the lifetime of a satellite is the amount of propellant in the satellite. The propellant mass budget is decided based on the orbit transfer requirements.

An extra or inefficient orbit transfer may decrease the lifetime of the satellite. Owing to the fact that the propellant is needed to be saved to increase the lifetime of a spacecraft, the orbit transfer should be performed at the correct point(s) of the orbit and the propulsion system should be operated for proper duration.

Numerous studies on orbit maintenance operations have been performed and these operations are turned to routines. Orbit maintenance procedures have reached to enough maturity and require less amount of propellant; therefore it is not critical to decrease the propellant consumption during orbit maintenance operations. Spacecrafts also utilize some amount of propellant during de-orbiting operations. The propellant consumption at the de-orbiting operations is much less than the propellant consumption during other orbit transfer and maintenance operations (Delft University faculty aerospace engineering). Similar to orbit maintenance, during de-orbiting it is also not so possible to save considerable amount of fuel because de-orbiting requires low amount of propellant. Typical propellant consumption percent of a geostationary earth orbit spacecraft is given in Table 1 (Delft University faculty aerospace engineering).

Table 1 Propellant consumption percent of a typical geostationary satellite

Operation	**Propellant consumption (%)**
Orbit transfer	70.0
Orbit maintenance	29.6
De-orbiting	0.4

Different from orbit maintenance and de-orbiting, orbit transfers require high amount of propellant consumption. The minimization of propellant consumption during orbit transfers is still a contemporary research area. In the case that spacecraft consumes less propellant during orbit transfers, it can use the remaining propellant for orbit maintenance purposes during the rest of its lifetime. Therefore, the service duration of the spacecraft increases. The design, production and insertion of a spacecraft to an orbit are very expensive tasks; because of these reasons increasing service time/lifetime of a spacecraft is very crucial and beneficial.

The classical orbit transfers suggest optimal results at specific conditions. At the most of the cases there exists no classical optimal orbit transfer strategy. While at some cases it is possible to use superposition of classical orbit transfers, they may not suggest the optimal result. Classical orbit transfers focus on changing the desired orbital element(s); however ignores the change at the other orbital elements which should be kept constant. Because of these reasons classical orbit transfers are not preferred.

The purpose of this research is to develop and implement an algorithm which provides optimum orbit transfer strategies for a spacecraft. The main function of this algorithm is to generate orbit transfer strategies which minimize propellant consumption. The algorithm should suggest solutions for different orbit transfer problems. These strategies should be applicable to real satellites and restartable last stage of launch vehicle which works under the exo-atmospheric region. The algorithm must work in the presence of constraints and it should find/obtain results for all types of orbit transfers.

1.3 LITERATURE SURVEY

Orbit transfer optimization is the changing of orbit of a spacecraft while minimizing/maximizing some parameters that prolong the service time of a spacecraft or increases the performance. It may be the minimization of fuel consumption and/or transfer duration or the maximization of payload to total weight ratio. For the Earth centered orbits, generally, the fuel consumption minimization is more critical.

In the orbit transfer optimization, initial and final orbits are state inputs and thrust forces are control inputs.

In this part, some applications of the orbit transfer optimization problem are summarized. Since orbit transfer optimization subject is very broad, only some of the critical steps and studies are summarized.

In the literature, it can be seen that different optimization methods can be used to minimize propellant consumption during orbit transfer. These studies are presented in the chronological order. Analytical solution methods for orbit transfer optimization are introduced at first. Then numerical optimization applications to orbit transfer are explained. Lastly the evolutionary optimization studies are presented.

The first type of orbit transfer optimization studies is analytical derivations. They include the first results of analytical studies. Hohmann and Lawden's studies fall into this category.

Hohmann transfer is an orbital transfer that realizes orbit transfers from one circular to another circular orbit. It was invented by a German scientist in 1925. It is the most fuel efficient way to get from one circular orbit to another circular orbit where the ratio of initial and final orbit radius is less than 11.94 (Hohmann, 1925). It is an analytical optimization method. (Barrar, 1963) proved the optimality of Hohmann transfer.

D.F. Lawden is also one of the earliest scientists who studied on optimal orbit transfers. In (Lawden, 1963), he described the analytical necessary conditions for the optimality of orbit transfer using his own theory "primer vector". In this study the motion is assumed to be confined to a plane and the time of transit is regarded as an optimization variable. He derived a necessary condition for the optimality of impulsive trajectories in terms of the magnitude of this vector. This study is the origin of the orbit transfer analytical optimization studies.

(Lion & Handelsman, 1968) derived gradients of the cost with respect to terminal impulse times and midcourse impulse times and positions using Lawden's primer vector theory. In this study the necessary conditions were developed to specify the applicability of improvement with an additional impulse, the effect of interior impulses of a multi-impulse; and the effect of initial and/or final coasts to the improvement of the trajectory. In the case of orbit transfers between coplanar circular orbits, a geometric interpretation was also given.

The gradients of the cost developed in (Lion & Handelsman, 1968) were then implemented in a nonlinear programming algorithm to iteratively improve a non-optimal solution and converge to an optimal trajectory (Jezewski & Rozandaal, 1968).

Lawden's primer vector theory has been applied to different orbit transfer problems such as spacecraft rendezvous, low thrust, etc. (Prussing, 1969) and (Prussing J. B., 1970) detailed fixed time rendezvous in the vicinity of a circular orbit. It is assumed that the terminal orbits lie close enough to an intermediate circular reference orbit that the linearized equations of motion can be used to describe the transfer. The linear problem for the rendezvous is then solved analytically. Carter and Pardis studied low thrust orbit transfers using primer vector theory (Carter & Pardis, 1996). In this study they assumed that the spacecraft thrusters can supply four level of thrust. The mathematical structure of the solution of the optimal rendezvous problem associated with this propulsion model is found. Computer simulations of rendezvous with a satellite in circular orbit are presented.

Although many scientists have tried to obtain analytical general solution for orbit transfers (mostly up to 1975), so far no closed form expressions have been obtained for optimal orbit transfers. Of course there have been many important optimal results obtained; however, these studies are valid only at specific conditions. During the evolution of spacecraft mission requirements, the orbit transfer requirements also change.

(Gobetz & Doll, 1969) collected the analytical results of orbit transfers available in the literature. The study was categorized depending on orbit types. The most common breakdown of these categories was according to geometrical features such as coplanar or noncoplanar boundary conditions, intersecting or nonintersecting orbits, type of conic section, etc. They also stated that three specific problem areas in which additional research is necessary have emerged in this investigation: fixed-time trajectories, optimal multi-impulse modes, and optimal rendezvous.

Since the boundary conditions of initial and final states and costates are specified, the orbit transfer optimization is a two point boundary value problem (TPBVP). As the analytical solution of TPBVP is not easy for this problem, the numerical methods are also used to obtain results for the orbit transfer optimization problem.

Numerical optimizations methods can be analyzed in two main parts; namely, indirect and direct optimization methods. The indirect optimization method uses necessary conditions of optimality and state equations to solve the problem. In direct optimization, the necessary conditions are not used to obtain results; instead the problem is converted to parameter optimization problem.

Many methods have been developed to solve the TPBVP that results from the indirect approach to the optimal trajectory problem. The list includes the method of gradients, quasilinearization, finite difference methods, and collocation techniques. For example (Dickmanns & Well, 1974) used the collocation scheme to solve the TPBVP of the indirect method.

(Conway, 2010) stated that the orbit transfer problem include nonlinearities and singularities. This means the solution is very sensitive to the initial point of some or all of costate variables. The indirect optimization method is not easy to implement this problem. A further difficulty is that the costate variables lack the physical significance of the state variables so that estimating the initial costates proves to be very difficult.

In direct methods, the continuous optimization problem is converted to parameter optimization problem, and it is tried to optimize the parameters while satisfying boundary conditions. (Hargraves & Paris, 1987) was obtained one of the biggest improvements in orbit transfer optimization. They suggested that it was not necessary to solve the all equations (state and costate equations); the costate variables could be ignored from the solution provided that discrete control variables were introduced as additional nonlinear programming parameter. This study enabled the reduction of size of the orbit transfer optimization problem.

After Hargraves and Paris studies, different direct methods are applied to this problem. Enright and Conway studied direct collocation and transcription methods for orbit transfer problem (Enright & Conway, 1991) & (Enright P.J., 1992). In these studies, they stated that the collocation method was found to have deficient accuracy, and an alternative method which discretizes the equations of motion by using an explicit Runge-Kutta parallel-shooting approach was developed. Both methods were applied to finite-thrust spacecraft trajectory problems. These are a three-burn rendezvous and a low-thrust transfer to the moon.

There also exist commercial software packages implementing direct methods for spacecraft and launch vehicle trajectory optimization, for example OTIS (Paris, 1992) and ASTOS (Well, Markl, & Mehlem, 1997) and (Wiegand, Mehlem, Steinkopf, & Ortega, 1999).

In the literature there are also methods that include both the principles of direct and indirect optimization methods. Zondervan et al. studied three impulse orbit transfer in ideal gravity (Zondercab, Wood, & Caughey, 1984). In Ilgen, 1994 (as cited in (Conway, 2010)) also a hybrid method to study orbit transfer problem is used. Low thrust orbit transfer problem is especially studied.

Evolutionary algorithms are also commonly used in orbit transfer optimization with the improvements on the computer technologies. They are also numerical optimizers, which mimic processes of nature. Evolutionary algorithms are stochastic optimization methods that obtain solution by trial and error process. They have important advantages over direct and indirect optimization methods because genetic algorithm does not require an initial prediction. Since in the orbit transfer optimization it is difficult to generate an initial prediction, the popularity of evolutionary algorithms is increased tremendously.

Genetic algorithm is one of the earliest evolutionary optimization methods that are applied at many different areas including orbit transfer problem. To date, genetic algorithm is accepted as one of the most powerful and mature evolutionary algorithms.

(Cacciatore & Toglia, 2008) studied impulsive orbit transfers using minimum fuel with a constraint in the transfer time. They selected genetic algorithm as an optimization method. Their studies also include the effects of genetic algorithm parameters to the solution accuracy and duration.

(Kim & Spencer, 2002) applied genetic algorithms to the coplanar and/or orbit rendezvous problems. They constructed the two impulse orbit transfer problem using six optimization parameters which are true anomaly and thrust components at the initial and final orbits. Their results are compatible with the analytical results.

Reichert also studied the coplanar orbit transfers but instead of six variables he used only three optimization variables (Reichert, 1999). These variables are the semimajor axis, eccentricity and orientation angle of the transfer orbit. He computed the required impulsive velocities from the initial and final orbit values.

(Abdelkhalik, 2005) obtained results for orbit selection and transfer using genetic algorithm. He compared orbit transfer solutions to known orbit transfers (optimal at some cases). He used Lambert problem in order to model the orbit transfer problem. In Lambert problem, the initial and final orbits are constant, so the solution may not be optimal, but there exists always a solution for an orbit transfer.

Denilson, Antonio and Guido studied Lambert problem using genetic algorithm (Santos, Prado, & Colasurdo, 2012). They obtained solution for four impulse rendezvous problem between coplanar circular orbits. In this paper, the approach is to assume that the problem is time-free (transfer duration is not important). In order to perform this task, an engine (kick motor or thruster) that can deliver four burns is assumed to be available. This assumption is used to represent a common constraint posed by real missions.

Particle swarm optimization is another heuristic optimization method that is applied to orbit transfer problem. One of the latest applications of particle swarm optimization to orbit transfer problem is the study of (Pontani & Conway, 2012). They solved the two impulse coplanar orbit transfer from circular to circular and elliptic to elliptic transfers. They also studied the noncoplanar orbit transfers briefly.

(Radice & Olmo, 2006) used ant colony optimization method to model orbit transfer from an Earth orbit to Mars orbit. They modeled the orbit transfer using two impulses. They obtained solutions to launch date and time for such a mission.

In the literature there also exist studies, which combine evolutionary algorithms with other optimization methods. One of the examples of this type is (Sentinella & Casalino, 2009). They suggested a hybrid optimization procedure that runs three different optimizers based on genetic algorithm, differential evolution and particle swarm optimization. They applied this method to the optimization of multiple-impulse rendezvous trajectories and of Earth-to-Mars round-trip missions.

1.4 SCOPE

In this study, a tool is developed to obtain optimal solutions for different type orbit transfer problems. This tool is mainly used at the orbit transfer problems since orbit maintenance and de-orbiting maneuvers do not require so much improvement. Hence orbit transfer is studied in this thesis. In the space, there may be perturbations and gravity effects. In this study the effects of perturbations and gravity are neglected. These are 3^{rd} body, atmospheric drag, solar pressure, Earth's nonspherical shape, and etc. disturbances. It is assumed that the spacecraft is only effected from the central body and the thrust force generated by its propulsion system. This assumption is valid for orbit transfer problems since Newton's gravitational force and thrust forces are much larger than perturbations and gravity effects.

The content of this thesis can be summarized as follows:

- Mathematical formulation of orbit transfer problem is defined.
- An optimization algorithm for orbit transfers is developed/applied using two optimization methods. These methods are:
    - Genetic algorithm
    - steepest descent algorithm
- The applicability of these optimization methods (genetic algorithm and steepest descent algorithm) is discussed.
- The orbit raising problem of Earth centered orbits (LEO, MEO, GEO, Molniya) are defined. The cost function coefficients for each case are obtained.
- Genetic algorithm based orbit transfer optimization and classical orbit transfer strategies are applied to LEO, MEO, GEO and Molniya raising problems. The performances of orbit transfer methods are compared.

- The effects of initial orbital values to the different orbit raising problems (LEO, MEO, GEO, and Molniya) are investigated.

The method that is developed in this study is expected to be used at the mission planning and sizing of:

- Turkey's satellite projects
- Turkey's launch vehicle projects: Note that only the mission planning and sizing of last stage of launch vehicle which is used at the out of atmospheric trajectories can be modeled using this tool.

1.5 THESIS OUTLINE

This thesis is organized as follows:

Chapter 1 is constructed to introduce the study. The introduction of the problem and motivation are explained. After that a short literature survey about orbit transfer optimization studies is given. Scope of the thesis is presented.

Chapter 2 focuses on the mathematical modeling of orbit transfers. The introduction of orbital elements, main governing equations of orbit transfers and orbit transfer models are detailed. At the end of chapter 2, orbit transfer force models are compared briefly.

Chapter 3 details two kinds of optimization methods (genetic and steepest descent algorithms) that can be applied to obtain orbit transfer strategies. The performance of these optimization methods are specified and compared using optimal orbit transfers.

Chapter 4 includes orbit transfer optimization results for different test cases. In this chapter the optimization results are compared to optimal orbit transfer results (optimal only at specific conditions). The effects of each orbital element to the solutions are also considered for all test cases.

Chapter 5 represents the conclusion of the whole study. It also includes a brief summary of the contributions of this thesis and recommendations for future research.

CHAPTER 2

MATHEMATICAL MODEL

2.1 ORBITAL ELEMENTS

A spacecraft's orbit can be defined using different sets of elements. The most common ones are Earth Centered Inertial (ECI) position and velocity vectors with components (r_x, r_y, r_z, v_x, v_y, v_z), perifocal frame (orbit frame) position and velocity vector components (r_p, r_q, r_w, v_p, v_q, v_w), and Keplerian elements. {X, Y, Z} is the axis triad of earth centered inertial frame and {P, Q, W} is the axis triad of the perifocal frame. ECI frame is the frame where the X-Y axes are at the Earth's equatorial plane, with X pointing along the intersection of the equator and the ecliptic (vernal equinox or line of Aries) direction. Z is along the Earth spin axis. Y completes the triad (Figure 4) (Orbit in Space Coordinate Frames and Time, 2012). Note that the effect of nutation of the Earth is ignored at the ECI frame. The perifocal frame is the frame p and q are unit vectors in the orbit plane with p directed to perigee; w along the angular momentum vector and q completes the triad. The ECI frame, perifocal frame and orbital elements are also shown in Figure 5.

The Keplerian orbital elements, sometimes called as classical or conventional elements, are generally used to define a spacecraft's orbit since they are defined around the whole space and make easier to define a spacecraft mission. In orbital elements, five of six variables are constant at an orbit and the remaining variable changes with time. The most commonly used Keplerian orbital element set is $\{a, e, i, \Omega, \omega, v\}$. There are also different set of elements which can be used to define the orbit. Two-line element sets are used by United States of America military and equinoctial elements preferred during perturbation analysis are also popular orbital element sets.

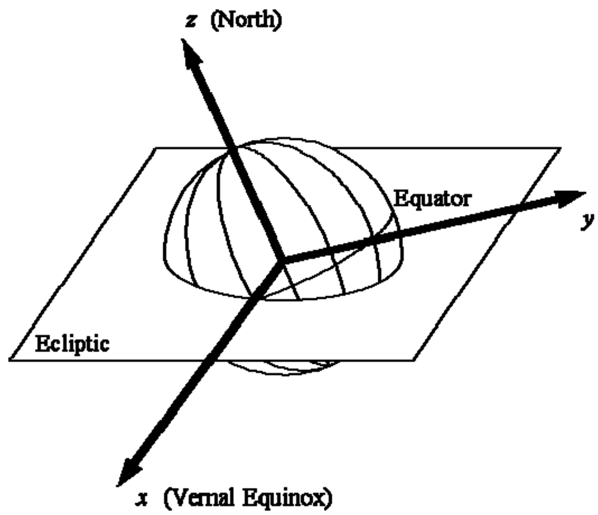

Figure 4 Earth Centered Inertial (ECI) frame

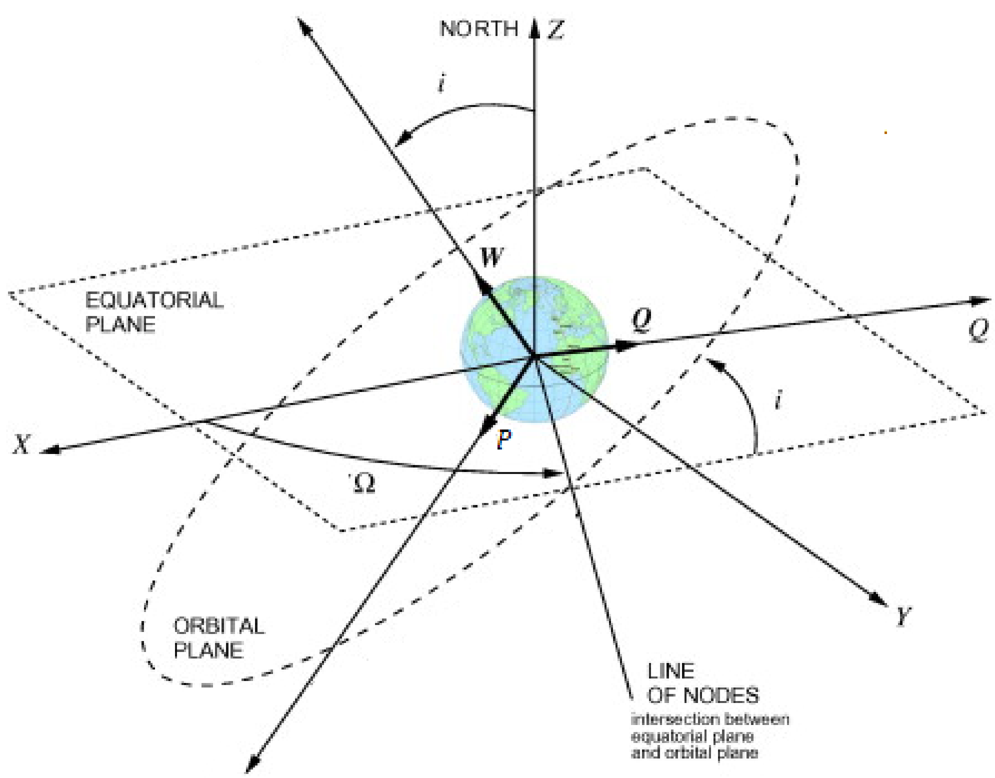

Figure 5 Orbit of a spacecraft (Casella & Lovera, 2008)

The short definitions of Keplerian orbital elements are tabulated in Table 2. Detailed descriptions of the orbital elements are given after Table 2 .

Table 2 Brief description of the Keplerian orbital elements

Keplerian Elements	**Symbol**	**Short Definition**
Semimajor Axis	a	describes the size of orbit ellipse
Eccentricity	e	describes the shape of orbit ellipse
Inclination	i	the angle between the orbit plane and Earth's equatorial plane.
Right Ascension of Ascending Node (RAAN)	Ω	the angle from the vernal equinox to the ascending node
Argument of Perigee	ω	the angle from the ascending node to the eccentricity vector (the vector which is defined in the direction from the Earth center to the perigee point with a magnitude of eccentricity) measured in the direction of satellite's motion
True Anomaly	v	indicates the position of the satellite in its orbit

This set of orbital elements, Keplerian, can be divided into two groups: the dimensional elements and the orientation elements. The dimensional elements specify the size and shape of the orbit and relate the position of spacecraft in the orbit to time; they are semimajor axis, eccentricity, and true anomaly. The orientation elements specify the orientations of the orbit in space which are: inclination, right ascension of ascending node, and argument of perigee (Chobotov, 2002). All Keplerian orbital elements are described briefly as follows:

1. *Semi-major axis (* a *):* This element is a geometrical parameter of an elliptical orbit. It can be computed from apogee and perigee distances as:

$$a = \frac{r_a + r_p}{2} \tag{2.1}$$

where:

r_a : apogee point (farthest point to the Earth)

r_p : perigee point (closest point to the Earth)

The planar geometry of an orbit is presented in Figure 6 where the position of a spacecraft with respect to the Earth is given. The Earth is at the one of the foci of the orbit ellipse.

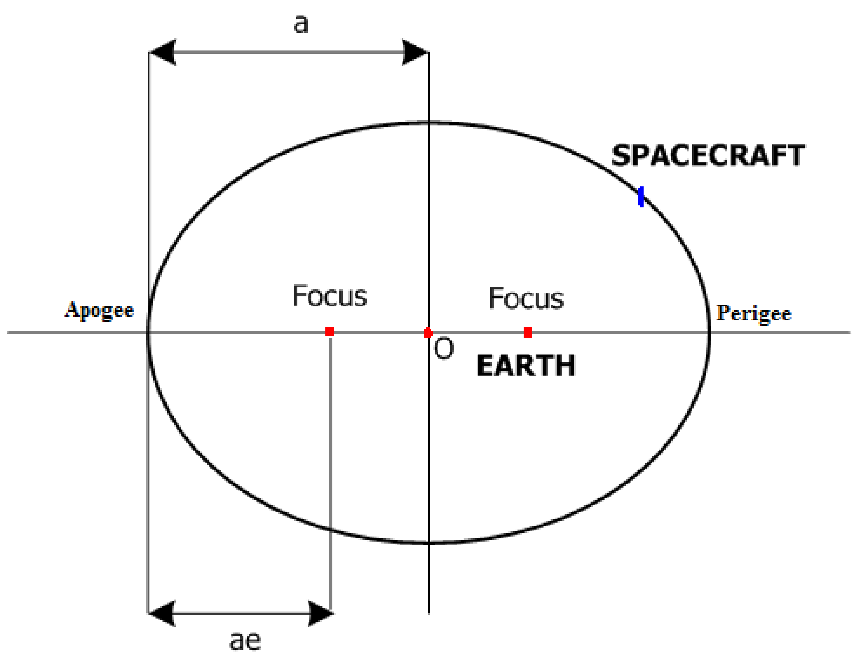

Figure 6 Planar Geometry of an Orbit

2. *Eccentricity (e):* The orbit eccentricity, e, is the ratio of the distance between the centers of the ellipse and the Earth to the semi-major axis of the ellipse. It can be computed applying the following expression;

$$e = \frac{r_a - r_p}{r_a + r_p} \tag{2.2}$$

3. *Inclination (i):* Inclination is the angle that the orbital plane of the spacecraft makes with the Earth's equatorial plane. The spacecraft orbit intersects with the equatorial plane at two points: the first one, called the descending node (N1), where the satellite passes from the northern hemisphere to the southern hemisphere, and the second one, called the ascending node (N2), where the satellite passes from the southern hemisphere to the northern hemisphere. Inclination is the angle between that half of the satellite's orbital plane containing the trajectory of the satellite from the descending node to the ascending node to that half of the Earth's equatorial plane containing the trajectory of a point on the equator from n_1 to n_2, where n_1 and n_2 are respectively the points vertically below the descending and ascending nodes (Figure 7) (Maini, Anil K.; Argawal, Varsha, 2011). Inclination is measured in the direction of counterclockwise and defined between 0° and 360°.

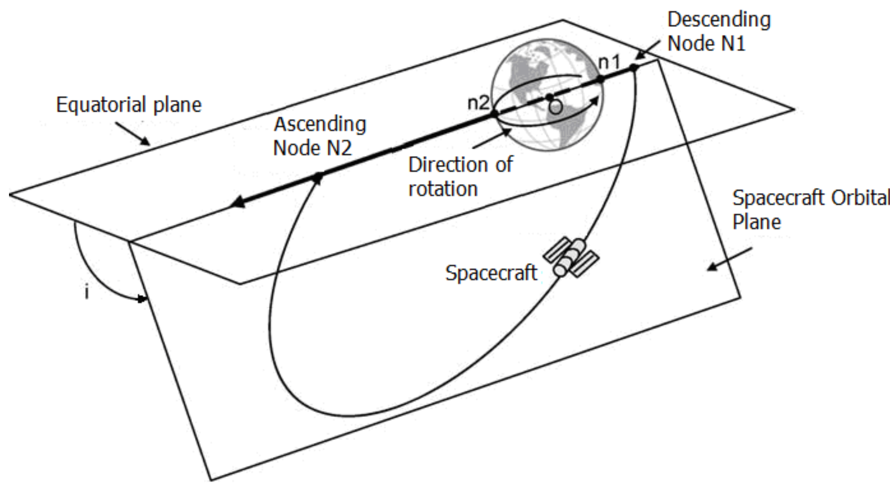

Figure 7 Inclination (i) of an orbit (Maini, Anil K.; Argawal, Varsha, 2011)

4. Right ascension of ascending node (Ω): Right ascension of ascending node is an angle measured counterclockwise in the equator plane, from the direction of the vernal equinox to the ascending node. It is defined between 0° to 360°. Vernal equinox direction is the direction from the center of the Earth to the intersection of the ecliptic and equatorial plane.

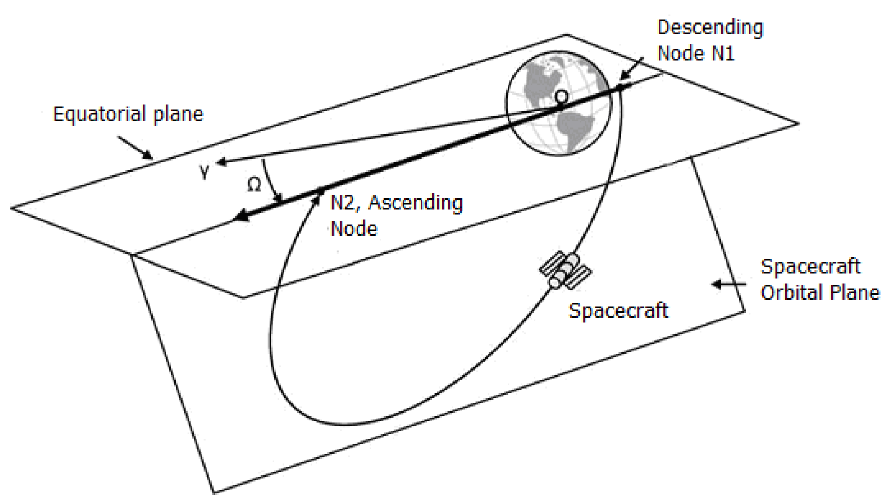

Figure 8 Right ascension of ascending node (Ω) of an orbit (Maini, Anil K.; Argawal, Varsha, 2011)

5. *Argument of perigee (ω):* This element specifies the orientation of the orbit in its plane, i.e. the location of the major axis at the orbit. It is measured as the angle ω between the line joining the perigee and the center of the Earth and the line of nodes from the ascending node to the descending node (Maini, Anil K.; Argawal, Varsha, 2011). Argument of perigee is defined in the direction of motion, from the ascending node to the perigee and defined between 0° and 360°.

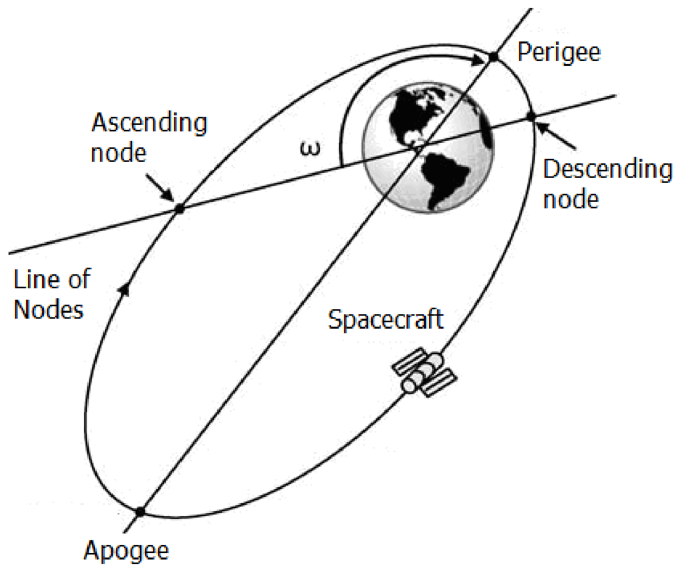

Figure 9 Argument of perigee (ω) of an orbit (Maini, Anil K.; Argawal, Varsha, 2011)

6. True anomaly (ν): This element is used to indicate the position of the satellite in its orbit. This parameter varies with time. True anomaly is defined as the angle formed by the line joining the perigee and the center of the Earth with the line joining the satellite and the center of the Earth (Maini, Anil K.; Argawal, Varsha, 2011). It is defined in the direction of motion, from the perigee to the satellite's position in the orbit and defined between 0° and 360°.

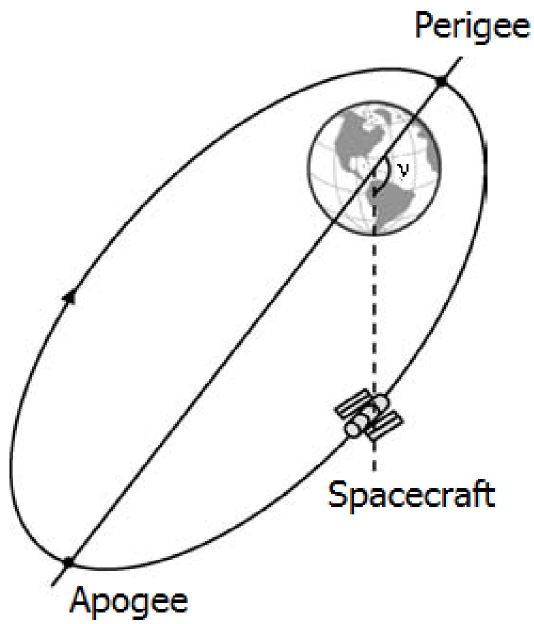

Figure 10 True anomaly (ν) of an orbit (Maini, Anil K.; Argawal, Varsha, 2011)

2.2 ORBITAL MOTION

The motion of a spacecraft, excluding the effect of drag and thrust, around Earth can be specified by the application of Newton's law of gravitational attraction and Kepler's laws. Kepler laws are the main governing laws that define the physical relations of spacecraft and Earth system.

2.2.1 Kepler's Laws

Johannes Kepler, well known Austrian mathematician and astronomer, stated three laws which describe the motions of the planets around the Sun. Although these laws are argued and derived for planetary motion, they are also valid for the motion of natural and artificial satellites around Earth or for any body revolving around another body. These laws can be described briefly as follows:

2.2.1.1 Kepler's First Law

Every planet moves in an orbit that is an ellipse, with the sun at one focus of the ellipse. It is applied to the satellite orbit as follows: The orbit of a satellite around Earth is elliptical with the center of the Earth lying at one of the foci of the ellipse (Figure 2.6). The elliptical orbit is characterized by semimajor axis (a) and eccentricity (e) (Chobotov, 2002).

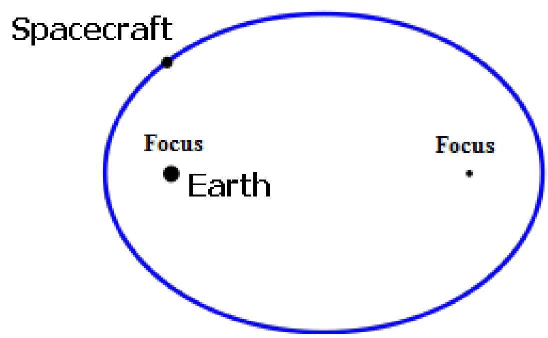

Figure 11 Kepler's First Law

2.2.1.2 Kepler's Second Law

A line joining a planet/comet and the Sun sweeps out equal areas in equal intervals of time. It is applied to a spacecraft orbit as follows: The line joining a spacecraft and the Earth sweeps out equal areas in equal intervals of time (Figure 12).

$$\frac{dA}{dt} = \frac{h}{2} \tag{2.3}$$

where:

h : specific angular momentum vector magnitude

A : sweep-out area

This means that angular momentum of a spacecraft on an orbit is constant at all points on the orbit (Figure 13):

$$\vec{\mathbf{h}} = \vec{\mathbf{r}} \times \vec{\mathbf{v}} \tag{2.4}$$

Note that γ angle between the direction of motion of the spacecraft and the local horizontal. Therefore the specific angular momentum can be stated as follows:

$$h = rv\cos\gamma \tag{2.5}$$

where:

$\vec{\mathbf{r}}$: position vector of a spacecraft

$\vec{\mathbf{v}}$: velocity vector of a spacecraft

$\vec{\mathbf{h}}$: specific angular momentum vector of a spacecraft

h : specific angular momentum vector magnitude of a spacecraft

r : position vector magnitude ($\vec{\mathbf{r}}$) of a spacecraft

v : velocity vector magnitude ($\vec{\mathbf{v}}$) of a spacecraft

γ : angle between the direction of motion of the spacecraft and the local horizontal

This means that spacecraft moves faster close to the Earth and slower far from the Earth on the orbit. Spacecraft reaches its maximum speed at the closest point to the Earth; perigee point and the satellite at the farthest point to the Earth reach its minimum speed which is named as apogee point.

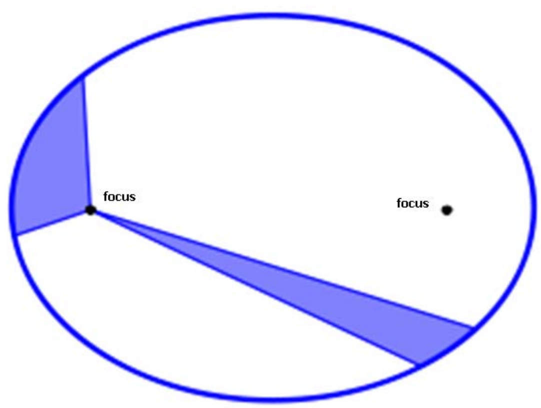

Figure 12 Kepler's Second Law

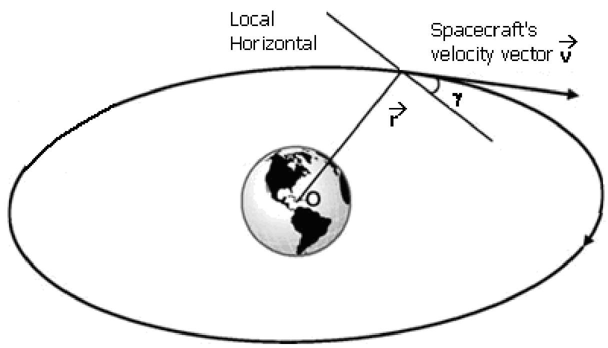

Figure 13 Spacecraft position and velocity vectors on an orbit

2.2.1.3 Kepler's Third Law

The squares of the periods of revolution of the planets are proportional to the cubes of the semimajor axes of their orbits. This law is applied to the spacecraft as follows: The squares of the time period of any spacecraft is proportional to the cube of the semi-major axis of its elliptical orbit.

The area of an ellipse is obtained in terms of its semimajor and semiminor axis by the formula:

$$A = \pi a b \tag{2.6}$$

To find the period T of the elliptical orbit, Kepler's second law is applied (Curtis, 2005):

$$\frac{dA}{dt} = \frac{h}{2} \tag{2.7}$$

For one complete revolution:

$$\Delta A = \pi a b \tag{2.8}$$

$$\Delta t = T \tag{2.9}$$

Thus the period can be stated as:

$$T = \frac{2\pi a b}{h} \tag{2.10}$$

Since semiminor axis is:

$$b = a\sqrt{1-e^2} \tag{2.11}$$

The period T can be stated as follows:

$$T = \frac{2\pi}{h} a^2 \sqrt{1-e^2} = \frac{2\pi}{h} \left(\frac{h^2}{\mu} \frac{1}{1-e^2} \right)^2 \sqrt{1-e^2} \tag{2.12}$$

$$T = \frac{2\pi}{\mu^2} \left(\frac{h}{\sqrt{1-e^2}} \right)^3 \tag{2.13}$$

since

$$h = \sqrt{\mu a \left(1-e^2\right)} \tag{2.14}$$

substituting equation 2.14 to 2.13

$$T = \frac{2\pi}{\sqrt{\mu}} a^{3/2} \tag{2.15}$$

where:

a : semimajor axis

$\mu = Gm_{earth} = 398600\ km^3/s^2$

T : period

h : specific angular momentum vector magnitude

2.2.2 Newton's Law of Gravitational Attraction

According to Newton's Law of Gravitational Attraction, represented in Figure 14, two particles irrespective of their masses are mutually attracted with equal and opposite force $\vec{\mathbf{F}}_{12}$ and $\vec{\mathbf{F}}_{21}$:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12} \tag{2.16}$$

$$\vec{\mathbf{F}} = -\frac{Gm_1 m_2}{r^2} \frac{\vec{\mathbf{r}}}{r} \tag{2.17}$$

where:

$\vec{\mathbf{r}}$: the vector between two particles

G : Newton's gravitational constant

m_1 : mass of the first body

m_2 : mass of the second body

$\vec{\mathbf{F}}_{12}$: force applied to mass-1 by body-2

$\vec{\mathbf{F}}_{21}$: force applied to mass-2 by body-1

This law is applied for a spacecraft and the Earth system:

$$\vec{\mathbf{F}} = -\mu \frac{m_1}{r^2} \frac{\vec{\mathbf{r}}}{r} \tag{2.18}$$

where:

m_1 : spacecraft mass

m_2 : Earth mass

$\mu = Gm_2 = 398600 \ km^3/s^2$

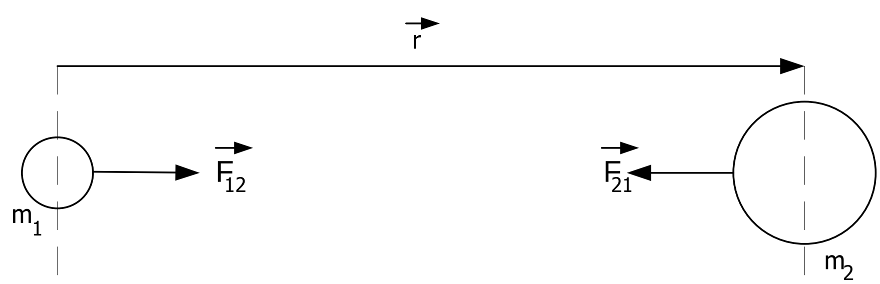

Figure 14 Newton’s Law of Gravitational Attraction

2.2.3 Two Body Problem

In classical mechanics, the two-body problem is used to determine the motion of two point mass particles that interact only with each other. Common examples include a planet orbiting a star, and a satellite orbiting a planet.

To develop a mathematical model for two body problem of an Earth orbiting spacecraft, it is assumed that (Vallado & McClain, 2007):

- The mass of the spacecraft is negligible compared to Earth. This is reasonable for artificial satellites.
- The bodies of the spacecraft and Earth are spherically symmetrical, with uniform density. This enables to treat Earth and satellite as a point mass.
- The coordinate system chosen for a particular problem is inertial.
- No other forces act on the system except for gravitational forces that act along a line joining the centers of the two bodies.

The orbiting motion of a spacecraft can be studied as a two body problem. In two body mathematical model, there exist only forces that are applied by Earth and spacecraft to each other.

Using Newton's second law, these forces can be obtained as follows:

$$\vec{\mathbf{F}} = -m_1 \frac{d^2\vec{\mathbf{r}}_1}{dt^2} = m_2 \frac{d^2\vec{\mathbf{r}}_2}{dt^2} \tag{2.19}$$

where:

$$\vec{\mathbf{F}} \equiv \vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12} \tag{2.20}$$

m_1 : mass of the first body

m_2 : mass of the second body

The vector $\vec{\mathbf{R}}$ denoting the position of the center of mass is defined as:

$$\vec{\mathbf{R}} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2} \tag{2.21}$$

Using the eqn (2.20) and eqn (2.21) the motion of center of masses can be written as follows:

$$(m_1 + m_2)\ddot{\vec{\mathbf{R}}} = 0 \tag{2.22}$$

Eqn (2.24) corresponds to the following equation:

$$\ddot{\vec{\mathbf{R}}} = 0 \tag{2.23}$$

This means that the velocity of the center of mass is constant.

The equation of motion of Earth (m_2)-spacecraft (m_1) system can be written after obtaining the relative position vector $\vec{\mathbf{r}}$ and the equivalent mass m_{eq}:

The relative position vector can be obtained as:

$$\vec{\mathbf{r}} \equiv \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1 \tag{2.24}$$

and the equivalent mass m_{eq} is defined as:

$$m_{eq} = \frac{m_1 m_2}{m_1 + m_2} \tag{2.25}$$

Using the above definition of the equivalent mass m_{eq}, the two-body equations of motion can be converted to an equivalent one-body problem

$$m_{eq}\ddot{\vec{\mathbf{r}}} = F\frac{\vec{\mathbf{r}}}{r} \tag{2.26}$$

The equation (2.27) describes the motion of a mass m under the action of force $\vec{\mathbf{F}}$. Inserting equation (2.19), Newtonian gravitational force, the equation of motion for the equivalent system becomes:

$$\ddot{\vec{\mathbf{r}}} = -\frac{G(m_1 + m_2)}{r^2}\frac{\vec{\mathbf{r}}}{r} \tag{2.27}$$

The motion of spacecrafts, which are not under the effect of thrust or perturbation, around Earth is governed by two forces. One of them is the centripetal force directed towards the center of the Earth caused by the gravitational force of attraction of Earth and the other is the centrifugal force that acts outwards from the center of the Earth. During the motion of a spacecraft under the effect of thrust, the problem can be examined as an extension of the two body problem. In this study, during the motion of a spacecraft both the force exerted by Earth and the thrust force are considered.

2.2.4 Rocket Equation

The rocket equation describes the motion of vehicles that follow the basic principle of a rocket: a device that can apply acceleration to itself (a thrust) by expelling part of its mass with high speed and move due to the conservation of momentum (Chobotov, 2002). The Rocket equation, also known as "Tsiolkovsky rocket equation", can be used during the motion of launch vehicles, satellites, space ships, etc. The rocket equation can be obtained as follows:

Let a system includes two masses: one of these masses is m with a velocity v and the other mass is Δm with a velocity v_1. The initial system momentum (in scalar form) is

$$P_1 = m\mathrm{v} + \Delta m \mathrm{v}_1 \tag{2.28}$$

where:

P_1 : initial system momentum

At a time Δt later, the two masses are joined and their combined momentum is:

$$P_2 = (m + \Delta m)(\mathrm{v} + \Delta \mathrm{v}) \tag{2.29}$$

Since the impulse is equal to change of momentum:

$$I = F\Delta t = P_2 - P_1 = m\Delta\mathrm{v} + \Delta m(\mathrm{v} - \mathrm{v}_1) + \Delta m \Delta \mathrm{v} \tag{2.30}$$

where:

I : impulse magnitude

F : force magnitude

Δt : force application duration

in the limit as $\Delta t \to 0$ and $\Delta m, \Delta\mathrm{v} \to 0$, the force F,

$$F = m\frac{dv}{dt} + \frac{dm}{dt}(\mathrm{v}\text{-}\mathrm{v}_1) \tag{2.31}$$

$$u = \mathrm{v}\text{-}\mathrm{v}_1 \tag{2.32}$$

$$F = m\frac{d\mathrm{v}}{dt} - u\frac{dm}{dt} \tag{2.33}$$

$$F = m\frac{d\mathrm{v}}{dt} - T \tag{2.34}$$

where:

$$T = -u\frac{dm}{dt} \tag{2.35}$$

where:

T : momentum thrust of a rocket

F : external force acting on the rocket (gravity, ,etc.)

u : exhaust velocity

assuming $F = 0$

$$dV = \frac{T\,dt}{m} = -\frac{u\,dm}{m} \tag{2.36}$$

after integration

$$\Delta v = u\,ln(\frac{m_i}{m_f}) = g_0\,I_{sp}\,ln(\frac{m_i}{m_f}) \tag{2.37}$$

where:

m_i : initial mass of rocket

m_f : final mass of rocket

I_{sp} : propellant specific impulse (thrust/propellant weight flow rate)

g_0 : gravitational constant at sea level

Δv : impulsive velocity vector magnitude

It should be noted that the consumed propellant is the difference between the initial and final mass:

$$m_{consumed} = m_f - m_i \tag{2.38}$$

where:

$m_{consumed}$: mass of consumed propellant

This shows that impulsive velocity vector magnitude and consumed propellant mass are directly related. As the level of required impulsive velocity vector magnitude decreases for an orbit transfer, the propellant consumption also decreases.

2.2.5 Orbit Transfer Force Models

Orbital maneuvers can be modeled in two ways depending on thrusting type; namely, impulsive and continuous thrusting. In continuous thrusting the variation of satellite position during maneuver is considered. This causes complexity at the orbit control issues. If it is not obligatory, the orbit transfer should not be modeled using continuous thrust model. On contrary to the continuous thrusting case, in impulsive force model equations are simple and the results are obtained fast and accurately. In this section both impulsive and continuous thrust models are detailed and compared. Impulsive model is valid if the maneuver duration is much less than orbit period. The assumption is ideal if the maneuver duration is zero (0).

2.2.5.1 <u>Continuous Force Model</u>

In continuous thrusting the variation of spacecraft position during transfer is also considered. Gauss Planetary Equation is a special solution to the orbit element variation equation problem where the position vector $\vec{\mathbf{r}}$, velocity vector $\vec{\mathbf{v}}$ and the acceleration column matrix $\overline{\mathbf{a}}$ expressed in R frame components. R frame is a very convenient rotating reference frame $R = \{i_r,\ i_\theta,\ i_h\}$used in orbital mechanics where i_r is along the orbit position vector, i_h is along the angular momentum vector and is the perpendicular to the previous two satisfying right hand rule. This frame is often referred to as Local Vertical Local Horizontal (LVLH) reference frame since it tracks the local horizontal plane.

By using Newton’s law of gravitation and variation of parameters, continuous force model of orbit transfer is obtained.

$$\frac{d(\overline{\mathbf{x}})}{d(t)} =$$

$$\begin{bmatrix} \frac{2a^2 e\sin(\nu)}{h} & \frac{2a^2 p}{hr} & 0 \\ \frac{p\sin(\nu)}{h} & \frac{(p+r)\cos(\nu)+re}{h} & 0 \\ 0 & 0 & \frac{r\cos(\omega+\nu)}{h} \\ 0 & 0 & \frac{r\sin(\omega+\nu)}{h} \\ \frac{-p\cos(\nu)}{he} & \frac{(p+r)\sin(\nu)}{he} & \frac{-r\sin(\omega+\nu)}{h\sin(i)} \\ \frac{b(p\cos(\nu)-2re)}{ahe} & \frac{-b(p+r)\sin(\nu)}{ahe} & 0 \end{bmatrix} \begin{bmatrix} a_r \\ a_\theta \\ a_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ n \end{bmatrix} \qquad (2.39)$$

where:

a = semimajor axis (meters)

e = eccentricity (dimensionless)

i = inclination (radian)

Ω = right ascension of the ascending node (radian)

ω = argument of perigee (radian)

n =mean motion (1/seconds)

$p = a\,(1-e^2)$ = semi-latus rectum (meters)

b = semiminor axis (meters)

h =specific angular momentum vector magnitude (meters per second)

a_r = radial component of acceleration vector $\vec{\mathbf{a}}$ (meters/second^2)

a_θ = tangential component of acceleration vector $\vec{\mathbf{a}}$ (meters/second^2)

a_n = normal component of acceleration vector $\vec{\mathbf{a}}$ (meters/second^2)

$\bar{\mathbf{x}}$ is the [6X1] classical orbital elements [a, e, i, Ω, ω, M], and $\vec{\mathbf{a}}$ is a [3X1] control acceleration column matrix.

2.2.5.2 Impulsive Force Model

In impulsive thrusting force model it is assumed that all the required energy is supplied to the spacecraft instantaneously. The position of the satellite is kept constant at the impulse time but the velocities of the satellite are changed. After the application of impulse, the position and velocity vector of a spacecraft is changed as follows:

$$r_x(t_0^+) = r_x(t_0^-) \tag{2.40}$$

$$r_y(t_0^+) = r_y(t_0^-) \tag{2.41}$$

$$r_z(t_0^+) = r_z(t_0^-) \tag{2.42}$$

$$\mathrm{v_x}(t_0^+) = \mathrm{v_x}(t_0^-) + \Delta\mathrm{v_x} \tag{2.43}$$

$$\mathrm{v_y}(t_0^+) = \mathrm{v_y}(t_0^-) + \Delta\mathrm{v_y} \tag{2.44}$$

$$\mathrm{v_z}(t_0^+) = \mathrm{v_z}(t_0^-) + \Delta\mathrm{v_z} \tag{2.45}$$

where:

r_x : x component of position vector (in ECI frame)

r_y : y component of position vector (in ECI frame)

r_z : z component of position vector (in ECI frame)

$\mathrm{v_x}$: x component of velocity vector (in ECI frame)

$\mathrm{v_y}$: y component of velocity vector (in ECI frame)

$\mathrm{v_z}$: z component of velocity vector (in ECI frame)

Since the initial and final orbits are needed to be defined in terms of Keplerian elements. Impulsive force model requires the conversion of position and velocity vectors to Keplerian elements and vice versa.

The position and velocity vectors (ECI) can be converted to Keplerian elements as follows (Vallado & McClain, 2007):

$$\xi = \frac{\mathrm{v}^2}{2} - \frac{\mu}{r} \tag{2.46}$$

where:

v : the magnitude of velocity vector

r : the magnitude of position vector

ξ : specific orbital energy

$$\mathrm{v} = |\vec{\mathbf{v}}| = \sqrt{\mathrm{v_x}^2 + \mathrm{v_y}^2 + \mathrm{v_z}^2} \tag{2.47}$$

$$r = |\vec{\mathbf{r}}| = \sqrt{r_\mathrm{x}^2 + r_\mathrm{y}^2 + r_\mathrm{z}^2} \tag{2.48}$$

$$a = -\frac{\mu}{2\xi} \tag{2.49}$$

where:

a : semimajor axis

Specific angular momentum vector of a spacecraft is calculated as follows:

$$\vec{\mathbf{e}} = \frac{(v^2 - \frac{\mu}{r})\,\vec{\mathbf{r}} - (\vec{\mathbf{r}}.\vec{\mathbf{v}})\,\vec{\mathbf{v}}}{\mu} \tag{2.50}$$

$$e = |\vec{\mathbf{e}}| \tag{2.51}$$

where:

e: eccentricity

The specific angular momentum vector of a spacecraft can be defined as:

$$\vec{\mathbf{h}} = \vec{\mathbf{r}} \times \vec{\mathbf{v}} \tag{2.52}$$

The cosine of inclination can be calculated using z-component and total magnitude of angular momentum:

$$\cos(i) = \frac{h_z}{|\vec{\mathbf{h}}|} \tag{2.53}$$

where:

h_z = the magnitude of $\vec{\mathbf{h}}$ vector in k direction

So the inclination (i) value can be calculated as follows:

$$i = a\cos\left(\frac{|h_z|}{|\vec{\mathbf{h}}|}\right) \tag{2.54}$$

The vector pointing to the node ($\vec{\mathbf{n}}$) is calculated as:

$$\vec{\mathbf{n}} = \vec{\mathbf{k}} \times \vec{\mathbf{h}} \tag{2.55}$$

Cosine of right ascension of ascending node (Ω) is obtained as follows:

$$\cos(\Omega) = \frac{|\vec{\mathbf{n}}_x|}{|\vec{\mathbf{n}}|} \tag{2.56}$$

Using the equation (2.57), right ascension of ascending node (Ω) can be calculated as:

$$\Omega = a\cos\left(\frac{|\vec{\mathbf{n}}_y|}{|\vec{\mathbf{n}}|}\right) \tag{2.57}$$

if $(|\vec{\mathbf{n}}_y| < 0)$ then $\Omega = 360° - \Omega$

The argument of perigee (ω) can be obtained using the vector pointing to the node ($\vec{\mathbf{n}}$) and eccentricity vector ($\vec{\mathbf{e}}$).

Cosine of argument of perigee (ω) is calculated as:

$$\cos(\omega) = \frac{\vec{\mathbf{n}}.\vec{\mathbf{e}}}{|\vec{\mathbf{n}}|.|\vec{\mathbf{e}}|} \tag{2.58}$$

So the argument of perigee (ω) is obtained as:

$$\omega = a\cos\left(\frac{\vec{\mathbf{n}}.\vec{\mathbf{e}}}{|\vec{\mathbf{n}}|.|\vec{\mathbf{e}}|}\right) \tag{2.59}$$

if ($e_z < 0$) then $\omega = 360° - \omega$

Cosine of true anomaly can be calculated using eccentricity and position vectors:

$$\cos(v) = \frac{\vec{\mathbf{e}}.\vec{\mathbf{r}}}{|\vec{\mathbf{e}}|.|\vec{\mathbf{r}}|} \tag{2.60}$$

So the true anomaly is:

$$v = a\cos\left(\frac{\vec{\mathbf{e}}.\vec{\mathbf{r}}}{|\vec{\mathbf{e}}|.|\vec{\mathbf{r}}|}\right) \tag{2.61}$$

if ($\vec{\mathbf{r}}.\vec{\mathbf{v}} < 0$) then $v = 360° - v$

Similarly, position and velocity vectors (perifocal frame) can be derived from Keplerian elements. :

$$\vec{\mathbf{r}}_{PQW} = \begin{bmatrix} \dfrac{a\,(1-e^2)\cos(v)}{1+e\cos(v)} \\ \dfrac{a\,(1-e^2)\sin(v)}{1+e\cos(v)} \\ 0 \end{bmatrix} \tag{2.62}$$

where:

$\vec{\mathbf{r}}_{PQW}$: position vector on the perifocal frame

$$\vec{\mathbf{v}}_{PQW} = \begin{bmatrix} -\sqrt{\dfrac{M}{a\,(1-e^2)}}\ \sin(v) \\ \sqrt{\dfrac{M}{a\,(1-e^2)}}\ (e+\cos(v)) \\ 0 \\ \end{bmatrix} \tag{2.63}$$

$$\vec{\mathbf{r}}_{IJK} = R_3(-\Omega)\,R_1(-i)\,R_3(-\omega)\,\vec{\mathbf{r}}_{PQW} \tag{2.64}$$

$$\vec{\mathbf{v}}_{IJK} = R_3(-\Omega)\,R_1(-i)\,R_3(-\omega)\,\vec{\mathbf{v}}_{PQW} \tag{2.65}$$

where:

$$R_3(-\Omega)\,R_1(-i)\,R_3(-\omega) = \begin{bmatrix} \cos(\Omega)\cos\omega - \sin(\Omega)\sin\omega\cos i & -\cos(\Omega)\sin\omega - \sin(\Omega)\cos\omega\cos i & \sin(\Omega)\sin i \\ \sin(\Omega)\cos\omega + c\,\mathrm{os}(\Omega)\sin\omega\cos i & -\sin(\Omega)\sin\omega - \cos(\Omega)\cos\omega\cos i & -\cos(\Omega)\sin i \\ \sin(\omega)\sin i & \cos(\omega)\sin i & \cos i \end{bmatrix} \tag{2.66}$$

In impulsive orbit transfer problem, parameters that are tried to be optimized are true anomaly values (the timing of thruster firings) and impulsive thrust vector to determine the direction and magnitude of the velocity vector that should be added to the spacecraft's velocity. Note that, actually the thrust is applied to the spacecraft and the thrust force provides the impulsive velocity vector.

A two-impulse orbit transfer computation flowchart is presented in Figure 15. First orbit values are stated in terms of Keplerian elements, and then position and velocity vector at the given impulse time are found. The thrust force is applied to spacecraft which provides the impulsive velocity at the impulse time. Position and velocity vectors are updated by considering the impulse/energy value added to spacecraft at the time of impulse. These position vector and velocity vector belong to new orbit, intermediate orbit. For the second impulse, similar procedure is applied and the spacecraft reaches to the final desired orbit. The flowchart of a two impulse orbit transfer is given in Figure 15.

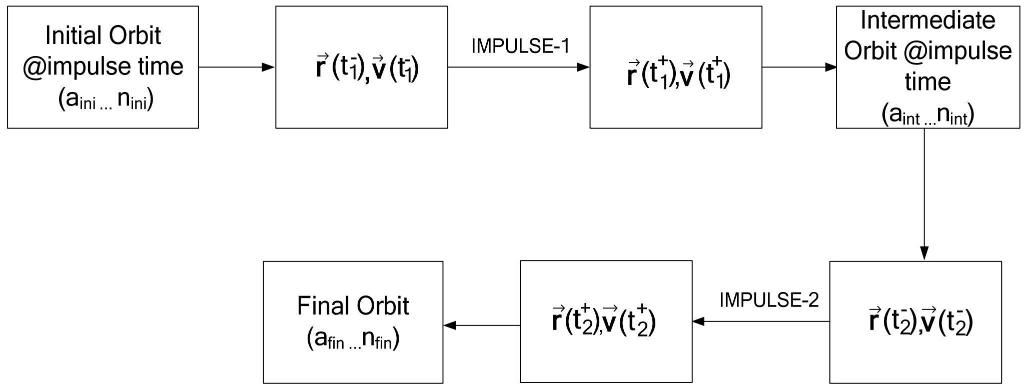

Figure 15 Two- impulse orbit transfer calculation flowchart

In this study, as orbit transfers of Earth-centered spacecrafts are examined, it is presumed that the impulsive model is valid. In the subsequent parts of the thesis, studies will be done under impulsive model assumption.

CHAPTER 3

ORBIT TRANSFER OPTIMIZATION PROBLEM

The orbit transfer optimization problem can be stated as the determination of a trajectory of a spacecraft which satisfies the initial and final conditions while minimizing some quantities (Conway, 2010). The most common objective is to minimize the required propellant or equivalently to maximize the fraction of the spacecraft that is not devoted to propellant (dry mass) to total mass.

In this chapter firstly the optimum orbit transfers are introduced. Although there is not a general optimal solution for orbit transfers, there exist optimal solutions for limited conditions. Then the optimization techniques, genetic algorithm and steepest descent methods are described briefly. These optimization methods are applied to obtain solutions for orbit transfer problems whose optimal solutions are known. Finally the performance of optimization method results is compared with optimal orbit transfer results and applicability of the optimization methods to orbit transfer problem are discussed.

3.1 OPTIMAL ORBIT TRANSFERS

In the literature different orbit transfer methods are suggested for different cases; however, most of these orbit transfer methods are not optimal. Among these orbit transfers, Hohmann and only inclination change orbit transfers suggest optimal results at specific conditions. It should be noted that there are some other optimal orbit transfers in the literature, in this section only two optimal orbit transfers are used. In this section Hohmann and only inclination change orbit transfers are described briefly which are used during the validation of optimization methods that are applied to orbit transfer problem.

3.1.1 Hohmann Orbit Transfer (Coplanar Orbit Transfer)

Hohmann orbit transfer is the most energy efficient two-impulse orbit transfer for transferring between two coplanar circular orbits sharing a common focus whose radius ratios ($r_{final}/r_{initial}$) is less than 11.94. The Hohmann transfer orbit is an elliptical orbit tangent to both circles at its apse line, as illustrated in Figure 16. The periapse and apoapse of the transfer ellipse are the radii of the inner and outer circles, respectively. Starting from A on the inner circle, a velocity increment $\Delta\vec{\mathbf{v}}_1$ in the direction of flight is required to transfer the spacecraft onto the higher-energy elliptical trajectory. After coasting from A to B, another forward velocity increment $\Delta\vec{\mathbf{v}}_2$ at the apogee of the transfer orbit places the vehicle onto the still higher-energy, outer circular orbit (Curtis, 2005).

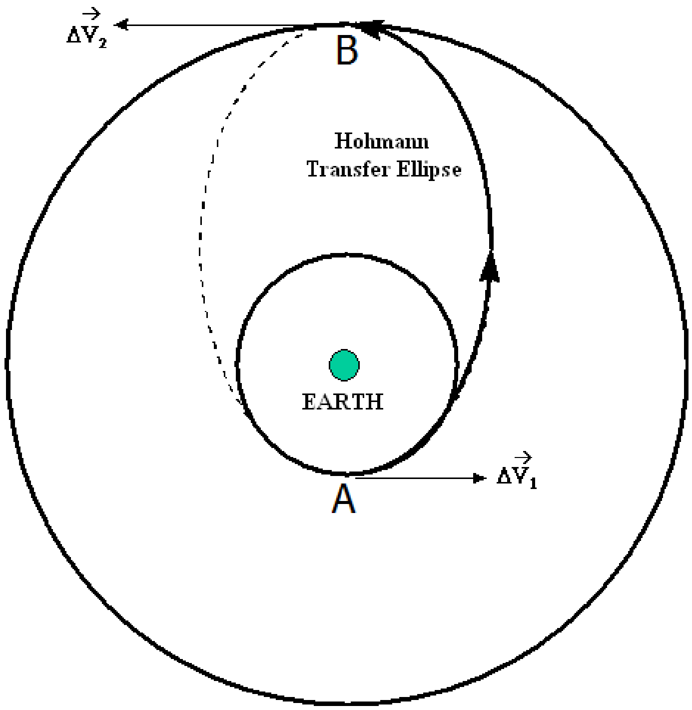

Figure 16 Hohmann Orbit transfer (Hale, 1994)

Hohmann orbit transfer impulsive velocity requirement can be obtained by using the following equations (Chobotov, 2002):

Firstly the radial ($\mathrm{v_r}$) and tangential velocities ($\mathrm{v_\theta}$) are defined:

$$\mathrm{v_r} = \sqrt{\frac{\mu}{a\,(1-e^2)}}\;\; e\sin\nu \tag{3.1}$$

$$\mathrm{v_\theta} = \sqrt{\frac{\mu}{a\,(1-e^2)}}\;\; (1+e\cos\nu) \tag{3.2}$$

Initial radial ($\mathrm{v_{ir}}$) and tangential ($\mathrm{v_{i\theta}}$) velocities are:

$$\mathrm{v}_{\mathrm{ir}} = 0 \tag{3.3}$$

$$\mathrm{v}_{\mathrm{i}\theta} = \sqrt{\frac{\mu}{a\,(1-e^2)}}\ (1+e) = \sqrt{\frac{\mu}{r_1}} \tag{3.4}$$

Note that radial distances are:

$$r_1 = r_i \tag{3.5}$$

$$r_2 = r_f \tag{3.6}$$

Using the vis-viva equation (orbital energy conservation equation) the impulsive velocity vector magnitude is obtained. The term, "vis-viva" is originated from the Latin, vis is force or power and viva is living. In other words "vis-viva" means living force. Vis-viva equation is the ability of a body to do work on its environment. It is used as to refer to the principle of energy conservation. This equation enables to calculate the orbital velocity of the spacecraft at a point on an elliptical orbit.

$$\xi = \frac{\mathrm{v}^2}{2} - \frac{\mu}{r} \tag{3.7}$$

The intermediate orbit velocity vector magnitude at the perigee point (ν_1) can be obtained as follows:

$$\mathrm{v}_1^2 = \mu\left[\frac{2}{r_1} - \frac{2}{r_1 + r_2}\right] \tag{3.8}$$

$$\mathrm{v}_1^2 = \frac{\mu}{r_1}\left[2 - \frac{2}{1 + (r_2/r_1)}\right] \tag{3.9}$$

$$\mathrm{v}_1^2 = \mathrm{v}_i^2\left[\frac{2 + 2\,(r_2/r_1) - 2}{1 + (r_2/r_1)}\right] \tag{3.10}$$

The ratio of intermediate orbit velocity vector magnitude at the perigee (ν_1) to the initial orbit velocity vector magnitude (v_i) is:

$$\frac{v_1}{v_i} = \left[\frac{2\ (r_2 / r_1)}{1 + (r_2 / r_1)} \right] \tag{3.11}$$

The required impulsive velocity vector magnitude at the first impulse (Δv_1) is:

$$\frac{\Delta v_1}{v_i} = \frac{v_1 \text{-} v_i}{v_i} = \sqrt{\frac{2\ (r_2 / r_1)}{1 + (r_2 / r_1)}} - 1 \tag{3.12}$$

$$\Delta v_1 = v_i \left(\sqrt{\frac{2\ (r_2 / r_1)}{1 + (r_2 / r_1)}} - 1 \right) \tag{3.13}$$

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{2\ (r_2 / r_1)}{1 + (r_2 / r_1)}} - 1 \right) \tag{3.14}$$

Again from the vis-viva equation, the intermediate orbit velocity vector magnitude at the apogee (v_2) is obtained as follows:

$$v_2^{\ 2} = \mu \left[\frac{2}{r_2} - \frac{2}{r_2 \left[1 + (1/(r_2 / r_1)) \right]} \right] \tag{3.15}$$

$$v_2^{\ 2} = \frac{\mu}{r_2} \left[2 - \frac{2\ (r_2 / r_1)}{1 + (r_2 / r_1)} \right] \tag{3.16}$$

$$v_f^{\ 2} = \frac{\mu}{r_2} \tag{3.17}$$

where:

v_f: final orbit velocity magnitude

The ratio of intermediate orbit velocity vector magnitude at the perigee point (v_1) to the initial orbit velocity vector magnitude (v_i) is:

$$v_f^{\ 2} = v_i^{\ 2} \frac{r_1}{r_2} \tag{3.18}$$

$$v_2{}^2 = v_f{}^2 \left[\frac{2 + 2\,(r_2/r_1) - 2(r_2/r_1)}{1 + (r_2/r_1)} \right] \qquad (3.19)$$

$$v_2{}^2 = \frac{v_i{}^2}{r_2/r_1} \left[\frac{2}{1 + (r_2/r_1)} \right] \qquad (3.20)$$

$$\frac{v_2}{v_i} = \sqrt{\frac{2}{(r_2/r_1)\,[1 + (r_2/r_1)]}} \qquad (3.21)$$

$$\frac{\Delta v_2}{v_i} = \frac{v_f - v_2}{v_i} = \sqrt{\frac{1}{r_2/r_1}} - \sqrt{\frac{2}{(r_2/r_1)\,[1 + (r_2/r_1)]}} \qquad (3.22)$$

The required impulsive velocity vector magnitude at the second impulse (Δv_2) is:

$$\Delta v_2 = v_i \left(\sqrt{\frac{1}{r_2/r_1}} - \sqrt{\frac{2}{(r_2/r_1)\,[1 + (r_2/r_1)]}} \right) \qquad (3.23)$$

$$\Delta v_2 = \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{1}{r_2/r_1}} - \sqrt{\frac{2}{(r_2/r_1)\,[1 + (r_2/r_1)]}} \right) \qquad (3.24)$$

The impulse moments (true anomalies) are:

$$v_1 = \text{free} \qquad (3.25)$$

$$v_2 = 180° \qquad (3.26)$$

The total required velocity vector magnitude is:

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 \qquad (3.27)$$

$$\Delta v_{total} = \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{1}{r_2/r_1}} - \sqrt{\frac{2}{(r_2/r_1)\,[1 + (r_2/r_1)]}} + \sqrt{\frac{2\,(r_2/r_1)}{1 + (r_2/r_1)}} - 1 \right) \qquad (3.28)$$

3.1.2 Only Inclination Change (OIC) Orbit Transfer

A spacecraft's semimajor axis (a), eccentricity (e), argument of perigee (ω), and true anomaly (v) can be changed by applying coplanar orbit transfers. On the other hand thrust normal to the orbit plane is applied when the orbit inclination (i), argument of perigee (ω) and the right ascension of ascending node (Ω) are desired to be changed. Plane change orbit transfers require more energy than planar orbit transfers since changing the orbit requires the change in the direction of the velocity vector, in other words, the direction of angular momentum. The change of velocity and specific angular momentum vectors during a noncoplanar orbit transfer are shown in Figure 17.

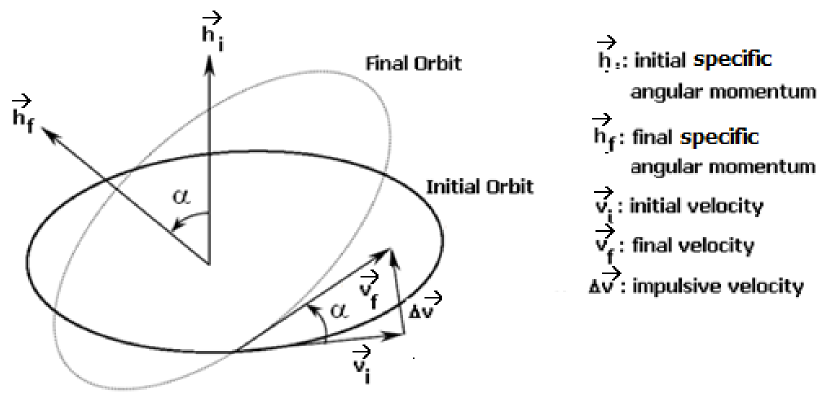

Figure 17 Velocity and specific angular momentum vector change during noncoplanar orbit transfer (Tewari, 2007)

Noncoplanar thrust changes inclination and right ascension of ascending node values as follows:

$$i = \frac{r\cos(\omega + v)}{h} a_n \tag{3.29}$$

$$\Delta i = \frac{r\cos(\omega + v)}{h} \Delta \mathrm{v}_\mathrm{n} \tag{3.30}$$

$$\Omega = \frac{r\sin(\omega + v)}{h\sin i} a_n \tag{3.31}$$

$$\Delta\Omega = \frac{r\sin(\omega + v)}{h\sin i} \Delta \mathrm{v}_\mathrm{n} \tag{3.32}$$

where:

$a_n = F_{normal} / m_{spacecraft}$ (specific normal force)

F_{normal} : normal force

$m_{spacecraft}$: spacecraft mass

Δv_n : impulsive velocity vector magnitude normal component

ω : argument of perigee

Sometimes it is required to change the plane of the orbit without changing its shape. Such an orbit transfer is called "only inclination change orbit transfer". At this orbit transfer, the flight path angle of the spacecraft at the impulse moment is zero (0). Neither the speed of the spacecraft nor the flight-path angle is modified during this orbit transfer. All orbital elements excluding inclination and true anomaly is kept constant. The thrust is supplied at a specific point on the orbit which enables only the change of inclination. The other Keplerian elements are not changed when the out-of-plane direction thrust is applied at this point. This specific point is the equatorial crossing (node) which corresponds to:

$$\omega + v = (2n)\,180° \quad n = 0,1,2,... \tag{3.33}$$

then using the continuous force model (eqn. 2.39), inclination and right ascension of ascending node changes are obtained as follows:

$$\Delta i = \frac{r}{h} \Delta v_n \tag{3.34}$$

$$\Delta \Omega = 0 \tag{3.35}$$

Analytical solution for this orbit transfer can be obtained using triangle vector geometry (Figure 18).

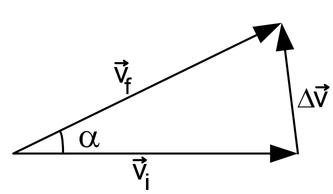

Figure 18 Velocity vector during only inclination change orbit transfer

From the triangle vector geometry:

$$\Delta v=\sqrt{v_i^2+v_f^2-2\ v_i\ v_f\ \cos\alpha} \tag{3.36}$$

since

$$v_i=v_f \tag{3.37}$$

no change in the magnitude of the velocities. Therefore the required impulsive velocity vector magnitude (Δv) is:

$$\Delta v=\sqrt{\ 2v_i^2\text{-}2v_i^2\ \cos\alpha} \tag{3.38}$$

using trigonometric half angle identity:

$$\Delta v=2\ v_i\ \frac{\sin\alpha}{2} \tag{3.39}$$

3.2 ORBIT TRANSFER OPTIMIZATION PROBLEM

The orbit transfer problem can be stated as the determination of a trajectory of a spacecraft which satisfies the initial and final conditions. During orbit transfer, the propellant consumption is minimized (Conway, 2010).

Optimization variables, sometimes called design variables, are the parameters that are desired to be optimized. At the orbit transfer problem the optimization variables are the impulsive velocity vector components and the timing (i.e. true anomaly) of each impulse. The number of optimization variables determines the dimension of the problem. For example for a two impulse orbit transfer problem the dimension of the problem is eight (8). The optimization parameters at an impulse are:

$$\begin{bmatrix} \Delta v_{x,i} \\ \Delta v_{y,i} \\ \Delta v_{z,i} \\ \nu_i \end{bmatrix}$$

where:

$\Delta v_{x,i}$: impulsive velocity vector x component magnitude

$\Delta v_{y,i}$: impulsive velocity vector y component magnitude

$\Delta v_{z,i}$: impulsive velocity vector z component magnitude

ν_i : impulse timing (true anomaly at impulse moment)

Orbit transfer is performed using thrust forces generated by the propulsion system of a spacecraft. For a spacecraft whose propulsion system only provides constant force, the orbit transfer optimization is the minimization of thrust application duration. To achieve this aim, the cost function should be constructed based on required impulsive velocity vector components and the difference between desired and calculated final orbits suggested by algorithm. In reality, the orbit of a spacecraft is not defined using constant orbital element values; instead a range is defined for all orbital elements. When spacecraft is at a position out of its defined tolerance limits, orbit maintenance maneuver is realized. The typical orbit maintenance values are used as a baseline to generate tolerance values for each Keplerian element. For instance, the maximum semimajor axis deviation of a spacecraft's orbit is defined as ±5 km during the design phase considering its mission. When spacecraft's position exceeds to out of this value, an orbit maintenance maneuver is applied. The orbit maintenance values/ranges may be defined as tolerances and they are strict which does not cause any problem for the mission of a spacecraft. In this study, the tolerances are defined for destination orbits using orbit maintenance values. The tolerance values are specified depending on final orbit and the mission of the spacecraft, in other words problem specific. If the final orbit values are in the tolerance limits of destination orbit, then, it can be stated that this orbit transfer is realized.

Orbit transfer problem may also include different physical restrictions. These physical restrictions should be defined as constraints and considered in the problem. In the orbit transfer optimization the most critical constraints are: transfer duration, impulse level per one operation of propulsion system, and total impulse capacity. The constraints specified here are only main constraints that are applied at the orbit transfer problem. In some cases there may be additional constraints such as the orbit of other spacecrafts, rendezvous, other celestial bodies, space junks, etc. Three main constraints are described briefly. In this study these constraints are not included in the problem.

Transfer duration: In some circumstances it is desired to realize an orbit transfer in a limited duration. The transfer duration is defined as a constraint for long duration orbit transfers. This constraint especially makes sense where the number of impulses is high and/or there is a spacecraft rendezvous problem. Transfer duration constrain can be described as follows:

$$\Delta t_{transfer} \leq \max(\Delta t_{transfer}) \tag{3.40}$$

where:

$\Delta t_{transfer}$: transfer duration

$\max(\Delta t_{transfer})$: maximum allowable transfer duration

Impulse level / operation of propulsion system : Thrusters of spacecraft propulsion system cannot generate an impulse less than a lower limit and more than an upper limit because of specific impulse (I_{sp}), propellant properties, etc. This property of propulsion subsystem should also be considered as a constraint. In orbit transfer problem terminology instead of impulse level, impulsive velocity vector magnitude is used which is:

$$I_i = F_i \, \Delta t_i = m \, \Delta \mathrm{v}_\mathrm{i} \tag{3.41}$$

$$\Delta \mathrm{v}_\mathrm{i} = \frac{I_i}{m} \tag{3.42}$$

where:

I_i : magnitude of ith impulse

$\Delta \mathrm{v}_\mathrm{i}$: impulsive velocity vector magnitude of ith impulse

m : spacecraft mass

The expected impulsive velocity vectors computed by algorithm are needed to be checked whether they are in the acceptable region or not. Acceptable impulsive velocity vector level constraint at one burn can be constructed mathematically as follows:

$$\min(\Delta \mathrm{v}_\mathrm{i}) \leq \Delta \mathrm{v}_\mathrm{i} \leq \max(\Delta \mathrm{v}_\mathrm{i}) \tag{3.43}$$

where:

$\min(\Delta \mathrm{v}_\mathrm{i})$: minimum impulsive velocity vector magnitude that can be supplied at the ith impulse

$\max(\Delta \mathrm{v}_\mathrm{i})$: maximum impulsive velocity vector magnitude that can be supplied at the ith impulse

Total impulse capacity: The amount of propellant in a spacecraft can also be limited. Since the propellant amount at a spacecraft is constant, the maximum level of impulse supplied by spacecraft propulsion system is limited. During orbit transfer analysis of a spacecraft, this constraint should be considered. This constraint may also be defined as total propellant capacity. Total impulse capacity constraint can be defined as:

$$I < I_{\max} \tag{3.44}$$

where:

I : Impulse requirement of an orbit transfer

$I_{\max}$: Total impulse capacity of a spacecraft

Adding constraints to the problem enables to obtain realistic results. If it is not mandatory, constraints should not be included to the problem since this causes elimination of some good results. In this study only the critical constraints are considered. These are the difference between the calculated and desired final orbital element values, the impulse level at one burn and the total impulse capacity.

It should be noted that true anomaly value can vary between 0^{o} and 360^{o}. Since it is not easy to generate a reasonable prediction for the true anomaly of an impulse, 0º for minimum and 360º for maximum are assumed:

$$0º < v < 360º \tag{3.45}$$

where:

v : true anomaly

The orbit transfer optimization problem may be stated mathematically as follows:

minimize:

$$f(x) = \sum_{i=1}^{n} \sqrt{(\Delta \mathrm{v_{ix}}^2 + \Delta \mathrm{v_{iy}}^2 + \Delta \mathrm{v_{iz}}^2)} \tag{3.46}$$

subject to:

$$a_{obtained} - \delta a \leq a_{final} \leq a_{obtained} + \delta a \tag{3.47}$$

$$e_{obtained} - \delta e \leq e_{final} \leq e_{obtained} + \delta e \tag{3.48}$$

$$i_{obtained} - \delta i \leq i_{final} \leq i_{obtained} + \delta i \tag{3.49}$$

$$\Omega_{obtained} - \delta \Omega \leq \Omega_{final} \leq \Omega_{obtained} + \delta \Omega \tag{3.50}$$

$$\omega_{obtained} - \delta \omega \leq \omega_{final} \leq \omega_{obtained} + \delta \omega \tag{3.51}$$

$$\min(\Delta \mathrm{v}_i) \leq \Delta \mathrm{v}_i \leq \max(\Delta \mathrm{v}_i) \tag{3.52}$$

$$I < I_{\max} \tag{3.53}$$

$$0^\circ < \nu < 360^\circ \tag{3.54}$$

$$\Delta t_{transfer} \leq \max(\Delta t_{transfer}) \tag{3.55}$$

where:

δa : Semimajor axis tolerance

δe : Eccentricity tolerance

δi : Inclination tolerance

$\delta \Omega$: Right ascension of ascending node tolerance

$\delta \omega$: Argument of perigee tolerance

n: number of impulse

$\Delta \mathrm{v}_{ix}$: x component impulsive velocity of the i^{th} impulse

$\Delta \mathrm{v}_{iy}$: y component impulsive velocity of the i^{th} impulse

$\Delta \mathrm{v}_{iz}$: z component impulsive velocity of the i^{th} impulse

$\min(\Delta \mathrm{v}_i)$: minimum impulsive velocity vector magnitude that can be supplied at the i^{th} impulse

$\max(\Delta \mathrm{v}_i)$: maximum impulsive velocity vector magnitude that can be supplied at the i^{th} impulse

I : Impulse requirement of an orbit transfer

$I_{\max}$: Total impulse capacity of a spacecraft

Orbit transfer problem is a constrained problem; it needs to be converted into an unconstrained one. The constrained optimization problem can be transformed into an unconstrained one by penalty functions method. The penalty function which is added to the objective function penalized the cost function when the constraints are violated. By this means, the solution that achieves a minimum value for the cost function is driven away from violating the constraints. The new objective function can now be written as follows:

$$J = c_v \sum_{i=1}^{n} \left|\Delta\vec{\mathbf{v}}_i\right| + c_a\, a_{error} + c_e\, e_{error} + c_i\, i_{error} + c_\Omega\, \Omega_{error} + c_\omega\, \omega_{error} \qquad (3.56)$$

where:

$a_{error} = a_{final} - a_{obtained}$

$e_{error} = e_{final} - e_{obtained}$

$i_{error} = i_{final} - i_{obtained}$

$\Omega_{error} = \Omega_{final} - \Omega_{obtained}$

$\omega_{error} = \omega_{final} - \omega_{obtained}$

c_v, c_a, c_e, c_i, c_Ω, c_ω: weighting factors chosen based on importance and magnitude of variable

The orbital element tolerances and cost function coefficients are tabulated in Table 3.

Table 3 Tolerances and cost function coefficients of orbital elements

Orbital Element	**Tolerance**	**Cost Function Coefficients**
a (km)	δa	c_a
e	δe	c_e
i (°)	δi	c_i
Ω (°)	$\delta \Omega$	c_Ω
ω (°)	$\delta \omega$	c_ω
ν (°)	N/A	N/A

The elements of cost function are needed to be multiplied by weighting factors since the importance; magnitude (order) and unit of each element are different. The semimajor axis cost function coefficients are assumed as unity (1) $c_a = 1$. Multiplication of semimajor axis tolerance value and corresponding cost function coefficient is obtained. This value is used as a reference value. To put differently, multiplication of each orbital element's cost function coefficient and tolerance are equalized to this value (C). Note that in the cost function the coefficients are needed to be scaled depending on the importance and value of orbital elements and impulsive velocity vector magnitude. In the literature there exist no direct cost function coefficient values. Some coefficients exist for specific cases. In this study it is assumed that the importance of orbital element is directly related to its tolerance. The cost function coefficients are also checked during different orbital elements and it is seen that this approach is applicable. With this approach depending on mission cost function coefficients can be obtained.

$$C = c_a\,\delta a = c_e\,\delta e = c_i\,\delta i = c_\Omega \delta\Omega = c_\omega\,\delta\omega \tag{3.57}$$

where:

C = constant

So the cost function coefficients of orbital elements are obtained as follows:

$$c_e = \frac{c_a\,\delta a}{\delta e} \tag{3.58}$$

$$c_i = \frac{c_a\,\delta a}{\delta i} \tag{3.59}$$

$$c_\Omega = \frac{c_a\,\delta a}{\delta\Omega} \tag{3.60}$$

$$c_\omega = \frac{c_a\,\delta a}{\delta\omega} \tag{3.61}$$

The cost function coefficient of impulsive velocity magnitude is needed to be at least 10 times of semimajor axis cost function coefficient (Kim & Spencer, 2002). Depending on the desired accuracy and available propellant, this value is defined. In this study this value is assumed to be 25. Note that although impulse level one burn and total impulse level are not included in the cost function, the applicability of solution is checked considering these constraints.

It should be noted that the cost function coefficients explained in this section are only typical values. The cost function coefficients can also be obtained by using different ways. In this study, the desired accuracy of each orbital element is used to define the cost function coefficients. The orbital elements which requires more accuracy is defined as more critical and its cost function coefficient is enlarged/highlighted. In the literature the cost function coefficients are specified depending on importance of orbital elements. The importance of orbital elements may change depending on the mission and the desired accuracy of spacecraft. Considering the accuracy values and the magnitudes of orbital elements, cost function coefficients are calculated.

In this study a generic cost function is used. Depending on mission, desired accuracy and the amount of propellant different cost function coefficients can be selected. In this study only typical values are selected. It is aimed to show the working principle of the orbit transfer optimization. At different situations different cost functions may be used.

3.3 OPTIMIZATION METHODS

The success of an optimization method greatly depends on choosing search directions/initial conditions. Optimization algorithms depending on search direction may be analyzed in two groups: those not requiring any information on the objective/cost function, and those based on the information provided by the gradient of the objective/cost function. The first group is non-gradient based optimization and the second group is gradient based optimization methods.

The convergence of gradient based optimization algorithms are mostly based on starting point (initial prediction/condition). Gradient-based algorithms may converge to a local optimum if a poor/bad starting point is given.

Non-gradient optimization algorithms do not use gradient information during the optimization process. Among non-gradient optimization methods, the evolutionary algorithms are one of the best optimization methods since they are global optimization methods and they do not require any initial prediction. With the evolution of computers, evolutionary algorithms become more popular. Genetic algorithm is one of the most advanced evolutionary methods. Several studies have been performed using genetic algorithm including orbit transfer optimization.

As can be seen in mathematical modeling section of the study, the problem includes discrete points (perigee & apogee) and nonlinearities (trigonometric relations). This means that linear optimization methods may not suggest good results. In orbit transfer problem it is hard to make reasonable prediction for an orbit transfer. Because of all these reasons the optimization method should pass local minimums and handle discrete points and nonlinearities. In this thesis, both gradient and non-gradient optimization methods are applied to orbit transfer problem. As a gradient method, steepest descent algorithm is used and as a non-gradient method genetic algorithm is selected and applied. The results of each optimization are compared with the optimality known orbit transfers.

3.3.1 Genetic Algorithm

The genetic algorithm (GA) is a stochastic global search method that mimics some aspects of natural biological evolution. The GA operates on a population of potential solutions by applying the principle of survival of the fittest to converge to an optimal solution (Kim & Spencer, 2002). The GA starts, like any other optimization algorithm, by defining the optimization variables and the cost function. It also ends like other optimization algorithms, by testing for convergence (Haupt & Haupt, 2004)

In genetic algorithm optimization globally optimal solution(s) are searched in a predefined search space. If the search space is wide, it is time consuming to reach a solution; on the other hand, if the search space is selected to be narrow, the optimal solutions may not be obtained. For this reason the search space should be large enough to be able to obtain optimal solutions and should be narrow enough to consume less time. The search space is defined by limiting the search interval for each optimization parameters.

The gene is the basic building block of genetic algorithm. Each optimization parameter is defined as a gene. There are usually two classes of genes: real, where a gene is a real number; and alphabetic, where a gene takes a value from an alphabet set. Common alphabet sets are the binary, octal, decimal, and hexadecimal sets (Qing, 2009). In this study the binary set is used to define a gene. The number of bits (Nb) defines the accuracy of a gene.

If the problem includes more than one optimization parameter, a multi-variable coding is constructed using as many single variables coding as the number of variables in the problem. A chromosome is the collection of genes which correspond to optimization parameters. A chromosome is the binary string collection that is suggested as a solution of an optimization problem.

Chromosome=[gene_1,gene_2,…., gene_{Nvar}]

where:

gene: an optimization parameter

Nvar: Number of optimization parameter (dimension of the problem)

An individual p is an aggregate of a chromosome and objective function (including constraint) values. The union of individuals is called population. The number of individuals (Ni) in a population is an important parameter of genetic algorithm.

Fitness is the measure of goodness of a chromosome. It is directly related to cost function values with a scaling operation (Qing, 2009). The individuals whose corresponding fitness values are more than others are closer to the optimal solution(s). Cost function is constructed as obtaining fittest individuals while using minimum effort. Main aim is to minimize the control effort while reaching the desired final boundary conditions. As an example, for a minimum path problem of a point mass (transfer of a point mass from an initial point to final point) the required force/energy ratio is minimized.

After the creation of initial population randomly, the corresponding cost function is computed for each individual. Later, in order to create a new generation selection, crossover and mutation operations are applied. These operations which are applied to model the evolutionary/biological process of the population are described below:

1. ***Selection:*** Selection (or reproduction) is an operator that makes more copies of better strings. Reproduction selects good strings. This is one of the reasons for the reproduction operation to be also known as the selection operator. Thus, in reproduction operation the process of natural selection causes those individuals that encode successful structures to produce copies more frequently. In reproduction, good strings in a population are probabilistically produced a larger number of copies and a mating pool is formed. It is important to note that no new strings are formed in the reproduction phase. Most commonly used selection methods are as follows:

   a. Ranking: A numerical rank based on fitness is assigned to each individual in the population. The disadvantage of this method is that it can prevent very fit individuals from gaining dominance early at the expense of less fit ones, which would reduce the population's genetic diversity and might hinder attempts to find an acceptable solution (Obitko, 1998).

   b. Roulette Wheel: The probabilities assigned to the chromosomes in the mating pool are inversely proportional to their cost. A chromosome with the lowest cost has the greatest probability of mating, while the chromosome with the highest cost has the lowest probability of mating. A random number determines which chromosome is selected. This type of weighting is often referred to as roulette wheel weighting (Haupt & Haupt, 2004).

c. Tournament: Another approach that closely mimics mating competition in nature is to randomly pick a small subset of chromosomes (two or three) from the mating pool, and the chromosome with the lowest cost in this subset becomes a parent. The tournament repeats for every parent needed. Roulette wheel and tournament selection make a nice pair, because the population never needs to be sorted. Tournament selection works best for larger population sizes because sorting becomes time-consuming for large populations (Haupt & Haupt, 2004).

2. ***Crossover* (Mating):** Mating is the creation of one or more offspring from the parents selected in the pairing process. A crossover operator is used to recombine two strings to get a better string. In crossover operation, recombination process creates different individuals in the successive generations by combining material from two individuals of the previous generation (Mathew).

   The crossover operation does not always occur. Sometimes, based on a set probability, no crossover occurs and the parents are copied directly to the new population (Skinner, 2001). (Oliver, Smith, & Holland, 1987) suggested that the probability should be between 0.60-0.90.

   The most common form of mating involves two parents that produce two offspring. The most common and most successful crossover methods are single point and two points crossover. Crossover operation is done at string level by randomly selecting two strings for crossover operations.

   a. One point crossover: A one site crossover operator is performed by randomly choosing a crossing site along the chromosome (string) and by exchanging all bits on the right side of the crossing site as shown in Table 4 (Mathew).

Table 4 One point crossover

	Before Crossover		After Crossover	
String-1	11	**1100**	11	**11001**
String-2	110	**11001**	11	**1100**

b. Two point crossover: Two point crossover is a variation of the one site crossover, except that two crossover sites are chosen and the bits between the sites are exchanged as shown in Table 5 (Mathew).

Table 5 Two point crossover

	Before Crossover			After Crossover		
String-1	011	**011**	00	011	**110**	00
String-2	110	**110**	01	011	**011**	01

3. ***Mutation:*** Mutation is the process of randomly disturbing genetic information. They operate at the bit level; when the bits are being copied from the current string to the new string, there is probability that each bit may become mutated. This probability is usually a quite small value, called as mutation probability. It is generally selected between 0.002 and 0.100 (Greenwell, Angus, & Finck, 1995).

The need for mutation is to create a point in the neighborhood of the current point, thereby achieving a local search around the current solution. The mutation is also used to maintain diversity in the population (Mathew).

After the application of selection, crossover, and mutation operators, a new generation is constructed. Note that for some genetic algorithms a process named as replacement is also defined and used. In this study this operator is not used. The corresponding cost function of new generation is calculated. This process is continued until the convergence criteria are met. The convergence criteria can be a value of a cost function, number of generations, etc. The number of generations (Ngen) is generally selected as a convergence criterion since the other criteria can be satisfied easier than the number of generation criteria.

The genetic algorithm working principle can be summarized as follows: the optimization parameters are decided and corresponding search space is defined. An initial population is constructed and the fitness of the individuals is evaluated. The optimization criterion is controlled, then biological processes (selection, crossover, mutation) are applied and a new iteration is started. The genetic algorithm flowchart is presented in Figure 19.

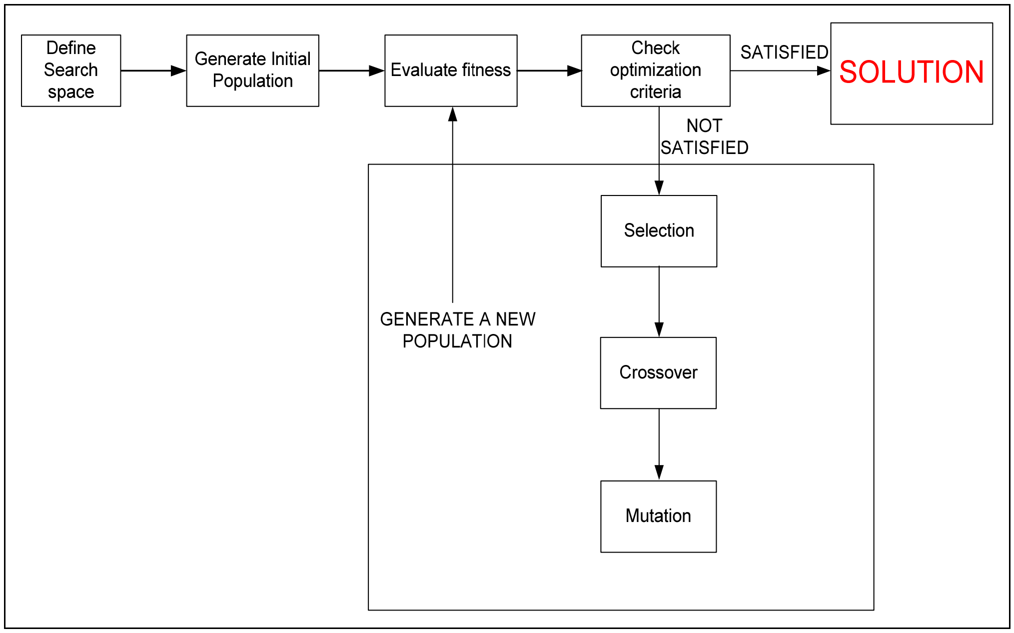

Figure 19 Genetic algorithms flowchart (Cao & H., 1999)

The following genetic algorithm optimization parameters are used during the orbit transfer problem. These parameters are selected based on the suggested ranges for each parameter in the literature and by trial & error process.

Ni (number of individuals): 400

Nb (number of bits): 7

Ngen (generation number): 600

Cp (crossover probability): 0.80

Mp (mutation probability): 0.08

The run time of a two impulse genetic algorithm based orbit transfer method at a computer with the following specifications: Intel® Core™2 Duo CPU E6550@2.33GHz, 2.00GB of RAM is approximately 620 seconds. The run time is directly related to number of impulses, number of individuals, and generation number.

3.3.2 Steepest Descent Optimization Method

Gradient based optimization methods use gradient of a function during optimization. These methods suggest fast and accurate results. However, gradient based optimization methods may not be good preference for functions that include discontinuities and/or local minimums/maximums.

In this section, a steepest descent optimization method is used.

3.4 COMPARISON OF OPTIMIZATION METHODS FOR ORBIT TRANSFER OPTIMIZATION

Orbit transfer optimization can be realized by using different type of optimization methods. In this study among these optimization techniques, genetic algorithm and steepest descent methods are selected and applied to orbit transfer problem. The method based on genetic algorithm is called as "genetic algorithm based orbit transfer method".

The performances of these methods are evaluated using the orbit transfer cases with known optimal results. These are coplanar circular to circular orbit transfers and only inclination change transfers. The optimal result of coplanar circular to circular orbit transfer can be obtained from Hohmann orbit transfer method. The result of only inclination change orbit transfer solution can also be obtained analytically. Coplanar circular to circular orbit transfer is defined as problem-1, and only inclination change orbit transfer is defined as problem-2.

3.4.1 Problem-1 Coplanar Circular-Circular Orbit Transfer

Coplanar circular to circular orbit transfer is a kind of orbit transfer that semimajor axis is changed while keeping other orbital elements constant. The optimal solution of coplanar circular-circular orbit transfer can be obtained using Hohmann orbit transfer. In this section the results of this orbit transfer is used to verify the applicability of optimization methods at the orbit transfer optimization problem. A test case is constructed to measure the performance of the optimization methods for orbit transfer problem (Table 6). This test case starts from low earth orbit altitudes and ends at geosynchronous earth orbit altitudes. This orbit transfer requires a large change in the semimajor axis value, in other words, the spacecraft performs a large change in orbit size. Therefore it can be stated that this test case requires larger change than real case orbit transfers of many spacecrafts.

Table 6 Problem-1 Initial and Final Orbit values

Orbital Element	**Initial Orbit**	**Final Orbit**
a (km)	7000.00	42164.00
e	0.00000	0.00000
i (°)	90.000	90.000
Ω (°)	0.00	0.00
ω (°)	N/A	N/A
ν (°)	free	free

Hohmann orbit transfer method is applied to obtain optimal solution to the problem given in Table 6. The required impulsive velocity vector magnitudes and corresponding true anomalies obtained by using Hohmann transfer are presented in Table 7. The total required impulsive velocity vector magnitude is 3770.70 m/s and the second impulse true anomaly is 180°. Note that since the initial orbit is circular, the first impulse true anomaly value is not important. The first impulse can be applied at any point of the initial orbit.

Table 7 Hohmann Orbit Transfer solution for Problem-1

HOHMANN ORBIT TRANSFER	**Impulsive Velocity Vector Magnitude (m/s)**	**Impulse True Anomaly (°)**
First Impulse	2336.80	Free
Second Impulse	1433.90	180
Total Impulse	3770.70	N/A

Cost Function for Problem-1

The thrust applied on the orbit plane of a spacecraft can only change semimajor axis (a), eccentricity (e), argument of perigee (ω) and true anomaly (ν). Right ascension of ascending node (Ω) and inclination (i) cannot be modified by the thrust applied on the orbit plane. True anomaly variation is generally not critical since it defines only the position of a spacecraft in an orbit. Argument of perigee is also not critical/defined at the circular orbits. Therefore, the cost function coefficients of true anomaly and argument of perigee is taken as zero.

Cost function coefficients of orbital elements for Problem-1 are as follows:

semimajor axis tolerance is (Satellite Programmes Overview:Satellite Orbits, 2012):

$$\delta_a = \pm 10.00 \text{ km} \tag{3.61}$$

eccentricity tolerance is (Satellite Programmes Overview:Satellite Orbits, 2012):

$$\delta_e = \pm 0.00024 \tag{3.62}$$

It is assumed that acceptable error for inclination is (Chao, 2005) :

$$\delta_i = \pm 0.100° \tag{3.63}$$

It is assumed that acceptable error for right ascension of ascending node is (Chao, 2005):

$$\delta_{\Omega} = \pm 0.20° \quad (3.64)$$

The tolerance values for each orbital element and the corresponding cost function coefficients for problem-1 are tabulated in Table 8.

Table 8 Tolerance values and cost function coefficients for problem-1

Orbital Element	**Tolerance Values**	**Cost Function Coefficient**
a (km)	±10.00	1.00
e	±0.00024	41666.67
i (°)	±0.100	100.00
Ω (°)	±0.20	50.00
ω (°)	N/A	N/A
ν (°)	N/A	N/A

so the cost function is constructed as:

$$\mathrm{J} = 25.00\sum_{i=1}^{n}\left|\Delta\vec{\mathbf{v}}_i\right| + a_{error} + 41666.67\, e_{error} + 100.00 i_{error} + 50.00\Omega_{error} \quad (3.65)$$

3.4.1.1 Application Of Genetic Algorithm Method To Problem-1

Genetic algorithm based orbit transfer optimization is applied to obtain solutions for Problem-1 (given in Table 6). In Table 9 the genetic algorithm based orbit transfer results are tabulated and in Table 10 the impulsive velocity vector magnitude and impulse timings (true anomaly) of the corresponding transfer is given.

Table 9 Genetic Algorithm Based Orbit Transfer Results for Problem-1

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	42164.00	42163.68	0.32	0.00
e	0.00000	0.00007	0.00007	*
i (°)	90.000	90.000	0.000	0.00
Ω (°)	0.00	0.00	0.00	0.00
ω (°)	N/A	N/A	N/A	N/A
ν (°)	Free	356.52	N/A	N/A

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

Table 10 Impulsive velocity vector magnitude comparison of optimal and genetic algorithm orbit transfer method results for problem-1

	Analytical Result	**Algorithm Result**	**Absolute Error**	**Error (%)**
first impulsive velocity vector magnitude (m/s)	2336.80	2340.31	3.49	0.15
second impulsive velocity vector magnitude (m/s)	1433.90	1434.29	0.39	0.03
total impulsive velocity vector magnitude (m/s)	3770.70	3774.60	3.90	0.10
first impulse true anomaly (°)	free	131.61	N/A	N/A
second impulse true anomaly (°)	180.00	180.18	0.18	0.10

As can be seen in Table 9, the orbital elements obtained by genetic algorithm orbit transfer strategy are compatible with the desired values. The semimajor axis absolute error is about 0.3 km and the percent error is 0.001%. The eccentricity absolute error is 0.00007 which is a negligible value since this value is within the predefined tolerances. As expected, the inclination and right ascension of ascending node errors are zero (0).

The genetic algorithm and analytical method results are compared considering total required impulsive velocity vector magnitude. As illustrated in Table 10, in the analytical method, the required impulsive velocity vector magnitude is 3770.70m/s and the impulsive velocity vector magnitude calculated by genetic algorithm based orbit transfer is 3774.60km/s. The difference between the impulsive velocity vector magnitude of analytical and genetic algorithm based orbit transfers is less than 1%. The second impulse true anomaly value obtained by genetic algorithm (180.18º) is also very close to the optimal result (180.00º).

Both final orbital elements and the required impulsive velocity vector magnitudes are very close to their desired/expected values. All the errors are within the tolerances that are presented in Table 8. It can be concluded that genetic algorithm based orbit transfer method can be used at the coplanar orbit transfer calculations.

3.4.1.2 Application Of Steepest Descent Algorithm To Problem-1

Gradient based numerical optimization methods are fast and robust, but they may converge to a local minimum instead of global minimum. In order to specify the performance of steepest descent method at the orbit transfer problem two initial predictions are generated: one close to the global minimum and one far to the global minimum for problem-1 (Table 6). In orbit transfer optimization main difficulty is to generate a good initial prediction. It is expected that the optimization method that is used at the orbit transfer optimization should pass over local minimums and reach to global minimum(s). If the steepest descent method reaches to a global minimum for these initial predictions, it can be concluded that this method can be used for orbit transfer optimization. Since the optimal solution of problem-1 is known (Hohmann transfer), problem-1 is used to test the performance of the steepest descent method. Note that the performance of genetic algorithm based orbit transfer is also tested using problem-1. The same cost function used for genetic algorithm is used at the steepest descent optimization method.

In the first case the initial prediction is done far from the optimal result and second case is devoted to the initial prediction close to the optimal result.

Case 1 Initial Prediction Far From Optimal Solution

The performance of steepest descent algorithm may change depending on initial prediction. Generally it is not easy to make a close prediction at the orbit transfer optimization problem. In this section an initial prediction that is far to the optimal solution is used to obtain solution to problem-1 (coplanar circular to circular orbit transfer). Initial prediction is generated as:

$$\text{Initial prediction:}\begin{bmatrix} \Delta v_{1x} \\ \Delta v_{1y} \\ \Delta v_{1z} \\ \nu_1 \\ \Delta v_{2x} \\ \Delta v_{2y} \\ \Delta v_{2z} \\ \nu_2 \end{bmatrix} = \begin{bmatrix} 1000.00 \\ 1000.00 \\ 1000.00 \\ 180.00 \\ 1000.00 \\ 1000.00 \\ 1000.00 \\ 180.00 \end{bmatrix}$$

Steepest descent algorithm is run for the initial prediction stated above. The obtained result is given in Table 11. There exist large errors in the semimajor axis, inclination, eccentricity and right ascension of ascending node. Steepest descent method suggests a highly elliptic (e=0.85) and retrograde (i>90º) orbit whereas it is aimed to reach circular or nearly circular and polar orbit. The errors are beyond the tolerances (Table 8) and not acceptable. In Table 12, for Problem-1 the required impulsive velocity vector magnitude and obtained orbit values of steepest descent method are compared to analytical results of coplanar circular-circular orbit transfer. Although the required total impulsive velocity vector magnitude is close to the optimal value, the distribution of impulsive velocity vector and second impulse true anomaly are not at the expected values. This shows that steepest descent algorithm cannot pass over a local minimum value at Problem-1 when far initial prediction is given as an initial prediction. Actually it is an expected situation since orbit transfer problem includes many local minimums (Conway, 2010). From these results it can be stated that the steepest descent method, when far initial prediction is selected, is not a proper optimization method for orbit transfer calculations.

Table 11 Steepest descent method (far initial prediction) result for Problem-1

Orbital Element	Desired Final Orbit	Calculated Final Orbit	Absolute Error	Error (%)
a (km)	42164.00	42124.00	40.00	0.11
e	0.00000	0.83280	0.83280	*
i (°)	90.000	95.238	5.238	*
Ω (°)	0.00	10.93	10.93	*
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	72.0	insignificant	insignificant

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

Table 12 Impulsive velocity vector magnitude comparison of analytical and steepest descent (far initial prediction) orbit transfer method results for Problem-1

	Analytical result	Algorithm result	Absolute Error	Error (%)
first impulsive velocity vector magnitude (km/s)	2336.80	2180.00	156.80	6.71
second impulsive velocity vector magnitude (km/s)	1433.90	1600.00	166.10	11.58
total impulsive velocity vector magnitude (km/s)	3770.70	3780.00	9.30	0.25
first impulse true anomaly (°)	Free	298.10	N/A	N/A
second impulse true anomaly (°)	180.00	82.94	97.06	53.92

Case 2 Initial Prediction Close To Optimal Solution

At the first case of application of steepest descent algorithm to problem-1 optimal solution cannot be obtained. In this section the optimal solution to the same problem (problem-1) is searched for an initial prediction that is close to the optimal solution.

Initial prediction is:

$$\text{Initial prediction:}\begin{bmatrix} \Delta v_{1x} \\ \Delta v_{1y} \\ \Delta v_{1z} \\ \nu_1 \\ \Delta v_{2x} \\ \Delta v_{2y} \\ \Delta v_{2z} \\ \nu_2 \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.00 \\ 2300.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ -1400.00 \\ 180.00 \end{bmatrix}$$

Steepest descent method is used for initial prediction given above and the corresponding result is tabulated in Table 13. The analytical and steepest descent algorithm based orbit transfer method results are close. The impulsive velocity vector magnitudes and true anomalies of this orbit transfer are presented in Table 14.

Table 13 Steepest descent algorithm (close initial prediction) result for Problem-1

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	42164.00	42164.00	0.00	0.00
e	0.00000	0.00067	0.00067	*
i (°)	90.000	89.997	0.003	*
Ω (°)	0.00	0.00	0.00	0.00
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	92.50	insignificant	insignificant

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

Table 14 Impulsive velocity vector magnitude comparison of analytical and steepest descent (close initial prediction) orbit transfer method results for Problem-1

	Analytical result	**Algorithm result**	**Absolute Error**	**Error (%)**
first impulsive velocity vector magnitude (km/s)	2336.80	2340.00	3.20	0.14
second impulsive velocity vector magnitude (km/s)	1433.90	1430.00	3.90	0.27
total impulsive velocity vector magnitude (km/s)	3770.70	3770.00	0.70	0.02
first impulse true anomaly (°)	Free	0.00	N/A	N/A
second impulse true anomaly(°)	180.00	180.18	0.18	0.10

3.4.2 Problem-2 Only Inclination Change Orbit Transfer

Only inclination change (OIC) orbit transfer is an orbit transfer that is applied to change only the inclination of the orbit. In order to perform this orbit transfer the thrust is needed to be applied at the equatorial crossing point. Details of this kind of orbit transfer can be found in section 3.1.2. In this section only inclination change orbit transfer is taken as baseline and the performance of genetic algorithm at the noncoplanar motion is evaluated. It should be noted that, since steepest descent optimization is found to be unsatisfactory for Problem-1 in 3.4.1.2 part, this method is not applied to only inclination change orbit transfer problem (Problem-2).

This kind of orbit transfer is a single impulse orbit transfer, to put differently, only one impulse is applied to spacecraft. To illustrate this situation, only inclination change orbit transfer problem is defined in Table 15. Note that the initial value of sum of argument of perigee and true anomaly is 360°, which is a prerequisite for this transfer in order not to modify right ascension of ascending node.

Table 15 Test scenario for only inclination change orbit transfer

Orbital Element	**Initial Orbit**	**Desired Final Orbit**
a (km)	42164.00	42164.00
e	0.00000	0.00000
i (°)	7.000	0.000
Ω (°)	300.00	300.00
ω (°)	N/A	N/A
ν (°)	0.0	free

The required impulsive velocity vector magnitude for the corresponding transfer is calculated:

$$\Delta v = 375.40 \text{ m/s} \tag{3.66}$$

The cost function is constructed similar to Problem 1 except the tolerances of inclination and right ascension of ascending node. Since only inclination change orbit transfer requires more accurate values for inclination and right ascension of ascending node, the acceptable errors are assumed to be as follows:

It is assumed that tolerance of inclination is (Chao, 2005):

$$\delta_i = \pm 0.050\,° \tag{3.67}$$

It is assumed that acceptable error for right ascension of ascending node is (Chao, 2005):

$$\delta\Omega = \pm 0.10\,° \tag{3.68}$$

The tolerance values for each orbital element and the corresponding cost function coefficients for Problem-2 are tabulated in Table 16.

Table 16 Tolerance values and cost function coefficients for problem-2

Orbital Element	Tolerance Values	Cost Function Coefficient
a (km)	±10.00	1.00
e	±0.00024	41666.67
i (°)	±0.050	200.00
Ω (°)	±0.10	100.00
ω (°)	undefined	undefined
ν (°)	insignificant	insignificant

so the cost function is constructed as:

$$\mathrm{J} = 25.00\sum_{i=1}^{n}\left|\Delta\vec{\mathbf{v}}_i\right| + 1.00a_{error} + 41666.67\,e_{error} + 200.00i_{error} + 100.00\Omega_{error} \tag{3.69}$$

3.4.2.1 Application Of Genetic Algorithm Method To Problem-2

In this section, the performance of genetic algorithm is evaluated by comparing analytical and genetic algorithm results for Problem-2.

The result of this strategy (genetic algorithm based orbit transfer) is tabulated in Table 17. The genetic algorithm based orbit transfer method provides a successful result; the spacecraft can be transferred to the final orbit as desired. All the final orbital element values are in their tolerances. Only the eccentricity value differs a little from the desired final value. Semimajor axis, inclination, and right ascension of ascending node are exactly at the desired orbit values.

Table 17 Genetic Algorithm orbit transfer method results for problem-2

Orbital Element	Desired Final Orbit	Calculated Final Orbit	Absolute Error	Error (%)
a (km)	42164.00	42164.00	0.00	0.00
e	0.00000	0.00003	0.00003	*
i (°)	0.000	0.000	0.000	0.00
Ω (°)	300.00	300.00	0.00	0.00
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	269.9	insignificant	insignificant

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

The required impulsive velocity magnitude is obtained by using genetic algorithm based orbit transfer for problem-2 is:

Δv =375.40 m/s.

This value is exactly the same as the analytical result. This shows that genetic algorithm based orbit transfer method's performance is good enough and can be used to search optimal orbit transfer solutions.

3.4.3 Optimization Method Comparisons For Problem-1 & 2

As can be seen in sections 3.4.1 and 3.4.2, it is possible to obtain solutions for orbit transfer optimization problems using different optimization methods. In this study two optimization methods; genetic algorithm and steepest descent algorithm, are used to optimize orbit transfers. The required impulsive velocity vector magnitudes of these optimization results and analytical results are presented in Table 18. The calculated final orbit values of problem-1 and 2 are given in Table 19 and Table 20 respectively.

It is seen that genetic algorithm provides optimal solutions for both problem-1 and problem-2 since the desired and calculated final orbit values are close and required impulsive velocity vector magnitudes are also close to analytical results.

Steepest descent optimization algorithm is run to obtain solution for problem-1. Two different initial predictions are generated: one is initial prediction far from the optimal solution and the other is close initial prediction to optimal solution. While the required impulsive velocity vector magnitudes are close to analytical result at the both cases (far and close to optimal solution), the calculated final orbit is different than the desired final orbit at the far initial prediction case.

Table 18 Impulsive velocity vector magnitude results of genetic and steepest descent algorithms for problem-1 & problem-2

	Total Impulsive Velocity Vector Magnitude (m/s)	
Optimization Method	**Problem-1 (coplanar circular-circular)**	**Problem-2 (inclination change only)**
Analytical result	3770.70	375.40
Genetic algorithm	3774.60	375.40
Steepest descent algorithm (far to optimal solution)	3780.00	Not calculated
Steepest descent algorithm (close to optimal solution)	3770.00	Not calculated

Table 19 Calculated final orbital element results of genetic and steepest descent algorithms for problem-1

	PROBLEM-1			
Orbital Element	**Desired Final Orbit**	**Genetic Algorithm**	**Steepest descent algorithm (far to optimal)**	**Steepest descent algorithm (close to optimal)**
a (km)	42164.00	42163.68	42124.00	42164.00
e	0.00000	0.00007	0.83280	0.00067
i (°)	90.000	90.000	95.238	89.997
Ω (°)	0.00	0.00	10.93	0.00
ω (°)	undefined	undefined	undefined	undefined
v (°)	insignificant	356.52	72.00	92.50

Since the optimal result for problem-1 cannot be obtained using steepest descent algorithm for a far prediction to the optimal solution, steepest descent algorithm is not used to search optimal result for problem-2.

Table 20 Calculated final orbital element results of genetic algorithm for problem-2

PROBLEM-2		
Orbital Elements	**Desired Final Orbit**	**Genetic Algorithm**
a (km)	42164.00	42163.68
e	0.00000	0.00007
i (°)	0.000	90.000
Ω (°)	300.00	0.00
ω (°)	undefined	undefined
v (°)	insignificant	356.52

It can be concluded from steepest descent method results of case-1 & 2; steepest descent optimization method can be used for orbit transfer optimization if a good initial prediction can be supplied. Poor starting prediction may cause the steepest descent method to stick in a local minimum. However, it is generally not very easy to make a good initial prediction for orbit optimization problem. Therefore, in orbit transfer optimization problem it is better to use a heuristic method rather than a gradient based method for optimization purposes.

CHAPTER 4

TEST CASES

The orbital elements of a spacecraft mainly depend on spacecraft mission, earth coverage area, the time distribution of spacecraft at specific areas, and the observation time.

Spacecrafts are generally inserted to orbits for Earth observation, scientific, meteorological, navigation and telecommunication purposes (97.8%) (Technical Issues: UCS Satellite Database, 2012). These types of spacecrafts have mission orbits where the Earth is at the one of the foci of the orbit ellipse. The spacecrafts which do not have Earth related missions have special and/or scientific missions. The mission analysis and orbit transfer problem of these spacecrafts have different requirements such as magnetic distribution, mass of central body, other perturbations, and etc. Since the orbit transfer of spacecrafts which Earth is not at the foci at their orbit are out of scope of this study, in this section these spacecrafts' orbit transfers are not analyzed.

In this section orbit transfer methods are developed for different orbit raising problem using classical orbit transfers and genetic algorithm based orbit transfer optimization. Classical orbit transfers are the transfers that exist in the literature and only some of these orbit transfers are optimal at specific cases. There exist no optimal solutions to the test case problems. In this section as a classical orbit transfer problem, if superposition of optimal orbit transfers are applied where applicable. For the cases where superposition of optimal orbit transfers is not applicable, nonoptimal orbit transfers are applied. The genetic algorithm based orbit transfer method is run for different number of impulses. These are two (2), three (3) and four (4) impulse orbit transfers. Among these the simplest orbit transfer strategy is the two impulse orbit transfer since it requires only two impulse. In the space industry, reliability is one of the most important parameter of spacecrafts. As the number of impulses increase, the reliability value decreases. If the two impulse orbit transfer strategy suggests good enough solution, it should be preferred. Similar to the two impulse case, among three and four impulse cases, three impulse should be selected if it suggests good enough results.

The results of genetic algorithm based orbit transfer and classical orbit transfer methods are compared. The genetic algorithm based orbit transfer optimization can be used to develop strategies for orbit transfer, orbit raising and de-orbiting. In this part only the orbit raising problems are detailed to illustrate the genetic algorithm orbit transfer method.

The test cases are selected based on realistic and popular missions. The most popular four Earth centered orbits, Sun Synchronous Low Earth Orbit (SS-LEO), Medium Earth Orbit (MEO), Geostationary Earth Orbit (GEO) and Molniya Orbit raising problems, are selected to be evaluated as test cases. These orbits are illustrated in Figure 20. The orbit at which spacecraft is separated from the launch vehicle is defined as initial orbit of the spacecraft. A launch vehicle can insert spacecraft with a limited accuracy. The initial orbit of a spacecraft is specified based on the worst performance of launch vehicle. It is assumed that the deviation from the final orbit of a launch vehicle is at its maximum (worst injection accuracy). In this section these test cases are introduced and orbit transfers are realized applying both classical orbit transfer methods and genetic algorithm based orbit transfer methods. For each test case, the classical and genetic algorithm based orbit transfer methods are compared where applicable. In all cases same initial and final orbital elements and genetic parameters are used.

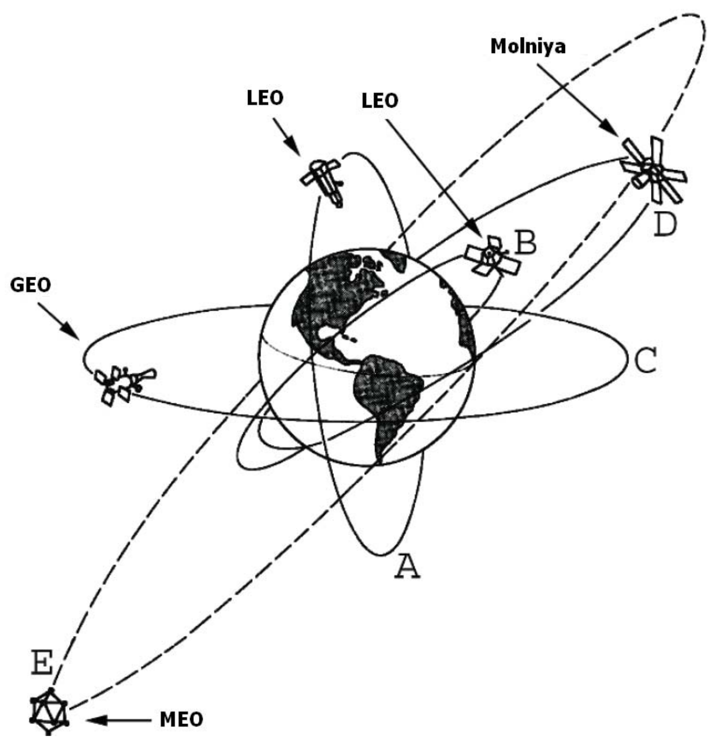

Figure 20 Test cases with respect to Earth (LEO, MEO, GEO, and Molniya orbit)

The performance of genetic algorithm based orbit transfer method for each test case is measured by using a cost function. The cost function includes difference between destination orbit elements and the algorithm results and the required energy/propellant to realize the algorithm suggested orbital maneuver. The coefficients of cost function are specified depending on the importance of each orbital element based on mission requirements and the importance of propellant/energy. The cost function does not include other constraints (transfer duration, impulse level per one burn, total impulse level, orbit of 3^{rd} bodies, etc.). Although impulse level one burn and total impulse level is not included in the cost function, the applicability of solution is checked considering these constraints.

For each test case, initial orbital element values, final orbital element values and the corresponding tolerances of each orbital element are defined. In each test case it is expected that each orbital element is to be within the predefined tolerances. If the algorithm result is not in the tolerance range, the result is assumed to be unsuccessful.

In this section classical and genetic algorithm based orbit transfer results are presented. The results of these methods are compared with respect to the impulsive velocity requirements for all test cases. Afterwards in order to investigate the effects of initial orbital elements to results, the algorithm is run for different orbital elements. The effects of each orbital element to the solutions are obtained for each case (LEO, MEO, GEO, and Molniya). These conclusions can also be used during the mission planning of both restartable last stage of launch vehicle which is used at the exoatmospheric region and spacecraft.

4.1 TEST CASE-1 SUN SYNCHRONOUS LOW EARTH ORBIT (SS-LEO) RAISING PROBLEM

The first test case is sun-synchronous Low Earth Orbit raising problem. Sun-synchronous Low Earth Orbit is a very popular orbit especially used for Earth observation and communication purposes. Today 190 of 994 operational satellites (19.1%) are sun-synchronous Low Earth Orbit satellites (Technical Issues: UCS Satellite Database, 2012). A sun-synchronous orbit is one that lies in a plane that maintains a fixed angle with respect to the Earth–sun direction. Therefore the inclination value is related to orbit altitude (semimajor axis and eccentricity). The relationship between altitude (for circular orbits) and the inclination value is given in Figure 21. Note that altitude is the difference between semimajor axis and radius of Earth for circular orbits. For sun synchronous orbit the inclination value is generally selected between 96°-104°. These properties ensure that:

- The satellite passes over a given location on Earth every time at the same local solar time, so guarantees almost the same illumination conditions [49].

- The satellite covers the whole surface of the Earth.

In Sun-synchronous Low Earth Orbit missions the critical orbital elements are semimajor axis, eccentricity, inclination, and right ascension of ascending node. At the SS-LEO raising problem includes both the correction of coplanar and noncoplanar orbit parameters. This disables the direct application of classical optimal orbit transfers, since there exist no optimal method in the literature that realizes orbit transfer like LEO raising.

In order to decrease the perturbation effects at the Sun Synchronous Low Earth Orbit (SS-LEO), eccentricity should not be exactly zero (0); instead it should be a close value to zero. Generally around 0.00112 values is selected (Larson & Wertz, 2005). Since SS-LEO mission requires almost circular orbit (i.e. e =0.00112), the argument of perigee is not critical/defined. True anomaly is not important for this mission; therefore it is not included in the cost function. Based on launch vehicle injection accuracies for LEO, the test Case-1 is constructed. It is assumed that the orbit that a spacecraft is separated from a launch vehicle (initial orbit) and it is expected that the spacecraft reaches its mission (final) orbit using its own propulsion system. Initial (separation) and final (mission) orbits of a typical SS-LEO satellite are tabulated in Table 21.

Table 21 Test Case-1 initial and final orbital element values

Orbital Element	**Initial Orbit** (PSLV user guide, 2005)	**Desired Final Orbit**
a (km)	7050.00	7080.00
e	0.00300	0.00112
i (°)	98.000	98.200
Ω (°)	0.00	0.00
ω (°)	N/A	N/A
ν (°)	free	free

The tolerance values of the orbital elements are the values that do not affect the mission of the spacecraft. Semimajor axis tolerance is specified at (Jensen, 1998):

$$\delta_a = \pm 1.00 \text{ km} \tag{4.1}$$

eccentricity tolerance is (Jensen, 1998):

$$\delta_e = \pm 0.00014 \tag{4.2}$$

Since this mission also requires sun synchronous orbit, the inclination value is needed to be fixed at an altitude (Figure 21).

$$i_{ss} = \cos^{-1}\left(-0.098922\,(1-e^{-1})^{2}\left(1+\frac{h}{R}\right)^{3.5}\right) \quad (4.3)$$

where:

i_{ss} : inclination value for sun synchronous mission

h : orbit altitude

R : Earth's radius (6378km)

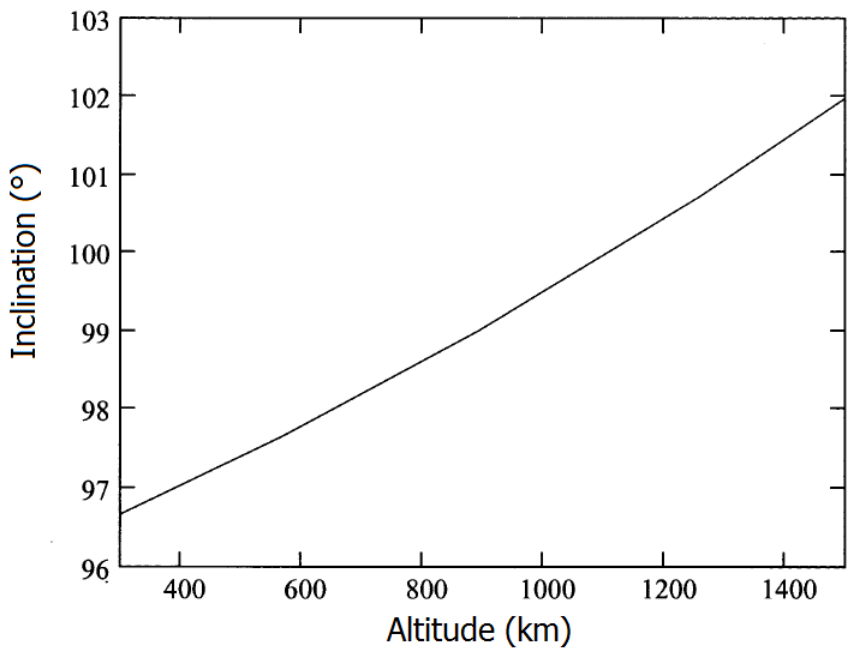

Figure 21 Orbital Altitude versus Orbital Inclination for Circular Sun Synchronous Earth Orbits (Boain, 2005)

The corresponding inclination tolerance for this case is given in (Jensen, 1998):

$$\delta_i = \pm 0.100° \quad (4.4)$$

where:

δ_i : tolerance value of inclination

It is assumed that 10 minutes local time error is acceptable for SS-LEO case. So

$$\delta_{\Omega} = (\pm 10/(24.60))360 = \pm 2.50° \quad (4.5)$$

where:

δ_{Ω} : tolerance value of right ascension of ascending node

The tolerance values and corresponding cost function coefficients of each orbital element for test case-1 are tabulated in Table 22. Note that these values are close to the values obtained in the literature. For example, USA's earth observation satellite Aquarius/Sac-D' has orbit accuracy values close to the values in Table 22 .

The cost function coefficients are calculated as follows: the multiplications of tolerance value and cost function coefficient are kept constant. Cost function coefficient for semimajor axis is assumed as unity (1) since the corresponding cost function coefficient of impulsive velocity is 25. During all cases the cost function coefficients of semimajor axis are kept constant and the other orbital elements cost function coefficients (eccentricity, inclination, right ascension of ascending node) are scaled depending on tolerance values. It should be noted that the tolerance values and cost function coefficients can be selected differently depending on the performance and accuracy requirements of a spacecraft. The tolerance and cost function values stated here are typical values and they can vary for different situations. The algorithm can be run for different tolerance and cost function values.

Table 22 Tolerance values and cost function coefficients of test case 1

Keplerian Elements	**Tolerance Values**	**Cost Function Coefficient**
a (km)	±1.00	1.00
e	±0.00014	7142.86
i (°)	±0.100	10.00
Ω (°)	±2.50	0.40
ω (°)	N/A	N/A
ν (°)	N/A	N/A

so the cost function is constructed as:

$$\mathrm{J} = 25.00\sum_{i=1}^{n}\left|\Delta\vec{\mathbf{v}}_i\right| + 1.00a_{error} + 7142.86e_{error} + 10.00i_{error} + 0.40\Omega_{error} \tag{4.6}$$

A spacecraft, which has sun synchronous LEO mission, can reach to its mission orbit by applying two or more orbital maneuvers. Since the initial and final orbits are not intersecting, the transfer cannot be realized by applying a single impulse. In the literature it is shown that applying more than four (4) maneuvers is not feasible (Gobetz & Doll, 1969). Since the required impulsive velocity vector magnitude of more than four impulse does not offer much better results than four or less impulse case and more than four impulse decreases the reliability factor of spacecraft, it is generally not preferred. Therefore in this study, the maximum number of impulse is assumed to be four (4).

Firstly, SS-LEO raising problem is performed applying classical orbit transfer methods. Afterwards, solution for SS-LEO raising problem is obtained using genetic algorithm based orbit transfer method to implement two, three, and four impulse orbit transfers. At the end of this part, the results of classical methods and genetic algorithm based orbit transfer methods for different number of impulses are compared.

4.1.1 Implementation of Classical Orbit Transfer Methods for SS-LEO Raising Problem

In this section a SS-LEO orbit raising problem whose initial and final values are given in Table 21 is studied using classical orbit transfer methods. Since it is possible to apply superposition of optimal orbit transfers such a SS-LEO raising problem, superposition of Hohmann and only inclination change orbit transfers are applied respectively. For this reason, the combination of optimal classical orbit transfers is applied to perform SS-LEO raising. Solutions for planar orbit transfers can be obtained using Hohmann transfer. This method is only valid between coplanar circular orbit transfers. Since the eccentricity values of initial and final orbit values are small, these orbits are assumed to be circular and Hohmann transfer is applied to perform necessary coplanar transfer. Only inclination change orbit transfer is used to make plane change corrections. Since the equations of Hohmann transfer and only inclination change orbit transfer is presented in chapter 3, it is not repeated again. In Table 23, the required impulsive velocity vector magnitude of a SS-LEO orbit raising problem is obtained using classical orbit transfer methods.

Table 23 Classical orbit transfer solution for SS-LEO orbit raising case

	Impulsive Velocity Vector Magnitude(m/s)
Hohmann Transfer	29.80
OIC	26.20
TOTAL	56.00

4.1.2 Implementation of Genetic Algorithm Orbit Transfer Methods for SS-LEO Raising Problem

The genetic algorithm based orbit transfer methods can suggest results better than classical orbit transfers. Since initial and final orbits are not intersecting at SS-LEO orbit raising case at least two impulse orbit transfer is needed to be applied. The solution to SS-LEO raising problem may also be obtained by applying more than two impulses. Solution for SS-LEO raising problem is searched by applying two, three, and four impulse genetic algorithm orbit transfer methods. The same cost function is used at each orbit transfer method for SS-LEO raising problem.

4.1.2.1 Two-Impulse SS-LEO Raising Strategy

The simplest SS-LEO raising strategy is the two impulse orbit transfer. The results of SS-LEO spacecraft orbit raising two impulse maneuvers are given in Table 24. The final orbit values are close to the destination orbit values. All the errors are less than 0.25%. The semimajor axis error is about 70 meters, the eccentricity and inclination errors are zero (0), and the largest error is observed at the right ascension of ascending node element which is only 0.03°. All these errors are in the defined tolerance range, in other words this transfer strategy can be applied to realize SS-LEO raising.

Table 24 SS-LEO spacecraft orbit raising result for two-impulse orbit transfer

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	7080.00	7079.93	0.07	0.23
e	0.00112	0.00112	0.00000	0.00
i (°)	98.200	98.200	0.000	0.00
Ω (°)	0.00	0.03	0.03	*
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	268.70	insignificant	insignificant

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 25. As seen, more than half of the required orbit change is realized at the initial impulse. Right ascension of ascending node is almost not changed.

Table 25 Two impulse SS-LEO spacecraft orbit raising intermediate orbit results

Orbital Element	**Initial Orbit**	**Intermediate Orbit**	**Final Orbit**
a (km)	7050.00	7076.99	7079.93
e	0.00300	0.00109	0.00112
i (°)	98.000	98.097	98.200
Ω (°)	0.00	359.99	0.03
ω (°)	undefined	undefined	undefined
ν (°)	173.92	39.15	268.70

The required impulsive velocity vector components in x, y, and z directions (Earth inertial) and corresponding true anomaly values for two impulse SS-LEO raising orbit transfer problem are given in Table 26. The total required impulsive velocity vector magnitude is about 33.97m/s. The largest component at the both impulse is the y axis which is about 14m/s.

Table 26 Impulse details of two impulse SS-LEO raising orbit transfer strategy

	X axis (m/s)	**Y axis (m/s)**	**Z axis (m/s)**	**Total Impulsive Velocity Magnitude (m/s)**	**Impulse True Anomaly (°)**
Impulse-1	-4.263	14.647	-12.064	19.449	173.92
Impulse-2	-1.433	-14.414	-0.940	14.516	246.17
TOTAL				33.97	

4.1.2.2 Three-Impulse SS-LEO Raising Strategy

In this section three impulse SS-LEO raising transfer results are presented. The results of three-impulse SS-LEO raising orbit transfer are tabulated in Table 27. As can be seen in Table 27, the semimajor axis, eccentricity, and inclination errors are much less than their tolerances. The largest error is observed at the right ascension of ascending node (0.01º) which is also much less than its tolerance (2.5º). Since all final orbital elements are close to the desired orbital values (in the tolerance range), three impulse orbit transfer strategy can also be applied to perform SS-LEO raising.

Table 27 SS-LEO spacecraft orbit raising result for three-impulse orbit transfer

Orbital Element	Desired Final Orbit	Calculated Final Orbit	Absolute Error	Error (%)
a (km)	7080.00	7080.00	0.00	0.00
e	0.00112	0.00113	0.00001	0.53
i (°)	98.200	98.200	0.000	0.00
Ω (°)	0.00	0.01	0.01	*
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	11.72	insignificant	insignificant

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 28. Most of the inclination error is corrected during the first and second impulse case. While eccentricity error is decreased to low value after the first impulse, eccentricity error is increased during second impulse. The eccentricity error is minimized at the third impulse.

Table 28 Three impulse SS-LEO spacecraft orbit raising intermediate orbit results

Orbital Element	Initial Orbit	Intermediate Orbit-1	Intermediate Orbit-1	Final Orbit
a (km)	7050.00	7064.81	7072.73	7080.00
e	0.00300	0.00122	0.00008	0.00113
i (°)	98.000	98.079	98.152	98.200
Ω (°)	0.00	0.01	0.06	0.01
ω (°)	N/A	51.51	343.20	undefined
ν (°)	free	135.72	127.99	11.72

The required impulsive velocity vector components and corresponding true anomaly values of this orbit raising strategy are given in Table 29. The total required impulsive velocity vector magnitude is 38.67m/s. Similar to the two impulse strategy, the largest vector components are at the y axis at the all impulses of three impulse strategy.

Table 29 Impulse details of three (3) impulse SS-LEO raising orbit transfer strategy

	X axis (m/s)	Y axis (m/s)	Z axis (m/s)	Total Impulsive Velocity Magnitude (m/s)	Impulse True Anomaly (°)
Impulse-1	-8.177	11.531	-7.459	15.983	187.23
Impulse-2	4.927	11.843	0.003	12.827	163.69
Impulse-3	4.133	-8.947	0.090	9.855	332.47
TOTAL				38.67	

4.1.2.3 Four-Impulse SS-LEO Raising Strategy

Four-impulse strategy can also be applied to perform SS-LEO orbit raising transfer. The results of four impulse SS-LEO orbit raising maneuver are tabulated in Table 30. The eccentricity and inclination are at the desired values. A small amount of error is observed at the semimajor axis (10 meters). The impulse details (impulsive velocity vector and true anomalies) are presented in Table 32.

Table 30 SS-LEO spacecraft orbit raising simulation result for four-impulse orbit transfer

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	7080.00	7079.99	0.01	0.03
e	0.00112	0.00112	0.00000	0.00
i (°)	98.200	98.200	0.000	0.00
Ω (°)	0.00	0.04	0.04	*
ω (°)	undefined	undefined	undefined	undefined
v (°)	insignificant	141.87	insignificant	insignificant

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 31. The inclination error is corrected linearly; however, semimajor axis error is corrected at the third and fourth impulse cases.

Table 31 Four impulse SS-LEO spacecraft orbit raising intermediate orbit results

Orbital Element	**Initial Orbit**	**Intermediate Orbit-1**	**Intermediate Orbit-2**	**Intermediate Orbit-3**	**Final Orbit**
a (km)	7050.00	7062.05	7055.09	7073.48	7079.99
e	0.00300	0.00126	0.00104	0.00230	0.00112
i (°)	98.000	98.058	98.088	98.141	98.200
Ω (°)	0.00	0.02	0.06	0.05	0.04
ω (°)	N/A	353.92	304.12	207.06	undefined
v (°)	free	156.97	107.14	33.15	141.87

Table 32 Impulse details of four (4) impulse SS-LEO raising orbit transfer strategy

	X axis (m/s)	Y axis (m/s)	Z axis (m/s)	Total Impulsive Velocity Magnitude (m/s)	Impulse True Anomaly (°)
Impulse-1	-0.880	8.853	-5.815	10.628	196.95
Impulse-2	2.611	-5.922	-3.541	7.377	57.33
Impulse-3	2.386	8.277	-9.000	12.458	229.79
Impulse-4	-4.750	-8.438	3.323	10.237	141.60
TOTAL				40.70	

The desired and algorithm results of orbital elements are close to each other (all in the tolerances), the maximum percent error is observed at the eccentricity which is less than 0.04%. The required impulsive velocity vector magnitude for four impulse genetic algorithm orbit transfer strategy is about 40.70m/s. At this strategy the largest component at each impulse is also at the y axis components.

4.1.3 Comparison of Transfer Alternatives for SS-LEO Raising Problem

It is possible to obtain solutions for SS-LEO raising problem with different orbit transfer strategies. All the strategies applied above are applicable to Sun Synchronous Low Earth Orbit (SS-LEO) raising problem since all results are in the tolerance limits. These strategies have advantages and disadvantages with respect to each other. Main difference between these strategies is the magnitude of the required impulsive velocity. Since the errors are negligibly small and all within their tolerances, there is no need to compare final orbital values to desired orbital values. In Table 33, the total required impulsive velocity vector magnitudes for each method that can be applied for SS-LEO raising are tabulated.

Table 33 Impulsive Velocity Requirement Comparison of SS-LEO raising strategies

SS-LEO raising strategy	Total Impulsive Velocity Vector Magnitude (m/s)
Classical Orbit Transfer Methods	56.00
Two-impulse Orbit Transfer Method	33.97
Three-impulse Orbit Transfer Method	38.67
Four-impulse Orbit Transfer Method	40.70

Although both the Hohmann and only inclination change orbit transfers are optimal for some conditions, the combination may not be optimal. This is actually the case that is encountered in SS-LEO raising orbit transfer problem. The solution of SS-LEO raising orbit transfer problem obtained by the superposition of optimal methods is the most fuel consuming method (56.00m/s). Two-impulse genetic algorithm strategy requires the least impulsive velocity magnitude (33.97m/s) which is approximately 39.3% less than the combination of optimal methods. Instead of two-impulse orbit transfer strategy, three and four-impulse strategies can also be applied for SS-LEO raising problem. However, three and four impulse strategies require more impulsive velocity than two-impulse strategy. Three and four impulse strategies should also not be preferred because of reliability issues. As can be seen from Table 33, the impulsive velocity requirements of three and four impulse are close, however the four-impulse orbit transfer strategy requires a little more.

4.1.4 Effect of Initial Orbital Elements to SS-LEO Raising Problem Solution

In this section the effects of initial orbital elements to SS-LEO raising problem is studied. The main critical parameters at the SS-LEO raising problem (defined in section 4.1) are semimajor axis (a), eccentricity (e), and inclination (i). Right ascension of ascending node is not critical for SS-LEO mission. Because of these reasons, the effects of initial semimajor axis, eccentricity, and inclination are studied.

The problem defined in Table 21 is taken as baseline and called as SS-LEO-1. The solutions are obtained for different initial conditions. For each orbital element, three different test cases are constructed.

In this section it is assumed that the cost function coefficients and tolerances are the same as the problem in section 4.1 since the cases are similar. The desired orbit for all SS-LEO raising problems are assumed to be the same (a=7080km, e=0.00112, i=98.200°, and Ω=0.00°)

4.1.4.1 Semimajor axis variation

The test problems and corresponding solutions for different initial semimajor axis values are given in Table 34.

At these test problems, eccentricity and inclination values are kept constant and only semimajor axis values are changed. Two, three and four impulse orbit transfer strategies are applied to obtain solutions for each test problems.

Table 34 Effect of initial semimajor axis value to SS-LEO raising problem

Test Case	Orbital Element			Total Impulsive Velocity Vector Magnitude (m/s)		
	a (km)	e	i (°)	Two Impulse Strategy	Three Impulse Strategy	Four Impulse Strategy
SS-LEO-1(+)	7050.00	0.00300	98.000	33.97	38.67	40.70
SS-LEO-2	7040.00	0.00300	98.000	37.69	38.94	44.04
SS-LEO-3	7020.00	0.00300	98.000	43.48	44.51	48.5
SS-LEO-4	7000.00	0.00300	98.000	54.54	53.74	51.91

(+) reference test case

As seen in Table 34, the initial semimajor axis values affect directly the required impulsive velocity vector magnitudes. In Figure 22, the required impulsive velocities of SS-LEO-1, 2, 3, and 4 are presented graphically.

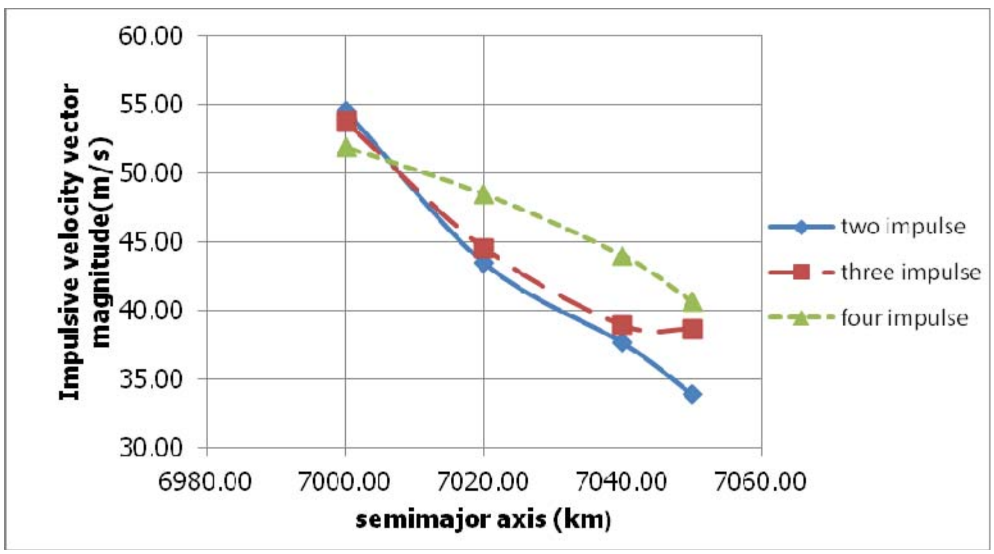

Figure 22 Effect of initial semimajor axis (km) to SS-LEO raising problem

As can be seen in Figure 22, as the initial semimajor axis value is further to the destination orbit semimajor axis value, the required impulsive velocity vector magnitude increases. The increment is close to linear at the two impulse case; however the increments are not linear at the three and four impulse cases. Note also that required impulsive velocity vector magnitudes are close. The minimum required total impulsive velocity magnitude for SS-LEO-4 test case (a=7000km) is 51.91m/s, for SS-LEO-1 test case (a=7050km) it is about 33.96m/s.

Two, three and four impulse orbit transfer strategy results are close at each test case; especially, two and three impulse strategy results.

At SS-LEO-4 the four impulse strategy requires minimum amount of propellant and the two impulse strategy requires maximum amount of propellant. However, the difference between results of these strategies is very small (less than 3m/s). Considering the reliability of a spacecraft, two impulse strategy should be selected instead of four impulse strategy. For the remaining test cases (SS-LEO-1, SS-LEO-2, SS-LEO-3), two impulse orbit transfer strategy offers the best solution. At these test cases, the required impulsive velocity vector magnitude increases as the number of impulses increases. Therefore, for these test cases two impulse orbit transfer strategy is also the orbit transfer strategy which requires minimum impulsive velocity vector magnitude.

4.1.4.2 Inclination variation

Inclination is one of the most critical and difficult to change orbital elements. The test cases based on different initial inclination values (SS-LEO-5, SS-LEO-6, and SS-LEO-7) are constructed and given in Table 35. The corresponding results of these test cases are also given in Table 35. The results are also presented graphically in Figure 23.

Table 35 Effect of initial inclination value to SS-LEO raising problem

	Orbital Element			Total Impulsive Velocity Vector Magnitude (m/s)		
Test Case	a (km)	e	i (°)	Two Impulse Strategy	Three Impulse Strategy	Four Impulse Strategy
SS-LEO-5	7050.00	0.00300	98.100	22.23	31.34	39.02
SS-LEO-1($^{+}$)	7050.00	0.00300	98.000	33.96	38.66	40.72
SS-LEO-6	7050.00	0.00300	97.900	47.46	53.64	56.86
SS-LEO-7	7050.00	0.00300	97.800	59.78	66.61	69.43

($^{+}$) reference test case

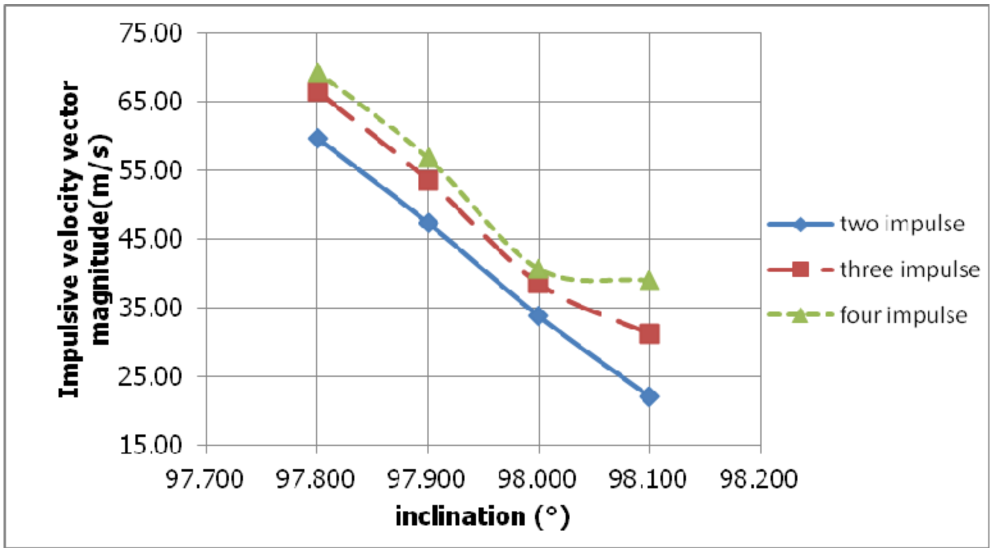

Figure 23 Effect of initial inclination (°) to SS-LEO raising problem

It can be stated that inclination changes require high amount of fuel. As seen in Figure 23, a 0.1° inclination change requires about 12m/s impulsive velocity vector magnitude at SS-LEO raising problems. This shows the importance of initial inclination value.

At all test cases that are constructed to study the effect of initial inclination value (SS-LEO-5, SS-LEO-6, SS-LEO-7), the orbit transfer strategies which require minimum energy are two impulse orbit transfer strategies. In these test cases as the number of impulses increases the required fuel also increases. Since the two impulse strategy is the least energy requiring strategy from the required impulse and reliability view, it should be selected at test cases SS-LEO-5, SS-LEO-6, SS-LEO-7.

Test cases SS-LEO-5, SS-LEO-6, and SS-LEO-7 showed that the inclination value is one of the most critical orbital elements. In order to prolong the lifetime of a spacecraft, inclination error should be minimized.

4.1.4.3 Eccentricity variation

The last studied orbital element at the SS-LEO raising problem is eccentricity. Eccentricity defines the shape of the orbit ellipse. The test cases which are constructed to study the effect of the initial eccentricity value at LEO raising are tabulated in Table 36. The corresponding results are also given in this table (Table 36). The results are graphically given in Figure 24.

Table 36 Effect of initial inclination value to SS-LEO raising problem

Test Case	Orbital Element			Total Impulsive Velocity Vector Magnitude (m/s)		
	a (km)	*e*	*i* (°)	Two Impulse Strategy	Three Impulse Strategy	Four Impulse Strategy
SS-LEO-8	7050.00	0.00200	98.000	32.16	33.85	38.12
SS-LEO-9	7050.00	0.00250	98.000	32.95	36.93	38.86
SS-LEO-1(+)	7050.00	0.00300	98.000	33.96	38.66	40.72
SS-LEO-10	7050.00	0.00350	98.000	35.38	45.72	46.21

(+) reference test case

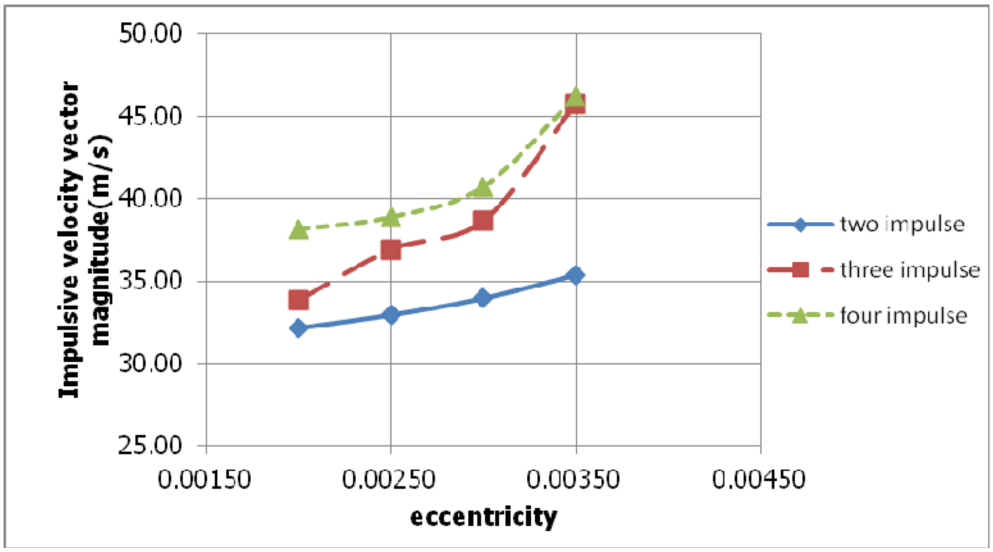

Figure 24 Effect of initial eccentricity to SS-LEO raising problem

As the initial eccentricity value is selected close to the desired (final) value, the required impulse level decreases. However the required impulsive velocity vector magnitudes are close. It can be stated that initial eccentricity value is not very critical.

While two impulse strategy shows linear behavior, three and four impulse strategies show climbing trend behavior.

The orbit transfer strategies which have minimum energy requirement are two impulse orbit transfer strategies for different initial eccentricity. As the number of impulses increases the required fuel also increases. Since the minimum energy requiring strategy is two impulse method and it also provides more reliable solution than three and four impulse orbit transfer strategies, it should be applied at cases similar to SS-LEO-8, SS-LEO-9, and SS-LEO-10.

4.2 TEST CASE-2 MEDIUM EARTH ORBIT (MEO) RAISING PROBLEM

Medium earth orbits are also popular orbits which are mainly used for navigation purposes. This mission requires more accuracy than SS-LEO missions. Because of this, the tolerance values are stricter than SS-LEO mission tolerances. Today 69 of 994 (6.9%) satellites have missions at the Medium Earth Orbit.

Similar to the sun synchronous Low Earth Orbit (SS-LEO) case, tolerances for MEO raising problem are defined. Since the eccentricity is almost zero, argument of perigee is not considered during calculations. For zero eccentricity (e=0), argument of perigee is undefined.

Table 37 Test case 2 initial and final orbital element values

Orbital Element	**Initial Orbit** (Proton Mission Planner's Guide, 2009)	**Final Orbit**
a (km)	19950.00	20000.00
e	0.00200	0.00100
i (°)	59.800	60.000
Ω (°)	0.00	0.00
ω (°)	undefined	undefined
v (°)	insignificant	insignificant

Similar to the SS-LEO raising case, the cost function coefficient of MEO raising is obtained by scaling coefficients with respect to the semimajor axis.

The tolerance values are obtained as follows:

semimajor axis tolerance is given in (Chao, 2005):

$$\delta_a = \pm 0.50\text{km} \tag{4.7}$$

eccentricity tolerance is given in (Chao, 2005):

$$\delta_e = \pm 0.00003 \tag{4.8}$$

The tolerance limits of inclination and right ascension of ascending node for a typical MEO mission (Chao, 2005):

$$\delta_i = \pm 0.010° \tag{4.9}$$

$$\delta_\Omega = \pm 0.10° \tag{4.10}$$

where:

δ_i : tolerance value of inclination

δ_Ω : tolerance value of right ascension of ascending node

So the tolerance values and cost function coefficients of orbital elements are obtained for MEO raising (Table 38).

Table 38 Tolerance values and cost function coefficients of test case 2

Orbital elements	Tolerance Values	Cost Function Coefficient
a (km)	±0.50	1.00
e	±0.00003	16666.67
i (°)	±0.010	50.05
Ω (°)	±0.10	5.05
ω (°)	undefined	undefined
ν (°)	insignificant	insignificant

The cost function for MEO raising are constructed as follows:

$$\mathrm{J} = 25.00\sum_{i=1}^{n}\left|\Delta\vec{\mathbf{v}}_i\right| + 1.00a_{error} + 16667.67\,e_{error} + 50.05i_{error} + 5.05\Omega_{error} \tag{4.11}$$

Note that this cost function is used in all MEO raising problem calculations.

Since the linear velocity vector magnitude of MEO spacecraft is less than LEO spacecraft, less impulse is expected to be required during orbit raising at MEO. Similar to SS-LEO orbit rising, MEO orbit rising also requires the correction of semimajor axis, eccentricity, and inclination.

Solutions for MEO raising problem can be obtained with the combination of optimal classical orbit transfers or genetic optimization based orbit transfer algorithm. MEO raising strategies are developed usng each method and at the end of this section the results are compared.

4.2.1 Implementation of Classical Orbit Transfer Methods for MEO Orbit Raising Problem

Similar to the SS-LEO raising case, there is no optimal orbit transfer strategy for MEO raising problem, but the superposition of optimal orbit transfers is possible. As a classical orbit transfer method, the superposition of optimal methods can be applied to obtain solution. As stated in section 3.1.1, Hohmann transfer is optimal for coplanar orbit transfers from circular to circular orbits for changing semimajor axis value. Only inclination change orbit transfer enables the change of only inclination. These are applied respectively to realize MEO raising problem (Table 39).

Table 39 Classical orbit transfer solution for MEO raising problem

	Impulsive Velocity Vector Magnitude (m/s)
Hohmann Transfer	11.40
OIC	15.60
TOTAL	27.00

4.2.2 Implementation of Genetic Algorithm Orbit Transfer Methods for MEO Orbit Raising Problem

In this section, the two, three, and four impulse genetic algorithm orbit transfer methods are applied to obtain solutions for MEO raising problem. It is expected that the genetic algorithm orbit transfer results suggest less impulse than combination of optimal orbit transfer methods since the combination of optimal orbit transfers may not be optimal.

4.2.2.1 Two Impulse MEO Raising Strategy

Similar to the SS-LEO raising, the initial and final orbits are not intersecting which means that MEO raising also cannot be performed with a single impulse. At least two-impulse is required to realize MEO raising. Two-impulse genetic algorithm based orbit transfer strategy results for MEO raising problem are given in Table 40 and impulse details of corresponding problem is tabulated in Table 42.

As can be seen in Table 40, the error values are very small (maximum 2%) and in the tolerance range. This strategy is applicable for MEO raising problem. The required impulsive velocity vector magnitude of this strategy is about 20.4 m/s (less than SS-LEO raising case). While the first impulse is about 2m/s, the second impulse is about 18.4m/s.

Table 40 MEO spacecraft orbit raising simulation result for two-impulse orbit transfer

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	20000.00	20000.00	0.00	0.00
e	0.00100	0.00100	0.00000	0.00
i (°)	60.000	59.996	0.004	2.00
Ω (°)	0.00	0.02	0.02	*
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	280.46	insignificant	N/A

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 41. As seen, the orbit transfers are almost equal. In other words about half of the required change is realized at the first impulse and about half of the remaining orbit transfers are realized in the second impulse.

Table 41 MEO spacecraft orbit raising intermediate orbit results

Orbital Element	**Initial Orbit**	**Intermediate Orbit**	**Final Orbit**
a (km)	19950.00	19978.48	20000.00
e	0.00200	0.00127	0.00100
i (°)	59.800	59.896	59.996
Ω (°)	0.00	0.05	0.02
ω (°)	undefined	undefined	undefined
ν (°)	insignificant	251.01	280.46

Table 42 Impulse details of two-impulse MEO raising orbit transfer strategy

	X axis (m/s)	**Y axis (m/s)**	**Z axis (m/s)**	**Total Impulsive Velocity Magnitude (m/s)**	**Impulse True Anomaly (°)**
Impulse-1	2.968	5.939	-5.997	8.95	203.01
Impulse-2	0.239	5.536	-5.999	8.16	225.21
TOTAL				17.11	

4.2.2.2 Three Impulse MEO Raising Strategy

MEO orbit raising transfer can also be realized by applying three-impulse strategy. The genetic algorithm three-impulse solution and the corresponding errors are tabulated in Table 43. There is no error observed at the eccentricity and inclination. At the semimajor axis, the difference between desired and obtained values is about 10 meters. The impulse details of three-impulse MEO raising orbit transfer strategy is presented in Table 45. The total required impulsive velocity magnitude is 18.22m/s.

Table 43 MEO spacecraft orbit raising simulation result for three-impulse orbit transfer

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	20000.00	19999.99	0.01	0.02
e	0.00100	0.00100	0.00000	0.00
i (°)	60.000	60.000	0.000	0.00
Ω (°)	0.00	0.020	0.020	*
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	100.06	insignificant	insignificant

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 44. Semimajor axis, eccentricity and inclination errors are decreased linearly. Since the impulse magnitudes are close, it is actually the expected situation.

Table 44 Three impulse MEO spacecraft orbit raising intermediate orbit results

Orbital Element	**Initial Orbit**	**Intermediate Orbit-1**	**Intermediate Orbit-2**	**Final Orbit**
a (km)	19950.00	19982.29	19985.52	19999.99
e	0.00200	0.00147	0.00126	0.00100
i (°)	59.800	59.855	59.923	60.000
W (°)	0.00	0.04	0.04	0.02
w (°)	undefined	undefined	undefined	undefined
n (°)	insignificant	90.80	128.93	100.06

Table 45 Impulse details of three-impulse MEO raising orbit transfer strategy

	X axis (m/s)	**Y axis (m/s)**	**Z axis (m/s)**	**Total Impulsive Velocity Magnitude (m/s)**	**Impulse True Anomaly (°)**
Impulse-1	3.255	3.442	-4.466	6.51	215.01
Impulse-2	-0.645	4.456	-2.944	5.38	229.52
Impulse-3	0.112	4.445	-4.500	6.33	224.62
TOTAL				18.22	

4.2.2.3 Four Impulse MEO Raising Strategy

Another MEO orbit raising strategy is the four-impulse orbit transfer. This strategy possibly increases the transfer duration and complexity. The results of four-impulse MEO orbit raising are given in Table 46. While there is no error observed at the inclination and eccentricity values, the semimajor axis error is about 0.04%. The impulse details of four-impulse orbit raising transfer are tabulated in Table 48. The magnitudes of impulsive velocity vector magnitudes are almost equal and about 5m/s.

Table 46 MEO spacecraft orbit raising result for four-impulse orbit transfer

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	20000.00	19999.96	0.04	0.04
e	0.00100	0.00100	0.00000	0.00
i (°)	60.000	60.000	0.000	0.00
Ω (°)	0.00	0.10	0.10	*
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	110.52	insignificant	insignificant

* The percent error is not calculated for this orbital element since the initial and final value of this element is the same.

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 47. All the errors are corrected linearly.

Table 47 Four impulse MEO spacecraft orbit raising intermediate orbit results

Orbital Element	**Initial Orbit**	**Intermediate Orbit-1**	**Intermediate Orbit-2**	**Intermediate Orbit-3**	**Final Orbit**
a (km)	19950.00	19960.20	19972.46	19986.66218	19999.96
e	0.00200	0.00173	0.00213	0.00147	0.00100
i (°)	59.800	59.851	59.906	59.958	60.000
Ω (°)	0.00	0.02	0.03	0.06	0.10
ω (°)	undefined	undefined	undefined	undefined	undefined
ν (°)	insignificant	144.14	11.00	151.26	110.52

Table 48 Impulse details of four-impulse MEO raising orbit transfer strategy

	X axis (m/s)	Y axis (m/s)	Z axis (m/s)	Total Impulsive Velocity Magnitude (m/s)	Impulse True Anomaly (°)
Impulse-1	2.182	3.304	-2.536	4.702	196.88
Impulse-2	-2.442	-3.411	2.983	5.148	31.31
Impulse-3	0.425	2.968	-3.500	4.609	200.41
Impulse-4	0.517	2.908	-3.402	4.505	226.10
TOTAL				18.96	

4.2.3 Comparison of Transfer Alternatives for MEO Raising Problem

As can be seen above, solution to MEO raising transfer can be found using different orbit transfer strategies. These orbit transfer strategies require different level of impulses. The required impulsive velocity vector magnitudes at all MEO raising transfer strategies are tabulated in Table 49. Since all the strategies suggest close results to the desired orbit (i.e. all results are in the tolerance limits), the strategies are not compared according to the closeness to the destination orbit. Classical orbit transfer method (superposition of Hohmann & only inclination change orbit transfer) requires the largest energy. The genetic algorithm based MEO raising strategy results are close but less than classical orbit transfer methods. Among all MEO raising orbit transfer methods, two-impulse orbit transfer method requires the minimum impulsive velocity vector magnitude. The increment of number of impulses increases the total required impulsive velocity magnitude in a small quantity. Since increasing the number of impulses can cause complexity, it is better to apply two impulse genetic algorithm based orbit transfer strategy for MEO raising problem.

Table 49 Impulsive Velocity Requirement Comparison of MEO raising strategies

MEO raising strategy	Total Impulsive Velocity Magnitude (m/s)
Classical Orbit Transfer Methods	27.00
Two-impulse Orbit Transfer Method	17.11
Three-impulse Orbit Transfer Method	18.22
Four-impulse Orbit Transfer Method	18.96

Another important observation is the difference between the level of required impulsive velocity magnitudes of LEO and MEO raising orbit transfers. As expected the required impulsive velocity vector magnitudes of MEO raising is less than SS-LEO raising orbit transfer. Main reason of this difference is that the velocity magnitude of spacecraft at MEO is less than LEO, therefore it is easier to change the velocity direction at MEO. While the minimum impulsive velocity vector magnitude of SS-LEO raising strategy requires 37.77

m/s, minimum impulsive velocity vector magnitude of MEO raising orbit transfer is only 20.36m/s.

4.2.4 Effect of Initial Orbital Elements to MEO Raising Problem Solution

In this section the effect of initial orbital elements to MEO raising problem is detailed. Medium Earth orbits are located about 14000-15000 km altitudes (a=20000km). Main critical orbital elements of Medium Earth Orbit are semimajor axis, eccentricity, and inclination. In this part of the study for MEO raising problem different test cases with different initial orbital values are constructed and examined. Afterwards the effects of orbital elements are evaluated.

It is assumed that the cost function coefficients, tolerances and final desired orbits are the same as the test case-2 (section 4.2, final orbit a =20000km, e =0.00100, i =60.000°). Test case-2 initial conditions are assumed to be baseline (Table 37). At each test case all orbital elements are selected as the same as test case-2 excluding the element whose effect is desired to be obtained. During the evaluation of initial orbital element values to the result, initial conditions of test case-2 are called as MEO-2 test case.

4.2.4.1 Semimajor axis variation

The final semimajor axis value is needed to be 20000km. The test cases are constructed as satellite may be separated from the launch vehicle between 19900 and 19950km. In order to specify the effect of initial semimajor axis value to the result of MEO raising problem, the test cases are specified. The initial orbital values and results of these test cases are tabulated in Table 50 and given graphically in Figure 25.

Table 50 Effect of initial semimajor axis value to MEO raising problem

Test Case	Orbital Element			Total Impulsive Velocity Vector Magnitude (m/s)		
	a (km)	e	i (°)	Two Impulse Strategy	Three Impulse Strategy	Four Impulse Strategy
MEO-1	19970.00	0.00200	59.800	16.34	17.77	17.39
MEO-2(+)	19950.00	0.00200	59.800	17.11	18.22	18.96
MEO-3	19925.00	0.00200	59.800	20.16	19.54	20.54
MEO-4	19900.00	0.00200	59.800	20.37	21.28	22.87

(+) reference test case

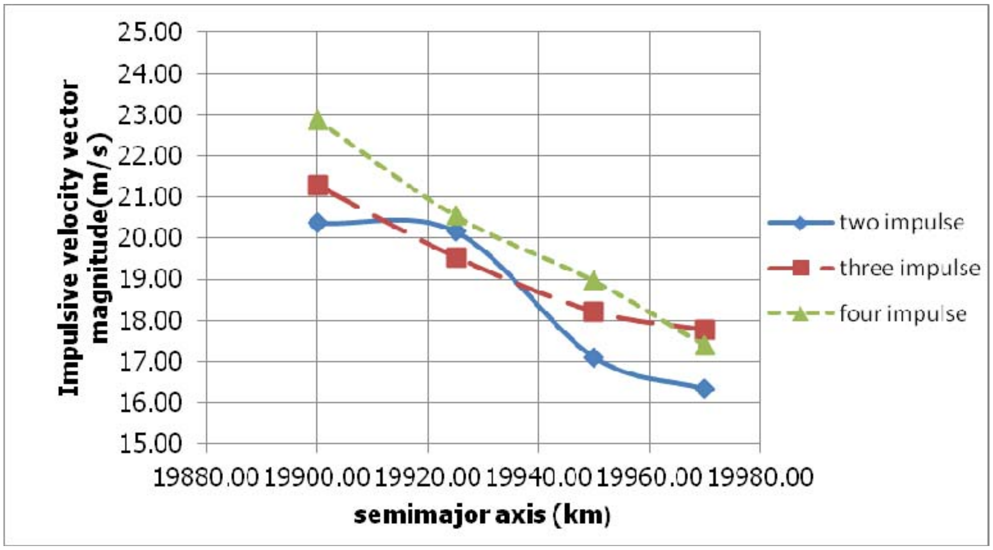

Figure 25 MEO raising results for different initial semimajor axis values

The test cases listed as close-far to final desired orbit from up to down. MEO-4 is the farthest orbit to the final orbit and MEO-1 is the closest orbit to the final desired orbit. When the initial orbit is selected close to the final desired orbit, then the required fuel decreases as expected.

At MEO-1, MEO-2 and MEO-4 cases the minimum energy requiring strategy is the two impulse orbit transfer strategy, while at the MEO-3 the minimum energy requiring strategy is three impulse strategy. The required impulsive velocity vector magnitudes are close at the two, three, and four impulse orbit transfer strategies.

At the test case shown in Figure 25 there is no single relationship observed between the number of impulse and required impulsive velocity vector magnitude. At MEO-1 and MEO-2 as the number of impulses increases the required impulsive velocity vector magnitude also increases. At MEO-2 and MEO-3 no relationship is observed between required impulse and number of impulses.

4.2.4.2 Inclination variation

Four different initial inclination values are selected to detail the initial inclination value effect to MEO raising problem and solutions for these cases are obtained. It is expected that in

MEO raising problem the effect of initial inclination value is important similar to LEO raising problem.

The test cases which are constructed to study the effect of initial inclination value are given in Table 51. Corresponding results are also given in this table (Table 51) and Figure 26.

Table 51 Effect of initial inclination value to MEO raising problem

Test Case	**Orbital Elements**			**Total Impulsive Velocity Vector Magnitude (m/s)**		
	a **(km)**	e	i **(°)**	**Two Impulse Strategy**	**Three Impulse Strategy**	**Four Impulse Strategy**
MEO-5	19950.00	0.00200	59.900	10.24	10.85	11.23
MEO-2(+)	19950.00	0.00200	59.800	17.11	18.22	18.96
MEO-6	19950.00	0.00200	59.700	25.33	27.53	29.91
MEO-7	19950.00	0.00200	59.600	32.11	34.46	36.68

(+) reference test case

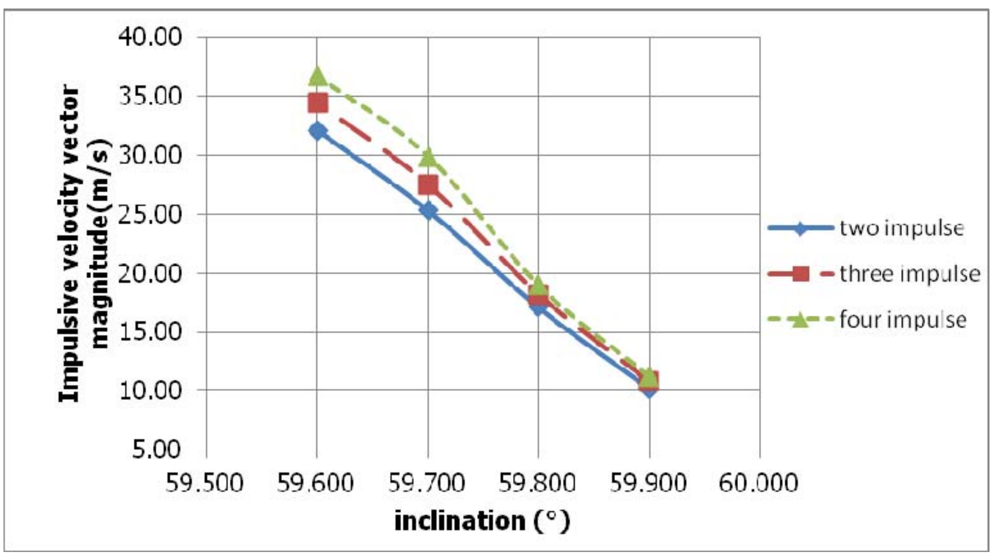

Figure 26 Effect of initial inclination value to MEO raising problem

As can be seen in, the initial inclination value is a critical orbital element since at MEO raising problem, 0.1° inclination change requires about 7m/s. Because of this inclination value is needed to be preferred as close as possible.

At the test cases which are constructed to the effect of initial inclination value there is a direct relationship between number of impulses and required impulsive velocity vector magnitude. As the number of impulse increases, the required fuel increases. The minimum impulsive velocity vector magnitude requiring strategy at these test cases is the two impulse orbit transfer strategy.

4.2.4.3 Eccentricity variation

Variation of initial eccentricity may also affect the solution of MEO raising problem. In this part of the study the effect of initial eccentricity value is studied. All orbital elements excluding eccentricity are kept constant and eccentricity is changed from 0.0015 to 0.0030. The test cases and corresponding results are tabulated in Table 52. The results are also given graphically in Figure 27.

Table 52 Effect of initial eccentricity value to MEO raising problem

	Orbital Element			Total Impulsive Velocity Vector Magnitude (m/s)		
Test Case	a **(km)**	e	i **(°)**	**Two Impulse Strategy**	**Three Impulse Strategy**	**Four Impulse Strategy**
MEO-8	19950.00	0.0015	59.800	16.73	18.00	18.48
MEO-2(+)	19950.00	0.0020	59.800	17.11	18.22	18.96
MEO-9	19950.00	0.0025	59.800	18.47	19.62	19.72
MEO-10	19950.00	0.0030	59.800	18.84	18.69	20.59

(+) reference test case

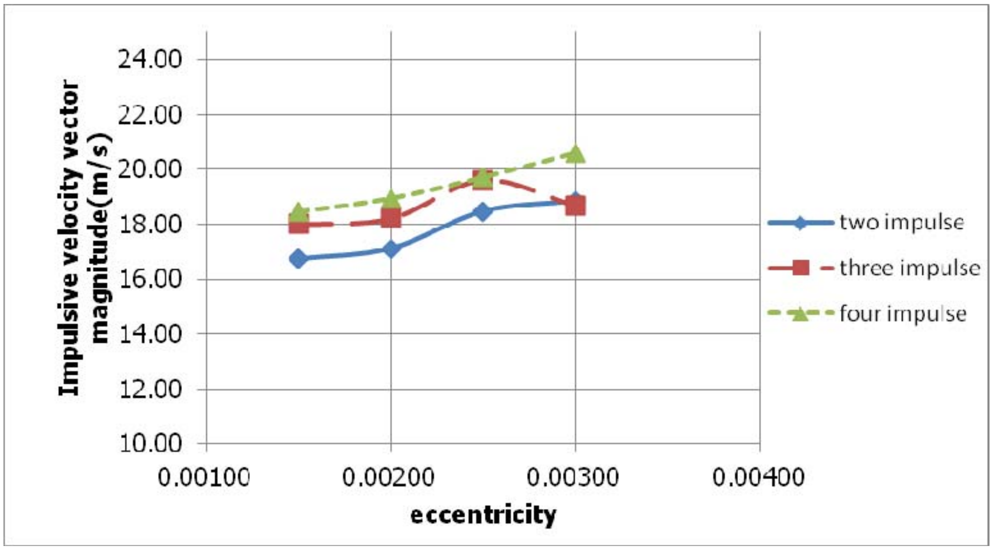

Figure 27 Effect of initial eccentricity to MEO raising problem

As expected the initial eccentricity value affect the solution. As the initial eccentricity is selected close to the desired orbit eccentricity, required amount of impulsive velocity decreases. It should also be noted that the required impulsive velocity vector magnitudes are close.

The least energy requiring strategy for all test cases constructed for eccentricity (MEO-1, MEO-8, MEO-9, and MEO-10) is the two impulse orbit transfer strategy. It is also observed that when the number of impulses increases, the required impulse increases. However it should also be noted that the required impulsive velocity values at different number of impulses are very close (less than 2m/s). Considering all these and reliability, two impulse strategy should be applied for cases similar to test cases given in Table 52.

4.3 TEST CASE-3 GEOSTATIONARY EARTH ORBIT (GEO) RAISING PROBLEM

A geostationary orbit, or Geostationary Earth Orbit (GEO), is a circular orbit at 35,786 km above the Earth's equator (zero inclination) and following the direction of the Earth's rotation. An object in such an orbit has an orbital period equal to the Earth's rotational period (one sidereal day), and thus appears motionless, at a fixed position in the sky, to ground observers. Communication and weather satellites are often inserted to geostationary orbits, so that the satellite antennas that communicate with them do not have to move to track them. Satellite antennas can be pointed permanently at the position in the orbit where they are positioned. A geostationary orbit is a particular type of geosynchronous orbit. 419 of 994 satellites (41.9%) are geostationary satellites.

In order to remain above the same point on the Earth' surface, a spacecraft must fulfill the following conditions (Maini, Anil K.; Argawal, Varsha, 2011):

- The orbit inclination should be zero.
- The orbit should be circular.
- The orbital period should be equal to 23 hours 56 minutes, which implies that the satellite must orbit at a height of 35786 km above the surface of the Earth.
- The satellite motion should be in the direction from west to east.

Most of the launch vehicles cannot insert the satellite to high altitude low inclination orbits directly; instead launch vehicle inserts spacecraft to a transfer orbit (Geostationary Transfer Orbit, GTO) from where spacecraft uses its own propulsion system to reach its mission orbit. This transfer orbit is a highly elliptical Earth orbit with apogee at about 35786 km, geostationary (GEO) altitude, and whose inclination is determined from launch site geographical parameters. In this test case a typical GEO raising problem (GTO to GEO transfer), is studied. Apart from case-1 and case-2, in this case the spacecraft is inserted to an orbit different from its mission orbit on purpose. This orbit change requires much more energy than the first and second cases. In Table 53, the GTO and GEO orbital elements are given. In this test case, Geostationary Transfer Orbit (GTO) is taken as initial orbit and Geostationary Earth Orbit (GEO) is the desired (final) orbit. The final orbit of Proton launch vehicle is defined as the initial orbit of a spacecraft (Proton Mission Planner's Guide, 2009). The semimajor axis, eccentricity, and inclination elements are needed to be modified. Since the inclination and eccentricity of final orbit (geostationary earth orbit) are zero (0), right ascension of ascending node and argument of perigee at the final orbit are undefined. Therefore the right ascension of ascending node and argument of perigee are not included in the cost function of GEO raising problem.

Table 53 Test case 3 initial and final orbital element values

Orbital Element	**Geostationary Transfer Orbit (GTO)**	**Geostationary Earth Orbit (GTO)**
a (km)	26331.10	42164.00
e	0.60130	0.00000
i (°)	23.200	0.000
Ω (°)	Free	N/A
ω (°)	Free	N/A
ν (°)	Free	N/A

The tolerance values are as follows:

semimajor axis tolerance is (Satellite Programmes Overview:Satellite Orbits, 2012):

$$\delta_a = \pm 5.00 \text{ km} \quad (4.12)$$

eccentricity tolerance is (Satellite Programmes Overview:Satellite Orbits, 2012):

$$\delta_e = \pm 0.00012 \quad (4.13)$$

The tolerance value for inclination is also assumed to be 0.10º (Delft University faculty aerospace engineering).

$$\delta_i = \pm 0.100° \quad (4.14)$$

Tolerance values and cost function coefficients for the orbit transfer problem from GTO to GEO are tabulated in Table 54.

Table 54 Tolerance values and cost function coefficients of test case 3

Orbital Element	Tolerance Values	Cost Function Coefficient
a (km)	±5.00	1.00
e	±0.00012	41666.67
i (°)	±0.10	50.00
Ω (°)	N/A	N/A
ω (°)	N/A	N/A
ν (°)	N/A	N/A

so the cost function is defined as follows:

$$\mathrm{J} = 25.00\sum_{i=1}^{n}\left|\Delta\vec{\mathbf{v}}_i\right| + 1.00a_{error} + 41666.67\,e_{error} + 50.00i_{error} \quad (4.15)$$

4.3.1 Implementation of Classical Orbit Transfer Methods for GEO Raising Problem

The orbit transfer from GTO to GEO cannot be realized by applying the combination of optimal methods since the Hohmann maneuver is optimal for the orbit transfers from circular orbit to circular orbit. As can be seen in Table 53 the eccentricity of initial orbit (GTO) is different from zero (about 0.73), therefore the Hohmann maneuver cannot provide an optimal solution to this problem. Because of this reason, the superposition of optimal methods for GTO to GEO orbit transfer problem is not applicable. In this part, the orbit transfer from GTO to GEO is realized applying classical orbit transfers which are not optimal. The classical orbit transfer for this case is the inclination change, altitude raising and circularization maneuvers. Firstly, inclination change is realized at the apogee point and then the semimajor axis and eccentricity values are altered. The required impulsive velocity vector magnitudes for each maneuver are given in Table 55.

Table 55 Classical GEO raising orbit transfer

	Impulsive Velocity Vector Magnitude (m/s)
Inclination correction	1529.62
Semimajor axis & eccentricity correction	1133.24
TOTAL	2662.86

4.3.2 Implementation of Genetic Algorithm Orbit Transfer Methods for GTO to GEO Transfer (Raising) Problem

In this section, the two, three and four impulse genetic algorithm orbit transfer methods are applied for GEO raising problem (from GTO to GEO).

4.3.2.1 <u>Two Impulse GEO Raising Strategy</u>

The simplest orbit transfer strategy for GTO to GEO transfer is the two impulse orbit transfer. The result of two impulse strategy is given in Table 56. As can be seen the error values are less than tolerances. The largest error is observed at the inclination which is 0.03 %. The impulse velocity requirements and timing of impulses are presented in Table 58. Total required impulsive velocity vector magnitude is 1530.70m/s.

Table 56 GEO raising results for two-impulse orbit transfer

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	42164.00	42165.00	1.00	0.01
e	0.00000	0.00002	0.00002	0.00
i (°)	0.000	0.007	0.007	0.03
Ω (°)	undefined	undefined	undefined	undefined
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	199.48	insignificant	insignificant

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 57. As seen, a small amount of change is realized at the semimajor axis and eccentricity values at the first impulse case.

Table 57 GEO spacecraft orbit raising intermediate orbit results

Orbital Element	**Initial Orbit**	**Intermediate Orbit**	**Final Orbit**
a (km)	26331.10	29428.84	42165.00
e	0.60130	0.43331	0.00002
i (°)	23.200	14.409	0.007
Ω (°)	Free	6.23	undefined
ω (°)	Free	358.24	undefined
ν (°)	Free	165.91	199.48

Table 58 Impulse details of two-impulse GEO raising orbit transfer strategy

	X axis (m/s)	**Y axis (m/s)**	**Z axis (m/s)**	**Total Impulsive Velocity Magnitude (m/s)**	**Impulse True Anomaly (°)**
Impulse-1	12.257	-477.214	198.634	517.048	170.07
Impulse-2	36.919	-833.350	575.893	1013.651	181.73
TOTAL				1530.70	

4.3.2.2 Three Impulse GEO Raising Strategy

The orbit transfer from GTO to GEO can also be performed applying three impulse orbit transfer strategy. The results of this orbit transfer strategy are given in Table 59. All the errors are below the corresponding tolerance values. The largest error is observed at the inclination which is 0.11%. Since the right ascension of ascending node and argument of perigee are not defined for circular zero inclination orbit (e=0, i=0°). The impulsive velocity vector components and timing of each impulse are tabulated in Table 61. Total magnitude of impulsive velocity vector is 1855.03m/s.

Table 59 GEO raising results using three-impulse orbit transfer method

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	42164.000	42165.75	1.75	0.01
e	0.00000	0.00001	0.00001	0.00
i (°)	0.000	0.025	0.025	0.11
Ω (°)	undefined	undefined	undefined	undefined
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	92.94	insignificant	insignificant

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 60.

Table 60 Three impulse GEO spacecraft orbit raising intermediate orbit results

Orbital Elements	**Initial Orbit**	**Intermediate Orbit-1**	**Intermediate Orbit-2**	**Final Orbit**
a (km)	26331.10	29353.17	37921.01	42165.75
e	0.60130	0.42909	0.12108	0.00001
i (°)	23.200	12.703	3.654	0.025
Ω (°)	Free	12.79	132.88	undefined
ω (°)	Free	352.72	199.54	undefined
ν (°)	Free	160.48	151.68	92.94

Table 61 Impulse details of three-impulse GEO raising orbit transfer strategy

	X axis (m/s)	**Y axis (m/s)**	**Z axis (m/s)**	**Total Impulsive Velocity Magnitude (m/s)**	**Impulse True Anomaly (°)**
Impulse-1	59.564	-530.046	284.164	604.355	165.42
Impulse-2	380.611	-629.360	610.210	955.675	174.88
Impulse-3	-46.230	-225.002	-185.094	294.996	160.12
TOTAL				1855.03	

4.3.2.3 Four Impulse GEO Raising Strategy

Four impulse strategy is also another alternative orbit transfer method for GTO to GEO transfer. Four impulse strategy results are given in Table 62. As can be seen in Table 62, the error is observed only at the inclination. The error is 0.02%. The remaining orbital elements are at the desired values. It should be noted that all the final orbital elements are retained in the tolerances. It can be said that the four impulse orbit transfer method is also applicable to GEO orbit raising (GTO-GEO).

Table 62 GEO raising results using three-impulse orbit transfer method

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	42164.000	42164.00	0.00	0.00
e	0.00000	0.00000	0.00000	0.00
i (°)	0.000	0.005	0.005	0.02
Ω (°)	undefined	undefined	undefined	undefined
ω (°)	undefined	undefined	undefined	undefined
ν (°)	insignificant	131.74	insignificant	insignificant

The impulsive velocity vector components and true anomalies of four (4) impulse GTO-GEO orbit transfer strategy are given in Table 64. The magnitudes of impulsive velocities are close and total required impulsive velocity magnitude is 2134.88m/s.

The intermediate orbit values of GEO raising problem are presented in Table 63.

Table 63 Four impulse GEO spacecraft orbit raising intermediate orbit results

Orbital Element	Initial Orbit	Intermediate Orbit-1	Intermediate Orbit-2	Intermediate Orbit-3	Final Orbit
a (km)	26331.10	31958.83168	36478.79594	33886.87212	42164.00
e	0.60130	0.46166124	0.2700989	0.24493916	0.00000
i (°)	23.200	15.71088658	9.27987072	3.89319074	0.005
Ω (°)	Free	348.3508392	17.55141678	329.9165093	undefined
ω (°)	Free	350.0941747	330.0854887	3.3535705	undefined
ν (°)	Free	137.1026156	150.4880462	67.4837099	131.74

Table 64 Impulse details of four-impulse GEO raising orbit transfer strategy

	X axis (m/s)	Y axis (m/s)	Z axis (m/s)	Total Impulsive Velocity Magnitude(m/s)	Impulse True Anomaly(°)
Impulse-1	313.219	-483.731	211.434	613.846	201.98
Impulse-2	-94.805	-471.544	283.304	558.214	159.06
Impulse-3	115.743	1.462	-500.000	513.224	53.35
Impulse-4	-161.175	-378.518	-181.330	449.593	176.70
TOTAL				2134.88	

4.3.3 Comparison of Transfer Alternatives for GEO Raising Problem

The orbit raising from GTO to GEO is one of the most important orbit transfer since the impulsive velocity requirement is high. While at LEO and MEO raising the required impulsive velocity vector magnitude is about 50-60 m/s; at GEO the impulsive velocity requirement is about 2000 m/s.

As can be seen above two, three and four impulse methods can be applied to realize orbit transfer from GTO to GEO. The total impulsive velocity requirements of these strategies are tabulated in Table 65. The most efficient method is two-impulse method which requires only 1588.88 m/s impulsive velocity vector magnitude. The worst method is classical method which requires 2662.89 m/s. The difference between the minimum and maximum impulsive velocity requiring strategies is about 1074 m/s.

The impulsive velocity requirement during orbit maintenance at GEO is about 50-60 m/s (Chao, 2005). If a spacecraft is raised to GEO by applying two impulse strategy instead of applying classical orbit transfer method the lifetime of spacecraft is prolonged about 12 years.

Table 65 Impulsive Velocity Requirement Comparison of GEO raising strategies

GEO raising strategy	Impulsive Velocity Vector Magnitude (m/s)
Classical Orbit Transfer Method	2662.86
Two-impulse Orbit Transfer Method	1588.88
Three-impulse Orbit Transfer Method	1820.61
Four-impulse Orbit Transfer Method	1880.68

4.3.4 Effect of Initial Orbital Elements to GEO Raising Problem Solution

Geostationary earth orbits (GEO) was one of the earliest orbit concept. The spacecrafts at this orbit have the same speed with Earth; therefore it remains at a fix position with respect to Earth. Geostationary Earth orbit (GEO) satellites are not directly inserted to their mission orbits. Launch vehicles carry these satellites up to a transfer orbit. This orbit is called as Geostationary transfer orbit (GTO). The apogee point of GTO is at 42164km (same as GEO), however apogee point is a close point to the Earth. Therefore semimajor axis and eccentricity values of initial orbit (GTO) are directly related.

In this section the effect of initial orbital element values to the orbit raising problem solution is studied. It is assumed that the final orbit and the cost function coefficients are the same as in test case-3 (section 4.3, final orbit a =42164km, e =0.00000, i =0.000°). Test case-3 (Table 53) is defined as baseline and other test cases are constructed depending on this test case. In this section test case-3 is named as GEO-3.

4.3.4.1 <u>Semimajor axis and eccentricity variation</u>

At the GTO semimajor axis and eccentricity values are dependent. Therefore the effect of initial values of semimajor axis and eccentricity to the GEO raising problem is studied together. The test cases which consist of different initial semimajor axis and eccentricity values are given in Table 66. The corresponding results are also given in Table 66 and Figure 28. The test cases are listed beginning from closest to farthest.

Table 66 Effect of initial semimajor axis and eccentricity values to GEO raising (GTO-GEO) problem

Test Case	Orbital Elements			Total Impulsive Velocity Vector Magnitude (m/s)		
	a (km)	e	i (°)	Two-Impulse Strategy	Three-Impulse Strategy	Four-Impulse Strategy
GEO-1	30177.14	0.40000	23.200	1371.01	1587.81	1918.92
GEO-2	28109.33	0.50000	23.200	1434.82	1673.10	2044.64
GEO-3(+)	26331.10	0.60130	23.200	1503.72	1855.04	2134.98
GEO-4	24802.35	0.70000	23.200	1747.16	2072.33	2776.74

(+) reference test case

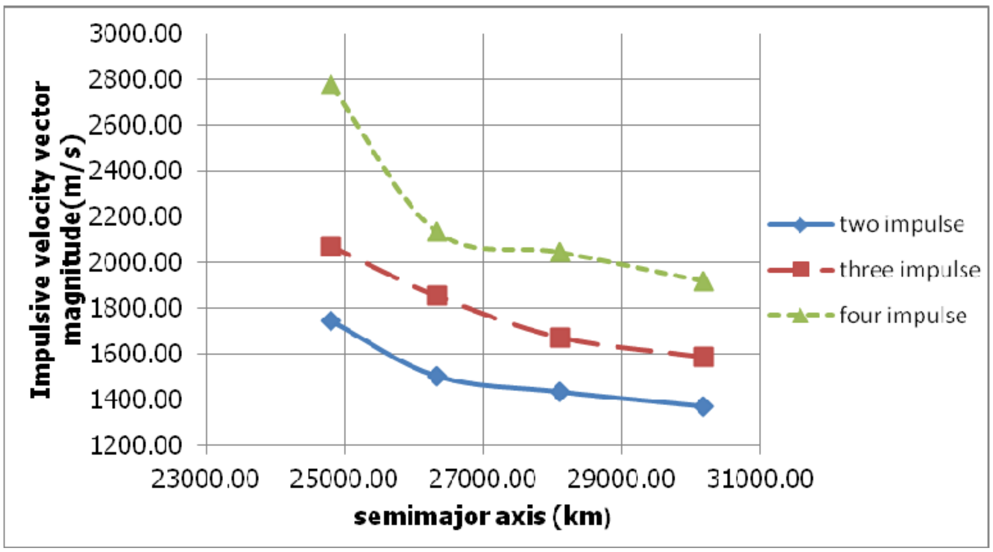

Figure 28 GEO raising problem results for different initial semimajor axis values

As expected the required impulsive velocity magnitudes change depending on initial semimajor axis and eccentricity values. While for GEO-2 the required impulsive velocity magnitude is about 1434m/s, for GEO-4 the required impulsive velocity magnitude is 1747m/s.

It is observed that the minimum energy requiring strategy for the test cases given in Table 66 are two impulse orbit transfer strategy. As the number of impulses increase, the required impulsive velocity magnitudes also increase in a growing trend (Figure 28). In other words the difference between four and three impulse strategies is much more than difference between three and two impulse strategies. Because of these reasons, four impulse strategy is not an applicable strategy from the required energy view.

4.3.4.2 Inclination variation

Initial inclination value is very critical since inclination is the most difficult orbital element to change. In Table 67 the test cases which are constructed to study the effect of initial inclination value at the GEO raising problem and the corresponding results are given. In Figure 29, the results are presented graphically.

Table 67 Effect of initial inclination value to GEO raising (GTO-GEO) problem

Test Case	Orbital Elements			Total Impulsive Velocity Vector Magnitude (m/s)		
	a **(km)**	***e***	***i*** **(°)**	**Two-Impulse Strategy**	**Three-Impulse Strategy**	**Four-Impulse Strategy**
GEO-5	26331.10	0.60130	20.000	1421.43	1516.67	2037.31
GEO-3(+)	26331.10	0.60130	23.200	1503.72	1855.04	2134.98
GEO-6	26331.10	0.60130	27.000	1835.71	1973.55	2631.92
GEO-7	26331.10	0.60130	30.000	1932.04	2243.62	2803.53

(+) reference test case

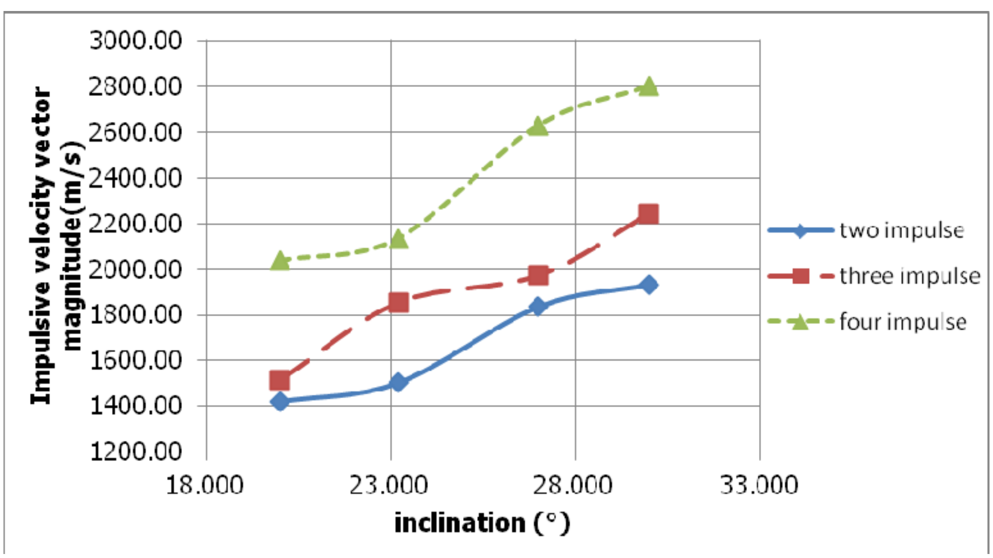

Figure 29 GEO raising problem results for different initial inclination values

As expected the initial inclination value affect the required impulsive velocity magnitude directly. While the required impulsive velocity magnitude of GEO-5 (i =20°) is 1421m/s, at GEO-7 (i =30°) this value is 1932m/s. It can be said that most of the propellant is consumed during the inclination change.

The least energy requiring strategy for the test cases given in Table 67 is the two impulse orbit transfer strategy. As the number of impulse increases, the required impulsive velocity vector magnitudes increase. Similar to the test cases in Table 66 (effect of initial semimajor axis and eccentricity values to GEO raising) the increment is growing type. It can be stated that at GEO raising problem, four impulse strategy is not a good orbit transfer alternative.

4.4 TEST CASE 4 MOLNIYA ORBIT RAISING PROBLEM

Geostationary communication spacecrafts are not proper for northern countries such as Russia since equatorial orbits do not offer good coverage of the poles or regions at very high latitudes. Instead, northern countries use spacecrafts in highly inclined orbits that can be easily seen from northern latitudes during the large part of their period. These orbits are highly elliptical and have their apogee near the North Pole. The spacecraft has a high speed as it moves near perigee and slowly near apogee, thus spending most of its time in the orbit over the northern hemisphere. This special orbit is known as Molniya orbit (Wright, Grego, & Gronlund, 2005).

Molniya orbits, a kind of Highly Eccentric Orbit (HEO), are highly elliptical, with a period of 12 hours and an inclination of 63.4° (critically inclined). At this inclination, the apogee remains over the same latitude in the northern (or southern) hemisphere, rather than precessing. Today 13 of 994 (1.3%) satellites are Molniya type satellites. A typical Molniya orbit spacecraft initial (launch vehicle separation orbit) and destination (final) orbit values are listed in Table 68 (Fortescue, Swinerd, & Stark, 2004).

Table 68 Initial and final orbit values of a typical Molniya Orbit spacecraft

Orbital Element	**Initial Orbit** (Soyuz from the Guiana Space Centre User's Manual, 2006)	**Final Orbit**
a (km)	26520.00	26554.00
e	0.72000	0.70000
i (°)	63.340	63.440
Ω (°)	free	free
ω (°)	270.00	270.00
ν (°)	free	N/A

It should be noted that classical orbit transfer method is not applicable for Molniya orbit transfer since the eccentricity value of initial and final orbits are different from zero (0) and there exist no optimal orbit transfer solution for argument of perigee correction. Although argument of perigee is not needed to be corrected, during the correction of semimajor axis, eccentricity, and inclination, the argument of perigee is also changed. At first and second case (SS-LEO, MEO) the final orbit is circular, therefore the argument of perigee is not critical; however, Molniya orbit is not circular; therefore, argument of perigee is also important. The semimajor axis, eccentricity, right ascension of ascending node and argument of perigee are needed to be corrected/kept accurately. Among these parameters, argument of perigee is one of the most critical elements since the location of perigee is critical for Molniya orbit missions. Because of this, a large value is initialized to the argument of perigee coefficient of the cost function.

The tolerance values for Molniya orbit transfers are as follows:

semimajor axis tolerance is defined in (Konstantinov, Popov, Obukhov, & Petukhov, 2005):

$$\delta_a = \pm 16.67 \text{ km} \tag{4.16}$$

eccentricity tolerance is defined in (Konstantinov, Popov, Obukhov, & Petukhov, 2005):

$$\delta_e = \pm 0.00019 \tag{4.17}$$

For Molniya orbit mission inclination is also an important orbital element. It needs to be at 63.44º (critically inclined). The tolerance value for inclination is assumed to be 0.05º (Konstantinov, Popov, Obukhov, & Petukhov, 2005):

$$\delta_i = \pm 0.050° \tag{4.18}$$

Argument of perigee is also another important orbital element for Molniya orbit mission. The tolerance value of argument of perigee is assumed to be 0.050° in (Konstantinov, Popov, Obukhov, & Petukhov, 2005)

$$\delta_\omega = \pm 0.05° \tag{4.19}$$

Tolerance values and cost function coefficients for Molniya orbit raising problem are tabulated in Table 69.

Table 69 Tolerance values and cost function coefficients of test case 4

Orbital Elements	**Tolerance Values**	**Cost Function Coefficient**
a (km)	±16.667	1.00
e	±0.00019	87737.84
i (°)	±0.050	333.40
Ω (°)	insignificant	insignificant
ω (°)	±0.05	333.40
ν (°)	insignificant	insignificant

So the cost function is defined as follows:

$$J = 25.00\sum_{i=1}^{n}\left|\Delta\vec{\mathbf{v}}_i\right| + 1.00a_{error} + 87737.84\,e_{error} + 333.34i_{error} + 333.34\Omega_{error} \quad (4.20)$$

4.4.1 Implementation of Classical Orbit Transfer Methods for Molniya Orbit Raising Problem

In the literature there is not an optimal orbit transfer strategy between highly elliptic orbits. Since argument of perigee value is also needed to be kept constant precisely, it is also not possible to realize Molniya orbit raising transfer using classical orbit transfer methods. Therefore in this section a classical orbit transfer strategy cannot be suggested to perform Molniya orbit raising.

4.4.2 Implementation of Genetic Algorithm Orbit Transfer Methods for Molniya Orbit Raising Problem

In this section Molniya orbit raising strategies are performed using two, three and four impulse genetic algorithm based orbit transfer methods. Since there exists no classical orbit transfer strategy for Molniya orbit raising, the results of genetic algorithm strategy results are compared to each other only.

4.4.2.1 Two Impulse Molniya Orbit Raising Strategy

Two impulse genetic algorithm orbit transfer strategy can be used to realize Molniya orbit raising. During this transfer semimajor axis, eccentricity and inclination values are needed to be corrected and argument of perigee is needed to be kept constant strictly.

Two impulse method solution of Molniya orbit raising problem is presented in Table 70. Two impulse strategy suggests very accurate results for the problem. There is no error observed, all orbital elements are at the desired values. The impulsive velocity vector components and impulse true anomalies are tabulated in Table 71. The magnitudes of required impulsive velocity vector magnitudes are close which are approximately 36m/s.

Table 70 Molniya orbit raising result of two-impulse orbit transfer

Orbital Element	Desired Final Orbit	Calculated Final Orbit	Absolute Error	Error (%)
a (km)	26554.00	26554.00	0.00	0.00
e	0.70000	0.70000	0.00000	0.00
i (°)	63.440	63.440	0.000	0.00
Ω (°)	insignificant	359.94	insignificant	insignificant
ω (°)	270.00	270.00	0.00	0.00
ν (°)	insignificant	181.25	insignificant	insignificant

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 71. As seen, at the first impulse semimajor axis is not corrected instead it is changed in the wrong direction. Inclination is also almost changed during the first impulse application.

Table 71 Molniya spacecraft orbit raising intermediate orbit results

Orbital Element	Initial Orbit	Intermediate Orbit	Final Orbit
a (km)	26520.00	26333.07	26554.00
e	0.72000	0.71418	0.70000
i (°)	63.340	63.437	63.440
Ω (°)	free	0.12	359.94
ω (°)	270.00	270.01	270.00
ν (°)	free	136.90	181.25

Table 72 Impulse details of two-impulse Molniya Orbit raising orbit transfer strategy

	X axis (m/s)	Y axis (m/s)	Z axis (m/s)	Total Impulsive Velocity Magnitude (m/s)	Impulse True Anomaly (°)
Impulse-1	-32.012	-12.457	-10.170	35.824	136.97
Impulse-2	-38.937	2.404	-5.120	39.345	181.16
TOTAL				75.17	

4.4.2.2 Three Impulse Molniya Orbit Raising Strategy

Three impulse orbit transfer can also be applied to realize Molniya orbit raising. The final orbit obtained by three impulse method is given in Table 73. The desired and calculated final orbits are close to each other. All the errors are within the tolerance limits; therefore, final orbit is sufficient for Molniya mission. The semimajor axis error is 20 meters, and inclination error is about 0.01°. The final orbit argument of perigee and eccentricity values are exactly at the desired values. It can be said that this orbit transfer strategy is applicable for Molniya orbit raising task. The impulse detail of this orbit transfer strategy is presented in Table 74 from where it can be seen that the total required impulsive velocity vector magnitude is about 83.63m/s.

Table 73 Molniya orbit raising results for three-impulse orbit transfer

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	26554.00	26553.98	0.02	0.06
e	0.70000	0.70000	0.00000	0.00
i (°)	63.440	63.443	0.003	3.00
Ω (°)	insignificant	359.45	insignificant	insignificant
ω (°)	270.00	270.00	0.00	0.00
ν (°)	insignificant	168.63	insignificant	insignificant

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 74. Note that as the eccentricity error is decreased during first and second impulse, other orbital element errors are increased. The semimajor axis and inclination errors are corrected at the last impulse.

Table 74 Three impulse Molniya spacecraft orbit raising intermediate orbit results

Orbital Element	**Initial Orbit**	**Intermediate Orbit-1**	**Intermediate Orbit-2**	**Final Orbit**
a (km)	26520.00	26690.67	26345.22	26553.98
e	0.72000	0.71138	0.70753	0.70000
i (°)	63.340	63.326	63.349	63.443
Ω (°)	free	359.86	359.96	359.45
ω (°)	270.00	270.48	270.33	270.00
ν (°)	free	172.86	15.35	168.63

Table 75 Impulse details of three-impulse Molniya orbit raising orbit transfer strategy

	X axis (m/s)	**Y axis (m/s)**	**Z axis (m/s)**	**Total Impulsive Velocity Magnitude (m/s)**	**Impulse True Anomaly(°)**
Impulse-1	-23.481	11.194	14.524	29.792	173.27
Impulse-2	-11.891	9.465	-14.219	20.813	344.55
Impulse-3	-19.702	2.141	-26.415	33.023	190.81
TOTAL				83.63	

4.4.2.3 Four Impulse Molniya Orbit Raising Strategy

Molniya orbit raising can also be performed by applying four impulse strategy. However it should be considered that as the number of impulses increases, the orbit transfer complexity also increases. The genetic algorithm result of four impulse case is tabulated in Table 76. The errors are less than tolerances. The impulse components for each impulse and the timing of maneuvers (true anomaly) are given in Table 76. Four impulse method requires 77.88m/s impulsive velocity magnitude to perform Molniya orbit raising.

Table 76 Molniya Orbit spacecraft orbit raising result for four-impulse orbit transfer

Orbital Element	**Desired Final Orbit**	**Calculated Final Orbit**	**Absolute Error**	**Error (%)**
a (km)	26554.00	26554.34	0.34	1.00
e	0.70000	0.70000	0.00000	0.00
i (°)	63.440	63.438	0.002	2.00
Ω (°)	insignificant	359.06	insignificant	insignificant
ω (°)	270.00	270.00	0.00	0.00
ν (°)	insignificant	28.74	insignificant	insignificant

The initial, intermediate and final orbits for this orbit transfer are tabulated in Table 78.

Table 77 Four impulse Molniya spacecraft orbit raising intermediate orbit results

Orbital Element	**Initial Orbit**	**Intermediate Orbit-1**	**Intermediate Orbit-2**	**Intermediate Orbit-3**	**Final Orbit**
a (km)	26520.00	26649.51	26736.68	26833.53	26554.34
e	0.72000	0.71441	0.70885	0.70327	0.70000
i (°)	63.340	63.381	63.377	63.398	63.438
Ω (°)	free	359.79	359.74	359.21	359.06
ω (°)	270.00	269.74	269.83	269.86	270.00
ν (°)	free	167.37	175.78	177.13	28.74

Table 78 Impulse details of three-impulse Molniya orbit raising orbit transfer strategy

	X axis (m/s)	**Y axis (m/s)**	**Z axis (m/s)**	**Total Impulsive Velocity Magnitude(m/s)**	**Impulse True Anomaly (°)**
Impulse-1	-15.680	-0.224	-12.877	20.291	192.28
Impulse-2	-15.409	3.423	-0.205	15.786	175.84
Impulse-3	-14.966	6.788	-16.000	22.936	182.66
Impulse-4	-11.142	-9.773	11.672	18.865	28.80
TOTAL				77.88	

4.4.3 Comparison of Transfer Alternatives for Molniya Orbit Raising Problem

Molniya orbit raising can be performed by applying two, three, and four impulse strategies. Since at Molniya orbit raising case the superposition of classical orbit transfer methods do not offer any solution, the genetic algorithm orbit transfer methods can only be compared with respect to each other.

The obtained results and impulse details for each strategy are given above. Since the errors are less than tolerance values for all strategies, these orbit strategies are only compared in terms of required impulsive velocity vector magnitudes. The required impulsive velocity vector magnitude of Molniya orbit raising task is given in Table 79. The minimum impulsive velocity requiring strategy for Molniya orbit raising is the two-impulse strategy (required impulsive velocity vector magnitude: 75.17m/s) and the worst strategy is the two-impulse strategy (83.63m/s).

Table 79 Impulsive Velocity Requirement Comparison of Molniya orbit raising strategies

Molniya Orbit Raising Strategy	**Total Impulsive Velocity Vector Magnitude (m/s)**
Two-impulse Orbit Transfer Method	75.17
Three-impulse Orbit Transfer Method	83.63
Four-impulse Orbit Transfer Method	77.88

There is no direct relationship observed between number of impulses and required impulsive velocity magnitudes. Because of this at the Molniya raising problem it can be stated that the minimum energy requiring solution is initial orbit dependent (not predictable).

4.4.4 Effect of Initial Orbital Elements to Molniya Raising Problem Solution

In this section the effect of initial orbital element values to Molniya orbit raising solution is studied. Semimajor axis, eccentricity, inclination and argument of perigee are critical for Molniya orbit missions. Argument of perigee can be obtained accurately at the initial orbit of a satellite. However the initial values of other orbital elements may be different than desired values. These orbital elements are needed to be corrected and argument of perigee is needed to be kept constant.

The test case-4 (Table 68) is assumed to be baseline and the solutions for different initial orbital elements (for semimajor axis, eccentricity, inclination) are obtained. Test case-4 is named as Molniya-2 test case. The cost function coefficients, tolerances and final orbits are assumed to be the same as test case-4 (section 4.4, final orbit a=26554.00km, e=0.70000, i=63.44°, ω=270.00°).

4.4.4.1 Semimajor axis variation

Semimajor axis is an important orbital element for Molniya orbit spacecrafts. In Table 80, different initial semimajor axis test cases and their corresponding results are given. In Figure 30 the results are also shown graphically. Two, three, and four impulse strategies are applied to obtain solutions to these problems.

Table 80 Effect of initial semimajor axis value to Molniya raising problem

Test Case	Orbital Elements			Total Impulsive Velocity Vector Magnitude (m/s)		
	a (km)	e	i (°)	Two-Impulse Strategy	Three-Impulse Strategy	Four-Impulse Strategy
Molniya-1	26540.00	0.72000	63.340	73.30	82.22	73.25
Molniya-2($^{+}$)	26520.00	0.72000	63.340	75.17	83.63	79.66
Molniya-3	26510.00	0.72000	63.340	76.67	86.42	88.63
Molniya-4	26500.00	0.72000	63.340	77.29	87.00	94.13

($^{+}$) reference test case

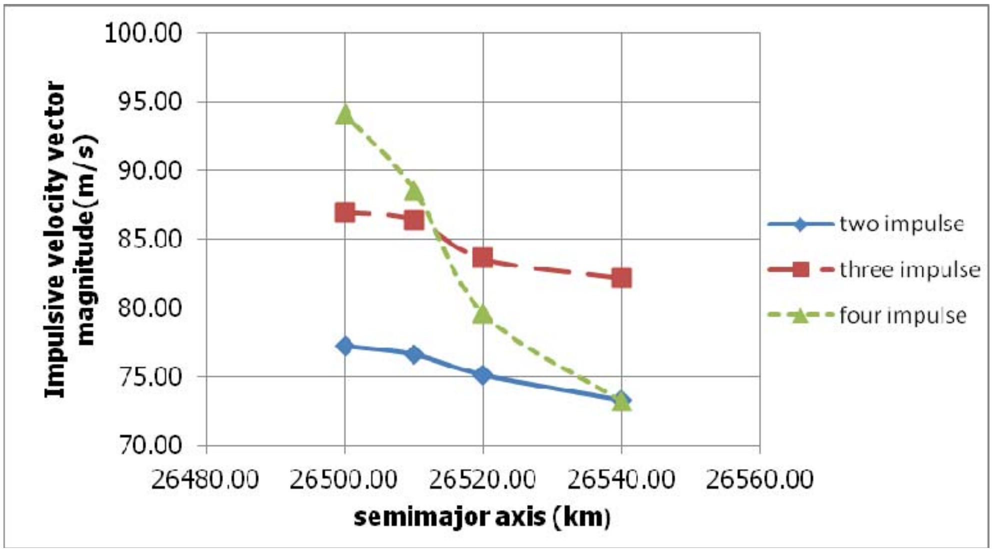

Figure 30 Molniya raising problem results for different initial semimajor axis values

As can be seen in Table 80, effect of initial semimajor axis is not critical at the Molniya orbit raising problem since the required impulsive velocity magnitudes are close. Note that the trend of orbit transfer strategies is close to linear. The impulsive velocity vector magnitudes are not changing dramatically at the different initial semimajor axis values.

Two impulse orbit transfer strategy requires the least impulsive velocity vector magnitude at the Molniya-2, 3, 4 cases. At Molniya-1 case the best orbit transfer strategy from the impulse requirement view is the four impulse orbit transfer strategy. Since the two and four impulse strategy values of Molniya-1 are close, the two impulse orbit transfer strategy can be applied instead of a four impulse orbit transfer strategy.

There is no direct relationship observed between number of impulses and required impulsive velocity vector magnitudes. While at Molniya-1 and 2 cases three impulse strategy requires more propellant than four impulse strategy, at Molniya-3 and 4 four impulse strategy requires less propellant than three impulse strategy.

4.4.4.2 Inclination variation

Molniya orbit is a critically inclined orbit which means that inclination value should be about 63.4°. Different inclination values are specified and orbit transfer solutions for these orbit raising problems are obtained. The test cases and their results are tabulated in Table 81. The graphical representations of results are given in Figure 31.

Table 81 Effect of initial inclination value to Molniya raising problem

Test Case	Orbital Elements			Total Impulsive Velocity Vector Magnitude (m/s)		
	a (km)	e	i (°)	Two-Impulse Strategy	Three-Impulse Strategy	Four-Impulse Strategy
Molniya-5	26520.00	0.72000	63.390	74.13	79.68	76.58
Molniya-2(+)	26520.00	0.72000	63.340	75.17	83.63	78.90
Molniya-6	26520.00	0.72000	63.240	79.61	87.54	82.45
Molniya-7	26520.00	0.72000	63.140	80.23	89.56	90.27
Molniya-8	26520.00	0.72000	63.040	84.48	91.52	96.24

(+) reference test case

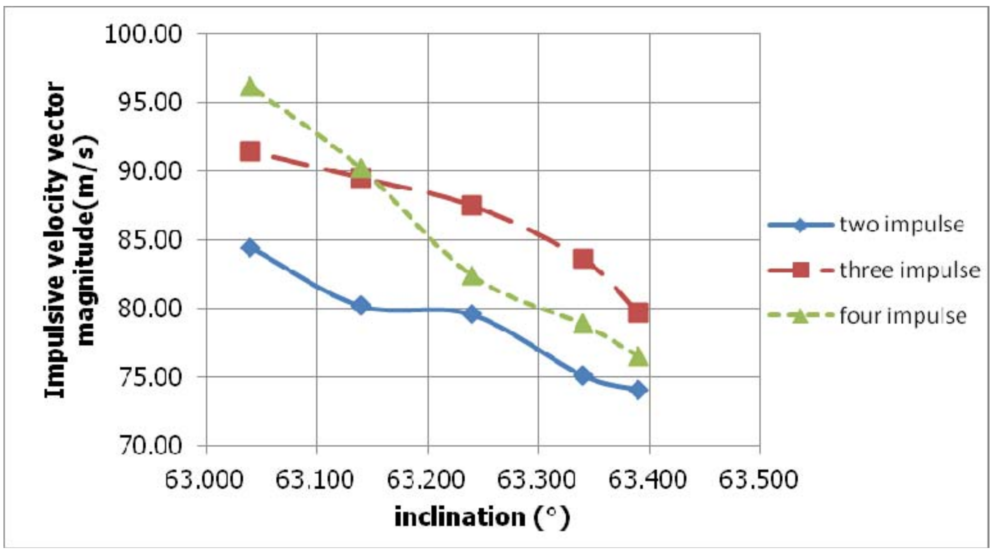

Figure 31 Molniya raising problem results for different initial inclination values

Initial inclination values affect the required impulsive velocity magnitude as expected. As the difference between the initial and final orbit inclination values increase, the required impulsive velocity magnitudes also increase. This increment trend is growing type. While at the Molniya-5 the required impulsive velocity magnitude is 74.13m/s, at the Molniya-8 test case this value is 84.48m/s.

Similar to semimajor axis case there is no direct relationship is observed between number of impulses and required energy. Four impulse strategy requires the maximum energy at the Molniya-2, 5 and 6 cases. On the other hand three impulse strategy requires the maximum energy at the Molniya-7 and 8 cases. It should be noted that the minimum energy requiring orbit transfer strategy is two impulse orbit transfer strategy between at all cases given in Table 81.

4.4.4.3 Eccentricity variation

Main difference of Molniya with respect to other popular orbits (LEO, MEO, and GEO) is that it is a highly elliptic orbit. Final eccentricity value is 0.70000. In this section the effect of initial eccentricity value to Molniya raising solution is studied. In Table 82 these test cases are prepared for the investigation of effects of initial eccentricity values to Molniya raising solution and their results are presented. The results are also given in Figure 32 graphically.

Table 82 Effect of initial eccentricity value to Molniya raising problem

Test Case	Orbital Elements			Total Impulsive Velocity Vector Magnitude (m/s)		
	a (km)	e	i (°)	Two-Impulse Strategy	Three-Impulse Strategy	Four-Impulse Strategy
Molniya-9	26520.00	0.68000	63.340	81.9	83.99	86.83
Molniya-10	26520.00	0.70000	63.340	47.99	52.40	49.65
Molniya-2(+)	26520.00	0.72000	63.340	75.17	84.20	78.90
Molniya-11	26520.00	0.74000	63.340	115.24	188.68	180.64
Molniya-12	26520.00	0.76000	63.340	256.42	333.32	270.36

(+) reference test case

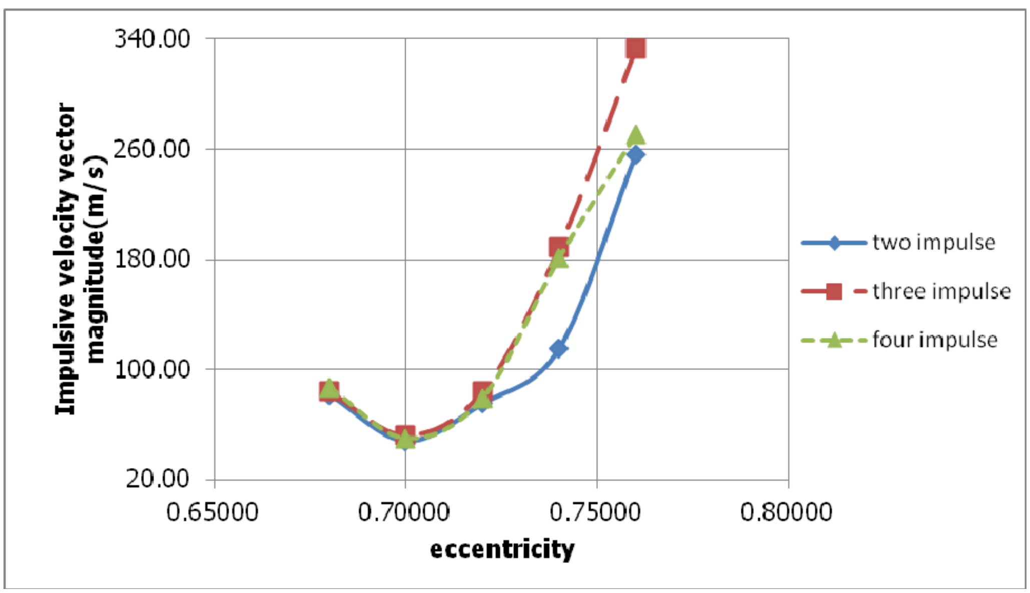

Figure 32 Molniya raising problem results for different initial eccentricity values

It is seen that the eccentricity value is the most critical orbital element at the Molniya raising problem. The initial eccentricity value directly affects the solution. Since the desired eccentricity value is 0.70000, the least impulse is required at Molniya-10 test case. Molniya-2 and 9 are approximately at the same distance to final orbit, therefore their required impulsive velocity magnitudes are close. As the distance to final orbit increases, the required impulsive velocity magnitude increases rapidly. The minimum required impulsive velocity magnitude at Molniya-1 (e =0.72000) is 75.17m/s, at Molniya-12 (e =0.76000) this value is 256.42m/s. Because of this reason initial eccentricity is very critical at Molniya orbit raising problems. Eccentricity should be as close as possible to final orbit for Molniya raising problem.

At test cases Molniya-2, 9, 10, 11 and 12 the best orbit transfer strategy is the two impulse orbit transfer strategy from the required total impulsive velocity vector magnitude view. Similar to semimajor axis and inclination test cases for Molniya, there is no direct relationship between number of impulses and required impulsive velocity magnitude. While for some cases three impulse strategy suggests less impulse at other cases four impulse strategy requires less impulse.

4.5 TEST CASE CONCLUSIONS

At all cases the required impulsive velocity vector magnitudes of genetic algorithm orbit transfer strategies are less than classical orbit transfer strategies. This means that applying genetic algorithm orbit transfer strategies prolong the service time of the spacecraft.

The required impulsive velocity vector magnitude difference between classical orbit transfers and genetic algorithm based orbit transfer methods for SS-LEO and MEO is about 39% and 36% respectively. The same difference at GEO raising case is 40%. It can be stated that application of genetic algorithm based orbit transfer method saves propellant and provides long service time. It should be noted that since there is no classical orbit transfer method to realize Molniya raising, it is not possible to compare the classical and genetic algorithm based orbit transfer methods for this case.

The effects of initial orbital element values to the results are also considered. For this purpose different initial conditions are set and results are obtained for these cases. This study is performed for LEO, MEO, GEO, and Molniya. Common and different conclusions have been obtained. Inclination is the most important and hard to change orbital element. At the LEO raising problem eccentricity is not critical. Similar results are also obtained for MEO raising problem. While inclination and semimajor axis are important, eccentricity is not as critical as inclination and semimajor axis. At the GEO raising problem, it is seen that all orbital elements are critical and change the result dramatically. Molniya raising problem results are different than other cases. Consequently, for the Molniya raising problem, it can be stated that inclination and eccentricity are critical and semimajor axis is not so critical.

The relationship between number of impulses and the required impulsive velocity vector magnitudes have also been examined. For this purpose two, three and four impulse strategies have been applied to all cases. It has been seen that different results are gained for a specific case depending on impulse number. Previous studies also show that while for some cases increasing impulse number is beneficial, for other cases decreasing the number of impulses are beneficial (Nakhjiri, 2011).

At LEO raising case generally two impulse strategy suggest the least amount of impulsive velocity vector magnitude. However, at some test cases of LEO raising problem different solutions are also obtained. At the cases constructed for MEO raising problem two impulse strategy suggest the best results (minimum energy). As the number of impulse increases, the required impulsive velocity magnitude generally increases. At some test cases four impulse strategy offers results that requires less amount of propellant. At the GEO raising problem, the least energy requiring strategy is the two impulse orbit transfer strategy. In this case, as the number of impulse increases, the required impulsive velocity increases dramatically. At GEO raising, four impulse orbit transfer strategy is not a good orbit transfer alternative. At the Molniya raising problem generally two impulse orbit transfer strategy offers the least energy requiring result. However sometimes three and four impulse strategies suggest better results.

To conclude, in this part of the study the orbit transfer methods of important satellite types are studied. These orbits are Earth centered orbits and contain 80% of all satellites. In other words almost all type satellites are studied. For the orbit raising cases, there exists no optimal solution in the literature. One way is to realize such transfers applying combination of optimal orbit transfers. While these transfers suggest an orbit transfer strategy, they are not optimal. The genetic algorithm based orbit transfer method introduced is applied to orbit raising problems examined in this section. This genetic algorithm based orbit transfer strategies can offer different solutions based on the number of impulses. For all cases, two, three and four impulse orbit transfer strategies are developed. Classical and genetic algorithm based orbit transfer strategies are compared. It is seen that genetic algorithm based orbit transfer method suggests much less propellant to realize orbit raising. This means that using the same amount of propellant, the satellite can service for a longer period.

CHAPTER 5

CONCLUSION

5.1 SUMMARY AND GENERAL DISCUSSIONS

In this thesis work, orbit transfer optimization problem is studied. The orbit transfers are modeled as realized by applying impulses. The magnitude and timing of impulses were aimed to be optimized to satisfy the specified objectives and constraints. In this study, main objective of this problem is to realize the desired orbit transfer using minimum energy/impulsive velocity vector magnitude while considering impulse level per one operation of propulsion system and total impulse capacity constraints. Since the orbits around only Earth are focused, the transfer duration is not considered as a constraint. The transfer duration becomes important at the missions which include rendezvous tasks.

Orbit transfers can be realized for different purposes. Among these the orbit transfer of a satellite starting from the separation from a launch vehicle ending at the mission orbit of a satellite is one of the most important orbit transfer problem. Since minimization of propellant consumption during this transfer enables to use the saved propellant during orbit maintenance. Therefore the service time of a satellite can be increased.

Classical orbit transfers are the orbit transfer methods that are suggested in the orbit transfer literature. While some of them are optimal at specific conditions, the remaining transfers are not optimal. Classical orbit transfer methods can be used to develop orbit transfer strategies but they have deficiencies. Main deficiencies of classical orbit transfers are:

- Some of the classical orbit transfers can only be used to change one or two orbital elements. Using classical orbit transfers the changes of six variables are not possible. At the orbit raising problem semimajor axis, eccentricity, inclination, and right ascension of ascending node (sometimes argument of perigee) is needed to be corrected. Classical orbit transfer methods do not offer any orbit transfer strategy for this problem. Only a superposition of classical orbit transfers can suggest a solution to this problem. The superposition of classical orbit transfers may not suggest optimal results.

- Classical orbit transfers focuses on only the changing of some orbital elements, some of the remaining orbital elements are also changed while they should be kept constant. Note that if the orbit is changed successive stages so that only one orbital element is changed at each stage, the energy requirement of orbit transfer increases.

Considering the deficiencies of classical orbit transfer methods and constraints such as transfer duration, impulse level per one burn, total impulse, rendezvous of orbit transfer problem, classical orbit transfer methods are generally not applicable. For these reasons, an orbit transfer optimization method is needed to be developed. In this study the orbit transfer problem optimization methods are developed. Genetic and steepest descent are used as optimization method. The closeness of final orbital elements and required impulsive velocity vector magnitudes are compared to optimal orbit transfers that are known to be optimal at specific conditions. These optimal orbit transfers are Hohmann and only inclination change orbit transfers. While genetic algorithm reaches to optimal solution, steepest descent method is not successful when a poor initial prediction is given. In other words steepest descent can only be used at orbit transfers where a strong intuition to optimal solution.

The genetic algorithm based orbit transfer method was applied to most common orbit transfer of satellites. The most popular satellite orbits are sun synchronous low earth orbit (SS-LEO), medium earth orbit (MEO), geostationary earth orbit (GEO) and Molniya orbit. In this study the orbit raising of these satellites are examined. The reason of orbit raising is the launch vehicle injection errors arising from final stage propulsion and orbit control systems excluding geostationary earth orbit raising. In geostationary earth orbit case a satellite is inserted to an orbit different from GEO on purpose since the capacity of launch vehicles are limited. This orbit is also known as geostationary transfer orbit (GTO). At this orbit satellite performs orbit transfers in order to reach GEO. Since the missions and orbits of SS-LEO, MEO, GEO, and Molniya satellites are different, the required accuracies of orbits are different. Therefore cost functions are specified based on cases.

The two, three, and four impulse genetic algorithm based orbit transfer strategies are obtained for all cases. The results of these strategies are compared to each other and classical orbit transfers. The synthesis of classical orbit transfer method requires the largest amount of impulse for each case. All genetic algorithm based methods requires less amount of impulse than superposition of classical orbit transfers. The genetic algorithm based orbit transfer method decreases the propellant consumption of satellites during orbit raising. The decrease in the propellant consumption directly increases the service lifetime of a satellite. The genetic algorithm based orbit transfer strategies offer different solutions based on the number of impulses.

The effects of initial orbital values to the orbit raising problems were also studied. For this purpose test cases were constructed and solutions for these test cases were obtained for SS-LEO, MEO, GEO, and Molniya. For all these problems, the critical orbital elements were specified. The least energy requiring strategy and relationship between number of impulses and required impulsive velocities were also obtained.

During these studies it is observed that there exist no single relationship between number of impulses and the required impulsive velocity vector magnitudes.

The obtained results prove that the genetic algorithm based orbit transfer optimization method is capable of finding an optimal orbit transfer strategy satisfying the user defined requirements and constraints. This plays a vital role in enhancing satellite service time and/or conceptual design of satellite since it reduces the effort and time to find out optimum orbit transfer strategy throughout a huge domain.

5.2 FUTURE WORK

In this study the genetic algorithm is used to obtain optimal solutions to orbit transfer problem. Although this thesis suggests important results for the specification of orbit transfer method, it needs improvements. It should be noted that while these improvements are out of scope of this thesis, they may be beneficial for the further studies.

1. The outputs of genetic algorithm may be used as the initial guess of a gradient based optimization method. In this case the generation number of genetic algorithm may be lessened since the genetic algorithm results are only used for initial guess. Therefore optimal orbit transfer strategy can be obtained faster than only genetic algorithm optimization.
2. The genetic algorithm based orbit transfer method presented in this thesis may be run considering more difficult constraints. The satellite propulsion system and attitude and orbit control system requirements should also be considered during orbit transfer calculations. The impulsive velocity vector enables to calculate the required amount of propellant at each direction.
3. Interplanetary trajectories are mainly used for scientific missions. The requirements of interplanetary orbit transfers are different from Earth centered orbit transfers. In interplanetary trajectories the timing of spacecraft is important; spacecraft should

be at a specific point at the defined time to benefit from the gravity assistance of third bodies. Interplanetary trajectories also are influenced from different perturbations in order to be at the specific point at the defined time and for escaping perturbations spacecraft should change its orbit. With the addition of all these parameters, the genetic algorithm based orbit transfer method can also be used for interplanetary trajectory planning and tracking tasks.

4. In this study the perturbation effects causing atmospheric drag, solar pressure, 3^{rd} body effects, etc are also not considered. The perturbations should also be included to the problem in order to obtain more realistic orbit transfer strategies. Note that during orbit transfers, the perturbations are not so critical since the orbit transfer duration is much less than the duration that perturbations affect the spacecraft.

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