VIBRATION ISOLATION OF
INERTIAL MEASUREMENT UNIT

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ABSTRACT

VIBRATION ISOLATION OF
INERTIAL MEASUREMENT UNIT

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Sensitive devices are affected by extreme vibration excitations during operation so require isolation from high levels of vibration excitations. When these excitation characteristics of the devices are well known, the vibration isolation can be achieved accurately. However, it is possible to have expected profile information of the excitations with respect to frequency. Therefore, it is practical and useful to implement this information in the design process for vibration isolation.

In this thesis, passive vibration isolation technique is examined and a computer code is developed which would assist the isolator selection process. Several sample cases in six degree of freedom are designed for a sample excitation and for sample assumptions defined for an inertial measurement unit. Different optimization methods for design optimizations are initially compared and then different designs are arranged according to the optimization results using isolators from catalogues for these sample cases.

In the next step, the probable designs are compared according to their isolator characteristics. Finally, one of these designs are selected for each case, taking into account both the probable location deviations and property deviations of isolators.

Keywords: Inertial Measurement Unit, Vibration Isolation Design, Six Degree of Freedom Vibration Analysis, Global Positioning System, Passive Vibration Isolation, Power Spectral Density, Frequency Response Function

ÖZ

ATALETSEL ÖLÇÜM BİRİMİNİN TİTREŞİM YALITIMI

Çınarel, Dilara
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Hassas cihazlar, cihazların kullanıldığı sistem yüksek seviyede titreşim etkileri içeriyorsa, cihazların titreşim yalıtımı gereklidir. Etkileşim karakteristikleri tam olarak biliniyorsa, titreşim yalıtımı başarılı bir şekilde gerçekleştirilir, ancak çoğu uygulamanın üzerine etki eden titreşim seviyeleri beklenmedik ve rastgele bir şekilde tezahür eder. Buna rağmen, tahrik seviyelerinin frekansa bağlı olarak değişim profilini elde etmek mümkündür. Bu sebeple, bu bilgiyi, titreşim yalıtımı tasarımında kullanmak uygun ve pratiktir.

Tez kapsamında, pasif titreşim yalıtımı tekniği incelenmiş ve sistem yalıtımı için izolatör seçiminde kullanılabilecek bir bilgisayar kodu geliştirilmiştir. Tez kapsamında, belirli bir etki altında ve ataletsel ölçüm birimine göre bazı kısıtlar göz önünde bulundurularak bazı örnek durumlar altı serbestlik derecesine göre tasarlanmıştır. Bunun yanında, değişik eniyileme yöntemleri karşılaştırılmış ve eniyileme sonuçlarına göre kataloglardan bazı gerçek izolatörler kullanılarak olası tasarımlar belirlenmiştir.

Bir sonraki adımda, bu olası tasarımlar izolasyon özelliklerine göre karşılaştırılmıştır. Son olarak, bu tasarımlardan biri, tasarımların olası yer ve özellik değişimlerine göre karşılaştırılmıştır.

Anahtar kelimeler: Ataletsel Ölçüm Birimi, Titreşim Yalıtımı Tasarımı, Altı Serbest Dereceli Titreşim Analizi, Küresel Konumlandırma Sistemi, Pasif Titreşim Yalıtımı, Güç Tayf Yoğunluğu, Frekans Cevap Fonksiyonu

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This thesis work has started in 2009 in my master study when I decided to work on vibration subject. Vibration isolation has always been my interest and all these work has been a study of interest.

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I also want to mention the infinite support and help my family has given in my work. I especially thank my mother and sister for their emotional support throughout this thesis work and in life. I am also deeply thankful to my father for logistical support he is offering to me in life.

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To my family,

with love and gratitude...

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LIST OF SYMBOLS

$[M]$	Mass matrix
$[K]$	Stiffness matrix
$[H]$	Damping matrix
k_{ij}	Stiffness of an isolator in i direction effective on j direction
m	Mass of system
I_{ij}	Moment of inertia components of system
ω	frequency
a_i	Distance magnitudes from CoG
$OBJ_{spectral}$	Spectral Objective
OBJ_{frf}	Frequency Response Objective
D_{res}	Response displacement
$Load_{vector}$	Load Vector
Acc_{res}	Acceleration Response
$Load_{static_{vector}}$	Static Load Vector
Dst_{res}	Static Displacement Response
Amp_{ratios}	Amplification Ratios
R_{unit}	Unit Response in One Direction
$F_{isolator}$	Force on One isolator
$ratio_{isolator}$	Axial to Radial Stiffness Ratio of an Isolator

LIST OF ABBREVIATIONS

IMU	Inertial Measurement Unit
PVI	Passive Vibration Isolation
AVI	Active Vibration Isolation
INS	Inertial Navigation Systems
MEMS	Micro-Electro Mechanical System
GPS	Global Positioning System
ISS	International Space Station
HEM	Hydraulic Engine Mount
CoG	Center of Gravity

CHAPTER 1

INTRODUCTION

1.1. Inertial Measurement Unit

In the 21st century; almost all of the mechanical systems need to work steadily. Exclusively, sensitive devices acquire strict isolation from high level and high frequency oscillations; namely excessive vibrations. Likewise, an Inertial Measurement Unit (IMU) contains sensitive electronic equipment that must be protected from vibrations and shocks during installation and operation. High frequency vibration isolation can be achieved through various means; principally such as passive or active vibration isolation methods. There are a lot of active vibration isolation methods and there are various passive methods.

In order to model a passive vibration isolation system; the number of discrete isolators that should be used and their characteristics, orientations and locations must be determined. The design is usually achieved through experiments via trial and error. Yet, an analytical groundwork is indispensable in order to be able to get a resolution. In this thesis work, the essential groundwork analyses will be summed up in a procedure for external vibration isolation designs in six degrees of freedom.

Shock excitation isolation of the IMU is not in the scope of this thesis directly and requires further work. On the other hand, external vibration isolation at high frequencies is going to be successfully achieved via the procedure described in this study.

1.2. Objective of Thesis Work

The objective of this thesis is about forming a vibration isolation design procedure according to the certain specified criteria in the design requirements. The objective of vibration isolation design is the main design requirement that must be taken into account and there are also specified design constraints to be considered. In the analysis of this thesis work, both the main design objective and the design constraints are represented by mathematical equations in order to make the analysis practical.

The design analysis is going to be achieved on six-degrees of freedom for a 3-D rigid prism. The first step in the design procedure is optimization in order to initiate the design process with parameters much close to perfect. On the second design step, vibration isolation designs are to be proposed with real isolators satisfying design requirements.

The initial optimization step is for the designer to be able to define the possible isolator characteristics ranges, the design constraints and the design objective since vibration isolation objective and constraints would change for each design. After optimization, the appropriate real isolators are going to be selected according to the optimization results, which is the second step in vibration isolation design. The vibration isolation system with the selected isolators are going to be demonstrated showing their effectiveness in the vibration isolation region as if the selected isolators from the catalog have the exact discrete characteristics as specified in the catalog.

On the other hand, isolator selection is just one part of isolation design since the locations of isolators should also be specified. The locations of the isolators are input with the information of the center of gravity location in the initial step of optimization for the procedure outlined in the thesis work. If the center of gravity of a solid object coincides with the elastic center of the system design, the cross responses between angular and translational directions becomes zero [1]. Since, the elastic center and center of gravity of the object to be isolated are rarely coincident, it is important to make them as close as possible. It is even more important for systems like IMU because IMU makes angular measurements, so the linear acceleration system exposed to should not create angular rotations on system due to vibration isolation. It should be checked while deciding on the locations of selected real isolators on system.

Following the second step, thus selecting appropriate real isolators for the isolation system, it follows a final controlling step for probable occasions. This final step is majorly divided into two parts. Thus the first part is in order to check the effects of deviations for probable designs and the second part is for the selection one of probable designs.

In real life, it is not always possible to design a sensitive system with the exact accurate parameters as intended in the design. The reason for this is that, the characteristics of isolators defined in catalogues vary on different isolators of the same type and mounting locations and angles of isolators may be different than it was intended in the design due to manufacturing errors, installing inaccuracies, etc. Even the variations may be small; they would change the characteristics of the vibration isolation system, the transmissibility and isolation characteristics. In order to analyze this situation, Monte Carlo simulation technique is going to be used in the third step of vibration isolation design. The selected design options in the previous step are to be eliminated in this part of the final step according to the simulation results.

The final decision is made in the second part of this last step between the remaining design options left from the previous steps according to the objective value and also the system natural frequencies. In this final step, vibration isolation objective is completed.

1.3. Motivation

The motivation for this thesis is about vibration isolation of sensitive devices the sensors of which are susceptible to deterioration when the devices are subjected to high frequency vibration levels. The inertial measurement unit has been taken as the example case of these throughout the thesis. Alongside with this example a general vibration isolation methods' investigation is also the target of this thesis study.

Vibration isolation has been a subject of interest in many areas. During manufacturing, sensitive equipment should be isolated from extreme levels of vibration [2, 3], spacecrafts should be isolated in order to protect them from launch loads [4], harsh-environment operating hard-disk drives are isolated from random and shock loads [5]. Other examples of vibration isolation are the engines in any kind of vehicles [6] and helicopter rotors [7].

The external vibration isolation of hard disk drives is the most similar example to the external vibration isolation of inertial measurement unit which is considered in this thesis [5]. The internal sensitive components of IMU can be isolated exclusively and the reliability of IMU can be increased internally, which is also achieved similarly for hard-disk drives. The Gimbals in an inertial measurement unit can be internally vibration isolated in order to decrease the expected negative effects of vibration on gimbals [8]. On the other hand, the appropriate external passive vibration isolation would be sufficient to increase reliability because it is known that without vibration isolation inertial measurement unit is not reliable [9] and reliability of IMU is critical in military applications such as guided missiles [10].

There are two types of disturbance to sensible parts in IMU. One type is due to high acceleration effects which are over the sensor acceleration capacity whereas the other type of disturbance is the failing of sensors due to high level of vibration at high frequencies. The sensors are damaged because of high level disturbances and the sensitivity of them decreases; as a result misleading information is obtained through the measurement system [9].

The vibration isolation system for an inertial sensor assembly with a ring shaped elastomeric member is suggested in a United States Patent [11], from which it is concluded that it is possible to isolate an inertial measurement system. It is also concluded that the vibration isolation system design must also be possible for an IMU using discrete commercial isolators without having to manufacture a ring shaped elastomeric member.

The problems that might be encountered using multiple discrete isolators are narrated in the patent, as of matching elastic center of the system with the mass center of gravity of the system which can be solved by only designing high accurate system which is not easy to obtain in reality. This can only be obtained by using certain types of isolators with the exact properties around the inertial navigation system on exact locations specified nevertheless it is not easy because the accuracy of the isolator parameters, isolator locations and orientations are never exactly certain. The uncertainty of these brings the motivation to do a Monte Carlo analysis which is going to be provided in order to study the effect of uncertainties in the vibration isolation design.

In the course of the thesis, the frequency domain approaches, in vibration isolation designs are addressed in order to manage the frequency response characteristics of a vibration isolation system in the broad frequency range. The time domain and frequency domain approaches are related through Fourier transform however frequency domain approach is selected to study because working in the frequency domain has critical advantages in computation speed and complexity [12].

The approach that is going to be utilized in this thesis depends on three-dimensional modeling of the isolated object as a rigid rectangular prism, the dimensions and the mass matrix of which is specified. The motivation of this work is then going to be realized step by step creating a vibration isolation design procedure.

1.4. Thesis Layout

The introduced thesis topic is going to be built up in six major chapters. First chapter is the introduction chapter.

Chapter 2 of the thesis has been devoted to the vibration isolation work in literature that has been done up to now. Vibration isolation concept and its exigency in mechanical designs in literature are going to be stated. The active and the passive vibration isolation techniques along with their requirements are going to be listed. Afterwards, the spring-mass-damper vibration isolation system design formulation is going to be demonstrated for single degree of freedom simple systems along with the examples of designs using this formulation. The vibration isolation system designs for IMUs in literature are going to be narrated at the end of this chapter. The optimization techniques and vibration isolator selection criteria are also given in this Chapter.

Chapter 3 of the thesis is based on problem formulation that is going to be utilized in the thesis. First of all, the mass, damping and stiffness matrices' formulations are shown. Then the eigenvalue problem is constructed for the system design in order to show the calculations of undamped natural frequencies and mode shapes. Following this, the calculations of frequency response functions are shown. Next, the spectral excitation and response formulations are shown.

In Chapter 4, optimization of system parameters is achieved using two different methods. The methods are introduced defining their objective functions and constraint functions. Then, four different sample cases were optimized using different methods or using different assumptions. After the optimization, in this chapter vibration isolation system designs with appropriate commercial isolators are also achieved. The objective value is compared between selected possible designs, the convenience of each isolation system according to constraints is then checked and initial eliminations between these possible designs have been made.

In Chapter 5, vibration isolation system design is achieved. In the design all different considerations were taken into account stated in Chapter 5. The decision is made finally from design options by varying characteristics of isolators via Monte Carlo simulation.

In Chapter 6, a summary of the thesis with attained conclusions are presented. Following this, the future work that may be fulfilled is suggested.

CHAPTER 2

LITERATURE SURVEY

In this chapter, the literature on vibration isolation is examined. There are two methods for vibration isolation, namely active and passive vibration isolation. These methods are going to be explained on the following sections. Next, the general spring-mass-damper system formulation is given for both single and multi degree of freedom systems. Afterwards, the vibration isolation systems for IMUs in literature are going to be narrated.

In literature, there are possible solutions to reduce excessive vibrations which can be classified into three general groups according to source, path or isolated object characteristics [12]. The first class of solutions is to mitigate the source of vibrations. This includes relocating the vibrating machine, replacing the machine with a one of higher quality, changing operating speed in order not to coincide with excitation frequencies, balancing system, adding a tuned vibration absorber or using active control. The second class of solutions is about the path characteristics. This is achieved by using isolators, inertia blocks or dampers. The last and third class of solutions is obtained by changing the system natural frequencies, or by adding structural damping.

For an IMU, for which the object is to mitigate vibrations eliminating the source of vibrations is not always possible since an IMU can be used in various applications such as in military applications. In such cases, the excitations causing vibrations are random, unexpected and inevitable. Therefore, it is not possible to use the first class of solutions meaning simply controlling the source of vibrations which has random characteristics. The third class of solutions is not also practical for an IMU which is about changing the system characteristics having various sensors and devices inside. However, some studies have been done for the vibration isolation of the components in IMU; hence protecting the whole system [8,11].

The second class of solutions can be used in the isolation of an IMU which is about mitigating vibrations along the path from the source to the receiver system. This is enhancing the sensitive system with the task of reducing excessive vibrations by placing vibration isolation elements along the path. This is simpler if vibration isolation path is along in one direction but more complex to design when the path is along more than one direction.

Vibration isolation system in multi-directions is required in spacecrafts for small launch vehicles [14]. The design is accomplished in three stages for these systems. The first stage is preliminary dynamic system analysis. It is followed by load analysis and isolator detailed design and final dynamic system analysis. In the analysis and design throughout this thesis a similar approach is going to be used. In the first stage, the dynamic excitation loads on the system are analyzed and vibration levels affecting the system are determined along with other geometric and system constraints. In the second stage, appropriate isolation system characteristics are determined and design is achieved. In the last stage the designed vibration isolation system is analyzed in order to check the success of vibration isolation design.

2.1. Active Vibration Isolation (AVI)

Active vibration isolation is effective in all possible situations. It is preferred because effective vibration isolation is necessary in order to obtain precise results in high-resolution manufacturing. In these systems signals are gathered by sensitive vibration detectors and vibration is reduced using a feedback system to drive electro-dynamic actuators [2]. Active systems are much more efficient than passive systems but active systems also have their physical limitations [3, 15]. The limitations can be satisfied using more than one vibration control device together, bringing an optimization problem for performance considering limitations.

AVI systems also have the following disadvantages compared to passive vibration isolation systems [16]:

- Increased direct cost
- Reduced reliability
- Dependence on a power supply
- High maintenance requirement for systems necessitating 3D isolation

In AVI design, six-degree-of-freedom rigid body model of a system is developed in order to determine the unknown parameters of unloaded system such as spring stiffness, damping or vertical position of center of mass via comparison of measured transfer functions with experiments or simulations [2]. This also helps to compare different strategies for actively controlling the determined parameters. The transmissibility curves are used in order to be able perform a comparison.

AVI systems have been widely used in space shuttle applications and in ground vehicle suspensions [17]. On International Space Station (ISS), active vibration isolation techniques are also used in order to be able to support experiments in free-fall environment with high accuracy. An active vibration isolation system consists of a stator, a flotor, an umbilical, several control actuators and several sensing devices as in Figure 2.1. Active vibration isolation can exist in both one directional and multi directional vibration isolation systems.

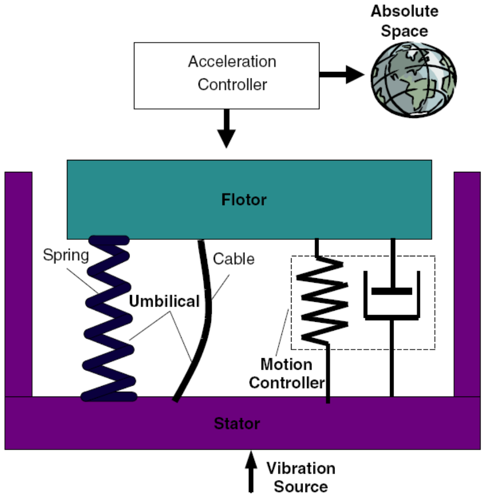

Figure 2.1. Active Vibration Isolation System [17]

A multi-directional active vibration isolation exist in six-axis vibration isolation tables vibration which are designed by using direct driven guide and ball contact mechanisms [18]. In active vibration isolation systems, the-cross coupling of actuators is undesired for the system working in multi-directions, because cross-coupling may lead to mechanical friction deteriorating the vibration isolation performance.

In multi-axis vibration isolation the initial consideration regarding frequencies have to be for natural frequencies in regarding axes and the effect of flexible body natural modes in addition to rigid body natural frequencies should also be considered because even they are much higher than rigid modes; they may cause an instability problem due to their interference with the control signal.

Another example of multi-axis active vibration isolation can be seen on flexible beam structures subjected to adaptive active vibration suppression [19], which is preferred because the amplitude reduction even at natural frequency of the system is aimed. Passive vibration isolation with constant properties for these kinds of flexible systems are only accepted when the design analysis is also performed in order to check the amplification of the isolation system at the system natural frequencies.

2.2. Passive Vibration Isolation - PVI

Passive vibration isolation systems are basic vibration isolation systems that do not require power input. Since they are useful even in the absence of power, they are usually preferred over active vibration isolation systems requiring power to be utilized even the passive systems also have their own disadvantages compared to the active systems.

The advantages of passive vibration isolation can be listed in terms of cost, reliability, maintenance and power. First of all, direct cost is less for passive vibration isolation systems. Secondly, the passive systems are more reliable and the feedback systems requiring high maintenance do not exist in passive systems. Finally, passive vibration isolation systems do not depend on a power supply most critically. Thus, they do not need external power input that also needs extra control concerns and maintenance cost.

Passive vibration isolation systems are usually introduced for a specific vibration frequency range. This characteristic can both be viewed as an advantage or a disadvantage. It is advantageous when vibration isolation only for a certain frequency range is required however, in real systems the vibration levels change continuously. In addition to this, passive vibration isolator characteristics change with respect to frequency and also with respect to some other environmental characteristics like temperature [16]. Therefore, the vibration isolator parameters predefined are not always exact and may have significant scatter during operation. The small deviations consequently are able to cause big failures in vibration and noise isolation [20].

The deviations or scatters may cause big problems for sensitive devices when passive vibration isolation is used. The requisition of better precision and minimization of production defects calls for improved isolation for sensitive devices. The improved passive vibration isolation design necessitates the dynamic characteristics of the object to be isolated in order to determine the ideal isolation characteristics for that object [16]. In addition to this, dynamic characteristics of disturbing vibration excitations must also be known for the improved passive vibration isolation design to be successful.

Although, these disadvantages and the necessity of requiring detailed information, the passive vibration isolation technique should not be underestimated. When properly designed, it is a powerful tool and this technique is also encountered in numerous applications. This method can also be thought of as the primary vibration isolation technique which is not as smart as active vibration isolation but is more reliable.

For example, on spacecrafts, some passive vibration isolation techniques are used in order to reduce the high amplitude and high acceleration excitation effects [4]. The most critical environment for a spacecraft is the launch environment and reduction of launch-induced dynamic loads transmitted to spacecraft would reduce the risk of failure of the spacecraft and the components of the spacecraft on orbit resulting in a more reliable system. The use of different vibration isolation techniques, on the frequency response of the spacecraft, the resonant peaks shift to lower frequencies and response levels reduce as in Figure 2.2.

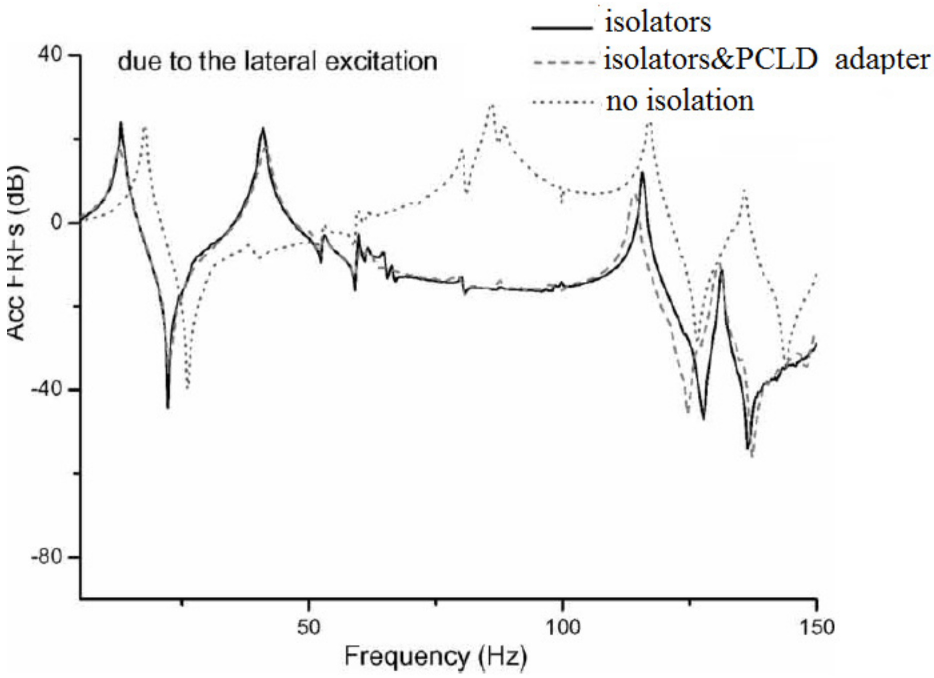

Figure 2.2. Frequency response function of spacecraft [4]

Another example is the passive vibration isolation used on hard disk drives, HDD, operating in harsh environments an effective vibration isolation system is required [5]. The vibration in HDD systems is being studied because they may be subjected to severe shock and vibration excitations whereas they are designed to work in stationary conditions. There are basically three methods to control shock and vibration problems in HDDs. The first two methods are about mechanical design robustness of the HDD which is widely used in academic or industrial research. The first method is to design a robust servo control mechanism in order to prevent read/write head error. The second is to design a robust slider/disk interface. The third is designing a suitable vibration isolation of HDD. There is some research on vibration isolation of components of the HDD and external vibration and shock isolation of HDD is also studied [5]. Similarly, vibration isolation of components in an IMU research also exists and external vibration isolation of IMU is to be studied in the scope of this thesis.

Passive vibration isolation system formulation changes according to its degree of freedoms considered. In literature, a general single degree of freedom formulation can be found which is similar to all other genres of vibration isolation formulation. The vibration isolation system that is generally made use of in literature is a single-degree-of-freedom system connected by vibration isolators shown by discrete stiffness and viscoelastic or structural damping values. The larger mass is assumed to be the foundation thus reducing the system to single degree of freedom as in Figure 2.3.

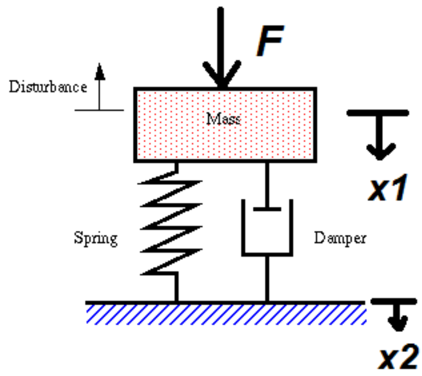

Figure 2.3. A Single degree of freedom passive vibration isolation system [21]

Equations of motion [21] for the system in Figure 2.3 are:

$$m\frac{d^2x_1}{dt^2}+c(\frac{dx_1}{dt}-\frac{dx_2}{dt})+k(x_1-x_2)=F \qquad (2.1)$$

$$m_f\frac{d^2x_2}{dt^2}+c(\frac{dx_2}{dt}-\frac{dx_1}{dt})+k(x_2-x_1)=0 \qquad (2.2)$$

These two equations are rewritten in complex form assuming x_1 and x_2 are magnitudes of displacements. Assuming that mass of the foundation m_f is much larger than the isolating mass itself eliminates Equation (2.2). In Equation (2.1), the force on system, F has been assumed to be zero and the equation is rewritten as in Equation (2.3).

$$x_1 / x_2 = (k + ic\omega) / ((k - m\omega^2) + ic\omega) \tag{2.3}$$

The six-degree-of-freedom vibration isolation formulation of a rigid object is similar and its formulation is going to be explained in the problem formulation chapter.

2.3. Isolator Damping Models

The formulation of vibration isolation systems is most practical if the discrete isolators are formulated by their stiffness characteristics and damping characteristics. Viscoelastic materials are usually used in vibration isolators but viscoelastic modeling of an isolator in analytical works is not practical and commercial isolators can be defined in terms of structural characteristics. Consequently, structural damping models are used in isolators.

The damping properties of viscoelastic materials are found out through experimentation making use of various methods. Through experimentation, isolators can be modeled using viscoelastic, viscous or structural damping. The isolator characteristics can be found through experimentation with respect to frequency and temperature [1]. Vibration isolation systems with the viscoelastic models taking into account the change of material characteristics as a function of frequency [22] would make the vibration analysis more complex than the analysis of isolation systems modeled with structural or viscous damping.

Viscoelastic materials as vibration dampers can be characterized experimentally [23]. There are two well known methods one of which is a standard test method for measuring vibration-damping properties of materials (ASTM method) and the other is the one based on indirect measures. Viscoelastic materials have the advantage of being able to provide the necessary damping characteristic. Some viscoelastic materials strengthen at high frequency levels which is not desired so new composite materials are designed from viscoelastic materials that soften at high frequencies. [24]. Viscoelastic modeling of an isolator is actually possible when the exact variation of the isolator property as a function of frequency is known [25].

This detailed viscoelastic design of vibration isolation systems would take longer times for analysis or optimization, and the results would not be more reliable if the characterization were not done with high accuracy. Getting reliable results by experimental characterization necessitates high number of experiments with good accuracy which would cause the process to have high cost and long time.

In dynamic analysis "damping force" in vibration isolation is an active research area. The "structural damping" model is the most commonly used one in which the damping matrix is assumed to be proportional to the stiffness matrix and "viscous damping" is the one that is damping properties are proportional to the instantaneous velocity across the damping element [26]. When viscoelastic and structural models are compared for isolators, structural models are more realistic for isolators. As a result, the assumed damping model in this thesis work is going to be the structural damping model in which the structural damping behavior of isolators in system is assumed to be as constant.

2.4. Inertial Measurement Unit – IMU

Inertial measurement unit (IMU) consists of acceleration sensors and gyroscopes that are used to measure linear acceleration, angular position and angular velocity in six degrees of freedom (DOF). The accelerometers are placed such that their measuring axes are orthogonal to each other. They measure inertial acceleration, also known as G-forces. Three gyroscopes are placed also in an orthogonal pattern, measuring rotational position in reference to an arbitrarily chosen coordinate system. The high level of exciting vibrations may adversely affect the performance of an IMU.

The inertial measurement unit can be represented by a box containing three accelerometers and three gyroscopes that are capable of measuring the required values. The IMU is the main component of inertial guidance systems used in air, space, and watercraft, including guided missiles. In this capacity, the data collected from the IMU's sensors allows a computer to track a craft's position using a method known as dead reckoning [30]. “Dead reckoning” is a system for navigation without feedback from a GPS.

The IMU’s are utilized on aerospace applications [9]. An IMU uses the information it gathers from guidance system in order to locate the vehicle it is installed on similar to a GPS navigation system. However, an IMU does not need to communicate with any external server gathering information from satellites. On the other hand, the major disadvantage of these kinds of systems is the accumulating error. In order to solve this problem, IMU’s should be as accurate as possible.

An IMU works by detecting the
1-Current rate of acceleration using one or more accelerometers,
2-Changes in rotational attributes like pitch roll and yawing using gyroscopes as shown in Figure 2.4.

An IMU is a unit consisting of sensors performing the integrated measurements in six DOF movement directions [29]. The position of an object free in space can be obtained in terms of three coordinates X, Y, Z and three angles in pitch, yaw and roll. Amongst the techniques utilized performing six DOF measurements, the IMU has been widely used in navigation systems for precise positioning because of its mobility advantage.

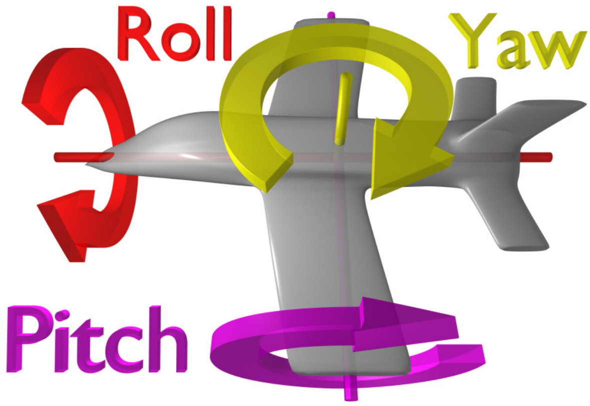

Figure 2.4. Roll, Pitch and Yaw directions [27]

In a navigation system, the data reported by the IMU is fed into a computer, which calculates its current position based on velocity and time. The computer should have the transmissibility characteristics of the vibration isolated IMU, for proper measurement.

IMUs are used in vehicle-installed inertial guidance systems. In these systems noise is not only characterized by engine noise. The environment the vehicle subjected to also changes the excitations. The IMU unit's primary mission is not sensing or measuring the excitation characteristics. But a high level of vibration excitation can make the IMU suffer from this [9]. The major disadvantage of an IMU is the accumulating error coming from the successive measurements.

The effect of shock is also very crucial in the correct operation of IMU [5]. MEMS (Micro-Electro-Mechanical System) inertial sensors are used in the design of isolation systems in order to minimize the effects of shock. The shock excitations must also be isolated from the IMU as well as high frequency vibration excitations because the shock or vibration energy may manifest themselves as linear and angular acceleration errors in the inertial data that is to be transferred to the navigational computer by the sensors.

In the latest technology, MEMS sensors are also preferred in designs because of their miniature size, low cost, reduced power consumption, and convenient integration with advantages of semiconductors/IC fabrication techniques [28]. These advantages bring up a disadvantage with it since MEMS sensors degrade in performance easily with shock or vibration effects. These undesirable effects are reported in several experimental observations [28]. Vibration leads to misleading device output and shock would cause the failure of sensitive devices. These inverse effects cannot easily be ceased electronically and they would ultimately reduce the service life of the MEMS sensors in IMU to an undesired level.

As mentioned before, the IMU also measures the angular displacements but if the isolation system is not properly aligned, the cross transmissibility effects may lead to incorrect measurements. These angular rotations in response to translational displacements should be made as close to zero as possible. High-precision gyroscopes are more expensive than accelerometers; hence, some researchers found gyro-free IMU systems consisting of only accelerometers. But, the sensing resolution of gyro-free inertial measurement units is worse than the IMUs consisting of both gyros and accelerometers. Accordingly, the inexpensive gyro-free IMUs are suitable for various applications requiring low-cost and medium performance such as car navigations or virtual realities. On the other hand, they are not suitable for systems that require high precision for long times without connection to a GPS.

The inertial measurement units (IMU) are also referred to as inertial navigation systems (INS) [31]. The *gyroscopes* in it have been used since the beginning of 19^{th} century even before the use of magnetic compass in it. *Navigation* is the determination of position with respect to a reference point. *Guidance* is orienting to the desired position and control means staying on the correct track. Especially in missile technologies all these three must be achieved simultaneously. As a result, the INS technology is developing parallel to guidance and control technology.

The inertial navigation systems (INS) have several advantages over other types of navigation systems like GPS:

1- It works autonomously without any external aid. It can be utilized anywhere even in tunnels or underwater.

2- It enables integrated navigation, guidance and control simultaneously.

3- It is suitable for tactical operations as it is not suitable for jamming and suits to stealth conditions.

The U.S. Air Force has determined three ranges for the performance of INS and inertial sensor as shown in Table 2.1.

Table 2.1. INS and inertial sensor performance ranges [31]

System or Sensor	Performance Units	Performance Ranges		
		High	Medium	Low
INS	CEP Rate (NMI/h)	$\leq 10^{-1}$	≈ 1	≥ 10
Gyros	deg/h	$\leq 10^{-3}$	$\approx 10^{-2}$	$\geq 10^{-1}$
Accelerometers	g^a	$\leq 10^{-7}$	$\approx 10^{-6}$	$\geq 10^{-5}$

[a] 1 $g \approx 9.8$ m/s/s.

On the other hand, INS has some disadvantages compared to GPS as well:

1- Mean-squared errors increase with time.

2- Acquisition, operation and maintenance have higher cost

3- INS has higher size and weight, higher power requirements and also has extra heat dissipation

The first disadvantage is due to calibration errors or corruption of sensors due to external effects [9]. If, it could be achieved to protect the unit from external disturbances, both the expected mean-square error and the maintenance cost would decrease. The third disadvantage almost diminished today in 2011 with increased technology.

In order to put up with the accumulating error, various methods are being developed. The first one of the methods is connecting the IMU system to a GPS system to make the main computer check periodically the accumulated error and adjust it. Other methods are minimizing the error genesis, evolving the IMU to become a reliable device even without a supporting GPS system. This is fatally important because it is not always possible to connect to the GPS system, and for outer space applications, connecting to or relying on a satellite system becomes infeasible. In addition to these, not having any error is always better than fixing accumulating error.

One of the requirements, minimizing the error genesis is high frequency vibration isolation of the IMU. The reason for the exigency of vibration isolation of the IMU is cited in a previous design work due some previous experimental experience [9]. The reasons of explanations given for isolation of IMU in literature are going to be lined up with introduction to its theoretical background in this section of the study.

The main cause highlighted in literature for the need of vibration isolation is the probability of the acceleration excitations exceeding the dynamic range of accelerometers. Gyroscopes' failing from vibration fatigue is given as another cause for mechanical vibration isolation of IMU. In order to enable adequate flight control, inertial measurement units are installed on isolation mounts for mechanical vibration isolation. Its outputs still have errors and they are analog filtered in order to discard them.

Furthermore, sensor electronics in IMU should be protected against strong accelerations at high frequencies so use of an isolation mount is inevitable due to the fact that without inadequate vibration isolation, the electronic sensors in IMU are likely to produce faulty readings [9].

In this thesis, theoretical formulation of vibration isolation of a 6-dof model of an IMU is going to be carried out by discrete modeling. Subsequently, the decision on vital isolators, thus a sample design is going to be shown forming know-how of the vibration isolation design.

2.5. Optimization Techniques for the Design of Vibration Isolation System (VIS)

Motion transmissibility between any two directions can be determined along any frequency range by using various analysis techniques. The motion transmissibility levels classified in four major areas [32] are given as:

1. Forbidden area
2. Dangerous area
3. Allowed working area: providing acceptable isolation performances level, used for common applications;
4. Optimum working area: providing high isolation performances level which is used for special applications.

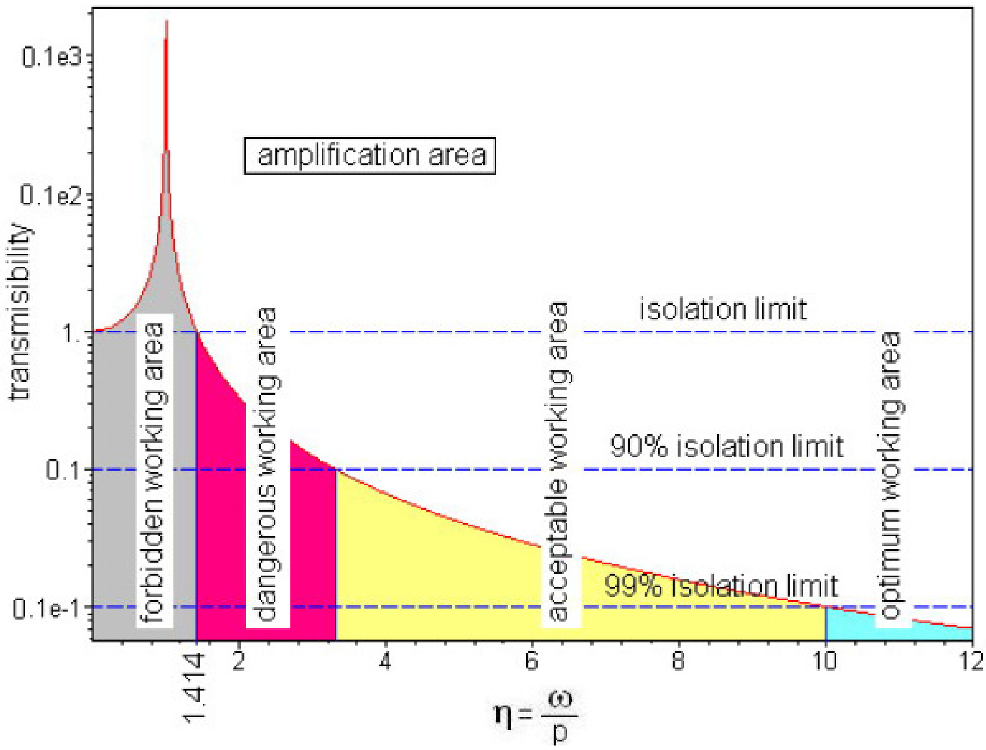

Figure 2.5. Classification of operating areas [32]

The forbidden area is where the transmissibility is larger than or equal to 1. Dangerous working area section is an isolation section but the isolation efficiency is at an acceptable order. This area represents where vibration isolation level is at most 90%. Acceptable working area is the vibration isolation section at frequency up to 99% vibration isolation level. Beyond this frequency is the optimum working area, where the vibration isolation is almost but not equal to 100%. The transmissibility is one of the measures that are used in the design optimization procedures.

The discrete optimization of isolator locations for vibration isolation systems can be achieved via random search, genetic optimization and continuous optimization by setting the objective as the transmissibility value at a single frequency [33]. The following constraints were taken into account in optimization such as alignment, stability, compression and static deflection constraints.

The optimization target may also be in order to have modal purity in the system which is the effect of a natural frequency in one direction not affecting other directions. For a multi degree of freedom system there is more than one natural frequency, one for each different degree of freedom. These natural frequencies should be far from each other in order to have modal purity so an excitation in one direction becomes independent of other directions. This objective in order to increase modal purity in one direction subjected to modal frequency ranges in other directions can be achieved by optimization using special commercial tools [34]. A similar procedure is also achieved in MATLAB using "*fmincon*" in optimization [6]. In these optimization processes the locations, considering the objective in terms of modal purity and constraints in terms of modal characteristics in different directions, orientations and characteristics of isolators were determined whereas the optimized characteristic intervals and location boundaries are specified.

Since the aim in inertial measurement unit vibration isolation is to decrease inverse effects of high level vibrations [9], the objective is determined as the reduction of the mean square responses of the system in translational directions. Because the responses in angular directions are going to be set as constraints, the response in translational directions can be considered in the objective. Sensitive devices present in an IMU let it to have a maximum acceleration level that it can be excited without causing performance degradation in the sensitive devices. The maximum acceleration response of the vibration isolation system is one other constraint in optimization that should be taken into account. Other optimization constraints are the free space available for the isolators, the maximum equipment motion possible or the maximum deflection an isolator can withstand without degradation.

After determination of the objective function and the constraints for the optimization problem mathematically, the problem can be solved numerically by various optimization methods. In this study, optimization routines in MATLAB such as fmincon, pattern search or genetic algorithm are used. On the other hand in literature there are different optimization techniques for vibration isolation. Reducing the total energy transmitted to isolated system or mean square motion of a single degree of freedom system under random excitations is called H_2 optimization, whereas minimization of amplitude at resonance is called H_∞ optimization [36]. In H_2 optimization, white noise excitation is assumed. H_2 optimization is chosen in the case of optimization under random vibration excitation.

Design optimization for systems with viscoelastic materials can also be achieved using genetic algorithm which is another optimization technique [37]. Maximizing the loss factor or minimizing the vibration energy of the system is the objective in design optimization. Design optimization may also be used for reduction of vibration level in rotor dynamics [39]. Optimization for the characteristics of the vibration isolation system is the objective in this work; hence, this can also be considered as design optimization.

The objective in vibration isolation may not always depend only on steady state responses and transient responses may also be important as well [24]. The transient response optimization is preferred in some industrial applications where the excitation is almost known with respect to time, like in automotive applications. The objective function may also be formed by contributions of both steady-state and transient responses when the excitation is estimated with respect to time and isolator characteristics are taken to be variable with respect to frequency.

In literature, optimization in time domain is also encountered in various other situations during design. For example, the characteristics of the advanced passive isolator, Hydraulic Engine Mount (HEM) may be decided upon a time-domain analysis [40]. Furthermore, a bladed rotor uses a passive vibration control scheme with piezoelectric devices on blades which are dynamically optimized via a homogenization procedure [41]. Optimal placement of active bars in design is called topological optimization. [43].

Multiple active trailing edges may also be used in design with optimization in order to reduce vibrations [7]. Reliability-based optimization can also be performed defining a reliability based objective function [42].

Before optimization, the bounds of the variables and initial starting guess values should be determined [45]. Bounds determine the lowest and highest value of a variable that is to be considered in optimization. Starting guess value is important in some optimization methods but some methods find global solutions without utilizing an initial starting point guess such as genetic algorithm method available in Matlab.

In some design problems, the objective is not one target but the objective may depend on more than one outcome. Then the multiple objectives can be united in a single objective mathematically. In multi-objective optimization problems, multiple objective functions are combined in a single function such as in Equation 2.4 [44].

$$F(x) = \sum_{i=1}^{k} \omega_i f_i(x)$$
$$\sum_{i=1}^{k} \omega_i = 1 \tag{2.4}$$

F(x): fitness function,

ω_i : weighting factors of *k* number of objective functions

$f_i(x)$: objective functions.

In literature; the number, characteristics and connecting angles of isolators have been studied in order to get the best performance from the isolation system [6]. In this thesis, a generalized formulation is going to be done for an object that is to be isolated from motion in 6 DOFs. Similarly, the number and locations of isolators and the characteristics of them are to be determined. The main objective in this thesis is to reduce the high level and high frequency vibration effects without affecting the sensing measurements of the IMU.

The vibration isolation system design process initiates using the optimization results. Once the isolation system is designed, the response characteristics of the system should be determined and introduced to IMU. As a result, the MEMS sensors in IMU would not be affected by high vibration amplitude levels as determined by previous experimental study [9] and IMU system would still be able to measure correct rotational position, velocity and linear acceleration. The aim is to transmit measurement forces within the bandwidth of measurement system region and filter the high frequency excitation components [35].

In order to obtain the optimum isolator characteristics, an optimization technique similar to the H_2 optimization is to be used since the random vibration excitation is assumed. At the same time, the maximum response at resonance which is the objective in H_∞ optimization technique is taken as one of the constraints in the optimization process. The target here is to find out the best isolator characteristics with the number of isolators specified in the specified locations. The optimization methods are explained further and exemplified in sample cases in Chapter 4.

2.6. Vibration Isolation System Design Criteria

In this section of the thesis, first of all the vibration isolation system design criteria [49] are listed and these criteria are going to be used for selecting vibration isolation design options after optimization on Chapter 4.

2.6.1. Isolator Loadings

The source of dynamic excitation to the vibration isolation system affects the type and direction of the isolation system loading. Dynamic vibration environment is listed in three different categories. [25]

1. Periodic vibration
2. Random vibration
3. Transient vibration, shock

The isolations to the above listed categories of vibration loadings are different. In addition, the solution to one of them may also be effective to another category as well. When the disturbance is random thus have no periodic behavior in time, random vibration isolation criteria is usually used. In order also to protect from shock disturbances, some additional mountings can be used over a vibration isolation system.

When the general characteristics of the above loading types are known, there are three rules that should be taken into account in decision of isolator selection in order to design a vibration isolation system.

1. Enough deflection space to accommodate for motions of the dynamic environment
2. Load carrying capacity of each isolator should not be exceeded under dynamic loads
3. The service life

The first and the second items listed above are taken into account in optimization and analysis process. In addition to the dynamic behavior of the system the static load carrying capacity of system must also be considered. On top of that, the natural frequency of each isolator individually is selected to be far from the system natural frequencies.

In terms of random vibration isolation in six degrees of freedom; the vibration is represented by the power spectral density. The power spectral density is represented by matrix distribution when the vibration problem is in six degrees of freedom. The diagonal elements show the actual excitations and the off-diagonal elements belong to the cross spectral densities showing the coherences of the excitations to system.

In terms of inertia measurement unit vibration isolation, the response to excitations should not be mixed and angular accelerations would only be allowed to affect the system negligibly. In theory, everything might seem perfect although there might be some deviations of characteristics which may result in unwanted responses. These must also be checked with an uncertainty analysis which is to be carried out in the next chapter. On the other hand, minimization of high frequency and high level vibration amplitudes is the target.

Disturbance type effects the isolator selection directly; the optimization analysis should be carried out according to whether the disturbance is periodic, random or shock type. In the previous section, the optimization code is developed according to the random spectral density of disturbances in different directions. The vibration isolation designs to a periodic excitation or shock are achieved by different methods.

2.6.2. Isolator Characteristics

Some basic rules should be used in order to determine the stiffness and damping characteristics of isolators in design process. Commercially, there are a variety of isolators each having different stiffness and damping characteristics. The stiffness characteristics of isolators in a vibration isolation system change the natural frequency of the isolation system and it also determines the load carrying capacity of the isolation system. The damping characteristics determine and limit the deflection space of the isolator.

Commercially available isolators are generally can be formulated by structural damping characteristics with good accuracy. The loss factor of a structurally damped isolator can easily be found by using the maximum transmissibility value that is associated with the transmissibility graph of the isolator. Selecting an isolator with an appropriate amount of “loss factor” is crucial in vibration isolation design. After deciding on the loss factor value, the next step is on selecting the isolators with appropriate stiffness values. This procedure may be repeated more than one with the use of different isolator combinations in order to meet the requirements.

Commercially available isolators should be mathematically formulated before the selection. The most usually selected formulation in optimization procedure is the linear stiffness and viscous or structural damping characteristic formulation whereas; there is also viscoelastic formulation of isolator materials. It is also dependent upon, the material characteristics used in forming the isolators.

Viscoelastic formulation of isolators is possible with experimental extraction of characteristics. The viscous formulation is usually used but the viscous formulation does not give characteristic results very close to structural isolators in the market. However, the structural formulation is very close to them.

In a vibration isolation system, isolators might also be used in combination in order to reach the desired characteristics because the commercial isolators have predefined characteristics so they might not be available having the desired characteristics directly. It might be advantageous to use isolators in series or in parallel in order to meet the stiffness requirements more closely. In parallel connection there is also the advantage of increasing static weight capacity.

The parallel isolator combination has advantages in a six-degree-of-freedom vibration isolation system. The stiffness characteristics of the isolation system can be increased by connecting isolators in parallel on one face as shown in Figure 2.6. and by Equation (2.5).

$$Total_{stiffness} = nk_{isolator} \tag{2.5}$$

where n is the number of isolators

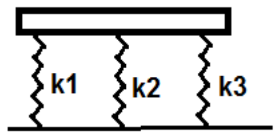

Figure 2.6. Isolators in parallel combination

The series connection may also be appropriate where the characteristics of the combination are calculated by Equation (2.6).

$$Total_{stiffness} = k_{isolator} / n \tag{2.6}$$

where n is the number of isolators in parallel

When, two or more different types of isolators are connected in series Equation (2.6) becomes Equation (2.7).

$$\frac{1}{Total_{stiffness}} = \frac{1}{k_{isolator1}} + ... + \frac{1}{k_{isolatorn}} \tag{2.7}$$

where n is the number of isolators

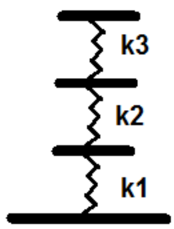

Figure 2.7. Isolators in series combination

In every different design case, there might be different solutions. Series combination may work for some cases while parallel would be better in others. For the sample cases demonstrated in this thesis work, isolators connected in parallel were only considered.

2.6.3. Allowable Limits for Isolators

Capabilities of vibration isolation systems are defined via the isolators it consists. Vibration isolation systems limit the disturbance that it can be exposed to, according to the capability of the isolators in it. The allowable response of the system must not be exceeded and the response depend both on stiffness and damping characteristics. The allowable response is a geometric constraint most of the time, but it can also be a system constraint depending on the capability of the sensitive equipment, the sensitive object that is to be isolated.

The space available for the isolated equipment motion is also very important in deciding the system parameters of the vibration isolation system. The motion that is to be considered is the sum of three response characteristics:

1- Static deflection due to weight of object to be isolated
2- Dynamic deflection due to environmental loads
3- Deflection due to steady state acceleration

All the above listed three motion amplitudes should be checked by the constraints in the analysis. If any one of these exceed the maximum allowable limit for the isolation system, the vibration isolation system may fail.

The space occupied by the isolators is limited by geometric considerations. The weights of isolators should also be low relative to the object that is being isolated. Isolator locations should be set such that, their elastic center should be close to the object's mass center. The elastic center calculation is approximately made using Equations (2.8) to (2.10). An assumption is made that the isolators are identical since the exact elastic center calculation can be made accurately when isolators are utilized for an object in three dimensional six degree of freedom space are identical [10]. Because of the assumption made, the elastic center can be approximated before assigning any stiffness value to isolators but just the locations of isolators. It is important to note that, in all the calculations, the elastic center and mass center coincidence is not a strict requirement but in initial design it is useful to select isolator locations to have an elastic center close to the mass center.

$$x_{elastic\_center} = \frac{(x_1 + x_2 ... x_n)}{n} \tag{2.8}$$

where x is the locations of n isolators on x axis

$$y_{elastic\_center} = \frac{(y_1 + y_2 ... y_n)}{n} \tag{2.9}$$

where y is the locations of n isolators on y axis

$$z_{elastic\_center} = \frac{(z_1 + z_2 ... z_n)}{n} \tag{2.10}$$

where z is the locations of n isolators on z axis
and n is the number of isolators

In the work carried out in this thesis, the isolators are located on four perpendicular surfaces, and the coordinates of the discrete isolators are determined such that the elastic center of the isolation system is close to the geometric center of the object determined in the previous section. This location adjustment is required in order to make cross transmissibility components as close to zero as possible.

For a 3-D object, like IMU for which the cross-transmissibility coefficients must be very close to zero between any two directions in order to make the measurements correct, the isolators should be located on at least two surfaces. The reason for this and the location determination procedure is going to be shown in this section.

In order to make sure that the locations are to be calculated without difficulty making the elastic center coincident with the geometric center in all directions, all the isolators may be designed have the same characteristics. One type of isolator use, leads the designer to be free to choose any number of the same isolator while making the elastic coordinate system coincident is with the mass center is simpler. This is an assumption, but different isolators can also be used in the same design.

Controlling not only the response amplitude is important for the design of an IMU but also the response characteristic in each direction is important. The response of the system in angular directions in response to linear acceleration should be made as low as it is possible in order to have exact measurements of angular velocity and angular acceleration from gyroscopes. So, the cross stiffness components between translational and angular directions should be as close to zero as possible.

The location information of each isolator, are stored in a matrix. The locations are stored in a vector of the form given in Equation (2.11).

$$LOC = \begin{bmatrix} x_{coordinates} \\ y_{coordinates} \\ z_{coordinates} \\ \alpha_{rotation} \\ \beta_{rotation} \\ \gamma_{rotation} \end{bmatrix} \tag{2.11}$$

The rotations might as well correspond to the mounting angles leading the designer to take into account them. When the isolators are mounted perpendicularly, the rotation angles should be taken equal to zero.

The characteristics of isolators may depend on environmental conditions even if isolator system is passive, the characteristics of isolators may be dynamic because the environment is unpredictable and dynamic. The isolators should be selected such that, they should not be affected inversely by environmental changes such as temperature or different level of vibration excitations. Thus, the operating environment of isolation system is critical in selecting the isolators with desired characteristics.

The selected isolators in this thesis are going to be from classes of isolators in AM001-AM005 series in LORD catalogue. They are approved to the environmental tests appearing in MIL-STD-810 or MIL-E-5400. And, since the BTR elastomeric is used, the maximum transmissibility of the single isolator is given as three. They are also assumed to be reliable in the temperature range from -50ºC to 145 ºC. Since they have linear deflection characteristics, the characteristics of the isolators are assumed to be constant. The variations possible in temperature or vibration levels are not taken into account since only negligible variation is expected.

CHAPTER 3

PROBLEM FORMULATION

3.1. Mathematical Model

The mathematical model is formed in this section by defining mass, stiffness and damping properties of the system in terms of proper matrices. The mass matrix is formed according to the inertial properties of the object to be isolated. The stiffness and damping matrices are defined according to the discrete isolator characteristics and their locations on the object to be isolated. The symbols used for these matrices are as follows:

$[M]$: mass matrix
$[K]$: stiffness matrix
$[H]$: structural damping matrix

The mass matrix is assumed to be constant because it solely depends on the inertial properties of the object to be isolated and independent of the isolation system since the vibration isolation equipment is assumed as massless. The reason for this is that, the object mass is so much greater than the isolator mass. On the other hand, the stiffness and damping matrices depend on the vibration isolation system properties such as isolator characteristics and locations. Also these depend on each other if the damping is proportional.

The characteristics are varied in this analysis and locations are assumed to be constant and determined according to the center of gravity location of the object. Along with the fixed positions of isolators, isolator characteristics are to be optimized in a defined range. Afterwards, the real isolators with close properties to the optimization results are going to be determined. The real isolators are going to be modeled by structural damping and selections are made according to the results of optimization made via structural damping formulation.

The determination of radial and axial stiffness coefficients as well as the loss factor is followed by stiffness and the damping matrix formation similarly by also making use of the isolator locations with respect to the elastic/mass center of the isolated system because it is also assigned as the coordinate origin. The calculated system matrices are going to be used in order to calculate the frequency response of the system to excitation from translational directions. Since, the geometric center is coincident with the elastic center; the angular rotations to translational excitations become zero [1]. In addition to this, if the elastic and geometric center locations are unequal but still close to each other than the corresponding rotations would be close to zero.

The isolators that are to be utilized in isolation system also have mass, even small; which also have effect on transmissibility of the system. If the mass effects of the isolators are neglected, the performance of the isolation system at high frequencies would be over estimated. Since this over-estimation would be present in all of the solutions, this effect is to be neglected. The isolators themselves have their own dynamics causing *internal resonances* [46]. This condition should be checked according to the design case such that the internal resonances should not coincide with system resonances of the object so it depends on the application area of the isolated object. In this thesis, the response of the system from three translational directions to the random vibration is going to be optimized along with restrictions of maximum sway amplitude and maximum acceleration transmission to object.

The stiffness and damping matrices are formed according to the six equations of motion in space, giving the motion in six degrees of freedom of a point thus the motion of a body that can be assumed to be rigid [21]. The equations of motion are three force equilibrium and three moment equilibrium equations [21], where the cross stiffness constants are also taken into account. The locations of the isolators are effective in these equations in terms of angular stiffness coefficients and also on the angular-translational cross stiffness coefficients. Force equation in x-direction is given by Equation (3.1) :

$$\begin{aligned}
&m\ddot{x} + \sum k_{xx}x + \sum k_{xy}y + \sum k_{xz}z + \\
&\sum (k_{xz}a_y - k_{xy}a_z)\alpha + \\
&\sum (k_{xx}a_z - k_{xz}a_x)\beta + \\
&\sum (k_{xy}a_x - k_{xx}a_y)\gamma = F_x
\end{aligned} \tag{3.1}$$

In Equation (3.1); x, y, z, α, β and γ represent motion in six degrees of freedom as shown in Figure 3.1. Similarly the other two force equations are given by Equations (3.2) and (3.3). A general schematic of system with three isolators can be seen in Figure 4.1.

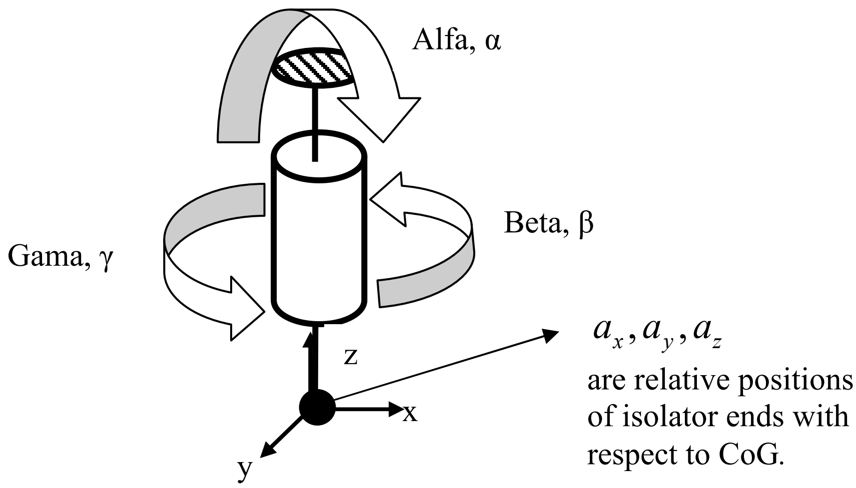

Figure 3.1. Schematic for the isolator movements

Where CoGis the "Center of Gravity" of the system to be isolated.

$$
\begin{aligned}
& m\ddot{y} + \sum k_{xy} x + \sum k_{yy} y + \sum k_{yz} z + \\
& \sum (k_{yz} a_y - k_{yy} a_z)\alpha + \\
& \sum (k_{xy} a_z - k_{yz} a_x)\beta + \\
& \sum (k_{yy} a_x - k_{xy} a_y)\gamma = F_y
\end{aligned}
\tag{3.2}
$$

$$
\begin{aligned}
& m\ddot{z} + \sum k_{xz} x + \sum k_{yz} y + \sum k_{zz} z + \\
& \sum (k_{zz} a_y - k_{yz} a_z)\alpha + \\
& \sum (k_{xz} a_z - k_{zz} a_x)\beta + \\
& \sum (k_{yz} a_x - k_{xz} a_y)\gamma = F_z
\end{aligned}
\tag{3.3}
$$

The three moment equations are represented by Equations (3.4) to (3.6). It is important here to note that the angular stiffness values of isolators commercially available are close to zero and can be neglected.

$$
\begin{aligned}
& I_{xx}\ddot{\alpha} - I_{xy}\ddot{\beta} - I_{xz}\ddot{\gamma} + \ldots \\
& \sum (k_{xz} a_y - k_{xy} a_z) x + \ldots \\
& \sum (k_{yz} a_y - k_{yy} a_z) y + \ldots \\
& \sum (k_{zz} a_y - k_{yz} a_z) z + \ldots \\
& \sum (k_{yy} a_z{}^2 + k_{zz} a_y{}^2 - 2k_{yz} a_y a_z + k_{\alpha\alpha})\alpha + \ldots \\
& \sum (k_{xz} a_y a_z + k_{yz} a_x a_z - k_{zz} a_x a_y - k_{xy} a_z{}^2 + k_{\alpha\beta})\beta + \ldots \\
& \sum (k_{xy} a_y a_z + k_{yz} a_x a_y - k_{yy} a_x a_z - k_{xz} a_y{}^2 + k_{\alpha\gamma})\gamma = M_x
\end{aligned}
\tag{3.4}
$$

$$
\begin{aligned}
& I_{yy}\ddot{\beta} - I_{xy}\ddot{\alpha} - I_{yz}\ddot{\gamma} + \\
& \sum (k_{xx} a_z - k_{xz} a_x) x + \\
& \sum (k_{xy} a_z - k_{yz} a_x) y + \\
& \sum (k_{xz} a_z - k_{zz} a_x) z + \\
& \sum (k_{xx} a_z{}^2 + k_{zz} a_x{}^2 - 2k_{xz} a_x a_z + k_{\beta\beta})\beta + \\
& \sum (k_{xz} a_y a_z + k_{yz} a_x a_z - k_{zz} a_x a_y - k_{xy} a_z{}^2 + k_{\alpha\beta})\alpha + \\
& \sum (k_{xy} a_x a_z + k_{xz} a_x a_y - k_{xx} a_y a_z - k_{yz} a_x{}^2 + k_{\beta\gamma})\gamma = M_y
\end{aligned}
\tag{3.5}
$$

$$
\begin{aligned}
& I_{zz}\ddot{\gamma} - I_{xz}\ddot{\alpha} - I_{yz}\ddot{\beta} + \\
& \sum (k_{xy}a_z - k_{xx}a_y)x + \\
& \sum (k_{yy}a_x - k_{xy}a_y)y + \\
& \sum (k_{yz}a_x - k_{xz}a_y)z + \\
& \sum (k_{xx}a_y{}^2 + k_{yy}a_x{}^2 - 2k_{xy}a_xa_y + k_{\gamma\gamma})\gamma + \\
& \sum (k_{xy}a_ya_z + k_{yz}a_xa_y - k_{yy}a_xa_z - k_{xz}a_y{}^2 + k_{\alpha\gamma})\alpha + \\
& \sum (k_{xy}a_xa_z + k_{xz}a_xa_y - k_{xx}a_ya_z - k_{yz}a_x{}^2 + k_{\beta\gamma})\beta = M_z
\end{aligned}
\tag{3.6}
$$

$k_{xx}, k_{xy}, k_{yy}, k_{yz}, k_{zz}$ are stiffness coefficients

$m, I_{xx}, I_{xy}, I_{xz}, I_{yy}, I_{yz}, I_{zz}$ are mass and inertial values

$x, y, z, \alpha, \beta, \gamma$ are coordinates

a_x, a_y, a_z are distance magnitudes of each isolator from mass CoG

In Equations (3.1) to (3.6), the cross translational stiffness values of commercially available isolators are also assumed to be zero if they are not placed in the vibration isolation system with inclined mounting angles and on non-precise locations. Inclined mounting of an isolator means that the isolator axial axis is not mounted perpendicularly to the surface but it makes an angle with the normal of the surface that it is being mounted. The possible angle between the normal of the surface and the isolator axial axis is going to be considered to be a small angle variable in the design procedure used in this thesis.

From these equations, the stiffness matrix of the whole 6-degree-of-freedom vibration isolation system can be derived. The damping matrix is formed the same way as the stiffness matrix for a structurally damped vibration isolation system. The damping in the isolation system is assumed to be structural in the problem formulation in this thesis.

3.1.1. Equations of Motion

The six equations of motion are represented in Equations (3.1) to (3.6) referencing literature. In this section, the equations are going to be summarized using matrix notation. Damping matrix with structural damping properties is also included in this section's formulation. The matrix and vector abbreviations are listed in the list of figures.

Three force and three moment, thus six equations of motion should be solved simultaneously in order to find the correct response of system to any excitation. For an object that is restricted in six degrees of freedom by isolators, the forcing to the object is via motion transformation.

Assume that the second coordinates represent the input coordinates and the first coordinates are at the object's center of gravity for a system with six degrees of freedom like the single degree of freedom system in Figure 2.3. The difference between Equations (2.1) to (2.3) is, here the movement is not only confined in a single direction. It is like connecting six single degree of freedom systems together so that, they also may affect one another. This effect is represented by the cross terms in right and left matrices. So, the equation of motions may be summarized in matrix form as in Equation (3.7). The right and left vectors, $\{q_1\}$ and $\{q_2\}$ denote the effect and the isolated response direction coordinates as given in the vector Equation (2.11).

$$([K]-\omega^2[M]+i[H])\{q_1\}=([K]+i[H])\{q_2\} \qquad (3.7)$$

The forcing vector should be determined by the relation on the right hand-side of the Equation (3.7). If the excitation on the system is defined spectrally as a matrix, the input vector can also be a two dimensional matrix. While in the case of two dimensional input, as in the case of random vibrations, for the sake of simplification, the modal model is obtained first from the defined geometric model and then the response model is obtained from the modal model. [47].

3.1.2. Mass Matrix

The mass matrix for a system is an input and it changes according to the characteristics of the object that is to be isolated. Mass matrix information can be got from appropriate computer codes. The mass matrix used in the sample case study in this thesis is supplied in the appendix.

The mass matrix contains the total mass value and the inertias of the object with respect to its center of mass location. The location of the center of mass must also be known in order to design an appropriate vibration isolation system.

The isolator masses would also be included in the mass matrix computation in order to take into account the inertial resonances. [46] They have small effect, and in theory just adding them simply to the mass matrix would not suffice; but since the mass ratio of the isolator and the object is small, the deviations expected are small enough to be ignored. And it is stressed that the deviation is going to exist for all options of isolators that are analyzed. Consequently, these deviations would not change the final solution but it is indispensable to use a safety factor in design. [16].

3.1.3. Stiffness and Damping Matrix Formation

The stiffness and damping matrices are formed according to the equations of motion given in Equations (3.1) to (3.6). They are both 6 x 6 square matrices and their components are given in Equations (3.8.a) to (3.8.u) for N number of isolators. K notation can be changed to H referring to the damping characteristics in order to form the damping equations similarly [21].

$$K_{11} = \sum_{i=1}^{N} k_{xx} \tag{3.8.a}$$

$$K_{12} = K_{21} = \sum_{i=1}^{N} k_{xy} \tag{3.8.b}$$

$$K_{13} = K_{31} = \sum_{i=1}^{N} k_{xz} \tag{3.8.c}$$

$$K_{14} = K_{41} = \sum_{i=1}^{N} k_{xz}a_y - k_{xy}a_z \tag{3.8.d}$$

$$K_{15} = K_{51} = \sum_{i=1}^{N} k_{xx}a_z - k_{xz}a_x \tag{3.8.e}$$

$$K_{16} = K_{61} = \sum_{i=1}^{N} k_{xy}a_x - k_{xx}a_y \tag{3.8.f}$$

$$K_{22} = \sum_{i=1}^{N} k_{yy} \tag{3.8.g}$$

$$K_{23} = K_{32} = \sum_{i=1}^{N} k_{yz} \tag{3.8.h}$$

$$K_{24} = K_{42} = \sum_{i=1}^{N} k_{yz}a_y - k_{yy}a_z \tag{3.8.i}$$

$$K_{25} = K_{52} = \sum_{i=1}^{N} k_{xy}a_z - k_{yz}a_x \tag{3.8.j}$$

$$K_{26} = K_{62} = \sum_{i=1}^{N} k_{yy}a_x - k_{xy}a_y \tag{3.8.k}$$

$$K_{33} = \sum_{i=1}^{N} k_{zz} \tag{3.8.l}$$

$$K_{34} = K_{43} = \sum_{i=1}^{N} k_{zz}a_y - k_{yz}a_z \tag{3.8.m}$$

$$K_{35} = K_{53} = \sum_{i=1}^{N} k_{xz}a_z - k_{zz}a_x \tag{3.8.n}$$

$$K_{36} = K_{63} = \sum_{i=1}^{N} k_{yz}a_x - k_{xz}a_y \tag{3.8.o}$$

$$K_{44} = \sum_{i=1}^{N} k_{yy} a_z^{\,2} + k_{zz} a_y^{\,2} - 2k_{yz} a_y a_z + k_{\alpha\alpha} \tag{3.8.p}$$

$$K_{45} = K_{54} = \sum_{i=1}^{N} k_{xz} a_y a_z + k_{yz} a_x a_z - k_{zz} a_x a_y - k_{xy} a_z^{\,2} + k_{\alpha\beta} \tag{3.8.q}$$

$$K_{46} = K_{64} = \sum_{i=1}^{N} k_{xy} a_y a_z + k_{yz} a_x a_y - k_{yy} a_x a_z - k_{xz} a_y^{\,2} + k_{\alpha\gamma} \tag{3.8.r}$$

$$K_{55} = \sum_{i=1}^{N} k_{xx} a_z^{\,2} + k_{zz} a_x^{\,2} - 2k_{xz} a_x a_z + k_{\beta\beta} \tag{3.8.s}$$

$$K_{56} = K_{65} = \sum_{i=1}^{N} k_{xy} a_x a_z + k_{xz} a_x a_y - k_{xx} a_y a_z - k_{yz} a_x^{\,2} + k_{\beta\gamma} \tag{3.8.t}$$

$$K_{66} = \sum_{i=1}^{N} k_{xx} a_y^{\,2} + k_{yy} a_x^{\,2} - 2k_{xy} a_x a_y + k_{\gamma\gamma} \tag{3.8.u}$$

The stiffness and damping matrices are not only formed with the components of characteristics in stiffness values but also they depend on the individual isolator locations. The relative positions of isolators are denoted by a_x, a_y, a_z throughout the thesis as given in Figure 3.1.

The optimization is going to be performed according to the defined excitation characteristics. The excitation defined on system may have definite or random characteristics with respect to frequency. The response level of the vibration isolation systems are calculated by modal analysis approach whereas the excitation response level with respect to frequency is calculated by using system dynamic receptance. The eigenvalue problem is the primary to be solved for a vibration response calculation.

3.2. Eigenvalue Problem

The eigenvalue problem associated with the undamped free vibration of the system is given in Equation (3.9) where U represents, the displacements in all six dof's represented in Equation (3.10).

$$\omega^2[M]\{U\} = [K]\{U\} \tag{3.9}$$

$$[U] = [U_1, U_2, U_3, U_4, U_5, U_6] \tag{3.10}$$

Natural frequencies of the undamped system can be found by solving Equation (3.11).

$$\left|[K] - \omega^2[M]\right| = 0 \tag{3.11}$$

Equation (3.11) is going to be solved in order to find the natural frequencies and once the natural frequencies are found, the mode shapes can be found by Equation (3.12).

$$([K] - \omega_i^2[M])\{U_i\} = 0 \tag{3.12}$$

For a six degree of freedom system, there would be six natural frequencies associated to six mode shapes. The un-normalized modal matrix can be formed like via Equation (3.12). Thus, the modal matrix becomes a 6 by 6 square matrix. The modal mass matrix then can be formed as in Equation (3.13).

$$[U]^T[M][U] = [M_r] \tag{3.13}$$

The mass matrix calculated by Equation (3.13) is the modal mass matrix that is diagonal. So the normalized modal matrix, mass and stiffness matrices are shown in Equations (3.14. a) to (3.14.d).

$$\phi_i = \frac{\{U_i\}}{\sqrt{[M_r]_{i,i}}} \tag{3.14.a}$$

$$\phi^T[M]\phi = [M_n] = [I] \tag{3.14.b}$$

$$\phi^T[K]\phi = [K_n] \tag{3.14.c}$$

$$\phi^T [H] \phi = [H_n] \tag{3.14.d}$$

The normalized stiffness and structural damping matrices can be expressed in terms of modal attributes that are shown in Equations (3.15.a) and (3.15.b).

$$[K_n] = \begin{bmatrix} \ddots & & 0 \\ & \omega_r^{\,2} & \\ 0 & & \ddots \end{bmatrix} \tag{3.15.a}$$

$$[H_n] = \begin{bmatrix} \ddots & & 0 \\ & \omega_r^{\,2}(i\gamma_r) & \\ 0 & & \ddots \end{bmatrix} \tag{3.15.b}$$

3.3. Frequency Response Functions

The frequency response functions are given by the Equation (3.16). [47,48]

$$G_r(\omega) = \frac{1/\omega_r^{\,2}}{1-(\omega/\omega_r)^2 + i\gamma_r} \tag{3.16}$$

The FRF matrix can be formed in Equation (3.17) for a six degree of freedom system using r=1…6 of previous Equation (3.16).

$$[FRF] = \begin{bmatrix} G_1(\omega) & 0 & 0 & 0 & 0 & 0 \\ 0 & G_2(\omega) & 0 & 0 & 0 & 0 \\ 0 & 0 & G_3(\omega) & 0 & 0 & 0 \\ 0 & 0 & 0 & G_4(\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & G_5(\omega) & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6(\omega) \end{bmatrix} \tag{3.17}$$

Equation (3.17) is used for random excitations by using Equation (3.15.a) and (3.15.b). This matrix seems to be diagonal but in calculating the response in Equation (3.23), the coordinates shall be transformed again. The frequency response of the system would be determined also for definite excitations by using $[K]$, $[H]$ and $[M]$ matrices directly by using Equation (3.18).

$$[FRF] = ([K] + i[H] - \omega^2 [M])^{-1}([K] + i[H]) \tag{3.18}$$

In order to determine the frequency response characteristics, the excitation should be determined along any direction in the form of an excitation vector. In the scope of the thesis, the highest expected level of excitations with respect to frequency is to be defined.

3.4. Excitation Functions

The excitation in most of the problems might be indefinite. When it is defined in terms of statistical characteristics, a spectral density method is used [47]. The excitation spectrum is usually defined in terms of acceleration power spectrum and it should be converted into force spectral density in Equation (3.19). The excitation spectral matrix associated with modal coordinates is given in Equation (3.20) [48].

$$F^2(\omega) = (([K] + i[H]))^2 \frac{S(\omega)}{(\omega^2)^2} = S_{FF}(\omega) \tag{3.19}$$

$$S_{QQ}(\omega) = \phi^T S_{FF}(\omega) \phi \tag{3.20}$$

In Equation (3.21), the acceleration spectrum is converted into forcing spectrum. The division by $(\omega^2)^2$ is to convert input acceleration spectrum density to input displacement spectrum density. If only output acceleration density is desired than no division would be required.

The excitation for the definite case is selected similar to the spectral excitation; thus response of the system to 1g excitation is investigated through the frequency range. Equation (3.18) is going to be multiplied with the excitation vector in order to get the frequency response of the system.

3.5. Response Spectral Matrix

The spectral matrix of the response is calculated by the expression given in Equation (3.21) [48].

$$S_{res}(\omega) = \phi[FRF^*(\omega)]S_{QQ}(\omega)[FRF(\omega)]\phi^T \tag{3.21}$$

The mean squares of the responses along any coordinate should be calculated via Equations (3.22.a) to (3.22.f) by replacing Q in Equation (3.21) with corresponding coordinate direction.

$$\sigma_x^{\ 2} = \int S_{xx}(\omega)d\omega \tag{3.22.a}$$

$$\sigma_y^{\ 2} = \int S_{yy}(\omega)d\omega \tag{3.22.b}$$

$$\sigma_z^{\ 2} = \int S_{zz}(\omega)d\omega \tag{3.22.c}$$

$$\sigma_\theta^{\ 2} = \int S_{\theta\theta}(\omega)d\omega \tag{3.22.d}$$

$$\sigma_\beta^{\ 2} = \int S_{\beta\beta}(\omega)d\omega \tag{3.22.e}$$

$$\sigma_\gamma^{\ 2} = \int S_{\gamma\gamma}(\omega)d\omega \tag{3.22.f}$$

The frequency response to force spectral density of the vibration isolation system has a similar behavior along the frequency range as a response spectral density. The response spectral density characteristics along different axes can be calculated as mean-square values between ranges of frequency

[47,48]. The objective function is going to be determined in order to decrease the mean square values along three translational axes.

CHAPTER 4

OPTIMIZATION

In this part of the thesis, the formulation for the optimization problem is given. The optimization is solved by a computer code developed in MATLAB and information on usage is given in the appendix. After the formulations, sample cases are going to be selected and optimized prior to design. The rules of thumb for the optimization are going to be determined. Then the optimized results are going to be manifested, showing their behavior in graphs and properties in tables.

In the first sub-section of this part, the optimization methods that are used in this study are explained. Then in the second sub-section, the constraints that are taken into account in optimization are listed and explained. In the third sub-section of this part the design specifications that are used in optimization are explained. Then in the last section of this part, sample cases are optimized via the pre-explained methods and optimization functions available in Matlab are compared with respect to the achieved objective value.

4.1. Optimization Methods

Two major optimization methods can be used in vibration isolation design. The first one of these methods is the spectral response minimization for the cases where the spectral characteristics of possible random excitations to the system are known. The second method is for the cases where the excitation characteristics to the system that needs to be isolated are known with respect to frequency and depends on the frequency response minimization of the system within the defined excitation range. .

4.1.1. Spectral Response Minimization

In this method, the spectral response of the isolation system to a spectral excitation is minimized by changing characteristics of the isolation system within the allowed ranges considering the constraints of the system as well.

In an optimization problem the heart of the solution depends on the objective function that is to be minimized. In order to minimize the spectral response of a system, the objective function is determined according to the power spectral density of vibrations acting on the isolated object which is the system that is concerned. The spectral response of the system is determined according to the defined PSD criteria. Then, by using the spectral response of the system, the ***grms*** value of the random excitations effecting on the system is calculated and included in the objective function. The ***grms*** value in a direction is obtained by calculating the area under the spectral density graph, then taking the square root of it.

The minimization can be achieved for responses to excitations in any direction. It can also be achieved in more than one direction simultaneously by taking into account the ***grms*** values in each direction via giving weights. The code formed in this thesis, takes equal weights if more than one direction is minimized.

The modal model can be obtained via the formulation given in Chapter 3. From the geometric model obtained, the eigenvalue problem for the undamped system is obtained and following that the modal matrix is extracted. From the modal model, following the procedure in Chapter 3; the response to random spectral vibration is obtained. Furthermore from mean square values in each direction objective value is calculated for the response to the spectral excitation defined.

The excitation spectra are also determined according to the isolation system specifications via Equation (3.19). The response of the system to the specified excitation spectra is calculated using Equation (3.20). The objective function is going to be determined via the mean values of response of the system given by Equation (3.22). The roots of the mean square values are calculated in order to obtain ***grms*** values.

The response spectral matrix components that are calculated by equations in (3.22) are utilized in the objective function. The spectral response mean-square values are to form the objective function with equal weights of 1/3 each when the excitation is defined in three directions. So a sample objective function is given in Equation (4.1) where the mean square calculations are given in Equations (3.22.a) to (3.22.f).

$$OBJ_{spectral} = \frac{1}{3}(\sqrt{\sigma_x^{\ 2}} + \sqrt{\sigma_y^{\ 2}} + \sqrt{\sigma_z^{\ 2}}) \tag{4.1}$$

Different optimization functions of MATLAB® are used in the optimization process. Upper and lower bounds for characteristics of isolators are defined initially to the optimization problem, and then the optimum characteristics of isolators ensuring the minimum objective function value for the problem confirming the constraint conditions are obtained.

The response of the isolated object depends on the isolation characteristics of the system and varies with the characteristics even the excitation is the same. Thus the characteristics of an isolation system should be decided upon an engineering phenomenon. The locations of isolators are input according to the center of mass information, assuming elastic center of isolators are coincident with it or they may be decided according to the problem case. The characteristics of isolators are optimization variables while the estimated response is used in forming the objective function.

A variety of optimization techniques can be used in order to find an optimum vibration isolation solution. The objective function used in optimization is formed according to the type and direction of the excitation.

In the isolation problem of the 6-dof object, the excitation can be along any of the three translational directions.

4.1.2. Frequency Response Minimization

In this method, the frequency response of the isolation system to a known excitation with respect to frequency is minimized by changing the isolation system characteristics within the allowed range considering the constraints of the system that need to be taken into account.

The heart of the solution in this method, thus the objective function is determined according to the area under the frequency response curves with respect to frequency. The frequency response of an excitation is found by multiplying the excitation vector by the FRF matrix given in Equation (3.20).

The objective function determination is similar but in this method, the calculated objective is not the ***grms*** but the area under the response curve so have the unit ***g.Hz.*** Thus the objective is formed by summation of the weighted responses of the object in all the excited directions as given in Equation (4.2).

$$OBJ_{frf} = \frac{1}{3}(Area_{Rx} + Area_{Ry} + Area_{Rz}) \tag{4.2}$$

This solution method should be preferred when the excitation is known accurately with respect to frequency but if the excitation is just known in terms of spectral quantities the previous method should be preferred.

4.2. Optimization Constraints

In vibration isolation design, the requirements of design should also be taken into account while minimizing the objective function of the design. The particular requirements of design form the constraints of the optimization problem which would give the initial design parameters for characteristics of isolators. The requirements may include the dynamic behavior of the system, the static behavior of the system, and the desired isolation characteristics of the system or the capacity of the isolators used in the system.

Some of the constraints due to dynamic behavior of system are calculated according to the maximum load effective on system. The dynamic maximum load information should be specified in order to be able to check the dynamic constraints. The static endurance of the system is confirmed according to the maximum static displacement the system is allowed to make. This constraint must also be appended with the maximum static load on the system.

The desired isolation characteristics of system are also requirements which come into the problem as constraints. The maximum isolation frequency limits the upper frequency where the isolation starts. The maximum direct or maximum cross amplification ratios limit the upper transmission ratio the system may experience.

The isolator capacity under static loads is another requirement for the system. These constraints also use the maximum static load on the system specified like the maximum static displacement constraint. It is useful to define this constraint separately for each isolator.

The constraints that are desired to be considered can be made active/inactive in the code supplied which will be described in detail in the sample cases.

The constraints are going to be taken into account during initial design optimization, primary design and final design stages.

4.2.1. Dynamic Behavior Constraints

The dynamic behavior of a vibration isolation system should be taken into account as the isolation system provides isolation for high frequencies whereas it provides increase of res at lower frequencies. And because the isolation system is much less stiff than the object isolated as an assumption, the excitations affecting the object are attenuated at low frequencies.

The attenuated excitations at low frequencies may lead to unwanted movements of the object in response to dynamic excitations or this may lead to unwanted effects on the object at low frequencies. The passive isolation ensures safe life of the isolated object at high frequencies but the low frequency behaviors of the system must also be kept at an acceptable level. So, the optimization problem can be completed with these constraints taken into account leading to a trade-off decision.

These constraints are taken into account by frequency response method calculated by matrix inversion technique. The maximum expected level of acceleration excitation is utilized in order to determine the maximum responses in defining the constraints the system must have to conform.

4.2.1.1. Sway (Dynamic Displacement) Constraint

This pre-mentioned constraint checks whether the system dynamic displacement exceeds the allowable dynamic displacement of the system or not. The dynamic displacement of the system is calculated starting from the stated frequency up to the frequency two times higher than the sixth natural frequency of the system.

The multiplier load vector is formed by the highest dynamic acceleration loads on system specified in each direction, represented by Equation (4.3).

$$\{Load_{vector}\} = \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad (4.3)$$

The dynamic displacement of the system is calculated by the Equation (4.4). This gives a vector of 6 x 1 where the first three rows indicate the displacement directions of the displacement.

$$[D_{res}] = ([K] + i[H] - \omega^2[M])^{-1}([K] + i[H])\{Load_{vector}\} / \omega^2 \qquad (4.4)$$

The load vector in Equation (4.4) has been divided by frequency square in order to find the displacement input to the system. The displacement response of system is estimated with respect to frequency on vector $[\mathrm{D}_{\mathrm{res}}]$. Since the first three rows of vector $[\mathrm{D}_{\mathrm{res}}]$ represent the displacement in three translational directions, the resultant displacement can be found by vector addition rule and the constraint can be represented as in Equation (4.5) by squaring and summing the first three rows of the response vector in order to compare the magnitude of the sway with the maximum sway the system is allowed to make.

$$\max((D_{res}(1))^2 + (D_{res}(2))^2 + (D_{res}(3))^2) < (sw_{\max})^2 \qquad (4.5)$$

If in the optimization or analysis of isolation system, this constraint is wanted to be considered, this constraint should be checked in the GUI window which will be discussed later.

4.2.1.2. Angular Dynamic Displacement Constraint

This constraint is also optional like the previous one in the code and this constraint also uses Equation (4.4) in order to find the dynamic response of the system. The last three rows of the response vector are taken into account in the calculation of this constraint.

The last three rows of the response vector has been squared and summed in order to compare the magnitude of the angular response with the maximum angular displacement the system is allowed to make as in Equation (4.6).

$$\max((D_{res}(4))^2 + (D_{res}(5))^2 + (D_{res}(6))^2) < (rot_{\max})^2 \tag{4.6}$$

This constraint is all about the stability of the isolated object in response to excitations. It is the rotational displacement that the object would have in case of translational acceleration input. This constraint is effective only if there are deviations in chosen characteristics and locations of isolators. Since the analytical design is precise and locations are determined in order to ensure the stability of the system, without deviations, this constraint is always conformed. On the other hand, if the deviations exist this constraint must also be checked given by Equation (4.6).

4.2.1.3. Maximum Acceleration Constraint

Maximum response at low frequencies is also limited according to the acceleration response. The acceleration response of the isolation system cannot exceed the maximum acceleration level the sensitive device which is the isolated object can withstand.

$$[Acc_{res}] = ([K] + i[H] - \omega^2[M])^{-1}([K] + i[H])\{Load_{vector}\} \tag{4.7}$$

The dynamic acceleration on the object is calculated by the Equation (4.7). This gives a vector of 6x1 where the first three rows indicate the translational accelerations in directions of the excitation. The resultant translational acceleration must not exceed the allowable acceleration level on the isolated object.

In Equation (4.8) the constraint of the object of maximum acceleration exposure is defined. The maximum acceleration acting on the object cannot exceed the maximum acceleration level the sensitive device can withstand.

$$\max((Acc_{res}(1))^2 + (Acc_{res}(2))^2 + (Acc_{res}(3))^2) < (Acc_{\max})^2 \quad (4.8)$$

4.2.2. Static Displacement Constraint

The static endurance of the system is checked according to the maximum static displacement the system is allowed to make. The multiplier load vector in Equation (4.9) for static load is formed similar to dynamic load vector given by Equation (4.3) is formed by the highest dynamic acceleration loads on system specified in each direction.

$$\{Load_{static_{vector}}\} = \begin{bmatrix} Ls_1 \\ Ls_2 \\ Ls_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4.9)$$

The maximum static displacement of the system is calculated by Equation (4.10).

$$[Dst_{res}] = ([K])^{-1} m \{Load_{static_{vector}}\} \quad (4.10)$$

The first three rows of the response vector has been squared and summed in order to compare the magnitude of the resultant static deflection with the maximum static deflection the system is allowed to make as in Equation (4.11).

$$((Dst_{res}(1))^2 + (Dst_{res}(2))^2 + (Dst_{res}(3))^2) < (sdef_{\max})^2 \qquad (4.11)$$

The calculation in Equation (4.12) is similar to the dynamic deflection calculation given in Equation (4.6). The static deflection is calculated once for the static case and the value does not vary with frequency as it was the case for dynamic loads. So, the maximum operator is not utilized in this constraint calculation.

4.2.3. Constraints from Required Isolation Characteristics

Every isolation system might not be possible for every set of constraints defined before. On the other hand, all the constraints may be conformed but the isolation may not be actually present in the system at desired frequencies even if the objective function has been made to have a minimum value. Another case is that the designer might desire the maximum transmission ratios to be kept at a predefined level instead of defining the previous constraints.

These constraints are calculated by obtaining the frequency response of the system from each translational direction, separately. The amplification ratio matrix is calculated initially in order to calculate the constraints in this section which is given by Equation 4.12.

$$[Amp_{ratios}] = ([K] + i[H] - \omega^2[M])^{-1}([K] + i[H]) \qquad (4.12)$$

4.2.3.1. Maximum Isolation Frequency Constraint

This constraint just checks whether the upper isolation frequency of the system is at a desired level or not. After the isolation frequency, all the responses of the system must be below the corresponding excitation level.

Since, the system is a multi degree of freedom vibration isolation system, there are more than one natural frequencies and it is not straightforward to find the damped natural frequency of the damped 6-dof isolated system. So the analytical method of multiplying the natural frequency with square root of two might lead to inaccurate results. So, a numerical method is used to estimate the isolation frequency instead of the traditional analytic method.

In order to find out the isolation frequency, the amplification ratio matrix multiplied by vectors having unit excitation for each translational direction. The responses for each case were checked whether the response is less than the unit excitation or not. Afterwards, the frequency range is checked starting from the lowest frequency. The frequency where all the responses from each direction have less than unit magnitude represents the isolation frequency.

An example approximation for the isolation frequency in one direction is represented in Equation (4.13). The responses of the system to unit excitations from other directions are calculated similarly.

$$R_{unit\_x} = [Amp_{ratios}][1,0,0,0,0,0] \tag{4.13}$$

4.2.3.2. Maximum Direct and Cross Amplification Ratios Constraints

This constraint checks both direct and cross amplification ratios of system on the defined frequency range. This is also found by Equation (4.12). After calculating the amplification ratios on the frequency range, the maximum direct and cross amplification ratios are calculated. These calculated values are checked whether they conform to the design requirements or not.

This constraint is not actually preventing the isolation system to be built or work but the design would not have desired characteristics in terms of amplification. This is important for sensors such as IMU because, the undesired high alterations of excitation responses may not be able to be corrected. For example, if the desired maximum amplification ratio is 200% but the system designed has maximum amplification ratio 300%, this constraint is not conformed. But if the design has direct maximum amplification ratio %190, this constraint is conformed.

4.2.4. The Isolator Static Load Constraint

This constraint is different from the static deflection constraint because when different types of isolators are used in an isolation design, even when the maximum effective deflection on each isolator is the same, the effective load on each isolator may differ. The maximum static load an isolator is capable of holding must be specified to the designer in order to check if it is possible to have a fail-safe operation of the vibration isolation system under high static loads.

This calculation is achieved by finding the maximum static displacement the isolation system makes under specified maximum static loads as in Equation (4.10). Then this static deflection vector is multiplied by the isolator stiffness matrices in order to find out the load acting on each of the isolators. This is given in Equation (4.14).

$$F_{isolator} = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix}_{isolator} \{Dst_{res}\} \tag{4.14}$$

Where isolator=1,…,number of isolators

4.2.5. The Isolator Stiffness Ratio Constraint

An isolator is defined with its axial and radial stiffness characteristics which define their dynamic properties. There are different isolators in the market having various dynamic properties, thus having different stiffness values on a broad range. A single isolator may have different axial and radial stiffness properties as well as these properties can also be the same. However, the axial and radial stiffness characteristics of an isolator often are related by a ratio defined in a range usually lower than two. In the optimization and analysis process so this was also considered as a constraint.

This constraint is going to be helpful for the designer when selecting appropriate real isolators from the market. If the selected isolator stiffness boundaries are on a broad range, optimization process may result in unrealistic results of isolator which cannot be found exactly or approximately from the isolators available on the market.

This condition is tested by calculating the ratio of each isolator's axial and radial stiffness ratio and comparing with the set lower and upper ratio limits as given in Inequality Equation (4.15). In Equation (4.16), the assumption used in this thesis is shown but instead of this radial to axial stiffness ratio can also be considered.

$$ratio_{low} < ratio_{isolator} < ratio_{up} \tag{4.15}$$

$$ratio_{isolator} = k_{axial} / k_{radial} \tag{4.16}$$

4.3. Design Specifications

The objective of the vibration isolation has been determined and the constraints are specified. In order to be able to make the calculations the mass matrix and the number and locations of isolators must also be supplied.

Along with the information specified, the intervals of characteristics that are going to be approximated must also be supplied. For any class of isolators available for selection in the market, there exist boundaries of characteristics in the selection. In the scope of this thesis, linear stiffness and structural damping properties are assumed. The numerical examples and the GUI for the code that has been written for these calculations are going to be shown in detail in the sample cases.

In the case studies, the lower and upper bounds for characteristics are going to be determined according to probable isolator selection lists. If the selected boundaries and the system requirements do not match, the procedure may not give any appropriate solution for selection. In the next part of the thesis, the design criteria that are to be taken into account are going to be explained in detail.

The LORD isolator characteristics are going to be used in the case studies [10]. In this catalogue and in similar ones from different companies, isolator characteristics can be found. The structural damping characteristic for the isolators in the catalogue are estimated according to the maximum transmissibility defined in the given transmissibility curve. The stiffness characteristics are defined in the catalogue.

There is another important decision designer has to make before optimization. One type of isolator use may be appropriate for most of the designs even when more than one isolator is used in the system. This would give direct and fast optimization and it is possible to perform further analyses faster than the case of utilizing multiple types of isolators. After the optimization step, probable designs are going to be determined according to optimization result.

Nevertheless, using more than one type of isolator would be more appropriate than only using a single type of isolator giving lower objective function value and leading to a safer design according to the constraints.

In vibration isolation system design considered in this thesis, the isolator characteristics on the determined locations are found by an optimization process in the previous chapter. In this chapter, the characteristics are going be determined with available isolators commercially. The recommended solutions with commercial isolators are then summarized. The isolators are selected from low-profile avionic mount series in LORD isolator catalogue.

The probable consequences, taking the isolator characteristics deviations into consideration are going to be analyzed in the next section. The standard deviation expectation is going to change the final recommended solution in the next section but the design options are going to be determined in this chapter.

The elastic enter of isolation system and the mass center of the object to be isolated should be coincident in order to have a stable system during operation. The C.G. location of the equipment must be determined initially, than according to the C.G. location, the isolator locations should be determined according to their specified characteristics.

The locations are assumed to be coincident with the mass center specified in the design as an initial estimation. But different locations for isolators may be defined via the GUI of the program by the designer during the calculations.

4.4. Case Studies

In the company of all the stated requirements and design specifications, the optimization problem is to be solved by Matlab. There are different numerical techniques in Matlab such as *fmincon*, *pattern search* or *ga*. The efficiencies of these methods are also going to be discussed for different design specifications in the sample cases presented utilizing the GUI formed for the code of optimization in this section. The code utilized in this section is supplied in the appendix.

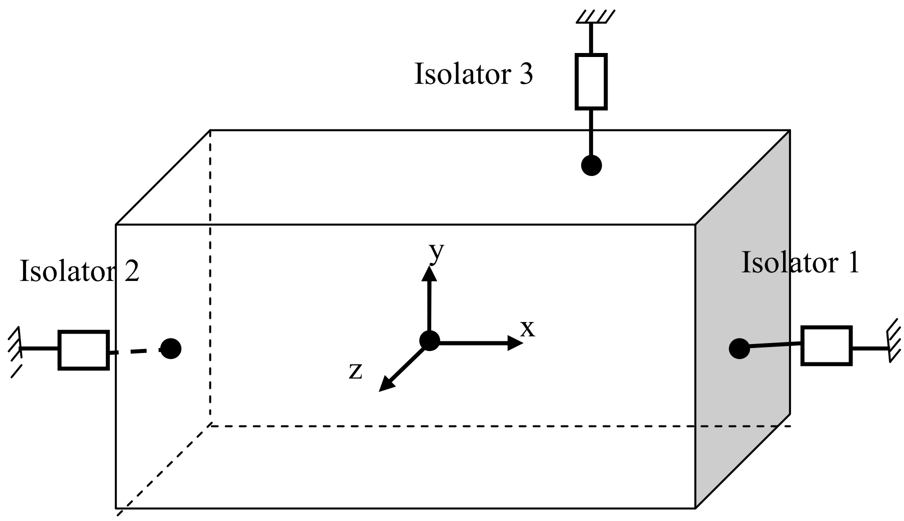

Figure 4.1. Schematic for the design in sample cases

The assumed 6-dof isolation system is shown schematically in Figure 4.1. The mass matrix properties are defined as an input to the program. The sample mass matrix calculation is explained in the appendix. When the exact object mass matrix properties are known, it should be used in the analysis. The calculations in detail are supplied in the appendix.

A single isolator representation is given in Figure 4.2 . In the figure, a more detailed schematic of a single isolator is shown. The translational sways can be in radial or axial directions. Radial and axial stiffness values are assigned to the isolator according to its mounting face of the prism. As it has also been shown in Figure 2.4, there can be mounting inaccuracies resulting in rotations in yaw, roll or pitch directions for an individual isolator.

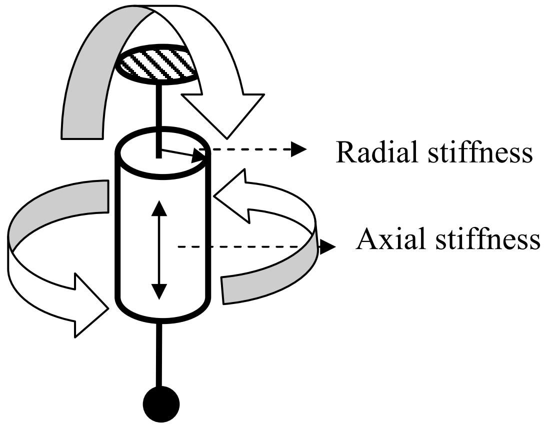

Figure 4.2. Schematic of an individual isolator

In all the sample cases, a random spectral density function distribution is used belonging to a jet aircraft cargo [50]. Different power spectrum density functions can also be used for each direction separately, via the use of the table option in the coded program. The input power spectral density graph is given in Figure 4.3.

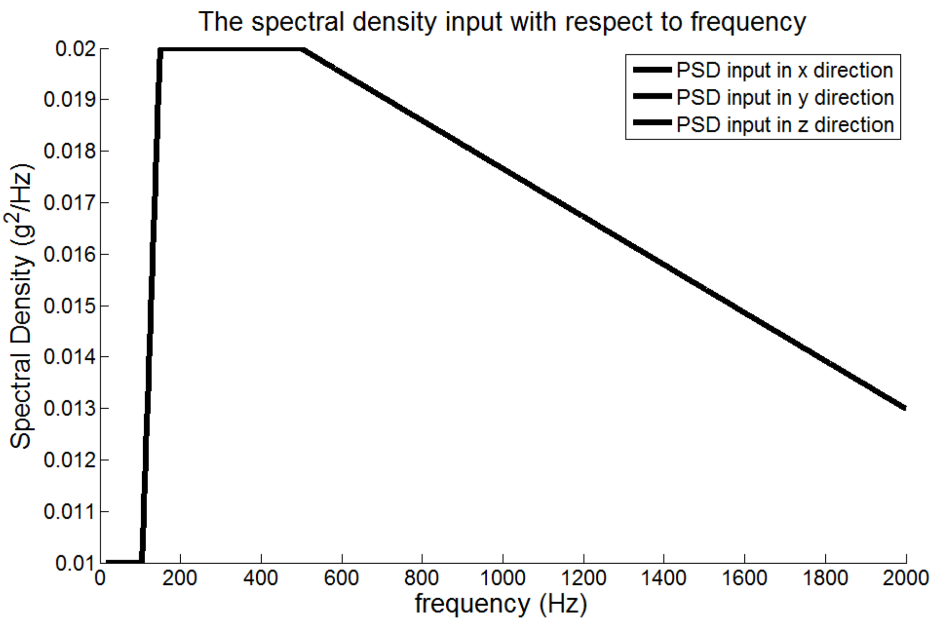

Figure 4.3. Power spectral density input for each direction

In frequency response optimization problems, 1g excitation spectrum is defined between 15Hz and 2000Hz. The input excitation is given in Figure 4.4.

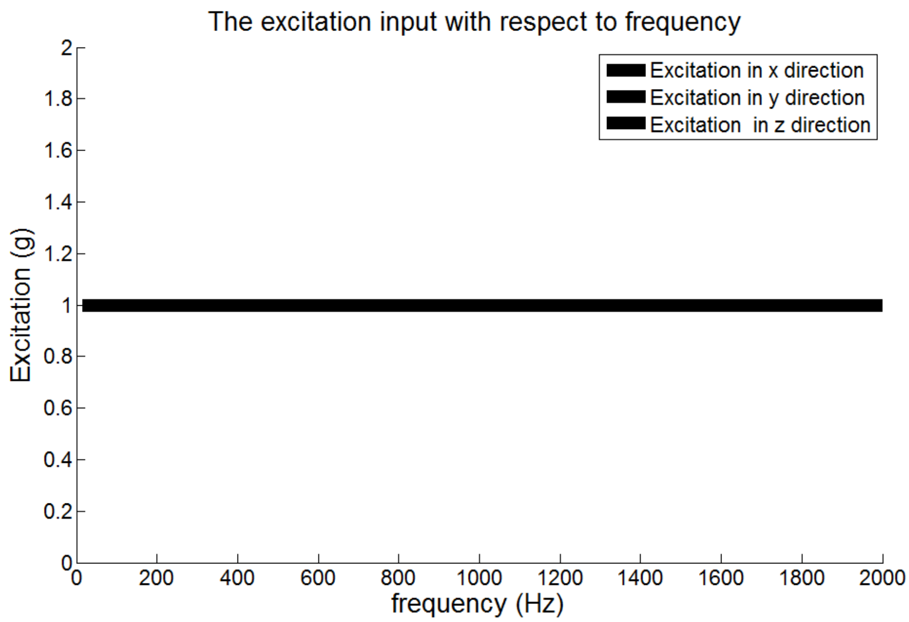

Figure 4.4. Excitation input from each direction with respect to frequency

Along with the isolation prism side length inputs, the number of isolators and the center of gravity location defined; the code proposes probable isolator locations, but the user may also input the locations of the isolators as an input to the program according to the design specifications.

Matlab can use different optimization techniques. In the scope of the thesis, three of the optimization methods are to be used. The comparison between these techniques is also going to be made. The techniques are linear minimization technique *fmincon* and genetic optimization techniques *ga* and *pattern search*. Only the *ga* technique does not use an initial estimation. When the techniques are used one after another, on the same window; the techniques using initial estimation uses the previous results as initial guess.

In the sample cases sample constraints were taken into account, which are listed in Table 4.1 These sample constraints are determined for sensitive equipment vibration isolation. Maximum displacement is taken as 2,5 mm because isolator maximum deflection is usually around this value. Maximum static deflection is deliberately taken to be short in order to let the remaining isolator sway distance to be occupied by dynamic loads and the static load on isolators are kept at lower levels this way.

Maximum rotation is desired to be kept at a minimum level and in the sample cases 1 degree of maximum rotation is considered. Maximum acceleration level is taken to be 6g, which is smaller than the sample excitation value but should not be exceeded because the sensitive components in the sample case are considered to deteriorate under loads higher than 6g. The maximum isolation frequency and direct-cross amplification ratios are also wanted to be kept at a desired maximum level. These are design requirements and they can be changed according to the requirements of the system. In literature around 100 Hz frequency is preferred to be the isolation value and the amplification ratio constraints are for the system not to have too much amplification at low frequencies because of vibration isolation. Static weight on one isolator is directly taken from the isolator catalogues to be used.

Table 4.1. Constraints for the vibration isolation design

Constraint	Value	Unit
Maximum Displacement	2,5	mm
Maximum Rotation	1	degree
Maximum Acceleration	6	g
Maximum Static Deflection	0,25	mm
Maximum Isolation Frequency	100	Hz
Maximum Direct Amplification Ratio	500	%
Maximum Cross Amplification Ratio	50	%
Maximum Static Weight on one Isolator	1,4	kg.g

The maximum excitation and loading levels in order to check the constraints given in Table 4.1 are presented in Table 4.2.

Table 4.2. Loadings in order to calculate the constraints

	x-dir	y-dir	z-dir
Maximum Static Load	2g	2g	2g
Maximum Dynamic Load *at minimum Frequency*	1g 15Hz	1g 15Hz	1g 15Hz

Assuming the geometric center of the prism is the origin, the locations of the isolators, the center of gravity location and the prism object sides are given in Table 4.3. From the table, it is seen that first isolator is on +x face, second on -x face and the third one is on +y face.

Table 4.3. Isolator locations, center of gravity location and prism object sides

	x (mm)	y mm	z mm
Isolator #1	100	-17.5	20
Isolator #2	-100	-17.5	-30
Isolator #3	90	50	-5
Center of Gravity	30	5	-5
Object Sides	200	100	100

Both of the presented optimization methods are going to be exhibited in the sample case solution. Afterwards, the effects of each constraint are going to be investigated. In addition to the constraints that were numerically taken into account there may be also physical constraints. In this section the main goal is to obtain the best optimization result and the real isolator selection is going to be exemplified in the following section considering all the criteria that must be taken into account in design of a vibration isolation system.

In optimization, both stiffness and damping characteristics are varied simultaneously in order to find out the best characteristics possible in a vibration isolation system. As explained in section 4.1; there can be two optimization methods; one minimizing the spectral response and the other minimizing the frequency response of an excitation. Both methods are going to be used for solutions. The sample cases are going to be solved by both “one type of isolator assumption” and for different types of isolators on each location.

The intervals of characteristics that are going to be used in optimization were also determined. The stiffness has been varied from 4N/mm to 100N/mm. And the loss factor has been varied from 0.25 to 0.35. The loss factor cannot be optimized accurately in a simple sense since it depends on material characteristics but the damping level can be determined approximately. So, after optimization each sample case is going to be designed with appropriate isolators according to design specifications given in section 4.3. Different design options are going to be selected for each case and the options are going to be compared considering the constraints and objective values. The number of design options selected here is for demonstration. In order to have the best design, the designer may provide any number of design options out of infinite possibilities. Even, a specified series of isolators is selected from a catalogue like LORD, in AM series there exist 135 types of isolators and even for a system consisting of three isolators there would be 135^3, i,e, 2,460,375, possible designs and it is indispensable to use an optimization step before the selection.

4.4.1. Sample Case 1

The first sample case to be presented is using the power spectral density response method, considering all the constraints. The three isolators are assumed to have the same characteristics in the first optimization. The center of gravity has been made coincident with the elastic center and the isolators are assumed to be identical; therefore, no rotation response on system is expected. The identical isolator use is recommended for highly sensitive device vibration isolation systems. The optimization is first run by genetic algorithm. Optimization with genetic algorithm is completed in approximately 24min, the results of which are given in Table 4.4.

Table 4.4. Optimization results using the genetic algorithm solution

Constraint	Value Unit	Allowed Values	Results	Value Unit
Maximum Displacement	2,03 mm	**2,5 mm**	**Axial Stiffness**	44,1 N/mm
Maximum Rotation	0,01 deg	**1 deg**	**Radial Stiffness**	45,9 N/mm
Maximum Acceleration	5,23 g	**6 g**	**Loss Factor**	0,35
Maximum Static Deflection	0,25 mm	**0,25 mm**	**Objective Value**	1,647 grms
Maximum Iso. Freq.	84 Hz	**100 Hz**	**Solution Time**	24 min
Maximum Direct Amp. Rat.	302 %	**500 %**	**Method**	G.A.
Maximum Cross Amp. Rat.	0,05 %	**150 %**		
Maximum Static Weight /Isolator	1,15 kg.g	**1,4 kg.g**		

The analysis has been continued in order to look for a better solution, using *fmincon* the objective value is not improved significantly. Then a different solution code *pattern search* has been used, giving the starting point as the mid-point of intervals of characteristics as the initial - point. The pattern search directly finds a similar solution in 85 seconds which is much shorter compared to the *ga* method for which the objective is the same as the one found for the genetic algorithm solution. The solution time for *pattern search* and the precision of this solution depends on the initial guess so it may not be preferable in all situations. The *fmincon* has been run again starting from the solution of pattern search, and the same results, as in the case of *genetic algorithm* solution, were obtained.

The found solution is the best one but since this problem case is not much complex, the solution can as well be obtained utilizing only *fmincon* in a short time, 29 seconds, taking the mid-point of intervals as the initial point.

The best result which can be directly obtained by *fmincon* is shown in Table 4.5.

Table 4.5. Optimization results using Fmincon solution

Constraint	Value Unit	Allowed Values	Results	Value Unit
Maximum Displacement	2,0 mm	**2,5 mm**	**Axial Stiffness**	45,30 N/mm
Maximum Rotation	0 deg	**1 degree**	**Radial Stiffness**	45,30 N/mm
Maximum Acceleration	5,24 g	**6 g**	**Loss Factor**	0,35
Maximum Static Deflection	0,25 mm	**0,25 mm**	**Objective Value**	1,647 grms
Maximum Iso. Freq.	83 Hz	**100 Hz**	**Solution Time**	29 sec
Maximum Direct Amp. Rat.	303 %	**500 %**	**Method**	Fmincon
Maximum Cross Amp. Rat.	0 %	**150 %**		
Maximum Static Weight /Isolator	1,15 kg.g	**1,4 kg.g**		

Since in this first sample case, the assumption of identical isolators was made, the number of variables does not depend on the number of isolators and three variables may well be solved only by *fmincon*. This situation may not be valid for every problem so it is recommended to use genetic algorithm initially to search for the best solution independent of initial guess.

The first sample case has the assumption of identical isolators on three locations. The best solution satisfying the constraints has been found such that each isolator has equal axial and radial stiffness values equal to 45.30 N/mm. The loss factor has been estimated to be 0.35, so the isolators having loss factor 0.35 is going to be used in the design instead of the isolators having 0.25 loss factor.

For this case study, the assumed three isolators are identical, the selected isolator should have properties equal to or greater than the optimized solution in order to assure the sway constraints are satisfied. The maximum isolation frequency and the maximum static load on isolator possible would increase so should be checked. From the mentioned low-profile avionic mount series from the AM001 series isolators of 0.35 loss factor, the one with 50N/mm axial stiffness and 41N/mm radial stiffness isolator might be appropriate. The other design possibilities are listed in Table 4.6.

Table 4.6. Probable design options for sample case 1

Design Option	*Isolator Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
1	**AM001**	1,4 kg.g	50 N/mm	41 N/mm	0,35
2	**AM003**	2,0 kg.g	49 N/mm	54 N/mm	0,35
3	**AM003**	2,0 kg.g	56 N/mm	62 N/mm	0,35
4	**AM005**	2,7 kg.g	62 N/mm	48 N/mm	0,35

In this step of the design, all probable options should be checked via the simulation code in order to control whether the selections are appropriate or not. The initial simulations are done without considering probable variance in characteristics or in locations and mounting positions. These variations are going to be taken into account in Chapter 6 in order to obtain a reliable design. The estimated critical constraints for each design are summarized in **Table 4.7**.

Table 4.7. Comparison of design options for sample case 1

Design Option	**Allowed Static Weight**	**Estimated Static Weight**	**Allowed Static Deflection**	**Estimated Static Deflection**
1	1,4 kg.g	1,17 kg.g	0,25 mm	0,26 mm
2	2,0 kg.g	1,16 kg.g	0,25 mm	0,22 mm
3	2,0 kg.g	1,16 kg.g	0,25 mm	0,19 mm
4	2,7 kg.g	1,18 kg.g	0,25 mm	0,22 mm

Analyzing **Table 4.7**, it is concluded that the first option would not be satisfying the maximum static deflection constraint. The other three options seem to work properly in the ideal design.

4.4.2. Sample Case 2

The second case is for the same problem in sample case 1 but this time the solution is going to be searched by using three different isolators. The stiffness characteristics for all isolators were determined separately for each but the loss factor for all of the isolators in the system were taken to be the same in order to make the system analytically solvable easily. Because when all the isolators have different loss factor values, the damping matrix $[H]$ would not be proportional to stiffness matrix $[K]$ and the formulations in Chapter 3 would not be correct. The calculation of damping matrix would take extra time. The same procedure has been followed as in sample case 1. Initially the genetic algorithm solution has been obtained. The results are given in Table 4.8.

Table 4.8. The results for the genetic algorithm solution for different isolators

Constraint	Value	Unit	Allowable Values		Results	Value	Unit
Maximum Displacement	2,0	mm	**2,5**	**mm**	**Axial Stiffness1**	46,1	N/mm
					Axial Stiffness2	44,8	N/mm
Maximum Rotation	0,01	deg	**1**	**degree**	**Axial Stiffness3**	48,2	N/mm
Maximum Acceleration	5,15	g	**6**	**g**	**Radial Stiffness1**	50,1	N/mm
					Radial Stiffness2	53,5	N/mm
Maximum Static Deflection	0,22	mm	**0,25**	**mm**	**Radial Stiffness3**	58,7	N/mm
Maximum Iso. Freq.	91	Hz	**100**	**Hz**	**Loss Factor**	0,35	
Maximum Direct Amp. Rat.	302	%	**500**	**%**	**Objective Value**	1,702	grms
Maximum Cross Amp. Rat.	1,74	%	**150**	**%**	**Solution Time**	16	min
Maximum Static Weight /Isolator	1,24	kg.g	**2**	**kg.g**	**Method**	G.A.	

When only *fmincon* is used, the result is not always optimum for this case because it gives only the local minimum according to the initial guess. This problem has a lot of local minimums because it has many variables. So, it is recommended to use *fmincon* after other optimization techniques for this problem. After obtaining *genetic algorithm* (GA) solution, it is improved further by using *fmincon* by using the genetic algorithm solution as the initial guess. This gives an objective value slightly less than one found in sample case 1 where the same problem has been solved with identical isolator assumption. This result is summarized in Table 4.9.

Table 4.9. Optimization results for genetic algorithm plus fmincon solution for different isolators

Constraint	Value	Unit	Allowed Values		Results	Value	Unit
Maximum Displacement	2,0	mm	**2,5**	**mm**	**Axial Stiffness1**	41,9	N/mm
					Axial Stiffness2	40,8	N/mm
Maximum Rotation	0,06	deg	**1**	**degree**	**Axial Stiffness3**	43,9	N/mm
Maximum Acceleration	5,11	g	**6**	**g**	**Radial Stiffness1**	40,0	N/mm
					Radial Stiffness2	47,6	N/mm
Maximum Static Deflection	0,25	mm	**0,25**	**mm**	**Radial Stiffness3**	54,9	N/mm
Maximum Iso. Freq.	86	Hz	**100**	**Hz**	**Loss Factor**	0,35	
Maximum Direct Amp. Rat.	302	%	**500**	**%**	**Objective Value**	1,641	grms
Maximum Cross Amp. Rat.	3.5	%	**150**	**%**	**Solution Time**	18	min
Maximum Static Weight /Isolator	1,3	kg.g	**2**	**kg.g**	**Method**	G.A. fmincon	

For sample case two, the same problem has been solved using different types of isolators, on three locations. The optimization results are given in table 4.6 for this design case. The recommended designs for this case are presented in Table 4.10.

Table 4.10. Probable design options for sample case 2

Design Option	*Isolator 1 Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
-1st -	**AM001**	1,4 kg.g	43 N/mm	36 N/mm	0,35
	AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35
	AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35
Design Option	*Isolator 2 Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
-2nd -	**AM001**	1,4 kg.g	50 N/mm	41 N/mm	0,35
	AM003	2,0 kg.g	43 N/mm	47 N/mm	0,35
	AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35
Design Option	*Isolator 3 Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
-3rd-	**AM002**	1,6 kg.g	40 N/mm	40 N/mm	0,35
	AM003	1,8 kg.g	40 N/mm	44 N/mm	0,35
	AM003	2,0 kg.g	56 N/mm	62 N/mm	0,35
Design Option	*Isolator 4 Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
-4th-	**AM001**	1,4 kg.g	50 N/mm	41 N/mm	0,35
	AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35
	AM003	2,0 kg.g	56 N/mm	62 N/mm	0,35

These recommendations have been made approximately close to the optimization results with real isolators. All the design options have been analyzed at this stage and it is seen that none of these options are violating any of the constraints. The most critical values close to the constraints are shown in Table 4.11.

Table 4.11. Comparison of design options for sample case 2

Design Option	**Allowed Static Weight**	**Estimated Static Weight**	**Allowed Static Deflection**	**Estimated Static Deflection**
1st	1,4 kg.g	0,95 kg.g	0,25 mm	0,24 mm
2nd	1,4 kg.g	1,08 kg.g	0,25 mm	0,24 mm
3rd	1,6 kg.g	0,98 kg.g	0,25 mm	0,24 mm
4th	1,4 kg.g	0,98 kg.g	0,25 mm	0,22 mm

4.4.3. Sample Case 3

In this sample case, the problem's objective function is different and so formulation is different as explained in Section 4.1.2, the frequency response method is used in order to find the objective value in response to an excitation defined in Figure 4.4. This problem, even by genetic algorithm is solved faster than the spectral response optimization. Optimization results for *G.A.* solution are summarized in

Table **4.12**.

Table 4.12. The results for the genetic algorithm solution of sample case 3

Constraint	Value Unit	Allowed Values	Results	Value Unit
Maximum Displacement	2,0 mm	**2,5 mm**	**Axial Stiffness**	41,7 N/mm
Maximum Rotation	0,02 deg	**1 degree**	**Radial Stiffness**	47,3 N/mm
Maximum Acceleration	5,16 g	**6 g**	**Loss Factor**	0,35
Maximum Static Deflection	0,25 mm	**0,25 mm**	**Objective Value**	176,05 g.Hz
Maximum Iso. Freq.	85 Hz	**100 Hz**	**Solution Time**	5 min
Maximum Direct Amp. Rat.	303 %	**500 %**	**Method**	G.A.
Maximum Cross Amp. Rat.	0,44 %	**150 %**		
Maximum Static Weight /Isolator	1,16 kg.g	**2 kg.g**		

Similar behavior has been observed as in sample case 1, when the solution has been tried to be improved by other optimization techniques. The *fmincon* solution gives the same result independent of the initial guess values for this case when all the designed isolators are taken to be the same.

Fmincon solution takes only 16 seconds. The other details of this result are given in Table 4.13.

Table 4.13. Optimization results for fmincon solution of sample case 3

Constraint	Value Unit	Allowed Values	Results	Value Unit
Maximum Displacement	2,0 mm	**2,5 mm**	**Axial Stiffness**	45,30 N/mm
Maximum Rotation	0 deg	**1 deg**	**Radial Stiffness**	45,30 N/mm
Maximum Acceleration	5,24 g	**6 g**	**Loss Factor**	0,35
Maximum Static Deflection	0,25 mm	**0,25 mm**	**Objective Value**	175,94 g.Hz
Maximum Iso. Freq.	83 Hz	**100 Hz**	**Solution Time**	16 sec
Maximum Direct Amp. Rat.	303 %	**500 %**	**Method**	Fmincon
Maximum Cross Amp. Rat.	0 %	**150 %**		
Maximum Static Weight /Isolator	1,15 kg.g	**1,4 kg.g**		

Therefore, it is again reconfirmed that the optimization problems with low number of variables are solved faster by basic methods. On the next case, this problem is also going to be solved with more variables, thus without identical isolators assumption. The solution time will get longer and the objective value is expected to decrease.

The third sample case has the assumption of identical isolators on three locations. The best solution satisfying the constraints has been found such that each isolator has equal axial and radial stiffness values equal to 45.30 N/mm. The loss factor has been estimated to be 0.35, so the isolators having loss factor 0.35 is going to be used in the design instead of the isolators having 0.25 loss factor. Since the assumed three isolators are identical, the selected isolator must have properties equal to or greater than the optimized solution in order to assure the sway constraints are satisfied. The maximum isolation frequency and the maximum static load on isolator possible would increase so should be checked.

From the mentioned low-profile avionic mount series from the AM001 series isolators of 0.35 loss factor, the one with 50N/mm axial stiffness and 41N/mm radial stiffness isolator might be appropriate. The other design possibilities are listed in Table 4.14.

Table 4.14. Probable design options for sample case 3

Design Option	*Isolator Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
1	**AM001**	1,4 kg.g	50 N/mm	41 N/mm	0,35
2	**AM003**	2,0 kg.g	49 N/mm	54 N/mm	0,35
3	**AM003**	2,0 kg.g	56 N/mm	62 N/mm	0,35
4	**AM005**	2,7 kg.g	62 N/mm	48 N/mm	0,35

In this step of the design, all probable options should be checked via the simulation code in order to control whether the selections are appropriate or not. The initial simulations are done without considering probable variance in characteristics or in locations and mounting positions. These variations are going to be taken into account in section 6 in order to obtain a reliable design.

The estimated critical constraints for each design are summarized in Table 4.15.

Table 4.15. Comparison of design options for sample case 3

Isolator Series	**Maximum Static Weight**	**Estimated Static Weight**	**Maximum Static Deflection**	**Estimated Static Deflection**
AM001	1,4 kg.g	1,17 kg.g	0,25 mm	0,26 mm
AM003	2,0 kg.g	1,16 kg.g	0,25 mm	0,22 mm
AM003	2,0 kg.g	1,16 kg.g	0,25 mm	0,19 mm
AM005	2,7 kg.g	1,18 kg.g	0,25 mm	0,22 mm

Analyzing Table 4.15, it is concluded that the first option would not be satisfying the maximum static deflection constraint. The other three options seem to work properly in the ideal design.

4.4.4. Sample Case 4

The fourth case is the same problem as in sample case 3 but the three isolators are considered to be different. The loss factor for all of the isolators in the system was taken to be the same again in order to make the system analytically solvable. Optimization results using genetic algorithm are given in Table 4.16.

Table 4.16. The results for the genetic algorithm solution of sample case 4

Constraint	**Value**	**Unit**	**Allowed Values**		**Results**	**Value**	**Unit**
Maximum Displacement	2,0	mm	**2,5**	**mm**	**Axial Stiffness1** **Axial Stiffness1**	41,1 38,7	N/mm N/mm
Maximum Rotation	0.03	deg	**1**	**deg**	**Axial Stiffness1**	41,6	N/mm
Maximum Acceleration	5,15	g	**6**	**g**	**Radial Stiffness** **Radial Stiffness**	50,8 42,1	N/mm N/mm
Maximum Static Deflection	0,25	mm	**0,25**	**mm**	**Radial Stiffness**	51,2	N/mm
Maximum Iso. Freq.	85	Hz	**100**	**Hz**	**Loss Factor**	0,35	
Maximum Direct Amp. Rat.	302	%	**500**	**%**	**Objective Value**	175,95	g.Hz
Maximum Cross Amp. Rat.	2,65	%	**150**	**%**	**Solution Time**	21	min
Maximum Static Weight /Isolator	1,23	kg.g	**2**	**kg.g**	**Method**	G.A.	

Afterwards, running fmincon using genetic algorithm results changes the final result a little more which is represented on Table 4.17. The result here seems close to the one found in sample case 3, but this point found by G.A. improves better by fmincon.

Table 4.17. The results for the genetic algorithm plus fmincon solution for different isolators

Constraint	Value Unit	Allowed Values	Results	Value Unit
Maximum			**Axial Stiffness1**	44,9 N/mm
Displacement	2,0 mm	**2,5 mm**	**Axial Stiffness1**	44,8 N/mm
Maximum			**Axial Stiffness1**	45,5 N/mm
Rotation	0,07 deg	**1 deg**		
Maximum			**Radial Stiffness**	55,6 N/mm
Acceleration	4,72 g	**6 g**	**Radial Stiffness**	32,0 N/mm
Maximum			**Radial Stiffness**	56,0 N/mm
Static Deflection	0,25 mm	**0,25 mm**		
Maximum Iso. Freq.	86 Hz	**100 Hz**	**Loss Factor**	0,35
Maximum Direct Amp. Rat.	301 %	**500 %**	**Objective Value**	174,44 g.Hz
Maximum Cross Amp. Rat.	9,74 %	**150 %**	**Solution Time**	2 min
Maximum Static Weight /Isolator	1,34 kg.g	**2 kg.g**	**Method**	Fmincon

It was expected to have a better objective value in this case study than the previous case study with assumptions. The difference is not so much but it would have been larger if there were more number of isolators bringing more number of variables. All of the optimization results obtained in this part are recommendations for real isolation system design initialization. In the next section, real isolator selection according to the optimization results for the sample cases and other physical recommendations are going to be achieved. The designs with the selected isolators are going to be demonstrated.

For sample case four, the same problem has been solved as the third sample case using different types of isolators, on three locations. The optimization results are given in Table 4.17 for this design case. The recommended designs for this case are presented in Table 4.18.

Table 4.18. Probable design options for sample case 4

Design Option	*Isolator 1 Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
-1st -	**AM003**	2,0 kg.g	43 N/mm	47 N/mm	0,35
	AM001	1,4 kg.g	50 N/mm	41 N/mm	0,35
	AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35
Design Option	*Isolator 2 Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
-2nd -	**AM003**	2,0 kg.g	49 N/mm	54 N/mm	0,35
	AM002	1,6 kg.g	40 N/mm	40 N/mm	0,35
	AM003	2,0 kg.g	56 N/mm	62 N/mm	0,35
Design Option	*Isolator 3 Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
-3rd-	**AM003**	2,0 kg.g	56 N/mm	62 N/mm	0,35
	AM001	1,4 kg.g	43 N/mm	36N/mm	0,35
	AM003	2,0 kg.g	43 N/mm	47 N/mm	0,35
Design Option	*Isolator 3 Series*	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**
-4th-	**AM003**	2,0 kg.g	56 N/mm	62 N/mm	0,35
	AM001	1,4 kg.g	50 N/mm	41N/mm	0,35
	AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35

All these options are appropriate for design when, deviations in characteristics and locations are not considered as shown in Table 4.19. The probable variations in characteristics and locations are going to be analyzed before making the final design decision.

Table 4.19. Comparison of design options for sample case 3

Design Option	Maximum Static Weight	Estimated Static Weight	Maximum Static Deflection	Estimated Static Deflection
1^{st}	1,4 kg.g	1,08 kg.g	0,25 mm	0,24 mm
2^{nd}	2,0 kg.g	1,38 kg.g	0,25 mm	0,23 mm
3^{rd}	2,0 kg.g	1,49 kg.g	0,25 mm	0,24 mm
4^{th}	1,4 kg.g	0,99 kg.g	0,25 mm	0,22 mm

Several designs are proposed in this section for the sample case assuming the characteristics in catalogue do not have any deviations. The deviations are going to be taken into account in the next chapter.

The important thing here is to note that the optimization results have come to be different from the sample case 2 in sample case 4. So the recommended designs have also become different.

Having the exact properties several designs are recommended seeming appropriate for the specified design constraints. In the next chapter, the probable deviations in characteristics of isolators and mounting location and orientation deviations are going to be taken into account before making the final decision in design. A design may seem to have the best isolation attributes but it is going to be demonstrated that “the best” solution is not always “the most reliable” one in the real world.

CHAPTER 5

DESIGN CHECK

The Monte Carlo simulation is used in order to determine the expected response of the isolation system in case of the assumed design parameters deviate from the original expected values. Both the spectral response characteristics and the frequency response characteristics are subject to change across the frequency range with the change of the system parameters.

The effects of change are going to be considered in two broad classes:

1- The deviation in the stiffness and damping characteristics
2- The deviations of orientations (in the locations and connection angles) of isolators

In the first class, the deviations in the isolator characteristics are going to be considered. This analysis should be carried out because the material used in isolators may deviate in both stiffness and damping characteristics. Then, in the second class, the probable deviations in the determined locations of isolators were considered, even the deviations are small, these deviations may lead to unwanted rotation displacements of the sensor equipment leading to inaccurate measurements. So the effect of deviations should be limited to a predicted level. In addition to this, even when the connection angles are meant to be perpendicular, they may not be so leading to different resultant characteristics of isolators in the system.

For the first class of deviations an effect of 10% standard deviation in properties are considered. If the isolators utilized have more or less accurate properties than expected, this value may be adjusted. For the second class of deviations finite deviation values are going to be considered which can be changed according to the design case. In the sample case explained in the thesis, 10 mm of maximum displacement standard deviation and 10 degrees of maximum orientation angle standard deviation are considered. The standard deviation magnitudes can be adjusted according to design case. The uniformly distributes pseudo numbers are used in simulation.

In order to specify the samples, characteristics are deviated the number of times required, and different sets of design characteristics are obtained for each design. These sets are simulated one by one and the results are stored, and in the end the stored results are displayed together. It is checked whether any one of the constraints is exceeded in any of the samples. If even with characteristic deviations, the constraints are satisfied, the selected design is assumed to be reliable.

The deviations are going to be tested according to the sample cases for the vibration isolation system. The test is going to be achieved by Monte Carlo simulation technique. The optimum isolators with precise characteristics are selected in the previous work, so in this section, the optimization of the objective function is far less significant than the conformity of the constraints in case of deviations in characteristics. Nevertheless, the design option having the least objective value from the design options having the conformity of constraints is going to be selected.

Throughout this section, the indispensability of using a safety factor in design as result of analysis of the vibration isolation system is going to be shown. Using the best solution for having the minimum objective function may lead to the violation of constraints in case of deviations. The deviations in characteristics in reality are going to be present, and sometimes they may deviate also with respect to frequency or time. The analysis work in this section is going to lead a fail-safe design by not using a simple safety factor, but simulating the worst cases.

5.1. Sample Case 1

The three remaining design options from the previous chapter are simulated by Monte Carlo simulation with 10% standard deviation in characteristics, 10 mm maximum deviation in locations and 10 degrees of mounting orientation angle of isolators for necessary number of times. 1000 cases were simulated and in the end of the analysis, the constraints are checked. The state of design options in case of deviations are summarized in Table 5.1.

Table 5.1. The state of design options in case of deviations

Design Option	*Constraints*	Max.OBJ	Avg.OBJ	Min.OBJ	
2	*satisfied*	1,790	1,698	1,610	**grms**
3	*not satisfied*	1,876	1,767	1,685	**grms**
4	*satisfied*	1,811	1,700	1,602	**grms**

The first option for this design has been eliminated in the previous section due to excessive static deflection. The third option is also eliminated at this stage because it would not satisfy the constraints in case of deviations in characteristics and locations as the results are presented in Table 5.2.

Table 5.2. The constraints of design options in case of deviations

Constraint	OPTION 2	OPTION 3	OPTION 4	max
Maximum Displacement	2,03 mm	2,01 mm	2,03 mm	**2,5 mm**
Maximum Rotation	0,18 deg	0,19 deg	0,22 deg	**1 deg**
Maximum Acceleration	5,67 g	5,74 g	5,52 g	**20 g**
Maximum Static Deflection	0,24 mm	0,21 mm	0,24 mm	**0,25 mm**
Maximum Iso. Freq.	94 Hz	***101 Hz***	97 Hz	**100 Hz**
Maximum Direct Amp. Rat.	330 %	332 %	309 %	**500 %**
Maximum Cross Amp. Rat.	31 %	42 %	37 %	**150 %**
Maximum Static	**2,0 kg.g**	**2,0 kg.g**	**2,7 kg.g**	**max**
Weight /Isolator	1,29 kg.g	1,30 kg.g	1,32 kg.g	

Investigating the constraint values for each design option for sample case 1, it is clear that option three is not also appropriate since the maximum isolation frequency constraint is not satisfied. The remaining design option attributes are given in **Table 5.3**.

Table 5.3. Possible Isolator Attributes for the Vibration Isolation System

Design Option	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**	**System Natural Frequencies (Hz)**
2	2,0 kg.g	49 N/mm	54 N/mm	0,35	43-62-63-64-92-109
4	2,7 kg.g	62 N/mm	48 N/mm	0,35	42-60-63-66-88-105

Both design options two and four are possible for the given design requirements. If the system is going to be used where there exists excitations at the isolation system's natural frequencies, then the design has to be changed. Here, since option two has lower "isolation frequency", 94Hz, it can be selected but if the maximum static weight increases, option 4 would be a better choice since its maximum static weight capacity is higher. The objective and other constraint changes in simulation for option 2 are presented in Figures 5.1 to 5.9. On the figures only 100 samples are shown for clarity in the figures.

In Figure 5.10, the effectiveness of the isolation system for high frequencies can be observed. After the isolation frequency, the response decreases and after two times the isolation frequency response reach to a value that is close to zero level.

The analyses were carried out up to 2000 Hz, but in Figures 5.10 to 5.13, the results have been shown up to 200 Hz in order to demonstrate the system's effect on response around the natural frequency. Moreover, since the isolation frequency is much lower than 200 Hz, response of the system is negligible beyond this frequency.

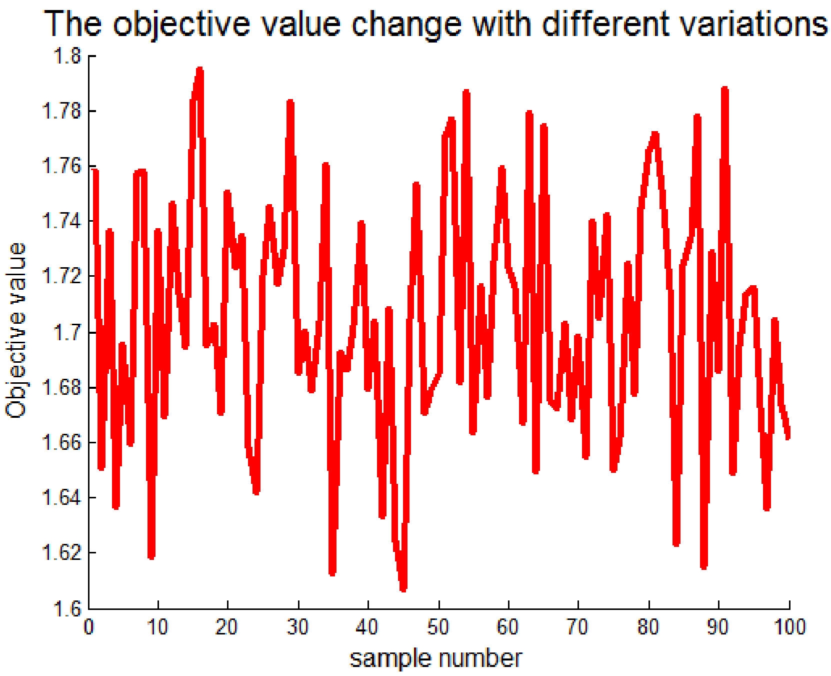

Figure 5.1. The objective value variation for the design samples

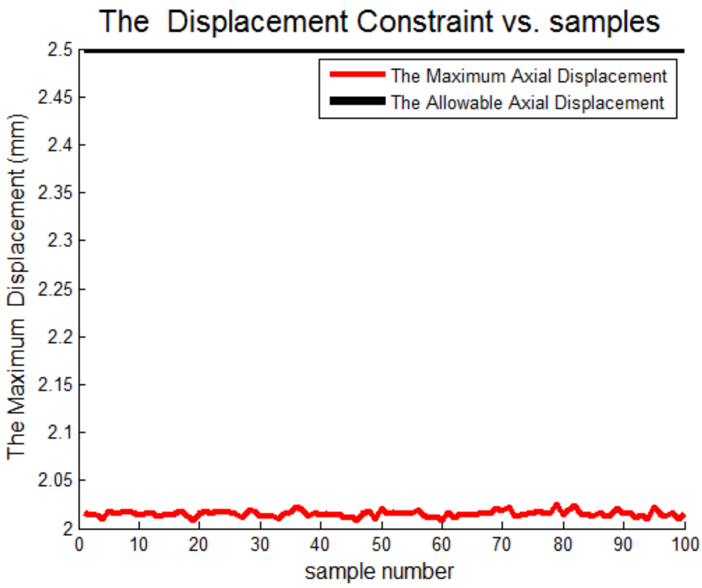

Figure 5.2. The maximum dynamic displacement of the design samples

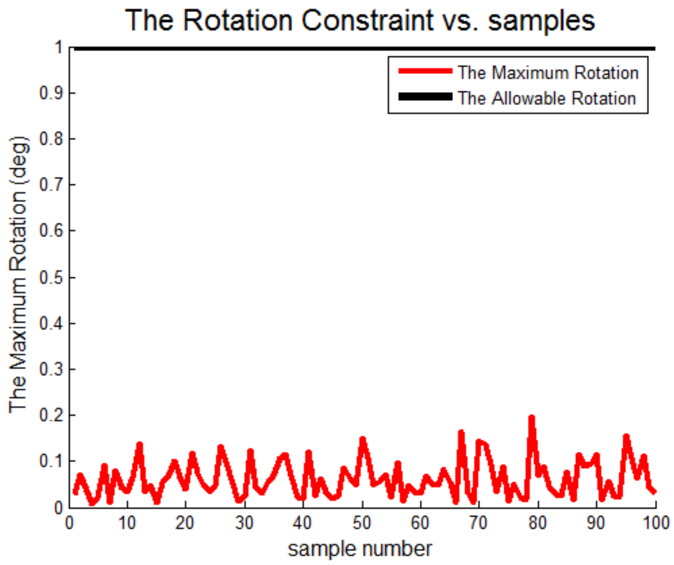

Figure 5.3. The maximum dynamic rotation of the design samples

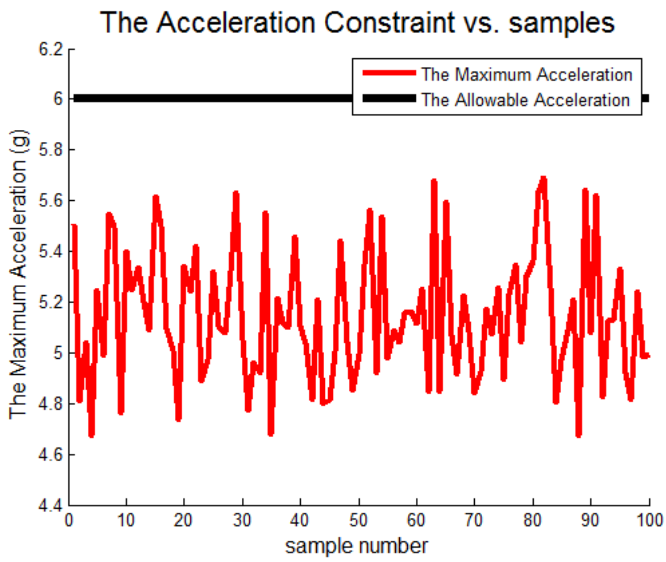

Figure 5.4. The maximum acceleration of the design samples

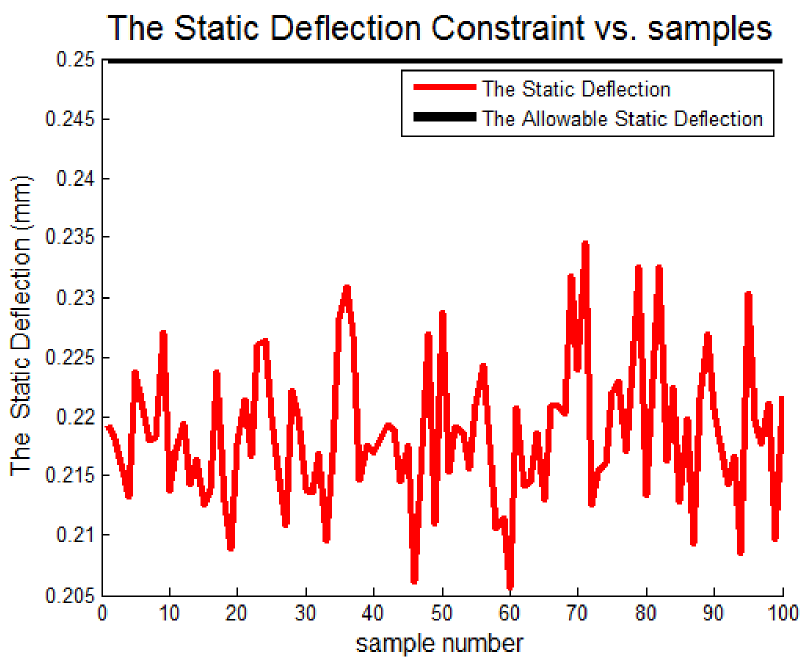

Figure 5.5. The maximum static deflection of the design samples

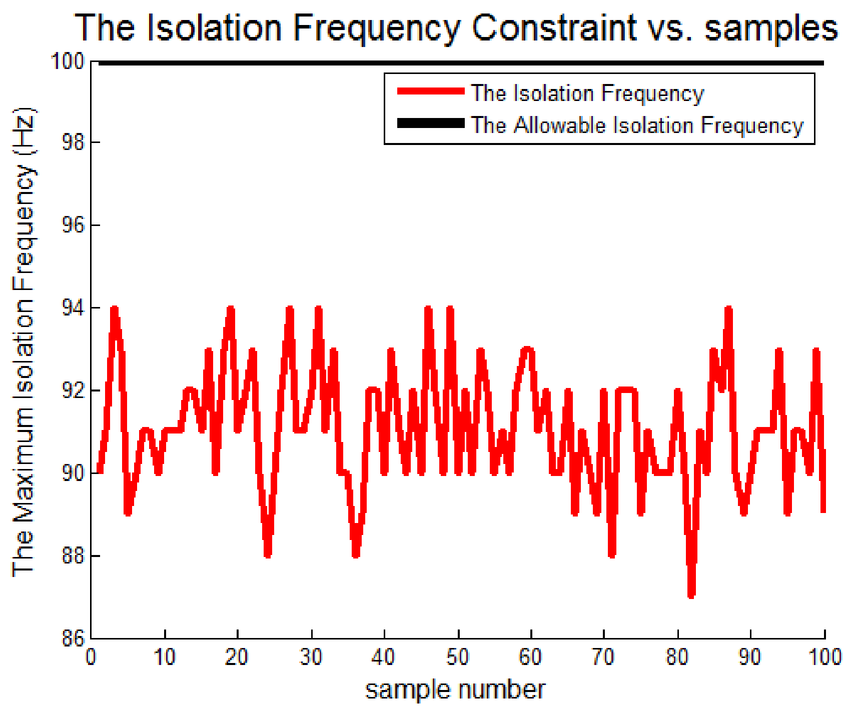

Figure 5.6. The maximum isolation frequency of the design samples

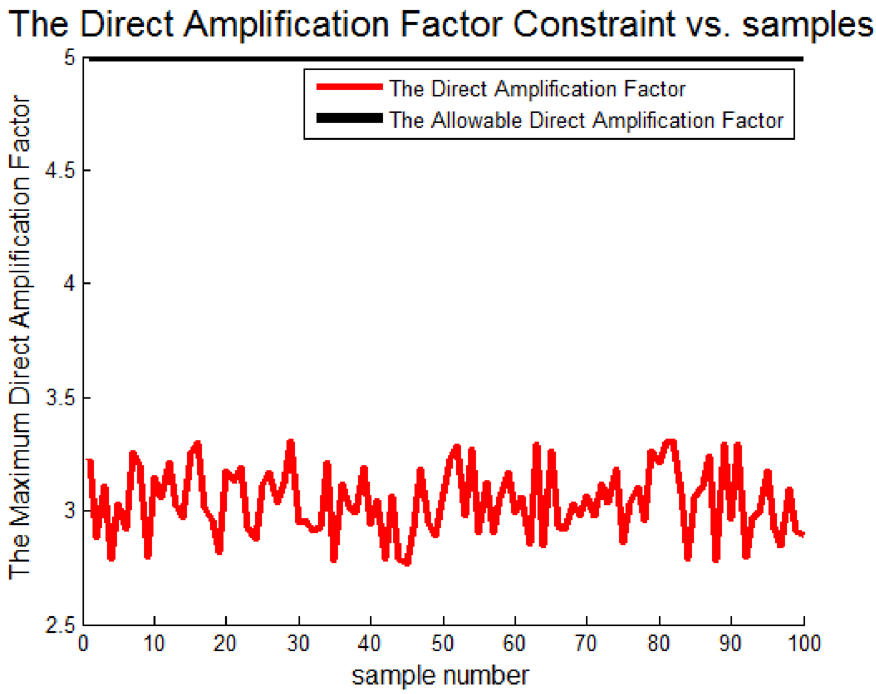

Figure 5.7. The direct amplification factor of the design samples

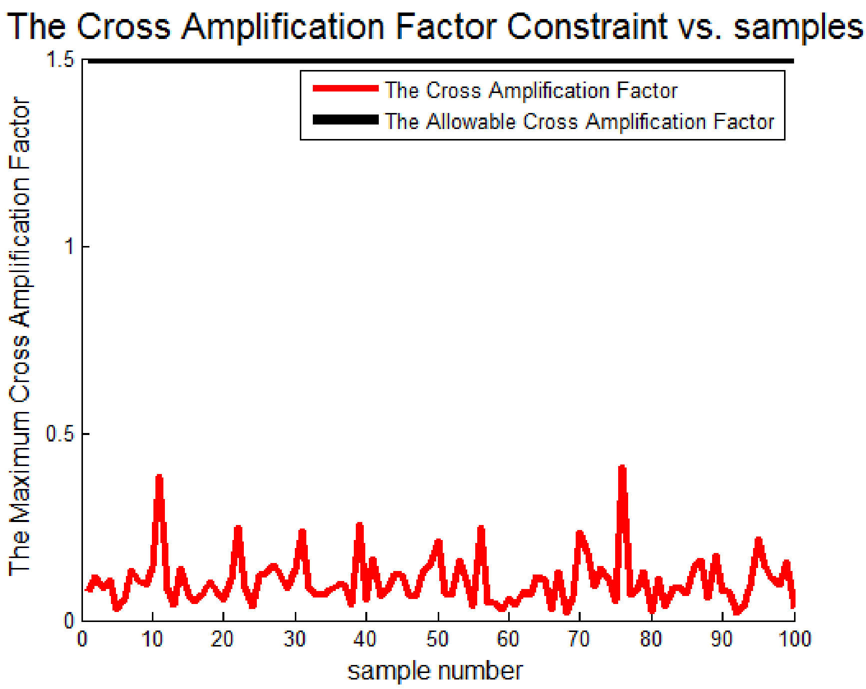

Figure 5.8. The cross amplification factor of the design samples

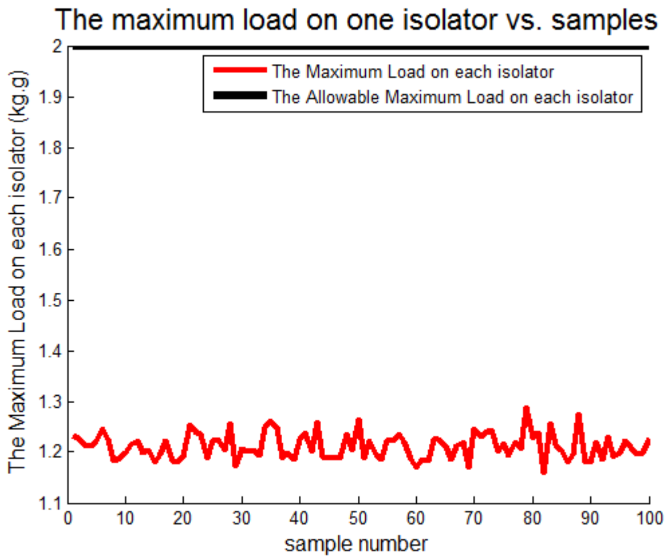

Figure 5.9. The maximum load on one isolator for the design samples

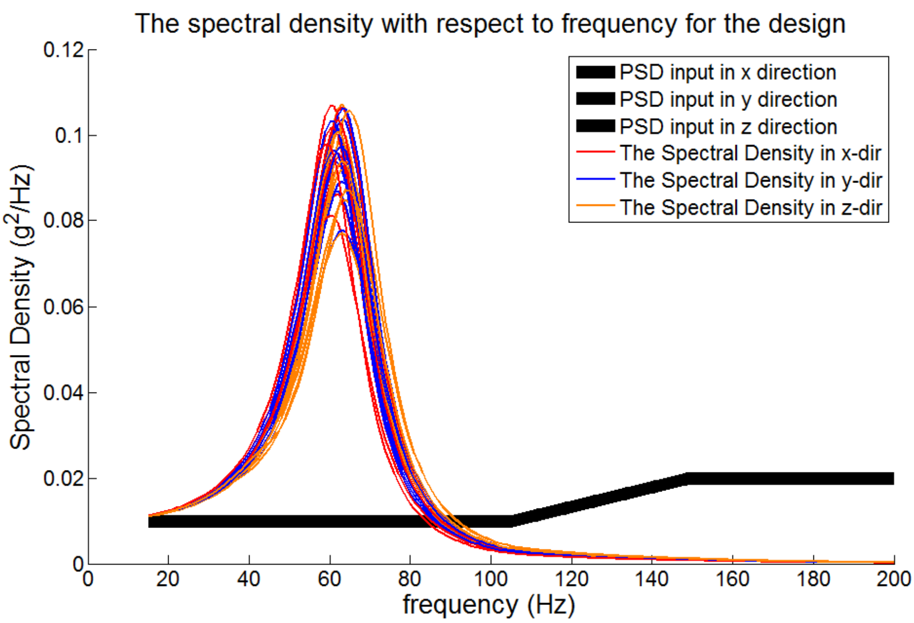

Figure 5.10. The spectral density with respect to frequency for the design samples of case 1 option 2

5.2. Sample Case 2

All four options are simulated by Monte Carlo simulation with 10% standard deviation in characteristics, 10 mm maximum deviation in locations and 10 degrees of mounting orientation angle of isolators for necessary number of times. In the end of the analysis, the constraints are checked. The state of design options in case of deviations are summarized in Table 5.4.

Table 5.4. The state of design options in case of deviations

OPTION	***Constraints***	**Max.OBJ**	**Avg.OBJ**	**Min.OBJ**	unit
1	*not satisfied*	1,732	1,638	1,550	**grms**
2	*not satisfied*	1,755	1,645	1,554	**grms**
3	*not satisfied*	1,760	1,639	1,552	**grms**
4	*satisfied*	1,797	1,691	1,600	**grms**

The first three options are eliminated at this stage because of maximum static deflection constraint. The constraint situations are presented in Table 5.5.

Table 5.5. The constraints of design options in case of deviations

Constraint	1	2	3	4	
Maximum Displacement	2,04mm	2,04mm	2,04mm	2,03mm	**2,5mm**
Maximum Rotation	0,25deg	0,26deg	0,25deg	0,28deg	**1deg**
Maximum Acceleration	5,57g	5,65g	5,72g	5,68g	**6g**
Maximum Static Deflection	*0,30mm*	*0,26mm*	*0,30mm*	0,24mm	**0,25mm**
Maximum Iso. Freq.	90Hz	90Hz	90Hz	93Hz	**100Hz**
Maximum Direct Amp. Rat.	329%	329%	331%	331%	**500%**
Maximum Cross Amp. Rat.	40%	44%	37%	46%	**150%**
Maximum Static Weight /Isolator	**1,4kg.g** 1,06kg.g	**1,4kg.g** 1,18kg.g	**2kg.g** 1,67kg.g	**1,4kg.g** 1,1kg.g	

The isolator attributes in the selected design option is given in Table 5.6.

Table 5.6. Isolator Attributes for the Vibration Isolation System

Isolator s	**Maximum SW**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**	**System Natural Frequencies (Hz)**
1-AM001	1,4 kg.g	50 N/mm	41 N/mm	0,35	41-61.6 63.6-63.9 91.5-109.2
2-AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35	
3-AM003	2,0 kg.g	56 N/mm	62 N/mm	0,35	

It is clear from the table that choosing the fourth option is the best for the system. The spectral density response behavior of the design is given in Figure 5.11.

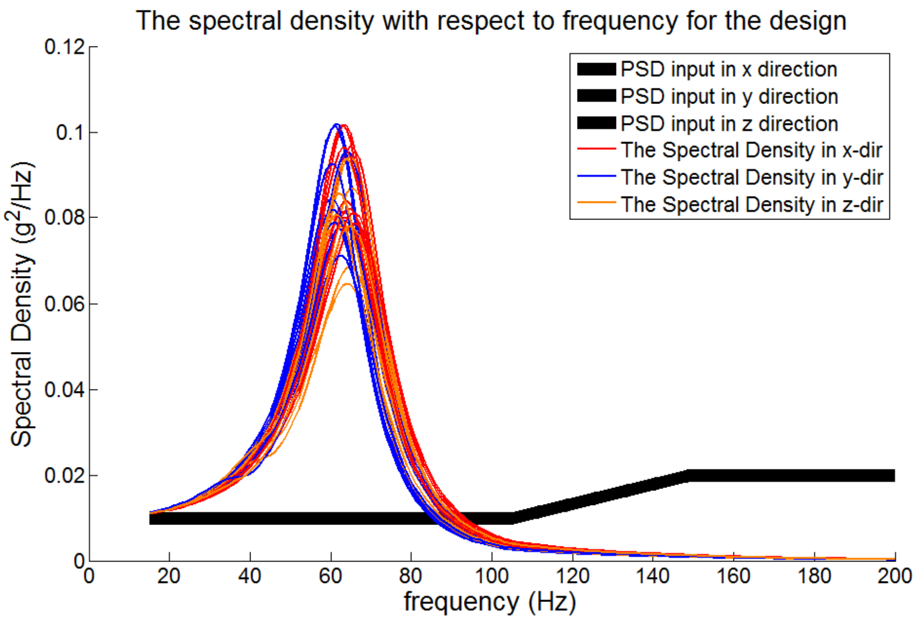

Figure 5.11. The spectral density with respect to frequency for case 2 option 4

5.3. Sample Case 3

All four options are simulated by Monte Carlo simulation with 10% standard deviation in characteristics, 10 mm maximum deviation in locations and 10 degrees of mounting orientation angle of isolators for necessary number of times. In the end of the analysis, the constraints are checked. The state of design options in case of deviations are summarized in Table 5.7.

Table 5.7. The state of design options in case of deviations

Design Option	***Constraints***	**Max.OBJ**	**Avg.OBJ**	**Min.OBJ**	
2	*satisfied*	199	189	180	**g.Hz**
3	*not satisfied*	214	203	190	**g.Hz**
4	*satisfied*	201	189	180	**g.Hz**

The first option for this design has been eliminated in the previous section due to excessive static deflection. The third option is also eliminated at this stage because it would not satisfy the constraints in case of deviations in characteristics and locations as the results are presented in Table 5.8.

Table 5.8. The constraints of design options in case of deviations

Constraint	**OPTION 2**	**OPTION 3**	**OPTION 4**	
Maximum Displacement	2,0 mm	2,0 mm	2,0 mm	**2,5 mm**
Maximum Rotation	0,20 deg	0,14 deg	0,21 deg	**1 deg**
Maximum Acceleration	5,66 g	5,32 g	5,5 g	**20 g**
Maximum Static Deflection	0,23 mm	0,21 mm	0,24 mm	**0,25 mm**
Maximum Iso. Freq.	94 Hz	*101 Hz*	97 Hz	**100 Hz**
Maximum Direct Amp. Rat.	329%	313%	330%	**500%**
Maximum Cross Amp. Rat.	29%	67%	38%	**150%**
Maximum Static	**2,0 kg.g**	**2,0 kg.g**	**2,7 kg.g**	
Weight /Isolator	1,29 kg.g	1,27 kg.g	1,31 kg.g	

Investigating the constraint values for each design option for sample case 3, it is clear that option three is not appropriate according to the requirements. The possible isolators of design attributes are given in Table 5.9.

Table 5.9. Possible Attributes for the Vibration Isolation System

Design Option	**Maximum Static Weight**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**	**System Natural Frequencies (Hz)**
2	2,0 kg.g	49 N/mm	54 N/mm	0,35	43-62-63-64-92-109
4	2,7 kg.g	62 N/mm	48 N/mm	0,35	42-60-63-66-88-105

This design has the same constraints as in the sample case 1 but the objective function is different from the sample case 1. The objective values are different for this case but the behaviors are the same, so similar to case 1, design option 2 can be selected because it has a lower objective value.

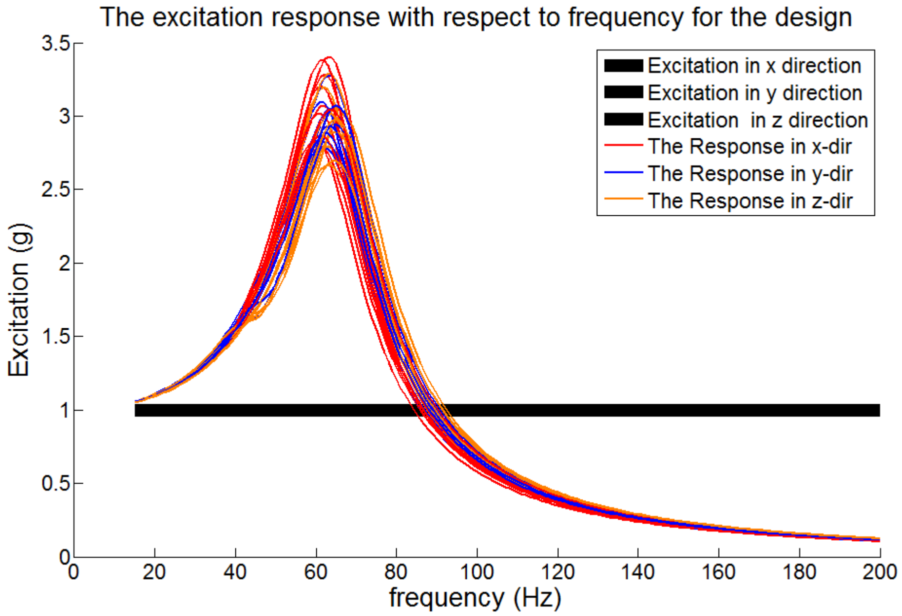

Figure 5.12. Excitation response with respect to frequency for case 3 option 2

5.4. Sample Case 4

All four options are simulated by Monte Carlo simulation with 10% standard deviation in characteristics, 10 mm maximum deviation in locations and 10 degrees of mounting orientation angle deviations of isolators for necessary number of times. In the end of the analysis, the constraints are checked. The state of design options in case of deviations are summarized in **Table 5.10**.

Table 5.10. The state of design options in case of deviations

OPTION	***Constraints***	**Max.OBJ**	**Avg.OBJ**	**Min.OBJ**	
1	*not satisfied*	188	178	169	**g.Hz**
2	*satisfied*	194	184	175	**g.Hz**
3	*not satisfied*	189	177	166	**g.Hz**
4	*satisfied*	198	187	176	**g.Hz**

The first and third options are eliminated at this stage because of constraints. The constraint situations are presented in Table 5.11.

Table 5.11. The constraints of design options in case of deviations

Constraint	**1**	**2**	**3**	**4**	
Maximum Displacement	2,04mm	2,03mm	2,04mm	2,03mm	**2,5mm**
Maximum Rotation	0,19deg	0,18deg	0,22deg	0,20deg	**1deg**
Maximum Acceleration	5,65g	5,65g	5,52g	5,66g	**6g**
Maximum Static Deflection	*0,26mm*	0,24mm	*0,27mm*	0,24mm	**0,25mm**
Maximum Iso. Freq.	90Hz	92Hz	89Hz	94Hz	**100Hz**
Maximum Direct Amp. Rat.	327%	332%	330%	332%	**500%**
Maximum Cross Amp. Rat.	33%	47%	47%	38%	**150%**
Maximum Static Weight /Isolator	**1,4kg.g** 1,18kg.g	**2,0kg.g** 1,52kg.g	**2kg.g** 1,74kg.g	**1,4kg.g** 1,10kg.g	

The isolator attributes in the remaining design options are given in Table 5.12.

Table 5.12. Probable Design's Isolator Attributes for the Vibration Isolation System

Isolator s Design 2	**Maximum SW**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**	**System Natural Frequencies (Hz)**
1-AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35	44.5-60.5 61.1-61.9 86.5-103
2-AM002	1,6 kg.g	40 N/mm	40 N/mm	0,35	
3-AM003	2,0 kg.g	56 N/mm	62 N/mm	0,35	
Isolator s Design 4	**Maximum SW**	**Axial Stiffness**	**Radial Stiffness**	**Loss Factor**	**System Natural Frequencies (Hz)**
1-AM001	2,0 kg.g	56 N/mm	62 N/mm	0,35	44.7-61.0 61.8-63.6 87.3-103.2
2-AM003	1,4 kg.g	50 N/mm	41 N/mm	0,35	
3-AM003	2,0 kg.g	49 N/mm	54 N/mm	0,35	

It is clear from the table that choosing the second option can be selected as well as the fourth option in this case. The second option has been selected because it has a lower objective value.

The excitation response for the selected design option is given in Figure 5.13.

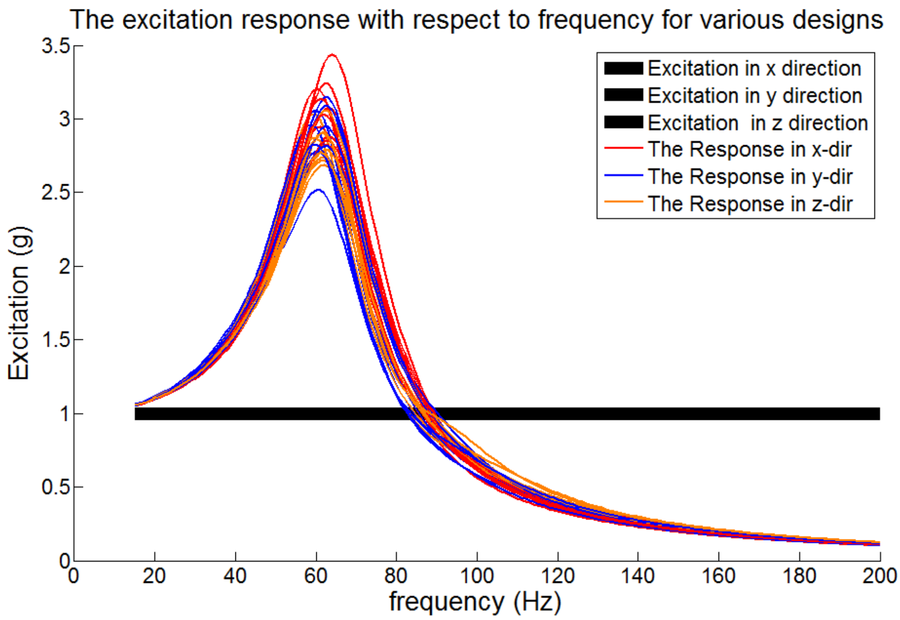

Figure 5.13. Excitation response with respect to frequency for case 4 option 2

CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1. Conclusion

Vibration isolation designs for different systems having different attributes and different constraints can be achieved by using the procedure outlined in this thesis. The input spectral density and excitation with respect to frequency should be defined with the design constraints. After the constraints and the effecting excitation are determined the vibration isolation system can be designed. If the isolated object is a measurement device, its calibration must be done with careful attention of frequency response behavior of the system after the isolation system is mounted on the object.

The inertial measurement unit is used as a sample case which would be exposed to high vibration levels during operation or on endurance periods. The optimization is carried out according to the response of the isolation system to an expected spectra an IMU may be exposed that is estimated. The object in optimization is reducing transmitted energy level by changing isolation characteristics while the system would still have the capability to respond to allowed vibration excitation levels without deterioration.

Vibration isolation of the IMU is studied and achieved in six degrees of freedom and the bad influence of high amplitude high frequency effects on the sensitive device IMU is decreased by the use of an appropriate vibration isolation system design. The vibration isolation system may have various characteristics depending on the characteristics and capacity of isolators, and the optimum solution should be determined according to the constraints of the system.

In vibration isolation design, isolators with viscous or structural damping and constant stiffness model or viscoelastic complex stiffness model lead to different solutions. In reality viscoelastic complex stiffness model for isolators is more valid. On the other hand, isolators characterized according to structural damping and linear stiffness values also lead to results close to reality.

The optimized characteristics are not always available for use directly in the market commercially. The defined characteristics were taken to be available and exact in the initial analysis. Different design options for the sample cases considering optimization results were determined, afterwards the deviations expected in the characteristics of isolators calculated and the possible results were simulated. The best of the options was chosen, making a trade off in the efficiency of the vibration isolation system with the uncertainties expected in the characteristics.

The response spectrum is going to differ from the excitation spectrum. In this thesis, the sample excitation spectra are affected on system in three translational directions in order to observe the response spectra characteristics. The response spectra must be consistent with the constraints while the response is to be minimized in the specified frequency range. The results are compared with respect to their corresponding response spectra. The results are tested with a Monte Carlo simulation since deviations in characteristics of isolators is expected in order to have a fail-safe operation.

The sensitive devices would be exposed to high vibration levels during operation or on endurance times. In addition to this, the sensitive devices may deteriorate when subjected to high vibration level. Via the utilization of a vibration isolation system design procedure explained, fail-safe designs are expected.

6.2. Future Work

In this thesis work, general formulation of a 6-dof system is achieved on a simple rectangular rigid prism having three dimensions. Following the same procedure, different geometries and even irregular shapes can also be formulated and analyzed improving this work.

There are some other simplifications in this thesis work such as the isolator masses have been ignored in mass matrix formulation. In a future study, the effects of isolator masses can also be taken into consideration in design specification. The effects of isolator masses would be significant for the isolation of lighter masses compared to the isolators.

There is also a simplification for modeling isolator characteristics. The linear stiffness and structural damping formulation is widely used in literature but there is also viscoelastic formulation of isolator which is sometimes considered to be better in defining the characteristics of isolators both in stiffness and damping. The isolators have been formulated using linear stiffness and constant structural damping characteristics in this thesis but it is also possible to use viscoelastic properties of isolators with proper determination of the characteristics. By using the exact viscoelastic properties of isolators, the analysis and response would be calculated closer to the real case.

In this study, isolator locations are considered as input to the system, but in a future study, isolator locations can as well be optimized. For example, the optimization for locations can be performed for possible discrete locations.

The vibration isolation system design should also be considering the fail-safe operation of the system. In this thesis work, the maximum deflection the isolator can handle without failure is considered. On the other hand, the fatigue life of isolators would be considered for long term use of the isolation system. The recommended designs in this thesis work are only effective to decrease vibration excitations. On the other hand, in case of shock excitations, the isolators may degrade in performance or fail which is undesired and fail-safe operation should be guaranteed. The “snubbers” can be used to protect the system in case of shock inputs. They also guarantee fail-safe operation of the isolated object in case the vibration isolation system failure. The shock isolation concept is not in the scope of this thesis work. Vibration isolation at high frequencies is also considered to be effective for shock isolation on system but shock isolation has its own parameters that can be considered in an extended study in a future work.

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APPENDIX A

MASS MATRIX FORMATION

For simplicity in this thesis, a block in 3D having eight parts with different masses is going to be used. This is selected for the sake of center of gravity position scatter tests.

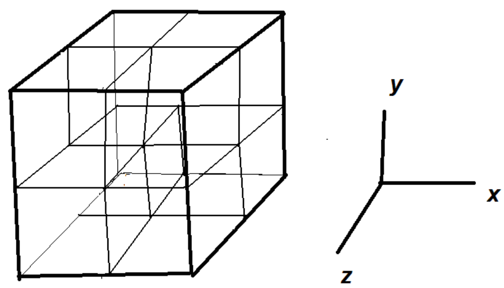


Figure A-1 - Mass Matrix Computation Assumption

By a mass vector, the masses of each cube in Figure A-1 are taken as:

$$Masses = [m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8] \tag{A.1}$$

The lengths of each side of the total prism are in a vector called side:

$$side = [length_x, length_y, length_z] \tag{A.2}$$

If the geometric center of the prism is taken to be the coordinate center. The sides of each 8 prisms are half of the sides of the predefined prism. And the coordinates for the center of mass locations are defined as:

$$a = side(1)/4 \quad \text{(A.3.a)}$$

$$b = side(2)/4 \quad \text{(A.3.b)}$$

$$c = side(3)/4 \quad \text{(A.3.c)}$$

And the center of gravity of the eight masses, are given with the following vectors:

$$X_a = [-a, +a, -a, +a, -a, +a, -a, +a] \quad \text{(A.4.a)}$$

$$Y_b = [+b, +b, +b, +b, -b, -b, -b, -b] \quad \text{(A.4.b)}$$

$$Z_c = [-c, -c, +c, +c, -c, -c, +c, +c] \quad \text{(A.4.c)}$$

Total mass of the system is given as:

$$M_{total} = m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8 \quad \text{(A.5)}$$

Subsequently, the center of gravity locations of each prism are:

$$X_g = (-m_1 + m_2 - m_3 + m_4 - m_5 + m_6 - m_7 + m_8)a / M_{total} \quad \text{(A.6.a)}$$

$$Y_g = (+m_1 + m_2 + m_3 + m_4 - m_5 - m_6 - m_7 - m_8)b / M_{total} \quad \text{(A.6.b)}$$

$$Z_g = (-m_1 - m_2 + m_3 + m_4 - m_5 - m_6 + m_7 + m_8)c / M_{total} \quad \text{(A.6.c)}$$

The mass matrix components are calculated as follows:

$$I_{11} = \sum_{i=1}^{8} 1/12 m_i 4(b^2 + c^2) + m_i[(Y_{b_i} - Y_g)^2 + (Z_{c_i} - Z_g)^2] \quad \text{(A.7.a)}$$

$$I_{22} = \sum_{i=1}^{8} 1/12 m_i 4(a^2 + c^2) + m_i[(X_{a_i} - X_g)^2 + (Z_{c_i} - Z_g)^2] \quad \text{(A.7.b)}$$

$$I_{33} = \sum_{i=1}^{8} 1/12 m_i 4(b^2 + a^2) + m_i[(Y_{b_i} - Y_g)^2 + (X_{a_i} - X_g)^2] \quad \text{(A.7.c)}$$

$$I_{12} = \sum_{i=1}^{8} -m_i (Y_{b_i} - Y_g)(X_{a_i} - X_g) \quad \text{(A.7.d)}$$

$$I_{13} = \sum_{i=1}^{8} -m_i (Z_{c_i} - Z_g)(X_{a_i} - X_g) \quad \text{(A.7.e)}$$

$$I_{23} = \sum_{i=1}^{8} -m_i (Z_{c_i} - Z_g)(Y_{b_i} - Y_g) \qquad \text{(A.7.f)}$$

And the mass matrix [M] of the whole system can be represented as follows in Equation (A.8):

$$[M] = \begin{bmatrix} M_{total} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{total} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{total} & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{11} & I_{12} & I_{13} \\ 0 & 0 & 0 & I_{12} & I_{22} & I_{23} \\ 0 & 0 & 0 & I_{13} & I_{23} & I_{33} \end{bmatrix} \qquad \text{(A.8)}$$

The MATLAB code, forming the mass matrix is supplied following this explanation. In the code, the mass matrix and center of gravity information are input separately.

APPENDIX B

THE GUI AND THE CODE IN MATLAB

B.1. Introduction Screen and Options

In this screen, there appears to be three buttons. The upper two buttons are for design and the lower button is just for comparison of different designs for the same isolator locations and constraints. (Figure B. 1)

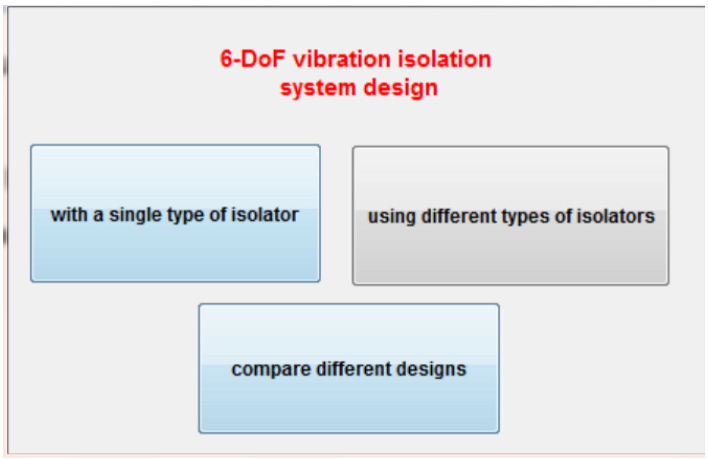


Figure B. 1. Intro screen of the GUI

As mentioned previously, the upper two buttons are in order to open design screens for the 6-dof vibration isolation system. Both can be utilized with the desired number of isolators. The first button uses the assumption of using identical isolators on each location. The second button takes each isolator characteristics independently from each other, thus creating a different solution.

The next button on the lower row opens up the screen that can be used to compare different design solutions according to objective and constraint values.

B.2. Design with a Single Type of Isolator

The screen for the design of an isolation system is given in Figure B. 2. Selecting this screen means that using identical isolators for the isolation system is already made.

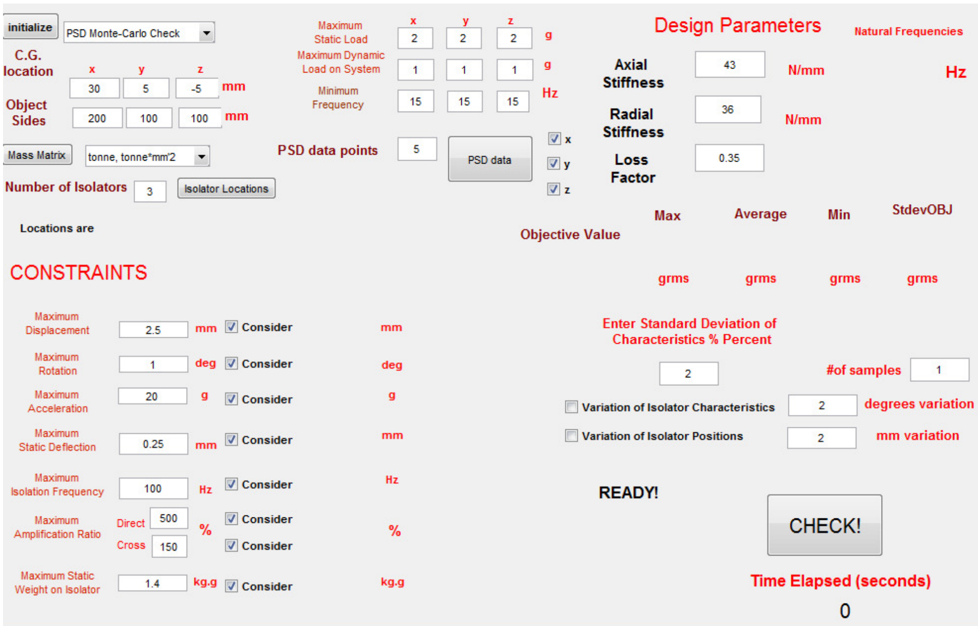


Figure B. 2. DoF Vibration Isolation System Design with a single type of isolator

On the upper left of the screen, the objective function type can be selected as well as choosing between monte-carlo simulation or optimization is possible. After the decision of one of the four options, the user should press the "Initialize" button in order to activate the necessary interface for the selected option.

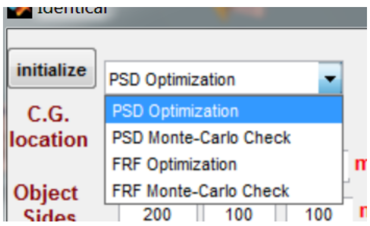


Figure B. 3. Initialization menu

Two of the options here are according to the “power spectral density” response of the isolator system to a specified “power spectral density” of excitation (Figure B. 4). The other two are according to the “frequency” response of the system to a specified excitation which will be described. On the same screen both optimization and Monte Carlo simulation can be achieved for both of the mentioned analyses. Then lower to the initialization menu, there are geometrical attributes input section.

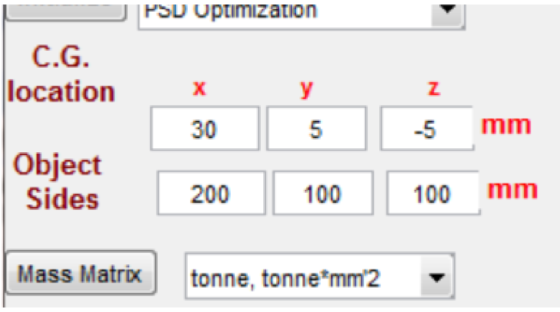
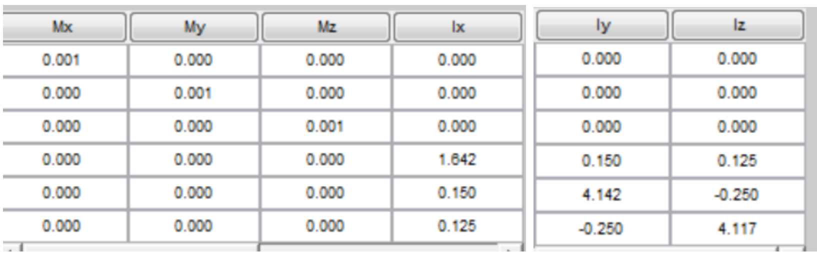


Figure B. 4. Geometrical Attributes Section

In this section, the prism object sides and center of gravity location of the object assuming the origin is at the prism geometric center should be input. After selecting the units, the mass matrix button should be clicked in order to input the mass matrix attributes. (Figure B. 5)



Mx	My	Mz	Ix	Iy	Iz
0.001	0.000	0.000	0.000	0.000	0.000
0.000	0.001	0.000	0.000	0.000	0.000
0.000	0.000	0.001	0.000	0.000	0.000
0.000	0.000	0.000	1.642	0.150	0.125
0.000	0.000	0.000	0.150	4.142	-0.250
0.000	0.000	0.000	0.125	-0.250	4.117

Figure B. 5. Mass Matrix Screen

The sample case, mass matrix that has been formed by the method in Appendix A is already input to the program. But this matrix can be edited.

The number of isolators and their locations according to the assumed origin on the prism geometrical center can be input on the number of isolators box and isolator locations screen.

Figure B. 6. Number of isolators box and locations button

x-coordinate	y-coordinate	z-coordinate
100.000	-17.500	20.000
-100.000	-17.500	-30.000
90.000	50.000	-5.000

Figure B. 7. Isolator locations screen

After the locations are input, the program calculates their elastic center assuming that they have identical properties and lets the user know if the elastic center coincide with the input center of gravity or not.

On the mid-section of the upper screen, the maximum static and dynamic loads that the system may be subjected to at a given minimum frequency information in terms of "g" units, corresponding to gravitational acceleration is input in all three translational directions, x, y and z.

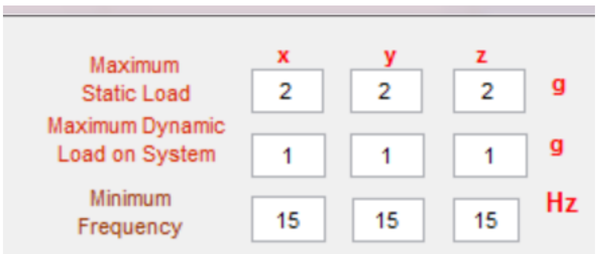


Figure B. 8. Maximum loads section

Then, according to whether the analysis input excitation is in terms of "power spectral density" or only an excitation in "g" units, there are "PSD data" or "excitation data" buttons.

	frequency (Hz)	x_psd g^2/Hz	y_psd g^2/Hz	z_psd g^2/Hz
1	15.000	0.010	0.010	0.010
2	105.000	0.010	0.010	0.010
3	150.000	0.020	0.020	0.020
4	500.000	0.020	0.020	0.020
5	2000.000	0.013	0.013	0.013

Figure B. 9. PSD data input screen

	frequency (Hz)	x_ex g	y_ex g	z_ex g
1	15.000	1.000	1.000	1.000
2	1000.000	1.000	1.000	1.000
3	2000.000	1.000	1.000	1.000

Figure B. 10. Excitation data input screen

In Figure B. 9 and Figure B. 10, the excitations shown in Figure 4.3 and Figure 4.4, data points are given. Some other type of excitations can also be input to program through these tables.

On the “constraints” section of the interface, all the constraint types defined in section 4.2 can be input numerically to program. If the selected option in “initialization menu” in Figure B. 3 is optimization, these constraints are used in the optimization problem but if the selected option is “monte-carlo analysis”, these constraints are input in order to check the limit values. The constraints screen is shown in Figure B. 11.

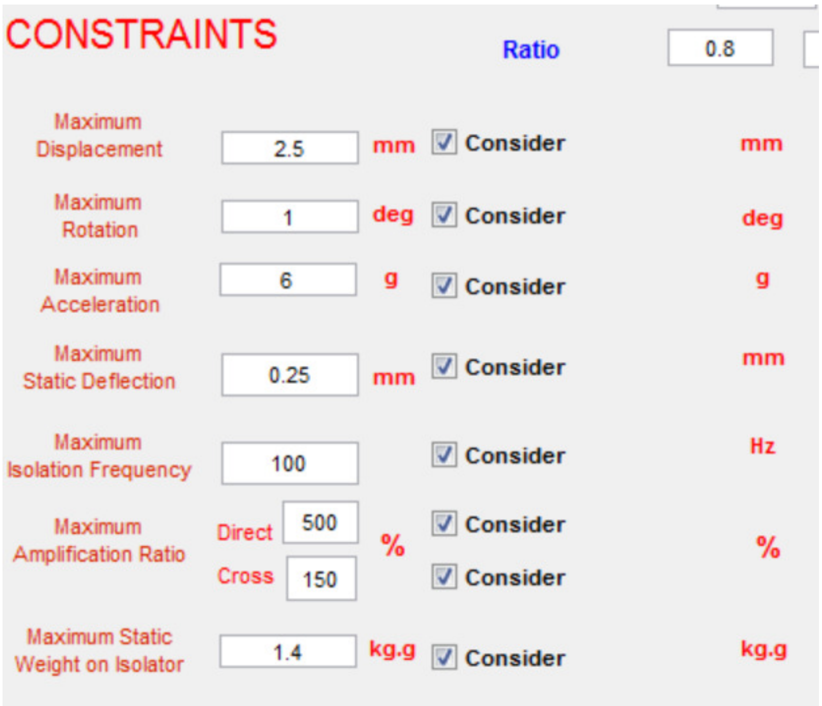


Figure B. 11. Vibration Isolation System Constraints Screen

On this screen, the limit values are entered in the boxes and the optimization results or the critical results of simulation are printed on the space right-hand side after the corresponding analysis is finished.

On the middle land of the screen, there are three pairs of boxes only visible for optimization analyses. The upper two boxes are for stiffness boundary values, the other pair on the middle corresponds to loss factor upper and lower limits. The lower pair corresponds to the probable to be selected isolators' axial/radial stiffness ratio upper and lower limit possible. These boxes that are only visible during optimization processes are shown in Figure B. 12.

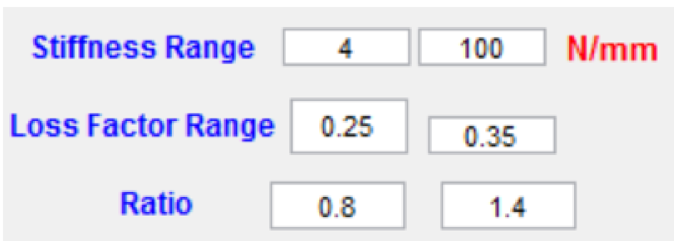


Figure B. 12. Characteristics Ranges and Ratio Ranges

On the left hand side of the screen, the results of optimization can be read, as stiffness values of both axial and radial directions, recommended loss factor value, the resulting objective value and the undamped natural frequencies of the resulting system (Figure B. 13).

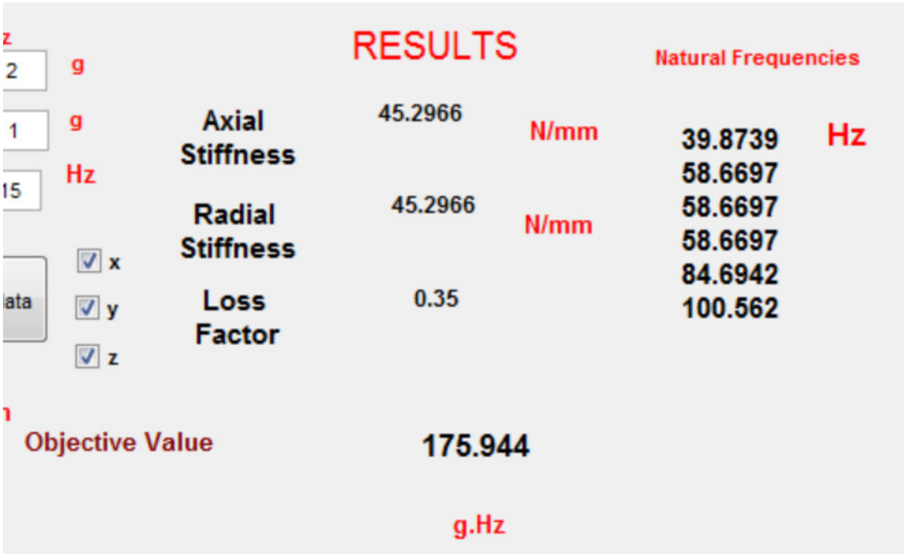


Figure B. 13. The screen for results

There also exists a menu for optimization cases enabling the user to be able to select the optimization method to be used in Matlab. The user can select one of the methods which are *Fmincon, Patternsearch* or *Genetic Algorithm*. Two successive optimizations would let the program use the result of one optimization as an initial guess for the second optimization. Two additional options are for users wanting to run the program without interruption, for *Fmincon* solution after either *patternsearch* or *genetic algorithm* (Figure B. 14).

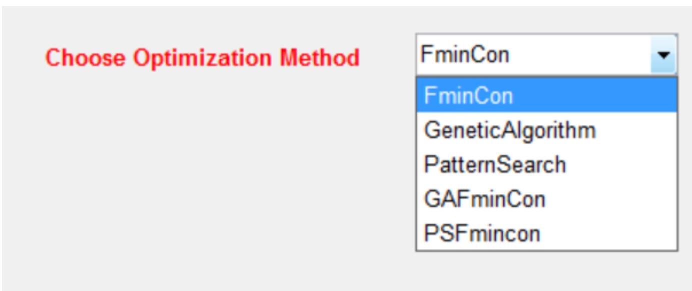


Figure B. 14. Optimization Method Selection Screen

There is another section, telling the user, how many seconds the analysis or simulation lasted and telling whether the design is appropriate or not. This prompt is read “running” through operation and “ready” when the analysis is ready (Figure B. 15).

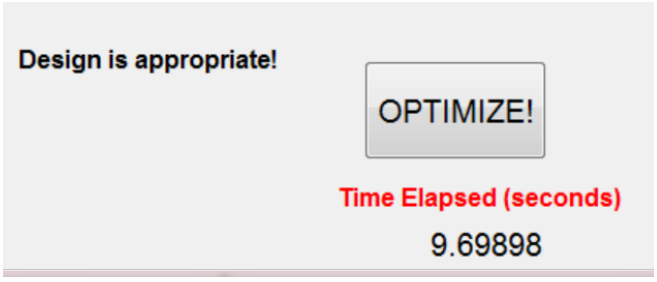


Figure B. 15. Communication screen

Monte-Carlo simulation is also conducted through this screen, with slight changes on the right hand-side. Design parameters are input in boxes and objective value statistical properties can be observed after the analysis is completed (Figure B. 16).

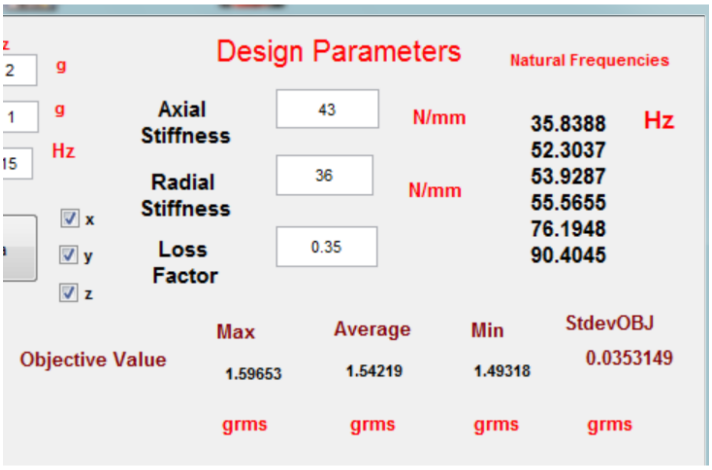


Figure B. 16. The design characteristics that is desired to be simulated

Below the “design parameters” screen, the simulation characteristics should be input. It is possible to determine the number of simulations, deviations in characteristics. And it is possible to select which variations are going to take place (Figure B. 17).

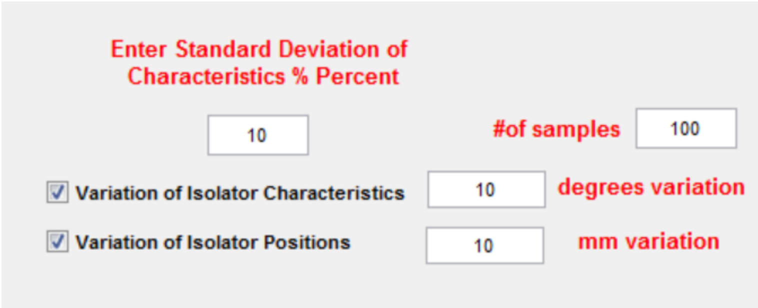


Figure B. 17. The deviations screen

B.3. Design with Different Types of Isolators

This screen is almost all the same, except this time the stiffness values are input and output not just in two boxes but with a table (Figure B. 18).

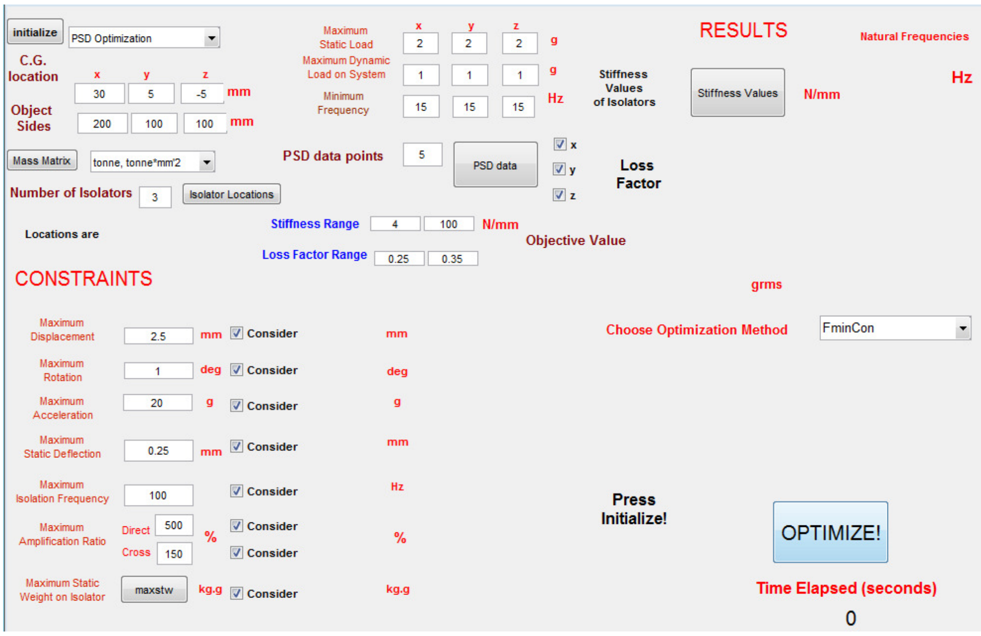
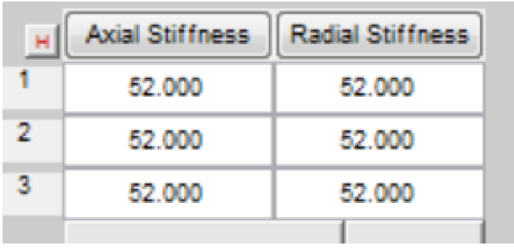


Figure B. 18. 6- DoF Vibration Isolation System Design with different types of isolators

The stiffness values of isolators are entered one by one starting from the first isolator to the last one respectfully, where the left column is for axial stiffness values and right column is for radial stiffness values (Figure B. 19).



	Axial Stiffness	Radial Stiffness
1	52.000	52.000
2	52.000	52.000
3	52.000	52.000

Figure B. 19. Different isolator properties input screen

B.4. Comparing Different Isolators

On this screen, different designs can be compared for a situation according to the loading situation of *psd response* or *frf response*. All one has to do is enter different stiffness properties of different designs respectfully.

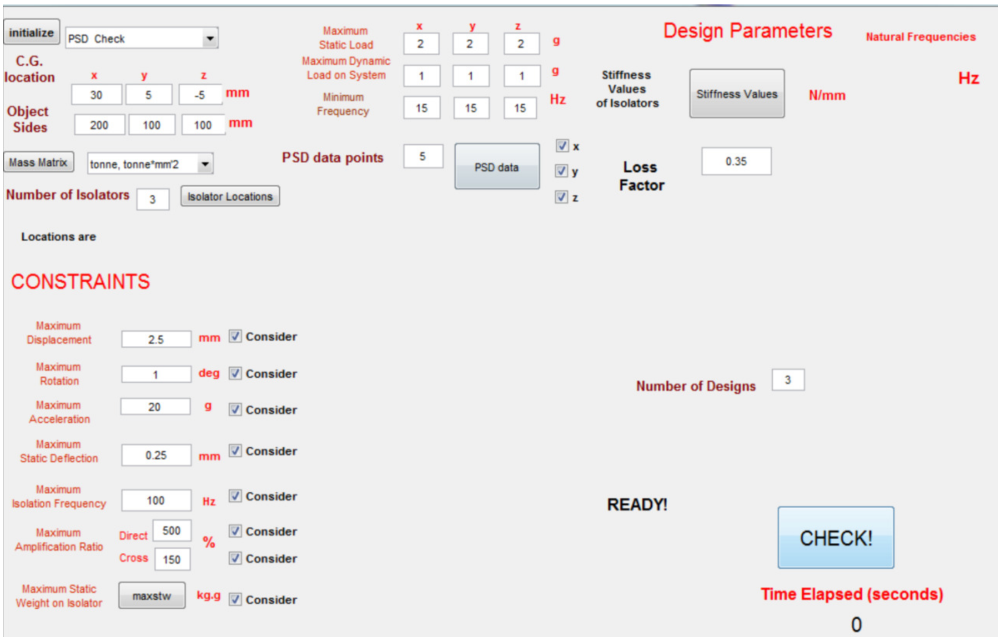


Figure B. 20. 6- DoF Vibration Isolation System Design Comparison Screen

The number of rows on the “stiffness values” screen equals to the multiplication of number of isolators and number of designs to be compared. (Figure B. 21)

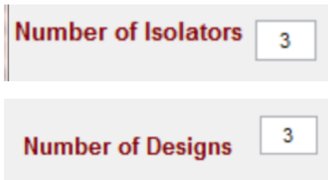
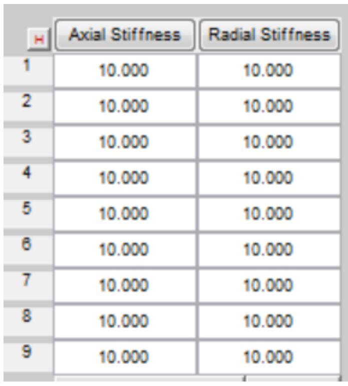


Figure B. 21. Number of Isolators and Designs Screens



	Axial Stiffness	Radial Stiffness
1	10.000	10.000
2	10.000	10.000
3	10.000	10.000
4	10.000	10.000
5	10.000	10.000
6	10.000	10.000
7	10.000	10.000
8	10.000	10.000
9	10.000	10.000

Figure B. 22. Isolator properties on each location for designs to be compare

For example, on Figure B. 22, three recommended designs' screen for an isolation design with three isolators can be seen. The stiffness values should be entered in order to make the comparison.