A COMPARATIVE ANALYSIS OF PERIMETER, AREA AND VOLUME TOPICS IN THE SELECTED SIXTH, SEVENTH AND EIGHTH GRADES MATHEMATICS TEXTBOOKS FROM TURKEY, SINGAPORE AND THE UNITED STATES

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ABSTRACT

A COMPARATIVE ANALYSIS OF PERIMETER, AREA AND VOLUME TOPICS IN THE SELECTED SIXTH, SEVENTH AND EIGHTH GRADES MATHEMATICS TEXTBOOKS FROM TURKEY, SINGAPORE AND THE UNITED STATES

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The purpose of this study was to compare selected sixth, seventh and eighth grades mathematics textbooks from Turkey, Singapore and the United States of America and explore their implications for presenting same opportunity to learn to the students at the same grade level. In this study, the selected books were analyzed in terms of whether they included perimeter, area and volume topics, how they presented the topics on the basis of the selected features and the complexity of to-be-solved mathematical problems related to the topics. Some similarities and differences were observed among the textbooks. It was found that the Turkish textbooks are inclusive in terms of subtopics related to perimeter, area and volume. However, the number of pages dedicated to present the topics is the highest in the Singaporean textbooks. That is, in comparison to the Turkish textbooks, the Singaporean textbooks include fewer number of subtopics related to perimeter, area and volume, but the subtopics are presented in a more detail manner. These books are also rich in terms of mathematically relevant illustrations that make the topics more understandable for students. While the U.S textbooks benefit heavily from technology to present the topics, especially by using three-dimensional shapes; the Turkish and Singaporean textbooks do not make use of technology. The textbooks do not show a difference in terms of complexity of to-be-solved problems. Since all of them mostly include the problems with moderate complexity. Despite there is not any difference among the textbooks in terms of the complexity of to-be-solved problems, there is a difference in terms of the number of to-be-solved problems in the textbooks. The Singaporean textbooks encompass more to-be-solved problems compared to others. The study was concluded by providing some useful suggestions to cover the perimeter, area and volume topics in a way that makes students' learning easier and to present same opportunity to learn to the students.

Keywords: Textbook Analyses, Mathematics Education, Comparative Analyses, Measurement.

ÖZ

TÜRKİYE, SİNGAPUR VE AMERİKA ÜLKELERİNDEN SEÇİLEN 6, 7 VE 8. SINIF MATEMATİK DERS KİTAPLARINDA ÇEVRE, ALAN VE HACİM KONULARININ KARŞILAŞTIRMALI BİR ANALİZİ

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Bu çalışmanın amacı Türkiye, Singapur ve Amerika ülkelerinden seçilen 6, 7 ve 8. sınıf matematik ders kitaplarını karşılaştırmak ve öğrencilere daha iyi öğrenme fırsatlarının sunulabilmesi için olanakları araştırmaktır. Bu çerçevede seçilen kitaplar; çevre, alan ve hacim konularını ne derece içerdikleri, belirlenen özellikler bazında konuların sunuluşları ve ilgili matematik problemlerinin güçlükleri açısından incelenmiştir. Ülkelerin ders kitapları arasında çeşitli benzerlikler ve farklılıklar gözlemlenmiştir. Yapılan çalışma sonucunda Türk ders kitaplarının çevre, alan ve hacme ilişkin konular açısından daha kapsamlı olduğu, ancak konuların sunumuna ayrılan sayfa sayısının en fazla Singapur ders kitaplarında olduğu bulunmuştur. Diğer bir deyişle, Türk ders kitaplarına oranla Singapur ders kitapları çevre, alan ve hacme ilişkin konuları daha az içermekte fakat ilgili konular Singapur kitaplarında daha detaylı şekilde sunulmaktadır. Singapur ders kitapları aynı zamanda öğrencilerin konuları anlamasını kolaylaştıran ilgili görseller açısından da zengindir. Amerikan kitapları özellikle

üç boyutlu şekillerin sunumunda teknolojiden fazlasıyla yararlanırken, Türk ve Singapur ders kitapları teknolojiden yararlanmamaktadır. Kitaplar problemlerin zorluk dereceleri açısından herhangi bir farklılık göstermemektedir. Çünkü kitapların hepsi orta zorluk derecesindeki problemleri içermektedir. Problemlerin zorluk seviyesi açısından kitaplar arasında fark olmamasına rağmen, problemlerin sayısı bakımından fark mevcuttur. Singapur ders kitapları diğer ülkelerin kitaplarına oranla en fazla sayıda problemi ihtiva etmektedir. Bu çalışma öğrencilerin çevre, alan ve hacim konularını öğrenmesini kolaylaştırıcı ve öğrencilere aynı seviyede öğrenme fırsatını tanımaya yönelik tavsiyelerle sonuçlandırılmıştır.

Anahtar Kelimeler: Ders Kitabı İncelemesi, Matematik Eğitimi, Karşılaştırmalı Analiz, Ölçme.

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I wish to express my gratitude to my parents Hikmet and Fatma and to my only sister, Sibel. Lastly, I wish to express my gratitude to my dear husband, Fatih. He always supported and motivated me throughout the desperate moments of writing this thesis.

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LIST OF ABBREVIATIONS

EARGED : Eğitimi Araştırma ve Geliştirme Dairesi (Educational Research and Development Directorate).

IEA : International Association for the Evaluation of Educational Achievement.

LYS : Lisans Yerleştirme Sınavı (University Placement Exam)

MEB : Milli Eğitim Bakanlığı (Ministry of National Education).

NAEP : National Assessment of Educational Progress

NCTM : National Council of Teachers of Mathematics.

SBS : Seviye Belirleme Sınavı (Level Determination Exam)

SIMMS : Second International Mathematics and Science Study

TIMSS : Third International Mathematics and Science Study.

TIMSS 1995: Third International Mathematics and Science Study

TIMSS 1999: Repeat of the Third International Mathematics and Science Study

CHAPTER 1

INTRODUCTION

1.1 Background of the study

Mathematics is used nearly in all fields of people's lives, so people give importance to learn mathematics undeniably (Li, 1999). In many countries, teachers and students mostly prefer to use textbooks in mathematics education (Haggarty & Pepin, 2002; Johansson, 2003; Pepin, 2001). It is reported that textbook usage is nearly 99% in Finland (Törnroos, 2004) and 75-90% in the United States of America (Tyson & Woodward, 1989). Also mathematics textbook usage is compulsory in Mexico (Santos, Macias, & Cruz, 2006).

Especially, textbooks are widely preferred by teachers in teaching mathematics since they help them to organize a class, to make daily instruction plan and to select activities (Ball & Feiman-Nemser, 1988; Freeman & Porter, 1989; Johansson, 2003, 2005; Pepin, 2001; Santos, Macias, & Cruz, 2006; Schmidt. et al., 2001). Besides, students use textbooks to review what they learned in the classroom through homework from textbooks (Reys, Reys, & Chavez, 2004; Tyson & Woodward, 1989).

Furthermore, textbooks are accepted as an important tool for the implementation of curricula and educational reforms (Amit & Fried, 2002; Haggarty & Pepin, 2002; Johansson, 2003; 2005), since they are seen as a tool to change mathematics education (Johansson, 2003).

After international comparative studies, like the Second International Mathematics Study (SIMMS), the Third International Mathematics Study (TIMMS) and the Evaluation of Educational Achievement (IEA); cross-national studies gained a ground on global scale to explain the difference in students' mathematics achievement. The countries in which students showed a poor performance in mathematics, like the USA, have examined the factors that are effective in this issue. The factors such as national curricula (Li, 2000), attitudes and beliefs of students (Randel, Stevenson, & Witruk, 2000) and involvement of parents in students' education (Stevenson, Lee, & Stigler, 1986) were identified as the some reasons behind the difference among students' performance in mathematics.

As the results of international comparative studies have indicated the textbooks have an important effect on students' achievement. This fact led many researchers to analyze the textbooks and their relationship with students' achievement (Fujita & Jones, 2003; Ginsburg & Leinwand, 2005; Haggarty & Pepin, 2002; Johansson, 2003; Li, 2000; Stevenson, 1985; Valverde et al., 2002; Zhu & Fan, 2004).

The results of TIMMS also put forward the positive correlation between usage of textbooks and students' achievement in mathematics (Foxman, 1999; Yeap, 2005). Yeap (2005) claimed that the textbooks which are rich in terms of pictorial representations and to-be-solved problems help students more in acquiring strong foundations in mathematics as well as enhance their creativity.

In a study where textbooks from America and Singapore compared, Ginsburg and Leinwand (2005) reported that giving in-depth information of mathematical topics in Singaporean elementary mathematics textbooks leads students' building deep understanding of mathematical concepts. However, American textbooks rarely get beyond definitions and formulas developing primary students' mechanical ability to apply mathematical concepts.

In a similar analysis which comparing American and Japanese textbooks, Stevenson (1985) found that the difficulty levels of mathematics problems in American textbooks are low, so all students can solve them easily. However, Japanese textbook include more complex problems that are not necessarily to be solved by every student. Furthermore, it was reported that Japanese textbooks cover more topics that are not covered in American textbooks such as measurement, decimals and probability. Also, more advanced topics such as calculus, statistics, equation and sets, functions, three-dimensional figures are emphasized in Japanese textbooks, whereas basic topics such as fraction, addition, subtraction and decimals are emphasized in American textbooks. Owing to a more comprehensive coverage of mathematical concepts in Japanese textbooks leads students showed a good performance. Moreover, the same study also put forward that American textbooks are very long, wordy and repetitive and this affects students' performance adversely. As a result, there are many differences among the textbooks.

For many researchers, the differences among the textbooks used in different countries are very effective in the existing worldwide differences among students' achievement in mathematics. That is, students given different learning opportunities in textbooks show different performance in mathematics (Haggarty & Pepin, 2002; Schmidt et al., 2001; Törnoos, 2004; 2005). Textbooks can be evaluated in terms of opportunity to learn on the basis of the following criteria: Coverage of topics, type of questions, included tools (computers, calculator) and used representational system (Floden, 2002; Herman, Klein, & Abedi, 2000; Törnoos, 2004).

Schmidt et al. (2001) reported positive connection between opportunity to learn a specific topic and students' mathematics achievement on that topic. Similarly, Törnoos (2004) found that students in Finland generally gave correct answers to questions if they were given opportunity to learn the related topic.

The results of international comparison studies also showed that students had poorer performance in the area of measurement in comparison to other areas (Mullis et al., 2000; Olkun & Aydoğdu, 2003), since they have important problems in comprehending the topics of perimeter, area and volume (Cavanagh, 2008; Olkun, 2003; Zacharos, 2006). Distinguishing between perimeter and area (Cavanagh, 2008; Clement & Stephan, 2004; D'Amore & Fandino Pinilla, 2006; Danielson, 2005), measuring area of figures (Cavanagh, 2008; Danielson, 2005; Kordaki & Potari, 1998; Zacharos, 2005; 2006), constructing relationship between geometric shapes (Olkun & Aydoğdu, 2003), calculating area of irregular shapes (Cavanagh, 2008; D'Amore & Fandino Pinilla, 2006; Kordaki & Potari, 1998), understanding of volume and volume formula (Olkun, 2003; Olkun & Sinoplu, 2008) and selecting a unit measurement and unit iteration (Clement & Stephan, 2004; Kordaki & Potari, 1998; Olkun & Aydoğdu, 2003) were identified as the most challenging points that the majority of students have a difficulty in understanding.

Students rely on same approaches used in the textbooks to calculate perimeter, area and volume despite the difference of the students' preference and capability to learning (Cavanagh, 2008), so more opportunities to learn should be presented to students in the textbooks to prevent students having difficulties (Olkun & Aydoğdu, 2003). That is, students can pick up the most favorable alternative compatible with their learning capacity from opportunities provided in the textbooks. Using technology (Clement & Stephan, 2004; Olkun, Altun, & Smith, 2005), student based activities (Cavanagh, 2008; Olkun, 2003; Olkun & Sinoplu, 2008; Outhred & Mitchelmore, 2000) and visuals of geometric shapes (Markus, 2001; Ozdemir, Duru, & Akgün, 2005) provide a positive difference on students' achievement by making easier students to understand the perimeter, area and volume topics. Thus, they can be utilized as the techniques that can be used to increase the opportunities to learn of the textbooks.

One of aims of mathematics education is to improve students' problem solving skills. Therefore, it can be useful to analyze textbooks in terms of their adequateness in developing problem solving skills (Olkun & Toluk, 2002). Existing differences among students in terms of solving problems can also stem from inadequateness of textbooks (Li, 2000). As a result, mathematical problems in the textbooks and also their effects on students' achievement in mathematics were analyzed by many researchers (Li, 2000; Lo, Cai, & Watanable, 2001; Mayer, Sim, & Tajika, 1995; Olkun & Toluk, 2003; Stigler, Fuson, & Ham, 1986).

Since learning mathematics is an experience that also has a great influence over daily lives of people, the results of cross-national studies on mathematics education should be followed carefully and be used as a guidance to make necessary reforms in this area (Li, 1999). Despite there are many cross-national studies focused on content analyses of textbooks (Törnoos, 2005; Schmidt et al.,1997) and placed on the analyses of the problems related to content areas other than measurement (Li, 1999; 2000; Zhu & Fan, 2004), there are still many areas that have not been explored yet (Johansson, 2003; Li, 2000; Mayer, Sims, & Tajika, 1995).

1. 2 Problem Statement and Research Questions

The purpose of this study was to compare selected sixth, seventh and eighth grades mathematics textbooks from Turkey, Singapore and the United States of America and explore their implications for presenting same opportunity to learn to the students at the same grade level. The selected books were analyzed in terms of whether they included perimeter, area and volume topics, how they presented the topics on the basis of the selected features and the difficulty levels of to-be-solved mathematical problems related to the topics. Through analyzing the textbooks, this study aimed to provide a detailed picture of how the measurement topics is conceptualized and treated across the Turkish, Singaporean and U.S textbooks.

The following research questions guided the study:

1. What are the similarities and differences among mathematics textbooks from Turkey, Singapore and the United States of America in terms of inclusion and emphases of perimeter, area and volume topics at the sixth, seventh and eighth grade levels?

2. What are the similarities and differences among mathematics textbooks from Turkey, Singapore and the United States of America in terms of presenting perimeter, area and volume topics on the basis of the determined content features at the sixth, seventh and eighth grade levels?

3. What are the similarities and differences among the to-be-solved problems of mathematics textbooks from Turkey, Singapore and the United States of America at the sixth, seventh and eighth grade levels on perimeter, area and volume topics?

1.3 Significance of the study

This study tried to contribute to the findings of previous cross-national studies by exploring the cross-national similarities and differences of perimeter, area and volume topics in terms of inclusion and emphases of the topics, presentation of the topics and the difficulty levels of to-be-solved problems in the selected sixth, seventh and eighth grades mathematics textbooks from Turkey, Singapore and the United States. In this way, the study provided an indirect window into the mathematics textbooks across three educational systems and explored their implications for presenting opportunity to learn mathematics.

Rather than attempting an overall survey of content coverage in the whole mathematics textbooks, the study focused on a common content area, particularly perimeter, area and volume topics, for an in-depth content analysis of the textbooks. However, this study did not only examine whether the topics are included or not, but also examined how they are presented.

Turkish Ministry of National Education (MEB) has reformed mathematics education curriculum for elementary (grades 1-8) and secondary (grades 9-12) schools. Thus, new mathematics textbooks based on these reformed curricula are different than the previous ones in many respects. There are rare studies (Ünal, 2006) in the current literature comparing the new Turkish mathematics textbooks with those of other countries. So, this study aimed to compare the new developing Turkish mathematics curriculum with the curriculum of the United States that has been experiencing new developments in curriculum area and with the curriculum of Singapore that takes attention with its high achievement in the national comparison studies. By this means, this study can provide information about strengths and weaknesses of the newly reformed Turkish curriculum.

Moreover, the studies by Olkun and Aydogdu (2003) and Ünal (2006) reported that Turkish students showed lower performance in measurement area in international comparison studies than Singaporean and American students. So this study aimed to explain the given opportunities to learn in the selected textbooks can be an important factor on understanding the differences in students' achievement, because Schmidt et al. (2001) reported positive connections in most cases between learning opportunities given in the textbook about a specific topic and mathematics achievement on that topic.

Many studies in the literature analyzed only one grade level of textbooks in comparing textbooks of different countries, but this study analyzed not only the eighth grade level mathematics textbooks but also the seventh and the sixth grade level of mathematics textbooks as a whole to give in-depth information.

Fuson et al. (1988) compared the grade placement of addition and subtraction topics in Japan, Mainland China, the Soviet Union, Taiwan and the United States to explore curricular influences on students' mathematics achievement. They claimed that although the topic placements among Japan, Mainland China, Taiwan and the Soviet Union textbooks have uniformity, there is a substantial difference in the topic placement between the U.S textbooks and the other countries textbooks. Fuson et al. (1988) think that the explaining factor of superior achievement of Japanese students and the other East countries' students can be the placement of topics in mathematics curriculum or in textbooks that represent the curriculum. Because if the topics were presented earlier in high achieving countries' textbooks, students would be given an opportunity to cover more topics and they may be able to learn more mathematics by comparable grade levels. In addition to inclusion of topics, this study analyzed the placement of perimeter, area and volume topics in the selected sixth, seventh and eighth grade level textbooks from Turkey, the United States and Singapore to explore the similarities and differences in terms of given opportunities to learn these topics.

Various researchers (e.g., Johansson, 2003; Li, 2000; Mayer, Sims, & Tajika, 1995) claimed that effects of mathematics textbooks are still unexplored field and there is a growing need for cross-national studies should be carried out to understand their role and influence on students understanding and achievement. Thus, this study aimed to make a contribution to previous cross-national research studies by examining differences in mathematics textbooks that imply national curricula and given opportunities to students.

Finally, examining the content, teaching methods and to-be-solved problems in textbooks provides information about how mathematics is taught in different regions (Lo, Cai, & Watanable, 2001), so this study aimed to give information to educators, teachers, pre-service teachers, textbook writers and researchers about similarities and differences existing in the textbooks and to suggest some ideas for improvements in teaching and learning mathematics by providing same opportunity to learn.

1.4 Definitions, Limitations and Delimitations:

1.4.1 Definitions:

- **Worked Examples:** In this study, worked examples are considered as questions appearing in content presentation parts in the textbooks analyzed. They can have complete solution or not.

- **To–be-solved Problems:** Problems that are presented for student practice and have no accompanying solution or answer (Li, 1999). They appear under headings such as "covering ideas", "review", "practices", "exercises" and "check your understanding".
- **Mathematically Relevant Illustration:** In this study, mathematically relevant illustrations are considered those such as pictures, drawings, icons, charts, models, diagrams and speech balloons that make clear mathematics content presentation and they are essential to solve mathematics problems.
- **Mathematically Irrelevant Illustration:** In this study mathematically irrelevant illustrations are considered those that are not related to mathematics content presentation and they are not essential to solve mathematics problems.

1.4.2 Delimitations:

The comparison of the chapters only on perimeter, area and volume measurement topics in mathematics textbooks from Turkey, Singapore and the United States in terms of content inclusion, content presentation and to-be-solved problems is one of delimitations of the study. Moreover, features of content presentation are delimited to the selected categories.

Furthermore, the mathematics textbooks that were examined in this study were delimited to student books. Teacher guidebooks and student workbooks were not examined.

1.4.3 Limitations:

This study is limited to only one series mathematics textbooks from each country selected. Moreover, analyzing textbooks from only three countries' textbooks is another limitation of the study. Finally, the determined criteria such as content inclusion, content presentation and to-be-solved problems are the other limitation of the study.

CHAPTER 2

LITERATURE

2.1 Textbooks and Textbook Usage

Teaching mathematics in the classrooms of many countries is provided thanks to prepared materials as worksheets, computer programs and especially textbooks (Haggarty & Pepin, 2002; Johansson, 2003). Textbook materials have been used as major resources for instruction in many countries throughout the world by teachers and students in especially mathematics education (Beaton, Mullis, & Martin, 1996; Freeman & Porter, 1989; Haggarty & Pepin, 2002; Johansson, 2003, 2005; Nicol & Crespo, 2006; Pepin, 2001; Schmidt et al., 1997). So, textbooks are inevitable parts of classroom instructions (Sosniak & Stodolsky, 1993).

Results of many studies confirmed that textbooks are used widely in mathematics education. For example, nearly 99% of the seventh grade students and teachers use mathematics textbooks in Finland (Törnroos, 2004) and 75-90% of classroom instruction in the United States of America is organized based on textbooks (Tyson & Woodward, 1989). Moreover, usage of mathematics textbooks in Mexico are compulsory for all children especially in elementary level since textbooks are very important in the national educational system (Santos, Macias, & Cruz, 2006). In Sweden, students and teachers are also dependent on textbooks in mathematics education (Johansson, 2005). Johansson (2005) reported that textbooks dictate content organization and preparation of the lessons. The analyses of answers given by Turkish teachers' answers for the teacher survey of TIMMS-R shows that percentage of textbook usage in Turkey mathematics education is also as high as in the countries mentioned above (Eğitim Araştırma ve Geliştirme Dairesi [EARGED], 2003).

Howson (1995) reported the importance role of textbooks on teaching and learning mathematics such that “despite the obvious powers of the new technology it must be accepted that its role in the vast majority of the world’s classrooms pales into insignificance when compared with that of textbooks and other written materials.” (p.21)

Textbooks are widely used by at least two groups: teachers and students. Many teachers use textbooks in teaching mathematics as a guideline to make a daily instruction plan by identifying “which topics will be instructed” in the lesson and “in which order the topics will be taught” (Freeman & Porter, 1989; Johansson, 2005; Nicol & Crespo, 2006; Pepin, 2001). For example, Reys, Reys and Chavez (2004) reported that American textbooks cover many different topics and repeated materials from earlier grades, so teachers have difficulties to design their instructions in the absence of textbooks.

Ball and Feiman-Nemser (1988) reported that teachers use textbooks as a class organizer, as a guideline and as a source of activities since they are unaware of the content to be taught. They added that especially inexperienced teachers use textbooks frequently because of their lack of self-confidence to design their own lesson and also school authority’s stress on the use of the textbook.

Teachers use textbooks to determine the homework that will be given for students (Pepin, 2001). Moreover, teachers use textbooks to specify which exercises and activities are suitable for students’ levels (Howson, 1995). Similarly, it is claimed that teachers in English, French and German classrooms use textbooks very frequently for exercises and worked examples and teachers rely mostly on textbooks to determine how they will teach (Santos, Macias, & Cruz, 2006; Schmidt et al., 2001). In other words, teachers’ pedagogical strategies are influenced by the instructional approaches of the textbooks (Haggarty & Pepin, 2002; Reys et al., 2003).

Kauffman (2002) carried out a study related to the use of mathematics curriculum materials by four elementary teachers, two of them were using a traditional math book and the other two were using a reform math textbook. He found that the works of these four teachers are depending on textbooks particularly. He also reported that these teachers used textbooks to make plan and deliver instruction. Love and Pimm (1996) summarizes this as follows:

> There is a good deal of evidence that many teachers like the security and freedom from responsibility that a text series provides….when using a text series, teachers need not involve themselves in ordering the topics, in ensuring that notation is consistent, nor in concerning themselves whether a student will have met the necessary pre-requisites for a new topic. (p.374)

In their analyses of relationships between teachers' use of textbooks and their instruction, Sun, Kulm and Capraro (2009) found that the relationship between teacher-textbook is affected from the curriculum and teachers' interpretation and reaction to students in classroom settings, so instructional variations among the teachers who used same textbooks were observed. Moreover, researchers reported that teaching strategies that engaged students and facilitated students thinking were used by teachers used standard-based textbooks that have six characteristics such as comprehensibility, coherence, development of ideas in depth, promotion of sense-making, engagement of students, and motivation for learning. However, teachers used non standard-based textbooks were not very effective to engagement students and to use representations. Finally, the study put forward that suggestions for activities, group work, representations in the textbooks were followed by many of teachers and also teachers used textbooks as a resource of their instruction.

Pehkonen (2004) interviewed with nine elementary school teachers about teacher-textbook relationship. She found that textbooks are important tools for teachers. Teachers use textbooks to maintain a uniform of quality of mathematics education. Moreover, it was reported that textbooks contain basic facts and tasks connected to everyday life, so they provide to see meaningfulness of mathematics for learners and teachers. Also, textbooks help teachers for their plans and choices by providing ready and sensible structures for lessons. Teachers have learnt new ideas and methods with the help of textbooks and their teacher guides.

Moreover, Collopy (2003) reported that teachers need to enact new teaching methods that are different from their own experience of mathematics instruction and they require having a deeper knowledge of mathematics than they have. It was pointed out that textbooks are important tools for teacher learning about subject matter content as well as pedagogy for instruction.

Remillard (2000) analyzed the textbooks’ role in teachers’ learning. It was found that teachers used textbooks as a resource for organizing mathematics content and for teaching the content. Moreover, more effective learning occurred when teachers used the tasks and activities in the textbooks. Using the activities developed newly in textbooks led teachers to decide how to act on them. This situation contributes to change in teachers’ ideas about mathematics learning. The teachers in the study used the tasks in the textbooks in a different way and this variation illustrates the role of teachers in mediating the textbooks’ contribution to enacted curriculum. Consequently, the analysis put forward that textbooks cannot shape curriculum directly.

Furthermore, textbooks are very important sources for students to repeat what they learned in the classroom by given homework from textbooks and workbooks (Reys, Reys, & Chavez, 2004; Tyson & Woodward, 1989). Moreover, textbooks motive students thank to its colorfulness and having different type of exercises (Pehkonen, 2004). Rezat (2008) carried out a study to explore the use of mathematical textbook by students. It was found that students use textbooks since they believe that worked examples in the textbook help them to solve tasks and problems. Moreover, students use textbooks to look up something, to follow the mathematical course-work, to develop mathematical concepts, rules and to repeat the tasks and exercises mediated by teacher.

The relationship among students, textbooks and teachers can be explained by the textbook use model of Rezat (2006) developed based on Vygoysky’s Activity Theory, subject- mediating artifacts-object. Firstly, a model was presented as a student- textbook- mathematical knowledge. In this model, students use textbooks as an instrument to acquire mathematical knowledge. However, it was realized that textbooks are also used and mediated by teachers, but teachers were not included in this model. The researcher took into consideration this situation and expanded the first model by combining with the activity theoretical model suggested by Newton (1990) such as student- teacher-textbook (See Figure 1).

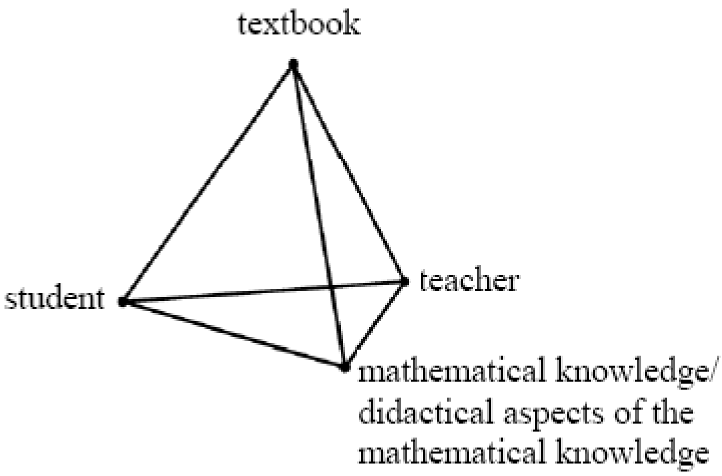


Figure 1: The model of textbook use (Rezat, 2006, p. 413)

According to this model of textbook use, it was reported that teachers mediate the use of textbooks, but students use textbooks without mediation by teacher. Moreover, textbook is seen as an instrument by students to access the mathematical knowledge. Textbook also mediates didactical aspects of presented mathematical knowledge for the teacher. In addition, teacher is also a mediator of the knowledge for students. As a result, teacher use textbooks in the lessons and to prepare their lessons and mediate textbooks for students' use and students learn from the textbooks. That is, textbook is interactive part within teaching and learning mathematics.

2.2 Textbooks and Curriculum

Education systems of countries have special curricula that are prepared on the basis of their cultural, economic and social backgrounds. Teachers have to follow this special curriculum during their instruction to maintain the equality of education in all areas of a country.

The three different curricula are been suggested as the intended, implemented and attained in TIMMS (See Figure 2).

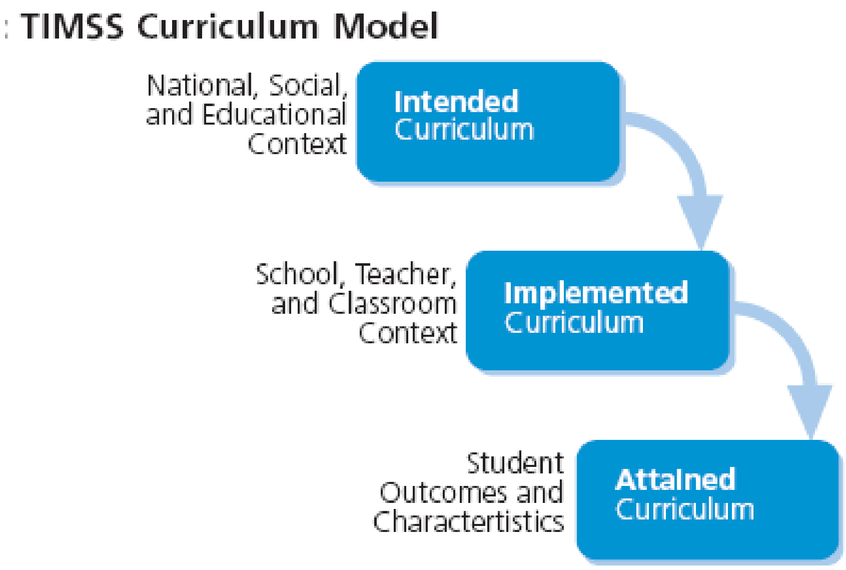


Figure 2: The Curriculum model utilized in TIMMS (Mullis et al., 2005, p.5)

In this model, intended curriculum exists in national policies and official documents that reflect social vision for educational objectives. The implemented curriculum is composed of the objectives of teacher and classroom activities. That is, it is very much related to teaching practices, classroom management, resources and characteristic of teachers. Finally, students' academic achievement, attitudes and beliefs are handled as an attained curriculum. (Schmidt et al., 1997, Robitaille et al., 1993)

A fourth level curriculum model, potentially implemented curriculum, was later added to this model. This model includes textbooks and other resource materials that are mediators between general intentions and classroom instructions (Johansson, 2003; Valverde et al., 2002). That is, textbooks are seen as a part of bridge between the intended and the implemented curriculum. Schmidt et al. (1997) claimed that "Textbooks served as intermediaries in turning intention into implementation. They make possible one or more potential implementation of mathematics curricular intentions." (p.178)

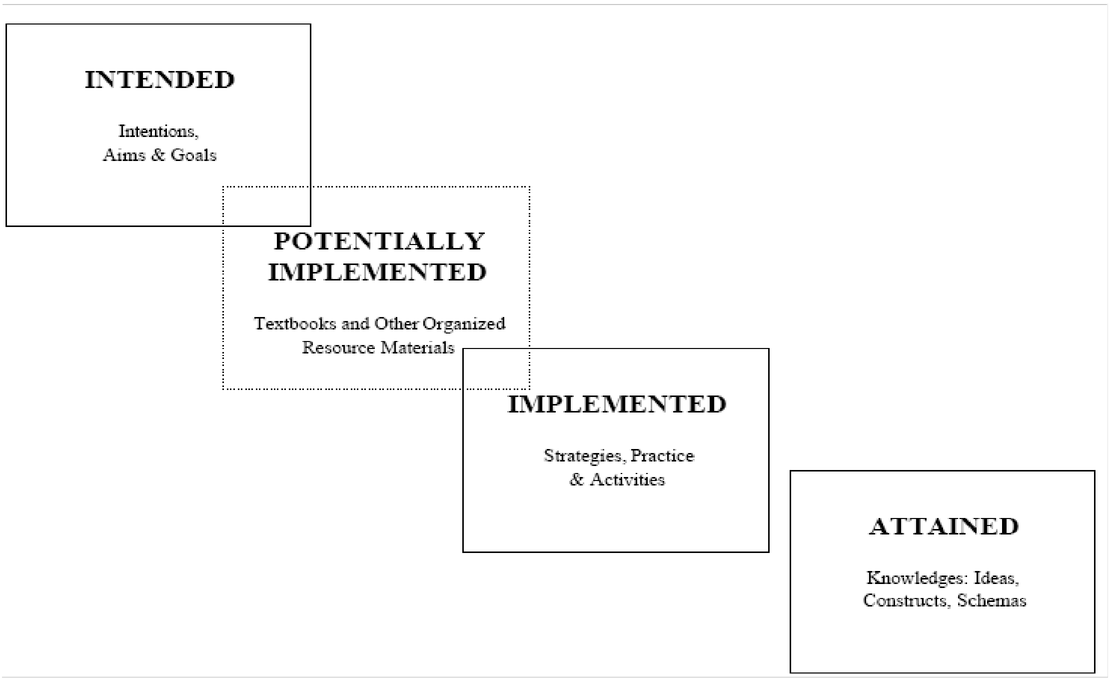


Figure 3: Textbooks and the tripartite model (Valverde et al., 2002. p. 13)

Textbooks are considered as the most important tool for the implementation of a new national curriculum in many countries (Haggarty & Pepin, 2002; Johansson, 2005), so textbooks are influenced by curriculum reforms made under the influence of technological and economical developments. For example; improvement of computers have created new applications in mathematics education and it has revolutionized the numeric mathematics and this leads to change the content of textbooks contents with curriculum reform (Johansson, 2003).

Johansson (2003) analyzed a Swedish textbook series to examine the link between intended curriculum and textbooks. His aim was to examine influence of curriculum reforms from 70’s to 90’s on the development of mathematics textbooks. In the analysis of the textbook series, he found that there is minor agreement between the objectives of mathematics (national curriculum) and the content of the textbooks. However, many researchers claim that textbooks have prominent role in a mathematics curriculum reform since they are easy way to change teaching. Yet, it should not be forgotten that many textbook publishers are only interested in marketing textbooks rather than following national curriculum

(Amit & Fried, 2002; Johansson, 2003; Santos, Macias, & Cruz, 2006). All in all, textbooks can also be conceptualized as a tool to reflect educational reforms to curriculum.

2.3 Studies Comparing Mathematics Textbooks

After the results of international comparative studies such as IEA, PISA and TIMMS, many countries dealt with the question of "why did not their students perform well at the desired level and had lower scores than expected?" That is, low achieving countries were concerned about the reasons behind mathematical success of the top countries and they researched where this high achievement comes from. Therefore, many comparison studies were carried out to uncover the reasons of the achievement differences between students of different countries (Ginsburg & Leinwand, 2005; Haggarty & Pepin, 2002; Lo, Cai, & Watanable, 2001; Mayer, Sims, & Tajika, 1995; Randel, Stevenson, & Witruk, 2000; Stevenson, Lee, & Stigler, 1986).

Many issues such as national curricula (Li, 1999), attitudes and beliefs of students (Randel, Stevenson, & Witruk, 2000), involvement of parents (Stevenson, Lee, & Stigler, 1986) and the other factors that affect students achievement were discussed in the comparison studies to explain the existing difference in students' performance. For example, Randel, Stevenson and Witruk (2000) conducted a study to explain the reasons of low scores of German students by comparing the abilities and attitudes of German and Japanese students. They found that Japanese students take high mathematics scores, because they have high motivation leading them to study hard. Moreover, German students' performance is poor since they have anxiety while taking test due to having lower self-satisfaction and self-ratings of ability.

Moreover, Beaton and Mullis (2002) reported that students who have study aids in the home including a dictionary, a computer and a study table got high scores in TIMMS. Additionally, parents' education has important role on their students' mathematics achievement. However, they claimed that the students who watched television very much had poor mathematics achievement.

Stevenson, Lee and Stigler (1986) carried out a study to reveal reasons why American students had low scores than Japanese and Chinese students. They tested American students on other achievement and cognitive tasks. Besides, they interviewed with mothers and finally they observed students in the classrooms. They found that students' success in mathematics and other subjects depend on greater awareness and increased willingness by American parents. Moreover, more cooperation and communication between the home and the school are needed to reach higher levels of achievement. Finally, they reported that American students spent the least time in practicing materials and on doing homework, so their achievement is lower than the students of Japan and China.

Moreover, international studies including TIMMS have demonstrated that textbooks are very important tools that affect the students' performance in the classrooms (Fujita & Jones, 2003; Valverde et al., 2002). Although Freeman and Porter (1989) claimed that textbooks have little effect on instruction and on what students learn, many researches were carried out to analyze the effect of textbooks on students' mathematics achievement and they wondered that their countries' mathematic textbooks resemble textbooks used in countries where students perform well on international mathematics assessments (Haggarty & Pepin, 2002; Li 1999; 2000; Mayer, Sims, & Tajika, 1995; Zhu & Fan 2004).

Foxman (1999) believes that the students who used textbooks in their classrooms got better TIMMS scores than the students who did not use. Similarly, Yeap (2005) reported that textbooks have important influences on Singaporean students' mathematics achievement and he supported his idea by pointing out the Singaporean students' high mathematics scores in TIMMS. Moreover, he claimed that the textbooks which are rich in terms of pictorial representations and to-be-solved problems help students more in acquiring strong foundations in mathematics as well as enhance their creativity.

Furthermore, a comparative study was carried out by Stevenson (1985) to analyze similarities and differences between American and Japanese textbooks through grade 1 to grade 12. As a result of the study, it was found that the difficulty level of mathematics problems in American textbooks is low, so all students can solve them easily. However, Japanese textbook include more complex problems that are not necessarily to be solved by every student. Furthermore, it was reported that Japanese textbooks cover more topics that are not covered in American textbooks such as measurement, decimals and probability. Also, more advanced topics such as calculus, statistics, equation and sets, functions, three-dimensional figures are emphasized in Japanese textbooks, whereas basic topics such as fraction, addition, subtraction and decimals are emphasized in American textbooks. Owing to a more comprehensive coverage of mathematical concepts in Japanese textbooks leads students showed a good performance. Moreover, the same study also put forward that American textbooks are very long, wordy and repetitive and this affects students' performance adversely.

The study of Ginsburg and Leinwand (2005) compared the mathematics systems and textbooks of U.S and Singapore to explore what the United States can learn from the Singaporean mathematics system that may help improve the mathematics performance of U.S students. They reported that Singaporean textbooks give in-depth treatment of mathematical topics. That is, the textbooks cover half the number of topics as U.S textbooks and they have cumulative topics presentation such that students use sum of angles and proportion to solve a pie chart problem. Moreover, a concept is first illustrated concretely, than pictorially and finally abstractly. All of the mentioned factors lead to build deep understanding of mathematical concepts, whereas American textbooks rarely go beyond definitions and formulas that are necessary for developing students' mechanical ability to apply mathematical concepts.

Mayer, Sims and Tajika (1995) analyzed addition and subtraction problems in the seventh grade textbooks of Japan and America. As a result of the study, it was found that Japanese textbooks present detailed information although they have fewer topics than American textbooks. Moreover, Japanese textbooks present more work-out examples by using different models and visuals relevant to given problems. American textbooks are insufficient in terms of using visuals, models and solving problems.

At this point, many researchers claimed that the differentiation between mathematics achievements of different countries' students is seen as a result of giving different opportunities to learn mathematics (Schmidt et al., 2001; Törnoos, 2004; 2005). That is, researchers started to give attention "opportunity to learn mathematics" as an explaining factor of international achievement results of students after TIMMS, SIMMS.

Opportunity to learn is defined by Husen (1967) such as "one of the factors which may influence scores and it is whether or not the students have had an opportunity to study a particular topic or learn how to solve a particular type of problem…" (p. 162)

Mathematics textbooks used in classroom have influence on students' opportunities to learn mathematics (Törnoos, 2004; 2005) since teachers are likely to present the topics included in the textbooks (Freeman & Porter, 1989; Reys et al., 2003). Opportunity to learn can be measured as how much emphasis a topic receives in textbook and as the time given to teach a topic during instruction (Floden, 2002) or it can be measured as textbook coverage of topics (Törnoos, 2005). Moreover, type of questions, representational systems as charts, tables, graphs, assessment types, availability of using suitable tools (calculator, computer technology) are some indicators of opportunity to learn in the textbooks (Herman, Klein, & Abedi, 2000).

Haggarty and Pepin (2002) analyzed popular selling mathematics textbooks and their usage relating to measure angles in lower secondary England, Germany and France. They found that there were clear differences among the textbooks. For example, technical vocabularies and mathematical angle notations are used extensively in France textbooks, whereas there is no notation in English textbooks. Moreover, English textbooks contain fewer examples than the textbooks of Germany and France. The questions in German textbooks are simpler than the questions in the textbooks of England and France. France textbooks make connection between angles and other mathematical topics. Furthermore, German textbooks correlate angles to everyday situation of students. As a result, students in England, Germany and France are offered different opportunities to learn measuring angles by textbook.

Another study claimed that students in Finland are suggested varying opportunities to learn mathematics since each textbook covers different topics (Törnoos, 2004). Moreover, Törnoos (2004) tried to analyze opportunity to learn and achievement of Finland students. He used different reference points to measure opportunity to learn such as textbook coverage of topics, the teachers' answers to questions on thirty-four topics telling whether they had taught the topics or not and finally the results of item-based analysis of textbooks. He found that textbooks seemed very well as measures of opportunity to learn.

Furthermore, the other study of Törnoos (2005) did not report a positive relationship between students' achievement and percentage of content coverage, but it was reported that opportunities to learn problem solving have a positive role on students' mathematics achievement. That is, students are very successful to solve mathematical problems as similar as the problems emphasized in their mathematics textbooks.

Schmidt et al. (2001) tried to analyze the connection between mathematics achievement scores on TIMMS 1995 items and learning opportunities provided for corresponding items in countries' textbooks. In many cases, he found positive connections between opportunity to learn a specific topic and mathematics achievement on that topic.

In Turkey, many students are given opportunity to learn the same mathematics topics at the same education year by using the textbooks supplied by Ministry of Education without considering abilities and achievement levels of students. However, students in Germany are allocated according to their wishes, abilities and school recommendations in the Hauptschule, the Realschule and the Gymnasium. Students use different mathematics textbooks and mathematics is offered in different ways in each school type (Haggarty & Pepin, 2002).

In the United States, each district decides which textbooks they will use, when they will teach the topics and how they will teach them by taking ideas of teacher, administrations and parents. However, The National Council of Teachers of Mathematics (NCTM) produced a general curriculum framework and nearly all textbooks in The United States are written based on NCTM standards. In such a way, the same opportunity to learn is wanted to be given to American students although they use different textbooks (Reys, Reys, & Chavez, 2004).

2.4 Perimeter, Area and Volume

Geometry is a branch of mathematics that concerns with the measurement and relationships between points, lines, curves and surfaces (Webster, n.d). People use it in all aspects of life as home, school, work and community to design a building, to measure attributes of physical objects and to measure time…etc.

Especially, geometry has many applications in solving practical problems (Royal Society, 2001) and it is used to introduce many important mathematical ideas. For example; area measurement is applied to many mathematical concepts like multiplication of fractions, enlargement, and similarity, number properties such as the commutative law of multiplication, integral calculus and algebraic results like the expansion of binomial expressions (Cavanagh, 2008; Kordaki & Potari, 1998).

As a result, geometry, especially measurement, is accepted as an important subject in school curriculum (Markus, 2001; Yeo, 2008), so achievement tests such as TIMMS, Lisans Yerleştirme Sınavı (LYS)[University Placement Exam], Seviye Belirleme Sınavı (SBS)[Level Determination Exam] include items related to measurement skills.

Geometry, as an important subject with wide applications, needs to have a strong visual capacity, so it is not an easy branch of mathematics to teach and to learn (Royal Society, 2001). For example, Olkun and Aydoğdu (2003) and Mullis et al. (2000) reported that 8^{th} grade Turkish students had lower scores in geometry area than the other areas such as number, algebra and data in TIMMS because of the fact that they have important difficulties in geometry area, especially in measurement. Not only Turkish students, but also students of the other countries have many difficulties in measurement content, especially perimeter, area and volume topics.

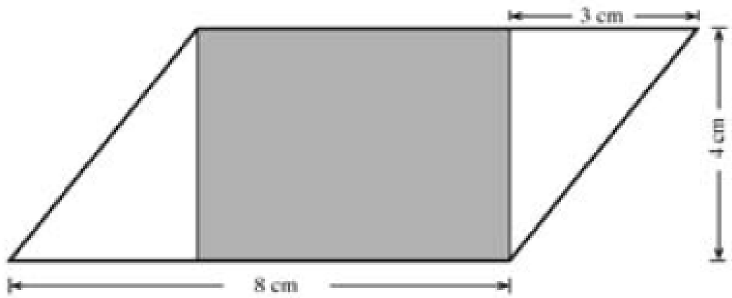
One of the most mentioned difficulties in the literature is that students confuse about perimeter and area in terminology (Cavanagh, 2008; Clement & Stephan, 2004; D'Amore & Fandino Pinilla, 2006; Danielson, 2005). Danielson (2005) reported that definitions and formulas of area and perimeter are given to students without a deep understanding of what they mean and how they are related to each other, so students confuse about perimeter and area.

Another difficulty pointed out in the literature is that students have many problems about measuring area of figures (Cavanagh, 2008; Danielson, 2005; Kordaki & Potari, 1998; Zacharos, 2005; 2006) since they use generally wrong strategies to find area of shapes such as "*area= height+ base*" and "*area= total lengths of sides of a figure*" (Zacharos, 2006). With the participation of 106 students in their last grade in elementary school, Zacharos (2006) found out that students had some problems stemming from inability to understanding the concept of area. Moreover, the study also revealed that students generally used the traditional area formula "*area = base × height*" to calculate the area of figures, but they were not able to explain the logic of the formula because focus is given on finding the relevant formula to calculate the area of a figure rather than dividing the area of the rectangle into units to understand the reasoning behind such formula.

Yeo (2008) recommended the use of squares to make traditional area formula understandable, because it is seen very easy to cover an area empirically with squares and then to use multiplication to show how many squares are used. Similarly, Zacharos (2006) supported the using of square units to find area of shapes. He carried out a study and found that students using square units to find area were more successful than students using rulers to calculate area with traditional formula.

Moreover, Cavanagh (2008) carried out a study about 7th grade students' understanding of area. It was observed that students rely mainly on approaches used in the textbooks and they have some misconceptions about area measurement. Students found the area of the rectangle easily by constructing a grid and counting the squares, but they had difficulty to find the area of a triangle. Some of them tried to divide a triangle into squares and to count these squares, but they were unsuccessful due to fractional parts of squares. Also, some of them used slant height of triangle instead of perpendicular height and some of them forgot the divide the product of base and height by two and the others multiplied all three side lengths together. Furthermore, it was reported that students have a limited understanding about the relationship between the area of a rectangle and a triangle. That is, students are not capable of explaining that if a rectangle and a triangle have a common base and a common perpendicular height; the area of the triangle is the half of the rectangle. As a result, he supported that needed importance about the relationship between geometric figures should be given.

Moreover, Olkun and Aydogdu (2003) reported that only 20% of 8th grade Turkish students gave correct answer to measuring area question in 1999 TIMMS, whereas 83% of Singapore students gave correct answer. They claimed that Turkish students have difficulties to find rectangular area by using the relationship between triangle, rectangular and parallelogram, so they had low scores in this question (See Figure 4). They suggested that geometric shapes should not be taught separately from each other, some problems should be presented to students in order to analyze and understand how the geometric shapes can be used in the same time in a problem and how they can translate one geometric shape to another geometric shape. That is, students should know to construct new shapes from given geometric shapes.



What is the area of the shaded rectangle?

Figure 4: The area question in TIMMS (Olkun & Aydoğdu, 2003, p. 29)

Constructing a relationship between geometric shapes is a necessary skill to be able to define a unit of measurement. The selections of the unit of measurement, the unit iteration, the counting of units of measurement are some important aspects of area measurement and students have also some difficulties about them (Clement & Stephan, 2004; Kordaki & Potari, 1998). For example; Olkun and Aydoğdu (2003) claimed that Turkish students have difficulties about the unit of measurement and the unit iteration because only small percent of the students in TIMMS gave correct answer to “how many triangles are needed to cover the rectangle?”

The researchers suggested that to know area formula may not needed to calculate area of a given figure if students understand the logic of area formulas rather than memorize the formulas. So, students should learn relationship between geometric figures to select a suitable unit measurement and unit iteration and student based activities should be covered in class instructions to provide students using and drawing geometric shapes very well.

Another difficulty students have is to calculate area of irregular shapes (Cavanagh, 2008; D'Amore & Fandino Pinilla, 2006; Kordaki & Potari, 1998) since using only standard geometric figures cause to students not accepting irregular shapes as geometric figures. Many of students attempt to find the perimeter instead of area of irregular shapes or they do not attempt to find the area (Cavanagh, 2008). Furthermore, students apply traditional area formula to find the area of irregular shapes (See Figure 5) and this cause students being unsuccessful to find the correct answer (Cavanagh, 2008; Zacharos, 2006).

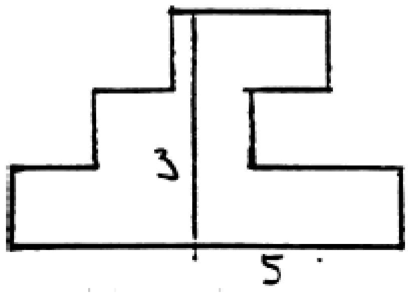


Figure 5: The "base x height" strategy (Zacharos, 2006, p.230)

The researchers claimed that students use the area of known geometric shape to calculate area of irregular shapes (D'Amore & Fandino Pinilla, 2006). It is observed that children in their study used generally rectangles whose area is measured by the multiplication of their dimensions to describe irregular areas (Kordaki & Potari, 1998). That is, the rectangular is used unit of measurement.

Students have difficulties to compare areas of geometric figures (Cavanagh, 2008; Kordaki & Potari, 1998). For example; students are not able to explain how a parallelogram and a rectangle can have same area or a triangle can have smaller area than the parallelogram (Cavanagh, 2008), so the construction relationship between geometric figures is also very important in comparison of areas. Zacharos (2006) advised to use Euclidean method to explain area comparison. That is, two figures occupied the same area are chosen and one of them can be divided into parts and then these parts are appropriately recomposed to form the other figure.

Moreover, Clement and Stephan (2004) claimed that conservation of area is also neglected in measurement instruction, so students have difficulty to understand that if a part of shape is cut and then rearranged to form another shape, the area remains the same. So they suggested that students should learn that decomposing and rearranging shapes do not affect their area.

D'Amore and Fandino Pinilla (2006) analyzed students and teachers beliefs about the relationship between area and perimeter. They found that students insistently apply "law of conservation". That is, if there is a reciprocal relationship between area and perimeter of figures, this relationship does not change even if one of them is changed. For example, students think that if perimeter of a figure is increased, its area is also increased. Of course, it is observed that this is not always a true case. Moreover, students have a misconception such that if a rectangle has bigger perimeter than the other, it has also bigger area than the other. They suggest that perimeter and area should be related to each other on the same figure and unit of perimeter (m) and unit of area (m^2) should be pointed by insisting on the differences. Moreover, transformations should be done on the figures in such a way to preserve perimeter and area in order to prevent students having misconceptions about the relationship between area and perimeter.

Students have difficulties about not only perimeter and area measurement but also volume measurement. For example; many students do not give any meaning to why the three numbers are being multiplied to find volume of geometric shapes "weight × length × height" (Olkun, 2003).

Curry and Outhred (2005) reported that understanding area measurement is a prerequisite for volume measurement, especially by packing. They also claimed that the connection between area and volume is not emphasized adequately. Moreover, they reported that a distinction between volume by filling and volume by packing should be made clearly since they believe that to learn measuring volume by filling is easier than to learn measuring volume by packing, so they suggested that volume by filling should be included much earlier in the curriculum.

Understanding of measurement of volume and volume formula can be provided by using unit small cubes in rectangular arrays (Olkun & Sinoplu, 2008; Olkun, 2001) but students have difficulty in finding the number of small unit cubes in rectangular arrays (Olkun, 2001; 2003). Olkun (2003) carried out a study that included 314 students from 4^{th}, 5^{th}, 6^{th} and 7^{th} grades to investigate students' success and strategies while finding the number of small cubes in rectangular solids. The results of the study explained that students in each grade had difficulty to determine the number of small unit cubes in rectangular solids. It was suggested that student-based activities that are designed based on using relevant concrete materials should be carried out during class instruction before the formal introduction of formulas are not given to students (Cavanagh, 2008) because it is believed that students can discover the formula by themselves after they understand the column and layers structures very well (Olkun, 2003). That is, students are needed some experiences with concrete materials to give meaning to volume and area formulas.

Battista and Clement (1996) carried out a study that analyzes students' solution strategies and errors dealing with 3-D cube arrays. They present a question to students (See Figure 6) and they found that students have difficulty to answer the question. The study reported that the students have errors to count the faces since they deal with picture as a two-dimensional object and counting the hidden cubes.

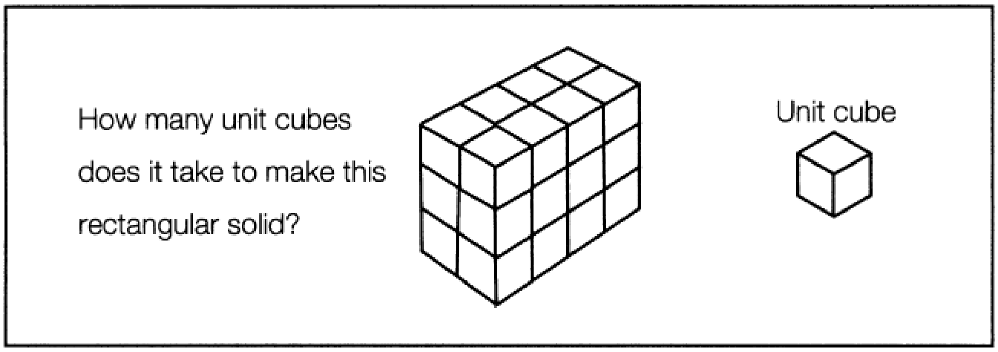


Figure 6: The cube enumeration question (Battista & Clement, 1996, p. 259)

Similarly, Olkun and Sinoplu (2008) claimed that students have a wrong understanding such as they count visible cubes rather than invisible cubes in figures and they find area of rectangular shapes by counting the number of cubes. That is, students have difficulty in visualizing the pictorial representation of 3-D shapes. So they agreed with Cavanagh (2008) and Olkun (2003) to provide the students-based activities including concrete materials.

Moreover, Olkun, Altun and Smith (2005) added those students who have experienced activities seem to have fewer problems with geometry problems. They advised that students should measure the circumference and diameter of many different circles and calculate the ratio of circumference to diameter. After than, students observe that this ratio is close to 3.2 or 3.1. This is called pi number. In such that, students develop clear understanding of pi number and they do not forget it.

Similarly, Outhred and Mitchelmore (2000) claimed that providing students dealing with hands-on activities such as constructing grids by hand and physically covering a region enable students understanding area measurement. Moreover, students observe that there must not be any gasps and overlaps during measurement. Addition, Olkun and Aydogdu (2003) reported that Turkish students were not successful in TIMMS 1999 to define a unit and iteration the unit for finding area, some tangram activities can be helpful to improve students' skills.

Maida and Maida (2006) used an activity for students to apply mathematical formulas while exploring the geometry of a doughnut. Their aim was to provide the students to develop more sophisticated thinking of surface area and volume by giving answers to discover questions. Moreover, students distinguished among one, two, three-dimensional measures during the activity. These support students to construction of personally meaningful strategies. Also, they observed that students recognize the geometry and measurement surrounding them thanks to activities, so they offer frequent and engaging activities for students.

Technology can be used to teaching measurement of perimeter, area and volume since McCoy (1996) thinks that technological tools, especially computers, are important tools in mathematics education since they can provide a variety of rich experiences that allow students to be actively involved with mathematics. Moreover, he explains that students who use computers can build their own learning.

For example; Dynamic Geometry Software and Sketchpad that are dynamic computer programs provide many educational and interactive opportunities for students (Clement, 1999; Hanna 2000; Olkun, Altun, & Smith, 2005) such as creating geometric figures and manipulating these figures and this situation yields positive results on students learning (Clement, 1999; Clement & Stephan, 2004). Similarly, NCTM (2003) claims that "Using dynamic geometry software, students can quickly generate and explore a range of geometric examples." (p. 311)

Kaput and Thompson (1994) mentioned about three aspects of technology that have important role on learning mathematics. One of them is interactivity. Another one is the controlling learning environment. And the last one is connectivity. It means "Technology link teachers to teachers, students to students, students to teachers, and perhaps most important, that link the world old education to the wider worlds of home…" (p. 679). Similarly, Drier, Dawson and Garofalo (1999) reported that technology helps teachers make connections among math topics and other disciplines by providing ready access to worthwhile data.

Olkun, Altun and Smith (2005) carried out a study to investigate how computer made a difference on students' geometry achievement and learning two-dimensional geometry. The experimental group solved computer based Tangram puzzles while the control group continued on their regular classes. After treatment period, a paper and pencil post-test consisting of 2-D geometry was applied to both of the groups. They reported that solving geometric puzzles with computer manipulative has a positive effect on students' geometric reasoning about 2-D geometric shapes since manipulating objects improve visual thinking.

Technology also provides visualization of geometric shapes. Visualization plays an important role on teaching and learning mathematics because it provides people learning the complex and abstract mathematical concepts and topics easily (Markus, 2001). That is, visualization by using figures and drawings help students to understand mathematics better. For example; Özdemir, Duru and Akgün (2005) think that if students learn the geometrical visualization of identities, they understand the meaning of identities and they do not forget them easily. Students can know x^2 means $x.x$, but if they do not draw visualization of this identity on a paper this means that they only memorized it without giving any meaning.

2.5 To-be-solved Problems in the Textbooks

Mathematics education aims to improve students' problem solving skills since students not only use their mathematical knowledge they already learned but also improve their knowledge to have a better mathematical insight in problem solving process (Olkun & Toluk, 2002).

Olkun and Toluk (2002) examined elementary school textbooks and they found that textbooks did not adequately represent all types of addition and subtraction problems. Giving different opportunities to learn causes students being unsuccessful on problem types underrepresented in textbooks due to lack of experience with different types of problem situations rather than the difficulty of problems.

As a result, textbook problems give an idea about the expectations from students for developing mathematical competence, so the existing differences between students are related to not only to differences in exposing students to the school mathematics but also to different experiences students have in solving to-be-solved problems (Li, 2000). For example, Stigler et al. (1986) compared addition and subtraction word problems in several American and Soviet elementary mathematics textbooks. The analysis indicated that American textbooks' problems have low mathematical and cognitive requirements. Moreover, there are not many types of word problems; problems are generally repetitive and only have two solution steps. However, the Soviet problems have many types, they are more complex and require high mathematical and cognitive requirement.

Another study (Fan & Zhu, 2007) analyzed the similarities and differences of textbooks of China, Singapore and America in terms of problems solving. In this study, a framework was developed based on Polya's steps and Schoenfeld's heuristic and this framework was used to analyze problems. The results showed that there are clear differences in "looking back" step. The explanation in this step is not adequate for Singapore textbooks. This situation is very amazing since the problems in the textbooks of Singapore are based on Polya's steps. In other way, there is a similarity in "devising a plan" step in the textbooks of three countries. They use similar strategies.

Mayer, Sims, and Tajika (1995) compared the textbooks of Japan and America to explore how textbooks teach mathematical problem solving. They analyzed the lessons on addition and subtraction of signed whole numbers in the three seventh grade Japanese and in the four seventh grade American mathematics textbooks. They found that Japanese textbooks contain many more worked-out examples and relevant illustrations than American books. Also, Japanese mathematics instruction tends to emphasize the process of problem solving more effectively than does America mathematics instruction. This may be an important determinant of Japanese students' high mathematical problem solving competence. Moreover, cognitive modeling of problem process is emphasized in Japanese textbooks whereas drill-practice on the product of problem solving is emphasized more in American textbooks.

Similarly, Li (2000) compared addition and subtraction integer problems presented in U.S and Chinese mathematics textbooks through three dimensions: mathematical feature, contextual feature and performance requirement. He found that U.S textbooks include more variety in problem requirement than Chinese textbooks. Moreover, American textbook problems emphasize conceptual understanding more than Chinese textbook problems. Also, problems in American and Chinese textbooks are similar in complexity in computation requirement and contextual features. Furthermore, Li (2000) noted that Chinese students are given more complex problems relating to addition and subtraction with rational numbers to at earlier stages.

In addition to introduction of ratio and proportion concepts in the selected series from China, Japan, Taiwan and America, Lo, Cai and Watanabe (2001) analyzed the types of ratio and proportion problems in the textbooks. They reported that Asian textbooks use contextual problems to support and to motivate the introduction of the concepts and procedures. However, these problems generally are very short and specific to concepts. Chinese series contain the most variety of problems that challenged students' thinking.

Stevenson and Stigler (1992) claimed that process of problem solving is important in Japanese schools although learning facts and rules to get correct answer is emphasized in the American schools. The explanation of high achievement of Japanese students could be the result of providing more verbal explanation; using concrete manipulative, providing critical feedback and keeping the focus on fewer problem types in more depth.

Mullis et al. (2000) reported that Turkish students had high performance on solving the problems required low or moderate cognitive behavior in TIMMS

1999, whereas their performance were not good on solving the high cognitive behavior problems. Similarly, Olkun and Aydoğdu (2003) analyzed the cognitive requirements of two released TIMSS 1999 geometry items on which Turkish students performed very poorly. They discussed on some possible reasons such as lack of some skills and suggest some activities to improve those skills. On the limitations of lower elementary Turkish textbook problems, Olkun and Toluk (2002) suggested teachers and prospective teachers are provided recent research on children's learning. It is seen from the literature that Turkish students have difficulties in solving problems partly resulting from limitations of the textbooks.

2.6 Summary of Literature

Textbooks are important tools in mathematics education. They are used by teachers as a guideline to make daily instruction plan, a class organizer and a source of activities, exercises, worked examples and homework. Moreover, students use textbooks to repeat what they learned in classroom, to look up something, to follow the mathematical course-work, to develop mathematical concepts and to repeat the tasks and exercises mediated by teacher. Some international studies showed that textbooks affect students' performance in mathematics, so many researchers compared the textbooks of different countries and they analyzed the content of textbooks, the mathematical problems in the textbooks and their effects on students' achievement. Moreover, opportunities to learn mathematics presented by textbooks cause the differentiation between mathematics achievements of different countries' students. How much emphasis a topic receives in textbook, the time given to teach a topic during instruction, coverage of topics, type of questions, representational systems as charts, tables, graphs, assessment types, availability of using suitable tools (calculator, computer technology) are some indicators of opportunity to learn in the textbooks. As a result, it is implied that textbook writers should take into consideration these indicators.

This study selected perimeter, area and volume topics. Many researchers reported that students have many difficulties about the topics such as distinguishing between perimeter and area, measuring area of figures, constructing relationship between geometric shapes, calculating area of irregular shapes, understanding of volume and volume formula, selecting a unit measurement and unit iteration. Moreover, the findings of these researches report why students have such difficulties and also some suggestions are given to prevent these difficulties. The content of the textbooks should be designed in a way preventing students having these difficulties.

Furthermore, many studies reported that using technology, student based activities and visuals of geometric shapes provided a positive difference on students’ achievement by making easier students to understand the perimeter, area and volume topics. Thus, the results of these studies imply that using technology, activities and visuals can be utilized as the techniques that can be used to increase the opportunities to learn of the textbooks.

CHAPTER 3

METHOD

3.1 Research Design

This is a qualitative study that used content analysis techniques to examine perimeter, area and volume measurement topics in terms of content inclusion, content presentation and to-be-solved problems in the sixth, seventh and eighth grade level mathematics textbooks from Turkey, Singapore and the United States. Using qualitative methods in this study permit us to gather the evidence more effectively since qualitative method permit the evaluator to study selected issues in deep and detailed manner (Patton, 1990).

Moreover, the number of to-be-solved problems, the number of irrelevant and relevant illustrations, the percentage of pages occupied by irrelevant and relevant illustrations and the number of pages devoted to perimeter, area and volume topics in the textbooks were used to have better picture of the selected contents.

3.2 Selection of the Mathematics Textbooks

Due to time and resource limitations, it was almost impossible to find and analyze all existing mathematics textbooks used in Turkey, Singapore and the United States. Thus, in this study a selected sample of mathematics textbook for the sixth, seventh and eighth grades were analyzed.

Turkey has a centralized education system. Although the textbooks are approved by Ministry of Education, there are several different textbooks series available in the education system of Turkey. Mathematics textbooks published by Ministry of Education are used in most of school in Turkey since the textbooks are given free to the students, so the sixth, seventh and eighth grade ministry mathematics textbooks were selected for this study to reach representative findings (See Table 1)

Similarly, Singapore has a centralized education system meaning that Ministry of Education sets all education standards. Several textbook series which are approved by the Ministry of Education of Singapore have been used in Singapore schools. “My Pals Are Here!” textbook series for sixth grade level was selected since it is the most widely used and highly rated textbook series in Singapore (Ministry of Education of Singapore, 2006). For the seventh and eighth grade levels, “New Syllabus Mathematics 1” series were selected since they are the best-selling textbooks in Singapore (New Syllabus Mathematics, n.d) and they are used in more than 80% of Singapore schools (Which Singapore math textbook, n.d) (See Table 1).

The selection of mathematics textbooks of the United States was more difficult. Unlike two other countries, the United States has a decentralized education system. There is no common used mathematics textbook for school education, so there is a wide range of commercial mathematics textbooks in the United States. Therefore, the textbooks selected from the United States needs to be ones that are frequently used and can present a general picture of the United States mathematics textbooks. “Everyday Mathematics” textbook developed by University of Chicago School Mathematics Project was selected for the sixth grade level analysis. “Transition Mathematics” is a series of Everyday Mathematics, so “Transition Mathematics” for grades 7 and 8 were selected for the study (See Table 1). Because they have a 20% market sharing and they represent mathematics standards in the United States (Malzahn, 2002).

Table 1: Textbooks analyzed in this study

Class	TURKEY	SINGAPORE	AMERICA
6	Aktaş, Ş., Atalay, A., Aygün, S. Ç., Aynur, N., Bilge, O., Çelik, M., et al. (2007). İlköğretim Matematik 6: Ders Kitabı (2. baskı). İstanbul: Milli Eğitim Bakanlığı.	Kheong, F. H., Ramakrishnan, C. & Soon, G.K. (2008). My Pals Are Here! Maths 6A & 6B, Singapore: Marshall Cavendish Education	Bell, M., Bell, J., Bretzlauf, J., Dillard, A., Flanders, J., Hartfield, R., Isaacs, A., Deborah, A. L., McBride, J., Pitvorec, K., & Saecker, P. (2007). *Everyday mathematics*. University of Chicago School Mathematics Project. Columbus, OH: McGraw-Hill Publishers
7	Aygun, S. Ç., Aynur, N., Çuha, S. S., K araman, U., Özçelik, U., Ulubay, M., & Ünsal, N. (2007). İlköğretim Matematik 7: Ders Kitabı (1. Baskı). İstanbul: Milli Eğitim Bakanlığı.	Seng, T. K., Yee, L.C. (2007). New Syllabus Mathematics 1 (6th Edition). Singapore: Shinglee Publishers.	Viktora, S. S., Cheung, E., Highstone, V., Capuzzi, C. R., Heeres, D., Metcalf, N. A., Sabrio, S., Jakucyn, N., & Usiskin, Z. (2008). *The University of Chicago School Mathematics Project: Transition Mathematics*. Chicago, IL: Wright Group/McGraw Hill.
8	Aygun, S. Ç., Aynur, N., Çuha, S. S., Karaman, U., Özçelik, U., Ulubay, M., & Ünsal, N. (2008). İlköğretim Matematik 8: Ders Kitabı (1. Baskı). İstanbul: Milli Eğitim Bakanlığı	Seng, T. K., Yee, L.C. (2007). New Syllabus Mathematics 1 (6th Edition). Singapore: Shinglee Publishers.	Viktora, S. S., Cheung, E., Highstone, V., Capuzzi, C. R., Heeres, D., Metcalf, N. A., Sabrio, S., Jakucyn, N., & Usiskin, Z. (2008). *The University of Chicago School Mathematics Project: Transition Mathematics*. Chicago, IL: Wright Group/McGraw Hill.

Table 2 shows the total page numbers of the textbooks. As seen in Table 2, the textbooks have different page numbers and the U.S textbooks cover more pages than Singaporean and Turkish textbooks.

Table 2: Page numbers of the textbooks in each grade. mathematics

	GRADE 6	GRADE 7	GRADE 8
TURKEY	228	222	208
SINGAPORE	252	405	382
AMERICA	370	424	364

3.3 Analyses of mathematics textbooks

This study analyzed the selected sixth, seventh and eighth grade mathematics textbooks from Turkey, Singapore and the United States of America in order to examine similarities and differences across the grade levels. This study focused on perimeter, area and volume measurement topics regarding content inclusion, the selected features of content presentation, and to-be-solved problems (See Table 3).

Table 3: Framework used for analyses of the textbooks

COMPARISON OF OPPURTUNITY TO LEARN OF TEXTBOOKS		
Inclusion and Emphases	**Content presentation**	**To-be-solved problems**
• Topics covered • Sequencing of topics • Average page number devoted to the topics	• Content presentation ▪ The ways presenting definitions, rules, concepts, procedures, principles, summary and worked examples ▪ The ways presenting the misconceptions and the difficulties of students mentioned in the literature about the topics • Technology usage ▪ The ways usage of technological materials such as computers, computer programs, overhead projectors and calculator usage. • Student-based Activities • Visualizations ▪ Mathematically Relevant Illustration ▪ Mathematically Irrelevant Illustrations ▪ Number of the relevant and irrelevant illustrations ▪ Percentage of page space occupied by the relevant and irrelevant illustrations	• Complexity of to-be-solved problems based on NAEP 2007 Framework ▪ Low ▪ Moderate ▪ High • Number of to-be-solved problems

3.3.1 Selection and specification of measurement content for perimeter, area and volume in the textbooks

The mathematics textbooks can be classified according to five main content areas such as number, geometry, measurement, algebra, probability and statistics (NCTM, 2000). This study focused on the analysis of measurement content presented in the selected mathematics textbooks because the results of international comparison studies showed that students had poorer performance in the area of measurement in comparison to other areas (Mullis et al., 2000; Olkun & Aydoğdu, 2003). Moreover, the studies by Olkun and Aydogdu (2003) and Ünal (2006) reported that Turkish students showed lower performance in measurement area in international comparison studies than Singaporean and American students. Although measurement content cover measuring angle, time, speed, perimeter, area and volume, only some of them were selected to be analyzed in the textbooks due to time and resources limitations. Perimeter, area and volume topics were selected since they are much more related to mathematical ideas and they are fundamental for high school topics. For example, area measurement is applied to many mathematical concepts like multiplication of fractions, enlargement, and similarity, number properties such as the commutative law of multiplication, integral and derivative calculus and algebraic results like the expansion of binomial expressions (Cavanagh, 2008).

Opportunity to learn in textbooks was measured as coverage and emphasizes of topics. The perimeter, area and volume topics from the textbooks of each country were identified and they were listed based on the subtopics in the textbooks after each textbook was gone over page by page carefully. General features of shapes were not included in the study. Also, measuring a length and the unit systems for lengths were not analyzed in the study. It was accepted that they are prerequisites for measurement of perimeter, area and volume. Moreover, some units' pages were not included such as "area model for multiplication" or "area model for commutative law" due to fact that they were accepted as topics of number content.

Although the textbooks of the United States present the selected topics under measurement chapter, the textbooks of Turkey and Singapore do not make such a distinction. A unit of Turkish textbooks has many topics from different content areas and a unit of Singaporean textbooks has only one topic under a chapter. That is, there is not a common system for chapter design in the textbooks, so numbers of chapters related to the selected topics were not accepted as being a comparison method for content emphasizes in the textbooks. Alternatively, the study compared the percentage of pages that are devoted to the topics in order to comprise their emphasis in the textbooks due to having limited page space in the textbooks. The pages only related to perimeter, area and volume topics were included in this study. Review questions in the textbooks are related to different content areas, so the review question pages were divided in proportion as 1/10 to calculate the pages devoted to measurement questions.

3.3.2 Features of Content Presentation:

Examining features of content presentation can give information about instructional approaches embedded in the textbooks of the countries. This study analyzed the feature of content presentation under five categories such as content presentation, technology, activities, problem solving and visualization developed based on literature (Li, 1999; Mayer et al., 1995; Shield, 2005) and based on the guide pages that present information at the beginning of the textbooks about how students will use them.

- **Category 1 – Content Presentation:** This category introduces mathematical content knowledge. It includes the textbooks units that are developed for the purpose of applying mathematical content being taught in the chapter. This category includes concepts, procedures, principles, summary and worked examples. The questions having answer or not in content presentation part were accepted as being worked examples rather than mathematical problems.

- **Category 2 – Activities**: This category includes student-based activities such as “Let’s work together”, “games” and “activities” that involve also group participation so that cooperative learning can take place.

- **<u>Category 3 – Problem solving practices</u>:** This category focused on teaching and learning of problem solving skills. It includes being solved problems under "quizzes", "self-tests", "practices", "review questions"… etc. This part will be mentioned together with to-be-solved problems part in the study.

- **<u>Category 4 – Technology Usage</u>**: This category is provided for computer or calculator instruction, activities and exercises.

- **<u>Category 5 – Visualization</u>:** This category consists of illustrations such as pictures, drawings, charts, photographs, graphics, figures, diagrams, models, icons and speech balloons. They were classified mathematically relevant illustrations and mathematically irrelevant illustrations whether they make clear mathematical explanations in content presentation and they are a part of mathematical problems or not.

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3.3.3 To-Be-Solved Problems

To-be-solved problems were also examined in this study. They are the problems at the end of content instruction or at the end of a chapter. They often appear under headings such as "covering ideas", "applying mathematics", "review", "practices", "subject review (*konu değerlendirme*)", "exercise" and "check your understandings". These parts include the problems given in textbooks as tests, quizzes or maintaining skills that are designed for relevant content unit. The questions have no answer in the instruction part were not included in this part. They are accepted as being worked examples.

The perimeter, area and volume problems were determined carefully. Some problems have more items. If items of a problem were not connected with each other, each item was accepted as being a separate problem. The selected problems were categorized such as low, moderate and high complexity based on the NAEP 2007 Framework (See Table 4).

NAEP framework was preferred to be used in the study since the complexity dimensions in the framework build not only on the dimensions of mathematical abilities (conceptual understanding, procedural knowledge and problem solving) but also on the dimensions of mathematical powers (reasoning, connections and communication).

10% percent of the problems from the each countries textbooks and from each grade level were selected separately by using MS Excel randomly page selection function. Two coders working independently coded 135 randomly selected problems based on NAEP Framework. One of the coders is the researcher and the other one is a research assistant in Middle East Technical University. She is very knowledgeable person about textbook analysis and NAEP Framework. After the initial coding, the inter-rater reliability was calculated as 95%. Each problem for which the coders did not agree was then discussed until an agreement was reached on how the problem would be coded. Since the inter-rater reliability score (95 %) is large enough (Li, 1999), the researcher coded all the problems of the textbooks.

Table 4: NAEP Framework used in this study (NAEP, 2007, p. 40)

HIGH COMLEXITY	MODERATE COMPLEXITY	LOW COMLEXITY
High complexity items make heavy demand on students who must engage in more abstract reasoning, planning, analysis, judgment and creative thought. The students are expected to think in abstract and sophisticated ways	Items in this category involve more flexibility of thinking and choice among alternatives. They require a response that has more than a single step. The students are expected to decide what to do, using informal methods of reasoning and problem solving strategies.	This category consists of the recall and recognition of previously learned concepts and principles. Students carry out some procedure that can be performed mechanically. Students are not expected to produce an original method or solution.
• Describe how different representations can be used for different purposes • Perform a procedure having multiple steps • Analyze similarities and differences between procedures and concepts • Generalize a pattern • Formulate an original problem given data • Solve a novel problem • Solve a problem in more than one way • Explain and justify a solution to a problem • Describe , compare and contrast solution methods • Formulate a mathematical model for a complex situation • Analyze the assumptions made in a mathematical model • Analyze or produce a deductive argument • Provide a mathematical justification.	• Represent a situation mathematically in more than one way • Select and use different representations ,depending on situation and purpose • Solve a problem requiring multiple steps • Compare figures or statements • Provide a justification for steps in a solution process • Interpret a visual representation • Extend a pattern • Retrieve information from a graph, table or figure and use it to solve a problem requiring multiple steps • Formulate a routine problem given data and conditions • Interpret a simple argument	• Recall or recognize a fact , term or property • Recognize an example of a concept • Compute a sum, difference, product or quotient • Recognize an equivalent representation • Perform a specified procedure • Evaluate an expression in an equation or formula for a given variable • Solve a one-step word problem • Draw or measure simple geometric figures • Retrieve information from a drawing table or graph

CHAPTER 4

RESULTS

4.1. Emphases of perimeter, area and volume topics in the selected sixth, seventh and eighth grade mathematics textbooks from Turkey, Singapore and the United States.

The selected sixth, seventh and eighth grade mathematics textbooks from Turkey, Singapore and the United States of America were examined page and page carefully and then it was determined objectively whether the selected textbooks include or not the topics and subtopics.

Table 5: A comparison of perimeter subtopics included in the textbooks

	T6	**T7**	**T8**	**S6**	**S7**	**S8**	**A6**	**A7**	**A8**
PERIMETER									
• Square	√			√			√		
• Rectangle	√			√			√		
• Triangle				√					
• Trapezoid				√					
• Hexagon	√								
Perimeter of Irregular Shapes	√			√			√		
Perimeter – Area Relationship		√							
Circumference of a circle		√		√			√		√

Note: T6, T7, T8: Turkish Textbooks for grade 6, grade 7 and grade 8 respectively

S6, S7, S8: Singaporean Textbooks for grade 6, grade 7 and grade 8

A6, A7, A8: American Textbooks for grade 6, grade7 and grade 8.

When Table 5 is examined, it is observed that the U.S textbooks have an interesting characteristic which distinguish it from Turkish and Singaporean textbooks. Perimeter is not presented at the seventh grade level textbook of the U.S. Additionally, area and volume topics are not included in the seventh grade level textbooks of U.S.

There are differences and similarities in terms of sequencing of perimeter, area and volume topics in the selected textbooks. Predominantly, the selected mathematics textbooks from Turkey, Singapore and the United States present the perimeter of geometric shapes at the sixth grade level. Although the sixth grade level mathematics textbook of the U.S gives attention to perimeter of square and rectangle as the Turkish textbook, the sixth grade level Singaporean textbook gives not only perimeter of square and rectangle but also perimeter of triangle and trapezium. All of the textbooks mention about how students find perimeter of irregular shapes. In addition, circumference is taught in the seventh grade level in Turkey, but students of Singapore and the U.S learn circumference in the sixth grade level.

Table 6: A comparison of area subtopics included in the textbooks

	T6	T7	T8	S6	S7	S8	A6	A7	A8
AREA									
• Units of area	√				√		√		
• Rectangle	√			√			√		√
• Parallelogram	√	√			√		√		√
• Rhombus		√							
• Triangle	√			√			√		√
• Circle		√		√			√		√
• Trapezoid		√		√	√				√
• Square	√			√			√		
Area of Irregular Shapes	√	√		√		√			
How changing dimension affects area									√

The sixth grade Turkish textbook and the seventh grade Singaporean textbook mention about unit of area measurement, whereas the any level of the U.S textbook does not.

Measuring areas of shapes is first mentioned at the sixth grade levels in the countries at first time and then it is continued to be taught at the seventh and eighth grade levels. Also, the students of each country have an opportunity to learn the area of irregular shapes. However, only the seventh grade Turkish textbook mentions about relationship between perimeter and area of shapes.

Table 7: A comparison of surface area subtopics included in the textbooks

	T6	**T7**	**T8**	**S6**	**S7**	**S8**	**A6**	**A7**	**A8**
SURFACE AREA									
• Rectangular Prism	√				√		√		√
• Triangular Prism			√		√				
• Square Prism	√								
• Circular Cylinder		√			√		√		√
• Pyramid			√			√			
• Cone			√			√			
• Cube	√								
• Sphere			√			√			√
Surface Area of Irregular Shapes	√								
2-D View of 3-D Shapes	√	√							√
Nets of 3-D Shapes	√	√		√					√

Surface area is included in the selected textbooks of all of the countries. Nearly, the all textbooks mention about the surface area of the same figures. However, it is observed that the surface areas of a square prism and a cube are only given in the sixth grade level Turkish textbooks. Moreover, the sixth grade level Turkish textbook gives information about how surface area of irregular shapes is measured, but the textbooks of the other countries do not. As seen in the

Table 5, nets of shapes are taught to students of all of the countries. 2D views of 3D figures are mentioned in the Turkish and U.S textbooks, but the Singaporean textbooks do not include this topic.

Table 8: A comparison of volume subtopics included in the textbooks

	T6	**T7**	**T8**	**S6**	**S7**	**S8**	**A6**	**A7**	**A8**
VOLUME									
▪ Units of volume	√						√		
▪ Cube	√								
▪ Rectangular Prism	√				√		√		√
▪ Triangular Prism			√		√		√		√
▪ Square Prism	√								
▪ Circular cylinder		√			√		√		√
▪ Pyramid			√			√	√		
▪ Cone			√			√	√		
▪ Sphere			√			√	√		√
Volume of Irregular Shapes				√					
Volume of Liquids	√			√	√				
How changing dimension affects volume									√

The sixth grade Singaporean textbook does not include anything about volume measurement since volume measurement predominantly is taught at the eighth-grade level in Singapore. Conversely, the students of U.S learn how volume of many shapes is measured at the sixth grade level and they repeat what they learned during the eighth grade level thanks to more examples. Additionally, the sixth grade level textbooks of Turkey and U.S present the unit of volume measurement, but it is not presented at any grade level textbooks of Singapore.

The sixth grade level Singaporean textbook of Singapore mention about measuring volume of irregular shapes, but the Turkish and U.S textbooks do not give any information about this issue. Also, in the U.S textbooks, there is a special and separate heading as “how changing dimension affect area and volume” and this is an important opportunity for American students.

The other emphasis point in this study is the percentage of the related pages in the textbooks. After the topics and subtopics related to perimeter, area and volume measurement were decided, the numbers of pages were determined by analyzing the selected sixth, seventh and eighth-grade mathematics textbooks of the countries page by page carefully. As mentioned, the pages related to features of shapes and pages related to commutative law, fractions, algebra and percentage that include area measurement were not included in this analyze.

Table 9: Average Number of Pages devoted to perimeter, area and volume in the textbooks

	Grade 6	**Grade 7**	**Grade 8**	**Total**
Turkey	30,1	24	31,1	85,2
Singapore	66	52,3	38,5	156,8
United States of America	16	0	73,8	89,8

As seen in Table 9, the Singaporean textbooks have the most pages devoted to presentation of perimeter, area and volume. Moreover, it is seen that although the seventh grade mathematics textbook of U.S does not include any related pages to perimeter, area and volume, the total number of the related pages in Turkish textbooks is nearly same with the total page number of U.S textbooks. However, when Table 2 and Table 9 are examined with together, it is observed that thirteen percentage of Turkish textbooks’ pages present perimeter, area and volume, whereas the percentage of the related pages in U.S textbooks is eight that is lower than of the other two countries have. Finally, the percentage of the related pages in Singaporean textbooks is fifteen and this means that although Singaporean textbooks have lower subtopics, they present the topics more detail.

4.2. Features of Content Presentation

4.2.1 Content Presentation

4.2.1.1 <u>General information about content presentations of the textbooks</u>

The sixth, seventh and eighth grade textbooks of Turkey, Singapore and the United States of America have some pages at the beginning of each chapter or unit with interesting visuals related to topics. Some questions are asked to student based on the given visuals in order to let students to perceive why this topic is taught and where they can use it. For example; the volume and surface area chapter in the eighth grade Singaporean textbook begins with a big ice-cream figure on a page and then it is asked that "how does the manufacturer determine the volume of ice-cream needed to fill up the cone completely?" (Seng & Yee, 2007, p. 196)

The volume and area chapter in the eighth-grade Turkish textbook has some pictures of world, moon and solar systems. Under the pictures, it is mentioned that world is a part of the solar system and the moon is a satellite of the world and then the textbook gives radius of the moon and the world. Finally, the textbook requires student to make a comparison between volume of the moon and volume of the world.

Similar to the textbook of Turkey, the volume and surface area chapter in the eighth-grade U.S textbook has pictures of the solar system and planets. It mentions that every object in the physical world takes up space and the sizes of these objects range from quarks and atoms to galaxies and cluster of galaxies. As a result of this situation, there are some basic questions regarding all objects such that "what is the shape of the objects" and "what is the size of object? /how much space does it take up" and finally "how much material does it use?" (Victoria et al., 2008, p. 673) All in all, these entry pages give information to students about what they will learn in this chapter or unit.

Content presentation in the Turkish textbooks starts by giving some information about peoples' daily lives or about social world and then it is continued with a question about what will be taught in this unit. For example; the eighth grade level textbook begins by giving information about what Egyptian Pyramids are, where they are and how they were built and then the textbook gives numeric values about pyramids and continues by asking questions how surface areas of these pyramids can be calculated.

The other example; the sixth grade Turkish textbook begins such that "Transporting companies use different vehicles. They transport furniture, foodstuffs, medicines and medical materials. Generally, boxes that are rectangular prism, square prism and cube are used during transportation. Why vertical prism boxes are preferred? Discuss with your classmates." (Aktaş et al., 2007, p. 231). Answers were not given to these questions in the Turkish textbooks.

The textbooks of Singapore and U.S do not begin by giving any information about daily life or world at the beginning of the unit. As different from Turkey, the textbooks of Singapore and U.S enter the unit directly. If topic is measurement of area, they begin by giving information about area of regular shapes or they begin with a worked example that shows how we can calculate area of a figure.

Content presentation in the eighth-grade U.S textbook begins by giving a big idea in a box at the beginning of the each unit. These big ideas give the main points of the unit. For example; the unit is surface area of prism and the big idea tell us that the surface area of prisms can be found by examining their two dimensional nets. Furthermore; the big idea in the volume of prisms and cylinder unit reports that the volume of prism and cylinders are found by multiplying the area of their base by their height. In this way, big ideas give students notice of what they will learn in that unit as the main objective. Later, these big ideas are represented by using mathematical language.

Any level of the Turkish and U.S textbooks does not have any summarization at the units' endings. However, the each grade textbook of Singapore summarizes what students have learnt at the end of the unit or chapter. The sixth grade textbook of Singapore orders the wanted objectives which students are to be learnt at the end of the unit under the head " Let's Wrap it up" and then gives some questions for students to test themselves whether they have learnt the objectives or not. The seventh and eighth grade Singaporean textbooks do not order objectives for summarization; they repeat important points and formulas what student have learnt in related unit.

4.2.1.2 Presentation of Perimeter in the textbooks

All of the selected textbooks mention about perimeter of figures in the sixth grade level. The sixth grade level Turkish textbook does not explain the meaning of perimeter and how perimeter of a figure is determined explicitly. That is, there is no a general perimeter formula in the textbook. Students are required to discover and understand what perimeter is and how perimeters of figures are calculated based on activities and worked examples.

The sixth grade Singaporean textbook directly begins with a worked example that shows how perimeter of a square is calculated. It has a speech balloon near this perimeter worked example. This speech balloon explains the meaning of perimeter and how perimeter of a square is calculated such as “Perimeter of closed figures with straight sides = sum of its sides” (Kheong, Ramakrishnan, & Soon, 2008, p. 46), but there is not any definite mathematical formula for perimeter of figures in the Singaporean textbook. The book requires students explore the definite formulas themselves based on worked examples.

Similarly, the sixth grade level U.S textbook explains the meaning of perimeter such as “the distance around a polygon” (Bell et al., 2007, p. 212) and also it explains how perimeters of figures are calculated as “add the lengths of all its sides” (Bell et al., 2007, p. 212) Moreover, the textbook gives mathematical perimeter formula of a square, a rectangle and any regular polygon. Also, the textbook presents some worked examples related to given formulas.

All three of the sixth grade textbooks analyzed explain the perimeter of irregular shapes through worked examples. There are only one or two worked examples in the Turkish and U.S textbooks about perimeter of irregular shapes and also these examples are similar to each other. However, the number of worked examples about perimeter of irregular shapes in the sixth grade level Singaporean textbook is more than the number of worked examples in the other countries’ textbooks. Additionally, the worked examples in the Singaporean textbook are seen more different than the examples in the other textbooks. The values of figure’s sides are given to students in the worked examples of Turkish and American textbooks, so students only will sum up the given values of sides to find perimeter of irregular shapes.

The shapes in the worked examples of Singaporean textbook include L-shapes and the combinations of other regular polygons. The sixth grade textbook of Singapore explains step by step how students can divide these irregular shapes into known regular shapes and also it explains how students can find perimeter of irregular shapes by using perimeter of known regular shapes. For example; it shows that L-shape figure can be divided as being a square and a rectangular and then perimeter of L-shaped can be calculated by using perimeter of a square and a rectangle (See Figure 7). Also, L-shape worked examples include missing values. Students have to find missing values of sides to calculate perimeter of L-shapes.

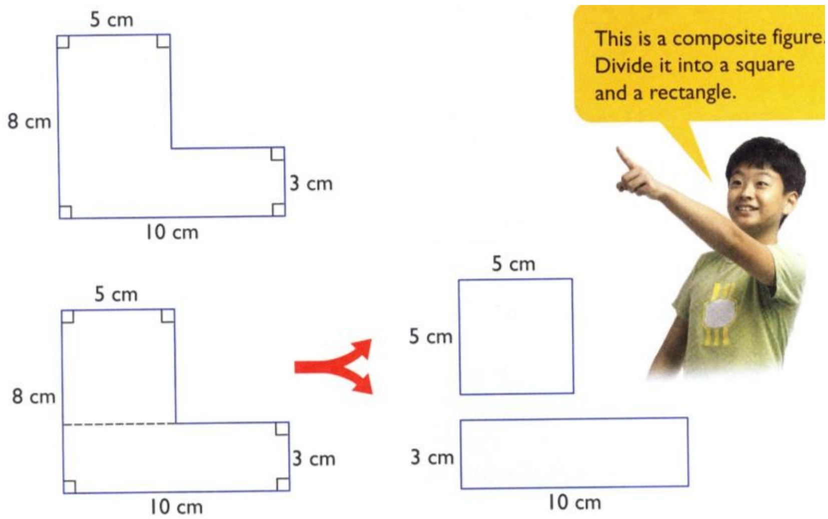


Figure 7: Area measurement strategy of irregular shapes from the sixth grade level Singaporean textbook (Kheong, Ramakrishnan, & Soon, 2008, p. 48)

Circumference is mentioned in the selected sixth grade level textbooks of Singapore and U.S and in the seventh grade textbook of Turkey. The seventh grade Turkish textbook does not explain the meaning of circumference and it directly gives traditional formula of circumference in a worked example without making any logical explanation about the formula. However, the sixth grade Singaporean and U.S textbooks firstly explain what the meaning of circumference. After explaining the meaning of circumference, the U.S textbook gives traditional formula of circumference directly. Similar to the Turkish textbook, there is no any logical explanation about traditional formula in the U.S textbook.

The sixth grade Singaporean textbook begins by giving a table that has numeric values of four circles' diameters, circumferences and circumference/ diameter. Then, the textbook tries student notice that ratio of circumference to diameter is always same regardless of different values of diameters and circumferences of any circles. In such that, the textbook explains what the meaning of pi is. After students understand how pi is obtained, the textbook gives "Circumference/ diameter = π and then cross product is made and Circumference = $\pi \times$ diameter is obtained" (Kheong, Ramakrishnan, & Soon, 2008, p. 12) That is, the textbook of Singapore explains the logic behind the traditional formula of circumference very well.

4.2.1.3 Presentation of area in the textbooks

The sixth grade Turkish textbook uses an activity to teach how students can find area of a regular shape and what the unit of area measurement is. The activity requires students to draw a square of sides 10 cm and then students will divide this square into small squares of sides 1 cm. The textbook presents some question about area of big square and areas of small squares, but it is not observed that there is no question that will provide students making a connection between area of big square and area of small squares. Although the textbook requires to teach that area of regular shapes can be found by using smaller squares and these smaller squares are accepted the unit of area measurement, it is not given any explicit information about this issue. After the activity, the textbook directly presents metric units to measure area and it gives how one metric unit can be diverted to another metric unit.

The sixth grade Turkish textbook presents definite traditional area formula of a square, a rectangle and a triangle to its students without using any activity and worked example. Worked examples in the content presentation of area measurement are very scant.

Similar to the Turkish textbook, the sixth grade Singaporean textbook gives general formula of a square, a rectangle and a triangle directly to its students. The textbook presents that area of any quadrilateral can be found with "length $\times$ breadth". Also, worked examples are as scant as in the textbook of Turkey. Furthermore, the area of a trapezium is taught to the students of Singapore at the sixth grade level with a worked example. There is no a general perimeter and area formula of a trapezium in the textbook. It is shown that combination of square and triangles forms a trapezium (See Figure 8). So, the area and perimeter of a trapezium can be found by adding the area of square and triangles. That is, students have opportunity to learn the relationship between figures.

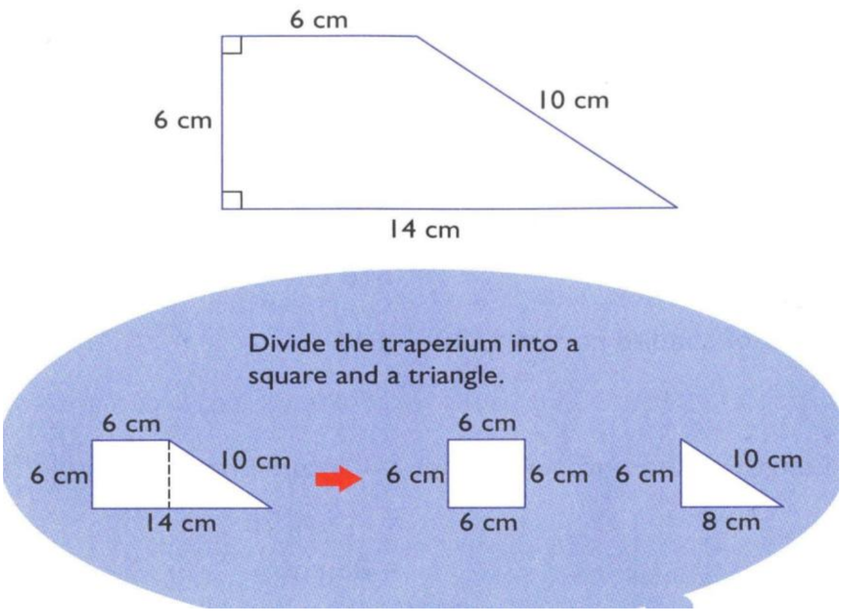


Figure 8: An example of measuring area of trapezium by using the relationship between figures from the sixth grade Singaporean textbook
(Kheong, Ramakrishnan, & Soon, 2008, p. 49)

The seventh grade Singaporean textbook also presents metric units of area measurement. It mentions that unit squares are used to measure area and a square of side 1 cm is used as a standard unit. The figure of this unit square is given to students. Later, the textbook mentions about square millimeter by dividing the square centimeter on a figure and then it explains how a connection there is between square centimeter and square millimeter. The other metric units are given to students with their values in each metric unit. There are many worked examples that enable students to understand metric units and converting to one another.

The sixth grade U.S textbook presents the metric units and the U.S' customary system at the beginning of the area chapter. The textbook reports that area can be found by counting the number of squares of a certain size that cover the region inside the boundary. Moreover, it is stressed that the entire region must be covered with the squares without having any overlap, gaps and without covering any surface outside of the boundary. The units of area are mentioned, but how students can convert one unit to another unit is not explained in the textbook.

The sixth grade textbook of the United States of America tries to explain the area formulas of shapes by using unit squares. It covers a row of a rectangle with unit squares and then it explain that each row of a rectangle contain the same number of squares. The numbers of squares are counted to find area of a rectangle (See Figure 9). Additionally, it is reported that the number of squares in one row is multiplied with the number of rows in order to find the area of a rectangle. This means that area is equal to length of a "base × height". Some worked examples about area of rectangle and square are given in the textbooks.

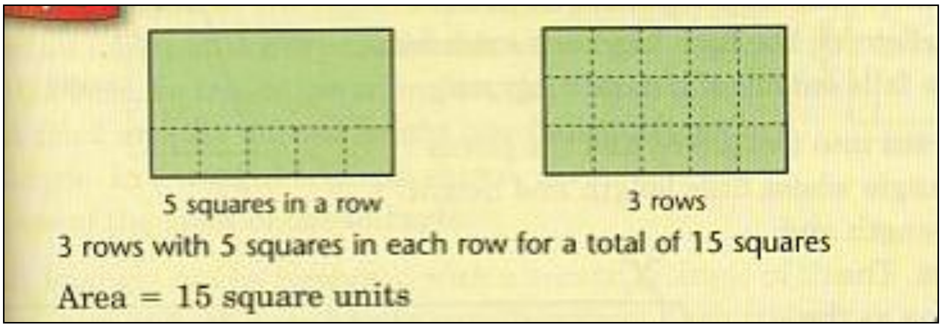


Figure 9: Area measurement strategy from the sixth grade U.S textbook
(Bell et al., 2007, p. 215)

After the area of a rectangle and a square, the area of a parallelogram is taught to the U.S students in the sixth grade level. The traditional area formula of a parallelogram is given as "base × height" and the logic of this formula is explained to students explicitly. The textbook reports that any parallelogram can be cut into two pieces and the pieces rearranged to form a rectangle whose base length and height are the same as the base length and height of a parallelogram (See Figure 10). The rectangle has the same area as the parallelogram. So, the area of a parallelogram can be found in the same way you find the area of the rectangle by multiplying the length of the base by the height. In such that, the textbook presents what is the meaning of the area formula of a parallelogram thanks to relationship between figures.

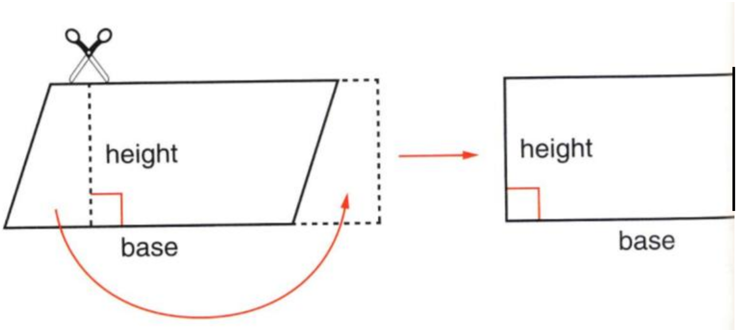


Figure 10: Area of Parallelogram (Bell et al., 2007, p. 216)

Similarly, the seventh grade Turkish textbook tries to give what the logic of area formula of a parallelogram, but one activity is wanted students to carry out rather than to give information directly. At the end of the activity, it is asked how a relation there is between areas of the parallelogram and the formed rectangle and then the traditional area formula of a rectangle is given to students. The aim of the textbook of Turkey is to provide students to find area of a parallelogram by using the relationship between figures. However, this is not guarantee since there is no explanation about this issue at the end of the activity.

Same procedure is followed in the content presentation of a parallelogram in the seventh grade Singaporean textbook. In addition, the textbook gives an additional method such that the area of a parallelogram can be obtained by cutting the parallelogram into two triangles and then using the formula of the area of a triangle. Similarly, area of a trapezium is explained based on the area of triangles in the seventh grade textbook of Singapore. Students are expected to explore the other possible methods of finding the area of a trapezium.

Moreover, the seventh grade Turkish textbook requires students to explore the meaning of area of a rhombus based on area of a rectangle and the meaning of area of a trapezium based on a parallelogram during the related activities. There is no information about what is expected from student to making connections between areas of the given shapes. At the end of the activities, area formulas of rhombus and trapezium are directly given to students with worked examples.

The sixth grade U.S textbook presents the area formula of a triangle finally. It teaches area formula of a triangle consciously after the area of a parallelogram due to fact that it uses area formula of a parallelogram to explain the meaning of the area of a triangle. It is given that a parallelogram is a combination of two same triangles. The triangles have same size bases and heights as the parallelogram. So the area of each triangle is half the area of the parallelogram. Therefore, the area of a triangle is half of the product of the base length multiplied by the height.

The sixth grade level Singaporean textbook does not mention about conversation of area in the subject part. It has an activity in the area and perimeter unit and it wants students to find out that an area of a shape is not changed even if the form of the shape is changed based on composite figures (See Figure 11).

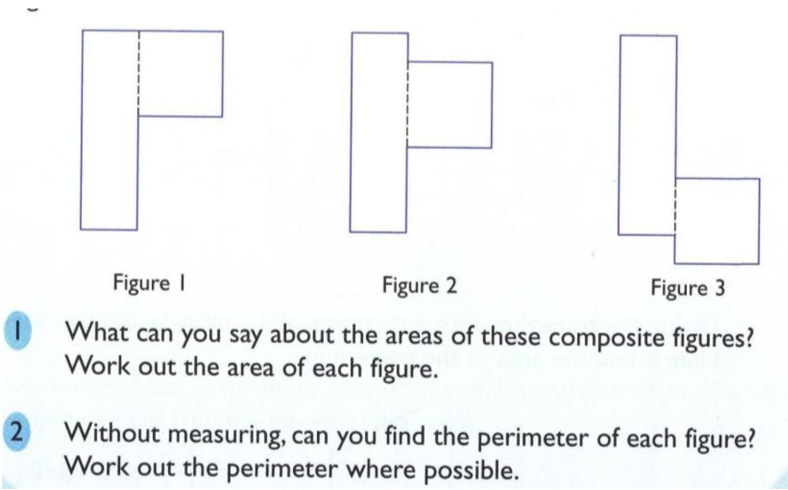


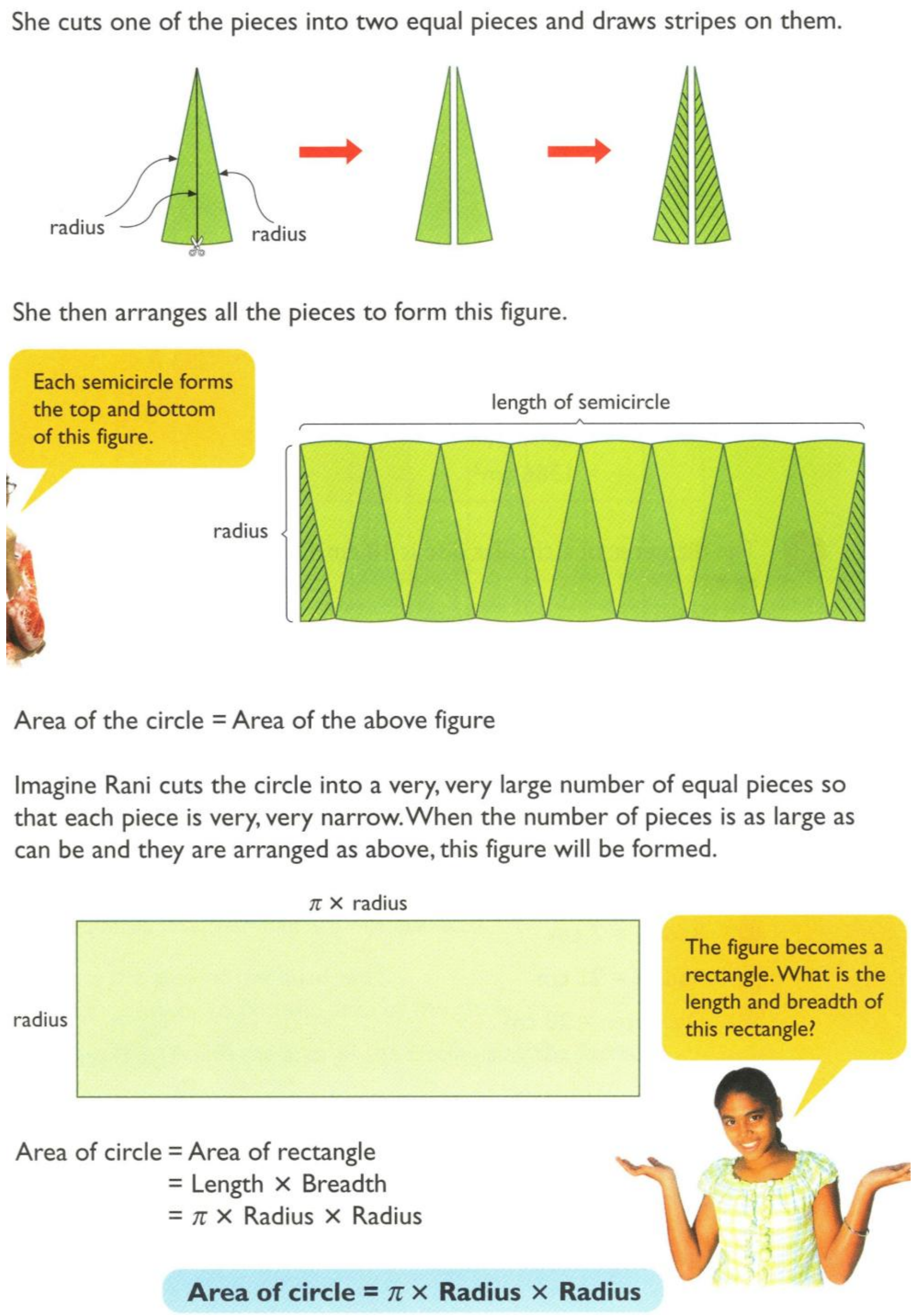
Figure 11: Conservation of area (Kheong, Ramakrishnan, & Soon, 2008, p. 51)

When the selected textbooks of Turkey, U.S and Singapore were analyzed, it was observed that the all students have an opportunity to learn area of a circle.

The Turkish textbook requires students understand the area formula of a circle by following given directions in the activity and by trying to give answers to asked questions during the activity. As per usual, the Turkish textbook gives general area formula of a circle as "area = $\pi \times r^2$" and it does not give an explanation of the formula. There is only one worked example related to area of a circle in the textbook of Turkey.

Similarly, the sixth grade U.S textbook gives only formula for the area of a circle with one worked example. However, it is observed that the area of a circle is taught in the eighth grade level U.S textbooks explicitly. Area formula of a circle is reminded to student and an activity that is as same as the activity in the Turkish textbook is wanted students to carry out to give meaning to area formula of a circle. Different from the Turkish textbook, the textbook of U.S finally points out what was the aim of the activity. It reports that a parallelogram is obtained if a circle is divided and then rearranged. The base of the parallelogram is the perimeter of the circle and the height of the parallelogram is the radius of the circle. The area of the parallelogram is found “Base × height = $\pi \times r^2$” and this is equal to area of the circle. These sentences explain the meaning of the area formula very well.

Similar to the U.S textbook, the sixth grade Singaporean textbook presents the area formula of a circle by explaining each step of the formula by using visuals (See Figure 12).


She cuts one of the pieces into two equal pieces and draws stripes on them.

She then arranges all the pieces to form this figure.

Area of the circle = Area of the above figure

Imagine Rani cuts the circle into a very, very large number of equal pieces so that each piece is very, very narrow. When the number of pieces is as large as can be and they are arranged as above, this figure will be formed.

Area of circle = Area of rectangle
= Length × Breadth
= π × Radius × Radius

Area of circle = π × Radius × Radius

Figure 12: Area of a circle (Kheong, Ramakrishnan, & Soon, 2008, p. 19)

Only the seventh grade Turkish textbook has a special heading to explain how a relation there is between area and perimeter of a shape. The textbook wants to emphasize that the shapes can have different areas although they have same perimeters vice versa the shapes can have different perimeters although they have the same areas. In such that, it is stressed that perimeter and area mean different things.

The textbooks of the countries present nets of solids. The textbooks of Singapore and the United States of America mention about the nets of all solids under only one heading, whereas the Turkish textbooks mention about the net of solids separately when it presents the surface area of each solid. The important point is that the sixth grade level Singaporean textbook gives all net forms of a solid although the Turkish and U.S textbooks give only one net form of a solid to their students.

The sixth grade level Singaporean textbook starts with the net of a cube and it uses a visual to explain how the net of a cube is taken due to providing students to visualize the net of a cube better. Moreover, it presents three net forms of a cube at the same page. Challenging practice of this unit gives the eighth net figures and asks to students to determine which given nets form a cube. In addition to square, the all forms of cuboids, prisms and pyramids nets are given to Singaporean students. The students are wanted to trace out the given nets in the textbook and then to fold these nets to form the solids.

The seventh grade level Singaporean textbook presents the surface area of the shapes based on the nets of the shapes. The textbook teaches not only volume and surface area of the general prisms such as rectangular, square and triangular but also it teaches the volume and surface area of the pentagonal and other right prisms.

The textbook of Singapore presents how Archimedes realized the length of a piece of twine that is coiled around a hemisphere with radius r is same with the length of a twine piece that is coiled around curved surface of a cylinder with base radius r and height r. Curved surface of this cylinder is $2\times\pi\times r^2$ and this equal to the surface area of hemisphere. So it is multiplied by two to obtain the surface area of a sphere. As a result, Archimedes discovered a formula for the surface area of a sphere (See Figure 13).

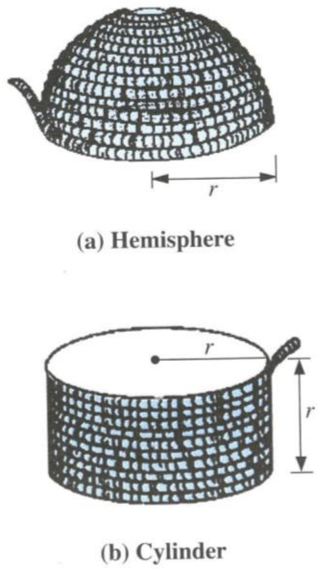


Figure 13: Surface area of a sphere from the eighth grade Singaporean textbook (Seng & Yee, 2007, p. 223)

The eighth grade Turkish textbook uses a different method to teach the surface area of a sphere. A circle of radius is equal to radius of the sphere is drawn on a paper and is cut out. The circle is divided into eighth equal parts and they are stuck on the sphere closely. It is observed that four circles are needed to cover the surface area of the sphere, so the surface area of a sphere is four times of the area of a circle (See Figure 14).

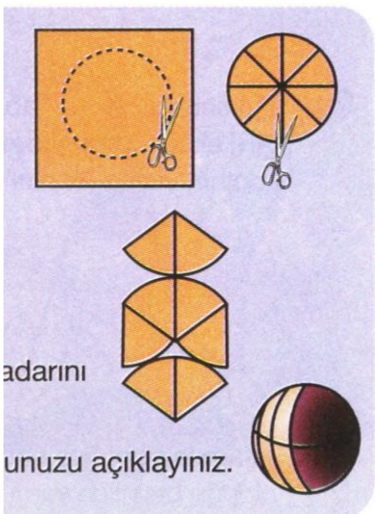


Figure 14: Surface area of a sphere from the eight grade Turkish textbook (Aygun et al., 2008, p.152)

The Turkish textbook uses the nets of the shapes to give surface area formulas. Also, the figures that are formed from small unit squares are given to students and it is required students to determine the surface area of the shape by using the surface area of the small unit squares.

4.2.1.4 Presentation of Volume in the textbooks

The sixth grade textbook of Singapore does not explain the meaning of the volume by using explicit sentences. It directly begins with a worked example that uses traditional volume formula of a cube and cuboids. Figures of the shapes in the worked examples are given to students near the examples. The shapes in the figures are divided into small cubes and the textbook requires students to explore that they can find the volume of the shape by counting the small cubes rather than by using the given traditional volume formula (See Figure 15).

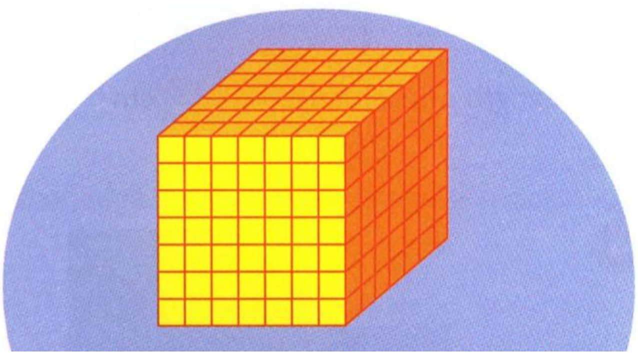
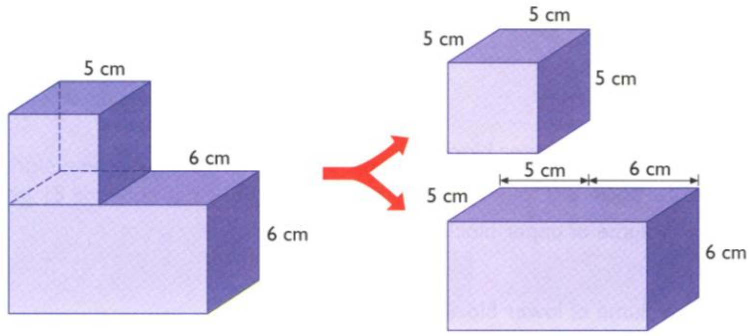


Figure 15: Volume of a cube by counting the small cubes
(Kheong, Ramakrishnan, & Soon, 2008, p. 65)

The Singaporean textbook requires students to find volume of a shape of sides are known and also it requires students to find missing values of sides of a shape of volume is known.

The volumes of irregular figures are mentioned in the sixth grade textbook of Singapore. The textbook teaches the volume of an irregular shape based on the volume of a cube and cuboids, so it firstly teaches the area of them. The textbook explain explicitly how students can divide a given irregular shape on some worked examples and related visuals. Also, it is teaches that how students can find the invisible sides of an irregular shape (See Figure 16).



The rectangular block has length 11 cm, breadth 5 cm and height 6 cm.

Volume of rectangular block $= 11 \times 5 \times 6$
$= 330 \text{ cm}^3$

Volume of cube $= 5 \times 5 \times 5$
$= 125 \text{ cm}^3$

Figure 16: An example of measuring volume of irregular shapes from the sixth grade Singaporean textbook (Kheong, Ramakrishnan & Soon, 2008, p. 67)

Volume of liquids (filling) is presented to Singaporean students at the sixth grade level. The content presentation implies that students have pre-information about the volume of liquids since there in not any information or formula about volume filling. However, the seventh grade Singaporean textbook mention about the concept of volume. It explains what the meaning of volume is with a few sentences and then it presents metric units of volume in addition to special units of fluids volume.

The textbook contain some examples that present to students how the volume of a shape can be calculated if its net is given. Students firstly should determine which shape is formed with the given net and then they determine height, base and length of the shape to calculate the volume of the shape.

The seventh grade textbook of Singapore uses small cubes to explain the volume of a shape in only exploration part. The book requires students to explore how many small cubes the shape has, how many small cubes the one face of the shape has and how many small cubes the invisible faces have. The book wants to give measurement of volume with the exploration examples, but there is no explicit remark related to this situation.

The textbook also presents the volume of the shapes with holes. It uses such method “external volume – internal volume” to find volume of shapes with holes rather than dividing the shape into prisms and finding volumes of these prisms and adding their volume to find the volume of shapes with holes.

The eighth-grade Singaporean textbook explains the logic of volume formula of a pyramid based on the volume of a prism that was taught in the seventh grade. The textbook uses some pictures of an activity to provide student to understand that it takes three times the volume of a pyramid to fill up the prism completely. This method is also used in the eight grade Turkish textbook to teach volume of a pyramid.

After the measurement of volume of a pyramid, the textbook of Singapore makes a comparison between pyramid and cone and it reports that a cone is quite similar to a pyramid. So the volume of a cone is similar to volume of a pyramid and then it gives the volume formula of a cone.

The eighth grade Turkish textbook prefers to present a different method to explain the volume formula of a cone. Students are required to fill up a cone by using a cylinder of radius and height is equal to radius and height of the cone. Students are asked “how many times did you fill up the cone by using the cylinder” and the traditional volume formula of a cone is given to students under the question.

The textbook of Singapore present an anecdote that belongs to Archimedes times such that:

> Archimedes is one of the three greatest mathematicians of all times. He lived during 287-212 BC in Greece. One day, the King asked a goldsmith to make him a gold crown. After the crown was made, the King doubted whether the crown was really made of pure gold. So he asked Archimedes to find out. It was easy to find the mass of the crown by weighing it. The problem was to find its volume. Archimedes thought for a few days but he still had no idea. Then, he went to take a bath. As he stepped into the bath full of water, the water overflowed. This gave Archimedes an idea of how to find the volume of the crown. He was so excited that he

> dashed out into the street shouting -Eureka - meaning - I have found it (Seng & Yee, 2007, p. 220).

After this anecdote was given, the textbook presents how Archimedes discovered a formula to calculate the volume of a sphere. A circular cylinder of radius r and height 2r was filled with the water and then the sphere of radius r was put into cylinder. He observed that the volume of water displaced was equal to 2/3 of the volume of the circular cylinder. Finally, the volume formula of a sphere is given to students.

Another method was observed to teach volume of a sphere in the textbook of Turkey. Students fill up the circular cylinder by using the sphere and they discuss the relation between the volume of the sphere and the volume of the circular cylinder. That is, volume of sphere is given based on the relationship between cylinder and sphere.

There is a unit related to forming shapes by using unit small squares in the Turkish textbook. Students firstly learn to draw shapes on the dot papers and then to draw front, right, left, back and top views of the shapes. The aim is to provide students to determine how many units small squares the shapes have. Also, students learn to visualize the invisible parts of the shapes.

The sixth grade Turkish textbook directly begins with an activity in order to teach volume measurement. Students use unit cubes to form a prism. Then, it is wanted students to count the unit cubes used to form the prism. Also, a formula is given to students such as “length × base ×height” in the same activity. The aim is to enable student to explore that the number of unit cubes used to form the prism is equal to the result of the formula, so the volume of a prism is found by using “length × base ×height” formula.

4.2. 2 Activities in the Textbooks

There are activities in the selected sixth, seventh and eighth grade textbooks of Turkey, Singapore and the United States.

The content presentation in the Turkish textbooks is depended on the activities. That is, the Turkish textbooks try to give information thanks to activities. As mentioned before, the textbooks begin by asking an interesting question related to topic. After the beginning question, there is certainly an activity about to be gained objective. For example; if topic is measurement of rectangular area, the textbook presents an activity related to rectangular area without giving any explanation or information about area. The aim of the textbooks of Turkey is that students have to build their own knowledge and they have to discover how they calculate area of rectangle thanks to activities.

The Turkish textbooks explain step by step what student has to make during activities such that “Let’s draw a figure and cut out it and then construct a new figure from the old one and estimate the area of new figure…etc”. Also, some questions are asked to students about important points of topics during the activities. That is, the important information and formulas about perimeter, area and volume are required to be discovered by students thanks to these activities. Generally there is not any knowledge about what are the objectives that students attain after the activity is completed in the Turkish textbooks.

The sixth grade level Singaporean textbook has a few student-based activities and these activities generally are presented at the end of content presentation under the head of “Let’s work together”. Similar to Turkish textbooks, each step of activities is given to students with exact details. The activities in the sixth grade Singaporean textbook have not a common approach. The aim of some of them is to provide students making a summarization of new learned knowledge and the aims of the others are to provide students noticing some important knowledge by using new learned knowledge. For example; there is an activity at the end of perimeter of figures topic. A rectangle and a square are given to student and students are wanted to construct some shapes by using these figures and a question is asked to students about the area comparison about the constructed shapes. The aim of the activity is to enable students notice the area of figures are same. That is, the areas of figures do not change if they are constructed by using the same shapes. This point is not mentioned during content instruction part.

There is not any student-based activity in the sixth grade U.S textbook, whereas the eighth grade U.S textbook includes many student-based activities. After the textbook gives some information about the topic, an activity is presented to students and each step of activities is explained to students very explicitly. The aim of these activities is to provide students understanding the topics better. Differently from the Turkish textbooks, the U.S textbook frequently mentions about what students have learnt at the end of the activity.

Moreover, the eighth-grade textbook of U.S include some games related to perimeter, area and volume measurement. The aim of these games is to provide students to reinforce their new knowledge. For example; there is a game “Shape Capture: 3D”. The game has two cards as being shape cards and property cards. Shape cards include pictures of some 3-D shapes including nets of 3-D shapes and top, front views of 3-D shapes. Property cards include names of some 3-D shapes and some properties of shapes. Students will select a two property cards and then they will guess which shape has these properties. The textbook explains how students will play this game step by step. The aim of the game is to give feedback to students whether they have learnt properties of 3-D shapes or not.

The activities in the sixth, seventh and eighth grade Turkish textbooks are generally based on individual work. A small number of them require group working. Similarly, the activities in the U.S textbooks are also based on individual work rather than group work.

When the activities in the sixth grade Singaporean textbooks were analyzed, it was observed that nearly all of them require to pair working and the textbook specifies this situation at the beginning of the each activity with “work in pairs” words. However, this situation is not the same in the seventh and eighth grade textbooks of Singapore.

4.2.3. Technology Usage in the Textbooks

The sixth, seventh and eighth grade textbooks of Turkey and Singapore do not require using any technological materials such as computers, calculators and overhead projector in the activities, in the worked examples or in the exercises related to perimeter, area and volume measurement topics.

Moreover, the sixth grade U.S textbook does not have any usage of technology, but it is observed that the eighth-grade U.S textbook is required students to use computers. Generally, dynamic geometry systems (DGS) are preferred to draw 3 dimensional figures in the U.S textbook. Students use DGS program during the activities and how students will use DGS is explained step by step in the activities. The textbook gives some samples about how students have an image on their computer screen while they are using DGS.

Activity 4

Step 1 Create a cube using the cube tool on a DGS 3-D.

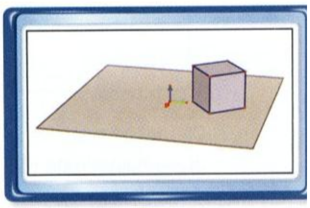
Step 2 Determine how to make a net from a cube on your DGS 3-D. On some DGS, you may need to choose the command OPEN POLYHEDRON. Then with the SELECT tool, open the net until it is flat.

Step 3 Create a net for the cube on another page of your document. On some DGS you will need to select the net and choose the command ADD NET PAGE.

Step 4 Print your net and fold it into a cube.

Step 5 Create another type of prism and its net in the same way you created the net for the cube. Print it out and fold it into the prism. How many bases does it have? What is their shape? How many lateral faces does it have? What shape are they? Give a precise name for your prism.

Step 6 Create a pyramid with a square base and then have the DGS 3-D create a net for the pyramid. Describe the shape of the base and the lateral faces.



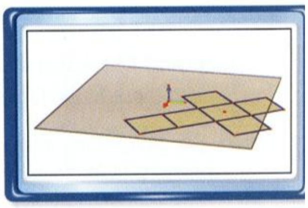


Figure 17: Technology usage in the eighth grade U.S textbook
(Victoria et al., 2008, p. 688)

DGS is also used when the textbook explains 2-dimensional nets for three dimensional shapes. For example; students explore what is the net of a cube by using DGS program. In addition, students create two triangles on DGS and they observe the areas of these triangles by changing their sides on DGS. The aim of usage DGS in this activity is that students are to be noticed how changing dimension affects area.

4.2.4 Visualization in the Textbooks

The related pages were examined page and page and then the related illustrations were determined. The illustrations were separated into two groups such as mathematically relevant illustrations and mathematically irrelevant illustrations. If illustrations provide students understand the mathematical explanations in content presentation better and they are a part of mathematical problems, they were accepted as being mathematically relevant illustrations (See Figure 18). The illustrations that are not related to content presentation and not required to solve the problems were accepted as being mathematically irrelevant illustration (See Figure 19).

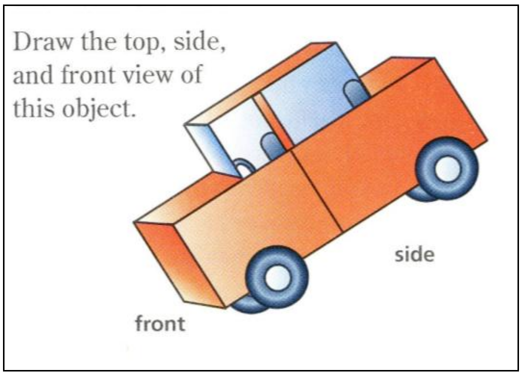


Figure 18: An example of mathematically relevant illustration from eighth grade U.S mathematics textbook (Victoria et. al., 2008, p.737)

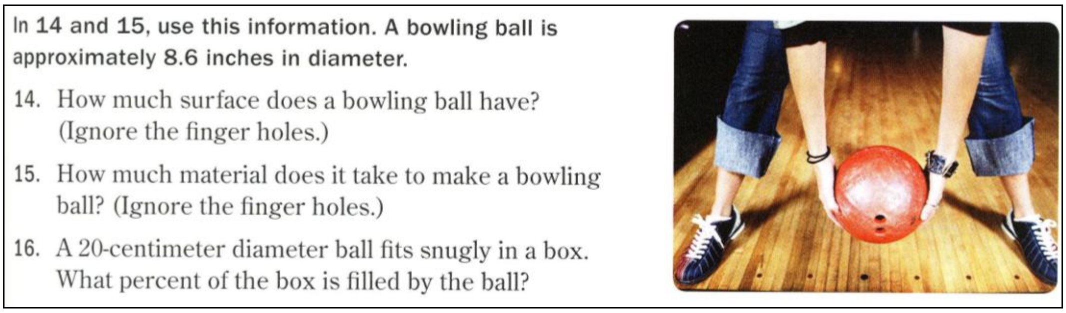


Figure 19: An example of mathematically irrelevant illustration from eighth grade U.S mathematics textbook (Victoria et. al., 2008, p. 717)

After illustrations were separated as being mathematically relevant or irrelevant, frequency of them and their total page space were calculated. In this part, researcher counted the frequency of the illustrations and calculated the percentage of pages occupied by illustration by measuring the page space area of the illustrations. No reliability-checking was performed for these procedures since these procedures were fairly objective.

Table 10: Frequency of the related perimeter, area and volume illustrations

	RELEVANT			TOTAL	IRRELEVANT			TOTAL
	Grade 6	**Grade 7**	**Grade 8**		**Grade 6**	**Grade 7**	**Grade 8**	
TURKEY	176	170	85	431	53	54	70	177
SINGAPORE	260	151	100	511	66	107	58	231
AMERICA	91	0	234	325	17	0	22	39

As seen in the Table 10, Singaporean textbooks have the most total number of illustrations. Moreover, a big percent of the illustrations in the textbooks of Singapore are mathematically relevant illustrations. However, the U.S textbooks have the least number of illustrations, but the interesting point is that there are a small number of mathematically irrelevant illustrations in the U.S textbooks.

Addition, it was observed that there is a decreasing about the total number of illustrations in the Turkish and Singaporean textbooks from grade 6 to grade 8. Also, the number of mathematically relevant illustrations decreases from grade 6 to grade 8 in the textbooks of the same countries. However, there is a increasing about the total number of illustrations and mathematically relevant illustrations in U.S. textbooks from grade 6 to grade 8 because the eighth grade U.S. textbook covers more perimeter, area and volume topics than the sixth grade U.S. textbook.

Table 11: Average Percentage (%) of pages that occupied by perimeter, area and volume illustrations.

	RELEVANT			TOTAL	IRRELEVANT			TOTAL
	Grade 6	**Grade 7**	**Grade 8**		**Grade 6**	**Grade 7**	**Grade 8**	
TURKEY	19	24	10	17	4	6	9	6
SINGAPORE	37	28	22	31	4	6	3	4
AMERICA	19	0	13	14	1	0	2	2

The measure of page space of illustrations is consistent with the frequency of them. That is, Singaporean textbooks cover the most page space related to mathematically relevant illustrations in addition to having the most number of relevant illustrations. However, although there is not a big difference between the frequencies of the relevant illustrations in the textbooks of the countries, the page space of the relevant illustration in the Singaporean textbooks is too much. The Singaporean textbooks present mathematics knowledge more visually than the Turkish and U.S textbooks.

4.3 To-be-solved Problems in the Textbooks

The perimeter, area and volume to-be-solved problems were determined carefully. Some problems have more items. If items of a problem were not connected with each other, each item was accepted as being a separate problem.

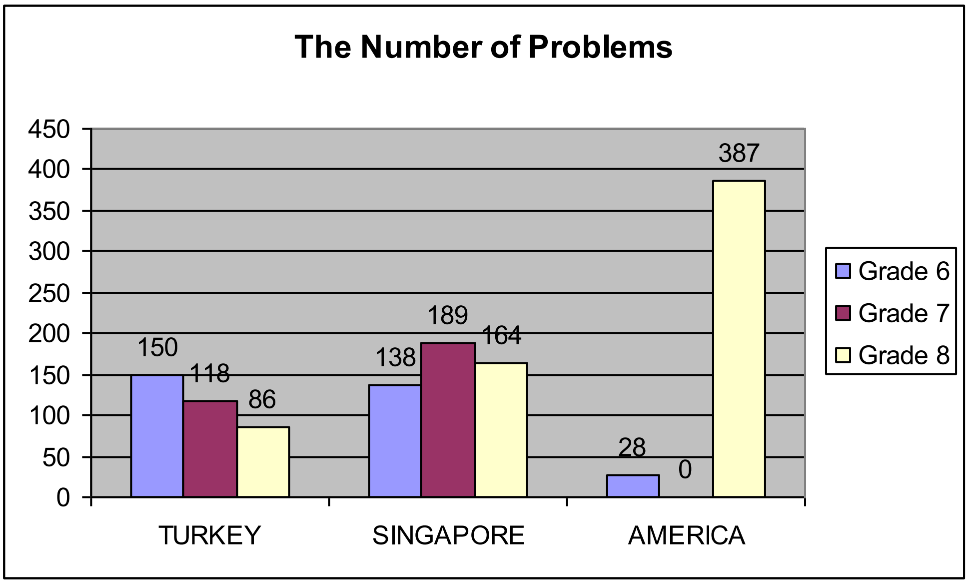


Figure 20: Number of perimeter, area and volume problems in the textbooks

Totally, the Singaporean textbooks have the most perimeter, area and volume measurement problems. However, there are the least perimeter, area and volume measurement problems in the Turkish textbooks. Also, there is a decline in the number of the problems from the sixth grade level to eighth grade level in the Turkish textbooks. The U.S textbooks include the most perimeter, area and volume problems in the eighth grade level.

10% percent of the problems from the each countries textbooks and from each grade level were selected separately by using MS Excel randomly page selection function. Two coders working independently coded 135 randomly selected problems based on NAEP Framework. One of the coders is the researcher and the other one is a research assistant in Middle East Technical University. After the initial coding, the inter-rater reliability was calculated as 95%. Each problem for which the coders did not agree was then discussed until an agreement was reached on how the problem would be coded. Since the inter-rater reliability score (95 %) is large enough (Li, 1999), the researcher coded all the problems of the textbooks.

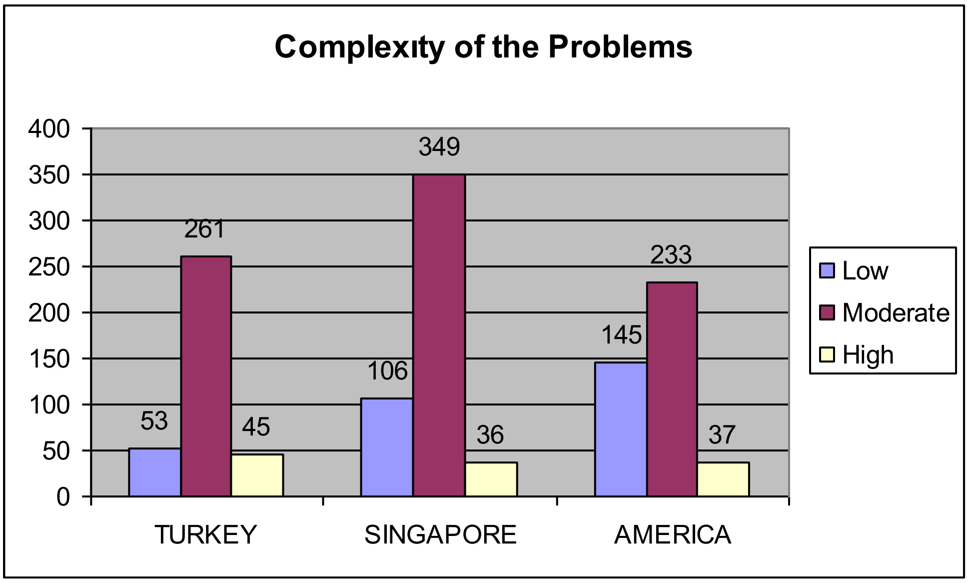


Figure 21: Complexity of to-be-solved problems in the textbooks

The interesting point is that all of the countries include more moderate problems than low and high complexity problems. Moreover, they include relatively the least number of high complexity problems.

The Turkish textbooks generally have high complexity problems that requires student to formulate an original problem to given a situation and to explain and justify a solution to a problem. The Singaporean textbooks have high complexity problems that ask students to perform a procedure having multiple steps and multiple decision point. Students need to provide a mathematical justification for the high complexity problems in the U.S textbooks.

Moreover, the moderate complexity problems in the Turkish textbooks require students to retrieve information from a graph, table or figure and use it to solve a problem requiring multiple steps and to solve a problem requiring multiple steps. The situation is same in the moderate complexity problems in the Singaporean and U.S textbooks.

Low complexity problems in the textbook of the countries generally require students to solve a one-step problem.

4.4 Summary of Results

Some similarities and differences were observed among the textbooks of the countries. They are nearly include the same topics, but the topics such as perimeter-area relationship and surface area of irregular shapes are only mentioned in Turkish textbooks and the topics as volume of irregular shapes is only presented in Singaporean textbooks and the topics as how changing dimensions affect area and volume is only given in American textbooks. Moreover, Turkish textbooks include the most number of perimeter, area and volume subtopics. However, Singaporean textbooks have the highest number of pages devoted to presentation of the selected topics. This means that Singaporean textbooks present the topics more detailed. The Turkish textbooks start to content presentation by asking a question related to daily lives of students to increase motivation, but the U.S textbooks give main ideas of topics in a box under big idea heading at the beginning of the presentation. None of the Turkish and U.S textbooks have a summarization of the new learned knowledge, whereas the Singaporean textbooks present a summarization at the end of the units. Moreover, more worked examples are included in the Singaporean textbooks than the textbooks of the other countries. Singaporean textbooks are also rich in terms of mathematically relevant illustrations that make the topics more understandable for students. The textbooks use different explanatory approaches to teach surface area of a sphere, volume of a cone, area of a parallelogram and volume of a sphere etc…Using the relationship between figures to teach area and volume were observed in all textbooks. Content presentation in Turkish textbooks is depended on activities, so they include more number of activities than the Singaporean and U.S textbooks have. While American textbooks benefit heavily from technology to present the topics, especially by using three dimensional shapes; Turkish and Singaporean textbooks do not make use of technology. The textbooks do not show a difference in terms of complexity of to-be-solved problems. Since all of them mostly include the problems with moderate complexity. Despite there is not any difference among the textbooks in terms of the complexity of to-be-solved problems, there is a difference in terms of the number of to-be-solved problems in the textbooks. Singaporean textbooks encompass more to-be-solved problems compared to others.

CHAPTER

DISCUSSION

The purpose of this study was to compare selected sixth, seventh and eighth grades mathematics textbooks from Turkey, Singapore and the United States of America and explore their implications for presenting same opportunity to learn to the students at the same grade level. In this study, the selected books were analyzed in terms of whether they included perimeter, area and volume topics, how they presented the topics on the basis of the selected features and to-be-solved mathematical problems related to the topics.

The selected textbooks from Turkey, Singapore and U.S generally cover the similar topics and subtopics. However, some important topics such as relationship between perimeter and area and measuring surface area of irregular figures are only presented in the Turkish textbooks. These represent important opportunity to learn for Turkish students. Additionally, only the Singaporean textbooks present an opportunity to learn to find volume of irregular shapes. Thus mathematics textbooks of different countries provide students different opportunities to learn perimeter, area and volume.

Furthermore, there are some differences among the textbooks about the placements of some topics such as circumference of a circle, area of a circle and volume of a prism. Fuson (1988) reported that if topics are presented earlier in textbooks, students would be given an opportunity to cover more topics and to learn mathematics by comparable grades. Is really the placement of topics affects students' opportunity to learn mathematics? How? More research is needed in this area.

Although the Turkish textbooks have more number of subtopics than the Singaporean textbooks, the page space of subtopics in the Singaporean textbooks is more than the page space of the subtopics in the Turkish textbooks. This means that the Singaporean textbooks present fewer topics by giving more information about the topics, whereas the Turkish textbooks present general and limited information about the topics. Which one present more opportunity to learn mathematics: giving depth knowledge by including fewer topics or giving less knowledge by including more topics? Reys, Reys and Chazen (2004) claimed that including more topics does not affect students' achievement positively. Maybe, the lower achievement of Turkish and American students can be explained by not giving deep information. Moreover, the greater performance of Singaporean students can be evaluated as an evidence of the fact that including less in number, but detailed in content topics in the textbooks more effective in enhancing the quality of mathematics education. This implies that developer of Turkish curriculum can bear in their mind this issue.

Furthermore, the Singaporean textbooks present the topics grade by grade. It does not repeat the same topics in different grades. The Turkish textbooks repeat some topics in different grades. However, the study supports the idea of Stevenson (1985) such that same topics are taught over different grades in U.S textbooks. Also, he reported repeating same topics affect students' performance poorly, but the achievement of Turkish students is lower than the achievement of American students in the international studies although American textbooks repeat the same topics grade over grade. So, more research is needed to report a positive or negative effect of repetition on students' performance. In addition, we can also argue that the way topics repeated has also influence over students' learning.

Students confuse area and perimeter in their terminology since definitions and formulas are given without a deep understanding of what they mean and how a relationship they have with each other (Cavanagh, 2008; Clement & Stephan, 2004; D'Amore & Fandino Pinilla, 2006; Danielson, 2005). The U.S and Singaporean textbooks explain the meaning of area and perimeter explicitly, whereas the Turkish textbooks do not. This situation can cause Turkish students' confuse perimeter and area in their terminology, so the meaning of area and perimeter in addition to traditional area and perimeter formula can be added to the content of perimeter, area and volume in the Turkish textbooks.

The U.S textbooks prefer to present the areas of figures by defining a unit and iterating this unit although the Turkish and Singaporean textbooks directly give traditional area formulas of figures without making any explanation the logic of the formulas. Zacharos (2008) revealed that students have problems about understanding of area concept, measuring area of regular and irregular figures because the traditional formula to find area is emphasized rather than the unit selection and iteration this unit to understand the formula reasoning. Also, the use of unit squares are recommended to make traditional area formula for understandable for students (Yeo, 2008), but the only U.S textbooks present opportunity to use squares to understand area measurement. So the Turkish and Singaporean textbooks are needed to present opportunities to learn defining a unit and iteration it to find area of figures in order to prevent their students' having difficulties about measuring area.

Although the Turkish textbooks do not explain the meaning of perimeter and area, they are the only textbooks that explain the relationship between perimeter and area. This is an important opportunity for the Turkish students since students have a misconception such that there is reciprocal relationship between area and perimeter of a figure. That is, if a figure has bigger perimeter than the other figures, it also have bigger area (D' Amore & Fandino Pinilla, 2006).

The textbooks of all the countries try to explain the relationship between the geometric figures to their students. The textbooks generally explain the logic of area formula of a figure by depending on the other figures. Although the Singaporean and U.S textbook explain these relationships explicitly, the Turkish textbooks require students to explore the existing relationship between figures themselves thanks to activities. However, Turkish students had difficulties to find the area of a rectangular by using the relationship between rectangular and the other figures, whereas the performance of Singaporean students was very good (Olkun & Aydoğdu, 2003). More emphasize can be given with an explicit explanation to the existing relationship between figures in the Turkish textbooks. Moreover, constructing relationship between figures also help to define a unit measurement, in such a way the difficulties about understanding and measurement area of figures also can be prevented.

Same explanatory approach is used in the Turkish, Singaporean and U.S textbooks to measure area of a circle. That is, the students of all of the countries have same opportunities to learn measurement area of a circle. However, the students are given different opportunities to learn measurement surface area of a sphere, volume of a cone and volume of a sphere since the textbooks of the countries use different explanatory approaches to present these topics. What are the roles of different explanatory approaches in textbooks? Moreover, students rely on approaches used in the textbooks (Cavanagh, 2008), so it is needed to be analyzed the roles of explanatory approached used in the textbooks on students' learning? Which one present more opportunity to learn mathematics?

In addition, many researchers (Haggarty & Pepin, 2002; Reys et al., 2003; Santos, Macias, & Cruz, 2006; Schmidt et al., 2001) reported that teachers' pedagogical strategies are influenced by the approaches in textbooks. Are textbooks a pedagogical means? How different explanatory approaches in the textbooks affect pedagogy of teachers? Moreover, how does this situation affect students' opportunity to learn mathematics?

Many teachers follow the activities in the textbooks during classroom instruction (Sun, Kulm & Capraro, 2009) since student-based activities have positive effects on students' learning perimeter, area and volume topics (Cavanagh, 2008; Maida & Maida, 2006; Olkun & Sinoplu, 2008; Outhred & Mitchelmore, 2000) by providing students to experience with concrete materials to give meaning to formulas (Olkun, 2003). Moreover, Olkun and Aydoğdu (2003) reported that Turkish students have difficulties to define a unit and iteration this unit for finding area of a figure, some tangram activities can be included in the Turkish textbooks to help to improve students' skills.

It was observed that the textbooks of the countries include student-based activities, but more number of activities is seen in the Turkish textbooks than the Singaporean and U.S textbooks since content presentation in the Turkish textbooks depends on the activities. That is, the Turkish textbooks require students discover knowledge themselves at the end of these activities rather than giving the knowledge and formulas directly to students, but there is not generally an explanation about the objectives that students attain after the activity is completed. So, how can we be sure about whether students gained the knowledge required in the textbooks or not? Also, the U.S textbooks mention about what students have learnt at the end of the activity since the aim of the activities is to help to students understanding the learned knowledge better. Moreover, the aim of the activities in the Singaporean textbooks is to make summarization of what students have learnt during classroom instruction. That is, the aims of the activities show differences among the textbooks, so which one is more effective on students' learning mathematics and which one presents more opportunities to learn mathematics for students are needed to be explored in the future studies. Furthermore, the activities in the Turkish and U.S textbooks are generally based on individual working, whereas group working is required in the activities included in the Singaporean textbooks. Which one is more effective on students' learning mathematics: individual working or group working?

Using technology affects students' mathematics learning positively (Clement, 1999; Clement & Stephan, 2004; Olkun, Altun, & Smith, 2005) since students have an opportunity as being active during their learning process and they are provided a variety of rich experiences by technological tools (McCoy, 1996). The study found that U.S textbooks give more opportunity to learn perimeter, area and volume by using computer programs that enables students to be involved with perimeter, area and volume in an active way, whereas the technology usage was not observed in the Turkish and Singaporean textbooks. Technology usage can be integrated into the Turkish textbooks by textbooks writers to provide students having more opportunity to learn and understand mathematics.

Moreover, the study found that Singaporean textbooks are rich in terms of illustrations and a huge number of these illustrations are mathematically relevant to given topics or problem. The relevant illustrations in the content presentation part of the Singaporean textbooks include a huge number of speech balloons, whereas there is not any speech balloon in the Turkish and U.S textbooks. What is the role of these speech balloons in the textbooks? Are they really an important factor for opportunity to learn mathematics? In addition, the most page space of the relevant illustrations belongs to the Singaporean textbooks. This means that the Singaporean textbooks present more opportunities to learn the topics depend on visuals. Maybe, this can be an important factor on high performance of Singaporean students since Markus (2001) and Özdemir, Duru and Akgün (2005) reported that presenting mathematical knowledge by using more visuals affects students' learning of mathematics positively since visuals provide students to learn complex and abstract mathematical concepts easily (Markus, 2001) and to acquire strong foundation in mathematics by enhancing students' creativity (Yeap, 2005). This implies that the important roles of visuals on students' learning can be taken into consideration through designing of textbooks and their contents.

When the problems in the textbooks were analyzed, it was found that the textbooks of the countries do not have more number of high complexity problems. All of them generally ask moderate complexity problems requiring retrieving information from a visual and than use it to solve a problem. Turkish students had poor performance in solving high complexity problems (Mullis et al., 2000) due to limitations of to-be-solved problems in the textbooks (Olkun & Aydoğdu, 2003). So, what is the ideal balance in terms of complexity of to-be-solved problems? Moreover, this study only analyzed the complexity of to-be-solved problems. A further study can analyze the different types of to-be-solved problems and their roles on students' opportunity to learn mathematics. It is reported that not giving opportunities to learn solving different types of mathematical problems underrepresented in textbooks causes students being unsuccessful due to the lack of experience rather than their difficulty (Olkun & Toluk, 2002).

Furthermore, the Singaporean textbooks include the most number of the to-be-solved perimeter, area and volume problems, whereas the Turkish textbooks have the least number of the problems. What is the optimum number of to-be-solved problems in the textbooks? How the complexity and the number of to-be-solved problems affect students' learning mathematics and achievement?

The results of the study put forward that the textbooks of the countries have some differences in terms of coverage of topics, presentation of content, technology usage, activities, visuals and to-be-solved problems. This implies that the textbooks of each countries present different opportunities to learn perimeter, area and volume to their students. However, does it mean that students have same opportunity to learn mathematics if there is not any difference among the books?

In other words, using same textbooks means having same opportunities to learn mathematics? Also, should students in the same or different country have the same opportunities to learn mathematics by not taking into consideration some factors such as abilities, characteristics and achievement levels of students, conditions of the school and social, economical and cultural features of a country?

Moreover, using same textbook means presenting same opportunities to learn mathematics by teachers? Since teachers who use same textbooks showed variations in their instructional approaches (Sun, Kulm, & Capraro, 2009), so teachers' textbook usage and its effects on students' opportunities to learn mathematics are needed to be explored in the future studies. Moreover, the other factors that affect students' opportunities to learn mathematics are needed to be explored. Also, the study only analyzed some indicators of opportunities in the textbooks such as technology, activity, visual and complexity of to-be-solved problems, so the other indicators of opportunities to learn such as used models, reading level and worked examples…etc can be studied in the other researchers.

All in all, the study implies that the Turkish textbooks can provide more opportunities to learn perimeter, area and volume topics when their strengths are combined with the strengths of the textbooks of the other countries. In the studies conducted in the future, the Turkish textbooks can be analyzed in terms of presented opportunities for the other content areas and topics out of perimeter, area and volume.

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