THE EFFECT OF INSTRUCTION WITH CONCRETE MODELS ON EIGHTH GRADE STUDENTS' PROBABILITY ACHIEVEMENT AND ATTITUDES TOWARD PROBABILITY

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Prof. Dr. Meliha ALTUNIŞIK
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Hamide ERTEPINAR
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Safure BULUT
Supervisor

Examining Committee Members

Prof. Dr. Ahmet ARIKAN (GAZI UNI., SSME)

Assoc. Prof. Dr. Safure BULUT (METU, SSME)

Assoc. Prof. Dr. Erdinç ÇAKIROĞLU (METU, ESME)

Assoc. Prof. Dr. Semra SUNGUR (METU, ESME)

Assoc. Prof. Dr. Esen UZUNTİRYAKİ (METU, SSME)

I hereby declare that all information in this document has been obtained and presented accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name **:** Fatmagül YAĞCI

Signature :

ABSTRACT

THE EFFECT OF OF INSTRUCTION WITH CONCRETE MODELS
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AND ATTITUDES TOWARD PROBABILITY

YAĞCI, Fatmagül

MS, Department of Elementary Science and Mathematics Education
Supervisor: Assoc. Prof. Dr. Safure BULUT

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The main purpose of the study was to investigate the effect of instruction with concrete models on eighth grade students' probability achievement and attitudes toward probability. Another aim was to examine students' views about instruction with concrete models. The study was conducted in a private school in a big city in Central Anatolia Region with 12 eighth grade students. Both quantitative and qualitative research designs were used. The treatment was applied by the mathematics teacher for 4 hours per week throughout 4 weeks. Probability Achievement Test and Probability Attitude Scale were administered to collect data. In order to analyze the data, Friedman and Wilcoxon tests were used. Also, the interview was carried out with 11 students to determine their views about the instruction.

It was found that there was a statistically significant change in probability achievement of eighth grade students participated in the instruction with concrete models across three time periods. In other words, it was found that there were statistically significant positive changes in students' probability achievement from pre-intervention through post-intervention and from pre-intervention through follow-up. It was also found that there was no statistically significant change in students' probability achievement from post-intervention through follow-up. The results also revealed that there was no statistically significant change in students' attitudes toward probability across three time periods. Moreover, according to findings of the interview it was determined that most of the students had positive views about the effects of instruction with concrete models on their cognitive processes and on their attitudes toward concrete models and probability lessons.

Keywords: concrete models, probability, achievement, attitude, views.

ÖZ

SOMUT MODELLERLE ÖĞRETİMİN 8. SINIF ÖĞRENCİLERİNİN OLASILIK BAŞARISINA VE OLASILIĞA YÖNELİK TUTUMLARINA ETKİSİ

YAĞCI, Fatmagül

Yüksek Lisans, İlkögretim Fen ve Matematik Alanları Egitimi
Tez Yöneticisi: Doç.Dr. Safure BULUT

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Çalışmanın ana amacı somut modellerle öğretimin 8. sınıf öğrencilerinin olasılık başarısına ve olasılığa yönelik tutumlarına etkisini araştırmaktı. Diğer amacı ise, öğrencilerin somut modellerle öğretim hakkında görüşlerini araştırmaktı. Çalışma, İç Anadolu Bölgesi'ndeki bir büyük şehirde bulunan bir özel ilköğretim okulunda 12 sekizinci sınıf öğrencisiyle yapılmıştır. Nicel ve nitel araştırma desenlerinin ikisi de kullanılmıştır. Veileri toplamak için olasılık başarı testi ve olasılığa yönelik tutum ölçeği uygulanmıştır. Uygulama öğrencilerin matematik öğretmeni tarafından haftada 4 saat olmak üzere 4 haftada yapılmıştır. Verileri analiz etmek için Friedman ve Wilcoxon testleri kullanılmıştır. Ayrıca, 11 öğrenci ile somut modellerle işlenen dersler hakkında görüşlerini araştırmak için görüşme yapılmıştır.

Somut modellere olasılık dersine katılan 8. sınıf öğrencilerinin olasılık başarısında 3 zaman periyodu arasında (uygulamadan önce, uygulamadan hemen sonra, belirli bir zaman sonra) istatistiksel olarak anlamlı bir değişim bulunmuştur. Diğer bir deyişle, öğrencilerin olasılık başarısında uygulama öncesinden uygulamanın hemen sonrasına ve uygulama öncesinden uygulamadan belirli bir zaman sonrasına kadar olan zamanda istatistiksel olarak anlamlı olumlu yönde bir değişim olduğu bulunmuştur. Ayrıca, uygulamanın hemen sonrasından uygulamadan belirli bir zaman sonrasına öğrencilerin olasılık başarısında istatistiksel olarak anlamlı bir fark bulunmamıştır. Sonuçlar öğrencilerin olasılığa yönelik tutumlarında 3 zaman periyodu arasında istatistiksel olarak anlamlı bir değişim olmadığını da göstermiştir. Ayrıca, görüşmenin bulgularına göre, çoğu öğrencinin somut modellerle öğretimin bilişsel süreçleri üzerinde ve somut modellere ve olasılık derslerine yönelik tutumları üzerinde olumlu etkileri olduğunu düşündükleri belirlenmiştir.

Anahtar sözcükler: somut modeller, olasılık, başarı, tutum, görüşler.

To my parents

Nezahat and Nurettin
YAĞCI

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ABBREVATIONS

PAT	: Probability Achievement Test
PAS	: Probability Attitude Scale
PKT	: Pre-requsite Knowledge and Skills Test
PC	: Permutation and Combination
BCP	: Basic Concepts of Probability
TP	: Types of Probability
DIE	: Dependent and Independent Events
MoNE	: Republic of Turkey Ministry of National Education
NCTM	: National Council of Teachers of Mathematics
M	: Mean
SD	: Standard Deviation
Max	: Maximum
Min	: Minimum
OS	: Out of Score
PISA	: Program for International Student Assessment
Post	: Post-intervention
Pre	: Pre-intervention
s	: Student

CHAPTER 1

INTRODUCTION

The concept of Probability has a great significance in scientific thinking and it has been a significant branch of mathematics, creating reactions in science, philosophy and daily human life for years (Fischbein, 1975). It is also real life mathematics (Fennel, 1990). People usually use the expressions of probability in daily lives (Hope & Kelly, 1983). Moreover, they often face situations which require probabilistic reasoning, knowledge and practice to make decisions. Questions related to probability of raining demonstrate that daily experiences are composed of probability (Horak & Horak, 1983). In addition to daily life, the concept of probability is used in different disciplines and occupations; such as quantum physics, law, insurance, etc (Lappan et al., 1987). Besides its importance in real life, the importance of probability in education is also emphasized. Research studies point out that an increased attention should be given to probability concept (Bulut, 2001) and it should be included in mathematics curriculum as a significant section (e.g. Fennel, 1990; Hope & Kelly, 1983; National Council of Teachers of Mathematics, 2000). Republic of Turkey Ministry of National Education (MoNE) (2009) also gives great importance to probability, starting from grades 4 to 8 in elementary mathematics. Probability instruction is important due to many reasons: to become a qualified citizen, consumer or worker, probabilistic reasoning is crucial (e.g. NCTM, 2000; MoNE, 2009). It also provides an enthusing and exciting foundation to the learning of basic issues in mathematics, especially to rational numbers (Fennel 1990) and it gives students opportunities to use and experience basic mathematics skills they learnt formerly (Horak & Horak,1983). Moreover, it provides a way for students to connect mathematics with other school subjects and with real life (e.g. NCTM, 2000; MoNE, 2009). However, there are problems in the instruction of probability. Many researchers emphasize that the probability concept can not be taught/learnt efficiently and students have difficulties in learning probability (e.g. Hope & Kelly, 1983; Garfield & Ahlgren, 1988; Baron & Or-Bach, 1988; Carpenter et. al., 1988; Bulut, 2001). Two of the reasons why students have difficulties in probability are as follows: it is taught in a formal and abstract way (Garfield & Ahlgren, 1988) and the appropriate materials in probability instruction are inadequate (Bulut, 1994).

Based on these problems, there has been growing research to enhance the probability instruction both in Turkey and abroad. These research studies were conducted to investigate effectiveness of different instructional methods to teach/learn probability (e.g. Cankoy, 1989; Bulut, 1994; Castro, 1998; Taylor, 2001; Yazıcı, 2002; Demir, 2005; Seyhanlı, 2007; Şengül & Ekinözü, 2007; Memnun, 2008; Ercan, 2008; Ünlü, 2008; Esen, 2009). Traditional method was found to be ineffective in the instruction of probability in almost all of these studies. However, the amount of these studies is not adequate and there should be much more research studies inspecting effects of different instructional tools or methods in teaching/learning process of probability. Therefore, the present study also takes the instruction of probability into consideration.

In the instruction of probability, the present study uses concrete models and presents various activities including concrete models. Current elementary school mathematics curriculum also emphasizes the use of concrete models in mathematics instruction (MoNE, 2009b). It is based on the understanding that “every child can learn”. (MoNE, 2009b, p.7) Mathematical concepts are inherently abstract; therefore, the perception of these abstract concepts is considerably difficult. In addition, children can learn meaningfully in learning environments where the information is obtained by concrete models. For this reason, it is considerably beneficial to use concrete models in mathematics instruction (MoNE, 2009b). Moreover, many researchers point out that use of concrete models is advantageous in mathematics instruction (e.g. Reys, 1971; Fennema, 1972; Fennema, 1973; Suydam & Higgins,1977; Driscoll, 1984; Heddens, 1986; Berman &Friederwitzer, 1989; Hartzhorn & Boren, 1990; Kober, 1991; Boling, 1991; Thompson & Lambdin, 1994; Heddens, 2005; MoNE, 2009b). An advantage is that concrete models help students make connections between real and abstract worlds (e.g. Fennema, 1973; Heddens, 1986; Berman & Friederwitzer, 1989; Kober, 1991). Furthermore, Granda and Lappan (1980) state that probability instruction through concrete activities is more effective than theoretical instruction. In this sense, various recommended concrete models for probability instruction are employed in the present study.

Although it is emphasized that using concrete models in mathematics instruction is beneficial, the number of research studies conducted to examine effectiveness of concrete models on mathematics achievement is not sufficient both abroad (e.g. Fennema, 1972; Suydam & Higgins,1977; Parham, 1983; Sowell, 1989; Leinenbach & Raymand, 1996; Hinzman, 1997; Daniel, 2007) and in Turkey (e.g. Bayram, 2004; Tutak, 2008; Sarı, 2010). Moreover, there are only a few research studies conducted to examine the effects of concrete models in probability instruction (e.g. Cankoy, 1989; Taylor, 2001). One of the aims of present study is to investigate the effect of instruction with concrete models on students' probability achievement.

Beside the use of concrete models, the present study also considers the environment in teaching/learning process. MoNE (2009b) also emphasizes the provision of environments in which children can discover, inquire and discuss the solution of problems. In this sense, it is important for children to discover the funny and aesthetical aspects of mathematics and to deal with mathematics while doing activities (MoNE, 2009b). The elementary school mathematics curriculum assigns responsibilities for children and teachers. Some of the roles of children are; participating actively in the learning process, asking questions, questioning, thinking, and discussing. Some of the roles of teachers are; making students question, ask questions, think and discuss, develop activities and apply them in the lessons and develop concrete materials (MoNE, 2009b). Therefore, the present study aims to provide an environment in which students can question, think, and discuss with each other and with the teacher while experiencing the concrete models.

Another concern of the present study is student attitudes toward probability. MoNE (2009b) states that while developing mathematical concepts and skills, the affective development of the students should be taken into consideration. For example, enjoying making mathematics, realizing the power and beauty of mathematics and developing self-concept toward mathematics are some of the roles of children related to affective domain (MoNE, 2009b). Moreover, Horak and Horak (1983) state that use of materials in probability activities increase student interest in probability and motivate them. The present study also aims to create an environment in which students can be motivated and enjoy the process of learning probability through using concrete models.

Although it is pointed out that students' affective development should be considered in learning environments, there are few research studies conducted to examine student attitudes toward probability (e.g. Bulut, 1994, Yazıcı, 2002; Demir; 2005; Tunç, 2006; Şengül & Ekinözü, 2006; Seyhanlı, 2007) and there are also limited research studies carried out to inspect the effect of concrete models on student attitudes towards mathematics (e.g. Sowell, 1989; Bayram, 2004; Tutak, 2008). However, it is not met any research studies in the literature which is designed to investigate the effect of concrete models on student attitudes toward probability. On that ground, other aims of the present study are to investigate the effect of instruction with concrete models on student attitudes toward probability and to examine their views about concrete models instruction.

In short, the purposes of the present study are (1) to investigate the effect of instruction with concrete models on eighth grade students' probability achievement and attitudes toward probability and (2) to inspect students' views about concrete models treatment. First of all, Pre-requisite Knowledge and Skills Test (PKT) was administered to 12 eighth grade students to determine their existing pre-requisite knowledge related to probability. The Probability Achievement Test (PAT) and Probability Attitude Scale (PAS) were administered three times during research period (pre-intervention, post intervention and follow up). The probability instruction was based on concrete models and it was administered 4 hours per week for 4 weeks. After the instruction, students were also interviewed to examine their views about instruction with concrete models.

1.1. Main and Sub-Problems of the Study and Associated Hypotheses

In this section main and sub-problems of the present study are stated.

The first main problem of the study is: "What is the effect of instruction with concrete models on eighth grade students' probability achievement and attitudes toward probability?"

It consists of two sub-problems. They are stated below:

S.1. What is the effect of the instruction with concrete models on eighth grade students' probability achievement?

S.2. What is the effect of the instruction with concrete models on eighth grade students' attitudes toward probability?

The second main problem of the present study is: "What are the eighth grade students' views about instruction with concrete models?"

In order to examine the first main problem, the two hypotheses given below are stated in the null form and tested at a significance level of 0.017 which was computed by dividing 0.05 with 3 according to the guidelines stated by Colman and Pulford (2006) since the test and scale were administered in 3 different time periods.

H_0. 1. There is no statistically significant change in eighth grade students' probability achievement scores across three time periods (pre-intervention, post-intervention and follow-up).

H_0. 2. There is no statistically significant change in eighth grade students' scores of attitudes toward probability within three time periods (pre-intervention, post-intervention and follow-up).

1.2. Definition of Terms

The important terms used in the study were explained below:

Probability achievement: It refers to the scores of students obtained from probability achievement test.

Attitude: Aiken (1970) defines attitude as "a learned predisposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or another person" (p.551).

***Attitude toward probability*:** It refers to the scores of students obtained from Probability Attitude Scale.

Concrete Model: It is defined as "a concrete model represents the mathematical idea by means of three-dimensional objects" (Fennema, 1972, p.635).

Views about instruction with concrete models: It refers to the answers of students obtained from interview questions which questioned student feelings and thoughts in concrete model process.

1.3. Significance of the Study

Probability gained little importance in the past in Turkey (Bulut, 2001). Before the implementation of current elementary school mathematics curriculum, probability was introduced in the eighth grade. In current elementary school mathematics curriculum, however, the subjects of probability and statistics are included in grades from 4 through 8 (e.g. MoNE, 2009a, MoNE, 2009b). However, students had low scores in questions related to probability and in general mathematics questions in international and national exams. For instance, in Third International Science and Mathematics Study (TIMSS), probability and data analysis were one of the five content areas. Turkey's probability and data analysis score was the 30th among 38 countries, and 31th in general mathematics (TIMSS, 1999). Similarly, according to Program for International Student Assessment (PISA) (2003) results, Turkish students' average scores of probability and general mathematics were under the international average. Furthermore, students obtained low scores in mathematics in level determining exam (SBS) which is a national exam that elementary students take in Turkey to be able to attain high qualified high schools. According to the data of General Management of Educational Technologies in MoNE (2009), the average of correct answers of 6^{th} grade students was 4.59 out of 16 questions in 2008. The average score of correct answers of 7^{th} grade students was 5.2 out of 18 questions and it was 3.7 out of 25 questions scored by 8^{th} grade students. These scores are considerably low. The scores achieved in 2007 were not different from the scores of 2008. Therefore, these results confirm that of Turkish education system needs revision.

The change in educational system in elementary grades and new improvements related to teaching/learning process has been brought by the current elementary school mathematics curricula which has been implemented to elementary grades from 1 to 5 since 2005-2006 (MoNE, 2009a), to 6^{th} grade since 2006-2007, to 7^{th} grade since 2007-2008 and to 8^{th} grade since 2008-2009 (MoNE, 2009b) in Turkey. The great significance has been given both to probability and use of materials in instruction. In this sense, the present study offers various activities including concrete models in the instruction of probability. This study also tries to offer an insight into implementation of current elementary school mathematics curriculum with the use of concrete models and to give considerable information to teachers and students on the subject of teaching/learning probability with concrete models.

Some studies were conducted in Turkey to determine the effectiveness of different instructional methods of teaching/learning probability; such as mathematics laboratory including concrete models (Cankoy, 1989), cooperative learning method and computer assisted instruction (Bulut, 1994), problem posing (Demir, 2005), discovery learning method (Yazıcı, 2002), graph theory based instruction (Seyhanlı, 2007), dramatization method (Şengül & Ekinözü, 2007), active learning method (Memnun, 2008), multiple intelligence theory based instruction (Ercan, 2008), cooperative learning method (Ünlü, 2008), and computer based instruction (Esen, 2009). The present study investigates the effect of instruction with concrete models on eighth grade students' probability achievement. Since the participants were tenth grade students and sixth grade students in studies of Demir (2005) and Esen (2009) respectively, the present study differs from these studies in terms of participant grade. The present study is also different from studies of Cankoy (1989), Yazıcı (2002), Seyhanlı (2007), Şengül and Ekinözü (2007), Ercan (2008), Ünlü (2008), Memnun (2008) since the participants are private school students. Another difference is that participants in the present study have been learning the subject of probability since they were 4th graders. The participants in previous research studies had received probability instruction in only 8th grade. In this respect, the present study is the first study in the literature which included participants who have been receiving probability instruction since 4th grade.

As mentioned before, the students in the present study had received probability lessons since they were in 4th grade. After the implementation of current elementary school mathematics curriculum, the probability unit took place in mathematics curriculum from grades 1 to 5 since 2005-2006 (MoNE, 2009a), in 6th grade since 2006-2007, in 7th grade since 2007-2008 and in 8th grade since 2008-2009 academic years (MoNE, 2009b). Before that time, it took place only in 8th grade mathematics curriculum. Therefore, although there are few research studies conducted to investigate the students' attitudes toward probability (e.g. Bulut, 1994; Yazıcı, 2002; Demir, 2005; Tunç, 2006; Seyhanlı, 2007), it is not met any research studies included participants who had probability courses from grades 4 to 8 in Turkey. Thus, the present study is different from the studies mentioned above in terms of including participants who received long time probability instruction and investigating the probability attitudes of eighth grade students who received probability instruction for a long time. Moreover, there are few research studies conducted to investigate the effects of instruction with concrete models on students' attitudes toward mathematics (e.g. Sowell, 1989; Bayram, 2004; Tutak, 2008). Also, it is not met any research study to our knowledge which is conducted to investigate the effects of instruction with concrete models on students' attitudes toward probability.

Consequently, this study is designed to investigate the effect of instruction with concrete models on eighth grade students' probability achievement and attitudes toward probability and to inspect eighth grade students' views about instruction with concrete models.

CHAPTER 2

LITERATURE REVIEW

The literature related to the present study was reviewed in this chapter. On the basis of the content and the main objectives of the study, the literature was composed of four sections: concrete models, discovery learning method, probability and attitude toward mathematics.

2.1. Concrete Models

In this section, firstly the theoretical background for concrete models was clarified and secondly the research studies on concrete models were stated. In some studies manipulative materials and concrete models are defined as they are different whereas in some studies they are defined as they are same. In the present study, concrete models and manipulative materials are dealt with as they are same.

2.1.1. Theoretical Background for Concrete Models

In mathematics classes, there are three kinds of models which are used as teaching aids. The first one is concrete model which is used as illustrating mathematical beliefs through three dimensional objects. The second one is symbolic model used as illustrating mathematical beliefs through generally admitted numbers and signs that indicates operations or relationships in mathematics. The third one is pictorial model sharing the properties of concrete and symbolic models and it serves as a bridge between concrete and symbolic models (Fennema, 1972). In addition, Sowell (1989) defines the words concrete, pictorial and abstract or symbolic as follows:

Concrete: The materials with which students study directly like bean sticks, Cuisenaire rods, geometric boards under the control of applier.
Pictorial: Animations that students watch, representations with concrete models that students observe or pictures in printed objects that students use.
Abstract or Symbolic: Doing paper and pencil study or reading from books, listening to lessons (Sowell, 1989).

There are definitions of concrete models which are used as teaching aids in classrooms. For example, Hynes (1986) defines manipulative materials as: "Concrete models that incorporate mathematical concepts, appeal to several senses, and can be touched and moved around by students."(p.11) Similarly, Reys (1971) describes that manipulative materials are "objects or things that the pupil is able to feel, touch, handle and move" (p.551). He also states that manipulatives correspond to various senses and students constitute them by attending actively in learning environments. In addition, Kober (1991) defines the manipulatives as objects addressing to many senses that can be touched by students. They rank from trading produces to daily life objects. Heddens (2005) states that manipulative materials are concrete materials including mathematics concepts and corresponding to many senses. Students can touch and handle them (Heddens, 2005).

The use of concrete models in the instruction of mathematics has had long antecedent. By the 1800s, the opinion of use of manipulative materials had been defended, and manipulative materials were involved in the activity elementary school mathematics curriculum by the 1930s (Sowell, 1989). In the half of 1960s, the importance was given to using concrete materials and pictorial presentations in the mathematics learning/teaching (Sowell, 1989). Similarly, Hartzhorn and Boren (1990) state that students' participation actively in teaching/learning period was supported by Pestalozzi and Mentessori in 19th and 20th centuries respectively. Since 1940, NCTM has promoted using manipulatives at all grade degrees (Hartzhorn & Boren, 1990). The activity based instruction involving the use of manipulative materials has become popular (Fennema, 1973).

The idea of using manipulative materials to improve teaching/learning mathematics has acquired prevalence by the learning theorists Piaget and Bruner (Fennema, 1973). Piaget introduced extensive cognitive improvement theory which covers individual growth from birth to adolescence (Fennema, 1972). He defines the cognitive developmental stages: sensorimotor, preoperational, concrete operational and formal operational. According to Piaget, all children should experience these stages. They can not pass the next stage by skipping the previous stage (Senemoğlu, 2005). The cognitive development stages of Piaget are as follow:

Sensimotor Stage: This stage develops between the ages 0 and 2. At this stage, children use their senses and motor skills to explore the world beyond them. First of all, children can not separate themselves from other objects. Then, they start to explore their own bodies and by interacting other objects they constitute new cognitive structures. Their reflexive behaviors turn into purposeful behaviors. The permanence of objects is obtained in this stage (Huitt & Hummel, 2003; Senemoğlu, 2005).

Pre-operational Stage: This stage develops between the ages 2 and 7. Children describe the objects with symbols. Their language use also improves at this stage. Children think in non-logical and nonreversible way. Egocentric thought is observed rarely at the end of this stage (Huitt & Hummel, 2003; Senemoğlu, 2005).

Concrete Operational Stage: This stage develops between the ages 7 and 12. Children can do reversible operations at this stage. They can think logically. Children can also do higher-up classification. They acquire the conversation of number, mass, area, volume, weight and they can order the objects according to their height, weight ect. Children can solve concrete problems even if they are complicated. However, they can not solve abstract problems (Huitt & Hummel, 2003; Senemoğlu, 2005).

Children's cognitive development improves at this stage. They are in need of activities including concrete models to acquire cognitive development (McBride & Lamb, 1986). Piaget supports the use of concrete models before symbolic instruction of mathematical issues (Fennema, 1972).

Formal Operational Stage: This stage develops at the age of 12 and goes on. The abstract thinking develops at this stage. Children can solve abstract problems in a scientific way (Huitt & Hummel, 2003; Senemoğlu, 2005).

Applying Piaget's theory in instruction demonstrates that there should be both concrete and symbolic models in learning surroundings for children at several developmental stages. The twelve years old students are in the concrete operational degree of cognitive growth and they can comprehend the abstract issues if they learnt the subjects through experiencing with concrete models before (Fennema, 1972). According to Piaget, in traditional instruction environments, children are passive and this situation is not suitable for their cognitive growth. Children should not be restricted. The learning environments should give opportunities to children for interacting with peers, teachers and objects. According to him, children should be active in teaching/learning process (Senomoğlu, 2005).

Likewise, Bruner states that children gather information through three different ways. They are enactive (concrete), iconic (pictorial) and symbolic (abstract).In enactive stage, children gather information through interacting with objects and concrete experiences. In iconic stage, pictorial representations gain importance. In symbolic stage, language and symbols are important. Children reach information by using symbols (Erden & Akman, 1991). According to Bruner, children become able to use pictures or symbols to obtain information only after they experience with concrete models (McBride & Lamb, 1986).

Furthermore, Schultz (1986) states that there are three kinds of learning behaviors. First kind of learning behavior involves listening, speaking and uses of concrete models and picture of objects. It is proper when learning concepts. In second kind, symbols are included as well as concrete and pictorial models. It is a transition from 1st kind to 3rd kind. Children try to apply their knowledge that they learnt in 1st kind to abstract issues. In 3rd kind, there are only symbols in the instruction. Only if understanding is provided in 2nd kind, the instruction can be solely abstract (Schultz, 1986).

Likewise Schultz (1986), many researchers advocate the use of concrete models before symbolic instruction. For instance, Boling (1991) emphasizes that when starting a new issue in mathematics, it should be thought about combining a concrete activity and a pictorial presentation with the presentation of symbolic mathematical exposition of the issue. He points out that such an application enables students who can not start at symbolic degree to engage in lecture and understand the issue. Moreover, issues learnt become permanent in students' minds. Clements (1999) also states that manipulatives are important in that they help students built up knowledge with meaning. Manipulatives must be used before symbolic teaching/ learning. Using manipulatives at the end of the instruction should be avoided. Similarly, Kober (1991) states that children's learning of mathematics is connected to their experience with concrete models. They can understand symbols, abstract issues, if only they begin to learn concepts experimenting through concrete tools. Fennema (1972) states that most of the elementary students need using concrete models to make abstract issues meaningful. Students can use symbols impressively if they experienced with concrete models before. If the concrete models are experienced before using symbols, it is more likely for children to learn mathematical issues meaningfully. Children can apply their knowledge to new cases and comprehend abstract issues of mathematics easily (Fennema, 1972).

Furthermore, many studies emphasize the usefulness of concrete models in mathematics instruction. For example, Hartzhorn and Boren (1990) state that experiential learning advocates the opinion that participation of students' actively in teaching/learning process improves their learning. Students' active participation can be provided by using manipulatives. MoNE (2009) also points out that students can learn meaningfully when they experience with concrete models. Mathematics concepts are inherently so abstract that children have difficulty in perceiving them. For this reason, use of concrete models is very useful in mathematics instruction.

Moreover, Hall (1998) states that concrete materials can be beneficial because the teacher can more easily explain operations through concrete materials than through symbols. Students also do not have difficulty in proceduralising issues accurately that they learnt by concrete materials. Moreover, teachers can benefit from materials to get opinion about students' cognitive configurations.

Reys (1971) also suggests widespread uses of manipulatives. Materials are used:

- to diversify educational activities.
- to provide practices in problem solving cases.
- to enable abstract issues presented concretely.
- to enable students to participate actively.
- to enable personal differences.,

- to enhance the motivation toward all mathematics topics (Reys, 1971).

Similarly, Fennema (1973) states that manipulatives increase students' motivation. Children need adequate motivation to learn mathematics. By giving children extrinsic awards, simple skills can be learnt by them. However, if the children do not have intrinsic motivation, they have trouble with learning abstract reciprocations of mathematics. Manipulative materials increase students' intrinsic motivation toward abstract issues of mathematics. Manipulatives also arouse students' interest and make them wonder about the issues to be learnt. Both situations are crucial elements for increasing intrinsic motivation (Fennema, 1973). Similarly, Kober (1991) states that manipulatives provide students to learn actively, motivate them and eliminate annoyance.

Fennema (1973) points out that children are different from each other in terms of capability, rate, learning types and pre-requisite knowledge. Namely, children have various differences. Hence, there should be variety of learning surroundings and materials enable such surroundings. By the use of materials children can learn more willingly than does usage of only symbols.

Some researchers have different beliefs related to effectiveness of concrete models in learning in terms of grade level. For example, Fennema (1972) states that children who are at an early stage of cognitive development can learn meaningfully through experiencing with concrete models. Children who are at an advanced stage of cognitive development can learn better through symbolic models. She also inspected the results of some studies and concluded that concrete models were effective in earlier grades. However, Suydam and Higgins (1977) examined the results of various studies conducted to inspect effectiveness of manipulatives at different grade degrees. They concluded that studies at all grade degrees advocate the use of materials and activity based lessons in mathematics instruction. Similarly, Driscoll (1984) states that students at all grade degrees are in need of using concrete models to conceive rational numbers. Furthermore, Hartzhorn and Boren (1990) point out that research studies conducted in mathematics instruction fields support a new idea about the use of manipulatives in all grade degrees. Kober (1991) states that manipulatives are effective teaching aids not only for elementary students but also for middle school students.

Heddens (2005) also mentions about the benefits of concrete models. He states that use of mathematics materials in the instruction let students:

1. cooperate with others in solving problems.
2. make arguments about mathematical conceptions.
3. word their mathematical ideas.
4. give representations to big groups.
5. symbolize problems in various ways.
6. construct a relationship between real world cases and mathematical symbols. (Heddens, 2005).

Heddens (1986) also states that most of the students can not make a linking between their real and symbolic worlds. He states that the cavity between concrete and abstract stages must be accepted as a whole. It is so important to help students fill this cavity. Teachers play an important role to do this. They should guide students and provide active involvement of students in the process. Using activities including pictures of things, textbook exemplifications, models can help students pass from concrete to abstract stage (Heddens, 1986). Similarly, Fennema (1973) mentions that use of manipulative materials is so significant that it makes abstract nature of mathematics understandable. This is done if children realize the relationship between symbols and real world by using materials. Moreover, research emphasizes that manipulatives are especially beneficial in helping students pass from the concrete to the abstract degree (Hartzhorn & Boren, 1990). Similarly, Kober (1991) emphasizes that students, learning mathematics through manipulatives, can make connection between concrete and abstract world of mathematics and can practice mathematical knowledge in their daily lives.

On the other hand, some researchers point out that using only concrete models is not adequate (e.g. Reys, 1971; Heddens, 1986; Thompson &Lambdin, 1994; Clements, 1999; Heddens, 2005) and does not ensure being successful (Thompson & Lambdin, 1994). Reys (1971) suggests some recommendations for teachers. According to him, students can not make generalizations or abstract conceptions if they used materials uniquely. So, teachers should provide various activities including concrete matipulatives. Teachers also should provide environments in which students interact with each other. While doing activities with manipulatives, they should ask questions to students and guide them. Heddens (1986) emphasizes that teachers should be guidance of students to improve their thought skills and they should question students systematically. So, students can start to improve their own thinking. Clements (1999) also supports the idea that concrete models should be used in the instruction with the lead of teacher. Thompson and Lambdin (1994) also point out that impressive use of concrete models should be taken into consideration in the learning surroundings. Both students and teachers should be aware of what they are teaching/learning by the use of concrete models. Boling (1991) emphasizes that how teachers present the subjects is more important than what they teach. There should be interesting and beneficial applications of topics. Upper mathematics topics should come after easy topics and they should be related to each other. Also, how these topics are taught to students are important. He states that teachers should use concrete models for introducing and strengthening concepts. When the concrete models are used with proper activities, students who have not passed from the concrete and semi concrete degrees to the abstract degree do not find them puerile. Moreover, students become more concerned in concrete activities than activities including books, papers and pencils only.

In addition, Heddens (2005) states that materials should be suitable for both concept which was developed and for growth stage of the students (Heddens, 2005).

In this sense, researchers support the idea that teachers are responsible for choosing appropriate materials. For instance, Hynes (1986) states that manipulative choosing in mathematics is important issue and it is the job of teachers. He suggests two criterions (pedagogical and physical) that must be taken into consideration when choosing manipulatives. Pedagogical criterion includes presenting mathematical thoughts clearly, suitability for students' developmental stage, interest and versatility. Physical criterion involves being durable, simple, attractive, functional and reasonable of cost of manipulatives (Hynes, 1986). Hartzhorn and Boren (1990) also emphasize that the most significant issue is availability in the use of manipulatives. Namely, manipulatives should be easily found by teachers and should be easily made. Moreover, Heddens (2005) states that the suitable materials should be chosen from students' daily lives that they encounter. Fine materials are substantive, simple, interesting (to attract students) and functional. Reys (1971) states that proper manipulatives should be chosen and they should be used properly. If the teachers fail to do this, students can not benefit from impressiveness of manipulatives.

To benefit from effectiveness of manipulatives, Suydam and Higgins (1977) suggest some recommendations as following:

- Manipulative materials should be often in the whole elementary school mathematics curriculum and related to the objectives of this curriculum.
- Manipulative materials should be used accompanied by the help of pictures, movies, charts and such like materials.
- The use of manipulative materials should be suitable for the mathematics content and the content should benefit from the uses of manipulatives.

- Manipulative materials should be used together with exploratory and inductive approchements.
- There should be a direct relationship between the simplest materials and Mathematical feature.
- Materials should help while organizing content (Suydam & Higgins, 1977).

In the present study, treatment was based on concrete models. In the literature, many researchers support the use of concrete models in mathematics instruction. Many advantages and how they should be used are also stated. It is pointed out that concrete models help students move from concrete to abstract level. Students can easily compose relationship between real world and abstract world of mathematics by the use of concrete models. Also, beginning to learn a new topic through using concrete models helps students learn the abstract issues easily. As emphasized in the literature, present study takes into consideration of these advantages of concrete models applies them in the probability instruction.

2.1.2. Research Studies on Concrete Models and Mathematics

While there are many studies utilizing the use of concrete models in the instruction of different branches of mathematics, it was met little research studies related to use of concrete models in probability instruction. These studies were conducted by Cankoy (1989) and Taylor (2001) and they are explained below:

In the study carried out by Cankoy (1989), the difference between traditional and mathematics laboratory based mathematics instruction in terms of achievement related to probability topic was investigated. In the mathematics laboratory based instruction, the concrete models were used.

The participants were 73 eighth grade students in Ankara. 36 of students were in control group and 37 students were in experimental group. Control group received instruction traditionally, whereas experimental group received mathematics laboratory based probability instruction. In the mathematics laboratory based instruction concrete models were used. Results demonstrated that there was a significant difference between the scores of two groups in favor of experimental group.

In another study conducted by Taylor (2001), effects of concrete manipulatives and computer simulations on learning skills and on students' experimental probability achievement were investigated. The participants were 83 fifth grade students. There were four groups. First group had instruction through concrete manipulatives. Second group had instruction through computer simulations. Third group had instruction through both concrete manipulatives and computer simulations. Fourth group was control group and received instruction traditionally. All groups answered pre-tests and post-tests. According to results of the study, there was no statistically significant difference between students who received instruction through concrete manipulatives and students who received instruction through computer simulations with respect to learning skills and concepts of experimental probability. There was also no significant difference between students who used concrete manipulatives and those who did not use concrete manipulatives. There was only significant difference between students who used computer manipulatives and those who did not use computer manipulatives in favor of computer manipulatives.

It was not met any other studies conducted to examine the effect of concrete models on probability achievement. Because of this reason, research studies performed use of concrete models in the instruction of other branches of mathematics are also inspected. These studies are explained below:

In the study of carried out by Bayram (2004), the impact of teaching/learning with concrete materials on students' geometry achievement and attitudes toward geometry was investigated. 106 eight grade students (51 girls, 55 boys) from one of private school of Ankara were included in her study. 72 students were in the experimental group and 34 students were in control group. In the same time, all students had instruction with same textbook and they learnt same mathematical subject. Experimental group had instruction by using concrete materials and control group had traditional instruction. According to results of her study, there was a statistically mean difference between students who had instruction with concrete materials and those who had instruction traditionally. Students in experimental group outperformed the students in control group.

Similar with the study of Bayram (2004), Sarı (2010) also carried out a study to investigate the effects of instruction with concrete models on 4th grade students' achievement of geometry. The participants were 32 fourth grade elementary school students. The design of the study was one group pretest-posttest. The treatment took for five hours per week throughout 10 weeks. She applied achievement test and interview to collect the data. The results demonstrated that there was a statistically significant change in students' geometry achievement after treatment. There was no statistically significant difference between students' post-intervention and retention scores. Moreover, according to interview results, most of the students had fun when concrete models were used.

In other study carried out by Tutak (2008), effects of instructions with concrete models and dynamic geometry software on fourth grade students' geometry achievement and their attitudes toward geometry were investigated. Sample consisted of three classes fourth grade students. First class had instruction though concrete materials. The second one had instruction through dynamic geometry software (Cabri). The third class had instruction traditionally. The design of the study was quasi-experimental. According to results of the study, students who had instruction with concrete models outperformed students who had instruction with dynamic geometry software. It was determined that attitudes toward geometry improved equally in both first and second classes after treatments.

In a study conducted by Leinenbach and Raymand (1996), the impacts of mathematics manipulatives on students' abilities to solve algebra questions were examined. The study had two phases. The first phase of the study took during 1994-1995 education year. In the first phase, during the first nine weeks the researchers did not teach with manipulatives. They used only textbook. After nine weeks, they implemented manipulative program during 26 lessons. The second phase took in 1995-1996 education year. The purpose of this phase was to investigate the retention effect of the study conducted in first phase. The subjects of the study were about 120 eighth-grade students. Data was collected by year survey, weekly student reflections, reflections and observations of teachers, samples of students' studies, scores of the tests and interview. According to the results of the study most of the students fulfilled better with use of manipulatives compared to text. Similarly, Hinzman (1997) examined impact of use of manipulatives and activities in algebra instruction. Results demonstrated that performances of students were increased by the use of manipulative materials.

In another similar study conducted by Daniel (2007) the effectiveness of uses of manipulatives on algebra achievement of fourth grades was examined. The subjects were 85 fourth grade students (53 regular education students, 32 gifted students) in this study. There were both experimental and control groups. The control group had instruction through activities including numbers and the textbook. The experimental group had instruction through manipulatives-based activities. Students were tested three weeks later. The results demonstrated that manipulatives improved algebra achievement of all fourth grade students (regular and gifted students).

In the literature there are research studies examined the effect of concrete materials on mathematics achievement by comparing the results of many studies. Fennema (1972) compared the results of various studies and found concrete materials useful when they were used in earlier grades. Contrary to findings of Fennema (1972), Suydam and Higgins (1977) also examined the results of different studies and found concrete materials beneficial at all grade levels.

In a similar study, Parham (1983) analyzed 64 studies which were conducted between the years 1960 and 1982. In these studies the including-not including of matipulatives were compared. Results demonstrated that students scored 85th percentile in manipulative used studies, whereas the students scored 50th percentile in manipulative non-used studies. Moreover, Sowell (1989), integrated the outcomes of 60 researches to examine the impact of manipulative materials on teaching/learning various mathematics subjects. The manipulative materials involved concrete and pictorial projections. Participants were in age kindergarteners to college students. They studied different kinds of mathematics subjects. Results demonstrated that mathematics achievement was improved by using concrete materials long period.

In a different study conducted by Moyer (2001), it was carried out a long year study to determine reasons of usage of manipulatives by teachers and how they used them in their classrooms. The participants were 10 of 18 middle grades mathematics teachers. Before the project, teachers had Middle Grades Mathematics Kit developed in cooperation with a trading supplier and the department of education of state in mathematics summer course for two weeks. The kit consisted of various concrete manipulatives such as base ten blocks, color tiles, cubes, dice, pattern blocks ect. In their classrooms, there were also hand-made materials. Teachers were observed and interviewed during the study. Results of observations and interviews showed that teachers perceived manipulative uses little more than divertissement in classes in case of being not able to represent the mathematics topics themselves. Teachers also stated that using manipulatives was funny, however instruction of mathematics did not necessitate using manipulatives.

In this section, some research studies carried out to examine the effectiveness of concrete models on students' mathematics achievement were explained. Most of these studies found concrete models effective on students' mathematics achievement. In the present study, the concrete models were used in the probability instruction and it also aims to investigate the effect of instruction with concrete models on students' probability achievement.

2.2. Discovery Learning Method

In this section the theoretical background and research studies on discovery learning method were explained. The aim of the present study is not to investigate the effect of instruction with discovery learning method on students' probability achievement and attitudes toward probability. Since most of the activities were based on discovery learning method, the information related to discovery learning method is given in this section.

2.2.1. Theoretical Background for Discovery Learning Method

Discovery learning method is named as Socratic method involving the conversation between teacher and a student until student achieves a favorable result by answering carefully prepared questions. It is as aged as formal education (Cooney, Davis & Henderson, 1975). Similarly, Willoughby (1963) states that the discovery learning method is as old as Socrates. However, it has gained much importance since 1960s. The discovery learning method was developed in 1960s. by Jerome Bruner (Erden &Akman, 1997).

Bruner is a supporter of student centered teaching/learning. He gives importance to students' being independent in learning surroundings. According to him, students can be independent, if the teachers allow students to discover and if they satisfy their curiosity. Teachers should not give answers to students. They should encourage students to solve problems and find answers by themselves. Students benefit from the things that they make rather than what teacher says (Senemoğlu, 2005). According to Bruner, in discovery the outcome is not important. Discovery should be viewed as a process. Discovery learning is learning how to explore. In discovery learning method, student confronts a problem and looks for the ways of solving it (Cooney, Davis & Henderson, 1975).

In discovery learning method, there are two approaches. They are guided discovery learning method and pure discovery learning method. In guided discovery method, the teachers organize the lesson. They guide students by helping them deducing generalizations through asking questions. In pure discovery learning, students find the solution of a problem by themselves in an unplanned way (Senemoğlu, 2005). Guided discovery is a student centered but teacher-leaded approach whereas pure discovery is both student centered and student leaded approach (Trowbridge, Bybee & Powell, 2004).

In addition, Senemoğlu (2005) states that guided discovery method is more effective than pure discovery with respect to retention and transformation of the knowledge. Similarly, according to Wittrock (1963) guided discovery is more efficient than pure discovery in terms of learning and transformation of the knowledge. Senemoğlu (2005) states that it is difficult to lead the activities in pure discovery lessons and students can not find any conclusion. Also, it requires more time than pure discovery method. Because of these reasons, the guided discovery learning method is more preferable than pure discovery method.

There are two methods of guided discovery. One of them is inductive discovery. The teacher questions students and leads them to make the generalization abstract. Inductive discovery lesson has two processes. They are making abstractions and generalizations. Students make the abstraction when they realize the commonality between the differences. Generalization happens when the students realizes that a relation in a sample is also correct for another related samples. In deductive discovery, Cooney, Davis and Henderson (1975) stated that teacher begins with a knowledge that students know and leads the students to conclude the generalization by asking efficient questions. Students make logical deductions from the knowledge that they have already known. In both discoveries, teacher makes guidance. In inductive discovery, teacher gives students examples in a carefully selected manner to make students to make the abstraction easily. In deductive discovery, teacher guides the students to deduce the generalization by asking questions in a manner (Cooney, Davis & Henderson, 1975).

In the literature there are advantages of discovery learning method. According to Bruner (1961), through discovery learning method, the transformation and the retainment of knowledge becomes better. It enhances students' intrinsic motivation which causes learning. In discovery learning lessons, the classroom ambiance is enthusing. It promotes students to participate and inquiry in the lessons. Students' abilities to learn new knowledge improve. Trowbridge, Bybee and Powell (2004) state that the biggest advantage of discovery learning method is that students actively participate in the process and it is student centered. Because of this reason, it motivates students. It incites students' development of thinking abilities. In discovery lessons, students have chances to practice the data analyze process and find the abstract issues from these data. According to Senemoğlu (2005), one of the advantages of discovery learning method is encouraging students to wonder and sustaining it until they deduce generalizations. Another advantage is directing students to solve problems independently. Students do not digest the information. Instead, they analyze, apply and synthesize the information. Furthermore, since students wonder about the indefiniteness created about the subject to be learnt, one advantage is that discovery learning method develops students' positive attitudes toward learning (Senemoğlu, 2005). Erden and Akman (1997) state that being successful, solving a problem independently, and discovering a new knowledge serve as reinforcements in the learning process. Cooney, Davis and Henderson (1975) state that students realize what they create through their own intellectual experiences and it is one of the experiences that is worth trying.

Additionally, it is vital for children to receive mathematics instruction in learning surroundings in which they are independent, they can discover the knowledge, discuss with each other and teacher, generalize what they learnt and implement their deductions in problem solving. Moreover, discovering the mathematical relationships and generalizing them provide students better perception of world around them (MoNE, 2009a, MoNE, 2009b).

However, there are also disadvantages of discovery learning method in the literature. According to Skinner (1968) and Trowbridge, Bybee and Powell (2004), it requires much time. Skinner (1968) also points out that teachers should be very experienced on this method. In discovery lessons, there should be small number of students to use this method effectively.

Although discovery teaching requires skillful questioning, practice and persistence, Trowbridge, Bybee and Powell (2004) state that teachers do not use discovery teaching because they do not feel comfortable with this method. However, they state that the results are worth applying this method in the lessons. Binter and Dewar (1968) state that while applying this method in the lessons teachers should understand the subject that they taught and how students think in different cases. They should also provide students materials to help them in discovery process and enable them to work effectively. Senemoğlu (2005) also states while appliying this method, the failure risk of students should be decreased and instruction should be appropriate for students as far as possible (Senemoğlu, 2005).

Most of the lessons were planned according to discovery learning method in the present study. As mentioned in the literature, there are many advantages of discovery learning method. In summary, it increases students' intrinsic motivation that causes learning. It is a student centered approach and provides students to participate in the lessons activiely. Since guided discovery learning is more effective than pure discovery learning, in the present study guided discovery learning is used. Moreover, present study aims to utilize the advantages of this method.

2.2.2. Research Studies on Discovery Learning Method in Mathematics Education

In the literature, there are research studies examined the effect of discovery learning method in mathematics instruction (e.g. Wittrock, 1963; Guthrie, 1967; Anthony; 1973). These studies emphasize the effectiveness of discovery learning method. In this section, recent studies related to discovery learning method are explained.

It was met only two research studies utilizing the discovery learning method in probability instruction (e.g. Bulut, 1994; Yazıcı, 2002). These studies are explained below:

In the study of conducted by Yazıcı (2002), effects of discovery learning method on students' probability achievement and attitudes toward probability were investigated. The participants were 8th grade students from two schools in Trabzon. There were two groups: control and experimental. The Permutation and Probability Achievement Test and Probability Attitude Scale were administered to obtain the data. According to the results of the study, there was a statistically significant mean difference between the scores of two groups in favor of experimental group. The discovery learning method also increased students' motivation and provided them to participate in lessons actively.

In another study of conducted by Bulut (1994), the impacts of cooperative learning method, computer based instruction and traditional method on eighth grade students' probability achievement and attitudes toward probability were investigated. In the treatment, the researcher uzilized from discovery leaning method in computer assisted tutorials. There were 29 students who had computer based instruction and 36 students who received instruction through cooperative learning method and 36 students who received traditional instruction. The measuring instruments were pre-requisite knowledge test, probability achievement test, questionnaire, probability and mathematics attitude scales. According to results of the study, there was a significant mean difference between groups who received instruction through cooperative learning method and traditional instruction with respect to achievement on PAT in favor of cooperative learning group. However, there was no statistical significant difference among the other pairs of groups with respect to achievement on PAT. There were also no significant mean differences on scores of probability attitude scale among all pairs of groups.

It was not met any other research studies including discovery learning method in probability instruction. So, research studies performed discovery learning method in the instruction of other branches of mathematics are also inspected.

In the study of Fidan (2009), it was examined the fifth grade students' geometric thinking degrees with respect to different variables and impacts of geometry instruction through discovery learning method on students' geometric thinking degrees. The experimental group consisted of 107 fifth grade students. The experimental group received instruction through discovery learning method, while control group received instruction through traditionally. According to results of the study, there was a statistically significant mean difference between experimental and control groups with respect to geometric thinking degrees in favor of experimental group.

Contrary to results of Fidan (2009), Ünlü (2007) found no significant results. The purpose of study was to determine effects of Web-based learning environment developed based on problem solving and discovery learning method on fractions achievement on fractions. The participants were 73 fifth grade students in an elementary school in Ankara. 38 students were in experimental group and 35 students were in control group. The experimental group received instruction through Web-based learning, the control group received instruction traditionally. Results revealed that Web-based learning environment developed based on problem solving and discovery learning method did not have a statistically significant mean difference on students' achievement.

In an other study conducted by Temizöz (2005), the beliefs of mathematics teachers related to implementation of discovery learning method were examined. The twenty five mathematics teachers of fourteen elementary schools in Ankara were the participants of the study. Data were obtained through observations, lesson plans that teachers applied in their classes and interviews. Results of the study demonstrated that most of the teachers taught mathematics traditionally. Furthermore, most of the teachers stated that discovery approach enabled visual and tactual learning surroundings; it was too difficult to implement in our country because of limited time and heavy mathematics curriculum.

In this section, some research studies which examined the effectiveness of discovery learning method in mathematics instruction were explained. Some of these studies found discovery learning method effective, some studies found no statistical changes in students' achievement. The present study also utilizes the discovery learning method in most of the activities.

2.3. Probabilistic Thinking

In this section, the theoretical background for probabilistic thinking and related research studies were explained.

2.3.1 Theoretical Background for Probability

In the literature there are various studies related to developmental stages of probabilistic thinking (e.g. Engel, 1966; Fischbein, Pampu & Manzat, 1970; Piaget and Inhelder, 1975; Fischbein, 1975; Carpenter et al., 1981; Fischbein & Gazit, 1984).

Piaget and Inhelder (1975) state that children's development of probabilistic thinking happens in three stages:

1. Stage: Sensory Motor (up to 7 years old): Children can not understand the probability concepts and tend to make unstable predictions in this stage.

2. Stage: Concrete-operational (approximately between ages 7 and 10): The concept of chance firstly improves in this stage.

3. Stage : Formal-operational (begins at approximately age 11): Children can totally understand the probability in adolescence, in third stage. They can also organize the probability concepts in this stage.

In the literature there are research studies advocating the results of Piaget and Inhelder. Fischbein and Gazit (1984) in the study with 5th, 6th and 7th grade students found that almost all of the concepts were quite difficult for the fifth grade students. However, approximately 60-70% of the sixth grade students and approximately 80-90% of the seventh grade students could understand and properly use many concepts included in the study. Carpenter and his colleagues (1981) in the study with the students at the age of 13 and 17 found that the number of correct answers of 17 year olds were higher than the number of correct answers of 13 year olds. The percentage of correct answers increased with age. Engel (1966) points out that all beginning secondary students do not have pre-cognitive concepts related to fundamental concepts of probability. They should be prepared for probability lessons.

Some research studies on probability contrary to findings of Piaget and Inhelder advocate that little children can understand the probability concepts even if it is slight (e.g. Fischbein, Pampu & Manzat, 1970; Fischbein 1975). Fischbein, Pampu and Manzat (1970) found that pre-school children are able to understand and get through the cases including chance. Moreover, students who are 9 to 10 year olds can accurately guess the chances by making comparisons between ratios. Fischbein (1975) states that there occurs difference between concept of chance and primary pre-cognition of chance. He states that primary pre-cognition of chance comes up at early ages before children enter the stage of concrete operational.

Beside the studies inspecting the development of probabilistic thinking in children, there are also studies emphasizing the importance of probability in real life. For instance, Lappan and his colleagues (1987) emphasize that probability is an important issue in real life. It is used in chance games. Furthermore, probability concepts provide people to make decisions in various areas as scientific studies, weather forecasts, martial operations, checking the design and standard of goods, making estimations about political issues, ect.

Moreover, the significance of probability in education is also stated. For example, In Turkey, the current elementary school mathematics curriculum has been implemented to grades 1 through 5 since 2005-2006 (MoNE, 2009a), to 6th grade since 2006-2007, to 7th grade since 2007-2008 and to 8th grade since 2008-2009 (MoNE, 2009b). This curriculum brings considerable differences in the instruction of probability. In previous elementary school mathematics curriculum, the probability subject took place only in 8th grade mathematics curriculum. However, in current elementary school mathematics curricula the probability topic took place from grades 4 to 8 (e.g. MoNE, 2009a, MoNE, 2009b). While in mathematics curriculum from grades 4 through 5 probability takes place in the "Data Learning Strand", grades 6 through 8 it is in the "Probability and Statistics Learning Strand". " They are related to children's' lives and provides children to become conscious citizens. It aims to provide students to apply required information related to probability in their lives and other school subjects. It also aims to provide students to realize the importance of this area for individuals, community, various science branches and different jobs (MoNE, 2009b).

Similarly, NTCM (1989) emphasizes that probability plays an important role in elementary school mathematics curriculum and suggests that mathematics curriculum should involve exploring the probability in real life cases. In grades 5-8, students should be able to determine probabilities by means of modeling cases through devising and conducting experiments and through building sample space. Students should also be able to make estimations related to theoretical and experimental probabilities. They should realize the widespread usage of probability in the real life. Furthermore, NTCM (2000) points out that if probability does not take place in the curriculum students can not improve probabilistic argument. Studying probability provides students to interrelate mathematics with school lessons and with real life. To be a qualified citizen, user, worker, students should learn probability. National Council of Supervisor of Mathematics (NCSM) (1989) also suggests that students should comprehend the concepts of elementary probability to detect the possibility of oncoming events. They should also understand that possibility of oncoming events is not affected by possibility of past events. They should be aware that they can estimate the results of polls, sports competitions, forecasts through use of probability. Fennel (1990) states that probability provides a enjoyable basis for understanding the fundamental notions in mathematics, especially for rational numbers. Probability activities enable students to improve their knowledge in rational numbers. NTCM (2000) suggests that mathematics curriculums from pre-kinder garden through 12th grade should provide students to understand and administrate fundamental notions of probability.

Although researchers emphasize that probability is an important curriculum issue, students have difficulties in learning probability. For instance, Garfield and Ahlgren (1988) point out that students' degree of definite mathematics competence and students' mental development have effects on the learning of probability. They also state that most of the students, at every grade, can not understand the fundamental concepts of probability. Because they have insufficient pre-requisite knowledge and can not understand the abstract issues. Carpenter and his colleagues (1981) state that students have some common intuitions of probability but they do not know the ways of reporting probability. Because students have difficulties with prerequisite knowledge which are fractions, percentages and decimals. Bar-On and Or-Bach (1988) also state that students have difficulties in comprehending the ratio sets topic. Students have difficulties in understanding the notion of independent events (Carpenter et al, 1981; Hope & Kelly, 1983). Hope and Kelly (1983) also state that students can not interrelate probability ideas to the real world. Garfield and Ahlgren (1988) point out that students dislike probability because the instruction of it is very abstract and formal. Ford and Kuhs (1991) emphasize that the development of language in children is crucial for comprehending the probability and can be acquired by discussing the accustomed situations.

Considering the difficulties in learning/ teaching probability, some researchers advice the use of materials in probability instruction. For instance, Lappan and Winter (1980) state that probability instruction with concrete experimentations fulfills more promise. Similarly, Hope and Kelly (1983) suggest that mathematics teachers should develop instruction to help students realize that probability is related to real world. Lappan and his colleagues (1987) also state that instruction of probability through concrete experiments provides desirable activities for children who did not achieved success in probability before, as well as all children.

Researchers also state what kinds of concrete materials can be used in probability instruction. For example, Shaw (1984) recommends the spinning the spinners and rolling the dices in probability activities. He states these kinds of activities enhance students' interest toward probability. They are multifunctional instructional supports that help users practice probability and study on fractions and proportions. Bruni and Silverman (1986) suggest that use of manipulative materials in probability activities provides great advantages. Activities including materials provide considerable motivation for students. Manipulative materials are useful for improving some basic concepts in probability and statistics when using ideas as to fractions, percents, ratios. They also recommend the use of cubes made of different materials and beans in probability activities. Bright (1989) recommends the use of simulations of dice rolls. He points out that simulations can not displace the use of concrete materials. Students should manipulate the real materials. Simulations are extensions of concrete models. Fennell (1990) points out that probability activities should involve physical objects and should provide students an environment for questioning, problem solving and discussing. Since probability is real life mathematics he recommends the activities including daily life implementations such as card games, newspapers involving weather forecasts. Horak and Horak (1983) recommend the use of bags, marbles, cards, dices, spinners, coins in the probability activities. They point out that these kinds of activities provide students an understanding that fundamental mathematics is related to real life situations. Also, using these materials in activities enhance students' interest and motivation.

Furthermore, Garfield and Ahlgren (1988) suggest some recommendations for teachers to overcome difficulties in probability. These recommendations are as following:

- Not using abstract issues in the presentation of topics. Presenting topics by activities.
- Trying to make students to realize that probability is related to reality, and is not composed of only symbols.
- Using visual examples.
- Developing students' rational number understanding by using appropriate strategies before probability instruction.
- Being aware of students' general faults in probabilistic thoughts.
- Building up cases providing probabilistic argument that fit with students' world opinion.

Related literature demonstrates that students have difficulties in learning probability. One of the reasons is its' abstract instruction. Moreover, students can not interrelate it with real life. Many researchers give recommendations how to overcome these difficulties, how to teach/learn probability effectively and they also point out that impressive instruction of probability develops understanding of probability. Based on these recommendations, the present study takes into consideration using concrete models in the instruction of probability. Also, it is given importance to using concrete models before symbolic instruction of probability. Moreover, the present study gives importance interrelating probability with real life.

2.3.2. Research Studies on Different Instructional Methods in Probability Instruction and Including Probability with respect to Different Variables

In the literature, there are some research studies conducted to examine the effectiveness of different instructional methods on students' probability achievement. The related studies which were met are explained below:

In one of these studies, Ünlü (2008) conducted a study to investigate the effects of cooperative learning method on 8th grade students' probability and permutation achievement. Also, the retention effect was investigated. There were 30 students in experimental group and 34 students in control group. The experimental and control group students were from different elementary schools. The students in experimental group received instruction through cooperative learning method, while students in control group received traditional instruction. Pre-tests, post-test and retention tests were applied to both experimental and control group. According to results of the study, there was a statistically mean difference between experimental and control groups in favor of experimental group. There was a statistically mean difference between experimental and control groups with respect to retention effect in favor of experimental group.

In an other study carried out by Esen (2009), effect of computer based instruction on 6th grade students' probability achievement was investigated. The participants were 316 6th grade students from two elementary schools. There were two groups as control and experimental groups. The experimental group received computer based instruction and control group received traditional instruction. Probability achievement test was applied to both experimental and control group as pre-test and post-test. According to results of the study, there was a significantly mean difference between control and experimental groups. The computer assisted instruction was significantly more effective than traditional instruction.

Similarly, Demir (2005) also found significant results. He conducted a study to examine the impacts of problem posing instruction on students' successes in probability and attitudes toward probability. The participants were 82 tenth grade students from two schools in Ankara. Twenty seven students served as experimental group and had instruction through problem posing. Fifty five students served as control group and received traditional instruction. Probability attitude scale, mathematics attitude scale and probability achievement test were applied to students to collect the data. According to results of the study, students who received probability instruction through problem posing outperformed those who received traditional probability instruction.

In another study conducted by Seyhanlı (2007), the effect of graph theory based instruction on 8th grade students' probability achievement and their attitudes toward probability was aimed to determine. The participants were 62 eighth grade students. The experimental group received graph theory based probability instruction. The control group received traditional instruction. Achievement test and attitude scale were applied as both pre and post-test to experimental and control group. According to results of the study the graph theory based instruction was more effective than traditional instruction. Also, students' attitudes toward probability improved in the experimental group.

In another study, Memnun (2008) conducted a study to investigate the effects of active learning method on 8th grade students' probability and permutation achievements in the implementation stage. The participants were 197 8th grade students in two elementary schools. There were both experimental and control groups. The experimental group received instruction through active learning method as well as innovative learning with games. The control group received traditional instruction. According to findings of the study, students who received probability and permutation instruction through active learning method outperformed those who received instruction traditionally.

In one of these studies, Ercan (2008) conducted a study to determine the effect of multiple intelligence theory based instruction on 8th grade students' probability and permutation achievements. The participants were 68 8th grade elementary students in an elementary school in Mersin. 34 students were in experimental group and 34 students were in control group. The experimental group received multiple intelligence theory based instruction, while control group received traditional instruction. The achievement test prepared by the researcher was applied as pre-test and post-test to both experimental and control groups. According to results of the study, the students in experimental group were more successful than those in control group. Also, most of the students in the experimental group stated that they had fun during the lessons.

In the study of Castro (1998), the effect of conceptual change and traditional teaching methods on students' performance when calculating probability, performance in probability reasoning were aimed to investigate. The participants were 136 14-18 years old secondary school students. There were 75 students in experimental group and 61 students in control group. The control group received traditional instruction and experimental group received instruction based on conceptual change. There were statistically mean differences between experimental and control groups with respect to teaching method in favor of experimental group. The conceptual change produced in experimental group was higher than that produced in control group. Students' skills in probability calculation improved in experimental group compared to control group. Students probability reasoning in experimental group improved compared to control group.

Şengül and Ekinözü (2007) conducted a study to investigate the effects of dramatization method on 8th grade students' probability achievement. Also, the retention effect of the dramatization method was determined. The participants were 70 8th grade students in an elementary school in Istanbul. 36 students were in experimental group and 34 students were in control group. Two different probability achievement tests were applied to students. One of them was applied as pre-test to experimental and control groups, another test were applied as post-test and retention test to both of groups. According to results of the study there was no statistically mean difference between experimental and control groups in terms of probability achievement. However, there was a statistically mean difference between experimental and control groups with respect to retention effect in favor of experimental method.

In the study of conducted by Bulut (1994), the effects of cooperative learning method, computer based instruction and traditional method on eighth grade students' probability achievement and attitudes toward probability were investigated. According to results of the study, there was a significant mean difference between groups who received instruction through cooperative learning method and traditional instruction with respect to achievement on PAT in favor of cooperative learning group. However, there was no statistical significant difference among the other pairs of groups with respect to achievement on PAT.

There are also research studies including probability with respect to different variables. These studies are explained below:

In the study of Mut (2003), students' probabilistic misconceptions in terms of grade level, previous instruction on probability and gender were examined. The participants were 885 5th, 6th, 7th, 8th, 9th and 10th grade students from different types of schools. The Probabilistic Misconception Test and a questionnaire were applied to students. According to results of the study, the frequencies of all misconception types changed according to grade levels. The percentages of students who had previous probability instruction were higher than those who did not have instruction with respect to misconceptions on Effect of Sample Size and Time Axis Fallacy. Moreover, the percentages of students who had probability instruction previously were lower than those who did not have instruction in terms of other misconception types. The frequencies of all misconception types changed according to grade level and gender.

In another related study, Özaytabak (2004) carried out a study to determine the factors which affect the opinions of preservice mathematics teachers related to probability teaching. Participants were 248 preservice mathematics teachers from three universities in Ankara. Results of the study indicated that the factors affecting preservice mathematics teachers' opinions about probability teaching were their attitudes toward probability, achievement in probability and misconceptions. Preservice teachers thought that gender would not be a factor affecting their opinions about probability teaching.

Tunç (2006) carried out a study to inspect the 8th grade students' achievement, attitudes toward probability and attitudes toward mathematics who were studying in public and private schools. Participants were 207 8th grade students from 2 private and 3 public schools. According to results of the study there was a significant mean difference between public school students and private school students with respect to probability achievement in favor of private school students.

In this section some research studies which examined the effects of different instructional methods on students' probability achievement and research studies which include probability with respect to different variables were explained. Most of the research studies found different instructional methods (cooperative learning method, computer based instruction, problem posing based instruction, graph theory based instruction, active learning method, multiple theory based instruction) effective on students' probability achievement. The present study also gives importance to instruction of probability. It aims to investigate the effect of instruction with concrete models on students' probability achievement.

2.4. Attitude toward Mathematics

In this section, the theoretical background for attitudes toward mathematics and related research studies were explained.

2.4.1. Theoretical Background for Attitude toward Mathematics

Hannula (2002) states that everyday thought of attitude is described as liking or disliking of someone of a known target.

According to Haladyna, Shaughnessy and Shaughnessy (1983), attitude toward mathematics is a common emotion that students dispose toward the mathematics in school. In general, a positive attitude toward mathematics is taken into consideration for the following factors: 1. A positive attitude is a crucial school product. 2. Attitude is usually positively, but a little, correlated with achievement. 3. A positive attitude toward mathematics may improve students' inclinations to choose mathematics courses or fields that are related to mathematics in the future. Aiken (1972) also states that it can be determined from students' attitudes toward mathematics whether they will take part in mathematics activities or choose mathematics courses in the future.

According to Neale (1969) research indicates that as students grow, the attitudes toward mathematics tend to decline. Aiken (1972) states that there is a low relation between achievement and mathematics attitude in elementary grades. Because, attitudes of elementary students tend to be less steady than in higher grades. Middleton and Spanias (1999) point out that students' mathematics motivations are formed at earlier grades and very constant for years.

Aiken (1972) states that many researchers found that there was a higher relation between attitude and achievement in mathematics than in other subjects including verbal issues. Similarly, Middleton and Spanias (1999) point out that students are more interested in their roles in mathematics than in other subjects. They have strong emotions about what they are able to do and they embrace these emotions in their self-concepts (Brassell, Petry & Brooks, 1980).

Mathematics self-concept and anxiety (Brassel, Petry & Brooks, 1980) motivation (Middleton & Spanias, 1999) which are the determinants of attitudes toward mathematics are correlated with mathematics achievement. Aiken (1976) also points out that there is dynamical relationship between attitude toward mathematics and mathematics competence. Being unsuccessful in mathematics could easily cause students to develop negative attitudes toward mathematics. Furthermore, the highest anxiety, the lowest self-concept and the least enjoyment for mathematics belong to students who are unsuccessful in mathematics. The special importance must be given to these students (Brassell, Petry & Brooks, 1980).

In addition, Neale (1969) states that positive attitudes toward mathematics cause students to learn mathematics. Also, not only positive attitudes toward mathematics cause learning but also learning leads to favorable attitudes. Students who are successful are awarded and students who are unsuccessful are not awarded, they are even penalized. So, successful students have a tendency to enjoy mathematics whereas unsuccessful students have a tendency to distaste or even hate mathematics (Neale,1969).

In this sense, some researchers suggest some recommendations for teachers to improve students' positive attitudes toward mathematics. For instance, Neale (1969) recommends the use of mathematics activities including discovery. Through such activities, students explore eagerly. Aiken (1972) suggests that considering interests, attitudes or anxiety level when preparing mathematics examinations and lessons can develop performances of students. Teachers can improve students' attitudes by connecting mathematics with things that students perceive as eligible, interesting and valuable. Also, teachers should demonstrate the benefits of mathematics in students' future careers and in daily lives (Aiken, 1972). Brassell, Petry and Brooks (1980) point out that teachers should enhance students' self-concepts and lessen their anxiety in mathematics lessons. Students who have low grades have great anxiety toward mathematics. Instructional methods should lessen these unsuccessful students' anxiety. Students perceive activities as funny and more activities may lessen anxiety.

Aiken (1972) states that attitudes and impressiveness of teachers determine the students' attitudes and achievement in mathematics. Moreover, providing teachers to develop positive attitudes toward mathematics may develop positive attitudes toward mathematics in students. The attitude toward mathematics may be formed by the students' attitude toward the teacher. Teachers should notice that at the important duration of attitude embodiment (the early high school stage) students become aware of teachers' attitudes toward mathematics and mathematics students. This may determine students' mathematics attitudes (Brassell, Petry & Brooks, 1980). Similarly, Ruffell, Mason and Allen (1998) and Middleton and Spanias (1999) points out that teachers' attitudes toward mathematics may affect students' attitudes toward mathematics.

One of the aims of the present study is to investigate the effect of instruction with concrete models on students' attitudes toward probability. In the literature, it is stated that there are many factors affecting students' attitudes toward mathematics. It is also emphasized that attitudes play important role in understanding mathematics concepts. More positive attitudes provide more learning. So, the present study gives importance to students' attitudes and aims to inspect effect of instruction with concrete models on students' attitudes toward probability.

2.4.2. Research Studies on Attitude toward Mathematics

It was not met any research studies inspecting the impacts of concrete models on students' attitudes toward probability, while there are studies examining the effectiveness of concrete models on students' attitudes toward other branches of mathematics. These studies are explained below:

Tutak (2008) suggested that there was improvement in fourth grade students' attitudes toward geometry. It was determined that attitudes toward geometry improved equally in both two classes who had instruction though concrete materials and through dynamic geometry software (Cabri) after treatments.

Contrary to results of Tutak (2008), Bayram (2004) found no statistically difference between 4th grade students who had geometry instruction through concrete models and students who had instruction traditionally with respect to attitudes toward geometry.

Sowell (1989) stated that according to results of her study, attitudes toward mathematics increased when students got instruction by teachers who were informed of using concrete materials.

There are also research studies examined the effects of different instructional methods on students' attitudes toward probability and mathematics. The related studies which were met are explained below:

In one of these studies, Seyhanlı (2007) carried out a study to determine the effects of graph theory based instruction on 8th grade students' probability achievement and their attitudes toward probability. The experimental group received graph theory based probability instruction. The control group received traditional instruction. According to results of the study students' positive attitudes toward probability improved in the experimental group.

Demir (2005) also found similar results. He carried out a study to determine the impacts of instruction through problem posing on probability achievements of tenth grade students and their attitudes toward probability. According to results of the study, students' attitudes toward both probability and mathematics improved significantly.

In a study carried out by Ekinözü and Şengül (2006), it was also found that students' attitudes toward probability improved in both experimental who received instruction through dramatization method and control groups. Furthermore, students' perceptions of profits of mathematics improved in the experimental group.

Contrary to results of studies explained above, in some studies students' attitudes toward probability did not change. For example, Bulut (1994) found no significant mean differences on scores of probability attitude scale among all pairs of groups who received instruction through computers, cooperative learning method and traditionally.

Similarly, in the study of Yazıcı (2002), although it was observed that students in experimental group developed more positive attitudes than those in control group, there was no statistically significant difference between probability attitude scores in two groups. Moreover, İdikut (2007) conducted a study determine the effects of mathematics history technique on students' attitudes toward mathematics. The participants were 85 (45 experimental group, 40 control group) seventh grade students. The control group received algebra instruction traditionally, the experimental group received algebra instruction through history technique. The mathematics attitude scale was applied to students to measure their attitudes toward mathematics. Results demonstrated that there was no statistically significant difference between control group and experimental group with respect to attitudes toward mathematics.

In another study conducted by Tunç (2006), the attitudes of public school students and private school students toward probability and mathematics were compared. According to results of the study, there was a significant mean difference between public schools students and private school students with respect to attitudes toward probability in favor of private school students. There was a significant mean difference between public school students and private school students with respect to attitudes toward mathematics in favor of private school study.

In this section, some research studies which examine the effects of concrete models and different instructional methods on students' attitudes toward mathematics or probability were examined. In some studies, students' positive attitudes toward mathematics improved after the instruction with concrete models, whereas in some studies students' attitudes toward mathematics did not differ. Moreover, some research studies found different instructional methods effective on students' attitudes toward probability or mathematics, whereas some studies did not. The present study also gives importance to students' attitudes and aims to investigate the effect of instruction with concrete models on students' attitudes toward probability.

2.5. Summary

In summary, probability, the importance of concrete models, attitudes, discovery learning method in mathematics instruction were discussed. Review of literature demonstrates that probability plays an important role in elementary mathematics curriculum and in real life. However, there are difficulties in teaching/learning probability both in Turkey and abroad. To overcome these difficulties, researchers recommend avoiding abstract issues in the instruction. Most of the research studies demonstrate that lessons including concrete models provide higher achievement than those not including concrete models. Research also indicates that there are better results in mathematics achievement when the instruction begins through experiencing with concrete models. In addition, it is pointed out that discovery learning method is a student centered approach providing students to learn independently. In the literature there are both advantages and disadvantages of discovery learning method. Many research studies reveal that discovery learning method increases students' mathematics achievement. Both abroad and in Turkey there are many studies conducted to examine the effects of different teaching/learning methods in the instruction of probability. Most of the results of these studies showed that students' probability achievement increased through the implementation of these methods. Traditional methods did not increase students' probability achievement. The literature related to concrete models and attitude points out that using concrete materials in teaching/learning probability enhances students' interest and motivation. In addition, in the literature it is stated that attitudes play an important role in learning mathematics. Moreover, it is stated that discovery leaning method motivates students and enable them to actively participate in the teaching/learning process eagerly. There are many research studies conducted to examine effects different teaching/learning methods on students' attitudes toward mathematics and probability. According to results of some studies, students developed positive attitudes toward mathematics, whereas in some studies attitudes toward mathematics did not differ. It was not met so many research studies conducted to examine the effects of concrete models on students' probability achievement. That is one of the aims of the present study

CHAPTER 3

METHOD OF THE STUDY

This chapter includes research design, participants, data collection instruments, treatment, variables, procedure, assumptions and limitations, internal and external validity of the present study.

3.1. Research Design of the Study

In the present study quantitative and qualitative research was performed. In quantitative research, one group pre-test, post-test and retention test design was used (Fraenkel & Wallen, 1996). The research design was shown in Table 3.1.

Table 3.2. Research Design of the Present Study

Pre-intervention measuring instruments	Treatment	Post-intervention measuring instruments	Follow-up measuring instruments
Pre-requisite knowledge and skills test	Treatment consisted of concrete models	Probability Achievement Test.	Probability Achievement Test.
Probability Achievement Test.		Probability Attitude Scale	Probability Attitude Scale

Table 3.1. (continued)

Pre-intervention measuring instruments	Treatment	Post-intervention measuring instruments	Follow-up measuring instruments
Probability Attitude Scale		Interview	

As seen in the Table 3.1, in quantitative part, Pre-requisite Knowledge and Skills Test was administered before the treatment to determine the pre-requsite knowledge and skills of students related to probability topic. The Probability Achievement Test (PAT) was administered across three time periods (pre-intervention, post-intervention and follow up). Probability Attitude Scale (PAS) was also administered across three time periods. The treatment was based on concrete models and most of the activities were prepared according to discovery learning method. The treatment with concrete models was performed for 4 weeks and 4 lesson hours per week by the students. To examine the retention effect, PAT and PAS were applied 5 weeks later after the treatment had finished. In qualitative part, interview was conducted with eleven students after treatment. The general purpose of the interview was to examine views of students about instruction with concrete models.

3.2. Participants of the study

The study was performed in one of the private schools in a big city in Central Anatolia Region in the first semester of 2008-2009 academic year. There were twelve 8th grade students in school and all students attended the study. The proper participants were selected for the study. All of the students were about the ages 13-14. Students were private school elementary students and their socioeconomic statuses were high. Three of the students had 3 points, four of them had 4 points and five of them had 5 points in last academic year in mathematics lesson. Two out of 12 students were girls; ten out of 12 students were boys in the present study.

Mathematics teacher of school was the instructor of the students and treatment had been performed by mathematics teacher.

3.3. Data Collection Instruments

In the present study the following data collection instruments were used:

1. Pre-requisite Knowledge and Skills Test (PKT)
2. Probability Achievement Test (PAT)
3. Probability Attitude Scale (PAS)
4. Interview

3.3.1. Pre-requisite Knowledge and Skills Test

Pre-requisite Knowledge and Skills Test (PKT) was prepared by the researcher by revising the test developed by Bulut (1994) (see Appendix A). Its aim was to assess the pre-requisite knowledge and skills of students related to probability topic. There were 11 questions Table of specification of PKT was prepared (see Appendix B).

The content of PKT was consisted of decimals, sets, fractions and data topics. Fractions knowledge was generally required for computing probability of an event and doing operations related to probability. For example, multiplication of fractions was required for computing probabilities of dependent and independent events. Sets knowledge was generally required for learning basic concepts of probability. For example, representing outcomes of sample space and outcomes of event requires sets knowledge. The decimals knowledge was generally required for representing probabilities as decimal and doing operations on probability. For example, some of the questions in PKT were related to multiplication of decimals. Multiplication of decimals knowledge was required for computing probability of dependent and independent events as decimals. The data topic was consisted of probability of a simple event, interpretation of table and percentage. The percentage knowledge was required for representing probabilities in percentages. The knowledge related to probability of a simple event was required for determining if students remembered computing probability of a simple event that they learnt in sixth grade levels.

There are 11 questions in PKT. The sixth and seventh questions have 2 items, the eighth question has 9 items. 4 questions are related to sets topic, 9 questions are related to fractions topic, 5 questions are related to decimals topic, 1 question is related to percentage topic, 1 question is related to probability of a simple event and 1 question is related to interpreting table. The minimum score of PKT was 0, and maximum score of PKT was 21. All of the questions were evaluated as 0, when the answers were wrong or there was no answer, and evaluated as 1 when the answer was correct.

The pilot study of PKT was conducted with 102 students in 3 elementary schools. The content validity of the PKT was checked by reviewing the course content, course objectives, table of specification and by mathematics educator and an elementary school mathematics teacher. The alpha reliability coefficient of PKT was found 0.926.

3.3.2. Probability Achievement Test

The probability Achievement Test was developed by the researcher to examine the students' probability achievement across three time periods (pre-intervention, post-intervention and follow-up) (see Appendix C). The test and lecture content and objectives were determined according to elementary mathematics curriculum of Ministry of National Education. The content of PAT consisted of basic concepts of probability, types of probability, dependent and independent events, permutation and combination. Table of specification was prepared (see Appendix D). An item bank was formed with 40 open-ended questions. Fifteen problems were chosen from the item bank by taking into consideration the table of specification and expert judgment. While developing PAT, some of the questions were quoted from different resources. For example the questions 4, 5, 7, 10 (see Appendix C) were quoted from Bulut and Ubuz (2001). In addition, the questions 9 and 10 were quoted from MoNE (2008).

There are 15 problems in PAT. The 7th and 12th problems have 2 items. 4 problems are related to basic concepts of probability, 4 problems are related to types of probability, 5 problems are related to dependent and independent events and 2 problems are related to permutation and combination. The rubric of PAT was prepared taking into consideration opinions of an elementary mathematics teacher and mathematic educator (see Appendix E). The answers of questions 1, 2, 3, 4, 5, 6, 13 and 15 were evaluated as 1 when the answer was correct and as 0 when the answer was wrong or there was no answer. The answers of questions 7a, 7b, 8, 9, 10, 11, 12a, 12b and 14 were evaluated as 0 when the answer was wrong or there was no answer, as 1 when the answer was correct but there was no explanation or there was an explanation but answer was wrong, as 2 when the answer was totally correct .The maximum score of PAT was 26. Its minimum was 0.

Pilot study of the PAT was conducted in 3 elementary schools in Ankara with 118 pupils of 8th grade students in the first semester of 2008-2009 academic year. The administration of the test took 40 minutes. Content validity of the instrument was checked by mathematics educator and an elementary mathematics teacher by reviewing the course content, course objectives, table of specification. The mathematics educator and the elementary mathematics teacher scored the answers of the test which was implemented in the pilot study. The correlation between two scorings was conducted to test the reliability of the instrument. The Pearson Product Moment correlation coefficient was computed to test the interrater reliability. It was found 0.950 with 17 items.

3.3.3. Probability Attitude Scale

Probability Attitude Scale (PAS) was developed by Bulut (1994) (see Appendix F). It was developed by applying the scale to 352 mathematics education students in METU. The 28-item PAS included 15 positive items and 13 negative items and was scaled on a six-point Likert Type scale: Strongly Agree, Agree, Tend to Agree, Tend to Disagree, Disagree, Strongly Disagree. The positively worded items were coded from Strongly Agree as 6, to strongly disagree as 1, and negatively worded items were inverted to a positive direction for scoring aims. This six-point scale was used to not allow the undecided answer in five-point scales. By using factor analysis, single factor was determined which was labeled as "general attitude toward probability". The alpha reliability coefficient of PAS was found 0.95 with SPSS.

162 students from 3 elementary schools in Ankara participated in the pilot study of Probability Attitude Scale in the first semester of 2008-2009 academic year. The alpha reliability coefficient was found 0.946 with 28 items for the present study. The total score of PAS was between 28 and 168.

3.3.4. Interview

The aim of the interview questions was to examine students' views about instruction with concrete models. Interview questions are as following:

1. What do you think about activities and concrete models in probability instruction?
2. How did you feel while using concrete models?
3. What did concrete models provide you?
4. What kind of mathematics instruction do you want to receive?

This interview was a structured interview. The interview questions were decided by an elementary mathematics teacher and a mathematics educator by taking into account the content of the study. The questions which examined students' views related to instruction with concrete models were determined. Results of the interview were also analyzed by another person to enable the reliability of this method. Students answered questions clearly. The codings of two people were 100% same so the reliability of coding the interview was enabled.

The interview was conducted with 11 students after treatment. Students were interviewed in school at classroom. Researcher told the purpose of the interview to students. All students wanted to participate in interview. They were told that each interview would be recorded and they accepted it. A speech recorder was used while interviewing to record the data under the permission of students. Each speech took about 3-6 minutes.

3.4. Teaching/ Learning Process

In this section, development of activities and treatment of the study are explained.

3.4.1. Development of Activities

In the present study, activities were planned with the use of concrete models. Also, most of the activities were prepared according to discovery learning method. As emphasized by Kober (1991), children's learning of mathematics is connected to their experience with concrete models. Symbols, abstract issues can be learnt, if learning begins with experimenting through concrete objects. Moreover, according to Clements (1999) manipulatives are important for students to create meaningful ideas. He also emphasizes that the uses of manipulatives must be before the symbolic teaching/ learning. They are not adequate alone. Teachers should guide students and make them actively participate in process by the use of manipulatives (Clements, 1999).

As pointed out by Fennema (1973), manipulatives increase students' motivation. Appropriate motivation is required for children to learn mathematics. By giving children extrinsic awards, simple skills can be learnt by them. However, if the children do not have intrinsic motivation, they have trouble with learning abstract interrelationships of mathematics. Manipulative materials increase students' intrinsic motivation toward abstract and difficult issues of mathematics. Probability is one of the abstract and difficult topics of mathematics (Fennema, 1973). As emphasized by Fennel (1990), probability activities should be active, include physical objects and provide opportunities for children to question, solve problems and discuss. Moreover, in the literature, many educators advocate the use of concrete models in the probability activities. Hence, the importance was given to use of concrete models in the present study. Moreover, discovery learning method makes classroom environment exciting, encourages students to participate in lessons, increases students' capability to learn a new topic, makes the knowledge transfer and retention better (Bruner, 1961), the discovery learning method was also used in the preparation of most of the activities in the present study . Moreover, in the literature, it was emphasized that guided discovery learning method was more effective than pure discovery learning in terms of facilitating students' discovery. Therefore, based on the literature, the instruction was planned through concrete models and most of the activities were prepared based on the guided discovery learning method.

Most of the activities and questions were prepared by the researcher. Some activities and questions were quoted the from eighth grade elementary school book of mathematics. Sample activities were given in Appendix G. All of the concrete materials were prepared by the researcher taking into consideration levels of the students. As supported in the literature related to probability activities, spinners, fair and unfair dices, several kinds of 3-d geometric object, pattern blocks, balls, coins, newspapers, cardboards, board markers, sugars, pockets and bags were used in the activities. Lesson plans (see Appendix H) were prepared by the researcher.

The name of the first activity was "Let's roll the cubes" and it was related to basic concepts of probability. Students were reminded what sample space, experiment, event, outcome were and wanted to express them and find out what the sample space, experiment, event and outcome were in the activity. The cubes were distributed to students. On each side of cubes, the numbers 1-6 were written. Teacher wanted students to roll the cubes. He stated that an experiment was a situation involving chance or probability that leaded to results called outcomes and asked what the experiment in this activity was. Students stated their answers that the rolling the cube was the experiment. Then, he wanted students to write possible numbers that could be on the upper side of the cube. Students wrote all of the six numbers. Teacher stated that these all numbers were possible outcomes and composed sample space. Teacher stated that an event was one or more outcomes of an experiment and these outcomes were favorable outcomes. Teacher wanted students to determine the event for the question "What is the probability of number 2 being on the upper side of the cube?" Students stated that number 2 being on the upper side of the cube was the event. Then, teacher wanted students to find the outcomes of the event. Students stated that number 2 is the outcome of the event. Teacher also stated that outcomes of event were favorable outcomes. Students stated their answers in written form and verbally. At the end of the activity, teacher summarized the procedure and conclusions.

The name of the second activity was "Letters of alphabet on stamps" students applied their knowledge that they learnt in the previous activity. Twenty nine stamps and one box were distributed to students. On each side of the stamps the letters in the alphabet were written. Then, teacher asked a question: What is the probability of drawing a vowel from the bag? Teacher wanted students to find out experiment, event, sample space, outcomes of the sample space, and outcomes of the event for this question. Students explained their answers in a written form and verbally.

The third activity "Fair or unfair?" was related to equally likely sample space. Activity plan is given in Appendix G. Teacher distributed students some unfair dices and wanted them to roll the dices. Students rolled the dices one more time. Teacher wanted students to determine the sample space and asked if occurrence of each number on the upper side of the dices was equal. Some students stated that there was always 1 on the upper side of the dice and some students stated that there was always 6 on the upper side of the dice. They discussed with each other and stated that dices were not fair and probabilities of outcomes of sample spaces were not equal. They also concluded that probabilities of sample space should be equal for a fair rolling with the guidance of teacher. Then, teacher summarized the procedure and conclusions.

The fourth activity “Let’s spin the spinner” was a kind of spinner activity. Activity plan is given in Appendix G. In this activity students were wanted to find out the probability formula by correlating with proportion. The spinners which were divided into four equal sectors (each part was colored red, yellow, blue and green) were distributed to students. Firstly, teacher wanted students to determine the proportion of yellow part to all parts. Then, they were wanted to determine proportion of blue part to all parts. Teacher also asked students the number of outcomes of sample space (possible outcomes). Then, for the question “What is the probability of needle landing on yellow part?”, students were wanted to determine the outcomes of event (favorable outcomes). After finding the number of outcomes of sample space and event, students were wanted to find the proportion of number of outcomes of event to number of outcomes of sample space. Students found it and teacher stated that they found the probability of landing on yellow part and asked students what the general probability formula of an event occurring was. Students explained their answers that the probability of an event occurring was equal to number of favorable outcomes/ number of possible outcomes in a written form and verbally. Then, teacher summarized the procedure and conclusion.

The fifth activity “Colored balls” was related to certain and impossible events. Activity plan is given in Appendix G. In the activity, there were blue and white balls and bags. Balls and bags were distributed to students. Teacher wanted students to put the white balls into the bag and asked what the probability of drawing a white ball from the bag was. Students computed the probability and also wrote it in decimal. They stated that drawing a white ball from the bag was certain and its probability was 1. Teacher also asked what the probability of drawing a blue ball was. Students stated that it was 0 and drawing a blue ball from the bag was impossible. They explained their answers in a written form and verbally. Then, teacher drew probability line on the board and wanted students to show the probabilities of these events on that line. At the end of the activity, teacher summarized the procedure and conclusions.

The sixth activity “colored cubes” was done to make students to apply their knowledge they learnt in pervious activities. Bags involving different colored cubes (4 red, 1 green, 5 blue) were distributed to students. Students were wanted to determine sample space, outcomes of sample space, event, outcomes of event, experiment and compute probability for the question: “What is the probability of drawing red cube from the bag?” Students were also wanted show the events of drawing pink cube, red cube, green cube, blue cube, one of the cubes and white cube on the probability line. Students drew probability line and showed all the probability of events on probability line. Also, they stated their conclusions in a written form and verbally related to activity. At the end of the activity, teacher summarized the conclusions.

The seventh activity “Let’s toss the coins” was related to experimental and theoretical probabilities. Each student was distributed coins. Before tossing, teacher asked the probabilities of heads or tails. Students discussed and stated that probabilities were ½ and equal. Teacher wanted students to write these probabilities as a fraction, decimal and percent. Then, students tossed their coins 5 times, 10 times and 20 times orderly and they recorded the results on tally-sheet and drew column graph to show results. Students were wanted to compare the probabilities before they tossed and after they tossed the coins. Students discussed with each other under the guidance of teacher and stated that before tossing the probabilities were equal and after they started to toss coins, each probability changed. Students were asked the reasons of these different probabilities. They stated their reasons as before tossing the coins they expected about the probabilities of heads and tails. But, the second probability was computed according to results of tossing. It was actually what happened. Finally, students were asked how they could define these probabilities. They stated their conclusions that first one is expected probability, the other one is experimental probability, because they found it according to results of coin tossing experiment. Also, the teacher stated that the expected probability was called theoretical probability. Students explained their conclusions in a written form and verbally. At the end of the activity, they realized what theoretical and experimental probabilities were and the difference between these two probabilities.

The eighth activity was called “Let’s spin the spinner” and was also related to experimental and theoretical probabilities. There was a spinner, divided into four parts equally. Each part was colored differently, yellow, blue, red and green. At the centre of the spinner there was a needle to spin. Spinners and tally-sheets were distributed to students. Before spinning the needle students were asked what theoretical probabilities for each color were. They stated their answers that it was ¼. They also stated their answers as a fraction, decimal and percent in a written form and verbally. Then, students were wanted to begin spinning the needle 20 times (section 1), 50 times (section 2) and 100 times (section 3) and record the outcomes on the tally sheet. After each section was completed, students wrote the experimental probabilities as a fraction, decimal and percent. After all sections were finished students were wanted to compare the experimental and theoretical probabilities in each section. They discussed with each other and deduced some conclusions under the guidance of teacher. They stated that from section 1 to 3, the experimental probability became closer to theoretical probability. In section 2, it started to be closer. In section 3, it became closest to theoretical probability. Finally, they stated that as the number of spinning increased, the experimental probability became closer to theoretical probability. Then, they explained their conclusion in a written form and verbally. Teacher also summarized the procedure and conclusions.

The ninth activity “Which team will win the match?” was related to subjective probability. Teacher distributed newspapers including sports articles of sports writers. Students read articles. Teacher wanted them to be careful about conjectures of football match results. They stated that the conjectures of sports writers about match results were different. At the end of the activity the teacher summarized the procedure, conclusion and its reason.

The tenth activity “King and vizier” was related to explaining the probability of the occurrence of an event by using their geometric knowledge. In this activity, matchboxes were used. Large surfaces of matchbox represented empty, medium surfaces represented vizier and small surfaces represented king. Students made groups of four. In each group students threw the matchbox orderly. The student who threw the box was king when the box standed on small face, and was vizier when the box standed on medium face and was punished when the box standed on large face. King decided the type of punishment and vizier punished the student. After punishment students continued the playing game. After the game, students compared the probabilities of king, vizier and empty and ordered them from high to low. They explained their conclusions and their reasons in a written form and verbally. They also discussed their conjectures and their reasons under the guidance of the teacher. Their conjectures was if the amount of favorable area was large, the probability of the occurrence of the event was high, if the favorable area was small, the probability was low. They explained their reasons for their conjectures that if the probability of the occurrence of the event was computed as the number of favorable outcomes of an event / the number of possible outcomes of an event, the geometric probability was found as the amount of favorable area / total amount of possible area. Finally, teacher summarized the procedure, conclusion and its reason.

The eleventh activity “Polygon” was related to calculating geometric probability. There was a throwing polygon on the board. Students made groups of four. Dimensions of geometric figures were given. Students calculated areas. The smallest area was yellow region. Areas of blue and purple regions were equal. The biggest area was red region. The area of orange region was smaller than blue and purple region. From each group one student threw board marker to polygon orderly. All students threw the board marker. Finally to decide which group would win the game, students were wanted to count and compare the number of dots on different regions. They stated that there were so many dots on the big regions, and fewer dots on the small regions. Teacher asked the reason for this situation. Students stated their reasons that areas of regions caused this situation. They also stated that the probability of throwing the board marker to big regions were higher than the probability of throwing board marker to small regions. Then, students were asked what the probability of throwing the board marker to red region was. In the former activity students realized that if the probability of the occurrence of the event was computed as the number of favorable outcomes of an event / the number of possible outcomes of an event, the geometric probability was found as the amount of favorable area / total amount of possible area. Then, they calculated the probability. Teacher also wanted students to compute all probabilities. Students explained their answers in written form and verbally. After that, to determine which team won the game teacher asked students how they could find the points of each region. Students expressed their conjectures that the biggest region should have smallest point because probability of throwing board marker to biggest region is highest and the smallest regions should have highest point because the probability of throwing board maker to the smallest region is the smallest. At the end of the activity, they decided the points. So, the red region was 1 point. Blue and purple regions were 2 points. The orange region was 3 points and the yellow region was 4 points. They calculated their points. According to results, two groups won the game. Finally, teacher summarized the procedure, conclusion and its reason.

The twelveth activity “Events in poeples’ lives” was prepared to make students to conceive dependent and independent events. The events were written on the strips. Event A: Today is birthday of Ayşe. Event B: Ayşe is so happy. Event C: Ayşe is very good at her lessons. Event D: There is a bandage on the left arm of Ali. Event E: Ali broke his left arm by falling from bicycle. Event F: Elif can not see the objects far away from her. Event G: Elif wears glasses. Event H: Today is rainy. Event I: Today, in the morning I took umbrella while I was leaving home. The teacher distributed strips to students. On each strip there were different events. Teacher wanted students to draw 2 events and wanted them to say if the occurrence of one of these two events was affected or not affected by occurrence of another event. Students made the activity for all events and while doing activity discussed their ideas under the lead of teacher. Students explained that if the occurrence of an event was affected by occurrence of another event they were dependent events, if occurrence of an event did not affect occurrence of another event they were independent events. Finally, teacher summarized the procedure and conclusion that if the probability of an event was affected by the probability of another event they were called dependent events, if the probability of an event did not affect the probability of another event they were called independent events.

The thirteenth activity “Let’s draw gifts” was also related to dependent and independent events. Each student was given 6 unit cubes in a bag on which there were written three kinds of gifts. (3 pencils, 2 notebooks, 1 fiction). Students drew cubes from bag orderly. Firstly, students were wanted to say the theoretical probabilities of drawing each gift and draw one cube. Students stated the theoretical probabilities of drawing cubes in written form and verbally. Then, they drew one cube and put it back into bag. Then, teacher wanted them to make second drawing. Then, students were asked if the probability of drawing second cube was affected by drawing the first cube. They discussed on this question and stated that it was not affected because when the first cube was put back into the bag, the total number of cubes did not decrease. After that, students drew one cube from bag but this time they did not put the cube back. For the second drawing, they were asked if the probability of drawing second cube was affected by drawing the first cube. Students discussed with each other and stated that because they did not put the first cube back into the bag the probability would change. They stated their reason that the number of cubes decreased. It meaned that the number of possible outcomes decreased. Students explained their conclusions in a written form and verbally and teacher summarized the activity.

The fouteenth activity "Who wants to eat lemony candies?" was prepared to make students to compute the probabilities of dependent and independent events. Activity plan is given in Appendix G. There were candies in a bag and only taste of them was different. 3 lemony candies and 5 minted candies. Kerim and Gülçin wanted to eat two lemony candies. As they could not decide who would eat lemony candies, they would draw the candies from bag. Candy drawing would be made two different ways:

1.case: Gülçin would draw first. After she drew first candy, she would put it into bag and would draw the second candy. Gülçin would draw the second candy after drawing and putting into the bag the first candy. If two of the candies were lemony Gülçin would be able to get lemony candies.

2. case: After Gülçin drew the first candy, she did not put it into bag and drew the second candy**.** If two of the candies were lemony, Gülçin would be able to get lemony candies.

Students were asked the probability of drawing the lemony candy for each drawing for the 1st case and what kind of these events were. Students stated that they were independent events and the probabilities were

$\frac{3}{8}$ for each drawing. Teacher stated that if A and B are independent events, probability of A and B was computed as $P(A \text{ and } B) = P(A) \bullet P(B)$. Then, he asked what the probability of these two candies drawn were lemony was in the 1st case. Students calculated it and stated their answers that it was $\frac{3}{8} \bullet \frac{3}{8}$ in a written form and verbally. Then, students were asked the probability of drawing the lemony candy for each drawing for the 2nd case and what kind of these events were. Students stated that they were dependent events and the probabilities were $\frac{3}{8}$ for first drawing and $\frac{2}{7}$ for second drawing. Then teacher stated that if A and B are dependent events, probability of A and B was computed as $P(A \text{ and } B) = P(A) \bullet P(B|A)$. Then, teacher asked what the probability of these two candies drawn was lemony was in the 2nd case. Students computed it and stated their answers that it was $\frac{3}{8} \bullet \frac{2}{7}$ in a written form and verbally. Then, teacher wanted students to make a tree diagram and compare the probabilities of the events in these two cases. All students made tree diagram with the guidance of teacher and found all possible outcomes and calculated all probabilities. Then, teacher summarized the procedure and conclusions.

The name of the fifteenth activity was “Let’s play the music” and it was related to permutation. In this activity, there were 4 students and 3 chairs in front of the class. Music would be played while doing activity. When the music stopped, students would try to sit on the chairs. Teacher played music on his phone and students started to turn around of the chairs. Music stopped and student who could not sit was out of the game and took one of the chairs with him. After that, there were 3 students and 2 chairs. The game went on until there was one student left. After game finished, teacher drew three chairs on the board and boxes under each chair. He asked how many students could sit on the first chair. Then, 4 students sat on the first chair one by one. Then, one of the four students stayed on the first chair and other three students sat on the second chair one by one. Students said that it could be sat on second chair 3 different ways. Then, one of the three students stayed on the second chair and other two students sat on the third chair one by one. Students stated that it could be sat on third chair 2 different ways. The teacher stated that the purpose of the activity was to find out how many different ways that 4 students could arrange three by three. Then, he solved the problem as 4.3.2=24.

Teacher stated that this problem could be solved by permutation formula. He stated that the number of ways of getting an ordered subset of r elements from a set of n elements is given by:

$$P\ (n, r) = \frac{n\,!}{(n-r)\,!}$$

Teacher stated that the answer for the question that how many different ways that 4 students could arrange three by three could be found by computing trio permutations of four. He stated that it was also the numbers of ordered subsets including 3 elements of a set including 4 elements. Then, students solved the problem by computing trio permutations of four.

The name of the sixteenth acvitity was “Let’s arrange the pattern blocks”. This activity was related to permutation. In this activity, 5 different pattern blocks were distributed to students. Students were wanted to compose patterns by arranging 3 figures orderly from 5 figures. While students were composing patterns, they were asked if each pattern they composed was different from other patterns. Students discussed with each other and stated that they were different from each other because changing the orders of blocks in a pattern changed the pattern. They realized that order was important while composing a pattern. They composed all the patterns and recorded them and stated that they composed 60 patterns. Teacher also asked students how they could find the result by using permutation formula.

Students stated that they could find it by computing P(5,3) . Then, students solved the problem by computing P(5,3) and stated their answers in a written form and verbally. At the end of the activity, teacher summarized the procedure and conclusions.

The name of seventeenth activity was “Let’s prepare sandwich” and it was related to combination. This activity was related to combination. There were cards and on each card there was a picture of food to prepare a sandwich (tomato, cheese, cucumber, salami). Cards were distributed to students. Teacher told students that they would prepare a sandwich by using 3 out of 4 food with these cards. Students started to prepare their sandwiches. While they were preparing their sandwiches, teacher asked students if the arrangement of food changed the sandwiches. Students discussed and stated their conjectures that the arrangements of food did not change the sandwiches because preparing sandwich with tomato, cheese and cucumber or with cheese, cucumber and tomato were not different. Then, students were asked how many different ways there were to prepare their sandwiches or how many different ways they could select 3 food from 4 different food. They stated their answers that they composed 4 different sandwiches. Then, teacher wanted students to determine the subsets including 3 elements from the set including 4 elements. Students stated their answers that there were 4 subsets including 3 elements and there were 4 ways to select 3 food from 4 food. Students also realized that the number of different selections is equal to the number of subsets of this set. Then teacher stated that A represented cheese, B represented tomato, C represented cucumber and D represented salami and wanted students to write the trio groups that could be composed with A, B, C and D. Students wrote on papers and teacher wrote on the board as follows:

I	II	III	IV
ABC	ABD	BCD	ACD

ACB	ADB	BDC	ADC
CBA	DBA	CDB	CDA
BCA	BDA	DCB	DCA
BAC	BAD	CBD	DAC
CAB	DAB	DBC	CAD

Teacher said that if the order was important 24 subsets could be composed with the three of A, B, C and D and it could be computed by the permutation formula as following: Trio permutations of four. He stated that because of the order was not important in the mixture, the trio groups in the columns of I, II, III, and IV specified the same cases. For example, in each column the 6 arrangements ABC, ACB, CBA, BCA, BAC, and CAB emphasized the same situation. He asked students how they could find the number of unordered lists. Students discussed with each other and stated that they could find it by dividing number of trio permutations of four by 6. Then, teacher asked students how they could formulate it by using permutation formula. Students discussed and stated that they could write 6 to the denominator in permutation formula. Teacher also asked students how they could define 6 as factorial. They explained their conclusion in a written form and verbally. Students solved the question by using the permutation formula.

Teacher emphasized that the combination was called the number of ways of picking r unordered outcomes from n possibilities. Also, each subset including r element of a set including n element is called combination with r of that set. Then he wanted them to write combination formula by using permutation formula. Students wrote the formula. Then, teacher wanted students to explain difference between permutation and combination mathematically. Students wrote the formulas and explained that the difference was r! in the denominator of combination. Because, it meant the number of orders in each list. Then, teacher summarized the procedure and conclusion.

3.4.2. Treatment of the Study

Probability activities were applied to 12 eighth grade elementary students at the first semester of 2008-2009 academic year. Activities were applied 2 days a week including 4 lesson hours and they took 4 weeks. Each lesson hour took 40 minutes. Treatment took 16 lesson hours. The lessons including basic concepts of probability took 4 lesson hours, the lessons including types of probability took 5 lesson hours, the lessons including types of events took 4 lesson hours and lastly the lessons including permutation and combination took 3 lesson hours. Lesson plans (see Appendix H) were prepared by the researcher. At the beginning of each topic, teacher asked students questions from their daily lives to catch their attention. In the lectures both inductive and deductive discovery learning methods were used. Teacher guided students through asking questions in activities. Students found generalizations by discussing on teacher's questions and each other while experiencing with concrete models. All of the students participated in the activities actively and answered teacher's leaded questions. At the end of the each topic, the evaluation questions were solved by students to apply generalizations and reinforce their learning. Before the treatment, researcher informed teacher about content of the treatment. Researcher gave information about the treatment to the teacher in detail. Lesson and activity plans were given to the teacher. After each activity, researcher and teacher consulted with each other.

Before the treatment, Pre-requisite Knowledge and Skills Test was administered to students to determine their pre-requisite knowledge and skills related to probability topic. Generally, students' deficient knowledge was in sets, fractions and decimals topics. For example, students had difficulties in determining intersection of two sets. They also had difficulties in determining universal set. In fractions topic, there was deficient knowledge in ordering the fractions. Also, in decimals topic, students had difficulties in multiplying the decimals. After determining these deficient knowledge and skills, teacher gave instruction to students related to these topics for 3 lesson hours. In the instruction, teachers also provide students to solve several problems. To test if the students learnt determining intersection of two sets and universal set, ordering fractions and multiplying decimals, teacher applied a quiz including questions related to these topics. After analyzing students' answers to the questions, it was seen that students learnt the knowledge and skills required for probability topic.

At the treatment, the first lecture was related to basic conceps of probability. It took two lesson hours. Firstly, the teacher took attention to history of probability. He talked about Cardano, Fermat and Pascal. He also talked about applications of probability in the biological, social and medical sciences. Students were asked where they heart about or used probability in their daily lives. Students' sample answers were about guessing about results of football matches, weather. Papers about daily life situations in which there were weather forecast report and global heating including probability concept were also distributed to students. Students were wanted to read papers and find where probabilities were used in the articles by discussing each other. After taking students' attention to the probability topic, students made the activity called "Let' roll the cubes" In this activity, students were reminded what sample space, experiment, event, outcome were. After the first activity finished, students made "Letters on the stamp" activity. They applied their knowledge that they learnt in the previous activity. After these two activities, students solved evaluation questions. After solving evaluation questions, students made another activity called "Fair or unfair?" activity. In this activity, students learnt equally likely sample space. At the end of the lesson, students solved evaluation questions.

The second lecture was related to probability of an event and certain and impossible events. It took two lesson hours. In this lesson, firstly students made “Let’s spin the spinner” activity. In the activity students found out the probability formula by correlating with proportion. After the activity, students solved related evaluation questions. Then, students made another activity called “Colored balls” and it was related to certain and impossible events. Before the activity, teacher asked students if every morning sun rised, if elephants could fly and how they could define these kinds of events. Students discussed on these questions and stated that the rising of sun every morning was certain and flying of elephants was impossible. Teacher also wanted students to give related examples from their daily lives. In this activity, students learnt what certain and impossible events and probabilities of these two events. After that, students made the activity called “Colored cubes”. In this activity, applied their knowledge they learnt in pervious activities. At the end of the lesson, students filled the table related to basic concepts of probability topic and solved related evaluation questions.

The third lecture was related to experimental and theoretical probabilities. It took two lesson hours. Lesson plan is given in Appendix H. In the beginning, teacher asked students where coin tossing was used to catch their attention to the subject. Students’ sample answer was that at the beginning of football matches, coin tossing was used to determine which team would start first. It was used for other games, also. Firstly students made “Let’s toss the coins” activity. In this activity, students learnt the theoretical and experimental probabilities and realized the different between these two probabilities. Then, students made another activiy called “Let’s spin the spinner”. In this activity, students realized that if the number of spinning increased, the experimental probability became closer to theoretical probability. At the end of the lesson, students solved related evaluation questions.

The fourth lecture was related to subjective probability. It took one lesson hour. At the beginning, students were asked what the probability of that it was going to be rainy was. Students discussed and stated their conjectures. Some students stated that it would be rainy with a probability of 90%. Some students stated that it would be rainy with a probability of 80%. Namely, students gave different answers. Then, students made the activity called “Which team will win the match?” In this activity, students realized that some probabilities could be personally. At the end of the lesson, students solved related evaluation questions.

The fifth lecture was related to geometric probability and it took two lesson hours. Lesson plan is given in Appendix H. In the lecture, the first activity “King and vizier” was about explaining the probability of the occurrence of an event by using their geometric knowledge. In this activity, students realized that if the probability of the occurrence of the event was computed as the number of favorable outcomes of an event / the number of possible outcomes of an event, the geometric probability was found as the amount of favorable area / total amount of possible area. Then, students made another activity called “Polygon”. In this activity, students calculated geometric probabilities of different colored regions. At the end of the lesson, students solved questions related to geometric probability.

The sixth lecture was related to dependent and independent events. It took two lesson hours. Lesson plan is given in Appendix H. At the beginning of the lesson, teacher asked students if they heart about dependent and independent events in their daily lives. After students gave the answers, students made “Events in peoples’ lives” activity. At the end of the activity, students explained that if the occurrence of an event was affected by occurrence of another event they were dependent events, if occurrence of an event did not affect occurrence of another event they were independent events. In this activity, students realized what dependent and independent events were. At the end of the lesson, students solved the questions related to dependent and independent events.

The seventh lecture was also related to calculating probabilities of dependent and independent events. It took two lesson hours. At the beginning of the lesson, students made “Let’ draw gifts” activity. In this activity, students realized that probabilities of dependent and independent events were different from each other. Then, students made another activity called “Who wants to eat lemony candies” activity. In this activity, students learnt how the probabilities of dependent and independent events were computed. At the end of the lesson, students solved related evaluation questions.

The eighth lecture was related to permutation. It took one lesson hour. At the beginning of the lesson, examples of real life situations about permutation were given to catch their attention. The teacher talked about genetic code and stated that although everyone’s DNA was composed of same protein enzymes, DNA was different for each person and asked what the reason for this situation was. Students discussed with each other and stated that arrangements of enzymes in DNA were different. Students also gave similar examples. Then, students made “Let’s play the music” activity. In this activity, students learnt what the permutation was and how the numbers of permutations could be found by using formula. After this activity, students made another activity called “Let’s arrange pattern blocks”. In this activity, students realized that order was important in permutation and they also found numbers of patterns by using permutation formula that they learnt in former activity. At the end of the lesson, students solved evaluation questions.

The last lesson was related to combination. It took two lesson hours. At the beginning of the lesson teacher asked questions to students about daily life uses of combination. Then, students made “Let’s prepare sandwich” activity. In this activity, students realized that order was not important in combination. Students also discovered the combination formula by using permutation formula that they learnt in previous lesson under the guidance of teacher. At the end of the lesson, students solved questions related to permutation and combination.

During the lessons, all of the students attended discussions and made conclusions with the guidance of the teacher. Almost at the end of each activity, they solved several problems to apply their knowledge that they learnt. As observed by the researcher, students were eager to experience with concrete models and they were motivated while manipulating with concrete models. Teacher's questions aroused students' curiosity. They discussed with each other to find out the answers.

3.5. Procedure

The study was conducted for 4 weeks with twelve 8th grade students in 2008-2009 academic year. Before the study, the necessary permissions were got from Ministry of National Education and school management. Two weeks before the study, the researcher introduced the content and activities to mathematics teacher. Then, the permissions of students and their families were obtained. Pre-requisite Knowledge and Skills Test, Probability Achievement Test and interview were prepared by the researcher by taking into consideration opinions of an elementary mathematics teacher and mathematics educator. Before the study, the Pre-requisite Knowledge and Skills Test (PKT) was administered to students. The content of the PKT consisted of topics of sets, fractions, decimals and data topics. According to results of the PKT, mathematics teacher gave course to students to remove their deficiencies of the subject to be learnt. PAT and PAS were administered to 12 eighth grade students across three time periods (pre-intervention, post-intervention and follow-up). Instruction consisted of concrete models was performed for two days per week throughout 4 weeks. Each day, the instruction took two lesson hours (40min+40min). Mathematics teacher of the students was the instructer and performed instruction. The researcher observed students, took notes and helped teacher in the activities. The recommended concrete models for probability topic were used in the activities. The content of the treatment included basic concepts of probability, types of probability, dependent and independent events, permutation and combination. Also, the interview was performed with 11 students after the treatment. Each interview took about 3-6 minutes. The purpose of the interview was to have students' views about instruction with concrete models. PAT and PAS were applied to students to examine the retention effect 5 weeks later.

3.6. Variables

The variables of the study could be categorized in three parts. The first part involves the variables of the first sub-problem-" What is the effect of instruction with concrete models on eighth grade students' probability achievement? They were the pre-intervention, post-intervention and follow-up test scores of the students obtained from the PAT.

The variables for the second sub-problem of the present study –" What is the effect of instruction with concrete models on eighth grade students' attitudes toward probability?" were the pre-intervention, post-intervention and follow-up test scores of the students obtained from the PAS.

The variable for second main problem of the study "What are the students' views about instruction with concrete models?" was students' views about treatment.

3. 7. Analysis of the Data

SPSS package program was used to analyze the data in the present study. To have means, medians, standard deviations and maximum and minimum values of students' PAS and PAT scores, the descriptive statistics was used. The hypotheses were analyzed by Friedman Test and Wilcoxon Signed Ranks Test. The sub-problems of the study will be examined by means of their associated hypotheses which are in the null form and tested at a significance level of 0.017(0.05 divided by 3) according to guidelines stated by Colman and Pulford (2006) because the data were obtained 3 different time periods. In addition, the reliability coefficient of PAT and PAS were computed. The pearson correlation was conducted to test reliability of PAT. The interrater reliability of PAT was found.

3.8 Assumptions and Limitations

In this section, there are assumptions and limitations for the present study.

3.8.1. Assumptions

The main assumptions of the present study are following:

1. Data collection instruments were administered under standard conditions.
2. The data collection instruments were answered by all subjects correctly and sincerely.
3. The participants were able to understand and comprehend the items accurately.
4. The items were comprehensible to the participants.
5. There was no outside event which could influence the results during the study.

6. The students were motivated enough to answer the questions in the tests.

7. The teacher was objective during the process.

9. The students answered interview questions honestly.

3.8.2. Limitations

The limitations of the present study are as listed below:

1. One of the limitations is sample size of the study. The nature of this study is limited to the data obtained from 12 eighth grade elementary school students in a private school in Ankara. Twelve students can not represent 8th grade students in Turkey.

2. The data obtained in the present study was analyzed by non-parametric test which is a weak method of analyzing data.

3. Design of the present study was weak. Because, there was no control group. So, it is difficult to state that there was an effect of concrete models on students' probability achievement and their attitudes toward probability directly.

4. In the present study time was limited for treatment.

5. One of the limitations may be the inexperience of researcher on how to interview.

3.9. Validity of the Study

In this section internal and external validity of the study is discussed.

3.9.1. Internal Validity of the Study

Internal validity of a study means that observed differences on the dependent variable, not due to some other unintended variable (Fraenkel & Wallen, 1996).

Subject characteristics, mortality, history, instrumentation, testing and implementation were possible threats to internal validity in the present study. In this part, ways of controlling these threats were explained.

First of all, subject characteristics are one of the possible threats to internal validity in the present study. The characteristics of subjects which might affect the internal valitidity were students' ages and their socioeconomic statuses. Students who participated in the present study were at the same grade level, so their ages were close to each other. Since the students were from a private school, socioeconomic statuses of their families were almost same. So, these characteristics did not influence the results accidentally.

Mortality is another threat which means the loss of subjects can be a threat to internal validity (Fraenkel & Wallen, 1996). In the present study, all of the students attended to all instrument interventions. So, mortality effect was eliminated.

The history is a threat to internal validity results from an event which is not belong to intervention and but can affect the students' performances (Fraenkel & Wallen, 1996). In the present study, there was not an event affected students' responses and study procedure. So, the history threat was eliminated.

The instrumentation threat is a threat to internal validity and it occurs to be if the data collection instruments are changed (Fraenkel &

Wallen, 1996). In the present study, all of the data collection instruments (PKT, PAT, PAS) were not changed during the study. So, instrumentation threat was controlled.

Testing threat is a threat to internal validity which means that higher scores on post-test may be due to having a pre-test (Fraenkel and Wallen, 1996). In the present study, there was four weeks break from pre-intervention through post-intervention and five weeks break from post-intervention through follow-up. These time intervals were enough long to prevent students to memorize the questions. So, testing threat was controlled.

Implementation threat is one of the threats to internal validity results from the changes in the applications of the treatment (Fraenkel & Wallen, 1996). In the present study, the teacher and researcher were different and also the treatment was applied by teacher throughout the study. So implementation threat was eliminated.

3.9.2. External Validity of the Study

External validity is extent to which the results of a study are could be generalized (Fraenkel & Wallen, 1996).

3.9.2.1. Population Validity

Fraenkel and Wallen (1996) state that population validity is the degree to which the results of a study can be generalized to an intended population. The sample size was very small in the present study. Because of this reason, to generalize of the results of the present study is very difficult. However, the participants in the present study can be generalized to the subjects having same characteristics.

3.9.2.2. Ecological Validity

Fraenkel and Wallen (1996) mention that the ecological validity is the degree to which results of a study can be extended to other settings and conditions. The treatment and tests were given in regular classroom settings in the present study. Therefore, the results of this study can be generalized to classroom settings similar to this study.

CHAPTER 4

RESULTS OF THE STUDY

In this chapter, the results of this study are explained in three sections. In the first section the results of descriptive statistics are presented. In the second section the results related to inferential statistics are explained.

4.1. The Results of Descriptive Statistics

In this section the descriptive statistics of the data are given. First of all, the results of descriptive statistics of Pre-requisite Knowledge and Skills Test scores are given. Secondly, the results of descriptive statistics of Probability Achievement Test scores are given.

4.1.1. The Results of Descriptive Statistics of PKT Scores

In this part, students' scores on Pre-requisite Knowledge and Skills Test (see Appendix A) will be examined. All questions were evaluated as 1 if the answer was correct and as 0 when the answer was wrong or it was blank.

The following table shows the students' scores on sets topic on pre-requisite knowledge and skills test.

Table 4.1. Students' Scores on Sets Topic on PKT

Questions	Wrong Answer		Correct Answer	
	n	%	n	%
1	1	8	11	92

Table 4.1. (continued)

Questions		Wrong Answer		Correct Answer	
		n	%	n	%
2		6	50	6	50
3		3	25	9	75
4		2	17	10	83
Total	f/%	11	23	36	75
	Out of		4		
	A.Mean		3.00		
	SD		1.206		

The questions 1, 2, 3 and 4 are related to sets topic. As seen in Table 4.1, 92% of students gave correct answers to 1st question, half of the students gave correct answers to 2nd question, 75% of students gave correct answers to 3rd question and 83% of students gave correct answers to 4th question related to sets topic. Moreover, as it is also seen in Table 4.1, 75% of students gave correct answers to questions related to sets topic. Also, arithmetic mean of "Sets scores" (SET) was high out of 4 points. (M_{SET}= 3.00; SD_{SET}=1.206)

The following table shows the students' scores on fractions topic on pre-requisite knowledge and skills test.

Table 4.2. Students' Scores on Fractions Topic on PKT

Questions	Wrong Answer		Correct Answer	
	n	%	n	%
5	2	17	10	83
6a	3	25	9	75
6b	3	25	9	75

Table 4.2. (continued)

Questions		Wrong Answer		Correct Answer	
		n	%	n	%
7a		1	8	11	92
7b		1	8	11	92
8a		2	17	10	83
8b		1	8	11	92
8c		0	0	12	100
8d		0	0	12	100
	f/%	13	12	95	88
Total	Out of		9		
	A.Mean		8.00		
	SD		1.595		

The questions 5, 6, 7a, 7b, 8a, 8b, 8c and 8d are related to fractions topic. The 5th question is related to modeling a fraction. As seen in Table 4.2, 17% of students gave wrong answer to this question. The questions 6a and 6b are related to arranging fractions and % 75 of students gave correct answers to these questions. The questions 7a and 7 b are related to numerator and denominator of a fraction and % 92 of the students gave correct answers to these questions. The questions 8a, 8c and 8d are related to subtraction and addition of fractions and 83% of students gave correct answers to the question 8a and all of the students gave correct answer to the questions 8c and 8d. In addition, 92% of the students gave correct answer to the question related to multiplication of fractions. Moreover, as it is also seen in Table 4.2, 88% of students gave correct answers to questions related to fractions topic. Also, arithmetic mean of "Fractions scores" (F) was high out of 9 points (M_F =8.00; SD_F = 1.595).

The following table shows the students' scores on decimals topic on pre-requisite knowledge and skills test.

Table 4.3. Students' Scores on Decimals Topic on PKT

Questions		Wrong Answer		Correct Answer	
		n	%	n	%
8e		1	8	11	92
8f		2	17	10	83
8g		6	50	6	50
8h		2	17	10	83
8i		6	50	6	50
Total	f/%	17	28	43	72
	Out of		5		
	A.Mean		3.58		
	SD		1.729		

The questions 8e, 8f, 8g, 8h and 8i are related to decimals topic. As seen in Table 4.3, 92% of students gave correct answer to the question related to addition of decimals and 83% of students gave correct answer to the question related to subtraction of decimals. The questions 8g, 8h and 8i are related to multiplication of decimals and half of the students gave correct answers the question 8g, most of the students gave correct answers to the question 8h and lastly half of the students gave correct answers to the question 8i. Moreover, as it is also seen in Table 4.3, 72% of students gave correct answers to questions related to decimals topic. Also, arithmetic mean of "Decimals scores" (D) was high out of 5 points ($M_D = 3.58$; $SD_D = 1.729$).

The following table shows the students' scores on data topic on pre-requsite knowledge and skills test.

Table 4.4. Students' Scores on Data Topic on PKT

Questions		Wrong Answer		Correct Answer	
		n	%	n	%
9		0	0	12	100
10		0	0	12	100
11		5	42	7	58
Total	f/%	5	14	31	86
	Out of		3		
	A.Mean	2.58			
	SD	1.372			

The questions 9, 10 and 11 are related to data topic. The question 9 is related to interpretation of table. As it is seen on Table 4.4, all of the students gave correct answers to this question. The question 11 is related to percentage and 58% of students gave correct answers to the question the question 11. In addition, the question 10 is related to probability of a simple event and all of the students gave correct answers to this question. Moreover, as it is also seen in Table 4.4, 86% of students gave correct answers to questions related to decimals topic. Also, arithmetic mean of "Data scores" (D) was high out of 3 points (M_D =2.58; SD_D =1.372).

Generally, students had difficulties in questions related to determining intersection of two sets, ordering fractions and multiplying decimals on Pre-requisite Knowledge and Skills Test. Only half of the students could solve these questions. Before the treatment, students received instruction related to these topics.

4.1.2. Results of Descriptive Statistics of PAT Scores

In this part results of descriptive statistics of PAT will be examined in two parts. In the first part, results of PAT scores will be examined with respect to each question and sub-categories. In the second part, total scores of PAT will be examined.

4.1.2.1. Results of Descriptive Statistics with respect to Each Question and Sub-Categories in PAT

In this part, students' scores for three time period (pre-intervention, post-intervention and follow-up) on Probability Achievement Test will be examined. The answers of questions 1, 2, 3, 4, 5, 6, 13 and 15 were evaluated as 1 when the answer was correct and as 0 when the answer was wrong or it was blank. The answers of questions 7, 8, 9, 10, 11, 12 and 14 questions were evaluated as 0 when the answer was wrong or it was blank, as 1 when the answer was correct but there was no explanation or there was an explanation but answer was wrong , as 2 when the answer was totally correct.

The following table demonstrates the students' scores on basic concepts of probability topic on probability achievement test.

Table 4.5. Students' Scores on Basic Concepts of Probability on PAT

Ques	f/ %	OS	Pre-Intervention			Post-Intervention			Follow-up		
			0	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
1	**f**	1	1	11		1	11		1	11	
	%		8	92		8	92		8	92	
11	**f**	2	11	0	1	6	1	5	8	1	3
	%		92	0	8	50	8	42	67	8	25

Table 4.5. (continued)

Ques	f/ %	OS	Pre-Intervention			Post-Intervention			Follow-up		
			0	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
14	**f**	2	4	7	1	0	0	12	0	4	8
	%		33	58	8	0	0	100	0	33	67
15	**f**	1	12	0		7	5		9	3	
	%		100	0		58	42		75	25	
Total	**f**	6	28	18	2	14	17	17	18	19	11
	%		58	38	4	29	35	35	38	39	23
	Mean		1.83			4.25			3.41		
	SD		1.029			1.288			1.378		

The questions 1, 11,14 and 15 are related to basic concepts of probability.

Question 1: The letters composing the word "ARKADAŞLIK" are written on papers and put it in a box. What is the probability of choosing a paper written the letter "A" on it?

As seen in Table 4.5, 8% of the students had 0 point and 92% of students had 1 point in pre-intervention in first question. This situation did not change in post-intervention and follow-up. Students' score on three time period were same in question 1.

Question 11: In the table below, there are points that 26 students had in mathematics. The names of students who had 5 point were written on table tennis balls, the other students' names were written on papers. These balls and papers were put in a bag and a drawing was made without looking. It was said that the probability of choosing the name of student whose point was 5 was 9/26. Explain if this result is correct or not with its reasons.

Distribution of Students According to Points

Point	*3*	*4*	*5*
Number of Students	*7*	*10*	*9*

As seen in Table 4.5, 92% of the students had 0 point and 8% of students had 2 points in pre-intervention in 11th question above. In post-intervention, the numbers of students who had 1 and 2 points increased. In follow-up, there was a slight decrease in correct answers compared to post-intervention. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in 11th question.

Question 14: Özlem threw a fair coin 4 times and all the results were tails. When the Özlem throws the coin for the 5th time, explain which one of the answers correct below, with its reasons.

a. The probability of tails is equal to the probability of heads.

b. The probability of tails is lower than the probability of heads.

c. The probability of tails is higher than the probability of heads.

As seen in table 4.5, 33% of the students had 0 point, 58% of students had 1 point and 8% of students had 2 points in 14th question in pre-intervention. In post-intervention, all of the students had 2 points in 14th question. In other words, there was an increase in totally correct answers. In follow-up, the numbers of students who had 1 point increased, number of students who had 2 points decreased but the numbers of incorrect answers did not change in 14th question. The numbers of correct answers in follow-up were more than those in pre-intervention in 14th question.

Question 15: By spinning the spinner, fractions are written which are composed of one digit numbers. What is the probability of value of fraction is higher than ½, when the spinner is spinned two times one after another?

As seen in Table 4.5, all of the students had 0 point in 15^{th} question, in pre-intervention. In post-intervention, there was an increase in correct answers. In follow-up, the numbers of correct answers decreased in 15^{th} question. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in 15^{th} question.

Consequently, the numbers of correct answers increased in post-intervention compared to pre-intervention in questions related to basic concepts of probability. Moreover, in follow-up, the numbers of correct answers slightly decreased in questions related to basic concepts of probability except for question 1. As it is also seen in Table 4.5, totally, 58% of the students had 0 point, 38% of students had 1 point and 4% of students had 2 points in pre-intervention in questions related to basic concepts of probability. In post-intervention, there was an increase in correct answers of 2 points in questions related to basic concepts of probability. In follow-up, there was a slight decrease in correct answers compared to post-intervention in questions related to basic concepts of probability. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in questions related to basic concepts of probability. As it is also seen in Table 4.5, arithmetic mean of " Basic Concepts of Probability scores" (BCP) in post-intervention was higher than arithmetic mean of BCP in pre-intervention and was slightly higher than arithmetic mean of BCP in follow-up ($M_{postBCP} = 4.25$; $SD_{postBCP} = 1.288$; $M_{PreBCP} = 1.83$; $SD_{PreBCP} = 1.029$; $M_{followupBCP} = 3.41$; $SD_{followupBCP} = 1.378$).

The following table shows the students' scores on permutation and combination topic on probability achievement test.

Table 4.6. Students' Scores on Permutation and Combination on PAT

Ques	f/ %	OS	Pre-Intervention			Post-Intervention			Follow-up		
			0	**1**	**2**	**0**	**1**	**2**	**0**	**1**	**2**
5	**f**	1	12	0		10	2		5	7	
	%		100	0		83	17		42	58	
12a	**f**	2	9	3	0	1	5	6	2	5	5
	%		75	25	0	8	42	50	17	42	42
12b	**f**	2	12	0	0	1	5	6	3	5	4
	%		100	0	0	8	42	50	25	42	33
Total	**f**	5	33	3	0	12	12	12	10	17	9
	%		92	8	0	33	33	33	28	47	25
	A.Mean		0.25			3.00			2.91		
	SD		0.452			1.414			1.831		

The questions 5, 12a and 12b are related to combination and permutation topic.

Question 5: How many different ways can 2 doctors, 3 nurses and 5 caregivers be chosen from 5 doctors, 6 nurses and 8 caregivers?

As seen in Table 4.5, all of the students had 0 point in 5th question, in pre-intervention. In post-intervention, there was a slight increase in correct answers. However, the correct answers in follow-up were more than those in pre-intervention and in post-intervention.

Question 12: Explain which subjects you would use with reasons while you are solving problems below.

a. A company will recruit two people, one of them is accountant, another is sales person. 18 people appealed for these two jobs. How many different ways that positions can be filled?

b. A company will recruit two landscape architects. These two people will do same job and earn same amount of money. 18 people appealed for these jobs. How many different ways that positions can be filled?

As seen in Table 4.6, 75% of the students had 0 point and 25 % of students had 1 point in the question 12a in pre-intervention. In post-intervention, the numbers of students who had 1 and 2 points increased. In follow-up, there was a slight decrease in the numbers of correct answers point of which is 2 in the question 12a. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in the question 12a.

As seen in Table 4.6, all of the students had 0 point in question 12b, in pre-intervention. In post-intervention, the numbers of students who had 1 and 2 points increased. In follow-up, there was a slight increase in incorrect answers in the question 12b. Nevertheless, the correct answers in follow-up were more than those in pre-intervention in the question 12b.

Consequently, the numbers of correct answers increased in post-intervention compared to pre-intervention in questions related to permutation and combination. Moreover, in follow-up, the numbers of correct answers slightly decreased. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in questions related to permutation and combination. These results were also consistent with the total results. As it is also seen in Table 4.6, totally, 92% of the students had 0 point, 8% of students had 1 point and none of the students had 2 points in pre-intervention in questions related to permutation and combination. In post-intervention, there was an increase in correct answers. In follow-up, there was a slight decrease in correct answers of two points compared to post-intervention. As it is also seen in Table 4.5, arithmetic mean of “Permutation and Combination scores” (PC) in post-intervention was higher than arithmetic mean of PC in pre-intervention and was also slightly higher than arithmetic mean of PC in follow-up (M_{PostPC} = 3.00; SD_{PostPC} = 1.414; M_{PrePC} = 0.25; SD_{PrePC} = 0.452; $M_{followupPC}$ = 2.91; $SD_{followupPC}$ = 1.831).

The following table shows the students' scores on types of probability topic on probability achievement test.

Table 4.7. Students' Scores on Types of Probability on PAT

Ques	**f/ %**	**OS**	**Pre-Intervention**			**Post-Intervention**			**Follow-up**		
			0	1	**2**	**0**	**1**	**2**	**0**	**1**	**2**
6	**f**	1	9	3		4	8		5	7	
	%		75	25		33	67		42	58	
8	**f**	2	7	3	2	1	1	10	0	3	9
	%		58	25	17	8	8	83	0	25	75
9	**f**	2	9	3	0	2	2	8	4	1	7
	%		75	25	0	17	17	67	33	8	58
10	**f**	2	10	2	0	9	0	3	6	0	6
	%		83	17	0	75	0	25	50	0	50
Total	**f**	7	35	11	2	16	11	21	15	11	22
	%		73	23	4	33	23	44	31	23	46
	A.Mean		1.25			4.41			4.58		
	SD		1.712			1.378			1.781		

The questions 6, 8, 9 and 10 are related to types of probability.

Question 6: A parachuter will abandon the aircraft and fall on a region on a day when the weather conditions are proper. The dimensions of region are 80 m and 40 m. At the center of the rectangular region, there is a circle region. The radius of this circle is 20 m. What is the probability of parachuter landing on the circle region? *(π=3)*

The 6th question above is related to geometric probability. As seen in Table 4.7, 75% of the students had 0 point and 25 % of students had 1 point in pre-intervention in 6th question. In post-intervention, more than half of the students had 1 point. In other words, there was an increase in correct answers in 6th question. In follow-up, similarly more than half of the students had 1 point in 6th question. Namely, there was a slight decrease in correct answers. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in 6th question.

Question 8: At the weekend there will be Galatasaray and Başiktaş match. According to Berk, Galatasaray's probability of winning the match is 70%, According to Sibel, Beşiktaş's probability of winning the match is 90%. Write the type of probability in this explanation with its reasons.

The 8th question is related to subjective probability. As seen in Table, 4.7, 58% of the students had 0 point, 25% of students had 1 point and 17% of students had 2 points in pre-intervention in 8th question. In post-intervention, most of the students had 2 points, and a few students had 1 point and 0 point in 8th question. Namely, the numbers of students who had 2 points increased. In follow-up, there was a slight decrease in correct answers point of which is 2. However, the numbers of incorrect answers decreased in 8th question. All of the students had 1 and 2 points in follow-up in 8th question.

Question 9: A computer program is written related to coin experiment. The probability of heads is calculated as 452/1000=0.452 at 1000th throwing and 48962/100000=0.48962 at 100000th throwing. Explain the relationship between number of throwing and values otbained, with its reasons.

The 9th question above is related to theoretical and experimental probabilities. As seen in Table 4.7, 75% of the students had 0 point 25% of students had 1 point in pre-intervention in 9th question. In post-intervention, the numbers of students who had 1 and 2 points increased in post-intervention in 9th question. In follow-up, there was a slight decrease in the numbers of students who had 1 and 2 points. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in 9th question.

Question 10: "There are not enough trees in Hüzünlü Village and methods of soil cultivation are wrong. However, in Yeşil Village, there are not negativities like in Hüzünlü Village." Write the probability type in this explanation with its reasons.

The 10th question above is related to experimental probability. As seen in Table 4.7, 83% of the students had 0 point and 17% of students had 1 point in pre-intervention in 10th question. In post-intervention, there was a slight increase in correct answers. However, there was an increase in correct answers in follow-up. The numbers of correct answers in follow-up were more than those in pre-intervention and post-intervention in 10th question.

Consequently, the numbers of correct answers increased in post-intervention compared to pre-intervention in questions related to types of probability. In follow-up, the numbers of incorrect answers decreased in questions 8 and 10 in questions related to types of probability. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in questions related to types of probability. As it is also seen in Table 4.7, totally, 73% of the students had 0 point, 23% of students had 1 point and 4% of students had 2 points in pre-intervention in questions related to types of probability. In post-intervention, there was an increase in correct answers. In follow-up, there was a slight increase in correct answers compared to post-intervention in questions related to types of probability. Furthermore, as it is seen in Table 4.6, arithmetic mean of "Types of Probability scores" (TP) in follow-up was higher than arithmetic mean of TP in pre-intervention and was slightly higher than arithmetic mean of TP in post-intervention ($M_{PostTP} = 4.41$; $SD_{PostTP} = 1.378$; $M_{PreTP} = 1.25$; $SD_{PreTP} = 1.712$; $M_{followupTP} = 4.58$; $SD_{followupTP} = 1.781$).

The following table shows the students' scores on dependent and independent event on probability achievement test.

Table 4.8. Students' Scores on Dependent and Independent Events on PAT

Ques	f/%	OS	Pre-Intervention			Post-Intervention			Follow-up		
			0	1	2	0	1	2	0	1	2
2	f	1	5	7		1	12		3	9	
	%		42	58		0	100		25	75	
3	f	1	9	3		6	6		2	10	
	%		75	25		50	50		17	83	
4	f	1	6	6		4	8		6	6	
	%		50	50		33	67		50	50	
7a	f	2	12	0	0	3	4	5	6	3	3
	%		100	0	0	25	33	42	50	25	25
7b	f	2	12	0	0	2	5	5	6	4	2
	%		100	0	0	17	42	42	50	33	17
13	f	1	8	4		3	9		4	8	
	%		67	33		25	75		33	67	
Total	f	8	52	20	0	19	44	10	27	40	5
	%		72	28	0	26	61	14	38	56	7
	A.Mean		1.66			5.33			4.16		
	SD		1.370			1.922			2.081		

The questions 2, 3, 4, 7a, 7b and 13 are related to dependent and independent events.

Question 2: Pelin took 2 hairpins from a box in which there were 15 pink, 12 red, 7 purple hairpins. She did not put these two hairpins back to the box. What is the probability of being purple of the first hairpin and being pink of the second hairpin?

The 2^{nd} question above is related to calculating probability of dependent events. As seen in Table 4.8, 42% of the students had 0 point and 58% of students had 1 point in pre-intervention in 2^{nd} question. In post-intervention, all of the students had 1 point in 2^{nd} question. Namely, there was an increase in correct answers. In follow-up, there was a decrease in numbers of correct answers. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in 2^{nd} question.

Question 3: What is the probability of being one girl and one boy from two children of a family?

The 3^{rd} question above is related to calculating probability of independent events. As seen in Table 4.8, 75% of the students had 0 point and 25% of students had 1 point in pre-intervention in 3^{rd} question. In post-intervention, half of students had 0 point and 1 point in 3^{rd} question. Namely, there was an increase in numbers of correct answers in 3^{rd} question. However, most of the students had 1 point in follow up in 3^{rd} question. Namely, there was an increase in numbers of correct answers. The numbers of correct answers in follow-up were more than those in pre-intervention and post-intervention in 3^{rd} question.

Question 4: A master student will choose randomly 3 people from 8 men and 16 women to collect data. What is the probability of being women of these three people?

The 4th question is related to calculating the probability of dependent events. As seen in table 4.8, half of the students had 1 point and half of the students had 0 point in pre-intervention in 4th question. In post intervention, more than half of the students had 1 point in 4th question. Namely, there was an increase in correct answers in 4th question. In follow-up, students had same scores in an in pre-intervention in 4th question. In other words, there was a decrease in correct answers. Moreover, students' scores in follow-up were equal to those in pre-intervention in 4th question.

Question 7: Write the kinds of events below with its reasons.

a. A county was made a city through a decision taken in Turkey. In this new city, new number plats will be given to vehicles. The number plate will be composed of code of city, two letters and 3 numbers. While composing two letters, same letters will be used more than one.

b. Ayhan uses a 3 digit password on his computer to avoid use of someone else. This password is composed of different numbers between 0 and 9(incl). What is the probability of predicting Ayhan's password correctly?

The question 7a above is related to expressing independent events. As seen in Table 4.8, all of the students had 0 point in pre-intervention in question 7a. In post-intervention, almost half of the students had 2 points in question 7a. In other words, there was an increase in correct answers. In follow-up, half of the students had 1 point and 2 points in question 7a. In other words, there was an increase in incorrect answers in follow-up.

Nevertheless, the correct answers in follow-up were more than those in pre-intervention in question 7a.

The question 7b above is also related to expressing dependent events. As seen in Table 4.8, all of the students had 0 point in pre-intervention in question 7b. In post-intervention, there was an increase in correct answers in question 7b. In follow-up, there was an increase in numbers of incorrect answers in follow-up. Nevertheless, the correct answers in follow-up were more than those in pre-intervention in question 7b.

Question 13: Aslı's probability of solving a problem is 0.8. Kerem' probability of solving a problem is 0.7. What is the probability of that problem solved by Aslı and Kerem at the same time?

The 13^{th} question above is related to calculating probability of independent events. As seen in Table 4.8, 67% of the students had 0 point and 33% of students had 1 point in pre-intervention in 13^{th} question. In post-intervention, most of the students had 1 point in 13^{th} question. Namely, there was an increase in correct answers. In follow-up, more than half of the students had 1 point in 13^{th} question. In other words, there was a slight decrease in correct answers in 13^{th} question. Nevertheless, the correct answers in follow-up were more than those in pre-intervention in 13^{th} question.

Consequently, the numbers of correct answers increased in post-intervention compared to pre-intervention in questions related to dependent and independent events. In follow-up, the numbers of incorrect answers decreased in 2 out of 4 questions. Nevertheless, the numbers of correct answers in follow-up were more than those in pre-intervention in questions related to dependent and independent events except for question 4. As it is also seen in Table 4.8, totally, 72% of the students had 0 point, 28% of students had 1 point and none of the students had 2 points in pre-intervention in questions related to dependent and independent events. In post-intervention, there was an increase in correct answers in post-intervention. In follow-up, there was a slight decrease in numbers of correct answers compared to post-intervention in questions related to dependent and independent events. Furthermore, as it is seen in Table 4.8, arithmetic mean of "Dependent and Independent Events scores" (DIE) in post-intervention was higher than arithmetic mean of DIE in pre-intervention and was slightly higher than arithmetic mean of DIE in follow-up ($M_{PostDIE} = 5.33$; $SD_{PostDIE} = 1.922$; $M_{PreDIE} = 1.66$; $SD_{PreDIE} = 1.370$; $M_{followupDIE} = 4.16$; $SD_{followupDIE} = 2.081$).

To sum up, post-intervention and follow-up scores were higher than pre-intervention scores in all questions except for questions 1 and 4. The follow up scores were equal to pre-intervention scores in question 4. The scores of question 1 stated constant across three time periods. The numbers of the correct answers in most of the questions increased in post-intervention compared to pre-intervention. Moreover, the numbers of correct answers in follow-up were slightly lower than in post-intervention. However, in some questions, the follow up scores were higher than post-intervention scores. It can be concluded that students' achievement increased in most of the questions. When the total scores were analyzed, it was seen that arithmetic means of "Basic Concepts of Probability scores", "Dependent and Independent Events scores", "Permutation and Combination scores" in post-intervention were higher than in pre-intervention However, arithmetic mean of "Types of Probability scores" in follow-up was higher than in pre-intervention and was slightly higher than in post-intervention.

4.1.2.2. Results of Descriptive Statistics of Total PAT Scores

Table 4.9 shows the arithmetic means and standard deviations, medians the minimum and maximum values of the PAT scores across three time periods such pre-intervention (preint), post-intervention (postint) and follow-up (followup).

Table 4.9. Arithmetic Means, Standard Deviations, Medians, Maximum and Minimum Values of PAT across Three Time Periods

Time	Arithmetic Mean	SD	Min	Max	Median
Pre-Intervention	5.00	3.219	2.00	11.00	4.00
Post-Intervention	17.00	3.668	11.00	23.00	17.50
Follow-up	15.00	4.981	6.00	22.00	15.50

As it is seen in Table 4.9, arithmetic mean of PAT scores at post-intervention is higher than arithmetic means of PAT scores in pre-intervention and in follow-up ($M_{postPAT} = 17.00$; $SD_{postPAT} = 3.668$; $M_{prePAT} = 5.00$; $SD_{prePAT} = 3.219$; $M_{followup\_PAT} = 15.00$; $SD_{followup\_PAT} = 4.981$). However, arithmetic mean of PAT scores in follow-up is lower than arithmetic mean score of PAT scores in post-intervention but higher than pre-intervention. In addition, arithmetic means of PAT scores in the post–intervention and follow up were close to each other. Furthermore, there was slightly decrease in the follow-up test score. However, the minimum value of the post-intervention PAT scores is higher than the minimum values of follow-up PAT scores and pre-intervention PAT Scores ($Min_{postPAT} = 11.00$; $Min_{followup\_PAT} = 6$; $Min_{prePAT} = 2$). The maximum value of the post–

intervention PAT scores is higher than maximum values of follow–up and pre-intervention PAT scores (Max $_{postPAT}$ = 23; Max $_{followup\_PAT}$ = 22; Max $_{prePAT}$ = 11).

The following figure shows the arithmetic mean scores of PAT across three time periods.

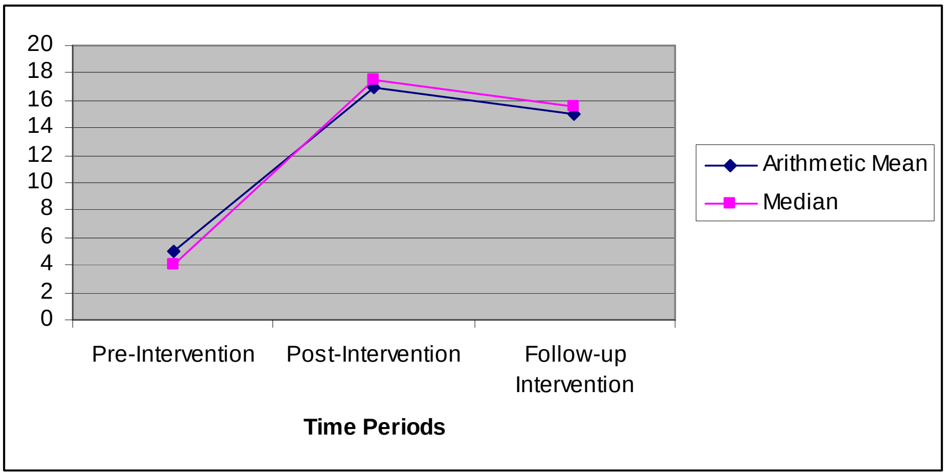

Figure 4.1. Arithmetic Means and Medians of PAT Scores across Three Time Periods

As seen in Figure 4.1 there was an increase in probability achievement from pre-intervention through post and follow-up interventions.

Table 4.10 shows the arithmetic means and standard deviations, medians the minimum and maximum values of the PAS scores across three time periods such pre-intervention (preint), post-intervention (postint) and follow-up (followup).

Table 4.10. Arithmetic Means, Standard Deviations, Medians, Maximum and Minimum Values of PAS across Three Time Periods

Time	Arithmetic Mean	SD	Min	Max	Median
Pre-Intervention	129.17	27.663	80	160	131.50
Post-Intervention	137.17	20.810	91	166	140.00
Follow-up	135.42	20.991	90	166	133.50

As it is seen in Table 4.10, arithmetic mean of PAS scores at post-intervention is slightly higher than arithmetic means of PAS scores in pre-intervention and follow-up ($M_{postPAS} = 137.17$; $SD_{postPAS} = 20.810$; $M_{prePAS} = 129.17$; $SD_{prePAS} = 27.663$; $M_{followup\_PAS} = 135.42$; $SD_{followup\_PAS} = 20.991$). Arithmetic mean of PAS scores in follow-up is slightly lower than arithmetic mean score of PAS scores in post-intervention but slightly higher than pre-intervention. In addition, arithmetic means of PAS scores in the post–intervention and follow up were close to each other. The minimum value of the post-intervention PAS scores is slightly higher than the minimum values of follow-up PAS scores and pre-intervention PAS Scores ($Min_{postPAS} = 91$; $Min_{prePAS} = 80$; $Min_{followup\_PAS} = 90$). The maximum value of the post–intervention PAS scores is equal to maximum values of follow–up. The maximum values of post-intervention scores is slightly higher than maximum values of pre-intervention PAS scores ($Max_{postPAS} = 166$; $Max_{followup\_PAS} = 166$; $Max_{prePAS} = 160$).

The following figure shows the arithmetic mean scores of PAS across time periods.

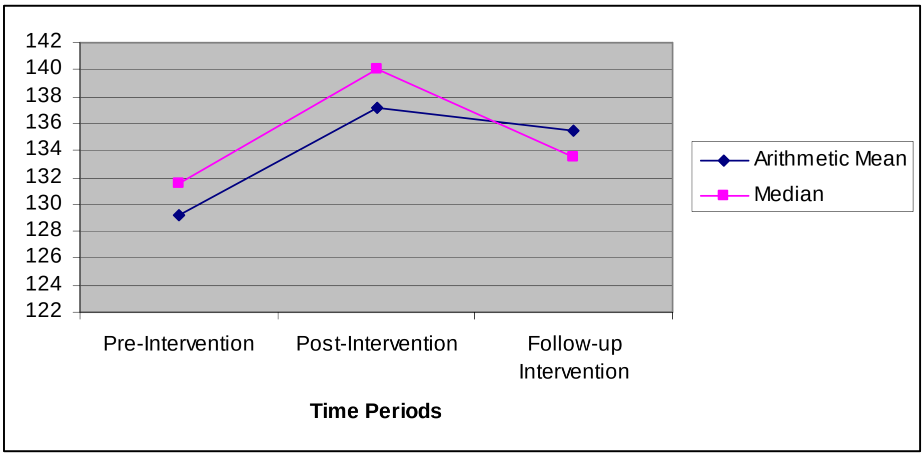

Figure 4.2. Arithmetic Means and Medians of PAS Scores across Three Time Periods

As seen in Figure 4.2, there was a slight increase in mean score of attitude toward probability from pre-intervention through post and follow-up interventions. However, this increase is not statistically significant.

4.2. The Results of Inferential Statistics

In this section, the sub-problems of the study will be examined by means of their associated hypotheses which are in the null form and since the data were obtained in different three time periods, hypotheses are tested at a significance level of 0.017. This level was determined as 0.05 was divided by 3 as stated by Colman and Pulford (2006) because the data were obtained 3 different time periods.

4.2.1. The Results of the First Main Problem

The first main problem is "What is the effect of instruction with concrete models on 8^{th} grade students' probability achievement and attitudes toward probability?"

The following hypothesis of the first sub-problem of the first main problem is “There is no statistically significant change in 8^{th} grade students’ probability achievement across three time periods (pre-intervention, post-intervention, and follow-up).” It is tested tested by using Friedman test at the level of significance 0.017. Because, the data was obtained in different three time preiods. After testing the hypothesis by using the Friedman test, it is found that there is a statistically significant change in students’ probability achievement scores across pre-intervention, post-intervention, and follow-up (Chi-Square= 18.957; df=2; Asymp.Sig= 0.000; $p<0.017$). In other words, there are significant mean rank differences in the PAT scores across the three time periods. The mean rank of PrePAT, PostPAT and Followup_PAT scores are 1.00, 2.58 and 2.42 respectively. To determine which mean ranks of the test scores cause this difference, the Wilcoxon test is used. The results are given in Table 4.11.

Table 4.11. Wilcoxon Signed Rank Test Results for PAT Scores of Students

Tests	Rank Types	n	Mean Rank	Sum of Ranks	Sig.
PostPAT-PrePAT	Negative	0	0.00	0.00	0.002
	Positive	12	6.50	78.00	
	Ties	0			
Followup_PAT-PrePAT	Negative	0	0.00	0.00	0.002
	Positive	12	6.50	78.00	
	Ties	0			

Table 4.11. (continued)

Tests	Rank Types	n	Mean Rank	Sum of Ranks	Sig.
Followup_PAT-PostPAT	Negative	6	6.00	36.00	0.384
	Positive	4	4.75	19.00	
	Ties	2			

As seen in Table 4.11 shows that there is a statistically significant difference between mean ranks of PostPAT scores and the PrePAT scores ($p < 0.017$). Moreover, the mean rank of PostPAT scores is statistically significantly greater than mean rank of PrePAT scores (Mean $\text{Rank}_{\text{PostPAT}}$= 2.58; Mean $\text{Rank}_{\text{PrePAT}}$=1.00). Another finding is that there is a statistically significant difference between mean ranks of Followup_PAT scores and the PrePAT scores ($p < 0.017$). The mean rank of Followup_PAT- scores is statistically significantly greater than mean rank of PrePAT scores (Mean $\text{Rank}_{\text{Followup\_PAT}}$ = 2.42; Mean $\text{Rank}_{\text{PrePAT}}$=1.00). The last finding is that there is no statistically significant difference between mean ranks of Followup_PAT scores and the PostPAT scores ($p > 0.017$). However, the mean rank of Followup_PAT scores is only less than mean rank of PostPAT scores (Mean Rank $\text{Rank}_{\text{Followup\_PAT}}$ = 2.42; Mean $\text{Rank}_{\text{PostPAT}}$=2.58).

The following hypothesis is stated for the second sub-problem of the first main problem as "There is no statistically significant change in 8^{th} grade students' attitudes toward probability across three time periods (pre-intervention, post-intervention, and follow-up)." It is tested by using Friedman test at the level of significance 0.017. After testing the hypothesis by using the Friedman test, it is found that there is no statistically significant change in students' attitude scores across pre-intervention, post-intervention, and follow-up (Chi-Square= 2.426; df=2; Asymp.Sig= 0.297; $p>0.017$). In other words, there are no significant mean rank differences in the PAS scores across the three time periods. The mean ranks of Pre-PAS, Post-PAS and Followup_PAS scores are 1.67, 2.29 and 2.04 respectively.

4.3. The Results of the Second Main Problem

The second main problem is "What are the eighth grade students' views related to instruction with concrete models?"

Eleven students were interviewed to test the second main problem. Two themes were determined according to codings of two coders of students' answers to the interview questions. They were affective domain and cognitive domain. While the affective domain has one main category which is called as emotion, the cognitive domain has two main categories as easiness and usefulness.

The main category of affective domain is called as "emotion". All of students' answers took part in this main category. It has 4 sub-categories: enjoyment, funny, interest and not boring.The results are given in Table 4.12.

Table 4.12. Results of Main Category of Affective Domain as Emotion

Main Category	Sub-category	n(%)	Student
Emotion	Enjoyment	11(100)	s1, s2, s3, s4, s5, s6, s7, s8, s9, s10, s11

Table 4.12. (continued)

Main Category	Sub-category	n(%)	Student
Emotion	Funny	10(91)	s1, s2, s4, s5, s6, s7, s8, s9, s10, s11
	Interest	1(9)	s3
	Not boring	2(18)	s8, s11

As seen in Table 4.12, all of students thought that they enjoyed through the use of concrete models. Students' sample explanations are as follow:

S*1: We used various concrete models. Obviously, I enjoyed.*

S4: I enjoyed so much. I mean, doing activities with concrete models were very enjoyable.

Student 5: I enjoyed so much. I felt myself good.

Also, as seen in Table 4.12, 91% of students said that they found the concrete model activities were funny. For example, student 10 expressed his views as follow:

S10: They were quite beautiful and funny.

S2: Probability lessons with concrete models were funnier than lessons that we received last year.

In addition, 9 % of students said that his interest toward lesson increased. Student 3 expressed his views as follow:

S3: My interest toward the subject has increased.

Moreover, 18% of students stated that they did no get bored during activities with concrete models. Sample student explanations are as follow:

S11: Concrete models were very enjoyable. I had a good time and did not get bored in the lessons.

S8: I had fun. I did not get bored because it was different from ordinary probability lessons.

The second main category labeled as "easiness" was emerged related to cognitive domain according to answers of 8 students' to the interview questions. It has three sub-categories: understanding, problem solving and retention. The table related with easiness is shown below.

Table 4.13. Results of Main Category of Cognitive Domain as Easiness

Main Category	Sub-category	n(%)	Student
Easiness	Understanding	6(54)	s1, s3, s5, s6, s9, s10
	Problem Solving	7(64)	s1, s2, s3, s4, s5, s6, s10
	Retention	3(27)	s1, s2, s9

As seen in Table 4.13, 54% of students said that they could understand the probability easily by the use of concrete models. Some expressions of students are as follow:

S3: I could understand the combination subject through concrete models easily.

S9: They provided me to understand a set of concepts related to probability easily.

Also, 64% of students said that they could solve probability problems easily. Some explanations of students are as follow:

S3: It provided me to solve the problems easily.

S10: In the past, I had difficulty in solving probability problems, now I can solve them easily.

In addition, 27% of students stated that they could remember the probability concepts easily because concrete models became permanent in their minds. For example, student 1 expressed his opinion as follows:

S1: The activities with concrete models came to my mind easily while I was solving problems so I could solve them easily.

S2: They helped me remember the subject in the exam. Firstly I remembered the activity then I remembered the result of it in the exam. So, they helped me remembering the subjects easily.

The third main category "usefulness" was emerged related to cognitive domain according to answers of 10 students to interview questions. It has 4 sub-categories which were "examination", "achievement", "learning" and "understanding". The results are given in Table 4.14.

Table 4.14. Results of Main Category of Cognitive Domain as Usefulness

Main Category	**Sub-category**	**n(%)**	**Student**
	Examination	2(18)	s1, s6
	Achievement	3(27)	s1, s11, s5
Usefulness	Understanding	4(36)	s2, s4, s5, s8
	Learning	3(27)	s7, s9, s10,

As seen in table 4.14, 18% of students stated that they found concrete models and activities useful in that they would meet concrete models in examinations. Student 6 expressed his related opinion as follow:

Student 6: I think they were useful for me. They would be used in SBS, in this sense they were useful for me.

As seen in table 4.14, 27% of students stated that concrete models and activities were useful for them in terms of increasing their achievement. Students' sample explanations are stated below.

Student 1: I think that concrete models and activities were very useful for me. They were so beautiful and contributed to my success.

Student5: I remember we learnt the probability last year. We did not do activities and we did not use concrete models. I was not successful in probability as much as now I am. It increased my achievement. They were useful for me.

Student11: They provided benefits for us. They were useful. My achievement increased. I had point 5 from the probability exam after the activities.

As seen in Table 4.14, 36% of students stated that they found concrete model activities useful in terms of understanding the probability. Students' sample explanations are stated below:

Student 5: I think concrete models were beneficial. They assisted me understanding the probability subject.

Student 4: We hold the concrete models and do activities with them by ourselves. So, I better understand the subject. I think this is so useful for us. In the past, teacher was used to tell the subject on the board, *now with concrete models and activities, I better understood the subject.*

Student 8: Concrete models were beneficial for me. I could understand the probability by means of them. I could not understand the probability before.

As seen in Table 4.14, 36% of students stated that they found cocnrete models and activities useful in terms of learning probability. Students' sample explanations are given below:

Student 7: I think they were educative for us. They were more useful for us and I think we learnt the subject well. It was better than only textbook instruction.

Student 9: Activities and concrete models were very useful for me. They provided me to learn the probability.

Consequently, most of the students had positive views about instruction with concrete models. Since they could solve problems related to probability, understand and remember the topic, most of the students had positive views about the effects of instruction with concrete models on their cognitive processes. Furthermore, most of them thought that the instruction had positive impacts on their affective processes in terms of developing positive attitudes toward concrete models instruction.

CHAPTER 5

DISCUSSION AND RECOMMENDATIONS

This chapter includes interpretation of results, conclusion of the study and recommendations for further studies. In the first part, discussion of findings is stated. The second part explains the conclusions and in the last part recommendations for further studies are given.

5.1. Discussion of Findings

In this section the findings related to students' probability achievement, their attitudes toward probability and their views about instruction with concrete models are discussed.

5.1.1. Discussion of Findings on Students' Probability Achievement

The present study had two main problems. The first main problem was the investigation of the effect of instruction with concrete models on eighth grade students' probability achievement and their attitudes toward probability. The second main problem was the investigation of the students' views about instruction with concrete models. It was found that there was a statistically significant change in students' probability achievement from pre-intervention through post-intervention, and from pre-intervention through follow-up in terms of the mean rank difference. However, there was no statistically significant change in students' probability achievement across post-intervention and follow-up in terms of the same criteria.

The results of the present study showed that there were statistically significant positive changes on the achievement from pre-intervention through post-intervention scores and from pre-intervention through follow-up scores after the instruction with concrete models. These results confirm the findings of research studies pointing out the effectiveness of concrete models on students' mathematics achievement (e.g. Parham, 1983; Cankoy, 1989; Leinebach & Raymand, 1996; Hinzman, 1997; Bayram, 2004; Daniel, 2007; Bayrak, 2008; Tutak, 2008; Sarı, 2010). In the study conducted by Sowell (1989), concrete models were found effective, depending on their use in long-term. The results of the present study also confirm the findings of Bayram (2004) who stated that instruction with concrete models was effective regardless of long-term or short-term use. Similarly, in the study conducted by Sarı (2010), it was found that the mean score of post-test were higher than the mean score of pre-test after the geometry instruction through concrete models. Moreover, Cankoy (1989) reported that concrete models are more effective in probability instruction when compared to traditional instruction. However, Taylor (2001) found that there was no significant mean difference between students who used concrete manipulatives and those who did not use concrete manipulatives in terms of probability achievement. In short, findings of previous studies suggest various results.

The difference between the findings of this study and the previous studies stems from the limitations. First of all, the design of the present study is weak when compared to the previous studies. Secondly, there was no control group. The third limitation is the data collection instruments which were applied as one group pre-test, post-test and retention test. Therefore, it is difficult to state that there was a direct effect of treatment on students' probability achievement. As a result, the reasons are not stated as certain reasons in this section. Instead, possible reasons of statistical significance changes across three time periods are stated.

The reason of statistical significance differences between pre-intervention and post-intervention scores and between pre-intervention and follow-up scores can be explained by the concrete models used in the present study. In this respect, Fennema (1973) emphasizes the importance of the use of concrete models as they make the abstract nature of mathematics understandable. Following Fennema, various concrete models for each sub-topic of probability and for each activity were prepared by the researcher as recommended. In addition, the concrete models chosen were suitable for both students' cognitive development and the subject of probability. Students manipulated with these concrete models on their own. They were encouraged to make a connection between concrete and abstract world of probability. Abstract issues were expected to become meaningful by using concrete models. Using these models, students were given the opportunity of applying real cases to the abstract issues of probability. Some of the students were also motivated and interested in using the concrete models.

Although Fennema (1972) found concrete models useful if only they were used in earlier grades, the findings of the present study support the results of Suydam and Higgins (1977) that concrete models were also beneficial in higher grades. There are also other researchers who support the use of concrete models at all grade levels (e.g. Driscoll, 1984, Harzhorn & Boren, 1990; Kober, 1991). In the present study, concrete models were found effective even though the participants were eighth grade students. Since probability is an abstract and difficult subject in mathematics, students might have understood the probability better through concrete models.

Moreover, beside the use of concrete models in the instruction of probability, an environment was created in which students could discuss with each other and teacher guided students with his leading questions in activities in the present study. Moreover, there was not simple activity. For each sub-topic, various activities were prepared to help students understand the topics easily. Furthermore, Suydam and Higgins (1977) report that students who receive activity based instruction succeed as well as or better than those who do not have this kind of instruction. In the present study, all of the students were involved actively in the teaching/learning process.

Furthermore, most of the activities were planned according to discovery learning method in the present study. This instructional method performed in the present study might have had an effect on statistically significant positive changes on the achievement from pre-intervention through post-intervention scores and from pre-intervention through follow-up scores. Senemoğlu (2005) points out that discovery learning method encourages students to wonder and maintains it until they find the answers. As it is pointed out by Bruner (1961), enabling students to participate in the process, discovery learning method might have developed students' intrinsic motivation and students might have learnt the topics by this means. In the present study, some of the students were motivated while they were trying to discover new knowledge. During the activities students questioned, discussed with each other, tried to discover the rules, made generalizations and applied these generalizations in solving the problems. Teacher's role was to increase their curiosity by asking leading questions.

Additionally, the increased results on the achievement test may be explained by students' existing adequate pre-requisite knowledge before the treatment. Since some researchers state that students have difficulty in learning probability because they have insufficient pre-requisite knowledge (e.g. Carpenter et al., 1981; Garfield & Ahlgren; 1988; Baron & Or-Bach; 1988), pre-requisite knowledge of the students participated in this study was measured by the pre-requisite knowledge and skills test and students' deficiencies were aimed to be removed before the application of the treatment.

5.1.2. Discussion of Findings on Students' Attitudes toward Probability

The second purpose of the present study was to examine the effect of instruction with concrete models on 8th grade students' attitudes toward probability. The present study revealed that there was no statistically significant change in students' attitudes toward probability over three time periods (pre-intervention, post-intervention and follow-up).

The literature is consisted of many research studies which were conducted to investigate the effect of instruction with concrete models on students' attitudes toward mathematics (e.g. Sowell, 1989; Bayram, 2004; Tutak, 2008). However, none of these research studies aimed to examine the effects of instruction with concrete models on students' attitudes toward probability. Sowell (1989) found that students' positive attitudes toward mathematics improved when they were given instruction by teachers who were experienced in the use of concrete models. Bayram (2004), on the other hand, found that there was no statistically significant mean difference between control and experimental groups with respect to attitudes toward geometry.

The results of the present study showed that there was no statistically significant change in eighth grade students' attitudes toward probability across three time periods. However, during the lessons, it was observed by the researcher that most of the students were interested and engaged in doing activities and using concrete models. They were eager to participate in the activities and use manipulative materials. The reason for this situation may be that the students in the study have been learning the probability topic since they were 4^{th} grade. They might have developed attitudes toward probability over the years, even though instruction was performed through concrete models and activities. Therefore, changing their ingrained attitudes toward probability was difficult in this short period of time. The statistical analysis shows a slight increase from pre-intervention through post-intervention and from pre-intervention through follow-up. Long-term application of probability instruction through concrete models might have changed their attitudes toward probability. Neale (1969) supports the idea that attitude can change in a long time. Results of Bayram (2004) and Şengül and Ekinözü (2006) also suggest the same idea. They stated that application of treatment lasted short and students' attitudes toward probability did not change significantly and added that attitudes could change in a long period of time. Moreover, students did not experience the particular method of instruction before. Because of the same reasons, in the present study students might not have internalized the instruction with concrete models and also they might not have got accustomed to use concrete models in a short period of time.

In the present study, the arithmetic mean of PAS scores in post-intervention was slightly higher than the arithmetic means of PAS scores in pre-intervention and follow up. Moreover, the arithmetic mean of PAS scores in follow up was slightly higher than the arithmetic mean of PAS scores in pre-intervention. Also, the arithmetic means of PAS scores in post-intervention and in follow up were close to each other. In other words, here was a slight increase from pre-intervention through post intervention, and from pre-intervention through follow-up. Also, there was a slight decrease from post-intervention through follow-up. These findings were the same with medians of scores across three time periods. These changes were not statistically significant. However, when the results of students' responses to some specific items were analyzed, half of the students agreed with the item "Probability topics are funny" in pre-intervention. In post-intervention, half of the students strongly agreed with this item and three of the students stated that they agreed with this it. Moreover, two of the students stated that they strongly agreed with the item "Probability topics are funny" and five of the students stated that they agreed with it in follow-up. In addition to this, three out of 12 students strongly disagreed with the item "I do not like probability topic” and five of the students stated that they did not agree with this item in pre-intervention. In post-intervention, five out of 12 students strongly disagreed with this item and four of the students stated that they did not agree with it. In follow-up, four out of 12 students strongly disagreed with the item “I do not like probability topic” and five of the students stated that they did not agree with this item. These findings confirm the results of the interview that most of the students’ views about probability instruction with concrete models were positive and they also stated that they enjoyed using concrete models and they found concrete models funny in probability instruction.

Additionally, the item index means of attitude scale scores were computed (i.e.item index mean = mean/number of items). The item index mean of attitude scale scores was found 4.61 for pre-intervention, 4.89 for post-intervention and 4.83 for follow-up. These values were close to the value of 5 which corresponded to agreement in the 6 likert type scale. In short, students’ responses to attitude scale were positive across three time periods. Therefore, these results confirm the findings of interview that students’ views about instruction with concrete models were positive.

5.1.3. Discussion of Findings on Students’ Views about Instruction with Concrete Models

The second purpose of the present study was to investigate the eighth grade students’ views about instruction with concrete models. Eleven students were interviewed to test the second main problem. Two themes, cognitive domain and affective domain, emerged according to students’ answers to interview questions. There were 3 main categories constituted according to students’ answers to the interview questions. The main category “emotions” was related to the affective domain, whereas the main categories “usefulness” and “easiness” were related to the cognitive domain.

The main category of “emotions” had 4 sub-categories which were “enjoyment”, “funny”, “interest”, “not boring”. According to answers, all of the students enjoyed while they were using concrete models and almost all of the students found concrete models and activities funny. Similarly, Bayram (2004) also stated that students in her study found concrete materials and activities enjoyable and they enjoyed concrete materials and activities. Moreover, Sarı (2010) found similar results that more than half of the students enjoyed the lesson through concrete models. In the present study, students used concrete models in activities. The activities were prepared to attract students and they were like games. Since the games play important role in children’s lives inherently, students might have enjoyed using concrete models.

The other main category “easiness” had 3 sub-categories which were “understanding”, “problem solving” and “retention”. More than half of the students reported that they could understand probability easily through concrete models in the present study. This finding confirms the idea of Berman and Friederwitzer (1989) who stated that children can understand the abstract issues through the use of concrete models. Similarly, Gürbüz (2007) also found that almost half of the students could understand the probability easily with concrete materials. Sarı (2010) found similar results with present study, in which students stated that problems became easier after experiencing concrete models. According to results of present study, more than half of the students declared that they could solve the probability problems easily through using concrete models. They stated that they could solve probability questions easily both after activities and after treatment. In the present study, some of the evaluation questions which students solved after activities were similar with activities. Students might have solved these questions easily. Moreover, the probability questions in several mathematics test resources include concrete models such as balls, cubes, spinners, dices, pattern blocks. In the present study, students had opportunities to use these kinds of concrete models several times that they did not face with before. By doing activities and experiencing with concrete models on their own, students might have understood what was asked in such questions and they might have solved them easily after treatment. Furthermore, some of students stated that they could remember the concept of probability easily by means of concrete models and activities in the present study. This result supports the view of Boling (1991) who indicated that issues learnt become permanent in students' minds by means of concrete models. In the interview, some of the students stated that while solving problems they remembered what they did with concrete models in the activities. Students experienced with several kinds of concrete models in the process. These experiences might have provided students to remember what they learnt. Similarly, Bayram (2004) also found that students in her study stated that they could remember the subject they learnt easily for they used concrete materials in the process.

The other main category "usefulness" had 4 sub-categories which were "examination", "understanding", "learning" and "achievement". In the present study, almost all of the students found concrete models useful. Similarly, Bayrak (2008) also found that most of the students in his study found activities including concrete materials useful. Also, Bayram (2004) found similar results that students in her study found concrete materials useful. Students in the present study stated different reasons for the usefulness of concrete models. Some of them stated that they found concrete models useful as they could understand probability by means of concrete models. In the present study, students had the probability instruction experiencing with concrete models on their own instead of listening to lesson. They were active in the process. They reached the knowledge by discussing each other while using concrete models. These experiences might have provided them more opportunuties than did just listening to the lesson. Because of these reasons, they might have found concrete models useful in terms of understanding. Moreover, some of the students found concrete models useful since they could learn probability through concrete models. Some of them found concrete models useful in that they would meet them in examinations. The educators and writers who prepare the mathematics tests, books and trial examinations have given weight to use of concrete models since the education system was changed in elementary grades. Since the students in the present study were preparing for level determination exam, they were aware of this situation. Because of this reason, they might have found concrete models useful in that they would them in examinations. Finally, some of the students stated that they found concrete models useful because their achievement increased by means of concrete models. Some students in the present study might have high points in the examination because they had 1st mathematics examination including probability topic after treatment. Moreover, activities including concrete models might have provided chance for students to solve probability questions that they did not face with before.

5.2. Conclusions of the Study

Each hypotheses of the present study was examined and following conclusions were gathered:

In the light of findings of the study, that there were statistically significant positive changes on the probability achievement from pre-intervention through post-intervention scores and from pre-intervention through follow-up scores after the instruction with concrete models. Therefore, it can be concluded that instruction with concrete models might have increased most of the students' probability achievement. Also, there was a slight increase in students' attitudes towards probability. However, there was no statistical change in students' attitudes towards probability. Moreover, according to findings of the interview, it was determined that most of the students had positive views about the effects of instruction with concrete models on their cognitive processes in terms of problem solving, understanding, learning, and remembering the subject. In addition to this, most of them thought that the instruction had positive impacts on their views in terms of developing positive attitudes toward concrete models and probability lessons.

5.3. Recommendations

In this section, recommendations are given for teachers, curriculum developers, teacher educators and further studies.

Teachers:

The results of this study demonstrated that use of concrete models in probability instruction increased students' probability achievement. It is suggested that teachers use concrete materials in their lessons. Various activities including concrete models related to probability topic were implemented during 4 weeks. These activities serve as a guide when the teachers perform probability topic in their lessons. These activities also can be varied and developed by teachers. The activities in the present study were prepared according to 8th grade students' cognitive level. Teachers should prepare activities according to students' levels.

Curriculum Developers:

The current elementary school mathematics curriculum has been implemented since 2005-2006 academic year. It was prepared based on student centered approaches. Also, the presentation of probability unit is activity based. However, probability activities suggested by this curriculum includes inadequate concrete models. Based on the findings of this study, it is suggested that probability activities should involve more concrete models. This study suggests various activities with concrete models related to probability topic. Curriculum developers can take into consideration these activities. The mathematics curriculum in secondary grades is not new. It is still based on traditional approaches. It is suggested that curriculum developers should revise and change secondary grades mathematics curriculum. Because, after elementary grades students may have difficulty in learning upper topics of probability through traditional instruction.

Teacher Educators:

Teacher educators are vital in the education system since they are responsible for raising future's teachers. In this sense, teacher educators should provide pre-service teachers many opportunities for the use of concrete models in the probability instruction. It is also suggested that there should be additional probability courses on how to teach probability efficiently. More opportunities regarding probability instruction should be given to pre-service teachers in material development lessons. Moreover, in-service training related to instruction with concrete models is suggested for teachers.

Further Studies:

This study was conducted with small sample size. Large sample size for further studies is recommended to increase the validity of the study. The present study was carried out with eighth grade students. The future studies can be conducted with students with different grade levels. Moreover, students in present study were from a private school. Future studies can be done with public school students. In the present study, instruction took short time. Because of this reason, there might not have been statistically significant change in students' attitudes toward probability across three time periods. The long time application of concrete models in probability instruction is recommended for further studies. The interview conducted in present study took very short time because of the students' limited time after school.

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APPENDICES

A.Pre-requisite Knowledge and Skills Test

Adınız Soyadınız:...........................

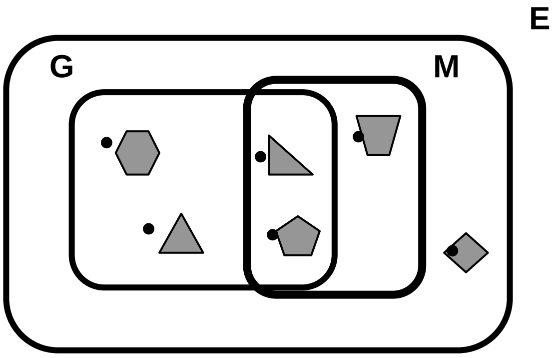
1. Aşağıdaki bilgileri Venn şeması çizerek gösteriniz

S={radyo, fotoğraf makinesi, video}

F={video, bilgisayar, yazıcı }

2. Bir sınıfta 10 öğrenci sadece sinemayı, 14 öğrenci sadece tiyatroyu, 8 öğrenci de hem sinemayı hem de tiyatroyu seviyor. Bu sınıftaki öğrenci sayısı kaçtır?

**** **3. ve 4. soruları aşağıdaki Venn Şemasını kullanarak cevaplayınız** ****



3. Evrensel kümeyi yazınız.

4. G $\cap$ M'nin elemanlarını yazınız

5. $\frac{3}{8}$ kesrini şekil çizerek gösteriniz.

6. Aşağıdaki rasyonel sayıları büyükten küçüğe sıraya koyunuz:

i) $\frac{3}{11}, \frac{5}{11}, \frac{2}{11}$ *ve* $\frac{8}{11}$　　ii) $\frac{2}{16}, \frac{3}{8}$ *ve* $\frac{5}{4}$

7. Aşağıda verilen cümlelerdeki boşlukları doldurunuz.

a) Bir kesirde kesir çizgisinin altında kalan sayı bütünün eşit parçalarının sayısını gösterir. Buna ___________________ denir.

b) Bir kesirde kesir çizgisinin üstünde kalan sayı bizim kullandığımız parça sayısını gösterir. Buna _____________________ denir.

8. Aşağıdakilerin her birini hesaplayınız.

i) $\frac{3}{4}+\frac{5}{6}=?$ ii) $\frac{3}{8}x\frac{4}{7}=?$ iii) $\frac{2}{5}+\frac{4}{5}=?$

iv) $\frac{3}{4}-\frac{2}{6}=?$ v) 0,4 + 0,8=? vi) 1,7 - 0,2 = ?

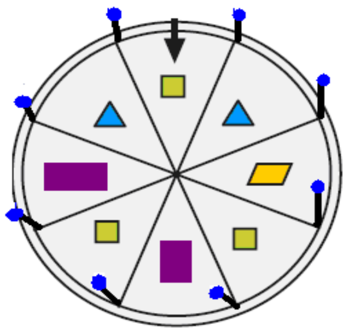
vii) 0,3 x 0,9 = ? viii) 2,34 x 2 = ? ix) 0,23 x 0,15 = ?

9.Aşağıdaki tabloda okulun boyasının rengini belirlemek için yapılan anket sonuçları verilmiştir. En çok tercih edilen renk hangisidir?

Tablo: Renk tercihlerine göre öğrenci dağılımı

Renkler	Pembe	Mavi	Sarı	Yeşil	Mor
Öğrenci Sayısı	58	158	99	56	32

10.Aşağıdaki çark bir kez çevrildiğinde okun üçgensel bölgede durma olasılığı nedir?



11. 200 kişilik bir okulun kantininden öğle yemeğinde 124 öğrenci tost satın almıştır. Kantinden tost alanların sayısı tüm öğrencilerin yüzde kaçıdır?

B. Table of Specification for PKT

		Demonstrates a set with different representation forms.	Does intersection, union, complementation operations in sets.	Determines the universal set	Determines the basic components of a fraction	Demonstrates a fraction by modeling.	Compares and arranges fractions and demonstrates them on number line	Does subtraction and addition with fractions	Does subtraction and addition with decimals.	Does multiplication with fractions.	Does multiplication with decimals.	Interprets the data given on the table.	Makes computation related to percentage	Computes probability of an event.	Total	
															n	%
Sets		1	2,4	3											4	19
Fractions					7a, 7b	5	6a,6b	8a, 8c, 8d		8b					9	43
Decimals									8e, 8f		8g, 8h, 8i				5	24
Data												9	11	10	3	15
Total	n	1	2	1	2	1	2	3	2	1	3	1	1	1		
	%	5	10	5	10	5	10	14	10	5	14	5	5	5		

C.Probability Achievement Test

Adınız Soyadınız: ....

1) "ARKADAŞLIK" kelimesini oluşturan hafler kağıtlara yazılıp torbaya atılıyor. Bu torbadan rasgele çekilen harfin "A" olma olasılığı nedir?

2) Pelin içinde 15 pembe, 12 kırmızı, 7 mor toka bulunan toka kutusundan iki toka almıştır. Iki tokayı da geri atmamıştır. Ilk tokanın mor, ikinci tokanın pembe gelme olasılığı nedir?

3) Bir ailenin 2 çocuğundan birinin kız diğerinin erkek olma olasılığı nedir?

4) Bir yüksek lisans öğrencisi veri toplamak amacıyla 8 bay 16 bayan arasından rastgele 3 kişi seçecektir. Bu seçilen kişilerin tümünün bayan olma olasılığı nedir?

5) 5 doktor, 6 hemşire ve 8 hastabakıcı arasından 2 doktor, 3 hemşire ve 5 hastabakıcıdan oluşan sağlık ekibi kaç farklı şekilde oluşturulabilir?

6) Bir paraşütçü hava koşullarının uygun olduğu bir günde uçaktan zemine atlayacaktır. Dikdörtgensel bölge şeklindeki zeminin kenar uzunlukları 80 m ve 40 m'dir. Zeminin ortasında daire şeklinde bir bölüm vardır. Bu dairenin yarı çapı 20 m.dir. Paraşütçünün dairenin içine inme olasılığı nedir?(π=3)

7) Aşağıdaki sorulardaki deneylerde yer alan olayların çeşitlerini nedenleri ile birlikte yazınız.

i) Türkiye'de bir ilçe alınan bir kararla il yapılmıştır. Bu yeni ilde araçlara yeni plaka verilecektir. Bu plaka ilin kodu, iki harf ve 3 rakamdan oluşacaktır. Plakadaki 2 harf oluşturulurken aynı harfler birden fazla kullanılabilmektedir.

ii) Ayhan işyerinde kullandığı bilgisayarı kendinden başka birisinin kullanmaması için 3 basamaklı şifre kullanmaktadır. Bu şifre sadece 0 ile 9(dahil) arasındaki farklı rakamlardan oluşmaktadır. Ayhan'ın bilgisayarının şifresinin doğru tahmin edilme olasılığı nedir?

8) "Hafta sonu Galatasaray- Beşiktaş maçı yapılacaktır. Berke göre Galatasaray'ın maçı kazanma olasılığı % 70 iken, Sibel'e göre Beşiktaş'ın maçı kazanma olasılığı %90'dır." Bu açıklamadaki olasılık çeşidini nedenleri ile birlikte yazınız.

9) Madeni para atma deneyi ile ilgili bir bilgisayar programı yazılmıştır. 1000 ve 100 000 kez para atıldığında "tura gelme" olasılıkları $\frac{452}{1000}$ = 0,452 ve $\frac{48962}{100000}$ = 0,48962 olarak hesaplanmıştır. Atış sayısı ile elde edilen değerler arasındaki ilişkiyi nedenleri ile birlikte açıklayınız.

10) "Hüzünlü Köyü'nde yeterince ağaç yoktur ve toprak işleme yöntemleri yanlıştır. Yeşil Köyü'nde ise Hüzünlü Köyü'ndeki olumsuzluklar yoktur." Bu açıklamadaki olasılık çeşidini nedenleri ile birlikte yazınız.

11) Aşağıdaki tabloda 26 öğrencinin matematik dersinden aldıkları notların dağılımı verilmiştir. Notu 5 olan öğrencilerin isimlerinin her biri ayrı ayrı masa tenisi toplarına yazılmıştır. Diğer notları olan öğrencilerin isimlerinin her biri ise ayrı ayrı küçük kağıtlara yazılmıştır. Bunlar torbaya atıldıktan sonra bakmadan bir çekiliş yapılmıştır. Beş alan öğrenciyi çekme olasılığı

$\frac{9}{26}$ olduğu söylenmiştir. Bu sonucun doğru olup olmadığını nedenleri ile birlikte açıklayınız.

Tablo: Notlara Göre Öğrenci Dağılımı

Alınan Not	3	4	5
Öğrenci Sayısı	7	10	9

12) Aşağıdaki soruları çözerken hangi konulardan yararlanarak çözebileceğinizi nedenleri ile birlikte açıklayınız.

i) Bir şirket, biri muhasebeci diğeri satış görevlisi olmak üzere iki kişiyi işe alacaktır. Her iki görev için 18 kişi başvurmuştur. Bu kadrolar kaç farklı şekilde doldurulabilir?

ii) Bir şirket, 2 tane peyzaj mimarını işe alacaktır. Bu kişiler aynı işi yapacak ve aynı ücreti alacaktır. Bu iş için 18 kişi başvurmuştur. Bu kadrolar kaç farklı şekilde doldurulabilir?

13) Aslı'nın bir problemi çözme olasılığı 0,8 iken Kerem'in çözme olasılığı 0,7 dir. Bu problemin aynı zamanda Aslı ve Kerem tarafından çözülme olasılığı nedir?

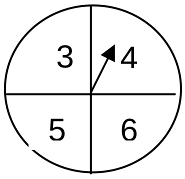
14) Özlem hilesiz bir madeni parayı 4 kez havaya atmış ve hepsinde yazı gelmiştir. Özlem 5. kez parayı havaya attığında aşağıdakilerden hangisinin doğru olduğunu nedenleri ile birlikte açıklayınız?

a) Yazı gelme olasılığı tura gelme olasılığına eşittir.

b) Yazı gelme olasılığı tura gelme olasılından küçüktür.

c) Yazı gelme olasılığı tura gelme olasılığından büyüktür.

15) Yandaki çarktaki ok çevrilerek birer basamaklı sayılardan oluşan kesirler yazılmaktadır.



Ok, 2 kez ard arda çevrildikten sonra elde edilen kesrin değerinin ½ den büyük olma

olasılığı nedir? (Not: Ok, çarkın üzerindeki çizgilerde durmayacak şekilde yapılmıştır.)

D. Table of Specification of PAT

		Computes the probability of an event	Explains dependent and independent events	Computes the probability of dependent and independent event.	Explains the experimental probability	Explains the subjective probability	Computes the probability of an event by using his/her geometric knowledge	Explains the difference between permutation and combination concepts.	Computes combitanion.	Total	
										n	%
Basic Concepts of Probability		1,11,14,15								4	24
Independent & Dependent Events			7a, 7b	2, 3, 4,13						6	35
Type of Probability Concepts					9,10	8	6			4	24
Permutation & Combination								12a, 12b	5	3	24
Total	n	4	2	4	2	1	1	2	1		
	%	24	12	24	12	6	6	12	6		

E.Rubric for PAT

Olasılık Başarı Testini Değerlendirme Kriteri

7a, 7b, 8, 9, 10, 11, 12a, 12b ve 14 Numaralı sorular için:

2 puan: Sorunun nedenleriyle birlikte doğru cevabı da verilmişse

1 puan: Sorunun doğru cevabı verilmiş fakat nedenler açıklanmaışsa veya sorunun nedenleri açıklanmış fakat cevabı verilmemişse

0 puan: Soruya herhangi bir cevap verilmemişse veya yanlış cevap verilmişse

1, 2, 3, 4, 5, 6, 13 ve 15 Numaralı sorular için;

1 puan: Soru tam ve doğru olarak çözülmüşse

0 puan: Soruya herhangi bir cevap verilmemişse veya yanlış cevap verilmişse

F. Probability Attitude Scale

Genel Açıklama: Aşağıda olasılığa ilişkin tutum cümleleri ile her cümlenin karşısında
"Tamamen Katılıyorum", "Katılıyorum", "Katılabilirim", "Katılmayabilirim",
"Katılmıyorum", "Hiç Katılmıyorum" olmak üzere altı seçenek verilmiştir. Lütfen cümleleri dikkatli
okuduktan sonra her cümle için kendinize uygun olan seçeneklerden birini işaretleyiniz.

	Tamamen Katılıyorum	Katılıyorum	Katılabilirim	Katılmayabilirim	Katılmıyorum	Hiç Katılmıyorum
01. Olasılık konularını severim.	O	O	O	O	O	O
02. Olasılık konuları sevimsizdir.	O	O	O	O	O	O
03. Olasılıkla ilgili konuları tartışmaktan hoşlanırım.	O	O	O	O	O	O
04. Olasılıkla ilgili bilgiler can sıkıcıdır.	O	O	O	O	O	O
05. Olasılıkla ilgili bilgiler zihin gelişmesine yardımcı olur	O	O	O	O	O	O
06. Olasılık konusu beni huzursuz eder.	O	O	O	O	O	O
07. Olasılıkla ilgili ders saatlerinin daha çok olmasını isterim	O	O	O	O	O	O
08. Olasılık konuları rahatlıkla/kolaylıkla öğrenilebilir.	O	O	O	O	O	O
09. Olasılıkla ilgili sınavlardan korkarım.	O	O	O	O	O	O
10. Olasılık konuları ilgimi çeker.	O	O	O	O	O	O
11. Olasılığın doğru karar vermemizde önemli rolü vardır.	O	O	O	O	O	O
12. Olasılık konuları aklımı karıştırır.	O	O	O	O	O	O
13. Olasılık konusunu severek çalışırım.	O	O	O	O	O	O
14. Olasılık konusunu, elimde olsa öğrenmek istemezdim.	O	O	O	O	O	O
15. Olasılık, ilginç bir konu değildir.	O	O	O	O	O	O
16. Olasılıkla ilgili ileri düzeyde bilgi edinmek isterim.	O	O	O	O	O	O

17. Olasılık hemen hemen her iş alanında kullanılmaktdır.	O	O	O	O	O	O
18. Olasılık konusunu çalışırken canım sıkılır.	O	O	O	O	O	O
19. Olasılık, kişiye düşünmesini öğretir.	O	O	O	O	O	O
20. Olasılığın adını bile duymak sinirlerimi bozuyor.	O	O	O	O	O	O
21. Olasılık konusundan korkarım.	O	O	O	O	O	O
22. Olasılık, herkesin öğrenmesi gereken bir konudur.	O	O	O	O	O	O
23. Olasılık konusundan hoşlanmam.	O	O	O	O	O	O
24. Olasılıkla ilgili bilgiler, kişinin tahmin (etme) yeteneğini artırır.	O	O	O	O	O	O
25. Olasılık konusu anlatılırken sıkılırım.	O	O	O	O	O	O
26. Olasılıkla ilgili bilgilerin, günlük yaşamda önemli bir yeri vardır.	O	O	O	O	O	O
27. Olasılık konusu okullarda öğretilmese daha iyi olur.	O	O	O	O	O	O
28. Olasılık konuları eğlencelidir.	O	O	O	O	O	O

G.Sample Activities

Etkinlik

Kazanım: Deney, çıktı, örnek uzay, olay, rastgele seçim ve eş olasılıklı terimlerini açıklar.

Araç ve gereçler: yüzlerinde 1' den 6'ya kadar rakamların yazılı olduğu küpler, üzerlerine alfabenin harflerinin yazılı olduğu pullar ve kutu.

1. Küp yere atıldığında üst yüze gelebilecek sayıları belirleyip liste yöntemiyle yazınız. Tüm durumların olasılıkları hakkında ne söyleyebilirsiniz? Açıklayınız.

Küp atıldığında üst yüzeyine 3'ten küçük sayı gelme durumundaki elemanları belirleyiniz. Bu elemanların oluşturduğu kümeyi yazınız.

2. Kutudan üzerinde alfabenin harflerinin yazılı olduğu pullardan çekildeğinde gelebilecek tüm harfleri belirleyip liste yöntemi ile yazınız. Tüm durumların olasılıkları hakkında ne söyleyebilirsiniz? Açıklayınız.

Kutudan bir pul çekildiğinde sesli harf gelme durumundaki elemanları belirleyiniz. Bu elemanların oluşturduğu kümeyi yazınız.

Sonuçlarını belirleyebildiğimiz olaylar deneydir. Deney sonucunda elde ettiğimiz her bir sonuca çıktı denir. Deneyden elde ettiğimiz tüm sonuçların yazıldığı kümeye örnek uzay denir. Örnek uzayın alt kümelerinin her birinin adına olay denir. O halde bu 2 durumun deneyini, çıktılarını, örnek uzayını ve olayını belirleyiniz.

Etkinlik

Araç ve gereçler: Hileli zarlar.

Zarlarınızı art arda atınız. Örnek uzayın çıktılarını belirleyiniz. Sizce örnek uzayın herbir çıktısının olma olasılıkları eşit midir? Arkadaşlarınızla tartışınız.

Etkinlik

Kazanım: Bir olayın olma olasılığını açıklar ve hesaplar.

Araç ve gereçler: 4 eş bölgeye ayrılmış çarklar

Elinizdeki çarklarda sarı bölgenin diğer bölgelere oranı nedir?

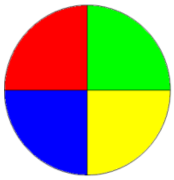
Oranı nasıl ifade edersiniz?

Çarkı çevirdiğinizde örnek uzayın eleman sayısı kaçtır?

"Çark çevrildiğinde sarı bölgede durma olasılığı nedir?" sorusu için olayın çıktı sayısı kaçtır?

Olayın çıktı sayısının örnek uzayın çıktı sayısına oranı nedir? Bulduğunuz değer çark çevrildiğinde sarı bölgede durma olasılığıdır.

Şimdi olasılığı genel olarak nasıl ifade edersiniz? Çark çevrildiğinde her bir bölgede durma olasılığını oran, kesir ve yüzde olarak hesaplayınız.



Daha sonra aşağıda tabloda verilen durumlar için tabloyu doldurunuz.

Çalışmanın Ismi	Örnek Uzay ve Olay Bilgileri	Oran/Kesir Ifadesi	Olasılık
1.Oyun zarı atma Üst yüze 3' ten küçük sayı gelme olayının gerçekleşebilme durumu nedir?	Deney: Örneklem uzay: s(ö)= Olay: s(o)=		
2.Dört eşit parçaya ayrılmış çarklar. İğnenin kırmızı rengi göstermesi olayının gerçekleşebilme durumu nedir?	Deney: Örneklem uzay: s(ö)= Olay: s(o)=		

3.Alfabedeki harfler. Sesli harf gelme olayının gerçekleşebilme durumu nedir?	Deney: Örneklemuzay: s(ö)= Olay: s(o)=		

Etkinlik

Kazanım: Kesin ve imkansız olayları açıklar.

Araç ve gereçler: Mavi ve kırmızı toplar, torba.

Kırmızı topları torbaya koyunuz. Torbadan bir tane kırmızı top çekme olasılığını hesaplayınız. Bulduğunuz olasılığı ondalık kesir olarak yazınız. Torbadan mavi top çekme olasılığını hesaplayınız. Torbadan kırmızı top çekme olayı ile mavi top çekme olayları nasıl olaylardır? Bulduğunuz olasılık değerlerini olasılık çizgisi üzerinde gösteriniz.

Değerlendirme Soruları

1. Farklı 2 tane hilesiz madeni paranın atılması sonucunda paraların en az bir tanesinin yazı gelme olasılığı nedir? Sorusunu göz önüne alarak bu sorunun deneyini, örneklem uzayını, örneklem uzayın eleman sayısını, istenen olayı ve istenen olayın olma sayısını belirleyiniz.

2. Hilesiz bir zar havaya atıldığında örneklem uzay eş olumlu mudur?

3. Bir torbada 3 tane kırmızı elma, 2 tane yeşil elma, 3 tane de yeşil erik bulunmaktadır. Torbadan rasgele bir meyve seçildiğinde örneklem uzay eş olumlu mudur?

4. A= (1, 2, 3, 4, 6, 8) kümesindeki rakamlar aynı özelliklere sahip kağıt parçaları üzerine yazılmıştır. Bu kağıt parçaları da bir torbaya konmuştur.

a. Bu torbadan bir kağıt parçasının çekilmesi sonucunda 10'dan küçük sayıların çekilmesi olasığını hesaplayınız.

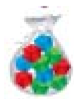
b. Yukarıda istenen olayın çeşidini yazınız.

[Note: This question was utilized from Aydın and Beşer (2008)]

5. A=(1, 3, 7, 11, 18, 20) sayıları ayrı ayrı kağıt parçalarına yazılarak bir torbaya atılmıştır. Bu torbadan 24'ten büyük çift sayı çekme olasılığını hesaplayınız. Bu olayın çeşidini yazınız.

[Note: This question was utilized from Aydın and Beşer (2008)]

6. Aşağıdaki poşetlerden içine bakılmadan birer küp çekiliyor. Buna göre aşağıdaki cümleleri doldurunuz.



Sarı küp çeklmesi olayı …… olaydır. Siyah küp çekilmesi olayı ………olaydır.

[Note: This question was utilized from Glencoe elementary mathematics book of Colins (1999)]

7. 678 932 sayısını oluşturan rakamlardan rastgele biri seçildiğinde bu rakamın 3'ün katı olma olasılığı yüzde kaçtır? (örnek uzayı ve olayın çıktılarını belirtiniz).

[Note: This question was utilized from Aydın and Beşer (2008)]

8. Bir torbada, üzerinde 5' ten küçük sayıların yazılı olduğu 4 mavi ve 4 siyah top vardır. Torbadan rasgele 2 top çekilmiştir. Çekilen toplardan birinin mavi ve diğerinin siyah olması koşuluyla oluşan sayı ikililerinin toplamlarının 5 olma olasılığı nedir?

[Note: This question was utilized from Aydın and Beşer (2008)]

Etkinlik

Kazanım: Deneysel ve teorik olasılığı açıklar.

Araç-gereçler: Madeni para, çetele tablosu.

Para atma deneyinden önce;

Madeni parayı attığınızda; Yazı gelme olasılığı nedir? Tura gelme olasılığı nedir? Bu olasılık değerlerini kesir, oran ve yüzde olarak yazınız.

Şimdi madeni paralarınızı 5, 10 ve 20 kere atınız ve sonuçlarınızı çetele tablosuna kaydediniz, sonuçlarınıza göre sütun grafiği oluşturunuz. Elde ettiğiniz sonuçlara göre; yazı gelme olasılığı nedir? Tura gelme olasılığı nedir?

- Parayı atmadan önce ve attıktan sonraki olasılık değerleriniz farklı mı? Niçin?
- Yaptığımız etkinliğe dayanarak bu olasılıkları nasıl adlandırırsınız?

ÇIKTI	ÇETELEME	SIKLIK SAYISI
Yazı		
Tura		

Etkinlik

Araç ve gereçler: 4 eş bölgeye ayrılmış çarklar, çetele tablosu.

Çarktaki iğneyi çevirmeden önce, iğnenin herbir renkte durma olasılıklarını söyleyiniz. Bu olasılıkları kesir, oran ve yüzde olarak ifade ediniz. Bu olasılıklar ne tür olasılıklar?

Çarktaki iğneyi 20, 50 ve 100 kere çevirin ve sonuçlarınızı sıklık tablosuna kaydedin.

Şimdi 20., 50. ve 100. çevirişlerdeki olasılıklarınızı hesaplayınız. Bu olasılıklar ne tür olasılıklar? Herbir bölümdeki olasılık değerlerinizi (deneysel olasılık), deneye başlamadan önceki olasılık değerlerinizle (teorik olasılık) karşılaştırınız. Ne söyleyebilirsiniz? Çevirme sayınız arttıkça elde ettiğiniz deneysel olasılık değeri ile teorik olasılık değeri arasında nasıl bir ilişki meydana geldi?

ÇIKTI	ÇETELE	FREKANS
kırmızı		
sarı		
mavi		
yeşil		

Değerlendirme Soruları

1. Bir madeni para 100 kez atılıyor. 42 kez tura, 58 kez yazı geliyor. Buna göre paranın üst yüzüne yazı gelme olasılığı nedir? Hesapladığınız olasılığın türü nedir?

1. Üzerinde alfabenin harflerinin yazılı olduğu pulların bulunduğu kutudan rastgele bir pul çekildiğinde sessiz harf çıkma olasılığı nedir? Hesapladığınız olasılığın türü nedir?

2. Üzerinde A, B, C, D harfleri yazılı olan dört bölmeli bir çarkı çevirme deneyi yapılmıştır. Deney süresince çark 60 kez çevrilmiş ve aşağıdaki tabloda bulunan verilere ulaşılmıştır. Elde edilen verilere göre çarkın şeklini çiziniz.

Harf	Veriler
A	20
B	15
C	10
D	15

3. 23' ten 33' e kadar olan sayıların (23 ve 33 dahil) yazılarak atıldığı bir torbadan hem 3'e hem de 2'ye bölünebilen bir sayıyı çekme olasılığını teorik olarak bulunuz.

Etkinlik

Kazanım: Bağımlı ve bağımsız olayların olma olasılığını hesaplar.

Etkinlik

Araç ve Gereçler: 6 tane üzerinde hediye yazılı topların (3 kalem, 2 not defteri, 1 roman) bulunduğu poşetler.

1. adım: Ilk olarak herbir hediyenin çekilme olasılığını söyleyerek poşetten 1 er top çekiniz. Olasılığını hesaplayınız. Daha sonra topu poşete geri atınız. Tekrar bir top çekiniz. Olasılığını hesaplayınız. Bu hesapladığınız olasılık değeri ilk çekilişten etkilendi mi? Niçin? Bu iki olay nasıl olaylardır?

2. adım: Poşetten bir adet top çekiniz. Olasılığını hesaplayınız. Çektiğiniz topu poşete geri atmadan ikinci bir top daha çekiniz. Olasılığını hesaplayınız. Bulduğunuz bu ikinci olasılık değeri ilk çekilişten etkilendi mi? Niçin? Bu iki olay nasıl olaylardır?

Etkinlik

Araç ve Gereçler: Limonlu ve naneli şekerler

Etkinliğin konusu şu şekildedir: "Bir torbada, tatları dışında aynı özelliklere sahip 3 limonlu ve 5 naneli şeker bulunmaktadır. Semra ve Aslıhan, 2 tane limonlu şeker yemek istemektedir. Kimin iki tane limonlu şeker yiyeceğine karar veremedikleri için şekerleri torbadan çekeceklerdir. ªeker çekme olayı iki farklı şekilde yapılacaktır.

1. durum: İşleme ilk önce Semra başlayacaktır. Semra, birinci şekeri çektikten sonra torbaya atarak ikinci şekeri çekecektir. Eğer çekilen her iki şeker limonlu ise Semra limonlu şekerleri alabilecektir. Çekilen iki şekerin de limonlu olma olasılığı nedir?

2. durum: Semra birinci çekilişten sonra çektiği şekeri torbaya atmadan ikinci kez torbadan şeker çekecektir. Eğer çekilen her iki şeker limonlu ise Semra limonlu şekerleri alabilecektir. Çekilen iki şekerin de limonlu olma olasılığı nedir?

[Note: This activity was utilized from MONE (2008)]

Etkinlikteki her iki durumu da gerçekleştiriniz. Her iki durumda da limonlu şeker çekilme olasılıklarını belirleyiniz. Olay çeşitlerini belirleyiniz. Eğer A ve B bağımsız olaylar ise, A ve B nin olma olasılığı ; *P*(*A* ve *B*) = P(A olayı) • P(B olayı) şeklinde hesaplanır. Eğer A ve B bağımlı olaylar ise, A ve B nin olma olasılığı ; *P*(*A* ve *B*) = *P*(A olayı) • P(A olayından sonra gelen B olayı) şeklinde hesaplanır. Buna göre olasılıkları hesaplayınız. Daha sonra bütün olasılıkları gösteren bir ağaç diyagramı çiziniz. Bu iki durumdaki olayların olma olasılıklarını karşılaştırınız.

Değerlendirme Soruları

Not: Aşağıdaki 1. ve 2. soruları 1. etkinlikten yararlanarak çözünüz.

1. Çekilen hediyeyi poşete geri kormak şartıyla, 1. çekilişte kalem 2. çekilişte de defter çekilme olasılığı nedir?

2. Çekilen hediyeyi geri koymadan, 1. çekilişte kalem, 2. çekilişte roman çekilme olasılığı nedir?

3.Yüzlerinde 6 tane geometrik şeklin (üçgensel, dörtgensel, beşgensel, altıgensel, yedigensel, sekizgensel bölgeler) resimlerinin bulunduğu iki eş küp yuvarlanıyor. Küpler durduğunda ikisinin de üst yüzlerinde üçgensel bölge olma olasılığı kaçtır?

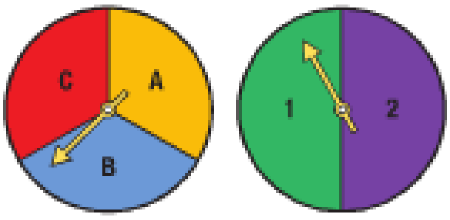
4. Bir kutuda 5' i bozuk olmak üzere 11 adet ampul vardır. Kutudan rasgele bir ampul çekilip kutuya geri bırakılmıyor ve tekrar rasgele bir ampul daha çekiliyor. Çekilen ampullerden 1. sinin bozuk, 2. sinin sağlam olama olasılığı nedir?

[Note: This question was utilized from Aydın and Beşer (2008)]

5. Aşağıdaki çarkların aynı anda çevrilmesi durumunda bütün çıktıları gösteren bir ağaç diyagramı yapınız.

Buna göre;

Atacın A ve 1 de durma olasılığı nedir?



[Note: This question was utilized from Glencoe elementary mathematics book of Colins (1999)]

H.Sample Lesson Plans

Dersin Adı: Matematik

Öğrenme Alanı: Olasılık ve Istatistik

Alt Öğrenme Alanı: Olasılık Çeşitleri

Kazanım: Deneysel, Teorik ve Öznel Olasılığı Açıklar

Kavramlar: Deneysel olasılık, teorik olasılık, öznel olasılık

Yöntem ve Teknikler: Buluş Yoluyla Öğrenme

Araç-Gereçler: madeni para, sıklık tabloları, çarklar

Süre: 3 ders saati

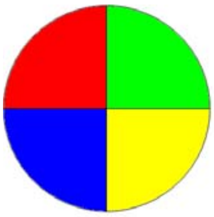
Dersin Işlenişi: Öğretmen dersin girişinde öğrencilerin dikkatlerini çekmek için madeni paraların günlük hayatımızda hangi durumlarda kullanıldığını sorar. Öğrencilerden cevaplar alındıktan sonra her öğrenciye bir madeni para dağıtılır. Öğretmen para atma deneyine geçmeden önce öğrencilere yazı veya tura gelme olasılıklarını sorar. Cevaplar alındıktan sonra öğretmen öğrencilerden bu olasılık ifadesini kesir, ondalık kesir ve yüzde olarak not etmelerini ister. Daha sonra etkinliğe geçilir. Öğretmen öğrencilerden madeni paraları ilk önce 5 kez, sonra 10 kez ve en son 20 kez atıp sonuçlarını sıklık tablosuna kaydetmelerini ister. 5, 10 ve 20 kez atışların her birindeki olasılık değerlerinin nasıl değiştiği sorulur. Öğretmen:

- Parayı atmadan önceki ve attıktan sonraki olasılık değerlerini nasıl yorumlarsınız?
- Para atma deneyi sonuçlarına göre elde edilen olasılık ile deney yapmadan elde edilen olasılık arasında neden farklılık vardır?
- Bu iki durum farklı olasılıklar olarak adlandırılabilir mi? Neden?
- Parayı attıktan sonra 5., 10. ve 20. kez atışlarda olasılık değerleri değişti mi?

- Siz bu olasılık değerlerine nasıl ulaştınız?

Sorularla öğrencilerden parayı atmadan önceki olasılığı tahmin ettiklerini söylemeleri beklenir. Parayı 5 kez, 10 kez ve 20 kez attıktan sonra ise olasılık değerlerinin değiştiği keşfettirilir. Bu olasılık değerlerine deney sonucunda ulaşıldığı için bunların deneysel olasılık oldukları keşfettirilir. Bir olasılık deneyi sonunda hesaplanan olasılığın deneysel olasılık, bir olasılık deneyinden teorik olarak beklenen olasılığın da teorik olasılık olduğu genellemesine ulaşılır.

Keşfetme gerçekleştikten sonra dersin 2. kısmında diğer etkinliğe geçilir. Bu etkinlikte öğrencilere 4 eş parçaya ayrılmış aşağıdaki çarklar ve sıklık tabloları dağıtılır.



Çarkın ortasında bulunan iğne çevrilerek deney yapılacaktır. Iğne çevrilmeden önce öğrencilere iğnenin herbir renkte durma olasılığı ve bunun hangi olasılık olduğu sorulur. Öğrencilerden beklenen cevap ¼ ve teorik olasılıktır. Öğrencilerden bu olasılık değerini kesir, ondalık kesir ve yüzde olarak yazmaları istenir. Daha sonra etkinliğe geçilir. Öğrencilerden iğneyi 20 kez, 50 kez ve 100 kez çevirmeleri ve sonuçlarını sıklık tablosuna kaydetmeleri istenir. Sonuçlar kaydedildikten sonra olasılık değerlerini kesir, ondalık kesir ve yüzde olarak yazmaları istenir. Deneyden sonra öğretmen öğrencilere aşağıdaki soruları sorar:

- Herbir atış bölümü tamamlandığında bunlar arasındaki olasılık değerleri arasında nasıl bir değişim gözlemlediniz?
- Gözlemlediğiniz değişim en fazla hangi bölümde oldu?
- Atış sayısı arttıkça elde edilen olasılık değerleri hakkında ne söyleyebilirsiniz?

(Burada amaç öğrencilerin atış sayısı arttıkça deneysel olasılık değerinin teorik olasılığa yaklaştığını keşfettirmektir). Keşfetme gerçekleştikten sonra öğrencilerden keşfettikleri bilgiyi kendi cümleleriyle genellemeleri istenir. Daha sonra öğrencilere çözmeleri için aşağıdaki değerlendirme soruları dağıtılır.

Değerlendirme Soruları:

1. Bir öğrenci 30 kişilik sınıflarında bir anket uygulamış ve arkadaşlarına en çok sevdikleri müzik türünü sormuş ve 14 kişiden "pop müzik" cevabını almıştır. Bu okuldaki 60 kişiye aynı soru sorulmuş olsaydı, bu gruptan rasgele seçilen bir öğrencinin "pop müzik" cevabını vermiş olma olasılığı ne olurdu? Hesapladığınız olasılığın türü nedir?
2. İçinde 3 tane kırmızı, 2 tane yeşil ve 4 tane mavi bilye bulunan bir poşetten rasgele bir bilye çekiliyor. Kırmızı bilye çekilme olasılığı nedir? Hesapladığınız olasılığın türü nedir?

Sorular çözüldükten ve yanıtlandıktan sonra 3. etkinliğe geçilir. Etkinlikten önce öğretmen öğrencilere bugün havanın yağmurlu olma olasılığını veya güneşli olma olasılığını sorar. Bütün öğrencilerden teker teker görüşleri alınır. Öğretmen öğrencilere aşağıdaki soruyu sorar:

- Bütün bu cevaplar (olasılık değerleri) doğru olabilir mi? Niçin?
- Bu olasılık değerleri neden farklıdır?

Burada öğrencilere bazı olasılık değerlerinin kişiden kişiye değişebileceği farkettirilir. Kişilerin kendi düşüncelerine göre karar verdikleri olasılıkların öznel olasılıklar olduğu sonucuna varılır. Daha sonra öğrencilere farklı spor yazarlarının maç tahminlerini içeren gazete kağıtları dağıtılır. Yorumlarda öznel olasılıkları bulmaları istenir.

Değerlendirme Soruları

1. Aslı gireceği SBS sınavında olasılık konusundan soru gelme olasılığını % 80, Hakan ise % 70 olarak tahmin etmektedir. Buradaki olasılık türü nedir?

2. Aşağıdaki cümlelerde geçen olasılık türlerini yazınız.

a. Hilesiz bir zar atıldığında üst yüze 5 gelme olasılığı nedir?
b. Hilesiz bir zar 10 kere atılmış ve 2 kez 5 gelmiştir. Bu sonuca göre 5 gelme olasılığı nedir?
c. Hilesiz bir zar atıldığında Ezgi'ye göre 5 gelme olasılığı %50, Ali'ye göre % 70 dir.

3. Üzerinde A, B, C, D harfleri yazılı olan dört bölmeli bir çarkı çevirme deneyi yapılmıştır. Deney süresince çark 60 kez çevrilmiş ve aşağıdaki tabloda bulunan verilere ulaşılmıştır. Elde edilen verilere göre çarkın şeklini çiziniz. Çark bir daha çevrildiğinde B harfinde durma olasılığı nedir? Hesapladığınız olasılığın türü nedir?

Harf	Veriler
A	20
B	15
C	10
D	15

4. 23' ten 33' e kadar olan sayıların (23 ve 33 dahil) yazılarak atıldığı bir torbadan hem 3'e hem de 2'ye bir sayıyı çekme olasılığı nedir? Hesapladığınız olasılığın türü nedir?

5. Bir kutuda birbirine eş 6 adet tenis topu vardır. Kutudan 30 kez top çekilmiş ve 25 kere beyaz top, 5 kere de siyah top gelmiştir. Buna göre aşağıdaki soruları cevaplayınız.

a. Kutuda kaç tane beyaz top olabilir? Açıklayınız.

b. Kutuda kaç tane siyah top olabilir? Açıklayınız.

Dersin Adı: Matematik

Öğrenme Alanı: Olasılık ve Istatistik

Alt Öğrenme Alanı: Olasılık Çeşitleri

Kazanım: Geometri bilgilerini kullanarak bir olayın olma olasılığını açıklar.

Kavramlar: Geometrik olasılık.

Yöntem ve Teknikler: Buluş Yoluyla Öğrenme

Araç-Gereçler: kutular, atış poligonu, tahta kalemi.

Süre: 2 ders saati.

Dersin Işlenişi: Dersin girişinde konuya dikkati çekmek ve motivasyonu arttırmak için bir oyun oynanır. Oyunun adı "padişah-vezir" dir. Öğrenciler dörderli gruplar oluştururlar. Gruplara kutular dağıtılır. Kutunun büyük yüzleri "boş" u , orta büyüklükteki yüzleri "vezir"i, küçük yüzleri de "padişah"ı temsil etmektedir. Bu oyunda kutu zar atılır gibi atılır, büyük yüz üzerinde durursa oyuncu cezalı olur, orta büyüklükteki yüz üzerinde durursa vezir olur, küçük yüz üzerinde durursa cezalı olur. Padişah cezaya karar verir, vezir de cezalı oyuncuyu cezalandırır. Cezalandırma bittikten sonra oyuna devam edilir.Oyundan sonra öğretmen öğrencilere aşağıdaki soruları sorar:

- Kaç kere kral, vezir ve cezalı oldunuz? Bu sayıların farklı olmasının sebebi nedir?
- Kaç kere kral oldunuz? Sizce bunun sebebi neydi?

- Padişah, vezir ve boş gelme olasılıklarını karşılaştırarak gerçekleşme olasılıklarını büyükten küçüğe sıralayınız.

[Note: This activity was utilized from Göğün (2008)]

Bulunan sonuçlar tartışılarak varılan sonuç yazılı ve sözlü olarak ifade ettirilir. Öğrencilerin burada kutu yüzeylerinin alanlarından bahsetmeleri beklenir. Eğer bahsetmezlerse öğretmen "sizce bu durum alanlarından kaynaklanabilir mi?" sorusunu sorabilir. Bu sorularla ve oyunda elde ettikleri sonuçlara dayanarak öğrencilere kutunun küçük yüz üzerinde durma olasılığının düşük, büyük yüz üzerinde durma olasılığının yüksek olduğu keşfettirilir. Daha sonra diğer etkinliğe geçilir.

2. etkinlikte tahtaya bir atış poligonu asılır. Bu atış poligonu aşağıdaki gibi parçalara ayrılmış geometrik şekillerden oluşmaktadır.

Ayrıt uzunlukları verilerek öğrencilerden bölgelerin alanlarını hesaplamaları istenir. Bölgeler alanlarına göre büyükten küçüğe kırmızı, mavi ve mor, turuncu, sarı bölgeler olarak sıralanmaktadır. Öğrencilerden dörderli grup oluşturmaları istenir. Gruplar sırasıyla tahta kalemiyle atışlarını yaparlar. Her grup atış yaptığı bölgeyi kaydeder. Öğretmen atışlar tamamlandıktan sonra öğrencilere aşağıdaki soruları sorar:

- En fazla hangi bölgeye atış yapıldı? Sizce bunun sebebi ne olabilir?
- En az atış yapılan bölge hangisiydi?
- Sizden kırmızı bölgeye atış yapma olasılığını bulmanız istense bunu nasıl hesaplayabilirsiniz?
- Bu tür olasılıklar ne tür olasılıklar olabilir?
- Daha önceki olasılık bilgilrinizden faydalanarak geometrik olasılığı tanımlayabilir misiniz?

Öğretmen yukarıdaki gibi yönlendirici sorular sorarak öğrencilerin alanı büyük olan bölgeye atış ihtimalinin yüksek olduğunu, alanı küçük olan bölgeye atış ihtimalinin düşük olduğunu, alandan yola çıkılarak bu tür olasılıkların geometrik olasılık olduklarını, gelen olasılık formülünden yola çıkarak

$$\text{Bir olayın olma olasılığı} = \frac{\text{istenen olayın toplam alanı}}{\text{mümkün olan tün alanların toplamı}}$$

Olduğunu keşfettirir. Bulunan formül sözlü ve yazılı olarak ifade edilir. Öğrenciler daha sonra herbir bölgenin alması gereken puanları kendileri belirleyerek kazanan grubu belirlerler. Öğretmen öğrencilerden tahta kaleminin herbir bölgeye atılma ihtimalini hesaplamalarını ve sonuçları yazılı ve sözlü olarak açıklamalarını ister. Dersin sonunda öğrencilerden aşağıdaki soruları çözmeleri istenir.

Değerlendirme

1. Bir televizyon kanalında Pazar günleri 12.00-13.00 saatleri arasında “Bilim Köşesi” isimli çocuk programı yapılmaktadır. Elementary school mathematics curiculumda ünlü bir bilim adamı ile ilgili açıklama yapılırken bir şifre ve bir telefon numarası verilmektedir. Bu telefon numarasına doğru ifreyi bildiren 10. kişiye ödül verilmektedir. Eylül, önümüzdeki Pazar günü bu programı seyretmeyi planlamaktadır. Fakat trafo bakımı nedeniyle Pazar günü 10.00-12.15 saatleri arasında elektrik kesintisi yapılacaktır. Bu durumda Eylül’ün şifreyi kaçırma olasılığı nedir? Not: Bir doğru parçası üzerinde saat 12.00 ile 13.00 arası 15’ er dakikalık aralıklara bölünerek problem çözülebilir.

2. Selim şekilde görülen dart tahtasına bir ok atıyor. 3 puan alma olasılığı nedir?

[Note: This question was utilized from Glencoe elementary mathematics book of Colins (1999)]

Dersin Adı: Matematik

Öğrenme Alanı: Olasılık ve Istatistik

Alt Öğrenme Alanı: Olay çeşitleri

Kazanım: Bağımlı ve bağımsız olayları açıklar.

Kavramlar: Olay, deney, bağımlı olay, bağımsız olay.

Yöntem ve Teknikler: Buluş Yoluyla Öğrenme

Araç-Gereçler: Olay kağıtları, birim küpler.

Süre: 2 ders saati.

Dersin Işlenişi: Dersin girişinde öğretmen öğrencilere bağımlı ve bağımsız olaylar hakkında ne bildiklerini, günlük hayatta böyle olaylarla karşılaşıp karşılaşmadıklarını sorar. Öğrencilerden cevaplar alınır ve kendi aralarında tartışmaları sağlanır. Daha sonra öğretmen konuyu fen ve teknoloji dersiyle ilişkilendirerek ekosistemdeki canlı ve cansız varlıkların birbirleriyle olan ilişkilerini anlatır. Örneğin, bir denizdeki balıkların yok olursa bu balıklarla beslenen diğer canlıların da yok olabileceğini veya sayısının azalabileceğini söyler. Öğretmen öğrencilere ekosistemdeki madde döngüleri hakkında ne düşündüklerini ve ekosistemdeki üyelerin hayatlarının birbirini etkileyip etkilemediğini sorar. Öğrencilerin sorular üzerinde düşünmeleri ve tartışmaları sağlanır. Derse giriş yapıldıktan sonra etkinliğe geçilir.
Öğrencilerin bağımlı ve bağımsız olayları kavrayabilmeleri için hazırlanmış bu etkinlikte öğrencilere aşağıdaki gibi değişik olayların yazılı olduğu kağıtlar dağıtılır.
OLAYLAR:

A Olayı: Bugün Ayşe'nin doğum günü olması.

B Olayı: Ayşe'nin çok mutlu olması .

C Olayı: Ayşe'nin derslerinde çok başarılı olması.

D Olayı: Ali'nin sol kolunun alçılı olması.

E Olayı: Ali'nin bisikletten düşerek sol kolunu kırması.

F Olayı: Elif'in uzağındaki cisimleri görememesi.

G Olayı: Elif'in gözlük kullanması.

H Olayı: Bugün havanın yağmurlu olması.

I Olayı: Bugün, sabah evden çıkarken Gamze'nin yanına şemsiyesini alması.

Öğrencilerden ikişer olay çekmeleri istenir ve bu olayların gerçekşeşmesinin birbirinden etkilenip etkilenmediği sorulur. Örneğin "Bugün havanın yağmurlu olması" ve "Bugün, sabah evden çıkarken Gamze'nin yanına şemsiyesini alması" olayları çekilmiş olsun. 1. olayın gerçekleşmesi sonucunda ortaya çıkan durumlar 2. olayın olma olasılığını etkilendiği için bu olaylar bağımlı olaylardır. Aynı şekilde "Ali'nin bisikletten düşerek sol kolunu kırması" ve "Elif'in gözlük kullanması" olayları çekilirse bu iki olayın gerçekleşmesi birbirinden etkilenmediği için bağımsız olaylardır. Diğer olaylar için bağımlı ve bağımsız olma durumları aynı şekilde yaptırılır. Bu etkinliğin amacı öğrencilere iki olaydan herhangi birinin gerçekleşmesi diğer olayın olma olasılığı etkiliyorsa bağımlı olaylar olduğunu, etkilenmiyorsa bağımsız olaylar olduğunu keşfettirmektir. Daha sonra öğrencilere aşağıdaki soru sorularak tartışmaları istenir.

Soru: Bir torbanın içinde üzerlerinde 2 bisiklet, 3 boya kalemi, 4 top yazan toplam 9 kâğıt parçası vardır. Birinci çekilişte bisiklet yazılı kâğıt çekilmiştir. Ikinci çekilişi yapacak kişi de bisiklet yazılı kâğıdı çekmeyi istemektedir. Bu kişi, birinci çekilişte çıkan bisiklet yazılı kâğıdı tekrar torbanın içine atarak mı, atmadan mı çekerse şansı daha fazla olur? Düşüncenizi gerekçeleriyle açıklayınız.

Öğretmen bu soruda olayların yer değiştirme sonucu ve yer değiştirmeme sonucu olasılıklarının değişip değişmeyeceğini sorar. Soru üzerinde tartışıldıktan sonra diğer etkinliğe geçilir. Bağımlı ve bağımsız olayların daha da pekiştirilmesini sağlayan bu etkinlikte öğrencilere poşet içinde üzerlerinde değişik hediye isimleri yazılı 6 adet birim küpler dağıtılır. 3 tanesinin üzerinde kalem, 2 tanesinin üzerinde defter, 1 tanesinin üzerinde roman yazmaktadır. Öğrencilerden herbir hediyenin çekilme olasılıklarını hesaplamaları istenir. Daha sonra çekiliş yaparlar. Çektikleri kübü torbaya geri atıp tekrar çekiliş yaparlar. Öğretmen öğrencilere aşağıdaki soruyu sorar:

-Bu durumda 2. kübün çekilme olasılığı 1. kübün çekilme olasılığından etkilenir mi? Niçin?

Daha sonra öğrencilerden bir küp çekmelerini, bu kez torbaya geri atmadan 2. kez çekiliş yapmaları istenir. Öğretmen öğrencilere aşağıdaki soruyu sorar:

-Bu durumda ikinci çekilen kübün olasılığı 1. çekilişten etkilenir mi? Niçin?

Öğrencilerin bu sorular üzerinde düşünmeleri sağlanır. Öğrencilere bu sorularla 1. de olasılıkların birbirinden etkilenmediği, 2. de ise 2. kübün çekilme olasılığının 1. çekilişten etkilendiği keşfettirilir. Daha sonra bağımlı ve bağımsız olaylar için genelleme yapmaları istenir. Dersin sonunda öğrencilerden aşağıdaki soruları çözmeleri istenir.

Değerlendirme:

1. Bir torbadan kalem çekme olayı için aşağıda verilen olayların bağımlı mı bağımsız mı olduğunu belirleyiniz.

a) Kalemi torbaya atmadan ikinci bir kalem çekme

b) Çekilen kalemi tekrar torbaya atarak ikinci bir kalem çekme

3.Bir piyango çekilişinde ilk çekilen rakam 6' dır. Ikinci çekilen rakamın 8 olma olasılığı ilk çekilişte 6 çekilmesinden veya çekilmemesinden etkilenir mi? Bu iki çekiliş nasıl olaylardır? Açıklayınız.

[Note: This question was utilized from Aydın and Beşer (2008)]

4. Aşağıdaki cümlelerde boş bırakılan alanları doldurunuz.

Berkay her iki eliyle zar atacaktır. Sol elinden attığı zarın 5 gelmesi ile sağ elinden attığı zarın 1 gelmesi olayları …………………….. olaylardır.

Bir çekmecede 5 adet siyah kalem ile 7 adet kırmızı kalem vardır. Aslı karanlıkta bu çekmeceden arka arkaya 2 adet kalem alıyor. Ilk alınan kalemin kırmızı olması olayı ile ikinci alınan kalemin siyah olması olayı ………………………. olaylardır.

[Note: This question was utilized from Aydın and Beşer (2008)]