MMSE BASED ITERATIVE TURBO EQUALIZATION FOR ANTENNA SWITCHING SYSTEMS

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ABSTRACT

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In this thesis, we study the performance of an antenna switching (AS) system in comparison to an Alamouti coded system. We analyze the outage probabilities and propose minimum mean-squared error based iterative equalizers for both systems. We see from the outage probability analysis of both systems that the AS system may achieve the same diversity order of the Alamouti coded scheme contingent on the transmission rate and constellation size. In the proposed receiver, MMSE equalization and channel decoding are jointly carried out in an iterative fashion. We use both hard and soft decision channel decoders in our simulations. It is observed that the Alamouti based scheme performs better when the channel state information is perfect. The Alamouti scheme also performs better than the AS scheme when the channel state information is imperfect in hard decision channel decoder case and a random interleaver is used. On the other hand, if a random interleaver is not used, AS scheme performs remarkably better than the Alamouti scheme in hard decision channel decoder case. In a soft decision channel decoder case, when the channel state information is imperfect, the AS scheme performs approximately a 2 dB better than the Alamouti scheme. Moreover, there is approximately a 3 dB performance gain if a soft decision channel decoder is used instead of

hard decision.

Keywords: Antenna switching, Alamouti, MMSE, turbo equalization

ÖZ

ANTEN ANAHTARLAMALI SİSTEMLER İÇİN MMSE TABANLI YİNELEMELİ TURBO DENKLEŞTİRİCİ

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Eylül 2010, 64 sayfa

Bu çalışmada, anten anahtarlama (AS) sisteminin performansı Alamouti kodlu sistem ile karşılaştırılmıştır. Her iki sistem içinde kanal kesinti olasılıkları araştırması yapılmış ve MMSE tabanlı yinelemeli denkleştirici önerilmiştir. Kanal kesinti olasılığı analizinde AS sisteminin Alamouti sistemiyle aynı çeşitleme kazanımına sahip olduğu gözlemlenmiştir. Önerilen alıcı yapısında, denkleştirici ve kanal kodçözücü beraber yinelemeli olarak çalıştırıldı. Benzetimler sırasında kanal kodçözücü olarak sert giriş sert çıkış ve yumuşak giriş yumuşak çıkış kodçözücüler kullanılmıştır. Kanal durum bilgilerinin alıcı tarafında tamamen bilindiği durumda Alamouti sistemi AS sistemine göre daha iyi bir performans göstermiştir. Ayrıca sert çıkış kanal koçözücü durumunda, verici tarafında serpiştirici kullanılırsa da Alamouti sisteminin performansının daha iyi olduğu gözlemlenmiştir. Kanal kodçözücü olarak yumuşak çıkış kodçözücü kullanıldığında ve kanal durum bilgisinin alıcı tarafında tamamen bilinmediği durumda AS sisteminin yaklaşık 2 dB daha iyi bir performans gösterdiği gözlemlendi. Bununla birlikte, yumuşak giriş yumuşak çıkış kanal kodçözücü kullanıldığında ise sert giriş sert giriş kanal kodçözücüye göre yaklaşık 3 dB performans kazancı sağladığı gözlemlenmiştir.

Anahtar Kelimeler: Anten anahtarlama, Alamouti, MMSE, turbo denkleştirici

to my wife Duygu

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CHAPTER 1

INTRODUCTION

The need for higher performance is an omnipresent driving motivation in wireless communication. Hence, increasing the capacity by mitigating the multipath interference of the channel or taking advantage of it has been sought in research. This can be achieved by using diversity at the transmitter or receiver or both. Antenna diversity is one of the most popular ways of diversity used in wireless communication. It is based on the principle of using more than one antennas either at the base station and/or at the mobile terminal. Using multiple antennas at the mobile are not feasible in general because of the size of the mobile terminals. Therefore, transmit diversity techniques are used mostly at base stations in practice to achieve diversity.

In literature, there are several ways to obtain transmit diversity for the case that no channel state information (CSI) is available at the transmitter. One approach is the delay diversity which creates an artificial ISI channel so that spatial diversity is transformed into frequency diversity, [1]. The possible need of multiple RF chains and a high complexity equalization are the main disadvantages of this system. Another method is phase sweeping transmitter diversity (PSTD) which was introduced in [2]. In PSTD, a single data stream is transmitted where each antenna transmit the signals with different time varying phases determined by the phase sweeping function. The superposition of the signals from multiple antennas forms a time-varying channel gain which creates temporal diversity and the analysis of this scheme was performed in [3].

Among all these techniques, the most well-known transmitter diversity technique to achieve high data rate under fading channels is the space-time block coding (STBC). In this technique, special signalling at the transmitter and its proper processing at the receiver provides diversity. The STBC technique has been recently adopted in WCDMA and CDMA 2000 which are third generation (3G) cellular standards [4]. The STBC idea is first introduced by Alamouti in [5] for two transmit and one receive antennas and this technique achieves full diversity and full data rate. In this technique there is a need, though, of full RF chains for both antennas. Moreover, both antennas are used at all times which creates inter-channel interference(ICI) and necessitates antenna synchronization.

In order to avoid the problem of using both transmit antennas at each symbol time a number of methods are proposed in literature. The first technique is spatial modulation which is introduced in [6], in which only one transmit antenna is active at any instant and the active transmit antenna number is an added source of information. The other technique is space shift keying proposed by [7] which is a special case of spatial modulation. In this technique the symbol itself does not contain any information, the information is transmitted by using the different antennas at different times. However it is stated in [6] and [7] that only one antenna is active at each time only with ideal rectangular pulse shape. Due to pulse shaping for bandlimiting purposes, the transmitted symbol will extend a few symbol periods. Therefore, in both techniques the number of RF chains should be set equal to the number of symbol periods in the transmitted pulse.

Another method proposed for using one antenna at each time is antenna switching (AS) technique which is first introduced in [8]. In this technique coded bits are transmitted from different antennas to gain diversity. The advantage of this technique is that only one RF chain is needed at the transmitter as opposed to the other techniques described above so that cost, complexity and power consumption are kept at minimum. In this thesis, the AS technique is studied in a two transmit and one receive antenna system and compared with the Alamouti based scheme which achieves full diversity and full data rate. The outline of the thesis is as follows.

In Chapter 2, channel models used in this work are introduced and then diversity and outage probability concepts are explained. We analyze the outage probability of the AS scheme in a block fading channel case and compare the results with that of the Alamouti coded system. Moreover, equalization methods proposed in literature for the Alamouti system are described.

In Chapter 3, the performance analysis of AS system in multipath fading channel is presented. The MMSE based iterative turbo equalizer for the AS scheme is explained and the performance comparisons are presented for the cases of hard and soft decision channel decoders.

The thesis concludes with Chapter 4.

CHAPTER 2

BACKGROUND INFORMATION

2.1 Channel Models

The wireless channel is susceptible to noise, interference and channel impediments. These impediments change over time and frequency unpredictably due to user movement and environment dynamics [9]. Channel impediments can be divided into two types:

Large-scale fading forms due to path loss and shadowing. Path loss is a theoretical attenuation of the transmitted signal which occurs under free line-of-sight conditions and it is a function of the distance between the transmitter and the receiver. Shadowing is caused by large objects such as hills and buildings between the transmitter and receiver. These obstacles can cause random variations on the received signal strength and this effect is referred as shadow fading and the shadowing.

Small-scale fading, results from the constructive and destructive addition of the multipath signal components. These multipath components are coming from different directions due to reflections from different objects. Thus, their travelling distances may be different from each other. Therefore, the multipath components are not in phase, and may reinforce or extinguish each other in a random fashion. The movement of the user causes continuous and unpredictable variations of the signal phases over time, thus the attenuation is very variable and at some points it is extremely high (deep fades).

We can write the baseband equivalent signal representation of the received signal coming from multiple reflection points as

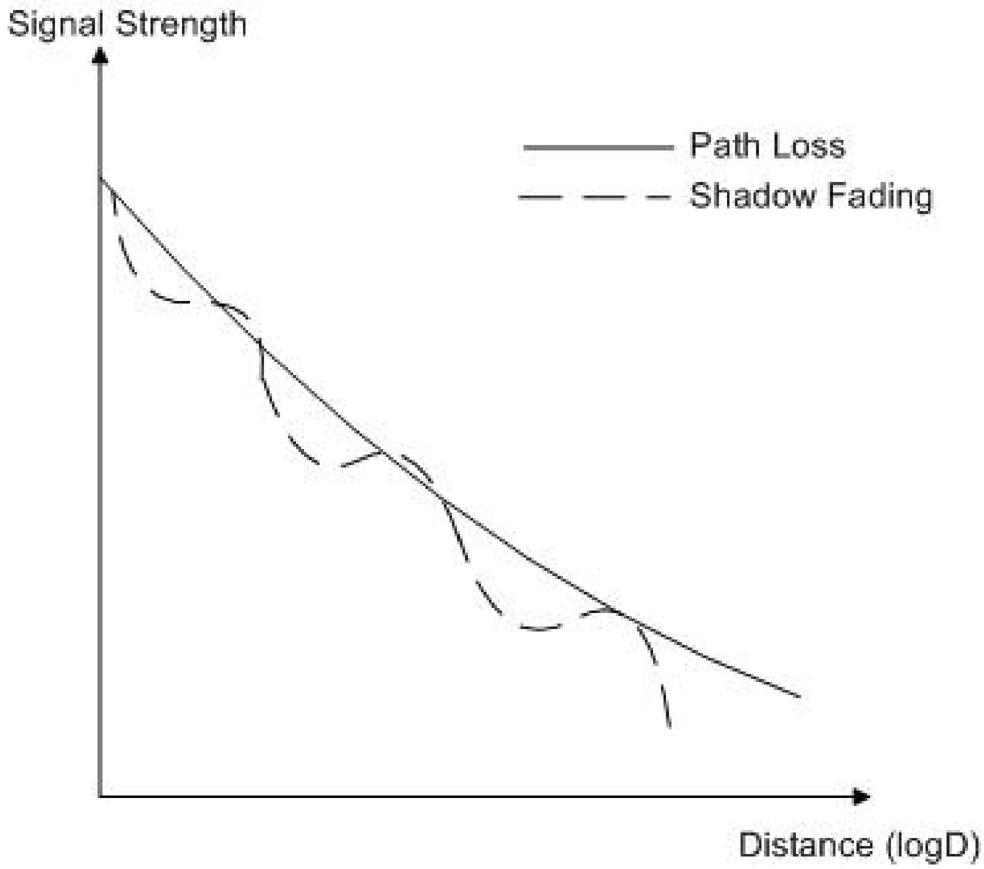

Figure 2.1: Path loss and shadow fading

$$r(t) = \sum_n \alpha_n(t) s(t - \tau_n(t)) \tag{2.1}$$

where α_n is the complex channel fading gain and τ_n is the delay between the 1^{st} and n^{th} paths. Then, one can write a model for this channel as

$$h(t, \tau) = \sum_n \alpha_n(t) \delta(t - \tau_n(t)). \tag{2.2}$$

The distribution of α_n is usually described statistically using the Rayleigh distribution. The probability density function (pdf) of Rayleigh distribution is given as

$$p(r) = \begin{cases} \frac{r}{\sigma^2} exp\left(\frac{-r^2}{2\sigma^2}\right) & 0 \leq r \leq \infty \\ 0 & r < 0 \end{cases} \tag{2.3}$$

where σ^2 is the average power of the received signal.

A Rayleigh fading channel can be either flat or frequency selective and slow or fast depending on the multipath structure of the channel and the relative movement in the medium. Flat fading occurs when the signal bandwidth is smaller than the coherence bandwidth of the channel which is the frequency separation necessary for the decorrelation of channel gain of a frequency selective channel. If the signal bandwidth is larger, then different frequencies undergo independent fading and inter-symbol-interference (ISI) forms.

The channel is said to be a slow fading channel if the symbol period of the transmitted signal is shorter than the coherence time. The coherence time of a fading channel is defined as the time necessary for the decorrelation of the channel gain, and it is calculated as $1/F_m = c/(vF_s)$ where F_m is the maximum Doppler frequency, c the speed of light, v the speed of the mobile and F_s is the carrier frequency of the transmitted signal [11].

In this work, we specifically focus on the static antennas scenario where there is no time variation of the channel gains.

2.1.1 Block Fading Channel

We consider the block fading channel with additive-white Gaussian noise in this section. If the channel has constant gain and linear phase response over the bandwidth which is larger than the bandwidth of the transmitted data, then the channel is flat fading. In a block fading channel model, the channel is flat in one block and change to another independent value at another block. This channel model is used in GSM, EDGE and multicarrier modulation using orthogonal frequencies (OFDM). The discrete time baseband equivalent form of the receive signal can be written as

$$y_k = gx_k + n_k \tag{2.4}$$

where x_k is the input, y_k is the output of the channel and n_k is an additive white Gaussian noise with zero mean and σ_n^2 variance. g is constant over some block length of N and changes to a new independent value according to a distribution $p(g)$. In this thesis the distribution of $p(g)$ is chosen as Rayleigh.

2.1.2 Time-Invariant Multipath Fading Channel

When the signal bandwidth is larger than the coherence bandwidth of the channel, then the channel is frequency selective. In such a channel the multipath delay spread is larger than the inverse of the transmitted data bandwidth and thus there are multiple copies of transmitted signal, which are attenuated, reflected or diffracted combinations of original signal, at the receiver. Stating alternately, the channel induces inter symbol interference (ISI) [12].

In this work, time-invariant frequency selective fading channel is used and the impulse response of the channel is modeled as a finite impulse response (FIR) filter and given as

$$h(t) = \sum_{m=0}^{L-1} a_m \delta (t - mT) \tag{2.5}$$

where L is the order of the channel and T is the symbol time. The complex tap gain a_m is varying block to block according to a Rayleigh density function but it is constant in one block.

Then, the baseband equivalent form of the received signal can be written as

$$y_k = \sum_{m=0}^{L-1} a_m x_{k-m} + n_k \tag{2.6}$$

where x_k, y_k and n_k are defined in the previous section.

2.2 Diversity

In wireless communication, transmitted signal commonly suffers from the power loss which is produced by the fading channels and this power loss deteriorates the performance significantly. In order to reduce the effect of fading, diversity is used. The idea behind the diversity concept is to generate multiple copies of the transmitted signal at the receiver which is made possible by transmitting the signal through more than one independent channels. In such a system, the probability of receiving a signal with an overall small energy equals the probability of having deep fades at all independent channels and this probability is very small. In order to explain the diversity gain that can be obtained from a multipath channel we will derive the probability of error that can be achieved at the receiver. Assume that two paths are received at each time;

$$r_1 = a_1 s + n_1 \tag{2.7}$$

$$r_2 = a_2 s + n_2 \tag{2.8}$$

where a_1 and a_2 are the channel responses of the first and second paths, respectively. Then, the received SNR at each time with optimal processing [13] is given as

$$\gamma_b = \frac{E_s}{N_0} \sum_{k=1}^{2} a_k^2 \tag{2.9}$$

where a_k is the amplitude of the k^{th} channel response.

The single branch average received SNR can be written as

$$\gamma_c = \frac{E_s}{N_0} E\left(a_k^2\right). \tag{2.10}$$

The probability of error of a fading channel is given in [13] as

$$P_e = \int_0^{\infty} P_e(\gamma_b)\, p(\gamma_b)\, d\gamma_b \tag{2.11}$$

Since a_k has a Rayleigh distribution then $\sum_{k=1}^{2} a_k^2$ has a Chi-square distribution with four degrees of freedom and the probability density function of γ_b for this case is given in [13] as

$$p(\gamma_b) = \frac{1}{\gamma_c^2} \gamma_b e^{-\gamma_b/\gamma_c} \tag{2.12}$$

Therefore, the closed form of the Eqn. (2.11) can be expressed by using [13] as

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^2 \sum_{k=0}^{1} \begin{pmatrix} k+1 \\ k \end{pmatrix} \left[\frac{1}{2}(1+\mu)\right]^k \tag{2.13}$$

where μ is defined as

$$\mu = \sqrt{\frac{\gamma_c}{1+\gamma_c}} \tag{2.14}$$

At high average SNR, the term $\left[\frac{1}{2}(1+\mu)\right] \approx 1$ and the term $\left[\frac{1}{2}(1-\mu)\right] \approx 1/4\gamma_c$. Therefore, the error probability at the high SNR region is given as

$$P_e \approx (1/4\gamma_c)^2 \tag{2.15}$$

If we have one fading path, then the probability of error at the receiver is $P_e \approx 1/4\gamma_c$, [13]. Therefore, there is a performance improvement if we use more than one independent channels.

There are many forms of diversity utilized in wireless communications. We will be interested in three of them in this thesis, time (temporal), frequency and antenna (spatial) diversity.

2.2.1 Time Diversity

The main idea of the time diversity method is to transmit the same signal at different time periods, as depicted in Figure 2.2. In order to make the transmitted paths independent from each other, the time difference between the transmitted signals, Δt in Figure 2.2, are larger than the coherence time of the fading channel [9]. To provide time diversity, interleavers and error control coding are used.

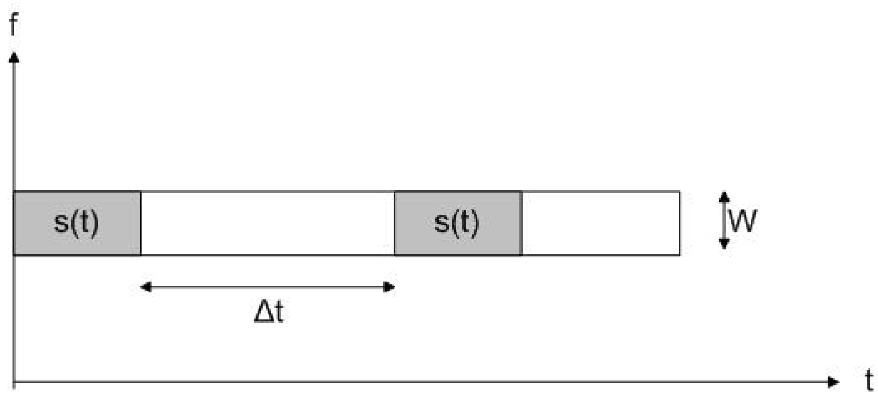

Figure 2.2: Time diversity

2.2.2 Frequency Diversity

Frequency diversity is an another diversity technique. In this technique, the signal is transmitted on different carriers. In order to obtain a diversity the frequency separation between the carriers, Δf in Figure 2.3, is larger than the coherence bandwidth of the channel. Therefore, the channel gains at distant carrier frequencies are uncorrelated and so the signals at the receiver from each replica. The most well-known systems that use this idea are frequency hopping, spread spectrum and OFDM systems.

The frequency diversity idea described above is mainly used for the frequency selective channels. There is not a performance improvement at flat fading channels, where the frequency response of the channel is constant over the whole transmission band [14].

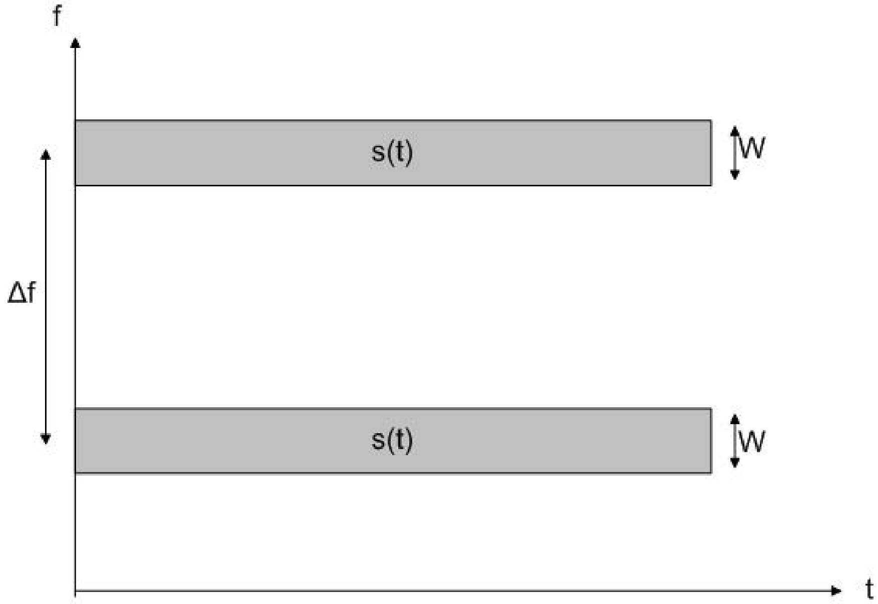

Figure 2.3: Frequency diversity

2.2.3 Antenna Diversity

Antenna diversity is used to improve the quality of the wireless link by the use of multiple antennas. If the distance between antennas are sufficiently far, then the channel gains between different antenna pairs are uncorrelated. Hence independent signals are obtained at the receiver. The required separation depends on the scattering environment and carrier frequency [15]. This technique can be divided into two groups: transmitter diversity shown in Figure 2.4, receiver diversity shown in Figure 2.5.

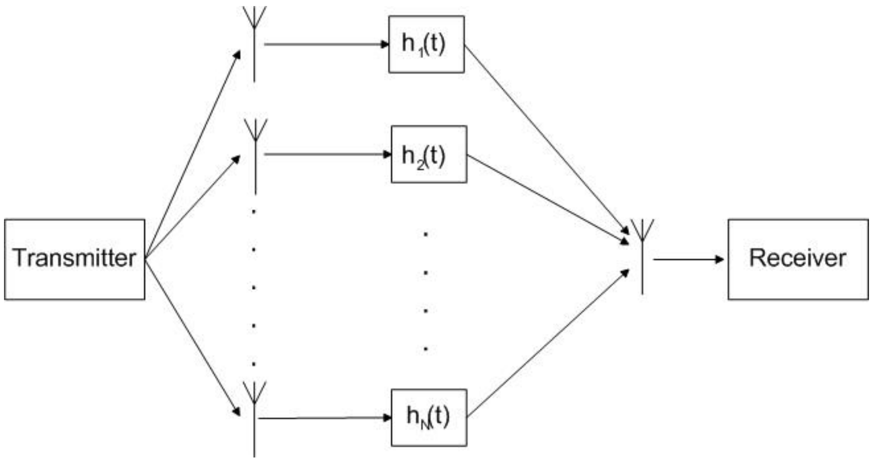

Figure 2.4: Transmit diversity

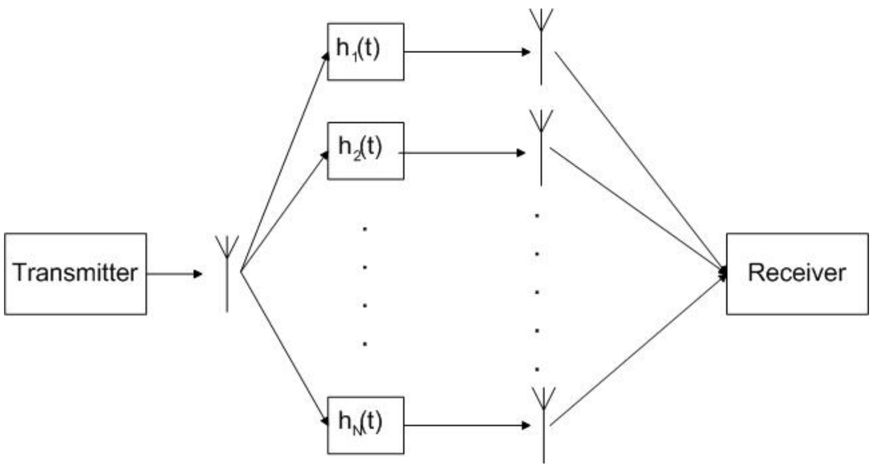

Figure 2.5: Receive diversity

2.2.3.1 Receiver Diversity

In receiver diversity, there are N receiver antennas and the signal that passed through independent fading paths are combined after these N antennas. Combining can be performed in several ways, such as selection combining, equal gain combining and maximum ratio combining, which are summarized below.

Selection Combining Selection combining is the simplest diversity combining method. In such a combining system, the signal with the largest instantaneous SNR is selected as the output signal as shown in Figure 2.6.

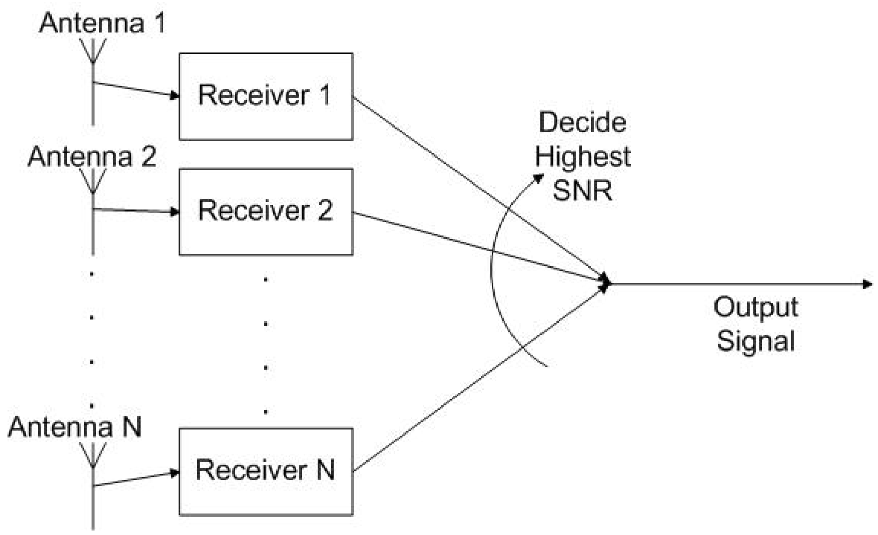

Figure 2.6: Selection Combining

Equal Gain Combining In selection combining technique, receiver uses only one of the incoming signal, but in this scheme, to increase the SNR of output signal, all receive signals are combined. Due to the fading effects, the phase of all signals may be different than each other. Therefore, combining is done by multiplying each signal with a phase of θ_i as shown in figure 2.7.

Maximal Ratio Combining In EGC, if one of the receive signal has a low SNR compared with the other branches, then this low SNR may decrease the output SNR. In order to overcome this effect, each branch is first multiplied with some weighting factor ω_i and then with

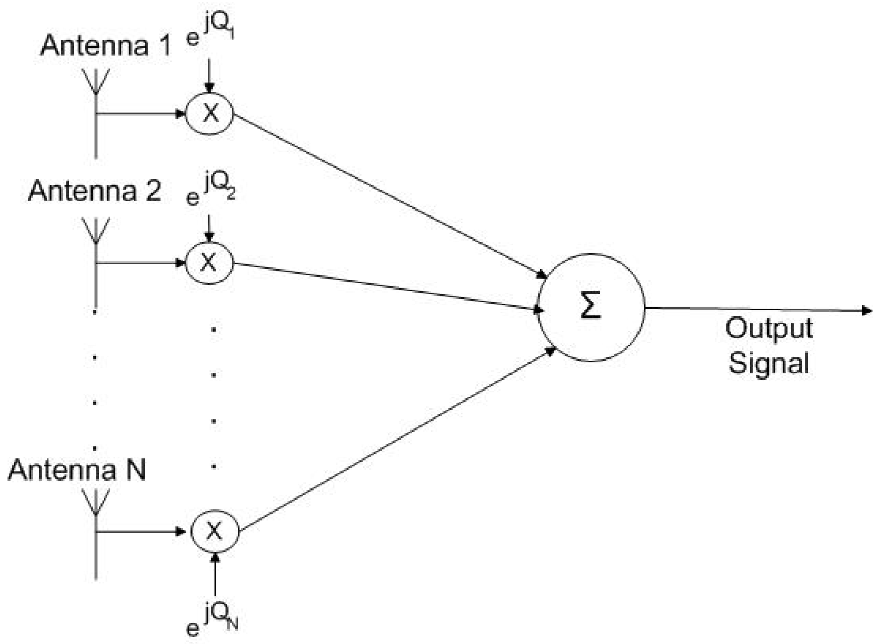

Figure 2.7: Equal Gain Combining

a phase θ_i as shown in Figure 2.8. To maximize the SNR of the output signal, the branch with the highest SNR is multiplied with a higher weighting value. The optimal weighting coefficients can be easily formed based on the matched filtering idea.

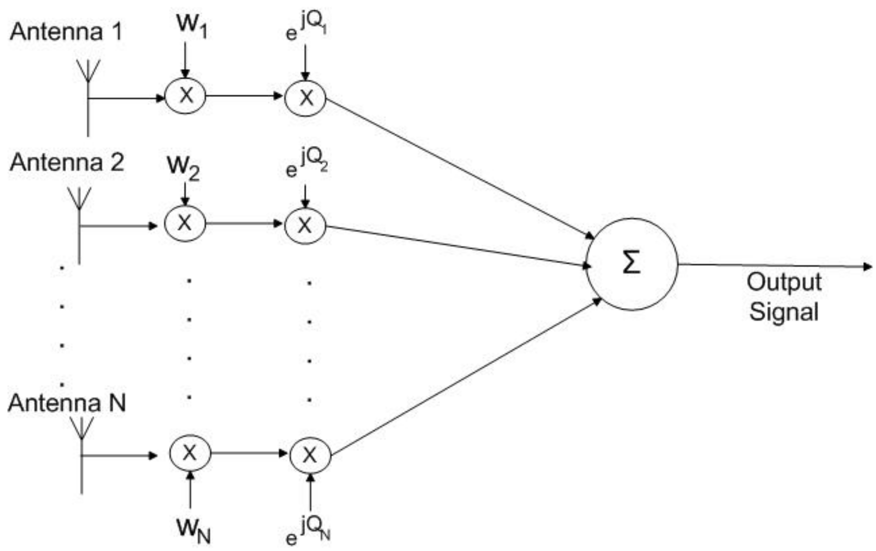

Figure 2.8: Maximal Ratio Combining

2.3 Transmit Diversity

In transmit diversity, the signal is sent by using more than one transmit antennas. If the spacing between the antennas is sufficient then the channel gains at time t between each transmit and receiver antenna pair, $h_i(t)$, are independent from each other. There are several transmitter diversity techniques in literature, but in this thesis the Alamouti and the antenna switching schemes will be investigated and these techniques will be explained in this section.

2.3.1 Alamouti System

Space time block coding, a well-known transmitter diversity technique, is first introduced by Alamouti in [5] for 2 transmit and 1 receive antennas and this system is known as the Alamouti scheme in literature.

In this scheme, the encoder takes 2 consecutive modulated symbols, s_1 and s_2 and encode them according to the matrix given by

$$\mathbf{S} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix}. \tag{2.16}$$

The first row is transmitted in the first symbol period and the second is transmitted in the next symbol period. During the first symbol period s_1 is transmitted from the first antenna and s_2 is transmitted from the second antenna. In the next symbol period s_1^* is transmitted from the second antenna and $-s_2^*$ is transmitted from the first. So, both space and time encoding are performed.

Since the rows and columns of the encoding matrix are orthogonal to each other, the receiver can decode the signals s_1 and s_2 by making a simple linear operation. This statement was validated in [16] which generalized the idea to more than two antennas.

If the channel gain to the receive antenna from the first transmit antenna is denoted by h_1 and from the second transmit antenna by h_2, the received signals in the discrete baseband equivalent at consecutive time epochs are

$$r_1 = h_1 s_1 + h_2 s_2 + n_1, \tag{2.17}$$

$$r_2 = -h_1 s_2^* + h_2 s_1^* + n_2 \tag{2.18}$$

where r_1 and r_2 are the received signal at time t and t+T respectively and

$$h_1 = \alpha_1 e^{j\theta_0}, \tag{2.19}$$

$$h_2 = \alpha_2 e^{j\theta_1}. \tag{2.20}$$

The combining operation is done by

$$\hat{s}_1 = r_1 h_1^* + r_2^* h_2, \tag{2.21}$$

$$\hat{s}_2 = r_1 h_2^* - r_2^* h_1. \tag{2.22}$$

So one obtains

$$\hat{s}_1 = \left(\alpha_1^2 + \alpha_2^2\right) s_1 + h_1^* n_1 + h_2 n_2, \tag{2.23}$$

$$\hat{s}_2 = \left(\alpha_1^2 + \alpha_2^2\right) s_2 + h_2^* n_1 - h_1 n_2^*. \tag{2.24}$$

Detection of the symbols s_1 and s_2 are directly done based on $\hat{s}_1$ and $\hat{s}_2$, respectively.

2.3.2 Antenna Switching

Antenna switching is an another transmitter diversity technique which was first proposed in [8]. In this scheme there is only one RF stage and the output of this is sent to a switch, which passes the signal to only one antenna at each time.

Antenna switching approach is used to transform the antenna diversity into the time diversity by the help of channel coding and interleaving together or precoding. A simple example that explains the operation of this scheme is transmitting a data frame twice, one from the first antenna and the other from the second antenna. A diversity of two can be attained by this scheme. This repetition code based operation can be generalized with other forms of coding.

The main idea in AS is to transmit different code bits of the same symbol from different antennas. In this thesis, we assume that there are 2 transmit antennas for AS scheme and first half of the block is transmitted using the first antenna and the other half is transmitted from the second antenna. In order to gain diversity in this scheme, interleaver is needed to transmit different code bits of the same user symbol from different antennas. One can say that diversity can be obtained without using interleaver by switching antennas in one bit period. But, in this case we need to switch antennas rapidly and this results in a bandwidth expansion of the transmitted signal and also increases the complexity of the receiver.

2.4 Outage Probability

Shannon in [17] defines the capacity of a channel as the maximum rate of communication at which transmitted data can be received with arbitrarily small probability of error and if data source, X, and received data, Y, are random variables, then the channel capacity is the maximum mutual information between them

$$C = \max I(X;Y) \tag{2.25}$$

where maximization is taken over all possible probability distributions $f_X(x)$ of X.

Consider a discrete-time additive-white Gaussian noise (AWGN) channel with the input/output relationship

$$y_k = x_k + n_k \tag{2.26}$$

where y_k is the channel output, x_k is the channel input and n_k is the white Gaussian noise. The capacity of an AWGN channel equals

$$I(X;Y) = log_2\left(1 + \frac{E_s}{N_o}\right) \tag{2.27}$$

where $\frac{E_s}{N_o}$ is the symbol signal-to-noise power ratio. The capacity achieving input distribution is Gaussian.

If the channel is a bandlimited channel with a bandwidth of B and a transmit power of P, then the channel SNR is constant and given by $\gamma = \frac{P}{N_o B}$ which also equals $\frac{E_s}{N_o}$. The capacity of this channel is given by Shannon's formula [17]:

$$C = B\log_2(1+\gamma) \quad bps/Hz. \tag{2.28}$$

The bandwidth B will be omitted in the remainder of this thesis since the rates will be considered in a normalized setting per Hertz.

Now, assume a fading channel for which channel state information is known only at the receiver. In this case there are two channel capacity definitions that are relevant to system design: Shannon capacity, also called ergodic capacity and outage capacity [9]. The Shannon capacity is defined as the maximum rate that can be transmitted over a channel with arbitrarily small probability of error. Since the transmitter does not know the CSI, the transmission rate need be taken constant. Thus, poor channel states reduce the Shannon capacity [9]. An alternative capacity definition for the fading channel with receiver CSI is outage capacity. The outage capacity is defined as the rate that can be transmitted with some outage probability where outage probability is defined as the probability that the transmitted sequence cannot be decoded with an arbitrarily small probability [9].

2.4.1 Ergodic Capacity

Ergodic capacity of a fading channel with receiver CSI is given in [9] as

$$C = \int_0^\infty \log_2(1+\gamma)\, p(\gamma) d\gamma. \tag{2.29}$$

The formula in (2.29) is equal to the Shannon capacity of an AWGN channel with SNR γ averaged over the distribution of γ.

2.4.2 Outage Capacity

The outage capacity is particularly used for the slowly fading channels where the channel gain is constant over a large number of transmitted symbols and change to a new independent value based on the fading distribution. In this case, if the received SNR is γ during a burst then data transmitted with a rate of $\log_2(1+\gamma)$ at that burst can be decoded with an arbitrarily small probability of error. But, since the transmitter does not know the CSI, the transmission rate must be fixed a value, R. Then, if the received SNR does not result in an instantenous capacity larger than the transmission rate, the received bits at that burst cannot be decoded correctly. The channel is then said to be in outage at that burst. Thus, the outage probability of a channel is defined as

$$P_{out} = P\left(\log_2(1+\gamma) < R\right) \tag{2.30}$$

We will give the outage probability analysis for the different systems in this section. We assume in here that channel state information is known at the receiver side.

2.4.2.1 Outage Probability of SISO Systems

In a SISO (single-input single-output) system, where there are one transmit and one receive antennas at the system, the received data can be written as

$$y = hx + n \tag{2.31}$$

where h is a zero mean Gaussian random variable with variance is 1. Then the outage probability of this system can be written as

$$P_{out} = P\left(\log_2\left(1 + SNR|h|^2\right) < R\right) \tag{2.32}$$

$$= P\left(|h|^2 < \frac{2^R - 1}{SNR}\right). \tag{2.33}$$

Since h has a Gaussian distribution, $|h|^2$ is exponentially distributed. By using the definition of exponential distribution in [19], one obtains

$$P_{out} = 1 - exp^{-\frac{2^R-1}{SNR}}. \tag{2.34}$$

Also, in high SNR region, Eqn. (2.34) becomes

$$P_{out} = \frac{2^R - 1}{SNR} \tag{2.35}$$

In Eqn. (2.35), outage probability is decaying as $1/SNR$ for a fixed rate of R. Therefore, the diversity of this channel is equal to one [20], where diversity order is usually defined as

$$d = -\lim_{SNR\to\infty} \frac{\log P_{out}}{\log SNR} \tag{2.36}$$

2.4.2.2 Outage Probability of SIMO Systems

In this case, there are multiple antennas, n_r, at the receiver side in order to increase the diversity order of the channel. The received vector in a SIMO, single-input multi-output, system can be written as

$$\mathbf{y} = \mathbf{h}x + \mathbf{n} \tag{2.37}$$

where $\mathbf{h}$ is a vector of n_r elements.

The optimal processing is based on the matched filtering which gives way to

$$P_{out} = P\left(\log_2\left(1 + SNR\,||\mathbf{h}||^2\right) < R\right) \tag{2.38}$$

which yields

$$P_{out} = P\left(||\mathbf{h}||^2 < \frac{2^R - 1}{SNR}\right) \tag{2.39}$$

Since the components of $\mathbf{h}$ has a Gaussian distribution, then $||\mathbf{h}||^2$ is a Chi-square distributed with $2n_r$ degree of freedom [21]. The probability density function of $||\mathbf{h}||^2$ can be written by using the definition in [13] and [20] as

$$f_{||\mathbf{h}||^2}(x) = \frac{1}{(n_r - 1)!} x^{n_r - 1} e^{-x} \quad x \geq 0. \tag{2.40}$$

Using this density, one can show at high SNR $\left(\frac{2^R-1}{SNR} << 1\right)$ that

$$P_{out} \approx \frac{\left(2^R - 1\right)^{n_r}}{n_r! SNR^{n_r}}. \tag{2.41}$$

In Eqn. (2.41), outage probability is decaying as $1/SNR^{n_r}$ for a fixed rate of R. Therefore, the diversity of this channel is equal to n_r [20].

2.4.2.3 Outage Probability of MISO Systems

In this case, there are multiple antennas, n_t, at the transmitter side. Received data in a MISO, multi-input single-output, system can be written as

$$y = \mathbf{h}\mathbf{x} + n \tag{2.42}$$

where $\mathbf{h}$ is $1 \times n_t$ and $\mathbf{x}$ is $n_t \times 1$ vectors.

When $\mathbf{h}$ is known at the transmitter side, the optimum $\mathbf{x} = \frac{\mathbf{h}}{|\mathbf{h}|} s$ where s is a scalar information bearing signal and $\frac{\mathbf{h}}{|\mathbf{h}|}$ is a normalized beamformer. The corresponding outage probability can be written as

$$P_{out} = P\left(\log_2\left(1 + SNR\,||\mathbf{h}||^2\right) < R\right) \tag{2.43}$$

which yields

$$P_{out} = P\left(||\mathbf{h}||^2 < \frac{2^R - 1}{SNR}\right). \tag{2.44}$$

The analysis follows from that of the SIMO case and

$$P_{out} \approx \frac{\left(2^R - 1\right)^{n_t}}{n_t!SNR^{n_t}}. \tag{2.45}$$

In Eqn. (2.45), outage probability is decaying as $1/SNR^{n_t}$ for a fixed rate of R. Therefore, the diversity of this channel is equal to n_t [20], when $\mathbf{h}$ is known at the transmitter side.

2.4.2.4 Outage Probability of Antenna Switching and Alamouti Schemes

We assume that the channel is a block fading channel and the signal model in the discrete time baseband equivalent form is

$$y_k = \mathbf{h}\mathbf{x}_k + n_k \tag{2.46}$$

where y_k is the received signal at time k, h is the $1 \times N_t$ channel gain array and x_k is the transmit signal vector and n_k is the zero mean circularly symmetric complex Gaussian (ZMC-SCG) white noise with variance N_0. We have assumed that a total transmitted average power constraint is imposed so that the trace of the input correlation matrix is equal to E_s.

In the Alamouti scheme, since each encoded symbol is transmitted through both antennas, the symbol energy must be set to $E_s/2$. Therefore, the received SNR is

$$\gamma = \frac{E_s}{2N_0} \|h\|^2 \tag{2.47}$$

If the distribution of the input sequence is Gaussian then the capacity of the Alamouti scheme is calculated as

$$C_{alamouti} = \log_2 (1 + \gamma) \tag{2.48}$$

for a given h.

Since no channel state information exists at transmitter, outage rate rather than the ergodic capacity is the parameter to consider [15]. The outage probability of the Alamouti scheme can be written as

$$P_{out} = P\,(C_{alamouti} < R) \tag{2.49}$$

where R is the transmission rate, which is a fixed value.

In the antenna switching (AS) scheme, there is a single signal produced at the output of the RF stages where the RF signal is fed into a switch before given to the antennas. The switch passes the signal to only one antenna at each time. One packet of data is divided into M subintervals equal in length and different antennas transmit in different subintervals. In our particular antenna switching scheme, M is equal to 2. Therefore, the first part of the frame is transmitted using one antenna and the other part is transmitted using the other antenna.

Based on [22], the instantaneous capacity of this scheme can be written for the Gaussian input sequence distribution as

$$C_{AS} = \frac{1}{2} \sum_{k=1}^{2} \log_2 \left(1 + \frac{E_s}{N_0} |h_k|^2\right) \tag{2.50}$$

where h_k is equal to the channel gain of the k^{th} antenna. It is easy to see by Jensen's inequality that (2.50) is bounded by the channel capacity. For a certain transmission rate R the outage probability is defined as

$$P_{out} = P(C_{AS} < R) \tag{2.51}$$

We will compare outage probabilities based on Monte Carlo simulations. We assumed Rayleigh fading channel where all channel gains are independently and identically distributed zero mean circularly symmetric complex Gaussian (ZMCSCG) with variance 1. When the received SNR in a block is less than the transmission rate, then an outage is recorded and at least 50 outage events have been recorded for each point in the figure.

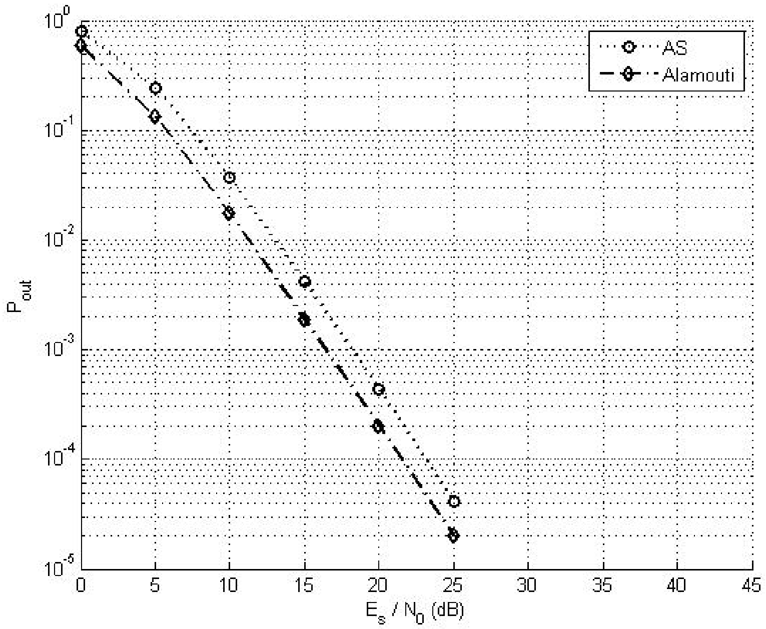

Figure 2.9: Block fading channel outage probability for a rate of R=1

We can see from Figure 2.9 and Figure 2.10 that when the transmission rate is larger, then the offset between the outage probabilities of AS and Alamouti schemes is larger. For the outage probability of 10^{-2}, when rate is equal to 1, the Alamouti system is 2 dB better than the AS system and when rate is equal to 4, the SNR offset between the Alamouti and AS systems is equal to 10 dB. This offset stems from the non-full-rate operation of AS scheme. However, it is observed that the Alamouti scheme is not significantly superior to AS scheme and AS also achieves full diversity when the input distrubition is Gaussian. The AS scheme has a transmitter with a single RF chain and it performs as good as the Alamouti scheme which has 2 RF chains.

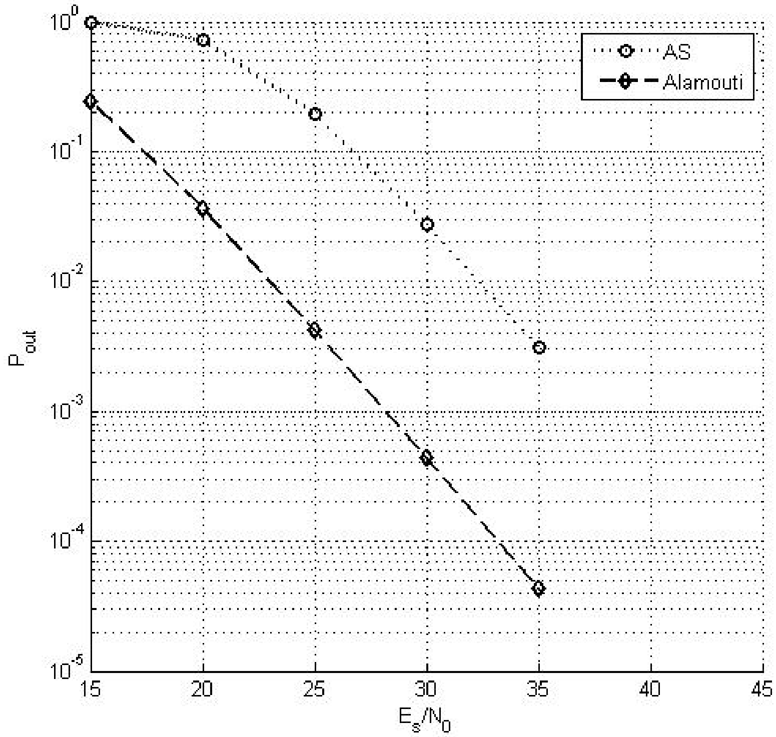

Figure 2.10: Block fading channel outage probability for a rate of R=4

The results given in Figure 2.9 and Figure 2.10 are obtained for the Gaussian input distrubitions. But, when the constellation size is finite, we have to look the rate-diversity tradeoff formula, which is given as [22]

$$d_F(R) \leq 1 + \lfloor F(1 - R/S) \rfloor \tag{2.52}$$

where $d_F(R)$ is the maximum coded diversity possible for a block fading channel with F independent blocks, R is the transmission rate in bits/symbol, S is the constellation size in logarithm-2. We know that Alamouti scheme has full diversity at all rates and constellation sizes. But in AS scheme, in order to achieve diversity of two, constellation size must be bigger than $2R$. For example, if the rate is equal to 1, then to attain diversity of two, we have to use QPSK or larger constellation sizes and also for the rate of 2, we have to use at least 16QAM.

2.5 Equalization

In a wireless communication, there are multiple copies of the transmitted signal at the receiver due to the multipath fading channel. Multiple copies may cause an irreducible error floor if the symbol period of the modulation type is on the same order of the channel time delay spread, which is defined as the time between the first arrival of the received signal component and the last arrival. To reduce the effect of ISI caused by delay spread, there are many techniques used at the transmitter and receiver sides. At the transmitter side, spread spectrum or multicarrier modulation can be used. Equalization can be used at receiver. The equalization concepts that are used for the Alamouti system will be introduced next.

2.5.1 Equalization Methods for Alamouti System

The Alamouti scheme has been introduced to combat the fading effects of wireless communication by providing antenna diversity as described in Section 2.3. But when the Alamouti coded signals are transmitted through a frequency selective fading channel which introduce ISI, there will be an error floor if a simple receiver designed for a flat fading channel is used [25]. Therefore, equalization is necessary in a such system. There are several equalization techniques proposed in the literature for the Alamouti scheme. We will give information about widely linear (WL) and MLSE equalizers and then we will derive the MMSE equalizer for the Alamouti system which is the equalizer model used in this thesis. Before giving information about equalizer techniques, the system model of the Alamouti system in a time-invariant multipath fading channel will be explained.

The signal s is first fed into the space-time encoder which generates the transmit sequence according to

$$s_1(k) = \begin{cases} s(2n) & k = 2n \\ -s^*(2n+1) & k = 2n+1 \end{cases} \tag{2.53}$$

$$s_2(k) = \begin{cases} s(2n+1) & k = 2n \\ s^*(2n) & k = 2n+1 \end{cases}. \tag{2.54}$$

The signals $s_1(k)$ and $s_2(k)$ are then passed through the multipath fading channel. The received signal at the receiving end becomes

$$r(n) = \sum_{k=1}^{2} \sum_{m=1}^{L} (h_k(m)\, s_k(n-m)) + z(n) \tag{2.55}$$

where L is order of the channel.

2.5.1.1 Widely Linear Equalizer

In a direct receiver model, the first channel is equalized into a ISI free channel and then space time decoding is performed, [23] and [24]. In this technique, linear equalization with transfer matrix of **F** is performed. The coefficients of this transfer matrix can be selected either based on the minimum-mean square error (MMSE) or the zero-forcing (ZF) criterion [25]. The ZF equalizer is stable if and only if [26]

$$\forall z \text{ with } |z| = 1 : rank(H(z)) = N_t \tag{2.56}$$

where H is the channel matrix and N_t is the number of the transmit antennas.

Eqn.(2.56) is not valid if there are multiple transmit antennas and a single receive antenna, which is the case in the Alamouti scheme [25]. Therefore, ZF equalization is not used in a direct approach model. Also, we know that MMSE equalization has a poor performance if the ZF equalization is not valid [25].

As an alternative approach to the direct approach, the channel and the ST encoder are modelled jointly as an equivalent channel and an equalizer filter matrix F whose coefficients are selected using the zero-forcing (ZF) approach is inflicted on the received signal. To derive the widely linear (WL) ZF equalizer, we use the approach in [27] that the received signal is cyclostationary. Time-invariant equalizer filters are obtained if a vector containing the polyphase components of each received sequence is taken as equalizer input [28]. Therefore, we have to construct a vector consisting of received symbols and their complex-conjugates.

$$\mathbf{r}(n) = [r(2n) \quad r^*(2n) \quad r(2n+1) \quad r^*(2n+1)]^T \tag{2.57}$$

The vector form of the received signal can be written in terms of the transmitted signal and the channel response as

$$\mathbf{r}(n) = \sum_{m=1}^{\tilde{L}} \left(\bar{\mathbf{H}}(m)\, \mathbf{s}(n-m) \right) + \mathbf{z}(n) \tag{2.58}$$

where $\tilde{L} = \lceil L/2 \rceil$ and

$$\mathbf{s}(n) = [s(2n) \quad s^*(2n) \quad s(2n+1) \quad s^*(2n+1)]^T, \tag{2.59}$$

$$\mathbf{z}(n) = [z(2n) \quad z^*(2n) \quad z(2n+1) \quad z^*(2n+1)]^T, \tag{2.60}$$

$$\bar{\mathbf{H}}(m) = \begin{bmatrix} h_1(2m) & h_2(2m-1) & h_2(2m) & -h_1(2m-1) \\ h_1(2m+1) & h_2(2m) & h_2(2m+1) & -h_1(2m) \\ h_2^*(2m-1) & h_1^*(2m) & -h_1^*(2m-1) & h_2^*(2m) \\ h_2^*(2m) & h_1^*(2m+1) & -h_1^*(2m) & h_2^*(2m+1) \end{bmatrix}. \tag{2.61}$$

The WL equalizer can be constructed with the equalizer filter $\mathbf{F}$ obtained through

$$\mathbf{F}\tilde{\mathbf{H}} = \mathbf{I}_4 \tag{2.62}$$

where $\tilde{\mathbf{H}}$ is a $4(N_1+N_2) \times 4\left(N_1+N_2+\tilde{L}\right)$ channel matrix and it is defined as

$$\tilde{\mathbf{H}} = \begin{bmatrix} \bar{\mathbf{H}}(0) & \bar{\mathbf{H}}(1) & .... & \bar{\mathbf{H}}\left(\tilde{L}\right) & 0 & .... & ..... & 0 \\ 0 & \bar{\mathbf{H}}(0) & .... & .... & \bar{\mathbf{H}}\left(\tilde{L}\right) & 0 & ..... & 0 \\ . & . & & & & & & . \\ . & . & & & & & & . \\ . & . & & & & & & . \\ 0 & 0 & .... & .... & \bar{\mathbf{H}}(0) & .... & ..... & \bar{\mathbf{H}}\left(\tilde{L}\right) \end{bmatrix} \tag{2.63}$$

2.5.1.2 Maximum-Likelihood Sequence Estimation

In a multipath fading channel, if the channel order is low, then an optimum Maximum-Likelihood Sequence Estimation (MLSE) filtering may be employed to the received signal. For a general MIMO transmission disturbed with white Gaussian noise the aim of the MLSE technique is to minimize the metric [29]

$$\Lambda = \sum_{n=1}^{N} \left\| r(n) - \left[\sum_{k=1}^{L} \mathbf{H}(k)\, s(k-n) \right] \right\|^2 \tag{2.64}$$

where N is the length of the block, L is the order of the channel, $\mathbf{H}$ is the channel matrix and s is the transmitted signal vector.

We again consider the polyphase components of the received sequences of all antennas. Because the optimum receiver minimizing the error probability given in [29] and [30] is now applied, a receiver extension in the sense of WL processing does not yield any performance gain and complex conjugates of the received signals are not used in here.

We define the received vector which includes two consecutive received symbols as

$$\mathbf{r}(n) = [r(2n) \quad r(2n+1)]^T \tag{2.65}$$

which satisfies

$$\mathbf{r}(n) = \sum_{m=0}^{\tilde{L}} \left(\bar{\mathbf{H}}(m)\, \mathbf{s}(n-m) \right) + \mathbf{z}(n) \tag{2.66}$$

where

$$\mathbf{s}(n) = [s(2n) \quad s(2n+1) \quad s^*(2n) \quad s^*(2n+1)]^T, \tag{2.67}$$

$$\mathbf{z}(n) = [z(2n) \quad z(2n+1)]^T, \tag{2.68}$$

$$\bar{\mathbf{H}}(m) = \begin{bmatrix} h_1(2m) & h_2(2m) & h_2(2m-1) & -h_1(2m-1) \\ h_1(2m+1) & h_2(2m+1) & h_2(2m) & -h_1(2m) \end{bmatrix}. \tag{2.69}$$

The maximum-likelihood metric given below have to be calculated in each symbol time, and a Viterbi algorithm may be used to find the most likely sequence for the transmitted signal recursively

$$\Lambda(\bar{s}) = \sum_{n=1}^{\frac{N}{2}-1} \left\| \mathbf{r}(n) - \sum_{k=1}^{\lceil L/2 \rceil} \bar{\mathbf{H}}(k)\, \mathbf{s}(k-n) \right\|^2 \tag{2.70}$$

where N is the length of the block and L is the order of the channel.

2.5.1.3 MMSE based Turbo equalization

The techniques explained above are restricted to non-iterative equalization. On the other hand, since channel coding is used to decrease the effect of random noise and fluctuations, performance of the receiver may be improved by using iterative equalizers compared to the non-iterative ones. Turbo equalization is an iterative equalization approach, in which MMSE equalization and MAP decoder are used together and exchange apriori probabilities. Initially, the computational complexity of turbo equalization techniques are very high when large block lengths or large constellations are used. But, a low-complexity MMSE based turbo equalization is proposed for the SISO case in [31] and for the SIMO case in [32]. Moreover, fractional turbo equalization is proposed for the MIMO channels in [33].

In this section, an MMSE based turbo equalization for the Alamouti system is explained. As stated in Section 2.5.1.1, if the channel and space-time encoders are processed seperately at the receiver, then the ZF solution is not valid [25] and MMSE equalization yields a poor performance as well. WL processing defined in Section 2.5.1.1 is used in here. As described before the received signal and its complex conjugate are processed together in a WL receiver. The vectors used in here are defined at Section 2.5.1.1 for received signal, transmit signal, noise and channel matrix, respectively.

We use sliding-window model defined in [34] for the MMSE equalization as

$$\tilde{\mathbf{r}}_n = \tilde{\mathbf{H}}\tilde{\mathbf{s}}_n + \tilde{\mathbf{z}}_n \tag{2.71}$$

where

$$\tilde{\mathbf{r}}_n = [\mathbf{r}^T\,(n+N_2)\ \ldots..\ \mathbf{r}^T\,(n)\ \ldots\ldots.\ \mathbf{r}^T\,(n-N_1)]^T, \tag{2.72}$$

$$\tilde{\mathbf{s}}_n = [\mathbf{s}^T\,(n+N_2)\ \ldots..\ \mathbf{s}^T\,(n)\ \ldots\ldots.\ \mathbf{s}^T\,(n-N_1-\lfloor L/2\rfloor)]^T, \tag{2.73}$$

$$\tilde{\mathbf{z}}_n = [\mathbf{z}^T\,(n+N_2)\ \ldots..\ \mathbf{z}^T\,(n)\ \ldots\ldots.\ \mathbf{z}^T\,(n-N_1)]^T \tag{2.74}$$

and a $4(N_1+N_2)\times 4\left(N_1+N_2+\tilde{L}\right)$ channel matrix is defined as

$$\tilde{\mathbf{H}}=\begin{bmatrix}\bar{\mathbf{H}}(0) & \bar{\mathbf{H}}(1) & .... & \bar{\mathbf{H}}\left(\tilde{L}\right) & 0 & .... & ..... & 0\\ 0 & \bar{\mathbf{H}}(0) & .... & .... & \bar{\mathbf{H}}\left(\tilde{L}\right) & 0 & ..... & 0\\ . & . & & & & & & .\\ . & . & & & & & & .\\ . & . & & & & & & .\\ 0 & 0 & .... & .... & \bar{\mathbf{H}}(0) & .... & ..... & \bar{\mathbf{H}}\left(\tilde{L}\right)\end{bmatrix} \tag{2.75}$$

where N_1 and N_2 are the lengths of the non-causal and causal parts of the equalizer filter.

In this equalizer technique MMSE equalization and channel decoding are jointly carried out at each iteration by using a priori probabilities from the previous iterations. At the beginning of each iteration (except in the first iteration), equalizer is provided by the mean, $\bar{s}(n)$, and variance, $\sigma^2_{\bar{s}}$, of the result of the previous iterations. These statistics are obtained from the a priori log likelihood ratio (LLR), $L(s(n))=\frac{P[s(n)=+1]}{P[s(n)=-1]}$, delivered by the channel decoder from the previous iteration as [34]

$$\bar{s}(n)=tanh\left(\frac{L(s(n))}{2}\right) \tag{2.76}$$

and

$$\sigma^2_{\bar{s}}=1-|\bar{s}(n)|^2\,. \tag{2.77}$$

A linear MMSE estimate, $\hat{\mathbf{s}}_{n,j}=[\hat{s}(2n+j)\,\hat{s}^*(2n+j)]^T$, of the transmitted symbol, $\tilde{\mathbf{s}}_{n,j}=[s(2n+j)\,s^*(2n+j)]^T$, is given by

$$\hat{\mathbf{s}}_{n,j}=\Psi^H_{n,j}\tilde{\mathbf{r}}_n+\mathbf{b}_{n,j} \tag{2.78}$$

where Ψ and $\mathbf{b}$ vary with n and in order to minimize the mean square error, $E\left\{|s(n)-\hat{s}(n)|^2\right\}$, Ψ and b are found as

$$\Psi_{n,j}=Cov\left(\tilde{\mathbf{r}}_n,\tilde{\mathbf{r}}_n\right)^{-1}Cov\left(\tilde{\mathbf{r}}_n,\tilde{\mathbf{s}}_{n,j}\right) \tag{2.79}$$

$$\mathbf{b}_{n,j} = E\left\{\tilde{\mathbf{s}}_{n,j}\right\} - \Psi_{n,j}^{H} E\left\{\tilde{\mathbf{r}}_{n}\right\}. \tag{2.80}$$

The proof of finding $\Psi_{n,j}$ and $\mathbf{b}_{n,j}$ are explained in [35] and given in Appendix A.

Therefore, the minimum mean square error estimate of transmitted symbol is found as

$$\hat{\mathbf{s}}_{n,j} = E\left\{\tilde{\mathbf{s}}_{n,j}\right\} + \Psi_{n,j}^{H}\left(\tilde{\mathbf{r}}_{n} - E\left\{\tilde{\mathbf{r}}_{n}\right\}\right) \tag{2.81}$$

In the turbo equalizer concept, to estimate the symbol s_k, a priori probability of that symbol from the previous iteration result is not used and the noise in Eqn. (2.71) is i.i.d. Hence,

$$E\left\{\tilde{\mathbf{r}}_{n}\right\} = \tilde{\mathbf{H}}\bar{\mathbf{s}}_{n,j} \tag{2.82}$$

where

$$\bar{\mathbf{s}}_{n,j} = \left[\bar{\mathbf{s}}^{T}\left(n + N_2\right) \;\ldots.. \;\bar{\mathbf{s}}_{j}^{T}\left(n\right) \;\ldots\ldots.\; \bar{\mathbf{s}}^{T}\left(n - N_1 - \lceil L/2\rceil\right)\right]^{T} \tag{2.83}$$

with

$$\bar{\mathbf{s}}\left(\tilde{n}\right) = \left[\bar{s}\left(2\tilde{n}\right) \;\; \bar{s}^{*}\left(2\tilde{n}\right) \;\; \bar{s}\left(2\tilde{n}+1\right) \;\; \bar{s}^{*}\left(2\tilde{n}+1\right)\right]^{T} \tag{2.84}$$

for $\tilde{n} \neq n$ and for $\tilde{n} = n$

$$\bar{\mathbf{s}}_{1}\left(n\right) = \left[0 \; 0 \; \bar{s}\left(2n+1\right) \;\; \bar{s}^{*}\left(2n+1\right)\right]^{T} \tag{2.85}$$

$$\bar{\mathbf{s}}_{2}\left(n\right) = \left[\bar{s}\left(2n\right) \;\; \bar{s}^{*}\left(2n\right) \;\; 0 \; 0\right]^{T} \tag{2.86}$$

It is noteworthy to mention that the term $\tilde{r}_n - E\left\{\tilde{r}_n\right\}$ in (2.81) is an ISI cancellation term.

The term $Cov\left(\tilde{\mathbf{r}}_n, \tilde{\mathbf{r}}_n\right)$ in Eqn. (2.79) can be found as

$$\begin{aligned} Cov\left(\tilde{\mathbf{r}}_n, \tilde{\mathbf{r}}_n\right) &= Cov\left(\tilde{\mathbf{H}}\tilde{\mathbf{s}}_{n,j} + \tilde{\nu}_n, \tilde{\mathbf{H}}\tilde{\mathbf{s}}_{n,j} + \tilde{\nu}_n\right) && (2.87)\\ &= \tilde{\mathbf{H}} Cov\left(\tilde{\mathbf{s}}_{n,j}, \tilde{\mathbf{s}}_{n,j}\right)\tilde{\mathbf{H}}^{H} + \sigma_{\tilde{\nu}}^{2}\mathbf{I}_{\tilde{N}_f} && (2.88)\end{aligned}$$

where

$$Cov\left(\tilde{\mathbf{s}}_{n,j}, \tilde{\mathbf{s}}_{n,j}\right) = diag\left[\upsilon\left(n + N_2\right) \;\ldots\ldots..\; \upsilon\left(n, j\right) \;\ldots...\; \upsilon\left(n - N_1 - \tilde{L}\right)\right] \tag{2.89}$$

with

$$\upsilon(\tilde{n}) = \begin{bmatrix} \sigma^2_{\bar{s}}(2\tilde{n}) & \sigma^2_{\bar{s}}(2\tilde{n}) & 0 & 0 \\ \sigma^2_{\bar{s}}(2\tilde{n}) & \sigma^2_{\bar{s}}(2\tilde{n}) & 0 & 0 \\ 0 & 0 & \sigma^2_{\bar{s}}(2\tilde{n}+1) & \sigma^2_{\bar{s}}(2\tilde{n}+1) \\ 0 & 0 & \sigma^2_{\bar{s}}(2\tilde{n}+1) & \sigma^2_{\bar{s}}(2\tilde{n}+1) \end{bmatrix}, \text{ and} \tag{2.90}$$

$$\upsilon(n,1) = \begin{bmatrix} \sigma^2_s & \sigma^2_s & 0 & 0 \\ \sigma^2_s & \sigma^2_s & 0 & 0 \\ 0 & 0 & \sigma^2_{\bar{s}}(2n+1) & \sigma^2_{\bar{s}}(2n+1) \\ 0 & 0 & \sigma^2_{\bar{s}}(2n+1) & \sigma^2_{\bar{s}}(2n+1) \end{bmatrix}, \text{ and} \tag{2.91}$$

$$\upsilon(n,2) = \begin{bmatrix} \sigma^2_{\bar{s}}(2n) & \sigma^2_{\bar{s}}(2n) & 0 & 0 \\ \sigma^2_{\bar{s}}(2n) & \sigma^2_{\bar{s}}(2n) & 0 & 0 \\ 0 & 0 & \sigma^2_s & \sigma^2_s \\ 0 & 0 & \sigma^2_s & \sigma^2_s \end{bmatrix}. \tag{2.92}$$

The term $\sigma^2_{\bar{s}}(\tilde{n})$ is the variance of the previous iteration results. In the first iteration since no a priori probabilities delivered by the channel decoder $\sigma^2_{\bar{s}}(\tilde{n})$ is equal to the σ^2_s for all $\tilde{n}$.

Moreover, the term $Cov\left(\tilde{\mathbf{r}}_n, \tilde{\mathbf{s}}_{n,j}\right)$ in Eqn. (2.79) can be found as

$$Cov\left(\tilde{\mathbf{r}}_n, \tilde{\mathbf{s}}_{n,j}\right) = \left\{\mathbf{0}_{2\times 4(N_2-\tilde{L})} \quad \phi_j \tilde{\mathbf{H}}^T\left(\tilde{L}\right) \quad ..... \quad \phi_j \tilde{\mathbf{H}}^T(0) \quad \mathbf{0}_{2\times 4N_1}\right\} \tag{2.93}$$

where

$$\phi_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \tag{2.94}$$

$$\phi_2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \tag{2.95}$$

Finally, the output of the MMSE equalizer is calculated as

$$\hat{\mathbf{s}}(2n) = \Psi_{n,1}^{H}\left(\tilde{\mathbf{r}}_n - \tilde{\mathbf{H}}\bar{\mathbf{s}}_{n,1}\right) \tag{2.96}$$

and

$$\hat{\mathbf{s}}(2n+1) = \Psi_{n,2}^{H}\left(\tilde{\mathbf{r}}_n - \tilde{\mathbf{H}}\bar{\mathbf{s}}_{n,2}\right). \tag{2.97}$$

The result of the MMSE equalizer consists of the estimated symbol and its complex conjugate. Therefore, the estimate of the transmitted symbol can be found as

$$\hat{s}(2n) = [1 \quad 0]\hat{\mathbf{s}}(2n) \tag{2.98}$$

and

$$\hat{s}(2n+1) = [1 \quad 0]\hat{\mathbf{s}}(2n+1). \tag{2.99}$$

CHAPTER 3

TURBO EQUALIZATION FOR ANTENNA SWITCHING SYSTEMS

In a frequency selective fading channel, the channel induces intersymbol interference and equalization is needed at the receiver to reduce the effect of this ISI. In this chapter, based on the technique described in Section 2.5.1.3, we will develop a MMSE based iterative equalizer for the AS scheme. We will inspect the performance of the AS scheme in two cases, one is hard decision channel decoder case and the other is soft decision channel decoder case. The performance of the AS scheme is then compared with the performance of the Alamouti scheme in both cases.

In this chapter, the system models of the Alamouti coded and AS schemes will be given. Then, the equalizers will be described. Two transmit antennas and one receive antenna exist for both systems.

3.1 Transmitter

An N_b-bit frame of user data, $b(n) \in \{0, 1\}$, is encoded using a convolutional encoder of constraint length v and rate R_c. The encoded sequence $s(n)$ is BPSK modulated as in

$$x(n) = \begin{cases} -1 & \text{if} \quad s(n) = 0 \\ 1 & \text{if} \quad s(n) = 1 \end{cases}. \tag{3.1}$$

The encoder and BPSK mapping are the same for both systems.

In the Alamouti system, shown in Figure 3.1, the modulated signal fed into the STBC encoder

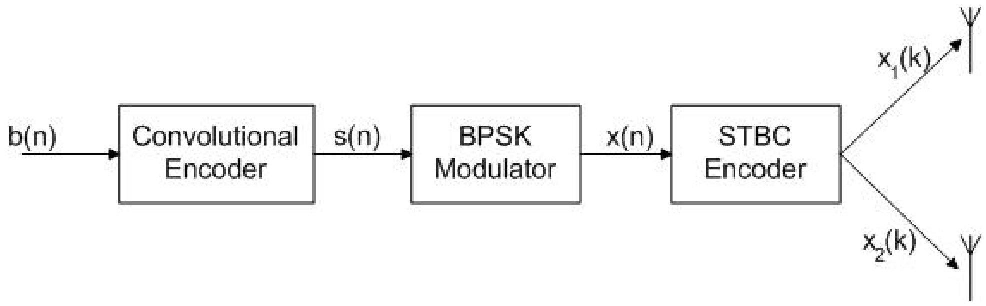

Figure 3.1: Transmitter of the Alamouti scheme

and output of the STBC encoder is given as

$$x_1(k) = \begin{cases} x(2n) & k = 2n \\ -x^*(2n+1) & k = 2n+1 \end{cases}, \tag{3.2}$$

$$x_2(k) = \begin{cases} x(2n+1) & k = 2n \\ x^*(2n) & k = 2n+1 \end{cases} \tag{3.3}$$

where $x_i(k)$ is the signal transmitted at time k from antenna i.

The main idea in AS is to transmit different code bits of the same symbol from different antennas. In this thesis, we assume that there are 2 transmit antennas and first half of the block is transmitted using the first antenna and the other half is transmitted from the second antenna. In order to gain diversity in this scheme, interleaver is needed to transmit different code bits of the same user symbol from different antennas. Therefore, the modulated signal is first interleaved for antenna switching purpose (3.5) and then sent from the transmit antennas with the rule described

$$x_1(n) = \begin{cases} a(n) & n \leq N/2 \\ 0 & n > N/2 \end{cases}, \quad x_2(n) = \begin{cases} 0 & n \leq N/2 \\ a(n) & n > N/2 \end{cases} \tag{3.4}$$

where $a(n)$ denotes the n^{th} signal of the interleaved sequence with the rule

$$a(n) = \begin{cases} x(2n-1) & n \leq N/2 \\ x(2(n-N/2)) & n > N/2 \end{cases}. \tag{3.5}$$

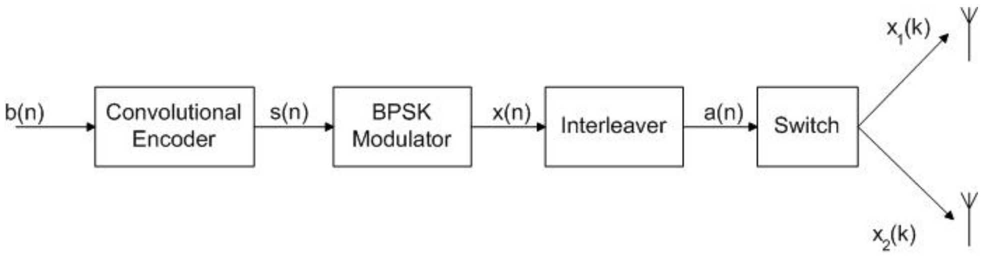

Figure 3.2: Transmitter of the AS scheme

3.2 Channel Model

The multipath channel between the transmit antenna i and receive antenna is modelled as a finite impulse response (FIR) filter

$$\mathbf{h}_i = [h_i(0)....h_i(L-1)] \tag{3.6}$$

It is assumed that the channel order between the transmit and receive antennas are the same for all subchannels and equal to L. It is also assumed that the multipath taps are time invariant in a block and independent from block to block. The discrete-time received signal of the Alamouti and AS systems is

$$r(n) = \sum_{l=0}^{l=L-1} (h_1(l)x_1(n-l) + h_2(l)x_2(n-l)) + \eta(n) \tag{3.7}$$

where $\eta(n)$ represents the noise term of circularly symmetric zero mean complex Gaussian distribution with variance σ_n^2.

3.3 Receiver

At the receiver we will inspect the performance of two cases, hard decision and soft decision channel cases, and we will explain these in this section.

3.3.1 Hard Decision Channel Decoder Case

The discrete-time receiver models of Alamouti and AS systems are given in Fig. 3.3 and 3.4, respectively. It can be seen from the figures that MMSE equalization and channel decoding are used together for both systems. The equalizer is provided with the mean and variance of the result of the previous iteration, $d(n)$, and these statistics are used to calculate the MMSE filter and the ISI cancellation terms. For the AS scheme, the MMSE equalizer output is first deinterleaved, then BPSK demodulated and then fed into the hard input hard output Viterbi algorithm for channel decoding. For the Alamouti system, the output of the MMSE equalizer is first demodulated, then decoded using the same channel decoder with the AS system. In this section, we will describe the MMSE equalizer for the Alamouti and AS systems for the case of hard decision channel decoder.

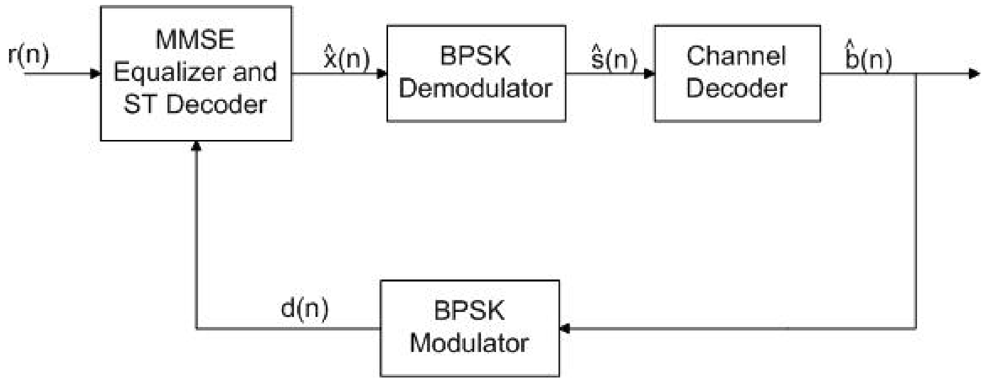

Figure 3.3: Discrete-time receiver model of the Alamouti system

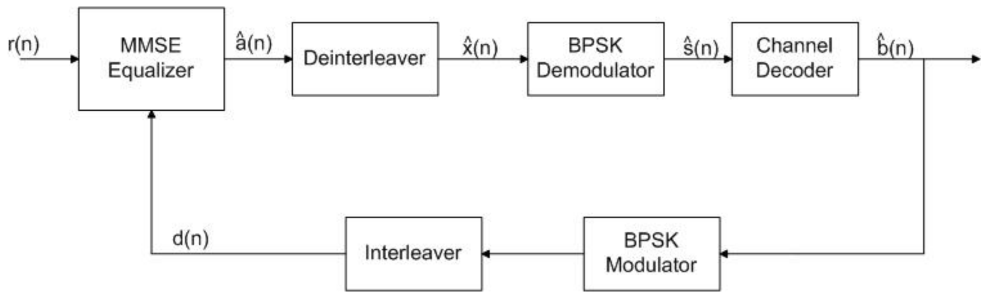

Figure 3.4: Discrete-time receiver model of the AS system

3.3.1.1 MMSE Equalizer for the Alamouti System

Based on the sliding window model defined in [34], we develop MMSE-based linear iterative equalizer for the Alamouti system. We use the equalization idea described in Section 2.5.1.3. The equalizer and hard decision channel decoder are used together in each iteration. At the beginning of each iteration, except for the first iteration, the equalizer is provided with the mean, $\bar{x}$, and variance, $v(n)$, of the results of the previous iterations. Since BPSK modulation and hard input hard output Viterbi algorithm is used, the mean, $\bar{x}$, is always ∓ 1 and variance, $v(n)$, is always 0.

The linear MMSE estimate of transmitted symbols, given in [31] to minimize the cost of $E\left\{|x_n - \hat{x}_n|^2\right\}$, is

$$\hat{\mathbf{x}}_{n,j} = \mathbf{g}_{n,j}^H \left[\tilde{\mathbf{r}}_n - E\left\{\tilde{\mathbf{r}}_n\right\}\right] \tag{3.8}$$

where

$$\mathbf{g}_{n,j} = Cov(\tilde{\mathbf{r}}_n, \tilde{\mathbf{r}}_n)^{-1} Cov(\tilde{\mathbf{r}}_n, \tilde{\mathbf{x}}_{n,j}) \quad \text{and} \tag{3.9}$$

$$E\left\{\tilde{\mathbf{r}}_n\right\} = \tilde{\mathbf{H}}\bar{\mathbf{x}}_{n,j} \tag{3.10}$$

$\tilde{\mathbf{r}}_n$, $\tilde{\mathbf{H}}$ and $\bar{\mathbf{x}}_{n,j}$ are defined in Section 2.5.1.3 for the Alamouti system explicitly, $\mathbf{g}_{n,j}$ is the linear equalizer filter.

It is also explained in Section 2.5.1.3 that the autocorrelation function of the received signal for the Alamouti scheme is

$$Cov(\tilde{\mathbf{r}}_n, \tilde{\mathbf{r}}_n) = \tilde{\mathbf{H}}\mathbf{R}_{xx}\tilde{\mathbf{H}}^H + \sigma_n^2 \mathbf{I}_{(N_1+N_2+1)} \tag{3.11}$$

where N_1 and N_2 are the lengths of the causal and noncausal parts of the equalizer and $\mathbf{R}_{xx}$ is the autocorrelation function of the channel decoder output of the previous iteration. Since we use hard decision channel decoder, $\mathbf{R}_{xx}$ is equal to

$$\mathbf{R}_{xx} = diag\left[\Theta(n+N_2) \;\; .... \;\; \Theta(n,j) \;\; ..... \; \Theta(n-N_1-L)\right]^T . \tag{3.12}$$

Let us define the matrices, $\Theta(\tilde{n})$ for $\tilde{n} \neq n$ and $\Theta(n, j)$ for $j = 1, 2$, as follows

$$\Theta(\tilde{n}) = \begin{bmatrix} v(2\tilde{n}) & v(2\tilde{n}) & 0 & 0 \\ v(2\tilde{n}) & v(2\tilde{n}) & 0 & 0 \\ 0 & 0 & v(2\tilde{n}+1) & v(2\tilde{n}+1) \\ 0 & 0 & v(2\tilde{n}+1) & v(2\tilde{n}+1) \end{bmatrix}, \tag{3.13}$$

and

$$\Theta(n,1) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.14}$$

$$\Theta(n,2) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \tag{3.15}$$

As explained before $v(n) = 0$ for all n. But in the first iteration, since no information is delivered to the equalizer, $v(n) = 1$ for all n.

It is also explained in Section 2.5.1.3 that, the correlation between the received signal and the output of the channel decoder in the previous iteration can be written as

$$Cov(\tilde{\mathbf{r}}_n, \tilde{\mathbf{x}}_{n,j}) = \left\{ \mathbf{0}_{2\times 4(N_2-\tilde{L})} \quad \phi_j \tilde{\mathbf{H}}^T(\tilde{L}) \quad ..... \quad \phi_j \tilde{\mathbf{H}}^T(0) \quad \mathbf{0}_{2\times 4N_1} \right\} \tag{3.16}$$

where

$$\phi_1 = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}, \tag{3.17}$$

$$\phi_2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}. \tag{3.18}$$

Therefore, the MMSE filter used in the Alamouti system will be

$$\mathbf{g}_{n,j} = \left[\tilde{\mathbf{H}} \mathbf{R}_{xx} \tilde{\mathbf{H}}^H + \sigma_n^2 \mathbf{I}_{2(N_1+N_2+1)} \right]^{-1} \cdot \left\{ \mathbf{0}_{2\times 4(N_2-\tilde{L})} \quad \phi_j \tilde{\mathbf{H}}^T(\tilde{L}) \quad ..... \quad \phi_j \tilde{\mathbf{H}}^T(0) \quad \mathbf{0}_{2\times 4N_1} \right\}^T \tag{3.19}$$

Finally, the MMSE equalizer outputs can be obtained by

$$\hat{\mathbf{x}}(2n) = \mathbf{g}_{n,0}^{H}[\tilde{\mathbf{r}}_n - \tilde{\mathbf{H}}\bar{\mathbf{x}}_{n,0}], \text{ and} \tag{3.20}$$

$$\hat{\mathbf{x}}(2n+1) = \mathbf{g}_{n,1}^{H}[\tilde{\mathbf{r}}_n - \tilde{\mathbf{H}}\bar{\mathbf{x}}_{n,1}]. \tag{3.21}$$

The result of the MMSE equalizer consists of the estimated symbol and its complex conjugate. Therefore, the estimate of the transmitted symbol can be found as

$$\hat{x}\,(2n) = [1 \quad 0]\hat{\mathbf{x}}\,(2n) \tag{3.22}$$

and

$$\hat{x}\,(2n+1) = [1 \quad 0]\hat{\mathbf{x}}\,(2n+1)\,. \tag{3.23}$$

After equalization, the signal is first BPSK demodulated as given in (3.24), and then fed into the channel decoder which employs a hard input hard output Viterbi algorithm.

$$\hat{s}(n) = \begin{bmatrix} 1 & if \;\; (\hat{x}(n) \geq 0) \\ 0 & if \;\; (\hat{x}(n) < 0) \end{bmatrix}. \tag{3.24}$$

3.3.1.2 MMSE Equalizer for the AS System

The MMSE equalizer used in the AS system is derived based on the equalizer model used for the Alamouti scheme. Since the complex conjugate of the signals are not used and each symbol is transmitted using one antenna at the transmitter part of the AS system, we have to change the MMSE equalizer model described for the Alamouti scheme to adapt to our system.

The sliding window model used in the AS system is given as

$$\tilde{\mathbf{r}}_n = \tilde{\mathbf{H}}\tilde{\mathbf{x}}_n + \tilde{\eta}_n \tag{3.25}$$

with

$$\tilde{\mathbf{r}}_n = [r(n+N_2) \,....\, r(n) \,.....\, r(n-N_1)]^T, \tag{3.26}$$

$$\tilde{\mathbf{x}}_n = [x_k(n+N_2) \,....\, x_k(n) \,....\, x_k(n-N_1-L)]^T, \tag{3.27}$$

$$\tilde{\eta}_n = [\eta(n+N_2) \,....\, \eta(n) \,....\, \eta(n-N_1)]^T \tag{3.28}$$

where k is the active antenna index. We assume that, first antenna is used in the first half of the block and the other is used in the second half. We also assume that there will be a sufficient guard interval between the first and second half of the packet so that there is no ISI between the symbols transmitted from different antennas.

The channel matrix, $\tilde{\mathbf{H}}$, is a Toeplitz matrix whose elements are $h_k(0).....h_k(L-1)$,

$$\tilde{\mathbf{H}} = \begin{bmatrix} h_k(0) & ..... & h_k(L-1) & 0 & .... & & 0 \\ 0 & h_k(0) & ..... & h_k(L-1) & 0 & .... & 0 \\ . & & & & & & . \\ . & & & & & & . \\ . & & & & & & . \\ 0 & ..... & & 0 & h_k(0) & ..... & h_k(L-1) \end{bmatrix} \tag{3.29}$$

where h_1 is used in the first half of the frame and h_2 is used in the second half.

Having explained the sliding-window model, we now define the equalizer used in the AS system. The idea used in the MMSE equalizer for the AS system is the same as the idea used in the Alamouti system.

The linear MMSE estimate of transmitted symbols, given in [31] to minimize the cost of $E\left\{|x_n - \hat{x}_n|^2\right\}$, is

$$\hat{x}_n = \mathbf{g}^H \left[\tilde{\mathbf{r}}_n - E\left\{\tilde{\mathbf{r}}_n\right\}\right] \tag{3.30}$$

where $\hat{x}_n$ is a scalar value. In Alamouti scheme, since complex conjugates of the signal is also transmitted, the equalizer must consider the complex conjugates, so the matrices used in

Alamouti scheme is larger than the AS scheme and therefore, the receiver complexity for the AS scheme is much less than the Alamouti scheme. And the equalizer filter is

$$\mathbf{g} = Cov(\tilde{\mathbf{r}}_n, \tilde{\mathbf{r}}_n)^{-1} Cov(\tilde{\mathbf{r}}_n, \tilde{\mathbf{x}}_n). \tag{3.31}$$

Based on the turbo principle, when coded symbol $x(n)$ is estimated, the information of the same coded symbol provided by the channel decoder is not used. This yields in (3.30),

$$E\{\tilde{\mathbf{r}}_n\} = \tilde{\mathbf{H}}\bar{\mathbf{x}}_n \tag{3.32}$$

with

$$\bar{\mathbf{x}}_n = [\bar{x}(n+N_2) \ \ ... \ \ \bar{x}(n+1) \ \ 0 \ \ \bar{x}(n-1) \ \ .... \ \ \bar{x}(n-N_1-L)] \tag{3.33}$$

where $\bar{x}(n)$ is the mean value of the result of the previous iteration and since we use hard decision channel decoder and BPSK modulation, $\bar{x}(n)$ is always equal to ∓ 1 for all n.

It is worth mentioning that the term $[\tilde{\mathbf{r}}_n - E\{\tilde{\mathbf{r}}_n\}]$ in (3.8) and (3.30) can be viewed as ISI cancellation.

The autocorrelation function of the received signal for the AS system is given as

$$Cov(\mathbf{r}_n, \mathbf{r}_n) = \tilde{\mathbf{H}}\mathbf{R}_{xx}\tilde{\mathbf{H}}^H + \sigma_n^2 \mathbf{I}_{(N_1+N_2+1)} \tag{3.34}$$

where $\mathbf{R}_{xx}$ is a block diagonal matrix of size $((N_1+N_2+L+1) \times (N_1+N_2+L+1))$ which is the autocorrelation function of the channel decoder output of the previous iteration and it is defined for the Alamouti system before. For the AS system, $\mathbf{R}_{xx}$ is constant for all iterations, except for the first iteration, and given as

$$\mathbf{R}_{xx} = diag\begin{bmatrix} v(n+N_2) & .... & v(n+1) & 1 & v(n-1) & ..... & v(n+N_1) \end{bmatrix}^T. \tag{3.35}$$

At each iteration, except in the first iteration, since hard decision channel decoder is used, the mean, $\bar{x}(n)$, of the result of the previous iteration is always ∓ 1, therefore the variance, $v(n)$, is 0 for all n. In the first iteration, since no information is delivered to the equalizer, $v(n) = 1$ for all n and $\mathbf{R}_{xx}$ will be an identity matrix.

It can also be shown that the cross-correlation between the received and transmitted signals for the AS system is,

$$Cov(\mathbf{r}_n, \mathbf{x}_n) = [\mathbf{0}_{1\times(N_2-L)} \ \ h_k(L-1) \ \ .... \ \ h_k(0) \ \ .... \ \ \mathbf{0}_{1\times N_1}]^T \tag{3.36}$$

where h_1 is used in the first half of the frame and h_2 is used in the second half.

Therefore the MMSE filter defined in (3.30) is

$$\mathbf{g} = \left[\tilde{\mathbf{H}}\mathbf{R}_{xx}\tilde{\mathbf{H}}^H + \sigma_n^2\mathbf{I}_{2(N_1+N_2+1)}\right]^{-1} \cdot \left[\mathbf{0}_{1\times(N_2-L)} \;\; h_k(L-1) \;\; .... \;\; h_k(0) \;\; .... \;\; \mathbf{0}_{1\times N_1}\right] \quad (3.37)$$

Using this filter we can obtain the estimate of the transmitted symbols as

$$\hat{x}(n) = \mathbf{g}^H \left[\tilde{\mathbf{r}} - E\left\{\tilde{\mathbf{r}}\right\}\right]. \quad (3.38)$$

After equalization, the signal is deinterleaved as given in

$$\hat{a}(n) = \begin{cases} \hat{x}(\frac{n+1}{2}) & if \; n \; odd \\ \hat{x}(N/2 + (\frac{n}{2})) & if \; n \; even \end{cases} \quad (3.39)$$

where $1 \leq n \leq N$.

The output of (3.39) is first BPSK demodulated as given in (3.40) and then fed into the hard input hard output Viterbi channel decoder.

$$\hat{s}(n) = \begin{bmatrix} 1 & if \;\; (\hat{a}(n) \geq 0) \\ 0 & if \;\; (\hat{a}(n) < 0) \end{bmatrix} \quad (3.40)$$

3.3.1.3 Simulation Results for Hard Decision Decoder

We investigate the average frame error rate (FER) performance of the proposed receiver for the AS system in multipath Rayleigh fading channels through simulations and compare the results with the Alamouti system. In simulations, we use data frames of length N_b data bits. The channel code used here is a convolutional code with rate $1/2$, constraint length 5, and generator polynomial of $(17, 35)_{octal}$. A 3-tap Rayleigh fading channel is used with equal average power. As mentioned before, it is assumed that subchannels are time-invariant in one transmitted block and independent from block to block.

In Figures 3.5 and 3.6, the FER performance of both systems are depicted. In the first iteration no information is delivered to the equalizer and in the second iteration the result of the previous iteration is used at the equalization part. Since we use hard decision channel decoder and BPSK modulation, the output of the channel decoder is always ∓ 1. Therefore, the performance of the receiver will not increase after two iterations. In Figures 3.5 and 3.6, we can see that the Alamouti system performs better than the AS system. Also, in high SNR region it is possible to get approximately 0.5 dB performance gain in both systems after 2 iterations compared to the 1st iteration.

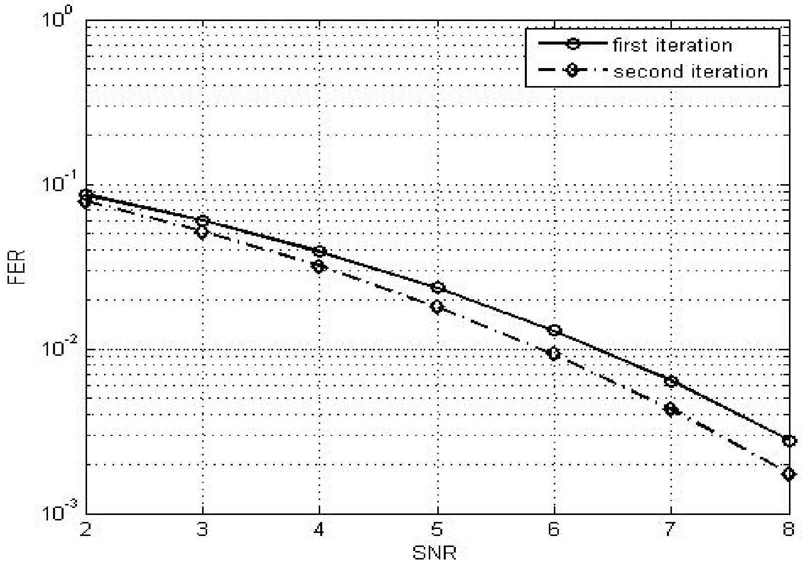

Figure 3.5: FER vs SNR of Alamouti system

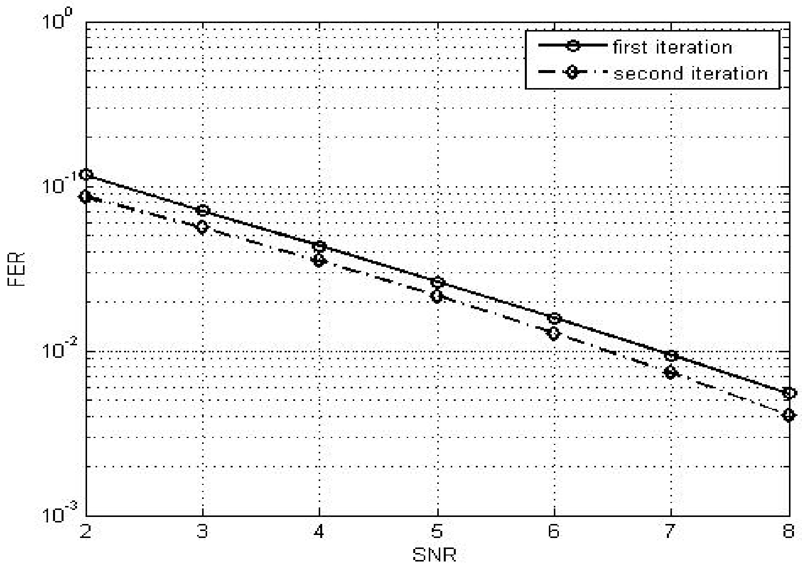

Figure 3.6: FER vs SNR of AS system

After observing the superiority of Alamouti-based system, one may wonder whether the same superiority carries to the imperfect channel state information case. To inflict imperfection, simulations are done with 10dB and 15dB channel estimation (CE) error levels at the receiver side. Since the subchannels are assumed to be time-invariant in a transmitted frame, we add a single noise component on it, i.e, $\bar{h}_k = h_k + n_k$ where $k \in \{1, 2\}$ is the subchannel number. The FER performance of both systems are given below.

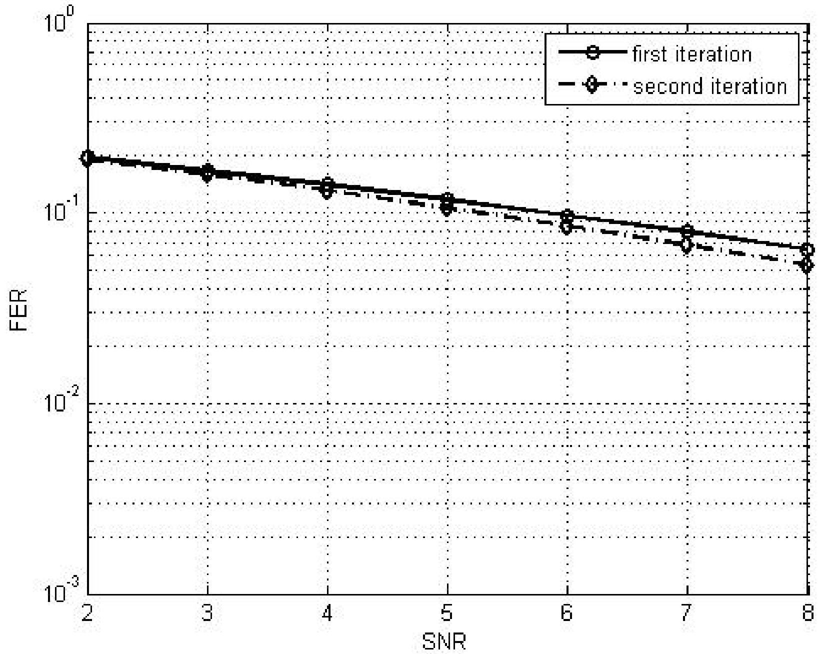

Figure 3.7: FER vs SNR of Alamouti system with 10dB CE noise

It can be seen from simulation results that there is still 0.5 dB performance gain in both systems after 2 iterations compared to the first iteration. Interestingly, if we compare the results of AS and Alamouti schemes, the AS system performs remarkably better than the Alamouti system when the channel state information is imperfect.

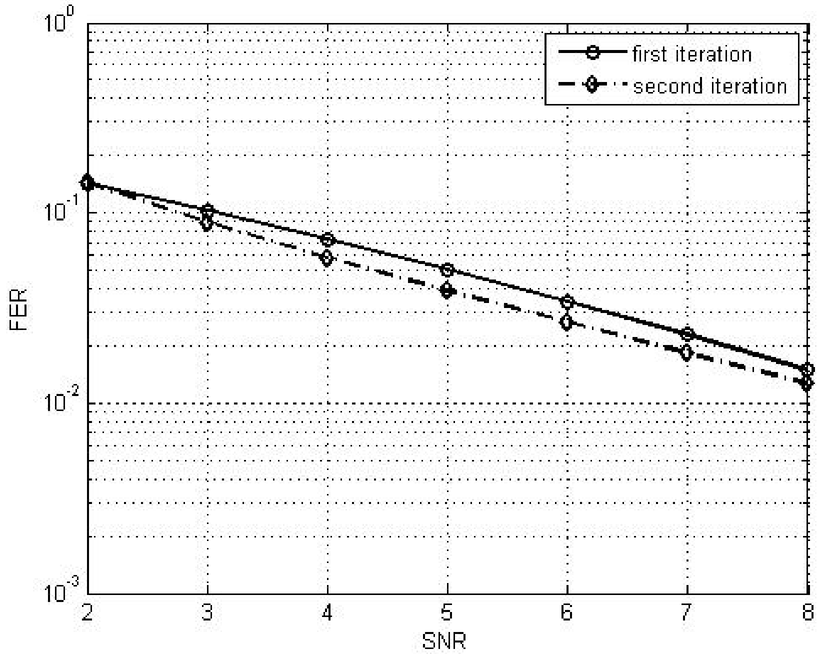

Figure 3.8: FER vs SNR of AS system with 10dB CE noise

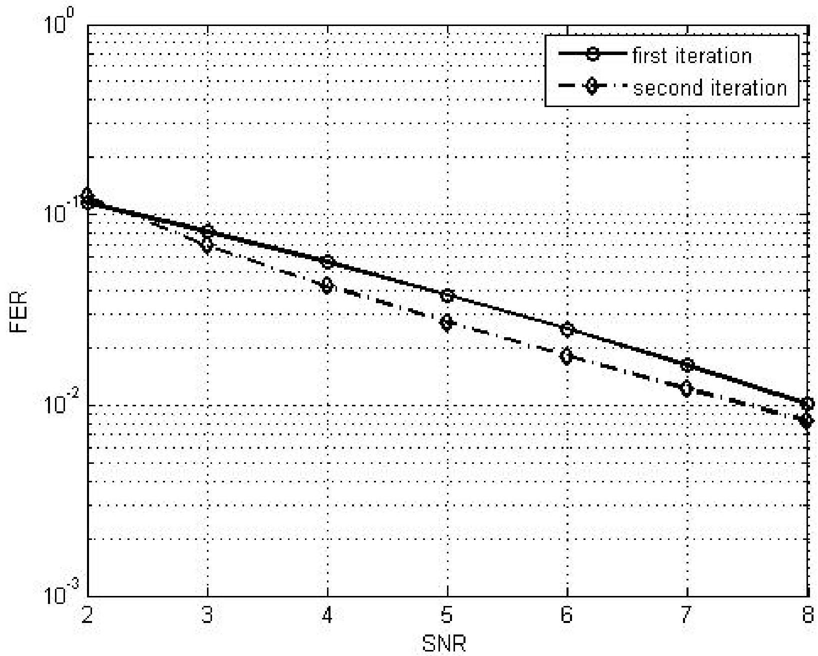

Figure 3.9: FER vs SNR of Alamouti system with 15dB CE noise

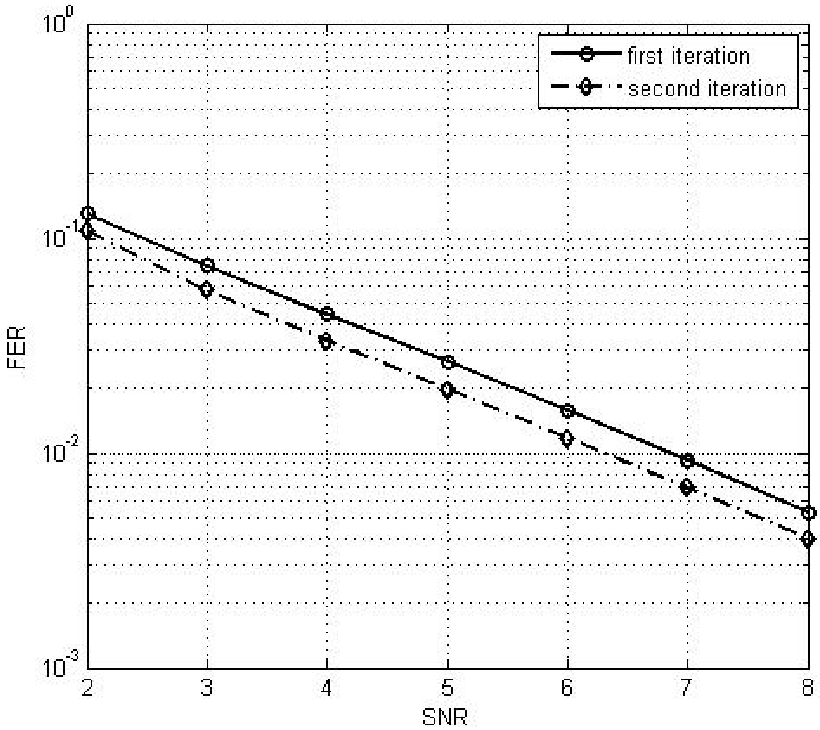

Figure 3.10: FER vs SNR of AS system with 15dB CE noise

Simulations given above are repeated in the case of a random interleaver used in the transmitter side of the both schemes. The random interleaver used in AS system interleave the blocks that will transmitted from different antennas separately. The simulation results are given in Figures 3.11 - 3.16.

We can see from results that after using a random interleaver the performance of both schemes increases. In the perfect channel estimation case, performance of the Alamouti system is still better than the AS system. In imperfect channel estimation case, before using an interleaver, the AS scheme performs remarkably better than the Alamouti scheme. But in this case, the Alamouti system performs better than the AS system especially in the first iteration but in the second iteration their performance are very close to each other.

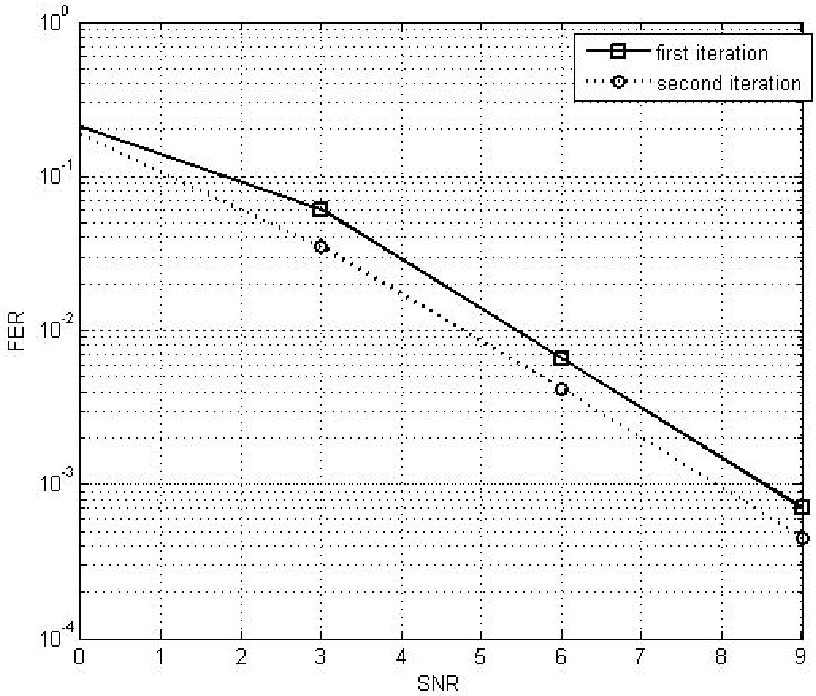

Figure 3.11: FER vs SNR of Alamouti system with random interleaver

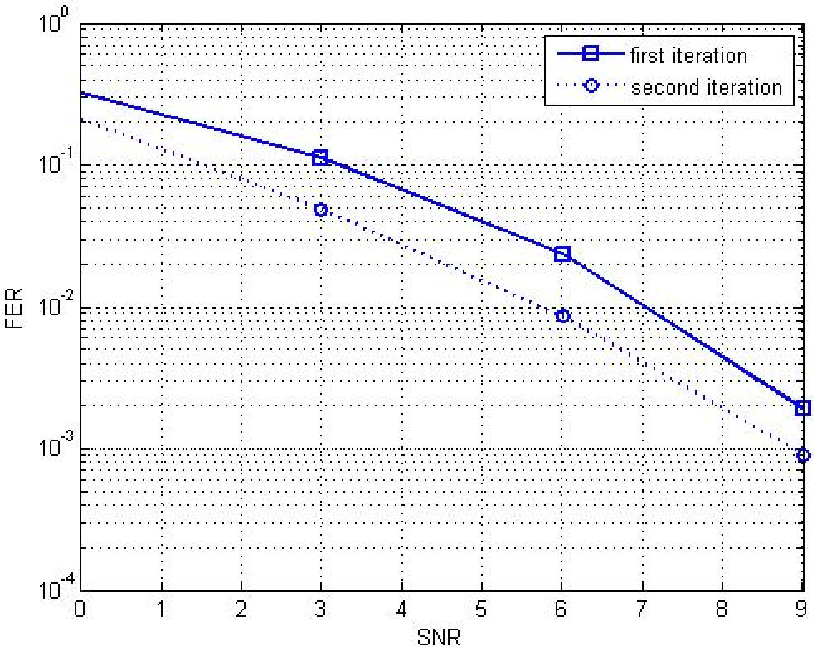

Figure 3.12: FER vs SNR of AS system with random interleaver

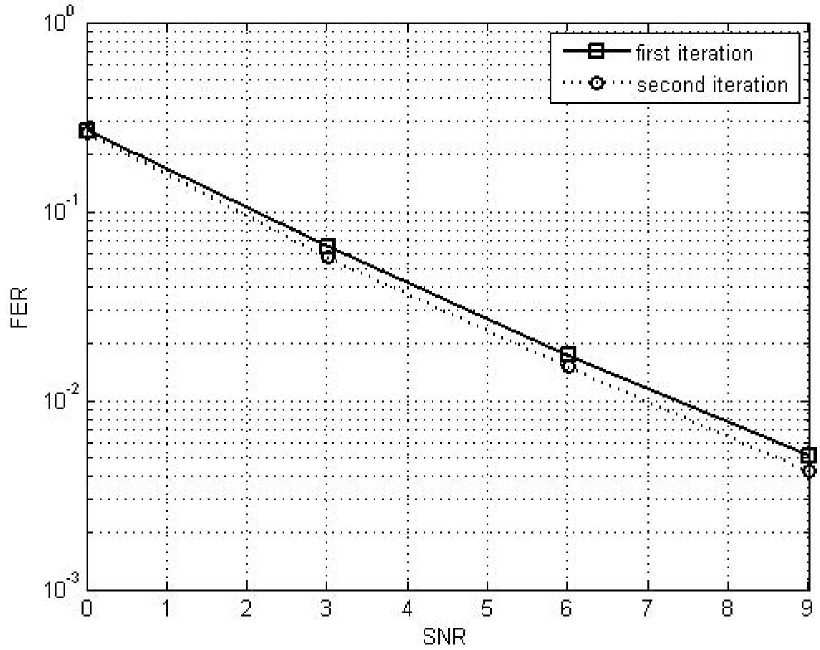

Figure 3.13: FER vs SNR of Alamouti system with random interleaver and 10dB CE noise

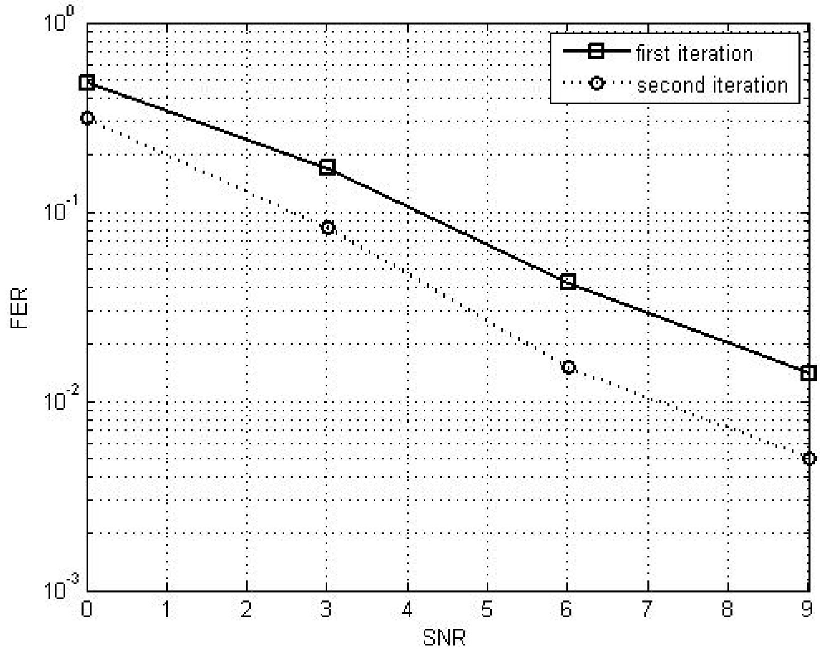

Figure 3.14: FER vs SNR of AS system with random interleaver and 10dB CE noise

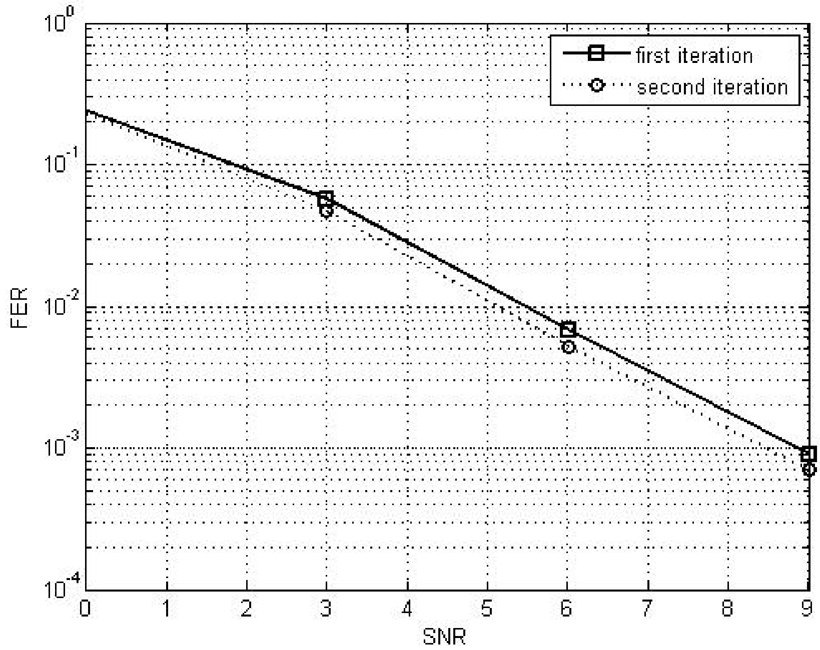

Figure 3.15: FER vs SNR of Alamouti system with random interleaver and 15dB CE noise

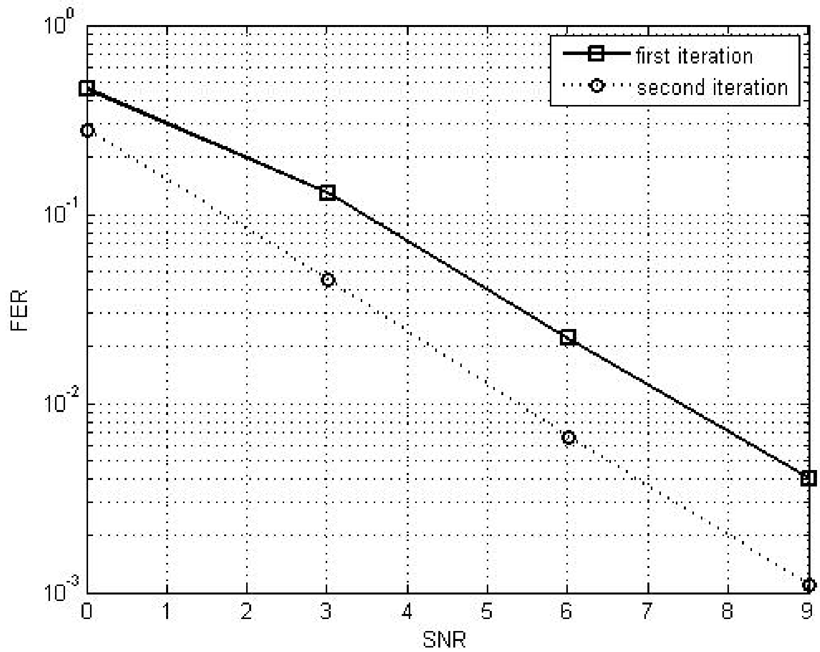

Figure 3.16: FER vs SNR of AS system with random interleaver and 15dB CE noise

3.3.2 Soft Decision Channel Decoder Case

In this case, the performance comparisons are done when a random interleaver is used in both schemes. The discrete-time receiver models of Alamouti and AS systems in the case of soft decision channel decoder are given in Fig. 3.17 and 3.18, respectively. In the same way as in the hard decision case, MMSE equalization and channel decoders are operated together. In both systems, the MMSE equalizer output is first deinterleaved, then BPSK demodulated and then fed into the MAP decoder for channel decoding.

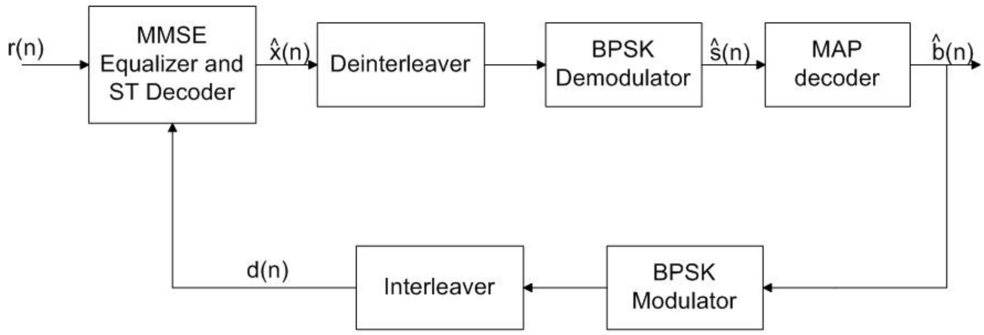

Figure 3.17: Discrete-time receiver model of Alamouti system

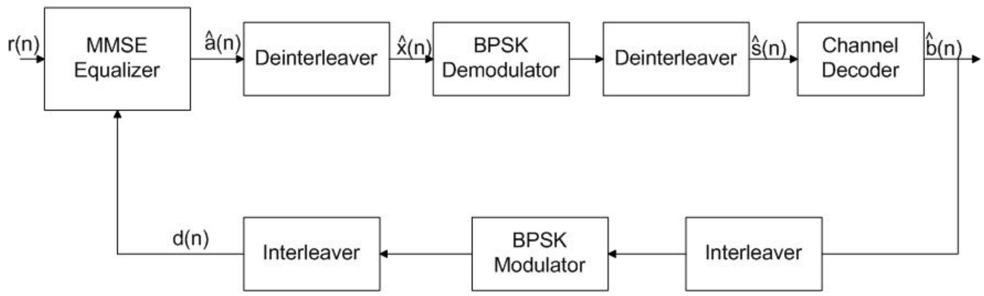

Figure 3.18: Discrete-time receiver model of AS system

The equalizer models of both system are the same as the equalizer models defined in Section 3.3.1. The differences here lie in the calculation of the autocorrelation function of the channel decoder output and the soft ISI cancellation term shown in (3.10). In Section 3.3.1, since hard decision decoding is used with BPSK modulation, the variance of the channel decoder output is always 0 which is used to calculate the autocorrelation function of the channel decoder output. In this case, the equalizer is provided with mean $\bar{x}(n)$ and variance $v(n)$ of every transmitted symbol $x(n)$. These statistics are obtained from the a priori probabilities $L_D^p(x(n))$ obtained from the output of the MAP decoder at the previous iteration:

$$L_D^p(x(n)) = \log \frac{P\,(x(n) = +1)}{P\,(x(n) = -1)}. \tag{3.41}$$

Based on (3.41), the mean and the variance of the transmitted symbol can be calculated as

$$\bar{x}(n) = \tanh\left(\frac{L_D^p(x(n))}{2}\right) \tag{3.42}$$

$$v(n) = 1 - |\bar{x}(n)|^2\,. \tag{3.43}$$

This information is used to calculate the ISI cancellation term and the autocorrelation function of the channel decoder output. The other operations are the same as the hard decision channel decoder case.

3.3.2.1 Simulation Results for Soft Decision Decoder

We investigate the average frame error rate (FER) performance of the proposed receiver for the AS system in multipath Rayleigh fading channels through simulations and compare the results with the Alamouti system. In simulations, we use data frames of length N_b data bits. The channel code used here is a convolutional code with rate $1/2$, constraint length 5, and generator polynomial of $(17, 35)_{octal}$. A 3-tap Rayleigh fading channel is utilized with equal average power. As mentioned before, it is assumed that subchannels are time-invariant in one transmitted block and independent from block to block.

We can see from Figures 3.19 and 3.20 that the Alamouti scheme performs a little better than the AS scheme and it is possible to get 1 dB performance gain in both systems after two iterations compared to the 1^{st} iteration. Moreover, it is observed that the performance does not enhance after 2 iterations in both systems.

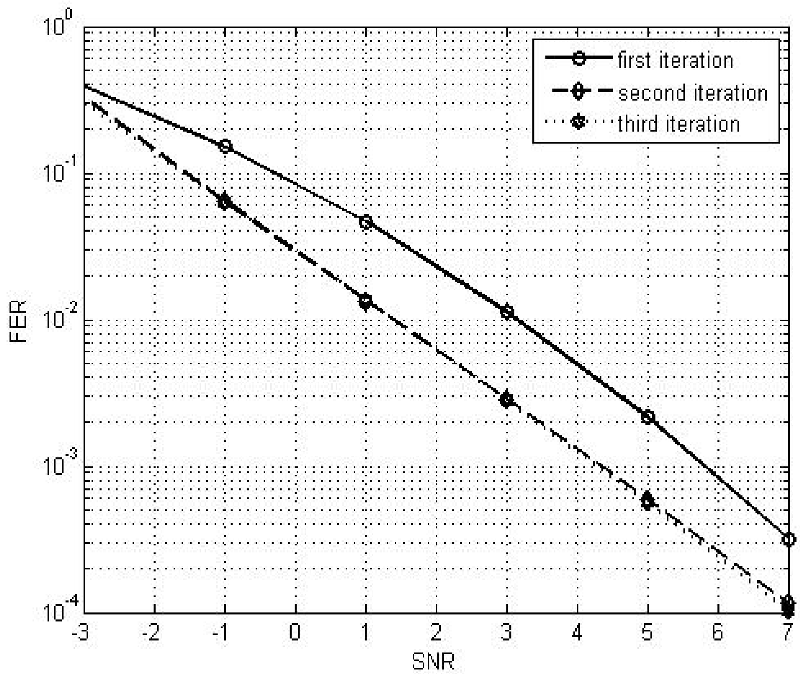

Figure 3.19: FER vs SNR of Alamouti system with soft decision channel decoder

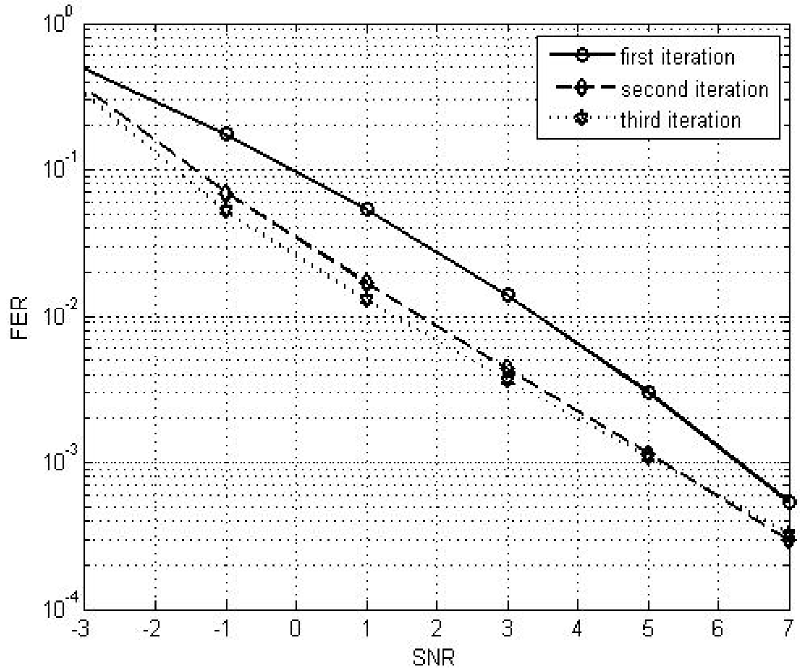

Figure 3.20: FER vs SNR of AS system with soft decision channel decoder

In hard decision channel decoder case, if the CSI is not exactly known at the receiver and a random interleaver is used than the Alamouti scheme performs better than the AS scheme. We repeat simulations with the soft decision channel decoder to see whether the Alamouti scheme still performs better or not. In order to inflict imperfect CSI at the receiver, simulations are performed with 10dB and 15dB channel estimation (CE) error levels at the receiver side. FER performance of both systems are shown in the Figures 3.21-3.24.

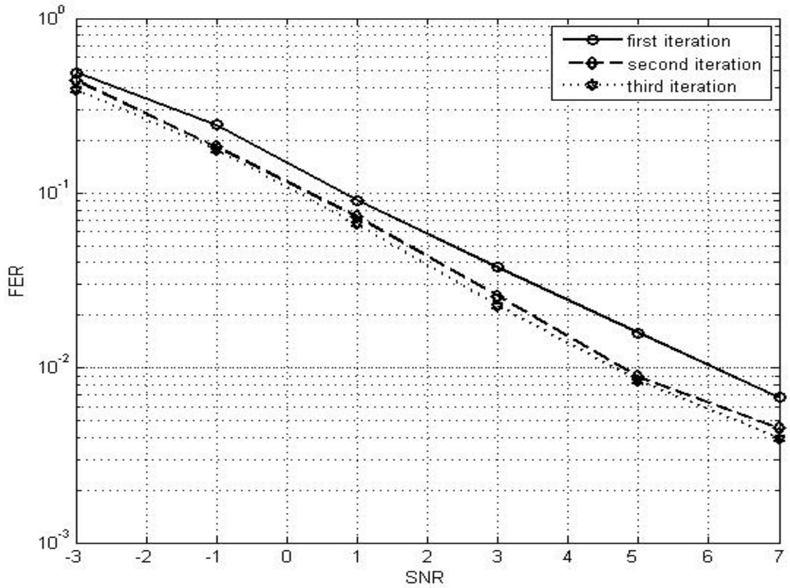

Figure 3.21: FER vs SNR of Alamouti system with 10dB CE noise and soft decision channel decoder

It can be seen from simulation results that the AS scheme performs significantly better than the Alamouti scheme. For the 10 dB and 15 dB CE cases, the AS scheme performs approximately 2 dB better than the Alamouti scheme. There is still approximately a 1 dB performance gain at second iteration compared to the first iteration and the performance does not enhance after 2 iterations.

In summary, if we compare the results of the AS and Alamouti schemes, the Alamouti scheme performs better than the AS scheme if CSI is perfect at the receiver in both hard and soft decision channel decoder cases. In imperfect channel estimation case in hard decision channel decoder case, without a random interleaver AS scheme performs better than the Alamouti scheme. But when we use a random interleaver the Alamouti system performs better than the AS system especially in the first iteration but in the second iteration their performance are

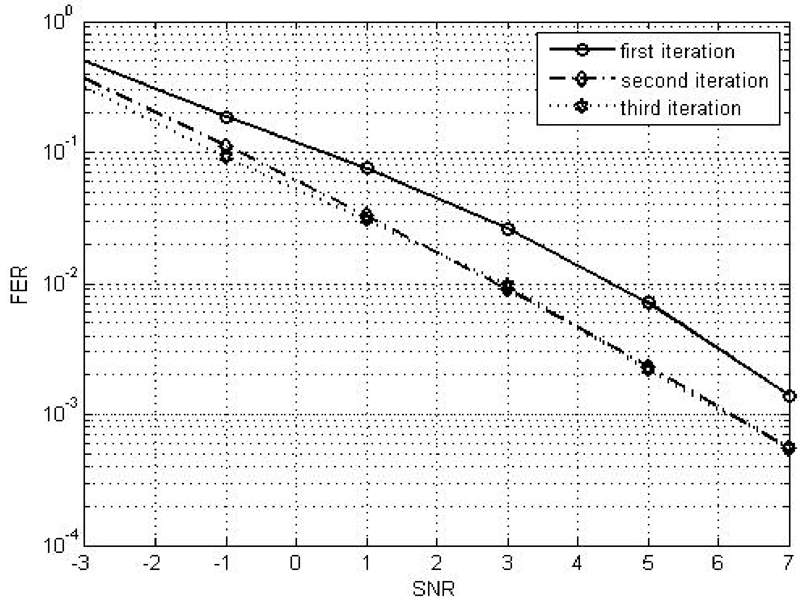

Figure 3.22: FER vs SNR of AS system with 10dB CE noise and soft decision channel decoder

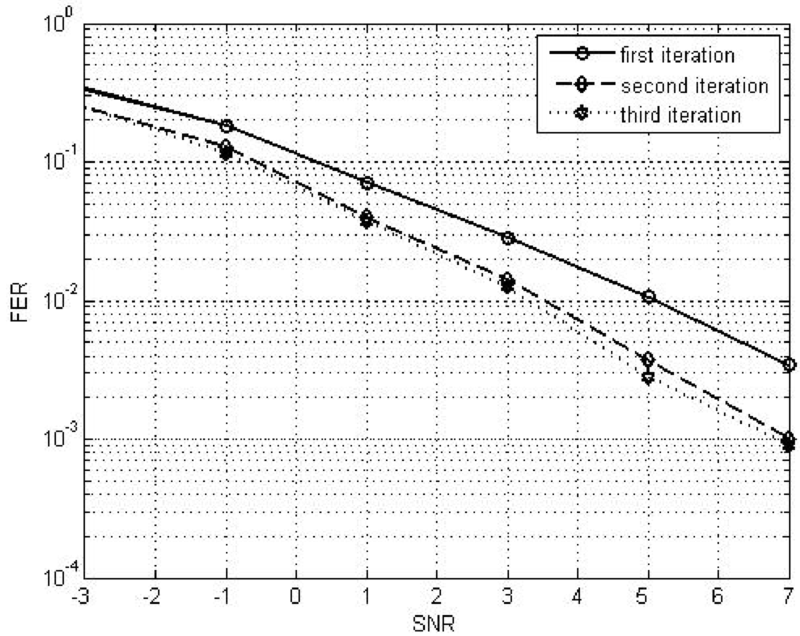

Figure 3.23: FER vs SNR of Alamouti system with 15dB CE noise and soft decision channel decoder

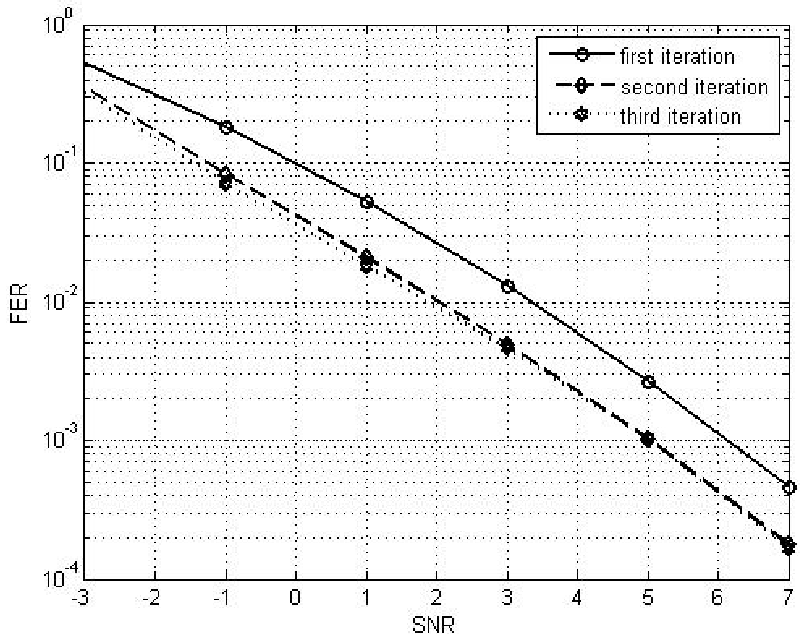

Figure 3.24: FER vs SNR of AS system with 15dB CE noise and soft decision channel decoder

very close to each other. Moreover, in soft decision decoder case, when the CSI is imperfect at the transmitter, then the AS system performs remarkably better than the Alamouti system.

CHAPTER 4

CONCLUSION AND FUTURE WORK

In this work, the antenna switching scheme for two transmit and one receive antennas was studied and a performance comparison with the Alamouti scheme was made. The outage probability analyses of both schemes under block fading and time-invariant multipath Rayleigh fading channels were performed. It is observed from the outage probability results in block fading that AS scheme also achieves full diversity order as the Alamouti scheme although there is an SNR offset between them. For the outage probability of 10^{-2}, when rate is equal to 1, the Alamouti system performs 2 dB better than the AS system and when rate is equal to 4, the SNR offset between the Alamouti and AS systems is equal to 10 dB. This SNR offset stems for the non-full rate operation of AS scheme and increases if the transmission rate increases.

Moreover, we compared the performances of Alamouti and AS based transmit diversity schemes over multipath Rayleigh fading channels. We made use of a minimum mean-squared error based iterative equalizer for both schemes. In the proposed receiver, equalization and channel decoding are jointly carried out. We observed from the simulation results that when the channel state information is perfect at the receiver, Alamouti system performs 0.5 dB better than the AS system. When the channel state information is imperfect in hard decision channel decoder case and a random interleaver is used, the Alamouti system still performs better than the AS system especially in the first iteration but in the second iteration their performance are very close to each other. On the other hand, if a random interleaver is not used, AS scheme performs remarkably better than the Alamouti scheme in hard decision channel decoder case. In a soft decision channel decoder case, when the channel state information is imperfect, the AS scheme performs approximately a 2 dB better than the Alamouti scheme. This performance improvement of the AS system in the imperfect channel estimation case may be the result of the larger matrix sizes at the equalization of the Alamouti scheme. In addition, since AS system uses one antenna at each time, the equalization complexity of the AS system is much less than that of Alamouti system. Based on the results presented, the antenna switching based transmit diversity turns out to be a better option than the Alamouti based system in practice.

In addition to the studies described in this thesis, some additional research and improvements are set as future research directions. These topics can be listed as follows:

- The estimation of the time invariant multipath fading channel can be performed.
- Time varying channel performance of the AS based system can be investigated.
- Theoretical performance analysis under imperfect channel estimation case can be performed in both hard and soft decision channel decoder cases.

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APPENDIX A

Proof of (2.79) and (2.80)

Suppose we have two sequences of random variables, Y_n and X_n. We observe Y_n some set of times $a \leq n \leq b$ and we wish to estimate X_t from these observations.

Let $\hat{X}_t$ is the estimate of X_t and it can be written as

$$\hat{X}_t = \sum_{n=a}^{b} h_{t,n} Y_n + c_t \tag{A.1}$$

where $h_{t,a}.....h_{t,b}$ and c_t are scalars.

In this proof, we assume that $E\left\{Y_n^2\right\} < \infty$ and $E\left\{X_n^2\right\} < \infty$ for all n. Also, H_a^b is the set of all estimates of the form (A.1).

Suppose that $\hat{X}_t \in H_a^b$. Then

- $E\left\{\hat{X}_t^2\right\} < \infty$; and
- if Z is a random variable satisfying $E\left[Z^2\right] < \infty$, then

$$E\left\{Z\hat{X}_t\right\} = \sum_{n=a}^{b} h_{t,n} E\left\{ZY_n\right\} Y_n + c_t E\left\{Z\right\} \tag{A.2}$$

The proof of this statements are obvious. To prove first property we use the inequality, $(x + y)^2 \leq 4(x^2 + y^2)$. We can write first property as

$$\hat{X}_t = \sum_{n=a}^{b} h_{t,n} Y_n + c_t + \left(\hat{X}_t - \sum_{n=a}^{b} h_{t,n} Y_n + c_t\right). \tag{A.3}$$

After using the inequality and taking expectations, we have

$$E\left\{\hat{X}_t^2\right\} \leq 4E\left\{\left(\sum_{n=a}^{b} h_{t,n}Y_n + c_t\right)^2\right\} + 4E\left\{\left(\hat{X}_t - \sum_{n=a}^{b} h_{t,n}Y_n + c_t\right)^2\right\}. \tag{A.4}$$

The first term is finite by the validity of first property, and second term converges to zero.

To prove the second property, we consider for $m < b$ the quantity

$$E\left\{Z\hat{X}_t\right\} - \sum_{n=m}^{b} h_{t,n}E\left\{XY_n\right\} - c_tE\left\{Z\right\} = \tag{A.5}$$

$$E\left\{Z\left(\hat{X}_t - \sum_{n=m}^{b} h_{t,n}Y_n - c_t\right)\right\}. \tag{A.6}$$

From the Schwarz inequality

$$\left|E\left\{Z\left(\hat{X}_t - \sum_{n=m}^{b} h_{t,n}Y_n - c_t\right)\right\}\right|^2 \tag{A.7}$$

$$\leq E\left\{Z^2\right\}E\left\{\left(\hat{X}_t - \sum_{n=m}^{b} h_{t,n}Y_n - c_t\right)\right\}. \tag{A.8}$$

By assumption that $E\left\{Z^2\right\} < \infty$ and the definition $E\left\{\left(\hat{X}_t - \sum_{n=m}^{b} h_{t,n}Y_n - c_t\right)\right\} \to 0$ as $m \to -\infty$. Thus (A.6) and (A.8) implies second property.

In MMSE estimation, we would like the solve the problem

$$\min_{\hat{X}_t \in H_a^b} E\left\{\left(\hat{X}_t - X_t\right)^2\right\}. \tag{A.9}$$

$\hat{X}_t \in H_a^b$ solves Eqn. (A.9) if and only if

$$E\left\{\hat{X}_t\right\} = E\left\{X_t\right\}, \tag{A.10}$$

$$E\left\{\left(\hat{X}_t - X_t\right)Z\right\} = 0 \quad \text{for all } Z \in H_a^b \tag{A.11}$$

and

$$E\left\{\left(\hat{X}_t - X_t\right)Y_l\right\} = 0 \quad \text{for all } a \leq l \leq b. \tag{A.12}$$

Using Eqn. (A.10), we can obtain

$$E\left\{\sum_{n=a}^{b} h_{t,n}Y_n + c_t\right\} = E\left\{X_t\right\} \tag{A.13}$$

from which using Eqn. (A.11) for $Z = 1$ we have

$$c_t = E\left\{X_t\right\} - \sum_{n=a}^{b} h_{t,n}E\left\{Y_n\right\} \tag{A.14}$$

From Eqn. (A.1) and (A.12), we get

$$E\left\{\left(\hat{X}_t - \sum_{n=m}^{b} h_{t,n}Y_n - c_t\right)Y_l\right\} = 0. \tag{A.15}$$

Substituting Eqn. (A.14) into Eqn. (A.15), we get

$$Cov\left(X_tY_l\right) = \sum_{n=a}^{b} h_{t,n}Cov\left(Y_nY_l\right) \tag{A.16}$$

which is known as Wiener-Hopf equation. For finite a and b the solution of this equation is quite easy. Eqn. (A.16) is a set of $b - a + 1$ linear equations with $b - a + 1$ unknowns. If we write this equation in a matrix form, we get

$$\mathbf{Cov}_{XY} = \mathbf{Cov}_{YY}\mathbf{h}. \tag{A.17}$$

We see from Eqn. (A.17) that the optimum estimator coefficients are given by

$$\mathbf{h} = \mathbf{Cov}_{YY}^{-1}\mathbf{Cov}_{XY}. \tag{A.18}$$