OFFICE RENT VARIATION IN ISTANBUL CBD: AN APPLICATION OF MAMDANI AND TSK-TYPE FUZZY RULE BASED SYSTEM

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ABSTRACT

OFFICE RENT VARIATION IN ISTANBUL CDB: AN APPLICATION OF MAMDANI AND TSK-TYPE FUZZY RULE BASED SYSTEM.

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Over the past decade, fuzzy systems have gained remarkable acceptance in many fields including control and automation, pattern recognition, medical diagnosis and forecasting. The fuzzy system application has also been accepted as a promising approach to dealing with uncertainty in real estate valuation analysis. This is mainly due to the necessity of coping with a large number of qualitative and quantitative variables that affect the value of a real property. The appraisers use a great deal of judgment to identify both the characteristics that contribute to property values and the relationships among these characteristics in order to derive estimates of market values. This thesis uses the two widely-used fuzzy rule-based systems; namely the Mamdani and Takagi-Sugeno-Kang (TSK) type fuzzy models in an attempt to examine the main determinants of office rents in Istanbul Central Business District (CBD). The input variables of the fuzzy rule-based systems (FRBS) comprise:

i) physical attributes of office spaces and office buildings,
ii) lease contract terms, and
iii) tenants' perception of the office rent determinants, tenants' location of residence, tenants' transportation modes, etc

and as the output the system proposes the office property's rental price. Obtaining office rent determinants is a significant issue for both practitioners and academics. While, practitioners use them directly in demand and sensitivity analyses, academics are more interested in the relative significance of these variables and their effect on the variation in office rent to forecast market behavior.

Our data set includes a detailed survey of 500 office spaces located in Istanbul CBD. We have carried out two Mamdani-type FRBS and two TSK-type FRBS for the office space and office building data sets. In these FRBS analyses, firstly the so-called representative office spaces are determined, then the average office space rents are estimated. Finally, the spatial variation in the average office rents across the CBD sub-districts, along with the Office space rent variations with respect to different clusters, like number of workers, number of floors and so on, have been analyzed. We believe that presenting the spatial variation in office rents will make a noteworthy contribution both to the real estate investors and appraisers interested in Istanbul office market.

Keywords: Real Estate Valuation, Office Rents, Fuzzy Rule-Based Systems, TSK Fuzzy Model, Istanbul-Turkey.

ÖZ

İSTANBUL MİA'DA OFİS KİRA DEĞİŞİMİ : MAMDANİ VE TSK BULANIK KURAL TABANLI MODEL UYGULAMASI

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Son on yıl içinde bulanık sistemler, kontrol ve otomasyon, görüntü tanıma, tıbbi teşhis ve tahmin gibi birçok alanda önemli ölçüde kabul görmüştür. Bulanık sistem uygulaması, gayrimenkul değerleme analizlerinde bulunan belirsizlik probleminin çözülmesinde umut verici bir yaklaşım olarak kabul edilmiştir. Bu durum esas olarak, bir taşınmaz değerini etkileyen çok sayıda nitel ve nicel değişkenleri göz önüne alma gerekliliğinden kaynaklanmaktadır. Eksperler piyasa değerini çıkarabilmek için gayrimenkulün değerine katkı sağlayacak özelliklerini ve bu özellikleri arasındaki ilişkiyi tanımlayabilmek amacıyla büyük oranda yargısal bir yaklaşım kullanmaktadır. Bu tez İstanbul Merkezi İş Alanı'nda (MİA) ofis kiralarının ana belirleyicilerini incelemek için yaygın olarak kullanılan bulanık kural tabanlı iki sistemi uygulamaktadır; Mamdani ve Takagi-Sugeno-Kang (TSK) bulanık modeller. Bulanık kural tabanlı sistemlerin (BKTS) girdi değişkenleri aşağıdakileri kapsamaktadır:

i) ofis alanları ve ofis binalarının fiziksel özellikleri
ii) kira sözleşmesi koşulları ve
iii) kiracıların office kira bedeli belirleyicileri ile ilgili algıları, kiracıların ikamet bölgeleri, kiracıların ulaşım yöntemleri vb.

sistem, çıktı olarak ise ofis kiralama fiyatı önermektedir. Ofis kira belirleyicilerinin elde edilmesi hem uygulayıcı hem de akademisyenler için önemli bir konudur. Uygulayıcılar bunları doğrudan talep ve duyarlılık analizlerinde kullanırken, akademisyenler daha çok piyasa davranışını tahmin etmek için bu değişkenlerin göreceli önemi ve bunların ofis kira değişimlerindeki etkisiyle ilgilenmektedirler.

Veri seti, İstanbul MİA'da bulunan 500 ofisin ayrıntılı anket bilgilerini içermektedir. Çalışma, ofis alanı ve ofis binası verileri için iki Mamdani yöntemi BKTS ve iki TSK yöntemi BKTS olmak üzere toplam dört bölümde yürütülmüştür. BKTS analizlerinde, öncelikle sözde temsilci ofis alanları belirlenmekte, sonra ortalama ofis kira bedeli tahmin edilmektedir. Son olarak, CBD alt bölgelerinde ortalama ofis kira bedellerindeki mekansal varyasyon ile işçi sayısı, kat sayısı gibi farklı değişken kümelerine bağlı olarak görülen ofis kira bedeli değişimleri analiz edilmiştir. Ofis kira bedellerinin mekana göre değişimi üzerine yapılan bu çalışmanın, İstanbul ofis piyasası ile ilgilenen gayrimenkul yatırımcıları ve eksperlerine önemli bir katkı sağlayacağına inanıyoruz.

Anahtar Kelimeler: Gayrimenkul Değerleme, Ofis Kira Bedelleri, Bulanık Kural Tabanlı Sistemler, TSK Bulanık Modeli, İstanbul-Türkiye.

To my beloved family

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I wish to express my sincere gratitude to my family; to my admirable father Nazim, to my lovely mother Azile, to my dear sister Könül for their love, for their help, for always trusting and supporting me.

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AZAR KARIMOV

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CHAPTER 1

INTRODUCTION

1.1 Motivation

Istanbul is a unique metropolitan city that stands on the crossroad between Europe and Asia continents. The European side emerges as the Central Business District (CBD) and the Asian side is mostly a residential part of the city.It is well documented that Istanbul is a rapidly growing city from early 1950s throughout late 1990s.

Eminönü and Beyoğlu districts, that are included within the borders of historical peninsula, are the areas where business activities were concentrated and identified as the CBD of the city until 1970's. After the construction of the bridges and peripheral highways over the Bosphorus and Golden Horn, the CBD moved towards the north of the city. Although the historical district of Eminönü and Beyoğlu retained their significance as a commercial center, starting from 1970's the business activities began to become concentrated along the Zincirlikuyu-Levent-Maslak axis in Şişli district (see Figure 1.1). This axis gained importance as the new CBD, where the professional office buildings, especially FIRE (finance, real estate and insurance) -sector offices, are concentrated.[1] The demand for high quality office space is mainly driven by the rising number of international companies, as well as the privatizations and corporate acquisitions.

Table 1.1 provides comparative information on the office usage space, vacancy rates and net rents for the Central, Eastern, South-Eastern European cities and Middle East and African cities in June 2009. Istanbul, as a South Eastern European city, has the second biggest office stock of 2.3 million m^2. Athens has the largest stock of office with approximately 5.4 million m^2. Among the South East European and Middle East and African cities, Istanbul has the second highest net rents, especially, for Top Class A offices, after Dubai. In comparison to the Central and Eastern European cities, Top Class A office rents in Istanbul are much higher than those of Budapest, Prague, S.Petersburg, Vienna and Warsaw. Office space under construction is nearly 73.4

[1] Central Business District (CBD) in Istanbul consists of the Historical Peninsula (Eminonu and Beyoğlu), Sisli, Besiktas and Kadikoy districts. This area can be called as the heart of Istanbul metropolitan area. The sub-centers like Bakirkoy, Kagithane, Avcilar, Uskudar, Umraniye, Kucukcekmece, Maltepe, Kartal and Pendik have strong connections with the CBD and serve for their own hinterlands.

thousand m^2, which is considerably small. As of June 2009, the vacancy rate for the Istanbul metropolitan area offices is 12.7%.

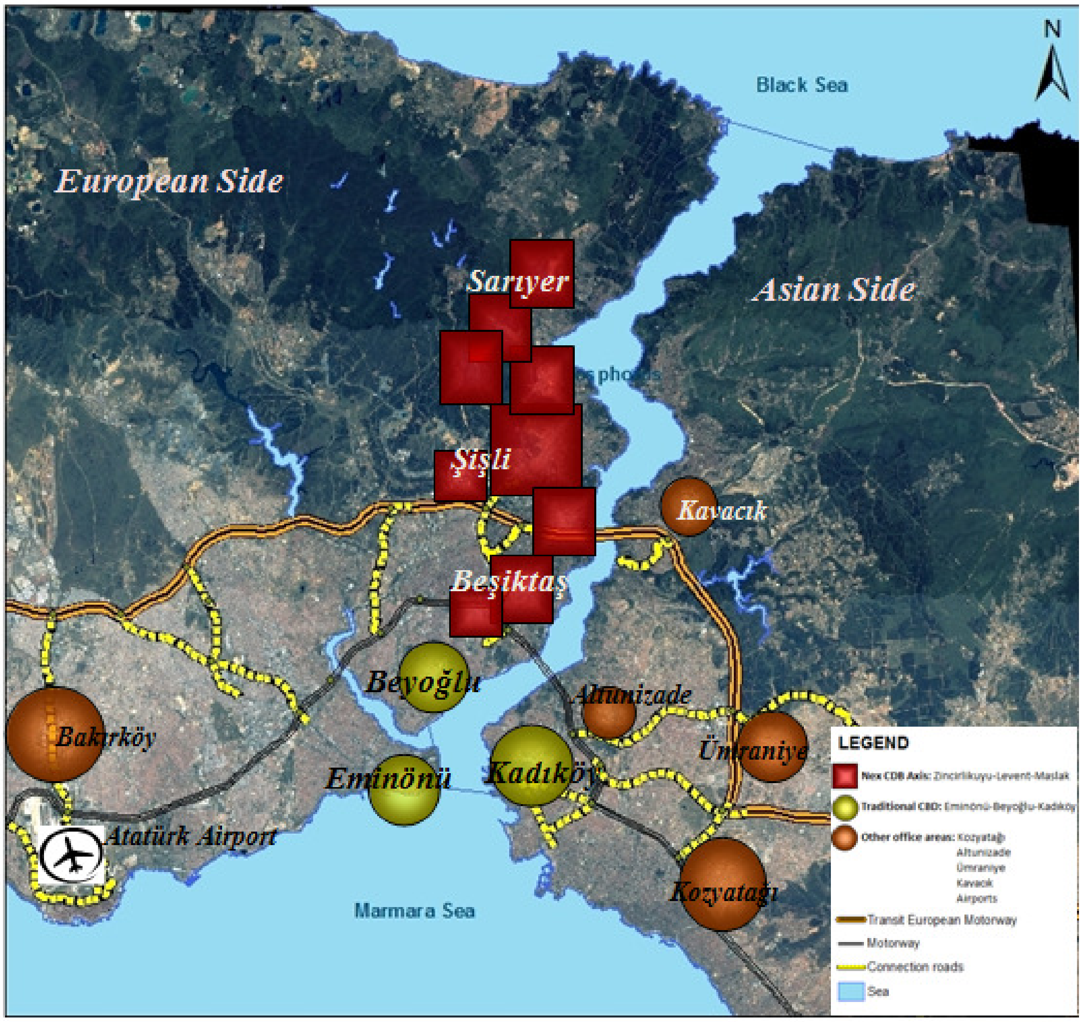

Figure 1.1: Map of Istanbul Central Business District (CBD)

Broadening the concept of real estate from a physical structure for residing to capital-gaining instrument and thereby emerging salient counter effects among the economic fundamentals and the price variations of the property has necessitated quantitative methods to delve into this application field to attribute an accurate price in the light of well-defined determinants. As a consequence of this progress, real estate valuation has become to be of great interest to different parties such as potential and actual homeowners, policy makers and the experts that deal with the appraisal.

Table 1.1: Office Usage Spaces (m^2), Vacancy Rates (%) and Net Rents for Central-Eastern& South Eastern Europe, Middle East and Africa: Mid-Year 2009

City	Existing Stock (m^2)	Under construction (m^2)	Vacancy (%)	Geographic area	CBD Class A Net Rent (€/m^2/month)	Top Class A Net Rent (€/m^2/month)
Central & Eastern Europe						
Budapest	2,214,823	332,157	18.0	Metro	12.50	19.00
Moscow	10,800,000	3,800,000	16.9	Metro	41.61	53.50
Prague	2,614,617	n/a	10.1	Metro	19.00	22.00
S. Petersburg	1,075,000	380,000	20.0	Metro	26.75	38.64
Vienna	10,820,000	170,000	5.0	Metro	18.00	24.00
Warsaw	3,152,962	253,000	5.7	Metro	22.50	26.00
South East Europe						
Athens	5,375,000	30,000	9.0	Metro	17.00	22.00
Belgrade	895,000	268,150	16.0	Metro	15.50	16.50
Istanbul	2,289,264	73,392	12.7	Metro	20.04	40.30
Sofia	965,000	864,000	13.5	Metro	17.00	30.00
Tirana	70,950	30,100	2.0	CBD	17.00	22.00
Zagreb	980,000	77,600	5.3	Metro	16.00	17.00
Middle East & Africa						
Cape Town	753,020	10,000	5.4	CBD	7.09	9.39
Dubai	3,688,303	2,517,030	20.0	Metro	43.70	61.01
Johannesburg	1,552,409	n/a	8.3	CBD	4.88	6.26
Riyadh	669,413	187,532	17.0	Metro	22.92	37.47
Tel Aviv	2,450,000	50,000	7.8	Metro	13.82	15.46

Source: Colliers International EMEA Market Overview: Office 2009 Mid-Year (**www.colliers.com**)

The expanding literature on this study, especially throughout last three decades, can be classified into two leading groups that one of which depends upon investigating the supply and demand dynamics of the real estate that leads to equilibrium price in the marketplace. On the other hand, the second group of studies analyze the interactions of the predetermined factors which can be enumerated as physical characteristics of the real estate and of the locational area, regional place transportation facilities and finally region based socio-economic conditions with the real estate price variations.

1.2 Aim of the Study

This thesis aims to gain additional insight into the office rent estimation and its variation within Istanbul CBD by using fuzzy rule-based system (FRBS), which has been recently accepted as an appropriate methodology in real estate valuation. As the basic rent determinants we use the physical attributes (both quantitative and qualitative characteristics) and lease contract terms of 500 office spaces in the European part of Istanbul CBD. The offices are located both in the historical peninsula (Eminonu and Beyoglu districts) and along the new CBD axis including Besiktas, Sisli and Sariyer districts in the northern part of the city (see Figure 1.1).

Obtaining office rent determinants is a significant issue for both practitioners and academics. While, practitioners use them directly in demand and sensitivity analyses, academics are more interested in the relative significance of these variables and their effect on the variation in office rent to forecast market behaviour.

Since the 1980s a considerable number of empirical studies have used the ***Hedonic Pricing models*** so as to model the determinants of office rents. Since the 1990s, several studies on the determinants of office and commercial property prices and/or rents have used different techniques in ***Geographic Information System (GIS)***, including visual mapping, querying or overlay and advanced spatial analysis. Over the past decade, the Theory of Fuzzy Sets and specifically ***Fuzzy Rule-Based System (FRBS)*** has been accepted as an appropriate methodology in real estate valuation. The present study uses the two widely-used fuzzy rule-based systems; namely the Mamdani and Takagi-Sugeno-Kang (TSK) type fuzzy models in an attempt to examine the main determinants of office rents in Istanbul CBD.

1.3 Contribution of the Thesis

To our knowledge, this thesis is a first attempt to use Mamdani-type and TSK-type fuzzy models with the aim of identifying basic determinants of office rents in a CBD and estimating the average office rents by examining the relationships among the rent-determinants. The thesis also examines the variation in the average office rents across different clusters, including the spatial clustering (the sub-districts of CBD), number of worker clustering, age of the building clustering, and so on.

Over the past decade, the Theory of Fuzzy Sets and specifically *fuzzy rule-based system (FRBS)* has been accepted as an appropriate methodology in real estate valuation. Although the fuzzy models provided promising empirical results in residential property valuation, no research has been applied FRBS to appraising office properties or analyzing office rent determinants.

Obviously, existing literature on the Turkish real estate market dynamics is very limited. In particular, the amount of academic research on the development of Turkish real estate sector, risk-return analysis of real estate investment tools like mortgages, REIT stocks and demand-supply analysis for real estate space in Turkey is scarce. Indeed, research on real estate markets requires interdisciplinary work with the collaboration of urban planning, economics, finance, statistics, mathematics, and information technologies. This thesis performs such a multidisciplinary work, which uses new methods and techniques in a comprehensive way, to analyze the office property rent determinants in Istanbul Metropolitan Area.

1.1 Scope of the Thesis

This thesis consists of 7 chapters in addition to the appendices and bibliography. Organization of the chapters is as follows;

Chapter 1, the present chapter, contains the motivation, aim and organization of the study as an introduction to the thesis.

In Chapter 2, empirical studies both in the Turkish and international literature are briefly given.

Chapter 3 includes descriptive statistics of data and brief explanation of methodology.

In Chapter 4, real estate appraisal methods are introduced within two main groups as traditional and advanced methods.

Chapter 5 represents the methodological background on Fuzzy Logic and Fuzzy Inference Systems.

In Chapter 6, application of the Fuzzy Logic to estimate the office space rental value and its empirical results are given.

Finally, Chapter 7 presents the concluding remarks identifying the most important contributions and also the suggestions for future research.

CHAPTER 2

LITERATURE REVIEW

Since the 1980s a considerable number of empirical studies have used the ***Hedonic Pricing Models*** so as to model the determinants of office rents.[2] The attempt has been to demonstrate through statistical inference the hedonic price of various spatial and non-spatial factors in the determination of office rents. Empirical results of the hedonic office rent/price models set forth that the variables defining physical characteristics of the office buildings are dominantly effective on the rents/prices. While the amenities provided by the building, height and size of the building, and rentable area in the building have explicitly positive effects on rents/prices, age of the building and the vacancy rate in the office building insert negative effects. Several studies find out the notable effects of locational variables on office rents. Especially, the distances from the CBD and the location adjacent to rail and highways have negative impacts on office rents. On the contrary, accessibility of the office building, employment in finance-insurance-real estate (FIRE) and retail sectors, and the concentration of offices in the district are positively affecting the rental prices. A limited number of studies examine the lease contract variables and find out that the length of lease has a positively significant effect on office rents. Besides, square feet of the lease, effective tax rate, and the rent escalation due to inflation rate are the significant determinants of office rent with negative impacts on it.

Several studies on the determinants of office and commercial property prices and/or rents have used different techniques in ***Geographic Information System (GIS)***, including visual mapping, querying or overlay and advanced spatial analysis, since the 1990s.[3] The "location" factor is comprehensively studied in GIS-based property valuation analyses. While the majority of studies have used "distance calculation and accessibility" analyses in an attempt to understand the locational factors for office rent determination, a few of them integrated the neighborhood's socio-economic and demographic structure, topographical characteristics or proximity and accessibility levels within the same spatial model for office rent determination in GIS environment. For instance, Desyllas (1998) concluded that spatial pattern of the streets (topographical characteristics) are highly correlated to office rents in Berlin. Zeng and Zhou

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[2] See Clapp (1980), Hough and Kratz (1983), Cannaday and Kang (1984), Brennan *et al.* (1984), Wheaton (1994), Vandell and Lane (1989), Glascock *et al.* (1990), Mills (1992), McDonald (1993), Wheaton and Torto (1994), Sivitanidou (1995), Webb and Fisher (1996), Dunse and Jones (1998), Gat (1998), Bollinger *et al.* (1998), Slade (2000), Nagai *et al.* (2000), Laverne and Winson-Geideman (2003), Oven and Pekdemir (2006) and Franz (2007).

[3] See Des Rosiers and Theriault (1992), Rodriguez *et al.* (1995), Wyatt (1997), Desyllas (1998), Bollinger *et al.* (1998), Clapp and Rodriguez (1998), Byrne (2005), Ryan (2005), Oven and Pekdemir (2006), Pillar *et al.* (2007) among others.

(2001) suggested that not only locational factors but also socio-economic indicators such as income level, population and household size insert significant impacts on property values.

Over the past decade, the Theory of Fuzzy Sets and specifically ***Fuzzy Rule-Based Systems (FRBS)*** has been accepted as an appropriate methodology in real estate valuation.

Zadeh (1971) states and as it is quoted here that "… As the complexity of a system increases, our ability to make precision and yet significant statements about the behavior diminishes until a threshold is reached beyond which precise and significance become almost mutually exclusive characteristics ...".

The concept of fuzziness was first proposed by Zadeh (1965). He aimed to describe complex and complicated systems by using fuzzy approximation and introduced fuzzy sets. "Generally, fuzzy logic can be considered as a logical system that provides a model for modes of human reasoning that are approximations rather than exact." as Rutkowska (2002) proclaimed.

Fuzzy logic systems have contributed eminent applications in wide variety of fields such as: automatic control, pattern recognition, signal processing, expert systems, communication, system identification and time series prediction (Czogala and Leski 2000).

The literature devoted specifically to real estate application on fuzzy logic is scarce. One of the reasons may be the difficulty inherited at the application process of this theory into that field.

The first appraisal practitioner and scholar to recognize the potential applications of fuzzy logic on real estate is Dilmore. In his paper, named "Fuzzy Set Theory: An Introduction to Its Application for Real Estate Analysts" dated to 1993, he makes several important highlights on fuzzy logic. Dilmore asserts that fuzzy logic is not loose thinking, rather it is a way of dealing with the lack of precision that one appeals in his great partition of daily decisions and evaluations (e.g., something is sweet, or salty, or neither). Secondly, reality is not precise. Dilmore quotes Einstein, who said, ''So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality.'' In his other study, to assess the different distance effects on real estate appraisal, Dilmore employed fuzzy logic with expert system and obtained more consistent estimates for these effects. Dilmore (1994) set forth that the comparable sales method of the adjustment techniques that applies fuzzy logic is better suited in case of insufficient information by providing more flexible means of membership.

McNeill and Freiberger (1993) cite many existing and potential applications of fuzzy logic. Among applications that have existed for several years is the system that controls subway trains in Sendai, Japan; computers that are able to recognize a person's handwriting; and ''intelligent'' washing machines, microwaves, cameras, camcorders and automobiles.

The research of Bagnoli and Smith (1998) can be considered as "tombstone" for the concept of Real Estate Valuation as well, and fuzzy set output is obtained by applying fuzzy logic method on a capital-gaining property. By the use of appropriate fuzzy input function, fuzzy sets gathered can be used to produce reasonable conclusions.

In the study of Bonissone and Cheetham (1997), the Property Financial Information Technology (PROFIT) is presented. The system enhances Case-Based Reasoning techniques with fuzzy predicates expressing preferences in determining similarities between subject and comparable properties. They guide the selection and aggregation process, leading to the final property value estimate. Fuzzy techniques are also used to generate a confidence value qualifying such estimate. In Mert and Yilmaz (2008) the values have been turned into scores and the scores found by the traditional method are compared to the scores found by the fuzzy logic method. It is observed that the traditional method has deficiency result in the abrupt changes in borderline cases. Because fuzzy logic method takes the whole effects into consideration in great details; more sensitive, exact and realistic results are obtained.

Lee, Ishii and Yeh (2004) applied Fuzzy linguistic logic to reduce the subjectivity of the appraiser in determining the weights of qualitative variables. The study focuses on the vagueness of the qualitative factors in linguistic form. The preference modeling used in this study provided the adjustment table based on multi-valued logic and fuzzy set theory for building the preference level modeling. The property valuation model with Quantification Theory I can also be integrated with the fuzzy linguistic form and give a more flexible adjustment for the appraiser to give imprecise information of the property.

Kuşan et al. (2010) employed fuzzy logic (FL) method to investigate the effect of diversified factors such as house factors, environmental factors, transportation factors and socio-economic factors on house unit prices (UPs). To measure the performance of the FL method on predicting the UPs, data sets composed by a questionnaire study on two hundred house residents from 40 different regions of Eskisehir city in Turkey are separated as training set and testing set and then compared the modeled Ups with that of real ones. The model they applied effectively captured the unit price trend of the houses in those 40 different regions.

Gonzalez and Formoso (2006) have shown that TSK rules are more convenient to real estate appraisal. The output of a fuzzy system is computed as a weighted average of the individual rule outputs, using the matching degree among inputs and the antecedent part of each rule. The small number of high errors, potential outliers, indicates the importance of data pre-processing. The use of different samples in modeling, or training, and in testing is important to increase the confidence in the models generated, and to avoid overfitting. The models presented are similar, with a small advantage to the fuzzy model based on location.

Krol, Lasota, Nalepa and Trawinski (2007) proposed the Mamdani fuzzy model for assisting the property appraisers' work. The model comprises 7 input variables referring to the attributes of a property evaluated. The results of learning rule base were promising, despite they were achieved for a relatively small number of generations performed by an evolutionary algorithm.

Hence, a considerable amount of research has applied fuzzy models to appraising residential properties. Although the FRBM analysis provided promising empirical results in residential property valuation, no research has been applied fuzzy rule-based systems to appraising office properties or analyzing office rent determinants.

The vast majority of Real Estate appraisal literature created in the twentieth century has been devoted to Artificial Neural Networks which is a rather broad discipline that requires in-depth expertise. One of the paramount studies in that field is conducted by Worzala, Lenk and Silva (1995). They compare the performance of two *Neural Networks* (*NN*) models in estimating the sales price of residential properties with a traditional multiple regression models and the results do not support previous findings which claim that NNs are superior tools for appraisal analyses. Significant problems such as inconsistent results between packages or between runs of the same package and long run times are encountered in the NN models.

The prediction models conducted by *Fuzzy Neural Networks* (*FNN*s) are also applied to estimate the appropriate price level of a new real estate (Liu, Zhang, and Wu, 2006). The experimental results demonstrates that applications of *artificial intelligence* methods on real estate prediction and *decision support systems* allow getting significantly improved predicting results and outperforms the classic prediction methods.

The broadest view of real estate appraisal can be represented by a brief review of three works first of which is *Real Estate Appraisal in a Nutshell* (Steward, 1967). In this research, the supply and demand forces of the market are tied to an informative overview of the different approaches to valuation. A more comprehensive work is found in Real Estate Appraisal and Investment

(Kahn and Case, 1977), a textbook covering all phases of real estate appraisal plus special sections on investment analysis of specific property types. The third work, *The Appraisal of Real Estate* (American Institute of Real Estate Appraisers, 1985), is a recently revised edition of the standard text used by the American Institute of Real Estate Appraisers for instructional purposes in appraisal fundamentals, and is a comprehensive treatment of valuation theory and practice. The eighth edition is probably the premier work on the basic subject matter of real estate appraisal.

While the foregoing literature is only a representation of the numerous substantive works on real estate appraisal, they provide an adequate review of the range of literature supporting the basic principles of fuzzy logic application in real estate.

CHAPTER 3

DATA AND METHODOLOGY

3.1 Data

The literature review has been made to identify the necessary data for the use in the given thesis. Furthermore, in order to gather the quantitative and qualitative characteristics of the Office Space and Office Building and the contract characteristics the survey[4] had been conducted. The survey had been consisting of the 2 part, one for the Office Space data and for the Office Building data. Below you may find the brief description of the data.

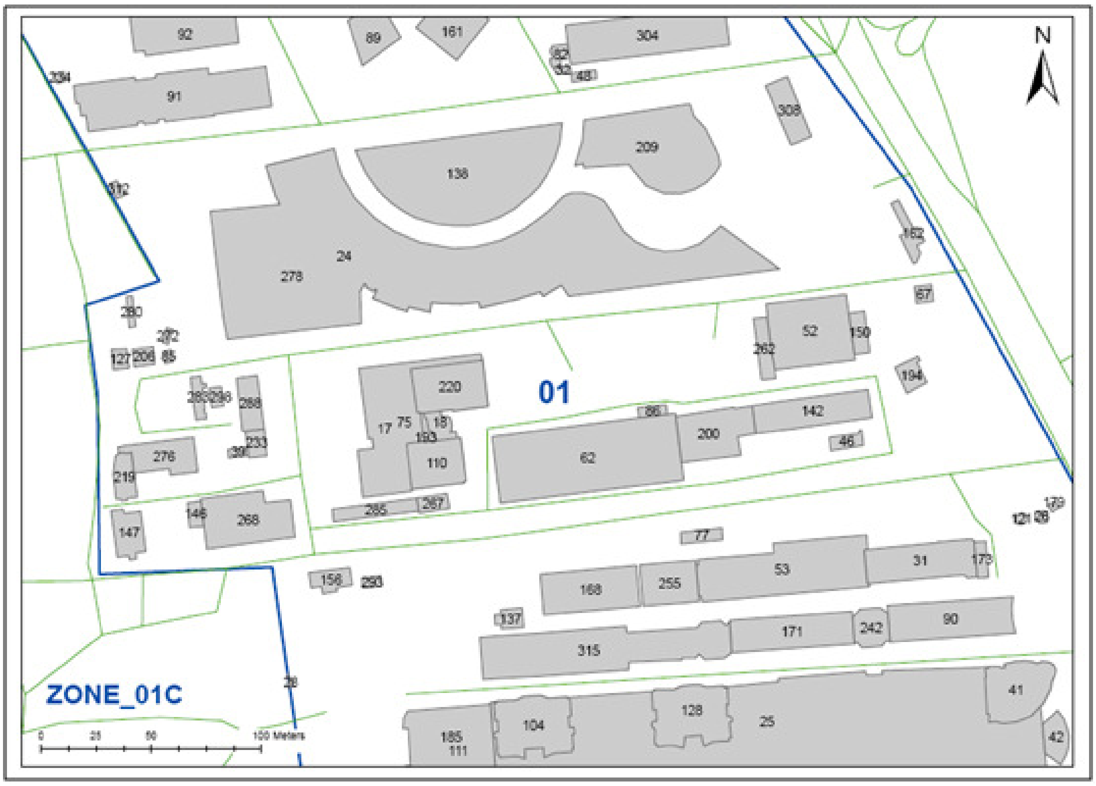


Figure 3.1: The map of the enumerated office and building created in the environment of the Geographical Information Systems.

[4] The survey forms can be found in the Appendix A and B

As mentioned earlier our data come from a number of sources, namely, from Office Space and Office Building data.

1. **Office Space Data Set:** 506 questionnaires have been conducted, with each questionnaire includes the following information:
   - General characteristics of the office units (10 questions),
   - Lease contract details (5 questions),
   - Tenants' perception of the office rent determinants, tenants' location of residence, tenants' transportation modes, etc. (5 questions).
2. **Building Data Set:** 177 questionnaires have been conducted, with each questionnaire includes the following information:
   - General characteristics of the buildings (5 questions),
   - Infrastructure details (7 questions),
   - Office related details (7 questions),
   - Construction material and design quality (4 questions),
   - Functions and locational parameters (2 questions),
   - Monthly expenses (1 question).

Table 3.1 provides the frequency distribution of sample data among the sub-districts of CBD. Out of 506 office spaces, 219 of them are located in the historical CBD (Eminönü and Beyoğlu) and make up of 43,3 percent of sample size. The new CBD axis, including Beşiktaş, Şişli and Sarıyer, comprises 287 offices by having 56,7 percent of the sample size. The frequency distribution of office usage types is exhibited in Table 3.2 and demonstrates that a considerable amount of office space – approximately 69 percent of sample data - belongs to the Finance-Insurance-Real Estate (FIRE) sector offices. Other usage types include transportation, communication, retail offices (13 percent), law offices (8 percent), engineering and architecture offices (5 percent); education offices (3 percent) and health offices (3 percent)

Table 3.1: Frequency distribution of sample data among sub-districts of the CBD

District	Frequency	Percent	Cumulative Percent
Eminönü	66	13,0	13,0
Beyoğlu	153	30,2	43,3
Beşiktaş	47	9,3	52,6

Şişli	230	45,5	98,0
Sarıyer	10	2,0	100,0
Total	506	100,0	

Table 3.2: Frequency distribution of sample data due to office usage type

Office usage type (Sectoral distribution)	Frequency	Percent	Cumulative Percent
Finance-Insurance-Real Estate offices	348	68.8	68.8
Health offices	14	2.8	71.6
Education offices	14	2.8	74.4
Engineering &Architecture offices	25	4.9	79.3
Law offices	40	7.9	87.2
Transportation-Communication- Retail offices	65	12.8	100
Total	506	100	

Table 3.3: Frequency distribution of sample data by office building category

Office Building Category	Frequency	Percent	Cumulative Percent
Office Plaza	5	2,82	2,82
Office Buildings(Is Hani)	130	73,45	76,27
Office in Apartment Building	41	23,16	99,44
Shopping Arcade(Pasaj)	1	0,56	100,00
Total	177	100	

Table 3.3 above shows the frequency distribution of the office building category. As it may be seen from the table the Office Buildings take the first place in the ranking of the buildings category with the 130 samples and the Shopping Arcade takes the last place with only 1 sample. From now on when we will say Office Building we will not refer to the special category but to the all the buildings.

If looked to the general physical characteristics of the office building, we can see that the majority of the offices locate in the 6-10 floors (58,19%), and the majority of the buildings had been constructed between 1951 and 1980 (42,37%).[5]

Table 3.4: Frequency distribution of sample data by nuber of the floors in the office building

Number of the Floor in Office Building	Frequency	Percent	Cumulative Percent
Between 3rd and 5th	61	34,46	34,46
Between 6th and 10th	103	58,19	92,66

[5] Frequency distribution of other variables can be found in the Appendix C.

Above 11th	12	6,78	99,44
No answer	1	0,56	100
Total	177	100	

Table 3.5: Frequency distribution of sample data by construction year of the building

Construction Year of the Building	Frequency	Percent	Cumulative Percent
Before 1950	32	18,08	18,08
1951 - 1980	75	42,37	60,45
1981 - 1990	34	19,21	79,66
1991 - 2000	21	11,86	91,53
2001 - After	8	4,52	96,05
No year given	7	3,95	100
Total	177	100	

3.2 Methodology

The classical (crisp) set theory suggests that objects are classified precisely and can either belong to or not belong to the set with the characteristic function, in which the value of one indicates the belonging and the value of zero indicates non-belonging. For the fuzzy set theory, the characteristic function can represent the various degree of belonging for the set (Zimmermann, 2001). That is why the fuzzy logic system can be viewed as the generalization of a crisp (non-fuzzy) set theory, which can deal with lexical uncertainty and imprecision effectively. Saying differently fuzzy logic tries to interpret the intuitive decision making when faced with uncertainty. Many fuzzy logic systems with multiple-components had been developed, based on fuzzy logic theory. For example, Ko and Cheng (2003) introduced an evolutionary fuzzy neural inference model for construction management. Perng et *al.* (2005) proposed a fuzzy logic inference system for sale-ratio evaluation of housing construction strategies in China. However, little has been done on decision making in real estate investment [26].

This thesis provides a way to capture the rental value moves of the office spaces as one commercial real estate type by using physical and locational rent determinants of the properties with imprecise and vague information. Our proposed fuzzy inference system consists of four main components, namely:

1. Fuzzification of crisp inputs,
2. Fuzzy inference engine,
3. Fuzzy Rule Base System; and
4. Defuzzification.

It is a multiple-input-single-output system, shown as in Figure 3.1.

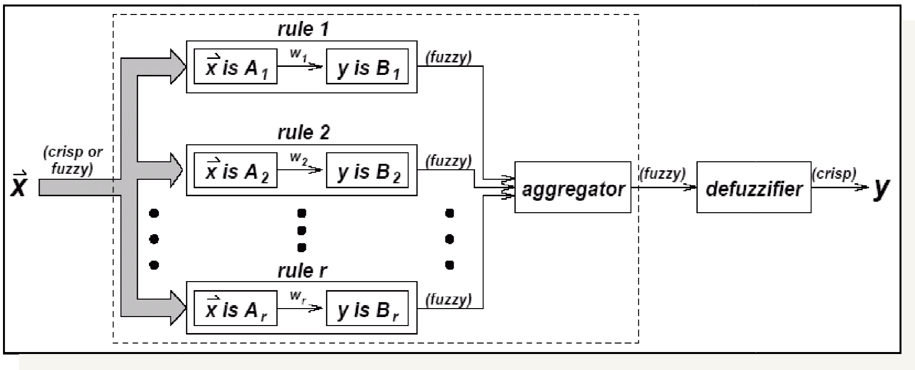


Figure 3.2 Fuzzy rule based system

The "Fuzzification" component is to transform crisp values into linguistic ones to compute outcomes. The "Fuzzy inference engine" aggregates the outcomes and performs decision making in accordance with fuzzy rules defined. The "fuzzy rule base system" contains a set of fuzzy rules (IF-THEN rules). Such system captures the relationship between the input data and anticipated results, and is supported by expert-knowledge. Finally, the fuzzy results will be converted into a crisp value by "Defuzzification."

The single output can represent the level of confidence to take a certain action, as defined by Dourra and Siy (2002). In order to obtain the whole picture of office market, several indicators from physical and locational factors of offices and buildings have been collected. By using backward stepwise regression first on Office Space and then on combined Office Space and Office Building variables the most significant ones have been chosen as system-input components. The detailed information about these input variables can be found in Chapter 6.

After selecting the input variables, all input indicators (crisp values) will be transformed into fuzzy values (suitable linguistic values) by fuzzification. The fuzzy rule-based system is also established with the necessary definitions for the linguistic rule-base, which will take on an important role in our system. Then the fuzzy inference engine can apply the fuzzy implication and linguistic rules to the case, which we consider. Consequently, defuzzification converts the output value into corresponding non-fuzzy value.

3.2.1 Fuzzy Set Theory

The fuzzy logic can be viewed as the extension of the crisp set theory and tries to simulate human behaviors.

The *membership function* (*MF*) represents the degree of belonging for input variable X in the corresponding set in the interval [0,1]. The value of zero implies that there is no membership. On the other hand, the value of one indicates that there is a complete membership. The MF can describe an imprecise statement or matter with a certain degree of certainty and capture vagueness exclusively. There are three types of MF that are widely used, namely triangular, trapezoidal, and generalized bell-shape. In this thesis, the triangular and trapezoidal MF mostly used.

In order to constitute a system framework with the fuzzy set theory, some basic operations of fuzzy set theory are required. For any given two fuzzy sets *A* and *B* with the corresponding membership functions μ_A and μ_B , the operations with fuzzy logic.AND operators is defined as the minimum of μ_A and μ_B:

$$\mu_{A\cap B}(x) = min\left(\mu_A(x), \mu_B(x)\right) = \mu_A(x) \wedge \mu_B(x)$$

whereas, OR operators is defined as the maximum of μ_A and μ_B:

$$\mu_{A\cup B}(x) = max\left(\mu_A(x), \mu_B(x)\right) = \mu_A(x) \vee \mu_B(x)$$

3.2.2 Fuzzy Rule Based System

In order to define an appropriate linguistic rule-base and assist in decision making, the fuzzy IF-THEN rule is adopted to characterize certain conditions in linguistic terms which are simulating the human cognitive process. In general, "if a set of conditions is satisfied, then a set of consequences can be produced" (Zimmermann, 2001). The general form of fuzzy rule can be expressed as:

$$R: \ If\ x\ is\ P\ then\ y\ is\ Q$$

where R represents "the fuzzy rule"; the former part " $x\ is\ P$ " is the premise/condition; the latter part " $y\ is\ Q$ " is the consequence. In the case of multiple antecedents, the fuzzy rule set can be generalized:

$$R_j: \ If\ x_j\ is\ P_j\ then\ y\ is\ Q$$

where $j = 1,2, \ldots, n$.

These rules can be grouped into 2 categories as follows:

For Mamdani FIS:

1. If Floor is low and Area is big and Number of Workers is few and Residence out of office district is low, Then Rent is Cls 4
2. If Floor is high and Area is big and Number of Workers is too much and Residence out of office district if medium, Then Rent is Cls 6
3. If Floor is low and Area is low and Number of Workers is few and Residence out of office district if low, Then Rent is Cls 2

For Takagi, Sugeno, and Kang (TSK) FIS

1. If Floor is low, Then Rent is Function 1
2. If Floor is medium, Then Rent is Function 2
3. If Floor is high, Then Rent is Function 3

3.2.3 Fuzzy Processing

In the case of a fuzzy inference engine, Mamdani's and Sugeno's methods are adopted for the inference of decision making, which was proposed by Mamdani and Assilian (1975) and M. Sugeno and G. T. Kang. (1988). A set of linguistic control rules, which are defined in a fuzzy rule based system, is applied with these two methods for expressing the human behavior for assisting in decision processes. The max-min operator is used for its composition:

$$\mu_{output}(x) = \max\left(\min\left(\mu_{A_j}(x), \mu_{B_j}(x)\right)\right),$$

where $j = 1,2, \dots, n$

After applying the two methods described, the result has to be defuzzified for converting the output value into a corresponding non-fuzzy value. The *Centroid of Area* (the expected values of probability distribution) the most widely adopted in defuzzification:

$$z_{COA} = \frac{\int_Z \mu_A(z) z \, dz}{\int_Z \mu_A(z) \, dz},$$

where z_{COA} is the crisp value; $\mu_A(z)$ is the membership function corresponding to the value z.

CHAPTER 4

REAL ESTATE VALUATION METHODS

4.1 Introduction

Recently, real estate appraisal is becoming an outstanding subject for numerous quantitative methods to assist to define an opinion regarding the value depending on the highest and best use of the property. The complexity of value attribution emerges from the need to consider so comprehensive factors while applying these models. As a long sight of determinants, these can be listed as all the interests, benefits, rights and obstacles inherent in the ownership of physical real estate and in addition the land together with all improvements that are permanently affixed to it and all attachments associated as well (Pagourtzi et al., 2003). As comes from their nature, each of any two real estates is dissimilar when their locations are compared at least.

The appraising process is a branch of fast developing service sector. The primary purpose in that process is to attain the maximum possible value for the highest and best use of the property while considering its possible level under the property's inherited constraints such as the legal issues affixed to it, the physical attributes it possesses, the residential qualifications of the local area and the structural conditions of the marketplace that affect to demand for and supply of the real estates.

To capture the most accurate estimate of the market price appraiser should also take into account the market culture, the conditions of the time and the existing fundamentals of the market. Without these factors are considered in the model, it cannot be concluded that the appraisal process is effective and that it reflects the maximum probable market price of the real estate.

Before identifying the quantitative appraisal methods existed in the literature, it is better to comprehend the value concept that can be found at different extents.

4.2 Types of Value

In this advancing service sector, various types of value of the real estate can be the concern at the attribution process for predetermined time by a number of different players in the marketplace based on the purpose of the use of the property.

Some of the common definitions of value sought by a real estate appraisal are listed as:

1. *Market Value* – Market value is the equilibrium price level that the real estate can be bought and sold in an open market. In a broader sense, it is the most probable price of the real estate for specified date that its rights are sold at a reasonable exposure in the market to the rational buyers that are willing to pay at. During the market value estimation process, the issues on legal interest, the structural characteristics of the building, timing through the market, and the potential buyers in that market are posited. This definition of the value is valid regardless of any national borders are concern
2. *Value-in-Use* - Another type of value that is predicted by the appraisers is value-in-use, or use value, which indicates the net present value (NPV) of the future cash flows that comes from the real estate itself. This value does not have to be at the exact level of the fair market value. This indicates the productivity of the real estate and shows the value of a specific usage of it.
3. *Investment Value* – In that definition of the value, the point is not the usage but the investment activity. The value is attained in order to satisfy the specific investment requirements of an entity. That value has not to be at the same level to the one at marketplace since it is rather needs of the individual or investor private. If these are totally typical of the market then two will be the identical.
4. *Going-Concern Value* – This type of value basically suits for the real estates that are used for commercial purposes like hotels, restaurants and so on. Therefore, the market value of these real estates also contains the business chain it encompasses including all the operational and business processes pursued, and human sources possessed that all contribute value to the enterprise.
5. *Assessed Value* – For the legal issues such due to the tax rolls, this value is calculated.
6. *Insurable Value* – In case the occurrence of any damage to real estate, it may be insured due to a predetermined value. This value may be above or below the fair market value of a property and named as the insurable value.

4.3 Valuation Methods

In their paper named "*Real estate appraisal: A Review of Valuation Methods*" Pagourtzi et al. (2003) proposed to divide the real estate valuation methods into two groups. The first one is called as *traditional methods* that rely upon some form of comparison to assess market value while the other one is called as *advanced valuation methods* which try to analyze the market by directly mimicking the thought processes of the players in the market in an attempt to estimate the point of exchange.

The subgroups of these two real estate valuation methods are given as follows[6]:

1. *Traditional valuation methods*:
   i. sale comparison method,
   ii. investment/income method,
   iii. residual method,
   iv. cost method,
   v. multiple regression method,
   vi. stepwise regression method;

2. *Advanced valuation methods*:
   i. hedonic pricing method,
   ii. spatial analysis methods,
   iii. artificial neural networks (ANNs),
   iv. fuzzy logic.

[6] Additional information about the real estate valuation methods can be found in the Appendix D.

CHAPTER 5

FUZZY LOGIC AND FUZZY INFERENCE SYSTEM

5.1 Introduction to Fuzzy Logic

The boundaries of a classical set are required to be drawn precisely and, therefore, set membership is determined with complete certainty. An individual is either definitely a member of the set or definitely not a member of it. However, most sets and propositions are not so neatly characterized in reality. For example, the set of tall people is a set whose exact boundary cannot be precisely determined. To overcome this limitation of classical set theory, the concept of a fuzzy set was introduced [62].

In 1965, L.A. Zadeh developed the concept of a *fuzzy set*. His purpose in developing the mathematics of fuzzy sets was to create a tool to model complex phenomena using human knowledge. The fuzzy set is based on a multi-valued logic which blurs the boundaries of subsets. Normally, a logic condition or expression can be either completely true or completely false. In the fuzzy set, all statements have some degree of truth between 0 and 1 inclusive and values range from 0 to 100% true or false [63].

The concept of a degree of membership in a set builds upon traditional logic and expands it to allow fuzzy sets to be defined qualitatively using linguistic terms such as tall, warm, short, cold, etc.. The elements of the sets are then assigned degrees of membership.

5.1.1 Crisp Sets

A set is any well defined collection of objects and an object contained by a set is called a *member*, or *element*.

The *universe of discourse* is the set of all allowable values for the given variable. Let us define a *crisp set* A in the universe of discourse X, [42]. We can define this set A by listing all of the members or by identifying the elements x that belong to this set, an element x that belong to the set A is shown as $x \in A$, and an element that does not belong to the set A is shown as $x \notin A$. Another way of defining the set A is by specifying the condition conditions for which $\in A$.

The general expression for this would be as follows:

$$A = \{x|\ x\ meets\ conditions\}\ .$$

If every element in the set A is also a member of set B, then A is a *subset* of B:

$$A \subseteq B.$$

If A is a subset of B and B is subset of A, then A and B are *equal sets*:

$$A = B.$$

If at least one element in A is not in B or at least one element in B is not in A, then A and B are *not equal:*

$$A \neq B.$$

If A is a subset of B but A and B are not equal, then set A is a proper subset of B :

$$A \subset B.$$

Alternatively, a zero-one membership function, also called a characteristic function, or indicator function, for A can be introduced, denoted as $\mu_A(x)$, which for every $x \subset X$, assigns a value that determines the strength of membership of each x in the set A such that

$$A \Rightarrow \mu_A(x) = \begin{cases} 1 & if\ x \in A, \\ 0 & if\ x \notin A. \end{cases}$$

Set A (which can also be treated as a subset of) is mathematically equivalent to its membership function $\mu_A(x)$ in the sense that knowing $\mu_A(x)$ is the same as knowing A itself.

Example 1: Consider the set, universe of discourse, X of all the humans in the Earth. The elements of X are individual persons, by they can be divided into the different types of subsets that can be established for X. One subset can be defined by the blood type of the person, one by the age and so on.

5.1.2 Operations on Classical Sets

There are various operations that can be performed in the classical or crisp sets. The results of the operation performed on the classical sets will be exact. Below we give a brief description of the operations that can be performed on the classical sets:
Consider two sets A and B defined on the universe of discourse X.

5.1.2.1 Union

The union of two classical sets A and B is denoted by $A \cup B$. It represents all the elements in the universe that reside in either the set A, the set B or both sets A and B. This operation is called the logical OR. In set theoretic form it is represented as

$$A \cup B := \{x |\ x \in A\ or\ x \in B\}.$$

In Venn diagram form it can be represented as shown in Figure 5.1.

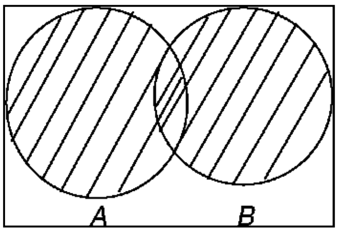


Figure 5.1 The union of two sets [57]

5.1.2.2 Intersection

The intersection of two sets A and B is denoted $A \cap B$. It represents all those elements in the universe X that simultaneously belongs to both sets A and B.
In set theoretic form it is represented as

$$A \cap B := \{x |\ x \in A\ and\ x \in B\}.$$

In Venn diagram form it can be represented as shown in Figure 5.2.

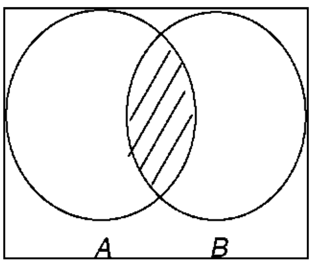


Figure 5.2 The intersection of two sets [57]

5.1.2.3 Complement

The complement of set A denoted $\bar{A}$, is defined as the collection of all elements in the universe that do not reside in the set A. In set theoretic form it is represented as

$$\bar{A} := \{x | x \notin A \text{ and } x \in X\}.$$

In Venn diagram form it is represented as shown in Figure 5.3.

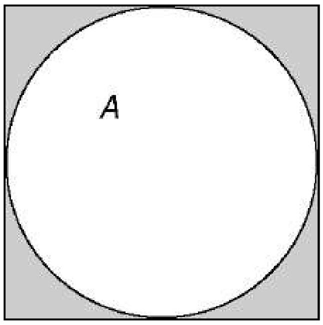


Figure 5.3 The complement of a set [57]

5.1.2.4 Difference

The difference of a set A with respect to B, denoted by $A|B$ is defined as collection of all elements in the universe that reside in A and that do not reside in B simultaneously.
In set theoretic form it is represented as

$$A|B := \{x | x \in A \text{ and } x \notin B\}.$$

In Venn diagram form it is represented as shown in Figure 5.4.

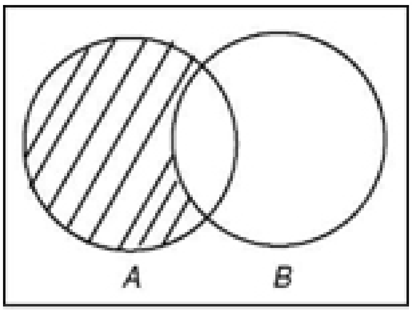


Figure 5.4 The difference of two sets [57]

5.1.3 Generalizing Crisp Set - Fuzzy Sets

In conventional set theory, an item is either element of a set or not. This means that something may be true or false, black or white. In his article dated to 1965, L.A. Zadeh suggested a modified set theory in which individual elements could have a degree of membership which ranged over a infinity values rather than 0 or 1 and named them fuzzy set. Fuzzy set theory allows degree of membership, so something maybe partially true and partially false at the same time, that means the gray color would be partly in black and partly white [28].

The characteristic function of classical set assigns a value of either 1 or 0 to each individual in the universal set of discourse, there by discriminating between members and nonmembers of the crisp set under consideration. In the new set theory the values assigned to the elements of the universal set fall within a specified range, in our case [0,1] and indicate the membership grade of these elements in the set. Larger values denote higher degrees of set membership. Such a function is called a membership function and the set is defined as a fuzzy set [25].

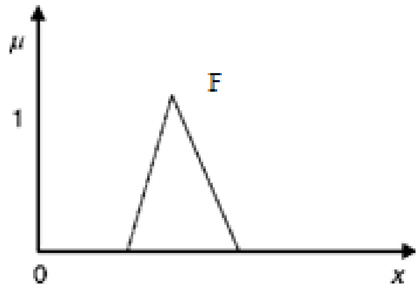


Figure 5.5 Fuzzy membership function [57]

Taking into account the above said we may assume that a *fuzzy set* $\mathcal{F}$ is a generalization of a crisp set. It is defined on a universe of discourse X and is characterized by a membership function $\mu_F(x)$ that takes on values in the interval [0, 1]. This membership function specifies the grade or degree to which any element x in X belongs to the fuzzy set $\mathcal{F}$. Larger values of $\mu_F(x)$ indicate higher degrees of membership. A membership function provides a *measure of the degree of similarity* of an element in X to the fuzzy set.

A fuzzy set $\mathcal{F}$ in X may also be represented as a set of ordered pairs of a generic element x and its grade of membership function, $\mu_F(x)$, i.e.,

$$\mathcal{F} = \{(x, \mu_F(x))|\, x \in X,\ \mu_F(x) \in [0,1]\}.$$

As we know a function is an ordered pair of elements no two of which have the same first element. Hence, strictly speaking, $(x, \mu_F(x))\ \forall\, x \in X$, denotes the membership function whereas $\mu_F(x)$ denotes the grade of the membership function. However, we will refer to $\mu_F(x)$ as the membership function, Figure 5.5 above.

When X is continuous (e.g., the real numbers), F is commonly written as

$$F = \int_X \mu_F(x)/x.$$

The integral sign does not denote integration. It simple denotes the collection of all points $x \in X$ with associated membership function $\mu_F(x)$. When X is discrete (e.g., the integers), F is commonly written as

$$F = \sum_X \mu_F(x)/x.$$

Again the summation sign does not denote arithmetic addition. It denotes the collection of all points $x \in X$ with associated membership function $\mu_F(x)$ hence, it denotes the set theoretic operation of union. The slash in the above formulas associates the elements in X with their membership grades, where $\mu_F(x) > 0$.

5.1.4 Linguistic Variables

Zadeh (1975, p. 201) states:

> *In retreating from precision in the face of overpowering complexity, it is natural to explore the use of what might be called linguistic variables, that is, variables whose values are not numbers but words or sentences in a natural or artificial language. The motivation for the use of words or sentences rather than numbers is that linguistic characterizations are, in general, less specific than numerical ones.*

Let u denote the name of a linguistic variable (e.g., Number of Offices). Numerical (measured) values of a linguistic variable u are denoted x, where $x \in X$. Sometimes x and u are used interchangeably, especially, when a linguistic variable is a letter, as is sometimes the case in engineering applications. A linguistic variable is usually decomposed into a set of terms which cover its universe of discourse.

Recently, Zadeh (1999, p. 107) has used the word *perception* to describe the terms associated with linguistic variables. For example, he states:

> *A fundamental difference between measurements and perceptions is that, in general, measurements are crisp numbers whereas perceptions are fuzzy numbers or, more generally, fuzzy granules, that is, clumps of objects in which the transition from membership to nonmembership is gradual rather than abrupt.*

5.1.5 Operations on Fuzzy Set

Considering three fuzzy sets A, B and C on the universe of discourse X. For a given element x of the universe, the following function theoretic operations for the set theoretic operations unions, intersection and complement are defined for A, B and C on X [57].

5.1.5.1 Union

The *union* of two fuzzy sets A and B is shown as follows:

$$\mu_{A \cup B}(x) := max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x)$$

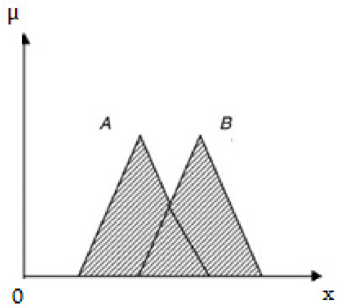


Figure 5.6 Union of two fuzzy sets [57]

5.1.5.2 Intersection

The *intersection* of two fuzzy sets A and B is shown as follows:

$$\mu_{A \cap B}(x) := min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x)$$

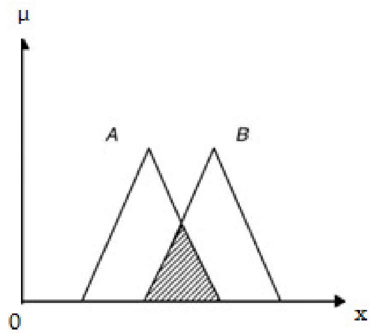


Figure 5.7 Intersection of two fuzzy sets [57]

5.1.5.3 Completion

The *completion* of fuzzy set A is shown as follows:

$$\mu_{\bar{A}}(x) := 1 - \mu_A(x)$$

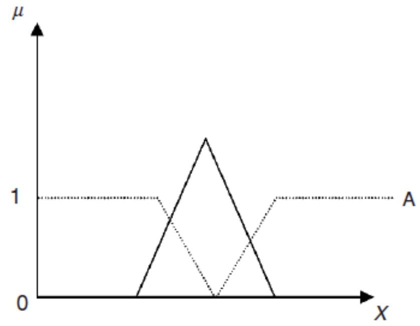


Figure 5.8 The complement of a fuzzy set [57]

5.1.6 Membership Functions

Fuzziness in a fuzzy set is characterized by its membership functions. It classifies the element in the set, whether it is discrete or continuous. We can form membership function using both functions as has been shown in the previous part and using graphical representation. The graphical representations may include different shapes. But, there are certain restrictions regarding the shapes used. The "shape" of the membership function is an important criterion that has to be considered. There are different methods to form membership functions, like triangular, trapezoidal and so on. This section discusses on the features and the various methods of arriving at membership functions [29].

The feature of the membership function is defined by three properties [42]. They are:

1. Core,
2. Support,
3. Boundary.

Figure 5.9 shown below defines the properties listed above. The membership function can take value between 0 and 1.

1. *Core* - If the region of universe is characterized by full membership, i.e. 1, in the set A then this gives the core of the membership function of fuzzy at A. The elements, which have the membership function equal to 1, are the elements of the core, i.e., $\mu_A(x) = 1$.

2. *Support* - If the region of universe is characterized by nonzero membership in the set A, this defines the support of a membership function for fuzzy set A. The support has the elements whose membership is greater than 0, i.e., $\mu_A(x) > 0$.
3. *Boundary* - If the region of universe has a nonzero membership but not full membership, this defines the boundary of a membership A: The boundary has the elements whose membership is between 0 and 1, i.e., $0 < \mu_A(x) < 1$.

These are the standard regions defined in the membership functions. They are:

1. *Crossover point* - The crossover point of a membership function is the elements in universe of discourse whose membership value is equal to 0.5, i.e., $\mu_A(x) = 0.5$.
2. *Height* - The height of the fuzzy set A is the maximum value of the membership function, $max\ (\mu_A(x))$.

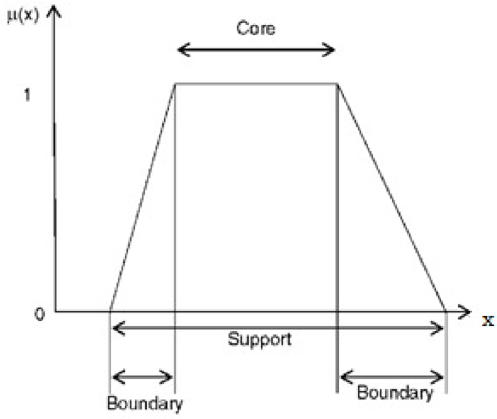


Figure 5.9 Boundary, Support and Core of the fuzzy membership function [57]

In rule-based applications of Fuzzy Logic, membership functions, $\mu_F(x)$, are associated with terms that appear in the antecedents or consequents of rules, or in phrases. The following can be given as the example of the rule and the associated membership function:

If Floor is low and Area is big and Number of Workers is few and Residence out of office district if low, Then Rent is Cls4 [$\mu_{Floor}(x)$, $\mu_{Number\ of\ Workers}(y)$, $\mu_{Area}(z)$, $\mu_{Residence\ out\ of\ Office\ District}(u)$]

Membership of a fuzzy set can be defined to be numerical or functional. *Numerical* membership expresses the degree of membership function of a fuzzy set as a vector of numbers. A *functional* definition defines the membership function in analytic expression which allows the degree of membership for each element, x, in the defined in the universe of discourse, X .

The most commonly used shapes for membership functions are triangular, bell-shaped, trapezoidal and so on. Membership functions can either be chosen by the user arbitrarily, based on the user's experience and, hence, the membership functions for two users could be quite different depending upon their experiences, perspectives, cultures, etc., or, they can be designed using optimization procedures (e.g., Horikawa et al. (1992), Jang (1992), Wang and Mendel (1992a, b).

5.1.6.1 Triangular Membership Function

A triangular membership function is defined as the function [31]

$$\alpha = \mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_M - a_1} & for\ a_1 \le x \le a_M, \\ \dfrac{x - a_2}{a_M - a_2} & for\ a_M \le x \le a_2, \\ 0 & otherwise, \end{cases}$$

where $[a_1, a_2]$ is the supporting interval and the point $(a_M, 1)$ is the peak:

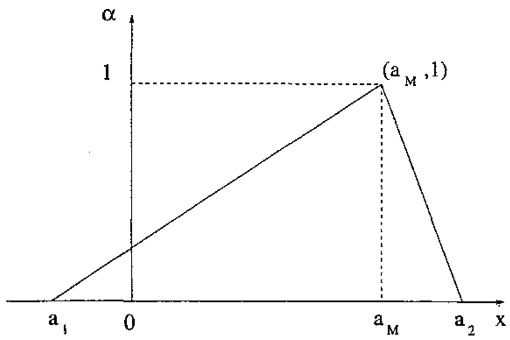


Figure 5.10 Triangular fuzzy membership function [57]

Often in the application the point $a_M \in (a_1, a_2)$ is located in the middle of the supporting interval, i.e.,

$$a_M = \frac{a_1 + a_2}{2}$$

Then the membership function takes the form

$$\alpha = \mu_A(x) = \begin{cases} 2\dfrac{x - a_1}{a_2 - a_1} & \text{for } a_1 \le x \le \dfrac{a_1 + a_2}{2} \\ 2\dfrac{x - a_2}{a_1 - a_2} & \text{for } \dfrac{a_1 + a_2}{2} \le x \le a_2 \\ 0 & \text{otherwise} \end{cases}$$

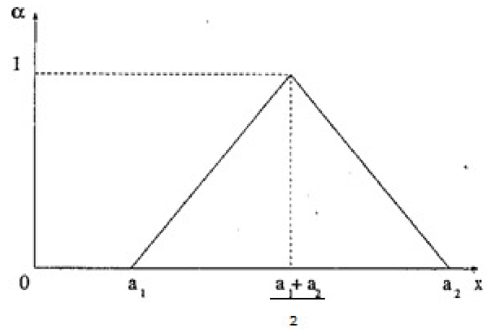


Figure 5.11 Symmetric triangular fuzzy membership function [57]

5.1.6.2 Bell-Shaped Fuzzy Membership Functions

There are several types of the bell-shaped fuzzy membership functions, but the most commonly used one is the bell-shaped fuzzy membership function. This function is constructed with the Fuzzy Normal distribution. The well-known Normal or Gauss distribution in probability is defined by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\eta}{\sigma})^2}, \quad -\infty < x < \infty \ (*)$$

where η is the mean and σ is the standard deviation of the distribution. The curve of $f(x)$ is bell-shaped. It is symmetric with respect to η. Figure below presents $f(x)$ for $\eta = 0$ and $\sigma = 0.25, 0.5, 1$. As σ gets smaller, the higher the peak becomes.

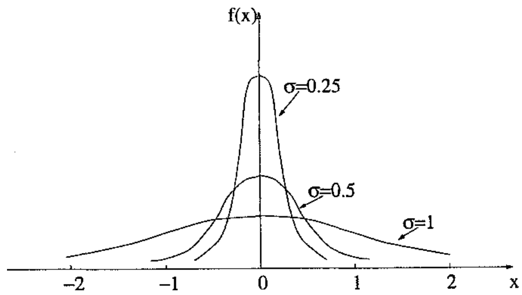


Figure 5.12 Bell-shaped fuzzy membership function

To construct a fuzzy number from a normal distribution we set in (*) $\sigma = \frac{1}{\sqrt{2\pi}}$ which gives a function with maximum 1,

$$\alpha = \mu_A(x) = e^{-\pi(x-\eta)^2}, \quad x \in (-\infty, \infty), \quad \alpha \in [0,1]$$

where η is a parameter which determines the shape of $\mu_A(x)$. No matter how η changes, the maximum of $\mu_A(x)$ is always 1, hence, the formula above represents the bell shaped membership function whose supporting interval $A \in (-\infty, \infty)$ is unbounded.

5.1.6.3 Trapezoidal Fuzzy Membership Functions

A trapezoidal membership function is defined as the function:

$$\alpha = \mu_A(x) = \begin{cases} \dfrac{x - a_1}{a_1^{(1)} - a_1} & \text{for } a_1 \le x \le a_1^{(1)} \\ 1 & \text{for } a_1^{(1)} \le x \le a_2^{(1)} \\ \dfrac{x - a_2}{a_2^{(1)} - a_2} & \text{for } a_2^{(1)} \le x \le a_2 \\ 0 & \text{otherwise} \end{cases}$$

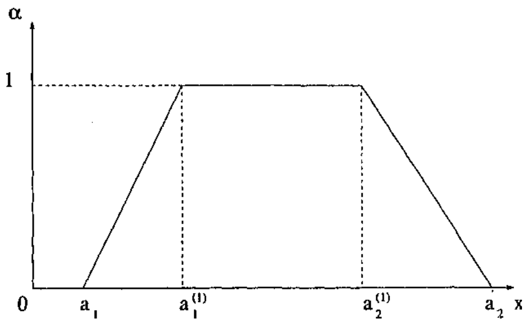


Figure 5.13 Trapezoidal fuzzy membership function [57]

The supporting interval is $[a_1, a_2]$ and the flat segment on level $\alpha = 1$ is $[a_1^{(1)}, a_2^{(1)}]$. If $a_1^{(1)} = a_2^{(1)}$ then the trapezoidal membership function reduces to a triangular membership function, so we can say that trapezoidal membership function is the generalized version of the triangular membership function. If $w[a_1, a_1^{(1)}] = w[a_2^{(1)}, a_2]$ where w is the width of an interval, the trapezoidal number is symmetrical with respect to the line $x = \frac{a_1^{(1)} + a_2^{(1)}}{2}$:

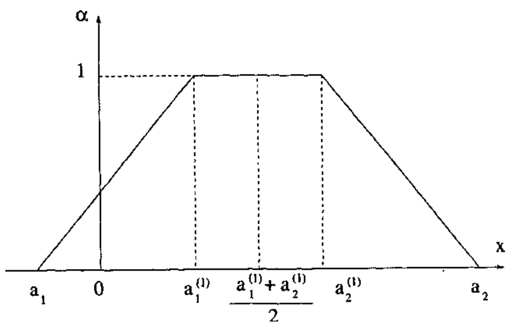


Figure 5.14 Symmetric trapezoidal fuzzy membership function [57].

The number of membership functions is up to us. By using more membership functions for the given variable we can achieve greater resolution but this resolution would be achieved at the price of the computational complexity. There is no need for membership functions to overlap, but one of the good sides of fuzzy logic is that membership functions can be made to overlap. This expresses the fact that "the glass can be partially full and partially empty at the same time." In this way, we are making fuzzy logic systems more vigorous by distributing our decision over more than one input class. Besides, there is nothing as membership function must take values between zero and one. Many researches do take the membership values between zero and one in order to normalize them. But even if the given membership function the values are not in between zero and one they can be normalized by simple techniques like, dividing $\mu_F(x)$ by the $sup_{x\in X}\mu_F(x)$.

5.1.7 Generalization of Fuzzy Intersection - T-norm

The intersection of two fuzzy sets A and B is specified in general by a function T: $[0,1]\text{x}[0,1] \rightarrow [0,1]$, which aggregates two membership grades as follows:

$$\mu_{A\cap B}(x) := T\big(\mu_A(x), \mu_B(x)\big) = \mu_A(x) \tilde{*} \mu_B(x),$$

where " $\tilde{*}$ " is a binary operator for the function T. This class of fuzzy intersection operators, which are usually referred to as *T-norm* (triangular norm) operators, meets the following basic requirements

A ***T-norm*** operator [14] is a two-place function $T(\cdot,\cdot)$ satisfying

1. *Boundary:* $T(0,0) = 0,\ T(a,1) = T(l,a) = a,$
2. *Monotonicity:* $T(a,b) \leq T(c,d) if\ a \leq c\ and\ b \leq d,$
3. *Commutativity:* $T(a,b) = T(b,a),$
4. *Associativity:* $T\big(a,T(b,c)\big) = T(T(a,b),c).$

The first requirement imposes the correct generalization to crisp sets. The second requirement implies that a decrease in the membership values in A or B cannot produce an increase in the membership value in $A \cap B$. The third requirement indicates that the operator is indifferent to the order of the fuzzy sets to be combined. Finally, the fourth requirement allows us to take the intersection of any number of sets in any order of pairwise groupings. The following example illustrates four of the most frequently encountered *T-norm* operators

Four of the most frequently used *T-norm* operators are:

1. *Minimum:* $T_{min}(a,b) = min(a,b) = a \wedge b,$
2. *Algebraic product:* $T_{ap}(a,b) = ab,$
3. *Bounded product:* $T_{bp}(a,b) = 0 \vee (a+b-1),$
4. *Drastic product:* $T_{dp}(a,b) = \begin{cases} a, & if\ b = 1, \\ b, & if\ a = 1, \\ 0, & if\ a,b < 1. \end{cases}$

We can mathematically easily verify that

$$T_{dp}(a,b) < T_{bp}(a,b) < T_{ap}(a,b) < T_{min}(a,b).$$

5.1.8 Generalization of Fuzzy Union - T-conorm

Like fuzzy intersection, the fuzzy union operator is specified in general by a function S: [0,1]x[0,1] → [0,1]. In symbols

$$\mu_{A\cup B}(x) := S\big(\mu_A(x), \mu_B(x)\big) = \mu_A(x) \tilde{\mp} \mu_B(x),$$

where "$\tilde{\mp}$" is a binary operator for the function S. This class of fuzzy union operators are often referred to as *T-conorm* (or *S-norm*) operators.

A ***T-conorm*** (or ***S-norm***) operator is a two-place function $S(\cdot,\cdot)$ satisfying

1. *Boundary*: $S(1,1) = 1,\ S(a,0) = S(0,a) = a,$
2. *Monotonicity*: $S(a,b) \leq S(c,d),\ if\ a \leq c\ and\ b \leq d,$
3. *Commutativity*: $S(a,b) = S(b,a)$,
4. *Associativity*: $S\big(a, S(b,c)\big) = S(S(a,b),c).$

The justification of these basic requirements is similar to that of the requirements for *T-norm* operators.

Corresponding to the four *T-norm* operators in the previous example, we have the following four *T-conorm* operators:

1. *Maximum:* $S_{max}(a,b) = max(a,b) = a \vee b,$
2. *Algebraic sum:* $S_{as}(a,b) = a + b - ab,$
3. *Bounded sum:* $S_{bs}(a,b) = 1 \wedge (a + b),$
4. *Drastic sum:* $S_{ds}(a,b) = \begin{cases} a, & if\ b = 0, \\ b, & if\ a = 0, \\ 1, & if\ a,b > 0. \end{cases}$

It can also be verified that

$$max(a,b) < S_{as}(a,b) < S_{bs}(a,b) < S_{ds}(a,b).$$

5.1.9 Advantage of Fuzzy Logic

Here is a list of general advantages about fuzzy logic [5]:

1. Fuzzy logic is conceptually easy to understand. The mathematical concepts behind fuzzy reasoning are very simple. What makes fuzzy nice is the "naturalness" of its approach and not its far-reaching complexity.
2. Fuzzy logic is flexible. With any given system, it is easy to massage it or layer more functionality on top of it without starting again from scratch.
3. Fuzzy logic is tolerant of imprecise data. Everything is imprecise if you look closely enough, but more than that, most things are imprecise even on careful inspection. Fuzzy reasoning builds this understanding into the process rather than tacking it onto the end.
4. Fuzzy logic can model nonlinear functions of arbitrary complexity. You can create a fuzzy system to match any set of input-output data. This process is made particularly easy by adaptive techniques like *ANFIS* (*Adaptive Neuro-Fuzzy Inference Systems*) which are available in the *Fuzzy Logic Toolbox*.
5. Fuzzy logic can be built on top of the experience of experts. In direct contrast to neural networks, which take training data and generate opaque, impenetrable models, fuzzy logic lets you stand on the shoulders of people who already understand your system.
6. Fuzzy logic can be blended with conventional control techniques. Fuzzy systems do not necessarily replace conventional control methods. In many cases fuzzy systems augment them and simplify their implementation.
7. Fuzzy logic is based on natural language. The basis for fuzzy logic is the basis for human communication. This observation underpins many of the other statements about fuzzy logic.

5.2 Introduction to Fuzzy Inference Systems

One of the popular computing framework that is based on the concepts of fuzzy set theory, fuzzy if-then rules, and fuzzy reasoning is *fuzzy inference system*. It was successfully used in a wide variety of fields, such as automatic control, data classification, decision analysis, expert systems, time series prediction, robotics, and pattern recognition. Because of its multidisciplinary nature, the fuzzy inference system is known by numerous other names, such as *fuzzy-rule-based system, fuzzy expert system* [30], *fuzzy model* [56, 59], *fuzzy associative memory* [31], *fuzzy logic controller* [35, 36, 41], and simply *fuzzy system* [28].

Rule base, database and reasoning mechanism are the basic structure components of the fuzzy inference system. A *rule base* contains a selection of fuzzy rules, *database* defines the membership functions used in the fuzzy rules, and a *reasoning mechanism* performs the inference procedure upon the rules and given facts to derive a reasonable output or conclusion.

As an inputs fuzzy inference system can take both fuzzy inputs or crisp inputs, but the outputs are nearly always fuzzy sets. Sometimes users need a crisp output, especially, in a situation where a fuzzy inference system is used as a controller. Therefore, we need a method of *defuzzification*, which extracts a crisp value that has a best representation of this fuzzy set. Figure 5.15 depicts a fuzzy inference system with a crisp output. The dashed line in this figure indicates a basic fuzzy inference system with fuzzy output and the defuzzification block serves the purpose of extracting the crisp single value from fuzzy set.

When the input and output values of the fuzzy inference system are crisp sets, then we may say that, fuzzy inference system implements a nonlinear mapping from its input space to output space. We may say that *fuzzy If – Then rules* play a role of functions in this mapping, with the antecedent parts of the rules defining fuzzy set of the input space and consequent part describing the output in the fuzzy set.

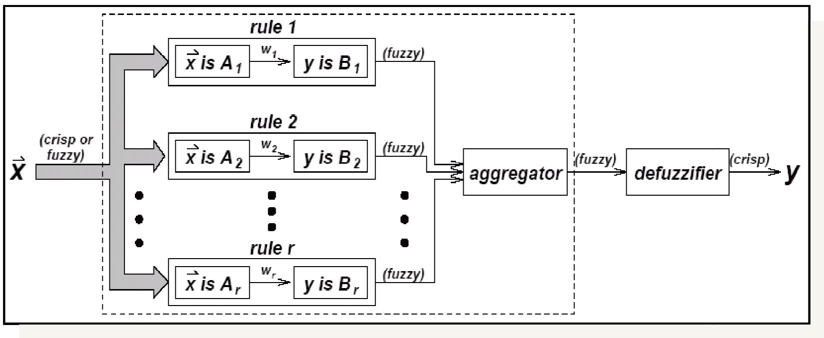


Figure 5.15: Block diagram for a fuzzy inference system.

In what follows, we shall first introduce three types of fuzzy inference systems that have been widely employed in various applications. The differences between these three fuzzy inference systems lie in the consequents of their fuzzy rules, and thus their aggregation and defuzzification procedures differ accordingly. Then we will introduce and compare three different ways of partitioning the input space; these partitioning methods can be adopted by any fuzzy inference system, regardless of the structure of the consequents of its rules. Finally, we will briefly address the features and the problems of fuzzy modeling, which is concerned with the construction of fuzzy inference systems for modeling a given target system, and give the comparison of the Mamdani and Sugeno fuzzy inference systems.

5.2.1 Mamdani-type Fuzzy Models

In their paper dated 1975, E. H. Mamdani and S. Assilian proposed the *Mamdani-type fuzzy inference system* as the first attempt to control a steam engine and boiler combination by a set of linguistic control rules obtained from experienced human operators. In Figure 5.16, we may see how a two-rule Mamdani fuzzy inference system works to derives the overall output $\boldsymbol{z}$ when subjected to two crisp inputs x and y**.**

Let us assume that *T-norm* and *T-conorm* operators are defined as max and algebraic product respectively, and let us also instead of the max-min composition use max-product composition. In this case Figure 5.17 will show the resulting fuzzy reasoning. Simple by changing the *T-norm* and *T-conorm* operators we can construct different fuzzy reasoning.

In the same paper of E. H. Mamdani and S. Assilian, two fuzzy inference systems were used as two controllers to generate the heat input to the boiler and throttle opening of the engine cylinder, respectively, to regulate the steam pressure in the boiler and the speed of the engine. Due to the fact that the plant takes only crisp values as inputs, we have to use a defuzzifier to convert a fuzzy set to a crisp value.

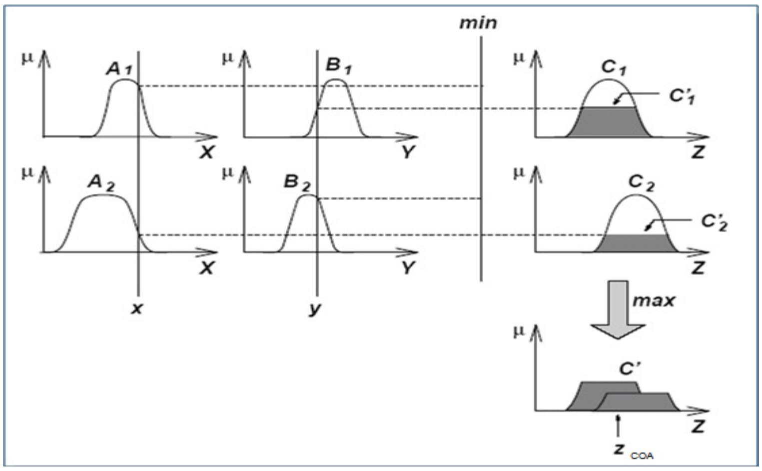


Figure 5.16: The Mamdani fuzzy inference system using min and max for *T-norm* and *T-conorm* operators, respectively [28].

5.2.1.1 Defuzzification

Defuzzification refers to the way a crisp value is extracted from a fuzzy set as a representative value. In general, there are five methods for defuzzifying a fuzzy set $\boldsymbol{A}$ of a universe of discourse Z, as shown in Figure 5.17. A brief explanation of each defuzzification strategy follows.

- *Centroid of area* z_{COA}:

$$z_{COA} := \frac{\int_Z \ \mu_A(z) z \, dz}{\int_Z \ \mu_A(z) \, dz},$$

where $\mu_A(z)$ is the aggregated output MF. This is the most widely adopted defuzzification strategy, which reminds of the calculation of expected values of probability distributions.

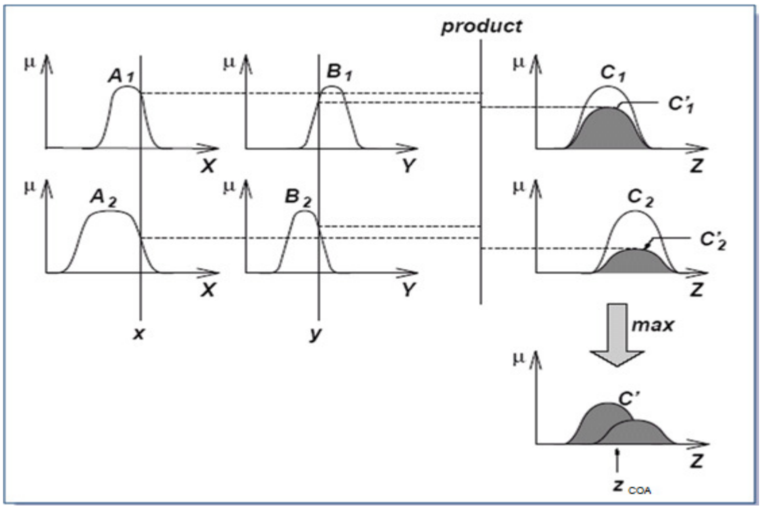


Figure 5.17: The Mamdani fuzzy inference system using product and *max* for *T- norm* and *T-conorm* operators, respectively [28].

- *Bisector of area* z_{BOA} : z_{BOA} satisfies

$$\int_{\alpha}^{z_{BOA}} \mu_A(z) \ dz = \int_{z_{BOA}}^{\beta} \mu_A(z) \ dz,$$

where $\alpha = min\{z|z \in Z\}$ and $\gamma = max\{z|z \in Z\}$. That is, the vertical line $z = z_{BOA}$ partitions the region between $\boldsymbol{z = \alpha, z = \beta, y = 0}$ and $y = \mu_A(z)$ into two regions with the same area.

- *Mean of maximum* z_{MOM}: z_{MOM} is the average of maximizing z at which the MF reach maximum μ^*. In symbols,

$$z_{MOM} := \frac{\int_{Z'} z \, dz}{\int_{Z'} dz},$$

where $Z' = \{z|\ \mu_A(z) = \mu^*\}$. in particular, if $\mu_A(z)$ has a single maximum at $z = z^*$ then $z_{MOM} = z^*$. Moreover, if $\mu_A(z)$ reaches its maximum whenever $\in [z_{left}, z_{right}]$, then

$$z_{MOM} = (z_{left} + z_{right})/2.$$

The mean of maximum is the defuzzification strategy employed in Mamdani's fuzzy logic controllers.

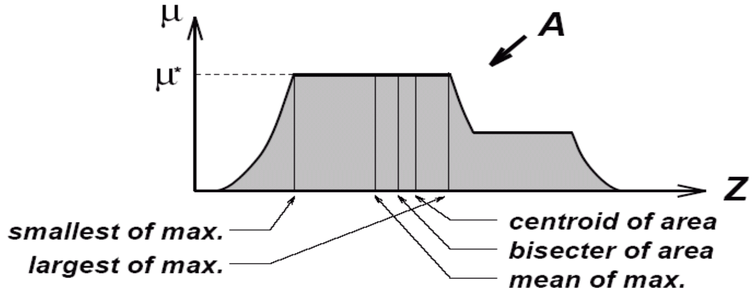


Figure 5.18: Various defuzzification schemes for obtaining a crisp output [28].

- *Smallest of maximum* z_{SOM}: z_{SOM} minimum (in terms of magnitude) of the maximizing z.
- *Largest of maximum* z_{LOM} : z_{LOM} is the maximum (in terms of magnitude) of the maximizing z. Because of their obvious bias, z_{SOM} and z_{LOM} are used as often as the other three defuzzification methods.

Calculations of these five defuzzification operations is time-consuming, so special hardware support must be used. Beside, studies on these defuzzification operations are based on experimental results.

5.2.1.2 Other Variants

Figures 5.16 and 5.17 conform to the fuzzy reasoning defined previously. However, in consideration of computation efficiency or mathematical tractability, a fuzzy inference system in practice may have a certain reasoning mechanism that does not follow the strict definition of the compositional rule of inference. Therefore, to completely specify the operation of a Mamdani fuzzy inference system, we need to assign a function for each of the following operators:

1. **AND operator** (usually T-norm) for calculating the firing strength, which is the degree to which the antecedent part of the rule is satisfied, of a rule with AND antecedents.

2. **OR operator** (usually *T-conorm*) for calculating the firing strength of a rule with OR antecedents.
3. **Implication operator** (usually *T-norm*) for calculating qualified consequent MFs based on given firing strength.
4. **Aggregate operator** (usually *T-conorm*) for aggregating qualified consequent MFs to generate an overall output MF.
5. **Defuzzification operator** for transforming an output MF to a crisp single output value.

One such example is to use product for the implication operator and point- wise summation (sum) for the aggregate operator. An advantage of this *sum-product composition* [31] is that the final crisp output via centroid defuzzification is equal to the weighted average of the centroids of consequent MFs, where the weighting factor for each rule is equal to its firing strength multiplied by the area of the consequent MF. Under sum-product composition, the output of a Mamdani fuzzy inference system with centroid defuzzification is equal to the weighted average of the centroids of consequent MFs, where each of the weighting factors is equal to the product of a firing strength and the consequent MF's area.

5.2.2 Sugeno-type (TSK) Fuzzy Models

In an effort to develop a systematic approach to generating fuzzy rules from a given input-output data set Takagi, Sugeno, and Kang in their papers dated 1985 and 1988 proposed Sugeno, or TSK, fuzzy model. A typical fuzzy rule in a Sugeno fuzzy model has the form

$$\text{If } x \text{ is } A \text{ and } y \text{ is } B \ \text{ Then } z = f(x, y)$$

where A and B are fuzzy sets in the antecedent, while $z = f(x, y)$ is a crisp function in the consequent. Usually, $f(x, y)$ can be any function as long as it can appropriately describe the output of the model within the fuzzy region specified by the antecedent of the rule, but in most cases, $f(x, y)$ is a polynomial in the input variables x and y. When $f(x, y)$ is a first-order polynomial, the resulting fuzzy inference system is called a first-order Sugeno fuzzy model. When $f(x, y)$ is a constant, we then have a zero-order Sugeno fuzzy model, which can be viewed either as a special case of the Mamdani fuzzy inference system, in which each rule's consequent is specified by a fuzzy singleton, or a special case of the Tsukamoto fuzzy model, which will be introduced in the next subsection, in which each rule's consequent is specified by an MF of a step function center at the constant.

As long as the neighboring MFs in the antecedent have enough overlap the output of a zero-order Sugeno model would be a smooth function of its input variables. But we does see any smoothing effect of the overlaps of MFs in the consequent of a Mamdani fuzzy inference model. We may say that it is the overlap of the antecedent MFs that determines the smoothness of the resulting input-output behavior in the Mamdani fuzzy inference model.

In Figure 5.19, the fuzzy reasoning procedure for a first-order Sugeno fuzzy model can be seen. We may see that the overall output is obtained via weighted average, because each rule has a crisp output. This helps to avoid the time-consuming process of defuzzification required in a Mamdani model. In practice, the weighted average operator is sometimes replaced with the weighted sum operator, that is, $\boldsymbol{z = w_1 z_1 + w_2 z_2}$ to reduce computation further, especially, in the training of a fuzzy inference system. Unless the sum of firing strengths (that is, $\sum_i \boldsymbol{w_i}$) is close to unity, this simplification could lead to the loss of MF linguistic meanings.

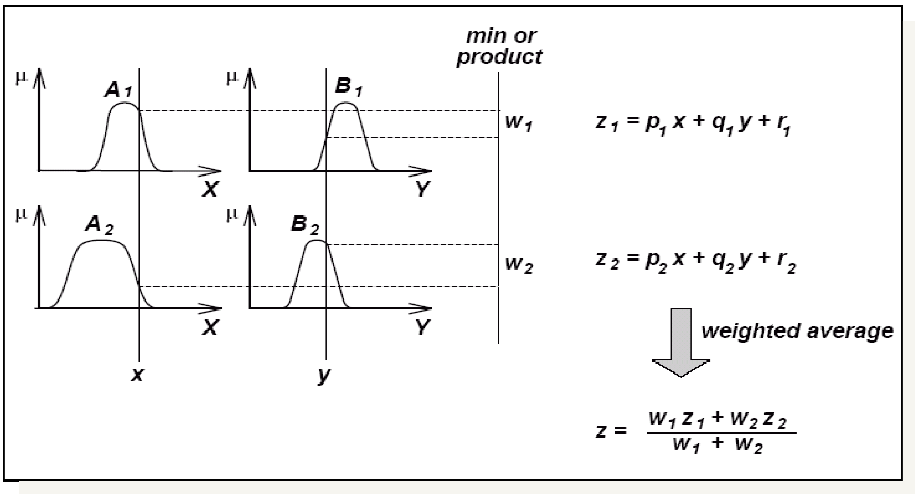


Figure 5.19: The Sugeno fuzzy model [28].

When instead of the crisp inputs, Sugeno fuzzy inference system gets fuzzy inputs some problems may arise, because Sugeno fuzzy system cannot strictly follow the compositional rule of inference in its fuzzy reasoning mechanism. In order to overcome this, we may use matching of fuzzy sets, to find the firing strength of each rule. But, as we know, not depending on the output method, weighted average or weighted sum, the output of the Sugeno model is crisp, which is against the nature of the fuzzy inference system, which transmits the fuzziness from inputs to outputs.

Without the time-consuming and mathematically intractable defuzzification operation, the Sugeno fuzzy model is by far the most popular candidate for sample-data-based fuzzy modeling.

5.2.3 Tsukamoto-type Fuzzy Models

Y. Tsukamoto in his article [61] dated 1979, proposed a new model, named after himself, *Tsukamoto fuzzy models*. In this model, the consequent of each fuzzy if-then rule is represented by a fuzzy set with a monotonical MF, as shown in Figure 5.20. As a result, the inferred output of each rule is defined as a crisp value induced by the rules' firing strength. The overall output is taken as the weighted average of each rules' output.

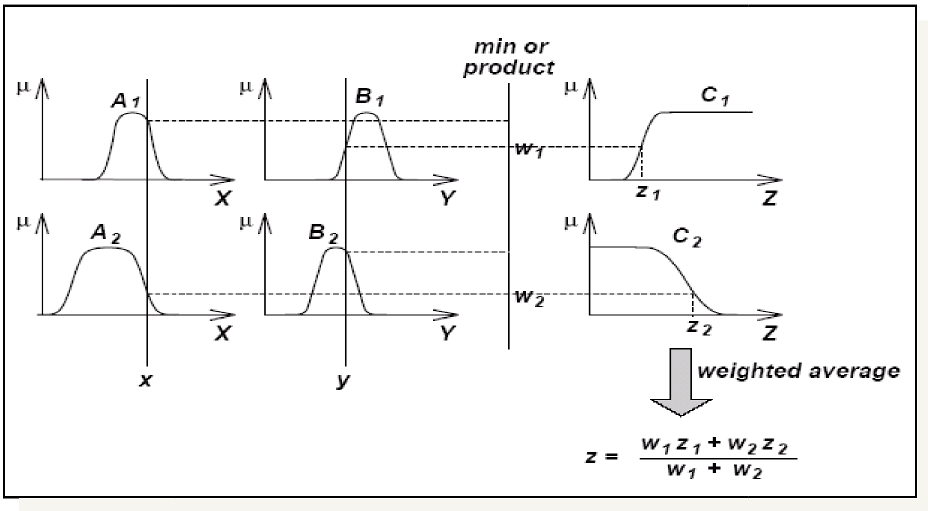


Figure 5.20: The Tsukamoto fuzzy model [28].

Since each rule infers a crisp output, the Tsukamoto fuzzy model like the Sugeno fuzzy logic aggregates each rules' output by the method of weighted average and thus avoids the time-consuming process of defuzzification. But since the Tsukamoto fuzzy model is not as transparent as either the Mamdani or Sugeno fuzzy models, it is not used often.

Since the reasoning mechanism of the Tsukamoto fuzzy model does not follow strictly the compositional rule of inference, the output is always crisp even when the inputs are fuzzy.

5.2.4 Input Space Partitioning

As we know the antecedent of a fuzzy rule defines a local fuzzy region, while the consequent describes the behavior within the region. The consequent generator can be a consequent MF (Mamdani and Tsukamoto fuzzy models), a constant value (zero-order Sugeno model), or a linear equation (first-order Sugeno model). Different consequent generators result in different fuzzy inference systems, but their antecedents are always the same. Therefore, the following discussion of methods of partitioning input spaces to form the antecedents of fuzzy rules is applicable to all three types of fuzzy inference systems [28].

5.2.4.1 Grid Partition

Figure 5.21(a) illustrates a typical grid partition in a two-dimensional input space. For this partition method the number of input and the number of membership functions for each of the given inputs must be small. However, we can also use this method when we have a moderately large number of inputs. For instance, a fuzzy model with 8 inputs and 3 membership functions on each input would result in $3^8 = 6561$ fuzzy if-then rules, which is prohibitively large. This problem can be alleviated by the other partition strategies.

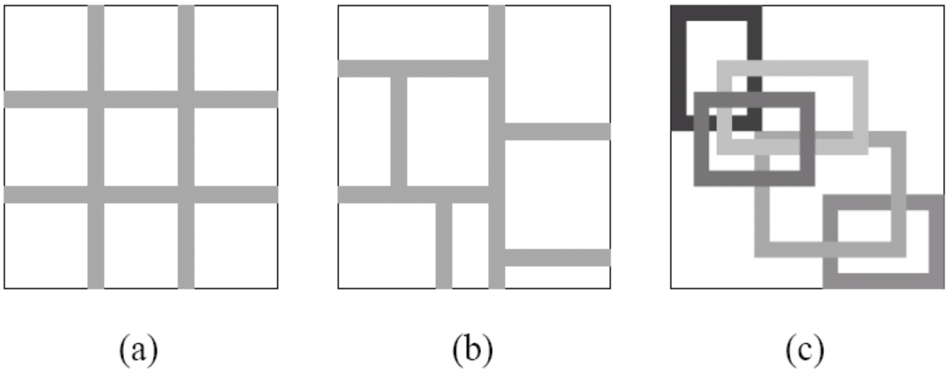


Figure 5.21: Various methods for partitioning the input space: *(a)* grid partition;*(b)* tree partition; *(c)* scatter partition [28].

5.2.4.2 Tree Partition

In Figure 5.21(b), a typical tree partition is depicted. In this partition method every region can be uniquely specified along a corresponding decision tree. The tree partition relieves the problem of an exponential increase in the number of rules, but more MFs for each input are needed to define these fuzzy regions.

5.2.4.3 Scatter Partition

As shown in Figure 5.21(c), by covering a subset of the whole input space that characterizes a region of possible occurrence of the input vectors, the scatter partition can also limit the number of rules to a reasonable amount. However, the scatter partition is usually dictated by desired input-output data pairs. This makes it hard to estimate the overall mapping directly from the consequent of each rules' output.

5.2.5 Fuzzy Modeling

As it may be clear from the previous section that we are nearly always designing a fuzzy inference system based on the past known behavior of a target system and the designed fuzzy system is expected to be able to reproduce the behavior of the target system.

Generally speaking, the standard method for constructing a fuzzy inference system, a process usually called *fuzzy modeling*, has the following features:

1. The rule structure of a fuzzy inference system makes it easy to incorporate human expertise about the target system directly into the modeling process. Namely, fuzzy modeling takes advantage of domain knowledge that might not be easily or directly employed in other modeling approaches.
2. When the input-output data of a target system is available, conventional system identification techniques can be used for fuzzy modeling. In other words, the use of numerical data also plays an important role in fuzzy modeling, just as in other mathematical modeling methods.

Conceptually, fuzzy modeling can be pursued in two stages, which are not totally disjoint. The *first stage* is the identification of the *surface structure,* which includes the following tasks:

1.(i). Select relevant input and output variables.
1.(ii). Choose a specific type of fuzzy inference system.
1.(iii). Determine the number of linguistic terms associated with each input and output variables. For a Sugeno model, determine the order of consequent equations.
1.(iv). Design a collection of fuzzy if-then rules.

We have to take account that to accomplish the preceding tasks, we rely on our own knowledge of the target system, information provided by human experts who are familiar with the target system, or simply trial and error.

After the first stage of fuzzy modeling, we obtain a rule base that can more or less describe the behavior of the target system by means of linguistic terms. The meaning of these linguistic terms is determined in the *second stage*, the identification of *deep structure,* which determines the MFs of each linguistic term and the coefficients of each rule's output polynomial if a Sugeno fuzzy model is used. Specifically, the identification of deep structure includes the following tasks:

2.(i). Choose an appropriate family of parameterized MFs.

2.(ii). Interview human experts familiar with the target systems to determine the parameters of the MFs used in the rule base.

2.(iii). Refine the parameters of the MFs using regression and optimization techniques.

Task 2.(i). and 2.(ii). assume the availability of human experts, while task 2.(iii). assumes the availability of a desired input-output data set.

5.2.6 Comparison Between Sugeno and Mamdani-type Methods

The main difference between Mamdani and Sugeno is that the Sugeno output membership functions can be constant, linear or any other function. Also the difference lies in the consequents of their fuzzy rules and, thus, their aggregation and fuzzy rules needed by the Sugeno fuzzy systems depend on the number and locations of the extreme of the function to be approximated. In Sugeno method, a large number of fuzzy rules must be employed to approximate periodic or highly oscillatory functions. The minimal configuration of the TSK fuzzy systems can be reduced and becomes smaller than that of the Mamdani fuzzy systems if nontrapezoidal or nontriangular input fuzzy sets are used. Sugeno controllers usually have far more adjustable parameters in the rule consequent and the number of the parameters grows exponentially with the increase of the number of input variables. Far fewer mathematical results exist for TSK fuzzy controllers than do for Mamdani fuzzy controllers. Mamdani is easy to form compared to Sugeno method. Below is the list of advantages and disadvantages of the Sugeno and Mamdani fuzzy inference systems:

1. *Advantages of the Sugeno Method*:
   - It is computationally efficient.
   - It works well with linear techniques (e.g., PID control).
   - It works well with optimization and adaptive techniques.
   - It has guaranteed continuity of the output surface.
   - It is well suited to mathematical analysis.
2. *Advantages of the Mamdani Method*:
   - It is intuitive.
   - It has widespread acceptance.
   - It is well suited to human input.
3. *Disadvantages of the Sugeno Fuzzy method*:
   - It is not intuitive.
   - When using the higher order Sugeno method, it is complex.
4. *Disadvantages of the Mamdani Fuzzy method*:

– It is too simple to control the process quickly and only suited to the long delay system, such as the temperature control system.

– When it controls the high frequent input system, it needs additional device to improve the efficiency.

Fuzzy inference system is the most important modeling tool based on fuzzy set theory. The FISs are built by domain experts and are used in automatic control, decision analysis, and various other expert systems, and especially, in financial mathematics.

CHAPTER 6

ANALYSIS OF FUZZY RULE-BASED MODEL (FRBM) RESULTS

6.1. Introduction

This thesis aims to gain additional insight into the office rent estimation and its variation within Istanbul Central Business District (CBD). The office data consist of the 502 offices located within Istanbul CBD. The offices are located both in the traditional CBD (Eminönü and Beyoğlu districts) and in the new CBD axis including Beşiktaş, Şişli and Sarıyer districts in the northern part of the city.

The majority of Fuzzy Rule-Based Systems (FRBS) use Mamdani or Takagi-Sugeno-Kang (TSK) rules. The main difference between them is that the TSK rules have a linear function in the consequent part and the Mamdani rules have a linguistic output such as “small”, “medium” and “large”. This chapter provides the empirical results both for the Mamdani-type and TSK-type FRBM analyses using “office space” and/or “office building” data set. In sum, the following four basic analyses are carried out, respectively:

1. Mamdani-type Fuzzy Rule-Based Model for Office Space Data,
2. TSK-type Fuzzy Rule-Based Model for Office Data,
3. Mamdani-type Fuzzy Rule-Based Model for Office Space and Office Building Data,
4. TSK-type Fuzzy Rule-Based Model for Office Space and Office Building Data.

6.2. Mamdani-Type Fuzzy Rule-Based Model for the Office Space Data

The office data consist of the 483 offices located within Istanbul CBD. A detailed questionnaire was prepared and carried out for each office space by a professional data-collection company. A survey of 20 questions including the physical attributes of the office spaces, lease terms, and the tenants’ residence and transportation preferences are displayed in Table 6.1. Twenty two different factors that affect office rental price possibly be highly correlated to each other and result in a multicollinearity problem in a multiple regression analysis. Accordingly, we first carry out a backward stepwise regression in order to reduce the number of explanatory variables, then we employ fuzzy rule-based analysis to the new data set.

Table 6.1: A survey of 22 questions for 506 office spaces in Istanbul CBD		
	Variable Description	Number of questions
Physical attributes of the office space	- office function (FIRE, health, etc.) - office floor - office area (m^2) - number of rooms - kitchen and wc facilities in the office - heating system - number of workers - monthly expenses - advantageous and prestigious view	10 questions
Lease terms and conditions of the contract	- beginning of lease term - term-to-maturity - rental increment - security deposit - increases in the rent	5 questions
Questions for the employees	- tenants' residence out of the office-district - transportation mode (home-to-work) - percentile of the workers coming from the Anadolu region - main factor that affect the office rent - new shopping center located in the office district	5 questions
TOTAL		20 questions

Stepwise regression is a method which can be used to sort out the relevant explanatory variables from a large set of candidate explanatory variables in order to allow the regression models to be computed. This method tries to find a reduced model that best explains the data. This study uses "backward-elimination" stepwise regression, which involves starting with all candidate variables (the maximum model), testing them one by one for statistical significance (using a sequence of t-tests) and removing any that are not significant. This procedure terminates when no more variables need to be removed from the model at a certain significance level.

Our results of backward stepwise regression show that 6 out of 22 variables are statistically significant in affecting the office rental prices. These variables are namely:

1. The Office Floor,
2. Office Usage Area,

3. Kitchen,
4. WC/Toilet,
5. Number of Workers,
6. Tenant’s Residence Out of the Office District,

Due to the fact that variables 3 and 4, namely kitchen and WC/toilet were dummy variables, they have been eliminated. In short, the input variables of the FRBM comprises the office floor, office usage area, number of workers, and tenants’ residence outside the office-district and as the output the model proposes the office property’s rental price. It is important to note that none of the variables defining lease contract terms are found to be effective on the rental prices.

Being the main determinants of office space rents, these variables are defined as the input variables of the fuzzy rule-based model. Then, we employ both the Mamdani-type and TSK-type fuzzy rule-based model for the whole data set and for each sub-district defined in Table 3.1 separately.[7] Specifically, we carried out five different Mamdani-type FRBMs as follows:

- One FRBM was created for the entire sample of 483 office spaces characterizing Istanbul CBD,
- One FRBM was created for Beşiktaş Sub-district of the CBD,
- One FRBM was created for Beyoğlu Sub-district of the CBD,
- One FRBM was created for Eminönü Sub-district of the CBD,
- One FRBM was created for Şişli Sub-district of the CBD.

6.2.1. Fuzzification of the Input and Output Variables for Office FRBM

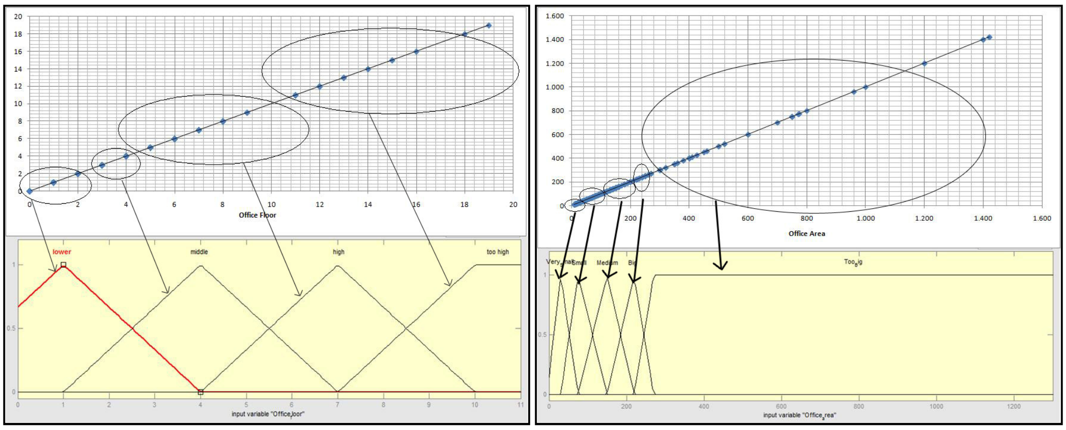
Mamdani type FRBM is constructed with 4 input variables and implemented by using the MATLAB Fuzzy Logic Toolbox. For each input variable, triangular and trapezoidal membership functions are defined with the corresponding linguistic variables over the defined interval values.

In detail, **“office floor” variable** describes on which floor the office space is located in the office building. The “office floor” variable is divided into three triangular and one trapezoidal fuzzy subset due to the distribution of the data. For the “office floor” input variable, the membership functions with four linguistic variables along with the mathematical expressions of the input

[7] We exclude Sariyer district in our fuzzy rule-based model analysis as the data set with only 10 observations is not sufficient to construct Mamdani rules and perform a FRBS analysis.

variable intervals are presented in Figure 6.1(a). Since the data clusters have centers around 1, 4, 7 and 10, four different fuzzy subsets are defined.

Similarly, **"office area" variable** gives office usage area in square meters. The membership functions with five linguistic variables along with the mathematical expressions of the input variable intervals are presented in Figure 6.1 (b). Based upon the distribution of data, the "office area" variable is divided into four triangular and one trapezoidal fuzzy subset. Figure 6.2 (b) shows that the data clusters have centers around 30, 75, 150, 220 and 270. Hence, five different fuzzy subsets are defined for this input variable.



(a) *(b)*

Figure 6.1: Fuzzification of the input variables *(a)* "office floor", and *(b)* "office usage area"

Mathematical expressions of variable *Office Floor*:

$$\mu_{lower}(x) = \begin{cases} \dfrac{x+2}{3}, & if \ -2 < x \le 1, \\ \dfrac{4-x}{3}, & if \quad 1 \le x \le 4, \end{cases} \qquad \mu_{middle}(x) = \begin{cases} \dfrac{x-1}{3}, & if \quad 1 \le x \le 4, \\ \dfrac{7-x}{3}, & if \quad 4 \le x \le 7, \end{cases}$$

$$\mu_{high}(x) = \begin{cases} \dfrac{x-4}{3}, & if \quad 4 \le x \le 7, \\ \dfrac{10-x}{3}, & if \quad 7 \le x \le 10, \end{cases} \qquad \mu_{too\ high}(x) = \begin{cases} \dfrac{x-7}{3}, & if \quad 7 \le x \le 10, \\ 1, & if \quad 10 \le x. \end{cases}$$

Mathematical expressions of variable *Office Usage Area*:

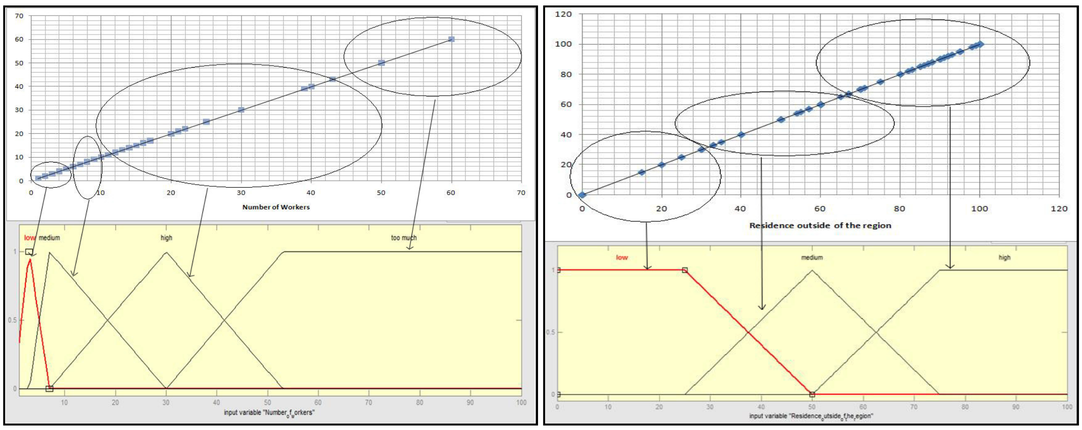
$$\mu_{very\ small}(x) = \begin{cases} \dfrac{x+5}{35}, & if \ -5 < x \le 30, \\ \dfrac{75-x}{45}, & if \quad 30 \le x \le 75, \end{cases} \qquad \mu_{small}(x) = \begin{cases} \dfrac{x-30}{45}, & if \quad 30 \le x \le 75, \\ \dfrac{150-x}{75}, & if \quad 75 \le x \le 150, \end{cases}$$

$$\mu_{medium}(x) = \begin{cases} \frac{x-75}{75}, & if \quad 75 \le x \le 150, \\ \frac{220-x}{70}, & if \quad 150 \le x \le 220, \end{cases} \qquad \mu_{big}(x) = \begin{cases} \frac{x-150}{70}, & if \quad 150 \le x \le 220, \\ \frac{270-x}{50}, & if \quad 220 \le x \le 270, \end{cases}$$

$$\mu_{too\ big}(x) = \begin{cases} \frac{x-220}{50}, & if \quad 220 \le x \le 270, \\ 1, & if \quad 270 \le x \le 3000, \\ \frac{3500-x}{500}, & if \quad 3000 \le x \le 3500. \end{cases}$$

The **"number of workers" variable** is the total number of employees in the office. The membership functions with four linguistic variables and the mathematical expressions of the input variable intervals are presented in Figure 6.2 (a). The "number of workers" variable is divided into three triangular and one trapezoidal fuzzy subset due to the distribution of the data. Figure 6.2 (a), which exhibits the data distribution for number of workers, illustrates that the data clusters have centers around 3, 7, 30 and 53. Thus, four different fuzzy are defined, accordingly.

"Tenants' residence out of the office-district" variable gives the percentage of employees reside out of the district where the office building is located. The membership functions with three linguistic variables along with mathematical expressions of the input variable intervals are presented in Figure 6.2 (b). This input variable is divided into one triangular and two trapezoidal fuzzy subsets due to the distribution of the data. As seen in Figure 6.2 (b), the data clusters have centers around 25, 50 and 75. Thus, three fuzzy subsets for the "tenants' residence outside the office-district" are defined.



(a) *(b)*

Figure 6.2: Fuzzification of the input variables *(a)* "number of workers", and *(b)* "Tenants' residence out of the office-district"

Mathematical expressions of variable *Number of Workers*:

$$\mu_{low}(x) = \begin{cases} \frac{x}{3}, & if \quad 0 < x \leq 3, \\ \frac{7-x}{4}, & if \quad 3 \leq x \leq 7, \end{cases} \quad \mu_{medium}(x) = \begin{cases} \frac{x-3}{4}, & if \quad 3 \leq x \leq 7, \\ \frac{30-x}{23}, & if \quad 7 \leq x \leq 30, \end{cases}$$

$$\mu_{many}(x) = \begin{cases} \frac{x-7}{23}, & if \quad 7 \leq x \leq 30, \\ \frac{53-x}{23}, & if \quad 30 \leq x \leq 53, \end{cases} \quad \mu_{too\ many}(x) = \begin{cases} \frac{x-30}{23}, & if \quad 30 \leq x \leq 53, \\ 1, & if \quad 53 \leq x \leq 220, \\ \frac{250-x}{30}, & if \quad 220 \leq x \leq 250. \end{cases}$$

Mathematical expressions of variable *Tenants' residence out of the office-district*:

$$\mu_{low}(x) = \begin{cases} 1, & if \quad x \leq 25, \\ \frac{50-x}{25}, & if \quad 25 \leq x \leq 50, \end{cases} \quad \mu_{medium}(x) = \begin{cases} \frac{x-25}{25}, & if \quad 25 \leq x \leq 50, \\ \frac{75-x}{25}, & if \quad 50 \leq x \leq 75, \end{cases}$$

$$\mu_{high}(x) = \begin{cases} \frac{x-50}{25}, & if \quad 50 \leq x \leq 75, \\ 1, & if \quad 75 \leq x. \end{cases}$$

Finally, **"monthly net rent for office spaces" variable** is the output of the fuzzy model. The membership functions with ten linguistic variables and the corresponding mathematical expressions of the output variable intervals are presented in Figure 6.3. The output variable is divided into nine triangular and one trapezoidal fuzzy subset due to the distribution of the data. As seen in Figure 6.3, the data clusters have centers around 350, 750, 1050, 1350, 2000, 2600, 5000, 8000, 15000 and 18000. Thus, ten fuzzy subsets for the output variable are defined.[8]

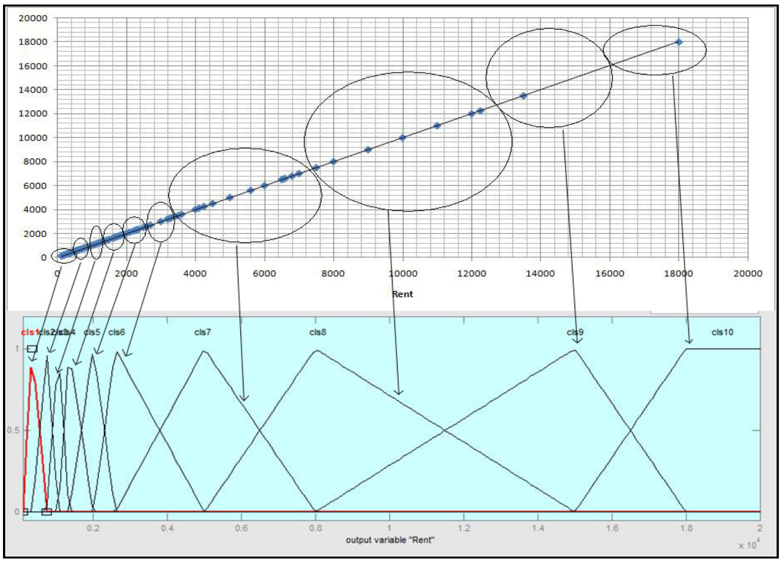


Figure 6.3: Fuzzification of the "office space rent" output variable

8 We note that, for the other Mamdani-type Fuzzy rule-based models the interval definitions and the membership functions would be similar.

Mathematical expressions of variable *Office Floor*:

$$\mu_{cls1}(x)=\begin{cases}\frac{x-100}{250}, & if \quad 100\le x\le 350,\\ \frac{750-x}{400}, & if \quad 350\le x\le 750,\end{cases} \quad \mu_{cls2}(x)=\begin{cases}\frac{x-350}{400}, & if \quad 350\le x\le 750,\\ \frac{1050-x}{300}, & if \quad 750\le x\le 1050,\end{cases}$$

$$\mu_{cls3}(x)=\begin{cases}\frac{x-750}{300}, & if \quad 750\le x\le 1050,\\ \frac{1050-x}{300}, & if \quad 1050\le x\le 1350,\end{cases} \quad \mu_{cls4}(x)=\begin{cases}\frac{x-1050}{300}, & if \quad 1050\le x\le 1350,\\ \frac{2000-x}{650}, & if \quad 1350\le x\le 2000,\end{cases}$$

$$\mu_{cls5}(x)=\begin{cases}\frac{x-1350}{650}, & if \quad 1350\le x\le 2000,\\ \frac{2600-x}{600}, & if \quad 2000\le x\le 2600,\end{cases} \quad \mu_{cls6}(x)=\begin{cases}\frac{x-2000}{600}, & if \quad 2000\le x\le 2600,\\ \frac{5000-x}{2400}, & if \quad 2600\le x\le 5000,\end{cases}$$

$$\mu_{cls7}(x)=\begin{cases}\frac{x-2600}{2400}, & if \quad 2600\le x\le 5000,\\ \frac{8000-x}{3000}, & if \quad 5000\le x\le 8000,\end{cases} ,\mu_{cls8}(x)=\begin{cases}\frac{x-5000}{3000}, & if \quad 5000\le x\le 8000,\\ \frac{15000-x}{7000}, & if \quad 8000\le x\le 15000,\end{cases}$$

$$\mu_{cls9}(x)=\begin{cases}\frac{x-8000}{7000}, & if \quad 8000\le x\le 15000,\\ \frac{18000-x}{3000}, & if \quad 15000\le x\le 18000,\end{cases} \quad \mu_{cls10}(x)=\begin{cases}\frac{x-15000}{3000}, & if \quad 15000\le x\le 18000,\\ 1, & if \quad 18000\le x\le 30000,\\ \frac{40000-x}{10000}, & if \quad 30000\le x\le 40000.\end{cases}$$

6.2.2. Production of the Rule Base

For the whole data set representing Istanbul CBD, we define four membership functions for the "office floor" input variable, five membership functions for "office area" variable, four membership functions for "number of workers" variable, and three membership functions for "residence outside the district". In total, 4×5×4×3 = 240 Fuzzy Inference System (FIS) rules are defined.

In the rule-definition process, firstly, rules are formulated by using data and appraisal practices, then, the contradicting rules are omitted intuitively. While the summation operation method is used for aggregation, the *Centroid of Area* is applied for defuzzification process of the modeling. Figure 6.4 illustrates the flow chart of the rule extraction process of FRBM.

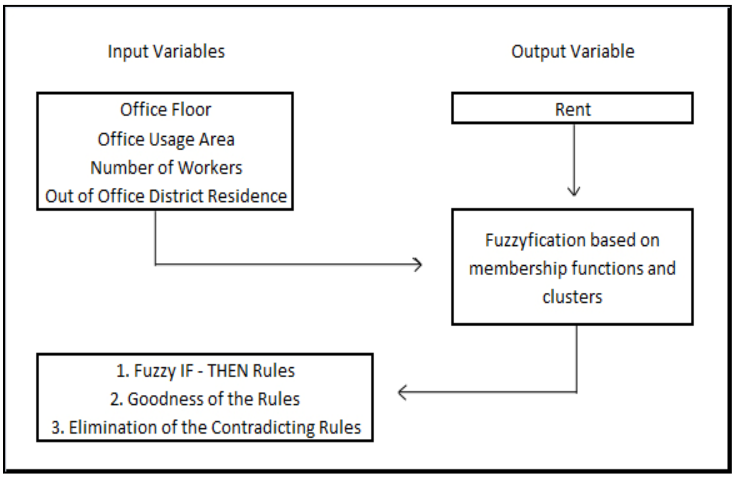


Figure 6.4: Flow chart of "fuzzy rule extraction" process

As seen from Figure 6.4, the relation between the input and the output variables of the fuzzy model is determined by fuzzy rules, which are obtained as follows:

Rule 1: If (Office_floor is low) **and** (Office_floor_area is very small) **and** (Number_of_workers is low) **and** (Tenants'_residence_out_of_the_office-district is low) **Then** (Office Rent is Cls1)
OR Rule 2: ......
……
OR Rule 240: If (Office_floor is too high) **and** (Office_floor_area is too big) **and** (Number_of_workers is too many) **and** (Tenants'_residence_out_of_the_office-district is high) **Then** (Office Rent is Cls7).

Table 6.2 presents the total number of FIS rules defined for each district separately, as well.

Table 6.2 : Number of Membership Functions for Input &Output Variables and the Total Number of FIS (Fuzzy Inference System) Rules for the whole data set and each sub-district

Sub-districts	Office Floor	Office area (m^2)	Number of workers	Tenants' residence out of the office-district (%)	Office Rent (TL)	Number of FIS Rules
Beşiktaş	2	3	3	3	3	54
Şişli	3	4	3	3	8	108
Beyoğlu	3	3	3	3	8	81
Eminönü	2	2	3	3	4	36
Whole Data Set	4	5	4	3	10	240

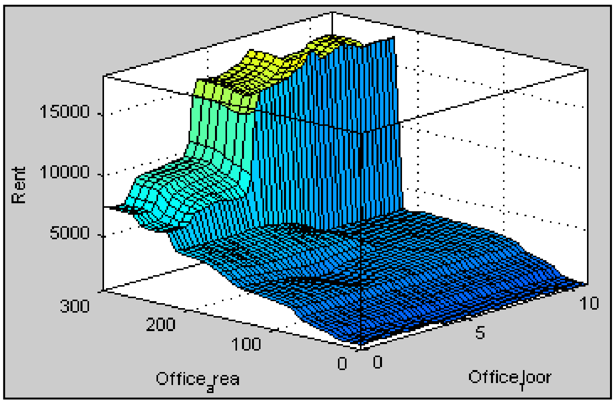
6.2.3. Empirical Results for the Mamdani-type Office Space FRBS

This subsection of the thesis reports our results first for the entire sample of 483 office spaces characterizing Istanbul CBD, then for each sub-district of CBD as the sub-samples of the study.

6.2.3.1. Analysis of Mamdani-type Fuzzy Rule-Based Model Results for the Entire Sample

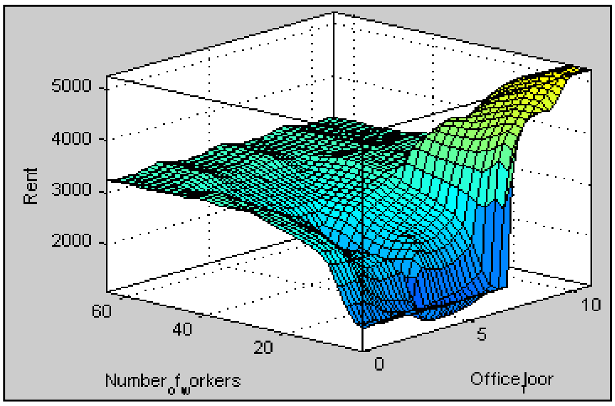
In an attempt to understand in which direction and magnitude do the physical attributes of office spaces and employees' residence preferences impact the office rents, we produce 3-Dimensional graphs for the entire sample of 483 office spaces. Another reason for presenting these 3-D figures is to demonstrate the smoothness of the fuzzy model solution over the defined surfaces. The relation between the inputs (physical attributes of office spaces; employees' residence preferences) and the output (office rental price) of the model determined by the FIS rules is a hypersurface that cannot be displayed in three dimensions. Hence, the input-output relation by using different input pairs can be visualized by bilateral surfaces.

Figure 6.5 displays the combined effect of "office floor" and "office area" input variables on office space rents. Clearly, office rents increase consistently as the office usage area (m^2) expands. For the smaller office usage areas (smaller than $100m^2$) on which floor the office space is located within the building does not affect the rents. Over the defined bilateral surface, the highest office rents (approximately 15,000TL to 18,000TL) are observed for high-rise office buildings with sizeable office usage areas.



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	NaN	NaN	33	50

Figure 6.5: Combined Effect of "Office Floor" and "Office Area" on Office Space Rents

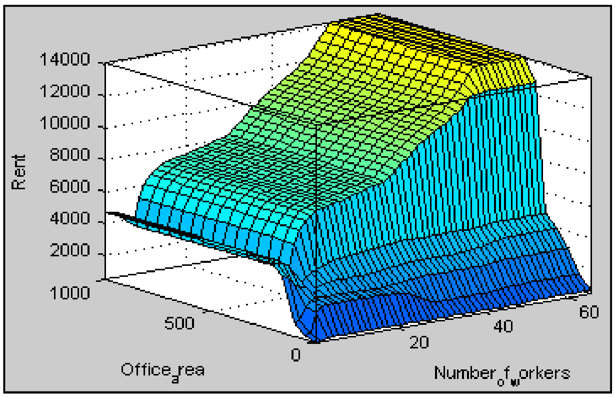


Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	NaN	100	NaN	50

Figure 6.6: Combined Effect of "Office Floor" and "Number of Employees" on Office Space Rents

The joint effect of "office floor" and "number of workers" variables on office space rents are exhibited in Figure 6.6. Increases in the number of employees lead to higher office rents. However, office space rents increase at a decreasing rate as the number of workers rises continually. Moreover, an increase in the level of office floor does not affect the office rents if the number of the workers at the office is approximately above 20. The highest levels of rents (approximately 5,200 TL) over the defined bilateral surface are seen for high-rise office buildings with reasonably small number of employees. Hence, our results suggest that, for Istanbul CBD, the highest levels of office rents are observed for the high-rise office building (higher than the 7th floor) with sizeable office spaces and small number of employees.

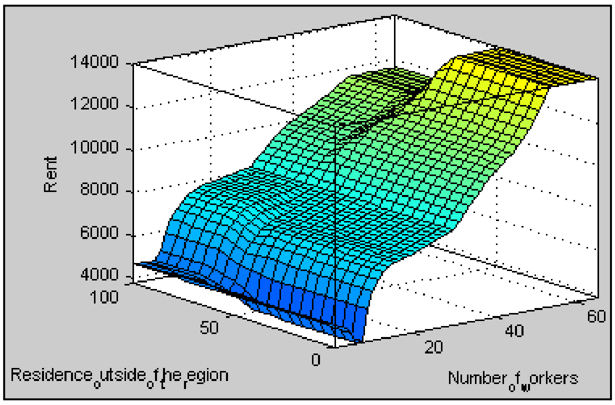
Figure 6.7 provides information on the combined effect of "number of employees" and "office area" on office rents. For the sizeable office spaces with higher than 250 m2 usage area – ignoring the impact of office floor – as the number of workers increases office rents steadily increase as well. In addition, when the office area is significantly small, the number of workers inserts negative impact on office rents.



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	5,5	NaN	NaN	50

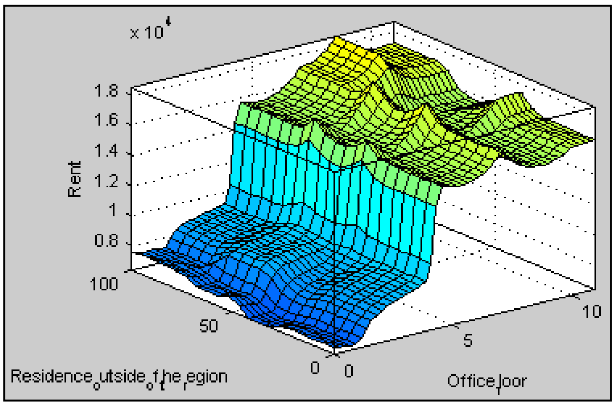
Figure 6.7: Combined Effect of "Number of Workers" and "Office Area" on Office Space Rents

The question of how the number of employees and their residence preferences jointly affect office rents is answered in Figure 6.8. The residence outside the office-district has only a slight effect on the office rents, when the number of workers is small. Noticeably, the highest rents are observed for the offices with considerably large number of employees, whose tenants mainly reside within the office-district. For the similar offices with many workers, tenants' preferences for living outside of the office-district significantly reduce the office rents. Hence, our results imply that as the tenants reside within the office-district, their demand for office space and correspondingly office rents are expected to increase in that particular district.



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	5,5	500	NaN	NaN

Figure 6.8: Combined Effect of "Number of Workers" and "Tenants' Residence out of the Office District" on Office Space Rents



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	NaN	500	33	NaN

Figure 6.9: Combined Effect of "Office Floor" and "Tenants' Residence out of the Office District" on Office Space Rents

Finally, Figure 6.9 provides information on the combined effect of "office floor" and "Tenants' Residence out of the Office District" on office rents. It is clear that, at the lower percentile of outside residences, tenants' residence outside the office-district has no significant effect on the overall rent value process. As the percentile of the residence outside of the region increases, office rent increases too but at a much lower rate. As office floor increases, the influence of the residence preference on the office rents increases simultaneously.

6.2.3.2. Mamdani-Type FRBM Based on Location: Analysis of Results for Each Sub-District in the CBD

This subsection of the thesis provides an analysis of spatial variation in the average office rents across the CBD sub-districts. We ask the question of "How office rents for a typical office space, with exactly the same attributes, vary among the CBD sub-districts?" In particular, given the certain characteristics of an office space such as the office floor, number of workers, office area, etc., we first calculate office rents for each district of Eminönü, Beyoğlu, Beşiktaş and Şişli, separately, then we discuss the variation of rents within the sub-districts CBD.

Examining the input data frequencies, we select five different representative office spaces with the most common characteristics (see Table 6.3). For instance, the first representative office space characterizes the offices located on the first floor of the building with 200 m^2 usage area and 5 employees. We assume that employees' preference for residing outside the office-district is 0 percent.

Table 6.3: Representative Office Spaces with the Highest Input Data Frequencies

Representative Offices	Office Floor	Office Usage Area	Number of Employees	Outside-district residence
# 1 – Table 6.3.1	1	200 m^2	5	0%
	4	200 m^2	5	0%
	7	200 m^2	5	0%
	10	200 m^2	5	0%
# 2 – Table 6.3.2	1	200 m^2	3	0%
	4	200 m^2	3	0%
	7	200 m^2	3	0%
# 3 – Table 6.3.2	10	200 m^2	3	0%
# 4 – Table 6.3.3	1	100 m^2	7	0%
	4	100 m^2	7	0%
# 5 – Table 6.3.3	7	100 m^2	7	0%
	10	100 m^2	7	0%

Table 6.3.1 displays the spatial variation of office rents for the first representative office space. Beyoğlu sub-district, as traditional part of the CBD, has the highest rent level of 5233 TL/month. Beşiktaş (2727 TL/month) and Şişli (1785 TL/month) sub-districts, including the new CBD axis of Zincirlikuyu-Levent-Maslak, have considerably lower rent levels in comparison to Beyoğlu district. Located in the southern part of traditional CBD within the borders of historical peninsula, Eminönü sub-district has the lowest rent levels; that is, less than 1500TL/month on average. Additionally, we keep our assumptions of 200 m^2 usage area, 5 employees and 0 percent residence outside the office-district constant and we examine rent differentiation for different floor levels (see Table 6.3 above). We find that for the 4^{th}, 7^{th}, and 10^{th} floor office spaces Beyoğlu sub-district has still the highest levels of rents. Beşiktaş, Şişli, and Eminönü sub-districts follow Beyoğlu, in that order. We also note that office rent variation across the sub-districts retains the same ranking when we examine larger offices with 300 m^2 usage area.

Table 6.3 shows that the second representative office space corresponds to the offices located on the first floor of the building with 200 m^2 usage area and 3 employees. We again assume that employees' preference for residing outside the office-district is 0 percent. Our results demonstrate that Beyoğlu district has the highest levels of office rents and that Beşiktaş, Şişli, and Eminönü sub-districts go behind Beyoğlu, respectively (see Table 6.3.2). The notable difference is observed in Eminönü district's rent levels. Specifically, the effect of smaller number of employees (3 instead of 5 employees) on the office rents is clearly observed in Eminönü district, which has now lower rent level of 1174 TL/month.

Table 6.3.2 also displays the spatial variation in office rents for the third representative office space. This office space group characterizes the offices located on the tenth floor of the building with 200 m^2 usage area and 3 employees. We keep constant the assumption of 0 percent residence outside the office-district (see Table 6.3). Analysis of this particular office space group enables us to understand how office rents for the high-rise office buildings with sizeable office spaces and small numbers of employees vary within the CBD. Once more, Beyoğlu district has the highest levels of office rents; that is, 5050 TL/month on average. Unlike the earlier results, Şişli district has significantly higher rents in comparison with Beşiktaş district. Eminönü (800 TL/month) and Beşiktaş (1383 TL/month) districts have the lowest rent levels less than 1500 TL/month. We also emphasize that office rent variation across the sub-districts retains the same ranking when we examine larger offices with 300 m^2 usage area.

The fourth and the fifth representative office spaces symbolize smaller-sized offices (100 m^2 usage area) with 7 employees. The spatial variation in office rents for the fourth and the fifth representative office spaces are presented in Table 6.3.3. While the fourth representative office space characterizes offices on the first-floor of the building, the fifth representative office space corresponds to the offices on the tenth-floor of the building (see Table 6.3 above). Obviously, for small-sized offices with 100 m^2 usage area, office rents are notably lower than the medium-sized offices (200 m^2 usage area) in all sub-districts within the CBD, except for Eminönü district (compare Table 6.3.2 with Table 6.3.3). Beyoğlu and Beşiktaş districts have relatively higher office rents in comparison with Şişli and Eminönü districts both for the first- and the tenth-floor offices. Within the new CBD axis, including Beşiktaş and Şişli districts, it is clear that Beşiktaş has higher rents for small-sized offices.

Overall the results of the model reveal that for all representative office spaces, while Beyoğlu district has the highest office rent levels Eminönü district has the lowest rent levels. These districts are two different parts of the traditional CBD. Within the new CBD axis, while Beşiktaş district has the highest rent levels for the small-sized offices, Şişli district has the highest rents for the high-rise office buildings with sizeable offices.

Table 6.3.1: CBD Office Rent Variation for a Typical Office Space

Office usage area is 200 m^2; number of employees is 5; employees' residence outside the office-district is 0%.

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	1	200	5	0	2727
09 (Beyoğlu)	1	200	5	0	5233
10 (Eminönü)	1	200	5	0	1441
23 (Şişli)	1	200	5	0	1785

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	2	200	5	0	2464
09 (Beyoğlu)	2	200	5	0	4675
10 (Eminönü)	2	200	5	0	1459
23 (Şişli)	2	200	5	0	1622

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	3	200	5	0	2353
09 (Beyoğlu)	3	200	5	0	4302
10 (Eminönü)	3	200	5	0	1364
23 (Şişli)	3	200	5	0	1622

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	4	200	5	0	2353
09 (Beyoğlu)	4	200	5	0	4363
10 (Eminönü)	4	200	5	0	1198
23 (Şişli)	4	200	5	0	1688

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	7	200	5	0	2352
09 (Beyoğlu)	7	200	5	0	4718
10 (Eminönü)	7	200	5	0	1136
23 (Şişli)	7	200	5	0	1785

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	10	200	5	0	2353
09 (Beyoğlu)	10	200	5	0	5050
10 (Eminönü)	10	200	5	0	1146
23 (Şişli)	10	200	5	0	1785

Table 6.3.2 : CBD Office Rent Variation for a Typical Office Space
Office usage area is 200 m^2; number of employees is 3; employees' residence outside the office-district is 0%.

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	1	200	3	0	2873
09 (Beyoğlu)	1	200	3	0	5200
10 (Eminönü)	1	200	3	0	1174
23 (Şişli)	1	200	3	0	1772

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	2	200	3	0	2558
09 (Beyoğlu)	2	200	3	0	4876
10 (Eminönü)	2	200	3	0	1174
23 (Şişli)	2	200	3	0	1609

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	3	200	3	0	2104
09 (Beyoğlu)	3	200	3	0	4471
10 (Eminönü)	3	200	3	0	1159
23 (Şişli)	3	200	3	0	1609

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	4	200	3	0	1383
09 (Beyoğlu)	4	200	3	0	3416
10 (Eminönü)	4	200	3	0	1100
23 (Şişli)	4	200	3	0	1669

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	7	200	3	0	1383
09 (Beyoğlu)	7	200	3	0	3200
10 (Eminönü)	7	200	3	0	800
23 (Şişli)	7	200	3	0	1771

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	10	200	3	0	1383
09 (Beyoğlu)	10	200	3	0	5050

10 (Eminönü)	10	200	3	0	800
23 (Şişli)	10	200	3	0	1772

Table 6.3.3: CBD Office Rent Variation for a Typical Office Space
Office usage area is 100 m^2; number of employees is 7; employees' residence outside the office-district is 0%.

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	1	100	7	0	2104
09 (Beyoğlu)	1	100	7	0	1981
10 (Eminönü)	1	100	7	0	1441
23 (Şişli)	1	100	7	0	823

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	2	100	7	0	2104
09 (Beyoğlu)	2	100	7	0	1767
10 (Eminönü)	2	100	7	0	1459
23 (Şişli)	2	100	7	0	823

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	3	100	7	0	2104
09 (Beyoğlu)	3	100	7	0	1575
10 (Eminönü)	3	100	7	0	1383
23 (Şişli)	3	100	7	0	823

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	4	100	7	0	2104
09 (Beyoğlu)	4	100	7	0	1050
10 (Eminönü)	4	100	7	0	1383
23 (Şişli)	4	100	7	0	823

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	7	100	7	0	2104
09 (Beyoğlu)	7	100	7	0	3250
10 (Eminönü)	7	100	7	0	1395
23 (Şişli)	7	100	7	0	1050

SIS District Code	*Office Floor*	*Office Usage Area (m^2)*	*Number of Employees*	*Outside-district residence (%)*	*Office Rent*
07 (Beşiktaş)	10	100	7	0	2104
09 (Beyoğlu)	10	100	7	0	5050
10 (Eminönü)	10	100	7	0	1383
23 (Şişli)	10	100	7	0	1050

6.3. Takagi, Sugeno, Kang Type FRBM for the Office Space Data

As we said earlier, the main difference between TSK and Mamdani-type fuzzy rule based systems (FRBS) is that the consequence part of the TSK-type FRBS can be constant or linear function, meaning that the output membership function of the TSK-type FRBS's can be linear or constant. In this section, we are going to construct the Takagi, Sugeno, Kang (TSK) fuzzy inference system (FIS). The input variables are the same as in subsection 6.2.1. Briefly, in order to construct the TSK-type fuzzy inference system we need to find the regression equation for the dependent variable "office space rent" as the function of the given 4 independent variable that have been found though the backward stepwise regression. So the only difference between different TSK-type FIS models in our case would the output membership function equations.
We carried out four different TSK-type FIS models for office space data with, namely:

1. For every Districts (Ilche);
2. For the intervals of the independent variable "Office Area";
3. For the intervals of the independent variable "Office Floor";
4. For the intervals of the independent variable "Number of Workers".

6.3.1. TSK-Type FRBS for the Office Space Data Regressed by the Districts

In order to construct the TSK-type model we divided the data into 5 parts as follows:

Table 6.4: Frequency distribution of the sample data among sub-districts of the CBD

Districts	Frequency	Percent	Cumulative Percent
Eminönü	59	12,22	12,22
Beyoğlu	147	30,43	42,65
Beşiktaş	44	9,11	51,76
Şişli	223	46,17	97,93
Sarıyer	10	2,07	100
Total	483	100	

For each part we made a linear regression, the given for input variables of the FIS model as independent variables and for the **"office space rent" variable** as the dependent one.
The outcome the regression can be seen in the Table 6.5.

Table 6.5: The regression coefficients for each district

Ditricts	Eminönü	Beyoğlu	Beşiktaş	Şişli	Sarıyer
(Constant)	249,92	405,42	-702,04	-1186,84	125,62
Office Floor	-11,73	-4,36	186,09	71,58	119,19
Office Usage Area (m2)	8,20	10,00	17,05	17,81	3,51
Number of the Workers	-21,55	32,47	-7,24	64,97	153,20
Tenant's Residence Out of the Office District(%)	0,31	0,04	1,21	0,50	-7,45

In order to construct the TSK-type FIS model regressed by the districts we put an additional 5 valued variable into the fuzzy inference system, namely **"district" variable**.
"District" variable describes the districts where the given office space is located. The "district" variable is divided into 5 nonintersecting triangular fuzzy subsets. Since given office building cannot be located in the two different districts at the same time we made the membership function nonintersecting. For this variable, the membership function with the five linguistic variables is presented in Figure 6.10.

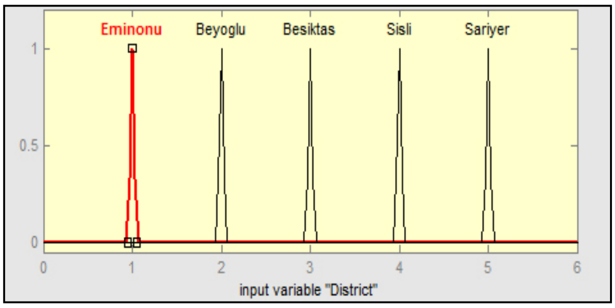


Figure 6.10: Fuzzification of the "district" input variable.

6.3.1.1. Production of the Rule Base

For the whole data set representing Istanbul CBD, we have defined five membership function for the **"district" variable** so we are going to define 5 Fuzzy Inference System rules. Using the regression coefficient we have to write five different functions for each district, which may be found in the Table 6.6. For simplicity we are going to name the input variables as x_1 for the "office floor", x_2 for the "ffice usage area", x_3 for the "number of workers" and x_4 for the "tenant's residence out of the office district" variable.

Table 6.6: The output membership function equations for TSK-type FIS model regressed for Sub-districts.

Function 1	$f(x) = 249{,}92 - 11{,}73x_1 + 8{,}2x_2 - 21{,}55x_3 + 0{,}31x_4$
Function 2	$f(x) = 405{,}42 - 4{,}36x_1 + 10x_2 + 32{,}47x_3 + 0{,}04x_4$
Function 3	$f(x) = -702{,}04 + 186{,}09x_1 + 17{,}05x_2 - 7{,}24x_3 + 1{,}21x_4$
Function 4	$f(x) = -1186{,}85 + 71{,}58x_1 + 17{,}81x_2 + 64{,}97x_3 + 0{,}50x_4$
Function 5	$f(x) = 125{,}62 + 119{,}19x_1 + 3{,}51x_2 + 153{,}20x_3 - 7{,}45x_4$

The relationship between input and output variables of the given model is determined by fuzzy rules, which may be found below:

Rule 1: If (District is Eminönü) **Then** (Office_Rent is Function 1),
Rule 2: If (District is Beyoğlu) **Then** (Office_Rent is Function 2),
Rule 3: If (District is Beşiktaş) **Then** (Office_Rent is Function 3),
Rule 4: If (District is Şişli) **Then** (Office_Rent is Function 4),
Rule 5: If (District is Sarıyer) **Then** (Office_Rent is Function 5).

6.3.2. TSK-Type FRBS for the Office Space Data Regressed by the Office Area

In order to construct the TSK-type model for office space data we divided the data into 5, using the parts as follows:

Table 6.7: Frequency distribution of the sample data by the Office Usage Area

	Frequency	Percent	Cumulative Percent
1st Interval	153	31,68	31,68
2nd Interval	134	27,74	59,42
3rd Interval	127	26,29	85,71
4th Interval	19	3,93	89,64
5th Interval	50	10,36	100
Total	483	100	

For each interval we made a linear regression, with the given for input variables of the FIS model as independent variables and for the **"office space rent" variable** as the dependent one.

The outcome of the regression can be seen in the Table 6.8.

Table 6.8: Regression coefficient for the "office usage area".

Office Usage Area					
Intervals	1	2	3	4	5
(Constant)	254,61	35,11	278,32	-4433,13	4199,73
Office Floor	-12,33	3,57	70,82	13,89	11,68
Office Usage Area (m2)	12,59	9,33	6,05	30,80	12,75
Number of the Workers	16,38	100,00	45,86	44,08	8,80
Tenant's Residence Out of the Office District(%)	-1,12	-1,08	3,52	10,20	-38,49

6.3.2.1. Production of the Rule Base

For the whole data set representing Istanbul CBD, we have defined five membership function for the **"office usage area" variable** so we are going to define 5 Fuzzy Inference System rules. Using the regression coefficient we have to write five different functions for each interval of the number of workers, as shown in Table 6.9.

Table 6.9: The output membership function equations for TSK-type FIS model regressed for Office Usage Area.[9]

Function 1	$f(x) = 254{,}61 - 12{,}33x_1 + 12{,}59x_2 + 16{,}38x_3 - 1{,}12x_4$
Function 2	$f(x) = 35{,}11 + 3{,}57x_1 + 9{,}33x_2 + 100x_3 - 1{,}08x_4$
Function 3	$f(x) = 278{,}32 + 70{,}82x_1 + 6{,}05x_2 + 45{,}86x_3 + 3{,}52x_4$
Function 4	$f(x) = -4433{,}13 + 13{,}89x_1 + 30{,}80x_2 + 44{,}08x_3 + 10{,}20x_4$
Function 5	$f(x) = 4199{,}73 + 11{,}68x_1 + 12{,}75x_2 + 8{,}80x_3 - 38{,}49x_4$

The relationships between input and output variables of the given model are determined by fuzzy rules, which may be found below:

Rule 1: If (Office_Usage_Area is Very Small) **Then** (Office_Rent is Function 1),
Rule 2: If (Office_Usage_Area is Small) **Then** (Office_Rent is Function 2),
Rule 3: If (Office_Usage_Area is Medium) **Then** (Office_Rent is Function 3),
Rule 4: If (Office_Usage_Area is Big) **Then** (Office_Rent is Function 4),

[9] Variables are the same as in the Subsection 6.3.1.1.

Rule 5: If (Office_Usage_Area is Too Big) **Then** (Office_Rent is Function 5).

6.3.3. TSK-Type FRBS for the Office Space Data Regressed by the Office Floor

In order to construct the TSK-type model for office space data, we divided the data into 5, using the parts as follows:

Table 6.10: Frequency distribution of the sample data by the Office Floor

Intervals	Frequency	Percent	Cumulative Percent
1st Interval	200	41,41	41,41
2nd Interval	165	34,16	75,57
3rd Interval	96	19,86	95,43
4th Interval	22	4,57	100
Total	483	100	

For each interval we made a linear regression, with the given for input variables of the FIS model as independent variables and for the **"office space rent" variable** as the dependent one. The outcome of the regression can be seen in Table 6.11.

Table 6.11: Regression coefficient for the "office floor".

Office Floor				
Intervals	1	2	3	4
(Constant)	126,85	313,35	237,14	-578,03
Office Floor	71,88	47,61	-182,54	184,02
Office Usage Area (m2)	10,97	2,75	20,09	13,88
Number of the Workers	27,57	125,36	39,44	-25,75
Tenant's Residence Out of the Office District(%)	0,86	-0,78	-3,24	-0,74

6.3.3.1. Production of the Rule Base

For the whole data set representing Istanbul CBD, we have defined four membership function for the **"office floor" variable** so we are going to define four Fuzzy Inference System rules. Using the regression coefficient we have to write five different functions for each interval of the number of workers; please see Table 6.12.

Table 6.12: The output membership function equations for TSK-type FIS model regressed for Office Floor[10]

Function 1	$f(x) = 126{,}85 + 71{,}88x_1 + 10{,}97x_2 + 27{,}57x_3 + 0{,}86x_4$
Function 2	$f(x) = 313{,}35 + 47{,}61x_1 + 2{,}75x_2 + 125{,}36x_3 - 0{,}78x_4$
Function 3	$f(x) = 237{,}14 - 182{,}54x_1 + 20{,}09x_2 + 39{,}44x_3 - 3{,}24x_4$
Function 4	$f(x) = -578{,}03 + 184{,}02x_1 + 13{,}88x_2 - 25{,}75x_3 - 0{,}74x_4$

The relationships between input and output variables of the given model are determined by fuzzy rules, which may be found below:

Rule 1: If (Office_Floor is Low) **Then** (Office_Rent is Function 1),
Rule 2: If (Office_Floor is Middle) **Then** (Office_Rent is Function 2),
Rule 3: If (Office_Floor is High) **Then** (Office_Rent is Function 3),
Rule 4: If (Office_Floor is Too high) **Then** (Office_Rent is Function 4).

6.3.4. TSK-Type FRBS for the Office Space Data Regressed by the Number of Workers

In order to construct the TSK-type model for office space data we divided the data into 4 parts using the number of workers intervals, as presented in the Table 6.13.

Table 6.13: Frequency distribution of the sample data by number of workers in the office

Intervals	Frequency	Percent	Cumulative percent
1st interval	300	62,11	62,11
2nd interval	98	20,29	82,4
3rd interval	64	13,46	95,86
4th interval	20	4,14	100
Total	483	100	

[10] Variables are the same as in the Subsection 6.3.1.1.

For each interval we made a linear regression, with the given for input variables of the FIS model as independent variables and for the **"office space rent" variable** as the dependent one.

The outcome of the regression can be seen in Table 6.14.

Table 6.14: Regression coefficient for the Number of Workers

Number of Workers				
Intervals	1	2	3	4
(Constant)	190,12	241,96	446,22	2593,82
Office floor	29,95	14,24	114,37	0,53
Office Area (m2)	10,04	9,02	7,67	14,17
Number of Workers	4,23	-38,92	-18,02	-8,52
Tenants' residence out of the office-district(%)	-1,61	2,74	1,31	-21,85

6.3.4.1. Production of the Rule Base

For the whole data set representing Istanbul CBD, we have defined four membership function for the **"number of workers" variable** so we are going to define 4 Fuzzy Inference System rules. Using the regression coefficient we have to write four different functions for each interval of the number of workers.

Table 6.15: The output membership function equations for TSK-type FIS model regressed for Number of Workers.[11]

Function 1	$f(x) = 190{,}12 + 29{,}95x_1 + 10{,}04x_2 + 4{,}23x_3 - 1{,}61x_4$
Function 2	$f(x) = 241{,}96 + 14{,}24x_1 + 9{,}02x_2 - 38{,}92x_3 + 2{,}74x_4$
Function 3	$f(x) = 446{,}22 + 114{,}37x_1 + 7{,}67x_2 - 18{,}02x_3 + 1{,}31x_4$
Function 4	$f(x) = 2593{,}82 + 0{,}53x_1 + 14{,}17x_2 - 8{,}52x_3 - 21{,}85x_4$

The relationships between input and output variables of the given model are determined by fuzzy rules, which may be found below:

Rule 1: If (Number_of_workers is Low) **Then** (Office_Rent is Function 1),
Rule 2: If (Number_of_workers is Medium) **Then** (Office_Rent is Function 2),
Rule 3: If (Number_of_workers is High) **Then** (Office_Rent is Function 3),
Rule 4: If (Number_of_workers is Too High) **Then** (Office_Rent is Function 4).

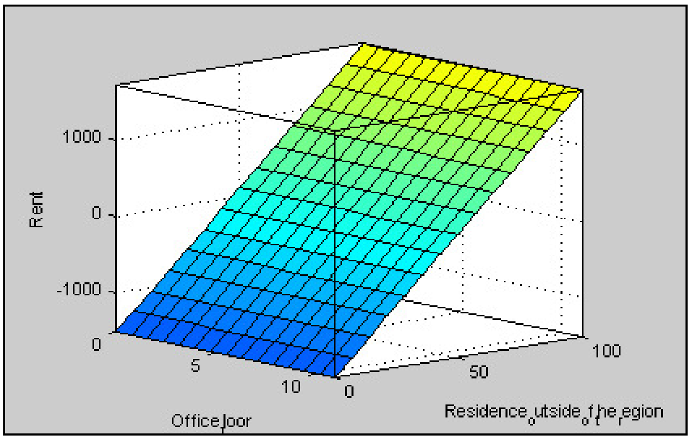
[11] Variables are the same as in the Subsection 6.3.1.1

6.3.5. Empirical Results of the TSK-type Fuzzy Rule Based System (FRBS) for Office Space Data

To understand the direction and magnitude of the effects of the physical attributes of office spaces and employees' residence preferences on the office rents and to show the smoothness of the fuzzy model solution, we produce 3-Dimensional graphs for the entire sample of 483 office spaces. The relation between the inputs (physical attributes of office spaces; employees' residence preferences) and the output (office rental price) of the model determined by the FIS rules is a 5 dimensional figure which cannot be displayed in three dimensions. Hence, the input-output relation by using different input pairs can be visualized by bilateral surfaces.

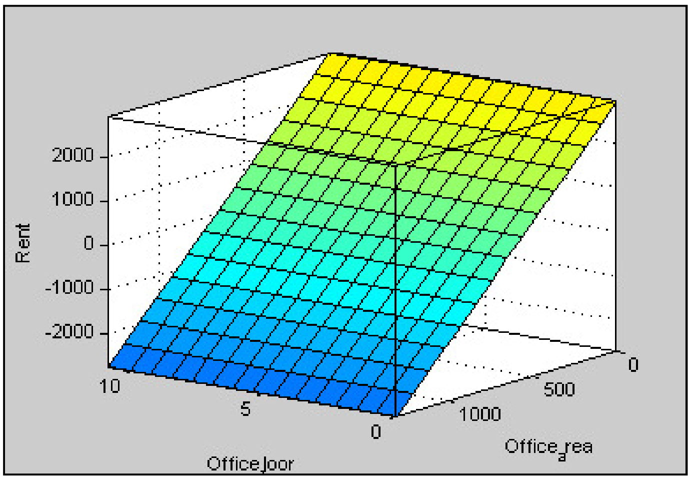
6.3.5.1. Empirical results for the TSK-type FRBS for the Office Space Data Regressed by Districts

Figure 6.11 to Figure 6.14 displays the combined effect of different input variables on office space rents for different sub-districts. If we will look to the district part of reference inputs in the Figure 6.11 and Figure 6.12, we will see that for the Eminönü and Beyoğlu the more than the half of the office rent values is at the negative part which shows that this model does not work good. We will not describe the Figure 6.11 and Figure 6.12 due to the reasons given above.



Reference input	District	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	1	NaN	650	50	NaN

Figure 6.11: Combined Effect of "Office Floor" and "Tenants' residence out of the office-district" on the Office Space Rent

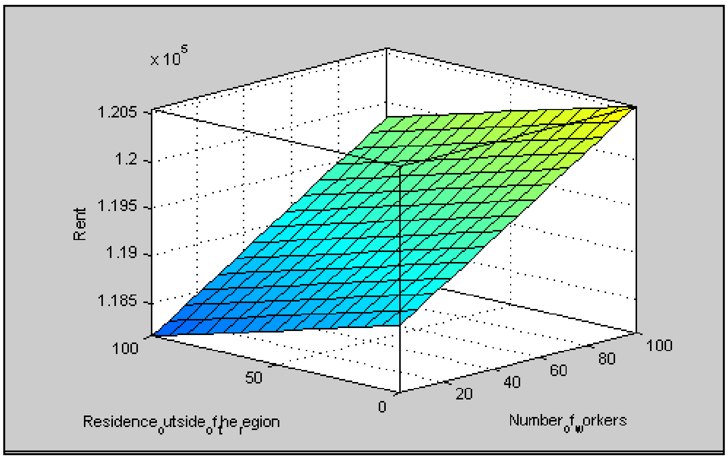


Reference input	District	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	2	NaN	NaN	50,5	50

Figure 6.12: Combined Effect of "Office Floor" and "Office Area" on the Office Space Rent

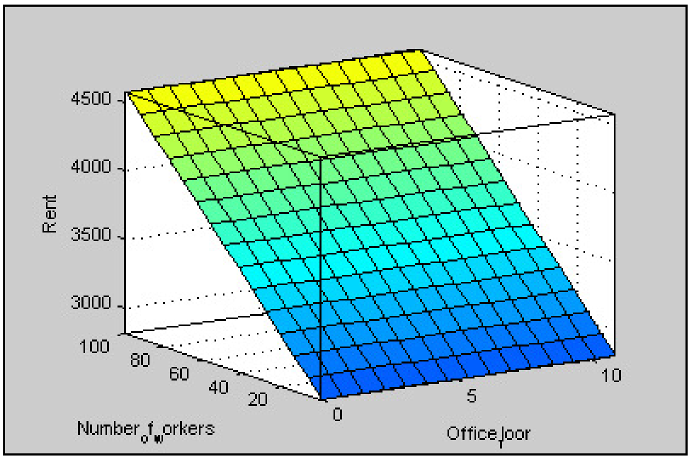
In Figure 6.13, we see the combined effect of "tenants' residence out of the office-district" and "number of workers" on the Office Space Rent in the Beşiktaş districts. From the given figure we may say that tenants' residence outside the office-district has only slight effect on the office rents. The highest rents are observed for the offices with zero percent of tenants mainly reside within the office-district, and considerably large number of employees. For the similar offices with many workers, tenants' preferences for living outside of the office-district significantly reduce the office rents. Therefore, we may say that as the tenants reside within the office-district, their demand for office space and correspondingly office rents are expected to increase in that particular district.

The joint effect of "office floor" and "number of workers" variables on office space rents are exhibited in Figure 6.14 for the Sisle sub-district. Increases in the number of employees lead to higher office rents. However, office space rents do not change as the office floor increases. The highest levels of rents (approximately 4,600 TL) over the defined bilateral surface are seen for large number of employees.



Reference input	District	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	3	5,5	650	NaN	NaN

Figure 6.13: Combined Effect of "Tenants' residence out of the office-district" and "Number of Workers" on the Office Space Rent



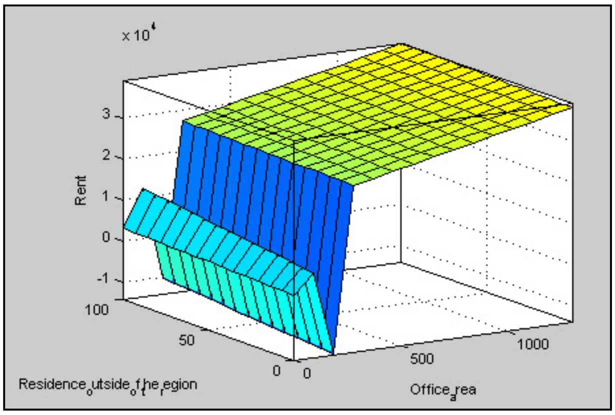
Reference input	District	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	4	NaN	60	NaN	50

Figure 6.14: Combined Effect of "Office Floor" and "Number of Workers" on the Office Space Rent

Due to the fact that this TSK-type fuzzy inference model gives negative office rent values we may conclude that model does not work properly and does not satisfy our needs.

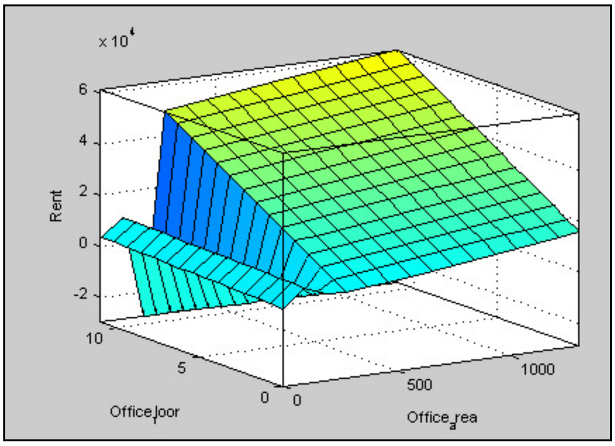
6.3.5.2. Empirical results for the TSK-type FRBS for the Office Space Data Regressed by Office Area

Figure 6.15 to Figure 6.17 display the combined effect of different input variables on office space rents. All the figures have four different joined subspaces. Figure 6.15 depicts the relationship between “Tenants’ residence out of the office-district” and “Office Area” with the Office Space Rent. We may see that for the offices with the area in the interval of $[51m^2, 200m^2]$ the office space rent goes to negative values.



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants’ residence out of the office-district
	5,5	NaN	15,5	NaN

Figure 6.15: Combined Effect of “Tenants’ residence out of the office-district” and “Office Area” on the Office Space Rent

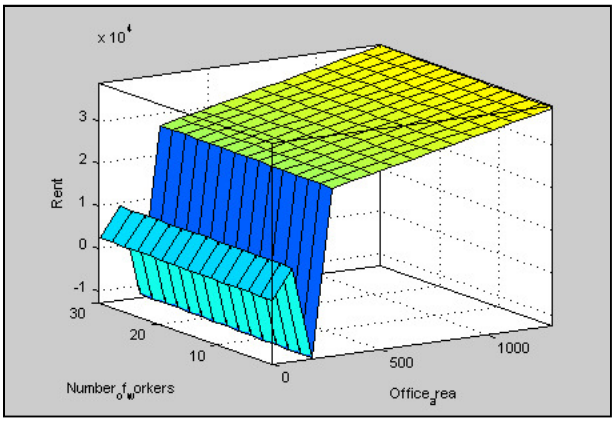


Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants’ residence out of the office-district
	NaN	NaN	15,5	50

Figure 6.16: Combined Effect of “Office Floor” and “Office Area” on the Office Space Rent

In Figure 6.16, we may see the combined effect of the “office floor” and “office area” on the “office space rent”. We may also see that when the “office area” is between the values of $51m^2$ and $200m^2$ then the “office space rent” shows negative values.

Figure 6.17 depicts the combined effect of the “number of workers” and “office area” on the “office space rent”. For the interval values of $[51m^2,200m^2]$ of the “office area” the “office space rent” shows negative values.



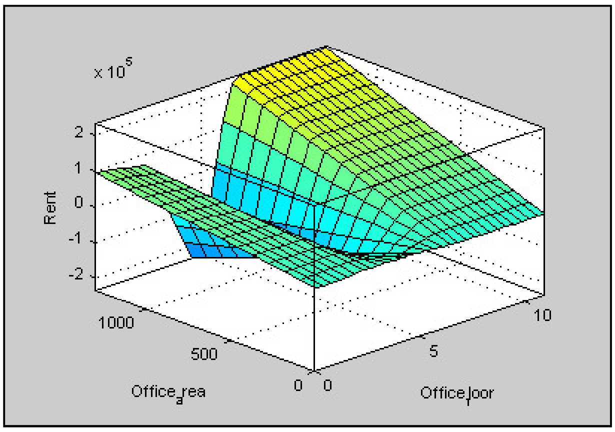
Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants’ residence out of the office-district
	5,5	NaN	NaN	50

Figure 6.17: Combined Effect of “Office Area” and “Number of Workers” on the Office Space Rent

Taking into account that the given output membership functions of TSK-type fuzzy inference system was been constructed by regressing the original data on the “office space rent” we may conclude that the TSK-type FIS model regressed by the Office Area does not work.

6.3.5.3. Empirical results for the TSK-type FRBS for the Office Space Data Regressed by Office Floor

Figure 6.18 to Figure 6.20 display the combined effect of different input variables on office space rents. Figure 6.18 depicts the relationship between “Office Floor” and “Office Area” with the Office Space Rent. We may see that for the offices located in the 3rd, 4th, and 5th floors, the “office space rent” tends to go to the negative values.

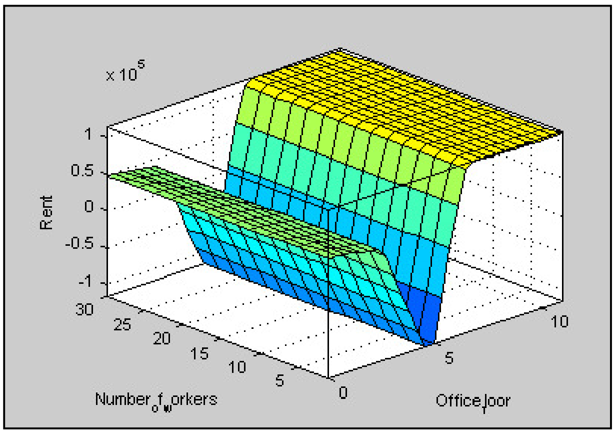


Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	NaN	NaN	15,5	50

Figure 6.18: Combined Effect of "Office Area" and "Office Floor" on the Office Space Rent

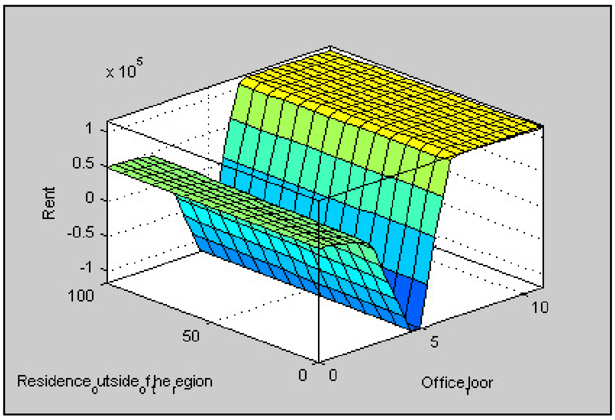
In Figure 6.19, the combined effect of the "office floor" and "number of workers" on the "office space rent" is depicted. For the values of the 3, 4, 5 and 6 of the "office floor" variable, the "office space rent" shows negative values.

Figure 6.20 depicts the combined effect of the "Tenants' residence out of the office-district" and "office area" on the "office space rent". As we said about the Figure 20 and Figure 21 for the values of 4,5 and 6 of the "office floor", the values of the "office space rent" have a negative sign.



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	NaN	650	NaN	50

Figure 6.19: Combined Effect of "Office Floor" and "Number of Workers" on the Office Space Rent



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	NaN	650	15,5	NaN

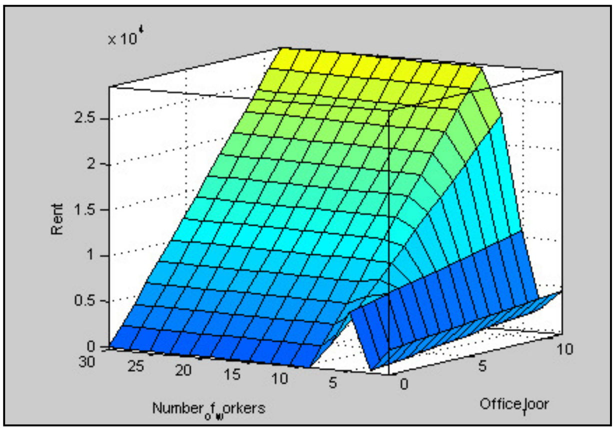
Figure 6.20: Combined Effect of "Tenants' residence out of the office-district" and "Office Floor" on the Office Space Rent

Taking into account the above said we may conclude that the TSK-type FIS model regressed by the Office Floor does not work.

6.3.5.4. Empirical Results for the TSK-Type FRBS for the Office Space Data Regressed by Number of Workers

Figure 6.21 to Figure 6.23 depict the combined effect of the "number of workers" with other three independent variables on the "office space rent".

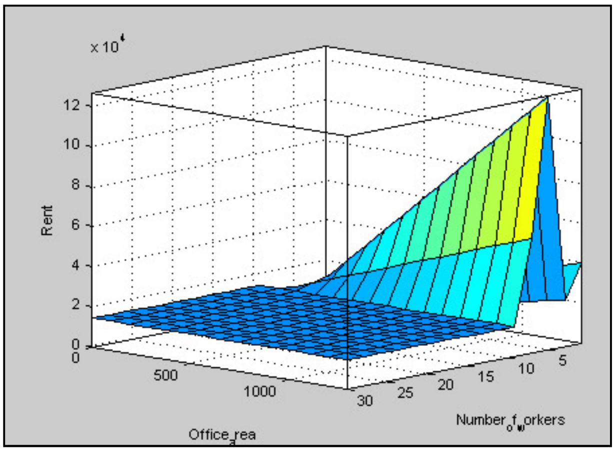
In Figure 6.21, we may see the joint effect of the "number of workers" and "office floor" on the "office space rent". Increases in the office floor lead to the increase in the office space rent. We may also see that for the values of less than 3 of the number of workers, the office rent decreases as the number of workers increases, for the values greater than 3 of the number of workers and for the values between 0 and 3 of the office floor the office rent decreases, for the values greater than 3 of the office floor the office space rent increases. The number of workers does not affect office rent price when it is greater than 10. For the office at low floors, the office space rent attains it is maximum at the point where number of workers is equal to 5 (approximately 5000TL), and for the high floor offices office space rent attains its maximum with high rise office buildings and large number of workers.



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	NaN	75	NaN	50

Figure 6.21: Combined Effect of "Office Floor" and "Number of Workers" on the Office Space Rent

Figure 6.22 depicts the combined effect of "office area" and "number of workers" on the office space rent. For the number of workers higher than 10, there is no change in the office space rent. For the small number of workers (between 1 and 3), the office space rent decreases as the number of workers increases. For the number of workers between 3 and 5, the office space rent increases with the steepest increase happening for the sizeable office spaces. When the number of workers is between 5 and 10, the office rent decreases. The highest level of office rent is observed for the sizeable office spaces and 5 workers.

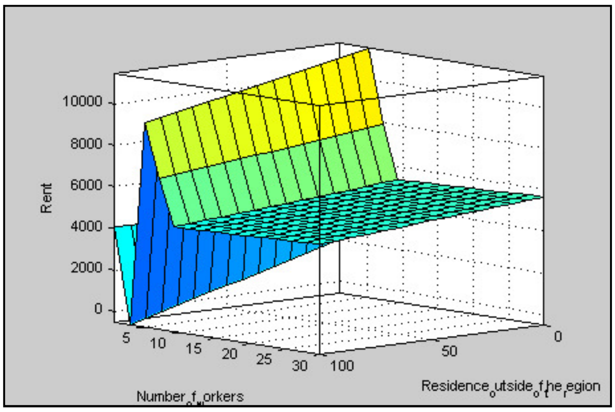


Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	5,5	NaN	NaN	50

Figure 6.22: Combined Effect of "Office Area" and "Number of Workers" on the Office Space Rent

The question of how the tenants' residence out of the office-district and number of workers jointly affect office space rents is answered in Figure 6.23. The tenants' residence out of the office-district has a big effect on the office rent, when the number of workers is small. As the number of workers increases the effect of the residence out of the office-district decreases. For the high in the office district residence the office rent is higher than for the out of the office district residence. The highest rent is observed for the value of zero percent residence out of the office-district variable and for the value of 5 of number of workers.

In the process of identifying the regression equation we came to conclusion that the regression equations for the Office Floor is not working properly and the regression equation for the Office Area and Districts works in some intervals but shows negative results in others. The regression equation for the Number of Workers works well. Below you may find the regression coefficients for the Number of Workers:



Reference input	Office Floor	Office Usage Area	Number of Workers	Tenants' residence out of the office-district
	5,5	650	NaN	NaN

Figure 6.23: Combined Effect of "Tenants' residence out of the office-district" and "Number of Workers" on the Office Space Rent

Table 6.16: Results of the regression by the Number of Workers

#	Floor	Area	Number of Workers = 3	Number of Workers = 5	Number of Workers = 7
1	1	100	2075,27	9370,65	6410,03
2	2	100	2304,27	9765,80	8144,80
3	3	100	2533,27	10160,95	9879,58

4	4	100	2762,28	10556,10	11614,35
5	7	100	3449,29	11741,56	16818,68
6	10	100	4136,30	12927,02	22023,00
7	1	150	2983,66	13837,55	8713,45
8	2	150	3212,66	14232,70	10448,22
9	3	150	3441,67	14627,86	12183,00
10	4	150	3670,67	15023,01	13917,77
11	7	150	4357,68	16208,46	19122,10
12	10	150	5044,69	17393,92	24326,43
13	1	200	3892,05	18304,45	11016,87
14	2	200	4121,06	18699,61	12751,65
15	3	200	4350,06	19094,76	14486,42
16	4	200	4579,06	19489,91	16221,20
17	7	200	5266,07	20675,37	21425,52
18	10	200	5953,08	21860,82	26629,85
19	1	300	5708,84	27238,26	15623,72
20	2	300	5937,84	27633,41	17358,50
21	3	300	6166,84	28028,57	19093,27
22	4	300	6395,85	28423,72	20828,05
23	7	300	7082,86	29609,17	26032,37
24	10	300	7769,87	30794,63	31236,70

6.4 Mamdani-Type Fuzzy Rule-Based Model for the Office Space and Office Building Data

The building data consist of the 173 buildings located within Istanbul CBD. A detailed questionnaire was prepared and carried out for each office space by a professional data-collection company. A survey of 26 questions including the general characteristics of the buildings, infrastructure details, and construction material and design quality and so on are displayed in Table 17. When combined office and building factors made total of 24. Sixty nine different factors that affect the office rental price possibly are highly correlated to each other and result in a multicollinearity problem in a multiple regression analysis. Accordingly, we first carry out a backward stepwise regression in order to reduce the number of explanatory variables, then we employ fuzzy rule-based analysis to the new data set.

Table 6.17: A survey of 26 questions for 173 office buildings in Istanbul CBD

	Variable Description	Number of questions
General	Building type	5 questions

characteristics of the buildings	Number of floors of the building Construction year Base area of the building Number of offices in the building	
Infrastructure details	Number of elevators in the building Common area of the building Availability of the shopping centers in the building, Availability of parking lot in the building. Size of the parking lot. Availability of the fire exit. Availability of the air conditioning system	7 questions
Office related details	Number of unused offices in the building Area of the unused offices in the building Number of the owners and contractors of the offices and shopping centers in the building Security system Heating system Cooling system Availability of the generators	7 questions
Construction material and design quality	Construction material of the building Physical condition of the building Architectural design quality Building's indoor functions	4 questions
Functions and locational parameters	Is the building in the prestigious location? Is there any internet connection in the building?	2 questions
Monthly expenses	The monthly expenses of the building	1 question
TOTAL		26 questions

Our results of backward stepwise regression show that 12 out of 24 variables are statistically significant in affecting the office rental prices. These variables are, namely:

1. Number of rooms in the office,
2. Number of workers in the office,
3. Office Expenses,
4. Number of floors in the building,
5. The age of the building,
6. Total area of the building,
7. Number of offices in the building,
8. Number of elevators,

9. Total area of the empty offices in the building,
10. Parking size of the building,
11. Total number of the offices in the building,
12. Number of the owners of the offices in the building.

The first 3 are office variables and the rest are building variables. In the course of the creating Mamdani FIS model couple of modification has been made to this 12 variables, which are as follows:

i. The input variables 11 and 12 have been combined to form new input variable - Ownership Ratio, which has been defined like (12/11),
ii. The input variables 6 and 9 have been combined to form the new input variable –Vacancy Ratio, which has been defined as (9/6).

So the new list of input variables is:

1. Number of rooms in the office,
2. Number of workers in the office,
3. Office Expenses,
4. Number of floors in the building,
5. The age of the building,
6. Number of offices in the building,
7. Number of elevators,
8. Vacancy ratio,
9. Parking size of the building,
10. Ownership ratio.
11.

Further we made a regression with and without the Vacancy Ratio and the results showed that the effect of this variable on the overall regression is practically zero, so we did not use it. The input variable number 3, namely "office expenses" has not been used in the construction of the Mamdani FIS model because both "office expenses" and "office space rent" are monetary values, and it does not make sense to have them on different sides of the equation. So after conducting all of these changes we have the following list of input variables on hand:

1. Number of rooms in the office,
2. Number of workers in the office,
3. Number of floors in the building,
4. Number of offices in the building,
5. Number of elevators,

6. The age of the building,
7. Parking size of the building,
8. Ownership ratio.

Being the main determinants of office space rents, these variables are defined as the input variables of the fuzzy rule-based model. Then, we employ both the Mamdani-type and TSK-type fuzzy rule-based model for the whole data set and for each sub-district defined in Table 3.1 separately.[12]

6.4.1. Fuzzification of the Input and Output variables for Office Space and Office Building FRBS

Mamdani type FRBM is constructed with 8 input variables and implemented by using the MATLAB Fuzzy Logic Toolbox. For each input variable, triangular and trapezoidal membership functions are defined with the corresponding linguistic variables over the defined interval values.

In detail, **"number of rooms in the office" variable**, briefly "number of rooms", describes how many rooms are in the office. The "number of rooms" is divided into two triangular and one trapezoidal fuzzy subset due to the distribution of the data. For the "number of rooms" input variable, the membership functions with three linguistic variables along the mathematical expressions of the input variable intervals are presented in Figure 6.24 *(a)*. Since the data clusters have centers around 1.5, 3 and 5, three different fuzzy subsets are defined.

Similarly, **"number of workers in the office" variable**, briefly "number of workers", gives the number of employees in the office. The membership functions with four linguistic variables along the mathematical expressions of the input variable intervals are presented in Figure 6.24 *(b)*. Based upon the distribution of data, the "number of workers" variable is divided into three triangular and one trapezoidal fuzzy subset. According to Figure 6.24 *(b)*, the data clusters have centers around 2, 3, 6 and 8. Hence, four fuzzy subsets are defined for this input variable.

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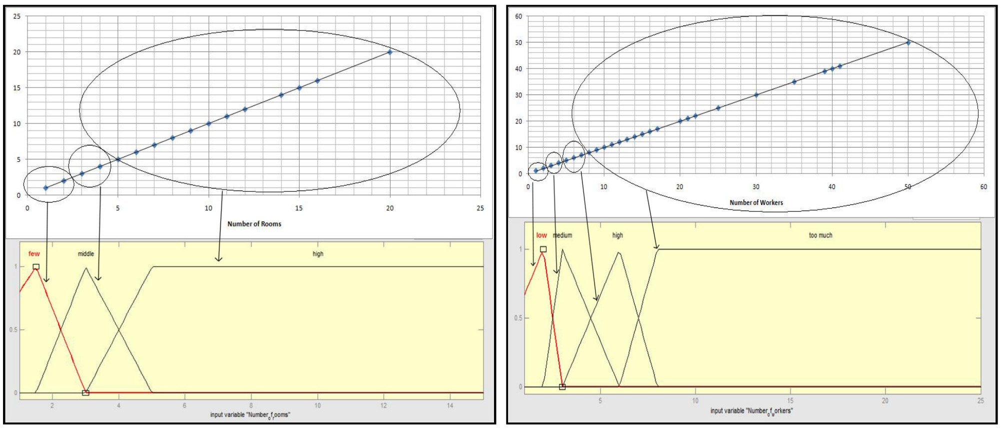
[12] We exclude Sariyer district in our fuzzy rule-based model analysis as the data set with only 10 observations is not sufficient to construct Mamdani rules and perform a FRBS analysis.



(a) *(b)*

Figure 6.24: Fuzzification of the input variables *(a)* "number of the rooms in the office", and *(b)* "number of the workers in the office"

Mathematical expressions of variable *Number of Rooms*:

$$\mu_{few}(x) = \begin{cases} \frac{x+1}{2.5}, & if \quad 0 < x \le 1{,}5, \\ \frac{3-x}{1.5}, & if \quad 1{,}5 \le x \le 3, \end{cases} \qquad \mu_{middle}(x) = \begin{cases} \frac{x-1.5}{1.5}, & if \quad 1{,}5 \le x \le 3\,, \\ \frac{5-x}{2}, & if \quad 3 \le x \le 5, \end{cases}$$

$$\mu_{high}(x) = \begin{cases} \frac{x-3}{2}, & if \quad 3 \le x \le 5, \\ 1, & if \quad 5 \le x \le 50, \\ \frac{8-x}{2}, & if \quad 50 \le x \le 60. \end{cases}$$

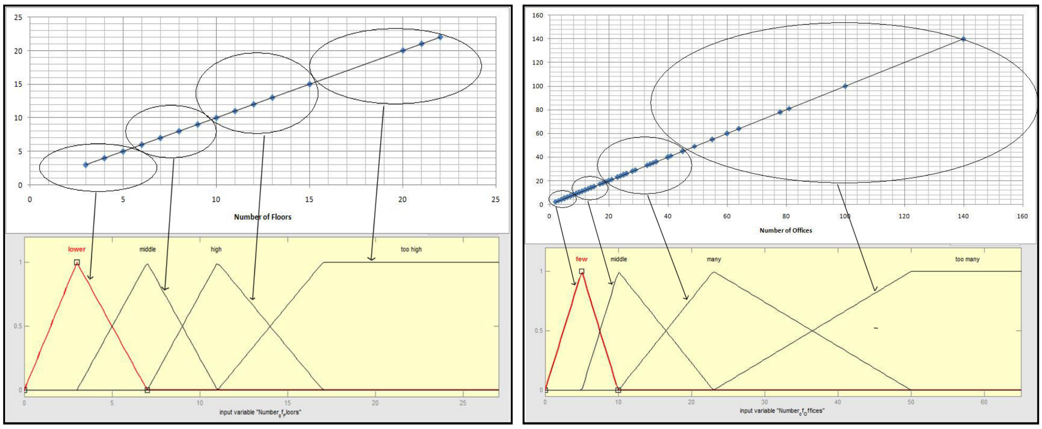
Mathematical expressions of variable *Number of Workers*:

$$\mu_{low}(x) = \begin{cases} \frac{x+1}{3}, & if \quad 0 < x \le 2, \\ 3-x, & if \quad 2 \le x \le 3, \end{cases} \qquad \mu_{medium}(x) = \begin{cases} x-2, & if \quad 2 \le x \le 3, \\ \frac{6-x}{3}, & if \quad 3 \le x \le 6, \end{cases}$$

$$\mu_{high}(x) = \begin{cases} \frac{x-3}{3}, & if \quad 3 \le x \le 6, \\ \frac{8-x}{2}, & if \quad 6 \le x \le 8, \end{cases} \qquad \mu_{too\ many}(x) = \begin{cases} \frac{x-6}{2}, & if \quad 6 \le x \le 8, \\ 1, & if \quad 8 \le x \le 40, \\ \frac{60-x}{20}, & if \quad 40 \le x \le 60. \end{cases}$$

The **"number of floors" variable** is the total number of the floors in the office building. The membership function with four linguistic variables and the mathematical expressions of the input variable intervals are presented in Figure 6.25 *(a)*. The "number of floors" variable is divided into three triangular and one trapezoidal fuzzy subset due to the distribution of the data. Figure 6.25 *(a)*, which exhibits the data distribution for number of workers, illustrates that the data clusters have centers around 3, 7, 11 and 17. Thus, four different fuzzy are defined, accordingly.

"Number of the offices" variable gives the total number of the offices in the given office building. The membership functions with four linguistic variables along with mathematical expressions of the input variable intervals are presented in Figure 6.25 *(b)*. This input variable is divided into three triangular and one trapezoidal fuzzy subsets due to the distribution of the data. As seen in Figure 6.25 *(b)*, the data clusters have centers around 5, 10, 50 and 105. Thus, three fuzzy subsets for the "number of the offices" are defined.



(a) *(b)*

Figure 6.25: Fuzzification of the input variables *(a)* "number of the floors in the building" and *(b)* "number of offices in the building"

Mathematical expressions of variable *Number of Floors*:

$$\mu_{few}(x) = \begin{cases} \dfrac{x}{3}, & if \quad 0 < x \le 3, \\ \dfrac{7-x}{4}, & if \quad 3 \le x \le 7, \end{cases} \qquad \mu_{middle}(x) = \begin{cases} \dfrac{x-3}{4}, & if \quad 3 \le x \le 7, \\ \dfrac{11-x}{4}, & if \quad 7 \le x \le 11, \end{cases}$$

$$\mu_{high}(x) = \begin{cases} \dfrac{x-7}{4}, & if \quad 7 \le x \le 11, \\ \dfrac{17-x}{6}, & if \quad 11 \le x \le 17, \end{cases} \qquad \mu_{too\ high}(x) = \begin{cases} \dfrac{x-11}{4}, & if \quad 11 \le x \le 17, \\ 1, & if \quad 17 \le x \le 40, \\ \dfrac{60-x}{20}, & if \quad 40 \le x \le 60. \end{cases}$$

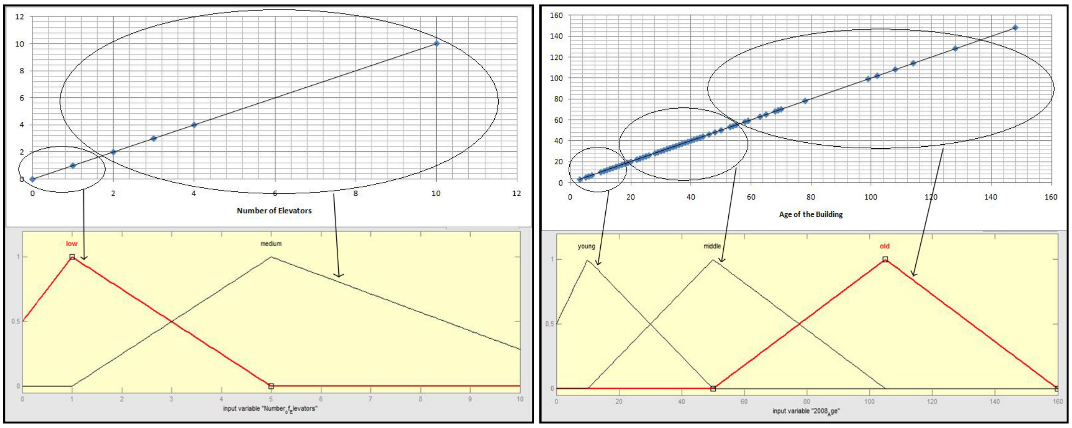
Mathematical expressions of variable *Number of Offices*:

$$\mu_{few}(x) = \begin{cases} \dfrac{x}{5}, & if \quad 0 < x \le 5, \\ \dfrac{10-x}{5}, & if \quad 5 \le x \le 10, \end{cases} \qquad \mu_{medium}(x) = \begin{cases} \dfrac{x-5}{5}, & if \quad 5 \le x \le 10, \\ \dfrac{23-x}{13}, & if \quad 10 \le x \le 23, \end{cases}$$

$$\mu_{many}(x) = \begin{cases} \dfrac{x-10}{13}, & if \ 10 \le x \le 23, \\ \dfrac{50-x}{27}, & if \ 23 \le x \le 50, \end{cases}, \qquad \mu_{too\ many}(x) = \begin{cases} \dfrac{x-30}{23}, & if \ 23 \le x \le 50, \\ 1, & if \ 50 \le x \le 220, \\ \dfrac{250-x}{30}, & if \ 220 \le x \le 250. \end{cases}$$

The **"number of elevators" variable** is the total number of elevators in the office building. The membership function with two linguistic variables and the mathematical expressions of the input variable intervals are presented in Figure 6.26 *(a)*. The "number of elevators" variable is divided into two triangular fuzzy subset due to the distribution of the data. Figure 6.26 *(a)*, which exhibits the data distribution for number of workers, illustrates that the data clusters have centers around 1 and 5. Thus, two different fuzzy are defined, accordingly.

"Age of the building" variable gives the age of the office building. The membership functions with three linguistic variables along the mathematical expressions of the input variable intervals are presented in Figure 6.26 *(b)*. Based upon the distribution of data, the "age of the building" variable is divided into three triangular fuzzy subsets. According to Figure 6.26 *(b)*, the data clusters have centers around 10, 50 and 105. Hence, three fuzzy subsets are defined for this input variable.



(a) *(b)*

Figure 6.26: Fuzzification of the input variables *(a)* "number of the elevators in the building" and *(b)* "age of the building"

Mathematical expressions of variable *Number of Elevators*:

$$\mu_{low}(x) = \begin{cases} \dfrac{x+1}{2}, & if \quad 0 < x \leq 1, \\ \dfrac{5-x}{4}, & if \quad 1 \leq x \leq 5, \end{cases} \qquad \mu_{medium}(x) = \begin{cases} \dfrac{x-1}{4}, & if \quad 1 \leq x \leq 5, \\ \dfrac{12-x}{7}, & if \quad 5 \leq x \leq 12. \end{cases}$$

Mathematical expressions of variable *Age of the Building*:

$$\mu_{young}(x) = \begin{cases} \dfrac{x+10}{20}, & if \quad 0 \leq x \leq 10, \\ \dfrac{50-x}{40}, & if \quad 10 \leq x \leq 50, \end{cases} \qquad \mu_{middle}(x) = \begin{cases} \dfrac{x-10}{40}, & if \quad 10 \leq x \leq 50, \\ \dfrac{105-x}{55}, & if \quad 50 \leq x \leq 105, \end{cases}$$

$$\mu_{old}(x) = \begin{cases} \dfrac{x-50}{55}, & if \quad 50 \leq x \leq 105, \\ \dfrac{160-x}{55}, & if \quad 105 \leq x \leq 160. \end{cases}$$

“Parking size” variable describes the size of the parking lot of the office building. The “parking size” is divided into two triangular and one trapezoidal fuzzy subsets due to the distribution of the data. For the “parking size” input variable, the membership functions with three linguistic variables along the mathematical expressions of the input variable intervals are presented in Figure 32. Since the data clusters have centers around 10, 20 and 50, three different fuzzy subsets are defined.

“Ownership ratio” variable gives the ratio of the total number of the offices in the office building to the number of the owners of the offices in the office building. The membership functions with three linguistic variables along with mathematical expressions of the input variable intervals are presented in Figure 33. This input variable is divided into one triangular and two trapezoidal fuzzy subsets due to the distribution of the data. As seen in Figure 33, the data clusters have centers around 10, 20 and 50. Thus, three fuzzy subsets for the “number of the offices” are defined.

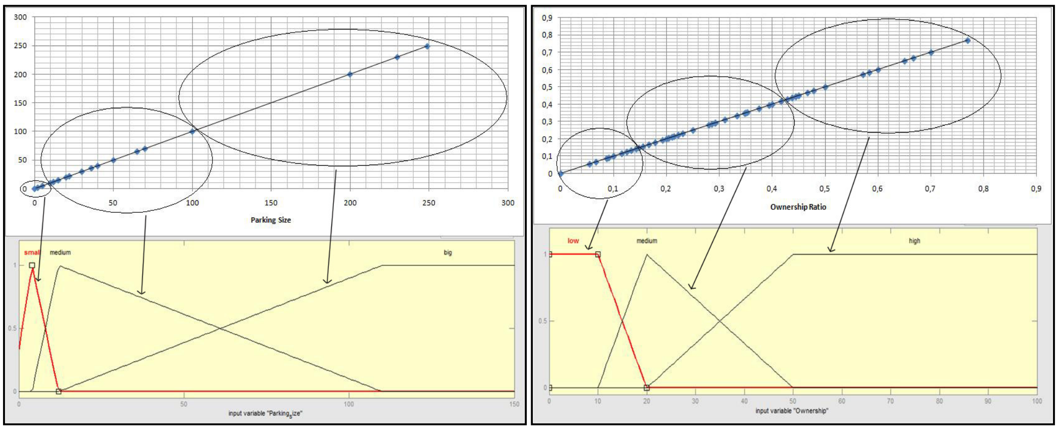


Figure 6.27: Fuzzification of the input variables *(a)* “parking size” and *(b)* “ownership ratio”

Mathematical expressions of variable *Parking Size*:

$$\mu_{low}(x) = \begin{cases} \dfrac{x+2}{6}, & if \quad 0 < x \leq 4, \\ \dfrac{12-x}{8}, & if \quad 4 \leq x \leq 12, \end{cases} \qquad \mu_{medium}(x) = \begin{cases} \dfrac{x-4}{8}, & if \quad 4 \leq x \leq 12, \\ \dfrac{110-x}{98}, & if \quad 12 \leq x \leq 110, \end{cases}$$

$$\mu_{high}(x) = \begin{cases} \dfrac{x-12}{98}, & if \quad 12 \leq x \leq 110, \\ 1, & if \quad 110 \leq x \leq 200, \\ \dfrac{300-x}{100}, & if \quad 200 \leq x \leq 300. \end{cases}$$

Mathematical expressions of variable *Ownership Ratio*:

$$\mu_{low}(x) = \begin{cases} 1, & if \quad 0 < x \leq 10, \\ \dfrac{20-x}{10}, & if \quad 10 \leq x \leq 20, \end{cases} \qquad \mu_{medium}(x) = \begin{cases} \dfrac{x-10}{10}, & if \quad 10 \leq x \leq 20, \\ \dfrac{50-x}{30}, & if \quad 20 \leq x \leq 50, \end{cases}$$

$$\mu_{high}(x) = \begin{cases} \frac{x-20}{30}, & if \quad 20 \leq x \leq 50, \\ 1, & if \quad 50 \leq x \leq 100. \end{cases}$$

The output variable "Office Space Rent" is the same as in Subsection 6.2.1

6.4.2. Production of the Rule Base

For the whole data set representing Istanbul CBD, we define three membership functions for the "number of rooms" input variable, four membership functions for "number of workers" variable, four membership functions for "number of floors" variable, four membership functions for the "number of offices" input variable, two membership functions for "number of elevators" variable, three membership functions for "age of the building" variable, three membership functions for "parking size" variable and three membership functions for "ownership ratio".

In order to create Mamdani FIS model for 8 variables, resulted from the backward stepwise regression, we have to create $(3 \times 4 \times 4 \times 4 \times 2 \times 3 \times 3 \times 3 =)$ 10368 rules. Due to the fact that this number of rules cannot be created, we regressed the variable Rent on the given 8 variables and took the variables that are most significant in the outcome equation. The regression outcome and significant ones may be seen in Table 6.18.

Table 6.18: Regression results of variable Rent on the given 8 variables

	Unstandardized Coefficients		Standardized Coefficients	t-stat	Sig.
		Std. Error	Beta		
(Constant)	-712,52	568,02		-1,25	0,210612
Number of Workers	140,06	9,54	0,52	14,68	1,07E-37
Number of Rooms	276,99	75,27	0,12	3,68	0,000273
Number of Floor	87,62	60,39	0,071	1,45	0,147763
Number of Offices	-22,91	3,65	-1,42	-6,27	1,15E-09
Number of Elevators	377,76	178,46	0,35	2,12	0,035043
Parking Size	12,15	3,299	1,08	3,68	0,000272
Ownership Ratio	550,80	772,43	0,02	0,71	0,476314
Age of the Building	-3,93	7,26	-0,01	-0,54	0,588588
Dependent Variable: Office Rent					

As it is seen from the table the variables that we are going to use are:

1. Number of Workers,
2. Number of Rooms,
3. Number of Offices,

4. Number of Elevators,
5. Parking size.

In total, $3 \cdot 4 \cdot 2 \cdot 4 \cdot 3 = 288$ Fuzzy Inference System (FIS) rules are defined.

In the rule-definition process, firstly, rules are formulated by using data and appraisal practices, then, the contradicting rules are omitted intuitively. While, the summation operation method is used for aggregation, the Center of Gravity is applied for defuzzification process of the modeling. Figure 6.28 illustrates the flow chart of the rule extraction process of FRBM.

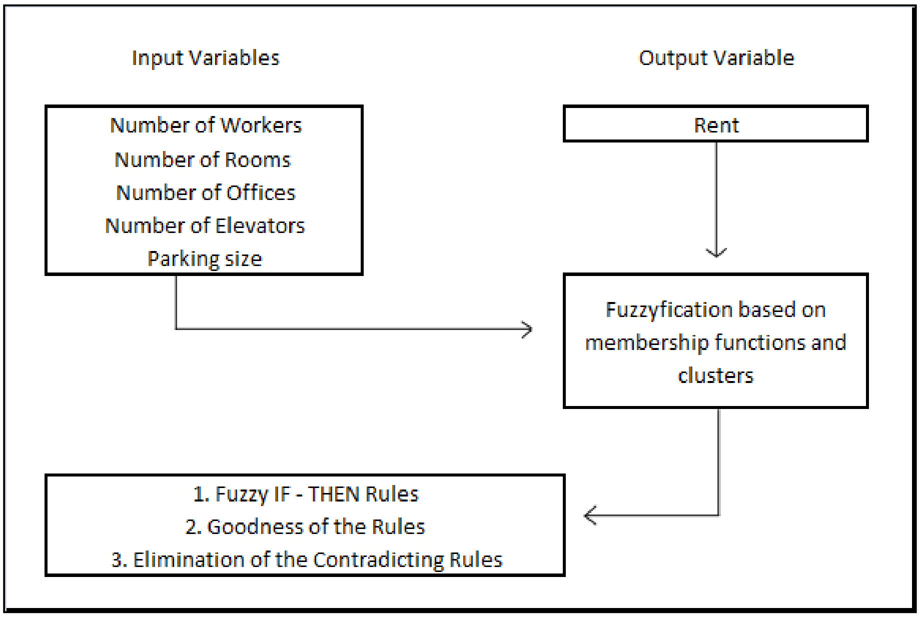


Figure 6.28: Flow chart of fuzzy rule extraction

As seen from Figure 6.28, the relation between the input and the output variables of the fuzzy model is determined by fuzzy rules, which are obtained as follows:

Rule 1: If (Number_of_Workers is Low) **and** (Number_of_Rooms is Few) **and** (Number_of_Offices is Few) **and** (Number_of _Elevators is Low) **and** (Parking_size is Small) **Then** (Office Rent is Cls1)

OR Rule 2: ......

.....

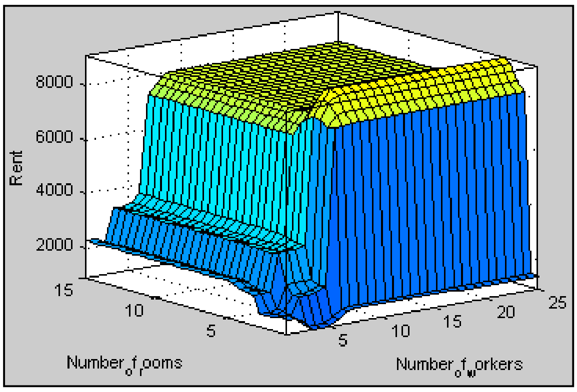
OR Rule 288: If (Number_of_Workers is Too Much) **and** (Number_of_Rooms is High) **and** (Number_of_Offices is Too Many) **and** (Number_of _Elevators is High) **and** (Parking_size is High) **Then** (Office Rent is Cls8).

6.5. Empirical Results of the Mamdani-type Fuzzy Rule-Based Model for Office Space and Office Building

This section of thesis reports our results first for the entire sample of 383 office spaces and office building data set characterizing Istanbul CBD.

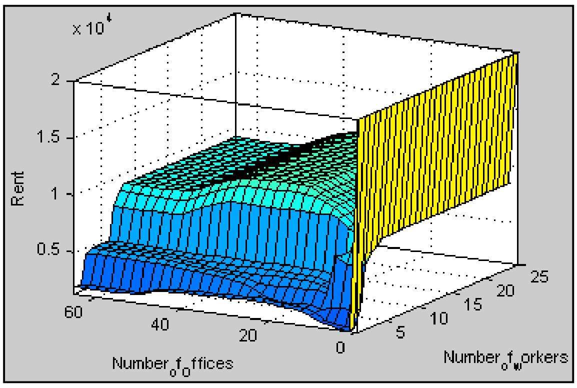
In an attempt to understand in which direction and magnitude the physical attributes of office spaces and office building characteristics impact the office rents, we produce 3-dimensional graphs for the entire sample of 383 office spaces and office building. The relation between the inputs (physical attributes of office spaces; physical attributes of the office building) and the output (office rental price) of the model determined by the FIS rules is a hypersurface that cannot be displayed in three dimensions. Hence, the input-output relation by using different input pairs can be visualized by bilateral surfaces.

Figure 6.29 displays the combined effect of "number of rooms" and "number of workers" input variables on office space rents. Clearly, office rents increase consistently as the number of workers increase. For the high number of workers, the number of rooms higher than 5 does not affect the rents. For the level that when number of rooms is higher than 5 and the number of workers exceeds 7, any positive change at these inputs does not affect the rent value. Over the defined bilateral surface, the highest office rents (approximately 8,000TL to 9,000TL) are observed for low number of rooms with moderate number of workers.



Reference input	Number of Workers	Number of Rooms	Number of Offices	Number of Elevators	Parking Size
	NaN	NaN	15	7	25

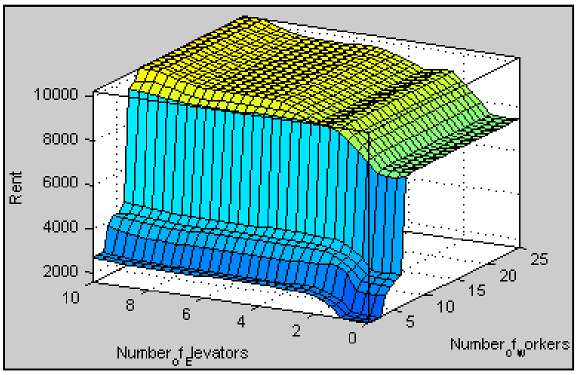
Figure 6.29: Combined Effect of "Number of Rooms" and "Number of Workers" on Office Space Rents



Reference input	Number of Workers	Number of Rooms	Number of Offices	Number of Elevators	Parking Size
	NaN	7	NaN	3	100

Figure 6.30: Combined Effect of "Number of Offices" and "Number of Workers" on Office Space Rents

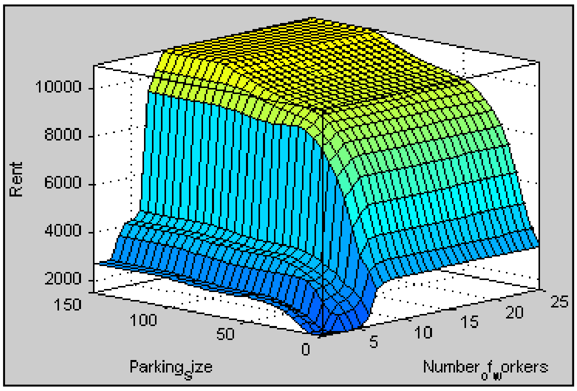
Figure 6.30 represents the impact of the number of offices at the building and the number of workers in these offices on the rent value process. The rise in the number of workers and number of offices at the related property to a specific point singly increases the rent value. Exceeding nearly 50 offices at the real estate, a raise in the number of workers cause no explicit change in the rent value. The highest office space rent is observed for the low number of offices and moderate number of workers.



Reference input	Number of Workers	Number of Rooms	Number of Offices	Number of Elevators	Parking Size
	NaN	10	15	NaN	50

Figure 6.31: Combined Effect of "Number of Elevators" and "Number of Workers" on Office Space Rents

Figure 6.31 illustrates how one of the physical facilities of the building and number of workers alter the rent value of that property. Upward movements of both of the number of elevators at the building and the number of employees at the office positively affect the value of that property to rent. At a certain degree, the case of capacity of almost 10 workers at the office, the office space rent value shows a significant jump. After exceeding this level, the positive change in the number of workers has no obvious contribution to the rent value, while each additive elevator slightly does. In the given bilateral surface the highest office rent is observed for the offices with moderate number of workers combined with the building with the large number of elevators.

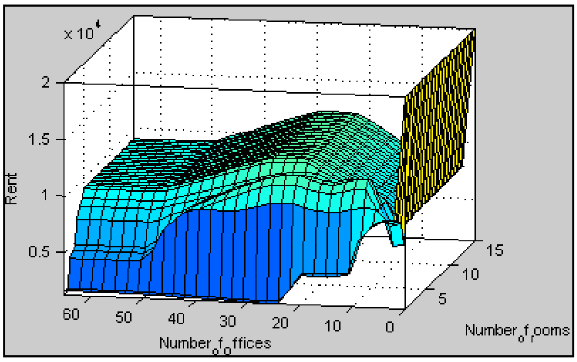


<table>
<tr><td rowspan="2">Reference input</td><td>Number of Workers</td><td>Number of Rooms</td><td>Number of Offices</td><td>Number of Elevators</td><td>Parking Size</td></tr>
<tr><td>NaN</td><td>5</td><td>15</td><td>5</td><td>NaN</td></tr>
</table>

Figure 6.32: Combined Effect of "Parking Size" and "Number of Workers" on Office Space Rents

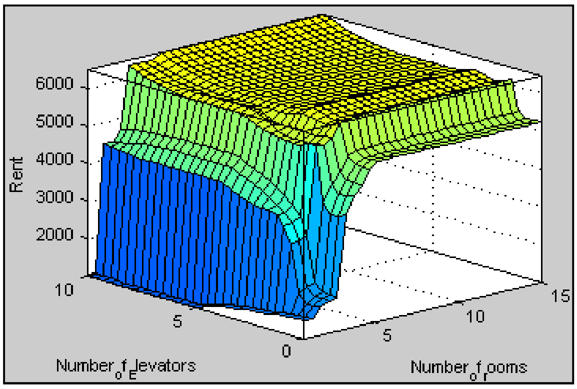
The visualization provided by Figure 6.32 above indicates positive contributions of the independent variables - the parking size and the number of workers - individually on the rental value of the real estate. Considering the minimum of 7-employee office, one additional employee increases rent value abruptly. Depending on the rising capacity of parking area for the automobiles, the rental value of the office continues to increase.

Figure 6.33 shows how the variation in the two reference inputs that define the physical attributes of the building, namely, the number of offices and number of rooms, affect the value process of that property. In a broad sense, the pattern of the process can be summarized as an increase in the office number at the building up to 20, adds further value to rental value. More than 20 offices at the building, does not change the rent value of the office and each additional office at the property, lowers the rental value of individual office. Following that amount, an increase in quantity does not have any effect on the rent value. The number of rooms at the office increases the rent value at a small degree, but the prices of the offices remain constant for the ones that have more than 3 rooms.



Reference input	Number of Workers	Number of Rooms	Number of Offices	Number of Elevators	Parking Size
	13	NaN	NaN	4	100

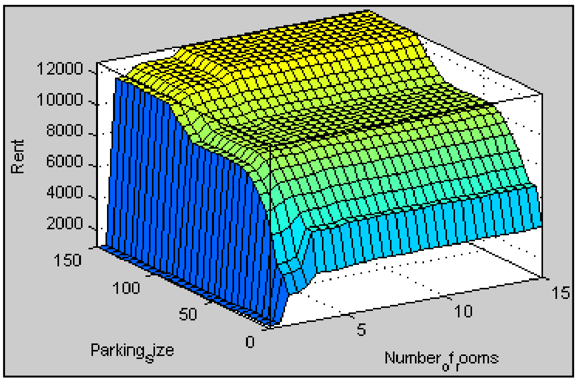
Figure 6.33: Combined Effect of "Number of Offices" and "Number of Rooms" on Office Space Rents



Reference input	Number of Workers	Number of Rooms	Number of Offices	Number of Elevators	Parking Size
	13	NaN	25	NaN	10

Figure 6.34: Combined Effect of "Number of Elevators" and "Number of Workers" on Office Space Rents

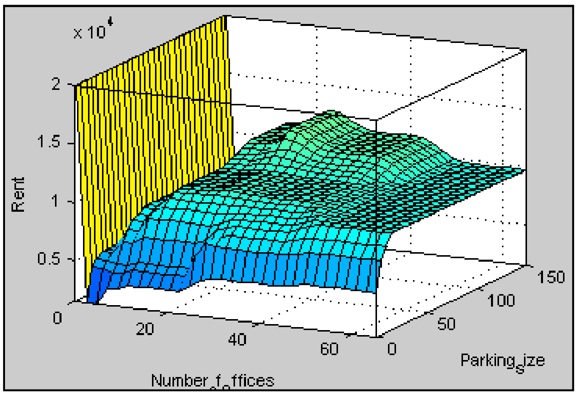
It can be observed from Figure 6.34 that when the number of rooms reaches to a specific level at different elevator numbers, a sharp increase in the rental value occurs. For more than 5-room offices this effect loses its steepness and the changes insignificantly alter the value process. A similar sort of relationship is valid for the change in number of elevators at the building, whereas that sharpness is not observed.



Reference input	Number of Workers	Number of Rooms	Number of Offices	Number of Elevators	Parking Size
	13	NaN	25	10	NaN

Figure 6.35: Combined Effect of "Number of Rooms" and "Parking Size" on Office Space Rents

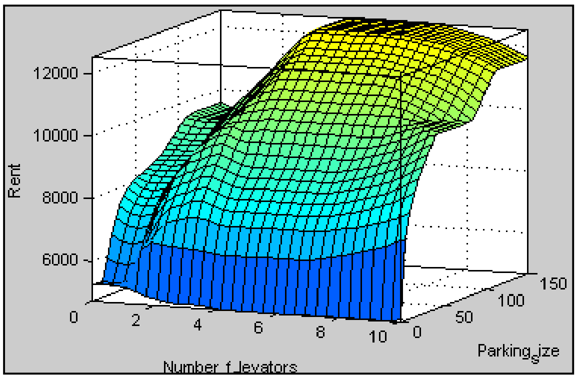
As Figure 6.35 indicates, each additional increase at not only parking size but also number of rooms raises the rental value of the property. The effect of the increasing parking size can be more explicitly observed on that value. Through this value process, at some ranges for the parking capacity for automobiles such as 50-80 and 120-150, the increase at this input does not reflect any change at the rental value of the office.



Reference input	Number of Workers	Number of Rooms	Number of Offices	Number of Elevators	Parking Size
	13	10	NaN	10	NaN

Figure 6.36: Combined Effect of "Number of Offices" and "Parking Size" on Office Space Rents

Figure 6.36 represents the relationship of the two of the reference inputs where the remaining ones are given as constants. As the number of offices at the building increases, the rent value increases as well. Increases in the parking size of the property, leads to the increase in rental value of the offices located in that property. However, the effect of the parking size on that value process is of a small magnitude at certain levels. From the Figure 6.36 we may also see that the when the Number of Offices approaches zero the Rent goes to infinity, meaning that this point is a pole of the function.



Reference input	Number of Workers	Number of Rooms	Number of Offices	Number of Elevators	Parking Size
	15	10	30	NaN	NaN

Figure 6.37: Combined Effect of “Number of Elevators” and “Parking Size” on Office Space Rents

Growing size of the parking area makes the value of the office increase continuously until this capacity reaches to 100-automobile ones. Then, this relationship is reshaped so that no apparent effect on the value process is observed. The other reference variable measured, the number of elevators the building has, has no effect on the rental value until the 2 elevators are concerned. Above this level, for each additional elevator facility the rental value seems to increase as well but does not follow a consistent pattern.

6.6. TSK-type FIS for the Office Space and Office Building Data

In this section we are going to construct the Takagi, Sugeno, Kang (TSK) fuzzy inference system (FIS). Similar to Section 6.3, there will be no change in the input variables, and we will use the input variables of the Mamdani-type FRBS for office space and building data, namely:

1. Number of rooms in the office,

2. Number of workers in the office,
3. Number of floors in the building,
4. Number of offices in the building,
5. Number of elevators,
6. The age of the building,
7. Parking size of the building,
8. Ownership ratio.

Again referring to the Sub-subsection 5.2.6, due to the differences between Mamdani-type and TSK-type fuzzy inference systems we have to regress the given 8 independent variable on the dependent variable, namely, “office space rent” variable, and use the regression equation as the output membership function.

We have carried out four TSK FIS model for the Office Space and Office Building data:

1. For every Districts (Ilche);
2. For the clusters of the independent variable “Age of the Building”;
3. For the clusters of the independent variable “Number of Workers”;
4. For the clusters of the independent variable “Number of Floors”.

6.6.1. TSK-Type FRBS for the Office Space and Office Building Data Regressed by the Districts

We separated the data into 5 groups one for every district. Because in the 5^{th} district, namely Sarıyer, we had few data point we did not make regression for this sub-district. The data partition can be found in the Table 6.19.

Table 6.19: Distribution of the data by the Districts

Districts	Frequency	Percent	Cumulative Percent
Eminönü	48	12,53	12,53
Beyoğlu	87	22,72	35,25
Beşiktaş	28	7,31	42,56
Şişli	211	55,09	97,65
Sarıyer	9	2,35	100
Total	383	100	

The regression coefficient for the Districts can be found below.

Table 6.20: Regression coefficients for the District TSK FIS

	Eminönü	Beyoğlu	Beşiktaş	Şişli
(Constant)	317,91	-1581,22	4229,28	-795,11
Number of Floors in the Building	-87,48	405,92	-136,81	-137,97
Age of the Building	-0,24	-2,05	-132,98	-27,60
Number of Offices in the Building	-2,69	-23,44	-83,05	-24,16
Number of Elevators	48,38	594,32	2493,85	741,45
Parking Size	-320,90	-14,01	101,77	9,99
Number of the Rooms in the Office	235,82	16,05	85,84	580,31
Number of Workers in the Office	151,43	109,28	25,47	152,42
Ownership Ratio	-115,94	-591,76	-1391,65	3645,83

As in Sub-subsection 6.3.5.1, in order to construct the TSK-type FIS model regressed by the districts, we put an additional 5 valued variable into the fuzzy inference system, namely, **“district” variable**.

“District” variable describes the districts where the given office space is located. The “district” variable is divided into 5 nonintersecting triangular fuzzy subsets. Since given office building cannot be located in the two districts, at the same time, we made the membership function nonintersecting. For this variable the membership function with the five linguistic variables are presented in Figure 6.38.

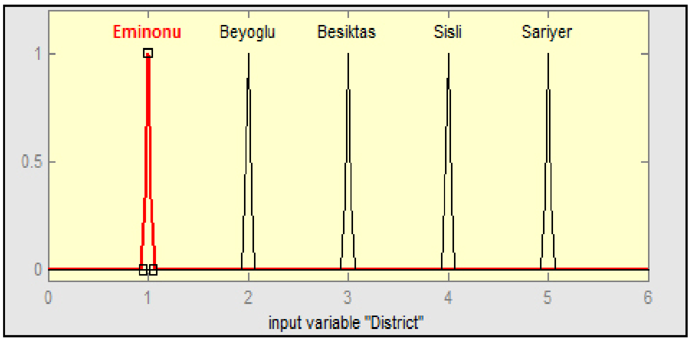


Figure 6.38: Fuzzification of the “district” input variable.

6.6.1.1. Production of the Rule Base

For the whole data set representing Istanbul CBD, we have defined five membership functions for the **“district” variable,** so we are going to define 5 Fuzzy Inference System rules. Using the regression coefficient we have to write five different functions for each district. For simplicity we are going to name the input variables as x_1 for the “number of floors”, x_2 for the “age of the building”, x_3 for the “number of offices”, x_4 for the “number of elevators” , x_5 for the “parking size”, x_6 for the “number of rooms”, x_7 for the “number of workers” and x_8 for the “ownership ratio” variable.

Table 6.21: The output membership function equations for TSK-type FIS model regressed for Districts.

Function 1	$f(x) = 371{,}91 - 87{,}48x_1 - 0{,}24x_2 - 2{,}69x_3 + 48{,}38x_4 - 320{,}9x_5 + 235{,}82x_6 + 151{,}43x_7 - 115{,}94x_8$
Function 2	$f(x) = -1581{,}22 + 405{,}92x_1 - 2{,}05x_2 - 23{,}44x_3 + 594{,}32x_4 - 14{,}01x_5 + 16{,}05x_6 + 109{,}28x_7 - 591{,}76x_8$
Function 3	$f(x) = 4229{,}28 - 136{,}81x_1 - 132{,}98x_2 - 83{,}05x_3 + 2393{,}85x_4 - 101{,}77x_5 + 85{,}84x_6 + 25{,}47x_7 - 1391{,}65x_8$
Function 4	$f(x) = -795{,}11 - 137{,}97x_1 - 27{,}60x_2 - 24{,}16x_3 + 741{,}45x_4 + 9{,}99x_5 + 580{,}31x_6 + 152{,}42x_7 + 3645{,}83x_8$

The relationship between input and output variables of the given model is determined by fuzzy rules, which may be found below:

Rule 1: If (District is Eminönü) **Then** (Office_Rent is Function 1),
Rule 2: If (District is Beyoğlu) **Then** (Office_Rent is Function 2),
Rule 3: If (District is Beşiktaş) **Then** (Office_Rent is Function 3),
Rule 4: If (District is Şişli) **Then** (Office_Rent is Function 4).

6.6.2. TSK-Type FRBS for the Office Space and Office Building Data Regressed by the Age of the Building

In order to construct the TSK-type model for office space and office building data, we divided the data into 3 parts as in the Table 6.22.

Table 6.22: Distribution of the data by the “age of the building” variable.

	Frequency	Percent	Cumulative Percent
Young	94	24,54	24,54
Middle Aged	255	66,58	91,12
Old	34	8,88	100
Total	383	100	

For each interval we made a linear regression, with the given for input variables of the FIS model as independent variables and for the **"age of the building" variable** as the dependent one.
The outcome of the regression can be seen in Table 6.23.

Table 6.23: Regression coefficient for the "age of the building".

	Young	Middle aged	Old
(Constant)	-2211,93	-677,56	2055,82
Number of Floors in the Building	1,90	223,66	-149,86
Age of the Building	-92,83	-1,61	-4,41
Number of Offices in the Building	-24,09	-6,31	1,88
Number of Elevators	-295,47	5,04	-374,98
Parking Size	20,72	6,16	-13,84
Number of the Rooms in the Office	1073,72	184,58	104,16
Number of Workers in the Office	171,26	102,28	78,47
Ownership Ratio	1110,50	-733,33	-561,22

6.6.2.1. Production of the Rule Base

For the whole data set representing Istanbul CBD, we have defined 3 membership functions for the **"age of the building" variable**, so we are going to define 3 Fuzzy Inference System rules. Using the regression coefficient, we have to write five different functions for each interval of the number of workers.

Table 6.24: The output membership function equations for TSK-type FIS model regressed for Age of the Building[13]

Function 1	$f(x) = -2211{,}93 + 1{,}9x_1 - 92{,}83x_2 - 24{,}09x_3 - 295{,}47x_4 + 20{,}72x_5 + 1073{,}72x_6 + 171{,}26x_7 + 1110{,}5x_8$
Function 2	$f(x) = -677{,}56 + 223{,}66x_1 - 1{,}61x_2 - 6{,}31x_3 + 5{,}04x_4 + 6{,}16x_5 + 184{,}58x_6 + 102{,}28x_7 - 733{,}33x_8$
Function 3	$f(x) = 2055{,}82 - 149{,}86x_1 - 4{,}41x_2 + 1{,}88x_3 - 374{,}98x_4 - 13{,}84x_5 + 104{,}16x_6 + 78{,}47x_7 - 561{,}22x_8$

[13] Variable definition is the same as in the Subsection 6.6.1.1

The relationship between input and output variables of the given model is determined by fuzzy rules, which may be found below:

Rule 1: If (Age_of _Building is Young) **Then** (Office_Rent is Function 1),
Rule 2: If (Age_of _Building is Middle Aged) **Then** (Office_Rent is Function 2),
Rule 3: If (Age_of _Building is Old) **Then** (Office_Rent is Function 3).

6.6.3. TSK-Type FRBS for the Office Space and Office Building Data Regressed by the Number of Floors

In order to construct the TSK-type model for office space data we divided the data into 5, using the parts as shown in Table 6.25.

Table 6.25: Distribution of the data by the "number of floors" variable.

	Frequency	Percent	Cumulative Percent
1st Interval	150	39,16	39,16
2nd Interval	177	46,21	85,37
3rd Interval	23	6,01	91,38
4th Interval	33	8,62	100
Total	383	100	

For each interval we made a linear regression, with the given for input variables of the FIS model as independent variables and for the **"office space rent" variable** as the dependent one. The outcome of the regression can be seen in Table 6.26.

Table 6.26: Regression coefficient for the "Number of Floors".

Intervals	**1**	**2**	**3**	**4**
(Constant)	-245,02	72,92	-6597,07	27589,62
Number of Floors in the Building	100,20	117,19	658,98	0
Age of the Building	-1,56	1,73	-13,83	0
Number of Offices in the Building	-4,96	-10,10	0	-523,63
Number of Elevators	-138,68	164,67	1112,51	0
Parking Size	3,64	-0,28	-12,06	-12,12
Number of the Rooms in the Office	222,98	91,50	54,83	1446,86
Number of Workers in the Office	102,07	98,37	116,22	156,35
Ownership Ratio	403,11	-671,54	2832,16	-27265,68

6.6.3.1. Production of the Rule Base

For the whole data set representing Istanbul CBD, we have defined four membership functions for the **"number of floors" variable,** so we are going to define four Fuzzy Inference System rules. Using the regression coefficient, we have to write four different functions, Table 6.27, for each interval of the number of workers.

Table 6.27: The output membership function equations for TSK-type FIS model regressed for Number of Floors.[14]

Function 1	$f(x) = -245{,}02 + 100{,}2x_1 - 1{,}56x_2 - 4{,}96x_3 - 138{,}68x_4 + 3{,}64x_5 + 222.98x_6 + 102{,}07x_7 - 403{,}11x_8$
Function 2	$f(x) = 72{,}92 + 117{,}19x_1 + 1{,}73x_2 - 10{,}10x_3 + 164{,}67x_4 - 0{,}28x_5 + 91{,}5x_6 + 98{,}37x_7 - 671{,}54x_8$
Function 3	$f(x) = -6597{,}07 + 658{,}98x_1 - 13{,}83x_2 + 1112{,}51x_4 - 12{,}06x_5 + 54{,}83x_6 + 116{,}22x_7 + 2832{,}16x_8$
Function 4	$f(x) = 27589{,}62 - 523{,}63x_3 - 12{,}12x_5 + 1446{,}86x_6 + 156{,}35x_7 - 27265{,}65x_8$

The relationship between input and output variables of the given model is determined by fuzzy rules, which may be found below:

Rule 1: If (Number_of_Floors is Low) **Then** (Office_Rent is Function 1),
Rule 2: If (Number_of_Floors is Middle) **Then** (Office_Rent is Function 2),
Rule 3: If (Number_of_Floors is High) **Then** (Office_Rent is Function 3),
Rule 4: If (Number_of_Floors is Too high) **Then** (Office_Rent is Function 4).

6.6.4. TSK-Type FRBS for the Office Space and Office Building Data Regressed by the Number of Workers

In order to construct the TSK-type model for office space data we divided the data into 4 parts using the number of workers intervals, as is in Table 6.28.

[14] Variable definition is the same as in the Subsection 6.6.1.1

Table 6.28: Distribution of the data by the "number of workers" variable.

	Frequency	Percent	Cumulative percent
1st Interval	105	27,42	27,42
2nd Interval	98	25,59	53,01
3rd Interval	73	19,06	72,07
4th Interval	107	27,93	100
Total	383	100	

For each interval we made a linear regression, with the given four input variables of the FIS model as independent variables and for the **"office space rent" variable** as the dependent one.

The outcome of the regression can be seen in the Table 6.29.

Table 6.29: Regression coefficient for the Number of Workers

Intervals	**1**	**2**	**3**	**4**
(Constant)	-169,14	38,93	282,38	-277,37
Number of Floors in the Building	139,55	78,40	-17,54	234,67
Age of the Building	-3,04	-4,69	2,48	-19,28
Number of Offices in the Building	-1,38	-5,62	-15,27	-82,36
Number of Elevators	-199,92	-52,32	160,31	-1413,51
Parking Size	2,62	4,22	8,79	50,41
Number of the Rooms in the Office	196,33	256,43	85,19	606,49
Number of Workers in the Office	46,09	102,86	227,72	105,75
Ownership Ratio	-104,35	-493,25	-563,86	488,10

6.6.4.1. Production of the Rule Base

For the whole data set representing Istanbul CBD, we have defined four membership functions for the **"number of workers" variable**, so we are going to define 4 Fuzzy Inference System rules. Using the regression coefficient, we have to write four different equations for each interval of the number of workers.

Table 6.30: The output membership function equations for TSK-type FIS model regressed for Number of Workers.[15]

Function 1	$f(x) = -169{,}14 + 139{,}55x_1 - 3{,}04x_2 - 1{,}38x_3 - 199{,}92x_4 + 2{,}62x_5 + 196{,}33x_6 + 46{,}09x_7 - 104{,}35x_8$

[15] Variable definition is the same as in the Subsection 6.6.1.1

Function 2	$f(x) = 38{,}93 + 78{,}4x_1 - 4{,}69x_2 - 5{,}62x_3 - 52{,}32x_4 + 4{,}22x_5 + 256{,}43x_6 + 102{,}86x_7 - 493{,}25x_8$
Function 3	$f(x) = 281{,}38 - 17{,}54x_1 + 2{,}48x_2 - 15{,}27x_3 + 160{,}31x_4 + 8{,}79x_5 + 85{,}19x_6 + 227{,}72x_7 - 563{,}86x_8$
Function 4	$f(x) = -277{,}37 + 234{,}67x_1 - 19{,}28x_2 - 82{,}36x_3 - 1413{,}51x_4 + 50{,}41x_5 + 606{,}49x_6 + 105{,}75x_7 + 488{,}1x_8$

The relationship between input and output variables of the given model is determined by fuzzy rules, which may be found below:

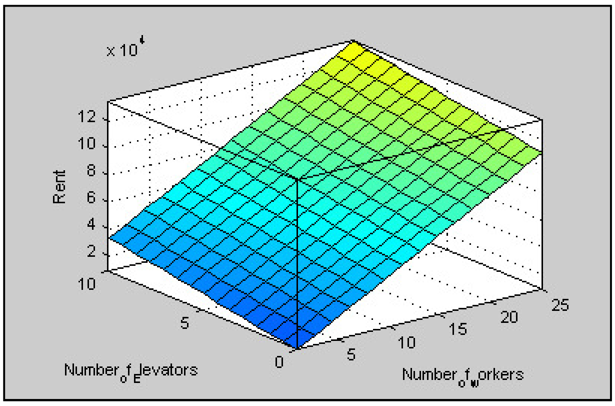
Rule 1: If (Number_of_workers is Low) **Then** (Office_Rent is Function 1),
Rule 2: If (Number_of_workers is Medium) **Then** (Office_Rent is Function 2),
Rule 3: If (Number_of_workers is High) **Then** (Office_Rent is Function 3),
Rule 4: If (Number_of_workers is Too High) **Then** (Office_Rent is Function 4),

6.7. Empirical Results of the TSK-type Fuzzy Rule-Based Model for Office Space and Office Building Data

In this section, we are going to discuss the results of the Takagi, Sugeno, Kang type fuzzy inference system implemented for the office space and office building data.

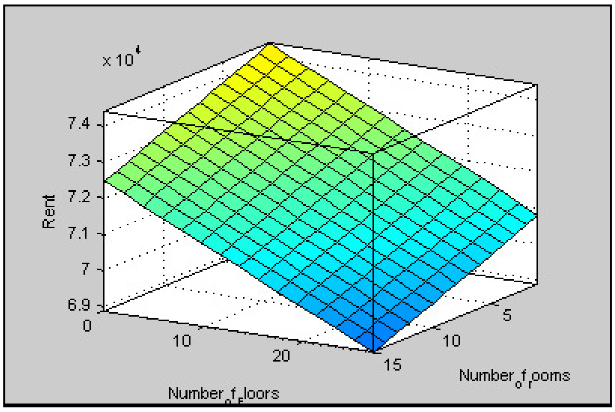
6.7.1. Empirical Results of the TSK-type Fuzzy Rule-Based Model for Office Space and Office Building Data Regressed by the Districts

Figure 6.39 displays the combined effect of "number of elevators" and "number of workers" input variables on office space rents. Clearly, office rents increase consistently as the number of workers increases. As the number of elevators increases, the office rent increases too. Over the defined bilateral surface, the highest office rents are observed for a large number of workers and a large number of elevators.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building	District
	NaN	8	13,5	32,5	NaN	75	50	80	3

Figure 6.39: Combined Effect of "Number of elevators" and "Number of Workers" on Office Space Rents

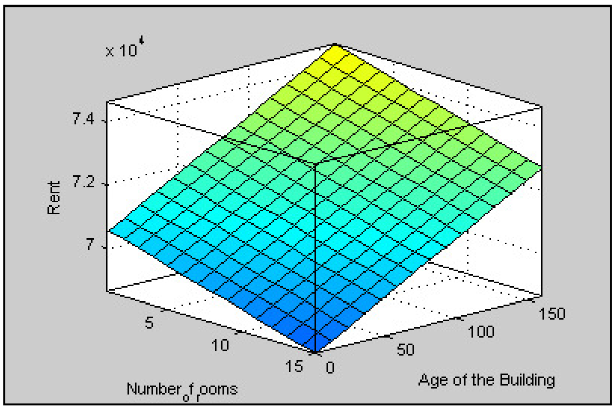


Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building	District
	13	NaN	NaN	32,5	5	75	50	80	3

Figure 6.40: Combined Effect of "Number of Floors" and "Number of Rooms" on Office Space Rents

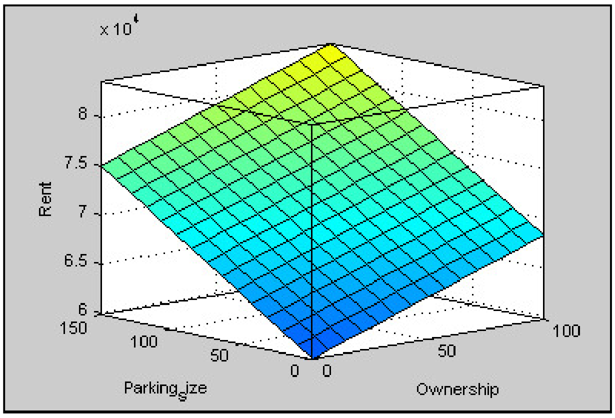
The joint effect of "number of floors" and "number of rooms" variables in office space rents are exhibited in Figure 6.40. The increases in the number of rooms lead to lower office rents. As the number of floors decreases, the office space rent increases, with the highest rent being observed for the small number of rooms and small number of floors.

Figure 6.41 provides information on combined effect of the "number of rooms" and "age of the building" on office rents. From the figure it is obvious that an increase in the number of rooms brings decrease in the office rent. It is also obvious that the office rent would be higher for the old buildings and lower for the new buildings. The highest office rent can be observed for the old building with the small number of rooms.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building	District
	13	NaN	13,5	32,5	5	75	50	NaN	3

Figure 6.41: Combined Effect of “Age of the Building” and “Number of Rooms” on Office Space Rents



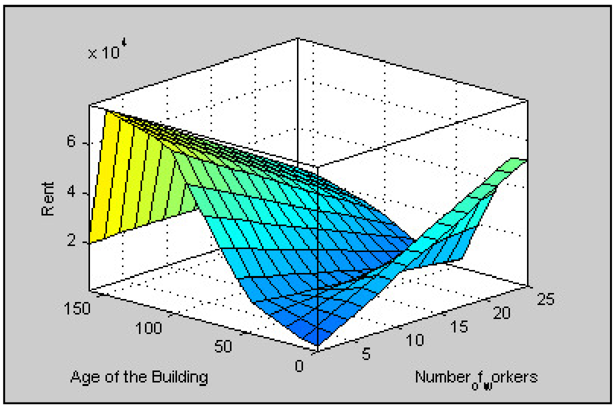
Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building	District
	13	8	13,5	32,5	5	NaN	NaN	80	3

Figure 6.42: Combined Effect of “Parking Size” and “Ownership” on Office Space Rents

Figure 6.42 provides information on the combined effect of the parking size and ownership ratio of the offices in the office space rent. The figure shows us that an increase in both the given input variables lead to an increase in the office rent. The highest rent is observed in the region of high ownership ratio and high parking size.

6.7.2. Empirical Results of the TSK-type Fuzzy Rule-Based Model for Office Space and Office Building Data Regressed by the Age of the Building

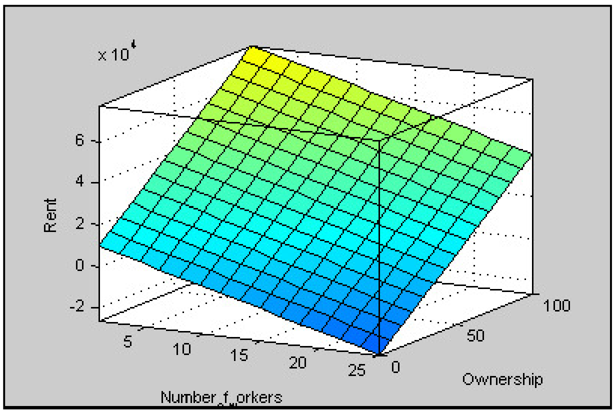
Figure 6.43 illustrates the combined effect of the "number of workers" and "age of the building" on the "office space rent" variable. As it may be seen from the figure for the building younger than 50 years the office rent increases with an increase in the number of workers. But as the building becomes older this increase in the office rent becomes less obvious, and as the 50 year boundary passes, the number of workers starts to negatively affect the office rent. For the small number of workers the age of the building positively affects the office rent. The highest rents are observed for the relatively small number of workers and for the buildings with 100 years of history.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	NaN	8	13,5	32,5	5	75	50	NaN

Figure 6.43: Combined Effect of "Age of the Building" and "Number of Workers" on Office Space Rents

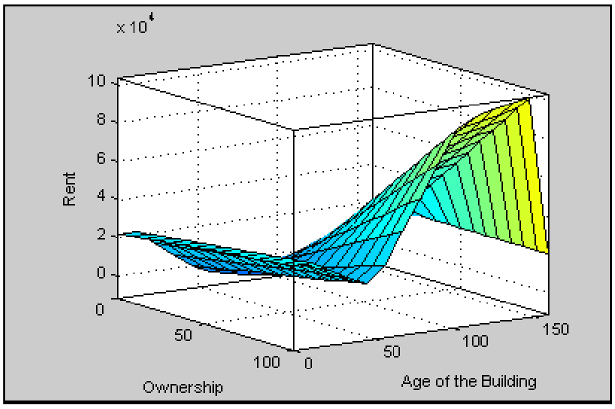
In Figure 6.44, the combined effect of the "ownership" and "number of workers" on office space rent is depicted. The figure shows that there is a negative relationship between the number of workers and office rent, it also shows that the highest office rents are in the buildings with the high ownership ratio. The highest rents can be observed for the high ownership ratio and relatively small number of workers.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	NaN	8	13,5	32,5	5	75	NaN	80

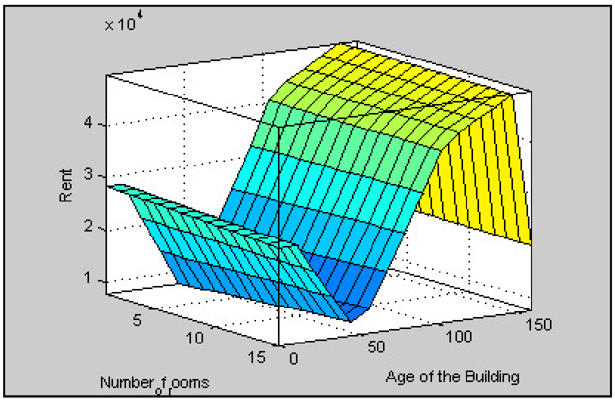
Figure 6.44: Combined Effect of "Ownership" and "Number of Workers" on Office Space Rents

The combined effect of the "age of the building" and "ownership" on the office rent can be seen in Figure 6.45. For the relatively young buildings (less that 50 years old) the ownership ratio has slightly little effect on the office space rent. For the older buildings this effect becomes quite visible. We may divide the effect of the age of building into three parts, from 0 to 50, from 50 to 130 and from 130 to 150. In the interval of 0 to 50 the age of the building has slightly negative effect on the office space rents.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	13	8	13,5	32,5	5	75	NaN	NaN

Figure 6.45: Combined Effect of "Age of the Building" and "Ownership" on Office Space Rents



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	13	NaN	13,5	32,5	5	75	50	NaN

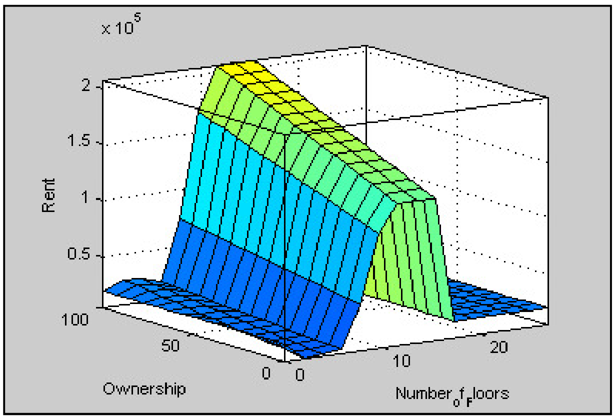
Figure 6.46: Combined Effect of "Age of the Building" and "Number of Rooms" on Office Space Rents

The joint effect of "age of the building" and "number of rooms" variables on the office space rent can be seen in Figure 6.46. As it is seen in figure, the number of rooms has little or no effect on the office rent. The behavior of the age of the building is nearly the same as we described it for the above figures.

6.7.3. Empirical Results of the TSK-type Fuzzy Rule-Based Model for Office Space and Office Building Data Regressed by the Number of Floors

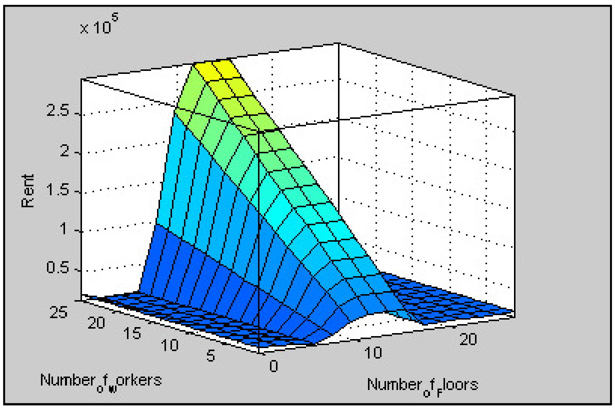
Figure 6.47 describe the combined effect of the "ownership ratio" and "number of floors" on the office space floor. From the figure we can derive that for the buildings with small number of floors the ownership ratio has slightly positive effect on the office space rent. As the number of floors increases, the effect of the ownership ratio increases too, having its maximum effect for the building with the number of floors between 13 and 15.

The question of how the "number of workers" and "number of floors" jointly affect office space rent is answered in Figure 6.48. For the low and high buildings the number of workers has no effect in the office space rent. For the buildings with a moderate number of the floors, the number of workers has a positive effect on the office rent. The highest rents are observed for the building with moderate number of floors and large number of workers.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	13	8	NaN	32,5	5	75	NaN	80

Figure 6.47: Combined Effect of "Ownership" and "Number of Floor" on Office Space Rents



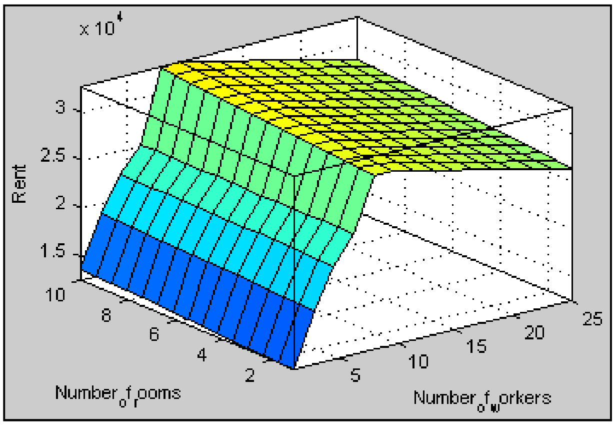
Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	NaN	8	NaN	32,5	5	75	50	80

Figure 6.48: Combined Effect of "Number of Workers" and "Number of Floor" on Office Space Rents

6.7.4. Empirical Results of the TSK-type Fuzzy Rule-Based Model for Office Space and Office Building Data Regressed by the Number of Workers

Figure 6.49 depicts the relationship between the Number of rooms, Number of Workers and Rent. As we see from the picture for the given number of workers as the number of rooms increase the rent increases too but slower, meaning that number of rooms has slight effect on the office space rent. For the given number of rooms as the number of workers increase the office space rent increases till the number of workers is equal to 9, afterwards the office rent starts to decrease.

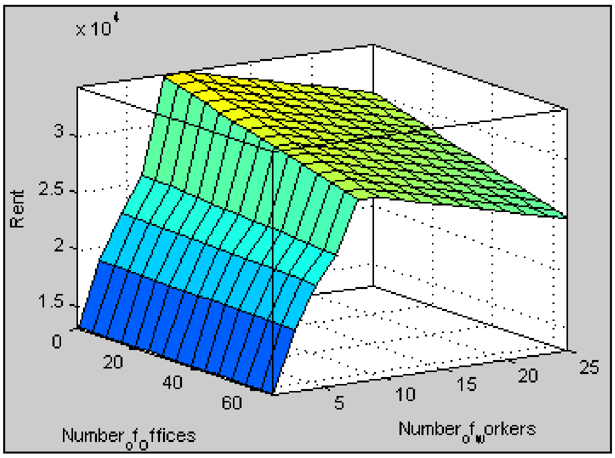
As we may see from Figure 6.50, for the given value of the input variable Number of Workers the Rent increases as the Number of Offices increases but it increases slightly for the small value of the Number of Workers and for the higher values of the Number of Workers the increase is seemingly fast. For the given value of the Number of Offices, the Rent increase initially as Number of Workers increases, but after some value of the Number of Workers the rent starts to decrease. We can also see that the achieved with the low Number of Offices and moderate Number of Workers.



Reference input	**Number of Workers**	**Number of Rooms**	**Number of Floors**	**Number of Offices**	**Number of Elevators**	**Parking Size**	**Ownership ratio**	**Age of Building**
	NaN	NaN	13,5	32,5	5	75	50	80

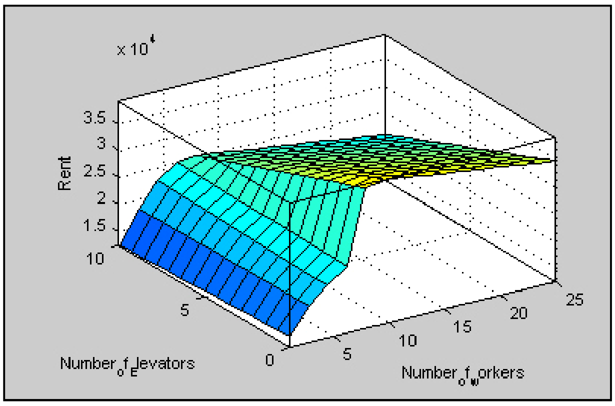
Figure 6.49: Combined Effect of “Number of Workers” and “Number of Rooms” on Office Space Rents

Figure 6.51 provides information on the combined effect of “number of elevators” and “number of workers” on office rents. The Number of Elevators has negative effect on the office space rent, but for the large number of workers this effect is stronger than for the small number of workers. As the number of workers increases, the office rent increases initially and then starts to decline.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	NaN	5,5	13,5	NaN	5	75	50	80

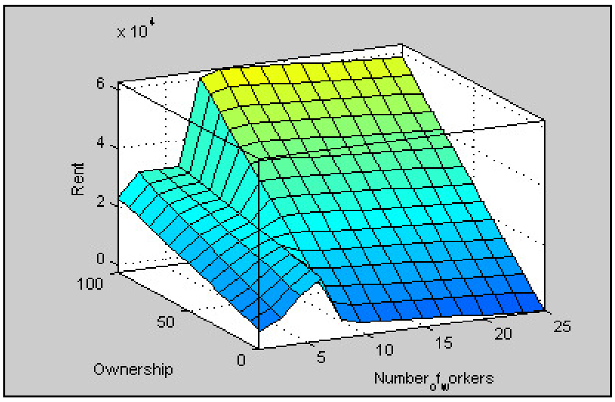
Figure 6.50: Combined Effect of “Number of Workers” and “Number of Offices” on Office Space Rents



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	NaN	8	13,5	32,5	NaN	75	50	80

Figure 6.51: Combined Effect of “Number of Workers” and “Number of Elevators” on Office Space Rents

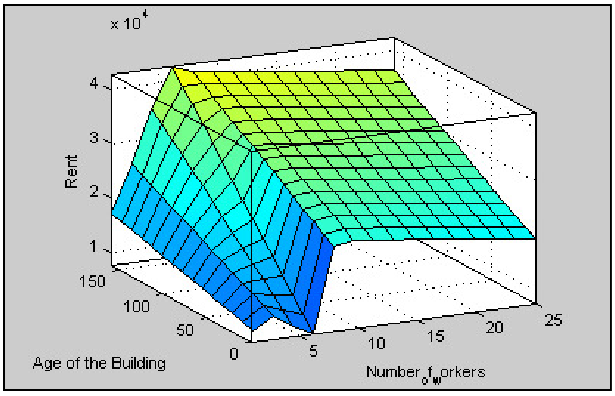
Figure 6.52 displays the combined effect of the “number of workers” and “ownership ratio” on office rent. Ownership ratio has a positive effect on the office rents. For the small number of workers ownership ratio has a slight positive effect on the office space rent. When the number of workers increase above 6, the office rents starts to decline but the effect of ownership ratio becomes more visible.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	NaN	8	13,5	32,5	5	75	NaN	80

Figure 6.52: Combined Effect of "Number of Workers" and "Ownership" on Office Space Rents

The question of the combined effect of the "age of the building' and "number of workers" on the office rents are answered in Figure 6.53. We may clearly see that old buildings require more office rents than younger ones. The highest office rents are observed for the old buildings with the moderate number of employees.



Reference input	Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of Building
	NaN	8	13,5	32,5	5	75	50	NaN

Figure 6.53: Combined Effect of "Number of Workers" and "Age of the Building" on Office Space Rents

From the Tables 31a and 31b we can derive that the highest Rent is in the Şişli district, then comes Eminönü afterwards Beyoğlu and Beşiktaş. Overall we may say that the change in one variable affects different districts differently. For example as the Number of Workers in the

Office increase we may see an increase in the Rent for Beşiktaş and Eminönü, and decrease in the Beyoğlu and Şişli. An increase in the Number of Rooms of the Office will lead to an increase Rent in the Beyoğlu, and decrease in other 3 districts. Furthermore, an increase in the Number of Floors and Number of Offices of the Building leads to the decrease in the Rent for all 4 districts.

From Table 31b it is obvious that as the Number of Elevators in the Building, Ownership Ratio and Age of the Building increase we may see an increase in the Rent for all 4 districts . An increase in the Parking Size will lead to an increase Rent in the Beyoğlu and Beşiktaş, and decrease Şişli and Eminönü districts.

From Table 32a it can be observed that the increase in Number of Workers leads to an increase in Rent. As the Number of Rooms increases the Rent increases too. An increase in the Number of Floors in the Building decreases the Rent , but the decrease is not so great, Rent differences between the 1 and 12 floor building is approximately 50 units. As the Number of Offices in the building increases the Rent decrease again.

From Table 32b it can be observed that the increase in Number of Elevators leads to a decrease in Rent. As the Parking Size increases, the Rent increases too. An increase in the Ownership Ratio increases the Rent. As the Age of the Building increases, the Rent increases again. The difference between 10 and 100 years old building is somewhat *9* folds.

Table 31a: Outcome of the District TSK FIS

Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of the Building	Rent by District			
								Beşiktaş (7)	Beyoğlu (9)	Eminönü (10)	Şişli (23)
1	2	3	10	2	10	0,1	60	5996,71	5413,55	6093,25	23973,83
2	2	3	10	2	10	0,1	60	6314,62	3832,32	10322,54	23178,71
3	2	3	10	2	10	0,1	60	6632,53	2251,1	14551,82	22383,6
4	2	3	10	2	10	0,1	60	6950,44	669,87	18781,11	21588,49
3	1	3	10	2	10	0,1	60	6720,02	1345,18	14688,63	22521,58
3	3	3	10	2	10	0,1	60	6545,05	2557,01	14415,01	22245,63
3	5	3	10	2	10	0,1	60	6370,08	3468,85	14141,4	21969,68
3	6	3	10	2	10	0,1	60	6282,59	3374,76	14004,59	21831,7
3	7	3	10	2	10	0,1	60	6195,11	4280,68	13867,78	21693,73
3	13	3	10	2	10	0,1	60	5570,2	6716,18	13046,93	20865,88
3	2	1	10	2	10	0,1	60	6633,01	2255,19	14817,79	22438,79
3	2	3	10	2	10	0,1	60	6632,53	2251,1	14551,82	22383,6
3	2	5	10	2	10	0,1	60	6632,05	2247,01	14285,85	22328,41
3	2	6	10	2	10	0,1	60	6631,81	2244,96	14152,87	22300,81
3	2	10	10	2	10	0,1	60	6630,85	2236,77	13620,93	22190,43
3	2	12	10	2	10	0,1	60	6630,37	2232,68	13354,96	22135,23
3	2	3	3	2	10	0,1	60	6651,35	2415,21	15133,18	22552,7
3	2	3	5	2	10	0,1	60	6645,97	2368,32	14967,08	22504,39
3	2	3	10	2	10	0,1	60	6632,53	2251,1	14551,82	22383,6
3	2	3	15	2	10	0,1	60	6619,09	2133,88	14136,57	22262,81
3	2	3	17	2	10	0,1	60	6613,72	2086,99	13970,47	22214,5
3	2	3	20	2	10	0,1	60	6605,65	2016,66	13721,32	22142,03

Table 31b: Outcome of the District TSK FIS

Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of the Building	Rent by District			
								Beşiktaş (7)	Beyoğlu (9)	Eminönü (10)	Şişli (23)
3	2	3	10	3	10	0,1	60	6680,91	2845,42	17045,67	23125,05
3	2	3	10	5	10	0,1	60	6777,67	4034,05	22033,38	24607,95
3	2	3	10	7	10	0,1	60	6874,43	5222,68	27021,08	26090,85
3	2	3	10	10	10	0,1	60	7019,56	7005,63	34502,64	28315,2
3	2	3	10	15	10	0,1	60	7261,46	9977,22	46971,89	32022,44
3	2	3	10	2	5	0,1	60	8237,04	2321,13	14042,97	22333,66
3	2	3	10	2	10	0,1	60	6632,53	2251,1	14551,82	22383,6
3	2	3	10	2	15	0,1	60	5028,02	2181,06	15060,68	22433,54
3	2	3	10	2	20	0,1	60	3423,52	2111,03	15569,53	22483,48
3	2	3	10	2	25	0,1	60	1819,01	2040,99	16078,38	22533,42
3	2	3	10	2	30	0,1	60	214,5	1970,96	16587,24	22583,35
3	2	3	10	2	10	0	60	6608,95	2249,49	14543,24	22325,57
3	2	3	10	2	10	0,1	60	6632,53	2251,1	14551,82	22383,6
3	2	3	10	2	10	0,3	60	6679,69	2254,31	14568,99	22499,66
3	2	3	10	2	10	0,5	60	6726,86	2257,52	14586,16	22615,72
3	2	3	10	2	10	0,7	60	6774,02	2260,73	14603,33	22731,79
3	2	3	10	2	10	1	60	6844,77	2265,54	14629,08	22905,88
3	2	3	10	2	10	0,1	40	3603,87	605,47	14042,49	19335,23
3	2	3	10	2	10	0,1	50	5118,2	1158,29	14297,16	20859,41
3	2	3	10	2	10	0,1	60	6632,53	2251,1	14551,82	22383,6
3	2	3	10	2	10	0,1	70	8146,86	3343,91	14806,49	23907,79
3	2	3	10	2	10	0,1	90	11175,52	5529,54	15315,83	26956,16
3	2	3	10	2	10	0,1	100	12689,85	6622,35	15570,49	28480,35

Table 32a: Outcome of the Workers TSK FIS

Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of the Building	Rent
3	1	3	10	1	230	0,1	10	1605,26
5	1	3	10	1	230	0,1	10	3997,03
7	1	3	10	1	230	0,1	10	7458,86
10	1	3	10	1	230	0,1	10	8366,84
15	1	3	10	1	230	0,1	10	6979,98
20	1	3	10	1	230	0,1	10	5593,11
3	1	3	10	1	230	0,1	10	1605,26
3	3	3	10	1	230	0,1	10	1762,06
3	5	3	10	1	230	0,1	10	1918,87
3	6	3	10	1	230	0,1	10	1997,27
3	7	3	10	1	230	0,1	10	2075,67
3	13	3	10	1	230	0,1	10	2546,08
3	5	1	10	1	230	0,1	10	1928,24
3	5	3	10	1	230	0,1	10	1918,87
3	5	5	10	1	230	0,1	10	1909,49
3	5	6	10	1	230	0,1	10	1904,81
3	5	10	10	1	230	0,1	10	1886,06
3	5	12	10	1	230	0,1	10	1876,69
3	5	3	3	1	230	0,1	10	1958,2
3	5	3	5	1	230	0,1	10	1946,96
3	5	3	10	1	230	0,1	10	1918,87
3	5	3	15	1	230	0,1	10	1890,77
3	5	3	17	1	230	0,1	10	1879,53
3	5	3	20	1	230	0,1	10	1862,67

Table 32b: Outcome of the Workers TSK FIS

Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of the Building	Rent
3	5	3	3	1	230	0,1	10	1958,2
3	5	3	3	3	230	0,1	10	1853,55
3	5	3	3	5	230	0,1	10	1748,9
3	5	3	3	7	230	0,1	10	1644,25
3	5	3	3	10	230	0,1	10	1487,28
3	5	3	3	15	230	0,1	10	1225,66
3	5	3	3	1	50	0,1	10	1197,82
3	5	3	3	1	75	0,1	10	1303,43
3	5	3	3	1	100	0,1	10	1409,04
3	5	3	3	1	150	0,1	10	1620,26
3	5	3	3	1	200	0,1	10	1831,47
3	5	3	3	1	250	0,1	10	2042,69
3	5	3	3	1	230	0	10	1932,56
3	5	3	3	1	230	0,1	10	1958,2
3	5	3	3	1	230	0,3	10	2009,49
3	5	3	3	1	230	0,5	10	2060,78
3	5	3	3	1	230	0,7	10	2112,06
3	5	3	3	1	230	1	10	2188,99
3	5	3	3	1	230	0,1	10	1958,2
3	5	3	3	1	230	0,1	30	4015,5
3	5	3	3	1	230	0,1	50	6072,79
3	5	3	3	1	230	0,1	70	8130,08
3	5	3	3	1	230	0,1	90	10187,37
3	5	3	3	1	230	0,1	100	11216,02

As it can be seen from Table 33a, an increase in the Number of Workers in the Office brings an increase in the Rent. Rent increased for a great amount when Number of Workers increased from 5 to 10. Table also shows us that an increase in the Number of Rooms of the Office results in decrease in the Rent. We may say that the Number of Floors in the Building has negative and small influence on Rent. Number of Offices in the Building has nearly the same effect on Rent, except an increase in the Number of Offices leads to an increase in the Rent.

Table 33b shows what effects have the Parking Size, Ownership Ratio and Age of the Building on Rent. It can be recognized that as Parking Size increases the Rent decreases. And it also can be seen that an increase in Ownership ratio and Age of the Building leads to an increase in the Rent. Former one has a little effect on the Rent, an increase from zero to 1 changed Rent for nearly 5%. An increase of 20 years in the latter increases the Rent for more that 50%.

It can be understood from Table 34a, an increase in the Number of Workers in the Office bring a decrease in the Rent. Table also shows us that an increase in the Number of Rooms of the Office results in increase in the Rent. We may say that the Number of Floors in the Building has a positive effect on Rent. Number of Offices in the Building has a negative and small effect on Rent. An increase of 17 offices resulted in the decrease of the Rent of less that 4.5%.

Table 34b shows the effects of Number of Elevators, Parking Size, Ownership Ratio and Age of the Building on Rent. An increase in the Number of Elevators leads to a decrease in the Rent. It can be learned that as Parking Size increases the Rent increases. Finally, it also can be understood that an increase in Ownership ratio and Age of the Building leads to an increase in the Rent.

Table 33a: Outcome of the Age of the Building TSK FIS

Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of the Building	Rent
1	5	3	3	1	230	0,1	10	2025,24
3	5	3	3	1	230	0,1	10	2086,39
5	5	3	3	1	230	0,1	10	6198,03
10	5	3	3	1	230	0,1	10	16477,12
15	5	3	3	1	230	0,1	10	26756,22
20	5	3	3	1	230	0,1	10	37035,31
3	1	3	3	1	230	0,1	10	2685,85
3	3	3	3	1	230	0,1	10	2386,12
3	5	3	3	1	230	0,1	10	2086,39
3	6	3	3	1	230	0,1	10	1936,53
3	7	3	3	1	230	0,1	10	1786,67
3	13	3	3	1	230	0,1	10	887,49
3	5	1	3	1	230	0,1	10	2095,2
3	5	3	3	1	230	0,1	10	2086,39
3	5	5	3	1	230	0,1	10	2077,58
3	5	6	3	1	230	0,1	10	2073,18
3	5	10	3	1	230	0,1	10	2055,56
3	5	12	3	1	230	0,1	10	2046,74
3	5	3	3	1	230	0,1	10	2086,39
3	5	3	5	1	230	0,1	10	2090,16
3	5	3	10	1	230	0,1	10	2099,58
3	5	3	15	1	230	0,1	10	2108,99
3	5	3	17	1	230	0,1	10	2112,76
3	5	3	20	1	230	0,1	10	2118,41

Table 33b: Outcome of the Age of the Building TSK FIS

Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of the Building	Rent
3	5	3	3	1	50	0,1	10	4577,51
3	5	3	3	1	75	0,1	10	4231,52
3	5	3	3	1	100	0,1	10	3885,53
3	5	3	3	1	150	0,1	10	3193,56
3	5	3	3	1	200	0,1	10	2501,58
3	5	3	3	1	250	0,1	10	1809,6
3	5	3	3	1	230	0	10	2075,98
3	5	3	3	1	230	0,1	10	2086,39
3	5	3	3	1	230	0,3	10	2107,23
3	5	3	3	1	230	0,5	10	2128,06
3	5	3	3	1	230	0,7	10	2148,89
3	5	3	3	1	230	1	10	2180,14
3	5	3	3	1	230	0,1	10	2086,39
3	5	3	3	1	230	0,1	30	3246,13
3	5	3	3	1	230	0,1	50	4882,17
3	5	3	3	1	230	0,1	70	8299,23
3	5	3	3	1	230	0,1	90	12719,56
3	5	3	3	1	230	0,1	100	15305,96

Table 34a: Outcome of the Floors TSK FIS

Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of the Building	Rent
1	5	3	3	1	230	0,1	10	2380
3	5	3	3	1	230	0,1	10	1889,97
5	5	3	3	1	230	0,1	10	1399,94
7	5	3	3	1	230	0,1	10	909,91
8	5	3	3	1	230	0,1	10	664,89
9	5	3	3	1	230	0,1	10	419,88
3	1	3	3	1	230	0,1	10	1489,17
3	3	3	3	1	230	0,1	10	1689,57
3	5	3	3	1	230	0,1	10	1889,97
3	6	3	3	1	230	0,1	10	1990,17
3	7	3	3	1	230	0,1	10	2090,37
3	13	3	3	1	230	0,1	10	2691,57
3	5	1	3	1	230	0,1	10	1893,1
3	5	3	3	1	230	0,1	10	1889,97
3	5	5	3	1	230	0,1	10	1545,31
3	5	6	3	1	230	0,1	10	1375,46
3	5	10	3	1	230	0,1	10	16696,96
3	5	12	3	1	230	0,1	10	19263,89
3	5	3	3	1	230	0,1	10	1889,97
3	5	3	5	1	230	0,1	10	1880,04
3	5	3	10	1	230	0,1	10	1855,22
3	5	3	15	1	230	0,1	10	1830,4
3	5	3	17	1	230	0,1	10	1820,47
3	5	3	20	1	230	0,1	10	1805,58

Table 34b: Outcome of the Floors TSK FIS

Number of Workers	Number of Rooms	Number of Floors	Number of Offices	Number of Elevators	Parking Size	Ownership ratio	Age of the Building	Rent
3	5	3	3	1	230	0,1	10	1889,97
3	5	3	3	3	230	0,1	10	1612,6
3	5	3	3	5	230	0,1	10	1335,24
3	5	3	3	7	230	0,1	10	1057,87
3	5	3	3	8	230	0,1	10	919,19
3	5	3	3	9	230	0,1	10	780,5
3	5	3	3	1	50	0,1	10	1235,54
3	5	3	3	1	75	0,1	10	1326,43
3	5	3	3	1	100	0,1	10	1417,32
3	5	3	3	1	150	0,1	10	1599,11
3	5	3	3	1	200	0,1	10	1780,9
3	5	3	3	1	250	0,1	10	1962,68
3	5	3	3	1	230	0	10	1867,67
3	5	3	3	1	230	0,1	10	1889,97
3	5	3	3	1	230	0,3	10	1934,57
3	5	3	3	1	230	0,5	10	1979,16
3	5	3	3	1	230	0,7	10	2023,76
3	5	3	3	1	230	1	10	2090,65
3	5	3	3	1	230	0,1	10	1889,97
3	5	3	3	1	230	0,1	30	3931,29
3	5	3	3	1	230	0,1	50	5972,6
3	5	3	3	1	230	0,1	70	8013,92
3	5	3	3	1	230	0,1	90	10055,23
3	5	3	3	1	230	0,1	100	11075,89

Table 6.35: The effects of the increases in an input variables on Rent.

	Rent						
	District TSK FIS				Workers TSK FIS	Age of the Buildings TSK FIS	Floors TSK FIS
	Beşiktaş	Beyoğlu	Eminönü	Şişli			
Number of Workers	+	-	+	-	+	+	-
Number of Rooms	-	+	-	-	+	-	+
Number of Floors	-	-	-	-	-	-	+
Number of Offices	-	-	-	-	-	+	-
Number of Elevators	+	+	+	+	-	-	-
Parking Size	-	-	+	+	+	-	+
Ownership Ratio	+	+	+	+	+	+	+
Age of the Building	+	+	+	+	+	+	+

From Table 6.35 we may conclude that different input variables behave differently under the different fuzzy inference system, except “ownership ratio” and “age of the building” which have a positive effect on the office space rent. For the District TSK FIS models theinput variables “number of floors in the building” and “number of officesin the building” have a negative effect on the office rent. Beside the “ownership ratio” and “age of the building”, the input variable “number of elevators in the building” has also a positive effect on the office rent for the District TSK FIS.

CHAPTER 7

CONCLUSION

As was said in the Chapter 1 of this thesis, we use Fuzzy Logic to capture the rental value moves of the office spaces as one commercial real estate type by using physical and locational rent determinants of the properties.

In the thesis, we carried out the following four basic analyses. The first analysis conducted was Mamdani-type fuzzy rule-based system (FRBS). It provides an analysis of spatial variation in the average office rents across the CBD sub-district and have been performed for the Office Space data set by using Mamdani-type FRBS. We have constructed 5 different Mamdani-type FRBS. The main question asked was “How office rents for a typical office space with exactly the same attributes, vary among CBD sub-districts? ”.

The results of the FRBS reveal that for all representative office spaces, while Beyoğlu district, as traditional part of the CBD, has the highest rent level, Eminönü district has the lowest rent levels. As already said, these districts are two different parts of the traditional CBD. Within the new CBD axis, while Beşiktaş district has the highest rent levels for the small-sized offices, Şişli district, including the new CBD axis of Zincirlikuyu-Levent-Maslak, has the highest rents for the high-rise office buildings with sizeable offices. The results are given in Table 7.1.

Table 7.1: The results of the Mamdani-type model for the Office Space data.

CBD	Traditional CBD		New CBD	
Level	Highest	Lowest	Highest	Lowest
District	Beyoğlu	Eminönü	Beşiktaş	Şişli

In the second analysis conducted is the Takagi, Sugeno, Kang (TSK) type FRBS for the same data set. We have constructed 4 different TSK-type FRBS. In order to insert the district into the FRBS, we created a new variable named “District”, but since the output, namely, office space rent, of the fuzzy inference system was negative for some values of inputs, namely, office floor, number of workers, office usage area and tenants’ residence outside of the office district, we marked these FISs as inapplicable.

The TSK-type FIS created using "number of workers" have showed good results. Since in the created fuzzy rule based system no district variables have been specified, the spatial analysis cannot be conducted. The overall results show that the number of workers does not have any effect on the office space rent, when it is greater than 10.

The next two analyses have been done using the office space and office building data. The outcomes of the backward stepwise regression have revealed that 12 variables out of the 69 characteristics are significant. The number of variables has been decreased to 8, after conducting some modification on the given 12 variables. Here, 2 out of the given 8 variables are office space, and the rest are office building variables. So we may understand that office building variables are dominant among the whole data set.

In the process of creating the Mamdani-type fuzzy inference system we made one more change to the given 8 variable, since it was impossible to write the necessary number of rules for the fuzzy rule-based system with 8 input variables. We regressed the given 8 variables on the output variable, namely office space rent, and picked 5 most significant ones. From the given 5 variables 2 have been selected from the office space data, and the remaining 3 ones from the office building data. Unlike the first subsection of the Chapter 6 we created only one Mamdani-type FIS, because of the small number of data points for the given FIS model.

The results of the Mamdani-type FRBS show that "number of workers" has no effect on the office rent, when it is higher than 5. We may also derive that "parking size" has a clear positive effect on the office space rent. It also shows that the input variable "number of elevators" has a slightly positive effect and that the "number of rooms in the office" has a negative effect, meaning that the more rooms the office has, the less office space rent will be paid. The input variable "number of offices in the building" has no clear effect on the office rent, but it has a positive effect until some value, afterwards it starts to negatively affect the office space rent.

In the last analysis, we used the office space and office building data to create the TSK-type FRBS. We have created 4 different TSK-type FRBSs by using 8 input variables we talked above. The spatial analysis performed on the office space and office building data show that, in the new CBD, Şişli has the highest office rent, and Beşiktaş the lowest. Within the traditional CBD, the highest office rent is observed in the Eminönü district, and with the lowest office rent in Beyoğlu. The results can be observed in Table 7.2.

Table 7.2: The results of the TSK-type model for the Office Space and Office Building data sets.

CBD	Traditional CBD		New CBD	
Level	Highest	Lowest	Highest	Lowest
District	Eminönü	Beyoğlu	Şişli	Beşiktaş

As the future work in this area we may suggest to combine the given two data sets, office space and office building, with the data sets acquired from the Turkish Statistics Committee. And we may also suggest to use the new regression methods like *Multivariate Adaptive Regression Splines (MARS)* or *CMARS* and *RCMARS*.

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APPENDIX A

SURVEY FORM FOR THE OFFICE BUILDING DATA

[X-1] Anket No:
[X-2] Liste No:

MİA (MERKEZİ İŞ ALANI) BİNA ANKETİ

İyi günler. Ben Veri Araştırma Şirketi'nin anketörüyüm. Orta Doğu Teknik Üniversitesi, İktisat Bölümü adına bir anket yapıyoruz. Çalışma alanlarının kullanım bedellerini belirleyen faktörleri analiz etmeye yönelik bu araştırmada "bina genel özellikleri", "çalışılan ofisin genel özellikleri", "kira kontratı değişkenleri", "kiracıların mahalle ve yakın çevre değişkenleri ile ilgili bilgisi" konularını içeren bilimsel bir araştırma yapmaktayız.Lütfen sorularımı cevaplar mısınız?

İlçe	Bölge ID	Bina ID	Kat	Birim No	ANKET ID
[X-3]	[X-4]	[X-5]	[X-6]	[X-7]	[X-8]

Bina Adresi
Bina No:........ Cadde/Sokak: ....

Semt/İlçe:...

Bina Yöneticisinin (Eğer bina yöneticisinden bilgi alınmışsa doldurulacak)
Çalıştığı Firma: ....

Adı soyadı: .... Ünvanı: ....

Telefonu: .... / ....

Görüşme tarihi : ...... / ...... / 2009 Saati: ...... / ....

1. BÖLÜM – BİNA GENEL ÖZELLİKLERİ

S1. Ofisin içinde bulunduğu bina: [X-9]
1( ) Plaza 2( ) İş Hanı/İş merkezi

S2. Bina kat adedi: .... [X-10]

S3. Bina yapım yılı: .... [X-11]

S4. Binanın taban alanı: .... m2 [X-12]

S5. Binada ofis sayısı: ....…….. [X-13]

S6. Bina içindeki asansör sayısı: ……..…….[X-14]

S7. Bina içindeki ortak alan: …………... m2 [X-15]

S8. Bina içinde alışveriş birimleri (dükkan/mağaza) var mı? [X-16]
1( ) Yok
2( ) Var ----------------→ **Bina içindeki konumu? [X-17]**
1( ) 1 Kat Pasaj
2( ) 2 Kat Pasaj
(...) Kat Pasaj
98( ) Yoldan cepheli

Kontrol: **[X-18] 6969**

S9. Binadaki kulanılmayan (boş) ofis sayısı: .... [X-19]

S10. Binadaki boş ofis alanı: .... m2 [X-20]

S11. Binada otopark var mı? [X-21]
1( ) Var 2( ) Yok

S12. Binadaki otopark kaç araçlık?............. [X-22]

S13. Binadaki ofis ve mağaza/dükkanlardaki kiracı ve mal sahibi sayısı:

	Kiracı sayısı	Mal sahibi sayısı	Toplam
Ofis	**[X-23]**	**[X-24]**	**[X-25]**
Mağaza/Dükkan	**[X-26]**	**[X-27]**	**[X-28]**

S14. Binada yangın çıkışı var mı? [X-29]
1( ) Evet 2( ) Hayır

S15. Binadaki havalandırma sistemi [X-30]
1( ) Merkezi 3( ) Karma
2( ) Bağımsız 4( ) Doğal(Teknik bir sistemin bulunmaması)

S16. Binadaki güvenlik sistemi [X-31]
1( ) Merkezi güvenlik 2( ) Ofise ait güvenlik 3( ) Güvenlik yok

S17a. Binadaki ısıtma sistemi [X-32]
1( ) Merkezi ısıtma 2( ) Bireysel ısıtma

S17b. Binadaki soğutma sistemi [X-33]
1( ) Merkezi soğutma 2( ) Bireysel soğutma

S18. Binada jeneratör var mı? [X-34]
1( ) Evet 2( ) Hayır

S19. Binanın yapı malzemesi [X-35]
1( ) Betonarme 4( ) Çelik yapı
2( ) Taş 5( ) Karma (Betonarme+Çelik, vb)
3( ) Kagir 6( ) Yığma

S20. Binanın fiziksel durumu [X-36] (ANKETÖR DİKKAT 5 VE 6. SEÇENEKLER İÇİN BİNAYA AİT TESCİL BELGESİ SORULMADLIDIR)
1( ) Yeni ve iyi durumda 4( ) Eski ve bakımsız durumda
2( ) Yeni ama orta kalite 5( ) Tarihi ve restore edilmiş
3( ) Eski ve iyi durumda 6( ) Tarihi ve bakımsız durumda

S21. Binanın mimari tasarım kalitesi yüksek mi? [X-37]
1( ) Evet 2( ) Hayır

S22. Bina inşaat kalitesi [X-38]
1( ) Lüks inşaat 2( ) 1. sınıf inşaat 3( ) 2. sınıf inşaat

S23. Bina içi fonksiyonlar [X-39] (Birden fazla şık işaretlenebilir)
1( ) Ofis 5( ) Atriyum/Avlu
2( ) Mağaza 6( ) Fitness- spor merkezi
3( ) Banka 7( ) Restaurant
4( ) Konferans odası ( ) Diğer ....

Kontrol: **[X-40] 6969**

S24. Bina prestijli bir lokasyonda mı? [X-41]

1( ) Evet --------------→ Açıklayın**[X-42]** (deniz manzarası vs..) ……………………

2( ) Hayır

S25. Binanın internet sistemi var mı? [X-43]

1( ) Yok

2( ) Var…………..> **[X-44]**Bağlanma şeklini belirtiniz(……………………………)

S26. Binanın aylık işletim harcamaları

(elektrik, su, gaz, güvenlik, vb.): ....………….........(TL) **[X-45]**

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Ad / Soyad **[X-46]** ___________________________

Bu görüşmeyi tanımadığım bir kişi ile Veri Araştırma tarafından verilen eğitime ve ESOMAR kurallarına göre yaptığımı taahhüt eder ve Saha Yönetmeni tarafından görüşmenin kısmen veya bütün olarak kontrol edileceğini kabul ederim.

İmza

APPENDIX B

SURVEY FORM FOR THE OFFICE SPACE DATA

[X-1] Anket No:
[X-2] Liste No:

MİA (MERKEZİ İŞ ALANI) KİRACI ANKETİ

İyi günler. Ben Veri Araştırma Şirketi'nin anketörüyüm. Orta Doğu Teknik Üniversitesi, İktisat Bölümü adına bir anket yapıyoruz. Çalışma alanlarının kullanım bedellerini belirleyen faktörleri analiz etmeye yönelik bu araştırmada "bina genel özellikleri", "çalışılan ofisin genel özellikleri", "kira kontratı değişkenleri", "kiracıların mahalle ve yakın çevre değişkenleri ile ilgili bilgisi" konularını içeren bilimsel bir araştırma yapmaktayız.Lütfen sorularımı cevaplar mısınız?

İlçe	Bölge ID	Bina ID	Kat	Birim No	ANKET ID
[X-3]	[X-4]	[X-5]	[X-6]	[X-7]	[X-8]

Kiracı Kişinin
Çalıştığı Firma: ....

Adı soyadı: ....   Ünvanı: ....

Telefonu: .... / ....

Görüşme tarihi : ...... / ...... / 2009   Saati: ...... / ....

Bina Adresi
Bina No:........   Cadde/Sokak: ....

Semt/İlçe:..

2. BÖLÜM –OFİS GENEL ÖZELLİKLERİ

(ANKETÖR DİKKAT OFİS KİRALIK İSE SORULACAK)

S27. Ofis fonksiyonu [X-9]

1( ) Banka   5( ) Mühendislik/Mimarlık   9( ) Eğitim
2( ) Sigorta şirketi   6( ) Mali müşavirlik   10( )Firma yönetim merkezi (head-quarter)
3( ) Gayrimenkul   7( ) Doktor
4( ) Finans   8( ) Avukat   ( ) Diğer...............……………………......

S28. Ofisin bulunduğu kat: .... [X-10]

S29. Ofis kullanım alanı: ....(m2) [X-11]

S30. Ofis, binanın prestijli/manzara avantajlı bir lokasyonunda mı bulunuyor? [X-12]
1( ) Evet 2( ) Hayır

S31. Ofisteki oda sayısı: .... [X-13]

S32. Ofiste mutfak var mı? [X-14]
1( ) Evet 2( ) Hayır

S33. Ofiste tuvalet/banyo var mı? [X-15]
1( ) Evet 2( ) Hayır

S34. Ofisin ısıtma sistemi [X-16]

1( ) Merkezi ısıtma   3( ) Doğal gaz sobası
2( ) Klima   4( ) Kat kaloriferi (kombi)
( ) Diğer..

S35. Ofisteki çalışan sayısı:................ [X-17]

Kontrol: **[X-18] 6969**

S36. Ofisin aylık işletim harcamaları

(elektrik, su, gaz, güvenlik, vb.): ....……………........(TL) **[X-19]**

3. BÖLÜM – KİRA KONTRATI İLE İLGİLİ SORULAR

S37. Kira kontratı başlangıç tarihi: ……. /…..... / .... [X-20]

S38. Kira miktarı: ....(aylık TL) **[X-21]**

S39. Kontrat süresi : ......…..... Yıllık ……....... Aylık **[X-22]**

S40. Kira miktarı artışı [X-23]
1( ) TÜFE/ÜFE'nin belirli bir yüzdesi
2( ) Kontratta belirtilen yüzde oranında
( )Diğer..................................

S41. Kiracı depozito ödüyor mu? [X-24]
1( ) Evet 2( ) Hayır

4. BÖLÜM – KİRACILARIN MAHALLE VE YAKIN ÇEVRE DEĞİŞKENLERİ İLE İLGİLİ BİLGİSİ

S42. Ofisin bulunduğu ilçenin dışında ikamet eden çalışan var mı? [X-25]
1( ) Yok
2( ) Var-------------------------→Belirtiniz **(ANKETÖR DİKKAT SORU 33 DE VERİLEN BİLGİYİ GÖZ ÖNÜNDE BULUNDURUN)**

Çalışanların İkâmet yeri	Toplam Çalışan Oranı	
1( ) İlçe içinde		[X-26]
2( ) İlçe dışında		[X-27]
TOPLAM	%100	

S43. Çalışanlarımızın % kaçı anadolu yakasından gelmektedir? %...................[X-28]

S44. Ev-ofis arası kullanılan ulaşım aracı [X-29] (Birden fazla şık işaretlenebilir)
1( ) Metro/tramvay
2( ) Servis
3( ) Deniz ulaşımı (vapur, vb)
4( ) Banliyö-Tren
5( ) Otobüs
6( ) Dolmuş
7( ) Minibüs
8( ) Özel araç

S45. Ofisinizin bulunduğu mahallede yeni inşaa edilen alışveriş merkezi var mı? [X-30]
1( ) Evet 2( ) Hayır

S46. Ofisinizin kirasını belirleyen ana faktörler sizce aşağıdakilerden hangisi veya hangileridir? [X-31]
(Birden fazla şık işaretlenebilir)
1( ) Ofisin prestijli semtlere yakın olması
2( ) Ulaşım kolaylığı
3( ) Bir alışveriş merkezine yakın olması
4( ) Etrafında diğer ofis alanlarının bulunması
5( ) Otopark olanaklarının bulunması
6( ) MİA'ya yakın olması
7( ) Deniz manzarasının olması
8( ) Binanın yeni olması
9( ) Yeme içme ve eğlence yerlere yakın olması
( ) Diğer...

SAHA GÖREVLİSİ

Ad / Soyad **[X-32]** ___________________________

Bu görüşmeyi tanımadığım bir kişi ile Veri Araştırma tarafından verilen eğitime ve ESOMAR kurallarına göre yaptığımı taahhüt eder ve Saha Yönetmeni tarafından görüşmenin kısmen veya bütün olarak kontrol edileceğini kabul ederim.

İmza

APPENDIX C

DESCRIPTIVE STATISTICS FOR THE OFFICE SPACE AND OFFICE BUILDING DATA

C.1. Descriptive Statistics for the Office Building Data

Table C.1: Frequency distribution of sample data due to executive floor.

Executive Floor	Frequency	Percent	Cumulative Percent
Ground and Sub-ground floors	52	29,10	29,10
1st and 2nd floors	56	31,30	60,30
Between 3rd and 5th floors	38	21,20	81,60
Between 6th and 10th floors	16	8,90	90,50
11th and upper floors	3	1,70	92,20
Outdoor Executive	14	7,80	100,00
Total	179	100,00	

Table C.2: Frequency distribution of sample data due to building basement area.

Building Basement Area	Frequency	Percent	Cumulative Percent
Less that 100	16	8,90	8,90
Between 100 and 200	52	29,10	38,00
Between 201 and 500	61	34,10	72,10
Between 501 and 1000	26	14,50	86,60
More than 1001	14	7,80	94,40
No answer	10	5,60	100,00
Total	179	100,00	

Table C.3: Frequency distribution of sample data due to number of offices in the building.

Number of Offices in the Building	Frequency	Percent	Cumulative Percent
Less than 5	31	17,30	17,30
Between 6 and 10	71	39,70	57,00
Between 11 and 20	30	16,80	73,70
Between 21 and 40	29	16,20	89,90
More than 41	18	10,10	100,00
Total	179	100,00	

Table C.4: Frequency distribution of sample data due to number of elevators in the building.

Number of Elevators in the Building	Frequency	Percent	Cumulative Percent
Buildings with no elevator	8	4,50	4,50
Buildings with only 1 elevator	128	71,50	76,00
Buildings with 2 elevators	22	12,30	88,30
Buildings with more than 3 elevators	11	6,10	94,40
No answer	10	5,60	100,00
Total	179	100,00	

Table C.5: Frequency distribution of sample data due to common area in the building.

Common Area in the Building (m^2)	Frequency	Percent	Cumulative Percent
Less than 25	51	28,5	28,5
Between 26 and 50	37	20,7	49,2
Between 51 and 250	37	20,7	69,8
More that 251	34	19	88,8
No answer	20	11,2	100
Total	179	100	

Table C.6: Dummy variable related to the availability of the shopping center in the building.

Are there any Shopping Centers in the Building?	Frequency	Percent	Cumulative Percent
Yes (1)	56	31,30	31,30
No (2)	123	68,70	100,00
Total	179	100,00	

Table C.7: Dummy variable related to the availability of the parking lot in the building.

Is there any Parking Lot in the Building?	Frequency	Percent	Cumulative Percent
Yes (1)	38	21,20	21,20
No (2)	141	78,80	100,00
Total	179	100,00	

Table C.8: Dummy variable related to the availability of the fire exit in the building.

Is there any Fire Exit in the Building?	Frequency	Percent	Cumulative Percent
Yes	70	39,10	39,10
No	108	60,30	99,40
No answer	1	0,60	100,00
Total	179	100,00	

Table C.9: Dummy variable related to the availability of the air conditioning system in the building.

Air Conditioning System in the Building	Frequency	Percent	Cumulative Percent
Central	39	21,80	21,80
Independent	38	21,20	43,00
Natural (no technical system)	101	56,40	99,40
No answer	1	0,60	100,00
Total	179	100,00	

Table C.10: Frequency distribution of sample data due to security systems in the building.

Security System in the Building	Frequency	Percent	Cumulative Percent
Central Security System	67	37,40	37,40
Office Specific Security	48	26,80	64,20
No Security	63	35,20	99,40
No answer	1	0,60	100,00
Total	179	100,00	

Table C.11: Frequency distribution of sample data due to cooling systems in the building.

Cooling System in the Building	Frequency	Percent	Cumulative Percent
Central Cooling	13	7,30	7,30
Individual Cooling	159	88,80	96,10
No Cooling	5	2,80	98,90
No answer	2	1,10	100,00
Total	179	100,00	

Table C.12: Dummy variable related to the availability of the power sources system in the building.

Availability of Power Sources in the Building	Frequency	Percent	Cumulative Percent
Yes	40	22,30	22,30
No	138	77,10	99,40
No answer	1	0,60	100,00
Total	179	100,00	

Table C.13: Frequency distribution of sample data due to construction material in the building.

Construction Material of the Building	Frequency	Percent	Cumulative Percent
Iron concrete	160	89,40	89,40
Masonry	4	2,20	91,60
Steel construction	1	0,60	92,20
Mixed (Iron concrete + Masonry, and so on)	3	1,70	93,90

Upset	11	6,10	100,00
Total	179	100,00	

Table C.14: Frequency distribution of sample data due to building quality.

Building Quality	Frequency	Percent	Cumulative Percent
New and in good condition	38	21,20	21,20
New and medium quality	14	7,80	29,10
Old and in good condition	114	63,70	92,70
Old and neglected	11	6,10	98,90
Historical and restored	1	0,60	99,40
Historical and neglected	1	0,60	100,00
Total	179	100,00	

Table C.15: Frequency distribution of sample data due to design quality of the building.

Is the Design Quality High for the Building?	Frequency	Percent	Cumulative Percent
Yes	107	59,80	59,80
No	69	38,50	98,30
Medium	2	1,10	99,40
No answer	1	0,60	100,00
Total	179	100,00	

Table C.16: Frequency distribution of sample data due to building construction quality.

Building Construction Quality	Frequency	Percent	Cumulative Percent
Luxury construction	19	10,60	10,60
1st class construction	112	62,60	73,20
2nd class construction	46	25,70	98,90
No answer	2	1,10	100,00
Total	179	100,00	

Table C.17: Dummy variable related to the availability of the prestigious location of the building.

Is the Building in the Prestigious Location?	Frequency	Percent	Cumulative Percent
Yes	144	80,40	80,40
No	35	19,60	100,00
Total	179	100,00	

Table C.18: Dummy variable related to the availability of the internet connection in the building.

Is there any Internet Connection in the Building?	Frequency	Percent	Cumulative Percent
Yes	122	68,20	68,20

No	57	31,80	100,00
Total	179	100,00	

Table C.19: Frequency distribution of sample data due to monthly operating costs of the building.

Monthly Operating Costs of the Building	Frequency	Percent	Cumulative Percent
Less than 500 TL	44	24,60	24,60
Between 501 and 1000TL	29	16,20	40,80
Between 1001 and 2000TL	37	20,70	61,50
Between 2001 and 5000TL	34	19,00	80,40
More than 5001TL	30	16,80	97,20
No answer	5	2,80	100,00
Total	179	100,00	

C.2. Descriptive Statistics for the Office Space Data

Table C.20: Frequency distribution of sample data due to office floor.

Office Floor	Frequency	Percent	Cumulative Percent
Ground and sub-ground	26	5,10	5,10
1st and 2nd floor Offices	181	35,80	40,90
3rd and 4th floor Offices	175	34,60	75,50
5th and 6th floor Offices	78	15,40	90,90
7th and higher floor Offices	46	9,10	100,00
Total	506	100,00	

Table C.21: Frequency distribution of sample data due to office usage area.

Office Usage Area	Frequency	Percent	Cumulative Percent
Less than 50 m2	158	31,20	31,20
Between 51 and 100 m2	136	26,90	58,10
Between 101 and 200 m2	136	26,90	85,00
Between 201 and 400 m2	44	8,70	93,70
Between 401 and 1000 m2	21	4,20	97,80
More than 1001 m2	10	2,00	99,80
No answer	1	0,20	100,00
Total	506	100,00	

Table C.22: Frequency distribution of sample data due to number of rooms in the office.

Number of Rooms in the Office	Frequency	Percent	Cumulative Percent
Offices with 1 or 2 rooms	215	42,5	42,5
Offices with 3 or 4 rooms	183	36,2	78,7

Offices with 5 or 6 rooms	57	11,3	89,9
Offices with 7 or 8 rooms	23	4,5	94,5
Offices with 11 or more rooms	13	2,6	97
No answer	15	3	100
Total	506	100	

Table C.23: Dummy variable related to the availability of the kitchen in the office.

Is there any Kitchen in the Office?	Frequency	Percent	Cumulative Percent
Yes	301	59,50	59,50
No	202	39,90	99,40
No answer	3	0,60	100,00
Total	506	100,00	

Table C.24: Dummy variable related to the availability of the toilets and/or bathroom in the office.

Are there any Toilets and/or Bathroom in the Office?	Frequency	Percent	Cumulative Percent
Yes	356	70,40	70,40
No	148	29,20	99,60
No answer	2	0,40	100,00
Total	506	100,00	

Table C.25: Frequency distribution of sample data due to the type of the heating system in office.

Type of the Heating System in the Office	Frequency	Percent	Cumulative Percent
Central Heating	280	55,30	55,30
Air Conditioning	50	9,90	65,20
Natural Gas Heater	3	0,60	65,80
Furnace	123	24,30	90,10
Heater	8	1,60	91,70
Electric Heater	40	7,90	99,60
Catalytic	1	0,20	99,80
No answer	1	0,20	100,00
Total	506	100,00	

Table C.26: Frequency distribution of sample data due to the type of number of workers in office.

Number of Workers in the Office	Frequency	Percent	Cumulative Percent
Offices with 1 or 5 workers	312	61,70	61,70
Offices with 6 or 10 workers	103	20,40	82,00
Offices with 11 or 25 workers	64	12,60	94,70
Offices with 26 or 100 workers	24	4,70	99,40

Offices with 101 or more workers	3	0,60	100,00
Total	506	100,00	

Table C.27: Frequency distribution of sample data due to the inception date of rent agreement.

Inception Date of Rent Agreement	Frequency	Percent	Cumulative Percent
1990 and before	23	4,50	4,50
1991 - 2000	42	8,30	12,80
2000	23	4,50	17,40
2001 - 2007	239	47,20	64,60
2008	104	20,60	85,20
2009	35	6,90	92,10
No answer	40	7,90	100,00
Total	506	100,00	

Table C.28: Frequency distribution of sample data due to amount of rent for the office.

Amount of Rent	Frequency	Percent	Cumulative Percent
Less than 500TL	117	23,10	23,10
Between 501 and 1000TL	115	22,70	45,80
Between 1001 and 2000TL	139	27,50	73,30
Between 2001 and 5000TL	75	14,80	88,10
Between 5001 and 10000TL	17	3,40	91,50
More than 10001TL	20	4,00	95,50
No answer	23	4,50	100,00
Total	506	100,00	

Table C.29: Frequency distribution of sample data due to duration of the rent.

Duration of Rent Agreement	Frequency	Percent	Cumulative Percent
12 months	441	87,20	87,20
24 months	9	1,80	88,90
36 months	9	1,80	90,70
48 months	1	0,20	90,90
60 months	15	3,00	93,90
120 months	4	0,80	94,70
No answer	27	5,30	100,00
Total	506	100,00	

Table C.30: Frequency distribution of sample data due to type of rent accrual.

Type of Rent Accrual	Frequency	Percent	Cumulative Percent
Specific percentage of TUFE / UFE	284	56,1	56,1

Specific rate at the agreement	52	10,3	66,4
Based on the decision of the proprietor	10	2	68,4
Based on mutual agreement	88	17,4	85,8
Based on foreign exchange index	10	2	87,7
Based on market structure	5	1	88,7
Based on inflation rate	10	2	90,7
No increase	6	1,2	91,9
Other	7	1,4	93,3
No answer	34	6,7	100
Total	506	100	

Table C.31: Dummy variable related to the payment of the deposit.

Does the Renter pay Deposit?	Frequency	Percent	Cumulative Percent
Yes	296	58,50	58,50
No	171	33,80	92,30
No answer	39	7,70	100,00
Total	506	100,00	

Table C.32: Dummy variable related to the residents outside of the office district.

Are there any Residents outside of the Office district?	Frequency	Percent	Cumulative Percent
Yes	102	20,20	20,20
No	404	79,80	100,00
Total	506	100,00	

Table C.33: Frequency distribution of sample data due to residence inside the office district.

Residence inside the Office District	Frequency	Percent	Cumulative Percent
Residents that totally live inside the district	240	47,40	47,40
Between %1 and 25	57	11,30	58,70
Between %25 and 50	86	17,00	75,70
Between %51 and 75	15	3,00	78,70
Between %76 and 99	4	0,80	79,40
Residents that totally live outside the district	102	20,20	99,60
No answer	2	0,40	100,00
Total	506	100,00	

Table C.34: Frequency distribution of sample data due to residence outside the office district.

Residence outside the Office District	Frequency	Percent	Cumulative Percent
Residents that totally live outside the district	102	20,20	20,20
Between %1 and 25	7	1,40	21,50

Between %25 and 50	48	9,50	31,00
Between %51 and 75	52	10,30	41,30
Between %76 and 99	55	10,90	52,20
Residents that totally live inside the district	240	47,40	99,60
No answer	2	0,40	100,00
Total	506	100,00	

Table C.35: Frequency distribution of sample data due to transportation alternatives between home and office.

Transportation Alternatives between Home and Office	Responses - N	Percent of Cases
Metro and/or Tram	272	53,80%
Private Buses	40	7,90%
Maritime Transport	163	32,20%
Suburban railway and/or Train	7	1,40%
Bus	373	73,70%
Shared Taxi	94	18,60%
Minibuses	126	24,90%
Automobile	227	44,90%
By Walking	17	3,40%
Metrobus	14	2,80%
Other	3	0,60%
No answer	2	0,40%
Total	1338	264,40%

Table C.36: Dummy variable related to the availability of the recently built shopping centers near the office.

Are there any recently built Shopping Centers close to the Office?	Frequency	Percent	Cumulative Percent
Yes	156	30,80	30,80
No	344	68,00	98,80
No answer	6	1,20	100,00
Total	506	100,00	

APPENDIX D

EXTENDED REAL ESTATE VALUATION METHODS

D.1 Traditional Valuation Methods

D.1.1 Sale Comparison Method

The focal point of the sales comparison method is the assumption that any prudent individual is not willing to pay more for a property if there exists another purchase opportunity in the marketplace for any other property that had the same attributes but at lower level of price. The concept behind this assumption is the economic substitution. The approach indicates that the prudent individual searches the market to gather information on the property prices and compares the outcomes to choose the best suitable one for his needs at the lowest existing cost.

To initiate this analysis, data on the recent sales of the properties that have the similar attributes with the one targeted to be purchased by the buyer, namely comparables, is collected. There are diversified sources of the data such as real estate publications, real estate brokers and/or agents, appraisers, and so on. The necessary adjustment is performed to erase the differences among the comparables and the subject property to better examine the lowest possible price for the alike characteristics contained. These differences may stem from the date of sale, location, style, amenities, square footage, site size, etc..

After this correction, the price of the subject property is estimated from the each comparable entities that have identical specifications. In case of the superiority of adjustment on the comparable on the subject property, a downward adjustment is needed to be applied; whereas if the comparable is inferior to the property concerned for analysis, an upward adjustment is required.

In summary, the following steps should be implemented at the Sale Comparison method:

1. Data on sales, listings, pending sales of the comparables that are similar to that of subject property should be gathered.
2. The accuracy of the obtained data should be verified by searching the marketplace.

3. The comparison units that are crucial to conduct that analysis at price attainment process should be decided. Following this process, relevant units of comparison should be identified to conduct a comparative analysis for each unit.
4. Analysis should be followed on the predetermined unit of comparisons of the subject and comparable properties as a result the proper adjustments should be applied for these real estates.
5. Unify all of the value assignments procured from each of the adjustments due to unit of comparisons into a single value indicator.

D.1.2 Investment/Income Capitalization Method

In this methodology, the property is much considered as an income generating instrument that how much potential it possesses in this process directly affects its value. Therefore, the investment/income capitalization method requires more specific knowledge than sales comparison method does that determines capital value directly. In case of the existence of similarities at a high degree in the marketplace in which the properties are compared, the comparison method may become more complex because of increasing adjustment quantities but not more than that of investment/income capitalization method.

The investment and commercial properties that are concern of the investment/income capitalization method analyzes, models and reflects the expectations and behaviors of the typical market participants. Provided that sufficient data on the market prevail, this method is the most applicable one for the income generating properties.

The two methods of investment/income capitalization approach that pursue different techniques to income capitalization to obtain an estimate of value are described below.

D.1.2.1 Direct Capitalization

Direct capitalization uses an appropriate capitalization rate, income rate or a factor to convert a single year's income generated from the property into an indication of value. The expectation generally gives the income of the first year of the holding period.

The rate used in the direct capitalization is obtained by the sales of the comparable properties that are similar to the subject property in such attributes as age, expense and net/gross income ratios, and the financing terms and conditions. When capitalization rate is derived from the net operating income, the rate is called as *overall capitalization rate*. To obtain an estimate value of the subject property the following is applied:

$$\frac{Net\ Operating\ Income}{Sales\ Price} = Overall\ Rate.$$

$$\frac{Net\ Operating\ Income\ of\ Subject\ Property}{Overall\ Capitalization\ Rate} = Value\ Estimate$$

The prominent advantage of this approach is that the capitalization rates and income multipliers is the direct result of market indications and represents the relationship between yearly income and the value of the property. Since single year's estimate of income is used to calculate both of the capitalization rates and income multipliers then, as it is posited definitely, expected changes in income will be same.

D.1.2.2 Yield Capitalization

In this approach, also known as discounted cash-flow analysis, primary objective is to convert all of the future benefits of the property for each year of a typical investment holding period into projected present value. These benefits can be any cash flow that is expected to realize during that holding period of time and discounted by using an appropriate discount rate. The discount rate applied is determined based on the rate of return that a typical investor requires from another investment in comparables instead of investment in the subject real estate.

Explicit determination of the future cash flows and income is the advantage of this model. Although the comparables have the same attributes and similar risk, any change in income and property that can be explicitly observed does not have to be the same for all the properties being analyzed and appraised. However, since the yield rates of the comparable risk should be the same by assumption, an appropriate yield rate can be chosen by the appraiser without directly observing on the comparable sales, presuming that it reflects the market's perceptions on the risk of the property rather than investing in other instruments.

The imperfections at the marketplace lead not identical value estimates of the subject property. Both techniques are applicable when data are available but yield capitalization would be much more reliable than direct capitalization when the information is little since appropriate yield rate can be selected by the researcher according to similar risk investments.

D.1.3 Residual Method

Real estates or land itself may be subject to any type of development projects that results in change in the value. Therefore it necessitates separating the cost components induced during the project implementation and value components added after the completion of that performed process. The value can be estimated just for the land that is subject to improvements, pre-project land value with the components or each of the components that belong to that land. As for the real estates, the components of the value like building and land /site vale should be determined.

Residual method is applied to estimate any change in the value of the property due to the development, expansion, modernization or other improvement projects that are to be implemented on the subject property. By this approach, it is possible to attain the maximum value that an investor is willing to pay at the current time envisaging all the benefits and costs that emerge from the investment project which is to be implemented. As it is mentioned, to capture the amount of change occurred after these projects, it is vital to identify the individual components

After the costs such as construction, design and monitoring, insurance, payoffs to be allocated to the responsible project attendants, etc. generated from the implementation of the development project are deducted from the value being contributed after termination, the remaining part gives the estimated future value of the property. Discounting this estimated future value with an appropriate discount rate, the appraiser obtains the value of the property in present time.

D.1.4 Cost Method

Cost method, namely replacement cost method, is carried out when the above mentioned methods cannot be implemented since the subject property does not generate any capital gain, or there is no such similar properties to be consider them as comparables at the marketplace, or the real estate cannot have any business operations. In this approach, the replacement cost of the real estate to make it continue to pursue its functions is the focal point to find out and therefore the unfeasibility of the comparable sales approach, investment and profits method is handled in the value estimation process. The market value will be the reconstruction costs of the real estate.

Following the calculated costs for reproducing or replacement of the improvements, then depreciation in these improvements are deducted from the cost of the newly constructed real

estate. Present value of physical parts of the property and all of the affixed improvements, amenities and land value are used to estimate the property's value.
In the process of the value determination, estimation of the current cost to reproduce the improvements, an estimate of the improvements' depreciation, and an estimate of the land value are all combined to access the most accurate estimation.

D.1.5 Multiple Regression Method

When the researchers investigate the influence of the several variables on the real estate value, multiple regression method is one of the techniques that are proper to be employed to conduct a research on these explanatory variables' cumulative and individual effect on the targeted dependent variable. This regression is especially efficacious when the only effect of the independent variables on the single dependent one is intended to be revealed. Moreover, statistical significance level, degree of confidence, of the relationship of explanatory variables and the dependent variable can also be assessed.

Multiple regression can be formulized as:

$$Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \cdots + \beta_k X_{k,i} + \varepsilon_i$$

where $Y_i, X_{1,i}, X_{2,i}, \dots, X_{k,i}$ are dependent and independent variables and indicate the i-th observations of each of the variables $Y, X_1, X_2, \dots, X_k$, respectively. The parameters in the regression $\beta_0, \beta_1, \beta_2, \dots, \beta_k$, are fixed, and error term ε_i, is a random variable which has $N(0, \sigma_\varepsilon^2)$ distribution.

The independent variables, $X_1, X_2, \dots, X_k$ can be considered as fixed numbers that there does not exist any error term, ε_i, in the multiple regression. If $X_1, X_2, \dots, X_k$ are given as random variables then there is no correlation between $X_1, X_2, \dots, X_k$ and ε_i terms. The i-th observations of the $X_1, X_2, \dots, X_k$ can take different values from each other for $i = 1,2, \dots, k$. Each of the ε_i terms are uncorrelated with the one another as well and $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$.

D.1.6 Stepwise Regression Method

In the case of large number of explanatory variables assumed to have effect on the dependent variable, it becomes complicating and costly to employ all possible regression models established with these variables. Therefore, stepwise regression is a valuable approach to assort the most relevant explanatory variables among them and eliminate the remaining from the system.

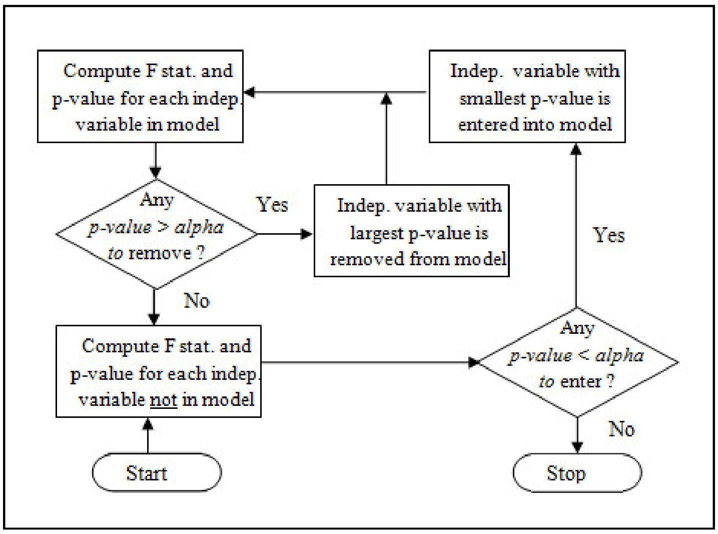

Figure D.1: Algorithm of the Stepwise Regression Method.

There are two main techniques to eliminate the less relevant variables applied by the stepwise regression:

1. *Forward selection procedure*: The system is initiated with no independent variable and each of these variables are analyzed based on $p - values$. Depending on whether these values are above or below the determined threshold $alpha - values$, the independent variable is inserted into the system as one at a time. Following this step, the independent variable with the largest $p - value$ is removed from the model. This step-by-step procedure is applied till a significant reduction in the error sum of squares (SSE) is achieved.

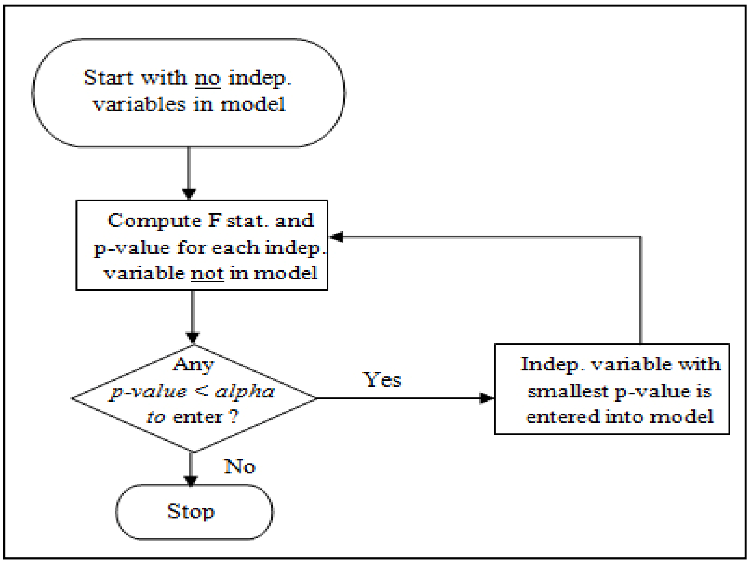

Figure D.2: Algorithm of the Forward Selection process.

2. *Backward elimination procedure*: In contrast to the forward selection procedure, this technique starts the analysis by inserting all of the independent variables into the system. If the $p-$value of each individual independent variable is less than the predetermined user-specified or default value, then one variable at each time is eliminated from the system. The removed independent variable cannot be inserted into the model at the succeeding processes.

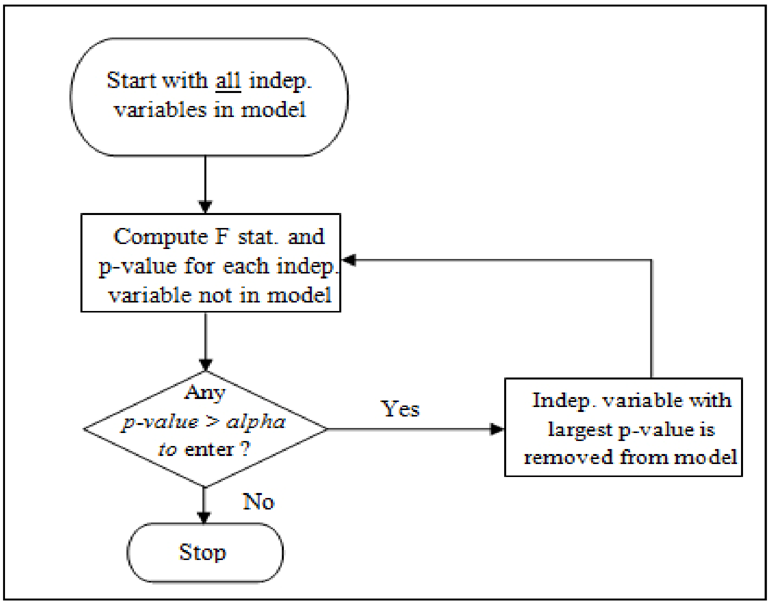

Figure D.3: Algorithm of the Backward Elimination Process.

The problems that can be encountered at the stepwise regression method are:

1. R-squared values when badly biased independent variables are in the system may result in at high level.
2. The claimed distributions by the F and chi-squared tests for each variable are not obtained.
3. The confidence intervals for the effects and the predicted values are too much narrow than it has to be.
4. $p-values$ given as a result of the procedure may have improper meaning and in turn correction effect for the each independent variable.
5. The resulting coefficients of the independent variables that are not subject to the stepwise procedure may be too large, that leads to be biased.
6. The methods that it uses such as $F-\ tests$ allow testing a prespecified hypothesis.
7. Higher magnitude of sample size does not improve the problems.

D.2 Advanced Valuation Methods

D.2.1 Hedonic Pricing Models

By the hedonic model, it is possible to price the differentiated goods like housing units which have a package of inherit individual characteristics. These latent attributes are represented by a vector of characteristics, each having an implicit price function that is predicted by the hedonic pricing equation. These attributes are categorized in real estate applications as structural, locational, neighborhood and environmental characteristics.
The hedonic equation is given as:

$$R = S\alpha + L\beta + N\gamma + E\tau + \varepsilon$$

where R is $(n \times 1)$ housing prices vector and structural, locational, neighborhood and environmental characteristics are represented by $S_{n \times k}, L_{n \times 1}, N_{n \times m}$ and , $E_{n \times 1}$ matrices respectively. Furthermore, α, β, γ and τ are the related parameters of these characteristics and ε is $(n \times 1)$ vector of random error term.
In the marketplace, a budget constraint for each attribute exists so that the optimal conditions for individual one has to be satisfied to maximize the overall utility of the market participant (Jim and Chen, 2009). The marginal implicit value is obtained by the partial derivative with respect to any of its arguments.

Some of the fundamental assumptions of hedonic pricing approach are:

1. Although it is arguable, house product is regarded as homogeneous. However, if differentiated attributes depend on some criteria such as locational, structural or neighborhood characteristics of the houses were considered in the model, it would be more accurate to apply the model for price estimation.
2. There are numerous buyers and sellers so that perfect competition is assumed to be valid in the marketplace. Therefore, an individual purchase does not significantly affect the price of the properties and it can be assumed as negligible. In addition, entry and exit the market is absolutely free for the buyers and the developers. No artificial constraints exist for demand and supply of the housing. Restrictions on house producing are not imposed.

The only fundamental requirement of the hedonic price approach are the certain data from the market regarding property price, house specifications, and the functional relationship between them. This may be denoted as the advantage of the model. The estimated price change derives from each of the additional attribute is gathered by forecasting parameters of these related attributes in the hedonic price equation.

D.2.2 Spatial Analysis Methods

The spatial analysis gives the appraiser a straightforward framework to relate the real estate's price moves with its spatial localization features by injecting this factor into the model.

Advanced systems like Geographical Information Systems (GIS) allows the researcher to identify and measure locational and access characteristics of a real estate in great detail so that it contributes and upgrades the analytical power of the spatial statistics methods. As well as these benefits, defining and integrating data into the system regarding the additional neighborhood factors improve the explanatory capacity of the market price fluctuations by the model in the light of spatial pattern analysis and autocorrelation analysis.

By spatial interpolation, the effects of the topography of the land that the subject property is locates in rather than of the fixed neighborhoods or composite submarkets on the property prices can be identified but requires more rigorous spatial analysis. The methodology of the spatial interpolation techniques is to determine a function that gives the relationship of the characteristics of the whole surface area with the price pattern of the property by using a set of data on discrete points for sub-areas. This obtained function may also be used to identify the price relationship of the property with other points or sub-areas.

The assumptions of the spatial analysis can be given as:

i. Since continuity of the surface of z-variable is posited, it is possible to estimate any lack of locational data from the existing sufficient information.
ii. The variable z is assumed to be dependent on the distribution of the spatial characteristics of the subject property. Therefore value at any specific location is related to the values of surrounding locations. This expedites to estimate the value pattern of the property at the surrounding locations by interpolating the variable value from the given spatial distribution.

Some of the types of spatial analysis are:

1. ***Point pattern analysis*** – this analysis studies the very detailed spatial characteristics of the locational points and derives the spatial distribution of these. From the distribution, the analysis leads the researcher to investigate the possible patterns and their randomness by applying hypothesis tests
2. ***Surface analysis*** – by this technique, surface area is reconstructed via the measured sample data.
3. ***Areal analysis*** – the information related to the locational specifications are obtained from the conducted survey data such as census, health statistics and real estate cadastre. The assumption at this point is that there exists homogeneity in the represented subgroups, therefore major differentiations in the subgroups are out of our concern.

D.2.3 Artificial Neural Networks (ANNs)

Artificial neural networks (*ANNs*) are systems that simulate the real world according to what they learn from the experience (Jones, 1994). The system's underlying approach is the connectionism within the interconnected artificial neurons that process information for further computation.

The neural network concept was inspired by and firstly introduced in the neuroscience applications which used artificial neurons in the relatively basic models. In biological applications, artificial neurons have a number of inputs, namely a cell body consisting of a summing node and semi-linear function node, and an output neuron which can be connected to a number of other artificial neurons. The system works as each of the connected artificial neurons in the network send signal regarding the information it perceived to another one. The artificial neurons have capability to adjust the strength or weight of the connections between the units due to the data injected into the system.

The fundamental components of the neural networks to perform are firstly the *neurons* and the *connections* among them; the *algorithm* for the training phase that makes the ANN model learn from the experience; the *testing phase* which is needed to interpret the response from the network.

The *input layer*, one or more *hidden layers* and the *output layer* are the units that compose the ANNs and perform the transmitting process. Before output is produced, a weight factor modifies the input value, and then it is transmitted to the neighbor neuron. The outcome is the signal that becomes the input of next neuron. The total signal that a neuron receives is the summation of all its input neurons.

Each individual output is constructed from a particular input, then the accuracy of the total is evaluated by the actual output value. The accuracy is controlled by the total mean square error and back propagation is used to eliminate this error value untill it is reduced below a threshold level. To achieve this elimination, neuron connection weights are modified.

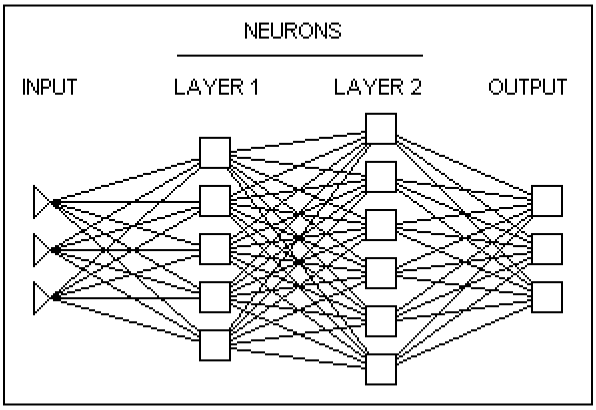

Figure D.4: Diagrammatic representation of a neural network [27].

D.2.4 Fuzzy Logic

The *Boolean logic* is binary, that is a certain element is true or false, an object belongs to a set or it does not. *Fuzzy logic*, introduced by Zadeh in 1965, permits the notion of refinement. The key to Zadeh's idea is to represent the similarity a point shares with each group with a function, membership function, whose values are between $0 < \mu < 1$. Each point will have a *membership* in every group: memberships close to unity signify a high degree of similarity between the point and a group, while membership close to zero implies little similarity between the point and that group. Additionally, the sum of the memberships for each point must be unity. Several types of membership functions can be utilized. The membership function reflects the knowledge for the specific object or event.

Another critical step in the fuzzy systems methodological approach is the definition of the rules, which connect the input with the output. These rules are based on the form "if … then … and". The knowledge in a problem-solving area can be represented by a number of rules.

In order to solve a problem with a *knowledge-based fuzzy system* it is necessary to describe and process the influencing factors in fuzzy terms and provide the result of this processing in a usable form. The basic elements of a knowledge-based fuzzy system are:

1. fuzzification,
2. knowledge base,
3. processing,
4. defuzzification.

The use of fuzzy logic for the analysis and the modeling of real estate could be a powerful tool in modern planning, as is pointed out by many researchers. The most important advantages of fuzzy modeling are:

a. It is a more realistic approach through the use of linguistic variables instead of numbers.
b. Hierarchical ranking of the objects and not an inclusion – exclusion list.
c. Fewer repetitions of the model.

Fuzzy inference is the process of formulating the mapping from a given input to an output using fuzzy logic. The mapping then provides a basis from which decisions can be made, or patterns discerned.

There are two widely used types of fuzzy inference systems: *Mamdani-type* and *Sugeno-type* that can be used in order to value the real estates.

Mamdani-type fuzzy inference method is the most commonly seen fuzzy methodology. Mamdani's method was among the first control systems built using fuzzy set theory. It was proposed in 1975 by Ebrahim Mamdani as an attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Mamdani's effort was based on Lotfi Zadeh's 1973 paper on fuzzy algorithms for complex systems and decision processes.

Mamdani-type inference expects the output membership functions to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification. It is possible, and in many cases much more efficient, to use a single spike as the output membership function rather than a distributed fuzzy set. This type of output is sometimes known as a singleton output membership function, and it can be thought of as a pre-defuzzified fuzzy set. It enhances the efficiency of the defuzzification process because it greatly simplifies the computation required by the more general Mamdani method, which finds the centroid of a two-dimensional function. Rather than integrating across the two-dimensional function to find the centroid, you use the weighted average of a few data points. Sugeno-type systems support this type of model. In general, Sugeno-type systems can be used to model any inference system in which the output membership functions are either linear or constant.