BLIND AND SEMI-BLIND CHANNEL ORDER ESTIMATION IN SIMO SYSTEMS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

SERKAN KARAKÜTÜK

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
ELECTRICAL AND ELECTRONICS ENGINEERING

SEPTEMBER 2009

Approval of the thesis:

BLIND AND SEMI-BLIND CHANNEL ORDER ESTIMATION IN SIMO SYSTEMS

submitted by **SERKAN KARAKÜTÜK** in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in Electrical and Electronics Engineering Department, Middle East Technical University by,

Prof. Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** ________________

Prof. Dr. İsmet Erkmen
Head of Department, **Electrical and Electronics Engineering** ________________

Prof. Dr. T. Engin Tuncer
Supervisor, **Electrical and Electronics Engineering Department, METU** ________________

Examining Committee Members:

Prof. Dr. T. Engin Tuncer
Electrical and Electronics , METU ________________

Prof. Dr. Kerim Demirbas
Electrical and Electronics , METU ________________

Assoc. Prof. Dr. Elif Uysal Bıyıkoğlu
Electrical and Electronics , METU ________________

Assist. Prof. Dr. Çağatay Candan
Electrical and Electronics , METU ________________

Assist. Prof. Dr. Yakup Özkazanç
Electrical and Electronics , Hacettepe University ________________

Date: ________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: SERKAN KARAKÜTÜK

Signature :

ABSTRACT

BLIND AND SEMI-BLIND CHANNEL ORDER ESTIMATION IN SIMO SYSTEMS

Karakütük, Serkan
Ph.D, Department of Electrical and Electronics Engineering
Supervisor : Prof. Dr. T. Engin Tuncer

September 2009, 138 pages

Channel order estimation is an important problem in many fields including signal processing, communications, acoustics, and more. In this thesis, blind channel order estimation problem is considered for single-input, multi-output (SIMO) FIR systems. The problem is to estimate the effective channel order for the SIMO system given only the output samples corrupted by noise. Two new methods for channel order estimation are presented. These methods have several useful features compared to the currently known techniques. They are guaranteed to find the true channel order for noise free case and they perform significantly better for noisy observations. These algorithms show a consistent performance when the number of observations, channels and channel order are changed. The proposed algorithms are integrated with the least squares smoothing (LSS) algorithm for blind identification of the channel coefficients. LSS algorithm is selected since it is a deterministic algorithm and has some additional features suitable for order estimation. The proposed algorithms are compared with a variety of different algorithms including linear prediction (LP) based methods. LP approaches are known to be robust to channel order overestimation. In this thesis, it is shown that significant gain can be obtained compared to LP based approaches when the proposed techniques are used. The proposed algorithms are also compared with the oversampled single-input, single-

output (SISO) system with a generic decision feedback equalizer, and better mean-square error performance is observed for the blind setting.

Channel order estimation problem is also investigated for semi-blind systems where a pilot signal is used which is known at the receiver. In this case, two new methods are proposed which exploit the pilot signal in different ways. When both unknown and pilot symbols are used, a better estimation performance can be achieved compared to the proposed blind methods. The semi-blind approach is especially effective in terms of bit error rate (BER) evaluation thanks to the use of pilot symbols in better estimation of channel coefficients. This approach is also more robust to ill-conditioned channels. The constraints for these approaches, such as synchronization, and the decrease in throughput still make the blind approaches a good alternative for channel order estimation. True and effective channel order estimation topics are discussed in detail and several simulations are done in order to show the significant performance gain achieved by the proposed methods.

Keywords: channel order estimation, channel identification, channel equalization

ÖZ

SIMO SİSTEMLERDE GÖZÜ KAPALI VE YARI KAPALI KANAL DERECESİ KESTİRİMİ

Karakütük, Serkan
Doktora, Elektrik Elektronik Mühendisliğ Bölümü
Tez Yöneticisi : Prof. Dr. T. Engin Tuncer

Eylül 2009, 138 sayfa

Kanal derecesi kestirimi sinyal işleme, haberleşme, akustik ve benzeri alanlar için önemli bir problemdir. Bu tezde, gözü kapalı kanal derecesi kestirimi problemi tek girişli çok çıktılı (SIMO) FIR sistemler için incelenmiştir. Problem, sadece gürültülü SIMO sistemi çıktılarının kullanılarak etkin kanal derecesinin kestirilmesidir. Kanal derecesi kestirimi için iki yöntem önerilmiştir. Önerilen yöntemlerin bilinen yöntemlere nazaran bir çok yararlı özelliği bulunmaktadır. Önerilen yöntemler gürültüsüz ortamda doğru kanal derecesi kestirimini garanti etmekte ve gürültülü ortamda da kayda değer şekilde daha iyi başarım göstermektedir. Bu algoritmalar kanal derecesi, kanal sayısı ve veri boyunun değişimine karşı tutarlı bir davranış göstermektedir. Önerilen algoritmalar, kanal katsayılarının gözü kapalı kestirimine yönelik olarak en küçük kareler yumuşatma (LSS) algoritması ile entegre şekilde çalışmaktadır. LSS algoritmasının seçilmesinin nedeni belirlenimci bir yöntem olması ve kanal derecesi kestirimi için uygun bazı ek özelliklerinin bulunmasıdır. Önerilen algoritmalar, doğrusal tahmin (LP) yöntemlerinin de yer aldığı değişik bir çok algoritma ile karşılaştırılmıştır. LP yaklaşımları kanal derecesinin üstten kestirimine karşı dirençlidir. Bu tezde, önerilen yöntemler kullanıldığında, LP tabanlı yöntemlere nazaran önemli oranda kazanç sağlanabileceği gösterilmiştir. Önerilen yöntemler, yüksek hızda örneklenmiş SISO sistemler için DFE yöntemi ile de karşılaştırılmış ve gözü kapalı durumda daha iyi hata kareleri ortalaması performansı elde edildiği görülmüştür.

Kanal derecesi kestirimi problemi, pilot sembollerin kullanıldığı sistemler için de incelenmiştir. Bunun için, pilot sembollerini farlı şekilde kullanan gözü yarı-kapalı kanal derecesi kestirimi algoritmaları önerilmiştir. Bilinen ve bilinmeyen semboller birlikte kullanıldığı zaman, gözü kapalı yöntemlere nazaran daha iyi kestirim performansı sağlanabilmektedir. Kanal kestirimi doğruluğu pilot sembollerin kullanımı ile arttığı için, gözü yarı-kapalı yaklaşım ile özellikle bit hata oranı (BER) açısından daha iyi başarım elde edilebilmektedir. Bu yaklaşımın diğer bir özelliği ise kötü durumlu (ill-conditioned) kanallara karşı daha dirençli olmasıdır. Öte yandan, eşzamanlama ve veri akışındaki düşüş gibi problemler nedeniyle gözü kapalı yöntemler hala kanal derecesi için iyi bir alternatif olarak gözükmektedir. Doğru ve etkin kanal derecesi kestirimi konuları ayrıntılı şekilde tartışılmış ve önerilen yöntemler ile elde edilen kayda değer başarım kazancını göstermek için çeşitli benzetimler yapılmıştır.

Anahtar Kelimeler: kanal derecesi kestirimi, kanal tanımlama, kanal eşitleme

To my love Gülay and my parents.

ACKNOWLEDGMENTS

It is a pleasure for me to express my sincere gratitude to my supervisor Prof. Dr. T. Engin Tuncer for his guidance, advice, sensitivity, encouragements and sharing all his thoughts.

I also want to thank to my colleagues in MIKES for their support. Moreover, I am thankful to my company MIKES for letting and supporting my thesis.

I would like to thank to METU Sensor Array and Multichannel Signal Processing Group for their useful comments and support during my thesis work.

I would also like to thank for the scholarship received from the Scientific and Technological Research Council of Turkey (TÜBİTAK).

I will also never forget to unending reliance, love and support of my family all the times.

Finally, I want to thank my love, Gülay Tuncer, who is always by my side and helped me to keep myself away from hopelessness.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

1.1 Motivation and Objectives

In this thesis, blind channel order estimation problem in linear time invariant (LTI) finite impulse response (FIR), single-input multiple-output (SIMO) systems is investigated. SIMO systems are observed when single-input single-output (SISO) system outputs are oversampled and polyphase representation is used or alternatively when multiple antennas and receivers are employed [1, 2]. Blind channel order estimation problem is defined as the estimation of the order of a FIR SIMO systems given the noisy observations.

Blind channel order estimation is not an easy problem to solve due to several reasons. In order to understand the problem better, the generic impulse response in Figure 1.1 can be considered. In this figure, the impulse response has large and small coefficients. Large coefficients can be assumed to be surrounded by the small coefficients which are called as the leading and trailing tails without loss of generality. The distinction between tails and the significant part of the channel coefficients can be made by considering the γ value. The coefficients whose magnitudes are above $\gamma/2$ may be defined as the significant part of the filter. Obviously the value of γ for determining the significant part depends on certain factors including SNR and the cost function or the measure used to define the significant part. When the SNR is very large, the true channel order can be defined to be the whole filter including the tail coefficients. When the SNR is low, it may not be possible to clearly identify the tail coefficients. It may also be meaningless to try to find those coefficients since the channel equalization performed over the noisy output samples does not give better MSE for the input samples when the tail coefficients are used due to noise amplification for the small channel coefficients [3]. There-

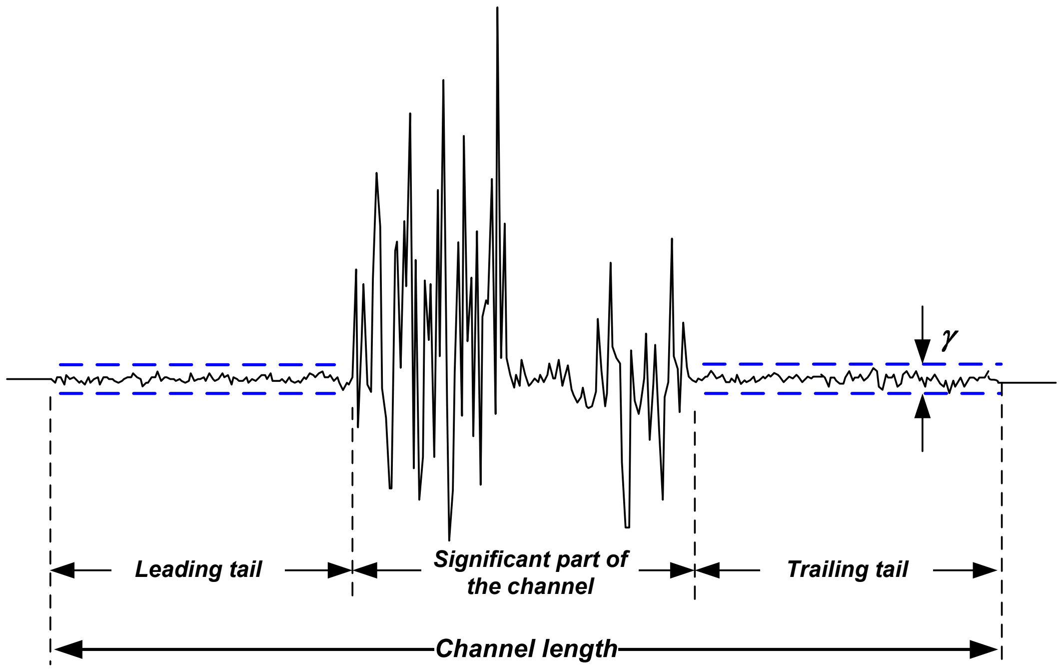


Figure 1.1: Channel impulse response showing the significant part and tail of the channels. Tail coefficients are uniformly distributed between $-\gamma/2$ and $+\ \gamma/2$.

fore in practice, the task is to find the effective channel order which corresponds to finding the significant part of the channel filter rather than the true channel order. In fact in blind problem, it may be impossible to find the true channel order from the noisy output samples. True channel order has meaning when the SNR is very large or when the channel order estimation algorithms are tested by assuming that the channel filter is known. In this thesis, different channel filters, including fixed and random long channels with tails and channels without the tails are considered. Note that when there are no tails, true channel order and effective channel order become same. It should also be pointed that when we consider the channels without the tails, the channel coefficients are generated randomly to obtain a Rayleigh distribution.

In many applications, channel order is required to characterize the linear system appropriately. For example, in room acoustic modeling, channel order is required to find the length of the FIR reverberation filter. In communications, minimum equalizer length is selected depending on the channel order. In addition, if the effective channel order is not used in equalization, MSE for the input signal is larger [4, 5]. In detection of the number of sources, channel order estimation techniques are valuable. For example, a single source with multipath reflectors in wideband direction of arrival estimation requires the detection of the number of reflecting points. Otherwise the DOA algorithms either do not work or report angles with large errors.

Channel order is an important parameter for the blind channel identification problem. In blind channel identification, the channel order is usually assumed to be known [1, 6, 7, 8, 10, 13] . In practice, it should be estimated. When the channel order is underestimated, the blind channel estimation algorithms fail. When the channel order is overestimated, their performance significantly degrades. Hence, best performance is achieved when the effective channel order is correctly estimated. Since previous algorithms for channel order estimation are not robust, this problem has been tried to be solved with linear prediction (LP) based channel estimation algorithms robust to channel order overestimation [14, 15, 16, 17, 18]. However, LP methods are based on statistical information of the channel outputs and therefore they require long observation data to obtain required performance. Furthermore, their performance is not as good as their deterministic alternatives such as subspace algorithm [6], when the channel order is known. On the other hand, the main drawback of deterministic algorithms is that their performance decreases dramatically when the channel order is not correctly estimated. Therefore the use of these algorithms is not practical as a consequence of absence of high performance channel order estimation algorithms. If an accurate and robust channel order estimation algorithm is used with deterministic channel estimator such as proposed in [6, 7, 8] for channel estimation, a better performance is obtained compared to LP techniques with an algorithm which has tendency to overestimate.

The main objective of the thesis is to find new blind channel order estimation algorithms for SIMO systems. The desired properties for a blind channel order estimation algorithms are as follows:

- It should have finite convergence property. That is channel order can be found correctly from finite number of sample in noiseless observations. Therefore, fast convergence can be established for time varying channels and high performance is guaranteed at high SNR.

- Channel order should also be correctly estimated with high probability at low SNR ranges. Correct channel order estimation is important to use deterministic channel estimators in an effective manner.

- It should be robust to SIMO channel parameters. That is, it should work properly for different receiver settings (i.e., number of antennas or oversampling rate) and physical channel parameters (i.e., channel length and channel impulse response).

In this thesis, channel order estimation problem in training based communication system is also considered. It is targeted to find channel order estimation algorithms, which use training data to obtain better performance compared to blind methods.

1.2 Previous Works

There are different algorithms for the channel order estimation in the literature. Minimum Description Length (MDL) [19] and Akaike Information Criteria (AIC) [20] algorithms are based on the information theoretic criteria. These algorithms require long observations for accurate extraction of the statistical parameters. It is known that MDL usually performs better than the AIC and AIC has a tendency for overestimation [4, 21]. Both of these algorithms are sensitive to colored noise [21] and deviation from idealized Gaussian white noise signal. In addition, they are very sensitive to SNR variations and data length [22]. Therefore they are not robust algorithms for practical applications and scenarios.

Joint channel order and channel estimation with LSS method (JLSS) is presented in [3]. It is shown that JLSS can find the true channel order from finite number of samples in case of noise free observations. The main disadvantage of the JLSS algorithm is its performance loss for noisy observations.

Most of the cost functions for order estimation decrease almost monotonically as the channel order increases, which makes it hard to find the true channel order. This problem is tried to be overcome by using an empirically chosen penalty coefficient [23]. This penalty term leads to over or underestimation in many of the information theoretic techniques. In [24], a new cost function is proposed. This cost function is obtained by combining two cost functions due to channel identification (ID) and channel equalization (EQ), and hence ID+EQ algorithm is obtained. The main feature of this cost function is its ”convex - like” shape. Therefore channel order estimation can be performed by finding the global minimum.

Previously, it is known that there are only two algorithms which are guaranteed to find the correct channel order from finite number samples in noise free case. These are the JLSS [3]

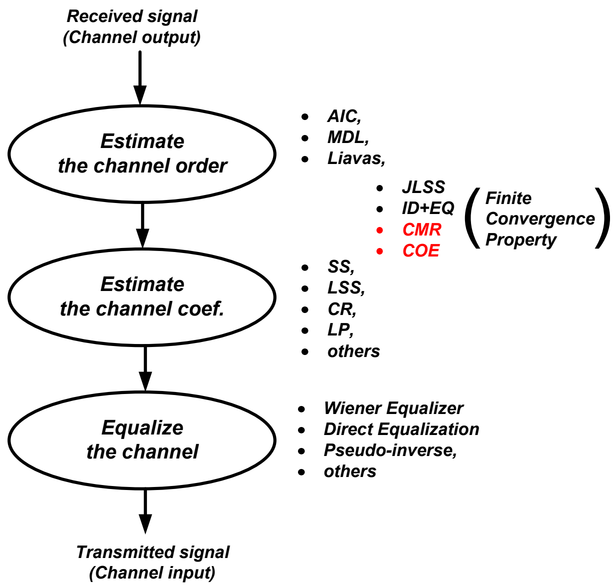


Figure 1.2: Blind channel input estimation steps starting from channel order estimation.

and ID+EQ [24] algorithms. The main disadvantage of the JLSS algorithm is its performance loss for noisy observations and its tendency to overestimation. While ID+EQ performs better than JLSS, it also suffers from performance loss in case of noisy observations. In this thesis, two new algorithms, Channel Output Error (COE) and Channel Matrix Recursion (CMR) are presented which find the true channel order from finite number of samples. In addition, it is shown that these algorithms perform significantly better than the alternatives in the estimation of the effective channel order.

In Figure 1.2, typical blind identification procedure is shown with the examples of algorithms that can be used at each step. The first step is the channel order estimation. Channel order estimation is followed by the estimation of channel coefficients. Channel order and filter can be estimated in a joint manner as in the case of JLSS, ID+EQ and the proposed methods. Channel estimation is followed by the equalization. Perfect equalization of SIMO system is possible if there are no common zeros between subchannels of the SIMO system. Equalization can also be done without the knowledge of the channel coefficients. Direct equalization algorithm [13] is an example for this type of algorithms which is employed in ID+EQ algorithm.

1.3 Contributions

Summary of main the contributions are as follows:

- Blind channel order estimation for SIMO systems is considered. Blind channel order estimation problem especially the effective channel order estimation problem is covered in detail. The comparisons and the results in this regard are unique in the literature.
- Two new blind channel order estimation algorithms are proposed, namely COE [27] and CMR [28].
  - These algorithms are proved to have the finite convergence property, i.e., they are guaranteed to find the true channel from finite number of observations for noise free case. Previously there were only two algorithms known in the literature with the same property. The presented algorithms have significantly better performance than the known algorithms for noisy observations. They find the effective channel order better than the alternative techniques.
  - Theorem-1 and Theorem-2 are defined for the proof of finite convergence property of the proposed algorithms. Lemma-1 and Lemma-2 are also defined and proved in order to show some properties of least squares algorithm. Lemma-3 explains the reason for better performance of the proposed algorithms for noisy observations.
  - It is shown that the proposed blind channel order estimation methods lead significant performance improvement compared to AIC or MDL when used with LP technique. This result also have its implications for SISO communication systems since they use an FIR equalizer for the estimation of input symbols which is related to the zero-forcing equalizer for LP techniques.
  - Proposed blind order estimation methods are compared with the decision feedback equalizer in an oversampled SISO system. It is shown that significant performance gain can be achieved in terms of BER when the proposed techniques are used.
- Channel order estimation for semi-blind case is considered where pilot symbols are used to estimate the channel.
  - Two algorithms for this case are proposed. One uses only the pilot symbols and

the other uses both the pilots and the unknown symbols. The second algorithm is shown to perform well in a variety of cases.

- Comparisons of several techniques are done for effective channel order estimation, channel and input estimation. The improvement achieved by proposed techniques are shown for variety of cases.

1.4 Organization of the Thesis

Organization of the thesis is as follows.

In Chapter 2, blind channel identification and channel order estimation problem is considered and previous works referenced throughout the thesis are summarized.

In Chapter 3, COE and CMR methods are introduced and their performance in true channel order estimation is analyzed and compared with the alternatives in the literature.

In Chapter 4, channel order estimation problem is considered for training based transmission systems. The ways of using training sequence in channel order estimation are investigated and two new semi-blind channel order estimation algorithms namely, channel input error with blind channel estimator (CIEB) and channel input error with semi-blind channel estimator (CIES), are proposed. Their performance in true channel order estimation is analyzed and compared with the blind algorithms.

In Chapter 5, effective channel order estimation is discussed and proposed methods are analyzed in terms of estimating the effective channel order. LP based methods [15, 17] robust to overestimation of the channel order and deterministic high performance channel estimators [6, 7] are evaluated with channel order estimation algorithms for comparison. It is shown that using COE and CMR with deterministic channel estimators performs much better than the case of using LP based methods in mean square error (MSE) and bit error rate (BER) sense.

In Chapter-6, the conclusion of the thesis is given.

CHAPTER 2

BLIND SYSTEM IDENTIFICATION

2.1 Introduction

Blind system identification is a fundamental signal processing topic aimed to retrieve unknown information for a system from its output only. The theory of blind system identification has a wide range of application areas including mobile communication, speech recognition, and blind image restoration.

In signal processing and communication societies, there have been an increasing interest to blind problem. The reason of interest may be the potential applications in wireless communication. Information signal is distorted during transmission because of the noise , interference of other users and the frequency selective characteristic of the channel. The distortion on the transmitted signal must be removed by processing at the receiver. Removing distortion is referred to as channel equalization. To facilitate compensation of distortion, in most cases, a training sequence is transmitted. With the help of the training sequence, which is also known at the receiver, the receiver determines the channel. After the identification of the channel, information transmission continues. The transmission of training sequence obviously decreases the channel capacity used for the information transmission. For time invariant channels, the loss is insignificant because only one training set is transmitted for all times. However for time varying channels, transmission of the training sequence must be repeated periodically. Each time the system has to converge to the varying channel impulse response and there will be even no time for data transmission. If the channel can be identified without training sequence, the time slot for training sequence can be used for information transmission so that efficiency can be increased significantly. Furthermore, there are various situations where the transmission of sequences to train receivers is either infeasible or undesirable. Communication intelligence (COMINT) and electronic intelligence (ELINT) systems are the examples of such systems used for military purposes to listen the environment. In these kind of applications, the receiver has no knowledge about the training sequence as a consequence of the nature of the problem. Therefore for the removal of the multipath effect, the use of blind algorithms is necessary. Another application area of blind system identification is the systems having synchronization problems. When the synchronization is not achieved, training based methods can not be used for channel estimation and equalization. In this case, blind methods are more practical.

In blind channel identification, channel impulse response is determined by using only the received signal. In blind equalization, the received signal is equalized without knowing the channel and the desired signal. There are two ways to equalize the channel. One way is to estimate the input signal directly. The second way is to first identify the channel and then determine the input signal using the estimated channel. In case of the second approach, the problem returns to the classical inverse problem after the estimation of the channel.

The solution of the blind identification problem depends on the system model. The structure of the communication channel can be single input single output (SISO), single input - multi output (SIMO) or multi input multi output (MIMO) depending on the application. For the equalization of the SISO channels, generally higher order statistics (HOS) based methods are used. However, the problem of convergence limits their applicability in practical settings. In [1], a new method is proposed to overcome these problems. The method uses cyclostationary property of the oversampled received signal, which enables the use of second order statistics (SOS) for the identification of channel. This work is a breakthrough and after that lots of algorithms were proposed using cyclostationary. In [1] the output covariance matrix is used. But main drawback of this kind of algorithms is the performance degradation due to the finite number of observations and model mismatch. Subspace (SS) based algorithms [6] allows channel identification from finite number observations, hence provides more data efficient algorithms for channel identification.

Generally, blind channel identification methods are classified into two main groups, statistical and deterministic methods. While statistical methods assume that the source is a random sequence with known second order structure, deterministic methods do not assume any specific statistical structure for the input signal. Perhaps a more striking property of the deterministic methods, such as SS [6], cross relation (CR) [8] and least squares smoothing (LSS) [3], is the finite convergence property. Namely, when there is no noise, the estimator produces the exact channel using only a finite number of samples, provided that the identifiability condition is satisfied. Therefore these methods are most effective at high SNR and for small data sample scenarios. Furthermore deterministic methods can be applied to a wide range of source signals. However, asymptotic performance might be affected due to the fact that statistical features are not used for the processing. Although deterministic methods have superior performances, they have some drawbacks. When the channel has common zeros, channel matrix singularity problems arise and subspaces can not be obtained truly. Hence these methods do not work. These methods assume the knowledge of the channel order and their performances decreases dramatically when the channel order is not known exactly.

Knowledge of the channel order is important for the blind channel identification algorithms to obtain required performance. Most of the channel estimation algorithms assumes that the channel order is exactly known. Some of the previously proposed methods are based on the exploiting eigenvalues of channel output covariance matrix. Minimum Description Length (MDL) [19] and Akaike Information Criteria (AIC) [20] algorithms are the examples of such algorithms and have some statistical assumptions on the received signal. [3] and ID+EQ [24] are the two examples of deterministic channel order estimation methods. They can estimate the channel order from finite number of samples in noise free case.

A detailed review of blind identification algorithms for multichannel systems is done in [25] and the references cited in that work would be helpful for the reader to see other works in that area. In this chapter, blind channel estimation algorithms SS, LSS, linear prediction [14] and channel order estimation algorithms MDL, AIC, Liavas, JLSS and ID+EQ algorithms which are referenced throughout the thesis are summarized for the completeness of the thesis.

2.2 System Model

Blind channel identification methods considered in this chapter require a multichannel representation of the communication system. In Figure 2.5 a common SIMO channel representation is shown. Basically a SIMO channel representation can be obtained with one or more of

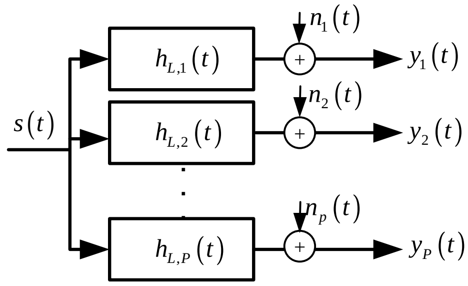


Figure 2.1: SIMO Channel Model

the following approaches:

- Sample an antenna array at symbol rate,
- Oversample the received signal with respect to the symbol rate,
- Interpolate the information signal before transmitting it through the channel.

In all of the above cases, a vector based system description can be obtained which is composed of P channels at baud rate.

SIMO model is composed of single-input P-output channel model as shown in Figure 2.5. Each subchannel is assumed to have a linear time-invariant (LTI) FIR filters. Channel order of a SIMO system is defined as the order of the filter which has the maximum filter order. For example, for a two channel SIMO system with filter orders three and two, the channel order is three. However, for the simplicity of the equations, it is assumed that each channel has equal order, L. Because of the frequency selective channel, inter block interference (IBI) occurs between the receiving blocks. So IBI must be taken into account as well as intersymbol interference (ISI). Assuming that IBI occurs only between two consecutive blocks (i.e. channel length $L+1$ is smaller than block length N), first L symbols in the received block are affected from the previous block. IBI can be removed by discarding the first L samples in a received block. After discarding the first L samples, the channel output vector can be written as

$$\mathbf{y}_1(t) = \sum_{k=0}^{L} \mathbf{h}_L(k)\, s(t-k) + \mathbf{n}_1(t) \tag{2.1}$$

where

$$\mathbf{y}_1(t) = \begin{bmatrix} y_1(t) & y_2(t) & \cdots & y_P(t) \end{bmatrix}^T \tag{2.2}$$

$$\mathbf{h}_L(k) = \begin{bmatrix} h_{L,1}(k) & h_{L,2}(k) & \cdots & h_{L,P}(k) \end{bmatrix}^T \tag{2.3}$$

$$\mathbf{n}_1(t) = \begin{bmatrix} n_1(t) & n_2(t) & \cdots & n_P(t) \end{bmatrix}^T \tag{2.4}$$

The $P \times 1$ vectors, $\mathbf{y}_1(t)$, $\mathbf{h}_L(k)$, and $\mathbf{n}_1(t)$ are the received signals, channel impulse response and additive noise respectively. $y_i(t)$, $h_i(k)$, and $n_i(t)$ are the scalar values of the output signal, channel impulse response and additive noise for the i^{th} channel respectively. The matrix formulation for the same model can be given as,

$$\mathbf{y}_1(t) = \mathbf{H}_1 \mathbf{s}_{L+1}(t) + \mathbf{n}_1(t) \tag{2.5}$$

where,

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{h}_L(0) & \mathbf{h}_L(1) & \cdots & \mathbf{h}_L(L) \end{bmatrix} \tag{2.6}$$

$$\mathbf{s}_{L+1} = \begin{bmatrix} s(t) & \cdots & s(t-L) \end{bmatrix}^T \tag{2.7}$$

System output can be modified to include M samples for each channel and the following equation can be written,

$$\mathbf{y}_M(t) = \mathbf{H}_M \mathbf{s}_{M+L}(t) + \mathbf{n}_M(t) \tag{2.8}$$

where

$$\mathbf{y}_M(t) = \begin{bmatrix} \mathbf{y}_1^T(t) & \cdots & \mathbf{y}_1^T(t-M+1) \end{bmatrix}^T \tag{2.9}$$

$$\mathbf{n}_M(t) = \begin{bmatrix} \mathbf{n}_1^T(t) & \cdots & \mathbf{n}_1^T(t-M+1) \end{bmatrix}^T \tag{2.10}$$

$$\mathbf{s}_{M+L}(t) = \begin{bmatrix} s(t) & \cdots & s(t-L-M+1) \end{bmatrix}^T \tag{2.11}$$

$$\mathbf{H}_M = \begin{bmatrix} \mathbf{h}_L(0) & \cdots & \mathbf{h}_L(L) & & \\ & \ddots & \cdots & \ddots & \\ & & \mathbf{h}_L(0) & \cdots & \mathbf{h}_L(L) \end{bmatrix} \tag{2.12}$$

The $MP \times (M+L)$ dimensional block Toeplitz matrix, $\mathbf{H}_M$, is channel matrix. Equation (3.8) can be written compactly as,

$$\mathbf{Y} = \mathbf{H}_M \mathbf{S} + \mathbf{N} \tag{2.13}$$

where,

$$\mathbf{Y} = \begin{bmatrix} \mathbf{y}_M(t) & \mathbf{y}_M(t+1) & \cdots & \mathbf{y}_M(t+N-1) \end{bmatrix} \tag{2.14}$$

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_{M+L}(t) & \mathbf{s}_{M+L}(t+1) & \cdots & \mathbf{s}_{M+L}(t+N-1) \end{bmatrix} \tag{2.15}$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{n}_M(t) & \mathbf{n}_M(t+1) & \cdots & \mathbf{n}_M(t+N-1) \end{bmatrix} \tag{2.16}$$

The goal is to estimate the unknown channel parameters from the observation data.

2.3 Channel Identifiability

The techniques used for the estimation of SIMO channel coefficients in a blind manner can at best identify the channel up to a complex scale factor. To ensure the channel identification in SIMO channels, channel diversity must be satisfied. When the channels are modeled as FIR filters, then channel diversity means that no common zeros exist, or in other words, they are coprime. If the channels are not coprime, there exists a common zero and that zero can not be distinguished from the zeros of the input. Hence, we can not identify the channel without knowing the input. The identifiability of the channel can also be defined through the channel matrix, $\mathbf{H}_M$. If the channel matrix is full column rank then the channel is said to be identifiable [2]. The channel matrix is full column rank if,

- The subchannels have no common zero.
- M is greater than $(M + L)/P$, so that the channel matrix is a tall matrix.
- At least one channel has an order of L and $h_{L,k} \neq 0 \; \forall k$.

The channel identifiability condition determines whether the channel can be obtained in a blind manner. There are also certain conditions that should be satisfied for the use of the blind channel identification algorithms. The key condition that is applicable to most of the algorithms is about the complexity of the input signal. If the outputs of the subchannels do not carry enough information , the channel filters can not be obtained. Such a case arises when a constant or a periodic signal is sent. Linear complexity [29] is one of the criteria in order to decide whether the signal carry enough information. Linear complexity measures the predictability of a finite-length deterministic sequence.

Definition:[7] The linear complexity of a sequence $\{s(t)\}_{t=0}^{n}$ is defined as the smallest value of c for which there exists $\{\lambda_j\}$ such that

$$s(t) = -\sum_{j=1}^{c} \lambda_j s(t-j) \qquad t = s, ..., n \tag{2.17}$$

Let us consider a Toeplitz matrix, $\mathbf{S}_c$, given by

$$\mathbf{S}_c = \begin{bmatrix} s(c) & s(c-1) & \cdots & s(0) \\ s(c+1) & s(c) & \cdots & s(1) \\ \vdots & \vdots & \cdots & \vdots \\ s(n) & s(n-1) & \cdots & s(n-c) \end{bmatrix} \tag{2.18}$$

If $s(t)$ has linear complexity c or greater, then $\mathbf{S}_c$ has full column rank. Hence the sample covariance of the vector $\mathbf{s}(t) = \begin{bmatrix} s(t) & s(t-1) & \cdots & s(t-c) \end{bmatrix}$ has full rank. On the other hand, if $s(t)$ has linear complexity less than c, $\mathbf{S}_c$ is rank deficient.

2.4 Subspace Method

The subspace method [6] exploits the low rank data model with the assumptions on noise and source signal characteristics to identify the unknown parameters. In a low rank data model, observation vectors belong to a certain subspace of the complex measurement space. Generated SIMO channel model has low rank structure when the channel matrix $\mathbf{H}_M$ has full column rank.

$$\mathbf{y}_M(t) \quad = \quad \mathbf{H}_M \mathbf{s}_{M+L}(t) + \mathbf{n}_M(t) \tag{2.19}$$

If the length of the temporal window, M, is chosen greater than $(L + 1 - P)/(P - 1)$, then channel matrix, $\mathbf{H}_M$, will have more rows than columns. And the columns of $\mathbf{H}_M$ are linearly independent if and only if the channels are coprime, in other words they do not have common zeros.

In the case of noiseless observations, the observation vectors, $\mathbf{y}_M(t)$, are exact linear combination of the columns of $\mathbf{H}_M$. Therefore the noiseless observation vectors are the elements of the vector space which is spanned by the columns of $\mathbf{H}_M$. Since $\mathbf{H}_M$ is a tall matrix, its columns do not span the overall measurement space. The vector space, which is spanned by the columns of channel matrix (i.e. range space of $\mathbf{H}_M$), is a subspace of complex measurement space and it is called as the signal subspace. The orthogonal complement of the signal subspace, which is the left null space of $\mathbf{H}_M$, is called noise subspace. So the measurement space is composed of these two orthogonal subspaces.

$$Measurement\ \ space = Noise\ \ subspace \overset{\perp}{\oplus} Signal\ \ subspace \tag{2.20}$$

The dimension of the signal subspace is equal to the number of linearly independent columns of $\mathbf{H}_M$. Since $\mathbf{H}_M$ has full column rank, it is equal to the number of columns which is $M + L$.

It is possible to determine the signal and noise subspaces by collecting a number of observation vectors at the receiver, i.e.,

$$\mathbf{Y} = \mathbf{H}_M\mathbf{S} \tag{2.21}$$

If the input data matrix, $\mathbf{S}$ (whose size is given as $(M+L)\times N$), is wide and full row rank, then $\mathbf{Y} = \mathbf{H}_M\mathbf{S}$ is a low rank factorization. This condition is provided by the assumption on the linear complexity of the input signal, which should be greater than $M + L$. The columns of $\mathbf{Y}$ are spanned by the columns of the channel matrix and it has nonempty nullspace with dimension of $MP - (M + L)$ as a result of low rank factorization. Denoting the $MP \times (MP - M - L)$ matrix $\mathbf{U_n}$ as the left singular vectors of $\mathbf{Y}$, which corresponds to the noise subspace,then

$$\mathbf{U}_n^H\mathbf{H}_M = \mathbf{0} \tag{2.22}$$

Where subindex n used in $\mathbf{U}_n$ is used to clarify that the matrix belongs to the noise subspace. For the signal subspace $\mathbf{U}_s$ is used. The channel matrix is identifiable up to a scalar complex factor from the above equation. Since the channel matrix is identifiable up to a complex factor, one parameter of $\mathbf{h}$ is fixed (i.e. say c). Under this consideration, vectorized channel matrix can be written as,

$$vec(\mathbf{H}_M) = \mathbf{\Phi}_h\mathbf{h}_c + \mathbf{a}_h \tag{2.23}$$

where $\mathbf{\Phi}_h$ is a selection matrix containing only zeros and ones, $\mathbf{h}_c$ is the parameter vector containing the channel parameters except the fixed parameter. $\mathbf{a}_h$ comes from the fixed parameter and it contains the fixed complex parameter and zeros. Applying (2.23) into (2.22) it is obtained that,

$$vec\left[\hat{\mathbf{U}}_n^H\mathbf{H}_M\right] = \mathbf{\Psi}_c\mathbf{h}_c + \mathbf{b}_h \tag{2.24}$$

where

$$\boldsymbol{\Psi}_c = (\mathbf{I} \otimes \hat{\mathbf{U}}_n^H)\boldsymbol{\Phi}_h, \quad \mathbf{b}_h = (\mathbf{I} \otimes \hat{\mathbf{U}}_n^H)\mathbf{a}_h \tag{2.25}$$

and $\otimes$ indicates the Kronecker product. An estimate of $\mathbf{h}_c$ in least square sense can be found by the following minimization,

$$\hat{\mathbf{h}_c} = arg\ \min_{\mathbf{h}_c}(\boldsymbol{\Psi}_c\mathbf{h}_c + \mathbf{b}_h)^T(\boldsymbol{\Psi}_c\mathbf{h}_c + \mathbf{b}_h) \tag{2.26}$$

Minimizing the cost function in (2.26) with respect to $\mathbf{h}_c$ results in the estimated channel coefficients.

$$\hat{\mathbf{h}}_c = -(\boldsymbol{\Psi}_c)^{\dagger}\,\mathbf{b}_h \tag{2.27}$$

When there is no noise, subspace method produces the exact channel using the finite number of samples. Therefore the subspace method has finite sample convergence property. It is an important property for blind channel identification methods in the sense of the speed of the convergence, especially in packet transmission systems where only a small number of samples is available for processing.

Subspace method is not robust against the modeling errors, especially when the channel matrix is approximately singular (i.e. Channel zeros are close to each other). Furthermore subspace method requires exact channel order. It is performance is not acceptable for under-overestimated channel orders. Therefore channel order must be estimated using one of the techniques in the literature.

2.5 Least Squares Smoothing Method

LSS method [7, 3] is based on the isomorphic relation between the input and output subspace. It is shown that the channel order and channel impulse response are uniquely determined by the least squares smoothing error when the isomorphism between the input and output subspaces is considered.

Since no assumption is made about the statistics of the input signal, LSS method is considered as a deterministic method. It has finite convergence property which is an important property especially for packet transmission system where the channel must be estimated in a limited time interval. The most striking property of the LSS over the other deterministic blind channel identification methods may be the estimation of the channel order jointly with the channel impulse response. Furthermore LSS method is more robust to modeling errors, i.e., when the channel matrix is approximately singular.

Considering the isomorphic relation between the input and output subspaces, we first consider the estimation of the channel from the input subspaces. By projecting the output data into the punctured input subspace Z, the channel is obtained from the least squares projection error. Since the input subspace is not directly available , the punctured input subspace Z is obtained from the output subspaces by exploiting the isomorphic relation between the input and output subspaces. When Z is constructed from the channel output by using the isomorphism between the input and output subspaces, this projection is called smoothing. In joint order detection and channel estimation, the smoothing error is minimized by jointly choosing the channel order and channel parameters.

2.5.1 Assumptions and Properties

There are two basic assumptions for the LSS [7, 3]. One is about the system, the other one is about the input signal. These are given below.

A1: *Channel disparity condition*: The subchannel transfer functions do not share common zeros, and there exists $M > W_0$ such that $\mathbf{H}_M$ has full column rank. W_0 is the smallest value of M which makes the channel matrix a tall matrix.

The following property reveals the equivalence of the input and output subspaces and it plays a critical role in smoothing approach for the channel estimation. Before that, let us define input and output subspaces spanned by p consecutive row (block row) vectors as

$$S_{t,p} = sp\left\{ \begin{matrix} \mathbf{s}_t & \cdots & \mathbf{s}_{t-p+1} \end{matrix} \right\} = R\left\{ \left(\begin{bmatrix} \mathbf{s}_t & \mathbf{s}_{t+1} & \cdots \\ \vdots & \vdots & \vdots \\ \mathbf{s}_{t-p+1} & \mathbf{s}_{t-p+2} & \cdots \end{bmatrix} \right) \right\} \tag{2.28}$$

$$X_{t,p} = sp\left\{ \begin{matrix} \mathbf{x}_t & \cdots & \mathbf{x}_{t-p+1} \end{matrix} \right\} = R\left\{ \left[\begin{bmatrix} \mathbf{x}_t & \mathbf{x}_{t+1} & \cdots \\ \vdots & \vdots & \vdots \\ \mathbf{x}_{t-p+1} & \mathbf{x}_{t-p+2} & \cdots \end{bmatrix} \right] \right\} \tag{2.29}$$

Where the row vector $\mathbf{s}_t$ is given as, $\mathbf{s}_t = \begin{bmatrix} s(t) & s(t+1) & \cdots \end{bmatrix}$, and noiseless observation matrix is given as $\mathbf{x}_t = \begin{bmatrix} \mathbf{x}_1(t) & \mathbf{x}_1(t+1) & \cdots \end{bmatrix}$. $R\{\mathbf{A}\}$ indicates the space spanned by the rows of a matrix $\mathbf{A}$. The above equations can also be written for spanning $|p|$ future data vectors in case of $p < 0$ such as,

$$S_{t+p,p} = S_{t,-p} \tag{2.30}$$

Property 1: Under disparity condition, there is an isomorphic relation between input and output (noiseless) subspaces

$$X_{t,M} = S_{t,M+L} \tag{2.31}$$

In other words, $X_{t,M}$ is isomorphic to $S_{t,M+L}$ with isomorphism $\mathbf{H}_M$.

In a linear system, input space may not be seen from the output space, that is some information may be lost. Therefore, in general $X_{t,M} \subseteq S_{t,M+L}$ for a fixed M. On the other hand, with A1, all the information of the input space is contained in the output space. Such a relation between the input and output subspaces enables us to use the output subspace instead of direct use of the input subspace to estimate the channel. An interesting point is that, even in case of common zeros $X_{t,M}$ may still be a good approximation of $S_{t,M}$. Therefore, it is more robust against common zeros compared to Subspace method [3].

Another important assumption is about the input signal. To ensure the channel identifiability, input signal must carry enough information. In other words, it must be sufficiently complex to identify the channel from the observation data. This requirement is imposed by the linear complexity of the signal.

A2: *Linear Complexity:* [3] The input sequence $s(t)$ has linear complexity greater than $2W_o + 2L$.

The reason of the assumption A2 will be clearer in the following section when the necessary number of input symbols to identify the channel is discussed.

2.5.2 Least Squares Smoothing Algorithm

In this section, we introduce the linear least squares smoothing channel estimation by exploiting the isomorphic relation between input and output (noiseless) spaces. First, the estimation of channel from the input subspace is considered.

By projecting the output data into the input subspace, the channel is obtained from the least squares projection error. But normally input subspace is not directly available from observation. Therefore, instead of using input subspace, output subspace can be used by exploiting the isomorphic relation between the input and output subspace. So, the second step will be to identify the channel from the output subspace.

2.5.2.1 Channel Identification from Input Subspace

Consider L+1 consecutive output block row vectors $\mathbf{x}_{t+L}, ..., \mathbf{x}_t$. From (A.1) we have,

$$\mathbf{x}_{t+L} = \mathbf{h}(0)\mathbf{s}_{t+L} + \mathbf{h}(1)\mathbf{s}_{t+L-1} + \cdots + \mathbf{h}(L)\mathbf{s}_t \tag{2.32}$$

$$\mathbf{x}_{t+L-1} = \mathbf{h}(0)\mathbf{s}_{t+L-1} + \cdots + \mathbf{h}(L-1)\mathbf{s}_t + \mathbf{h}(L)\mathbf{s}_t \tag{2.33}$$

$$\vdots \qquad \ddots \tag{2.34}$$

$$\mathbf{x}_t = \mathbf{h}(0)\mathbf{s}_t + \cdots + \mathbf{h}(L)\mathbf{s}_t \tag{2.35}$$

The aim is to identify $\mathbf{h}(0), ..., \mathbf{h}(L)$ up to a scaling factor from $\mathbf{x}_{t+L}, ..., \mathbf{x}_t$. One way is to

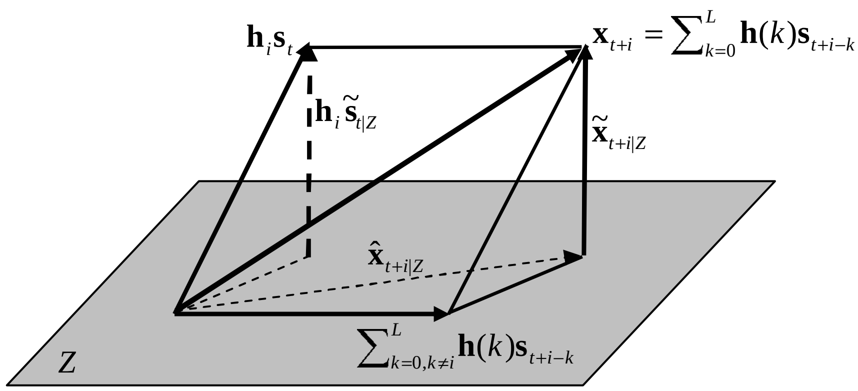


Figure 2.2: Projection of output data onto projection subspace Z

eliminate all terms in $\mathbf{x}_{t+i}$ except the ones associated with $\mathbf{s}_t$. This can be done by projection of $\mathbf{x}_{t+i}$ into the subspace, Z, which is spanned by the input row vectors except $\mathbf{s}_t$. The input subspace, Z, satisfies the following properties.

C1: $\{\mathbf{s}_{t+L}, \cdots, \mathbf{s}_{t+1}, \mathbf{s}_{t-1}, \cdots, \mathbf{s}_{t-L}\} \subset Z$

C2: $\mathbf{s_t} \notin Z$

As illustrated in figure 2.2, $\mathbf{x}_{t+i}$ is composed of two components: one is inside the punctured subspace Z, which equals to $\sum_{k=0,k\neq i}^{L} \mathbf{h}(k)\mathbf{s}_{t+i-k}$, the other one is outside the Z space which equals to $\mathbf{h}_i\mathbf{s}_t$. The projection error of $\mathbf{x}_{t+i}$ into Z space denoted by $\tilde{\mathbf{x}}_{t+i|Z}$ is

$$\tilde{\mathbf{x}}_{t+i|Z} = \mathbf{h}_i\tilde{\mathbf{s}}_{t|z} \tag{2.36}$$

Consequently, we have

$$\mathbf{E} = \begin{bmatrix} \tilde{\mathbf{x}}_{t+L|Z} \\ \vdots \\ \tilde{\mathbf{x}}_{t|Z} \end{bmatrix} = \mathbf{h}\tilde{\mathbf{s}}_{t|z} \tag{2.37}$$

Note that $\mathbf{E}$ is a rank one matrix whose columns and rows are spanned by $\mathbf{h}$ and $\mathbf{s}_{t|Z}$ respectively. The channel can be identified from the projection error matrix $\mathbf{E}$ by several ways. One way is to obtain the singular value decomposition (SVD) of $\mathbf{E}$ or the sample covariance of $\mathbf{E}$. The eigenvector corresponding to the maximum eigenvalue spans the column space of $\mathbf{E}$ and so $\mathbf{h}$. Therefore, this eigenvector can be taken as an estimate of $\mathbf{h}$.

2.5.2.2 Channel Identification from Output Subspace

In the previous section, it is shown that channel can be identified from the projection errors of $\mathbf{x}_{t+L}, ..., \mathbf{x}_t$ into the projection subspace Z satisfying the properties C1 and C2. Using the isomorphic relation between the input and output subspaces, the input subspace, Z, can be constructed from the output subspace, so that direct use of the input sequence is avoided. Under C1 and C2, the projection subspace is defined as,

$$Z = S_{t-1,p} \cup S_{t+1,-p}$$

for any $p \geq L$. With the isomorphic relation between the input and output subspaces described in property 1, we have

$$\begin{aligned} Z &= S_{t-1,M+L} \cup S_{t+1,-(M+L)} = X_t \\ X_t &= X_{t-1,M} \cup X_{t+L+1,-M} \end{aligned} \tag{2.38}$$

In figure 2.3 isomorphic relation between input and output subspaces is illustrated. Projection of $\mathbf{x}_{t+L}, ..., \mathbf{x}_t$ into input subspace Z is converted into the projection of the current data $\mathbf{x}_{t+L}, ..., \mathbf{x}_t$ into the output subspace which is spanned by block row vectors of past and future

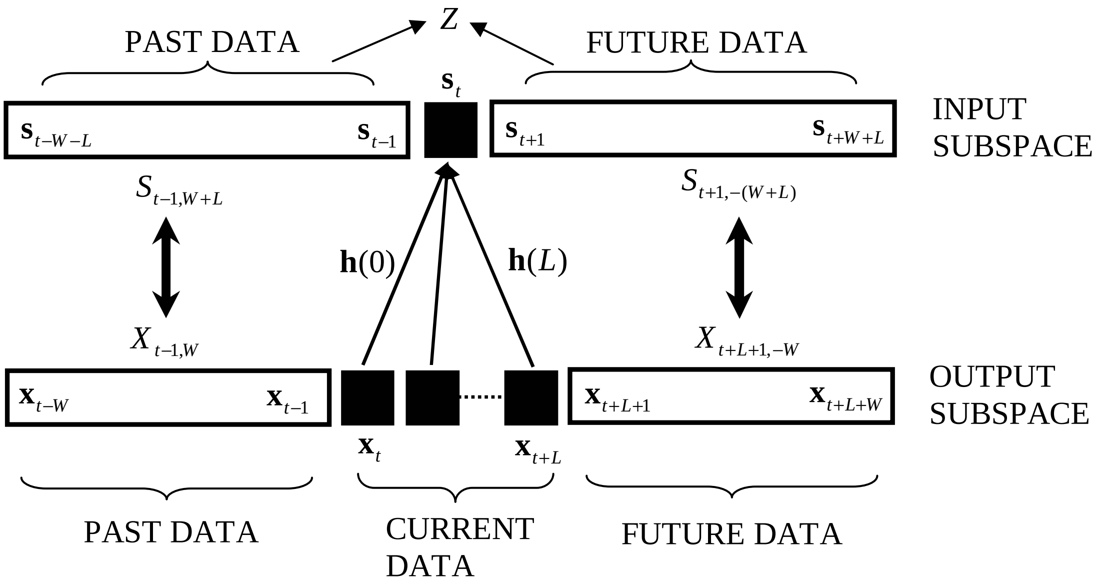


Figure 2.3: isomorphism between input and output subspaces

data. This type of projection is called as smoothing, since it uses the future data besides the past data. The smoothing error matrix, **E**, can be obtained from the observation data.

$$\mathbf{E} = \begin{bmatrix} \tilde{\mathbf{x}}_{t+L|X_t} \\ \vdots \\ \tilde{\mathbf{x}}_{t|X_t} \end{bmatrix} = \mathbf{h}\tilde{\mathbf{s}}_{t|X_t} \tag{2.39}$$

The price paid for avoiding the direct use of input sequence is that more input symbol is required to identify the channel. From Figure 2.3, it is seen that the projection subspace Z and current data span a $(2M + L + 1)$-dimensional input subspace denoted as V are given as,

$$V = sp\{\mathbf{s}_{t-w-L}, \mathbf{s}_{t-w-L+1}, \ldots, \mathbf{s}_{t+w+L}\} \tag{2.40}$$

$$= R\left\{\begin{pmatrix} s(t+w+L) & s(t+w+L+1) & \cdots & s(N) \\ \vdots & & Toeplitz & \\ s(t-M-L) & & & \end{pmatrix}\right\} \tag{2.41}$$

To ensure that the input data span the whole input subspace, V, the length of row vector of input signal must be at least $2M + 2L$ and the input signal complexity must be greater than $2M + 2L$. Satisfying these conditions, minimum number of input symbols required to identify the channel is $4M + 4L + 1$.

2.5.3 General Formulation of LSS

Up to now, channel order is assumed to be known. In this section a general formulation is given when the channel order is not known. For this purpose, projection space, Z, is redefined according to an arbitrary channel order l as follows.

$$\begin{aligned} Z_l &= X_{t-1,M} \cup X_{t+l+1,-M} \\ &= X_{t-1,M} \cup X_{t+l+M,M} \end{aligned} \tag{2.42}$$

Because of the isomorphic relation between the output and input subspaces,

$$Z_l = S_{t-1,L+M} \cup S_{t+l+M,L+M} \tag{2.43}$$

Therefore,

$$Z_l = \begin{cases} sp\{\mathbf{s}_{t-L-M}, ..., \mathbf{s}_t, ..., \mathbf{s}_{t+l+M}\} & , l < L \\ sp\{\mathbf{s}_{t-L-w}, ..., \mathbf{s}_{t-1}\} \cup sp\{\mathbf{s}_{t+l-L+1}, ..., \mathbf{s}_{t+l+M}\} & , L \leq l \end{cases}$$

Projecting $\mathbf{x}_{t+i}$, $i = 0, ..., l$, into Z_l, following results are obtained through Theorem-1 in [3].

Let $\mathbf{E}_l$ be least squares smoothing error matrix defined by

$$\mathbf{E}_l = \begin{bmatrix} \tilde{\mathbf{x}}_{t+l|Z_l} \\ \vdots \\ \tilde{\mathbf{x}}_{t|Z_l} \end{bmatrix} \tag{2.44}$$

then,

$$\mathbf{E}_l = \begin{cases} 0 & , l < L \\ H_l(\mathbf{h}) \begin{bmatrix} \tilde{\mathbf{s}}_{t+l-L|Z_l} \\ \\ \tilde{\mathbf{s}}_{t|Z_l} \end{bmatrix} & , L \leq l \end{cases} \tag{2.45}$$

where

$$H_l(\mathbf{h}) = \begin{bmatrix} \mathbf{h}(L) & & \\ \vdots & \ddots & \\ \mathbf{h}(0) & \ddots & \mathbf{h}(L) \\ & \ddots & \vdots \\ & & \mathbf{h}(0) \end{bmatrix} \tag{2.46}$$

The above result is the center of the approach especially when the channel order is unknown. In the case of $l < L$ (Figure 2.4.a), the projection space Z_l includes $\mathbf{s}_t$. Since $\mathbf{x}_t, ..., \mathbf{x}_{t+l}$ all

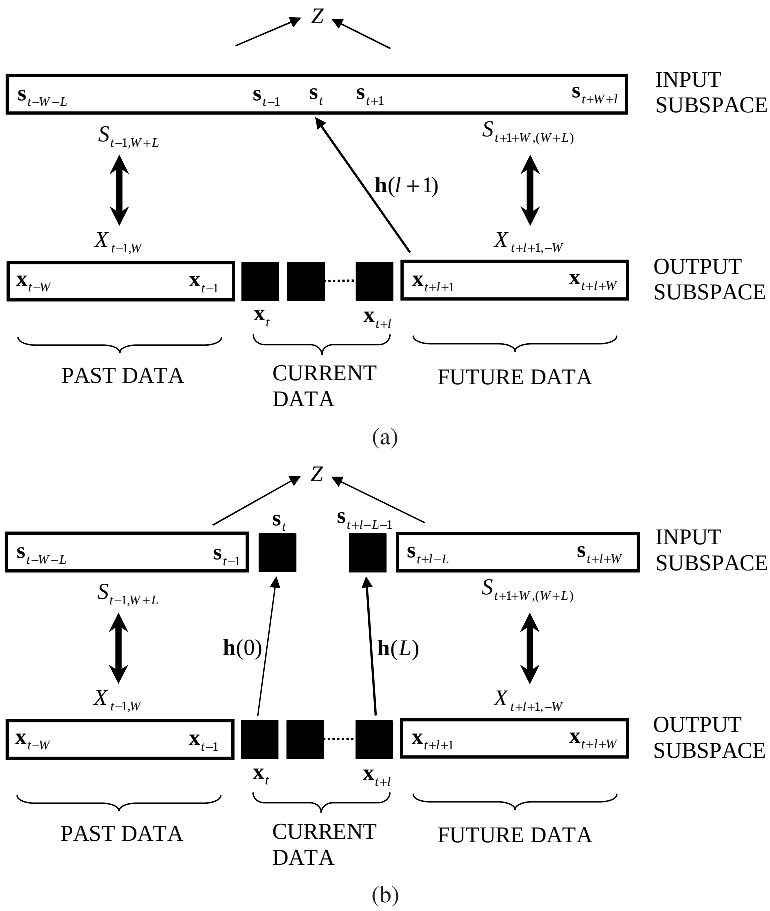


Figure 2.4: Isomorphism between input and output subspaces for $l \neq L$ (a) $l < L$ (b) $l > L$

includes $\mathbf{s}_t$, they also lie in the projection space. As a result, when $\mathbf{x}_t, ..., \mathbf{x}_{t+l}$ are projected onto Z_l, no projection error exists, i.e., $\mathbf{E}_l = \mathbf{0}$. When $l = L$, we have the case described in previous section, where the channel vector spans the column space of $\mathbf{E}_l$. When we choose the channel order greater than L (Figure 2.4.b), $\mathbf{s}_{M+1}, .., \mathbf{s}_{M+l-L}$ are not in Z_l. Since each of $\mathbf{x}_t, ..., \mathbf{x}_{t+l}$ does not lie in Z_l, they contribute the least squares smoothing error which is formulated in (2.45).

2.5.4 Joint Order Detection and Channel Estimation by Using LSS (LSS) [3]

The idea here is to fit the smoothing error matrix, $\mathbf{E}_l$, by jointly choosing the channel order and channel impulse response. With a fixed L_u as the upper bound of the channel order L, from equation (2.45) under assumption of linear complexity, A2, we have

$$C\{\mathbf{E}_{L_u}\} = C\{H_{L_U}\} \tag{2.47}$$

Let $\mathbf{Q}=\begin{bmatrix} \mathbf{Q}_0 & \cdots & \mathbf{Q}_{L_u} \end{bmatrix}$ be the matrix whose row vectors are orthogonal to the column space of $\mathbf{E}_l$ with dimension $(P(L_u + 1) - L_u + L - 1) \times P(L_u + 1)$. $\mathbf{Q}_i$ $i = 1, .., Lu$ are the partitioned matrix of $\mathbf{Q}$ each having P columns.

$$\begin{bmatrix} \mathbf{Q}_0 & \cdots & \mathbf{Q}_{L_u} \end{bmatrix} \mathbf{E}_{L_u} = \mathbf{0} \tag{2.48}$$

which implies

$$\begin{bmatrix} \mathbf{Q}_0 & \cdots & \mathbf{Q}_L \\ & Block & \vdots \\ & Hankel & \\ & & \mathbf{Q}_{L_u} \end{bmatrix} \begin{bmatrix} \mathbf{h}(L) \\ \vdots \\ \mathbf{h}(0) \end{bmatrix} = T_L(Q)\mathbf{h} = \mathbf{0} \tag{2.49}$$

The remaining is to show that the solution of the homogenous linear equation $T_L(Q)\mathbf{h} = \mathbf{0}$ is unique up to a complex scaling factor. Assume that k is treated as the estimated channel order, then we have

$$\begin{bmatrix} \mathbf{Q}_0 & \cdots & \mathbf{Q}_k \\ & Block & \vdots \\ & Hankel & \\ & & \mathbf{Q}_{L_u} \end{bmatrix} \mathbf{z} = T_k(Q)\mathbf{z} = \mathbf{0}, \quad 1 \le k < L_u \tag{2.50}$$

The homogenous equation $T_k(\mathbf{Q})\mathbf{z} = \mathbf{0}$ has unique non-trivial solution $\mathbf{z} = \alpha\mathbf{h}$ when $k = L$, (i.e., the dimension of null space of $T_k(\mathbf{Q})$ is one and the basis for $N(T_k(\mathbf{Q}))$ is a scaled vector of **h**). Otherwise there is only trivial solutions (i.e., since $T_k(\mathbf{Q})$ is full column rank, only element in null space of $\mathrm{T}_k(Q)$ is null vector and as a result there is only one solution $\mathbf{z} = \mathbf{0}$), [3]. Under this statement the criteria is,

$$\{L, \mathbf{h}\} = \underset{k,||\mathbf{h}||=1}{\arg\min} ||T_k(\mathbf{Q})\mathbf{h}|| \tag{2.51}$$

The above equation has a closed form solution involving the left singular vector of $T_k(\mathbf{Q})$ corresponding to the smallest singular value.

2.5.5 Algorithm:

1) Obtain the projection subspace from the observation data for an upper bound on channel order, L_u. For this purpose first construct the data structure below.

$$Z_{M,L_u} = \begin{bmatrix} y(2M+L_u+1) & \cdots & N \\ \vdots & \mathbf{F}_{M,L_u} & \\ y(M+L_u+2) & \cdots & \\ -------- & -- & -- \\ y(M+L_u+1) & & \\ \vdots & \mathbf{Y}_{M,L_u} & \\ y(M+1) & \cdots & \\ -------- & -- & -- \\ y(M) & \cdots & \\ \vdots & \mathbf{P}_{M,L_u} & \\ y(1) & \cdots & \end{bmatrix} \tag{2.52}$$

where overall data matrix $\mathbf{Z}_{M,L_u}$ is defined under a fixed predictor size $M \geq M_0$ and upper bound for channel order L_u. $\mathbf{F}_{M,L_u}$, $\mathbf{P}_{M,L_u}$, $\mathbf{Y}_{M,L_u}$ are the future past and current data matrices respectively. Based on the data matrix, $\mathbf{Z}_{M,l}$, determine the $2M + 2L_u$ orthogonal basis $\{\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_{2M+2L_u}\}$ which spans the row space of future-past data matrix, $\mathbf{D}_{w,l} = \begin{bmatrix} F_{M,L_u} \\ P_{M,k} \end{bmatrix}$.

2) Obtain the projection error matrix of $\mathbf{Y}_{M,L_u}$ onto $sp\{\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_{2M+2L_u}\}$

$$\mathbf{E}_{L_u} = \mathbf{Y}_{M,L_u} - \mathbf{Y}_{M,L_u}\mathbf{U}^H\mathbf{U}\ , \quad \mathbf{U} = \begin{bmatrix} \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{2M+2L_u} \end{bmatrix} \tag{2.53}$$

3) For each $1 \leq k \leq L_u$, treated as the estimated channel order, let $\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_0 & \cdots & \mathbf{Q}_l \end{bmatrix}$ be a matrix whose rows are the smallest $P(L_u+1) - L_u + k - 1$ left singular vectors of E_{L_u} and form

$$T_k(\mathbf{Q}) = \begin{bmatrix} \mathbf{Q}_0 & \cdots & \mathbf{Q}_k \\ & \mathit{Block} & \vdots \\ & \mathit{Hankel} & \\ & & \mathbf{Q}_{L_u} \end{bmatrix}$$

4) Joint order and channel estimation:

$$\{L, \mathbf{h}\} = \arg\min_{k,||\mathbf{h}||=1} ||T_k(\mathbf{Q})\mathbf{h}||$$

Left singular vector of $T_k(\mathbf{Q})$ corresponding to the smallest singular value can be taken as a solution.

2.6 Linear Prediction Method

Linear prediction method is first proposed by Slock [14], it is based on the fact that moving average (MA) SIMO channel output can also be represented as AR process, whose innovation is the SIMO channel input. Yule-Walker (YW) equations are solved to obtain zero delay zero forcing equalizer. Channel impulse response is derived from equalizer equations. It uses SOS to construct the YW equations and needs pseudoinverse of the covariance matrix. LP algorithm uses statistical characteristic of the inputs and based on the second order statistics. It assumes that, the channel input signal is Gaussian distributed white signal. Therefore it is not a deterministic algorithm as opposed to SS, CR and LSS algorithms. Therefore in noise free case it does not give the exact channel coefficients from finite number of samples. As cited in [30] the most striking property of the LP algorithm is the robustness to channel order overestimation, since the m_a order AR process can be treated as m_b^{th} order AR process when $m_a > m_b$. However in [4] and [18], it is claimed that it is not the case when the estimated SOS is used and the channel order is over estimated more then two degrees. It is understood that this technique is sensitive to observation noise and its performance depends on the channel statistics [16]. Therefore a new algorithm which is claimed to be robust to channel order overestimation is proposed in [16]. It turns out that this algorithm outperforms the LP algorithm in [14]. In [17] SNR optimum linear prediction equalizer is proposed and it is shown that [17] and [16] have the same asymptotically behavior whereas [17] performs better especially for short data lengths. In this section, a review of the original LP method [15] and the modified LP method (MLP) [17] is given.

Referring to the section 2.2, the channel output vector, $\mathbf{y_1}(t)$, which is composed of single samples from each channel outputs, is given as follows in noise free case.

$$\mathbf{y}_1(t) = \sum_{k=0}^{L} \mathbf{h}_L(k)\, s(t-k). \tag{2.54}$$

This equation can also be written as follows.

$$\mathbf{y}_1(t) = \mathbf{H}_1(z)s(t) \tag{2.55}$$

Where, $\mathbf{H}_1(z) = \sum_{k=0}^{L} \mathbf{h}_L(k)z^{-k}$. Under channel identifiability condition, according to the Bezout Identity [37], there exists inverse filter matrix $\mathbf{G}(z)$ such that,

$$\mathbf{G}(z)\mathbf{H}_1(z) = \mathbf{I} \tag{2.56}$$

i.e.,

$$\mathbf{G}(z)\mathbf{y}_1(t) = s(t) \tag{2.57}$$

Where $\mathbf{G}(z) = \sum_{k=0}^{K+1} \mathbf{g}_k z^{-k}$ and $\mathbf{g}_k = \begin{bmatrix} g_1(k) & \cdots & g_P(k) \end{bmatrix}^T$. $g_i(k)$ is the k^{th} coefficient of the i^{th} equalizer filter.

The equation (2.57) implies that $\mathbf{y}_1(t)$ is an AR process. Therefore linear prediction filter exits for $\mathbf{y}_1(t)$. For the K^{th} order linear prediction, the estimated channel output vector $\hat{\mathbf{y}}_1(t)$ is written as follows,

$$\widehat{\mathbf{y}}_1(t) = \sum_{k=1}^{K+1} \mathbf{P}_k \mathbf{y}_1(t-k) \tag{2.58}$$

where $\mathbf{P}_k$ are the prediction filter coefficient vector. The prediction filter coefficients can be

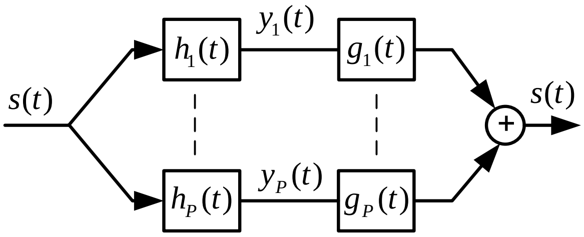


Figure 2.5: Linear prediction algorithm. Zero forcing equalization.

determined by minimizing the mean square of the prediction error,

$$\begin{aligned}
\mathbf{e}(t) &= \mathbf{y}_1(t) - \widehat{\mathbf{y}}_1(t) && (2.59)\\
&= \mathbf{y}_1(t) - \sum_{k=1}^{K+1} \mathbf{P}_k \mathbf{y}_1(t-k) && (2.60)\\
&= \mathbf{y}_1(t) - \begin{bmatrix} \mathbf{P}_1 & \cdots & \mathbf{P}_{K+1} \end{bmatrix} \mathbf{y}_K(t-1) && (2.61)
\end{aligned}$$

where $\mathbf{y}_K(t-1) = \mathbf{H}_K \mathbf{s}_{K+L}(t-1)$. The orthogonality principle leads to $E\left\{\mathbf{e}(t)\mathbf{y}_K^H(t-1)\right\} = \mathbf{0}$, hence the following equation can be written to obtain linear prediction coefficients.

$$\begin{aligned}
\begin{bmatrix} \mathbf{P}_1 & \cdots & \mathbf{P}_{K+1} \end{bmatrix} \mathbf{R}_K &= -\begin{bmatrix} \mathbf{r}(1) & \cdots & \mathbf{r}(K+1) \end{bmatrix} && (2.62)\\
\begin{bmatrix} \mathbf{P}_1 & \cdots & \mathbf{P}_{K+1} \end{bmatrix} &= -\begin{bmatrix} \mathbf{r}(1) & \cdots & \mathbf{r}(K) \end{bmatrix} \mathbf{R}_K^{\dagger} && (2.63)
\end{aligned}$$

where,

$$\mathbf{r}(k) = E\left\{\mathbf{y}_1(t)\mathbf{y}_1^H(t-k)\right\} \tag{2.64}$$

$$\mathbf{R}_K = E\left\{\mathbf{y}_K(t)\mathbf{y}_K^H(t-k)\right\} = \begin{bmatrix} \mathbf{r}(0) & \mathbf{r}(1) & \cdots & \mathbf{r}(K) \\ \mathbf{r}^H(1) & \mathbf{r}(0) & & \\ \vdots & & \ddots & \\ \mathbf{r}^H(K) & & & \mathbf{r}(0) \end{bmatrix} \tag{2.65}$$

Under this solution, the prediction error is found as follows,

$$\mathbf{e}(t) = \mathbf{h}_L(0)s(t) \tag{2.66}$$

$\mathbf{e}(t)$ is a rank one process and the input signal $\mathbf{s}(t)$ can be obtained by multiplying $\mathbf{e}(t)$ from left by a vector $\mathbf{f}^{\mathbf{T}}$ which is the singular vector associated the largest singular value of the covariance matrix of the prediction error, $\mathbf{D} = E\left\{\mathbf{e}(t)\mathbf{e}^H(t)\right\} = \mathbf{r}(0) + \sum_{k=1}^{K+1} \hat{\mathbf{P}}_k \mathbf{r}(k)$.

$$\mathbf{f}^T\mathbf{e}(t) = s(t) \tag{2.67}$$

Using the above equation, equalization filter $\mathbf{G}(z)$ can be obtained as follows.

$$\begin{aligned}
\mathbf{f}^T\mathbf{e}(t) &= s(t) && (2.68)\\
\mathbf{f}^T\left(y_1(t) - \sum_{k=1}^{K+1}\hat{\mathbf{P}}_k\mathbf{y}_1(t-k)\right) &= s(t) && (2.69)\\
\mathbf{f}^T\hat{\mathbf{P}}(z)\mathbf{y}_1(t) &= s(t) && (2.70)
\end{aligned}$$

Hence,

$$\hat{\mathbf{G}}(z) = \mathbf{f}^T\hat{\mathbf{P}}(z) \tag{2.71}$$

where, $\hat{\mathbf{P}}(z) = I - \sum_{k=1}^{K+1}\hat{\mathbf{P}}_k z^{-k}$.

The channel coefficients can be obtained as a follows,

$$\hat{\mathbf{h}}(k) = E\{\mathbf{y}_1(t)s(t-k)\} = E\left\{\mathbf{y}_1(t)\hat{\mathbf{G}}(z)\mathbf{y}_1(t-k)\right\} \tag{2.72}$$

in matrix form,

$$\hat{\mathbf{h}} = \begin{bmatrix} \mathbf{r}(0) & \mathbf{r}(1) & \cdots & \mathbf{r}(K) \\ \mathbf{r}(1) & \mathbf{r}(2) & & \mathbf{0} \\ \vdots & & & \vdots \\ \mathbf{r}(K) & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \hat{\mathbf{G}}^T \tag{2.73}$$

The main problem of LP method is that prediction error increases when the first channel coefficient is small. With modified LP, [17] this problem is solved in a certain extent and SNR optimum LP solution is obtained. The main difference from LP stems from obtaining $\mathbf{f}$ and the matrix $\mathbf{P}$, which is the matrix form of $\mathbf{P}(z)$. In MLP, $\mathbf{P}$ matrix obtained in LP is modified such that its columns are orthogonal. In the same manner, $\mathbf{f}$ is the singular vector associated the largest singular value of $\mathbf{P}^H\mathbf{R}_K\mathbf{P}$. MLP methods is more robust channel order overestimation and has better performance when the channel has small tail coefficients.

2.7 Channel Order Estimation

Blind channel order estimation is a critical task required for blind system identification. Deterministic channel estimation algorithms such as SS, CR, and LSS require exact channel order information. There are several algorithms for the channel order estimation in the literature. Minimum Description Length (MDL) [19] and Akaike Information Criteria (AIC) [20] algorithms are based on the information theoretic criteria. These algorithms require long observations for accurate extraction of the statistical parameters. It is known that both of these algorithms are very sensitive to deviation from idealized Gaussian white noise signal, number of observations and variations in SNR [4]. Therefore they are not robust approaches and their performances are not consistent and reliable in realistic scenarios.

In practice, channel impulse response contains long leading and trailing tails as in the case of microwave channels. Usually the channel is modeled to contain only the significant part of the impulse response for those cases. The order of the significant part is defined as the effective channel order whereas the true channel order indicates the whole channel filter including the tails. In [22], it is shown that when the channel model includes a part of the tail, the estimation performance of the subspace and least squares algorithms decrease dramatically. In [5] a channel order estimation algorithm is proposed for the effective channel order estimation. This algorithm is based on numerical analysis arguments and essentially consider the gap between the two consecutive eigenvalues of the estimated covariance matrix.

Most of the cost functions derived for channel order estimation monotonically decreases as the channel order increases, which makes it hard to find the channel order. This problem is tried to be overcome by using an empirically chosen penalty coefficient [23]. This penalty term leads to over or underestimation in many of the information theoretic techniques. In [24], a new cost function is proposed. This cost function is obtained by combining two cost functions due to channel identification (ID) and channel equalization (EQ), and hence ID+EQ algorithm is obtained. The main feature of this cost function is its ”concave - like” shape. Therefore channel order estimation can be performed by finding the global minimum. ID+EQ algorithm is a deterministic method and has finite convergence property as JLSS. It is claimed that it olso perform well in finding the effective channel order.

In this section, channel order estimation algorithms MDL, AIC, Liavas, and ID+EQ algorithms which are referenced throughout the thesis are summarized.

2.7.1 Akaiki Information Criteria (AIC) [20]

AIC is basically a rank determination method. It uses the covariance matrix to determine the dimension of the signal or noise subspace to identify the channel length. It assumes that the channel input is a Gaussian distributed white signal. Under this assumption, correlation matrix of channel the output can be written as follows:

$$\mathbf{R}_y = E\left\{\mathbf{y}_M(t)\mathbf{y}_M^H(t)\right\} = \sigma_s^2\mathbf{H}_M\mathbf{H}_M^H + \sigma_v^2\mathbf{I} \tag{2.74}$$

The SVD of $\mathbf{R}_y$ is given as follows, (assuming that channel identifiability condition is hold),

$$\mathbf{R}_y = \begin{bmatrix} \mathbf{U}_s & \vdots & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \lambda_1+\sigma_v^2 & & & & & \\ & \ddots & & & & \\ & & \lambda_{M+L}+\sigma_v^2 & & & \\ & & & \sigma_v^2 & & \\ & & & & \ddots & \\ & & & & & \sigma_v^2 \end{bmatrix} \begin{bmatrix} \mathbf{U}_s \\ \mathbf{U}_n \end{bmatrix} \tag{2.75}$$

where σ_s^2 and σ_v^2 are the channel input signal and noise variances respectively. $\mathbf{U}_s$ is the eigenvector matrix corresponding the eigenvalues $\lambda_1 + \sigma_v^2 \ldots \lambda_{M+L} + \sigma_v^2$ and its columns span the signal space. $\mathbf{U}_n$ is the eigenvector matrix corresponding eigenvalues equal to noise variance, σ_v^2 and its columns span the noise space. If one can find the dimension of one of these spaces, channel order can be found. In noise free case, the number of nonzero eigenvalues give the dimension of the signal space, which is equal to the number of columns of the channel matrix. Subtracting the value of M (which is a known value) from the found signal subspace dimension, the channel order is determined. However in noisy case, it is not so easy to separate the signal and noise spaces from each other, and the true data covariance matrix is not accessible in practice from finite number of samples. AIC algorithm uses statistical information to find the dimension of signal/noise spaces or equivalently the rank of the channel output matrix. AIC finds the channel order via maximum likelihood (ML) optimization and selects the model that minimizes,

$$AIC = -2\log\left(f\begin{pmatrix} \mathbf{y}_M(t) & \cdots & \mathbf{y}_M(t+N-1) & |\hat{\theta}\end{pmatrix}\right) + 2k \tag{2.76}$$

where $f\begin{pmatrix} \mathbf{y}_M(t) & \cdots & \mathbf{y}_M(t+N-1) & |\hat{\theta}\end{pmatrix}$ is the parameterized probability densities and $\hat{\theta}$ is the ML estimate of the parameter vector θ. Assuming that the observed vectors $\{\mathbf{y}_M(t+k)\}_{k=1}^N$

are zero mean i.i.d. Gaussian random vectors (note that, this is not the case for the channel identification problem, because the channel output samples are correlated as a consequence of FIR channel impulse response.), AIC obtains the dimension of signal space by minimizing the cost function given below [20], when the empirical covariance matrix $\hat{\mathbf{R}}_y = \frac{1}{N}\mathbf{Y}_M\mathbf{Y}_M^H$ is used.

$$AIC(k) = -2\log\left[\frac{\prod_{i=k+1}^{p}\lambda_i^{1/(MP-k)}}{\frac{1}{MP-k}\sum_{i=k+1}^{MP}\lambda_i}\right]^{(MP-k)N} + 2k(2MP-k) \tag{2.77}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{MP}$ are the eigenvalues of covariance matrix $\mathbf{R}_y$. MP is the size of the observation vector $\mathbf{y}_M(t)$ for P channel SIMO system. Channel order is the value of k that minimizes the cost function $AIC(k)$.

2.7.2 Minimum Description Length (MDL) [19]

MDL is similar to the AIC and selects the model that minimizes,

$$MDL = -\log\Big(f\Big(\begin{matrix}\mathbf{y}_M(t) & \cdots & \mathbf{y}_M(t+N-1)\end{matrix}\ |\hat{\theta}\Big)\Big) + \frac{1}{2}k\log N \tag{2.78}$$

which results the following cost function to find the dimension of signal space.

$$MDL(k) = -\log\left[\frac{\prod_{i=k+1}^{p}\lambda_i^{1/(MP-k)}}{\frac{1}{MP-k}\sum_{i=k+1}^{MP}\lambda_i}\right]^{(MP-k)N} + \frac{1}{2}k(2MP-k)\log N \tag{2.79}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{MP}$ are the eigenvalues of the empirical covariance matrix $\hat{\mathbf{R}}_y$. Channel order is the value of k that minimizes the cost function $MDL(k)$.

MDL is an asymptotically consistent algorithm in contrast to the AIC. When the number of snapshots is increased, probability of wrong detection of the channel order goes to zero [23]. It is more robust to deviation from the Gaussian assumption on the observation vectors.

2.7.3 Liavas Algorithm [4]

Liavas algorithm is proposed for the estimation of effective channel order from channel outputs when the channel impulse is long and includes small long channel tails. In that case, true channel and effective channel order are different and using effective channel order to model the system is more practical. Effective rank of the covariance matrix of the received signal $\mathbf{R}_M = E\left\{\mathbf{y}_M(t)\,\mathbf{y}_M(t)^T\right\}$ is used. $\mathbf{y}_M(t)$ is the channel output vector formed via stacking M samples received from each subchannels. Effective rank, $r(k)$ is defined as $\bar{k} = \arg\min_k r(k)$ and

$$r(k) = \begin{cases} \frac{1}{\frac{\lambda_k}{\lambda_{k+1}}-2} & , if \frac{\lambda_k}{\lambda_{k+1}} \geq 3 \\ 1 & otherwise \end{cases} \tag{2.80}$$

where λ_k is the k^{th} eigenvalue of the matrix $\mathbf{R}_M$, such that $\lambda_k \geq \lambda_{k+1}$. It is also possible to write,

$$\bar{k} = \arg\max_k \frac{\lambda_k}{\lambda_{k+1}} \tag{2.81}$$

Then the effective channel order is gives as $L_e = \bar{k}$.

2.7.4 ID+EQ Algorithm [24]

ID+EQ algorithm minimizes a combination of a blind channel cost function, which decreases with the channel order and a blind equalization cost function, which increases with estimated channel order. As a blind channel estimator, CR algorithm proposed in [8], and as an equalizer the method proposed in [13] for direct equalization of channel are used. In noise free case, it was shown that the identification term is zero when the channel order is exact or over estimated and the equalization term is zero when the channel order is exact or underestimated. These two cost functions are summed to produce a cost function that has minimum at the true channel order. ID+EQ is a deterministic method and has finite convergence property. In the following sections, we summarize the algorithm by giving the identification and the equalization cost functions used in the algorithm.

2.7.4.1 Identification Part

CR algorithm [8] obtains the channel order that minimizes the following cost function.

$$J_{id}\left(\hat{L}\right) = \frac{1}{2}\sum_{k,m=1}^{P}\left\|\mathbf{Y}_k\left(\hat{L}\right)\hat{\mathbf{h}}_m - \mathbf{Y}_m\left(\hat{L}\right)\hat{\mathbf{h}}_k\right\|^2, \quad k = 1,\ldots,P \tag{2.82}$$

where

$$\mathbf{h}_m = \begin{bmatrix} h_{\hat{L},m}\left(\hat{L}\right), & \cdots & h_{\hat{L},m}(0) \end{bmatrix}^T \tag{2.83}$$

$$\mathbf{Y}_m(L) = \begin{bmatrix} y_m(t) & y_m(t+1) & \cdots & y_m\left(t+\hat{L}\right) \\ y_m(t+1) & y_m(t+2) & \cdots & y_m\left(t+\hat{L}+1\right) \\ \vdots & \vdots & \ddots & \vdots \\ y_m\left(t+N-\hat{L}-1\right) & y_m\left(t+N-\hat{L}\right) & \cdots & y_m(t+N-1) \end{bmatrix} \tag{2.84}$$

The cost function can also be written as follows,

$$J_{id}\left(\hat{L}\right) = 1 - \sum_{\substack{k,m=1 \\ k\neq m}}^{P} \hat{\mathbf{h}}_m^H \mathbf{R}_{km} \hat{\mathbf{h}}_k \tag{2.85}$$

where $\mathbf{R}_{km}\left(\hat{L}\right) = \mathbf{Y}_k^H\left(\hat{L}\right)\mathbf{Y}_m\left(\hat{L}\right)$. The key point is that, when the channel order is overestimated, the channel transfer functions obtained via CR algorithm are in the following form,

$$\hat{h}_{k,\hat{L}}(t) = h_{k,L}(t) * c_{\hat{L}-L}(t) \tag{2.86}$$

That is the overestimated channel order generates common zeros besides the true channel response $\hat{h}_{k,\hat{L}}(t)$.

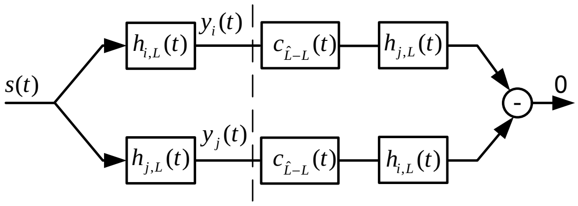


Figure 2.6: CR blind channel identification for SIMO channel.

In figure 2.6 this situation can be seen for two channel case. Since the extra zeros are common, two branches of the channels are equal as in the case of exact channel order. Therefore the cost function will be zero for the overestimated channel orders.

The channel vector, $\hat{\mathbf{h}} = \begin{bmatrix} \hat{\mathbf{h}}_1^T & \cdots & \hat{\mathbf{h}}_P^T \end{bmatrix}^T$, that minimizes the cost function under the constraint $\sum_{\substack{k,m=1 \\ k\neq m}}^{P} \left\|\hat{\mathbf{Y}}_k\hat{\mathbf{h}}_m\right\|^2 = 1$ (which is used to avoid nontrivial solutions) is given as the eigenvector associated the largest eigenvalue of the following generalized eigenvalue (GEV) problem:

$$\mathbf{R}\left(\hat{L}\right)\hat{\mathbf{h}} = \left(1 - J_{id}\left(\hat{L}\right)\right)\mathbf{D}\left(\hat{L}\right)\hat{\mathbf{h}} \tag{2.87}$$

where

$$\mathbf{R}\left(\hat{L}\right) = \begin{bmatrix} \mathbf{0} & \mathbf{R}_{21}\left(\hat{L}\right) & \cdots & \mathbf{R}_{P1}\left(\hat{L}\right) \\ \mathbf{R}_{12}\left(\hat{L}\right) & \mathbf{0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{R}_{P(P-1)}\left(\hat{L}\right) \\ \mathbf{R}_{1P}\left(\hat{L}\right) & \cdots & \mathbf{R}_{(P-1)P}\left(\hat{L}\right) & \mathbf{0} \end{bmatrix} \tag{2.88}$$

$$\mathbf{D}\left(\hat{L}\right) = \begin{bmatrix} \sum_{k=2}^{P} \mathbf{R}_{kk}\left(\hat{L}\right) & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \sum_{k=2}^{P-1} \mathbf{R}_{kk}\left(\hat{L}\right) \end{bmatrix} \tag{2.89}$$

When the channel order is underestimated channel identification is not possible and $J_{id}\left(\hat{L}\right) > 0$. When the channel order is overestimated, the cost function is equal to zero. When the correct channel order is known, channel is identified up to a complex scalar and the cost function is equal to zero.

2.7.4.2 Equalization Part

Direct equalization method proposed in [13] is used. In that method, equalization is realized without the estimated channel coefficients. We will review the direct equalization algorithm and give equalization cost function in this section. For this purpose, let us start with the definition of row vectors $\tilde{\mathbf{h}}(n) = \begin{bmatrix} h_{L,1}(n) & \cdots & h_{L,P}(n) \end{bmatrix}$, $\tilde{\mathbf{y}}(n) = \begin{bmatrix} y_1(n) & \cdots & y_P(n) \end{bmatrix}$

and matrices shown below.

$$\tilde{\mathbf{Y}}_k\left(\hat{L}\right) = \begin{bmatrix} \tilde{\mathbf{y}}(K+k) & \cdots & \tilde{\mathbf{y}}(k) \\ \tilde{\mathbf{y}}(K+k+1) & \cdots & \tilde{\mathbf{y}}(k+1) \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{y}}\left(N-\hat{L}-K+k-1\right) & \cdots & \tilde{\mathbf{y}}\left(N-\hat{L}-2K+k-1\right) \end{bmatrix} \tag{2.90}$$

$$\tilde{\mathbf{S}}_k\left(\hat{L}\right) = \begin{bmatrix} s(K+k) & \cdots & s(k-L) \\ s(K+k+1) & \cdots & s(k-L+1) \\ \vdots & \ddots & \vdots \\ s\left(N-\hat{L}-K+k-1\right) & \cdots & s\left(N-\hat{L}-2K+k-L-1\right) \end{bmatrix} \tag{2.91}$$

$$\mathrm{T}_h = \begin{bmatrix} \tilde{\mathbf{h}}(0) & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \tilde{\mathbf{h}}(0) & \ddots & \vdots \\ \tilde{\mathbf{h}}(L) & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{h}}(L) & & \tilde{\mathbf{h}}(0) \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \tilde{\mathbf{h}}(L) \end{bmatrix} \tag{2.92}$$

In noiseless case, SIMO system outputs can be written as follows,

$$\tilde{\mathbf{Y}}_k\left(\hat{L}\right) = \tilde{\mathbf{S}}_k\left(\hat{L}\right)\mathrm{T}_h \tag{2.93}$$

If T_h is full row rank, there exists a matrix,$\mathbf{W}(L) = \begin{bmatrix} \mathbf{w}_0(L) & \cdots & \mathbf{w}_{K+L}(L) \end{bmatrix}$ such that,

$$\tilde{\mathbf{Y}}_k\left(\hat{L}\right)\mathbf{W}(L) = \tilde{\mathbf{S}}_k \tag{2.94}$$

and for $k, m = 0, \ldots, K+L$, this matrix will satisfy

$$\tilde{\mathbf{Y}}_k\left(\hat{L}\right)\mathbf{w}_k(L) = \tilde{\mathbf{Y}}_m\left(\hat{L}\right)\mathbf{w}_m(L) \tag{2.95}$$

This equation is used to drive the equalization cost function which is given as,

$$J_{eq}\left(\hat{L}\right) = \frac{1}{2(K+L)}\sum_{k,m=0}^{K+\hat{L}}\left\|\tilde{\mathbf{Y}}_k\left(\hat{L}\right)\mathbf{w}_k(L) - \tilde{\mathbf{Y}}_m\left(\hat{L}\right)\mathbf{w}_m(L)\right\|^2 \tag{2.96}$$

$$= 1 - \sum_{\substack{k,m=0 \\ k\neq m}}^{K+\hat{L}}\mathbf{w}_k^H\left(\hat{L}\right)\mathbf{R}_{km}\mathbf{w}_m\left(\hat{L}\right) \tag{2.97}$$

and with a constraint $\sum_{k=0}^{K+\hat{L}}\left\|\tilde{\mathbf{Y}}_k\left(\hat{L}\right)\mathbf{w}_k\left(\hat{L}\right)\right\|^2 = 1$, and $\mathbf{R}_{km}\left(\hat{L}\right) = \tilde{\mathbf{Y}}_k^H\left(\hat{L}\right)\tilde{\mathbf{Y}}_m\left(\hat{L}\right)$. The solution that minimizes the cost function is given by the eigenvector associated largest eigenvalue

value of GEV:

$$\frac{1}{K+\hat{L}}\mathbf{R}\left(\hat{L}\right)\hat{\mathbf{w}}\left(\hat{L}\right)=\left(1-J_{eq}\left(\hat{L}\right)\right)\tilde{\mathbf{D}}\left(\hat{L}\right)\hat{\mathbf{w}}\left(\hat{L}\right) \tag{2.98}$$

where $\hat{\mathbf{w}}\left(\hat{L}\right)=\left[\begin{array}{ccc}\hat{\mathbf{w}}_0^T\left(\hat{L}\right) & \cdots & \hat{\mathbf{w}}_{K+\hat{L}}^T\left(\hat{L}\right)\end{array}\right]$ and,

$$\mathbf{R}\left(\hat{L}\right) = \left[\begin{array}{cccc}\mathbf{0} & \tilde{\mathbf{R}}_{01}\left(\hat{L}\right) & \cdots & \tilde{\mathbf{R}}_{0(K+\hat{L})}\left(\hat{L}\right)\\ \tilde{\mathbf{R}}_{10}\left(\hat{L}\right) & \mathbf{0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \tilde{\mathbf{R}}_{(K+\hat{L}-1)(K+\hat{L})}\left(\hat{L}\right)\\ \tilde{\mathbf{R}}_{(K+\hat{L})0}\left(\hat{L}\right) & \cdots & \tilde{\mathbf{R}}_{(K+\hat{L})(K+\hat{L}-1)}\left(\hat{L}\right) & \mathbf{0}\end{array}\right] \tag{2.99}$$

$$\mathbf{D}\left(\hat{L}\right) = \left[\begin{array}{ccc}\tilde{\mathbf{R}}_{00} & \cdots & \mathbf{0}\\ \vdots & \ddots & \vdots\\ \mathbf{0} & \cdots & \tilde{\mathbf{R}}_{(K+\hat{L})(K+\hat{L})}\end{array}\right] \tag{2.100}$$

In [24] It is shown that, when the channel order is overestimated equalization is not possible and $J_{eq}\left(\hat{L}\right) > 0$. When the channel is underestimated, there are infinite number of solutions and the input signal is obtained as $\hat{s}(n) = \left[\begin{array}{ccc}a_0 & \cdots & a_{L-\hat{L}}\end{array}\right] * s(n)$ in noise free case ($*$ indicates the convolution operation). Therefore, the equalization cost function is equal to zero for underestimated channel orders. When the channel order is known, channel is equalized perfectly and equalization cost function is equal to zero in noise free case.

2.7.4.3 Combined Cost Function

The determined channel order, equalization and identification cost function are summed to obtain a single cost function. Since both of the cost functions for the equalization and identification are limited between 0 and 1 as a result of the constraints defined on them, summation with equal weighting is enough to construct the combined cost function for the channel order estimation. Therefore overall cost function is,

$$J(\hat{L}) = J_{eq}\left(\hat{L}\right) + J_{id}\left(\hat{L}\right) \tag{2.101}$$

To find the channel order, the cost function defined in (2.101) is determined for the channel orders in the search set. The channel order that produces the minimum cost is taken as the true channel order.

CHAPTER 3

BLIND CHANNEL ORDER ESTIMATION

3.1 Introduction

Channel order estimation is an important problem in many signal processing applications. In this thesis, this problem is considered for finite impulse response (FIR) SIMO systems. In blind estimation for SIMO systems, the main problem is to obtain the channel coefficients or the input signal given the SIMO system outputs. The input and channel coefficients can be estimated accurately when the channel order is known.

Incorrect channel order estimation results performance loss especially for the channel estimation algorithms with finite convergence property such as SS [6], LSS [7] and cross relation (CR) [8]. On the other hand, there are methods that work in a robust manner in case of overestimated channel order [32, 14, 15, 33]. The main disadvantage of such techniques is that their performance is not as good as the SS or LSS algorithm when the true channel order is supplied to those algorithms [15]. Therefore channel order estimation is an important problem and it determines the performance of the channel estimation algorithms.

There are different algorithms for the channel order estimation in the literature. Minimum Description Length (MDL) [19] and Akaike Information Criteria (AIC) [20] algorithms are based on the information theoretic criteria. These algorithms require long observations for accurate extraction of the statistical parameters. It is known that MDL usually performs better than the AIC and AIC has a tendency for overestimation [4, 21]. Both of these algorithms are sensitive to colored noise [21].

Joint channel order and channel estimation with LSS method (JLSS) is presented in [3]. It is shown that JLSS can find the true channel order from finite number of samples in case of noise free observations. Joint least squares smoothing (JLSS) [3] and ID+EQ [24] are the two algorithms that can find the true channel order from finite number of samples in noise free case. Until now, JLSS and ID+EQ were the only algorithm known to have finite convergence property for the channel order estimation.

In this work, two new channel order estimation algorithms are proposed, namely the channel output error (COE) and channel matrix recursion (CMR) algorithms. Both of these algorithms have the finite convergence property. In other words, they find the true channel order by using finite number of samples for noise free case. In addition, they have several distinct features which make them practically the most effective algorithms known in the literature. Their performances are significantly better than the alternatives for noisy observations. They are robust to different parameters such as the number of channels, channel order and the number of input samples. COE algorithm is computationally demanding. In return, its performance gets better than CMR with the increase in the number of channels and the channel order. On the other hand, CMR performs better when the number of channels and the channel order is small. CMR has better computational efficiency than the COE algorithm. In this respect, COE and CMR complement each other nicely.

Most of the cost functions monotonically decrease as the channel order increases, which makes it hard to find the true channel order. This problem is tried to be overcome by using an empirically chosen penalty coefficient [23]. This penalty term leads to over or underestimation in many of the information theoretic techniques. In [24], a new cost function is proposed. This cost function is obtained by combining two cost functions due to channel identification (ID) and channel equalization (EQ), and hence ID+EQ algorithm is obtained. The main feature of this cost function is its ”convex - like” shape. Therefore channel order estimation can be performed by finding the global minimum. The motivation in this thesis is to construct a similar cost function, which allows us to obtain the channel order from the global minimum. In this respect, channel output error is chosen as one of the cost functions. In order to compute the COE, channel coefficients and the input signal are estimated for a given channel order. Then the channel output is regenerated and compared with the observed channel output. The cost function is the norm of the difference between the estimated and the observed channel outputs. It is proved that the proposed cost function has a global minimum at the true channel order for noise free case. COE has a ”convex-like” shape due to two main reasons. These are the generation of common channel zeros by the LSS algorithm [7] for the overestimated channel order and the unstacking operation for the regeneration of the input sequence. While a similar cost function is used for the channel estimation and frame synchronization in [34], through the use of pilot symbols, COE is not employed for the channel order estimation before.

As in the COE algorithm, CMR is also based on the properties of the LSS algorithm. Channel matrix is estimated using LSS algorithm for a range of channel orders. The relation between the channel matrices with consecutive channel orders are used to obtain a new cost function for channel order estimation. It has the finite convergence property. Therefore CMR finds the true channel order by using finite number of samples in noise free case. In addition, it has several distinct features which make it one of the most effective algorithm in the literature. Its performance is very good for noisy observations. Furthermore, CMR is robust to different parameters such as the number of channels, channel order and the number of samples.

Several experiments are done in order to compare the proposed methods with the alternative techniques such as MDL, AIC, Liavas [5], ID+EQ [24] and JLSS [3]. These algorithms are evaluated for random channels. It is shown that the proposed methods perform significantly better for noisy observations when different number of channels and channel orders are considered.

The organization of the chapter is as follows. Section 3.2 describes the system model and the problem. Proposed methods COE and CMR are presented in Section 3.3 and Section 3.4 respectively. The proof of the Lemmas and Theorems in these sections are given at the Appendix. The performances of the proposed methods are evaluated in Section 3.5. Finally, conclusion is given in section 3.6.

3.2 System Model and Problem Definition

The structure for a SIMO system is shown in Figure 3.1. $s(t)$ is the input signal, and there are P channels with channel order L. The channel output vector can be written as,

$$\mathbf{y}_1(t) = \sum_{k=0}^{L} \mathbf{h}_L(k)\, s(t-k) + \mathbf{n}_1(t) \tag{3.1}$$

where

$$\mathbf{y}_1(t) = \begin{bmatrix} y_1(t) & y_2(t) & \cdots & y_P(t) \end{bmatrix}^T \tag{3.2}$$

$$\mathbf{h}_L(k) = \begin{bmatrix} h_{L,1}(k) & h_{L,2}(k) & \cdots & h_{L,P}(k) \end{bmatrix}^T \tag{3.3}$$

$$\mathbf{n}_1(t) = \begin{bmatrix} n_1(t) & n_2(t) & \cdots & n_P(t) \end{bmatrix}^T \tag{3.4}$$

The $P \times 1$ vectors, $\mathbf{y}_1(t)$, $\mathbf{h}_L(k)$, and $\mathbf{n}_1(t)$ are the received signals, channel impulse response and additive noise respectively. $y_i(t)$, $h_i(k)$, and $n_i(t)$ are the scalar values of the output signal, channel impulse response and additive noise for the i^{th} channel respectively. The matrix formulation for the same model can be given as,

$$\mathbf{y}_1(t) = \mathbf{H}_1\mathbf{s}_{L+1}(t) + \mathbf{n}_1(t) \tag{3.5}$$

where,

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{h}_L(L) & \mathbf{h}_L(L-1) & \cdots & \mathbf{h}_L(0) \end{bmatrix} \tag{3.6}$$

$$\mathbf{s}_{L+1} = \begin{bmatrix} s(t-L) & \cdots & s(t) \end{bmatrix}^T \tag{3.7}$$

System output can be modified to include M samples for each channel and the following equation can be written,

$$\mathbf{y}_M(t) = \mathbf{H}_M\mathbf{s}_{M+L}(t) + \mathbf{n}_M(t) \tag{3.8}$$

where

$$\mathbf{y}_M(t) = \begin{bmatrix} \mathbf{y}_1^T(t-M+1) & \cdots & \mathbf{y}_1^T(t) \end{bmatrix}^T \tag{3.9}$$

$$\mathbf{n}_M(t) = \begin{bmatrix} \mathbf{n}_1^T(t-M+1) & \cdots & \mathbf{n}_1^T(t) \end{bmatrix}^T \tag{3.10}$$

$$\mathbf{s}_{M+L}(t) = \begin{bmatrix} s(t-L-M+1) & \cdots & s(t) \end{bmatrix}^T \tag{3.11}$$

$$\mathbf{H}_M = \begin{bmatrix} \mathbf{h}_L(L) & \cdots & \mathbf{h}_L(0) & & \\ & \ddots & \cdots & \ddots & \\ & & \mathbf{h}_L(L) & \cdots & \mathbf{h}_L(0) \end{bmatrix} \tag{3.12}$$

The $MP \times (MP+L)$ matrix, $\mathbf{H}_M$, is block Toeplitz with M block rows and with the first row equal to $\begin{bmatrix} \mathbf{H}_1 & 0_{P\times(M-1)} \end{bmatrix}$. $\mathbf{H}_M$ is called as the channel matrix. Equation (3.8) can be written compactly as,

$$\mathbf{Y} = \mathbf{H}_M\mathbf{S} + \mathbf{N} \tag{3.13}$$

where,

$$\mathbf{Y} = \left[\begin{array}{cccc} \mathbf{y}_M(t) & \mathbf{y}_M(t+M) & \cdots & \mathbf{y}_M\left(t+(N-1)M\right) \end{array}\right] \quad (3.14)$$

$$\mathbf{S} = \left[\begin{array}{cccc} \mathbf{s}_{M+L}(t) & \mathbf{s}_{M+L}(t+M) & \cdots & \mathbf{s}_{M+L}(t+(N-1)M) \end{array}\right] \quad (3.15)$$

$$\mathbf{N} = \left[\begin{array}{cccc} \mathbf{n}_M(t) & \mathbf{n}_M(t+M) & \cdots & \mathbf{n}_M(t+(N-1)M) \end{array}\right] \quad (3.16)$$

Note that the convolution expression $\mathbf{H}_M\mathbf{S}$ in equation (3.13) requires $\mathbf{S}$ to have a special form. In other words, the last L rows of a given column of $\mathbf{S}$ are the same as the first L rows of the following column.

Our goal is to estimate the unknown channel order L from the observations in a blind manner. The following assumptions are used in sequel.

A1. The subchannel transfer functions do not share common zeros .

A2. Input signal, $s(t)$, has a linear complexity greater than $2M + 2L$, where M is chosen such that the channel matrix is a tall matrix.

3.3 Channel Output Error (COE) Algorithm

COE algorithm is based on a cost function which has a global minimum at the true channel order. The generation of the cost function is presented in Figure 3.1. The channel coefficients are estimated with the LSS algorithm by assuming $\hat{L} = L_{min}, \ldots, L_{max}$. The operations are repeated for each value of $\hat{L}$. The input signal matrix, $\hat{\mathbf{S}}$, is obtained by the Moore-Penrose pseudoinverse of the channel matrix. Data unstacking is applied and the input signal, $\hat{s}(t)$, is extracted from $\hat{\mathbf{S}}$, which should have a special structure. The last L rows are the same as the first L rows for a given column for noise-free case. In case of noisy observations, data unstacking should be used to properly extract the input signal, $s(t)$, from $\mathbf{S}$. The data unstacking in Figure 3.1 corresponds to removing the last L rows of the input signal matrix, $\mathbf{S}$, and vectorizing the data. This step is important since the cost function becomes zero for overestimation, if $\hat{\mathbf{S}}$ is directly used to regenerate the system output. Once the channel coefficients and the input signal are available, SIMO system output is regenerated. The estimated output, $\hat{\mathbf{Y}}$, is then compared with the observed SIMO output, $\mathbf{Y}$. The channel output error is defined as the difference between these two terms, i.e.,

$$COE\left(\hat{L}\right) = \left\|\hat{\mathbf{Y}} - \mathbf{Y}\right\|_2, \quad \hat{L} = L_{min}, \ldots, L_{max} \quad (3.17)$$

The estimated channel order, L_{est} is found as,

$$L_{est} = \arg\min_{\hat{L}} \left\{ COE\left(\hat{L}\right) \right\} \tag{3.18}$$

In the following parts, we describe the steps for the proposed method in detail.

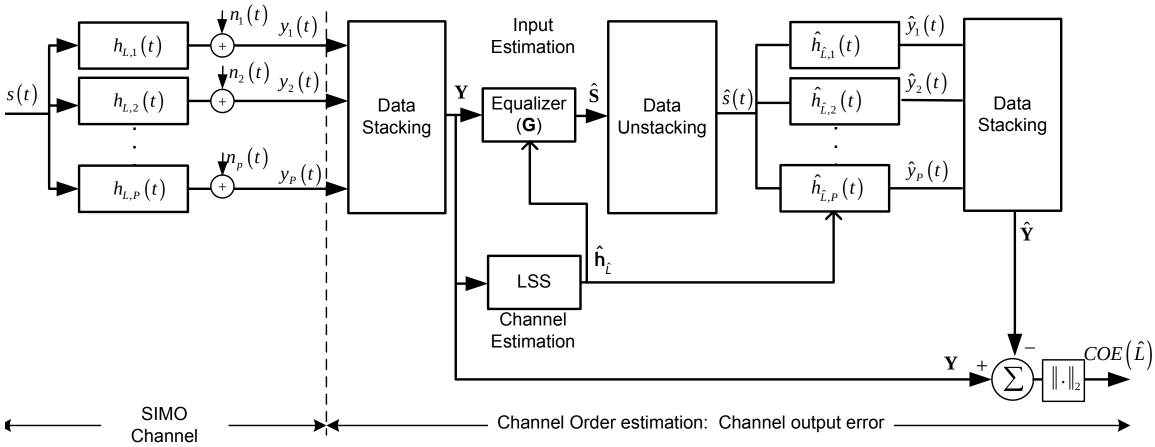


Figure 3.1: Channel output estimation for channel order $\hat{L}$ in COE algorithm.

3.3.1 Blind Channel Estimation

LSS algorithm [7, 3] is used to estimate the channel coefficients. LSS is a deterministic method which uses the isomorphic relation between the input and output signal spaces. An important property of this algorithm is that it has finite convergence property [7]. Therefore, it can work with small data packets and gives exact result in noise-free case. We have selected the LSS algorithm for channel estimation, since it generates common channel zeros, when the channel order is overestimated. In that case, channel matrix is not full column rank and COE cost function generates a non-zero value for the overestimation. Lemma-1 describes this property of the LSS algorithm.

Lemma-1: LSS algorithm [7, 3] generates common channel zeros for noise free case, when the channel order is overestimated. The remaining zeros are the true channel zeros. If the true channel order is L and the overestimated channel order is $\hat{L}$, the number of common zeros is $\hat{L} - L$.

Proof: The proof is given in Appendix.

Lemma-2 describes the condition of the channel matrix in case of common channel zeros.

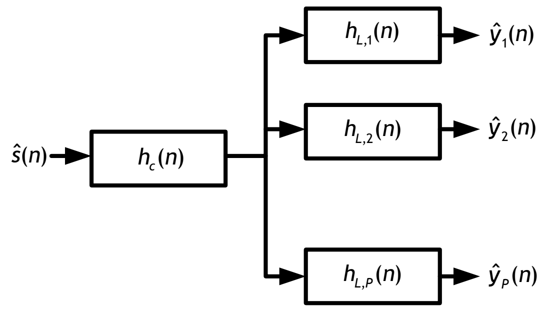


Figure 3.2: Estimated SIMO channel with common channel zeros.

While this is known in the literature, it is presented for the completeness of the discussion.

Lemma-2: If the FIR channels of SIMO system have common zeros, then the channel matrix, $\mathbf{H}_M$ whose row size is greater than the column size, is not full column rank.

Proof: The proof is given in Appendix.

As a result of Lemma-1, the channel transfer functions estimated by using LSS method include common zeros besides the true channel zeros, when the channel is overestimated. Then, the estimated channel is combination of a SIMO channel and a SISO channel, $h_c(n)$, whose zeros are the common zeros as shown in Figure 3.2.

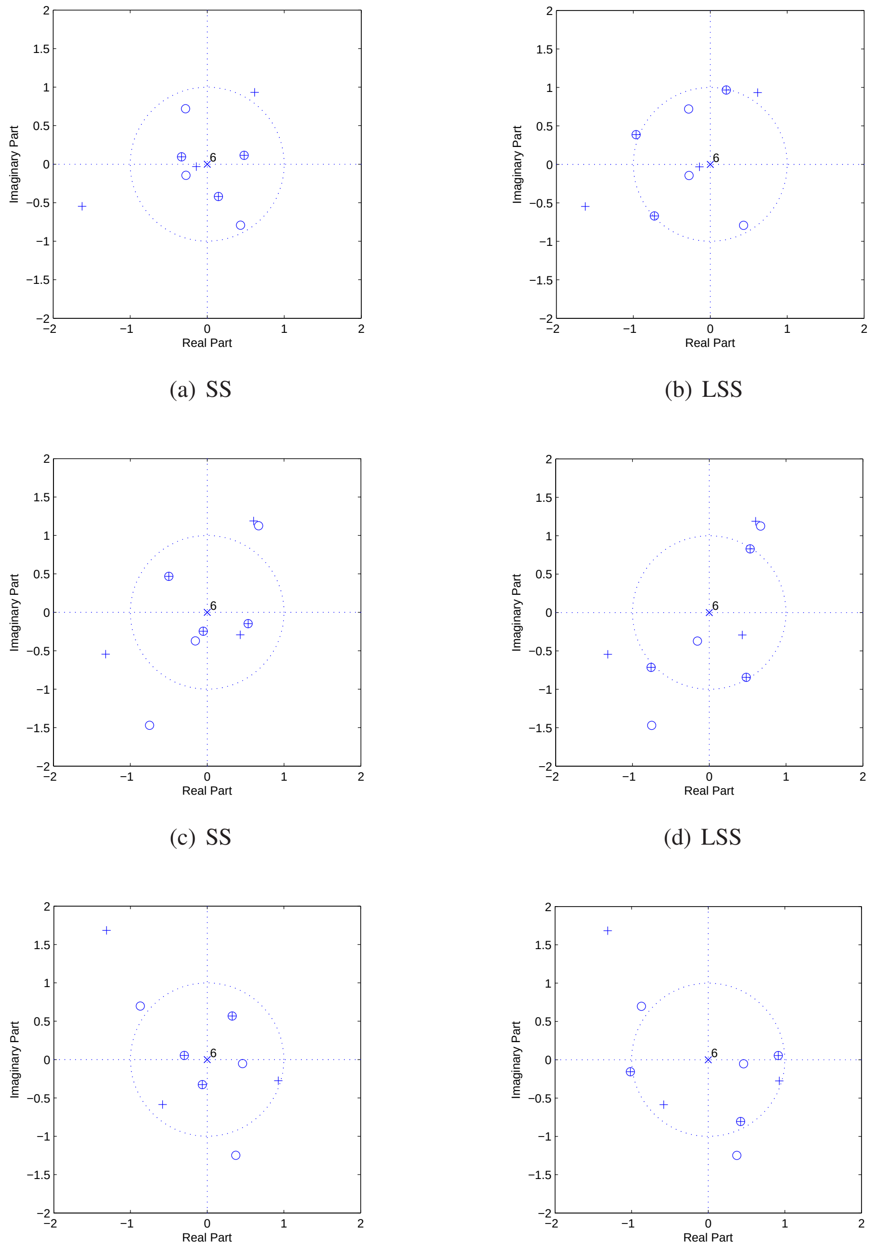
When we want to equalize the channel with the estimated channel coefficients, common channel transfer function, $h_c(n)$, is also equalized with Wiener equalizer besides the true channel. Wiener equalizer is a linear equalizer and perfect equalization of a FIR modeled SISO channel is not possible. As a result, channel input cannot be estimated truly when the channel order is not known.

Another important property of the LSS algorithm is that, the generated common zeros are located on the unit circle. This property of LSS algorithm is described by Lemma-3, and the proof is done for the overestimated channel order by one because of the complexity of the problem. This property is not shared by other blind algorithm, such as SS and CR, which also generate common zeros. When the common channel zeros are located on the unit circle, the inverse channel transfer function does not decay to zero as time index go to infinity. Therefore FIR equalization is not possible. Equalization error is large and increases as the input length increases due to stability problems. Hence, the error on the estimated channel input due to the equalization of the common channel zeros is higher compared to common zeros located inside the unit circle.

Lemma-3: When the channel order is overestimated by one, the common zero generated by the LSS algorithm is located on the unit circle.

Proof: The proof is given in Appendix.

In Figure 3.3, pole-zero plots are given for the estimated channels by SS and LSS, when the channel order is overestimated. The true channel order is $L = 3$ and the number of subchannels is $P = 2$. Channel order is taken as $\hat{L} = 6$ in SS and LSS algorithms. As shown in Figure 3.3, common channel zeros are generated by both of the algorithms. However, common zeros in LSS are close to the unit circle. Therefore, this gives a good idea about the performance of the LSS algorithm when it is used within the proposed channel order estimation methods. Since the equalization of channels with zeros on unit circle is harder, equalization error is higher for LSS case when the channel order is overestimated.



(a) SS (b) LSS

(c) SS (d) LSS

(e) SS (f) LSS

Figure 3.3: Pole-zero plots for the estimated channels by SS and LSS algorithms. At each row of the figure a different channel is used. True channel order $L = 3$ and the number of subchannels is $P = 2$. o and $+$ indicates the zeros of the first and second subchannels of the SIMO system.

3.3.2 Channel Equalization and Input Estimation

Once the channel coefficients are available, the input signal for the SIMO system can be obtained by using Moore-Penrose pseudoinverse of the channel matrix. The input signal matrix, $\mathbf{S}$, is obtained as $\hat{\mathbf{S}} = \mathbf{G}\mathbf{Y}$, where $\mathbf{G}$ is the pseudoinverse of $\hat{\mathbf{H}}_M$, $\mathbf{G} = \hat{\mathbf{H}}_M^{\dagger}$. Note that the pseudoinverse can be found even when $\hat{\mathbf{H}}_M$ is not full column rank. In that case, pseudoinverse uses singular value decomposition and singular values below a certain level are taken as zero for the inverse operation [35]. (Built in pseudoinverse function in MATLAB is directly used. Default threshold value defined in the Matlab function is used.)

3.3.3 Data Unstacking

For noise free case, SIMO system output can be obtained through a convolution operation which can be written as a matrix equation given below,

$$\mathbf{Y} = \mathbf{H}_M \mathbf{S} \tag{3.19}$$

This equation represents a true convolution operation only if the input signal data matrix, $\mathbf{S}$, is constructed appropriately as explained after the equation (3.13). When there is noise in the observations and $\mathbf{S}$ is estimated by the pseudoinverse of the channel matrix as $\hat{\mathbf{S}} = \mathbf{H}_M^{\dagger}\mathbf{Y}$, the structure of the $\mathbf{S}$ matrix is not preserved in $\hat{\mathbf{S}}$. Data unstacking is used to obtain the input signal, $\hat{s}(t)$ from $\hat{\mathbf{S}}$ appropriately. Then a new input signal data matrix which has the similar structure like $\mathbf{S}$ can be constructed from $\hat{s}(t)$, or $\hat{s}(t)$ can be directly used in the convolution operation as shown in Figure 3.1. Note that, if data unstacking is not performed and $\hat{\mathbf{S}}$ is directly used to generate an estimate of the channel output, $\hat{\mathbf{Y}} = \hat{\mathbf{H}}_M\hat{\mathbf{S}}$, COE becomes zero independent of the value of the overestimated channel order, $\hat{L}$, for noise free case. Therefore data unstacking results nonzero COE for overestimation and we obtain a ”convex-like” cost function for the channel order estimation. Data unstacking deletes the last L rows of $\hat{\mathbf{S}}$ and $\hat{s}(t)$ is obtained from this matrix through a vectorization operation. A toy example for data unstacking is given below for $L = 2$.

$$\begin{bmatrix} a & d & g & j \\ b & e & h & k \\ c & f & i & l \\ d & g & j & m \\ e & h & k & n \end{bmatrix} \rightarrow \begin{bmatrix} a & d & g & j \\ b & e & h & k \\ c & f & i & l \end{bmatrix} \rightarrow [a, b, c, d, e, ...]$$

Data unstacking plays an important role in the COE algorithm. The effect of the data unstacking can be included to the convolution equation in order to prove that the proposed algorithm has the finite convergence property. The matrix formulation is reorganized in Appendix-A for this purpose and it is shown that the convolution operation $\mathbf{H}_M\mathbf{S}$ in equation (3.13) can be written in a different way by taking the special structure of $\mathbf{S}$ into account. The formulation in the Appendix-A can be used to prove the relation between the observed, $\mathbf{Y}$, and the estimated channel output, $\hat{\mathbf{Y}}$.

3.3.4 Channel Output Error

Once the channel coefficients and the channel input are available, the channel output is regenerated as shown in Figure 3.1. The difference between the observed and the estimated channel outputs is defined as the channel output error. COE is used as the cost function to find the channel order. In Theorem 1, it is shown that COE has a global minimum at the true channel order for noise free case.

Theorem-1 : It is assumed that a SIMO system is given as in Figure 3.1. For a range of channel order values, $\hat{L} = L_{min}, \ldots, L_{max}$, the channel coefficients are estimated by the LSS algorithm. The input signal matrix is obtained through pseudoinverse, $\hat{\mathbf{S}} = \hat{\mathbf{H}}_{\mathbf{M}}^{\dagger}\mathbf{Y}$. After the data unstacking operation on $\hat{\mathbf{S}}$, input signal, $\hat{s}(t)$, is obtained. Given $\hat{s}(t)$ and the estimated channel coefficients, $\hat{h}_{\hat{L},i}(t)$, $i = 1, ..., P$, $t = 0, ..., \hat{L}$, SIMO system output is regenerated and $\hat{\mathbf{Y}}$ is obtained. The channel output error is defined as

$$COE\left(\hat{L}\right) = \left\|\hat{\mathbf{Y}} - \mathbf{Y}\right\|_2, \quad \hat{L} = L_{min}, \ldots, L_{max} \tag{3.20}$$

COE has a global minimum at the true channel order $\hat{L} = L$ for noise free case.

Proof: The proof is given in Appendix.

In Figure 3.4, Theorem-1 is verified for noisy observations through simulations. Channel order is selected as $L = 5$ and the number of channels is $P = 3$. Channel coefficients are complex values chosen randomly from a zero mean unit variance Gaussian set and change in each trial. For different values of channel orders, COE is computed for 200 trials and average of the COE's are taken as the final COE for a given channel order. The results also include the wrong channel order estimation cases, that is outliers are not discarded. As shown in Figure 3.4, COE has a global minimum at the true channel order, $L = 5$.

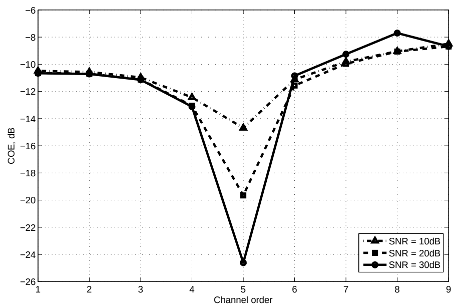


Figure 3.4: Channel output error (COE) for noisy observations.

3.4 Channel Matrix Recursion Algorithm (CMR) Algorithm

Channel matrix recursion algorithm is based on the estimation channel matrix via LSS algorithm for different channel orders. Before the explanation of the algorithm details, it is better to summarize some important properties of the LSS algorithm for noise free case.

- When the channel order is overestimated, LSS results common zeros besides the true channel zeros.
- When the channel order is overestimated, the estimated channel matrix is not full column rank as a result of common zeros.

Properties of the LSS algorithm given above are stated by Lemma-1 and Lemma-2 in this chapter.

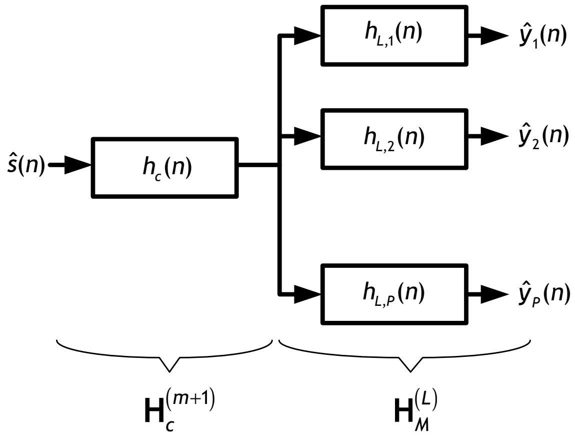


Figure 3.5: Overestimated channel order results common channel zeros in LSS.

As a result of Lemma-1, the channel matrix with overestimated channel order, i.e., $\hat{L} = L+m$, $m > 0$, can be written as follows.

$$\mathbf{H}_M^{(L+m+1)} = \mathbf{H}_M^{(L)}\mathbf{H}_c^{(m+1)} \tag{3.21}$$

where $\mathbf{H}_c^{(m+1)}$ is a Toeplitz matrix with first row $[c_{m+1}(0)\cdots c_{m+1}(m+1)\,0\cdots\,0]$ and first column equal to $[c_{m+1}(0)\;\; 0\cdots 0]^T$. $c_{m+1}(k)$ are the coefficients of the transfer function of $m+1$ common zeros. $\mathbf{H}_M^{(L)}$ is a block Toeplitz convolution matrix. $\mathbf{H}_c^{(m+1)}$ is Toeplitz convolution matrix only if the channel order correctly estimated. Consider that the channel matrices with consecutive channel orders $\hat{L} = L+m$ and $\hat{L} = L+m+1$ are estimated with the LSS algorithm. The following relation between these two matrices are assumed to be hold,

$$\mathbf{H}_M^{(\hat{L}+1)} = \mathbf{H}_M^{(\hat{L})}\mathbf{A}_m \tag{3.22}$$

The equation (3.22) is satisfied perfectly only when $\hat{L} = L$ (i.e., $m = 0$). In this case, $\mathbf{A}_0 = \mathbf{H}_c^1$ is a Toeplitz matrix which can be easily seen from (3.21). When $\hat{L} < L$ (which corresponds to underestimation) or $\hat{L} > L$ (which corresponds to overestimation), (3.22) is not perfectly satisfied. In CMR algorithm $\hat{L}$ is changed between $[L_{min}, L_{max}]$ range and the value of $\hat{L} = L$ is searched based on (3.22) and the fact that $\mathbf{A}_m$ is a Toeplitz matrix only when $\hat{L} = L$. The estimation of $\mathbf{A}_m$ is done using the following equation.

$$\hat{\mathbf{A}}_m = \mathbf{F} \odot \left(\left(\mathbf{H}_M^{(\hat{L})}\right)^{\dagger} \mathbf{H}_M^{(\hat{L}+1)}\right) \tag{3.23}$$

where, $\odot$ is the Hadamard product and $(M+L+m)\times(M+L+m+1)$ matrix $\mathbf{F}$ is a Toeplitz matrix with first row equal to $[1\ \ 1\ \ 0\cdots0]$ and first column equal to $[1\ \ 0\cdots0]^T$. $(.)^\dagger$ indicates the Moore-Penrose pseudoinverse. There are two functions of $\mathbf{F}$ in (3.22). The first one is a filtering action which selects the diagonals from the matrix in parenthesis to generate the expected matrix structure in (3.22) for $m=0$. The second is to generate error for overestimation. In the following theorem, CMR cost function is defined and its main characteristics are described.

Theorem-2 : It is assumed that a SIMO system is given as in Figure 3.1. For a range of channel order values, $\hat{L}=L+m=L_{min},\cdots,L_{max}$, the channel coefficients are estimated by the LSS algorithm. Let the estimated channel matrix is given by $\mathbf{H}_M^{\hat{L}}$ for the channel order $\hat{L}$. M is chosen such that the channel matrix is a tall matrix. The cost function is defined as,

$$
\begin{aligned}
E_{CMR}\left(\hat{L}\right) &= \left\|\mathbf{H}_M^{(\hat{L}+1)}-\mathbf{H}_M^{(\hat{L})}\hat{\mathbf{A}}_m\right\|_2 / \left\|\mathbf{H}_M^{(\hat{L}+1)}\right\|_2 &\quad (3.24)\\
\hat{\mathbf{A}}_m &= \mathbf{F}\odot\mathbf{B}_m &\quad (3.25)\\
\mathbf{B}_m &= \left(\mathbf{H}_M^{(L+m)}\right)^\dagger \mathbf{H}_M^{(L+m+1)} &\quad (3.26)
\end{aligned}
$$

has global minimum at true channel order, $\hat{L}=L$, in the noise free case.

Proof : The proof is given at the Appendix.

In Figure 3.6, Theorem-2 is verified for noisy observations through simulations. Channel order is selected as $L=5$ and the number of channels is $P=3$. Channel coefficients are complex values chosen randomly from a zero mean unit variance Gaussian set and change in each trial. For different values of the channel orders, E_{CMR} is calculated for 200 trials and average of the E_{CMR}'s are taken as the final E_{CMR} for a given channel order. The results also include the wrong channel order estimation cases. As shown in Figure 3.6, CMR has a global minimum at the true channel order, $L=5$.

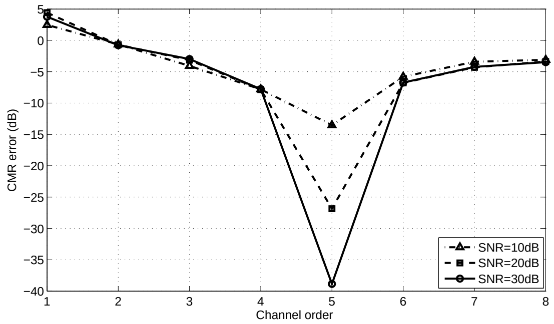


Figure 3.6: Cost function, E_{CMR}, for channel matrix recursion (CMR) for noisy observations.

3.5 Evaluation and Comparison of the COE and CMR Algorithms

Proposed methods are evaluated for random channels, signals and noise sequences in order to show the robustness of the algorithms. In order to improve the performance of the COE algorithm in noisy observations, we used the Wiener formulation instead of the pseudoinverse, namely,

$$\hat{\mathbf{S}} = \hat{\sigma}_s^2 \hat{\mathbf{H}}_M^H \left(\hat{\mathbf{H}}_M \hat{\mathbf{H}}_M^H + \hat{\sigma}_v^2 \mathbf{I} \right)^{-1} \mathbf{Y} \tag{3.27}$$

Note that the Wiener expression $\hat{\sigma}_s^2 \hat{\mathbf{H}}_M^H \left(\hat{\mathbf{H}}_M \hat{\mathbf{H}}_M^H + \hat{\sigma}_v^2 \mathbf{I} \right)^{-1}$ and Moore Penrose pseudoinverse $\left(\hat{\mathbf{H}}_M^H \hat{\mathbf{H}}_M \right)^{-1} \hat{\mathbf{H}}_M^H$ are the same for noise free case ($\hat{\sigma}_v^2 = 0$) up to a scale factor, if the singular value decomposition is used for the matrix inversion, and the singular values below a certain level are set to zero in the inverse operation [35]. Wiener expression in (3.27) requires the knowledge of the signal and noise powers. For this purpose, the method proposed in [36] is used. It is based on the noise and signal subspaces. The space spanned by the columns of the channel matrix is the signal subspace. The projection matrix, $\mathbf{P}_h$, is generated for estimating the noise variance as,

$$\mathbf{P}_h = \mathbf{I} - \mathbf{H}_M \mathbf{H}_M^- \tag{3.28}$$

where,

$$\mathbf{H}_M^- = \left(\mathbf{H}_M^H \mathbf{H}_M \right)^\dagger \mathbf{H}_M^H \tag{3.29}$$

The noise variance can be found as,

$$\hat{\sigma}_v^2 = \frac{1}{PM + M + \hat{L}} tr \left(\mathbf{P}_h \hat{\mathbf{R}}_y \right) \tag{3.30}$$

where $\hat{\mathbf{R}}_y = \frac{1}{MP}\mathbf{Y}\mathbf{Y}^H$. Signal power is found by removing the noise part from the output correlation matrix, $\hat{\mathbf{R}}_y$, i.e.,

$$\hat{\mathbf{R}}_s = \mathbf{H}_M^{-}\left(\mathbf{R}_y - \sigma_v^2\mathbf{I}\right)\left(\mathbf{H}_M^{-}\right)^H \tag{3.31}$$

$$\hat{\sigma}_s^2 = tr\left(\hat{\mathbf{R}}_s\right)/\left(M + \hat{L}\right) \tag{3.32}$$

The SNR for the evaluation of the algorithms is is computed as,

$$SNR = \frac{1}{P\sigma^2}E\left\{\sum_{j=1}^{P}\left|y_j(k)\right|^2\right\} \tag{3.33}$$

where P is the number of outputs, σ^2 is the noise variance and $y_j(k)$ is the noiseless samples of the j^{th} output.

In the simulations, complex channel coefficients are selected from a zero mean unit variance Gaussian distribution. Input is a QPSK modulated sequence with 100 samples and 200 trials are performed to report the average of these trials. The value of M in (3.8) is chosen as $M = \hat{L}$.

Figure 3.7 shows the approximate probability density functions (pdf) of different channel order estimation algorithms and the proposed methods for a $SNR = 15dB$. COE and CMR give the best distribution in terms of finding the true channel order. COE and CMR have small underestimate tails but they do not have overestimation. MDL has both underestimate and overestimate tails but it has the second best characteristics after COE and CMR. ID+EQ has a significant tendency for underestimation. AIC has a strong overestimation characteristic mostly missing the true channel order due to the fact that it works properly only at high SNR ($SNR > 25dB$). Liavas also works properly at high SNR and its distribution is not satisfactory for low SNR.

Table 3.1 and Table 3.2 summarize the performance of COE, CMR, MDL and AIC algorithms for different number of channels and channel orders. The robustness of the proposed algorithms for different SIMO parameters can be seen easily. For COE and CMR algorithms, estimation performance is improved as the number of channels increases. Performance decreases as the channel order is increased. Overall, COE and CMR algorithms return the best performance almost all of the cases considered in the tables. CMR shows a better performance than COE when the channel order ($L < 5$) and the number of channels is small ($P < 5$). COE is the best algorithm when the channel order and number of channels is large. There is a small part ($L > 7$, $P > 4$) where MDL seems to perform better than CMR. If the number of samples for the input signal is increased to 150 samples, the performance of CMR increases. As a result, it becomes better than MDL for all cases. In this case, COE is better than CMR when $L > 7$ and $P > 2$. Therefore, the increase in number of samples has more positive effect for the CMR algorithm. However further increase do not change the performance significantly. Liavas algorithm has the same tendency as the COE. In other words, its performance increases with the number of channels and it decreases as the channel order is increased. AIC and MDL show somehow mixed and opposite characteristics. While AIC performance has a tendency to fall as the number of channels increases, MDL performance mostly improves except for $L = 2$.

The probability of correct channel order estimation is presented in Figures 3.8, 3.9 and 3.10 for ($L = 3$, $P = 3$); ($L = 5$, $P = 3$) and ($L = 5$, $P = 5$) respectively. As shown in these figures, the proposed algorithms outperform the alternatives in all SNR ranges. The characteristics of the algorithms observed in Table 3.1 and 3.2 are also verified for different SNR values in these figures. CMR is the best one when the channel order and the number of channels is small. As the channel order or the number of channels increases, the performance of the COE algorithm gets better than the CMR approximately after $SNR > 6dB$.

3.6 Conclusion

Two new channel order estimation algorithms, COE and CMR are proposed for FIR SIMO systems. Both of these algorithms are based on the LSS algorithm which generates common channel zeros for the overestimated channel order. In COE, channel coefficients and the input signal are estimated to regenerate the SIMO system output in order to compare it with the observed output. It is shown that COE has a global minimum at the true channel order for noise free case. CMR uses the relation between the channel matrices for the overestimated channel orders. It also finds the true channel order for the noise free case. CMR is a computationally efficient alternative to the COE algorithm. It performs better than COE algorithm especially when the number of channels and channel order is small. Both COE and CMR perform significantly better than the alternative algorithms for a variety of cases.

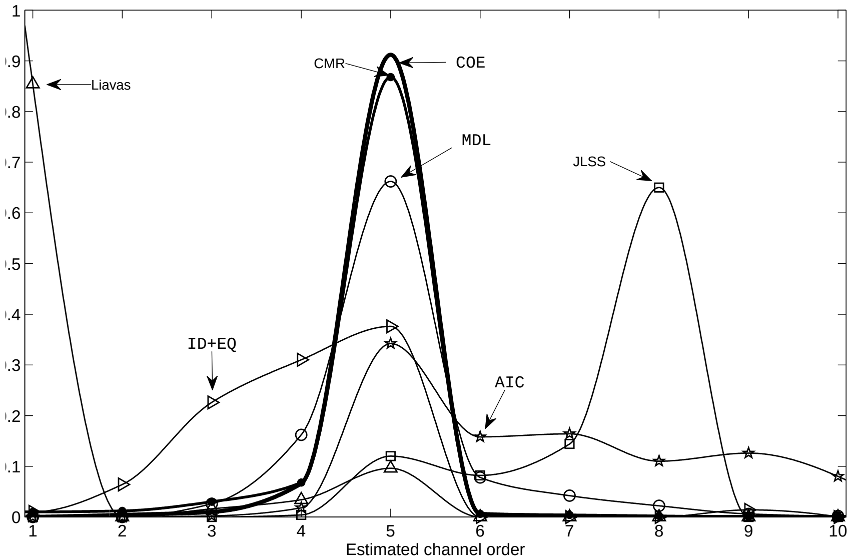


Figure 3.7: Probability density functions (with shape-preserving curve fitting) of the channel order estimation algorithms: MDL, AIC, Liavas, JLSS, ID+EQ and proposed methods COE and CMR. $SNR = 15dB$, the true channel order is $L = 5$ and $P = 3$.

Table 3.1: AIC, Liavas and MDL performances (percentage of true channel order estimate) for different channel order and number of channels. $SNR = 15dB$, input length = 100.

Channel order (L)	Number of channels, (P)																	
	AIC						Liavas						MDL					
	2	3	4	5	6	7	2	3	4	5	6	7	2	3	4	5	6	7
2	52.6	23.4	14.0	7.8	6.0	4.8	18.6	44.2	62.4	82.2	90.0	92.2	67.8	66.2	69	65.4	68.8	65.6
3	44.2	29.2	17.4	10.0	11.2	10.0	7.4	30.4	45.0	63.8	73.6	80.6	49.8	69.0	76.2	74.4	80.0	79.4
4	44.0	35.6	19.4	16.8	11.8	8.0	3.2	17.2	33.8	44.2	58.8	68.8	35.0	67.4	76	83.4	79.8	85.4
5	36.4	34.2	23.0	17.4	13.6	10.2	1.6	9.6	25.4	37.6	46.8	60.0	21.8	66.2	82.8	88.0	90.8	91.8
6	32.2	40.2	27.6	21.4	14.2	12.0	0.2	6.8	14.4	23.6	32.0	41.4	11.0	57.2	77.8	86.0	90.2	89.4
7	22.6	40.0	30.2	22.2	16.0	12.2	0	2.0	6.6	14.2	19.2	26.0	5.4	46.6	74.2	85.8	89.8	91.4
8	24.8	39.0	31.6	22.2	20.2	15.2	0	0.6	2.6	8.6	14.0	16.6	2.6	38.4	71.6	85.0	93.2	93.6
9	16.4	42.6	33.0	24.6	19.0	13.4	0	0.2	2.6	5.4	8.0	12.6	1.0	27.6	61.2	82.6	89.0	94.4
10	0	38.6	34.0	23.0	20.4	17.0	0	0	0.8	1.6	5.0	6.8	0	16.0	51.8	73.4	85.2	90.8

Table 3.2: COE and CMR performances (percentage of true channel order estimate) for different channel order and number of channels. $SNR = 15dB$, input length = 100.

Channel order (L)	Number of channels, (P)											
	COE						CMR					
	2	3	4	5	6	7	2	3	4	5	6	7
2	84.8	96.4	99.0	99.2	100	100	92.2	99.4	99.8	99.8	100	100
3	71.0	93.6	97.8	99.6	99.0	99.4	80.4	95.4	98.8	98.6	99.4	99.4
4	68.2	91.2	97.4	98.6	99.4	99.4	74.0	92.8	98.2	98.4	98.8	99.6
5	59.2	91.2	95.6	98.0	98.4	99.6	66.6	86.8	93.4	96.8	98.4	98.4
6	50.0	81.6	93.6	98.4	98.0	98.6	54.8	79.8	90.6	93.8	97.2	97.8
7	39.0	79.6	91.2	98.4	97.6	98.6	42.2	69.8	83.8	90.4	93.4	94.6
8	42.6	69.0	86.8	93.6	95.2	94.6	37.0	62.4	78.6	86.6	90.8	93.0
9	39.2	66.2	82.6	91.2	96.2	95.6	30.4	55.4	69.4	79.4	84.8	90.2
10	27.6	64.0	79.4	90.0	94.2	96.8	22.0	43.4	61.8	70.2	77.4	83.8

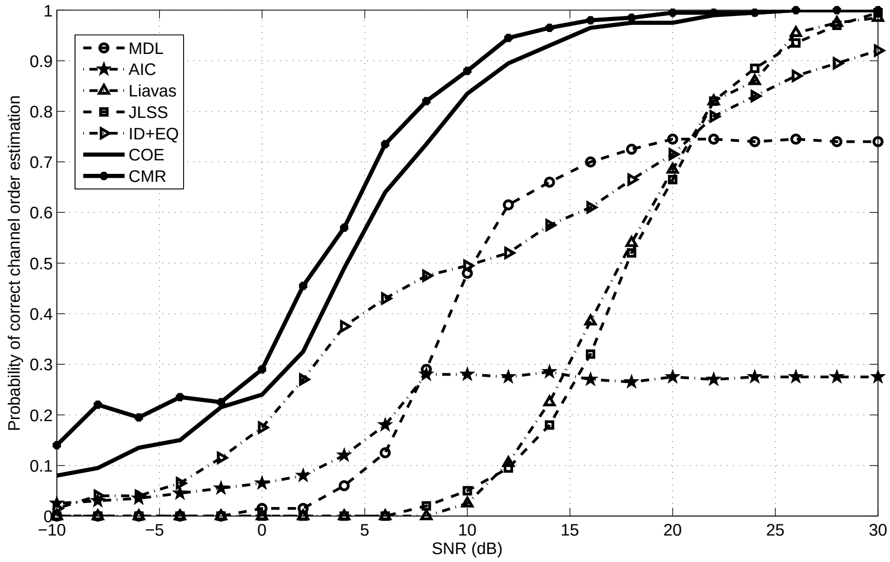


Figure 3.8: Channel order estimation performance for L = 3, P = 3 and input length = 100.

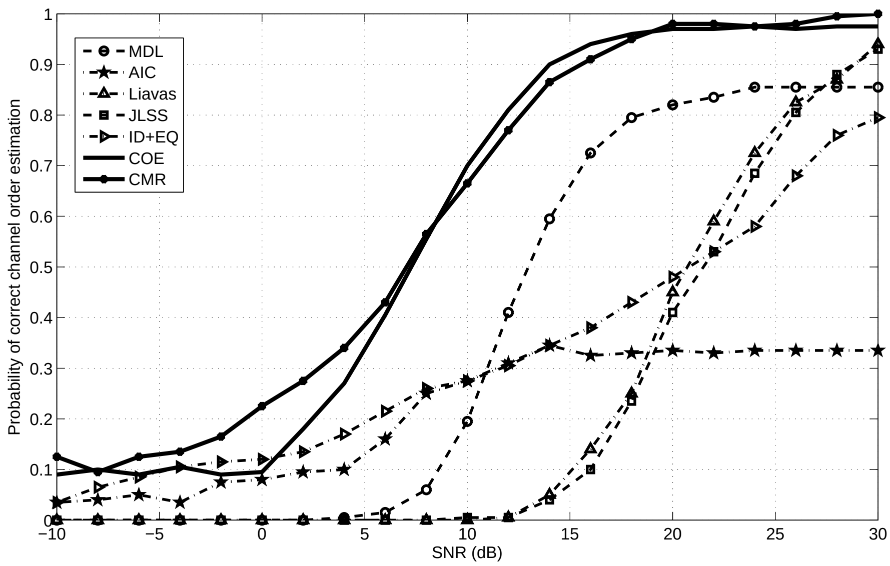


Figure 3.9: Channel order estimation performance for L = 5, P = 3 and input length = 100.

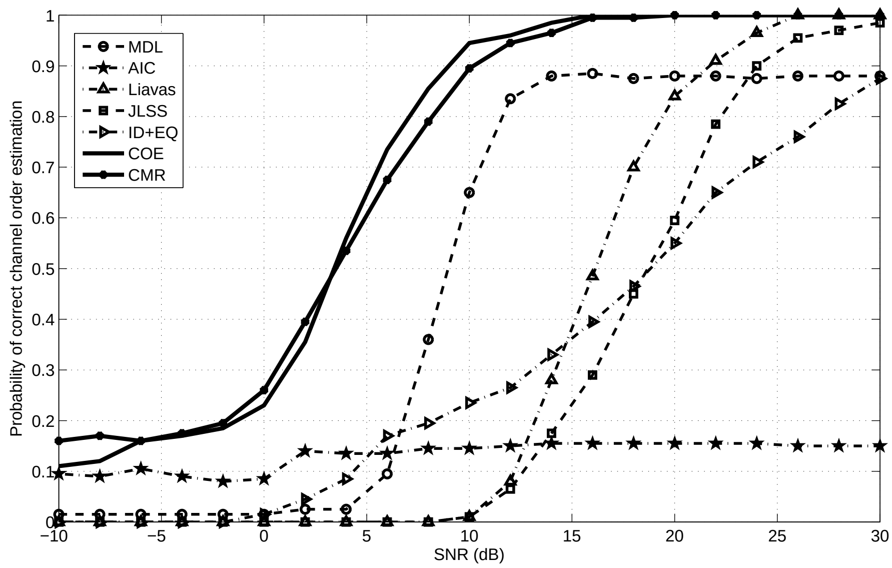


Figure 3.10: Channel order estimation performance for $L = 5$, $P = 5$ and input length = 100.

CHAPTER 4

CHANNEL ORDER ESTIMATION USING TRAINING DATA

4.1 Introduction

In this chapter, channel order estimation is considered for the systems which uses training sequences. The motivation here is to understand and compare the order estimation performance with the blind techniques. Blind methods do not use training data to identify/ equalize the channel. Therefore they are more bandwidth efficient methods. Blind methods are preferred for the systems which have synchronization problem, because they do not have to synchronize to the training data for channel identification. On the other hand,blind methods can identify the channel up to a scale factor for SIMO systems, while the training based methods can identify the channel exactly.

Training based methods estimate channel coefficients using only the received data containing the known symbols. All other observations are ignored. Therefore synchronization should be achieved for channel identification. Otherwise, expected performance cannot be obtained. Synchronization should be achieved before the equalization or jointly with the channel identification, since the channel coefficients cannot be found without knowing the placement of the known symbols at received signal. In [34], synchronization and channel identification is achieved in a joint manner using a single cost function which has a global minimum.

Another problem to be solved in training based methods is the channel order estimation as in the case of blind methods. In training based methods, channel order is usually assumed to be known as in [34] or they use overestimated channel order employing algorithms like MDL and AIC. The robustness of training based methods for overestimated channel order is low at low SNR.

The bit error rate (BER) performance of the training based methods is better than the blind methods when the synchronization is achieved and the channel order is known. However if one of them is not properly estimated, their performance decreases and even becomes worse than the blind methods. The performance of blind methods can be increased by using training data beside the unknown data. This can also be thought from the side of training based methods. With the help of blind methods, which enable the use of whole received data, it is possible to obtain the same performance with smaller number of training data. This type of methods are called as the semi-blind methods and have better performance than the training only and blind only methods.

To our knowledge, there is no algorithm proposed for channel order estimation using only the training data or using the benefit of training data in channel order estimation. In this chapter, two new channel order estimation algorithms namely Channel Input Error with Semiblind Channel Estimator (CIES) and Channel Input Error with Blind Channel Estimator (CIEB) are proposed. They use the training data to obtain the channel order. In both of the algorithms, channel input error, which is the difference between the estimated and known pilot symbols, is used as the cost function to obtain the true channel order. The difference between CIEB and CIES is due to the method used in channel estimation. In CIEB, channel is estimated with LSS algorithm in a blind manner. In CIES, channel is estimated by means of a semi blind method, which is proposed in this thesis. CIES is obtained by the modification of the method [38] which combines SS and least squares fit on training sequence (LST) in a single cost function to obtain the channel coefficients. In the modification, LSS algorithm is used instead of the SS algorithm. In this way, a better algorithm is obtained in terms estimating the channel order in noisy observations. This is due to the fact that when the channel is estimated by LSS with overestimated channel order, common zeros produced by the LSS algorithm are placed on unit circle. This leads to high error in the equalization step of CIES and generates deeper valleys at true channel order. Both of the algorithms require synchronization to find the location of training symbols. In CIES, synchronization and channel order estimation is achieved in a joint manner with multidimensional search for the channel order and synchronization point. Therefore it has high computational complexity. In CIEB, synchronization is achieved after channel equalization and on the ISI removed signal by using the optimum synchronization method proposed in [39]. Proposed algorithms CIES and CIEB are compared with blind channel order estimation algorithms in estimating the channel order and in BER. CIES performs between COE and CMR in channel order estimation. BER performances of CMR, CIES and CIEB are close. However when the number of channel is high, CIES performs slightly better than other proposed blind methods.

4.2 Training Based Channel Estimation

In training based techniques, least-squares minimization is used to obtain the channel coefficients by only using the data corresponding to the training sequence. In this section, the performance of training based LS method (LST) for channel identification/equalization is analyzed with respect to channel order and synchronization. For the SIMO systems, channel output can be written as follows,

$$\mathbf{y_M}(t) = \mathbf{H}_M \mathbf{s}_{M+L}(t) \tag{4.1}$$

where $MP \times (M + L)$ block Toeplitz matrix $\mathbf{H}_M$ is the channel matrix and $\mathbf{s}_{M+L}(t)$ is the input data vector for P channel SIMO system. This equality can also be written as follow, by stacking the unknown channel coefficients in single vector, $\mathbf{h}_L = \begin{bmatrix} \mathbf{h}_L^T(L) & \mathbf{h}_L^T(L-1) & \cdots & \mathbf{h}_L^T(0) \end{bmatrix}^T$, called as channel vector, and rearranging the matrix multiplication.

$$\mathbf{y}_M(t) = \mathbf{S}_{LP}(t)\mathbf{h}_L \tag{4.2}$$

Where,

$$\mathbf{S}_{LP}(t) = \mathbf{S}_L(t) \otimes \mathbf{I}_{P\times P} \tag{4.3}$$

$$\mathbf{S}_L(t) = \begin{bmatrix} s(t) & s(t+1) & \cdots & s(t+L) \\ s(t+1) & s(t+2) & \cdots & s(t+L+1) \\ \vdots & \vdots & \ddots & \vdots \\ s(t+M-1) & s(t+M) & \cdots & s(t+M+L-1) \end{bmatrix}_{M\times(L+1)} \tag{4.4}$$

and $\otimes$ indicates the Kronecker product. $\mathbf{I}_{P\times P}$ is $P \times P$ identity matrix. In training based methods, a part of the input data is known at the receiver side. A typical training data transmission strategy is to send training data at the start of each frame for synchronization and channel estimation purposes as shown in Figure 4.1. Training data starts at time index $t + \mu$, where μ is the frame offset reference to starting point of received data. If the synchronization is provided, (i.e., μ is known), channel coefficients can be found via LS minimization from the received signal that contains the training data as follows,

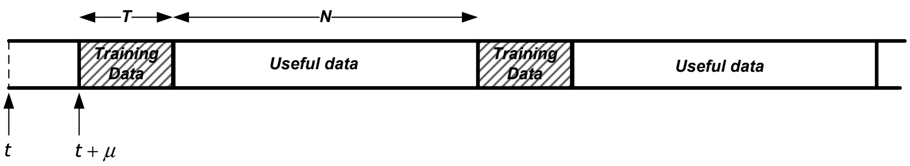


Figure 4.1: Training sequence is send at start of each frame. T is the length of training sequence and N is the frame length.

$$\hat{\mathbf{h}}_L = \arg\min_{\hat{\mathbf{h}}} \left\| \mathbf{y}_M(t) - \mathbf{S}_{LP}(t)\hat{\mathbf{h}} \right\| \tag{4.5}$$

where $\mathbf{S}_{LP}(t)$ is the matrix only containing the training data (i.e., training data starts at the time index t with length $M + L$). The solution of the LS minimization problem is given as follows.

$$\hat{\mathbf{h}}_L = \left(\mathbf{S}_{LP}^H(t)\mathbf{S}_{LP}(t)\right)^{-1} \mathbf{S}_{LP}^H(t)\mathbf{y}_M \tag{4.6}$$

Exact solution exists in noise free case, if $\mathbf{S}_L(t)$ has full column rank. $\mathbf{S}_L(t)$ is of full column rank, when the length of training data is greater than $2L+1$ and it has linear complexity greater than L. Otherwise, there is infinite number of solutions and exact channel coefficients cannot be obtained.

A number of simulations are done to see the performance of training based least-squares minimization (LST). In the first simulation, the synchronization delay, μ and the channel order, L is assumed to be known for the LST, and compared with the CMR method which obtains channel coefficients and channel order in a blind way (CMR does not require synchronization for channel estimation.). Wiener equalizer is used for the equalization of the channel. The number of frames is $K = 5$, frame length is $N = 100$ and 15 bit randomly generated sequence is used as a training sequence. The training sequence is changed in each trial. Synchronization point or the starting time index of the useful data, μ, is selected as 3. BPSK is used as the modulation waveform. Channel order is $L = 4$ and the number of sub-channels is $P = 3$. Channel coefficients are complex values randomly selected from a zero mean unit variance set. At each SNR level, 200 trials are realized in the simulations. As seen from the Figure 4.2, LST method is better than the CMR, as expected.

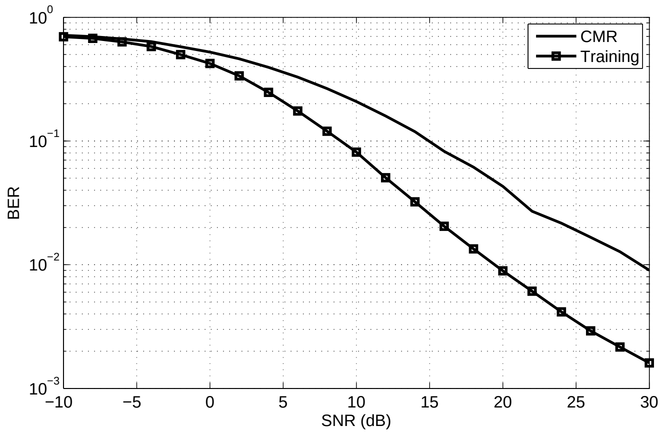


Figure 4.2: BER performance of training based method and CMR method. Channel order is, $L = 4$, and it is assumed to be known in training based LS method.

When the channel order, L ,is overestimated as $\hat{L} = L + 3 = 7$ and synchronization point, μ is known, the BER graph is given in Figure 4.3. As shown from the Figure 4.3, the performance of the training based methods decreases and becomes worse than CMR. That is when the channel order is given as overestimated to the training based method, it performance is poor and worse than the blind method, CMR.

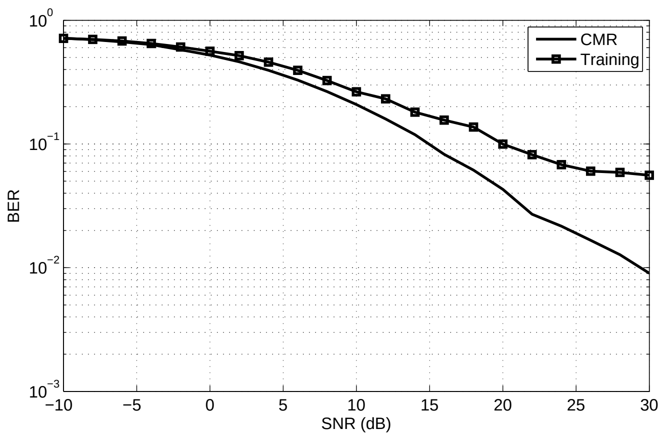


Figure 4.3: BER performance of training based method CMR method. Channel order is overestimated in training based method, $\hat{L} = L + 3$.

In Table 4.1, BER performance of training based LS method is tested against the channel order and code length. SNR is $10dB$ and the true channel order is $L = 4$. Training sequence with the specified length in Table 4.1 are randomly generated, and changed in each trial. For the comparison, CMR method is used. Since, the channel order is estimated in a blind way, and training sequence is not used in CMR, a table for CMR is not given. For these settings, BER performance of the CMR method is measured between 0.033 and 0.036. As shown from Table 4.1, when the channel order is known, the best performance is achieved with the training based method. When the channel order is underestimated it does not work, i.e., channel coefficients cannot be estimated, since BER values are smaller than 0.033 in general. When the channel order is overestimated by few orders, it works when the code length is sufficiently long. Its performance is similar to the CMR, when the code length is greater than 15, and channel order is less than 7.

Table 4.1: BER performance of training based method against estimated channel order and different code lengths. True channel order is $L = 4$ and $SNR = 10dB$.

Channel order, $\hat{L}$	Length of training sequence, $T = M + L$						
	10	15	20	25	30	35	40
1	0.403	0.393	0.376	0.367	0.375	0.369	0.358
2	0.309	0.281	0.281	0.284	0.276	0.269	0.277
3	0.249	0.202	0.170	0.158	0.154	0.159	0.160
4	0.068	0.010	0.009	0.009	0.011	0.009	0.008
5	0.221	0.044	0.048	0.025	0.041	0.037	0.028
6	0.372	0.075	0.032	0.029	0.030	0.026	0.031
7	0.501	0.137	0.070	0.023	0.034	0.030	0.057
8	0.560	0.238	0.055	0.035	0.058	0.037	0.035
9	0.704	0.382	0.090	0.067	0.035	0.038	0.052

Up to now synchronization is assumed to be provided. If the synchronization is not provided, performance of LST method decreases dramatically. This case is shown in Figure 4.4. In the simulations, there is synchronization mismatch up to one sample. The similar simulations setting are used as in Figure 4.2.

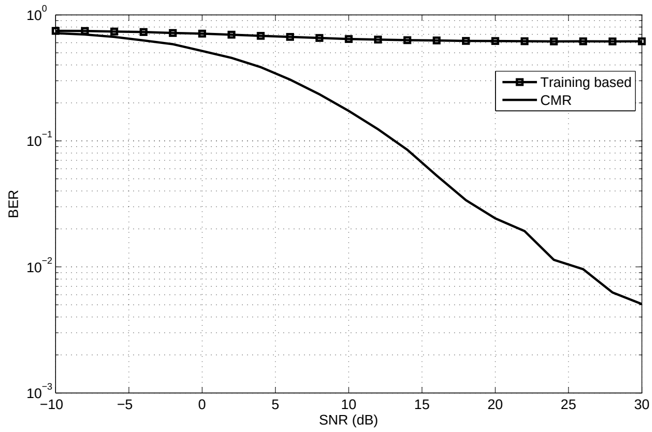


Figure 4.4: Channel order estimation performance when the synchronization is not achieved.

When the synchronization is not achieved, the LST cost (equation (4.5)) increases. In other words, LST cost has a minimum when the synchronization is achieved. Using this fact, in [34] Serpedin method is proposed where channel estimation and channel equalization is done in a joint manner. The basic principle is to find the best delay at the channel output that gives the minimum cost at the LS minimization defined in (4.5) for the channel identification. If the starting point of the training data is $t + \mu$, then the cost function for the joint synchronization and channel identification can be defined as follows.

$$\hat{\mathbf{h}}_L, \hat{\mu} = \arg\min_{\hat{\mathbf{h}}, \hat{\mu}} \left\| \left(\mathbf{y}_M \left(t + \hat{\mu} \right) - \mathbf{S}_{LP}(t + \hat{\mu})\hat{\mathbf{h}} \right) \right\| \tag{4.7}$$

where,

$$\mathbf{y}_M(t + \hat{\mu}) = \begin{bmatrix} \mathbf{y}_1^T \left(t + \hat{\mu} \right) & \mathbf{y}_1^T \left(t + \hat{\mu} + M - 1 \right) \end{bmatrix}^T \tag{4.8}$$

$$\mathbf{y}_1 \left(t \right) = \begin{bmatrix} y_1 \left(t \right) & \cdots & y_P \left(t \right) \end{bmatrix}^T \tag{4.9}$$

This minimization can also be realized over more than one frame. When the number of frames is increased, the performance of the method increases.

$$\hat{\mathbf{h}}_L, \hat{\mu} = \arg\min_{\hat{\mathbf{h}}, \hat{\mu}} \left\| \sum_{k=0}^{K-1} \left(\mathbf{y}_M \left(t + \hat{\mu} + kN \right) - \mathbf{S}_{LP}(t + \hat{\mu})\hat{\mathbf{h}} \right) \right\| \tag{4.10}$$

where N is the frame length, K is the number of frames, and $T = M + L$ is the length of the training data. As seen from the formulation and the structure of the matrices, channel order is assumed to be known. In case of channel order is overestimated, i.e., $\hat{L} = L + m$, the solution is given as,

$$\hat{\mathbf{h}}_{L+\hat{\mu}}, \hat{\mu} = \arg\min_{\hat{\mathbf{h}},\hat{\mu}} \left\| \sum_{k=0}^{K-1} \left(\mathbf{y}_M (t + \hat{\mu} + kN) - \mathbf{S}_{(L+m)P}(t + \hat{\mu})\hat{\mathbf{h}} \right) \right\| \tag{4.11}$$

where

$$\mathbf{S}_{(L+m)P}(t + \hat{\mu}) = \left[\begin{array}{c} \mathbf{S}_{LP}(t + \hat{\mu})\mathbf{S}_{mP}(t + L + \hat{\mu} + 1) \end{array} \right] \tag{4.12}$$

First $(L + 1)P$ columns of $\mathbf{S}_{(L+m)P}(t + \mu)$ are identical with $\mathbf{S}_{LP}(t + \mu)$, therefore last mP columns are eliminated by setting the last mP elements of the channel vector as zero. Hence the solution in noise free case is $\hat{\mathbf{h}}_{L+m} = [\mathbf{h}_L^T \quad \mathbf{0}_{1\times mP}]^T$. In this solution, synchronization is assumed to be provided. If it is not the case, there will be more than one solution. For example, if $\mu = 3$ and $m = 2$, then $\hat{\mu} = 3$, $\hat{\mathbf{h}}_{L+2} = [\mathbf{h}_L^T \;\; \mathbf{0}^T \;\; \mathbf{0}^T]^T$; $\hat{\mu} = 2$, $\hat{\mathbf{h}}_{L+2} = [\mathbf{0}^T \;\; \mathbf{h}_L^T \;\; \mathbf{0}^T]^T$ and $\hat{\mu} = 1$, $\hat{\mathbf{h}}_{L+2} = [\mathbf{0}^T \;\; \mathbf{0}^T \;\; \mathbf{h}_L^T]^T$ are the solutions for the channel vector. In noiseless case, summing the number of leading zeros in the channel vector and $\hat{\mu}$, the final delay for synchronization can be calculated. However at low SNR, small coefficients at the tails lead errors on the channel equalization and furthermore it is not easy to obtain the number of zeros in channel vector especially when the first element of the true channel vector is small.

4.3 Channel Order Estimation Using Blind Channel Estimator and Training Data, CIEB

In the previous section, LST method, which is training only method, is introduced and it is shown that its performance is affected from channel order and synchronization mismatch. Serpedin method [34] solves the channel estimation and synchronization problem in a joint manner. However it assumes that the channel order is known, does not provide a solution for the channel order estimation as discussed in the previous section. Therefore, an algorithm is needed to obtain channel order in training based systems considering the synchronization problem. In this section, we have proposed a new channel order estimation method, called as CIEB, which uses training data for the channel order estimation. CIEB also obtains the channel coefficients and solve the synchronization problem. This method is similar to the COE. In CIEB, the cost function is computed by considering the input pilot samples. COE uses the channel output for the cost function. Channel coefficients are obtained with the blind LSS method and channel is equalized with the obtained channel coefficients. The difference

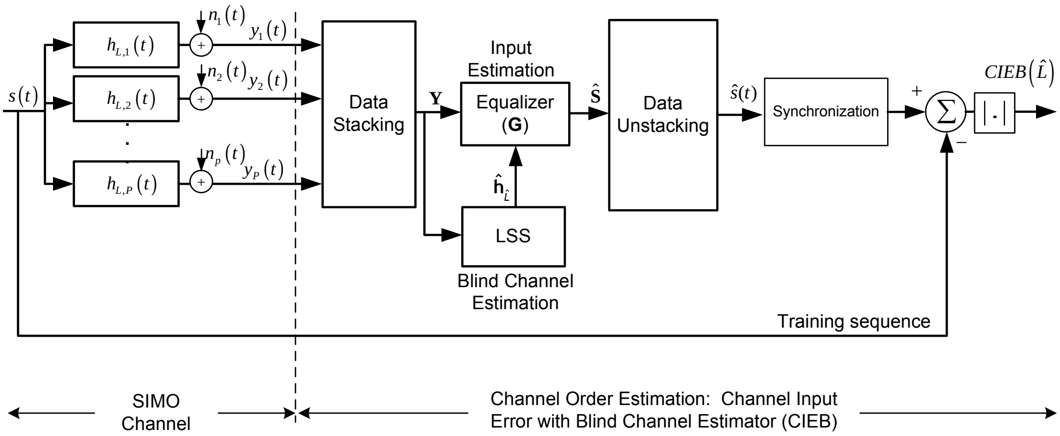


Figure 4.5: Channel order estimation with CIEB method.

between the known training data and estimated input is taken as the cost function. To find the position of training data on the equalized channel, a synchronization step is required. For the synchronization, Massey method, [39], is used. Proposed method solves synchronization, channel order estimation and channel identification in a joint manner.

In Figure 4.5, the system structure of CIEB algorithm is given. The details of the algorithm are explained in the following sections.

4.3.1 Channel Estimation

As a channel estimator, LSS method is used. However, one can ask that why training based least-squares minimization method (LST), whose channel estimation performance is better than the blind methods, is not used. The answer is that, the defined cost function for the channel order estimation does not have not a global minimum when the channel is estimated from training data via LS minimization. In noiseless case, channel is equalized perfectly for the overestimated channel orders and therefore LST cost is zero for $\hat{L} > L$. Therefore the cost function does not have a single minimum and the true channel order cannot be detected. However in LSS method the situation is different. As a result of Lemma-1, the channel transfer functions estimated by using LSS methods includes common zeros besides the true channel zeros, when the channel is overestimated. Then, the estimated channel is combination of a SIMO channel and a SISO channel, $h_c(n)$, whose zeros are the common zeros as shown in Figure 4.6.

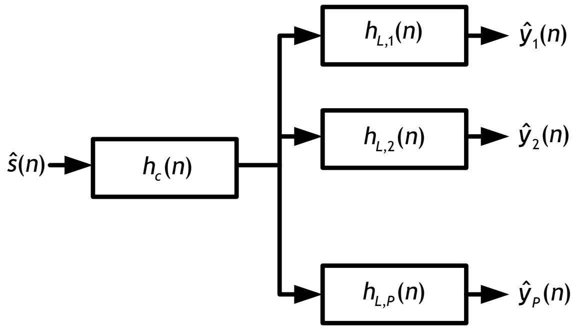


Figure 4.6: Estimated SIMO channel with common channel zeros.

When we want to equalize the channel with the estimated channel coefficients, common channel transfer function, $h_c(n)$, is also equalized with Wiener equalizer besides the true channel. Wiener equalizer is a linear equalizer and perfect equalization of a FIR modeled SISO channel is not possible. As a result, channel input cannot be estimated truly when the channel order is not known.

Another important property of the LSS algorithm is that, the generated common zeros are located on unit circle (Lemma-3, see the appendix). This property is not shared by other blind algorithm, such as SS and CR, which also generate common zeros. When the common channel zeros are located on the unit circle, the inverse channel transfer function does not decay to zero as time index go to infinity. Therefore FIR approximation is not possible and equalization error is high. Hence, the error on the estimated channel input signal due to the equalization of common channel zero is higher compared to common zeros located inside the unity circle. We want to obtain high input estimation error, when the channel order is overestimated. So that, the true channel order can be detected more easily in noisy observations by determining the channel input error, which takes its minimum at true channel order in noise free case. Therefore LSS algorithm is preferred as the blind channel estimator instead of SS and CR.

4.3.2 Channel Equalization

Channel input signal is estimated by using Wiener equalizer. For FIR SIMO modeled systems, perfect equalization is possible in noise free case, when the channel has no common zeros. Pseudo inverse of the channel matrix can be used as the zero forcing equalizer. For better performance in noisy observations, MSE optimum Wiener equalizer is used. Wiener equalizer is given as follows.

$$\hat{\mathbf{S}} = \hat{\sigma}_s^2 \hat{\mathbf{H}}_M^H \left(\hat{\mathbf{H}}_M \hat{\mathbf{H}}_M^H + \hat{\sigma}_v^2 \mathbf{I} \right)^{-1} \mathbf{Y} \tag{4.13}$$

Wiener expression in (4.13) requires the knowledge of the signal and noise powers. For this purpose, the method proposed in [36] is used.

4.3.3 Data Unstacking

By using Wiener equalization matrix, $\mathbf{G}$, channel input matrix $\mathbf{S}_M(t)$ is estimated. Channel input sequence, $\hat{s}(t)$ is extracted from $\hat{\mathbf{S}}_M(t)$ by means of data unstacking operation. In data unstacking operation, last L rows of the channel output matrix is deleted and resulted matrix is vectorized.

4.3.4 Synchronization

Synchronization method varies depending on the frame transmission strategies (i.e periodic, aperiodic, burst frame strategies), channel model (i.e., additive noise channel, flat fading channel and frequency selective channel) and where the synchronizer is applied at the receiver (i.e., before or after the equalizer). The most interesting work about frame synchronization was done Massey, [39], for a continuous stream of BPSK data with a periodically inserted known frame synchronization pattern. He developed the maximum likelihood (ML) frame synchronization rule. The result is that classical correlators are not optimum way of synchronization in AWGN in contrast common belief. ML rule consisted of a standard correlator followed by a non-linear correction term accounting for the presence of random data surrounding the known synchronization pattern. In the same work, the high- and low-SNR approximations were also proposed to generated ML rules for practical systems. The ML criteria for Massey is given as follows.

$$\hat{\mu} = \max_{\hat{\mu}} \left(\left| \sum_{t=1}^{T} \rho_t s(t + \hat{\mu}) \right| - \sum_{t=1}^{T} |s(t + \hat{\mu})| \right) \tag{4.14}$$

where ρ_t, $t = 1 \ldots T$, are the samples of the synchronization word. s is the received data samples , and μ is the frame offset.

4.3.5 CIEB Cost Function

After estimating the input sequence and frame offset, the difference between the estimated training data and training data is used as the cost function to determine the channel order. Channel order that minimizes the cost function given below is taken as the channel order.

$$\hat{L} = \arg\min_{\hat{L}} \sum_{k=1}^{N} \left(\sum_{t=1}^{T} \left| \hat{s}_{\hat{L}} \left(t + \hat{\mu} + kN\right) - s\left(t + \mu\right) \right|^2 \right) \tag{4.15}$$

where $\hat{s}_{\hat{L}}$ indicates the estimated channel input sequence for the channel order, $\hat{L}$. T is the length of training sequence, N is the frame length and K is the number frames. The cost function has a global minimum at the true channel order. When the channel order is underestimated, channel cannot be estimated by the LSS algorithm, hence the estimated channel input signal is erroneous and cost function has a value greater than zero. When the channel order is overestimated, common zeros are generated by the LSS algorithm in addition to the true channel zeros. As mentioned before, this leads to errors on the estimated channel input signal. Therefore the cost function is greater than zero when the channel order is overestimated. When the channel order is estimated correctly, channel coefficients are exactly estimated by the LSS algorithm and channel input is estimated approximately. As a result, the cost function has a global minimum at true channel order in noise free case.

4.4 Channel Order Estimation Using Semi-blind Channel Estimator and Training Data, (CIES)

A new method, CIES, is proposed for channel order estimation. CIES is the modified version of CIEB to obtain a better performance in channel order estimation. The performance improvement is achieved by using a semi-blind channel estimation method instead of blind one. For this purpose, semi-blind channel estimation method proposed in [38] is modified by using LSS algorithm instead of SS method. Since a semi-blind method is used for channel estimation, synchronization cannot be achieved as in CIEB. Here the synchronization is done after the channel equalization. Synchronization problem in CIES is solved by jointly minimizing the channel input error against channel order and frame offset. We show that the channel input error is minimum when the channel order and frame offset is truly estimated.

The system structure of CIES method is given in Figure 4.7. The steps of the algorithms are

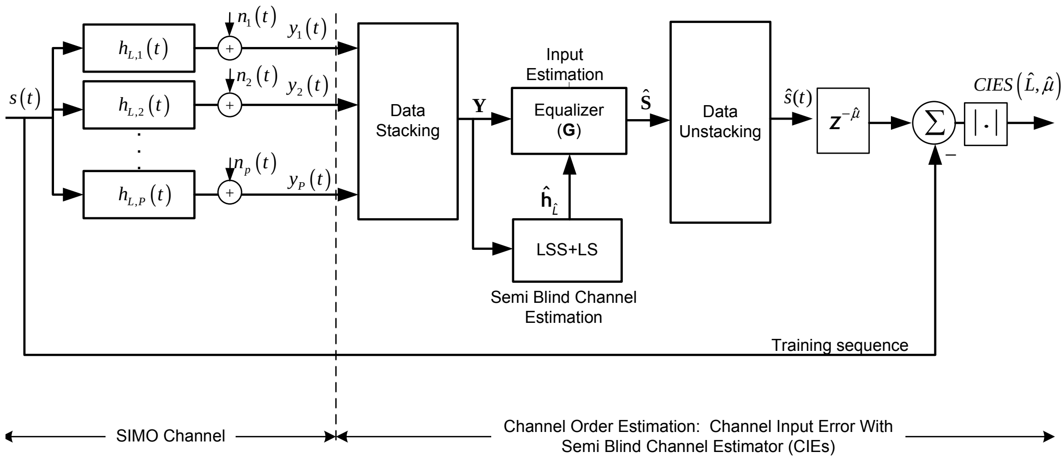


Figure 4.7: Channel order estimation with CIES method.

as follows. For a given channel order, $\hat{L}$ and frame offset $\hat{\mu}$, channel is estimated by means of a semi-blind channel estimator. Estimated channel coefficients are used to equalize the channel. The norm difference between the estimated channel input and training sequence is taken as the cost function for the given channel order and frame offset. The cost function, $CIES(\hat{L}, \hat{\mu})$, is calculated for different values of channel order and frame offset. The channel order and frame offset given the minimum cost is taken as the true channel order and frame offset. The details of the algorithms are explained in the following sections.

4.4.1 Semi-blind Channel Estimation

In [38], a semi-blind method for channel estimation is proposed that combines the cost function of LST and SS [1] in single cost function. The cost function is given as follows,

$$\arg\min_{\hat{\mathbf{h}}_L} \left\| \mathbf{y}_M(t+\mu) - \mathbf{S}_L(t+\mu)\hat{\mathbf{h}}_L \right\| + \alpha \left\| \mathbf{U_n}\hat{\mathbf{h}}_L \right\| \tag{4.16}$$

where μ is the frame offset and $\mathbf{U}_n$ is the noise subspace matrix. α is the weighting coefficient for the contribution of the blind technique. If α is higher, blind subspace method is more effective on the result. The value of α is adjusted to obtain the best result by using an ad-hoc approach. The channel that minimizes the cost function is given as follows for a given α and μ. (Note that μ is assumed to be known.)

$$\hat{\mathbf{h}}_L = \left(\mathbf{S}_L^H(t+\mu)\mathbf{S}_L(t+\mu) + \alpha\mathbf{U}_n^H\mathbf{U}_n\right)^{\dagger}\mathbf{S}_L^H(t+\mu)\mathbf{y}_M(t+\mu) \tag{4.17}$$

Semi-blind channel estimation algorithm proposed in [38] is modified by using LSS algorithm instead of the SS method. In this way, a better algorithm is obtained in terms of estimating the channel order in noisy observations. This is due to the fact that when the channel is estimated by LSS with overestimated channel order, common zeros produced by the LSS algorithm are placed on the unit circle. This leads to high error in the equalization step of CIES and generates deeper valleys at the true channel order at the cost function defined for CIES.

LSS algorithm minimizes the cost function below to find the channel coefficients up to a scale factor, i.e.,

$$\arg\min_{\hat{\mathbf{h}}_L} \left\|\mathbf{Q}\hat{\mathbf{h}}_L\right\| \tag{4.18}$$

where $\mathbf{Q}$ is constructed from the null space of the smoothing error matrix, $\mathbf{E}$, in LSS method. (For the detail refer to the Chapter 2.) LST minimizes the following the cost function using only the received signal which contains the training sequence, i.e.,

$$\arg\min_{\hat{\mathbf{h}}_L} \left\|\mathbf{y}_M(t+\mu) - \mathbf{S}_L(t+\mu)\hat{\mathbf{h}}_L\right\| \tag{4.19}$$

where $\mathbf{S}_L(t+\mu)$ is constructed from the training sequence. The combined cost function is defined as follows,

$$\arg\min_{\hat{\mathbf{h}}_L} \left\|\mathbf{y}_M(t+\mu) - \mathbf{S}_L(t+\mu)\hat{\mathbf{h}}_L\right\| + \alpha\left\|\mathbf{Q}\hat{\mathbf{h}}_L\right\| \tag{4.20}$$

with the solution given as follows,

$$\hat{\mathbf{h}}_L = \left(\mathbf{S}_L^H(t+\mu)\mathbf{S}_L(t+\mu) + \alpha\mathbf{Q}^H\mathbf{Q}\right)^{\dagger}\mathbf{S}_L^H(t+\mu)\mathbf{y}_M(t+\mu) \tag{4.21}$$

The value of α determines the effect of LSS on the channel estimation. It should be adjusted to obtain the best result. When $\alpha = 0$, it is a training based LS method and channel order cannot be found properly in noisy observations. When $\alpha = \infty$, it becomes to the CIEB method. By adjusting α, the channel estimation performance can be increased beyond LSS.

4.4.2 CIES Cost Function

CIES uses the channel input error, $s(t) - \hat{s}(t)$, for the estimation of channel order and frame offset and given as follows.

$$CIES(\hat{L}, \hat{\mu}) = \sum_{k=1}^{K} \left(\sum_{t=1}^{T} \left| \hat{s}_{(\hat{L})}(t + \hat{\mu} + kN) - s(t + \mu) \right|^2 \right) \tag{4.22}$$

$$\hat{L}, \hat{\mu} = \arg\min_{\hat{L}, \hat{\mu}} CIES(\hat{L}, \hat{\mu}) \tag{4.23}$$

$$\tag{4.24}$$

where T is the length of the training sequence, N is the frame length and K is the number frames. The defined cost function, CIES, has global minimum at the true channel order and frame offset. Semi-blind method used in CIES is a deterministic method and channel coefficient can be found exactly from finite number of samples in noise free case when the channel order and frame offset is known. Therefore, CIES cost function is zero when the channel order and frame offset is correctly estimated. When the synchronization is not achieved, LST does not work and channel cannot be estimated. Therefore synchronization is a pre-request. When the channel order is underestimated, LSS and LST cannot estimate the channel coefficients. When the channel order is overestimated, LSS finds the channel transfer function with additional common channel zeros. This leads equalization error in estimated channel input. Therefore, the cost function will be greater than zero when the channel order is overestimated. As a result, the cost function is zero only if the channel order and frame offset is correctly estimated.

4.4.3 Simulations

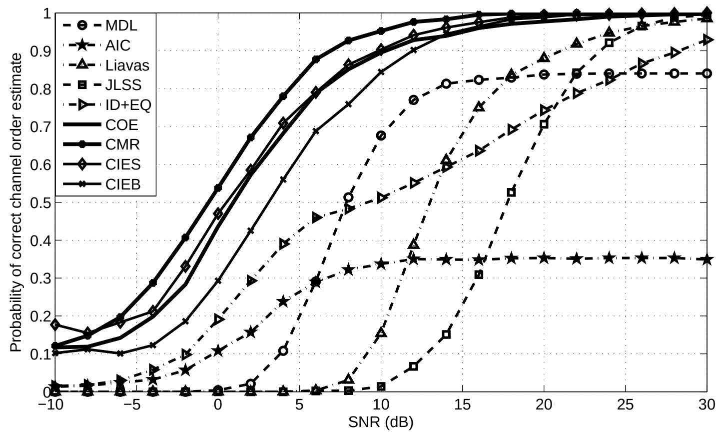
Several experiments for different channel orders and number of sub-channels were realized for the performance comparison. The common simulation settings are as follows. , the number of frames is $K = 2$ and frame length is 100. As a training sequence (which is also synchronization word) 13 bit Barker code is used. As a modulation waveform, BPSK is used. For a fair comparison, after the channel order is estimated by the algorithms, channel is estimated with LSS and equalized with Wiener equalizer for all blind channel order estimation algorithms. In CIES algorithm, the value of α is set to $\alpha = 100$. Frame offset is $\mu = 3$. Frame offset is searched in the set $\hat{\mu} = 1 \ldots 5$ for frame synchronization. Channel order is searched in the set $\hat{L} = 1 \ldots L + 6$ in all algorithms. 1000 trials are evaluated at each SNR level to compute the probability of correct channel order estimation and BER. SNR is computed as follows,

$$SNR = \frac{1}{P\sigma^2} E \left\{ \sum_{j=1}^{P} \left| y_j(k) \right|^2 \right\} \tag{4.25}$$

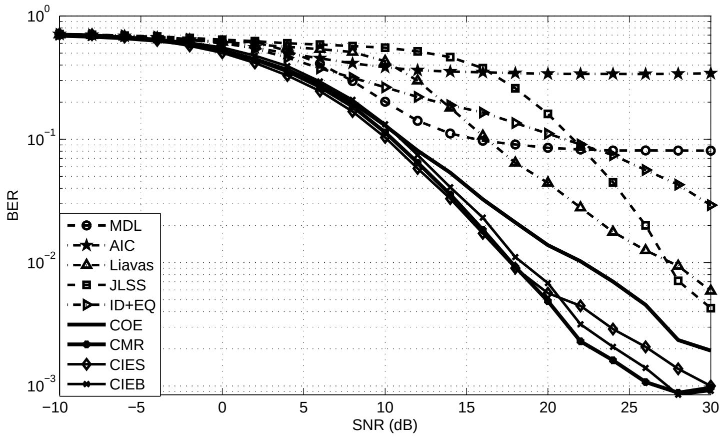
where P is the number of channel outputs, σ^2 is the noise variance and $y_j(k)$ is the noiseless samples of the j^{th} output.

In Figure 4.8, probability of correct channel order estimation and BER is plotted against SNR for $L = 3$ and $P = 3$. As seen from the Figure 4.8, proposed methods are better than the blind methods except CMR and COE in a wide SNR range. Performance of CIES is close to COE and worse than CMR in channel order estimation. On the other hand, COES has lower BER than COE when $SNR > 12dB$. Although CIEB performs worse than CIES, CMR and COE in channel order estimation, resulted BER in CMR, CIEB and CIES are nearly same.

The same experiment is done for $L = 5$, $P = 3$; and $L = 5$, $P = 5$ in Figure 4.9 and Figure 4.10 respectively. Channel order estimation performance of CIES increases when the number of channel increases and became similar with the CMR, which is the best one. Performance increment is also affected the BER of CIES and lowest BER is obtained by this method, when $SNR > 12dB$. Channel order estimation of CIEB also increases with the number of channel; however this increment is no enough to make it better than COE and CMR. On the other hand, BER performance is nearly same with CMR and better than COE when the channel order is also increased.

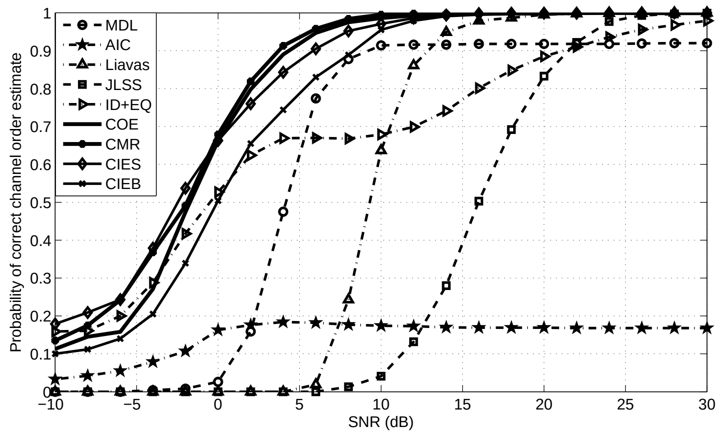


(a)

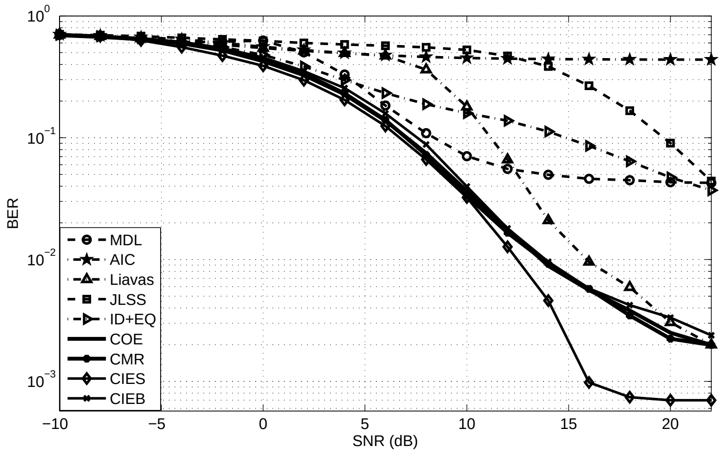


(b)

Figure 4.8: (a) Probability of correct channel order estimation versus SNR, (b)BER versus SNR for $L = 3$, $P = 3$.

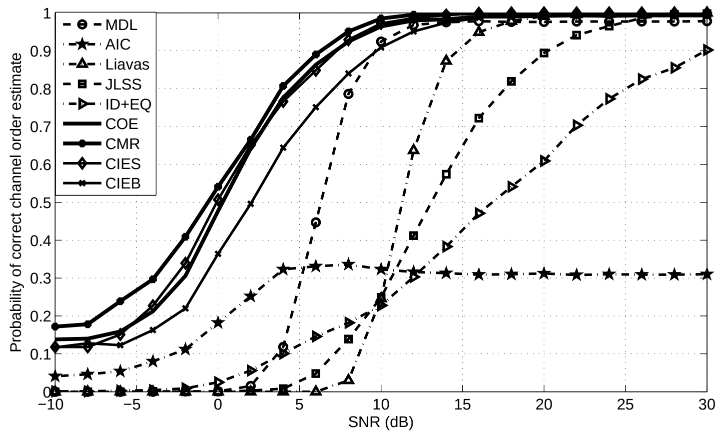


(a)

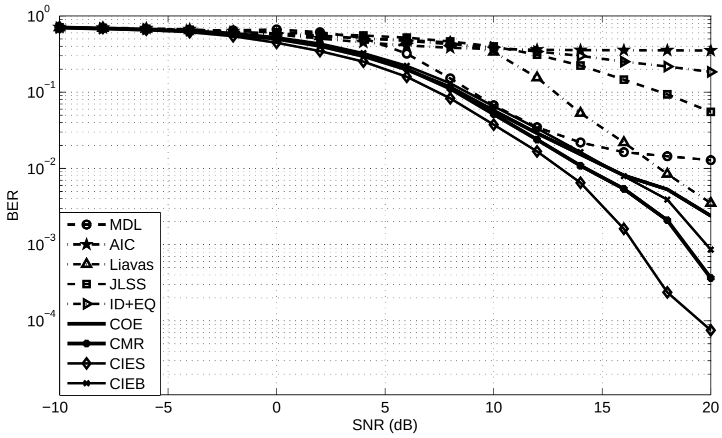


(b)

Figure 4.9: (a) Probability of correct channel order estimation versus SNR, (b)BER versus SNR for $L = 3$, $P = 5$.



(a)



(b)

Figure 4.10: (a) Probability of correct channel order estimation versus SNR, (b)BER versus SNR for $L = 5$, $P = 5$.

4.5 Conclusion

The use of training sequence for channel order estimation and channel identification is considered. For this purpose two new algorithms, CIEB and CIES, are proposed. Both of them compares the estimated channel input with the known training sequence to obtain a cost that has global minimum at true channel order. Their basic difference is in channel estimation. CIEB uses the LSS algorithm while CIES uses a semi-blind algorithm to estimate the channel coefficients. CIES performs better than CIEB as expected, as a result of using semi-blind algorithm. However, CIES has higher computational complexity due to multidimensional search done for joint frame synchronization and channel order estimation. When the training based methods are compared with the blind methods COE and CMR, there is no significant improvement for the channel order estimation. However, resulted BER in CIES is lower when $SNR > 12dB$ and improvement in BER becomes more significant when the number of channel is increased.

CHAPTER 5

EFFECTIVE CHANNEL ORDER ESTIMATION

5.1 Introduction

Effective channel order estimation is the task of finding the significant part of the channel filter. Figure 5.1 shows a generic channel filter impulse response. In this figure, the impulse response has large and small coefficients. Large coefficients can be assumed to be surrounded by the small coefficients which are called as leading and trailing tails without loss of generality. The distinction between tails and the significant part of the channel coefficients can be made by considering a threshold amplitude $\gamma/2$. The coefficients whose magnitudes are above $\gamma/2$ may be defined as the significant part of the filter. Obviously the value of $\gamma/2$ for determining the significant part depends on certain factors including SNR and the cost function or the measure used to define the significant part. When the SNR is very large, the true channel order can be defined to be the whole filter including the tail coefficients. When the SNR is low, it may not be possible to clearly identify the tail coefficients. It may also be meaningless to try to find those coefficients since the channel equalization performed over the noisy output samples does not give better MSE for the input samples when the tail coefficients are used due to noise amplification for the small channel coefficients. Therefore in practice, the task is to find the effective channel order which corresponds to finding the significant part of the channel filter rather than the true channel order. In fact in blind problem, it may be impossible to find the true channel order from the noisy output samples. True channel order has meaning when the SNR is very large or when the channel order estimation algorithms are tested by assuming that the channel filter is known. In this thesis, different channel filters, including fixed and random long channels with tails and channels without the tails are considered. Note that when there are no tails, true channel order and effective channel order become same. It should also be pointed that when we consider the channels without tails, the channel coefficients are generated randomly to obtain a Rayleigh distribution.

In practice, finding the effective channel order is more important than finding the true channel order, especially for the cases when the channel has leading and trailing tail coefficients. This is mainly due to the fact that channel matrix becomes ill-conditioned, when the channel includes small tail coefficients. In [22], the effect of small leading and trailing tails for subspace (SS) channel identification/equalization methods are investigated. It is shown that when the channel order is overestimated (i.e., the estimated channel includes a part of tail/head coefficients besides the significant part of the channel), the performance of LS and SS methods in channel estimation degrades dramatically. Therefore estimating the effective channel order is important for the improvement of the performance of blind channel estimation algorithms and obtaining better BER.

In [5], a channel order estimation algorithm is proposed for the effective channel order estimation. This algorithm is based on numerical analysis arguments and essentially consider the gap between the two consecutive eigenvalues of the estimated covariance matrix. In [4], information theoretic algorithms are analyzed and compared with [5] where the algorithm is shown to work well at high SNR. In [47], a criteria is presented to determine the effective channel order when the channel filter is known. In this thesis, this criteria is used to determine the effective channel order for a given channel filter. In addition, bit-error-rate (BER) is used as a measure to test the performance of the order estimation algorithms. The proposed methods for channel order estimation are analyzed under effective channel order concept. It is shown that COE and CMR are significantly better than other blind methods especially at low SNR and they tend to estimate the effective channel order when the search set does not include the true channel order. When the search set includes the true channel order, they tend to find true channel order at high SNR. The SNR range, where the effective channel order is mostly estimated, is larger than the alternative algorithms and better BER is obtained.

Channel order is an important parameter for channel estimation and equalization. Best performance is achieved when the effective channel order is correctly estimated. However, due to non existence of high performance of channel order estimation algorithms, this problem has been solved with the channel estimation algorithms robust to channel order overestimation or with non-linear adaptive equalizer such as DFE. It is known that their performance is not good as the their deterministic alternatives such as SS, CR and LSS which has superior performance when the channel order is known. On the another hand, the main drawback of blind deterministic algorithms is that their performances decreases dramatically when the channel order is not correctly estimated. Therefore the use of these algorithms is not practical as a consequence of absence of high performance channel order estimation algorithms. In this thesis, it is shown that when the proposed algorithms COE and CMR are used, deterministic blind algorithms can be used effectively and significant performance improvement can be achieved compared to LP based algorithms and fractionally spaced DFE which not require exact knowledge of the channel order.

Proposed semi-blind algorithms, CIES and CIEB are also evaluated and compared with the blind algorithms to see the gain of using training symbols in effective channel order estimation. In chapter 4, it is seen that the training based techniques do not offer much when the true channel order is considered. This is not the case, when the effective channel order estimation is considered. It is shown that CIES performs better than blind algorithms due to better channel estimation with pilots, when the final BER is considered. However, the main drawback of this algorithm is the high computational cost due to the multidimentional search for synchronization.

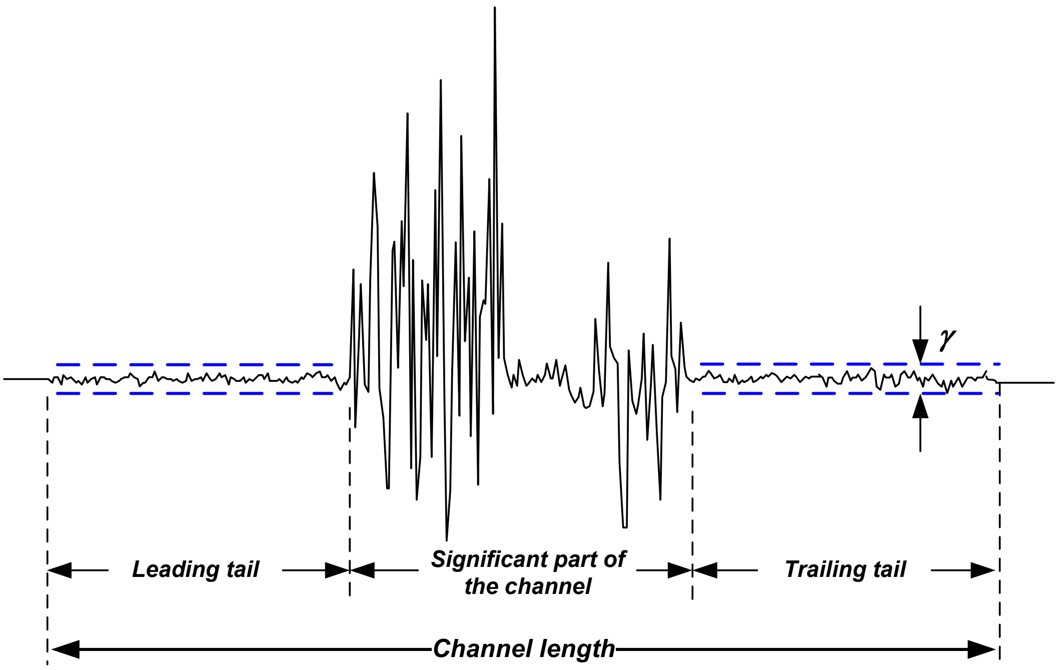


Figure 5.1: Channel impulse response showing the significant part and tail of the channels. Tail coefficients are assumed to be distributed uniformly between $-\gamma/2$ and $+\gamma/2$.

Organization of this chapter is as follows. Effective channel order concept is introduced and the measurement method given in [47] is summarized. Then several simulations are evaluated to see the performance of the proposed methods in estimating the effective channel order in different conditions.

5.2 Effective Channel Order

The measure in [47] is based on the effect of small tails on the estimation performance of the subspace methods. The estimation performance of those algorithms is measured by the closeness of the channel estimate to the significant part of the channel.

Suppose that the channel length is $L+1$ and the significant part of the channel coefficients are grouped at the middle of channel impulse response with length L_e+1 starting from the m_{th} index. In this case, the entire channel parameter vector is given as follows.

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}(0)^T & \cdots & \mathbf{h}(m-1)^T & \mathbf{h}(m)^T & \cdots & \mathbf{h}(m+L_e-1)^T & \mathbf{h}(m+L_e)^T & \cdots & \mathbf{h}(L)^T \end{bmatrix} \tag{5.1}$$

Channel parameter vector can be written as the sum of the significant part, $\mathbf{h}_t$, and residuals containing the tails, $\mathbf{h}_t$, as follows.

$$\mathbf{h} = \mathbf{h}_s + \mathbf{h}_t \tag{5.2}$$

where,

$$\begin{aligned}
\mathbf{h_s} &= \begin{bmatrix} 0^T & \cdots & 0^T & \mathbf{h}(m)^T & \cdots & \mathbf{h}(m+L_e)^T & 0^T & \cdots & 0^T \end{bmatrix} \\
\mathbf{h}_t &= \begin{bmatrix} \mathbf{h}\left(0^T\right) & \cdots & \mathbf{h}(m-1)^T & 0^T & \cdots & 0^T & \mathbf{h}(m+L_e-1)^T & \cdots & \mathbf{h}(L)^T \end{bmatrix} \\
\mathbf{h}(m) &= \begin{bmatrix} h_1(m) & \cdots & h_P(m) \end{bmatrix}^T
\end{aligned} \tag{5.3}$$

The significant part of the channel is defined as the $L_e + 1$ successive terms, $\mathbf{h}(m), \cdots, \mathbf{h}(m + L_e)$, which contain the most energy. Hence m is chosen as,

$$m = \arg\max_{l} \sum_{k=l}^{l+L_e} \|\mathbf{h}(k)\|^2 \tag{5.4}$$

Now let $\mathbf{H}_M(\mathbf{f})$ denote the channel convolution matrix with MP rows constructed from the channel parameter vector $\mathbf{f}$. M is temporal window length for the received data.

$$H_M(\mathbf{f}) = \begin{bmatrix} \mathbf{f}(0) & \mathbf{f}(1) & \cdots & \mathbf{f}(L) & & & \\ & \mathbf{f}(0) & \mathbf{f}(1) & \cdots & \mathbf{f}(L) & & \\ & & \ddots & \ddots & & \ddots & \\ & & & \mathbf{f}(0) & \mathbf{f}(1) & \cdots & \mathbf{f}(L) \end{bmatrix}_{MP\times(M+L)} \tag{5.5}$$

Using this notation, channel convolution matrix can be written as the sum of the convolution matrix constructed from the significant channel coefficients, $H_M(\mathbf{h}_s)$ and the convolution matrix constructed from the tail coefficients, $H_M(h_t)$.

$$H_M(\mathbf{h}) = H_M(\mathbf{h}_s) + H_M(\mathbf{h}_t) \tag{5.6}$$

SS and LS based channel estimation methods use the data auto-correlation matrix. If the channel input is white Gaussian process with unit variance, the data covariance matrix can be written as follows.

$$\mathbf{R}_M = H_M(\mathbf{h})\, H_M(\mathbf{h})^H \tag{5.7}$$

$$= H_M(\mathbf{h}_s)\, H_M(\mathbf{h}_s)^H + \mathbf{E}_M \tag{5.8}$$

where $\mathbf{E_M}$ is the covariance matrix including the effect of tails. If the tails are zero and the order of the significant part is known, SS and LS algorithms estimate the channel coefficients correctly in noise free case. Addition of zero tails does not change the covariance matrix. However small leading and trailing tails add noisy terms to covariance matrix, which has nearly the same effect to the algorithms with colored input sequence [22]. Assuming that the significant part is to be estimated, in [22] it is found that the estimation error is bounded by

$$\left\|\mathbf{h}_s - \hat{\mathbf{h}}_s\right\| \leq 2\sqrt{2}\frac{\varepsilon_{L_e}}{\sigma_{2L_e+1}} \tag{5.9}$$

for two channel SIMO system in the case of $M = L_e + 1$. σ_{2L_e+1} is the non-zero smallest singular value of the convolution matrix $H_M(\mathbf{h}_s)$ and ε_{L_e} is the norm of the convolution matrix $H_M(\mathbf{h}_t)$. The value of σ_{2L_e+1} identifies the diversity of the channel. Hence the estimation error depends on the diversity of the channel and the size of the tails. If the diversity of the channel is large and the magnitude of the tails is small, then LS/SS methods estimate the channel impulse response closer to the significant part of the channel. Concluding from that, the following criteria is selected to determine the effective channel order, which is generalized to the P channel SIMO system, i.e.,

$$\sigma_{M+L_e}/\varepsilon_{L_e} \tag{5.10}$$

where σ_{M+L_e} is the non-zero smallest singular value of the matrix $\mathbf{H}_M$, which is built from the significant part of the channel. ε_{L_e} is the norm of the matrix $\mathbf{H}_M$, built from tails. The effective channel order L_e, is selected as the value that corresponds to the maximum of $\sigma_{M+L_e}/\varepsilon_{L_e}$.

Note that, when the selected value of L_e is equal to the true channel order, L, then $\mathbf{h}_t = 0$ and $\varepsilon_{L_e} = 0$. The measure given in equation (5.10) takes the maximum value and the effective channel order and true channel order becomes the same. Therefore, true channel order should be excluded from the search set, $L_{min} < L_e < L_{max}$. This requires knowledge of the range for the possible effective channel orders.

In practical cases, the channel impulse response is not known and the effective channel order should be estimated. In [4], a method, which is called as Liavas algorithm in this thesis, is proposed to obtain the effective channel order from the channel outputs. Effective rank of the covariance matrix of the received signal $\mathbf{R}_M = E\left\{\mathbf{y}_M(t)\,\mathbf{y}_M(t)^H\right\}$ is used. $\mathbf{y}_M(t)$ is the channel output vector formed via stacking M samples received from each subchannels. Effective rank, $r(k)$ is defined as $\bar{k} = \arg\min_k r(k)$ and

$$r(k) = \begin{cases} \frac{1}{\frac{\lambda_k}{\lambda_{k+1}}-2} & , if \frac{\lambda_k}{\lambda_{k+1}} \geq 3 \\ 1 & otherwise \end{cases} \tag{5.11}$$

where λ_k is the k^{th} eigenvalue of the matrix $\mathbf{R}_M$, such that $\lambda_k \geq \lambda_{k+1}$. It is also possible to write ,

$$\bar{k} = \arg\max_k \frac{\lambda_k}{\lambda_{k+1}} \tag{5.12}$$

since the minimization of $r(k)$ corresponds to the maximization of $\frac{\lambda_k}{\lambda_{k+1}}$. Then the effective channel order is given as follows,

$$L_e = \bar{k} - M \tag{5.13}$$

5.3 Evaluation of The Performance of Channel Order Estimation Algorithms in Estimating The Effective Channel Order

5.3.1 Fixed Channel

In this section, the proposed algorithms are tested with fixed channels having small leading and trailing tails. In the simulations, the same channel coefficients are used at each trial. For this purpose, the channel given at [3] and measured microwave channel impulse responses obtained from "*http://spib.rice.edu/spib/microwave.html*" are used. Subchannels are obtained via sampling at twice the baud rate; hence they fit into the two channel SIMO model. The impulse responses of the channels are plotted in Figure 5.2 for [3] and Figure 5.3 and Figure 5.4 for microwave channels (chan2.mat and chan10.mat).

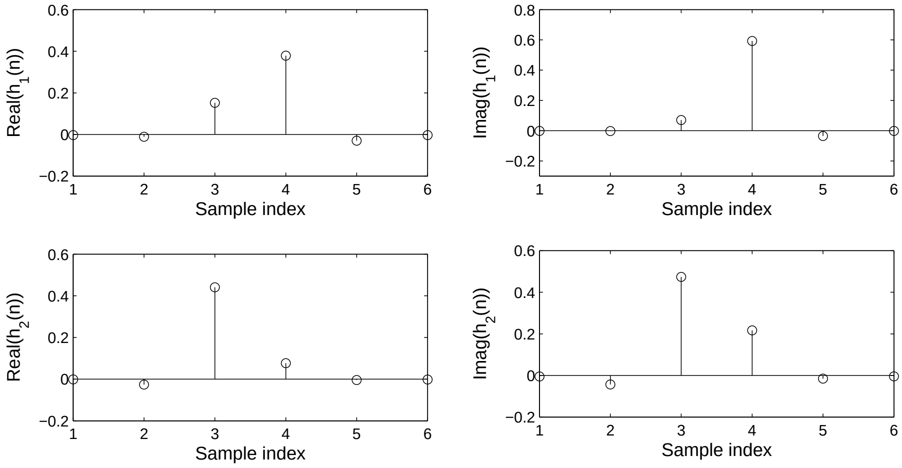


Figure 5.2: Channel impulse response for Channel-1. $h_1(n)$ and $h_2(n)$ are the impulse responses of two channel SIMO system ($P = 2$).

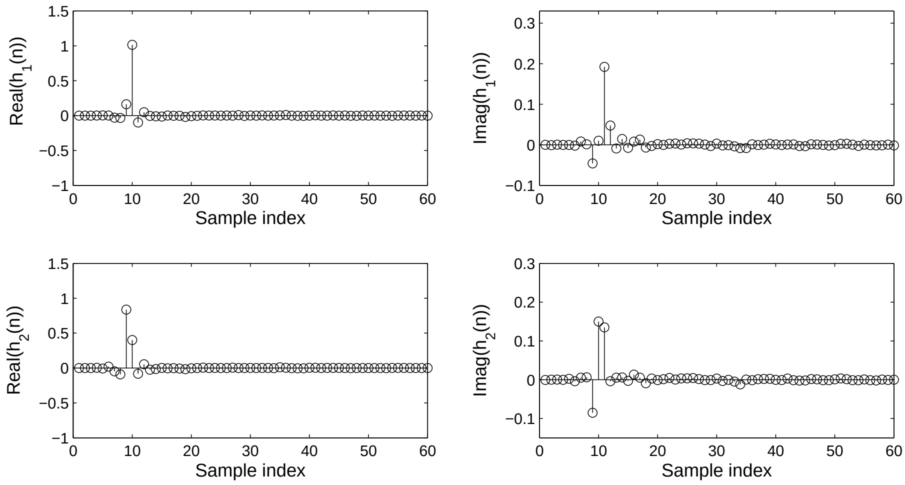


Figure 5.3: Channel impulse response for the microwave channel, Channel-2, is shown for only 60 samples for clarity. The total number of samples for each channel is 115. $h_1(n)$ and $h_2(n)$ are the impulse responses of two channel SIMO system ($P = 2$).

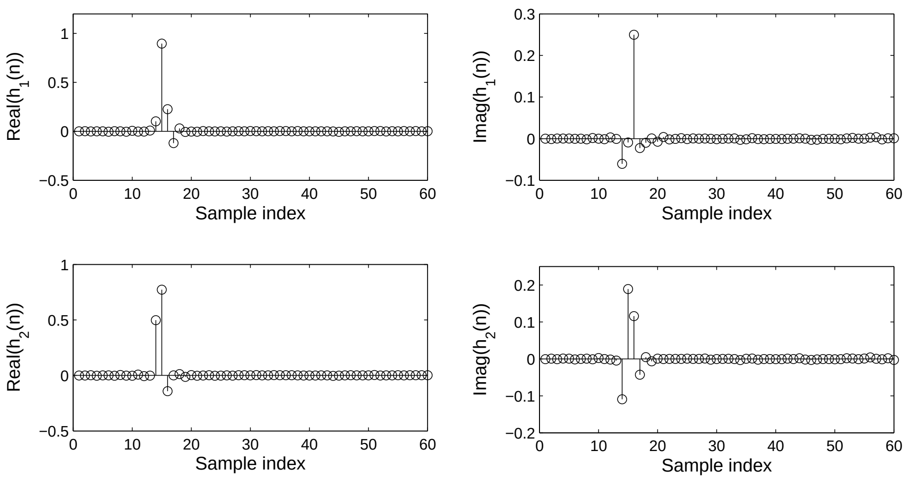
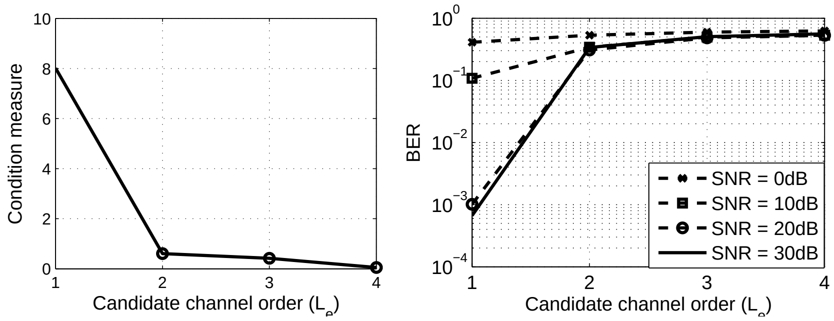
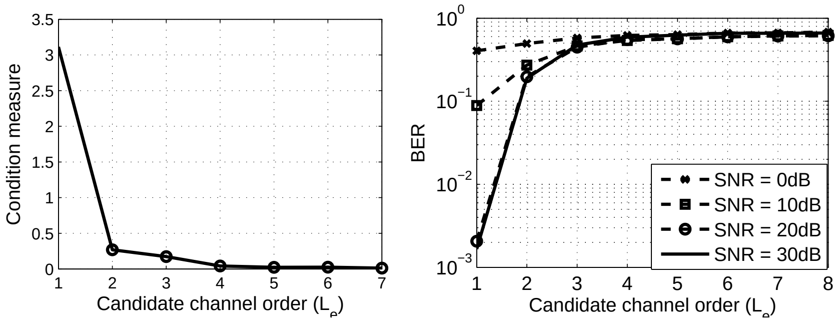


Figure 5.4: Channel impulse response for the microwave channel, Channel-3, is shown for only 60 samples for clarity. The total number of samples for each channel is 150. $h_1(n)$ and $h_2(n)$ are the impulse responses of two channel SIMO system ($P = 2$).

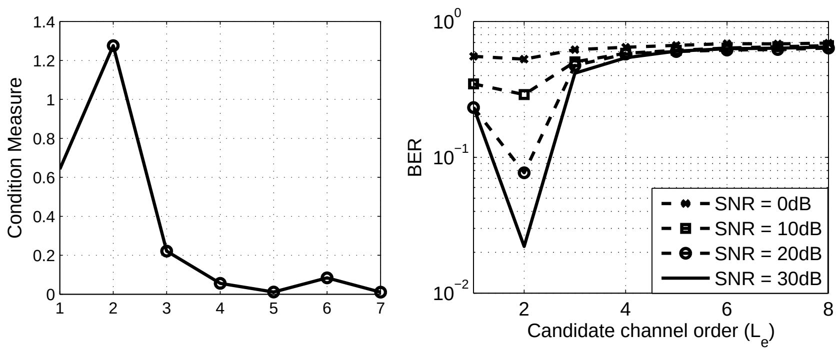
The effective channel order is determined using the criteria $\frac{\sigma_{2L_e+1}}{\varepsilon_{L_e}}$ for the selected channels. In Figure 5.5, the value of the criteria given by equation 5.10 (also called as condition measure) is plotted against the channel order. The index which gives the maximum value is taken as the effective channel order. As seen from the Figure 5.5, the effective channel order is one for the Channel-1 and Channel-2, and two for Channel-3. Condition measure not only gives information about the effective length of the significant part. But it also shows whether the channel is ill-conditioned or not. When the condition measure is lower, the signal and noise spaces are closer and it is hard to separate them in noisy measurements. Therefore channel estimation error increases in subspace based methods. This is also related to the rank of the channel matrix. The channel matrix should be of full column rank for blind channel estimation algorithms. If the diversity is zero, which is the case when the condition measure is zero, channel matrix is not of full column rank. Hence, proper channel estimation is not possible. As a result, when the condition measure is low the estimation of effective channel order and the channel coefficient is problematic. In this respect, Channel-3 can be said to be an ill-conditioned channel compared to the others. The condition measure is defined originally for the SS based methods, and finds the channel order which leads to maximumly separable subspaces. Therefore, effective channel order measured through condition measure may be different for other blind channel estimation algorithms, such as LSS. To clarify this point for the LSS method, BER performance of LSS algorithm is also plotted against channel order for different SNR values, when the Wiener equalizer is used. In the simulations, uniformly distributed uncorrelated QPSK modulated input signal with length 113 is used. 1000 trials are made and the channel is fixed, while the input signal and noise change in each trial. As shown in Figure 5.5, minimum BER is obtained at the determined effective channel order in all channels. Hence the effective channel order determined by means of condition measure is also valid for LSS.



(a)



(b)



(c)

Figure 5.5: Condition measure, $\frac{\sigma_{2L_e+1}}{\varepsilon_{L_e}}$, versus candidate channel order and BER versus candidate channel order for (a) Channel-1. (b) Channel-2, and (c) Channel-3.

After investigating the properties of the channels, we have tested the proposed methods. For this purpose, the probability of correct effective channel order estimation of the algorithms and BER at the equalizer output is plotted against the SNR. Common simulation settings are as follows. Input signal length is 113, and first 13 elements are the training samples used in CIES and CIEB. In blind methods, these samples are assumed to be unknown. As a modulation waveform QPSK is used. For a fair comparison, after the channel order is estimated by the algorithms, channel is estimated with LSS and equalized with Wiener equalizer for all blind channel order estimation algorithms. The channel order search set does not include the true channel order. Therefore, the effective channel order is the channel order to be found. In CIES algorithm, the value of α is set to $\alpha = 100$. 1000 trials are evaluated at each SNR level to compute the probability of correct channel order estimation and BER. SNR is computed as follows.

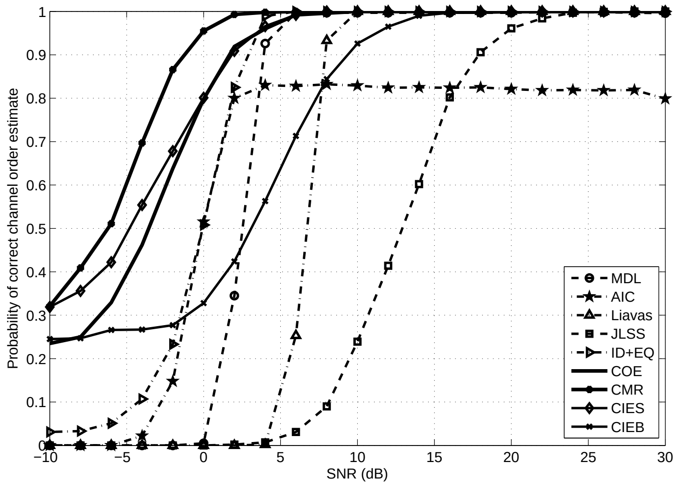
$$SNR = \frac{1}{P\sigma^2} E\left\{ \sum_{j=1}^{P} \left| y_j(k) \right|^2 \right\} \tag{5.14}$$

where P is the number of outputs, σ^2 is the noise variance and $y_j(k)$ is the noiseless samples of the j^{th} channel output.

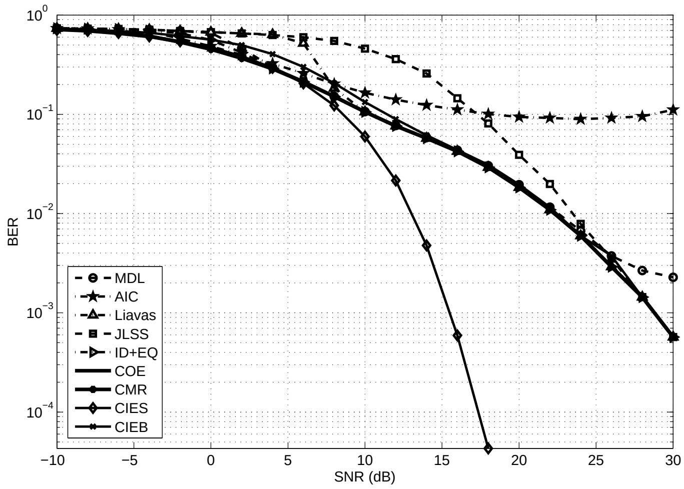
For Channel-1, the probability of correct effective channel order estimation versus SNR and BER versus SNR graphs are given at Figure 5.6. The proposed blind algorithms COE and CMR are the best ones in both BER and for the estimation of effective channel order. ID+EQ and MDL follow them. Although ID+EQ and MDL are worse than the proposed method in order estimation, their BER is approximately same. The reason is that, the main error source is the additive channel noise at low SNR. Since all blind algorithms find the channel using the same algorithm, their BER performances are the same when they detect the effective channel order with high probability. Semi-blind method, CIES, is different than others in that respect. It obtains the channel coefficients via proposed semi-blind channel estimation method. Therefore, it has the best BERperformance, although it has performance between COE and CMR in order estimation. The reason is that, channel coefficients are estimated more correctly in CIES because of using training sequence in channel estimation. The gain obtained in channel estimation compensates the error in channel order estimation. CIEB is not as good as CMR, COE and CIES in order estimation when $SNR < 15dB$. However it does not lead high BER performance difference. When $SNR > 15dB$, there is no BER difference with COE and CMR. This is because, it estimates the effective channel order correctly with high probability and channel coefficients are found by using the same algorithm (LSS).

For Channel-2, similar results are obtained for COE, CMR, ID+EQ and CIES as shown in Figure 5.7. Different from the Channel-1, MDL and AIC can not determine the effective channel order at high SNR anymore. This leads high BER compared to Channel-1 for these algorithms. CIEB performs worse when compared to Channel-1. It detects the effective channel order with high probability when $SNR > 20dB$.

Channel-3 is the most problematic channel for the algorithms. As discussed earlier, Channel-3 is an ill conditioned channel. The performance graphs are given at Figure 5.8 for Channel-3. MDL seems to be the best blind algorithm for this channel and CMR follows it. COE performance is poor. COE can detect the effective channel order when $SNR > 25dB$. ID+EQ algorithm seems to be the worst one when we look at the channel order estimation performance. Although JLSS is better than ID+EQ in channel order estimation for that channel, this is not the case in BER. This is because, ID+EQ tends to estimate the effective channel order as one, which is the second best point obtained in condition measure result in Figure 5.5.c. On the other hand, JLSS mostly overestimates the channel order, i.e., $\hat{L}_e > 2$. This means that estimating effective channel order as one is better than overestimating it for Channel-3, if the true effective channel order (2) can not be estimated. Semi-blind algorithm, CIES, is the best one both in channel order estimation and BER for Channel-3. The reason is that, semi-blind algorithms are more robust to the ill-condition channels in channel estimation because of the use of training. One of the most important gain of using training sequence in channel estimation is that, there is no need for the SIMO channel conditions. It can be ill-condition or it can include common zeros. Channel identifiability conditions forced for blind methods are not required for training based methods. By changing the value of α, robustness to ill-conditioned channel can be increased.

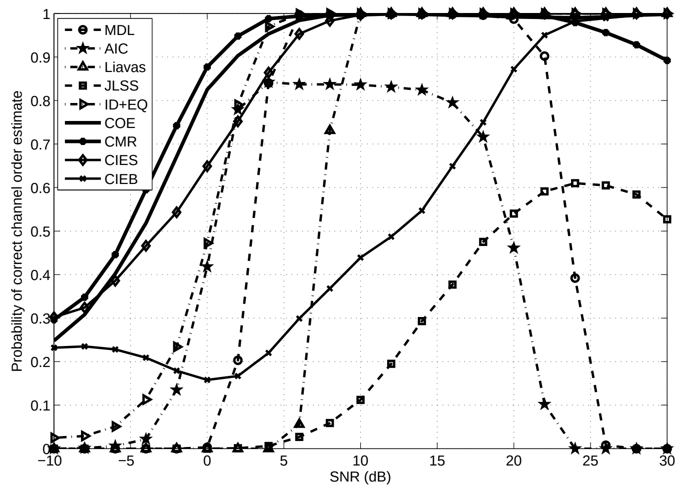


(a)

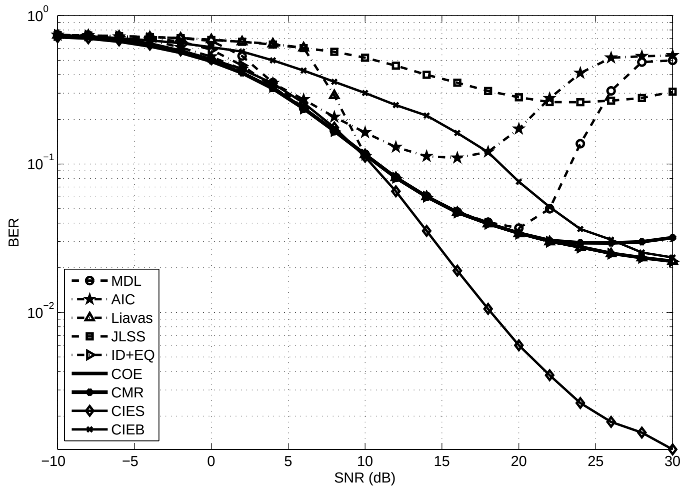


(b)

Figure 5.6: (a) Probability of correct effective channel order estimation for Channel-1. Effective channel order is assumed to be one. (b) BER versus channel order for Channel-1.

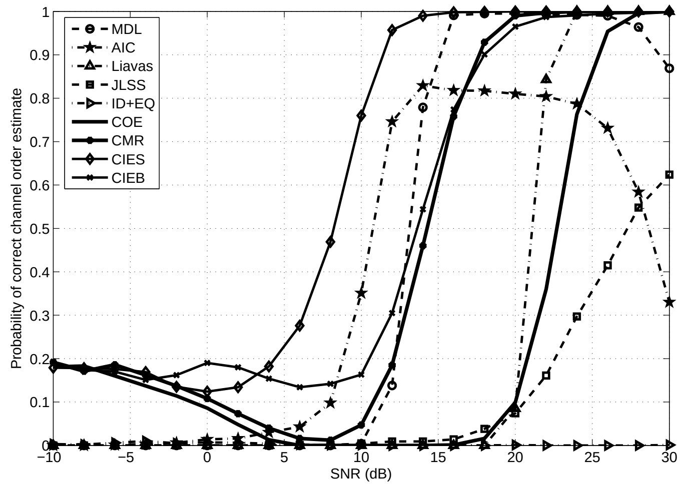


(a)

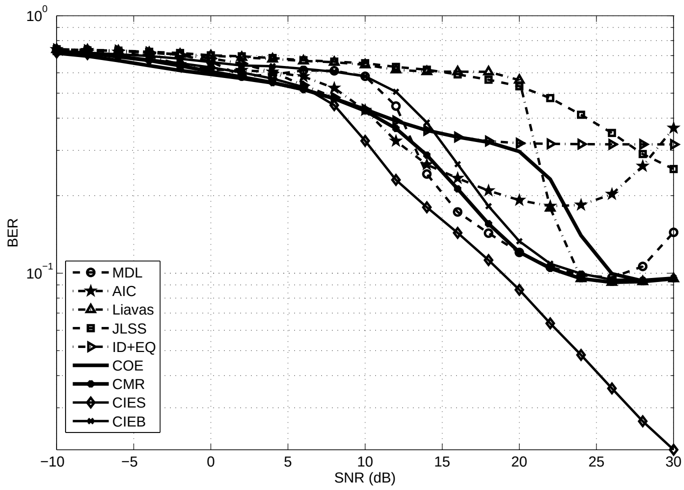


(b)

Figure 5.7: (a) Probability of correct effective channel order estimation for Channel-2. Effective channel order is assumed to be one. (b) BER versus channel order for Channel-2.



(a)



(b)

Figure 5.8: (a) Probability of correct effective channel order estimation for Channel-3. Effective channel order is assumed to be two. (b) BER versus channel order for Channel-3.

5.3.2 Random Channel

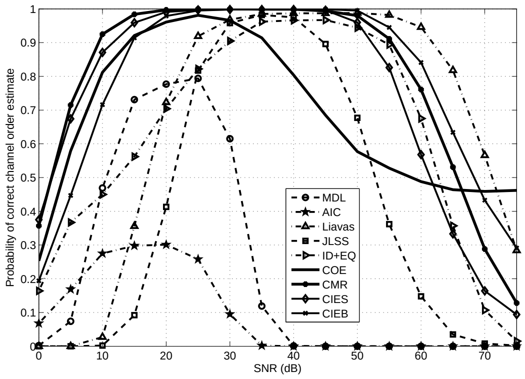
The proposed algorithms are tested for a channel having small leading and trailing tails. In the simulations in this section, channel coefficients are randomly generated. Condition measure in (5.10) is used to calculate the effective channel order and only the channels satisfying the given effective channel order are used. Channel coefficients in significant part of the channel are complex values randomly chosen from unit variance zero mean Gaussian set. The channel coefficients in the tails are complex values and uniformly distributed between $-\gamma/2$ and $+\gamma/2$. The value of γ is related to the energy of the tails and hence ε_{L_e}. When γ is increased keeping σ_{M+L_e} constant, the condition measure decreases. In this case, it will be harder to obtain the effective channel order calculated via condition measure. Therefore it is expected that the performance of the algorithms decreases. Robustness to this change in γ is important for the algorithms. Therefore, we have also tested the algorithms for different values of γ.

Common simulation settings are as follows. Input signal length is 113, and first 13 elements are the training samples used in CIES and CIEB. In blind methods, these samples are assumed to be unknown. As a modulation waveform QPSK is used. Number of channels is $P = 3$ and the effective channel order is $L_e = 3$. For a fair comparison, after the channel order is estimated by the algorithms, channel is estimated with LSS algorithm and equalized with Wiener equalizer for all blind channel order estimation algorithms. In CIES algorithm, the value of α is set to $\alpha = 100$. The channel order search set does not include the true channel order. Therefore, the effective channel order is the channel order to be found. 1000 trials are evaluated at each SNR level to compute the probability of correct channel order estimation and BER.

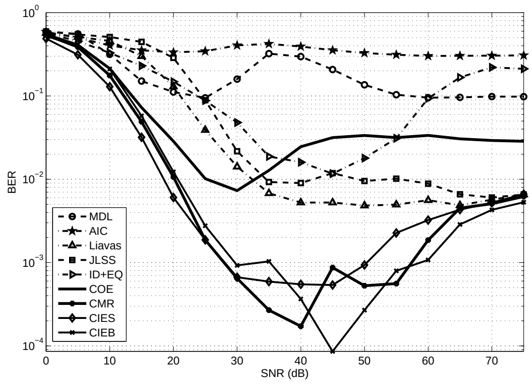
Two cases are considered. In the first case, channel length is short, $L = 8$. Channel order is searched between 1 and 9. In that way, true channel order, $L = 8$, and it is included in the search set. In the second case, channel length is long, $L = 18$. Channel order is searched between 1 and 9. Hence the true channel order ,$L = 18$, is excluded from the search set. Effective channel order is the channel order that should be estimated in this case. In Figure 5.9.a and 5.9.b channel order estimation and BER performance of the algorithms are shown for $\gamma = 0.05$. In this simulation, short channel case is considered. As shown from the Figure 5.9.a, algorithms find the effective channel order in mid SNR ranges, while they tend to estimate the true channel order at high SNR ranges. CMR is the best one among the blind algorithms in estimating the channel order when $SNR < 50dB$. When $SNR > 50dB$ it also estimates the true channel order. The SNR range for estimating the effective channel order is larger for CMR. BER is lower when the effective channel order is detected, as shown in Figure 5.9.b. COE algorithm is not good as CMR. COE is better than other blind algorithms when $SNR < 32dB$. Semi-blind algorithms, CIES and CIEB, show similar performance with CMR.

In Figure 5.9.c and 5.9.d, channel order estimation and BER performance of the algorithms are shown for $\gamma = 0.05$ with longer tails. In this simulation, long channel case is considered. As shown from the Figure 5.9.c, CMR, COE, CIES, Liavas and ID+EQ tends to estimate the effective channel order even at high SNR ranges. When Figure 5.9.b and Figure 5.9.d are compared, it is seen that better BER is obtained when the effective channel order is detected even at high SNR ranges. When $SNR > 30dB$, BER does not decrease any more as SNR increases. This is because, in that region the main error source is the tail coefficients, which acts as a colored noise in channel estimation. CMR is the best one among the blind algorithms and it is followed by COE in both channel order estimation and BER. Semi-blind algorithm, CIES performs significantly better than blind algorithms both in channel order estimation and BER. CIEB is worse than CIES and has similar performance with COE.

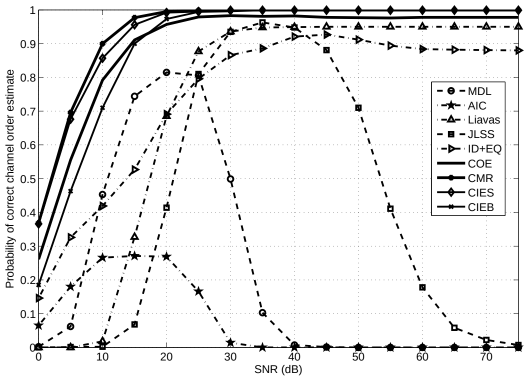
The same experiments are repeated for $\gamma = 0.1$, and 0.2. The performance plots are shown in Figure 5.10 and Figure 5.11 for $\gamma = 0.1$ and 0.2 respectively. As expected, the performance of the algorithms decreases as γ increases. On the other hand, the performance difference between the proposed algorithms and others increases with increasing γ when the channel length is $L = 18$, i.e., search set does not include the true channel order. For the short channel case, BER performances are closer when $SNR > 55dB$. At lower SNR ranges, CMR is the best one among blind algorithms. The size of SNR range, where the effective channel order is mostly estimated, decreases with increasing γ. In that respect, the best algorithms are CIES, CIEB and CMR. Blind algorithms except COE and CMR can not detect the effective channel order even for the long channel case when $\gamma = 0.2$. In that respect proposed algorithms COE, CMR, CIES and CIEB are more robust to the value of γ.



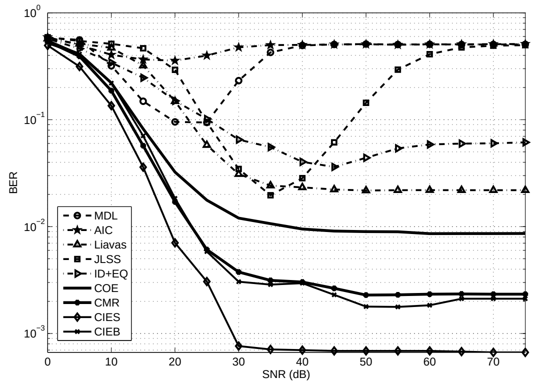
(a) Short channel case, L=8



(b) Short channel case, L=8

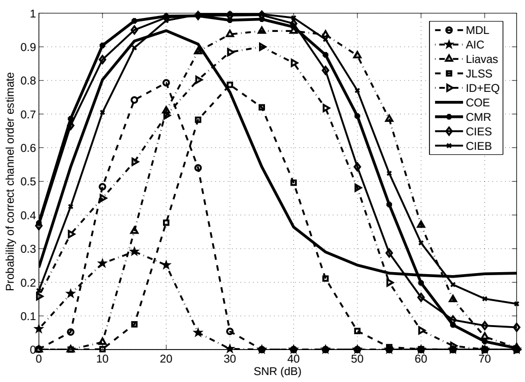


(c) Long channel case, L=18

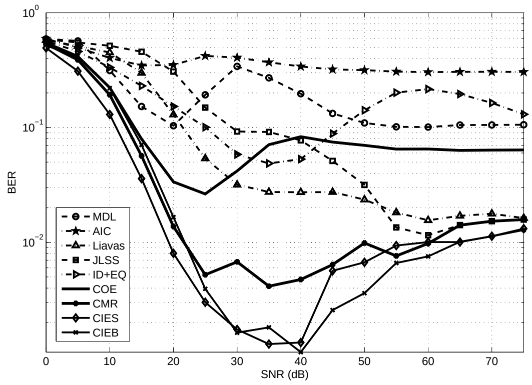


(d) Long channel case, L=18

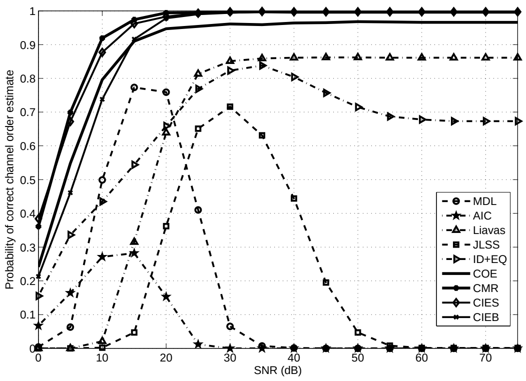
Figure 5.9: Channel order estimation performances for $\gamma = 0.05$, $L_e = 3$ and $P = 3$.



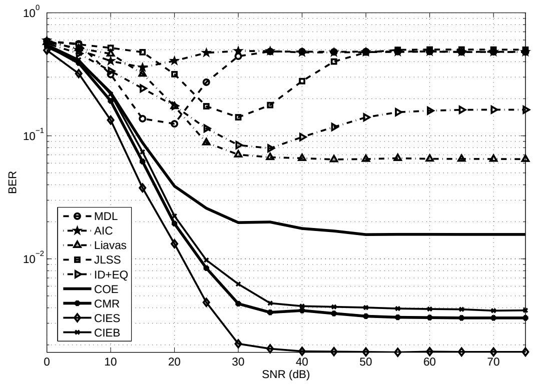
(a) Short channel case, L=8



(b) Short channel case, L=8

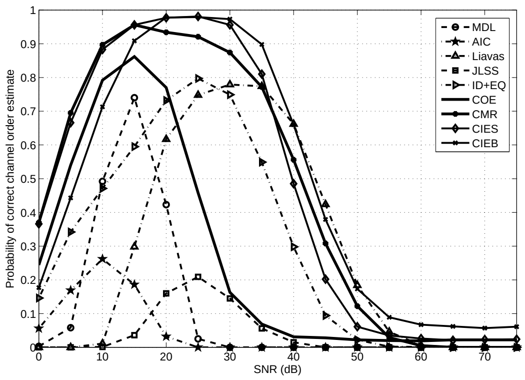


(c) Long channel case, L=18

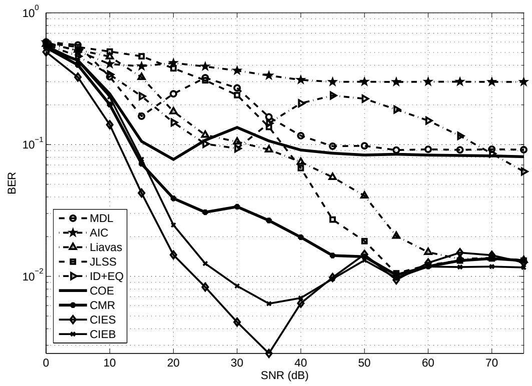


(d) Long channel case, L=18

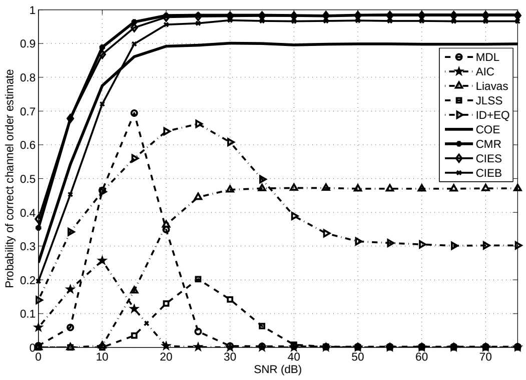
Figure 5.10: Channel order estimation performances for $\gamma = 0.1$, $L_e = 3$ and $P = 3$.



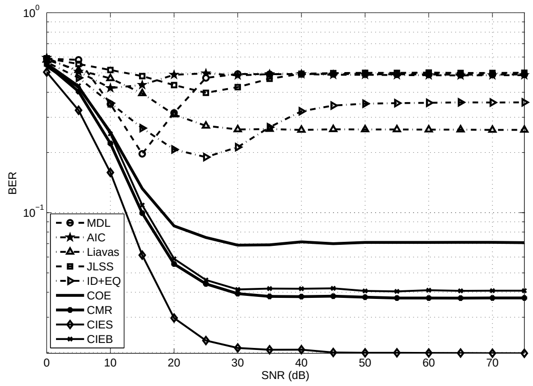
(a) Short channel case, L=8



(b) Short channel case, L=8



(c) Long channel case, L=18



(d) Long channel case, L=18

Figure 5.11: Channel order estimation performances for $\gamma = 0.2$, $L_e = 3$ and $P = 3$.

5.4 BER When Different Channel Estimation Algorithms are Used

In blind identification, it is possible to use a channel order estimation algorithms which has a tendency to overestimate like MDL and AIC. Then algorithms robust to overestimation such as LP [14, 15] and MLP [17] can be employed.

Therefore, correct channel order estimation may not be seen as a critical issue. However the performance of such algorithms is low compared to deterministic methods such SS [6] and LSS [7, 3] when the channel order is correctly estimated. On the other hand, SS and LSS algorithms are not robust to channel order overestimation and their performances are dramatically reduced when the channel order is not correctly known [22]. With the help high performance channel order estimator they can be used effectively. Therefore, COE and CMR can be a solution for this problem. In order to determine the best approach in blind identification, LP based approach and the proposed blind algorithms are compared by using extensive simulations. For that purpose LP [14, 15] and MLP [17] algorithms are used as the algorithms robust to overestimation of the channel order and SS and LSS algorithms are used as high performance deterministic algorithms in the comparison.

5.4.1 Performance of The Channel Estimation Algorithms in Case of Channel Order Mismatch

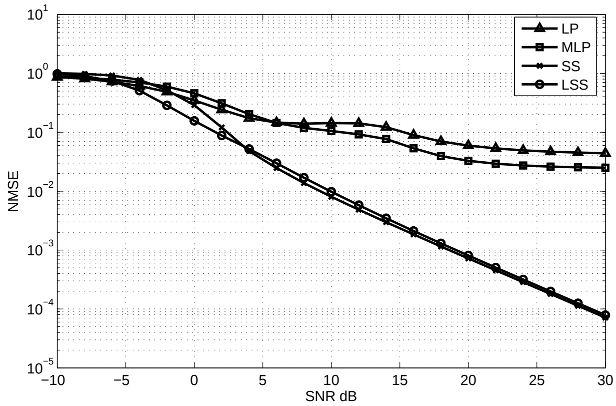
Channel estimation performances of LP, MLP, SS and LSS algorithms are compared in case of channel order mismatch. The channel given in [15] is used in the simulations. It is a four channel ($P = 4$) SIMO channel with channel order $L = 5$. Channel coefficents are given in Table 5.1 Length of the input signal is 200 and QPSK is used as a modulation waveform. 500 trials are realized for each simulation and at each trial input signal and channel noise is randomly generated. When SS and LSS algorithms are used, channel is equalized with Wiener equalizer. MLP algorithm obtains the inputs signal besides the channel coefficients. Channel estimation performances is measured by the normalized mean square error (NMSE) . NMSE is defined as,

$$NMSE = \frac{\sum_{n=1}^{N_r} \left\|\hat{\mathbf{h}}_n - \mathbf{h}\right\|^2}{N_r \left\|\mathbf{h}\right\|^2} \tag{5.15}$$

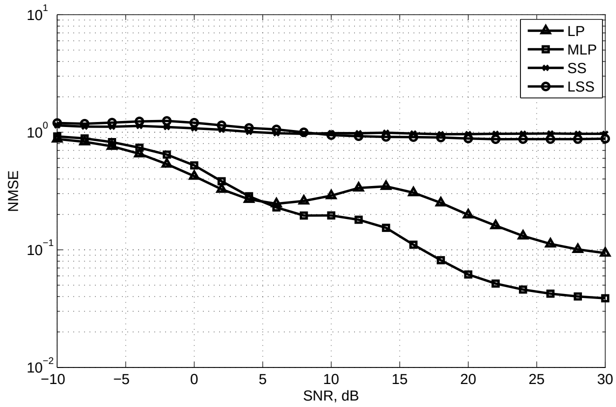
where N_r is the number of trials. In Figure 5.12.a and Figure 5.12.b channel estimation performance of LP, MLP, LSS and SS against SNR are compared when the channel order is known and overestimated by two respectively. As shown form the Figure 5.12.a, LSS and SS algorithms outperform the MLP algorithm in all SNR ranges and the performance gap between SS/LSS and LP/MLP increases with increasing SNR. LP and MLP are statistical methods and their performance remains constant due to the finite number of observations after a certain point. When the channel order is overestimated, which is the case in Figure 5.12.b, performance of SS and LSS algorithms dramatically reduces. On the other hand, the performance of MLP algorithm does not change significantly when the channel order is overestimated and in this case it works better than SS and LSS. However the performance difference is not so much as compared to the case when the channel order is correctly estimated. In Figure 5.13, normalized channel estimation error is plotted against the channel order for a fixed SNR level, $SNR = 15dB$. In this case, the channel order is fed to each algorithm. As shown from the Figure 5.13, LSS and SS are not robust algorithms against channel order mismatch, and MLP is robust for channel order overestimation. However channel estimation error is higher for MLP when the channel order is correctly estimated compared to the SS and LSS.

Table 5.1: Channel coefficients of the channel given in [15], L=5, P=4.

$\mathbf{h}_1$	-0.0419	-0.2993	-1.2808	-0.5301	0.1417	-0.2624
$\mathbf{h}_2$	0.9097	-0.2021	-0.4401	-1.0153	-0.5364	-0.0817
$\mathbf{h}_3$	-1.1836	0.4906	-0.3093	0.4011	0.1269	-1.8522
$\mathbf{h}_4$	1.2965	0.0525	0.3410	-0.0260	0.3991	0.8817



(a)



(b)

Figure 5.12: Channel estimation error versus SNR when (a) channel order is correctly estimated $\hat{L} = L$ and (b) channel order is over estimated, $\hat{L} = L + 2$.

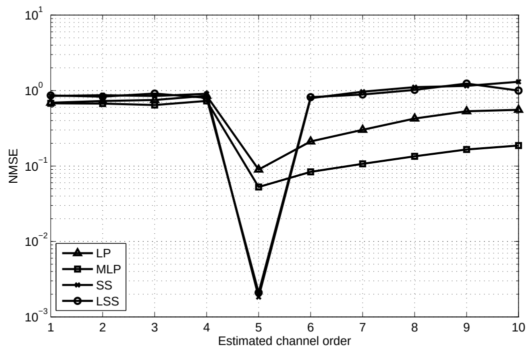
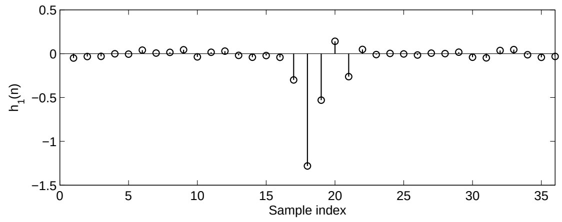


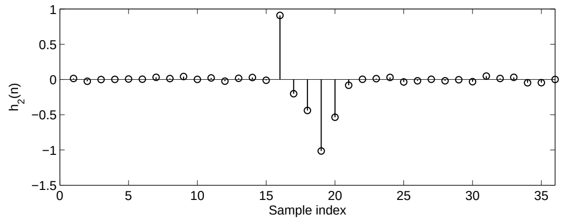
Figure 5.13: Channel estimation error versus channel order when $SNR = 15dB$.

The same comparison is also done when the channel has small leading and trailing tails. For that purpose, leading and trailing tail coefficients are added to the channel given in [15]. The channel condition measure is used to ensure that effective channel order is $L_e = 5$. Channel impulse responses for each subchannel are shown in Figure 5.14. In Figure 5.15.a and Figure 5.15.b channel estimation error is plotted against SNR when channel order is correctly estimated and overestimated by two respectively. In Figure 5.16, channel estimation errors of the algorithms are shown for fixed SNR level. As shown from the figures, the LP and MLP are robust to over estimation of the channel order. In addition they have the best result at the effective channel order, $L_e = 5$. SS and LSS algorithms are not robust to channel order overestimation and their performance is better than LP and MLP when the effective channel order is correctly estimated.

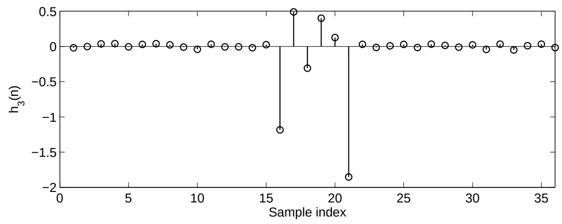
As a result of these simulations, we understand that the LP and MLP are robust to overestimation of channel order and MLP is better than original the original LP algorithm in this respect. SS and LSS perform significantly better than LP and MLP in case of correct channel order estimation. On the other hand their performance is not acceptable when the channel order is overestimated.



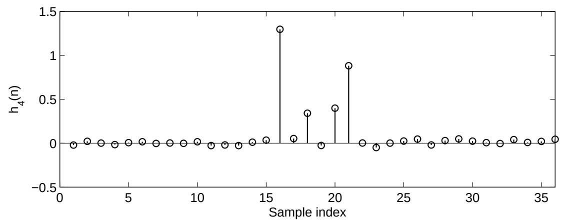
(a)



(b)

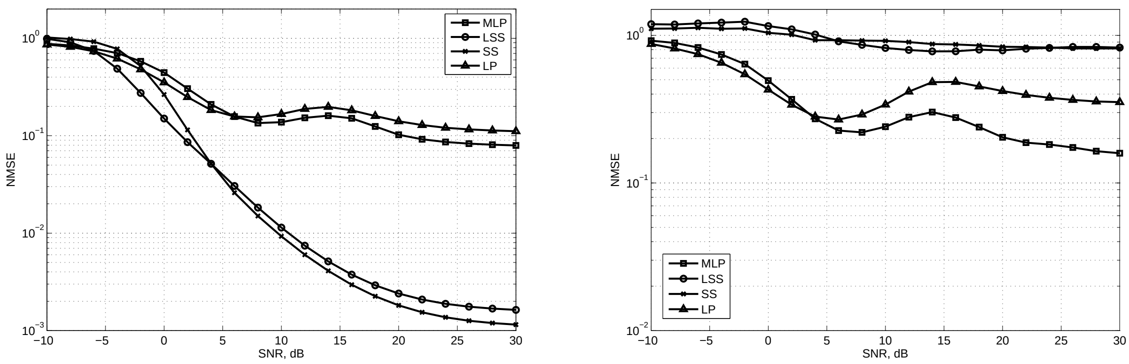


(c)



(d)

Figure 5.14: Channel impulse responses of four channel SIMO system.



(a) (b)

Figure 5.15: Channel estimation error versus SNR when (a) channel order is correctly estimated $\hat{L} = L$ and (b) channel order is over estimated, $\hat{L} = L + 2$.

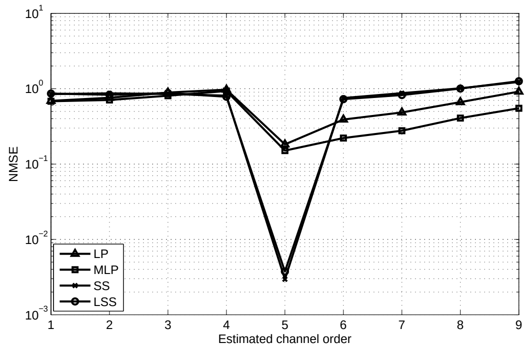


Figure 5.16: Channel estimation error versus channel order when $SNR = 15dB$.

5.4.2 Performance of Channel Estimators, MLP and LSS, with Different Channel Order Estimation Algorithms

In this section, MLP and LSS algorithms are evaluated with different channel order estimation algorithms. For the comparison, BER graphs are used. Channel order is first estimated with the channel order estimation algorithms, COE, CMR, AIC, MDL, JLSS, Liavas and ID+EQ. Then, channel coefficients are estimated by MLP and LSS. Wiener equalizer is used to estimate the channel input for the LSS algorithms. MLP finds the input sequence jointly. The simulation settings are same as defined in the previous section. Simulations are divided into two parts. In the first part, the channel given in [15] is used. For this channel, effective channel order and true channel order are equal. In the second part, the same simulations are repeated for the channel which has small leading and trailing tails. This channel is obtained by adding small tail coefficients to the beginning and end of the subchannels. The effective channel order is $L_e = 5$ and it is checked with channel condition measure defined in equation 5.10. In the simulations, BER and normalized mean-square-error (NMSE) used for performance comparisons. Blind methods find the channel input signal up to complex scale factor, α. Therefore it is needed to estimate α in order to calculate the BER and NMSE for the estimated channel input signal. An optimum value of α is given as,

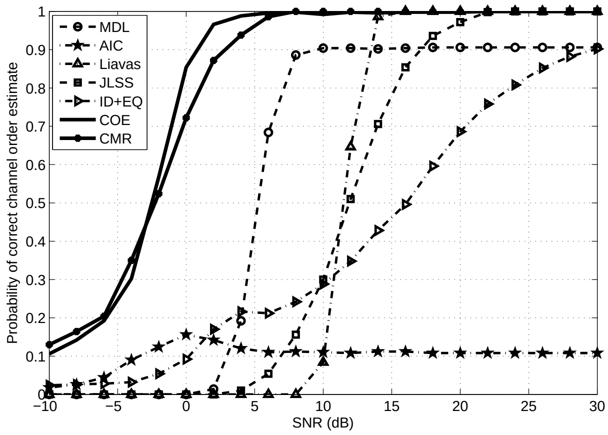
$$\alpha = \frac{\hat{\mathbf{s}}^H \mathbf{s}}{\hat{\mathbf{s}}^H \hat{\mathbf{s}}} \tag{5.16}$$

where $\hat{\mathbf{s}}$ is the estimated channel input vector and $\mathbf{s}$ is the channel input vector.

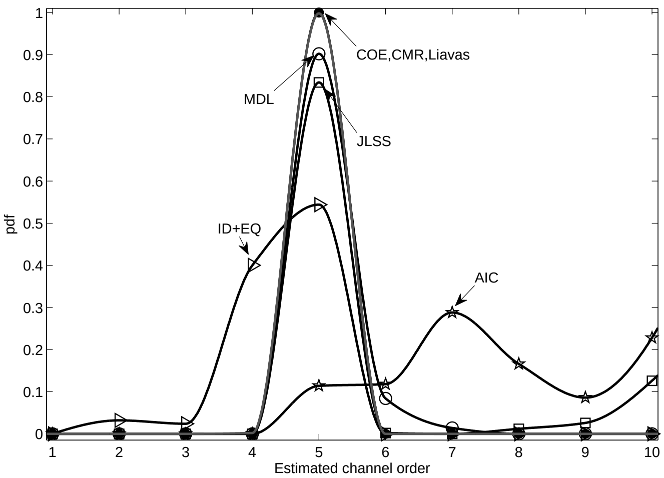
Part-1:

In this part, the channel in [15] is used where effective and true channel order are same. In Figure 5.17.a, probability of correct channel order estimation for different channel order estimation algorithms are plotted against SNR. As shown from the Figure 5.17.a COE and CMR perform significantly better than their alternatives and channel order is correctly estimated when $SNR > 6dB$. MDL and AIC algorithms tend to overestimate the channel order at $SNR = 20dB$ as shown in Figure 5.17.b.

In Figure 5.18.a-b, BER and NMSE performances are plotted against SNR when the LSS algorithm is used as a channel estimator and Wiener equalizer is used for input estimation for MDL, AIC and Liavas algorithms. The same simulation is repeated in Figure 5.18.c-d where MLP is used for both channel and input estimation for MDL, AIC and Liavas. Other channel order estimation methods include channel estimators in their algorithms. As shown from the figure, COE and CMR obtain the best results in both of the cases. AIC works better with MLP, since it tends to overestimate the channel order. MDL performs better with LSS channel estimator and Wiener equalizer. Figure 5.18 also shows that the equalization in MLP algorithm is not satisfactory in general and a better equalization algorithm can be used once the channel coefficients are found. For this purpose, Wiener equalizer is used after estimating the channel coefficients for the MLP algorithm. The results are shown in Figure 5.19 for this case. As shown from the figure, better results are obtained for MDL and AIC compared to case of using MLP. However their performances are still far away from the performances of proposed methods.

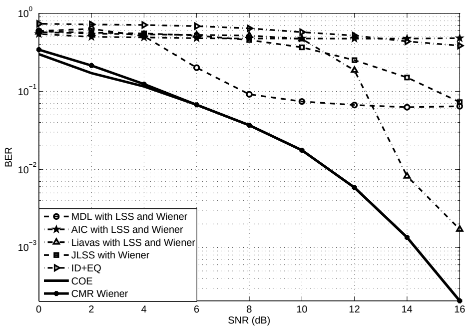


(a)

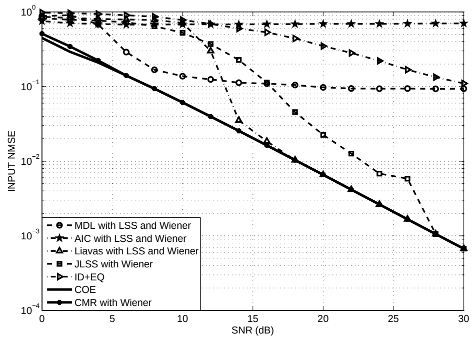


(b)

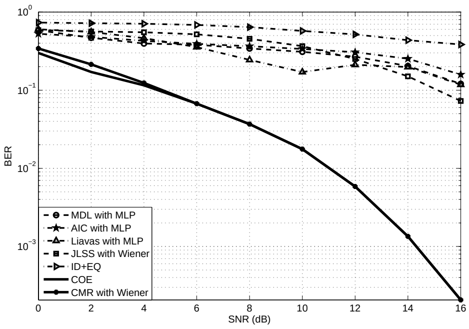
Figure 5.17: Channel order estimation performances of different algorithms. (a) Probability of correct channel order estimation, (b) Probability density functions of estimated channel orders with different algorithms when $SNR = 15dB$. PDF is obtained from the histogram of the estimated channel orders with curve fitting technique.



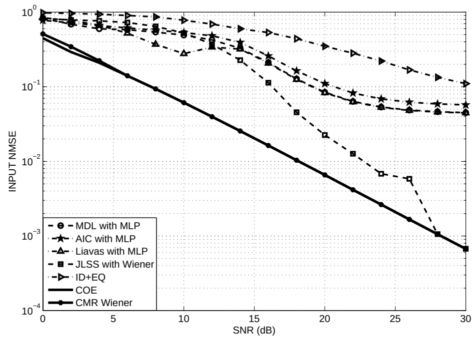
(a)



(b)

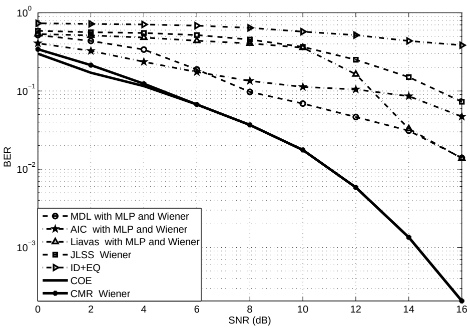


(c)

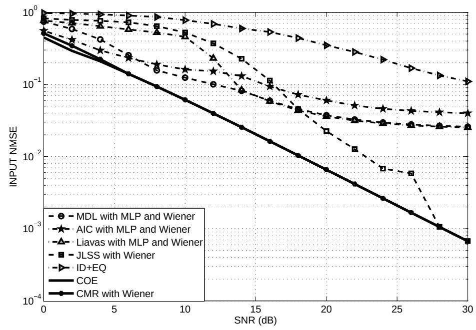


(d)

Figure 5.18: BER and NMSE versus SNR when the true channel order is estimated with different channel order estimation algorithms (a)-(b) LSS and Wiener equalizer are used to estimate the channel and input signal for MDL, AIC and Liavas. (c)-(d) MLP is used to estimate the channel and input signal for MDL, AIC and Liavas.



(a)



(b)

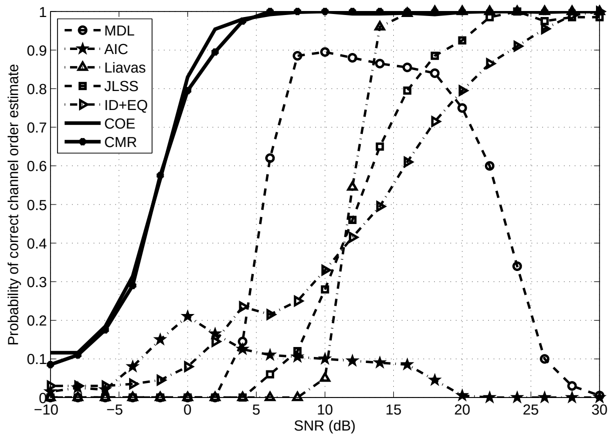
Figure 5.19: BER and NMSE versus SNR when the true channel order is estimated with different channel order estimation algorithms. MLP is used with Wiener equalizer in MDL, AIC and Liavas.

Part-2:

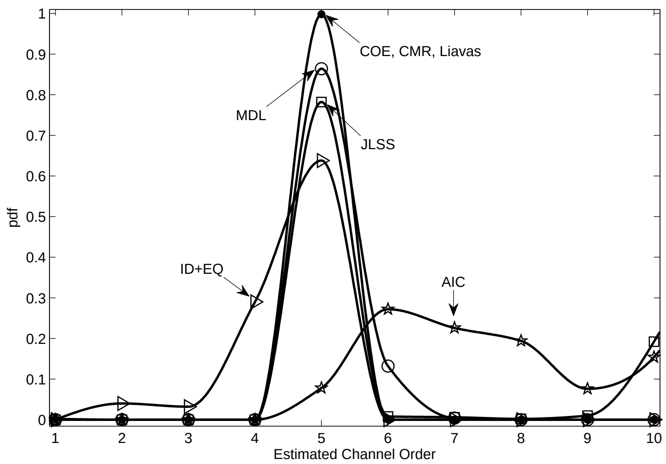
In this part, evaluations for a channel with tail coefficients are done. In Figure 5.20.a, the probability of effective channel order estimation performance of the channel order estimation algorithms are plotted against SNR. As shown from the figure 5.20, COE and CMR perform significantly better than others and the effective channel order is correctly estimated when $SNR > 6dB$. MDL and AIC algorithms tend to overestimate the channel order at $SNR = 15dB$ as shown in Figure 5.20.b.

In Figure 5.21.a-b, BER and NMSE performances are plotted against SNR when the LSS algorithm is used as a channel estimator for MDL, AIC and Liavas. In this case Wiener equalizer is employed. The same simulation is repeated in Figure 5.21.c-d where MLP is used for both channel and input estimation for MDL, AIC and Liavas. AIC and MDL algorithms works better with MLP as a result of their tendency to overestimation. However, this performance improvement is not enough and COE and CMR are significantly better than these algorithms. Liavas algorithm shows better performance with LSS algorithm and has similar performance with COE and CMR, when the SNR is high. To increase the performance of MLP, Wiener equalizer is used by omitting the equalization step in MLP in Figure 5.21.e-f. In this case, better results are obtained for MDL, AIC and Liavas compared to the case of using MLP. However, even in this case COE and CMR outperform MDL and AIC.

When we compare Figure 5.18 and 5.21, it is seen that the COE and CMR perform better for channels with tail coefficients. In other words, they estimate the effective channel order more accurately compared to the alternative techniques. When we compare Part-I and Part-II, MDL and AIC performances decreases when the channel has small tail coefficients. This leads higher BER. However, COE and CMR performances are not much affected from the small tail coefficients. Therefore, the performance gap between the cases of using MLP+ MDL/AIC and using LSS+COE/CMR increases more when the effective channel order is considered.



(a)



(b)

Figure 5.20: Channel order estimation performances of different algorithms. (a) Probability of correct channel order estimation, (b) Probability density functions of estimated channel orders with different algorithms when $SNR = 15dB$. PDF is obtained from the histogram of the estimated channel orders with curve fitting technique.

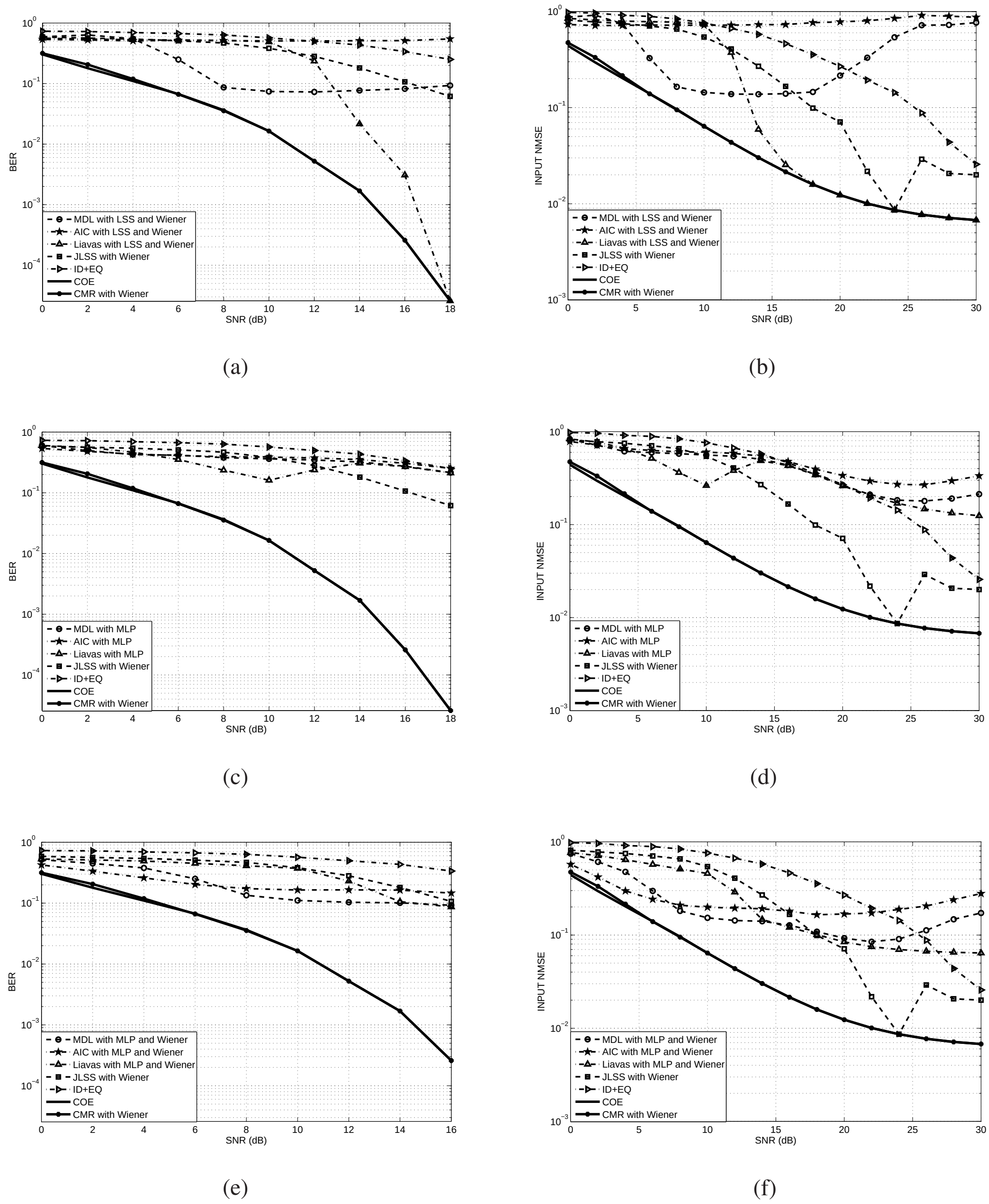


Figure 5.21: BER and NMSE versus SNR when the effective channel order is estimated with different channel order estimation algorithms (a)-(b) LSS and Wiener equalizer are used to estimate the channel and input signal for MDL, AIC and Liavas. (c)-(d) MLP is used to estimate the channel and input signal for MDL, AIC and Liavas. (e)-(f) MLP is used with Wiener equalizer in MDL, AIC and Liavas.

5.5 Performance Comparison of Proposed Methods with DFE in Fractionally Spaced SISO Channels

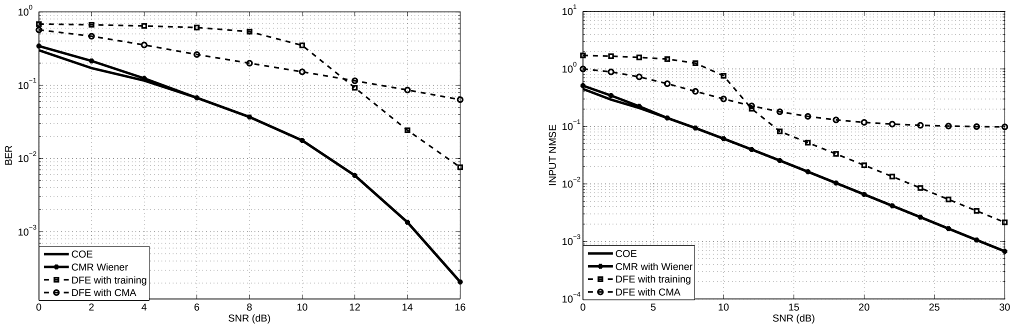
In SISO systems, equalizer performance can be significantly improved by introducing nonlinearity into the equalizer structure [41, 45]. One of the best example of nonlinear equalizers is the decision feed back equalizer (DFE) [45]. The nonlinearity of DFE comes from the nonlinear characteristics of the detector that provides an input to the feedback filter. The main idea is that if the previously detected symbols are known, the ISI introduced on the currents symbol can be removed by the subtraction of previously detected symbols with appropriate weighting [41]. The advantage of DFE over linear equalizer is the feedback filter which removes residual ISI after feedforward filter. DFE feedback loop remains stable as long as correct decisions are done at the decision ruler (or slicer). If an error occurs at the decision, the DFE starts generating errors on the equalized signal, and it cause more decision errors and hence error propagation. This is the main drawback of DFE systems. The convergence problem is another drawback of the algorithm especially for time varying channels. In this case, for fast convergence, training data should be repeated occasionally. Training based DFE system starts in training mode. After the filter coefficients converged, it switches to decision directed mode. For proper operation of decision directed mode, eye diagram of the signal at decision input should be sufficiently open. Training sequence is used to open the eye diagram. Blind methods can also be used for this purpose. Constant Modulus Algorithm (CMA) (or Godard) method [30] is one of the most popular blind methods used with DFE.

DFE is a suboptimum method considering that it assumes that past decision are correct. The optimum solution is the maximum likelihood sequence estimator (MLSE). If the feedforward filter were infinitely long, DFE would be a perfect zero-forcing equalizer.

Fractionally spaced equalization is used to provide immunity to sampling errors. Fractionally spaced system corresponds to oversampling of the received signal. This receiving structure can also be implemented by a SIMO system by means of using a polyphase structure. Therefore, blind methods proposed for SIMO systems can be used in this system structure. In SIMO systems, perfect identification and equalization is possible with the help deterministic blind methods such as SS, CR, and LSS. In practice, blind deterministic methods are not preferred due to their computational complexity and the lack of robustness to channel order. In this thesis, COE and CMR are proposed to solve the robustness problem. In turns out that proposed blind approach performs better in case of blind problem setting. To verify this, we have compared the DFE method with the proposed methods, COE and CMR, for oversampled SISO channels. In the simulations, the channel given in [15] used. For this channel, oversampling ratio is 4 and the channel order is $L = 5$. QPSK is used as modulation waveform and the length of the transmitted data is 1000. Fractionally spaced DFE with constant modulus algorithm and DFE with training are compared with the channel order estimation methods COE and CMR. In CMR, channel is equalized with Wiener equalizer. DFE uses 25 tap feedforward filter and two tap feedback filter; the step size is 0.0005. 500 trials are used in the simulations. The result of DFE is compared after it is converged. In training based DFE first 500 samples are used for training purpose and recursive least squares (RLS) algorithm is used to update filter coefficient with a forgetting factor of 0.98. Note that DFE with training is considered only to see the performance in comparison. Since other techniques are completely blind and do not take advantage of training sequence.

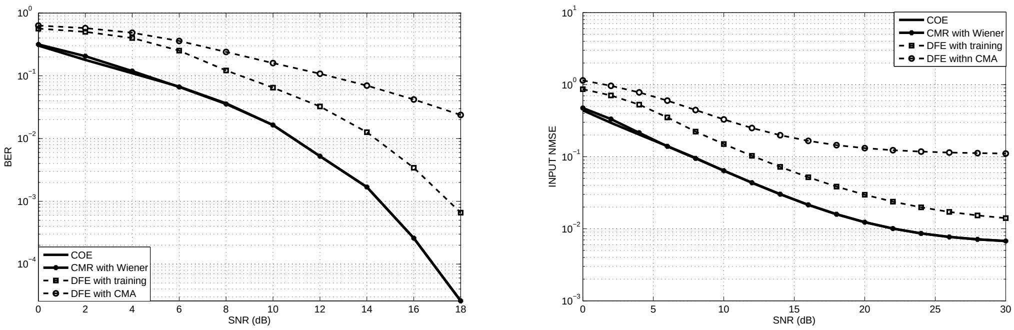
In Figure 5.22.a and Figure 5.22.b, MSE and BER performance of the algorithms are presented. As shown from the figures, proposed methods, COE and CMR perform significantly better than DFE with CMA especially at high SNR ranges, due to the accuracy in channel order estimation and finite convergence of the LSS algorithm. On the other hand, the performance of DFE with training is close to the COE and CMR but not better. In Figure 5.23.a and Figure 5.23.b, BER and MSE performance of the algorithms are presented, when the channel includes long tail coefficients. Transfer functions of the channels are given in Figure 5.14. As shown from Figure 5.23.a-b,, the similar results are obtained in case of a channel with tails.

In Figure 5.24, performance of the algorithms are tested with the microwave channel given in "*http://spib.rice.edu/spib/microwave.html*" as chan3.mat, whose impulse response is plotted in Figure 5.3. As shown from the figures, DFE with training shows the best performance. However, CME and CMR are still better than the blind DFE significantly. The microwave channel has two taps per channel, and DFE can estimate the true equalizer coefficients better using the same number of filter coefficients. On the other hand COE and CMR performances decrease when the number of channels in SIMO system is decreased.



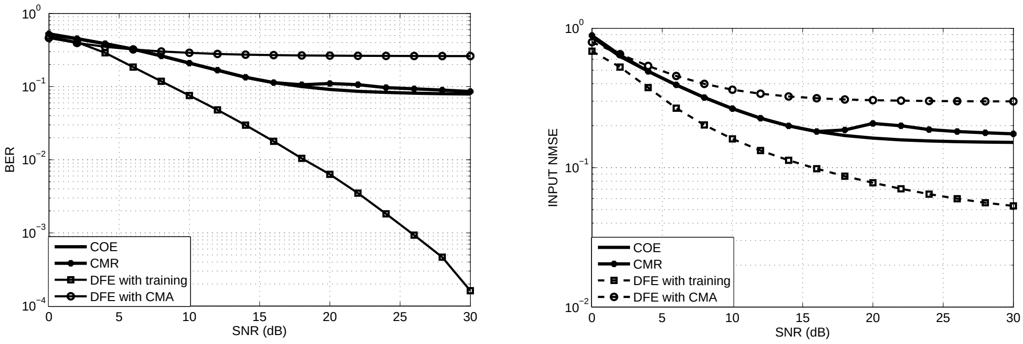
(a) (b)

Figure 5.22: MSE and BER performance of COE, CMR and DFE methods. The channel given in [15] is used without tail coefficients.



(a) (b)

Figure 5.23: MSE and BER performance of COE, CMR and DFE methods. The channel given in [15] is used with tail coefficients.



(a) (b)

Figure 5.24: MSE and BER performance of COE, CMR and DFE methods. Two channel case considered with a microwave channel given in "*http://spib.rice.edu/spib/microwave.html*" as chan3.mat.

5.6 Conclusion

In this chapter, effective channel order estimation performances of the proposed algorithms are considered. There are two cases to be handled. In the first case, channel is comparably short and the search set includes the true channel order. In the second case, channel is long and the channel order search set does not include the true channel order. The long channel case is more practical one considering the long microwave channels. The effective channel order should be taken into account in channel order estimation. It is shown that, algorithms tend to estimate the true channel order at high SNR instead of the effective channel order. However, BER performance loss occurs even when the true channel order is used. This is because the channel is ill conditioned as a result of small tail coefficients and this leads to channel estimation error. COE and CMR are the best blind algorithms in obtaining effective channel order. The performance difference between COE/CMR and their alternatives increases when the energy of tails increases. Semi-blind method, CIES, performs better than all blind methods and BER performance is significantly better than the others for all cases. This is due to the fact that CIES takes advantage of the training sequence. However, it has high computational complexity due to the multidimensional search to handle synchronization and channel order estimation in a joint manner.

LP based methods are robust to overestimation of the channel order and therefore they do not require high performance channel order estimators. On the other hand, it is seen that their performances are not good as the deterministic channel estimation methods such as SS and LSS when the channel order is correctly estimated. SS and LSS require correct channel order estimation to work well. COE and CMR have superior channel order estimation performance and the robustness problem of the SS and LSS algorithms for channel order overestimation is handled by using COE and CMR. Much better BER is obtained by using COE and CMR with LSS compared to the case of using LP based algorithm as a channel estimator. COE and CMR are investigated with the LSS algorithm in order to estimate channel coefficients. LSS algorithm has certain properties which make it ideal choice in order estimation. Proposed methods are also compared with DFE, which does not require channel order, in a fractional spaced SISO systems. It is shown that COE and CMR outperform blind DFE especially at high SNR ranges. On the other hand, DFE with training shows better performance than COE and CMR with the loss in bandwidth efficiency, when the number of channels is small.

CHAPTER 6

CONCLUSION

Blind system identification is an important topic in signal processing with many applications in different fields including communications, radar, acoustics, speech and more. The main problem in this context is to find the system impulse response and the input by using only the output samples. This problem can be considered in SISO, SIMO and MIMO systems. In SISO systems, there are techniques which can solve the blind identification problem under certain conditions [30, 42, 43]. In SIMO systems, blind identification problem is shown to be solvable with fewer constraints. In addition, it is possible to solve the problem perfectly in noise free case, when there is no common channel zero between subchannels of SIMO system. The problem in MIMO setting is the hardest one to solve due to many reasons.

In this thesis, blind channel order estimation problem is considered for SIMO communication systems. While the importance of the estimation and use of channel order is widely known in signal processing community, this thesis shows that the implications of the effective channel order estimation are more significant than the common expectations.

In this thesis, two new blind channel order estimation algorithms are presented, namely COE and CMR. These algorithms are proved to have the finite convergence property, i.e., they are guaranteed to find the true channel order from finite number of observations for noise free case. Previously there were only two algorithms known in the literature with the same property. Proposed algorithms work in completely different ways but their cost functions have similarities. In fact, they have a "convex like" shape which allows one to pick the channel order corresponding to the minimum value of the cost function. Therefore there is no need to define and use a threshold value. COE algorithm is based on the estimation of channel output. The difference between the estimated and observed channel outputs is taken as the cost function. In estimating the channel output, LSS algorithm as a channel estimator and Wiener equalizer are used. The superior performance of the COE method on estimating the channel order is based on the properties of LSS algorithms which are derived in this thesis. It is shown that the LSS algorithm generates common channel zeros besides the true channel zeros in noise free case when the order is overestimated. Furthermore, these common zeros are located close to the unit circle. When the common channel zeros are located on the unit circle, the inverse channel transfer function does not decay to zero rapidly. Perfect FIR equalization is not possible, and equalization error is large. Therefore equalization error is higher for the LSS case when the channel order is overestimated. The convex like cost function of COE is much deeper compared to the case where other blind estimators such as SS and CR are used. CMR method also uses the LSS algorithm. It is based on generating a relation in terms of the channel matrix. It turns out that the channel matrix relation is established by a matrix which has a Toeplitz structure only if the channel order is overestimated by one. The deviation from the Toeplitz structure is used to obtain a cost function for the channel order estimation. The estimation of common channel zeros requires the inverse of the estimated channel matrix. When the channel matrix includes common channel zeros positioned on the unit circle, the deviation from the Toeplitz structure increases and a convex like function with a deeper valley at the true channel order is obtained. It should also be noted that the common channel zeros found by the LSS algorithm has no relation to the input or channel in general.

While the finite sample convergence property is important, the performance of the order estimation algorithms for noisy observations is critical. Especially when the channel filter has some leading and trailing tails, estimation of the effective channel order rather than the true channel order becomes important. Deterministic channel estimation methods require the knowledge of the channel order. In the case of the small tail coefficients, channel identification is more problematic due to the reason that tail coefficients have the effect of colored noise on the channel. In [47] the condition measure is defined for the calculation of the effective channel order for a given channel. It also measures the degree of the ill conditioning of the channel. Estimation of effective channel order is important to obtain the best performance in blind channel identification. Proposed methods are tested with channels having small leading and trailing tails. In the simulations, randomly generated channels as well as measured microwave channels are used for complete evaluation. COE and CMR shows significantly better performance in estimating the effective channel order in a wide SNR range. When COE and

CMR are compared, CMR is slightly better than COE for order estimation. However this difference leads to considerable BER difference, since LSS method is not robust to over or under estimation of the channel order. Proposed algorithms are robust to different parameters such as the number of channels, channel order and the number of input samples. Hence, they are applicable for the applications having different receiver settings and channel conditions.

Proposed methods COE and CMR use the LSS algorithm which is integrated to the channel order estimation. Therefore these two algorithms estimate the channel coefficients in a joint manner. Since LSS is one of the best channel estimator, the combination performs well in blind identification.

The research on blind channel order estimation and system identification can be grouped in two main categories. In the first group, there is a significant amount of work devoted to the estimation of true and/or effective channel order. In the second group, there is an alternative approach where it is sufficient to have a rough idea about the channel in order to have an overestimated channel order. The leading technique in this group is the blind channel equalization and estimation by linear prediction. While these two approaches are alternatives of each other, none of the previous works compare these two approaches in order to have a good idea about the best possible technique for blind channel order estimation. It is believed that this thesis sheds a light on these alternative techniques by comparing them for a variety of cases.

LP based methods are claimed to be robust to overestimation of channel order. This property makes them popular in the solution of channel equalization problem when the channel order is not exactly known. LP method is first proposed by Slock [14],and it is based on the fact that moving average (MA) SIMO channel output can also be represented as an AR process, whose innovation is the SIMO channel input. LP is originally developed by assuming that the first channel coefficient is different from zero [14]. It is known that if this condition is not satisfied and $h(0)$ is close to zero, prediction error increases. In [17], this problem is solved to a certain extend. LP algorithm uses statistical characteristic of the inputs and it is based on the second order statistics. It assumes that, the channel input signal is Gaussian distributed white signal which is not perfectly satisfied in practical applications. Therefore it is not a deterministic algorithm as opposed to SS, CR and LSS algorithms. Being a non-deterministic method is the main disadvantage of the LP algorithm, since it is not possible to obtain channel input or channel coefficients without error even in noise free case. LP channel/input estimation error increases as the overestimation increases. While this increase is relatively small compared to LSS, SS and CR, LP has the best performance at true or effective channel order. LP can be considered as a robust algorithm for overestimation, but its equalization performance is poor. If Wiener equalizer is used instead of its known equalizer, performance can be improved. Even so it is still much worse than the CMR and COE techniques. As a result, using AIC or MDL for channel order estimation and LP for channel estimation and equalization does perform poorly compared to the proposed methods COE and CMR. This shows the value of accurate channel order estimation. In this respect, the best performance in blind channel and input estimation can be obtained by the proposed techniques in case of unknown channel order.

SIMO systems can be obtained in different ways. In one case, SIMO system is obtained by employing single transmit, multiple receive antennas. In the second case, the SISO system output is oversampled and polyphase structure is used to obtain the SIMO system. Many communication systems are in SISO structure. They use either symbol rate equalizers or fractionally spaced equalizers. It is known that if there is unknown channel filter, best performance is obtained by employing a fractionally spaced equalizer [44]. In this case, it is possible to obtain an equivalent SIMO system. In this thesis, blind decision feedback equalizer for fractionally spaced equalizer is shown to perform worse than the SIMO system equalization by employing the proposed techniques even when the comparison is done after DFE is converged. Obviously the above result is achieved when there is no common zeros for the equivalent SIMO system. However, it should also be pointed that proposed technique is relatively robust to closely spaced channel zeros compared to the alternative techniques.

Training sequences are widely used in traditional communication systems for the purpose of channel equalization and synchronization. With the help of training sequence, channel can be identified more accurately without any restriction on the channel. However their bandwidth efficiency is less than the blind methods and requires synchronization for channel identification. The number of training samples required for the channel identification depends on the channel length, therefore at least an upper bound is required for the estimation of the channel. The best performance is obtained when the channel order is known. Synchronization is another problem to be solved for training based methods. The use of training sequence in channel identification requires knowledge of the location of training sequence in the received data stream. Therefore synchronization should be achieved before channel identification or synchronization and channel estimation problems should be solved in a simultaneous way as in [34]. To our knowledge, there is no previous work which consider the semi-blind channel order estimation, channel estimation and synchronization problems simultaneously in the literature. Channel order is mostly assumed to be known or an upper limit for the channel order is used. The upper limit for channel order is estimated by MDL or AIC methods which mostly overestimate the channel order. Recognizing that the best performance can be achieved with correct channel order estimation, there is a need of channel order estimation algorithm with high accuracy. For this purpose, blind methods COE and CMR can be used without the need for the synchronization. In this thesis, semi-blind methods are presented in order to obtain performance improvement over CMR and COE. In this respect, two new channel order estimation algorithms, CIES and CIEB, are proposed. CIES and CIEB are based on the cost function which is obtained by taking the difference of the estimated input training sequence and known training sequence. The cost function has global minimum at the true channel order. The difference between two semi-blind methods is due to how they estimate the channel input. In CIEB, blind channel estimation algorithm LSS is used. In CIES, semi-blind channel estimation method, which is the combination of LSS and training based least squares method (LST). Semi-blind channel estimation methods combine the advantages of blind and training only methods and have better performance than blind and training only methods. Therefore, it is not surprise that CIES perform better than CIEB. However, CIES has large computational complexity due to multidimensional search to achieve synchronization and channel order estimation jointly. Several simulations are done for the performance comparisons. CIES and CIEB have similar performance with COE and CMR in true and effective channel order estimation. No improvement is obtained in this respect. However, CIES has better BER performance than other methods, due to the reason that it uses semi-blind channel estimator, which estimates the channel more accurately. As a result, semi-blind method CIES can achieve better BER performance with a loss in bandwidth efficiency and increase in computational complexity.

REFERENCES

[1] L. Tong, G. Xu, and T. Kailath, *A new approach to blind identification and equalization of multipath channel*, Proc. of the 25^th Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, pp. 856 – 860, November 1991.

[2] Elisabeth de Carvalho, Dirk T. M. Slock, *Blind and semi-blind FIR Multichannel Estimation: (global) identifiability conditions*, IEEE Trans. on Signal Proc., Vol.52, No.4, pp. 1053 – 1064, April 2004.

[3] L. Tong and Q. Zhao, *Joint order detection and blind channel estimation by least squares smoothing*, IEEE Trans. on Signal Proc., Vol. 47, pp. 2345 – 2355, Sept. 1999.

[4] A. P. Liavas, P. A. Regalia, *On the behaviour of information theoretic criteria for model order selection*, IEEE Trans. on Signal Proc., Vol. 49, No. 8, pp. 1689 – 1695, Aug. 2001.

[5] A.P. Liavas, P.A. Regalia, J.-P. Delmas, *Blind channel approximation: effective channel order determination*, IEEE Trans on Signal Proc., Vol. 47, No.12, pp. 3336 – 3344, December 1999.

[6] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, *Subspace methods for blind identification of multichannel FIR filters,* IEEE Trans. on Signal Proc., SP-43(2):516-525, February 1995.

[7] L. Tong and Q. Zhao, "Blind channel estimation by least squares smoothing," in Proc. Int. Conf. Acoust. Speech Signal Process., Seattle, vol. IV, pp. 2121 – 2124, WA, 1998.

[8] G. Xu, H. Liu, L. Tong and T. Kailath, *A least-squares approach to blind channel identification*, IEEE Trans. on Signal Proc., Vol. 43, pp. 2982 – 2993, Dec 1995.

[9] A. Scaglione, G. B. Giannakis, S. Barbarossa, *Redundant filterbank precoders and equalizers Part I : Unification and optimum design*, IEEE Trans on Signal Processing, vol.47, pp. 1988-2006, Jul. 1999.

[10] A. Scaglione, G. B. Giannakis, S. Barbarossa, *Redundant filterbank precoders and equalizers Part II : Blind channel estimation, syncronization and direct equalization*, IEEE Trans on Signal Processing, vol.47, pp. 2007 – 2042, Jul.1999.

[11] S. Barbarossa, A. Scaglione, G. B. Giannakis, *Performance analysis of a deterministic channel estimator for block transmission systems with null guard intervals*, IEEE Trans on Signal Processing, vol.50, pp. 684 – 695, Mar. 2002.

[12] B. Muquet, Wang Zhengdao, G. B. Giannakis, M. de Courville, P: Duhamel, *Cyclic Prefixing or zero padding for wireless multicarrier transmission?*, IEEE Trans. Commun., vol. 50, pp. 2136 – 2148, Dec. 2002.

[13] Georgios B. Giannakis, Cihan Tepedenlioglu, *Direct blind equalizers of multiple FIR channels: A deterministic approach.*, IEEE Trans. on Signal Proc., Vol.47, No.1, pp. 62 – 74, Jan. 1999.

[14] D. Slock, *Blind fractionally-spaced equalization, perfect-reconstruction filter banks and multichannel linear prediction*, Proc. IEEE ICASSP Conf., Adalaide, Australia, pp. $IV/585 - IV/588$, Apr. 1994.

[15] K. Abed-Meraim, R. Moulines, and P. Loubaton, *Prediction error method for second order blind identification*, IEEE Trans. Signal Proc., Vol.45, pp. 694 – 705, Mar. 1997.

[16] Houcem Gazzah, Phillip A. Regalia, Jean-Pierre Delmas, and Karim Abed-Meraim,*A Blind Multichannel Identification Algorithm Robust to Order Overestimation*, IEEE Trans. on Signal Processing, vol.50, no. 6, pp. 1449 – 1458, June 2002.

[17] Houcem Gazzah, *Optimum blind multichannel equalization using the linear prediction algorithm*, IEEE Trans. on Signal Processing, vol.54, no. 8, pp. 3242 – 3247, June 2006.

[18] A. Gorokhov and P. Loubaton, *Blind identification of MIMO-FIR systems: A generalized prediction approach,* Signal Process., vol. 73, pp. 105 – 124, 1999.

[19] J. Rissanen, *Modelling by shortest data description* Automatica, Vol.14, pp. 465 – 471, 1978.

[20] H. Akaike, *A new look at the statistical model identification*, IEEE Trans. Automat. Contr., Vol. AC-19, pp. 716 – 723 Dec. 1974.

[21] Q.-T. Zhang, K. M.Wong, P. C. Yip, and J. P. Reilly, *Statistical analysis of the performance of information theoretic criteria in the detection of the number of signals in array processing,* IEEE Trans. Acoust. Speech S gnal Processing, vol. 37, pp. 1557 – 1567, Oct. 1989.

[22] A.P. Liavas, P. A. Regalia, and J. P. Delmas, *Robustness of least-squares and subspace methods for blind channel identification/equalization with respect to effective channel undermodelling/overmodelling*, IEEE Trans. on Signal Proc., Vol. 47, No. 6, pp. 1636 – 1645, Jun. 1999.

[23] P. Stoica, Y. Selen, *Model order selection: A review of information criterion rules*, IEEE Signal Proc. Magazine, Jully 2004.

[24] Javier Via, Ignacio Santamaria, Jesus Perez, *Effective channel order estimation based on combined identification/equalization*, IEEE Trans. on Signal Proc., Vol. 54, No. 9, pp. 3518 – 3526, Sept. 2006.

[25] L. Tong , S. Perreau, *Multichannel Blind Identification: From Subspace to Maximum Likelihood Methods,* Proceedings of the IEEE, vol. 86, no. 10, pp. 1951 – 1968, October 1998

[26] S. Karakutuk, T. E. Tuncer, *Effects of Modulation Type on Blind SIMO Communications Systems*, Signal Processing and Communication Applications, 2007. SIU 2007, Vol.15, pp. 1 – 4, 11-13 June 2007.

[27] S. Karakutuk, T. E. Tuncer, *Channel output error (COE) for channel order estimation*, Signal Processing and Communication Applications, 2009. SIU 2009, pp. 57 – 60, April 2009.

[28] S. Karakutuk, T. E. Tuncer, *A new channel order estimation algorithm for FIR SIMO channels*, 17th European Signal Processing Conferece, EUSIPCO-2009, August 2009.

[29] R.E. Blahut, *Algebraic methods for signal processing and communication coding*, New York: Springer-verlag, 1992.

[30] D. Godard, *Selfrecovering equalization and carrier tracking in two dimnetional data communication systems*, IEEE Trans. on Communications, Value:28, Issue:11, pp. 1867 – 1875, Nov. 1980.

[31] T. Kailath, *Linear systems,* Prentice hall, Englewood Cliffs, New Jersey, 1980.

[32] Houcem Gazzah, Phillip A. Regalia, Jean-Pierre Delmas, Kerim Abed-Meraim, *A blind multichannel identification algorithm robust to order overestimation.*, IEEE Trans. on Signal Proc., Vol.50, No.6, pp. 1449 – 1458 June 2002.

[33] Z. Ding, *Matrix outer product decomposition method for blind multiple channel idetification*, IEEE Trans. Signal Proc., Vol. 45, pp. 3053 – 3061, Aug. 2000.

[34] Y. Wang, K. Shi, and E. Serpedin, *Continuous-mode frame synchronization for frequency-selective channels*, IEEE Trans. on Vehicular Technology, vol. 53, no. 3, pp. 865 – 874, May 2004.

[35] Adi. Ben-Isreal and Thomas N.E. Greville, *Generalized inverses. Theory and applications.*, 2nd ed. New York, NY:Springer, 2003.

[36] T. Kaya Yasar, T. Engin Tuncer, *Wideband DOA estimation for nonuniform linear arrays with Wiener array interpolation*, SAM-2008, Darmstadt, Germany, July 2008.

[37] T. Kailath, *Linear systems*, Prentice hall, Englewood Cliffs, New Jersey, 1980.

[38] Vincent Buchoux, Oliver Cappe, Eric Moulines, Alexei Gorokhov, *On the performance of semi-blind subspace-based channel estimation,* IEEE Trans. on Signal Proc., vol. 48, No. 6, pp. 1750 – 1759, June 2000.

[39] James L. Massey, *Optimum frame synchronization*, IEEE Trans. on Communications, Vol. Com-20, No. 2 , pp. 115 – 119, April 1972.

[40] Sergio Barbarossa, Anna Scaglione, Georgios B. Giannakis, *Performance analysis of a deterministic channel estimator for block transmission systems with null guard intervals*, IEEE trans on Signal Proc., Vol. 50, No.3, pp. 684 – 695, March 2002.

[41] John G. Proakis, *Digital Communications*, McGraw-Hill, 2000.

[42] Y. Sato, *A method of self-recovering equalization for multi-level amplitude modulation,* IEEE Trans- on Communications, Volume 23, Issue 6, pp. 679 – 682, Jun 1975.

[43] O. Shalvi, E. Weinstein, *New criteria for blind deconvolution of nonminimum phase systems(channels)*, IEEE Trans. on Information Theory, Volume 36, Issue 2, pp. 312 – 321, Mar 1990

[44] Simon Haykin, *Communication Systems*, John Wiley and Sons, 2001.

[45] Z. Ding and Y.Li, *Blind equalization and identification*, New York, Marcel Dekker, 2001.

[46] R.L. Kashyap, *Inconsistency of the AIC rule for estimating the order of AR models.,* IEEE Trans. Automat. Contr., Vol. 25, no. 5, pp. 996 – 998, Oct. 1980.

[47] G.B. Giannakis, Y. Hua, P. Stoica and L. Tong, *Signal Processing Advances in Wireless and Mobile Communications, Volume 1: Trends in Channel Estimation and Equalization*, Prentice Hall, 2001.

[48] Quing Zhao and Lang Tong, *Adaptive blind channel estimation by least squares smoothing,* IEEE Trans on Signal Proc., Vol. 47, No.11, pp. 3000 – 3012, November 2002.

APPENDIX A

PROOFS

The proofs in this section are presented for noise free case.

A.1 Reorganized Convolution Equation

The matrix formulation of a convolution equation is given as $\mathbf{Y} = \mathbf{H_M S}$. In this formulation, $\mathbf{S}$ has a special form and the last L rows of the $\mathbf{S}$ is included in the first L rows of $\mathbf{S}$. Convolution equation can be modified to eliminate this redundancy and properly generate the COE cost function. The manipulated convolution equation for channel order L is obtained by decomposing the output matrix $\mathbf{Y}$ into two components as $\mathbf{Y} = \mathbf{Y}_a + \mathbf{Y}_b$, where $\mathbf{Y}_a$ and $\mathbf{Y}_b$ are defined as follows.

$$\begin{aligned}
\mathbf{Y_a} &= \mathbf{H}_a \mathbf{S} \mathbf{P}_{sa} && \text{(A.1)}\\
&= \begin{bmatrix} \overbrace{\mathbf{h}_L(L) \;\cdots\; \mathbf{h}_L(0) \qquad\qquad}^{M} & \overbrace{\mathbf{0} \;\cdots\; \mathbf{0}}^{L} \\ \qquad \mathbf{h}_L(L) \;\cdots\; \mathbf{h}_L(0) & \vdots \qquad \vdots \\ \qquad\qquad \mathbf{h}_L(L) \;\; \vdots & \\ \qquad\qquad\qquad \mathbf{h}_L(L) & \mathbf{0} \;\cdots\; \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{s}^T_{M+L}(t) \\ \mathbf{s}^T_{M+L}(t+M) \\ \vdots \\ \mathbf{s}^T_{M+L}(t+(N-2)M) \end{bmatrix}^T && \text{(A.2)}\\
\mathbf{Y_b} &= \mathbf{H}_b \mathbf{S} \mathbf{P}_{sb} && \text{(A.3)}\\
&= \begin{bmatrix} \overbrace{\mathbf{0} \qquad \cdots \qquad}^{L} & \overbrace{\qquad\qquad \mathbf{0}}^{M} \\ \mathbf{h}_L(0) \quad \mathbf{0} \quad \cdots & \\ \vdots \qquad \ddots \quad \ddots & \\ \mathbf{h}_L(L-1) \;\cdots\; \mathbf{h}_L(0) & \mathbf{0} \;\cdots\; \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{s}^T_{M+L}(t+M) \\ \mathbf{s}^T_{M+L}(t+2M) \\ \vdots \\ \mathbf{s}^T_{M+L}(t+(N-1)M) \end{bmatrix}^T && \text{(A.4)}
\end{aligned}$$

$$\begin{aligned}
\mathbf{H_a} &= \mathbf{H}_M \mathbf{P}_{ha} && \text{(A.5)}\\
\mathbf{H_b} &= \mathbf{H}_M \mathbf{P}_{hb} && \text{(A.6)}
\end{aligned}$$

$$\mathbf{Y} = \mathbf{H}_M \mathbf{P}_{ha} \mathbf{S} \mathbf{P}_{sa} + \mathbf{H}_M \mathbf{P}_{hb} \mathbf{S} \mathbf{P}_{sb} \tag{A.7}$$

where,

$$\mathbf{P}_{ha} = \begin{bmatrix} \mathbf{I}_M & \mathbf{0}_{M\times L} \\ \mathbf{0}_{L\times M} & \mathbf{0}_{L\times L} \end{bmatrix}, \mathbf{P}_{hb} = \begin{bmatrix} \mathbf{0}_{M\times L} & \mathbf{0}_{M\times M} \\ \mathbf{I}_{L\times L} & \mathbf{0}_{L\times M} \end{bmatrix} \tag{A.8}$$

$$\mathbf{P}_{sa} = \begin{bmatrix} \mathbf{I}_{(N-1)} \\ \mathbf{0}_{1\times(N-1)} \end{bmatrix}, \mathbf{P}_{sb} = \begin{bmatrix} \mathbf{0}_{1\times(N-1)} \\ \mathbf{I}_{N-1} \end{bmatrix} \tag{A.9}$$

In equation (3.13), all of the samples of **S** are used. On the other hand, in equation (A.7) last L rows, which are included in the first L rows, are not used due to redundancy and multiplication by zero. In this way, data unstacking operation is represented by the matrix operations. We will use this formulation in the following parts.

A.2 Lemma 1

The proof of the lemma is based on Theorem-1 defined in [3]. According to this theorem smoothing error matrix, $\mathbf{E_l}$ is given by equation A.10 for different channel orders l, when the true channel order is L.

$$\mathbf{E}_l = \begin{cases} 0 & , l < L \\ H_l(\mathbf{h}) \begin{bmatrix} \tilde{\mathbf{s}}_{t+l-L|Z_l} \\ \\ \tilde{\mathbf{s}}_{t|Z_l} \end{bmatrix} & , L \leq l \end{cases} \tag{A.10}$$

where

$$H_l(\mathbf{h}) = \underbrace{\begin{bmatrix} \mathbf{h}_L(L) & & \\ \vdots & \ddots & \\ \mathbf{h}_L(0) & \ddots & \mathbf{h}_L(L) \\ & \ddots & \vdots \\ & & \mathbf{h}_L(0) \end{bmatrix}}_{l-L+1\ columns} \tag{A.11}$$

When the channel order is known, i.e., $l = L$, the dimension of column space of $\mathbf{E}_l$ is one and spanned by the channel vector $\mathbf{h_L}$. Therefore, the eigenvector corresponding the maximum eigenvalue of $\mathbf{E}_l$ can be taken as the solution for channel estimation.

In the case of overestimated channel order, i.e., $l = L + m$ and $m > 0$, the dimension of the columns space of $\mathbf{E}_l$ is $m + 1$ and spanned by the columns of $H_l(\mathbf{h})$.

$$C\{\mathbf{E}_l\} = C\{H_l(\mathbf{h})\} \tag{A.12}$$

Since the channel order is not known, the overestimated channel order is treated as the true channel order and the solution used in the case of known channel order is considered. In other words, the eigenvector corresponding the maximum eigenvalue of $\mathbf{E}_l$ is taken as the solution when $l > L$. Let us the channel estimate be $\hat{\mathbf{h}}_l$ as a result of this solution. $\hat{\mathbf{h}}_l$ is in the range space of $\mathbf{E}_l$, which is spanned by the columns of $H_l(\mathbf{h})$.

Therefore $\hat{\mathbf{h}}_l$ can be written as a linear combination of columns of $H_l(\mathbf{h})$. i.e.,

$$\hat{\mathbf{h}}_l = \alpha_0 \begin{bmatrix} \mathbf{h}_L(L) \\ \vdots \\ \mathbf{h}_L(0) \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \alpha_1 \begin{bmatrix} 0 \\ \mathbf{h}_L(L) \\ \vdots \\ \mathbf{h}_L(0) \\ 0 \\ 0 \end{bmatrix} + \cdots + \alpha_m \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \mathbf{h}_L(L) \\ \vdots \\ \mathbf{h}_L(0) \end{bmatrix} \tag{A.13}$$

Let $\mathbf{v} = \begin{bmatrix} a_0 & \cdots & a_N \end{bmatrix}^T$ be a vector, then its z-transform is given as $v(z) = \sum_{n=0}^{N} a_n z^{-k}$. Hence, z-transform of the estimated i^{th} channel, $\hat{h}_{l,i}(z)$, can be written as,

$$\begin{aligned} \hat{h}_{l,i}(z) &= \alpha_0 h_{L,i}(z) + \alpha_1 z^{-1} h_{L,i}(z) + \cdots + \alpha_m z^{-m} h_{L,i}(z) \\ &= \left(\alpha_0 + \alpha_1 z^{-1} + \cdots \alpha_m z^{-m}\right) h_{L,i}(z) \end{aligned} \tag{A.14}$$

where

$$h_{L,i}(z) = \sum_{k=0}^{L} h_{L,i}(L-k) z^{-k} \tag{A.15}$$

and i refers the i^{th} channel and $i = 1, \ldots, P$. As shown from (A.14), channels have a common product $\left(\alpha_0 + \alpha_1 z^{-1} + \cdots \alpha_m z^{-m}\right)$ which corresponds to m common zeros for the channels.

A.3 Lemma 2

Let the channel order be $\hat{L} = L + m$ and we have m common zeros. In that case, channel matrix can be written as a product of two matrices,

$$\mathbf{H}_M = \mathbf{H}'_M \mathbf{A} \tag{A.16}$$

and

$$\begin{aligned} &\underbrace{\begin{bmatrix} \mathbf{h}_{L+m}(L+m) & \cdots & \mathbf{h}_{L+m}(0) & & \\ & \ddots & \ddots & & \ddots \\ & & \mathbf{h}_{L+m}(L+m) & \cdots & \mathbf{h}_{L+m}(0) \end{bmatrix}}_{PM\times(M+L+m)} \\ &= \underbrace{\begin{bmatrix} \mathbf{h}'_L(L) & \cdots & \mathbf{h}'_L(0) & & \\ & \ddots & \ddots & \ddots & \\ & & \mathbf{h}'_L(L) & \cdots & \mathbf{h}'_L(0) \end{bmatrix}}_{PM\times(M+L)} \underbrace{\begin{bmatrix} \alpha_0 & \cdots & \alpha_m & & \\ & \ddots & \ddots & \ddots & \\ & & \alpha_0 & \cdots & \alpha_m \end{bmatrix}}_{(M+L)\times(M+L+m)} \end{aligned} \tag{A.17}$$

where, the roots of the polynomial, whose coefficients are given as α_k, are the common zeros. Using the property of the rank of the matrix product, following inequality can be written,

$$rank\,(\mathbf{H}_M\mathbf{A}) \le \min\{rank\,(\mathbf{H}_M)\,, rank\,(\mathbf{A})\} \tag{A.18}$$

Rank of the matrices are

$$\begin{aligned} rank\,(\mathbf{H}_M) &\le M + L && \text{(A.19)}\\ rank\,(\mathbf{A}) &\le M + L && \text{(A.20)} \end{aligned}$$

As a result of inequality (A.18), the rank of $\mathbf{H}_M$ is equal or less than $M + L$, which is smaller than the number of columns of $\mathbf{H}_M$. Therefore the channel matrix, $\mathbf{H}_M$, is not full column rank.

A.4 Lemma-3

In Lemma-1, it is shown that, $\hat{\mathbf{h}}_l$ can be written as a linear combination of the columns of $H_l(\mathbf{h})$. Consider that $l = L + 1$ and define $\mathbf{h}_1$ and $\mathbf{h}_2$ as the two columns of $H_l(\mathbf{h})$. Then,

$$\hat{\mathbf{h}}_l = a\mathbf{h}_1 + b\mathbf{h}_2 \tag{A.21}$$

where,

$$\mathbf{h}_1 = \begin{bmatrix} \mathbf{h}_L(L) \\ \vdots \\ \mathbf{h}_L(0) \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{h}_2 = \begin{bmatrix} \mathbf{0} \\ \mathbf{h}_L(L) \\ \vdots \\ \mathbf{h}_L(0) \end{bmatrix} \tag{A.22}$$

and $a + bz^{-1}$ is the transfer function of common zero. If $|a| = |b|$, then the common zero is located on the unit circle.

Let the eigenvectors spanning the columns of $H_l(\mathbf{h})$ or $\mathbf{E}_l$ be $\mathbf{e}_1$ and $\mathbf{e}_2$ with corresponding eigenvalues $\lambda_1 > \lambda_2$. Since $\mathbf{h}_1$ and $\mathbf{h}_2$ are also in the column space of $\mathbf{E}_l$, they can be written as a linear combination of $\mathbf{e}_1$ and $\mathbf{e}_2$ as follows.

$$\begin{aligned} \mathbf{h}_1 &= \alpha_1\mathbf{e}_1 + \beta_1\mathbf{e}_2 \\ \mathbf{h}_2 &= \alpha_2\mathbf{e}_1 + \beta_2\mathbf{e}_2 \end{aligned} \tag{A.23}$$

In LSS algorithm, channel estimate for the channel order, l is given as the eigenvector corresponding the maximum eigenvalue of $\mathbf{E}_l\mathbf{E}_l^H$. Therefore, $\mathbf{h}_{,l} = \mathbf{e}_1$ if the scale factor is assumed to be one. If the equation (A.21) is multiplied by $\mathbf{e}_1^H$ from left, it is obtained that $a = \alpha_1$. In the same manner, if the equation (A.21) is multiplied by $\mathbf{e}_2^H$ from left, it is obtained that $b = \alpha_1$. Hence if we show that $|\alpha_1| = |\alpha_2|$, the proof will be completed.

$H_l(\mathbf{h})H_l(\mathbf{h})^H$ can be written in terms of eigenvectors $\mathbf{e}_1$ and $\mathbf{e}_2$ as follows.

$$H_l(\mathbf{h})H_l(\mathbf{h})^H = \lambda_1\mathbf{e}_1\mathbf{e}_1^H + \lambda_2\mathbf{e}_2\mathbf{e}_2^H \tag{A.24}$$

Another equation can be obtained from the structure of $\mathbf{h}_1$ and $\mathbf{h}_2$.

$$\begin{aligned} \|\mathbf{h}_1\| &= \|\mathbf{h}_2\| && \text{(A.25)} \\ \mathbf{h}_1^H\mathbf{h}_1 &= \mathbf{h}_2^H\mathbf{h}_2 && \text{(A.26)} \end{aligned}$$

Replacing (A.23) in (A.26),

$$|\alpha_1|^2 + |\beta_1|^2 = |\alpha_2|^2 + |\beta_2|^2 \tag{A.27}$$

Multiplying (A.24) from left by $\mathbf{e}_1^H$ and from right by $\mathbf{e}_1$,

$$|\alpha_1|^2 + |\alpha_2|^2 = \lambda_1 \tag{A.28}$$

Multiplying (A.24) from left by $\mathbf{e}_2^H$ and from right by $\mathbf{e}_2$,

$$|\beta_1|^2 + |\beta_2|^2 = \lambda_2 \tag{A.29}$$

Multiplying (A.24) from left by $\mathbf{e}_1^H$ and from right by $\mathbf{e}_2$,

$$\frac{\alpha_1}{\alpha_2} = -\frac{\beta_1}{\beta_2} = c \tag{A.30}$$

As a summary, we have the following equations to be used for the proof of lemma. We want to show that $|\alpha_1| = |\alpha_2|$.

$$\begin{aligned} |\alpha_1|^2 + |\beta_1|^2 &= |\alpha_2|^2 + |\beta_2|^2 && \text{(A.31)} \\ |\alpha_1|^2 + |\alpha_2|^2 &= \lambda_1 && \text{(A.32)} \\ |\beta_1|^2 + |\beta_2|^2 &= \lambda_2 && \text{(A.33)} \\ \frac{\alpha_1}{\alpha_2} = -\frac{\beta_1}{\beta_2} &= c && \text{(A.34)} \end{aligned}$$

From (A.34), it is obtained that $\alpha_1 = c\alpha_2$. By replacing it in (A.32),following equality is obtained.

$$\left(1 + |c|^2\right)|\alpha_2|^2 = \lambda_1 \tag{A.35}$$

In the same manner, using (A.34) and (A.32) the following equality is obtained.

$$\left(1 + |c|^2\right)|\beta_2|^2 = \lambda_2 \tag{A.36}$$

Summing side by side of the equations (A.35) and (A.36),

$$\left(1 + |c|^2\right)\left(|\alpha_2|^2 + |\beta_2|^2\right) = \lambda_1 + \lambda_2 \tag{A.37}$$

$$\left(|\alpha_2|^2 + |\beta_2|^2\right) = \frac{\lambda_1 + \lambda_2}{1 + |c|^2} \tag{A.38}$$

Summing side by side of the equations (A.32) and (A.33),

$$\left(|\alpha_1|^2 + |\beta_1|^2\right) + \left(|\alpha_2|^2 + |\beta_2|^2\right) = \lambda_1 + \lambda_2 \tag{A.39}$$

By using (A.31) in (A.39), the following equality is obtained.

$$\left(|\alpha_2|^2 + |\beta_2|^2\right) = \frac{\lambda_1 + \lambda_2}{2} \tag{A.40}$$

Using the equality of (A.40) and (A.38), it is obtained that,

$$|c| = \frac{|\alpha_1|}{|\alpha_2|} = 1 \tag{A.41}$$

It means that, the common zero is on the unit circle with an arbitrary phase. Hence the proof is completed.

A.5 Theorem 1

The proof of this theorem is organized in three parts. True channel order, overestimation and underestimation cases are investigated separately.

A.5.1 Correct Channel Order Estimation

Consider that the channel order is correctly estimated. LSS algorithm gives the exact channel coefficients for noise free case [7, 3]. Therefore, $\hat{\mathbf{H}}_M = \mathbf{H}_M$. We want to show that the estimated output matrix, $\hat{\mathbf{Y}}$, and observed SIMO channel output, $\mathbf{Y}$, are equal to each other.

Using equation (A.7), $\hat{\mathbf{Y}}$, can be written as,

$$\begin{aligned}
\hat{\mathbf{Y}} &= \hat{\mathbf{Y}}_a + \hat{\mathbf{Y}}_b && \text{(A.42)}\\
&= \mathbf{H}_M\mathbf{P}_{ha}\hat{\mathbf{S}}\mathbf{P}_{sa} + \mathbf{H}_M\mathbf{P}_{hb}\hat{\mathbf{S}}\mathbf{P}_{sb} && \text{(A.43)}\\
&= \mathbf{H}_M\mathbf{P}_{ha}\mathbf{G}\mathbf{Y}\mathbf{P}_{sa} + \mathbf{H}_M\mathbf{P}_{hb}\mathbf{G}\mathbf{Y}\mathbf{P}_{sb} && \text{(A.44)}\\
&= \mathbf{H}_M\mathbf{P}_{ha}\left(\mathbf{H}_M^H\mathbf{H}_M\right)^{-1}\mathbf{H}_M^H\mathbf{H}_M\mathbf{S}\mathbf{P}_{sa} + \mathbf{H}_M\mathbf{P}_{hb}\left(\mathbf{H}_M^H\mathbf{H}_M\right)^{-1}\mathbf{H}_M^H\mathbf{H}_M\mathbf{S}\mathbf{P}_{sb} && \text{(A.45)}\\
&= \mathbf{H}_M\mathbf{P}_{ha}\mathbf{S}\mathbf{P}_{sa} + \mathbf{H}_M\mathbf{P}_{hb}\mathbf{S}\mathbf{P}_{sb} && \text{(A.46)}\\
&= \mathbf{Y} && \text{(A.47)}
\end{aligned}$$

In equation (A.46), the equality, $\left(\mathbf{H}_M^H\mathbf{H}_M\right)^{-1}\mathbf{H}_M^H\mathbf{H}_M = \mathbf{I}$, is used. The equality holds due to the fact that the channel has no common zeros. Hence,

$$COE\,(L) = \left\|\hat{\mathbf{Y}} - \mathbf{Y}\right\|_2 = 0 \qquad \text{(A.48)}$$

A.5.2 Overestimated Channel Order

Assume that the overestimated channel order is $\hat{L} = L + m$. LSS algorithm results m common zeros besides the true channel zeros by Lemma-1. Therefore estimated channel matrix can be written as follows,

$$\begin{aligned}
\hat{\mathbf{H}}_M &= a_0\begin{bmatrix} \mathbf{H}_M & \mathbf{0}_{PM\times m} \end{bmatrix} + a_1\begin{bmatrix} \mathbf{0}_{PM\times 1} & \mathbf{H}_M & \mathbf{0}_{PM\times(m-1)} \end{bmatrix} + \cdots \\
&+ a_m\begin{bmatrix} \mathbf{0}_{PM\times m} & \mathbf{H}_M \end{bmatrix} && \text{(A.49)}\\
&= a_0\mathbf{H}_M\mathbf{P}_0 + a_1\mathbf{H}_M\mathbf{P}_1 + \cdots + a_m\mathbf{H}_M\mathbf{P}_m && \text{(A.50)}\\
&= \mathbf{H}_M\sum_{k=0}^{m} a_k\mathbf{P}_k && \text{(A.51)}\\
&= \mathbf{H}_M\mathbf{P}_c && \text{(A.52)}
\end{aligned}$$

where, $\mathbf{P}_k = \begin{bmatrix} \mathbf{0}_{(M+L)\times k} & \mathbf{I}_{(M+L)} & \mathbf{0}_{(M+L)\times(m-k)} \end{bmatrix}$ and a_k are the coefficients of the transfer function whose zeros are the common channel zeros. In the following equations, the singular value decomposition of the channel matrix is required. The singular value decomposition of the channel matrix $\mathbf{H}_M$, is given as follows.

$$\mathbf{H}_M = \mathbf{U}\mathbf{\Sigma}_h\mathbf{V}^H \qquad \text{(A.53)}$$

Since the channel matrix is full column rank due to the assumption A1, $\mathbf{\Sigma}_h$ has the following form,

$$\mathbf{\Sigma}_h = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_{M+L} \\ \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{\Lambda_h} \\ \mathbf{0} \end{bmatrix} \tag{A.54}$$

Estimated output signal, $\hat{\mathbf{Y}}$, is composed of $\hat{\mathbf{Y}}_a$ and $\hat{\mathbf{Y}}_b$, which are defined in equations (A.1) and (A.3) respectively. It will be easier to determine $\hat{\mathbf{Y}}_a$ and $\hat{\mathbf{Y}}_b$ separately and then compute $\hat{\mathbf{Y}} = \hat{\mathbf{Y}}_\mathbf{a} + \hat{\mathbf{Y}}_\mathbf{b}$.

$$\begin{aligned}
\hat{\mathbf{Y}}_a &= \hat{\mathbf{H}}_M\mathbf{P}_{ha}\hat{\mathbf{S}}\mathbf{P}_{sa} && \text{(A.55)}\\
&= \hat{\mathbf{H}}_M\mathbf{P}_{ha}\mathbf{G}\mathbf{Y}\mathbf{P}_{sa} && \text{(A.56)}\\
&= \hat{\mathbf{H}}_M\mathbf{P}_{ha}\left(\hat{\mathbf{H}}_M^H\hat{\mathbf{H}}_M\right)^{\dagger}\hat{\mathbf{H}}_M^H\mathbf{H}_M\mathbf{S}\mathbf{P}_{sa} && \text{(A.57)}\\
&= \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{ha}\left(\mathbf{P}_c^H\mathbf{H}_M^H\mathbf{H}_M\mathbf{P}_c\right)^{\dagger}\mathbf{P}_c^H\mathbf{H}_M^H\mathbf{H}_M\mathbf{S}\mathbf{P}_{sa} && \text{(A.58)}\\
&= \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{ha}\left(\mathbf{P}_c^H\mathbf{V}\mathbf{\Sigma}_h^H\mathbf{U}^H\mathbf{U}\mathbf{\Sigma}_h\mathbf{V}^H\mathbf{P}_c\right)^{\dagger}\mathbf{P}_c^H\mathbf{V}\mathbf{\Sigma}_h^H\mathbf{U}^H\mathbf{U}\mathbf{\Sigma}_h\mathbf{V}^H\mathbf{S}\mathbf{P}_{sa} && \text{(A.59)}\\
&= \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{ha}\left(\mathbf{P}_c^H\mathbf{V}\mathbf{\Lambda}_h^2\mathbf{V}^H\mathbf{P}_c\right)^{\dagger}\mathbf{P}_c^H\mathbf{V}\mathbf{\Sigma}_h^H\mathbf{\Sigma}_h\mathbf{V}^H\mathbf{S}\mathbf{P}_{sa} && \text{(A.60)}\\
&= \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{ha}\left(\mathbf{P}_c\right)^{\dagger}\left(\mathbf{V}\mathbf{\Lambda}_h^2\mathbf{V}^H\right)^{-1}\left(\mathbf{P}_c^H\right)^{\dagger}\mathbf{P}_c^H\mathbf{V}\mathbf{\Lambda}_h^2\mathbf{V}^H\mathbf{S}\mathbf{P}_{sa} && \text{(A.61)}\\
&= \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{ha}\left(\mathbf{P}_c\right)^{\dagger}\mathbf{V}\mathbf{\Lambda}_h^{-2}\mathbf{V}^H\mathbf{V}\mathbf{\Lambda}_h^2\mathbf{V}^H\mathbf{S}\mathbf{P}_{sa} && \text{(A.62)}\\
&= \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{ha}\left(\mathbf{P}_c\right)^{\dagger}\mathbf{V}\mathbf{\Lambda}_h^{-2}\mathbf{\Lambda}_h^2\mathbf{V}^H\mathbf{S}\mathbf{P}_{sa} && \text{(A.63)}\\
&= \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{ha}\left(\mathbf{P}_c\right)^{\dagger}\mathbf{S}\mathbf{P}_{sa} && \text{(A.64)}
\end{aligned}$$

where $\mathbf{P}_c = \sum_{k=0}^{m} a_k\mathbf{P}_k$ and $\mathbf{G} = \left(\hat{\mathbf{H}}_\mathbf{M}^\mathbf{H}\hat{\mathbf{H}}_\mathbf{M}\right)^{\dagger}\hat{\mathbf{H}}_\mathbf{M}^\mathbf{H}$. $\mathbf{P}_{ha}$, $\mathbf{P}_{hb}$; and $\mathbf{P}_{sa}$ and $\mathbf{P}_{sb}$ are the matrices in equations (A.8) and (A.9) for the channel order, $L + m$, respectively. In equation (A.61), the following property of Moore-Penrose pseudoinverse is used. *If* $\mathbf{A}$ *is full column rank and* $\mathbf{B}$ *is full row rank then;* $(\mathbf{AB})^{\dagger} = \mathbf{B}^{\dagger}\mathbf{A}^{\dagger}$. Note that $\mathbf{P_c}$ is a full row rank and $\mathbf{V}\mathbf{\Lambda}_h^2\mathbf{V}^H$ is a full rank matrix.

In the same manner $\hat{\mathbf{Y}}_b$ can be found as,

$$\hat{\mathbf{Y}}_b = \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{hb}\left(\mathbf{P}_c\right)^{\dagger}\mathbf{S}\mathbf{P}_{sb} \tag{A.65}$$

Then the output $\hat{\mathbf{Y}}$ is written as,

$$\hat{\mathbf{Y}} = \hat{\mathbf{Y}}_\mathbf{a} + \hat{\mathbf{Y}}_\mathbf{b} = \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{ha}\left(\mathbf{P}_c\right)^{\dagger}\mathbf{S}\mathbf{P}_{sa} + \mathbf{H}_M\mathbf{P}_c\mathbf{P}_{hb}\left(\mathbf{P}_c\right)^{\dagger}\mathbf{S}\mathbf{P}_{sb} \tag{A.66}$$

In order to compare $\mathbf{Y}$ and $\hat{\mathbf{Y}}$, equation (A.66) should be modified. In this respect, $\mathbf{P}_c\mathbf{P}_{ha}(\mathbf{P}_c)^{\dagger}$ and $\mathbf{P}_c\mathbf{P}_{hb}(\mathbf{P}_c)^{\dagger}$ in equation (A.66) are written as follows,

$$\mathbf{P}_c\mathbf{P}_{ha}(\mathbf{P}_c)^{\dagger} = \begin{bmatrix} \mathbf{I}_{M\times M} & \mathbf{0}_{M\times(L+m)} \\ \mathbf{A}_{L\times M} & \mathbf{B}_{L\times(L+m)} \end{bmatrix} \tag{A.67}$$

$$\mathbf{P}_c\mathbf{P}_{hb}(\mathbf{P}_c)^{\dagger} = \begin{bmatrix} \mathbf{0}_{M\times(M+L+m)} \\ \mathbf{C}_{(L+m)\times(M+L+m)} \end{bmatrix} \tag{A.68}$$

where $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are non-zero and non-identity matrices and $m > 0$. Furthermore $\mathbf{Y}$, $\mathbf{H}_M$, and $\mathbf{S}$ matrices are written in the following form,

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_a \\ \mathbf{S}_b \end{bmatrix}, \quad \mathbf{Y}_a = \begin{bmatrix} \mathbf{Y}_{a1} \\ \mathbf{Y}_{a2} \end{bmatrix}, \quad \mathbf{H}_M = \begin{bmatrix} \mathbf{H}_x & \mathbf{0} \\ \mathbf{H}_y & \mathbf{H}_z \end{bmatrix} \tag{A.69}$$

where $\mathbf{S}_a$, $\mathbf{S}_b$, $\mathbf{Y}_{a1}$, $\mathbf{Y}_{a2}$, $\mathbf{H}_x$, $\mathbf{H}_y$ and $\mathbf{H}_z$ are $M\times N$, $(L+m)\times N$, $P(M-L-m)\times N$, $P(L+m)\times N$, $P(M-L-m)\times M$, $P(L+m)\times M$, $P(L+m)\times L$ matrices respectively. Output matrix, $\mathbf{Y}_a$, can be written with these matrix formations as follows,

$$\begin{aligned}
\mathbf{Y}_a &= \mathbf{H}_M\mathbf{P}_{ha}\mathbf{S}\mathbf{P}_{\mathbf{sa}} && \text{(A.70)}\\
&= \begin{bmatrix} \mathbf{H}_x & \mathbf{0} \\ \mathbf{H}_y & \mathbf{H}_z \end{bmatrix}\begin{bmatrix} \mathbf{I}_{M\times M} & \mathbf{0}_{M\times L} \\ \mathbf{0}_{L\times M} & \mathbf{0}_{L\times L} \end{bmatrix}\mathbf{S}\mathbf{P}_{sa} = \begin{bmatrix} \mathbf{H}_x & \mathbf{0} \\ \mathbf{H}_y & \mathbf{H}_z \end{bmatrix}\begin{bmatrix} \mathbf{S}_a \\ \mathbf{0} \end{bmatrix}\mathbf{P}_{sa} && \text{(A.71)}\\
&= \begin{bmatrix} \mathbf{H}_x\mathbf{S}_a\mathbf{P}_{sa} \\ \mathbf{H}_y\mathbf{S}_a\mathbf{P}_{sa} \end{bmatrix} && \text{(A.72)}
\end{aligned}$$

In the same manner, $\hat{\mathbf{Y}}_a$ can be written as,

$$\begin{aligned}
\hat{\mathbf{Y}}_a &= \mathbf{H}_M\left[\mathbf{P}_c\mathbf{P}_{ha}(\mathbf{P}_c)^{\dagger}\right]\mathbf{S}\mathbf{P}_{\mathbf{sa}} && \text{(A.73)}\\
&= \begin{bmatrix} \mathbf{H}_x & \mathbf{0} \\ \mathbf{H}_y & \mathbf{H}_z \end{bmatrix}\begin{bmatrix} \mathbf{I}_{M\times M} & \mathbf{0}_{M\times\hat{L}} \\ \mathbf{A}_{L\times M} & \mathbf{B}_{L\times L} \end{bmatrix}\mathbf{S}\mathbf{P}_{sa} = \begin{bmatrix} \mathbf{H}_x & \mathbf{0} \\ \mathbf{H}_y & \mathbf{H}_z \end{bmatrix}\begin{bmatrix} \mathbf{S}_a \\ \mathbf{A}\mathbf{S}_a + \mathbf{B}\mathbf{S}_b \end{bmatrix}\mathbf{P}_{sa} && \text{(A.74)}\\
&= \begin{bmatrix} \mathbf{H}_x\mathbf{S}_a\mathbf{P}_{sa} \\ \mathbf{H}_y\mathbf{S}_a\mathbf{P}_{sa} + \mathbf{H}_z(\mathbf{A}\mathbf{S}_a + \mathbf{B}\mathbf{S}_b)\mathbf{P}_{sa} \end{bmatrix} && \text{(A.75)}
\end{aligned}$$

From equations (A.72) and (A.75), it is seen that, $\hat{\mathbf{Y}}_{a1} = \mathbf{Y}_{a1}$ and $\hat{\mathbf{Y}}_{a2} \neq \mathbf{Y}_{a2}$.

The similar equations can be written for $\mathbf{Y}_b$ and $\hat{\mathbf{Y}}_b$. Now rearrange the matrices in the following way to write $\mathbf{Y}_{\mathbf{b}}$,

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_b \\ \mathbf{S}_c \end{bmatrix}, \quad \mathbf{Y}_b = \begin{bmatrix} \mathbf{Y}_{b1} \\ \mathbf{Y}_{b2} \end{bmatrix} \tag{A.76}$$

where $\mathbf{S}_b$, $\mathbf{S}_c$, $\mathbf{Y}_{b1}$, $\mathbf{Y}_{b2}$ are $(L+m)\times N$, $M\times N$, $P(M-L-m)\times N$, $P(L+m)\times N$ matrices respectively. Using these matrix formations, output matrix $\mathbf{Y}_b$ can be written as,

$$\mathbf{Y}_b = \mathbf{H}_M \mathbf{P}_{hb} \mathbf{S} \mathbf{P}_{sb} \tag{A.77}$$

$$= \begin{bmatrix} \mathbf{H}_x & \mathbf{0} \\ \mathbf{H}_y & \mathbf{H}_z \end{bmatrix} \begin{bmatrix} \mathbf{0}_{M\times L} & \mathbf{0}_{M\times M} \\ \mathbf{I}_{L\times L} & \mathbf{0}_{L\times M} \end{bmatrix} \mathbf{S}\mathbf{P}_{sb} = \begin{bmatrix} \mathbf{H}_x & \mathbf{0} \\ \mathbf{H}_y & \mathbf{H}_z \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{S}_b \end{bmatrix} \mathbf{P}_{sb} \tag{A.78}$$

$$= \begin{bmatrix} \mathbf{0}_{(M-L-m)\times N} \\ \mathbf{H}_z \mathbf{S}_b \mathbf{P}_{sb} \end{bmatrix} \tag{A.79}$$

In the same manner, $\hat{\mathbf{Y}}_b$ can be written as,

$$\hat{\mathbf{Y}}_b = \mathbf{H}_M \left[\mathbf{P}_c \mathbf{P}_{hb} (\mathbf{P}_c)^{\dagger} \right] \mathbf{S}\mathbf{P}_{sb} = \begin{bmatrix} \mathbf{H}_x & \mathbf{0} \\ \mathbf{H}_y & \mathbf{H}_z \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{C} \end{bmatrix} \mathbf{S}\mathbf{P}_{sb} = \begin{bmatrix} \mathbf{0}_{(M-L-m)\times N} \\ \mathbf{H}_z \mathbf{C} \mathbf{S}_b \mathbf{P}_{sb} \end{bmatrix} \tag{A.80}$$

From equations (A.79) and (A.80) it is seen that, $\hat{\mathbf{Y}}_{b1} = \mathbf{Y}_{b1}$ and $\hat{\mathbf{Y}}_{b2} \neq \mathbf{Y}_{b2}$.

The result is that, when $m > 0$, first $M - L - m$ rows of $\hat{\mathbf{Y}}$ are identical with the channel output $\mathbf{Y}$. When $M \geq L + m$, $\hat{\mathbf{Y}}$ samples will be erroneous and the cost function is greater than zero, i.e.,

$$COE(L+m) = \left\| \hat{\mathbf{Y}} - \mathbf{Y} \right\|_2 \tag{A.81}$$

$$= \| \mathbf{H}_z (\mathbf{A}\mathbf{S}_a + \mathbf{B}\mathbf{S}_b) \mathbf{P}_{sa} + \mathbf{H}_z (\mathbf{C} - \mathbf{I}) \mathbf{S}_b \mathbf{P}_{sb} \|_2 > 0 \tag{A.82}$$

A.5.3 Underestimated Channel Order

When the channel order is underestimated, channel coefficients can not be estimated because of the reason that the smoothing error matrix, $\mathbf{E_l} = \mathbf{0}$ [3, pg.234, eq.25]. Therefore channel coefficients are not correctly estimated and there is always a nonzero error. This leads to nonzero error in channel equalization and $\left\| \hat{\mathbf{Y}} - \mathbf{Y} \right\|_2 > 0$.

A.6 Theorem-2

For the proof of the theorem, correctly estimated channel order, overestimated channel order and under estimated channel order cases are studied separately to show that the cost function defined in (3.24) is zero only when the channel order is correctly estimated in noise free case.

A.6.1 Correct Channel Order Estimation

Consider that the channel order is correctly estimated, i.e. $\hat{L} = L$. LSS algorithm finds the true channel coefficients in the noise free case, when the true channel order is given [7, 3]. Therefore, $\hat{\mathbf{H}}_M^{(L)} = \mathbf{H}_M^{(L)}$. We want to show that the cost function defined through equation (3.24) is equal to zero when $\hat{L} = L$, i.e. $E_{CMR}(L) = 0$.

According to Lemma-1, when the channel order is overestimated by one, one common zero is added to true channel transfer function. In this case, channel matrix $\mathbf{H}_M^{(L+1)}$ can be written as the product of the true channel matrix, $\mathbf{H}_M^{(L)}$, and the matrix, $\mathbf{H}_c^{(1)}$.

$$\mathbf{H}_M^{(L+1)} = \mathbf{H}_M^{(L)}\mathbf{H}_c^{(1)} \tag{A.83}$$

Since the channel coefficients are estimated exactly for the true channel order, channel matrix, $\mathbf{H}_M^{L}$, is full column rank under assumption that there is no common zeros between SIMO channels. Therefore,

$$\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L)} = \mathbf{I} \tag{A.84}$$

The matrix, $\hat{\mathbf{A}}$, can be written as follow.

$$\begin{aligned}
\hat{\mathbf{A}} &= \mathbf{F} \odot \left(\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L+1)}\right) && \text{(A.85)}\\
&= \mathbf{F} \odot \left(\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L)}\mathbf{H}_c^{(1)}\right) && \text{(A.86)}\\
&= \mathbf{F} \odot \left(\mathbf{H}_c^{(1)}\right) && \text{(A.87)}\\
&= \mathbf{H}_c^{(1)} && \text{(A.88)}
\end{aligned}$$

Replacing, $\hat{\mathbf{A}}$, in equation (3.24), the cost function is obtained as follows,

$$\begin{aligned}
E_{CMR}(L) &= \left\|\mathbf{H}_M^{(L+1)} - \mathbf{H}_M^{(L)}\mathbf{H}_c^{(1)}\right\|_2 / \left\|\mathbf{H}_M^{(L+1)}\right\|_2 && \text{(A.89)}\\
&= \left\|\mathbf{H}_M^{(L+1)} - \mathbf{H}_M^{(L+1)}\right\|_2 / \left\|\mathbf{H}_M^{(L+1)}\right\|_2 && \text{(A.90)}\\
&= 0
\end{aligned}$$

A.6.2 Overestimated Channel Order

Consider that the channel order is overestimated, i.e., $\hat{L} = L + m$ and $m > 0$. As noted before, the function of $\mathbf{F}$ is to extract the Toeplitz form from the matrix $\mathbf{B}_m$. If $\mathbf{B}_m$ has Toeplitz matrix with first row equal to $[b_m(0)\ \ b_m(1)\ \ 0\cdots 0]$ and first column equal to $[b_m(0)\ \ 0\cdots 0]^T$, then $\hat{\mathbf{A}}_m = \mathbf{B}_m$. If this is the case,

$$
\begin{aligned}
\mathbf{H}_M^{(L+m)}\hat{\mathbf{A}}_m &= \mathbf{H}_M^{(L+m)}\mathbf{B}_m && \text{(A.91)}\\
&= \mathbf{H}_M^{(L+m)}\left(\mathbf{H}_M^{(L+m)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} && \text{(A.92)}\\
&= \mathbf{H}_M^{(L)}\mathbf{H}_c^{(m)}\left(\mathbf{H}_M^{(L)}\mathbf{H}_c^{(m)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} && \text{(A.93)}\\
&= \mathbf{H}_M^{(L)}\mathbf{H}_c^{(m)}\left(\mathbf{H}_c^{(m)}\right)^{\dagger}\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} && \text{(A.94)}\\
&= \mathbf{H}_M^{(L)}\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} && \text{(A.95)}\\
&= \mathbf{H}_M^{(L)}\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L)}\mathbf{H}_c^{(m+1)} && \text{(A.96)}\\
&= \mathbf{H}_M^{(L)}\mathbf{H}_c^{(m+1)} && \text{(A.97)}\\
&= \mathbf{H}_M^{(L+m+1)} && \text{(A.98)}
\end{aligned}
$$

and $E_{CMR}(\hat{L})$ in (3.24) becomes zero. $\mathbf{H}_c^{(m)}$ and $\mathbf{H}_M^{(L)}$ are full row rank and full column rank matrices respectively. Therefore, $\left(\mathbf{H}_M^{(L)}\mathbf{H}_c^{(m)}\right)^{\dagger} = \left(\mathbf{H}_c^{(m)}\right)^{\dagger}\left(\mathbf{H}_M^{(L)}\right)^{\dagger}$ in (A.93) as result of properties of Moore-Penrose pseoudeinverse. In the same manner, $\mathbf{H}_c^{(m)}\left(\mathbf{H}_c^{(m)}\right)^{\dagger} = \mathbf{I}$ in equation (A.94) and $\left(\mathbf{H}_M^{(L)}\right)^{\dagger}\mathbf{H}_M^{(L)} = \mathbf{I}$ in equation (A.96). $E_{CMR}(\hat{L})$ is different than zero as long as $\hat{\mathbf{A}}_m$ is not equal to $\mathbf{B}_m$ or $\mathbf{B}_m$ is not a Toeplitz matrix with two coefficients. In this proof, it is shown that $\mathbf{B}$ can not become a Toeplitz matrix for $m > 0$.

The proof will be by contradiction. So let us first assume that $\mathbf{B}$ is Toeplitz matrix with the formation described in the previous paragraph. In (A.98) it is also shown that $\mathbf{H}_M^{(L+m+1)} = \mathbf{H}_M^{(L+m)}\mathbf{B}_m$. Replacing $\mathbf{H}_M^{(L+m+1)}$ in (3.26),

$$
\begin{aligned}
\mathbf{B}_m &= \left(\mathbf{H}_M^{(L+m)}\right)^{\dagger}\mathbf{H}_M^{(L+m+1)} && \text{(A.99)}\\
&= \left(\mathbf{H}_M^{(L+m)}\right)^{\dagger}\mathbf{H}_M^{(L+m)}\mathbf{B}_m && \text{(A.100)}\\
&= \mathbf{P}\mathbf{B}_m && \text{(A.101)}
\end{aligned}
$$

where $\mathbf{P} = \left(\mathbf{H}_M^{(L+m)}\right)^{\dagger}\mathbf{H}_M^{(L+m)} \neq \mathbf{I}$, because $\mathbf{H}_M^{(L+m)}$ is not full column rank as a result of Lemma-1 and Lemma-2. When $\mathbf{B}_m$ is a Toeplitz matrix, $\mathbf{P}\mathbf{B}_m$ can not be a Toeplitz matrix with same formation, i.e. $\mathbf{P}\mathbf{B}_m \neq \mathbf{B}$. Equality only holds when $\mathbf{P} = \mathbf{I}$, which is not the case when $m > 0$, because channel matrix is not full column rank. Hence it contradicts with the assumption about $\mathbf{B}_m$, which states that $\mathbf{B}_m$ can not be a Toeplitz matrix with two coefficients. Therefore $E_{CMR}(L + m) > 0$ for $m > 0$.

A.6.3 Underestimated Channel Order

When the channel order is underestimated, the channel coefficients are not correctly estimated [3] and this leads to nonzero cost function for underestimated channel orders.

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Karakütük, Serkan
Nationality: Turkish (TC)
Date and Place of Birth: 1978 , Ankara
Phone: +90 542 560 71 61
email: serkankarakutuk@gmail.com

EDUCATION

Degree	Institution	Year of Graduation
MS	METU, Electrical and Electronics Engineering, Ankara	2003
BS	Gazi University, Electrical and Electronics Engineering, Ankara	2000
High School	Arı Science High School, Ankara	1995

WORK EXPERIENCE

Year	Place	Enrollment
2005-	MIKES	System Engineer
2003-2005	Education and Research Foundation of METU Prof. Dr. Mustafa N. PARLAR	RD Engineer
2000-2003	Başkent University	Research Assistant

FOREIGN LANGUAGES

Advanced English

PUBLICATIONS

1. S. Karakutuk, T. E. Tuncer, *Effects of Modulation Type on Blind SIMO Communications Systems*, Signal Processing and Communication Applications, 2007. SIU 2007, Vol.15, pp. 1 – 4, 11-13 June 2007.
2. S. Karakutuk, T. E. Tuncer, *Channel output error (COE) for channel order estimation*, Signal Processing and Communication Applications, 2009. SIU 2009, pp. 57 – 60, April 2009.
3. S. Karakutuk, T. E. Tuncer, *A new channel order estimation algorithm for FIR SIMO channels*, 17th European Signal Processing Conferece, EUSIPCO-2009, August 2009.