INVESTIGATION OF SUPERDIRECTIVE ANTENNA ARRAYS

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YASEMİN BAKTIR

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submitted by **YASEMİN BAKTIR** in partial fulfillment of the requirements for the degree of **Master of Science in Electrical and Electronics Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen
Dean, Gradute School of **Natural and Applied Sciences** _______________________

Prof. Dr. İsmet Erkmen
Head of Department, **Electrical and Electronics Engineering Dept.** _______________________

Assoc. Prof. Dr. Seyit Sencer Koç
Supervisor, **Electrical and Electronics Engineering Dept., METU** _______________________

Examining Committee Members

Prof. Altunkan HIZAL
Electrical and Electronics Engineering Dept., METU _______________________

Assoc. Prof. Seyit Sencer KOÇ
Electrical and Electronics Engineering Dept., METU _______________________

Prof. Gülbin DURAL
Electrical and Electronics Engineering Dept., METU _______________________

Assist. Prof. Lale ALATAN
Electrical and Electronics Engineering Dept., METU _______________________

MSc. EE. Erdinç ERÇİL
REHİS Division, ASELSAN _______________________

Date: 09 September 2009

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name : YASEMİN BAKTIR

Signature :

ABSTRACT

INVESTIGATION OF SUPERDIRECTIVE ANTENNA ARRAYS

Baktır, Yasemin

M.S., Department of Electrical and Electronics Engineering

Supervisor : Assoc. Prof. Dr. Seyit Sencer Koç

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In some antenna applications, having high directivity while keeping the antenna dimensions small is desired, which can be obtained by use of superdirective arrays. Superdirective arrays have been popular in academic world since a superdirective array provides higher directivity than the uniformly excited antenna array of same length.

In this thesis, superdirective arrays are investigated by making high precision numerical computations. Superdirective array element excitations, array factors and directivities are inspected for different number of elements. Superdirective array pattern and directivity features are compared to uniformly excited array pattern and directivities. Superdirective array tolerance is investigated by examination of array element excitation sensitivities. Bandwidth of superdirective arrays is also inspected. Multiple Precision Toolbox is used during numerical computations in Matlab.

Key Words: Superdirecitivity, Uniform array, Directivity, Array Factor, Multiple Precision Toolbox

ÖZ

SÜPER YÖNLENDİRİCİ ANTEN DİZİLERİNİN İNCELENMESİ

Baktır, Yasemin

Y. Lisans, Elektrik and Elektronik Mühendisliği Bölümü

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Bazı anten uygulamalarında küçük boyutlu ve yüksek yönlendiriciliği olan antenler kullanmak ihtiyacı olmaktadır. Süper yönlendirici diziler bu amaçla kullanılabilirler. Süper yönlendirici dizilerin akademik dünyada popüler olmasının sebebi, süper yönlendirici bir dizinin aynı boyutlardaki düzgün anten dizisinden daha yönlü olmasıdır.

Bu tez kapsamında, süper yönlendirici diziler yüksek doğruluklu sayısal hesaplamalar yapılarak incelenmiştir. Süper yönlendirici dizi eleman katsayıları, dizi faktörleri ve yönlülüğü farklı sayıda elemana sahip diziler için araştırılmıştır. Süper yönlendirici dizi anten örüntüsü ve yönlülüğü düzgün dizi anten örüntüsü ve yönlülüğü ile karşılaştırılmıştır. Süper yönlendirici dizilerin toleransı, eleman katsayılarının hassaslığı incelenerek ortaya konmuştur. Çalışma kapsamında, süper yönlendirici

dizilerin bant genişliği de incelenmiştir. Matlab ortamında yapılan sayısal hesaplamalarda yüksek doğruluk sağlayan Multiple Precision Toolbox kullanılmıştır.

Anahtar Kelimeler: Süper Yönlülük, Düzgün Anten Dizisi, Yönlülük, Dizi Faktörü, Multiple Precision Toolbox

To My Family,

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CHAPTER 1

INTRODUCTION

1.1. Thesis Scope

With the developing antenna and microwave technology, array antennas have been popular in antenna applications. Especially radars using electronic scanning arrays are common in recent years. Antennas with high directivity are usually required in these applications.

As a common guiding principle, higher directivities are obtained by enlarging the antenna aperture. On the other hand, dimension of the system is critical if the total size, weight and allocation of the system are considered. Superdirective antennas come to the stage exactly at that point since superdirective antennas offer higher directivity than uniformly excited arrays with the same array length although uniformly excited arrays are generally considered to give the optimum directivity.

The superdirectivity phenomenon has attracted great attention among the academic society since the first studies in the first half of 20th century. The reason of the academic curiosity is that superdirectivity contradicts with the common experience which is an increase in directivity is provided by an increase in the aperture size, [1].

A useful operational definition of antenna array superdirectivity is directivity higher than that obtained with the same array length and elements uniformly excited (constant amplitude and linear phase), [2].

In the scope of the thesis, while computing superdirective array coefficients, it is observed that high accuracy is required in order to obtain correct results. So Multiple Precision Toolbox that provide computation with high number of digits in Matlab is used. Thus, the array coefficients are found accurately. Then patterns for superdirective arrays are obtained. Uniform and superdirective arrays are compared in array factor and directivity considerations. Then tolerance of the superdirective array pattern against the perturbations in array coefficients is also observed. Lastly, bandwidth of superdirective arrays is investigated.

1.2. Motivation and Objective of the Thesis

Motivation of this thesis is to determine if very narrow antenna beams can be realized by superdirective arrays, since it will bring advantages in antenna array applications because of relatively small lengths.

On the other hand, in the literature it is stated that superdirective arrays are very sensitive and high precision is required through the numerical computations. Our question is to investigate if superdirective arrays can be realized in microwave frequencies with the help of developing technology.

In order to answer the questions of “how precise?” and “is it practical?”, the order of precision required in numerical computations is inspected and practical aspects like tolerance and bandwidth are also examined.

Through the thesis, different aspects of superdirective arrays are examined and they are given below:

- Superdirective array coefficients are computed
- Directivity versus distance between the elements is observed
- Superdirective array antenna patterns are obtained
- Uniform arrays and superdirective arrays are compared
- The tolerance of superdirective array coefficients is examined

- The bandwidth of superdirective arrays is investigated

1.3. Thesis Organization

First chapter of the thesis is the introduction part in which the scope, objective and organization of the thesis are given. In the second chapter, background is given by some definitions in the antenna field and basic concepts related to matrices which are used throughout the thesis. Short history of superdirectivity including a literature survey is also included in this chapter. 3rd chapter consists of theoretical optimization concept, the problem encountered while doing the calculations and the solution ways of the problem. Chapter 4 includes the simulations done by using Multiple Precision Toolbox. In this chapter computations using the toolbox, directivity and array antenna pattern issues are discussed. In Chapter 6, superdirective and uniform arrays are compared. In Chapter 7, tolerance of the superdirective array is investigated. Chapter 8 is the conclusion part of the thesis.

CHAPTER 2

BACKGROUND

2.1. Basic Antenna Definitions

2.1.1. Antenna

An antenna is a structure which is used to radiate and receive radio frequencies. Antennas are used in several applications from radars to wireless communication. Antennas have various types including wire, aperture, microstrip, reflector, lens and array antennas.

2.1.2. Antenna pattern

Antenna pattern is the characteristics of the antenna which shows the radiation property of the antenna with respect to the coordinates.

The antenna which has equal radiation in all directions hypothetically is called an isotropic antenna. A directional antenna is the antenna which transmits or receives more effectively in a specific direction. An omnidirectional antenna is a type of directional antenna which is nondirectional in one plane, but directional in the orthogonal plane.

2.1.3. Directivity

Directivity of an antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

$$D(\theta,\varphi) = \frac{power\ radiated\ per\ unit\ solid\ angle\ in\ (\theta,\varphi) direction}{average\ power\ radiated\ per\ unit\ solid\ angle} \qquad [2.\ 1]$$

$$= \frac{dP_r/d\Omega}{P_r/4\pi} \qquad [2.\ 2]$$

$$= 4\pi \frac{dP_r/d\Omega}{P_r} \qquad [2.\ 3]$$

where P_r is the total radiated power and Ω is the solid angle.

Directivity of an antenna is a measure of how effectively the antenna is radiating in a given direction with respect to an isotropic antenna.

2.1.4. Gain

Gain of an antenna is related to antenna directivity. The total input power to the antenna rather than the total radiated power is used as a reference.

$$G(\theta,\varphi) = 4\pi \frac{power\ radiated\ per\ unit\ solid\ angle\ in\ (\theta,\varphi) direction}{input\ power} \qquad [2.\ 4]$$

$$= 4\pi \frac{dP_r/d\Omega}{P_{in}} \qquad [2.\ 5]$$

$$= \eta D(\theta,\varphi) \qquad [2.\ 6]$$

where

$$\eta = \frac{P_r}{P_{in}} \qquad [2.\ 7]$$

is the efficiency.

2.1.5. Array antenna

Radiation pattern of an electrically small single element is usually wide and so directivity of the single element is relatively low. Since directive antennas are required in many applications, this can be provided by increasing the electrical size of the individual element. Another way of increasing the directivity is to form an assembly of radiating elements instead of increasing the dimensions of the individual element. This new antenna of multielements is referred to as an array, [3].

Arrangement of the array elements can be such that radiation from array elements add up in particular desired direction.

Below are the parameters that can shape the array antenna pattern:

- the geometrical configuration of the overall array,
- the relative displacement of the elements,
- the excitation amplitude of the individual elements,
- the excitation phase of the individual elements and
- the relative pattern of the individual elements

Different array antenna configurations are given in Figure – 2.1.

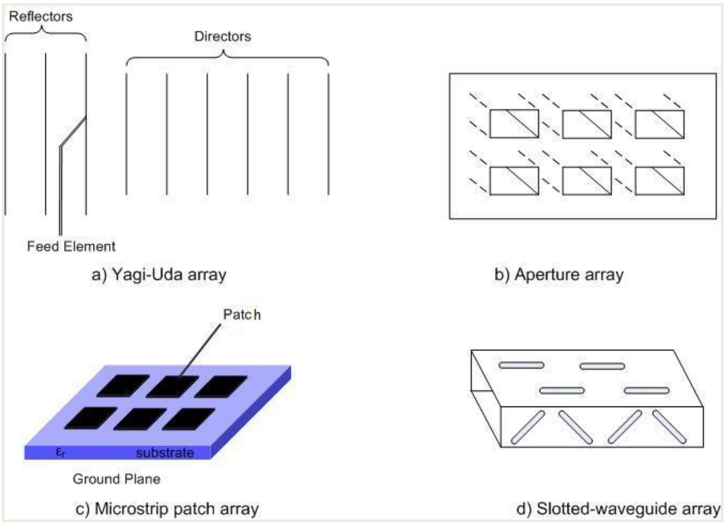

Figure - 2. 1: Typical wire, aperture and microstrip array configurations

2.1.6. Array Factor

Array factor is a function of number of array elements, the geometrical configuration of the array, amplitude of the elements, relative phase of the elements and the spacing, [4].

Since array factor does not depend on the directional characteristics of the individual elements, by replacing the elements with isotropic point sources the array factor can be derived. When the array factor is obtained, the total field of the actual array is found by multiplication of radiation field of the reference antenna and the array factor.

$$\mathrm{E(total)} = [\mathrm{E(single\ element\ at\ reference\ point]x[array\ factor]} \qquad [2.\ 8]$$

In Figure - 2. 2, an antenna array of 2 elements is given. In the far field, the rays from the antenna elements become parallel.

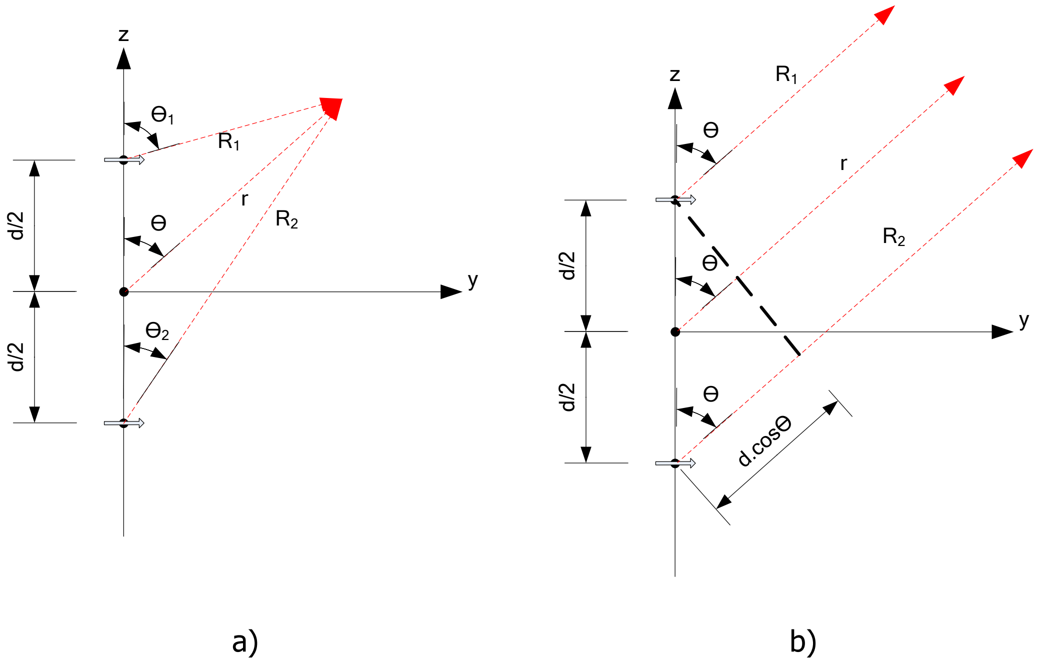

Figure - 2. 2: a) Two infinitesimal dipoles, b) Far-field observations

The distance form the ith antenna to far-field point is $\mathrm{R_i} = \mathrm{r} - \boldsymbol{a_r} \cdot \boldsymbol{r_i}$.

So the phase delay of the i^{th} antenna will be $k_0 \boldsymbol{a_r} \cdot \boldsymbol{r_i}$ smaller than that of the reference antenna at the origin. There does not need to be an antenna at the origin necessarily, but it constitutes a reference.

$\boldsymbol{r_i}$ vector represents the position of the i^{th} antenna.

$$\begin{aligned} E(r) &= \sum_{i=1}^{N} \left(c_i \, e^{j\alpha_i} \, f(\theta,\varphi) \frac{e^{-jk_0(r-\boldsymbol{a_r}\cdot \boldsymbol{r_i})}}{4\pi r} \right) \\ &= \sum_{i=1}^{N} f(\theta,\varphi) \, \frac{e^{-jk_0 r}}{4\pi r} \left(c_i \, \frac{e^{j\alpha_i + jk_0 \boldsymbol{a_r}\cdot \boldsymbol{r_i}}}{4\pi r} \right) \end{aligned} \qquad [2.\ 9]$$

$f(\theta,\varphi)$ is the electric field radiation of the single array element.

This derivation uses the assumption that all antennas in the array have the same radiation field. This is not correct because of the mutual coupling between the antenna elements. But the effect in radiation pattern of the antennas is often small enough to be neglected. Therefore, pattern multiplication principle is assumed to have good accuracy [4].

While studying on arrays, usually attention is paid on the array factor since individual antenna elements have a broader pattern in a high directivity array and so the array factor dominates.

2.1.7. Broadside array

In some applications, it can be desired to have the maximum radiation directed normal to the axis of the array, $\theta = 90°$. If the array factor and also the maxima of single element are directed toward $\theta = 90°$, the design will be optimized. By arranging the seperation and excitation of individual radiators, array factor maxima in the desired direction can be accomplished.

2.1.8. End-fire array

In some applications, it may be desired to direct the maximum radiation along the axis of the array ($\theta = 0°$ or $\theta = 180°$), instead of normal to the array axis.

This type of arrays are called end-fire arrays.

2.1.9. Inverse of a Matrix

The inverse of a square matrix $\mathbf{A}$ is matrix $\mathbf{A^{-1}}$ such that

$$\mathbf{AA^{-1}} = \mathbf{I} \qquad [2.\ 10]$$

where $\mathbf{I}$ is the identity matrix.

If a matrix has an inverse, then the matrix is a square matrix. The existance of matrix inverse is determined by determinant of that matrix such that,

$$\mathbf{A^{-1}}\ exists\ \Leftrightarrow \det(\mathbf{A}) \neq 0 \qquad [2.\ 11]$$

Assume that $\mathbf{A}$ is a $\mathrm{n} \times \mathrm{n}$ square matrix and $\mathbf{x}$, $\mathbf{b}$ are $\mathrm{n} \times 1$ vectors. For known $\mathbf{A}$ and $\mathbf{b}$, let us have

$$\mathbf{Ax} = \mathbf{b} \qquad [2.\ 12]$$

$\mathbf{x}$ can be found and it is unique, if matrix $\mathbf{A}$ has an inverse:

$$\mathbf{x} = \mathbf{A^{-1}}\,\mathbf{b} \qquad [2.\ 13]$$

Otherwise $\mathbf{x}$ will either not exist or will not be unique.

As one of the elements in the matrix varies, the determinant of the matrix also varies. Most of the time, the determinant will be nonzero and inverse of the matrix exists. Sometimes, the determinant will be very close to zero and inverse of the matrix exists but it will be so sensitive to the original matrix that it will be hard to compute. In this case, the matrix is called an ill-conditioned matrix. Rarely, the determinant is exactly zero and the inverse just does not exist. In this case, matrix is a singular matrix. [5]

In Matlab, $\det(\mathbf{A})$ command will compute the determinant where $\mathrm{inv}(\mathbf{A})$ command returns the inverse. However, for ill-conditioned matrices, $\mathrm{inv}()$ will not give the correct inverse.

If $\mathbf{A}$ in equation [2.12] is nonsingular, then $\mathbf{Ax} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$.

If $\mathbf{A}$ is singular (determinant of $\mathbf{A}$ is zero), then $\mathbf{Ax} = \mathbf{b}$ has either zero solution or infinitely many solutions [6]:

- $\mathbf{Ax} = \mathbf{b}$ has no solution for some nonzero $\mathbf{b}$ vector in the nullspace of $\mathbf{A}^\mathbf{T}$ such that $\mathbf{Ax} = \mathbf{b}$ is unsolvable.

$$\mathbf{Ax} = \mathbf{b} \quad \Rightarrow \quad 0 = \mathbf{b}^\mathbf{T}\mathbf{Ax} = \mathbf{b}^\mathbf{T}\mathbf{b} > 0 \qquad [2.\ 14]$$

- If $\mathbf{Ax} = \mathbf{b}$ is solvable, then it has infinitely many solutions:

$$\mathbf{A}(\mathbf{x} + \mathrm{t}\mathbf{v}) = \mathbf{b} \qquad [2.\ 15]$$

For all scalar t and all nonzero vectors $\mathbf{v}$ in the nullspace of $\mathbf{A}$.

2.1.9.1. Condition Number

Assume $\mathbf{A}$ is nonsingular (i.e. invertible) and $\mathbf{Ax} = \mathbf{b}$.

If $\mathbf{b}$ is changed to $\mathbf{b} + \Delta\mathbf{b}$, the solution becomes $\mathbf{x} + \Delta\mathbf{x}$ such that

$$\mathbf{A}(\mathbf{x} + \Delta\mathbf{x}) = \mathbf{b} + \Delta\mathbf{b} \qquad [2.\ 16]$$

Where the change in $\mathbf{x}$ is

$$\Delta\mathbf{x} = \mathbf{A}^{-1}\Delta\mathbf{b} \qquad [2.\ 17]$$

The condition number is the maximum ratio of the relative error in $\mathbf{x}$ divided by the relative error in $\mathbf{b}$.

If the relative error in $\mathbf{x}$ is not much greater than the relative error in $\mathbf{b}$, matrix $\mathbf{A}$ is a well-conditioned matrix and condition number of the matrix is close to 1.

If the relative error in $\mathbf{x}$ is much greater than the relative error in $\mathbf{b}$, matrix $\mathbf{A}$ is badly conditioned or ill-conditioned matrix and condition number of the matrix is large.

In numerical analysis, the condition number associated with a problem is a measure of that problem's amenability to digital computation, that is, how numerically well-conditioned the problem is.

For example, the condition number associated with the linear equation $\mathbf{Ax} = \mathbf{b}$ gives a bound on how inaccurate the solution $\mathbf{x}$ will be after approximate solution. Note that this is before the effects of round-off errors are taken into account. Conditioning is a property of the matrix, not the algorithm or floating point accuracy of the computer used to solve the corresponding system. In particular, it can be thought that the condition number is very roughly the rate at which the solution, $\mathbf{x}$, will change with respect to a change in $\mathbf{b}$. Thus, if the condition number is large, even a small error in $\mathbf{b}$ may cause a large error in $\mathbf{x}$.

2.1.10. Eigenvalue Problem

Eigenvalue problem is the problem where it is aimed to find the pairs $(\lambda, \mathbf{x})$ that satisfy the linear equation:

$$\mathbf{Ax} = \lambda\mathbf{x} \qquad [2.\ 18]$$

The set of λ for which corresponding $\mathbf{x}$ exist are the eigenvalues of the matrix.

The $\mathbf{x}$ that correspond to each eigenvalue form the corresponding eigenvector space. The members of the space are called eigenvectors. The problem is to find the eigenvalues and then using the eigenvalues to find the eigenvectors.

The eigenvalue equation can be expressed as

$$\mathbf{Ax} - \lambda\mathbf{Ix} = 0 \qquad [2.\ 19]$$

This can be written as

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{x} = 0 \qquad [2.\ 20]$$

If there exists the inverse $(\mathbf{A} - \lambda\mathbf{I})^{-1}$ then both sides will be zero and the trivial solution $\mathrm{x} = 0$ will be obtained.

In order not to have the trivial solution, the determinant should be equal to zero:

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \qquad [2.\ 21]$$

If the determinant is expanded, a polynomial is obtained:

$$\mathrm{p}(\lambda) = \det(\lambda\mathbf{I} - \mathbf{A}) = \sum_{k=1}^{n} a_k \lambda^k \qquad [2.\ 22]$$

$\mathrm{p}(\lambda)$ is called the characteristic polynomial. The eigenvalues can be obtained by finding the roots of the characteristic polynomial.

In Matlab, eigenvalue problems are usually solved by using the function $\mathrm{eig}()$.

$$[\mathbf{V}, \mathbf{D}] = \mathrm{eig}(\mathbf{A}) \qquad [2.\ 23]$$

produces a diagonal matrix $\mathbf{D}$ of eigenvalues and a full matrix $\mathbf{V}$ whose columns are the corresponding eigenvectors.

2.2. A Short History of Superdirectivity

First study on superdirectivity is probably done by Oseen in 1922 regarding the early references. Franz, K. was also one of the early contributers, [7]. In 1943 Schelkunoff, S.A. put forward a mathematical theory for linear arrays, [8]. In the same year, La Paz and Miller asserted that a given aperture would allow a maximum directivity, [9]. In 1946, Bouwkamp and De Brujin declared that they have corrected their error and there was no limit on theoretical directivity, [10]. It is not easy to get the original copies of these early studies. Information about these studies can be acquired by the later contributors in the literature.

Bouwkamp and De Brujin showed that the directivity of a linear current distribution of fixed length may be made arbitrarily large by the suitable choice of current distribution, [11].

As an example to one of the early famous contributions, approach of Bouwkamp and De Brujin can be explained shortly here. In the paper, it is shown that for any given continuous function $g(t)$on the closed interval $(-b \leq t \leq b)$, there exists a continuous function $f(x)$ such that

$$\left| g(t) - \int_{-a}^{a} f(x) e^{jxt} dx \right| < \epsilon \qquad [2.\ 24]$$

for all t in the interval and for any $\epsilon > 0$.

Bouwkamp and De Bruijn approximated $g(t)$ uniformly by a polynomial of sufficiently high degree which is always possible regarding the famous theorem of Weierstrass. Under different assumptions they show that the patterns may be approximated with any required accuracy in the form of integrals. So, they prove that the observed patterns may be made arbitrarily directive by a suitable choice of the current distribution.

After a while, Bloch, Medhurst and Pool studied on optimum directivity of linear antenna arrays by using the impedance relations between the elements. [12]

Tai C.T. investigated uniformly spaced broadside arrays of dipoles, [13]. He gives the formulation for directivity in superdirective limit case. In "Numerical Computation" part he states that "when n is greater than two, and D is less than π, the computation becomes rather difficult even with the aid of an IBM-7090 computer. Accurate results may be obtained only if a multiple precision program for evaluation determinant and its adjoint is used". In his paper, Tai included set of curves showing the optimum directivity for different types of broadside arrays. He also compared superdirective array directivity to the directivity of a uniformly excited array.

In his study, Tai mentions that for number of elements smaller than and equal to 6, $N \leq 6$, the optimum directivity G_N is,

$$G_N = \left(\frac{3}{2} \cdot \frac{5}{4} \cdot \frac{7}{6} \cdots \frac{N}{N-1}\right)^2, \qquad N = odd, \qquad D \to 0$$

$$G_N = \left(\frac{3}{2} \cdot \frac{5}{4} \cdot \frac{7}{6} \cdots \frac{N-1}{N-2}\right)^2, \qquad N = even, \qquad D \to 0 \qquad [2.\ 25]$$

Where N denotes the number of elements.

He states that from the Peirce-Foster table it can be found that

$$\frac{4n}{\pi} > \left(\frac{3}{2} \cdot \frac{5}{4} \cdots \frac{2n-1}{2n-2}\right)^2 > \frac{2(2n-1)}{\pi} \qquad [2.26]$$

where $N = 2n - 1 = \text{number of elements for } N \text{ odd},$

$N = 2n = \text{number of elements for } N \text{ even}.$

Then he states that the mean value of $4n/\pi$ and $2(2n-1)/\pi$ may be used as an approximate value of G_N, the optimum directivity, as the spacing between the elements goes to zero for large values of N.

$$G_N \doteq \frac{2(2n - \frac{1}{2})}{\pi} = \frac{2N+1}{\pi}, \qquad N = odd, \qquad d \to 0 \qquad [2.27]$$

where $D = kd$, k is the wave number and d is the spacing between the elements.

Tai describes the formula 2.27 as the asymptotic value of optimum directivity G_N for large values of N.

Stearns (1965) studied on "Computed Performance of Moderate Size, Super-gain, End-Fire Antenna Arrays". He encountered to the problem of ill-conditioned matrix for a decrease in spacing between the elements or an increase in number of elements, [14].

Dawoud and Hansen (1978-1986) used optimized polynomial technique to generate superdirective array functions. Legendre polynomials were used to study the effects of changing number of elements and array length on the different array parameters, [15].

Liu Yuan, Deng Weibo and Xu Rongqing examined the application of superdirectivity in HF band to see whether it is feasible or not. In their study, the three difficulties that superdirectivity suffer from (low efficiency, narrow bandwidth and tolerance) are discussed for HF band, [16].

While low efficiency is the key problem that makes superdirectivity unpractical, in HF band it is no longer a problem. It is explained below.

The system SNR can be written as:

$$SNR = \frac{\eta S_{ext} D(\theta_M, \phi_M)}{\eta N_{ext} + N_i} \quad [2.\ 28]$$

where η is the antenna efficiency, N_{ext} is the external noise level where N_i is the internal noise. S_{ext} is the external signal level received from the direction (θ_M, ϕ_M). $D(\theta_M, \phi_M)$ is the directive gain.

ηN_{ext} is the power at the receiver port due to external noise. If ηN_{ext} is much greater than the internal noise in the receiver port, then:

$$\eta N_{ext} \gg N_i \quad [2.\ 29]$$

And SNR can be reduced as:

$$\mathrm{SNR} = \frac{S_{ext} D(\theta_M, \phi_M)}{N_{ext}} \quad [2.\ 30]$$

As seen from the above equation, SNR is independent of efficiency η, but proportional to directive gain. Under proper restrictions on efficiency, superdirective antenna will reduce external noise on average by the same factor that it reduces the signals and the system SNR will be proportional to the directive gain.

On the other hand, it is stated that the problem of very narrow bandwidth still remained in HF band. Lastly, about the tolerance sensitivity, the requirement on precision of superdirective antenna excitation is mentioned as being critical also in HF band. However it is stated that the excitation requirements can be satisfied with the developments in Digital Beam Forming (DBF).

Don Barrick of CODAR Ocean Sensors Ltd, suggests concepts in order to make High Frequency Surface Wave Radar (HFSWR) efficient and inexpensive, [17]. The idea behind the invention of CODAR which reduces the size and cost is the use of superdirective arrays. According to Barrick, since the sum of the signals from the array elements is low when compared to single element signal, superdirectivity seems disadvantegous at first stage. It is stated that in microwave this may be true, but it is not correct for HF frequencies. Because external noise exceeds internal noise at HF. So, while signal from the target is decreasing external noise also decreases. The important point is defined as to stop before internal noise dominates on external noise.

CHAPTER 3

FINDING ARRAY COEFFICIENTS OF SUPERDIRECTIVE BROADSIDE UNIFORM LINEAR ARRAYS

3.1. Optimization of Directivity

Assume a uniform linear array of 2N+1 isotropic antenna array elements located on z axis as in Figure 3.1.

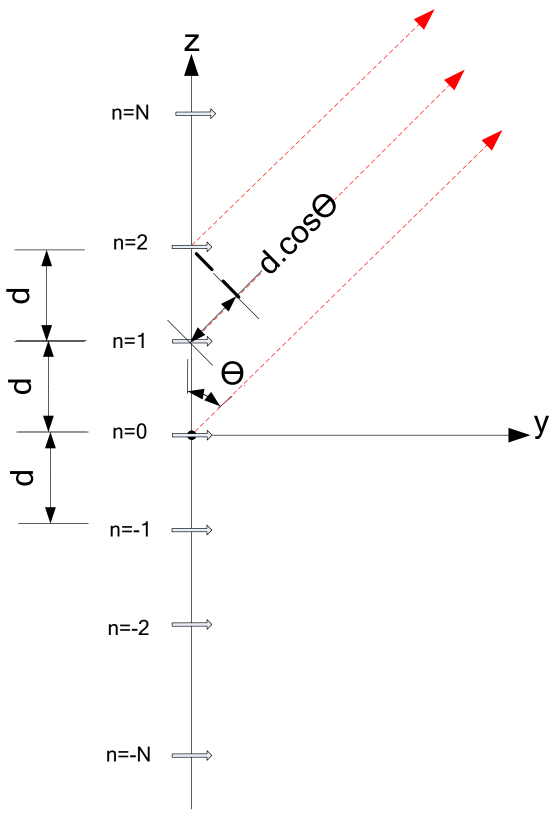

Figure - 3. 1: Array of 2N+1 elements lying on z axis

The antennas are labeled from $n = -\mathrm{N}$ to $n = +\mathrm{N}$ and they are located from $\mathrm{z} = -\mathrm{N}d$ to $\mathrm{z} = \mathrm{N}d$ since the distance d between the elements is fixed, [18].

The excitation coefficients of the elements are complex in general and can be written in the form of

$$c_n = a_n e^{j\alpha_n} \qquad [3.\ 1]$$

where a_n is the excitation amplitude and α_n is the excitation phase of the n^{th} element.

Then the array factor can be written as,

$$F(\theta) = \frac{e^{-jkr}}{4\pi r}\sum_{n=-N}^{N} c_n e^{j\boldsymbol{k}\cdot\boldsymbol{d_n}} = \frac{e^{-jkr}}{4\pi r}\sum_{n=-N}^{N} c_n e^{jnkd\cos\theta} \qquad [3.\ 2]$$

As seen in the formulation, the array factor is independent of φ, azimuth variation.

In Equation 2.5 the directivity is defined as:

$$D(\theta,\varphi) = 4\pi\frac{dP_r/d\Omega}{P_r}$$

Radiation intensity $dP_r/d\Omega$ is obtained by multiplying the Poynting vector flux density by r^2 where P_r is the total radiated power and Ω is the solid angle. Thus,

$$\frac{dP_r}{d\Omega} = \frac{1}{2}r^2\mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\}\cdot\mathbf{a_r} \qquad [3.\ 3]$$

So,

$$P_r = \int_0^{2\pi}\int_0^{\pi}\frac{1}{2}r^2\mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\}\cdot\mathbf{a_r}d\Omega \qquad [3.\ 4]$$

$d\Omega$ = element of solid angle = $\sin\theta\ d\theta\ d\emptyset$

Since there is no variation in φ direction, $\mathrm{d}\Omega = 2\pi\sin\theta\ d\theta$ and

$$P_r = \pi \int_0^{\pi} r^2 Re\{\mathbf{E} \times \mathbf{H}^*\} \cdot a_r \sin\theta \, d\theta \qquad [3.\ 5]$$

Then ,

$$D = \frac{4\pi max_\theta |F(\theta)|^2}{2\pi \int_{-\pi/2}^{\pi/2} |F(\theta)|^2 \sin\theta d\theta} \qquad [3.\ 6]$$

For broadside arrays for which the pattern has its maximum value at $\theta = \pi/2$, we get,

$$D = \frac{4\pi |F(\pi/2)|^2}{2\pi \int_0^{\pi} |F(\theta)|^2 \sin\theta d\theta} = \frac{4\pi \left|\sum_{n=-N}^{N} c_n\right|^2}{2\pi \int_0^{\pi} |\sum_{n=-N}^{N} c_n \, e^{jknd\cos\theta}|^2 \sin\theta d\theta} \qquad [3.\ 7]$$

$$= \frac{2\,(\sum_{n=-N}^{N} c_n)(\sum_{m=-N}^{N} c_m{}^*)}{\int_0^{\pi} (\sum_{n=-N}^{N} c_n e^{jknd\cos\theta})(\sum_{m=-N}^{N} c_m{}^* \, e^{-jkmd\cos\theta}) \sin\theta \, d\theta} \qquad [3.\ 8]$$

$$= \frac{2 \sum_{n=-N}^{N} \sum_{m=-N}^{N} c_n c_m{}^*}{\sum_{n=-N}^{N} \sum_{m=-N}^{N} c_n c_m{}^* \int_0^{\pi} e^{jk(n-m)d\cos\theta} \sin\theta d\theta} \qquad [3.\ 9]$$

$$= \frac{\sum_{n=-N}^{N} \sum_{m=-N}^{N} c_n c_m{}^*}{\sum_{n=-N}^{N} \sum_{m=-N}^{N} c_n c_m{}^* \dfrac{\sin(n-m)kd}{(n-m)kd}} \qquad [3.\ 10]$$

we define,

$$\mathbf{c} = [c_{-N} \quad c_{-N+1} \quad \cdots \quad c_N]^T \qquad [3.\ 11]$$

$$\mathbf{A} = [a_{nm}],\ a_{nm} = \frac{\sin(n-m)kd}{(n-m)kd} \qquad [3.\ 12]$$

$$\mathbf{O} = [1] \qquad [3.\ 13]$$

where $\mathbf{O}$ is a nxn matrix and all entries 1.

Then directivity can be written in matrix form as,

$$\mathrm{D} = \frac{\mathbf{c}^{\mathrm{H}} \mathbf{O} \mathbf{c}}{\mathbf{c}^{\mathrm{H}} \mathbf{A} \mathbf{c}} \qquad [3.\ 14]$$

$()^{\mathrm{H}}$ denotes the Hermitian that is to say complex conjugate transpose of a matrix.

If derivative is taken and equated to zero,

$$[(\mathbf{c}^{\mathrm{H}}\mathbf{Ac})\mathbf{O} - (\mathbf{c}^{\mathrm{H}}\mathbf{Oc})\mathbf{A}]\mathbf{c} = 0 \qquad [3.\ 15]$$

Equivalently

$$\left[\mathbf{O} - \frac{\mathbf{c}^{\mathrm{H}}\mathbf{Oc}}{\mathbf{c}^{\mathrm{H}}\mathbf{Ac}}\mathbf{A}\right]\mathbf{c} = 0$$
$$[\mathbf{O} - \mathbf{DA}]\mathbf{c} = 0 \qquad [3.\ 16]$$

From equation 3.14, it can be seen that if coefficients are scaled by a scalar directivity will not change. To resolve that, we can insert the constraint that the radiated power is unity such that

$$\mathbf{c}^{\mathrm{H}}\mathbf{Ac} = 1 \qquad [3.\ 17]$$

Matrix $\mathbf{A}$ is also a real symmetric Toeplitz matrix since

$$a_{n+1,m+1} = \frac{\sin(n+1-m-1)kd)}{(n+1-m-1)kd} = \frac{\sin((n-m)kd)}{(n-m)kd} = a_{nm} \qquad [3.\ 18]$$

and

$$a_{mn} = \frac{\sin((m-n)kd)}{(m-n)kd} = \frac{\sin((n-m)kd)}{(n-m)kd} = a_{nm} \qquad [3.\ 19]$$

which implies that the array coefficients for maximum directivity are also symmetric, i.e.,

$$c_{-n} = c_n \qquad [3.\ 20]$$

The problem of determining the stationary values of the equation 3.14 subject to the constraint $\mathbf{c}^{\mathrm{H}}\mathbf{Ac} = 1$ is equal to finding the characteristic values of the matric equation $[\mathbf{O} - \boldsymbol{\lambda}\mathbf{A}]\mathbf{c} = 0$. That is to finding the roots of $|\mathbf{O} - \boldsymbol{\lambda}\mathbf{A}| = 0$.

The solution is called a stationary solution since it is found by equating the first derivative to zero.

Nontrivial solutions of $[\mathbf{O} - \boldsymbol{\lambda}\mathbf{A}]\mathbf{c} = 0$ will give the excitation coefficients. The largest characteristic value gives the largest stationary value of the directivity.

Since the matrix is of rank 1, it must be noticed that only one of the characteristic values will be nonzero.

In order to convert the equation to an eigenvalue problem, multiply with identity of $(\mathbf{A}^{-1}\mathbf{A})$:

$$[\mathbf{O} - \boldsymbol{\lambda}\mathbf{A}]\mathbf{c} = 0 \qquad [3.\ 21]$$

$$[\mathbf{O} - \boldsymbol{\lambda}\mathbf{A}]\mathbf{A}^{-1}\mathbf{A}\mathbf{c} = 0 \qquad [3.\ 22]$$

define $\mathbf{v}$ such that $\mathbf{v} = \mathbf{A}\mathbf{c}$.

$$(\mathbf{O}\mathbf{A}^{-1} - \boldsymbol{\lambda}\mathbf{I})\mathbf{v} = 0 \qquad [3.\ 23]$$

since $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$.

Above equation is now an eigenvalue problem where $\boldsymbol{\lambda}$ represents eigenvalue of $\mathbf{O}\mathbf{A}^{-1}$, $\mathbf{v}$ the eigenvector and largest eigenvalue will give the stationary coefficients.

So, we choose the eigenvector that corresponds to the largest eigenvalue.

After finding eigenvector $\mathbf{v}$, we find the stationary coefficients $\mathbf{c}$ by the relation of

$$\mathbf{v} = \mathbf{A}\mathbf{c} \qquad [3.\ 24]$$

$$\mathbf{c} = \mathbf{A}^{-1}\mathbf{v} \qquad [3.\ 25]$$

Finding $\mathbf{c}$ gives the stationary coefficients for $2N + 1$ elements for a given kd.

Matlab command to find eigenvalues and eigenvectors in an eigenvalue problem was mentioned in equation 2.23. By using the similar command, expression in 3.26 will be used in this case,

$$[\mathbf{V}, \mathbf{D}] = \mathrm{eig}(\mathbf{O}^{*}\mathrm{inv}(\mathbf{A})) \qquad [3.\ 26]$$

When continued to examining the problem, a second approach for solving the problem is obtained as below explained.

The equation ($[(\mathbf{c}^{\mathrm{H}}\mathbf{A}\mathbf{c})\mathbf{O} - (\mathbf{c}^{\mathrm{H}}\mathbf{O}\mathbf{c})\mathbf{A}]\mathbf{c} = 0$) can be written in the form

$$d\sum_{m=-N}^{N} c_m - p\sum_{m=-N}^{N} a_{nm}c_m = 0, \qquad n = -N, \dots, N \qquad [3.\ 27]$$

where $d = \mathbf{c}^{\mathrm{H}}\mathbf{A}\mathbf{c}$ and $p = \mathbf{c}^{\mathrm{H}}\mathbf{O}\mathbf{c}$.

$n = 0$ equation can be subtracted from the others. Since the equations for $n' = -n$ will be identical to those for n, they are not used and then equation 3.28 is obtained,

$$-p\sum_{m=-N}^{N}(a_{nm} - a_{0m})c_m = 0, \qquad n = 1,2,\dots,N \qquad [3.\ 28]$$

The above equation can be rearranged by using the fact that $a_{0,-m} = a_{0m}$ and $c_{-m} = c_m$ as,

$$\left.\begin{array}{l} \sum_{m=-N}^{-1}(a_{nm} - a_{0m})c_m + (a_{n0} - a_{00})c_0 + \sum_{m=1}^{N}(a_{nm} - a_{0m})c_m = 0 \\ \sum_{m=1}^{N}(a_{n,-m} - a_{0,-m})c_{-m} + (a_{n0} - a_{00})c_0 + \sum_{m=1}^{N}(a_{nm} - a_{0m})c_m = 0 \\ (a_{n0} - a_{00})c_0 + \sum_{m=1}^{N}(a_{n,-m} - 2a_{0m} + a_{nm})c_m = 0 \end{array}\right\} \quad n = 1,2,\dots,N \qquad [3.\ 29]$$

By rearranging equation 3.28, last row in equation 3.29 is obtained. When those N equations are used with the constraint $\mathbf{c}^{\mathbf{H}}\mathbf{A}\mathbf{c} = 1$, there will be $N + 1$ equations in the computation. Therefore, $N + 1$ coefficients c_m will be determined for $\mathrm{m} = 0,1,\dots,\mathrm{N}$.

c_m for negative values of m are found by using the symmetry.

3.2. Difficulty of the Problem

Using the defined approach in the previous part, optimum directivity and corresponding array element coefficients can be obtained using Matlab.

Directivity versus number of elements while the total length of the array fixed, is considered first. As the number of elements increases, the distance between the elements reduces.

The variation is given in Figure-3.2 as below.

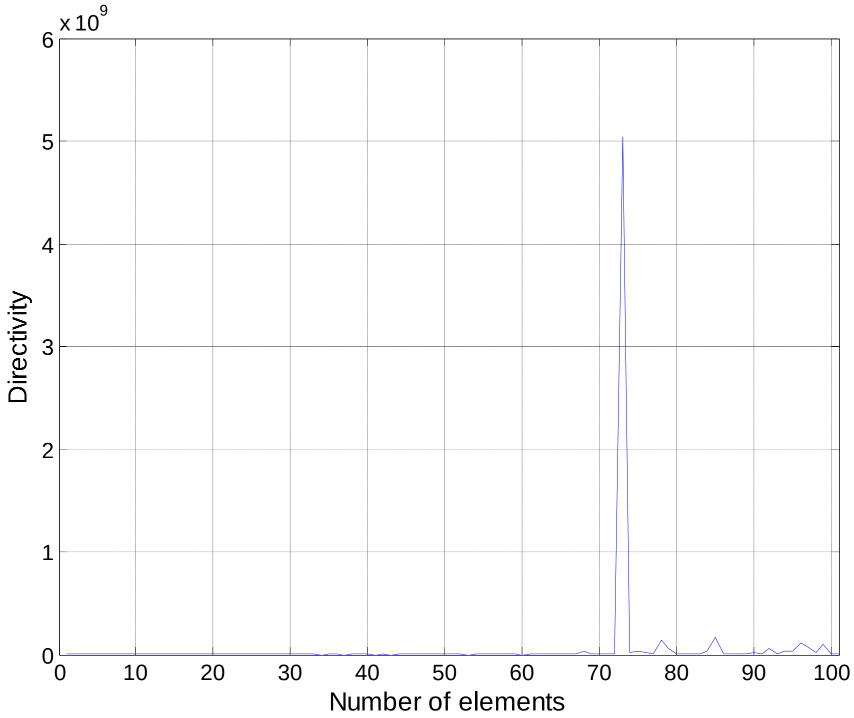

Figure - 3.2: Directivity vs Number of elements for fixed array length

Directivity versus number of elements characteristic does not seem realistic since the directivity gets so high values as the spacing gets smaller.

Let us examine the array factor for 73-element array where the directivity seems to be maximum and determine if the array factor supports directivity data. In Figure - 3.3, array factor for 73 elements is given.

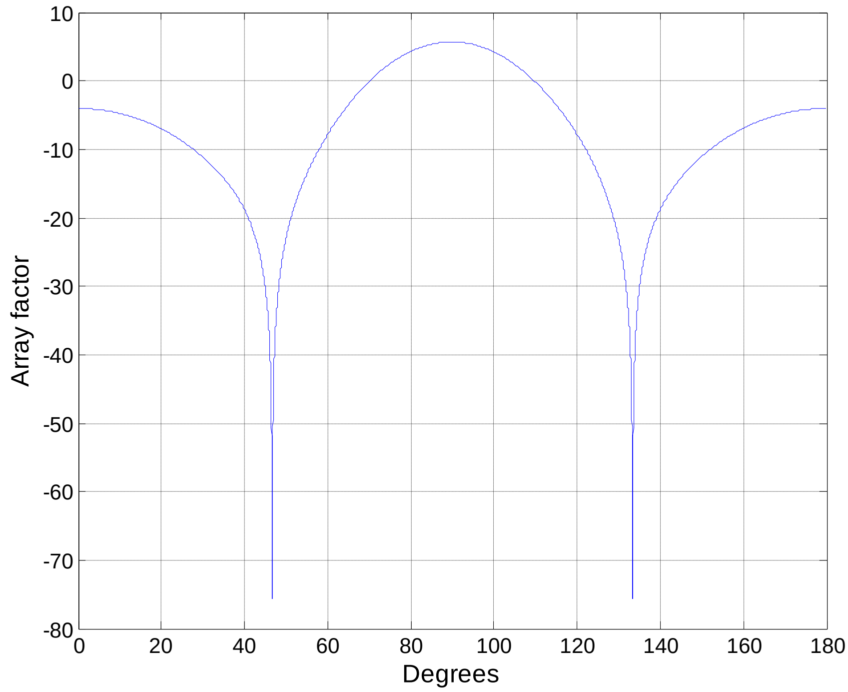

Figure - 3.3: Array factor (dB) for 73 elements

As seen in the above figure, beam is not so directive and it contradicts previously found directivity versus number of elements graph.

Regarding the two characteristics, directivity versus number of elements is neither as expected nor realistic. Moreover directivity and array factor graphs contradict with each other. When the reason of that unexpected result is inspected, it is observed that inverse of matrix $\mathbf{A}$ is used through the calculations.

Now let us look through matrix $\mathbf{A}$ if it is the source of the problem encountered.

Matrix $\mathbf{A}$ is composed of the elements $a_{nm} = \mathrm{sinc}(\frac{(n-m)kd}{n})$ as defined in Equation 3.19.

When the behavior of matrix $\mathbf{A}$ is examined, it is realized that the condition number of matrix $\mathbf{A}$ becomes very high as the spacing between the elements gets smaller. Condition number of matrix $\mathbf{A}$ versus kd is given in Figure – 3.4, where d is the distance between the elements and k is the wave number.

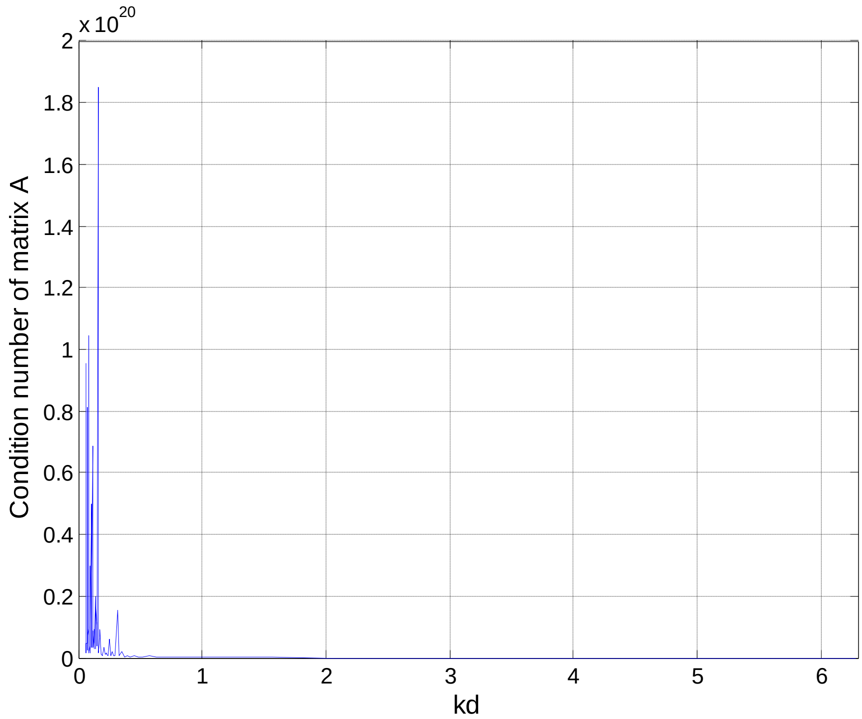

Figure - 3.4: Condition number of matrix A vs kd

As explained in Chapter 2 Section 2.2.1.1, condition number is a measure of how precisely the numerical computation is done and how accurate the results are. Since the condition number of matrix $\mathbf{A}$ is very high and it is an ill-conditioned matrix, it can be concluded that the results are inaccurate.

Matrix $\mathbf{A}$ is an ill conditioned matrix and it means that computing inverse of matrix $\mathbf{A}$ is very susceptible to floating point errors. Since the results show that accuracy of the numeric computation is insufficient, solution of the problem is to increase the floating point accuracy.

Hence, it is observed that $\mathrm{inv}()$ function used during Matlab computations is poor in order to find inverse of matrix $\mathbf{A}$. Therefore, methods which compute superdirective array coefficients and array factor more precisely are required in order to do the calculations accurately.

3.3. How to Resolve the Problem

Ways tried in order to overcome the problem are explained in following sections.

3.3.1. pinv() Function

The computation is based on singular value decomposition method. Any singular value less than a tolerance are treated as zero and so replaced by zero. The matrix obtained is called the $\mathrm{pseudoinverse\ matrix}$.

Matlab command to compute the pseudoinverse using SVD method is $\mathrm{pinv}()$.

The default tolerance is

$$tol = \max\big(size(\mathbf{A})\big) * \mathrm{norm}(\mathbf{A}) * eps \qquad [3.\ 30]$$

This tolerance can be overridden by using

$$X = \mathrm{pinv}(\mathbf{A}, tol) \qquad [3.\ 31]$$

Let us see how directivity versus number of elements change when $\mathrm{pinv}()$ command is used. Array length is 2λ and fixed. Number of elements are increased up to 100. The characteristic is given in Figure 3.5.

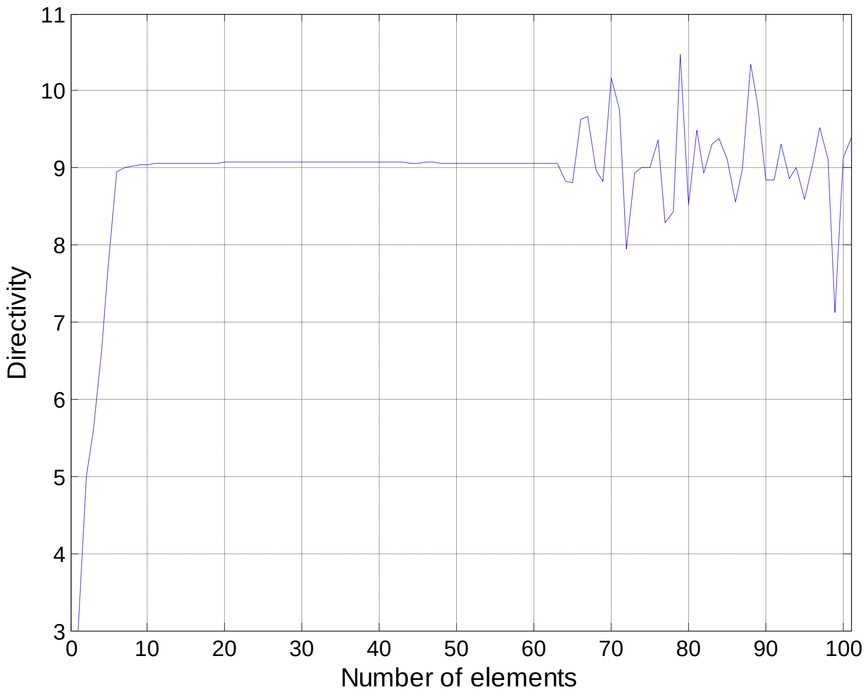

Figure - 3.5: Directivity vs Number of elements obtained by using pinv()

Directivity versus distance between the elements is given in below Figure 3.6.

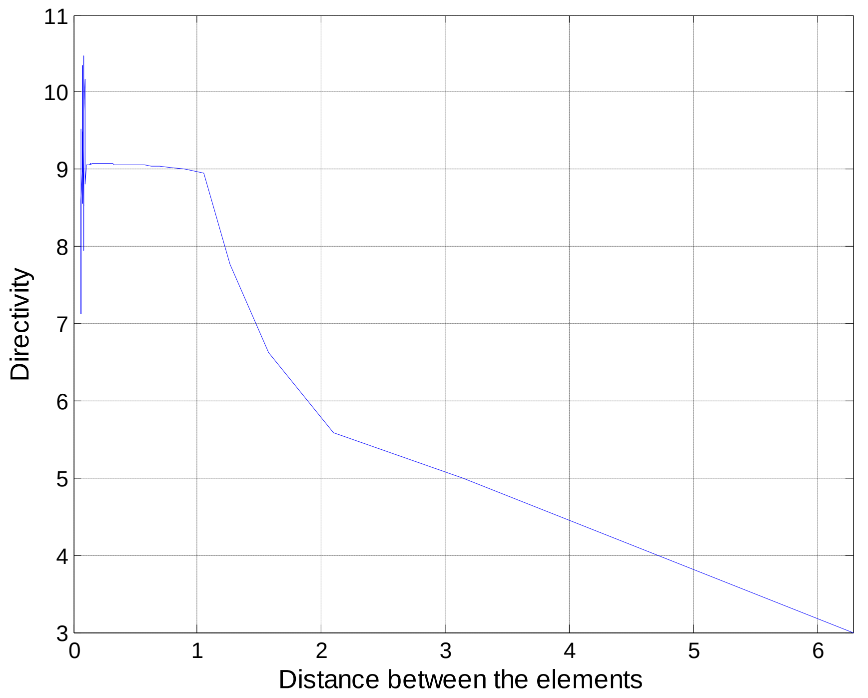

Figure - 3.6: Directivity vs Distance between the elements obtained by using pinv()

If $\mathrm{pinv}(\mathbf{A}, \mathrm{tol})$ command is used with a tolerance value of 1e-13, Figure 3.7 is obtained.

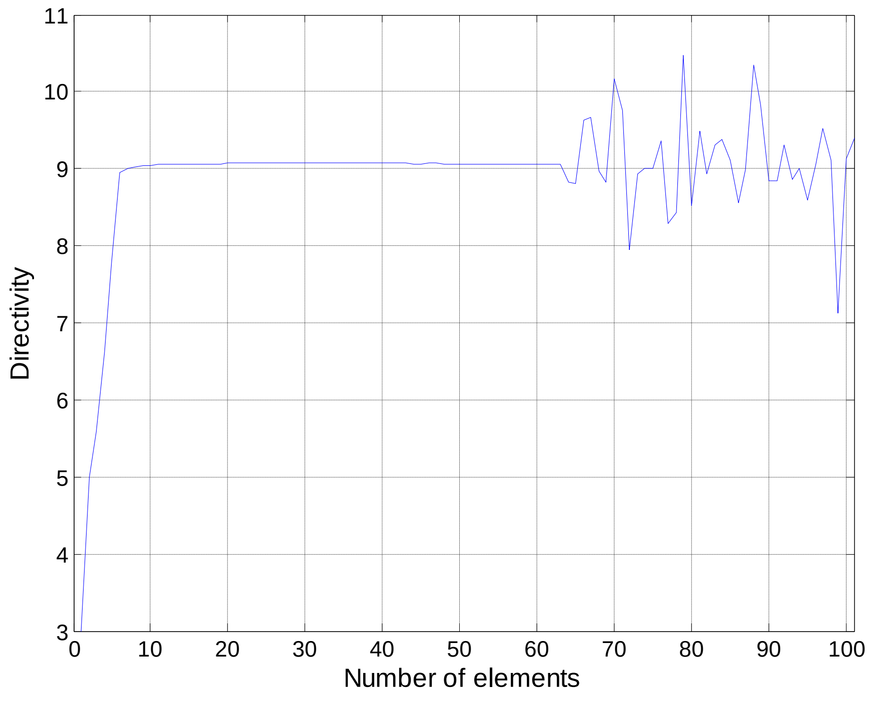

Figure - 3.7: Directivity vs Number of elements obtained by using pinv(,1e-13)

If $\mathrm{pinv}(\mathbf{A}, \mathrm{tol})$ command is used with a tolerance value of 1e-7, Figure 3.8 is obtained.

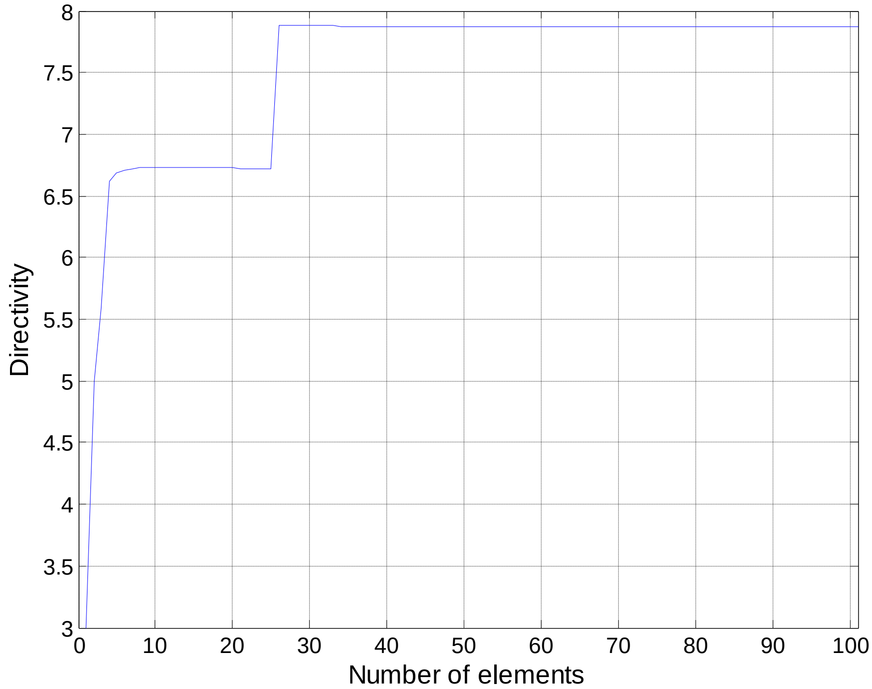

Figure - 3.8: Directivity vs Number of elements obtained by uisng pinv(,1e-7)

If $\mathrm{pinv}(\mathbf{A}, \mathrm{tol})$ command is used with a tolerance value of 1e-3, following figure is obtained, Figure 3.9.

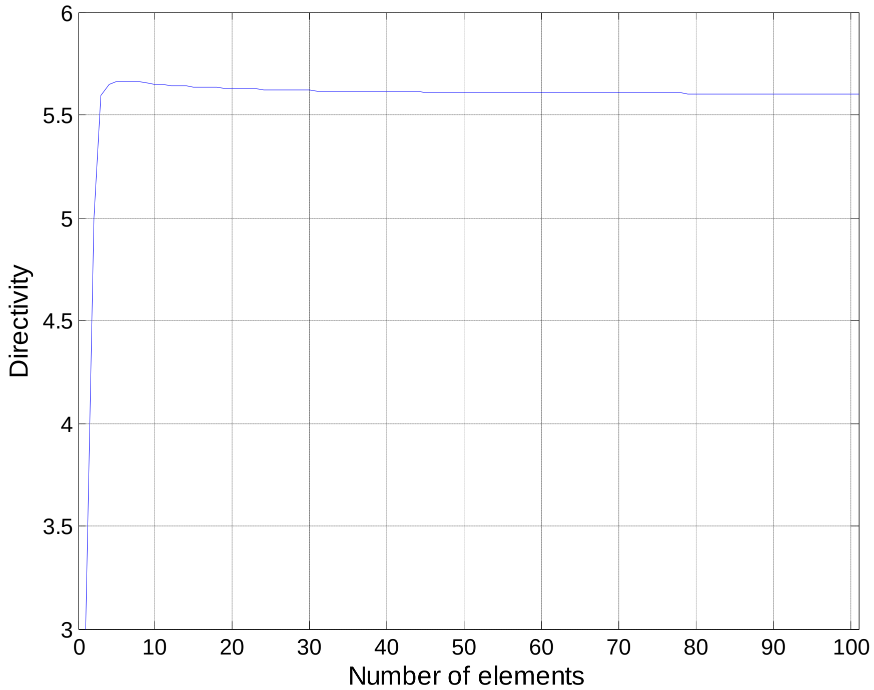

Figure - 3.9: Directivity vs Number of elements obtained by uisng pinv(,1e-3)

The results obtained by using $\mathrm{pinv}()$ and $\mathrm{pinv}(\ ,\mathrm{tol})$ commands are not accurate enough since while number of elements is increasing there is increase in directivity only for number of elements smaller than 10. For higher number of elements, the directivity seems to be saturated.

In next case, the number of elements will be fixed and the directivity spacing between the elements will be examined.

For 11 elements, Figure 3.10 shows directivity versus distance between the elements in terms of d/λ.

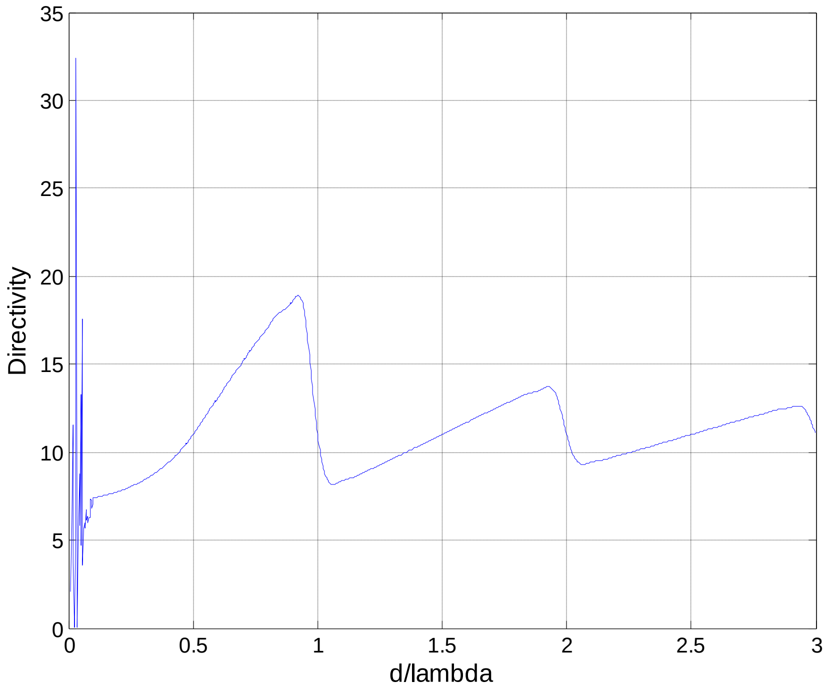

Figure - 3.10: Direcitivy vs d/λ for number of elements 11 obtained by using inv()

In Figure – 3.10, inv() command is used and it is seen that directivity can not be computed accurately when the spacing between the elements get closer than 0.15λ.

In the following 2 figures, Figure 3.11 and 3.12, pinv( ) and pinv(,$1e^{-3}$) commands are used respectively.

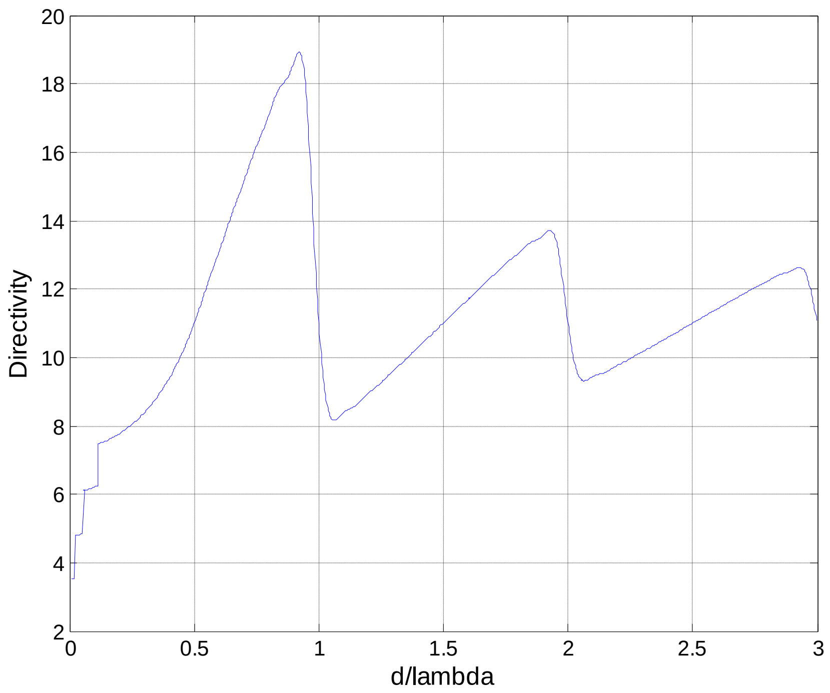

Figure - 3.11: Direcitivy vs d/λ for number of elements 11 obtained by using pinv()

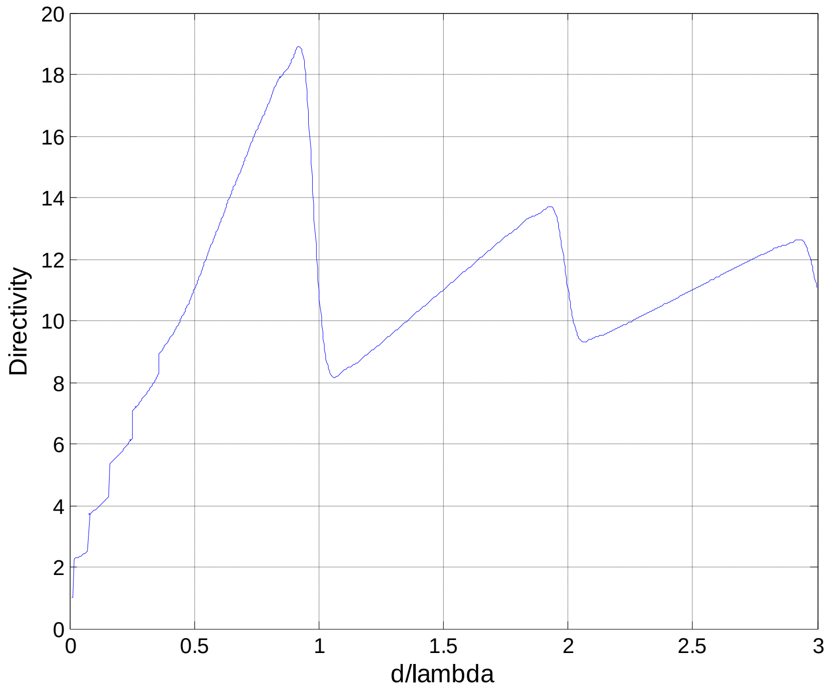

Figure - 3.12: Direcitivy vs d/λ for number of elements 11 obtained by using pinv(,1e-3)

Inspecting the figures, it can be stated that accuracy is still not sufficient to have precise results when directivities for small spacings are considered.

Regarding Tai's study [13], directivity versus spacing between the elements characteristic (for fixed number of elements) is expected to reach the limit value as the spacing gets closer, (Section 2.3). $\mathrm{pinv}()$ and $\mathrm{pinv}(\ ,\mathrm{tol})$ commands in Matlab are not able to solve the problem precisely.

3.3.2. An Iterative Method: bicg( ) Function

$\mathbf{x} = \mathrm{bicg}(\mathbf{A}, \mathbf{b})$ is a Matlab command to solve the system of linear equations $\boldsymbol{A}\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$.

$\mathrm{bicg}()$ command converges to the result by making iterations.

If $\mathrm{bicg}()$ fails to converge after the maximum number of iterations, it shows a warning message that includes the relative residual and the iteration number at which the method stopped or failed.

Tolerance of the method can be specified. Otherwise default value of 1e-6 is used.

Maximum number of iterations can also be specified. Otherwise bicg uses the default value.

$\mathrm{bicg}()$ command which makes iterations while numerical computing is used to resolve the accuracy problem.

In Figure 3.13, directivity versus distance between the elements in terms of λ is given for fixed 11 elements. Accuracy problem still exists for spacings smaller than 0.4λ. So, iterative solution way does not work, either.

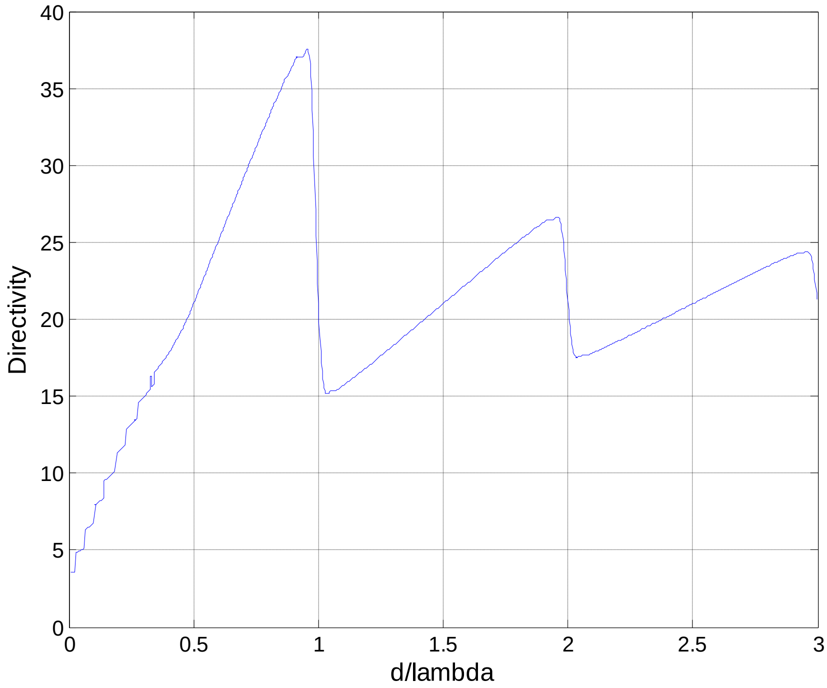

Figure - 3.13: Direcitivy vs d/λ for number of elements 11 obtained by using bicg()

3.3.3. Multiple Precision Toolbox

Multiple Precision Toolbox was created by Benjamin Barrows to increase numerical precision of computations done in Matlab. The dll-files for Microsoft Windows were compiled by Carlos Lopez who also generated some of the functions in the toolbox [19].

Benjamin Barrow defined a new class, the mp class, which holds arbitrary precision quantities. The class is saved in the Matlab Work directory. Many common numerical functions are overloaded for this class and therefore no modification is needed to source code while using mp class. When @mp directory under the MATLAB directory is looked through, a list of mp supported functions can be found. If the function is not specifically written for mp objects, it still may work if the function in question relies only on functions in the @mp directory.

Multiple Precision Toolbox for Matlab increases numerical precision of the computations by making use of hundreds or even thousands of decimals. Examples for mp toolbox applications are given as solving for high order polynomial roots, eigenvector algorithms, numerical analysis of convergence and errors, and when solving differential equations. π and e can also be computed with thousands of decimals by using mp files. Precision parameter M is used in mp functions and it is the number of digits used through the Matlab computations.

Let us see if multiple precision toolbox will work or not while obtaining directivity versus distance between the elements characteristic for 11 array elements accurately.

Figure 3.14 gives the characteristic obtained by using mp toolbox with precision M=165.

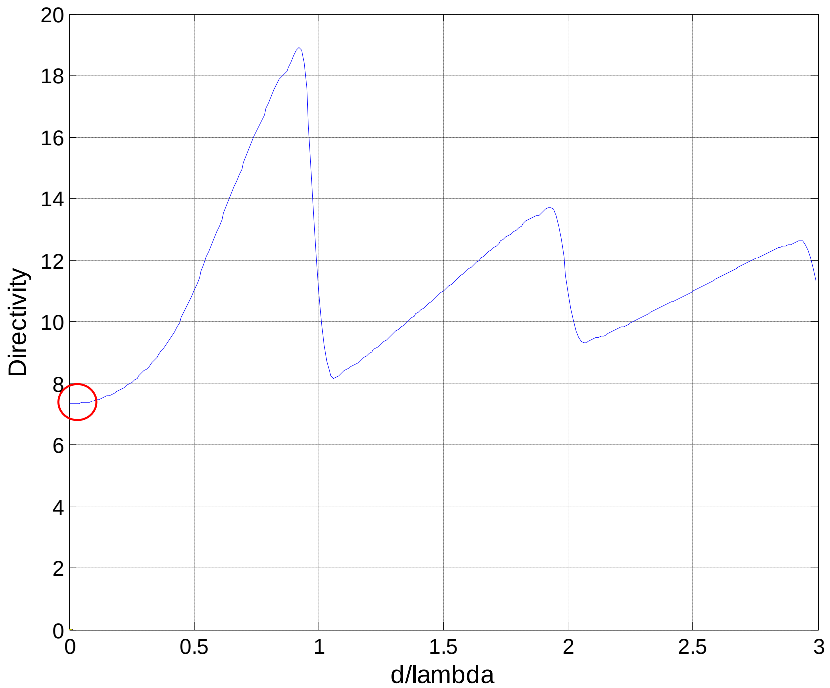

Figure - 3.14: Direcitivy vs d/λ for number of elements 11 obtained by using mp toolbox

As seen in Figure 3.14, Multiple Precision Toolbox provides required numerical precision in order to solve the numeric problem since directivity is computed accurately for small spacings.

Tai formulated the limit directivity in superdirective case as in Equation 2.27. Regarding that, limit directivity for 11 elements is 7.32. Figure - 3.14 shows that, directivity goes to that limit value while the spacing between the elements is reduced.

Theoretically, spacing between the elements can go to zero. However, precision required in order to make numerical computations for zero spacing is infinity. Since this is not practical, our aim is to find the reasonable spacing which still provides the superdirectivity in Tai's formulation. As the spacing between the elements is reduced, directivity converges to Tai's limit directivity. At some specific distance, limit directivity is obtained and spacings smaller than this distance constitute the superdirective region.

By using precision M=165, for 11 elements case in Figure – 3.14, superdirectivity is obtained at spacing d=0.006λ and spacings smaller than that give the superdirectivity region.

As seen in Figure – 3.15 stationary solution has advantage over uniform solution in superdirective region. As spacing is reduced below 0.5λ, uniform directivity decreases and the ratio of superdirectivity to uniform directivity increases.

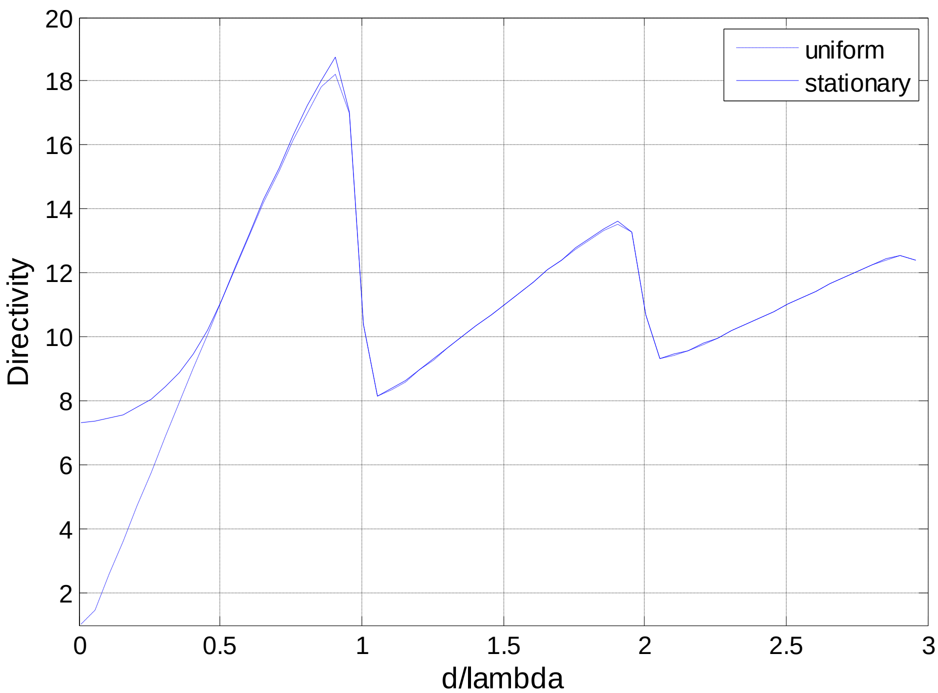

Figure - 3.15: Stationary and uniform directivity curves for 11 elements

Multiple Precision Toolbox is used hereafter throughout the thesis. In Chapter 4, directivity as spacing goes to zero is investigated for number of elements up to 37. As the number of elements is increased for a fixed array length, beam gets narrower and directivity increases. In order to see the order of change in directivity, results obtained for number of elements up to 37 are given in the thesis. In that chapter, spacings that provide superdirectivity and required precision are found for different number of elements and those values are used in numerical computations in the rest of the thesis.

CHAPTER 4

SIMULATIONS USING MULTIPLE PRECISION TOOLBOX

4.1. Using Multiple Precision Toolbox in Numerical Computations

After Mp toolbox is downloaded in Work directory of Matlab as mentioned in Chapter 3, superdirectivity concept is examined by using the mp functions.

In order to find superdirective array coefficients, the spacing between the elements that gives the limit directivity mentioned by Tai is determined first. It is observed in Figure 3.14 that directivity converges to a limit value as the spacing between the elements goes to zero. Since it is not possible to make the spacing zero or very near to zero while doing the numerical computation, spacing between the elements that allows superdirectivity is inspected.

While doing the study the parameters changed are number of elements, distance between the elements and number of digits. Since in theory spacing between the elements goes to zero for superdirectivity and it is not practical, relatively larger spacing that still provides superdirectivity is investigated.

Secondly, the smallest precision, i.e. number of digits, used through mp functions that still provide the required accuracy is examined.

After finding distance between the elements and required precision, superdirective array coefficients are computed for different number of elements. Superdirective array patterns are found by using corresponding spacing and precision values.

Odd number of elements are used in the study.

To visualize the whole picture, directivity versus spacing between zero and 3λ for 3 elements for precision M=55 is given in Figure – 4.1. In Section 4.2, spacing close to zero will be investigated to find the spacing and precision values that provide superdirectivity.

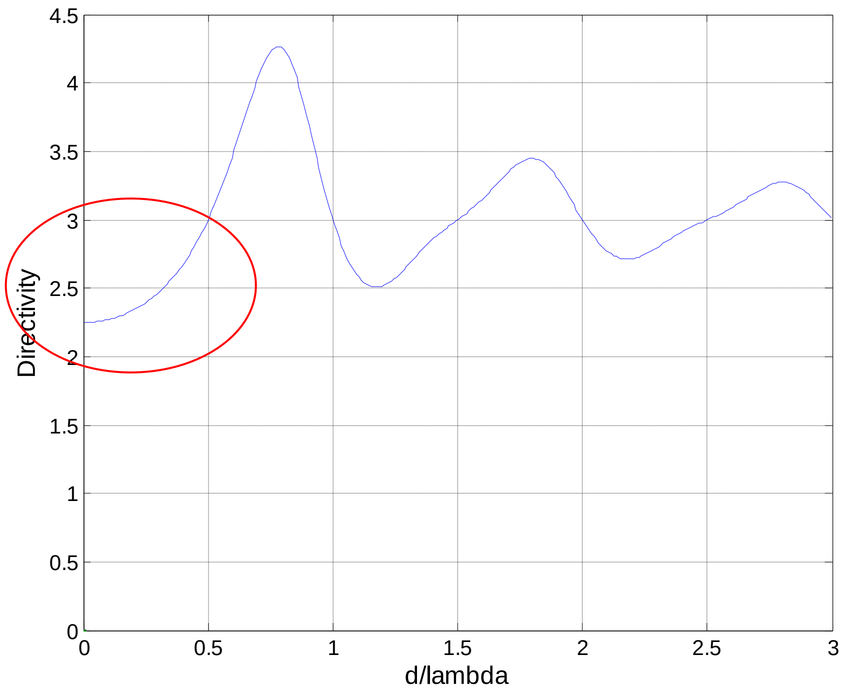

Figure - 4. 1: Directivity versus distance between the elements for 3-element array

4.2. Directivity Versus Distance Between the Elements

In order to determine which spacing between the elements provide superdirectivity and corresponding precision required for accurate calculations, directivity versus distance between the elements is observed for different number of elements.

4.2.1. 3-Element Array

For an array of 3 elements, let us first examine how directivity changes when spacing is close to zero, where spacing is smaller than 0.5λ.

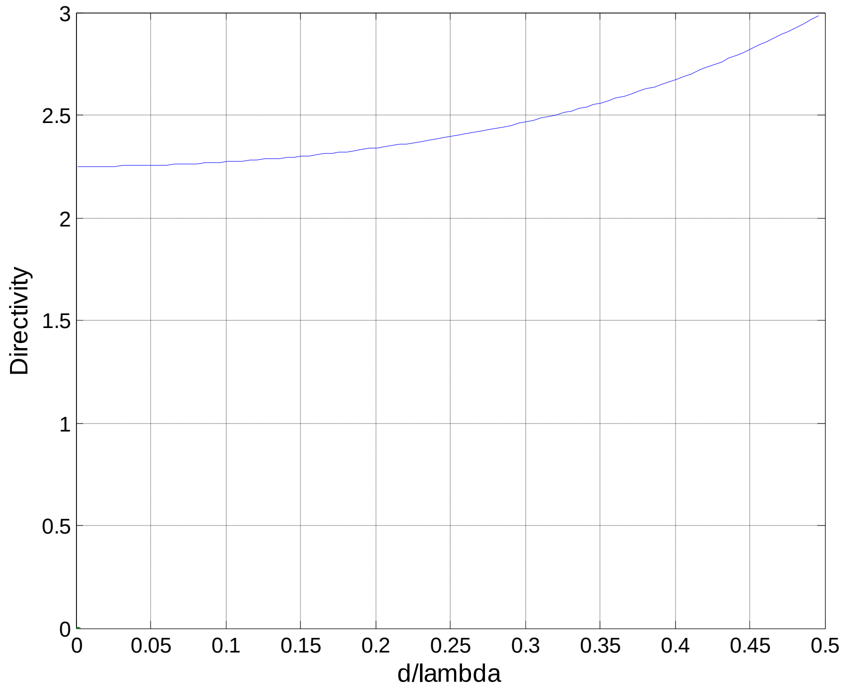

Figure - 4. 2: Directivity vs spacing for 3-element array

Limit directivity for the case of 3 elements is 2.25 as calculated from Tai's formula in Equation 2.25.

When distance between the elements is 0.001λ and precision of mp toolbox is 55, limit directivity is achieved and so superdirectivity limit is achieved.

In this case coefficients are obtained as:

$$c_1 = -63325.93$$

$$c_0 = +12665.03$$

$$c_{-1} = -63325.93$$

The coefficients are symmetric about c_0, as mentioned in Equation 3.20. For $d = 0.001\lambda$, total length of the array is 0.002λ.

Array factor for $d = 0.001\lambda$ is given in Figure – 4.3

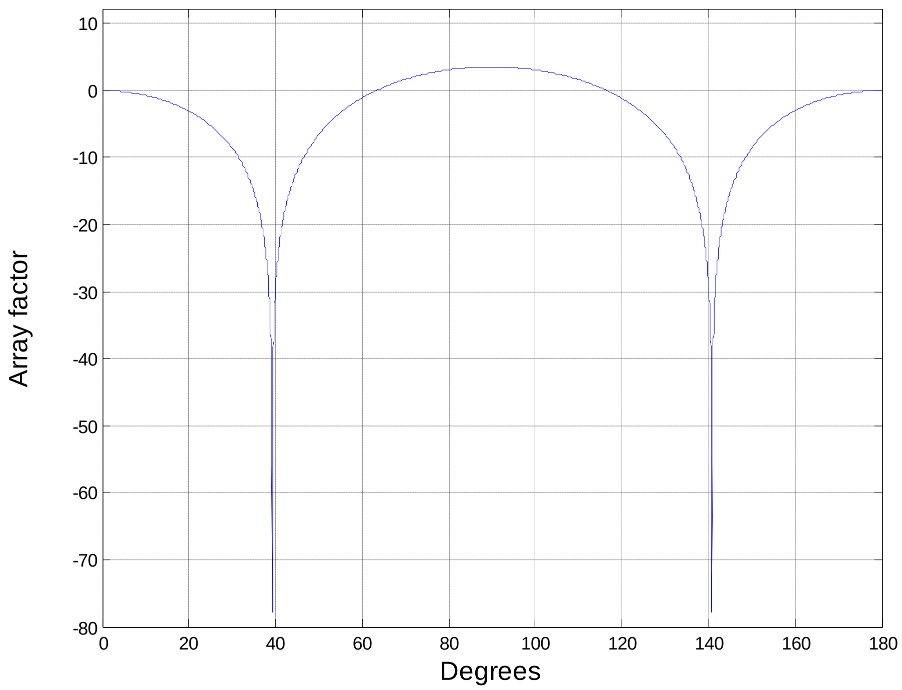

Figure - 4. 3: Array factor (dB) for 3-element superdirective array

As seen in the figure, array is a broadside array.

4.2.2. 5-Element Array

For an array of 5 elements, let us first examine how directivity changes when spacing is close to zero.

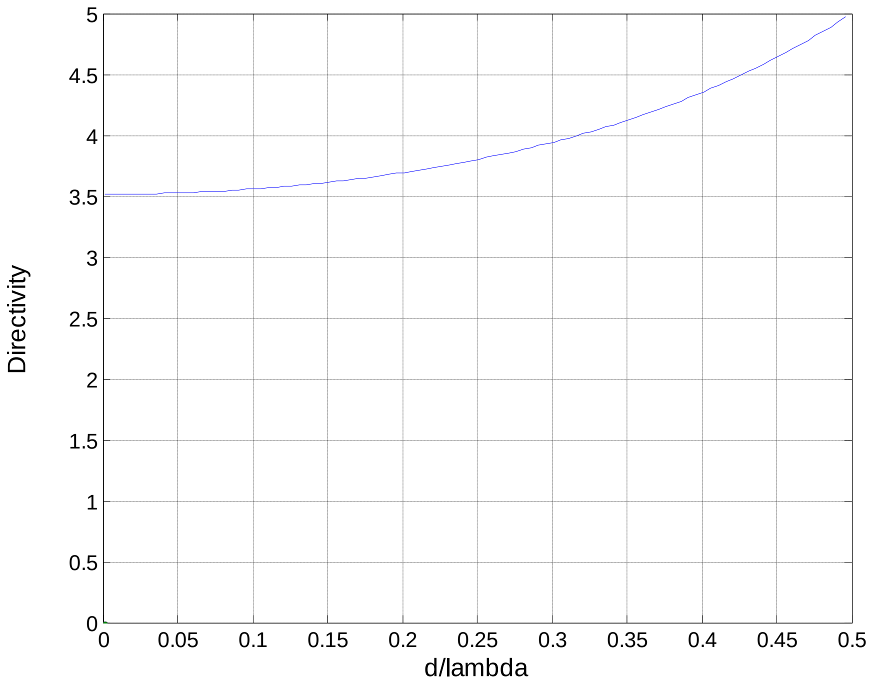

Figure - 4. 4: Directivity vs spacing for 5-element array

Limit directivity for the array of 5 elements is 3.5156 as calculated from Tai's formula in Equation 2.25.

When distance between the elements is 0.004λ and precision of mp toolbox is 75 superdirectivity limit is achieved.

When $d = 0.004\lambda$, total length of 5-elements array is $L = 0.016\lambda$.

For that array, array coefficients are as below:

$$c_2 = +19739462.79$$

$$c_1 = -78943997.91$$

$$c_0 = +11840907.21$$

$$c_{-1} = -78943997.91$$

$$c_{-2} = +19739462.79$$

The coefficients obtained for 5 elements are very large as seen above.

Array factor of superdirective array which has coefficients obtained is shown in Figure-4.5.

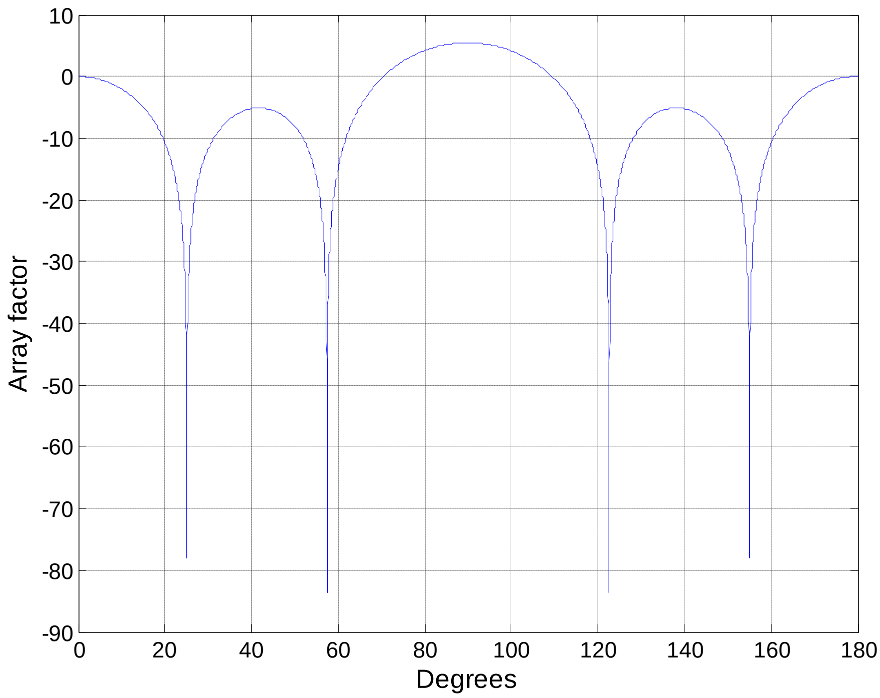

Figure - 4. 5: Array factor (dB) for 5-element superdirective array

4.2.3. 7-Element Array

For the array of 7 elements, directivity versus spacing when spacing is close to zero, is given in Figure – 4.6.

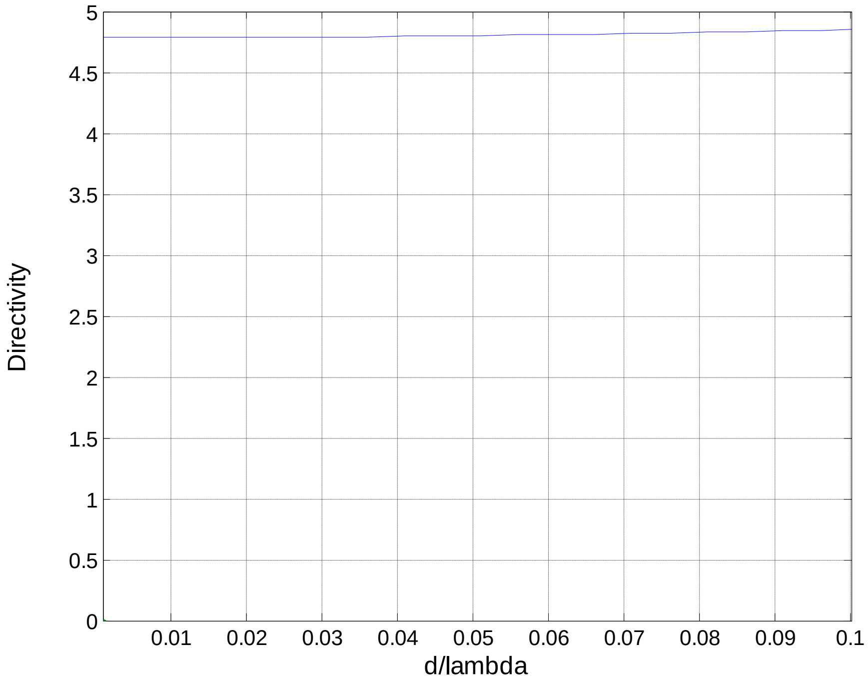

Figure - 4. 6: Directivity vs spacing for 7-element array

Limit directivity for the array of 7 elements is 4.78.

When distance between the elements is 0.004λ and precision of mp toolbox is 95 superdirectivity limit is provided.

When $d = 0.004\lambda$, total length of 7-elements array is $L = 0.024\lambda$.

For that array, array coefficients are as below:

$$c_3 = -436178283.56 \qquad c_{-1} = -653155610.64$$

$$c_2 = +261428891.75 \qquad c_{-2} = +261428891.75$$

$$c_1 = -653155610.64 \qquad c_{-3} = -436178283.56$$

$$c_0 = +870689094.29$$

The array factor for $d = 0.004\lambda$ is given in Figure – 4.7.

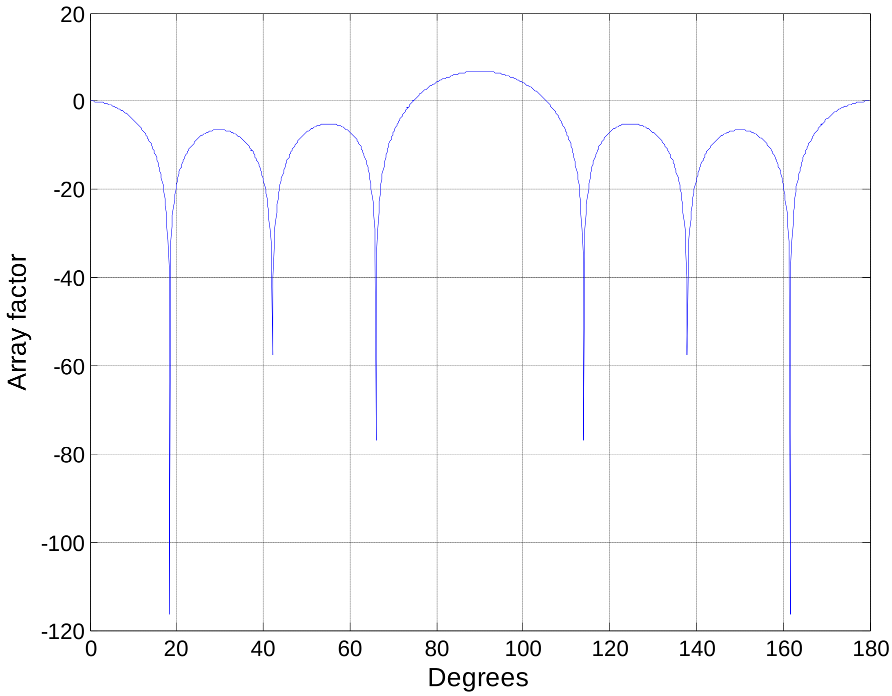

Figure - 4. 7: Array factor (dB) for 7-element superdirective array

4.2.4. 9-Element Array

For the array of 9 elements, directivity versus spacing when spacing is close to zero is obtained in Figure – 4.8.

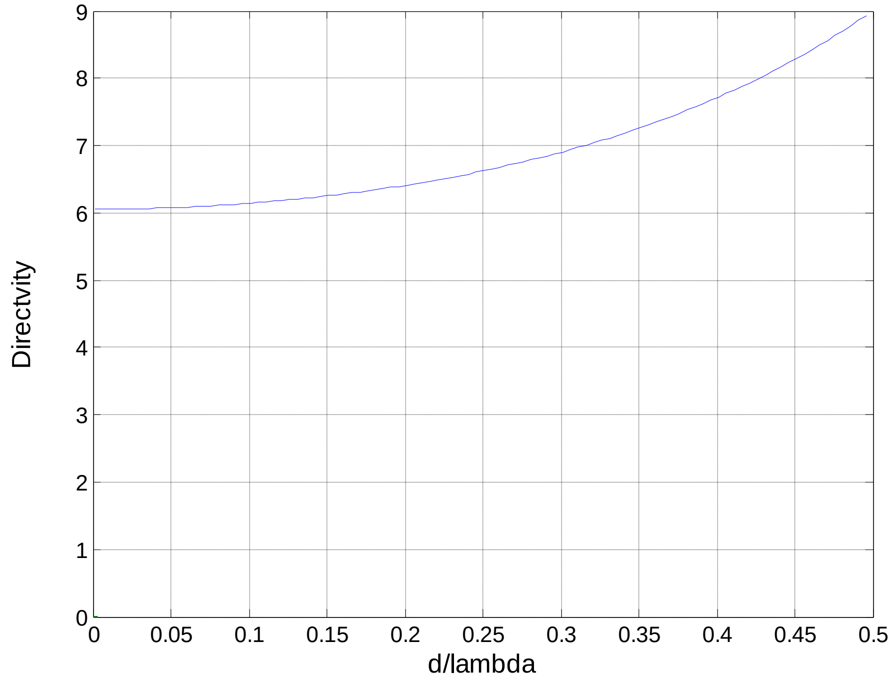

Figure - 4. 8: Directivity vs spacing for 9-element array

Limit directivity for the array of 9 elements is 6.05.

Another observation here is that, for example when the spacing of 0.01λ is taken as the distance, precision of $M = 105$ is enough where sufficient precision for $d = 0.005\lambda$ is $M = 155$.

Hence, it can be said that when the spacing gets smaller, precision required increases.

When $d = 0.005\lambda$, precision required is $M = 155$ and total length of 9-element array is $d = 0.04\lambda$.

For that array, array coefficients are as below:

$$c_4 = +391414118369.12$$

$$c_3 = -312804167708.96$$

$$c_2 = +109399767325.57$$

$$c_1 = -218701577211.84$$

$$c_0 = +273336170946.97$$

c_{-n} can be found by the relation $c_{-n} = c_n$, Equation 3.20.

The array factor for $d = 0.005\lambda$ is given in Figure – 4.9.

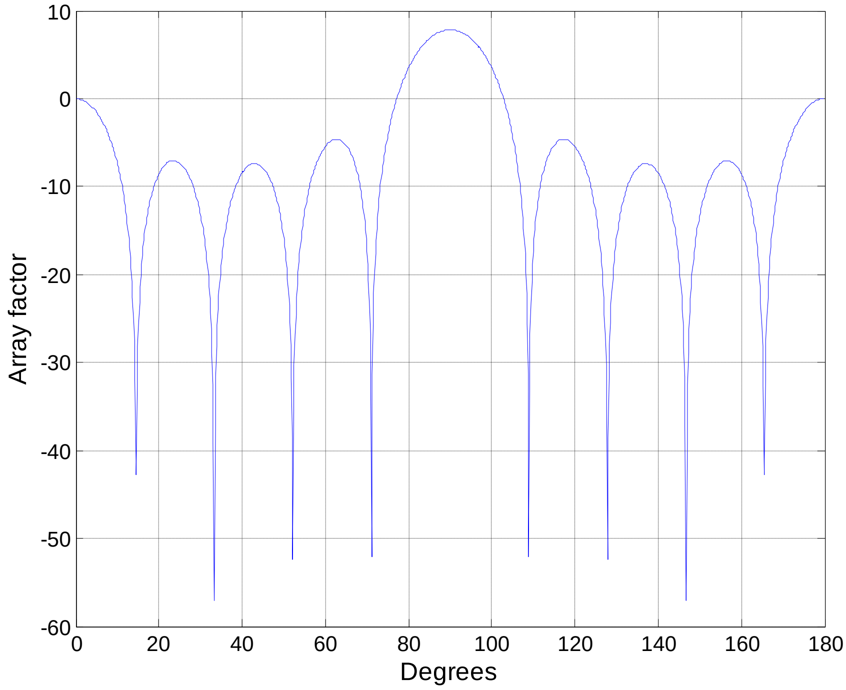

Figure - 4. 9: Array factor (dB) for 9-element superdirective array

4.2.5. 11-Element Array

For the array of 11 elements, directivity versus spacing when spacing is close to zero is given in Figure – 4.10.

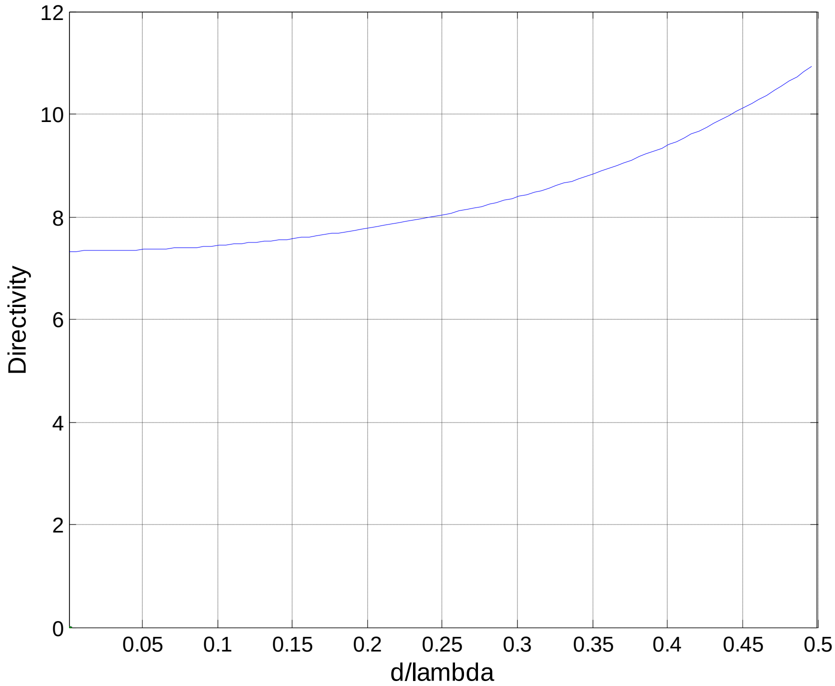

Figure - 4. 10: Directivity vs spacing for 11-element array

Limit directivity for the array of 11 elements is 7.32.

When distance between the elements is smaller than 0.006λ and precision of mp toolbox is 165, superdirectivity limit is provided.

When $d = 0.006\lambda$, total length of 11-elements array is $L = 0.06\lambda$.

For that array, array coefficients are as below:

$$c_5 = -0.59438303646027859979 \text{ x e}17$$

$$c_4 = +0.59416181744650876852 \text{ x e}18$$

$$c_3 = -0.26729542097814946474 8\text{ x e}19$$

$$c_2 = +0.71264040918859176608 \text{ x e}19$$

$$c_1 = -\,0.12469659967503703891 92\text{ x e}20$$

$$c_0 = +0.14962973143197599937 03\text{ x e}20$$

c_{-n} can be found by the relation $c_{-n} = c_n$, Equation 3.20.

As seen from the coefficients, when number of elements get larger, coefficients er also get increased.

For above found coefficients, array factor is obtained as in Figure – 4.11.

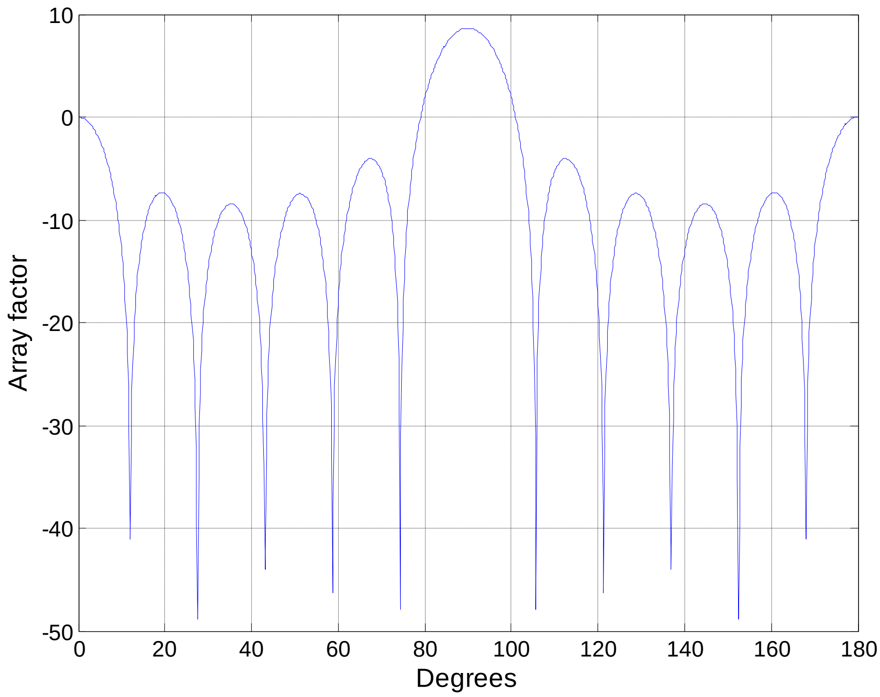

Figure - 4. 11: Array factor (dB) for 11-element superdirective array

When the array factor is observed for different number of elements, it can be said that if number of elements increases array factor becomes more directive.

4.2.6. 13-Element Array

For the array of 13 elements, directivity versus spacing when spacing is close to zero is obtained as in Figure – 4.12.

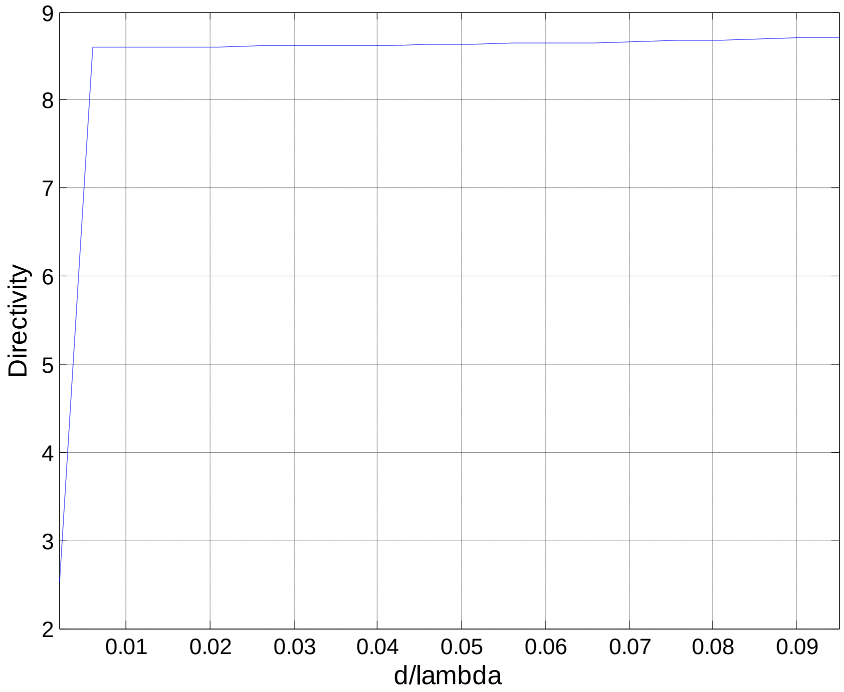

Figure - 4. 12: Directivity vs spacing for 13-element array

Limit directivity for the array of 13 elements is 8.59 by Equation 2.25. as seen in Figure - 4.12 that directivity is achieved.

For spacing $d = 0.006\lambda$ and precision of 205, superdirectivity limit is provided. For this spacing, limit directivity is achieved and computations can be done accurately. But as seen in the figure, as spacing gets closer to zero (when d is smaller than 0.005 λ), directivity can not be computed accurately by mp toolbox. Increasing the precision does not work and for smaller spacing than that distance, directivity can not be computed accurately.

When $d = 0.006\lambda$, total length of 13-element array is $L = 0.072\lambda$.

For that array, array coefficients are as below:

$$c_6 = +0.154169553407558597536 00 \text{ x e21}$$

$$c_5 = -0.184935109833149849170992 \text{ x e22}$$

$$c_4 = +0.101683562607101454184 4575 \text{ x e23}$$

$$c_3 = -0.338865516247821783949 0832 \text{ x e23}$$

$$c_2 = +0.76231936343271804164877 28 \text{ x e23}$$

$$c_1 = -0.12195880738579458495710246 \text{ x e24}$$

$$c_0 = +0.14228049590303750732572628 \text{ x e24}$$

c_{-n} can be found by the relation $c_{-n} = c_n$.

The array factor for $d = 0.006\lambda$ is given in Figure – 4.13.

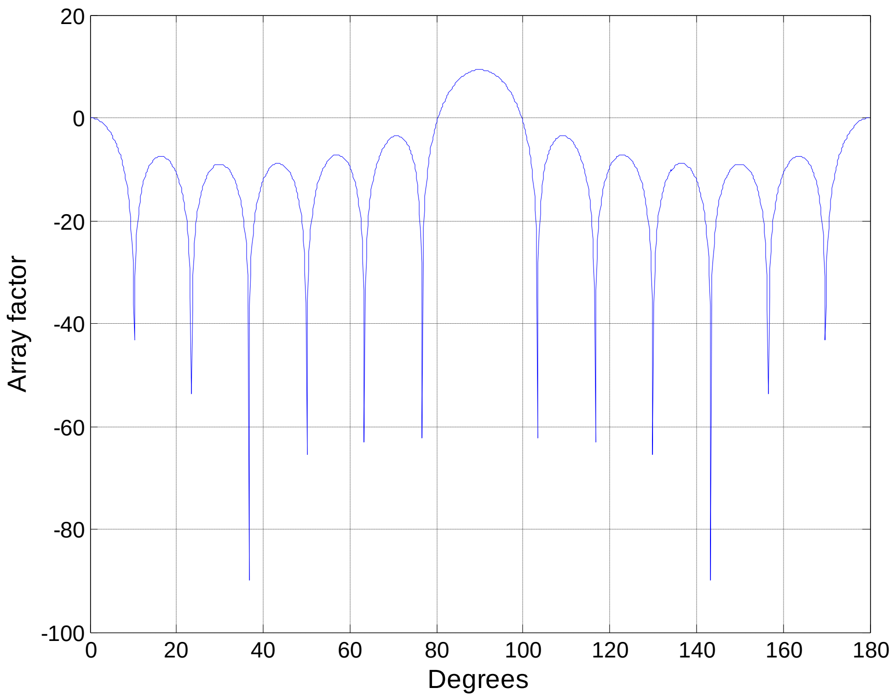

Figure - 4. 13: Array factor (dB) for 13-element superdirective array

4.2.7. 15-Element Array

For the array of 15 elements, directivity versus spacing when spacing is close to zero is obtained below:

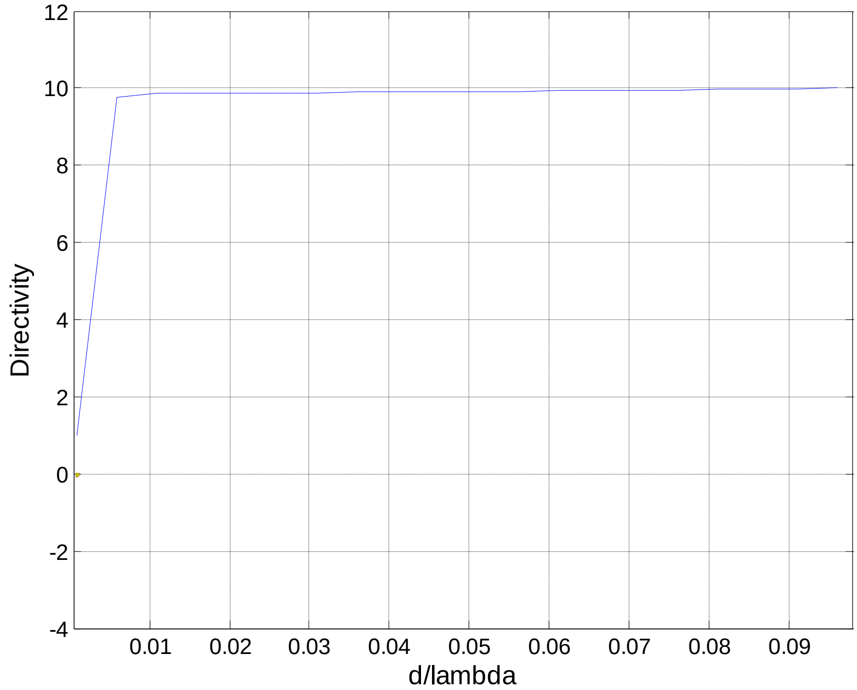

Figure - 4. 14: Directivity vs spacing for 15-element array

Limit directivity for the array of 15 elements is 9.87 by Equation 2.25.

For spacing $d = 0.007\lambda$ and precision of 235, superdirectivity limit is provided.

It can be seen in the figure that, directivity again can not be computed accurately when spacing is below $d = 0.007\lambda$.

When $d = 0.007\lambda$, total length of 15-elements array is $L = 0.098\lambda$.

For that array, array coefficients are as below:

$c_7 = -0.4674589309754404351667432 \text{ x e}23$ $c_3 = -0.4672066147037584066035237 87 \text{ x e}26$

$c_6 = +0.6541151457974857432585518 4 \text{ x e}24$ $c_2 = +0.9342334569567523953784040110 \text{ x e}26$

$c_5 = -0.4249948895616704597397089 74 \text{ x e}25$ $c_1 = -0.1401188417624687366194989 8564 \text{ x e}27$

$c_4 = +0.1699390863228988317007379847 \text{ x e}26$ $c_0 = +0.1601296570955924349391847 5090 \text{ x e}27$

c_{-n} can be found by the relation $c_{-n} = c_n$.

The array factor is obtained as in Figure – 4.15.

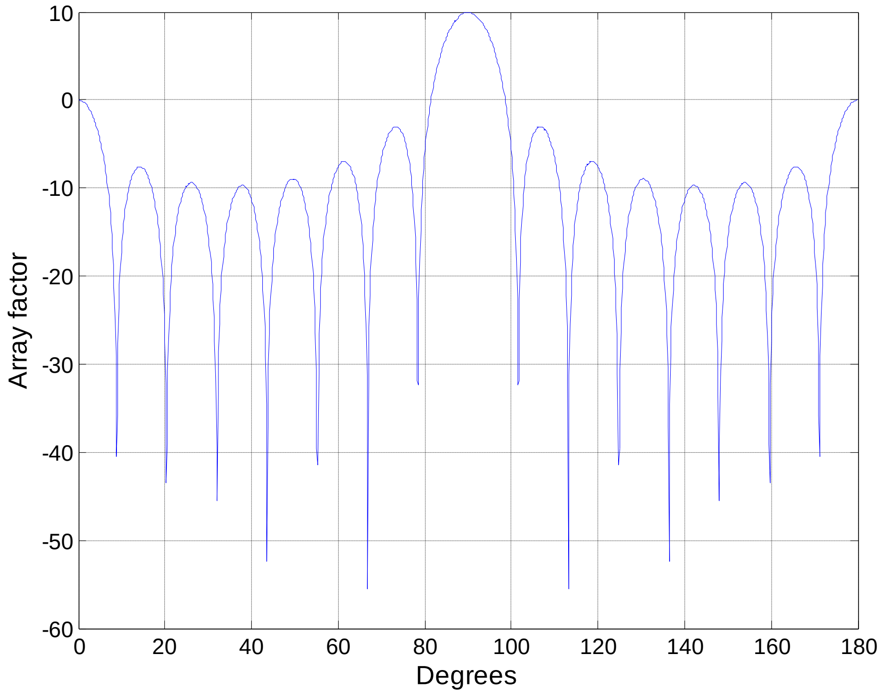

Figure - 4. 15: Array factor (dB) for 15-element superdirective array

4.2.8. 17-Element Array

For the array of 17 elements, directivity versus spacing when spacing is close to zero is obtained as in Figure – 4.16.

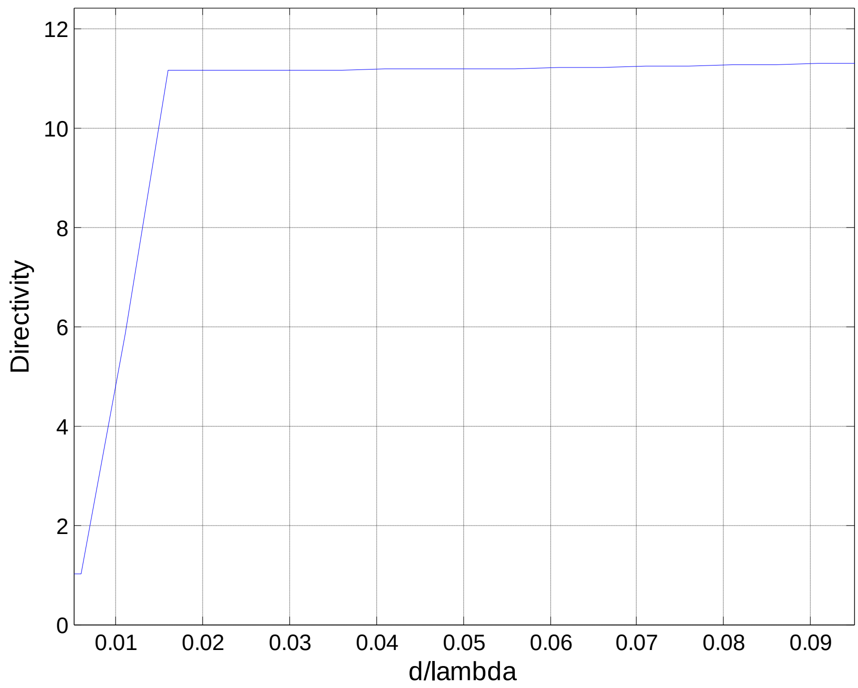

Figure - 4. 16: Directivity vs spacing for 17-element array

Limit directivity for the array of 17 elements is 11.14 by Equation 2.2.

For spacing $d = 0.016\lambda$ and precision of 245, superdirectivity limit is achieved.

As seen from previous computations, the spacing where directivity can not be computed precisely increases when the number of elements incerases.

When $d = 0.016\lambda$, total length of 17-elements array is $L = 0.256\lambda$.

For that array, array coefficients are as below:

$c_8 = +0.16466769158343227838764 \text{ e21}$

$c_7 = -0.26278301461422445924651 9 \text{ x e22}$

$c_6 = +0.19664299318346959568888 09 \text{ x e23}$

$c_5 = -0.91591696943526606058008 01 \text{ x e23}$

$c_4 = +0.29720846049479290609019493 \text{ x e24}$

$c_3 = -0.71243444036800221979024336 \text{ x e24}$

$c_2 = +0.13049972153339014640764663 0 \text{ x e25}$

$c_1 = -0.18633116729129674494358264 1 \text{ x e25}$

$c_0 = +0.20958619950640275157252153 7 \text{ x e25}$

c_{-n} can be found by the relation $c_{-n} = c_n$.

The array factor for the superdirective array of 17 elements is obtained in Figure – 4.17.

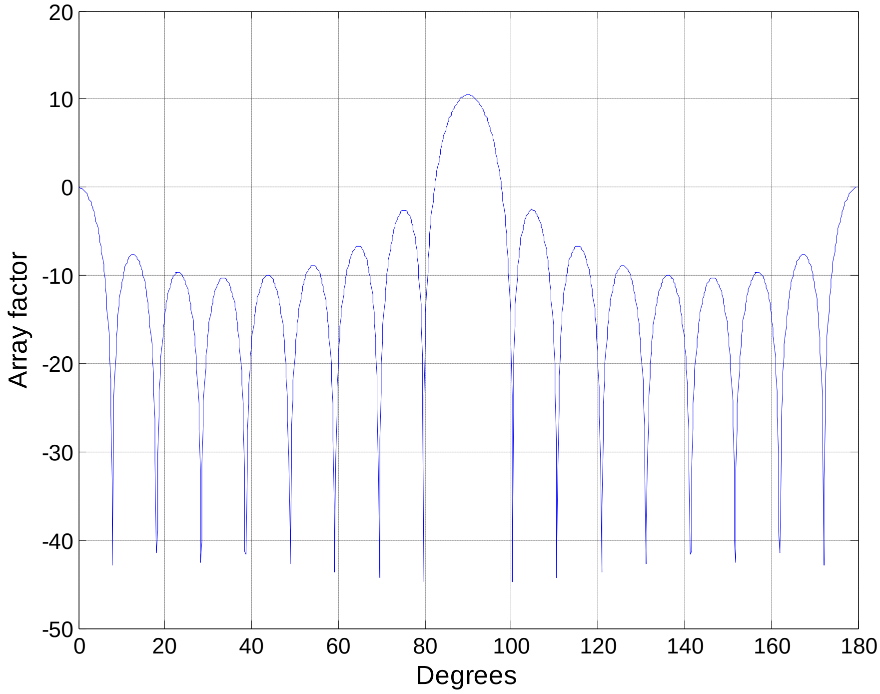

Figure - 4. 17: Array factor (dB) for 17-element superdirective array

4.2.9. 19-Element Array

For the array of 19 elements, directivity versus spacing when spacing is close to zero is obtained in Figure – 4.18.

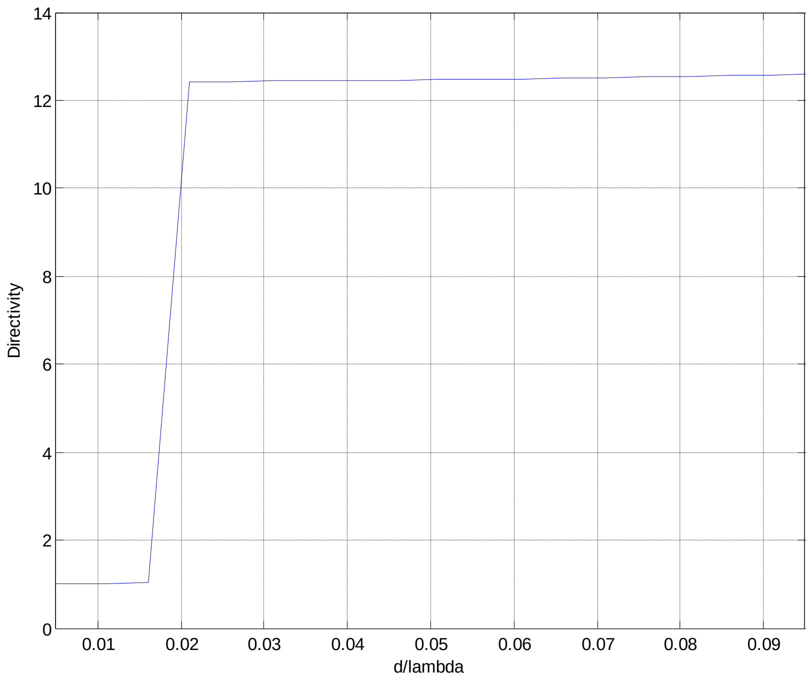

Figure - 4. 18: Directivity vs spacing for 19-element array

Limit directivity for the array of 19 elements is 12.42.

For spacing $d = 0.022\lambda$ and precision of 245, superdirectivity limit is provided. Superdirectivity limit is achieved before mp toolbox accuracy becomes insufficient as seen in Figure-4.18.

When $d = 0.022\lambda$, total length of 19-element array is $L = 0.396\lambda$.

For that array, array coefficients are as below:

$c_9 = -0.2044031356407281416434 7 \text{ x e21}$ $c_4 = +0.1718799662264234672129275 02 \text{ x e25}$

$c_8 = +0.3661251957348617796229 64 \text{ x e22}$ $c_3 = -0.3716560539891718755807474 92 \text{ x e25}$

$c_7 = -0.3098627461269043349082 853 \text{ x e23}$ $c_2 = +0.6362073376735997125492813 18 \text{ x e25}$

$c_6 = +0.1646417396018945132290 5644 \text{ x e24}$ $c_1 = -0.8740292294473498572264283 97 \text{ x e25}$

$c_5 = -0.6154515836241023870480 3050 \text{ x e24}$ $c_0 = +0.9708638130356351896209770 72 \text{ x e25}$

c_{-n} can be found by the relation $c_{-n} = c_n$.

For that array, the array factor is obtained as:

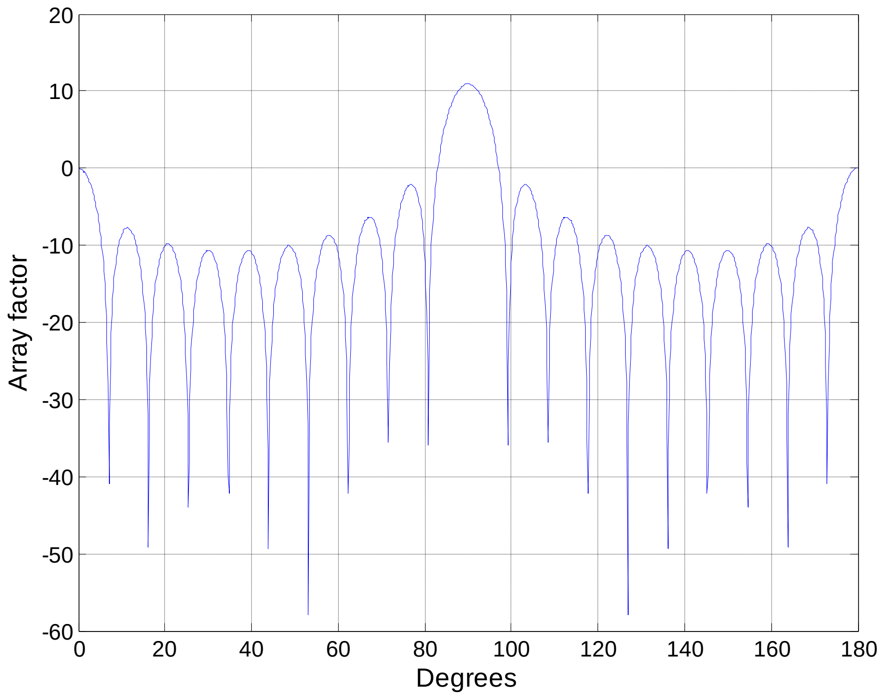

Figure - 4. 19: Array factor (dB) for 19-element superdirective array

4.2.10. 21-Element Array

For the array of 21 elements, directivity versus spacing when spacing is close to zero, smaller than 0.1λ, is obtained below:

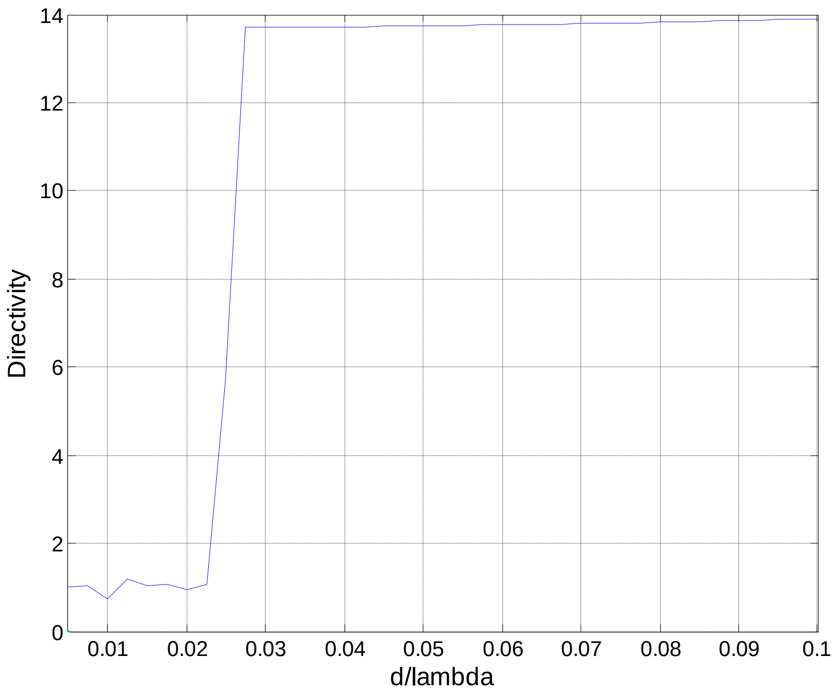

Figure - 4. 20: Directivity vs spacing for 21-element array

Limit directivity for the array of 21 elements is 13.69.

For spacing $d = 0.027\lambda$ and precision of 255, superdirectivity limit is provided. When the precision is observed, it can be mentioned that precision required increases as the number of elements increases.

When $d = 0.027\lambda$, total length of 21-element array is $L = 0.54\lambda$.

For that array, array coefficients are as below:

$c_{10} = +0.6465504037756648937185 6 \text{ x e21}$

$c_9 = -0.12835925185011928816045 44 \text{ x e23}$

$c_8 = +0.12111390697757662661614 0249 \text{ x e24}$

$c_7 = -0.72261532890607828560736785 \text{ x e24}$

$c_6 = +0.30556643349663538911768 0269 \text{ x e25}$

$c_5 = -0.97364963584438118198540 9803 \text{ x e25}$

$c_4 = +0.24256439962019889663122 1843 \text{ x e26}$

$c_3 = -0.48381401617326473633468 19866 \text{ x e26}$

$c_2 = +0.78467545752009877537093 34837 \text{ x e26}$

$c_1 = -0.10450180939175573193795 8432132 \text{ x e27}$

$c_0 = +0.11490744590488288916651 337496 \text{ x e27}$

c_{-n} can be found by the relation $c_{-n} = c_n$.

For that array, array factor is obtained as:

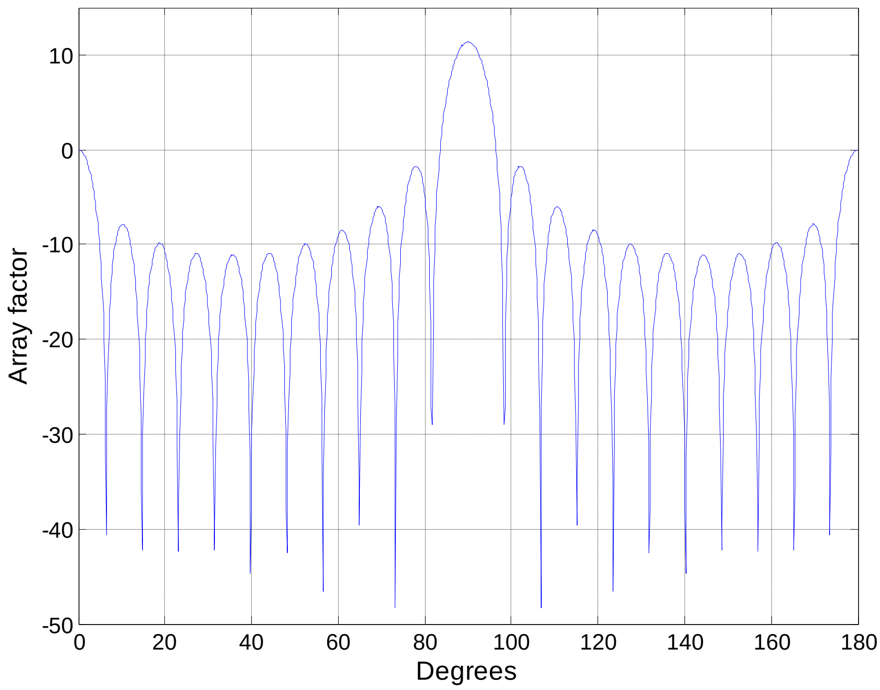

Figure - 4. 21: Array factor (dB) for 21-element superdirective array

4.2.11. 23-Element Array

For the array of 23 elements, directivity versus spacing when spacing is close to zero, smaller than 0.5λ, is obtained in Figure – 4.22.

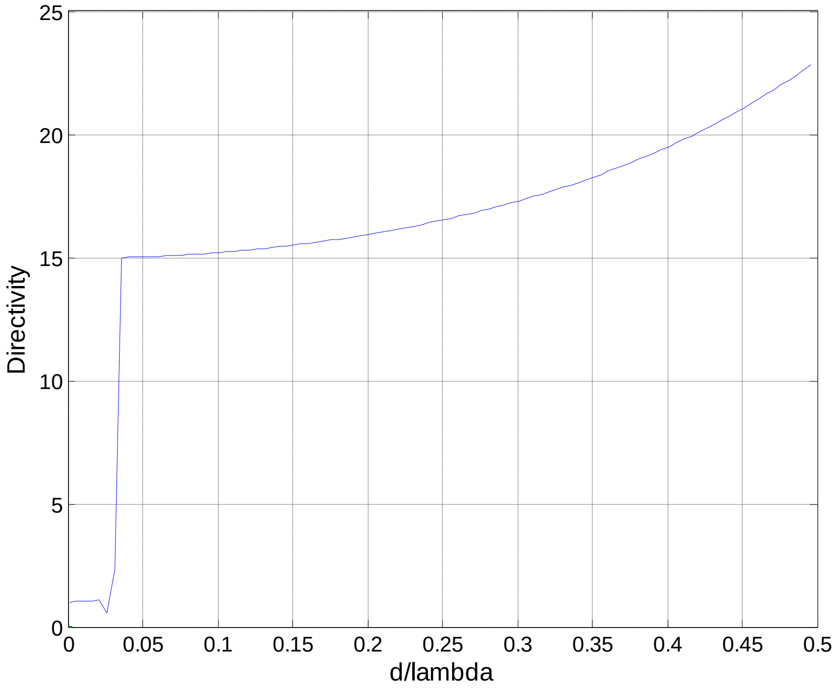

Figure - 4. 22: Directivity vs spacing for 23-element array

Limit directivity for the array of 23 elements is 14.96.

For spacing $d = 0.0386\lambda$ and precision of 255, superdirectivity limit is provided.

When $d = 0.0386\lambda$, total length of 23-element array is $L = 0.85\lambda$.

For that array, array coefficients are as below:

$$c_{11} = -0.354621295095183967 7214 \text{ x e}20$$

$$c_{10} = +0.76849571866331777860098 \text{ x e}21$$

$$c_{9} = -0.79599976180569106497059 6 \text{ x e}22$$

$$c_{8} = +0.52424058488155860882707 31 \text{ x e}23$$

$$c_{7} = -0.24635347486705605528409142 \text{ x e}24$$

$$c_{6} = +0.87865823775259606767713806 \text{ x e}24$$

$$c_{5} = -0.247001629116307897057563321 \text{ x e}25$$

$$c_{4} = +0.560952989351205183640561689 \text{ x e}25$$

$$c_{3} = -0.1046536348829453261423557902 \text{ x e}26$$

$$c_{2} = +0.162213789981401497998719363 \text{ x e}26$$

$$c_{1} = -0.2104263049600841790264687562 \text{ x e}26$$

$$c_{0} = +0.2293919905293807017834531164 \text{ x e}26$$

c_{-n} can be found by the relation $c_{-n} = c_n$.

The array factor is obtained as in Figure – 4.23.

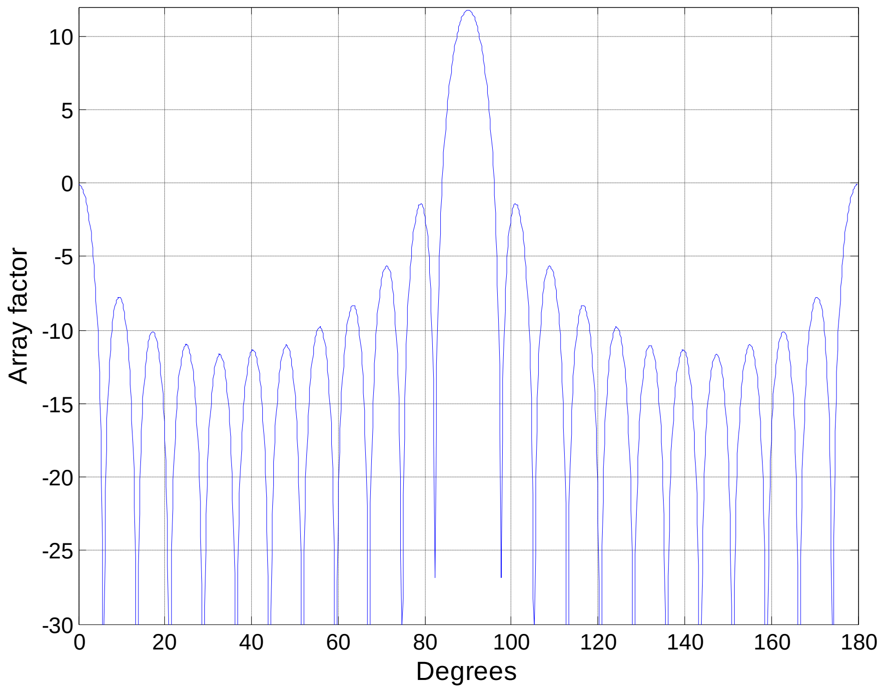

Figure - 4. 23: Array factor (dB) for 23-element superdirective array

4.2.12. 25-Element Array

For 25-element array, directivity versus spacing when spacing is close to zero, smaller than 0.1λ, is obtained in Figure – 4.24.

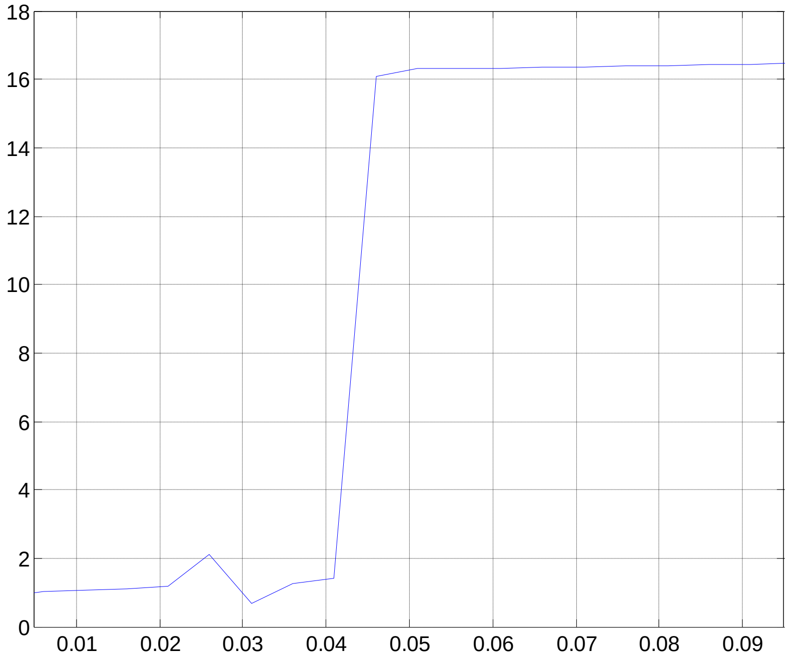

Figure - 4. 24: Directivity vs spacing for 25-element array

Limit directivity for the array of 25 elements is 16.23.

For spacing $d = 0.047\lambda$ and precision of 285, superdirectivity limit is provided.

When $d = 0.047\lambda$, total length of 25-element array is $L = 1.128\lambda$.

For that array, array coefficients are as below:

$c_{12} = +0.21648304142062194118 49 \text{ x e20}$

$c_{11} = -0.5080831921781567832 2200 \text{ x e21}$

$c_{10} = +0.5725132050296342803 90001 \text{ x e22}$

$c_9 = -0.4121831590869716471 900058 \text{ x e23}$

$c_8 = +0.2128633701583729717 8635779 \text{ x e24}$

$c_7 = -0.8391860658078447345 0300949 \text{ x e24}$

$c_6 = +0.8391860658078447345 0300949 \text{ x e24}$

$c_5 = -0.6676677698538491058 61686589 \text{ x e25}$

$c_4 = +0.1406515033490976239 717310109 \text{ x e26}$

$c_3 = -0.2483630048112127180 786054027 \text{ x e26}$

$c_2 = +0.3707510453794744238 185457625 \text{ x e26}$

$c_1 = -0.4705009560527070842 088872050 \text{ x e26}$

$c_0 = +0.5092178070199618808 912645588 \text{ x e26}$

c_{-n} can be found by the relation $c_{-n} = c_n$.

The array factor for $d = 0.047\lambda$ is given in Figure – 4.25.

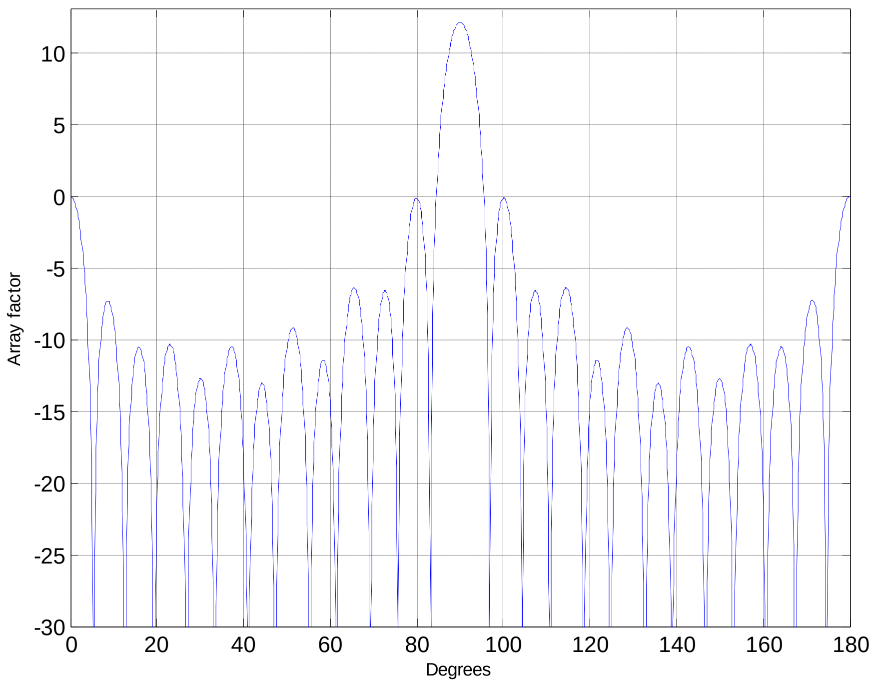

Figure - 4. 25: Array factor (dB) for 25-element superdirective array

4.2.13. 27-Element Array

For the array of 27 elements, directivity versus spacing when spacing is close to zero, smaller than 0.1λ, is obtained as in Figure – 4.26.

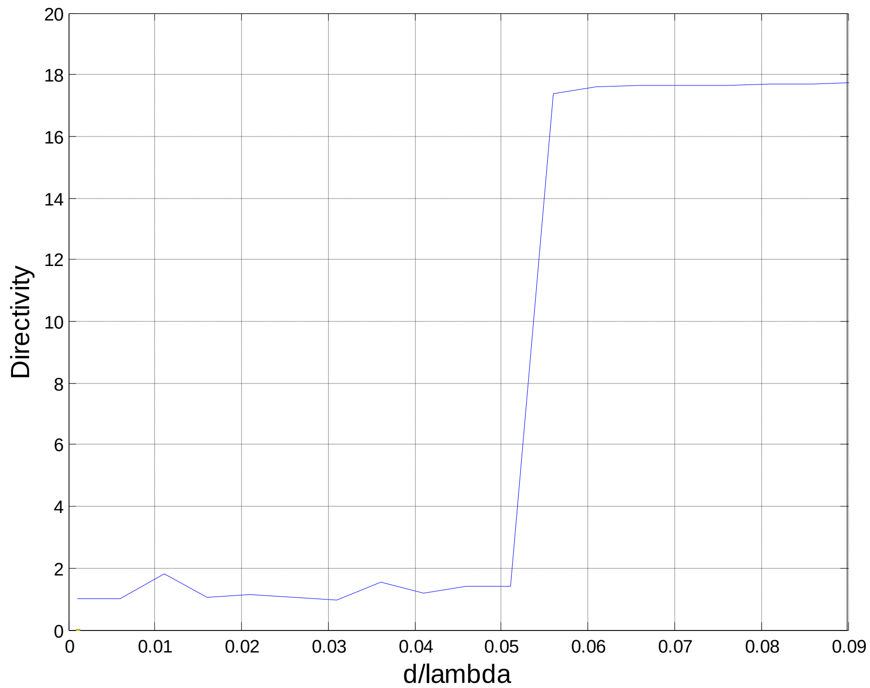

Figure - 4. 26: Directivity vs spacing for 27-element array

Limit directivity for the case of 27 elements is found as 17.50 by Equation 2.25.

For spacing $d = 0.0578\lambda$ and precision of 295, superdirectivity limit is provided.

When $d = 0.0578\lambda$, total length of 27-element array is $L = 1.5\lambda$.

For that array, array coefficients are as below:

$$c_{13} = -0.4676468507141982516 90 \text{ x e19}$$

$$c_{12} = +0.117547504449875677332 50 \text{ x e21}$$

$$c_{11} = -0.142442657240167509072609 \text{ x e22}$$

$$c_{10} = +0.110773552415409752324 1881 \text{ x e23}$$

$$c_9 = -0.620861384631100603124 9402 \text{ x e23}$$

$$c_8 = +0.267004467174109449228 78909 \text{ x e24}$$

$$c_7 = -0.915871445284407525631 46124 \text{ x e24}$$

$$c_6 = +0.257150809236392055555942961 \text{ x e25}$$

$$c_5 = -0.601788087686067551846672925 \text{ x e25}$$

$$c_4 = +0.118914262226837419655234868 91 \text{ x e26}$$

$$c_3 = -0.200267089603058458931288710 8 \text{ x e26}$$

$$c_2 = +0.289353315304179660555006215 8 \text{ x e26}$$

$$c_1 = -0.360241588029539087552193446 5 \text{ x e26}$$

$$c_0 = +0.387433402147389000067973612 87 \text{ x e26}$$

c_{-n} can be found by the relation $c_{-n} = c_n$.

Array factor of that array is given in Figure – 4.27.

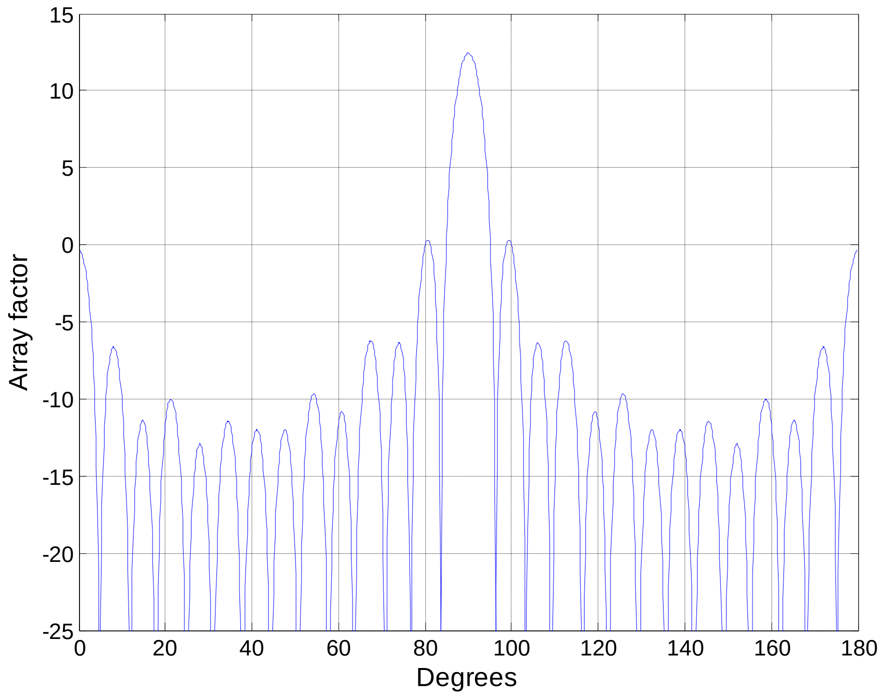

Figure - 4. 27: Array factor (dB) for 27-element superdirective array

4.2.14. 29-Element Array

For the array of 29 elements, directivity versus spacing when spacing is close to zero is given in Figure – 4.28.

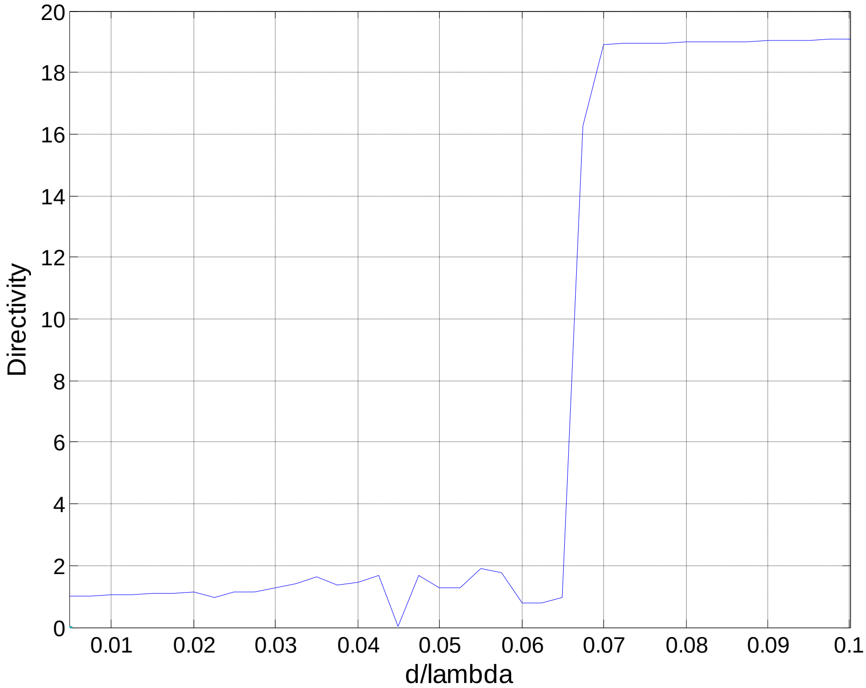

Figure - 4. 28: Directivity vs spacing for 29-element array

Limit directivity for the array of 29 elements is 18.78 by using Equation 2.25.

For spacing $d = 0.0678\lambda$ and precision of 305, superdirectivity limit is provided.

When $d = 0.0678\lambda$, total length of 29-element array is $L = 1.9\lambda$.

For that array, array coefficients are as below:

$$c_{14} = +0.1593574031705815100068 \text{ x } e19$$

$$c_{13} = -0.4259074651357246504326 \text{ x } e20$$

$$c_{12} = +0.5507662124208767173 1756 \text{ x } e21$$

$$c_{11} = -0.4588412054171976707 62412 \text{ x } e22$$

$$c_{10} = +0.276632035089966571318 3841 \text{ x } e23$$

$$c_9 = -0.12853274349113685204552140 \text{ x } e24$$

$$c_8 = +0.47859064398003645758299610 \text{ x } e24$$

$$c_7 = -0.1466090967447775122064 44228 \text{ x } e25$$

$$c_6 = +0.376405259803237263587407425 \text{ x } e25$$

$$c_5 = -.820914341079527494729043234 \text{ x } e25$$

$$c_4 = +0.15360021593698121760174 04712 \text{ x } e26$$

$$c_3 = -0.24836756835460193070896 63780 \text{ x } e26$$

$$c_2 = +0.3488722665592067276852400383 \text{ x } e26$$

$$c_1 = -0.4271948778127659346850280244 \text{ x } e26$$

$$c_0 = +0.4569307137269001229630625571 \text{ x } e26$$

c_{-n} can be found by the relation $c_{-n} = c_n$.

Array factor is obtained in Figure – 4.29.

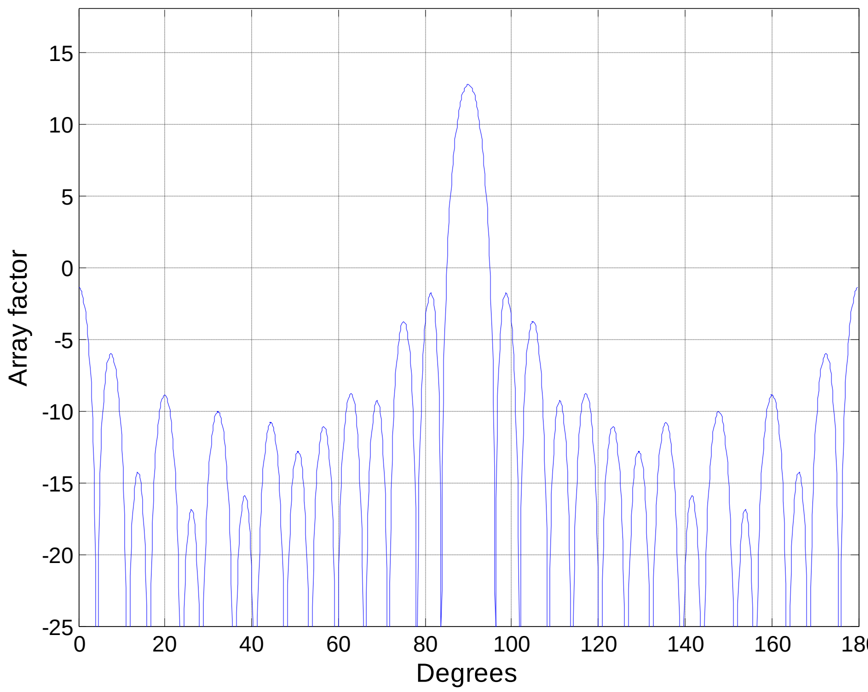

Figure - 4. 29: Array factor (dB) for 29-element superdirective array

4.2.15. 31-Element Array

For the array of 31 elements, directivity versus spacing when spacing is close to zero, is obtained in Figure – 4.30.

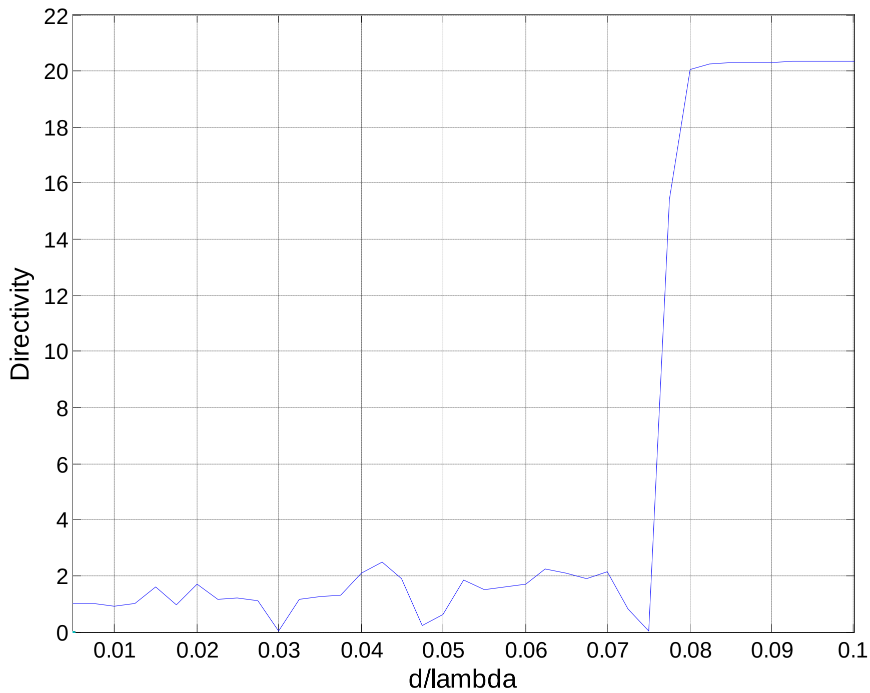

Figure - 4. 30: Directivity vs spacing for 31-element array

Limit directivity for the case of 31 elements is 20.05 by Equation 2.25.

For spacing $d = 0.08\lambda$ and precision of 405, superdirectivity limit is achieved.

As seen in Figure – 4.30, limit spacing that accuracy is insufficient becomes close to 0.1 when the number of elements incerases.

When $d = 0.08\lambda$, total length of 31-element array is $L = 2.4\lambda$.

For that array, array coefficients are as below:

$c_{15} = -0.26487104289973458636 \text{ x e18}$

$c_{14} = +0.74465023985002290787 6\text{ x e19}$

$c_{13} = -0.10165468688571480013119 \text{ x e21}$

$c_{12} = +0.89736083271863632434280 \text{ x e21}$

$c_{11} = -0.57551462107946725716716 0\text{ x e22}$

$c_{10} = +0.28563780686929925528535 20\text{ x e23}$

$c_9 = -0.11410905185324422684257363 \text{ x e24}$

$c_8 = +0.37678961732514012141202251 \text{ x e24}$

$c_7 = -0.1047957859999432950241 02479\text{ x e25}$

$c_6 = +0.24892296629359359071582 3156\text{ x e25}$

$c_5 = -0.5102144404749913151282 85714\text{ x e25}$

$c_4 = +0.9094538175055123341847 48342\text{ x e25}$

$c_3 = -0.14179513164628911311617 08943\text{ x e26}$

$c_2 = +0.19418385252098790483360 10826\text{ x e26}$

$c_1 = -0.19418385252098790483360 10826\text{ x e26}$

$c_0 = +0.24931148216700089850794 08181\text{ x e26}$

c_{-n} can be found by the relation $c_{-n} = c_n$.

Array factor is given in Figure – 4.31.

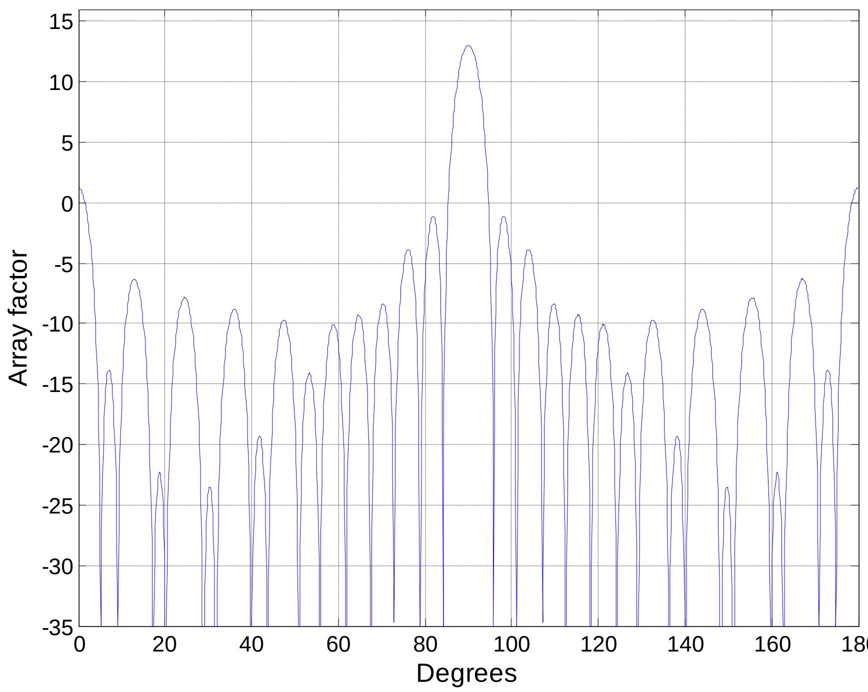

Figure - 4. 31: Array factor (dB) for 31-element superdirective array

4.2.16. 33-Element Array

For the array of 33 elements, directivity versus spacing when spacing is close to zero, smaller than 0.15λ, is obtained in Figure - 4.32.

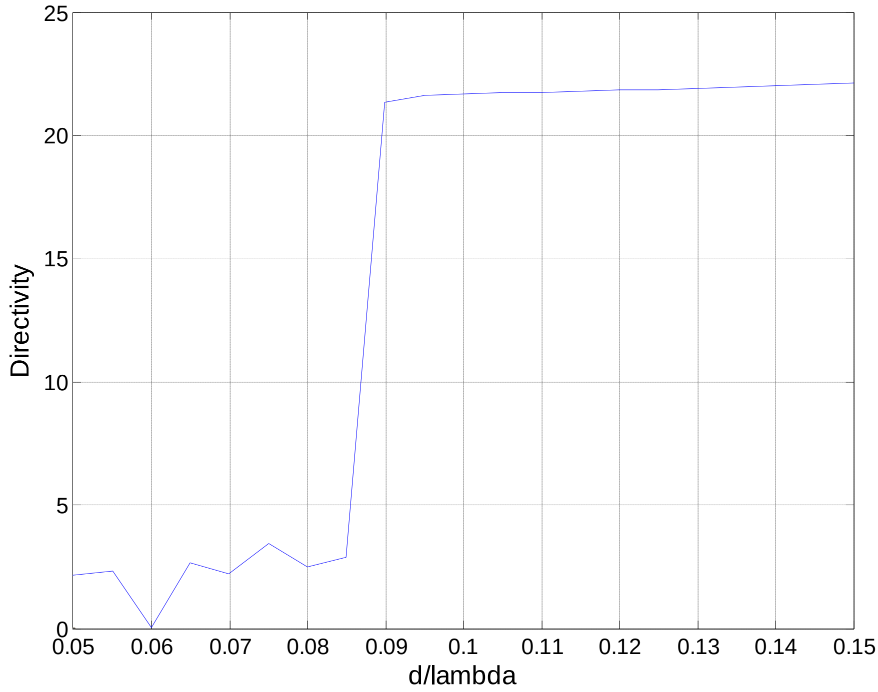

Figure - 4. 32: Directivity vs spacing for 33-element array

Limit directivity for the case of 33 elements is 21.32.

For spacing $d = 0.09\lambda$ and precision of 425, superdirectivity limit is provided.

When $d = 0.09\lambda$, total length of 33-element array is $L = 2.88\lambda$.

For that array, array coefficients are as below:

$$c_{16} = +0.10318807997188412039 \text{ x e18}$$

$$c_{15} = -0.30409670445804197095 6 \text{ x e19}$$

$$c_{14} = +0.43649792161397584656 20 \text{ xe20}$$

$$c_{13} = -0.40644953547924290669979 \text{ x e23}$$

$$c_{12} = +0.27588985320069799516 9708 \text{ x e22}$$

$$c_{11} = -0.14543340751474934754 97478 \text{ x e23}$$

$$c_{10} = +0.61937091144306316473 44954 \text{ x e23}$$

$$c_9 = -0.21888693747303965062589470 \text{ x e24}$$

$$c_8 = +0.65429058321424937081276228 \text{ x e24}$$

$$c_7 = -0.16777845381888582988 8043283 \text{ x e25}$$

$$c_6 = +0.37303288157972129843 2134221 \text{ x e25}$$

$$c_5 = -0.72500032437026771695 1069492 \text{ x e25}$$

$$c_4 = +0.12393910821623151029 86345195 \text{ x e26}$$

$$c_3 = -0.18723735146937614344 22359833 \text{ x e26}$$

$$c_2 = +0.25082828701709020404 90050152 \text{ x e26}$$

$$c_1 = -0.29865794784084652321 67913473 \text{ x e26}$$

$$c_0 = +0.31650117633281304174 41832158 \text{ x e26}$$

c_{-n} can be found by the relation $c_{-n} = c_n$.

The array factor is obtained as in Figure – 4.33.

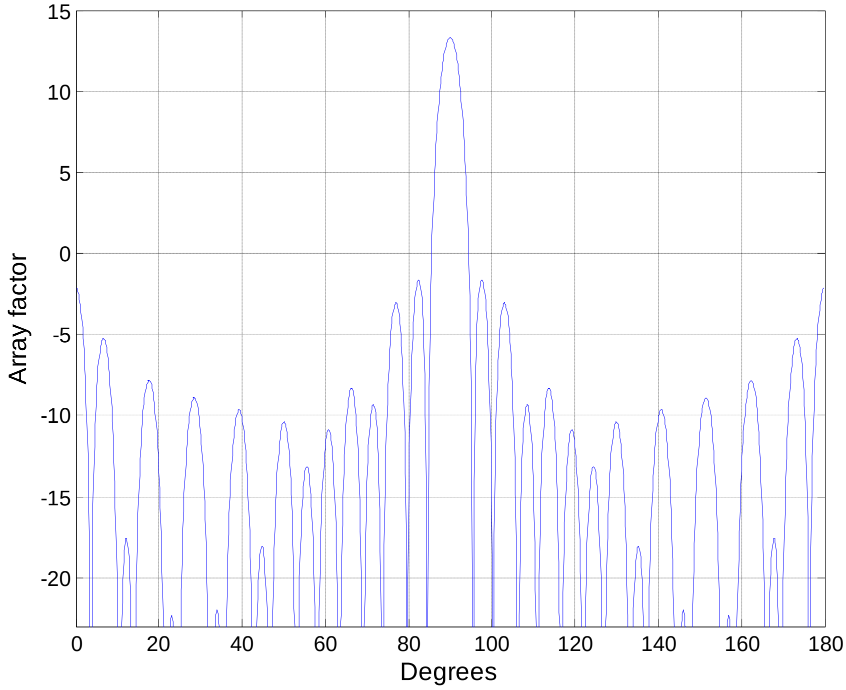

Figure - 4. 33: Array factor (dB) for 33-element superdirective array

4.2.17. 35-Element Array

For the array of 35 elements, directivity versus spacing when spacing is close to zero, smaller than 0.15λ, is given in Figure – 4.35.

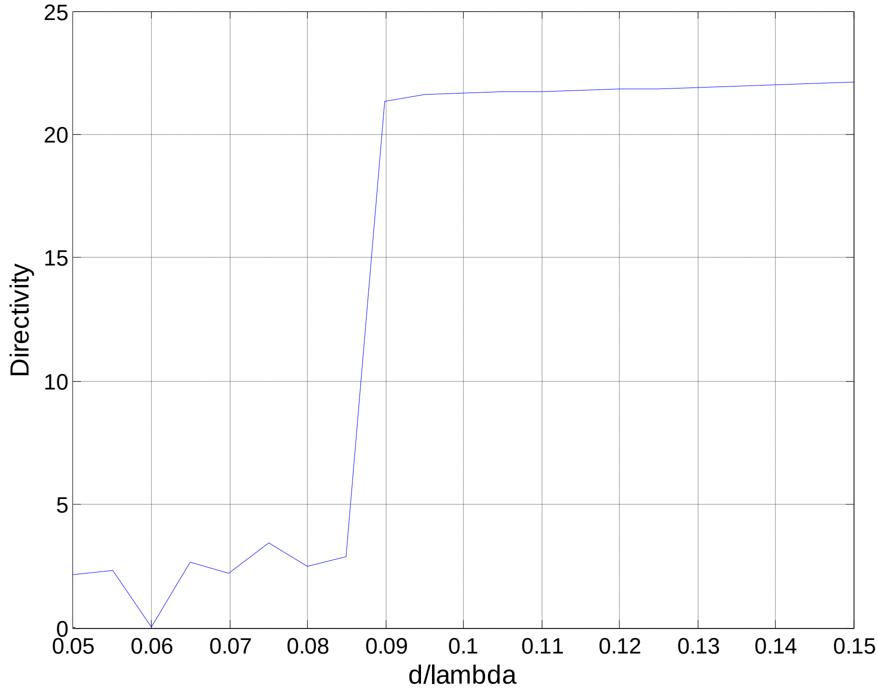

Figure - 4. 34: Directivity vs spacing for 35-element array

Lmit directivity for the case of 35 elements is found as 22.6 by Equation 2.25.

For spacing $d = 0.102\lambda$ and precision of 425, superdirectivity limit is provided.

When $d = 0.102\lambda$, total length of 35-element array is $L = 3.47$.

For that array, array coefficients are as below:

$c_{17} = -0.2262318754473156605 \text{ x e17}$

$c_{16} = +0.69173683720993931249 \text{ x e18}$

$c_{15} = -0.10334324054521653348 09 \text{ x e20}$

$c_{14} = +0.1004814274542343097 0259 \text{ x e21}$

$c_{13} = -0.7145814547201521687 5306 \text{ x e21}$

$c_{12} = +0.3960322270525143953 63881 \text{ x e22}$

$c_{11} = -0.1779690897446296951 265762 \text{ x e23}$

$c_{10} = +0.6661781855648257557 120956 \text{ x e23}$

$c_9 = -0.2117627172438287839 6059978 \text{ x e24}$

$c_8 = +0.5798933971907947288 7652662 \text{ x e24}$

$c_7 = -0.1383004294896413175 87617428 \text{ x e25}$

$c_6 = +0.2896889441190558259 98581797 \text{ x e25}$

$c_5 = -0.5364282393257568964 23244802 \text{ x e25}$

$c_4 = +0.8825879287856234568 25894598 \text{ x e25}$

$c_3 = -0.1295217022603429661 037494457 \text{ x e26}$

$c_2 = +0.1700180252847886024 454294294 \text{ x e26}$

$c_1 = -0.2000131113045079244 8861177453 \text{ x e26}$

$c_0 = 0.2111181762921742049 136833338 \text{ x e26}$

c_{-n} can be found by the relation $c_{-n} = c_n$.

The array factor is given in Figure – 4.35.

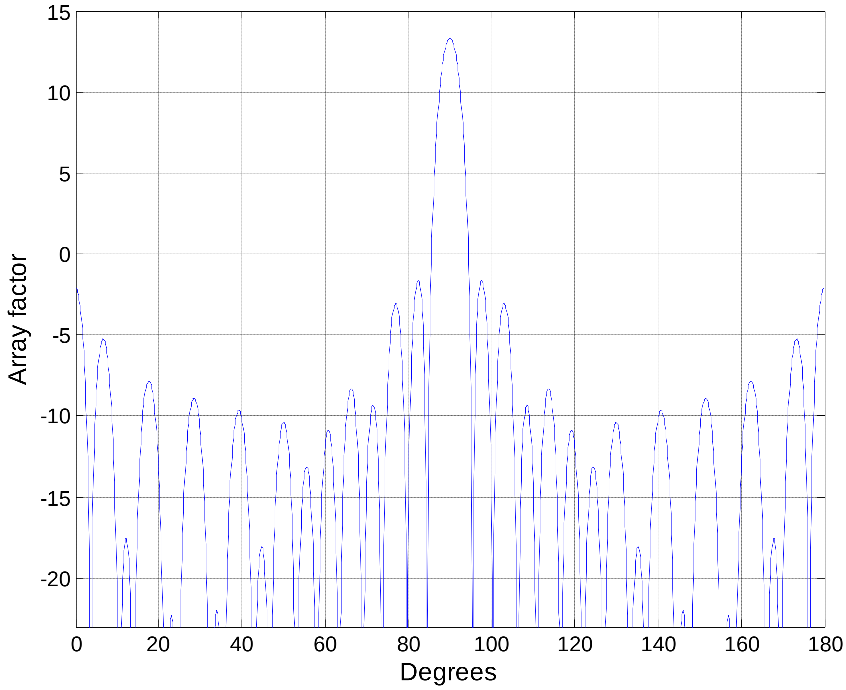

Figure - 4. 35: Array factor (dB) for 35-element superdirective array

4.2.18. 37-Element Array

For the array of 37 elements, directivity versus spacing when spacing is close to zero, smaller than 0.15λ, is obtained as in Figure 4.36.

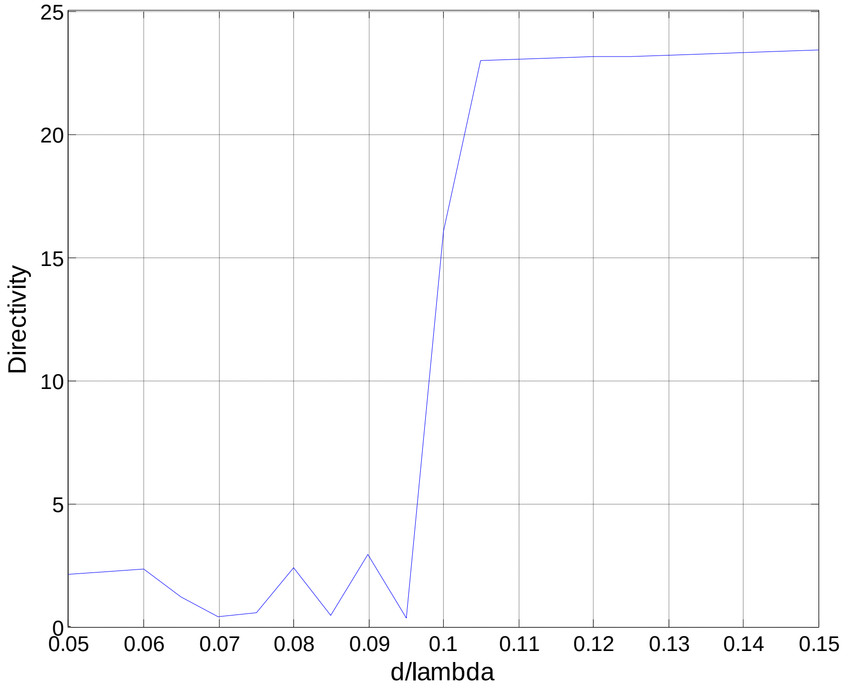

Figure - 4. 36: Directivity vs spacing for 37-element array

Limit directivity for the case of 37 elements is 23.87.

For spacing $d = 0.113\lambda$ and precision of 435, superdirectivity limit is provided.

When $d = 0.113\lambda$, total length of 37-element array is $L = 4.07\lambda$.

For that array, array coefficients are found as below:

$c_{18} = +0.570054232857297763 \text{ x e}16$

$c_{17} = -0.18007436177609530047 \text{ x e}18$

$c_{16} = +0.278734872988923294552 \text{ x e}19$

$c_{15} = -0.2816277050867093159526 \text{ x e}20$

$c_{14} = +0.20875717848020318959739 \text{ x e}21$

$c_{13} = -0.120971578794692185614909 \text{ x e}22$

$c_{12} = +0.570260215393659661850595 \text{ x e}22$

$c_{11} = -0.2246794969791489615441570 \text{ x e}23$

$c_{10} = +0.7543969720285256822887052 \text{ x e}23$

$c_9 = -0.21901884307964328676741346 \text{ x e}24$

$c_8 = +0.55593963753081247624594042 \text{ x e}24$

$c_7 = -0.124448140400931124771217085 \text{ x e}25$

$c_6 = +0.247348196537177983589435834 \text{ x e}25$

$c_5 = -0.438839542428322697906656550 \text{ x e}25$

$c_4 = +0.697895734506304015462324782 \text{ x e}25$

$c_3 = -0.998052437862158137985262 1197 \text{ x e}25$

$c_2 = +0.1286539578625341646973692509 \text{ x e}26$

$c_1 = -0.1497299182001519284632747020 \text{ x e}26$

$c_0 = 0.1574797858907219496484067100 \text{ x e}26$

c_{-n} can be found by the relation $c_{-n} = c_n$.

The array factor is obtained as in Figure – 4.37.

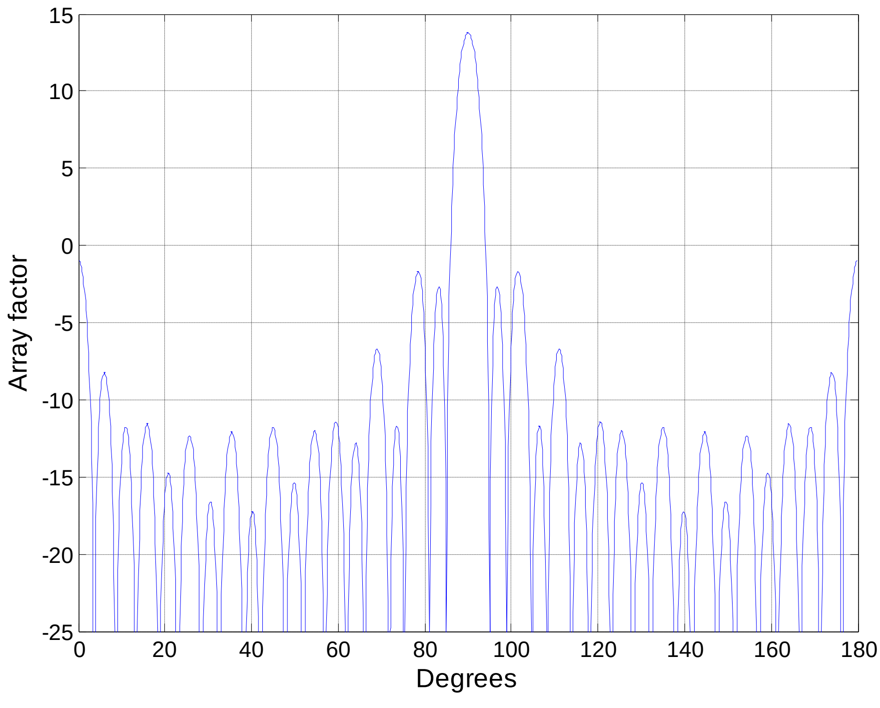

Figure - 4. 37: Array factor (dB) for 37-element superdirective array

4.2.19. Summary of the Results

By using the observations done, it can be concluded that increase in number of elements or decrease in spacing between the array elements demand more precision. Figure 4.38 summarizes precision required during numerical computations versus number of elements.

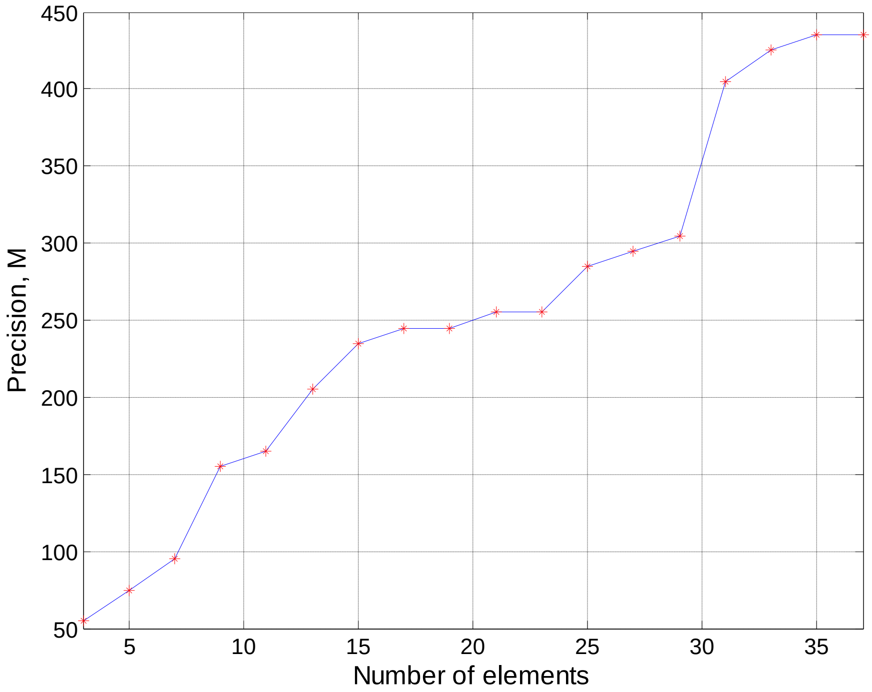

Figure - 4. 38: Precision required vs Number of elements

It is also observed that, while the spacing between the elements goes to zero, even the accuracy provided by Multiple Precision Toolbox does not allow for accurate calculations below a value. Therefore smallest spacing where the calculations can be done precisely is obtained for arrays composed of up to 37 elements. Figure-4.39 shows how that spacing (the smallest spacing that allows numerical computations) changes while number of elements increases.

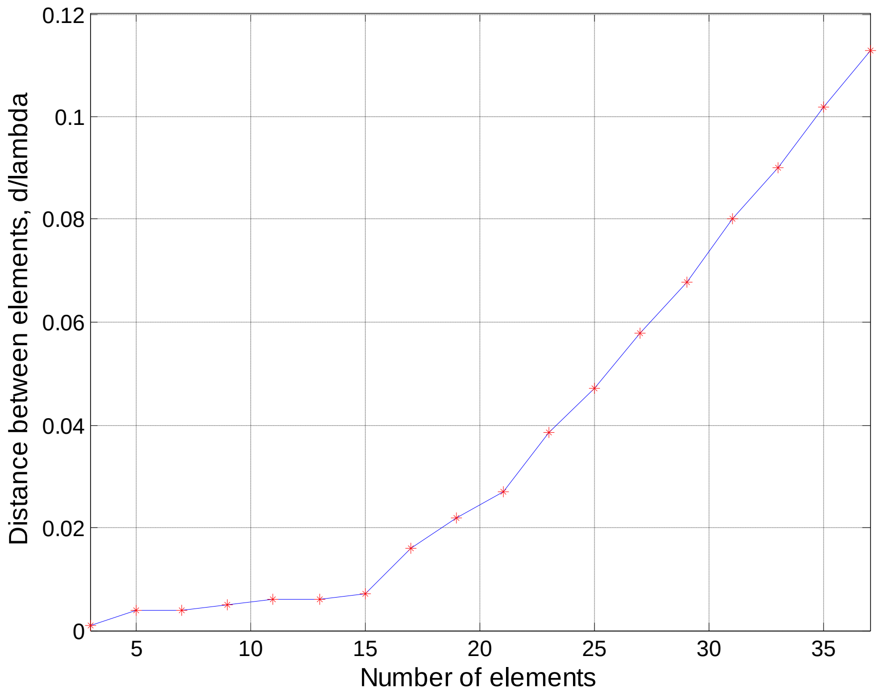

Figure - 4. 39: Minimum spacing for accurate calculations vs Number of elements

As seen in Figure – 4.39 when number of array elements increases the numerical limit distance gets larger. For large number of elements, precision does not allow to make superdirectivity matrix calculations at small spacings. So, spacing between the array elements has to be increased when the number of elements is increased in order to make the Matlab calculations accurately.

Moreover it is observed that, while accuracy required increases with increase in number of elements, corresponding superdirective element coefficients also increase.

Another observation is that coefficients are symmetric around the element in the center and there is 180° phase difference between two adjacent array elements.

Lastly, it is observed that array factor becomes more directive as the number of elements increase. Figure – 4.40 shows corresponding array factors for 3, 11, 17, 23, 29 and 37 elements.

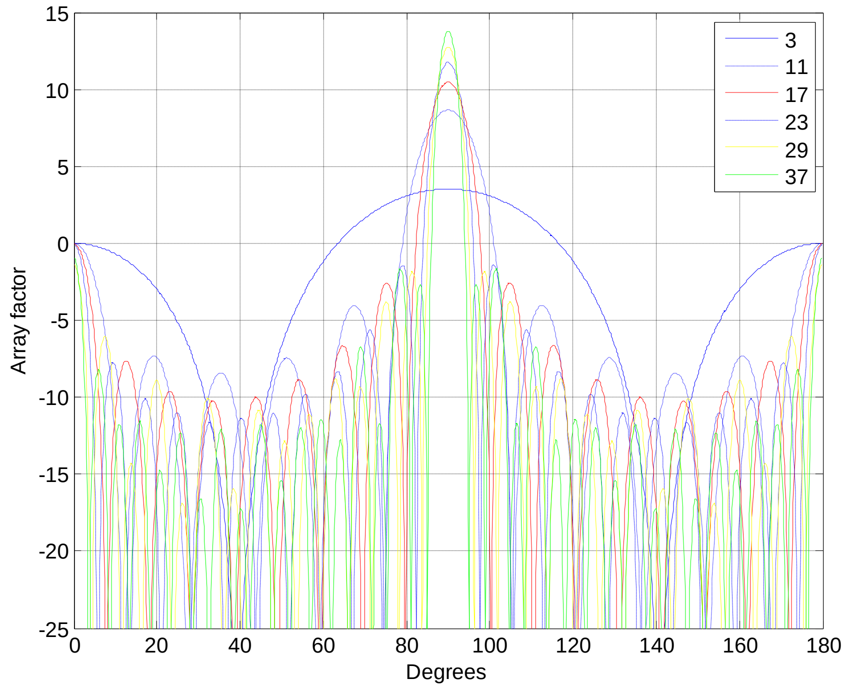

Figure - 4. 40: Array factors (dB) for 3,11,17,23,29 and 37 elements

CHAPTER 5

COMPARISON OF SUPERDIRECTIVE AND UNIFORM ARRAYS

The name "superdirective array" implies that the array has directivity greater than that of a uniform array for the same total length and same number of elements.

In this chapter, the directivity and antenna patterns for superdirective and uniform arrays are compared.

For superdirective arrays with different number of elements, spacing between the elements and precision values are found in Chapter 4. These spacing and precision values are used in this chapter while comparing superdirective and uniform arrays.

5.1. 3-Element Array

For the array of 3 elements, firstly directivity versus spacing will be observed for superdirective and uniform arrays. Precision used is 55 as found in 4.2.1.

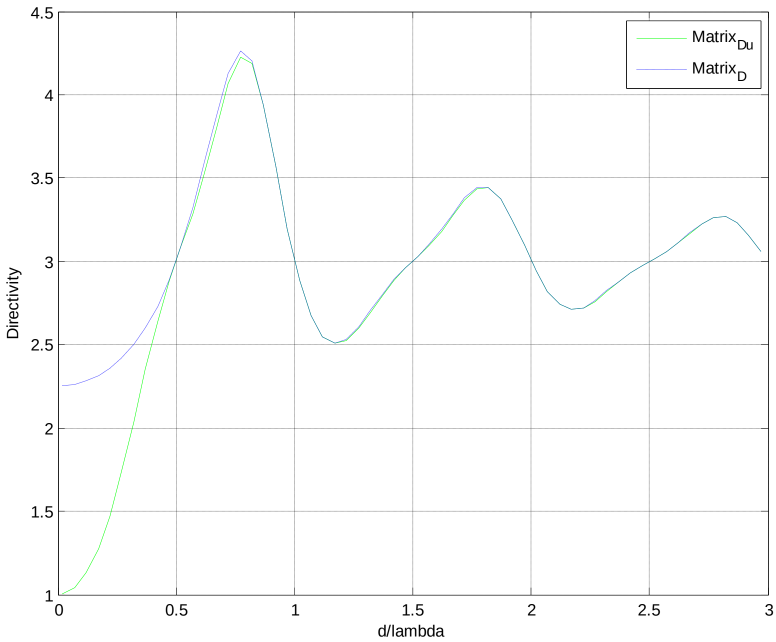

Figure - 5. 1: Directivity vs spacing for 3-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

Figure – 5.1 shows the directivity of a uniform linear array and the directivity of the stationary array as calculated by solving Equation 3.15 for a 3 element array, as a function of the array element spacing. The dashed curve represents the stationary solution and the solid curve is the uniform array.

As can be seen from the above figure, the two curves meet at d=0.5λ, i.e kd=π, and they are very close to each other when d>0.5λ, i.e. kd>π.

In d<0.5λ region, while the spacing is getting close to zero, superdirectivity is obtained. In this region, the advantage of superdirectivity over uniform array appears.

If the array factor for uniform and superdirective arrays of length 0.002λ (length is found in 4.2.1) and composed of 3 elements are compared, Figure – 5.2 is obtained.

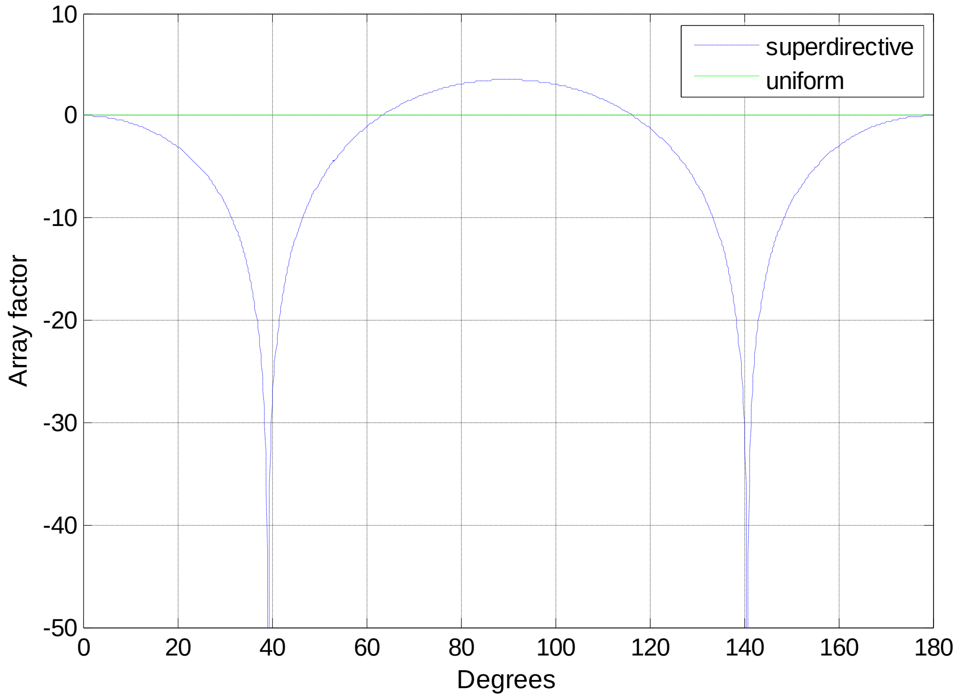

Figure - 5. 2: Array factors (dB) for 3-element array

When two array factors are compared, it can be seen that uniform array is almost isotropic for this case.

For that 3-element array of length 0.002λ, directivities are compared as:

$$Superdirectivity: 2.25$$
$$Uniform\ directivity: 1.0000088$$

5.2. 5-Element Array

For the array of 5 elements, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.3. Precision used is 75 as found in 4.2.2.

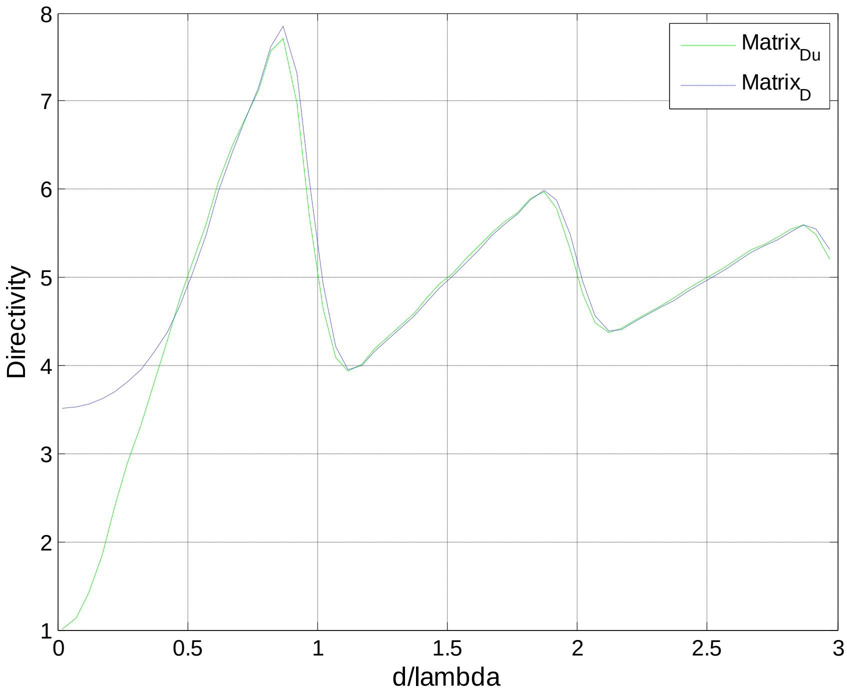

Figure - 5. 3: Directivity versus spacing for 5-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If we compare the dashed curves in 3-element and 5 –element directivity figures in d<0.5λ region, we can see that 5–element curve is more advantageous over the uniform directivity curve. Comparison of stationary directivity curves for different number of elements will be given in "Summary of the Results" part, 5.19.

If the array factor for uniform and superdirective arrays of length 0.016λ (length is found in 4.2.2) and composed of 5 elements are compared, Figure – 5.4 is obtained.

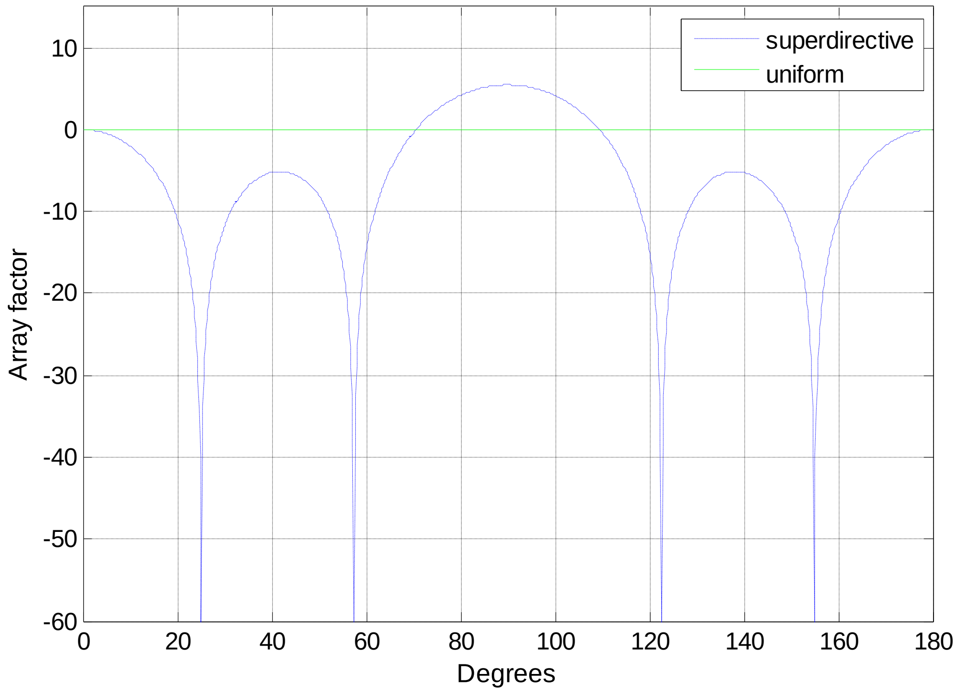

Figure - 5. 4: Array factors (dB) for 5-element array

For the 5-element array of length 0.016λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 3.5156$$
$$Uniform\ directivity: 1.0004$$

5.3. 7-Element Array

For the array of 7 elements, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.5. Precision used is 95 as found in 4.2.3.

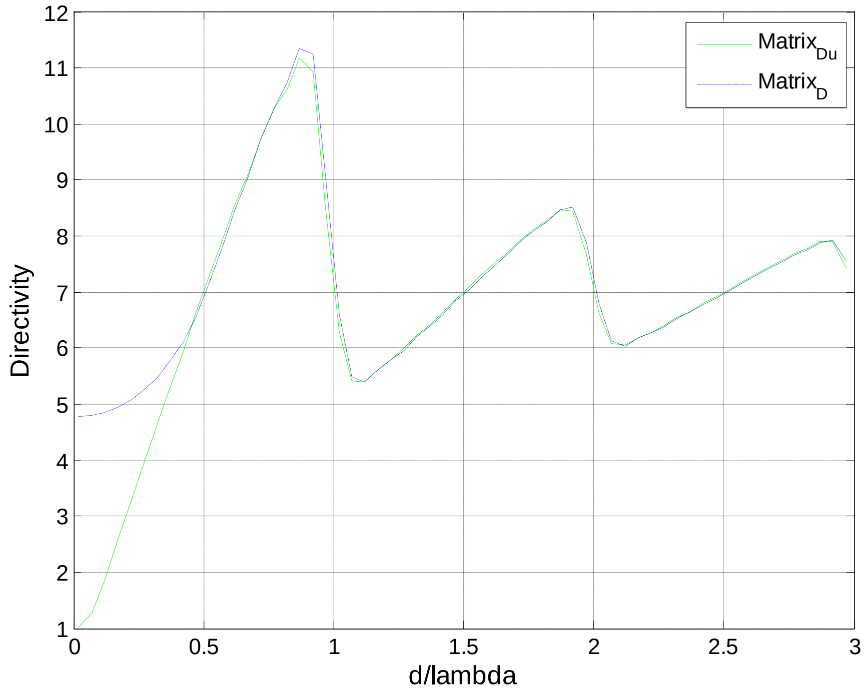

Figure - 5. 5: Directivities vs spacing for 7-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If the array factor for uniform and superdirective arrays of length 0.024λ (length is found in 4.2.3) and composed of 7 elements are compared, Figure – 5.6 is obtained.

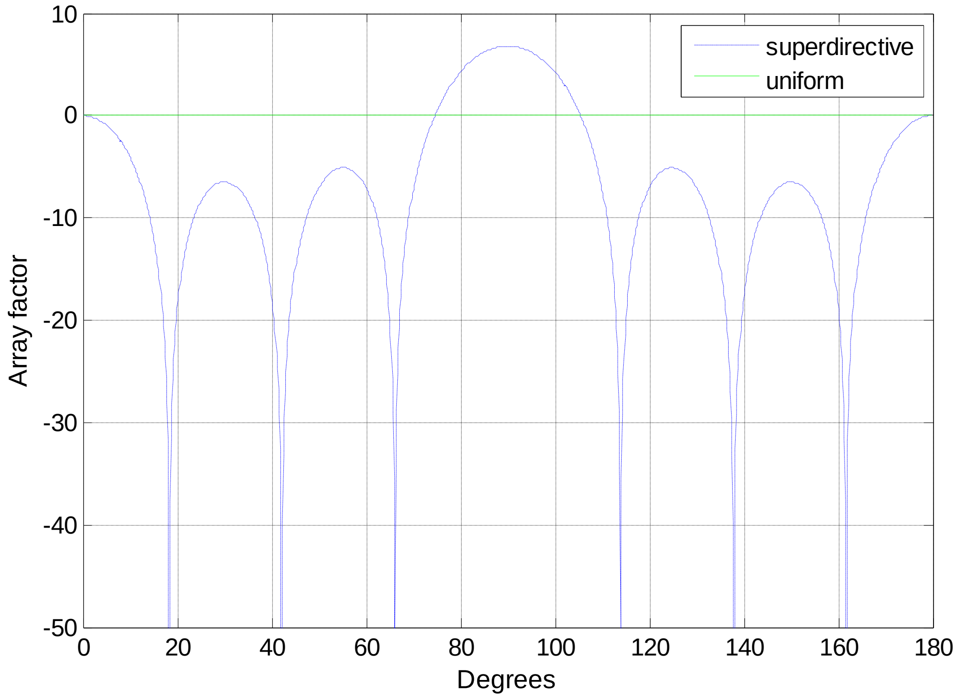

Figure - 5. 6: Array factors (dB) for 7-element array

For that 7-element array of length 0.024λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 4.78$$
$$Uniform\ directivity: 1.0008$$

As seen in the figure, uniform array is still not directive but almost isotropic for 7-element L=0.024λ length array when compared to the superdirective beam.

5.4. 9-Element Array

For the array of 9 elements, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.7. Precision used is 155 as found in 4.2.4.

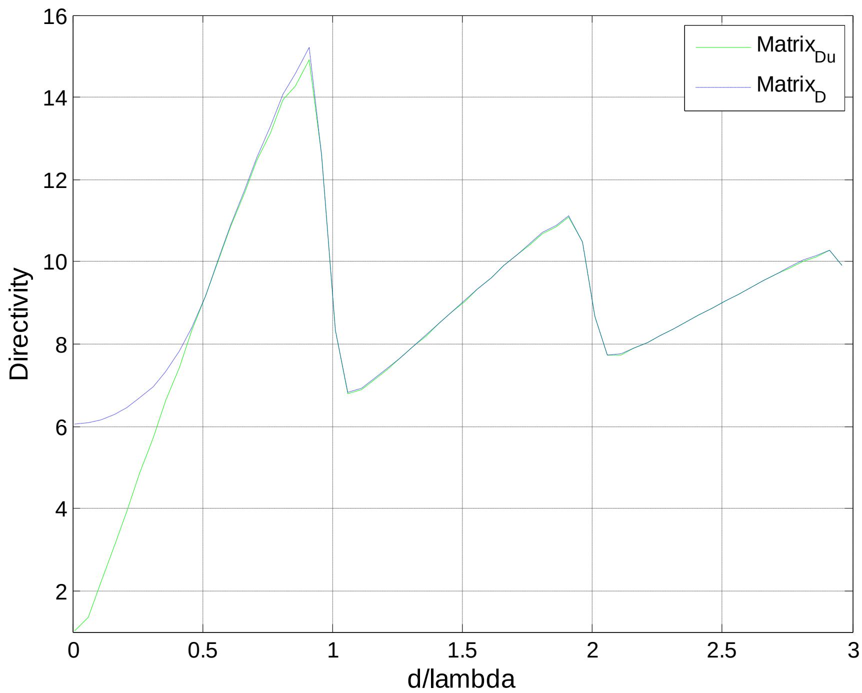

Figure - 5. 7: Directivities vs spacing for 9-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If the array factor for uniform and superdirective arrays of length 0.04λ (length is found in 4.2.4) are compared, Figure – 5.8 is obtained.

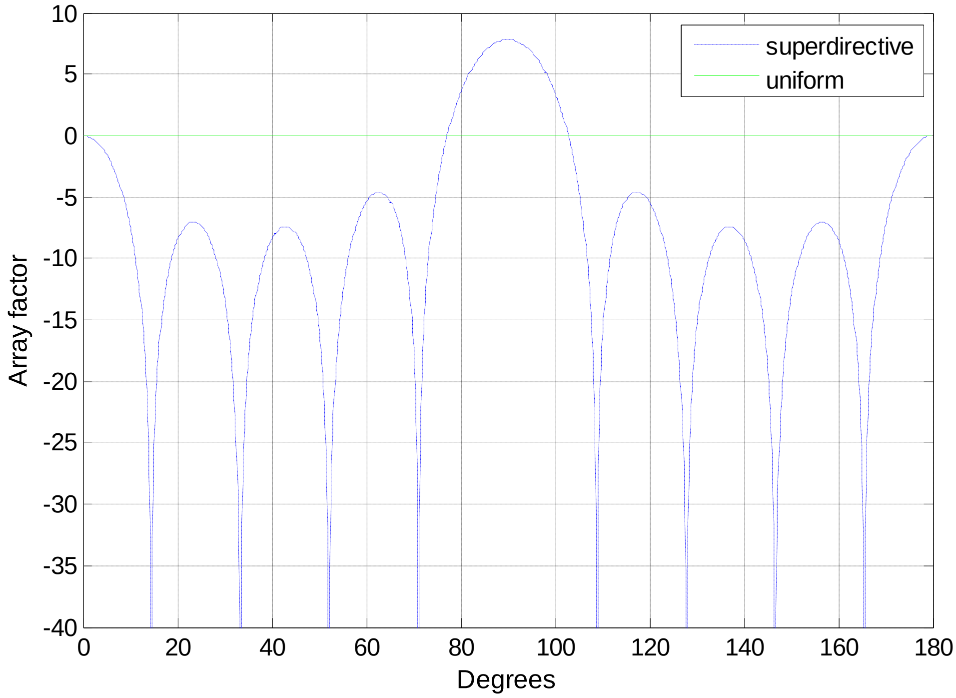

Figure - 5. 8: Array factors (dB) for 9-element array

As seen in the figure, uniform array is still not directive but almost isotropic for 9-element L=0.04λ length array when compared to the superdirective beam.

For the 9-element array of length 0.04λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 6.05$$
$$Uniform\ directivity: 1.0022$$

5.5. 11-Element Array

For number of 11 elements case, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.9. Precision used is 165 as found in 4.2.5.

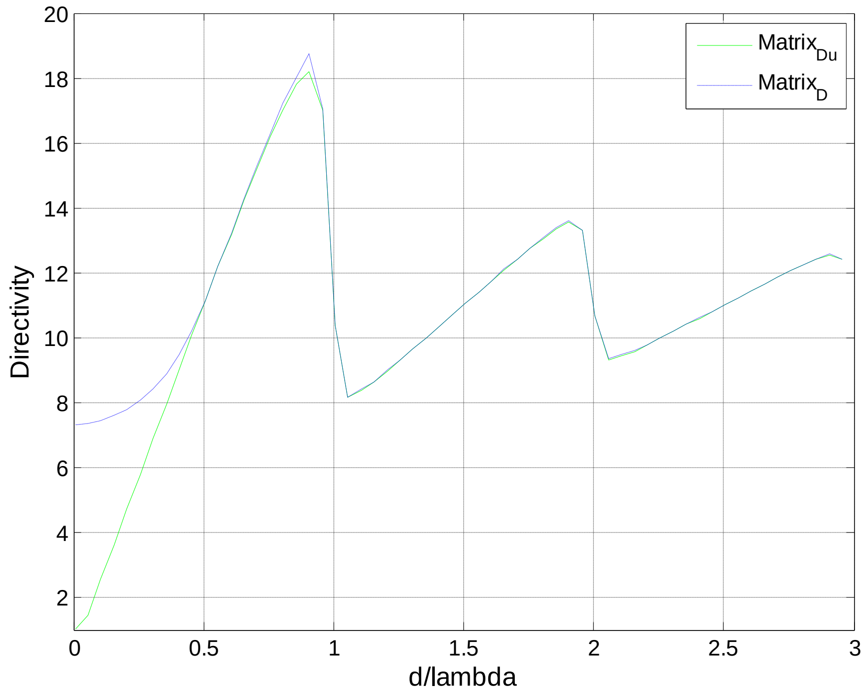

Figure - 5. 9: Directivites vs spacing for 11-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If the array factor for uniform and superdirective arrays of length 0.06λ (length is found in 4.2.5) are compared, Figure – 5.10 is obtained.

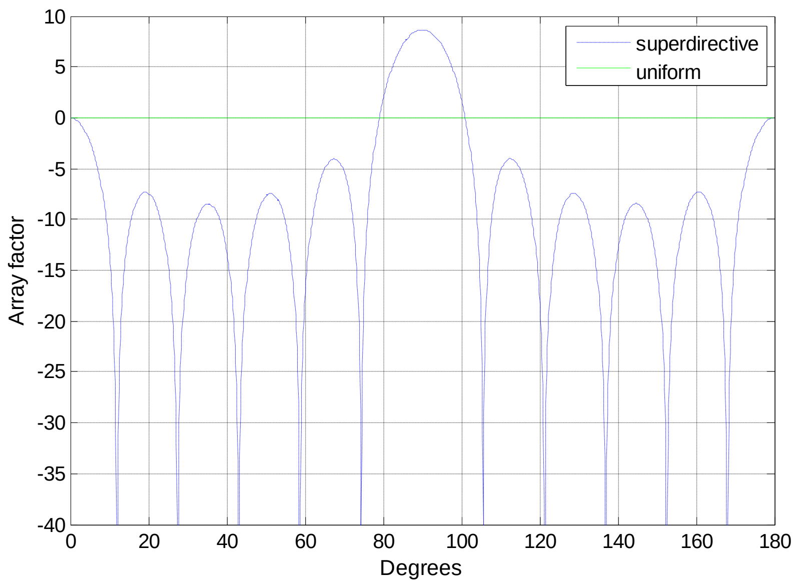

Figure - 5. 10: Array factors (dB) for 9-element array

For that 11-element array of length 0.06λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 7.32$$
$$Uniform\ directivity: 1.0047$$

5.6. 13-Element Array

For the array of 13 elements, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.11. Precision used is 205 as found in 4.2.6.

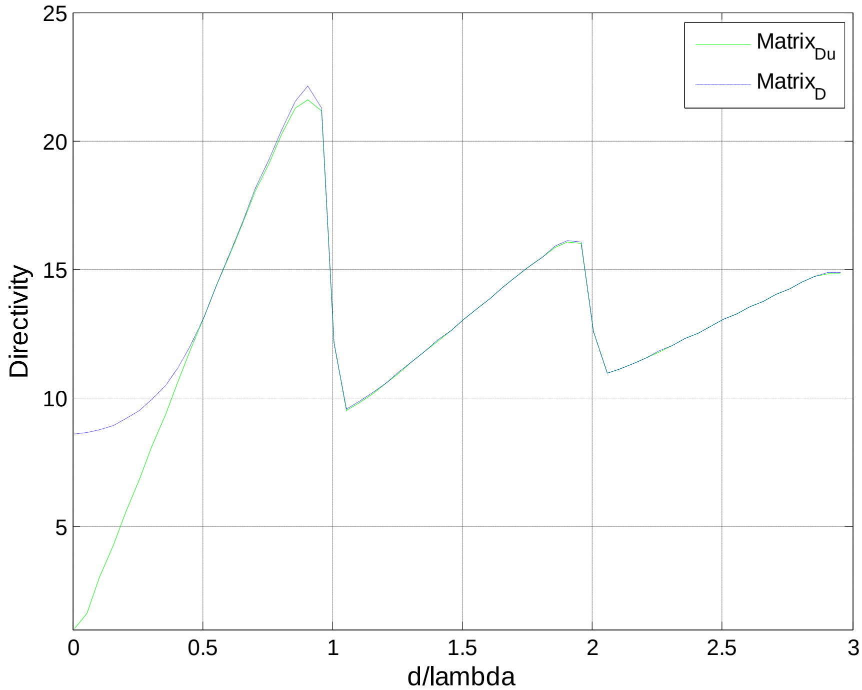

Figure - 5. 11: Directivities vs spacing for 13-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If the array factor for uniform and superdirective arrays of length 0.072λ (length is found in 4.2.6) are compared, Figure – 5.12 is obtained.

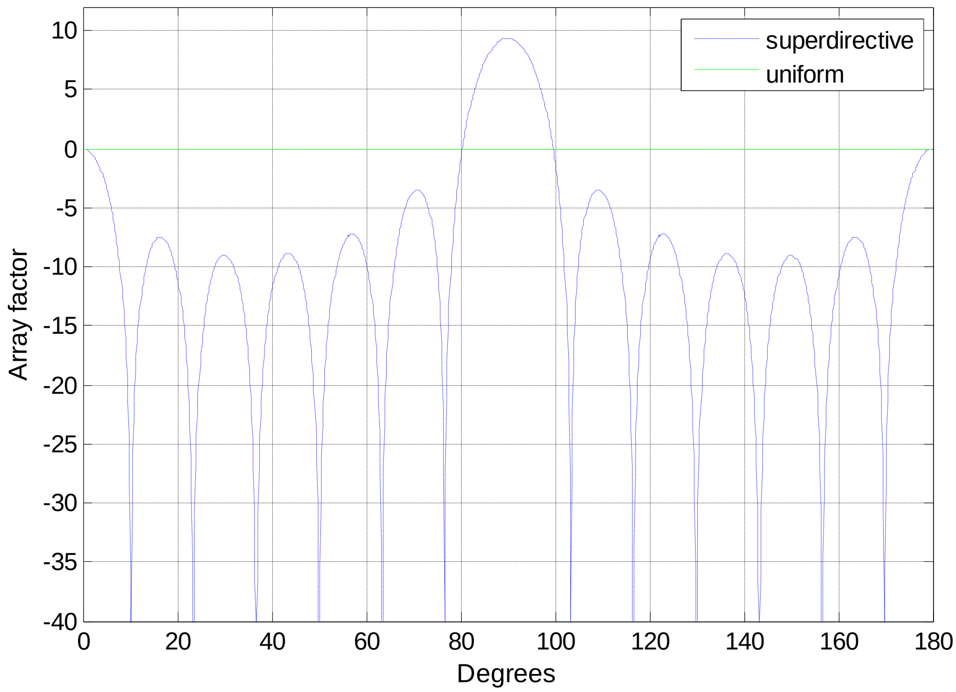

Figure - 5. 12: Array factors (dB) for 13-element array

As seen in the figure, uniform array is still not directive but almost isotropic for 13-element L=0.072λ length array when compared to the superdirective beam.

For that 13-element array of length 0.072λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 8.59$$

$$Uniform\ directivity: 1.006$$

5.7. 15-Element Array

For the array of 15 elements, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.13. Precision used is 235 as found in 4.2.7.

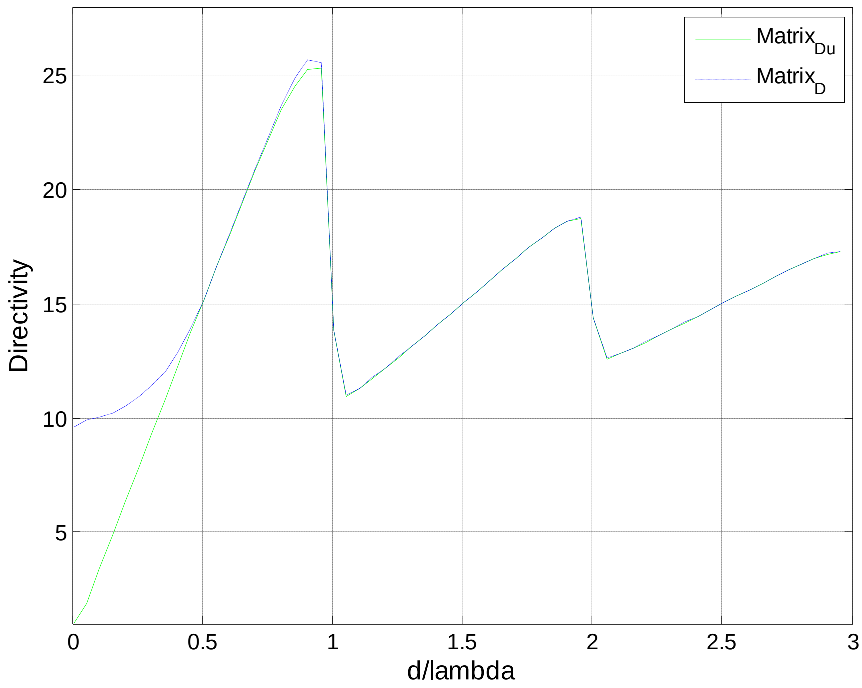

Figure - 5. 13: Directivities vs spacing for 15-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If the array factor for uniform and superdirective arrays of length 0.098λ (length is found in 4.2.7) are compared, Figure – 5.14 is obtained.

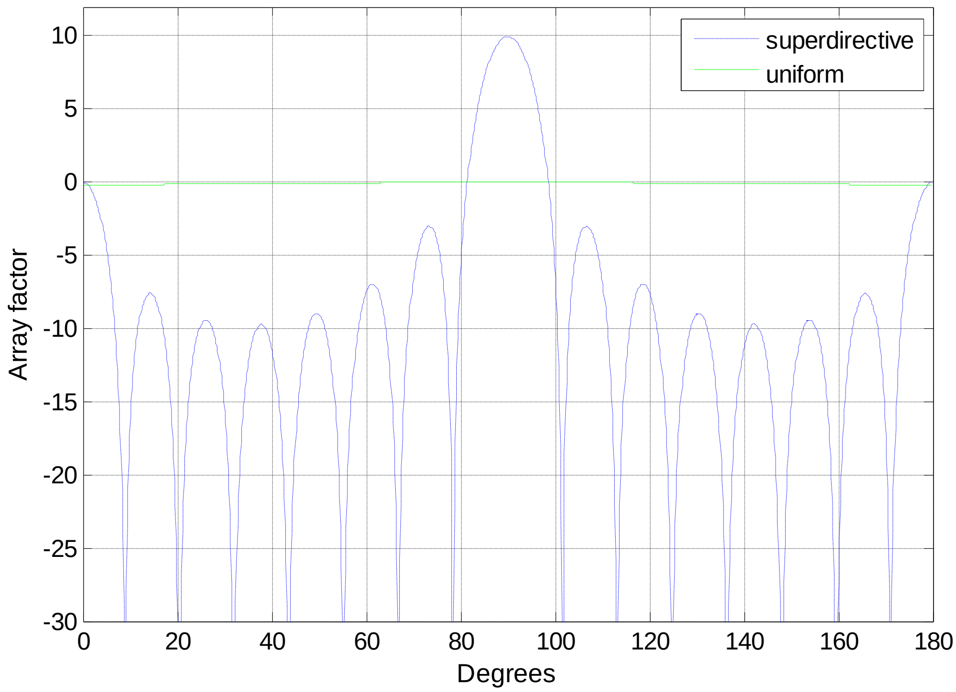

Figure - 5. 14: Array factors (dB) for 15-element array

As seen in the figure, uniform array is still not directive but almost isotropic for 15-element L=0.098λ length array when compared to the superdirective beam.

For the 15-element array of length 0.098λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 9.87$$

$$Uniform\ directivity: 1.012$$

5.8. 17-Element Array

For the array of 17 elements, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.15. Precision used is 245 as found in 4.2.8.

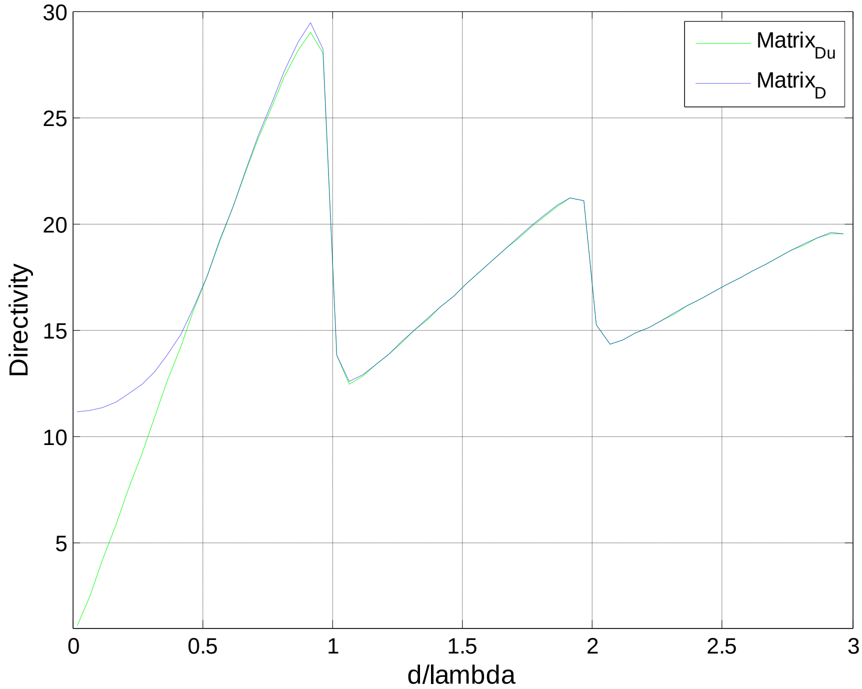

Figure - 5. 15: Directivities vs spacing for 17-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If the array factor for uniform and superdirective arrays of length 0.256 λ (length is found in 4.2.8) are compared, Figure – 5.16 is obtained.

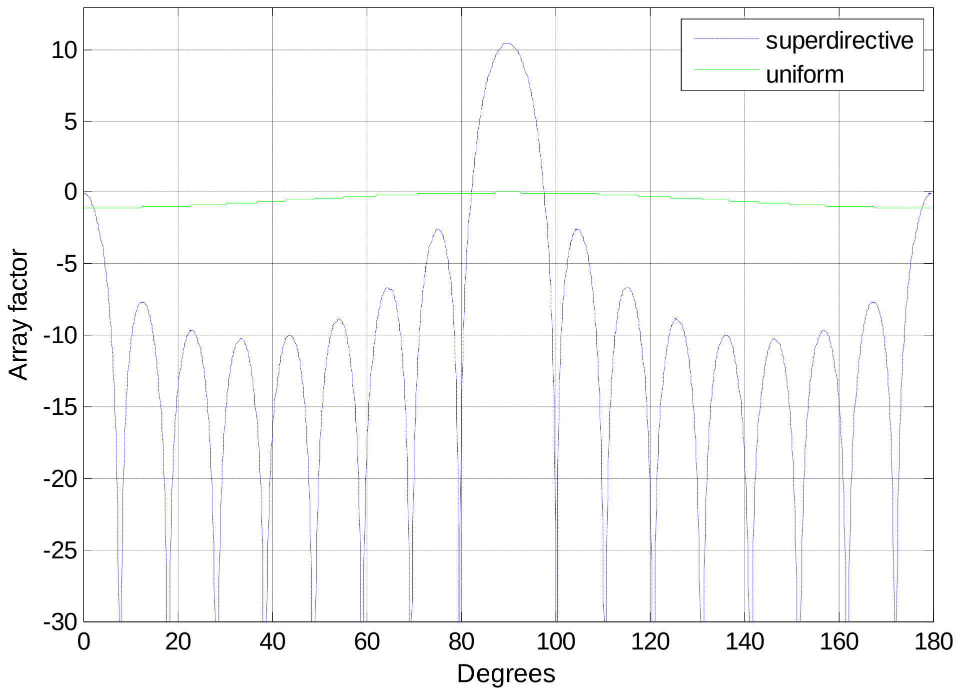

Figure - 5. 16: Array factors (dB) for 17-element array

For that 17-element array of length 0.256λ., superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 11.14$$
$$Uniform\ directivity: 1.082$$

5.9. 19-Element Array

For the array of 19 elements, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.17. Precision used is 245 as found in 4.2.9.

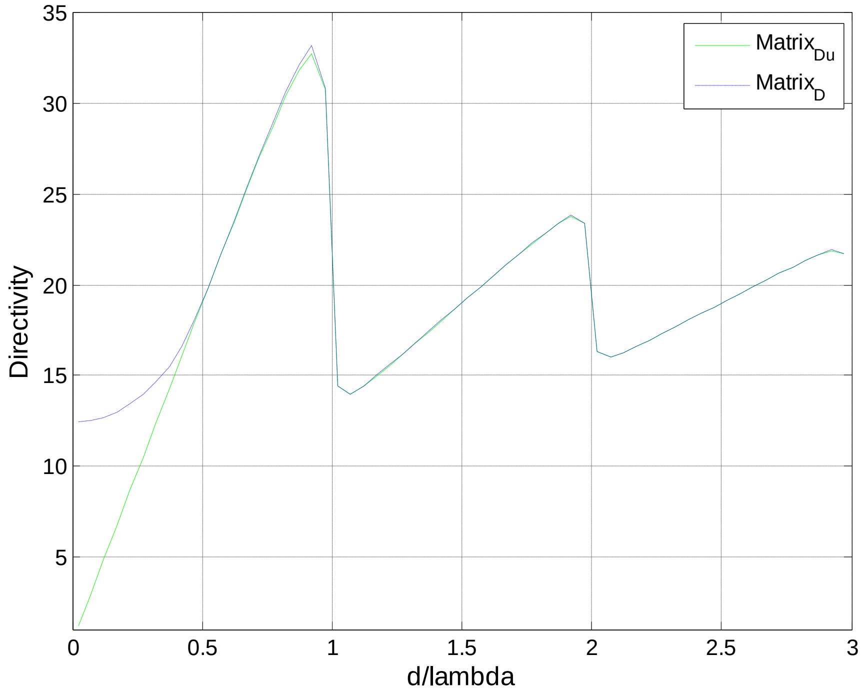

Figure - 5. 17: Directivites vs spacing for 19-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If the array factor for uniform and superdirective arrays of length 0.396 λ (length is found in 4.2.9) are compared, Figure – 5.18 is obtained.

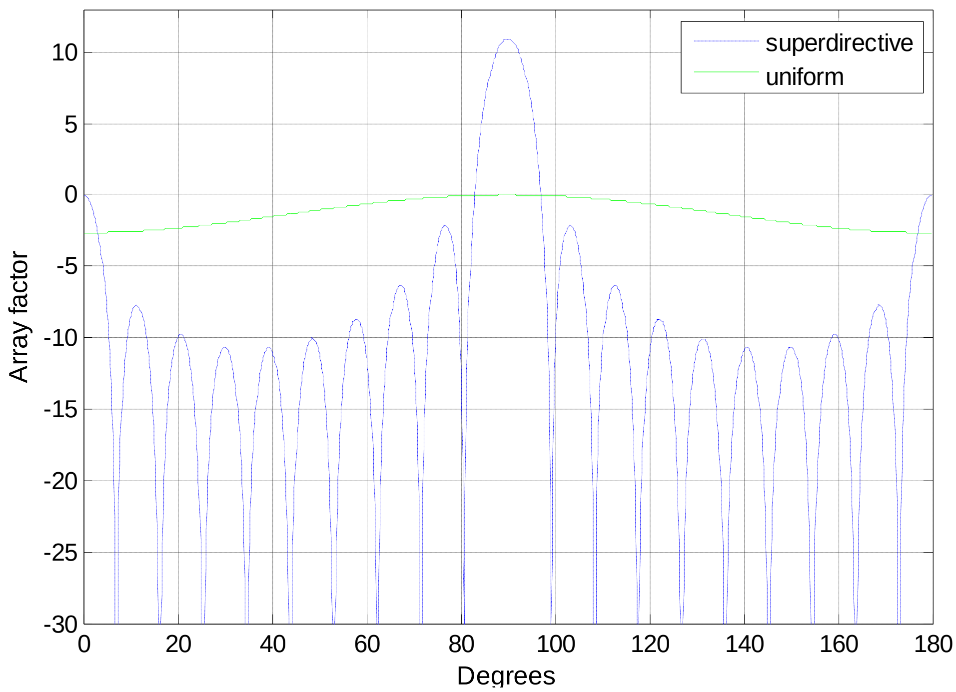

Figure - 5. 18: Array factors (dB) for 19-element array

For that 19-element array of length 0.396λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 12.42$$
$$Uniform\ directivity: 1.2$$

5.10. 21-Element Array

For the array of 21 elements, directivity versus spacing for superdirective and uniform arrays is given in Figure – 5.17. Precision used is 255 as found in 4.2.10.

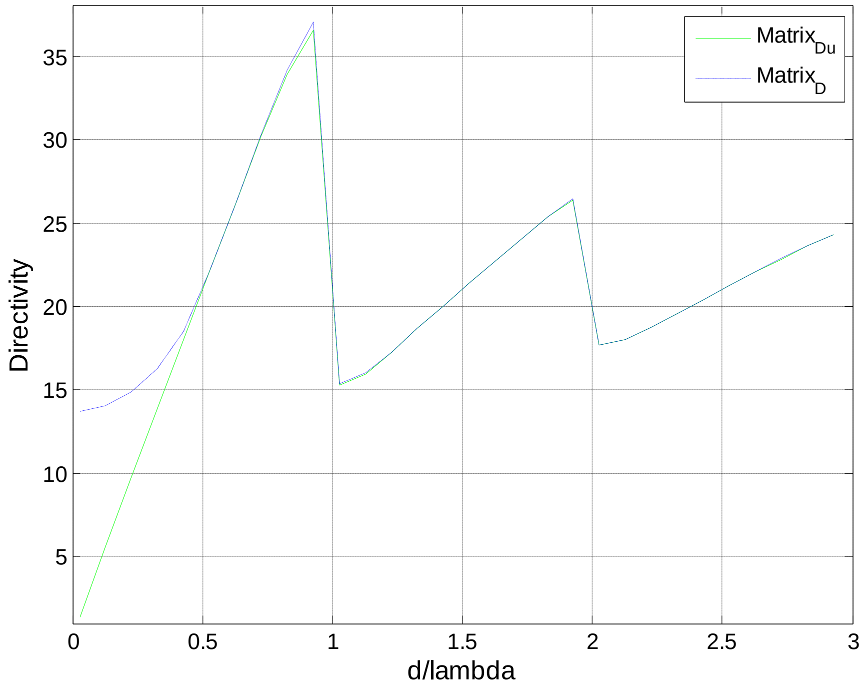

Figure - 5. 19: Directivites vs spacing for 21-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If the array factors for uniform and superdirective arrays of length 0.396 λ (length is found in 4.2.9) are compared, Figure – 5.20 is obtained.

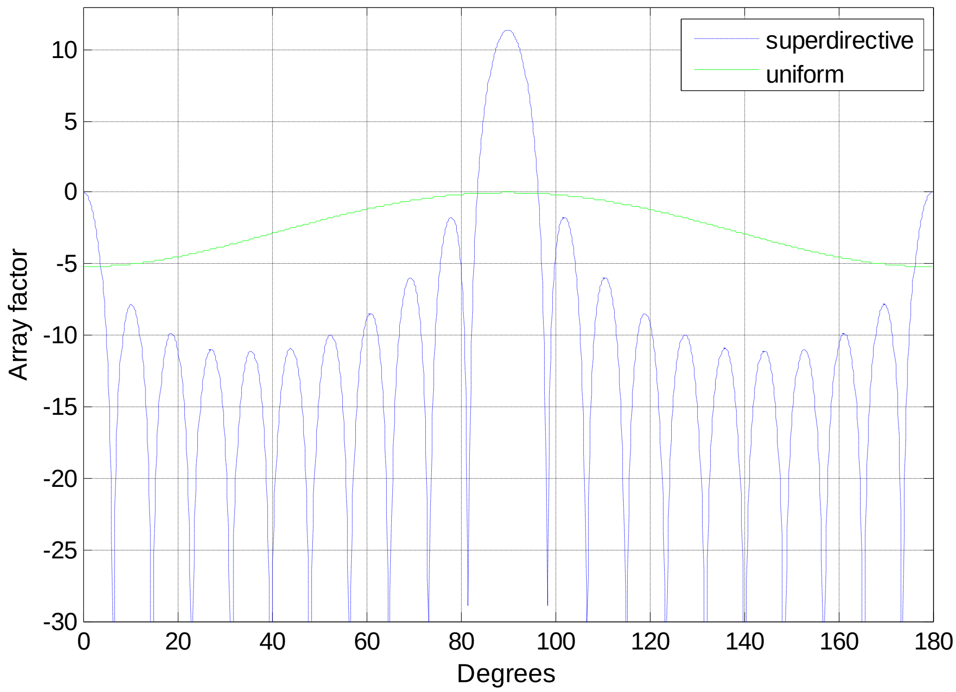

Figure - 5. 20: Array factors (dB) for 21-element array

For the 21-element array of length 0.54λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 13.69$$
$$Uniform\ directivity: 1.38$$

5.11. 23-Element Array

For the array of 23 elements, let us observe how directivity changes for stationary solution and uniform arrays when the spacing between the elements changes.

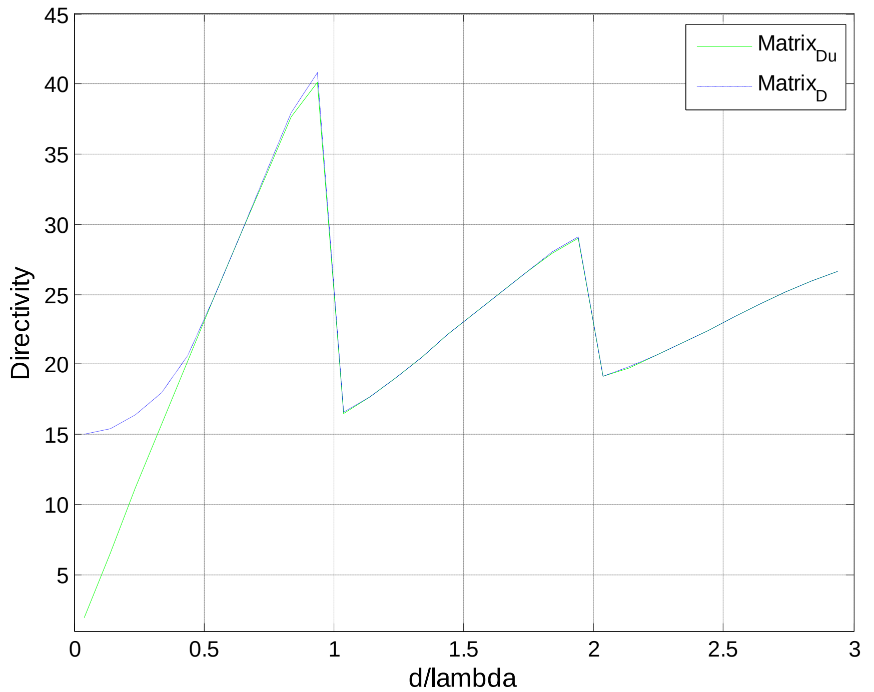

Figure - 5. 21: Directivites vs spacing for 23-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If we compare the array factor for uniform and superdirective arrays of the same length and composed of 23 elements:

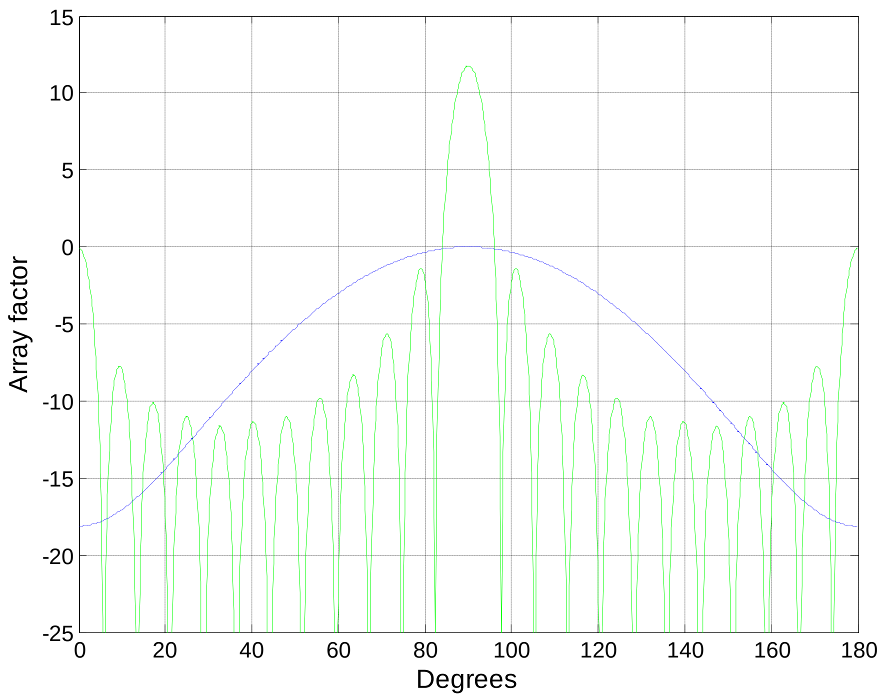

Figure - 5. 22: Array factors (dB) for 23-element array

For a 23-element array of length 0.85λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 14.96$$
$$Uniform\ directivity: 1.96$$

5.12. 25-Element Array

For number of 25 elements case, let us observe how directivity changes for stationary solution and uniform arrays when the spacing between the elements changes.

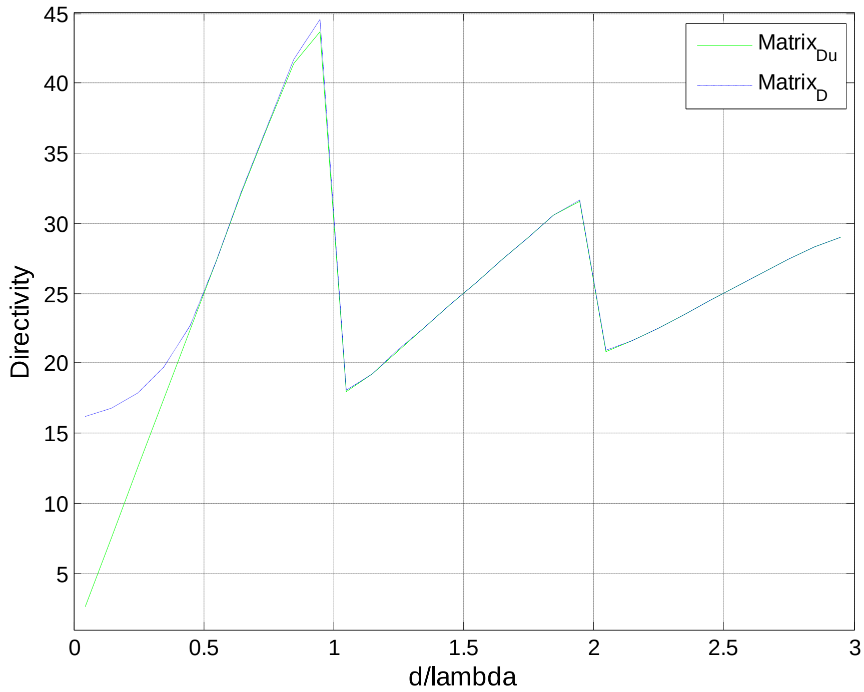

Figure - 5. 23: Directivities vs spacing for 25-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If we compare the array factor for uniform and superdirective arrays of the same length and composed of 25 elements, we obtain Figure 5.24.

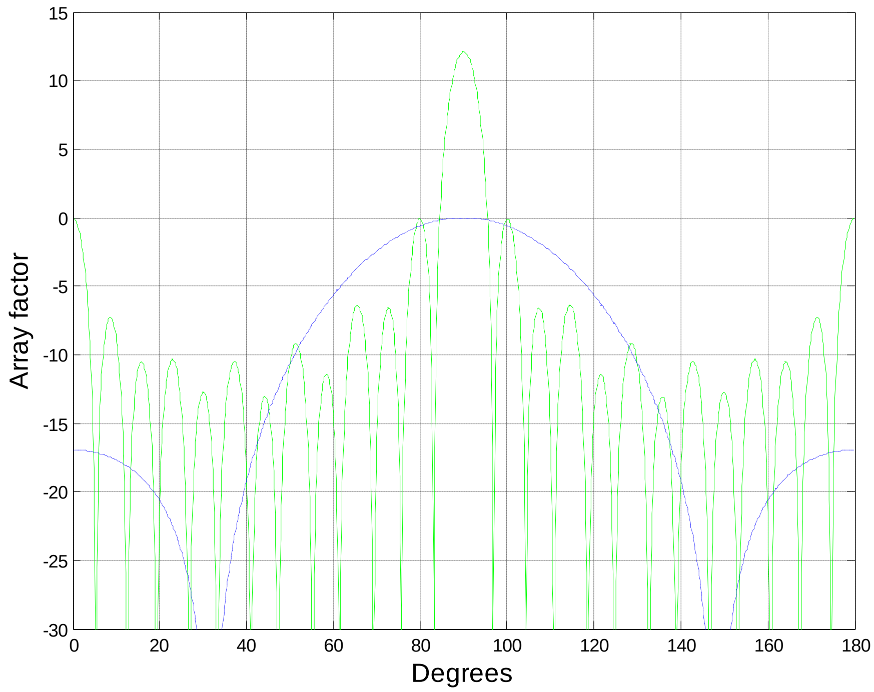

Figure - 5. 24: Array factors (dB) for 25-element array

(Dashed curve represents uniform array, solid line represents superdirective array)

For a 25-element array of length 1.128λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 16.23$$
$$Uniform\ directivity: 2.59$$

5.13. 27-Element Array

For number of 27 elements case, let us observe how directivity changes for stationary solution and uniform arrays when the spacing between the elements changes.

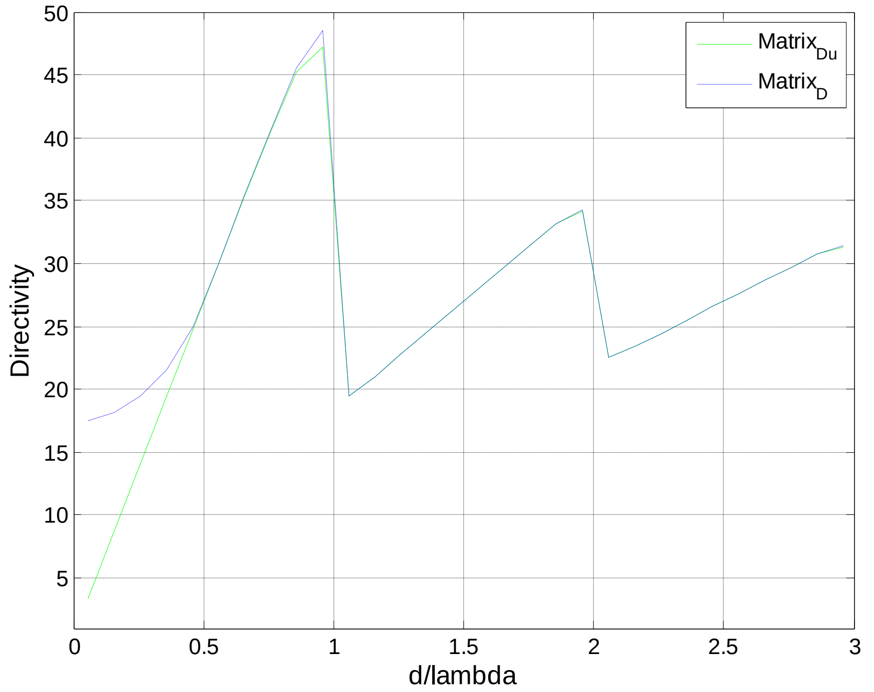

Figure - 5. 25: Directivites vs spacing for 27-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If we compare the array factor for uniform and superdirective arrays of the same length and composed of 27 elements:

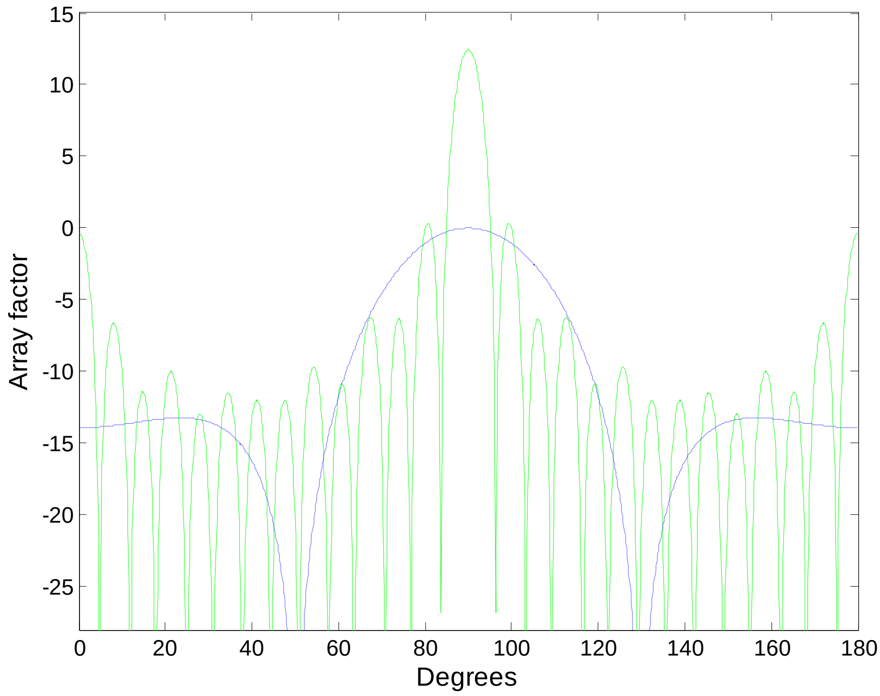

Figure - 5. 26: Array factors (dB) for 27-element array

For a 27-element array of length 1.5λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 17.50$$
$$Uniform\ directivity: 3.33$$

5.14. 29-Element Array

For number of 29 elements case, let us observe how directivity changes for stationary solution and uniform arrays when the spacing between the elements changes.

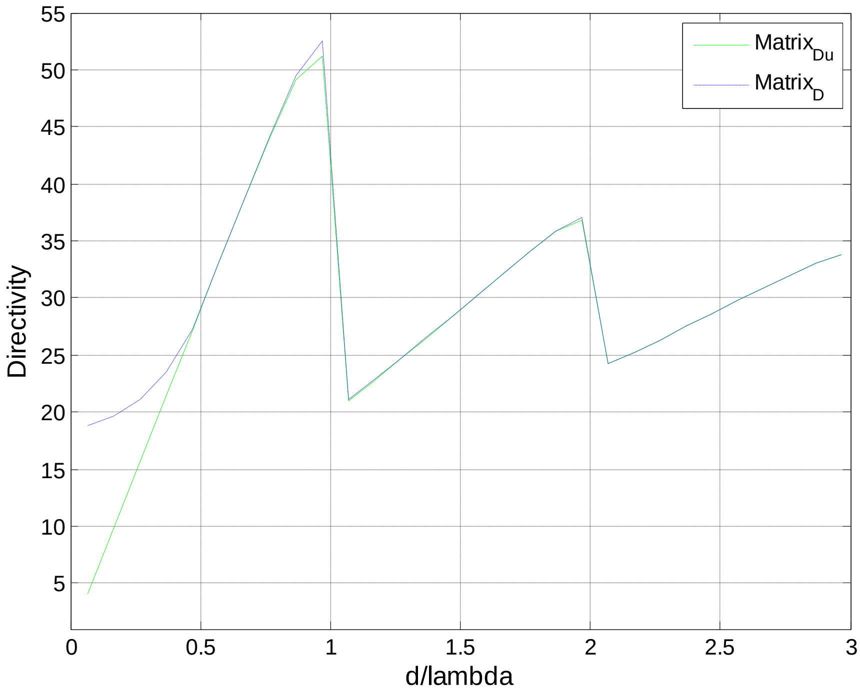

Figure - 5. 27: Directivites vs spacing for 29-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If we compare the array factor for uniform and superdirective arrays of the same length and composed of 29 elements:

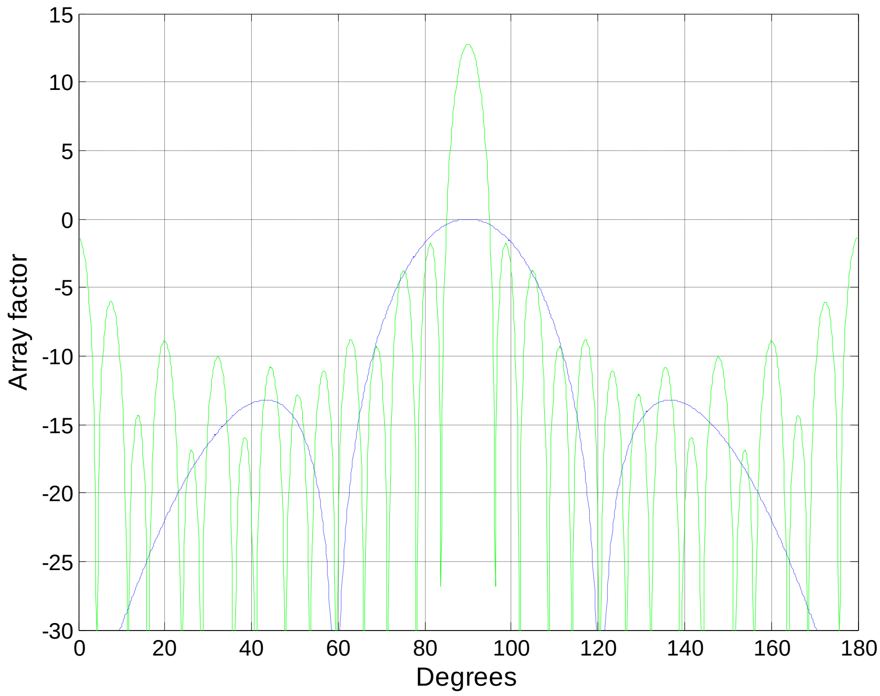

Figure - 5. 28: Array factors for 29-element array

(Dashed curve represents uniform array, solid line represents superdirective array)

For a 29-element array of length 1.9λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 18.78$$

$$Uniform\ directivity: 4.13$$

5.15. 31-Element Array

For number of 31 elements case, let us observe how directivity changes for stationary solution and uniform arrays when the spacing between the elements changes.

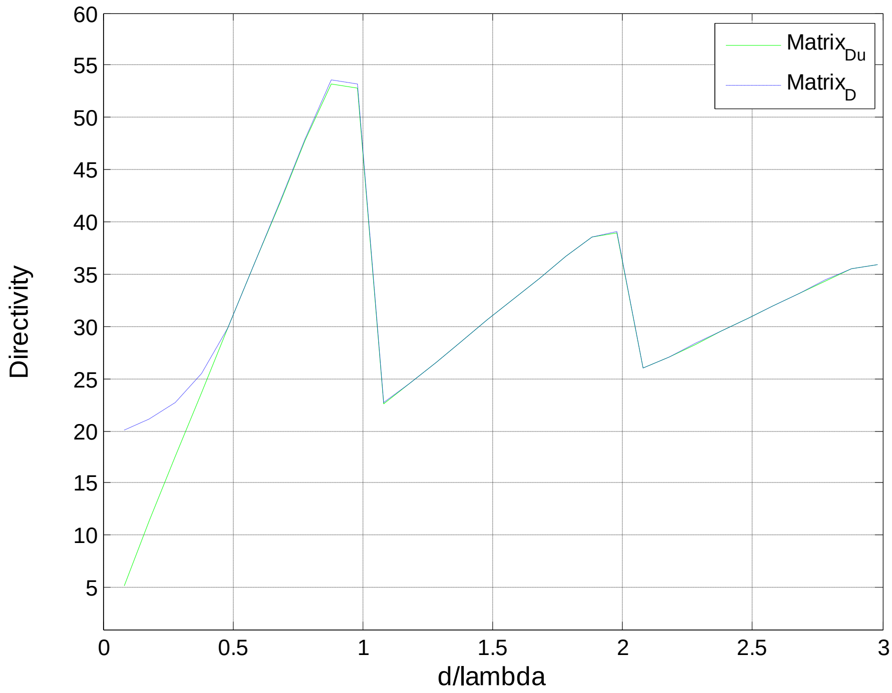

Figure - 5. 29: Directivites vs spacing for 31-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If we compare the array factor for uniform and superdirective arrays of the same length and composed of 31 elements:

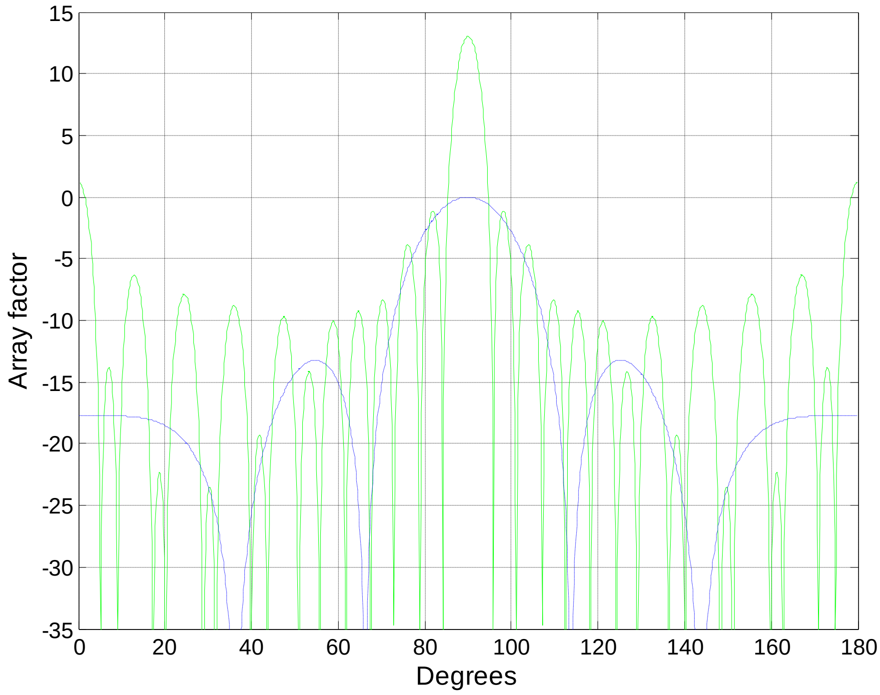

Figure - 5. 30: Array factors for 31-element array

For a 31-element array of length 2.4λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 20.05$$
$$Uniform\ directivity: 5.17$$

5.16. 33-Element Array

For number of 33 elements case, let us observe how directivity changes for stationary solution and uniform arrays when the spacing between the elements changes.

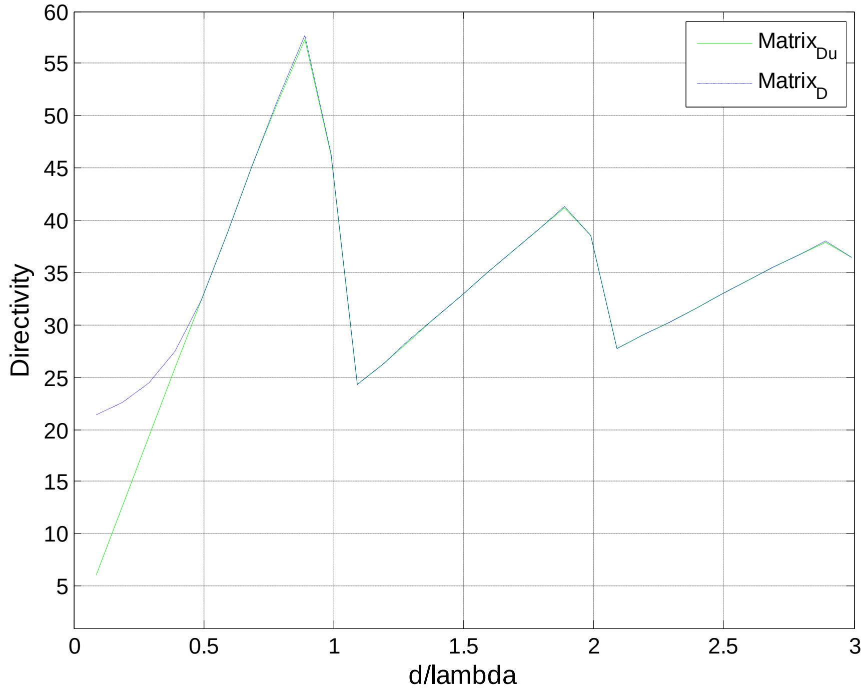

Figure - 5. 31: Directivites vs spacing for 33-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If we compare the array factor for uniform and superdirective arrays of the same length and composed of 33 elements:

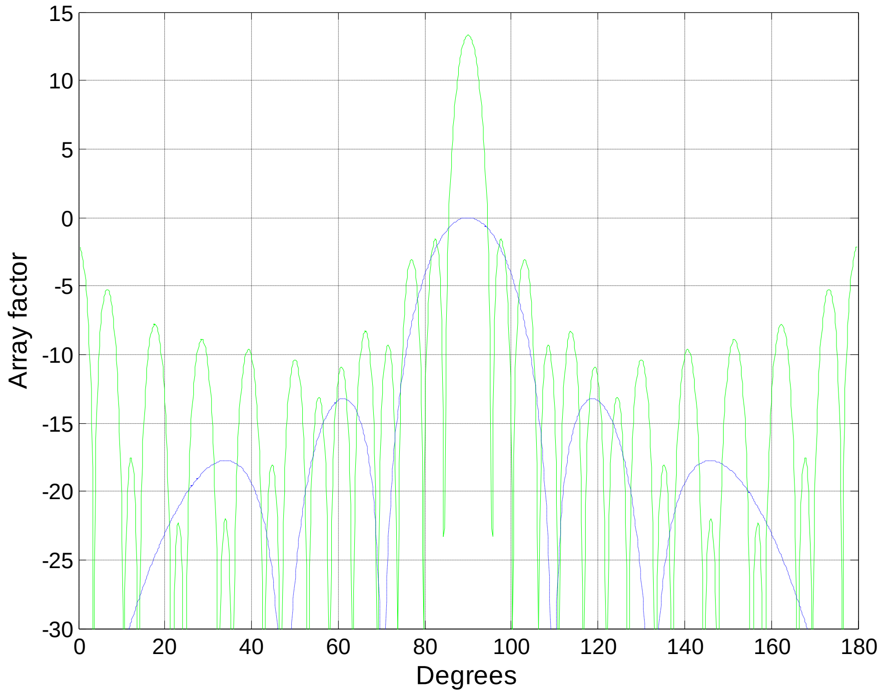

Figure - 5. 32: Array factors for 33-element array

(Dashed curve represents uniform array, solid line represents superdirective array)

For a 33-element array of length 2.4λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 21.32$$

$$Uniform\ directivity: 6.14$$

5.17. 35-Element Array

For number of 35 elements case, let us observe how directivity changes for stationary solution and uniform arrays when the spacing between the elements changes.

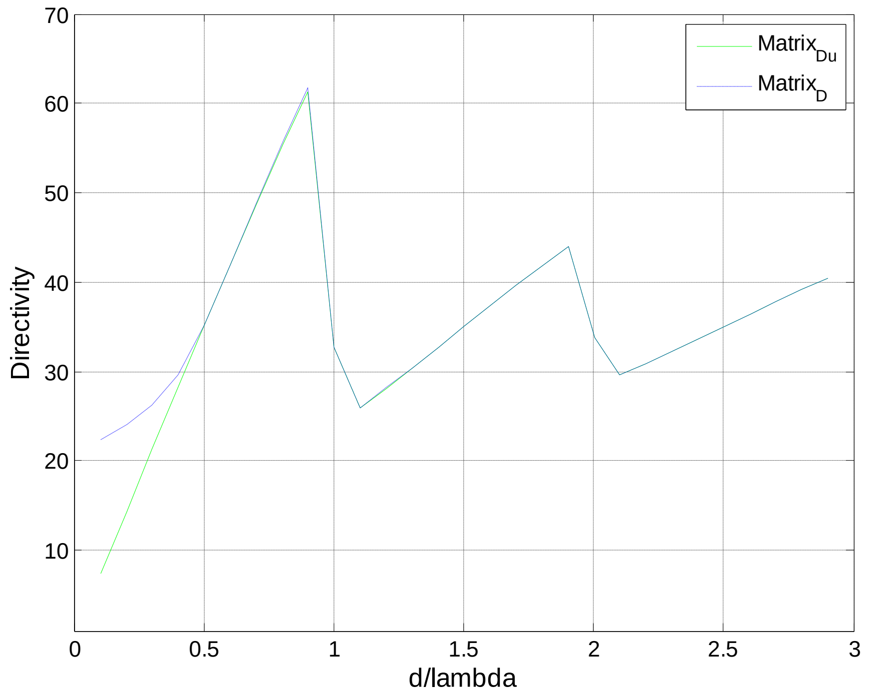

Figure - 5. 33: Directivities vs spacing for 35-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

If we compare the array factor for uniform and superdirective arrays of the same length and composed of 35 elements:

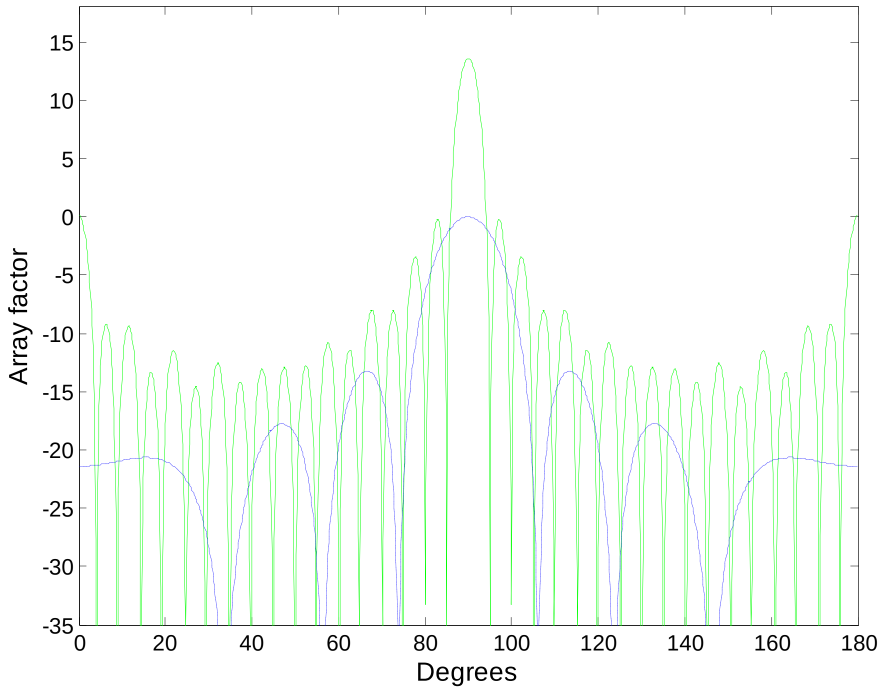

Figure - 5. 34: Array factors for 35-element array

For a 35-element array of length 3.47λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 22.6$$
$$Uniform\ directivity: 7.34$$

5.18. 37-Element Array

For number of 37 elements case, let us observe how directivity changes for stationary solution and uniform arrays when the spacing between the elements changes.

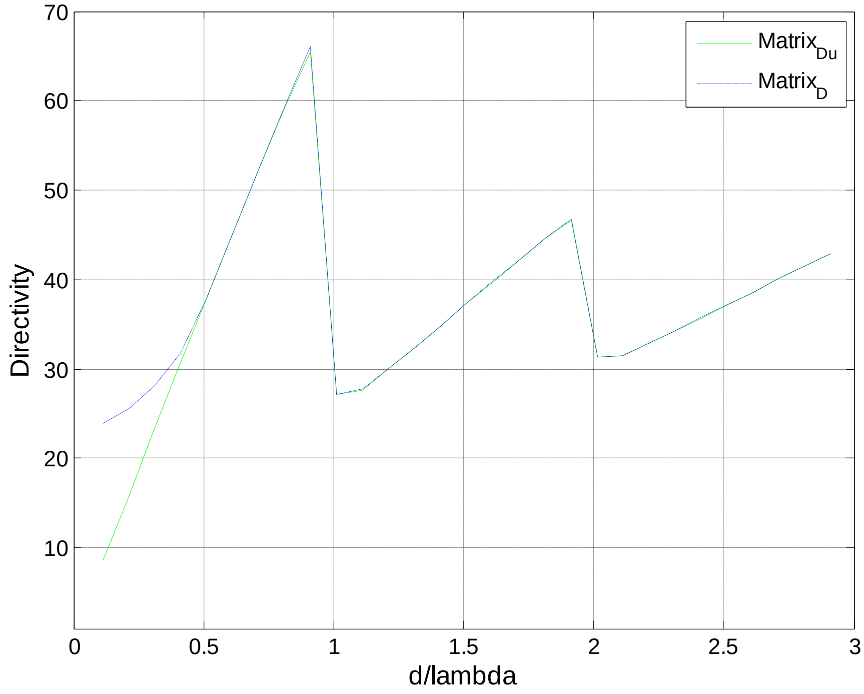

Figure - 5. 35: Directivities vs spacing for 37-element array

(Dashed curve represents stationary directivity, solid line represents uniform directivity)

We can see from the figure that stationary directivity is more advantageous over uniform directivity as spacing gets closer to zero when compared to the previous cases.

If we compare the array factor for uniform and superdirective arrays of the same length and composed of 37 elements:

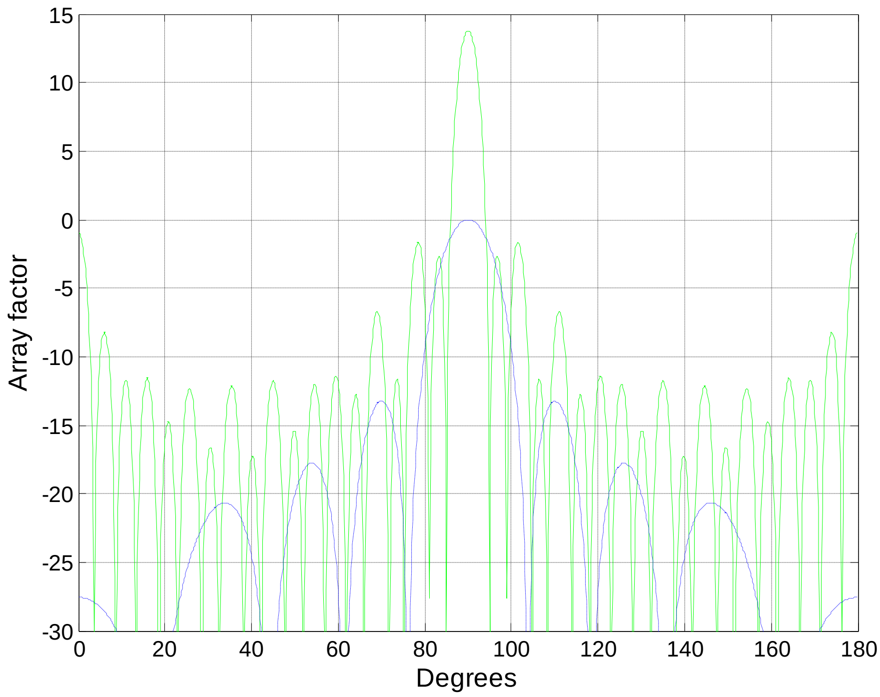

Figure - 5. 36: Array factors for 37-element array

(Dashed curve represents uniform array, solid line represents superdirective array)

For a 37-element array of length 4.07λ, superdirectivity and uniform directivities are compared as:

$$Superdirectivity: 23.87$$
$$Uniform\ directivity: 8.57$$

5.19. Summary of the Results

In "directivity versus distance between the elements" graphs, it is observed that directivity curves for stationary solution and uniform arrays meet at d=0.5λ, i.e kd=π, and they are very close to each other when d>0.5λ, i.e. kd>π.

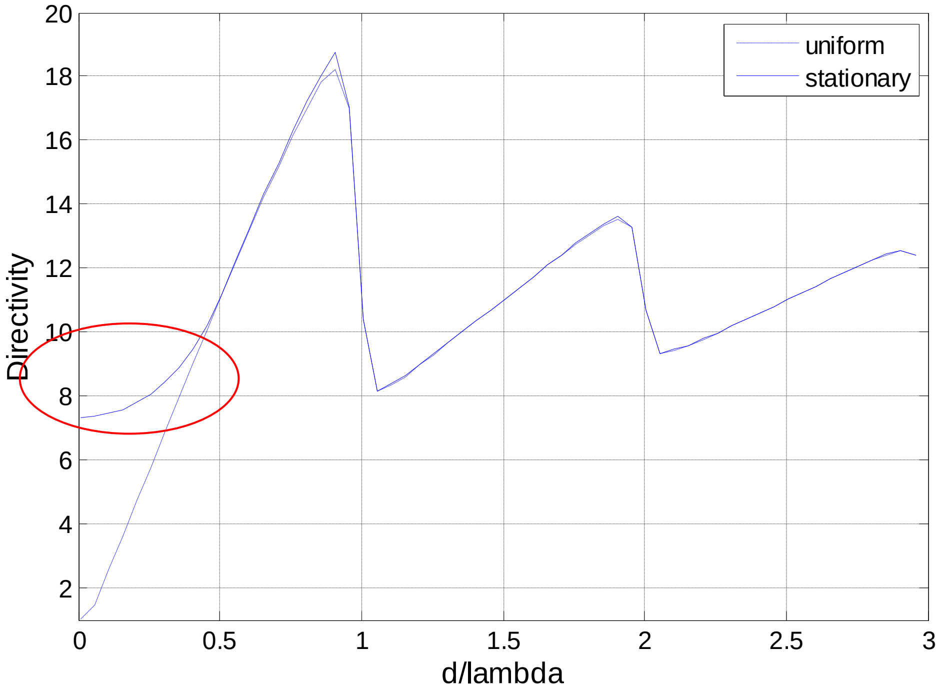

Figure - 5. 37: Stationary and uniform directivities for 11-element array

In d<0.5λ region, while the spacing gets closer to zero, superdirectivity is obtained. In this region, the advantage of superdirectivity over uniform array appears.

If we compare superdirectivity for number of elements 3, 5, 7,9 and 11, Figure – 5.38 is obtained. In the figure, it can be seen that when number of elements increases, superdirectivity also increases.

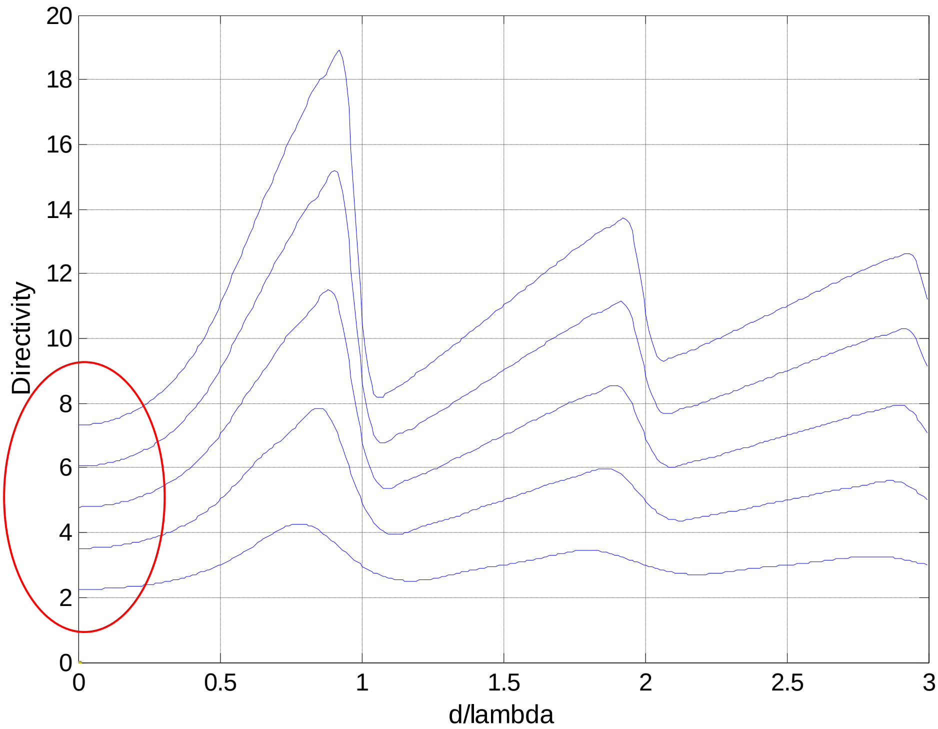

Figure - 5. 38: Directivity vs spacing for 3, 5, 7, 9 and 11 elements

(The bottom curve is for 3-element case and the top is for 11 elements)

At the begining the question was "how much narrow" an array beam can be obtained by using superdirectivity. Therefore number of elements up to 37 is investigated. However, it is observed that if number of elements is increased, spacing between the elements also has to be increased in order to make the calculations accurately. Due to the increase in spacing between the elements, uniform array directivity also increases and so for large number of elements superdirectivity starts to loose its advantage over uniform directivity. According to the observations, advantage of superdirective array over uniform array starts to decrease for number of elements greater than 17 for precision M=435. Figure - 5.39 summarizes that conclusion.

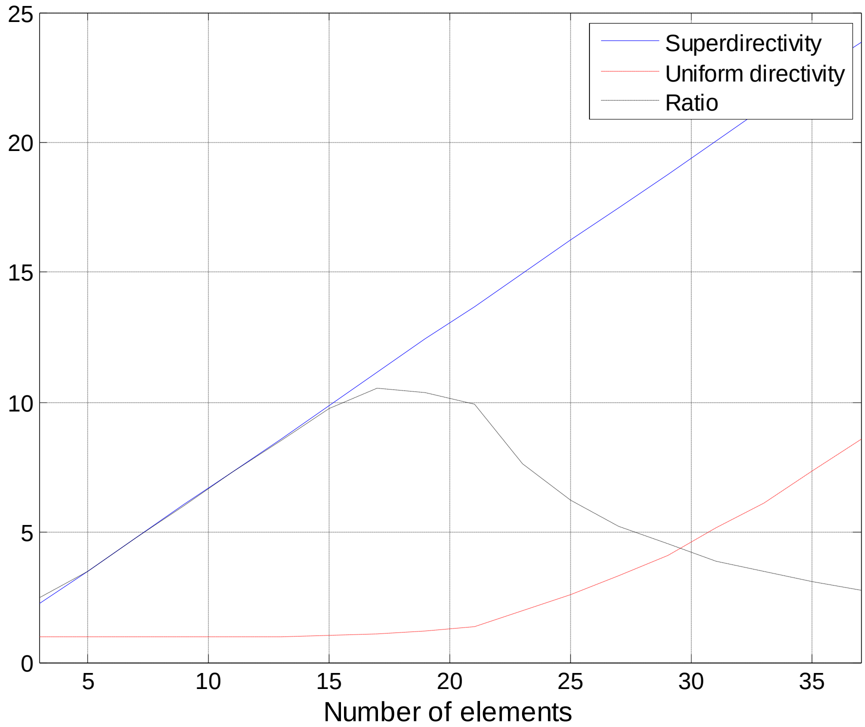

Figure - 5. 39: Comparison of Superdirectivity and Uniform directivities, precision M=435

As seen in Figure – 5.39, for small number of elements uniform directivity is close to 1. However, when the number of elements is increased uniform array directivity also increases due to the increase in spacing. Figure – 5.40 shows the spacing required between the elements versus number of elements.

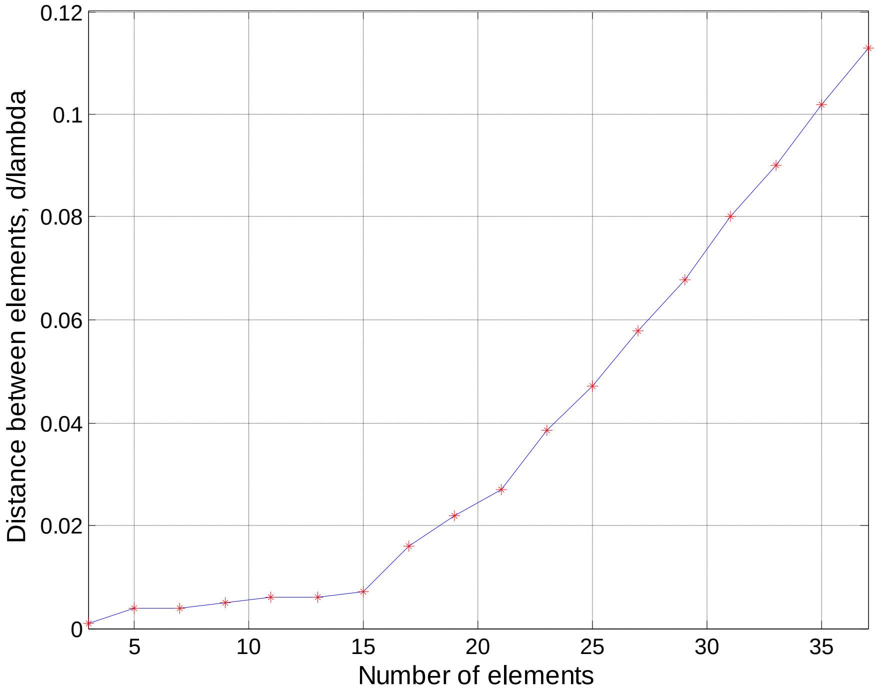

Figure - 5. 40: Distance between the elements vs Number of elements

The two figures Figure -5.39 and 5.40 give the feeling that if number of elements is increased more, limit spacing where the numerical calculations can be done accurately will get close to 0.5 λ, i.e.$\mathrm{kd} = \pi$. At that point, the two curves meet and superdirectivity and uniform directivity get the same value.

The results show that even if we have the precision of 435 digits, using a superdirective array of 17 elements provides the maximum advantage over the same length uniform array.

5.20. An Example

Let us compare superdirective and uniform arrays in an example.

Assume a superdirective array composed of 17 elements and the distance between the elements is $d = 0.016\lambda$.

Let the operating frequency be 1 GHz and so wavelength $\lambda = 30\ cm$. Therefore,

$$d = 0.48\ cm$$

$$L = (N - 1)d = 7.68\ cm$$

That superdirective array has the directivity of 11.54.

In order to obtain the same directivity with the same number of elements by a uniform array, the uniform array shall have the specifications below:

$$d = 0.325\lambda = 9.75\ cm$$

$$L = (N - 1)d = 156\ cm$$

It shows that the uniform array will be 20.32 times longer than the superdirective array with the same number of elements in order to obtain the same directivity.

In this example, spacing between the superdirective array elements is 0.48 cm. It may be possible to realize such small lengths for the array elements by using RF Micro Electro-Mechanical Systems (RF MEMS) technology, [20].

CHAPTER 6

TOLERANCE OF SUPERDIRECTIVE ARRAYS

Tolerance sensitivity is stated as one of the difficulties in superdirective arrays' practical application in literature, [16]. In this chapter, tolerance of superdirective arrays to perturbations in element coefficients and order of sensitivity for different number of elements will be investigated.

In order to see the influence of random change in array coefficients, array factor will be constituted by new coefficients and then see how the perturbation in coefficients effect the array factor of the array.

Superdirective array coefficients and directivity was found by taking the derivative of directivity expression in Equation 3.14 and equating it to zero as in Equation 3.15.

If we examine the second derivative of directivity, below equations are obtained.

Directivity expression is given in Equation 3.14 as:

$$\mathrm{D} = \frac{\mathbf{c}^{\mathbf{H}}\mathbf{O}\mathbf{c}}{\mathbf{c}^{\mathbf{H}}\mathbf{A}\mathbf{c}} \qquad [3.\ 32]$$

If first derivative is taken and equated to zero:

$$\mathrm{D}' = \left[\left(\mathbf{c}^{\mathbf{H}}\mathbf{A}\mathbf{c}\right)\mathbf{O} - \left(\mathbf{c}^{\mathbf{H}}\mathbf{O}\mathbf{c}\right)\mathbf{A}\right]\mathbf{c} = 0 \qquad [3.\ 33]$$

If we take the second derivative, below equation is obtained:

$$\mathbf{D}'' = \frac{2\mathbf{O}}{\mathbf{c}^{\mathbf{H}}\mathbf{A}\mathbf{c}} + \frac{-4\left(\mathbf{O}\mathbf{c}\mathbf{c}^{\mathbf{H}}\mathbf{A}^{\mathbf{H}} + \mathbf{A}\mathbf{c}\mathbf{c}^{\mathbf{H}}\mathbf{O}^{\mathbf{H}} + \frac{1}{2}\mathbf{c}^{\mathbf{H}}\mathbf{O}\mathbf{c}\mathbf{A}\right)}{\left(\mathbf{c}^{\mathbf{H}}\mathbf{A}\mathbf{c}\right)^{2}} + \frac{8\left(\mathbf{c}^{\mathbf{H}}\mathbf{O}\mathbf{c}\right)\left(\mathbf{A}\mathbf{c}\mathbf{c}^{\mathbf{H}}\mathbf{A}^{\mathbf{H}}\right)}{\left(\mathbf{c}^{\mathbf{H}}\mathbf{A}\mathbf{c}\right)^{3}}$$

If the second derivative is examined for different number of elements, it is found that second derivative is negative.

Since first derivative of the directivity is zero ($D'=0$) and second derivative is smaller than zero ($D''<0$), it can be concluded that directivity D has a local maximum at that point.

6.1. 3-Element Array

For the array of 3 elements, if we perturbate one of the array elements c_{-1} by 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5} and 10^{-6} relatively, Figure – 6.3 is obtained.

(Perturbating by 10^{-1} means 10% change in coefficient value)

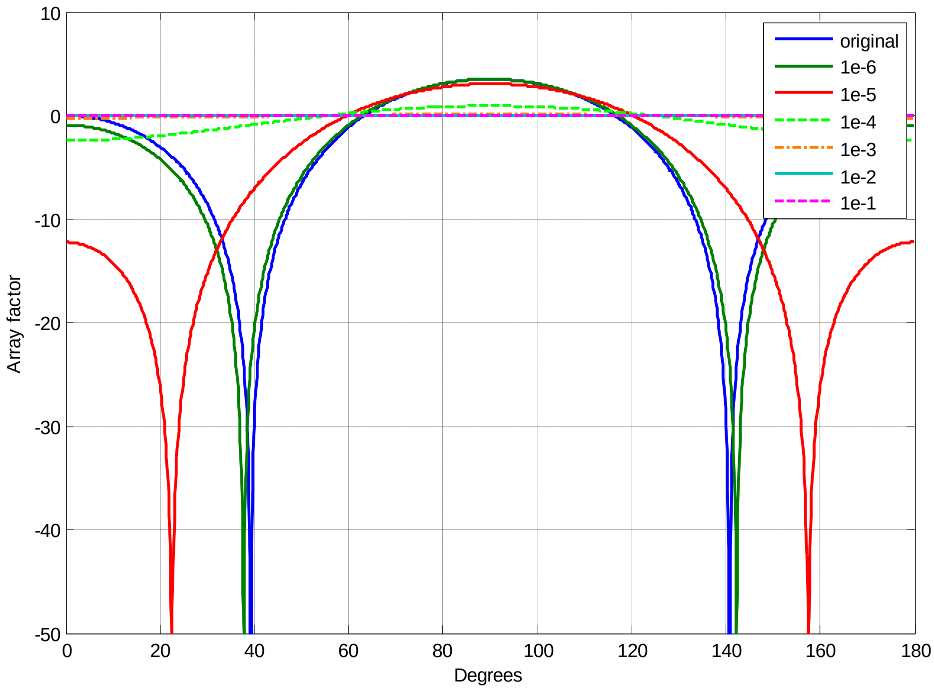

Figure - 6. 1: Perturbated Array factors (dB) for 3-element array

Corresponding directivities are given in Table – 6.1.

Table - 6. 1: Directivites for different perturbation values

Perturbation	Directivity
10^{-6}	2.245
10^{-5}	2.027
10^{-4}	1.238
10^{-3}	1.026
10^{-2}	1.0026
10^{-1}	1.00026

Original curve in Figure - 6.3 is the array factor for array coefficients which provide optimum directivity. As seen in Table – 6.1 if coefficient c_{-1} is perturbated, directivity starts to decrease and array factor (Figure – 6.3) becomes like an isotropic antenna pattern. In order to find acceptable tolerance values, perturbation values which do not cause change in array factor more than 1 dB are investigated.

For the array of 3 elements, if we perturbate one of the array elements c_{-1} with a random number smaller than 10^{-5}, change in array factor will be smaller than 1 dB as in Figure – 6.1.

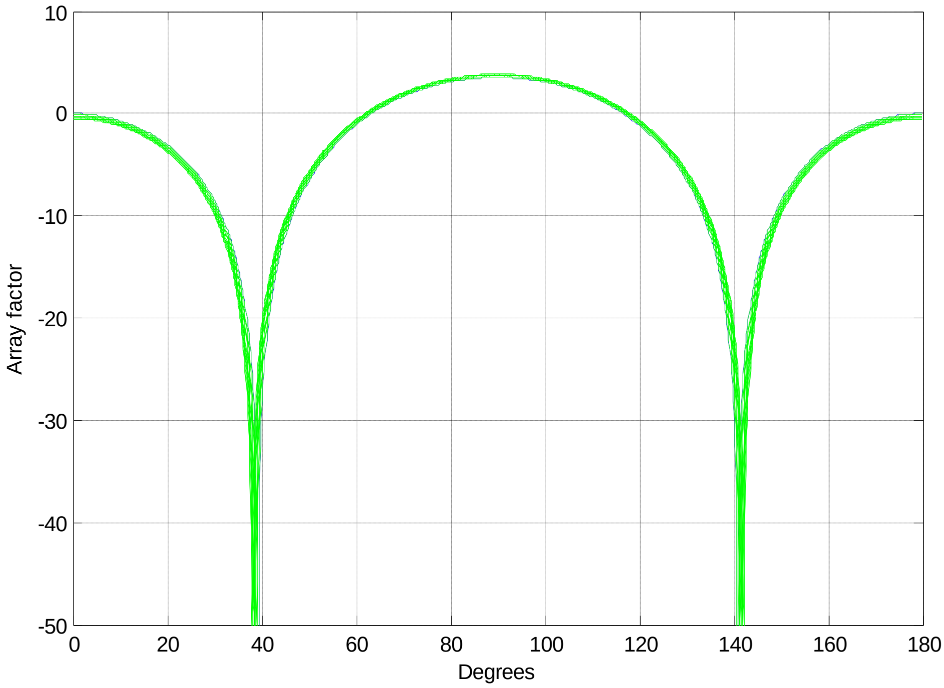

Figure - 6. 2: Perturbated Array factors (dB) for 3-element array

If all of the elements are perturbated by random numbers smaller than 10^{-6}, Figure – 6.2 is obtained. Change in array factor is smaller than 1 dB.

This means that if the change in the array factor is restricted to 1 dB, the relative error in array coefficients should not exceed 10^{-6}.

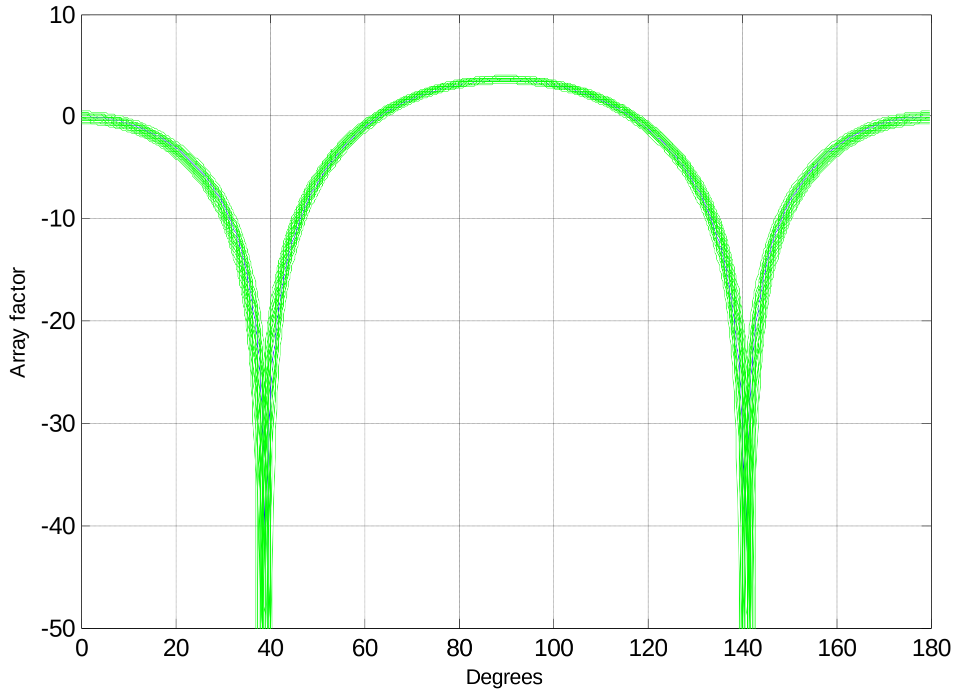

Figure - 6. 3: Perturbated Array factors (dB) for 3-element array

6.2. 5-Element Array

For the array of 5 elements, if all of the elements are perturbated by random numbers smaller than 10^{-9}, Figure – 6.2 is obtained. Change in array factor main lobe is smaller than 1 dB.

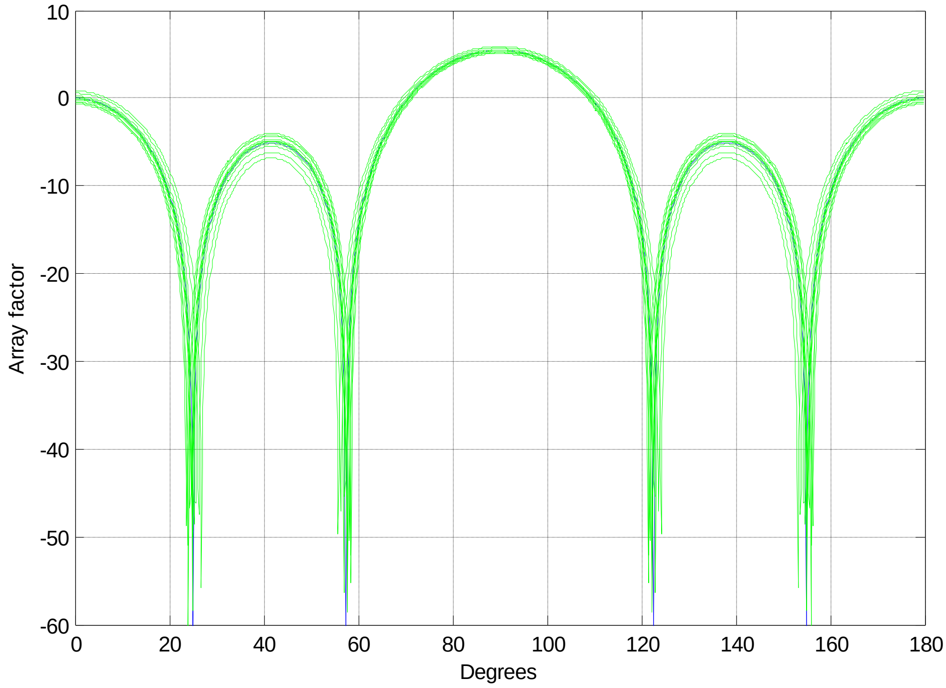

Figure - 6. 4: Perturbated Array factors (dB) for 3-element array

It can be stated that the array of 5 elements is tolerable to perturbations less than 10^{-9}. When we compare it to the 3 number of elements case, the array become less tolerable to perturbations.

6.3. 7-Element Array

For the array of 7 elements, if all of the elements are perturbated by random numbers smaller than 10^{-13}, Figure – 6.2 is obtained. Change in array factor is smaller than 1 dB in main lobe.

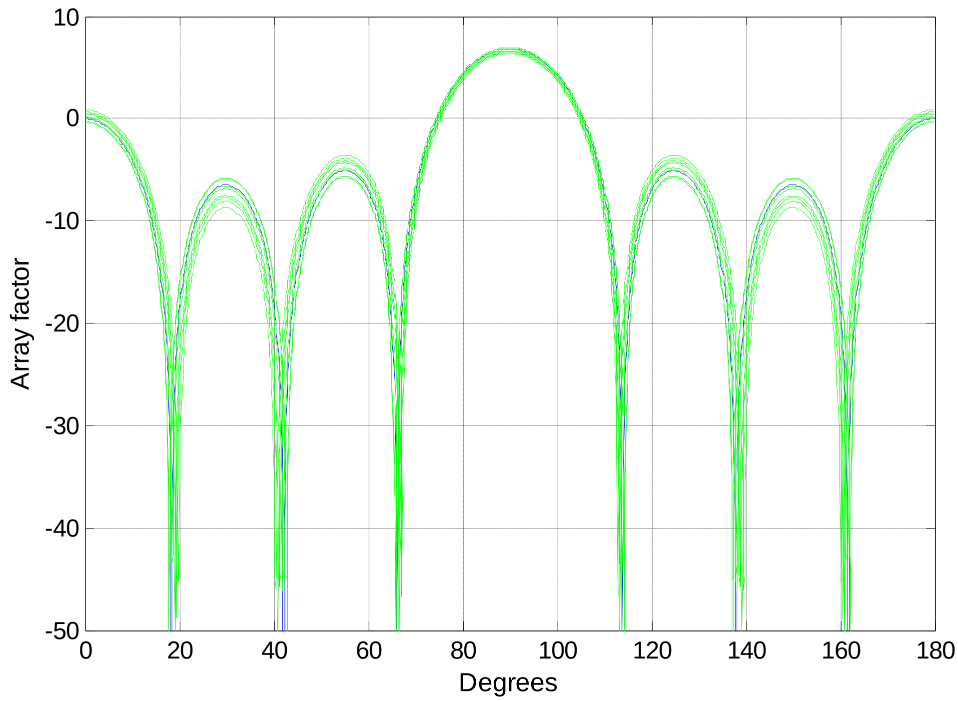

Figure - 6. 5: Perturbated Array factors (dB) for 7-element array

As seen in the figure, the array of 7 elements is tolerable to perturbations less than 10^{-11} %. 7-element array is less tolerable then 5-element array to perturbations in coefficients.

Hence, it can be concluded that as the number of elements increase array becomes less tolerable to perturbations in the coefficients. After 7 elements, sensitivity becomes very high.

CHAPTER 7

BANDWIDTH

Balanis defines bandwidth of an antenna as "the range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard", [3]. He also states that antenna characteristics like input impedance, pattern, gain, polarization, etc. do not necessarily vary in the same manner or even critically affected by the frequency. Therefore there is no unique characterization of the bandwidth. The specifications are set in each case to meet the needs of the particular application.

In this chapter, bandwidth of superdirective arrays will be investigated such that change in array factor will be observed when the operating frequency is changed. This observation will be done for different number of elements in order to examine the bandwidth when the number of elements is increased.

7.1. 3-Element Array

For the array of 3 elements, for the case of $d = 0.001\lambda$ precision required is $M = 55$. Let us assume that the frequency changed such that new frequency is 1% higher than the original frequency, i.e.

$$f_2 = f_1 * 1.01$$

Then,

$$\lambda_2 = \frac{c}{f_2} = \frac{c}{1.01 * f_1} = \frac{\lambda_1}{1.01}$$

So, the distance between the elements of 3-element array becomes

$$d = 0.001\lambda_1 = 0.001 * 1.01\lambda_2 = 0.00101\lambda_2$$

Now, let us see how the array factor differs:

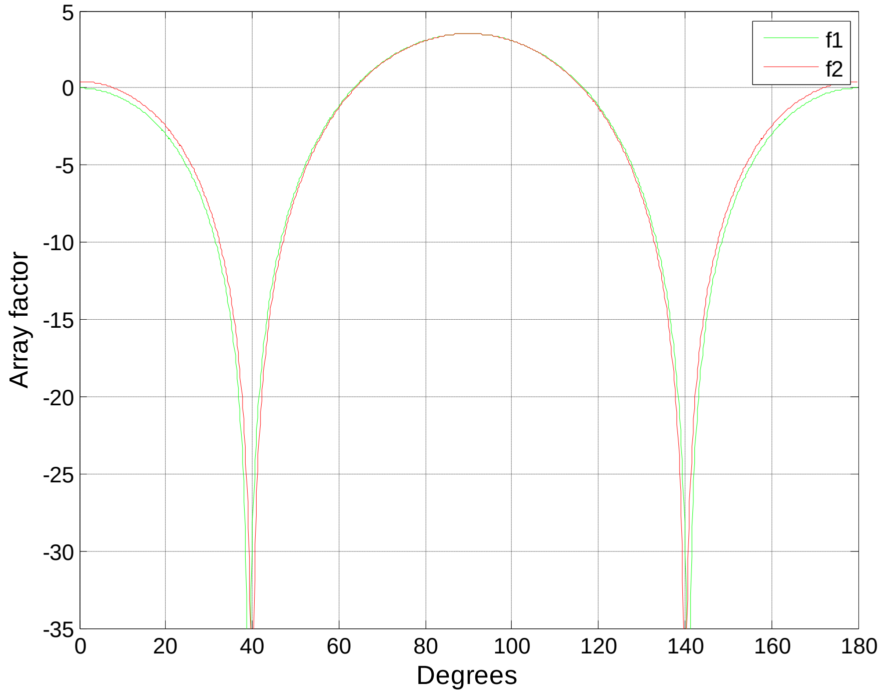

Figure - 7. 1: Array factor (dB) for 1% frequency change for 3-element array

Directivity for 1% frequency change is 2.248 where the optimum directivity is 2.25.

Now let us change the frequency such that new frequency is 10% higher than the original frequency, i.e.

$$f_2 = f_1 * 1.1$$

Then,

$$\lambda_2 = \frac{\lambda_1}{1.1}$$

So, the distance between the elements of 3-element array becomes

$$d = 0.001\lambda_1 = 0.001 * 1.1\lambda_2 = 0.0011\lambda_2$$

Corresponding array factors are given in Figure – 7.2.

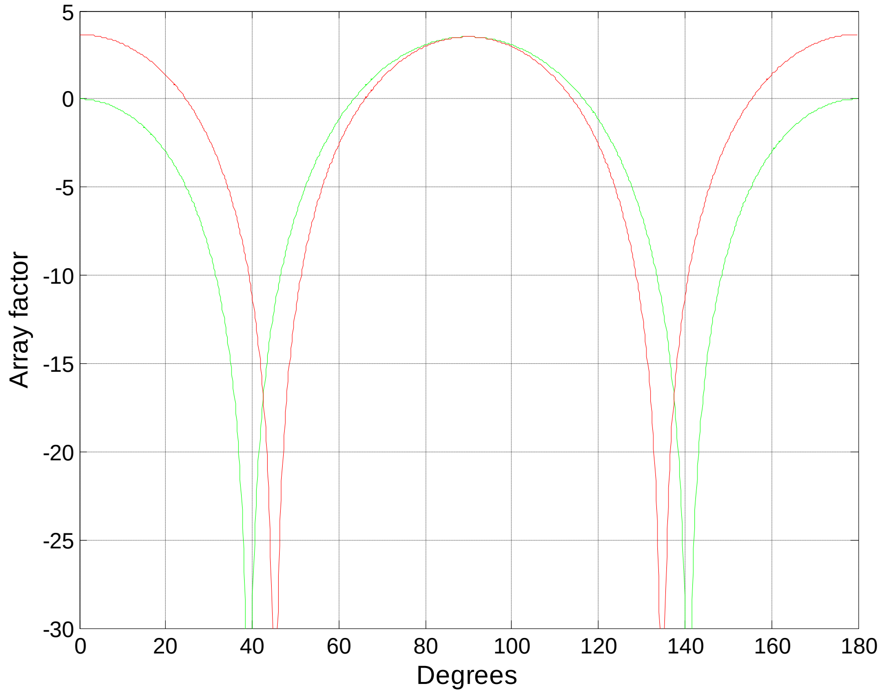

Figure - 7. 2: Array factor (dB) for 10% frequency change for 3-element array

Directivity for 10% frequency change is 2.132 where the optimum directivity is 2.25.

As seen in the figure, sidelobes start increasing.

In order to have a change in sidelobe smaller than 1 dB, there should be a random frequency change of smaller than 1%, as seen in Figure – 7.3.

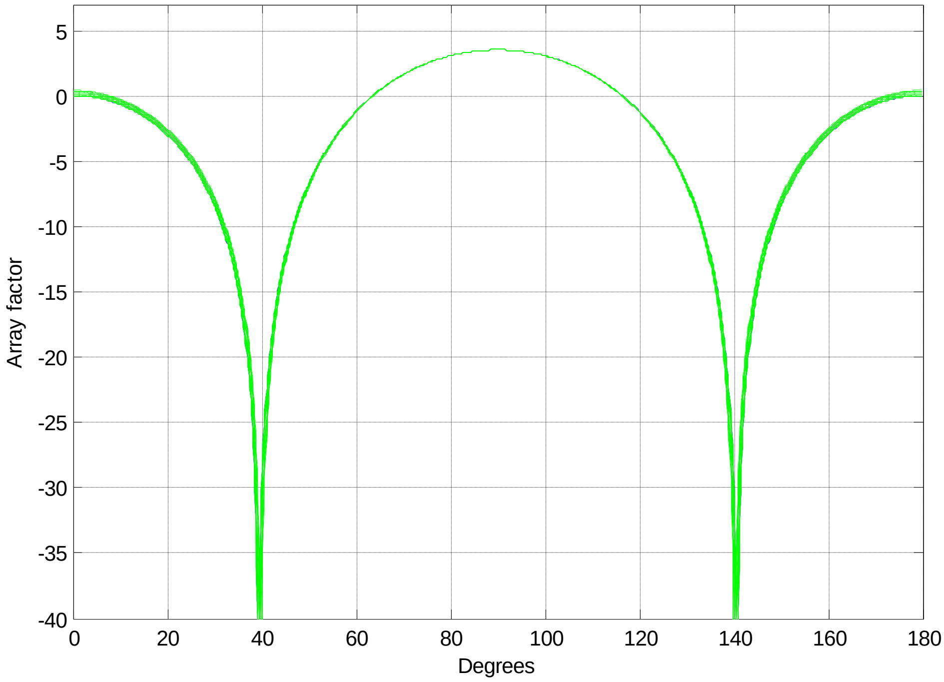

Figure - 7. 3: Array factor (dB) for smaller than 1% frequency change for 3-element array

7.2. 5- element Array

For 5-element array case, the frequency increased by %1, %10 and %50 of the original frequency relatively and the change in the array factor is observed:

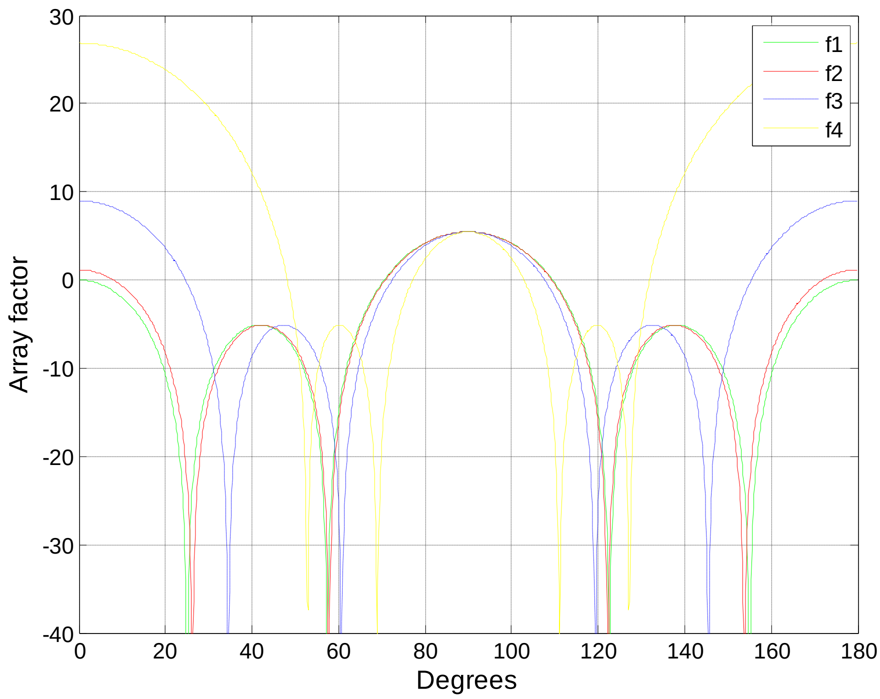

Figure - 7. 4: Array factor (dB) for %1, %10, %50 frequency changes for 5-element array

In order to have a change in sidelobe smaller than 1 dB, there should be a random frequency change of smaller than 0.5%.

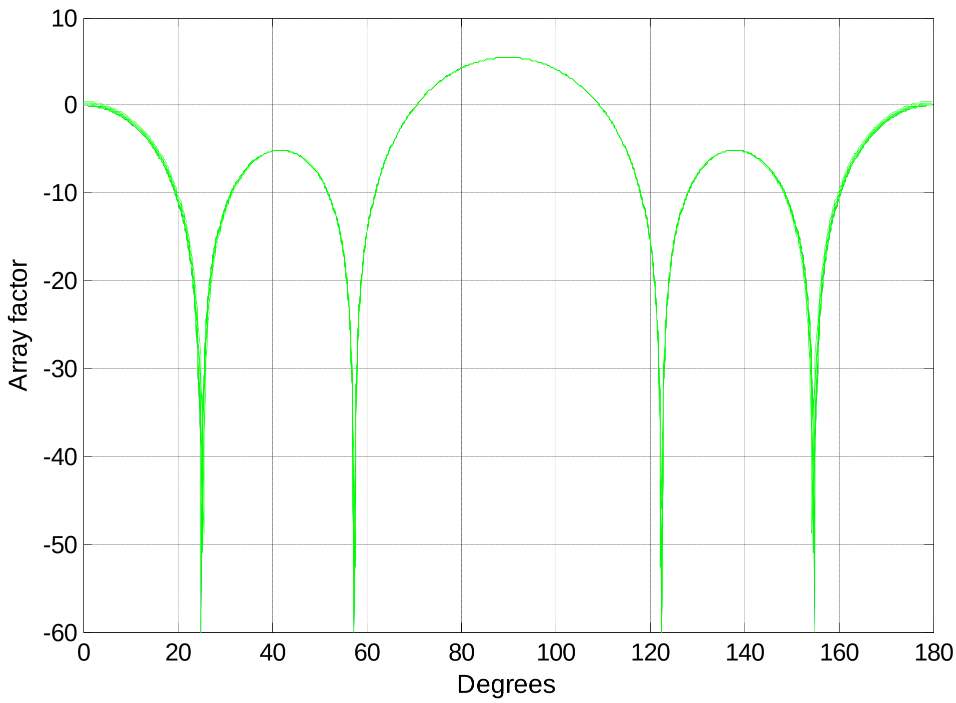

Figure - 7. 5: Array factor (dB) for smaller than 0.5% frequency change for 5-element array

Corresponding directivites are given in Table – 7.1.

Table - 7. 1: Directivites for different frequency changes

Frequency change	Directivity
Original (optimum)	3.5156
%0.5	3.514
%1	3.510
%10	2.83
%50	0.091

7.3. 7-Element Array

For 7-element array case, the frequency increased by %1, %10 and %50 of the original frequency relatively and the change in the array factor is observed:

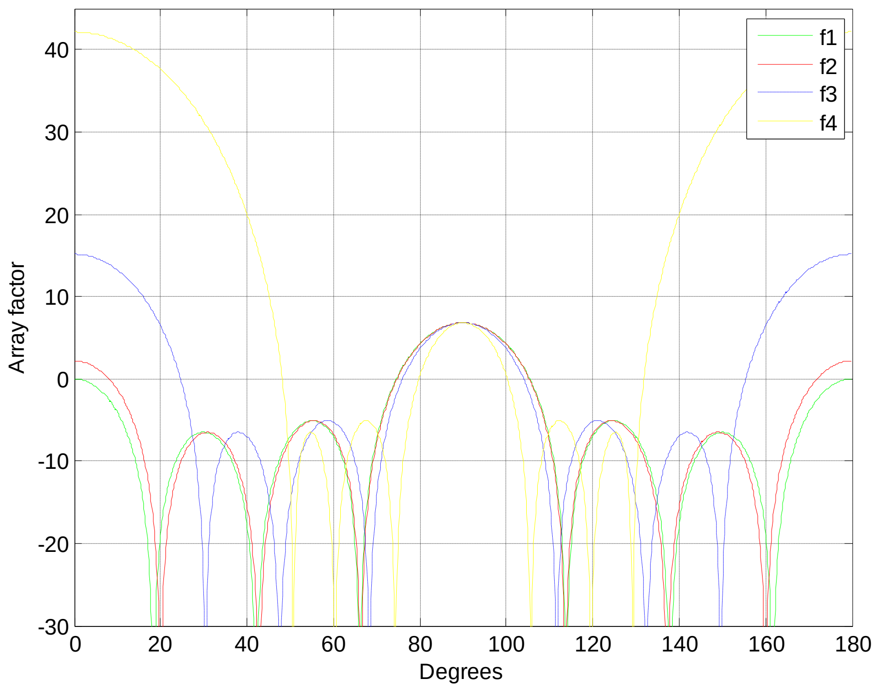

Figure - 7. 6: Array factor (dB) for %1, %10, %50 frequency changes for 7-element array

In order to have a change in sidelobe smaller than 1 dB, there should be a random frequency change of smaller than 0.3%, as seen in Figure – 7.7.

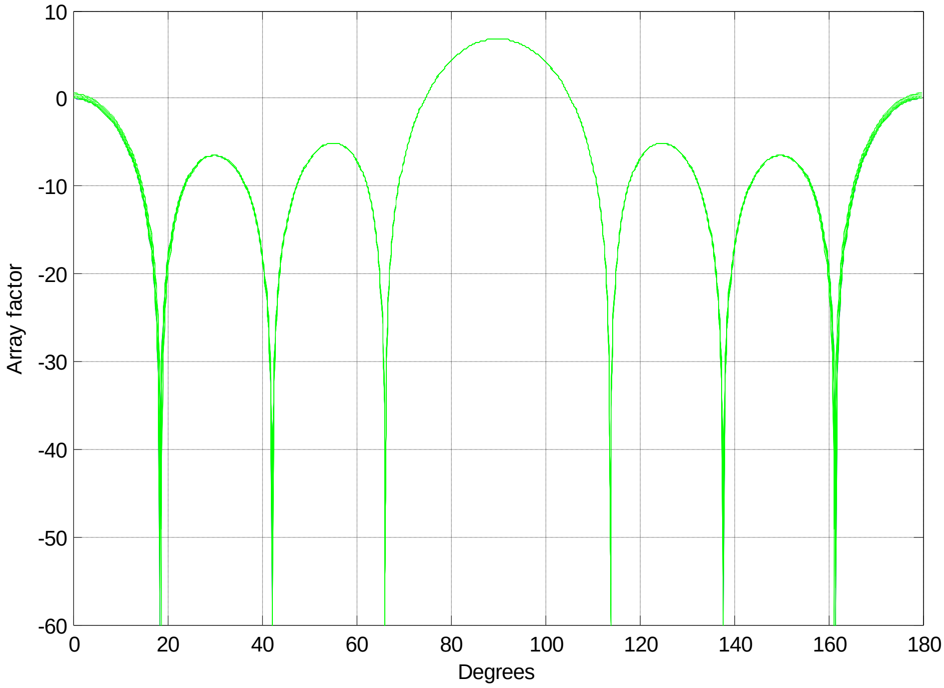

Figure - 7. 7: Array factor (dB) for smaller than 0.3% frequency change for 7-element array

7.4. 9-Element Array

For 9-element array case, the frequency increased by %1, %10 and %50 of the original frequency relatively and the change in the array factor is observed:

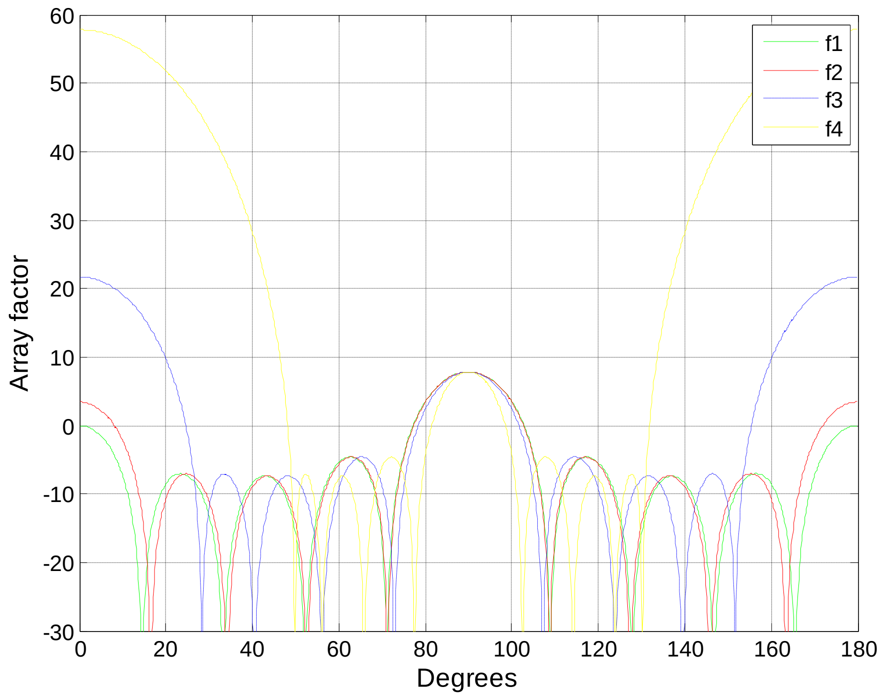

Figure - 7. 8: Array factor (dB) for %1, %10, %50 frequency changes for 7-element array

In order to have a change in sidelobe smaller than 1 dB, there should be a random frequency change of smaller than 0.1%, as seen in Figure – 7.9

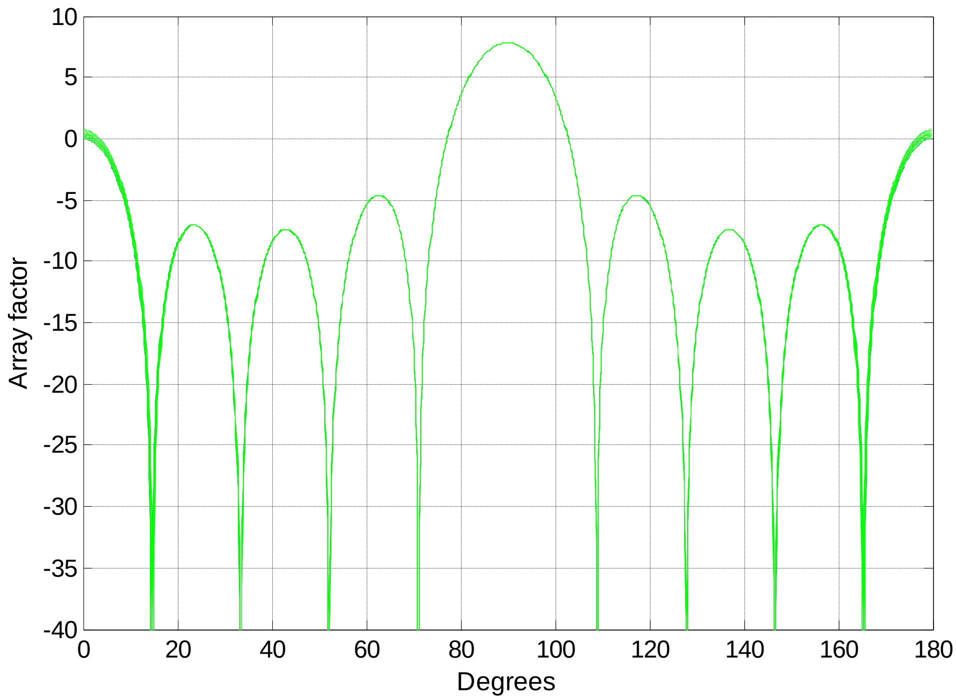

Figure - 7. 9: Array factor (dB) for smaller than 0.1% frequency change for 9-element array

From the observations done, it can be stated that when the number of elements are increased, sidelobe is increased more in case of frequency changes.

It can be concluded that, superdirective arrays are very sensitive to the frequency changes since the distance between the elements is critical in superdirectivity view. As a result, superdirective arrays are narrowband array antennas.

CHAPTER 8

CONCLUSION

In this thesis Superdirective arrays are investigated in element coefficient, array factor, directivity, tolerance, bandwidth aspects and compared to uniform arrays. Since high accuracy is required while doing the calculations, Multiple Precision (Mp) Toolbox is used through the numerical computations done in Matlab.

It is concluded that, superdirective arrays provide more directivity when compared to the same length uniformly excited arrays. Increasing number of elements increases the advantage of superdirectivity over uniform directivity in theory. However, increase in number of elements demands more precision. Up to 37 elements are investigated in order to see how much directivity can be obtained from a superdirective array by using precision provided by Mp Toolbox. However, it is seen that after some point (17-element array case, precision used is 435) the advantage of superdirective array over uniform array starts to decrease since the numerical calculations could not be done for small spacings by even using Mp Toolbox.

It is also observed that superdirective array factor is very sensitive to perturbations in element coefficients. As the number of elements increase, tolerance sensitivity of elements also increase.

Lastly, superdirective arrays are narrowband antennas since they are very sensitive to frequency changes. Moreover, as the number of elements increases, bandwidth of superdirective arrays decreases.

In conclusion, superdirective arrays provide more directivity when compared to uniformly excited arrays. However tolerance and bandwidth of superdirective array antennas prevent them from practical use even with today’s high precision Digital Beam Forming tools and techniques.

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