# THE RELATIONSHIP BETWEEN 6th GRADE STUDENTS' PROBLEM SOLVING ACHIEVEMENT AND MATHEMATICS ACHIEVEMENT SCORES AFTER COMPLETING INSTRUCTION ON PROBLEM SOLVING

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

DİLEK KARAOĞLAN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
THE DEPARTMENT OF ELEMENTARY SCIENCE AND MATHEMATICS EDUCATION

AUGUST 2009

Approval of the Graduate School of Social Sciences

_____

Prof. Dr. Sencer AYATA
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science

_____

Prof. Dr. Hamide ERTEPINAR
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science

_____

Assist. Prof. Dr. Mine IŞIKSAL
Supervisor

**Examining Committee Members**

Assoc. Prof. Dr. Erdinç ÇAKIROĞLU (METU,ELE) _____

Assist. Prof. Dr. Mine IŞIKSAL (METU,ELE) _____

Dr. Cengiz Savaş AŞKUN (METU,CEIT) _____

**I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.**

Last Name, Name: KARAOĞLAN, Dilek

Signature:

## ABSTRACT

THE RELATIONSHIP BETWEEN 6th GRADE STUDENTS' PROBLEM SOLVING ACHIEVEMENT AND MATHEMATICS ACHIEVEMENT SCORES AFTER COMPLETING INSTRUCTION ON PROBLEM SOLVING

Karaoğlan, Dilek

M.S., Department of Elementary Science and Mathematics Education

Supervisor: Assist. Prof. Dr. Mine Işıksal

August 2009, 102 pages

The purpose of this study is to examine the relationship between 6th grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores related to Least Common Multiple (LCM), Greatest Common Factor (GCF), Sets and Whole Numbers topics obtained throughout the semester. In addition, the relationship between 6th grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from Level Determination Exam (SBS) was investigated. In total, 170 sixth grade students from a private school in Istanbul participated in the study. The data were collected via three sources namely; Problem Solving Achievement Tests (PSATs), Mathematics Achievement Tests (MATs) and SBS exam. Quantitative methods were utilized to examine the research questions and a correlational design was used. The results of the statistical analysis showed that there was a significant positive correlation between students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics. In addition, the findings of the analysis showed that there

was a significant large positive correlation between the problem solving achievement scores after completing instruction on problem solving and students' actual mathematics net scores obtained from SBS.

Keywords: Mathematics Education, Problem Solving Achievement, Mathematics Achievement, Elementary Students, Level Determination Exam (SBS)

# ÖZ


6. SINIF ÖĞRENCİLERİNİN PROBLEM ÇÖZMEYE DAYALI ETKİNLİKLER SONRASI PROBLEM ÇÖZME BAŞARILARI İLE MATEMATİK BAŞARILARI ARASINDAKİ İLİŞKİ

Karaoğlan, Dilek

Yüksek Lisans, İlköğretim Fen ve Matematik Alanları Eğitimi Bölümü

Tez Yöneticisi: Yrd. Doç. Dr. Mine Işıksal

Ağustos 2009, 102 sayfa

Bu çalışmanın amacı 6. sınıf öğrencilerinin EBOB-EKOK, kümeler ve doğal sayılar konularında problem çözmeye dayalı etkinlikler sonrası problem çözme başarıları ile matematik başarıları arasındaki ilişkinin araştırılmasıdır. Bu çalışmada ayrıca öğrencilerin problem çözme başarı puanları ile Seviye Belirleme Sınavındaki (SBS) matematik netleri arasındaki ilişki de incelenmiştir. Çalışma İstanbul'da özel bir okulda öğrenim gören 170 altıncı sınıf öğrencisi ile gerçekleştirilmiştir. Çalışmanın verileri, Problem çözme başarı testleri (PSATs), Matematik başarı testleri (MATs) ve SBS sınav sonuçları kullanılarak elde edilmiştir. Araştırma sorularının incelenmesinde nicel yöntemlerden faydalanılmış ve ilişkisel model kullanılmıştır. İstatistiksel analiz sonuçları, 6. sınıf öğrencilerinin EBOB-EKOK, kümeler ve doğal sayılar konularında problem çözmeye dayalı etkinlikler sonrası aldıkları problem çözme başarı puanları ile ortalama matematik başarı puanları arasında anlamlı pozitif bir ilişki olduğunu göstermiştir. Ayrıca, öğrencilerin SBS sınavındaki matematik netleri ile problem çözmeye dayalı etkinlikler sonrası aldıkları problem çözme başarı puanları arasında da anlamlı pozitif bir ilişki bulunmuştur.

Anahtar Sözcükler: Matematik Eğitimi, Problem Çözme Başarısı, Matematik Başarısı, İlköğretim Öğrencileri, Seviye Belirleme Sınavı (SBS)

**To my parents**

Raziye and Kadir KARAOĞLAN

**ACKNOWLEDGEMENTS**

I would like to thank first my advisor Mine IŞIKSAL for her valuable comments, support and encouragement at every step of my thesis. Also, I would like to express my deepest gratitude to her for her wonderful guidance although she is pregnant and a new mother throughout the duration of my thesis. I also want to thank my previous advisor Yusuf KOÇ for his help and encouragement all the time.

My Family…I would like to express special thanks to my parents Raziye and Kadir Karaoğlan and my sisters Ayşegül and Memnune for their love, trust and support in all my life.

Orhan Ustaoğlu…Special thanks to him for being with me all the time throughout the duration of my thesis and sharing his love and support.

I would like to thank my colleagues, Eser Saraçoğlu, Ayfer Dürnel, Sultan Tuncel, Erim Koçyiğit, Derya Gündüz Sefer and Oktay Cavlun for their valuable help.

I also would like to express my deepest gratitude to my friends Funda Barutçu and Nilüfer Ünal for their help and moral support. Thank you for sharing Ankara days with me.

I also want to thank Sedanur Çakmak and Sümeyra Doğan for sharing their experiences with me, Işıl İşler for her support whenever I need and Ağılönü Family for their support all the time.

Finally, I want to thank my committee members Erdinç Çakıroğlu and Cengiz Savaş Aşkun for their willingness to serve in my committee and for their valuable comments, contributions and suggestions.

**TABLE OF CONTENTS**

## LIST OF TABLES

TABLES

## LIST OF FIGURES

FIGURES

## LIST OF ABBREVIATIONS

ABBREVIATIONS

MoNE: Ministry of National Education

NCTM: National Council of Teachers of Mathematics

LCM: Least Common Multiple

GCF: Greatest Common Factor

PSATs: Problem Solving Achievement Tests

MATs: Mathematics Achievement Tests

SBS: Level Determination Exam

# CHAPTER 1

# INTRODUCTION

*"There should be no more conflict between content and pedagogy than between one's right foot and left foot. They should work in tandem toward the same end and avoid tripping each other*"

David Klein

There has been a rapid reformist curriculum change wave in education in all over the world and Klein (2003) states that the main reason underlying the curricular reforms from the past to present is the struggle between *content* (what to teach) and *pedagogy* (how to teach). He saw content and pedagogy as the right and left foot of people and the quote above clearly indicates that not only the content but also the pedagogy is indispensable parts of education.

In Turkey's attempt to construct a compatible 'left and right foot', a curriculum reform that was not markedly different from what was done in other countries was accomplished. Bulut (2004) identifies the common points of the new curriculums as, its student-centered structure which encourages students' active participation in the teaching-learning process and its major emphasis to aesthetic and enjoyable parts of mathematics. Koc, Isıksal and Bulut (2007) stated that the content of the new curriculum in Turkey is not so different from the old one. However, unlike the small revisions in the area of content, there are revisions in relation to pedagogy.

Mathematics is one of the content areas included in curriculum change in Turkey and the new mathematics curriculum reform began on the basis that "Every child can learn mathematics" (Ministry of National Education [MoNE], 2005). In addition to content standards, the new curriculum also gives importance to some

process standards. *Problem Solving* that the theoretical framework of this study is centered on it is one of these process standards and major changes exist in the new mathematics curriculum that related to problem solving. The other four process standards are as follows: reasoning and proof, communication, connections and representation (NCTM, 2000).

There are various *problem* definitions in the literature. Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier and Human (1997) defined a problem as a situation where the students do not have any specific recommended rules or ways of solution. In addition, students do not perceive there is a particular solution method for each problem. Furthermore, Grouws (1996) defined the problem as a situation where there is something to be found or shown and the way to find or show it is not obviously explained. For a mathematical situation to be a problem; various knowledge and skills should be utilized together and there should be no routine solution to the problem. Moreover, the problem should be relevant to students' experiences and it should be interesting for them. In addition, students should feel the need of solving it. In such a case, mathematical knowledge and skills gained by students will be more meaningful for students and transferring this knowledge to various situations will be easier (MoNE, 2005).

Although there are various *problem* definitions in the literature (Grouws, 1996; Hiebert et al., 1997) the general consensus is that there is no direct way of solving a problem and all problem solving performances requires much more than simple coming back of facts or the use of well-learned procedures (Lester, 1994). When problem solving is specifically investigated, it is obvious that problem solving should be a main goal of all mathematics instruction; hence, it should be an essential component of all mathematical activities (NCTM, 1989). Studying with selected problems enables students to learn about, and deepen their understanding of mathematical concepts. These selected problems not only provide students with opportunities to apply mathematics to their contexts, but are also useful in developing or deepening students' understanding of important mathematical ideas (NCTM, 2000). Thus, teaching through problem solving is of great importance and problem solving is seen not as an isolated part of the mathematics program but as an integral part of all mathematics learning (NCTM, 2000). Similarly, Van De Walle

(2004) sees problem solving as a principle instructional strategy since teaching through problem solving is one of the best ways of teaching significant mathematics concepts and procedures.

Moreover, most of the previous studies have emphasized the importance of teaching through problem solving and researchers have discussed that when students were provided with suitable teaching and learning environments, there were statistically significant improvements not only on their problem solving performances but also on their various mathematical abilities like modeling and their attitudes towards problem solving (Jitendra, Griffin, Deatline, & Sczesniak, 2007; Lester, Garofalo & Kroll, 1989; Van Haneghan, Barron, Young, Williams, Vye, &Bransford 1992; Verschafell, De Corte, Lasure, Vaerenbergh, Bogaerts, &Ratinckx, 1999). In his study, Santos-Trigo (1998) discussed instructional qualities of a successful mathematical problem solving class based on the analysis of parts from a mathematical problem solving course. In order to evaluate the potential use of mathematical problem-solving instruction, the author discusses three issues. The first issue is the type of learning activities; the second issue is tasks to help students to engage in mathematical discussion in the classroom, and the last one is type of evaluation to assess students' progress in mathematical problem solving.

Some studies also conducted in Turkey on problem solving for both elementary and secondary schools, and for prospective teachers. Problem solving behaviors (Altun, 1995), learning and utilization of problem solving strategies (Arslan & Altun, 2006; Karatas &Güven, 2004; Okur, 2008; Özkaya, 2002; Özsoy, 2007;Yazgan &Bintas, 2005), problem designing and solving activities in the process of teaching mathematics (Albayrak, Ipek & Isık, 2006; Iskenderoglu, Akbaba & Olkun, 2004) were examined. These studies showed that training had a positive effect on students' learning and utilization of problem solving strategies (Arslan & Altun, 2007; Karatas &Güven, 2004; Okur, 2008; Özkaya, 2002; Özsoy, 2007; Yazgan &Bintas, 2005). Furthermore, Altun (1995) examined the problem solving behaviors of the elementary school students in the 3$^{rd,}$ 4$^{th}$ and 5$^{th}$ grades and determined what differences are displayed by students who are successful and unsuccessful in problem solving according to these behaviors. In another study, Iskenderoglu, Akbaba and Olkun (2004) investigated elementary school students'

success in choosing the correct arithmetic operation for different types of standard word problems. Results of the study showed that students generally used key words like “and, more, increase, decrease, minus” when they were solving word problems. Students usually matched the key words with specific operations. In addition, Albayrak, Ipek and Isık (2006) designed a study that the participants were teachers and prospective teachers. The researchers found that teachers were inadequate in their application of problem designing and solving activities in the process of teaching basic operation skills and prospective teachers had not sufficiently acquired the skills required to design and solve problems in the process of teaching basic operation skills.

Mathematics achievement is of great importance in the literature and various studies performed to increase it. It is also a prominent variable of this study. Many studies have been conducted to investigate the relationship of various variables with mathematics achievement (Husen, 1967; Keeves 1976; McKnight, Crasswhite, Dossey, Kifer, Swafford, Travers, &Cooney, 1987; Schmidt,McKnight & Raizen, 1997). These studies showed that there is a strong positive relationship between students’ opportunity to learn scores and mean student achievement scores in mathematics (Husen, 1967; McKnight et al., 1987; Schmidt, McKnight & Raizen, 1997); there is a positive relationship between total time allocated to mathematics and general mathematics achievement (Keeves 1976) and there is a significant improvement on students’ achievement and understanding when teachers know about how students construct knowledge, are acquainted with solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics (Grouws & Cebulla, 2000).

Regarding mathematics achievement, some studies also conducted in Turkey. Effects of different teaching methods on mathematics achievement (Duatepe-Paksu & Ubuz, 2009; Pilli, 2008; Şişman, 2007; Teltik-Başer, 2008; Yıldız, 2008) and effects of teacher and class characteristics on mathematics achievement (Akyüz, 2006) were examined. Researchers have discussed that different teaching methods like drama-based instruction and 5E learning model had a significant effect on students’ mathematics achievement in comparison with traditional teaching (Duatepe-Paksu & Ubuz, 2009; Teltik-Başer, 2008). Moreover, in her study Akyüz,

(2006) found that the classes of male teachers were more successful and teacher experience, time spent on tests and quizzes, use of textbooks, disciplined class climate and class mean of home educational resources had positive significant effect on student achievement.

There were also studies in the literature that investigate the effects of problem solving approach on achievement in various areas of mathematics and the relationship among two variables (Ceylan, 2008; Fidan, 2008; Özsoy, 2007; Yıldız, 2008). Ceylan (2008) investigated the relationship between the scores of the 6$^{th}$ grade primary school students at daily-life problem solving inventory and their performance at mathematical problem solving. The results of the study showed that there is a highly significant correlation between mathematics test scores and the scores of daily-life problem-solving inventory. Moreover; Fidan (2008) examined the effect of the teaching through problem posing on students' problem solving success. According to the statistical analysis of results, a positive significant difference was found in favor of experimental group students' problem solving success. Ozsoy (2007) is another researcher who found positive significant effect of being thought to use metacognition strategies on 5$^{th}$ grade students' mathematical problem solving achievement. Lastly; Yıldız (2008) examined the change in 6$^{th}$ grade students' problem solving abilities and found that instruction based on Polya's steps significantly affected students' problem solving abilities in a positive way.

Nevertheless, while both problem solving and mathematics achievement have been studied in depth over the years, there have been very few research studies completed on the possible connection between two in relation with the new curriculum. The current study aimed to make contribution to problem solving and mathematics achievement researches in the context of a major elementary mathematics curriculum change initiated in Turkey in 2004-2005 academic year.

### 1.1. Purpose of the Study

The purpose of this research study was to investigate the relationship between 6$^{th}$ grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores related to Least Common Multiple (LCM), Greatest Common Factor (GCF), Sets and Whole

Numbers topics obtained throughout the semester. In addition, the relationship between 6th grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from SBS exam was investigated.

### 1.2. Research Questions

The specific research questions addressed in this study were:

1. What is the relationship between 6th grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics?

2. What is the relationship among 6th grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from Level Determination Exam (SBS)?

### 1.3. Definitions of Important Terms

*Problem Solving:* refers to engaging in a task for which the solution method is not obvious (NCTM, 2000).

*Classes designed for problem solving:* refers to classes where the students were provided with real-life context mathematical problems and wanted to solve them according to the Polya's problem solving steps.

*Problem solving achievement score*: refers to the scores gained from Problem Solving Achievement Tests (PSATs). Problem Solving Achievement Tests (PSATs) included open-ended problems involving scenarios taken from real-life situations related to Whole Numbers, Sets and Least Common Multiple (LCM) and Greatest Common Factor (GCF).

*Mathematics achievement mean scores*: refers to the mean scores gained from Mathematics Achievement Tests (MATs) related to contents of Whole Numbers, Sets and Least Common Multiple (LCM) and Greatest Common Factor (GCF). Mathematics Achievement Tests (MATs) included multiple choice questions having four choice alternatives.

*Actual mathematics net scores obtained from SBS*: refers to the net score gained from mathematics subtest of Level Determination Exam **(**SBS). Level Determination Exam **(**SBS) is a nationwide achievement test included multiple choice questions having four choice alternatives. Net scores obtained after three wrong answers cancel one correct answer.

### 1.4. Significance of the Study

Since problem solving "lies at the heart of doing mathematics" (Lester, 1980, p.29) and mathematics achievement is a prominent variable that many studies have been done to investigate the effect of various variables on it; there were also some studies conducted about problem solving and mathematics achievement in Turkey (Ceylan, 2008; Fidan, 2008; Özsoy, 2007; Yıldız, 2008). These studies have showed that problem solving approach have been found to have various effects on achievement in various areas of mathematics. For instance, instruction based on Polya's problem solving steps significantly affected students' problem solving abilities in a positive way (Yıldız, 2008) and teaching through problem posing had significant positive effect on students' problem solving success (Fidan, 2008). In addition, there were also studies conducted to investigate the relationship between various variables of problem solving approach and achievement. For instance, Ceylan (2008) found a highly significant correlation between students' test scores of daily-life problem-solving inventory and their performance at mathematical problem solving.

Nevertheless, while several research studies about problem solving and mathematics achievement have been conducted in Turkey, there have been very few research studies completed on the possible connection between two in relation with the new curriculum. Thus, the current study contributes to the literature in the context

of the problem solving studies embedded in the new curriculum which investigates the relationship between students' problem solving achievement scores that were assessed by an alternative assessment technique and their mathematics achievement scores. On the other hand, this study also aimed to investigate the relationship between students' problem solving achievement scores and their actual mathematics achievement net scores obtained from SBS to see the relation among students' problem solving success with their achievement on a nationwide exam.

Finally; this study includes five chapters. In the first chapter revealed above, introduction and significance of this study are explained. The second chapter gives information about the review of relevant literature related to curriculum reform in elementary education, new elementary mathematics curriculum in Turkey, theoretical framework and research studies on problem solving and mathematics achievement. Chapter three is about the research design, population and sample, data collection instruments, reliability and validity of the study, data collection procedure, analyses of data, assumptions and limitations, and lastly the internal and external validity of the study. The results of the statistical analyses and findings are explained in chapter four. Lastly, chapter five consists of the discussion of the findings and the recommendations for future research studies.

# CHAPTER 2

# LITERATURE REVIEW

The purpose of this chapter is to give information about the review of relevant literature related to curriculum reform in elementary education, new elementary mathematics curriculum in Turkey, theoretical framework and research studies on problem solving and mathematics achievement respectively.

## 2.1. Curriculum Reform in Elementary Education

There have been educational reform movements in all over the world and Turkey is also attending these reform movements and changing the nature of new elementary school curriculum. Babadoğan and Olkun (2006) claim that the changes in the content of the elementary school curriculum in Turkey is somewhat similar to those in the US, UK, Singapore, Ireland and Holland.

Bulut (2004) identifies the common points of the new curriculums in USA, Canada, Ireland and France as, its student-centered structure which encourages students' active participation in the teaching-learning process and its major emphasis to aesthetic and enjoyable parts of mathematics. Koc, Isıksal and Bulut (2007) stated that the content of the new curriculum in Turkey is not so different from the old one. Although there are small revisions in the area of content, immense revisions are designed in relation to pedagogy.

Ministry of National Education (MoNE) started to implement the recent curriculum reform in Turkey in 2003. It was piloted in the academic year 2005-2006 through the grades 1-5. Then, with the academic year 2008-2009, all grades ($1^{st}$-$8^{th}$ grade) in Turkish elementary schools were offered instruction under the light of the new curriculum. This curriculum reform includes five content areas which are;

mathematics, science, social science, life science and Turkish. Social, individual, economical, historical and cultural aspects are the common basic principles that all of the content areas cover (Koc, Isiksal & Bulut, 2007).

Many factors initiated and promoted the changes made in the curriculum. Sustainable development of Republic of Turkey and the aim of reaching ideal international standards of education is the first reference point that triggers curriculum reforms in Turkey (MoNE, 2003; Koc, Isiksal & Bulut, 2007). Moreover, Turkish students' academic performances both in national and international exams, development of information technology, teaching episodes and connections with European Union countries motivates the change in the curriculum (MoNE 2003, Koc, Isiksal & Bulut, 2007).

This chapter gives information about curriculum reform in elementary education and in the next chapter new elementary mathematics curriculum will be explained.

**2.2. New Elementary Mathematics Curriculum**

As previously mentioned, mathematics is one of the content areas included in curriculum change in Turkey. In this section, I specifically mention about the new mathematics curriculum in Turkey and make a comparison between the old and new elementary mathematics curricula.

The new mathematics curriculum reform began on the basis that "Every child can learn mathematics" and students' active participation to learning environments should be encouraged (MoNE, 2005). Interdisciplinary connections among different subject areas, enriching the learning environment with the use of instructional technology and other instructional manipulative are other important characteristics of the new curriculum. Furthermore, starting teaching with concrete experiences, aiming to provide meaningful learning, providing students with communication with mathematical knowledge, making connections, considering motivation of the students', utilizing instructional technology effectively, enhancing learning through cooperative learning, organizing the teaching- learning process according to appropriate steps during the instruction are of great importance in the application process of the new curriculum (MoNE, 2005, p.18-21). Along with these

changes in the new mathematics curriculum, as the skills like estimation and problem solving gain significance, the importance given to the pencil-paper calculations decrease (MoNE, 2005).

Ministry of National Education (MoNE) explains the differences between the new curriculum initiated in 2003 and the previous one as follows. First of all, the old elementary school mathematics curriculum for the 1st -5th grades includes 1249 behavioral objectives. Textbooks which are based on these objectives are uniform and this uniform structure is believed to be restricted both the author and the teacher. However, the new curriculum includes 368 outcomes which also contain skills for students to develop. Since the structure of the outcomes requires student-centered teaching activities, both the teacher and the author have the flexibility they need. In addition, the new curriculum includes learning areas that maintain content standards as cited by the National Council of Teachers of Mathematics (NCTM) such as the inclusion of numbers, geometry, measurement, statistics and probability and algebra and sub-learning areas like fractions and angles.

Secondly, the content of the fourth and seventh grades old mathematics curriculum was immensely dense and took account of the cumulative structure of the 8 year elementary school curriculum. In the new mathematics curriculum, however, the math content is evenly distributed through the grades and unnecessary repetitions of the contents were eliminated. Moreover replacements of outcomes among different grades in the application process of the new curriculum have taken place in light of revision studies. For instance, positive and negative numbers, and addition and subtraction operations with these numbers are firstly taught in 6th grades according to the new mathematics curriculum developed in 2005 but a decision to handle these topics in the seventh grade was taken since many research studies revealed that sixth grade new mathematics curriculum is too dense to be covered in the intended time (MoNE, 2008). Moreover, unlike the prior mathematics curriculum, the new one includes examples of applications that emphasize student-centered teaching methods, techniques and strategies. In the teacher-centered structure of the old curriculum, the passive transfer of the knowledge from teachers to students created an environment for rote learning. However, students experience meaningful leaning in the new curriculum since the outcomes require utilization of activities with

concrete experiences and exploration. In fact, contrary to the old one, the new curriculum gives great importance to the usage of materials and manipulative that can be easily reached (Babadoğan & Olkun, 2006).

In addition to these changes explained above, some modifications were made in the context of the subject areas. Even though many of the old topics took their place in the new curriculum, the old curriculum lacked a meaningful connection between the subjects. The synchronous change in the curriculums of other subject areas enabled curriculum developers to select those overlapping contents through different subject areas and make connections within the contents in other subject domains.

Changes were also made to the nature of assessment as part of the curriculum reform. The comparison of the old and new curriculum reveals that alternative assessment techniques, extracurricular activities, research studies and projects are used in the new curriculum to assess students' performances. Moreover, the structure of the old curriculum expected all students to exhibit same performances without paying sufficient attention to individual and environmental differences. However, the new curriculum gives great importance to individual abilities and different performances. Activities suitable to various learning environments are planned to develop students' skills like independent thinking, reasoning and decision making. Lastly, more effort is expended in the new curriculum to enable students to develop positive attitudes towards mathematics. The active participation of students to learning activities enables them to appreciate the aesthetic and enjoyable parts of mathematics.

Finally, the new elementary mathematics curriculum has five content standards: Number and Operations, Algebra, Geometry, Measurement and Data Analysis and Probability. In addition to these content standards, it also gives importance to some process standards. *Problem Solving* is one of these process standards and it will be discussed in the next section in detail since the theoretical framework of this study is centered on it.

## 2.3. Theoretical Framework of Problem Solving

In this section, theoretical framework of problem solving will be explained.

### 2.3.1. What is Problem and Problem Solving?

One of the major changes in the new mathematics curriculum is related to problem solving. There are various *problem* definitions in the literature. Hiebert et al. (1997) defined a problem as a situation where the students do not have any specific recommended rules or ways of solution. In addition, students do not perceive there is a particular solution method for each problem. Grouws (1996) defined the problem as a situation where there is something to be found or shown and the way to find or show it is not obviously explained. Lastly, a problem is explained by Polya (1962) as the task that needs conscious action in order to reach obviously conceived but not directly reachable aim.

Although there are various definitions of problem, the general consensus is that there is no direct way of solving a problem and all problem solving performances requires much more than simple coming back of facts or the use of well-learned procedures (Lester, 1994). For a mathematical situation to be a problem; various knowledge and skills should be utilized together and there should be no routine solution to the problem. Moreover, the problem should be relevant to students' experiences and it should be interesting for the student. In addition, students should feel the need of solving it. In such a case, mathematical knowledge and skills gained by students will be more meaningful for students and transferring this knowledge to various situations will be easier (MoNE, 2005).

National Council of Teachers of Mathematics (NCTM) listed five process standards in *Principles and Standards for School Mathematics* in 2000. The process standards mean mathematical processes though which students attain and use mathematical knowledge (Van De Walle, 2004). *Problem Solving* is one of these process standards and it is explained as engaging in a situation where there is no direct solution (NCTM, 2000). The other four process standards are as follows: reasoning and proof, communication, connections and representation (NCTM, 2000). Reys, Robinson, Sconiers and Mark (1999) report that the aim of the standards is to make students mathematically literate persons who are able to hypothesize,

investigate, reason logically and solve problems by utilizing various mathematical methods. Along with the new conceptual approach, the Turkish curriculum has the aim of encouraging students to construct mathematical meaning and do abstractions by making use of their concrete experiences and intuitions. Furthermore, in addition to developing mathematical concepts, the curriculum aims to develop some important skills such as problem solving, communication, reasoning, and connection (MoNE, 2005, p.8).

### 2.3.2. Problem Solving and Learning Mathematics

When problem solving is specifically investigated, it is obvious that problem solving should be a main goal of all mathematics instruction; hence, it should be an essential component of all mathematical activities (NCTM, 1989). NCTM underlined the importance of teaching through problem solving in *Principles and Standards for School Mathematics* and problem solving is seen not as an isolated part of the mathematics program but as an integral part of all mathematics learning (NCTM, 2000). It is stated that studying with selected problems enables students to learn about, and deepen their understanding of mathematical concepts. These selected problems not only provide students with opportunities to apply mathematics to their contexts, but are also useful in developing or deepening students' understanding of important mathematical ideas (NCTM, 2000). In addition, NCTM (2000) proposed with the problem solving standard that instructional programs from pre-kinder garden through grade 12 should enable all students to build new mathematical knowledge through problem solving, solve problems that arise in mathematics and in other contexts, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem solving. Similarly, Van De Walle (2004) stated that the first goal given above, explains problem solving as a vehicle that helps children to develop mathematical ideas. He sees problem solving as a principle instructional strategy since teaching through problem solving is one of the best ways of teaching significant mathematics concepts and procedures.

In addition to teaching through problem solving, there are situations in which teachers teach *about* problem solving. Teaching about problem solving has an

important restriction since it enables teachers and textbook writers to see problem solving as a separated unit of curriculum rather than an integral part in which mathematics is learned and applied (Schroeder & Lester, 1989). Schroeder and Lester added that there are also situations in mathematics classrooms in which teachers teach other mathematical content in order to make students solve problems. Here, the aim of the teaching mathematics is to teach solving problems. Teachers usually solve an example story problem and then wait for students to solve very similar problems whose solutions can be acquired by simply following the same patterns that teacher has already used (Schroeder & Lester, 1989). Van De Walle (2004) states that teach-then-solve paradigm distinguishes problem solving from the learning process. Children are used to teachers' telling them the rules of solving a problem and as a result, these children are unable to solve problems for which solution methods have not been provided. If the purpose is to make students understand mathematics, then understanding can be seen as an outcome of problem solving process rather than something that can be taught directly (Hiebert et al., 1997). In other words, learning mathematics is an outcome of solving problems. Mathematical ideas are results of the problem solving experience rather than elements that must be taught before problem solving (Hiebert et al., 1997).

Van De Walle (2004) explained the reasons that give great value to teaching *through* problem solving. He claimed that problem solving places the focus of the students' attention on ideas and sense making, develops the belief in students that they are capable of doing mathematics and that mathematics makes sense, provides ongoing assessment data that can be used to make instructional decisions, helps students succeed, and informs parents, develops mathematical power, and allows an entry point for a wide range of students. Moreover; problem-based approach engages students so that there are fewer discipline problems and it is enjoyable. Problem solving is of great importance in the literature since it is a goal for mental development, a skill to be taught and a teaching method used in mathematics (Brown, 2003; Giganti, 2004; Jonassen, 2004; Lester, 1980; Manuel, 1998; Martinez, 1998; Polya, 1953; Schoenfeld, 1989; Willoughby, 1985).

Although the nature of the new mathematics curriculum give great importance to constructing new mathematical knowledge through problem solving

(NCTM, 2000); it also "speaks to the need to develop problem solving strategies and processes, metacognitive habits of monitoring and regulating problem solving activity and a positive disposition toward mathematical problem solving" (Van De Walle, 2004, p.37). Hence, if problem solving is the vehicle to teach other mathematical ideas, than how to use this vehicle should be taught to students. Charles, Lester and O'Daffer (1987)'s seven goals for teaching problem solving aims at developing students' thinking skills, students' abilities to select and use problem-solving strategies, helpful attitudes and beliefs about problem solving, students' abilities to use related knowledge, students' abilities to monitor and evaluate their thinking and progress while solving problems, students' abilities to solve problems in cooperative learning situation, and at developing students' abilities to find correct answers to a variety of types of problems (p.7). Similarly, the new mathematics curriculum in Turkey aims to encourage students to use problem solving to learn mathematics, to develop awareness about the contribution of problem solving to learning, to use problem solving skills in their lives, in other subject areas and in new situations they meet in mathematics, to carry out the problem solving steps meaningfully, to construct new problems in addition to solving problems, to feel self-confidence in problem solving processes, and to have a positive sense and feelings towards problem solving (MoNE, 2005, p.14).

While teaching through problem solving, however, there are some important features that should be taken into consideration. First of all, teachers should provide students with a stress-free learning environment in which they can express what they understand from the problem easily and determine whether there is more or less knowledge given to solve the problem. Moreover; students should construct a plan to solve the problem by themselves. They should become aware that one problem can be solved with different strategies or more than one strategy can be used to solve a problem. Thus, they should be taught to value different ways to solve a problem. Lastly, students should share the solutions and methods of solution with peers and the teacher (MoNE, 2005).

Polya's problem solving steps are of great importance in problem solving literature of mathematics education. Thus, in the next chapter I will explain his famous steps in detail.

### 2.3.3. Polya's Problem Solving Stages

With his famous book "How to solve it", Polya (1957) stated four steps to solve a problem. In addition, he reminds that the order of the steps should be same in all problem solving activities. The first step is "understanding the problem". To understand a problem one has to find the given data and decide on the unknown. Moreover, whether the given knowledge is sufficient or not to solve the problem need to be considered. In the second step, "devising a plan"; the problem solver makes connections between the given data and the unknown. Moreover, a plan is made to solve the problem and there are strategies used to reach the solution of the problem.

The word strategies need to be defined to understand the qualities of the second step thoroughly. Van De Walle (2004) defines *strategies* as the recognizable methods of accessing a task that are entirely free from specific topic and content area. Possamentier and Krulik (1998) states ten major strategies in order to help solving problems. These are intelligent guessing and testing, finding a pattern, accounting for all possibilities, making a drawing (visual representation), working backwards, logical reasoning, solving simpler analogy problems, organizing data, adopting a different point of view and considering extreme cases.

Each of Possamentier and Krulik's (1998) strategies can be elaborated. *Intelligent guessing and testing* is about organizing the way of guessing in the light of evaluation results of previous guesses to reach an answer. *Finding a pattern*, on the other hand, includes determining a pattern or extending it. A pattern is systematic and predictable repetition of numeric, visual or behavioral data. Moreover, while using *accounting for all possibilities* strategy all options that can be useful for solving the problem are considered and the problem is solved by eliminating them in a systematic way. It is obvious that visual representations help us to describe and follow the problem solving process. Thus, *making a drawing strategy which* is the use of drawings to solve a problem enables us to find the solution in an easier way by the use of visuals. Another strategy is *working backwards* which makes the problem solver start with the end results and reverses the steps until s/he reaches the answer of the problem. In addition, *logical reasoning* is one of these strategies that is, in fact, formally used in most problem solving processes since it requires logical and

organized thoughts to produce an answer. Furthermore, *solving a simpler analogies problem* is the strategy that enables us to solve the problem by altering it into an easier one and solving the main problem by utilizing the solution of the simplest one. Using *Organizing data* strategy provide us with the advantage of reorganization of data to reach the solution of the problem in a practical way. When we talk about another problem solving strategy which is *adopting a different point of view,* we can easily say that this strategy does not have an exact methodology like most of the other ones. In general, you want to reach a target but something prevents you doing this. By using this strategy, you redefine the problem and try to solve this new one to reach an answer. The last strategy explained by Possamentier and Krulik (1998) is *considering extreme cases*. It is the changing of some variables to extreme cases while the other variables remain constant. Thinking about extreme cases of the given situation makes problem solving really easier in some problems.

The third step is "carrying out the plan" in which previously decided plan is followed to have a solution of the problem and the last step is "looking back and extend". This refers to going back to the problem, checking the plan and the result and then trying to develop an ability to generate another problem.

I explained Polya's problem solving steps above and now in the next section I will reveal information about assessment process of problem solving. More significantly, I will explain alternative assessment in Problem Solving.

### 2.3.4. Alternative Assessment in Problem Solving

Following the reform of instructional processes, there are also important changes in assessment practices in the new curriculum since assessment is an inevitable part of classroom instructions (NCTM, 2000). The purpose of the assessment and evaluation in teaching and learning processes is to determine students' achievement, assign deficiencies, understand efficiency of instructional strategies, and to pinpoint the strong and weak parts of the curriculum (MoNE, 2005). The goal of the assessment systems explained by the new curriculum is not only assessing the product but also the process (MoNE, 2005; Miller and Linn, 2005). In addition, alternative assessment enables teachers to regulate the instruction according to results since assessment is an integral part of the instruction.

McQuade and Champagne (1996) explain that in "alternative" assessments, students have more to do than simply providing an answer to a question asked by the instructor and how and what students' learn are greatly influenced from the way that students used to demonstrate what they have learned. According to them, a curriculum planned to provide students with 'understanding' might not make students ready for tests of broad general knowledge and vice versa. McQuade and Champagne added that understanding, critical thinking and creativity should be fostered by school culture whereas broad general knowledge and factual recall are emphasized much more by tests. They advised an eclectic approach to assessment which includes explanations to assess understanding, problems to assess problem solving, recall questions to assess memorization, investigations to assess inquiry, voluntary participation to assess motivation, and portfolios to assess habits of mind.

Actually, the new curriculum in Turkey makes use of both traditional and alternative assessment strategies in the attempt to develop an assessment system. In addition to standardized and classroom tests, alternative assessment techniques like portfolios, checklists, projects are included in the new curriculum (Koc, Isiksal & Bulut, 2007).

Baki and Gökçek (2005), on the other hand, looked at the assessment change in the new curriculum from a different perspective and they defined assessment system as norm-referenced in the former curriculum and criterion-based in the latter one. The shift of emphasis from the product to process fosters giving importance to not only the skills such as problem solving, reasoning and communication but also the way these metacognitive processes are assessed (MoNE, 2005). It is obviously apparent that performance-based assessment techniques are much more successful on following up the students' progress than the traditional assessment tools such as paper and pencil tests (Linn and Miller, 2005).

When the assessment of problem solving is specifically analyzed, it is interesting to see that students tend to do calculations with no interpretations. However; pure calculations often fail to reveal the adequate nature of problem solvers' work and thinking (Szetela & Nicol, 1992). Similarly, Harskamp and Suhre (2007) mentioned that copying the teacher's method of solution during exercise may provide students with success in standard exercises; however, these students have

great difficulty in solving non-standard problems. Furthermore, to be more successful in the assessment of problem solving, methods for providing students with better communication of thinking should be planned. Problem solving entails substantial thinking but even students are able, they are not disposed to communicate their thinking (Szetela & Nicol, 1992). It is obvious that better understanding of students' knowledge and thinking enables teachers to instruct more effectively and as a result, students have a better learning of higher-order skills necessary to be a good problem solver (Szetela & Nicol, 1992). The reformist philosophy adapted by the Turkish curriculum developers encourage active construction of students' own knowledge by means of thought-provoking processes like problem solving, exploration, reflection and communication which also require high level cognitions (Stein et al, 1996).

Charles, Lester and O'Daffer (1994) argue that it is of great importance to evaluate not only students' problem solving performances, but also their attitudes and beliefs about problem solving. They advise the use of some techniques to evaluate these two significant outcomes. These techniques are observing and questioning students, using self-assessment data from students, using holistic scoring techniques and using multiple choice and completion tests. In the first technique, students can be observed and questioned informally, and recording techniques like observation checklists can be used or a structured interview can be applied to observe and question students. In the second technique, student reports and inventories can be used to collect self-assessment data from students. Furthermore, there are a various number of holistic scoring techniques that can be used. *Analytic scoring* which includes a scale to assign points to specific phases of the processes, *focused holistic scoring* which provides us with a numerical score to the total solution of the problem based on the criteria of special thinking processes or *general impression scoring* in which the evaluator uses his/her general impression to rate the total solution are the various techniques that may be implemented. Lastly, a multiple choice test or a completion test can be used to evaluate problem solving performances.

Moreover, which evaluation technique is to be used should be based on such factors as the type of problem solving skill or outcome being measured, the numbers of students being evaluated, the time available for evaluation, the experience of the

teacher in teaching and evaluating problem solving, how the teacher intends to use the results of the evaluation and the availability of evaluation materials (Charles, Lester & O'Daffer, 1994). In this research study, Problem Solving Achievement Tests (PSATs) which include open-ended questions were prepared and used to collect data. Moreover, each of the PSATs was assessed by using a rubric. Thus, the properties mentioned above were utilized by the researcher during the data collection process. Details of data collection process will be discussed in the next chapter.

Alternative assessment in problem solving is explained above and now in the next section the research studies conducted on problem solving will be listed.

## 2.4. Research Studies on Problem Solving

In this section, recent research studies on problem solving not only in other countries but also in Turkey will be explained.

### 2.4.1. Problem Solving Studies in Other Countries

Jitendra, Griffin, Deatline, and Sczesniak (2007) designed two classroom experiments in Pennsylvania and Florida with 3rd grade students. Firstly, they investigated the effects of schema-based instruction (SBI) on the acquisition of skills for solving mathematical word problems. Secondly, they examined the effect of word problem-solving instruction on the acquisition of computational skills where word problems play a significant role in the development of number operations. The former study was applied to two 3rdgrade classrooms one of which was a low-ability classroom and the other one was a special education classroom. Results of this study indicated mean score improvements from pretest to posttest on word problem solving and computation fluency measures. The latter study was conducted with a heterogeneous (high-, average-, and low-achieving) sample of 3rd graders and results of the second study also revealed student improvement on the word problem solving and computation fluency measures. However, the outcomes of study two were not as positive as those in study one.

Similarly, Verschafell, De Corte, Lasure, Vaerenbergh, Bogaerts, Ratinckx (1999) conducted a design experiment in which a learning environment for teaching and learning how to model and solve mathematical application problems was

developed. The study was applied to 4 classes of 5th graders and pupils of 7 control classes pursued the regular mathematics classes. A pretest, posttest and retention test was applied to both the experimental and control group in order to determine implementation and effectiveness of the experimental learning environment. Results of the study showed that the intervention had a positive effect on different aspects of students' mathematical modeling and problem solving abilities.

In another study, Van Haneghan et al. (1992) designed an experimental study in which video technology was used to teach authentic, complex problem situations in realistic contexts providing opportunities for problem posing, modeling, self-regulation and interpretation. Participants of the study were 5th grade students who were classified as above-average. Both the experimental group and the control group were given three teaching sessions. However, while the experimental group was studying on the authentic problems, the control group studied on traditional word problems. Two tests were administered to both groups before and after instruction. The first one is a traditional word problem solving test and the second one is a videotape-based test. The results of the study revealed that no differences were found between both groups on traditional word problem solving test. However, the experimental group performances were significantly higher than those from the control group on the videotape-based test.

Furthermore, Cai (2003) investigated Singaporean students' mathematical thinking in problem solving and problem posing in an exploratory study. He explained the purpose of the study as providing some information about Singaporean students' mathematical thinking and reasoning, and discussing the findings from a cross-national comparative perspective to understand why Singaporean students were successful in international assessments. Participants of the study were 155 4th graders, 167 5th graders, and 150 6th graders from four Singaporean elementary schools which represented distinct levels according to students' overall academic performance. The results of this study showed that most of the students were able to select appropriate solution strategies to solve the given problems and represent their solutions clearly in appropriate ways. Moreover, most of them were able to pose problems beyond the initial figures in the pattern. The results of this study also revealed that as the grade level increased, a higher percentage of students in that

grade level showed evidence of providing correct answers. Nevertheless, it is interesting that there were no statistically significant differences between fifth and sixth grade students. However, there were significant differences between fourth and fifth grade students' performances.

Moreover, Lester, Garofalo and Kroll (1989) designed a teaching experiment with 7th graders and they investigated the effects of strategy, awareness and self-regulation training on mathematical problem solving of an instructional program. In this program students were provided with *strategy training* in which they practiced the use of some important heuristics, *awareness training* in which they learned to express and reflect on their problem-solving strategies and *self-regulation training* in which they learned to monitor their problem solving activities. The program was applied in a regular level and in an advanced level 7th grade class for 15 hours in 12 weeks. A test including five non-routine application problems was administered before and after the intervention. Results of the study showed that both the regular and advance classes acquired a substantial gain in the total score from pretest to posttest but the progress was not as large as expected. In addition, the results of clinical interviews express no considerable difference between students' regulatory activities before and after instruction.

In his study, Santos-Trigo (1998) discussed instructional qualities of a successful mathematical problem solving class based on the analysis of parts from a mathematical problem solving course taught at university level. He explained aspects related to implementation of problem-solving activities in the classrooms. In order to evaluate the potential use of mathematical problem-solving instruction, the author discusses three issues. The first issue is the type of learning activities; the second issue is tasks to help students to engage in mathematical discussion in the classroom, and the last one is type of evaluation to assess students' progress in mathematical problem solving. The course based on problem solving included various problem solving activities in which participants were provided with tasks including diverse challenges, discussing the importance of using various strategies, participating in small and whole group discussions, reflecting on feedback and challenges that emerge from interactions with the teacher or class-mates, communicating their ideas in written or oral forms and searching for connections or extensions of the problems.

The results of the study revealed that students who took the course make significant progress in the development of their problem solving abilities.

Lastly, Jonassan (2003) designed a research-based instruction for story problems. The author having studied on story problem-solving instruction articulated a model for designing learning environments in order to help students learn how to solve story problems. He explained that in order to solve story problems effectively, students construct a conceptual model of the problem. It includes structural relationships between the sets in the problem that define the class of problem, situational (story) characteristics of the problem context, reconciliation of the structural and situational characteristics, and processing operations required to solve the problem based on the structural characteristics. According to him, students must classify problems and construct conceptual models of problems before using formulas to solve problems.

### 2.4.2. Problem Solving Studies in Turkey

Iskenderoglu, Akbaba and Olkun (2004) investigated elementary school students' success in choosing the correct arithmetic operation for different types of standard word problems. First of all, 80 students from 3$^{rd}$, 4$^{th}$ and 5$^{th}$ grades of an elementary school located in a mid-low socioeconomic area in Bolu were presented with 20 problems and 9 of the students (3 of each grade) were selected to be clinically interviewed according to their solutions of the problems. Results of the study showed that students generally used key words like "and, more, increase, decrease, minus" when they were solving word problems. Students usually matched the key words with specific operations. Furthermore, some students applied wrong operations since they made a false generalization of matching the operation and key word. The authors claim that memorization may cause this situation. Interestingly, students usually chose the correct operation to solve the problems in which no key words were included in the problem.

In another study, Altun (1995) examined the problem solving behaviors of the elementary school students in the 3$^{rd,}$ 4$^{th}$ and 5$^{th}$ grades and determined what differences are displayed by students who are successful and unsuccessful in problem solving according to these behaviors. Moreover, he also investigated the relationship

between 3rd to 5th grade students' attitudes towards mathematics and their success of problem solving. A high number of the following behaviors were identified in the 3rd, 4th and 5th grade students: writing what is given and asked, making a drawing related to the problem, writing operations that will be used in the solution respectively, doing operations respectively and solving the problem. However, they rarely showed the following behaviors: guessing the result of the problem, checking the result, and writing a similar problem. Another result is that students rarely performed behaviors such as writing a brief summary of the problem and solving the problem in alternative ways. After determining the frequencies of these behaviors, the researcher carried out an experimental study and applied treatment on these behaviors which are critical for problem solving but not performed by students. He found that the behaviors; writing what is given and asked, writing a brief summary of the problem, telling which operations will be used in the solution respectively, doing operations respectively and solving the problem are critical behaviors for 3rd grade students and they are able to learn these behaviors. In addition to these behaviors, making a drawing related to the problem is critical for 4th grade students and these students can learn this behavior. Furthermore, solving the problem in alternative ways is critical for 5th grade students and they can also learn this behavior. The last result of the study displays that the relationship between students' attitudes towards mathematics and their success of problem solving increases from 3rd to 4th grade but it decreases from 4th to 5th grades.

Similarly, Yazgan and Bintaş (2005) investigated 4th and 5th grade students' learning and utilization of problem solving strategies. It was an experimental study and the participants of the study were chosen from 4th and 5th grade students in Bursa. The researchers worked on specific strategies which are guess and check, looking for a pattern, making a drawing, working backward, simplifying the problem and making a systematic list. While students in the control group continued their traditional courses, these strategies were taught to the experimental group and students were asked to solve each problem by using these strategies. In addition, a pretest, posttest and retention test were carried out during the study. The results showed that 4th and 5th grade students can informally use problem solving strategies

without any training and 4th and 5th grade students can learn strategies. Moreover; training had a positive effect on students' problem solving success.

In another study, Altun and Arslan (2006) designed an experimental study in which a learning environment is developed in order to help 7th and 8th grade students acquire metacognitive strategies to solve nonroutine mathematical problems. The aim of the study was to investigate which strategies can be learned and at which level by 7th and 8th grade students. Specifically, "Simplify the Problems", "Guess and Check", "Look For a Pattern", "Drawing a Picture", "Making a Systematic List" and "Working Backward" strategies were chosen and taught according to Polya's problem solving stages. Classroom activities included a short whole-class introduction, heterogeneous group studies and a final whole-class discussion on the given problem. The researchers analyzed the results strategy by strategy and calculated the frequency at which 7th and 8th grade students utilized these strategies before and after treatment. The results are similar to what Yazgan and Bintaş found in 2005 for 4th and 5th grade students. They found that 4th and 5th grade students can informally use problem solving strategies without any training and 4th and 5th grade students can learn strategies. Similarly, 7th and 8th grade students informally use problem solving strategies without any training and training had a positive effect on students' implementation of problem solving strategies. Thus, the authors claim that problem solving strategies can be learned at these ages.

More recently, Okur (2008) explored students' strategies, episodes and metacognitions in the context of PISA 2003 mathematical literacy items. He investigated the problem solving strategies, episodes and metacognition of five fairly new Turkish graduates from elementary school and examined the effect of these factors on success of problem solving. The results of the study reveal that problem solving behaviors of the students in correspondence to their academic success levels and the problem solving success is too complex to be explained by a unique property or a behavior of the problem solver. Since problem solving necessitates different obstacles to reach a successful answer, the researcher claims that in addition to prerequisites like adequate mathematical knowledge and experiences with different problem solving strategies, students should know when and how to use these

strategies and they could manage their problem solving processes by using their metacognitive skills.

In her thesis, Özkaya (2002) investigated tenth grade students' problem solving strategies in geometry. She examined the effect of gender and cognitive style on students' problem solving strategies and the relationship between the attitude towards problem solving and students' problem solving strategies. The results showed that students mostly preferred to use conventional strategies like trial-and-error or direct calculation in open-ended geometry problems, but they prefer unconventional strategies like estimation or deriving an equation or procedure according to the results of the Problem Solving Strategy Preference Scale (PSSPS). Moreover; results revealed that there is a significant difference in conventional strategy use favoring boys in geometry tests including open-ended geometry problems while there is a significant difference favoring girls in PSSPS. Furthermore, there was a significant relationship between unconventional strategy use in geometry tests and attitudes towards problem solving while there was no significant relationship between neither conventional nor unconventional strategy use in PSSPS and attitudes towards problem solving.

Albayrak, Ipek and Isık (2006) investigated at what level primary teachers use problem designing and solving activities in the process of teaching basic operation skills (addition, subtraction, multiplication, division and fractions) and determined prospective teachers skills in the related topic. The researchers carried out their investigation on 108 prospective teachers and the teachers working in the classes the prospective teachers were observing and teaching at. They analyzed their results in the light of properties defined by Marton (1955). According to Marton (1955) a good problem must contain the following properties. The first property is *reality* which says a problem should fit students' level and experiences in their daily-life. The second property is *attention* which maintains that a problem should trigger curiosity of students. The third one is *language* which explains that both written and verbal expressions of a problem should match with students' level. The last property is *using basic skills* which assert that a problem should enable students to use basic skills previously acquired. Albayrak, Ipek and Isık (2006) found that teachers were inadequate in their application of problem designing and solving activities in the

process of teaching basic operation skills and prospective teachers had not sufficiently acquired the skills required to design and solve problems in the process of teaching basic operation skills.

Some studies conducted in our country on problem solving for both elementary and secondary schools, and for prospective teachers. Problem solving behaviors (Altun, 1995), learning and utilization of problem solving strategies (Okur, 2008; Arslan & Altun, 2007; Özsoy, 2007; Karatas &Güven, 2004; Yazgan &Bintas, 2005; Özkaya, 2002), problem designing and solving activities in the process of teaching mathematics (Albayrak, Ipek & Isık, 2006; Iskenderoglu, Akbaba & Olkun, 2004) were examined. However; not many studies have been conducted about the relationship of problem solving approach explained in the new curriculum and mathematics achievement. Thus, the current study contributes to the literature in the context of the problem solving studies embedded in the new curriculum which investigates the relationship between students' problem solving achievement scores that were assessed by an alternative assessment technique and their mathematics achievement scores.

### 2.5. Mathematics Achievement

Mathematics achievement is a prominent variable and many studies have been conducted to investigate the affect of many variables on it. Some studies have also been conducted to examine the relationship of various variables with mathematics achievement. In the following paragraphs, some of these studies and their results will be shared.

First of all, the extent of the students' opportunity to learn mathematics content has been found to support their mathematics achievement (Grouws & Cebulla, 2000). In other words, there is a strong positive relationship between students' opportunity to learn, scores and mean student achievement scores in mathematics (Husen, 1967; McKnight et al, 1987; Schmidt, McKnight & Raizen, 1997). In fact, students' achievement improve when students are given opportunities to discover and invent new knowledge and practice what they have learned (Grouws & Cebulla, 2000).

Secondly, there is a positive relationship between total time allocated to mathematics and general mathematics achievement (Keeves 1976). Furthermore; there are also research studies that indicate a strong relationship between student achievement and the mathematics courses they attended at secondary school level (Grouws & Cebulla, 2000). NAEP mathematics reports (1992) illustrate that 'the number of advanced mathematics courses taken was the most powerful predictor of students' mathematics performance after adjusting for variations in home background'.

Moreover, research studies support that there is a significant improvement on students' achievement and understanding when teachers know about how students construct knowledge, are acquainted with solution methods that students use when they solve problems, and utilize this knowledge when planning and conducting instruction in mathematics (Grouws & Cebulla, 2000). Grouws and Cebulla (2000) also revealed that when the instruction is given formed around carefully chosen problems, students were allowed to interact with the problem and they were given opportunities to share their solution methods, achievement on problem-solving measures increases. Significantly, in addition to these gains, there is no loss of achievement in the skills and concepts measured on standardized achievement tests.

Similarly, in Turkey, there are some studies on mathematics achievement. First of all; effects of different teaching methods on mathematics achievement were examined (Duatepe-Paksu & Ubuz, 2009; Pilli, 2008; Şişman, 2007; Teltik-Başer, 2008; Yıldız, 2008).

Pilli (2008) investigated the effects of a computer software on 4$^{th}$ grade student's mathematics achievement. An experimental study was designed by the researcher where the control group consisted of 26 students and the experimental group consisted of 29 students from a primary school in Gazimagusa, North Cyprus. While the control group was taught using a lecture-based traditional instruction; the experimental group was taught using educational software, namely *Frizbi Mathematics 4*. The study was conducted in 2006-2007 academic year and included three units, Multiplication of Whole Numbers, Division of Whole Numbers, and Fractions. The results of the study indicated significant difference between the groups on the post achievement tests in favor of experimental group.

In another study, Yıldız (2008) investigated the effect of learning "Rate, Proportion and Percentage" unit with Project Based Learning (PBL) at 7th grades to mathematic success and behaviour. This study was implemented with 70 7th grade students of a primary school in Eminönü district in İstanbul. It was an experimental study and while the control group was taught classical method, the experimental one was instructed by Project Based Learning. The results of the study depending on the findings were summarized by the researcher as fallowing: PBL Approach is more effective than Classical Approach in students' success of mathematics and gaining positive behaviors towards mathematics. Moreover; in the teaching of "Rate, Proportion and Percentage" unit PBL Approach is more effective than Classical Approach in gaining target behaviors of the unit and lastly the results showed that the use of PBL Approach don't differentiate according to student sex.

Duatepe-Paksu and Ubuz (2009) are other researchers who studied with 7th grade students and investigated the effect of drama-based instruction on students' geometry achievement, geometric thinking level, attitudes toward mathematics and geometry, and retention of achievement, in comparison with traditional teaching. The sample involved 102 7th grade students from a public school. The results of the study revealed that drama-based instruction had a significant effect on students' achievement, retention of achievement, thinking level, and attitudes, regardless of gender, mathematics grade in previous year, and prior attitudes and thinking levels. Thus, Drama-based instruction made learning easy and understanding better since students were provided with the opportunity to contextualize geometric concepts and problems, act as a character (role playing), and communicate and study in a collaborative learning environment.

In her thesis, Teltik-Başer (2008) aimed to compare the application of teaching activities of 5E learning model based on constructivist approach with traditional teaching methods for teaching of circle and cylinder topics. 52 7th grade students from a primary school of Keçiören in Ankara participated in the study. It was an experimental study and while the control group was instructed by traditional teaching methods, the experimental group was taught by activities planned according to 5E learning model through 6 weeks. Results of the study revealed positive significant difference of achievement in favor of experimental group.

Regarding mathematics achievement; some other studies were also conducted in Turkey. For instance; Akyüz (2006) investigated the effects of teacher and class characteristics on mathematics achievement across Turkey, European Union countries by analyzing the data collected from student and teacher background questionnaires and mathematics achievement test in the Third International Mathematics and Science Study (TIMSS-R). Results of the study showed that the factors that had significant effect on mathematics achievement were different across countries and mean of home educational resources was the only factor that had positive significant effect on students' mathematics performance in all the countries. In Turkey, the classes of male teachers were more successful and teacher experience, time spent on tests and quizzes, use of textbooks, disciplined class climate and class mean of home educational resources were found to have positive significant effect on student achievement.

There were also studies in the literature that investigate the effects of problem solving approach on achievement in various areas of mathematics and the relationship among two variables (Ceylan, 2008; Fidan, 2008; Özsoy, 2007; Yıldız, 2008).

Ceylan (2008) investigated the relationship between the scores of the 6$^{th}$ grade primary school students at daily-life problem solving inventory and their performance at mathematical problem solving. The study was carried out in four different schools in Keçiören, Çankaya and Yenimahalle districts of Ankara. Two of the schools were state-owned while the other two were private schools. It was a correlational study and 209 students participated in the study. Two tests were utilized during the data collection. One of these tests was Problem Solving Achievement Test which includes 30 multiple-choice questions and developed by the researcher. The second one is the inventory developed by Heppner and Petersen (1982). The results of the study showed that there is a highly significant correlation between mathematics test scores and the scores of daily-life problem-solving inventory.

Ozsoy (2007) is another researcher who studied with 5$^{th}$ grade students and investigated the effect of being thought to use metacognition strategies on 5$^{th}$ grade students' mathematical problem solving achievement. A pretest-posttest control group experimental study design was used in the study in which experimental group

group was taught by metacognitive problem solving activities for nine weeks to improve their metacognitive knowledge and the control group continued to their current lessons. The results of the study indicated that students in the experimental group significantly improved in both problem solving achievement and metacognitive skills, and this improvement was more than the control group's either problem solving achievement or metacognitive skills. Moreover, "devising a plan" scores of problem solving achievement increased more than the other subcategories of problem solving process for the experimental group. The students in the control group did not show any improvement significantly.

Moreover, Fidan (2008) investigated the effect of the teaching through problem posing on students' problem solving success. In addition, she examined the effect of problem posing studies on students' success in Polya's problem solving steps. The researcher designed an experimental study and 48 $5^{th}$ grade students from an elementary school in Emirdağ district of Afyon participated in the study. According to the statistical analysis of results, a positive significant difference was found in favor of experimental group students' problem solving success. However; the results revealed that there was no significant difference among experimental and control group results according to Polya's problem solving steps.

In another study, Yıldız (2008) investigated the change in $6^{th}$ grade students' problem solving abilities after mathematics instruction based on Polya's problem solving steps. It was a weak experimental study since there was no control group. 53 students from an elementary school in İstanbul participated in the study. These students were instructed according to Polya's problem solving steps and the researcher found that instruction based on Polya's steps significantly affected students' problem solving abilities in a positive way.

Some studies which investigated the effects of problem solving approach on achievement in various areas of mathematics and the relationship among two variables (Ceylan, 2008; Fidan, 2008; Özsoy, 2007; Yıldız, 2008) were mentioned above. However, while both problem solving and mathematics achievement have been studied in depth over the years, there have been very few research studies completed on the possible connection between two in relation with the new curriculum. Thus, the current study contributes to the literature in the context of the

problem solving studies embedded in the new curriculum which investigates the relationship between students' problem solving achievement scores that were assessed by an alternative assessment technique and their mathematics achievement scores. In addition, this study also aimed to investigate the relationship between students' problem solving achievement scores and their actual mathematics achievement net scores obtained from SBS to see the relation among students' problem solving success with their achievement on a nationwide exam. On the other hand, one of the aims of investigating the relationship between students' problem solving achievement scores and their actual mathematics achievement net scores obtained from SBS was to see the retention of problem solving achievement scores.

Next section will give detailed information about the nature of the nationwide exam (SBS) mentioned above.

### 2.5.1. Level Determination Exams (SBS)

In Turkey, although the names and structures of exams display differences, a national exam has been applied at the end of the elementary school to select and place students to secondary education institutions. OKS is one of these national level determination exams that was last applied in 2008. OKS is a two-hour multiple choice exam that is administered at the end of the academic year to $8^{th}$ grade students. In October 2007, Turkish Ministry of Education declared that OKS would be replaced by three Level Determination Exams (SBS) rather than the one shot OKS which would be applied at the end of $6^{th}$, $7^{th}$ and $8^{th}$ grades. The most significant reason for the change was announced as to keep step with the curriculum reforms being applied since 2003. It was explained that the new perspective of the curriculum requires the assessment of learning processes; however, only the products had been assessed by OKS (MoNE, 2007). In addition, OKS is a selection exam but it is announced that the aim of SBS is not selecting and ranking students; it would be an exam that would be used to determine the level at which students had achieved the objectives of that years' curriculum (MoNE, 2007).

Other reasons have also been reported explaining the reason for the change from OKS to SBS. For instance, OKS is defined as a one-session exam that does not enable students to have any alternatives. If this two-hour exam finishes successfully,

the students have the chance to go a better secondary school. If it does not, there is no alternative. Unlike LGS (the national student selection and placement exam for secondary schools applied before OKS) and SBS; OKS is an exam in which students are selected and placed in a secondary school only according to the score they get from the exam. In LGS; students' weighted average points taken from $4^{th}$ grade to $7^{th}$ grade partially affected the total score taken from the exam but in OKS this situation did not continue. This situation is believed to make both students and parents see the exam as their only target and cause anxiety, stress and tension on them (MoNE, 2007). The three level structures of the SBS are supposed to prevent parents and students from these negative sensations. Moreover, it is claimed that SBS will trigger the system to focus on school education instead of the exam (MoNE, 2007). Ministry of Education makes such a comment because not only the score taken from the SBS-exam, but also the academic achievement and behaviors of students at school affect the total score of that year's SBS (MoNE, 2007). In addition, students usually go to Test Preparation Centers (*dersane)* which are private education institutions that provide students with courses and lots of multiple choice questions to prepare them for the national exams like OKS. Since their only target is generally to obtain a high score from the OKS exam, students give more importance to studies and instruction offered at these centers. However, with the properties mentioned above it is expected that the structure of SBS will reduce the importance of out-of school institutions like test preparation centers and students will pay more attention to school lessons and the grades they obtain from school exams (MoNE, 2007). Another difference among OKS and SBS is that OKS contains only questions from 4 subject areas which are Turkish, Mathematics, Science and Social Science. However, SBS includes questions from foreign language lessons in addition to questions to the mentioned four content areas.

The nature of the SBS is explained above. Moreover; this chapter presented the related literature about problem solving and mathematics achievement. In the next chapter, the methodology of the study is briefly described.

# CHAPTER 3

# METHODOLOGY

The goal of this chapter is to give information about the research design, population and sample, data collection instruments, reliability and validity of the study, data collection procedure, analyses of data, assumptions and limitations, and lastly the internal and external validity of the study.

## 3.1. Design of the Study

The purpose of this research study was to investigate the relationship between $6^{th}$ grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores related to LCM, GCF, Sets and Whole Numbers topics obtained throughout the semester. In addition, the relationship between $6^{th}$ grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from SBS exam was investigated. In this research design, variables that are $6^{th}$ grade students' problem solving achievement scores after completing instruction on problem solving, their mathematics achievement mean scores obtained throughout the semester related to LCM, GCF, Sets and Whole Numbers topics, and their actual mathematics net scores obtained from SBS exam were studied without any attempt to influence them (Fraenkel & Wallen, 2006). Thus, quantitative methods were used to examine the research questions and a correlational design was used. In correlational research there is no manipulation of variables. A correlation is believed to exist between two variables, if the scores within a particular range on one variable are associated with scores within a particular range on another variable (Fraenkel & Wallen, 2006).

### 3.2. Population and Sample

In this study, all 6th grade private school students taking the SBS exam at the end of the academic year in Turkey were identified as the target population. The accessible population of this study was determined as all 6th grade private school students taking the SBS exam at the end of the academic year in Istanbul. Convenience sampling method was used to obtain the sample of the population who are 6th grade students of a private school in Istanbul. Since the purpose of the research study was to investigate the relationship among 6th grade students' problem solving achievement scores with their mathematics achievement mean scores and their actual mathematics scores obtained from SBS, the attendance of participants to all Problem Solving Achievement Tests (PSATs), all Mathematics Achievement Tests (MATs) and the SBS exam were needed. Thus, the students who did not attend at least one of these tests were eliminated from the sample. As a result; 170 out of 256 6th grade students in this private school constituted the participants of this study. In other words; 66 % of the sixth grade students of this private school participated in the study.

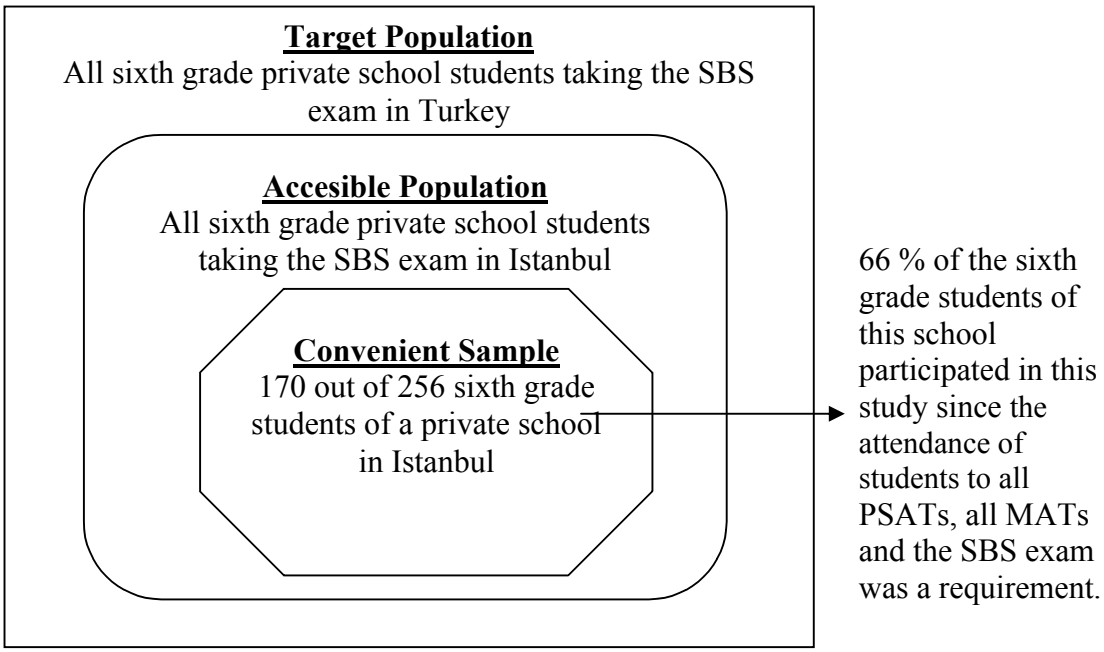



**Figure 3.1** Sampling Procedure of the Study

Moreover, the gender distribution of sample shows that the sample includes 87 male (51 %) and 83 female (49 %) sixth grade students. As can be seen there was an almost equal distribution between male and female participants in the study.

Table 3.1 given below also shows gender distributions of all students of each class and the number of students who did not participate in the study because he/she did not attend at least one of the PSATs or MATs. No elimination needed to be done according to students' attendance of SBS exam because all of the students attended SBS exam at the end of the year. As can be seen from the Table 3.1, 86 students who did not participate in the study were from different classes. Thus, we could conclude that students who participated in the study represent the whole 6th graders in this private school. The subjects were lost from not only one class, but all classes.

**Table 3.1** Distributions of Students from Each Class in terms of Gender

| Class | Total Number of Boys | Total Number of Girls | Number of students **not** participating in the study |
|---|---|---|---|
| 6A | 16 | 6 | 5 |
| 6B | 10 | 13 | 6 |
| 6C | 9 | 14 | 6 |
| 6D | 16 | 7 | 10 |
| 6E | 10 | 9 | 9 |
| 6F | 8 | 9 | 7 |
| 6G | 9 | 8 | 5 |
| 6H | 19 | 5 | 10 |
| 6I | 12 | 10 | 10 |
| 6J | 11 | 12 | 6 |
| 6K | 5 | 16 | 5 |
| 6L | 16 | 6 | 7 |
| Total | 141 | 115 | 86 |

Additionally, the school where the participants of this study were selected is a private school. Thus, socio-economic status of families of the participants can be rated as very high. As it is mentioned before, 170 out of 256 6th grade students in this private school participated in the study where 201 of them were students of the

same school from 5th grade and 55 of them started the school at the beginning of the 6th grade having performed successfully in the exam applied by the school.

### 3.3. Instruments

As mentioned above, the purpose of this study was to investigate the relationship between 6th grade students' problem solving achievement scores having attended classes designed for problem solving and their mathematics achievement mean scores obtained throughout the semester related to LCM, GCF, Sets and Whole Numbers Topics. Furthermore, the relationship between 6th grade students' Problem Solving Achievement Scores and their actual SBS scores was investigated. Thus, the data were collected via three sources namely; Problem Solving Achievement Tests (PSATs), Mathematics Achievement Tests (MATs) and SBS exam.

#### 3.3.1. Problem Solving Achievement Tests (PSATs)

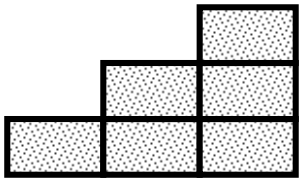
Problem Solving Achievement Tests (PSATs) were used to measure problem solving achievement scores of 6th grade students after completing instruction on problem solving in the contents of Whole Numbers, Sets and Least Common Multiple (LCM) and Greatest Common Factor (GCF). These contents were chosen for study because they were the first three subjects that 6th grade students were taught through problem solving according to the yearly plan. Three Problem Solving Achievement Tests (PSATs) were prepared by four mathematics teachers working in the private school who were teaching 6th graders during the 2007-2008 academic year and with an expert from the assessment and evaluation unit of the same school. The same group of people where one of the mathematics teachers is the researcher of the study designed the class activities for problem solving in the topics of Whole Numbers, Sets and LCM and GCF in their routine meetings held twice a week. The classes were prepared in the light of Polya's problem solving steps. Students were provided with mathematical problems involving scenarios taken from real-life situations during the lessons and they were expected to solve the problems according to Polya's problem solving steps. All of the PSATs were implemented just after completing instruction on problem solving. Details of problem solving classes will be discussed in the data collection section but it is important to mention that PSAT-1

was implemented after problem solving classes related to Whole Numbers; PSAT-2 was applied after problem solving classes related to Sets and PSAT-3 was carried out after problem solving classes related to LCM and GCF. Students were provided with open-ended real life problems in PSATs and they were expected to solve the problems according to Polya's problem solving steps. Details of the PSATs are explained below.

#### 3.3.1.1. Problem Solving Achievement Test-1 (PSAT-1)

PSAT-1 was the test prepared for measuring students' problem solving achievement scores after completing instruction on problem solving related to Whole Numbers. PSAT-1 includes 4 real-life context problems and students were expected to solve these problems according to Polya's problem solving steps. Figure 3.2 illustrates one of the problems from this test.

**Problem:** Yandaki şekilde Mert'lerin kapısındaki merdivenin ilk 3 basamağı görülüyor. Bu merdiven 9 basamaklı olduğuna göre; merdivenin yapımında **toplam** kaç tuğla kullanılmıştır?



- ❖ Problemi Anlayalım
- ❖ Plan Yapalım
- ❖ Planı Uygulayalım
- ❖ Kontrol Edelim

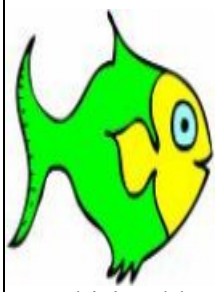
**Figure 3.2** An Example Problem from PSAT-1

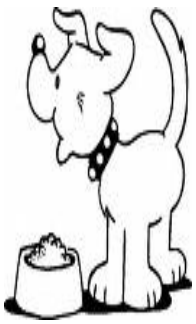
Students are expected to use *finding a pattern* and *making drawing strategies* to solve the problem given in Figure 3.2. While preparing the problems, it was taken into consideration that each problem could be solved by using different problem solving strategies or a combination of them.

### 3.3.1.2. Problem Solving Achievement Test-2 (PSAT-2)

PSAT-2 was the test prepared for measuring students' problem solving achievement scores after completing instruction on problem solving related to Sets. PSAT-2 includes two real-life context problems and students were expected to solve these problems according to Polya's problem solving steps. Figure 3.3 illustrates one of the problems from this test.

**Problem:** Arzu; 28 kişilik sınıfında evcil hayvanlar ile ilgili bir anket uygular. Anketin sonuçlarına göre; bu sınıfta köpek veya balık dışında beslenen evcil hayvan yoktur. Evinde köpek besleyen 9 kişi, hem köpek hem balık besleyen 6 kişi vardır. Evinde hiç hayvan beslemeyen 10 kişi olduğuna göre, bu sınıfta evinde sadece balık besleyen kaç kişi bulunmaktadır?





- ❖ Problemi Anlayalım
- ❖ Plan Yapalım
- ❖ Planı Uygulayalım
- ❖ Kontrol Edelim

**Figure 3.3** An Example Problem from PSAT-2

Students are expected to use visual representations (Venn Diagrams) to solve the problems in PSAT-2. Thus, they are expected to use *Making a drawing* strategy in order to devise a plan as part of finding the solution.

### 3.3.1.3. Problem Solving Achievement Test-3 (PSAT-3)

PSAT-3 was the test prepared for measuring students Problem Solving Achievement Scores after they had attended classes designed for problem solving related to Least Common Multiple (LCM) and Greatest Common Factor (GCF).

PSAT-3 includes four problems and like in PSAT-1 and PSAT-2, students were asked to solve the problems in PSAT-3 according to Polya's problem solving steps. Figure 3.4 illustrates one of the problems of this test.

**Problem:** Dikdörtgen şeklindeki bir havuzun tabanının kenar uzunlukları 6m ve 8m'dir. Havuzun tabanı mümkün olan **<u>en büyük kare şeklindeki taşlarla</u>** döşenecektir. Havuzun tabanını döşemek için kaç tane taş gereklidir?

- ❖ Problemi Anlayalım
- ❖ Plan Yapalım
- ❖ Planı Uygulayalım
- ❖ Kontrol Edelim

**Figure 3.4** An Example Problem from PSAT-3

Students are expected to use *making a drawing* and *logical reasoning* strategies to solve the problem given in figure 3.4. Moreover; as part of the devising a plan part of the solution they are expected to decide whether they would use LCM or GCF to solve the problem or not.

Each of the PSATs was assessed by using a rubric developed by four mathematics teachers and an expert from the assessment and evaluation unit of the school. Students' problem solving achievement scores are the mean score of the scores obtained from PSAT-1, PSAT-2 and PSAT-3.

The assessment process of each PSAT contains two parts. In the first part, four teachers randomly take 1 exam paper from each class (totally 12 papers) and assess these papers together just after the application of each PSAT. PSAT-1 and PSAT-3 which contain 4 problems were assessed out of 100. PSAT-2 contained 2 problems; as a result, each problem was scored over a grade of 50. In the second part, the rest of the papers were assessed by the teacher of the class based on the rubric prepared by

the teachers. However, if a teacher met with an exceptional situation that could not be related to the answer sheet, she did not assess it herself. Instead, it was discussed by the other teachers and scored upon collaborative consensus. Furthermore, not only the total scores, but also the partial scores obtained from the problem solving steps of each student were recorded according to the rubric given in Appendix A. The partial score distribution of each problem of PSATs is illustrated in table 3.2 given below.

**Table 3.2** The Partial Score Distribution of Each Problem in PSATs

| Problem Solving Steps | Score |
|---|---|
| Understanding the problem | 6 |
| Devising a plan | 2 |
| Carrying out the plan | 15 |
| Looking Back (Checking) | 2 |
| Total | 25 |

Thus, the total point gained from each PSAT is the sum of the partial points taken from the problem solving steps of each problem and as it is mentioned before, students' problem solving achievement scores are the mean score of the scores obtained from PSAT-1, PSAT-2 and PSAT-3. All of these data were recorded on the rubric prepared on an excel sheet.

Additionally, some formulas were defined on the excel sheet to convert students' partial scores taken from each problem solving step to scores 0, 1 and 2. This process was repeated just after all problem solving instructions based on the topics Whole Numbers, Sets and LCM, GCF respectively. The main purpose of this conversion is to determine the cognitive differences occur from the beginning of the instruction to the end of it based on problem solving steps. Thus, teachers could easily follow not only individual cognitive differences but also whole class' or whole participants' cognitive differences by utilizing the rubric and also graphs generated from these data. For instance, the graph given in chapter 4.1.1 illustrates the cognitive differences process of whole participants based on problem solving steps on different topics.

### 3.3.2. Mathematics Achievement Tests (MATs)

Achievement tests are used to measure the knowledge or skills of a particular person in a specific content area where schools usually use these tests to measure students' learning and determine the effectiveness of instruction (Fraenkel & Wallen, 2006). Mathematics Achievement Tests (MATs) used in this study were the mathematics part of the specific general achievement tests applied in a private school where the`study was carried out.

In the 2007-2008 academic year, 8 general achievement tests were applied in this school. Each general achievement test contains four main content areas namely; Turkish Achievement Test (TAT), Mathematics Achievement Test (MAT), Social Science Achievement Test (SSAT), and Science Achievement Test (SAT). Moreover, each area consists of multiple choice questions each with four choice alternatives. These tests were prepared by the content area teachers and specialists from the assessment and evaluation unit of the school. Table 3.3 illustrates the number of questions in each area of the general achievement test.

**Table 3.3** The Number of Questions in Each Area of the Achievement Tests

| Achievement Tests | The number of questions |
|---|---|
| Turkish Achievement Test | 19 |
| Mathematics Achievement Test | 16 |
| Social Science Achievement Test | 16 |
| Science Achievement Test | 16 |

In this study, Mathematics Achievement Tests (MATs) were used to measure $6^{th}$ grade students' mathematics achievement mean scores obtained throughout the semester related to the Whole Numbers, Sets, LCM, and GCF units. The PSATs that were mentioned above included open-ended real-life context problems and students were expected to solve these problems according to Polya's problem solving steps. However; MATs consisted of multiple choice questions having four choice alternatives. The aim of the application of MATs is to determine students' achievement on Whole Numbers, Sets, LCM, and GCF unit problems with a multiple choice exam. Furthermore, they were applied to see whether the students who are

successful in open-ended tests like PSATs would be successful in multiple choice exams like MATs. Like PSATs, MATs are also prepared by four mathematics teachers and a specialist from the assessment and evaluation part of the school.

As indicated in Table 3.3 MATs consist of 16 questions. Eight MATs were applied throughout the semester and these tests include questions from all the mathematics topics that students learned until that achievement test. Thus, not all of the MATs consist of questions from all the contents involving Whole Numbers, Sets, LCM and GCF. Students were taught Whole Numbers, Sets, LCM and GCF respectively according to the yearly plan. Thus, the previous MATs did not include questions from the following content areas. Table 3.4 shows both the application date of MATs and the number of questions that each MATs include related to Whole Numbers, Sets, LCM, and GCF units. Figure 3.5, on the other hand, illustrates some example questions of MATs related to Whole Numbers, Sets, LCM, and GCF units.

**Table 3.4** The Number of Questions MATs Include

| Application Data | Test Name | Total # of Questions | # of Questions in Specific Content Areas | | | Total # of Questions related to topics Whole Numbers, Sets, LCM and GCF |
|---|---|---|---|---|---|---|
| | | | Whole Numbers | Sets | LCM &GCF | |
| 02.11.2007 | MAT-1 | 16 | 5 | 2 | - | 7 |
| 03.12.2007 | MAT-2 | 16 | 2 | 1 | - | 3 |
| 26.12.2007 | MAT-3 | 16 | 2 | 2 | - | 4 |
| 21.01.2008 | MAT-4 | 16 | 1 | 1 | - | 2 |
| 03.03.2008 | MAT-5 | 16 | 1 | 1 | - | 2 |
| 03.04.2008 | MAT-6 | 16 | 1 | - | 1 | 2 |
| 05.05.2008 | MAT-7 | 16 | - | - | 1 | 1 |
| 04.06.2008 | MAT-8 | 16 | - | 1 | - | 1 |

1) Meltem Hanım, marketten ton balığı alacaktır. Ton balıkları aşağıdaki gibi farklı seçeneklerde satışa sunulmuştur. **Meltem Hanım hangi seçeneği tercih ederse <u>en ucuz</u> ton balığını almış olur?**

A) 
100 gr
4 YTL

B) 
2 x 50 gr
6 YTL

C) 
3 x 50 gr
6 YTL

D) 
4 x 50 gr
6 YTL

2) Bir çiftçi, çiftlikten aldığı bir sepet yumurta ile yürürken, yolda bir adam ona çarpar ve tüm yumurtaları kırılır.

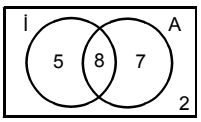

**Buna göre sepette kaç yumurta vardı?**

A) 60 B) 172 C) 180 D) 190

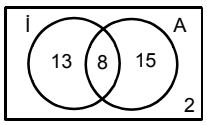
**Figure 3.5** Example Problems from MATs

3) Bir takımdaki futbolcuların 13'ü İngilizce, 15'i Almanca, 8'i her iki dili de bilmektedir. **Bu futbolculardan 2'si iki dili de bilmediğine göre, aşağıdaki gösterimlerden hangisi doğrudur?**

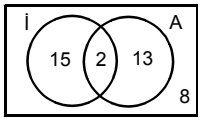
A)




B)




C)




D)

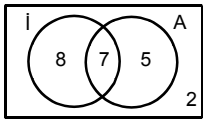



**Figure 3.5 (continued)** Example Problems from MATs

As mentioned before, Mathematics Achievement Tests (MATs) were used to measure 6$^{th}$ grade students' mathematics achievement mean scores obtained throughout the semester related to the Whole Numbers, Sets, LCM, and GCF units. The assessment process of each MATs contains two parts. In the first part, each of the MATs was assessed by using an optical reader since they consisted of multiple choice questions each following with four choice alternatives. In the second part, each student's achievement percentage of each topic was determined for each test. For example, MAT-1 contains 5 questions related to Whole Numbers and 2 questions related to Sets. If student A answered 4 of the 5 questions related to Whole Numbers correctly, his/her achievement was accepted to be 80 % for Whole Numbers and if he/ she answered all of the questions related to Sets correctly, his/her achievement was accepted as 100 % for Sets. This process was completed for each MATs and at the end of the application of 8 MATs throughout the semester, achievement mean scores of each student were determined for each topic. Moreover, students' general mathematics achievement scores related to the Whole Numbers, Sets, LCM, and

GCF units were determined by calculating the mean score obtained from the three tests separately.

### 3.3.3. Level Determination Exam (SBS)

In this study, Mathematics subtest of Level Determination Exam **(**SBS) was used to measure 6th grade students' actual mathematics net scores at the end of the semester. The SBS is a national three Level Determination Exam which is applied at the end of the 6th , 7th and 8th grades to select and place students into secondary education institutions. The SBS exam contains questions from five content areas which are Turkish, Mathematics, Social Sciences, Science, and Foreign Language. These are all multiple choice items with four choice alternatives. The number of questions each sub-test involves is illustrated in Table 3.5. At the end of each academic year middle grade students enter SBS exams and receive a score from this exam. Moreover, not only the score obtained from the SBS-exam, but also the academic achievement and behaviors of students at school affect the total score of that year's SBS total score (MoNE, 2007).

**Table 3.5** The Number of Questions Each Subtest of SBS Contains

| Subtest | The number of questions |
|---|---|
| Turkish Achievement Test | 19 |
| Mathematics Achievement Test | 16 |
| Social Science Achievement Test | 16 |
| Science Achievement Test | 16 |
| Foreign Language Achievement Test | 13 |

The net score which can be taken from Mathematics test of SBS range from -5,33 to 16. If the student answers all the questions correctly, he/she would receive a score of 16. While determining the net score, three wrong answers cancel one correct answer and thus if a student answers all the questions wrongly he receive a score of -5.33 from the exam.

As previously indicated, in this study the Mathematics subtest of Level Determination Exam **(**SBS) was used to measure the 6th grade students' actual

mathematics net scores at the end of the semester since one of the aims of the study was investigating the relationship between 6th grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from the SBS exam.

### 3.4. Reliabilty and Validity Issues

In the following sections reliability and validity issues of PSATs, MATs and SBS exam will be discussed.

#### 3.4.1 Reliabilty and Validity of PSATs

Validity is appropriate, meaningful, correct and useful interpretations of any measurement (Fraenkel & Wallen, 2006). Thus, it is about the goal of the test and what it measures. Content-related evidence of validity is about how appropriate and comprehensive the test is. In addition to adequacy of sampling, it is concerned about the appropriate format of the test (Fraenkel & Wallen, 2006). Additionally, Construct-related evidence of validity is about the nature of psychological construct or characteristic being measured by the instrument. In this study, PSATs were used to measure problem solving achievement scores of 6th grade students after completing instruction on problem solving related to Whole Numbers, Sets and Least Common Multiple (LCM) and Greatest Common Factor (GCF) units. Table 3.6 illustrates the table of specification of each PSAT. In addition, the problems of the PSATs were written by four mathematics teachers and two other people one of which is a specialist on assessment and evaluation and the other is an instructor on mathematics education checked the questions according to appropriateness of objectives. Thus, it can be deduced that PSATs have content and construct related evidence of validity.

**Table 3.6.** Table of Specification of PSATs

| Unit | Objectives | | |
|---|---|---|---|
| | Solve and construct problems entail doing operations in Whole Numbers | Perform union, intersection, difference and complement operations in Sets and solve problems by using these operations | Determine common multiples and common factors of Whole Numbers and apply to problems |
| Whole Numbers | 1,2,3,4 | | |
| Sets | | 1,2 | |
| LCM and GCF | | | 1,2,3,4 |

Reliability is about consistency of the scores obtained from the instrument (Fraenkel & Wallen, 2006 ). Inter-rater reliability is the degree of agreement among raters and it gives a score of how much consensus is supplied by raters which is called scoring agreement (Fraenkel & Wallen, 2006). As mentioned before, in the evaluation process of PSATs, four teachers prepared a rubric together and randomly chosen 1 paper from each class (a total of 12 papers) and these 12 papers were evaluated together. The rest of the papers were assessed by the teacher of that class. All assessment was done according to the rubric prepared by the four teachers and exceptional situations were assessed through the 100 % consensus of four mathematics teachers. Thus, it can be deduced that PSATs have inter-rater reliability.

### 3.4.2 Reliability and Validity of MATs

It is important to use valid instruments because valid instruments provide the researcher with valid data. Thus, with the use of valid data, the researcher can reach valid conclusions and inferences (Fraenkel & Wallen, 2006). Like PSATs, MATs were also written by four mathematics teachers and a specialist on assessment and evaluation and an instructor on mathematics education checked the questions according to appropriateness of objectives. Thus, it can be deduced that MATs have content related evidence of validity. Moreover, internal consistency is a method of estimating reliability. It shows how different items of an instrument are alike or different (Fraenkel & Wallen, 2006). Cronbach's alpha coefficient is one of the most

common indicators of internal consistency (Pallant, 2001). Table 3.7 below illustrates reliability values of each MAT.

**Table 3.7.** Reliability of MATs

| | MAT-1 | MAT-2 | MAT-3 | MAT-4 | MAT-5 | MAT-6 | MAT-7 | MAT-8 |
|---|---|---|---|---|---|---|---|---|
| Reliability (Cronbach's alpha) | 0.79 | 0.76 | 0.72 | 0.70 | 0.69 | 0.82 | 0.80 | 0.80 |

The reliability of MATs range from .69 to .80. Fraenkel and Wallen (2006) explained that reliability values which are above .70 can be accepted as relatively high in social sciences. Thus, it can be said that reliability of MATs were high.

### 3.4.3 Reliability and Validity of SBS

As mentioned above, SBS is a nationwide three Level Determination Exam which is applied at the end of $6^{th}$, $7^{th}$ and $8^{th}$ grades to select and place students into secondary education institutions. Since it is a nationwide exam, the test is accepted as reliable and valid.

## 3.5. Data Collection Procedure

The purpose of this research study is to investigate the relationship between $6^{th}$ grade students' problem solving achievement scores following classes designed for problem solving and their mathematics achievement mean scores obtained throughout the semester related to LCM, GCF, Sets and Whole Numbers topics. In addition, the relationship between $6^{th}$ grade students' problem solving achievement scores following classes designed for problem solving and their actual mathematics net scores obtained from SBS exam are investigated.

The data were collected from $6^{th}$ grade students of a private school in Istanbul during the fall semester of the 2007-2008 academic year. First of all, official permissions were taken from the school and content areas to study were determined as Whole Numbers, Sets and LCM and GCF. As mentioned above, these content areas were chosen to study because they were the first three content areas that $6^{th}$

grade students were taught through problem solving according to the yearly plan. Teaching through real-life context mathematical problems according to the Polya's problem solving steps was the main purpose of the problem solving classes. The data obtained from PSATs was collected during mathematics lessons; the data obtained from MATs was collected during the application of these tests on periods determined by the school and lastly the data obtained from SBS was collected at the end of the semester.

There were 12 sixth grade classes and 4 mathematics teachers were teaching $6^{th}$ graders during the 2007-2008 academic year. The classes for problem solving were designed by these four mathematics teachers with an expert from the assessment and evaluation unit of the same school and applied to all sixth grades. The researcher is one of these mathematics teachers and she is also responsible for the curriculum development studies for $6^{th}$ grades during the whole academic year. In curriculum development studies, the responsible teacher and the expert on assessment and evaluation prepare several learning activities, materials, and their assessment instrument in their routine meetings held twice a week.

Before these meetings, all mathematics teachers teaching the same grade level at that academic year determined the content areas and objectives of the learning activities that would be prepared for each unit on the basis of previous years' experiences and assessment results. In other words, they determined the content areas and objectives that students would be compelled to learn. Then, the responsible teacher and the expert on assessment and evaluation started to work on these objectives and prepared learning activities, materials and assessment instruments according to these objectives. During the preparation process, all drafts were shared with the other teachers in their routine meetings held twice a week. Teachers' opinions were always taken into consideration and necessary revisions were done according to the consensus of the teachers since all teachers were expected to implement the same lesson plans in all classes.

Problem solving is one of these content areas that teachers identified as a content area to study in curriculum development studies. The activities, stories, materials and work sheets that were used during problem solving classes were

prepared in these meetings and applied in all classes. A time schedule for the application of problem solving activities is given in Table 3.8.

**Table 3.8.** A Time Schedule for the Data Collection Process from Problem Solving classes

| **Unit** | **Objective** | **Activity** | **Number of Lesson Hours** | **Application Date** |
|---|---|---|---|---|
| Whole Numbers | Solve and construct problems entail doing operations in Whole Numbers | "Math Problems in Daily Life" activity | 1 | 24.09.2007-06.10.2007 |
| | | The story called "Kızıl Ejderha" and worksheet | 2 | |
| | | "Problem Çözüyorum" worksheet | 5 | |
| | | PSAT-1 | 1 | |
| Sets | Perform union, intersection, difference and complement operations in Sets and solve problems by using these operations | "Kümeleri öğreniyorum" activity | 1 | 30.10.2007-09.11.2007 |
| | | "Kümelerle İşlem Yapıyorum" worksheet | 2 | |
| | | "Problem Çözüyorum" worksheet | 3 | |
| | | PSAT-2 | 1/2 | |
| LCM and GCF | Determine common multiples and common factors of Whole Numbers and apply to problems | "İp kesiyoruz" activity | 1 | 03.03.2008-13.03.2008 |
| | | "Çubuklarla Öğreniyoruz activity | 1 | |
| | | LCM-GCF Stations | 1 | |
| | | "Problem Çözüyorum" | 3 | |
| | | PSAT-3 | 1 | |

As shown in Table 3.8, problem solving classes for the Whole Numbers lasted 8 lesson hours, for Sets 6 lesson hours and LCM and GCF lasted for 7 lesson hours. In total, 21 lesson hours of application was implemented in all $6^{th}$ grade classes by four mathematics teachers. The details of problem solving classes are explained in the following section.

### 3.5.1. Problem Solving Classes

In the first hour of the problem solving classes related to Whole Numbers, students were asked "What does the word 'problem' represent for you?". Most of the students talked about problem solving in mathematics, doing operations, and their feelings and experiences about problem solving in mathematics. In addition, in some classes some students talked about daily life problems. If no one talked about daily life problems, the teacher asked questions such as "What about daily life problems? Don't we have any problems in daily life? What are they? " and continued "Are the problems that we are solving in mathematics lessons different from the ones that we meet in daily life?" A few minutes were given to students to think and discuss but the teacher made no comments herself. She guided the discussion with questions such as "What do we do to solve daily life problems? Do mathematics and learning to solve mathematics problems in class help us to deal with daily life problems?" Furthermore, the teacher wrote on the board that; "The sixth grade students of this school will go to a trip to Kapadokya. How do you think they will go?" Then she asked questions like "Is this a daily life problem or a mathematics problem? What do we need to know to solve this problem?" She wrote the answers of the students on the board. Students said they needed information such as, the number of the students, to decide on the vehicle of transportation, and the passenger capacity of the transportation vehicle, the number of teachers coming. Then the teacher wanted students to express this problem mathematically. They filled the figure illustrated in Figure 3.6 together on the board.

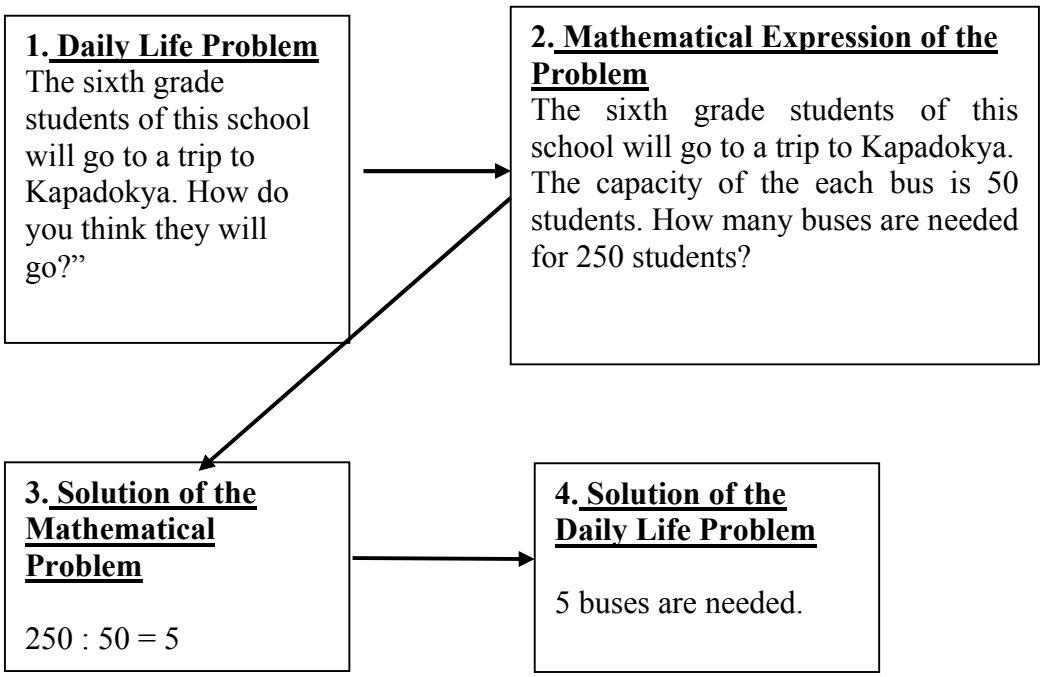



**Figure 3.6** Mathematical expression of a daily life problem

In the next step, students formed groups of two and were distributed a sheet of A3 paper. They were asked to find a daily life problem, express this problem mathematically and find the solution of their daily life problem with the help of a mathematical solution. Each group shared their work with the whole class. The aim of the "Math Problems in Daily Life" activity is make students be aware that mathematical problems are not independent from the problems that they meet in their daily lives and they could benefit from mathematical knowledge and skills to deal with real life problems. Figure 3.7 shows an example of students' work in this activity.

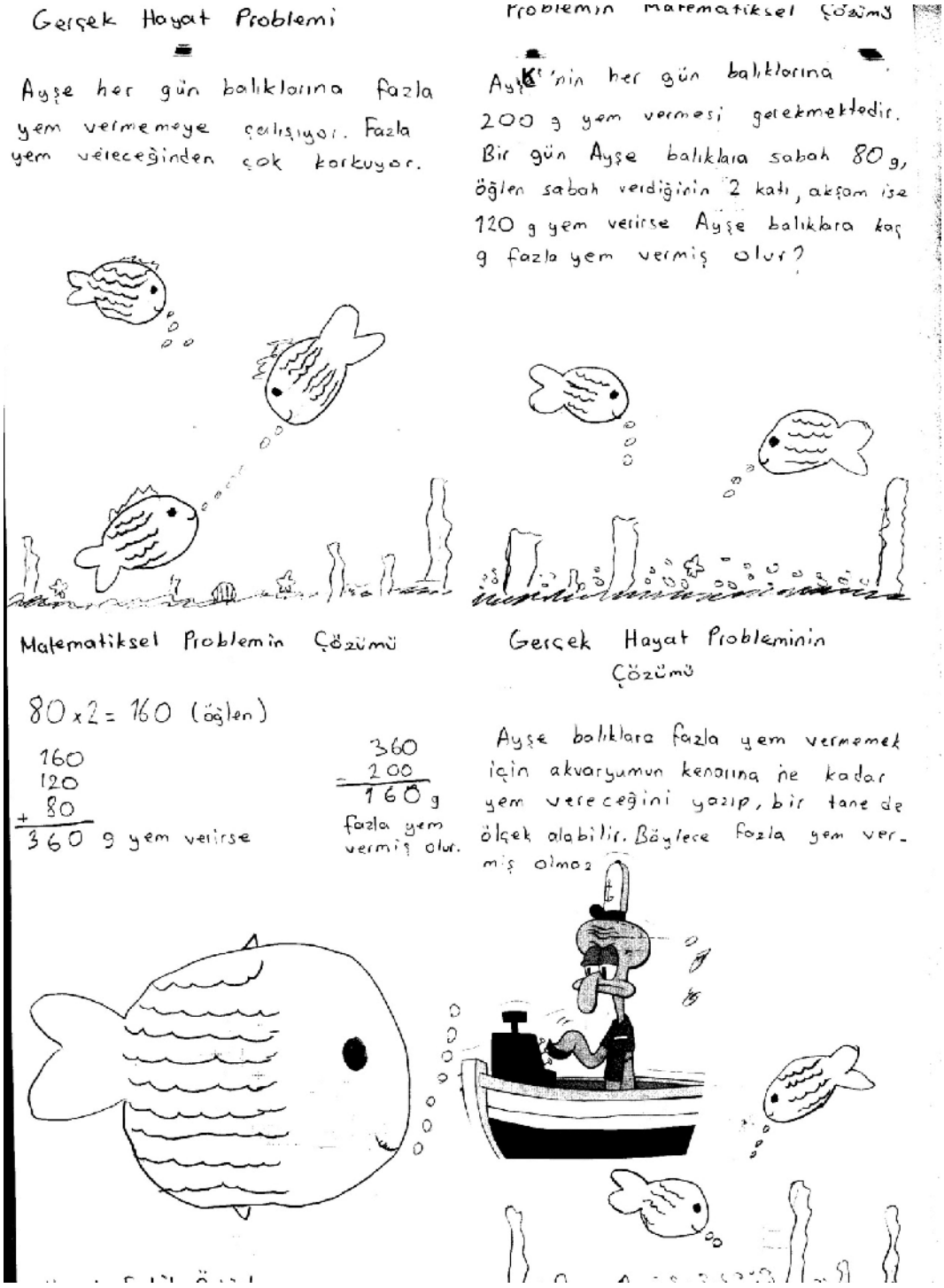



**Figure 3.7** An Example of Students' Work in"Math Problems in Daily Life" Activity

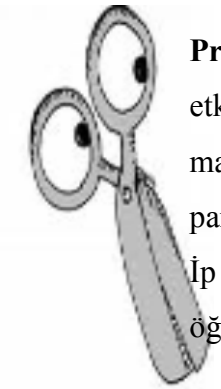
In the next two hours of the problem solving classes on Whole Numbers, students were provided with a story called "Kızıl Ejderha" and asked to fill a worksheet according to the story given in appendix B. In this story, students met with a mysterious situation like in Agahta Christine's books. In the story, there is a detective trying to solve a problem. He is investigating the unexpected disappearance of an over-aged businessman. The detective goes to the hotel room where the over aged businessman was last seen and talks with the manservant and business partner of the missing man. In addition, he observes all the things around and asks questions about them. At the end of the story, he takes into custody the manservant and business partner. Students were requested to determine why the detective takes into custody the men and solve the problem by following the steps given in the worksheet. In the first step, students were asked to fill the table given by determining the names of the characters and their jobs. In the second step, they were asked to find mathematical clues given in the story that would be used to solve the problem. In the third step, students solved the problem by using the clues found in the previous step and lastly they constructed a mathematical problem with the clues they determined. Thus, the first two steps were related with understanding a problem, the third step is about solving a problem and the last step is constructing a problem. Each step was assessed by using the rubrics with one dimension and each rubric was given just below the related step.

This activity is applied just before the "Problem Çözüyorum" activities that students were provided with open-ended real-life context mathematical problems and were asked to solve these problems according to Polya's problem solving steps. The aim of this activity is to make students become aware that understanding a problem is of great importance and it requires determining the clues that will help them to solve the problem. Moreover, the activity aims to teach students that mathematical knowledge and skills can be used in various situations to solve daily life problems. At the end of the activity, a discussion about the importance of solving problems and learning to solve problems was conducted and it was concluded that we can not meet all kinds of real-life context problems in mathematics lessons but if we can develop some strategies to solve problems in mathematics lessons, we could apply these previously developed strategies to various real-life context problems.

In the last five hours of the problem solving classes on Whole Numbers, students were provided with a "Problem Çözüyorum" worksheet which contained 10 mathematical problems and were asked to solve these problems according to Polya's problem solving steps. While preparing the problems, it was paid attention to write problems that can be solved by using different strategies or a combination of them. Moreover, some problems that enable students to determine the missing or excessive data were prepared.

In "Problem Çözüyorum" worksheet, a template which contains the Polya's problem solving steps was given just below each problem and students were informed about what they were expected to do at each step. The first step was "understanding the problem" and they were requested to write the given data and decide on the unknown. In the second step which was "devising a plan", they were asked to make a plan to solve the problem. At first, they did not want to write the name of the strategy they would plan to use because they had just encountered with the 'Strategy' concept for the first time at the Whole Numbers problem solving classes. In each problem, students were given a few minutes to read the problem and fill the "understanding the problem" part of the template. Then the teacher asked how they planned to solve the problem and wanted them to share their ideas with the whole class. Sometimes, she waited until students solved the problem and shared their solutions. The teacher guided students to become aware that there are strategies used to reach the solution of problems. Most of the time, unconsciously they used these strategies to solve the problems and one of the aim of these problem solving classes is to make them develop awareness of strategies that they already use while solving problems and to teach them some other strategies that they did not know. In most cases, while explaining their solution plan or sharing their solutions, students talked about the specific strategy(s) that the teacher had aimed at teaching for that particular problem. Then, the teacher helped them to define the name of the strategy that they used. If they did not solve the problem by applying the intended strategy, the teacher guided them to think about other ways to solve the problem. In this worksheet, intelligent guessing and testing, finding a pattern, accounting for all possibilities, making a drawing (visual representation), working backwards, logical reasoning, solving a simpler analogies problem and adopting a different point of

view strategies which were defined by Possamentier and Krulik (1998) were aimed at being taught in order to help the students to solve the problems. Figure 3.8 illustrates one of the problems given in this worksheet.



**Problem:** Erdal öğretmen matematik dersinde bir etkinlik yapacaktır. Bunun için elindeki ip parçasını, 25 makas darbesiyle eş parçalara ayırır ve elindeki parçaları hiç artmayacak şekilde öğrencilerine dağıtır. İp almayan öğrenci kalmadığına göre; Erdal öğretmenin bu sınıfta kaç öğrencisi vardır?

| | |
|---|---|
| 1.Problem Anlayalım | |
| 2.Plan Yapalım | Strateji :…… |
| 3.Planı Uygulayalım | |
| 4.Kontrol Edelim | |

**Figure 3.8** An Example Problem from "Problem Çözüyorum" Worksheet on Whole Numbers

Students tried to solve the problem given in Figure 3.8 by using different strategies. Some of them used the making a drawing strategy. They drew a line and pointed 25 cutting points on the line, and then counted the pieces of the line. The teacher asked these students questions like "Can you use this strategy if the problem asked for 152 cutting points?". They answered they could do it but it would be harder

as the number of the cutting point increases. Some other students used finding a pattern strategy to solve the problem. First of all, they looked for 1 cutting point and obtained 2 pieces; then, they tried to cut it at 2 cutting points and had 3 pieces, 3 cutting points and 4 pieces…etc. Thus, they concluded that the number of the pieces obtained is one more than the number of the cutting points. Since the problem is asking for 25 cutting points, they concluded that the number of strips cut would be 26. Although, students used these two strategies in their solutions, they did not know the name of the strategies they used. The teacher helped them to define the name of the strategy they used. They talked about making a drawing and finding a pattern strategy. In addition, they talked about solving a simpler analogies problem strategy and the advantages of using it in some problems like this one. Problem solving classes for Whole Numbers were completed with "Problem Çözüyorum" activity and PSAT-1 was applied at the end of the class.

Like the problem solving classes for Whole Numbers, students were provided with various learning activities and materials in Problem solving classes for Sets and LCM-GCF. They were expected to solve these problems according to Polya's problem solving steps and the related PSAT were applied just after the classes. Activity plans for Whole Numbers of problem solving classes are given in appendix B.

In this section the data collection procedure was explained in detail. The following section gives information about the analysis of the data.

### 3.6. Data Analysis

In this study, quantitative research methodologies were used. In order to answer the research questions, quantitative data analysis technique was utilized. In data analysis, both descriptive and inferential statistics were applied. All statistical analysis was carried out by using SPSS 15.0 windows program. In terms of descriptive statistics, mean and standard deviations of PSATs were calculated and frequency and percentages were used to describe the data. In addition, for the inferential statistics, scatterplots and the Pearson Product-Moment Coefficient of correlation were utilized to determine the correlation among $6^{th}$ grade students' problem solving achievement scores, their mathematics achievement mean scores,

and their actual mathematics scores obtained from SBS. There are different correlation coefficients that are used for particular situations. The Pearson Product-Moment Correlation Coefficient (Pearson r) is one of them and it is appropriate to use it when the data for both variables are interpreted in terms of quantitative scores (Fraenkel & Wallen, 2006 ).

This section gives information about the data analysis. The next section gives information about assumptions and limitations of the study.

### 3.7. Assumptions and Limitations

In this part, the basic assumptions and limitations of the research study are explained. First of all, it was assumed that all of the participants took problem solving activities according to Polya's problem solving steps for the first time and answered the questions in Problem Solving Achievement Tests (PSATs), Mathematics Achievement Tests (MATs) and Level Determination Exam (SBS) seriously and accurately.

On the other hand, in this study, convenience sampling method was used to obtain the sample of the population. A convenience sample is a group of individuals who (conveniently) are available for study and in general convenience samples can not be considered representative of any population (Fraenkel & Wallen, 2006). Thus, the sampling method of the study limits the generalizability of the research findings to the broader context. Moreover, the sample did not consist of participants from public schools and only $6^{th}$ graders participated in the study. This also limits the generalizability of the research findings to the broader population.

Since the purpose of the research study was to investigate the relationship among $6^{th}$ grade students' problem solving achievement scores, their mathematics achievement mean scores and their actual mathematics scores obtained from SBS, the attendance of participants to all Problem Solving Achievement Tests (PSATs), all Mathematics Achievement Tests (MATs) and the SBS exam were needed. Thus, the students who did not attend at least one of these tests were eliminated from the sample. At the end, 86 participants who represent 34 % of sample could not participate in the study because of this criterion.

Lastly, since the study contained three topics namely, Whole Numbers, Sets, LCM and GCF, the results of the study can not be generalized to other topics.

### 3.8. Internal and External Validity of the Study

Validity of the results of a study is dependent upon both internal and external validity threats. Both of these threats of this study are discussed in the following section.

#### 3.8.1. Internal Validity

Internal validity gives information about the degree to which observed differences on the dependent variable is aroused from the independent variable (Fraenkel & Wallen, 2006). Thus, if the results of the study are not related to the dependent variable itself or in other words they are related with some other unintended variables, internal validity threats occur. Each research designs have different internal validity threats. Subject characteristics, location, instrumentation (instrument decay, data collector characteristics, and data collector bias), testing and mortality are internal validity threats of a correlational study (Fraenkel & Wallen, 2006). In correlational studies, there is no intervention. Thus, internal validity threats like implementation, attitude of subjects or regression can not be applied to these studies. Importance needs to be given to possible threats and it is necessary to control them in order to reach valid results of the study (Fraenkel & Wallen, 2006).

In this study data was collected from sixth grade students of a private school. Some characteristics of the subjects might affect the internal validity of this study. These are socioeconomic status and prerequisite knowledge of students. Since the data was collected from a private school, socio-economic status of families of the all participants can be rated as very high. Thus, this characteristic affect was controlled since the socio-economic statuses of families were very similar. Moreover, prerequisite knowledge of students might affect the internal validity of this study. However, it should be mentioned that students were taught the topics Sets and LCM-GCF in $6^{th}$ grade for the first time. Hence, they do not have any prerequisite knowledge about these topics. Nevertheless, students were familiar to the topic of Whole Numbers, but this characteristic affect was controlled since 170 out of 256

sixth grade students in this private school participated in this study and 201 of them were students of the same school from 5th grade. Only 55 of them started the school at the beginning of the 6th grade and they all showed an expected degree of success in the exam applied by the school.

Location threat is another internal validity threat of this study and it is possible to occur when the application of each instrument occurs at a particular location but it differs from subject to subject (Fraenkel & Wallen, 2006). The data of this study was collected from only one school and in this school there were specific classes for each lesson. There were 6 mathematics classes in this school and 2 of them were used by 6th grade students. These 2 classes were very similar to each other and not only problem solving classes but also PSATs were applied to all students in these classes. Moreover, during the application of MATs, each class was in their own classrooms. Thus, each class testing conditions were almost the same since the classrooms were very similar to each other. Lastly, it should also be mentioned that students entered SBS exam in different exam places but the standards of the classes were very similar to each other since it was a nationwide exam. In nationwide exams, common standards and rules are applied to each classroom. Thus, although students from 12 distinct classes participated in the study, location threat was controlled since all participants completed the instruments in similar testing conditions.

Additionally, a different interpretation of results depending on the scorers or the time makes instrumentation decay an internal threat for research studies (Fraenkel & Wallen, 2006). The MATs and SBS exam were multiple-choice tests so instrument decay in scoring procedure did not occur. In addition, PSATs were assessed by using a rubric and four teachers conducted the assessment as mentioned above. Thus, instrumentation decay is not a threat for this study.

Data collectors or scorers might unconsciously distort the results of the study that would support the hypothesis of the researcher (Fraenkel & Wallen, 2006). Not only the researcher but also the other data collectors were aware of the purpose of the study, so data collector characteristics and bias were important threats in this study. However, these teachers (data collectors) were mathematics teachers of a private school and in this school the teachers who are teaching the same grade level always prepared lesson plans and assessment tools together. In addition, all the teachers

were expected and did use the same lesson plans in their classes. Thus, they got used to standardizing all procedures and performed common actions on situations. This property helped to control the data collector characteristics and bias internal threats.

Testing is one of the internal threats of correlational studies. However, it is not an internal threat for this study since none of the instruments were applied twice.

Lastly, mortality threat which means the loss of subjects is considered to be an internal threat in studies. However, mortality is not an important internal threat for correlational studies because lost subjects were excluded from the study since correlation could not be calculated if there were no scores for both variables (Fraenkel & Wallen, 2006). In this study, attendance of students to all PSATs, all MATs and the SBS exam were needed to participate in the study and as mentioned above, to control this threat the students who did not attend at least one of these tests were eliminated from the sample.

### 3.8.2. External Validity

External validity of the study is defined as "the extent to which the results of a study can be generalized from a sample to a population" (Fraenkel & Wallen, 2006, p.108). Population generalizability and ecological generalizability are two dimensions of external validity.

Population generalizability is about a sample's degree of representativeness of an intended population (Fraenkel & Wallen, 2006). The target population of this study was all sixth grade private school students sitting the SBS exam at the end of the academic year in Turkey. The accessible population of this study was all sixth grade private school students sitting the SBS exam at the end of the academic year in Istanbul and the sample of the population were sixth grade students of a private school in Istanbul. In this study, convenience sampling method was used to obtain the sample of the population and a convenience sample is a group of individuals who (conveniently) are available for study and in general convenience samples can not be considered representative of any population (Fraenkel & Wallen, 2006). Thus, the sampling method of the study limits the population generalizability of the research findings.

Moreover, the term ecological generalizability refers to “the extent to which the results of a study can be generalized to conditions or settings other than those that prevailed in a particular study” (Fraenkel & Wallen, 2006, p.108). This study was applied in a private school and results could be generalized to the students in other private schools that have similar conditions with the school that the data was collected.

# CHAPTER 4

# RESULTS AND CONCLUSIONS

This chapter aims to present the results of the study in two main parts. In the first part descriptive statistics of students' problem solving achievement scores, mathematics achievement scores and actual mathematics achievement net scores obtained from SBS will be explained. In the second part, inferential statistics obtained by the statistical analysis will be presented. Scatterplots and the Pearson Product-Moment Coefficient of correlation were utilized to determine the correlation among $6^{th}$ grade students' problem solving achievement scores, their mathematics achievement mean scores, and their actual mathematics scores obtained from SBS. Also this chapter presents the conclusion section regarding the results of the study.

## 4.1. Descriptive Statistics

Descriptive statistics concerning the participants' problem solving achievement scores, mathematics achievement scores and actual mathematics net scores obtained from SBS are presented in this section.

### 4.1.1. Descriptive Statistics of Problem Solving Achievement Scores

In terms of descriptive statistics; minimum-maximum scores, mean, standard deviations, and skewness and kurtosis values of problem solving achievement scores are calculated and used to describe the data. As it is mentioned before, students' problem solving achievement scores are the mean score of the scores obtained from PSAT-1, PSAT-2 and PSAT-3. Thus, this section summarizes the descriptive statistics of not only problem solving achievement scores but also the scores obtained from PSAT-1, PSAT-2 and PSAT-3 separately. These scores are related to contents of Whole Numbers, Sets and LCM-GCF respectively.

Totally, 170 participants took all of the Problem Solving Achievement Tests. Table 4.1 summarizes minimum-maximum scores, mean, standard deviation, and skewness and kurtosis values of the scores for each test separately.

**Table 4.1**. Descriptive Statistics of Scores Obtained from PSAT-1, PSAT-2, PSAT-3, and PSAT

| | PSAT1 | PSAT2 | PSAT3 | PSAT |
|---|---|---|---|---|
| N | 170 | 170 | 170 | 170 |
| Mean | 65.33 | 69.44 | 72.98 | 69.28 |
| Std. Deviation | 19.99 | 24.67 | 20.40 | 16.72 |
| Minimum | 12.00 | 8.00 | 20.00 | 18.00 |
| Maximum | 100.00 | 100.00 | 100.00 | 100.00 |
| Skewness | -0.31 | -0.45 | -0.55 | -0.39 |
| Kurtosis | -0.39 | -0.91 | -0.53 | -0.23 |

As given in Table 4.1, although the maximum scores taken from PSAT-1, PSAT-2 and PSAT-3 are the same (*max* = 100), the minimum scores vary from test to test. Thus, the difference between the minimum and maximum scores takes the biggest value in PSAT-2 scores since the minimum score taken from the test is 8. Considering the problem solving achievement score which is one of the main variables in this study, the maximum score is 100 and the minimum score is 18, with a mean of 69.28 (*SD* = 16.72). In addition, Table 4.1 also presents the skewness and kurtosis values of problem solving achievement scores. These values provide information about the nature of the distribution of the scores. Based on these values, distribution of problem solving achievement scores are regarded as normally distributed.

Additionally, the figure 4.1 given below illustrates the differences in cognitive processes of the whole participants based on problem solving steps related to the topics of Whole Numbers, Sets and LCM-GCF.

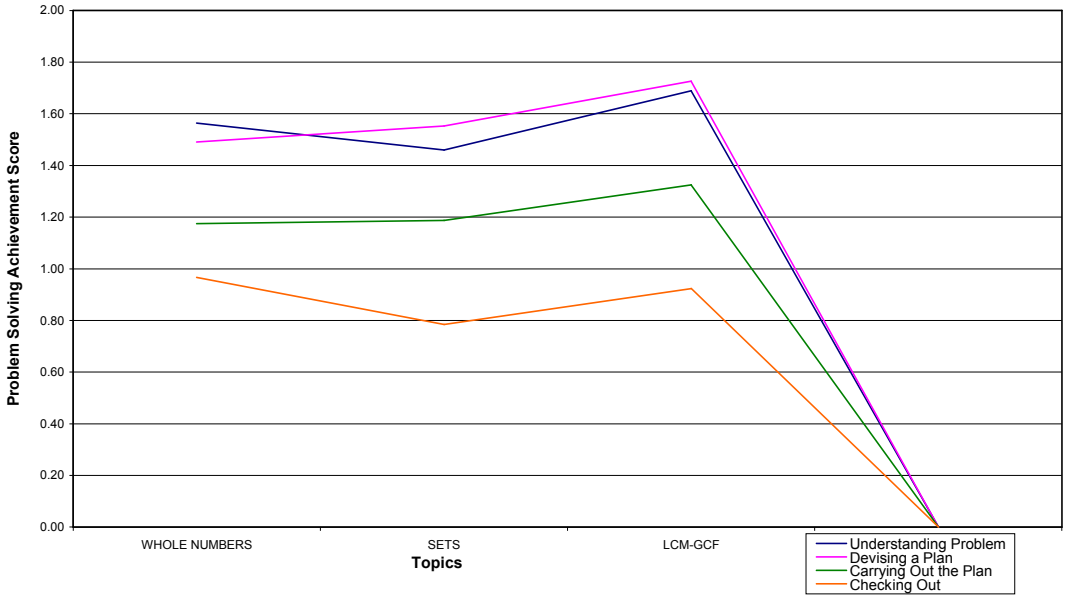



**Figure 4.1.** The Differences in Cognitive Processes of the Whole Participants Based on Problem Solving Steps Related to the Topics of Whole Numbers, Sets and LCM-GCF.

According to the figure above, students' understanding the problem, devising a plan and carrying out the plan scores increase from the beginning of the instruction with the topic Whole Numbers to the end of it with the topics LCM-GCF. However, there was a small decrease in checking out scores from the beginning of the process to the end of it.

### 4.1.2. Descriptive Statistics of Mathematics Achievement Scores

In this study, Mathematics Achievement Tests (MATs) were used to measure 6$^{th}$ grade students' mathematics achievement mean scores obtained through out the semester related to the Whole Numbers, Sets, LCM, and GCF units. Students' mathematics achievement mean scores were determined by calculating the mean score of each separate unit scores. Table 4.2 gives information about the descriptive statistics of students' mathematics achievement mean scores.

**Table 4.2.** Descriptive Statistics of Scores obtained from MATs Considering Units

| | N. Numbers | Sets | LCM-GCF | Mathematics Achievement Mean Score |
|---|---|---|---|---|
| N | 170 | 170 | 170 | 170 |
| Mean | 73.32 | 73.28 | 57.06 | 67.89 |
| Std. Deviation | 21.12 | 17.97 | 31.86 | 18.04 |
| Minimum | 3.33 | 16.67 | 0.00 | 21.67 |
| Maximum | 100.00 | 100.00 | 100.00 | 100.00 |
| Skewness | -0.83 | -0.71 | -0.13 | -0.10 |
| Kurtosis | 0.20 | 0.17 | -0.57 | -0.59 |

As given in Table 4.2, mathematics achievements mean scores which is one of the main variables in this study has the maximum score of 100 and minimum score of 21.67 ; with a mean of 67.89 (*SD* = 18.04). Considering the units separately, the minimum and maximum scores are respectively 3.33 and 100 in Whole Numbers; 16.67 and 100 in Sets and 0 and 100 in LCM and GCF. The mean of the scores related to Whole Numbers is 73.32 (*SD* = 21.12); related to Sets is 73.28 (*SD* = 17.97); related to LCM and GCF is 57.06 (*SD* = 31.86). Table 4.2 also shows the skewness and kurtosis values of mathematics achievements mean scores. Those values indicated that the distribution of mathematics achievements mean scores is normal.

### 4.1.3. Descriptive Statistics of Actual Mathematics Achievement Net Scores Obtained from SBS

In this study, Mathematics subtest of Level Determination Exam **(**SBS) was used to measure 6$^{th}$ grade students' actual mathematics achievement net scores at the end of the semester. The net score which can be taken from Mathematics test of SBS range from -5.33 to 16. If the student answers all the questions correctly, he/she would take score of 16. While determining the net score, three wrong answers cancel one correct answer and thus if a student answer all the questions wrongly he would take score of -5.33 from the exam.

All of the students of this private school took the SBS exam. However; 170 out of 256 students constituted the participants of this study since the students who did not attend at least one of the tests (PSATs or MATs) were eliminated from the sample. Table 4.3 summarizes minimum-maximum scores, mean, standard deviation, and skewness and kurtosis values of the participants' actual mathematics achievement net scores that they obtained from SBS.

**Table 4.3.** Descriptive Statistics of Scores Obtained from Level Determination Exam **(**SBS)

| | N | Mean | Std. Deviation | Minimum | Maximum | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| SBS | 170 | 11.50 | 3.15 | 1.00 | 16.00 | -0.74 | 0.11 |

As given in Table 4.3, actual mathematics achievement net score which is one of the main variables in this study has the maximum score of 16 and minimum score of 1.00; with a mean of 11.50 (SD = 3.15). Furthermore, the skewness and kurtosis values indicated that students' actual mathematics achievement net scores are normally distributed.

Descriptive statistics of students' problem solving achievement scores, mathematics achievement scores and actual mathematics achievement net scores obtained from SBS were explained in this section. In the following section, inferential statistics obtained by the statistical analysis will be presented.

### 4.2. Inferential Statistics

In terms of inferential statistics; the Pearson Product-Moment Coefficient of correlation was utilized to determine the correlation among $6^{th}$ grade students' problem solving achievement scores, their mathematics achievement mean scores, and their actual mathematics achievement net scores obtained from SBS.

### 4.2.1. The Relationship between Problem Solving Achievement Scores and Mathematics Achievement Mean Scores

To investigate the relationship between Problem Solving Achievement Scores and Mathematics Achievement Mean Scores, correlation analysis was utilized. It is used to explain the direction and the strength of the linear relationship between two variables (Pallant, 2001). There are different correlation coefficients that are used for particular situations. The Pearson Product-Moment Correlation Coefficient (Pearson r) is one of them and it is appropriate to use it when the data for both variables are interpreted in terms of quantitative scores (Fraenkel & Wallen, 2006). Pearson correlation coefficient can take the values from -1 to +1 (Pallant, 2001). The results of the analysis are presented with research questions in the following paragraphs.

#### 4.2.1.1. Research Question 1

*What is the relationship between 6$^{th}$ grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics?*

Before calculating Pearson product-moment correlation, preliminary analyses were done to guarantee that there was no violation of the assumptions of normality, linearity and homoscedasticity. Moreover; scatterplots were also utilized to have idea about the nature of the relationship of the variables. As it was mentioned in Descriptive Statistics section, the normality of the distribution of PSAT and MAT scores was met regarding the corresponding skewness and kurtosis values. More specifically, in order to calculate correlation coefficients precisely, the relationship between the two variables is required to be linear (Pallant, 2001). Figure 4.2 shows the scatterplot generated to investigate the relationship between the problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics.

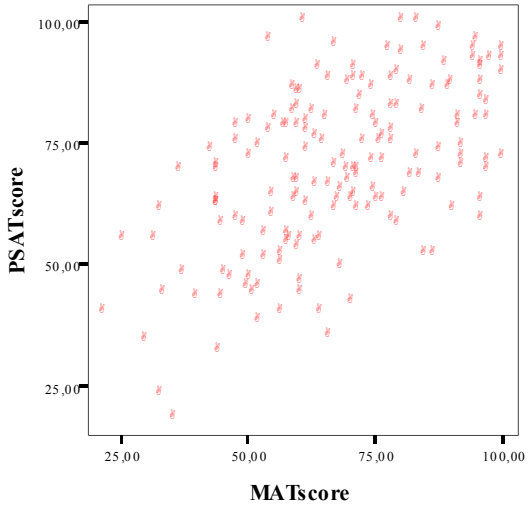



**Figure 4.2.** Scatterplot of Problem Solving Achievement Scores and Mathematics Achievement Mean Scores

The distribution of the scores on the scatterplot revealed that the relationship between the variables was linear. Furthermore, it was shown in the scatterplot that the scores are almost arranged in a cigar shape. Thus, the homoscedasticity assumption was also ensured (Pallant, 2001). The results of Pearson product-moment correlation are presented in Table 4.4.

**Table 4.4.** Results of the Bivariate Correlations of Problem Solving Achievement Scores and Mathematics Achievement Mean Scores

| | | PSATscore | MATscore |
|---|---|---|---|
| PSATscore | Pearson Correlation | 1 | 0.56(**) |
| | Sig. (2-tailed) | | 0.00 |
| | N | 170 | 170 |
| MATscore | Pearson Correlation | 0.56(**) | 1 |
| | Sig. (2-tailed) | 0.00 | |
| | N | 170 | 170 |

The results revealed that there was a significant large positive correlation between the problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained through

out the semester related to LCM, GCF, Sets and Whole Numbers Topics [$r = .563$, $n = 170$, $p < .05$]. More specifically, 6th grade students having high problem solving achievement scores after completing instruction on problem solving have higher mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics. In addition, results revealed that students' problem solving achievement scores explain nearly 32 percent of the variance in students' mathematics achievement mean scores ($r^2 = 0.32$). That is, a large effect size in terms of practical significance according to guidelines of Cohen (1988) is obtained.

### 4.2.2. The Relationship between Problem Solving Achievement Scores and Actual Mathematics Achievement Net Scores obtained from SBS

To investigate the relationship between Problem Solving Achievement Scores and Actual Mathematics Scores, The Pearson Product-Moment Correlation Coefficient (Pearson r) was utilized.

#### 4.2.2.1. Research Question 2

*What is the relationship among 6th grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from SBS?*

First of all, to check the linearity and homoscedasticity assumptions a scatterplot was generated. As it was mentioned in Descriptive Statistics section, the normality of the distribution of PSAT and SBS scores was met regarding the corresponding skewness and kurtosis values. Figure 4.2 shows the scatterplot generated to investigate the relationship between the problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from SBS.

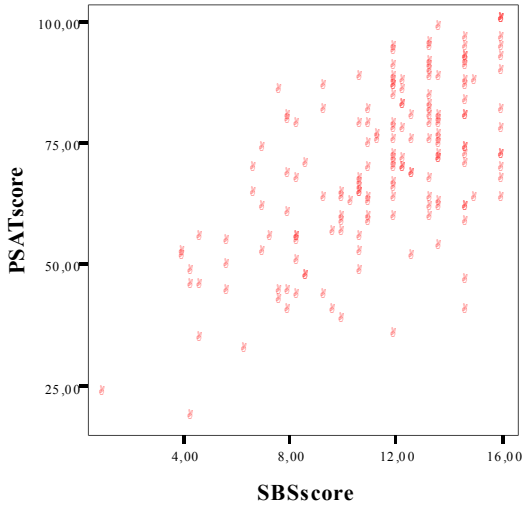



**Figure 4.3.** Scatterplot of Problem Solving Achievement Scores and Actual Mathematics Net Scores obtained from SBS

The distribution of the scores on the scatterplot indicated that the relationship between the variables was linear. In addition, it was shown in the scatterplot that the scores are almost arranged in a cigar shape. Thus, not only the linearty but also the homoscedasticity assumptions were ensured (Pallant, 2001). The results of Pearson product-moment correlation are presented in Table 4.5 below.

**Table 4.5.** Results of the Bivariate Correlations of Problem Solving Achievement Scores and Actual Mathematics Net Scores obtained from SBS

| | | PSATscore | SBSscore |
|---|---|---|---|
| PSATscore | Pearson Correlation | 1 | 0.63(**) |
| | Sig. (2-tailed) | | 0.00 |
| | N | 170 | 170 |
| SBSscore | Pearson Correlation | 0.63(**) | 1 |
| | Sig. (2-tailed) | 0.00 | |
| | N | 170 | 170 |

The results revealed that there was a significant large positive correlation between the problem solving achievement scores after completing instruction on problem solving and actual mathematics net scores obtained from SBS [$r$ = .627, $n$ =

170, $p < .05$]. More specifically, 6th grade students having high problem solving achievement scores after completing instruction on problem solving have higher mathematics net scores from SBS. In addition, results revealed that students' problem solving achievement scores explain nearly 39 percent of the variance in students' mathematics net scores obtained from SBS ($r^2 = 0.39$). That is, a large effect size in terms of practical significance according to guidelines of Cohen (1988) is obtained.

### 4.3. Summary

First of all; the results of the statistical analyses explored the relationship between 6th grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics. The findings of the analysis showed that there was a significant large positive correlation between the problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics.

Another conclusion of this study was about the relationship among 6th grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from SBS. The findings of the analysis showed that there was a significant large positive correlation between the problem solving achievement scores after completing instruction on problem solving and students' actual mathematics net scores obtained from SBS.

# CHAPTER 5

# DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The purpose of this research study is to explore the relationship between 6th grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores related to LCM, GCF, Sets and Whole Numbers topics obtained throughout the semester. In addition, the relationship between 6th grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics achievement net scores obtained from SBS exam is investigated. The previous chapter explains the results of the statistical analysis of the study. This chapter deals with conclusions based on findings and reasoning about the results of the study. Furthermore; comparing those of the studies in the literature and implications and recommendations for practice and further studies are stated in this chapter.

## 5.1. Relationship between Problem Solving Achievement Scores and Mathematics Achievement Mean Scores

As mentioned in method chapter, problem solving achievement scores were determined by utilizing Problem Solving Achievement Tests (PSATs) which included open-ended real-life context problems. Moreover, students' mathematics achievement mean scores were determined by utilizing Mathematics Achievement Tests (MATs) that consisted of multiple choice questions having four choice alternatives. The results of the study indicated that that there was a significant large positive correlation between the problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics. Thus, it could be inferred from the results of the study that students

who were successful in problem solving classes were also successful in the mathematics achievement tests applied through out the semester. In addition, although the structures of the tests were different from each other, students could use their problem solving abilities in various environments through out the semester. Thus, it could be inferred that problem based instruction was effective and students internalized it. Moreover; this type of instruction might enable students to have permanent learning. Furthermore, the findings of this study are in agreement with the results of Özsoy's (2002) study who pointed out that there is a positive significant relationship between students' mathematics achievement scores and scores obtained from Polya's problem solving steps in problem solving ability test. In addition, Ceylan (2008) found a highly significant correlation between 6$^{th}$ grade students' test scores of daily-life problem-solving inventory and their performance at mathematical problem solving. Moreover, the literature includes many studies where problem solving approach has been found to have various effects on achievement in various areas of mathematics. For example, instruction based on Polya's problem solving steps significantly affected students' problem solving abilities in a positive way (Yıldız, 2008) and teaching through problem posing had significant positive effect on students' problem solving success (Fidan, 2008). Additionally, in Töre's (2007) study, it was stated that instruction according to Polya's problem solving steps had positive effective on students' problem solving abilities. Lastly, Verschafell and others (1999) stated that intervention had a positive effect on different aspects of students' mathematical modeling and problem solving abilities. These studies emphasized the role of problem solving in mathematics education.

As it is mentioned before, the results of this study indicated that 6$^{th}$ grade students having high problem solving achievement scores after completing instruction on problem solving have higher mathematics achievement mean scores obtained through out the semester related to LCM, GCF, Sets and Whole Numbers Topics. The reason of high correlation might be due to the instruction given in classes designed for problem solving. As mentioned in chapter 3, in these problem solving classes students were provided with various activities, stories, materials and work sheets which include real-life context mathematical problems and wanted to solve them according to the Polya's problem solving steps. In other words, it can be

said that the instruction was planned according to the aspects of new elementary mathematics curriculum. Thus, this type of instruction might provide students with permanent learning. In their study, Karataş and Güven (2004) mentioned that in order to make students acquire new knowledge, providing students with learning environments which includes visual and electronic materials are of great importance in the new program based on the constructivist approach. Similarly, Tandoğan and Akınoğlu (2006) explained that daily-life expressions of problems might lead students to participate actively. Moreover, daily-life expressions might enable students to remove their worries about problem solving and enhance their learning (Yıldız, 2008). In other words, instruction based on problem solving might attract students' interest and might make them deal more with the problems and involve in mathematical tasks. Thus, students who took high problem solving achievement scores after completing instruction on problem solving also took high mathematics achievement mean scores. Moreover; instruction due to Polya's problem solving steps (understanding the problem, devising a plan, carrying out the plan, looking back) might have affect this high correlation. In this study, students' problem solving scores were determined by utilizing Problem Solving Achievement Tests (PSATs) which included open-ended problems and assessed according to a rubric. In the step of "devising a plan", students made a plan to solve the problem and they used different strategies to reach the solution of the problem. Using strategies like *finding a pattern* or *making a drawing* might have affect students' problem solving performances. These findings were consistent with the results of Ozsoy's (2007) study who found positive significant effect of being thought to use metacognition strategies on 5$^{th}$ grade students' mathematical problem solving achievement. In addition, Yazgan and Bintaş (2005) investigated 4$^{th}$ and 5$^{th}$ grade students' learning and utilization of problem solving strategies. The results showed that 4$^{th}$ and 5$^{th}$ grade students can informally use problem solving strategies without any training and 4$^{th}$ and 5$^{th}$ grade students can learn strategies. Moreover; training had a positive effect on students' problem solving success.

Most of the studies in the literature investigate the effects of problem solving approach on achievement (Ceylan, 2008; Fidan, 2008; Özsoy, 2007; Yıldız, 2008). However, there were few studies conducted to see retention of problem

solving ability through out the year. In addition, although there are several research studies performed with students and teachers regarding problem solving abilities and mathematics achievement, very few research studies completed on the possible connection between two in relation with the new curriculum. Therefore, this study contributes to the literature in this context.

Another result of this study showed that there was a significant large positive correlation between the problem solving achievement scores after completing instruction on problem solving and students' actual mathematics net scores obtained from SBS. The SBS is a nationwide exam and contains multiple choice items with four choice alternatives. Thus, students' actual mathematics net scores obtained from SBS were also achievement scores and this finding of the study is in agreement with the results of studies which were mentioned above (Ceylan, 2008; Fidan, 2008; Özsoy, 2007; Töre, 2007; Yıldız, 2008). Similarly, the reason of high correlation might be due to the instruction given in classes designed for problem solving. Students used their problem solving abilities in different environments and SBS is one of them. Students might be successful in SBS exam because the type of questions asked in SBS is also not traditional.

### 5.2. Implications and Recommendations

This study focused on the relationship between 6$^{th}$ grade students' problem solving achievement scores after completing instruction on problem solving and their mathematics achievement mean scores related to Least Common Multiple (LCM), Greatest Common Factor (GCF), Sets and Whole Numbers topics obtained throughout the semester. In addition, the relationship between 6$^{th}$ grade students' problem solving achievement scores after completing instruction on problem solving and their actual mathematics net scores obtained from SBS exam was investigated. Based on the analysis of the data, some recommendations for further researches could be offered.

As it is mentioned in method chapter, convenience sampling method was used to obtain the sample of this study who are sixth grade students of a private school in Istanbul. Thus, the sampling method of the study limits the generalizability of the research findings to the broader context. Moreover, the sample did not consist

of participants from public schools and only 6th graders participated in the study. Thus, replication of the present study in not only other private schools but also public schools is recommended to determine whether the results will be similar. In addition, this study could be performed with different grade level students and comparisons of the results according to different grade levels could be done.

Present study contained three topics namely, Whole Numbers, Sets, LCM and GCF. Thus, the results of the study can not be generalized to other topics but in further studies other topics could be selected. In addition, experimental studies which investigate the effect of problem solving approach on achievement related to these topics could be done.

In this study, it was revealed that instruction given in problem solving classes had important benefits on students' problem solving skills. Moreover, significant large positive correlation was found between students' mathematics achievement scores and problem solving achievement scores not only in school exams but also in SBS. Taking into account of these results, it was seen that implication of real-life context problem solving instruction according to Polya's problem solving steps was not difficult. Thus, great importance should be given to problem solving instruction as it is mentioned in the new curriculum. In order to be successful in exams like SBS, problem based instruction is necessary. Teachers should avoid from traditional methods which students solve hundreds of questions and memorize the solution way of various particular questions. The instruction given in problem solving classes might enable students to use their abilities not only in school exams but also in a nationwide exam like SBS. In addition, a learning environment should be planned and prepared in such a way that students learn with concrete experiences, utilize from various activities, stories and materials. Lastly, cooperative learning should be used and students' active participation to learning environments should be encouraged.

## REFERENCES


Akyüz, G. (2006). Investigation of the effect of teacher and class characteristics on mathematics achievement in Turkey and European Union Countries. *Elementary Education Online, 5*(2), 75-86. Retrieved from June, 4, 2009 http://ilkogretim-online.org.tr/

Altun, M. (1995). *İlkokul 3., ve 5. sınıf öğrencilerinin problem çözme davranışları üzerine bir çalışma*. Unpublished doctoral dissertation, Hacettepe Üniversitesi, Ankara, Türkiye.

Albayrak, M., Ipek, A. S., & Isık, C. (2006). Temel işlem becerilerinin öğretiminde problem kurma-çözme çalışmaları, *Erzincan Eğitim Fakültesi Dergisi, 8*(2), 1-11.

Altun, M., & Arslan, Ç. (2006). İlköğretim öğrencilerinin problem çözme stratejilerini öğrenmeleri üzerine bir çalışma. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 19*(1), 1-21.

Babadogan, C., & Olkun, S. (2006). Program development models and reform in Turkish primary school mathematics curriculum. *International Journal for Mathematics Teaching and Learning*. Retrieved from May, 15, 2009 http://www.cimt.plymouth.ac.uk/journal/default.htm

Baki, A., & Gökçek, T. (2005). Comparision of the development of elementary mathematics curriculum studies in Turkey and the U.S.A, *Educational Sciences:Theory & Practice, 5*(2), 579-588.

Bulut, S. (2004). İlköğretim programlarında yeni yaklaşımlar, Matematik (1-5), *Bilim ve Aklın Aydınlığında Eğitim Dergisi,* 54-55.

Brown, N.M. (2003). A Study of elementary teachers' abilities, attitudes, and beliefs about problem solving. *Dissertation Abstracts International*, *64*(10), 3620. (UMI No. 3108818).

Cai, J. (2003). Singaporean students' mathematical thinking in problem solving and problem posing: an exploratory study, *International Journal of Mathematics Education in Science and Technology, 34*(5), 719-737.

Charles, R., Lester, F., & O'Daffer, P. (1987). *How to evaluate progress in problem solving.* Reston: National Council of Teachers of Mathematics.

Ceylan, F. (2008). *İlköğretim 6. sınıf öğrencilerinin günlük hayat problemlerini çözme envanteri puanları ile matematik problemlerini çözme başarıları arasındaki ilişki.* Unpublished master's thesis. Gazi Üniversitesi, Ankara, Türkiye.

Duatepe-Paksu, A., & Ubuz, B. (2009). Effects of drama-based geometry instruction on student achievement, attitudes, and thinking levels. *The Journal of Educational Research. 102* (4), 272-286.

Fidan, S. (2008). *İlköğretim matematik dersinde öğrencilerin problem kurma çalışmalarının problem çözme başarısına etkisi.* Unpublished master's thesis. Gazi Üniversitesi, Ankara, Türkiye.

Fraenkel, J. R., & Wallen, N. E. (2006). *How to design and evaluate research in education*, New York: McGraw-Hill Inc.

Giganti, P. (2004). Mathematical Problem Solving. *Book Links*, *14*, 15-17.

Grouws, D. A. (1996). *Critical issues in problem solving instruction in mathematics.* In D. Zhang, T. Sawada, & J. P. Becker (Eds.), Proceedings of the China-Japan-U.S. seminar on mathematical education (pp. 70-93). Carbondale, IL: Board of Trustees of Southern Illinois University.

Grouws, D. A., & Cebulla, K.J. (2000). *Improving student achievement in mathematics.* IAE Educational practices series, Chicago: University of Illinois

Harskamp, E., & Suhre, C. (2007). Schoenfeld's problem solving theory in a student controlled learning environment. *Computers & Education ,49*, 822–839.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., and Human, P. (1997) *Making Sense: Teaching and Learning Mathematics with Understanding.* Portsmouth, NH: Heinemann.

Husen, T. (1967). *International study of achievement in mathematics.* New York: Wiley

İskenderoglu, T., Akbaba-Altun, S., & Olkun, S. (2004). İlköğretim 3., 4. ve 5. sınıf öğrencilerinin standart sözel problemlerde işlem seçimleri. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 27,* 126-134.

Jitendra, A.K., Griffin, C.C., Deatline, A., & Sczesniak, E. (2007) Mathematical word problem solving in third-grade classrooms. *The Journal of Educational Research*, *100*(5), 283-302.

Jonassen, D. H. (2003). Designing research-based instruction for story problems. *Educational Psychology Review, 15*(3), 267-296.

Jonassen, D. H. (2004). *Learning to Solve Problems: An Instructional Design Guide*. San Francisco, CA: Jossey-Bass.

Karatas, D., & Güven, B. (2004). 8. Sınıf oğrencilerinin problem cozme becerilerinin belirlenmesi: Bir ozel durum calısması. *Milli Eğitim Dergisi*, 163.

Keeves, J.P. (1976). Curriculum factors influencing school learning. *Studies in Educational Evaluation, 2,* 167-184.

Klein, D. (2003). A brief history of american K-12 mathematics education in the 20th century. In J. Royer (Ed.), *Mathematical Cognition* (pp. 175-225). Greenwich: Information Age Publishing.

Koc, Y., Isiksal, M., & Bulut, S. (2007). Elementary school curriculum reform in Turkey. *International Education Journal, 8*(1)*,* 30-39.

Lester, F. K. (1980). Problem Solving: Is it a Problem?. In M. M. Lindsquist (Ed.), *Selected Issues in Mathematics* (pp. 29-45). NCTM, Reston VA.

Lester, F. K. (1994). Musings about mathematical problem solving research: 1970-1994. *Journal for Research in Mathematics Education*, *25*(6), 660-675.

Lester, F.K., Garofalo, J., Kroll, D.L. (1989). *The role of metacognition in mathematical problem solving: A study of two grade seven students*. A Project Report: School of Education, Indiana University.

Linn. R.L. and Miller, M.D. (2005). *Measurement and Assessment in Teaching (9th edition)*. Englewood Cliffs, NJ: Prentice Hall.

Manuel, S. T. (1998). Instructional Qualities of a Successful Mathematical Problem Solving Class. *International Journal of Mathematics Education in Science & Technology*, *29*(5), 631-645.

Martinez, M. E. (1998). What is problem solving?. *Phi Delta Kappan*, *79* (8), 605-609.

Marton, R.L. (1955). *Teaching aritmetic in the elementary school.* Çev. Yakalıoğlu, A., İstanbul: Maarif Basımevi.

McKnight, C., Crosswhite, F.J., Dossey, J.A., Kifer, E., Swafford, J.O., Travers, K.J., Cooney, T.J. (1987). *The underachieving curriculum: Assessing U.S. school mathematics from an international perspective. A national report on the Second International Mathematics Study, International Association for the Evaluation of Education Achievement.* Champaign, IL: Stipes Publishing Company.

McQuade, F., & Champagne, D.W. (1996). Assessment : There is no alternative. *Ideas & Perspectives, 21*(7), 153-186.

Milli Egitim Bakanlıgı [MEB]. (2005). *İlkögretim matematik dersi ögretim programı 1–5. sınıflar.* Ankara: MEB.

Milli Egitim Bakanlıgı [MEB] (2007). *İlkögretim matematik dersi ögretim programı 6-8. sınıflar.* Ankara: MEB.

Milli Egitim Bakanlıgı [MEB]. (2007). *64 Soruda Ortaoğretime Gecis Sistemi*. Retrieved January 27, 2009 from oges.meb.gov.tr/docs/64_soru.pdf

Milli Egitim Bakanlıgı [MEB]. (2008). *Revizyon: ilköğretim matematik(1-8.sınıflar)dersi öğretim programlarında bazı değişiklikler.* Retrieved January 15, 2009 from http://ttkb.meb.gov.tr/

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM.

Okur, S. (2008). *Students' strategies, episodes and metacognitions in the context of PISA 2003 mathematical literacy items.* Unpublished master's thesis. Orta Doğu Teknik Üniversitesi, Ankara, Türkiye.

Özkaya, S. (2000). *Investigation of tenth grade students' problem solving strategies in geometry.* Unpublished master's thesis. Orta Doğu Teknik Üniversitesi, Ankara, Türkiye.

Özsoy, G. (2002). *İlköğretim beşinci sınıfta matematik dersi genel başarısı ile problem çözme becerisi arasındaki ilişki.* Unpublished master's thesis, Gazi Üniversitesi, Ankara, Türkiye

Özsoy, G. (2007). *İlköğretim beşinci sınıfta üstbiliş stratejileri öğretmenin problem çözme başarısına etkisi.* Unpublished doctoral dissertation, Gazi Üniversitesi, Ankara, Türkiye.

Pallant, J. (2001). *SPSS Survival Manual: A step by step guide to data analysis using SPSS (Version 11).* Open University Press.

Pilli, O. (2008). *The effects of computer-assisted instruction on the achievement, attitudes and retention of fourth grade mathematics course.* Unpublished doctoral dissertation. Orta Doğu Teknik Üniversitesi, Ankara, Türkiye.

Polya, G. (1953).On Teaching Problem Solving. In H. F. Fehr (Ed.),*The Learning of Mathematics: Its theory and practice* (pp. 228-270). 21st yearbook of the NCTM. Reston, VA: NCTM.

Polya, G. (1957). *How to solve it.* Princeton, New Jersey: Princeton University Press.

Polya, G. (1962). *Mathematical Discovery: On understanding, teaching, and learning problem solving*. New York: John Wiley.

Posamentier, A. S., & Krulik, S. (1998). *Problem-Solving Strategies for Efficient and Elegant Solutions.* California: Corwin Press,Inc.

Reys, B., Robinson, E., Sconiers, S., & Mark, J. (1999). Mathematics curricula based on rigorous national standards : What, Why, and How?. *Phi Delta Kappan, 2,* 454-456.

Santos-Trigo, M. (1998). Instructional qualities of a mathematical problem-solving class. *International Journal of Mathematical Education in Science & Technology, 29*(5), 631-646.

Schmidt,W.H., McKnight, C.C., Raizen, S.A. (1997). *A splintered vision: an investigation of U.S. science and mathematics education.* Dordrecht, Netherlands: Kluwer Academic Publishers.

Schoenfeld, A. H. (1989). Explorations of Students' Mathematical Beliefs and Behavior. *Journal for Research in Mathematics Education*, *20*(4), 338-355.

Schroeder, T. & Lester, F. K. (1989). Developing Understanding in Mathematics via Problem Solving. In P. R. Trafton (Ed.), *New Directions for Elementary School Mathematics* (pp. 31- 56). Reston, VA: National Council of Teachers of Mathematics.

Szetela, W., & Nicol, C. (1992). Evaluating problem solving in mathematics. *Educational Leadership, 49*(8), 42-45.

Şişman, M. (2007). *İlköğretim 8. sınıf matematik dersi çarpanlara ayırma ve özdeşlikler konusunun yapılandırmacı öğrenme yaklaşımına uygun olarak öğretiminin öğrenci başarısına etkisi*, Unpublished master's thesis. Gazi Üniversitesi, Ankara, Türkiye.

Tandoğan, Ö.R., & Akinoglu, O.(2007). The Effects of problem-based active learning in science education on students' academic achievement, attitude and concept learning. *Eurasia Journal of Mathematics, Science & Technology Education*, *3*(1), 71-81.

Teltik-Başer, E. (2008). *5E modeline uygun öğretim etkinliklerinin 7. sınıf öğrencilerinin matematik dersindeki akademik başarılarına etkisi.* Unpublished master's thesis. Gazi Üniversitesi, Ankara, Türkiye

Van de Walle, J. A. (2004). *Elementary and Middle School Mathematics Teaching Developmentally*. (5th ed.). Boston: Allyn and Bacon.

Van Haneghan, J., Vye, N., Bransford, J., Furman, L., Barron, B., Montavon, E.,Young, M., & Barron, L. (1989, April). *An analysis of students' mathematical problem solving in real world settings*. Paper presented at the meeting of the American Educational Research Association, San Francisco, CA.

Verschaffel, L., De Corte, E., Lasure, S.,Van Vaerenbergh, G., Bogaerts, H.& Ratinckx, E. (1999). Learning to Solve Mathematical Application Problems: A Design Experiment with Fifth Graders, *Mathematical Thinking and Learning*, *1*(3), 195-229.

Willoughby, S. S. (1985, April). How to teach problem solving. *Educational Leadership*, *42*, 90-92.

Yazgan, Y., & Bintaş, J. (2005). İlköğretim dördüncü ve beşinci sınıf öğrencilerinin problem çözme stratejilerini kullanabilme düzeyleri: Bir öğretim deneyi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 28,* 210-218.

Yıldız, F. (2008). "*Oran, orantı ve yüzdeler" ünitesinin proje tabanlı öğrenme ile öğrenilmesinin matematik dersindeki başarıya ve tutuma etkisi,* Unpublished master's thesis. Marmara Üniversitesi, Ankara, Türkiye.

Yıldız, V. (2008). *Investigation of the change in sixth grade students' problem solving abilities, attitude towards problem solving and attitude toward mathematics after mathematics instruction based on Polya's Problem Solving Steps*, Unpublished master's thesis. Orta Doğu Teknik Üniversitesi, Ankara, Türkiye.

# APPENDICES

## APPENDIX A

## PROBLEM ÇÖZME BECERİSİ DEĞERLENDİRME FORMU

**MATEMATİK DERSİ**

**DOĞAL SAYILAR KONUSU**

**PROBLEM ÇÖZÜYORUM ETKİNLİĞİ**

**Problem Çözme Becerisi Değerlendirme Formu**

Problem çözme becerisi değerlendirme ölçütlerine uygun olarak öğrencinin yanıtlarını değerlendirerek puan değerini ilgili boşluğa yazınız.

| PROBLEM ÇÖZME BASAMAKLARI VE ÖLÇÜTLER | | P. |
|---|---|---|
| PROBLEMİ ANLAMA | Problemde verilenleri ve istenilenleri kendi ifadesiyle doğru bir şekilde özetlemiş. Problemin çözümü için gerekli ipuçlarını bulabilmiş. | 2 |
| | Problemde verilenleri ve istenilenleri kısmen doğru özetlemiş ve/veya kendi ifadesiyle özetlememiş. Problemin çözümü için gerekli ipuçlarını kısmen bulabilmiş. | 1 |
| | Problemde verilenleri ve istenilenleri özetleyememiş ve/veya hatalı bulmuş. | 0 |
| PLAN YAPMA | Problemde verilen bilgilerin dışında çözüm için kendisine gerekli olanları (bilgiler, işlemler vb.) ifade etmiş ve problemin çözümü için uygun bir strateji belirlemiş. | 2 |
| | Problemde verilen bilgilerin dışında çözüm için kendisine gerekli olanları (bilgiler, işlemler vb.) ifade etmiş veya problemin çözümü için uygun bir strateji belirlemiş. | 1 |
| | Problemde verilen bilgilerin dışında çözüm için kendisine gerekli olanları (bilgiler, işlemler vb.) ifade etmemiş ve problemin çözümü için uygun olmayan bir strateji belirlemiş. | 0 |
| PLANI UYGULAMA | Çözüm için kullandığı işlemleri hatasız olarak yapmış. Yaptığı işlemlerin sonucunda hangi bilgileri bulduğunu belirtmiş. | 2 |
| | Çözüm için kullandığı işlemleri çoğunlukla hatasız olarak yapmış. Yaptığı işlemlerin sonucunda hangi bilgiyi bulduğunu bazıları için yazmış. | 1 |
| | Çözüm için kullandığı işlemlerin çoğunun ya da tümünün sonucunu yanlış bulmuş. Yaptığı işlemlerin sonucunda hangi bilgileri bulduğunu yazmamış. / Hiç işlem yapmamış. | 0 |
| KONTROL ETME | Bulduğu sonucun doğruluğunu kontrol etmiş. | 2 |
| | Sonucu kontrol ettiği halde işlem hatalarını tespit edememiş. | 1 |
| | Bulduğu sonucun doğruluğunu kontrol etmemiş. | 0 |

Tarih: ... / ... / 200.

Sınıf: 6 Şube: G

| | | 1 | | | | 2 | | | | 3 | | | | 4 | | | | 5 | | | | TOPLAM | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Problemi Anlama | Plan Yapma | Planı Uygulama | Kontrol etme | Problemi Anlama | Plan Yapma | Planı Uygulama | Kontrol etme | Problemi Anlama | Plan Yapma | Planı Uygulama | Kontrol etme | Problemi Anlama | Plan Yapma | Planı Uygulama | Kontrol etme | Problemi Anlama | Plan Yapma | Planı Uygulama | Kontrol etme | Problemi Anlama | Plan Yapma | Planı Uygulama | Kontrol etme |
| No | Adı Soyadı | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | | | | | | | | | | | | 0 | 0 | 0 | 0 |
| | ORTALAMA | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | ### | 0 | 0 | 0 | 0 |

## APPENDIX B

### PROBLEM ÇÖZME ETKİNLİĞİ-1

**DERS** : MATEMATİK
**ETKİNLİĞİN ADI** : Doğal Sayılarda Problem Çözme
**SINIF DÜZEYİ** : 6
**ÖĞRENME ALANI** : Sayılar
**ALT ÖĞRENME ALANI** : Doğal Sayılar
**KAZANIMLAR:**
-Doğal sayılarla işlem yapmayı gerektiren problemleri çözer ve kurar.
**GEREKLİ MATERYALLER:**

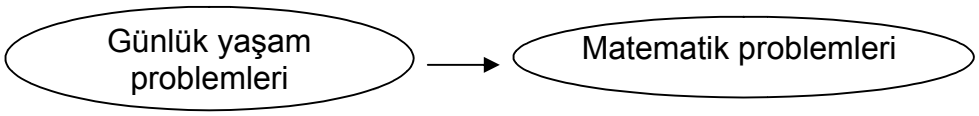
- "Günlük Yaşamda Matematik Problemleri" adlı etkinlik kağıdı
- "Kızıl Ejderha" adlı çalışma kağıdı
- "Problem Çözüyorum" adlı çalışma yaprağı

**İŞLENİŞ:**

- "Günlük Yaşamda Matematik Problemleri" adlı etkinlik yapılır.
- "Kızıl Ejderha" adlı çalışma kağıdı öğrencilere dağıtılır ve öğrencilerden hikayeyi okuduktan sonra arkasındaki soruları yanıtlamaları istenir.
- Problem çözmenin önemi üzerinde durulur.
- "Problem Çözüyorum" adlı çalışma yaprağı öğrencilere dağıtılır.
- Problem Çözme basamakları (Problemi Anlayalım, Plan Yapalım, Planı Uygulayalım, Kontrol Edelim, Problem Kuralım) ve farklı problem çözme stratejileri (Örüntü arama, Şekil çizme..vb) üzerinde durulur. Öğrencilerin problemleri bu basamaklara uygun olarak ve farklı stratejiler kullanarak çözmeleri sağlanır. Bu stratejiler sınıfça tartışılır.

# Günlük Yaşamda Matematik Problemleri

**Giriş :** Öğrencilere **“** Problem deyince aklınıza ne geliyor? “ sorusu yöneltilir.




- “Sizce sınıfta çözdüğümüz matematik problemleri ile günlük yaşamda karşımıza çıkan problemler aynı mıdır?” sorusu öğrencilere yöneltilir ve sınıfça kısa bir süre tartışılır. Sorunun yanıtı öğrencilere verilmez, aşağıdaki etkinlik ile sonuca kendilerinin ulaşması sağlanır.

Okulumuzun 6. sınıf öğrencileri Kapadokya’ ya gideceklerdir. Sizce nasıl giderler ?

- Bu bir günlük yaşam problemi midir yoksa matematik problemi midir? Neden?
- Bu problemi matematiksel olarak nasıl ifade edersiniz?

Bu problemi çözmek için hangi bilgilere ihtiyaç vardır?

- ❖ 6. sınıflar öğrenci sayısı (250)
- ❖ Ne ile yolculuk yapacakları (Otobüs, Uçak....vb)
- ❖ Bir otobüsün kaç kişi taşıyabileceği (40 Kişilik veya 50 kişilik otobüs)

Matematiksel ifadesi ( Bu bir matematik problemi olsaydı nasıl sorardık ? )

**1. Günlük yaşam Problemi**

Siz 6. sınıflar olarak Kapadokya’ya bir gezi düzenleyeceksiniz. Nasıl?

**2. Problemin Matematiksel Anlatımı**

Okulumuzun 6. sınıf öğrencileriyle Kapadokya’ya bir gezi düzenlenecektir. 250 öğrenci 50 kişi taşıyabilecek araçlarla yolculuk edeceklerdir. Bu gezi için kaç araç gereklidir?

**3. Matematiksel Problemin Çözümü**

250 : 50 = 5

**4. Günlük Yaşam Probleminin Çözümü**

5 araçla gideriz.

Yukarıdaki şema tahtaya çizilerek öğrencilere matematik problemlerinin yakın çevrelerinde ve günlük yaşamda karşılaşılan durumlar olduğu öğrencilere fark ettirilir.

**Görev :**

- Sınıf ikişer kişilik gruplara ayrılır.
- Her gruba 1'er tane A3 kağıt verilir.
- Öğrencilerden ;önce A3 kağıdı ikiye katlamaları istenir.
- Daha sonra; bu bölümleri aşağıdaki gibi dörde bölmeleri istenir.
- Her gruptan iki adet gerçek yaşam problemleri kurmaları, problemlerin matematiksel anlatımını yazmaları ve problemleri çözmeleri istenir.
- Çözümler sınıfla paylaşılır.Sınıfça seçilen 3 problem poster haline getirilmesi için hazırlayan öğrencilere ödev verilir.

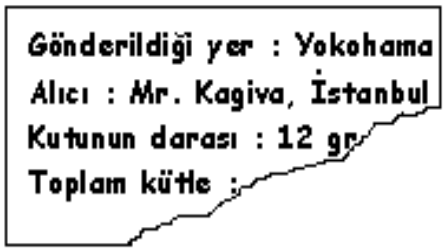
| **1) Gerçek Hayat Problemi** | **2) Problemin Matematiksel Anlatımı** |
|---|---|
| **3) Matematiksel Problemin Çözümü** | **4) Gerçek hayat Probleminin Çözümü** |

# KIZIL EJDERHA

**BÖLÜM I**

Yaşlı Japon Bankacı Naga Kagiva ortalıktan kaybolalı 5 saat olmuştu.Dedektif Selim Güçkan Hilton otelinin 506 nolu odasında zengin bankacının hizmetkarı Mr.Kyoto ve ortağı Can Açıkeli'yi sorguluyordu..

"Dün akşam yatağına kendi ellerimle yatırdım." Dedi, Kyoto,
"İlaçlarını alması için bir bardak suyu da başucuna koydum."
"Elbiseleri, valizi her şeyi kaybolmuş" dedi, Can Açıkeli.
"Önce kim fark etti ?" diye sordu, Selim Güçkan.
-Can Bey haber verdi. Odaya girdiğinde Mr.Kagiva'yı bulamamış, ben de kahvaltısını hazırlamak için aşağı restorana iniyordum.
*Selim Güçkan:* Odaya nasıl girdiniz? Kapıda zorlanma olmamış, size kapıyı kim açtı?
*Can Açıkeli* : Kapı açıktı. Garipliği zaten ilk anda fark ettim.
*Selim Güçkan* (odadaki polislere dönüp): Otel personelini sorguladınız mı? Nereye gittiğini gören var mı?
*Polis Memuru Sarp* : Tüm personelle tek tek görüştük, ne bir gören, ne de bir duyan var.
Dedektif Güçkan odayı incelemeye başladı, yaşlı Japonun gözlükleri yatağının yanı başındaydı, yerde bir Uzakdoğu halısı, yanında da büyükçe bir kutu vardı.
*Selim Güçkan:* Bu kutunun içinde ne var Mr.Kyoto?
*Mr.Kyoto:* Mr Kagiva bu taşları hep yanında taşırdı. Benim uğurlu taşlarım derdi, Bay Güçkan...Size de dün göstermişti ya Bay Açıkeli!.
*Can Açıkeli:* ??? evet bir ara gösterdi, basit siyah mermer taşlar...
*Selim Güçkan* :Şu kutuyu bir inceleyelim bakalım...
Kutunun üzerinde Japon Havayollarının etiketi vardı.



**BÖLÜM II**

Etiketin son kısmı yırtılmıştı, yırtık kısım kutunun diğer kapağında Kalmıştı. Selim Güçkan kutunun içine baktı, siyah mermer küpler… Gözleriyle hızlı bir şekilde küpleri saydı 20 tane…üzerlerindeki kızıl ejder resimleri dikkatini çekti, bir tanesini eline alıp resme yakından baktı. “Kayda değer bir şey yok.” dedi. Kutuyu kapatırken etiketin diğer kısmına göz ucuyla baktı: “41kg 12 g”

yazıyordu. “ Bugünlük bu kadar yeter beyler ,çıkalım ” Selim Güçkan koridora çıktı, asansöre yöneldi, kafasını kurcalayan bir şey vardı. Etikette onu rahatsız eden bir şey…Yanındaki polise döndü: “Çabuk bana bir terazi bulun, basit bir şey…”

**BÖLÜM III**

Selim Güçkan taşın bir tanesini tarttı, tam 2 kg geldi. Başka bir taşı tarttı o da 2kg’dı. Bütün taşların ağırlığı 2kg’dı.Taşın üzerindeki kızıl ejderhanın üzerine parmağıyla bastırdı, taşta küçük bir delik açıldı, taşın içi boştu. Diğer taşı aldı, ondaki ejderhaya bastırdı, o deliğin içi de boştu. “Hiç şaşırmadım” dedi. Kyoto ve Can Açıkeli’ye döndü.

*Selim Güçkan:* İkinizi de soruşturma dahilinde tutukluyorum Beyler!

**Selim Güçkan’ı geriye döndüren , Kyoto ve Can Açıkeliyi tutuklattıran delili aşağıdaki adımları izleyerek bulunuz.**

**<u>1.ADIM</u> Hikayede belirtilen kişilerin tablosunu oluşturunuz.** (20 puan)

| **Hikayedeki Karakterin İsmi** | **Görevi** |
|---|---|
| | |

**<u>2.ADIM</u> Hikayedeki problemin çözümünde kullanılacak sayısal ipuçlarını yazınız. (**20 puan)

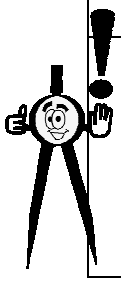
**<u>İpucu1</u>:**……

**<u>İpucu2</u>:**……

**<u>İpucu3</u>:**……

**<u>İpucu4</u>:**……

***Öğretmeniniz tarafından doldurulacaktır!!!***

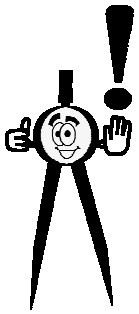


| **Problem Çözme Basamakları** | **Ölçüt** | **1.Adım** | **2.Adım** | **Toplam Puan (40 üzerinden)** | **Puan Aralığı karşılığı** |
|---|---|---|---|---|---|
| Problemi Anlama | Verilenleri Bulma | | | | |
| | Çözüm için gerekli ipuçlarını bulma | | | | |

**Not:** ***Puan Aralığı Karşılığı:***

| **0 (0-10 puan)** | **1 (11-30 Puan)** | **2 (31-40 Puan)** |
|---|---|---|
| Problemi tamamen yanlış anlamış. | Problemin bir kısmını yanlış anlamış veya yanlış yorumlamış. | Problemi tamamiyle doğru anlamış. |

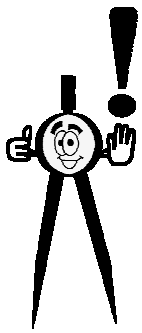
**<u>3.ADIM</u> Bulduğunuz ipuçlarından faydalanarak problemi çözünüz.**



***Öğretmeniniz tarafından doldurulacaktır!!!***

| | | Ölçütler | Puan |
|---|---|---|---|
| **Problemi Çözme** | 0 | Hiç yanıtlamamış yada tamamiyle yanlış çözmüş. İşlem basamaklarını göstermeden sadece doğru yanıtı yazmış. | |
| | 1 | İşlem hatası yapmış. Soruyu yanlış anladığı için yanlış yanıtlamış. Soruyu kısmen doğru yanıtlamış. | |
| | 2 | Soruyu tamamiyle doğru yanıtlamış. | |

**<u>4.ADIM</u> Bu hikayedeki ipuçlarından faydalanarak bir matematik problemi oluşturunuz.**



***Öğretmeniniz tarafından doldurulacaktır!!!***

| | | Ölçütler | Puan |
|---|---|---|---|
| **Benzer bir problemi kurma** | 0 | Hiç kurmamış veya tamamiyle yanlış bir problem kurmuş. | |
| | 1 | Kısmen doğru bir problem kurmuş | |
| | 2 | Tamamiyle doğru bir problem kurmuş | |

- Problem çözmenin önemi üzerinde durulur.

“**Problem çözmek neden bu kadar önemli?”**

**“Neden problem çözmeyi öğreniyoruz?”** gibi sorular öğrencilere yöneltilir.

- Öğrencilerden **“Çağımız problem çözme çağıdır.”** cümlesinden ne anladıklarını iki üç cümleyle defterlerine yazmaları istenir.
- Aşağıdaki konular üzerinde durulur.,

.

| |
|---|
| Problem çözme; ne yapılacağının bilinmediği durumlarda yapılması gerekeni bulmaktır |

Günlük yaşam problemleri kişisel de olabilir, tüm toplumu ilgilendiren bir problem de olabilir. Örneğin;

- İstanbul’daki su sıkıntısı var ve bu problem küresel ısınmayla birlikte günden güne artmakta. Su sıkıntısı **günlük yaşam problemi, tüm toplumu ilgilendiriyor, hayati önemi var**.
- Bir adada kalan adam için adada nasıl hayatta kalacağı bir problem.
- Yemeğe koyulan yağ, tuz miktarı sağlıklı beslenme, damak tadımız vs. açısından bir problem.

Matematik dersinde günlük yaşamda karşımıza çıkabilecek her türlü problemi çözemeyiz ama problem çözmeyi öğrendiğimizde, burada öğrendiklerimizi günlük yaşamda, farklı alanlardaki problemleri çözmede kullanabiliriz. Bilgisayar oyunlarında basit bir oyunu oynamak için geliştirdiğimiz stratejileri daha zor ve karmaşık bir oyunu oynarken kullandığımız gibi. Problem çözmenin basamaklarından bahsedilir.

# PROBLEM ÇÖZÜYORUM

**1)** Ali Baba çiftliğinde inek ve ördek beslemektedir. Ali baba'nın çiftliğinde 54 tane hayvanı vardır ve tüm hayvanların ayak sayısının 122 olduğu bilinmektedir. Buna göre, Ali Baba'nın çiftliğinde kaç tane ördek vardır?

➢ **Problemi Anlayalım:**

•*Verilenler:*

•Ali Baba ördek ve inek beslemektedir.
•54 hayvan
•122 ayak

•İstenenler:
Ördek sayısı=?

➢ **Plan Yapalım:**

Ördek ve inek sayısını tahmin ederim.
Bir tablo yapar, tahminlerimden yola çıkarak ayak sayılarını belirlerim.
Doğru cevabı buluna kadar mantıklı çıkarımlarda bulunurum.

Tahmin ve Kontrol | Akıl Yürütme | Tablo Yapma

➢ **Planı Uygulayalım:**

| İnek Sayısı | Ördek Sayısı | Ördek Ayak Sayısı | İnek Ayak Sayısı | Toplam Ayak Sayısı (122) | Kontrol |
|---|---|---|---|---|---|
| 30 | 24 | 48 | 120 | 168 | çok |
| 20 | 34 | 68 | 80 | 148 | çok |
| 10 | 44 | 88 | 40 | 128 | çok |
| 8 | 46 | 92 | 32 | 124 | çok |
| 7 | 47 | 94 | 28 | 122 | ☺ |

**Kontrol Edelim:**

47 x 2 =94 (Ördek Ayak Sayısı)
7 x 4 = 28 (İnek Ayak Sayısı)  84 +28= 122 (Toplam Ayak Sayısı)

**2)** ATV Haber Spikeri Şenol Yazıcı Pazar akşamı; “Pazartesi günü 17$^{o}$C olan hava sıcaklığı her gün iki derece artacak.” haberini vermiştir. Buna göre Cuma günü hava sıcaklığı kaç derece olacaktır?

I.Problemi Anlayalım


II.Plan Yapalım

Strateji :……..

III.Planı Uygulayalım

IV.Kontrol Edelim

**3)** Erdal öğretmen matematik dersinde bir etkinlik yapacaktır. Bunun için elindeki ip parçasını, 25 makas darbesiyle eş parçalara ayırır ve elindeki parçaları hiç artmayacak şekilde öğrencilerine dağıtır. İp almayan öğrenci kalmadığına göre; Erdal öğretmenin bu sınıfta kaç öğrencisi vardır?

I.Problemi Anlayalım

II.Plan Yapalım

Strateji :……..

III.Planı Uygulayalım

IV.Kontrol Edelim

**4) Cem :** Benim 19 tane oyuncak arabam var.
**Sinan :** Oooo ne kadar çokmuş, benimkilerin 3 katının 4 fazlası…

Yukarıdaki diyaloğa göre Sinan'ın kaç arabası vardır?

I.Problemi Anlayalım

II.Plan Yapalım

Strateji :……..

III.Planı Uygulayalım

IV.Kontrol Edelim

**5)** Barış Kırtasiyede 3, 5 ve 10 TL' lik kalemler satılmaktadır. Okul çıkışı cebindeki 12TL ile kırtasiyeye giden Ceren en fazla kaç kalem alabilir?

I.Problemi Anlayalım

II.Plan Yapalım

Strateji :……..

III.Planı Uygulayalım

IV.Kontrol Edelim

**6)** Şimdi de size bir mantık sorusu:

Bir kutuda bulunan 6 eş görünümlü topun 5 inin ağırlığı birbirine eşit diğerininki daha ağırdır. Ağır olan topu en az kaç tartıda ayırabilirsiniz?

I.Problemi Anlayalım

---

II.Plan Yapalım

Strateji :…….

---

III.Planı Uygulayalım

---

IV.Kontrol Edelim

**7)** Oya,aşağıdaki örüntüyü devam ettirmek için sizden yardım istemektedir.Ona yardımcı olur musunuz?

**a)** F satırında kaç harf vardır?

**b)** Örüntünün hangi satırında 19 harf vardır?

A
BBB
CCCCC
ÇÇÇÇÇ

I.Problemi Anlayalım

---

II.Plan Yapalım

Strateji :……..

---

III.Planı Uygulayalım

IV.Kontrol Edelim

**8)** Mert, eşi ve iki çocuğu her hafta sonu tiyatro, sinema, sergi vb. etkinliklere katılırlar. Bu hafta sonu " Ferhat ile Şirin" adlı eseri izlemek için tiyatroya gidecekler.Tiyatroya giriş ücreti yetişkinler için 6 TL, çocuklar için 4 TL dir.

**a)** Yukarıdaki verileri kullanarak bir soru cümlesi yazınız.

**b)** Tiyatronun bir günlük toplam hasılatının bulunabilmesi için hangi bilgiye ihtiyaç vardır?

**9)** **"**Üç kardeşin yaşları toplamı 36'dır. Bu kardeşler 4'er yıl ara ile doğduklarına göre en büyükleri kaç yaşındadır?" Problemin çözümünde boş bırakılan basamakları doldurunuz.

a) Adım:

b) Adım: 36-12= 24

c) Adım:

d) Adım: 8+8=16

**10)** Şahin Bey, işe geç kaldığı için evinin önünden taksiye biner, taksimetrenin açılış ücreti 1,50 TL dir. Taksimetre her 200 metrede bir 30 Kr yazar. Şahin Bey, taksiciye 1 TL de bahşiş bırakarak toplam 10 TL verir. Şahin Bey'in işyeri evinden kaç kilometre uzaklıktadır?

Problem çözüyorum çalışma yaprağından seçeceğiniz bir problem için aşağıdaki formu doldurunuz.

**PROBLEM ÇÖZME ÖĞRENCİ ÖZ DEĞERLENDİRME FORMU**

İsim:..... Tarih:.../.../.....

Problem:

.....

.....

Problemi nasıl çözdüm:

.....

.....

Çözüm yolum işe yaradı mı?

.....

.....

Bir sonraki sefer benzer bir problemi nasıl çözerim:

.....

.....

Kullanabileceğim diğer stratejiler:

.....

.....

Ne öğrendim?

.....

.....

**Bu problemin çözümünde kullandığınız strateji/stratejileri işaretleyiniz.**

| | | | |
|---|---|---|---|
| 1. Bir şekil çizme. | ☐ | 11. Problemi basitleştirme | ☐ |
| 2. Bir model yapma. | ☐ | 12. Benzer bir problemi çözme | ☐ |
| 3. Bir örüntü arama | ☐ | 13. İşlem seçme | ☐ |
| 4. Tahmin ve kontrol etme. | ☐ | 14. Denklem kurma | ☐ |
| 5. Sayı cümleleri | ☐ | 15. Matematiksel iletişim | ☐ |
| 6. Matematik cümlesi olarak ifade etme | ☐ | 16. Deneme yanılma | ☐ |
| 7. Bir liste, grafik ve tablo yapma | ☐ | 17. Varsayımları kullanma | ☐ |
| 8. Problemi bölümlere ayırma | ☐ | 18. Matematiksel ilişkilendirme | ☐ |
| 9.Geriye dönerek çalışma | ☐ | 19. Diğer derslerle ilişkilendirme | ☐ |
| 10. Akıl yürütme | ☐ | 20. Strateji kullanmadım. | ☐ |