ASSESSMENT OF SECOND-ORDER ANALYSIS METHODS PRESENTED IN DESIGN CODES

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Approval of the thesis:

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ABSTRACT

ASSESSMENT OF SECOND-ORDER ANALYSIS METHODS PRESENTED IN DESIGN CODES

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The main objective of the thesis is evaluating and comparing *Second-Order Elastic Analysis Methods* defined in two different specifications, AISC 2005 and TS648 (1980). There are many theoretical approaches that can provide exact solution for the problem. However, approximate methods are still needed for design purposes. Simple formulations for code applications were developed, and they are valid as acceptable results can be obtained within admissible error limits. Within the content of the thesis, firstly background information related to second-order effects will be presented. The emphasis will be on the definition of geometric non-linearity, also called as P-δ and P-Δ effects. In addition, the approximate methods defined in AISC 2005 (B_1 – B_2 Method), and TS648 (1980) will be discussed in detail. Then, example problems will be solved for the demonstration of theoretical formulations for members with and without end translation cases. Also, the results obtained from the structural analysis software, SAP2000, will be compared with the results acquired from the exact and the approximate methods. Finally, conclusions related to the study will be stated.

Keywords: Second-order elastic analysis, beam-column, P-delta effects, AISC 2005, TS648 (1980)

ÖZ

TASARIM ŞARTNAMELERİNDEKİ İKİNCİ MERTEBE ANALİZ METOTLARININ DEĞERLENDİRİLMESİ

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Mart 2009, 81 Sayfa

Bu tezin amacı, AISC 2005 ve TS648 (1980) tasarım şartnamelerinde tanımlanan ikinci mertebe elastik analiz yöntemlerini değerlendirmek ve karşılaştırmaktır. Problemin çözümü için bazı teorik yaklaşımlar bulunmaktadır. Fakat tasarım yapmak amacıyla bazı yaklaşık yöntemler ve varsayımlara ihtiyaç vardır. Şartname uygulamaları için belirli hata limitleri içinde kabul edilebilir sonuçlar verebilen basit yöntemler geliştirilmiştir. Tez kapsamında, öncelikle ikinci mertebe etkiler hakkında ön bilgi sunulacaktır. Özellikle geometrik doğrusal olmayan etkiler (Kuvvet-Deplasman Etkileri) vurgulanacaktır. İlave olarak, AISC 2005 (B1-B2 Metodu) ve TS648 (1980) tasarım şartnamelerinde yer alan yaklaşık yöntemler detaylı olarak irdelenecektir. Sonraki bölümde yer alan örneklerde, düğüm noktalarının ötelenmesine müsaade edilmeyen ve yanal deplasmanın mümkün olduğu çubuk elemanlar ve çerçeveler için teorik yaklaşımlardan elde edilen gerçek sonuçlar hesaplanacaktır. Ayrıca, SAP2000 Yapısal Analiz Programı ile elde edilen sonuçlar, gerçek değerlerle ve şartnamelerin yaklaşık yöntemleriyle elde edilen sonuçlarla karşılaştırılacaktır. Son kısımda, bu çalışma kapsamında edinilen sonuçlar özetlenecektir.

Anahtar Sözcükler: İkinci mertebe elastik analiz, kiriş-kolon, kuvvet-deplasman etkileri, AISC 2005, TS648 (1980)

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LIST OF SYMBOLS

Symbol	**Definition**
A	Cross-sectional area
A_f	Amplification factor to multiply with the first-order moments to determine the second-order moments
B_1, B_2	Factors used in determining M_u for combined bending and axial forces when first-order analysis is employed, P-δ and P-Δ amplification factors, respectively
C_m	*Moment reduction factor* for members braced against joint translation with transverse loading between supports, *equivalent moment factor* for braced members subjected to end moments only
E	Modulus of elasticity
F_e'	Euler stress divided by a factor of safety of 23/12 (AISC 1969)
I	Moment of inertia in the plane of bending
K_1	Effective length factor in the plane of bending, calculated based on the assumption of no lateral translation
K_2	Effective length factor in the plane of bending, calculated based on a side-sway buckling analysis
L	Length of the member
M_c	Available flexural strength
M_{lt}	First-order moment using LRFD or ASD load combinations caused by lateral translation of the frame
M_{nt}	First-order moment using LRFD or ASD load combinations, assuming there is no lateral translation of the frame
M_r	Required second-order flexural strength using LRFD or ASD load combinations
$M_{z,max1}$	Maximum first-order elastic moment
$M_{z,max2}$	Maximum second-order elastic moment

M_0	Maximum first-order moment within the member due to transverse loading
M_1, M_2	The smaller and larger moments, respectively, calculated from a first-order analysis at the ends of that portion of the member unbraced in the plane of bending under consideration
P	Applied axial force
P_c	Available axial compressive strength
P_{e1}	Elastic critical buckling resistance of the member in the plane of bending, calculated based on the assumption of zero sidesway
P_{lt}	First-order axial force using LRFD or ASD load combinations, assuming there is no lateral translation of the frame
P_{nt}	First-order axial force using LRFD or ASD load combinations caused by lateral translation of the frame only
P_r	Required second-order axial strength using LRFD or ASD load combinations
W	Transverse point load
f_a	Computed axial stress (AISC 1969)
i_b	Radius of gyration about an axis perpendicular to the bending plane (TS648 - 1980)
s_b	Unbraced span length (TS648 - 1980)
w	Transverse uniformly distributed load
δ_0	Maximum deflection due to transverse loading
σ_a	Yield stress of the material (TS648 - 1980)
σ_B	Bending stress permitted in the absence of axial force (TS648 - 1980)
σ_b	Computed bending stress (TS648 - 1980)
σ_{bem}	Axial stress permitted in the absence of bending moments (TS648 - 1980)
σ_{eb}	Computed axial stress (TS648 - 1980)
σ_{ex}', σ_{ey}'	Critical elastic buckling stresses about *x*- and *y*-axes divided by a factor of safety of 2.5 (TS648 - 1980)
Ω_0	Horizontal seismic overstrength factor

Δ_H	First-order interstory drift due to lateral forces. Where Δ_H varies over the plan area of the structure, Δ_H shall be the average drift weighted in proportion to vertical load or, alternatively, the maximum drift
$\sum H$	Story shear produced by the lateral forces used to compute Δ_H
ΣP_{e2}	Elastic critical buckling resistance for the story determined by sidesway buckling analysis
ΣP_{nt}	Total vertical load supported by the story using LRFD or ASD load combinations, including gravity column loads

CHAPTER 1

INTRODUCTION

1.1 Background on Second-Order Effects

Generally, the analysis of most conventional structure type of buildings is done by using linear elastic analysis methods. However, the second-order effects should be considered in the design. According to Mashary & Chen (*1990*), main second-order effects are listed below:

- Geometric non-linearity, P-δ and P-Δ effects
- Column axial shortening (Bowing effect)
- Semi-rigid behavior of connections rather than a fully rigid / ideally hinged condition
- Panel-zone effect
- Differential settlement of foundation
- Non-uniform temperature effects
- Out-of-straightness and out-of-plumbness effects
- Residual stresses & other imperfections
- Column or beam yielding
- Redistribution effect

The main emphasis of the thesis is on the geometric non-linearity, P-δ and P-Δ effects. In the following lines, "second-order effects" term will be used for only geometric non-linearity of P-δ and P-Δ effects.

The issue occurs mainly in the element that is subjected to both bending and axial compression known as "beam-column". Also, second-order effects can be significant for the members having initial imperfections. That is why the design of a member by considering only the axial compression is prohibited by the design specifications.

1.2 Definition of P-Delta Effects

There are various definitions of P-delta effects from the projection of different aspects. According to Chen & Lui (1991), two types of secondary effects can be identified: The P-δ (P-small delta) effect and the P-Δ (P-big delta) effect. These secondary effects cause the member to deform more and induce additional stresses in the member. As a result, they have a weakening or destabilizing effect on the structure.

In addition, P-delta effects are defined in AISC Specification (2005). P-δ is the effect of loads acting on the deflected shape of a member between joints and nodes, whereas P-Δ is the effect of loads acting on the displaced location of joints or nodes in a structure *(Figure 1.1)*.

According to White & Hajjar (1991), P-δ effect is the influence of axial force on the flexural stiffness of individual members (member curvature effect); however P-Δ effect is the influence of gravity loads on the side-sway stiffness (member chord rotation effect).

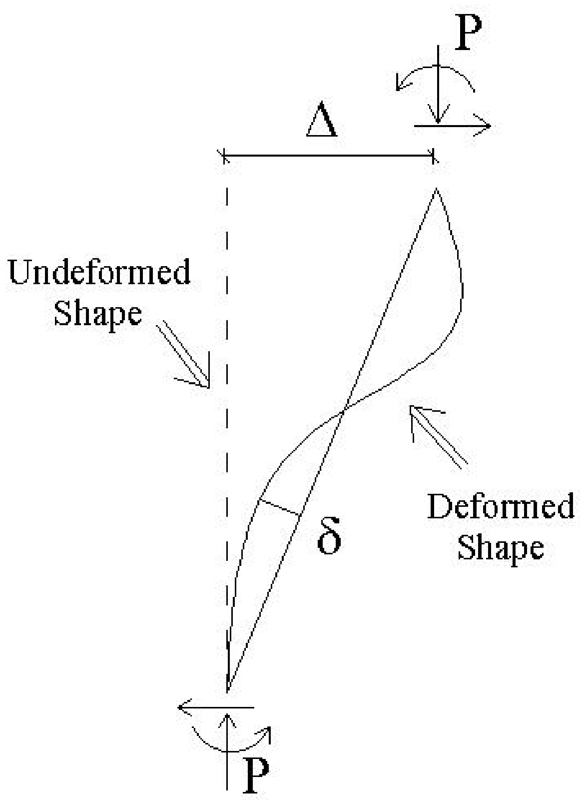


Figure 1.1: P-Δ and P-δ Effects

Equilibrium is formulated on the undeformed geometry in linear elastic analysis. However, in geometrically nonlinear or second-order elastic analysis, equilibrium is formulated based on the deformed configuration of the structure. The exact solution of second-order analysis is based on *"the differential equation approach"* in which the formulations are founded on the natural deformed shape of the element.

Second-order matrix analysis methods have been developed for taking the second-order effects into account by using the advanced computer technology at the present time. Despite the opposite arguments taking part in the articles published in early 1990's, the computer technology is so wide today that even the rigorous problems are solved within seconds in the personal computers. The main matrix structural analysis methods are *geometric matrix approach (finite element / geometric stiffness approach)* and *stability functions approach*.

On the other hand, approximate methods have been developed based on the assumptions and simplifications which are used mainly in design applications and software algorithms. Second-order effects are considered in design codes by using the recommended *"strength interaction equations"* that express a safe combination of axial force and bending moments that the member can sustain (*Chen & Lui, 1991*). Different approaches have been proposed within the content of the specifications. Besides, the equations are revised frequently within the new editions of the specifications parallel to the trends in the computer technologies. The main criterion is the applicability of the method in design by using simplified approaches. The method must represent a wide range of various conditions by presenting reasonable results within acceptable safety limits without exceeding the feasibility of practical design applications.

1.3 Structural Analysis Software Used in the Thesis, SAP2000

SAP2000 is a practical general purpose structural program used widely on the market. It is capable of performing the wide variety of analysis and design options including Step-by-Step Large Deformation Analysis, Multiple P-Delta, Eigen and Ritz Analyses, Cable Analysis, Tension or Compression Only Analysis, Buckling Analysis, etc. The powerful user interface provides convenience for modeling and evaluation of complicated structural systems.

SAP2000 provides pre-processing, analysis, and post-processing capabilities. Pre-processing options include definition of structural geometry, support conditions, application of loads, and section properties. The analysis routines provide opportunity to perform first- or second-order elastic or inelastic analyses of two- or three-dimensional frames and trusses subjected to static loads. Post-processing capabilities include the interpretation of structural behavior through deformation and force diagrams, printed output, and so on.

Furthermore, it should be noted that SAP2000 is capable of analyzing both of the P-Delta effects; the first due to the overall sway of the structure and the second due to the deformation of the member between its ends, in other words P-Δ and P-δ effects are referenced, respectively. However, it is recommended that former effect be accounted for in the SAP2000 analysis, and the latter effect be accounted for in design by using the applicable building-code moment-magnification factors. This is how the SAP2000 design processors for steel frames and concrete frames are set up (CSI Analysis Reference Manual, 2008). Besides, P-δ effects can be taken into account by dividing the single member into several pieces.

In addition, AISC 2005 states that the second-order internal forces cannot be normally combined by superposition since second-order amplification depends, in a nonlinear fashion, on the total axial forces within the structure. Therefore, a separate second-order analysis must be conducted for each load combination considered in the design.

Several structural analyses were performed by SAP2000 Advanced v.11.0.0 throughout the thesis. Basically, first-, and second-order elastic analysis options were used in 2-dimensional problems.

1.4 AISC (2005) Provisions

According to AISC 2005, second-order effects defined in *Section 1.1* must be considered in design. However, some of these effects may be neglected by professional judgment of the designer when they are insignificant. Specifically, P-delta effects must be taken into account in the analysis part of the design process according to AISC 2005 methodology, since the interaction equations for beam-columns were calibrated implying this phenomenon. Interaction equations for doubly and singly symmetric members are presented in *Equations (1.1) & (1.2).*

$$For\ \frac{P_r}{P_c} \geq 0.2 \qquad \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \tag{1.1}$$

$$For\ \frac{P_r}{P_c} < 0.2 \qquad \frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \tag{1.2}$$

Fundamentally, three different methodologies were specified to account for the stability of the structural systems in AISC 2005. These are Effective Length Method, First-Order Analysis Method, and Direct Analysis Method. Effective Length Method is the classical methodology used ever since the first AISC/LRFD Specification published in 1986. On the other hand, First-Order Analysis Method and Direct Analysis Method were set in AISC 2005 for the first time. Second-order analysis is required for Effective Length Method and Direct Analysis Method, whereas first-order analysis is sufficient for First-Order Analysis Method when some special conditions are satisfied.

In the content of the thesis, second-order analysis procedures defined in AISC 2005 will be evaluated. According to AISC 2005, any second-order elastic analysis method considering both P-Δ and P-δ effects may be used, including a direct second-order analysis performed by using structural analysis software. Besides, an approximate procedure is specified as *"Second-Order Analysis by Amplified First-Order Elastic Analysis",* which is also called *B_1-B_2 Method*. In this procedure, P-delta effects are taken into account by the amplification of first-order moments and axial forces in members to obtain secondary forces. The B_1 and B_2 factors are the P-δ and P-Δ moment amplification factors, respectively (*Chen & Lui, 1991*). The following formulations are specified in AISC 2005 to consider second-order effects.

$$M_r = B_1 M_{nt} + B_2 M_{lt} \tag{1.3}$$

$$P_r = P_{nt} + B_2 P_{lt} \tag{1.4}$$

Second-order effects are considered by calculating the contribution of sway and no-sway components, separately. B_1 and B_2 factors are defined comprehensively in the following subsections. Also, it should be noted that Allowable Stress Design (ASD) practice defined in AISC 2005 will be used instead of Load and Resistance Factor Design (LRFD), since TS648 (1980) formulations are based on ASD.

In applying B_1-B_2 Method, two first-order analyses are required. In the first-analysis, artificial supports are introduced to brace the frame against lateral translation *(Fig. 1.2b)*. The moments obtained from this analysis are designated as M_{nt}. In the second analysis, the reactions induced in the artificial supports are applied in the reverse direction to the frame *(Fig. 1.2c)*. The moments obtained from this analysis are designated as M_{lt} (Chen & Lui, 1991).

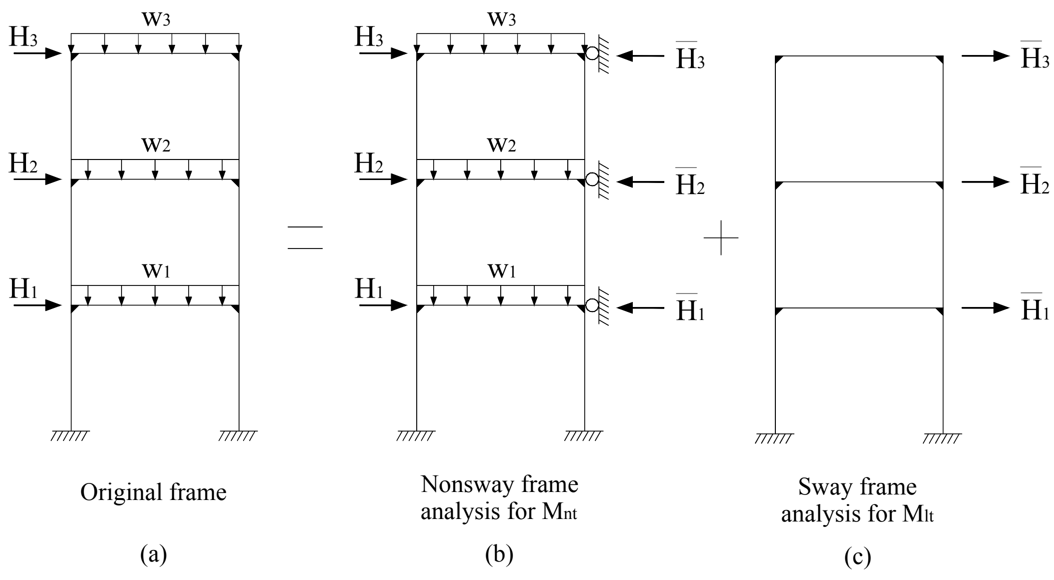


Figure 1.2: Determination of M_{nt} and M_{lt} (Chen & Lui, 1991)

1.4.1 B_1 Coefficient

B_1 coefficient is an amplifier to account for the second-order effects caused by displacements between brace points (AISC 2005), which is also known as "*P–δ amplification factor*". The formulation is given as follows:

$$B_1 = \frac{C_m}{1 - \alpha \frac{P_r}{P_{e1}}} \geq 1 \qquad (1.5)$$

In which α=1.0 for LRFD or 1.6 for ASD.

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} \qquad (1.6)$$

The P–δ amplification factor, B_1, is directly proportional to the axial load level that is represented by the term, P_r/P_{e1}, in *Equation (1.5)*. C_m factor and axial thrust level are the main factors concerning the magnification of no-sway part of the first-order elastic moments.

At that point, *C_m factor* is needed to be defined and examined in detail. C_m coefficient is called *moment reduction factor* for members braced against joint translation with transverse loading between supports, whereas it is referred as *equivalent moment factor* for members subjected to end moments only (Chen & Lui, 1987).

Effect of transverse loading on the magnification level of the first-order moments is taken into account by C_m factor as defined below:

$$C_m = 1 + \psi \left(\frac{\alpha P_r}{P_{e1}} \right) \qquad (1.7)$$

In which, ψ is given for simply supported members in the formulation presented as follows:

$$\psi = \frac{\pi^2 \delta_0 EI}{M_0 L^2} - 1 \qquad (1.8)$$

The definition for ψ in *Eq. (1.8)* is applicable only for cases in which the maximum primary moment occurs at or near mid-span. If this condition is not satisfied, ψ must be redefined

(Chen & Lui, 1991). The rigorous solutions for the fixed-ended members were presented by Iwankiw (1984) as shown in *Table 1.1* which is quoted from AISC 2005. By this way, C_m factor can be selected from *Table 1.1* without dealing with the calculation of ψ term for frequently encountered loading conditions.

In the current version of AISC Specification (AISC 2005), usage of *Eq. (1.8)* to obtain the ψ term is limited for only simply supported members. However, the same formula was erroneously used for fixed-ended members in AISC manuals until the revised updated edition published in 1978 (AISC 1978). The table ignoring the amplification of the first-order elastic moments at the fixed-ends was published in AISC Manual (1969) which is also given in *Table 1.2*. The same error occurred in the specifications that share the same philosophy of design with AISC. The same table takes its part in the current Turkish Standard, TS648 (1980), with the wrong C_m factors for fixed-ended members.

Moreover, according to AISC 2005 *C_m factor* can be conservatively taken as 1.0 for all since α rarely will exceed about 0.3 (Salmon & Johnson, 1996) . In the previous editions of AISC Manual, C_m = 0.85 was used for members with restrained ends, which can sometimes result in a significant under-estimation of internal moments.

Table 1.1: Amplification Factors, ψ and C_m (AISC 2005)

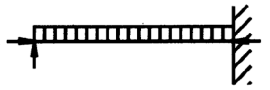
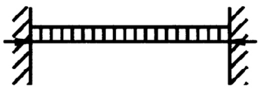
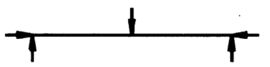
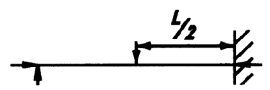
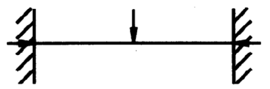
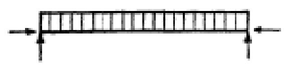
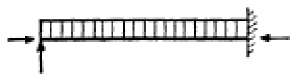
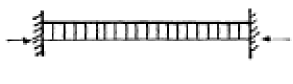
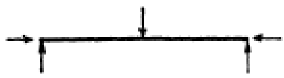
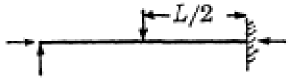
Case	Ψ	C_m
	0	1.0
	–0.4	$1-0.4\frac{P_u}{P_{e1}}$
	–0.4	$1-0.4\frac{P_u}{P_{e1}}$
	–0.2	$1-0.2\frac{P_u}{P_{e1}}$
	–0.3	$1-0.3\frac{P_u}{P_{e1}}$
	–0.2	$1-0.2\frac{P_u}{P_{e1}}$

Table 1.2: Amplification Factors, ψ and C_m (AISC 1969)

Case	ψ	C_m
	0	1.0
	−0.3	$1 - 0.3\frac{f_a}{F'_e}$
	−0.4	$1 - 0.4\frac{f_a}{F'_e}$
	−0.2	$1 - 0.2\frac{f_a}{F'_e}$
	−0.4	$1 - 0.4\frac{f_a}{F'_e}$
	−0.6	$1 - 0.6\frac{f_a}{F'_e}$

In addition to the cases with transverse loading specified in the preceding paragraphs, *Eq. (1.9)* is presented for the beam-columns subjected to end moments without transverse loading in AISC 2005.

$$C_m = 0.6 - 0.4(M_1/M_2) \qquad (1.9)$$

where M_1 and M_2, calculated from a first-order analysis, are the smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature, negative when bent in single curvature. Member slenderness effect is ignored in *Eq. (1.9)* because of the relatively small effect on C_m for design purposes (SSRC, 1988).

1.4.2 B_2 Coefficient

When lateral forces, ∑H, act on a frame, the frame will deflect laterally until the equilibrium position is reached *(Figure 1.3a)*. The corresponding lateral deflection calculated based on the undeformed geometry is denoted by Δ_I. If in addition to ∑H, vertical forces ∑P are acting on the frame, these forces will interact with lateral displacement Δ_1 caused by ∑H to drift the frame further until a new equilibrium position is reached. The lateral deflection that corresponds to the new equilibrium position is denoted by Δ *(Figure 1.3b)*.

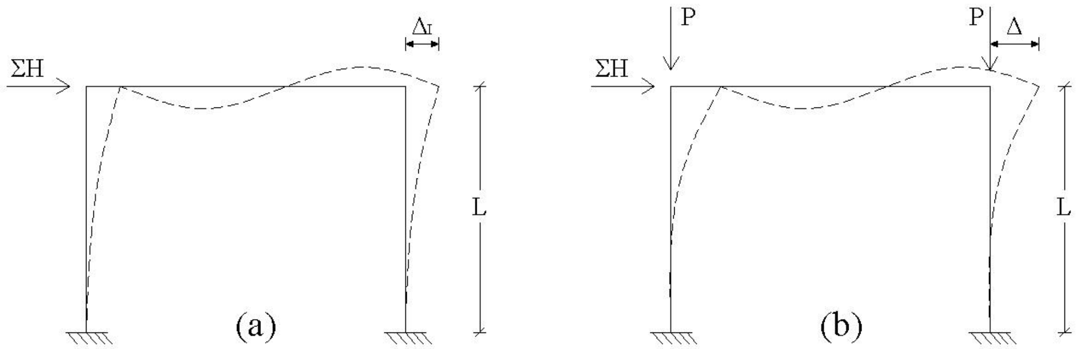


Figure 1.3: P-Δ Effect (Chen & Lui, 1991)

The phenomenon by which the vertical forces, ∑P, interact with the lateral displacement of the frame is called the *P-Δ Effect*. The consequences of this effect are an increase in drift and an increase in overturning moment (Chen & Lui, 1991).

B_2 is an amplifier to account for second-order effects caused by displacements of brace points (AISC 2005), which is also known as "*P-Δ amplification factor*" *(Equation 1.10)*.

$$B_2 = \frac{1}{1 - \alpha \frac{\Sigma P_{nt}}{\Sigma P_{e2}}} \geq 1 \qquad (1.10)$$

For moment frames, where sidesway buckling effective length factors K_2 are determined for the columns, it is permitted to calculate the elastic story sidesway buckling resistance as specified in *Eq. (1.11)*.

$$\Sigma P_{e2} = \Sigma \frac{\pi^2 EI}{(K_2 L)^2} \tag{1.11}$$

For all types of lateral load resisting systems, it is permitted to use *Eq. (1.12)*.

$$\Sigma P_{e2} = R_M \frac{\Sigma HL}{\Delta_H} \tag{1.12}$$

R_M coefficient specified in *Eq. (1.12)* can be taken as 1.0 for braced-frame systems; however it should be assumed as 0.85 for moment-frame and combined systems, unless a larger value is justified by analysis.

The procedure defined in *Eq. (1.11)* is based on *"Multiple Column Magnifier Method"* defined by Chen & Lui (1991) in detail. When instability is to occur in a story, all columns in that story will become unstable simultaneously. Thus, the term P/P_E can be replaced by the term $\sum(P/P_E)$, where the summation is carried through all columns in a story.

As an alternative procedure for the determination of P-Δ amplification factor, B_2, *"Story Magnifier Method"* was proposed by Rosenblueth et al. (1965). The fundamental assumptions are that each story behaves independently of other stories, and the additional moment in the columns caused by P-Δ effect is equivalent to that caused by a lateral force of $\sum P(\Delta/L)$. The procedure is summarized in the figure shown below.

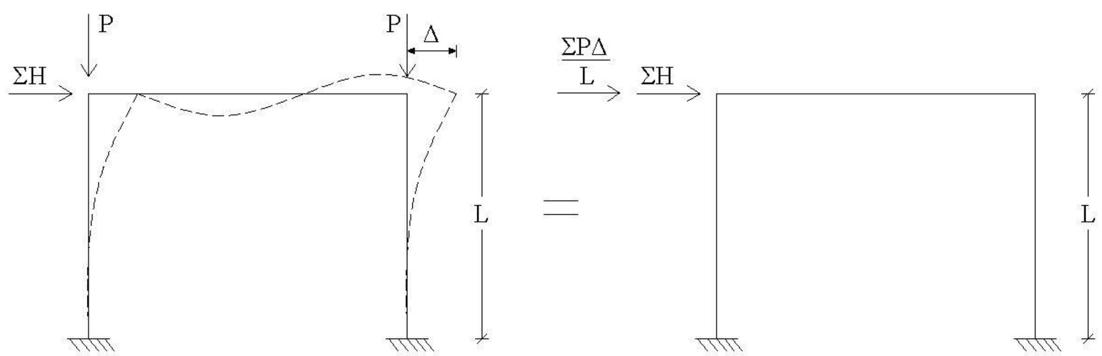


Figure 1.4: Story Magnifier Method

If P-Δ effect is small, the methods will give similar results. *Story Magnifier Concept* gives slightly better results for large P-Δ effect. Nevertheless, *Multiple Column Magnifier Concept* is simpler to use since B_2 can be evaluated without the need to perform a first-order elastic analysis on the structure. However, the effective length factor *K* is required in Story Magnifier Method for each column in the story (Chen & Lui, 1991).

1.5 TS648 (1980) Provisions

Currently, Turkish Standard, TS648 - Building Code for Steel Structures published in December 1980 is valid in Turkey for the design of steel buildings. Allowable Stress Design (ASD) is the main principle of TS648 (1980). Second-order effects are considered in TS648 (1980) within the content of the stability equation shown in *Eq. (1.13)* for the case of $\sigma_{eb}/\sigma_{bem}>0.15$.

$$\frac{\sigma_{eb}}{\sigma_{bem}}+\frac{C_{mx}\cdot\sigma_{bx}}{\left(1.0-\frac{\sigma_{eb}}{\sigma_{ex}'}\right)\cdot\sigma_{Bx}}+\frac{C_{my}\cdot\sigma_{by}}{\left(1.0-\frac{\sigma_{eb}}{\sigma_{ey}'}\right)\cdot\sigma_{By}}\leq 1.0 \qquad (1.13)$$

In which

$$\sigma_e'=\frac{\pi^2 E}{(K\cdot s_b/i_b)^2}\cdot\frac{1}{2.5}=\frac{8{,}200{,}000}{(K\cdot s_b/i_b)^2} \qquad (1.14)$$

Amplification of first-order moments is performed by the multiplication of the bending term with the coefficient found in the formulation presented as follows.

$$A_f=\frac{C_m}{1.0-\frac{\sigma_{eb}}{\sigma_e'}} \qquad (1.15)$$

Since a lower limit is not specified for the amplification factor in order not to be less than unity, an additional strength equation is required as given in the formulation below for the case of $\sigma_{eb}/\sigma_{bem}>0.15$.

$$\frac{\sigma_{eb}}{0.6\sigma_a}+\frac{\sigma_{bx}}{\sigma_{Bx}}+\frac{\sigma_{by}}{\sigma_{By}}\leq 1.0 \qquad (1.16)$$

In lieu of using *Equations (1.13)* and *(1.16)*, the following formulation is proposed by TS648 (1980) when σ_{eb}/σ_{bem} ratio is below 0.15.

$$\frac{\sigma_{eb}}{\sigma_{bem}}+\frac{\sigma_{bx}}{\sigma_{Bx}}+\frac{\sigma_{by}}{\sigma_{By}}\leq 1.0 \tag{1.17}$$

Obviously, moment amplification factor is equal to unity for $\sigma_{eb}/\sigma_{bem}\leq 0.15$. Nevertheless, TS648 (1980) underestimates the P-delta effects in some circumstances since a lower limit is not specified for the moment amplification factor. This phenomenon was exemplified in *Section 2.4*.

The approach is basically similar to AISC Manual published in 1969 with some modifications. The fundamental difference when compared with AISC Specification (2005) for covering the second-order effects is that there is no distinction between P-δ and P-Δ effects. The magnification is directly applied in the strength-interaction equation without amplification of the first-order elastic moments separately as used in AISC Manual (2005) defined in *Equations (1.3)* and *(1.4)*. Therefore, the moment amplification factor defined in *Eq. (1.15)* gives a coarse approximation of the true second-order effects (White et al., 2006). Also, change in axial forces in the columns caused by overturning moments, is disregarded in TS648 (1980) approach.

The C_m factor is defined as the coefficient accounts for end moments, span moments and support conditions.

- For unbraced frames, $C_m = 0.85$,
- For braced frames with only end moments without transverse loading,

$$C_m = 0.6 - 0.4\cdot\frac{M_1}{M_2}\geq 0.4 \tag{1.18}$$

- For braced frames with transverse loading without end moments,

$$C_m = 1+\psi\left(\frac{\sigma_{eb}}{\sigma_e{}'}\right) \tag{1.19}$$

First of all, it is seen that *Equation (1.18)* is exactly same for braced frames with only end moments when compared with AISC 2005 proposal, given in *Eq. (1.9)*. The only difference is the lower limit on C_m that is evaluated as being very conservative approach. So, the 0.4 lower limit is omitted in the new specifications. The AISC/LRFD Specification (1993) and

AISC/ASD Specification (1989) do not have the lower limit on C_m (Salmon & Johnson, 1996).

Equation (1.19) can be used for the selection of ψ and C_m factors. Also, ψ term can be determined from *Equation (1.8)* for simply supported beam-columns. As specified in *Section 1.4*, ψ and C_m values are not calculated properly since the amplification for negative moments is disregarded. The amplification parameters, ψ and C_m factors, for different transverse loading cases are the same as given in *Table 1.2* for TS648 (1980).

1.6 Aim of the Study

The focus of this study is to evaluate the second-order analysis methods presented in AISC 2005 and TS648 (1980) specifications. In *Chapter 2*, members with no lateral translation are studied. *Chapter 3* is devoted to members with end translation. Finally, conclusions based on the solution of practical cases are presented in *Chapter 4*.

CHAPTER 2

EVALUATION OF SECOND-ORDER EFFECTS FOR MEMBERS WITHOUT END TRANSLATION

In this chapter, the focus will be on the differences between the two specifications under consideration, AISC 2005 and TS648 (1980), with respect to the way that P-δ effects are taken into account. Also, the results obtained from the structural analysis software, SAP2000 will be presented. Since SAP2000 has a wide commercial usage in the analysis and design of structures, its applicability on covering P-δ effects will be discussed by comparing with the solutions obtained from exact formulations and code applications.

In the first subsection, reasons for using different C_m factors for beam-columns subjected to transverse loading between supports in AISC 2005 and TS648 (1980) will be discussed, and then the results will be compared with the exact results, and within each other. Basically, six loading cases encountered frequently in practical applications are specifically defined in AISC 2005 and TS648 (1980), as presented in *Tables 1.1 & 1.2*. However, different C_m factors are proposed for three cases, which will be investigated in *Section 2.1*.

Then, applicability of ψ formulation presented in *Eq. (1.8)* will be discussed in *Section 2.2*. Fundamentally, the ψ formulation is valid according to both of the two specifications, AISC 2005 and TS648 (1980), as a general formulation to account for P-δ effects in the case of transverse loading. Nevertheless, application of the specified equation is limited to simply-supported members in AISC 2005, as it should be, whereas misinterpretation of the ψ formulation by applying it to the fixed-ended members may cause deviation from the exact results as in TS648 (1980).

Finally, P-δ effects on the members subjected to end moments in combination with an axial thrust without transverse loading will be investigated in *Section 2.3*. Principally, the approximate formulations given in *Equations (1.9) & (1.18)* are valid in AISC 2005 and TS648 (1980), respectively. The only difference is the lower limit of 0.4 on the C_m formulation proposed in TS648 (1980). So, the code applications will be compared with the exact solutions, and within each other. Finally, a braced frame example will be provided in which an unconservative result was obtained by the application of TS648 (1980).

2.1 Comparison of AISC 2005 and TS648 (1980) Approaches in the Presence of Transverse Load

Fundamentally, C_m values are used to represent the P-δ effect on the magnification of first-order moments to obtain second-order moments for sidesway-inhibited members. The only exception is that C_m is taken as 0.85 for sidesway-permitted cases according to TS648 (1980). On the other hand, the numerator of B_2 formulation given in *Eq. (1.10)* is specified as 1.0, instead of highlighting a specific value for C_m according to AISC 2005 approach, whereas C_m factor is still used in determination of B_1 factor to account for P-δ effects.

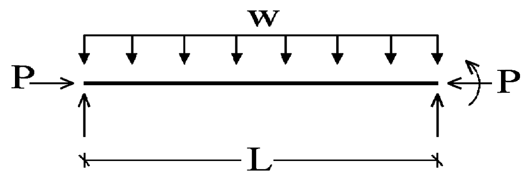
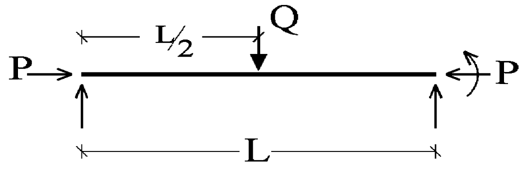
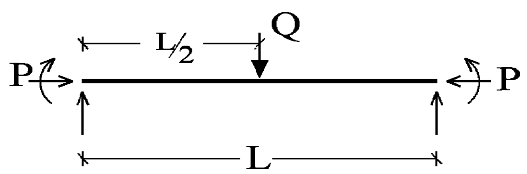
It should be stated that value of transverse load does not affect the rate of the amplification factor; however type of transverse loading is a significant parameter in the calculation of the amplification ratio, which is expressed as C_m factor in the numerator of P-δ amplification formulations specified in *Equations (1.5) & (1.15).*

Furthermore, C_m value is proposed to be taken conservatively as 1.0 in the presence of transverse loading after a *"rational analysis"* according to AISC 2005, which is valid for the practical cases without overestimating the results in design of real structural members subjected to low axial load levels. On the other hand, a formulation is proposed in *Eq. (1.7)* for obtaining a more precise C_m factor. Additionally, six specific loading cases encountered frequently are defined in *Tables 1.1 & 1.2* for AISC 2005 and TS648 (1980), respectively.

In the content of this section, three of the cases presented in *Table 2.1* in which C_m values differ between the tables specified in AISC 2005 and TS648 (1980) will be compared. Additionally, SAP2000 solutions will be provided in order to investigate the usage of computer applications for handling P-δ effect. Also, conservatism level by taking C_m factor as 1.0 according to AISC 2005 will be examined in the following problems.

It is not possible to cover P-δ effect in the member with a single frame element when finite element methods are under consideration. This is conducted in the same manner in most structural analysis programs capable of performing geometrically non-linear analysis, likewise the software used throughout the thesis, SAP2000.

Table 2.1: Theoretical Formulations for the Problems Specified in Section 2.1

SECTION	DEFINITION	CASE	MAXIMUM FIRST-ORDER ELASTIC MOMENT	MOMENT AMPLIFICATION FACTOR		
			$M_{z,max1}$	Theoretical	AISC 2005	TS648 (1980)
2.1.1	Propped Cantilever with Uniformly Distributed Transverse Load		$\frac{1}{8}wL^2$	$\frac{2(\tan\mu-\mu)}{\mu^2\left(\frac{1}{2\mu}-\frac{1}{\tan 2\mu}\right)}$ $\mu=\frac{L}{2}\sqrt{\frac{P}{EI}}$	$\frac{1-\mathbf{0.4}(P/P_{cr})}{1-(P/P_{cr})}$	$\frac{1-\mathbf{0.3}(P/P_{cr})}{1-(P/P_{cr})}$
2.1.2	Propped Cantilever with Point Load at the Mid-Span		$\frac{3}{16}QL$	$\frac{4\mu(1-\cos\mu)}{3\mu^2\cos\mu\left(\frac{1}{2\mu}-\frac{1}{\tan 2\mu}\right)}$	$\frac{1-\mathbf{0.3}(P/P_{cr})}{1-(P/P_{cr})}$	$\frac{1-\mathbf{0.4}(P/P_{cr})}{1-(P/P_{cr})}$
2.1.3	Fixed-Ended Beam-Column with Point Load at the Mid-Span		$\frac{1}{8}QL$	$\frac{2(1-\cos\mu)}{\mu\sin\mu}$	$\frac{1-\mathbf{0.2}(P/P_{cr})}{1-(P/P_{cr})}$	$\frac{1-\mathbf{0.6}(P/P_{cr})}{1-(P/P_{cr})}$

Since the studies in this section are based on braced member behavior, the beam-column was divided into 100 elements for computer applications to improve accuracy of the results. Section and material properties required for further analyses are presented in table given below. It should be noted that the values given in *Table 2.2* represent any set of consistent units.

Table 2.2: Data for the Problems Specified in Section 2.1

E	L	I	A
10	100	1000	120

2.1.1 Propped Cantilever with Uniformly Distributed Transverse Load

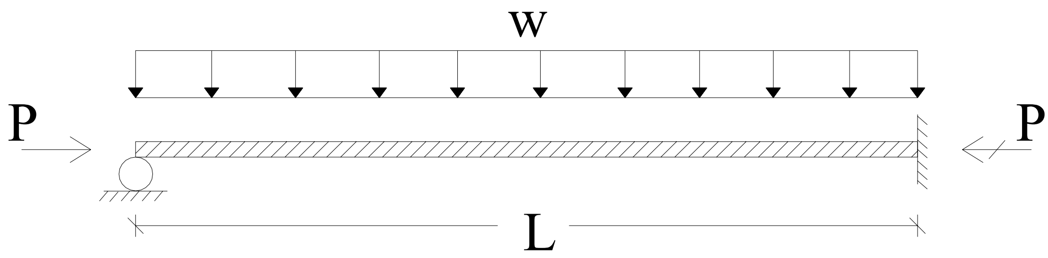


Figure 2.1: Propped Cantilever with Uniformly Distributed Transverse Load

The beam-column subjected to uniformly distributed transverse load in combination with an applied axial compressive load was considered as shown in *Figure 2.1.* The maximum first-order elastic moment occurring at the fixed-end can be calculated from the equation provided in *Table 2.1.* Exact and approximate solutions of the maximum second-order moment can be found by multiplying the first-order moment with the amplification factor provided in the same table.

Uniformly distributed transverse load, w, was taken as equal to 0.0008 to obtain unity as the first-order elastic moment with compatible units. Euler elastic buckling load for the member was calculated as P_{e1} = 20.14 for the specified problem. Results obtained from subsequent analyses are summarized in *Table 2.3*, and expressed graphically in *Figure 2.2*.

First of all, exact solutions should be evaluated with respect to axial load level which is referenced to elastic buckling load (P_{e1}). Second-order moments can be detrimental for high axial load values if second-order effects are disregarded in analysis and design. For instance, member loaded with an axial compressive load of $0.6 \cdot P_{e1}$ can be subjected to an internal moment value of 96% more than the value obtained from first-order elastic analysis. On the other hand, axial load level is below $0.4 \cdot P_{e1}$ in most practical cases. Since, still the amplification of first-order moment can reach up to 43%; geometric non-linearity should be an important parameter for the design.

Then, second-order moment values obtained from SAP2000 were reported as being accurate for all axial thrust levels.

Eventually, second-order moments obtained from TS648 (1980) Method were deviated from exact results more than AISC 2005, since erroneously **ψ = -0.3** was used instead of **ψ = -0.4** in the calculation of C_m factor. Nevertheless, conservative results were acquired by carrying out TS648 (1980) Method when compared with AISC 2005 Method and exact solution.

Finally, C_m can be taken as 1.0 conservatively according to AISC 2005, as stated in *Section 1.4*. This procedure was reported as being conservative for the specified case.

Table 2.3: Comparison of Maximum Second-Order Moments Occurring at Fixed-End

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 with ψ = -0.4				TS648 (1980) with ψ = -0.3			AISC 2005 with C_m = 1.0	
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.00	1.000	**1.000**	0.00	1.00	**1.000**	0.00	**1.000**	0.00
0.1	2.014	**1.074**	**1.074**	0.00	0.96	1.067	**1.067**	-0.69	0.97	**1.078**	0.34	**1.111**	3.45
0.2	4.028	**1.166**	**1.166**	0.00	0.92	1.150	**1.150**	-1.33	0.94	**1.175**	0.81	**1.250**	7.25
0.3	6.043	**1.282**	**1.282**	0.01	0.88	1.257	**1.257**	-1.91	0.91	**1.300**	1.43	**1.429**	11.47
0.4	8.057	**1.434**	**1.435**	0.00	0.84	1.400	**1.400**	-2.40	0.88	**1.467**	2.24	**1.667**	16.19
0.5	10.071	**1.646**	**1.646**	0.00	0.80	1.600	**1.600**	-2.79	0.85	**1.700**	3.29	**2.000**	21.52
0.6	12.085	**1.959**	**1.959**	0.00	0.76	1.900	**1.900**	-3.02	0.82	**2.050**	4.64	**2.500**	27.60
0.7	14.099	**2.475**	**2.475**	0.00	0.72	2.400	**2.400**	-3.03	0.79	**2.633**	6.39	**3.333**	34.68
0.8	16.114	**3.493**	**3.493**	0.00	0.68	3.400	**3.400**	-2.66	0.76	**3.800**	8.79	**5.000**	43.14
0.9	18.128	**6.482**	**6.482**	0.00	0.64	6.400	**6.400**	-1.26	0.73	**7.300**	12.62	**10.000**	54.27

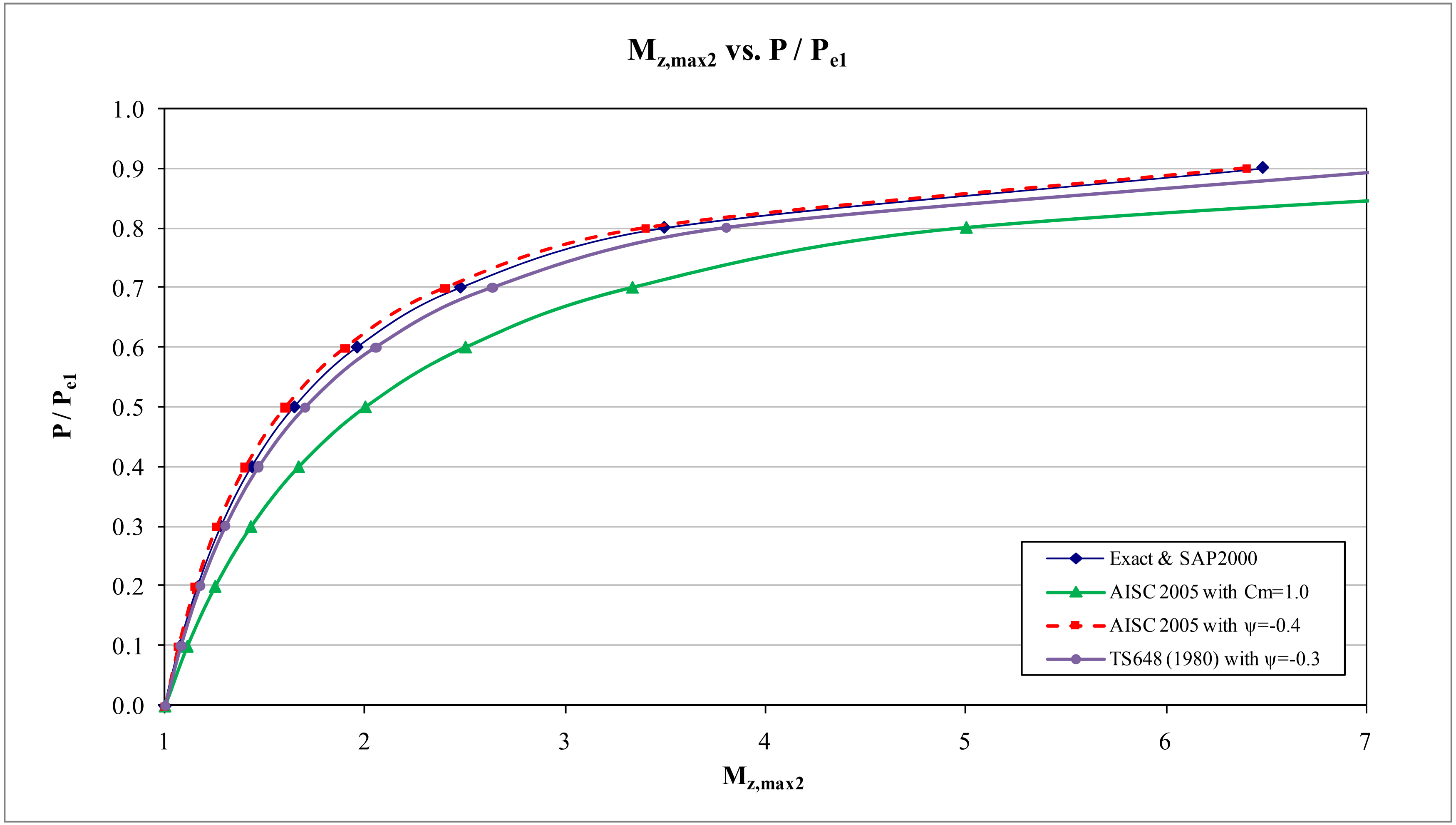


Figure 2.2: Comparison of Maximum Second-Order Moments Occurring at Fixed-End

2.1.2 Propped Cantilever with Point Load at the Mid-Span

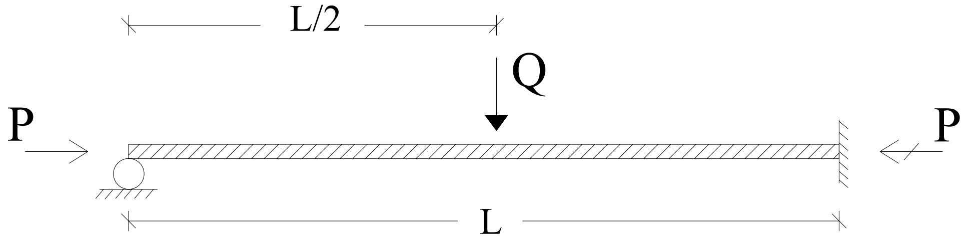


Figure 2.3: Propped Cantilever with Point Load at the Mid-Span

The propped cantilever subjected to axial compressive load in combination with an applied point load at the mid-span, as shown in the figure given above, was investigated. Maximum second-order moments occurring at the fixed-end were calculated by multiplying the first-order moment with the amplification factor defined in *Table 2.1*.

Euler elastic buckling load for the member with specified boundary conditions was computed as P_{e1} = 20.14. The transverse point load, Q, was selected as equal to 4/75 to obtain unity as the first-order moment.

Results obtained from the successive analyses were summarized in *Table 2.4*, and then presented graphically in *Figure 2.4*. It is obvious that SAP2000 results were exactly fitted to theoretical solutions.

Second-order moments were magnified up to 7.1 times of the first-order moment according to exact results. Also, accurate results within admissible error limits (1.47% maximum deviation from the exact solution for an axial load level of $0.5 \cdot P_{e1}$) were obtained by applying the B_1 Method proposed by AISC 2005. Then, second-order moments obtained by carrying out TS648 (1980) Method were deviated from exact results unconservatively, since erroneously **ψ = -0.4** was used instead of **ψ = -0.3** in the calculation of C_m factor. However, results obtained from TS648 (1980) Method were within acceptable limits for practical cases (6% error for an axial load level of $0.4P_{e1}$) in spite of being inconsistent with respect to assumptions and limitation of the approximate ψ formulation.

As a final result, if C_m was taken as equal to 1.0 conservatively according to AISC 2005, the procedure was reported as being conservative and acceptable for practical load cases.

Table 2.4: Comparison of Maximum Second-Order Moments Occurring at Fixed-End

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 with ψ = -0.3				TS648 (1980) with ψ = -0.4			AISC 2005 with $C_m = 1.0$	
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.00	1.000	**1.000**	0.00	1.00	**1.000**	0.00	**1.000**	0.00
0.1	2.014	**1.083**	**1.083**	-0.06	0.97	1.078	**1.078**	-0.52	0.96	**1.067**	-1.54	**1.111**	2.56
0.2	4.028	**1.186**	**1.185**	-0.07	0.94	1.175	**1.175**	-0.94	0.92	**1.150**	-3.05	**1.250**	5.38
0.3	6.043	**1.317**	**1.316**	-0.06	0.91	1.300	**1.300**	-1.27	0.88	**1.257**	-4.52	**1.429**	8.50
0.4	8.057	**1.488**	**1.487**	-0.06	0.88	1.467	**1.467**	-1.45	0.84	**1.400**	-5.93	**1.667**	11.98
0.5	10.071	**1.725**	**1.724**	-0.06	0.85	1.700	**1.700**	-1.47	0.80	**1.600**	-7.27	**2.000**	15.92
0.6	12.085	**2.076**	**2.075**	-0.07	0.82	2.050	**2.050**	-1.27	0.76	**1.900**	-8.49	**2.500**	20.41
0.7	14.099	**2.653**	**2.651**	-0.07	0.79	2.633	**2.633**	-0.75	0.72	**2.400**	-9.54	**3.333**	25.64
0.8	16.114	**3.790**	**3.788**	-0.05	0.76	3.800	**3.800**	0.27	0.68	**3.400**	-10.28	**5.000**	31.94
0.9	18.128	**7.122**	**7.118**	-0.05	0.73	7.300	**7.300**	2.49	0.64	**6.400**	-10.14	**10.000**	40.40

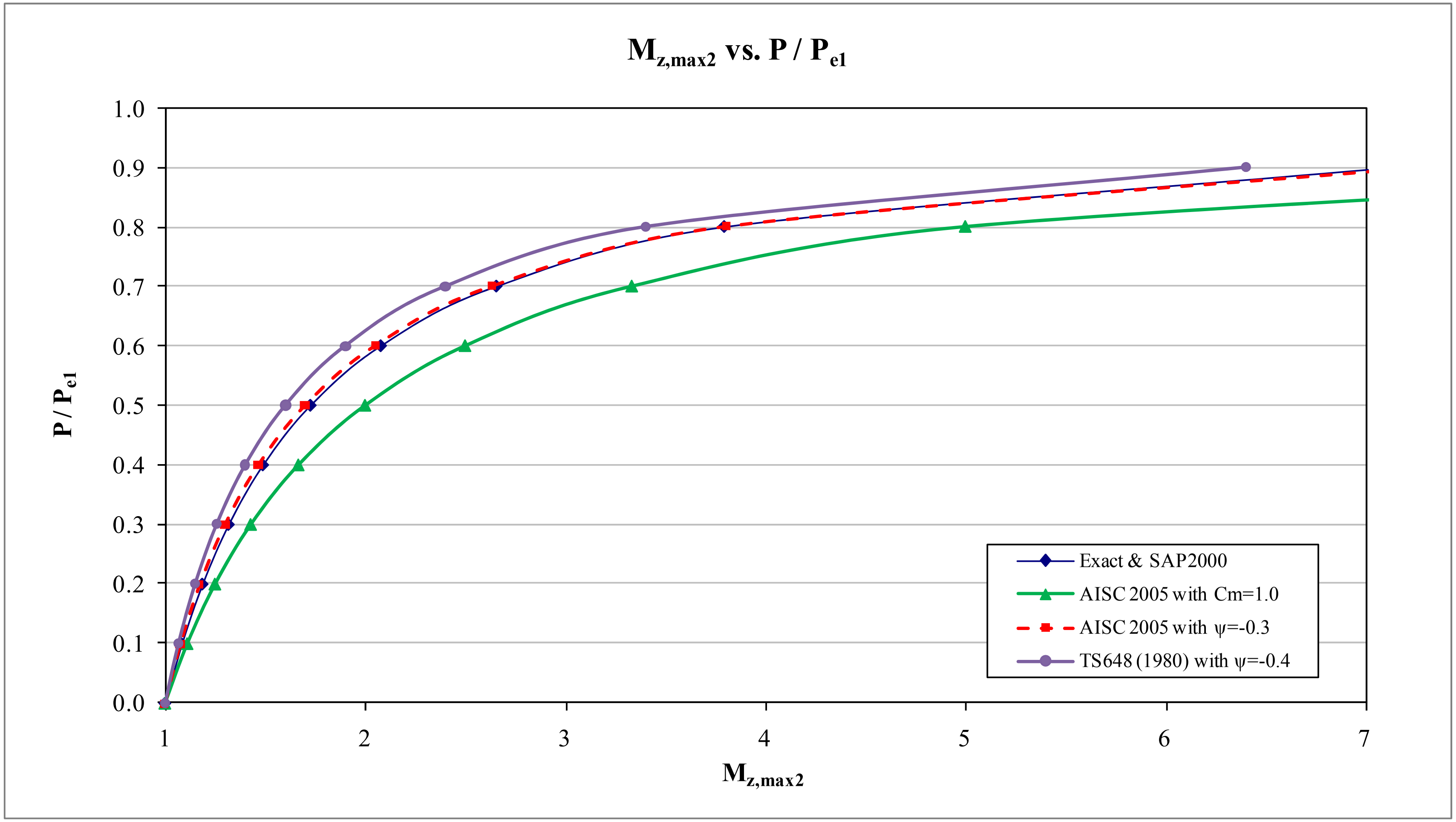


Figure 2.4: Comparison of Maximum Second-Order Moments Occurring at Fixed-End

2.1.3 Fixed-Ended Beam-Column with Point Load at the Mid-Span

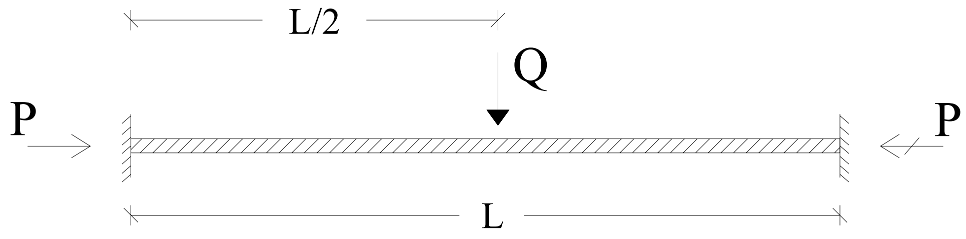


Figure 2.5: Fixed-Ended Beam-Column with Point Load at the Mid-Span

The fixed-ended beam-column shown in the figure above is subjected to axial compressive load in combination with an applied point load at the mid-span. The maximum moment value occurring at the mid-span and fixed-ends simultaneously can be computed from the formulations specified in *Table 2.1*.

Euler elastic buckling load for the beam-column was calculated as P_{e1} = 39.48. Also, a point load, Q, with a value of 0.08 according to compatible units was used to obtain unity as the first-order elastic moment.

Results obtained from successive first-, and second-order elastic analyses were summarized in *Table 2.5 & Figure 2.6*. Results obtained by performing second-order elastic analysis with SAP2000 were reported to be accurate.

Second-order moments reached up to 8.3 times of first-order moments when exact results were under consideration. Generally, stable results within admissible error limits (1.29% maximum deviation from the exact solution for an axial load level of $0.9 \cdot P_{e1}$) were obtained by applying B_1 Method proposed by AISC 2005. On the other hand, second-order moments obtained from TS648 (1980) Method were deviated from the exact results since erroneously **ψ = -0.6** was proposed instead of **ψ = -0.2** as specified in AISC 2005 for the calculation of C_m factor. So, error of the results obtained from TS648 (1980) Method were reached up to 44% unconservatively. According to this study, it may be concluded that the error can be tolerated with safety factors used in design for practical load cases (18% error for an axial load level of $0.4P_{e1}$). Also, if the value of C_m was taken as 1.0 according to AISC 2005 procedures, conservative results were obtained with a maximum deviation of 20% when compared with exact results.

Table 2.5: Comparison of Maximum Second-Order Moments Occurring at Fixed-Ends

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 with ψ = -0.2				TS648 (1980) with ψ = -0.6			AISC 2005 with C_m = 1.0	
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.00	1.000	**1.000**	0.00	1.00	**1.000**	0.00	**1.000**	0.00
0.1	3.948	**1.091**	**1.091**	0.00	0.98	1.089	**1.089**	-0.22	0.94	**1.044**	-4.29	**1.111**	1.82
0.2	7.896	**1.205**	**1.205**	0.00	0.96	1.200	**1.200**	-0.42	0.88	**1.100**	-8.72	**1.250**	3.73
0.3	11.844	**1.351**	**1.351**	0.00	0.94	1.343	**1.343**	-0.61	0.82	**1.171**	-13.30	**1.429**	5.74
0.4	15.791	**1.545**	**1.545**	0.00	0.92	1.533	**1.533**	-0.78	0.76	**1.267**	-18.03	**1.667**	7.85
0.5	19.739	**1.817**	**1.817**	0.00	0.90	1.800	**1.800**	-0.93	0.70	**1.400**	-22.94	**2.000**	10.08
0.6	23.687	**2.223**	**2.223**	0.00	0.88	2.200	**2.200**	-1.05	0.64	**1.600**	-28.04	**2.500**	12.44
0.7	27.635	**2.900**	**2.900**	0.00	0.86	2.867	**2.867**	-1.16	0.58	**1.933**	-33.34	**3.333**	14.93
0.8	31.583	**4.253**	**4.253**	0.00	0.84	4.200	**4.200**	-1.24	0.52	**2.600**	-38.86	**5.000**	17.57
0.9	35.531	**8.307**	**8.308**	0.01	0.82	8.200	**8.200**	-1.29	0.46	**4.600**	-44.62	**10.000**	20.38

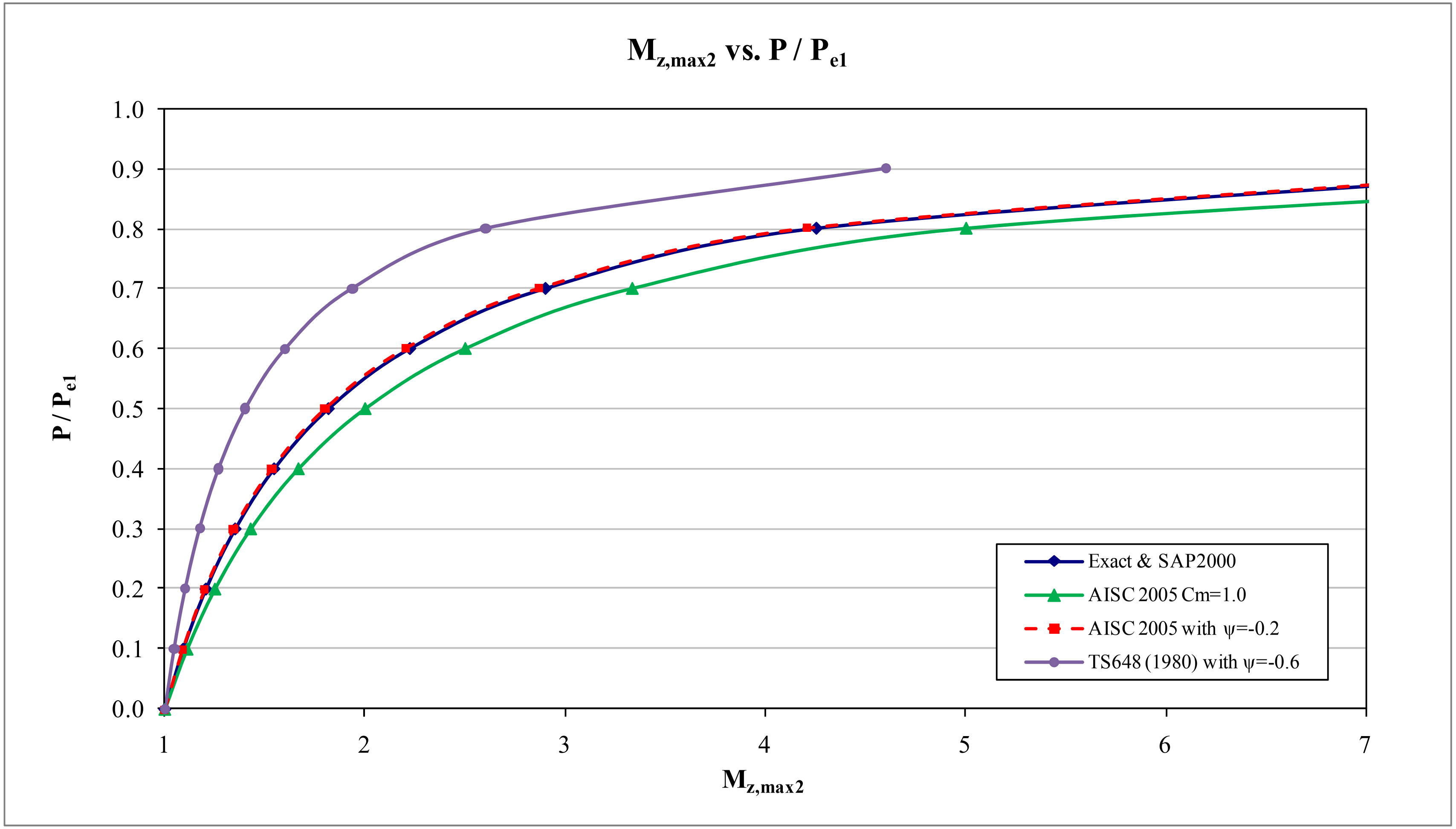


Figure 2.6: Comparison of Maximum Second-Order Moments Occurring at Fixed-Ends

2.2 Evaluation of ψ Coefficient

Basically, ψ coefficient accounts for the effect of transverse loading on the amplification of first-order moment. It is used in the numerator of C_m formulation given in *Eq. (1.7)*. Same formulation for obtaining ψ coefficient is specified in both of the two specifications, AISC 2005 and TS648 (1980) as shown in *Eq. (1.8)*. The only difference is that ψ formulation is limited for only simply supported beam-column case according to AISC 2005. Since TS648 (1980) is based on AISC 1969 formulations, ψ coefficient was erroneously used for fixed-ended beam-columns, also.

The ψ formulation is based on the multiplication of elastic buckling load with the maximum deflection at the span due to transverse loading. Then, the multiplied value is divided to maximum moment occurring at or near the mid-span. Since, in most cases maximum moment occurs at the support for fixed-ended frames, it is obvious that multiplication of span deflection with fixed-end moment is inappropriate.

In *Section 2.1*, it was concluded that deviation from the exact result can reach up to 44% unconservatively by using theoretically wrong ψ factors given in TS648 (1980). On the other hand, TS648 (1980) is currently valid in practice of Turkish steel structure construction. ψ coefficients were calculated for each loading case, and given in the tables containing the summary of the analyses.

Three problems will be investigated in the following sub-sections. Beam-columns with different support conditions were loaded with a transverse point load along the span in combination with an axial compressive load. In first part of the analysis, the point load, Q, was applied at a distance of 0.1L from the support. Then, axial compressive load was increased step-by-step from $0.1P_{e1}$ to $0.9P_{e1}$, with an increment of $0.1P_{e1}$. After that, same procedure was repeated by moving the transverse point load to the distances of 0.2L, 0.3L, 0.4L, and 0.5L from the support, respectively. In *Section 2.2.1*, a simply supported member will be taken into account, whereas a fixed-ended beam-column will be under consideration in *Section 2.2.2*. Furthermore, the propped cantilever was investigated in *Section 2.2.3* by moving the transverse point load throughout the frame from a distance of a = 0.1L to a = 0.9L, since the system is not symmetrical around the mid-span.

The member was divided into 100 subdivisions to improve the precision of the results acquired with the application of finite element methods using the structural analysis software, SAP2000. Data required for further analyses was defined in *Table 2.2*.

2.2.1 Simply Supported Beam-Column with Point Load at Span

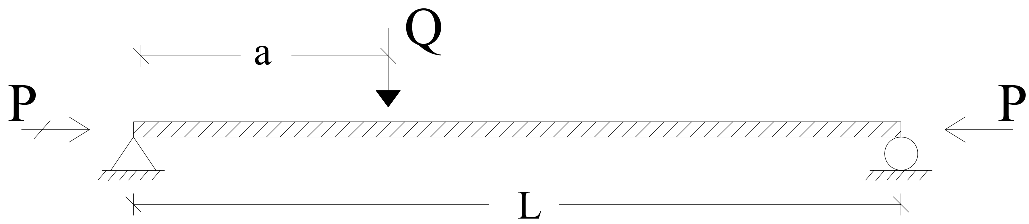


Figure 2.7: Simply Supported Beam-Column with Point Load at Span

A transverse point load was applied on the simply supported beam-column as shown in *Figure 2.7*, in combination with an axial thrust. Theoretical solution of the maximum second-order elastic moment is presented in the equation below:

$$M_{z,max2} = EI\,\frac{Qk'\sin(k'a)\cdot\sin[k'(L-x)]}{P\sin(k'L)} \tag{2.1}$$

Euler elastic buckling load was calculated as P_{e1} = 9.87 for the specified problem. According to the results presented in *Table 2.6*, accurate second-order moment values were obtained by performing successive second-order elastic analyses with SAP2000 (The maximum error was 1.6% in unconservative side).

As explained in the beginning of *Section 2.1*, application of ψ formulation is valid for this problem according to both AISC 2005 and TS648 (1980) Specifications, since the frame system is determinate. So, conservative results were obtained as expected. It should be noted that, deviation from the exact results increased as the point load approaches from mid-span to the support, which can be observed in *Figure 2.8*.

Table 2.6: Comparison of Maximum Second-Order Moments Occuring at the Span

a = 0.1L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.307)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.00	1.000	**1.000**	0.00
0.1	0.987	**1.032**	**1.030**	-0.19	0.97	1.077	**1.077**	4.36
0.2	1.974	**1.071**	**1.068**	-0.28	0.94	1.173	**1.173**	9.55
0.3	2.961	**1.118**	**1.115**	-0.27	0.91	1.297	**1.297**	16.01
0.4	3.948	**1.207**	**1.200**	-0.58	0.88	1.462	**1.462**	21.13
0.5	4.935	**1.385**	**1.368**	-1.23	0.85	1.693	**1.693**	22.24
0.6	5.922	**1.692**	**1.665**	-1.60	0.82	2.040	**2.040**	20.54
0.7	6.909	**2.238**	**2.238**	0.00	0.79	2.617	**2.617**	16.93
0.8	7.896	**3.368**	**3.368**	0.00	0.75	3.772	**3.772**	12.00
0.9	8.883	**6.822**	**6.822**	0.00	0.72	7.237	**7.237**	6.08

a = 0.2L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.256)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.00	1.000	**1.000**	0.00
0.1	0.987	**1.058**	**1.053**	-0.47	0.97	1.083	**1.083**	2.33
0.2	1.974	**1.128**	**1.124**	-0.35	0.95	1.186	**1.186**	5.14
0.3	2.961	**1.216**	**1.211**	-0.41	0.92	1.319	**1.319**	8.46
0.4	3.948	**1.331**	**1.323**	-0.60	0.90	1.496	**1.496**	12.40
0.5	4.935	**1.520**	**1.501**	-1.25	0.87	1.744	**1.744**	14.74
0.6	5.922	**1.847**	**1.819**	-1.52	0.85	2.116	**2.116**	14.56
0.7	6.909	**2.431**	**2.431**	0.00	0.82	2.736	**2.736**	12.55
0.8	7.896	**3.641**	**3.641**	0.00	0.80	3.976	**3.976**	9.20
0.9	8.883	**7.337**	**7.337**	0.00	0.77	7.696	**7.696**	4.89

a = 0.3L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.215)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.00	1.000	**1.000**	0.00
0.1	0.987	**1.076**	**1.069**	-0.65	0.98	1.087	**1.087**	1.04
0.2	1.974	**1.170**	**1.164**	-0.51	0.96	1.196	**1.196**	2.24
0.3	2.961	**1.290**	**1.283**	-0.54	0.94	1.336	**1.336**	3.60
0.4	3.948	**1.447**	**1.437**	-0.69	0.91	1.523	**1.523**	5.28
0.5	4.935	**1.665**	**1.646**	-1.14	0.89	1.785	**1.785**	7.21
0.6	5.922	**2.006**	**1.977**	-1.45	0.87	2.178	**2.178**	8.55
0.7	6.909	**2.617**	**2.618**	0.04	0.85	2.832	**2.832**	8.20
0.8	7.896	**3.886**	**3.886**	0.00	0.83	4.140	**4.140**	6.54
0.9	8.883	**7.763**	**7.763**	0.00	0.81	8.065	**8.065**	3.89

Table 2.6 (continued)

a = 0.4L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.188)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.00	1.000	**1.000**	0.00
0.1	0.987	**1.087**	**1.079**	-0.74	0.98	1.090	**1.090**	0.30
0.2	1.974	**1.196**	**1.189**	-0.59	0.96	1.203	**1.203**	0.59
0.3	2.961	**1.336**	**1.327**	-0.67	0.94	1.348	**1.348**	0.90
0.4	3.948	**1.520**	**1.507**	-0.86	0.92	1.541	**1.541**	1.40
0.5	4.935	**1.778**	**1.754**	-1.35	0.91	1.812	**1.812**	1.91
0.6	5.922	**2.163**	**2.131**	-1.48	0.89	2.218	**2.218**	2.54
0.7	6.909	**2.803**	**2.803**	0.00	0.87	2.895	**2.895**	3.27
0.8	7.896	**4.107**	**4.107**	0.00	0.85	4.248	**4.248**	3.43
0.9	8.883	**8.095**	**8.095**	0.00	0.83	8.308	**8.308**	2.63

a = 0.5L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.178)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.00	1.000	**1.000**	0.00
0.1	0.987	**1.091**	**1.082**	-0.85	0.98	1.091	**1.091**	0.01
0.2	1.974	**1.205**	**1.197**	-0.67	0.96	1.206	**1.206**	0.04
0.3	2.961	**1.351**	**1.342**	-0.67	0.95	1.352	**1.352**	0.09
0.4	3.948	**1.545**	**1.532**	-0.86	0.93	1.548	**1.548**	0.17
0.5	4.935	**1.817**	**1.792**	-1.37	0.91	1.822	**1.822**	0.28
0.6	5.922	**2.223**	**2.190**	-1.50	0.89	2.233	**2.233**	0.43
0.7	6.909	**2.900**	**2.901**	0.02	0.88	2.918	**2.918**	0.61
0.8	7.896	**4.253**	**4.253**	0.01	0.86	4.288	**4.288**	0.83
0.9	8.883	**8.307**	**8.310**	0.04	0.84	8.398	**8.398**	1.10

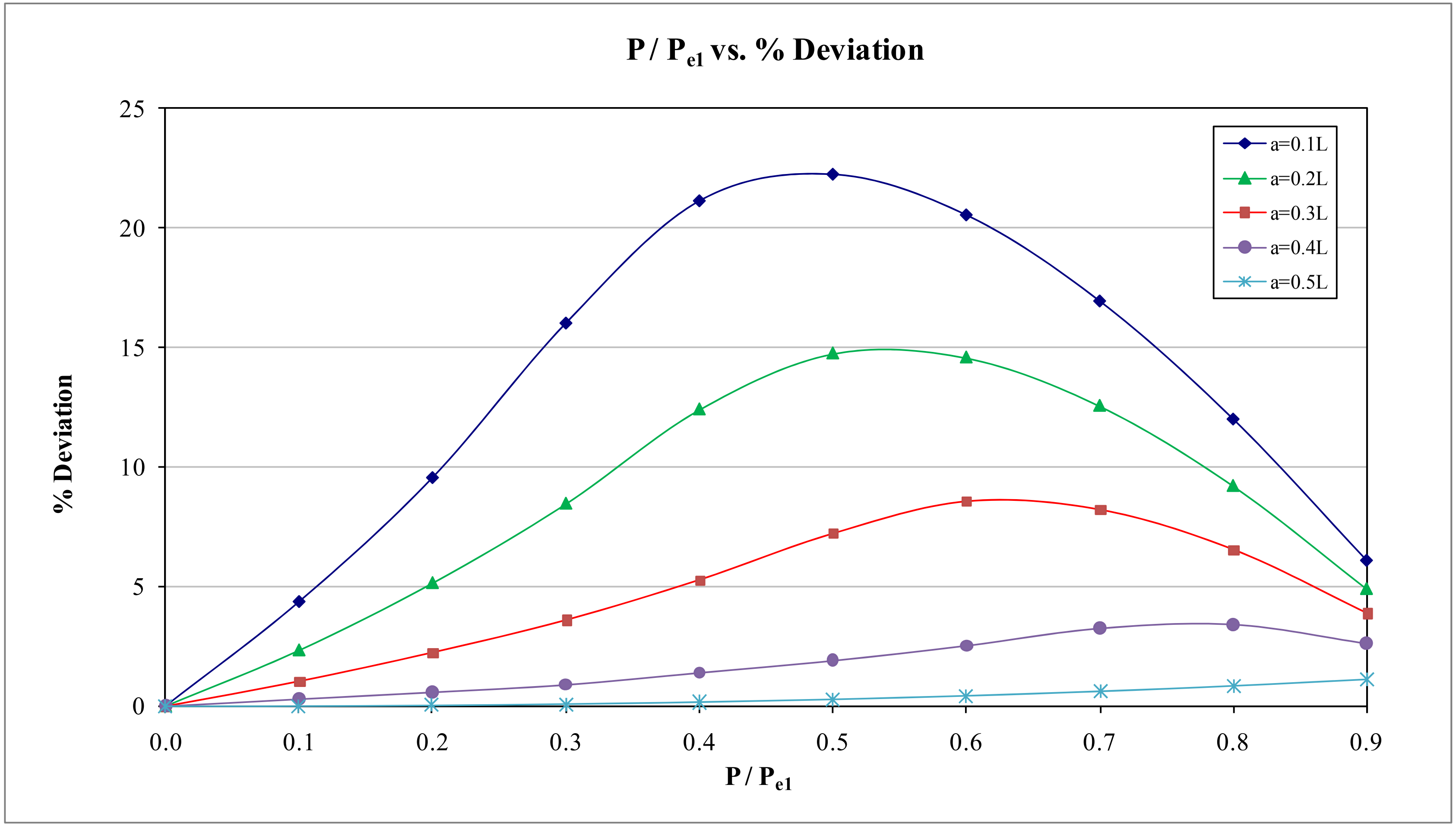


Figure 2.8: Deviation of Results Obtained by Approximate Method from Exact Solutions

2.2.2 Fixed-Ended Beam-Column with Point Load at Span

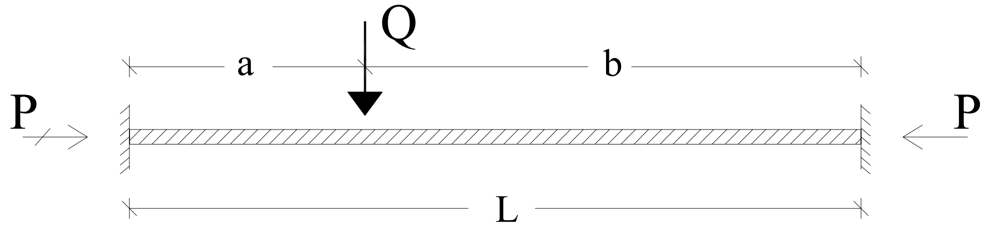


Figure 2.9: Fixed-Ended Beam-Column with Point Load at Span

The fixed-ended beam-column was loaded with a transverse point load in combination with an applied axial compressive load. Exact solution of the maximum second-order elastic moment presented by Chen & Lui (1991) is expressed in the formulation presented below:

$$M_{z,max2} = \frac{QL}{d} \cdot \left[\frac{2\mu b}{L}\cos 2\mu - 2\mu \cos\frac{2\mu b}{L} - \sin 2\mu + \sin\frac{2\mu a}{L} + \sin\frac{2\mu b}{L} + \frac{2\mu a}{L}\right] \quad (2.2)$$

In which

$$\mu = \frac{k'L}{2} = \frac{L}{2}\sqrt{\frac{P}{EI}} \quad (2.3)$$

and

$$d = 2\mu(2 - 2\cos 2\mu - 2\mu \sin 2\mu) \quad (2.4)$$

Accurate results were obtained by performing successive second-order elastic analyses by using SAP2000. The maximum error value of 3.4% was found out to be applicable when compared with the exact results.

Since ψ formulation is only applicable for simply supported beam-columns, it obvious that the approximate results deviate from the exact solutions. However, there is no limitation on the usage of ψ coefficient for fixed-ended members according to TS648 (1980). So, the approximate solutions obtained by using ψ factor should be examined in detail.

Results obtained from the analyses corresponding to theoretical formulations and approximate methods were given in *Table 2.7*. Then, unconservative moment values were acquired by carrying out approximate method with ψ formulation (A maximum deviation of 43% was reported from the theoretical solution).

Table 2.7: Comparison of Maximum Second-Order Moments Occurring at Fixed-End

a = 0.1L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.924)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.000	1.000	**1.000**	0.00
0.1	3.948	**1.026**	**1.024**	-0.20	0.908	1.008	**1.008**	-1.72
0.2	7.896	**1.056**	**1.055**	-0.12	0.815	1.019	**1.019**	-3.53
0.3	11.844	**1.092**	**1.087**	-0.48	0.723	1.033	**1.033**	-5.46
0.4	15.791	**1.136**	**1.122**	-1.26	0.630	1.051	**1.051**	-7.54
0.5	19.739	**1.193**	**1.178**	-1.27	0.538	1.076	**1.076**	-9.82
0.6	23.687	**1.271**	**1.248**	-1.84	0.446	1.114	**1.114**	-12.38
0.7	27.635	**1.391**	**1.344**	-3.39	0.353	1.177	**1.177**	-15.37
0.8	31.583	**1.613**	**1.533**	-4.95	0.261	1.304	**1.304**	-19.15
0.9	35.531	**2.235**	**2.235**	-0.01	0.168	1.684	**1.684**	-24.66

a = 0.2L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.844)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.000	1.000	**1.000**	0.00
0.1	3.948	**1.048**	**1.045**	-0.33	0.916	1.017	**1.017**	-2.97
0.2	7.896	**1.106**	**1.103**	-0.27	0.831	1.039	**1.039**	-6.05
0.3	11.844	**1.176**	**1.173**	-0.25	0.747	1.067	**1.067**	-9.28
0.4	15.791	**1.264**	**1.252**	-0.96	0.662	1.104	**1.104**	-12.67
0.5	19.739	**1.381**	**1.363**	-1.28	0.578	1.156	**1.156**	-16.27
0.6	23.687	**1.545**	**1.513**	-2.10	0.494	1.234	**1.234**	-20.15
0.7	27.635	**1.805**	**1.752**	-2.92	0.409	1.364	**1.364**	-24.42
0.8	31.583	**2.296**	**2.296**	0.02	0.325	1.624	**1.624**	-29.26
0.9	35.531	**3.703**	**3.703**	0.00	0.240	2.404	**2.404**	-35.08

a = 0.3L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.760)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.000	1.000	**1.000**	0.00
0.1	3.948	**1.067**	**1.066**	-0.08	0.924	1.027	**1.027**	-3.77
0.2	7.896	**1.148**	**1.143**	-0.42	0.848	1.060	**1.060**	-7.66
0.3	11.844	**1.249**	**1.243**	-0.44	0.772	1.103	**1.103**	-11.67
0.4	15.791	**1.378**	**1.370**	-0.59	0.696	1.160	**1.160**	-15.83
0.5	19.739	**1.553**	**1.539**	-0.91	0.620	1.240	**1.240**	-20.16
0.6	23.687	**1.806**	**1.775**	-1.73	0.544	1.360	**1.360**	-24.71
0.7	27.635	**2.213**	**2.171**	-1.91	0.468	1.560	**1.560**	-29.52
0.8	31.583	**3.001**	**3.001**	0.01	0.392	1.960	**1.960**	-34.68
0.9	35.531	**5.297**	**5.298**	0.02	0.316	3.160	**3.160**	-40.34

Table 2.7 (continued)

a = 0.4L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.674)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.000	1.000	**1.000**	0.00
0.1	3.948	**1.081**	**1.081**	-0.02	0.933	1.036	**1.036**	-4.16
0.2	7.896	**1.181**	**1.180**	-0.10	0.865	1.082	**1.082**	-8.44
0.3	11.844	**1.308**	**1.300**	-0.58	0.798	1.140	**1.140**	-12.84
0.4	15.791	**1.473**	**1.462**	-0.76	0.730	1.217	**1.217**	-17.37
0.5	19.739	**1.701**	**1.681**	-1.19	0.663	1.326	**1.326**	-22.06
0.6	23.687	**2.037**	**2.011**	-1.29	0.596	1.489	**1.489**	-26.91
0.7	27.635	**2.588**	**2.588**	0.00	0.528	1.761	**1.761**	-31.97
0.8	31.583	**3.672**	**3.672**	0.00	0.461	2.304	**2.304**	-37.25
0.9	35.531	**6.881**	**6.882**	0.01	0.393	3.934	**3.934**	-42.83

a = 0.5L

P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648 (1980) (For ψ = -0.589)			
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00	1.000	1.000	**1.000**	0.00
0.1	3.948	**1.091**	**1.090**	-0.12	0.941	1.046	**1.046**	-4.18
0.2	7.896	**1.205**	**1.204**	-0.09	0.882	1.103	**1.103**	-8.49
0.3	11.844	**1.351**	**1.342**	-0.67	0.823	1.176	**1.176**	-12.95
0.4	15.791	**1.545**	**1.532**	-0.86	0.764	1.274	**1.274**	-17.56
0.5	19.739	**1.817**	**1.804**	-0.71	0.706	1.411	**1.411**	-22.34
0.6	23.687	**2.223**	**2.189**	-1.55	0.647	1.617	**1.617**	-27.30
0.7	27.635	**2.900**	**2.900**	-0.01	0.588	1.959	**1.959**	-32.45
0.8	31.583	**4.253**	**4.253**	0.01	0.529	2.644	**2.644**	-37.83
0.9	35.531	**8.307**	**8.308**	0.01	0.470	4.699	**4.699**	-43.43

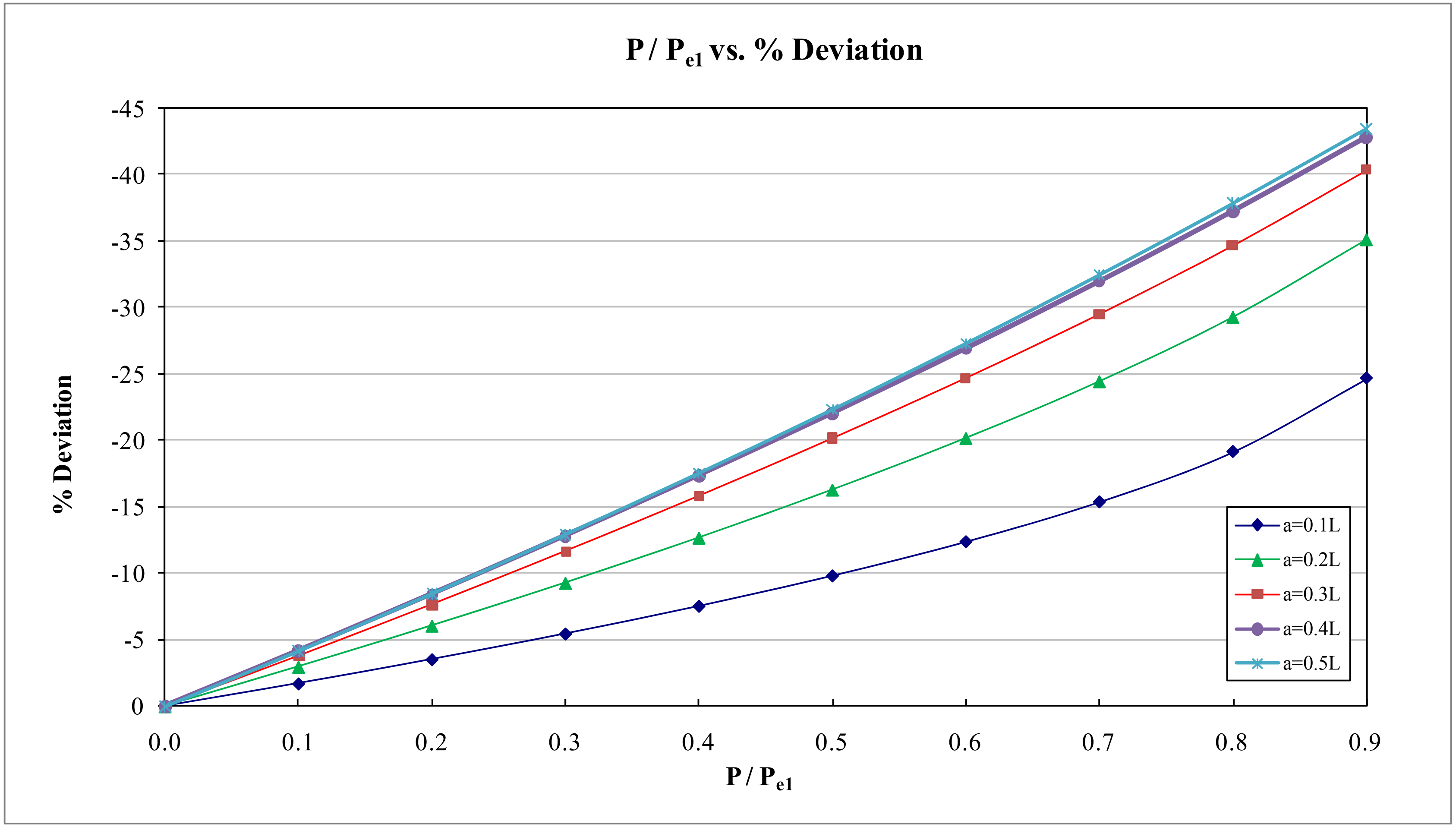


Figure 2.10: Deviation of Results Obtained by Approximate Method from Exact Solutions

On the other hand, %20 error level was reported for practical load cases with low axial compressive level. Also, error values dropped down when the transverse point load was located adjacent to the support. This phenomenon can be clearly followed in *Figure 2.10*.

2.2.3 Propped Cantilever Beam-Column with Point Load at Span

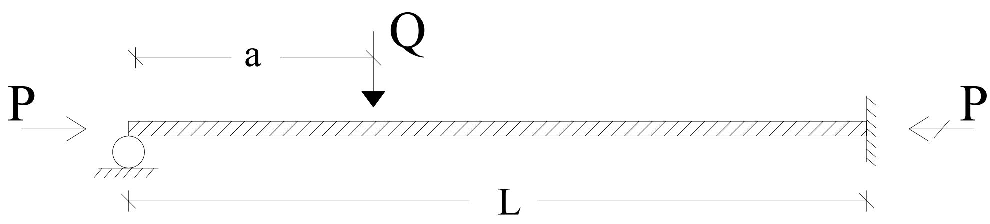


Figure 2.11: Propped Cantilever Beam-Column with Point Load at Span

The propped cantilever was subjected to a point load applied transversely in combination with an axial thrust as shown in the figure above.

Results obtained by performing successive second-order elastic analyses with the software, SAP2000, were presented with the approximate method of ψ coefficient in *Table 2.8*. Accurate results were obtained from SAP2000 in the previous loading cases so that it was accepted as the reference for the ones computed by approximate method with ψ coefficient. Also, deviation of approximate solutions from SAP2000 results was summarized graphically in *Figure 2.12*.

It should be noted that maximum second-order elastic moment values occur at the span for the case of $a \leq 0.4L$. Since the maximum moments were calculated as being at the fixed-end for whole other cases, amplification of first-order moments by using approximate ψ coefficient was theoretically inapplicable for this case. However, deviation from the theoretical solutions according to TS648 (1980) will be investigated in the following lines.

As the transverse point load becomes closer to the fixed-end, more accurate results were detected. Generally, unconservative results were obtained with executing the approximate method. Deviation from theoretical values was reported as 10% for practical loading cases.

Table 2.8: Comparison of Maximum Second-Order Moments

a = 0.1L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.580)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.039**	0.942	1.047	**1.047**	0.74
0.2	4.028	**1.090**	0.884	1.105	**1.105**	1.38
0.3	6.043	**1.152**	0.826	1.180	**1.180**	2.43
0.4	8.057	**1.232**	0.768	1.280	**1.280**	3.90
0.5	10.071	**1.380**	0.710	1.420	**1.420**	2.90
0.6	12.085	**1.649**	0.652	1.630	**1.630**	-1.15
0.7	14.099	**2.171**	0.594	1.980	**1.980**	-8.80
0.8	16.114	**3.151**	0.536	2.680	**2.680**	-14.95
0.9	18.128	**6.320**	0.478	4.780	**4.780**	-24.37

a = 0.2L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.528)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.071**	0.947	1.052	**1.052**	-1.73
0.2	4.028	**1.153**	0.894	1.118	**1.118**	-3.04
0.3	6.043	**1.263**	0.842	1.202	**1.202**	-4.81
0.4	8.057	**1.405**	0.789	1.315	**1.315**	-6.43
0.5	10.071	**1.595**	0.736	1.472	**1.472**	-7.71
0.6	12.085	**1.894**	0.683	1.708	**1.708**	-9.82
0.7	14.099	**2.474**	0.630	2.101	**2.101**	-15.06
0.8	16.114	**3.622**	0.578	2.888	**2.888**	-20.27
0.9	18.128	**7.082**	0.525	5.248	**5.248**	-25.90

a = 0.3L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.480)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.087**	0.952	1.058	**1.058**	-2.69
0.2	4.028	**1.196**	0.904	1.130	**1.130**	-5.52
0.3	6.043	**1.335**	0.856	1.223	**1.223**	-8.40
0.4	8.057	**1.519**	0.808	1.347	**1.347**	-11.35
0.5	10.071	**1.775**	0.760	1.520	**1.520**	-14.37
0.6	12.085	**2.156**	0.712	1.780	**1.780**	-17.44
0.7	14.099	**2.787**	0.664	2.213	**2.213**	-20.58
0.8	16.114	**4.039**	0.616	3.080	**3.080**	-23.74
0.9	18.128	**7.779**	0.568	5.680	**5.680**	-26.98

Table 2.8 (continued)

a = 0.4L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.440)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.090**	0.956	1.062	**1.062**	-2.55
0.2	4.028	**1.202**	0.912	1.140	**1.140**	-5.16
0.3	6.043	**1.346**	0.868	1.240	**1.240**	-7.88
0.4	8.057	**1.537**	0.824	1.373	**1.373**	-10.65
0.5	10.071	**1.804**	0.780	1.560	**1.560**	-13.53
0.6	12.085	**2.202**	0.736	1.840	**1.840**	-16.44
0.7	14.099	**2.863**	0.692	2.307	**2.307**	-19.43
0.8	16.114	**4.178**	0.648	3.240	**3.240**	-22.45
0.9	18.128	**8.117**	0.604	6.040	**6.040**	-25.59

a = 0.5L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.510)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.083**	0.949	1.054	**1.054**	-2.64
0.2	4.028	**1.186**	0.898	1.123	**1.123**	-5.35
0.3	6.043	**1.317**	0.847	1.210	**1.210**	-8.12
0.4	8.057	**1.488**	0.796	1.327	**1.327**	-10.84
0.5	10.071	**1.725**	0.745	1.490	**1.490**	-13.62
0.6	12.085	**2.076**	0.694	1.735	**1.735**	-16.43
0.7	14.099	**2.653**	0.643	2.143	**2.143**	-19.21
0.8	16.114	**3.790**	0.592	2.960	**2.960**	-21.90
0.9	18.128	**7.123**	0.541	5.410	**5.410**	-24.05

a = 0.6L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.605)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.071**	0.940	1.044	**1.044**	-2.53
0.2	4.028	**1.157**	0.879	1.099	**1.099**	-5.03
0.3	6.043	**1.266**	0.819	1.169	**1.169**	-7.64
0.4	8.057	**1.407**	0.758	1.263	**1.263**	-10.21
0.5	10.071	**1.599**	0.698	1.395	**1.395**	-12.76
0.6	12.085	**1.882**	0.637	1.593	**1.593**	-15.38
0.7	14.099	**2.341**	0.577	1.922	**1.922**	-17.91
0.8	16.114	**3.240**	0.516	2.580	**2.580**	-20.37
0.9	18.128	**5.859**	0.456	4.555	**4.555**	-22.26

Table 2.8 (continued)

a = 0.7L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.704)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.056**	0.930	1.033	**1.033**	-2.19
0.2	4.028	**1.124**	0.859	1.074	**1.074**	-4.45
0.3	6.043	**1.207**	0.789	1.127	**1.127**	-6.64
0.4	8.057	**1.315**	0.718	1.197	**1.197**	-8.95
0.5	10.071	**1.460**	0.648	1.296	**1.296**	-11.23
0.6	12.085	**1.669**	0.578	1.444	**1.444**	-13.48
0.7	14.099	**2.007**	0.507	1.691	**1.691**	-15.76
0.8	16.114	**2.661**	0.437	2.184	**2.184**	-17.93
0.9	18.128	**4.552**	0.366	3.664	**3.664**	-19.51

a = 0.8L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.805)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.039**	0.920	1.022	**1.022**	-1.67
0.2	4.028	**1.086**	0.839	1.049	**1.049**	-3.43
0.3	6.043	**1.143**	0.759	1.084	**1.084**	-5.20
0.4	8.057	**1.215**	0.678	1.130	**1.130**	-7.00
0.5	10.071	**1.310**	0.598	1.195	**1.195**	-8.78
0.6	12.085	**1.447**	0.517	1.293	**1.293**	-10.68
0.7	14.099	**1.664**	0.437	1.455	**1.455**	-12.56
0.8	16.114	**2.079**	0.356	1.780	**1.780**	-14.38
0.9	18.128	**3.268**	0.276	2.755	**2.755**	-15.70

a = 0.9L

P/P_{e1}	P	SAP2000	TS648 - 1980 *(For ψ = -0.904)*			
		$M_{z,max2}$	C_m	B_1	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	1.000	1.000	**1.000**	0.00
0.1	2.014	**1.021**	0.910	1.011	**1.011**	-1.01
0.2	4.028	**1.045**	0.819	1.024	**1.024**	-2.01
0.3	6.043	**1.073**	0.729	1.041	**1.041**	-2.97
0.4	8.057	**1.109**	0.638	1.064	**1.064**	-4.06
0.5	10.071	**1.156**	0.548	1.096	**1.096**	-5.19
0.6	12.085	**1.220**	0.458	1.144	**1.144**	-6.23
0.7	14.099	**1.314**	0.367	1.224	**1.224**	-6.85
0.8	16.114	**1.460**	0.277	1.384	**1.384**	-5.21
0.9	18.128	**1.968**	0.186	1.864	**1.864**	-5.28

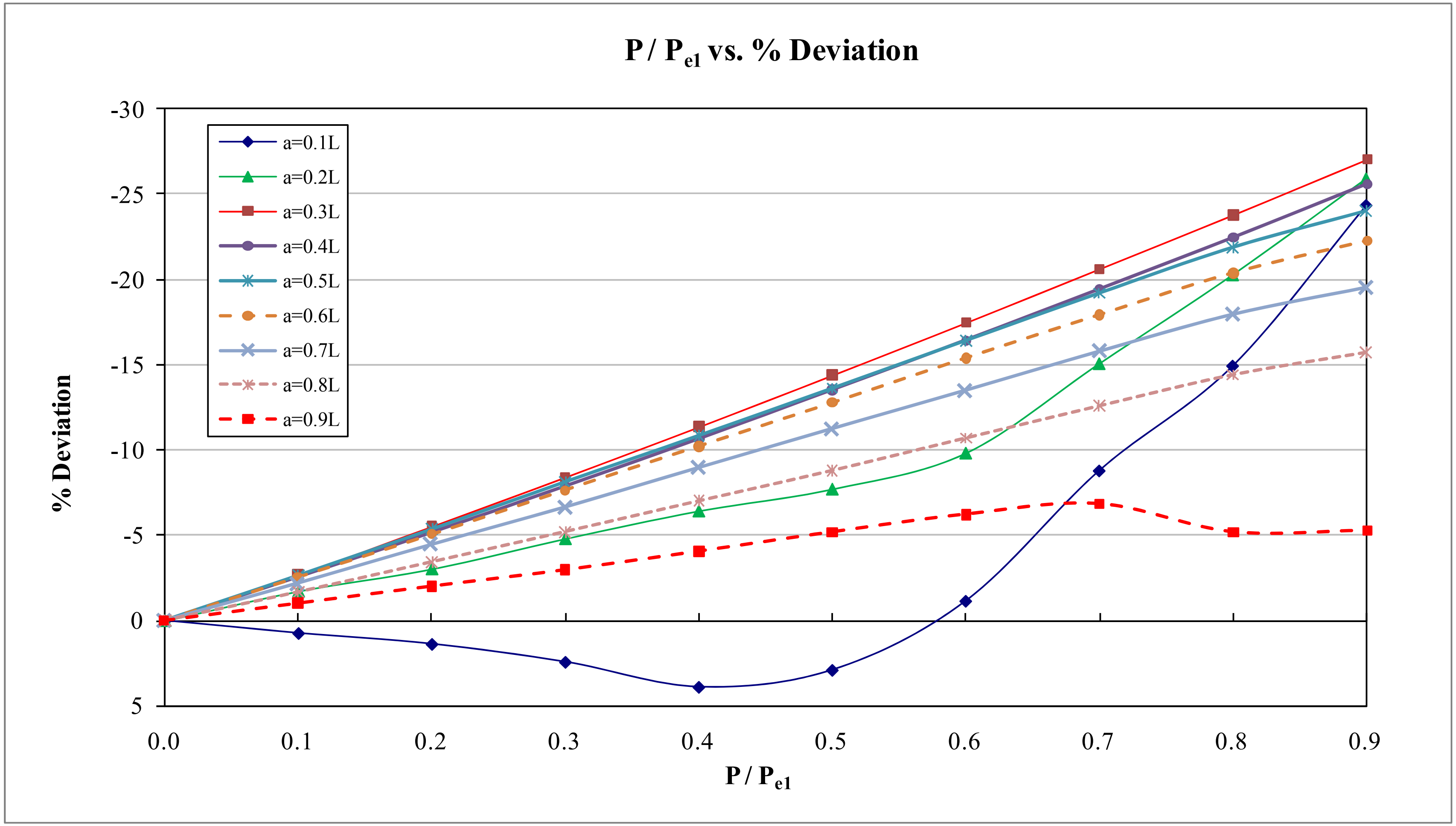


Figure 2.12: Deviation of Results Obtained by Approximate Method from SAP2000 Solutions

2.3 Members with End Moments Only

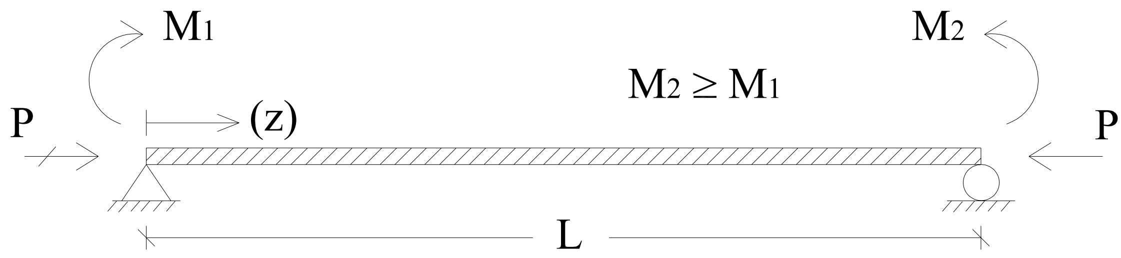


Figure 2.13: Simply Supported Beam-Column with End Moments Only

The first-order elastic maximum moment is equal to *"M_2"* which occurs at the end. The exact solution of the second-order moment as a function of distance *(z)* presented by Salmon & Johnson (1996) is given in *Equation (2.5)*.

$$M_{z,2} = \left(\frac{M_2 - M_1 \cos k'L}{\sin k'L}\right) \cdot \sin k'z + M_1 \cdot \cos k'z \qquad (2.5)$$

If derivative of *Eq. (2.5)* is taken with respect to *(z)*, and equated to zero, the location of the maximum moment is determined. *Equation (2.6)* is obtained to determine the maximum second-order elastic moment by inserting the value of *(z)* into *Eq. (2.6)*.

$$M_{z,max2} = M_2 \sqrt{\frac{1 - 2(M_1/M_2)\cos k'L + (M_1/M_2)^2}{\sin^2 k'L}} \qquad (2.6)$$

Equation (2.6) can be used for single curvature cases in which the maximum moment value occurs in the span. However, the maximum moment is obtained from the moment variation derived from *Eq. (2.5)* for double curvature cases.

Equation (2.6) does not fully cover the double-curvature cases in which M_1/M_2 lies between -0.5 and -1.0. The actual failure of members bent in double curvature with bending moment ratios -0.5 to -1.0 is generally one of *"unwinding"*, as shown in *Figure 2.13*, through from double to single curvature in a sudden type of buckling (Salmon & Johnson, 1996).

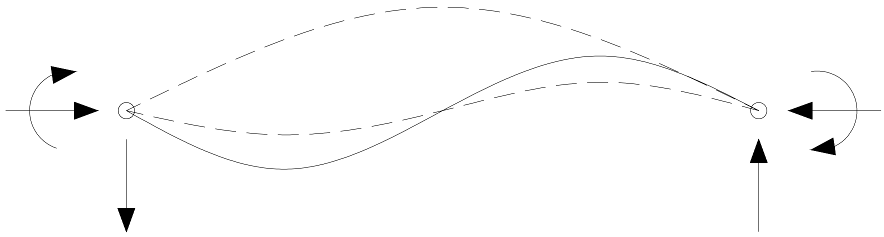


Figure 2.13: Unwinding

Several second-order elastic analyses were performed with SAP2000 for various end moment ratios. Required data for further analyses was given in *Table 2.2*.

Euler elastic buckling load for the simply-supported beam-column was calculated as P_{e1} = 9.87 *(=π^2)*. The member was divided into 100 pieces in the model created with SAP2000 to improve the precision of the results. Solutions obtained from the successive analyses are compared with the exact results, AISC 2005 and TS648 (1980) methods in *Tables 2.9 & 2.10*, and expressed graphically in *Figure 2.14.*

According to *Table 2.9*, in which analysis results of single curvature cases were presented, second-order moments were amplified up to 12.4 times of the first-order moment according to exact results. The reason for that case is that the C_m formulation given in *Eq. (1.9)* is an approximation of the exact solution, which is independent of any other parameter like axial thrust level except the moment gradient. Nevertheless, it is applicable despite a maximum deviation of 10% on the unconservative side was reported. As well, the C_m formulation shown in *Eq. (1.9)* is used in the reinforced concrete design manuals.

Generally, stable results within admissible error were obtained by performing second-order elastic analysis by using SAP2000 algorithm when compared with B_1 Method proposed by AISC 2005. Basically, unconservative moment values were obtained for single curvature cases; whereas conservative values were acquired for double curvature cases. Also, the solutions of AISC 2005 and TS648 (1980) Methods differ in double curvature cases since a lower limit of 0.4 is specified in TS648 (1980). The lower bound for the C_m formulation in the case of end moments was removed in AISC/ASD Manual (1978), since it was found out to be over-conservative for high axial load levels, which is observed in TS648 (1980) results presented in *Table 2.10* if the M_1/M_2 ratio is between 0.5 and 1.0.

Table 2.9: Comparison of Maximum Second-Order Moments (Single Curvature)

			$M_1/M_2 = -1.0$							$M_1/M_2 = -0.8$						
P/P_{e1}	P	k'	Exact Result	SAP2000		AISC 2005 & TS648			Exact Result	SAP2000		AISC 2005 & TS648				
			$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.	$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.		
0.0	0.000	0.0000	**1.000**	**1.000**	0.00		**1.000**	0.00	**1.000**	**1.000**	0.00		**1.000**	0.00		
0.1	0.987	0.0099	**1.137**	**1.137**	0.00		**1.111**	-2.32	**1.045**	**1.045**	0.00		**1.022**	-2.18		
0.2	1.974	0.0140	**1.310**	**1.310**	0.00		**1.250**	-4.59	**1.189**	**1.189**	0.00		**1.150**	-3.30		
0.3	2.961	0.0172	**1.533**	**1.533**	0.00		**1.429**	-6.83	**1.386**	**1.386**	0.00		**1.314**	-5.20		
0.4	3.948	0.0199	**1.832**	**1.832**	0.00	1.00	**1.667**	-9.03	**1.653**	**1.653**	0.00	0.92	**1.533**	-7.26		
0.5	4.935	0.0222	**2.252**	**2.252**	0.01		**2.000**	-11.20	**2.030**	**2.030**	0.00		**1.840**	-9.36		
0.6	5.922	0.0243	**2.884**	**2.884**	0.01		**2.500**	-13.32	**2.598**	**2.598**	0.01		**2.300**	-11.47		
0.7	6.909	0.0263	**3.941**	**3.941**	0.01		**3.333**	-15.41	**3.548**	**3.548**	0.01		**3.067**	-13.57		
0.8	7.896	0.0281	**6.058**	**6.059**	0.02		**5.000**	-17.46	**5.453**	**5.454**	0.02		**4.600**	-15.64		
0.9	8.883	0.0298	**12.419**	**12.424**	0.04		**10.000**	-19.48	**11.178**	**11.182**	0.04		**9.200**	-17.69		

Table 2.9 (continued)

			$M_1/M_2 = -0.5$							$M_1/M_2 = 0$						
P/P_{e1}	P	k'	Exact Result	SAP2000		AISC 2005 & TS648			Exact Result	SAP2000		AISC 2005 & TS648				
			$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.	$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.		
0.0	0.000	0.0000	**1.000**	**1.000**	0.00		**1.000**	0.00	**1.000**	**1.000**	0.00		**1.000**	0.00		
0.1	0.987	0.0099	**1.001**	**1.000**	-0.15		**1.000**	-0.15	**1.000**	**1.000**	0.00		**1.000**	0.00		
0.2	1.974	0.0140	**1.056**	**1.056**	0.00		**1.000**	-5.31	**1.000**	**1.000**	0.00		**1.000**	0.00		
0.3	2.961	0.0172	**1.196**	**1.196**	0.00		**1.143**	-4.47	**1.011**	**1.011**	0.00		**1.000**	-1.12		
0.4	3.948	0.0199	**1.406**	**1.406**	0.00	0.80	**1.333**	-5.18	**1.093**	**1.093**	0.00	0.60	**1.000**	-8.53		
0.5	4.935	0.0222	**1.712**	**1.712**	-0.01		**1.600**	-6.54	**1.257**	**1.257**	-0.01		**1.200**	-4.52		
0.6	5.922	0.0243	**2.180**	**2.180**	0.00		**2.000**	-8.24	**1.537**	**1.537**	-0.01		**1.500**	-2.44		
0.7	6.909	0.0263	**2.967**	**2.966**	-0.01		**2.667**	-10.11	**2.037**	**2.037**	-0.01		**2.000**	-1.82		
0.8	7.896	0.0281	**4.550**	**4.549**	-0.02		**4.000**	-12.10	**3.071**	**3.071**	-0.02		**3.000**	-2.31		
0.9	8.883	0.0298	**9.318**	**9.305**	-0.14		**8.000**	-14.14	**6.230**	**6.225**	-0.08		**6.000**	-3.69		

Table 2.10: Comparison of Maximum Second-Order Moments (Double Curvature)

		$M_1/M_2 = 0.5$						$M_1/M_2 = 0.8$								
P/P_{e1}	P	Exact Result	SAP2000		AISC 2005 & TS648			Exact Result	SAP2000		AISC 2005			TS648		
		$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.	$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.
0.0	0.000	**1.000**	**1.000**	0.00		**1.000**	0.00	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.1	0.987	**1.000**	**1.000**	0.00		**1.000**	0.00	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.2	1.974	**1.000**	**1.000**	0.00		**1.000**	0.00	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.3	2.961	**1.000**	**1.000**	0.00		**1.000**	0.00	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.4	3.948	**1.000**	**1.000**	0.00	0.40	**1.000**	0.00	**1.000**	**1.000**	0.00	0.28	**1.000**	0.00	0.40	**1.000**	0.00
0.5	4.935	**1.008**	**1.009**	0.08		**1.009**	0.08	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.6	5.922	**1.076**	**1.077**	0.06		**1.077**	0.07	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.7	6.909	**1.254**	**1.254**	-0.03		**1.254**	-0.02	**1.010**	**1.011**	0.05		**1.000**	-0.99		**1.333**	32.0
0.8	7.896	**1.693**	**1.694**	0.08		**1.695**	0.11	**1.095**	**1.095**	0.03		**1.400**	27.85		**2.000**	82.6
0.9	8.883	**3.189**	**3.193**	0.14		**3.196**	0.21	**1.535**	**1.536**	0.05		**2.800**	82.41		**4.000**	160.6

Table 2.10 (continued)

			$M_1/M_2 = 1.0$								
P/P_{e1}	P	k'	Exact Result	SAP2000		AISC 2005			TS648		
			$M_{z,max2}$	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.	C_m	$M_{z,max2}$	% diff.
0.0	0.000	0.0000	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.1	0.987	0.0099	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.2	1.974	0.0140	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.3	2.961	0.0172	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.4	3.948	0.0199	**1.000**	**1.000**	0.00	0.20	**1.000**	0.00	0.40	**1.000**	0.00
0.5	4.935	0.0222	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.6	5.922	0.0243	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.000**	0.00
0.7	6.909	0.0263	**1.000**	**1.000**	0.00		**1.000**	0.00		**1.333**	33.3
0.8	7.896	0.0281	**1.000**	**1.000**	0.00		**1.000**	0.00		**2.000**	100.0
0.9	8.883	0.0298	**1.000**	**1.000**	0.00		**2.000**	100.00		**4.000**	300.0

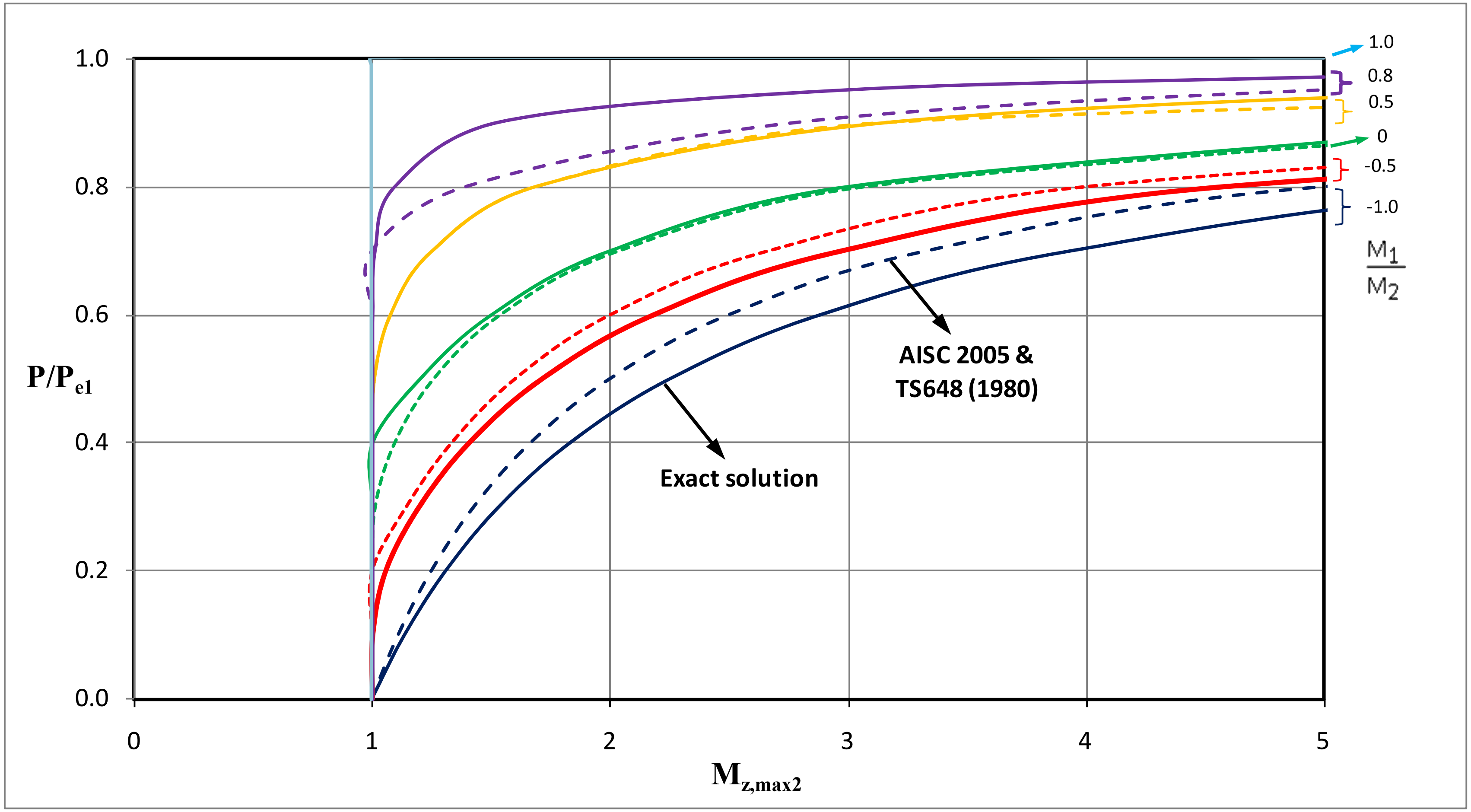


Figure 2.14: Comparison of Second-Order Moments for Beam-Columns Subjected to Applied End Moments

2.4 Braced Frame Example

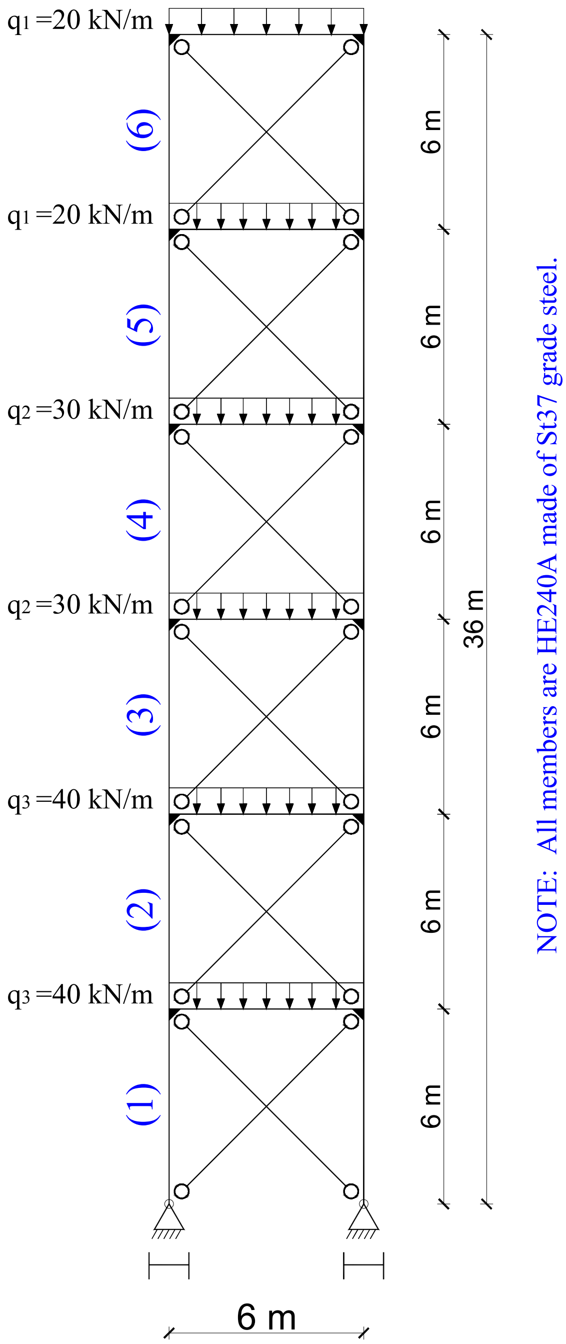


Figure 2.15: Braced Frame Example

A braced frame subjected to uniformly distributed gravity load was investigated to evaluate the methods of considering P-δ effect according to AISC 2005 and TS648 (1980). The problem was presented in *Figure 2.15*. The columns are connected to the foundation with pin-supports in the plane-of-bending, while the columns are braced out-of-plane, i.e. effective length factor is K=1.0.

Data obtained from the linear elastic analysis performed by SAP2000 is presented in the table given below.

Table 2.11: First-Order Elastic Analysis Results

Column #	**FIRST-ORDER ELASTIC ANALYSIS**			
	P_{axial} (kN)	M_{bottom} (kN·m)	M_{top} (kN·m)	M_{max} (kN·m)
(1)	424.8	0.0	35.1	35.1
(2)	351.2	-61.7	56.2	61.7
(3)	248.1	-46.0	40.3	46.0
(4)	173.5	-37.6	40.4	40.4
(5)	100.0	-34.6	25.4	34.6
(6)	48.6	-28.7	41.4	41.4

An approximate second-order analysis was carried out in accordance with AISC 2005, i.e. B_1 amplifier was computed. Despite the calculated B_1 factors for each member were smaller than 1, it was taken as unity because of the lower limit specified in AISC 2005, as specified in *Eq. (1.5)*. Thus, deamplification of first-order moments was prevented when dealing with second-order effects. So, the second-order moments and axial thrust values were taken as equal to first-order analysis results. Also, it should be noted that all columns were bent in double curvature except the one labeled as (1).

Then, design checks for the beam-columns were applied according to AISC 2005 and TS648 (1980) as presented in *Tables 2.12 & 2.13*. Basically, behavior of column (2) should be examined further. Despite the high safety factors included in TS648 (1980) beam-column formulations, an unconservative result was obtained when compared with AISC 2005 beam-column formulations, since there is no lower limit for the moment amplification factor.

Table 2.12: Design Check for Columns according to AISC 2005 Formulations

COLUMN (1)	$\frac{P_r}{P_c} = \frac{425}{657} = 0.647 \; > 0.2$ $\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{425}{657} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{35.1}{104.8}}_{0.335} = 0.647 + 0.298 = \mathbf{0.945} \; \mathbf{< 1.0 \; O.K.}$
COLUMN (2)	$\frac{P_r}{P_c} = \frac{351}{657} = 0.534 \; > 0.2$ $\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{351}{657} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{61.7}{104.8}}_{0.589} = 0.534 + 0.523 = \mathbf{1.058} \; \mathbf{> 1.0}$ **OVERSTRESSED**
COLUMN (3)	$\frac{P_r}{P_c} = \frac{248}{657} = 0.377 \; > 0.2$ $\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{248}{657} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{46.0}{104.8}}_{0.439} = 0.377 + 0.390 = \mathbf{0.768} \; \mathbf{< 1.0 \; O.K.}$
COLUMN (4)	$\frac{P_r}{P_c} = \frac{174}{657} = 0.265 \; > 0.2$ $\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{174}{657} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{40.4}{104.8}}_{0.385} = 0.265 + 0.343 = \mathbf{0.608} \; \mathbf{< 1.0 \; O.K.}$
COLUMN (5)	$\frac{P_r}{P_c} = \frac{100}{657} = 0.152 \; < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{100}{2 \cdot 657} + \frac{34.6}{104.8} = 0.076 + 0.330 = \mathbf{0.406} \; \mathbf{< 1.0 \; O.K.}$
COLUMN (6)	$\frac{P_r}{P_c} = \frac{49}{657} = 0.075 \; < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{49}{2 \cdot 657} + \frac{41.4}{104.8} = 0.037 + 0.395 = \mathbf{0.432} \; \mathbf{< 1.0 \; O.K.}$

Table 2.13: Design Check for Columns according to TS648 (1980) Formulations

COLUMN (1)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{55.3}{70.7} = 0.782 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{55.3}{70.7} + \underbrace{\frac{52.0}{141}}_{0.368} \cdot \underbrace{\frac{0.6}{1 - 55.3/264.3}}_{0.759} = 0.782 + 0.280 = \mathbf{1.062} \; \mathbf{> 1.0}$ **OVERSTRESSED** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{55.3}{141} + \frac{52.0}{141} = 0.392 + 0.368 = 0.761 < 1.0$ O.K.
COLUMN (2)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{45.7}{70.7} = 0.647 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{45.7}{70.7} + \underbrace{\frac{91.3}{141}}_{0.648} \cdot \underbrace{\frac{0.4}{1 - 45.7/306.1}}_{0.470} = 0.647 + 0.305 = 0.951 < 1.0$ O.K. $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{45.7}{141} + \frac{91.3}{141} = 0.324 + 0.648 = \mathbf{0.972} \; \mathbf{< 1.0}$ **O.K.**
COLUMN (3)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{32.3}{70.7} = 0.457 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{32.3}{70.7} + \underbrace{\frac{68.1}{141}}_{0.483} \cdot \underbrace{\frac{0.4}{1 - 32.3/306.1}}_{0.447} = 0.457 + 0.216 = 0.673 < 1.0$ O.K. $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{32.3}{141} + \frac{68.1}{141} = 0.229 + 0.483 = \mathbf{0.712} \; \mathbf{< 1.0}$ **O.K.**
COLUMN (4)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{22.6}{70.7} = 0.319 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{22.6}{70.7} + \underbrace{\frac{59.9}{141}}_{0.425} \cdot \underbrace{\frac{0.4}{1 - 22.6/306.1}}_{0.432} = 0.319 + 0.184 = 0.503 < 1.0$ O.K. $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{22.6}{210} + \frac{59.9}{141} = 0.108 + 0.425 = \mathbf{0.532} \; \mathbf{< 1.0}$ **O.K.**
COLUMN (5)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{13.0}{70.7} = 0.184 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{13.0}{70.7} + \underbrace{\frac{51.2}{141}}_{0.363} \cdot \underbrace{\frac{0.4}{1 - 13.0/306.1}}_{0.418} = 0.184 + 0.152 = 0.336 < 1.0$ O.K. $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{13.0}{210} + \frac{51.2}{141} = 0.062 + 0.363 = \mathbf{0.425} \; \mathbf{< 1.0}$ **O.K.**
COLUMN (6)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{6.3}{70.7} = 0.090 < 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} = \frac{6.3}{70.7} + \frac{61.4}{141} = 0.090 + 0.435 = \mathbf{0.525} \; \mathbf{< 1.0}$ **O.K.**

The amplification factor was calculated as 0.470 for column (2) according to TS648 (1980), which is highlighted on *Table 2.13*. So, second-order effects were underestimated in TS648 (1980) beam-column formulations. Also, as a result of this study, it is recommended to take the amplification factor out of the stability equation *(Eq. 1.13)* as specified in *Eq. (1.15)*, and a lower limit equal to 1 should be defined in TS648 (1980). By this way, underestimation of second-order effects can be prevented. Thus, procedure should be carried out properly as specified in the table below.

Table 2.14: Proposed Design Check for Column (2) according to TS648 (1980) Formulations

COLUMN (2)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{45.7}{70.7} = 0.647 > 0.15$ $\quad A_f = \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{0.4}{1 - 45.7/306.1} = \mathbf{0.470 < 1.0} \rightarrow \mathbf{A_f = 1.0}$
	$\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot A_f = \frac{45.7}{70.7} + \frac{91.3}{141} \cdot \mathbf{1.0} = 0.647 + 0.648 = \mathbf{1.295 > 1.0}$
	$\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{45.7}{141} + \frac{91.3}{141} = 0.324 + 0.648 = 0.972 < 1.0$ O.K.

CHAPTER 3

EVALUATION OF SECOND-ORDER EFFECTS FOR MEMBERS WITH END TRANSLATION

In this chapter, the main emphasis will be on the differences between the two specifications under consideration, AISC 2005 and TS648 (1980), with respect to the way that P-Δ effects are taken into account. Basically, explicit permission is given to the designer in AISC 2005 to perform a second-order analysis with any method considering both P-δ and P-Δ effects. So, the results obtained from the structural analysis software, SAP2000 will be presented and used in the moment interaction check formulations. Also, an approximate second-order analysis method by amplified first-order elastic analysis, i.e. B_1-B_2 Method, is suggested by AISC 2005.

In contrast, P-δ and P-Δ effects are taken into account together in the beam-column formulation given in *Eq. (1.13)* by the amplification of the first-order bending stress in the right-side of the formulation with the amplification factor presented in *Eq. (1.15)* according to TS648 (1980) approach. Obviously, second-order effects are estimated roughly, unlike the methodology used by AISC 2005, in which additional effects of geometric nonlinearity on the structural systems are clearly defined. Generally, AISC 2005 approaches were found out to be contemporary without disregarding the improvements and wide usage of computer technology in structural analysis.

As well, C_m value is taken as 0.85 for sidesway-permitted cases according to TS648 (1980). On the other hand, the numerator of B_2 formulation defined in *Eq. (1.10)* is specified as unity in AISC 2005, instead of highlighting a specific value for C_m. However, C_m factor is still used in determination of B_1 factor. Since two different amplification factors are specified in AISC 2005 to account for P-delta effects, as B_1 and B_2 amplifiers for no-translation and lateral-translation portions, respectively, it is so complicated to establish a direct comparison methodology. Instead, the overall amplification can be compared to obtain a general opinion.

3.1 Lean-on Systems

3.1.1 Example 1

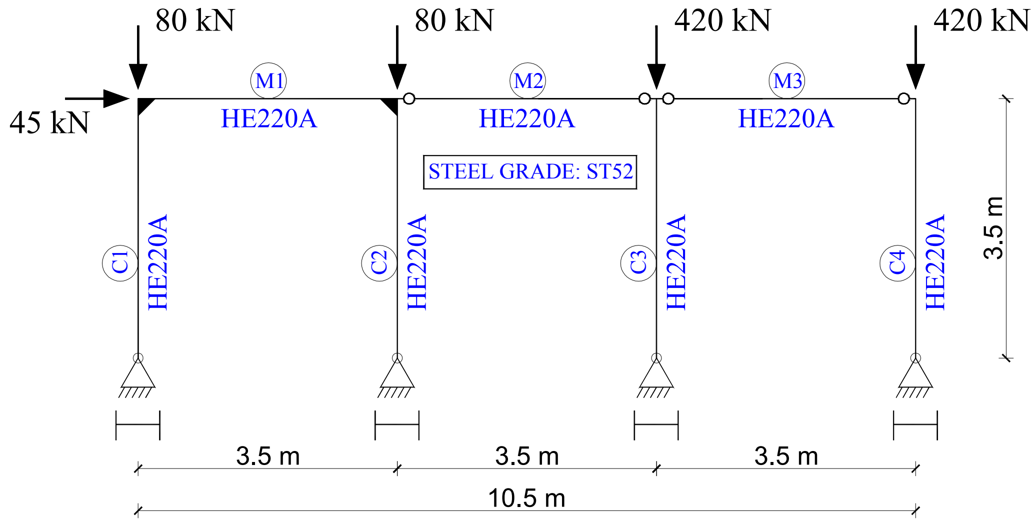


Figure 3.1: Example 1 for Lean-on Systems

The one-story frame presented in the figure above was considered for investigation of second-order effects on lean-on systems. Whole frame system consists of wide-flange section, HE220A, which is made of St52 grade steel. Lateral stability of the leaner columns (C3 & C4) is provided by the moment frame system composed of the columns C1 and C2, and the beam M1. The columns are connected to the foundation with pin-supports in the plane-of-bending, while the columns are braced out-of-plane, i.e. effective length factor is K=1.0.

Since the frame is side-sway permitted type, P-Δ effect is expected to be more significant rather than P-δ effect for this case. The approximate B_1-B_2 Amplification Method was carried out for the consideration of geometrical non-linearity according to principles specified in AISC 2005. Since an explicit permission is given to the designer in AISC 2005 for selecting the methodology to cover second-order effects, a second-order analysis was performed by using SAP2000, and the results obtained from the computational methods will be used in moment interaction check. Then, design of the two columns, specified as C1 and

C2, was checked according to ASD formulations proposed by AISC 2005 and TS648 (1980) specifications for comparison.

Three options were used to consider geometrical non-linearity effects on design moments and axial forces according to AISC Manual (2005). B_1-B_2 Method was carried out to amplify the first-order elastic analysis results with approximate amplifiers, B_1 and B_2, according to two different procedures, Multiple-Column Magnifier Method, and Story Magnifier Method, as described in *Section 1.4* in detail. These amplified internal forces forming in the members will be compared with the outputs obtained from elastic second-order frame analysis performed with the software to account for secondary internal forces directly. The analysis results are presented in *Tables 3.1 & 3.2,* whereas design checks of the columns C1, and C2 were summarized for AISC 2005 and TS648 (1980) in *Table 3.3*. It should be noted that available flexural and compressive strengths for the columns C1 and C2 were calculated as equal to M_c = 119.14 kN·m and P_c = 734.4 kN, respectively. Also, available tensile strength was computed as equal to P_c = 13,476 kN for the specified columns.

SAP2000 results were accepted as a reference point for further evaluation, and both of the columns were reported as being overstressed according to moment interaction check of AISC 2005. Also, results obtained by B_1-B_2 Method were evaluated as being conservative when compared with the software outputs. Moreover, Story Magnifier Method was found out to be slightly more conservative than Multiple-Column Magnifier Method. It should be kept in mind that design of column C1 was checked according to beam-column methodology, since it was subjected to axial compression and bending. However, since the member was internally under tension in combination with bending moment, the design check was performed by considering this phenomenon.

On the other hand, since the beam-column design procedure in TS648 (1980) is based on the magnification of first-order internal moments roughly with the embedded amplification factor *(Eq. 1.15)* in the stability interaction equation *(Eq. 1.13)*, it resulted in an increase in axial compressive force in both of the columns, C1 and C2. Additionally, the change in axial forces in the members by accounting for second-order effects was disregarded in the left hand side of the stability interaction equation given in *Eq. (1.13)*. The columns passed through the design check corresponding to TS648 (1980) as presented in *Table 3.3*. So, the structural behavior of the columns was not taken into account properly by TS648 (1980).

Table 3.1: SAP2000 Analysis Results for Example 1 of Lean-On Systems

SAP2000 ANALYSIS RESULTS						
Column #	**FIRST-ORDER E.A.**		**SECOND-ORDER ELASTIC ANALYSIS**			
	$P_{first-order}$ (kN)	**$M_{first-order}$ (kN·m)**	**P_r (kN)**	**M_r (kN·m)**	**$A_{F,axial}$**	**$A_{F,moment}$**
(1)	35 (C)	78.78	3.11 (T)	**146.05**	*Deampl.*	*1.854*
(2)	125 (C)	78.72	163.13 (C)	**144.85**	*1.305*	*1.840*

Table 3.2: Summary of Second-Order Analysis by Amplified First-Order Elastic Analysis Defined in AISC 2005

SECOND-ORDER ANALYSIS BY AMPLIFIED FIRST-ORDER ELASTIC ANALYSIS												
Column #	**MULTIPLE COLUMN MAGNIFIER METHOD**						**STORY MAGNIFIER METHOD**					
	ΣP_{nt} (kN)	**ΣP_{e2} (kN)**	**B_2**	**B_1**	**M_r (kN·m)**	**$A_{F,moment}$**	**ΣH (kN)**	**Δ_H (mm)**	**ΣP_{e2} (kN)**	**B_2**	**M_r (kN·m)**	**$A_{F,moment}$**
(1)	1000	3,104	*2.064*	1.0	**162.60**	*2.064*	45	44.79	2,988	*2.153*	**169.59**	*2.153*
(2)				1.0	**162.48**	*2.064*					**169.46**	*2.153*

Table 3.3: Design Checks for Columns according to AISC 2005 and TS648 (1980)

AISC-ASD 2005	**COLUMN (1)**	$\frac{P_r}{P_c} = \frac{3.11}{13476} = 0.000 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{3.11}{2 \cdot 13476} + \frac{146.05}{119.14} = 0.000 + 1.226 = 1.226 > 1.0$ **OVERSTRESSED**
	COLUMN (2)	$\frac{P_r}{P_c} = \frac{163.13}{734.4} = 0.222 > 0.2$ $\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{163.1}{734.4} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{144.85}{119.14}}_{1.216} = 0.222 + 1.081 = 1.303 > 1.0$ **OVERSTRESSED**
TS648 (1980)	**COLUMN (1)**	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{5.44}{94.4} = 0.058 < 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} = \frac{5.44}{94.4} + \frac{153.0}{210} = 0.058 + 0.728 = 0.786 < 1.0$ **O.K.**
	COLUMN (2)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{19.4}{94.4} = 0.206 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{19.4}{94.4} + \underbrace{\frac{152.85}{210}}_{0.728} \cdot \underbrace{\frac{0.85}{1 - 19.4/96.5}}_{1.064} = 0.206 + 0.775 = 0.981 < 1.0$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{19.4}{210} + \frac{152.9}{210} = 0.093 + 0.728 = 0.820 < 1.0$ O.K.

3.1.2 Example 2 - The SAC Model Building

The buildings defined in the content of the SAC Model Buildings Project were previously analyzed and designed according to seismic experiences gained after the 1994 Northridge Earthquake (USA). As a part of the SAC steel project, three consulting firms were commissioned to perform code designs for 3-, 9-, and 20-story model buildings, following the local code requirements for the following three cities: Los Angeles (*UBC 1994*), Seattle (*UBC 1994*), and Boston (*BOCA 1993*).

In the content of the thesis, 9-story typical Post-Northridge Earthquake Design Building in Los Angeles specified in Appendix B of FEMA-355C (2000) was selected for the first-, and second-order analyses *(Figure 3.2)*. P-Δ effect was significant rather than P-δ effect for this example. Also, the buildings were designed to conform story drift limits, in order to reduce second-order effects.

One of the exterior moment-resisting frames was modeled in the content of the thesis. Additionally, a leaner column was attached to the five-bay moment frame to completely account for the second-order effects caused by the loading on the leaner columns (*Figure 3.3)*. Also, column nomenclature can be found in *Figure 3.2*.

The building has a single-level basement and has pin-supported restraints on the foundation level, whereas the basement floor is modeled with lateral only support conditions. Furthermore, one of the exterior bays has only one moment-resisting connection to avoid bi-axial bending in the corner column. The building is a standard office building situated on stiff soil (Soil type S2 as per *UBC 1994*). It should be noted that St37 grade structural steel was considered for whole members.

The story loads were calculated with respect to tributary areas corresponding to each column-beam connection, and applied on each node. The lateral seismic load was computed as 4% of the total weight of the building according to *UBC 1994*, and applied on the system in combination with the vertical loads shown in *Figure 3.4*. The 1.0D + 0.75L + 0.7E load combination was used according to *Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-05 (2005)*, published by *the American Society of Civil Engineers*. Also, the building was required to conform to a drift limit of h/400, where "*h*" is the story height.

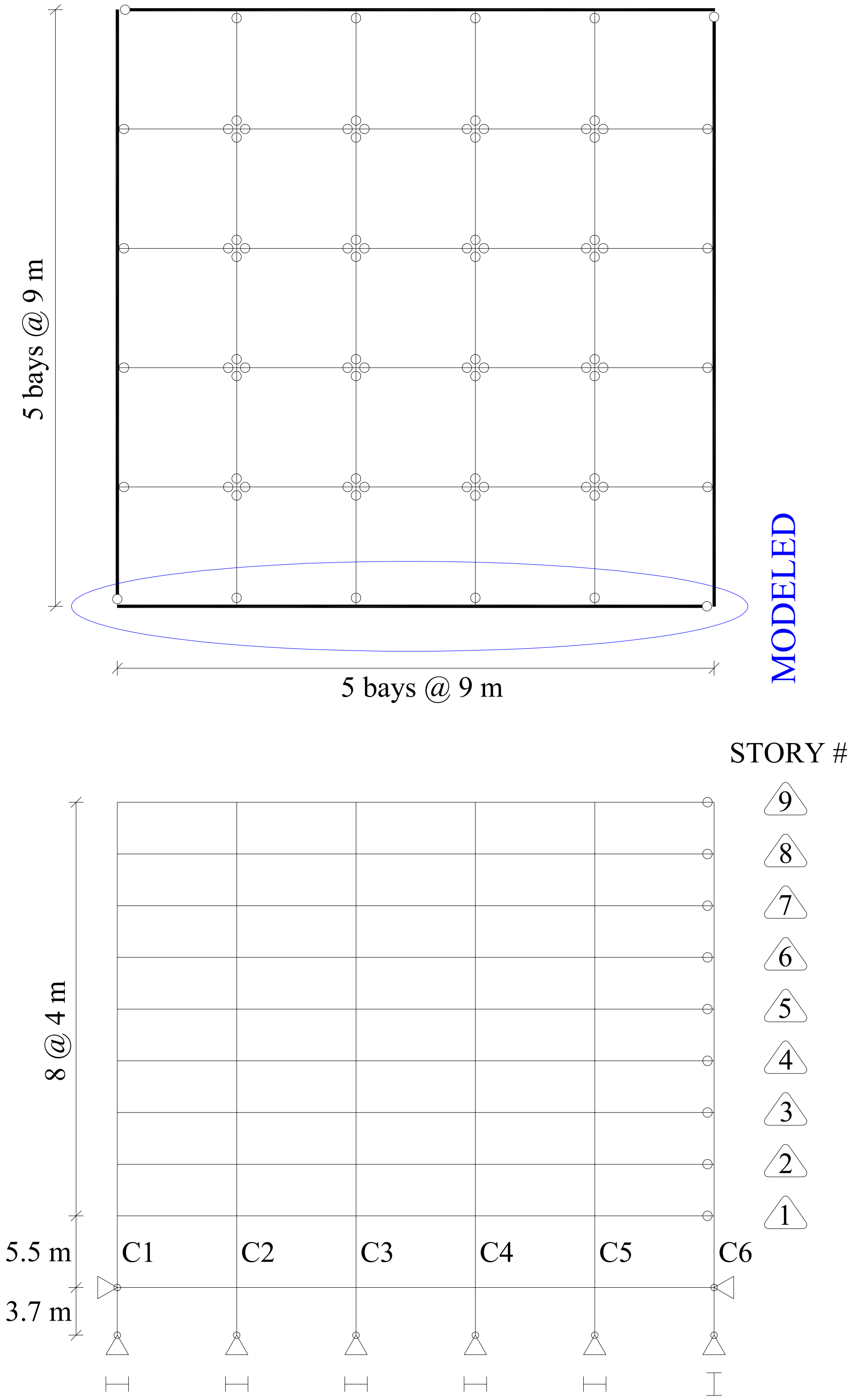


Figure 3.2: Floor Plan and Elevation for 9-Story Model Building

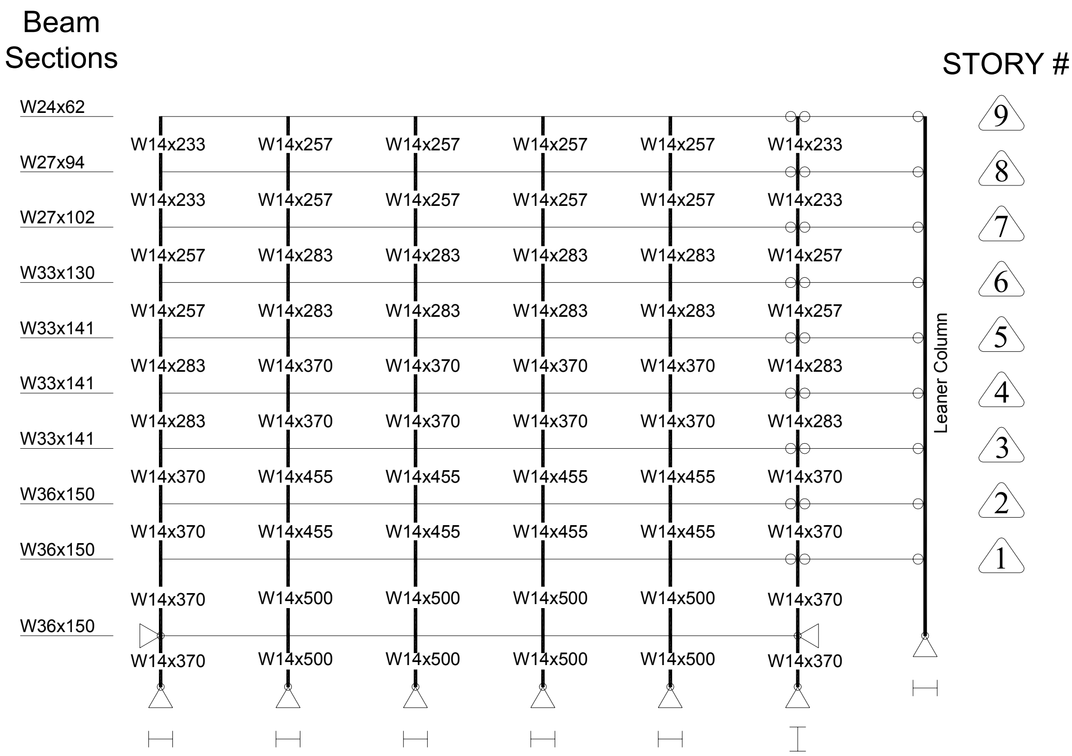


Figure 3.3: Frame Sections

First of all, a first-order elastic analysis was carried out by using the software, SAP2000, which will be used in the design checks according to TS648 (1980). Then, two approximate second-order analysis methods specified in AISC 2005 Manual, *Story Magnifier Method and Multiple Column Magnifier Method*, were applied. Also, additional first-order elastic analyses were carried out to be used in the approximate methods, which are specified as Second-Order Analysis by Amplified First-Order Elastic Analysis, in other words, B_1-B_2 Amplification Method according to AISC 2005. Finally, a second-order elastic analysis was performed to be used in AISC 2005 formulations, in lieu of the results obtained from approximate methods. It should be noted since ASD formulations of AISC 2005 are under consideration, the second-order elastic analysis with SAP2000 was carried out under 1.6 times the ASD load combination, and the results were divided by 1.6 to obtain the required strengths.

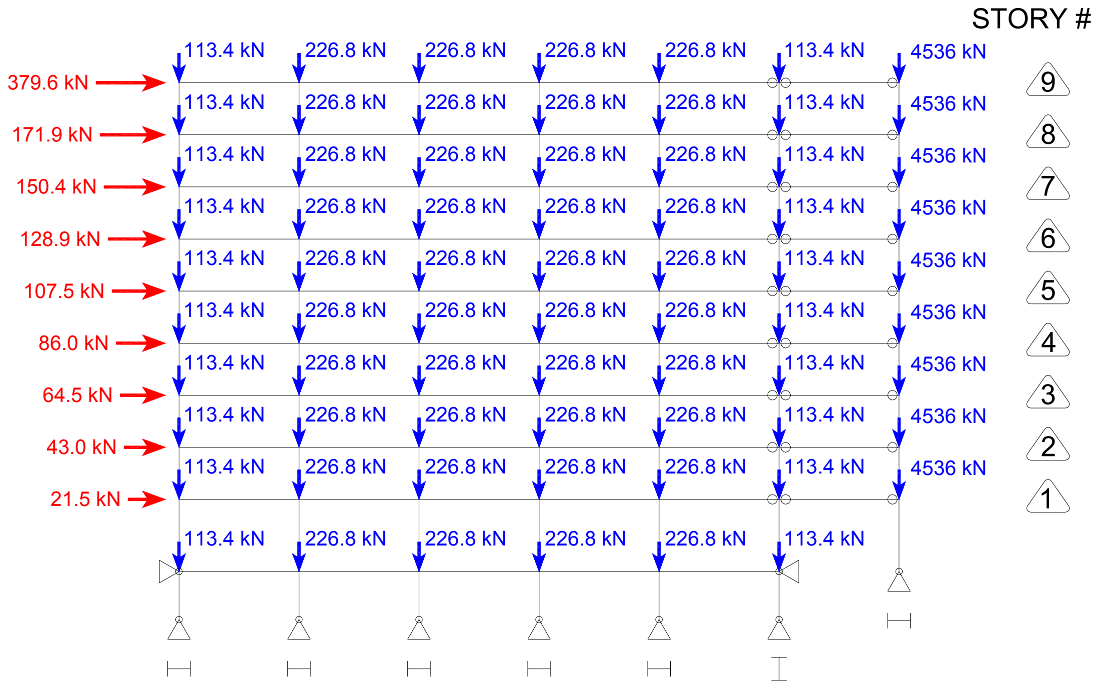


Figure 3.4: Applied Loads

Results obtained from the successive analyses are presented in *Table 3.3*, including steps in the analyses. The increase in second-order moments was reported as being between %8 and %24. Definitely, the amplification was based on the deflected shape of the frame between the nodes, i.e. P-Δ effect was significant rather than P-δ effect. Even, B_1 amplifier was calculated as being equal to 1.0 because of the lower limit defined in AISC 2005.

Generally, *Story Magnifier Method* was found out to be more conservative when compared with *Multiple Column Magnifier Method*, except for the columns in the first-story. SAP2000 solutions obtained from the second-order elastic analysis were parallel to the ones acquired from the approximate B_1-B_2 Method. Basically, the amplification factor decreases for upper stories.

Moreover, design of first-story columns were checked according to AISC-ASD 2005 and TS648 (1980) for completeness of the subject after the comparison of first-, and second-order results. Internal forces occurring in the beam-columns obtained from first-order elastic analysis were used in TS648 (1980) formulations, whereas second-order moments and axial forces were considered in the application of AISC 2005 methodology. Calculations related to moment interaction checks are summarized in *Tables 3.4 & 3.5*.

As a result, beam-column formulations specified in TS648 (1980) Method were found out to be conservative when compared with AISC-ASD 2005 formulations, despite not considering effects of geometric non-linearity on the frame. Since the beam-columns are lightly-loaded, strength equation governed when the stability equation is inadequate in the case of $\sigma_{eb}/\sigma_{bem} > 0.15$, as observed in the columns entitled by C2, C3, and C4. Another reason for the conservatism of TS648 (1980) formulations is the high safety factors used in the derivation of the beam-column interaction equations.

Table 3.3: Comparison of First- and Second-Order Moments for the Columns

Story	Column #	First-Order E.A. SAP2000 Moment (kN·m)	First-Order E.A. SAP2000 Axial Load (kN)	Second-Order SAP2000 Moment (kN·m)	Second-Order SAP2000 Moment Amplifier	Story Magnifier ΣH	Story Magnifier L	Story Magnifier Δ_H	Story Magnifier ΣP_{e2}	Story Magnifier ΣP_{nt}	Story Magnifier Moment (kN·m)	Story Magnifier B_2 Factor	Multiple Column K_2	Multiple Column P_{e2} (kN)	Multiple Column ΣP_{e2}	Multiple Column Moment (kN·m)	Multiple Column B_2 Factor
1st STORY	C1	517	276	592	1.146	1,092 kN	5.5 m	10.54 mm	484,355 kN	49,993 kN	619	1.198	1.70	50,750	416,746 kN	640	1.238
	C2	757	2,019	868	1.146						907		1.53	95,257		938	
	C3	709	2,051	813	1.147						849		1.53	95,257		878	
	C4	705	2,036	810	1.150						844		1.53	95,257		872	
	C5	701	2,575	807	1.150						840		1.67	79,955		868	
2nd STORY	C1	273	295	312	1.141	1,138 kN	4.0 m	7.04 mm	549,602 kN	44,437 kN	314	1.149	1.78	88,167	699,963 kN	304	1.113
	C2	536	1,784	609	1.135						616		1.52	159,804		597	
	C3	546	1,822	618	1.133						627		1.52	159,804		608	
	C4	551	1,817	625	1.133						634		1.52	159,804		614	
	C5	450	2,255	510	1.133						517		1.67	132,386		501	
3rd STORY	C1	269	299	298	1.108	1,081 kN	4.0 m	6.84 mm	537,339 kN	38,882 kN	304	1.131	1.80	86,218	664,719 kN	296	1.103
	C2	515	1,560	572	1.110						583		1.56	151,713		568	
	C3	529	1,595	586	1.107						599		1.56	151,713		584	
	C4	526	1,592	581	1.106						594		1.56	151,713		580	
	C5	425	1,942	468	1.102						480		1.73	123,362		468	

Table 3.3 (continued)

	Column #	First-Order E.A. SAP2000 Moment (kN·m)	First-Order E.A. SAP2000 Axial Load (kN)	Second-Order Elastic Analysis (AISC - ASD 2005) SAP2000 Moment (kN·m)	SAP2000 Moment Amplifier	Story Magnifier Method ΣH	L	Δ_H	ΣP_{e2}	ΣP_{nt}	Moment (kN·m)	B_2 Factor	Multiple Column Magnifier Method K_2	P_{e2} (kN)	ΣP_{e2}	Moment (kN·m)	B_2 Factor
4th STORY	C1	242	289	269	**1.111**	999 kN	4.0 m	7.19 mm	472,264 kN	33,327 kN	273	**1.127**	1.72	66,653	521,312 kN	270	**1.114**
	C2	466	1,338	516	**1.107**						525		1.53	119,333		519	
	C3	482	1,368	533	**1.105**						543		1.53	119,333		537	
	C4	475	1,366	525	**1.106**						535		1.53	119,333		529	
	C5	386	1,639	427	**1.106**						435		1.70	96,660		430	
5th STORY	C1	208	271	229	**1.102**	935 kN	4.0 m	7.00 mm	453,949 kN	27,773 kN	230	**1.109**	1.67	70,704	567,643 kN	225	**1.085**
	C2	440	1,118	483	**1.097**						488		1.47	129,273		478	
	C3	459	1,140	502	**1.095**						509		1.47	129,273		498	
	C4	455	1,139	498	**1.095**						504		1.47	129,273		493	
	C5	376	1,341	411	**1.093**						417		1.60	109,120		408	
6th STORY	C1	209	247	226	**1.085**	812 kN	4.0 m	7.19 mm	383,978 kN	22,218 kN	230	**1.102**	1.68	61,859	450,108 kN	226	**1.086**
	C2	372	897	405	**1.088**						410		1.39	102,058		404	
	C3	388	912	421	**1.086**						428		1.39	102,058		421	
	C4	383	912	416	**1.087**						422		1.39	102,058		416	
	C5	321	1,049	348	**1.085**						353		1.55	82,075		348	

Table 3.3 (continued)

Story	Column #	First-Order E.A. SAP2000 Moment (kN·m)	First-Order E.A. SAP2000 Axial Load (kN)	Second-Order Elastic Analysis (AISC - ASD 2005) SAP2000 Moment (kN·m)	SAP2000 Moment Amplifier	Story Magnifier Method ΣH	L	Δ_H	ΣP_{e2}	ΣP_{nt}	Moment (kN·m)	B_2 Factor	Multiple Column Magnifier Method K_2	P_{e2} (kN)	ΣP_{e2}	Moment (kN·m)	B_2 Factor
7th STORY	C1	183	213	196	**1.071**	667 kN	4.0 m	7.40 mm	306,459 kN	16,664 kN	200	**1.095**	1.83	52,134	359,461 kN	197	**1.080**
	C2	344	674	370	**1.074**						377		1.53	84,235		372	
	C3	356	685	382	**1.073**						390		1.53	84,235		384	
	C4	350	685	376	**1.074**						384		1.53	84,235		378	
	C5	293	767	314	**1.073**						321		1.90	54,622		316	
8th STORY	C1	119	156	129	**1.086**	491 kN	4.0 m	7.15 mm	233,483 kN	11,110 kN	129	**1.082**	2.06	36,423	280,379 kN	127	**1.068**
	C2	251	452	268	**1.070**						271		1.65	64,129		268	
	C3	268	457	286	**1.068**						290		1.65	64,129		286	
	C4	263	457	280	**1.068**						284		1.65	64,129		281	
	C5	216	499	230	**1.067**						233		1.84	51,569		230	
9th STORY	C1	109	91	110	**1.016**	269.7 kN	4.0 m	6.32 mm	145,253 kN	5,555 kN	116	**1.065**	2.18	32,524	263,158 kN	113	**1.035**
	C2	197	227	205	**1.043**						210		1.70	60,412		204	
	C3	192	228	201	**1.048**						205		1.70	60,412		199	
	C4	180	228	189	**1.052**						191		1.70	60,412		186	
	C5	145	240	153	**1.053**						154		1.88	49,398		150	

Table 3.4: Design Check for First-Story Columns according to AISC 2005 Formulations

COLUMN C1	$\frac{P_r}{P_c} = \frac{198}{8645} = 0.023 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{198}{2 \cdot 8645} + \frac{592}{1697} = 0.011 + 0.349 = \mathbf{0.360 < 1.0}$ **O.K.**
COLUMN C2	$\frac{P_r}{P_c} = \frac{2020}{11846} = 0.171 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{2020}{2 \cdot 11846} + \frac{868}{2421} = 0.085 + 0.359 = \mathbf{0.444 < 1.0}$ **O.K.**
COLUMN C3	$\frac{P_r}{P_c} = \frac{2051}{11846} = 0.173 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{2051}{2 \cdot 11846} + \frac{813}{2421} = 0.087 + 0.336 = \mathbf{0.423 < 1.0}$ **O.K.**
COLUMN C4	$\frac{P_r}{P_c} = \frac{2035}{11846} = 0.172 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{2035}{2 \cdot 11846} + \frac{810}{2421} = 0.086 + 0.335 = \mathbf{0.421 < 1.0}$ **O.K.**
COLUMN C5	$\frac{P_r}{P_c} = \frac{2630}{11846} = 0.222 > 0.2$ $\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{2630}{11846} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{807}{2421}}_{0.333} = 0.222 + 0.296 = \mathbf{0.518 < 1.0}$ **O.K.**

Table 3.5: Design Check for First-Story Columns according to TS648 (1980) Formulations

COLUMN C1	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{3.93}{112.7} = 0.035 < 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} = \frac{3.93}{112.7} + \frac{51.9}{141} = 0.035 + 0.369 = \mathbf{0.404 < 1.0\ O.K.}$
COLUMN C2	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{21.3}{115.3} = 0.185 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{21.3}{115.3} + \underbrace{\frac{55.1}{141}}_{0.391} \cdot \underbrace{\frac{0.85}{1 - 21.3/486}}_{0.889} = 0.185 + 0.348 = 0.533 < 1.0\ \text{O.K.}$ $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{21.3}{141} + \frac{55.1}{141} = 0.151 + 0.391 = \mathbf{0.542 < 1.0\ O.K.}$
COLUMN C3	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{21.6}{115.3} = 0.187 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{21.6}{115.3} + \underbrace{\frac{51.6}{141}}_{0.366} \cdot \underbrace{\frac{0.85}{1 - 21.6/486}}_{0.890} = 0.187 + 0.326 = 0.513 < 1.0\ \text{O.K.}$ $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{21.6}{141} + \frac{51.6}{141} = 0.153 + 0.366 = \mathbf{0.519 < 1.0\ O.K.}$
COLUMN C4	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{21.5}{115.3} = 0.186 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{21.5}{115.3} + \underbrace{\frac{51.4}{141}}_{0.365} \cdot \underbrace{\frac{0.85}{1 - 21.5/486}}_{0.889} = 0.186 + 0.325 = 0.511 < 1.0\ \text{O.K.}$ $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{21.5}{141} + \frac{51.4}{141} = 0.152 + 0.365 = \mathbf{0.517 < 1.0\ O.K.}$
COLUMN C5	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{27.2}{115.3} = 0.236 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{27.2}{115.3} + \underbrace{\frac{51.1}{141}}_{0.362} \cdot \underbrace{\frac{0.85}{1 - 27.2/330}}_{0.926} = 0.236 + 0.335 = \mathbf{0.571 < 1.0\ O.K.}$ $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{27.2}{141} + \frac{51.1}{141} = 0.193 + 0.362 = 0.555 < 1.0\ \text{O.K.}$

3.2 Regular Framing

3.2.1 Example 1

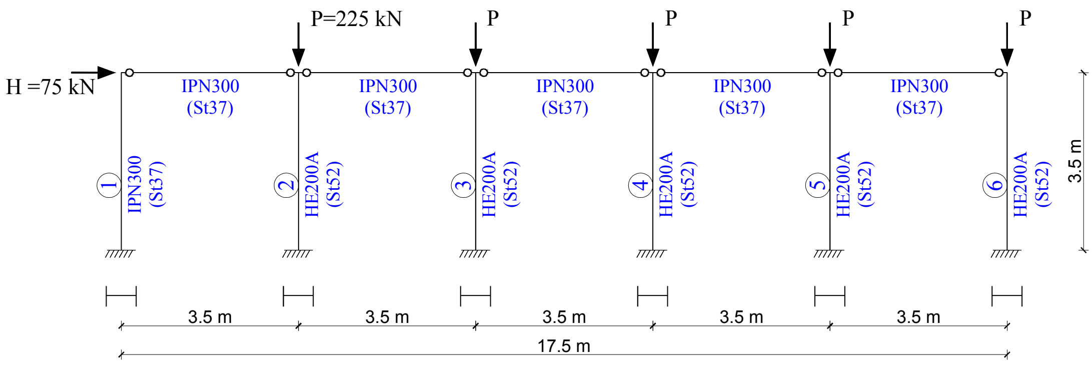


Figure 3.5: Example 1 for Regular Framing

Second-order effects were investigated on the one-story five-bay frame given above. Column nomenclature is specified on the figure. The columns are fixed-supported at the base in the plane-of-bending, while the columns are braced out-of-plane, i.e. effective length factor is K=1.0.

In the first step, second-order moments and axial forces were taken directly from the analysis performed with SAP2000 including the geometrical non-linearity effects on the system. Then, first-order moments and axial forces were amplified with B_1 and B_2 factors to consider second-order effects according to AISC-ASD 2005. Output data obtained from the successive structural analyses was summarized in *Tables 3.6 & 3.7*. Finally, the design checks were carried out to examine the level of applicability according to TS648 (1980) and AISC-ASD 2005 methodology.

First-order moments were amplified 14% in the beam-columns, referred as (2) to (6) according to direct second-order analysis performed by SAP2000. On the other hand, 18% amplification was reported for the member labeled as (1). Despite not subjecting any axial thrust, first-order moment is amplified because of the additional lateral drift caused by P-Δ effect.

Table 3.6: SAP2000 Analysis Results for Example 1 of Regular Framing

SAP2000 ANALYSIS RESULTS						
Column #	Section	FIRST-ORDER E.A.		SECOND-ORDER E.A.		
		$P_{first\text{-}order}$ (kN)	$M_{first\text{-}order}$ (kN·m)	P_r (kN)	M_r (kN·m)	$A_{F,moment}$
(1)	IPN300	0	91.9	0	**108.6**	*1.182*
(2)	HE200A	225	34.4	225	**39.1**	*1.136*
(3)	HE200A	225	34.3	225	**38.9**	*1.134*
(4)	HE200A	225	34.1	225	**38.8**	*1.138*
(5)	HE200A	225	34.0	225	**38.7**	*1.139*
(6)	HE200A	225	34.0	225	**38.7**	*1.139*

Table 3.7: Summary of Second-Order Analysis by Amplified First-Order Elastic Analysis Defined in AISC 2005

SECOND-ORDER ANALYSIS BY AMPLIFIED FIRST-ORDER ELASTIC ANALYSIS													
Column #	MULTIPLE COLUMN MAGNIFIER METHOD							STORY MAGNIFIER METHOD					
	ΣP_{nt} (kN)	P_{e2} (kN)	ΣP_{e2} (kN)	B_2	B_1	M_r (kN·m)	$A_{F,moment}$	ΣH (kN)	Δ_H (mm)	ΣP_{e2} (kN)	B_2	M_r (kN·m)	$A_{F,moment}$
(1)	1125	3948	11,380	1.19	1.0	**109.1**	*1.188*	75.01	19.14	11,659	1.18	**108.7**	*1.183*
(2)		1486			1.0	**40.9**	*1.188*					**40.7**	*1.183*
(3)		1486			1.0	**40.8**	*1.188*					**40.6**	*1.183*
(4)		1486			1.0	**40.5**	*1.188*					**40.3**	*1.183*
(5)		1486			1.0	**40.4**	*1.188*					**40.2**	*1.183*
(6)		1486			1.0	**40.3**	*1.188*					**40.1**	*1.183*

Table 3.8: Design Check for Columns according to AISC 2005 Formulations

COLUMN (1)

* Member behaves like a beam.

* Lateral torsional buckling is not a limit state. $M_r = 108.6$ kN·m

$M_n = M_p = 179.1$ kN·m $\quad M_c = M_n / \Omega_b = 179.1 / 1.67 = 107.2$ kN·m

$M_r / M_c = 108.6 / 107.25 =$ **1.013 > 1.0 ACCEPTABLE**

COLUMN (2)

$$\frac{P_r}{P_c} = \frac{225}{663} = 0.339 > 0.2$$

$$\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{225}{663} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{39.1}{89.9}}_{0.435} = 0.339 + 0.387 = \mathbf{0.726} < \mathbf{1.0}\ \text{O.K.}$$

Table 3.9: Design Check for Columns according to TS648 (1980) Formulations

COLUMN (1)	* Member behaves like a beam. * Lateral torsional buckling is not a limit state. $\sigma_B = 141$ N/mm^2 $\sigma_b = \frac{M_{first\text{-}order}}{S_{elastic}} = \frac{9.19E+07}{6.53E+05} = 140.7$ N/mm^2 $\sigma_b \;/\; \sigma_B = 140.7 \;/\; 141 = \mathbf{0.998} \; \mathbf{< 1.0}$ **O.K.**
COLUMN (2)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{41.8}{101.6} = 0.411 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{41.8}{101.6} + \underbrace{\frac{88.4}{210}}_{0.421} \cdot \underbrace{\frac{0.85}{1 - 41.8/110.5}}_{1.367} = 0.411 + 0.576 = \mathbf{0.987} \; \mathbf{< 1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{41.8}{210} + \frac{88.4}{210} = 0.199 + 0.421 = 0.620 < 1.0$ O.K.

As well, approximate amplified first-order elastic analysis defined in AISC 2005, in other words B_1-B_2 Method, was found out to be conservative when compared with SAP2000 results.

Then, demand/capacity ratios were checked according to AISC-ASD 2005 and TS648 (1980), and the calculations are presented in *Tables 3.8 & 3.9*. Basically, TS648 (1980) does not take into account the increase in moment on column (1). However, second-order moment was considered according to AISC 2005 approach. On the other hand, design of column (1) is acceptable for AISC 2005 similarly as TS648 (1980) formulations, since AISC 2005 uses a lower factor of safety. If it uses the same level of safety as TS648 (1980), then column (1) would be overstressed.

Finally, TS648 (1980) beam-column methodology was reported as being over-conservative in the design of column (2) when compared with AISC 2005 approach.

3.2.2 Example 2

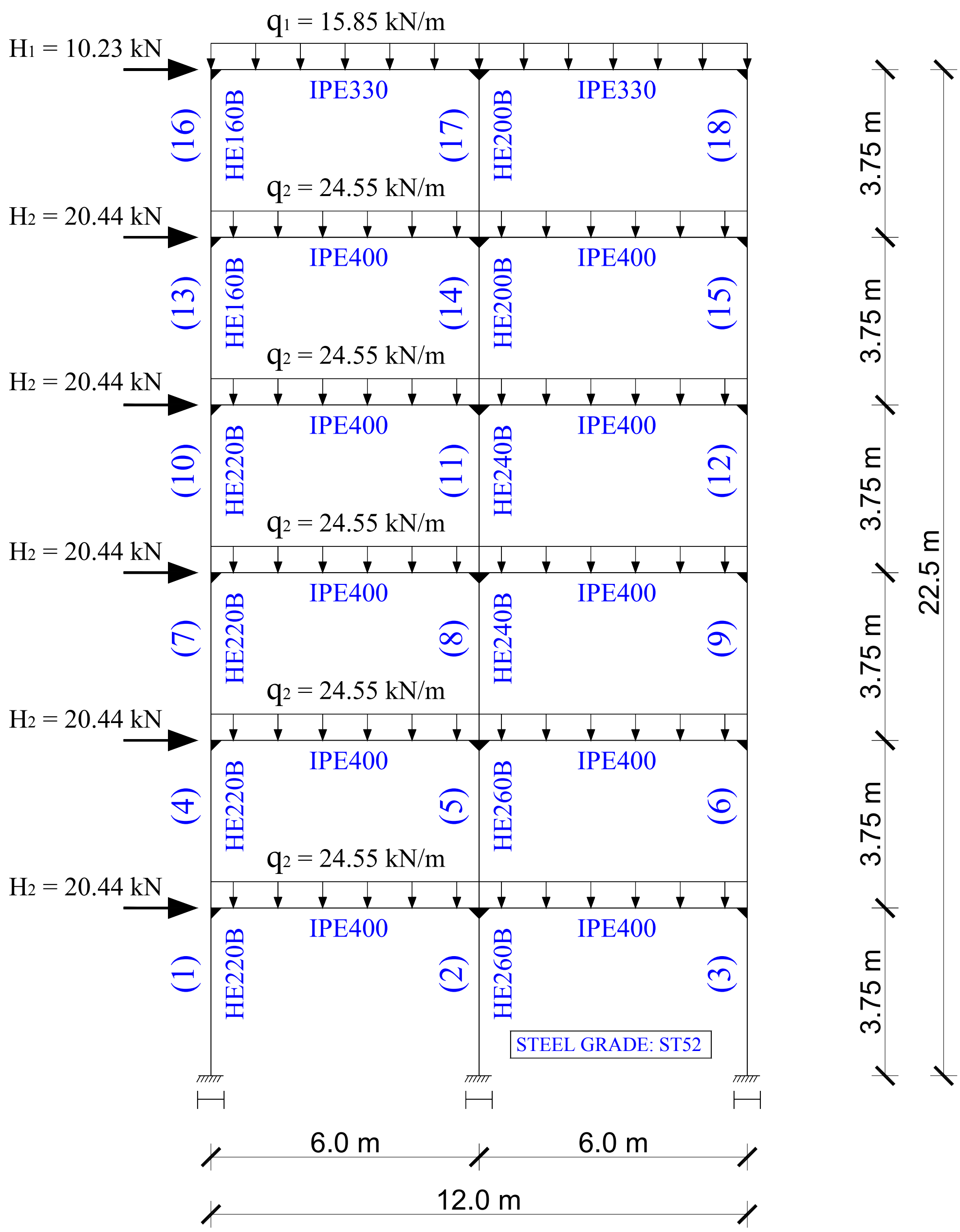


Figure 3.6: Example 2 for Regular Frame Systems

Second-order effects were investigated on the six-story two-bay frame shown in *Figure 3.6*. All frame sections are made of St52 grade steel. The supports under the columns are fixed-type in the plane-of-bending, while the columns are braced out-of-plane, i.e. effective length factor is K=1.0. Also, column nomenclature is specified on the figure given above.

Several structural analyses were carried out by using the software, SAP2000, which are summarized in *Table 3.10*. First-order elastic analysis results were used in the design checks according to TS648 (1980). Basically, all vertical and horizontal loads were applied on the structural model, and then first-order elastic analysis was performed with combination of all available loads. Also, additional linear elastic analyses were carried out in two steps which were used in B_1-B_2 Amplification Method to perform an approximate second-order internal forces. Finally, a direct second-order analysis was carried out by considering P-delta effects, which was used in AISC 2005 formulations, also.

First-order moments were amplified up to 1.3 times according to second-order analysis performed by SAP2000. It should be noted that excessive deviations reported for columns (10) and (13) are not representative for the evaluation, since the moment values are small, and slight differences cause extreme amplification, or deamplification ratios. This phenomenon can be observed in *Table 3.10*. In general, results obtained from the approximate B_1-B_2 Amplification Method were in well agreement with the ones obtained from direct second-order analysis. On the other hand, *Story Magnifier Method* was found out to be conservative relative to *Multiple Column Magnifier Method*.

In addition, data obtained from successive analyses was taken into account in design checks according to AISC-ASD 2005 and TS648 (1980). The applied procedures were given in *Tables 3.11 & 3.12*. Since TS648 (1980) formulations are based on the amplification of first-order moments roughly without considering sway and no sway cases separately as in AISC 2005, it was reported to be over-conservative in some cases such as in column (2). The member design was satisfactory according to AISC 2005 with a demand/capacity ratio of 0.881, whereas it was overstressed with a PM ratio of 1.046 along with TS648 (1980). Also, high safety factors used in the derivation of TS648 (1980) formulations are responsible on the over-conservatism of the results.

Table 3.10: SAP2000 Analysis Results for Example 2 of Regular Framing

Story	Column #	FIRST-ORDER E.A. $P_{first-order}$ (kN)	FIRST-ORDER E.A. $M_{first-order}$ (kN·m)	SECOND-ORDER ELASTIC ANALYSIS P_r (kN)	SECOND-ORDER ELASTIC ANALYSIS M_r (kN·m)	$A_{F,axial}$	$A_{F,moment}$	FIRST-ORDER E.A. FOR B_1-B_2 METHOD P_{nt} (kN)	ΣP_{nt} (kN)	P_{lt} (kN)	$M_{nt,bottom}$ (kN·m)	$M_{nt,top}$ (kN·m)	$M_{lt,bottom}$ (kN·m)	$M_{lt,top}$ (kN·m)
1st Story	(1)	285	55.3	**277**	**60.7**	*0.971*	*1.097*	379.4	1663	-94.5	-8.1	18.0	63.5	-45.4
	(2)	904	116.5	**905**	**125.7**	*1.000*	*1.079*	904.2		0.0	0.5	-0.4	116.0	-85.5
	(3)	474	72.5	**482**	**77.7**	*1.016*	*1.072*	379.7		94.5	8.6	-18.4	63.9	-45.7
2nd Story	(4)	247	18.0	**241**	**22.5**	*0.978*	*1.249*	312.5	1369	-65.4	-26.3	25.6	42.3	-43.6
	(5)	743	88.7	**744**	**97.5**	*1.000*	*1.099*	743.4		0.0	0.1	-0.2	84.3	-88.5
	(6)	378	69.3	**383**	**73.7**	*1.013*	*1.062*	312.7		65.7	26.5	-25.8	42.2	-43.5
3rd Story	(7)	203	12.0	**200**	**15.5**	*0.986*	*1.286*	243.7	1074	-40.4	-25.5	25.7	34.8	-37.8
	(8)	587	64.7	**587**	**70.2**	*1.000*	*1.085*	586.5		0.0	-0.1	0.1	59.0	-64.8
	(9)	284	63.3	**287**	**66.6**	*1.009*	*1.052*	243.8		40.4	25.4	-25.5	34.8	-37.8
4h Story	(10)	152	5.5	**151**	**4.1**	*0.991*	*0.750*	173.7	779	-21.3	-28.5	32.0	23.0	-27.7
	(11)	432	46.4	**432**	**49.4**	*1.000*	*1.064*	431.9		0.0	0.3	-0.3	43.0	-46.2
	(12)	195	60.1	**196**	**61.8**	*1.006*	*1.028*	173.8		21.3	28.8	-32.4	22.9	-27.7
5th Story	(13)	98	0.2	**97**	**1.0**	*0.995*	*6.583*	105.8	485	-8.2	-12.9	14.4	12.8	-14.3
	(14)	273	31.5	**273**	**33.7**	*1.000*	*1.067*	273.2		0.0	-0.2	0.2	29.8	-31.7
	(15)	114	28.5	**114**	**29.4**	*1.003*	*1.029*	105.8		8.2	12.8	-14.2	12.8	-14.3
6th Story	(16)	39	13.2	**39**	**13.0**	*0.998*	*0.985*	41.1	190	-1.7	-16.8	18.1	4.5	-5.0
	(17)	108	10.4	**108**	**10.8**	*1.000*	*1.034*	108.0		0.0	0.1	-0.1	8.8	-10.3
	(18)	43	23.2	**43**	**23.3**	*1.001*	*1.006*	41.1		1.7	16.8	-18.2	4.5	-5.0

SAP2000 ANALYSIS RESULTS

Table 3.11: Summary of Second-Order Analysis by Amplified First-Order Elastic Analysis Defined in AISC 2005

		SECOND-ORDER ANALYSIS BY AMPLIFIED FIRST-ORDER ELASTIC ANALYSIS (B_1-B_2 METHOD)															
		MULTIPLE COLUMN MAGNIFIER METHOD							**STORY MAGNIFIER METHOD**								
	Column #	ΣP_{e2} (kN)	B_2	P_r (kN)	$A_{F,axial}$	B_1	M_r (kN·m)	$A_{F,moment}$	ΣH (kN)	Δ_H (mm)	ΣP_{e2} (kN)	B_2	P_r (kN)	$A_{F,axial}$	B_1	M_r (kN·m)	$A_{F,moment}$
	(1)			**273**	*0.960*	1.0	**63.0**	*1.138*					**275**	*0.966*	1.0	**61.9**	*1.119*
1st Story	**(2)**	24,727	1.121	**904**	*1.000*	1.0	**130.4**	*1.120*	112.0	12.56	28,413	1.103	**904**	*1.000*	1.0	**128.4**	*1.103*
	(3)			**486**	*1.024*	1.0	**80.2**	*1.106*					**484**	*1.021*	1.0	**79.1**	*1.091*
	(4)			**241**	*0.976*	1.0	**22.3**	*1.236*					**239**	*0.970*	1.0	**23.3**	*1.291*
2nd Story	**(5)**	24,666	1.097	**743**	*1.000*	1.0	**97.3**	*1.097*	91.8	14.33	20,426	1.120	**743**	*1.000*	1.0	**99.3**	*1.120*
	(6)			**385**	*1.017*	1.0	**73.6**	*1.061*					**386**	*1.021*	1.0	**74.5**	*1.075*
	(7)			**200**	*0.983*	1.0	**15.2**	*1.263*					**199**	*0.980*	1.0	**15.9**	*1.322*
3rd Story	**(8)**	22,224	1.084	**587**	*1.000*	1.0	**70.1**	*1.084*	71.7	12.36	18,493	1.102	**587**	*1.000*	1.0	**71.3**	*1.103*
	(9)			**288**	*1.012*	1.0	**66.5**	*1.050*					**288**	*1.015*	1.0	**67.2**	*1.061*
	(10)			**174**	*1.140*	1.0	**4.3**	*0.771*					**151**	*0.990*	1.0	**3.8**	*0.689*
4h Story	**(11)**	24,165	1.054	**432**	*1.000*	1.0	**49.0**	*1.054*	50.8	8.96	18,054	1.074	**432**	*1.000*	1.0	**49.9**	*1.074*
	(12)			**196**	*1.006*	1.0	**61.6**	*1.025*					**197**	*1.008*	1.0	**62.1**	*1.034*
	(13)			**97**	*0.994*	1.0	**1.0**	*6.838*					**97**	*0.993*	1.0	**1.1**	*7.352*
5th Story	**(14)**	10,999	1.076	**273**	*1.000*	1.0	**34.0**	*1.077*	30.9	9.53	10,322	1.081	**273**	*1.000*	1.0	**34.1**	*1.082*
	(15)			**115**	*1.005*	1.0	**29.6**	*1.038*					**115**	*1.006*	1.0	**29.7**	*1.041*
	(16)			**39**	*0.999*	1.0	**13.0**	*0.990*					**39**	*0.998*	1.0	**13.0**	*0.986*
6th Story	**(17)**	11,920	1.026	**108**	*1.000*	1.0	**10.7**	*1.027*	10.1	3.78	8,542	1.037	**108**	*1.000*	1.0	**10.8**	*1.038*
	(18)			**43**	*1.001*	1.0	**23.3**	*1.006*					**43**	*1.001*	1.0	**23.4**	*1.008*

Table 3.12: Design Check for Columns according to AISC 2005 Formulations

Story	Column	Design check
1st STORY	COLUMN (1)	$\frac{P_r}{P_c}=\frac{277}{1366}=0.203\ >0.2$ $\frac{P_r}{P_c}+\frac{8}{9}\cdot\frac{M_r}{M_c}=\frac{277}{1366}+\underbrace{\frac{8}{9}}_{0.889}\cdot\underbrace{\frac{60.7}{173.3}}_{0.350}=0.203+0.311=\mathbf{0.514<1.0\ O.K.}$
	COLUMN (2)	$\frac{P_r}{P_c}=\frac{905}{1943}=0.466\ >0.2$ $\frac{P_r}{P_c}+\frac{8}{9}\cdot\frac{M_r}{M_c}=\frac{905}{1943}+\underbrace{\frac{8}{9}}_{0.889}\cdot\underbrace{\frac{125.7}{268.9}}_{0.467}=0.466+0.416=\mathbf{0.881<1.0\ O.K.}$
	COLUMN (3)	$\frac{P_r}{P_c}=\frac{482}{1366}=0.353\ >0.2$ $\frac{P_r}{P_c}+\frac{8}{9}\cdot\frac{M_r}{M_c}=\frac{482}{1366}+\underbrace{\frac{8}{9}}_{0.889}\cdot\underbrace{\frac{77.7}{173.3}}_{0.448}=0.353+0.399=\mathbf{0.751<1.0\ O.K.}$
2nd STORY	COLUMN (4)	$\frac{P_r}{P_c}=\frac{241}{1366}=0.176\ <0.2$ $\frac{P_r}{2P_c}+\frac{M_r}{M_c}=\frac{241}{2\cdot1366}+\frac{22.5}{173.3}=0.088+0.130=\mathbf{0.218<1.0\ O.K.}$
	COLUMN (5)	$\frac{P_r}{P_c}=\frac{744}{1943}=0.383\ >0.2$ $\frac{P_r}{P_c}+\frac{8}{9}\cdot\frac{M_r}{M_c}=\frac{744}{1943}+\underbrace{\frac{8}{9}}_{0.889}\cdot\underbrace{\frac{97.5}{268.9}}_{0.363}=0.383+0.322=\mathbf{0.705<1.0\ O.K.}$
	COLUMN (6)	$\frac{P_r}{P_c}=\frac{383}{1366}=0.280\ >0.2$ $\frac{P_r}{P_c}+\frac{8}{9}\cdot\frac{M_r}{M_c}=\frac{383}{1366}+\underbrace{\frac{8}{9}}_{0.889}\cdot\underbrace{\frac{73.7}{173.3}}_{0.425}=0.280+0.378=\mathbf{0.658<1.0\ O.K.}$
3rd STORY	COLUMN (7)	$\frac{P_r}{P_c}=\frac{200}{1366}=0.146\ <0.2$ $\frac{P_r}{2P_c}+\frac{M_r}{M_c}=\frac{200}{2\cdot1366}+\frac{15.5}{173.3}=0.073+0.089=\mathbf{0.163<1.0\ O.K.}$
	COLUMN (8)	$\frac{P_r}{P_c}=\frac{587}{1675}=0.350\ >0.2$ $\frac{P_r}{P_c}+\frac{8}{9}\cdot\frac{M_r}{M_c}=\frac{587}{1675}+\underbrace{\frac{8}{9}}_{0.889}\cdot\underbrace{\frac{70.2}{220.7}}_{0.318}=0.350+0.283=\mathbf{0.633<1.0\ O.K.}$
	COLUMN (9)	$\frac{P_r}{P_c}=\frac{287}{1366}=0.210\ >0.2$ $\frac{P_r}{P_c}+\frac{8}{9}\cdot\frac{M_r}{M_c}=\frac{287}{1366}+\underbrace{\frac{8}{9}}_{0.889}\cdot\underbrace{\frac{66.6}{173.3}}_{0.384}=0.210+0.342=\mathbf{0.552<1.0\ O.K.}$

Table 3.12 (continued)

4th STORY	COLUMN (10)	$\frac{P_r}{P_c} = \frac{151}{1366} = 0.111 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{151}{2 \cdot 1366} + \frac{4.1}{173.3} = 0.055 + 0.024 = \mathbf{0.079} < \mathbf{1.0}$ **O.K.**
	COLUMN (11)	$\frac{P_r}{P_c} = \frac{432}{1675} = 0.258 > 0.2$ $\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{432}{1675} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{49.4}{220.7}}_{0.224} = 0.258 + 0.199 = \mathbf{0.457} < \mathbf{1.0}$ **O.K.**
	COLUMN (12)	$\frac{P_r}{P_c} = \frac{196}{1366} = 0.143 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{196}{2 \cdot 1366} + \frac{61.8}{173.3} = 0.072 + 0.357 = \mathbf{0.428} < \mathbf{1.0}$ **O.K.**
5th STORY	COLUMN (13)	$\frac{P_r}{P_c} = \frac{97}{602} = 0.161 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{97}{2 \cdot 602} + \frac{1.0}{74.2} = 0.081 + 0.013 = \mathbf{0.094} < \mathbf{1.0}$ **O.K.**
	COLUMN (14)	$\frac{P_r}{P_c} = \frac{273}{1091} = 0.250 > 0.2$ $\frac{P_r}{P_c} + \frac{8}{9} \cdot \frac{M_r}{M_c} = \frac{273}{1091} + \underbrace{\frac{8}{9}}_{0.889} \cdot \underbrace{\frac{33.7}{134.8}}_{0.250} = 0.250 + 0.222 = \mathbf{0.472} < \mathbf{1.0}$ **O.K.**
	COLUMN (15)	$\frac{P_r}{P_c} = \frac{114}{602} = 0.189 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{114}{2 \cdot 602} + \frac{29.4}{74.2} = 0.095 + 0.396 = \mathbf{0.491} < \mathbf{1.0}$ **O.K.**
6th STORY	COLUMN (16)	$\frac{P_r}{P_c} = \frac{39}{602} = 0.065 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{39}{2 \cdot 602} + \frac{13.0}{74.2} = 0.032 + 0.175 = \mathbf{0.208} < \mathbf{1.0}$ **O.K.**
	COLUMN (17)	$\frac{P_r}{P_c} = \frac{108}{1091} = 0.099 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{108}{2 \cdot 1091} + \frac{10.8}{134.8} = 0.049 + 0.080 = \mathbf{0.130} < \mathbf{1.0}$ **O.K.**
	COLUMN (18)	$\frac{P_r}{P_c} = \frac{43}{602} = 0.071 < 0.2$ $\frac{P_r}{2P_c} + \frac{M_r}{M_c} = \frac{43}{2 \cdot 602} + \frac{23.3}{74.2} = 0.036 + 0.314 = \mathbf{0.350} < \mathbf{1.0}$ **O.K.**

Table 3.13: Design Check for Columns according to TS648 (1980) Formulations

1st STORY	COLUMN (1)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{31.3}{126.9} = 0.247 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{31.3}{126.9} + \underbrace{\frac{75.2}{210}}_{0.358} \cdot \underbrace{\frac{0.85}{1 - 31.3/280.1}}_{0.957} = 0.247 + 0.343 = \mathbf{0.589} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{31.3}{210} + \frac{75.2}{210} = 0.149 + 0.358 = 0.507 < 1.0$ O.K.
	COLUMN (2)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{76.6}{141.8} = 0.540 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{76.6}{141.8} + \underbrace{\frac{101.3}{210}}_{0.482} \cdot \underbrace{\frac{0.85}{1 - 76.6/403.0}}_{1.049} = 0.540 + 0.506 = \mathbf{1.046} > \mathbf{1.0}$ **OVERSTRESSED** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{76.6}{210} + \frac{101.3}{210} = 0.365 + 0.482 = 0.847 < 1.0$ O.K.
	COLUMN (3)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{52.1}{126.9} = 0.411 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{52.1}{126.9} + \underbrace{\frac{98.5}{210}}_{0.469} \cdot \underbrace{\frac{0.85}{1 - 52.1/321.6}}_{1.044} = 0.411 + 0.490 = \mathbf{0.900} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{52.1}{210} + \frac{98.5}{210} = 0.248 + 0.469 = 0.717 < 1.0$ O.K.
2nd STORY	COLUMN (4)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{27.1}{126.9} = 0.214 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{27.1}{126.9} + \underbrace{\frac{24.5}{210}}_{0.117} \cdot \underbrace{\frac{0.85}{1 - 27.1/273.1}}_{0.944} = 0.214 + 0.110 = \mathbf{0.324} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{27.1}{210} + \frac{24.5}{210} = 0.129 + 0.117 = 0.246 < 1.0$ O.K.
	COLUMN (5)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{63.0}{141.8} = 0.444 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{63.0}{141.8} + \underbrace{\frac{77.1}{210}}_{0.367} \cdot \underbrace{\frac{0.85}{1 - 63.0/411.7}}_{1.004} = 0.444 + 0.368 = \mathbf{0.813} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{63.0}{210} + \frac{77.1}{210} = 0.300 + 0.367 = 0.667 < 1.0$ O.K.
	COLUMN (6)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{41.6}{126.9} = 0.328 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{41.6}{126.9} + \underbrace{\frac{94.2}{210}}_{0.449} \cdot \underbrace{\frac{0.85}{1 - 41.6/273.1}}_{1.003} = 0.328 + 0.450 = \mathbf{0.778} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{41.6}{210} + \frac{94.2}{210} = 0.198 + 0.449 = 0.647 < 1.0$ O.K.

Table 3.13 (continued)

3^{rd} STORY	COLUMN (7)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{22.3}{126.9} = 0.176 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{22.3}{126.9} + \underbrace{\frac{16.3}{210}}_{0.078} \cdot \underbrace{\frac{0.85}{1 - 22.3/273.1}}_{0.926} = 0.176 + 0.072 = \mathbf{0.248} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{22.3}{210} + \frac{16.3}{210} = 0.106 + 0.078 = 0.184 < 1.0$ O.K.
	COLUMN (8)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{55.3}{134.9} = 0.410 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{55.3}{134.9} + \underbrace{\frac{68.9}{210}}_{0.328} \cdot \underbrace{\frac{0.85}{1 - 55.3/369.3}}_{1.000} = 0.410 + 0.328 = \mathbf{0.738} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{55.3}{210} + \frac{68.9}{210} = 0.263 + 0.328 = 0.591 < 1.0$ O.K.
	COLUMN (9)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{31.2}{126.9} = 0.246 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{31.2}{126.9} + \underbrace{\frac{86.0}{210}}_{0.410} \cdot \underbrace{\frac{0.85}{1 - 31.2/273.1}}_{0.960} = 0.246 + 0.393 = \mathbf{0.639} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{31.2}{210} + \frac{86}{210} = 0.149 + 0.410 = 0.558 < 1.0$ O.K.
4^{th} STORY	COLUMN (10)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{16.8}{126.9} = 0.132 < 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} = \frac{16.8}{126.9} + \frac{7.5}{210} = 0.132 + 0.036 = \mathbf{0.168} < \mathbf{1.0}$ **O.K.**
	COLUMN (11)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{40.7}{134.9} = 0.302 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{40.7}{134.9} + \underbrace{\frac{49.5}{210}}_{0.236} \cdot \underbrace{\frac{0.85}{1 - 40.7/399.5}}_{0.946} = 0.302 + 0.223 = \mathbf{0.525} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{40.7}{210} + \frac{49.5}{210} = 0.194 + 0.236 = 0.430 < 1.0$ O.K.
	COLUMN (12)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{21.4}{126.9} = 0.169 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{21.4}{126.9} + \underbrace{\frac{81.6}{210}}_{0.389} \cdot \underbrace{\frac{0.85}{1 - 21.4/298.2}}_{0.916} = 0.169 + 0.356 = \mathbf{0.524} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{21.4}{210} + \frac{81.6}{210} = 0.102 + 0.389 = 0.490 < 1.0$ O.K.

Table 3.13 (continued)

5th STORY	COLUMN (13)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{18.0}{89.9} = 0.200 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{18.0}{89.9} + \underbrace{\frac{0.5}{210}}_{0.002} \cdot \underbrace{\frac{0.85}{1 - 18.0/186.9}}_{0.941} = 0.200 + 0.002 = \mathbf{0.202} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{18}{210} + \frac{0.5}{210} = 0.086 + 0.002 = 0.088 < 1.0$ O.K.
	COLUMN (14)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{35.0}{116.9} = 0.299 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{35.0}{116.9} + \underbrace{\frac{55.3}{210}}_{0.263} \cdot \underbrace{\frac{0.85}{1 - 35.0/303.8}}_{0.961} = 0.299 + 0.253 = \mathbf{0.552} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{35.0}{210} + \frac{55.3}{210} = 0.167 + 0.263 = 0.430 < 1.0$ O.K.
	COLUMN (15)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{21.0}{89.9} = 0.234 > 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} \cdot \frac{C_m}{1-\sigma_{eb}/\sigma_e'} = \frac{21.0}{89.9} + \underbrace{\frac{91.7}{210}}_{0.437} \cdot \underbrace{\frac{0.85}{1 - 21.0/186.9}}_{0.958} = 0.234 + 0.418 = \mathbf{0.652} < \mathbf{1.0}$ **O.K.** $\frac{\sigma_{eb}}{0.6 \cdot \sigma_a} + \frac{\sigma_b}{\sigma_B} = \frac{21.0}{210} + \frac{91.7}{210} = 0.100 + 0.437 = 0.537 < 1.0$ O.K.
6th STORY	COLUMN (16)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{7.3}{89.9} = 0.081 < 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} = \frac{7.3}{89.9} + \frac{42.3}{210} = 0.081 + 0.201 = \mathbf{0.283} < \mathbf{1.0}$ **O.K.**
	COLUMN (17)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{13.8}{116.9} = 0.118 < 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} = \frac{13.8}{116.9} + \frac{18.3}{210} = 0.118 + 0.087 = \mathbf{0.205} < \mathbf{1.0}$ **O.K.**
	COLUMN (18)	$\frac{\sigma_{eb}}{\sigma_{bem}} = \frac{7.9}{89.9} = 0.088 < 0.15$ $\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_b}{\sigma_B} = \frac{7.9}{89.9} + \frac{74.6}{210} = 0.088 + 0.355 = \mathbf{0.443} < \mathbf{1.0}$ **O.K.**

CHAPTER 4

CONCLUSIONS

- In the current version of AISC Specification (2005), usage of *Equation (1.8)* to calculate the ψ term is limited for only simply supported members. However, the same formula was erroneously used for fixed-end support condition in AISC Specifications until the edition published in 1978. The table ignoring the amplification of the first-order elastic moments at the fixed-ends was published in AISC Manual (1969) which is also given in *Table 1.2*. The same error occurred in the current Turkish Standard, TS648 (1980) that shares the same philosophy of design with AISC Manual (1969). Therefore, C_m values in TS648 (1980) should be revised.

- C_m coefficient is called *equivalent moment factor* for beam-columns subjected to end moments without any transverse loading on the span, and the formulation specified in *Eq. (1.18)* is used for this case in TS648 (1980). The lower limit of 0.4 should be removed from the equation, since it was found out to be over-conservative and eliminated not only in AISC-LRFD Manuals, but also AISC-ASD Specifications published in 1978 & 1989.

- There is no limitation for the applicability of ψ formulation given in *Eq. (1.8)* in TS648 (1980), whereas usage of the equation is limited for only simply supported members according to AISC 2005. So, it is applicable to all braced members in TS648 (1980). However, inappropriate application of ψ formulation may cause deviation from the exact result, even in the unconservative side as studied in *Chapter 2*. This inconsistency should be restored by defining the ψ term clearly in TS648 (1980). In addition, Chen & Lui (1991) state that definition for ψ in *Eq. (1.8)* is applicable only for cases in which the maximum primary moment occurs at or near mid-span, and this expression was supported the problem investigated in *Section 2.2.1*. Even limiting the usage of ψ formulation on simply-supported members as in AISC 2005 is not sufficient. A definition such as presented by Chen & Lui (1991) should be placed in AISC 2005 to prevent errors when the maximum moment is not at the mid-span.

- The beam-column interaction formulation specified in TS648 (1980) as *Eq. (1.13)*, i.e. the stability equation, should be revised in order not to underestimate second-order effects on the bending part of the stability equation. P-delta effects are taken into account by using the amplification factor given in *Eq. (1.15)* in TS648 (1980) approach, which is a rough estimate of possible second-order effects obtained by the magnification of first-order elastic analysis results. As a result, unconservative results were reported in *Sections 2.4 & 3.1* for TS648 (1980) formulations even high safety factors are included. So, the amplification factor should be separated from the stability equation *(Eq. 1.13)* as shown in *Eq. (1.15)*, and a lower limit of unity should be specified to prevent the underestimation of second-order effects, even with a value below the first-order moments as exemplified in *Sections 2.4*.

- On the other hand, high safety factors may cause over-conservatism in regular framing as studied in *Section 3.2*, which may not be feasible economically since the application of structural steel structures in Turkish market is expensive. The high safety factors may be decreased by applying refined analysis techniques, which are not a big deal with the development of computer technology up to 2009.

- TS648 (1980) methodology for considering second-order effects was reported as being over-conservative for the problem specified in *Section 3.2.2,* since the moments caused by gravity loading were amplified with the side-sway amplification factor, $0.85/(1-\sigma_{eb}/\sigma_e')$, unnecessarily. An approximate method considering the P-δ and P-Δ effects separately, as in B_1-B_2 Method defined in AISC 2005, or a direct second-order analysis using structural software may be reasonable for this purpose.

- Since beam-column interaction formulations in TS648 (1980) are based on amplification of the moments obtained from first-order elastic analysis with a rough magnification factor *(Eq. 1.15)*, usage of member forces acquired from a second-order elastic analysis does not seem applicable, since the first-order effects would be amplified twice. On the other hand, exact structural behavior of a frame system may be different from the classical linear elastic analysis when a second-order elastic analysis is performed. This phenomenon was illustrated in the problem considered in *Section 3.1.1.* Column C1 was subjected to axial compressive force as a result of first-order elastic analysis, whereas it is under tension when second-order effects were taken into account. Also, the change in axial member forces is considered in B_1-B_2 Method specified in AISC 2005, which is disregarded in TS648 (1980) approach.

- Lean-on frame systems should be defined in TS648 (1980) such as specified in AISC 2005, since a second-order analysis is required for proper evaluation of such kind of systems, as illustrated in *Section 3.1*.

- AISC 2005 approach for considering the second-order effects is contemporary; because of the development in the computer technology was not neglected. AISC 2005 gives explicit permission to the designer for the direct second-order analysis. On the other hand, for conventional structure type buildings, an approximate method, B_1-B_2 Method, is still recommended. When B_1-B_2 Method is used, two first-order elastic analyses are performed, and by manipulating the first-order effects, second –order internal forces and displacements can be determined. Also, as a result derived from the analyses done in the paper, generally conservative and reasonable results are obtained by the application of B_1-B_2 Method, since the method was updated in AISC Specifications reviewed periodically. The drawbacks and limitations of B_1-B_2 Method are clearly stated in AISC 2005, and alternative methods are proposed.

- Second-order effects should be defined explicitly in Turkish Standard for Steel Buildings, TS648 (1980). Still, the methodology presented in previous AISC specifications published before 1980 is used. Since the design philosophy developed by AISC was reviewed, updated, and cancelled, respectively, by the same institution, TS648 procedures should be revised parallel to modern approaches, unless the application of the current method was approved by further research on the subject. Also, the revision and update process of national specifications should be continuously, and periodically in the manner of AISC, Eurocode, etc.

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