OPTIMIZATION MODELS FOR PUBLIC DEBT MANAGEMENT

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

MUSTAFA UĞUR ALVER

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER IN SCIENCE
IN
INDUSTRIAL ENGINEERING

FEBRUARY 2009

Approval of the thesis:

OPTIMIZATION MODELS FOR PUBLIC DEBT MANAGEMENT

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ABSTRACT

OPTIMIZATION MODELS FOR PUBLIC DEBT MANAGEMENT

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February 2009, 114 pages

Management of public debt is crucial for every country. Public debt managers make efforts to both minimize the cost of borrowing and to keep debt stock at sustainable levels. However, due to competition for funds in the continuously changing and developing financial markets, new threats and opportunities appear constantly.

Public debt managers construct borrowing policies in order to minimize the cost of borrowing and also to decrease risk by using various borrowing instruments. This thesis presents a mathematical model to determine the borrowing policy that minimizes the cost of borrowing in line with future projections and then seeks to extend it to construct risk sensitive policies that allow minimizing the effects of changes in the market on the cost of borrowing.

The model's application results for determining the borrowing strategies of Turkish Treasury for 100 month horizon have been evaluated through the study.

Keywords: Public Debt Management, Risk Management, Non-linear Programming

ÖZ

KAMU BORÇ YÖNETİMİ İÇİN OPTİMİZASYON MODELLERİ

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Tez Yöneticisi: Prof.Dr. Çağlar Güven

Şubat 2009, 114 sayfa

Kamu borcunun yönetilmesi tüm ülkeler için büyük önem taşımaktadır. Kamu borç yöneticileri gerek borçlanmanın maliyetini azaltmak, gerekse de borç stoklarını yönetilebilir seviyelerde tutmak için gayret göstermektedir. Ancak, sürekli değişen ve gelişen finansal piyasalarda, fonlara erişim için yaşanan rekabetçiliğin de etkisiyle, ülkelerin finansmanı için yeni fırsatlar ve tehditler oluşmaktadır.

Kamu borç yöneticileri, ellerindeki çok sayıdaki borçlanma enstrümanını kullanarak hem borçlanma maliyetini azaltmak hem de riski düşürmek amacıyla borçlanma politikaları geliştirmektedir. Bu tez çalışmasında ileriye yönelik projeksiyonlar doğrultusunda, borçlanma maliyetini en aza indiren politikayı belirlemek amacıyla matematiksel bir model kurulmuş ve daha sonrasında bu model finansal koşullarda meydana gelebilecek değişimlerin borçlanma maliyetleri üzerindeki etkisini düşürecek şekilde risk yönetimi esaslı bir borçlanma politikası oluşturmak üzere geliştirilmeye çalışılmıştır.

Modelin, Türkiye Cumhuriyeti Hazine'sinin 100 aylık borçlanma stratejilerini belirlemek amacıyla uygulanması neticesinde bulunan sonuçlar çalışma kapsamında değerlendirilmiştir.

Anahtar kelimeler: Kamu Borç Yönetimi, Risk Yönetimi, Doğrusal Olmayan Programlama

To My Family

ACKNOWLEDGEMENTS

In this thesis I've worked with a great number of people whose contribution to me is inevitable to mention. It is a pleasure for me to express my thanks to them all in my acknowledgement.

First and foremost, I would like to express my sincere gratitude to my supervisor Prof.Dr. Çağlar Güven for his continuous guidance, advice and insight throughout this study. Without his contribution, this work could never be completed.

I gratefully acknowledge Sedef Meral for her encouragement and advise to finish this study. She also helped me to concentrate on my thesis in my difficult times. I also thank for Banu Lokman for her supports in GAMS.

I gratefully thank Emre Balıbek for his comments on my thesis and kindness in his advices. I also thank İsmail Serdar Bakal and Serhan Duran for their constructive comments and accepting to be members of the examining committee.

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CHAPTER 1

INTRODUCTION

In order to increase the welfare of the nation and meet future targets, governments aim to strengthen the national economy in all countries. The strength of the economy depends on many factors, but foremost on the sustainability of economic development. In the last few centuries, many governments tried to accelerate economic development by borrowing; therefore, government debt increased all over the world.

As debt stocks increased across the world, debt management became more and more important. Legal and institutional frameworks have been established in all countries to better manage debt issues. Public debt management (PDM) can be defined as raising funds for government financing needs and managing government liabilities in accordance with government policies. These financing needs can be met from markets using various types of borrowing instruments which differ in terms of the type of currency, interest rate, coupon type etc. The ultimate target of debt management is to meet financing needs of the government at the least borrowing cost. Governments try to achieve this by using an appropriate combination of borrowing instruments.

The complex nature of this problem can be understood by viewing this as an inventory management problem in which the inventory is cash. There is a certain demand (liabilities) and supply (borrowing) over several periods, where cost is determined by interest rates. The challenging part of the debt management problem is to make the right selection from a very broad range of instruments with many dimensions.

Borrowing instruments can be denominated in local or foreign currency, with varying interest rate and type. Also the coupon types and maturity of the bonds can be subject to different options. Time of borrowing and the amounts are other matters of concern. The problem may be modeled as a multi-period linear inventory management programming model. However, fluctuations in market conditions make the problem stochastic. Interest rates, market demand and future expectations in the financial sector are always subject to change. As a result of globalization, financial markets have become very volatile, so debt managers have to consider risk in all decisions.

When risk is taken into account the debt management problem becomes a multi period multiple criteria problem. In many cases achieving least cost may mean facing more risk, so debt managers have to decide on a policy, where there exists a reasonable borrowing cost at an appropriate risk level.

In modern debt management practice, debt management offices and Treasuries issue different kinds of borrowing instruments in order to diversify the associated risks. Diversification absorbs the effect of shocks in certain parameters. Strategic benchmarks and fiscal rules are defined by the governments in order to manage debt issues. Some important indicators, such as the debt stock over GDP, ratio of maturing bonds and interest re-fixing bonds in one year in the debt stock are closely monitored by debt managers.

After the 2001 banking sector crisis, the Turkish Treasury has established a strong debt management framework supported by legislation regarding debt management issues. The Turkish Treasury is the only borrowing authority for central government, and issues bonds and bills in the domestic market and Eurobonds in foreign markets. Alongside issuing bonds and bills, program and project financing from foreign creditors is also managed by the Treasury.

In this study, we explain how the government's borrowing requirement arises and then explain the various borrowing methods which are available. General definitions about external and domestic debt are followed by information about borrowing instruments. Risks on the borrowing are then categorized and the cost versus risk dilemma is explained. After explaining the general problem in detail about debt management, we summarize some of the related work addressing debt management issues.

Second, debt management problem is narrowed down for the Turkish Treasury case. Therefore we develop a cash management model for the Turkish Treasury that can be used as an instrument to compute and evaluate alternative borrowing schemes that aim to minimize the total borrowing cost. We aim to define an issuance strategy using a model extending over a 100-month horizon. We seek an answer to the question, "which borrowing instruments should the Treasury issue so as to minimize the borrowing cost at a reasonable risk level?" All publicly available data have been used in the study. Also, we have introduced and generated parameters such as monthly primary surplus, the interest rate and the exchange rate etc.

Borrowing requirement, principal payments, interest payments over the planning horizon are calculated by the model. In the base case scenario, debt stock over GDP ratio declines nearly from 39 to 30 percent at the end of the period, where interest payments over GDP is stabilized fewer than 5 percent. This represents a desirable sustainability performance.

Sensitivity of the cash flows and debt stock figures to parametrical changes in financial conditions are evaluated. For the sake of constructing a more robust debt composition to shocks, sensitivities to various such factors are analyzed. We apply stress tests to the model by changing only one parameter at a time and generate new solutions under shock scenarios of primary surplus, interest rates and the yield curve. In this way, the sensitivity of objective function and important debt management indicators to parametric changes are evaluated and trade off graph between risks and costs are constructed.

Afterwards, dependency between some internal decision variables in debt management is inserted to the model, in order to better reflect actual practice. For this purpose the model is extended to explain the liquidity conditions of the market and differentiates between the volumes of the instruments. The extended model is also capable of evaluating the cost of borrowing and setting an issuance strategy for 100 months horizon. The extended model assumes a perfect equation between interest rate and volume of borrowing. Actually, this area is very broad and auctions of Treasury can be a topic for another thesis study.

This study may provide insight for debt managers about the dynamics of debt management. In current practice, simulation analysis is preferred by debt managers when evaluating debt sustainability. In this study, we define strategies based on a linear programming model in which all feasible strategies can be included. Although, it is impossible to construct a model that represents the actual case exactly, the model provides better insight than a simulation model that can handle only a number of strategies.

In practical case, effect of changes in the financial conditions is calculated roughly, because the effect of change is measured only on the base borrowing strategy. But, in our study, a new strategy is generated for every scenario, so more reliable sensitivity analysis is acquired. Therefore, this study may give more accurate insights to the debt managers about the effect of changes in the financial conditions on debt sustainability.

In chapter 2, general information about Public Debt Management is provided. In this chapter, studies of academicians, government offices and international institutions are also summarized.

In chapter 3, we introduce Turkish Treasury debt management system and established a linear programming model for Turkish Treasury debt management.

In Chapter 4, we run the model and perform the sensitivity analysis under some shock scenarios. We also discuss the results at the end of the chapter.

In chapter 5, we try to develop the model by adding the dependency between borrowing and interest rates. Penalty rates for the interest rates have been added to certain instruments in order to analyze the effect of high volume borrowing on the debt management strategies.

In this study we could manage to develop an effective cash management model enabling to minimize the cost of borrowing for Turkish Treasury under a certain period. The model has important strengths on calculating the borrowing requirement and all payments, and issuing the appropriate instruments in line with the parameters given by the performer. Under given scenarios and stress tests, the model is capable of constructing all future cash flows and generates important figures for debt sustainability. Also the model gives us, how a change in the financial conditions can affect the debt sustainability figures.

CHAPTER 2

PUBLIC DEBT MANAGEMENT

2.1 Public Finance

One of the most important issues for a government is to finance required needs of the public sector. Thanks to the collected taxes from citizens, governments have a reasonable level of income. These revenue and expenditures are issues of government budget, which should be in balance in order to well manage the economy. In fact, for most governments, revenues and expenditures are not equaled annually, which means that there is a budget deficit or surplus.

As a result of inequalities in the conditions of the countries regarding geographical situation, natural and human resources, history, demographic conditions etc., some governments spend more than their income and for some other vice versa. Simply, the deficit between the expenditure and revenue is financed by borrowing. Indeed, the explanation of borrowing requirement is not that easy to summarize as deficit between expenditure and revenue. Addition to that, in some cases, governments also borrows money in order to finance a future investment with regard to its growth policy.

A budget deficit occurs when an entity (often a government) spends more money than it takes in. The opposite of a budget deficit is a budget surplus. Debt is essentially an accumulated flow of deficits. In other words, a deficit is a flow and debt is a build up stock. As stated in the definition, continuous budget deficits cause government debt through government borrowing that aims to finance the gap between expenditure and revenue.

2.1.1 Budget Finance

Government budget is a legal document that determines the expense and revenues of the government in a certain period. Revenues and expenditures are the main components of the budget.

Government revenues depend mainly on taxes collected from citizens and to a lesser extent on revenues from non-tax revenues such as dividend, special fund revenues etc. In order to have zero balance for the government budget, revenues and expenditures should be equal.

If $G(t)$ represents government spending in year t, and $T(t)$ represents government revenues in year t, then:

$$Budget\ balance_t = T(t) - G(t)$$

If budget balance is greater than zero than there is budget surplus, otherwise there is a budget deficit. A typical budget sheet of Turkish Treasury is given in Table 1.

Table 1: Central Government Budget Figures

CENTRAL GOVERNMENT BUDGET REALIZATION AND FINANCING							
(Million YTL)	2008						
	Jan	Feb	Mar	Apr	May	Jun	Jul
Central Government Revenues	**15,781**	**17,628**	**13,779**	**17,562**	**19,212**	**18,544**	**17,920**
Tax Revenues	14,159	15,177	10,920	12,902	16,902	12,691	13,778
-Direct Taxes	4,782	6,812	2,866	3,637	7,485	3,117	4,051
-Indirect Taxes	9,377	8,365	8,055	9,265	9,417	9,574	9,727
Nontax Revenues	1,623	2,452	2,858	4,660	2,309	5,852	4,142
Central Government Expenditures	**16,306**	**16,608**	**18,649**	**18,638**	**15,823**	**14,566**	**21,237**
Personnel Expenditure	4,841	3,856	3,816	4,014	3,996	3,984	4,171
Social Security Contributions	622	483	480	488	494	485	496
Purchase of Goods and Services	1,059	1,193	1,528	1,833	1,813	1,735	2,065
Interest Payments	4,283	4,714	4,753	4,166	1,681	1,142	7,436
Current Transfers	5,136	5,643	6,735	6,531	6,243	4,936	4,650
Capital Expenditure	16	117	797	996	1,034	1,719	1,655
Capital Transfers	29	281	236	140	109	330	416
Lending	319	320	303	470	452	234	349
Contingencies	0	0	0	0	0	0	0
Central Government Balance	**-524**	**1,021**	**-4,870**	**-1,076**	**3,389**	**3,978**	**-3,317**

Regarding the budget finance, primary balance is a key issue in modern public debt management. Primary surplus is defined as the difference between government receipts and the sum of primary expenditure. Primary expenditure is simply the expenditures of the government that exclude interest payments for outstanding debt stock.

If *D(t)* represents primary balance and *F(t)* represents interest payments, we can write the equation as follows:

$$D(t) = T(t) - (\, G(t) \; - F(t)\,)$$

In modern debt management practice, interest payments are considered separately from other expenses. Indeed, expenses other than interest payments should be compensated by government income, if a government economy is sustainable. Otherwise, a government, which cannot pay for its expenses other than interest payments by its income, will always have to increase its debt stock.

Maintaining a reasonable level of primary surplus is very important for debt sustainability. A typical budget sheet showing the primary surplus of Turkish Treasury is given in Table 2.

Table 2: Central Government Primary Balance Figures

CENTRAL GOVERNMENT BUDGET REALIZATION AND FINANCING							
(Million YTL)	2008						
	Jan	Feb	Mar	Apr	May	Jun	Jul
Central Government Revenues	**15,781**	**17,628**	**13,779**	**17,562**	**19,212**	**18,544**	**17,920**
Tax Revenues	14,159	15,177	10,920	12,902	16,902	12,691	13,778
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Social Security Contributions	622	483	480	488	494	485	496
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Interest Payments	4,283	4,714	4,753	4,166	1,681	1,142	7,436
Current Transfers	5,136	5,643	6,735	6,531	6,243	4,936	4,650
Capital Expenditure	16	117	797	996	1,034	1,719	1,655
Capital Transfers	29	281	236	140	109	330	416
Lending	319	320	303	470	452	234	349
Contingencies	0	0	0	0	0	0	0
Central Government Primary Balance	**3,759**	**5,735**	**-117**	**3,090**	**5,070**	**5,120**	**4,119**
Central Government Balance	**-524**	**1,021**	**-4,870**	**-1,076**	**3,389**	**3,978**	**-3,317**

2.1.2 Cash Management

The budget deficit does not mean that government needs that amount of cash. Cash requirement of the government is different from budget deficit, because in some cases a receivable may not been collected yet or a payment not been made. A payment that has been planned to be paid in the budget, but not paid in its planned period is called a deferment. On the other hand, if a payment has been planned to be paid in the following periods, but paid before its planned period is called an advance. Advance and deferments result in a difference between budget balance and cash balance. The cash balance can be formulated as follows:

Cash balance(t) = T(t) – G(t) – Advance(t) + Deferment(t)

Actually, advance and deferment payments are nonzero numbers for periods. But it is impossible to produce projections for advance and deferred payments for this thesis study. Therefore for this study we will assume that advance and deferred payments are equal to zero for all periods. Central government cash balance figures for Turkey is given in Table 3. Under this assumption, we can write the formula as follows:

Cash balance(t) = Budget balance(t)

Table 3: Central Government Cash Balance Figures

Central Government Balance	-524	1,021	-4,870	-1,076	3,389	3,978	-3,317
Deferred Payments	1,510	-2,315	-67	-23	19	-500	292
Other Deferred Payments	-2,087	952	-911	867	243	271	-70
Advances	1,686	-162	-636	-45	-151	-106	70
Central Government Cash Balance	585	-504	-6,483	-277	3,500	3,643	-3,026

2.1.3 Borrowing Requirement

In any year, the cash deficit is financed by finding extra resources. In some cases, revenues from privatization of public assets or collecting a receivable may finance the deficit, but in most cases this extra finance is supplied by government borrowing. Indeed all debt has to be paid in some day and a government has to repay its previous borrowings at the maturity date. Therefore, government should borrow in a period for both financing the cash deficit and repaying the previous principal on the maturity day. Moreover, the government has to maintain a certain level of cash reserve.

If $B(t)$ represents borrowing requirement of the government, $C(t)$ represents principal to be repaid and $X(t)$ represents cash reserve level in period t; then:

$$B(t) + Cash\ balance(t) = C(t) + (X(t) - X(t-1))$$

Keeping in mind that cash deficit has been assumed to be equal to the budget deficit, than the following formulation can be written:

$$B(t) + T(t) - G(t) = C(t) + (X(t) - X(t-1))$$

A positive value for the borrowing requirement means that the government needs extra finance to balance its cash condition. Actually, it is nearly impossible for a government to have a zero borrowing requirement, which has to repay a reasonable amount of principal.

Primary surplus has a key role in the cash management. The above equation represents the cash management of Treasury. By using the primary surplus numbers, we can reach the debt management equation. As it is stated in page 19, primary balance is the difference between government revenues and government expenditures excluding interest payments. So borrowing requirement can be represented by debt management figures as follows:

Considering;

$$D(t) = T(t) - (G(t) - F(t)) \Longrightarrow D(t) - F(t) = T(t) - G(t);$$

Hence;

$$B(t) + T(t) - G(t) = C(t) + (X(t) - X(t-1)) \text{ where } D(t) - F(t) = T(t) - G(t)$$

Therefore we can reach the ultimate equation for the financial balance of the government cash flow is below:

$$B(t) + D(t) = C(t) + F(t) + (X(t) - X(t-1))$$

Actually interest payments represent cost of borrowing to finance previous and current year cash deficits. Decreasing the debt stock can be achieved by both decreasing the borrowing requirement and also by decreasing interest payments. Lowering the interest payments can only be achieved by lowering the interest rate (which is not a government controlled variable) or decreasing borrowing requirement. Therefore a strong primary surplus is crucial for lowering the borrowing requirement.[1]

The ratio of borrowing over sum of interest and principal payment is very crucial for the debt sustainability. Therefore this ratio is defined as roll over ratio which is a powerful indicator to show the relation among these three. In calculation of roll over ratio cash reserve changes are not taken into account.

$$\text{Roll over ratio} = B(t) \, / \, (C(t) + F(t))$$

A roll over ratio less than one "1" means that the government can pay some of its liabilities without borrowing. The opposite case is a very bad sign for debt sustainability, because a government that borrows more than its liabilities will always increase its debt stock spirally. Primary surplus and roll over ratio has an important relation, a positive primary surplus will yield a ratio less than one.

$$\text{Since; } D_t + B_t = F_t + C_t$$
$$\text{If } D_t > 0 \text{ ; then Roll over ratio} < 1$$

2.1.4 Types of Borrowing

Governments borrow cash through different borrowing methods. As we shall explain, borrowing methods matter and are important in managing costs and risks. Governments mainly borrow cash by issuing bills and bonds in various financial markets.

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[1] Public Debt Management Report, Turkish Treasury April 2003

The difference between a bond and a bill has to do with the maturity of the security; if the maturity is less than one year, it is called **bill,** otherwise it is called a **bond**. These bonds and bills can be issued in domestic and foreign financial markets.

These bonds and bills have different attributes defined at issuance. Bonds and bills are securities that pay the principal on the maturity day and pays interest in coupon periods.

Most important attributes of bonds or bills are related with their interest (coupon) payment type, currency denomination and interest (re)fixing structure. A summary of the borrowing types are given in Table 4.

Types of Borrowing with Respect to Interest Payment Timing

Bonds and bills are classified into two groups with respect to their interest payment time. One group namely **zero-coupon** (discounted) bonds and bills has only one payment, which is at the maturity time, both principal and interest. Whereas the **coupon bonds** and bills pays interest through the life of bonds in certain periods and pays the principal at the maturity time of the bonds. Schematic diagram for zero coupon and coupon bonds are shown in Figure 1 and Figure 2 respectively.

Table 4: List of Borrowing Types

Classification Type	Instrument Type
Maturity time	Bonds Bills
Interest Payment Timing	Zero coupon bills/bond Coupon bonds
Type of Interest Rate	Fixed interest rate bills/bonds Floating Interest rate
Type of Currency	Local currency bills/bonds Foreign Currency bills/bonds
Type of Issuance Market	Domestic market bills/bonds Foreign market bonds

Zero coupon bonds

Figure 1: Cash flow diagram for a zero coupon bond

Coupon bonds

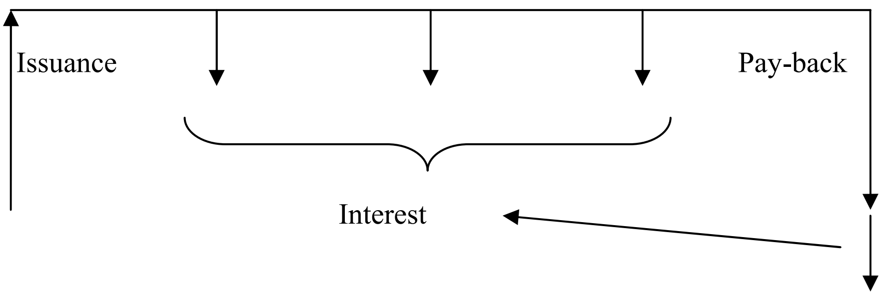

Figure 2: Cash flow diagram for a coupon bond

Borrowing Types with Respect to Interest Type

Bonds are classified into two groups with respect to their interest type. If the interest of a bond is certain through its life to maturity and it is defined in the issue date, then this type of bond is called **fixed rate-coupon bond**, otherwise it is called **floating rate coupon bond**.

Types of Borrowing with Respect to Currency

Government borrowings can be classified into two groups with respect to its currency. The mostly used method is **local currency borrowing**, in which government borrows local currency and pays principal and interest both in local currency. In the **foreign currency borrowing**, government borrows in a foreign currency and pays principal and interest in the same foreign currency.

Types of Borrowing with Respect to the Market

If a government sells its bonds and bills in the domestic market, it is called **domestic borrowing.** If the bonds or bills are sold in the foreign markets or cash money is borrowed from foreign governments, foreign banks or international institutions, it is called **foreign borrowing**. The currency of debt does not have any effect on the definition. For example a dollar currency bond may be sold in domestic market is a type of domestic borrowing, whereas a Turkish citizen buys a Eurobond which is sold in foreign market is a type of foreign borrowing.

To conclude, governments can find finance to compensate its borrowing requirement in many ways. The above stated attributes gives many dimensions to the instruments. A government and investors can decide on buying or issuing the bonds with respect to their needs. The complicated structure of the instruments give decision makers to have a broad range of activity to raise funds suitable for their needs.

2.2 The Debt Management Problem

Governments need some level of borrowing which can be implemented through different methods. These methods have been developed in financial markets in order to serve both lenders and borrowers. The wide range of the instruments enables investors and borrowers to reflect their needs to their portfolios. For example, if a lender wants to earn equal amount of income from interest payments, then he/she will buy a fixed coupon bond. On the other hand, an investor may want to secure the US Dollar equivalent value of his capital, then he will buy a dollar currency denominated bond.

Broadly speaking, there are two important issues for debt management problem. First one is the cost of borrowing which is the total interest payment of the outstanding stock. Second one is the risk level of the borrowing which is caused by the change in the interest rates.

"In a broader macroeconomic context for public policy, governments should seek to ensure that both the level and rate of growth in their public debt is fundamentally sustainable, and can be serviced under a wide range of circumstances while meeting cost and risk objectives. Sovereign debt managers share fiscal and monetary policy advisors' concerns that public sector indebtedness remains on a sustainable path and that a credible strategy is in place to reduce excessive levels of debt. Debt managers should ensure that the fiscal authorities are aware of the impact of government financing requirements and debt levels on borrowing costs. Examples of indicators that address the issue of debt sustainability include the public sector debt service ratio, and ratios of public debt to GDP and to tax revenue."[2]

2.2.1 Cost of Borrowing

Money has a time value, so if someone needs money, the lender will give the money provided he will get more when he receives his money back. This is same for government borrowing; if a government borrows some money from financial markets, government will pay back more.

Under the turbulent condition of global economy many investors are reluctant to extend long term credit over hence interest rates are effectively floating.

Cost of borrowing has two bones, one is the interest paid on the debt stock, and the second is the cost caused by foreign currency appreciation.

2.2.2 Cost and Risk Dilemma

Interest payments may be fixed or floating or indexed to other variables such as inflation rate.

Interest payments are not certain in all cases. Due to the floating or indexed structure of the rates, the interest amount to be paid is prone to some level of changes. In fact, price level, currency of supply of money can change in the financial market. When an investor gives money to an issuer, he wants to be free of risks or demands for high profit in case of bearing the risks. Therefore, in the issuer side, less cost means more risks (such as floating rate or foreign currency indexed bonds) and less risks means more cost in general.

Therefore issuers and investors independently analyze their situations and prioritize their needs. So, they come into a common point to share the costs and risks in a manner where the cost and risk of borrowing as well as profit and risk of investing have been defined.

As so many times repeated in this study, the financial markets include a number of risks. These risks can be categorized into groups which are explained below:

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[2] IMF Guidelines

2.2.3 Liquidity Risk

"There are two types of liquidity risk. One refers to the cost or penalty investors face in trying to exit a position when the number of transactions has markedly decreased or because of the lack of depth of a particular market. This risk is particularly relevant in cases where debt management includes the management of liquid assets or the use of derivatives contracts. The other form of liquidity risk, for a borrower, refers to a situation where the volume of liquid assets can diminish quickly in the face of unanticipated cash flow obligations and/or a possible difficulty in raising cash through borrowing in a short period of time."[3]

2.2.4 Market Risk

"Refers to the risks associated with changes in market prices, such as interest rates, exchange rates, commodity prices, on the cost of the government's debt servicing. For both domestic and foreign currency debt, changes in interest rates affect debt servicing costs on new issues when fixed-rate debt is refinanced and on floating-rate debt at the rate reset dates. Hence, short- duration debt (short-term or floating-rate) is usually considered to be more risky than long-term, fixed rate debt. (Excessive concentration in very long-term, fixed rate debt also can be risky as future financing requirements are uncertain.) Debt denominated in or indexed to foreign currencies also adds volatility to debt servicing costs as measured in domestic currency owing to exchange rate movements. Bonds with embedded put options can exacerbate market and rollover risks."[4]

2.2.5 Rollover Risk

"The risk that debt will have to be rolled over at an unusually high cost or, in extreme cases, cannot be rolled over at all. To the extent that rollover risk is limited to the risk that debt might have to be rolled over at higher interest rates, including changes in credit spreads, it may be considered a type of market risk. However, because the inability to roll over debt and/or exceptionally large increases in government funding costs can lead to, or exacerbate, a debt crisis and thereby cause real economic losses, in addition to the purely financial effects of higher interest rates,

[3] IMF Guidelines

[4] IMF Guidelines

it is often treated separately. Managing this risk is particularly important for emerging market countries."[5]

2.3 Literature Review on Public Debt Management Strategy Formulation

Literature on public debt management has a wide range. Due to the relevance of public debt management with both macroeconomics and financial aspects, there are numerous studies on public debt management. Some of the studies are focused mostly on the macroeconomic side, generally dealing with debt sustainability supported with econometric analysis. Effects of public debt to the welfare and growth have been researched by numerous academicians. But in this study, macroeconomic effects of public debt are not analyzed; only the sustainability of public debt is a matter of concern. Therefore, literature about designing the public debt management, in terms of managing the cash flows regarding the risk and cost aspects, with setting maturity and type of instrument will be given.

All over the world, the institutions in charge with the debt management, such as debt management agencies, Ministry of Finance or Treasuries have a reasonable number of studies on debt management. Swedish National Debt Office publishes Central Government Debt Management Reports (2000 – 2005) where the proposed guidelines are being explained. In Swedish Central Government Debt Management Report (2005) proposed benchmarks are defined under numerous scenarios in order to evaluate the debt composition and maturity of borrowing.

Barro (1995) has analyzed optimal debt management under varying tax revenue collection cases. He studies on optimal taxation that will yield a tax smoothing in debt management perspective. "His strategy in his paper was to establish a dynamic optimal taxation in a tractable equilibrium model"[6]. He proposes that the general recommendation for public debt management is to issue price level indexed bonds with long maturity, in order to insulate future government budget from real interest rate changes. His study was written after a period that higher inflation rates have appeared because of non independent Central Banking system in the world. He also noted in 1995 that, the trend of independent central banks with the aim of price

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[5] IMF Guidelines

[6] Barro, R., (1995). Optimal Debt Management Working Paper

stability will result in a structure that, nominal debt will be close to real debt and the difference between the indexed and conventional bonds will be less.

Additionally, in his latter study, Barro (2003) stresses again on the indexed (price level, interest rate etc) and long term for optimal policy, and adds that nominal government bonds should rely on short-run macroeconomics.

The studies of Barro have mostly concentrated on the revenue side. The revenue aspects in this study are reflected to the primary surplus.

Alesina (1990) stresses that long maturity debt stocks which are equally distributed among the periods, have a key role on public confidence and has a cushion role to prevent confidence crisis.

Leong (1999) has analyzed UK debt management strategies. She briefly explained the debt management policy of UK. Leong argued the debt management theory, with analyzing risk measurement techniques, minimization of cost policies and allocating of risk in line with the credibility of the government debt policy. She underlines the important differences between literature and practice of debt management policies. She imposes that literature is mostly depending on the risk reduction whereas in practice, debt managers aims to reduce the actual cost. At the end of the study, improvement strategies of public debt management are summarized. Her study mostly depends on verbal comparisons among the theory and practices rather than a mathematical modeling approach.

On the other hand, IMF and World Bank have important concerns about debt management. Numerous surveys and papers are published by these institutions. The analysis of Bretton Woods institutions are mostly depending on the plausible scenarios rather than simulation analysis. For the sake of performing a sound analysis, quantification of costs and risks are a matter of concern. Valendia (2001) studied on an Asset Liability Management Framework, in order to integrate government assets in the model. He also proposes a methodology for risk quantification. His study depends on econometric analysis trying to find the macroeconomic feed back mechanisms.

Balıbek (2008) focused on Multi Criteria Decision Making (MCDM) problem using a multi-period stochastic programming model that takes into account sequential decisions concerned with debt issuance policies. He inserted quarterly cash flows into a stochastic model on a decision tree basis where numerous scenarios are simulated in order to form a multi criteria decision tool where cost and risks are compared.

As stated above, debt management has a wide range of literature. Due to the boundaries of this thesis, only the mathematical modeling based studies that focus on maturity and instruments allocations are underlined in this part. Broadly speaking, the studies are clustering in one point that is the ultimate aim of finding the minimum cost at a reasonable risk level. Some of the academicians try to measure the risk on the revenues side, whereas some others try to explain in the expenditure side. Due to the stochastic nature of the problem, the possibilities are inserted to the models with simulation, scenario generation etc.

On the other hand, not in the debt management literature, but in the portfolio management approaches, an approach called chance constraints, is used in order to reflect the stochastic nature of financial conditions.

Simply, "Chance constrained programming admits random data variations and permits constraint violations up to specified probability limits. Different kinds of decision rules and optimizing objectives may be used so that, under certain conditions, a programming problem (not necessarily linear) can be achieved that is deterministic—in that all random elements have been eliminated."[7]

Haneveld and Van Der Vlerk (2002) developed a new concept called "Integrated Chance Constraints" which provides quantitative alternatives for traditional chance constraints. They derived explicit polyhedral descriptions for convex feasible sets induced by ICCs, where the underlying distribution is discrete. Haneveld and Van Der Vlerk (2005) applied their studies in an Asset Liability Management Problem for Dutch Pension Funds in order to analyze the Value at Risk.

[7] Charnes, A., Cooper W.W., (1963) Deterministic Equivalents for Optimizing and Satisficing under Chance Constraints

CHAPTER 3

A LINEAR PROGRAMMING MODEL FOR TURKISH TREASURY DEBT MANAGEMENT

3.1 Background Information on Turkish Treasury Debt Management

In this chapter we develop a general debt management model for the Turkish Treasury that can be used as an instrument to compute and evaluate alternative borrowing schemes that aim to minimize the total borrowing costs.

Turkish Treasury is responsible for cash and debt management of Turkish government. All tax or non-tax revenues are transferred to Treasury accounts by Ministry of Finance and all expenditures of central government is made by Treasury. The first responsibility for Treasury is to manage all cash flows.

Mainly Turkish Treasury finances its cash deficit and principal payment by borrowing. In the last few years, thanks to the strong primary surplus policy, borrowing requirement have been decreased. On the other hand, privatization revenues and receipts from indebted institutions have been important anchors in financing the cash requirement of Treasury. General black-box diagram for Treasury cash management activity is given in Figure 3:

As mentioned in the previous chapter,

$$D(t) + B(t) + X(t-1) = F(t) + C(t) + X(t) \;;$$

Borrowing of Treasury has some certain rules defined by a Turkish Law No:4749. According to the Law, Treasury borrows cash from both domestic and international markets through bills or bonds. Also other than the bills and bonds there are other kind of borrowing such as project finance. Types of bonds and bills have been mentioned in the previous chapters, but in this part the common instruments of Turkish Treasury will be introduced.

In Turkish Treasury, the most common instrument in domestic borrowing is the local currency zero coupon bonds with the maturity periods between 12 and 24 months. This instrument has a wide range of demand in the Turkish financial market, due to the high liquidity in secondary market. Treasury re-issues same instrument in consecutive months for the sake of implementing a benchmark issuance strategy.

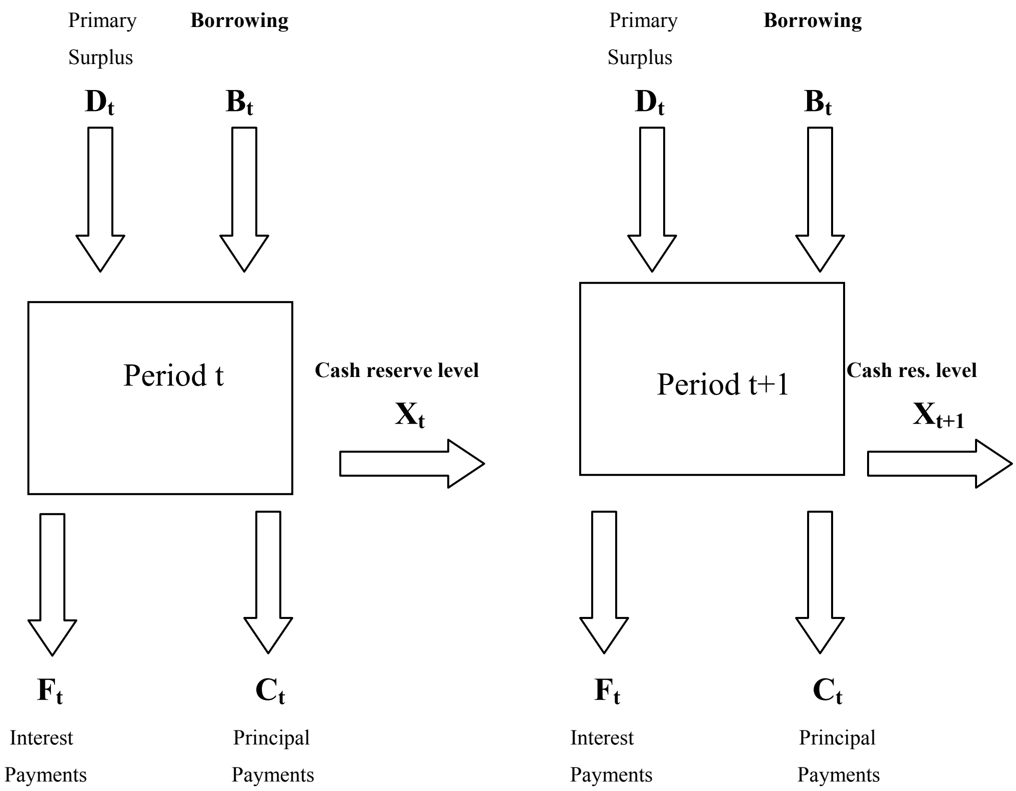


Figure 3: Black box diagram for borrowing requirement

Other issued instruments in domestic market are floating rate and fixed rate coupon bonds. The fixed rate coupon bond's coupon period is 6 months with the six months term interest rate fixed at the issuance date. Floating rate coupon bonds' coupon period is 3 or 6 months with the variable interest rate at each coupon payments. Also with a low percentage, inflation indexed bonds are issued by Treasury with 6 months coupon period. There are also foreign currency denominated bond issuance in domestic market, but they have a very small ratio in domestic borrowing.

Foreign borrowing is done through Eurobond issuance in the international financial markets. Eurobonds are a kind of fixed rate coupon bonds which is foreign currency denominated and issued in international financial markets. Coupon period for Eurobonds is 6 months. Eurobonds are not the only source for foreign borrowing. Program and project financing is a kind of foreign borrowing without a bond or bill issuance.

Bills and bond issuances are done through Treasury auctions. Debt managers in Treasury decide to establish a monthly auction schedule. In the monthly auction schedule, Treasury announces which type of bond will be issued in the following month. Maturity, interest payment type, coupon structure of the bonds are written in the announcement. Treasury debt managers also have a planned volume of issuance for each item in the announcement. However, the exact volume of issuance is determined after the auction.

A debt manager has several instruments to choose from and a wide range of decision making chance for determination of medium-long term issuances. The decision stages are as follows:

When: A debt manager can decide on the date of borrowing. For example by using cash from the cash reserve accounts, he can postpone a borrowing requirement for a few months or he can borrow in advance.

How much: A debt manager can decide on how much to borrow in a certain period within the auction demand. He can increase or decrease the amount as allowed by cash reserve.

Which type: The most important decision is to choose the instrument type. A debt manager can decide on the currency, interest rate and coupon type of a bond in order to manage the medium-long term program and minimize the cost of borrowing.

How long: A debt manager can also choose the maturity period at the time of issuance process.

Hence the instruments that are issued by the debt managers, constitute the decision variables of the system. In this way, a debt manager also decides on the interest and principal payments indirectly, by deciding on the instruments. Broadly speaking, debt managers actively manage the instruments and passively manage the payments of the system. The schematic diagram for the decision process is given in Figure 4.

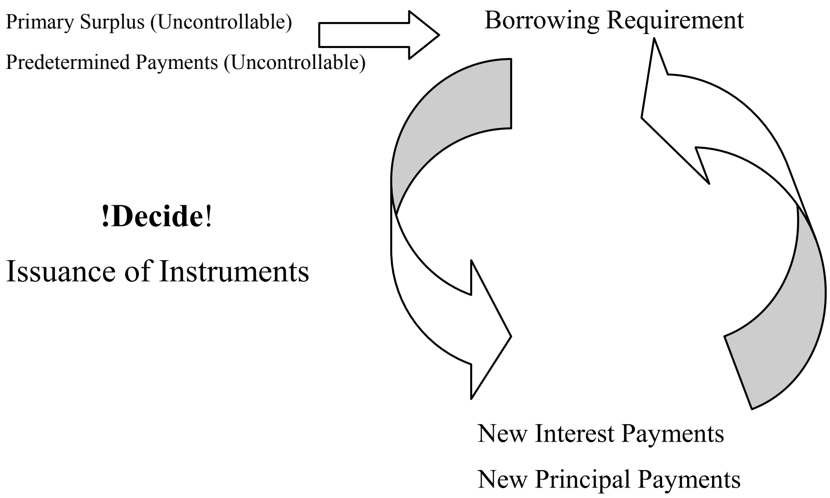


Figure 4: Decision process for issuance of instruments

3.2 Turkish Treasury Linear Programming Model for Debt Management

Debt management procedures in Treasury have lots of details. All interest and principal payments structures, market restrictions should be explained. But explaining the system verbally is not very easy for Treasury debt management because of the complicated structure. For the sake of explaining the model in an easy way, we will first give the model and all details will be explained after.

We have constructed an LP model that can be used as an instrument to perform both cash management and medium-long term borrowing cost minimization. The model given below is performed for a 100 month horizon.

Please see the Appendix A before reading the LP Model close form.

3.2.1 LP Model Summary of Close Formulation

Objective Function

Min $\sum_{t=1}^{100}(ai_t + ask_t + adk_t + aek_t + ae_t + ap_t)$

Subject to

Cash flow constraints:

1) $c(t) + f(t) + x(t-1) = b(t) + D(t) + x(t) \quad for \quad \forall t \in [1{,}100]$............ General cash balance in period t

2) $b(t) = i(t) + sk(t) + dk(t) + ek(t) + e(t) + p(t)$

$for \quad \forall t \in [1{,}100]$ ....Borrowing distribution in per. t

3) $c(t) = ci(t) + csk(t) + cdk(t) + cek(t) + ce(t) + cp(t) + KC(t) + KCD(t) * KUR(t)$

$for \quad \forall t \in [1{,}100]$ .... Principal pay. dist. in per. t

4) $f(t) = fi(t) + fsk(t) + fdk(t) + fek(t) + fe(t) + fp(t) + KF(t) + KFD(t) * KUR(t)$

$for \quad \forall t \in [1{,}100]$ .... Interest pay. dist. in per. t

Instrument constraints – Only zero-coupon bonds equations are shown below!

5) $i(t) = \sum_{vi=12}^{24} ix(t, vi) \quad for \quad \forall t \in [1{,}100]$............ Dist. of zero-coupon bond iss.

6) $ci(t) = \sum_{vi=12}^{24} ix(t - vi, vi) \quad for \quad \forall t \in [1{,}100]$....Calc. of zer-cop. originated pri. pay.

7) $fi(t) = \sum_{vi=12}^{24} ix(t - vi, vi) * r(t - vi, vi) \quad for \quad \forall t \in [1{,}100]$.....Calc. of zer-cop. originated int. pay.

8) $ai(t) = \sum_{vi=12}^{24} ix(t - vi, vi) * r(t - vi, vi) / PAYDA(vi)$

$for \quad \forall t \in [1{,}100]$....Calc. of zer-cop. originat. accrued int.

Debt stock constraints - Only local currency denominated stock equation is shown below!

9) $tlstok(t) = tlstok(t-1) + i(t) + sk(t) + dk(t) + ek(t) - ci(t) - csk(t) - cdk(t) - cek(t) - KC(t)$

$for \quad \forall t \in [1{,}100]$.....Calc. of local cur. denom. debt stock

Market and Borrowing policy constraints - Only zero coupon bond equations are shown below!

10) $i(t) \le 12000 \quad for \quad \forall t \in [1{,}100]$............ Max. level for zero coupon bond iss. in period t

11) $i(t) \le 0.6 * b(t) \quad for \quad \forall t \in [1{,}100]$......Max. percentage level for zer-cop.in total bor. in per. t

3.2.2 Detailed Lp Model – Definition of Sets, Parameters and Variables

SETS

t	represents periods	1 - 100
v	represents maturity of instruments	1 - 240

Subsets for v

vi(v)	maturity of zero-coupon bonds	/12-24/
vsk(v)	maturity of fixed-coupon bonds	/36,48,60,72/
vdk(v)	maturity of floating-coupon bonds	/36,48,60,72,84/
vek(v)	maturity of inflation-coupon bo/36,48,60,72/	
ve(v)	maturity of Eurobonds	/60,120,180,240/
vp(v)	maturity of project finance	/24,36 …180/
z(v)	coupon periods	/6,12,…,240/

PARAMETERS

r(t,v)	Term interest rates for TL borrowing in **period t** for **maturity v**	
re	Six month term int. rate for Eurobonds for all periods	/0.050/
rp	Six month term int. rate for project fin. for all periods	/0.040/
drate	Monthly discount rate for all periods	/0.0087/

D(t)	Primary surplus for **period t**
GSYH(t)	GDP value at the end of **period t**
ENFL(t)	Inflation index in **period t**
KUR(t)	Exchange rate for USD/TL **in period t**
KC(t)	Predetermined local currency principal payment in **period t**
KCD(t)	Predetermined foreign cur. den. principal payment for **period t**
KF(t)	Predetermined local currency interest payment in **period t**
KFD(t)	Predetermined foreign cur. den. interest payment for **period t**

m(v)	Maturity of bonds for **maturity v**
PAYDA(v)	Accrued interest discount factor for **maturity v**

PAY(v)	Accrued interest coupon discount factor for **maturity v**

VARIABLES

General equilibrium variables

c(t)	Total principal payment in **period t**
f(t)	Total interest payment in **period t**
b(t)	Borrowing requirement in **period t**
x(t)	Cash reserve level at the end of **period t**

Instrument issuance variables

i(t)	Total volume of zero-coupon issuance in **period t**
sk(t)	Total volume of fixed-coupon issuance in **period t**
dk(t)	Total volume of floating-coupon issuance in **period t**
ek(t)	Total volume of inflation indexed-coupon issuance in **period t**
e(t)	Total volume of Eurobond issuance in **period t**
p(t)	Total volume of project/program finance in **period t**

ix(t,vi)	Volume of zero-coupon bond iss. in **period t** with **maturity vi**
skx(t,vsk)	Volume of fixed-coupon bond iss. in **period t** with **maturity vsk**
dkx(t,vdk)	Volume of floating-cou. bond iss. in **period t** with **maturity vdk**
ekx(t,vek)	Volume of inflation-cou. bond iss. in **period t** with **maturity vek**
ex(t,ve)	Volume of eurobond iss. in **period t** with **maturity ve** in USD
px(t,vp)	Volume of prj/prog. finance in **period t** with **maturity vp** in USD.

Debt stock variables

tlstok(t)	Total local currency denominated debt stock in **period t**
fxstok(t)	Total foreign currency denominated debt stock in **period t** in TL
stok(t)	Total debt stock in **period t**
brcstk(t)	Total debt stock over GDP in **period t**

Principal Payment variables

ci(t)	Principal payment of zero-coupon bond in **period t**
csk(t)	Principal payment of fixed-coupon bond in **period t**

cdk(t)	Principal payment of floating-coupon bond in **period t**
cek(t)	Principal payment of inflation-coupon bond in **period t**
ce(t)	Principal payment of Eurobond in **period t**
cp(t)	Principal payment of project/program finance in **period t**

Interest payment variables

fi(t)	Interest payment of zero-coupon bond in **period t**
fsk(t)	Interest payment of fixed-coupon bond in **period t**
fdk(t)	Interest payment of floating-coupon bond in **period t**
fek(t)	Interest payment of inflation-coupon bond in **period t**
fe(t)	Interest payment of eurobond in **period t**
fp(t)	Interest payment of project/program finance in **period t**

Present worth of accrued interest variables

ai(t)	**A**ccrued interest caused by zero coupon bonds in **period t**
ask(t)	**A**ccrued interest caused by fixed coupon bonds in **period t**
adk(t)	**A**ccrued interest caused by floating coupon bonds in **period t**
aek(t)	**A**ccrued interest caused by inflation coupon bonds in **period t**
ae(t)	**A**ccrued interest caused by eurobonds in **period t.**
ai(t)	**A**ccrued interest caused project/program finance in **period t**

w	Total present worth of all accrued interests in all periods ;

3.2.3 Lp Model Formulation

Objective Function

Min $\sum_{t=1}^{100}(ai_t + ask_t + adk_t + aek_t + ae_t + ap_t)$

Subject to

Cash flow constraints:

1) $c(t) + f(t) + x(t-1) = b(t) + D(t) + x(t)$ $\quad for \quad \forall t \in [1,100]$............ General cash balance in period t

2) $b(t) = i(t) + sk(t) + dk(t) + ek(t) + e(t) + p(t)$

$for \quad \forall t \in [1,100]$ ....Borrowing distribution in per. t

3) $c(t) = ci(t) + csk(t) + cdk(t) + cek(t) + ce(t) + cp(t) + KC(t) + KCD(t) * KUR(t)$

$for \quad \forall t \in [1,100]$ .... Principal pay. dist. in per. t

4) $f(t) = fi(t) + fsk(t) + fdk(t) + fek(t) + fe(t) + fp(t) + KF(t) + KFD(t) * KUR(t)$

$for \quad \forall t \in [1,100]$ .... Interest pay. dist. in per. t

Equation related with instruments – Distribution of the borrowing instruments in period t

5) $i(t) = \sum_{vi=12}^{24} ix(t, vi)$ $\quad for \quad \forall t \in [1,100]$............ Dist. of zero-coupon bond iss.

6) $sk(t) = \sum_{vsk=36}^{72} skx(t, vsk)$ $\quad for \quad \forall t \in [1,100]$............ Dist. of fixed-coupon bond iss.

7) $dk(t) = \sum_{vdk=36}^{84} dkx(t, vdk)$ $\quad for \quad \forall t \in [1,100]$............ Dist. of FRN-coupon bond iss

8) $ek(t) = \sum_{vek=36}^{72} ekx(t, vek)$ $\quad for \quad \forall t \in [1,100]$............ Dist. of inf.ind.-coupon bond iss

9) $e(t) = \sum_{ve=60}^{240} ex(t, ve) * KUR(t)$ $\quad for \quad \forall t \in [1,100]$............ Dist. of Eurobond issuance

10) $p(t) = \sum_{vp=24}^{180} px(t, vp) * KUR(t)$ $\quad for \quad \forall t \in [1,100]$............ Dist. of project finance

Equation related with instruments – Distribution of principal payment in period t

11) $ci(t) = \sum_{vi=12}^{24} ix(t - vi, vi)$ $\quad for \quad \forall t \in [1,100]$........Calc. of zer-cop. originated pri. pay.

12) $csk(t) = \sum_{vsk=36}^{72} skx(t - vsk, vsk)$ $\quad for \quad \forall t \in [1,100]$...... Calc. of fixed-cop. ori. pri. pay.

13) $cdk(t) = \sum_{vdk=36}^{84} dkx(t - vdk, vdk)$ $\quad for \quad \forall t \in [1,100]$......... Calc. of FRN-cop. ori. pri. pay.

14) $cek(t) = \sum_{vek=36}^{72} ekx(t - vek, vek)$ $\quad$ *for* $\forall t \in [1,100]$............ Calc. of inf-cop. ori. pri. pay.

15) $ce(t) = \sum_{ve=60}^{240} ex(t - ve, ve) * KUR(t)$ $\quad$ *for* $\forall t \in [1,100]$............ Calc. of Eurobond ori. pri. pay.

16) $cp(t) = \sum_{vp=24}^{180} px(t - vp, vp) * KUR(t)$ $\quad$ *for* $\forall t \in [1,100]$............ Calc. of Prj. fin. ori. pri. pay.

Equation related with instruments – Distribution of interest payment in period t

17) $fi(t) = \sum_{vi=12}^{24} ix(t - vi, vi) * r(t - vi, vi)$ $\quad$ *for* $\forall t \in [1,100]$.....Calc. of zer-cop. originated int. pay.

18) $fsk(t) = \sum_{vsk=36}^{72} skx(t - vsk, vsk) * r(t - vsk, '6')$ $\quad$ *for* $\forall t \in [1,100]$.....Calc. of fixed-cop. originated int. pay.

19) $fdk(t) = \sum_{vsk=36}^{84} dkx(t - vdk, vdk) * r(t - 6, '6')$ $\quad$ *for* $\forall t \in [1,100]$.....Calc. of FRN-cop. originated int. pay.

20) $fek(t) = \sum_{vek=36}^{72} ekx(t - vek, vek) * ENF(t) / ENF(t - 6) * 0.08$

for $\forall t \in [1,100]$.....Calc. of inf-cop. originated int. pay.

21) $fe(t) = \sum_{ve=60}^{240} ex(t - ve, ve) * re * KUR(t)$ $\quad$ *for* $\forall t \in [1,100]$.....Calc. of Eurobond originated int. pay.

22) $fp(t) = \sum_{vp=24}^{180} px(t - vp, vp) * rp * KUR(t)$ $\quad$ *for* $\forall t \in [1,100]$.....Calc. of Prj. fin. originated int. pay.

Equation related with instruments – Distribution of interest payment in period t

23) $ai(t) = \sum_{vi=12}^{24} ix(t - vi, vi) * r(t - vi, vi) / PAYDA(vi) / (1 + drate)^t$

for $\forall t \in [1,100]$....Calc. of zer-cop. originat. accrued int.

24) $ask(t) = \sum_{vsk=36}^{72} skx(t - vsk, vsk) * r(t - vsk, '6') * PAY(vsk) / PAYDA(vsk) / (1 + drate)^t$

for $\forall t \in [1,100]$....Calc. of fixed-cop. ori. accrued int.

25) $adk(t) = \sum_{vdk=36}^{84} dkx(t - vdk, vdk) * r(t - 6, '6') * PAY(vdk) / PAYDA(vdk) / (1 + drate)^t$

for $\forall t \in [1,100]$....Calc. of FRN-cop. ori. accrued int.

26)

$$aek(t) = \sum_{vek=36}^{72} ekx(t-vek, vek) * ENF(t) / ENF(t-6) * 0.08 * PAY(vek) / PAYDA(vek) /(1+drat$$

for $\forall t \in [1,100]$....Calc. of inf-cop. ori. accrued int.

27) $$ae(t) = \sum_{ve=60}^{240} ex(t-ve, ve) * re * PAY(ve) / PAYDA(ve) /(1+drate)^t$$

for $\forall t \in [1,100]$....Calc. of Eurobond ori. accrued int.

28) $$ap(t) = \sum_{vp=24}^{180} px(t-vp, vp) * rp * PAY(vp) / PAYDA(vp) /(1+drate)^t$$

for $\forall t \in [1,100]$....Calc. of prj. fin. ori. accrued int.

Equation related with debt stock

29)

$$tlstok(t) = tlstok(t-1) + i(t) + sk(t) + dk(t) + ek(t) - ci(t) - csk(t) - cdk(t) - cek(t) - KC(t)$$

for $\forall t \in [1,100]$.....Calc. of local cur. denom. debt stock

30)

$$fxstok(t) = fxstok(t-1) * KUR(t) / KUR(t-1) + e(t) + p(t) - ce(t) - cp(t) - KCD(t) * KUR(t)$$

for $\forall t \in [1,100]$.....Calc. of for. cur. denom. debt stock

31) $stok(t) = tlstok(t) + fxstok(t)$ *for* $\forall t \in [1,100]$.....Calc. of total debt stock in local cur.

32) $brcstok(t) = stok(t) / GSYH(t) * 100$ *for* $\forall t \in [1,100]$.....Calc. of debt stock over GDP ratio

Equation related with market restrictions

33) $i(t) \leq 12000$ *for* $\forall t \in [1,100]$............ Max. level for zero coupon bond iss. in period t

34) $sk(t) \leq 7000$ *for* $\forall t \in [1,100]$............ Max. level for fixed coupon bond iss. in period t

35) $dk(t) \leq 7000$ *for* $\forall t \in [1,100]$............ Max. level for FRN coupon bond iss. in period t

36) $ek(t) \leq 4000$ *for* $\forall t \in [1,100]$............ Max. level for inf. coupon bond iss. in period t

37) $e(t) \leq 1500 * KUR(t)$ *for* $\forall t \in [1,100]$............ Max. level for inf. Eurobond iss. in period t

38) $p(t) \leq 800 * KUR(t)$ *for* $\forall t \in [1,100]$............ Max. level for inf. Project fin. in period t

39) $i(t) \leq 0.6 * b(t)$ *for* $\forall t \in [1,100]$......Max. percentage level for zer-cop.in total bor. in per. t

40) $sk(t) + sk(t+1)... + sk(t+11) \leq 0.25 * [b(t) + b(t+1).... + b(t+11)]$

$for \ \ \forall t \in [1,100]$......Max. percentage level for fix-cop.in total annual borrowing.

41) $e(t) + e(t+1) + e(t+2) \leq 1500 * KUR(t)$

$for \ \ \forall t \in [1,100]$......Max. percentage level for Eurobond in total quarterly borr.

42) $p(t) + p(t+1) + p(t+2) \leq 1000 * KUR(t)$

$for \ \ \forall t \in [1,100]$......Max. percentage level for prj. fin. in total quarterly borr.

43) $b(t) \leq 30000$ $for \ \ \forall t \in [1,100]$........... Max. level for total borrowing. in period t

44) $x(t) \geq 500$ $for \ \ \forall t \in [1,100]$........... Max. level for cash reserve level in period t

Since the LP model is very complicated, we decided to explain the Treasury debt management system through the model we have generated.

3.3 General Assumptions about LP Model

The LP model has been constructed on a monthly basis. In practice, issuances of bonds are performed in different dates and their maturity period is represented in days. For example in the issuance date, a zero coupon bond is said to be 600 days, not 20 months. But in order to simplify the problem, we had to construct the model on the monthly basis. Any issuance in one month is assumed to be done in the same time and this is also valid for all types of payments. In the cash management manner, there are daily cash flows in practice, but in this study we assume that all cash flows is done at the same time in a month. The parameters which represent the financial conditions are also assumed to be constant through the month. Therefore a period in the thesis is equivalent to one month and the model is horizon is set to be 100 months. In the LP model, the periods are represented by set item t which is from 1 to 100.

Another assumption about the LP model is related with the type of instruments. Local currency denominated instruments have been categorized under four different bonds, namely zero-coupon, fixed coupon, floating rate coupon and inflation indexed coupon bonds. Maturity period for these bonds have been restricted parallel to the real case. For this reason zero coupon bond maturities are between 12 and 24 months, whereas coupon bonds maturity between 36 to 72 months for fixed and inflation coupon bonds and 36 to 84 months for floating rate coupon bonds.

vi(v) maturity of zero-coupon bonds /12*24/

vsk(v) maturity of fixed-coupon bonds /36,48,60,72/

vdk(v) maturity of floating-coupon bonds /36,48,60,72,84/

vek(v) maturity of inflation-coupon bonds /36,48,60,72/

On the other hand, all types of foreign currency denominated bonds have been combined under Eurobonds. In practice, foreign currency denominated domestic bond issuance may occur with a very low percentage in total borrowing, but in this study we assume that there is no domestic foreign currency denominated bond issuance. Since maturities in international financial markets is higher, maturity period of Eurobonds are assumed to be 60,120,180 or 240 months. Program and project finance issue is a little bit complicated in practice, but in this thesis study they have been assumed to have the same cash flow structure with Eurobonds. Actually, project finance structure is very complicated in practice, but due to the very low proportion in borrowing and has a wide range, efforts to model them completely in this model parallel to real case will be inefficient. Maturity for project finance is chosen to be between 24 to 180 months. All foreign financing is assumed to be done on USD currency in the model.

ve(v) maturity of Eurobonds /60,120,180,240/

vp(v) maturity of project finance /24,36,48,60,72,84,96,108,120,132,144 ,156,168,180/

On the other hand, all coupon bonds coupon period is assumed to be 6 months. In practice this is valid for Eurobonds, Project/Program finance, Inflation indexed bonds and fixed coupon bonds. Floating rate coupon bonds have two alternatives, namely 3 months coupon payment or 6 months coupon payment, but this is not reflected to the model. All types of coupon bonds interest payments have been assumed to be done in 6 months.

3.4 Parameters of LP Model

Main issues of debt and cash management are borrowing, interest and principal payments. Amount of borrowing or the realization of interest payment is subject to change with respect to other factors. On the other hand, there are some external factors affecting the amount of payments and borrowing requirement in the system. In this thesis study these factors will be accepted as parameters given from market and it will be assumed that debt managers have no power to change them.

3.4.1 Parameters about Financial Conditions:

<u>Interest Rate for TL:</u>

Interest rate figures are assumed to be uncontrollable variables for the debt manager. In practice, interest rates are defined in the market with respect to some financial factors such as inflation rate expectation, Central bank interest rate, risk expectation etc. All interest payments of local currency bonds in the model are dependent to TL interest rates.

Interest rate values are sensitive to the maturity, so interest rate for 6 months is not equal to 12 months. The relation between interest rate and maturity is explained in yield curve. Yield curve shows all interest rate values for all valid maturities. Yield curve for TL interest rates in various dates are shown in Figure 5:

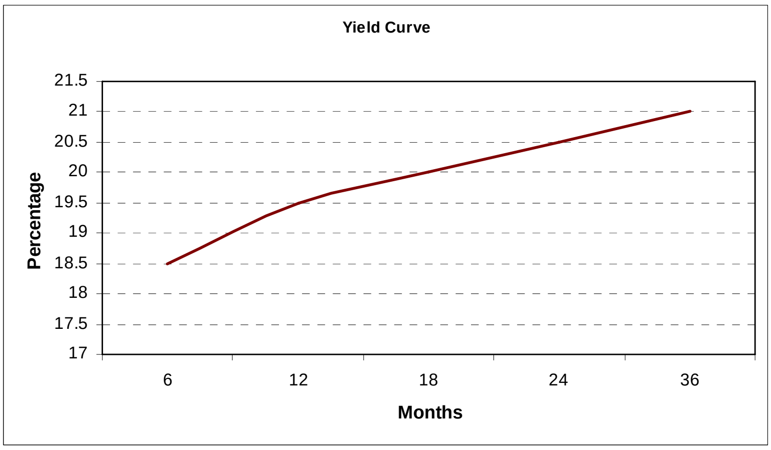


Figure 5: Yield curve for TL interest rates

The figures shown in the yield curve is the compounded 12 months equivalent interest rates for the given maturities. In the study we calculated the term interest rates from the yield curve for all possible maturities. The calculation of term interest rate formula is given below:

If v represents the maturity in months, $r_{annual}(v)$ represents the 12 month compounded interest rate for maturity v and $r_{term}(v)$ represents the term interest rate for maturity v, then:

$$r_{term}(v) = (1 + r_{annual}(v)/100)^{(v/12)} - 1$$

In the study we insert the future projections of TL interest rates for maturities between 1 and 84 months for the following 100 months horizon.

It is very important to underline here that; the aim of this study is neither to forecast the future interest rates nor to construct a proper yield curve. This thesis study's future estimations are not based on econometric analysis. The aim of the study is to construct a borrowing program that borrows through defined instruments and pays every debt in their exact payment date. The aim is to construct a model that tries to minimize the cost at given interest rates, currency etc.

TL interest rates are represented in the model with r(t,v), where t represents the period and v represents the maturity. The table is constructed with two dimensions and for a bond issued in period t' with maturity v', will pay the interest payment subject to that certain interest rate.

r(t,v) Term interest rates in period t for maturity v.

<u>Interest Rate for Foreign Currency:</u>

In financial markets every currency has their own interest rates. Therefore, foreign currency denominated bonds pay the interest subject to their interest rates. In this study we assumed that all foreign currency denominated bonds are in USD currency, so we will use only USD interest rates for Eurobonds and program/project financing.

As mentioned above Eurobonds and program/project finance interest payments are done in every six months. Therefore we conclude that they are only subject to 6 months USD interest rate in all periods. In this study, due to lack of predicting the future interest rate value for USD 6 months interest rate for 100 months, we will assume that this value will be constant for Eurobonds as 5 percent and for program/project finance as 4 percent.

re six month interest rate for Eurobond /0.050/

rp six month interest rate for project finance /0.040/

<u>Discount rate:</u>

In order to calculate the present worth of the accrued interests in the model we have to calculate a discount factor. Therefore the monthly GDP deflator change in year 2007 has been accepted to be discount factor. The value has been found as 0.0087 for a month period.

drate monthly discount rate /0.0087/

<u>Inflation Index:</u>

Inflation indexed bonds pay the interest with respect to the inflation index. Inflation index is calculated by adding the monthly inflation rate to the previous month's index. The index is announced by Turkey Statistics Agency (TÜİK) every month. In the study we insert the future projections of the inflation index with respect to Turkey Republic Central Bank inflation report for the following 60 months (five years). The remaining 40 months data have been assumed to be equal to the fifth years data.

ENFL(t) Inflation index in period t /

<u>Gross Domestic Product:</u>

The total market value final goods and services that are produced in a country in a period, investment and government spending and value of exports, minus the value of imports are defined as the Gross Domestic Product (GDP) of that country. GDP is generally accepted as an important indicator for economic power of a country. Financial and economic figures of a country such as expenses, revenues, debt stock are compared with the GDP generally.

In this study, for the sake of measuring the debt stock level over GDP, we have to generate GDP series for 100 months period. The GDP series are constructed by multiplying the growth estimation that has been announced in Medium Term Financial Program (OVMÇ). Again OVMÇ is announced for 60 months data, so the remaining part has been calculated by assumption in the inflation rate part.

GSYH(t) GDP value at the end of period t /

USD Exchange Rates:

As mentioned before, foreign currency denominated bonds are in USD currency. All cash transactions in the model are done in TL currency, so the issuance volumes and payments should be converted into TL in the periods. In order to calculate the currency changes, exchange rate of USD/TL is required for the following 100 months period. On the other hand, exchange rate is essential for total debt stock calculation.

In fact, it is very difficult to construct future estimations for USD/TL exchange rate. Here we have to remind that, this thesis study's future estimations are not based on econometric analysis. The aim of the study is to construct a borrowing program that borrows through defined instruments and pays every debt in their exact payment date. The aim is to construct a model that tries to minimize the cost at given interest rates, currency etc.

KUR(t) Exchange rate for USD/TL

3.4.2 Parameters about Cash Flows:

In the previous part, the parameters about the financial conditions have been explained. Other than the financial conditions, the model we have to insert some parameters related with the cash flows. These parameters are cash flows that are independent from the decision variables. These items are primary surplus and predetermined payments.

Primary Surplus:

As mentioned in the previous chapters, primary surplus has a direct effect on borrowing requirement. In fact, for any period t, interest and principal payments are financed from borrowing and primary surplus. Although, governments can set some rules about primary surplus, we accept that debt managers of government are outside of this primary surplus setting process. For this reason a debt manager should make his calculation by forecasting primary surplus values in the future, with respect to government policies, market and growth rate etc. The primary surplus series in this study have been prepared parallel to government Medium Term Financial Program (OVMÇ) announced in May 2008. But the values in the program have been calculated annually and then it has been distributed among months parallel to the previous year realizations. Primary surplus value for period t is represented by d(t) in the LP model.

D(t) primary surplus for period t /

<u>Predetermined Payments:</u>

Predetermined payments are result of outstanding debt stock. Previous borrowings cause some level of interest and principal payments in today and future, so we will assume that debt manager have nothing to change them. Actually, in modern debt management practices debt managers can change the profile with buy back or switching auctions, but this can not be applied to the payments more than %3 in practice. Therefore, it will be assumed that these are parametric variables that a debt manager can not have authority to make a decision on these payments.

Predetermined payments should be divided into groups, in order to insert their effects accurately in the model. First, predetermined payments should be divided into two groups as local currency predetermined payments and foreign currency predetermined payments.

Predetermined local currency payments include predetermined principal and interest payments. Local currency predetermined principal payments and interest payments originated from fixed or zero coupon bonds are exactly known and they are not subject to change. However, predetermined interest payments originated by floating rate or inflation indexed bonds are subject to change with respect to the interest rate and inflation index changes. But in our model we do not insert the distribution of outstanding stock, so it is impossible to calculate the change in predetermined interest payments. So in this study, all local currency predetermined payments are assumed to be deterministic parameters.

KC(t) predetermined local currency principal payment for period t/

KF(t) predetermined local currency interest payment for period t /

On the other hand, foreign currency predetermined payments are affected from exchange rates. Indeed, their foreign currency payments are exact but their equivalent value to TL is subject to change. So both foreign currency predetermined principal and interest payments are inserted to model by a different parameter other than the local currency payments. The values in the model which represents the predetermined foreign currency payments are in USD figures. In the stock calculation and summation of payments step, they are multiplied with the exchange rate of that period. This separation between foreign and local currency plays a key role in accurately calculating the debt stock.

KCD(t) predetermined foreign currency denominated principal payment for period t/

KFD(t) predetermined foreign currency denominated interest payment for period t/

3.5 Decision Variables of LP Model

Parallel to the practical case, all types of cash flows in the cash and debt management are dependent to each other. For example, if a debt manager decides to issue a bond in period t with maturity v; it means that at the same time he decides to pay a principal amount at period t+v. Moreover, the principal payment in period t+v will increase the borrowing requirement in that period and at the same time it means that the debt manager had also decided to increase the borrowing requirement in period t+v by issuing the bond in period t with maturity v.

Therefore, all kinds of issuances, payments and debts stock realizations are decision variables of the LP model, whether they are dependent or independent to each other. Due to the complicated structure of the LP model, the decision variables will be introduced under groups for the sake of better explaining the model.

3.5.1 Decision Variables about Instruments:

The ultimate aim of the LP model is to minimize the borrowing cost by setting an optimum issuance strategy among the available instruments. In order to explain the model in an effective way, first we will introduce the details about the instruments. From issuance to maturity, each instrument has their characteristics, so their issuance, interest and principal payments are evaluated separately. As mentioned before there are six type instruments in the LP model, namely zero-coupon bond, fixed rate-coupon bond, floating rate-coupon bond and inflation indexed coupon bond in local currency plus Eurobonds and program/project finance in foreign currency.

Zero coupon bonds:

Zero coupon bonds pay the principal and interest at the maturity date. So there are only two cash flows for this kind of bonds, first is positive at the issuance day, second is negative at the maturity date including both principal and interest payment.

In LP model, available zero coupon bond maturity is set to be between 12 and 24 months. Therefore in any period t, total issuance of zero-coupon bond is equal to the sum of all issuance of zero-coupon bonds with various maturities in period t. If ix(t,vi) represents the volume of zero-coupon bond issued in period t with maturity vi, and i(t) represents the total volume of zero-coupon bond issuance in period t, then;

$$i(t) = \sum_{vi=12}^{24} ix(t, vi) \qquad for \ \ \forall t \in [1,100]$$

In the payment side, first we have to explain the calculation of zero-coupon bonds interest payment. Simply, the interest payment of the zero-coupon bond is equal to the term interest rate times the principal. If r(t,v) represents the TL term interest rate for maturity v, and fi(t+v,v) represents the interest payment of a zero coupon bond with a maturity v in period t+v, then;

fi(t,v) = ix(t,v) * r(t,v)

Since the LP model is based monthly cash flows, we have to calculate the monthly total interest payments originated from zero coupon bonds in order to insert the monthly summation into the LP model. If fi(t) represents total interest payment originated from zero-coupon bonds in period t, then;

$$fi(t) = \sum_{vi=12}^{24} ix(t - vi, vi) * r(t - vi, vi) \quad for \ \ \forall t \in [1,100]$$

Principal payment is similar to interest payment calculation except for the interest rate multiplication. In period t, summation of all principal payments originated from zero-coupon bonds should be calculated. If ci(t) represents total principal payments originated from zero-coupon bonds in period t, then;

$$ci(t) = \sum_{vi=12}^{24} ix(t - vi, vi) \qquad for \ \ \forall t \in [1,100]$$

<u>Fixed rate-coupon bonds:</u>

Fixed rate coupon bonds are similar to zero-coupon bonds in terms of interest fixing in the issuance date. The interest rate is fixed all life time of the bond and all coupon payments are equal. Cash flow structure of fixed rate-coupon bonds are different than the zero coupon bonds. The payments in the fixed rate coupon bonds are not bullet type as in zero-coupon bonds. In this kind of bond, there are coupon (interest) payments in every six months till the maturity. The last coupon is paid with the principal payment at the maturity date.

In the LP model, there are four available maturities for fixed rate coupon bonds parallel to real case as 36, 48, 60 and 72 months. So, in period t, there can be at most four different instruments to be issued as a fixed rate coupon bond. Therefore total fixed rate coupon bond issuance in period t is the summation of the volumes of all available fixed rate coupon bonds issued in period t. If skx(t,vsk) represents the volume of fixed rate-coupon bond issuance in period t with maturity vsk and sk(t) represents the total volume of fixed rate coupon bond issuance in period t, then;

$$sk(t) = \sum_{vsk=36}^{72} skx(t, vsk) \qquad for \ \ \forall t \in [1,100]$$

Coupon (interest) payments of fixed rate coupons can be calculated easily by multiplying the six month TL term interest rate with the original principal amount. Since, interest rate is fixed at the issuance date, all coupon payments would be equal for fixed rate coupon bonds. If fsk(t,vsk) represents the coupon payment of a fixed rate coupon bond with maturity vsk in period t, t’ represents the issuance period of that fixed rate coupon bond and r(t’,’6’) represents the six months TL term interest rate for six months in period t’, then;

fsk(t,vsk) = skx(t’,vsk) * r(t’,’6’)

Since the model is based on the monthly cash flows, so we have to calculate the total coupon payments originated from fixed rate coupon bonds in period t. If fsk(t) represents the total coupon payments originated from fixed rate coupon bonds in period t, then;

$$fsk(t) = \sum_{vsk=36}^{72} skx(t - vsk, vsk) * r(t - vsk, '6') \qquad for \;\; \forall t \in [1,100]$$

the above formulation gives us the interest payments of fixed rate coupon bonds in period t. In fact, coupon bonds are not always issued at the par (face) value in the market, but in this study in order not to make the problem more complicated we assumed that all coupon bonds are issued at face value. But in real case, all fixed coupon bonds have a higher risk premium compared to floating rate coupon bonds, because of taking the risk of interest rate. So, for the sake of catching a parallel structure to the real case, 2 bp. risk premium is added six months term interest rates to the fixed rate coupon bonds

Principal payment of fixed coupon bonds is similar to the zero coupon bonds principal payment. If csk(t) represents the total principal payment originated from fixed rate coupon bonds in period t, then;

$$csk(t) = \sum_{vsk=36}^{72} skx(t - vsk, vsk) \qquad for \;\; \forall t \in [1,100]$$

Floating rate-coupon bonds:

Floating rate coupon (FRN) bonds are similar to fixed rate coupon bonds except for interest fixing structure. Different than the fixed rate coupon bonds, in this type of bond, interest rate changes in all coupon payments. Total volume of issuance calculation is similar to the previous instruments. If dkx(t,vdk) represents the volume of FRN bond issuance with maturity vdk in period t, and dk(t) represents the total volume of FRN bond issuance in period t, then;

$$dk(t) = \sum_{vdk=36}^{84} dkx(t, vdk) \qquad for \ \ \forall t \in [1{,}100]$$

Coupon payments are slightly different than the fixed rate coupon bonds, since the interest rate changes in all coupon payments. The effective interest rate in the coupon payment for an FRN bond is the six months before (time of the previous coupon payment) 6 months term interest rate. So if fdk(t,vdk) represents the coupon payment of a FRN bond with maturity vdk in period t, t' represents the issuance period of that floating rate coupon bond and r(t-6,'6') represent the TL term interest rate for six months in period t-6, then;

fdk(t,vdk) = dkx(t',vdk) * r(t-6,6')

Therefore, total interest payment originated from FRN bonds in period t can be calculated as follows. If fdk(t) represents total interest payment originated from FRN bonds in period t, then;

$$fdk(t) = \sum_{vsk=36}^{84} dkx(t - vdk, vdk) * r(t-6, '6') \quad for \ \ \forall t \in [1{,}100]$$

Principal payments of FRN bonds are similar to earlier mentioned instruments. So if cdk(t) represents the total principal payment in period t originated from FRN bonds, then;

$$cdk(t) = \sum_{vdk=36}^{84} dkx(t - vdk, vdk) \qquad for \ \ \forall t \in [1{,}100]$$

<u>Inflation indexed-coupon bonds:</u>

Inflation indexed coupon bonds are similar to earlier mentioned coupon bonds, except for the coupon payments, since their coupon payments are indexed to inflation rate. So to sum up, if ek(t) total volume of inflation indexed bonds issuance and ekx(t,vek) represents the volume of issued inflation indexed bond with maturity vek in period t; then;

$$ek(t) = \sum_{vek=36}^{72} ekx(t, vek) \qquad for \ \ \forall t \in [1,100]$$

Calculation of principal payments of inflation indexed coupon bonds is similar to other coupon bonds. If cek(t) represents the total principal payment originated from inflation indexed coupon bonds in period t, then;

$$cek(t) = \sum_{vek=36}^{72} ekx(t - vek, vek) \qquad for \ \ \forall t \in [1,100]$$

Coupon payments of inflation indexed bonds are a little bit complicated. These bonds coupon rates are also floating, but the coupon rate is not the interest rate. The coupon rate is defined by the changes in the inflation index. The spread over the inflation indexed bonds will be assumed to be 8 bp.

Interest payment of an inflation indexed bond is calculated by multiplying the inflation index rate of period t with defined spread of the paper, over inflation index rate of period t-6 times. So if fek(t,vek) represents the coupon payment of an inflation indexed bond in period t with maturity vek, enfl(t) represents the inflation index rate in period t and s represents the defined spread then;

fek(t,vek) = ekx(t,vek) * enfl(t) / enfl(t-6) * s

Total coupon payment of inflation indexed coupon bonds can be calculated in the formulation below. If fek(t) represents the total interest payment originated from inflation indexed bonds in period t, then;

$$fek(t) = \sum_{vek=36}^{72} ekx(t - vek, vek) * ENF(t) / ENF(t-6) * 0.08$$

$$for \ \ \forall t \in [1,100]$$

<u>Eurobonds:</u>

Obviously, Eurobonds are typical examples of fixed rate coupon bonds. The only difference for the Eurobonds is their issuance market. Eurobonds are issued in international markets in foreign denominated currency. In this study all Eurobonds are assumed to be issued in USD currency. In practical case, Eurobonds are issued with longer currencies than the local currency bonds. In this study we assume that Eurobonds are issued for 5, 10, 15 and 20 years maturity. If ex(t,ve) represents the volume of Eurobonds issuance in USD currency in period t with maturity ve, e(t) represents the total volume of Eurobonds issuance in period t in TL currency, and kur(t) represents the USD/TL exchange rate in period t, then;

$$e(t) = \sum_{ve=60}^{240} ex(t, ve) * KUR(t) \qquad \textit{for } \forall t \in [1,100]$$

Principal payment of Eurobonds is similar to other coupon bonds. If ce(t) represents the principal payments originated from Eurobonds in period t; then

$$ce(t) = \sum_{ve=60}^{240} ex(t - ve, ve) * KUR(t) \qquad \textit{for } \forall t \in [1,100]$$

Interest payments of Eurobonds are similar fixed rate coupon bonds except for the interest rate. As we mentioned before all currencies have their interest rates. In this study we assumed that the six month USD interest rate will be 5 percent through 100 months period. So the coupon payment of a Eurobond is the USD interest rate times the principal amount in every six months. If fe(t) represents the interest payment originated from Eurobonds in period t, re represents the USD 6 months interest rate, then;

$$fe(t) = \sum_{ve=60}^{240} ex(t - ve, ve) * re * KUR(t) \qquad \textit{for } \forall t \in [1,100]$$

<u>Project/Program Finance:</u>

Program and project financing is unique in the government borrowing. All previously mentioned instruments were kind of bonds, at the issuance they provide cash for Treasury. The previous bonds' financing were called budget finance, because the money is not tied up to any disbursement. In project finance government sings a loan agreement with a creditor and the creditor pays the cash for the disbursements that are defined in the loan agreement. Broadly speaking project financing is used by the government in order to finance a project.

On the other hand, some international institutions such as International Monetary Fund (IMF), World Bank etc. supplies program financing. Again in program financing there is a loan agreement, but in this type the financing directly enters to Treasury accounts as budget finance.

This topic has a broad range whereas their percentage in the government borrowing is very low. Especially the project finance disbursements, interest payments and principal payments are slightly different than the bonds. Contrary to their low percentage, their complicated structure is hard to model and estimate. Therefore in this study we assume that they are issued similar to Eurobonds with a lower interest rate parallel to practical case. All interest and principal payment are assumed to be similar with Eurobonds. The only difference is the available maturity for project finance in the model, since available maturities for program/project finance has been defined to between 4 to 15 years.

If, p(t) represents total cash supplied from program/project finance in period in TL, px(t,vp) represents the finance supplied from program/projects finance in period t with maturity vp in USD, then;

$$p(t) = \sum_{vp=24}^{180} px(t, vp) * KUR(t) \qquad for \;\; \forall t \in [1,100]$$

Principal and interest payments of the program and project financing are similar to Eurobonds. So if cp(t) represents the principal payments originated from program/project finance in period t in TL, fp(t) represents the interest payments originated from program/project finance in period t in TL and rp represents the 6 months USD interest rate for program/project financing, then;

$$cp(t) = \sum_{vp=24}^{180} px(t - vp, vp) * KUR(t) \quad for \;\; \forall t \in [1,100]$$

$$fp(t) = \sum_{vp=24}^{180} px(t - vp, vp) * rp * KUR(t) \qquad for \;\; \forall t \in [1,100]$$

3.5.2 Decision Variables about Total Cash Flows:

Figure 6 explains the complicated model in an easy way including all cash flows of the system. As mentioned in the previous part, the micro level decision variable is the issued instruments in the periods with various maturities.

Borrowing

These variable are represented by ix(t,vi), skx(t,vsk), dkx(t,vdk), ekx(t,vek), ex(t,ve), px(t,vp) notations in the model. For example, the summation of ix(t,vi) variables give the total volume of zero coupon bonds issuance in period t. This is valid for all other available instruments used in LP model. Summation of volume of all available instruments in period t, gives us the total borrowing made by Treasury in that period. If b_t represents the total borrowing of Treasury in period t, then;

$$b(t) = i(t) + sk(t) + dk(t) + ek(t) + e(t) + p(t) \qquad \textit{for} \;\; \forall t \in [1,100]$$

Principal Payment

Obviously all issued instruments should be paid back in their maturity date. In the previous part, we have already introduced the formulation of principal payment for each instrument. In the model all types of principal payments should be inserted to the equations under one variable, say c_t. Obviously, the only principal payments of Treasury is not originated by the instruments those are issued by the model. As mentioned before, there are predetermined principal payments both in local or foreign currency source from outstanding debt stock. Total principal payment in period t is the summation of predetermined payments to be paid period t and principal payments originated from the issued instruments by the model. It is obvious that, in the beginning of the model horizon predetermined payments form a higher proportion in the principal payment, since the principal payment of newly instruments would be made in the following horizons.

The last explanation about the principal payment is related with the foreign currency denominated principal payments. As underlined in the previous part, principal payment of newly issued Eurobonds and program/project financing is converted into local currency in the related equation. Since these payments are in local currency we can directly add them to other kind of local currency payments. On the other hand, foreign currency denominated predetermined payments is given in foreign currency in the model. So we have to convert them into local currency before we add up to total principal payments part. If c_t represents total principal payment in period t, then;

$$c(t) = ci(t) + csk(t) + cdk(t) + cek(t) + ce(t) + cp(t) + KC(t) + KCD(t) * KUR(t)$$

$$\text{for} \quad \forall t \in [1,100]$$

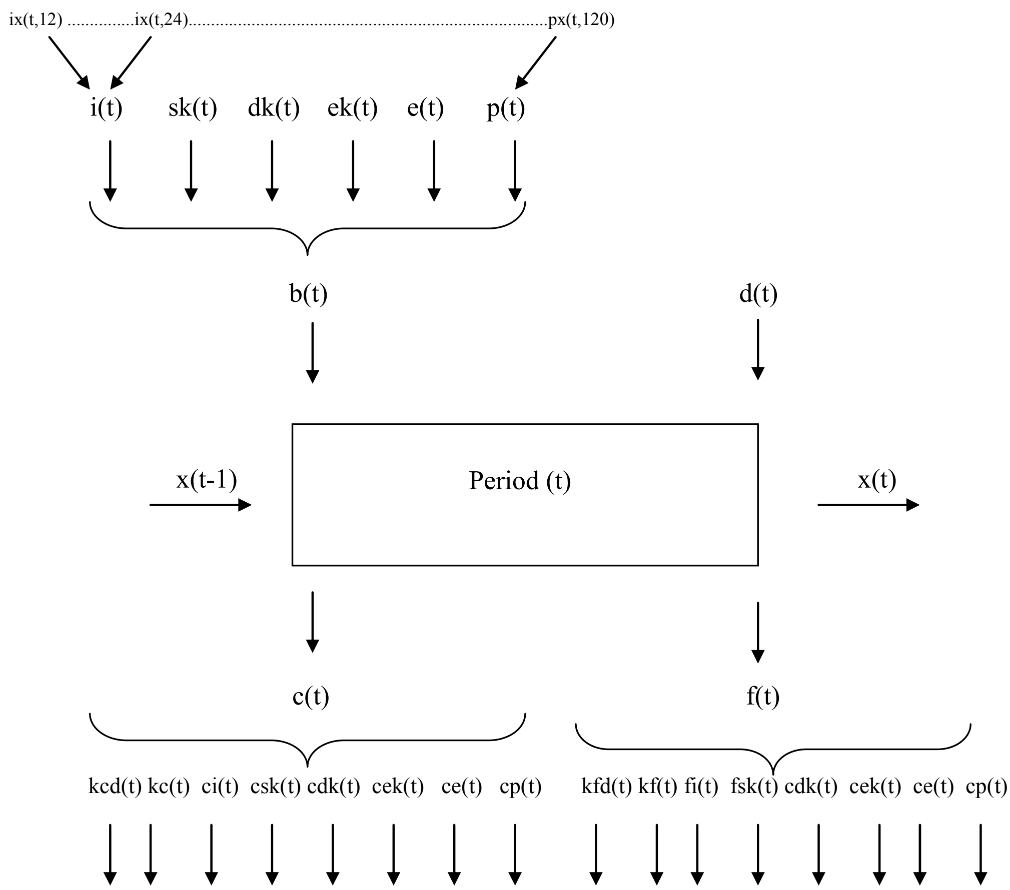


Figure 6: Diagram for All Cash Flows in LP Model

Interest Payment

Total interest payments calculation is similar to the total principal payment. Again foreign currency denominated predetermined payments should be converted into local currency and all interest payments shall be summed up.

$$f(t) = fi(t) + fsk(t) + fdk(t) + fek(t) + fe(t) + fp(t) + KF(t) + KFD(t) * KUR(t)$$

$$for \quad \forall t \in [1,100]$$

General Cash Balance Equation

For every period Treasury should meet the financing needs, so borrowing plus primary surplus plus previous cash reserve level should be equaled to total principal and interest payments plus new cash reserve level. We assume that at the beginning Treasury has 10 billion TL in cash reserves. If x_t represents the cash reserve level at the end of period t, then the general cash equilibrium can be stated as follows:

$$c(t) + f(t) + x(t-1) = b(t) + D(t) + x(t) \qquad for \quad \forall t \in [1,100]$$

3.5.3 Decision Variables about Debt Stock:

All borrowings of the government are recorded as an obligation in the government debt statistics. For that reason debt stock is an important indicator for debt sustainability. For example, a government can excessively borrow for ten years maturity and this may not affect its payments too much in short term, but debt stock shows the effect of this excessive borrowing. Therefore debt stock is an indicator showing the sustainability of government debt management. Simply debt stock is the summation of total principal payments that will be paid by the government in future.

The obligations of the government can be in different currencies. If total debt stock is calculated in local currency, then foreign currency denominated obligations should be converted to local currency. Otherwise, local currency obligations should be converted into foreign currency in order to calculate total debt stock in foreign currency. In this study total debt stock is calculated in local currency, so the formulations enabling to calculate the total debt stock are given below.

Simply, if government pays principal from its obligations, its debt stock decreases as the amount paid. On the other hand, if government borrows then the debt stock will increase. In the bellow part, the formulation of the total debt stock will be given. But first, we have to calculate the debt stock of foreign currency liabilities and local currency liabilities. If tlstok(t) represents the total local currency denominated debt stock, then;

$$tlstok(t) = tlstok(t-1) + i(t) + sk(t) + dk(t) + ek(t) - ci(t) - csk(t) - cdk(t) - cek(t) - KC(t)$$

$$for \quad \forall t \in [1,100]$$

Obviously, local currency debt stock can be calculated by adding the previous month's local currency debt stock to current months volume of local currency bond issuance and subtracting principal payments of current month's in local currency including the predetermined local currency principal payments. In this study for the starting horizon local currency denominated debt stock has been assumed to be 248 billion TL as of announced in September 2008.

On the other hand, foreign currency debt stock calculation is similar to the local currency debt stock except for the exchange rate effects. If fxstok(t) represents the total foreign currency denominated debt stock in local currency, then;

$$fxstok(t) = fxstok(t-1) * KUR(t) / KUR(t-1) + e(t) + p(t) - ce(t) - cp(t) - KCD(t) * KUR(t)$$

$$for \quad \forall t \in [1,100]$$

Since all total cash flows of foreign currency denominated bonds and financing have been converted into local currency in previous steps, we can easily sum up the issuances and subtract the principal payments. The only cash flow in USD is predetermined foreign currency principal payment, so it is converted into TL currency with the period's currency. In this study for the starting horizon foreign currency denominated debt stock in TL has been assumed to be 130 billion TL as of announced in September 2008.

After explaining the debt stock details analytically, we can combine them in the total debt stock figure. As both local and foreign currency denominated debt stock are represented in local currency, the total debt stock in local currency is the summation of these two. If stok(t) represents the total debt stock in period t, then;

$$stok(t) = tlstok(t) + fxstok(t) \qquad for \quad \forall t \in [1,100]$$

As mentioned before, total debt stock over GDP is a very important indicator for debt sustainability. Therefore in this study, we calculate the ratio for every period. If brcstk(t) represents the total debt stock over GDP ratio, then;

$$brcstok(t) = stok(t) / GSYH(t) * 100 \textit{ for } \forall t \in [1,100]$$

3.6 Objective of the LP Model

So far now, only cash flow and stock side of debt management have been explained. The LP model is capable of calculating all cash flows and debt stock of the government. But this is not sufficient to construct an LP model to minimize the cost of borrowing, because there is still an important matter of concern. We should first answer the question that what is the cost for Treasury in its borrowings.

As mentioned in previous chapters that cost of borrowing for Treasury is the interest payments. Also, cost of currency depreciation can be considered as cost of borrowing, but this will be neglected in the study. But if we accept that the objective of this model is to minimize the total interest payments in the horizon, we should face an **end-of horizon effect**. Since, especially for the zero-coupon bonds the total interests are paid at the end of the bond. Consider, a zero coupon bond issued in period 80 with maturity 24 months with a high volume. This borrowing will obviously cause some cost for Treasury, but our LP model shall evaluate that this borrowing has zero cost for Treasury because the interest payments is out of the horizon, if we would construct the model's objective to minimize the interest payments.

In order to eliminate the end of horizon effects and accurately measure the cost of borrowing, we shall minimize the **accrued interest** of the bonds. Accrued interest can be defined as the accumulated interest due to the principal investment.

For example, assuming with zero discount rate, if lender will earn 200 TL for a zero-coupon bond with 20 months maturity, the accrued interest for each month is equal to (200/20) 10 TL. This money is not paid at the end of each month in practice, but this interest is accumulated.

So the objective of the LP model is to minimize the total present worth of all accrued interests that are originated from the Treasury borrowing. For the sake of avoiding end of horizon effects, an accruement based objective function should be constructed. Even though interest payments are made in the usual way, the real cost of borrowing in the form of the accrued interests are computed separately.

Calculation of the accruement of interest payment is a little bit complicated, because each instrument has different types of accrued interest. Let's consider two bonds issued in the same date with different maturities and coupon types. Although they may have the same interest rate, they will have different accruement of interest due to various maturity.

For a zero coupon bond that is paying the interest at the maturity as a bullet payment, calculation of the accrued interest will be explained. If ai(t,v) represents the accrued interest of a zero coupon bond that has been issued in a certain period between t-v and t-1, drate represents the discount rate and the fi(t,v) represents the interest payment of this bond, then;

$$fi(t,v) = ix(t\text{-}v,v) * r(t\text{-}v,v)$$

The system can be explained in a standard zero coupon bond as follows:

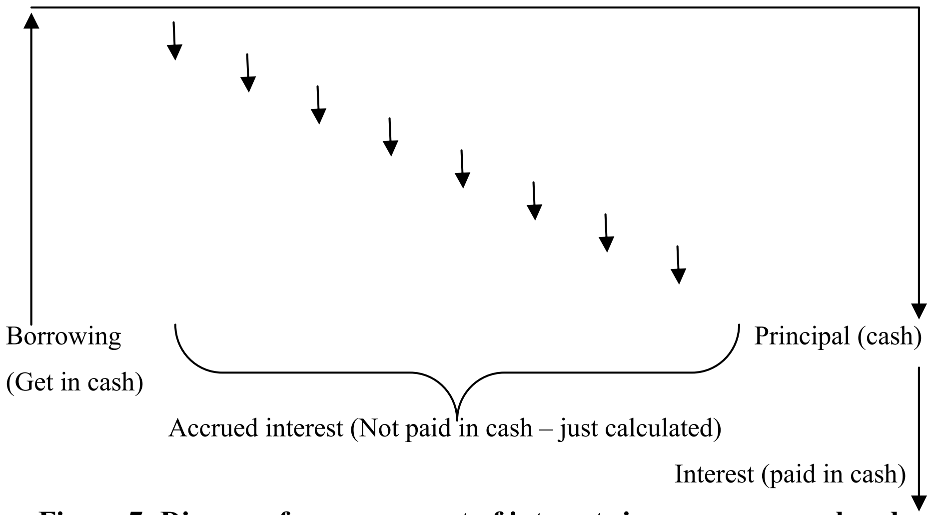


Figure 7: Diagram for accruement of interests in a zero coupon bond

Total future worth of all accrued interest of that bond should be equal to the interest payment of that bond at the maturity date, so;

$$fi(t,v) = ai(t\text{-}v\text{+}1,v) * drate^{v-1} + ai(t\text{-}v\text{+}2,v) * drate^{v-2} + \ldots.. + ai(t,v) * drate^{0}$$

As a general assumption in financial calculation, monthly accrued interests of a bond are equal for a bond in its lifetime. Hence;

$$fi(t,v) = ai(t\text{-}v+1,v) * (drate^{v-1} + drate^{v-2} + \ldots\ldots + drate^{1} + drate^{0})$$

is the equation for the accrued interest of zero coupon bond issued in period t-v with maturity v. Indeed, there can be other zero coupon bond's accrued interest at period "t-v+1". For the periods between the issuance and maturity of bond, there will be an accrued interest for that period for that bond.

A zero coupon bond can only generate accrued interest in period "t" under the following conditions:

i) Issued before period "t"

ii) Does not have maturity before period "t".

Only the third zero coupon bond generates accrued interest among the three zero coupon bonds. So if fai(t) represents total accrued interest originated from zero coupon bonds in period t, then by induction we can calculate fai(t) as follows. In period t, all zero coupon bonds that have been issued before period t and does not have the maturity before t should be taken into account. If we use ***α*** for parameters **drate**, then;

$$\left.\begin{array}{l}
ix_{t-1,v} * r_{t,v} = fai_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \ldots + \alpha^{0}) \\
ix_{t-2,v} * r_{t-2,v} = fai_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \ldots + \alpha^{0}) \\
ix_{t-3,v} * r_{t-3,v} = fai_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \ldots + \alpha^{0}) \\
. \\
. \\
. \\
ix_{t-v,v} * r_{t-v,v} = fai_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \ldots + \alpha^{0}) \\
+\overline{}
\end{array}\right\} \textit{V times equations}$$

$$fai_{t,v} = [ix_{t-1,v} * r_{t-1,v} + ix_{t-2,v} * r_{t-2,v} + \ldots + ix_{t-v,v} * r_{t-v,v}] \,/\, (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \ldots + \alpha^{0})$$

The formulation gives the total accrued interest of a zero coupon bond in period "t+1". Moreover, in this study we aim to minimize the present worth of total accrued interest. Therefore if ai(t) represents the present worth (value at period 1) of total accrued interest originated from zero coupon bonds in period t, then;

$ai_{t,v} = [ix_{t-1,v} * r_{t-1,v} + ix_{t-2,v} * r_{t-2,v} + \ldots + ix_{t-v,v} * r_{t-v,v}] / (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \ldots + \alpha^{0}) / (1+drate)^{t}$

For simplification in the calculation, we provide a formula for the part related with discount rate (α). Since the values related with the discount rate are the summation from zero to v, we provide the following equation for the set of maturities. In the parameters part of the LP model, the simple value for the discount rates is given below:

$$payda(v) = \alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \ldots + \alpha^{0}$$

Hence;

$$ai_{t,v} = [ix_{t-1,v} * r_{t-1,v} + ix_{t-2,v} * r_{t-2,v} + \ldots + ix_{t-v,v} * r_{t-v,v}] / payda(v) / (1+drate)^{t}$$

Accrued interest of coupon bonds has different calculation because of the coupon payments. For a coupon bond, not only the maturity date but also the coupon payment date values should be calculated for the accrued interest of the bond.

The problem with the coupon bonds accrued interest is originated from multiple interest payments. In coupon bonds, also the future worth of all coupon payments should be calculated. For ease of calculation, first we will explain the structure on fixed-rate coupon bonds. If β represents the coupon payments of a fixed coupon bond issued in period t-v, with maturity v, then;

$$\textit{Coupon payment} = skx_{t-v,v} * r_{t-v,v} = \beta$$
$$\textit{Future worth of all coupon payments} = \beta * \alpha^{v-6} + \beta * \alpha^{v-12} + \ldots + \beta * \alpha^{0}$$
$$\textit{Future worth of all coupon payments} = \beta * (\alpha^{v-6} + \alpha^{v-12} + \ldots + \alpha^{0})$$

The accruement system can be explained in Figure 8 for a standard coupon bond as follows:

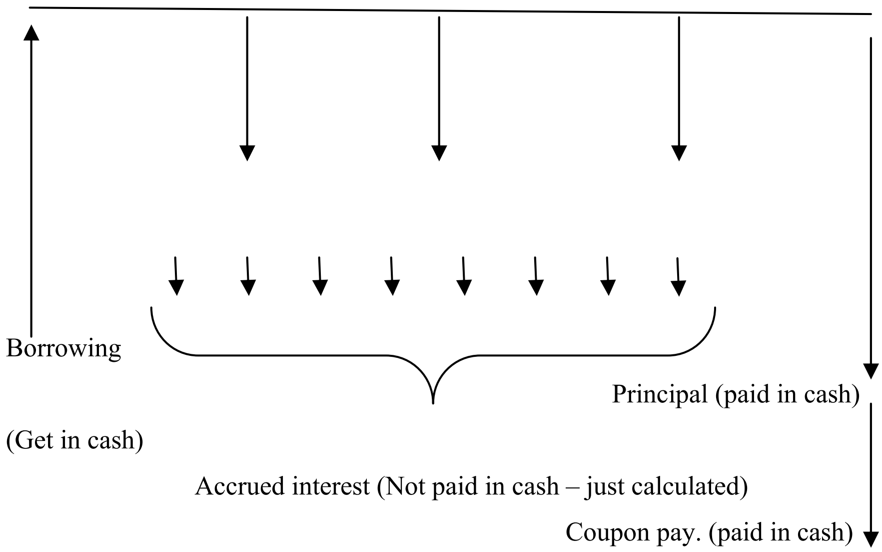


Figure 8: Diagram for accruement of interest in a coupon bond

Therefore the future worth of the coupon payments should be equal to the future worth of all accrued interest of that coupon bond within the maturity period. If fask(t,v) represents the accrued interest originated from fixed coupon bond, then;

$$\beta * (\alpha^{v-6} + \alpha^{v-12} + \dots + \alpha^{0}) = fask_{t+1,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \dots + \alpha^{0})$$

Hence,

$$skx_{t-1,v} * r_{t-1,v} * (\alpha^{v-6} + \alpha^{v-12} + \dots + \alpha^{0}) = ask_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \dots + \alpha^{0})$$

From the induction;

$$skx_{t-1,v} * r_{t-1,v} * (\alpha^{v-6} + \alpha^{v-12} + \dots + \alpha^{0}) = fask_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \dots + \alpha^{0})$$

$$skx_{t-2,v} * r_{t-2,v} * (\alpha^{v-6} + \alpha^{v-12} + \dots + \alpha^{0}) = fask_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \dots + \alpha^{0})$$

$$skx_{t-3,v} * r_{t-3,v} * (\alpha^{v-6} + \alpha^{v-12} + \dots + \alpha^{0}) = fask_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \dots + \alpha^{0})$$

.

.

$$skx_{t-v,v} * r_{t-v,v} * (\alpha^{v-6} + \alpha^{v-12} + \dots + \alpha^{0}) = fask_{t,v} * (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \dots + \alpha^{0})$$

+____

$fask_{t,v} = [skx_{t-1,v} * r_{t-1,v} + skx_{t-2,v} * r_{t-2,v} + \ldots + sk_{t-v,v} * r_{t-v,v}] / (\alpha^{v-1} + \alpha^{v-2} + \alpha^{v-3} + \ldots + \alpha^{0}) * (\alpha^{v-6} + \alpha^{v-12} + \ldots + \alpha^{0})$

The general accrued interest calculation for coupon bonds is given above. Therefore the only difference between zero coupon and coupon bonds is the ($\alpha^{v-6} + \alpha^{v-12} + \ldots + \alpha^{0}$) coefficient. In order to insert this coefficient to the LP model for the coupon bonds, we produced a new parameter called pay(v).

$pay(v) = \alpha^{v-6} + \alpha^{v-12} + \ldots + \alpha^{0}$ where v>6;

Hence, if ask(t) represents the present worth (value at period 1) of total accrued interest originated from fixed coupon bonds in period t, then;

$ask(t) = [skx_{t-1,v} * r_{t-1,v} + \ldots + sk_{t-v,v} * r_{t-v,v}] * pay(v) / payda(v) / (1+drate)^{t}$

After calculating this data, all of the accrued interest rates should be collected in the objective function. Therefore, we can directly sum the present worth of all accrued interest rates in the periods. Therefore the objective function of the model is very simple:

$$\text{Min} \sum_{t=1}^{100} (ai_t + ask_t + adk_t + aek_t + ae_t + ap_t)$$

3.7 LP Model Constraints

Unfortunately, debt managers are not free of financial markets. They have to do all of their movements in the market which has some boundaries. Issuance of bonds is performed in a supply-demand system. A debt manager can only sell the instruments which have buyers in the market. No where in the world, debt managers are free to decide on the instrument only parallel to their expectations. As being a game theory issue, a debt manager tries to minimize its cost but an investor tries to maximize his profit, so the decision map has some boundaries for debt managers.

Obviously, it is impossible to mathematically define all restrictions of the market and it need not be, but a rough restriction map can be defined for instruments that will be issued in a period. These can be stated as follows:

Due to the investor choices, every instrument has different demands in the financial markets. Therefore, each instrument type has a rough upper limits for monthly, quarterly and annually issuances. Zero coupon bonds have the highest demand in the market, but also they have an upper limit for borrowing because their interest payment increases the total interest payment at the time of maturity very sharply. Therefore for zero coupon bonds, the monthly maximum volume of issuance is 12 billion TL and up to sixty percent of borrowing can be supplied by zero coupon bonds in a month.

$$i(t) \leq 12000 \qquad for \quad \forall t \in [1,100]$$

$$i(t) \leq 0.6 * b(t) \qquad for \quad \forall t \in [1,100]$$

For the fixed coupon bonds, there is more restriction due to the insufficient demand in the market. In Turkey financial market, investors do not want to invest their money for a long term at a fixed rate, so the upper limit for fixed coupon bonds are less than zero and FRN coupon bonds. Therefore, the upper limit for a fixed coupon bond is 7 billion TL and thirty five percent of borrowing in that month. Moreover there is an annual limit for fixed coupon bonds, up to twenty five percent of annual borrowing can be supplied through fixed rate coupon bonds.

$$sk(t) + sk(t+1)... + sk(t+11) \leq 0.25 * [b(t) + b(t+1).... + b(t+11)] \qquad for \quad \forall t \in [1,100]$$

$$sk(t) \leq 7000 \qquad for \quad \forall t \in [1,100]$$

$$sk(t) + sk(t+1)... + sk(t+11) \leq 0.25 * [b(t) + b(t+1).... + b(t+11)] \qquad for \quad \forall t \in [1,100]$$

For FRN bonds, we have assumed to be only one limit that is the monthly upper limit due to the liquidity conditions. Therefore up to 7 billion TL, Treasury can issue FRN in one month. Moreover, inflation indexed bonds have the same conditions with the upper limit of only 4 billion TL in one month.

$$dk(t) \leq 7000 \qquad for \ \ \forall t \in [1,100]$$

$$ek(t) \leq 4000 \qquad for \ \ \forall t \in [1,100]$$

For the foreign currency denominated bonds, there is both monthly and quarterly ceilings due to the limited demand in the international markets. When the issuance type of Eurobonds is analyzed it is understood that generally the issuance are done once a quarter. Therefore the following restrictions exist for foreign currency denominated financing and bonds.

$$e(t) \leq 1500 * KUR(t) \qquad for \ \ \forall t \in [1,100]$$

$$p(t) \leq 800 * KUR(t) \qquad for \ \ \forall t \in [1,100]$$

$$e(t) + e(t+1) + e(t+2) \leq 1500 * KUR(t) \qquad for \ \ \forall t \in [1,100]$$

$$p(t) + p(t+1) + p(t+2) \leq 1000 * KUR(t) \qquad for \ \ \forall t \in [1,100]$$

Finally, Treasury has an upper limit of borrowing in a month because of liquidity conditions in the market. Obviously it is nearly impossible to define certain limits, but for Turkish Treasury finding 30 billion TL in one month by borrowing is nearly impossible. In order to converge the practical case, we have defined an upper limit for Treasury as 30 billion TL in one month. Also Treasury should have to have a minimum amount of money in the cash accounts. In this study very lower than the practical case we defined to have at least 0,5 billion TL in the reserves.

$$b(t) \leq 30000 \qquad for \ \ \forall t \in [1,100]$$

$$x(t) \geq 500 \qquad for \ \ \forall t \in [1,100]$$

CHAPTER 4

COMPUTATIONAL RESULTS AND SENSITIVITY ANALYSIS

In this chapter, the LP model is performed under base case scenario model, of which estimations have been given in previous chapters. After that, in order to measure the sensitivity of the debt management figures to the parametrical changes in the financial conditions, we have tested the results under various scenarios. The model output includes all cash flows of Treasury in the following 100 months. For every period, borrowing instruments have been issued in order to finance the borrowing requirements. The model evaluates the interest payments and principal repayments of both existing stock and newly borrowed instruments.

4.1 The Base Case Scenario

In the base case scenario, the required data has been inserted to the model parallel to the information given in the previous chapters. The interest rates have been assumed to be stable till end of horizon, meaning the same yield curve for all periods. The yield curve of TL borrowing in the model is accepted to be positively sloped. It is assumed that there is no yield curve for foreign borrowing, the semi annual rates is 0,05 percent for Eurobond and 0,04 for Project/program financing as previously mentioned.

After performing the model, successful results have been gained from model. Under the base case scenario for the following 100 months, total value of present worth of accrued interests have been minimized to 259,9 billion TRL.

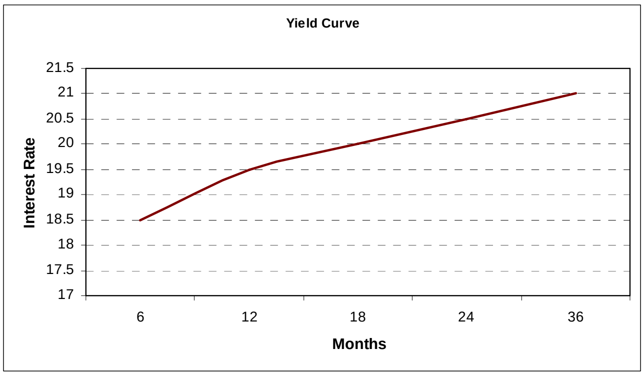


Figure 9: Yield Curve of TL Borrowing

In the programming horizon, borrowing requirement of Treasury has increased year to year parallel to the growing economy. Monthly borrowing requirement of Treasury has some fluctuations due to the varying primary surplus and predetermined payment values. On the other hand, annual borrowing requirement has been smoothed with eliminating yearly important fluctuations in the model.

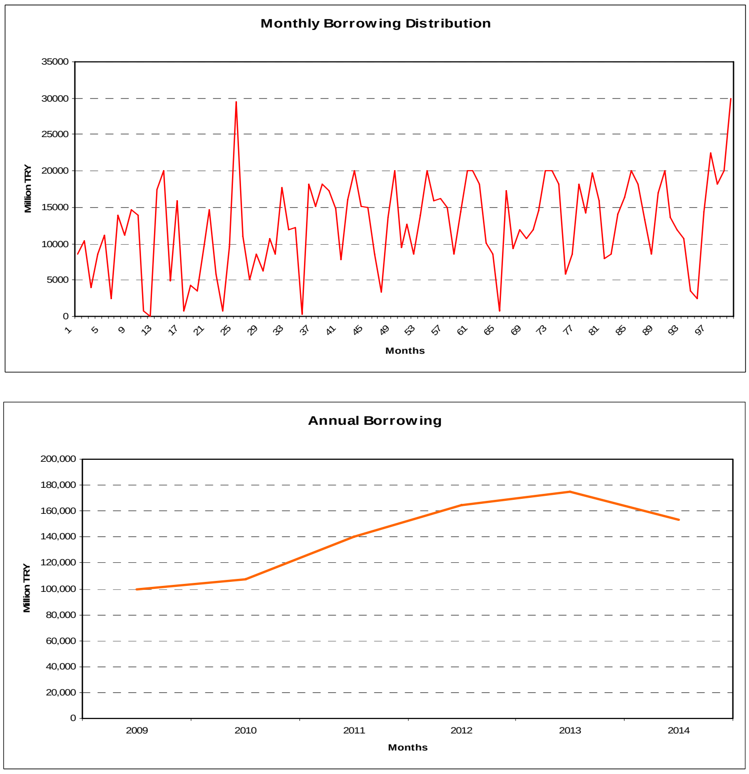
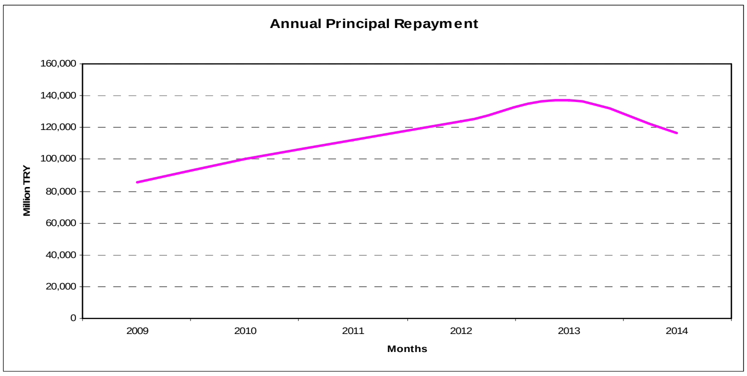


Figure 10: Monthly and Annual Borrowing Distribution of Treasury

Parallel to the borrowing requirement, debt service increases year to year due to the rising of nominal debt stock. In fact, upper shift in borrowing volumes causes a rise both in principal and interest payments. In the model especially in year 2011, there is a sharp rise in principal and interest payments originating from the issued 20-24 month zero coupon bonds which will be matured in 2011. After year 2011 the monthly amortization schedule has been smoothed since the effect of predetermined payments decreases after 2011.



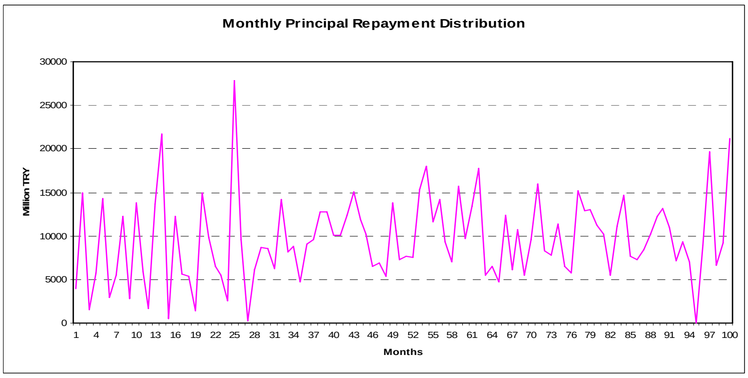
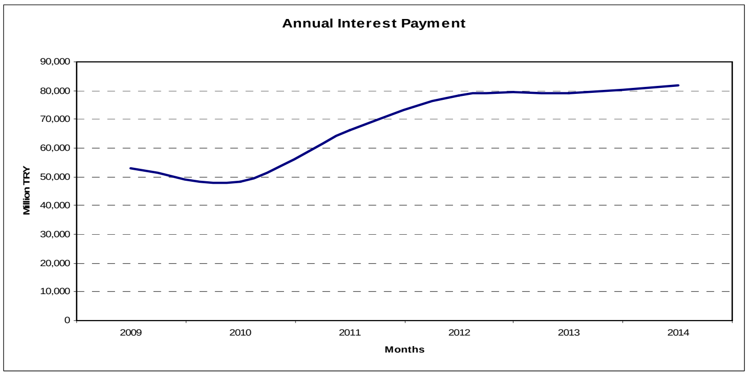


Figure 11: Annual and Monthly Principal Repayment

Annual interest payments have been stabilized in a level which means that the interest payments will be kept in a limit. Monthly interest payments have a curvy structure due to the zero coupon interest payment allocation. Consequently, monthly **total debt service** level has been kept between 10 billion and 25 billion TL for the programming horizon.



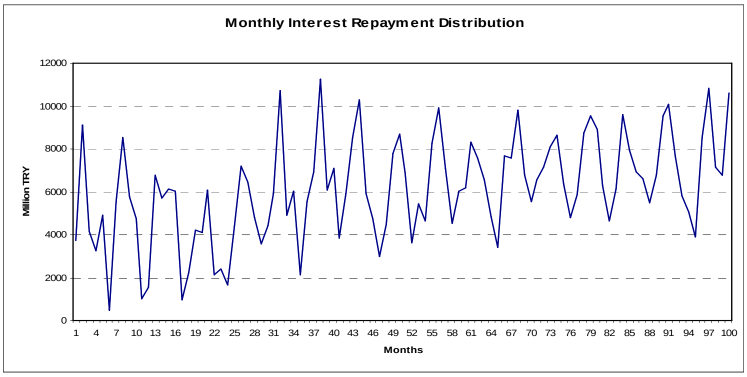


Figure 12: Annual and Monthly Interest Payments

After explaining the cash flow details, the roll over ratios can be examined. As mentioned in the above chapters low roll-over ratios are vital for effective debt management. Roll over ratios of Treasury is stabilized within 75 – 85 percentage bands, enabling to pay some level of interest without borrowing.

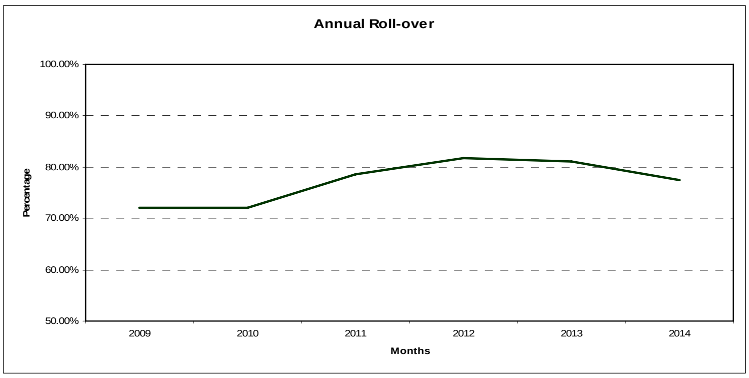


Figure 13: Annual Roll-over ratios

The above parts were related with effective cash flow management, but in order to analyze the effective debt management in sustainability perspective, we have to analyze some ratios. The most common ratios are accepted as debt stock over GDP and interest payments over GDP. Within the given financial conditions, sustainability of Treasury debt management has been tested in the model. The debt stock/GDP has been declined in the six year period and interest payments have been stabilized enabling the sustainability of the stock.

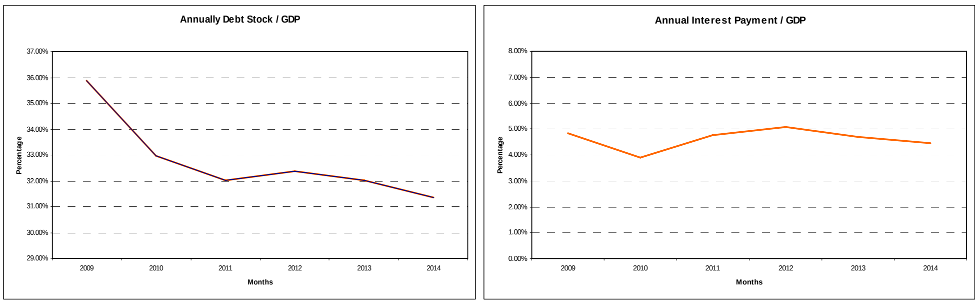


Figure 14: Debt Stock /GDP and Interest Payments /GDP

The borrowing profile of the solution gives us that; most of the borrowing has been made by zero coupon bonds (nearly 60 percent) with maturities between 17-24 months regarding the market conditions and yield curve. On the other hand, fixed coupon bonds and floating rate bonds have been equally distributed because of the market restrictions and deterministic-stable interest rate profiles. The maturity choice of the model is the longest available maturity for both fixed and floating rate instruments, respectively 72 and 84 months.

Inflation indexed bonds have not been chosen to be issued by the model since it yields more costs to Treasury compared to other instruments. In practice, Treasury issues this instrument in order increase the investor base and also due to the parallel structure in the increase of inflation and government incomes, the inflation indexed bonds are hedged to the risks. However, the model has no risk perception yet and also the attraction of investors is not a subject of this thesis study. Therefore, evaluating only the cost perspective the inflation indexed bonds have not been chosen to be issued in the model.

Eurobond and project financing have been supplied in the upper bound of the market limits. Both instruments have been chosen to be issued in the longest available maturity, respectively 20 and 15 years. Consequently, the principal repayments of the issued Eurobond and project/program finance credits have not been repaid in the model horizon enabling to decrease the roll-over rates.

The final discussions on the model result are about the cash reserve levels. As mentioned in the above parts, Treasuries have a certain level of cash reserve in order to decrease the level of refinancing risk. In the model, cash reserve level has been set as 500 million TL representing a minimum amount of cash reserve level. According to the model results, most of the months cash reserve level does not exceed the minimum amount level excluding a few number of months which have high level of debt service because of fluctuated predetermined payments.

Table 5: Results of Base Case Scenario

	2009	2010	2011	2012	2013	2014
Total Borrowing	99,614	107,017	140,173	164,852	174,943	153,503
TL Borrowing	84,602	91,008	123,165	146,843	155,933	134,498
FX Borrowing	15,000	15,987	16,980	17,986	18,977	18,977
USD	10,000	9,992	9,988	9,992	9,988	9,988
Total Debt Service	138,146	148,345	178,473	201,797	215,962	198,303
Principal	85,290	99,950	112,362	123,630	136,941	116,474
Interest	52,856	48,395	66,111	78,167	79,021	81,829
TL Principal	68,672	72,856	100,250	113,720	129,598	105,547
TL Interest	45,125	39,853	56,857	67,291	66,392	67,863
FX Principal	16,621	27,099	12,114	9,907	7,344	10,927
FX Interest	7,733	8,549	9,260	10,885	12,640	16,565
Debt Stock	392,324	407,949	443,623	492,998	539,598	576,624
Stock / GDP	35.87%	32.98%	32.00%	32.37%	32.03%	31.34%
Roll Over	72.11%	72.14%	78.54%	81.69%	81.01%	77.41%
Interest/GDP	4.83%	3.91%	4.77%	5.07%	4.69%	4.45%

The deterministic model has strengths in smoothing the amortization schedule and minimizing the cost of interest payments. However, robustness of the model results should be tested in a manner to analyze the effects of changes in the financial conditions. The financial conditions that have the probability to vary can be listed as primary surplus, interest rate, exchange rate and yield curve structure. In Appendix C more explanatory figures can be found about base case scenario results.

4.2 Stress Test under Various Scenarios

In this part, LP model will be tested under various scenarios. In each scenario, only one financial parameter will be changed, and the effect of this change to the model results will be analyzed.

4.2.1 Effect of Changes in Primary Surplus

In order to identify the effects of primary surplus changes to the robustness of model, the model will be tested under 10 and 20 percent discounted primary surplus values. All other financial conditions will be assumed to be not changed (not sensitive to the primary surplus changes) in the planning horizon.

The objective function value has been found as 280,5 billion TL in 10 percent reduction case. A reduction in primary surplus directly affects the borrowing requirement, since a reduction in primary surplus is compensated by borrowing. The rise in the borrowing requirement results in a rise in debt service of following years. The total effect of primary surplus increase is a rise in all debt service and debt stock values. Since the FX borrowing has been supplied in its upper limit in the base case scenario, the primary surplus reduction put pressure on TL borrowing. Since more borrowing causes more debt service, the following year's borrowing requirement increases consecutively. The increase in the principal and interest is given in the table below.

Table 6: Effect of 10 percent reduction in primary surplus on borrowing requirement, interest and principal repayment

	2009	2010	2011	2012	2013	2014
Total Borrowing	2.64%	4.81%	4.24%	8.47%	9.30%	10.67%
TL Borrowing	3.11%	5.65%	4.83%	9.51%	10.43%	12.17%
FX Borrowing	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
USD	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Total Debt Service	0.04%	0.24%	1.37%	5.09%	5.63%	6.00%
Principal	0.00%	0.00%	1.12%	5.30%	5.59%	5.67%
Interest	0.11%	0.72%	1.79%	4.75%	5.71%	6.46%

Debt service change is less than the borrowing requirement because of the issuance of instruments that have maturities longer than one or more years. Higher volumes of borrowing also yield a higher debt stock ratio. Interest payments over GDP ratios slightly increase in this case. Debt stock over GDP ratio can not be decreased below 33 percent level in this scenario and roll over ratios increases 3 percent point compared to base case scenario. Cash reserve levels remain unchanged, because the effect of primary surplus reduction directly reflects on the monthly distribution figures.

Table 7: Effect of 10 percent reduction in primary surplus on debt stock and roll over ratios

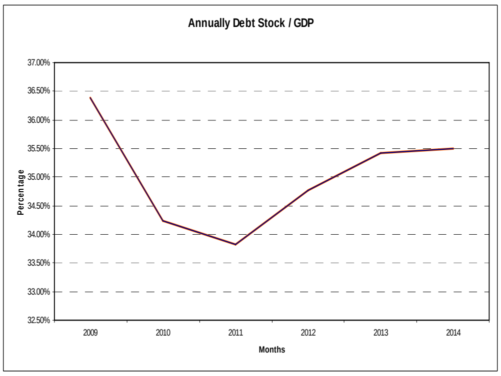
	2009	2010	2011	2012	2013	2014
Debt Stock	0.67%	1.91%	2.81%	4.03%	5.28%	6.63%
Stock / GDP	0.24%	0.63%	0.90%	1.23%	1.69%	2.08%
Roll Over	1.87%	3.29%	2.23%	2.63%	2.81%	3.41%
Interest/GDP	0.01%	0.03%	0.09%	0.24%	0.27%	0.29%

When the 20 percent reduction of primary surplus case is analyzed, the effects on the debt stock and debt service is approximately doubled, whereas the effect on borrowing requirement is slightly less than them since the nominal reduction in primary surplus is comparatively less than the total borrowing requirement of the model. Objection function value has been found as 301 billion TL in this scenario. The results of 20 percent reduction in primary surplus are given in Table 8.

As a final assessment regarding the primary surplus, the borrowing requirement increases first, then feeding mechanisms force debt service and debt stock figures to increase. However, neither shares of zero coupon bonds and coupon bonds nor foreign borrowing instruments change. Regarding the debt sustainability the figures seems to be worsened, especially the debt stock over GDP and interest payments over GDP figures shows the negative effects on sustainability of debt management.

Table 8: Effect of 20 percent reduction in primary surplus to the general debt management figures

	2009	2010	2011	2012	2013	2014
Total Borrowing	5.53%	9.10%	8.57%	16.60%	19.50%	21.14%
TL Borrowing	6.51%	10.69%	9.76%	18.63%	21.88%	24.12%
Total Debt Service	0.09%	0.11%	2.94%	9.90%	12.00%	11.84%
Principal	0.00%	-0.40%	2.23%	10.24%	12.30%	11.32%
Interest	0.24%	1.15%	4.13%	9.36%	11.48%	12.59%
Debt Stock	1.41%	3.84%	5.67%	8.08%	10.59%	13.25%
Stock / GDP	0.50%	1.26%	1.81%	2.40%	3.39%	4.15%
Roll Over	3.92%	6.48%	4.30%	4.98%	5.43%	6.43%
Interest/GDP	0.01%	0.04%	0.20%	0.47%	0.54%	0.56%



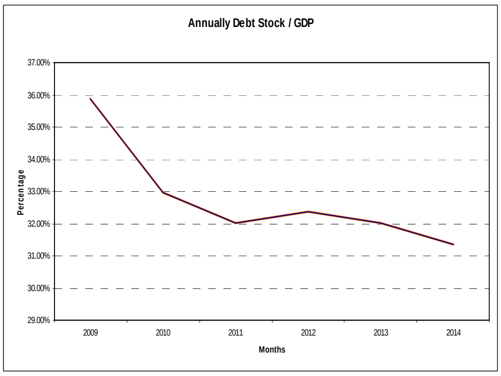
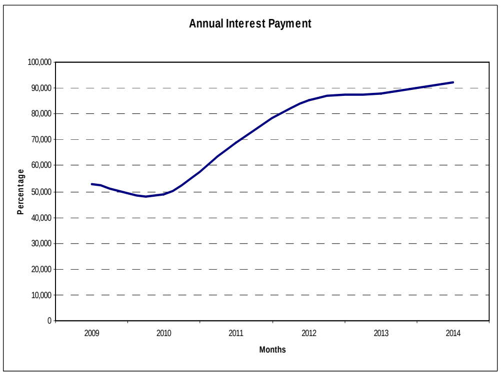


Figure 15: Comparison of Debt stock / GDP ratios in Base vs 20 percent PS reduction (Base on the right)



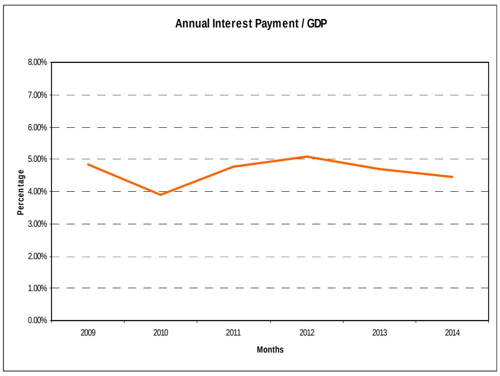


Figure 16: Comparison of Interest payments in Base vs 20 percent reduction (Base on right)

4.2.2 Effect of Changes in the TL Interest Rates

As mentioned above, the most influential factor on the cost of borrowing is the interest rates since the cost in the system is originated from the interest payments. The robustness of the model with respect to the interest rate changes is very important for debt sustainability analysis. In the base case scenario, a yield curve has been established and the interest rates have been defined as between 18,5 on the short side to 20,5 on the long side. In the sensitivity analysis of interest rates, 2 and 4 points shifts in the yield curve will be tested. It should be noted that, this is not related with the yield curve structure; just it is related with the level of yield curve.

First, a 2 point shock in the interest rates is tested in the model. 2 points shift in the yield curve made objective function value 300 billion TL. Unlike, the primary surplus effects, interest rate shifts directly affect the interest payments, and then the rising of interest payments increases the borrowing requirement. 2 points increase in the interest rates has been chosen since it is nearly equivalent to 10 percent increase in the interest rates similar to primary surplus stress test. However, the effect of 10 percent changes in the interest rates has a greater size impact on the objective function value.

In the 2 points interest rate shock scenario, in the first years borrowing requirements do not increase sharply, but because of higher interest payments in the following years, borrowing requirement starts to rise. This is same for principal repayments, but they are less sensitive to the interest rate shock, because of long maturity instruments. On the other hand, interest payments are subject to the highest impact starting from the first year and the variance from the base case scenario exceeds 12 percent in the 2 points shock scenario.

On the debt stock manner, the effects of interest rate change is a little bit lower than the primary surplus values. In fact, the roll-over ratios does not change as much as in the primary surplus case, but the debt stock realizations are close to the primary surplus scenario.

Table 9: Effect of 2 points increase in interest rates on borrowing requirement, interest and principal repayment

	2009	2010	2011	2012	2013	2014
Total Borrowing	0.09%	0.80%	3.98%	6.41%	11.27%	8.36%
TL Borrowing	0.10%	0.93%	4.53%	7.19%	12.64%	9.54%
Total Debt Service	0.07%	0.53%	3.16%	5.23%	9.13%	6.47%
Principal	0.00%	0.00%	0.86%	2.37%	6.16%	1.89%
Interest	0.18%	1.62%	7.06%	9.77%	14.28%	12.99%

Table 10: Effect of 2 points increase in interest rates on debt stock figures

	2009	2010	2011	2012	2013	2014
Debt Stock	0.02%	0.23%	1.25%	2.67%	4.53%	6.09%
Stock / GDP	0.01%	0.08%	0.40%	0.77%	1.45%	1.91%
Roll Over	0.01%	0.19%	0.63%	0.91%	1.59%	1.37%
Interest/GDP	0.01%	0.06%	0.34%	0.49%	0.67%	0.58%

In the next step, effect of 4 points change in the interest rates is examined. The objective function value rises to 344 billion TL which yields really important rise compared to primary surplus results. As mentioned above the most tremendous effect of interest rate increase is on interest payments. Then the higher level of interest payments increases the borrowing requirement and so on. Debt stock figures also increases because of increasing level of borrowing requirement.

General structure of instrument issuance does not change in the solution; still zero coupon bonds have the highest share in the portfolio. Additionally, the maturity profile of the issued instruments is same in the base case scenario. Increase in the interest rates has negative effects on debt sustainability. Therefore, debt sustainability over GDP and interest payments over GDP ratios has important distortions. Roll-over structure is not affected too much, because the borrowing requirement and payments increases parallel in this scenario.

Table 11: Effect of 4 points increase in the interest rates to the general debt management figures

	2009	2010	2011	2012	2013	2014
Total Borrowing	0.17%	1.52%	6.60%	15.42%	16.14%	13.04%
TL Borrowing	0.20%	1.78%	7.52%	17.30%	18.11%	14.88%
Total Debt Service	0.14%	0.97%	5.28%	12.59%	13.08%	10.09%
Principal	0.00%	-0.09%	0.46%	7.09%	5.86%	-0.89%
Interest	0.36%	3.15%	13.48%	21.30%	25.59%	25.74%
Debt Stock	0.04%	0.46%	2.40%	5.53%	8.80%	11.89%
Stock / GDP	0.02%	0.15%	0.77%	1.48%	2.82%	3.73%
Roll Over	0.02%	0.40%	0.98%	2.05%	2.20%	2.07%
Interest/GDP	0.02%	0.12%	0.64%	1.08%	1.20%	1.14%

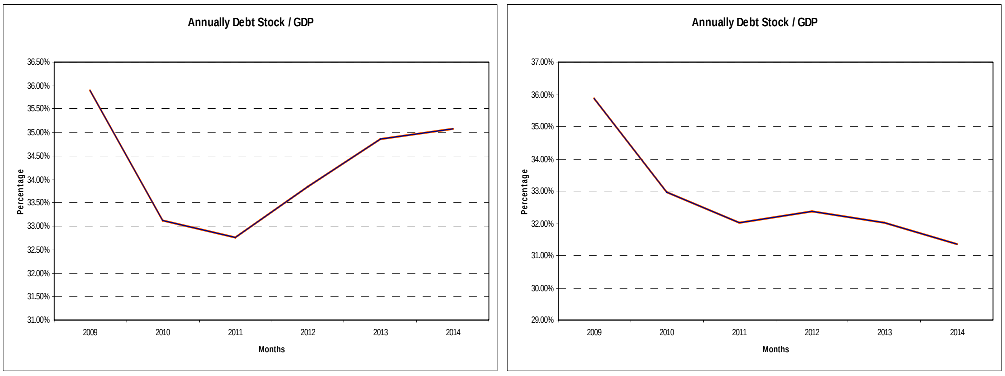


Figure 17: Comparison of Debt stock / GDP ratios in Base vs 4 points interest rates increase (Base on the right)

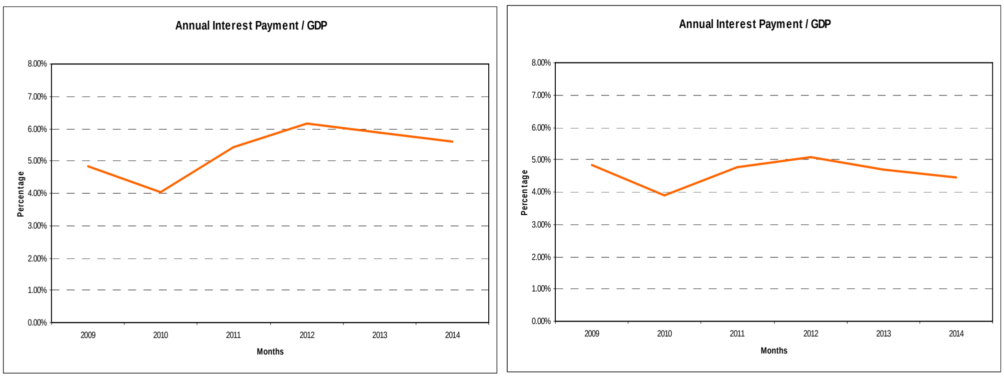


Figure 18: Comparison of Interest payments in Base vs 4 points interest rates increase (Base on right)

4.2.3 Effect of Changes in USD Exchange Rate

One of the most effective financial figures on the debt sustainability is the exchange rates. Exchange rates changes affect the debt management figures on the interest and principal payments of which are foreign currency type. With the increase of exchange rates, the foreign currency type interest and principal payments increases. In fact, the increase in the principal side does not reflect on the interest payments part, it just increases the borrowing requirement when the time of maturity. On the other hand, exchange rates increases may have positive effects on debt profiles, such as borrowing amount of foreign currency can be equivalent to a greater amount in local currency compared to previous period. If the interest rates of foreign currency are less than the local currency interest rates, then the government could achieve more borrowing from the lower interest rates. However, the effect is limited and therefore an increase in the exchange rates yields an increase in the debt stock figures.

First, 10 percent rise in the projection of exchange rates has been tested in the model. The objective function value has been found to be just 267,8 billion TL. The interest and principal payments have risen because of foreign type predetermined payments, also the coupon payments of newly borrowed Eurobonds and project financing. Total debt service increases because of similar increases in interest and principal repayments. Parallel to the debt service increase, the borrowing requirement also increases in the model.

Table 12: Effect of 10 percent increase in currency rates on borrowing requirement, interest and principal repayment

	2009	2010	2011	2012	2013	2014
Total Borrowing	2.28%	3.92%	2.67%	4.43%	3.49%	5.33%
TL Borrowing	0.92%	2.85%	1.66%	3.75%	2.70%	4.67%
Total Debt Service	1.75%	2.68%	2.14%	3.62%	2.83%	4.13%
Principal	1.95%	3.08%	2.25%	3.89%	2.75%	4.43%
Interest	1.43%	1.87%	1.94%	3.20%	2.98%	3.70%

On the other hand, debt stock figures did not vary much because of cheap foreign currency borrowing compared to local currency borrowing. Roll over ratios and debt stock figures do not fluctuate compared to base case scenario because of the limited portion of currency denominated debt stocks. Increase in the currency rates increases the nominal total debt stock in TL, but the TL equivalent borrowing in foreign currency, which is cheaper because of lower interest rate, acts to decrease the debt stock. The outputs about roll over and debt stock values in ten percent exchange rates increase scenario are given below.

Table 13: Effect of 10 percent increase in currency rates on debt stock figures

	2009	2010	2011	2012	2013	2014
Debt Stock	0.16%	0.42%	0.64%	1.08%	1.42%	1.85%
Stock / GDP	0.06%	0.14%	0.21%	0.33%	0.45%	0.58%
Roll Over	0.38%	0.87%	0.41%	0.64%	0.52%	0.90%
Interest/GDP	0.07%	0.07%	0.09%	0.16%	0.14%	0.16%

When the 20 percent shift in exchange rates scenario is analyzed, the total value of present worth's of accrued interests are found to be 275,7 billion TL. On the other hand, this scenario has the lowest impact on debt figures when compared to 20 percent primary surplus or 4 points interest rate scenarios. On the long run, debt stock over GDP ratios has not been deeply affected. The changes in the cash flow items are also lower compared to previous two scenarios.

Table 14: Effect of 20 percent increase in Currency rates on general debt management figures

	2009	2010	2011	2012	2013	2014
Total Borrowing	4.57%	7.87%	4.92%	9.17%	7.93%	9.20%
TL Borrowing	1.84%	5.73%	2.84%	7.84%	6.46%	7.68%
Total Debt Service	3.49%	5.41%	3.93%	7.49%	6.42%	7.12%
Principal	3.89%	6.20%	4.05%	8.08%	6.50%	7.43%
Interest	2.85%	3.78%	3.71%	6.56%	6.29%	6.68%
Debt Stock	0.31%	0.84%	1.27%	2.16%	2.89%	3.65%
Stock / GDP	0.11%	0.28%	0.40%	0.65%	0.93%	1.15%
Roll Over	0.75%	1.68%	0.75%	1.28%	1.15%	1.50%
Interest/GDP	0.14%	0.15%	0.18%	0.33%	0.30%	0.30%

Consequently, changes in exchange rates have a limited effect on the debt management figures. Since change in exchange rates is related with only foreign denominated borrowing, local currency borrowing instruments have not been affected in maturity and allocation manner. Impact of changes in exchange rates on debt sustainability is given in Figure 19 and 20.

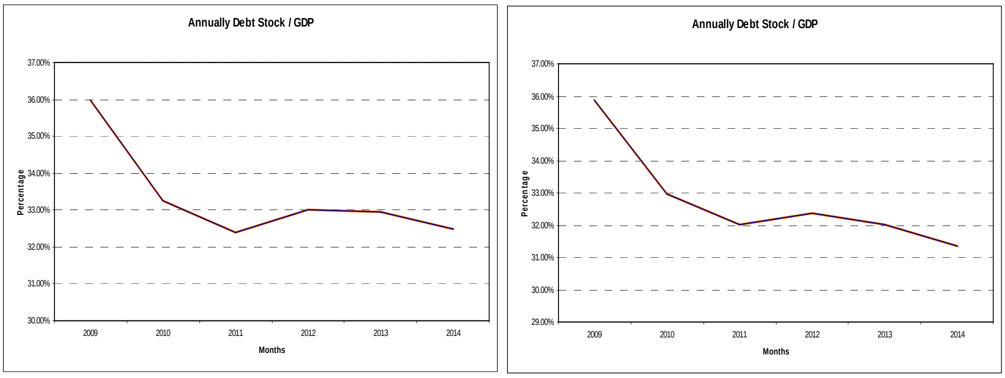


Figure 19: Comparison of Debt stock / GDP ratios in Base vs 20 percent increase in currency (Base on the right)

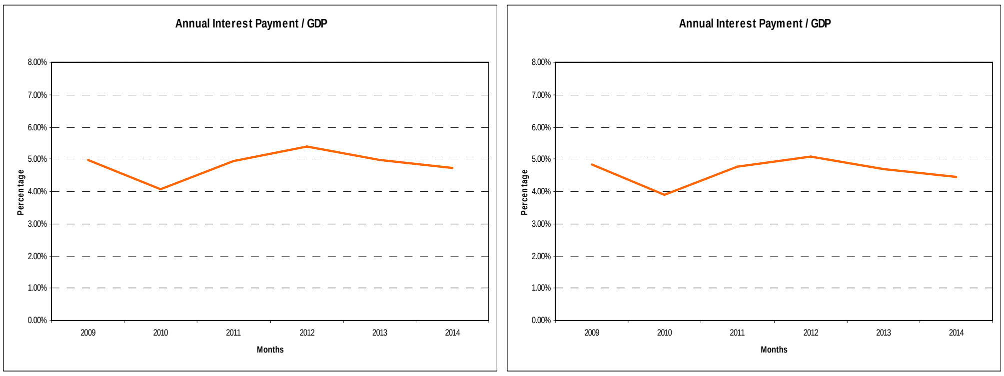


Figure 20: Comparison of Interest payments in Base vs 20 percent increase in currency (Base on right)

4.2.4 Effect of Changes in TL Yield Curve Structure

Structure of yield is also an important factor that affects the debt management figures. Structure of yield curve simply tells the behaviors of investors among investing on the various periods. If investors generally think that tomorrow's conditions will be worse than today's conditions than they will seek for higher interest rates for longer maturities. Moreover, as said before, in the long term there is a higher risk premium since predicting tomorrow is more difficult than predicting today. In the base case scenario, a positive sloped yield curve has been chosen reflecting the risk premium of the investors. In order to analyze the impact of structural changes in yield curve, first a flat yield curve and second a steeper yield curve will be tested.

In the flat yield curve analysis, the interest rates for all durations have been fixed at 19.5 percent for TL currency. This will have important affect on zero coupon and coupon bonds allocation and interest payments. Since coupon bonds are prone to the six months duration interest, but zero coupons are prone to their own maturity interest rates.

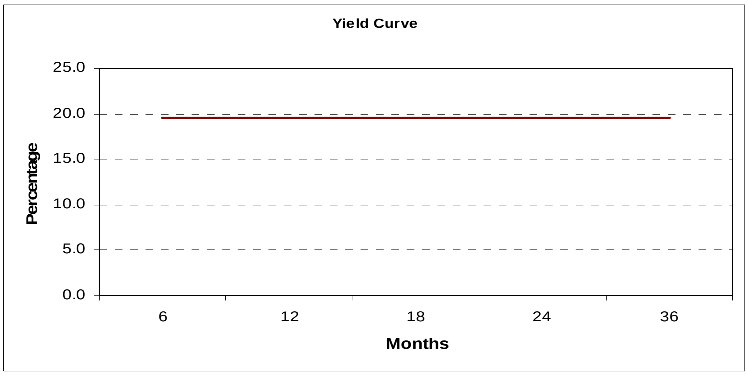


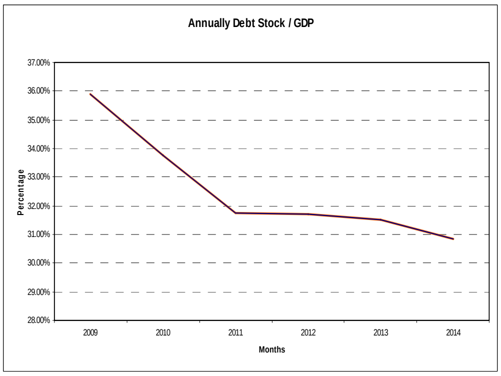
Figure 21: Graph of flat yield curve

The objective function value has found to be 261 billion TL in the flat yield curve case, with increasing borrowing requirement in the first years then a reduction in borrowings in future. Therefore, in the short run the higher interest rates on the short side of yield curve resulted interest payments of coupon bonds to raise reflecting a higher amount of interest payments to be paid. On the other hand, longer term maturity zero coupon bonds can be issued less costly, enabling to decrease the borrowing requirement in the following years. Furthermore, the impact of flat yield curve is increasing the cash flow in the short term, but reducing in the long term.

Additionally, increasing the maturity of the instruments decreases the roll over ratio and this yield to decrease the debt stock over GDP in the long run. As interest payment of coupon bonds increases in this case, the present worth of accrued interests slightly increase which resulting a higher value in objective function compared to base case.

Table 15: Effect of flat yield curve (19.5) on the debt management figures

	2009	2010	2011	2012	2013	2014
Total Borrowing	0.04%	8.99%	-14.48%	-18.65%	-12.29%	-13.04%
TL Borrowing	0.05%	10.58%	-16.48%	-20.93%	-13.79%	-14.88%
Total Debt Service	0.03%	0.11%	-6.07%	-15.23%	-9.95%	-10.09%
Principal	0.00%	0.00%	-6.09%	-19.80%	-16.72%	-16.81%
Interest	0.08%	0.32%	-6.04%	-8.01%	1.76%	-0.53%
Debt Stock	0.01%	2.37%	-0.86%	-2.04%	-1.61%	-1.58%
Stock / GDP	0.00%	0.78%	-0.27%	-0.67%	-0.51%	-0.49%
Roll Over	0.01%	6.41%	-7.03%	-3.29%	-2.10%	-2.53%
Interest/GDP	0.00%	0.01%	-0.29%	-0.41%	0.08%	-0.02%



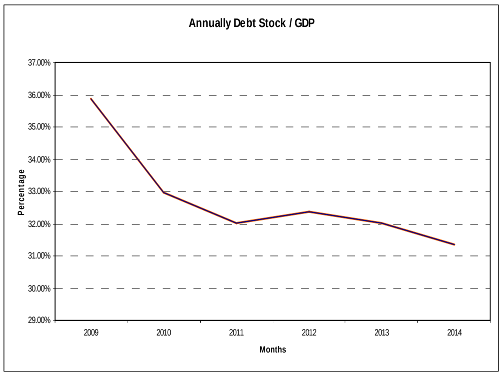
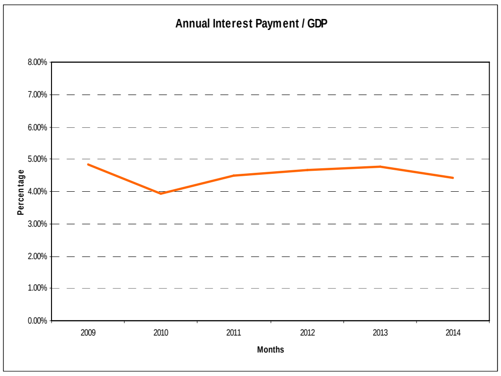


Figure 22: Comparison of Debt stock / GDP ratios in Base vs Flat yield curve (Base on the right)



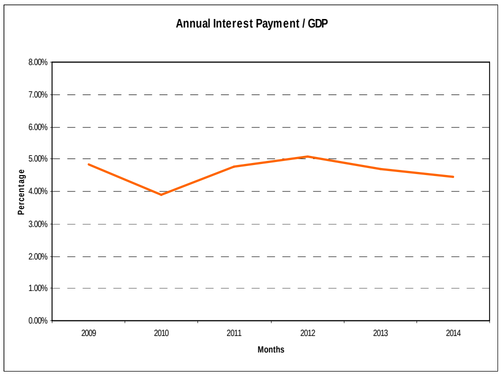


Figure 23: Comparison of Interest payment / GDP in Base vs Flat yield curve (Base on right)

Contrast, impact of a steeper yield curve on debt management figures has been analyzed in the study. In the steeper curve analysis, interest rates on the short side have been reduced whereas interest rates on the long side have been increased. Therefore, a yield curve with a higher slope compared to base case yield curve has been formed. The interest rates are varying between “17.5 to 24” percent.

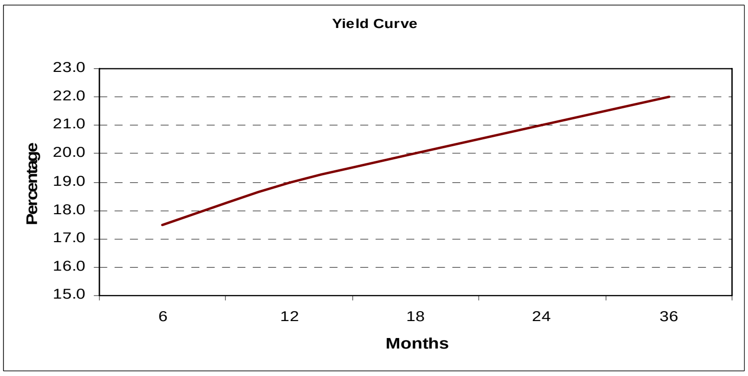


Figure 24: Graph of steeper yield curve

A steeper yield curve results in a 246,5 billion TL present worth of accrued interests. The reason behind the reduction in the objective function is the reduction in the 6 month interest rates which are applied to the coupon bonds. Moreover, the zero coupon bonds’ maturity have been reduced by the model to borrow opportunistically in the market. Cash flows in the borrowing and debt service side reduce starting from the first year to the end period, except of some years in which zero coupon maturities are higher. The reduction in cash flows also yield a reduction in the roll over and debt stock figures. Consequently, a structural change in yield curve not only affects the cash flows and debt stock figures, but also affects the share of borrowing instruments as well as their maturities.

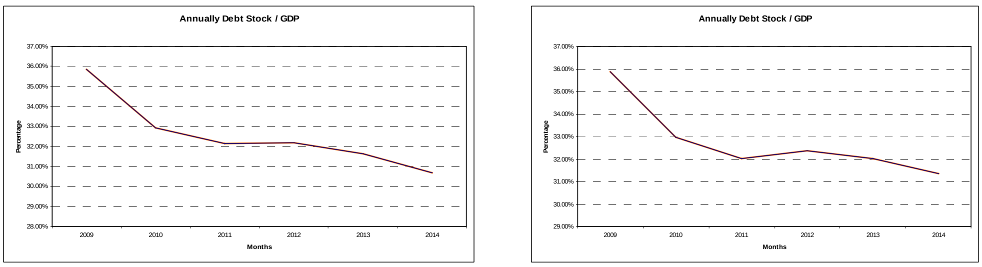


Figure 25: Comparison of Debt stock / GDP ratios in Base vs Steeper yield curve (Base on the right)

Table 16: Effect of steeper yield curve (17.5 - 24) on debt management figures

	2009	2010	2011	2012	2013	2014
Total Borrowing	-0.05%	-0.33%	3.07%	-18.12%	-11.22%	-13.76%
TL Borrowing	-0.06%	-0.39%	3.50%	-20.34%	-12.58%	-15.70%
Total Debt Service	-0.04%	-0.21%	2.39%	-14.80%	-9.08%	-10.65%
Principal	0.00%	0.00%	1.63%	-19.90%	-11.77%	-13.28%
Interest	-0.10%	-0.66%	3.70%	-6.74%	-4.43%	-6.91%
Debt Stock	-0.01%	-0.10%	0.47%	-0.65%	-1.24%	-2.14%
Stock / GDP	0.00%	-0.03%	0.15%	-0.19%	-0.40%	-0.67%
Roll Over	-0.01%	-0.08%	0.52%	-3.18%	-1.90%	-2.69%
Interest/GDP	0.00%	-0.03%	0.18%	-0.34%	-0.21%	-0.31%

Structural changes in the yield have also impact on debt sustainability; those are given below in the graphs.

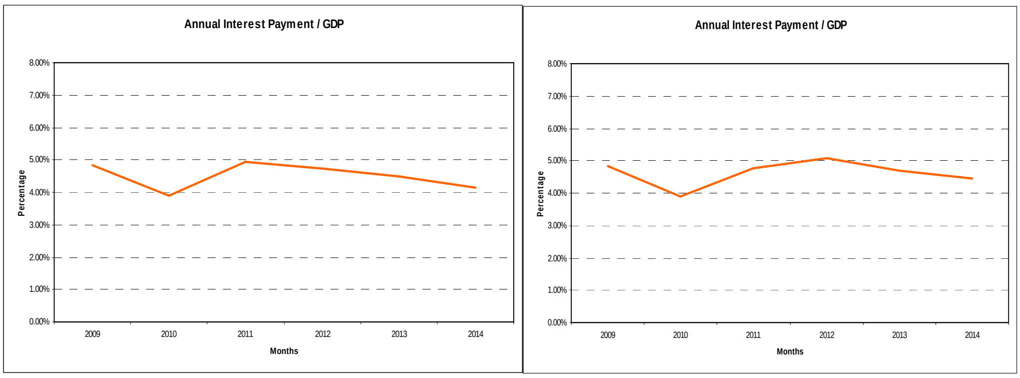


Figure 26: Comparison of Interest payment / GDP in Base vs Steeper yield curve (Base on right)

4.2.5 Effect of Changes in Interest Rates Path

In the above part regarding the change in interest rates, only an upward shift among all the period's predictions have been analyzed. Thus, an increase in interest rates for all horizons has surely different impact, than an increase on a periodic rise in the interest rates. In this part, an upward and a downward path for interest rates will be analyzed. Obviously, this analysis can be made for primary surplus and currency rates, but the affect would be less than the interest rates.

In these scenarios the structure of yield curve has not been changed. As mentioned before, interest rates for the horizon have been assumed to be stable till the end of the horizon. In this part, we will change the interest rate path. In the first case, a %0,5 reduction have been inserted to the monthly interest rates, which resulted a yield curve between 11.3 to 12.8 percent in 100th month. The most remarkable results have been found in the interest rates path changes. The objective function value has been found as 198,4 billion TL meaning a dramatic reduction in the present worth of accrued interests. On the other hand, the share of issued instruments has fluctuated much relative to the other scenarios analyzed above. Fixed coupon bonds issuance has been lowered by the model, in order not to pay high interest rates in the future due to the decreasing path in the future. Moreover, contrast to the above analyzed scenarios, maturity of zero coupon bonds have been lowered by the model enabling to borrow more zero coupons in future from lower interest rates. Interest payments have reduced dramatically in the consecutive years as a result of lower interest rates; however principal payments have not been affected too much. Borrowing requirement reduces because of both for reduction in interest payments and issuance of longer maturity floating rate bonds.

Table 17: Effect of decreasing interest rates path on debt management figures

	2009	2010	2011	2012	2013	2014
Total Borrowing	-0.11%	5.71%	-5.94%	-0.60%	-15.51%	-16.68%
TL Borrowing	-0.13%	6.72%	-6.76%	-0.67%	-17.40%	-19.03%
Total Debt Service	-0.08%	-0.52%	-0.81%	-0.49%	-12.56%	-12.91%
Principal	0.00%	0.00%	1.40%	2.97%	-7.06%	-3.72%
Interest	-0.22%	-1.61%	-4.56%	-5.98%	-22.10%	-25.99%
Debt Stock	-0.03%	1.47%	-0.88%	-1.74%	-4.83%	-8.21%
Stock / GDP	-0.01%	0.48%	-0.28%	-0.54%	-1.55%	-2.57%
Roll Over	-0.02%	4.52%	-4.07%	-0.09%	-2.73%	-3.35%
Interest/GDP	-0.01%	-0.06%	-0.22%	-0.30%	-1.04%	-1.16%

Regarding the debt sustainability aspects, debt stock over GDP ratios has important gaining because of lowered borrowing requirement. Additionally, interest payment over GDP ratios has been lowered tremendously.

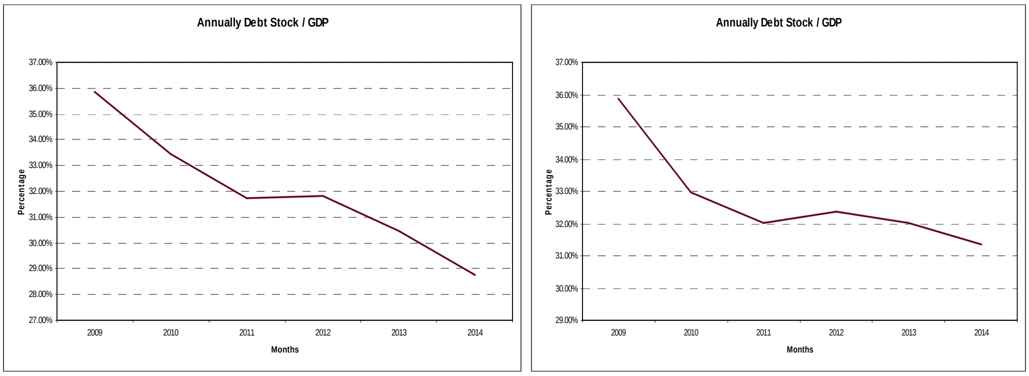


Figure 27: Comparison of Debt stock / GDP ratios in Base vs Decreasing Interest Rate path (Base on the right)

Figure 28: Comparison of Interest payment / GDP in Base vs Decreasing Interest Rate path (Base on right)

Contrast, a path including an upward scenario for TL interest rates have oppositely impact the results of the model. Similarly in the above case, a %0,5 increase has been made for the monthly interest rate figure without changing the structure of yield curve. Ultimately, the yield curve has been formed between 30.3 and 34.4 for TL interest rates in the 100th month.

The objective function value has been found to be 342,7 billion TL meaning a dramatic change in the present worth of accrued interest. Share of issued instruments have changed tremendously, thus, floating rate coupon bonds share has been lowered too much. This reduction was compensated by issuing more fixed rate coupon bonds which fixes the interest rates to their issuance time rates. Additionally, the maturity of zero coupon bonds have been increased by the model in order to be affected from the interest rate changes as late as possible.

Borrowing requirement allocation varies dramatically from the base case scenario, since the model borrows in advance in the first years from the lower interest rates. But, in the following years because of the increase of interest payments which are dependent to the interest rates, borrowing requirement of government increases. Principal payments have been lowered by the model, because of the increasing maturity. Consequently, the upward path of interest rates results in a very higher interest payments relative to the base case scenario.

Table 18: Effect of increasing interest rates path on debt management figures

	2009	2010	2011	2012	2013	2014
Total Borrowing	0.00%	15.60%	-12.48%	-5.32%	-1.73%	-1.00%
TL Borrowing	0.01%	18.34%	-14.21%	-5.98%	-1.94%	-1.14%
Total Debt Service	0.01%	4.86%	-4.49%	-4.35%	-1.40%	-0.77%
Principal	0.00%	5.43%	-5.89%	-10.00%	-12.03%	-17.42%
Interest	0.01%	3.68%	-2.13%	4.60%	17.03%	22.93%
Debt Stock	0.00%	2.76%	0.09%	0.81%	3.23%	6.28%
Stock / GDP	0.00%	0.91%	0.03%	-0.15%	1.04%	1.97%
Roll Over	0.00%	7.38%	-6.57%	-0.83%	-0.27%	-0.18%
Interest/GDP	0.00%	0.14%	-0.10%	0.23%	0.80%	1.02%

Increased interest payments and decreased principal payments result in a higher debt stock. Debt sustainability figures are badly affected from the upward path scenario of interest rates. Especially, interest payments over GDP ratio have important distortions in the case of an upward path of interest rates.

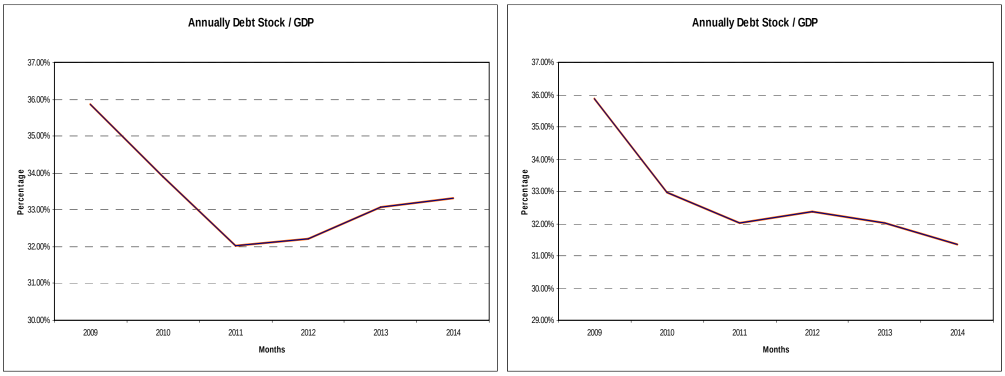


Figure 29: Comparison of Debt stock / GDP ratios in Base vs Increasing Interest Rate path (Base on the right)

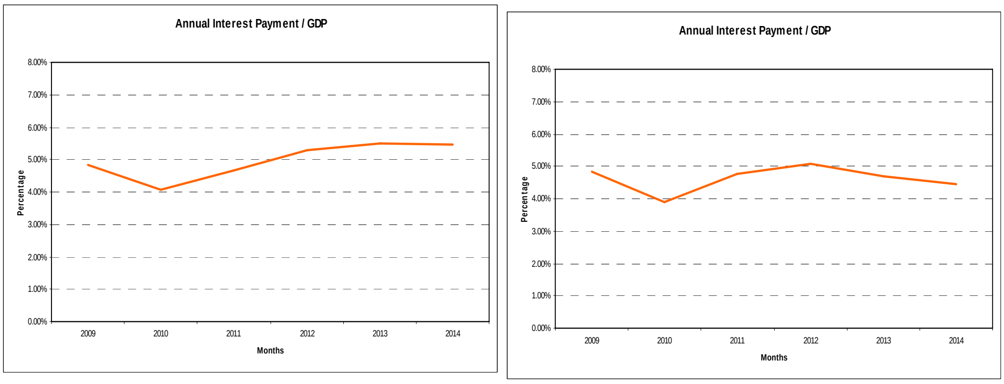


Figure 30: Comparison of Interest payment / GDP in Base vs Increasing Interest Rate path (Base on right)

4.3 Sensitivity of Debt Management Figures to the Tested Scenarios

Impacts of changes in financial conditions to the debt management figures have been given in previous parts. The general sensitivity of the figures are summarized in the below table. It is important to understand that changes in the financial conditions do not affect the model in the same way. Some of the changes directly affect borrowing requirement or payments, whereas some of them affect the share of the instruments.

As analyzed in detail in the above part, the debt management figures can easily be affected from the changes in the market conditions. Since, the concept of the model is very broad, including amount, time and type in three dimensions (currency, coupon, fixed or float) the result of the model dramatically changes parallel to the conditional changes in the market. As said before, share of coupon bonds to maturity of the bonds may vary, which result in a tremendous changes in the cash flow.

Table 19: Sensitivity of Debt Management Figures to Selected Financial Condition Changes

Financial Condition	Shock	Change: Objective function values	Total borrowing	Total Principal Payments	Total Interest Payments	Total Debt Stock	Change in Share of Instruments
Primary Surplus	- 10%	7.90%	8.99%	4.93%	5.58%	9.27%	Neglectable
	-20%	15.83%	17.64%	9.48%	11.07%	18.45%	Neglectable
Interest Rates (Percent)	+ 2*	15.50%	6.98%	1.65%	11.67%	10.16%	Neglectable
	+ 4*	32.72%	13.83%	2.07%	24.99%	21.76%	Neglectable
Currency	- 10%	3.05%	3.72%	3.03%	2.85%	2.46%	Neglectable
	-20%	6.09%	7.40%	6.03%	5.68%	4.90%	Neglectable
Yield Curve Slope	+%100	-5.15%	-7.52%	-7.04%	-4.33%	-3.77%	Moderate
	-%100	0.43%	-5.88%	-8.33%	1.01%	0.88%	Moderate
Interest Rates Path	* (0.955)	-23.65%	-15.63%	-7.52%	-20.14%	-17.53%	Dramatic
	* (1.005)	31.87%	10.64%	-3.00%	26.40%	22.99%	Dramatic

* Given increase s are equivalent to 10 and 20 percent increases in interest rates

To conclude, the deterministic model seems to be a useful tool for debt managers to analyze the future cash flows and type of borrowing instruments parallel to the future estimations.

The model can be useful for Treasury debt managers to generate their issuance strategies. In practical case, due to the benchmark issuance, the maturity profile of the debt stock has important fluctuations. According to the LP model results, the maturity profile of the Treasury becomes smoother. The effect of the LP model can be better understood from Figure 31. Since the ratio of pre-determined payments is higher in the first beginning periods, Treasury redemption profile is still fluctuating. But after the ratio of predetermined payments decreases the LP model starts to decrease fluctuations in the model.

Smoothing the payments means smoothing the borrowing requirement. So this will also be useful for Treasury mitigate the risks.

The accrued interest insight of the model is useful to measure the effectiveness of the model in the medium-long term. For example, a zero coupon bond issuance with a high volume can raise the funds for the Treasury. But at the same time it may produce a high volume of principal and interest payments in the future. This means extra borrowing for that month, which will yield a demand shock for Treasury issuances. The LP model eliminates the debt service fluctuations by allocating the payments to the periods in a more efficient way.

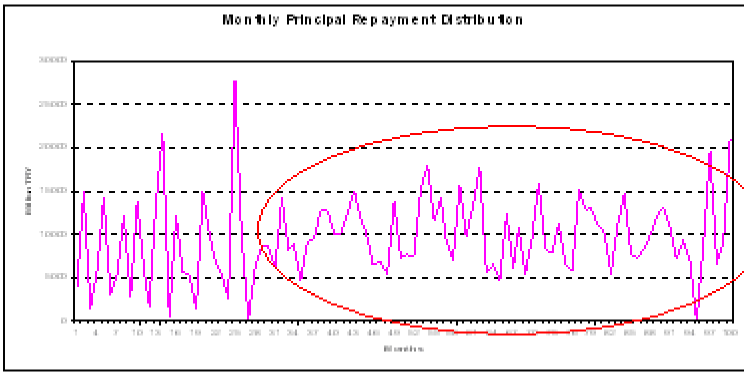


Figure 31: Monthly Principal Payment Distribution according to LP Model Results

This insight may be beneficial for Turkish Treasury, since it may be more accurate to calculate the cost of borrowing by this LP model. In practice, Turkish Treasury generally measures the cost of borrowing by the annual interest payments. But as we mentioned before, cash flows may be illusive. For example in order to raise enough funds for year 2010, government may borrow excessively with zero coupon bonds which have 24 months maturity. Perhaps this will be beneficial for year 2009, but in year 2011 Treasury will face a huge repayment problem. Of course, there is a strong risk management system in Turkish Treasury to measure these side effects and these risks are managed by simulation models in line with the strategic benchmarks, but this model may also be helpful to add some contribution to measure risks more effectively.

On the other hand, it is impossible to say that the future estimations will be hundred percent accurate in the future as predicted in the model. Indeed, it need not be. The model is useful to give insight to debt managers about future cash flows and debt stock figures that are parallel to their estimations. In Treasuries, the future cash flow calculation is made with some computer programs based on Excel or Matlab. In modern practices, the future cash flows are calculated with respect to certain possible strategies, and the cash flows are generated parallel to them. Our LP model evaluates the minimum cost solution by evaluating all feasible cases, so the model is useful to construct new solution for varying financial conditions. For example, under the real case, if an estimation about the exchange rate changes, only effect of that can be measured under the defined strategy. But our LP model performs to calculate a new issuance strategy under the change of exchange rate. This also appears to be beneficial for accurately calculating the effects of changes due to the flexible strategy defining under various conditions.

For example, in practical case, the effect of a downward change in the interest rate can only be measured by calculating the interest payments with respect to new figures only. But our LP model manages to construct a new issuance strategy under a downward change in interest scenario by increasing the level of floating rate coupon bond issuances in the model.

In the above parts we have mentioned that the financial conditions are very volatile. Therefore, setting a static 100 month strategy may not be very useful for debt managers. However, for the long term, perhaps not the result of LP, but its process may be used as a useful tool, since it gives important insights about the sensitivity of the result to changes in financial conditions. By the help of the model, debt managers can accurately answer the question “what level of change” are the debt figures prone to change under various conditions.

It gives managers the chance to analyze the future debt sustainability within the predictions. Moreover, by the help of deterministic model, debt managers can perform stress tests to measure the impact of changes in financial conditions to analyze how debt sustainability will be affected if condition(s) change(s) in the financial system. Consequently, the analysis made so far, is useful for testing the impact of conditional changes to the debt sustainability and understanding the general dynamics effecting the debt management.

CHAPTER 5

POSSIBLE EXTENSIONS OF THE LP MODEL

In practical case, financial conditions are dependent. For example, if interest rates increase, this will have effects on currencies, inflation rate etc. In order to simplify our LP model, all decision variables are assumed to independent. But, we can relax some of the assumptions in order to better reflect the practical case.

In the LP model the effect of volume of issuance on interest rates has been neglected. We assumed that up to the highest limit of borrowing, interest rate remain unchanged. However, in practical case if Treasury demand for issuance increases, then interest rate of TL borrowing also rises. In second stage this dependency will be inserted to the model.

5.1 A Non Linear Model Explaining the Dependency between Interest Rates and Volume of Issuances

So far now, the model has been performed as all market conditions are parametric and are independent. On the contrary, in practice financial conditions are dependent and have a high level of correlation. The change in one indicator consecutively affects the other conditions. For example, a change in the currency can be balanced with a change in the interest rates. However, the purpose of this study is not to analyze the macroeconomic relations in the market. But, the effect of borrowing on the interest rates should be modeled in order to analyze the system better.

In the LP model, in line with the normal market conditions, there were some ceiling values for bond auctions and yearly demands. Also it is assumed that Treasury can not borrow more than a limit. Actually, in practice, the market operates different than the assumptions in the model. Sometimes Treasury can borrow more than the limit (perhaps a month in five years period), in this case this will be more costly to Treasury, but better than default.

5.1.1 Inserting the Flexibility in Borrowing Limits

The deterministic model used to perform with certain limits in borrowing manner. In practice, Treasuries can not exceed some market limits in borrowing because of insufficient market demand. The market demand for different instruments changes instrument to instrument. For example, Treasuries may not issue fixed coupon bonds over a limit, because the investors will never be persuaded to buy such amount of fixed coupon bond in order not get such level of interest rate risk. However, Treasuries can increase the upper limit of floating rate bonds by accepting to pay a higher rate of interest rate. In practice, the upper limits defined for borrowing and payments are set in order to absorb the pressure on the interest rates. But if finding a high level of cash is compulsory for a certain period, than debt managers will accept to issue those bonds with a higher interest rate than the market conditions. The trick in this aspect is related with the instruments type, such that Treasuries have different range of rooms to increase the issuance amount for varying instruments.

For the sake of well explaining the mechanism, brief information about the Treasury auctions should be given. In practice, Treasury issues the bonds by special auctions. A bond auction is announced to the public and especially primary dealers and banks, even normal citizens can enter the auctions by bidding in the auction day (just between 10 A.M. – 12.00 A.M.). The auction type in Treasury auctions is the multi price bid auction, in which the investors gets the interest that they accepted, in case they win the auction. The auction is ended by Treasury top managers by cutting the price in a level that will enable to supply Treasury sufficient cash in that auction. Actually, the multi price bid auction has a broad literature, but this is not the topic of this thesis study.

The only topic about the Treasury auction is related with the volume of the issuance. In normal conditions, each instrument has average demands in the market. Regarding the banking sector structure, duration of deposits in the banks is very short. Therefore banks, buyers of the Treasury bonds, always have an attitude to mitigate long term interest rate risk. So floating rate bonds have higher demand compared to fixed coupon bonds. Zero coupon bonds are also preferred by the market because of their pricing advantage in the secondary market.

Considering the multi price bid auction structure, if Treasury wants to borrow more than a usual limit, then the managers should cut the auction rate from a higher interest rate point, which means that the average interest rate for the auction will rise.

This relation can be modeled in a mathematical form as follows[8]:

$$\theta(t) = \begin{cases} 0 & \text{if} \quad i(t) <= \mu \\ (\, i(t) - \mu \,) / \Omega & \text{if} \quad i(t) > \mu \end{cases}$$

where θ(t) represents the increase in the interest rate in case of exceeding the usual limit in the auction, i(t) represents the volume of zero coupon issuance in period t, μ represents the usual limit in the auction and Ω is the constant of the increase with the assumption that the increase in the interest rate is linearly dependent to the volume.

Obviously, there are and there should be some assumptions in these equations. Since auction is also very complicated, it would be impossible to measure the exact dependency between interest rate and volume. Also the dependency between these two variables may change period to period in line with the liquidity conditions. But in order to simplify the case, some important assumptions have been held.

The first assumption is related with the usual limits. Monthly issuance of Treasury has been analyzed and some average values have been found for each kind of instruments. Also the demand for auctions has been analyzed and in order to reflect the structure to the model, different rates for each instrument have been produced.

Increase in the interest rates has been modeled as follows:

$$i(t) \leq u(i) + \Omega(i) * \theta(i)$$

enabling that if I_t is equal or less than the usual limit no penalty for interest rates will be given, but if the usual limit is exceeded θi_t penalty increase will be applied. The interest rates for period t will be calculated as follows after the penalty cost summation.

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[8] A Stochastic Simulation Framework for the Government of Canada's Debt Strategy

$$r(t,v) = \begin{cases} r(t,v) & \text{if} \quad i(t) <= \mu \\ r(t,v) + \theta(t,v) & \text{if} \quad i(t) > \mu \end{cases}$$

Obviously, the model calculates the penalty cost for period t, but interest rates have been used parametrically in the model with a sloped yield curve. Therefore the increase in the interest rates should be distributed to the term rates, called $\theta_{t,v}$.

Important point about this dependency is the difference between the average issuance of each instruments and their interest elasticity to volumes.

5.1.2 Zero Coupon Bond Case

Zero coupon bonds have the highest share in the Treasury portfolio; nearly fifty percent of issuance is done by zero coupon bonds. Generally speaking, except for the liquidity conditions, there is sufficient demand for these auctions.

μi = 6000 TL

Ωi = 2000

In order to insert the structure into the model, the following revisions should be done in the model:

i(t) <= 6000 + 2000 * ietk(t);

where ietk(t) represents the penalty increase in the interest rate in case of a exceed in the zero coupon usual limit. Penalty increase should be inserted to the interest payments and accrued interest calculations as follows.

$$fi(t) = \sum_{vi=12}^{24} ix(t-vi,vi) * \left[r(t-vi,vi) + ietk(t-vi,vi) \right]$$

where ietk(t,v) represents the term interest rate for period v, that is originated from ietk(t) penalty interest rate. Also accrued interests are affected from the penalty interest rate. Therefore the equations should be revised as follows:

ai(t) = **[** r(t',v) **+ ietk(t',v)]** * i(t', v) / PAYDA(vi)

5.1.3 Fixed Coupon Bond Case

As mentioned above, fixed coupon bonds have not a very high demand in the market, since the investor should bear the interest rate risk till the maturity of the bond. For this reason, auctions of the fixed coupon bonds have the lowest demand enabling a higher rise in the interest rates if issued volume is increased.

µsk = 1000 TL
Ωsk = 500

Again in order to insert the structure into the model, the following revisions will be done in the model:

sk(t) <= 1000 + 500 * setk(t);

where setk(t) represents the penalty increase in the interest rate in case of a exceed in the fixed coupon usual limit. Penalty increase should be inserted to the interest payments and accrued interest calculations as follows.

$$fsk(t) = \sum_{vsk=36}^{72} skx(t-vsk, vsk) * \left[r(t-vsk, '6') + setk(t-vsk, vsk) \right]$$

where setk(t,v) represents the term interest rate for period v, that is originated from setk(t) penalty interest rate. Also accrued interests are affected from the penalty interest rate. Therefore the equations should be revised as follows:

ask(t) = **[** r(t',v) **+ setk(t',v)]** * sk(t', v) * PAY(vsk) / PAYDA(vsk)

5.1.4 Floating Rate Coupon Bond Case

Floating rate coupon bonds have comparatively higher demand than fixed coupon bonds.

μdk = 1500 TL

Ωdk = 2000

Again in order to insert the structure into the model, the following revisions will be done in the model:

dk(t) <= 1500 + 2000 * detk(t);

where detk(t) represents the penalty increase in the interest rate in case of a exceed in the floating coupon usual limit. Penalty increase should be inserted to the interest payments and accrued interest calculations as follows.

$$fdk(t) = \sum_{vsk=36}^{84} dkx(t - vdk, vdk) * \left[r(t-6,'6') + \det k(t - vdk, vdk) \right]$$

where detk(t,v) represents the term interest rate for period v, that is originated from detk(t) penalty interest rate. Also accrued interests are affected from the penalty interest rate. Therefore the equations should be revised as follows:

Adk(t) = **[** r(t',v) **+ detk(t',v)]** * dk(t', v) * pay(v) / payda(v)

where pay(v) and payda(v) have been calculated in Section 4.6.

5.1.5 Solution of the NLP Model

Since, the penalty interest rate and volume of issuance are both decision variables, the multiplication of them yields non-linearity in the model. Therefore interest rate – volume function has been assumed to be linear in order to simplify the problem. In practice, it is nearly impossible to define a function between interest rate and issuance volume, because these are also dependent to the liquidity conditions, macroeconomic expectations and some other financial conditions. But, the purpose of these dependency constraints is to explain that borrowing of Treasury in all levels is not same at all. If a debt manager tries to find more money, then he should accept to pay more interest.

The model has been performed under the dependency constraints with the base case scenario assumptions. All growth, currency, inflation, interest rate figures have been used in the dependency related model in line with the deterministic model discussed in chapter 4.

After inserting the linear dependency between the volume of borrowing and interest rates, the model has smoothed the borrowing amounts for some of the issued instruments. Different than the deterministic model, the variance between the monthly borrowings has considerably decreased.

Total present worth of accrued interest has been found as 271 billion TL, 12 billion TL more than the deterministic model base case scenario. The increase in the present worth of accrued interest is due to the penalty interest rates.

Second model yields a higher floating rate coupon bonds ratio than the first model. Obviously the maturity period for the floating rate coupon bonds is longer than the zero coupon bonds, therefore the average maturity of the borrowing increases in the second model. Increasing the maturity means that the principal payments of these bonds will be made later, that means that the refinancing of the bond will be later. This has an overall decreasing effect on cash flows. Therefore the second model results in a lower principal payments and borrowing than the first model, although the objective function value is absolutely higher.

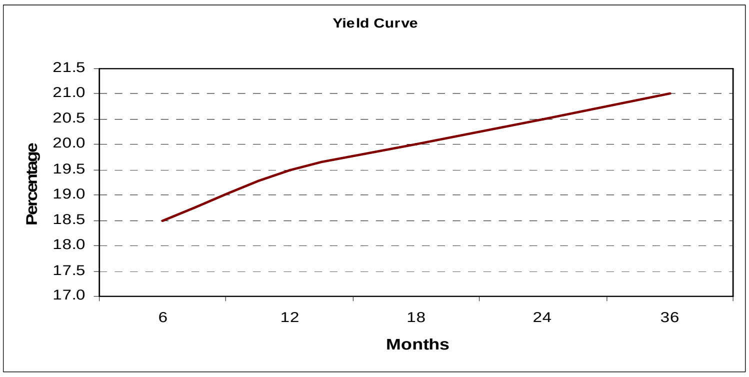


Figure 32: Yield Curve of TL Borrowing

Due to the penalty interest rates the second model decreases the fluctuations in the monthly distributions of the borrowings. Annual borrowing is also smoothed in the second model. Blue line represents the deterministic model whereas red line represents the dependent model.

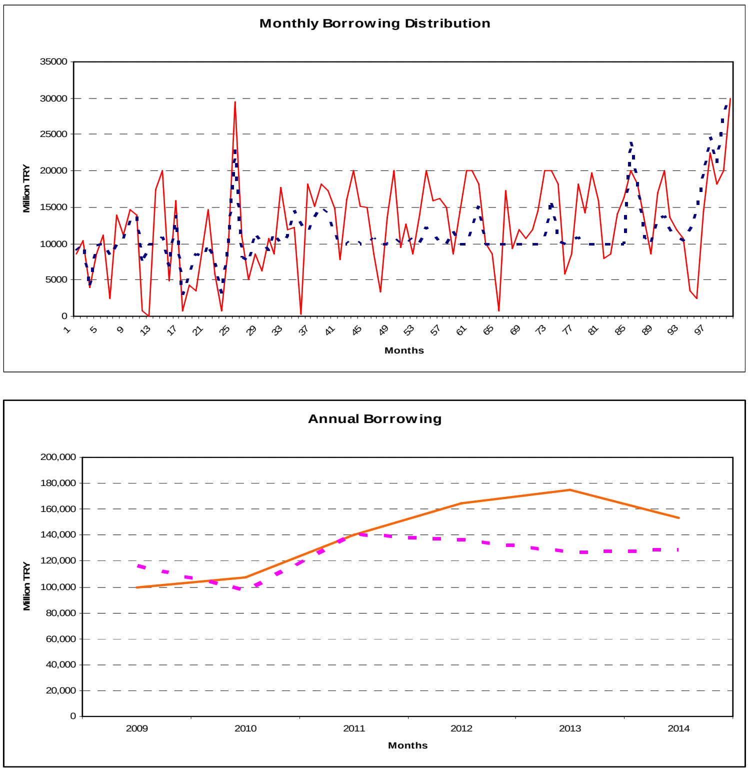
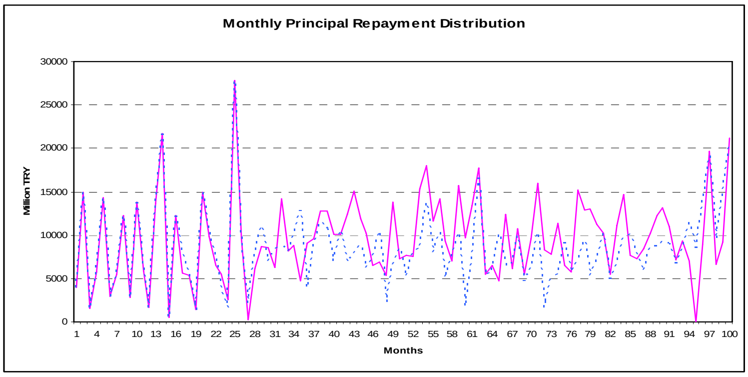


Figure 33: Differences between Dependent and Deterministic Model in Monthly and Annual Borrowing Distribution of Treasury

For the same reason stated above, principal payments have decreased in the dependent model due to the longer maturity. Fluctuations still exist in the dependent model.



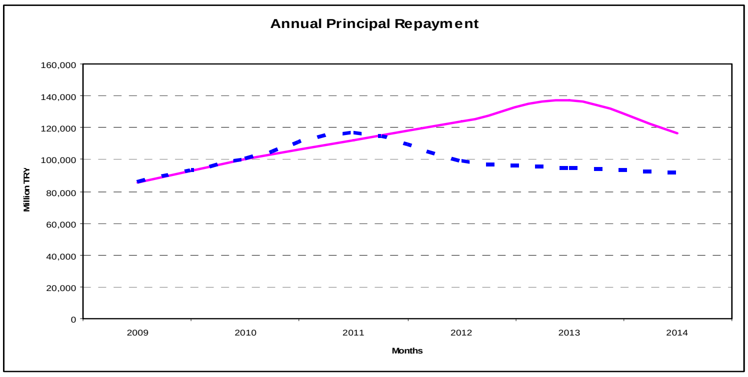
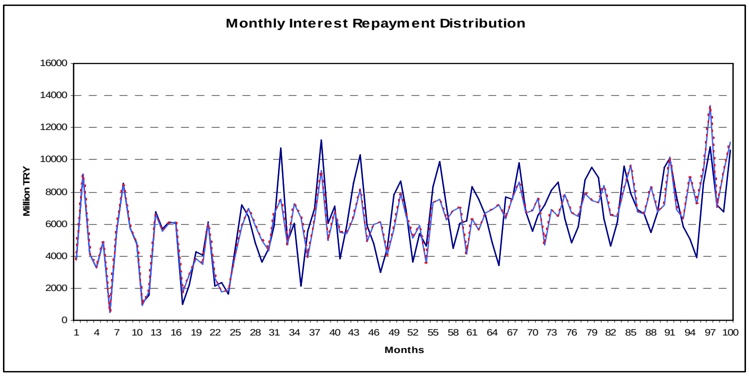


Figure 34: Differences between Dependent and Deterministic Model in Annual and Monthly Principal Repayment

Interest payments are one of the most important indicators for the models. At the beginning borrowings can be managed under a certain level in the dependent model, but increased stock starts to yield a higher interest payments. This means that the borrowing requirement increases over the usual limit levels. In the first graph below shows the penalty effects on the interest payments after period 60, where the floating rate coupon bonds starts to be repaid.



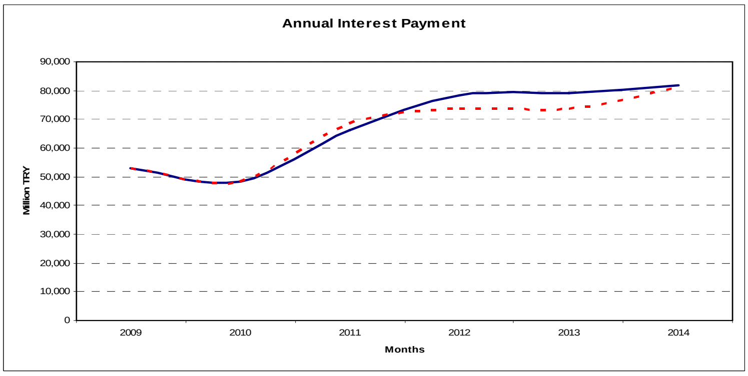


Figure 35: Differences between Dependent and Deterministic Model in Annual and Monthly Interest Payments

Dependent model results should also be analyzed with respect to the instruments type. In the dependent type model zero coupon bonds issuance has been slightly lowered where the ratio of floating rate bonds have increased. The most important reason for this is the interest fixing differences among the bonds. Zero coupon bonds are a kind of fixed coupon bond where the interest is fixed at the issuance date. Therefore an interest rate that has the penalty cost is valid till maturity of the bond. On the other hand, floating rate bonds, which is assumed to be issued at face value in the model, re fixes the interest rate every six months and the penalty interest rate lost its effect on the bond. Foreign type borrowing instruments are not affected by the model changes since they are all issued in their highest limit. The summary of the results are given in the table below for the dependent model.

Table 20: Results of Base Case Scenario of Dependent Model

	2009	2010	2011	2012	2013	2014
Total Borrowing	115,985	97,214	140,321	135,519	126,706	127,859
TL Borrowing	100,977	81,208	123,311	117,509	107,697	108,853
FX Borrowing	15,000	15,987	16,976	17,978	18,970	18,970
USD	10,000	9,992	9,986	9,988	9,984	9,984
Total Debt Service	138,280	148,631	184,766	172,459	167,726	172,654
Principal	85,290	100,152	116,217	98,669	94,032	91,562
Interest	52,990	48,479	68,549	73,790	73,694	81,092
TL Principal	68,672	73,057	104,102	88,762	86,688	80,636
TL Interest	45,261	39,939	59,297	62,915	61,066	67,127
FX Principal	16,621	27,099	12,114	9,907	7,344	10,927
FX Interest	7,733	8,549	9,260	10,885	12,640	16,565
Debt Stock	408,694	414,315	446,284	491,283	532,556	568,846
Stock / GDP	37.37%	33.49%	32.20%	32.00%	31.61%	30.92%
Roll Over	83.88%	65.41%	75.95%	78.58%	75.54%	74.06%
Interest/GDP	4.85%	3.92%	4.95%	4.78%	4.37%	4.41%

The deterministic model has strengths in smoothing the amortization schedule and minimizing the cost of interest payments. However, robustness of the model results should be tested in a manner to analyze the effects of changes in the financial conditions. The financial conditions that have the probability to vary can be listed as primary surplus estimations, interest rate figures, currency, inflation rate and yield curve structure.

CHAPTER 6

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this study, we have constructed a model that decides on issuance of the Treasury borrowing instruments on a monthly basis in order to minimize the total present worth of accrued interests over the planning horizon. The model calculates all the cash flows on a monthly basis and produces a detailed payback schedule.

The model is successful in choosing to issue the available instrument that will minimize the borrowing cost for Treasury. Also, at the same instant it calculates the effects of that borrowing by producing all payments of that instrument. As a result, for a long horizon the issuance strategy of the Treasury is defined and outputs of the strategy have been analyzed. Different than the practical case, the model may evaluate all possible feasible strategies for the Treasury rather than concentrating on a number of strategies in the simulation analysis.

The model may also be used for stress testing. Effects of changes in financial conditions to the cost of borrowing have been tested under various shock scenarios. The model is useful in giving insights to the debt managers for understanding the sensitivity of cash flows, debt stock and roll over. By the help of the model, a debt manager can have a better understanding on the debt dynamics and he can understand that at what levels can the values rise if a change occurs in parameter(s). In the study, we did not test multiple parametrical change scenarios, but by using the model all scenarios can be tested.

In order to set an issuance strategy for the long term, we inserted to the model the financial conditions of the future. Since this study's aim is not to produce an accurate estimate for the future financial conditions, we can not say that the conditions used in the model is the most probabilistic scenario. Fortunately, the LP model is very easy to use and different scenarios can easily be inserted to the model. For that reason, future researches can use this model with their financial conditions estimations.

In order to strengthen the model, we developed it further to take account of the interdependency between volume of issuance and the interest rate. The extension about the LP model includes the interdependency between the volume of borrowing and TL interest rates. After inserting the interdependency, the borrowing and payment profile becomes more smooth. Parallel to the practical case, the NLP model force Treasury to hold more cash reserve in the accounts.

The study can give insight to the debt managers in Treasury in understanding the debt dynamics. In practice, debt sustainability analysis is made in macro level. But this study gives power to managers to make this analysis in micro level.

On the other hand, this model can be extended to evaluate multiple scenarios at once. This may increase the strength of the model in reflecting the practical case. Obviously, financial conditions are very volatile and setting an issuance strategy for long term under multiple scenarios may better evaluate the debt dynamics. Our LP model has been designed to be easily extended to insert multiple scenarios, but we did not produce multiple scenarios for the future estimations, because this may be a topic of another research area. Therefore this study may be developed to analyze the system in multiple scenarios in a stochastic environment.

Finally, this study can be an source for new studies about debt management, new researchers especially those make econometric analysis for future conditions can benefit from the models.

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APPENDIX A

MANUAL FOR ABBREVIATIONS IN THE MODEL

In the LP model, the name of the parameters and variables are given in a systematic manner enabling to understand the complex structure easily.

The basic of the names depends on the instruments, so the instruments' names will be explained first.

Table 21: Abbreviation List for Instruments

Abbreviation	Instrument	Origin of the Name
i	Zero-coupon	Turkish name "**İskontolu**" first letter
sk	Fixed rate-coupon	Turkish name "**S**abit **K**uponlu" first letters
dk	Floating rate-coupon	Turkish name "**D**egişken **K**uponlu" first letters
ek	Inflation ind.-coupon	Turkish name "**E**nflasyon **K**uponlu" first letters
e	Eurobond	Turkish name "**E**urobond" first letter
p	Project finance	Turkish name "**P**roje" first letter

All other variables are constructed over this framework. For example the name of principal payment of zero coupon bond is defined as **ci,** where c represents the principal payment and i represents the zero coupon bond. Please see the table below for detailed explanation.

Table 22: Abbreviations List for Cash Flows

Abbreviation	Type of Flow	Origin of the Name
c	Principal payment	English word "**C**apital" first letter
f	Interest payment	Turkish name "**F**aiz" first letter
b	Borrowing req.	Turkish name "**B**orçlanma" first letter
a	Accrued interest	English word "**A**ccrued" first letter
x	Cash reserve level	N/A

After giving the general framework, we can give explanation about construction of the variables. Please see the examples below:

i(t) Total volume of zero coupon bond issuance in **period t.**

c(t) Total principal payment in **period t.**

If we want to represent the **principal payment** originated from **zero coupon bond** in **period t**, then;

C + **İ** = **ci**(t)

represents the **principal payment** originated from **zero-coupon bonds** in **period t**. This is same for other kinds of variables that have the examples below:

F + **SK** = **fsk(**t)

represents the interest payment originated from fixed-coupon bonds in period t.

A + **DK** = **adk(t)**

represents the accrued interest originated from inflation indexed bonds in period t.

The latter comment about the variables is related with the issuance of a single bond in period t. As remembered, i(t) represents the **total volume** of issuance of **zero coupon bond** in period t. However, in period t, ix(t,vi) represents the **volume** of issuance of a **zero coupon** bond issuance in period t with **maturity vi.** So this abbreviation is constructed as follows:

SK + X = skx(t,vsk)

represents the **volume** of issuance of a **fixed rate coupon bond** issuance in period t, with **maturity vsk.**

Note that, in the sets part the **maturities** are represented with notation **v ("V**ade**")**. Therefore the **subsets t**hat represents the available maturity for a bond type is constructed as follows:

V + DK = vdk

represents the set of all available maturities for floating rate bonds.

APPENDIX B

PARAMETERS REGARDING FINANCIAL CONDITIONS

Tablo 23: Annual Simple Interest Rate Projections for YTL

Period / Maturity	6	12	18	24	36	48	60
1	18.5	19.5	20.0	20.5	21.0	21.0	21.0
2	18.5	19.5	20.0	20.5	21.0	21.0	21.0
3	18.5	19.5	20.0	20.5	21.0	21.0	21.0
4	18.5	19.5	20.0	20.5	21.0	21.0	21.0
5	18.5	19.5	20.0	20.5	21.0	21.0	21.0
6	18.5	19.5	20.0	20.5	21.0	21.0	21.0
7	18.5	19.5	20.0	20.5	21.0	21.0	21.0
8	18.5	19.5	20.0	20.5	21.0	21.0	21.0
9	18.5	19.5	20.0	20.5	21.0	21.0	21.0
10	18.5	19.5	20.0	20.5	21.0	21.0	21.0
11	18.5	19.5	20.0	20.5	21.0	21.0	21.0
12	18.5	19.5	20.0	20.5	21.0	21.0	21.0
13	18.5	19.5	20.0	20.5	21.0	21.0	21.0
14	18.5	19.5	20.0	20.5	21.0	21.0	21.0
15	18.5	19.5	20.0	20.5	21.0	21.0	21.0
16	18.5	19.5	20.0	20.5	21.0	21.0	21.0
17	18.5	19.5	20.0	20.5	21.0	21.0	21.0
18	18.5	19.5	20.0	20.5	21.0	21.0	21.0
19	18.5	19.5	20.0	20.5	21.0	21.0	21.0
20	18.5	19.5	20.0	20.5	21.0	21.0	21.0
21	18.5	19.5	20.0	20.5	21.0	21.0	21.0
22	18.5	19.5	20.0	20.5	21.0	21.0	21.0
23	18.5	19.5	20.0	20.5	21.0	21.0	21.0
24	18.5	19.5	20.0	20.5	21.0	21.0	21.0
25	18.5	19.5	20.0	20.5	21.0	21.0	21.0
26	18.5	19.5	20.0	20.5	21.0	21.0	21.0
27	18.5	19.5	20.0	20.5	21.0	21.0	21.0
28	18.5	19.5	20.0	20.5	21.0	21.0	21.0
29	18.5	19.5	20.0	20.5	21.0	21.0	21.0
30	18.5	19.5	20.0	20.5	21.0	21.0	21.0
31	18.5	19.5	20.0	20.5	21.0	21.0	21.0
32	18.5	19.5	20.0	20.5	21.0	21.0	21.0
33	18.5	19.5	20.0	20.5	21.0	21.0	21.0
34	18.5	19.5	20.0	20.5	21.0	21.0	21.0
35	18.5	19.5	20.0	20.5	21.0	21.0	21.0
36	18.5	19.5	20.0	20.5	21.0	21.0	21.0

Table 23: Annual Simple Interest Rate Projections for YTL (Continued)

Period / Maturity	6	12	18	24	36	48	60
37	18.5	19.5	20.0	20.5	21.0	21.0	21.0
38	18.5	19.5	20.0	20.5	21.0	21.0	21.0
39	18.5	19.5	20.0	20.5	21.0	21.0	21.0
40	18.5	19.5	20.0	20.5	21.0	21.0	21.0
41	18.5	19.5	20.0	20.5	21.0	21.0	21.0
42	18.5	19.5	20.0	20.5	21.0	21.0	21.0
43	18.5	19.5	20.0	20.5	21.0	21.0	21.0
44	18.5	19.5	20.0	20.5	21.0	21.0	21.0
45	18.5	19.5	20.0	20.5	21.0	21.0	21.0
46	18.5	19.5	20.0	20.5	21.0	21.0	21.0
47	18.5	19.5	20.0	20.5	21.0	21.0	21.0
48	18.5	19.5	20.0	20.5	21.0	21.0	21.0
49	18.5	19.5	20.0	20.5	21.0	21.0	21.0
50	18.5	19.5	20.0	20.5	21.0	21.0	21.0
51	18.5	19.5	20.0	20.5	21.0	21.0	21.0
52	18.5	19.5	20.0	20.5	21.0	21.0	21.0
53	18.5	19.5	20.0	20.5	21.0	21.0	21.0
54	18.5	19.5	20.0	20.5	21.0	21.0	21.0
55	18.5	19.5	20.0	20.5	21.0	21.0	21.0
56	18.5	19.5	20.0	20.5	21.0	21.0	21.0
57	18.5	19.5	20.0	20.5	21.0	21.0	21.0
58	18.5	19.5	20.0	20.5	21.0	21.0	21.0
59	18.5	19.5	20.0	20.5	21.0	21.0	21.0
60	18.5	19.5	20.0	20.5	21.0	21.0	21.0
61	18.5	19.5	20.0	20.5	21.0	21.0	21.0
62	18.5	19.5	20.0	20.5	21.0	21.0	21.0
63	18.5	19.5	20.0	20.5	21.0	21.0	21.0
64	18.5	19.5	20.0	20.5	21.0	21.0	21.0
65	18.5	19.5	20.0	20.5	21.0	21.0	21.0
66	18.5	19.5	20.0	20.5	21.0	21.0	21.0
67	18.5	19.5	20.0	20.5	21.0	21.0	21.0
68	18.5	19.5	20.0	20.5	21.0	21.0	21.0
69	18.5	19.5	20.0	20.5	21.0	21.0	21.0
70	18.5	19.5	20.0	20.5	21.0	21.0	21.0
71	18.5	19.5	20.0	20.5	21.0	21.0	21.0
72	18.5	19.5	20.0	20.5	21.0	21.0	21.0
73	18.5	19.5	20.0	20.5	21.0	21.0	21.0
74	18.5	19.5	20.0	20.5	21.0	21.0	21.0
75	18.5	19.5	20.0	20.5	21.0	21.0	21.0
76	18.5	19.5	20.0	20.5	21.0	21.0	21.0
77	18.5	19.5	20.0	20.5	21.0	21.0	21.0
78	18.5	19.5	20.0	20.5	21.0	21.0	21.0
79	18.5	19.5	20.0	20.5	21.0	21.0	21.0
80	18.5	19.5	20.0	20.5	21.0	21.0	21.0
81	18.5	19.5	20.0	20.5	21.0	21.0	21.0
82	18.5	19.5	20.0	20.5	21.0	21.0	21.0
83	18.5	19.5	20.0	20.5	21.0	21.0	21.0
84	18.5	19.5	20.0	20.5	21.0	21.0	21.0

Table 23: Annual Simple Interest Rate Projections for YTL (Continued)

Period / Maturity	6	12	18	24	36	48	60
85	18.5	19.5	20.0	20.5	21.0	21.0	21.0
86	18.5	19.5	20.0	20.5	21.0	21.0	21.0
87	18.5	19.5	20.0	20.5	21.0	21.0	21.0
88	18.5	19.5	20.0	20.5	21.0	21.0	21.0
89	18.5	19.5	20.0	20.5	21.0	21.0	21.0
90	18.5	19.5	20.0	20.5	21.0	21.0	21.0
91	18.5	19.5	20.0	20.5	21.0	21.0	21.0
92	18.5	19.5	20.0	20.5	21.0	21.0	21.0
93	18.5	19.5	20.0	20.5	21.0	21.0	21.0
94	18.5	19.5	20.0	20.5	21.0	21.0	21.0
95	18.5	19.5	20.0	20.5	21.0	21.0	21.0
96	18.5	19.5	20.0	20.5	21.0	21.0	21.0
97	18.5	19.5	20.0	20.5	21.0	21.0	21.0
98	18.5	19.5	20.0	20.5	21.0	21.0	21.0
99	18.5	19.5	20.0	20.5	21.0	21.0	21.0
100	18.5	19.5	20.0	20.5	21.0	21.0	21.0

Table 24: Primary Surplus, Exchange Rate and Inflation Index Future Projections

Primary Surplus Projections (Million YTL)		USD / TL Exchange Rates Projections		Inflation Index Projections	
1	3325	1	1.5	1	121.57
2	-52	2	1.5	2	122.25
3	1722	3	1.5	3	123.16
4	1502	4	1.5	4	124.74
5	6892	5	1.5	5	126.19
6	908	6	1.5	6	125.51
7	-2804	7	1.5	7	125.88
8	9593	8	1.5	8	125.92
9	-6084	9	1.5	9	126.30
10	4702	10	1.5	10	128.79
11	6071	11	1.5	11	129.80
12	5748	12	1.5	12	130.07
13	1576	13	1.6	13	129.40
14	6330	14	1.6	14	129.53
15	1780	15	1.6	15	129.87
16	2475	16	1.6	16	131.10
17	6610	17	1.6	17	131.64
18	2538	18	1.6	18	131.47
19	3880	19	1.6	19	131.08
20	7707	20	1.6	20	131.00
21	1312	21	1.6	21	132.10
22	2833	22	1.6	22	133.67
23	7161	23	1.6	23	135.02
24	-4009	24	1.6	24	135.42
25	1449	25	1.7	25	136.29
26	5821	26	1.7	26	136.63
27	1637	27	1.7	27	136.99
28	2276	28	1.7	28	138.21
29	6078	29	1.7	29	138.29
30	2334	30	1.7	30	137.58
31	3567	31	1.7	31	136.79
32	7087	32	1.7	32	136.54
33	1206	33	1.7	33	137.79
34	2605	34	1.7	34	139.53
35	6585	35	1.7	35	140.73
36	-3686	36	1.7	36	140.84
37	1449	37	1.8	37	142.13
38	5820	38	1.8	38	142.57
39	1637	39	1.8	39	143.09
40	2275	40	1.8	40	143.59

Table 24: Primary Surplus, Exchange Rate and Inflation Index Future Projections (Continued)

Primary Surplus Projections (Million YTL)		USD / TL Exchange Rates Projections		Inflation Index Projections	
41	6077	41	1.8	41	143.29
42	2333	42	1.8	42	141.90
43	3567	43	1.8	43	141.66
44	7086	44	1.8	44	141.91
45	1206	45	1.8	45	143.25
46	2604	46	1.8	46	144.95
47	6583	47	1.8	47	146.23
48	-3686	48	1.8	48	146.47
49	1609	49	1.9	49	147.64
50	6462	50	1.9	50	149.55
51	1818	51	1.9	51	149.82
52	2526	52	1.9	52	151.16
53	6748	53	1.9	53	151.68
54	2591	54	1.9	54	151.23
55	3960	55	1.9	55	150.02
56	7868	56	1.9	56	149.07
57	1339	57	1.9	57	149.61
58	2892	58	1.9	58	150.86
59	7310	59	1.9	59	152.52
60	-4092	60	1.9	60	152.37
61	1757	61	1.9	61	153.59
62	7057	62	1.9	62	155.57
63	1985	63	1.9	63	155.86
64	2759	64	1.9	64	157.25
65	7369	65	1.9	65	157.79
66	2829	66	1.9	66	157.32
67	4325	67	1.9	67	156.06
68	8592	68	1.9	68	155.07
69	1462	69	1.9	69	155.63
70	3158	70	1.9	70	156.94
71	7983	71	1.9	71	158.67
72	-4469	72	1.9	72	159.94
73	-4092	73	1.9	73	162.00
74	1757	74	1.9	74	162.30
75	7057	75	1.9	75	163.75
76	1985	76	1.9	76	164.31
77	2759	77	1.9	77	163.83
78	7369	78	1.9	78	162.51
79	2829	79	1.9	79	161.48
80	4325	80	1.9	80	162.06

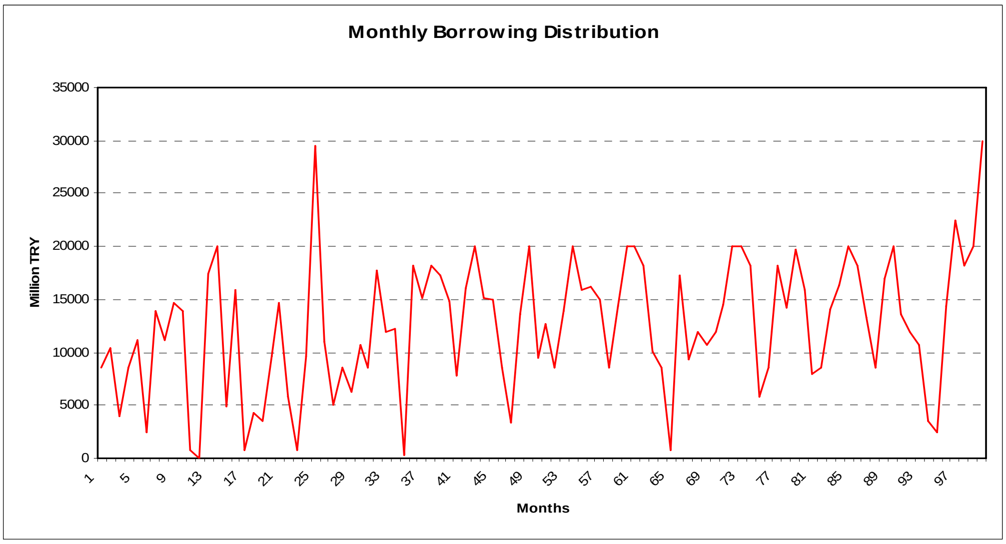
Table 24: Primary Surplus, Exchange Rate and Inflation Index Future Projections (Continued)

Primary Surplus Projections (Million YTL)		USD / TL Exchange Rates Projections		Inflation Index Projections	
81	8592	81	1.9	81	163.43
82	1462	82	1.9	82	165.22
83	3158	83	1.9	83	165.06
84	7983	84	1.9	84	166.38
85	-4469	85	1.9	85	168.53
86	-4092	86	1.9	86	168.83
87	1757	87	1.9	87	170.35
88	7057	88	1.9	88	170.93
89	1985	89	1.9	89	170.42
90	2759	90	1.9	90	169.06
91	7369	91	1.9	91	167.98
92	2829	92	1.9	92	168.59
93	4325	93	1.9	93	170.01
94	8592	94	1.9	94	171.88
95	1462	95	1.9	95	171.71
96	3158	96	1.9	96	173.08
97	7983	97	1.9	97	175.31
98	-4469	98	1.9	98	175.63
99	-4092	99	1.9	99	177.21
100	1757	100	1.9	100	177.81

APPENDIX C

APPLICATION RESULTS OF THE MODEL UNDER VARIOUS SCENARIOS

Figure 36: Selected Results for Base Case Scenario



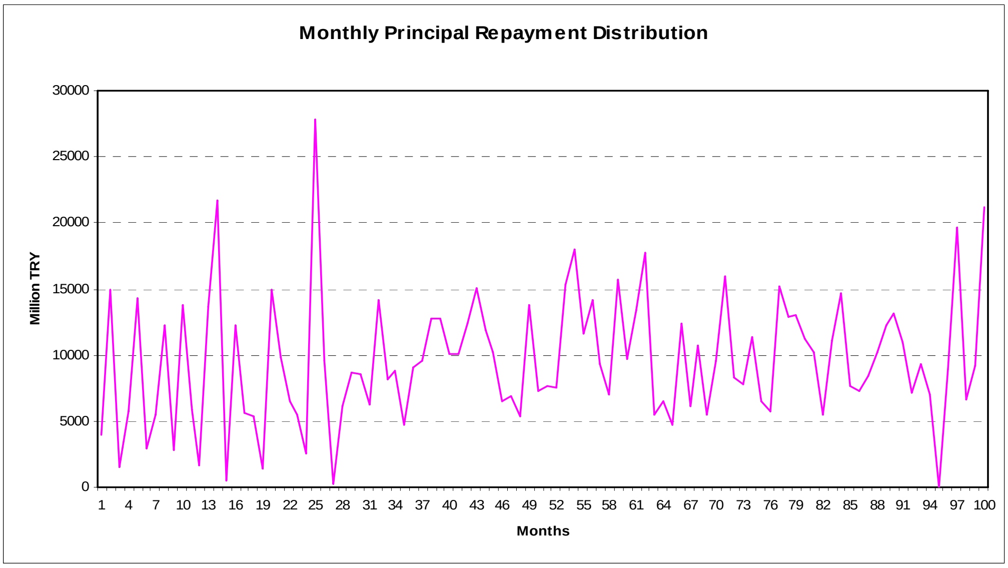
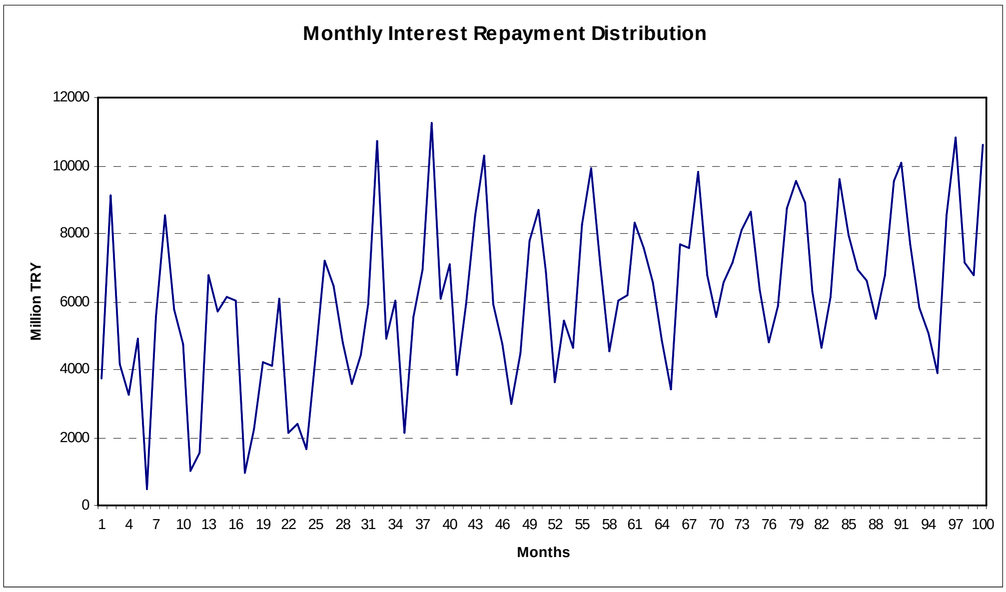


Figure 36: Selected Results for Base Case Scenario (Continued)



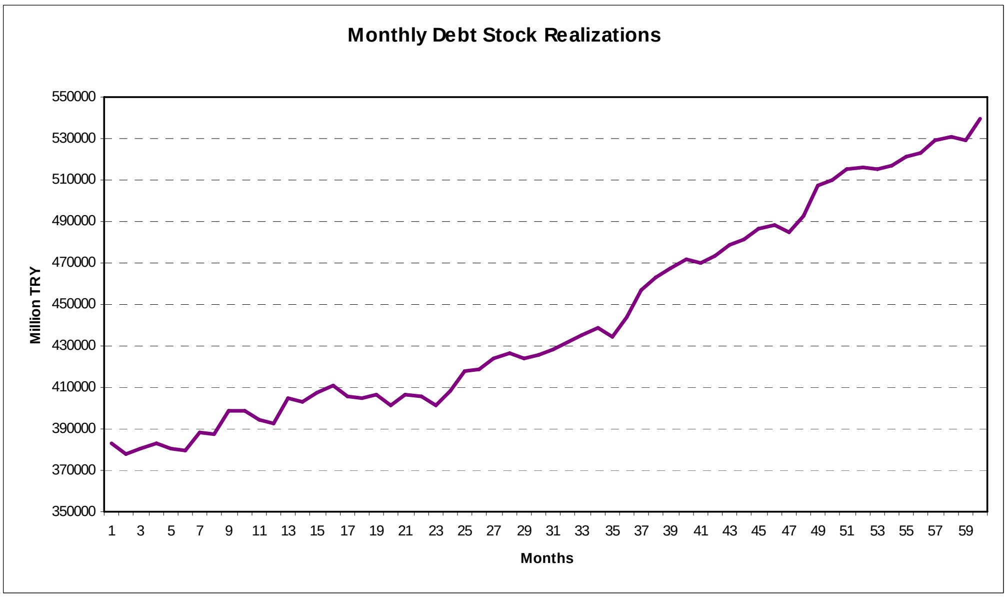
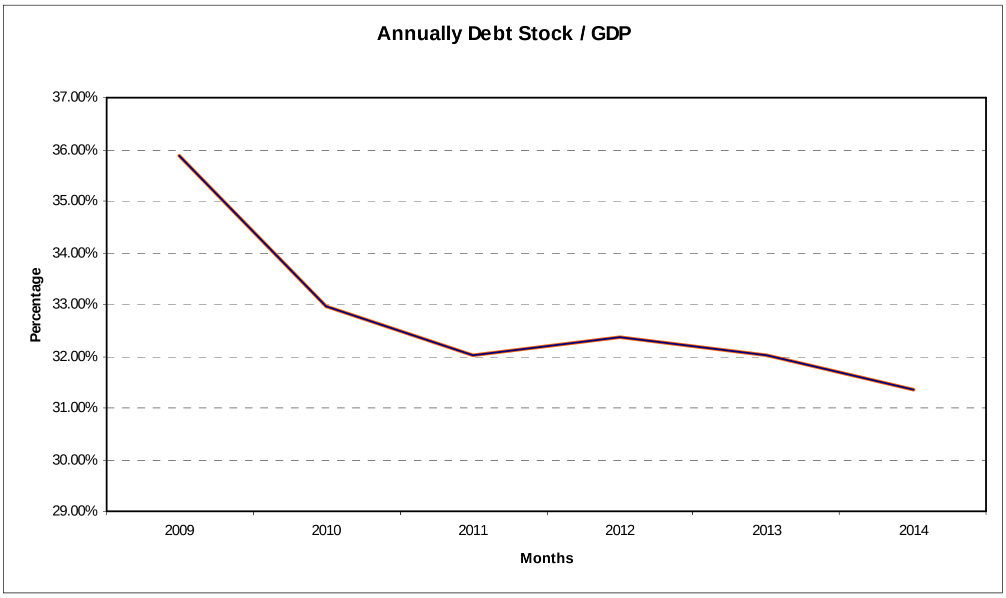


Figure 36: Selected Results for Base Case Scenario (Continued)



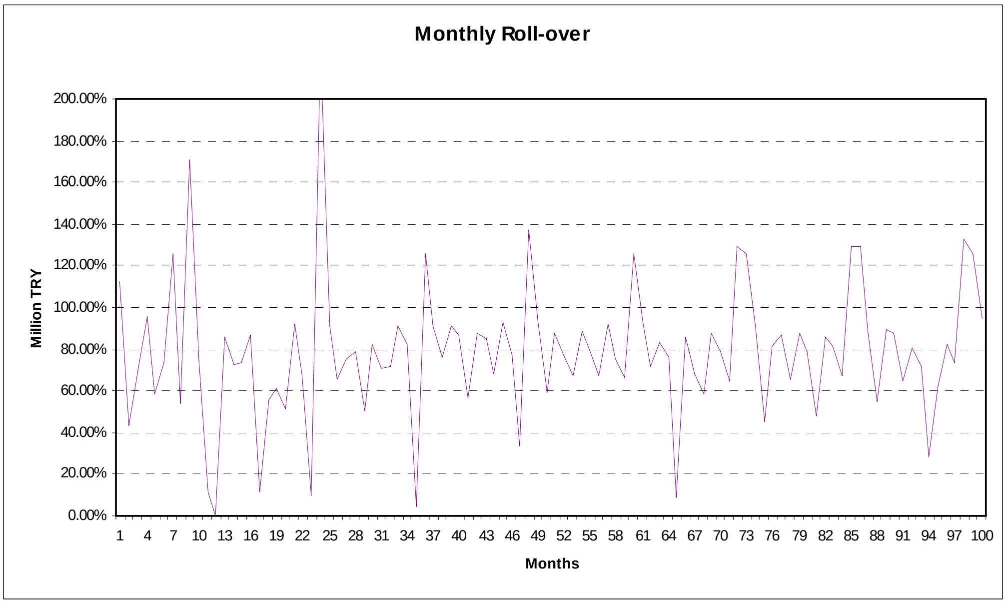
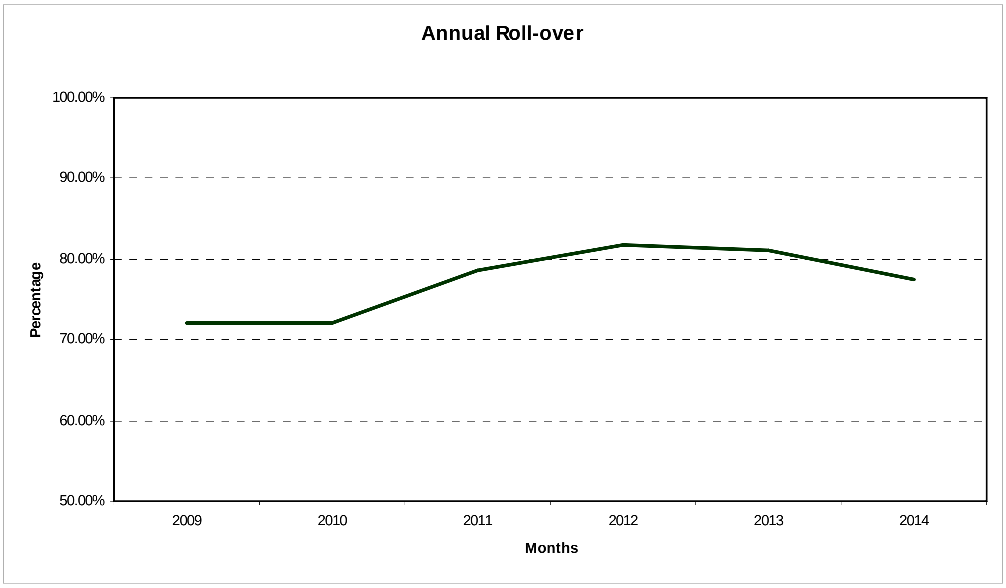


Figure 36: Selected Results for Base Case Scenario (Continued)



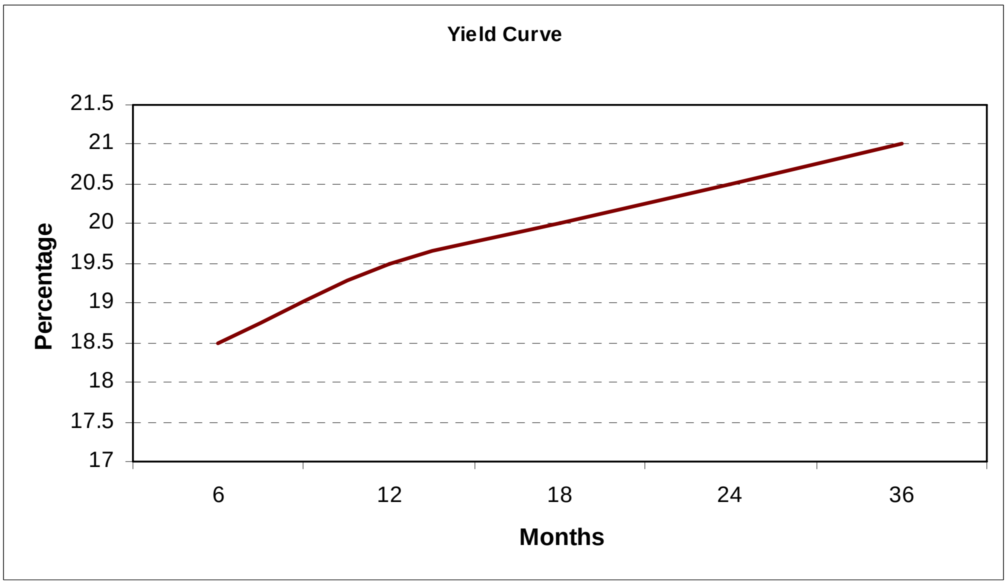
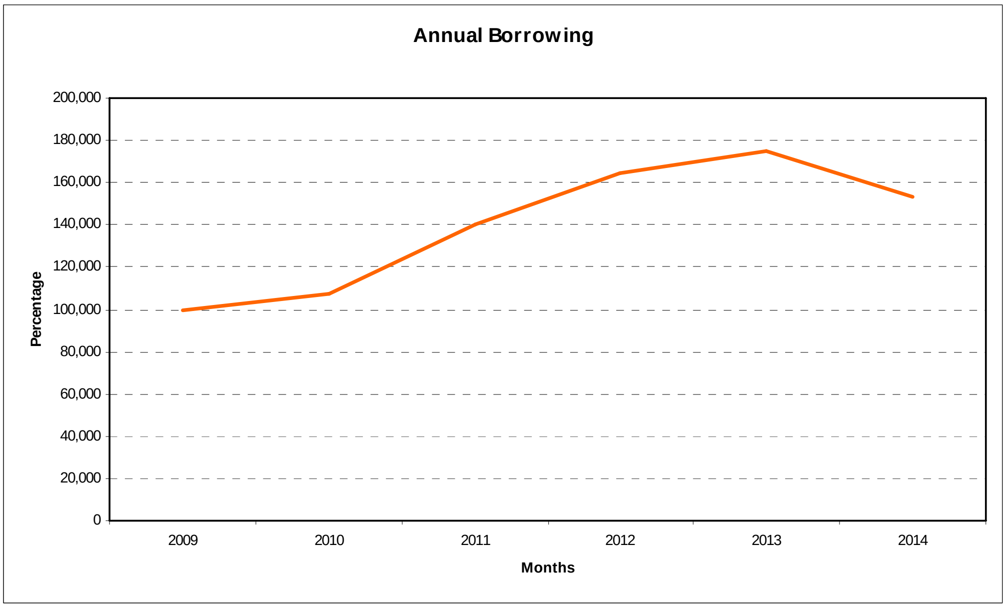


Figure 36: Selected Results for Base Case Scenario (Continued)



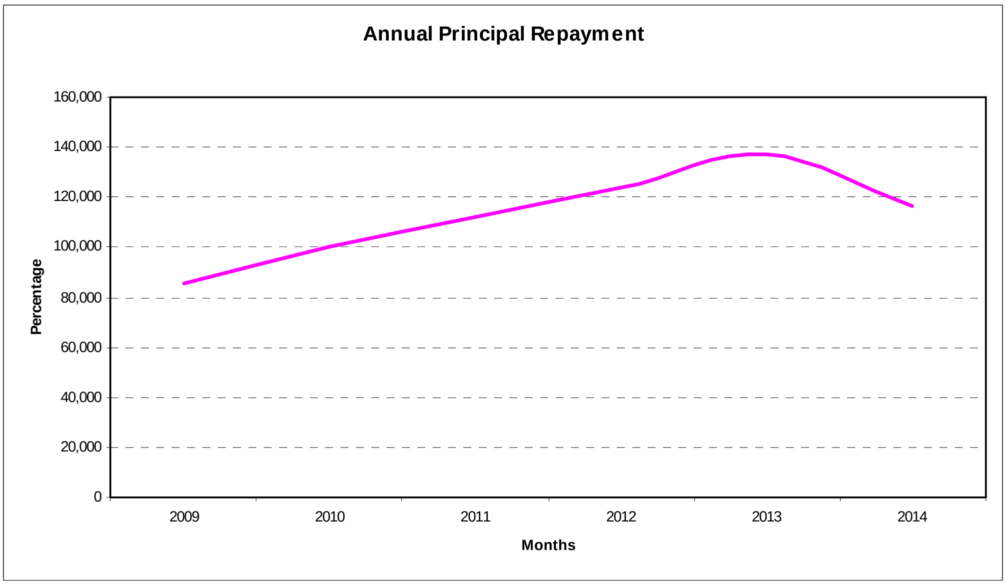
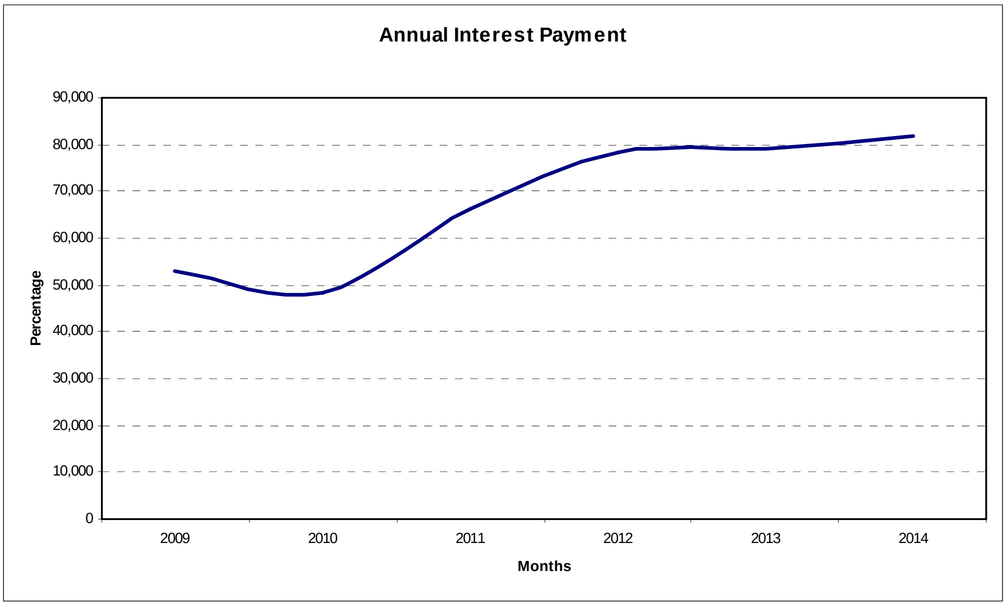


Figure 36: Selected Results for Base Case Scenario (Continued)



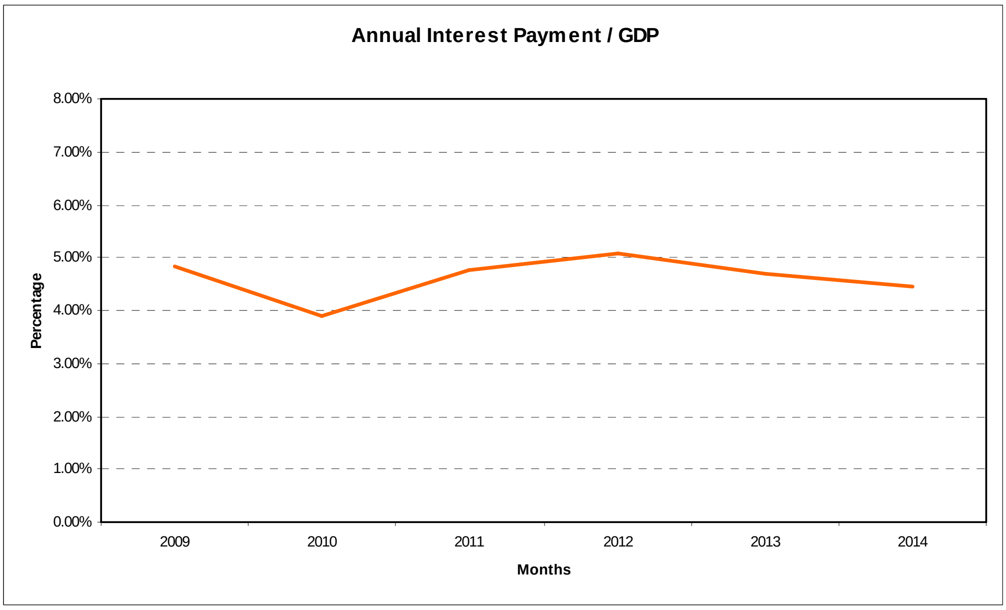


Figure 36: Selected Results for Base Case Scenario (Continued)