SCALAR MESONS IN RADIATIVE PHI-MESON DECAYS INTO CHARGED K-MESON STATES

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ABSTRACT

SCALAR MESONS IN RADIATIVE PHI-MESON DECAYS INTO CHARGED K-MESON STATES

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The role of $f_0(980)$ and $a_0(980)$ scalar meson intermediate states in the mechanism of radiative $\phi(1020)$ meson decay into two charged $K(494)$ mesons and a photon $\phi \rightarrow K^+ + K^- + \gamma$ is investigated. For the contribution of scalar meson intermediate state two models are considered. In the kaon-loop model, the scalar meson intermediate state couples the final state to the initial ϕ meson through a charged kaon-loop. The second model, called no-structure model, consist of point-like coupling of intermediate scalar meson state to the initial state. It is found that in the kaon-loop model, scalar meson intermediate state results in a considerable modification of the pure Bremsstrahlung photon spectrum.

Keywords: Radiative decays, ϕ meson, Bremsstrahlung, Scalar mesons, Kaon-loop model, No-structure model.

ÖZ

PHI-MEZONUN YÜKLÜ K-MEZON DURUMLARINA IŞINSAL BOZUNMALARINDA SKALAR MEZONLAR

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Haziran 2008, 50 sayfa

$\phi(1020)$ mezonunu iki yüklü K(494) mezona ve bir fotona ışınsal bozunma $\phi \rightarrow K^+ + K^- + \gamma$ tepkime mekanizmasında ortaya çıkan $f_0(980)$ ve $a_0(980)$ skalar mezon ara durumlarının rolü araştırıldı. Skalar mezon ara durumlarının katkıları için iki model gözönüne alındı. Kaon ilmek modelinde, skalar mezon ara durumları yüklü bir kaon ilmeği aracılığı ile ϕ mezonunu son duruma bağlarlar. Yapısız model olarak adlandırılan ikinci model de ise, skalar mezon ara durumu başlangıç durumuna noktasal olarak bağlanır. Kaon ilmek modelinde, skalar mezon ara durumlarının Bremsstrahlung foton spektrumunda önemli oranda değişikliğe neden olduğu bulundu.

Anahtar Kelimeler: Işınsal bozunmalar, ϕ mezonu, Bremsstrahlung, Skalar mezonlar, Kaon ilmek modeli, Yapısız model.

For my family and friends, who offered me unconditional love and support throughout the course of this thesis.

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CHAPTER 1

INTRODUCTION

The low-mass scalar mesons with masses $M \leq 1\ GeV$ and with vacuum quantum numbers $J^{PC} = 0^{++}$ have a fundamental importance in hadron physics. On the one hand, they play an essential role in understanding the confinement problem and the realization of chiral symmetry and symmetry breaking mechanisms in the low-energy region in Quantum Chromodynamics (QCD) which is the currently accepted theory of strong interactions. Moreover, the nature and the quark substructure of these scalar mesons have not been established yet, and this question have been a subject of controversy in hadron spectroscopy over the years. On the other hand, the scalar mesons play an important role in the mechanism of different reactions in hadron physics, in particular in reactions in hadron electrodynamics. Two such categories of reactions in hadron electrodynamics are the photoproduction of vector mesons on nucleons in the near threshold region, and the radiative decay processes of vector mesons into a pair of pseudoscalar mesons and a photon.

The light scalar mesons isoscalar $f_0(980)$ and isovector $a_0(980)$ have been well established experimentally. Recent experimental and theoretical analyses have found evidence for the existence of an isoscalar $\sigma(600)$ [1] and an isodoublet $\kappa(900)$ [2] scalar mesons. The properties of most of the known pseudoscalar and vector mesons can be understood within the framework of naive quark model [3]. In this model, they are classified according to the representations of the SU(3) symmetry group. Their mass patterns and quantum numbers are consistent with their assignment to SU(3) multiplets formed from a quark (q) and an antiquark ($\bar{q}$). Vector mesons are identified as the spin triplet $S = 1$ ground states $L = 0$ of $q\bar{q}$ states and they form a nonet in which the mass splittings can be understood by the number of strange quarks (s) present in each state. For example, $\phi(1020)$ has the structure $s\bar{s}$, and it is heavier than ${K^*}^0(896)$ which has one light down quark (d) and one anti-strange quark ($\bar{s}$) with the structure ($d\bar{s}$). K^* vector mesons are then heavier than the other vector mesons $\rho(770)$ and $\omega(782)$ which are formed from the light down (d) and light up quarks (u). Similarly pseudoscalar mesons in the quark model are formed from u,d and s quarks as $q\bar{q}$ states with relative orbital angular momentum $L = 0$ and total quark spin $S = 0$, and they show the similar mass patterns as in the case of vector mesons depending on their quark content. It is therefore natural to assign the scalar mesons to a nonet in the quark model as well [4]. In this model, the scalar mesons are expected to be $q\bar{q}$ states with relative angular momentum $L = 1$ so that the parity of the state will be $(-1)^{L+1} = 1$ taking into account the opposite intrinsic parities of q and $\bar{q}$. However, it is then expected that such $q\bar{q}$ states in p-wave state will be more or less in the same mass range as the other p-wave multiplets in the quark model which have masses more than 1 GeV. Therefore, the low masses of scalar mesons cannot be explained in this model. Furthermore, equality of the masses of the isoscalar $f_0(980)$ and isovector $a_0(980)$ scalar mesons poses a serious problem in quark model assignment of scalar mesons. The $f_0(980)$ scalar meson has strong coupling to K-meson, or kaon, system ($K\bar{K}$), and thus it can be interpreted as an $s\bar{s}$ state. However, this then does not explain the mass degeneracy between $f_0(980)$ and $a_0(980)$ because the quark model assignment to $a_0(980)$ consistent with its quantum numbers is a $(u\bar{u} - d\bar{d})/\sqrt{2}$ state. It was also suggested that scalar mesons may also form a multiplet with $qq\bar{q}\bar{q}$ ($q^2\bar{q}^2$) quark structure with zero relative orbital angular momentum within the context of the MIT bag model which incorporates the confinement phenomenologically in a Lorentz covariant manner [5]. The other commonly discussed possibility suggests that the scalar mesons can be considered to be mesonic molecules, that is they are taken to be the bound states of hadrons [6]. This proposal was originally formulated for the structure of $f_0(980)$ and $a_0(980)$ scalar mesons suggesting that they can be considered as $K\bar{K}$ molecules with spatially extended structures. In contrast, although the quark content of scalar mesons is the same in $(q^2\bar{q}^2)$ model, scalar mesons in $(q^2\bar{q}^2)$ are spatially compact objects.This difference stems from the fact that in $(q^2\bar{q}^2)$ case the multiquark system is confined within the scalar meson state with radius of the order of Λ_{QCD}^{-1}, where Λ_{QCD} sets the distance scale for confinement in QCD, forming a compact system, on the other hand in the mesonic molecule model the two pseudoscalar mesons are spread over a region with radius of the order of $\sqrt{\mu E}$ forming a bound state, where μ is the reduced mass of the mesonic molecule and E is binding energy of the mesons which is significantly larger than Λ_{QCD}^{-1}, thus the resulting bound state is a spatially extended system.

One category of reactions in hadron electrodynamics where scalar mesons play a decisive role is the photoproduction of vector mesons on nucleons (N). For the vector meson (V) photoproduction on nucleons $\gamma + N \rightarrow V + N$ in the near threshold region several mechanisms have been considered for the reaction, these are pseudoscalar meson and scalar meson exchanges in the t-channel, one-nucleon exchange in (s+u) channels, and pomeron exchange [7]. In the analyses of these reactions different combinations of these contributions to reaction mechanism are usually considered. The essential ingredient in these analyses is the coupling constants of scalar mesons to hadron states and to hadron and photon states. These coupling constants reflect the quark substructure of the scalar mesons.

The other category of reactions involving scalar mesons in a critical way in the reaction mechanisms in hadron electrodynamics are the radiative decays of vector mesons into a pair of neutral pseudoscalar (P^0) mesons and a photon $V \rightarrow P^0 + P^{0'} + \gamma$ [8]. These reactions have been studied over the years by the SND [9] and CMD-2 [10] groups at Novosibirsk and by the KLOE Collaboration at Frascati [11]. The theoretical studies of these reactions have resulted in the understanding that the scalar meson intermediate states play an important role in the reaction mechanisms along with the vector meson dominance contribution.

Moreover, it has also been established that the mechanism of the scalar meson production in the radiative decays $\phi \to f_0+\gamma$ and $\phi \to a_0+\gamma$ are the one-loop, or charged kaon-loop, mechanism $\phi \to K^+K^- \to f_0+\gamma$ and $\phi \to K^+K^- \to a_0+\gamma$ [12]. Therefore, the experimental data on radiative ϕ-decays $\phi \to \pi^0+\pi^0+\gamma$ and $\phi \to \pi^0+\eta+\gamma$ are described by the $\phi \to (\gamma f_0+\pi^0\rho) \to \pi^0+\pi^0+\gamma$ and $\phi \to (\gamma a_0+\pi^0\rho) \to \pi^0+\eta+\gamma$ models for the reaction mechanism. Furthermore, the confrontation of the theoretical analyzes with the experimental data for the radiative decays $\phi \to \pi^0+\pi^0+\gamma$ and $\phi \to \pi^0+\eta+\gamma$ along with the decays $\phi \to f_0+\gamma$ and $\phi \to a_0+\gamma$ have been used to provide evidence for the structure of scalar f_0 and a_0 mesons. Although not generally accepted yet, these analyzes seem strongly to suggest the $q^2\bar{q}^2$ structure for scalar mesons [13].

In this thesis, we study the radiative $\phi \to K^++K^-+\gamma$ decay in order to analyze the reaction mechanism for this reaction. For the description of the contribution of the scalar f_0 and a_0 mesons in the intermediate state to the amplitude of the reaction we employ two different approaches. One is the commonly used kaon-loop model (KL) where the scalar f_0 and a_0 mesons are coupled to the ϕ-meson through a charged kaon loop [14]. The other one is the recently proposed no structure (NS) formulation where the coupling describing the $\phi f_0\gamma$ and $\phi a_0\gamma$ vertex is pointlike [15]. We calculate the branching ratio in two approaches and we analyze the contributions of scalar meson intermediate states to the total bremsstrahlung amplitude for different physical observables.

CHAPTER 2

FORMALISM

In this chapter, we present the theoretical framework that we employ in our study of the radiative decay of $\phi(1020)$ vector meson into a K^+K^- pair of charged $K(494)$ pseudoscalar mesons and a single photon. We first give the general expressions for the calculation of the decay rate of the radiative $\phi \to K^+ + K^- + \gamma$ decay. Next, we present a discussion of the bremsstrahlung mechanism for this decay. We then outline the kaon-loop (KL) mechanism for the contribution of scalar mesons to this radiative decay and obtain the relevant expressions. In the last section, we discuss the recently proposed no structure (NS) model for the calculation of the contribution of the scalar meson intermediate state to the amplitude of the radiative $\phi \to K^+ + K^- + \gamma$ decay.

2.1 General Considerations

The radiative $\phi(p) \to K^+(q_1) + K^-(q_2) + \gamma(q)$ decay, where we indicate the four-momenta of the particles, can be represented by the general Feynman diagram shown in Fig. 2.1.

The invariant amplitude $\mathcal{M}$ representing this decay can be taken to be a function of K^+ meson energy E_1 and photon energy E_γ in the rest frame of the decaying ϕ meson utilizing four-momentum conservation $p = q_1 + q_2 + q$. In order to calculate the invariant amplitude $\mathcal{M}(E_1, E_\gamma)$ we have to consider two mechanisms that can contribute to this radiative decay. The first one is internal bremsstrahlung where one of the charged K mesons from the decay $\phi \to K^+ + K^-$ emits a

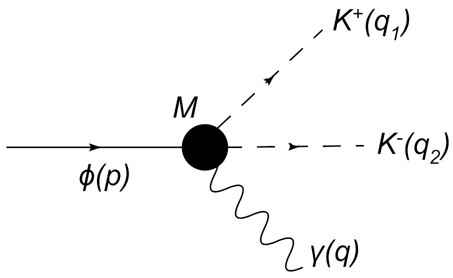

Figure 2.1: General Feynman diagram representing the $\phi \rightarrow K^+ + K^- + \gamma$ decay.

photon, the amplitude of which is well described by quantum electrodynamics [16]. The second mechanism is the structural radiation which is caused by the internal transformation of the ϕ-meson quark structure. The contribution to the structural radiation amplitude results from the scalar $f_0(980)$ and $a_0(980)$ meson intermediate states. In order to calculate the contribution of scalar meson intermediate states we employ two different models. The first model is the kaon-loop model where the scalar meson couples to ϕ meson through a charged kaon loop [14]. The second one is the recently proposed no structure (NS) model, which should more properly be called point coupling model, where the coupling of scalar meson to ϕ meson is considered to be point-like [15].

In terms of the invariant amplitude $\mathcal{M}(E_1, E_\gamma)$ the differential decay probability for an unpolarized ϕ meson at rest is given by

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\frac{1}{8M_\phi}|\mathcal{M}|^2 \tag{2.1}$$

where we perform an average over the spin states of ϕ meson and a sum over the polarization states of the photon. The decay width $\Gamma(\phi \rightarrow K^+ + K^- + \gamma)$ is obtained by integration

$$\Gamma = \int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \quad . \tag{2.2}$$

The maximum photon energy $E_{\gamma,max}$ is given as $E_{\gamma,max} = (M_\phi^2 - 4M_K^2)/2M_\phi$

which is equal to $E_{\gamma,max} = 31.5\ MeV$. The minimum photon energy is kinematically equal to zero, however in our calculation we consider the experimentally detected minimum photon energy which we use as $E_{\gamma,min} = 10\ MeV$. The maximum and minimum values for K^+ meson energy E_1 are given by

$$E_{1max,1min} = \frac{1}{2(2E_\gamma M_\phi - M_\phi^2)} \left\{ -2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3 \right.$$
$$\left. \pm E_\gamma \sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_K^2)} \right\} \quad (2.3)$$

The above relations given in Eq. 2.1, 2.2 and 2.3 have been discussed previously on several occasions [12], and we present the derivations in Appendix A for completeness.

For a particular reaction the invariant amplitude $\mathcal{M}$ can be obtained from the Feynman diagrams describing the mechanism of the given reaction [16]. The intermediate scalar meson states in these diagrams are represented by the scalar meson propagator $D(q) = i/(q^2 - M_s^2 + i\epsilon)$ in the corresponding amplitude. Since the scalar mesons f_0 and a_0 are unstable with a finite lifetime and they are broad, we use the Breit- Wigner prescription in the propagators of these resonances. In the scalar meson propagators we make the replacement $q^2 - M_s^2 \rightarrow q^2 - M_s^2 + i\sqrt{q^2}\,\Gamma_s$ where the energy dependent widths for these scalar resonances are given as

$$\begin{aligned} \Gamma_{f_0}(q^2) &= \frac{g^2_{f_0 K^+ K^-}}{16\pi\sqrt{q^2}} \sqrt{1 - \frac{4M^2_{K^+}}{q^2}}\,\theta(\sqrt{q^2} - 2M_{K^+}) \\ &+ \frac{g_{f_0 K^0 \bar{K}^0}}{16\pi\sqrt{q^2}} \sqrt{1 - \frac{4M^2_{K^0}}{q^2}}\,\theta(\sqrt{q^2} - 2M_{K^0}) \\ &+ \frac{2}{3}\Gamma_{f_0} \frac{M_{f_0}}{\sqrt{q^2}} \frac{\sqrt{1 - \frac{4M^2_{\pi^0}}{q^2}}}{\sqrt{1 - \frac{4M^2_{\pi^0}}{M^2_{f_0}}}}\,\theta(\sqrt{q^2} - 2M_{\pi^0})\ , \end{aligned} \quad (2.4)$$

$$
\begin{aligned}
\Gamma_{a_0}(q^2) &= \frac{g^2_{a_0K^+K^-}}{16\pi\sqrt{q^2}}\sqrt{1-\frac{4M^2_{K^+}}{q^2}}\theta(\sqrt{q^2}-2M_{K^+}) \\
&+\frac{g_{a_0K^0\bar{K}^0}}{16\pi\sqrt{q^2}}\sqrt{1-\frac{4M^2_{K^0}}{q^2}}\theta(\sqrt{q^2}-2M_{K^0}) \\
&+\Gamma_{a_0}\frac{M_{a_0}}{\sqrt{q^2}}\frac{\sqrt{\left[1-\frac{(M_{\pi^0}+M_\eta)^2}{q^2}\right]\left[1-\frac{(M_{\pi^0}-M_\eta)^2}{q^2}\right]}}{\sqrt{\left[1-\frac{(M_{\pi^0}+M_\eta)^2}{M^2_{a_0}}\right]\left[1-\frac{(M_{\pi^0}-M_\eta)^2}{M^2_{a_0}}\right]}} \\
&\times\ \theta(\sqrt{q^2}-(M_{\pi^0}+M_\eta))\ , \qquad (2.5)
\end{aligned}
$$

and we use the experimental values for the widths Γ_{f_0} and Γ_{a_0} of the scalar resonances f_0 and a_0 in the above expressions [17].

2.2 Internal Bremsstrahlung Mechanism

The interaction of charged pseudoscalar K-mesons, and vector ϕ-meson is described in lowest order of chiral perturbation theory by the following Lagrangian

$$
\mathcal{L}_{\phi K^+K^-} = -ig_{\phi K^+K^-}\phi^\mu(K^-\partial_\mu K^+ - K^+\partial_\mu K^-) \qquad (2.6)
$$

where $g_{\phi K^+K^-}$ is the coupling constant characterizing the ϕK^+K^--vertex and the fields for the relevant particles are denoted by the same symbols as the respective particles. Interaction with the electromagnetic field A_μ is introduced with the replacement $\partial_\mu \rightarrow \partial_\mu + iqA_\mu$, where q is the charge of the particle destroyed by the field on which the operator ∂_μ acts in the above and in the free Lagrangian of K-meson system, and this way we obtain the gauge invariant Lagrangian describing the interaction of charged K-mesons, ϕ-meson and the photon as

$$\mathcal{L}_I = -i(eA^\mu + g_{\phi K^+K^-}\phi^\mu)(K^-\partial_\mu K^+ - K^+\partial_\mu K^-) + 2eg_{\phi K^+K^-}\phi^\mu A_\mu K^+K^- \quad . \tag{2.7}$$

We determine the coupling constant $g_{\phi K^+K^-}$ by employing experimental information for the ϕ-meson decays. The main decay mode of ϕ meson is the $\phi \to K^+ + K^-$ decay. The decay rate for the $\phi \to K^+ + K^-$ decay that results from the Lagrangian given in Eq. 2.6 can be calculated as

$$\Gamma(\phi \to K^+ + K^-) = \frac{g^2_{\phi K^+K^-}}{48\pi} M_\phi \left[1 - \left(\frac{2M_K}{M_\phi}\right)^2\right]^{\frac{3}{2}} \quad . \tag{2.8}$$

The derivation of this decay rate and the other two-body decay rates that we use in this thesis are presented in Appendix B for completeness. We then use the experimental value for the branching ratio $BR(\phi \to K^+ + K^-) = 0.491$ for the decay $\phi \to K^+ + K^-$ [17] and this way we determine the coupling constant $g_{\phi K^+K^-}$ as $g_{\phi K^+K^-} = 4.43$.

The Feynman diagrams that describe the internal bremsstrahlung that follow from the Lagrangian given in Eq. 2.7 are shown in Fig. 2.2. The first two terms in this figure are not gauge invariant and they are supplemented by the direct term shown in Fig. 2.2(c) that results from the minimal coupling prescription.

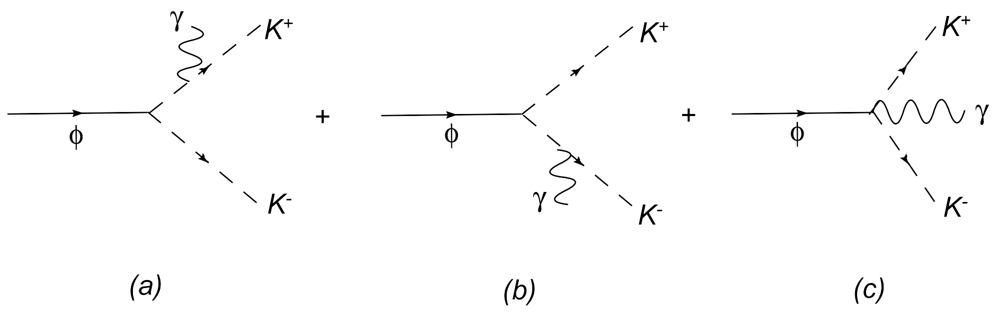

Figure 2.2: Feynman diagrams for internal bremsstrahlung for the decay reaction $\phi \to K^+ + K^- + \gamma$ decay reaction.

We denote the corresponding amplitudes of the Feynman diagrams for internal bremsstrahlung as $\mathcal{M}_{Ba}$, $\mathcal{M}_{Bb}$ and $\mathcal{M}_{Bc}$, and write the amplitude for the internal bremsstrahlung process as $\mathcal{M}_B = \mathcal{M}_{Ba} + \mathcal{M}_{Bb} + \mathcal{M}_{Bc}$. We note that although each amplitude is not gauge invariant, the total amplitude $\mathcal{M}_B$ is gauge invariant. We give the amplitude $\mathcal{M}_B$ and $|\mathcal{M}_B|^2$ explicitly in Appendix C.

2.3 Kaon-Loop Mechanism For The Scalar Meson Contribution

The kaon-loop model was suggested for the radiative decays $\phi \to f_0 + \gamma$ and $\phi \to a_0 + \gamma$ of $\phi(1020)$ vector meson into isoscalar $f_0(980)$ and isovector $a_0(980)$ scalar mesons and a photon through the theoretical analyses of experimental investigations [18]. Although these decays have been studied extensively [12], we present a brief discussion of these decays through the kaon-loop mechanism for completeness. The interaction of vector ϕ-meson, charged K-mesons and the photon is described by the gauge invariant chiral Lagrangian given Eq. 2.7. The interaction of the scalar meson S, where S stands for either $f_0(980)$ or $a_0(980)$ meson, with the pseudoscalar K^+ and K^- mesons is described by the effective Lagrangian

$$\mathcal{L}_{SK^+K^-} = -g_{SK^+K^-} K^+ K^- S \quad . \tag{2.9}$$

The Feynman diagrams that describe the radiative decays $\phi \to S + \gamma$ through the kaon-loop mechanism are shown in Fig. 2.3.

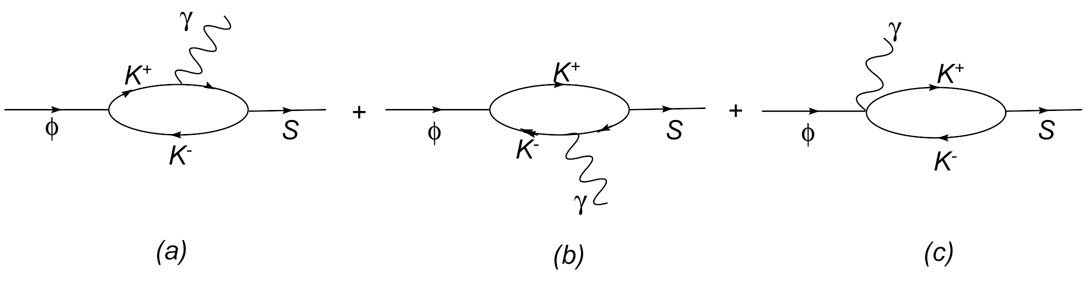

Figure 2.3: Feynman diagrams for the $\phi \to S + \gamma$ decay.

The vertices in these Feynman diagrams can be read off from the Lagrangian given in Eqs. 2.7 and 2.9, and we then obtain the amplitude for the radiative decay of ϕ meson into a scalar meson and a single photon as

$$\mathcal{M}_{KL}(\phi \rightarrow S + \gamma) = u^{\mu}\epsilon^{\nu}(p_{\nu}q_{\mu} - g_{\mu\nu}p \cdot q)\frac{e g_{\phi K^+K^-} g_{SK^+K^-}}{i2\pi^2 M_k^2}\mathcal{I}(a,b) \tag{2.10}$$

where (u,p) and (ϵ, q) are the polarization vector and four momentum of the ϕ meson and the photon, respectively. In this expression, the parameters a and b are given by $a = M_{\phi}^2/M_K^2$ and $b = M_s^2/M_K^2$. In this case the unstable scalar meson then decays into the final state K^+K^-, the factor M_s^2 in the parameter b must be replaced by the square of the invariant mass of the K^+K^- final state. It should be noted that the amplitude $\mathcal{M}(\phi \rightarrow S + \gamma)$ has the structure required by gauge invariance. The invariant function $\mathcal{I}(a,b)$ is given in terms of Feynman integrals appropriate to the Feynman diagrams shown in Fig. 2.3. These integrals have been evaluated previously in different works several times by different methods [19], and the function $\mathcal{I}(a,b)$ is obtained as

$$\mathcal{I}(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right)\right] + \frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right)\right] \tag{2.11}$$

$$f(x) = \begin{cases} -\left[\arcsin(\frac{1}{2\sqrt{x}})\right]^2 \ , & x > \frac{1}{4} \\ \frac{1}{4}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right]^2 \ , & x < \frac{1}{4} \end{cases}$$

$$g(x) = \begin{cases} (4x-1)^{\frac{1}{2}} \arcsin(\frac{1}{2\sqrt{x}}) \ , & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{\frac{1}{2}}\left[\ln(\frac{\eta_+}{\eta_-}) - i\pi\right] \ , & x < \frac{1}{4} \end{cases}$$

$$\eta_{\pm} = \frac{1}{2x}\left[1 \pm (1-4x)^{\frac{1}{2}}\right] \ . \tag{2.12}$$

We present a derivation of the above result in Appendix D.

The decay rate $\Gamma(\phi \rightarrow S + \gamma)$ of the radiative decay $\phi \rightarrow S + \gamma$ of the vector ϕ meson into a scalar meson S, which is $f_0(980)$ or $a_0(980)$, and a single photon can then be obtained from the corresponding amplitude $\mathcal{M}_{KL}(\phi \to S+\gamma)$ given in Eq. 2.10 in a straightforward fashion which is outlined in Appendix B as

$$\Gamma(\phi \to S\gamma) = \frac{\alpha g^2_{\phi K^+K^-} g^2_{SK^+K^-}}{3(2\pi)^4} \frac{\omega}{M^2_\phi} \mid (a-b)I(a,b) \mid^2 \tag{2.13}$$

where α is the fine structure constant and $\omega = (M^2_\phi - M^2_S)/2M_\phi$ is the energy of the emitted photon. The coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ can be therefore obtained using the above formula for the decay rate $\Gamma(\phi \to S+\gamma)$ and by utilizing the experimental values of the branching ratios $BR(\phi \to f_0+\gamma)$ and $BR(\phi \to a_0+\gamma)$. These radiative decays have been studied by different experimental groups. We use the recent results $BR(\phi \to f_0+\gamma) = (4.40 \pm 0.21) \times 10^{-4}$ and $BR(\phi \to a_0+\gamma) = (0.76 \pm 0.06) \times 10^{-4}$ reported by the KLOE collaboration [11], and we obtain these coupling constants as $g_{f_0K^+K^-} = (5.14 \pm 1.2)$ and $g_{a_0K^+K^-} = (2.13 \pm 0.8)$.

In the kaon-loop model for the radiative decay of the ϕ-meson into a pair of charged K^+K^- mesons and a single photon there are two contributions to the decay mechanism. One is the internal bremsstrahlung contribution discussed in section 2.2, and the other one is the contribution coming from the scalar mesons taken into account through the coupling of the scalar mesons to the ϕ-meson by a charged kaon-loop. Therefore, in the kaon-loop model, the mechanism of the radiative decay $\phi \to K^+ + K^- + \gamma$ is represented by the Feynman diagrams shown in Fig. 2.4. The amplitude $\mathcal{M}(\phi \to K^+ + K^- + \gamma)$ can thus be written as $\mathcal{M} = \mathcal{M}_B + \mathcal{M}_{KL}$ where the internal bremsstrahlung amplitude $\mathcal{M}_B$ is obtained from the Feynman diagams shown in Fig. 2.4.(a,b,c) and the amplitude $\mathcal{M}_{KL}$ representing the contribution coming to the decay from the scalar mesons is obtained from the Feynman diagrams shown in Fig 2.4.(d,e,f).

The individual amplitudes as well as $|\mathcal{M}|^2$ calculated from these contributions are presented in Appendix E.

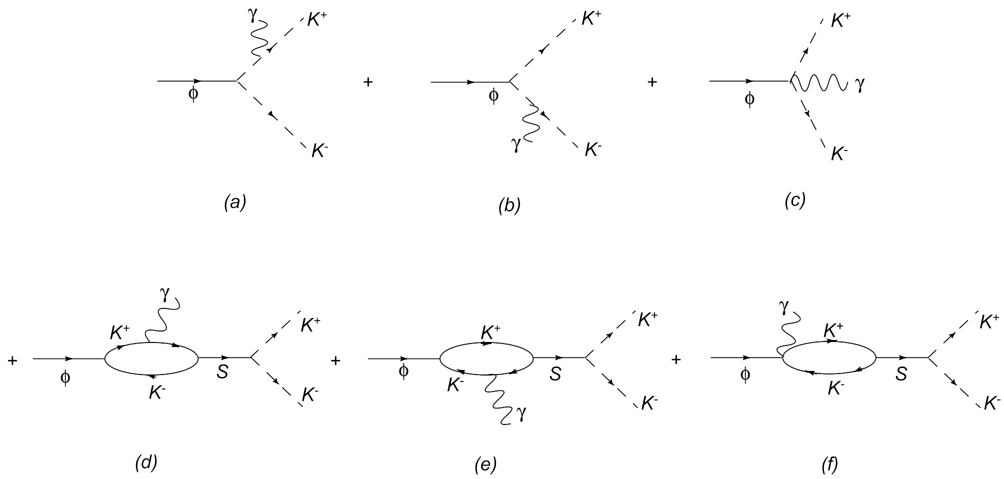

Figure 2.4: Feynman diagrams for the decay $\phi \to K^+ + K^- + \gamma$ in the kaon-loop mechanism, where S denotes the scalar f_0 or a_0 meson.

2.4 No-Structure Model For The Scalar Meson Contribution

No-structure model, or point-coupling model, has been recently introduced to study the basic Φ-factory observables, such as $e^+ + e^- \to P + P' + \gamma$ cross sections, where P, P' denote pseudoscalar mesons, because an accurate and possibly model-independent description of all the components of these reactions are very valuable in order to obtain reliable information about the low-mass scalar meson sector of QCD [15]. In this model, the contribution of the scalar meson to the radiative decay $\phi \to K^+ + K^- + \gamma$ is characterized by the Feynman diagram shown in Fig. 2.5 where S denotes f_0 or a_0 scalar meson.

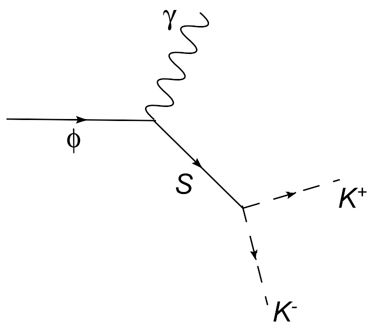

Figure 2.5: Feynman diagrams for the contribution of scalar meson in the no-structure model.

In the Feynman diagram describing the contribution of the scalar meson intermediate state to the radiative decay $\phi \to K^+ + K^- + \gamma$, the SK^+K^--vertex is described by the effective Lagrangian given in Eq. 2.9. We describe the $\phi S\gamma$-vertex by the gauge invariant effective Lagrangian

$$\mathcal{L}_{\phi S\gamma} = \frac{e}{M_\phi} g_{\phi S\gamma} \partial^\mu \phi^\nu [\partial_\mu A_\nu - \partial_\nu A_\mu] S \tag{2.14}$$

where ϕ^μ denotes the ϕ-meson field, A^μ is the photon field, S denotes the scalar meson field, and $g_{\phi S\gamma}$ is the coupling constant characterizing the point-coupling of the ϕ-meson, the scalar meson and the photon. The decay rate $\Gamma(\phi \to S+\gamma)$ for the radiative decay of the ϕ-meson into a scalar meson and a photon resulting from this Lagrangian is given as

$$\Gamma(\phi \to S+\gamma) = \frac{\alpha}{24\pi} \frac{\left(M_\phi^2 - M_s^2\right)^3}{M_\phi^5} g_{\phi S\gamma}^2 \; . \tag{2.15}$$

We then use the experimental branching ratios $BR(\phi \to f_0 + \gamma)$ and $BR(\phi \to a_0 + \gamma)$ determined by the KLOE collaboration [11] and obtain the coupling constants $g_{\phi f_0 \gamma}$ and $g_{\phi a_0 \gamma}$ as $g_{\phi f_0 \gamma} = -3.72$, $g_{\phi a_0 \gamma} = -1.87$. We note that if isoscalar σ-meson, isoscalar f_0-meson and isovector a_0-meson are assigned to a unitary $SU(3)$ nonet, then it follows that $g_{\phi f_0 \gamma} < 0$ and $g_{\phi a_0 \gamma} < 0$ [16].

The amplitude of the radiative decay $\phi \to K^+ + K^- + \gamma$ has contributions coming from the internal bremsstrahlung mechanism and from the scalar meson intermediate state as described by the point coupling mechanism in the no-structure model for this radiative decay. Therefore, the mechanism of the radiative $\phi \to K^+ + K^- + \gamma$ decay in the no-structure model is described by the Feynman diagrams shown in Fig. 2.6. The amplitude $\mathcal{M}$ of the decay is then obtained as $\mathcal{M} = \mathcal{M}_B + \mathcal{M}_{NS}$ where the amplitude $\mathcal{M}_B$ characterizing the internal bremsstrahlung mechanism is calculated using the Feynman diagrams given Fig. 2.6 (a,b,c), and the amplitude $\mathcal{M}_{NS}$ characterizing the contribution of the intermediate state scalar mesons f_0 and a_0 are calculated from the Feyn-

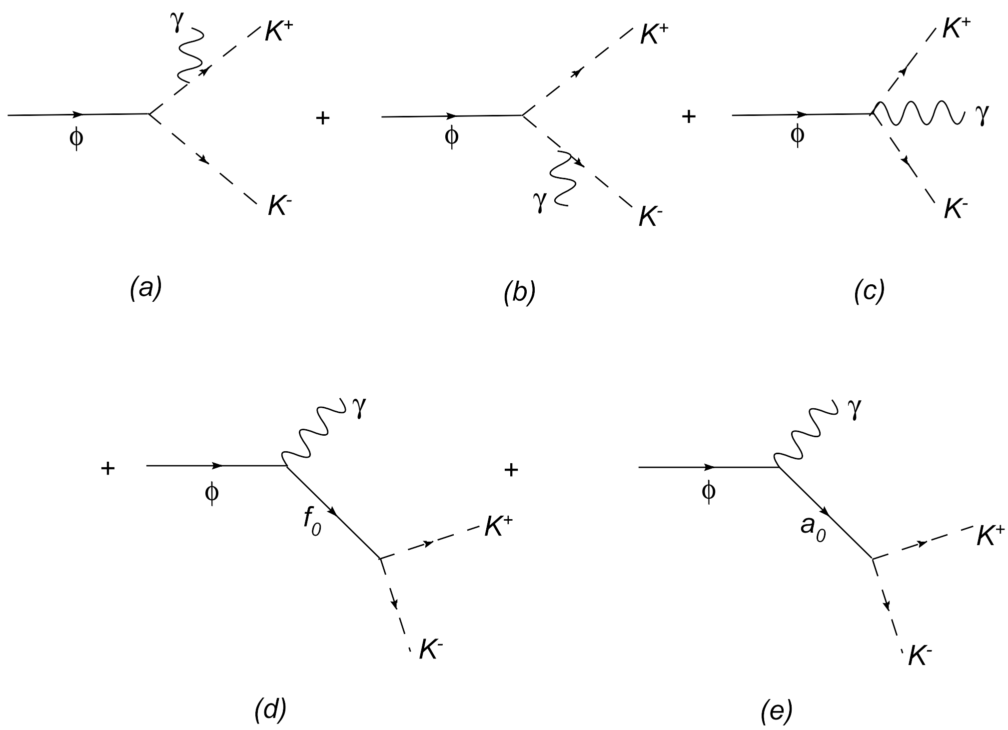

Figure 2.6: Feynman diagrams for the radiative decay $\phi \rightarrow K^+ + K^- + \gamma$ in the no-structure model.

man diagrams shown in Fig. 2.6 (d,e). The bremssrahlung amplitudes $\mathcal{M}_B$ and scalar meson contribution amplitudes $\mathcal{M}_{Nf_0}$ *and* $\mathcal{M}_{Na_0}$ as well as $|\mathcal{M}|^2$ calculated using these amplitudes are given and discussed in Appendix F.

CHAPTER 3

RESULTS AND DISCUSSION

In this chapter, we present our results and our discussions about the role of the scalar mesons in the mechanism of the radiative $\phi \rightarrow K^+ + K^- + \gamma$ decay.We compare the results we obtained for the two different models, namely the kaon-loop model and the no-structure model, to include the scalar meson intermediate states into the mechanism of the vector ϕ-meson radiative $\phi \rightarrow K^+ + K^- + \gamma$ decay. It should be mentioned that the radiative decay reactions of the type $V \rightarrow P + P' + \gamma$ where V denotes a vector meson, P and P' denote pseudoscalar mesons have been analyzed in order to obtain information about the nature and the quark substructure of the scalar mesons contributing in the intermediate states to the amplitudes of these reactions. However, our discussion will not be about the structure of the scalar mesons but about their role in the mechanism of the $\phi \rightarrow K^+ + K^- + \gamma$ reaction. In our phenomenological approach, the coupling constants of scalar mesons to different hadron and photon states reflecting their structure are taken from the relevant experimental quantities.

3.1 Kaon-Loop Model

In our numerical calculation of the decay width of the radiative $\phi \rightarrow K^+ + K^- + \gamma$ decay reaction as well as in our analysis of the contributions of the internal bremsstrahlung and of the scalar mesons calculated using kaon-loop mechanism to the decay width of this reaction we use the mass values $M_{K^+} = 494\ MeV$, $M_\phi = 1020\ MeV$, $M_{f_0} = 980\ MeV$, $M_{a_0} = 984.7\ MeV$, and the coupling constants $g_{\phi K^+K^-} = 4.47$, $g_{f_0 K^+K^-} = 5.24$, and $g_{a_0 K^+K^-} = 2.30$. In our calculation, the minimum photon energy is taken as $E_\gamma = 10\ MeV$. For the decay width and consequently for the branching ratio of the $\phi \to K^+ + K^- + \gamma$ radiative decay reaction we obtain the values $\Gamma(\phi \to K^+ + K^- + \gamma) = 2.21 \times 10^{-4}$ MeV, and $BR(\phi \to K^+ + K^- + \gamma) = 5.18 \times 10^{-5}$, respectively. On the other hand, for minimum photon energy $E_{\gamma,min} = 5$ MeV these values become $\Gamma(\phi \to K^+ + K^- + \gamma) = 4.56 \times 10^{-4}$ MeV and $BR(\phi \to K^+ + K^- + \gamma) = 1.07 \times 10^{-4}$. If we consider the contribution of scalar meson intermediate state only, we then obtain for the branching ratio the value $BR(\phi \to S\gamma \to K^+ + K^- + \gamma) = 9.94 \times 10^{-6}$ for $E_{\gamma,min} = 5$ MeV, and $BR(\phi \to S\gamma \to K^+ + K^- + \gamma) = 9.71 \times 10^{-6}$ for $E_{\gamma,min} = 10$ MeV. If we calculate the branching ratio by considering the bremsstrahlung amplitude only, the resulting branching ratio is $BR_{Brem}(\phi \to K^+ + K^- + \gamma) = 9.07 \times 10^{-5}$ for $E_{\gamma,min} = 5$ MeV and $BR_{Brem}(\phi \to K^+ + K^- + \gamma) = 3.55 \times 10^{-5}$ for $E_{\gamma,min} = 10$ MeV. In the figures, we show the results for minimum photon energy $E_{\gamma,min} = 10$ MeV. The photon spectra for the decay width of the de-

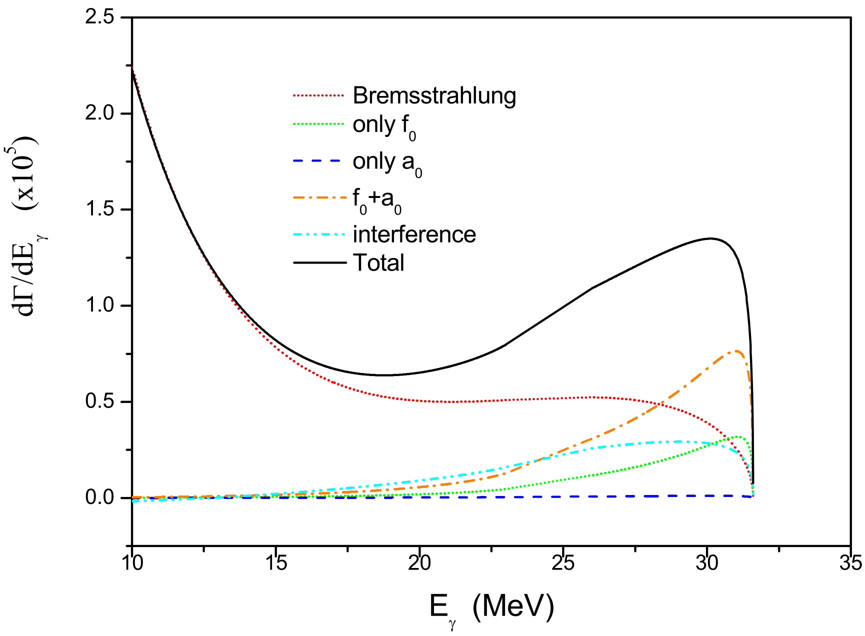

Figure 3.1: The photon spectra for the decay width of $\phi \to K^+ + K^- + \gamma$ in the kaon-loop model. The contributions of different terms are indicated.

cay $\phi \to K^+ + K^- + \gamma$ is plotted in Fig. 3.1 as a function of photon energy E_γ. The contributions of bremsstrahlung and structural radiation amplitude calculated with the f_0- and a_0- scalar meson intermediate states calculated in the kaon-loop model, the interference contribution of the bremsstrahlung amplitude with the total structural radiation amplitude of the both scalar mesons are shown as a function of photon energy. The contribution of the scalar mesons amplitude becomes increasingly important in the region of high photon energies. The contribution coming from the interference term is constructive, therefore in the region of high photon energies shape of the photon spectra curve is considerably modified and it is quite different from that obtained using only the bremsstrahlung amplitude.

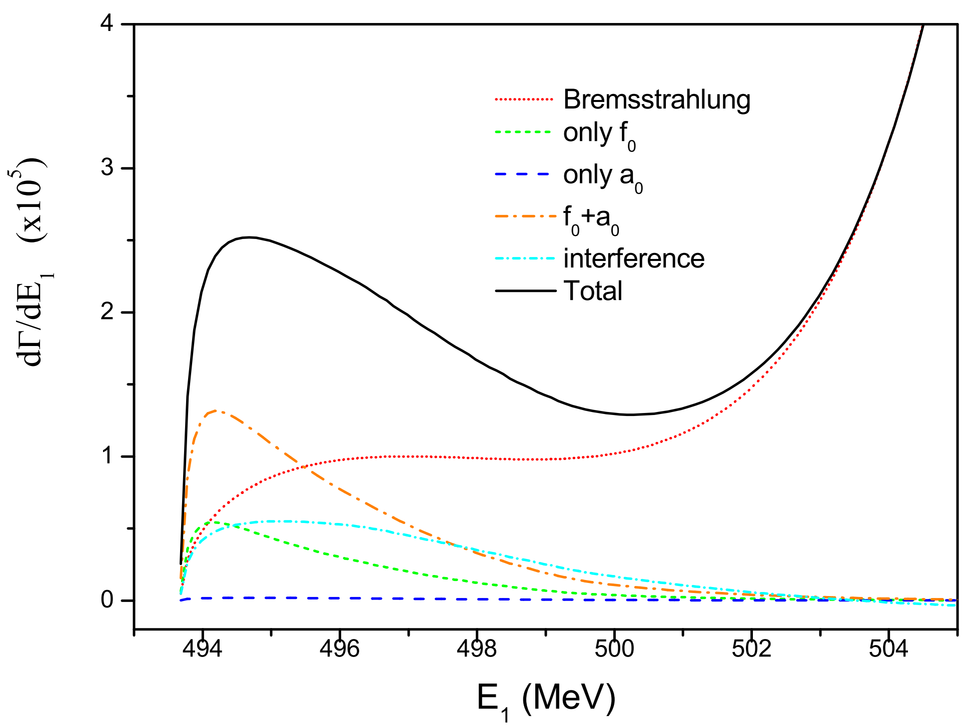

Figure 3.2: The kaon energy spectra for the decay width of $\phi \to K^+ + K^- + \gamma$ in the kaon-loop model. The contributions of different terms are indicated.

The kaon energy spectra for the decay width of the $\phi \to K^+ + K^- + \gamma$ reaction is shown in Fig. 3.2 where the contribution of different terms are also indicated.

The effect of including the scalar meson contributions is again significant in the region of low kaon energies which becomes quite insignificant for high values of kaon energy where the spectra is determined solely by the bremsstrahlung amplitude. We show the dependence of the decay width of the radiative $\phi \to$

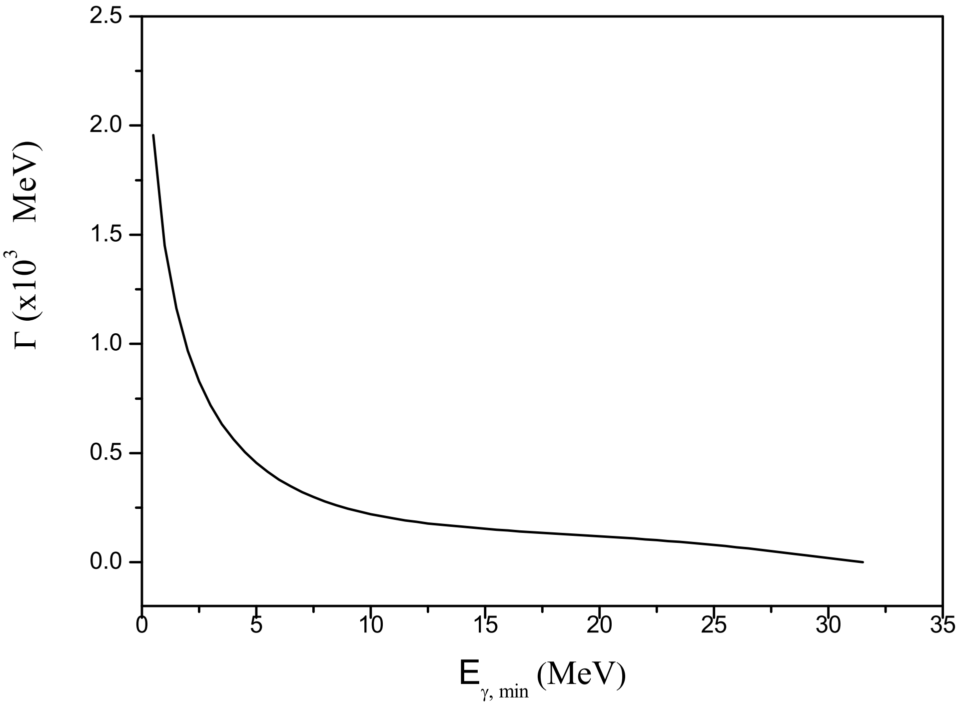

Figure 3.3: The decay width of $\phi \to K^+ + K^- + \gamma$ as a function of minimum detected photon energy in the kaon-loop model.

$K^+ + K^- + \gamma$ decay on the minimum detected photon energy which results in our calculation in Fig. 3.3. This dependence is quite strong, however since there is no experimental study of this reaction we use the minimum detected photon energy as $E_\gamma = 10\ MeV$ in our calculations.

Furthermore, in order to provide an experimental test for the magnitude of the structural radiation mechanism involving scalar mesons in the future experimental studies of the radiative $\phi \to K^+ + K^- + \gamma$ reaction, we plot the ratio $\mathcal{R}_\beta$ as a function of β which we show Fig. 3.4.

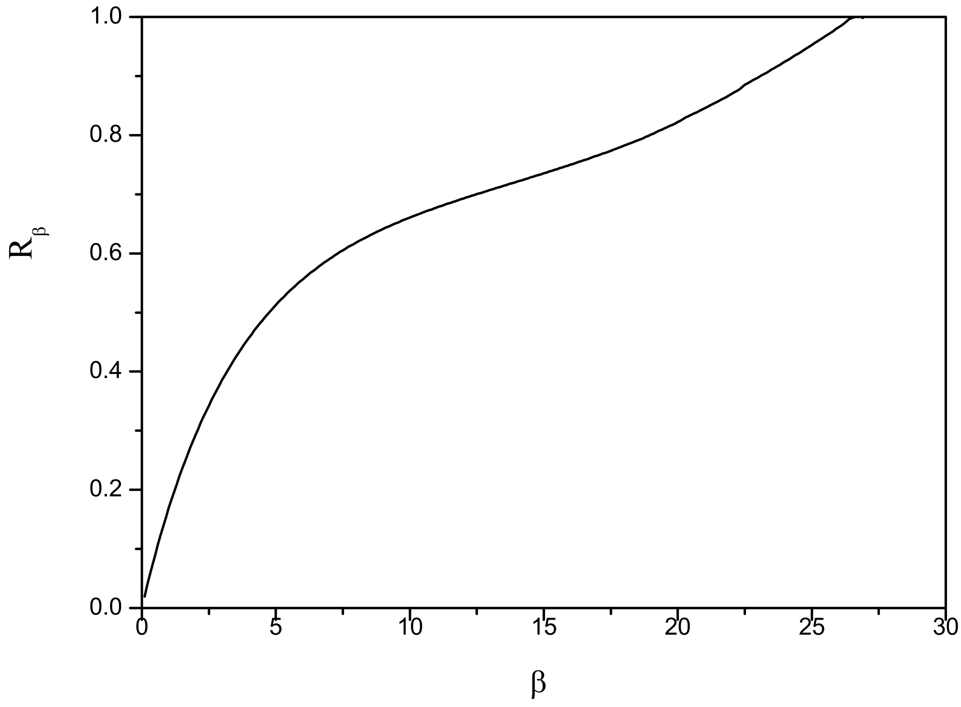

Figure 3.4: The ratio $\mathcal{R}_\beta = \frac{\Gamma_\beta}{\Gamma_{tot}}$ as a function of β for the decay reaction $\phi \to K^+ + K^- + \gamma$ in the kaon-loop model.

The ratio $\mathcal{R}_\beta$ is defined by

$$\mathcal{R}_\beta = \frac{\Gamma_\beta}{\Gamma_{tot}(\phi \to K^+ + K^- + \gamma)} \ , \tag{3.1}$$

where the terms in the numerator and in the denominator of this ratio are given as

$$\Gamma_\beta = \int_{10}^{10+\beta} \quad , \quad \Gamma_{tot} = \int_{10}^{E_\gamma, max} dE_\gamma \frac{d\Gamma}{dE_\gamma} \tag{3.2}$$

The shape of this curve indicates through its derivation from that of pure bremsstrahlung mechanism the contributions of scalar meson amplitudes and shows that the dependence of the decay width in the contribution of the structural radiation characterized by the scalar meson amplitudes is quite important in the region of high photon energies.

3.2 No-Structure Model

In our calculation of the decay width of radiative $\phi \to K^+ + K^- + \gamma$ decay in the framework of the no-structure model the additional parameters that are needed are the coupling constants characterizing the point couplings of vector meson ϕ, scalar mesons f_0 and a_0, pseudoscalar mesons K^+ and K^-, and the photon. The coupling constant $g_{\phi S\gamma}$ which characterizes the vector ϕ-meson, scalar S-meson ($S = f_0$ *or* a_0) and the photon point vertex is determined by employing the effective Lagrangian given in Eq. 2.14 and using the experimental values of the branching ratios $BR(\phi \to f_0 + \gamma)$ and $BR(\phi \to a_0 + \gamma)$. The coupling constants obtained this way are $g_{\phi f_0\gamma} = -3.71$, and $g_{\phi a_0\gamma} = -1.87$. However, since there are no direct experimental information that we can employ to determine the point coupling of scalar mesons to a charged kaons K^+K^- final state, we use the results for these coupling constants obtained through QCD sum rules techniques. The values of the coupling constants $g_{f_0K^+K^-}$ and $g_{a_0K^+K^-}$ determined through studies using light cone QCD sum rules method are $g_{f_0K^+K^-} = 7.14$ [21] and $g_{a_0K^+K^-} = -5.08$ [22]. It should be noted that the relative minus sign between these coupling constants which is consistent with the $q^2\bar{q}^2$ model for the structure of the scalar mesons has a pround effect for the contribution of scalar meson amplitudes to the total amplitude for the decay width of the radiative $\phi \to K^+ + K^- + \gamma$ reaction because these amplitudes are linear in these coupling constants in the no-structure model. On the other hand, the scalar meson amplitudes are quadric in these coupling constants in the kaon-loop model for the contribution of scalar mesons to the decay amplitude of $\phi \to K^+ + K^- + \gamma$ reaction. For the decay width and consequently for the branching ratio of the $\phi \to K^+ + K^- + \gamma$ radiative decay reaction in the no-structure model we obtain the values $\Gamma(\phi \to K^+ + K^- + \gamma) = 4.36 \times 10^{-6}$ MeV, and $BR(\phi \to K^+ + K^- + \gamma) = 1.02 \times 10^{-6}$, respectively. On the other hand, for minimum photon energy $E_{\gamma,min} = 5$ MeV these values become $\Gamma(\phi \to K^+ + K^- + \gamma) = 2.26 \times 10^{-4}$ MeV and $BR(\phi \to K^+ + K^- + \gamma) = 5.30 \times 10^{-5}$. If we consider the contribution of scalar meson intermediate state only, we then obtain for the branching ratio the value $BR(\phi \to S \to K^+ + K^- + \gamma) = 1.76 \times 10^{-6}$ for $E_{\gamma,min} = 5$ MeV, and $BR(\phi \to S\gamma \to K^+ + K^- + \gamma) = 1.69 \times 10^{-6}$ for $E_{\gamma,min} = 10$ MeV. If we calculate the branching ratio by considering the bremsstrahlung amplitude only, the resulting branching ratio is $BR_{Brem}(\phi \to K^+ + K^- + \gamma) = 9.07 \times 10^{-5}$ for $E_{\gamma,min} = 5$ MeV and $BR_{Brem}(\phi \to K^+ + K^- + \gamma) = 3.55 \times 10^{-5}$ for $E_{\gamma,min} = 10$ MeV, as before. In the figures, we show the results for minimum photon energy $E_{\gamma,min} = 10$ MeV. In Fig. 3.5 we show the photon spectra for

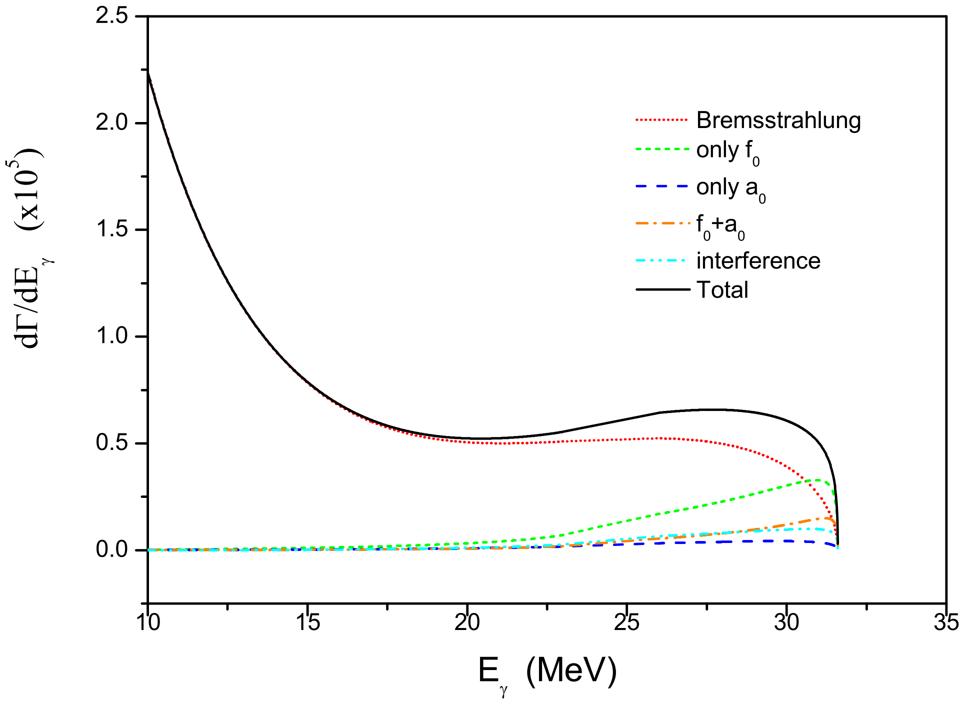

Figure 3.5: The photon spectra for the decay width of $\phi \to K^+ + K^- + \gamma$ in the no-structure model. The contributions of different terms are indicated.

the decay width of the radiative $\phi \to K^+ + K^- + \gamma$ decay calculated using the no-structure model for including the effects of intermediate scalar mesons to the total amplitude of the reaction where we also show the contributions of the different amplitudes in the mechanism of the reaction. It can be observed from this figure that the scalar meson amplitude again contributes in the region of high photon energies, however this contribution is not as pronounced as in the kaon-loop model so that the shape of the photon spectra does not deviate significantly from that of the bremsstrahlung mechanism. This is due to the relative minus sign between the coupling constants $g_{f_0 K^+ K^-}$ and $g_{f_0 K^+ K^-}$ which results in a cancelation of some degree of the contributions coming from the f_0- and a_0-scalar meson intermediate state amplitudes.

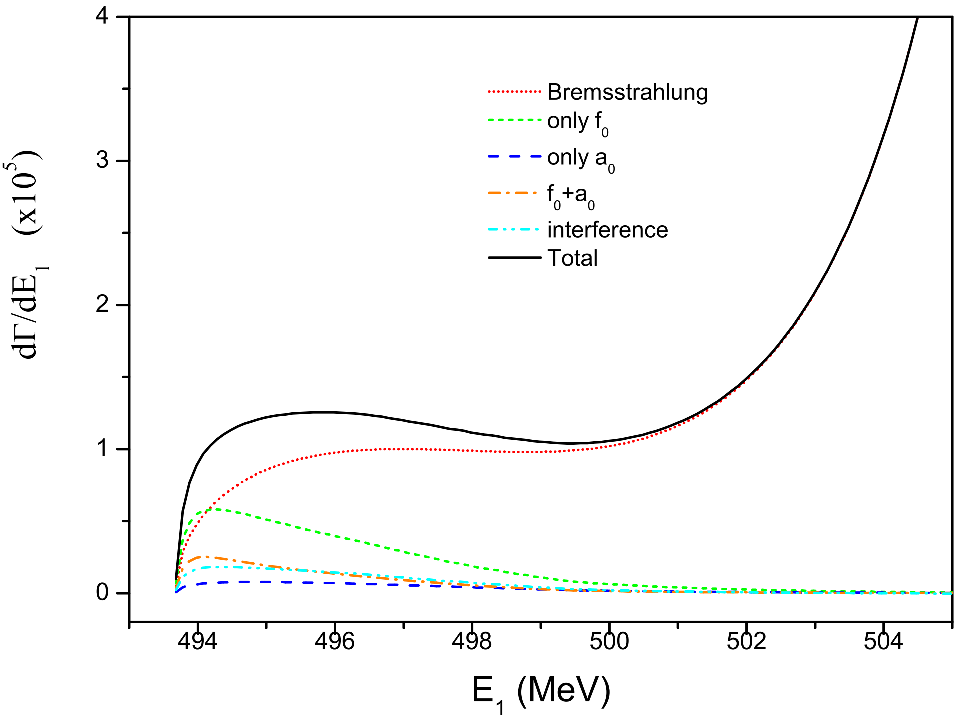

Figure 3.6: The kaon energy spectra for the decay width of $\phi \to K^+ + K^- + \gamma$ in the no-structure model. The contributions of different terms are indicated.

In Fig. 3.6 we show the energy spectra of the final state kaon for the decay width of $\phi \to K^+ + K^- + \gamma$ decay as a function of kaon energy where the contributions of bremsstrahlung amplitude, the contributions of scalar meson intermediate state amplitudes as well as the contribution of the interference term are shown together with the result obtained using the total amplitude. We again observe that the contributions of scalar mesons are significant only in the region of high photon energies, however they are small as compared to the contribution of the bremsstrahlung amplitude so that the energy spectra of the total amplitude essentially show the bremsstrahlung character.

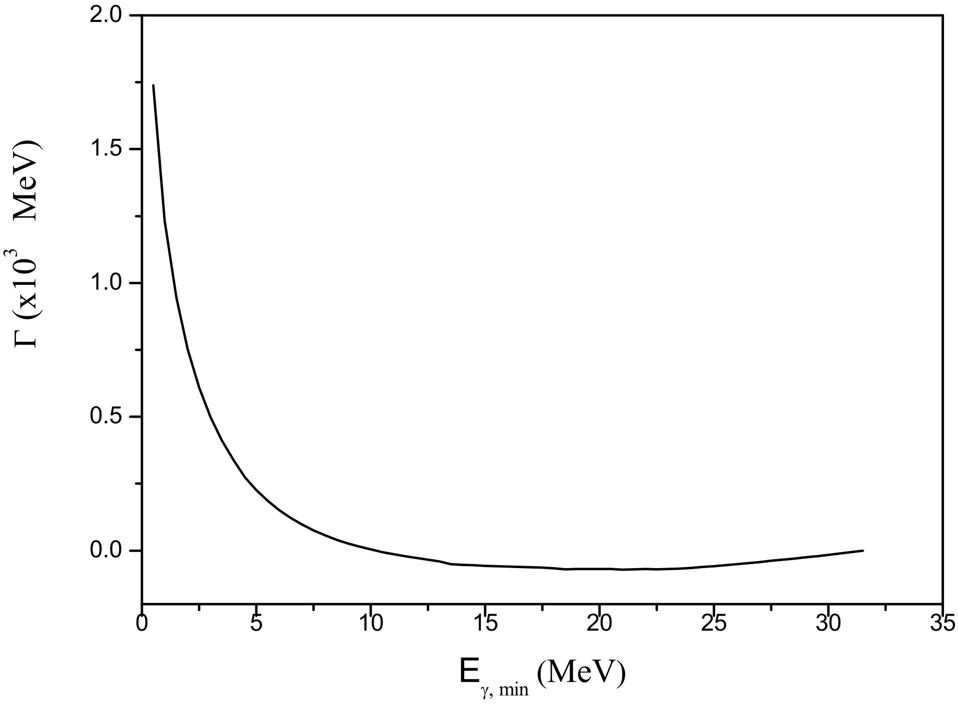

Figure 3.7: The decay width of $\phi \to K^+ + K^- + \gamma$ as a function of minimum detected photon energy in the no-structure model.

In Fig. 3.7 we show the decay width of the radiative $\phi \to K^+ + K^- + \gamma$ decay as a function of minimum detected photon energy. Since it is observed from Fig. 3.5 and Fig. 3.6, that the scalar meson intermediate state contribution is not pronounced enough to significantly change the bremsstrahlung spectra, the dependence of the decay width of the radiative $\phi \to K^+ + K^- + \gamma$ decay as a function of minimum detected photon energy is more critical as compared to the previous analysis where the kaon-loop was used for the scalar meson contribution which is shown in Fig. 3.3.

Finally, we show the ratio $\mathcal{R}_\beta = \frac{\Gamma_\beta}{\Gamma_{tot}}$ as a function of β in Fig. 3.8 for the no-structure model calculation of the scalar meson contributions to the $\phi \to$

$K^+ + K^- + \gamma$ radiative decay. It is to be observed that the modification of this ratio in the region of high photon energies is not as significant as in Fig. 3.4 calculated using the kaon-loop model for the intermediate scalar meson

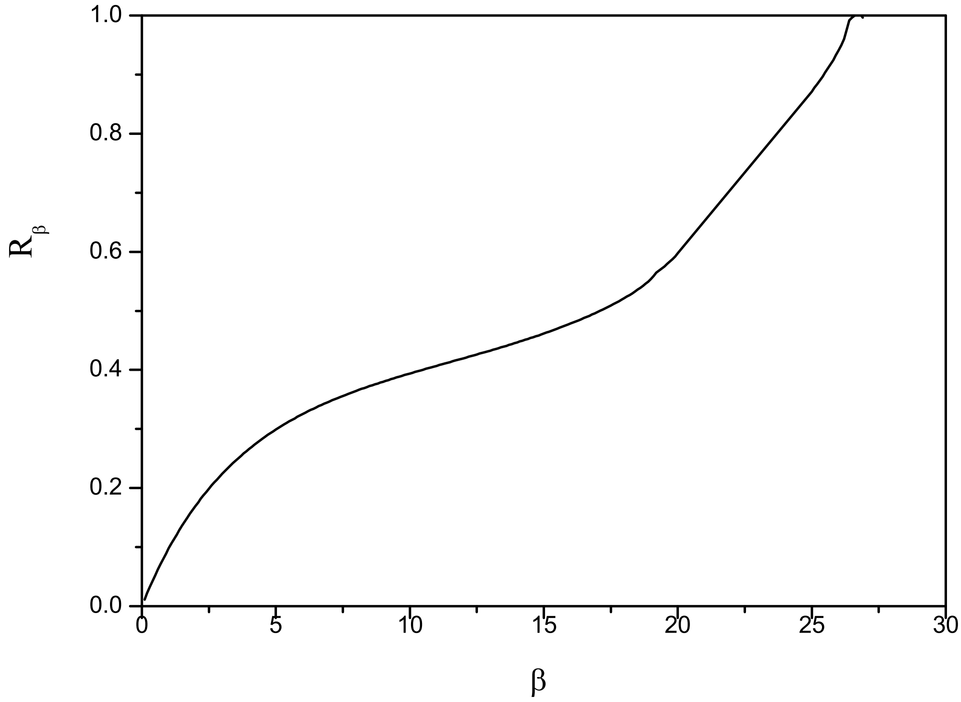

Figure 3.8: The ratio $\mathcal{R}_\beta = \frac{\Gamma_\beta}{\Gamma_{tot}}$ as a function of β for the reaction $\phi \rightarrow K^+ + K^- + \gamma$ in the no-structure model.

contributions.

3.3 Conclusion

In this thesis, we study the role of scalar mesons in hadron physics. Although still the controversial question of the nature and structure of scalar mesons is very important in hadron spectroscopy, our concern is to elucidate the role that the scalar mesons play in the mechanisms of reactions in hadron electrodynamics. It has been realized some time ago that the radiative decay processes where a vector meson decays into a pair of pseudoscalars and a photon are the source of valuable information about the nature of scalar mesons as well as the role they play in the reaction amplitudes.

We consider the particular radiative decay reaction where a vector ϕ-meson decays into a charged kaon pair K^+K^- and a photon. Our aim is to study the role that the scalar f_0- and a_0- mesons play in the mechanism of this reaction. We consider two models to investigate the f_0- and a_0- scalar meson intermediate states in the mechanism of this reaction. The first model is called the kaon-loop model where the scalar meson intermediate state couples to the initial ϕ-meson through a charged kaon-loop. In the second model the coupling of the intermediate scalar meson states to the initial ϕ-meson is point-like which is called no-structure model. In these models, we calculate the decay width of the $\phi \to K^+ + K^- + \gamma$ radiative reaction and we analyze the photon spectra for the decay width of this reaction as a function of photon energy and the final kaon energy spectra for the decay width as a function of kaon energy. We conclude that of when the contribution of intermediate scalar meson states in the mechanism of the radiative $\phi \to K^+ + K^- + \gamma$ reaction is taken into account using the kaon-loop model the photon spectra and the kaon energy spectra are modified considerably in the high photon energy regions whereas if we use the no-structure model for the contribution of scalar mesons the modifications are insignificant. Therefore, although the phase space for this reaction is small thus the experiment is difficult to perform, future experimental studies of the $\phi \to K^+ + K^- + \gamma$ radiative decay reaction may shed light on the role the scalar mesons play in the reaction mechanism of this decay in particular and also on their importance in hadron electrodynamics.

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APPENDIX A

THREE BODY FINAL STATE

We consider the decay of a vector ϕ-meson at rest to a three body final state consisting two pseudoscalar mesons and a photon. We denote the relevant four momenta as $V(p) \to P(q_1) + P'(q_2) + \gamma(q)$. Since the decaying particle is at rest, the invariant matrix element can be taken to be a function of E_1 and E_γ as $\mathcal{M}_{fi}(E_1, E_\gamma)$. The differential decay rate is given by [23]

$$d\Gamma = \frac{1}{(2E_p)(2\pi)^5}\,\delta^{(4)}(p - q_1 - q_2 - q)\frac{d^3q_1}{(2E_1)}\frac{d^3q_2}{(2E_2)}\frac{d^3q}{2E_\gamma}\overline{|\mathcal{M}_{fi}|}^2 \quad . \tag{A.1}$$

The absolute square of the invariant matrix element of the decay $\overline{|\mathcal{M}_{fi}|}^2$ is obtained by performing an average over the spin states of the initial vector meson and a sum over the polarization states of the photon. Since the vector meson is at rest, we can write the δ-function as

$$\delta^{(4)}(p - q_1 - q_2 - q) = \delta(M_\phi - E_1 - E_2 - E_\gamma)\delta^{(3)}(\vec{q_1} + \vec{q_2} + \vec{q}) \quad . \tag{A.2}$$

Three-momentum δ-function can be eliminated by performing the integral over d^3q_2. Since

$$\frac{d^3q}{2E_\gamma} = \frac{|\vec{q}|^2 dq d\Omega_\gamma}{2E_\gamma} = \frac{1}{2}E_\gamma dE_\gamma d\Omega_\gamma \quad , \tag{A.3}$$

$$\frac{d^3q_1}{2E_1} = \frac{|\vec{q_1}|^2 dq_1 d\Omega_1}{2E_1} = \frac{1}{2}|\vec{q_1}|dE_1 d\Omega_1 \quad , \tag{A.4}$$

and

$$E_2 = \sqrt{(\vec{q} + \vec{q_1})^2 + M_2^2}\ , \tag{A.5}$$

from the equation of the differential decay rate we obtain

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{|\vec{q_1}|E_\gamma \overline{|\mathcal{M}_{fi}|^2}}{16M(2\pi)^5}\int d\Omega_\gamma d\Omega_1 \frac{\delta(E_\gamma + E_1 - M + \sqrt{(\vec{q}+\vec{q_1})^2 + M_2^2})}{\sqrt{(\vec{q}+\vec{q_1})^2 + M_2^2}} \quad . \tag{A.6}$$

We then consider the integral I defined by

$$I = |\vec{q_1}|E_\gamma \int d\Omega_\gamma d\Omega_1 \frac{\delta(M - E_\gamma - E_1 + \sqrt{(\vec{q}+\vec{q_1})^2 + M_2^2})}{\sqrt{(\vec{q}+\vec{q_1})^2 + M_2^2}} \quad , \tag{A.7}$$

where $\qquad (\vec{q} + \vec{q_1})^2 = E_\gamma^2 + E_1^2 - M_1^2 + 2E_\gamma|\vec{q_1}|cos\theta$.

The angular integrals can be performed which results in

$$I = 8\pi^2 \int_{-1}^{1} d(\cos\theta)|\vec{q_1}|E_\gamma \frac{\delta\left(E_\gamma + E_1 - M + \sqrt{E_1^2 + E_\gamma^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2}\ \right)}{\sqrt{E_1^2 + E_\gamma^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2}} \quad , \tag{A.8}$$

We then make a change of variables defined by

$$\rho = \sqrt{E_1^2 + E_\gamma^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2} \quad , \tag{A.9}$$

and obtain the integral as

$$I = 8\pi^2 \int d\rho\ \delta(M - E_\gamma - E_1 - \rho) = 8\pi^2 \quad , \tag{A.10}$$

where the condition $M - E_\gamma - E_1 - \rho = 0$ is satisfied. Therefore we obtain the double differential decay rate as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\frac{1}{8M}\overline{|\mathcal{M}_{fi}|^2} \quad , \tag{A.11}$$

and consequently the decay rate is given by

$$\Gamma = \int_{E_\gamma,min}^{E_\gamma,max} dE_\gamma \int_{E_1,min}^{E_1,max} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \quad . \tag{A.12}$$

The limits of integral are obtained using the contain $M - E_\gamma - E_1 - \rho = 0$ as

$$(M - E_\gamma - E_1)^2 = E_1^2 + E_\gamma^2 - M_1^2 + 2E_\gamma|\vec{q_1}|\cos\theta + M_2^2 \quad . \tag{A.13}$$

Since $cos\theta$ satisfies the inequality $-1 \leq cos\theta \leq 1$, we obtain

$$-1 \leq \frac{(M - E_\gamma - E_1)^2 - E_\gamma^2 - E_1^2 + M_1^2 - M_2^2}{2|\vec{q}||\vec{q_2}|} \leq 1 \quad , \tag{A.14}$$

which can also be written in the form

$$-1 \leq \frac{(M - E_\gamma - E_1)^2 - E_\gamma^2 - E_1^2 + M_1^2 - M_2^2}{2E_\gamma\sqrt{E_1^2 - M_1^2}} \leq 1 \quad , \tag{A.15}$$

since $E_\gamma^2 = |\vec{q}^2|$ and $E_1^2 = |\vec{q_1}^2| + M_1^2$. We then solve this equation and find two roots for E_1 as

$$E_{1min} = \frac{1}{2(2E_\gamma M_\phi - M_\phi^2)}\Big\{-2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3 + E_\gamma\sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_K^2)} \ \Big\} \tag{A.16}$$

$$E_{1max} = \frac{1}{2(2E_\gamma M_\phi - M_\phi^2)}\Big\{-2E_\gamma^2 M_\phi + 3E_\gamma M_\phi^2 - M_\phi^3 - E_\gamma\sqrt{(-2E_\gamma M_\phi + M_\phi^2)(-2E_\gamma M_\phi + M_\phi^2 - 4M_K^2)} \ \Big\} \tag{A.17}$$

APPENDIX B

TWO BODY DECAY RATES

We consider the decay of a particle of mass M and four momentum p into two particles of masses M_1 and M_2 with four momenta p_1 and p_2, respectively. The differential decay rate is given by [24]

$$d\Gamma = \frac{1}{2E_p}\overline{|\mathcal{M}_{fi}|}^2 d\Phi_2 \tag{B.1}$$

where the two body phase space is

$$d\Phi_2 = \delta^{(4)}(p - p_1 - p_2)\frac{d^3p_1}{(2\pi)^3(2E_1)}\frac{d^3p_2}{(2\pi)^3(2E_2)} \ . \tag{B.2}$$

Since in the rest frame of decaying particle $\vec{p}_1 = -\vec{p}_2 \equiv \vec{p}$ and $M = E_1 + E_2$, then the phase space becomes

$$d\Phi = \frac{1}{(2\pi)^6}\frac{\delta(\vec{p}_1 + \vec{p}_2)\delta(E_1 + E_2 - M)}{4E_1E_2}\ d^3p_1 d^3p_2 \tag{B.3}$$

We can eliminate the first δ-function by performing d^3p_2 integration and obtain

$$d\Phi_2 = \frac{1}{(2\pi)^6}\frac{\delta(E_1 + E_2 - M)}{4E_1E_2}\ d^3p_1 \tag{B.4}$$

where $\quad d^3p_1 = p_1^2\ d|\vec{p}_1|\ d\Omega_1 = |\vec{p}_1|\ d\Omega\ \frac{E_1E_2\ d(E_1+E_2)}{E_1+E_2}$.

We then eliminate the second δ-function by integrating over $(E_1 + E_2)$ which yields

$$d\Gamma = \frac{1}{32\pi^2 M^2} |\mathcal{M}_{fi}|^2 |\vec{p}| d\Omega \quad . \tag{B.5}$$

After a final integration of both sides the decay rate is obtained as

$$\Gamma = \frac{1}{8\pi^2 M^2} |\mathcal{M}_{fi}|^2 |\vec{p}| \quad . \tag{B.6}$$

In the rest frame of the decaying particle

$$|\vec{p}| = \frac{1}{2M} \sqrt{[M^2 - (M_1 + M_2)^2][M^2 - (M_1 - M_2)^2]} \quad . \tag{B.7}$$

Therefore for two body decay $M \to M_1 + M_2$ where $M_1 = M_2$

$$|\vec{p}| = \frac{1}{2} M \sqrt{1 - \left(\frac{2M_1}{M}\right)^2} \quad . \tag{B.8}$$

In addition, for the decay $M \to M_1 + \gamma$

$$|\vec{p}| = \frac{1}{2} M \left[1 - \left(\frac{M_1}{M}\right)^2\right] \quad . \tag{B.9}$$

For the decay $\phi \to K^+ + K^-$ the invariant matrix element is obtained from the effective Lagrangian

$$\mathcal{L}_{\phi K^+ K^-} = -i g_{\phi KK} \phi^\mu (K^- \partial_\mu K^+ - K^+ \partial_\mu K^-) \tag{B.10}$$

as $\mathcal{M}(\phi \to K^+ + K^-) = -i g_{\phi K^+ K^-} (2q_1 - p)_\mu u^\mu$, where (p,u) are the four-momenta and polarization of the decaying ϕ-meson and q_1 is the four-momenta of the plus signed kaon. As a result, the decay rate is

$$\Gamma(\phi \to K^+ + K^-) = \frac{g^2_{\phi K^+K^-}}{48\pi} M_\phi \left[1 - \left(\frac{2M_K}{M_\phi}\right)^2\right]^{3/2} \quad . \tag{B.11}$$

Furthermore, for the decay $\phi \to S + \gamma$, where $S = f_0$ or a_0, the amplitude in the kaon-loop model is obtained as

$$\mathcal{M}(\phi \to S + \gamma) = u^\mu \epsilon^\nu (q_\mu p_\nu - g_{\mu\nu} q \cdot p) \frac{e g_{\phi K^+K^-}(g_{SK^+K^-} M_S)}{i2\pi^2 M_K^2} I(a,b) \quad , \tag{B.12}$$

where (u, p) denotes the polarization and four-momentum of the decaying vector meson and (ϵ, q) of the photon, respectively, and $a = M_\phi^2/M_k^2$, $b = M_s^2/M_k^2$. Thus, the decay rate is

$$\Gamma(\phi \to S + \gamma) = \frac{\alpha}{6(2\pi)^4} \frac{M_\phi^2 - M_S^2}{M_\phi^3} g^2_{\phi K^+K^-}(g_{SK^+K^-} M_S)^2 |(a-b) I(a,b)|^2 \tag{B.13}$$

Finally, for the decay $S \to K^+ + K^-$ described by the effective Lagrangian,

$$\mathcal{L}_{SK^+K^-} = -g_{SK^+K^-} K^+ K^- S \quad , \tag{B.14}$$

the invariant amplitude is determined as $\mathcal{M}(S \to K^+K^-) = -i g_{SK^+K^-} M_S$, and then the decay rate is obtained as

$$\Gamma(S \to K^+K^-) = \frac{g^2_{SK^+K^-}}{16\pi} M_S \left[1 - \left(\frac{2M_K}{M_S}\right)^2\right]^{1/2} \quad . \tag{B.15}$$

APPENDIX C

INVARIANT BREMSSTRAHLUNG AMPLITUDE OF THE RADIATIVE $\phi \to K^+ + K^- + \gamma$ DECAY

For bremsstrahlung processes of the radiative decay $\phi(p) \to K^+(q_1) + K^-(q_2) + \gamma(q)$ the invariant amplitude $\mathcal{M}_B$ is expressed as

$$\mathcal{M}_B = \mathcal{M}_{Ba} + \mathcal{M}_{Bb} + \mathcal{M}_{Bc} \quad , \tag{C.1}$$

where $\mathcal{M}_{Ba}$, $\mathcal{M}_{Bb}$, and $\mathcal{M}_{Bc}$ are the invariant amplitudes obtained from the diagrams (a), (b), and (c) in Fig. 2.2 as

$$\mathcal{M}_{Ba} = 4i(eg_{\phi K^+K^-}) \left[\frac{q_{2\mu} q_{1\nu}}{(p-q_2)^2 - M_K^2} \right] u^\mu \epsilon^\nu \tag{C.2}$$

$$\mathcal{M}_{Bb} = 4i(eg_{\phi K^+K^-}) \left[\frac{q_{1\mu} q_{2\nu}}{(p-q_1)^2 - M_K^2} \right] u^\mu \epsilon^\nu \tag{C.3}$$

$$\mathcal{M}_{Bc} = 2i(eg_{\phi K^+K^-}) u_\mu \epsilon^\mu \quad . \tag{C.4}$$

Using the gauge $\epsilon \cdot p = 0$, we then obtain the total invariant amplitude as

$$\mathcal{M}_B = 2i(eg_{\phi K^+K^-}) \epsilon^\nu \left[2\frac{q_{2\mu} q_{1\nu} u^\mu}{(p-q_2)^2 - M_K^2} + 2\frac{q_{1\mu} q_{2\nu} u^\mu}{(p-q_1)^2 - M_K^2} + u_\mu g_{\mu\nu} \right] \ . \tag{C.5}$$

Although each single amplitude is not gauge invariant, the total amplitude $\mathcal{M}_B$ is gauge invariant. We present a brief discussion of this point. When any external photon polarization vector in the amplitude is replaced by the four-momentum of the corresponding photon, the amplitude must vanish. The amplitude can be writen as, $\mathcal{M} = \epsilon^\nu \mathcal{M}_\nu$ corresponding to a process involving one external photon [19]. If ϵ^ν is replaced by q^ν ($\epsilon^\nu \rightarrow q^\nu$) and then we must have $q^\nu \mathcal{M}_\nu = 0$. We apply this prescription to $\mathcal{M}_B$ and obtain

$$\begin{aligned} q^\nu (\mathcal{M}_B)_\nu &= 4i(eg_{\phi K^+K^-}) \left[(q_2 \cdot u) \frac{1}{p^2 - 2p \cdot q_2 + q_2^2 - M_K^2} (q_1 \cdot q) \right. \\ & \left. + (q_1 \cdot u) \frac{1}{p^2 - 2pq_1 + q_1^2 - M_K^2} (q_2 \cdot q) \right] + 2i(eg_{\phi K^+K^-})(u \cdot q) \ . \end{aligned} \tag{C.6}$$

We then note that $q_1^2 = q_2^2 = M_K^2$ and $p - q = q_1 + q_2$, and since ϕ-meson is at rest

$$\begin{aligned} p \cdot q_1 &= M_\phi E_1 \\ p \cdot q_2 &= M_\phi E_2 \\ p \cdot q &= M_\phi E_\gamma \quad , \end{aligned} \tag{C.7}$$

$$\begin{aligned} q_1 \cdot q &= \frac{1}{2} \left(M_\phi^2 - 2M_\phi E_2 \right) \\ q_2 \cdot q &= \frac{1}{2} \left(M_\phi^2 - 2M_\phi E_1 \right) \quad . \end{aligned} \tag{C.8}$$

We use these relations and obtain $q_\nu \mathcal{M}_B^\nu = 0$.

The complex invariant amplitude is parameterized with $\mathcal{M}_B = i(\mathcal{M}'_{Ba} + \mathcal{M}'_{Bb} + \mathcal{M}'_{Bc})$. Therefore, the square of the invariant amplitude is obtained as

$$|\mathcal{M}_B|^2 = |\mathcal{M}'_{Ba}|^2 + |\mathcal{M}'_{Bb}|^2 + |\mathcal{M}'_{Bc}|^2 + 2(\mathcal{M}'^{*}_{Ba} \mathcal{M}'_{Bb} + \mathcal{M}'^{*}_{Ba} \mathcal{M}'_{Bc} + \mathcal{M}'^{*}_{Bb} \mathcal{M}'_{Bc}) \tag{C.9}$$

As mentioned above, bremsstrahlung diagrams are not separately gauge invariant. If the diagram is gauge invariant, then we can use

$$\sum_{\lambda} \varepsilon_{\alpha}(q,\lambda)\varepsilon_{\beta}(q,\lambda) = -g_{\alpha\beta} \tag{C.10}$$

However, we perform calculations for bremsstrahlung diagrams which are not separately gauge invariant. Therefore, in the calculation either we can use the following relation

$$\sum_{\lambda} \varepsilon_{\alpha}(q,\lambda)\varepsilon_{\beta}(q,\lambda) = -g_{\alpha\beta} - \frac{1}{(qn)^2}\left[q_{\alpha}q_{\beta} - (qn)\left(q_{\alpha}n_{\beta} + q_{\beta}n_{\alpha}\right)\right] \ , \tag{C.11}$$

where n is the four-vector $n^{\mu} = (1,0,0,0)$ [24], or we can choose a suitable gauge. Therefore, we use the gauge $\epsilon \cdot p = 0$ in our calculations. In this gauge, there is no time component of ϵ_{α}, thus

$$\sum_{\lambda} \epsilon_i \ (q,\lambda) \ \ \epsilon_j \ (q,\lambda) = \delta_{ij} - \hat{q}_i\hat{q}_j \ . \tag{C.12}$$

Moreover, since decaying vector meson ϕ is at rest, its polarization vector has no time component since $u^{\mu} \ p_{\mu} = 0$. In this case, the polarization sum becomes

$$\sum_{\lambda'} u_l(p,\lambda')u_m(p,\lambda') = \delta_{lm} \ . \tag{C.13}$$

On the other hand, for vector meson we can use the polarization sum as

$$\sum_{\lambda'} u_{\alpha}(p,\lambda') \ \ u_{\beta}(p,\lambda') = -\left(g_{\alpha\beta} - \frac{p_{\alpha}p_{\beta}}{M_{\phi}^2}\right) \ . \tag{C.14}$$

The terms in Eq. C.9 are evaluated in the above framework. The absolute square of $\mathcal{M}_{Ba}$ amplitude is written as

$$\begin{aligned}
|\mathcal{M}_{Ba}|^2 &= \left[4eg_{\phi K^+K^-}q_{2\alpha}u^*_{\alpha}(p,\lambda')\frac{1}{(p-q_2)^2 - M_K^2}q_{1\beta}\epsilon^*_{\beta}(q,\lambda)\right] \\
&\times \left[4eg_{\phi K^+K^-}q_{2\mu}u_{\mu}(p,\lambda')\frac{1}{(p-q_2)^2 - M_K^2}q_{1\nu}\epsilon_{\nu}(q,\lambda)\right]
\end{aligned}$$

$$
\begin{aligned}
&= 16(eg_{\phi K^+K^-})^2 (q_{2\alpha}\; q_{1\beta}\; q_{2\mu}\; q_{1\nu})\; \overline{u_\alpha^*(p,\lambda')\; u_\mu(p,\lambda')}\; \sum_\lambda \epsilon_\beta^*(q,\lambda)\; \epsilon_\nu(q,\lambda) \\
&\quad \times \left[\frac{1}{(p-q_2)^2 - M_K^2}\right]^2 \qquad \text{(C.15)}
\end{aligned}
$$

Using the gauge $\epsilon \cdot p = 0$ and in ϕ-meson rest frame we have

$$
\begin{aligned}
|\mathcal{M}_{Ba}|^2 &= 16(eg_{\phi K^+K^-})^2 \left[\frac{1}{(p-q_2)^2 - M_K^2}\right]^2 (q_{2i}\; q_{1j}\; q_{2k}\; q_{1\ell}) \\
&\quad \times \overline{u_i^*(p,\lambda')\; u_k(p,\lambda')}\; \sum_\lambda \epsilon_j^*(q,\lambda)\; \epsilon_\ell(q,\lambda) \qquad \text{(C.16)}
\end{aligned}
$$

where

$$
\begin{aligned}
&\overline{u_i^*(p,\lambda')\, u_k(p,\lambda')} = \frac{1}{3}\sum_{\lambda'} u_i(p,\lambda') u_k(p,\lambda') = \frac{1}{3}\delta_{ik}\ , \\
&\sum_\lambda \epsilon_j^*(q,\lambda)\epsilon_l(q,\lambda) = \delta_{jl} - \hat{q}_j \hat{q}_l\ . \qquad \text{(C.17)}
\end{aligned}
$$

We then obtain

$$
|\mathcal{M}_{Ba}|^2 = \left(\frac{16}{3}\right)(eg_{\phi K^+K^-})^2 \left[\frac{1}{(p-q_2)^2 - M_K^2}\right] \vec{q_2}^{\,2} \left[\vec{q_1}^{\,2} - (\vec{q_1}\cdot\hat{q})^2\right] \qquad \text{(C.18)}
$$

where $\vec{q_1}^{\,2} = E_1^2 - M_K^2$, $\vec{q_2}^{\,2} = E_2^2 - M_K^2$, and $\vec{q_1}\cdot\hat{q} = \left(2E_1 E_\gamma - M_\phi^2 + 2M_\phi E_2\right)/2E_\gamma$.

The absolute square of $\mathcal{M}_{Bb}$ amplitude is obtained by the same way

$$
\begin{aligned}
|\mathcal{M}_{Bb}|^2 &= \left[4eg_{\phi K^+K^-} q_{1\alpha} u_\alpha^*(p,\lambda') \frac{1}{(p-q_1)^2 - M_K^2} q_{2\beta}\epsilon_\beta^*(q,\lambda)\right] \\
&\quad \times \left[4eg_{\phi K^+K^-} q_{1\mu} u_\mu(p,\lambda') \frac{1}{(p-q_1)^2 - M_K^2} q_{2\nu}\epsilon_\nu(q,\lambda)\right] \\
&= \left(\frac{16}{3}\right)(eg_{\phi K^+K^-})^2 \left[\frac{1}{(p-q_1)^2 - M_K^2}\right] \vec{q_1}^{\,2} \left[\vec{q_2}^{\,2} - (\vec{q_2}\cdot\hat{q})^2\right] \\
&\qquad \text{(C.19)}
\end{aligned}
$$

where $\vec{q_2}\cdot\hat{q} = \left(2E_2 E_\gamma - M_\phi^2 + 2M_\phi E_1\right)/2E_\gamma$.

The other amplitude becomes

$$
\begin{aligned}
|\mathcal{M}'_{Bc}|^2 &= \left[2eg_{\phi K^+K^-} u_\alpha^*(p,\lambda')\epsilon_\alpha^*(q,\lambda)\right]\left[2eg_{\phi K^+K^-} u_\mu(p,\lambda')\epsilon_\mu(q,\lambda)\right] \\
&= \left(\frac{8}{3}\right)(eg_{\phi K^+K^-})^2\ . \qquad \text{(C.20)}
\end{aligned}
$$

The remain amplitudes in Eq. C.9 are found as

$$
\begin{aligned}
|\mathcal{M}'_{Ba}{}^{*}\mathcal{M}'_{Bb}| &= \left[4eg_{\phi K^+K^-}q_{2\alpha}u^*_\alpha(p,\lambda')\frac{1}{(p-q_2)^2-M_K^2}q_{1\beta}\epsilon^*_\beta(q,\lambda)\right] \\
&\quad \times\left[4eg_{\phi K^+K^-}q_{1\mu}u_\mu(p,\lambda')\frac{1}{(p-q_1)^2-M_K^2}q_{1\nu}\epsilon_\nu(q,\lambda)\right] \\
&= \left(\frac{16}{3}\right)(eg_{\phi K^+K^-})^2\left\{\frac{\vec{q_1}\cdot\vec{q_2}\,[\vec{q_1}\cdot\vec{q_2}-(\vec{q_1}\cdot\hat{q})(\vec{q_2}\cdot\hat{q})]}{[(p-q_1)^2-M_K^2]\,](p-q_2)^2-M_K^2]}\right\}
\end{aligned}
\tag{C.21}
$$

$$
\begin{aligned}
|\mathcal{M}'_{Ba}{}^{*}\mathcal{M}'_{Bc}| &= \left[4eg_{\phi K^+K^-}q_{2\alpha}u^*_\alpha(p,\lambda')\frac{1}{(p-q_2)^2-M_K^2}q_{1\beta}\epsilon^*_\beta(q,\lambda)\right] \\
&\quad \times\left[2eg_{\phi K^+K^-}u_\mu(p,\lambda')\epsilon_\mu(q,\lambda)\right] \\
&= \left(\frac{8}{3}\right)(eg_{\phi K^+K^-})^2\left[\frac{(\vec{q_1}\cdot\vec{q_2})-(\vec{q_1}\cdot\hat{q})(\vec{q_2}\cdot\hat{q})}{(p-q_2)^2-M_K^2}\right]
\end{aligned}
\tag{C.22}
$$

$$
\begin{aligned}
|\mathcal{M}'_{Bb}{}^{*}\mathcal{M}'_{Bc}| &= \left[4eg_{\phi K^+K^-}q_{1\alpha}u^*_\alpha(p,\lambda')\frac{1}{(p-q_1)^2-M_K^2}q_{2\beta}\epsilon^*_\beta(q,\lambda)\right] \\
&\quad \times\left[2eg_{\phi K^+K^-}u_\mu(p,\lambda')\epsilon_\mu(q,\lambda)\right] \\
&= \left(\frac{8}{3}\right)(eg_{\phi K^+K^-})^2\left[\frac{(\vec{q_1}\cdot\overrightarrow{q_2})-(\vec{q_1}\cdot\hat{q})(\vec{q_2}\cdot\hat{q})}{(p-q_1)^2-M_K^2}\right]
\end{aligned}
\tag{C.23}
$$

where $\vec{q_1}\cdot\vec{q_2}=\left(2E_1E_2-M_\phi^2+2M_\phi E_\gamma+2M_K^2\right)/2$.

APPENDIX D

THE DERIVATION OF THE FUNCTION $\mathcal{I}$(a,b)

In this Appendix, we outline a derivation of the function $\mathcal{I}(a, b)$ that appear in the amplitude of the $\phi \to S + \gamma$ decay in the kaon-loop model. the amplitude of this radiative decay is given Eq. 2.10 as

$$\mathcal{M}_{KL}(\phi \to S + \gamma) = u^\mu \epsilon^\nu (p_\nu q_\mu - g_{\mu\nu} p \cdot q) \frac{e g_{\phi K^+K^-} g_{SK^+K^-}}{2i\pi^2 M_K^2} \mathcal{I}(a, b) \ . \quad \text{(D.1)}$$

We consider the diagram shown in Fig. 2.3.(a) and denote the momentum assignments in Fig. C-1.

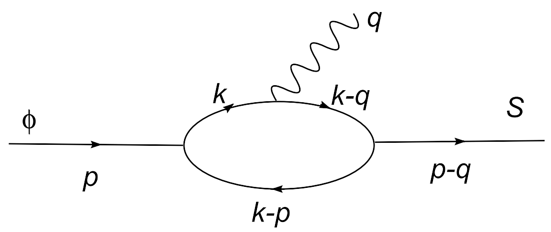

Figure D.1: Momentum assignments in the loop diagram.

We can write the contribution of this diagram in the form $\mathcal{M} = u^\mu \epsilon^\nu \mathcal{M}_{\mu\nu}$ where

$$\mathcal{M}_{\mu\nu} = e g_{\phi K^+K^-} g_{SK^+K^-} \int \frac{d^4k}{(2\pi)^4} \frac{(2k - p)_\mu (2k - q)_\nu}{(k^2 - M_K^2)[(k - q)^2 - M_K^2][(k - p)^2 - M_K^2]} \quad \text{(D.2)}$$

We then use the Feynman technique to combine the denominator and obtain

$$\mathcal{M}_{\mu\nu} = eg_{\phi K^+K^-}g_{SK^+K^-}\ 8\int_0^1 dz \int_0^{1-z} dy \int_{-\infty}^{\infty} \frac{1}{2\pi}^4 \frac{d^4k\ k_\mu k_\nu}{[(k-qy-pz)^2 - c + i\epsilon]^3}\ , \tag{D.3}$$

where $c = M_k^2 - z(1-z)M_\phi^2 - zy(M_s^2 - M_\phi^2)$. We then perform the shift $k \to k + qy + pz$ in the integral. We note that the form of the amplitude $\mathcal{M}(\phi \to S + \gamma)$ given Eq. C.1 is the one required by gauge invariance. We therefore consider the coefficient of the $p_\nu q_\mu$ term that we obtain after the shift in the integral. There is a similar term coming from the diagram where the photon is emitted by K^--meson in the loop shown in Fig. 2.3.(b). The contact term shown in Fig. 2.3.(c) does not make any contribution to the $p_\nu q_\mu$ term, we thus obtain

$$\mathcal{I}(a,b) = \int_0^1 \int_0^{1-z} dy\ yz[M_k^2 - 2(z-1)M_\phi^2 - zy(M_S^2 - M_\phi^2)]^{-1}\ . \tag{D.4}$$

If we define $a = M_\phi^2/M_K^2$, $b = M_s^2/M_K^2$ and note that $a > 4$, $b < 4$, we obtain

$$\begin{aligned}\mathcal{I}(a,b) &= \frac{1}{(a-b)}\int_0^1 \frac{dz}{z}\left[z(1-z) - \frac{(1-z(1-z)a)}{(a-b)} ln\left(\frac{1-z(1-z)b}{1-z(1-z)a}\right)\right] \\ &\quad - \frac{i\pi}{(a-b)}\int_{1/\eta^+}^{1/\eta^-} (1-z(1-z)a)\frac{dz}{z}\ ,\end{aligned} \tag{D.5}$$

where $\quad \eta_\pm = \frac{1}{2}a(1 \pm \alpha)$ with $\alpha = \sqrt{1 - 4/a}$.

After a final integration we obtain the expression for $\mathcal{I}(a,b)$ given in Eq. 2.10.

APPENDIX E

INVARIANT AMPLITUDE OF THE $\phi \to K^+ + K^- + \gamma$ DECAY IN KAON-LOOP MODEL

For the radiative decay $\phi(p) \to K^+(q_1)+K^-(q_2)+\gamma(q)$ contribution of the scalar meson intermediate state to the invariant amplitude $\mathcal{M}$ in kaon-loop model is expressed in the form

$$\mathcal{M}_{KL} = \mathcal{M}_{KLa} + \mathcal{M}_{KLb} + \mathcal{M}_{KLc} \quad , \tag{E.1}$$

where $\mathcal{M}_{KLa}$, $\mathcal{M}_{KLb}$, and $\mathcal{M}_{KLc}$ are the invariant amplitudes obtained from the diagrams (a), (b), and (c) in Fig. 2.2 as

$$\begin{aligned}
\mathcal{M}_{KLa} &= \mathcal{M}_{KLb} \\
&= e g_{\phi K^+K^-} (g_{SK^+K^-})^2 \\
&\quad \times \int \frac{d^4k}{(2\pi)^4} \frac{(2k)_\mu u^\mu (2k)_\nu \epsilon^\nu}{(k^2 - M_K^2)[(k-p)^2 - M_K^2][(k-q)^2 - M_K^2]} \\
&\quad \times \frac{1}{[(p-q)^2 - M_S^2 + i\Gamma_S M_S]}
\end{aligned} \tag{E.2}$$

$$\begin{aligned}
\mathcal{M}_{KLc} &= -2e g_{\phi K^+K^-} (g_{SK^+K^-})^2 \int \frac{d^4k}{(2\pi)^4} \frac{u_\mu \epsilon^\mu}{(k-q)^2 - M_K^2)[(k-p)^2 - M_K^2]} \\
&\quad \times \frac{1}{[(p-q)^2 - M_S^2 + i\Gamma_S M_S]} \quad ,
\end{aligned} \tag{E.3}$$

where (u,p) and (ϵ, q) are the polarization and four momenta of the ϕ meson and the photon, respectively. Using the relations $u^\nu p_\nu = 0$ and $\epsilon^\mu q_\mu = 0$, we then obtain the invariant amplitude

$$
\begin{aligned}
\mathcal{M}_{KL} &= 2eg_{\phi K^+K^-}(g_{SK^+K^-})^2 \frac{1}{[(p-q)^2 - M_S^2 + i\Gamma_S M_S]} \\
&\times u^\mu \epsilon^\nu \int \frac{d^4k}{(2\pi)^4} \frac{4k_\mu k_\nu - g_{\mu\nu}}{(k^2 - M_K^2)[(k-q)^2 - M_K^2][(k-p)^2 - M_K^2]} \\
&= eg_{\phi K^+K^-}(g_{SK^+K^-})^2 \frac{(p-q)^2 - M_S^2 - i\Gamma_S M_S}{\left\{[(p-q)^2 - M_S^2]^2 + (\Gamma_S M_S)^2\right\}} \\
&\times \left\{ \frac{1}{2\pi^2 M_K^2} I(a,b)[(\epsilon \cdot u)(q \cdot p) - (\epsilon \cdot p)(q \cdot u)] \right\} \qquad \text{(E.4)}
\end{aligned}
$$

where $a = M_\phi^2/M_k^2$, $b = (p-k)^2/M_K^2 = (M_\phi^2 - 2M_\phi E_\gamma)/M_K^2$. The invariant function I(a,b) is discussed in Appendix D.

The complex invariant amplitude is parameterized with

$$\mathcal{M}_{KL} = \mathcal{M}''_{KL} + i\mathcal{M}'_{KL} \qquad \text{(E.5)}$$

where $\mathcal{M}''_{KL}$ and $\mathcal{M}'_{KL}$ are

$$
\begin{aligned}
\mathcal{M}'_{KL} &= \frac{1}{2\pi^2 M_K^2} eg_{\phi K^+K^-}(g_{SK^+K^-})^2 [(\epsilon \cdot u)(q \cdot p) - (\epsilon \cdot p)(q \cdot u)] \\
&\times \left\{ [(p-q)^2 - M_S^2] ImI(a,b) - (\Gamma_S M_S) ReI(a,b) \right\} \Delta_S^0(p-q) \\
\mathcal{M}''_{KL} &= \frac{1}{2\pi^2 M_K^2} eg_{\phi K^+K^-}(g_{SK^+K^-})^2 [(\epsilon \cdot u)(q \cdot p) - (\epsilon \cdot p)(q \cdot u)] \\
&\times \left\{ [(p-q)^2 - M_S^2] ReI(a,b) + (\Gamma_S M_S) ImI(a,b) \right\} \Delta_S^0(p-q)
\end{aligned}
\qquad \text{(E.6)}
$$

and

$$\Delta_S^0(q) = \frac{1}{(q^2 - M_S^2)^2 + (\Gamma_S M_S)^2} \; . \qquad \text{(E.7)}$$

The absolute value of the square of the invariant amplitude is obtained as $\mid \mathcal{M}_{KL} \mid^2 = |\mathcal{M}''|^2 + |\mathcal{M}'|^2$. The squares of the real and imaginary parts become

$$
\begin{aligned}
|\mathcal{M}'_{KL}|^2 &= \left\{\frac{1}{2\pi^2 M_K^2} e g_{\phi K^+K^-}(g_{SK^+K^-})^2\right\}^2 \\
&\times\left\{\left\{[(p-q)^2 - M_S^2] ImI(a,b) - (\Gamma_S M_S) ReI(a,b)\right\}\Delta_S^0(p-q)\right\}^2 \\
&\times[\epsilon_\alpha u_\alpha q.p - \epsilon_\alpha p_\alpha q_\beta u_\beta]^*[\epsilon_{\alpha'} u_{\alpha'} q.p - \epsilon_{\alpha'} p_{\alpha'} q_{\beta'} u_{\beta'}] \qquad \text{(E.8)}
\end{aligned}
$$

$$
\begin{aligned}
|\mathcal{M}''_{KL}|^2 &= \left\{\frac{1}{2\pi^2 M_K^2} e g_{\phi K^+K^-}(g_{SK^+K^-})^2\right\}^2 \\
&\times\left\{\left\{[(p-q)^2 - M_S^2] ReI(a,b) + (\Gamma_S M_S) ImI(a,b)\right\}\Delta_S^0(p-q)\right\}^2 \\
&\times[\epsilon_\alpha u_\alpha q.p - \epsilon_\alpha p_\alpha q_\beta u_\beta]^*[\epsilon_{\alpha'} u_{\alpha'} q.p - \epsilon_{\alpha'} p_{\alpha'} q_{\beta'} u_{\beta'}]\ . \qquad \text{(E.9)}
\end{aligned}
$$

Using the relations $\sum_\lambda \epsilon^*_\alpha(q,\lambda)\epsilon_{\alpha'}(q,\lambda) = -g_{\alpha\alpha'}$ and $\overline{u^*_\alpha(p,\lambda')u_{\alpha'}(p,\lambda')} = -\frac{1}{3}g_{\alpha\alpha'}$, we evaluate the polarization terms as

$$
\begin{aligned}
&[\epsilon_\alpha u_\alpha q\cdot p - \epsilon_\alpha p_\alpha q_\beta u_\beta]^*[\epsilon_{\alpha'} u_{\alpha'} q\cdot p - \epsilon_{\alpha'} p_{\alpha'} q_{\beta'} u_{\beta'}] \\
&\quad = \sum_\lambda \epsilon^*_\alpha(q,\lambda)\epsilon_{\alpha'}(q,\lambda)\ \overline{u^*_\alpha u_{\alpha'}}\ (q\cdot p)^2 \\
&\qquad\qquad - \sum_\lambda \epsilon^*_\alpha(q,\lambda)\epsilon_{\alpha'}(q,\lambda)\ \overline{u^*_\alpha u_{\beta'}}\ (q\cdot p) p_{\alpha'} q_{\beta'} \\
&\quad = -\sum_\lambda \epsilon^*_\alpha(q,\lambda)\epsilon_{\alpha'}(q,\lambda)\ \overline{u^*_\beta u_{\alpha'}}\ (q.p) p_\alpha q_\beta \\
&\qquad\qquad + \sum_\lambda \epsilon^*_\alpha(q,\lambda)\epsilon_{\alpha'}(q,\lambda)\ \overline{u^*_\beta u_{\beta'}}\ p_\alpha q_\beta p_{\alpha'} q_{\beta'} \\
&\quad = \frac{1}{3}[4(q\cdot p)^2 - (q\cdot p)^2 - (q\cdot p)^2 + p^2 q^2] \\
&\quad = \frac{2}{3}(q\cdot p)^2\ . \qquad \text{(E.10)}
\end{aligned}
$$

We then obtain

$$
\begin{aligned}
|\mathcal{M}'_{KL}|^2 &= \left\{\frac{1}{2\pi^2 M_K^2} e g_{\phi K^+K^-}(g_{SK^+K^-})^2\right\}^2 \frac{2}{3}(q\cdot p)^2 \\
&\times\left\{\left\{[(p-q)^2 - M_S^2] ImI(a,b) - (\Gamma_S M_S) ReI(a,b)\right\}\Delta_S^0(p-q)\right\}^2
\end{aligned}
$$
(E.11)

$$|\mathcal{M}''_{KL}|^2 = \left\{\frac{1}{2\pi^2 M_K^2} e g_{\phi K^+K^-}(g_{SK^+K^-})^2\right\}^2 \frac{2}{3}(q\cdot p)^2 \times \left\{\left\{[(p-q)^2 - M_S^2] ReI(a,b) + (\Gamma_S M_S) ImI(a,b)\right\} \Delta_S^0(p-q)\right\}^2 \ . \tag{E.12}$$

We note that in the rest frame of ϕ meson $p\cdot q = M_\phi E_\gamma$ and $(p-q)^2 = (q_1+q_2)^2 \equiv M_{KK}^2$.

Since both f_0 and a_0 resonances make a contribution to the decay $\phi \to K^+ + K^- + \gamma$ the complex amplitudes are parameterized with

$$\mathcal{M}_{KL} = \left[\mathcal{M}''_{f_0} + \mathcal{M}''_{a_0}\right] + i\left[\mathcal{M}'_{f_0} + \mathcal{M}'_{a_0}\right] \ . \tag{E.13}$$

The absolute square of the invariant amplitude is now obtained as
$\mid \mathcal{M}_{KL} \mid^2 = \left[\mathcal{M}''_{f_0} + \mathcal{M}''_{a_0}\right]^2 + \left[\mathcal{M}'_{f_0} + \mathcal{M}'_{a_0}\right]^2$.

The interference term of the bremsstrahlung amplitude and the kaon-loop scalar meson amplitude becomes

$$\begin{aligned} \mathcal{M}_{int} &= 2\left(\mathcal{M}'^*_B \mathcal{M}'_{KL}\right) \\ &= 2\left\{4(e g_{\phi K^+K^-})\left[\frac{(q_{2\mu}\ q_{1\nu}\ u^\mu\ \epsilon^\nu)^*}{(p-q_2)^2 - M_K^2} + \frac{(q_{1\mu}\ q_{2\nu}\ u^\mu\ \epsilon^\nu)^*}{(p-q_1)^2 - M_K^2}\right]\right. \\ &\qquad \left. + 2(e g_{\phi K^+K^-})(\epsilon^\mu\ u^\mu)^*\right\} \times [p_\alpha\ u_\beta(q_\alpha\ \epsilon_\beta - q_\beta\ \ \epsilon_\alpha)] \\ &\quad \times \left\{\frac{(e g_{\phi K^+K^-})(g_{SK^+K^-})^2}{2\pi^2 M_K^2}\left[\left((p-q)^2 - M_S^2\right)\mathcal{I}_I - (\Gamma_S M_S)^2 \mathcal{I}_R\right]\right. \\ &\qquad \left. \times\ \Delta_S^0(p-q)\right\} \end{aligned} \tag{E.14}$$

where $\mathcal{I}(a,b) = Re\mathcal{I}(a,b) + i\ Im\mathcal{I}(a,b) = \mathcal{I}_R + i\ \mathcal{I}_I$.

In the $p\cdot\epsilon = 0$ gauge, we have $(q_{2\mu}\ q_{1\nu}\ u^\mu\ \epsilon^\nu)^* \left[p_\alpha\ u_\beta(q_\alpha\ \epsilon_\beta - q_\beta\ \ \epsilon_\alpha)\right] = \left(\frac{1}{3}\right)(p\cdot q)\left[(\vec{q_1}\cdot\vec{q_2}) - (\vec{q_1}\cdot\hat{q})(\vec{q_2}\cdot\hat{q})\right]$.

We finally obtain

$$
\begin{aligned}
\mathcal{M}_{int} &= \left(\frac{8}{3}\right)\frac{(eg_{\phi K^+K^-})^2(g_{SK^+K^-})^2}{(2\pi^2 M_K^2)}\Big\{[(p-q)^2 - M_S^2]\mathcal{I}_I - (\Gamma_S M_S)\mathcal{I}_R\Big\} \\
&\quad \times \left\{\frac{[(\vec{q_1}\cdot\vec{q_2}) - (\vec{q_1}\cdot\hat{q})(\vec{q_2}\cdot\hat{q})]}{(p-q_2)^2 - M_K^2} + \frac{[(\vec{q_1}\cdot\vec{q_2}) - (\vec{q_1}\cdot\hat{q})(\vec{q_2}\cdot\hat{q})]}{(p-q_1)^2 - M_K^2} + 1\right\} \\
&\quad \times \Delta_S^0(p-q)(p\cdot q)
\end{aligned}
\tag{E.15}
$$

APPENDIX F

INVARIANT AMPLITUDE OF THE $\phi \to K^+ + K^- + \gamma$ DECAY IN NO-STRUCTURE MODEL

For the radiative decay $\phi(p) \to K^+(q_1)+K^-(q_2)+\gamma(q)$ contribution of the scalar meson intermediate state in no-structure model to the invariant amplitude of the reaction is parameterized as

$$\mathcal{M}_{NS} = \mathcal{M}''_{NS} + i\mathcal{M}'_{NS} \ ,$$

$$\mathcal{M}'_{NS} = -\frac{e}{M_\phi} g_{\phi S\gamma} g_{SK^+K^-} \left\{ \frac{(p-q)^2 - M_S^2}{\left[(p-q)^2 - M_S^2\right]^2 + (\Gamma_S M_S)^2} \right\} [(\epsilon \cdot u)(q \cdot p) - (\epsilon \cdot p)(q \cdot u)] \tag{F.1}$$

$$\mathcal{M}''_{NS} = \frac{e}{M_\phi} g_{\phi S\gamma} g_{SK^+K^-} \left\{ \frac{\Gamma_S M_S}{\left[(p-q)^2 - M_S^2\right]^2 + (\Gamma_S M_S)^2} \right\} [(\epsilon \cdot u)(q \cdot p) - (\epsilon \cdot p)(q \cdot u)] \ . \tag{F.2}$$

The absolute square of the amplitude then becomes

$| \mathcal{M}_{NS} |^2 = |\mathcal{M}''_{NS}|^2 + |\mathcal{M}'_{NS}|^2$

where

$$\begin{aligned} |\mathcal{M}'_{NS}|^2 &= \left(-\frac{e}{M_\phi}\right)^2 (g_{\phi S\gamma})^2 (g_{SK^+K^-})^2 \left\{ \frac{(p-q)^2 - M_s^2}{\left[(p-q)^2 - M_S^2\right]^2 + (\Gamma_s M_s)^2} \right\}^2 \\ &\quad \times \left[p_\alpha \, u_\beta \, (q_\alpha \, \epsilon_\beta - q_\beta \ \ \epsilon_\alpha)\right]^* \left[p_\alpha \, u_\beta \, (q_\alpha \, \epsilon_\beta - q_\beta \ \ \epsilon_\alpha)\right] \ . \end{aligned} \tag{F.3}$$

Using the result

$$[p_\alpha\ u_\beta\,(q_\alpha\ \epsilon_\beta - q_\beta\ \ \epsilon_\alpha)]^*\,[p_\alpha\ u_\beta\,(q_\alpha\ \epsilon_\beta - q_\beta\ \ \epsilon_\alpha)] = \frac{2}{3}(p\cdot q)^2 \qquad \text{(F.4)}$$

we then obtain

$$\begin{aligned} |\mathcal{M}'_{NS}|^2 &= \left(\frac{2}{3}\right)\left(-\frac{e}{M_\phi}\right)^2 (g_{\phi S\gamma})^2\,(g_{SK^+K^-})^2\,(p\cdot q)^2 \\ &\times \left\{\frac{(p-q)^2 - M_S^2}{[(p-q)^2 - M_S^2]^2 + (\Gamma_S M_S)^2}\right\}^2 \end{aligned} \qquad \text{(F.5)}$$

$$\begin{aligned} |\mathcal{M}''_{NS}|^2 &= \left(\frac{e}{M_\phi}\right)^2 (g_{\phi S\gamma})^2\,(g_{SK^+K^-})^2 \left\{\frac{\Gamma_S M_S}{[(p-q)^2 - M_S^2]^2 + (\Gamma_S M_S)^2}\right\}^2 \\ &\times [p_\alpha\ u_\beta\,(q_\alpha\ \epsilon_\beta - q_\beta\ \ \epsilon_\alpha)]^*\,[p_\alpha\ u_\beta\,(q_\alpha\ \epsilon_\beta - q_\beta\ \ \epsilon_\alpha)] \\ &= \left(\frac{2}{3}\right)\left(\frac{e}{M_\phi}\right)^2 (g_{\phi S\gamma})^2\,(g_{SK^+K^-})^2\,(p\cdot q)^2 \\ &\times \left\{\frac{\Gamma_S M_S}{[(p-q)^2 - M_S^2]^2 + (\Gamma_S M_S)^2}\right\}^2 \end{aligned} \qquad \text{(F.6)}$$

with $p\cdot q = E_\gamma M_\phi$ and $(p-q)^2 = M_\phi^2 - 2M_\phi E_\gamma$.

The complex amplitudes are parameterized with

$$\mathcal{M}_{NS} = \left[\mathcal{M}''_{f_0} + \mathcal{M}''_{a_0}\right] + i\left[\mathcal{M}'_{f_0} + \mathcal{M}'_{a_0}\right]\ . \qquad \text{(F.7)}$$

The absolute square of the invariant amplitude is now obtained as

$$|\mathcal{M}|^2 = |\mathcal{M}_{Brem}|^2 + |\mathcal{M}_{NS}|^2 + 2(\mathcal{M}'^*_{Brem}\mathcal{M}'_{NS}) \qquad \text{(F.8)}$$

where $|\ \mathcal{M}_{NS}\ |^2 = \left[\mathcal{M}''_{f_0} + \mathcal{M}''_{a_0}\right]^2 + \left[\mathcal{M}'_{f_0} + \mathcal{M}'_{a_0}\right]^2$.

Interference terms in no structure model become

$$\mathcal{M}_{int} = 2(\mathcal{M}'_{Brem}\mathcal{M}'_{NS})$$

$$
\begin{aligned}
&= 2\left\{4(eg_{\phi K^+K^-})\left[\frac{(q_{2\mu}\ q_{1\nu}\ u^\mu\ \epsilon^\nu)^*}{(p-q_1)^2-M_K^2}+\frac{(q_{1\mu}\ q_{2\nu}\ u^\mu\ \epsilon^\nu)^*}{(p-q_2)^2-M_K^2}\right]\right.\\
&\qquad\left.+\,2(eg_{\phi K^+K^-})(\epsilon^\mu\ u^\mu)^*\right\}[p_\alpha\ u_\beta(q_\alpha\ \epsilon_\beta-q_\beta\ \ \epsilon_\alpha)]\\
&\quad\times\left\{(-\frac{e}{M_\phi})(g_{\phi S\gamma})(g_{SK^+K^-})\left[(p-q)^2-M_S^2\right]\Delta_s^0(p-q)\right\}\ . \qquad (F.9)
\end{aligned}
$$

In the $p\cdot\epsilon=0$ gauge we finally obtain

$$
\begin{aligned}
\mathcal{M}_{int} &= -\frac{8}{3}\,(eg_{\phi K^+K^-})\,\frac{e}{M_\phi}(g_{\phi S\gamma}\ g_{SK^+K^-})(p\cdot q)\Delta_s^0(p-q)\left[(p-q)^2-M_S^2\right]\\
&\quad\times\left[\frac{(\vec{q_1}\cdot\vec{q_2})-(\vec{q_1}\cdot\hat{q})(\vec{q_2}\cdot\hat{q})}{(p-q_2)^2-M_K^2}+\frac{(\vec{q_1}\cdot\vec{q_2})-(\vec{q_1}\cdot\hat{q})(\vec{q_2}\cdot\hat{q})}{(p-q_1)^2-M_K^2}+1\right].
\end{aligned}
\qquad (F.10)
$$