A STUDY ON THE PROBLEM OF LOGICAL CONSTANTS AND THE PROBLEM'S SOLUTION CRITERIA

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ABSTRACT

A Study on the Problem of Logical Constants and the Problem's Solution Criteria

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In this thesis I study the problem of logical constants with respect to logical truth and logical consequence. In order to do that, I focused on the following two questions. First, what is a logical constant and what kind of relation there is between a logical truth, logical consequence and logical constant? Second, what are the solutions to the problem and to what extent these criteria can solve it?

The main argument of my thesis is to determine that all of the examined systems are satisfactory to considerable level still none of these is completely acceptable.

Keywords: Logical constants, Logical truth, Logical consequence, Tarski, Quine.

ÖZ

Mantıksal Değişmezler Sorunu ve bu Sorunun Çözüm Önerileri Üzerine Bir Çalışma

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Bu tezdeki amacım mantıksal değişmezler sorununu, mantıksal doğru ve mantıksal içerme kapsamında ele almaktır. Bunu yaparken de odak noktamı şu sorular oluşturmaktadır: Öncelikle, mantıksal değişmezler nelerdir, mantıksal doğru, mantıksal içerme ve mantıksal değişmezler arasında ne tür bir ilişki bulunmaktadır? İkinci olarak da, mantıksal değişmezler sorununun çözüm önerileri nelerdir ve bu tür önerilerle ne derece bir çözüm sağlanabilir?

Tezimde, incelenmiş sistemlerin tümünün belli bir ölçüde tatmin edici olsa da, bu sistemlerin hiçbirinin bütünüyle kabul edilemeyeceği sonucuna vardım.

Anahtar Kelimeler: Mantıksal değişmezler, Mantıksal doğru,
Mantıksal içerme, Tarski, Quine.

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To My Family

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CHAPTER 1

INTRODUCTION

There are several definitions to the term logical constant and there are various criteria offered in order to solve the problem of logical constants. The primary problem of logical constants appears twofold:

1. Reasons making a logical constant problematic
2. In what respect the criteria for demarcating logical constants are formulated.

A logical constant becomes problematic on the way of searching logical truth. Logical truth can be formulated with logical consequence of terms. But this logical consequence necessitates clear-cut definitions of logical constants. Although there are plenty of definitions, none of them is complete.

In chapter two I will make a slight historical review of the term logical constant, trying to explain how it appeared, how it is used and when has become essential. I will try to show that being 'problematic' and 'vague' are the only properties of logical constants accepted by all logicians. Since the idea of logical constants originated from Tarski's theory of logical consequence while searching for the definition of logical constants, it is necessary to give an explanation to the terms of logical consequence and logical truth as well.

After the description of the problem I will define logical truth, logical consequence and logical constants separately in order to clarify their connection with each other. The problem unveils after understanding why these logical concepts necessitate each other. Then I will list the main definitions of logical constant, together with the reasons which logicians prefer.

In chapter three I will list the systems offered to demarcate the logical constants and to solve this problem. Most of the subtitles that I will use in this chapter are those used by MacFarlane, in order to group different solution systems.

In section 3.1, I will present the group of logicians who name logical constants as "syncategorematic terms", which are meaningless when used alone but which combine meaningful terms.

In section 3.2, I will show the group of logicians defining logical constants as "grammatical particles", which build complex sentences from atomic ones.

In section 3.3, I will outline Grünberg's demarcation of logical constants in terms of analyticity.

In section 3.4, I will mention "Davidsonian approach" according to which logical constants play a structural role in a systematic theory of meaning for a language.

In section 3.5, I will indicate logicians who build up their systems according to "topic-neutrality". In this approach logic is a universal canon for reasoning. It is not about anything in particular, but applicable everywhere. Also topic-neutrality is examined under two distinct headlines

1. expression specific to a certain domain
2. universal applicability

Section 3.6, is a study of "permutation invariance". Logical constants' properties invariant under arbitrary permutations of the domain of objects. These are insensitive to particular identities of objects. Constants defined in this approach can behave differently on domains with different kinds of objects.

Under the title “inferential characterizations”, in section 3.7, I will examine logical constants characterized totally in terms of inferential rules.

Finally, in section 3.8, I will mention “pragmatic demarcations”. Sections from 3.1 to 3.7 will take into account analytic demarcation which tries to identify some favored property as a necessary and sufficient condition to any expression to be a logical constant in a holistic approach. Although, section 3.8 will illustrate a framework for the deductive systematization of scientific theories, everything mentioned in this section depends on the current state of scientific and mathematical theory.

Consequently, all of the following criteria propose some basic solutions in order to overcome the problem of logical constants. Although these criteria offer satisfactory solutions, none of them can solve the problem sufficiently.

CHAPTER 2

THE PROBLEM OF LOGICAL CONSTANTS

This chapter is a research of the meaning of logical constants and a study of the problem of logical constants. The problem appears exactly during the demarcation of logical constants. Although there are various definitions of logical constants, it is not yet commonly specified what makes a term logical constant and what are the criteria of a good theory of logical constants. Another vital question is how logical consequence and logical truth are necessary for determining logical constants? This is a vital problem, because any solution proposed to it, not only will define logical constants, but also will determine "the nature and special status of logic". Correspondingly, the role of logic is explained as a study of the properties of the arguments "in virtue of their logical forms or structures". Therefore, the problem necessitates a detailed definition of logically true arguments, in order to be able to differentiate them from the rest non-logical ones.

McCarthy defines the problem of logical constants as one of the most important problems of philosophy of logic, since it is the characterization of "the notion of *logical truth* and the related notion of *logical rule of inference*" (1981, p. 499). In order to solve this problem, McCarthy pretends "to characterize a class Γ of semantically primitive expressions of [language] **L** whose semantic interpretations are regarded as fixed; the elements of Γ are called the *logical constants* of **L**" (1981, p. 499).

In accordance with topic-neutrality, McCarthy, tries to show the characterization of logical constants. For instance, "the extensional predicate modifier 'red' characterized by the satisfaction rule

(4) $\forall s$(s satisfies ⌜red ϕx_i⌝ iff (s satisfies ⌜ϕx_i⌝ $\wedge$ the ith component of s is red))

affords a counterexample" (McCarthy, 1981, p. 508). In this example, 'red' is treated extensionally; therefore it is not a logical constant. "…no reasonable understanding of the notion of topic-neutrality could a modifier so characterized count as topic-neutral" (1981, p. 508). Accordingly, "a demarcation of logical expressions in the context of extensional theories of satisfaction", appear to be problematic.

In Gentzen's system, "the introduction rules for the logical constants ought to be considered as… the definitions of the constants…, as what gives the constants in question their meaning" (Martin-Löf, 1987, p. 410). One way of constructing the truth conditions is the Tarskian method. According to Gentzen's formulation the introduction rules for the logical constants are explicated in the same way as the truth conditions, just the way like in the following example (Martin-Löf, 1987, p. 411):

$$\frac{A \text{ is true}}{A \vee B \text{ is true}} \qquad \frac{B \text{ is true}}{A \vee B \text{ is true}} \qquad (\text{A is true})$$

$$\frac{A \text{ is true} \quad B \text{ is true}}{A \,\&\, B \text{ is true}} \qquad \frac{B \text{ is true}}{A \supset B \text{ is true}}$$

$$\frac{A(a) \text{ is true}}{(\exists x)A(x) \text{ is true}} \qquad \frac{A(x) \text{ is true}}{(\forall x)A(x) \text{ is true}}$$

In this example the table of truth conditions is turned counterclockwise by one right angle, turning vertical lines into horizontal, placing the terms of the left column below the horizontal line, and placing the terms of the right column on the same line of the left column, in order to satisfy "the conditions under which the proposition is true". To sum up, “the explanation of a proposition as the expression of its truth conditions is no different from Gentzen's explanation to the effect that the meaning of a proposition is determined by its introduction rules.” (Martin-Löf, 1987, p. 411)

Martin-Löf states that, while explaining the meaning of the logical constants the important point is not “whether a proof of a proposition was direct or indirect”. Wittgenstein also declares that “a proposition is the expression of its truth conditions”, is “the official intuitionistic explanation of the notion of propositions” (Martin-Löf, 1987, p. 411).

Warmbrod, from the minimalist point of view, “dictates” that logical constants should be as small as possible. Similarly, in order to formulate such a systematization of logical constants, a theory must be “simple”, “modest in its assumptions” and “flexible in providing a conceptual apparatus”. Then, “if a given body of theory can be systematized without recognizing logical constants such as "contains water" and "contains hydrogen atoms", then it is preferable to do so” (Warmbrod, 1999, p. 521). Due to the same limitations, terms like “contains more water than” and “contains less water than” are not defined as logical constants, accepting “only the smallest set of constants such as those of first order logic” (1999, p. 521). Warmbrod maintains that, a logical theory requires the recognition of “truth-functional connectives as logical”, in order to be sufficient for “the purposes of deductive systematizations” (1999, p. 525).

Conversely, there is a group of logicians some of which are Bolzano and Etchemendy, who define the problem of logical constants as “a pseudoproblem”. For this group, logic is involved in “validity *simpliciter*, not just validity that holds in virtue of a limited set of logical forms” (MacFarlane, 2005, p. 18). “The logician's *method* for studying validity is to classify arguments by their forms, but these forms (and the logical constants that in part define them) are logic's *tools*, not its subject matter" (2005, p. 18). Instead of the expressions like 'and', 'or', and 'not', the topic of investigation of logicians must be 'validity', 'consequence', 'consistency', 'proof', etc.

Etchemendy criticizes Tarski's criterion about consequence. He realizes that, "Tarski's theory … fails in certain respects to capture pre-theoretic modal intuitions about necessary (*a priori*, etc.) relations between sentences". Tarski's greatest mistake is done in adopting the idea that "an account of consequence *needs* to conform to pre-theoretic intuitions about necessity, apriority and form" (Warmbrod, 1999, p. 522). Etchemendy explains Tarski's weakness as "an inappropriate choice of logical constants" (McGee, 1996, p. 379).

Tarski is deeply involved with the definition of logical truth and logical consequence "in purely mathematical terms". According to Tarski an argument is valid if "there is no interpretation of its nonlogical constants on which the premises are true and the conclusion false" (MacFarlane, 2005, p. 19). He emphasizes that, "if *every* expression of a language counted as a logical constant, logical validity would reduce to material truth preservation" (MacFarlane, 2005, p. 19).

Hacking in his proposal defines *do-it-yourself* semantics, which is in need for "the constants that are thus introduced". "Analytic truths are those whose proofs may be traced back to 'general logical laws'. But these general laws were not to be primitive propositions written down in the object language" (Hacking, 1979, p. 318). He asserts that, analytic truths are "not about particular logical ideas, such as the quantifier or the conditional", but "the nature of the semantic framework itself" (Hacking, 1979, p. 318). Hacking suggests that, "A logical truth is a truth in which only logical constants occur essentially" (Hacking, 1979, p. 318). While looking for a definition of logical constant by using the deducibility theory, Hacking realizes that "although the existence of particular theorems of logic may be explained in terms of rules that define individual constants, the notion of logical truth depends on the notion of truth for a language" (1979, pp. 318-319). To sum up,

> If a nonstandard logic is possible, in a way that is not parasitic upon classical logic, then a non-classical notion of truth and consequence is possible. But if a nonstandard logic must ultimately be explained using classical logic, then indeed we would have found something that 'our thought can overflow, but never displace'. (1979, p. 319)

Gómez-Torrente explains the problem of logical constants as "the problem of demarcating in some principle-based, non-arbitrary-looking way the set of expressions that logic should deal with as directly responsible for the logical correctness of arguments" (2002, p. 2). He claims that logical constants are expressions that satisfy these essentials. Although being indefinite and complicated, logical constants are "principle-based" (2002, p. 4).

This problem appeared "in the work of some logicist authors, interested in giving an explanatory theory of the analyticity and apriority of logic", who also acknowledge the logical constants need to fulfill a "version of hypothesis" (Gómez-Torrente, 2002, p. 30). Moreover, these logicians believe that, the problem of logical constants is "of giving a theory of the semantic and epistemic properties of the logical constants which could serve to ground that hypothesis" (2002, p. 30). However, Gómez-Torrente reminds us the reality that this type of study “was never carried out to the satisfaction of all concerned, because no satisfactory theory of the semantics and epistemology of logical constants was ever produced” (2002, p. 30).

Actually Tarski’s effort for making a characterization of logical constants is a necessity for him, since he tries to get rid of “the usual concept from his mathematical explication of logical truth and logical consequence” (Gómez-Torrente, 2002, p. 30). Gómez-Torrente interprets Tarski’s obligation as “an extensionally correct characterization of the traditional set of logical constants, but he required it to be given in terms of logical and mathematical concepts” (2002, p. 30).

Consequently, Gómez-Torrente maintains that all existent “philosophical conceptions of the problem of logical constants”, formulated are “unsolvable versions of the problem”. Therefore, “if the project is hopeless, then all the versions of the problem generated by these conceptions will be unsolvable” (2002, pp. 31-32). Similar to Tarski’s characterization, Hacking’s characterization is also based on the idea that “the (extensional) semantics of logical constants ought to be "simple", obey some simple (in fact mathematical) semantic laws” (Gómez-Torrente, 2002, p. 32). There is no such a characterization that “seems to give sufficient conditions for membership in the intended set of logical expressions” (Gómez-Torrente, 2002, p. 32).

According to Tarski, “logical consequence presupposes the distinction between logical and extra-logical constants” (1986, p. 143). He believes that a notion can be logical “if it is invariant under all possible one-one transformations of the world onto itself” (1986, p.149). In this sense, Tarski accepts that the notions of *Principia Mathematica* are logical notions, and for that matter any other familiar system of logic, is invariant under every one-one transformation of the 'world' or 'universe of discourse' onto itself" (Tarski,1986, p. 150).

The term "logical constant" was primarily mentioned by Russell in 1903 in *Principia Mathematica* as:

> The fact that all mathematical constants are logical constants, and that all the premises of mathematics are concerned with these, gives, I believe, the precise statement of what philosophers have meant in asserting that mathematics is *a priori*. (p. 8)

Although the usage of the terms is based on mathematical grounds, for Gomez-Torrente the main idea could be "reduced in the logicist fashion" as well (2002, p. 6). According to Gómez-Torrente, Russell in his thesis says that truths containing only logical constants (and variables) must be *a priori*. He believes that **"**a true proposition containing only non-empirical notions with which we are intimately acquainted must be knowable non-empirically" (Gómez-Torrente, 2002, p. 6). Instead of being used in early nineties, the nature of logical constants was seriously questioned after the problems which appeared during Tarski's search for logical truth.

On the other hand, the definition of logical constants is necessary "for a satisfactory theory of logical truth, since it seems impossible to analyze the latter concept without using the concept of a logical constant" (Pap, 1950, p. 378). Otherwise, logical truth definitions were "easily shown to be unsatisfactory".

Tarski on his way of searching logical truth needed the selection of logical constants:

> No objective grounds are known to me which permit us to draw a sharp boundary between [logical and non-logical terms]. It seems possible to include among logical terms some which are usually regarded by logicians as extra-logical without running into

> consequences which stand in sharp contrast to ordinary usage. (Tarski, 1936, pp. 418-419)

The selection of logical constants mentioned by Tarski is related completely or partially "on the needs at hand, as long as the resulting consequence relation reflects the essential features of the intuitive, pre-theoretic concept of logical consequence" (McKeon, 2006a, pp. 25-26). At that point the importance of the notion of logical consequence becomes more apparent. Since "the primary aim of logic is to tell us what follows logically from what", "logical consequence" appears as a very important "central" term for logic (McKeon, 2006b, p. 1). From Tarski's point of view, in a given language L, x is a logical consequence of a set of sentences K, only when "it is not possible for all of the sentences in K to be true with x false" (McKeon, 2006b, p. 3). Moreover, the knowledge of "x is a logical consequence of K, is not based on empirical grounds, but on an "ordinarily employed" concept of logical consequence which can be reflected by "an adequate response". Then, "an adequate account of logical consequence must reflect the formality and necessity of logical consequence, and must also reflect the fact that knowledge of what follows logically from what is *a priori*"(McKeon, 2006b, p. 6).

In order to understand what is the logical consequence of a language L, McKeon alleges that a class of constants must be selected, a class which "determines a formal consequence relation that is both necessary and known, if at all, *a priori*" (2006b, p. 6). These "essential determinants of the logical consequence relation" are logical constants (McKeon, 2006b, p. 12).

Gila Sher believes that Tarski needs the notion of logical consequence for "preservation of truth", which at the same time necessitates "a general understanding of truth". According to Sher's definition truth "is based on correspondence between language and the world", and is "the simple Aristotelian principle that to say of what is that it is or of what is not that it is not is true, while to say of what is that it is not or of what is not that it is, is false" (2002, pp. 10-11). On the other hand, logical consequence is defined as "the relation that connects a given claim or set of claims with those things that follow logically from it: to say that B is a logical consequence of A is simply to say that B follows logically from A" and such a relation of logical consequence is nothing but "truth-preserving" (Blanchette, 2001, pp. 1-2).

The relation of logical truth and logical consequence is more apparent in Sher's paper, *Logical Consequence: An Epistemic Outlook.* According to Sher, "the bounds of logic are… the bounds of logical constants" which are stipulated by "*enumeration,* i.e., dogmatically, without grounding or explanation" (Sher, 2002, p. 1). Sher states that,

> … my analysis of the role logical constants play in producing logical consequences led me to arrive at a criterion of logical constanthood whose 1st-order extension far exceeds the standard selection. More specifically, I showed that if we characterize logical consequence as necessary, formal, topic neutral, indifferent to differences between individuals, etc., then this characterization, restricted to languages of the 1st-level, is not adequately systematized by the standard 1st-order system. A richer system (or family of systems), with new logical constants, is required to fully capture it (Sher, 2002, p. 1).

Sher aims to formulate an account of logic, which modifies and is supported by "a broader epistemology". Parallel to this, Sher defines truth and logical truth as a "correspondence with reality" and "a particular type of truth, exhibits a particular kind of correspondence", respectively (2002, p. 10). In order to understand this correspondence, it is necessary to examine the relation of logical consequence: "Logical consequence is a particular kind of consequence and consequence relations in general are relations of preservation, or transmission, of truth" (2002, p. 10).

Thus, to clarify the everlasting relation between 'logical truth', 'logical consequence' and 'logical constants', it is necessary to write down the separate definitions of each of these concepts.

2.1 Logical Truth

Truth of a sentence is the resemblance of the 'fact' by its 'meaning'. As Quine says in his book *Philosophy of Logic* the sentence "Snow is white" is true in case "meaning matches the fact" (1970, p. 1). Since logic is concerned not with 'fact' but with 'meaning', then "meanings of sentences are exalted as abstract entities in their own right, under the name of propositions" (Quine, 1970, p. 2). Replacing a word with another, usually keeps the truth value of the context of a sentence. That is to say, it is "turning truths into truths and falsehoods into falsehoods". Philosophers like Wittgenstein, who favors propositions, "have said that propositions are needed because truth is intelligible only of propositions, not of sentences… sentences are true whose meanings are true propositions" (Quine, 1970, p. 10). Also, they support the idea that "truth should hinge on reality, not language". So, "no sentence is true but reality makes it so". For instance "the sentence 'Snow is white' is true, as Tarski has thought us, if and only if real snow is really white" (1970, p. 10).

Quine defines logical truth as "a truth in which only logical constants occur essentially". Whereas, Tarski primarily assertes that "there is no delineation of the logical constants". For example, "as if we could characterize the concept *planet* of *the sun* only by reciting Mars, Venus, Earth, etc., and could not tell by any general principle whether the heavenly body epsilon is a planet or not " (Hacking, 1979, p. 287). Hence, Hacking maintains that, "We have a laundry list of logical constants, but no characterization of what a logical constant is–except the circular one, that logical constants are those which occur essentially in analytic truths" (1979, p. 287).

According to Hacking the truths related with individual integers, "such as 5 + 7 = 12, are analytic, but arithmetic as a whole is synthetic. The concept number is not explicable by logical constants alone. Hence, even sentences of the form, 'For every natural number ***x,*** then ...,' are not in general analytic, for the numerical quantifier is not a logical one" (1979, pp. 287-288).

The difference between analytic truth and logical truth must not be neglected. Although both of the truths are concerned with the meaning, still logical truth is concerned with the meanings of minor groups such as, the logical constants like "and", "or", "not", "for all", "exists", perhaps "=", and "terms definable in terms of these"(Blanchette, 2001, p. 24). For Tarski, "the lack of an account of the distinction between logical and extralogical terms" is his main problem (McGee, 1996, p. 379).

According to Gómez-Torrente, Tarski in 1936 constructed such a logical truth that "all the propositions of the same form are true, by means of his celebrated model-theoretic method of definition, whose *general description* (but not the particular definitions given rise to by the method) uses the notion of a logical constant: roughly" (2002, p. 10).

> Nothing in Tarski's writings suggests the idea that a sentence is true if and only if it is known or believed or held or accepted or felt to be true. Occasionally, Tarski uses the expression 'asserted sentence' to refer to theorems, or provable sentences (see [26, p. 1661). By the same token, it is clear that when Tarski defines validity, he intends an objective, pure ontic property; for any given argument-text, either it is valid or it is invalid, regardless of whether the argument-text is known, believed, held, accepted or felt to be valid. (Saguillo, 1997, p. 217)

According to Wittgenstein "some classes of logical truth are a "by-product" of facts about the use of logical constants". Nevertheless, "[t]his fact is a by-product of rules for the introduction of the logical constants" (Hacking, 1979, p. 228). Wittgenstein emphasizes that "without bothering about sense or meaning, we construct the logical proposition out of others using only rules that deal with signs" (Hacking, 1979, p. 288). Consequently, "The fact that these logical truths are a by-product of rules for signs is taken to explain the necessary, apodictic, and *a priori* character of some of the truths that we call logically necessary " (Hacking, 1979, pp. 288-289).

2.2 Logical Consequence

Tarski's proof theoretic characterization of logical consequence is deficient, at the same time "conceptually inadequate". This is because, he acknowledges that, "it is possible to construct and add additional rules to the system ad infinitum", therefore, "different methods are needed to properly characterize the consequence relation" (Schoubye, 2005, pp. 3-4).

For Kuhn, "a sentence (in the artificial language) is said to be a *logical consequence* of some other sentences if the argument with the latter as premises and the former as conclusion remains correct after any sensible substitution for those expressions which aren't logical. Moreover, a sentence is a logical consequence of another sentences only when the argument with the latter as premises and the former as conclusion remains correct after any sensible change in the denotation of the nonlogical expressions" (Kuhn, 1981, p. 488).

Warmbrod defines the notion of logical consequence as "truth-preserving, and the theory should provide a complete proof procedure" (1999, p. 518). He proposes a definition of the consequence in terms of the notion of truth:

> If one can then define the consequence relation in terms of the notion of truth, one can argue from the theory of truth and the definition of "consequence" that if p is a consequence of $\mathbf{\Gamma}$ and all members of $\mathbf{\Gamma}$ are true, p will be true. Since the class of sentences is infinite, the most plausible way to assign a truth condition to each sentence is for the semantic theory to parse sentences into structural components and assign meanings to the components in a way that allows one to derive a truth condition for each sentence. (1999, p. 518)

According to him, this definition states a starting point for the connection of "consequence relation to truth". Then, something is a logical consequence relation only if "all permitted assignments that make a premise true also make a certain conclusion true" (Warmbrod, 1999, p. 519). "The claim that the consequence relation is truth-preserving thus inherits its plausibility from the theory of truth and does not depend on any assumption that the relation is necessary." (1999, p. 519)

2.3 Logical Constants

In mathematics a term with a fixed, determined and unchanging meaning is a constant. For example "number", "zero", "+", "=", etc. are all mathematical constants. In logic also there are terms and symbols with fixed and determined meaning called constants (Mourant, 1967, p. 23). In general, all formula-maker constants are defined as logical constants: “~”, “→”, “↔”, “.”, “∀” etc. (Kalish et al. 1980, p. 400). These logical constants are not the same as variables, because variables do not have any determinate and indeterminate meaning. Logical constants “are basic to the propositional calculus, for without them we could not put our propositions together and form a calculus” (Mourant, 1967, p. 23). Logical constants promote the relation and connection among propositions.

Kuhn defines “logical truth” as a truth which “is true under all category-preserving substitutions for lexical atoms” (Kuhn, 1981, p. 488). The logical expressions called constants are “particles which occur essentially in logical truth” (Kuhn, 1981, p. 493). He defines constants as expressions which render “the basis for a highly successful science of reasoning” and “that are held fixed or whose meanings are held fixed while the others are substituted for or reinterpreted” (Kuhn, 1981, p. 488). He also explains that

> There are exactly seven logical constants, corresponding roughly to the English expressions 'and', 'or', 'not', 'if', 'if and only if', 'all', and 'some'. The reasons that logical consequence and logical truth can be characterized in a convenient and useful sense by these seven constants is not completely understood–there is undoubtedly an element of arbitrariness… in the choice. (Kuhn, 1981, p. 488)

Lycan defines logical truth as “a sentence or formula that is true under any admissible reinterpretation of its nonlogical terms; nonlogical terms, that is, as opposed to *logical constants*” (Lycan, 1989, pp. 392-393). Therefore, logical constants necessarily appear to be different from the rest of the constituents of logic. Yet, this assumption is not completely accepted, since it causes certain difficulties (which are mentioned in Chapter 3).

However, in the *Tractatus*, logical constants cannot represent the logic of facts, since there are no such representatives at all (Cheung, 1999, p. 395). In *Tractatus*, the sign of any logical constant, is nothing but "a punctuation mark, or a pair of brackets, in all relevant propositional contexts and thus no one would believe it denotes" (Cheung, 1999, p. 396). Wittgenstein states that "there are no 'logical objects' or 'logical constants'". According to him,

> The reason is that the results of truth-operations on truth-functions are always identical whenever they are one and the same truth-function of elementary propositions. (TLP 5.41) Since logical constants are symbols of operations, the 'reason' amounts to saying that different results of combining propositions by means of logical constants are identical if and only if they are one and the same truth-function of elementary propositions or, given the analyticity thesis, one and the same proposition. (Cheung, 1999, p. 397).

Wittgenstein needs to explain that "the sign of a logical constant does not denote in all propositional contexts in which it occurs" (Cheung, 1999, p. 398). He also states that punctuation marks are the signs of logical operations. Therefore, "for if the sign of a logical constant is just like a pair of brackets indicating… only the order and scope of application, then no one is going to believe that it denotes or has an independent meaning" (Cheung ,1999, p. 402).

Wiredu gives a method for distinguishing the logical constants in a symbolic formula: "subtract from the symbols the variables and count any symbol remaining (discounting logical punctuation) as a logical constant" (Wiredu, 1975, p. 312).

Gentzen's system of "natural deduction and sequent calculi" is based on the formulation of "*separate* rules for each constant". This proposed Gentzen "the idea that the meaning of a constant did not reside in the whole body of postulates of the calculus, but rather in the *rules* for introducing it into (or eliminating it from) discourse" (Paoli, 2003, p. 536).

It is possible to differentiate some facets of the meaning of a logical constant *c*, in case of the "adoption of proof theoretic semantics". One of these aspects is "*operational meaning*", which gives knowledge about "how to use *c* in a deduction does not mean being able to recognize, or to assent to, a correct inference involving *c*" (Paoli, 2003, p. 537). Another aspect is "a *global meaning*", which is "specified by the class of the system's theorems (provable sequents) containing *c*."(Paoli, 2003, p. 537)

According to Quine's system "logical connectives are *immanent*, not *transcendent*". That is to say "no pretheoretical fact of the matter a theory of logical constants must account for" (Paoli, 2003, p. 542). But, Quine's distinction between the *transcendent* and *immanent* is defined as such:

> Conceptions of truth that lead one to built truth-theories covering no sentences beyond one's home language are *immanent*. Conceptions of truth which require one to develop a truth-theory applying beyond one's own language are *transcendent*… (Shapiro, 2003, p. 115)

Quine's immanency thesis could be questioned on the ground that it destroys each and every connection between the logical constants and the natural language particles which correspond to them; following this line of reasoning, it might be suggested that the classical truth table for negation translates into precise truth conditions the notions of "opposition", "denial" and the like, that dictionaries assume as constitutive of the meaning of the English sentential particle “not” (Paoli, 2003, p. 543).

For Fennell, “‘and’, ‘not’, ‘if . . . then’, and so on, terms whose proprieties of use are expressed in logical laws” are examples of logical constants, and “color terms like ‘red’, material object terms like ‘table’, natural kind terms such as ‘acid’, ‘rain’, and ‘rabbit’, and mathematical expressions like ‘ + ’” are examples of non-logical constants (Fennell, 2003, p. 265).

According to Bonnay’s Inferential Role Thesis, the basis for reasoning due to “the role they play in building inferences”, are the logical words. Thus, “their meaning should be at least determined by the way they can be used in reasoning” (Bonnay-Simmenauer, 2005, p. 33). For criteria defending the syntactic rule in the definition of logical constants similar to Benlap’s restriction are “to add only connectives that preserve good properties of the deducibility relation `, *i.e.* connectives such that adding them to a previously given system yields a conservative extension of that system” (Bonnay-Simmenauer, 2005, p. 35).

Peregrine makes the definition of “‘natural’ logical constants”. According to him, those logical constants are “delimitable inferentially (presumably the intuitionist ones), and that the classical ones are their artificial adjustments available only after metalogical reflections and through explicit tampering with the natural meanings” (2006, p. 23).

Warmbrod pays attention on the possibility of systematizing “the truth-functional connectives and first-order quantifiers as constants, treating "=" as an ordinary predicate, and adopting appropriate axioms for identity” (1999, p.521). Hence, the identity predicate is not recognized as a part of “the minimal conceptual apparatus needed to deductively systematize scientific theories. To treat "=" as if it were needed as a constant of core logic–the theory of deductive systematization–is thus simply a mistake" (Warmbrod, 1999, pp. 521-522). A systematization adopting topic-neutrality or generality, will semantically require '=' as a constant, but it is not the case with minimalism (Warmbrod, 1999, p. 522).

Quine (1970, p. 11) rejects the definition of '=' as a logical constant, giving the following example:

Tom is mortal.
Dick is mortal.
___
All men are mortal.

Tom is Tom.
Dick is Dick.
0 is 0.
___
Everything is itself.

Tom is mortal or Tom is not mortal.
Snow is white or snow is not white.
___
Every sentence of the form 'p or not p' is true.

"What prompts this ascent is . . . the oblique way in which the instances over which we are generalizing are related to one another" (McKeon, 2004, p. 211). If we take 'man' and 'mortal' as logical terms, then "no non-logical terminology occurs in 'All men are mortal'", which brings us to the point of "all sentences of the form 'All men are mortal' are true would be equivalent to saying All men are mortal since any sentence of the form ‘All men are mortal’ just is ‘All men are mortal’” (McKeon, 2004, p. 211). Finally, for Quine, ‘man’ and ‘mortal’ are not logical constants, “in so far as ‘All men are mortal’ is not true in virtue of form on Quine’s understanding of logical form” (McKeon, 2004, p. 211). Despite all problems, Quine characterizes ‘=’ as a logical constant. And, the logical validity of the open sentence ‘‘x=x’’ is of course uncontested, and therewith the logical truth of the closed sentence ‘‘(x)(x=x)’’’(McKeon, 2004, p. 217). However, it becomes apparent that, “‘=’ is a primitive logical constant and at least some existential sentences are logical truths on Quine’s account, which Quine does not want” (McKeon, 2004, p. 217).

CHAPTER 3

CHIEF APPROACHES TO ADJUDICATE THE PROBLEM OF LOGICAL CONSTANTS

This chapter consists of the main approaches to determine the problem of logical constants. In the following section there are seven subtitles which also exist in MacFarlane's 2005 article "logical constants". These subtitles represent the different systems proposed by various logicians, from various philosophical aspects.

3.1 Logical constants as syncategorematic terms

This is the approach which recognizes logical constants with "the languages *syncategorematic* terms". These signs "signify nothing by themselves, but serve to indicate how independently meaningful terms are combined" (MacFarlane, 2005, p. 2). They illustrate "the relation between subject and predicate or between two distinct subject-predicate propositions" also known as *syncategorematic* words. For instance, "'only', 'every', 'necessarily', and 'or' " are syncategorematic. Such words mostly show "the *structure* or *form* of the proposition", instead of "its matter".

Still this approach confronts some difficulties, since "it is not clear how the distinction between categorematic and syncategorematic terms, so natural in the framework of a term logic, can be extended to a post-Fregean function/argument conception of propositional structure" (MacFarlane, 2005, p. 3). Although not being so much popular, the effect of the syncategorematic terms still appears in "Wittgenstein's insistence that the logical constants are like punctuation marks", in

Russell's words "logical constants indicate logical form and not propositional constituents", and in Quine's and Dummett's "logical constants of a language can be identified with its grammatical particles" (MacFarlane, 2005, p. 3).

3.2 Logical constants as grammatical particles

In this group, philosophers like Quine define logical constants in a language as "the expressions by means of which complex sentences are built up, step by step, from atomic ones—while non-logical expressions are the simple expressions of which atomic sentences are composed" (MacFarlane, 2005, p. 3). According to Quine, "[l]ogic studies the truth conditions that hinge solely on grammatical constructions" (1980, p. 17), therefore *all* operators and connectives are "paradigm logical constants".

One of the main restrictions of this grammatical criterion is that, it "will not impose significant constraints on what counts as a logical constant unless it is combined with some principle for limiting the languages to which it applies… and privileging some regimentations of their grammars over others" (MacFarlane, 2005, p. 4). With Quine's grammatical demarcation it is possible to determine "the logical constants with members of small, "closed" lexical categories: for example, conjunctions and determiners" (MacFarlane, 2005, p. 4). Accordingly, prepositions in English are defined as logical constants as well.

Nonetheless, MacFarlane states that "if a distinction that plays an important role in a theory of linguistic competence should turn out to coincide (in large part) with our traditional distinction between logical and nonlogical constants, then this fact would stand in need of explanation" (2005, p. 4).

3.3 Logical constants as analyticity-preserving

Teo Grünberg is one of the latest logicians who published a criterion of demarcation of logical constants and of logical truth, in terms of analyticity. He starts his thesis by a conjecture that "the largest analyticity – preserving set of constants … constitutes the set of logical constants, at least in the case of the usual language. Such a conjecture provides a criterion of demarcation for the logical constants" (Grünberg, 2005, pp. 27-28). Grünberg's thesis consists of four criteria of demarcation (CD). The first three of the criteria are abandoned due to counter-examples, but the fourth criterion is "analyticity-generating and strongly non-idle" (Grünberg, 2005, p. 37). Grünberg (2005, p. 37) further writes: "[In case] this fails, we can still adopt in place of CD4 [a] restricted and weakened criterion of demarcation [i.e., CD4']" (Grünberg, 2005, p. 37). Grünberg's basic characterization of the logical constants is given in the following passage:

> According to Quine, Carnap's criterion of demarcation ultimately avoids the difficulty… by using Cartesian co-ordinates. Carnap assigns to each object (or event) E a set K_E of quadruples of real numbers which are the spatio temporal co-ordinates of the point-events constituting that object. Let K_E [t] be defined as such:
>
> $$K_E[t] = \{ < x, y, z > : < x, y, z, t > \in K_E \}$$
>
> K_E [t] characterizes the momentary state at time t of object E. Now the constants of the form K_E and K_E [t] are logical proper names. We shall add these new constants to the vocabulary of any language as *auxiliary logical constants*. We denote the set of these constants by ***K***. Then the set of logical constants of the extended language is L U ***K.*** The members of L [viz., the set of

> logical constants of the non-extended language] are the *proper* logical constant (Grünberg, 2005, p. 35).

On the other hand, the exact formulation of the demarcation criteria CD4 and CD4' are given respectively in Grünberg, 2005, p. 35 and p. 37.

3.4 Logical constants with respect to their linguistic meanings

In this approach, Davidson characterizes logical constants as "the expressions that play a privileged, "structural" role in a systematic grammatical theory for a language" (MacFarlane, 2005, p. 5). According to Davidson, "[t]he logical constants may be identified as those iterative features of the language that require a recursive clause in the characterization of truth or satisfaction" (1984, p. 71). In this criterion "different truth theories can be given for the same language, and they can agree on the truth conditions of whole sentences while differing in which expressions they treat in the recursive clauses" (MacFarlane, 2005, p. 5). Moreover, the lack of further limitations on "the theory of meaning", raises difficulties in formulating an explicit criterion for logical constancy.

Davidson believed that "the logical constants of $\mathbf{L_o}$ are those of $\mathbf{L_o}$ which are referred to in the phrasal axioms of TM" (Edwards, 2002, p. 252). Accordingly, he would use TM to define '$\vee$', '$\sim$', and '$\forall$' as logical constants of $\mathbf{L_o}$, since they are referred to phrasal axioms, but not 'Tom' or 'is a man' (Edwards, 2002, p. 252). He legitimize that "the interpretations of '$\vee$' and '$\sim$' are to remain *fixed*". Davidson's theory suggests "the transcendent account of the sentential connectives and quantifiers" (Edwards, 2002, p. 271).

3.5 Topic-neutrality in logical constants

Logic is convenient anywhere, "no matter what we are reasoning about", therefore it is supposed that logical constants are "topic-neutral". Accordingly, logic is accepted as "a *universal* canon for reasoning, one that is applicable not just to reasoning about this or that domain, but to all reasoning" (MacFarlane, 2005, p. 6). For example, arithmetic is topic-neutral. In other words "anything can be counted, so the theorems of arithmetic will be useful in any field of inquiry". But it confronts the problem of "*antinomy of topic-neutrality*". George Boolos emphasizes that this problem occurs also in the case of logical constants: "it might be said that logic is not so 'topic-neutral' as it is often made out to be: it can easily be said to be about the notions of negation, conjunction, identity, and the notions expressed by 'all' and 'some', among others …" (MacFarlane, 2005, p. 6). Hence, the reason of the "antinomy" is the uncertainty of the notion of the topic-neutrality.

In order to clarify the topic-neutrality state, MacFarlane gives an example of a person who comprehends English weakly and hears these words:

> blah blah blah *and not* blah blah blah *because it* blah blah blah *to be* blah blah blah *and was always* blah blah blah. *But every* blah blah *is* blah blah, *although a few* blah blah *might be* blah.(MacFarlane, 2005, p. 7)

Although this person is unable to understand the paragraph as a whole, he catches some of the tricky words like "Because", "It", "was always", "every", "a few"…etc., the meanings of which are known to him. "Perhaps some of these words are not topic-neutral and should not be included in the domain of logic", still ruling out *all* of these is an unpreferable situation (MacFarlane, 2005, p. 7). Actually, the problem with this criterion is that, it “gives no guidance about where to draw the line”. Furthermore, it could be thought that, “there is no line, and that topic neutrality is a matter of degree, truth-functional expressions being more topic-neutral than quantifiers, which are more topic-neutral than tense and modal operators, which are more topic-neutral than epistemic expressions, and so on” (MacFarlane, 2005, p. 7).

On the other hand, there is a fact that, no account is appropriate for the demarcation of logical constants. That is to say, “if there is any point to invoking topic neutrality in demarcating logic, it is presumably to distinguish the logical truths from a wider class of necessary propositions, some of which are subject matter-specific” (MacFarlane, 2005, p. 7). As was mentioned before, MacFarlane accepts the idea that the “capacity to *discriminate* between different individuals”, makes an expression special to a definite subject.

For instance, “the monadic predicate "is a thing", the dyadic predicate "is identical with", and the quantifier "everything" do not distinguish between Lucky Feet and the Statue of Liberty” (MacFarlane, 2005, p. 8). In that case, there is no distinction between two objects. In fact, “expressions with this kind of indifference to the particular identities of objects might reasonably be said to be topic-neutral” (MacFarlane, 2005, p. 8).

Different from this “discrimination” approach there is another type of “topic neutrality of logic”, which has a “universal applicability”. As MacFarlane states, “logic is useful for the guidance and criticism of reasoning about any subject… because it is intimately connected somehow with the very conditions for thought or reasoning” (2005, p. 8).

As mentioned above there are two notions of topic-neutrality; one of which is insensitivity (permutation invariance) criterion, and the other universal applicability (inferential) criterion. To sum up, both of these approaches of topic-neutrality are incomplete, since each of them "support one side of the antinomy" (2005, p. 8).

Peacocke is focused on the discussion about "expressions for which the appropriate specification of meaning is an account of their contribution to truth conditions of sentences" (1976, p. 221). He thought that theories beginning with the idea that "*a!* is a logical constant just in case there are sentences containing *a!* that remain true under uniform substitutions for their parts other than" are poor because of the fact that they "isolate the logical constants via properties of whole sentences or arguments in which they occur and which at the same time attempts thereby to pick out what is fundamentally distinctive of those constants" (Peacocke, 1976, p. 221).

To get rid of the problem, Peacocke states that it is needed "to prevent the truth of all instances of 'Everything that is *F* and is blue is colored', 'If *a* knows that *p*, then *p'*, 'If *a* prevented it from being the case that *p*, then *~p'* from making all of 'blue', 'colored', 'knows', and 'prevents' into logical constants". He emphasizes that,

> We do not have a conception of the *validity* of arguments in advance of a selection of the logical constants; we have only a notion of truth-preservingness and of an argument's necessarily being truth-preserving. (1976, p. 222).

For example, if the premise of this argument is true, then necessarily its conclusion is true.

$$\frac{\text{John drank some water}}{\therefore \text{ John drank some } H_2O}$$

Yet, applying different "one-place predicate in place of occurrences of 'water' ", it changes its property.

Peacocke's characterization of logical constancy is this (1976, pp. 225-226):

> α is a logical constant iff α is noncomplex and, where the syntactic category of α is $\tau/\sigma_1, \ldots, \sigma_n$, for any expressions $\beta_1, \ldots, \beta_n$ of
>
> categories $\sigma_1, \ldots, \sigma_n$, respectively, given knowledge of
>
> (a) which sequences satisfy those β_i which have satisfaction conditions, and of
>
> (b) which object each sequence assigns to those β_i which are input to the assignment function, and of
>
> (c) the satisfaction condition or assignment clause for expressions of the form $\alpha(\gamma_1, \ldots, \gamma_n)$
>
> one can know a priori which sequences satisfy the expression $\alpha(\beta_1, \ldots, \beta_n)$ of category τ, or which object any given sequence assigns to $\alpha(\beta_1, \ldots, \beta_n)$, in particular without knowing the properties and relations of the objects in the sequences.

In fact, Peacocke claims that an expression in order to be a logical constant requires a "necessary and sufficient" condition of topic-neutrality. Explicitly this understanding is illustrated in Peacocke's criterion given above.

On Peacocke's account, "the truth theories are formulated using finite sequences of objects in the satisfaction relation" (1976, p. 223). Explicitly this criterion accepts '~' as a logical constant, since "given a knowledge of which sequences satisfy *A* and knowledge that any sequence satisfies '~*A*' iff it does not satisfy *A*, one can know a priori which sequences satisfy '~*A*', viz., just those which fail to satisfy *A*; so '~' is a logical constant by the criterion" (Peacocke, 1976, p. 223). On the other hand, Peacocke's determination theory decides that '~' is faintly truth functional, because 'in case *A* is foiled then '~*A*' is contented' (Hodes, 2004, p. 161).

Hodes primarily ponder logical constants "with respect to their sense". Yet, for a *determination theory* –as Peacocke calls it 'a theory of sense'– the semantic value of the expression needs to be determined. That is to say, "a determination theory to characterize how the sense of an expression, or better, the conditions for grasp of that sense, contribute to determining the expression's 'referent' " is necessary (Hodes, 2004, p. 157).

Moreover, checking whether "in the past (*'P')*" is a logical constant or not, it should be asked "whether *s* now satisfies *A*", "whether it did in the past", and "whether it will do so in the future". If it satisfies, then '*P*' is a logical constant. Accepting "knowledge of whether *s* now satisfies *A*", will be "an error", because "there would then be an unmotivated asymmetry of treatment between the quantifiers and the temporal operators" (Peacocke, 1976, p. 224). Then, '*P*' would not be counted as a logical constant. Additionally, "the father of ξ" is not a logical constant. Truly the missing point is "a proper name *c* of some language such that we can truly say in that language that the believer can infer that *s* assigns *c* to 'the father of *β*' " (Peacocke, 1976, p. 226).

Following the same method in his reasoning Peacocke proposes certain reasons, which will be sufficient to name '□' as a logical constant. Nevertheless, '□'s "introduction rules" and "elimination rules" are "consistent with the supposition that '□*A*', where *A* does not contain '□', is true only if *A* is a logical truth" (Peacocke, 1976, p. 231). Furthermore, a system is logical if "at least that one property of a system that cut elimination implies, namely that every derivable sequent σ has a proof in which occur only sub-formulas of formulas in *σ*" (Peacocke, 1976, pp. 231-232).

Peacocke in order to check whether identity is a logical constant or not gives this example: "if anything is a donkey cannot but be a donkey and if anything that is not a donkey could not be a donkey, then, for any given sequence s_o of objects, we can say either that

$$\Box\ (s_0{*}t = a\ \&\ \forall s(\text{sats}(s, \ulcorner \text{donkey}(t_1)\urcorner) \equiv \text{donkey}\ (s{*}t_1))\ .\supset \text{sats}(s_0, \ulcorner \text{donkey}(t)\urcorner))$$

or that

$$\Box\ (s_0{*}t = a\ \&\ \forall s(\text{sats}(s, \ulcorner \text{donkey}(t_1)\urcorner) \equiv \text{donkey}\ (s{*}t_1))\ .\supset \sim \text{sats}(s_0, \ulcorner \text{donkey}(t)\urcorner))$$

for some proper name or variable a , according as it names or it assigned a donkey or not" (1976, p. 234).

Definitely 'is a donkey' is not a logical constant, "since one cannot ever know *a priori* of a given object whether or not it is a donkey". In such a state, for Peacocke, "what to count as *logical* necessity should fall out as a consequence of a theory of the logical constants and logical truth, and not be a resource presupposed by the theory "(1976, p. 234).

According to Peacocke, Quine advocates that identity "will not be a primitive expression that is handled by some axiom of the truth theory for the language" (1976, p. 234). Thus identity is not a logical constant.

Still Peacocke legitimize "the concept of truth in all models in which the identity sign is assigned the identity relation on the domain of the model… as rather arbitrary" (1976, p. 235). That is to say, "the denotations of individual constants are allowed there to vary between models, ‘c = *d’* is not valid unless ‘c’ is‘d’, and so all sentences valid on that notion can be known to be satisfied by all sequences *a priori* given knowledge of the satisfaction conditions of the components, without further requirements being placed on the imagined knower's knowledge” (Peacocke, 1976, p. 235). This legitimization is valid not only for identity, but also for the expressions like ‘a few’, ‘many’, ‘most’.

In Peacocke’s system, ‘→’is a logical constant in case, “no *a posteriori* truths enter the methods of transformation of proofs of A that one admitted”. On the contrary, “if ‘∀x(~Fx ∨ Gx)’ were assertible but a posteriori… and if this were taken as sufficient for asserting that ‘∃xFx →∃xGx ’ even when neither ‘∃xGx’ is assertible nor ‘∃xFx’ reducible to decidable falsity” (1976, p. 239). On the whole, ‘→’is not a logical constant, because

> on the interpretation that does make ‘→’ a logical constant, there will be no sentences of the form ‘A→B’ dealing with purely a posteriori subject matter that both are assertible and are not logical consequences of decidable sentences that are assertible. (Peacocke, 1976, p. 239)

Peacocke’s opinion about the general criterion for logical-constanthood is “to observe how it can help to explain the plausibility of the operation of the Principle of Charity in a way that (at least) guarantees that there will be no assent to the negations of certain uncontentious logical truths” (1976, p. 240). He explains the Principle of Charity as a method of “identifying where we can as negation, conjunction . . . devices of another language on the basis solely of the conditions under which wholes containing these devices are assented to or dissented from by speakers of the other language on the basis of assent or dissent to the expressions on which they operate" (1976, p. 240). Consequently, during the empirically possible cases, it could be possible for "assent and dissent" to be wrong especially if the Charity is applied for logical constants. Therefore, "Charity and Constancy are inseparable".

Lycan holds that, Peacocke is precise in suggesting that "topic-neutrality is intuitively the key desideratum" (Lycan, 1989, p. 395). He asserts that there is no restriction in the case of truth-functional connectives. According to him, "quantification theory is… a domain of individuals and either sets or properties defined on that domain" (1989, p. 395). That's why in his account "only the truth-functional connectives are genuinely logical constants, and not even the quantifiers qualify". Peacocke's differentiation can be plausible, since it is "providing for some purposes a good meaning for 'logical constant', without committing oneself to the claim that, as a matter of fact, all and only expressions meeting the Peacocke condition are true logical constants" (Lycan, 1989, p. 395).

On the other hand, McCarthy criticizes Peacocke's criterion by pointing out the lack of "the existence of a truth-table representation that *invariantly* specifies the truth value of the truth-functional compound in terms of those of its immediate subsentences" (McCarthy, 1981, p. 518). What makes a truth-functional connective a logical constant, is the presence of "a truth-table description that represents that connective in *every* state of information normal for that connective with respect to the language in question" (1981, p. 519). McCarthy's idea for completing this missing point is to provide "appropriate generalizations of the notions of valuation, truth table, representation, and representation in a state of information" (1981, p. 519).

3.6 Logical constants invariance under random permutations

MacFarlane states that, one group of the philosophers who adopt topic-neutrality in their principles, decided that what is peculiar to logical constants is "their *invariance* under arbitrary permutations of the domain of objects". Explicitly, logical constants are insensitive to "the particular identities of objects" (MacFarlane, 2005, p. 8).

In case *p* is a permutation of objects on a domain *D,* the *p*-transform function *p** is defined as such:

- if *x* is an object in *D*, $p^*(x) = p(x)$.
- if *x* is a set, then $p^*(x)$ = {*y* : □*z*(*z* □ *x* & *y* = $p^*(z)$)} (that is, the set of objects to which *p** maps members of *x*).
- if *x* is an ordered *n*-tuple <x_1, …, x_n>, then $p^*(x)$ = <$p^*(x_1)$, …, $p^*(x_n)$> (that is, the *n*-tuple of objects to which *p** maps x_1, …, x_n) (MacFarlane, 2005, p. 9).

What is implied with this permutation condition is that, it "allows that a permutation-invariant constant might behave differently on domains containing different kinds of objects".

According to McGee "every permutation-invariant operation can be defined in terms of operations with an intuitively logical character"(MacFarlane, 2005, p. 10), thus he defines every operation as "permutation invariant". Besides, Tarski holds that the notations which are steady under the largest possible group of transformations "the group of *permutations* of the elements in the domain" are logical notions (1986, p. 149). Further, these notions are "the end point of a chain of progressively more abstract, "formal," or topic-neutral notions defined by their invariance under progressively wider groups of transformations of a domain" (MacFarlane, 2005, p. 10). At this point, for MacFarlane, "one might demand that logical constants be insensitive not just to permutations of the domain of objects, but to permutations of the domain of possible worlds and the domain of times" (2005, p. 10).

Equally, Feferman's account of "similarity invariance" defines "the truth-functional operators and first-order existential and universal quantifiers" as logical constants, except "identity, the first-order cardinality quantifiers, or the second-order quantifiers". Indeed, his criterion shows the borders between "logic and mathematics much closer to the traditional boundary than the permutation invariance criterion does". However, "it is not clear that any compelling reason has yet been given for taking any *one* of them to mark the line between the logical and the non-logical" (MacFarlane, 2005, p. 11). Different from Feferman, McCarthy makes another definition: "the logical status of an expression is not settled by the functions it introduces, independently of how these functions are *specified*" (McCarthy, 1981, p. 516). As an illustration, the meaning of "≈" is given as such:

> "$x \approx y$" is true on an assignment a just in case the object that a assigns to x and the object that a assigns to y have exactly the same mass.

So far it is discussed that ""≈" is a logical constant just in case its extension on every domain is invariant under every permutation of that domain". Accordingly, "if there is *no* domain containing two objects with exactly the same mass, "≈" counts as a logical constant, and "□x ($x \approx x$)" as a logical truth". Yet it appears that "the logical status of "≈"

and "□x ($x \approx x$)" should depend on a matter of contingent fact: whether there are distinct objects with identical mass". MacFarlane summarizes the problem as,

> A natural response to this kind of objection would be to require that the extension of a logical constant on every *possible* domain of objects be invariant under every permutation of that domain, or, more generally, that a logical constant satisfy the permutation invariance criterion as a matter of *necessity*. But this would not get to the root of the problem. For consider the unary connective "#", defined by the clause ⌜#φ⌝ is true on an assignment *a* just in case φ is not true on *a* and water is H_2O (2005, p. 11).

MacFarlane claims that "even if it is metaphysically necessary that water is H_2O, there are presumably epistemically possible worlds, or information states, in which water is not H_2O". Additionally he states that, if a logical constant is defined as "a matter of epistemic necessity (or *a priori*), "#" does not count as a logical constant" (2005, p. 11). In brief, the notion of a logical constant can be explicated "in terms of an obscure primitive notion of logical necessity", which could not be explicated "by reference to logical constants" (2005, p. 12).

Sher's definition of truth for L_c:

> Let L_c be the formalized language of the calculus of classes whose primitive symbols are the individual variables 'x_1', 'x_2, … , the non-logical constant '$\subseteq$', the logical constant '~', 'V', '∀', and the auxiliary symbols '(' and ')'. (Sher, 2001, p. 196)

The pattern of Sher's logical structure is "generated by "highlighting" the logical constants of a given well-formed expression and "dimming" its non-logical constants, and to understand the connection between logical structure and truth" these points must be clarified: (i) "who" the logical terms are, and (ii) how they behave semantically" (2001, p. 200). According to Sher, there are three main tasks of an informative theory of the influence of logical content of truth:

> (A) Formulate an informative criterion of logical constants (based on their content);
> (B)Give a systematic characterization of the satisfaction conditions of logical constants based on (A) and show how their influence on truth extends to complex logical structures;
> (C) Explain how the special connection between logical structure and truth gives rise to logical inference" (2001, p. 203)

Under her proposal, Sher says that, "a logical term is *identified* with the (class)-function that assigns to every (set)-universe the denotation of the term in the universe: logical terms are identified with their (actual) extensions; so that in the metatheory the definitions of logical terms are rigid. (...) Their (actual) extensions determine one and the same formal function over models, and this function is a legitimate logical operator". Therefore, "the meaning of a term used as a logical constant" can only be understood by reading it rigidly and formally, i.e.; "to identify it with the mathematical function that semantically defines it" (Gómez-Torrente, 2002, p. 18).

In Sher's systematization, similar to Tarski's, "the bounds of the standard logical constants are specified by *enumeration,* i.e., dogmatically, without grounding or explanation" (2002, p. 555). She introduces a criterion of logical constanthood, which is based on "the role logical constants play in producing logical consequences" (Sher, 2002, p. 555). Subsequently, a greater system with another logical constants is needed to seizure it completely.

Selection of logical constants is an "ultimately pragmatic matter", for Hanson (1997, p. 365). He indicates that “terms usually called ‘logical’ (that is, the usual connectives and quantifiers plus identity), are those that exist in discourse on almost all subject matter. The which terms which are being used as logical constants “has the effect of promoting further generality in logic” (Hanson, 1997, p. 375). According to him, “choosing ubiquitous terms as logical constants ensures that nontrivial argument forms will be ubiquitous” and makes the argument to be “widely applicable”.

Although considering ubiquitous terms as logical constants, Hanson is not willing to allege that “ubiquity is a necessary condition for being a logical constant” (1997, p. 376). Hanson’s suggestion for appointing the logical constants is to assure some terms designated as such and “also to include among them at least some ubiquitous terms”, still “if no terms are designated as logical constants, logic will lose the generality that comes with the ability to isolate and study argument forms” (1997, p. 376). His designation also avoids the idea that “there is an inherent property of logicality that some terms have and others lack”. Hanson maintains two restriction points for the selection of logical constants (1997, p. 378):

i. It must designate some terms as logical constants, and it will include among them some that appear in discourse on a wide variety of subjects.

ii. Taken together, the terms it chooses must allow us to distinguish arguments exhibiting the resulting relation of logical consequence from those that do not exhibit it in a strictly a priori manner, to the extent that we can make.

Therefore he proposes the pragmatic way for selecting the logical constants. Still, “logical constants can be chosen in different ways, and… there is nothing written in stone about which terms are logical" (Hanson, 1997, p. 381).

On the contrary, Hanson criticizes Sher's idea on the possibility of specifying "isomorphism conditions that, if satisfied by the semantic definition of a term, make that term logical" (Hanson, 1997, p. 390). He proposes that, by using "simple first-order quantifiers" this access can be exemplified. "These quantifiers can be thought of as functions from models to subsets of the power set of a model's domain"(Hanson, 1997, p. 390). Then the existential quantifier adopts the function "that takes each model into the set of all nonempty subsets of the domain of that model" (Hanson, 1997, p. 390). Sher's work makes it possible to "classify some of the quantifiers, predicates, and functors that are logical by her criterion as logical constants", most of them which are ubiquitous. Thus, "taking them to be logical constants will promote generality" which will be convenient "as long as the terms we classify as logical do not rob logical consequence of its rightful share of apriority. But to expand the scope of logic in this way is still to make a practical, and somewhat arbitrary, decision" (Hanson, 1997, p. 394).

Sher claims that ""logical terms are identified with their (actual) extensions," so that "#", "%", and "¬" are just different notations for the same term". That is to say, "if these expressions are used the way a logical constant must be used—as rigid designators of their semantic values—then they can be identified with the operation of Boolean negation and hence with each other" (MacFarlane, 2005, p. 12). "Sher's proposal can only be understood as a *stipulation* that if one of a pair of coreferential rigid designators counts as a logical constant, the other does too" (2005, p. 12). Gómez-Torrente criticizes Sher's account for not being apparent and for pertaining some kind of “counterintuitive consequences” like “that "$P \vee \#P$" is a logical truth, at least when "#" is used rigidly” (2002, p. 19).

3.6 Inferential characterization

MacFarlane states that, one another group of the philosophers who adopt topic-neutrality in their principles, decided that “the logical constants are just those expressions that can be characterized by a set of purely inferential introduction and elimination rules” (2005, p.13). For example, one will be able to understand the meaning of the conjunction connective ‘&’, by learning its rules

$$\frac{A, B}{A \,\&\, B} \qquad \frac{A \,\&\, B}{A} \qquad \frac{A \,\&\, B}{B}$$

So, understanding the significance of “the horizontal line in an inference rule”, clarifies the meaning of ‘&’. On the whole, various “inferential characterization approach” formulate various conclusions about these cases and “these differences affect which constants get certified as ‘logical’ ” (MacFarlane, 2005, p. 13).

Gómez-Torrente declares that the nature of logical constants consists of *a priori* knowledge. In addition to this, he asserts that “someone who makes use of this ability a sufficient number of times, applying it to more and more complex subparts of a logical truth, will eventually obtain *a priori* knowledge of that logical truth” (2002, p. 23).

In fact, “there is no explanation in the theory of why acquaintance with the meaning of the logical constants puts one in the position of attaining certain kinds of a *priori* knowledge, and in particular *a priori* knowledge of logical truths" (Gómez-Torrente, 2002, p. 24). He takes another characterization of logical constancy into account, which "offer[s] a certain kind of explanation of the acquaintance with the meaning of the logical constants and of how this acquaintance produces *a priori* knowledge". But these cannot accomplish "the minimal requirement of extensional adequacy".

For Gómez-Torrente, Hacking's characterization of logical constants is one of the best. Hacking says that "a logical constant is a constant that can be introduced, characterized, or defined in a certain way". Gómez-Torrente, justifies this way parallel to Kneale's system, such that: "a logical constant is a constant that can be introduced by operational rules like those of Gentzen". And, these operational rules introducing a constant are defined in two ways:

(i) have the subformula property
(ii) be conservative with respect to the basic facts of deducibility [rules for the reflexivity, transitivity and monotonicity of the relation of deducibility] (Gómez-Torrente, 2002, p. 26)

On the whole Gomez-Torrente summarizes Hacking's proposal:

> one is in a certain sense able to read off the semantics of the logical constants from the operational rules. Given the underlying notions of truth and logical consequence, the syntactic rules determine a semantics. (...) I claim (...) that the operational rules "fix the meanings of the logical connectives" in the sense of giving a semantics ([12, p. 3001). The semantics that Hacking is talking about is an extensional semantics which gives instructions for assigning truth values to sentences dominated by logical constants, instructions determined by a procedure applied to the introduction and elimination rules for those constants (see [12, pp. 312ff.l). A technical evaluation of the merits of Hacking's procedure is out of place in this paper. It suffices to say that he claims that the procedure

> determines for the logical constants the extensional semantics that we antecedently attribute to them. In this sense, according to Hacking, the Gentzenian operational rules for the logical constants, when viewed as sense-constituting, "characterize" the logical constants or "fix their meaning" (2002, p. 27).

Gómez-Torrente defines Hackings theory of logical constancy as being “compatible with a classical formal view of logical consequence as truth preservation, and thus of logical truth as truth in all interpretations of the non-logical constants” (2002, p. 28). The problem of Hacking’s theory is its inadequacy due to “the reasons why 'unicorn' poses problems for Tarski's theory and Sher's theory, 'heptahedron' poses problems for McCarthy's theory and 'male widow' poses problems for McGee's theory” (Gómez-Torrente, 2002, p. 29).

3.8 Pragmatic demarcations

All of the previously discussed proposals are analytical demarcations. They are searching some specific property, “as a necessary and sufficient condition for an expression to be a logical constant” (MacFarlane, 2005, p. 16). Warmbrod’s type of demarcation is a kind of pragmatic demarcation, which describes ‘a job of logic’ as a “framework for the deductive sytematization of scientific theories” (1999, p. 516). This kind of demarcation depends on “requirements of minimalism”. According to minimalism, “the set of terms recognized as logical constants should be as small as possible”. Parallel to this view, Warmbrod defines his logical theory “as simple, as modest in its assumptions, and as flexible as possible given the goal of providing a conceptual apparatus adequate for the project of systematization” (MacFarlane, 2005, p. 16). Concerning his theory, Warmbrod argues that “the theory of identity” and “modal operators” are not part of logic. This is because his logic can exist without these notions.

As a result, on ‘a pragmatic demarcation’, logic “may depend on the current state of scientific and mathematical theory” (MacFarlane, 2005, p. 17). Moreover, “If the advance of science results in an increase or decrease in the resources needed for deductive systematization of science (or whatever is the favored task of logic), what counts as logic changes accordingly” (Warmbrod, 1999, p. 533). On his account, Warmbrod claims that “the terms whose meaning assignments are held fixed while the assignments to other terms vary through some admissible range of assignments” (1999, p. 505).

In Sher’s criterion, “‘Most x’, ‘Exactly 5 things x are such that ...’, ‘An even number things x are such that ...’, ‘Uncountably many things x are such that ...’ and many other generalized quantifiers will be accepted to be logical constants” (Warmbrod, 1999, p. 507).

Hacking defines logical constants with respect to Gentzen-style rules of inference, which “introduce a term determine the truth conditions of sentences containing the term” or “determine the meaning of the term” (Warmbrod, 1999, p. 509). On the other hand, this method contributes, “a reason for saying that the meanings of such terms should be held constant in a semantic theory”. That is to say, a term is a logical constant, “if a term's meaning is determined by inference rules, and the rules do not change across interpretations, then the meaning should not change either” (Warmbrod, 1999, p. 509). The problem with Hacking is that, “it is unclear why it is a mistake to recognize logical constants that do not satisfy his criteria” (Warmbrod, 1999, p. 510). One of the Warmbrod’s criterion for characterizing logical constants is flexibility, which “betrays weakness in the arguments that were meant to establish the criterion” (1999, p. 511).

Gentzen's proof-theoretic work is important because, "he had the idea that his operational rules actually define the logical constants that they introduce" (Hacking, 1979, p. 296). Hacking emphasizes the "conservativity in definitions", especially when logical constants are defined. He takes the proof of cut-elimination as a factor to prove that "the operational rules are conservative definitions". For Hacking, Gentzen's rules are the characteristic definitions and asserts that "cut-elimination, dilution-elimination, and identicals elimination (all for complex formulas) are necessary conditions for this" (1979, p. 298).

According to Hacking, "[l]ogic is concerned with the preservation of truth". Therefore, "the peculiarity of the logical constants resides precisely in this: that, given a certain pure notion of truth and consequence, all the desirable semantic properties of the constants are determined, by their syntactic properties" (1979, p. 299). "If operational rules could be regarded as definitions, then they would have to be conservative" and "these rules could not define the constants for a being that lacked all logical concepts" (Hacking 1979, p. 299).

> One must understand something like conjunction to apply the conjunction rule, and one must have some surrogate for some sort of quantifier to apply the rule for universal quantification. This kind of consideration particularly influences workers trying to find predicative or other constructive foundations for branches of mathematics. (Hacking, 1979, p. 299)

However, "the operational rules at most *characterize* the logical constants in a certain way for a person that already has some logical ideas". For Hacking the operational rules are unsuccessful in characterizing the logical constants (1979, p. 299). To sum up, rules which are defining the logical constants consist of two parts, "syntactic part” and “inferential part”. The syntactic part adjusts “the rules of well-formedness for the constant” and the inferential part adjusts “the rules of derivability of statements in which the constant occurs” (Hacking, 1979, p. 303).

On the otherhand, Hacking in his definition of logical constants supports the usability of Gntzen’s operational rules. Hacking (1979, p. 303) says that: “a logical constant is a constant that can be introduced by operational rules like those of Gentzen”, so, these operational rules acquire “the subformula property” and are “conservative with respect to the basic facts of deducibility”. That is to say, they resemble the “provability of the elimination theorems”. (1979, pp. 303-304). Hacking maintains that the rules of logic must be conservative, because of the following reasons (1979, p. 304):

> (A) The demarcation should give the "right" logicist class of logical constants and theorems. That is, it should include the traditional (and consistent) core of what logicists said was logic and should exclude what they denied to be logic.
>
> (B) Since the demarcation is couched in terms of how logical constants are characterized, it should provide the semantics for the constants called "logical."
>
> (C) It should explicate why logic is important to the analytic program.

In *Tractatus*, it is easily seen how the truth conditions for complex sentences may be explained in terms of simpler sentences. These words are important “to display something about the logical connectives–"they are words of this peculiar sort, that one can imagine them being planted in the language in this way" ” (Hacking, 1979, p. 314). In summary, Hackings demarcation “enables one to characterize the logical constants without being forced to say what is on the other side of the dichotomy". Therefore, there is no need for a separate definition for the "descriptive constants". So, a logical constant is "a constant that can be added to any language of a certain sort" (Hacking, 1979, p. 314).

As was mentioned above, Hacking defines the classical logical constants as logical constants that "can be introduced by operational rules which have the sub-formula property and which are conservative with respect to a given background deducibility relation" (Peacocke, 1981, p. 169). Also the conditions which are satisfactory for a deducibility relation, Hacking states, are *reflexivity*, *dilution*[1] and *transitivity*.

Peacocke raises an objection to Hacking's criterion taking into account the following example:

> (S) Rockefeller is wealthy, and if every element of ζ is true, some element of ζ is true.

Peacocke decides an imaginary operator $ and applies on it similar rules to conjunction. If one adopts Hacking's theory of 'deducibility relations', then $ is accepted as a classical logical constant. Peacocke claims that, it is possible to infer,

> given the rules governing $, that ***'A*** $ *B'* is true if Rockefeller is wealthy and ***A*** and B are both true, and is false if Rockefeller is wealthy and either ***A*** or *B* is false. The rules determine nothing about the true value of ***'A*** $ *B'* in the case in which Rockefeller is not

[1] Hacking (p. 293) explains the term *dilution* as such: "If $\Gamma \vdash \Theta$, then $\Gamma, A \vdash \Theta$ and $\Gamma \vdash A, \Theta$. Adding a possibly irrelevant premise or consequent does not affect the deducibility relation… this rule is here asserted only for the classical case. We may call this the *stability of classical deducibility* under the addition of arbitrary sentences. Note that it provides an essential contrast between deductive and inductive reasoning; for the introduction of a new premise may spoil an inductive inference."

wealthy. So the meaning of ***'A*** $ *B'* is specified by a partial truth function of the truth values of ***A,*** *B,* and 'Rockefeller is wealthy' (1981, p.169).

Hence, he defends that “$ may be said to be a logical constant”, but “relative to the supposition that Rockefeller is wealthy”. Peacocke proposes four solutions to the problem appeared with Hacking’s system.

i. The generality proposed by Hacking that “a logical constant is one that can be added to any language of a certain sort”, cannot solve the problem. This is because, “for any class of pairs of sets of sentences satisfying reflexivity, dilution, and transitivity, the standard left- and right- introduction rules for conjunction formulated for $ will not fail to preserve truth in the required way”.
ii. It is impossible to attest “the relation specified by (S) is a deducibility relation without using nonlogical resources”.
iii. One can say: “If Rockefeller is wealthy, then ***'A*** $ ***B'*** is true iff ***'A*** and ***B’*** is true. So if Rockefeller is wealthy, $ is a logical constant. Since Rockefeller is wealthy, $ is a logical constant, and there is no objection to Hacking's theory”. However, an expressions being a logical constant or not depends only on the meaning of that expression. “Any conception of logical-constant-hood on which that status is not a matter of meaning fails to supply directly an account of what is distinctive of the meaning of the logical constants.”
iv. “There is a different way of understanding the conjunction-like rule for $ which would make ***'A*** $ ***B’*** equivalent to ***' A*** & ***B’*** even if Rockefeller were not wealthy[2]”. (Peacocke, 1981, pp. 169-170).

In such a case where truths or *a priori* knowledge are not needed, “an account which requires logical truths to be necessary or a priori will have the result that there are no logical truths”. Similarly there will be no sentences which can “logically imply other sentences if logical implication is required to be necessary or a priori”. As a result, nothing literal would leave behind for logicians to talk about (Warmbrod, 1999, p. 513).

[2] (Peacocke introduces & as another operator for conjunction, similar to $)

Gómez-Torrente criticizes Warmbrod's criterion as follows:

> there is nothing inherent in the idea of a set of necessary and sufficient conditions for constancy which guarantees an answer to the critical question, namely, why should terms satisfying the criterion, and only those terms, have their meanings held constant while the meanings of other terms vary [in a Tarskian test for logical truth or logical consequence]. (...) Assuming that no obvious catastrophe results from assigning a fixed meaning to a new term, consideration surely must be given to the benefits achieved by treating the new term as a constant and whether, indeed, such treatment furthers the fundamental purposes of logical theory (2002, p. 33).

As mentioned above, Warmbrod suggests that something in order to be a 'logical expressions' needs to be incomplete also should "leave out some pretheoretic intuition about the idea of an expression that logic should deal with"(Gómez-Torrente, 2002, p. 33). Accordingly, Gómez-Torrente declare Warmbrod as "misidentified the source of the difficulties facing typical characterizations of logical constancy", also "the logical constants have certain properties, even if they don't have them *because* they are logical constants: roughly, properties that imply the analyticity and apriority of some logical truths and which thus make (versions of) (*) true[3]" (2002, p. 34).

The problem appears while trying to define other logical constants, especially ∀, ~, and →. Diez with his criterion tries to overcome the problem of impredicativity, which is "the construction which is being defined and which proves $p \rightarrow q$ must be able to transform any possible proof of p into a proof of q" (2000, p. 410). Such that, "the definition of a proof of $p \rightarrow q$ appeals to a totality of proofs, with some of which the very proof of $p \rightarrow q$ could be intimately related".

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[3] Where (*) refers to Bolzanian hypothesis "if a proposition *p* is formally true, i.e., if it is true by virtue of its form, i.e., if all the results of replacing/reinterpreting uniformly its non-logical constants are true, then *p* is analytic/ *a priori.*"(Gómez-Torrente, 2002, p. 7)

Another problematic point is that "this definition of → has the effect of converting the proof relation induced into a non-decidable relation", then "will be unable to decide whether… the original construction *c* is a proof of $p \rightarrow q$ or not" (Diez, 2000, p. 410). Diez's definition of → causes decidability problem for V and ∃ but not for ∀ (2000, p. 419). Also, he faces an "analogous" problem, which compels him "to re-define all the other logical constants in terms of a finite set of free variables". The problem appears immediately after attempting to do this, and he misses "the inductive character of the corresponding definitions of disjunction and the existential quantifier" (Diez, 2000, p. 419). That is to say,

> the intuitionistic negation is defined as a conditional sentence which has as consequent a basic self-evident absurdity (a construction which is obviously impossible to carry out), and as antecedent the sentence negated. Hence, all that has been said about the conditional applies to negation directly. Conjunction is a straight 'sum' of the proofs (or performing constructions) corresponding to each conjunct: its definition does not create any problem at all" (Diez, 2000, p. 420).

Consequently, Diez's criterion explains that there are "two groups of intuitionistic logical constants: {→, ~, ∀ } and {V, ∃} are *really* opposed to each other : →, ~, and ∀ pose a decidability problem, which V and ∃ normally do not; and the attempts to resolve it produces a loss of inductiveness *either* in the definitions of →, ~, and ∀, *or* in those of V and ∃" (Diez, 2000, p. 422). Yet, taken all the systems into consideration the solutions are insufficient to solve the problem.

CHAPTER 4

CONCLUSION

The basic aim of this thesis is to examine the concept of logical constants. The key focus of the thesis is to find out the reasons causing the problem of logical constants and to make internal analysis of the criteria formulated in order to reach a solution of the problem. In this work I mainly investigated the place of logical constants in logic, and their relation with logical truth and logical consequence, the definition of logical constants, and the proposed solutions to the problem of logical constants.

The problem emerged during the search of an explanatory theory of the analyticity and apriority of logic. More specifically the problem appeared with the characterization of logical truth that is one of the most important problems of philosophy of logic. Nevertheless, the function of logical constants is described under different titles. Most of the definitions defend the minimalist point of view, according to which logical constants are defined as small, simple and flexible expressions existing only within a simple systematization. Another group of logicians name the problem as a pseudoproblem, since they claim that logical constants are not the subject matter of logic, but only tools necessary for the classification of arguments.

Kuhn maintains that, in logic there are seven logical constants constituting the basis for reasoning, which are the logical connectives 'and', 'or', 'not', 'if', 'if and only if', 'all', and 'some'. While some logicians such as Lycan dedicate unique properties to the logical constants, some others like Wittgenstein define logical constants as nothing but a punctuation mark without any representative property in logic. Additionally in Gentzen's view, that problem can be solved by formulating separate rules for each constant.

Since the need for logical constants appeared with the search for logical truth, the definition of logical truth also carries certain clues for the definition of logical constants. Both for Quine and Tarski, logical truth needs logical constants, since logical truth is a kind of truth which occurs only with logical constants. Therefore a definition of logical constants was necessary for an acceptable theory of logical truth. Also logical constants are the vital determinants of the logical consequence relation. In Tarski's conception, logical consequence is a truth-preserving term which distinguishes between logical and non-logical constants. Thus, logical constants are steady in all transformations. Furthermore, logical consequence represents the knowledge of what follows logically from what and the a priority of this knowledge is one of the central terms of logic.

In order to overcome the problem of logical constants, various solution models were proposed. In order to investigate these solution models, I preferred to follow MacFarlane's classification. Due to MacFarlane's explanation, the criteria were grouped according to their interpretation of the logical constants. Different definitions of the logical constants caused different solutions. However, all of the maintained definitions appeared to be incomplete, or in other words defective. The problem of each solution group is as such:

In section 3.1, the "syncategorematic terms" were accepted as the logical constants. As defined in that section categorematical terms are words used as subject or predicate (e.g. cat, John, box,..) and syncategorematic terms are words whose functions are to show the relation between subject and predicate (e.g. every, or, necessary,…).

Yet, there is no clear distinction between categorematic and syncategorematic terms.

In section 3.2, Quine defined all singular terms and predicates as non-logical constants, and all operators and connectives as paradigm logical constants. Nevertheless, the conflict between linguistic theory and traditional logic causes difficulty in the distinction of logical and non-logical constants.

In section 3.3, Grünberg's last demarcation criterion, though conjectures that it is immune of counterexamples, it does not give such a guarantee.

In section 3.4, Davidson's system reflects the similar problem as the one in Quine. That is to say, there is no definite criterion for logical constancy.

In section 3.5, the problem is named as *antinomy of topic neutrality*. This problem is caused by the vagueness of the problem of logical constants, since there are no clear-cut cases in the criterion. Also there are no an exact distinction line between the topic-neutral and other terms. This is the problem emerging from the Peacocke's system.

In section 3.6, the problem in the "permutation invariance" case can be visualized in Sher's, Tarski's and McGee's systems. This type of criterion attends only to the level of senses, but cannot attend to the level of meaning.

In section 3.7, different inferential characterization approaches make different decisions about the matters and these differences affect the choice selection of constants described as 'logical'.

In the last section 3.8, logician's like Warmbrod and Gödel emphasize that what counts as logic necessitates a certain state of scientific or mathematical theory. In other words this point of view is problematic, because logic itself is inadequate for this type of demarcation.

At the end of the considerations, I am in accordance with the thoughts of Gómez-Torrente, expounded in details in chapter 2, that there is not such a system which solves the problem. However, definitions of logical constants and the criteria constructed to cope with the problem of logical constants are promising. As the problematic regions profoundly investigated, the expected solutions will become more satisfactory and effective.

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