PERIODIC-REVIEW INVENTORY SYSTEMS WITH EXOGENOUS AND ENDOGENOUS REPLENISHMENT LEAD TIMES

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ABSTRACT

PERIODIC-REVIEW INVENTORY SYSTEMS WITH EXOGENOUS AND ENDOGENOUS REPLENISHMENT LEAD TIMES

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In this thesis, two-echelon systems with exogenous and endogenous lead times are studied for the orders placed by the retailer(s) from the supplier. The retailer(s) employ periodic-review base-stock policy, namely (R, S) policy. For the case the demand during review period is i.i.d. and the probability distribution is Normal for each review period, a new method is proposed for exogenous lead time case under stationary policy. The results of the proposed method is then compared with the results of the existing methods in the literature and it is concluded that the proposed method provides service levels sufficiently close to target levels whereas the existing methods do not necessarily provide target levels. We use the simulation to study the endogenous replenishment lead time case. The proposed method is modified when the retailer employs stationary policy and it is seen that the proposed method gives no-stockout probabilities close to target levels.

Moreover, the impacts of using adaptive policy on the performance of the retailer are studied for endogenous replenishment lead time case. It is concluded that

updating of the order-up-to-level deteriorates the performance of the retailer. Finally, it is questioned whether it is beneficial for a retailer to use adaptive policy in a supply chain with two retailers. Simulation results show that the deterioration in the performance of the retailer handling stationary policy is larger compared to the other retailer handling adaptive policy and the deteriorations get larger in the case of an increase in update frequency or in utilization of the supplier.

Keywords: Periodic-review, base-stock policy, exogenous and endogenous lead times, adaptive, lead time syndrome.

ÖZ

İÇ VE DIŞ KAYNAKLI TEDARİK SÜRELERİ İÇİN ARALIKLI GÖZLEMLENEN ENVANTER SİSTEMLERİ

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Bu tezde, iki kademeli tedarik zinciri sistemleri iç ve dış kaynaklı tedarik süreleri için analiz edilmektedir. Tedarikçiler aralıklı gözlemlenen baz-stok politikası kullanmaktadır. Stok gözden geçirmeleri arasındaki talep, bağımsız ve özdeş kabul edilmekte ve olasılık dağılımının normal olduğu varsayılmaktadır. Hedeflenen stoksuz kalmama olasılığı için sipariş seviyesini belirlemekte kullanılan metotlar verilmekte ve durağan politikada dış kaynaklı tedarik süreleri içeren sistemler için yeni bir metot önerilmektedir. önerilen metodun sonuçları, literatürde var olan metotların sonuçlarıyla karşılaştırılmakta ve önerilen metotla elde edilen stoksuz kalmama olasılıklarının hedef seviyelere yeterince yakın olduğu, fakat var olan metotların hedef seviyelere yeterince yakın sonuçlar vermediği fark edilmektedir.

İç kaynaklı durumu incelemek için simülasyon kullanılmaktadır. Tedarikçi durağan sipariş politikası kullandığı durumlarda üst sipariş seviyesinin belirlenmesinde kullanılmak üzere önerilen metot yeniden düzenlenmektedir. Önerilen metodun hedef seviyelere yeterince yakın stoksuz kalmama olasılıkları verdiği görülmektedir.

Ayrıca bu çalışmada, durağan talep ve iç kaynaklı tedarik süreleri için değişken parametreli sipariş politikası kullanılmasının tedarikçinin performansı üzerindeki etkileri üzerine çalışılmaktadır. Çift tedarikçili bir tedarik zincirinde değişken parametreli sipariş politikası kullanmanın, tedarikçinin yararına olup olmadığı sorgulanmaktadır. Sonuçta durağan politika kullanan tedarikçinin performansındaki gerileme değişken sipariş politikası kullanan tedarikçinin performansındaki gerilemeden daha fazladır. Sipariş politikasının parametrelerinin güncellenme sıklığı arttıkça veya üretim sisteminin kullanım yüzdesi arttıkça performanstaki düşüş artmaktadır.

Anahtar Kelimeler: Aralıklı envanter, baz-stok politikası, dış ve iç kaynaklı tedarik süresi, değişken sipariş politikası, tedarik süresi sendromu.

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Chapter 1

INTRODUCTION

Simchi-Levi *et al.* (2000) define Supply Chain Management as follows:

> ”Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize system-wide costs while satisfying service level requirements.”

Supply chain management deals with the coordination of the activities contributing directly or indirectly to the production and distribution of the finished products that the customers are willing to get. The challenge is that the members in a supply chain usually have conflicting objectives and an effective balance between these objectives is hard to achieve. In a supply chain, a member observes the demands of the members at the lower echelons and generates demands for the members at the higher echelons by placing orders. The retailers, who observe demands of the end customers, are at the lowest echelon in a typical supply chain. By placing orders, these retailers generate demands for the suppliers at a higher echelon. Then, an inventory control problem arises to determine ordering policies of the retailers depending on the supply mechanism of the suppliers.

In this thesis, two-echelon inventory systems are analyzed for the cases of endogenous and exogenous replenishment lead times. The motivation is the relation between the ordering policies of the retailers and the supply mechanism or production system of the supplier that should definitely be taken into consideration while determining inventory control policies to be employed by the retailers. If the amount and frequency of the orders do not change the replenishment lead times (time lag between the points in time an order is placed by the retailers and delivered by the supplier) observed by the retailer, the replenishment lead times are said to be exogenous. In other words, there is no impact of the retailers' orders on the production plan of the supplier. Exogenous replenishment lead times can be explained by the operating system (e.g., keeping high stock levels) and/or high capacity of the supplier. If the production capacity of the supplier is much higher as compared to the amount demanded by the retailers or if the supplier operates keeping high inventory levels for finished goods to offset fluctuating demands of the retailers, then changes in the ordering policies of the retailers would not have an impact on the replenishment lead times.

However, it is expected that there is an impact of the ordering policies of the retailers on the supply mechanism of the supplier when the orders should firstly be produced by the supplier. In this case, the replenishment lead times are endogenous to the retailer. That is, the replenishment lead times would be endogenous if ordering decisions have an impact on the production planning decisions of the supplier. As an extreme example of the systems with endogenous replenishment leads times, capacitated suppliers employing make-to-order production policy can be considered. In this case, an arriving order has to wait in the queue of the orders to be processed until processing all orders already in the system is completed. Thus, replenishment lead time for a large (small) order quantity would be longer (shorter). If the retailers start working with large (small) order quantities as compared to the previous order quantities, then they observe an increase (decrease) in the replenishment lead times. The impact in the replenishment lead time would be similar for increases (decreases) in the frequencies of the orders placed by the retailers. That is, the order frequency and quantity would have a joint impact on the corresponding replenishment lead times. In the case there exists congestion in the production system of the supplier resulted from high demand periods, the retailers would work with larger order quantities. But, as observed by Selçuk et al. (2006), there is no use of increasing order quantities in the case of congestion if the supply process is stationary. It is resulted from the fact that more erratic replenishment lead times are observed which eventually causes the performance of the retailers to worsen. The solution to this case is simple: decrease next order quantity and wait until the congestion in the supplier queue ends. This solution would work because there are only one retailer's orders in the queue of the supplier. The solution approach is well-known in the literature and called as 'order smoothing' (Dejonckheere et al., 2003; Balakrishnan et al., 2004; Disney et al., 2006).

In order to study the systems with exogenous replenishment lead times, replenishment lead times are assumed either constant or distributed according to a stationary probability distribution in the models studied in this thesis. For the cases with endogenous replenishment lead times, a capacitated make-to-order system is considered as in the studies due to Boute (2006) and Selçuk (2007). The supplier is modeled as a single-server queuing model with first-come-first-served service discipline. The items are produced one by one by the single server. Once all of the items in an order placed by a retailer are processed, the items are shipped to the retailer. In this setting, the replenishment lead times observed by the retailer would be longer (shorter) if the current workload of the supplier is at a high (low) level. Then, the retailers should decide whether to update planned lead times according to their observations for the changes in the replenishment lead times. Especially for the case of more than one retailer working with the same retailer, this would be an important decision for each retailer to have an advantage over its competitor(s).

Since the demand of the end customers is the driving force of a supply chain, the primary objective of the retailers is to provide good service to these customers. However, there is a tradeoff between the amount of inventory kept and the service level provided. Higher inventory levels result in higher service level but also higher inventory holding costs. Therefore, inventory planning and control decisions should lead to the best balance between holding costs and the costs of unsatisfied demand. In this thesis, service models are considered to determine the minimizing inventory holding costs subject to a constraint to satisfy a target service level. Different performance measures can be used for the constraints on the customer service levels. The following are widely used measures: fraction of cycles without stockout (α-service measure), fraction of demand satisfied immediately upon customer arrival (fill rate, β-service measure), fraction of time the demand is satisfied immediately upon customer arrival (ready rate), 1 minus the average backorder divided by the mean demand rate per time unit (modified fill rate, γ-service measure). In this study, we work with the models subject to a constraint on α-service level. Although we do not work with fill rate as the service measure in our models, in the numerical experiments, the fill rates are also obtained and given for comparison purposes.

For both exogenous and endogenous lead times, we consider the case the retailers place orders according to periodic-review base-stock policy denoted as (R, S) policy, where R is the review period and S is the order-up-to-level. The base-stock policy is an optimal inventory policy in systems where there is no fixed ordering cost, stockouts are backordered and both holding and shortage costs are linear as shown in the study due to Clark and Scarf (1960) and the book due to Zipkin (2000). In the studies due to Silver *et al.* (1998) and Eppen and Martin (1988), S is calculated for a given R value by working with the distribution of the demand during the period, which we call as the risk period. Risk period covers the review period and the following replenishment lead time just after placing an order upon a review. In both of the references due to Silver *et al.* (1998) and Eppen and Martin (1988), the objective is to find S that is minimizing inventory holding costs while satisfying a target α-service level. In the book due to Silver *et al.* (1998), it is assumed that the distribution of the demand during risk period is Normal. Eppen and Martin (1988) show that the distribution of the demand during risk period is not Normal even if the distribution of the demand during review period is Normal. Then, Eppen and Martin (1988) propose a method to calculate order-up-to-level based on the assumption that the distribution of risk period demand is Normal for each possible value of replenishment lead time. This assumption is valid if the distribution of the demand during review period is Normal. However, it is numerically shown in this study that the order-up-to-levels calculated by the method given by Eppen and Martin (1988) do not necessarily provide target service levels even if the distribution of the demand during review period is Normal.

For the case of exogenous replenishment lead time, an alternative approach is proposed in this thesis to solve service models with α-service measure under stationary (R, S) policy based on the assumption that the distribution of inventory-on-order is Normal for each possible number of outstanding orders. This assumption is also valid if the distribution of the demand during review period is Normal. We consider the case all of the parameters are time-stationary parameters and work with the long-run average inventory costs implied by constant S and long-run α-service measure. We assume that R is a given time-stationary (constant) parameter. Probability distribution function of the inventory-on-order is determined to be used in the calculation of S by the proposed method. In a similar way, the distribution of inventory-on-order is used to calculate S for endogenous replenishment lead times in the study due to Boute (2006). The proposed approach in this thesis extends his findings for the exogenous case by using the method due to Eppen and Martin (1988). This approach is compared with the existing approaches in the literature for different scenarios by simulation analysis. In these scenarios, the distribution of the demands during review periods is assumed to be i.i.d. and Normal. Further, we assume that the distribution of the replenishment lead times is known by the retailers. For the use of stationary ordering policies by the retailers with time-stationary (constant) R and S values, the comparisons based on simulation analysis shows that the order-up-to-levels calculated by the proposed method give α-service measures (no-stockout probabilities) which are very close to the target levels when long-run measures are considered. However, the existing methods in the literature give no-stockout probabilities larger than the target levels in most of the cases, especially for high target service levels. This results in higher inventory holding costs as compared to the inventory holding costs incurred by the order-up-to-levels we propose.

In all of studies mentioned above and most of the other existing studies on periodic-review systems, replenishment lead time is assumed to be exogenous, and even constant. However, most of the time this is not the case in real life due to randomness of many factors or simply due to changes in the environment. Endogenous replenishment lead time would naturally be variable especially when the order quantities change over time. In the study due to Boute (2006), it is noted that working with lead times as if they are always exogenous is not enough since the correlation between the order quantity and its lead time cannot be simply ignored. In the case of endogenous replenishment lead time, Boute (2006) studies a two-echelon system when the retailer at the lower echelon uses periodic-review base-stock ordering policy and the supplier at the higher echelon has a capacitated make-to-order production system. In the numerical studies given by Boute (2006) for the case of endogenous replenishment lead time, it is shown that working as if the replenishment lead times are exogenous seriously underestimates the required safety stock, which causes customer service to degrade dramatically. In his study, an efficient procedure is developed based on Markov chain formulations and matrix analytic techniques to compute the distribution of the replenishment lead time explicitly by taking the correlation between demand and lead times into account. Then, he calculates the order-up-to-level providing a given target level on the fill rate (β-service level) by working with the distribution of inventory-on-order. In this thesis, we also use a similar setting to calculate order-up-to-levels for a given target level on the stockout probability in a review period (α-service measure). As oppose to the method given by Boute (2006), we use simulation to obtain order-up-to-levels because of the difficulties for an exact theoretical analysis. As long as there is no change in the supply process, and in the environment, it would make sense to use a stationary (R, S) policy for endogenous replenishment lead time. The proposed method given for the exogenous replenishment lead time case is modified to determine order-up-to-level when the retailer employs stationary (R, S) policy. The results of the proposed method is, then, compared with the results of the existing methods in the literature and it is concluded that α-service measures (no-stockout probabilities) obtained by the proposed method are sufficiently close to target levels whereas the existing methods do not necessarily provide target levels.

On the other hand, demand and replenishment lead times are not always known before they are observed by the retailers and have to be estimated somehow. Then, the parameters of ordering policy employed by the retailer should be updated based on the most recent information about customer demand and replenishment lead time. In the literature, the impact of updating demand forecasts on the ordering policy is studied extensively under the heading of bullwhip effect. Among many others, the related studies for periodic-review systems are due to Chen *et al.* (2000a, 2000b), Lee *et al.* (1997), Zhang (2004). In these studies, supply chains are evaluated for different demand patterns and forecasting mechanisms. The main findings in these studies are that the adjustments in parameters of the ordering policy based on the demand forecasting cause erratic responses and eventually increase the supply chain costs. In this study, we do not aim to study the impacts of demand forecasting and focus on updating estimates for planned lead times. For this reason, the demands during review periods are assumed to be time-stationary and independently and identically distributed in the models under consideration.

Compared to the studies discussing the impact of updating demand forecasts, the impact of updating the estimates for replenishment lead time on the performance of the supply chains is studied less in the literature. There are a few papers discussing the issue under the heading of 'lead time syndrome'. Mather and Plossl (1978) are the first who describe lead time syndrome as a vicious cycle between lead time update and order release decisions. It is argued that closing the gap between the planned lead time and the actual order flow times by updating the lead time results in uncontrolled order release pattern. As the lead time gets longer, orders must be released earlier to cover increased expected demand during the longer lead time, leading to longer queues of production backlog. Thus, flow times get longer, which causes again a longer lead time. This results from the fact that, in releasing the orders, the impact of the ordering decisions on future lead times and on future orders is ignored. It is suggested that the lead time syndrome causes instability, and should be avoided. Selçuk (2007) considers a simple setting for updating planned lead times and report their analytical observations for lead time syndrome. Selçuk (2007) argues that trying to close the gap between planned lead times and actual lead times by updating the planned lead times frequently results in uncontrolled order release patterns. That is, the variability in the order quantity and replenishment lead times increase in such an environment. To the best of our knowledge, the study due to Boute (2006) is the only one that discusses lead time syndrome issue in the supply chain context. In his study, it is stated that order release mechanism is similar to the inventory control systems such that release quantities can be considered as the order quantities placed by the retailer to the supplier and flow times can be seen as the replenishment lead times corresponding to these order quantities. But, this relation is not clearly stated in his study since he does not study the impacts of using an adaptive ordering policy based on the update of planned lead times. In this thesis, the impact of using adaptive (R, S_t) policy on the performance of the system is studied for time-stationary demand and endogenous replenishment lead times. To the best of our knowledge, lead time syndrome is not studied for such an inventory system before. Evaluating the system by the simulation analysis, it is concluded that updating the order-up-to-level based on the updated estimate of replenishment lead times has an adverse impact on the performance of the system.

Finally the analysis is extended to the models with two retailers for the case of endogenous replenishment lead times. In the case of multiple retailers, an order placed by one of the retailers must wait until the completion of the other outstanding orders to be processed. Since ordering policies of the retailers determine the replenishment lead time of the supplier, each retailer's replenishment is dependent on the ordering policies of the other retailer. In such an environment, a change in the ordering policy of a retailer has an impact on the performance of other retailer even if the other retailer does not consider any change in its own ordering policy. Then, it is questioned whether it is beneficial for a retailer to use an adaptive (R, S_t) policy in a supply chain with two retailers. To the best of our knowledge, there is not any other work in the literature on the analysis of updating ordering policy parameters based on the changes in replenishment lead times in a setting with multiple retailers. Simulation results show that both retailers perform worse even if only one of them uses an adaptive policy. The deterioration in the performance of the retailer employing stationary (R, S) policy is higher than the performance of the other retailer employing adaptive (R, S_t) policy. When both retailers use adaptive (R, S_t) policies, this leads to a situation both retailer perform worse compared to the case both retailers employ stationary policies and to the case one retailer employs adaptive policy and the other employs stationary policy. We also observe that the deterioration in performance measures gets larger in the case of an increase in the update frequency or in the utilization of the supplier.

The outline of the thesis is as follows. In Chapter 2, the related literature are reviewed and the relation of the literature to our work is stated. The models with single retailer and single supplier are studied for exogenous and endogenous lead times in Chapter 3. Chapter 4 extends the analysis to the case with two retailers. Finally, Chapter 5 concludes this thesis with a summary of the main findings and a discussion of future research directions.

Chapter 2

RELATED STUDIES

There exist extensive studies related to our work in the literature on supply chain management, specifically under the heading of inventory management in supply chains. However, the models considered in these studies differs in many points such as: number of echelons, number of members in a echelon, ordering policies employed, demand patterns and number of the customers, inventory review mechanisms, supply mechanisms, etc. In this thesis, we investigate two-echelon inventory systems with retailer(s) at the lower echelon and a supplier at the higher echelon. We restrict our analysis to the cases with single retailer and two retailers. In the models considered, we work with both exogenous and endogenous replenishment lead times. For both exogenous and endogenous lead times, we consider the case the retailers place orders according to periodic-review base-stock policy. Moreover, customer demand that the retailer observes is assumed to be time-stationary, independently and identically distributed for each review period and the distribution of the demand during the review period is assumed to be Normal.

In the studies existing in the literature, different performance measures are used and different problems are try to be resolved. In this thesis, we work with the models minimizing inventory holding costs while providing a target service level for no-stockout probability in a review period. And, we deal with the following problem domains: bullwhip effect and lead time syndrome. In this context, we investigate the effects of using adaptive ordering policy on the performance of the retailer(s) and obtain different measures to identify whether bullwhip effect and lead time syndrome are present in our models. In the following parts of this chapter, the related studies are reviewed and the position of our work among the related studies are explained based on the mentioned points above.

In the case of exogenous replenishment lead times, computations of the optimal order-up-to-level, S, are extensively discussed in the literature. In the studies due to Silver *et al.* (1998) and Eppen and Martin (1988), S is calculated to satisfy a given target level on the stockout probability in a review period by working with the demand during the risk period, which is the sum of a review period and replenishment lead time just after placing an order. In the afore mentioned studies, the demands during the review periods are assumed to be i.i.d. and time-stationary. Also, the replenishment lead times are assumed to be exogenous. In the book due to Silver *et al.* (1998), it is assumed that the distribution of the demand during risk period is Normal. Eppen and Martin (1988) show that the distribution of the demand during risk period is not Normal even if the distribution of the demand during review period is Normal. Then, Eppen and Martin (1988) propose a method to calculate order-up-to-level based on the assumption that the distribution of risk period demand is Normal for each possible value of replenishment lead time. This assumption is valid if the distribution of the demand during review period is Normal. However, it is numerically shown in Section 3.1 in this thesis that the order-up-to-levels calculated by the method given by Eppen and Martin (1988) do not necessarily provide the target service levels even if the distribution of the demand during review period is Normal. For the same system setting given by Eppen and Martin (1988), we propose a new method by working with the distribution of inventory-on-order and show that the service levels obtained by simulation analysis are sufficiently close to target service levels. The details of the proposed method for the order-up-to-levels found by the proposed method are explained in Section 3.1.

On the other hand, the number of the studies for endogenous replenishment lead times is very limited when the retailer employs (R, S) policy. Nevertheless, the study due to Boute (2006) fills this gap by giving a comprehensive analytical study for endogenous replenishment lead times. The investigation in his thesis is for two-echelon production/inventory systems with endogenous and exogenous replenishment lead times. Boute (2006) notes that working with exogenous lead times is incomplete since the correlation between the order quantity and its lead time cannot be simply ignored. In the case of endogenous replenishment lead time, Boute (2006) studies two-echelon systems when the retailer uses periodic-review base-stock ordering policy and the supplier has a capacitated make-to-order production system. In the numerical studies given by Boute (2006) for the systems where replenishment lead time is endogenous, it is shown that the assumption of exogenous lead times for determining order-up-to-level to be used seriously underestimates the required safety stock, which causes customer service to dramatically degrade. In his study, an efficient procedure is developed working with Markov chain formulations and using matrix analytic techniques to compute the distribution of the replenishment lead time explicitly by taking the correlation between demand and lead times into account. Then, he calculates the order-up-to-level providing a given target level on the *fill rate* (β-service level) by working with the distribution of inventory-on-order. In this thesis, we also use a similar method to calculate order-up-to-levels for a given target level on the *stockout probability in a review period* (α-service level). As oppose to the method given by Boute (2006), we use simulation to obtain order-up-to-levels because of the difficulties of an exact theoretical analysis. The details of this method are given in Section 3.2.

As in the study due to Boute (2006), we also restrict our analysis to two-echelon systems with retailer(s) at the lower echelon and a supplier at the higher echelon. The retailer holds a finished goods inventory to meet a random consumer demand and a single supplier produces the retailer's orders on a make-to-order basis. In order to study endogenous replenishment lead times, we model the supplier's capacitated production system by a single-server queuing model that processes the orders using first-come-first-served queueing discipline. Once processing on order quantity is completed, the retailer's inventory is replenished. Ordering policy used by the retailer is periodic-review base-stock policy.

Also, there exist extensive studies in the literature on the calculation of order-up-to-levels for the use of periodic-review base-stock policy when the demands during review periods are correlated and replenishment lead time is exogenous. When the demands are correlated, a forecasting mechanism for the demand should be employed as stated in the studies due to Chen *et al.* (2000a, 2000b) and Kim and Ryan (2002). When R is given explicitly and assumed constant, the optimal policy is an adaptive (R, S_t) policy with the order-up-to-level, S_t, used to determine order quantity at review period t. Order-up-to-levels of the adaptive policy are updated based on the demand forecasts. Most of these studies are given under the heading of bullwhip effect and discussed in the following parts where the related literature to the bullwhip effect explained.

In this thesis, we also aim to investigate the impacts of using adaptive ordering policy on the system performance. The systems analyzed in this context are related to studies in supply chain literature with two domains: bullwhip effect and lead time syndrome. Bullwhip effect that is studied extensively in literature is defined as the amplification in demand variability as one moves from the lowest echelon (retailer) to the highest echelon (supplier) in the supply chain. As stated by Chopra and Meindl (2007) bullwhip effect is a costly phenomenon because it reduces the profitability of a supply chain by making it more expensive to provide a given level of product availability. One of the reasons is that it is difficult to achieve smooth production levels under the existence of bullwhip effect. Then, inventory holding costs for raw materials and costs caused by capacity adjustments would be higher on the supplier side. For the retailer, costs due to unsatisfied customer demand would be higher since the supplier operates at lower service levels in this case.

Lee *et al.* (1997) identify five major causes of the bullwhip effect: demand signal processing, lead time, order batching, price fluctuations, rationing and shortage gaming. Demand signal processing is the adjustment of the parameters of ordering policy based on demand forecasting. The impacts of demand forecasting on the ordering policy are studied extensively in literature under the heading of bullwhip effect. Among many others, the related studies are due to Chen *et al.* (2000a, 2000b), Lee *et al.* (1997), Luong (2007), Zhang (2004). The main findings in these studies are that the adjustments in the parameters of ordering policy based on the demand forecasts cause erratic responses in the replenishments to the retailer and eventually increase the supply chain costs. In this study, we do not aim to study the impacts of demand forecasting and focus on the update of planned lead times. For this reason, the demands during review periods are assumed to be time-stationary, independent and identically distributed in the models under consideration.

As another cause of the bullwhip effect, lead time is also studied extensively in the literature. Lee *et al.* (1997) and Zhang (2004) show that the increase in bullwhip effect is magnified with an increase in lead time. In these and many of other existing studies in the literature, replenishment lead times are assumed to be exogenous, constant and known. On the other hand, there are only a few studies on the analysis of variable lead times. In the study due to Hayya *et al.* (2006), the replenishment lead times are variable, yet exogenous. In the study due to So and Zheng (2003), the systems with both exogenous and endogenous replenishment lead times under an adaptive ordering policy and correlated demand are analyzed. They conclude that supplier's variable lead time can greatly increase the order quantity variability of the retailer, and the compounded effect of demand correlation and variable endogenous replenishment lead time on the amplification of order quantity variability is much more than the sum of each of the individual effects of demand correlation and variable endogenous lead time alone. However, they do not give information on the service levels achieved in such an environment although their model tries to provide a given target α-service level. In this thesis, we try to close this gap by obtaining the service levels achieved by the retailer. Also, Boute (2006) studies adaptive periodic-review base-stock policy based on the updated demand forecasts. He shows that the bullwhip effect caused by demand forecasting increases mean and variance of replenishment lead times observed by the retailer when the demand is positively correlated even if an optimal forecasting method is used. Boute (2006) states that these increased replenishment lead times inflate the inventory holdings of the retailer.

Also lead time syndrome is related to the bullwhip effect as explained in the following parts. Mather and Plossl (1978) are the first to describe the lead time syndrome as a vicious cycle between lead time update and order release decisions. It is argued that closing the gap between the lead time and the order flow times by updating the lead time results in uncontrolled order release pattern.

As the lead time gets longer, orders must be released earlier to cover increased expected demand during the longer lead time, leading to longer queues of production backlog and thus, flow times get longer, which causes again a longer lead time. Lead time syndrome results from the fact that in releasing the orders, the effects on future lead times and on future orders are ignored. It is suggested that the lead time syndrome causes instability, and should be avoided. This reasoning has become one of the main arguments for controlling flow times within predetermined norms instead of forecasting flow times. De Kok and Fransoo (2003) and Pahl *et al.* (2005) present an extensive overview of the literature on the workload control in production planning.

Selçuk (2007) presents an analytical evaluation of the lead time syndrome and provides stability conditions for systems with updated planned lead times. However, Selçuk (2007) discuss the issue in workload control domain. To the best of our knowledge, the study due to Boute (2006) is the only one that discusses lead time syndrome issue in the supply chain context. In his study, order release mechanism is comparable to the inventory control polices employed systems in a supply chain such that release quantities can be considered as order quantities requested by the retailer from the supplier and flow times at the supplier can be seen as the replenishment lead times corresponding to these order quantities. But, this relation is not clearly stated in his study since he does not study the impacts of using an adaptive ordering policy based on the update of planned lead times as explained previously. In this thesis, we study bullwhip effect and lead time syndrome issues under an adaptive ordering policy based on the update of planned lead times.

To summarize, the topics covered in Sections 3.3 and 4.2 are related to the studies due to Boute (2006) and So and Zheng (2003) and Selçuk (2007). Firstly, our study and all of these studies analyze two-echelon systems with endogenous lead times. Next, we consider the case the retailer uses periodic-review base-stock policy as in the studies due to Boute (2006) and So and Zheng (2003). Moreover, in order to study systems with endogenous replenishment lead times, we consider a capacitated make-to-order production system and model the production system of the supplier as a single-server queuing model with constant service time per item and first-come-first-served service discipline where the items are produced one by one. Similarly, Boute (2006) and Selçuk (2007) use the same model setting for the production system of the supplier but with different distributions for the service time per item. Finally, we update planned lead times depending on changes in lead time in similar to the update mechanisms used by Selçuk (2007) and So and Zheng (2003). All of these studies analyze the supply chains with single retailer single supplier. In Section 4.2, we extend our analysis to the case with two retailers and single supplier. In such an environment, a change in the ordering policy of a retailer has an impact on the other retailer's performance even if the other retailer does not consider any change in its own ordering policy. Then, it is questioned whether it is beneficial for a retailer to use an adaptive (R, S_t) policy in this setting. To the best of our knowledge, there is no work in the literature on the analysis of updating ordering policy parameters based on the changes in replenishment lead times in a setting with multiple retailers.

Chapter 3

SINGLE-RETAILER CASE

In this chapter, we consider two-echelon models with single retailer at the lower echelon and single supplier at the higher echelon. Customer demand that the retailer observes is assumed to be time-stationary, and independently and identically distributed for each review period. Note that we work with cumulative demand during a review period. That is, only total demand during a review period is assumed to be known and there is no information about exactly when and in what amount the demand is realized during a review period. The distribution of the demand during the review period is assumed to be Normal and the probability of observing no demand during the review period is assumed to negligible. In this case, it would make sense for the retailer to use a stationary ordering policy as long as the supply process does not change over time. For the models considered in this chapter, the retailer holds finished goods inventory to meet customer demand and places orders according to a specified periodic-review base-stock policy. Unsatisfied demand is completely backordered. Here, we investigate the use of periodic-review base-stock policy, namely (R, S) policy with stationary parameters R and S. R is the review period in time units and S is the order-up-to-level (base-stock level). Review period R is the time that elapses between two consecutive moments at which the retailer monitors the inventory position, IP, which is the sum of the inventory-on-hand and the inventory-on-order (items ordered but not delivered yet due to replenishment lead time). In the case of backordering, inventory position is determined by subtracting the number of the backorders from the number items available to meet demand. If the inventory position is below S at the review point, an order is placed to raise the inventory position to the order-up-to-level S, which determines the order quantity. Employment of the (R, S) policy is suggested when there is no fixed ordering cost and both holding and shortage costs are linear functions of the amount of on-hand inventory or backorder. The related references are due to Nahmias (1997) and Zipkin (2000).

A typical illustration of an inventory system with (R, S) policy is shown in Figure 3.1 where t_0 and $(t_0 + R)$ are two consecutive review points. Assuming that L denotes the replenishment lead time, $(t_0 + L)$ and $(t_0 + R + L)$ are the replenishment points for the orders placed at t_0 and $(t_0 + R)$, respectively. For the inventory systems where the retailer employs (R, S) policy, the following definition of 'risk period' is taken from the study due to Boute (2006).

Definition 1. Risk period (the time between placing a replenishment order until receiving the subsequent replenishment order) is equal to the review period plus the replenishment lead time.

Based on this definition, $(R + L)$ *time units* following each review, say to, are considered to constitute a risk period because the subsequent replenishment is not realized before $(t_0 + R + L)$. Hence, S is the amount available to satisfy the demand during the risk period between t_0 and $(t_0 + R + L)$. Although the demand is assumed continuous in Figure 3.1, the decision maker would work with cumulative demand of each review period since the inventory is reviewed periodically every R time units. Under the employment of stationary (R, S) policy by the retailer, Sections 3.1 and 3.2 are devoted to the analysis of the

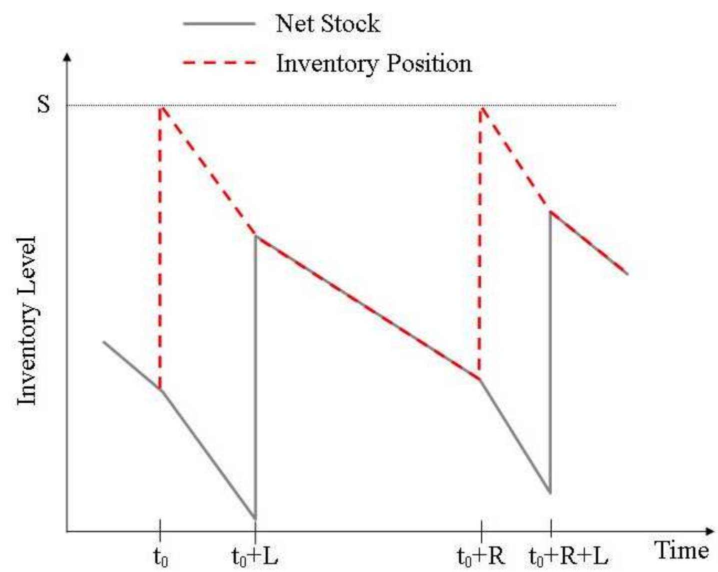

Figure 3.1: Inventory levels under (R,S) policy.

cases with exogenous and endogenous replenishment lead times, respectively.

Replenishment lead times are not always known before they are observed, then, the decision makers should work with lead time estimates for planning purposes. This would be case especially when the replenishment lead time is endogenous. In this case, as the random demands and replenishment lead times are observed and the corresponding estimates and/or probability functions are updated, the parameters of the (R, S) policy employed should also be updated. That is, an adaptive policy would be used.

In Section 3.3, the ordering policy parameters are updated for the case of time-stationary demand and endogenous supply process. In the models in Section 3.3, the demand distribution is assumed to be known and only planned lead time is updated according to the observations for the replenishment lead times. Since supply process is endogenous to the retailer, the replenishment lead times would be determined by the workload in the supplier. That is, the replenishment lead times would be longer (shorter) for the high (low) utilization periods.

We will use the following notation throughout the thesis: D_t is the random variable for demand in review period t, L is the random variable for replenishment lead time, W_t is the random variable for demand during risk period $(R + L)$ following an order placement at review period t. $E(.)$ and $Var(.)$ denote the expectation and variance of the random variable under consideration.

In this thesis, we work with the models minimizing inventory costs while providing a target service level for no-stockout probability in a review period. The fraction of the cycles (review periods) in which a stockout does not occur is called as α-service level due to the definition given by Nahmias (1997). A stockout is realized when the inventory position is negative. Note that a cycle can be considered as the time between two consecutive reviews at which IP is raised to S. As explained in Zipkin (2000), α-service measure is used in accordance with the cost incurred for each stockout occasion regardless of the duration of stockout or amount of backordered demand. We assume that R is a given constant and a linear holding cost h is incurred for each unit of inventory carried from one review period to the next. Then, the optimization would be to determine the best order-up-to-level S working with the mathematical model given below.

$$
\begin{aligned}
&Minimize\ h \cdot \overline{I}(S) \\
&subject\ to \\
&A(S) \geq \alpha, \\
&S \geq 0,
\end{aligned}
$$

where $\overline{I}(S)$ denotes average inventory-on-hand in the long run and $A(S)$ is the no-stockout probability in a review period. Note that the planning horizon is infinity.

Instead of $\overline{I}$, we prefer to consider S in the objective function. This allow us avoiding the difficulties of formulating $\overline{I}$ as a function of S but resolving the trade-off between inventory (base-stock) investment and service measure. Note that, $\overline{I}$ increases (decreases) as S increases (decreases). Then, the mathematical model becomes

$$
\begin{aligned}
MODEL\,1: \quad & Minimize\ h \cdot S \\
& subject\ to \\
& A(S) \geq \alpha, \\
& S \geq 0.
\end{aligned}
$$

In Section 3.1, different approaches are explained to determine the relation between S and α, that is, to define $A(S)$.

For periodically reviewed inventory systems, the inventory level is only known at the review points. Then, it is not possible to keep track of stockout occasions and the total number of backorders at the time of a replenishment unless review and replenishment are at the same point in time. Because of this situation, most of the studies in the literature works with one of the following assumptions:

Assumption 1. *Replenishment lead times are integer multiples of review periods.*

Assumption 2. *Demand during a review period is satisfied at the end of the review period.*

If *Assumption 1* is valid, then stockout occasions can always be tracked since review and replenishment are at the same point in time. In our models, orders are placed at the end of each review period and inventory of the retailer is replenished at the beginning of a review period after a duration of replenishment lead time is spent when the replenishment lead times are integer multiples of review periods. On the other hand, Boute (2006) does not restrict himself to replenishment lead times that are integer multiples of review periods by working with *Assumption 2*. Under *Assumption 2*, the replenishments during the review period can be used to fulfill the demand during the current review period. Hence, a replenishment in a review period is evaluated as if it is realized at the beginning of the review period and risk periods can be rounded down to the nearest integer multiples of review periods to be used in the ordering policy formulations. In this study, we will utilize one of these assumptions to be able to keep track of the stockout occasions.

In the study due to Boute (2006), the fraction of stockout occasions is measured at the end of each review period just after an order is placed instead of measuring it just before a replenishment point. Both ways of measurement give the same results as long as the replenishment lead times are integer multiples of review periods since review and replenishment are at the same point in time. However, the result turns out to be different in the case that the replenishment lead time is not integer multiples of the review periods. In this case, it is not possible to measure the fraction of stockout occasions just before a replenishment point. We distinguish between two ways of the measurement for α-service level since different methods are developed to calculate the order quantities for each measurement type. In Section 3.1, the situation is further explained.

Fill rate (β-service level) is the fraction of customer demand that is met from inventory, see the definition given by Nahmias (1997). In the case there is a cost incurred per shortage regardless of the time taken to satisfy backordered demand, fill rate should be considered as the corresponding service measure. In this study, we do not consider fill rate as the service measure. However, the fill rates in the numerical studies are obtained and given for the comparison purposes.

If the demand during risk period is distributed according to a Normal distribution, then the order-up-to-level for the (R, S) policy is determined as follows in most of the studies in the literature:

$$S = E(W) + z_\alpha \cdot \sqrt{Var(W)} \tag{3.1}$$

where z_α is standard normal inverse for a given α-service level; that is,

$$Pr(\frac{W - E(W)}{\sqrt{Var(W)}} \geq z_\alpha) = \alpha.$$

From (3.1), order-up-to-level S is the sum of the mean demand during the risk period and the safety stock for the variation in the demand during the risk period. Safety stock is calculated by assuming the distribution of the demand during the risk period is Normal. That is, safety stock is equal to $z_\alpha \cdot \sqrt{Var(W)}$.

Note that the formula in (3.1) is obtained based on the measurement of no-stockout probability just before replenishment rather than the measurement at the end of each review period just after an order is placed. Both ways of measurement give the same results as long as the replenishment lead times are integer multiples of review periods since reviews and replenishments are simultaneous in this case. In the following definition, notation used is introduced for two different ways of measuring no-stockout probability.

Definition 2.

a) α_{rep} is the probability of no-stockout just before replenishment.

b) α_{rev} is the probability of no-stockout probability at the end of each review period just after an order is placed.

3.1 Exogenous Replenishment Lead Times with Stationary (R, S) Policy

In this section, the models with exogenous replenishment lead times are considered. The probability distribution function of the replenishment lead times is assumed to be exogenous and known by the retailer. Moreover, we work with *Assumption 1* in the models considered in this section. For the given problem setting, different methods are given to calculate order-up-to-levels that are providing target α-service levels. Then, these methods are compared for relevant performance results for different performance measures that are obtained by simulation analysis. In this section, three different methods are given: *Method 1, Method 2, Method 3*. In *Method 1* due to Silver *et al.* (1998), it is assumed that the distribution of the demand during risk period $(R + L)$ is i.i.d. and Normal. However, Eppen and Martin (1988) show that the distribution of the demand during the risk period is not Normal even if the distribution of the demand during review period is Normal. Then, Eppen and Martin (1988) propose *Method 2* which is based on the convolutions of the review period's demands over the risk period. In this study, we propose *Method 3* which is based on the distribution of inventory-on-order and show that service levels obtained by simulation analysis are sufficiently close to target service levels while the order-up-to-levels calculated by using *Method 1* and *Method 2* do not necessarily provide target service levels. In a similar way, Boute (2006) propose a method to calculate order-up-to-level by obtaining the distribution of inventory-on-order for endogenous replenishment lead time case. *Method 3* extends his approach to the exogenous replenishment lead time case by using the findings due to Eppen and Martin (1988).

Method 1. (due to Silver *et al.*, 1998) This method is based on the investigation of the demand during the replenishment lead time plus review period, which is the risk period demand. In a periodic-review base-stock policy, an order is placed at every review point assuming that there is negligible probability of no demand during a review period. Then, demand during risk period $(R + L)$ following an order placement at review period t, W_t, is given by the following equation:

$$W_t = \sum_{n=1}^{L+1} D_{t+n}. \tag{3.2}$$

Recall that the assumption here is that the replenishment lead times are integer multiples of the review periods (*Assumption 1*). However, (3.2) is still applicable for the non-integer replenishment lead times if the demand during a review period is assumed to be satisfied or backordered at the end of the review period (*Assumption 2*). In this case, replenishments in the review period can be used to fulfill the demand during the current review period. Then, the value of the risk period can be rounded down to the nearest integer value and used in (3.2). The explanation below for rounding is due to Boute (2006).

> "For instance, suppose that an order placed at the end of period t has a production lead time of 0.8 periods. This order quantity will be added to the inventory in the next period $t + 1$ and can be used to satisfy demand in period $t + 1$. Therefore, the replenishment lead time is 0 periods since the order can immediately be used to satisfy

next period's demand."

When the assumption above does not hold in the case of non-integer replenishment lead times, the demand pattern during a review period should be known to calculate the demand during the risk period correctly. However, this is not possible without reviewing inventory continuously. This will be discussed later in Section 3.1.1).

Since D_ts are i.i.d. random variables, we will proceed by dropping subscripts of D_t and W_t. According to (3.2), expectation and variance of the demand during risk period are given as follows: $E(W) = [E(L) + 1] \cdot E(D)$ and $Var(W) = \sum_{n=1}^{L+1} Var(D_n)$. By using these in (3.1), the order-up-to-level is given as

$$S \;=\; [E(L) + 1] \cdot E(D) + z_\alpha \cdot \sqrt{\sum_{n=1}^{L+1} Var(D_n)}. \tag{3.3}$$

For the variable replenishment lead time case, (3.3) becomes

$$S = [E(L) + 1] \cdot E(D) + z_\alpha \cdot \sqrt{[E(L) + 1] \cdot Var(D) + [E(D)]^2 \cdot Var(L)}. \tag{3.4}$$

The second term on the right hand side for safety stock part is equal to standard deviation of the random sum of the random variables multiplied by standard normal inverse value for a given α-service level. When L is constant, we have $Var(L) = 0$ and (3.4) reduces to

$$S = [L + 1] \cdot E(D) + z_\alpha \cdot \sqrt{[L + 1] \cdot Var(D)}. \tag{3.5}$$

Method 2. (due to Eppen and Martin, 1988) In the formulations used for *Method 1*, safety stock is calculated by assuming that the distribution of the demand during risk period is Normal. However, this assumption would not hold true for most of the cases even if the demand of the review period is distributed according to a Normal distribution, especially when the replenishment lead time can take a wide range of values. Eppen and Martin (1988) show how the order-up-to-level increases as the replenishment lead time is halved in the 50% of the replenishments as a result of Normal distribution assumption of the lead time demand. We will illustrate this with a numerical example.

Numerical Example 1. *Consider the case replenishment lead time is constant and equal to* 4 *review periods. Distribution of the demand per review period is Normal with a mean of* 100 *and a variance of* 100*. Then,* Method 1*, equation (3.5), gives the following order-up-to-level to satisfy a probability of stockout during the risk period not greater than 5%:*

$$S = [4+1] \cdot 100 + 1.65 \cdot \sqrt{[4+1] \cdot 100} = 537.$$

Now, consider the case replenishment lead time is equal to 2 *or* 4 *review periods with equal probabilities. That is,*

$$L = \begin{cases} 2 & \textit{with probability } \frac{1}{2}, \\ 4 & \textit{with probability } \frac{1}{2}. \end{cases}$$

In this case, $E(L) = 3$ *and* $Var(L) = 1$*. Then, Method 1 gives*

$$S = [3+1] \cdot 100 + 1.65 \cdot \sqrt{[3+1] \cdot 100 + [100] \cdot 1} = 568$$

working with (3.4) which is used for the variable replenishment lead time. It is obvious that $S = 568$ *provides a lower stockout probability than* $S = 537$ *since the replenishment lead time is halved with probability* 0.5*. Since the retailer employs stationary* (R, S) *policy, the average inventory obtained by using* $S = 568$ *is* $568 - 537 = 31$ *units higher than using* $S = 537$*. That is, average inventory is*

%5.77 higher for S calculated by Method 1 compared to Method 2. Recalling that our objective is to minimize $h \cdot \overline{I}$ as given by the mathematical model previously, the inventory holding cost will also be %5.77 higher if the retailer uses Method 1. $\square$

Method 2 is also based on the distribution of the demand during the risk period. That is, the probability of being no-stockout in a review period is found by the probability that the demand during the risk period is less than order-up-to-level. Then, the mathematical model is defined as

$$
\begin{aligned}
MODEL\,2: \quad & Minimize\ h \cdot S \\
& subject\ to \\
& Pr(W < S) \geq \alpha_{rep}, \\
& S \geq 0.
\end{aligned}
$$

Note that MODEL 2 is obtained by defining $A(S)$ in MODEL 1 according to the given stockout probability definition. In the case that the replenishment lead time is not integer multiples of the review periods, we should

Eppen and Martin (1988) show that the distribution of the demand for the risk period is Normal for each possible value of the replenishment lead time if the distribution of the demand during the review period is Normal and independent of the demands in other periods. Therefore, they propose a method to calculate order-up-to-level for variable replenishment lead times. They estimate service level by working with the convolutions of Normal distributions for the demands during the review periods in the risk period. Although Eppen and Martin (1988) assume that replenishment lead times are integer multiples of the review periods, that is *Assumption 1* is valid, we can use the same method to calculate order-up-to-levels for non-integer replenishment lead times proceeding with the assumption that the demand during the review period can be fulfilled at the end of the review period (*Assumption 2*). Then, the order-up-to-level that ensures a given α-service level is the smallest S value such that

$$Pr(W < S) = \sum_{l} Pr(W < S|L = l) \cdot Pr(L = l) \geq \alpha_{rep}. \tag{3.6}$$

See the study due to Eppen and Martin (1988) for the detailed explanation.

In (3.6), $Pr(W < S)$ is given as convex combinations of the probabilities that W is less than S for each possible value of replenishment lead time l. When the distribution of demand during the review period is Normal with mean of $E(D)$ and variance of $Var(D)$, the distribution of the demand during risk period is Normal for each possible replenishment lead time l with mean $(l+1) \cdot E(D)$ and variance $(l+1) \cdot Var(D)$. Referring to (3.6),

$$Pr(W < S) = \sum_{l} F_Z(\frac{S - (l+1) \cdot E(D)}{\sqrt{(l+1) \cdot Var(D)}}) \cdot Pr(L = l)$$

where Z is the Standard Normal random variable with mean 0 and variance 1, $F_Z(.)$ is the cumulative distribution function of Z. Then, the following equation should be solved for order-up-to-level S providing service level α_{rep}:

$$\sum_{l} F_Z(\frac{S - (l+1) \cdot E(D)}{\sqrt{(l+1) \cdot Var(D)}}) \cdot Pr(L = l) \quad = \quad \alpha_{rep}. \tag{3.7}$$

Method 3. (Proposed method) All the formulations given for *Method 1* and *Method 2* are to calculate the order-up-to-level ensuring a given no-stockout probability level measured just before replenishment. Recall that this measure is called as α_{rep}. The order-up-to-level ensuring a given no-stockout probability level measured at the end of each review period (α_{rev}) can be calculated by working with the distribution of inventory-on-order. This is shown by Boute (2006) for the stationary systems with endogenous replenishment lead times. Boute (2006) observes the inventory position IP_t at the end of every review period t just after the demand during the review period t (D_t) is satisfied and an order of size Q_t has been placed. Then, it is stated that the inventory position IP_t just after an order is placed is equal to the initial inventory-on-hand (NI_0) plus all replenishment orders received so far minus total customer demand observed. That is,

$$IP_t = NI_0 + \sum_{n=K(t)+1}^{t} Q_{t-n} - \sum_{n=0}^{t} D_{t-n}, \tag{3.8}$$

$$IP_t = S + \sum_{n=K(t)+1}^{t-1} Q_{t-n} - \sum_{n=0}^{t} D_{t-n}, \tag{3.9}$$

where random variable $K(t)$ denotes the number of outstanding orders that have not been delivered yet at the end of review period t just after an order is placed. Note that $NI_0 + Q_0 = S$.

When the order-up-to-level is constant, size of the order placed at the end of the review period t is equal to the demand during the review period t. The reason is that inventory position IP_t is raised to S at period t using the following order size: $Q_t = S - IP_t$. Also, IP_t is equal to $(S - D_t)$ since inventory position is raised to S at review point $(t-1)$. Then, the order quantity calculated at the end of review period t is equal to $Q_t = S - (S - D_t) = D_t$. Hence, using Q_t in (3.9), $NI_t = S - \sum_{n=0}^{K(t)} D_{t-n}$ is obtained. Since S is constant, the distribution of net inventory can be determined by inventory-on-order IO_t at the end of period t as follows:

$$IO_t = S - NI_t = \sum_{n=0}^{K(t)} D_{t-n}, \tag{3.10}$$

Then, $Pr(NI_t < 0) = Pr(IO_t > S)$.

Hence, the probability of being no-stockout in a review period is equal to the probability that inventory-on-order at the end of a review period is less than order-up-to-level. Then, the mathematical model for *Method 3* becomes

$$\begin{aligned} MODEL\,3: \ & Minimize\ h \cdot S \\ & subject\ to \\ & Pr(IO < S) \geq \alpha_{rev}, \\ & S \geq 0. \end{aligned}$$

where IO denotes average inventory-on-order in the long run.

Hence, we just need to know the distribution of inventory-on-order for each value of the number of outstanding orders at the end of a review period to calculate the order-up-to-level S that ensures service level α_{rev}. If it is known, then the order-up-to-level is the smallest S value such that

$$Pr(IO < S) = \sum_k Pr(IO < S | K = k) \cdot Pr(K = k) \geq \alpha_{rev}. \tag{3.11}$$

The distribution of inventory-on-order is equal to the convolution of the demands during K review periods as seen in (3.10). Note that K is a discrete random variable. For the case the demands during the review periods are i.i.d. random variables and the distribution for these random variables is Normal, the distribution of inventory-on-order is Normal with mean $k \cdot E(D)$ and variance $k \cdot Var(D)$ for each possible k for random variable K. Moreover, the probability of having k outstanding orders at the end of review period t just after an order is placed is equal to the probability that the orders placed at or before t have not been delivered yet. Denote L_{t-n} as the replenishment lead time for the order placed at the end of review period $(t-n)$ where $0 \leq n \leq t$. Then, the number of the outstanding orders at the end of review period t just after an order is placed is equal to $K(t) = \sum_{n=0}^{t} 1_{\{L_{t-n}>n\}}$ where

$$1_{\{L_{t-n}>n\}} = \begin{cases} 1 & \text{if } L_{t-n} > n, \\ 0 & \text{otherwise.} \end{cases}$$

If the probability distributions for L_t are known, then the probability distribution of $K(t)$ is found as follows:

$$Pr(K(t) = k) = Pr(\sum_{n=0}^{t} 1_{\{L_{t-n}>n\}} = k). \tag{3.12}$$

After obtaining the probability distribution of random variable K, that result from (3.12), the following equation can be used to find the value of S providing service level α_{rev}:

$$\sum_{k} F_Z(\frac{S - k \cdot E(D)}{\sqrt{k \cdot Var(D)}}) \cdot Pr(K = k) \quad = \quad \alpha_{rev}. \tag{3.13}$$

The methods given in this section are evaluated by using simulation analysis. The simulation code is written in SIMAN language and the runs are performed by using ARENA Simulation Program. A generic simulation code used for the models studied in this thesis is given in Appendix A. Simulation time is 100,000 review periods and 30 replications are considered. For the models with different settings and/or parameters (different number of retailers, different demand patterns and utilization levels), different performance measures are obtained. The results of the first 5,000 review periods are not taken into consideration to eliminate the impact of the initial state. Since the demand for the retailer is random and generated during the simulation run, different seeds for the generation of the demands are used in each replication to have independent replications. In Sections 3.1.1 and 3.1.2, we give numerical examples, and simulation results for each of the methods under consideration. Section 3.1.1 and Section 3.1.2 devoted to the cases of constant and variable exogenous lead times, respectively.

3.1.1 Constant Replenishment Lead Time

Consider the case replenishment lead time is constant and equal to a value that is integer multiples of the review period, that is *Assumption 1* is valid. Since replenishment lead time takes only one value (*L review periods*), the distribution of the demand during the risk period is Normal with mean $(L+1) \cdot E(D)$ and variance $(L+1) \cdot Var(D)$. Thus, the assumption that the distribution of the demand during the risk period is Normal holds for *Method 1*. Then, *Method 1* and *Method 2* are equivalent. Moreover, α_{rep} and α_{rev} are equal to each other since reviews and replenishments are at the same moments in time. The number of outstanding orders at the end of the period t is equal to $(L+1)$ just after an order is placed. These are the orders placed at review periods (t-L), ..., (t-1), t. Then, the distribution of the inventory-on-order is also Normal with mean $(L+1) \cdot E(D)$ and variance $(L+1) \cdot Var(D)$. As far as replenishment lead time is constant and integer multiples of the review period, also *Method 3* gives the same S value.

Numerical Example 2. *Consider the case replenishment lead time L is constant and equal to 2 review periods. For a service level of* $\alpha = 95\%$, z_α *is equal* *to* 1.65*. The distribution of the demand during a review period is Normal with mean 100 and variance 900. Then, the distribution of the demand during the risk period is Normal with mean* $300 = (2+1) \cdot 100$ *and variance* $2700 = (2+1) \cdot 900$*.* Method 1 *gives the following order-up-to-level for service level* $\alpha_{rep} = 95\%$*:*

$$\begin{aligned} S &= [L+1] \cdot E(D) + z_\alpha \cdot \sqrt{(L+1) \cdot Var(D_n)}, \\ S &= [2+1] \cdot 100 + 1.65 \cdot \sqrt{(2+1) \cdot 900}, \\ S &= 385.47. \end{aligned}$$

The order-up-to-level for a target service level $\alpha_{rep} = \%95$ *is also calculated as 385.47 by Method 2 by solving the following equation:*

$$F_Z(\frac{S - 3 \cdot 100}{\sqrt{3 \cdot 900}}) \cdot 1 = 0.95.$$

The number of outstanding orders at the end of the period t *is equal to 3 just after an order placed (orders of periods* $(t-2), (t-1), t)$*. Then, the distribution of inventory-on-order is Normal with mean* $300 = 3 \cdot 100)$ *and variance* $2700 = 3 \cdot 900$*. Hence, Method 3 gives S=385.47 for* $\alpha_{rev} = 95\%$ *by solving*

$$F_Z(\frac{S - 3 \cdot 100}{\sqrt{3 \cdot 900}}) \cdot 1 = 0.95.$$

□

The simulation for *Numerical Example 2* gives the results in Table 3.1. Using $S = 385.47$, service level of the retailer is obtained as $\alpha = 95.02\%$ with a half-width of 0.05% and this value is sufficiently close to the target level of 95%.

Now, consider the case replenishment lead time L is constant but has a value which is not integer multiples of the review period, that is *Assumption 1* is not

Table 3.1: Simulation results for constant replenishment lead time case.

	Average	***Half-width***	***Minimum***	***Maximum***
Mean Demand	100.0	0.0	99.8	100.2
Variance of Demand	898.8	1.8	886.8	907.2
Mean Order Quantity	100.0	0.0	99.8	100.2
Variance of Order Quantity	898.8	1.8	886.8	907.2
Mean Replenishment Lead Time	2.00	0.00	2.00	2.00
Variance of Rep. Lead Time	0.00	0.00	0.00	0.00
Mean Net Inventory	185.5	0.1	185.1	185.9
Variance of Net Inventory	1798.4	5.0	1768.8	1815.7
Average Inventory-on-hand	185.5	0.1	185.1	185.9
Variance of Inventory-on-hand	1798.4	5.0	1768.7	1815.6
Fill Rate (β)	98.94%	0.01%	98.88%	98.99%
No-stockout Probability (α)	95.02%	0.05%	94.84%	95.22%

valid. In this case, review and replenishment points are at different points in time. Thus, we are not able to keep track of the stockout occasions at the replenishment points to determine α_{rep}. However, it is still possible to observe the stockout occasions at the end of the review periods and α_{rev} can be determined as in the case L is integer multiples of the review period. If demand is assumed to be satisfied at the end of the review period (replenishments during the period can be used to fulfill the demand of the current period), that is *Assumption 2* is valid, then L can be treated as $\lfloor\lfloor L \rfloor\rfloor$ which denotes the largest integer smaller than or equal to L. Under *Assumption 2*, α_{rev} is equal to α_{rep}. If *Assumption 2* does not hold, α_{rev} is not equal to α_{rep}. If demand during $L - \lfloor\lfloor L \rfloor\rfloor$ is strictly greater than zero, then for a given value of the order-up-to-level S: $\alpha_{rev} < \alpha_{rep}$.

3.1.2 Variable Replenishment Lead Time

In this section, replenishment lead times can take more than one value. It is shown that the methods given for the calculation of order-up-to-level result in different S values even if replenishment lead time is integer multiples of the review period. In order to see the differences of the methods, the following numerical example is considered.

Numerical Example 3. *For* Numerical Example 2*, consider the case replenishment lead time is equal to 2 and 4 review periods with equal probabilities. That is,*

$$L = \begin{cases} 2 & \text{with probability } \frac{1}{2}, \\ 4 & \text{with probability } \frac{1}{2}. \end{cases}$$

Moreover, L and D_ts are statistically independent. Then, the mean and variance of replenishment lead time is calculated as $E(L) = 3$ and $Var(L) = 1$ and the distribution of the demand during review period is Normal with mean 100, variance 900. Method 1 *gives the following order-up-to-level for a service level of $\alpha = 95\%$:*

$$S = [3+1] \cdot 100 + 1.65 \cdot \sqrt{[3+1] \cdot 900 + [100]^2 \cdot 1} = 591.82.$$

In Method 1*, distribution of the demand during risk period is assumed to be Normal. However, the distribution of the demand during risk period is not Normal when L is variable. Instead it is Normal for each possible value of the replenishment lead times as illustrated in Figure 3.2. This distribution turns out to be Normal with mean $(l+1) \cdot E(D)$ and variance $(l+1) \cdot Var(D)$ for $l = 2, 4$ with* *equal probabilities. That is,*

$$W = \begin{cases} \sum_{t=1}^{2+1} D_t & \textit{with probability } \frac{1}{2}, \\ \sum_{t=1}^{4+1} D_t & \textit{with probability } \frac{1}{2}. \end{cases}$$

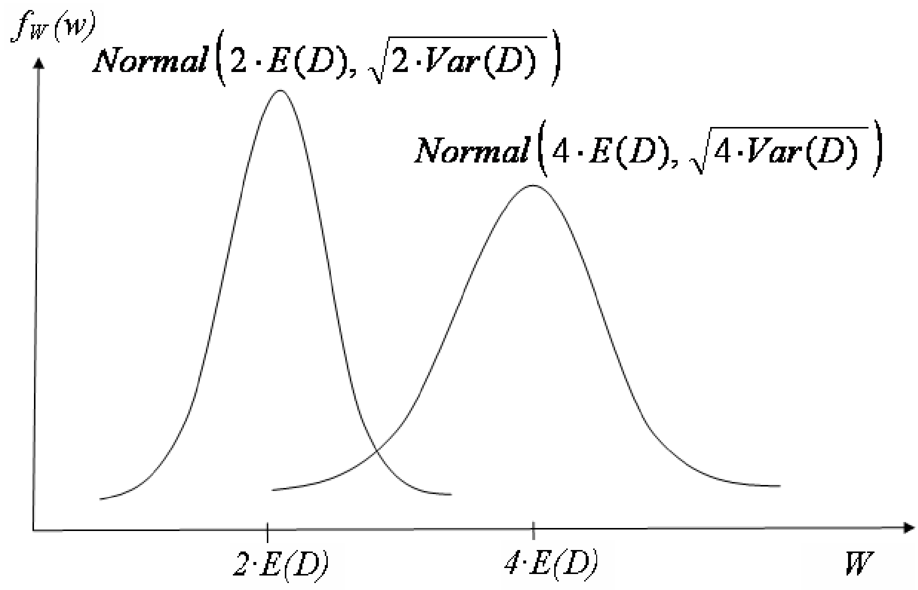

Figure 3.2: Probability density function of the demand during risk period for variable replenishment lead time, Numerical Example 3.

Then, the order-up-to-level giving a service level of $\alpha = 95\%$ *is calculated as* 585.97 *by solving the following equation, recall (3.7) for* Method 2*:*

$$F_Z\left(\frac{S - (2+1) \cdot 100}{\sqrt{(2+1) \cdot 900}}\right) \cdot (.5) + F_Z\left(\frac{S - (4+1) \cdot 100}{\sqrt{(4+1) \cdot 900}}\right) \cdot (.5) = 0.95.$$

Hence, the order-up-to-levels calculated by using Method 1 and Method 2 are different. For the use of Method 3, we need to determine the distribution of *inventory-on-order. That is, we should find the distribution of the number of outstanding orders just after an order is placed. This number can take any integer value between* $(L_{min}+1)$ *and* $(L_{max}+1)$ *where* L_{min} *is the minimum replenishment lead time and* L_{max} *is the maximum replenishment lead time since all orders placed before period* $(t-L_{max})$ *are delivered at or before* t*. In this example,* L_{min} *and* L_{max} *are 2 and 4 review periods, respectively. Then, the number of outstanding orders at the end of a review period* t *can take the values of 3, 4, 5. The number of outstanding orders at the end of review period* t *just after an order is placed is 3 if there are 2 outstanding orders. Since* L_{max} *is 4, all orders placed before period* $(t-4)$ *are delivered at or before* t*. Then, the orders placed at the end of the review periods* $(t-n)$ *are not replenished until* t *with probability* $Pr(L_{t-n} \geq n) \quad for \quad n = 1,2,3,4$*. For example,* $Pr(K(t)=3)$ *is equal to the probability that exactly 2 of the last 4 review periods' orders are not replenished until* t*. In Table 3.2, all possible combinations of the replenishment lead times for the last 4 review periods are listed with the corresponding probabilities. After finding probability distribution of the random variable* K*, the order-up-to-level giving a service level of* $\alpha = 95\%$ *is calculated as* 558.47 *by solving the following equation for Method 3:*

$$F_Z\left(\frac{S-3\cdot 100}{\sqrt{3\cdot 900}}\right)\cdot 0.25 + F_Z\left(\frac{S-4\cdot 100}{\sqrt{4\cdot 900}}\right)\cdot 0.5 + F_Z\left(\frac{S-5\cdot 100}{\sqrt{5\cdot 900}}\right)\cdot 0.25 = 0.95.$$

□

When we use the simulation model for $S = 558.47$, a service level of $\alpha = 94.98\%$ is observed. However, $S = 591.82$ and $S = 585.97$ obtained by *Method 1* and *Method 2*, respectively, give service levels of almost 97%. Simulation results are tabulated in Tables 3.3 and 3.4. In Table 3.5, order-up-to-levels calculated by all of the methods are given for different target service levels.

Table 3.2: Probability distribution of random variable K for Numerical Example 3

Replenishment Lead Times						
t-4	**t-3**	**t-2**	**t-1**	**Probability**	**K**	**Pr(K=k)**
2	2	2	2	0.0625	3	0.25
2	2	4	2	0.0625		
2	2	2	4	0.0625		
2	2	4	4	0.0625		
4	2	2	2	0.0625	4	0.5
2	4	2	2	0.0625		
2	4	4	2	0.0625		
4	2	4	2	0.0625		
4	2	2	4	0.0625		
2	4	2	4	0.0625		
2	4	4	4	0.0625		
4	2	4	4	0.0625		
4	4	2	2	0.0625	5	0.25
4	4	4	2	0.0625		
4	4	2	4	0.0625		
4	4	4	4	0.0625		

Simulation results show that calculating order-up-to-level by estimating the demand during the risk period as in *Method 2* does not necessarily provide the specified target service levels. This turns out to be the case even when the replenishments and reviews occur at the same time as in *Numerical Example 3*. Recall that service models are considered to determine the minimizing inventory holding costs subject to a target no-stockout probability in the methods under consideration. Based on the results obtained by the simulation analysis, we can conclude that *Methods 1* and *2* give higher no-stockout probabilities almost for all of the cases since S values calculated by these methods are higher. This is especially the case for high target service levels. However, higher S values result in higher inventory levels. Since we defined our objective function as $h \cdot S$ in the mathematical model given at the beginning of this chapter, we can compare the increase in the inventory cost by comparing S values. For example, the inventory costs obtained when target service level is 95% are almost 5% higher for S values calculated by Method 1 and 2 compared to Method 3.

To see that the mean and variance of the demand during the risk period are estimated correctly, the replenishment lead times are observed and the demand during $R+L$ are obtained by simulation. As a result of the simulation analysis, observed replenishment lead times turn out to be 2 in 49.87% of the time and 4 50.13% of the time. The corresponding $E(W)$ and $Var(W)$ values are observed to be 300 and 2666.87 for $L = 2$ and 500 and 4420.04 for $L = 4$. The mean and variance of the demand during the risk period obtained by simulation and analytical formulations are almost the same, which numerically shows that the values obtained analytically are correct. The histogram of the demand during risk period is given in Figure 3.3, which shows that the distributions of the demand during risk period is Normal for each possible value of replenishment lead time.

The number of outstanding orders ($K(t)$) are also observed by the simulation. Then, $K(t)$ values are observed as 3 in 25.04% of the time, 4 in 49.75% of the time and 5 in 25.21% of the time and these values are almost same with the calculated values given in Table 3.2. The corresponding $E(IO)$ and $Var(IO)$ values are observed to be 300 and 2660.81 for $k = 3$, 400 and 3528.09 for $k = 4$, 400 and 4444.56 for $k = 5$. Again, the simulation results are almost the same as the values obtained analytically.

Another observation based on the simulation results is that demands during the review periods and order quantities placed by the retailer have the same means and variances as shown in Table 3.2. The reason is that the order-up-to-level is constant and the demand pattern and order pattern are equal in this case, that is $Q_t = D_t$. This means that there is no change in the variance of the

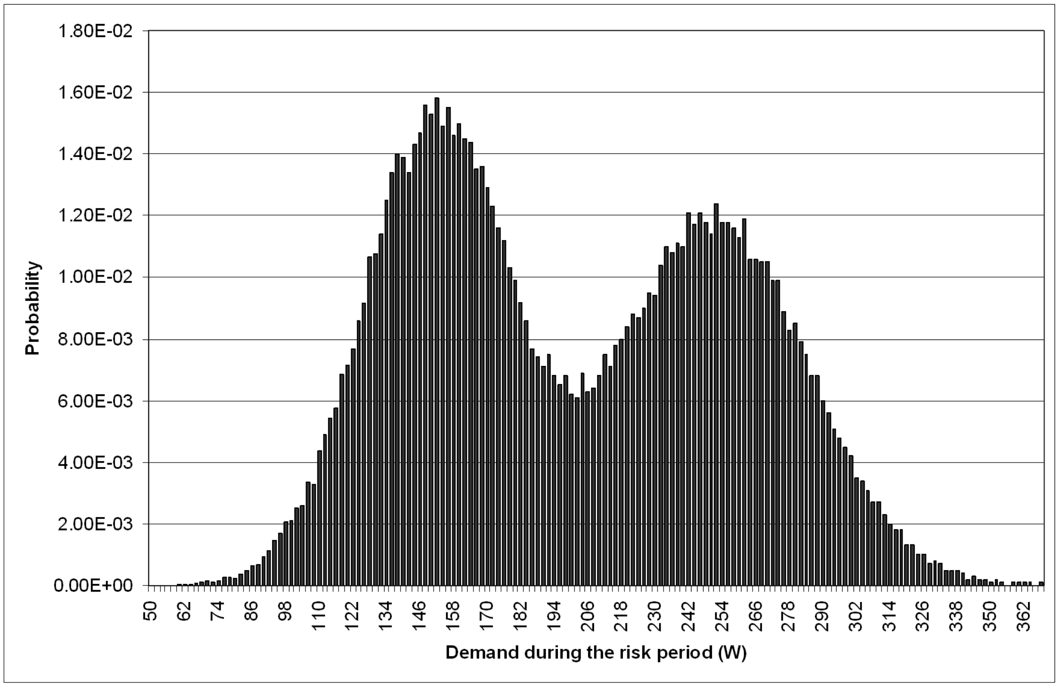

Figure 3.3: Histogram for the demand during risk period.

order quantity as far as the variance of the demand is the same. Thus, any S calculated by the given methods provide the same variance of the order quantity.

Table 3.3: Simulation results for $S = 558.47$ obtained by *Method 3*.

	Average	***Half-width***	***Minimum***	***Maximum***
Mean Demand	100.0	0.0	99.8	100.2
Variance of Demand	900.0	1.5	894.4	907.4
Mean Order Quantity	100.0	0.0	99.8	100.2
Variance of Order Quantity	900.0	1.5	894.4	907.4
Mean Replenishment Lead Time	3.00	0.00	3.00	3.00
Variance of Rep. Lead Time	1.00	0.00	1.00	1.00
Mean Net Inventory	258.3	0.2	257.4	259.0
Variance of Net Inventory	7721.3	16.8	7642.4	7796.2
Average Inventory-on-hand	258.4	0.2	257.4	259.0
Variance of Inventory-on-hand	7709.7	16.6	7631.6	7780.9
Fill Rate (β)	98.21%	0.02	98.10%	98.27%
No-Stockout Probability (α)	94.98%	0.03	94.75%	95.09%

Table 3.4: Comparison of the simulation results for Methods 1, 2, 3.

	Method 1	**Method 2**	**Method 3**
Order-up-to-level (S)	***591.82***	***585.97***	***558.47***
Mean Demand	100.0	100.0	100.0
Variance of Demand	899.9	900.1	900.0
Mean Order Quantity	100.0	100.0	100.0
Variance of Order Quantity	899.9	900.1	900.0
Mean Replenishment Lead Time	3.00	3.00	3.00
Variance of Rep. Lead Time	1.00	1.00	1.00
Mean Net Inventory	291.7	285.8	258.3
Variance of Net Inventory	7721.8	7722.2	7721.3
Average Inventory-on-hand	291.7	285.8	258.4
Variance of Inventory-on-hand	7719.8	7719.4	7709.7
Fill Rate (β)	99.34%	99.21%	98.21%
No-Stockout Probability (α)	97.82%	97.44%	94.98%

Table 3.5: Order-up-to-levels for different target service levels and simulation results for *Method 3*.

	Target α			
	50%	*75%*	*90%*	*95%*
Method 1	400.00	478.66	549.45	591.82
Method 2	387.31	500.02	556.47	585.97
Method 3	397.53	463.32	523.88	558.47
α	50.01%	74.99%	89.96%	94.98%
Half-width	0.10%	0.07%	0.04%	0.03%

3.2 Endogenous Replenishment Lead Times with Stationary (R, S) Policy

In this section, the production system of the supplier is modelled as a single-server queuing model that processes the orders of the retailer with first-come-first-served discipline. Then, replenishment lead times are determined endogenously by the supplier's capacitated production system depending on the orders placed by the retailer. To ensure stability of the system, capacity of the single server should be sufficient to process the orders placed by the retailer. Here, we express utilization of the production facility as mean production rate of the supplier per review period divided by mean demand during the review period and assume that it is strictly smaller than one. Since the production system of the supplier is modelled as a single server queue, the service time of a single item should be defined. The relationship between utilization level u, mean service time $E(T)$ and mean period demand $E(D)$ is given by

$$u = \frac{E(D)}{1/E(T)}.$$

Note that time unit is 'review period'. That is T is expressed in terms of review periods. Therefore, $1/E(T)$ is mean production rate of the supplier per review period. Although the mean service time is calculated by the equation above, the distribution of service time should be defined also. A constant service time is used in our experiments, which is set as $E(T)$.

The models considered in this section have the same setting given in Section 3.1 except that the replenishment lead times are endogenous. Moreover, demand is assumed to be satisfied at the end of each review period in the models considered in this section. That is *Assumption 2* is valid. Then, α_{rev} is equal to α_{rep} under *Assumption 2* as explained in Section 3.1. Finding replenishment lead times for the model outlined above is an analytically complex task. To the best of our knowledge, the analysis performed by Boute (2006) is the only one in the literature in this context. Boute (2006) models the system with single retailer and single supplier as a discrete-time Markov chain and obtains the steady-state probabilities for replenishment lead time under stationary (R, S) policy employed by the retailer.

Theoretical analysis due to Boute (2006) is very complex. Here, we prefer to use simulation to calculate the order-up-to-levels. As in Section 3.1, we distinguish two different ways of measuring the probability of no-stockout: the probability of no-stockout measured just before a replenishment (α_{rep}) and the probability of no-stockout measured just after an order is placed (α_{rev}). We should find the distribution of the demand during risk period to calculate the order-up-to-level by using *Method 2* to satisfy the constraint on the stockout probability in a review period. In Section 3.1, the distribution of the demand during risk period are obtained by using the distribution of the replenishment lead times which is assumed to be exogenous and known. In this section, we estimate the necessary probability distributions working with the simulation results since the distribution of the replenishment lead time is not known explicitly. However, the method we use is the same: observe replenishment lead times during the simulation runs and estimate the probabilities of possible replenishment lead times. Then, *Method 2* can be used to find order-up-to-level ensuring a given target service level α_{rep}.

On the other hand, distribution of the inventory-on-order is needed to find order-up-to-level by using *Method 3* for a service level giving no-stockout probability measured at the end of each period (α_{rev}). In Section 3.1, the distribution of inventory-on-order for each possible value of the number of outstanding orders (denoted by k) is equal to the convolution of k periods' demands which is Normal with mean $k \cdot E(D)$ and variance $k \cdot Var(D)$. However, this is not the case for endogenous replenishment lead time. The reason is that order the size of the orders and corresponding replenishment lead times are correlated in this case. Then, the number of outstanding orders k is higher for high demand periods, that is during the congestion periods. Thus, the observed demand during last k periods is higher than $k \cdot E(D)$ for high values of k. Hence, we cannot use the equations given for *Method 3* to calculate order-up-to-level for a given target service level α_{rev} for an endogenous replenishment lead time environment. Then, we should modify *Method 3* for the case of endogenous replenishment time.

In this section, the distribution of inventory-on-order to be used in *Method 3* is determined working with the results of a preliminary simulation run. Mean and variance of inventory-on-order for each possible value of k are estimated as $E(IO|K = k)$ and $Var(IO|K = k)$, respectively. Then, the following equation is solved for order-up-to-level S to satisfy a given target service level α_{rev}:

$$\sum_k F_Z\left(\frac{S - E(IO|K = k)}{\sqrt{Var(IO|K = k)}}\right) \cdot Pr(K = k) \quad = \quad \alpha_{rev}. \qquad (3.14)$$

Note that $E(IO|K = k) = k \cdot E(D)$ and $Var(IO|K = k) = k \cdot Var(D)$ in the case of exogenous replenishment lead time.

Suppose r is the number of the replications and o_i is the number of observations (review periods) for the i^{th} replication where $i = 1, ..., r$. Moreover, $k(ij)$ and $IO(ij)$ are defined as the number of the outstanding orders and the inventory-on-order for the j^{th} observation (review period) in the i^{th} replication, respectively. Then, the mean estimate of the inventory-on-order when there are k outstanding orders is equal to the division of the sum of the inventory-on-orders when there are k outstanding orders to the number of the observations that there are k outstanding orders. That is,

$$E(IO|K = k) = \frac{\sum_{i=1}^{r} \sum_{j=1}^{o_i} IO(ij) \cdot 1_{ij}(k)}{n_k} \tag{3.15}$$

where

$$1_{ij}(k) = \begin{cases} 1 & \text{if } k(ij) = k, \\ 0 & \text{otherwise,} \end{cases}$$

and $n_k = \sum_{i=1}^{r} \sum_{j=1}^{o_i} 1_{ij}(k)$ is the number of the observations that there are k outstanding orders.

The estimate for the variance of the inventory-on-order when there are k outstanding orders is the sample variance of all observations. That is,

$$Var(IO|K = k) = \frac{\sum_{i=1}^{r} \sum_{j=1}^{o_i} (IO_i - E(IO|K = k))^2 \cdot 1_{ij}(k)}{n_k - 1}. \tag{3.16}$$

Note that the inventory-on-orders for the consecutive review periods are correlated since the inventory-on-order for the current review period is dependent on the inventory-on-order of the previous review periods.

Based on an initial simulation run, $E(IO|K = k)$ and $Var(IO|K = k)$ are calculated by (3.15) and (3.16). But, we need to set S to an initial level for the initial run. Fortunately, order/replenishment pattern, and thus IO values, are the same for any constant value of order-up-to-level S. Recall that $Q_t = D_t$ for each review period t as shown in Section 3.1. So, we can arbitrarily select an initial order-up-to-level to find the estimates $E(IO|K = k)$ and $Var(IO|K = k)$.

Table 3.6: Simulation results for $S = 100$.

(a)

l	*Frequency*
0	42.3%
1	53.5%
2	3.9%
3	0.3%

(b)

k	*Frequency*	$E(IO\|K = k)$	$Var(IO\|K = k)$
1	42.3%	100.1	894.5
2	53.5%	216.9	1518.1
3	3.9%	324.0	2662.3
4	0.3%	426.1	3508.0

Numerical Example 4. *The distribution of demand during the review period is Normal with mean and variance of 100 and 900, respectively. Then, mean service time giving* $u = 0.90$ *is calculated as*

$$E(T) = \frac{u}{E(D)} = \frac{0.90}{100} = 0.009 \textit{ review periods.}$$

S *is selected as* 100 *for the initial simulation run, and* 30 *replications are performed. The observed frequencies of the replenishment lead times are summarized in Table 3.6(a). These frequencies are used as the estimator for* $Pr(L = l)$ *for each possible value of* l*, which are then used in* Method 2. *The number of outstanding orders and the corresponding inventory-on-orders are also observed at the end of the each review period just after an order is placed. Then,* $E(IO|K = k)$ *and* $Var(IO|K = k)$ *for each possible number of outstanding orders are calculated using (3.15) and (3.16). The results are tabulated in Table 3.6(b).*

For Method 2*, the estimates for the replenishment lead time probabilities are used to find the order-up-to-level S for a target service level α_{rep} = %95 by solving the following equation:*

$$
\begin{aligned}
F_Z(\frac{S-(0+1)\cdot 100}{\sqrt{(0+1)\cdot 900}})\cdot(.423)+F_Z(\frac{S-(1+1)\cdot 100}{\sqrt{(1+1)\cdot 900}})\cdot(.535) \\
+F_Z(\frac{S-(2+1)\cdot 100}{\sqrt{(2+1)\cdot 900}})\cdot(.039)+F_Z(\frac{S-(3+1)\cdot 100}{\sqrt{(3+1)\cdot 900}})\cdot(.003) &= 0.95.
\end{aligned}
$$

S is obtained as 278.0*. For* Method 3*, the estimates for the probabilities of the number of outstanding orders and the corresponding inventory-on-orders by (3.15) and (3.16) are used to find the order-up-to-level for a target service level α_{rev} = %95 by solving the following equation:*

$$
\begin{aligned}
F_Z(\frac{S-100.1}{894.5})\cdot(.423)+F_Z(\frac{S-216.9}{1518.1})\cdot(.535) \\
+F_Z(\frac{S-324.0}{2662.3})\cdot(.039)+F_Z(\frac{S-426.1}{3508.0})\cdot(.003) &= 0.95.
\end{aligned}
$$

S is found to be 288.7*.* □

For the given example, $S = 278.0$ obtained by using *Method 2* is smaller than $S = 288.7$ obtained by using *Method 3*. Then, $S = 278.0$ results in lower no-stockout probability and average inventory level compared to $S = 288.7$ since higher(lower) S values result in higher(lower) inventory levels. S values obtained by *Method 2* and *Method 3* are then used in the simulation models to investigate performances of these S values. Simulation results are tabulated in Table 3.7. For the methods under consideration, recall that we work with *Model 1* at where the objective is to find S that is minimizing inventory holding costs defined as $h \cdot S$ while satisfying a target α-service level. Although $S = 278.0$ obtained by using *Method 2* provides lower inventory costs, the service level obtained by simulation is 93.45%, which is less than target level 95%. On the other hand, the simulation results show that $S = 288.7$ obtained by using *Method 3* gives a service level of 95.03% that is almost equal to target service level 95%.

Order-up-to-levels obtained by *Method 3* and the simulation results for these order-up-to-levels are given are given in Table 3.8 for different utilization levels. Moreover, the order-up-to-levels are calculated for different target service levels at a given utilization level of 90% and the simulation results are given in Table 3.9. It is seen that the service levels obtained by using *Method 3* are sufficiently close to 95% target level.

Order-up-to-levels obtained by using *Method 2* for different service and utilization levels are also given in Tables 3.10 and 3.11. Especially for high utilization levels, order-up-to-levels obtained by using *Method 2* are much lower than the order-up-to-levels obtained by using *Method 3*. Hence, *Method 2* cannot satisfy target no-stockout probabilities in these cases since we know that order-up-to-levels obtained by using *Method 3* gives no-stockout probabilities that is almost equal to target service levels. Recall that the situation is reverse for exogenous replenishment lead time as given in Section 3.1.

The other observations based on the simulation results are given below.

- *Method 2* and *Method 3* give the same means and variances of the order quantities. The reason is that the order-up-to-level is constant and the demand pattern and order pattern are exactly the same in this case.

- As utilization of the supplier gets higher, mean and variance of the replenishment lead time get higher since service times are higher for higher utilization levels.

Table 3.7: Simulation results for $S = 278.0, 288.71$; $u = 90\%$ and $\alpha = \%95$.

	Method 2	**Method 3**
Order-up-to-level (S)	278.0	288.71
Mean Demand	100.0	100.0
Variance of Demand	898.5	898.5
Mean Order Quantity	100.0	100.0
Variance of Order Quantity	898.5	898.5
Mean Replenishment Lead Time	1.13	1.13
Variance of Rep. Lead Time	0.19	0.19
Mean Net Inventory	156.7	167.5
Variance of Net Inventory	5235.3	5232.2
Average Inventory-on-hand	157.9	168.4
Variance of Inventory-on-hand	4766.9	4852.1
Fill Rate (β)	97.28%	97.83%
No-Stockout Probability (α)	93.45%	95.03%

Table 3.8: Simulation results for Method 3, $\alpha = 95\%$.

	Utilization of the Supplier (*u*)			
	50%	**75%**	**90%**	**95%**
Order-up-to-level (S)	149.4	260.34	288.7	413.8
Mean Demand	100.0	100.0	100.0	100.0
Variance of Demand	898.5	898.5	898.5	898.5
Mean Order Quantity	100.0	100.0	100.0	100.0
Variance of Order Quantity	898.5	898.5	898.5	898.5
Mean Replenishment Lead Time	0.50	0.77	1.13	1.61
Variance of Rep. Lead Time	0.02	0.05	0.19	0.74
Mean Net Inventory	94.9	176.4	167.5	244.0
Variance of Net Inventory	3380.3	3366.4	5232.2	11288.3
Average Inventory-on-hand	95.4	176.6	168.4	246.6
Variance of Inventory-on-hand	3264.7	3277.1	4852.1	9526.4
Fill Rate (β)	99.35%	98.91%	97.83%	96.91%
No-Stockout Probability (α)	94.97%	95.02%	95.03%	95.01%

Table 3.9: Simulation results for Method 3, $u = 90\%$.

	Target Service Level (α)				
	50%	**75%**	**85%**	**90%**	**95%**
Order-up-to-level (S)	176.3	227.7	248.8	263.3	288.7
Mean Demand	100.0	100.0	100.0	100.0	100.0
Variance of Demand	898.5	898.5	898.5	898.5	898.5
Mean Order Quantity	100.0	100.0	100.0	100.0	100.0
Variance of Order Quantity	898.5	898.5	898.5	898.5	898.5
Mean Replenishment Lead Time	1.13	1.13	1.13	1.13	1.13
Variance of Rep. Lead Time	0.19	0.19	0.19	0.19	0.19
Mean Net Inventory	55.1	106.5	127.5	142.1	167.5
Variance of Net Inventory	5229.3	5234.8	5231.2	5237.9	5232.2
Average Inventory-on-hand	67.1	110.7	130.0	143.7	168.4
Variance of Inventory-on-hand	2638.7	3977.7	4397.0	4610.8	4852.1
Fill Rate (β)	72.70%	90.87%	94.62%	96.21%	97.83%
No-Stockout Probability (α)	49.98%	74.98%	85.07%	90.07%	95.03%

Table 3.10: Order-up-to-levels calculated by using Methods 2 and 3, $u = 90\%$.

	Target Service Level (α)				
	50%	*75%*	*85%*	*90%*	*95%*
Method 2	158.73	211.76	234.41	249.83	278.00
Method 3	176.31	227.72	248.76	263.32	288.71

Table 3.11: Order-up-to-levels calculated by using Methods 2 and 3, $\alpha = 95\%$.

	Utilization of the Supplier			
	50%	*75%*	*90%*	*95%*
Method 2	149.44	221.32	278.00	399.34
Method 3	149.40	260.34	288.70	413.80

3.3 Endogenous Replenishment Lead Times with Adaptive (R, S_t) Policy

There is a considerable impact of the changes in the retailer's ordering policy on the inventory system with endogenous replenishment lead times because replenishment lead times are highly dependent on the order quantities. A large order quantity results in longer replenishment lead time whereas a small order quantity results in shorter replenishment lead time. Hence, replenishment lead times are higher (lower) for high (low) demand periods. The retailer would prefer to take action and change the ordering policy parameters when replenishment lead time increases in order to avoid stockouts beyond a certain level. That is , an adaptive ordering policy is employed by the retailer in that case. Then, the retailer would work with higher order quantities in such a situation. However, this causes more congestion in the system and replenishment lead times continue to increase in the following periods. This phenomenon is well-known and called as 'Lead Time Syndrome' in the literature, see the related reference due to Selçuk (2007). In a similar way, the parameters of ordering policy may be considered to be updated in the case of low demand periods to operate at lower inventory levels. In that case, the risk of being stockout increases since smaller order quantities are placed at consecutive periods.

Here, we consider the case with single retailer and single supplier. When there is congestion, it is obvious that increase in order quantity causes the retailer to perform worse in the long run. The reason is that the more congestion in the system is created after order quantities increase and the replenishment lead times get higher which result in more stockout occasions. In this case, also for the supplier order quantities become more variable and production cannot be smoothed, which in turn increases supplier's production costs. The solution to this problem is obvious: smoothing order quantities or at least not increasing order quantities during congestion periods until the congestion ends. This would work because there is only one retailer in the supply chain that determining the supplier's replenishment lead times. However, this solution approach may not work in the case of multiple retailers since replenishment lead times result from all of the retailers' ordering policies.

In this section, the question addressed is the following: how does the performance of the retailer affected in the case that the retailers prefer to use an adaptive policy when the demand observed by the retailer and the production system of the supplier is time-stationary. As in Section 3.2, production system of the supplier is modelled as a single-server queuing system. However, an adaptive periodic-review base-stock policy (R, S_t) is used by the retailer instead of a stationary (R, S) policy. In (R, S_t) policy, the order-up-to-level for review period t, S_t, is determined depending on the changes in the demand forecasts and the estimates of the replenishment lead time.

Use of adaptive policies based on demand forecasting is frequently studied in the literature. However, this is not the case for the use of adaptive policies based on the update of replenishment lead time estimates. Our study focuses on the update of replenishment lead time estimates and than the policy parameters for the retailer. Replenishment lead time may change in time and this change is either temporary or permanent. A temporary change may naturally arise from the randomness of the production time or delivery time. However, a permanent change may be due to the change in the state of supplier's operations such as production facilities, delivery modes. For example, capacity of the supplier may increase/decrease because of the change in the amounts of resources available. If the retailer decides that a change in the replenishment lead time should be reflected to the ordering policy parameters, then S_t should be updated to ensure the same target service level under the new conditions. In the models under consideration in this section, the production system of the supplier is time-stationary. Since we also work with time-stationary demand, the changes in the replenishment lead times is temporary in our models. As stated previously, the long run performance measures worsen when an adaptive policy employed by the retailer. In this thesis, we aim to measure the decreases in the performance measures caused by the use of adaptive policy based on the update of the replenishment lead time estimates. Moreover, we investigate whether there is gain in the short run measures by the use of adaptive policy.

There is an important point in our inventory model related with the calculation of order-up-to-level. Since our objective is to find order-up-to-level providing a target service level α_{rev}, the probabilities of the replenishment lead times are not considered directly or explicitly in the equations to determine order-up-to-level. As explained in the previous sections, the distributions of the number of outstanding orders and inventory-on-order should be obtained to calculate the order-up-to-level. However, the analytical formulation of the relation between the distribution of inventory-on-order and replenishment lead times is involved mathematically when the replenishment lead times are endogenous. Such an analysis is performed by Boute (2006) by modelling the system as a discrete-time Markov chain. Because of the difficulty of the mathematical analysis, the approach in Section 3.2 is used to calculate S except that the estimates should now be updated frequently. In this section, we only work with *Method 3* to calculate the order-up-to-levels since it is shown in Section 3.2 that *Method 2* does not give order-up-to-levels satisfying target service levels. Again, mean and variance of the inventory-on-order for each possible number of outstanding orders are estimated by a simulation model and used in the calculation of the order-up-to-level. Thus, mean and variance of inventory-on-order given that there are k outstanding orders should be estimated at the end of each update period, which is the duration between two consecutive update of the ordering policy parameters in time.

The estimates $E(IO|K = k)$ and $Var(IO|K = k)$ are obtained exactly as in

Section 3.2 except that r is the number of the replications for each update period and $o_i = o$. Then, S is calculated using 3.14 at the end of each update period. Determining the length of the update period is an issue for the use of adaptive policies. In our inventory analysis, a simple update mechanism is considered. As seen above, S is updated at the end of a constant update period of o *review periods*. Then, the order-up-to-level used to determine order quantities in the n^{th} update period is defined as $S_{(n)}$ and the same order-up-to-level is used for all review periods in the n^{th} update period. That is, $S_{(n-1)\cdot o+t} = S_{(n)}$ for all $t = 1, ..., o$. Actually, the parameters of the ordering policy should have been updated if the retailer decides that a change in the estimates of replenishment lead time should be reflected to the ordering policy parameters. And, this decision can be affected by many factors such as the performance measures obtained by the retailer in the last update period, the forecasts for future periods' demands, the state of the supplier.

Numerical Example 5. *The same setting used in Numerical Example 4 is used to analyze the case of adaptive (R, S_t) policy. Again, the service time of the single server is assumed to be constant and equals to* 0.009 *review periods for an utilization level of* $u = 0.90$. *The distribution of the demand during review period is assumed to be Normal with mean* 100 *and variance* 900 *and target α-service level is set as 95%. Finally, update periods of 10, 25, 50 and 100 review periods are considered for this numerical example.* □

For the numerical setting given in Numerical Example 5, simulation results for each value of update period and the results are given in Table 3.12. The following observations have been derived from the given results.

- Mean and variance of the order-up-to-level increase as update periods get shorter. Also, variance of the order quantities increases in a similar way.

This means that more erratic order pattern is observed when the update frequency for S is increased.

- Mean and variance of the replenishment lead time increase when the update frequency for S_t is increased. That is, more erratic replenishment pattern is observed by the supplier.

- Means and variances of inventory position and inventory-on-hand increase when the update frequency for S is increased. This is in accordance with the observation in the first item above and shows that higher levels of safety stock are held for lower update periods.

- Fill rate and the probability of no-stockout in the long-run decrease as the update frequency increases. No-stockout probabilities obtained by the simulation runs are below the target level of 95% and the difference between target level and observed values gets larger as update frequency gets higher. Thus, the delivery performance of the retailer deteriorates when the order-up-to-level is updated frequently.

- Variance of the order quantity is higher than the variance of the demand for all cases. These amplification in variance get larger as update frequency increases.

These observations have allow us to come up with the following interpretation. As replenishment lead time gets longer during the congestion periods, order-up-to-levels would get higher and then, orders with larger quantities are placed to cover increased expected demand during the longer lead time, leading to longer queues in production system of the supplier. Thus, replenishment lead times get longer and this causes again a longer lead time. Eventually, erratic ordering and replenishment behaviors are observed in the system, resulting in larger variances on inventory levels and replenishment lead times. This phenomenon

Table 3.12: Simulation results for different update periods, u=0.90.

	Update Period (*Review Periods*)			
	10	*25*	*50*	*100*
Mean Demand	100.0	100.0	100.0	100.0
Variance of Demand	898.5	898.5	898.5	898.5
Mean Order-up-to-level	304.7	294.6	291.3	290.0
Variance of Order-up-to-level	2158.4	639.7	245.4	114.4
Mean Order Quantity	100.0	100.0	100.0	100.0
Variance of Order Quantity	993.1	921.9	905.2	900.4
Mean Replenishment Lead Time	1.17	1.14	1.13	1.13
Variance of Rep. Lead Time	0.26	0.21	0.20	0.19
Mean Net Inventory	178.6	172.2	169.7	168.7
Variance of Net Inventory	7028.3	5844.0	5481.1	5360.6
Average Inventory-on-hand	179.5	173.1	170.7	169.6
Variance of Inventory-on-hand	6600.5	5438.8	5091.7	4970.9
Fill Rate (β)	97.75%	97.80%	97.82%	97.81%
No-Stockout Probability (α)	94.53%	94.79%	94.91%	94.96%

is called as 'lead time syndrome'. Lead time syndrome should be avoided since it causes the production cost of the supplier to increase by preventing smooth production levels. Also, the customer service of the retailer deteriorates in the existence of lead time syndrome because of the increases in mean and variance of the replenishment lead time.

The observation in the last item above shows the existence of the bullwhip effect, which is the amplification in demand variability as one moves from the lowest echelon (retailer) to the highest echelon (supplier) in the supply chain. Thus, updating the replenishment lead time result in bullwhip effect. Similar to lead time syndrome, bullwhip effect should be avoided because it reduces the profitability of a supply chain by making it more expensive to provide a given level of product availability as stated by Chopra and Meindl (2007). Moreover, the observation that there is an increase in the mean of inventory-on-hand caused by frequent update of the ordering policy parameters shows that the inventory holding cost gets higher in this case. Although more inventory is kept, that is safety stock gets higher, the service level of the retailer decreases.

Another interesting question to be answered is the following: what happens if the utilization level of the supplier gets higher? The following numerical example is considered to answer this question.

Numerical Example 6. *The same setting used in Numerical Example 5 is used except that the utilization level of the supplier is 95%. This is achieved by setting the service time of the single server to* 0.0095 *review periods. Again, update periods of 10, 25, 50 and 100 review periods are considered for this numerical example.* □

The results obtained by simulating the system in Numerical Example 6 are given in Table 3.13. When the results given in Tables 3.12 and 3.13 for $u = 90\%$ and $u = 95\%$, respectively, are compared, the followings are observed for the same values of update period:

- Means and variances of the order-up-to-level and order quantity are higher for 95% utilization level. (See Figure 3.4)
- Mean and variance of the replenishment lead time are higher for 95% utilization level. (See Figure 3.5)
- Mean and variance of inventory-on-hand is higher for 95% utilization level. This observation shows that safety stock to achieve the same service level is higher for 95% utilization level.
- Decreases in fill rate and no-stockout probability are larger for 95% utilization level.

- Increase in the amplification in variance is bigger for 95% utilization level.

The main conclusion from these observations is that all of the performance measures deteriorate for 95% utilization level when compared to 90% utilization level. By considering the results obtained using stationary (R, S) policy in Section 3.2, we can state that stationary policy leads to the lowest cost situation for both 90% and 95% utilization levels. However, replenishment lead times are not always known before they are realized and have to be estimated somehow. Then, an adaptive ordering policy should be inevitably employed by the retailer although it is not optimal policy.

Table 3.13: Simulation results for different update periods, u=0.95.

	Update Duration (Review Periods)			
	10	25	50	100
Mean Order Up To Point	606.63	491.00	441.13	425.22
Variance of Order Up To Point	122279.58	34628.28	9482.48	3898.91
Mean Order Quantity	100.01	100.01	100.01	100.01
Variance of Order Quantity	1426.36	1124.65	978.40	928.39
Mean Replenishment Lead Time	2.6123	1.9363	1.7055	1.6437
Variance of Rep. Lead Time	6.1824	1.9480	1.0153	0.8220
Mean Net Inventory	332.31	287.23	261.51	252.11
Variance of Net Inventory	63858.61	34474.27	19344.06	14843.17
Average Inventory-on-hand	336.31	290.61	264.51	254.90
Variance of Inventory-on-hand	60430.55	31915.70	17224.29	12940.31
Fill Rate (β)	95.53%	96.07%	96.50%	96.67%
No-stockout Probability (α)	92.90%	93.65%	94.29%	94.57%

Note that the performance measures given in the above tables are evaluated in the long run. That is, for such steady state measures, naturally use of stationary policy makes sense in a stationary setting. Next, we investigate whether there is gain in the short run by the use of adaptive policy. Then, α-service levels obtained in each update period are observed by the simulation analysis and the results for the mean and standard deviation of α-service level are tabulated in Table 3.14. The results show that more variable α-service levels are observed for more frequent updates. That is, the variation in α-service level increases significantly as update period gets shorter although mean α-service level in the long run decreases slightly. Figures 3.7 and 3.8 illustrate the situation. It is seen in these figures that more smooth α-service levels are observed when the update period is 100 *review periods* compared to the case that the update period is 25 *review periods*. The reason is due the fact that the system is more responsive when the update period gets shorter. Then, this responsive pattern results in higher (lower) S values when the replenishment lead times get higher (lower) and eventually these S values result in higher (lower) α-service levels for the consecutive review periods. Figures B.1 and B.2 in Appendix B illustrate this situation by giving S values calculated at the end of each update period and α-service levels obtained in each update period. Moreover, Figures B.3 and B.4 in Appendix B show the order quantities placed by Retailer 2 when update period is 10 *and*100 *review periods*, respectively. As it is seen in these figures, more variable order quantities are placed when update period is 10 *review periods* compared to 100 *review periods*.

Table 3.14: Simulation results for α-service levels, $u = 95\%$.

	Update Period			
	100	**50**	**25**	**15**
Average	94.57%	94.29%	93.65%	92.100%
Standard Deviation	11.85%	14.79%	17.38%	19.29%

Figure 3.4: Means and variances of the order quantity, $u = 90\%, 95\%$.

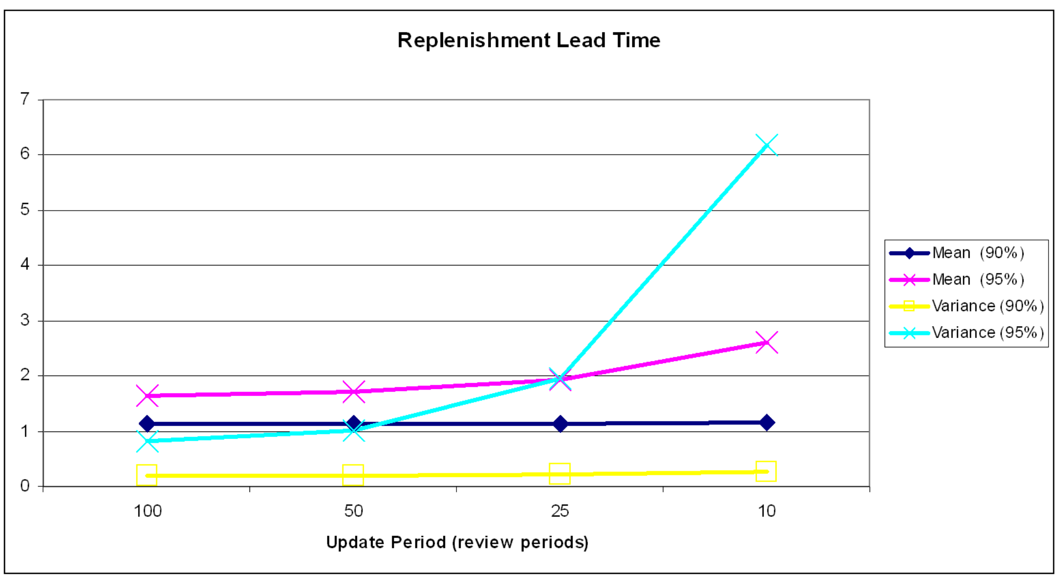

Figure 3.5: Means and variances of the replenishment lead time, $u = 90\%, u = 95\%$.

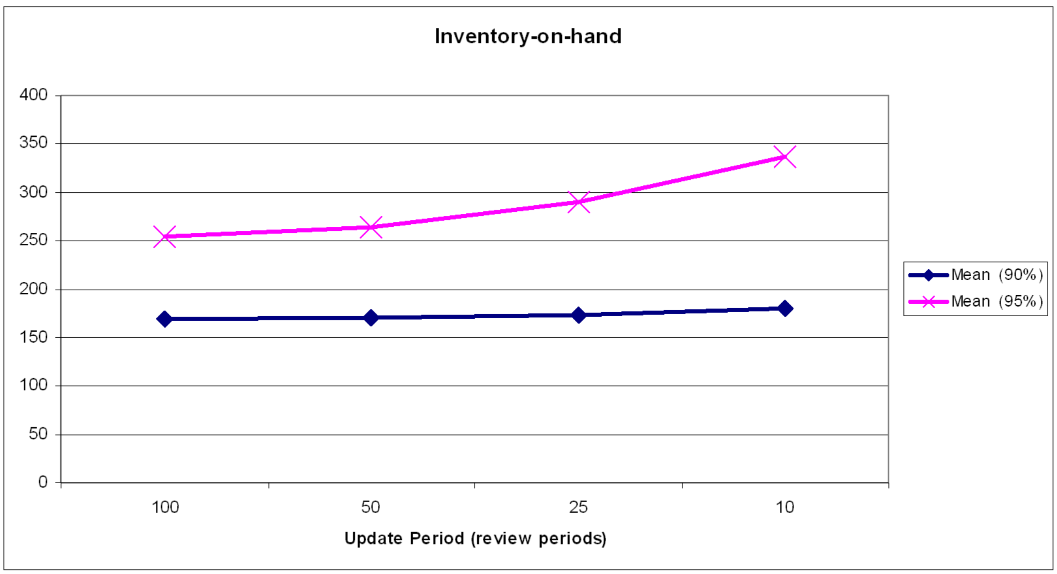

Figure 3.6: Means and variances of inventory-on-hand, $u = 90\%, 95\%$.

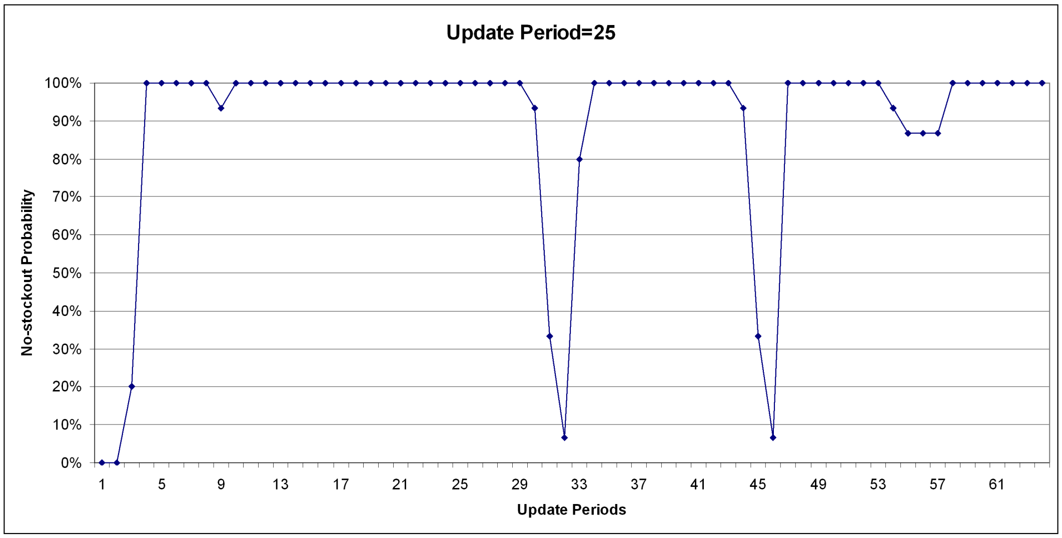

Figure 3.7: No-stockout probabilities for the update period of 25 *review periods*, $u = 95\%$.

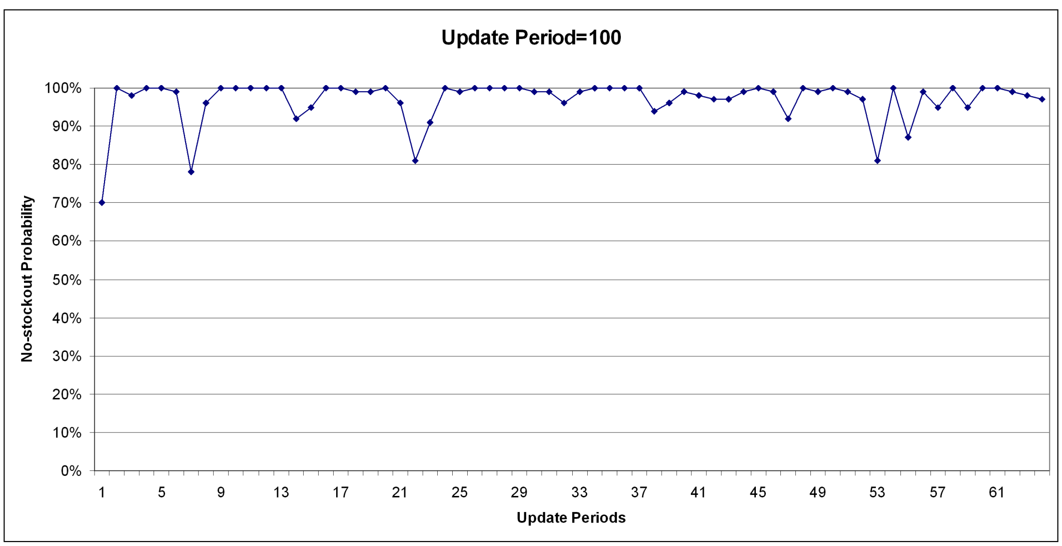

Figure 3.8: No-stockout probability for the update period of 100 *review periods*, $u = 95\%$.

Chapter 4

TWO-RETAILER CASE

In this chapter, we analyze the models with two retailers and single supplier. Results of the models with exogenous replenishment lead times are the same as the ones in Section 3.1 because exogenous replenishment lead times are independent of the ordering policies of the retailers. However, endogenous replenishment lead times are determined by the ordering policies of both retailers because the retailers place orders from the same supplier having a capacitated production system. In this case, a change in the ordering policy of a retailer has an impact on the other retailer's performance even if the other retailer does not make any change in its own ordering policy. That is, a retailer has to update its own ordering policy in order to obtain the target service level if the other retailer changes its ordering policy. At first glance, it does not make sense for the retailers to employ adaptive policies in a time-stationary setting. However, the retailers would tend to increase order quantities when higher replenishment lead times are observed because of a congestion in the system. In this chapter, the question addressed is the following: how do the retailers performs when one or both of the retailers prefer to use an adaptive policy. In the models of this chapter, we assume the retailers do not collaborate to share information. That is, the case we study is the case of strict competition. Demands of the retailers are assumed i.i.d and independent of each other. This assumption is realistic if they operate in different markets. Also, the distribution of the demand in a review period is assumed to be Normal as in Chapter 3.

Recall that we work with the models minimizing inventory costs while providing a target service level for no-stockout probability in a review period. For two-retailer case, a mathematical model similar to the given models in Chapter 3 are presented below.

$$
\begin{aligned}
MODEL\,4: \;\; & Minimize \;\sum_{R=1}^{2} h \cdot \overline{I}^{(R)}(S^{(R)}) \\
& subject\ to \\
& A^{(R)}(S^{(R)}) \geq \alpha^{(R)} \qquad for\ R = 1, 2, \\
& S^{(R)} \geq 0 \qquad for\ R = 1, 2,
\end{aligned}
$$

where superscript (R) is used to differentiate the retailers with $R = 1$ defining the first retailer and $R = 2$ defining the second retailer, $S^{(R)}$ is the order-up-to-level used in the ordering policy by Retailer R. Moreover, $\overline{I}^{(R)}(S^{(R)})$ denotes average inventory-on-hand in the long run and $A^{(R)}(S^{(R)})$ is the no-stockout probability in a review period for the given retailer. As in Chapter 3, we prefer to consider $S^{(R)}$ in the objective function Instead of $\overline{I}^{(R)}(S^{(R)})$. Then, the mathematical model becomes

$$
\begin{aligned}
MODEL\,5: \;\; & Minimize \;\sum_{R=1}^{2} h \cdot S^{(R)} \\
& subject\ to \\
& Pr(IO^{(R)} < S^{(R)}) \geq \alpha^{(R)}_{rev} \qquad for\ R = 1, 2, \\
& S^{(R)} \geq 0 \qquad for\ R = 1, 2,
\end{aligned}
$$

where $IO^{(R)}$ denotes average inventory-on-order in the long run for Retailer R. In the following sections, we show how to calculate $S^{(R)}$ values for the given mathematical models. In Section 4.1, the case that both retailers employ stationary (R, S) policy is studied. In Section 4.2, it is questioned whether it is beneficial for a retailer to use an adaptive (R, S_t) policy in a supply chain with two retailers.

4.1 Endogenous Replenishment Lead Times with Stationary (R, S) Policy

We aim to investigate the use of the stationary (R, S) policy in this section for both retailers. As explained in Sections 3.2 and 3.3, exact analysis to calculate the order-up-to-level for the case of endogenous replenishment lead times would be very complex. Thus, the solution approach considered in Section 3.2 is used also in this section. The initial simulation run is for arbitrary S values and then, the order-up-to-levels that ensure target service levels of the retailers are calculated using the simulation results. Again, the order and demand patterns are the same for any value of the order-up-to-level S as long as both retailers use stationary (R, S) policies. Hence, we choose two arbitrary initial order-up-to-level for each of the two retailers and observe the number of outstanding orders and the corresponding inventory-on-orders at the end of the each review period just after an order is placed. Mean and variance of inventory-on-order for each possible number of outstanding orders are estimated as in Chapter 3. Then, *Method 3* is used to calculate order-up-to-levels of each retailer.

For the numerical experiments, service time of the supplier that is required to achieve a predetermined utilization level should be determined. Here, we assume the service times are constant as in Chapter 3. Then, the utilization level u is given by the following equation in terms of mean service time $E(T)$ and mean review period demands $E(D^{(1)})$ and $E(D^{(2)})$) of retailer 1 and 2, respectively:

$$u = \frac{E(D^{(1)}) + E(D^{(2)})}{1/E(T)}. \tag{4.1}$$

Numerical Example 7. *Assuming that the distribution of the demands of the retailers are Normal with mean* 100 *and variance* 900*, mean service time giving* $u = 0.90$ *is calculated as*

$$E(T) = \frac{u}{E(D^{(1)}) + E(D^{(2)})} = \frac{0.90}{100 + 100} = 0.0045 \textit{ review periods.}$$

$S^{(R)}$ *values used in the initial simulation run is chosen as 100 for both retailers. During the initial simulation run, the number of outstanding orders with the corresponding inventory-on-orders are observed at the end of the each review period just after an order is placed and* $E(IO|K = k)$ *and* $Var(IO|K = k)$ *for each possible* k *are obtained by using (3.15) and (3.16), respectively. The estimates obtained by the initial simulation run are given in Table 4.1.*

Table 4.1: Estimates obtained by initial simulation run with $S^{(R)} = 100$ for $R = 1, 2$.

k	Frequency		$E(IO\|K = k)$		$Var(IO\|K = k)$	
	Retailer 1	Retailer 2	Retailer 1	Retailer 2	Retailer 1	Retailer 2
1	95.78%	95.78%	100.03	100.04	895.38	895.53
2	4.22%	4.22%	222.36	222.36	1783.74	1782.91

For the symmetric numerical setting such that parameters are the same for both of the retailers, simulation results show that estimates of the retailers are the *same. Then, the following equation is solved to find* $S^{(1)}$ *for Retailer 1:*

$$F_Z(\frac{S-100.03}{895.38}) \cdot (.9578) + F_Z(\frac{S-222.36}{1783.74}) \cdot (.0422) = 0.95.$$

Solution of the above equation gives $S^{(1)} = 166.97$ *for Retailer 1. In a similar way,* $S^{(2)}$ *for Retailer 2 is calculated as* 166.98. □

Simulation results for $S^{(R)}$ values found above are given in Table 4.2. Simulation results are almost the same for each retailer since they employ the same ordering policy with nearly equal $S^{(R)}$ values. The average of α-service levels obtained by 30 replications turn out to be 94.97% and 94.98% when target service levels are 95%. Half-widths for 95% confidence level are 0.08% and 0.07% for α-service levels obtained for Retailer 1 and 2, respectively. $\alpha = 95\%$ is between upper and lower limits. Hence, we conclude that $S^{(R)}$ values calculated by *Method 3* provides the target service levels in a 95% confidence level.

Moreover, the simulation results show that means and variances of the order quantity and demand are equal to each other. The reason is that the order-up-to-level is constant and the demand pattern and order pattern are equal in this case. This observation shows bullwhip effect, which is the amplification in demand variability as one moves from the lowest echelon (retailer) to the highest echelon (supplier) in the supply chain, does not observed in this case.

The next question to be answered is that what happens if different utilization levels for the supplier are considered. Then, the results for 75% and 95% utilization levels with a target service level of 95% are obtained by simulation analysis and given in Table 4.3. Again, the results show that α values obtained by the simulation are sufficiently close to the target levels. It is observed from the sim-

Table 4.2: Simulation results, $u = 90\%$.

	Retailer-1		Retailer-2	
	Average	Half-width	Average	Half-width
Mean Demand	100.0	0.0	100.0	0.0
Variance of Demand	899.2	1.4	899.2	1.5
Mean Order Quantity	100.0	0.0	100.0	0.0
Variance of Order Quantity	899.2	1.4	899.2	1.5
Mean Replenishment Lead Time	0.57	0.00	0.57	0.00
Variance of Rep. Lead Time	0.05	0.00	0.05	0.00
Mean Net Inventory	106.3	0.1	106.3	0.1
Variance of Net Inventory	3485.4	6.7	3487.2	7.0
Average Inventory-on-hand	106.9	0.1	106.9	0.1
Variance of Inventory-on-hand	3328.9	2.9	3330.0	3.1
Fill Rate (β)	97.52%	0.05%	97.52%	0.04%
No-Stockout Probability (α)	94.98%	0.08%	94.97%	0.07%

ulation results that more inventory should be kept to satisfy the same service level for higher utilization levels. Figure 4.1 illustrates the situation.

Table 4.3: Simulation results for different utilization levels.

	Utilization Level		
	75%	**90%**	**95%**
Order-up-to-level (S)	149.34	166.97	255.45
Mean Demand	100.0	100.0	100.0
Variance of Demand	899.2	899.2	899.2
Mean Order Quantity	100.0	100.0	100.0
Variance of Order Quantity	899.2	899.2	899.2
Mean Replenishment Lead Time	0.39	0.57	0.80
Variance of Rep. Lead Time	0.01	0.05	0.18
Mean Net Inventory	85.4	106.3	170.7
Variance of Net Inventory	3119.3	3485.4	4784.9
Average Inventory-on-hand	107.8	85.7	171.5
Variance of Inventory-on-hand	3021.1	3328.9	4443.5
Fill Rate (β)	99.39%	97.52%	98.09%
No-Stockout Probability (α)	94.98%	94.98%	95.00%

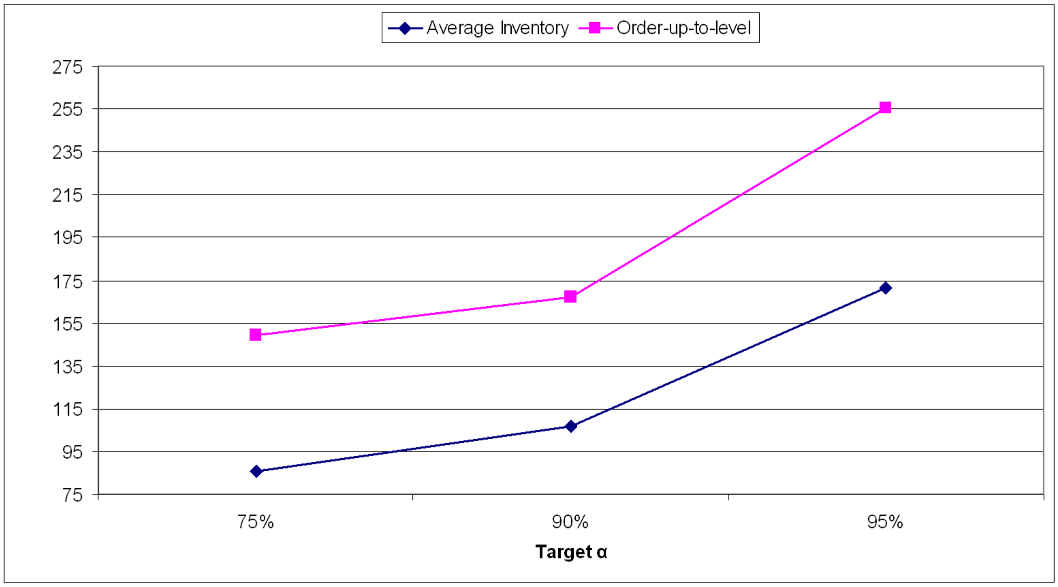

Figure 4.1: Plot of target no-stockout probability vs. average inventory obtained by the simulation analysis.

4.2 Endogenous Replenishment Lead Times with Adaptive (R, S_t) Policy

In this section, use of adaptive ordering policies is questioned for the case of two retailers. In the case of single retailer in Section 3.3, we observe that there is no gain of updating the ordering policy parameters frequently when long-term measures are used. The situation is now different since it is possible for each of the retailers to have an impact on the performance of the other retailer by causing a change in the status of the supplier's production system. For example, a large order quantity placed by one of the retailers results in longer replenishment lead time observed by the other retailer for the future orders.

The approach considered in Section 3.3 is used also in this section. Parameters of the ordering policies are updated by observing the inventory system during the last update period. Again, the length of the update period is kept constant.

Numerical Example 8. *The setting is the same as in Numerical Example 7. Service time of the single server is set as* 0.0045 *review periods to obtain an utilization level* $u = 0.90$*. The demand distributions observed by the retailers are assumed Normal with mean* 100 *and variance* 900*. In the first simulation runs, a target service level* α *is chosen as* 95% *for both retailers. Retailer 2 uses an adaptive* (R, S_t) *policy for the update periods of 10, 25, 50 review periods and Retailer 1 uses a stationary* (R, S) *policy with* $S = 166.97$ *which is the order-up-to-level calculated in Numerical Example 7 where both retailers use stationary* (R, S) *policies and target service level* 95%*.* □

Simulation analysis for the given setting in *Numerical Example 8* gives the results for long-run measures in Table 4.4. For a confidence level of 95%, it cannot be concluded that use of the adaptive (R, S_t) policy by Retailer 2 instead of the stationary (R, S) policy with $S = 166.98$ causes a decrease in α-service levels for the update periods of 25, 50 *review periods* although average α-service levels obtained for the update periods of 25 and 50 *review periods* is lower than α-service levels obtained for the use of the stationary (R, S) policy by Retailer 2. However, for the update period of 10 *review periods*, there is a decrease in α-service levels from 94.97% to 94.47% for Retailer 1 and from 94.98% to 94.72% for Retailer 2. Upper and lower limits on no-stockout probability based on 95% confidence level are shown in Figures 4.2 and 4.3 for Retailers 1 and 2, respectively. When the update period of Retailer 2 is 10 *review periods*, the decreases in α-service levels are 0.51% and 0.25% for Retailer 1 and 2, respectively and Retailer 1 observes lower service level than Retailer 2 based on the limits for 95% confidence level. That is, the deterioration in the service level of the retailer who uses a stationary (R, S) policy is larger than the retailer who uses an adaptive policy when the update period is 10 *review periods*. For the other update periods, there is no statistical evidence to come up to the same conclusion.

Another observation based on the simulation results is that mean and variance of the inventory carried are higher for Retailer 2 compared to Retailer 1 for all values of the update period (See Figure C.1 in Appendix C). For example, when the update period is 10 *review periods*, mean inventory carried is equal to 106.32 and 117.09 for Retailers 1 and 2, respectively. That is, there is almost 10% increase in mean inventory carried for Retailer 2 compared to Retailer 1. According to MODEL 4, this result means that inventory holding costs are 10% higher for the retailer using adaptive policy in this case. Although more inventory is kept by Retailer 2, α-service level observed by Retailer 2 is 94.72%, which is slightly below the target level 95%. Hence, the constraint on the service level in MODEL 4 is not satisfied. However, the service level observed by Retailer 2 is 0.26% higher than the service level observed by Retailer1. In other words, the service level observed by the retailer using adaptive policy is higher compared to the service level observed by the retailer using stationary policy but this comes up with an increase in the inventory holding costs.

Under these findings, we cannot answer the question that whether or not it is beneficial to use an adaptive policy. The answer completely depends on the trade-off between the service level provided to the customer and the inventory holding costs charged to the retailer. Note that we consider only mean inventory carried to compare the results in a cost view. However, variance of inventory carried also increases considerably for the retailer using adaptive policy which may directly or indirectly contribute to the costs of the retailer. Moreover, the results show that the decrease in α-service level obtained by the retailer using stationary policy is faster compared to α-service level obtained by the retailer using adaptive policy. Conversely, the increase in mean inventory carried by the retailer using stationary policy is slower compared to mean inventory carried by the retailer using adaptive policy.The other observations have been derived from the given results are itemized below.

- For Retailer 2, mean and variance of the order-up-to-level increase as update periods get shorter. Variance of the order quantities increases in a similar way and is higher than the variance of the demand for all cases. These amplification in variance gets bigger as update frequency increases. Hence, bullwhip effect is present and ordering policy of Retailer 2 contributes to the bullwhip effect observed in the system. For Retailer 1, variance of the order quantity is equal to variance of the demand for any value of the update period. That is, Retailer 1 do not contribute to the bullwhip effect. Figure C.2 in Appendix C illustrates these findings.

- Means and variances of the replenishment lead time for both retailers increase when the update frequency increases. That is, more erratic replenishment pattern is observed by the supplier. Actually, these increases in mean and variance of the replenishment lead time causes the service levels observed by the retailers to decrease as update frequency increases. The graphical representation of these results is given in Figures C.3 and C.4 in Appendix C. These results can be explained by lead time syndrome observed in the system as discussed in Section 3.3. Note that the means of the replenishment lead time observed by both retailers are almost equal to each other for all update periods. However, the increase in variance of the replenishment lead time is faster for Retailer 2 compared to Retailer 1 as update frequency increases.

Table 4.4: Simulation results with adaptive and stationary policies, $u = 90\%$.

Retailer	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**
S for stationary (R, S) policy	166.97	166.98	166.97	-	166.97	-	166.97	-
Update Period	-	-	-	50	-	25	-	10
Mean Demand	100.00	100.02	100.00	100.02	100.00	100.02	100.00	100.02
Variance of Demand	899.20	899.18	899.20	899.18	899.20	899.18	899.20	899.18
Mean Order Up To Level	-	-	-	169.67	-	171.96	-	178.09
Variance of Order Up To Level	-	-	-	144.32	-	304.38	-	821.22
Mean Order Quantity	100.00	100.02	100.00	100.02	100.00	100.02	100.00	100.02
Variance of Order Quantity	899.20	899.18	899.20	904.01	899.20	915.31	899.20	971.80
Mean Replenishment Lead Time	0.5656	0.5657	0.5661	0.5661	0.5672	0.5671	0.5727	0.5724
Variance of Rep. Lead Time	0.0477	0.0477	0.0479	0.0480	0.0485	0.0488	0.0516	0.0528
Mean Net Inventory	106.31	106.30	106.34	109.00	106.21	111.15	105.67	116.52
Variance of Net Inventory	3485.43	3487.16	3486.97	3628.74	3492.56	3778.40	3519.94	4229.08
Average Inventory Carried	106.87	106.85	106.89	109.56	106.78	111.71	106.32	117.09
Variance of Inventory Carried	3328.94	3330.02	3329.55	3469.45	3330.61	3615.89	3336.59	4061.24
Fill Rate (β)	97.52%	97.52%	97.50%	97.58%	97.46%	97.63%	97.23%	97.72%
No-Stockout Probability (α)	94.98%	94.97%	94.95%	94.91%	94.88%	94.85%	94.47%	94.72%
Half-width for 95% Confidence Level	0.08%	0.07%	0.07%	0.08%	0.08%	0.07%	0.08%	0.06%

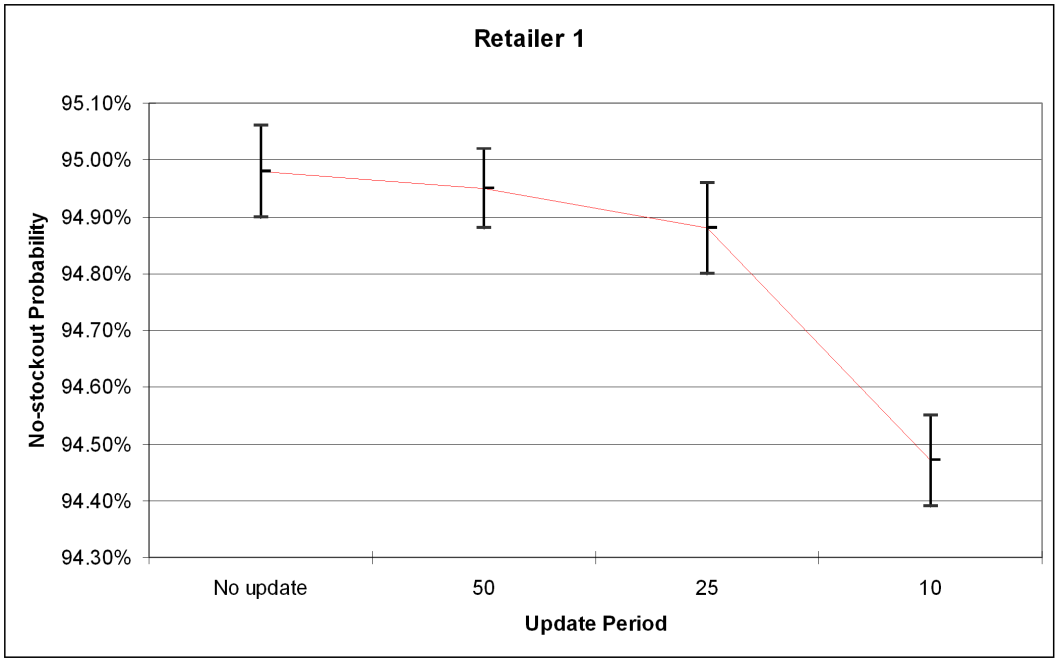

Figure 4.2: Upper and lower limits on no-stockout probability based on 95% confidence level, Retailer 1.

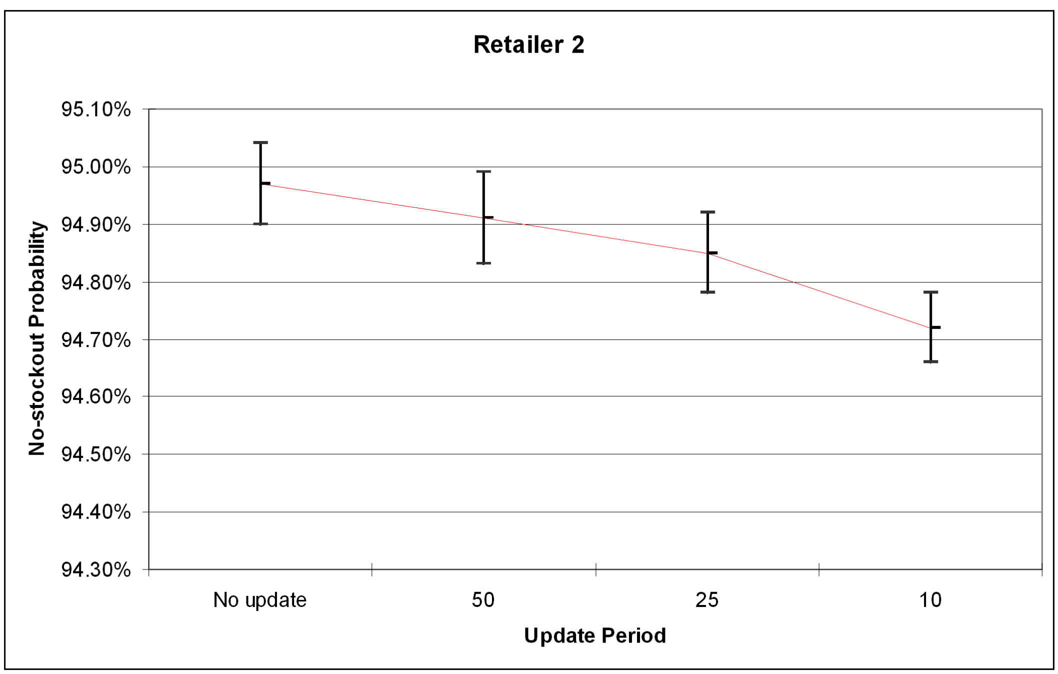

Figure 4.3: Upper and lower limits on no-stockout probability based on 95% confidence level, Retailer 2.

Numerical Example 9. *Continuing with the same setting in Numerical Example 8, we investigate the case both retailers use adaptive* (R, S_t) *policies. Again, the target service level is chosen as* 95% *for both retailers.* □

Simulation results on long-run measures for Numerical Example 9 are given in Table 4.5. As it is seen in Figures 4.4 and 4.5, it can be concluded that the use of the adaptive (R, S_t) policies by both of the retailers causes decreases in α-service levels and the decrease gets larger as update frequency increases. The graph in Figure D.2 in Appendix D compares α-service levels obtained by Retailer 2 in Numerical Example 8, where only Retailer 2 uses adaptive ordering policy, and in Numerical Example 9, where both retailers use adaptive ordering policies. As it is seen in this figure, the decrease in α-service level is higher when both retailers use adaptive ordering policies and the gap between α-service levels increases as update period gets shorter. Moreover, for the same value of update period, Retailer 2 keeps more inventory when both retailers use adaptive policies as it is illustrated in Figure D.3 in Appendix D.

The comparisons for the other performance measures are given in the figures given in Appendix D. The other observations based on the simulation results are as follows:

- Average and variance of the inventory carried get higher when both retailers use adaptive ordering policy. Hence, the inventory holding costs defined in the objective function of MODEL 4 increase even though the constraint on no-stockout probability cannot be satisfied.

- Variance of the order quantity for both retailers also increases when both retailers use adaptive ordering policy. Thus, the bullwhip effect is present and both retailers contribute equally to the bullwhip effect.

- Both mean and variance of the replenishment lead times are higher when both retailers use adaptive ordering policy. Thus, the effect of lead time syndrome on the system is bigger for this case.

Based on the above observations, we conclude that all of the performance measures deteriorate when both retailers use adaptive (R, S_t) policy compared to only one retailer uses adaptive policy and the deterioration gets larger as the update period gets shorter. The following numerical example is considered to answer what happens if the utilization level of the supplier is higher than 90%.

Table 4.5: Results of the simulations when both retailers uses adaptive policy, $u = 90\%$.

Retailer	**1**	**2**	**1**	**2**	**1**	**2**
Update Period	50	50	25	25	10	10
Mean Demand	100.00	100.02	100.00	100.02	100.00	100.02
Variance of Demand	899.20	899.18	899.20	899.18	899.20	899.18
Mean Order Up To Level	170.44	170.58	174.96	175.49	187.58	189.22
Variance of Order Up To Level	166.38	171.88	443.02	476.55	1454.22	1653.86
Mean Order Quantity	100.00	100.02	100.00	100.02	100.00	100.02
Variance of Order Quantity	904.13	904.15	918.83	919.87	1003.37	1015.84
Mean Replenishment Lead Time	0.5669	0.5669	0.5712	0.5711	0.5964	0.5963
Variance of Rep. Lead Time	0.0485	0.0486	0.0512	0.0515	0.0691	0.0705
Mean Net Inventory	109.70	109.82	113.73	114.21	123.52	124.93
Variance of Net Inventory	3649.30	3656.60	3906.03	3946.29	4752.66	4961.12
Average Inventory Carried	110.25	110.37	114.29	114.77	124.12	125.56
Variance of Inventory Carried	3489.75	3496.45	3740.94	3779.01	4566.00	4762.42
Fill Rate (β)	97.59%	97.58%	97.64%	97.62%	97.68%	97.58%
No-Stockout Probability (α)	94.91%	94.90%	94.80%	94.78%	94.42%	94.31%
Half-width for 95% Confidence Level	0.08%	0.07%	0.07%	0.07%	0.06%	0.06%

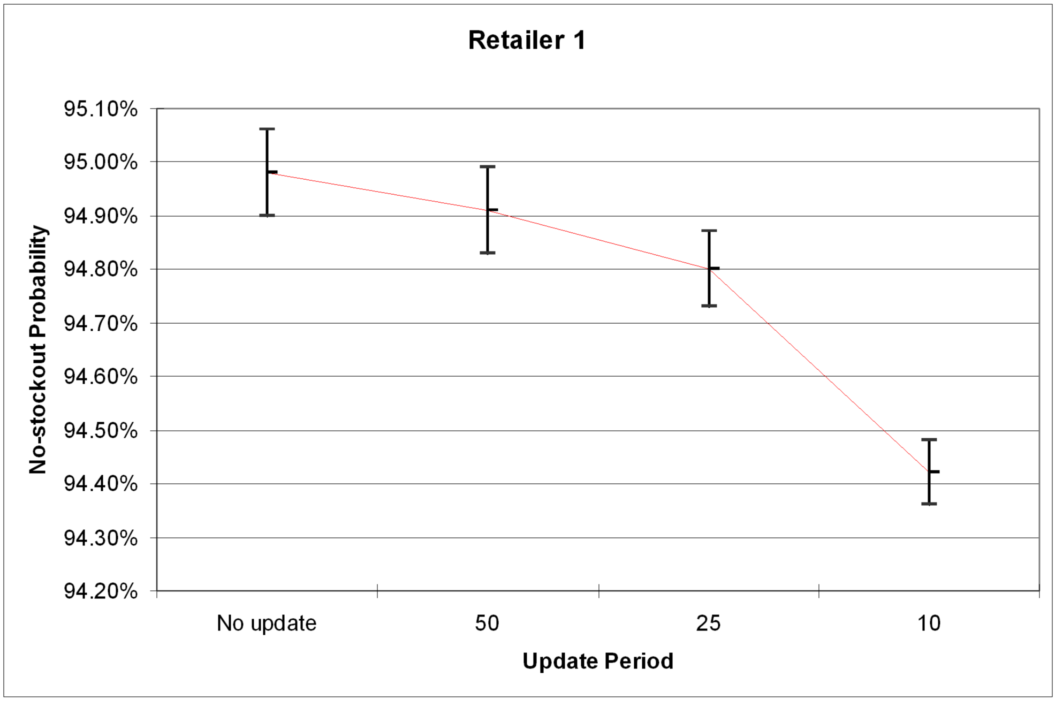

Figure 4.4: Upper and lower limits on no-stockout probability for different update periods based on 95% confidence level, Retailer 1, Numerical Example 9.

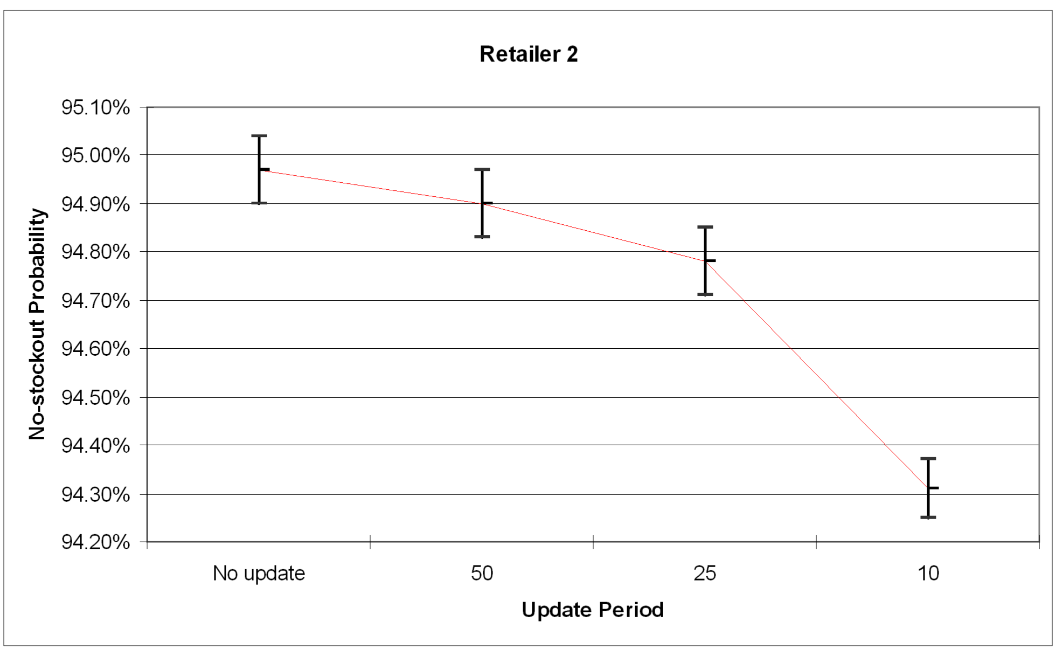

Figure 4.5: Upper and lower limits on no-stockout probability for different update periods based on 95% confidence level, Retailer 2, Numerical Example 9.

Numerical Example 10. *In this example, the same settings in Numerical Examples 8 and 9 are used except that the utilization level of the supplier is* 95%*. Again, the target service level is chosen as* 95% *for both retailers.* □

The results obtained by simulating the system with $u = 95\%$ is given in Table 4.6 for the case only Retailer 2 uses adaptive ordering policy. As it is seen in Figures 4.6 and 4.7, we cannot conclude that α-service levels obtained by one of the retailers are higher or lower than α-service levels obtained by the other retailer. Mean inventories carried by both retailers are compared in Figure 4.8 and we observe that mean inventory carried decreases slightly for Retailer 1 and increases slightly for retailer 2 as update period gets shorter but there is no significant difference between mean inventories carried by the retailers. Based on these observations, we conclude that there is no gain for Retailer 2 by employing adaptive ordering policy instead of stationary ordering policy. For the case both retailers use adaptive ordering policy, the simulation results are given in Table 4.7. Similar to the simulation results of Numerical Example 9, there are worsening in the performance measures for both retailers. As it is seen in Figures 4.9 and 4.9, these deteriorations are much higher for $u = 95\%$ when compared to the simulation results for $u = 90\%$.

Table 4.6: Simulation results when only Retailer 2 uses adaptive ordering policy, $u = 95\%$.

Retailer	**1**	**2**	**1**	**2**	**1**	**2**	**1**	**2**
S for stationary (R, S) policy	255.45	255.45	255.45	-	255.45	-	255.45	-
Update Duration (review periods)	-	-	-	50	-	25	-	10
Mean Demand	100.00	100.02	100.00	100.02	100.00	100.02	100.00	100.02
Variance of Demand	899.20	899.18	899.20	899.18	899.20	899.18	899.20	899.18
Mean Order Up To Level	-	-	-	256.71	-	257.87	-	259.80
Variance of Order Up To Level	-	-	-	288.99	-	549.77	-	1147.86
Mean Order Quantity	100.00	100.02	100.00	100.02	100.00	100.02	100.00	100.02
Variance of Order Quantity	899.20	899.18	899.20	906.15	899.20	917.39	899.20	957.66
Mean Replenishment Lead Time	0.8050	0.8050	0.8071	0.8072	0.8113	0.8113	0.8228	0.8227
Variance of Rep. Lead Time	0.1810	0.1810	0.1835	0.1837	0.1895	0.1900	0.2038	0.2051
Mean Net Inventory	170.67	170.65	170.49	171.69	170.07	172.37	168.92	172.96
Variance of Net Inventory	4784.93	4785.63	4811.22	5033.89	4874.76	5229.59	5024.04	5645.96
Average Inventory	171.46	171.43	171.30	172.49	170.95	173.16	169.94	173.76
Variance of Average Inventory	4443.52	4445.46	4461.32	4691.37	4494.20	4887.96	4576.21	5303.51
Fill Rate (β)	98.09%	98.08%	98.05%	98.02%	97.96%	97.99%	97.75%	97.90%
No-Stockout Probability (α)	95.00%	94.98%	94.93%	94.79%	94.80%	94.70%	94.47%	94.52%
Half-width for 95% Confidence Level	0.14%	0.14%	0.13%	0.14%	0.14%	0.14%	0.15%	0.13%

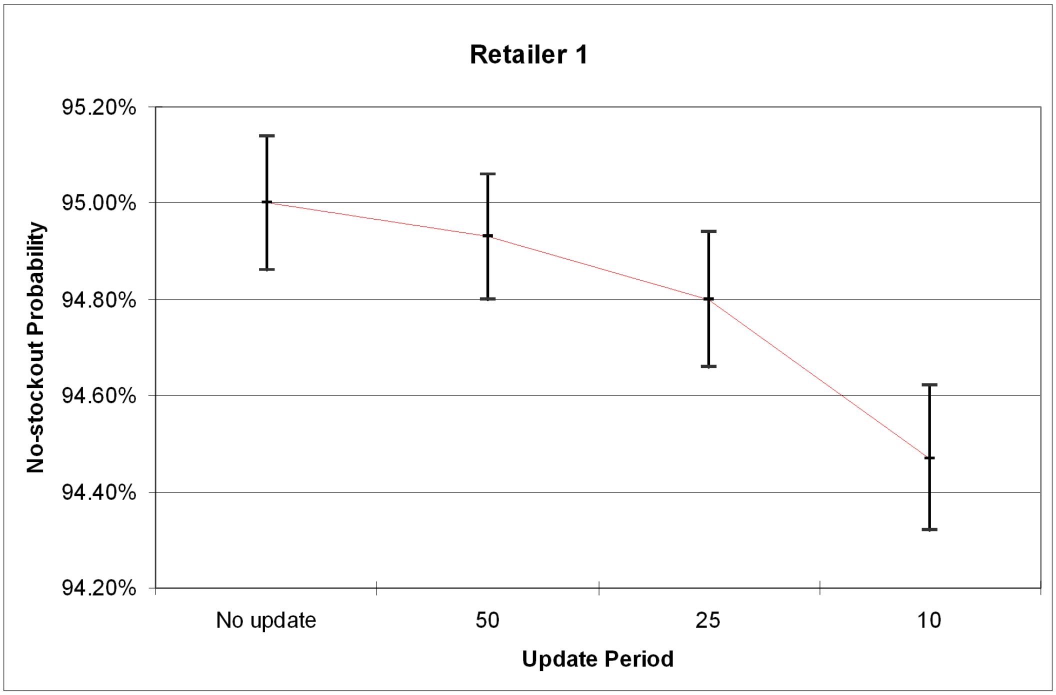

Figure 4.6: No-stockout probabilities for Retailers 1 when only Retailer 2 uses adaptive ordering policy.

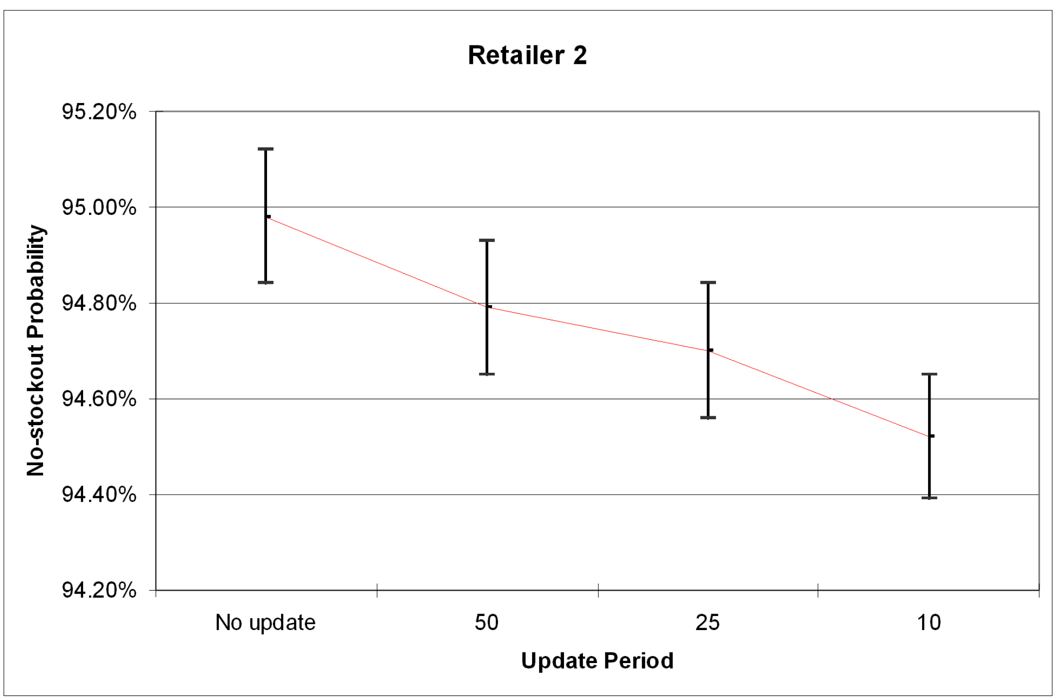

Figure 4.7: No-stockout probabilities for Retailers 2 when only Retailer 2 uses adaptive ordering policy.

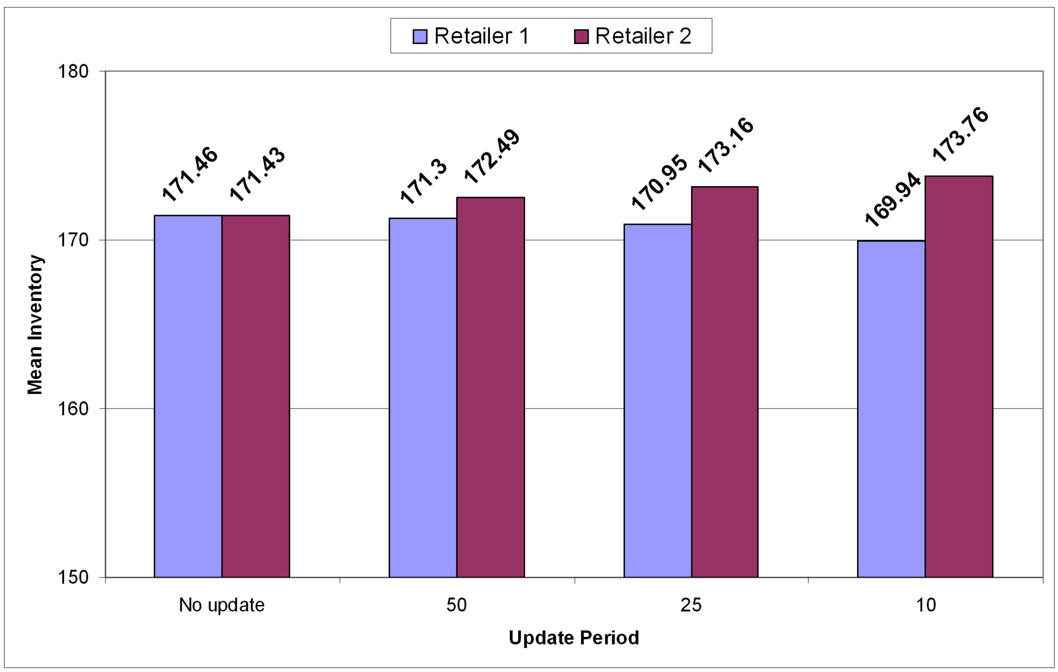

Figure 4.8: Mean inventory carried for Retailers 1 and 2 when only Retailer 2 uses adaptive ordering policy.

Table 4.7: Simulation results when both retailers use adaptive policy, $u = 95\%$.

Retailer	**1**	**2**	**1**	**2**	**1**	**2**
Update Duration (review periods)	50	50	25	25	10	10
Mean Demand	100.00	100.02	100.00	100.02	100.02	100.02
Variance of Demand	899.20	899.18	899.20	899.18	899.18	899.18
Mean Order Up To Level	257.75	258.03	261.84	262.16	297.99	297.99
Variance of Order Up To Level	380.16	388.57	1053.42	1078.40	10244.70	10244.70
Mean Order Quantity	100.00	100.02	100.00	100.02	100.01	100.01
Variance of Order Quantity	907.24	907.23	925.33	925.11	1080.50	1080.50
Mean Replenishment Lead Time	0.8122	0.8122	0.8297	0.8296	1.0154	1.0154
Variance of Rep. Lead Time	0.1903	0.1905	0.2165	0.2170	0.6015	0.6015
Mean Net Inventory	172.24	172.47	174.49	174.72	190.75	190.75
Variance of Net Inventory	5137.71	5151.39	5703.92	5728.00	11064.31	11064.31
Average Inventory	173.07	173.30	175.37	175.60	191.99	191.99
Variance of Average Inventory	4780.78	4794.44	5318.91	5343.42	10470.38	10470.38
Fill Rate (β)	97.99%	97.99%	97.88%	97.88%	97.16%	97.16%
No-Stockout Probability (α)	94.78%	94.77%	94.60%	94.59%	93.58%	93.58%
Half-width for 95% Confidence Level	0.14%	0.14%	0.14%	0.15%	0.13%	0.13%

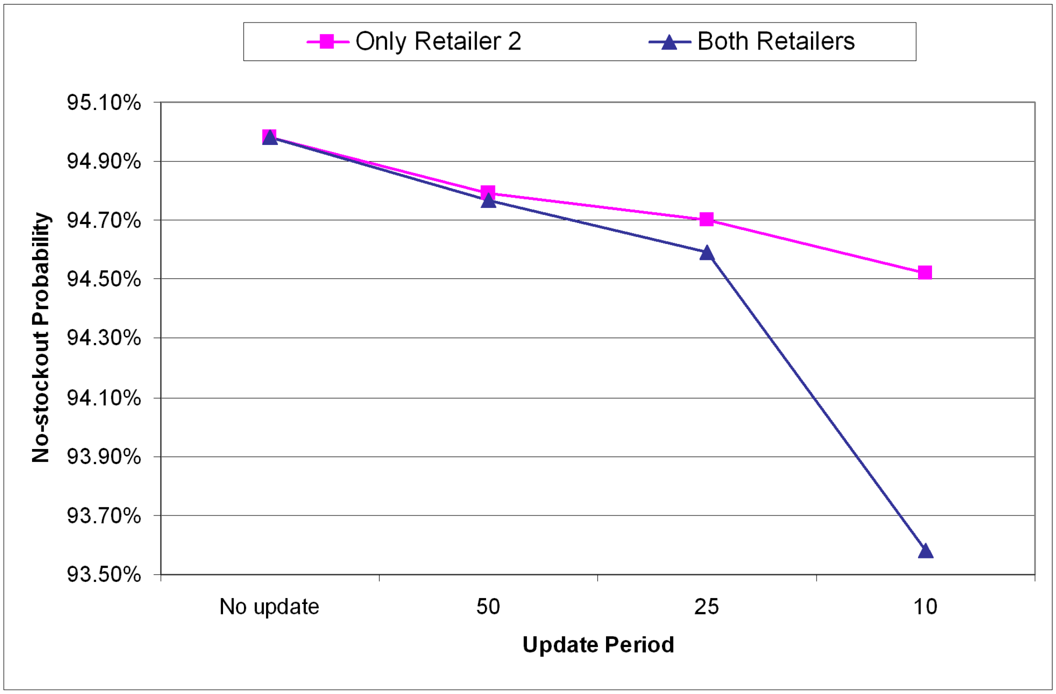

Figure 4.9: Comparison of no-stockout probabilities for the cases only Retailer 2 or both retailers use adaptive policies, Retailer 2.

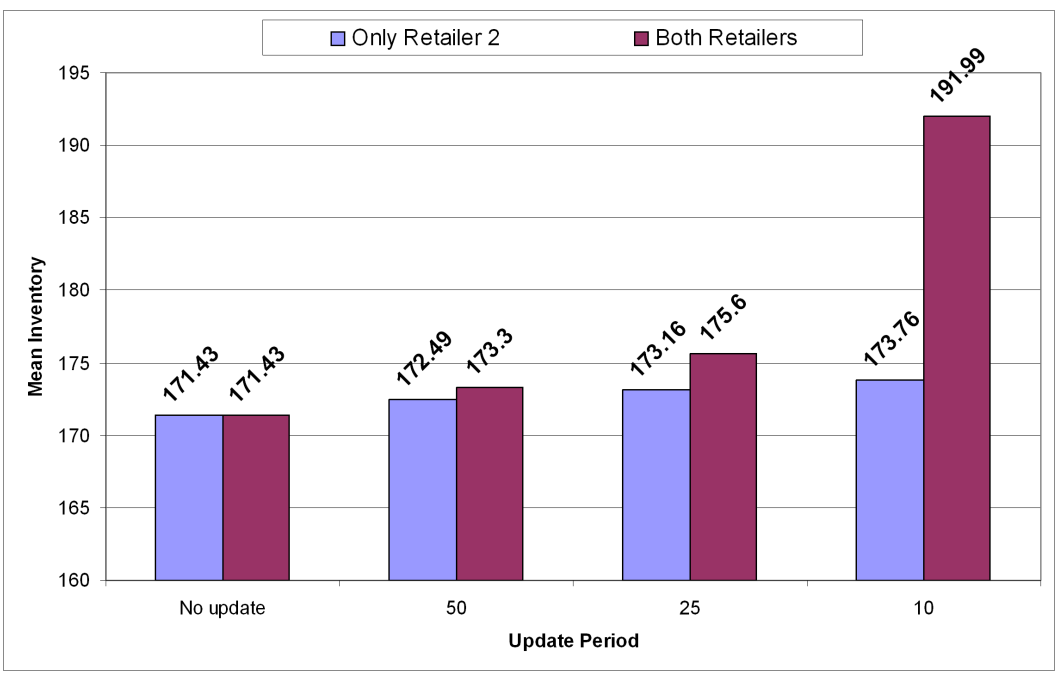

Figure 4.10: Comparison of mean inventory carried for the cases only Retailer 2 or both retailers use adaptive policies, Retailer 2.

For the case only Retailer 2 uses adaptive ordering policy, the simulation results given in Table 4.6 for $u = 95\%$ are compared with the simulation results given in Table 4.4 for $u = 90\%$. First of all, α-service levels obtained by Retailer 1 and 2 for $u = 95\%$ are smaller than the ones obtained for $u = 90\%$ as it can be seen in Figures 4.11 and 4.12. However, the difference between α-service levels obtained by Retailer 2 gets larger as update frequency increases and eventually Retailer 2 observes slightly higher α-service levels when compared to Retailer 1 as update period is 10 *review period*. Recall this is not the case for $u = 90\%$. Secondly, mean inventory carried by both retailers is higher for $u = 95\%$ compared to $u = 90\%$ for all update periods. Figures E.1 and E.2 in Appendix E illustrates this situation. But, oppose to the observations for $u = 90\%$, there is almost no increase in mean inventory carried by both retailers as update period gets shorter for $u = 95\%$. The comparisons for variance of the order quantities and the order-up-to-levels are given in Figures E.3 and E.4 in Appendix E, respectively. Interestingly, variance of the order quantity for $u = 90\%$ is higher compared to $u = 95\%$ when update period is 10 *review periods* although the situation is reverse for variance of the order-up-to-level.

For the case both retailers use adaptive ordering policy, it is seen that all performance measures in the long-run deteriorate for 95% utilization level when compared to 90% utilization level. Figures E.5, E.6, E.7 in Appendix E compare the simulation results for α-service levels, mean inventory levels and variance of the order quantity, respectively. When the results in Table 4.5 and 4.7 for $u = 90\%$ and $u = 95\%$, respectively, are compared, we come up with the followingobservations for the same values of update period.

- Means and variances of the order-up-to-level and order quantity are higher for 95% utilization level.

- Mean and variance of the replenishment lead time are higher for 95% utilization level.
- Means and variances of inventory-on-hand are higher for 95% utilization level. This observation shows that safety stock to achieve the same service level is higher for 95% utilization level.
- Decreases in fill rate and no-stockout probability are higher for 95% utilization level.

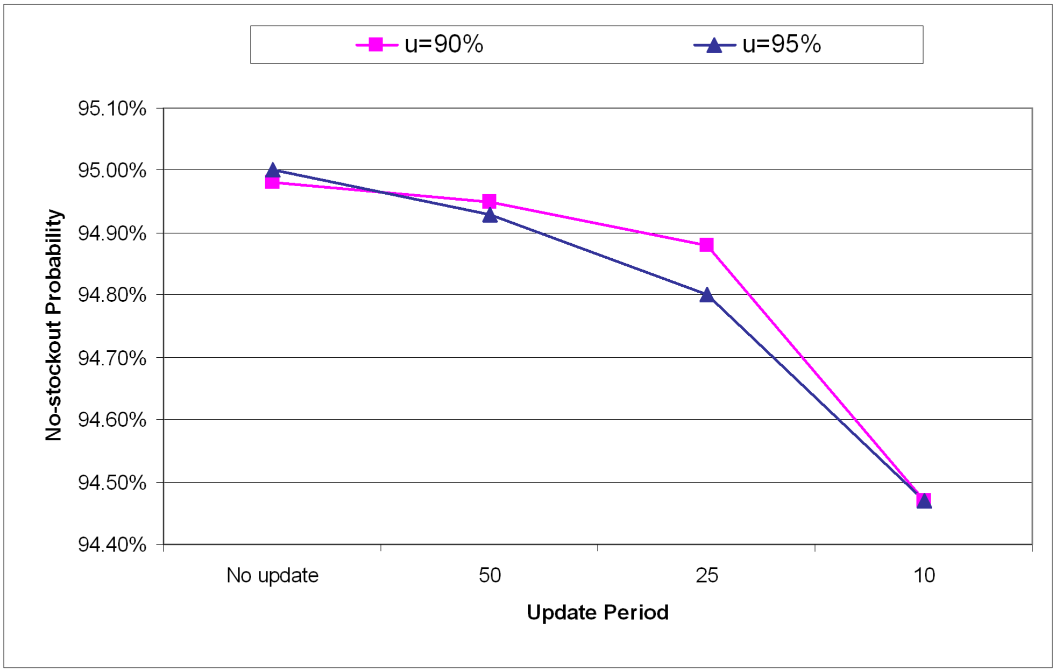

Figure 4.11: Comparison of no-stockout probabilities for $u = 90\%$ and $u = 95\%$ when only Retailer 2 uses adaptive ordering policy, Retailer 1.

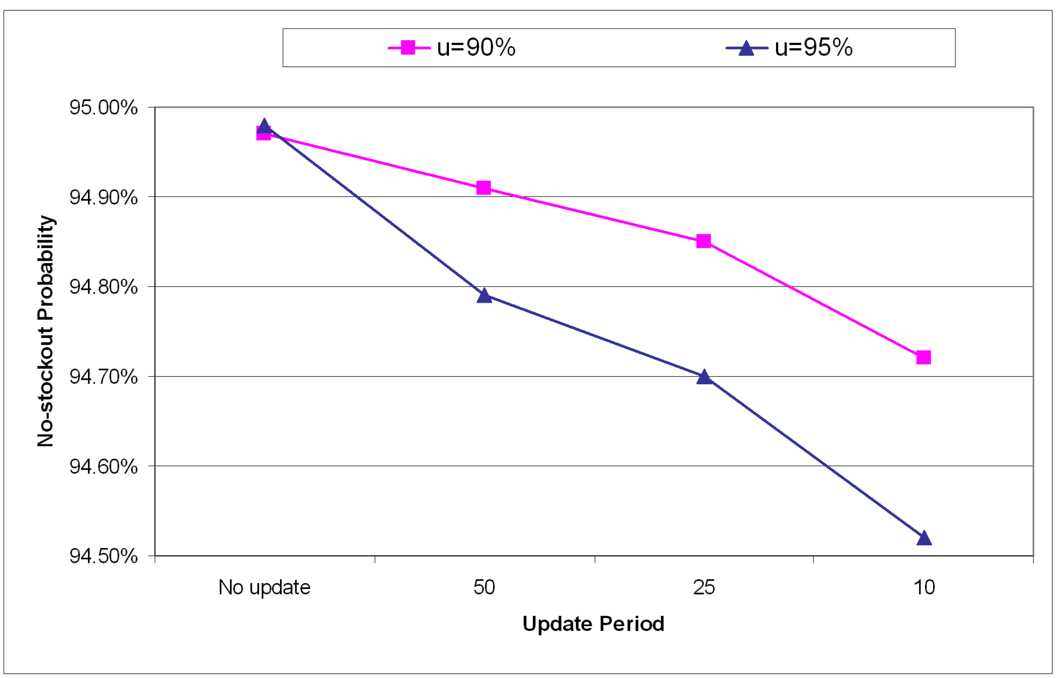

Figure 4.12: Comparison of no-stockout probabilities for $u = 90\%$ and $u = 95\%$ when only Retailer 2 uses adaptive ordering policy, Retailer 2.

Chapter 5

CONCLUSION

In this study, we analyze two-echelon supply chain systems with the retailer(s) at the lower echelon and the supplier at the upper echelon. Degree of the dependence between ordering policy of the retailer(s) and the production system of the supplier is varied in our models. The models with exogenous and endogenous replenishment lead times are studied for both single-retailer and two-retailer cases.

In Chapter 3, the models with single retailer are considered. For stationary (R, S) policy, the existing methods to calculate order-up-to-level for a given no-stockout probability during the review period are reviewed in Section 3.1 and a new method is proposed for exogenous replenishment lead time. The proposed method is based on the use of the distribution of inventory-on-order while the methods in the literature are mainly based on the use of the distribution of the demand during the risk period. Simulation results show that the order-up-to-level calculated by using the proposed method provides service levels that are almost equal to the target service levels. This cannot be ensured by using the existing methods. In the case of endogenous replenishment time, use of simulation is preferred avoiding the difficult exact analysis to obtain the conditional distributions of the inventory-on-order given the number of outstanding orders. Then, order-up-to-levels are calculated using the estimates obtained by the initial simulation run. The proposed method given for exogenous replenishment lead time can be used to determine order-up-to-level for endogenous replenishment lead time. Based on the simulation results presented in Section 3.2, it is seen that the proposed method give no-stockout probabilities very close to target levels for endogenous replenishment lead time.

The impacts of using an adaptive (R, S_t) policy on the performance of the retailer are questioned in Section 3.3. The experimental results indicate that the updating of the order-up-to-level causes the performance of the retailer to get worse as update frequency increases.

In Chapter 4, the analysis is extended to the models with two retailers for the case of endogenous replenishment lead times. Then, it is questioned whether it is beneficial for a retailer to use an adaptive (R, S_t) policy in a supply chain with two retailers. Simulation results show that performances of both retailers get worse even if only one of them uses an adaptive policy. However, the deterioration in the performance of the retailer using stationary (R, S) policy is larger than the performance of the other retailer handling adaptive (R, S_t) policy. We also observe that the deterioration gets larger in the case of an increase in the update frequency or in the utilization of the supplier. Moreover, based on the observations obtained in Section 4.2, we conclude that all of the performance measures deteriorate when both retailers use adaptive (R, S_t) policy and the deterioration gets larger as the update period gets shorter.

The models analyzed in this thesis can be extended in several ways. In the periodic-review base-stock policies discussed in this thesis, we assume that an order is replaced at every point that the inventory is reviewed and this is the optimal ordering policy since there is no fixed ordering cost in the models considered in this thesis. In the case there exists fixed ordering costs, the ordering policy employed by the retailer can be extended such that orders are not placed at every review point to avoid incurring excessive ordering costs. Moreover, the distribution of the demand during a review period is assumed to be Normal in the studied models. Then, the models with different demand distributions can be considered as a future research direction.

For two-retailer case, we studied the models where the retailers operates in independent markets and observe equivalent customer demands. In the case the retailers share same market or operate in independent markets with different sizes of the customer demand, the use of adaptive ordering policies by one or both of the retailers should be reinvestigated. In that case, the retailer having larger market share or observing higher customer demand may take advantage of using adaptive ordering policy since the orders placed by this retailer dominates the production system of the supplier. Another restriction in our models is the assumption for constant service time of the supplier for each unit in endogenous replenishment lead time case. Then, the models can be extended to have service times resulted from different probability distributions rather than to be constant.

REFERENCES

Balakrishnan A., Geunes J. and Pangburn M., 2004. Coordinating supply chains by controlling upstream variability propagation. *Manufacturing & Service Operations Management*, 6(2), pp. 163-183.

Boute, R., 2006. The impact of replenishment rules with endogenous lead times on supply chain performance. *PhD thesis*, Faculteit der Economische en Toegepaste Economische Wetenschappen, Katholieke Universiteit Leuven.

Chen, F., Drezner, Z., Ryan, J.K., and Simchi-Levi, D., 2000a. Quantifying the bullwhip effect in a simple supply chain. *Management Science* 46(3), pp. 436-443.

Chen, F., Drezner, Z., Ryan, J.K., and Simchi-Levi, D., 2000b. The impact of exponential smoothing forecasts on the bullwhip effect. *Naval Research Logistics* 47(4), pp. 269-286.

Chopra S. and Meindl P., 2007. *Supply chain management: Strategy, planning and operation,* 3^{rd} edition. Pearson Prentice Hall, NJ.

Clark, A.J., and Scarf, H. 1960. Optimal policies for a multi-echelon inventory problem. *Management Science*, 50(12), pp. 17821790.

De Kok, A.G., and Fransoo, J.C. 2003. Planning supply chain operations: Definition and comparison of planning concepts. Pages 597675 of: De Kok, A.G., and Graves, S.C. (eds), *Handbook in Operations Research and Management Science, Volume 11: Design and Analysis of Supply Chains*. Amsterdam: Elsevier.

Dejonckheere, J., Disney, S. M., Lambrecht, M. R. and Towill D. R., 2003. Measuring and avoiding the bullwhip effect: A control theoretic approach. *European Journal of Operational Research,* 147(3), pp. 567-590.

Disney, S.M., Farasyn, I., Lambrecht, M., Towill, D.R., and de Velde, W.V., 2006. Taming the bullwhip effect whilst watching customer service in a single supply chain echelon. *European Journal of Operational Research* 173, 151-172.

Eppen, G.D., Martin, R.K., 1988. Determining safety stock in the presence of stochastic lead time and demand. *Management Science,* 34, pp. 1380-1390.

Hayya, J. C, Kim J. G., Disney, S.M., Harrison T. P. and Chatfield D., 2006. Estimation in supply chain inventory management. *International Journal of Production Research* 44(7), pp. 13131330.

Kim H., Ryan, J.K., 2002. The Cost Impact of Using Simple Forecasting Techniques in a Supply Chain. *Naval Research Logistics*, vol. 50, pp. 388-411.

Lee, H.L., Padmanabhan, V., Whang, S., 1997. The bullwhip effect in supply chains. *Sloan Management Review,* 38(3), pp. 9-102

Luong, H. T., 2007. Measure of bullwhip effect in supply chain with autoregressive demand process. *European Journal of Operational Research* 180, pp. 1086-1097.

Mather, H., and Plossl, G.W. 1978. Priority fixation versus throughput planning. *Production and Inventory Management*, 3rd Quarter, pp. 2751.

Nahmias, S., 1997. *Production and Operation Analysis*, 3rd Edition. McGraw-Hill.

Pahl, J., Vo, S., and Woodruff, D.L. 2005. Production planning with load dependent lead times. *4OR*, 3, pp. 257302.

Selçuk, B., De Kok, A. G., and Fransoo, J. C., 2006. The effect of updating lead times on the performance of hierarchical planning systems. *International Journal of Production Economics* 104, pp. 427-440.

Selçuk, B., 2007. Dynamic Performance of Hierarchical Planning Systems: Modeling and Evaluation with Dynamic Planned Lead Times. PhD thesis, *Technische Universiteit Eindhoven*.

Silver, E. A., Pyke, D. F. and Peterson, R., 1998. *Inventory Management and Production Planning and Scheduling,* 3rd Edition. John Wiley & Sons, New York.

So, K.C., Zheng, X., 2003. Impact of suppliers lead time and forecast demand updating on retailers order quantity variability in a two-level supply chain. *International Journal of Production Economics* 86, pp. 169179.

Song, J.S. 1994. The effect of lead time uncertainty in a simple stochastic inventory model. *Management Science* 40, pp. 603-613.

Zhang, X., 2004. The impact of forecasting methods on the bullwhip effect. *International Journal of Production Economics* 88, pp. 15-27.

Zipkin, P.H., 2000. *Foundations of Inventory Management*. McGraw-Hill.

Appendix A

Simulation Code

SIMAN PROJECT ENVIRONMENT

```
PROJECT,     ”Unnamed Project”,”METU”,,,No,Yes,Yes,Yes,No,No,Yes;

ATTRIBUTES:
    1,CustomerType,0:
    2,QuantityToDuplicate,0:
    3,TimeIntoManufacturer,0:
    4,QuantityToDuplicate_2,0:
    5,Order Period,0:
    6,Order Period_2,0:
    OUT_Calculated,77.85:
    OUT_Calculated_2,78.41;

VARIABLES:
    AverageTimes(50,25),CLEAR(System):
    OnOrderInventory_2(30),CLEAR(System),0:
    OOI_2(30,50001),CLEAR(System),0:
    LA(30,50001),CLEAR(System),0:
    NumberInManufacturing(30),CLEAR(System),0:
    Demand(30),CLEAR(System),0:
    PeriodControl(30),CLEAR(System),0:
    VariableFlowTime(50),CLEAR(System),0.095:
    StockoutPeriod_2(30),CLEAR(System),0:
    NA(30,50001),CLEAR(System),0:
    Order Quantity_2(30),CLEAR(System),0:
    Waiting Orders(30),CLEAR(System),0:
    PeriodControl_2(30),CLEAR(System),0:
    OnHandInventory(30),CLEAR(System),50:
    NumOfTimes_2(50,25),CLEAR(System):
    OP(30,50001),CLEAR(System),0:
    OP_2(30,50001),CLEAR(System),0:
    NLT(30),CLEAR(System),0:
    Demand_2(30),CLEAR(System),0:
    OnOrderNumber(30,50001),CLEAR(System),0:
```

```
    AverageTimes_2(50,25),CLEAR(System):
    DA_2(30,50001),CLEAR(System),0:
    NLT_2(30),CLEAR(System),0:
    OnHandInventory_2(30),CLEAR(System),50:
    Period(30),CLEAR(System),0:
    Stockout_2(30),CLEAR(System),0:
    LA_2(30,50001),CLEAR(System),0:
    DA(30,50001),CLEAR(System),0:
    NA_2(30,50001),CLEAR(System),0:
    Order Up To Point(30),CLEAR(System),100:
    Waiting Orders_2(30),CLEAR(System),0:
    OnOrderNumber_2(30,50001),CLEAR(System),0:
    VarianceTimes(50,25),CLEAR(System):
    Order Quantity(30),CLEAR(System),0:
    OOI(30,50001),CLEAR(System),0:
    LeadTimeDemand(30),CLEAR(System),0:
    StockoutPeriod(30),CLEAR(System),0:
    VarianceTimes_2(50,25),CLEAR(System):
    Period_2(30),CLEAR(System),0:
    LeadTimeDemand_2(30),CLEAR(System),0:
    StockoutControl(30),CLEAR(System),0:
    Rep_Lead_Time_2(30),CLEAR(System),0:
    Order Up To Point_2(30),CLEAR(System),81.29:
    OnOrderInventory(30),CLEAR(System),0:
    Rep_Lead_Time(30),CLEAR(System),0:
    Stockout(30),CLEAR(System),0:
    NumOfTimes(50,25),CLEAR(System):
    StockoutControl_2(30),CLEAR(System),0;

QUEUES:
     1,WaitingQueue,FirstInFirstOut:
    2,ShopQueue,FirstInFirstOut;

FAILURES:
     1,Failure 1,Time(EXPO(100),EXPO(2),);

RESOURCES:
     1,Waiting,Capacity(1),,Stationary,COST(0.0,0.0,0.0):
    2,Machine,Capacity(1),,Stationary,COST(0.0,0.0,0.0);

COUNTERS:
     1,TotalDemand,,Replicate,"C:\Total Demand.dat":
    2,TotalStockout,,Replicate,
    "C:\Total Stockout.dat":
    3,ValueLookCount,,Replicate:
    4,StockoutPeriodCount,,Replicate:
    5,CountedPeriod,,Replicate:
    6,CountedAlphaPeriod,,Replicate:
```

```
7,StockoutAlphaPeriodCount,,Replicate:
8,TotalAlphaStockout,,Replicate:
9,UpdateTimes,,Replicate:
101,TotalDemand_2,,Replicate,
"C:\Total Demand_2.dat":
102,TotalStockout_2,,Replicate,
"C:\Total Stockout_2.dat":
103,ValueLookCount_2,,Replicate:
104,StockoutPeriodCount_2,,Replicate:
105,CountedPeriod_2,,Replicate;

TALLIES:
 1,DemandTally,"C:\Demand.dat":
 2,OrderUpToPoint,"C:\OrderUpToPoint.dat":
 3,OrderQuantity,"C:\OrderQuantity.dat":
 4,RealizedLeadTime,"C:\RealizedLeadTime.dat":
 6,Rep_Lead_Time_Tally:
 7,LTDemandTally,"C:\LTDemandTally.dat":
 8,Net Stock Tally,"C:\NetStockTally.dat":
 9,On Order Inventory Tally,
 "C:\OnOrderInventoryTally.dat":
 10,telly_tally,"C:\telly_tally.dat":
 101,DemandTally_2,"C:\Demand_2.dat":
 102,OrderUpToPoint_2,"C:\OrderUpToPoint_2.dat":
 103,OrderQuantity_2,"C:\OrderQuantity_2.dat":
 104,RealizedLeadTime_2,
 "C:\RealizedLeadTime_2.dat":
 106,Rep_Lead_Time_Tally_2:
 107,LTDemandTally_2,"C:\LTDemandTally_2.dat":
 108,Net Stock Tally_2,"C:\NetStockTally_2.dat":
 109,On Order Inventory Tally_2,
 "C:\OnOrderInventoryTally_2.dat":
 110,telly_tally_2,"C:\telly_tally_2.dat":
 Cum Lead Times_2:
 Cum Outstanding Orders:
 Cum Lead Time Demand_2:
 Cum Outstanding Orders_2:
 Cum Lead Times,"C:\000\Cum Lead Times.dat":
 Cum Inventory On Order_2:
 Cum Lead Time Demand,
 "C:\000\Cum Lead Time Demand.dat":
 Cum Inventory On Order;

DSTATS:
 1,Max(0, OnHandInventory(1)),Average Inventory,
 "C:\Average Inventory.dat":
 2,Max(0, -OnHandInventory(1)),Average Backlog,
 "C:\Average Backlog.dat":
```

```
    3,OnHandInventory(1),Net Stock,”C:\Net Stock.dat”:
    101,Max(0, OnHandInventory_2(1)),Average Inventory_2,
    ”C:\Average Inventory_2.dat”:
    102,Max(0, -OnHandInventory_2(1)),Average Backlog_2,
    ”C:\Average Backlog_2.dat”:
    103,OnHandInventory_2(1),Net Stock_2,
    ”C:\Net Stock_2.dat”:
    1000,NumberInManufacturing(1),Number In Manufacturing,
    ”C:\NumberInManufacturing.dat”;

OUTPUTS:
     1,TAVG(DemandTally),,Mean Demand:
    2,TSTD(DemandTally)**2,,Variance of Demand:
    3,TAVG(OrderUpToPoint),,Mean Order Up To Point:
    4,TSTD(OrderUpToPoint)**2,,Var. of Or. Up To Point:
    5,TAVG(OrderQuantity),,Mean Order Quantity:
    6,TSTD(OrderQuantity)**2,,Var. of Order Quantity:
    7,TAVG(RealizedLeadTime)/PeriodLong,,Mean Real. Lead Time:
    8,(TSTD(RealizedLeadTime)**2)/(PeriodLong**2),,Var. of Real. Lead Time:
    9,DAVG(Net Stock),,Mean Net Stock:
    10,DSTD(Net Stock)**2,,Var. of Net Stock:
    11,DAVG(Average Inventory),,Average Inventory:
    12,DSTD(Average Inventory)**2,,Var. of Average Inventory:
    14,DAVG(Average Backlog),,Average Backlog:
    15,DSTD(Average Backlog)**2,,Var. of Average Backlog:
    16,NC(TotalDemand),,TotalDemand:
    17,NC(TotalStockout),,Total Stockout:
    18,1-NC(TotalStockout)/NC(TotalDemand),,Percent Stockout:
    19,1-NC(StockoutPeriodCount)/NC(CountedPeriod),,% Stockout Period:
    22,TAVG(Rep_Lead_Time_Tally):
    23,TSTD(Rep_Lead_Time_Tally)**2:
    24,TAVG(LTDemandTally):
    25,TSTD(LTDemandTally)**2:
    101,TAVG(DemandTally_2),,Mean Demand_2:
    102,TSTD(DemandTally_2)**2,,Variance of Demand_2:
    103,TAVG(OrderUpToPoint_2),,Mean OUT Point_2:
    104,TSTD(OrderUpToPoint_2)**2,,Var. of OU To Point_2:
    105,TAVG(OrderQuantity_2),,Mean Order Quantity_2:
    106,TSTD(OrderQuantity_2)**2,,Var. of Order Quan_2:
    107,TAVG(RealizedLeadTime_2)/PeriodLong_2,,Mean Real. Lead Time_2:
    108,TSTD(RealizedLeadTime_2)**2/(PeriodLong_2**2),,Var. of Real. LeadTime_2:
    109,DAVG(Net Stock_2),,Mean Net Stock_2:
    110,DSTD(Net Stock_2)**2,,Var. of Net Stock_2:
    111,DAVG(Average Inventory_2),,Average Inventory_2:
    112,DSTD(Average Inventory_2)**2,,Var. of Avg. Inventory_2:
    114,DAVG(Average Backlog_2),,Average Backlog_2:
    115,DSTD(Average Backlog_2)**2,,Var. of Avg. Backlog_2:
    116,NC(TotalDemand_2),,TotalDemand_2:
```

```
    117,NC(TotalStockout_2),,Total Stockout_2:
    118,1-NC(TotalStockout_2)/NC(TotalDemand_2),,Percent Stockout_2:
    119,1-NC(StockoutPeriodCount_2)/NC(CountedPeriod_2),,% Stockout Period_2:
    122,TAVG(Rep_Lead_Time_Tally_2):
    123,TSTD(Rep_Lead_Time_Tally_2)**2:
    124,TAVG(LTDemandTally_2):
    125,TSTD(LTDemandTally_2)**2:
    998,DAVG(Number In Manufacturing),,Num. In Manuf. Mean:
    999,DSTD(Number In Manufacturing)**2,,Num. In Manuf. Variance;

REPLICATE,     1,0.0,500000,Yes,Yes,10000,,,24.0,Hours;

EXPRESSIONS:
     1,PeriodLong,10:
    2,MeanDemand,100:
    3,ErrorStdDev,30:
    4,Ro,0:
    5,Mu,MeanDemand * (1-Ro):
    6,StdDev,SQRT ( (ErrorStdDev**2) * (1- (Ro**2) ) ):
    7,zed,1.644853476:
    8,SmoothFactor,1:
    9,FlowTimeMean,0.09:
    10,FlowTimeStdDev,0.00000001:
    11,Alpha,0.01:
    12,UpdateFreq,100:
    13,ForecastPeriod,40000:
    14,LTorLTD,0:
    15,SLevel,1:
    16,WhichOne,2:
    101,PeriodLong_2,10:
    102,MeanDemand_2,100:
    103,ErrorStdDev_2,30:
    104,Ro_2,0:
    105,Mu_2,MeanDemand_2 * (1-Ro_2):
    106,StdDev_2,SQRT ( (ErrorStdDev_2**2) * (1- (Ro_2**2) ) ):
    107,zed_2,1.644853476:
    108,SmoothFactor_2,1:
    109,FlowTimeMean_2,0.0045:
    110,FlowTimeStdDev_2,0.00000001:
    111,Alpha_2,0.01:
    112,UpdateFreq_2,25000:
    113,ForecastPeriod_2,20000:
    114,LTorLTD_2,3:
    115,SLevel_2,1:
    116,WhichOne_2,2:
    Single_Process,0:
    LimitNumber,125:
    Limit,0:
```

```
    FlowChange,0.05:
    First_Creation_2,55555555;
```

SIMAN MODEL ENVIRONMENT

```
0$    CREATE,    1:PeriodLong:NEXT(1$);

1$    ASSIGN:    Period(1)=Period(1)+1:
        CustomerType=1:
        Demand(1)=max (0, (Mu+ Ro * (Demand(1) ) + NORM(0,StdDev,1) ) ):
        DA(1,Period(1))=Demand(1);
5$    TALLY:    DemandTally,Demand(1),1;
2$    COUNT:    TotalDemand,Demand(1);
3$    ASSIGN:    Stockout(1)= (OnHandInventory(1)<=0)*Demand(1) +
(OnHandInventory(1)>0)*Max(0, (Demand(1) - OnHandInventory(1)) ):
        StockoutPeriod(1)=(Stockout(1)>0)*1:
        OnHandInventory(1)=OnHandInventory(1) - Demand(1);
38$    TALLY:    Net Stock Tally,OnHandInventory(1),1;
25$    COUNT:    CountedPeriod,1;
24$    COUNT:    StockoutPeriodCount,StockoutPeriod(1);
4$    COUNT:    TotalStockout,Stockout(1);
6$    ASSIGN:    max ( 0,(SmoothFactor* (Order Up To Point(1) -
(OnHandInventory(1)+OnOrderInventory(1))) ) ):
        Order Period=Period(1):
        OnOrderInventory(1)=OnOrderInventory(1) + Order Quantity(1);
41$    BRANCH,    1:
        If,Order Quantity(1)>0,40$,Yes:
        If,Order Quantity(1)==0,43$,Yes;
40$    ASSIGN:    Waiting Orders(1)=Waiting Orders(1)+1;
43$    ASSIGN:    OnOrderNumber(1,Period(1) )=Waiting Orders(1):
        OOI(1,Period(1) )=OnOrderInventory(1):NEXT(95$);

95$    IF:    (Period(1)>50);
89$    ASSIGN:    StockoutControl(1)=StockoutControl(1)+StockoutPeriod(1):
        PeriodControl(1)=PeriodControl(1)+1;
96$    ENDIF;
90$    IF:    ( Period(1)>=(UpdateFreq+50) ) * ( MOD(Period(1),updateFreq)==0 );
99$    TALLY:    telly_tally,1 - (StockoutControl(1)/PeriodControl(1) ),1;
97$    VBA:    1,vba;
98$    DUPLICATE:    1,XXX:NEXT(94$);

94$    COUNT:    UpdateTimes,1;
93$    ENDIF;
91$    ENDIF:    NEXT(7$);

7$    TALLY:    OrderUpToPoint,Order Up To Point(1),1;
8$    TALLY:    OrderQuantity,Order Quantity(1),1;
39$    TALLY:    On Order Inventory Tally,OnOrderInventory(1),1;
42$    BRANCH,    1:
```

```
        If,Order Quantity(1)>0,32$,Yes:
        If,Order Quantity(1)==0,31$,Yes;
32$     ASSIGN:     QuantityToDuplicate=Order
Quantity(1)-1:MARK(TimeIntoManufacturer):NEXT(62$);

62$     ASSIGN:     NumberInManufacturing(1)=NumberInManufacturing(1) +
(QuantityToDuplicate + QuantityToDuplicate_2);
68$     IF:     NumberInManufacturing(1)>=LimitNumber;
69$     ASSIGN:     VariableFlowTime(1)=FlowTimeMean-FlowChange;
71$     ELSEIF:     NumberInManufacturing(1)<LimitNumber;
72$     ASSIGN:     VariableFlowTime(1)=FlowTimeMean;
70$     ENDIF;
60$     QUEUE,     WaitingQueue;
61$     SEIZE,     1,Other:
        Waiting,1:NEXT(73$);

73$     BRANCH,     1:
        If,Single_Process==1,84$,Yes:
        If,Single_Process==0,74$,Yes;
84$     BRANCH,     1:
        If,CustomerType==1,88$,Yes:
        If,CustomerType==2,85$,Yes;
88$     DUPLICATE:     AINT(QuantityToDuplicate):NEXT(77$);

77$     QUEUE,     ShopQueue;
78$     SEIZE,     1,Other:
        Machine,1:NEXT(79$);

79$     BRANCH,     1:
        If,CustomerType==1,80$,Yes:
        If,CustomerType==2,83$,Yes;
80$     DELAY:     FlowTimeMean,,Other:NEXT(81$);

81$     RELEASE:     Machine,1;
86$     BRANCH,     1:
        If,CustomerType==1,82$,Yes:
        If,CustomerType==2,87$,Yes;
82$     GROUP,     ,Permanent:AINT(QuantityToDuplicate)+1,First:NEXT(67$);

87$     GROUP,     ,Permanent:AINT(QuantityToDuplicate_2)+1,First:NEXT(67$);

83$     DELAY:     FlowTimeMean_2,,Other:NEXT(81$);

85$     DUPLICATE:     AINT(QuantityToDuplicate_2):NEXT(77$);

67$     RELEASE:     Waiting,1;
64$     BRANCH,     1:
        If,CustomerType==1,65$,Yes:
```

```
        If,CustomerType==2,66$,Yes;
65$     DELAY:    0,,Other:NEXT(63$);

63$     ASSIGN:    NumberInManufacturing(1)=NumberInManufacturing(1) -
(QuantityToDuplicate + QuantityToDuplicate_2)
        :NEXT(9$);

66$     DELAY:    0,,Other:NEXT(63$);

74$     BRANCH,    1:
        If,CustomerType==1,75$,Yes:
        If,CustomerType==2,76$,Yes;
75$     DELAY:    FlowTimeMean * (QuantityToDuplicate+1),,Other:NEXT(67$);

76$     DELAY:    FlowTimeMean_2 * (QuantityToDuplicate_2+1),,Other:NEXT(67$);

9$     BRANCH,    1:
        If,CustomerType==1,10$,Yes:
        If,CustomerType==2,23$,Yes;
10$     TALLY:    RealizedLeadTime,INT(TimeIntoManufacturer),1;
28$     ASSIGN:    Rep_Lead_Time(1)=AINT( TVALUE(RealizedLeadTime) / PeriodLong):
        NLT(1)=NLT(1)+1:
        LA(1,NLT(1) )=Rep_Lead_Time(1):
        OP(1,NLT(1) )=Order Period;
29$     TALLY:    Rep_Lead_Time_Tally,Rep_Lead_Time(1),1;
50$     VBA:    3,vba;
30$     TALLY:    LTDemandTally,LeadTimeDemand(1),1;
56$     TALLY:    Cum Lead Times,Rep_Lead_Time(1),1;
57$     TALLY:    Cum Lead Time Demand,LeadTimeDemand(1),1;
11$     ASSIGN:    OnHandInventory(1)=OnHandInventory(1) + (QuantityToDuplicate+1):
        OnOrderInventory(1)=OnOrderInventory(1) - (QuantityToDuplicate+1):
        Waiting Orders(1)=Waiting Orders(1)-1;
12$     DISPOSE:    No;

23$     TALLY:    RealizedLeadTime_2,INT(TimeIntoManufacturer),1;
34$     ASSIGN:    Rep_Lead_Time_2(1)=AINT( TVALUE(RealizedLeadTime_2) /
PeriodLong_2 ):
        NLT_2(1)=NLT_2(1)+1:
        LA_2(1,NLT_2(1) )=Rep_Lead_Time_2(1):
        OP_2(1,NLT_2(1) )=Order Period_2;
35$     TALLY:    Rep_Lead_Time_Tally_2,Rep_Lead_Time_2(1),1;
51$     VBA:    4,vba;
36$     TALLY:    LTDemandTally_2,LeadTimeDemand_2(1),1;
58$     TALLY:    Cum Lead Times_2,Rep_Lead_Time_2(1),1;
59$     TALLY:    Cum Lead Time Demand_2,LeadTimeDemand_2(1),1;
37$     ASSIGN:    OnHandInventory_2(1)=OnHandInventory_2(1) +
(QuantityToDuplicate_2+1):
        OnOrderInventory_2(1)=OnOrderInventory_2(1) - (QuantityToDuplicate_2+1):
```

```
        Waiting Orders_2(1)=Waiting Orders_2(1) - 1:NEXT(12$);

31$     DISPOSE:    Yes;


13$     CREATE,     1,First_Creation_2:PeriodLong_2:NEXT(14$);

14$     ASSIGN:     Period_2(1)=Period_2(1)+1:
        CustomerType=2:
        Demand_2(1)=max (0, (Mu_2+ Ro_2 * (Demand_2(1) ) + NORM(0,StdDev_2,25) ) ):
        DA_2(1,Period_2(1) )=Demand_2(1);
18$     TALLY:    DemandTally_2,Demand_2(1),1;
15$     COUNT:     TotalDemand_2,Demand_2(1);
16$     ASSIGN:     Stockout_2(1)= (OnHandInventory_2(1)<=0)*Demand_2(1) +
(OnHandInventory_2(1)>0)*Max(0, (Demand_2(1) - OnHandInventory_2(1)) ):
        StockoutPeriod_2(1)=(Stockout_2(1)>0)*1:
        OnHandInventory_2(1)=OnHandInventory_2(1) - Demand_2(1);
44$     TALLY:    Net Stock Tally_2,OnHandInventory_2(1),1;
27$     COUNT:     CountedPeriod_2,1;
26$     COUNT:     StockoutPeriodCount_2,StockoutPeriod_2(1);
17$     COUNT:     TotalStockout_2,Stockout_2(1);
19$     ASSIGN:     Order Quantity_2(1)= max ( 0, (SmoothFactor_2* (Order Up To
Point_2(1) - (OnHandInventory_2(1)+OnOrderInventory_2(1))) ) ):
        Order Period_2=Period_2(1):
        OnOrderInventory_2(1)=OnOrderInventory_2(1) + Order Quantity_2(1);
46$     BRANCH,     1:
        If,Order Quantity_2(1)>0,45$,Yes:
        If,Order Quantity_2(1)==0,47$,Yes;
45$     ASSIGN:     Waiting Orders_2(1)=Waiting Orders_2(1)+1;
47$     ASSIGN:     OnOrderNumber_2(1,Period_2(1) )=Waiting Orders_2(1):
        OOI_2(1,Period_2(1) )=OnOrderInventory_2(1):NEXT(103$);

103$     IF:     (Period_2(1)>50);
100$     ASSIGN:     StockoutControl_2(1)=StockoutControl_2(1)+StockoutPeriod_2(1):
        PeriodControl_2(1)=PeriodControl_2(1)+1;
104$     ENDIF;
101$     IF:     ( Period_2(1)>=(UpdateFreq_2+50) ) * (
MOD(Period_2(1),updateFreq_2)==0 );
109$     TALLY:     telly_tally_2,1 - (StockoutControl_2(1)/PeriodControl_2(1) ),1;
107$     VBA:     2,vba;
108$     DUPLICATE:     1,YYY:NEXT(106$);

106$     ENDIF;
102$     ENDIF:     NEXT(21$);

21$     TALLY:     OrderUpToPoint_2,Order Up To Point_2(1),1;
22$     TALLY:     OrderQuantity_2,Order Quantity_2(1),1;
49$     TALLY:     On Order Inventory Tally_2,OnOrderInventory_2(1),1;
```

```
48$   BRANCH,   1:
      If,Order Quantity_2(1)>0,33$,Yes:
      If,Order Quantity_2(1)==0,20$,Yes;
33$   ASSIGN:   QuantityToDuplicate_2=Order Quantity_2(1)-1:
MARK(TimeIntoManufacturer):NEXT(62$);

20$   DISPOSE:   Yes;

XXX   DELAY:   0.01,,Other:NEXT(53$);

53$   VBA:   5,vba;
54$   DISPOSE:   No;

YYY   DELAY:   0.01,,Other:NEXT(52$);

52$   VBA:   6,vba;
55$   DISPOSE:   No;
```

VISUAL BASIC APPLICATION CODE

```
Private Function normCDF (x As Double) As Double

    y = Math.Abs(x)
    p = 0.2316419
    b1 = 0.31938153
    b2 = -0.356563782
    b3 = 1.781477937
    b4 = -1.821255978
    b5 = 1.330274429
    z = (1 / (Math.Sqr(2 * 3.141592654)) * Math.Exp(-(y ^ 2) / 2))
    t = 1 / (1 + p * y)
    normCDF = 1 - z * (b1 * t + b2 * t ^ 2 + b3 * t ^ 3 + b4 * t ^ 4 + b5 * t ^ 5)
    If x < 0 Then
      normCDF = 1 - normCDF
    End If
End Function
Private Sub VBA_Block_1_Fire()
   Dim s As SIMAN
   Set s = ThisDocument.Model.SIMAN
   curDEPer = s.VariableArrayValue(s.SymbolNumber("Period", 1))
   updFreq = s.ExpressionValue(s.SymbolNumber("UpdateFreq"))
   curLTPer = s.VariableArrayValue(s.SymbolNumber("NLT", 1))
   SC = s.VariableArrayValue(s.SymbolNumber("StockoutControl", 1))
   PC = s.VariableArrayValue(s.SymbolNumber("PeriodControl", 1))
   Dim numTimes(25) As Long
   Dim fracTimes(25) As Double
   Dim sumDTimes(25) As Long
   Dim aveDTimes(25) As Long
   Dim LTD(25, 1000000) As Long
```

```
Dim squareSum(25) As Double
Dim varDTimes(25) As Double

avePer = PC
Dim onHandTimes(1000000) As Integer
Dim leadTimes(1000000) As Integer
Dim r As Double
Dim z(25) As Double
Dim sum As Double
Dim mean As Double
Dim std As Double
mean = s.ExpressionValue(s.SymbolNumber("MeanDemand"))
std = s.ExpressionValue(s.SymbolNumber("ErrorStdDev"))
' replace with 102, 103
WhichOne = s.ExpressionValue(s.SymbolNumber("WhichOne"))
LTorLTD = s.ExpressionValue(s.SymbolNumber("LTorLTD"))
' replace with WhichOne_2, LTorLTD_2
r = 1

If (updFreq > 987654) Then

If (WhichOne = 1) Then
  For i = 0 To (avePer - 1)

    leadTimes(i) = s.VariableArrayValue(s.SymbolNumber("LA", 1, (curLTPer -
(LTorLTD) - i)))
    numTimes(leadTimes(i)) = numTimes(leadTimes(i)) + 1
    fracTimes(leadTimes(i)) = numTimes(leadTimes(i)) / avePer
    sumDTimes(leadTimes(i)) = sumDTimes(leadTimes(i)) +
s.VariableArrayValue(s.SymbolNumber("NA", 1, (curLTPer - (LTorLTD) - i)))
    LTD(leadTimes(i), (numTimes(leadTimes(i)) - 1)) =
s.VariableArrayValue(s.SymbolNumber("NA", 1, (curLTPer - (LTorLTD) - i)))
    ' replace with LA_2, NA_2, NA_2

  Next

  If (LTorLTD = 0) Then

    Do While sum < 0.95
      sum = 0
      For i = 1 To 25
        z(i - 1) = (r - i * mean) / (Math.Sqr(i) * std)
        sum = sum + fracTimes(i - 1) * normCDF(z(i - 1))
      Next
      r = r + 1
    Loop

    r = r - 2
```

```
sum = 0

Do While sum < 0.95
   sum = 0
   For i = 1 To 25
      z(i - 1) = (r - i * mean) / (Math.Sqr(i) * std)
      sum = sum + fracTimes(i - 1) * normCDF(z(i - 1))
   Next
   r = r + 0.01
Loop

ElseIf (LTorLTD = 3) Then

   For i = 0 To 24

      If (numTimes(i) >= 1) Then
         aveDTimes(i) = sumDTimes(i) / numTimes(i)

         For j = 0 To (numTimes(i) - 1)
            squareSum(i) = squareSum(i) + ((LTD(i, j) - aveDTimes(i)) ^ 2)
         Next

      ElseIf (numTimes(i) = 0) Then
         aveDTimes(i) = 0

      End If


      If (numTimes(i) > 1) Then
         varDTimes(i) = squareSum(i) / (numTimes(i) - 1)

      ElseIf (numTimes(i) <= 1) Then
         varDTimes(i) = 0

      End If

   Next

   Do While sum < 0.95
      sum = 0
      For i = 0 To 24
         If (numTimes(i) >= 2 And varDTimes(i) <> 0) Then
            z(i) = (r - aveDTimes(i)) / (Math.Sqr(varDTimes(i)))
            sum = sum + fracTimes(i) * normCDF(z(i))
         End If

      Next
      r = r + 1
```

```
      Loop

      r = r - 2
      sum = 0

      Do While sum < 0.95
        sum = 0
        For i = 0 To 24
          If (numTimes(i) >= 2 And varDTimes(i) <> 0) Then
            z(i) = (r - aveDTimes(i)) / (Math.Sqr(varDTimes(i)))
            sum = sum + fracTimes(i) * normCDF(z(i))
          End If
        Next
        r = r + 0.01
      Loop

    End If

  ElseIf (WhichOne = 2) Then

    For i = 0 To (avePer - 1)

      onHandTimes(i) = s.VariableArrayValue(s.SymbolNumber("OnOrderNumber", 1,
(curDEPer - i)))
      numTimes(onHandTimes(i)) = numTimes(onHandTimes(i)) + 1
      sumDTimes(onHandTimes(i)) = sumDTimes(onHandTimes(i)) +
s.VariableArrayValue(s.SymbolNumber("OOI", 1, (curDEPer - i)))
      LTD(onHandTimes(i), (numTimes(onHandTimes(i)) - 1)) =
s.VariableArrayValue(s.SymbolNumber("OOI", 1, (curDEPer - i)))
      ' replace with OnOrderNumber_2, OOI_2, OOI_2

    Next

    For i = 0 To 24

      If (numTimes(i) >= 1) Then
        aveDTimes(i) = sumDTimes(i) / numTimes(i)
        fracTimes(i) = numTimes(i) / avePer

        For j = 0 To (numTimes(i) - 1)
          squareSum(i) = squareSum(i) + ((LTD(i, j) - aveDTimes(i)) ^ 2)
        Next

      ElseIf (numTimes(i) = 0) Then
        aveDTimes(i) = 0

      End If
```

```
        If (numTimes(i) > 1) Then
            varDTimes(i) = squareSum(i) / (numTimes(i) - 1)

        ElseIf (numTimes(i) <= 1) Then
            varDTimes(i) = 0

        End If

    Next


    For i = 0 To 24
        If (varDTimes(i) > 0) Then
                sumOfFrac = sumOfFrac + fracTimes(i)
        End If
    Next


    Do While sum < 0.95
        sum = 0
        For i = 0 To 24
            If (numTimes(i) >= 2 And varDTimes(i) <> 0) Then
                z(i) = (r - aveDTimes(i)) / (Math.Sqr(varDTimes(i)))
                sum = sum + (fracTimes(i) / sumOfFrac) * normCDF(z(i))
            End If

        Next
        r = r + 1
    Loop

    r = r - 2
    sum = 0

    Do While sum < 0.95
        sum = 0
        For i = 0 To 24
            If (numTimes(i) >= 2 And varDTimes(i) <> 0) Then
                z(i) = (r - aveDTimes(i)) / (Math.Sqr(varDTimes(i)))
                sum = sum + (fracTimes(i) / sumOfFrac) * normCDF(z(i))
            End If

        Next
        r = r + 0.01
    Loop

End If
```

```
    End If

End Sub
```

Private Sub VBA_Block_2_Fire()

```
    Dim s As SIMAN
    Set s = ThisDocument.Model.SIMAN
    curDEPer = s.VariableArrayValue(s.SymbolNumber("Period_2", 1))
    updFreq = s.ExpressionValue(s.SymbolNumber("UpdateFreq_2"))
    curLTPer = s.VariableArrayValue(s.SymbolNumber("NLT_2", 1))
    ' replace with Period_2, UpdateFreq_2, NLT_2
    SC = s.VariableArrayValue(s.SymbolNumber("StockoutControl_2", 1))
    PC = s.VariableArrayValue(s.SymbolNumber("PeriodControl_2", 1))
    ' replace with StockoutControl_2, PeriodControl_2
    Dim numTimes(25) As Long
    Dim fracTimes(25) As Double
    Dim sumDTimes(25) As Long
    Dim aveDTimes(25) As Long
    Dim LTD(25, 1000000) As Long
    Dim squareSum(25) As Double
    Dim varDTimes(25) As Double

    avePer = PC
    Dim onHandTimes(1000000) As Integer
    Dim leadTimes(1000000) As Integer
    Dim r As Double
    Dim z(25) As Double
    Dim sum As Double
    Dim mean As Double
    Dim std As Double
    mean = s.ExpressionValue(s.SymbolNumber("MeanDemand_2"))
    std = s.ExpressionValue(s.SymbolNumber("ErrorStdDev_2"))
    ' replace with MeanDemand_2, ErrorStdDev_2
    WhichOne = s.ExpressionValue(s.SymbolNumber("WhichOne_2"))
    LTorLTD = s.ExpressionValue(s.SymbolNumber("LTorLTD_2"))
    ' replace with WhichOne_2, LTorLTD_2
    r = 1

    If (updFreq > 987654) Then

    If (WhichOne = 1) Then
        For i = 0 To (avePer - 1)

            leadTimes(i) = s.VariableArrayValue(s.SymbolNumber("LA_2", 1, (curLTPer -
(LTorLTD) - i)))
            numTimes(leadTimes(i)) = numTimes(leadTimes(i)) + 1
            fracTimes(leadTimes(i)) = numTimes(leadTimes(i)) / avePer
            sumDTimes(leadTimes(i)) = sumDTimes(leadTimes(i)) +
s.VariableArrayValue(s.SymbolNumber("NA_2", 1, (curLTPer - (LTorLTD) - i)))
```

```
      LTD(leadTimes(i), (numTimes(leadTimes(i)) - 1)) =
s.VariableArrayValue(s.SymbolNumber("NA_2", 1, (curLTPer - (LTorLTD) - i)))
      ' replace with LA_2, NA_2, NA_2

   Next

   If (LTorLTD = 0) Then

      Do While sum < 0.95
         sum = 0
         For i = 1 To 25
            z(i - 1) = (r - i * mean) / (Math.Sqr(i) * std)
            sum = sum + fracTimes(i - 1) * normCDF(z(i - 1))
         Next
         r = r + 1
      Loop

      r = r - 2
      sum = 0

      Do While sum < 0.95
         sum = 0
         For i = 1 To 25
            z(i - 1) = (r - i * mean) / (Math.Sqr(i) * std)
            sum = sum + fracTimes(i - 1) * normCDF(z(i - 1))
         Next
         r = r + 0.01
      Loop

   ElseIf (LTorLTD = 3) Then

      For i = 0 To 24

         If (numTimes(i) >= 1) Then
            aveDTimes(i) = sumDTimes(i) / numTimes(i)

            For j = 0 To (numTimes(i) - 1)
               squareSum(i) = squareSum(i) + ((LTD(i, j) - aveDTimes(i)) ^ 2)
            Next

         ElseIf (numTimes(i) = 0) Then
            aveDTimes(i) = 0

         End If


         If (numTimes(i) > 1) Then
            varDTimes(i) = squareSum(i) / (numTimes(i) - 1)
```

```
ElseIf (numTimes(i) <= 1) Then
    varDTimes(i) = 0

End If

Next

Do While sum < 0.95
    sum = 0
    For i = 0 To 24
        If (numTimes(i) >= 2 And varDTimes(i) <> 0) Then
            z(i) = (r - aveDTimes(i)) / (Math.Sqr(varDTimes(i)))
            sum = sum + fracTimes(i) * normCDF(z(i))
        End If

    Next
    r = r + 1
Loop

r = r - 2
sum = 0

Do While sum < 0.95
    sum = 0
    For i = 0 To 24
        If (numTimes(i) >= 2 And varDTimes(i) <> 0) Then
            z(i) = (r - aveDTimes(i)) / (Math.Sqr(varDTimes(i)))
            sum = sum + fracTimes(i) * normCDF(z(i))
        End If
    Next
    r = r + 0.01
Loop

End If

ElseIf (WhichOne = 2) Then

For i = 0 To (avePer - 1)

onHandTimes(i) = s.VariableArrayValue(s.SymbolNumber("OnOrderNumber_2", 1,
(curDEPer - i)))
numTimes(onHandTimes(i)) = numTimes(onHandTimes(i)) + 1
sumDTimes(onHandTimes(i)) = sumDTimes(onHandTimes(i)) +
s.VariableArrayValue(s.SymbolNumber("OOI_2", 1, (curDEPer - i)))
LTD(onHandTimes(i), (numTimes(onHandTimes(i)) - 1)) =
s.VariableArrayValue(s.SymbolNumber("OOI_2", 1, (curDEPer - i)))
' replace with OnOrderNumber_2, OOI_2, OOI_2
```

```
Next

For i = 0 To 24

   If (numTimes(i) >= 1) Then
      aveDTimes(i) = sumDTimes(i) / numTimes(i)
      fracTimes(i) = numTimes(i) / avePer

      For j = 0 To (numTimes(i) - 1)
         squareSum(i) = squareSum(i) + ((LTD(i, j) - aveDTimes(i)) ^ 2)
      Next

   ElseIf (numTimes(i) = 0) Then
      aveDTimes(i) = 0

   End If


   If (numTimes(i) > 1) Then
      varDTimes(i) = squareSum(i) / (numTimes(i) - 1)

   ElseIf (numTimes(i) <= 1) Then
      varDTimes(i) = 0

   End If

Next


For i = 0 To 24
   If (varDTimes(i) > 0) Then
         sumOfFrac = sumOfFrac + fracTimes(i)
   End If
Next


Do While sum < 0.95
   sum = 0
   For i = 0 To 24
      If (numTimes(i) >= 2 And varDTimes(i) <> 0) Then
         z(i) = (r - aveDTimes(i)) / (Math.Sqr(varDTimes(i)))
         sum = sum + (fracTimes(i) / sumOfFrac) * normCDF(z(i))
      End If

   Next
   r = r + 1
Loop
```

```
    r = r - 2
    sum = 0

    Do While sum < 0.95
      sum = 0
      For i = 0 To 24
        If (numTimes(i) >= 2 And varDTimes(i) <> 0) Then
          z(i) = (r - aveDTimes(i)) / (Math.Sqr(varDTimes(i)))
          sum = sum + (fracTimes(i) / sumOfFrac) * normCDF(z(i))
        End If

      Next
      r = r + 0.01
    Loop

  End If

  End If
End Sub
```

Private Sub VBA_Block_3_Fire()

```
  Dim s As SIMAN
  Set s = ThisDocument.Model.SIMAN

  Dim LTD As Integer
  Dim prevLT As Integer

  curLTPer = s.VariableArrayValue(s.SymbolNumber("NLT", 1))
  curDEPer = s.VariableArrayValue(s.SymbolNumber("Period", 1))
  ' replace with NLT_2, Period_2

  If (curLTPer > 3) Then
    prevLT = s.VariableArrayValue(s.SymbolNumber("LA", 1, (curLTPer - 3)))
    orderPer = s.VariableArrayValue(s.SymbolNumber("OP", 1, (curLTPer - 3)))
    ' replace with LA_2, OP_2

    For i = 0 To (prevLT)
     perDemand = s.VariableArrayValue(s.SymbolNumber("DA", 1, (orderPer + 1 + i)))
     ' replace with DA_2
     LTD = LTD + perDemand

     Next

    s.VariableArrayValue(s.SymbolNumber("NA", 1, (curLTPer - 3))) = LTD
    s.VariableArrayValue(s.SymbolNumber("LeadTimeDemand", 1)) = LTD
    ' replace with NA_2, LeadTimeDemand_2, son satırı sil
  Else
    prevLT = 1
```

```
   End If
End Sub
Private Sub VBA_Block_4_Fire()
   Dim s As SIMAN
   Set s = ThisDocument.Model.SIMAN

   Dim LTD As Integer
   Dim prevLT As Integer

   curLTPer = s.VariableArrayValue(s.SymbolNumber("NLT_2", 1))
   curDEPer = s.VariableArrayValue(s.SymbolNumber("Period_2", 1))
   ' replace with 115, 101

   If (curLTPer > 3) Then
      prevLT = s.VariableArrayValue(s.SymbolNumber("LA_2", 1, (curLTPer - 3)))
      orderPer = s.VariableArrayValue(s.SymbolNumber("OP_2", 1, (curLTPer - 3)))
      ' replace with LA_2, OP_2

      For i = 0 To (prevLT)
       perDemand = s.VariableArrayValue(s.SymbolNumber("DA_2", 1, (orderPer + 1 + i)))
       ' replace with DA_2
       LTD = LTD + perDemand

       Next

      s.VariableArrayValue(s.SymbolNumber("NA_2", 1, (curLTPer - 3))) = LTD
      s.VariableArrayValue(s.SymbolNumber("LeadTimeDemand_2", 1)) = LTD
      ' replace with NA_2, LeadTimeDemand_2, son satırı sil
   Else
      prevLT = 1
   End If
End Sub
```

Appendix B

Figures for Numerical Example 6

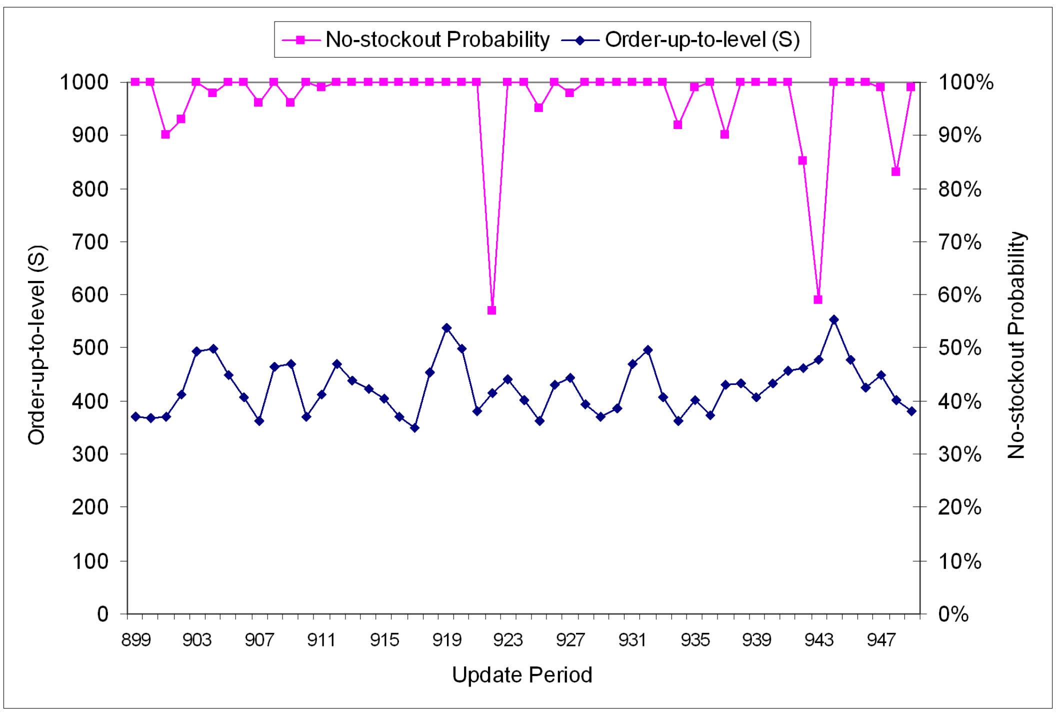

Figure B.1: Plot of no-stockout probability vs. order-up-to-level for the update period of 25 *review periods*, $u = 95\%$.

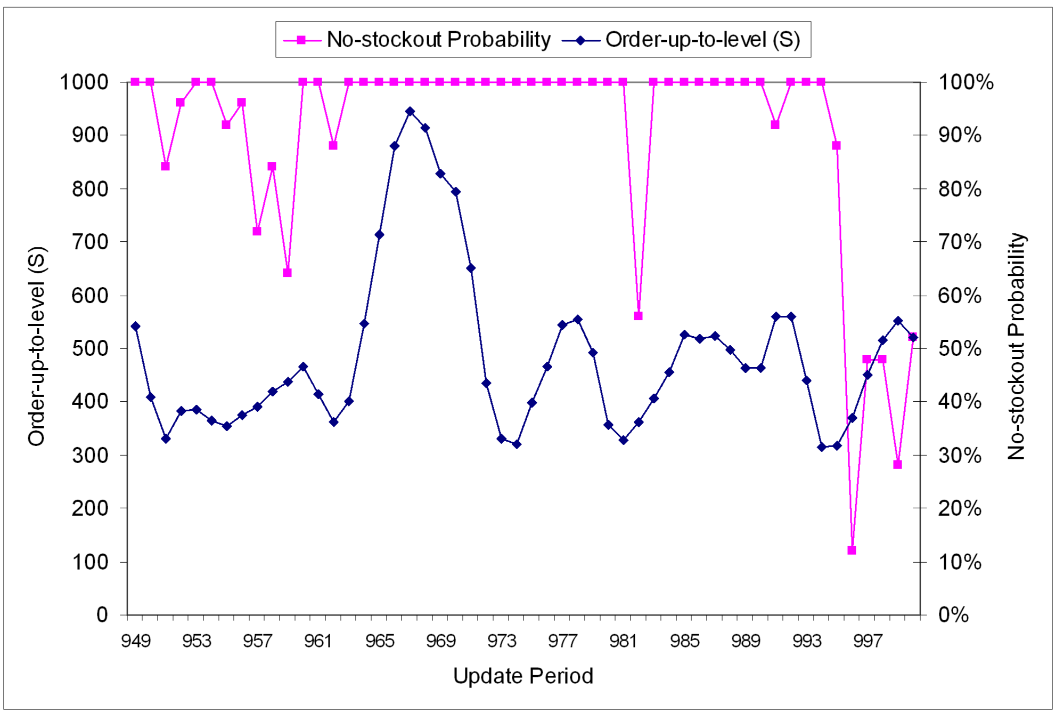

Figure B.2: Plot of no-stockout probability vs. order-up-to-level for the update period of 100 *review periods*, $u = 95\%$.

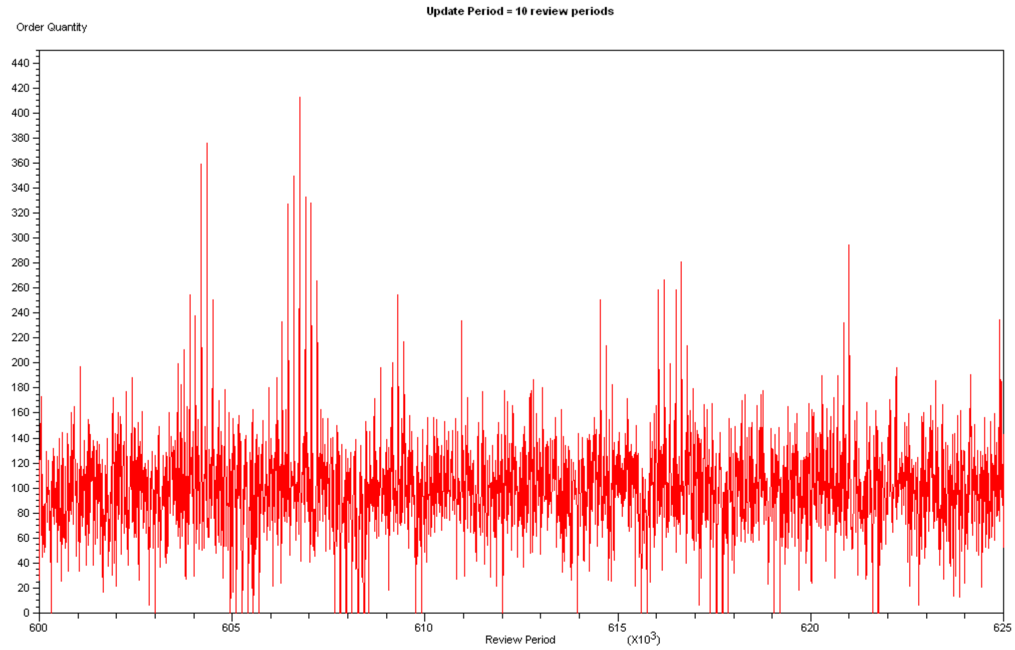

Figure B.3: Plot of order quantity for the update period of 10 *review periods*, $u = 95\%$.

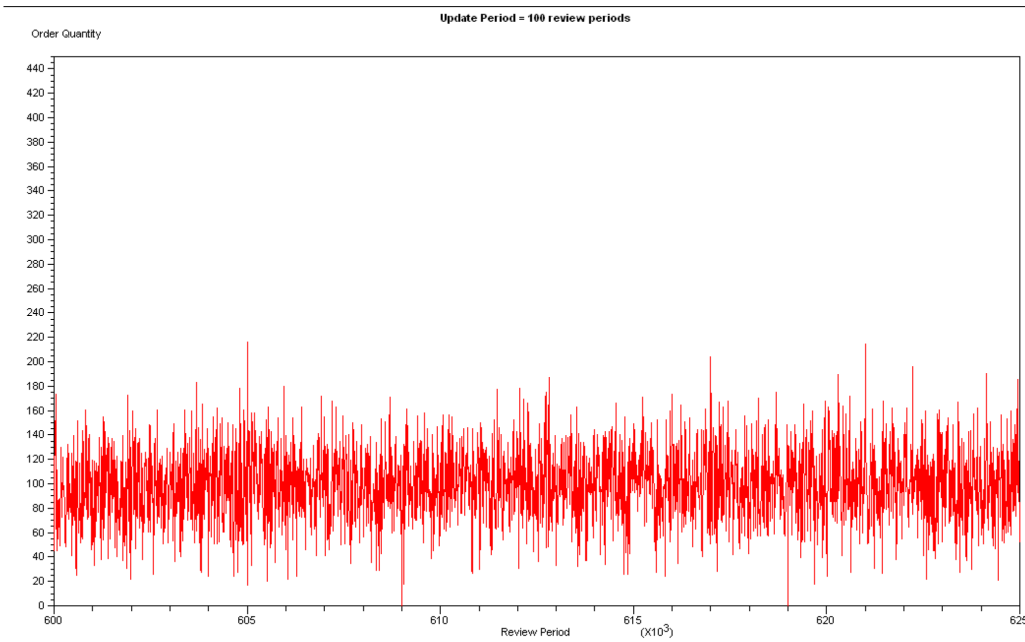

Figure B.4: Plot of order quantity for the update period of 100 *review periods*, $u = 95\%$.

Appendix C

Figures for Numerical Example 8

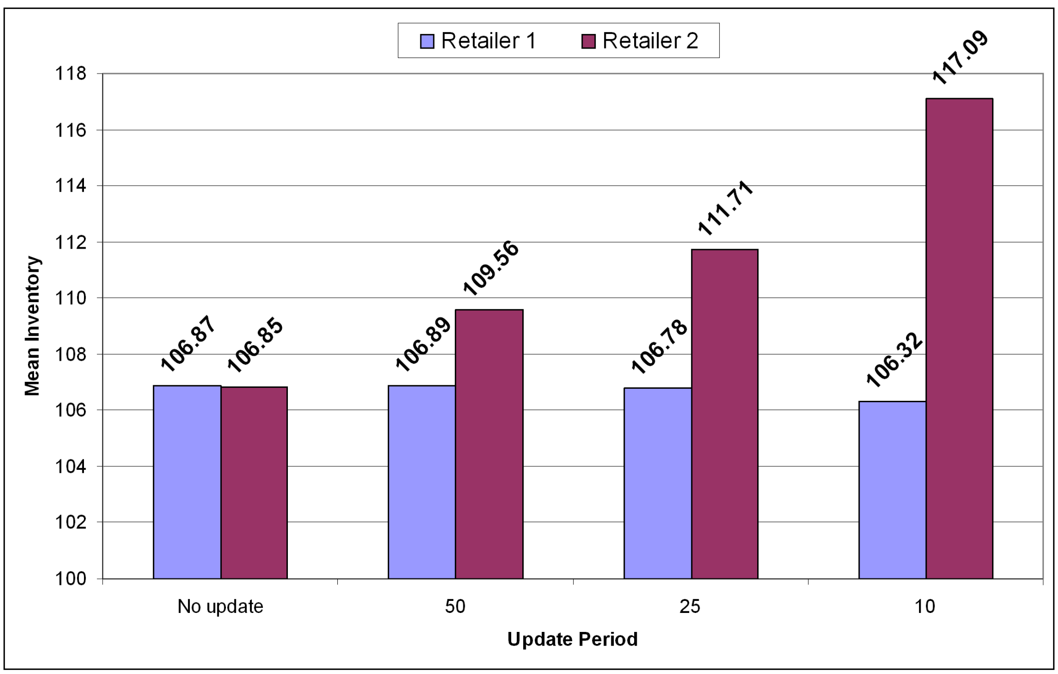

Figure C.1: Simulation results for mean inventory carried.

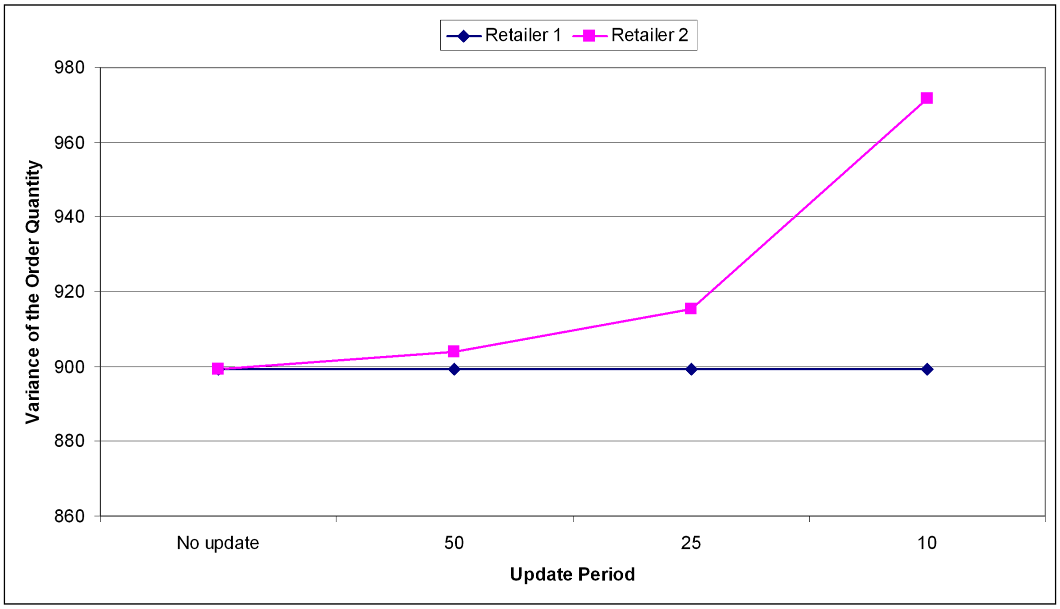

Figure C.2: Simulation results for variance of the order quantity.

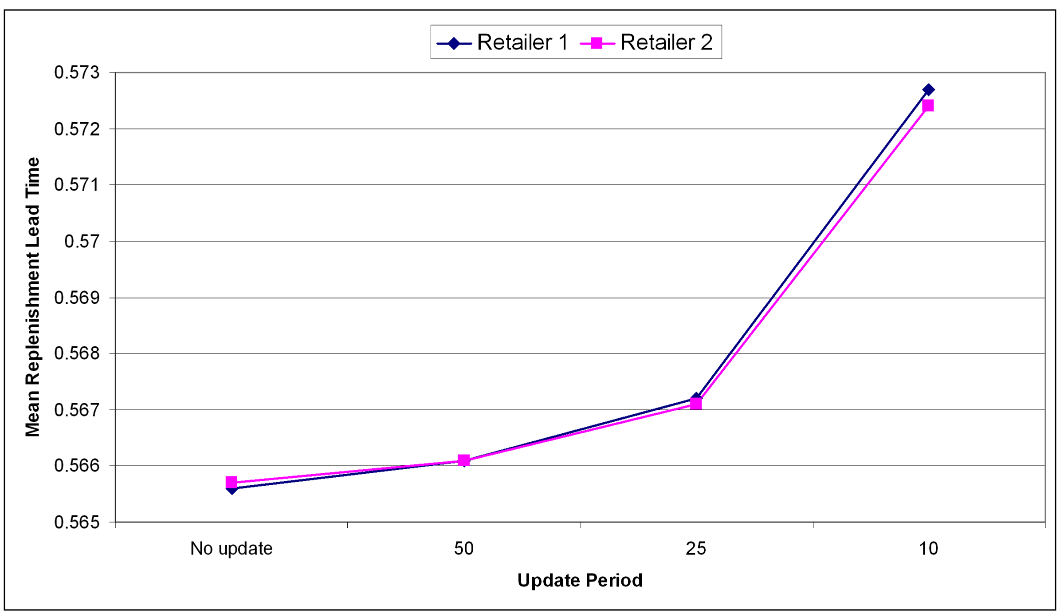

Figure C.3: Simulation results for mean of the replenishment lead time.

Figure C.4: Simulation results for variance of the replenishment lead time.

Appendix D

Figures for Numerical Example 9

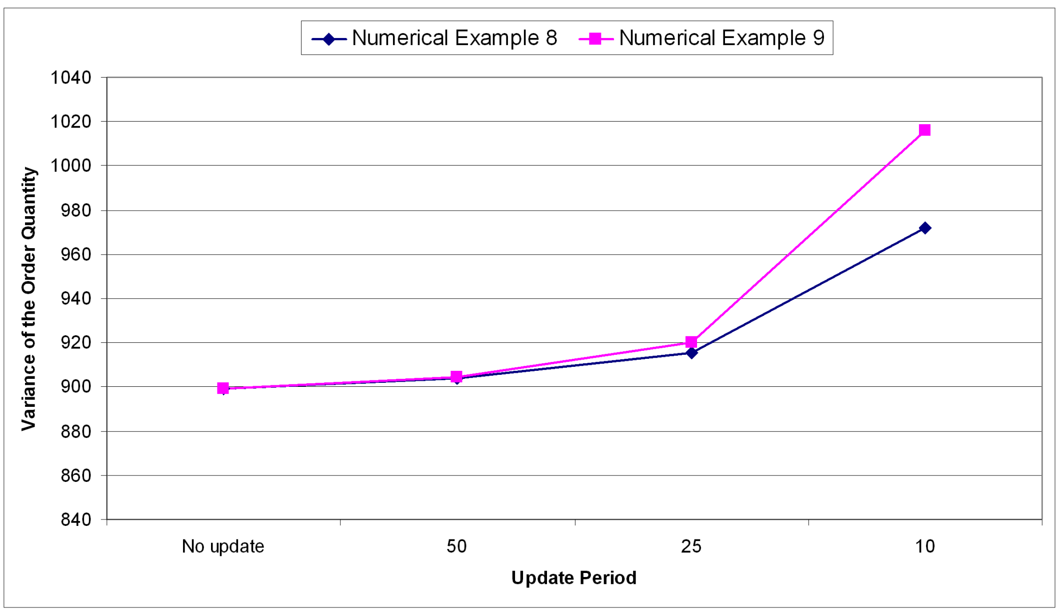

Figure D.1: Simulation results for variance of the order quantity, Retailer 2.

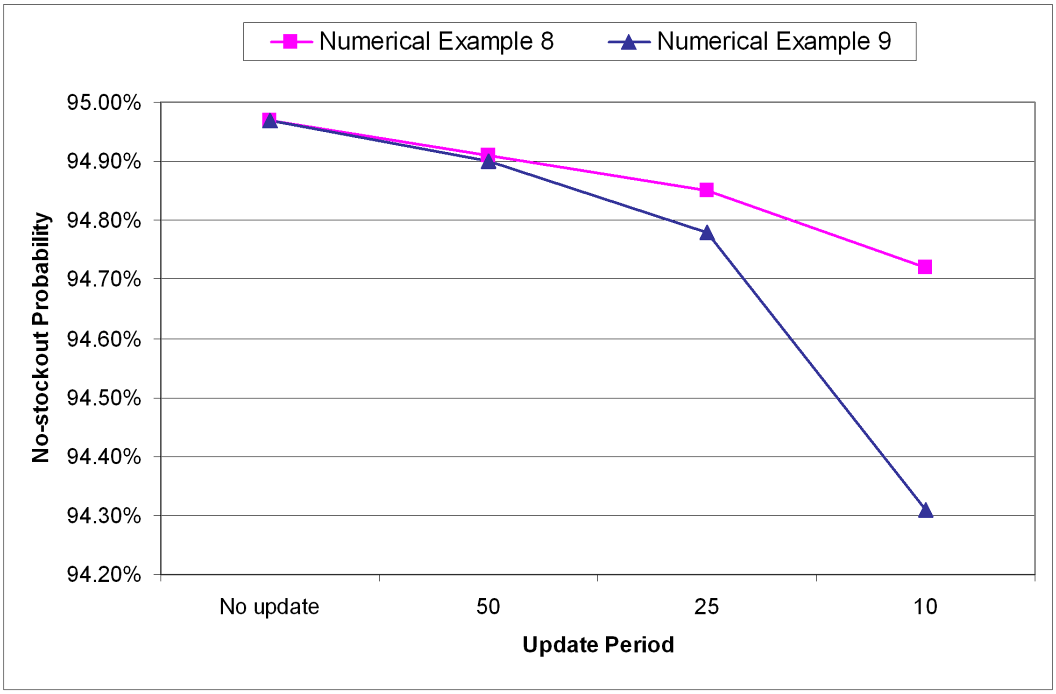

Figure D.2: No-stockout probabilities for Retailer 2.

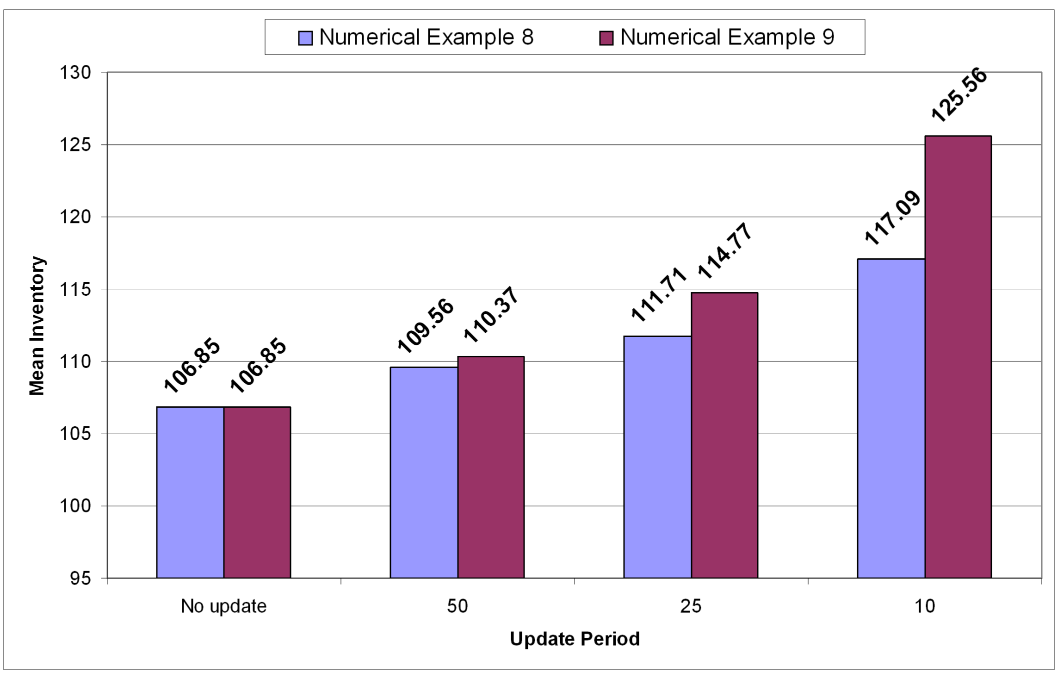

Figure D.3: Simulation results for mean inventory carried by Retailer 2.

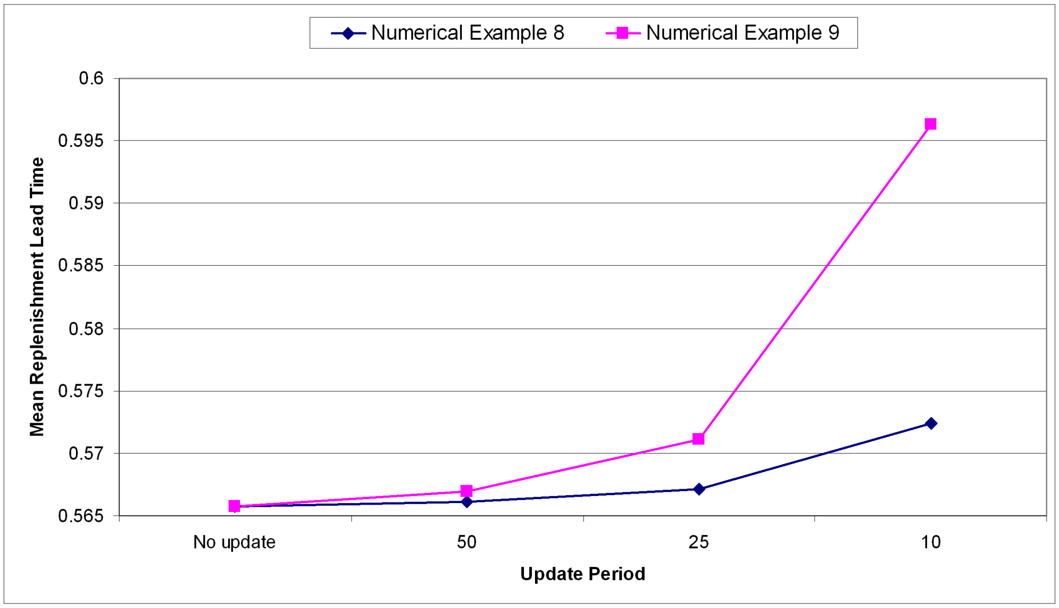

Figure D.4: Simulation results for mean of the replenishment lead time, Retailer 2.

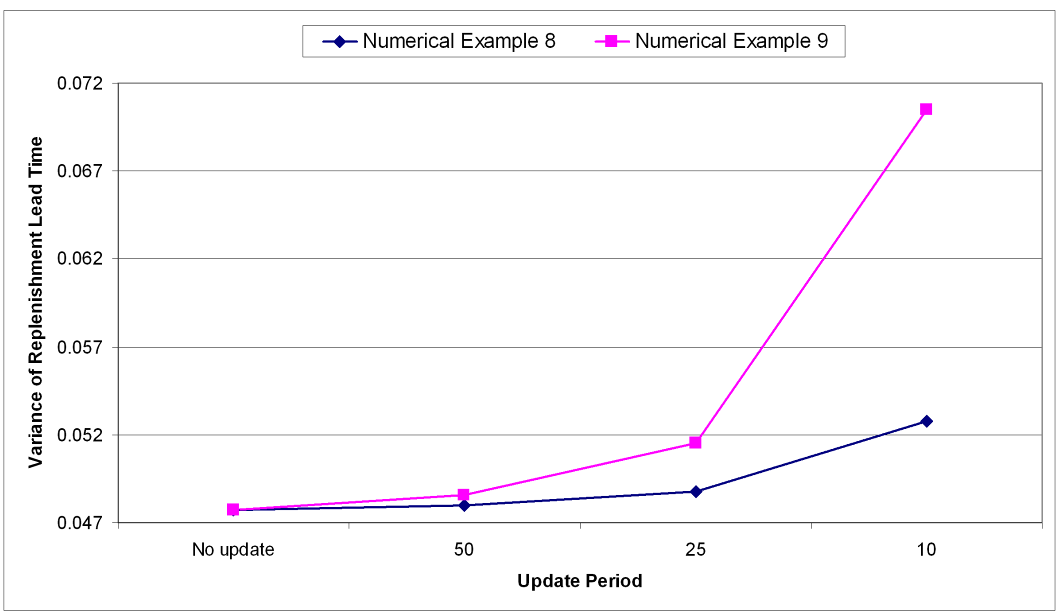

Figure D.5: Simulation results for variance of the replenishment lead time, Retailer 2.

Appendix E

Figures for Numerical Example 10

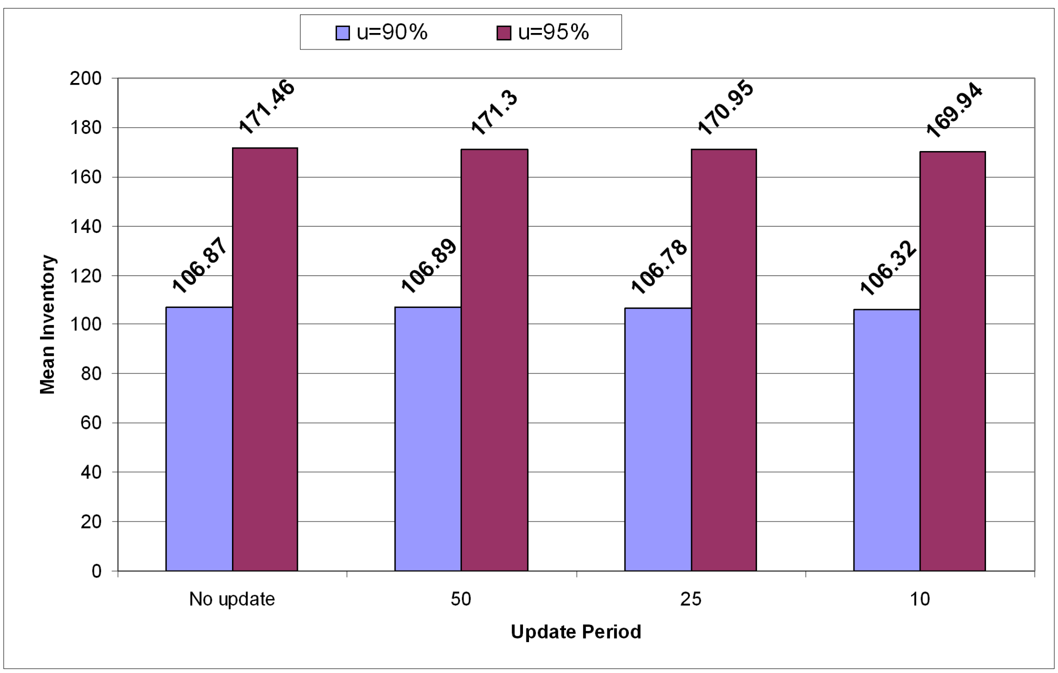

Figure E.1: Simulation results for mean inventory carried by Retailer 1 when only Retailer 2 uses adaptive ordering policy.

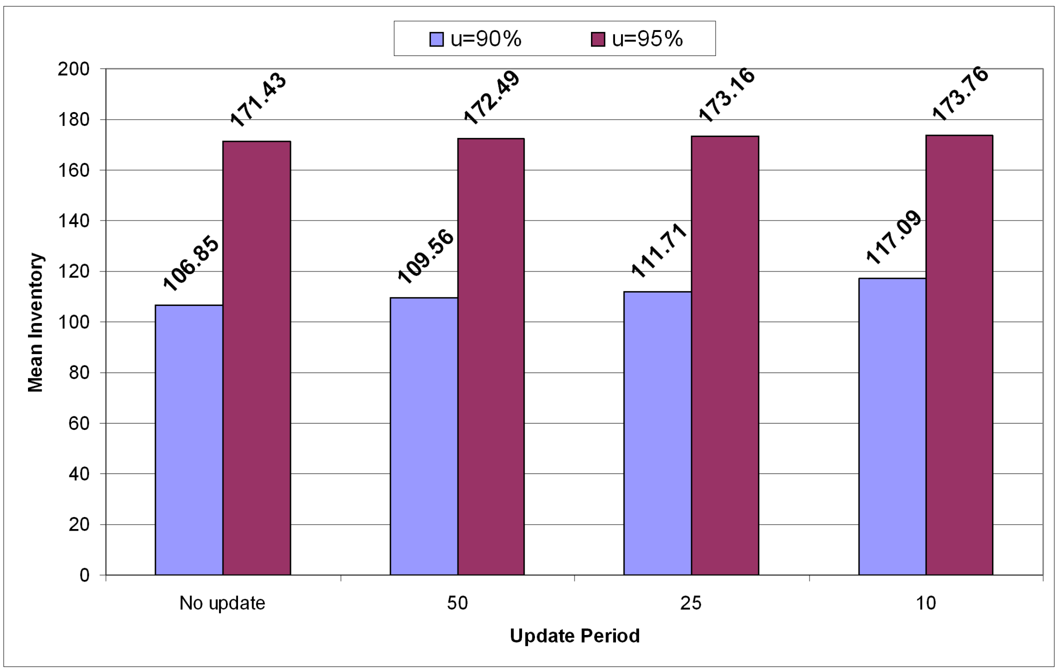

Figure E.2: Simulation results for mean inventory carried by Retailer 2 when only Retailer 2 uses adaptive ordering policy.

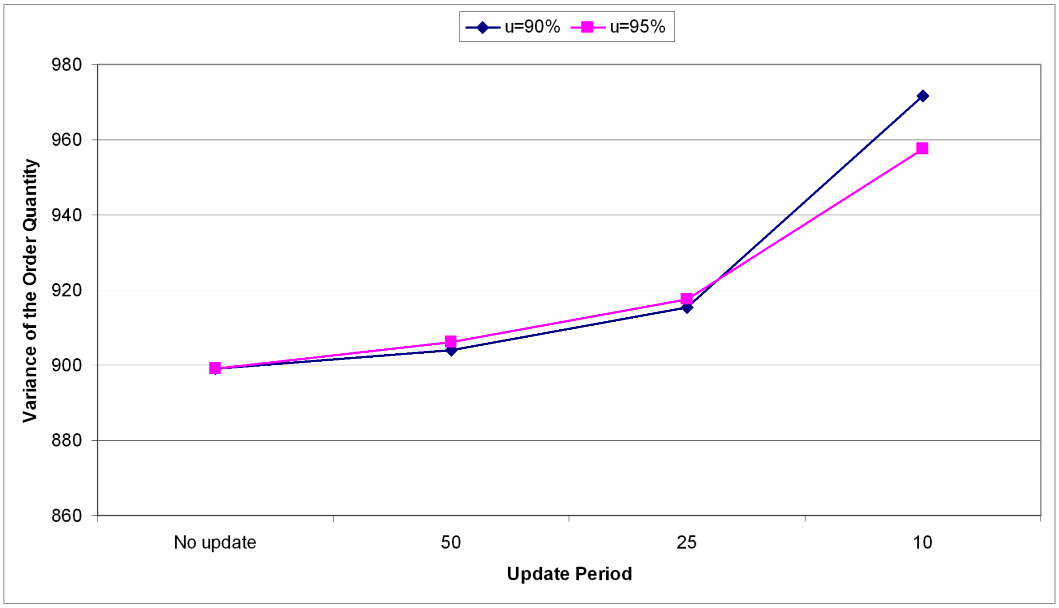

Figure E.3: Comparison of variance of the order quantity for $u = 90\%$ and $u = 95\%$ when only Retailer 2 uses adaptive ordering policy.

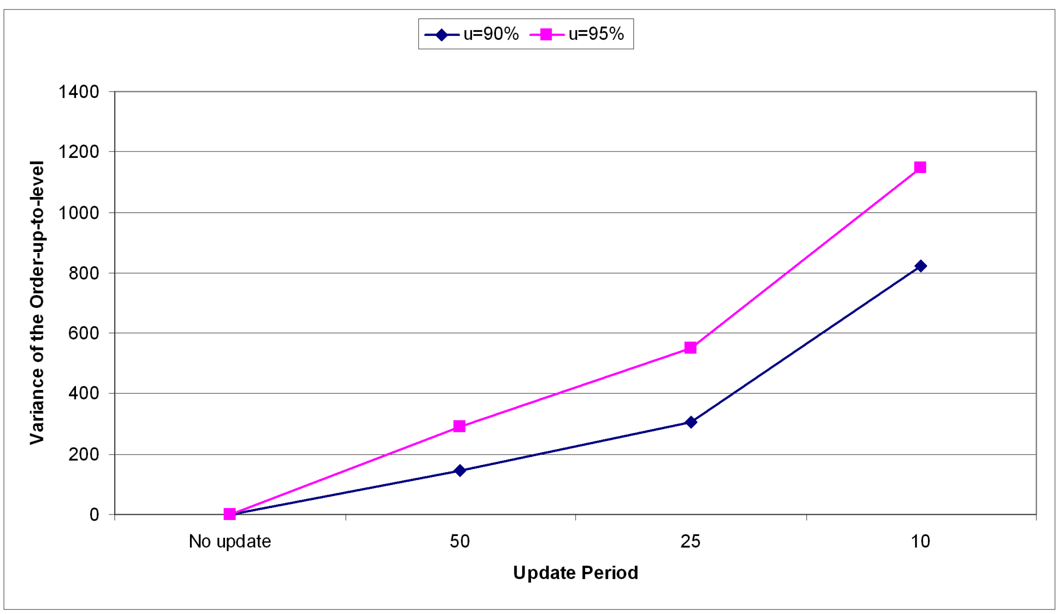

Figure E.4: Comparison of variance of the order-up-to-level for $u = 90\%$ and $u = 95\%$ when only Retailer 2 uses adaptive ordering policy.

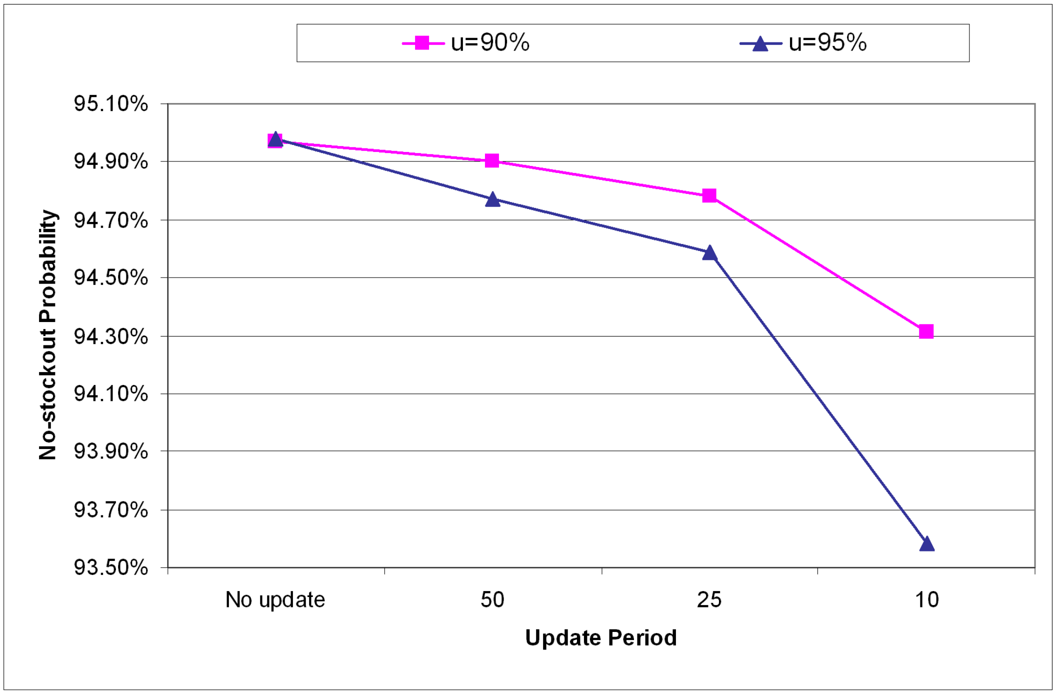

Figure E.5: Comparison of no-stockout probabilities for $u = 90\%$ and $u = 95\%$ when only Retailer 2 uses adaptive ordering policy, Retailer 2.

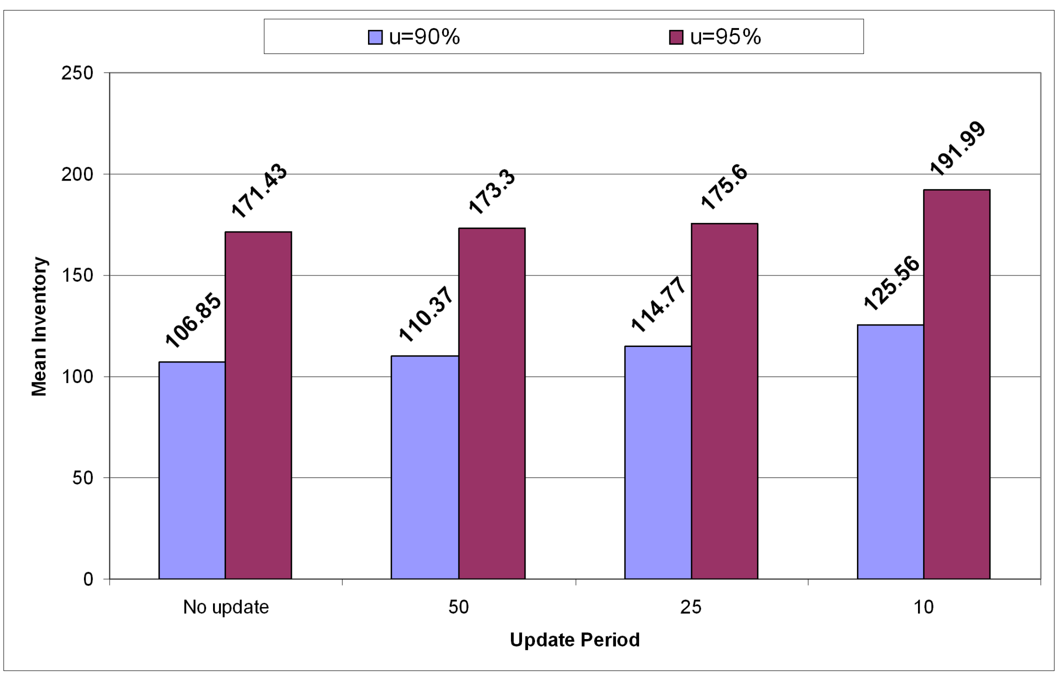

Figure E.6: Simulation results for mean inventory carried by Retailer 2 when only Retailer 2 uses adaptive ordering policy.

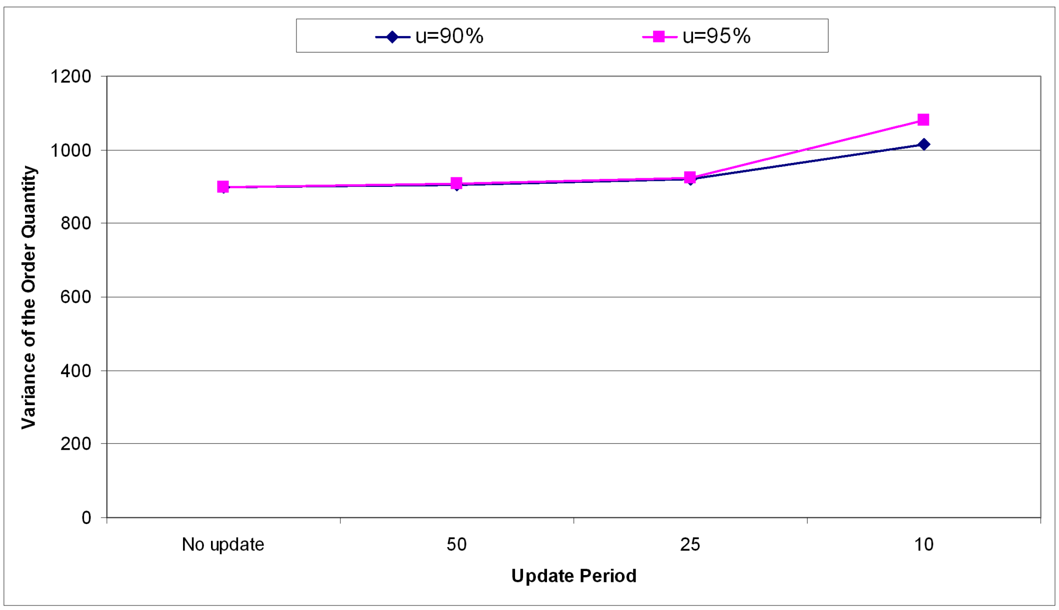

Figure E.7: Comparison of variance of the order quantity for $u = 90\%$ and $u = 95\%$ when both retailers use adaptive ordering policies.