THE BIOBJECTIVE TRAVELING SALESMAN PROBLEM WITH PROFIT

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ABSTRACT

THE BIOBJECTIVE TRAVELING SALESMAN PROBLEM WITH PROFIT

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The traveling salesman problem (TSP) is defined as: given a finite number of cities along with the cost of travel between each pair of them, find the cheapest way of visiting all the cities only once and returning to your starting city. Some variants of TSP are proposed to visit cities depending on the profit gained when the visit occurs. In literature, these kind of problems are named TSP with profit. In TSP with profit, there are two conflicting objectives, one to collect profit and the other to decrease traveling cost. In literature, TSP with profit are addressed as single objective, either two objectives are combined linearly or one objective is constrained with a specified bound. In this study, a multiobjective approach is developed by combining ε-constrained method and heuristics from the literature in order to find the efficient frontier for the TSP with profit. The performance of approach is tested on the problems studied in the literature. Also an interactive software is developed based on the multiobjective approach.

Keywords: TSP with Profit, ε-constrained method, Multiobjective Approach

ÖZ

İKİ AMAÇLI KAR GETİREN GEZGİN SATICI PROBLEMİ

Ömür, Şimşek
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Gezgin Satıcı Problemi (GSP) belirli sayıda şehri en kısa şekilde dolaşacak turun bulunmasıdır. Her yere gitmek yerine gidilecek şehirlerin elde edilecek kazançlara göre seçildiği literatürde problemlere Kar getiren GSP (KGSP) denir. KGSP probleminde, kazancın artırılması ve dolaşılan mesafesinin kısaltılması olarak tanımlanan iki amaç vardır. Literatürde KGSP'ler, iki amacın ağırlıklarla birleştirilmesi ya da amaçlardan birinin belirli bir sınırla kısıt olarak ifade edilmesi suretiyle tek amaçlı problemler olarak çözülmüştür. Bu çalışmada KGSP problemi iki amaçlı bir problem olarak ele alınmış ve literatürdeki sezgisel yöntemler, çok amaçlı bir yaklaşım olan ε-kısıt yöntemiyle birleştirilerek etkin sınırın (efficient frontier) bulunması amaçlanmıştır. Bu yaklaşımın performansı literatürdeki çeşitli problemlerle test edilmiştir. Aynı zamanda çok amaçlı yaklaşımı temel alan kullanıcı etkileşimli bir yazılım hazırlanmıştır.

Anahtar Kelimeler: Kar Getiren GSP, ε- Kısıt Yöntemi, Çok Amaçlı Yaklaşım

To my lovely family

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CHAPTER 1

INTRODUCTION

Traveling Salesman Problem (TSP) is one of the most widely studied combinatorial optimization problems. This has led to numerous extensions and modifications of the basic TSP. In many studies, the number of cities is given and every city has to be visited. This is, however, not always realistic. Consider the example demonstrated in Figure 1, which shows 14-city problem and its optimum TSP tour.

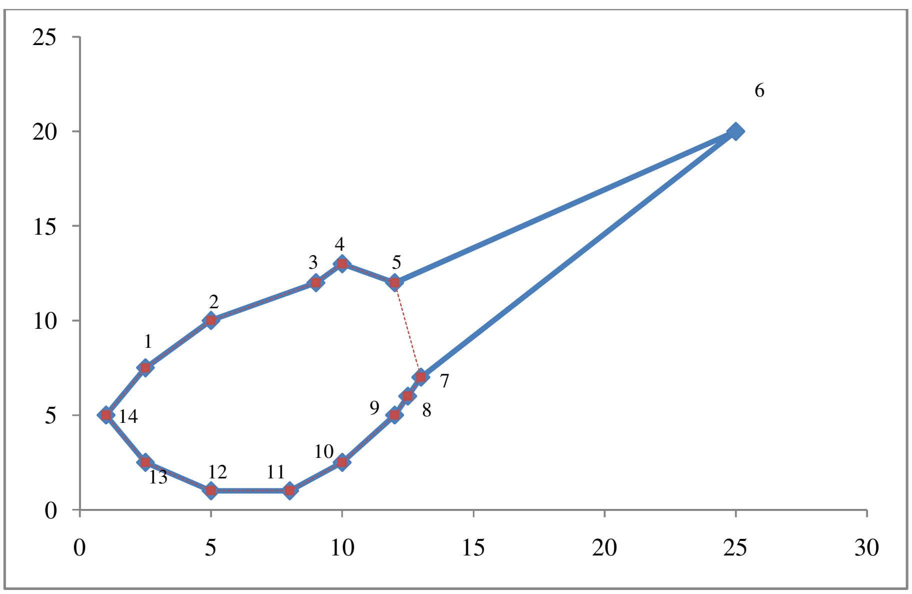

Figure 1. Example of a TSP route

Assume that every city has associated with some profit and a visiting cost is charged when travelling between cities. Figure 1 shows that city 6 is quite isolated from the rest of the cities. A decision whether city 6 should be included in or excluded from the route could depend on the trade-off relationship between the profit and the visiting cost associated with city 6.

The problem in which cities are selected to be visited depending on the profit associated with them is called Traveling Salesman Problem with Profit (TSP with profit). TSP with profit is encountered in many different situations. For instance, it may not be possible to visit every city in a TSP application. In this kind of application some constraints can enforce selection of cities to be visited. Gensch (1978) and Pekny et al. (1990) studied such problems in steel and chemical industry, respectively. Balas and Martin (1985) introduce the scheduling of daily operations of a steel rolling mill, which is an application of TSP with profit. This problem gives rise to a Prize Collecting Traveling Salesman Problem (Prize Collecting TSP) with penalty terms in the objective function.

Orienteering game is another application of TSP with profit. It is introduced by Tsiligirides (1984). In orienteering, competitors start from a control point and have to reach another control point within a prescribed time limit. The aim is collecting as many points as possible within the time limit. Since it is not possible to visit all the points, a selection of points to be visited has to be done. The optimal route, which maximizes the points collected, is obtained by solving the Orienteering Problem (OP).

Some other applications of TSP with profit can also be encountered in the literature. Ramesh and Brown (1991) propose an application in control theory. Fischetti and Toth (1988) notice that TSP with profit arises when a factory needs a given amount of product, which can be provided by a set of suppliers with given amounts and costs.

TSP with profit sometimes appears as subproblems in solution procedures devoted to the different kinds of complex problems. Göthe-Lundgren et al. (1995, 1996) address such subproblems in the context of vehicle routing cost allocation problems. Noon et al. (1994) propose a heuristic procedure for the solution of VRP, based on the iterative solution of TSP with profit.

There are varieties of TSP with profit. The two most studied problems among them are: (i) Selective TSP (or Orienteering), (ii) Prize Collecting TSP. These problems can be considered as the dual of each other. It can easily be recognized that it is actually a biobjective problem where one objective is maximizing the profit to be collected by visiting as many cities as possible, and the other objective is keeping the visiting costs at minimum. If two objectives can be defined in commensurable terms, then they can be combined in a single objective function and can be solved as a single objective problem. Yet, in many cases (e.g. one objective is maximizing the profit but the other objective is minimizing time) it is not possible to combine two objectives. Then, a study of the trade-off relation between two objectives may be of interest.

In literature, TSP with profit is studied as a single objective problem. The only attempt to solve TSP with profit as a biobjective problem is done by Keller and Goodchild (1988). The main difference of biobjective approach compared to a single objective approach is finding not only one, but Pareto optimal solutions. By finding more solutions, the trade-off among them can be analyzed to make better decision. The purpose of this study is to develop a multiobjective approach for the biobjective TSP with profit in order to obtain the Pareto optimal solutions.

The organization of the thesis is as follows: In Chapter 2, the formal definition of the problem is presented and the mathematical representation of the problem is given. A brief review of the related literature is presented in Chapter 3. The related studies are classified according to solution approaches. The solution approach is discussed in Chapter 4. ε-constrained method is presented in detail after discussing the properties of Pareto optimal solutions. In Chapter 5, the analysis methods for the Pareto optimal solutions are discussed. In Chapter 6, the performance of the ε-constrained method and the results of extensive computational experiment are presented. Chapter 7 describes the interactive software developed for the biobjective TSP with profit. Finally, the thesis is concluded with possible future research directions.

CHAPTER 2

PROBLEM DEFINITION AND MODEL

In this chapter, a formal presentation of TSP with profit is provided and the mathematical model of the problem is presented. In section 2.1 the definition of the problem is given and the mathematical model of the problem is given in section 2.2.

2.1 TSP with Profit

TSP is finding the shortest route for a given number of cities. It is one of the most widely studied combinatorial optimization problems (Guttin and Punnen 2002; Toth and Vigo 2001). The main characteristics of TSP are that every city has to be visited and no profit is associated to the cities. In Figure 2 a sample of TSP solution is given. In this figure, the problem has 33 cities and the optimal solution that visits each city once is shown.

A variant of TSP where a profit value is associated to each city and cities are selected depending on their profit are proposed in the literature. Feillet et al. (2005) define these kinds of problems as TSP with profit.

TSP with profit is actually a multiobjective problem with two conflicting objectives, one is to collect the maximal profit and the other is to minimize the travel cost. As a multiobjective problem, solving TSP with profit should result non-dominated solution set, a set of feasible solutions such that neither objective can be improved without deteriorating the other (Feillet et al. 2005).

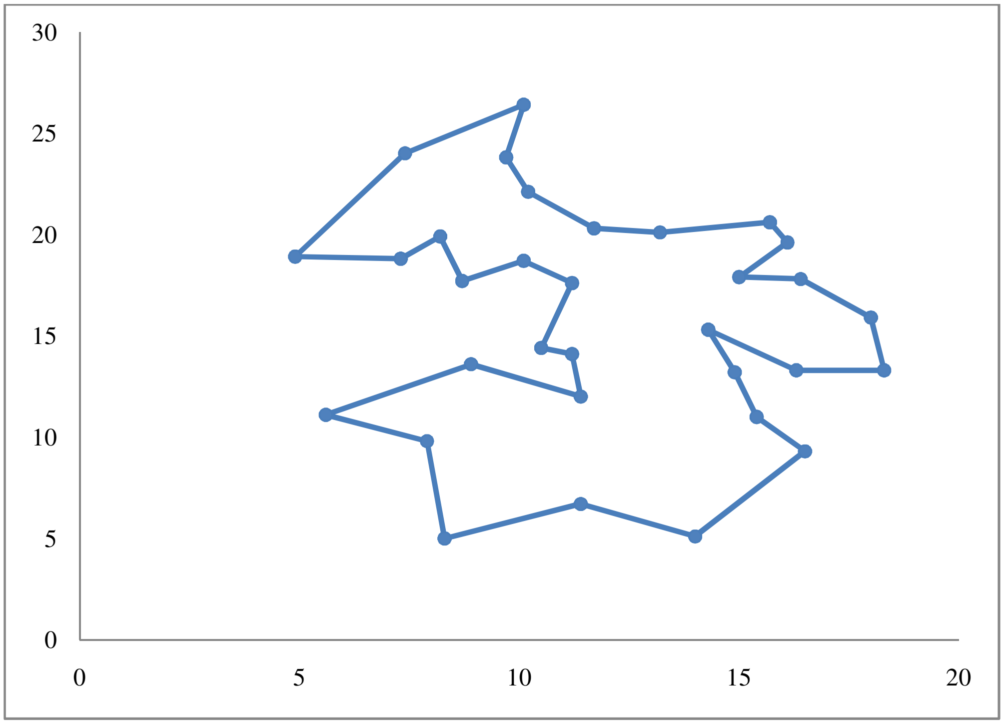

Figure 2. Illustration of TSP solution

2.2 Mathematical Formulation

In this section, integer model for TSP with profit and its variants are given. The indices, parameters and decision variables for the problem are given below.

Indices

i, j : city indices, 1, … , n

Parameters

c_{ij} : the cost of visiting city *j* after city *i*

p_i : the profit that is associated to city *i*

Decision Variables

$x_{ij} =$ 1 if city *j* is visited after city *i*, 0 otherwise

$y_i =$ 1 if city *i* is visited, 0 otherwise

The mathematical model of TSP with profit is

$$Max \sum_i p_i y_i \tag{2.1}$$

$$Min \sum_i \sum_j c_{ij} x_{ij} \tag{2.2}$$

$$subject\ to$$

$$\sum_{j \neq i} x_{ij} = y_i \qquad \forall\, i \tag{2.3}$$

$$\sum_{i \neq j} x_{ij} = y_j \qquad \forall\, j \tag{2.4}$$

$$sub - route\ elimination\ constraints \tag{2.5}$$

$$x_{ij} \in \{0, 1\} \qquad y_j \in \{0, 1\} \tag{2.6}$$

The first objective function of the model expressed in (2.1) is the sum of the profits collected from the cities that belong to the solution route. The second objective function of the model expressed in (2.2) is the total route cost. Constraint set (2.3) ensures that if city *i* is visited then another city has to be visited after city *i*. Constraint set (2.4) guarantees that if city *j* is visited then another city has to be visited before city j. Both constraint sets (2.3) and (2.4) ensure that if city i is arrived, then it must be leaved. Constraint set (2.5) is the set of sub-route elimination constraints that guarantees single tour along the all cities visited. Finally, constraint set (2.6) sets up the binary restrictions for x_{ij} and y_i variables.

In most of the research on TSP with profit, the problem is studied as a single objective problem, either it is maximizing the profit and the route cost is constrained by an upper bound or it is minimizing the route cost and the route profit is constrained by a lower bound.

The single objective variant of TSP with profit in which the objective is maximizing the profit is called Selective Traveling Salesman Problem (Selective TSP). The mathematical formulation of Selective TSP is given below

$$Max \sum_i p_i y_i \tag{2.7}$$

subject to

$$\sum_i \sum_j c_{ij} x_{ij} \leq C \tag{2.8}$$

and (2.3) – (2.6)

On the other hand, the single objective variant of TSP with profit in which the objective is minimizing the route cost is called Prize Collecting Traveling Salesman Problem (Prize Collecting TSP). The mathematical formulation of Prize Collecting TSP is given below

$$Min \sum_{i} \sum_{j} c_{ij} x_{ij} \tag{2.9}$$

$subject\ to$

$$\sum_{i} p_i y_i \ \geq P \tag{2.10}$$

and (2.3) – (2.6)

Intuitively, the biobjective TSP with profit is NP-hard, because TSP is NP-hard and a TSP instance can be stated as a TSP with profit instance by defining very large profits on vertices, therefore it is also NP-hard.

To compute all the solutions in the solution space one has to solve O many TSPs where

$$O = \sum_{i=0}^{n} Com(n,i) \qquad n = number\ of\ cities$$

To better understanding, let $n = 10$. In Table 1, for each i, combination of $n\ and\ i$, $Com(n,i)$, is calculated. As seen for n = 10, one has to solve 1024 TSPs to find all the solutions in the solution spaces. In Table 2, O is given for various n.

Suppose TSP can be solved in one operation and the computer can make 5,000,000 operations in a second. For $n = 100$, it will take 2,53530120045646E+23 seconds or 4,22550200076077E+21 minutes or 70,425,033,346,012,800,000 hours or 2,934,376,389,417,200,000 days or 8,039,387,368,266,300 years.

Table 1. Number of TSPs for $\boldsymbol{n}$ = 10

i	$Com(n,i)$
0	1
1	10
2	45
3	120
4	210
5	252
6	210
7	120
8	45
9	10
10	1
O	1024

Table 2. Number of TSPs for various $\boldsymbol{n}$

n	O
10	1024
20	1048555
50	1.13E+15
100	1.27E+30

Studies of TSP with profit and its single objective variants are discussed in Chapter 3.

CHAPTER 3

LITERATURE REVIEW

The traveling salesman problem is defined as: given a finite number of cities along with the cost of travel between each pair of them, find the cheapest way of visiting all the cities only once and returning to your starting city. The problem can be defined on an undirected complete graph G = (V, E), where V represents the nodes located at the city points and the starting city, and E represents the edges between the nodes. For every edge $\{i, j\} \in E$, there is a cost c_{ij} associated with it. We refer to the books of Gutin and Punnen (2002) and Lawyer et al. (1985) for TSP literature.

This chapter, focusing on the well known variants of TSP, Prize Collecting TSP and Orienteering Problem (Selective TSP), provides a literature survey of solution approaches. These problems can be considered as the dual of each other. When we consider them it can easily be recognized that the problem is actually biobjective problem where the one objective is maximizing the profit to be collected by visiting as many cities as possible, and the other objective is keeping cost to a minimum.

In section 2.1, Selective TSP and Orienteering Problem literatures are presented since Orienteering Problem is a special case of Selective TSP and it is more widely studied. In section 2.2, Prize Collecting TSP literature is presented. Finally, the only approach for the biobjective TSP with profit is presented in section 2.3.

3.1 Selective TSP

There has been work on exact methods. Laporte and Martello (1990) present a branch-and-bound scheme with linear programming (LP) relaxation. They solve the problem where 0-1 constraints are relaxed, through linear programming and the violated conditions are gradually solved through a branch-and-bound process. Leifer and Rosenwein (1993) relax the 0-1 constraints and drop the connectivity constraints. Thereafter, certain valid inequalities are added to the model. After solving the LP relaxation, a cutting plane algorithm is added and the LP is solved again. Fischetti et al. (1998) and Gendreau et al. (1998a) quickly tighten the bounds with valid inequalities all along the search tree (in branch and cut procedures). Ramesh et al. (1992) use Lagrange relaxation along with improvement procedures within a branch and bound method. Gendreau et al. (1998b) extend it to the insertion of clusters. Although these approaches have yielded solutions to smaller sized problems, as in other NP-hard problems, the computational limitations of exact algorithms encourage the exploration of heuristic procedures.

The first heuristics, the S-algorithm and the D-algorithm, were proposed by Tsiligirides (1984). In the S-algorithm, Tsiligirides defines a new term, desirability measure. Points are added to the path depending on this desirability measure. In the D-algorithm, Tsiligirides divides the area into sectors and routes are built up within the sector. In these papers, Tsiligirides also devises the most well known test problems for the OP, which has 21, 32 and 33 cities.

Golden, Levy and Vohra (1987) propose a procedure with three steps: path construction using a greedy method, path improvement and center of gravity which guides the next search step. Golden, Wang, and Liu (1988) incorporate the center of gravity idea and desirability concepts, along with the learning capabilities. An artificial neural network approach is proposed by Wang et al.

(1995). Ramesh and Brown (1991) propose a four-phase heuristic consists of node insertion, cost improvement, node deletion and maximal insertions.

Chao, et al. (1996) introduce a two-step heuristic to solve the OP. In the first step, initialization, by using the starting and ending nodes as the two foci of an ellipse and the route cost constraint as the length of the major axis, several routes are generated and the one with the highest score is the initial solution. The initial route is then improved by a 2-node exchange in the cheapest-cost way, and then improved by a 1-node improvement that tries to increase the total score. They apply this algorithm to Tsiligirides (1984) problems and 40 new test problems. The authors also point out a mistake in Tsiligirides data set and suggest the correction.

Tasgetiren and Smith (2000) propose a genetic algorithm (GA) to solve the orienteering problem. Four test sets, the three originally from Tsiligirides (1984) and the one corrected by Chao, et al. (1996), are used. Tasgetiren results are competitive to the best known heuristics, though the computational time is relatively high. Liang and Smith (2001) recently proposed a standard ant colony algorithm, hybridized with local search, for the OP. They apply this algorithm to Tsiligirides (1984) problems and the one corrected by Chao, et al. (1996). Their results are competitive to the best known heuristics, too.

3.2 Prize Collecting TSP

"PCTSP was introduced by Balas and Martin (1985) as a model for scheduling the daily operations of a steel rolling mill and the same optimization problem was successively addressed by Balas and Martin (1991). Also, structural properties of the PCTSP related to the TSP polytope and to the knapsack polytope were presented by Balas (1989) and Balas (1995)." (Dell'Amico et al. 1998)

Fischetti and Toth (1988) use a Lagrangian relaxation for the generalized covering constraint and solve assignment problem which is resulted by a subtour relaxation. The solution of this assignment problem provides a bound. Dell'Amico et al. (1995) also uses bounding procedure based on different relaxation. Bienstock et al. (1993) studied undirected Prize Collecting TSP. They use linear programming relaxation. Goemans and Williamson (1995) improve the above algorithm. Göthe-Lundgren et al. (1995) propose another approach based on Lagrangian decomposition to obtain a bound. Balas (1999) introduces ordering constraints for which the PCTSP becomes polynomially solvable. In the same way, Kabadi and Punnen (1996) extend results on polynomially solvable cases of the TSP to the PCTSP.

Dell'Amico et al. (1998) present two heuristic procedures for the PCTSP. In the first one, the Lagrangian relaxation is used for route construction. Insertion is then used to attain feasibility of the route. Afterward, extension and collapse are applied iteratively to improve the route. Extension applies insertion as long as insertions are over a computed average ratio. Collapse carries out the replacement of a chain by a single vertex. The second heuristic uses the same components, but in a different order.

3.3 The Biobjective TSP with Profit

The only study for the biobjective TSP with profit is Keller and Goodchild (1988). They uses Tsiligirides' (1984) algorithm for the multi-objective vending problem (MVP) to solve the OP. A path construction phase uses a measure identical to that of the S-algorithm. This is followed by a three step improvement phase that uses node insertion and identification of node clusters. They used 25 cities located in West Germany. Bonn was used as the depot and terminal node. The populations of cities were treated as profit associated with each city.

CHAPTER 4

PROPOSED MULTIOBJECTIVE APPROACH

As mentioned earlier there are numerous studies that address TSP with profit as a single objective problem, either the two objectives are weighted and combined linearly, or one of the objectives is constrained with a specified bound value. The only attempt to solve the true multiobjective problem is Keller and Goodchild (1988). In this chapter, a new multiobjective approach is presented to solve the biobjective TSP with profit.

TSP with profit is studied as Selective TSP or Prize Collecting TSP in the literature. A multiobjective approach, which scalarized the TSP with profit, can be easily used since Selective TSP and Prize Collecting TSP are scalarized TSP with profit and there are numerous studies about them. ε-constraint method, which is a multiobjective solution approach based on scalarization, where one of the objective functions is minimized while all other objective functions are bounded by means of additional constraints, is selected in this research. In section 3.1 some definitions and notations are given. ε-constraint method and adaptation of ε-constraint method to the biobjective TSP with profit that generates two new subproblems are discussed in sections 3.2 and 3.3. Finally, the heuristic approach used to solve generated subproblems and the entire proposed method are presented in section 3.4.

4.1 Definitions and Notations

Multi-objective Optimization, multi-criteria optimization, vector optimization, or multi-criteria decision making is an optimization with regard to multiple objective functions, aiming at a simultaneous improvement of the objectives.

Let R^n and R^m be vector spaces referred to as the decision space and the objective space. Let $X \subset R^n$ be a non-empty and compact feasible set and let f be a vector valued objective function $f : R^n \rightarrow R^m$ composed of m real-valued continuous objective functions, $f = (f_1, \ldots, f_m)$, where $f_k : R^n \rightarrow R$ for $k = 1, \ldots, m$. A multi-objective problem (MOP) can be modeled as

$$\min\big(f_1(x), \ldots, f_m(x)\big) \tag{4.1}$$

$$subject\ to$$

$$x \in X \tag{4.2}$$

For MOP only minimization term is used for the objectives, because max $f(x)$ can be easily converted to min $-f(x)$.

It is usually assumed that X is given implicitly in the form of constraints, i.e., $X : \{x \in R^n : g_j(x) \leq 0, j = 1, \ldots, l;\ h_j(x) = 0, j = 1, \ldots, m\}$. The set of all attainable points or objective vectors for all feasible solutions $x \in X$ in the objective space is defined as $Y := f(X) \subset R^m$ (Ehrgott and Ruzika, 2005).

A continuous sample problem is illustrated. Let R^2 be euclidean vector space referred to as the decision space and the objective space. Let $X \subset R^2$ and bounded by

$$3x_1 + x_2 \geq 12$$

$$x_1 + 3x_2 \geq 12$$

$$x_1 + x_2 \geq 9$$

Let $f = (f_1, f_2)$ where $f_1(x) = x_1$ and $f_2(x) = x_2$.

This simple MOP (*S3.1*) can be modeled as

$\min x_1$

$\min x_2$

$subject\ to$

$$3x_1 + x_2 \geq 12 \quad (4.3)$$

$$x_1 + 3x_2 \geq 12 \quad (4.4)$$

$$x_1 + x_2 \geq 9 \quad (4.5)$$

This problem is referred as *S3.1*. The decision space and the objective space are illustrated in Figure 3 and Figure 4, respectively. The decision space is bounded by equations 3.3, 3.4 and 3.5. The bound between (0, 12) and (1.5, 7.5) is generated by equation 3. The bound between (1.5, 7.5) and (7.5, 1.5) is generated by equation 5 and the bound between (7.5, 1.5) and (12, 0) is generated by equation 4.

Figure 3. Illustration of decision space for the sample MOP

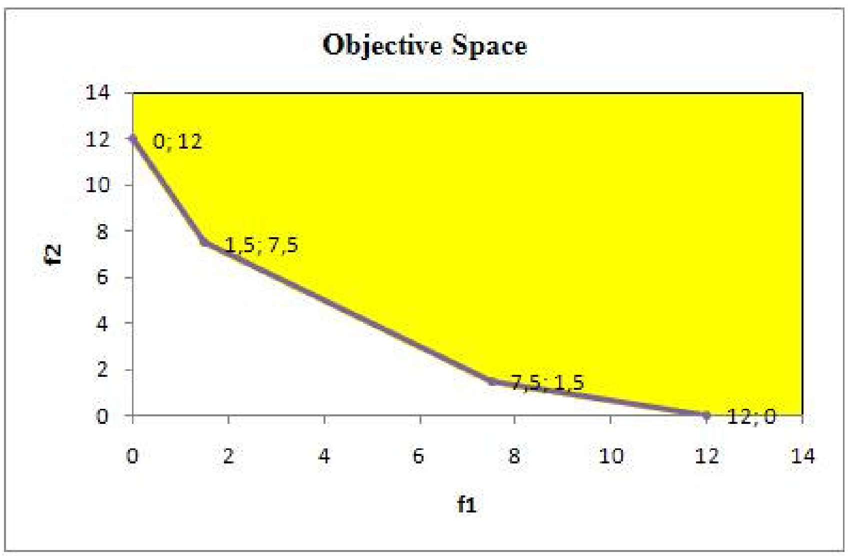

Figure 4. Illustration of objective space for the sample MOP

For S3.1, it can easily seen that the decision space and the objective space have same points and the bounds of decision space are also the bounds of objective space, since $f_1(x) = x_1$ and $f_2(x) = x_2$. But this is not the case for most of the MOPs and it is hard to find the bounds for the objective space.

The objective functions are usually conflicting in MOPs. As the objective function contradicts, no point can be optimal for all m objective functions simultaneously. Thus the optimality concept used in scalar optimization must be replaced by a new one, called *Pareto Optimality*.

Pareto Optimality is an optimality criterion for MOPs. A solution x^a is said to be Pareto optimal, if there is no other solution x^b dominating the solution x^a with respect to a set of objective functions. A solution x^a dominates a solution x^b, if x^a is better than x^b in at least one objective function and not worse with respect to all other objective functions.

$x^a \in X$ is Pareto optimal if and only if there exists no $x^b \in X$ such that $f_k(x^b) \leq f_k(x^a)$ for all $k = 1, \dots, m$ with $f_k(x^b) < f_k(x^a)$ for at least one k.

$x^a \in X$ dominates $x^b \in X$ if and only if $f_k(x^a) \leq f_k(x^b)$ for all $k = 1, \dots, m$ with $f_k(x^a) < f_k(x^b)$ for at least one k.

Table 3. Sample solution set for S3.1

	x_1	x_2	$f_1(x)$	$f_2(x)$
x^1	12	0	12	0
x^2	11	6	11	6
x^3	7.5	1.5	7.5	1.5
x^4	6	6	6	6
x^5	4.5	4.5	4.5	4.5
x^6	2	8	2	8
x^7	1.5	7.5	1.5	7.5
x^8	0	12	0	12

For the sample solution given in Table 3, dominations and Pareto optimality is explained below.

x^3 dominates x^2 since $7.5 < 11$ and $1.5 < 6$.

x^4 dominates x^2 since $6 = 6$ and $6 < 11$.

x^5 dominates x^4 since $4.5 < 6$ and $4.5 < 6$.

x^5 also dominates x^2.

x^7 dominates x^6 since $1.5 < 2$ and $7.5 < 8$.

x^1 is also Pareto optimal, since there is no solution that dominates x^1.

For the sample solution set, solutions x^2, x^4 and x^6 are dominated by solutions x^3, x^5 and x^7, respectively. Solutions x^1, x^3, x^5, x^7 and x^8 are Pareto optimal, since there is no solution that dominates these solutions. Pareto optimal solutions and dominated solutions are illustrated in Figure 5.

In the literature other terms have also been used instead of Pareto optimal, including non-dominated, non-inferior, efficient, functional-efficient and EP-optimal (Edgeworth-Pareto optimal) solutions.

The set of solutions satisfying the criterion of Pareto optimality is called *Pareto Set*, or Pareto front or efficient frontier. In S3.1, the set of solutions x^1, x^3, x^5, x^7 and x^8 is called Pareto set, since they are Pareto optimal solutions. Since the decision space for S3.1 is continuous the Pareto set is continuous for S3.1. In Figure 6, the line that connects solutions x^1, x^3, x^5, x^7 and x^8 contains all the Pareto optimal solutions.

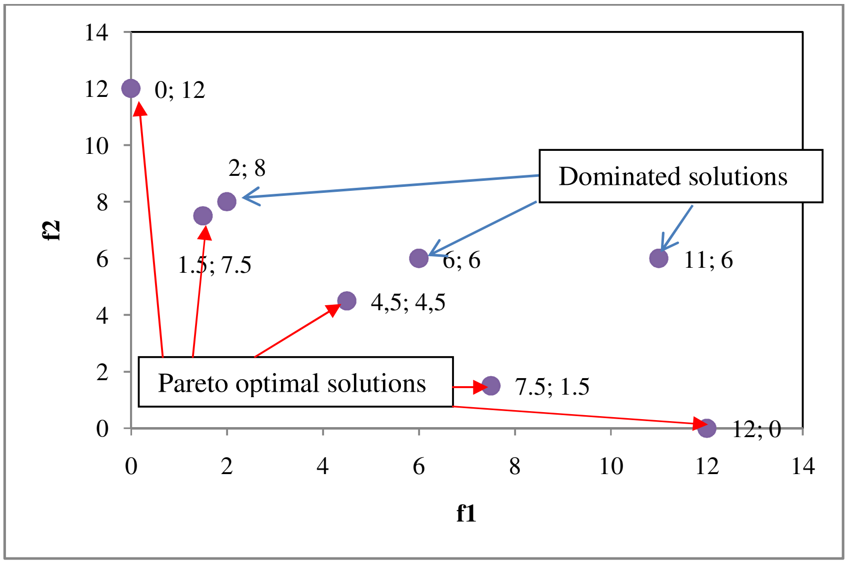

Figure 5. The illustration of sample solutions for S3.1

The exact number of Pareto optimal solutions depends on the type of the decision space. If the decision space is continuous, the number of Pareto optimal solutions is mostly infinite. If it is discrete, the number of Pareto optimal solutions is mostly finite.

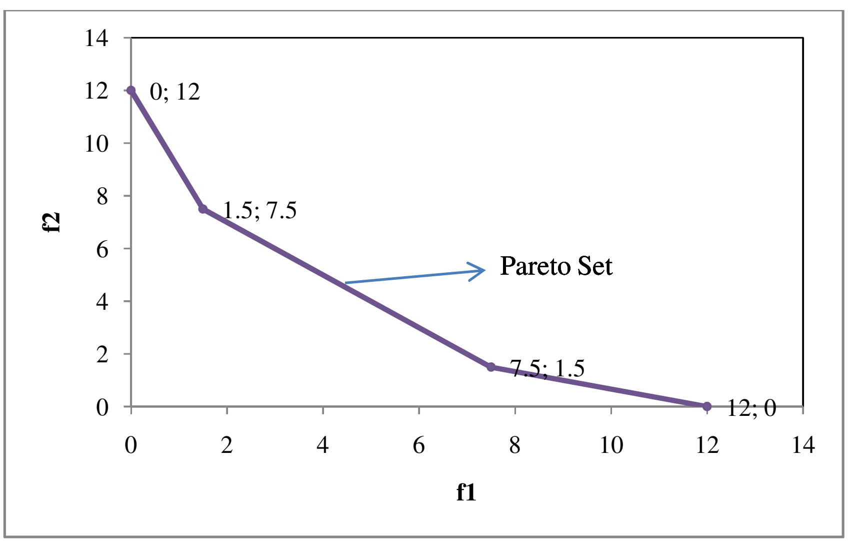

Figure 6. Continuous Pareto set for S3.1

For a discrete decision space Pareto set may be illustrated as in Figure 7.

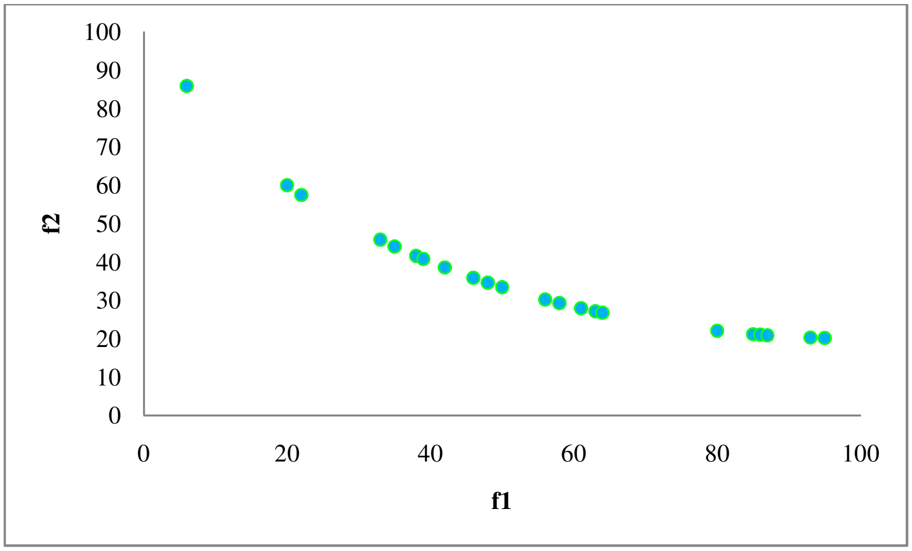

Figure 7. Illustration of discrete Pareto set

4.2 ε-constraint Method

The ε-constraint method is a multi-objective optimization technique, proposed by Haimes et al. (1983), for generating Pareto optimal solutions. It is based on a scalarization where one of the objective functions is choosen as a scalar objective to be minimized and other objective functions are transformed into constraints. For transforming the multi-objective problem into several single-objective problems with constraints, it uses the following procedure.

$$\min f_k(x) \tag{4.6}$$

$subject\ to$

$$f_i(x) \leq \varepsilon_i \qquad i \neq k \tag{4.7}$$

$$x \in X$$

In equation (4.6), objective function $f_k(x)$ is choosen to be minimized and other objective functions $f_i(x)\ i \neq k$ are constrained by upper bounds $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_{k-1}, \varepsilon_{k+1} \ldots, \varepsilon_m)$ in equation (4.7). The vector of upper bounds, $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_{k-1}, \varepsilon_{k+1} \ldots, \varepsilon_m)$, defines the maximum value that each objective can have. In order to obtain a subset of the Pareto optimal set (or even the entire set, in case this set is finite), one must vary the vector of upper bounds along the efficient frontier for each objective, and perform a new optimization process for each new vector.

An implementation of ε-constraint method for S3.1 can be modeled in two different ways since it is a bicriteria problem. Either $f_1(x) = x_1$ or $f_2(x) = x_2$ is choosen to be minimized and $f_2(x) = x_2$ or $f_1(x) = x_1$ is constrained, respectively.

In the first model $f_1(x) = x_1$ is choosen to be minimized and $f_2(x) = x_2$ is choosen to be minimized in the second model.

Let say *S3.1.1* as it is the first single objective version of *S3.1*.

$$\min x_1$$

$$subject\ to$$

$$x_2 \leq \varepsilon_2^i$$

$$3x_1 + x_2 \geq 12$$

$$x_1 + 3x_2 \geq 12$$

$$x_1 + x_2 \geq 9$$

Let say *S3.1.2* as it is the second single objective version of *S3.1*.

$$\min x_2$$

$$subject\ to$$

$$x_1 \leq \varepsilon_1^i$$

$$3x_1 + x_2 \geq 12$$

$$x_1 + 3x_2 \geq 12$$

$$x_1 + x_2 \geq 9$$

For a sample $\varepsilon_2^i = \{12, 10, 8, 6, 4, 2, 0\}$ where $\varepsilon_2^i = \varepsilon_2^{i-1} - 2$ and $\varepsilon_2^1 = 12$ for $i = 1, \ldots, 7$, the solution procedure for S3.1.1 starts by solving the model by $\varepsilon_2^1 = 12$,

$$\min x_1$$

$$subject\ to$$

$$x_2 \leq 12$$

$$3x_1 + x_2 \geq 12$$

$$x_1 + 3x_2 \geq 12$$

The solution of this problem is (0, 12). The next step is updating the model with and solving the new model. This continues in this manner until there is no feasible solution or all models of are solved. The Pareto optimal solutions founded by are illustrated in Figure 8.

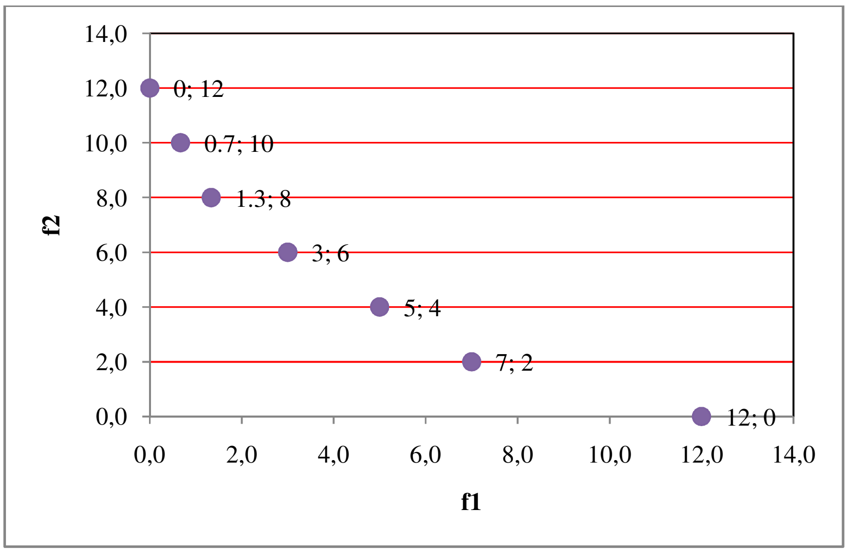

Figure 8. Pareto optimal solutions of S3.1.1 found by ε-constraint method

Exact algorithms, heuristics, or meta-heuristics could be used to solve single objective problems generated by ε-constraint method (ε-MOP). The solution method used for ε-MOP could generate dominated solutions, as it improves not all objectives, only the scalar objective. To eliminate these dominated solutions, ε-constraint method has to be modified. There are two possible modification ways (it is assumed that all the objective functions are minimization):

i. For example, if m=2, one additional scalar problem must be solved to weep out a possible weak solution that is not Pareto optimal. Here the earlier constrained objective is put into the objective function and the former objective function is removed to form an equality constraint where the allowable limit is the optimum solution of the first problem.

ii. The constrained objectives are added to the scalar objective by a set of appropriate weights. The objective function equals to the sum of k^{th} objective function and the negative weighted constrained objectives.

The first modification needs more computations as there are subproblems needed to solve. The second modification is choosen in order to eliminate dominated solution in our implementation. The ε-constraint method algorithm is given in Figure 9.

Step 1. Set Pareto Set $= \emptyset$, $l = 0$
Step 2. Choose the k^{th} objective, $f_k(x)$, to be minimized,
Step 3. Initialize $\varepsilon = (\varepsilon_1, \dots, \varepsilon_{k-1}, \varepsilon_{k+1} \dots, \varepsilon_m)$
Step 4. Constrain objectives $f_i(x)$ $i \neq k$ by using upper bounds ε
Step 5. Solve ε-MOP
Step 6. Set $S_l =$ solution of ε-MOP. If there is no feasible solution, then stop
Step 7. Set S_l $\in$ Pareto Set
Step 8. Set $l = l + 1$
Step 9. Update ε, return Step 4

Figure 9. ε-constraint method algorithm

4.3 Adaptation of ε-constraint Method

Implementation of the ε-constrained method to the biobjective TSP with profit arises two different problems depending on the main single objective:

- The objective is increasing the profit while the route cost is upper bounded as an additional constraint
- The objective is decreasing the route cost while the profit is lower bounded as an additional constraint

The first problem is known as STSP, or Orienteering Problem, as discussed earlier. The ε-constrained problem (it is referred as *ε-BTSP with profit* $^{(1)}$) is modeled as

ε-BTSP with profit $^{(1)}$

$$Max \sum_i p_i y_i - \theta_1 \sum_i \sum_j c_{ij} x_{ij} \tag{4.8}$$

$$subject\ to$$

$$\sum_i \sum_j c_{ij} x_{ij} \leq \varepsilon_1^k \tag{4.9}$$

$$\sum_{j \neq i} x_{ij} = y_i \qquad \forall\, i \tag{4.10}$$

$$\sum_{i \neq j} x_{ij} = y_j \qquad \forall\, j \tag{4.11}$$

$$sub-route\ elimination\ constraints \tag{4.12}$$

$$x_{ij} \in \{0,1\} \quad y_j \in \{0,1\} \tag{4.13}$$

Objective function (4.8) maximizes the sum of total profit and negative weighted route cost. Equation (4.9) ensures that the route cost is upper bounded by ε_1^k.

The second problem is known as PCTSP as discussed earlier. The ε-constrained problem (it is referred as *ε-BTSP with profit* [(2)]) is modeled as

ε-BTSP with profit [(2)]

$$Min \sum_i \sum_j c_{ij} x_{ij} + \theta_2 \sum_i p_i y_i \tag{4.14}$$

subject to

$$\sum_i p_i y_i \geq \varepsilon_2^k \tag{4.15}$$

(4.10) – (4.13)

Objective function (4.14) minimizes the sum of route cost and weighted total profit. Equation (4.15) ensures that the total profit is lower bounded by ε_2^k.

As mentioned earlier there are solution methods for both of the single objective problems in the literature. This means that one can implement ε-constrained method for both of the problems. Considering that STSP is more widely studied than PCTSP in the literature, we choose to solve *ε-BTSP* with profit [(1)] in this study.

In *ε-BTSP* with profit [(1)], θ_1 has to guarantee that the optimal solution to the *ε-BTSP* with profit[(1)] is the solution with the highest profit and with the least route cost if there exist any other solutions with the highest profit. Let $C_{max} = \max\{c_{ij}\}$, $C_{min} = \min\{c_{ij}\}$ and $TMAX_0 = n \times C_{max}$. It is natural that C_{min} is the lower bound and $TMAX_0$ is the upper bound for the route cost. Let P_1 be the maximum profit gained for the ε-$BTSP$ with an upper bound $TMAX_0$. Let $TMAX_0 \geq TMAX_1 \geq TMAX_2$.

$$P_1 - \theta_1 \times TMAX_2 \geq P_1 - \theta_1 \times TMAX_1 \qquad (4.16)$$

$$\theta_1 \leq TMAX_1/TMAX_2$$

Let $f_2(P_1, TMAX_2)$ be the solution with profit P_1 and cost $TMAX_2$ and $f_1(P_1, TMAX_1)$ be the solution with profit P_1 and cost $TMAX_1$. Expression (4.16) implies that $f_2(P_1, TMAX_2)$ dominates $f_1(P_1, TMAX_1)$. Then the solution of the ε-$BTSP$ has to be $f_2(P_1, TMAX_2)$ which means that the objective function value of $f_2(P_1, TMAX_2)$ would be higher than the objective function value of $f_1(P_1, TMAX_1)$ as written in equation (4.16). If $\theta_1 \leq TMAX_1/TMAX_2$, it is guaranteed that among the solutions with the same profit, the solution with the least route cost is chosen.

On the other hand, θ_1 also has to satisfy that the solution with the highest profit is choosen instead of the solution with the lower profit but also the lower route cost. Let P_2 be the profit gained for Pareto optimal solution where the route cost is $TMAX_0$ and $P_2 - h$, where h is a integer small number (Let $h = 1$), be the profit for the solution with the lower route cost, C_{min}. θ_1 also satisfies equation (4.17)

$$P_2 - \theta_1 \times TMAX_0 \geq P_2 - h - \theta_1 \times C_{min} \qquad (4.17)$$

$$\theta_1 \leq h/(TMAX_0 - C_{min})$$

Lower and upper bounds used in equation (4.17) to obtain θ_1 that guarantee for any route cost, the solution with the highest profit is chosen. Since $1/TMAX_0 \leq h/(TMAX_0 - C_{min})$, $\theta_1 = 1/TMAX_0$ could be used.

In ε-$BTSP$ with profit [(1)], there is only one constrained objective, upper bounded by $\varepsilon_1^k = \varepsilon_1^{k-1} - \varepsilon$. For $k = 0$, $\varepsilon_1^0 = TMAX_0$ since $TMAX_0$ is the upper bound for route cost, there is no route whose cost higher than $TMAX_0$. Since ε_1^0 is obtained, one can calculate other ε_1^k where $k = 1, \ldots, K$ depending on ε. There is no way to obtain a value of ε such that every Pareto optimal solutions are found. In this study $\varepsilon = 0.0001$ is choosen.

The ε-constraint method algorithm for the biobjective TSP with Profit is given in Figure 10.

The performance of the ε-constraint method depends on the method used to solve the ε-MOP (ε-BTSP in this study). The ε-constraint method only guarantees to obtain exact Pareto optimal solutions if the solution methodology of ε-$BTSP$ ($SM\varepsilon$-$BTSP$) can find the global optimum of the ε-constraint problem. Otherwise, Pareto optimal solutions found by ε-constraint method are near Pareto optimal solutions. Also, the number of Pareto optimal solutions depends on not only ε, but on SMε-BTSP. In literature, there are good SMε-BTSPs for ε-BTSP. The best SMε-BTSPs are of Ramesh and Brown (1991), Chao et al. (1996a), Golden et al. (1988) and Wang et al. (1995) as discussed in Chapter 3.

CGW heuristic is developed by Chao et al. (1996a) and it is simple, fast, and effective heuristic. The results of CGW heuristic are the best results obtained so far in the literature (Tasgetiren and Smith, 2000). Therefore, CGW heuristic is used as SMε-BTSP for ε-BTSP in this study.

Step 1. Set Pareto Set $= \emptyset$, $l = 0$

Step 2. Define C_{max} and set $TMAX_0 = n \times C_{max}$

Step 3. Set $\theta_1 = 1/TMAX_0$, $\varepsilon_1^0 = TMAX_0$, $\varepsilon = 0.0001$

Step 4. Set objective function as $\sum_i p_i y_i - \theta_1 \sum_i \sum_j c_{ij} x_{ij}$

Step 5. Add $\sum_i \sum_j c_{ij} x_{ij} \leq \varepsilon_1^l$ as a constraint to form ε-$BTSP$ with profit [(1)]

Step 6. Solve $\varepsilon BTSP$ with profit [(1)]

Step 7. Set $S_l =$ solution of $\varepsilon BTSP$ with profit [(1)]. If there is no feasible solution, then stop

Step 8. Set $S_l \in$ Pareto Set

Step 9. Set $l = l + 1$

Step 10.Calculate $\varepsilon_1^l = \varepsilon_1^{l-1} - \varepsilon$, return Step 5

Figure 10. ε-constraint method algorithm for $\boldsymbol{\varepsilon}$-$\boldsymbol{BTSP}$ with profit [(1)]

4.4 CGW Heuristic Method

CGW heuristic basically consists of initialization and improvement steps. In the initilization step, L solutions are generated by a greedy method. In the improvement step, first, two-point exchange is applied to the initial solution on a record-to-record improvement basis. Then one point movement is applied to the current solution generated by two-point exchange procedure. Finally, 2-opt procedure is applied to the current solution to decrease the length of the current solution (Taşgetiren et al., 2002). This procedure is repeated until M loops. At the end of M loops reinitialization step is applied. The loop that contains M loops and reinitialization step is repeated until K loops. The best tour found so far is the result of the heuristic.

In sections 4.4.1 and 4.4.2, set-up process and initilization of the heuristic in which paths constructions are done in a greedy way are discussed. In sections, 4.4.3, 4.4.4 and 4.4.5, improvement steps, two-point exchange, one point movement, and 2-opt are described, respectively. Finally, re-initialization step is discussed in section 4.4.6.

4.4.1 Set – Up Process of CGW

Let n be the number of cities for a given problem instance and c_0 be starting city and c_{n-1} be ending city. Let T_{max} be the upper bound for the constrained objective function, $\sum_i \sum_j c_{ij} x_{ij}$ and $d(i,j)$ be the distance between cities i and j, c_i and c_j. The procedure is initialized by calculating the sum of distances of the city i to c_0 and c_{n-1}, d_i, for all $i \neq 0, n-1$, where

$$d_i = d(i,0) + d(i,n-1)$$

Cities where $d_i \leq T_{max}$ for all $i \neq 0, n-1$ are used for the next steps of heuristic and cities where $d_i > T_{max}$ for all $i \neq 0, n-1$ are eliminated.

In other words, if $c_0 \neq c_{n-1}$, an ellipse is constructed over the entire set of cities by using starting and ending cities as the foci of the ellipse and the upper bound T_{max} as the length of the major axis, as seen in Figure 11. If $c_0 = c_{n-1}$, a circle is constructed over the entire set of cities by using starting (ending) city as the center of the circle and the upper bound T_{max} as the diameter of the circle, as seen in Figure 12. Only the cities that are within the ellipse (or circle) are considered for generating the routes.

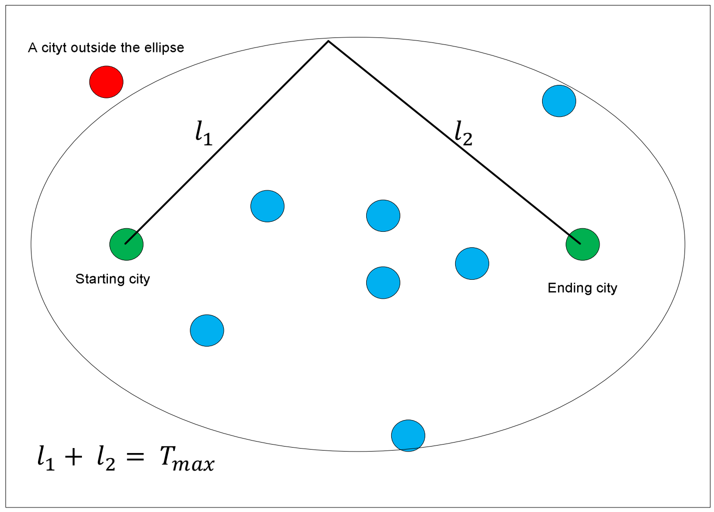

Figure 11. Illustration of set-up process of CGW heuristic by ellipse

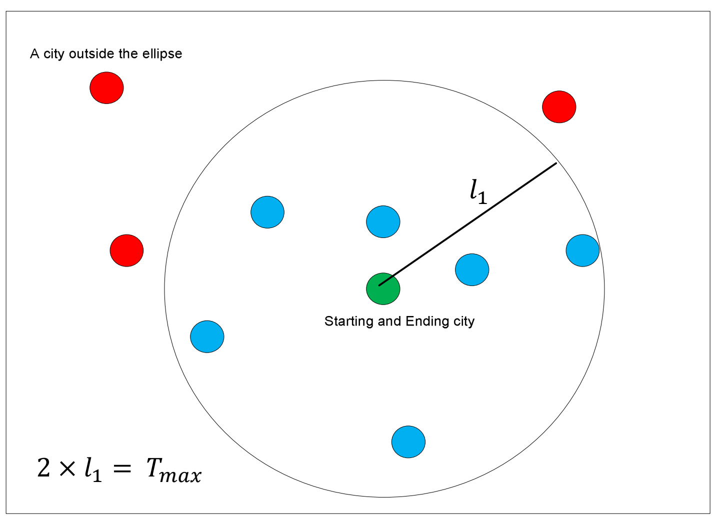

Figure 12. Illustration of set-up process of CGW heuristic by circle

4.4.2 Initialization

Let F_c be the set of cities where $d_i \leq T_{max}$ for all $i \neq 0, n-1$ and Seq^c be the set of cities $\in F^c$ where $Seq^c(k) = c_i$ where $d_i = max\{d_j\}$ for $c_j \in F_c \setminus \{Seq^c(m)\}$ for $m = 0, \dots, k-1$ and $Seq^c(0) = \emptyset$. Let $U_c = \emptyset$ be the set of cities used to construct routes in solution sets.

In initialization step, L solution sets, where L is $\min(10, s(F^c))$ where $s(F^c)$ is the number of cities in F^c, are constructed. To construct l^{th} solution, city marked as $Seq^c(l)$ is added between c_0 and c_{n-1} to generate the route $c_0 - Seq^c(l) - c_{n-1}$. Since $Seq^c(l)$ is used it is added to U_c, $U_c = \{Seq^c(l)\}$. Afterwards city $j \in F_c \setminus U_c$, which minimizes the increase in the route cost $\sum_i \sum_j c_{ij} x_{ij}$, is inserted in the route. The city insertion continues until $F_c \setminus U_c = \emptyset$ or inserting a city violates the route cost constraint. If inserting a city violates the route cost constraint when $F_c \setminus U_c \neq \emptyset$, the remaining cities are inserted by minimum increase in the route cost rule to generate new route. This process continues until all the cities $\in F_c$ are on a route and the $(l+1)^{th}$ solution is generated in the same way by setting $U_c = \emptyset$.

Let $n = 9$ for a given problem instance and c_0 and c_8 be the starting and ending cities. Let $Seq^c(3) = c_4$ and generate the 3^{rd} solution. c_4 is inserted between c_0 and c_8 and the remaining cities are added by minimum increase in the route cost rule. The generated route and set U_c is given in Table 4.

Table 4. First route generation by initialization

generated route	$c_0 - c_3 - c_4 - c_5 - c_8$
U_c	c_3, c_4, c_5
$F_c \setminus U_c$	c_1, c_2, c_6, c_7

As $F_c \setminus U_c \neq \emptyset$ and route cost constraint is violated, remaining cities are inserted between c_0 and c_8 to generate new routes. 3 solutions (solution set 3) are given in Table 5.

Table 5. Solution set 3

generated route 1	$c_0 - c_3 - c_4 - c_5 - c_8$
generated route 2	$c_0 - c_1 - c_2 - c_8$
generated route 3	$c_0 - c_6 - c_7 - c_8$

Among L solution sets generated by initialization process, the solution set which has the highest objective value, $\sum_i p_i y_i - \theta_1 \sum_i \sum_j c_{ij} x_{ij}$, is choosen as the initial set of routes and the objective value is set as *Record*. Within the initial solutions, the route with the highest objective value is denoted as $route_{op}$ and the other routes are denoted as $route_{nop}$.

4.4.3 Two-point Exchange

Chao et al. (1996) apply two-city exchange procedure to improve $route_{op}$. A city *i* is selected from $route_{op}$ and inserted into one of the routes in $route_{nop}$ and a city *j* is selected from one of the routes in $route_{nop}$ and inserted into $route_{op}$. The selection of cities is done arbitrary. The insertions are performed by considering the minimum increase in route cost rule, and the feasibility of routes is maintained. If any city insertion is not possible in $route_{nop}$ then a new route that includes city *i,* has to be generated and added to $route_{nop}$. If the objective function value associated with a route in $route_{nop}$ has a higher value than the objective function value of $route_{op}$, $route_{op}$ is updated and the previous $route_{op}$ is placed into $route_{nop}$.

Let r^1 be the initial route and r^2 be the updated route obtained by removing city m and inserting city n. Let $\sum_i \sum_j c_{ij}x_{ij}{}^1$ be the route cost associated with r^1. To check the route feasibility of r^2 the following expression is used.

$$\sum_i \sum_j c_{ij}x_{ij}{}^1 - \left(d\left(c_m, c_{f_m}\right) + d\left(c_m, c_{p_m}\right) - d\left(c_{p_m}, c_{f_m}\right)\right) + min_{k=1,n}\left\{d\left(c_n, c_{p_k}\right) + d(c_n, c_k) - d\left(c_k, c_{p_k}\right)\right\} \quad (4.18)$$

where c_{p_m} is the city precedes city m, c_{f_m} is the city follows city m and c_{p_k} is the city precedes city k. If the distance calculated by Expression (12) is less than T_{max}, then the generated route is feasible; otherwise, it is infeasible. In expression (4.18), $\left(d\left(c_m, c_{f_m}\right) + d\left(c_m, c_{p_m}\right) - d\left(c_{p_m}, c_{f_m}\right)\right)$ is the savings by removing city m and $min_{k=1,n}\left\{d\left(c_n, c_{p_k}\right) + d(c_n, c_k) - d\left(c_k, c_{p_k}\right)\right\}$ is the cost incurred by inserting city n onto path r^1.

If the city exchange increases the objective function value, the exchange is performed immediately. On the other hand, if there is no city exchange that increases the objective function value, then the exchanges that decrease the objective function value by acceptable amounts are considered and the city exchange that results the minimum decrease in the objective function value is performed. This approach is based on *record-to-record improvement* (Dueck, 1990). In Figure 13, Two- point exchange algorithm is given.

Step 1. Set the route with the highest objective funtion value = $route_{op}$

Step 2. Set other routes = $route_{nop}$

Step 3. Set the $city_{best\_exchange} = 0$ and $record_{best\_exchange} = 0$

Step 4. For m = the first to the last city in $route_{op}$

Step 5. For n = the first to the last city in the first to the last route in $route_{nop}$

Step 6. If exchanging city m and city n is feasible and the objective function value increases, then do the exchange and go step 6.1, else go step 7

Step 6.1 If the objective function value associated with a route in $route_{nop}$ has a higher value than the objective function value of $route_{op}$, then update $route_{op}$, $route_{nop}$ and $record$ and go step 4, else go step 7

Step 7. If the objective function value $\geq record_{best\_exchange}$

Step 7.1 Set $city_{best\_exchange} = n$ and $record_{best\_exchange}$ = the objective function value

Step 8. If n = number of cities in $route_{nop}$, then go step 9, else go step 5

Step 9. If $record_{best\_exchange} \geq 10\% \times record$, then exchange city m with $city_{best\_exchange}$ and update $route_{op}$ and $route_{nop}$ and set $city_{best\_exchange} = 0$ and $record_{best\_exchange} = 0$

Step 10. If m = number of cities in $route_{op}$, then exit, else go step 4

Figure 13. Two-point exchange algorithm

4.4.4 One Point Movement

In one point movement, one city is moved from one route to other route at a time and movement is performed by the first feasible insertion rule. City i within the ellipse or circle is inserted between cities in the first edge of route r, then the second edge of path p, and so on, where route r is a route that does not contain city i. The movement is performed whenever it is feasible, it is referred as *first feasible insertion rule*, and the objection function value increases. If there is no movement that increases the objection function value, then the city movements that decrease the route profit by acceptable amounts are considered and the city movement that has the minimum decrease in the objection function value is performed. The feasibility of insertion is checked by Equation (4.19).

$$\sum_i \sum_j c_{ij} x_{ij} - \left(d\left(c_m, c_{f_m}\right) + d\left(c_m, c_{p_m}\right) - d\left(c_{p_m}, c_{f_m}\right)\right) + \left(d\left(c_n, c_{p_k}\right) + d(c_n, c_k) - d\left(c_k, c_{p_k}\right)\right) \tag{4.19}$$

where c_{p_m} is the city preceding city m, c_{f_m} is the city following city m and c_{p_k} is the city preceding city k. If the distance calculated by expression (4.19) is less than T_{max}, then the generated route is feasible; otherwise, it is infeasible. In Expression (4.19), $\left(d\left(c_m, c_{f_m}\right) + d\left(c_m, c_{p_m}\right) - d\left(c_{p_m}, c_{f_m}\right)\right)$ is the savings by removing city m and $\left(d\left(c_n, c_{p_k}\right) + d(c_n, c_k) - d\left(c_k, c_{p_k}\right)\right)$ is the cost incurred by inserting city n onto path r^1.

One point movement algorithm is given in Figure 14.

Step 1. Set the $city_{best\_movement} = 0$ and $record_{best\_movement} = 0$

Step 2. For m = the first to the last city in ellipse or circle (say city m is in route q)

Step 3. For n = the first to the last city in the first to the last route (route p) in both $route_{op}$ and $route_{nop}$ $(q \neq p)$

Step 4. If inserting city m in front of city n is feasible and the objective function value increases, then make the movement and go step 4.1, else go step 5

> Step 4.1 If the objective function value associated with a route in $route_{nop}$ has a higher value than the objective function value of $route_{op}$, then update $route_{op}$, $route_{nop}$ and $record$ and go step 2, else go step 5

Step 5. If the objective function value $\geq record_{best\_movement}$

> Step 5.1 Set $city_{best\_movement} = n$ and
> $record_{best\_movement}$ = the objective function value

Step 6. If n = number of cities in ellipse or circle - 1, then go step 7, else go step 3

Step 7. If $record_{best\_movement} \geq 10\% \times record$, then insert city m in front of $city_{best\_movement}$ and update $route_{op}$ and $route_{nop}$ and set $city_{best\_movement} = 0$ and $record_{best\_movement} = 0$

Step 8. If m = number of cities in ellipse or circle, then exit, else go step 2

Figure 14. One point movement algorithm

4.4.5 2 - Opt

For a given route with n cities, if $d(c_i,\ c_{i+1}) + d(c_j,\ c_{j+1}) \geq d(c_i,\ c_j) + d(c_{i+1},\ c_{j+1})$, then the sequence of cities are changed to improve the route cost as in Figure 16. In 2-opt algorithm, sequence of cities are changed based on $\max\{d(c_i,\ c_{i+1}) + d(c_j,\ c_{j+1}) - d(c_i,\ c_j) + d(c_{i+1},\ c_{j+1})\}$ for $i = 1, \ldots, n$. 2-opt algorithm is given in Figure 15.

Step 1. Set the $best_{2-opt} = 0$, $bestm = 0$, $bestn = 0$

Step 2. For *m* = the first to the last city in route *p*

Step 3. For *n* = the first to the last city in route *p*, $n \neq m$

Step 4. Calculate $d(c_m,\ c_{m+1}) + d(c_n,\ c_{n+1}) - d(c_m,\ c_n) + d(c_{m+1},\ c_{n+1})$

Step 5. If $d(c_m,\ c_{m+1}) + d(c_n,\ c_{n+1}) - d(c_m,\ c_n) + d(c_{m+1},\ c_{n+1}) \geq best_{2-opt}$, then set $best_{2-opt} = d(c_m,\ c_{m+1}) + d(c_n,\ c_{n+1}) - d(c_m,\ c_n) + d(c_{m+1},\ c_{n+1})$ and $bestm = m$, $bestn = n$

Step 5. If n = number of cities in *p*, then go step 6, else go step 3

Step 6. If m = number of cities in *p*, then go step 7, else go step 2

Step 7. If $best_{2-opt} > 0$, then change the sequence of cities in route by $bestm$ and $bestn$, and return step 1, else exit

Figure 15. 2-opt algorithm

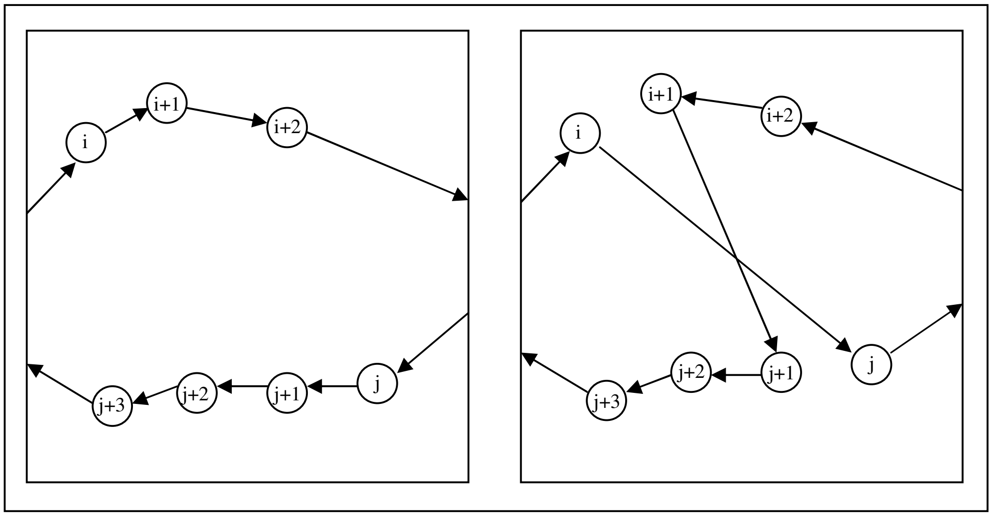

Figure 16. 2-opt illustration

4.4.6 Reinitialization

For finding a route that yields a larger objective function value, k cities are removed from $route_{op}$ and inserted into routes on $route_{nop}$ by the first feasible insertion rule. Cities are chosen based on the smallest ratio

$$p_i/(c_{i,i-1} + c_{i,i+1})$$

where p_i is the profit associated with city i and $c_{i,i-1}$ and $c_{i,i+1}$ are insertion costs of city i before city $i-1$ and after city $i+1$, respectively.

For better understanding of CGW heuristic, a problem instance with 32 cities is used to demonstrate how CGW heuristic works. The coordinates of cities are given in Table 6. Let city 0 be the starting point and city 31 be the ending city.

Table 6. Co-ordinates of cities

i	$x(i)$	$y(i)$	$Profit(i)$
0	10.5	14.4	0
1	18	15.9	10
2	18.3	13.3	10
3	16.5	9.3	10
4	15.4	11	10
5	14.9	13.2	5
6	16.3	13.3	5
7	16.4	17.8	5
8	15	17.9	5
9	16.1	19.6	10
10	15.7	20.6	10
11	13.2	20.1	10
12	14.3	15.3	5
13	14	5.1	10
14	11.4	6.7	15
15	8.3	5	15
16	7.9	9.8	10
17	11.4	12	5
18	11.2	17.6	5
19	10.1	18.7	5
20	11.7	20.3	10
21	10.2	22.1	10
22	9.7	23.8	10
23	10.1	26.4	15
24	7.4	24	15
25	8.2	19.9	15
26	8.7	17.7	10
27	8.9	13.6	10
28	5.6	11.1	10
29	4.9	18.9	10
30	7.3	18.8	10
31	11.2	14.1	0

The illustration of cities is given in Figure 17.

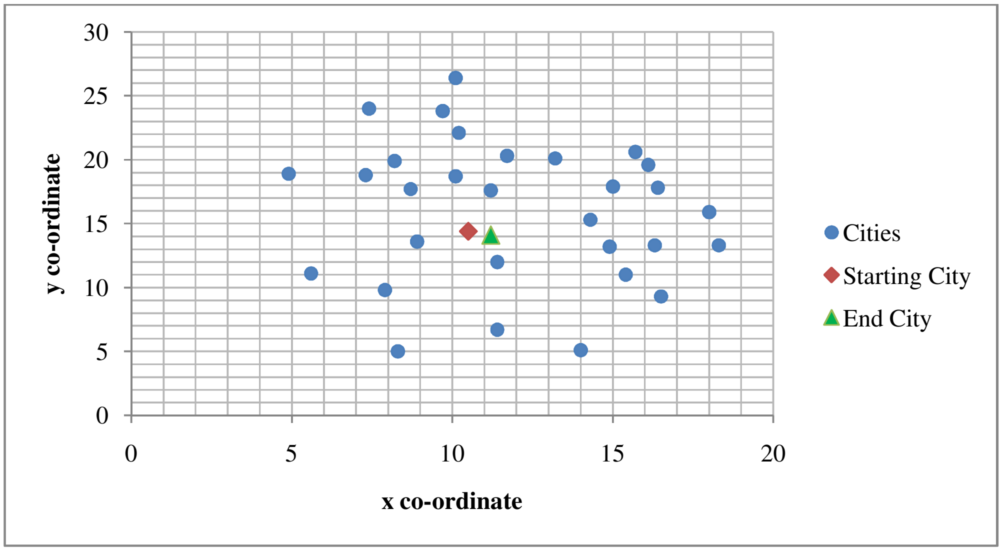

Figure 17. Illustration of cities for the sample problem

TMAX is set as 50 and all the cities are within the ellipse. The next step is calculating the sum of the distance between each city and starting city and the distance between each city and ending city and ordering the cities based on their total distances. In Table 7, the sorted distances and the corresponding cities are given.

Table 7. Sorted cities from maximum distance to minimum distance

$City\ i$	d_i	$City\ i$	d_i	$City\ i$	d_i
23	24.36	1	14.68	8	11.07
24	20.69	9	15.01	30	11.55
13	19.36	29	15.10	16	10.70
15	19.20	7	13.19	5	8.37
22	19.25	11	12.63	19	9.05
10	16.00	20	12.24	12	7.23
2	15.02	4	11.18	26	8.14
3	15.03	25	12.49	18	6.78
14	15.16	28	12.26	17	4.67
21	15.77	6	11.07	27	4.14

As discussed in the initialization step, the solution sets are constructed and the solution set that includes the solution with the highest profit is choosen as the initial solution set. In Table 8, the solution sets are given and the initial solution set is highlighted. In Figure 18, initial solution set is illustrated.

Table 8. Solution sets

	Profit	Cost	Path
Solution Set 1	160	47	0 27 26 19 30 25 21 22 23 24 20 11 10 9 7 8 12 18 31
	115	49	0 28 16 15 14 13 17 4 3 2 1 6 5 31
	10	15	0 29 31
Solution Set 2	160	49	0 27 26 30 25 24 22 21 20 11 19 18 8 10 9 7 12 6 5 17 31
	110	49	0 29 28 16 15 14 13 3 4 2 1 31
	15	24	0 23 31
Solution Set 3	130	49	0 27 17 16 15 14 13 3 4 2 1 7 8 6 5 12 31
	145	45	0 26 30 29 25 19 21 24 23 22 20 11 10 9 18 31
	10	12	0 28 31
Solution Set 4	130	46	0 27 28 16 15 14 13 3 4 2 1 6 5 12 17 31
	155	46	0 26 30 29 25 19 21 24 23 22 20 11 10 9 7 8 18 31
Solution Set 5	165	49	0 17 27 26 19 30 25 24 23 22 21 20 11 10 9 7 8 12 18 31
	110	43	0 28 16 15 14 13 4 3 2 1 6 5 31
	10	15	0 29 31
Solution Set 6	155	47	0 27 18 26 30 25 19 21 22 20 11 10 9 7 8 12 6 5 4 17 31
	100	48	0 29 28 16 15 14 13 3 2 1 31
	30	26	0 24 23 31
Solution Set 7	155	48	0 12 8 18 26 30 25 19 20 11 10 9 7 1 2 6 5 4 17 27 31
	90	48	0 21 29 28 16 15 14 13 3 31
	40	26	0 24 23 22 31

Table 8. Solution sets (Continued)

Solution Set 8	130	48	0 27 17 3 4 2 1 7 9 10 11 20 19 18 8 6 5 12 31
	130	48	0 14 16 28 29 30 25 24 23 22 21 26 31
	25	25	0 15 13 31

Solution Set 9	130	46	0 27 28 16 15 14 13 3 4 2 1 6 5 12 17 31
	155	46	0 26 30 29 25 19 21 24 23 22 20 11 10 9 7 8 18 31

Solution Set 10	165	49	0 17 27 26 19 30 25 24 23 22 21 20 11 10 9 7 8 12 18 31
	110	43	0 28 16 15 14 13 4 3 2 1 6 5 31
	10	15	0 29 31

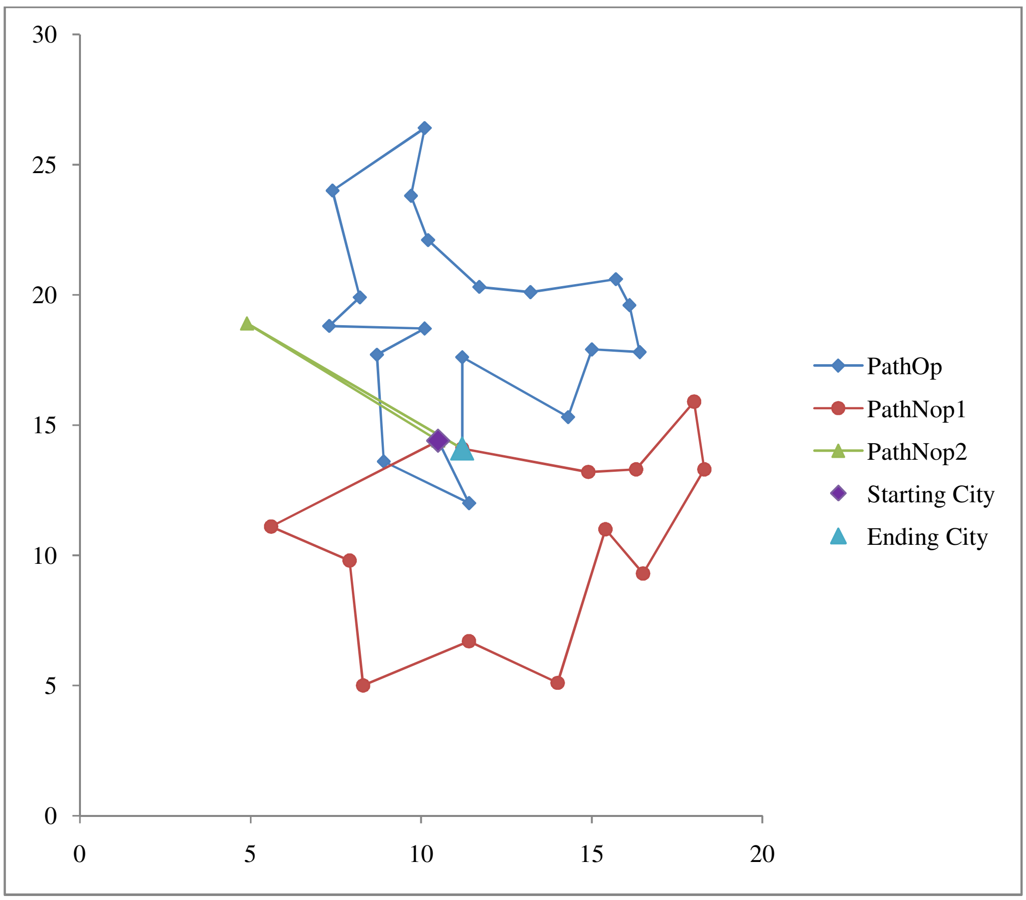

Figure 18. Illustration of initial solution set

Two-point exchange is implemented to the initial solution set. It is implemented so that some cities are moved from existing paths and inserted into other paths. For instance, cities 18, 28, 27 and 9 are moved from $path_{op}$ and inserted onto paths in $paths_{nop}$ and cities 3, 7, 5 and 6 are moved from paths in $paths_{nop}$ and inserted onto $path_{op}$. Resulted routes are illustrated in Figure 19 and are shown in Table 9.

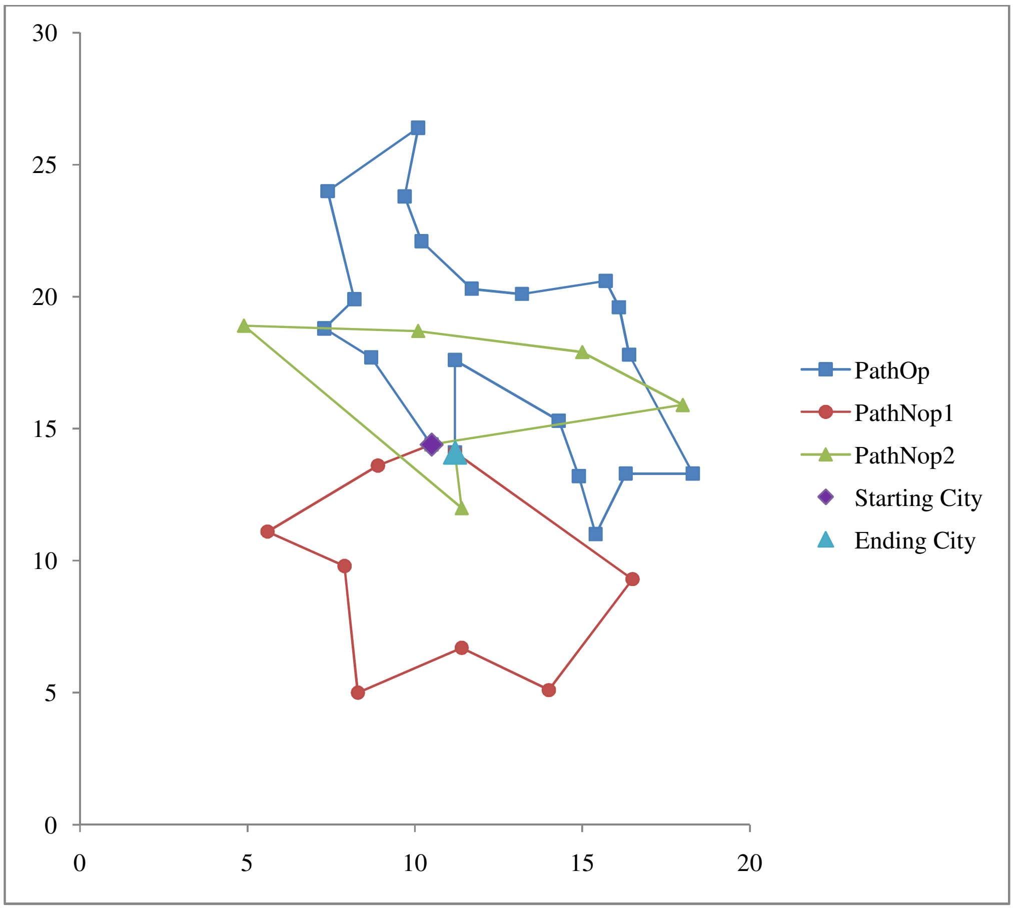

Figure 19. Two-point exchange implementation

Table 9. Generated routes by two-point exchange

Profit	Cost	Path
170.0	49,62	0 26 30 25 24 23 22 21 20 11 10 9 7 2 6 4 5 12 18 31
80.0	32,014	0 27 28 16 15 14 13 3
35.0	33,011	0 1 8 19 29 17 31

Resulted routes of One Point Movement are illustrated in Figure 20 and are shown in Table 10.

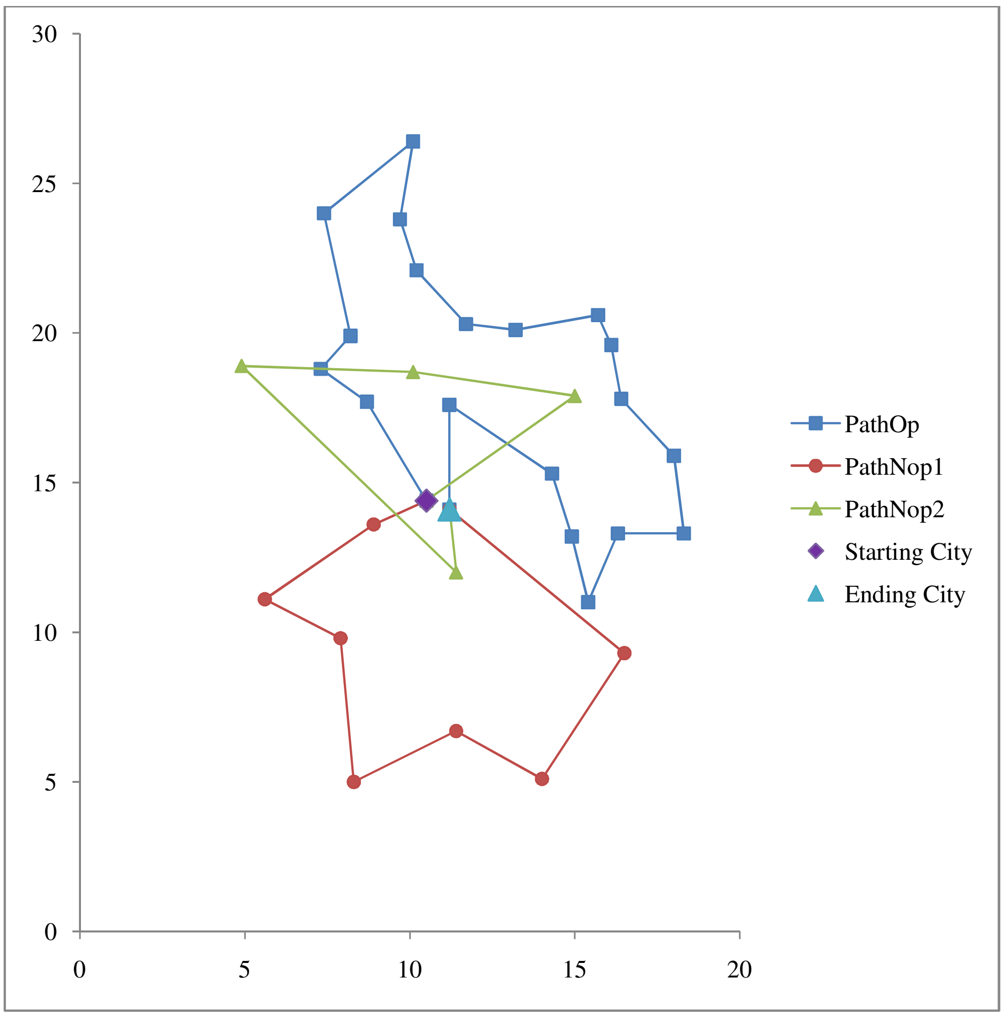

Figure 20. One point movement implementation

Table 10. Generated routes by one point movement

Profit	Cost	Path
180.00	49.83	0 26 30 25 24 23 22 21 20 11 10 9 7 1 2 6 4 5 12 18 31
80.00	32.01	0 27 28 16 15 14 13 3 31
25.00	27.46	0 8 19 29 17 31

2-opt implementation is improved the route cost but not the profit. The cost of best route decreases from 49.83 to 47.81. The illustration of 2-opt is given in Figure 21 and the generated routes are shown in Table 11.

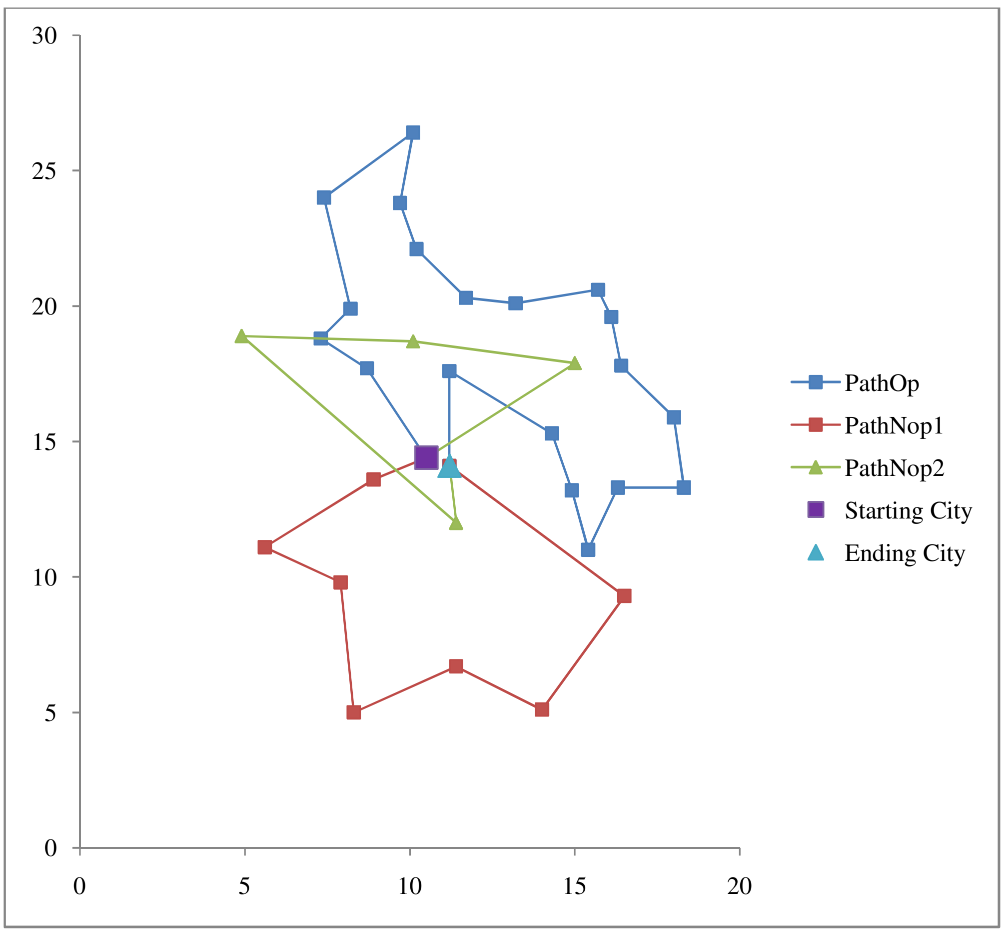

Figure 21. 2-opt implementation

Table 11. 2-opt implementation

Profit	Cost	Path
180.00	47.81	0 18 26 30 25 24 23 22 21 20 11 20 9 7 1 2 6 4 5 12 31
80.00	32.01	0 27 28 16 15 14 13 3 31
25.00	27.46	0 8 19 29 17 31

The overall proposed algorithm is given in Figure 22.

Step 1. Set $Pareto\_Set = \emptyset$, $l = 0$, $Feasible\_Cities = \emptyset$, $Feasible\_Cities^{(*)} = \emptyset$, $Solution\_Set^{(d)} = \emptyset$

Step 2. Define C_{max} and set $TMAX_0 = n \times C_{max}$

Step 3. Set $\theta_1 = 1/TMAX_0$, $\varepsilon_1^0 = TMAX_0$, $\varepsilon = 0.0001$

Step 4. Set objective function as $\sum_i p_i y_i - \theta_1 \sum_i \sum_j c_{ij} x_{ij}$

Step 5. Add $\sum_i \sum_j c_{ij} x_{ij} \leq \varepsilon_1^l$ as a constraint

Step 6. Define the starting and the ending cities

Step 7. If the starting and the ending cities are the same city, then go step 7.1, else go step 8

> Step 7.1. Calculate $d_i = 2 \times d_{(i,starting\ city)}$ for $i = 1, \dots, n$ and $i \neq$ starting city, if $d_i \leq \varepsilon_1^l$ add city i to $Feasible\_Cities$. Go step 9.

Step 8. Calculate $d_i = d_{(i,starting\ city)} + d_{(i,starting\ city)}$ for $i = 1, \dots, n$ and $i \neq$ the starting city and the ending city, if $d_i \leq \varepsilon_1^l$ add city i to $Feasible\_Cities$.

Step 9. Set n_l = number of cities in the $Feasible\_Cities$ and $Feasible\_Cities^{(*)} = Feasible\_Cities$, if $n_l = 0$ stop, else continue

Step 10. Find $min(10,\ n_l)$, set $a_{max} = min\ (10,\ n_l)$, Set $a = 1$

Step 11. Find the city i with a^{th} largest d_i in $Feasible\_Cities$, set $b = 1$

Step 12. Insert city i between the starting and the ending cities, p_a^b, update $Feasible\_Cities = Feasible\_Cities \setminus$ city i

Step 13. If $Feasible\_Cities = \emptyset$, then go step 17, else continue

Step 14. Insert the city i, in $Feasible\_Cities$, that increases the route cost minimum, to p_a^b

Step 15. If route cost of $p_a^b \leq \varepsilon_1^l$, then update $Feasible\_Cities = Feasible\_Cities \setminus$ city i and return step 13, else continue

Step 16. Move city i from p_a^b, set $b = b + 1$, return step 14.

Figure 22. The proposed algorithm

Step 17. Add p_a^c where $c = 1, \dots, b$ to Solution Set$^{(a)}$, set $a = a + 1$, if $a > a_{max}$, then go step 18, else set $Feasible\_Cities = Feasible\_Cities^{(*)}$ and return step 11,

Step 18. Calculates $\sum_i p_i y_i - \theta_1 \sum_i \sum_j c_{ij} x_{ij}$ for all routes in $Solution\_Set^{(a)}$ $a = 1, \dots, a_{max}$, take $Solution\_Set^{(j)}$ that contains the solution with highest objective function value, as initial solution

Step 19. Set $record =$ highest objective function value

Step 20. Set $deviation = 10\% \times record$

Step 21. For g = 1,…,10

Step 22. For h = 1,…, 10

Step 23. Perform Two-Point Exchange

Step 24. Perform One Point Movement

Step 25. Perform 2-Opt

Step 26. If a new better solution has been obtained, then go step 26.1 else set $h = 1$

Step 26.1 Update $record$ and $deviation$, return step 23

Step 27. Perform Reinitialization, set $g = g + 1$

Step 28. If $g \leq 10$, then return step 22, else continue

Step 29. Set S_l = solution of $\varepsilon BTSP$ with profit $^{(1)}$

Step 30. Set $S_l \in$ Pareto Set

Figure 22. The proposed algorithm (Continued)

CHAPTER 5

SOLUTION SET ANALYSIS

Multiobjective optimization can be regarded as a systematic sensitivity analysis of the most important value judgments. An essential feature in the multiobjective approach is the generation of several good alternatives (i.e. Pareto optimal solutions) and the comparison of them with each other. If there is only one alternative, like an optimal solution of a scalar problem, then the only decision is if that solution is acceptable or not. A real decision making becomes possible only if there are several alternatives which should be judged in order to pick up the best one. One approach to find the best solution, is finding the set of good solutions by calculating distances of Pareto optimal solutions to a given infeasible alternative. The distance formulation includes the parameter $\delta = 1, \dots, \infty$. Base on the chosen value of parameter δ, the distance value changes. The solutions which have the minimum distance for at least one of δ value for $\delta = 1, \dots, \infty$ constructs the set of effective solutions. Then trade-off concept can be applied in choosing the best solution among good solutions. In section 5.1, some definitions used in this chapter are described, and then experimental computations and the procedure to find the good solutions are explained and some illustrations are shown in section 5.2. Finally, trade-off concept is described in section 5.3.

5.1 Some Definitions

An ideal point (ideal solution) is generally an infeasible alternative consisting of the best value for each objective function. Each objective function is optimized subject to the given constraints, separately in order to obtain ideal point.

On the other hand, *goal point* is also an infeasible alternative, but not the best solution for all the objective functions. The basic characteristic of the goal point is that it is specified by decision maker (DM). For the above example, $z = \{8, 12,7\}$ could be a goal point, if $z \notin Y$. The ideal point and goal points of a biobjective problem are illustrated in Figure 23.

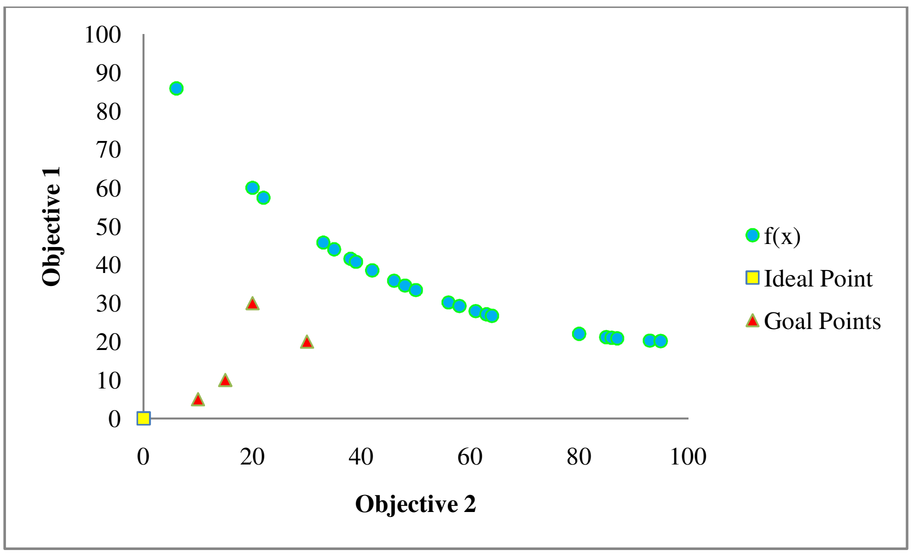

Figure 23. Illustration of ideal point and goal points

The distance of the solutions on the efficient frontier has to be measured in some way. While measuring the distance, *weights* may be used. Like the goal point, the basic characteristic of the weights is that it is specified by DM. Let d_i be the distance of $f_i(x)$ from the ideal point (or goal point) and w_i stands for the weight associated with d_i. Then the total distance of $f(x)$ could be measured as

$$\sum_i w_i d_i \tag{5.1}$$

where $\sum_i w_i = 1$.

5.2 Distance Formulation

Let x and $y \in R^m$, $x : x_i \in R$ and $y : y_i \in R$ for $i = 1, \ldots, m$. The distance between x and y could be described as

$$d_\delta = \left\{ \sum_{k=1}^{m} |x_k - y_k|^\delta \right\}^{1/\delta} \tag{5.2}$$

Where $\delta \geq 1$ which means that δ may take any value from 1 to $+\infty$.

For $\delta = 2$, Equation (5.2) calculates *Euclidean* distance between two points. For a given points x and y $\in R^2$, Equation (5.2) becomes

$$d_2 = \sqrt[2]{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

For $\delta = 1$ and $+\infty$, Equation (5.2) calculates *Manhattan* and *Tchebycheff* distances between two points, respectively. The distances d_1 and d_∞ represent bounds on the distance between any two points

$$d_1 \geq d_\delta \geq d_\infty \tag{5.3}$$

For the weighted case, Equation (5.2) is modified as

$$d_\delta = \left\{ \sum_{k=1}^{m} w_k |x_k - y_k|^\delta \right\}^{1/\delta} \tag{5.4}$$

where $\sum_i w_i = 1$.

For a given Pareto optimal set, an ideal point (or a goal point) and weights, one could find a subset of efficient solutions, effective solutions, solutions that has the minimum distance for one of δ value for $\delta = 1, \ldots, \infty$, by the below procedure.

For each solution in the efficient frontier, calculate $d_1\big(f(x)\big)$ and determine $f_k(x)$ where

$$d_1\big(f_k(x)\big) = \min\Big(d_1\big(f(x)\big)\Big)$$

For each solution in the efficient frontier, calculate $d_\infty\big(f(x)\big)$ and determine $f_l(x)$ where

$$d_\infty\big(f_l(x)\big) = \min\Big(d_\infty\big(f(x)\big)\Big).$$

The solutions between and including $f_k(x)$ and $f_l(x)$ are subset of efficient solutions. In Table 12, a sample solution set and an ideal point is given. In Table 13 the calculated unweighted distances for $\delta = (1, \infty)$ is given. The graphical illustration of effective solutions is illustrated in Figure 24.

Table 12. A sample solution set

No	$f_1(x)$	$f_2(x)$	No	$f_1(x)$	$f_2(x)$
1	60	20	12	29	58
2	57	22	13	28	61
3	46	30	14	27	63
4	44	34	15	27	64
5	43	37	16	22	80
6	42	39	17	21	85
7	39	41	18	21	86
8	36	46	19	21	87
9	35	48	20	20	93
10	33	50	21	20	95
11	30	56	Ideal Point	0	0

Table 13. Distance table

1	80	60	12	87	58
2	79	57	13	89	61
3	76	46	14	90	63
4	78	44	15	91	64
5	80	43	16	102	80
6	81	42	17	106	85
7	80	41	18	107	86
8	82	46	19	108	87
9	83	48	20	113	93
10	83	50	21	115	95
11	86	56			

is equal to where and is equal to where . The solutions between and including and solutions are the effective solutions.

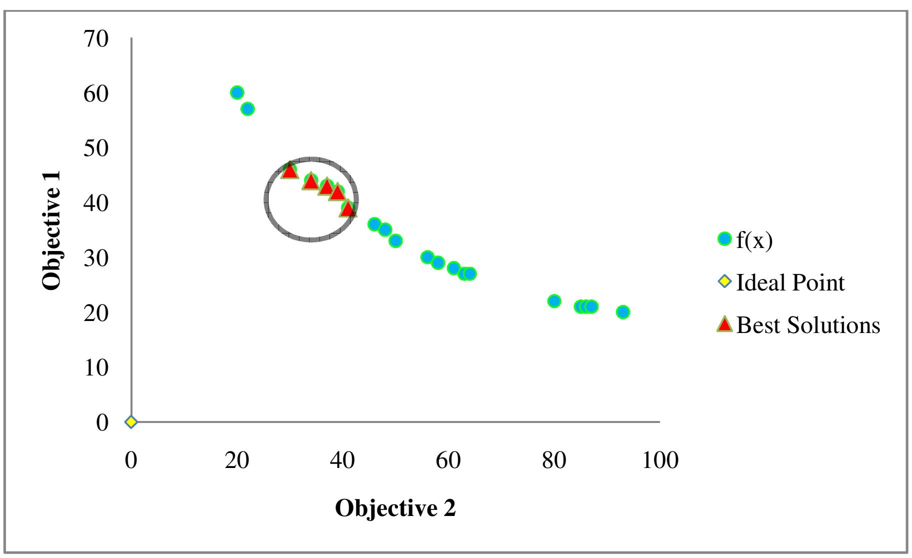

Figure 24. Illustration of effective solutions of the efficient frontier

More detailed numerical calculations and graphical illustrations are performed in the next section.

For better understanding about distance calculations and effective solutions, an example is illustrated for an ideal point and given goal points and weights. In Table 14 sample solution space is given.

Table 14. Solution space

No	$f_1(x)$	$f_2(x)$	No	$f_1(x)$	$f_2(x)$
1	86	6	7	35	45
2	73	12	8	30	56
3	60	20	9	27	63
4	55	25	10	22	80
5	46	33	11	20	95
6	41	39			

In the first part, distances are calculated based on the ideal point (0, 0). In Table 15 the distances are given and minimum values are colored and in Figure 25 the effective solutions are illustrated.

Table 15. Distance based on ideal point

	$d_1\big(f_k(x)\big)$	$d_\infty\big(f_k(x)\big)$
1	92	86
2	85	73
3	80	60
4	80	55
5	79	46
6	80	41
7	80	45
8	86	56
9	90	63
10	102	80
11	115	95

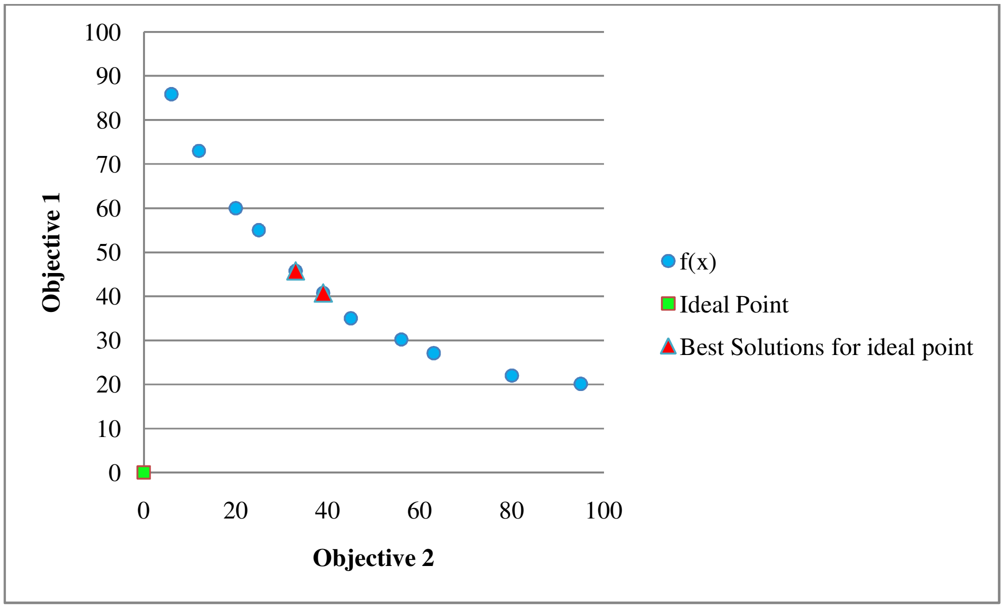

Figure 25. Illustration of effective solutions based on ideal point

In the second part, distances are calculated based on the goal points (60, 0) and (0, 60). In tables 4.7 and 4.8 the distances are given and minimum values are colored and in Figure 26 and Figure 27 the effective solutions are illustrated for the goal points (60, 0) and (0, 60), respectively.

Table 16. Distance based on goal point (60, 0)

	$d_1(f_k(x))$	$d_\infty(f_k(x))$
1	32	26
2	25	13
3	20	20
4	30	25
5	47	33
6	58	39
7	70	45
8	86	56
9	96	63
10	118	80
11	135	95

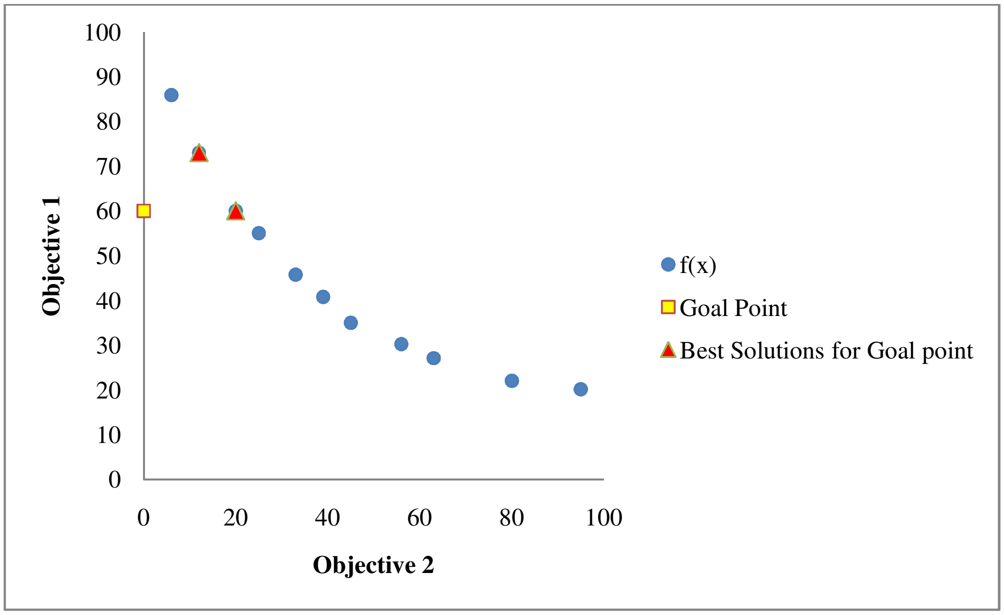

Figure 26. Illustration of effective solutions based on goal point (60, 0)

Table 17. Distance based on goal point (0, 60)

	$d_1(f_k(x))$	$d_\infty(f_k(x))$
1	140	86
2	121	73
3	100	60
4	90	55
5	73	46
6	62	41
7	50	35
8	34	30
9	30	27
10	42	22
11	55	35

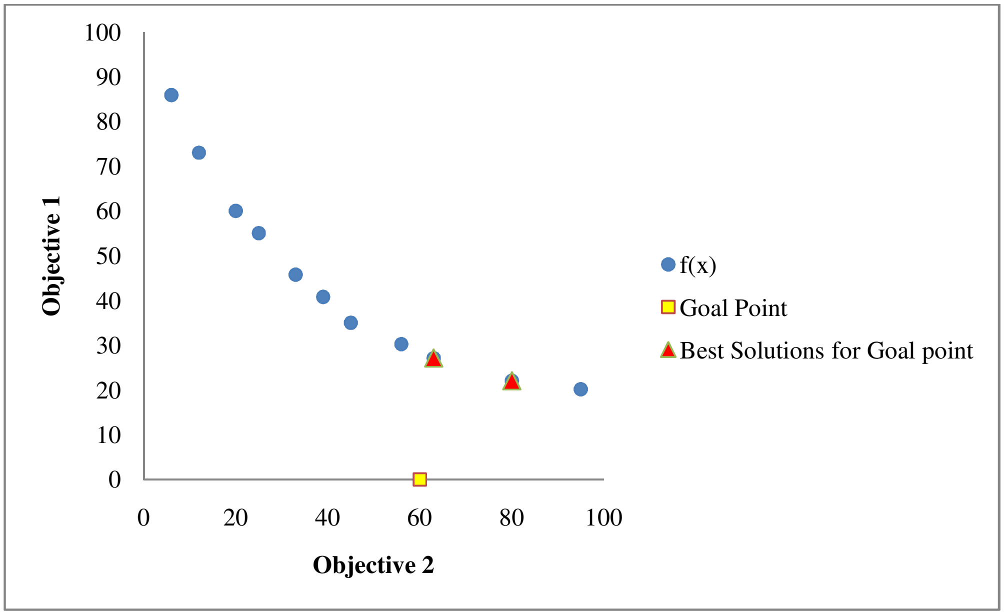

Figure 27. Illustration of effective solutions based on goal point (0, 60)

In the third part distances are calculated based on the ideal point (0, 0) and weights. Let w_i stands for the weight associated with $f_i(x)$. $w_1 = (0.25\ ,\ 0.75, 0.40)$ and $w_2 = (0.75\ ,\ 0.25, 0.60)$ are used to calculate distances for the tables 4.9, 4.10 and 4.11 and effective solutions for the Figure 28, Figure 29 and Figure 30, respectively.

Table 18. Weighted Distance based on ideal point (0, 0) with weights (0.25, 0.75)

	$d_1(f_k(x))$	$d_\infty(f_k(x))$
1	26	21
2	27	18
3	30	15
4	33	19
5	36	25
6	39	29
7	43	34
8	50	42
9	54	47
10	66	60
11	76	71

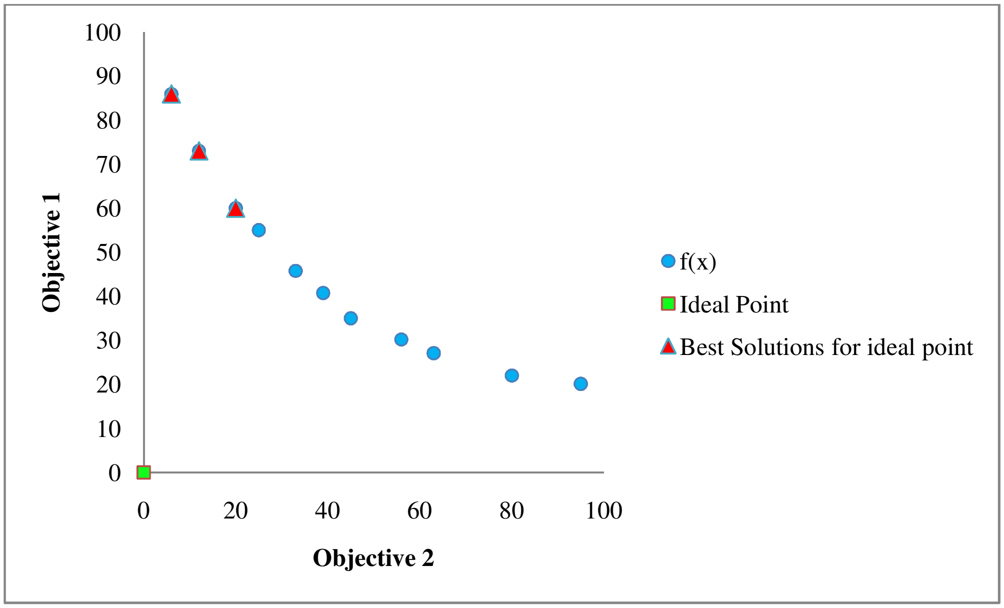

Figure 28. Illustration of effective solutions based on ideal point (0, 60) with weights (0.25, 0.75)

Table 19. Weighted Distance based on ideal point (0, 0) with weights (0.75, 0.25)

	$d_1(f_k(x))$	$d_\infty(f_k(x))$
1	66	64
2	58	55
3	50	45
4	48	41
5	43	34
6	40	31
7	38	26
8	37	23
9	36	20
10	37	20
11	39	24

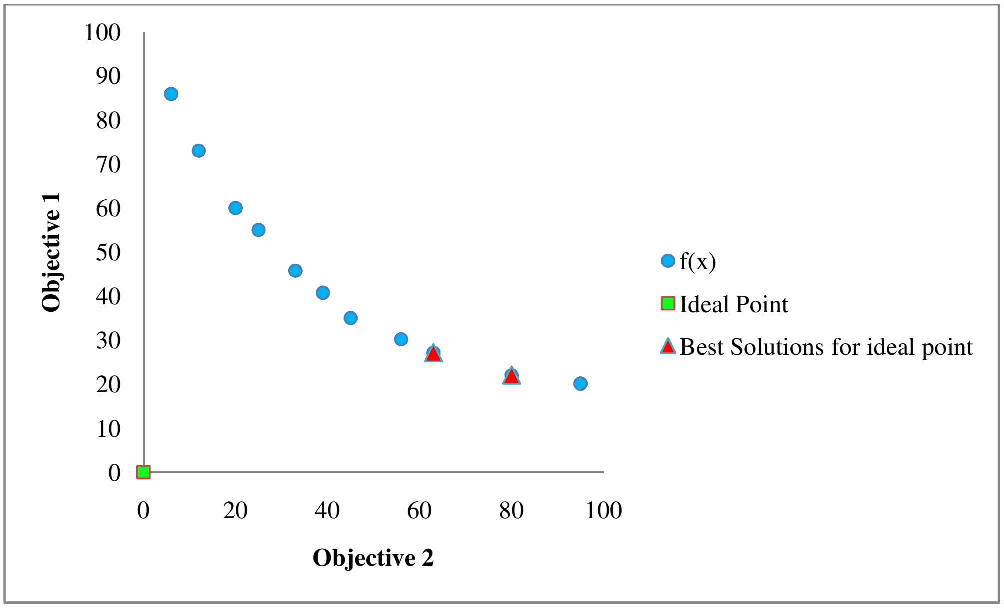

Figure 29. Illustration of effective solutions based on ideal point (0, 60) with weights (0.25, 0.75)

Table 20. Weighted Distance based on ideal point (0, 0) with weights (0.40, 0.60)

	$d_1(f_k(x))$	$d_\infty(f_k(x))$
1	38	34
2	36	29
3	36	24
4	37	22
5	38	20
6	40	23
7	41	27
8	46	34
9	49	38
10	57	48
11	65	57

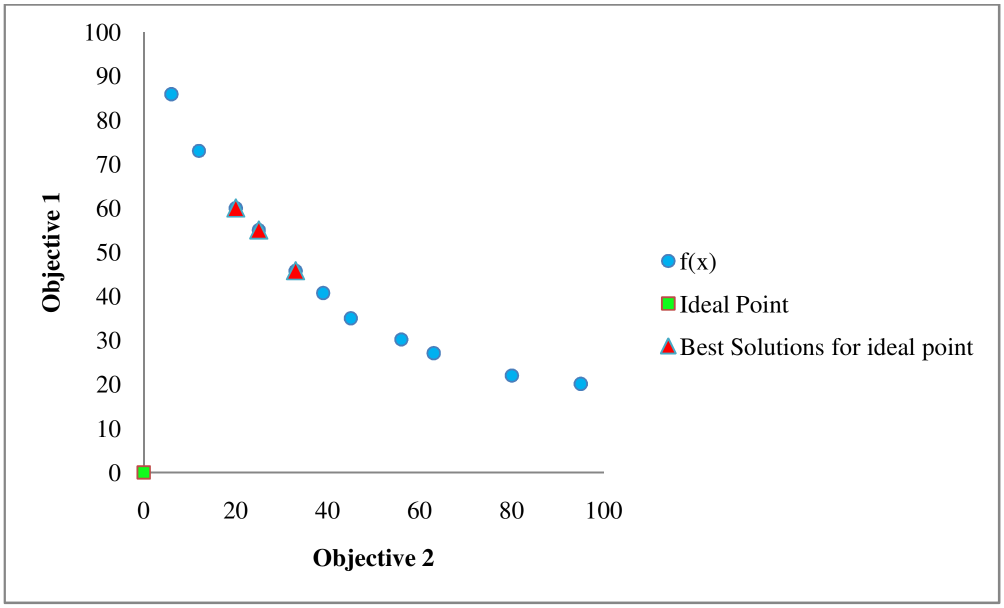

Figure 30. Illustration of effective solutions based on ideal point (0, 60) with weights (0.40, 0.60)

5.3 Trade-Off Concept

Trade-off is a frequently used concept in multiobjective optimization. It is defined as the amount of one objective that must be sacrificed to gain an unit improvement in another criterion. DM imposes its own trade-offs by stating the deterioration of one criterion which it accepts in order to improve the other criterion by one unit. So there are two trade-offs, the first associated with the properties of the minimal surface at a Pareto optimum under consideration, and the second associated with the preferences of the decision maker.

For Biobjective TSP with profit the trade-off formulation for solution routes r_i and r_j, can be given as

$$t_{off_i} = \left(\sum_{k \in r_i} p_k y_k - \sum_{k \in r_j} p_k y_k\right) \Bigg/ \left(\sum_{k \in r_i} \sum_{l \in r_i} c_{kl} x_{kl} - \sum_{k \in r_j} \sum_{l \in r_j} c_{kl} x_{kl}\right) \quad (5.5)$$

where is the profit associated with city k and is the cost associated with the route between city and city . Equation (1) means that one unit of cost objective can be sacrificed to gain unit improvement in profit objective by choosing instead of .

Let , and , for a given solution routes and . Then

One unit of cost objective can be sacrificed to gain unit improvement in profit objective by choosing instead of . If the preference of DM is choosing instead of which means the trade off is acceptable for DM, is set as candidate best solution. This process continues until there is no trade-off, which DM accepted, for the candidate best solution or another termination condition is guaranteed. Then candidate best solution is set as best solution.

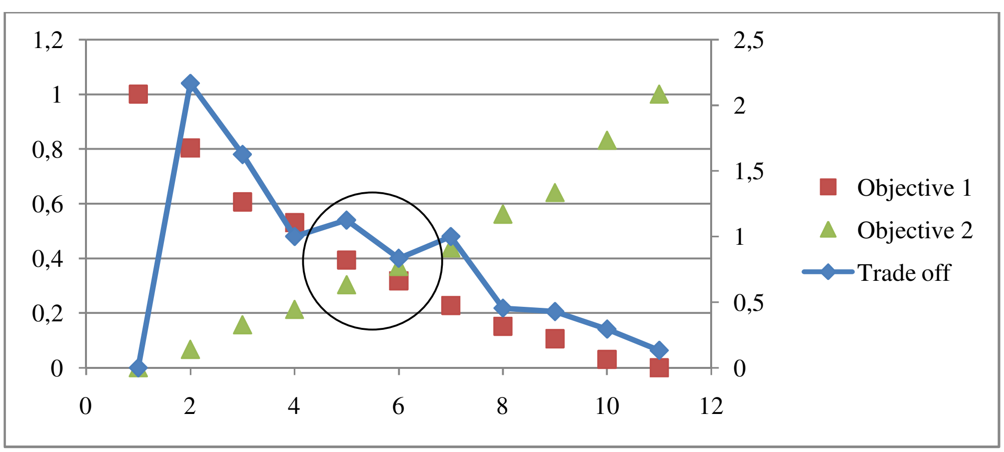

Figure 31. Trade off diagram for the sample solution set in Table 14

In Figure 31, the trade off diagram for the sample solution set in Table 14 is shown. The trade off values for solution x is the trade of value for solution $x - 1$ and solution x. Because of that the trade off value for solution 1 is represented as 0. Base on the information given by trade off diagram DM can easily see that the trade off between solution 1 and solution 2 is high, one unit increase in objective 2 causes two unit decrease in objective 1. For the high level picture DM can prefer solution 2 or he can check other solutions depending on the trade off level he can accept.

The marked area is the effective solutions set for the ideal point (0,0). Moving from solution 5 to solution 6 causes 0.83 unit decrease in objective 1 for 1 unit increase in objective 2. If DM can not accept the trade offs smaller than 1 unit decrease in objective 1 for 1 unit increase in objective 2 he would choose solution 5. If it is acceptable he will choose solution 6.

CHAPTER 6

COMPUTATIONAL RESULTS

The performance of the ε-constrained method is evaluated by applying the method to 5 problem sets taken from the literature and comparing the solutions with the results in the literature. Also, the method is applied to 9 problem sets taken from the literature and the solutions are compared not fully by partially with the published results, since these problem sets are solved partially.

In section 6.1, the problem sets are given. In section 6.2, the performance of the ε-constrained method is discussed. Finally, the solutions of the problem sets out of problem sets used for performance evaluation are given in section 6.3.

6.1 Problem Sets

We considered 2 different classes of test problems. The first problem class includes 5 instances from the OP literature and Biobjective TSP with Profit literature. Problems OP21, OP32, and OP33 are OP instances introduced by Tsiligirides (1984). Problem OP32-1- instance is introduced by Chao et al. (1998). In the literature OP problem sets are solved with fixed parameters for the right hand side of route cost constraint. Problem K25 instance is introduced by Keller and Goodchild (1986). Keller and Goodchild (1986) use only K25 problem set for their algorithm.

The second problem class includes 9 instances from Vehicle Routing Problem (VRP) literature. Problems ATT48, EIL30, EIL33, EIL51, EIL76, and EIL101 are VRP instances taken from the Traveling Salesman Problem Library TSPLIB of

Reinelt (2007). Problems CMT101 and CMT121 are VRP instances from Christofides, Mingozzi and Toth (1979). The problem sets and related papers are given in Table21. Fischetti et al. (1998) solve these problem sets for three values of right hand side of route cost constraint. In Table 21, problem sets and related papers are given.

Table 21. Problem sets and related papers

Paper Name	Authors	Problem Sets	Definition
The multiobjective vending problem: A generalization of the traveling salesman problem	Keller, C. P., M. Goodchild. 1988	K25	Keller and Goodchild test the performance of their proposed method by just one problem set
An ant colony approach to the orienteering problem	Liang, Y.-C., A. E. Smith. 2001	OP21, OP31, OP32	Liang et al. test the performance of ant colony approach by OP21, OP31 and OP32 for specific TMAX values
A genetic algorithm for the orienteering problem	Tasgetiren, F. M., A. E. Smith. 2000	OP21, OP31, OP32	Tasgetiren et al. test the performance of ant colony approach by OP21, OP31 and OP32 for specific TMAX values
Solving the orienteering problem through branch-and-cut	Fischetti, M., J. J. Salazar González, P. Toth. 1998	OP21, OP31, OP32, EIL30, EIL33, EIL51, EIL76, EIL101, CMT101, CMT121	Fischetti et al. test the performance of branch and bound algorithm by problem sets in the left cell for 3 specific TMAX values

6.2 Computational Results

An interactive program that solves the biobjective TSP with profit was written in Java. As mentioned earlier only Keller and Goodchild (1988) study the biobjective TSP with profit and they use one problem set in their study. We evaluate our solutions with Keller and Goodchild (1988). We evaluate our Pareto optimal solutions with their Pareto optimal solutions and number of Pareto optimal solutions. For OP21, OP32, and OP33 we evaluate our Pareto optimal solutions for the fixed parameters used by Taşgetiren et al. (2000), Liang et al. (2001) and Fischetti et al. (1998) solutions. Also we look for the Pareto optimal solutions out of fixed values because they do not generate all the Pareto optimal solutions. We also look for the Pareto optimal solutions we found where they have same profits as the solutions Fischetti et al. found but the route costs are less than their solutions. By this we can show that we can found better solutions for the fixed values. The other computational analyze is trade off relations. By trade-off relations we mean showing that for a small increase in the route cost if we can generate solutions with higher profits then their solutions.

On the other hand, K25, OP21, OP32, OP33, and OP32-1- problem sets are also used to demonstrate and evaluate the performance of the ε-constraint method. The results show that ε-constraint method found all solutions for specific TMAX values published in the literature.

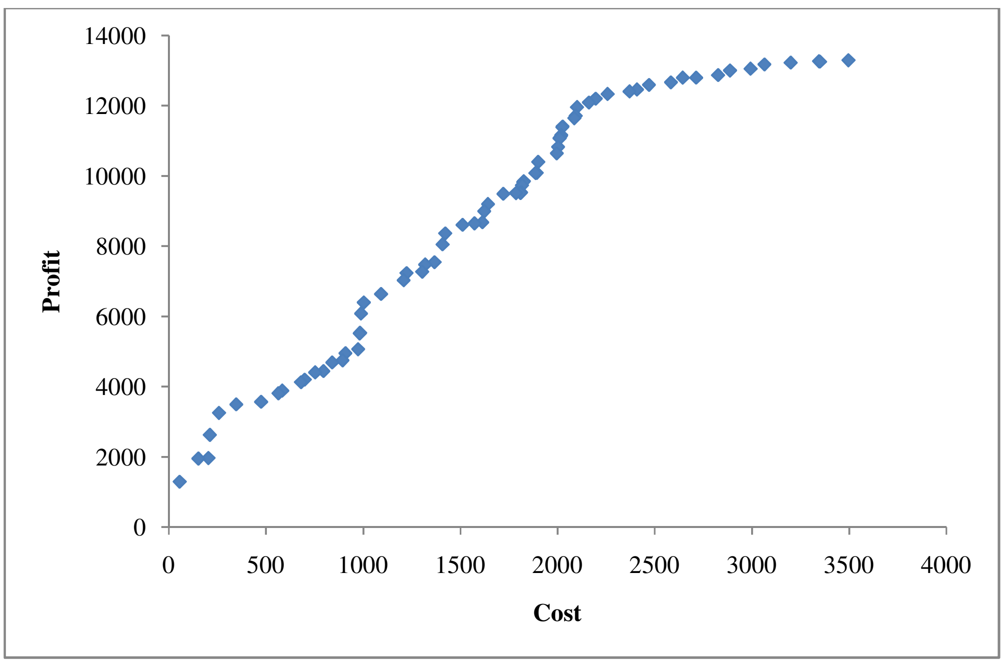

Figure 32. Pareto optimal solutions for K25 problem set

In literature, the only true attempt for solving the biobjective TSP with profit was Keller and Goodchild (1988). Keller and Goodchild (1988) test the performance of their proposed method by just one problem set that includes 25 cities located in West Germany. Bonn was used as the depot and terminal node. The populations of cities were treated as profit associated with each city.

The number of solutions obtained by Keller was 27 which is less than 71 solutions that we obtained. 18 solutions obtained by our method are also obtained by Keller. Keller could not obtain 44 solutions that we obtain. 9 solutions dominated 9 Keller solutions. In Figure 31, the Pareto optimal solutions are given for K25. In Figure 32, the trade-off is shown for K25. In this figure and in all trade-off figures, route profits and costs are scalarized between 0 – 1 and shown by left vertical axis. Also, trade-offs are calculated for each solution. From left to right, trade-off values means decrease in profit for one unit decrease in cost. It is reverse from right to left. The values for trade-offs are shown by right vertical axis.

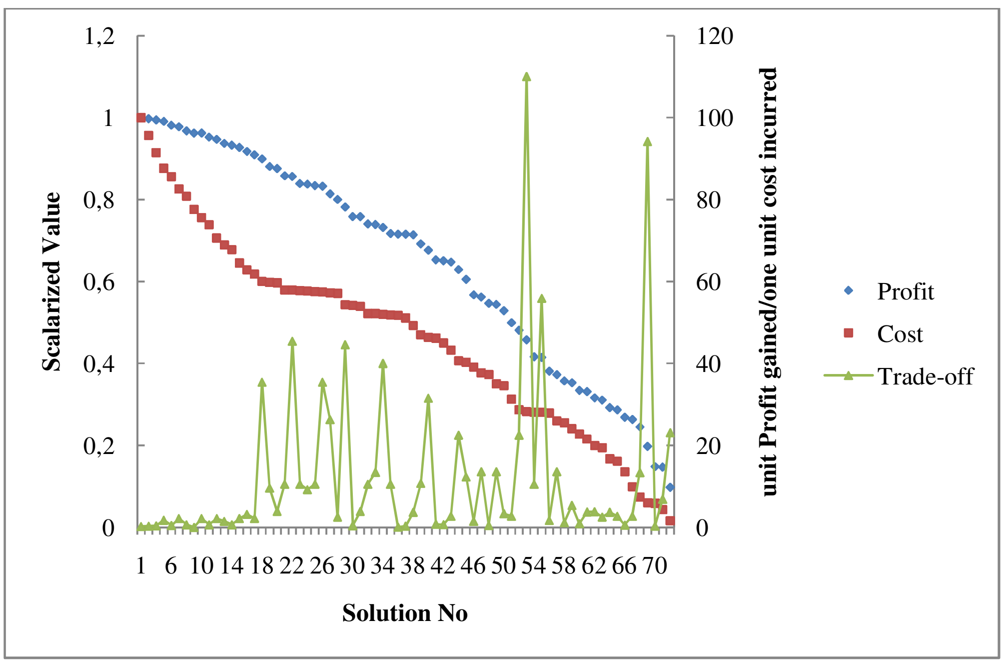

Figure 33. Scalarization of profit and cost, and trade-off for K25 problem set

For the Pareto optimal solutions, the robustness is analyzed by an arc matrix. In arc matrix the number of arcs between cities are shown. In Table 22, the arc matrix of the solutions of K25 is given. As an illustration, the arc between city 5 and city 18 (traveling from city 5 to city 18), arc 5 – 18, is included in 55 solutions and the arc 18 – 5 (traveling from city 18 to city 5) is included in 17 solutions. The matrix shows that the main structure of the solutions does not change so much.

Table 22. Number of arcs in the Pareto optimal solutions for K25

		Cities																								
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
Cities	**1**					2			21										2							
	2																				2					20
	3											1			6	5						31				
	4						12	10				1				16			1							
	5	2							5			9							55							
	6				5						2				36	3		1	2							
	7				40		3			16		1				7										
	8	4				5				51		1							7							
	9					2		50	16																	
	10														1			3								
	11				1	42										2				6		1				
	12																								4	
	13		1																							
	14			29			9	2			1					6		8								
	15			1	3	1	24	1				2			10				4							
	16											7				11										
	17			9							1				2											
	18	19			2	17	1	4	26	1						1										
	19					2						26													3	4
	20		19											1								2				4
	21			5								1									13			11		6
	22																				2					
	23																				9		2			
	24											3								5						
	25		2									1	4							24		2			1	

The overall results for OP21, OP32, and OP33 are compared with Tasgetiren et al. (2000) and Liang et al. (2001). In all problem sets, our method finds all the published results for the specific upper bounds. For OP21, we found 47 more solutions and for OP32, OP33, and OP33-1-, we found 37, 57 and 35 more solutions respectively. In Figures 33-40, the Pareto optimal solutions and scalarized profits and costs and trade off values are given for OP21, OP32, OP33, and OP33-1-, respectively. The results for all problem sets except Keller are compared with the results published by Fischetti et al. (1998). The branch and bound algorithm is used by Fischetti et al. (1998) and they found good solutions. Comparison with Fischetti et al. (1998) is done base on the $TMAX$ values. The solutions we found near to this $TMAX$ value is tabled and shown.

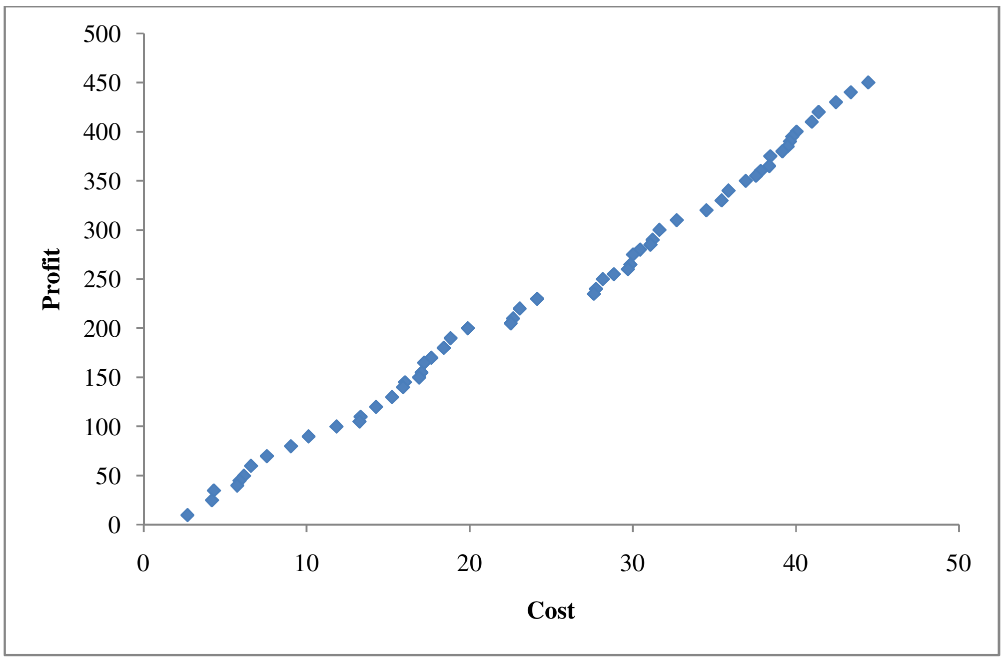

Figure 34. The Pareto optimal solutions for OP21

For OP21, the proposed method generates 58 Pareto optimal solutions. The number of solutions obtained by Taşgetiren and Liang was 11, because they use fix parameters. Liang and Taşgetiren dominated our 0 solutions. 8 solutions obtained by our method are also obtained by Liang and 10 solutions obtained by our method are also obtained by Taşgetiren. We dominated 3 Liang solutions and

1 Taşgetiren solutions. Taşgetiren and Liang could not obtain 47 solutions that we obtain.

Fischetti et al. (1998) uses 11.50, 22.99, and 34.49 for $TMAX$. Based on these TMAX values our solutions and neighborhood solutions are given in Table 23.

For route cost 34.49 the published result is better than the generated result but for route cost 34.51 the generated solution has a profit value of 320 which is higher than 315, which means for 0.51 unit increase in route cost we gain 5 unit of profit. For route cost 22.99 the generated solution is better than the published result. Also, for 23.06 the generated solution has a profit value of 220, which means for 0.06 unit increase in route cost we gain 15 unit of profit. For route cost 11.50 the generated solution is same with the published result but our solution has a lower route cost. Also for 11.81 the generated solution has a profit value of 100. In Table 24, the arc matrix of the solutions of OP21 is given. As it can be seen that some arcs are included in most of the solutions and most of the arcs are not included in the solutions. For instance, arcs 9 – 10, 10 – 11, and 11 – 13 are included in most of the solutions and arcs 1 – 4, 8 – 2, and 11 – 8 are not included in any solutions. The main structure of the solutions does not change so much.

Table 23. Published solutions and neighborhood solutions for OP21 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	315	34.49	205	22.99	90	11.50
Generated Solutions	340	35.86227	230	24.128345	105	13.231623
	330	35.446358	220	23.063688	100	11.818177
	320	34.514474	210	22.647777	90	10.103147
	310	32.690083	205	22.513478	80	9.0272882
	300	31.625426	200	19.879525	70	7.5491731

Table 24. Number of arcs in the Pareto optimal solutions for OP21

		Cities																				
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**
Cities	**1**						1	39				2	13	2	1							
	2			3					24				1									
	3		17		16																	
	4			4																	21	
	5		7	23	9								2									
	6		2			41							1									
	7						43						6									
	8									27								9				
	9										44											
	10											44			4							
	11										4			42	8							
	12		2					10	2			8		1								
	13														32							
	14													2								
	15									9								13				
	16															22						
	17								10	8												
	18																	5				
	19																		18			
	20			3																18		
	21																					

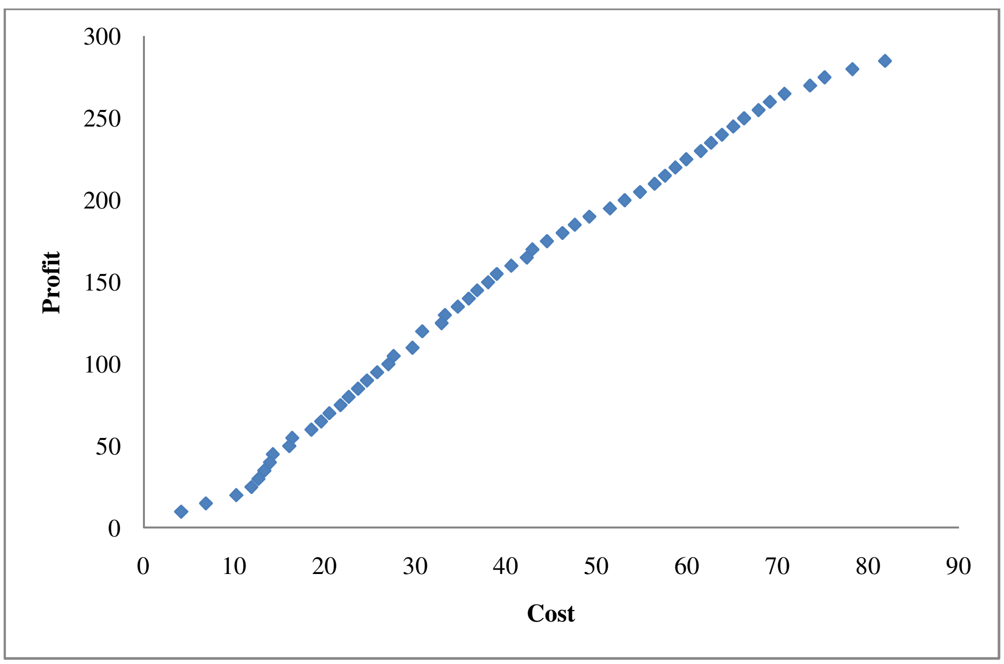

Figure 35. The Pareto optimal solutions for OP32

For OP32, the proposed method generates 55 Pareto optimal solutions. The number of solutions obtained by Taşgetiren and Liang was 18. Liang dominated our 3 solutions and Taşgetiren dominated our 2 solutions. 11 solutions obtained by our method are also obtained by Liang and 7 solutions obtained by our method are also obtained by Taşgetiren. We dominated 4 Liang solutions and 9 Taşgetiren solutions. Taşgetiren and Liang could not obtain 37 solutions that we obtain.

Fischetti et al. (1998) uses 20.64, 41.27, and 61.91 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 25.

For all $TMAX$ the published and generated results are same but the route costs of our solutions are lower than the published results. Also there are no good trade offs, which means for one unit increase in route cost there is no solution where profit increases 5 units. In Table 26, the arc matrix of the solutions of OP21 is given. As it can be seen that some arcs are included in most of the solutions and most of the arcs are not included in the solutions.

Table 25. Published solutions and neighborhood solutions for OP32 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	70	20.64	160	41.27	230	61.91
Generated Solutions	80	22.62657	170	42.90821	240	63.82241
	75	21.73011	165	42.29618	235	62.63256
	70	20.49183	160	40.57741	230	61.4916
	65	19.59537	155	38.97382	225	59.88802
	60	18.5229	150	38.01586	220	58.69817

Table 26. Number of arcs in the Pareto optimal solutions for OP32

Cities																
Cities	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1													2	1		
2			19					9								
3		9		1			18									
4					11									8		
5				8		12	1									
6					7		8						2			
7			9	1	5	11							1			
8		19							6	5			1			
9								8		4			8			
10								14	14		9					
11										28		9				
12											28					
13						1								1		
14				7											8	
15														7		
16															7	
17																
18				2												
19																
20																
21												32				
22																
23																
24																
25																
26																
27																
28																
29																
30																
31																
32																

Table 26. Number of arcs in the Pareto optimal solutions for OP32 (Continued)

Cities	Cities															
	17	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
1			2								18	35				
2																
3																
4																
5		2														
6																7
7																
8																
9																
10																
11																
12			3	1												
13																11
14																
15		1														
16	7															
17												5	2			
18																6
19				2												22
20			17								2					2
21			1	3		9										
22							9									
23						27		9								
24							16		9							
25							11	16		9						
26																
27																
28																
29																
30																
31																
32																

For OP33, the proposed method generates 77 Pareto optimal solutions. The number of solutions obtained by Taşgetiren and Liang was 20. Liang dominated our 1 solution and Taşgetiren dominated our 1 solution. 10 solutions obtained by our method are also obtained by Liang and 8 solutions obtained by our method are also obtained by Taşgetiren. We dominated 9 Liang solutions and 11 Taşgetiren solutions. Taşgetiren and Liang could not obtain 57 solutions that we obtain.

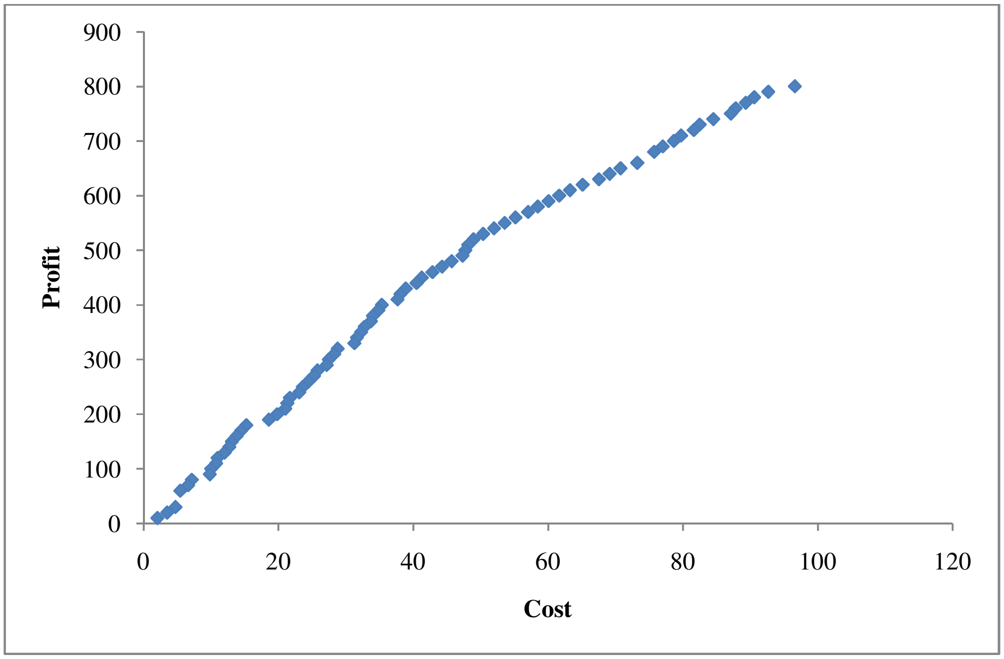

Figure 36. The Pareto optimal solutions for OP33

Fischetti et al. (1998) uses 24.39, 48.78, and 73.17 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 27.

For route cost 24.39 the published result and generated result are same but generated result has lower route cost. Also for route cost 24.45 the generated solution has a profit value of 260, which means for 0.06 increase in route cost there is 10 unit increase in profit. For route cost 48.78 the generated solution is better than the published result. Also, for route cost 48.93 the generated solution has a profit value of 520, which means for 0.15 increase in route cost there is 10 unit increase in profit. For route cost 73.17 the published result is better than the generated result but for route cost 73.21 the generated solution has a profit value of 660.

There is no published result for OP32-1-. The Pareto optimal solutions are shown in Figure 36. For the other problem sets tables are given for comparison. If the published result is better than the generated result, trade-offs can be checked.

Table 27. Published solutions and neighborhood solutions for OP33 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	250	24.39	500	48.78	660	73.17
Generated Solutions	280	25.775957	530	50.367091	680	75.717808
	270	25.249041	520	48.935549	660	73.212922
	260	24.458365	510	48.144873	650	70.738358
	250	23.606682	500	47.727167	640	69.125037
	240	23.079766	490	47.289638	630	67.525893

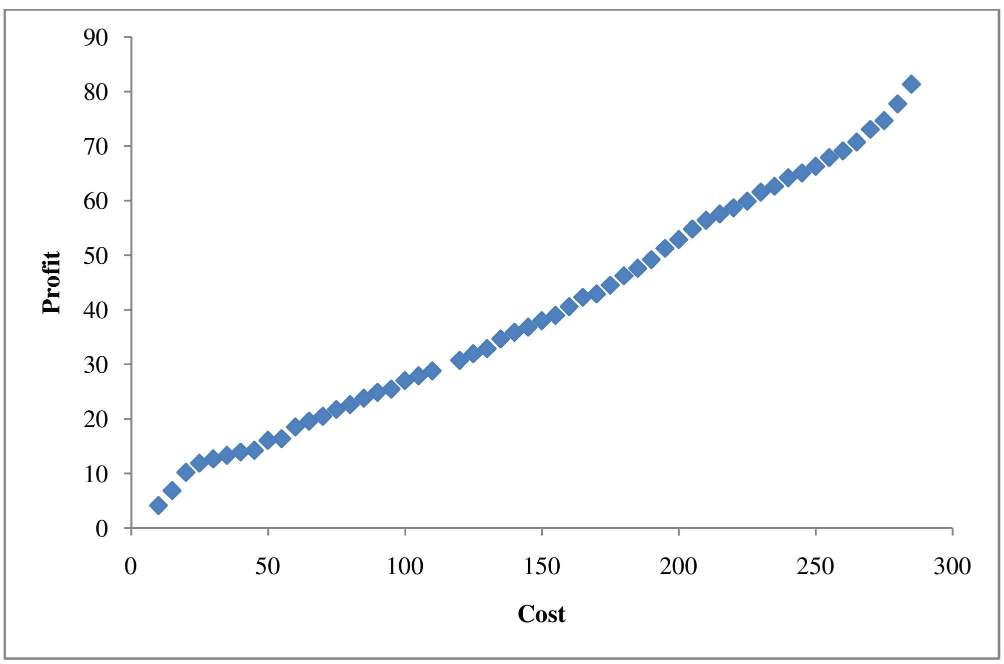

Figure 37. The Pareto optimal solutions for OP32-1-

On the other hand, the aim of this study is to show importance of the trade-offs between solutions. In the literature, the single objective forms of the biobjective TSP with profit is solved by bounds. To illustrate the trade off, the solutions for OP21 are given in Table 28. In OP21, the published results for upper bounds 40 and 30 of the route cost objective, are 395 and 265, respectively. Our method find solutions where profit is 400 and cost is 40.05 and profit is 275 and cost is 30.01.

Table 28. The solutions for OP21

Solution No	Route Profit	Route Cost	Route
1	450	44.4377	1 12 7 6 5 2 3 4 20 19 18 16 15 17 8 9 10 11 13 14 21
2	440	43.373	1 7 6 5 2 3 4 20 19 18 16 15 17 8 9 10 11 13 14 21
3	430	42.4576	1 12 7 6 5 3 4 20 19 18 16 15 17 8 9 10 11 13 14 21
4	420	41.393	1 7 6 5 3 4 20 19 18 16 15 17 8 9 10 11 13 14 21
5	410	40.977	1 7 6 5 3 4 20 19 18 16 15 17 8 9 10 11 14 21
6	400	40.0452	1 7 6 5 3 4 20 19 18 16 15 17 9 10 11 13 14 21
7	395	39.7781	1 7 6 5 3 4 20 19 18 16 15 17 8 9 10 11 13 21
8	390	39.6293	1 7 6 5 3 4 20 19 18 16 15 17 9 10 11 14 21
9	385	39.495	1 12 7 6 5 3 4 20 19 18 16 15 17 9 10 11 13 21
10	380	39.1715	1 7 6 5 4 20 19 18 16 15 17 9 10 11 13 14 21
11	375	38.4303	1 7 6 5 3 4 20 19 18 16 15 17 9 10 11 13 21
12	365	38.3661	1 6 5 3 4 20 19 18 16 15 17 9 10 11 13 21
13	360	37.8423	1 7 6 5 2 3 4 20 19 18 17 8 9 10 11 13 14 21
14	355	37.5566	1 7 6 5 4 20 19 18 16 15 17 9 10 11 13 21
15	350	36.9269	1 12 7 6 5 3 4 20 19 18 17 8 9 10 11 13 14 21
16	340	35.8623	1 7 6 5 3 4 20 19 18 17 8 9 10 11 13 14 21
17	330	35.4464	1 7 6 5 3 4 20 19 18 17 8 9 10 11 14 21
18	320	34.5145	1 7 6 5 3 4 20 19 18 17 9 10 11 13 14 21
19	310	32.6901	1 12 7 6 5 3 2 8 17 16 15 9 10 11 13 14 21
20	300	31.6254	1 7 6 5 3 2 8 17 16 15 9 10 11 13 14 21
21	290	31.2095	1 7 6 5 3 2 8 17 16 15 9 10 11 14 21
22	285	31.0752	1 12 7 6 5 3 2 8 17 16 15 9 10 11 13 21
23	280	30.4432	1 7 6 5 2 8 17 16 15 9 10 11 13 14 21
24	275	30.0106	1 7 6 5 3 2 8 17 16 15 9 10 11 13 21
25	265	29.8491	1 7 6 2 8 17 16 15 9 10 11 13 14 21
26	260	29.6983	1 7 12 2 8 17 16 15 9 10 11 13 14 21
27	255	28.8283	1 7 6 5 2 8 17 16 15 9 10 11 13 21
28	250	28.1525	1 7 6 5 4 20 3 2 8 9 10 11 13 14 21
29	240	27.7366	1 7 6 5 4 20 3 2 8 9 10 11 14 21
30	235	27.6023	1 12 7 6 5 4 20 3 2 8 9 10 11 13 21
31	230	24.1283	1 12 7 6 5 4 3 2 8 9 10 11 13 14 21
32	220	23.0637	1 7 6 5 4 3 2 8 9 10 11 13 14 21
33	210	22.6478	1 7 6 5 4 3 2 8 9 10 11 14 21
34	205	22.5135	1 12 7 6 5 4 3 2 8 9 10 11 13 21
35	200	19.8795	1 12 7 6 5 3 2 8 9 10 11 13 14 21
36	190	18.8149	1 7 6 5 3 2 8 9 10 11 13 14 21
37	180	18.399	1 7 6 5 3 2 8 9 10 11 14 21

Table 28. The solutions for OP21 (Continued)

38	170	17.6326	1 7 6 5 2 8 9 10 11 13 14 21
39	165	17.2	1 7 6 5 3 2 8 9 10 11 13 21
40	155	17.0385	1 7 6 2 8 9 10 11 13 14 21
41	150	16.8878	1 7 12 2 8 9 10 11 13 14 21
42	145	16.0178	1 7 6 5 2 8 9 10 11 13 21
43	140	15.9043	1 7 6 5 3 2 12 11 13 14 21
44	130	15.2238	1 7 12 8 9 10 11 13 14 21
45	120	14.2488	1 12 8 9 10 11 13 14 21
46	110	13.2963	1 7 6 5 12 11 10 14 21
47	105	13.2316	1 7 6 12 11 10 14 13 21
48	100	11.8182	1 7 6 5 12 11 13 14 21
49	90	10.1031	1 7 12 11 10 14 13 21
50	80	9.02729	1 7 12 11 10 14 21
51	70	7.54917	1 7 12 11 13 14 21
52	60	6.57422	1 12 11 13 14 21
53	50	6.14422	1 11 13 14 21
54	45	5.87326	1 12 13 14 21
55	40	5.72831	1 11 14 21
56	35	4.29402	1 13 14 21
57	25	4.18154	1 14 21
58	10	2.67915	1 13 21

For another illustration, solution 22 in Table 28 has a route cost 31.07 with a profit 285. If one set TMAX 31, solution 23 with profit 280 would be found. Solutions 6, 24, 32, 40, 42, and 50 are same as solution 22.

As mentioned earlier, Selective TSP and Prize Collecting TSP are scalarized versions of the biobjective TSP with profit. To solve these problems, they have to be bounded by some value. The solution would depend on these bounds and a solution with a good trade-off (means a small increase in cost but a high increase in profit) could not be generated. By trade-off figures, one can analyze the trade off relation between solutions. In Figure 37, the trade-offs between solutions of OP21 is given. Also the scalarized cost and profits are shown in Figure 37. The slope for scalarized profit is constant but the slope of scalarized cost is changeable. For the selected area in Figure 37, one can choose the solution with the highest profit since the slope of profit is more vertical than the slope of cost.

Trade-off curve for OP32, OP33 and OP32-1- are given in Figures 38 - 40, respectively. In Figure 38, it can be easily seen that for the solutions having higher profits than the marked solutions, the decrease for route cost is higher than the decrease for the route profit and for the solutions having lower profits than the marked solutions, the decrease for route cost is less than the decrease for the route profit. The DM has a high level view with the help of this information so that he can give more precise decisions.

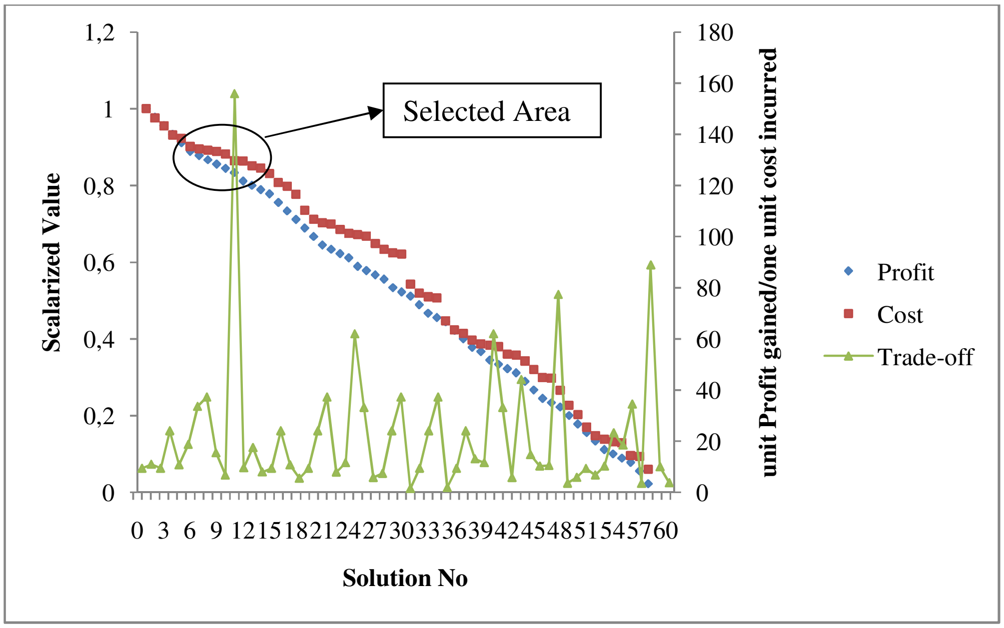

Figure 38. Scalarization of profit and cost, and trade-off OP21 problem set

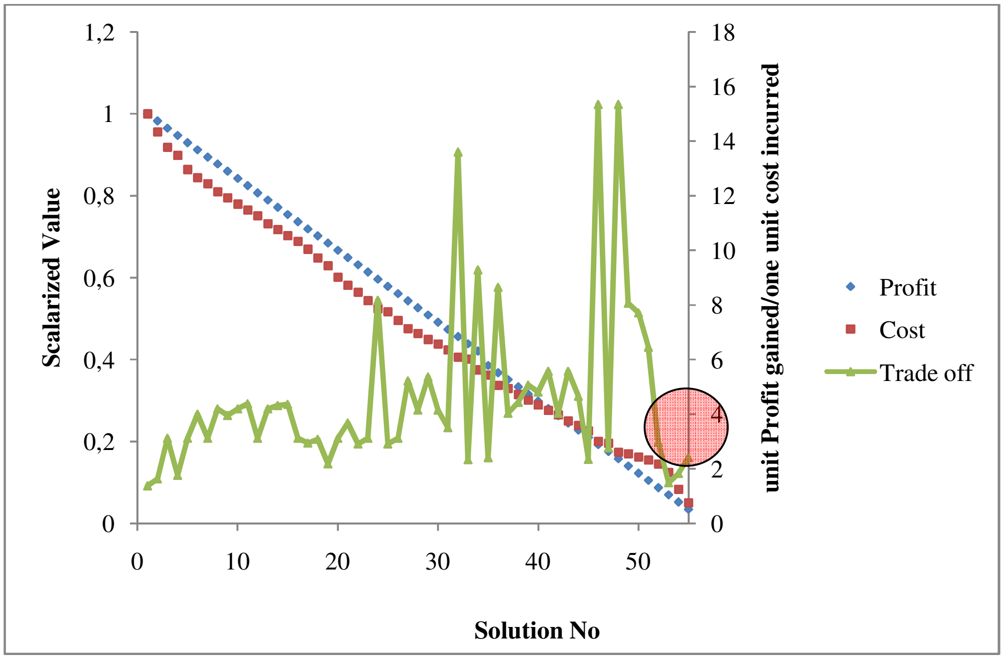

Figure 39. Scalarization of profit and cost, and trade-off for OP32 problem set

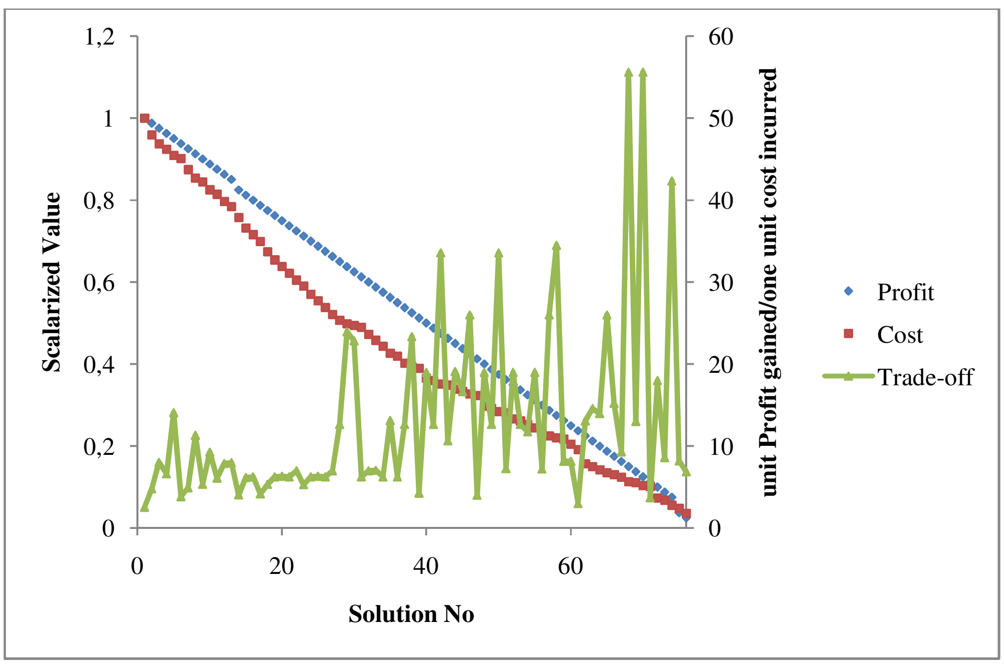

Figure 40. Scalarization of profit and cost, and trade-off for OP33 problem set

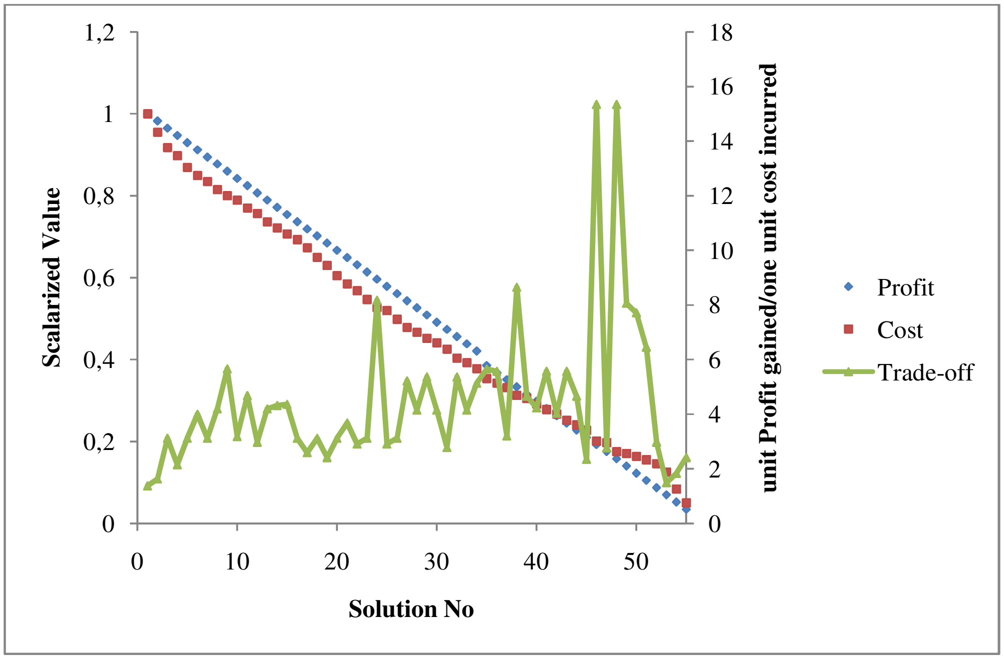

Figure 41. Scalarization of profit and cost, and trade-off for OP32-1- problem set

The Pareto optimal solutions, trade-off curves and scalarized costs and profits for ATT48, EIL30, EIL31, EIL33, EIL51, EIL76, EIL101, CMT101, AND CMT121 are shown in Figure 41 – Figure 56.

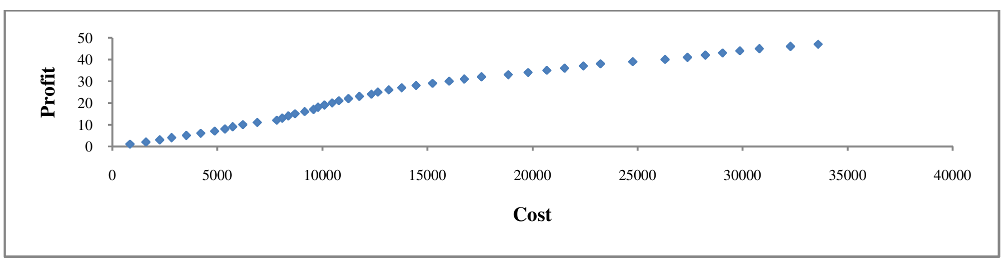

Figure 42. The Pareto optimal solutions for ATT48

For ATT48, the proposed method generates 47 Pareto optimal solutions. Since the distances are calculated as euclidean space, there could be no comparison for the solutions.

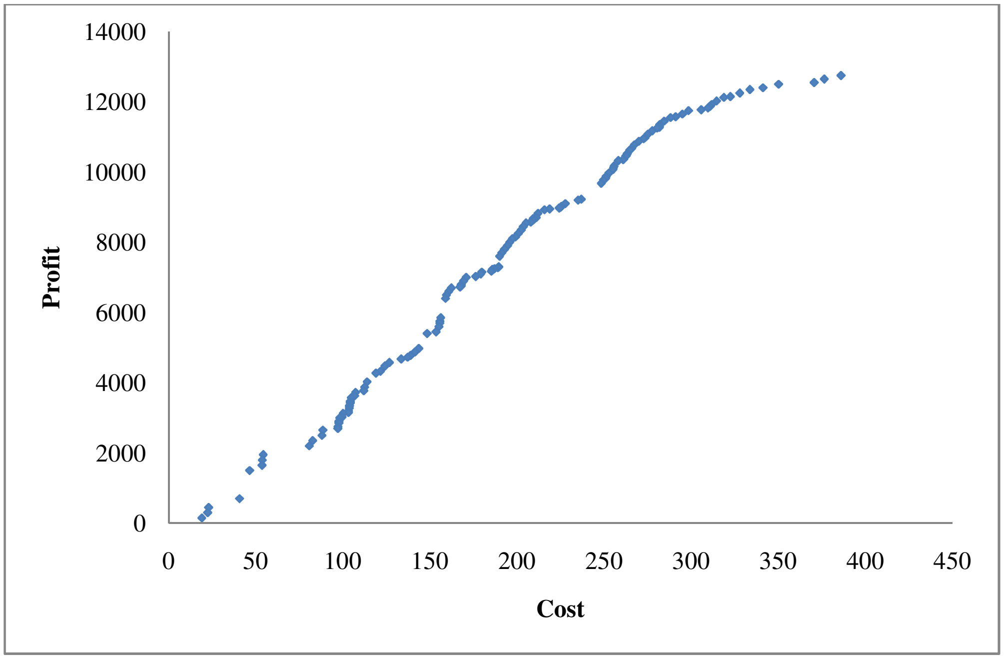

Figure 43. The Pareto optimal solutions for EIL30

For EIL30, the proposed method generates 134 Pareto optimal solutions. Fischetti et al. (1998) uses 96, 191, and 286 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 29.

Table 29. Published solutions and neighborhood solutions for EIL30 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	2650	96	7600	191	11550	286
Generated Solutions	2750	97.126218	7800	192.74108	11575	290.98106
	2700	97.044642	7700	191.21443	11550	287.97805
	2650	88.483477	7600	189.90826	11450	284.30868
	2500	87.940916	7300	189.43731	11350	281.75621
	2350	82.624585	7275	188.89561	11300	281.67188

For 96, the published result and generated result is same but for 97.04 the generated solution has a profit value of 2700. For 191 the published result and generated result is same but for 191.21 the generated solution has a profit value of 7700. For 286 the published result is better than the generated result but for 287.97 the generated solution has a profit value of 11550. In Figure 44, the trade-off curve is given for EIL30.

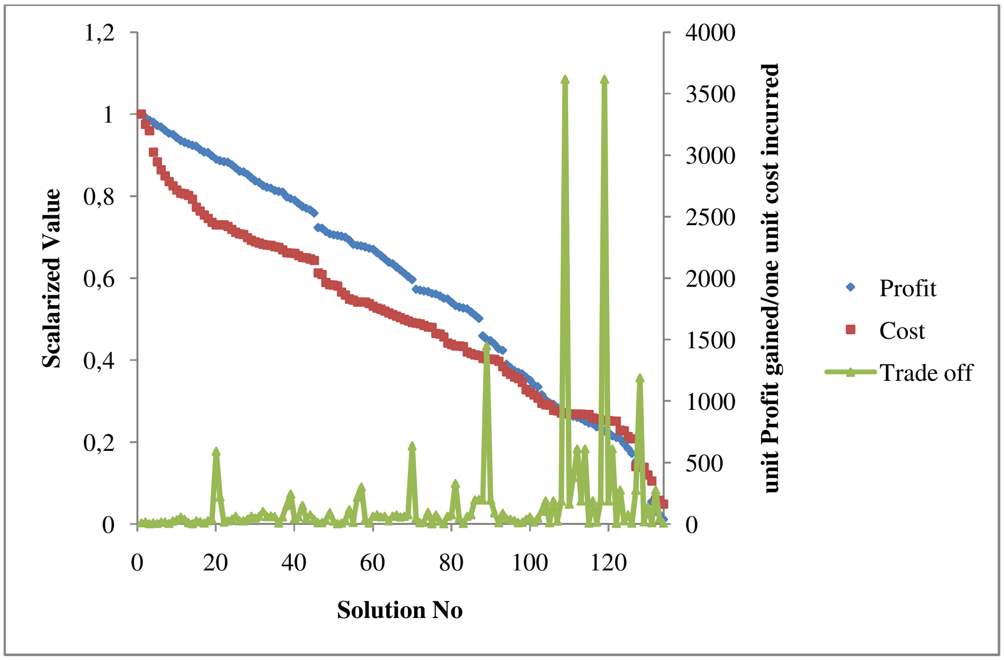

Figure 44. Scalarization of profit and cost, and trade-off for EIL30 problem set

Table 30. Published solutions and neighborhood solutions for EIL33 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	800	111	16220	221	26380	331
Generated Solutions	2520	131.15174	16380	222.2998	26420	333.28922
	2500	115.94826	16340	221.89242	26380	331.08552
	1600	113.78666	16180	220.34592	26280	330.92903
	1200	96.664368	16070	220.32606	26200	330.4569
	400	68.876701	15990	219.91842	26100	330.23758

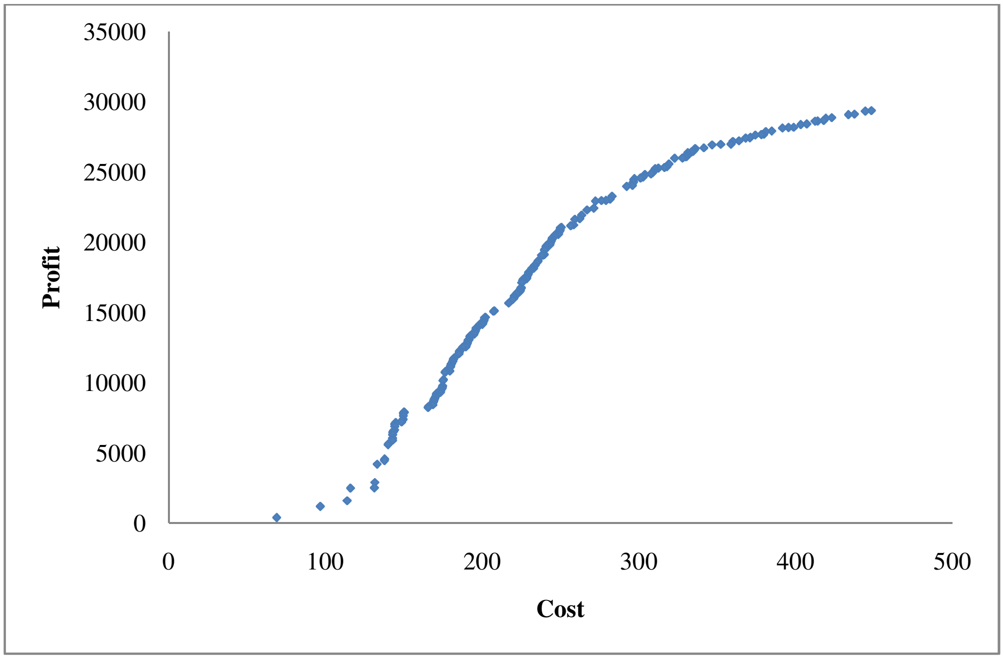

Figure 45. The Pareto optimal solutions for EIL33

For EIL33, the proposed method generates 259 Pareto optimal solutions. Fischetti et al. (1998) uses 111, 221, and 331 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 30.

For 111 the generated result is better than the published result. For 221 the published result is better than the generated result but for 221.89 the generated solution has a profit value of 16340. For 331 the published result is better than the generated result but for 331.08 the generated solution has a profit value of 26380. In Figure 46, the trade-off curve is shown for EIL33.

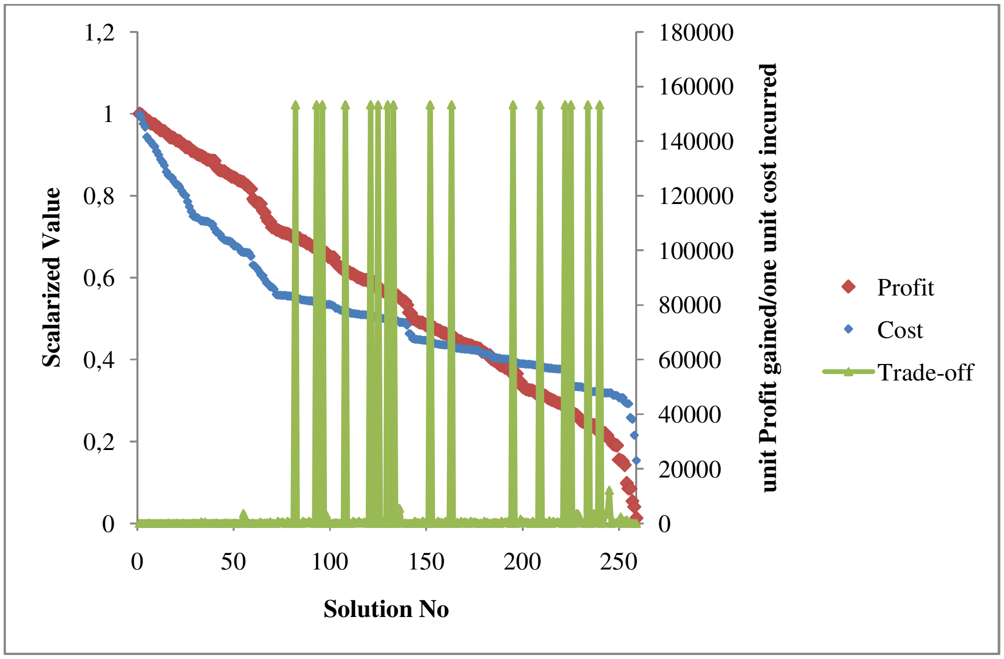

Figure 46. Scalarization of profit and cost, and trade-off for EIL33 problem set

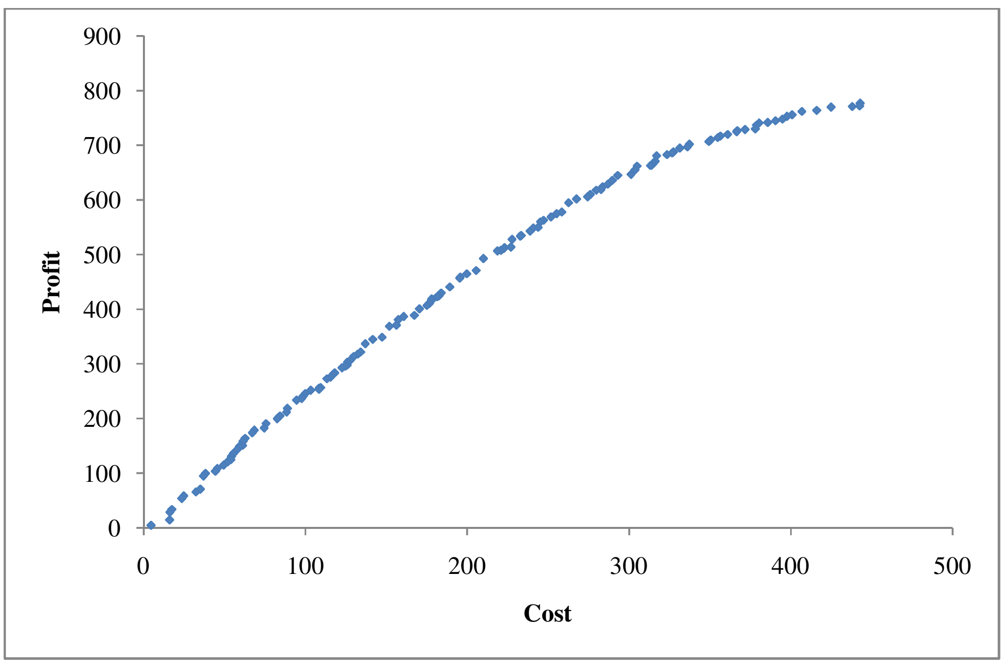

Figure 47. The Pareto optimal solutions for EIL51

The proposed method generates 134 Pareto optimal solutions. Fischetti et al. (1998) uses 107, 213, and 320 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 31.

Table 31. Published solutions and neighborhood Solutions For EIL51 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	264	107	508	213	690	320
Generated Solutions	256	108.37382	508	220.79259	686	326.61919
	254	108.31128	507	218.54225	683	323.35949
	252	103.22058	493	209.92902	681	316.95439
	246	99.85429	471	205.41212	671	316.09264
	241	98.398678	465	199.63184	667	314.93473

For all $TMAX$ values the published result is better than generated result and there are no good trade-offs. In Figure 48, the trade-off curve is shown for EIL51.

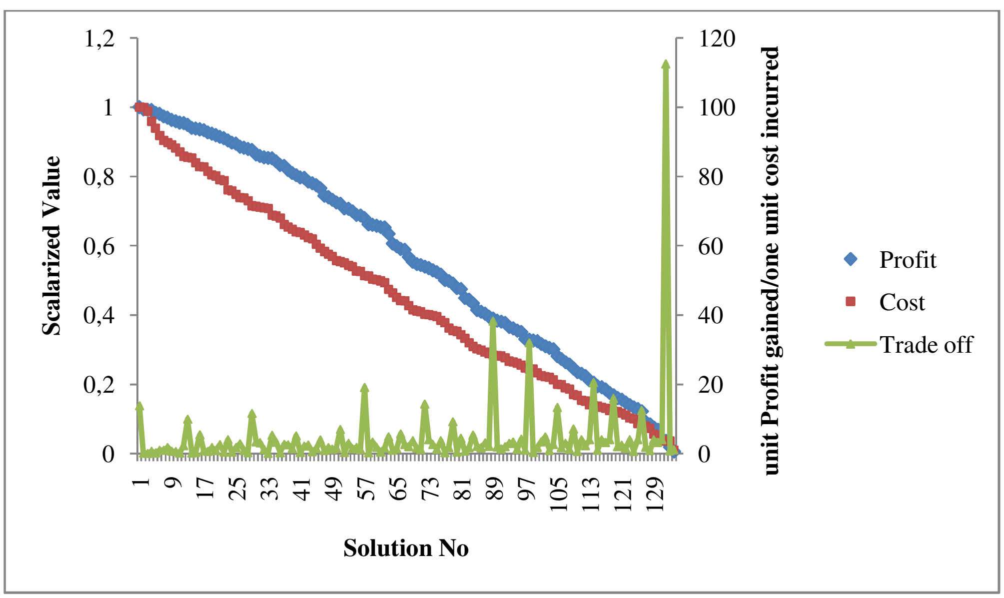

Figure 48. Scalarization of profit and cost, and trade-off for EIL51 problem set

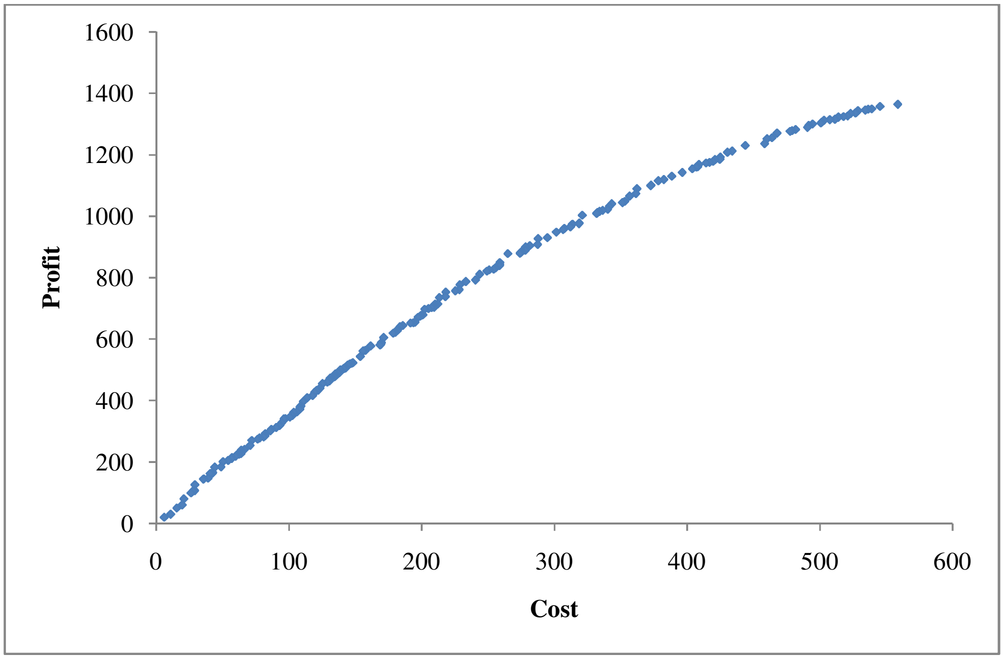

Figure 49. The Pareto optimal solutions for EIL76

For EIL76, the proposed method generates 204 Pareto optimal solutions. Fischetti et al. (1998) uses 135, 269, and 404 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 32.

Table 32. Published solutions and neighborhood Solutions For EIL76 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	490	135	907	269	1186	404
Generated Solutions	491	138.06092	881	274.45006	1161	408.02379
	490	136.90173	879	274.03737	1160	406.97744
	486	134.84544	878	264.899	1154	403.95529
	478	134.72891	849	258.87176	1142	396.26543
	477	134.22723	843	258.79781	1130	388.43885

For all $TMAX$ values the published results are better than generated results. For 135, the solution with cost 136.9 has a profit value of 490 same as the published solution. For 269 and 404, there are no good trade-offs. In Figure 50, the trade-off curve is shown for EIL76.

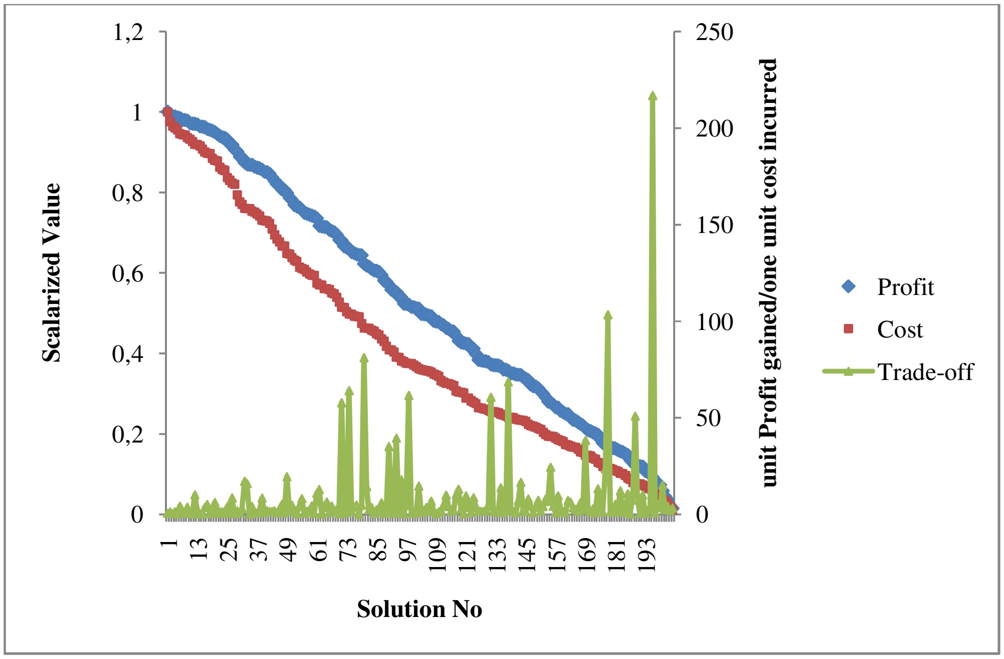

Figure 50. Scalarization of profit and cost, and trade-off EIL76 problem set

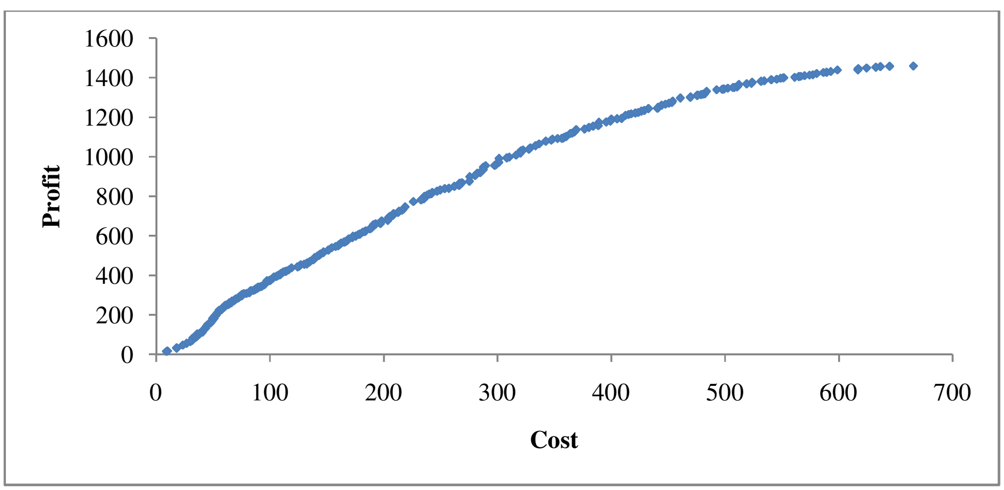

Figure 51. The Pareto optimal solutions for EIL101

For EIL101, the proposed method generates 284 Pareto optimal solutions. Fischetti et al. (1998) uses 158, 315, and 472 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 33.

Table 33. Published solutions and neighborhood Solutions For EIL101 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	572	158	1049	315	1336	472
Generated Solutions	555	160.79282	1017	320.04665	1154	383.91961
	549	159.96388	1007	316.17854	1147	380.23928
	545	157.68423	997	310.26072	1139	376.46407
	539	154.27274	993	308.02565	1136	369.04711
	527	151.23029	990	301.26317	1124	367.35119

For all $TMAX$ values the published results are better than generated results and there are no good trade-offs. In Figure 52, the trade-off curve is shown for EIL101.

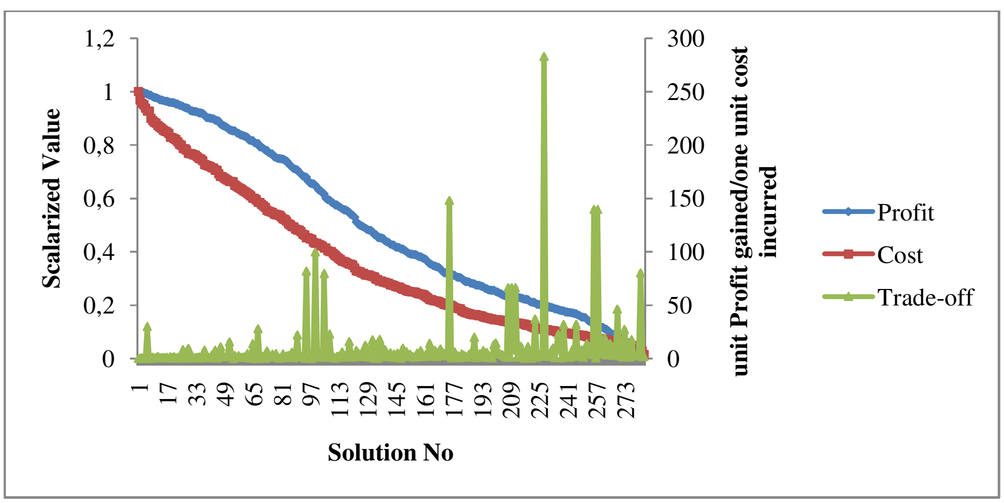

Figure 52. Scalarization of profit and cost, and trade-off for EIL101 problem set

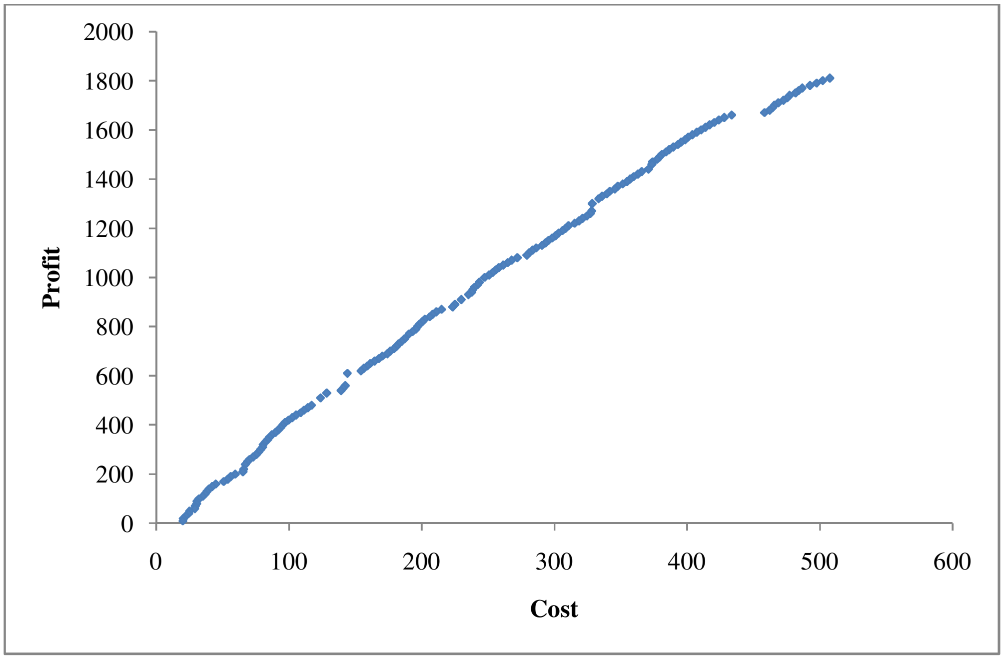

Figure 53. The Pareto optimal solutions for CMT101

For CMT101, the proposed method generates 165 Pareto optimal solutions. Fischetti et al. (1998) uses 127, 253, and 379 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 34.

Table 34. Published solutions and neighborhood Solutions For CMT101 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	530	127	1030	253	1480	379
Generated Solutions	540	139.19579	1030	255.60207	1500	380.96178
	530	128.36219	1020	253.47795	1490	379.23895
	510	123.69834	1010	250.87766	1480	377.4826
	480	116.95925	1000	247.37919	1470	374.03386
	470	114.13082	980	243.38521	1460	373.54206

For 127, the published solution is better than the generated solution but for 128.36 the generated solution has a profit value of 530. For 253, the published solution is better than the generated solution but for 253.47 the generated solution has a profit value of 1020, still worse but a good trade-off. For 379 the solutions are same but for 379.23 the generated solution has a profit value of 1490. In Figure 54, the trade-off curve is shown for CMT101.

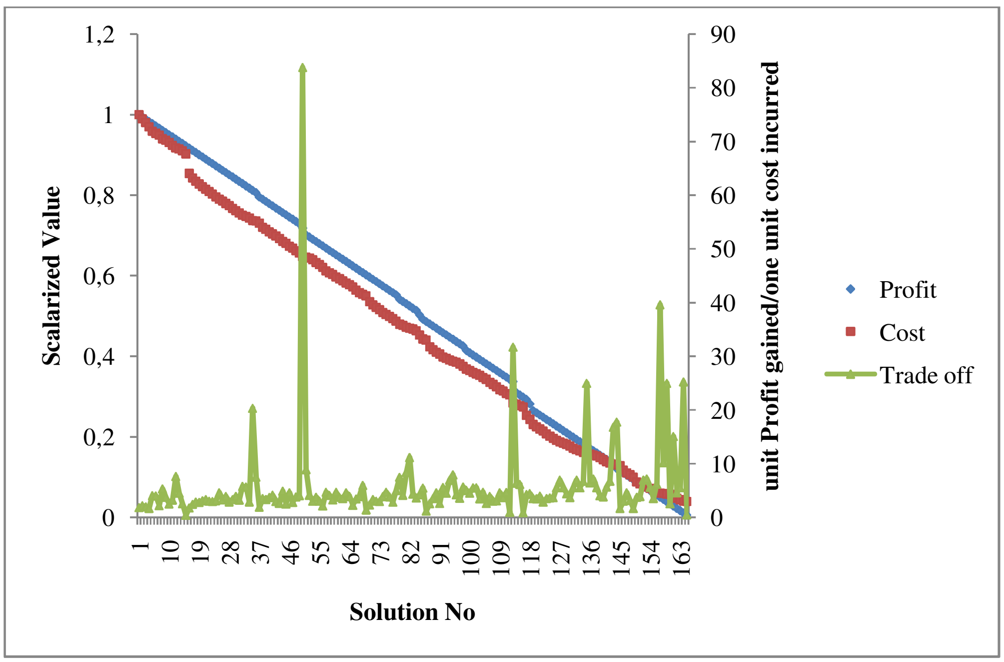

Figure 54. Scalarization of profit and cost, and trade-off for CMT101 problem set

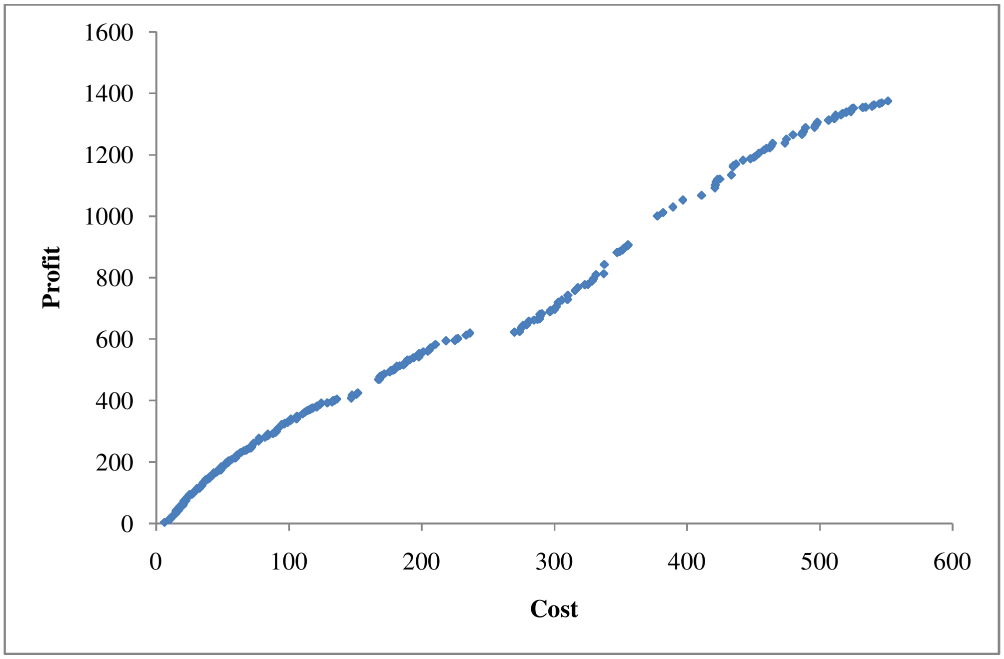

Figure 55. The Pareto optimal solutions for CMT121

For CMT121, the proposed method generates 240 Pareto optimal solutions. Fischetti et al. (1998) uses 137, 273, and 409 for $TMAX$. Based on these $TMAX$ values our solutions and neighborhood solutions are given in Table 35.

Table 35. Published solutions and neighborhood Solutions For CMT121 for the given $TMAX$ values

	Profit	Cost	Profit	Cost	Profit	Cost
Fischetti et al. Solutions	412	137	715	273	1134	409
Generated Solutions	418	147.59638	635	274.86314	1092	420.98125
	408	146.96524	625	273.82584	1068	410.97865
	405	136.12438	623	269.99263	1053	396.8431
	401	134.60443	620	236.35082	1030	389.37166
	400	133.34101	613	233.45321	1012	381.90041

For all $TMAX$ the published solutions are better than generated solutions and there are no good trade-offs. In Figure 56, the trade-off curve is shown for CMT121.

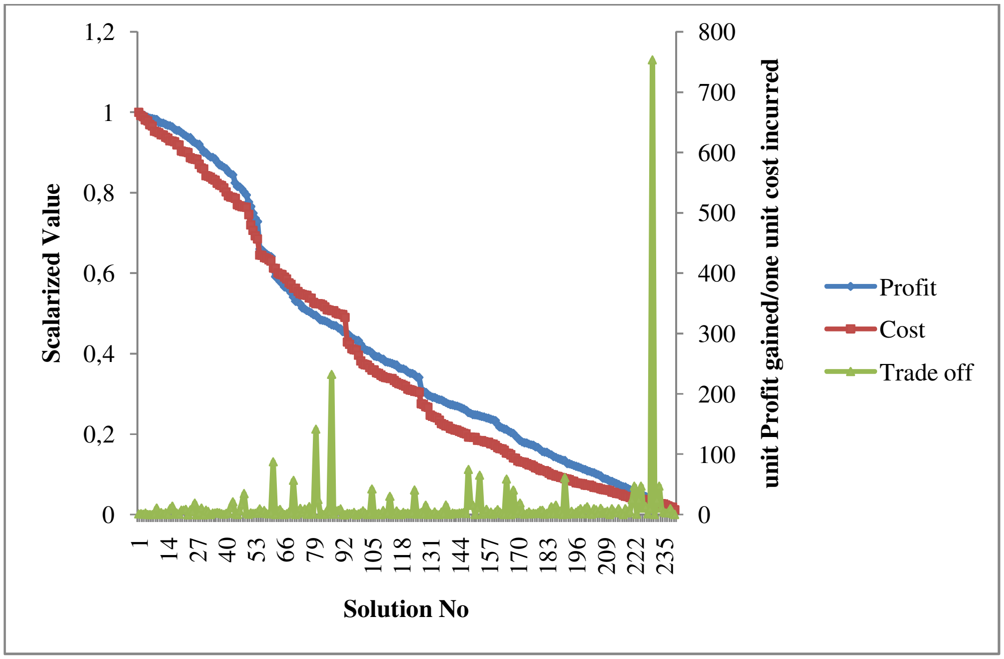

Figure 56. Scalarization of profit and cost, and trade-off for CMT121 problem set

The generated routes and route profits and costs for K25, OP21, OP32, OP32-1-, OP33, EIL30, EIL33, EIL51 and EIL76 are in Appendix A.

6.3 Performance Measures

As discussed, there is no attempt to solve the biobjective TSP with profit, except Keller and Goodchild (1988). So that, there are no performance measures for the proposed method. However, we define some measurements for the performance of our method.

i. The first performance measure is number of Pareto optimal solutions, number of dominated solutions and the percentage of these solutions in order to measure how effective the method find the Pareto optimal solutions.

ii. The second performance measure is the time to find a single solution for the various of number of cities and TMAX for each problem set.

iii. The third performance measure is the total runtime for the problem sets.

For the first performance measure, the number of Pareto optimal solutions and dominated solutions generated during the solution process for the problem sets. In Table 36, the first column is the name of the problem sets which already includes the number of cities for the given problem set. In the second column and third column, the number of Pareto optimal solutions and dominated solutions are given, respectively. In the fourth and fifth column, the percentage of dominated solutions to Pareto optimal solutions and the percentage of Pareto optimal solutions and total solutions are presented, respectively.

The maximum percentage is for $DS\ /\ POS$ 0.61,CMT101. The number of Pareto optimal solutions and number of dominated solutions does not depend on the size of the problem set. On the other side, 62% of the generated solutions is Pareto optimal solutions. CMT101 seems to be bottleneck, so one can conclude that at least 62% of the generated solutions are Pareto optimal solutions. This table could be used to evaluate future works.

Table 36. Pareto optimal and dominated solution analyze table

	Pareto Optimal Solutions	Dominated Solutions	DS /POS	POS / (DS+POS)	
OP32	55	27	0.49	0.67	82
OP21	58	4	0.07	0.94	62
OP33	77	24	0.31	0.76	101
OP32-1-	55	23	0.42	0.71	78
K25	69	3	0.04	0.96	72
ATT48	47	6	0.13	0.89	53
EIL30	134	48	0.36	0.74	182
EIL33	259	48	0.19	0.84	307
EIL51	99	35	0.35	0.74	134
EIL76	152	52	0.34	0.75	204
EIL101	237	48	0.20	0.83	285
CMT101	103	63	0.61	0.62	166
CMT121	170	70	0.41	0.71	240

For the second performance measure, times are calculated with $TMAX_0$ and $TMAX_0/2$. The single runs are done for 10 times for each problem set and the maximum value of the runtimes for each step is selected. Times are in milliseconds.

For $TMAX_0$ it is interesting that the most time consuming processes are 2-opt and initialization steps. 80% of the time the algorithm process 2-opt step and 15% of the time the algorithm process initialization step. For $TMAX_0$ the problem behaves as pure TSP so that the dominated step is 2-opt and the main steps like two point exchange and one point movement becomes ineffective. It is expected to see that as the number of given cities increases the runtime increases. In Table 37, runtimes for each step is given. In Figure 57, the percentages are illustrated.

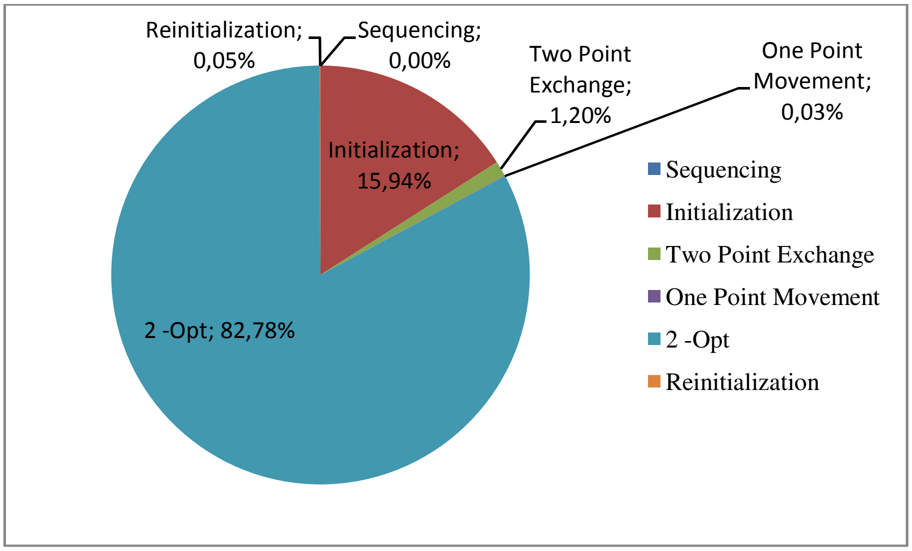

Figure 57. Time percentages of steps for $\boldsymbol{TMAX_0}$

Table 37. Runtimes for each step for each problem set for a single run with $TMAX_0$

	Sequencing	Initialization	Two Point Exchange	One Point Movement	2 -Opt	Reinitialization	Total
OP32	0	94	30	0	284	0	408
OP21	0	31	46	16	94	16	203
OP33	0	94	15	0	313	0	422
OP32-1-	0	47	32	0	186	16	281
K25	0	32	14	0	95	0	141
ATT48	0	313	30	0	1017	0	1360
EIL30	0	62	32	0	593	0	687
EIL33	0	47	32	0	343	0	422
EIL51	0	250	62	0	1626	0	1938
EIL76	0	1219	156	0	8531	0	9906
EIL101	0	3578	250	0	23797	0	27625
CMT101	0	3484	218	0	17313	0	21015
CMT121	0	7063	313	15	30531	16	37938
Total	0	16314	1230	31	84723	48	102346
Percentage	0	0.15940047	0.0120181	0.0003029	0.8278096	0.000468997	1

For $TMAX_0/2$ two point exchange becomes effective and the time percentage of it increases while the time percentage of 2-opt decreases but still effective. The other point is that initialization step becomes more dominated then $TMAX_0$. Now, 50% of time the proposed method processes two point exchange step and 30% of time it processes initialization step. In Table 38, runtimes for each step is given. In Figure 58, the percentages are illustrated.

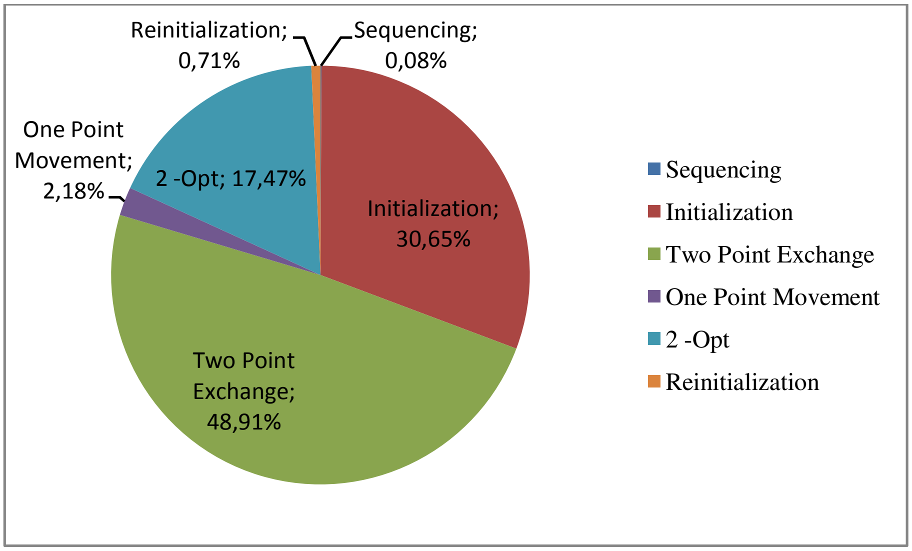

Figure 58. Time percentages of steps for $TMAX_0/2$

Table 38. Runtimes for each step for each problem set for a single run with $TMAX_0/2$

	Sequencing	Initialization	Two Point Exchange	One Point Movement	2 -Opt	Reinitialization	Total
OP32	0	16	172	15	47	0	250
OP21	0	15	79	31	15	0	140
OP33	0	31	109	0	48	0	188
OP32-1-	0	16	94	46	16	16	188
K25	0	16	16	16	16	0	64
ATT48	0	188	374	0	220	0	782
EIL30	0	16	110	15	0	0	141
EIL33	0	16	63	30	0	0	109
EIL51	0	110	359	48	45	16	578
EIL76	0	594	1061	0	220	31	1906
EIL101	0	1844	2171	48	890	63	5016
CMT101	16	1468	2656	62	1376	16	5594
CMT121	0	1781	2488	124	591	0	4984
Total	16	6111	9752	435	3484	142	19940
Percentage	0.0008024	0.30646941	0.4890672	0.0218154	0.1747242	0.007121364	1

These measurements can be used to understand the behavior of the heuristic and improve it. Also one can evaluate the single runtimes for performance.

Out of steps there are some extra process in the proposed algorithm needs time. Also one single runtime cannot show the whole picture. The total runtimes for each problem set are collected and presented in Table 39. Times are in milliseconds.

Table 39. Total runtimes for each problem set

OP32	8547
OP21	1703
OP33	11750
OP32-1-	8496
K25	3438
ATT48	27109
EIL30	13829
EIL33	30718
EIL51	15694
EIL76	933203
EIL101	9632532
CMT101	4708250
CMT121	13534015

From Table 39, it seems that runtimes increase exponentially as the city numbers increase. Even runtime depends on the problem structure, city number is a dominated facto for runtime. In Figure 59, the relationship between runtime and the city number can be seen explicitly.

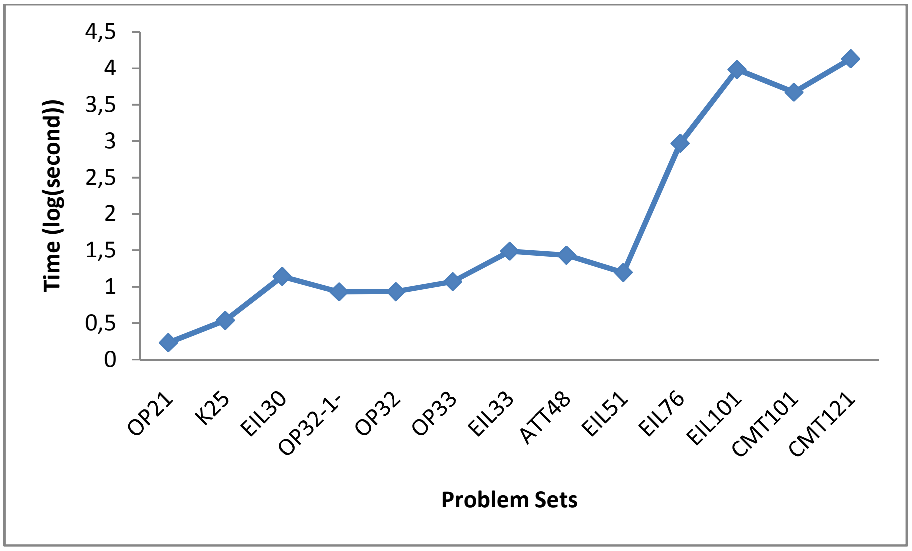

Figure 59. Runtimes for the problem sets

The proposed method solves small problem sets easily and efficiently. But as the number of city increases the runtime increases rapidly. The maximum number of cities that the proposed algorithm solved is 121.

CHAPTER 7

INTERACTIVE SOFTWARE

An interactive software is developed to implement the proposed method. The software uses data files that contain information about city locations or city distances and city profits. User can choose starting and ending cities. The software solves the problem with the choosen starting and ending cities and illustrate the Pareto optimal solutions in a graphic with an ideal point and subset of efficient solutions based on the distance formulation and default weights $w_{profit} = 0.5$ and $w_{cost} = 0.5$. Also, only the subset of efficient solutions is illustrated in another graphic in order to give detail view to the subset of efficient solutions. In another graphic the trade offs are illustrated between Pareto optimal solutions. User can define new goal point or weights. The nearest solutions are changed based on the defined goal point or weights. Detailed information is given about the interactive software in section 6.1.

7.1 Interactive Software

Software has only one frame, main frame. Main frame contains a map that cities are illustrated, three graphics that illustrate Pareto optimal solutions, subset of Pareto optimal solutions and scalarization graphic. In Figure 60, main frame is shown.

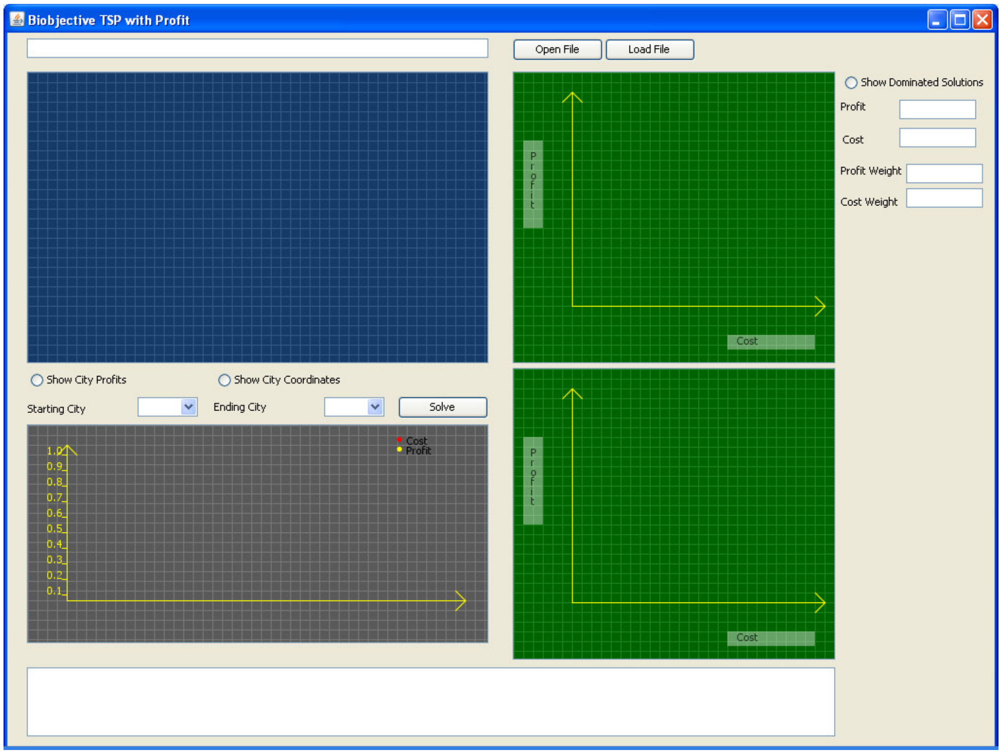

Figure 60. Main frame of interactive software

To open a problem data set, user has to click "Open File" button. When user clicks "Open File" button, a file chooser window opens to choose a problem file as shown in Figure 61. File to load has to have a specific structure. In the first line of the file number of cities in the problem set has to be written and in the $i+1^{th}$ line x-coordinate, y-coordinate and profit for city i has to be written for each city. The sample structure of a file is given in Figure 62. When a file is choosen, the path of the file is shown in text line and when "Load File" button is clicked, cities are illustrated. If "Show City Profits" and "Show City Coordinates" are selected, city profits and city coordinates are presented in the screen with white and yellow background, respectively. The illustrated cities, city coordinates and city profits are shown in Figure 63.

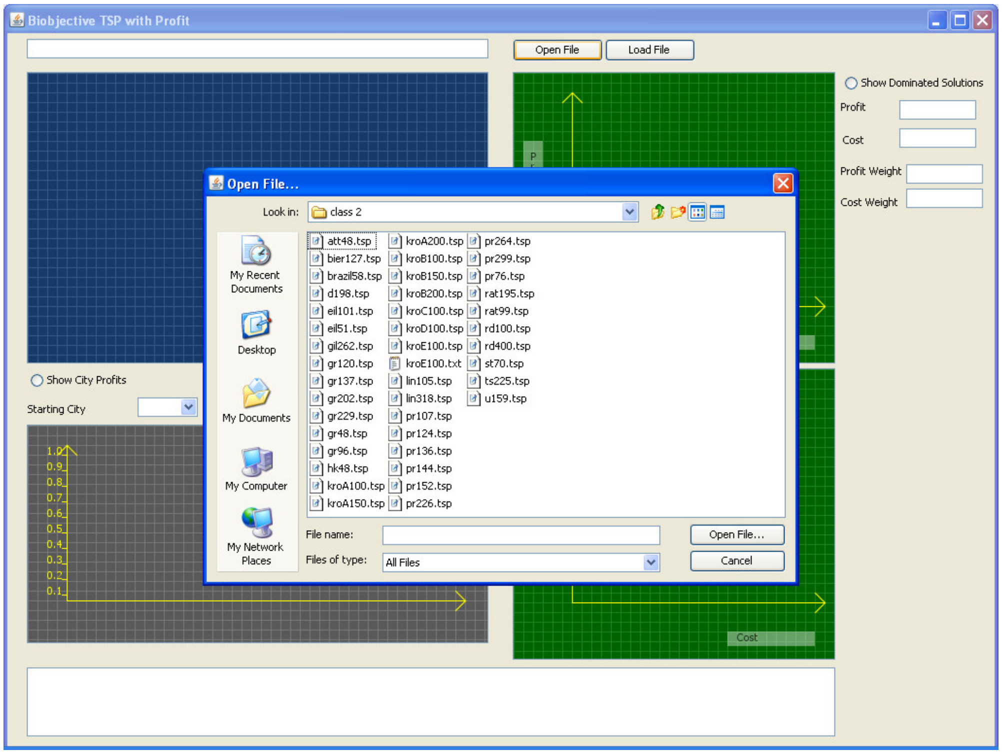

Figure 61. File chooser window

10			→ number of cities
10.5	14.4	0	→ x and y coordinates and profit for city 0
18	15.9	10	→ x and y coordinates and profit for city 1
18.3	13.3	10	→ x and y coordinates and profit for city 2
16.5	9.3	10	→ x and y coordinates and profit for city 3
15.4	11	10	→ x and y coordinates and profit for city 4
14.9	13.2	5	→ x and y coordinates and profit for city 5
16.3	13.3	5	→ x and y coordinates and profit for city 6
16.4	17.8	5	→ x and y coordinates and profit for city 7
15	17.9	5	→ x and y coordinates and profit for city 8

Figure 62. Data file sample

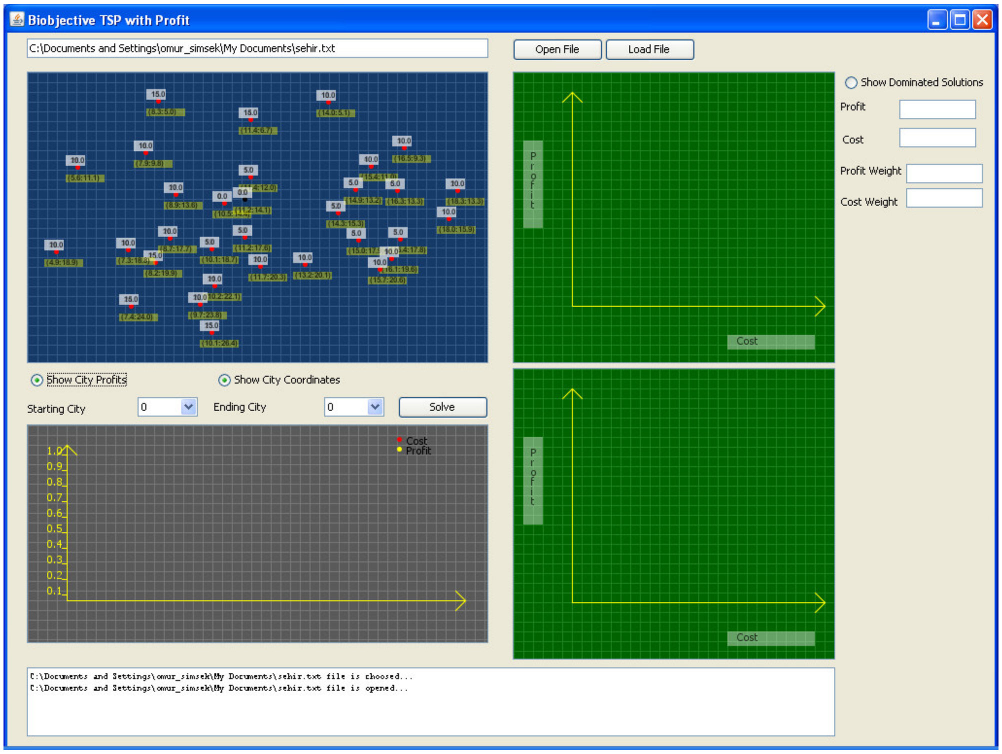

Figure 63. Cities and their coordinates and profits

User is now ready to select "Starting City" and "Ending City". After this selection "Solve" button is clicked and the proposed method runs. The solutions are generated in the run and when run ends, the solutions are illustrated in the graphs. If user checks "Show Dominated Solutions", dominated solutions are also shown. The generate Pareto optimal solutions and dominated solutions are shown in Figure 64. The black point in the solution space is either an ideal point or a goal point and the circles not red are nearest solutions.

User can define a goal point either clicking a point in the solution space or changing the value of "Profit" and "Cost" texts. User can define new weights by changing either the value of "Profit Weight" or "Cost Weight". Whenever one of them is changed the other one is calculated by the formula

$$w_{profit} + w_{cost} = 1$$

The new goal point and weights are illustrated in Figure 65. The nearest solutions are changed based on the goal point and weights.

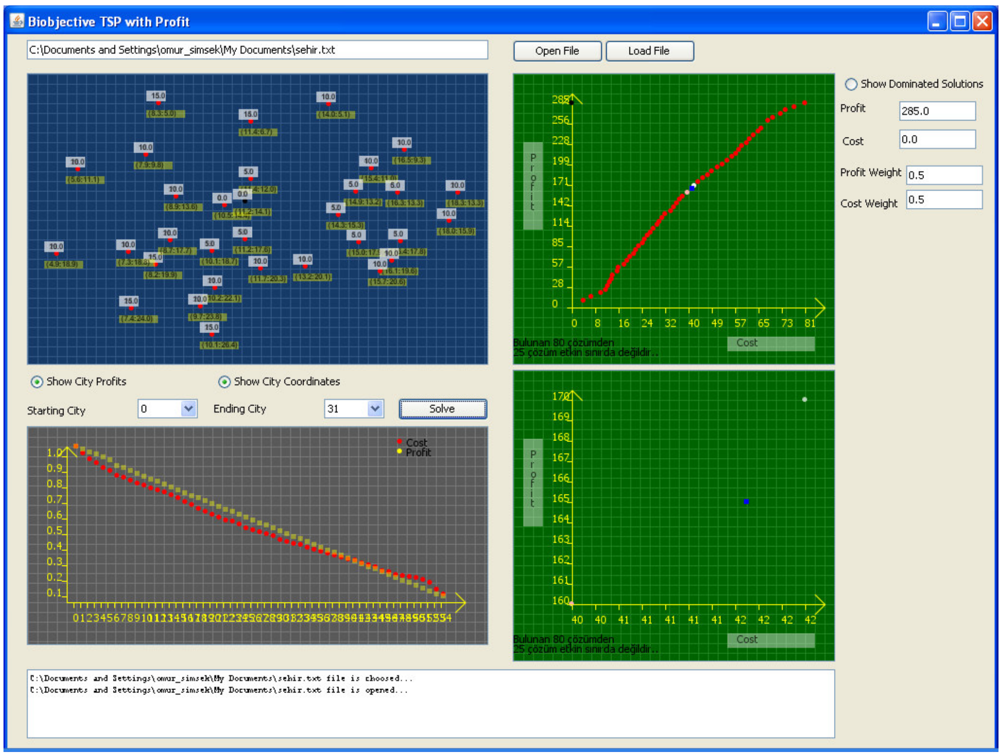

Figure 64. Pareto optimal solutions, nearest solutions and ideal point

If user clicks a Pareto optimal solution in the graph, the route of the solution is generated on the map as shown in Figure 66.

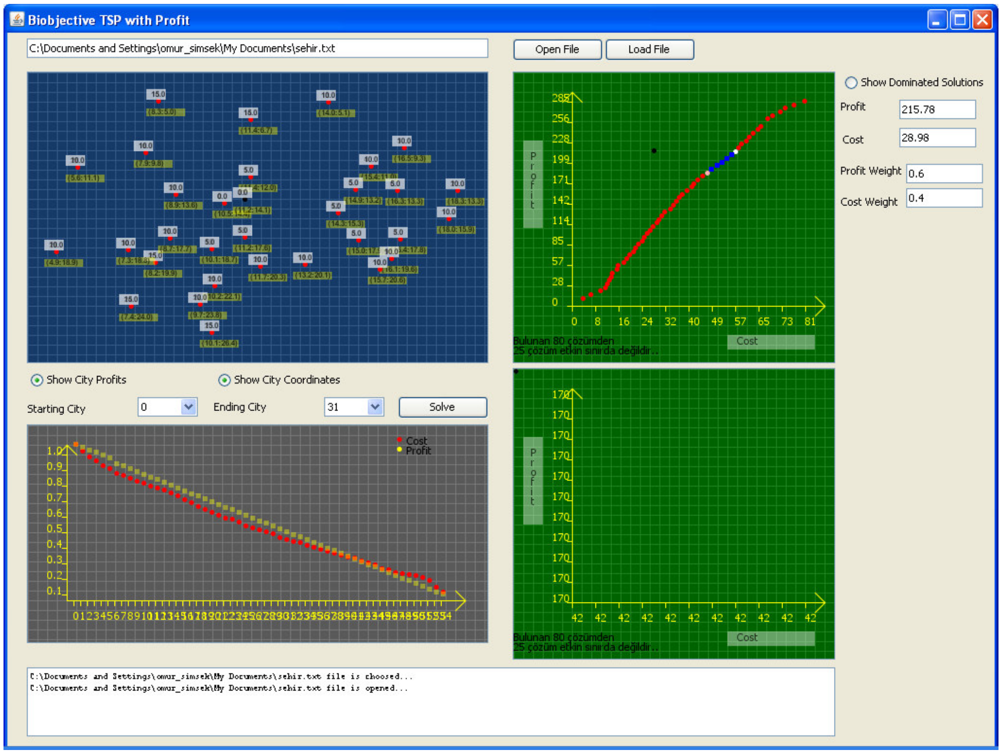

Figure 65 Updated goal point, weights and nearest solutions

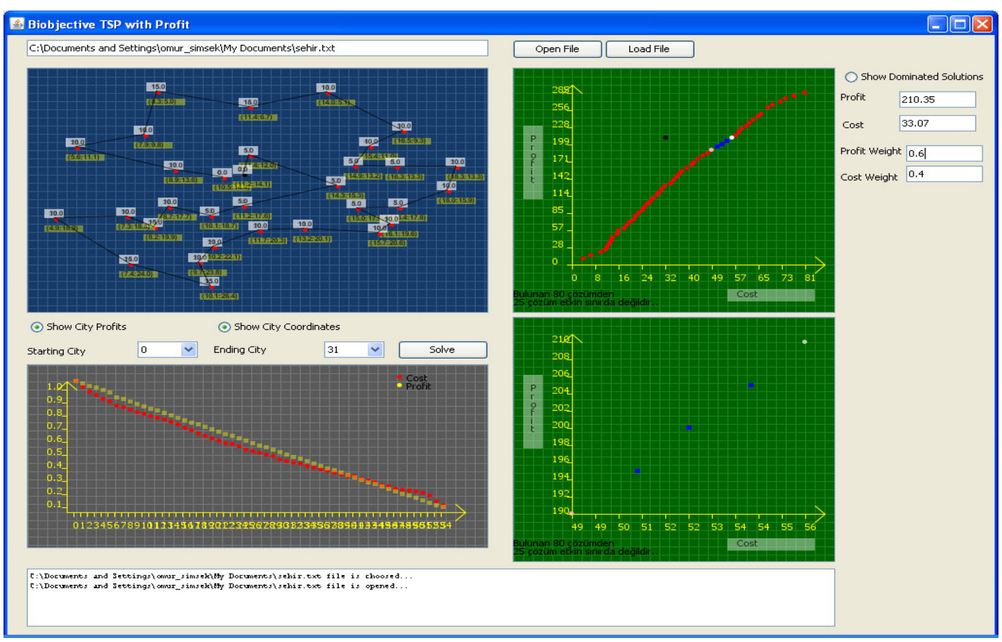

Figure 66 Route generation on the map

CHAPTER 8

CONCLUSION

TSP with profit is naturally biobjective problem where objectives are contradictory. In the literature single objective TSP with profits are studied. The only attempt to solve the biobjective TSP with profit is Keller and Goodchild (1988). In this study we developed a multiobjective approach based on ε-constraint method to solve biobjective TSP with profit. ε-constraint method is chosen because it transforms the problem into single objective problem that is studied widely in the literature. Since Selective TSP is more widely studied than Prize Collecting TSP, Selective TSP version of the scalarization is choosen to study. CGW heuristic, which is fast and effective heuristic, is one of the best solution methods that solve Selective TSP. To solve the single objective problem, CGW heuristic is choosen. The computational analysis show that proposed method performs well. Our study shows that Keller's algorithm, the only multiobjective approach for biobjective TSP with Profit, is not good enough to find Pareto optimal solutions.

An interactive software is developed based on the proposed method. The aim to develop an interactive software is to give a better understanding about Pareto optimal solutions to the user. User can change goal points or weights and see how the subset of efficient solutions changes. Also user can analyze the slope of decreases or increases of profit and cost by scalarization graphic.

For the future work, the proposed method can store all the solutions that is generated not only best solutions but the bad solutions generated for specific upper bounds. The bad solutions for a specific upper bound can be Pareto optimal solutions for other upper bounds. Also the solution set including best solution and other solutions for a specific upper bound can be used an initial solution for the updated upper bound.

The interactive software tries to generate all the Pareto optimal solutions. Instead of this, to increase the performance of the proposed method, subsets of Pareto optimal solutions can be generated. Considering these solutions and the goal point and weights defined by user, Pareto optimal solutions between specific limits of upper bound can be generated. So that the user only gets the information he needs. Also, the proposed method does not try to generate all the Pareto optimal solutions. The runtime of the propose method is expected to decrease, especially for the large problems.

The algorithm mostly uses 2-opt when the problem is almost TSP. So the performance of the algorithm decreases. One can use Concorde (or any other TSP solver) instead of 2-opt procedure in the algorithm to improve the solution.

REFERENCES

Balas, E. 1989. The prize collecting traveling salesman problem. *Networks* 19(6) 621–636.

Balas, E. 1995. The prize collecting traveling salesman problem. II: Polyhedral results. *Networks* 25(4) 199–216.

Balas, E. 1999. New classes of efficiently solvable generalized travelingsalesman problems. *Ann. Oper. Res.* 86 529–558.

Balas, E., G. Martin. 1985. *Roll-a-Round: Software Package for Scheduling the Rounds of a Rolling Mill*. Balas and Martin Associates.

Bienstock, D., M. Goemans, D. Simchi-Levi, D. Williamson. 1993. A note on the prize collecting traveling salesman problem. *Math. Programming* 59 413–420.

Chao, I.-M., B. L. Golden, E. A. Wasil. 1996. A fast and effective heuristic for the orienteering problem. *Eur. J. Oper. Res.* 88(3) 475–489.

Dell'Amico, M., F. Maffioli, A. Sciomachen. 1998. A Lagrangian heuristic for the prize-collecting travelling salesman problem. *Ann. Oper. Res.* 81 289–305.

Dell'Amico, M., F. Maffioli, P. Värbrand. 1995. On prize-collecting tours and the asymmetric travelling salesman problem. *Internat.Trans. Oper. Res.* 2(3) 297–308.

Dueck, G. 1990. New optimization heuristics: The great deluge algorithm and the record-to-record travel. *J. Comput. Physics* 104(1) 86–92.

Ehrgott, M. 2000. Approximation algorithms for combinatorial multicriteria optimization problems. *Internat. Trans. Oper. Res.* 7(1) 5–31.

Feillet, D., Dejax P., Gendreau, M. 2005. Traveling Salesman Problems with Profits. *Transportation Science* 39(2) 188-205

Fischetti, M., P. Toth. 1988. An additive approach for the optimal solution of the prize-collecting traveling salesman problem.

Fischetti, M., J. J. Salazar González, P. Toth. 1998. Solving the orienteering problem through branch-and-cut. *INFORMS J. Comput.*10(2) 133–148.

Gendreau, M., G. Laporte, F. Semet. 1998a. A branch-and-cut algorithm for the undirected selective traveling salesman problem. *Networks* 32 263–273.

Gendreau, M., G. Laporte, F. Semet. 1998b. A tabu search heuristic for the undirected selective traveling salesman problem. *Eur. J.Oper. Res.* 106 539–545.

Golden, B. L., L. Levy, R. Vohra. 1987. The orienteering problem. *Naval Res. Logist.* 34(3) 307–318.

Golden, B. L., Q. Wang, L. Liu. 1988. A multifaceted heuristic for the orienteering problem. *Naval Res. Logist.* 35 359–366.

Göthe-Lundgren, M., K. Jörnsten, P. Värbrand. 1996. On the nucleolus of the basic vehicle routing game. *Math. Programming* 72 83–100.

Göthe-Lundgren, M., F. Maffioli, P. Värbrand. 1995. A Lagrangian decomposition approach for a prize collecting traveling salesman type problem. Technical Report LiTH-MATH-R-1995-10, Linköping Institute of Technology, Linköping, Sweden.

Gutin, G., A. Punnen, eds. 2002. *Traveling Salesman Problem and Its Variations*. Kluwer Academic Publishers, Dordrecht, The Netherlands. Helmberg, C. 1999. The m-cost ATSP. *Lecture Notes Comput. Sci.* 1610 242–258.

Kabadi, S. N., A. Punnen. 1996. Prize-collecting traveling salesman problem. *INFORMS Washington Conf.*, Washington, D.C.

Keller, C. P., M. Goodchild. 1988. The multiobjective vending problem: A generalization of the traveling salesman problem. *Environ. Planning B: Planning Design* 15 447–460.

Laporte, G., S. Martello. 1990. The selective traveling salesman problem. *Discrete Appl. Math.* 26 193–207.

Lawler, E.L., Lenstra, J.K., Rinnooy Kan, A.H.G., Shmoys, D.B. (eds,), 1985. *The traveling salesman problem*, Wiley, Chichester

Liang, Y.-C., A. E. Smith. 2001. An ant colony approach to the orienteering problem. Technical report. Department of Industrial and Systems Engineering, Auburn University, Auburn, AL.

Ramesh, R., K. M. Brown. 1991. An efficient four-phase heuristic for the generalized orienteering problem. *Comput. Oper. Res.* 18(2) 151–165.

Ramesh, R., Y.-S. Yoon, M. H. Karwan. 1992. An optimal algorithm for the orienteering tour problem. *ORSA J. Comput.* 4(2) 155–165.

Tasgetiren, F. M., A. E. Smith. 2000. A genetic algorithm for the orienteering problem. *Proc. 2000 Congress Evolutionary Comput.*, San Diego, CA, 1190–1195.

Tsiligirides, T. 1984. Heuristic methods applied to orienteering. *J. Oper. Res. Soc.* 35(9) 797–809.

Wang, Q., X. Sun, B. L. Golden, J. Jia. 1995. Using artificial neural networks to solve the orienteering problem. *Ann. Oper. Res.* 61 111–120

APPENDIX

SOLUTIONS OF THE EXPERIMENTAL PROBLEMS

Table A1. The solutions for K25

Route Profit	Route Cost	Route
13298	3496	5 18 1 8 9 7 4 16 15 6 10 17 14 3 21 23 22 20 13 2 25 12 24 19 11 5
13268	3345	5 18 1 8 9 7 4 16 15 6 17 10 14 3 21 23 22 20 2 25 12 24 19 11 5
13258	3350	5 18 1 8 9 7 4 16 15 6 14 10 17 3 21 23 20 13 2 25 12 24 19 11 5
13228	3199	5 18 1 8 9 7 4 16 15 6 14 10 17 3 21 23 20 2 25 12 24 19 11 5
13178	3064	5 18 1 8 9 7 4 16 15 6 14 17 3 21 23 20 2 25 12 24 19 11 5
13054	2993	5 18 1 8 9 7 4 16 15 6 10 17 14 3 21 23 20 2 25 24 19 11 5
13004	2887	5 18 1 8 9 7 4 16 15 6 14 17 3 21 23 20 2 25 19 24 11 5
12873	2826	5 18 1 8 9 7 4 16 15 6 14 17 3 21 20 2 25 19 24 11 5
12800	2713	5 18 1 8 9 7 4 15 6 14 17 3 21 23 20 2 25 19 24 11 5
12799	2644	5 18 1 8 9 7 4 16 15 6 14 17 3 21 23 20 2 25 19 11 5
12668	2583	5 18 1 8 9 7 4 16 15 6 14 17 3 21 20 2 25 19 11 5
12595	2470	5 18 1 8 9 7 4 15 6 14 17 3 21 23 20 2 25 19 11 5
12464	2409	5 18 1 8 9 7 4 15 6 14 17 3 21 20 2 25 19 11 5
12407	2370	5 18 1 8 9 7 4 16 15 6 14 3 21 20 2 25 19 11 5
12334	2257	5 18 1 8 9 7 4 15 6 14 3 21 23 20 2 25 19 11 5
12203	2196	5 18 1 8 9 7 4 15 6 14 3 21 20 2 25 19 11 5
12092	2161	5 18 8 9 7 4 15 6 14 3 21 23 20 2 25 19 11 5
11961	2100	5 18 8 9 7 4 15 6 14 3 21 20 2 25 19 11 5
11713	2093	5 18 8 9 7 4 15 6 14 3 21 20 25 19 11 5
11646	2086	5 11 19 25 2 20 21 3 14 6 15 7 9 8 18 5
11411	2026	5 11 19 25 2 20 21 3 14 6 4 7 9 8 18 5
11390	2024	5 18 8 9 7 4 15 14 3 21 20 2 25 19 11 5
11163	2019	5 18 8 9 7 4 6 14 3 21 20 25 19 11 5
11142	2017	5 18 8 9 7 4 15 14 3 21 20 25 19 11 5
11096	2012	5 18 8 9 7 6 14 3 21 20 2 25 19 11 5
11075	2010	5 18 8 9 7 15 14 3 21 20 2 25 19 11 5
10827	2003	5 18 8 9 7 15 14 3 21 20 25 19 11 5
10643	1996	5 18 1 8 9 7 4 15 6 14 3 21 25 19 11 5
10401	1900	5 11 19 25 21 3 14 6 15 4 7 9 8 18 5
10089	1893	5 18 8 9 7 4 15 6 14 3 21 25 11 5
10086	1886	5 18 8 9 7 15 6 14 3 21 25 19 11 5
9851	1826	5 18 8 9 7 4 6 14 3 21 25 19 11 5
9830	1824	5 11 19 25 21 3 14 15 4 7 9 8 18 5
9736	1817	5 11 21 3 14 6 15 4 7 9 8 1 18 5
9536	1812	5 18 8 9 7 6 14 3 21 25 19 11 5
9515	1810	5 18 8 9 7 15 14 3 21 25 19 11 5
9514	1786	5 18 8 9 7 4 6 14 3 15 16 11 19 5
9494	1721	5 18 8 9 7 4 15 6 14 3 21 11 5

Table A1. The solutions for K25 (Continued)

9202	1642	5 18 8 9 7 4 6 14 3 15 16 11 5
8998	1623	5 18 8 9 7 4 15 6 14 3 11 5
8683	1613	5 18 8 9 7 6 14 3 15 11 5
8652	1573	5 18 8 9 7 4 6 14 3 16 11 5
8609	1511	5 1 8 9 7 4 6 14 3 15 18 5
8367	1422	5 8 9 7 4 6 14 3 15 18 5
8052	1408	5 8 9 7 15 3 14 6 18 5
7547	1367	5 18 8 9 7 4 6 14 15 16 11 19 5
7477	1319	5 18 1 8 9 7 4 6 14 15 16 11 5
7273	1304	5 11 15 14 6 4 7 9 8 1 18 5
7235	1223	5 11 16 15 14 6 4 7 9 8 18 5
7031	1208	5 11 15 14 6 4 7 9 8 18 5
6642	1092	5 1 8 9 7 4 6 14 15 18 5
6400	1003	5 8 9 7 4 6 14 15 18 5
6085	989	5 8 9 7 15 14 6 18 5
5535	984	5 18 6 14 7 9 8 5
5514	982	5 18 15 14 7 9 8 5
5067	974	5 18 7 4 6 14 15 5
4956	909	5 18 8 9 7 4 15 16 11 5
4752	894	5 18 8 9 7 4 15 11 5
4692	841	5 8 9 7 15 6 4 18 5
4444	795	5 18 1 8 9 7 4 11 5
4406	753	5 18 8 9 7 4 16 11 5
4202	699	5 11 4 7 9 8 18 5
4129	680	5 18 1 8 9 7 11 5
3887	584	5 18 7 9 8 11 5
3813	564	5 18 4 7 9 8 1 5
3571	475	5 18 4 7 9 8 5
3498	347	5 18 7 9 8 1 5
3256	258	5 18 7 9 8 5
2628	211	5 18 8 9 5
1969	204	5 18 9 5
1954	152	5 18 8 5
1295	56	5 18 5

Table A2. The solutions for OP21

Route Profit	Route Cost	Route
450	44.44	1 12 7 6 5 2 3 4 20 19 18 16 15 17 8 9 10 11 13 14 21
440	43.37	1 7 6 5 2 3 4 20 19 18 16 15 17 8 9 10 11 13 14 21
430	42.46	1 12 7 6 5 3 4 20 19 18 16 15 17 8 9 10 11 13 14 21
420	41.39	1 7 6 5 3 4 20 19 18 16 15 17 8 9 10 11 13 14 21
410	40.98	1 7 6 5 3 4 20 19 18 16 15 17 8 9 10 11 14 21
400	40.05	1 7 6 5 3 4 20 19 18 16 15 17 9 10 11 13 14 21
395	39.78	1 7 6 5 3 4 20 19 18 16 15 17 8 9 10 11 13 21
390	39.63	1 7 6 5 3 4 20 19 18 16 15 17 9 10 11 14 21
385	39.49	1 12 7 6 5 3 4 20 19 18 16 15 17 9 10 11 13 21
380	39.17	1 7 6 5 4 20 19 18 16 15 17 9 10 11 13 14 21

Table A2. The solutions for OP21(Continued)

375	38.43	1 7 6 5 3 4 20 19 18 16 15 17 9 10 11 13 21
365	38.37	1 6 5 3 4 20 19 18 16 15 17 9 10 11 13 21
360	37.84	1 7 6 5 2 3 4 20 19 18 17 8 9 10 11 13 14 21
355	37.56	1 7 6 5 4 20 19 18 16 15 17 9 10 11 13 21
350	36.93	1 12 7 6 5 3 4 20 19 18 17 8 9 10 11 13 14 21
340	35.86	1 7 6 5 3 4 20 19 18 17 8 9 10 11 13 14 21
330	35.45	1 7 6 5 3 4 20 19 18 17 8 9 10 11 14 21
320	34.51	1 7 6 5 3 4 20 19 18 17 9 10 11 13 14 21
310	32.69	1 12 7 6 5 3 2 8 17 16 15 9 10 11 13 14 21
300	31.63	1 7 6 5 3 2 8 17 16 15 9 10 11 13 14 21
290	31.21	1 7 6 5 3 2 8 17 16 15 9 10 11 14 21
285	31.08	1 12 7 6 5 3 2 8 17 16 15 9 10 11 13 21
280	30.44	1 7 6 5 2 8 17 16 15 9 10 11 13 14 21
275	30.01	1 7 6 5 3 2 8 17 16 15 9 10 11 13 21
265	29.85	1 7 6 2 8 17 16 15 9 10 11 13 14 21
260	29.70	1 7 12 2 8 17 16 15 9 10 11 13 14 21
255	28.83	1 7 6 5 2 8 17 16 15 9 10 11 13 21
250	28.15	1 7 6 5 4 20 3 2 8 9 10 11 13 14 21
240	27.74	1 7 6 5 4 20 3 2 8 9 10 11 14 21
235	27.60	1 12 7 6 5 4 20 3 2 8 9 10 11 13 21
230	24.13	1 12 7 6 5 4 3 2 8 9 10 11 13 14 21
220	23.06	1 7 6 5 4 3 2 8 9 10 11 13 14 21
210	22.65	1 7 6 5 4 3 2 8 9 10 11 14 21
205	22.51	1 12 7 6 5 4 3 2 8 9 10 11 13 21
200	19.88	1 12 7 6 5 3 2 8 9 10 11 13 14 21
190	18.81	1 7 6 5 3 2 8 9 10 11 13 14 21
180	18.40	1 7 6 5 3 2 8 9 10 11 14 21
170	17.63	1 7 6 5 2 8 9 10 11 13 14 21
165	17.20	1 7 6 5 3 2 8 9 10 11 13 21
155	17.04	1 7 6 2 8 9 10 11 13 14 21
150	16.89	1 7 12 2 8 9 10 11 13 14 21
145	16.02	1 7 6 5 2 8 9 10 11 13 21
140	15.90	1 7 6 5 3 2 12 11 13 14 21
130	15.22	1 7 12 8 9 10 11 13 14 21
120	14.25	1 12 8 9 10 11 13 14 21
110	13.30	1 7 6 5 12 11 10 14 21
105	13.23	1 7 6 12 11 10 14 13 21
100	11.82	1 7 6 5 12 11 13 14 21
90	10.10	1 7 12 11 10 14 13 21
80	9.03	1 7 12 11 10 14 21
70	7.55	1 7 12 11 13 14 21
60	6.57	1 12 11 13 14 21
50	6.14	1 11 13 14 21
45	5.87	1 12 13 14 21
40	5.73	1 11 14 21
35	4.29	1 13 14 21
25	4.18	1 14 21
10	2.68	1 13 21

Table A3. The solutions for OP32

Route Profit	Route Cost	Route
285	81.83325	1 19 20 27 31 30 26 25 24 23 22 21 12 11 10 9 8 2 3 7 13 6 5 4 14 15 16 17 29 28 18 32
280	78.22436	1 19 20 27 31 30 26 25 24 23 22 21 12 11 10 9 8 2 3 7 6 5 4 14 15 16 17 29 28 18 32
275	75.15988	1 28 29 17 16 15 14 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 30 31 27 20 19 32
270	73.55630	1 28 29 17 16 15 14 4 5 6 7 3 2 8 10 11 12 21 22 23 24 25 26 30 31 27 20 19 32
265	70.73088	1 28 29 17 16 15 14 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 31 27 20 19 32
260	69.12729	1 28 29 17 16 15 14 4 5 6 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
255	67.87824	1 28 17 16 15 14 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 31 27 20 19 32
250	66.27465	1 28 17 16 15 14 4 5 6 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
245	65.08480	1 28 17 16 15 14 4 5 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
240	63.82241	1 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 6 5 4 14 15 16 17 28 32
235	62.63256	1 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 5 4 14 15 16 17 28 32
230	61.49160	1 27 31 26 25 23 22 21 12 11 10 9 8 2 3 7 6 5 4 14 15 16 17 28 32
225	59.88802	1 27 31 26 25 23 22 21 12 11 10 8 2 3 7 6 5 4 14 15 16 17 28 32
220	58.69817	1 27 31 26 25 23 22 21 12 11 10 8 2 3 7 5 4 14 15 16 17 28 32
215	57.54153	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 5 4 14 15 18 32
210	56.39486	1 28 18 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 31 27 20 19 32
205	54.79128	1 28 18 4 5 6 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
200	53.09447	1 28 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 7 4 5 6 13 32
195	51.47582	1 28 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 4 5 6 13 32
190	49.19262	1 28 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 7 6 5 18 32
185	47.58904	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 6 5 18 32
180	46.23056	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 5 6 32
175	44.51179	1 28 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 7 6 32
170	42.90821	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 6 32
165	42.29618	1 28 27 31 26 25 23 22 21 12 11 10 8 2 3 7 5 6 32
160	40.57741	1 28 27 31 26 25 23 22 21 12 11 10 9 8 2 3 7 6 32
155	38.97382	1 28 27 31 26 25 23 22 21 12 11 10 8 2 3 7 6 32
150	38.01586	1 28 27 31 26 25 24 23 22 21 12 11 10 8 9 13 32
145	36.81231	1 28 27 31 26 25 24 23 22 21 12 11 10 9 13 32
140	35.88112	1 27 31 26 25 24 23 22 21 12 11 10 8 9 13 32
135	34.67757	1 27 31 26 25 24 23 22 21 12 11 10 9 13 32
130	33.25029	1 28 27 31 26 25 23 22 21 12 11 10 8 13 32
125	32.88271	1 28 27 31 26 20 21 12 11 10 8 2 3 7 6 32
120	30.74318	1 27 31 26 25 23 22 21 12 11 10 9 13 32
110	29.66690	1 28 27 31 26 25 23 22 21 12 20 19 32
105	27.59313	1 28 27 31 26 22 21 12 11 10 9 13 32
100	27.01490	1 28 27 31 26 25 23 22 21 20 19 32
95	25.77662	1 27 31 26 25 23 22 21 12 19 32
90	24.65207	1 27 31 26 20 21 12 11 10 9 13 32
85	23.66489	1 27 31 26 21 12 11 10 9 13 32
80	22.62657	1 28 27 31 26 22 21 12 19 32
75	21.73011	1 28 27 31 26 22 21 20 19 32
70	20.49183	1 27 31 26 22 21 12 19 32
65	19.59537	1 27 31 26 22 21 20 19 32
60	18.52290	1 27 31 26 22 21 19 32
55	16.39830	1 28 27 31 26 20 19 32
50	16.07236	1 28 27 31 26 20 32
45	14.26356	1 27 31 26 20 19 32

Table A3. The solutions for OP32(Continued)

40	13,93762	1 27 31 26 20 32
35	13,31795	1 27 26 20 19 32
30	12,66983	1 28 27 20 19 32
25	11,89573	1 28 27 19 32
20	10,21758	1 27 28 32
15	6,86652	1 28 18 32
10	4,14257	1 28 32

Table A4. The solutions for OP33

Route Profit	Route Cost	Route
800	96.587694	1 24 25 9 10 18 19 11 30 26 29 12 31 8 2 6 3 13 15 16 21 17 20 4 14 28 5 7 22 27 23 32 33
790	92.648736	1 24 25 9 10 18 19 11 30 26 29 12 31 8 2 6 13 15 16 21 17 20 3 4 14 28 5 7 22 27 23 33
780	90.562161	1 24 25 9 10 18 19 11 30 29 12 31 8 2 6 3 13 15 16 21 17 20 4 14 28 5 7 22 27 23 33
770	89.313188	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 11 19 18 10 9 30 29 26 32 33
760	87.800956	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 29 30 11 19 18 10 9 25 33
750	87.09097	1 24 22 7 5 14 4 20 17 21 16 15 13 3 6 2 8 31 12 11 19 18 10 9 30 29 26 33
740	84.492442	1 24 25 9 10 18 19 11 30 26 29 12 31 8 2 6 3 13 15 16 17 20 4 14 28 5 7 22 27 23 33
730	82.471335	1 24 25 9 10 18 19 11 30 29 12 31 8 2 6 3 13 15 16 17 20 4 14 28 5 7 22 27 23 33
720	81.589049	1 25 9 10 18 19 11 30 29 12 31 8 2 6 3 13 15 16 17 20 4 14 28 5 7 22 24 23 33
710	79.71013	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 29 30 11 19 18 10 9 25 33
700	78.62956	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 11 19 18 10 30 29 26 32 23 33
690	76.988317	1 24 25 9 10 18 19 11 12 31 8 2 6 3 13 15 16 17 20 4 14 28 5 7 22 27 23 33
680	75.717808	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 29 30 11 10 9 25 33
660	73.212922	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 11 18 10 30 29 26 32 23 33
650	70.738358	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 11 10 9 30 29 26 32 23 33
640	69.125037	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 11 30 29 26 32 23 33
630	67.525893	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 11 30 29 26 32 33
620	65.114579	1 24 23 27 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 29 30 26 32 33
610	63.235954	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 29 30 26 32 23 33
600	61.63681	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 29 30 26 32 33
590	60.057061	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 29 26 32 23 33
580	58.457917	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 29 26 32 33
570	57.026375	1 24 22 7 5 28 14 4 20 17 21 16 15 13 3 6 2 8 31 12 29 26 33
560	55.145128	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 29 30 26 32 23 33
550	53.545984	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 29 30 26 32 33
540	51.966235	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 29 26 32 23 33
530	50.367091	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 29 26 32 33
520	48.935549	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 31 12 29 26 33
510	48.144873	1 24 22 7 5 14 4 20 17 16 15 13 3 6 2 8 31 12 29 26 33
500	47.727167	1 24 22 7 5 14 4 20 17 16 15 13 6 2 8 31 12 29 26 32 33
490	47.289638	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 29 26 32 23 33
480	45.690493	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 29 26 32 33
470	44.258952	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 29 26 33
460	42.841787	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 32 23 33
450	41.242643	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 8 32 33
440	40.479983	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 32 23 33
430	38.880839	1 24 22 7 5 28 14 4 20 17 16 15 13 3 6 2 32 33

Table A4. The solutions for OP33(Continued)

420	38.090163	1 24 22 7 5 14 4 20 17 16 15 13 3 6 2 32 33
410	37.661943	1 24 22 7 5 28 14 4 20 17 16 15 13 6 3 23 33
400	35.317936	1 24 22 7 5 28 20 17 16 15 13 3 4 14 27 23 33
390	34.79102	1 24 22 7 5 28 14 4 20 17 16 15 13 3 23 33
380	34.000344	1 24 22 7 5 14 4 20 17 16 15 13 3 23 33
370	33.702295	1 24 22 7 5 14 4 20 17 16 15 13 3 33
360	32.764666	1 24 22 7 5 28 20 17 16 13 3 4 14 27 23 33
350	32.241088	1 24 22 27 14 4 20 17 16 15 13 3 23 33
340	31.640763	1 24 22 7 5 28 20 17 13 6 3 4 14 27 23 33
330	31.256251	1 24 22 7 5 28 20 17 13 6 3 4 14 23 33
320	28.769841	1 24 22 7 5 28 20 17 13 3 4 14 27 23 33
310	28.242925	1 24 22 7 5 28 14 4 20 17 13 3 23 33
300	27.452249	1 24 22 7 5 14 4 20 17 13 3 23 33
290	27.1542	1 24 22 7 5 14 4 20 17 13 3 33
280	25.775957	1 24 22 7 5 28 20 13 3 4 14 27 23 33
270	25.249041	1 24 22 7 5 28 14 4 20 13 3 23 33
260	24.458365	1 24 22 7 5 14 4 20 13 3 23 33
250	23.606682	1 24 22 7 5 28 20 3 4 14 27 23 33
240	23.079766	1 24 22 7 5 28 14 4 20 3 23 33
230	21.691966	1 24 22 7 5 28 20 4 14 27 23 33
220	21.307454	1 24 22 7 5 28 20 4 14 23 33
210	21.017543	1 24 22 5 28 20 4 14 27 23 33
200	19.792061	1 24 22 7 5 28 14 4 3 27 23 33
190	18.566418	1 24 22 7 5 28 14 4 3 23 33
180	15.230463	1 24 22 7 5 28 4 14 27 23 33
170	14.467041	1 24 22 7 5 14 4 27 23 33
160	13.782064	1 24 22 7 5 14 4 23 33
150	13.065388	1 24 22 7 5 14 27 23 33
140	12.680876	1 24 22 7 5 14 23 33
130	12.022808	1 24 22 27 14 4 23 33
120	10.9462	1 24 22 7 5 27 23 33
110	10.766276	1 24 22 7 5 27 33
100	9.9974719	1 24 22 7 27 23 33
90	9.8175483	1 24 22 7 27 33
80	7.1432002	1 24 22 27 23 33
70	6.5876393	1 24 22 23 33
60	5.4237028	1 22 24 33
30	4.7152542	1 24 27 23 33
20	3.4965295	1 24 23 33
10	2.0421677	1 24 33

Table A5. The solutions for OP32-1-

Route Profit	Route Cost	Route
285	81.334087	1 19 20 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 7 13 6 5 4 14 15 16 17 29 30 28 18 32
280	77.725194	1 19 20 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 7 6 5 4 14 15 16 17 29 30 28 18 32
275	74.660714	1 28 30 29 17 16 15 14 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 31 27 20 19 32
270	73.05713	1 28 30 29 17 16 15 14 4 5 6 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
265	70.730878	1 28 29 17 16 15 14 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 31 27 20 19 32
260	69.127294	1 28 29 17 16 15 14 4 5 6 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
255	67.878237	1 28 17 16 15 14 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 31 27 20 19 32
250	66.274654	1 28 17 16 15 14 4 5 6 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
245	65.084802	1 28 17 16 15 14 4 5 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
240	64.201533	1 28 17 15 14 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 31 27 20 19 32
235	62.632557	1 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 5 4 14 15 16 17 28 32
230	61.566164	1 28 17 16 15 14 4 5 6 7 3 2 8 10 11 12 21 22 23 25 26 31 27 19 32
225	59.88802	1 27 31 26 25 23 22 21 12 11 10 8 2 3 7 6 5 4 14 15 16 17 28 32
220	58.698169	1 27 31 26 25 23 22 21 12 11 10 8 2 3 7 5 4 14 15 16 17 28 32
215	57.541531	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 5 4 14 15 18 32
210	56.394861	1 28 18 4 5 6 7 3 2 8 9 10 11 12 21 22 23 24 25 26 31 27 20 19 32
205	54.791278	1 28 18 4 5 6 7 3 2 8 10 11 12 21 22 23 24 25 26 31 27 20 19 32
200	52.864974	1 28 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 7 6 5 4 18 32
195	51.261391	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 6 5 4 18 32
190	49.192619	1 28 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 7 6 5 18 32
185	47.589035	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 6 5 18 32
180	46.230565	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 5 6 32
175	44.511795	1 28 27 31 26 25 24 23 22 21 12 11 10 9 8 2 3 7 6 32
170	42.908211	1 28 27 31 26 25 24 23 22 21 12 11 10 8 2 3 7 6 32
165	42.296177	1 28 27 31 26 25 23 22 21 12 11 10 8 2 3 7 5 6 32
160	40.577407	1 28 27 31 26 25 23 22 21 12 11 10 9 8 2 3 7 6 32
155	38.973823	1 28 27 31 26 25 23 22 21 12 11 10 8 2 3 7 6 32
150	38.015862	1 28 27 31 26 25 24 23 22 21 12 11 10 8 9 13 32
145	36.812312	1 28 27 31 26 25 24 23 22 21 12 11 10 9 13 32
140	35.881122	1 27 31 26 25 24 23 22 21 12 11 10 8 9 13 32
135	34.677572	1 27 31 26 25 24 23 22 21 12 11 10 9 13 32
130	32.877924	1 28 27 31 26 25 23 22 21 12 11 10 9 13 32
125	31.946733	1 27 31 26 25 23 22 21 12 11 10 8 9 13 32
120	30.743183	1 27 31 26 25 23 22 21 12 11 10 9 13 32
110	28.796684	1 28 27 31 26 22 21 12 11 10 8 9 13 32
105	27.911361	1 28 27 31 26 25 23 22 21 12 19 32
100	27.014901	1 28 27 31 26 25 23 22 21 20 19 32
95	25.458394	1 27 31 26 22 21 12 11 10 9 13 32
90	24.880161	1 27 31 26 25 23 22 21 20 19 32
85	23.80769	1 27 31 26 25 23 22 21 19 32
80	22.626571	1 28 27 31 26 22 21 12 19 32
75	21.730111	1 28 27 31 26 22 21 20 19 32
70	20.491831	1 27 31 26 22 21 12 19 32
65	19.595371	1 27 31 26 22 21 20 19 32
60	18.5229	1 27 31 26 22 21 19 32
55	16.398301	1 28 27 31 26 20 19 32
50	16.07236	1 28 27 31 26 20 32
45	14.263561	1 27 31 26 20 19 32
40	13.93762	1 27 31 26 20 32
35	13.317947	1 27 26 20 19 32

Table A5. The solutions for OP32-1- (Continued)

30	12.66983	1 28 27 20 19 32
25	11.895729	1 28 27 19 32
20	10.217585	1 27 28 32
15	6.8665211	1 28 18 32
10	4.1425748	1 28 32

Table A6. The solutions for EIL30

Route Profit	Route Cost	Route
12750	385.86423	1 19 24 11 12 13 16 17 14 8 18 10 9 15 22 20 27 29 28 30 26 25 7 2 6 3 5 4 23 21 1
12650	376.28171	1 19 24 11 12 13 16 17 14 8 18 10 9 15 22 23 4 5 3 6 2 7 25 26 30 29 27 21 20 1
12550	370.45049	1 19 24 11 12 13 16 17 14 8 18 10 9 15 22 23 4 3 6 2 7 25 26 30 29 27 21 20 1
12500	350.08104	1 19 24 11 12 13 16 17 14 8 18 10 9 15 22 20 27 30 26 25 7 2 6 3 5 4 23 21 1
12400	341.11548	1 19 24 11 12 13 16 17 14 8 18 10 9 15 22 21 23 4 3 6 2 7 25 26 30 27 20 1
12350	333.60228	1 19 24 11 12 13 16 17 14 18 10 9 15 22 23 4 5 3 6 2 7 25 26 30 27 21 20 1
12250	327.77106	1 19 24 11 12 13 16 17 14 18 10 9 15 22 23 4 3 6 2 7 25 26 30 27 21 20 1
12150	322.41079	1 19 24 11 12 13 16 17 14 18 10 9 15 22 21 4 3 6 2 7 25 26 30 27 20 1
12125	318.59429	1 19 24 11 12 13 16 17 14 18 10 9 15 22 23 3 6 2 7 25 26 30 27 21 20 1
12025	314.51346	1 19 24 11 12 13 16 17 14 18 10 9 15 22 3 6 2 7 25 26 30 27 21 20 1
11925	311.65174	1 19 24 11 12 13 16 17 14 10 9 15 22 3 6 2 7 25 26 30 27 21 20 1
11875	310.77783	1 19 24 22 15 9 10 13 12 11 14 17 16 27 30 26 25 2 6 3 23 21 20 1
11825	309.54573	1 19 24 22 15 9 10 18 13 12 11 16 27 30 26 25 7 2 6 3 23 21 20 1
11775	305.66071	1 19 24 22 15 9 10 13 12 14 17 16 27 30 26 25 2 6 3 23 21 20 1
11750	298.38275	1 19 24 11 12 13 16 17 14 10 9 15 22 3 6 2 25 26 30 7 4 21 20 1
11650	294.79622	1 19 24 11 16 12 13 18 10 9 15 22 3 6 2 25 26 30 7 4 23 21 20 1
11575	290.98106	1 19 24 11 16 12 13 10 9 15 22 21 23 3 6 2 25 26 30 27 20 1
11550	287.97805	1 19 24 11 16 12 13 10 9 15 22 3 6 2 25 26 30 7 4 23 21 20 1
11450	284.30868	1 19 24 11 16 12 13 10 9 15 22 3 6 2 25 26 30 7 4 21 20 1
11350	281.75621	1 19 24 16 12 13 10 9 15 22 3 6 2 25 26 30 7 4 21 20 1
11300	281.67188	1 19 24 11 16 12 13 10 9 15 22 3 6 2 25 26 30 4 21 20 1
11275	281.55864	1 19 24 11 16 12 13 10 9 15 22 23 3 6 2 25 26 30 21 20 1
11250	280.22956	1 19 24 16 12 13 10 9 15 22 3 5 2 25 26 30 7 4 21 20 1
11175	277.47781	1 19 24 11 16 12 13 10 9 15 22 3 6 2 25 26 30 21 20 1
11075	274.92534	1 19 24 16 12 13 10 9 15 22 3 6 2 25 26 30 21 20 1
10975	273.39869	1 19 24 16 12 13 10 9 15 22 3 5 2 25 26 30 21 20 1
10950	272.52643	1 19 24 16 12 13 9 15 22 3 5 2 25 26 30 7 4 21 20 1
10875	269.77467	1 19 24 11 16 12 13 9 15 22 3 6 2 25 26 30 21 20 1
10775	267.22221	1 19 24 16 12 13 9 15 22 3 6 2 25 26 30 21 20 1
10675	265.69556	1 19 24 16 12 13 9 15 22 3 5 2 25 26 30 21 20 1
10625	264.71492	1 20 21 30 26 25 2 6 3 22 9 13 12 16 24 19 1
10525	263.18828	1 20 21 30 26 25 2 5 3 22 9 13 12 16 24 19 1
10475	262.70806	1 20 30 26 25 2 6 3 22 15 9 13 12 16 24 19 1
10450	262.30094	1 20 21 4 30 26 25 2 6 3 22 15 9 13 12 11 24 19 1
10375	261.18141	1 20 30 26 25 2 5 3 22 15 9 13 12 16 24 19 1
10350	260.77429	1 20 21 4 30 26 25 2 5 3 22 15 9 13 12 11 24 19 1
10325	258.10687	1 20 21 30 26 25 2 6 3 22 15 9 13 12 11 24 19 1
10175	255.59958	1 20 21 30 26 25 2 6 3 22 9 13 12 11 24 19 1
10125	255.27405	1 20 21 30 26 25 2 3 22 15 9 13 12 11 24 19 1
10075	255.07084	1 20 21 30 26 25 2 6 3 22 9 13 12 24 19 1

Table A6. The solutions for EIL30 (Continued)

9975	252.76676	1 20 21 30 26 25 2 3 22 9 13 12 11 24 19 1
9875	251.08543	1 20 30 26 25 2 6 3 22 9 13 12 11 24 19 1
9825	250.7599	1 20 30 26 25 2 3 22 15 9 13 12 11 24 19 1
9775	249.55879	1 20 30 26 25 2 5 3 22 9 13 12 11 24 19 1
9675	248.25262	1 20 30 26 25 2 3 22 9 13 12 11 24 19 1
9225	236.76308	1 19 24 22 21 23 4 3 6 2 7 25 26 30 27 20 1
9200	234.93793	1 20 27 30 26 25 2 7 5 6 3 23 21 22 24 19 1
9100	227.58631	1 20 27 30 26 25 7 2 6 3 23 21 22 24 19 1
9025	225.33986	1 20 21 23 4 5 7 30 26 25 2 6 3 22 24 19 1
9000	225.07311	1 20 27 30 26 25 2 7 5 3 23 21 22 24 19 1
8975	224.10715	1 20 27 30 26 25 2 6 3 4 21 22 24 19 1
8950	218.59976	1 20 27 30 26 25 2 6 3 23 21 22 24 19 1
8925	215.59675	1 20 21 23 4 7 30 26 25 2 6 3 22 24 19 1
8825	211.92738	1 20 21 4 7 30 26 25 2 6 3 22 24 19 1
8700	210.87012	1 20 21 7 30 26 25 2 6 3 22 24 19 1
8675	209.29058	1 20 21 4 30 26 25 2 6 3 22 24 19 1
8650	209.17734	1 19 24 22 23 3 6 2 25 26 30 21 20 1
8625	209.09456	1 19 24 22 3 2 25 26 30 7 4 21 20 1
8575	207.76394	1 19 24 22 3 5 2 25 26 30 4 21 20 1
8550	205.09651	1 19 24 22 3 6 2 25 26 30 21 20 1
8450	203.56986	1 19 24 22 3 5 2 25 26 30 21 20 1
8350	202.26369	1 19 24 22 3 2 25 26 30 21 20 1
8250	200.58236	1 20 30 26 25 2 6 3 22 24 19 1
8150	199.05572	1 20 30 26 25 2 5 3 22 24 19 1
8100	197.25523	1 20 21 30 26 25 2 6 3 22 1
8000	195.72858	1 20 21 30 26 25 2 5 3 22 1
7900	194.42241	1 22 3 2 25 26 30 21 20 1
7800	192.74108	1 22 3 6 2 25 26 30 20 1
7700	191.21443	1 22 3 5 2 25 26 30 20 1
7600	189.90826	1 22 3 2 25 26 30 20 1
7300	189.43731	1 22 3 25 26 30 20 1
7275	188.89561	1 20 27 30 26 25 7 2 6 3 4 23 21 1
7250	187.07046	1 20 27 30 26 25 2 7 5 6 3 23 21 1
7225	185.74028	1 20 27 30 26 25 2 6 3 5 4 23 21 1
7175	185.22623	1 21 4 3 6 2 7 25 26 30 27 20 1
7150	179.71884	1 20 27 30 26 25 7 2 6 3 23 21 1
7100	179.07046	1 20 27 30 26 25 2 5 6 3 23 21 1
7025	176.23968	1 20 27 30 26 25 2 6 3 4 21 1
7000	170.73229	1 20 27 30 26 25 2 6 3 23 21 1
6900	169.20564	1 20 27 30 26 25 2 5 3 23 21 1
6800	167.89947	1 20 27 30 26 25 2 3 23 21 1
6750	167.7487	1 20 7 30 26 25 2 5 3 23 21 1
6725	167.21311	1 20 4 30 26 25 2 5 3 23 21 1
6700	162.19607	1 21 23 3 6 2 25 26 30 20 1
6600	160.66943	1 20 30 26 25 2 5 3 23 21 1
6500	159.36326	1 20 30 26 25 2 3 23 21 1
6400	158.85519	1 20 30 26 25 2 3 21 1
5850	156.12363	1 19 24 22 3 23 21 20 1
5750	155.61556	1 19 24 22 3 21 20 1
5700	155.58107	1 24 22 3 23 21 20 1
5600	155.073	1 20 21 3 22 24 1
5450	153.45495	1 21 23 3 22 24 19 1

Table A6. The solutions for EIL30 (Continued)

5400	148.28235	1 22 3 23 21 20 1
4975	143.54802	1 22 15 9 10 18 14 17 16 13 12 11 24 19 1
4875	141.28633	1 22 15 9 10 13 12 14 17 16 11 24 19 1
4775	138.73386	1 22 15 9 10 13 12 14 17 16 24 19 1
4725	137.08109	1 22 15 9 10 13 12 17 16 11 24 19 1
4675	133.4304	1 22 15 9 10 18 13 12 16 11 24 19 1
4575	126.61223	1 22 15 9 10 13 12 16 11 24 19 1
4475	124.05976	1 22 15 9 10 13 12 16 24 19 1
4325	121.55248	1 22 9 10 13 12 16 24 19 1
4275	118.9091	1 22 15 9 13 12 16 11 24 19 1
4025	113.84935	1 22 9 13 12 16 24 19 1
3875	112.43714	1 22 9 10 13 12 11 24 19 1
3775	111.9084	1 22 9 10 13 12 24 19 1
3725	107.24129	1 22 15 9 13 12 11 24 19 1
3625	106.71255	1 22 15 9 13 12 24 19 1
3575	104.73401	1 22 9 13 12 11 24 19 1
3475	104.20526	1 22 9 13 12 24 19 1
3425	104.19145	1 22 9 13 12 11 24 1
3350	103.72879	1 22 9 12 24 19 1
3325	103.6627	1 22 9 13 12 24 1
3275	103.58113	1 22 9 13 12 11 19 1
3200	103.18623	1 22 9 12 24 1
3150	103.10466	1 22 9 12 11 19 1
3125	100.0993	1 22 13 12 11 24 19 1
3025	99.57056	1 22 13 12 24 19 1
3000	98.197521	1 22 12 11 24 19 1
2900	97.668779	1 19 24 12 22 1
2850	97.65496	1 22 12 11 24 1
2750	97.126218	1 22 12 24 1
2700	97.044642	1 19 11 12 22 1
2650	88.483477	1 19 24 22 21 20 1
2500	87.940916	1 20 21 22 24 1
2350	82.624585	1 19 24 22 20 1
2200	80.642195	1 22 21 20 1
1950	54.230936	1 19 24 22 1
1800	53.688375	1 24 22 1
1650	53.562273	1 19 22 1
1500	46.389654	1 22 1
700	40.616006	1 20 21 1
450	22.903241	1 24 19 1
300	22.36068	1 24 1
150	18.973666	1 19 1

Table A7. The solutions for EIL33

Route Profit	Route Cost	Route
29370	448.10668	1 31 32 2 14 15 16 18 26 27 28 29 30 17 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 5 1
29330	444.25998	1 31 32 2 14 15 16 18 26 27 28 29 30 17 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 5 1
29120	437.38696	1 31 32 2 15 16 18 26 27 28 29 30 17 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 5 1
29080	433.54027	1 31 32 2 15 16 18 26 27 28 29 30 17 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 5 1
28870	422.91531	1 31 32 2 14 15 16 18 26 27 28 29 17 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 5 1
28830	419.06862	1 31 32 2 14 15 16 18 26 27 28 29 17 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 5 1
28670	417.69627	1 31 32 2 14 15 16 18 26 27 30 29 28 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 5 1
28630	413.84957	1 31 32 2 14 15 16 18 26 27 30 29 28 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 5 1
28620	412.1956	1 31 32 2 15 16 18 26 27 28 29 17 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 5 1
28420	406.97655	1 31 32 2 15 16 18 26 27 30 29 28 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 5 1
28380	403.12986	1 31 32 2 15 16 18 26 27 30 29 28 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 5 1
28180	398.58868	1 31 32 2 15 16 18 26 27 28 29 17 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 5 1
28170	395.31592	1 31 32 2 14 15 16 18 26 27 29 28 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 5 1
28130	391.46922	1 31 32 2 14 15 16 18 26 27 29 28 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 5 1
27920	384.5962	1 31 32 2 15 16 18 26 27 29 28 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 5 1
27880	380.74951	1 31 32 2 15 16 18 26 27 29 28 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 5 1
27680	379.5218	1 31 32 2 15 16 18 26 27 29 28 25 24 23 21 22 19 11 10 9 8 7 33 12 13 3 4 5 1
27670	378.00535	1 31 32 2 14 15 16 18 26 27 28 29 17 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 1
27630	374.15866	1 31 32 2 14 15 16 18 26 27 28 29 17 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 1
27480	370.98928	1 31 32 2 15 16 18 26 27 29 28 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 5 1
27430	370.58261	1 31 32 2 14 15 16 18 27 30 29 28 26 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 1
27420	367.90571	1 31 32 2 15 16 18 26 27 28 29 17 25 24 23 21 22 20 19 11 10 9 33 8 7 6 12 13 3 4 1
27220	363.70959	1 31 32 2 15 16 18 27 30 29 28 26 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 1
27180	359.8629	1 31 32 2 15 16 18 27 30 29 28 26 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 1
26980	358.63519	1 31 32 2 15 16 18 27 30 29 28 26 25 24 23 21 22 19 11 10 9 8 7 33 12 13 3 4 1
26970	352.04896	1 31 32 2 14 15 16 18 27 29 28 26 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 1
26930	346.55926	1 31 32 2 14 15 16 18 26 27 29 28 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 1
26720	341.32924	1 31 32 2 15 16 18 27 29 28 26 25 24 23 21 22 20 19 11 10 9 8 7 6 33 12 13 3 4 1
26680	335.83955	1 31 32 2 15 16 18 26 27 29 28 25 24 23 21 22 20 19 11 10 9 8 7 33 12 13 3 4 1
26600	335.36741	1 31 32 2 15 16 18 26 27 29 28 25 24 23 21 22 20 19 11 10 9 8 33 12 13 3 4 1
26480	334.61184	1 31 32 2 15 16 18 26 27 29 28 25 24 23 21 22 19 11 10 9 8 7 33 12 13 3 4 1
26420	333.28922	1 31 32 2 15 16 18 26 27 29 28 25 24 21 23 20 19 11 10 9 8 7 6 33 12 13 3 4 1
26380	331.08552	1 31 32 2 15 16 18 27 29 28 26 25 24 21 23 20 19 11 10 9 8 7 33 12 13 3 4 1
26280	330.92903	1 31 32 2 15 16 18 27 29 28 26 25 24 23 22 20 19 11 10 9 8 7 33 12 13 3 4 1
26200	330.4569	1 31 32 2 15 16 18 27 29 28 26 25 24 23 22 20 19 11 10 9 8 33 12 13 3 4 1
26100	330.23758	1 31 32 2 15 16 18 27 29 28 26 25 24 21 23 19 11 10 9 8 33 12 13 3 4 1
26080	329.70133	1 31 32 2 15 16 18 27 29 28 26 25 24 23 22 19 11 10 9 8 7 33 12 13 3 4 1
26020	328.08332	1 31 32 2 15 16 18 27 29 28 26 25 24 23 20 19 11 10 9 8 7 6 33 12 13 3 4 1
26000	327.58619	1 31 32 2 15 16 18 26 27 29 28 25 24 23 22 19 11 10 9 8 33 12 13 3 4 1
25980	322.59363	1 4 3 13 12 33 7 8 9 10 11 19 20 23 24 25 28 29 27 26 18 16 15 2 32 31 1
25580	319.02646	1 31 32 2 15 16 18 26 27 29 28 25 23 22 20 19 11 10 9 8 7 33 12 13 3 4 1
25380	317.79875	1 31 32 2 15 16 18 26 27 29 28 25 23 22 19 11 10 9 8 7 33 12 13 3 4 1
25320	316.18075	1 4 3 13 12 33 6 7 8 9 10 11 19 20 23 25 28 29 27 26 18 16 15 2 32 31 1
25280	312.33405	1 4 3 13 12 33 7 8 9 10 11 19 20 23 25 28 29 27 26 18 16 15 2 32 31 1
25230	310.28629	1 4 3 13 12 33 7 8 9 10 11 19 20 22 21 23 26 28 29 27 18 16 15 2 32 31 1
25150	309.81415	1 4 3 13 12 33 8 9 10 11 19 20 22 21 23 26 28 29 27 18 16 15 2 32 31 1
25030	309.05858	1 4 3 13 12 33 7 8 9 10 11 19 22 21 23 26 28 29 27 18 16 15 2 32 31 1
24950	308.58645	1 4 3 13 12 33 8 9 10 11 19 22 21 23 26 28 29 27 18 16 15 2 32 31 1
24870	307.57947	1 4 3 13 12 33 6 7 8 9 10 11 19 20 22 23 26 28 29 27 18 16 15 2 32 31 1
24830	303.73277	1 4 3 13 12 33 7 8 9 10 11 19 20 22 23 26 28 29 27 18 16 15 2 32 31 1

Table A7. The solutions for EIL33 (Continued)

24750	303.26064	1 4 3 13 12 33 8 9 10 11 19 20 22 23 26 28 29 27 18 16 15 2 32 31 1
24630	302.50507	1 4 3 13 12 33 7 8 9 10 11 19 22 23 26 28 29 27 18 16 15 2 32 31 1
24570	300.88706	1 4 3 13 12 33 6 7 8 9 10 11 19 20 23 26 28 29 27 18 16 15 2 32 31 1
24530	297.04037	1 4 3 13 12 33 7 8 9 10 11 19 20 23 26 28 29 27 18 16 15 2 32 31 1
24450	296.56823	1 4 3 13 12 33 8 9 10 11 19 20 23 26 28 29 27 18 16 15 2 32 31 1
24300	296.52725	1 4 3 12 33 8 9 10 11 19 20 23 26 28 29 27 18 16 15 2 32 31 1
24250	296.19242	1 4 3 13 12 33 8 9 10 11 19 23 26 28 29 27 18 16 15 2 32 31 1
24050	295.83589	1 3 13 12 33 8 9 10 11 19 20 23 26 28 29 27 18 16 15 2 32 31 1
23980	292.1097	1 4 3 13 12 33 7 8 9 10 11 19 20 23 26 28 29 27 16 15 2 32 31 1
23270	282.70354	1 31 32 2 15 16 18 27 26 23 22 20 19 11 10 9 8 7 6 33 12 13 3 4 1
23070	281.47584	1 31 32 2 15 16 18 27 26 23 22 19 11 10 9 8 7 6 33 12 13 3 4 1
22980	278.92574	1 31 32 2 15 16 27 28 26 23 20 19 11 10 9 8 7 33 12 13 3 4 1
22970	276.01114	1 31 32 2 15 16 18 27 26 23 20 19 11 10 9 8 7 6 33 12 13 3 4 1
22930	272.16445	1 31 32 2 15 16 18 27 26 23 20 19 11 10 9 8 7 33 12 13 3 4 1
22420	271.08047	1 31 32 2 15 16 27 26 23 20 19 11 10 9 8 7 6 33 12 13 3 4 1
22300	266.76164	1 31 32 2 15 16 27 26 23 20 19 11 10 9 8 33 12 13 3 4 1
21930	263.32708	1 31 32 2 15 27 26 23 20 19 11 10 9 8 7 33 12 13 3 4 1
21670	262.10729	1 31 32 15 27 28 26 18 16 2 14 12 33 11 10 9 8 7 6 13 3 4 1
21630	258.96541	1 31 32 2 15 16 18 27 26 20 19 11 10 9 8 7 33 12 13 3 4 1
21230	258.22835	1 31 32 15 27 26 23 20 19 11 10 9 8 7 33 12 13 3 4 1
21180	256.42202	1 31 32 2 15 27 26 18 20 19 11 10 9 8 7 33 12 13 3 4 1
21070	250.41532	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 7 6 13 3 4 1
21030	249.87276	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 7 13 3 4 1
20950	249.77546	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 13 3 4 1
20880	249.50907	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 7 3 4 1
20840	249.44526	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 6 3 4 1
20800	249.35121	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 3 4 1
20780	248.95505	1 31 32 15 27 26 18 16 2 12 33 11 10 9 8 7 13 3 4 1
20700	248.85776	1 31 32 15 27 26 18 16 2 12 33 11 10 9 8 13 3 4 1
20590	248.52755	1 31 32 15 27 26 18 16 2 12 33 11 10 9 8 6 3 4 1
20550	248.4335	1 31 32 15 27 26 18 16 2 12 33 11 10 9 8 3 4 1
20520	246.39597	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 7 6 4 1
20480	246.39571	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 7 4 1
20440	245.98833	1 31 32 15 27 26 18 16 2 14 12 33 11 10 9 8 6 4 1
20320	244.98439	1 31 32 15 27 26 18 16 2 14 12 33 9 10 8 7 6 13 3 4 1
20280	244.44183	1 31 32 15 27 26 18 16 2 14 12 33 9 10 8 7 13 3 4 1
20200	244.34454	1 31 32 15 27 26 18 16 2 14 12 33 9 10 8 13 3 4 1
20090	244.01433	1 31 32 15 27 26 18 16 2 14 12 33 9 10 8 6 3 4 1
20030	243.52412	1 31 32 15 27 26 18 16 2 12 33 9 10 8 7 13 3 4 1
19950	243.42683	1 31 32 15 27 26 18 16 2 12 33 9 10 8 13 3 4 1
19840	243.09662	1 31 32 15 27 26 18 16 2 12 33 9 10 8 6 3 4 1
19800	243.00257	1 31 32 15 27 26 18 16 2 12 33 9 10 8 3 4 1
19770	240.96504	1 31 32 15 27 26 18 16 2 14 12 33 9 10 8 7 6 4 1
19730	240.96478	1 4 7 8 10 9 33 12 14 2 16 18 26 27 15 32 31 1
19690	240.5574	1 4 6 8 10 9 33 12 14 2 16 18 26 27 15 32 31 1
19520	240.04733	1 4 6 7 8 10 9 33 12 2 16 18 26 27 15 32 31 1
19480	240.04707	1 4 7 8 10 9 33 12 2 16 18 26 27 15 32 31 1
19440	239.63969	1 4 6 8 10 9 33 12 2 16 18 26 27 15 32 31 1
19130	239.5448	1 4 3 13 7 8 10 9 33 12 14 18 26 27 15 32 31 1
19120	239.47521	1 4 3 13 6 7 8 9 10 11 33 12 14 2 15 16 27 32 31 1
19080	238.93265	1 4 3 13 7 8 9 10 11 33 12 14 2 15 16 27 32 31 1

Table A7. The solutions for EIL33 (Continued)

19070	237.78709	1 4 6 7 8 10 9 33 12 14 16 18 26 27 15 32 31 1
18670	235.56852	1 4 3 13 6 7 8 9 10 11 33 12 14 2 15 27 32 31 1
18570	235.13019	1 4 6 7 8 9 10 11 33 12 14 2 16 27 15 32 31 1
18370	233.71862	1 4 3 13 6 7 8 10 9 33 12 14 2 16 27 15 32 31 1
18330	233.17606	1 4 3 13 7 8 10 9 33 12 14 2 16 27 15 32 31 1
18140	232.74856	1 4 3 6 8 10 9 33 12 14 2 16 27 15 32 31 1
18120	231.54917	1 4 6 7 8 9 10 11 33 12 14 2 15 27 32 31 1
18080	231.54891	1 4 7 8 9 10 11 33 12 14 2 15 27 32 31 1
18040	231.14153	1 4 6 8 9 10 11 33 12 14 2 15 27 32 31 1
17920	230.13759	1 4 3 13 6 7 8 10 9 33 12 14 2 15 27 32 31 1
17880	229.59503	1 4 3 13 7 8 10 9 33 12 14 2 15 27 32 31 1
17800	229.49774	1 4 3 13 8 10 9 33 12 14 2 15 27 32 31 1
17690	229.16753	1 4 3 6 8 10 9 33 12 14 2 15 27 32 31 1
17630	228.67733	1 4 3 13 7 8 10 9 33 12 2 15 27 32 31 1
17550	228.58003	1 4 3 13 8 10 9 33 12 2 15 27 32 31 1
17490	228.37392	1 4 6 8 10 9 33 12 2 16 27 15 32 31 1
17480	228.31364	1 4 3 7 8 10 9 33 12 2 15 27 32 31 1
17440	228.24983	1 4 3 6 8 10 9 33 12 2 15 27 32 31 1
17400	228.15577	1 4 3 8 10 9 33 12 2 15 27 32 31 1
17370	226.11824	1 4 6 7 8 10 9 33 12 14 2 15 27 32 31 1
17330	226.11798	1 4 7 8 10 9 33 12 14 2 15 27 32 31 1
17290	225.7106	1 4 6 8 10 9 33 12 14 2 15 27 32 31 1
17120	225.20053	1 4 6 7 8 10 9 33 12 2 15 27 32 31 1
16770	224.76921	1 4 6 7 8 9 33 12 14 2 15 27 32 31 1
16730	224.76895	1 4 7 8 9 33 12 14 2 15 27 32 31 1
16690	224.36157	1 4 6 8 9 33 12 14 2 15 27 32 31 1
16630	224.25261	1 4 3 13 7 8 10 9 33 12 14 2 15 16 27 31 1
16550	224.15532	1 4 3 13 8 10 9 33 12 14 2 15 16 27 31 1
16520	223.8515	1 4 6 7 8 9 33 12 2 15 27 32 31 1
16480	223.85124	1 4 7 8 9 33 12 2 15 27 32 31 1
16440	223.44386	1 4 6 8 9 33 12 2 15 27 32 31 1
16420	222.30006	1 31 27 15 2 14 12 33 11 10 9 8 7 6 4 1
16380	222.2998	1 4 7 8 9 10 11 33 12 14 2 15 27 31 1
16340	221.89242	1 4 6 8 9 10 11 33 12 14 2 15 27 31 1
16180	220.34592	1 4 3 13 7 8 10 9 33 12 14 2 15 27 31 1
16070	220.32606	1 4 3 6 7 8 10 9 33 12 14 2 15 27 31 1
15990	219.91842	1 4 3 6 8 10 9 33 12 14 2 15 27 31 1
15930	219.42822	1 4 3 13 7 8 10 9 33 12 2 15 27 31 1
15670	216.86913	1 4 6 7 8 10 9 33 12 14 2 15 27 31 1
15120	207.67262	1 31 32 15 16 2 14 12 33 11 10 9 8 7 6 13 3 4 1
15080	207.13006	1 31 32 15 16 2 14 12 33 11 10 9 8 7 13 3 4 1
14670	202.02069	1 4 3 13 6 7 8 9 10 11 33 12 14 2 15 32 31 1
14630	201.47813	1 4 3 13 7 8 9 10 11 33 12 14 2 15 32 31 1
14550	201.38083	1 4 3 13 8 9 10 11 33 12 14 2 15 32 31 1
14440	201.05063	1 4 3 6 8 9 10 11 33 12 14 2 15 32 31 1
14400	200.95657	1 4 3 8 9 10 11 33 12 14 2 15 32 31 1
14380	200.56042	1 4 3 13 7 8 9 10 11 33 12 2 15 32 31 1
14300	200.46313	1 4 3 13 8 9 10 11 33 12 2 15 32 31 1
14190	200.13292	1 4 3 6 8 9 10 11 33 12 2 15 32 31 1
14150	200.03887	1 4 3 8 9 10 11 33 12 2 15 32 31 1
14120	198.00134	1 4 6 7 8 9 10 11 33 12 14 2 15 32 31 1

Table A7. The solutions for EIL33 (Continued)

14080	198.00108	1 4 7 8 9 10 11 33 12 14 2 15 32 31 1
14040	197.5937	1 4 6 8 9 10 11 33 12 14 2 15 32 31 1
13920	196.58976	1 4 3 13 6 7 8 10 9 33 12 14 2 15 32 31 1
13880	196.0472	1 4 3 13 7 8 10 9 33 12 14 2 15 32 31 1
13800	195.9499	1 4 3 13 8 10 9 33 12 14 2 15 32 31 1
13690	195.6197	1 4 3 6 8 10 9 33 12 14 2 15 32 31 1
13650	195.52564	1 4 3 8 10 9 33 12 14 2 15 32 31 1
13630	195.12949	1 4 3 13 7 8 10 9 33 12 2 15 32 31 1
13550	195.0322	1 4 3 13 8 10 9 33 12 2 15 32 31 1
13440	194.70199	1 4 3 6 8 10 9 33 12 2 15 32 31 1
13370	192.57041	1 4 6 7 8 10 9 33 12 14 2 15 32 31 1
13330	192.57015	1 4 7 8 10 9 33 12 14 2 15 32 31 1
13290	192.16277	1 4 6 8 10 9 33 12 14 2 15 32 31 1
13120	191.6527	1 4 6 7 8 10 9 33 12 2 15 32 31 1
13070	191.49892	1 4 3 13 6 7 8 9 10 11 33 12 14 2 32 31 1
13030	190.95636	1 4 3 13 7 8 9 10 11 33 12 14 2 32 31 1
12950	190.85907	1 4 3 13 8 9 10 11 33 12 14 2 32 31 1
12880	190.59267	1 4 3 7 8 9 10 11 33 12 14 2 32 31 1
12840	190.52886	1 4 3 6 8 9 10 11 33 12 14 2 32 31 1
12800	190.43481	1 4 3 8 9 10 11 33 12 14 2 32 31 1
12780	190.03866	1 4 3 13 7 8 9 10 11 33 12 2 32 31 1
12700	189.94136	1 4 3 13 8 9 10 11 33 12 2 32 31 1
12630	189.67497	1 4 3 7 8 9 10 11 33 12 2 32 31 1
12590	189.61116	1 4 3 6 8 9 10 11 33 12 2 32 31 1
12550	189.5171	1 4 3 8 9 10 11 33 12 2 32 31 1
12520	187.47957	1 4 6 7 8 9 10 11 33 12 14 2 32 31 1
12320	186.06799	1 31 32 2 14 12 33 9 10 8 7 6 13 3 4 1
12280	185.52543	1 31 32 2 14 12 33 9 10 8 7 13 3 4 1
12200	185.42814	1 31 32 2 14 12 33 9 10 8 13 3 4 1
12090	185.09793	1 31 32 2 14 12 33 9 10 8 6 3 4 1
12050	185.00388	1 31 32 2 14 12 33 9 10 8 3 4 1
11770	182.04864	1 31 32 2 14 12 33 9 10 8 7 6 4 1
11690	181.641	1 4 6 8 10 9 33 12 14 2 32 31 1
11650	181.52146	1 4 8 10 9 33 12 14 2 32 31 1
11520	181.13094	1 4 6 7 8 10 9 33 12 2 32 31 1
11370	180.65242	1 4 3 13 6 7 8 10 9 33 12 32 31 1
11330	180.10986	1 4 3 13 7 8 10 9 33 12 32 31 1
11250	180.01257	1 4 3 13 8 10 9 33 12 32 31 1
11180	179.74617	1 4 3 7 8 10 9 33 12 32 31 1
11140	179.68236	1 4 3 6 8 10 9 33 12 32 31 1
11100	179.58831	1 4 3 8 10 9 33 12 32 31 1
10840	179.37426	1 4 6 8 9 33 12 2 32 31 1
10820	176.63307	1 4 6 7 8 10 9 33 12 32 31 1
10780	176.63281	1 4 7 8 10 9 33 12 32 31 1
10740	176.22543	1 4 6 8 10 9 33 12 32 31 1
10220	175.28404	1 4 6 7 8 9 33 12 32 31 1
10140	174.8764	1 31 32 12 33 9 8 6 4 1
9780	174.65569	1 4 3 13 7 8 10 9 33 12 14 2 32 1
9700	174.55839	1 4 3 13 8 10 9 33 12 14 2 32 1
9630	174.292	1 4 3 7 8 10 9 33 12 14 2 32 1
9590	174.22819	1 4 3 6 8 10 9 33 12 14 2 32 1

Table A7. The solutions for EIL33 (Continued)

9550	174.13413	1 4 3 8 10 9 33 12 14 2 32 1
9530	173.73798	1 4 3 13 7 8 10 9 33 12 2 32 1
9450	173.64069	1 4 3 13 8 10 9 33 12 2 32 1
9340	173.31048	1 4 3 6 8 10 9 33 12 2 32 1
9300	173.21643	1 4 3 8 10 9 33 12 2 32 1
9270	171.1789	1 4 6 7 8 10 9 33 12 14 2 32 1
9230	171.17864	1 32 2 14 12 33 9 10 8 7 4 1
9190	170.77126	1 32 2 14 12 33 9 10 8 6 4 1
9150	170.65171	1 32 2 14 12 33 9 10 8 4 1
9020	170.26119	1 32 2 12 33 9 10 8 7 6 4 1
8940	169.85355	1 32 2 12 33 9 10 8 6 4 1
8870	169.78268	1 32 12 33 9 10 8 7 6 13 3 4 1
8830	169.24012	1 32 12 33 9 10 8 7 13 3 4 1
8750	169.14282	1 32 12 33 9 10 8 13 3 4 1
8680	168.87643	1 32 12 33 9 10 8 7 3 4 1
8640	168.81262	1 32 12 33 9 10 8 6 3 4 1
8600	168.71856	1 32 12 33 9 10 8 3 4 1
8430	168.50778	1 32 12 33 9 10 8 7 13 3 1
8320	165.76333	1 32 12 33 9 10 8 7 6 4 1
8280	165.76307	1 32 12 33 9 10 8 7 4 1
8240	165.35569	1 32 12 33 9 10 8 6 4 1
7920	150.17296	1 3 13 12 33 11 10 9 8 7 6 4 1
7880	150.1727	1 3 13 12 33 11 10 9 8 7 4 1
7840	149.76532	1 3 13 12 33 11 10 9 8 6 4 1
7800	149.64578	1 3 13 12 33 11 10 9 8 4 1
7650	149.6048	1 3 12 33 11 10 9 8 4 1
7400	149.532	1 13 12 33 11 10 9 8 4 1
7320	149.04986	1 3 13 12 33 11 9 8 7 6 4 1
7240	148.64222	1 3 13 12 33 11 9 8 6 4 1
7200	148.52267	1 3 13 12 33 11 9 8 4 1
7170	144.74203	1 3 13 12 33 9 10 8 7 6 4 1
7130	144.74177	1 3 13 12 33 9 10 8 7 4 1
7090	144.33439	1 3 13 12 33 9 10 8 6 4 1
7050	144.21485	1 3 13 12 33 9 10 8 4 1
6900	144.17387	1 3 12 33 9 10 8 4 1
6650	144.10107	1 13 12 33 9 10 8 4 1
6570	143.393	1 3 13 12 33 9 8 7 6 4 1
6530	143.39274	1 3 13 12 33 9 8 7 4 1
6490	142.98536	1 3 13 12 33 9 8 6 4 1
6450	142.86582	1 3 13 12 33 9 8 4 1
6300	142.82484	1 3 12 33 9 8 4 1
6050	142.75204	1 13 12 33 9 8 4 1
5900	142.73969	1 12 33 9 8 4 1
5670	140.2699	1 3 13 12 33 8 7 6 4 1
5630	140.21485	1 4 8 7 33 12 13 3 1
5590	139.86226	1 4 6 8 33 12 13 3 1
4570	137.7572	1 3 13 12 7 8 6 4 1
4530	137.63766	1 3 13 12 7 8 4 1
4450	137.60376	1 3 13 12 8 4 1
4200	132.9353	1 31 32 1
2900	131.46373	1 4 31 1

Table A7. The solutions for EIL33 (Continued)

2520	131.15174	1 4 6 7 8 1
2500	115.94826	1 31 1
1600	113.78666	1 4 5 1
1200	96.664368	1 5 1
400	68.876701	1 4 1

Table A8. The solutions for EIL51

Route Profit	Route Cost	Route
777	442.7971	1 47 6 39 12 33 2 28 7 49 24 8 44 25 15 26 14 42 41 20 43 45 16 46 34 40 31 35 22 30 21 36 37 4 29 32 27 9 23 3 17 51 10 50 11 38 18 5 19 48 13 1
772	442.43448	1 47 12 39 6 50 40 31 35 22 51 10 17 3 30 21 36 37 4 29 32 27 9 23 2 33 28 7 49 24 8 44 25 15 26 19 14 42 41 20 43 45 46 34 16 38 18 5 48 13 1
771	437.8986	1 47 6 50 11 40 31 35 22 17 51 10 39 12 33 2 23 3 30 21 36 4 29 32 9 27 8 44 25 24 49 28 7 15 26 19 14 42 41 20 43 45 46 34 16 38 18 5 48 13 1
770	424.76107	1 47 6 50 11 40 31 35 22 17 51 10 39 12 3 30 21 36 37 4 29 32 27 9 23 2 33 28 7 49 24 8 44 25 15 26 14 42 20 43 45 46 34 16 38 18 5 19 48 13 1
764	415.93619	1 47 28 2 33 12 39 10 31 35 51 17 22 30 3 21 36 4 23 29 32 27 9 49 7 24 8 44 25 15 26 19 14 42 20 43 45 16 46 34 40 11 50 6 38 18 5 48 13 1
762	406.72664	1 28 2 33 12 39 6 50 11 40 31 35 22 51 10 17 3 30 21 36 37 4 29 32 27 9 49 7 24 8 44 25 15 26 14 42 20 43 45 46 34 16 38 18 5 19 48 13 47 1
756	400.67107	1 47 28 7 49 24 8 44 25 15 26 19 5 14 42 20 43 45 38 16 46 34 40 31 35 22 51 17 3 21 36 37 4 29 32 27 9 23 2 33 12 39 10 50 6 13 48 1
753	397.59233	1 47 13 6 50 10 39 12 33 2 23 9 27 32 29 4 36 21 3 17 51 22 35 31 40 34 46 16 38 18 45 43 20 42 14 5 48 19 26 15 25 44 8 24 49 7 28 1
748	394.7762	1 28 2 33 12 39 6 50 11 40 31 35 51 10 17 3 30 21 36 4 29 32 9 27 8 44 25 24 49 7 15 26 19 14 42 20 43 45 46 34 16 38 18 5 48 13 47 1
745	390.48818	1 28 2 33 12 17 3 21 36 4 29 32 9 27 8 44 25 24 49 7 15 26 19 14 42 20 43 45 16 46 34 40 31 35 22 51 10 50 39 6 38 18 5 48 13 47 1
742	385.66634	1 28 2 33 12 39 6 50 11 40 31 35 51 10 17 3 21 36 4 29 32 27 9 49 7 24 8 44 25 15 26 19 14 42 20 43 45 46 34 16 38 18 5 48 13 47 1
741	380.36083	1 28 7 49 24 8 44 25 15 26 19 5 14 42 20 43 45 38 16 46 34 40 31 35 51 17 3 30 21 36 4 29 32 9 23 2 33 12 39 10 50 6 13 48 47 1
737	378.63702	1 28 7 49 24 8 44 25 15 26 19 5 14 42 20 43 45 38 16 46 34 40 31 35 51 17 3 21 36 4 29 32 27 9 23 2 33 12 39 10 50 6 13 48 1
730	377.9321	1 7 28 2 33 12 39 6 50 11 40 31 35 51 10 17 3 21 36 4 29 32 9 49 24 8 44 25 15 26 19 14 42 20 43 45 46 34 16 38 18 5 48 13 1
729	371.63812	1 28 7 49 24 8 44 25 15 26 19 5 14 42 20 43 45 16 46 34 40 11 31 35 51 17 3 30 21 36 4 29 32 9 2 33 12 39 10 50 6 13 48 47 1
727	366.78258	1 28 2 33 12 17 3 21 36 4 29 32 9 49 7 24 8 44 25 15 26 19 5 14 42 20 43 45 38 16 46 34 40 31 35 51 10 50 39 6 13 48 47 1
725	366.41557	1 28 2 33 12 17 3 21 36 4 29 32 27 9 49 7 24 8 44 25 15 26 19 5 14 42 20 43 45 16 46 34 40 31 35 51 10 50 39 6 13 48 47 1
720	360.88377	1 28 33 12 17 3 21 36 4 29 32 9 49 7 24 8 44 25 15 26 19 5 14 42 20 43 45 38 16 46 34 40 31 35 51 10 50 39 6 13 48 47 1
717	356.42505	1 47 6 50 10 39 12 33 28 7 25 44 8 24 49 9 32 29 4 36 21 30 3 17 51 35 31 40 34 46 16 45 43 20 42 14 26 15 19 5 48 13 1
714	354.71154	1 28 2 33 12 3 21 36 4 29 32 9 49 8 24 7 15 26 19 5 14 42 20 43 45 38 16 46 34 40 31 35 22 17 51 10 50 39 6 13 48 47 1

Table A8. The solutions for EIL51 (Continued)

710	350.36999	1 47 48 13 6 50 11 40 34 46 16 45 43 20 42 14 19 26 15 25 24 8 27 9 32 29 4 36 21 3 17 51 35 31 10 39 12 33 2 28 49 7 1
707	349.20993	1 28 33 12 39 6 50 11 40 31 35 51 10 17 3 21 36 4 29 32 9 49 8 24 7 15 26 19 14 42 20 43 45 46 34 16 38 18 5 48 13 47 1
702	337.18144	1 28 2 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 19 14 42 20 43 45 46 34 16 38 18 5 48 13 47 1
697	335.93656	1 28 2 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 19 14 42 20 43 45 46 34 16 38 18 5 48 13 1
695	331.28263	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 19 14 42 20 43 45 46 34 16 38 18 5 48 13 47 1
688	327.32798	1 47 28 33 12 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 19 14 42 20 43 45 16 46 34 40 31 35 51 10 50 39 6 13 48 1
686	326.61919	1 47 13 48 18 38 16 34 46 45 43 20 42 14 19 26 15 7 49 24 8 9 32 29 4 36 21 3 17 51 35 31 10 50 6 39 12 33 2 28 1
683	323.35949	1 28 33 12 39 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 19 14 42 20 43 45 16 46 34 40 31 35 51 10 50 6 13 48 1
681	316.95439	1 28 7 24 8 49 9 32 29 4 36 21 3 17 10 51 35 31 40 34 46 16 45 43 20 42 14 26 15 19 48 13 6 50 39 12 33 47 1
671	316.09264	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 19 14 42 20 43 45 46 34 16 38 13 48 1
667	314.93473	1 28 33 12 3 21 36 4 29 32 9 49 8 24 7 15 26 19 14 42 20 43 45 16 46 34 11 50 10 31 35 51 17 39 6 13 48 1
664	313.98828	1 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 19 14 42 20 43 45 46 34 16 38 18 48 13 47 1
663	313.16432	1 33 12 39 6 50 10 31 35 51 17 3 21 36 37 4 29 32 9 27 8 24 49 28 7 15 26 14 42 20 43 45 16 38 18 5 19 48 13 1
662	304.88319	1 28 2 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 14 42 20 43 45 16 38 18 5 19 48 13 47 1
655	303.56619	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 19 14 42 20 43 45 16 38 18 5 48 13 47 1
647	301.14923	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 19 14 42 20 43 45 38 18 5 48 13 1
645	292.97062	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 14 42 20 43 45 38 18 5 19 48 13 47 1
636	289.62139	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 8 24 49 7 15 26 14 42 20 43 45 18 5 19 48 13 47 1
629	286.88721	1 28 33 12 39 6 50 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 14 42 20 43 45 38 18 5 19 48 13 1
624	283.55297	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 14 42 20 43 5 19 48 13 47 1
619	282.58251	1 47 33 12 39 6 50 31 35 51 17 3 21 36 4 29 32 9 8 24 49 7 15 26 14 42 20 43 45 38 18 5 19 48 13 1
618	279.64138	1 28 2 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 14 42 20 5 19 48 13 47 1
610	275.96457	1 47 13 48 19 5 20 42 14 26 15 7 24 8 49 9 32 29 4 36 21 30 3 17 51 35 31 10 50 6 39 12 33 28 1
606	274.35867	1 28 33 12 39 6 50 31 35 51 17 3 21 36 4 29 32 9 8 24 49 7 15 26 14 42 20 43 5 19 48 13 47 1
602	267.43138	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 27 8 24 49 7 15 26 14 42 5 19 48 13 47 1
595	262.58971	1 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 14 42 5 19 48 13 47 1
578	258.30206	1 33 12 39 6 50 31 35 51 17 3 21 36 4 29 32 9 8 24 49 7 15 26 14 42 20 5 19 48 13 47 1
575	255.09862	1 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 14 42 5 19 48 13 1
569	251.7157	1 47 33 12 39 6 50 31 35 51 17 3 21 36 4 29 32 9 49 8 24 7 15 26 14 42 5 19 48 13 1
563	247.0328	1 33 12 39 6 50 10 31 35 51 17 3 21 4 29 32 9 49 8 24 7 15 26 14 42 5 19 48 13 47 1
560	245.1679	1 47 28 33 12 39 6 50 10 31 35 51 17 3 21 36 4 29 32 9 49 7 15 26 14 42 5 19 48 13 1

Table A8. The solutions for EIL51 (Continued)

550	243.7326	1 47 33 12 39 6 50 10 31 35 51 17 3 21 4 29 32 9 49 7 15 26 14 42 20 43 5 19 48 13 1
549	240.7966	1 47 33 12 39 6 50 31 35 51 17 3 21 36 4 29 32 9 49 28 7 15 26 14 42 5 19 48 13 1
543	238.79986	1 47 33 12 39 6 50 31 35 51 17 3 21 4 29 32 9 49 8 24 7 15 26 14 42 19 48 13 1
535	233.26861	1 28 33 12 39 6 50 31 35 51 17 3 21 36 4 29 32 9 49 7 15 26 14 42 19 48 13 1
534	232.71929	1 47 33 12 39 6 50 10 31 35 51 17 3 21 4 29 32 9 49 28 7 15 26 14 42 19 48 13 1
528	227.611	1 47 33 12 39 6 50 10 31 35 51 17 3 21 4 29 32 9 49 7 15 26 14 42 5 19 48 13 1
514	226.87765	1 47 12 39 6 50 31 35 51 17 3 21 4 29 32 9 8 24 7 15 26 14 42 19 48 13 1
513	223.05339	1 47 12 39 6 50 31 35 51 17 3 21 36 4 29 32 9 49 7 15 26 14 42 19 48 13 1
511	222.45613	1 47 12 39 6 50 31 35 51 17 3 21 4 29 32 9 49 28 7 15 26 14 42 19 48 13 1
508	220.79259	1 13 48 19 14 26 15 7 49 9 32 29 4 36 21 3 17 51 35 31 50 6 39 12 33 28 1
507	218.54225	1 47 12 39 6 50 10 31 35 51 17 3 21 4 29 32 9 8 24 49 28 7 15 26 19 48 13 1
493	209.92902	1 47 33 12 39 6 50 31 35 51 17 3 21 4 29 32 9 49 8 24 7 15 26 19 48 13 1
471	205.41212	1 47 6 50 31 35 51 17 3 21 4 29 32 9 49 7 15 26 14 42 5 19 48 13 1
465	199.63184	1 47 13 48 19 26 15 7 49 9 32 29 4 21 3 17 51 35 31 50 6 12 33 2 28 1
459	195.60747	1 13 48 19 26 15 7 49 9 29 4 36 21 3 17 51 35 31 50 6 39 12 33 1
457	195.25544	1 28 33 12 3 17 51 35 31 10 50 39 6 38 45 43 20 42 14 26 15 19 48 13 47 1
441	189.17203	1 28 33 12 3 17 51 35 31 50 39 6 38 45 43 20 42 14 26 15 19 48 13 1
430	183.79534	1 47 12 3 17 51 35 31 10 50 39 6 38 45 43 20 42 14 26 15 19 48 13 1
425	182.46343	1 47 33 12 3 17 51 35 31 10 50 39 6 13 48 19 42 14 26 15 7 28 1
423	181.23005	1 28 7 15 26 14 42 5 19 48 13 6 39 50 31 35 51 17 3 12 33 47 1
419	177.93989	1 47 13 48 19 26 15 7 49 9 29 4 21 3 17 51 35 31 10 39 12 33 1
416	177.73007	1 33 12 3 17 51 35 31 50 6 38 45 43 20 42 14 26 15 19 48 13 47 1
411	176.48519	1 33 12 3 17 51 35 31 50 6 38 45 43 20 42 14 26 15 19 48 13 1
407	174.99032	1 7 15 26 14 42 5 19 48 13 6 39 50 10 31 35 51 17 3 12 47 1
401	170.39204	1 47 6 50 31 35 51 17 3 21 4 29 9 49 7 15 26 19 48 13 1
389	167.19946	1 28 49 7 15 26 14 19 48 13 6 50 31 35 51 17 3 12 33 47 1
387	160.6282	1 33 12 3 17 51 35 31 10 50 39 6 13 48 19 26 15 7 49 28 1
381	157.24528	1 28 49 7 15 26 19 48 13 6 39 50 31 35 51 17 3 12 33 47 1
371	156.15685	1 13 48 19 26 15 7 49 28 33 3 17 51 35 31 10 50 39 12 47 1
369	151.82065	1 28 49 7 15 26 19 48 13 6 39 50 31 35 51 17 3 12 47 1
349	147.29808	1 7 15 26 19 48 13 6 50 31 35 51 17 3 39 12 33 47 1
345	141.49834	1 7 15 26 19 48 13 6 50 10 31 35 51 17 3 12 33 47 1
337	136.88881	1 28 7 15 26 19 48 13 6 50 31 35 51 17 3 12 47 1
322	134.08919	1 47 12 3 17 51 35 31 50 39 6 13 48 19 26 15 1
318	132.22821	1 47 12 3 17 51 10 50 39 6 13 48 19 26 15 7 28 1
314	129.88022	1 7 15 26 19 48 13 6 50 10 35 51 17 3 12 47 1
307	127.64856	1 15 26 19 48 13 6 50 31 35 51 17 3 12 47 1
303	125.78758	1 28 7 15 26 19 48 13 6 50 10 51 17 3 12 47 1
298	125.65683	1 12 39 10 51 35 31 50 6 13 48 19 26 15 7 1
296	124.60636	1 47 12 3 17 39 6 13 48 19 26 15 7 49 28 1
293	122.47387	1 28 7 15 26 19 48 13 6 50 10 17 3 12 47 1
284	117.99831	1 19 48 13 6 39 50 10 31 35 51 17 3 12 47 1
281	116.98231	1 19 48 13 6 50 10 31 35 51 17 3 12 33 47 1
276	115.48307	1 47 33 3 12 39 6 13 48 19 26 15 7 28 1
273	113.15978	1 19 48 13 6 39 50 31 35 51 17 3 12 47 1
257	109.33321	1 19 48 13 6 50 10 35 51 17 3 12 33 1
256	108.37382	1 7 15 26 19 48 13 6 39 12 3 33 1
254	108.31128	1 19 48 13 6 50 10 31 35 51 17 39 12 47 1
252	103.22058	1 28 7 15 26 19 48 13 6 50 39 12 47 1
246	99.85429	1 47 33 12 39 6 13 48 19 26 15 7 28 1
241	98.398678	1 28 7 15 26 19 48 13 6 39 12 33 1
237	97.566946	1 7 15 26 19 48 13 6 50 39 12 47 1

Table A8. The solutions for EIL51 (Continued)

234	94.42966	1 28 7 15 26 19 48 13 6 39 12 47 1
219	88.776029	1 7 15 26 19 48 13 6 39 12 47 1
212	88.250697	1 28 49 24 7 15 26 19 48 13 47 1
205	84.15885	1 47 13 48 5 19 26 15 7 49 28 1
200	82.493944	1 47 6 13 48 19 26 15 7 28 1
191	75.550207	1 28 49 7 15 26 19 48 13 1
183	74.42276	1 28 7 15 26 19 5 48 13 1
179	68.303873	1 28 7 15 26 19 48 13 47 1
174	67.058995	1 28 7 15 26 19 48 13 1
164	62.650242	1 7 15 26 19 48 13 47 1
159	61.405364	1 13 48 19 26 15 7 1
151	61.016039	1 28 7 15 19 48 13 47 1
149	59.063628	1 15 26 19 48 13 47 1
144	57.81875	1 15 26 19 48 13 1
136	55.362408	1 7 15 19 48 13 47 1
131	54.11753	1 7 15 19 48 13 1
125	53.74012	1 28 7 19 48 13 1
121	51.775794	1 15 19 48 13 47 1
115	49.331368	1 7 19 48 13 47 1
109	45.497982	1 47 13 48 5 19 1
104	44.253104	1 13 48 5 19 1
100	38.134217	1 47 13 48 19 1
95	36.889339	1 13 48 19 1
71	34.962644	1 47 48 19 1
66	32.261062	1 48 19 1
59	24.741117	1 48 13 47 1
54	23.496239	1 48 13 1
34	17.369394	1 13 47 1
29	16.124515	1 13 1
15	16	1 28 1
5	4.472136	1 47 1

Table A9. The solutions for EIL76

Route Profit	Route Cost	Route
1364	558,69987	1 27 68 35 47 53 28 5 46 30 49 31 3 75 29 63 74 34 2 44 64 17 4 45 33 51 26 56 19 25 50 24 57 42 43 65 23 62 22 48 37 70 72 61 71 21 38 6 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 32 11 59 73 40 10 41 13 18 52 7 69 76 1
1357	545,35283	1 27 68 35 47 53 28 46 30 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 51 19 25 50 4 45 33 10 40 73 59 13 41 18 52 17 64 34 74 2 44 24 57 42 43 65 23 63 75 29 62 22 48 37 70 72 61 71 21 38 6 49 31 3 7 69 76 5 1
1349	539,14696	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 19 51 33 10 40 73 59 13 41 18 52 7 69 31 3 63 74 2 34 64 17 4 45 25 50 24 57 42 44 43 65 23 62 29 75 22 48 37 72 61 71 21 38 58 16 6 49 30 46 5 76 1
1348	536,41506	1 27 68 35 47 53 28 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 59 11 32 40 10 26 19 51 33 45 4 41 13 18 52 7 34 64 17 50 25 24 57 42 65 43 44 2 74 63 23 62 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1345	534,20018	1 27 68 35 47 53 28 46 30 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 11 59 13 41 18 52 17 4 45 33 10 40 32 26 51 19 25 57 42 44 43 65 23 2 64 34 74 63 29 62 22 48 37 70 72 61 71 21 38 6 49 31 75 3 7 69 5 76 1

Table A9. The solutions for EIL76 (Continued)

1343	528,68262	1 27 68 35 47 53 28 46 30 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 32 11 59 73 40 10 26 51 19 25 50 4 45 33 41 13 18 52 17 64 34 74 2 44 57 42 43 65 23 63 29 62 22 48 37 72 61 71 21 38 6 49 31 75 3 7 69 5 76 1
1337	527,28155	1 27 68 35 47 53 28 46 30 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 51 19 25 4 45 33 10 40 73 59 13 41 18 52 17 64 34 74 63 2 24 57 42 44 43 65 23 29 75 22 48 37 70 72 61 71 21 38 6 49 31 3 7 69 5 76 1
1336	526,7812	1 27 68 35 47 53 28 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 51 19 25 50 17 4 45 33 10 40 73 59 13 41 18 52 7 69 31 3 63 74 2 34 64 24 57 42 44 43 65 23 29 62 22 48 37 61 71 21 38 6 49 30 46 5 76 1
1334	523,03017	1 27 68 35 47 53 28 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 59 11 32 26 10 40 73 13 41 18 52 7 34 64 17 4 45 33 51 19 25 50 24 57 42 44 43 65 23 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1326	520,89483	1 27 68 35 47 53 28 58 14 55 20 9 36 8 54 15 60 67 12 39 59 11 32 40 10 26 51 19 25 4 45 33 41 13 18 52 17 64 24 57 42 44 43 65 23 2 34 74 63 29 62 22 48 37 72 61 71 21 38 6 49 30 46 31 75 3 7 69 5 76 1
1324	517,71374	1 27 68 35 47 53 28 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 11 59 73 40 10 26 51 19 25 50 4 45 33 41 13 18 52 7 34 17 64 24 57 42 44 43 65 23 62 29 63 2 74 3 31 75 22 49 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1323	513,97763	1 27 68 35 47 53 28 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 51 19 25 4 45 33 10 40 73 59 13 41 18 52 17 64 34 74 2 44 57 42 43 65 23 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 7 76 1
1321	513,7805	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 59 11 32 26 51 19 25 4 45 33 10 40 13 41 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 23 62 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1315	511,38269	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 67 12 39 11 59 73 40 10 26 19 51 33 45 4 41 13 18 52 7 69 3 34 64 17 50 25 24 57 42 65 43 44 2 74 63 23 62 29 75 31 49 22 48 37 72 61 71 21 38 58 16 6 30 46 5 76 1
1314	507,45147	1 27 68 35 47 53 28 58 14 55 20 9 36 8 54 15 60 12 67 66 39 59 11 32 40 10 26 51 19 25 4 45 33 41 13 18 52 17 64 34 74 2 44 57 42 43 65 23 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 7 76 1
1312	503,03648	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 51 19 25 50 17 4 45 33 10 40 73 59 13 41 18 52 7 34 64 24 57 42 44 43 65 23 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1306	501,41998	1 27 68 35 47 53 28 16 58 14 55 20 9 36 8 54 15 60 12 67 66 39 59 11 32 26 51 19 25 4 45 33 10 40 13 41 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 76 1
1303	500,84081	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 51 19 25 4 45 33 10 40 73 59 13 41 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 70 72 61 71 21 38 6 30 46 5 69 76 1
1300	494,53867	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 51 19 25 50 17 4 45 33 10 40 73 59 13 41 18 52 7 69 3 34 64 24 57 42 65 43 44 2 74 63 29 75 31 49 22 48 37 72 61 71 21 38 6 30 46 5 76 1
1295	491,64539	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 32 26 51 19 25 4 45 33 10 40 73 59 13 41 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1289	490,67337	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 69 31 3 34 64 17 50 25 24 57 42 65 43 44 2 74 63 23 62 29 75 22 49 48 37 72 61 71 21 38 6 30 46 5 76 1
1282	481,75578	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 59 73 40 10 26 51 19 25 4 45 33 41 13 18 52 7 69 31 3 34 17 64 24 57 42 65 43 44 2 74 63 23 29 75 22 49 48 37 72 61 71 21 38 6 30 46 5 76 1
1278	479,00593	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 59 73 40 10 26 51 19 25 50 4 45 33 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 23 63 29 75 3 31 49 22 48 37 61 71 21 38 6 30 46 5 69 76 1
1276	477,59075	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 67 12 39 11 59 73 40 10 26 51 19 25 4 45 33 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 62 22 48 37 72 61 71 21 38 6 49 30 46 31 75 3 69 5 76 1
1270	467,65748	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 59 73 40 10 26 51 19 25 4 45 33 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1255	463,94326	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 59 73 40 10 26 19 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 23 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1252	460,25649	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 12 67 66 39 11 59 73 40 10 26 51 19 25 4 45 33 41 13 18 52 17 64 34 74 2 44 57 42 43 65 23 63 29 75 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 3 7 76 1

Table A9. The solutions for EIL76 (Continued)

1236	458,38833	1 68 27 18 13 41 10 40 32 11 59 39 66 67 12 60 15 54 8 36 9 47 35 53 28 46 30 49 6 38 21 71 61 72 37 48 22 62 29 63 74 34 64 2 44 43 65 42 57 25 19 51 33 45 4 17 52 7 69 3 31 5 76 1
1230	443,7298	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1212	433,90337	1 68 27 8 36 9 47 35 53 28 14 55 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 64 17 50 25 57 42 65 43 44 2 74 63 29 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 76 1
1208	430,32314	1 68 27 8 36 9 47 35 53 28 14 55 20 15 54 12 67 66 39 11 59 73 40 10 26 51 19 25 4 45 33 41 13 18 52 7 34 17 64 24 57 42 43 44 2 74 63 29 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1192	424,97807	1 27 68 35 47 9 20 36 8 54 15 60 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 1
1186	424,62843	1 27 68 35 47 9 8 36 20 15 54 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 58 16 6 30 46 28 53 5 69 76 1
1185	424,56567	1 27 68 47 8 36 9 20 15 54 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 17 50 25 24 57 42 65 43 44 2 34 74 63 29 75 22 48 37 72 61 71 21 38 6 49 30 46 28 53 35 5 31 3 7 69 76 1
1184	420,90985	1 27 68 35 47 9 8 36 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 2 44 24 57 42 43 65 23 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 69 76 1
1178	419,45023	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 67 12 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 64 24 57 42 65 43 44 2 74 63 3 31 75 29 22 48 37 72 61 71 21 38 6 49 30 46 5 76 1
1175	416,88285	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 60 67 12 39 11 59 73 40 10 26 19 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 44 2 74 63 29 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 76 1
1173	414,30918	1 27 68 35 47 9 36 8 54 15 60 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 69 76 1
1168	408,84486	1 27 68 47 35 28 53 9 8 36 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 76 1
1161	408,02379	1 27 68 35 47 53 28 14 55 20 9 36 8 54 15 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 64 24 57 42 65 43 44 2 63 29 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 76 1
1160	406,97744	1 27 68 35 47 9 36 8 54 15 12 67 66 39 11 59 73 40 10 26 19 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 1
1154	403,95529	1 27 68 35 47 9 36 8 54 15 60 67 12 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 1
1142	396,26543	1 27 68 47 35 28 53 9 8 36 20 15 54 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 69 76 1
1130	388,43885	1 27 68 35 47 8 9 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 1
1119	382,46053	1 27 68 47 35 28 53 9 36 8 54 15 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 43 44 2 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 5 76 1
1115	378,30552	1 27 68 35 47 9 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 1
1101	372,8532	1 27 68 35 47 9 8 54 15 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 43 44 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 1
1099	372,47875	1 27 68 35 47 9 36 8 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 52 17 64 24 57 42 43 44 2 34 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 69 7 1
1089	362,17265	1 68 35 47 9 20 15 54 12 67 66 39 11 59 73 40 10 33 45 4 17 64 24 57 42 43 44 2 34 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 69 7 52 18 41 13 27 1
1073	361,35463	1 27 68 47 9 36 8 54 15 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 44 2 74 63 29 75 3 31 49 22 48 37 21 38 6 30 46 28 53 35 5 76 1
1065	356,60698	1 27 68 35 47 9 36 8 54 15 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 64 24 57 42 44 2 63 29 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 1
1048	353,03282	1 27 68 47 35 28 53 9 36 54 15 60 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 44 2 74 63 29 75 3 31 49 22 48 6 30 46 5 76 1
1044	351,19867	1 68 27 8 47 35 28 53 9 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 44 2 74 63 29 75 3 31 49 22 48 6 30 46 5 76 1

Table A9. The solutions for EIL76 (Continued)

1040	343,17741	1 27 68 35 47 9 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 43 44 2 74 63 29 75 3 31 49 22 48 6 30 46 28 53 5 76 1
1032	341,35757	1 27 68 35 47 9 20 15 54 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 52 7 34 17 64 24 57 42 65 43 44 2 74 63 29 75 3 31 49 22 48 6 30 46 28 53 5 76 1
1022	340,13719	1 27 68 47 35 28 53 9 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 41 13 18 52 7 34 17 64 24 57 42 44 2 63 29 75 3 31 49 22 48 6 30 46 5 69 76 1
1019	336,2957	1 27 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 13 18 52 7 34 17 64 24 57 42 44 2 63 29 75 3 31 49 22 48 37 21 38 6 30 46 28 53 5 76 1
1015	333,83919	1 27 68 47 35 28 53 9 8 36 20 15 54 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 64 24 57 42 44 2 63 29 75 3 31 49 22 48 6 30 46 5 76 1
1012	332,43696	1 27 68 35 47 53 9 20 15 54 12 67 66 39 11 59 73 40 10 26 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 44 2 74 63 29 75 3 31 49 22 48 6 30 46 5 76 1
1009	331,75846	1 27 68 47 35 28 53 9 36 8 54 15 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 17 64 24 57 42 44 2 63 29 3 31 49 22 48 6 30 46 5 76 1
1003	320,98301	1 18 41 13 27 68 5 46 28 53 35 47 9 8 36 20 15 54 12 67 66 39 11 59 73 40 10 33 45 4 17 64 24 57 42 44 2 34 74 63 29 75 22 48 6 49 31 3 7 69 76 1
978	318,70724	1 68 35 47 9 8 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 52 7 34 17 64 2 44 42 43 65 23 29 63 3 31 49 22 48 6 30 46 28 53 5 76 1
975	318,47432	1 27 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 51 33 45 41 13 18 52 7 69 3 34 74 2 44 42 43 65 23 63 29 22 48 6 49 31 46 28 53 5 76 1
974	313,60538	1 27 68 35 47 9 36 8 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 52 7 34 2 63 29 75 3 31 49 22 48 37 61 71 21 38 6 30 46 28 53 5 76 1
973	313,32358	1 27 68 35 47 9 8 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 52 7 34 2 74 63 29 75 3 31 49 22 48 37 72 61 71 21 38 6 30 46 28 53 5 76 1
965	312,08304	1 27 68 35 47 53 9 36 54 15 12 67 66 39 11 59 73 40 10 51 33 45 41 13 18 52 7 34 2 44 42 43 65 23 63 29 75 3 31 49 22 48 6 30 46 5 76 1
960	307,45065	1 27 68 35 47 9 36 8 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 18 52 7 34 2 44 42 43 65 23 29 63 3 31 49 22 48 6 30 46 28 53 5 76 1
956	306,55241	1 27 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 18 52 17 64 34 2 44 42 43 65 23 29 63 3 31 49 22 48 6 30 46 28 53 5 76 1
948	301,43822	1 27 68 35 47 9 36 8 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 52 17 64 34 2 74 63 29 75 3 31 49 22 48 6 30 46 28 53 5 69 7 76 1
930	294,69962	1 27 68 35 47 9 36 20 15 54 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 69 31 3 34 2 74 63 29 22 48 49 6 30 46 28 53 5 76 1
927	287,66358	1 68 27 18 13 41 45 33 10 40 73 59 11 39 66 67 12 54 15 20 9 47 35 53 28 46 30 6 48 22 49 31 3 75 29 63 74 2 34 64 17 52 7 69 5 76 1
908	287,33499	1 76 69 7 3 34 2 74 63 29 22 48 49 6 30 46 31 5 28 53 35 47 9 20 15 54 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 27 68 1
904	281,47461	1 27 68 5 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 51 33 45 41 13 18 52 17 34 2 63 29 22 48 6 49 31 3 7 69 76 1
900	278,23247	1 76 69 7 34 74 63 29 3 31 49 22 48 6 30 46 28 53 35 47 9 36 8 54 15 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 27 68 5 1
889	278,06079	1 76 7 52 17 64 34 3 63 29 22 48 6 49 31 30 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 27 68 5 1
885	275,47011	1 27 68 35 47 53 9 20 15 54 12 67 66 39 11 59 73 40 10 51 33 45 4 41 13 18 52 7 34 74 63 3 31 75 29 22 48 49 6 30 46 5 76 1
881	274,45006	1 27 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 51 33 45 4 41 18 52 17 34 74 63 29 75 3 31 49 22 48 6 30 46 28 53 5 76 1
879	274,03737	1 18 41 13 27 68 35 47 53 9 36 54 15 12 67 66 39 11 59 73 40 10 51 33 45 4 17 64 34 7 69 3 63 29 22 48 6 49 31 46 5 76 1
878	264,899	1 68 27 18 13 41 4 45 33 10 40 73 59 11 39 66 67 12 15 54 8 36 9 47 35 53 28 46 30 6 49 48 22 29 63 74 34 7 69 3 31 5 76 1

Table A9. The solutions for EIL76 (Continued)

849	258,87176	1 27 68 35 47 53 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 13 18 52 7 34 2 63 29 3 31 49 22 48 6 30 46 5 76 1
843	258,79781	1 27 68 35 47 53 9 36 54 15 12 67 66 39 11 59 73 40 10 51 33 45 41 18 52 7 34 74 63 29 3 31 49 22 48 6 30 46 5 76 1
839	258,52054	1 68 35 47 9 20 15 54 12 67 66 39 11 59 73 40 10 33 45 41 13 18 52 17 64 34 7 3 63 29 75 31 49 22 48 6 30 46 5 76 1
832	255,67373	1 27 68 35 47 53 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 18 52 17 64 34 7 3 63 29 22 48 6 49 31 46 5 76 1
827	254,28605	1 27 68 35 47 53 9 36 8 54 15 12 67 66 39 11 59 73 40 10 33 45 41 13 18 7 3 34 74 63 29 22 48 6 49 31 46 5 76 1
825	250,86948	1 76 5 69 7 3 34 74 63 29 22 48 6 49 31 46 28 53 35 47 9 36 54 15 12 67 39 11 59 73 40 10 33 45 4 41 18 27 68 1
821	249,16524	1 68 35 47 53 9 20 15 54 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 52 7 34 3 31 75 29 22 48 49 6 30 46 5 76 1
811	243,65247	1 68 27 18 13 41 45 33 10 40 73 59 11 39 66 67 12 15 54 36 9 47 35 53 28 46 30 6 48 22 29 75 49 31 3 7 69 5 76 1
792	240,5138	1 76 5 69 7 34 3 31 75 29 22 48 49 6 30 46 28 53 35 68 47 9 20 15 54 12 67 66 39 11 59 73 40 10 33 41 18 1
787	233,20621	1 76 5 31 3 34 2 74 63 29 22 48 49 30 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 18 27 68 1
777	228,75638	1 68 27 18 41 4 45 33 10 40 73 59 11 39 66 67 12 15 54 36 9 47 35 53 28 46 30 6 49 48 22 29 75 3 31 5 76 1
761	228,30114	1 27 68 35 47 9 36 54 12 67 66 39 11 59 73 40 10 41 33 45 4 17 64 34 74 63 29 3 31 49 22 48 6 30 46 5 76 1
757	225,36668	1 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 18 52 7 34 74 63 29 75 22 48 49 6 30 46 5 76 1
753	218,13477	1 68 27 13 41 33 10 40 73 59 11 39 66 67 12 15 54 36 9 47 35 53 28 46 30 6 49 48 22 29 63 3 31 5 76 1
738	217,755	1 76 5 31 3 29 22 48 49 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 13 18 27 68 1
735	213,29624	1 76 5 31 3 29 22 48 49 6 30 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 13 27 68 1
714	212,12557	1 76 69 3 63 29 22 48 6 49 31 46 5 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 13 27 68 1
713	209,99273	1 76 69 7 34 74 63 29 22 48 49 30 46 5 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 18 1
710	209,36998	1 76 69 3 63 29 22 48 49 6 30 46 5 28 53 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 41 13 27 68 1
703	209,2564	1 76 5 46 30 6 49 31 3 34 7 52 18 41 45 33 10 40 73 59 11 39 66 67 12 15 54 36 9 53 47 35 68 1
702	207,34179	1 76 5 69 7 34 74 63 29 75 31 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 18 27 68 1
699	204,99318	1 68 27 13 41 33 10 40 73 59 11 39 66 67 12 54 15 20 9 47 35 53 28 46 30 49 31 3 34 7 69 5 76 1
697	202,22487	1 76 5 31 49 22 48 6 30 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 13 27 68 1
679	201,00026	1 76 69 3 63 29 22 48 49 6 30 46 5 68 35 47 53 9 36 54 12 67 66 39 11 59 73 40 10 33 41 18 1
678	200,19591	1 76 69 3 63 29 22 48 49 6 30 46 5 68 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 41 18 27 1
677	199,53271	1 5 76 7 3 31 49 30 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 13 18 27 68 1
674	198,69937	1 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 18 7 34 74 63 29 22 48 49 31 5 76 1
671	197,24789	1 68 35 47 9 20 15 54 12 67 66 39 11 59 73 40 10 33 41 18 52 7 34 74 63 29 3 31 46 5 76 1
655	194,92001	1 76 69 31 3 7 52 18 41 33 10 40 73 59 11 39 66 67 12 15 54 8 9 47 35 53 28 46 5 68 27 1
653	193,64837	1 68 35 47 9 36 54 15 12 39 11 59 73 40 10 33 41 18 52 7 34 74 63 29 3 31 49 30 46 5 76 1
652	191,6341	1 76 69 7 3 31 49 30 46 5 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 18 27 1
644	185,70113	1 76 5 31 49 30 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 18 27 68 1
640	183,65598	1 76 5 31 49 30 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 13 27 68 1
632	182,07172	1 76 69 3 31 5 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 13 27 68 1
628	181,67182	1 76 5 69 3 31 30 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 13 27 68 1
622	180,2042	1 76 5 3 31 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 13 27 68 1
619	178,53599	1 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 18 52 7 3 31 49 30 46 5 76 1
605	171,27725	1 76 5 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 4 41 13 18 27 68 1
588	169,63981	1 76 5 31 46 28 53 35 47 9 8 54 15 12 67 66 39 11 59 73 40 10 33 41 18 27 68 1
583	169,25772	1 76 69 3 31 46 5 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 18 27 68 1
581	168,52783	1 68 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 41 13 18 52 7 69 3 31 46 5 76 1

Table A9. The solutions for EIL76 (Continued)

578	161,53024	1 76 5 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 45 41 18 27 68 1
577	161,4266	1 76 5 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 13 18 27 68 1
563	157,57102	1 76 5 46 28 53 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 45 41 13 18 27 68 1
561	155,75568	1 76 5 46 28 53 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 18 27 68 1
543	153,6788	1 18 41 33 10 40 73 59 11 39 66 67 12 15 54 36 9 47 68 35 53 28 46 5 76 1
523	148,18322	1 27 18 41 33 10 40 73 59 11 39 66 67 12 15 54 36 9 53 47 35 68 5 76 1
522	147,69106	1 18 41 45 33 10 40 73 59 11 39 66 67 12 15 54 36 9 53 47 35 68 5 76 1
520	146,42824	1 76 5 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 13 18 27 68 1
519	145,79012	1 68 27 18 41 33 10 40 73 59 11 39 66 67 12 54 8 47 35 53 28 46 5 76 1
515	144,25772	1 76 5 46 28 53 35 47 9 36 54 15 12 39 11 59 73 40 10 33 41 18 27 68 1
508	142,60562	1 76 5 68 35 47 9 20 15 54 12 67 66 39 11 59 73 40 10 33 45 41 18 1
506	142,57265	1 76 5 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 45 41 13 18 27 68 1
505	141,9165	1 18 41 33 10 40 73 59 11 39 66 67 12 15 54 36 9 53 47 35 68 5 76 1
504	141,11215	1 27 18 41 33 10 40 73 59 11 39 66 67 12 15 54 36 9 47 35 68 5 76 1
500	138,71217	1 76 5 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 13 27 68 1
491	138,06092	1 18 41 45 33 10 40 73 59 11 39 66 67 12 54 36 9 53 47 35 68 5 76 1
490	136,90173	1 76 5 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 45 41 18 27 68 1
486	134,84544	1 76 5 68 35 47 9 36 54 15 12 67 66 39 11 59 73 40 10 33 41 18 1
478	134,72891	1 27 13 41 33 10 40 11 39 66 67 12 15 54 36 9 47 35 68 5 76 1
477	134,22723	1 76 5 68 35 47 9 36 54 15 12 67 39 11 59 73 40 10 33 41 18 1
474	132,28636	1 76 5 68 35 47 53 9 36 54 12 67 66 39 11 59 73 40 10 33 41 18 1
473	131,12717	1 76 5 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 41 18 27 68 1
472	130,98985	1 76 5 68 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 45 41 18 1
464	130,50734	1 76 5 68 35 47 9 36 54 15 12 67 66 39 11 40 10 33 41 18 1
460	128,88868	1 76 5 68 35 47 9 8 54 12 67 66 39 11 59 73 40 10 33 41 18 1
455	125,21529	1 76 5 68 35 47 9 36 54 12 67 66 39 11 59 73 40 10 33 41 18 1
440	123,34747	1 76 5 68 35 47 9 36 54 15 12 39 11 59 73 40 10 33 41 18 1
434	122,02554	1 76 5 68 35 47 9 36 54 12 67 66 39 11 73 40 10 33 41 18 1
433	120,87719	1 76 5 68 35 47 9 36 54 12 67 66 39 11 40 10 33 41 18 1
427	119,30904	1 18 41 10 40 73 59 11 39 66 67 12 54 36 9 47 35 68 5 76 1
416	117,8028	1 76 5 68 35 47 53 9 36 54 12 67 66 39 11 59 13 41 18 1
409	113,71733	1 76 5 68 35 47 9 36 54 12 39 11 59 73 40 10 33 41 18 1
397	110,73173	1 76 5 68 35 47 9 36 54 12 67 66 39 11 59 13 41 18 1
382	108,86391	1 18 41 13 59 11 39 12 15 54 36 9 47 35 68 5 76 1
379	108,41309	1 68 35 47 53 28 46 30 6 49 48 22 29 63 3 31 5 76 1
372	108,12621	1 18 41 13 59 11 39 12 54 15 9 47 35 68 5 76 1
363	105,99047	1 27 68 35 47 53 28 46 30 6 49 48 22 29 75 31 5 76 1
361	103,57456	1 76 5 31 3 29 22 48 49 6 30 46 28 53 47 35 68 1
351	102,54346	1 18 41 13 27 68 35 47 53 28 46 30 49 31 3 5 76 1
346	100,68505	1 27 59 11 39 66 67 12 54 36 9 47 35 68 5 76 1
341	97,250008	1 68 35 47 53 28 46 30 6 48 22 29 3 31 5 76 1
340	96,198416	1 68 35 47 53 28 46 30 49 48 22 29 3 31 5 76 1
328	94,539091	1 68 35 47 53 28 46 30 49 48 22 29 3 69 5 76 1
320	93,285881	1 68 35 47 53 28 46 30 49 31 3 34 7 69 5 76 1
318	92,706794	1 68 35 47 53 28 46 30 49 48 22 29 3 5 76 1
312	90,379369	1 18 41 13 27 68 35 47 53 28 46 49 31 5 76 1
306	86,728639	1 68 35 47 53 28 46 30 6 48 22 49 31 5 76 1
301	85,903816	1 76 5 46 28 53 47 35 68 27 13 41 18 52 7 1
292	82,151015	1 18 41 13 27 68 35 47 53 28 46 31 5 76 1
285	81,96819	1 68 35 47 53 28 46 30 6 49 31 3 5 76 1
282	80,946494	1 27 68 35 47 53 28 46 30 49 31 3 5 76 1
278	77,784368	1 68 35 47 53 28 46 30 6 48 49 31 5 76 1

Table A9. The solutions for EIL76 (Continued)

274	76,42434	1 68 35 47 53 28 46 30 49 31 3 69 5 76 1
270	71,94028	1 76 5 46 28 53 47 35 68 27 13 41 18 1
254	70,735711	1 76 5 46 28 53 47 35 68 27 41 18 1
241	66,54027	1 68 35 47 53 28 46 31 3 69 5 76 1
238	64,083665	1 68 35 47 53 28 46 30 49 31 5 76 1
232	63,829816	1 18 41 13 27 68 35 47 53 5 76 1
227	63,781461	1 27 68 35 47 9 53 28 46 5 69 76 1
225	62,427949	1 68 35 47 53 28 46 49 31 5 76 1
223	60,554046	1 27 68 35 47 53 28 46 31 5 76 1
218	59,609473	1 68 35 47 53 28 46 30 31 5 76 1
214	56,956328	1 27 68 35 47 53 28 30 46 5 76 1
211	56,710393	1 27 68 35 47 53 28 46 5 69 76 1
205	54,199594	1 68 35 47 53 28 46 31 5 76 1
201	50,343312	1 27 68 35 47 53 28 46 5 76 1
184	48,761354	1 27 68 35 47 53 46 5 76 1
183	43,98886	1 68 35 47 53 28 46 5 76 1
166	42,406902	1 68 35 47 53 46 5 76 1
164	42,367627	1 68 47 53 28 46 5 76 1
163	42,232847	1 27 68 35 47 53 5 76 1
162	40,564544	1 68 35 47 53 28 5 76 1
147	39,077465	1 68 47 35 46 5 76 1
145	35,878395	1 68 35 47 53 5 76 1
144	35,431917	1 27 68 47 35 5 76 1
126	29,077465	1 68 47 35 5 76 1
107	28,989874	1 76 5 47 68 1
106	27,763368	1 5 35 47 68 1
99	26,226562	1 76 5 35 68 1
80	20,77033	1 76 5 68 1
60	19,456233	1 68 5 1
50	15,456233	1 76 5 1
30	10,77033	1 68 1
20	6	1 76 1