# STUDY OF $D_{SJ}(2317)$ AND $D_{SJ}(2460)$ MESON PROPERTIES WITHIN THE QUARK MODEL AND QCD SUM RULES

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**STUDY OF THE $D_{sJ}$(2317) AND $D_{sJ}$(2460) MESON PROPERTIES WITHIN THE QUARK MODEL AND QCD SUM RULES**

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# ABSTRACT

STUDY OF $D_{SJ}(2317)$ AND $D_{SJ}(2460)$ MESON PROPERTIES WITHIN THE QUARK MODEL AND QCD SUM RULES

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The recently discovered $D_{sJ}(2317)$ and $D_{sJ}(2460)$ mesons had stimulated many theoretical and experimental studies due to their unexpected properties. In this thesis, we make a review of the predictions on the properties of these mesons using the quark model and QCD Sum Rules. We studied different models about the structure of these mesons, which are suggested because of their unexpected properties. Moreover, using the quark model which implies that the structure of $D_{sJ}$ meson as $c\bar{s}$ and QCD Sum Rules method, we investigated the semileptonic decay $D_{sJ}(2317) \to D_0 \ell \nu$.

Keywords: $D_{sJ}(2317)$ meson , $D_{sJ}(2460)$meson , QCD Sum Rules, Quark model.

# ÖZ

$D_{SJ}(2317)$ VE $D_{SJ}(2460)$ MEZONLARININ ÖZELLİKLERİNİN KUVARK MODELİ VE QCD TOPLAM KURALLARI İLE İNCELENMESİ

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Son yıllarda keşfedilen $D_{sJ}(2317)$ ve $D_{sJ}(2460)$ mezonları beklenmeyen özelliklerinden dolayı pek çok teorik ve deneysel çalısmalara konu olmuştur. Bu tezde, bu mezonların iç yapıları hakkındaki kuvark modeli ve QCD toplam kurallarının tahminlerinin bir derlemesini yaptık. Söz konusu mezonların beklenmeyen özelliklerinden dolayı önerilen değişik modelleri inceledik. Buna ek olarak, $D_{sJ}$ mezonunun iç yapısını $c\bar{s}$ olarak öneren kuvark modeli ve QCD toplam kurallarını kullanarak yarı-leptonik $D_{sJ}(2317) \rightarrow D_0 \ell \nu$ bozunmasını inceledik.

Anahtar Kelimeler: $D_{sJ}(2317)$ mezonu , $D_{sJ}(2460)$ mezonu, QCD Toplam Kuralları, Kuvark model.

*To my mother*

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# CHAPTER 1

# INTRODUCTION

In 1934, Yukawa proposed the first significant theory of the strong force. He claimed that in the nucleus proton and neutron are attracted by a field [1]. Like photon in Quantum Electrodynamics, there is a particle mediating the strong force. Yukawa calculated its mass and found that its mass should be nearly 300 times the mass of the electron or about 1/6 of the mass of a proton. This particle is called meson which means middle-weight. However, after the discovery of mesons whose masses are greater than a proton, it was understood that not all the meson masses are between electron and proton. In 1964, Gell-Mann and Zweig argued that the hadrons are actually composed of more elementary constituents, called "quarks". After defining the quark content, one can define the mesons as bound states of strongly interacting quarks and anti-quarks.

In understanding the nature of the strong interaction, the mesons consisting of one heavy and one light quark play an important role [2]. This situation became more clear by the discoveries of the mesons $D_{sJ}(2317)$ and $D_{sJ}(2460)$.

In April 2003, the BaBar Collaboration at SLAC observed a narrow peak in the $D_s^+\pi^0$ channel denoted as $D_{sJ}(2317)$ [3]. Also they saw a peak near 2460 MeV in the $D_s^+\pi^0\gamma$ channel but they did not declare this peak as a new state. In May 2003, CLEO Collaboration confirmed the BaBar's observation and also observed a second narrow peak in the $D_s^+\pi^0\gamma$ channel denoted as $D_{sJ}(2460)$ [4]. The existence of these two mesons is also confirmed by the Belle Collaboration.

Possible quantum numbers $J^P$ of these two mesons $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are $0^+$ and $1^+$, respectively [3, 4]. These quantum numbers are assigned according to the experimental results. For $D_{sJ}(2317)$, $J^P = 0^+$ is assigned according to the "angular distribution of the meson decay with respect to its direction in the $e^+ - e^-$ centre of mass frame" [5]. Also, BaBar Collaboration and CLEO Collaboration could not find an evidence for the decay of $D_{sJ}(2317)$, namely $D_{s0}$, in the $D_s\gamma$ and $D_s\gamma\gamma$ channel. This is also an evidence of the quantum number of $D_{sJ}(2317)$, since these channels are forbidden for a scalar due to the angular momentum and parity conservation ($D_s$ is a pseudoscalar and $\gamma$ is a vector). Also, the observed channel $D_s^+\pi^0$ supports the assigned quantum number ( $D_s^+$and $\pi^0$ are both pseudoscalar). Similarly, for $D_{sJ}(2460)$, the observed channel, $D_{sJ}(2460) \to D_s^*\pi^0$, supports the assigned quantum number ( $D_s^{*+}$ is a vector and $\pi^0$ is a pseudoscalar). At the same time, the $D_s^+\pi^0$ channel is not observed which is forbidden for an axial-vector particle.

The discovery of these mesons caused speculations about their structure. In quark model calculations [6, 7], it was thought that they are very good candidates for the missing two states of j=1/2 of $c\bar{s}$, where j is the total angular momentum of the s-quark [8]. Most of the quark models explained the reason of absence of these j=1/2 doublets by arguing the broadness of the decay widths. The other two members (j=3/2) $(D_{sJ}(2536), D_{sJ}(2573))$ of the L=1 multiplet have already been observed. However, the expected and measured masses of $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are not consistent. Their measured masses are about 100-150 MeV below the predicted ones. The predicted masses are calculated using the potential-based quark models [6, 7]. For $D_{sJ}(2317)$, the predicted mass range was 2450-2500 MeV [6]. So it was thought that this meson would decay mostly in the DK channel. However, in the experiments, it was realized that its mass is below the threshold of the DK mode. The only observed channel for $D_{s0}$ meson in the BaBar and CLEO Collaboration's experiments is the iso-spin breaking $D_s^+\pi^0$ mode. Similarly, for $D_{sJ}(2460)$, it was expected that it would decay into the $D^*K$ channel. However, its mass is lower than the threshold of the $D^*K$ channel. Moreover, since their measured masses are below the threshold of these channels, they decay into the suppressed channels [8]. So their decay widths are narrower than the expected ones.

As it was mentioned above, prior experimental observations, the masses of the mesons of concern are calculated using the quark models. Most of the quark models [6, 7, 9, 10, 11, 13, 14, 15, 16] give higher values for the masses than the observed ones. Also lattice QCD calculations [14, 15] give higher masses than the measured ones. Since most of the quark models predict the masses of these two new states higher than the measured ones, some new ideas appeared. It is suggested that these two states might be composed of multi-quarks. For $D_{sJ}(2317)$, it is suggested that it could be a DK molecule [17]. It is also suggested that it could be a $D\pi$ atom [18] or a four-quark state [19, 20, 21, 22].

In this thesis, these two new mesons $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are studied. In chapter 2, the Godfery-Isgur potential-based quark model, which is used for the prediction of the mass of these mesons and some other alternatives to the quark model are explained. Chapter 3 deals with QCD Sum Rules method, which can be used for calculating the coupling constant of $D_{sJ}(2317) \rightarrow D_0 \ell \nu$ decay. Finally, in chapter 4, already studied radiative transitions and strong decays of $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are examined. And also some parts of the calculation of the coupling constant of the decay $D_{sJ}(2317) \rightarrow D_0 \ell \nu$ are outlined.

# CHAPTER 2

# GODFREY-ISGUR POTENTIAL-BASED QUARK MODEL AND SOME SUGGESTIONS FOR $\mathbf{D_{sJ}(2317)}$ AND $\mathbf{D_{sJ}(2460)}$

In this chapter, firstly, one of the most important reason,which makes the $D_{sJ}(2317)$ and $D_{sJ}(2460)$ so interesting, namely Godfrey-Isgur potential-based quark model is explained. In the first section, Godrey and Isgur's paper [6] is taken as reference. In the following section, instead of the quark model, some other suggestion for the structure of the mesons of concern are considered.

## 2.1 Godfrey-Isgur Potential-Based Quark Model

After the discovery of the charmonium state, it was understood that heavy-quark systems can be described by nonrelativistic potential models. However, while considering the light-quark systems, it was understood that these nonrelativistic quark models become ineffective. To solve this problem, Godfrey and Isgur [6] stated that all mesons can be described in a unified framework. The meson spectra and meson couplings are calculated within this model.

In this model, a Coulomb+linear potential, like almost all the quark potential models [9, 10, 11, 13, 14, 15, 16], is used. Although some other alternatives were also tried, it was found out that these alternatives are not necessary.

An example for a Coulomb+linear potential is (Cornell potential).

$$V(r) = \frac{a}{r} + br \tag{2.1}$$

The a/r part is the Coulombic part and the br part is the linear part. The

Coulombic part appears in the equation due to one gluon exchange like the photon exchange in QED. QCD is believed to show another phenomenon called confinement. Confinement can be taken into account phenomenologically by a linear potential, hence one includes a linear potential term (br part in this case) in the potential.

In Godfrey-Isgur potential based quark model, the Schrödinger equation is used as the basic equation [6].

$$H|\psi\rangle = (H_0 + V)|\psi\rangle = E|\psi\rangle \tag{2.2}$$

where

$$H_0 = \sqrt{p^2 + m_q^2} + \sqrt{p^2 + m_{\bar{q}}^2} \tag{2.3}$$

And the suggested potential is:

$$V_{ij}(\overrightarrow{p}, \overrightarrow{r}) = H_{ij}^{conf} + H_{ij}^{hyp} + H_{ij}^{so} + H_A \tag{2.4}$$

where $H_{ij}^{conf}$ includes confinement and Coulomb-type interactions, $H_{ij}^{hyp}$ includes the color hyperfine interaction and $H_{ij}^{so}$ contains the spin-orbit interaction with the color magnetic interaction and Thomas-precession terms. And $H_A$ is the annihilation interaction term for $q\bar{q}$, which is unnecessary for the $D_{sJ}$ mesons.

$$H_{ij}^{conf} = -(\frac{3}{4}c + \frac{3}{4}br - \frac{\alpha_s(r)}{r})F_i \cdot F_j \tag{2.5}$$

$$H_{ij}^{hyp} = -\frac{\alpha_s(\vec{r})}{m_i m_j}\left[(\frac{8\pi}{3}\vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}) + \frac{1}{r^3}(\frac{3\vec{S}_i \cdot r\vec{S}_j \cdot \vec{r}}{r^2} - \vec{S}_i \cdot \vec{S}_j)\right]\vec{F}_i \cdot \vec{F}_j \tag{2.6}$$

$$H_{ij}^{so} = H_{ij}^{so(cm)} + H_{ij}^{so(tp)} \tag{2.7}$$

where

$$H_{ij}^{so(cm)} = -\frac{\alpha_s(\vec{r})}{r^3}\left(\frac{1}{m_i} + \frac{1}{m_j}\right)\left(\frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j}\right) \cdot \vec{L}(\vec{F}_i \cdot \vec{F}_j) \tag{2.8}$$

$$H_{ij}^{so(tp)} = -\frac{1}{2r}\frac{\partial H_{ij}^{conf}}{\partial r}(\frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2}) \cdot \vec{L} \tag{2.9}$$

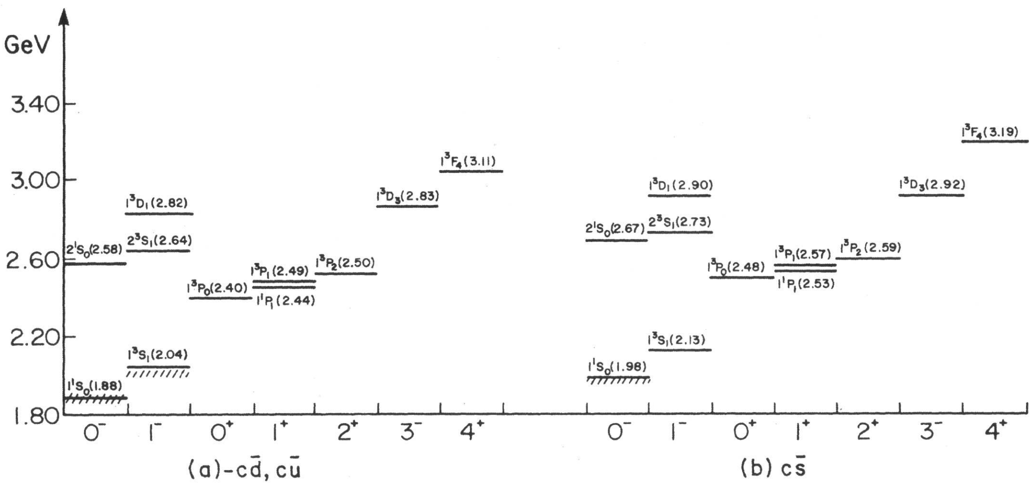


Figure 2.1: Predicted masses of $c\overline{d}, c\overline{u}$ and $c\overline{s}$

for all these equations: $\vec{L} = \vec{r} \times \vec{p}$ , $\alpha_s(\vec{r})$ is the running coupling constant of QCD and

$$F_i = \begin{cases} \frac{\lambda_i}{2} \text{ for quarks} \\ \frac{\lambda_i^c}{2} = -\frac{\lambda_i^*}{2} \text{ for antiquarks} \end{cases} \tag{2.10}$$

are the Gellmann matrices.

The predicted masses of $c\overline{u}$ and $c\overline{d}$ states are very precise. For example, the measured masses of $D^0$ and $D^+$ mesons, which are the $0^-$ $c\overline{u}$ and $c\overline{d}$ states, are $1864.1 \pm 1$ MeV and $1869.62 \pm 0.2$ MeV, respectively. These values are consistent with the predicted one, which is 1.88 GeV [23]. Also the experimental masses of $1^-$ $c\overline{u}$ and $c\overline{d}$ states are $2006.97 \pm 0.19$ and $2010.27 \pm 0.17$, respectively [24], while the predicted mass is 2.04 GeV. Moreover, the measured masses of $0^+$ and $1^+$ $c\overline{u}$ and $c\overline{d}$ states are $2352 \pm 50$ MeV and $2422.3 \pm 1.3$ MeV, respectively [24] whereas the predicted masses are 2.40 GeV and 2.44 GeV.

As it can be seen in Eq.(2.5), there are only two free parameters in this model. These two parameters b and c can be chosen to fit two masses, which is not a prediction. However, with this model, one can predict more than two masses successfully. In general, the Godfrey-Isgur potential based quark model is a very successful model. Therefore, the inconsistency between the measured and calculated masses of $D_{sJ}(2317)$ and $D_{sJ}(2460)$ makes the situation more interesting. As it is shown in the Fig. (1.1), the predicted masses using this model of $0^+$ and $1^+$ states of $c\bar{s}$ are 2.48GeV and 2.53GeV, compared to the experimental values, 2317 GeV and 2460 GeV. This difference in the predicted and measured values of the masses leads to new ideas about the structure of these two mesons.

### 2.2 Four-Quark State, DK Molecule and D$\pi$ atom

With the motivation of the inconsistency between the measured and predicted mass of $D_{sJ}(2317)$ calculated using potential models, it is suggested that either the potential models are renewed or its quark content should be assumed different than the traditional quark model description, i.e $q\bar{q}$. The four-quark structure for $D_{sJ}(2317)$ was firstly suggested by Lipkin [25]. The difference between the observed and expected masses might be explained by a strong interaction between $qq$ and $\overline{qq}$ in the P-wave four-quark state [19]. From this point of view, the scalar $c\overline{n}n\overline{s}$ ($n = u, d$) can be lighter than the P-wave $c\overline{s}$ state and so it can be below the threshold DK molecule and the dominance of the $D_s^+\pi^0$ channel can be explained.

Another motivation to suggest the four-quark structure is the decay width. According to the results of the CLEO Collaboration [4], the decay widths ratio of electroweak and iso-spin violating channels is:

$$\frac{\Gamma(D_{s0}^+(2317) \longrightarrow D_s^{*+}\gamma)}{\Gamma(D_{s0}^+(2317) \longrightarrow D_s^+\pi^0)} < 0.059 \tag{2.11}$$

Actually, electroweak interactions and isospin violating interactions are equally probable. However, as it can be seen from Eq. (2.11), for $D_{s0}^+$, the electroweak decay channel is suppressed with respect to isospin violating one [26]. From the exotic meson point of view, this situation can be explained because of the fact that with this structure, the $D_{s0}^+(2317) \longrightarrow D_s^+\pi^0$ decay becomes a strong decay instead of isospin violating decay. In this explanation, the $D_{s0}^+$ should be an iso-triplet ($I = 1, I_z = 0$). The suggested four-quark structure for $D_{s0}^+$ meson is $(c\overline{n}n\overline{s})^+$ [20].

For the $D_{sJ}(2460)$ meson, the four-quark structure is also suggested with the same motivations [27]. It might also be a P-wave $c\overline{n}n\overline{s}$ (where $n = u, d$) state and has the same problems with $D_{sJ}(2317)$.

Another suggestion for the structure of the $D^+_{sJ}(2317)$ meson is DK molecule as mentioned before [28]where K is a strange, pseudoscalar meson. Hadronic molecules are weakly bounded color-singlet hadrons. There are some signatures for hadronic molecules, which are [29]:

1) $J^{PC}$ and flavor quantum number of an $L = 0$ hadron pair

The binding force between the molecules, which is nuclear force, has a short range. So $L > 0$ molecules cannot be in light hadronic systems, although there are some exceptions.

2)A binding energy of at most about 50-100 MeV

For the hadrons to keep their separate identities in a hadron molecule, the separation between them has to be at least 1 fm. From the uncertainty principle, this corresponds to a binding energy of approximately 50 MeV.

3)Strong coupling to constituent channels

For example, the very large coupling of $f_0(975)$ to $K\overline{K}$

4)Unexpectedly large EM couplings relative to expectations for conventional quark model states

For example, $f_0(975)$ has a very small decay width in the $\gamma\gamma$ channel, which is between 0.2 to 0.6 KeV, while the naive quark model's prediction is 3 KeV.

In the paper of Barnes, Close and Lipkin [28], these four conditions are examined and it is claimed that $D_{s0}(2317)$ is a good candidate for DK molecule. The existence of the decay channel $D^+_s\pi^0$ and the absence of the decay channel $D^{*+}_s\pi^0$ confirms the first signature. The DK thresholds ($D^0K^+$, $D^0K^0$, $D^+K^+$ and $D^+K^0$) are 2358 MeV , 2362 MeV, 2363 MeV and 2367 MeV, respectively. These values are calculated with considering the following condition: $m_{D_{sJ}} > m_D + m_K$. So DK molecule at 2.32 GeV would have a binding energy about 40 MeV, which confirms the second signature. The third condition is problematic for $D_{sJ}(2317)$ because only observed channel for this meson is $D^+_s\pi^0$. And to confirm the fourth one, one needs to know the coupling strengths of $D^+_{s0}$ to both DK and $D^{*+}_s$. The transition rate of $D^+_{s0}(2317) \longrightarrow Ds^{*+}\gamma$ calculated using quark model is 2 KeV, so the fourth condition can be easily verified while knowing the necessary coupling strengths.

Owing to the same reasons, Szczepaniak [30] suggests another four-quark structure, which is $D\pi$ atom, where $\pi$ is a light unflavored pseudoscalar meson. $D\pi$ atom may exists because of the strong flavor-singlet attraction between the pion and the $D_{s0}^{+}$ meson.

Also for the structure of the $D_{sJ}(2460)$ meson some new suggestions are made. Four-quark structure and $D^{*}K$ [31] molecule are the suggestions for the $D_{sJ}(2460)$ meson with similar arguments.

Besides these different suggestions for the structure of the $D_{s0}(2317)$ and $D_{sJ}(2460)$ mesons, QCD sum rule analysis is consistent with the quark-antiquark structure [32].

# CHAPTER 3

# QCD SUM RULES

Hadrons, which is the general name of mesons and baryons, are subatomic particles which experiences nuclear force. They are bound states of quarks. To have some information about hadrons, one can use QCD sum rules method, which is based on spontaneous symmetry breaking, duality and asymptotic freedom [33]. While using this method, as one moves from short distances to the long distances, the confinement mechanism becomes important.

## 3.1 Asymptotic Freedom and Confinement Mechanism

The coupling constant between quarks is not constant, on the contrary it depends on the distance between quarks. Therefore, it is called sometimes as running coupling constant. Unlike the electric charges, the running coupling constant of quarks gets smaller while the distance decreases. This phenomena is called asymptotic freedom, discovered by David Gross and Frank Wilczek (1973) [34] and David Politzer (1973) [35].

The quantity of an electric charge in a dielectric medium depends on how far the measurement is done. The nearer is the distance in which the measurement is done, the less is the screening. Instead of the electron-photon vertices in QED case, in QCD, there are quark-gluon vertices and in addition to this, there are also gluon-gluon vertices. In QCD, the strong coupling constant is proportional to the factor a,

$$a = 2f - 11n \tag{3.1}$$

where f is the quark number and n is the color number. If a is positive, then

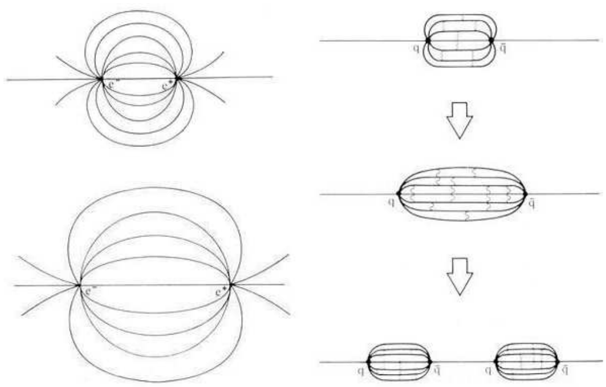

Figure 3.1: QED field lines in comparison to field lines QCD

the coupling constant increases at short distances, as in the QED case. If a is negative, then the coupling constant decreases at short distances. In QCD, there are 6 quarks and 3 colors, i.e. $f = 6$ and $n = 3$. So a is equal to -21 in QCD. This is the origin of asymptotic freedom [34]. Owing to the asymptotic freedom, one can use the perturbation theory at very small separation, since quarks are almost non-interacting free particles there.

Unlike the photon in QED, the gluons in QCD carry the charge of the corresponding force. Hence, they also interact with themselves. It is believed that, due to the interactions of the gluons with each other, the field lines of color are confined within tubes. If one tries to pull a bound quark-antiquark pair apart, the flux tube binding them together grows and hence the energy stored in this flux tube increases. At some point, the energy stored in the flux tube becomes so large that it becomes energetically more favorable for the system to create a quark-antiquark pair and break the flux tube. In other words, the energy given to separate the quark-pair, is used in pair production. So free quarks cannot be produced. Although there does not exist a proof that QCD implies confinement, it is believed that it has to do so.

### 3.2 Correlation Function

QCD sum rules method was suggested in 1978 by M.A. Shifman, A.I. Vainstein and V.I. Zakharov [36]. This method can be used to extract the hadron properties, such as the masses and coupling constants. In this method, hadrons are represented by their interpolating quark currents instead of constituent quarks, consequently QCD sum rules are model-independent [37]. For example, one can use this method for four-quark structure model and for naive quark model.

The QCD Lagrangian is

$$L_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_q \overline{\psi_q}(i\gamma^\mu D_\mu - m_q)\psi_q \tag{3.2}$$

where $G^a_{\mu\nu}$ is the gluon field-strength tensor, $G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - igf^{akl}G^k_\mu G^l_\nu$, D is covariant derivative, $D_\mu \psi^q = (\partial_\mu - igG_\mu)\psi_q$ and $\psi_q$'s are the quark fields. Although this Lagrangian governs all attributes of hadrons and hadronic processes involving the strong force only, it can be used directly only within limited framework of perturbation theory [37]. While dealing with highly virtual processes, which can be achieved in hard scattering process, not only the perturbative effects but also the non-perturbative effects have to be considered. In the processes in which small momentum transformations are considered, the coupling constant takes values greater than 1. So perturbation theory cannot be applied. However, in order to calculate the hadronic properties, the region where coupling constant is greater than 1 has to be considered. Therefore, non-perturbative methods are needed. QCD sum rules is one of these non-perturbative methods.

In QCD sum rules, one starts by considering a suitable correlation function such as,

$$\Pi(q^2) = \int d^4x e^{iq\cdot x} \langle 0|T\{J(x)\overline{J}(0)\}|0\rangle \tag{3.3}$$

where T is the time ordered product operator and $J(x)$ and $J(0)$ are interpolating quark currents. To write down the interpolating quark current, the quark content of the hadron might be used. Eq. (3.3) is a two point correlation function that can be used to calculate the mass of the baryon under consideration.

The correlation function given in Eq. (3.3) can be calculated in two different ways: using the operator product expansion (OPE) in terms of QCD parameters, and using a phenomenological expansion, in terms of hadron parameter. After calculating the correlation function in two different ways and applying both sides called Borel transformation, which enhances single resonance contributions and equating these two representations, one can get sum rule. In the next two sections a brief explanation of these two parts of the sum rules are presented.

### 3.3 Operator Product Expansion (OPE)

Operator Product Expansion is based on Wilson operator expansion [38, 33]. OPE is a very convenient tool at high virtualities since the short and long distances are considered separately [37].

$$i\int dx e^{iq\cdot x} T\{j(x)j(0)\} = C_I I + \sum_n C_n(x^2) O_n \tag{3.4}$$

where $C_n$'s are Wilson coefficients and $O_n$'s are local operators constructed from the quark and gluon fields. And the correlation function becomes

$$\Pi(q^2) = \sum_n C_n(q^2)\langle 0|O_n|0\rangle \tag{3.5}$$

$O_n$'s are ordered by their dimension and $C_n$'s decrease by increasing powers of $q^2$ [33]. So, at high virtualities, lowest dimensional operators dominate. The unit operator with dimension zero represents the perturbative part, which is expected to be the dominant part. In OPE, only the spin-zero operators are considered because only the vacuum expectation values are considered. Effects of the operators having dimension higher than 6 are considered as negligible [36]. The operators whose spin is zero and dimension is equal to or less than 6 are [36]

$$\begin{aligned}
&I(unit\,operator) && (d=0),\\
&m\bar{q}q && (d=4),\\
&G^a_{\mu\nu}G^{\mu\nu a} && (d=4),\\
&\bar{q}\Gamma_1 q\bar{q}\Gamma_2 q && (d=6),\\
&m\bar{q}\sigma_{\mu\nu}\tfrac{\lambda^a}{2}qG^{\mu\nu a} && (d=6),\\
&f_{abc}G^a_{\mu\nu}G^{\nu b}_{\sigma}G^{\sigma\mu c} && (d=6)
\end{aligned} \tag{3.6}$$

where $\lambda^a$ are Gell-Mann matrices and $G^a_{\mu\nu}$ is the gluon field tensor.

In perturbative vacuum, the vacuum expectation values of these operators are zero. However, since the QCD vacuum contains condensates due to the non-perturbative effects, the vacuum expectation values of these operators are not zero anymore. The vacuum expectation values of the operators with dimension $d \neq 0$, called as vacuum condensates, are associated with non-perturbative effects.

As it can be seen from Eq. (3.4), OPE is defined in coordinate space. Actually, it is a taylor expansion around $x \sim 0$, where $x \simeq 0$. By applying taylor expansion, the contributions coming from short distances are included in $C_n$ and long distances are included in $O_n$. There is a cut off $1/\mu$, which separates long and short distances. $C_n$ are calculated perturbatively. If $q^2 \ll 0$, then the main contribution to the integral comes from $x \sim 0$ region [37], where OPE can be applied.

### 3.4 Analytic Continuation and Phenomenological Representation

As has already mentioned above, OPE ,in QCD, factorizes short and long distance effects, i.e. Wilson coefficients are calculated perturbatively and long distance effects are parameterized by vacuum expectation values of local operators. In order to express the correlation function in terms of the hadronic parameters, unitary relation might be used. After inserting a complete set of hadronic states into Eq. (3.3), the correlation function becomes [37]:

$$\Pi = \frac{\langle 0|j(x)|h\rangle\langle h|j(0)|0\rangle}{q^2 - m_h^2} + \frac{\langle 0|j(x)|h_1\rangle\langle h_1|j(0)|0\rangle}{q^2 - m_{h_1}^2} + higher\ mass\ states \quad (3.7)$$

where $\langle 0|j(x)|h\rangle = f_h m_h$ and $f_h$ and $m_h$ are leptonic decay constant and mass of the corresponding hadron, respectively and they are known phenomenologically. The correlation function can be written for a two- point correlation function as:

$$\Pi = \sum_h \frac{\lambda_h^2}{q^2 - m_h^2} \quad (3.8)$$

where $\lambda_h$ are defined by the matrix element of the current j between the vacuum and meson states [37]. And using Eq. (3.8), the spectral density of phenomenological side can be written as:

$$\rho^{(Phen)}(s) = -\sum_h \lambda_h^2 \delta(s - m_h^2) = \rho_0^{Phen}(s) + \rho^{higher}(s) \quad (3.9)$$

where $\rho_0^{Phen}(s) = -\lambda_0^2 \delta(s - m_0^2)$.

As mentioned above, $\Pi(q^2)$ can be treated in perturbative QCD at large negative $q^2$, while at $q^2 > 0$ region it has a decomposition in terms of hadronic observables [37]. The $q^2 > 0$ region can be connected to the $q^2 < 0$ region by analytical continuation. By using Cauchy integral formula with the contour in Fig. [3.1] [37], One gets;

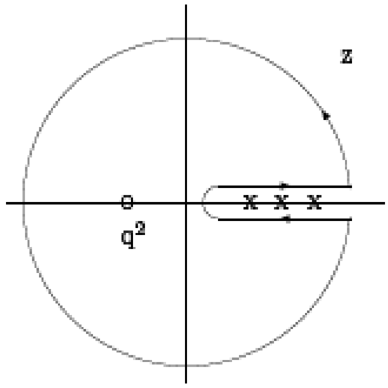


Figure 3.2: The contour in complex plane

$$\begin{aligned}\Pi(q^2) &= \frac{1}{2\pi i}\oint_c dz \frac{\Pi(z)}{z-q^2} \\ &= \frac{1}{2\pi i}\oint_{|z|=R} dz \frac{\Pi(z)}{z-q^2} + \frac{1}{2\pi i}\int_0^R dz \frac{\Pi(z+i\epsilon)-\Pi(z-i\epsilon)}{z-q^2}\end{aligned} \tag{3.10}$$

where $R \rightarrow \infty$. The first term on the r.h.s. is equal to the polynomials in $q^2$ since;

$$\begin{gathered}\frac{1}{s-q^2} = \sum_{n=0}^{\infty} \frac{(q^2)^n}{s^{n+1}} \\ and\ for\ sufficiently\ large\ N \\ \oint_{|z|=R} ds \frac{\rho(s)}{s^{N+1}} \rightarrow 0\ as\ R \rightarrow 0\end{gathered} \tag{3.11}$$

And by using Schwartz reflection principle, other term on the r.h.s can be written as $\Pi(q^2+i\epsilon) - \Pi(q^2-i\epsilon) = 2iIm\Pi(q^2)$. After inserting this equation into the contour integral in Eq. (3.10), one gets the following dispersion relation.

$$\Pi(q^2) = \frac{1}{\pi}\int_0^{\infty} ds \frac{Im\Pi(s)}{s-q^2} + substraction\ terms \tag{3.12}$$

### 3.5 Borel Transformation

As it can be seen from the above discussion, there are some substraction terms in both OPE and phenomenological side. To eliminate these unknown terms, one needs to apply Borel transformation to both sides [37].

$$\Pi(M^2) = B_{M^2}\Pi(q^2) = \lim_{(-q^2,n\to\infty)(-q^2/n=M^2)} \frac{(-q^2)^{(n+1)}}{n!}(\frac{d}{dq^2})^n\Pi(q^2) \tag{3.13}$$

Two important Borel transformations are:

$$\begin{aligned} B_{M^2}(q^2)^k &= 0, \\ B_{M^2}(\tfrac{1}{(m^2-q^2)^k}) &= \tfrac{1}{(k-1)!}\tfrac{\exp(-m^2/M^2)}{M^{2(k-1)}} \end{aligned} \tag{3.14}$$

where M is the Borel mass, which is completely arbitrary and the physical parameters should be independent of $M^2$. On the other hand, in calculations, one may need to chose a suitable region for $M^2$ where the physical parameters are almost independent of $M^2$.

Since both substraction terms of OPE and phenomenological sides are polynomials in $q^2$, after applying Borel transformation, these unknown terms are eliminated.

After applying Borel transformation to both sides, one need to use quark-hadron duality, which is the assumption that the spectral function that is calculated phenomenologically and the one that is calculated using OPE are approximately equal to each other when s is above some threshold value $s_0$:

$$\int_{s_0^h}^{\infty} \rho^{higher\ states}(s)e^{-\frac{s}{M^2}} \equiv \int_{s_0}^{\infty} \rho^{OPE}(s)e^{-\frac{s}{M^2}} \tag{3.15}$$

where $s_0^h$ is the threshold of the lowest continuum state.

The motivation of quark-hadron duality is that higher states are at high energies and OPE can be used at high energies, then they should be equal to each other. The sum rules are obtained from:

$$\int_0^{s_0} dse^{-\frac{s}{M^2}}\rho_0^{Phen}(s) = \int_0^{s_0} dse^{-\frac{s}{M^2}}\rho^{OPE}(s) \tag{3.16}$$

# CHAPTER 4

# $\mathbf{D_{sJ}(2317)}$ AND $\mathbf{D_{sJ}(2460)}$ MESONS IN QCD SUM RULES

In this chapter, some decay calculations of $D_{sJ}(2317)$ and $D_{sJ}(2460)$ using QCD Sum Rules are investigated. Moreover, the steps of the calculation of the coupling constant of semileptonic decay $D_{sJ}(2317) \rightarrow D_s \ell \nu$ is studied.

## 4.1 The Radiative Transitions and Strong Decays of $\mathbf{D_{sJ}(2317)}$ and $\mathbf{D_{sJ}(2460)}$

In [40], the radiative transitions of the considered mesons are studied by using light-cone QCD sum rules. As in Eq. (3.4), OPE is actually represented in coordinate space.In light-cone QCD sum rules (LCSR) [41, 42, 43], this representation is considered by considering the vacuum to hadron matrix element. This is also an expansion around $x \sim 0$. However, in LCSR, this representation is reorganized and turn out to be an expansion around $x^2 \sim 0$. With this method, both hard and soft scattering contributions can be taken into account [44]. Another difference between SVZ sum rules and LCSR is that while in SVZ sum rules in correlation function, the vacuum expectation value of a T-product of currents are used, in LCSR vacuum to on-shell state correlation function is used [45, 46]. Also in this case, non-local operators are considered, because point of interest is OPE near light-cone. OPE near the light cone is made over twist (difference between dimension and spin) of the operator while in QCD sum rules, OPE is made over the dimension of the operators [47]. To investigate the structure of non-local operators, one can expand it in terms of local operators around $x = 0$ [37] and resum certain infinite subsets to convert it to an expansion around $x^2 \simeq 0$. Following this expansion, as an example, the following matrix element takes the form as:

$$\langle \gamma(q)|\overline{u}(x)\gamma_\mu\gamma_5 u(0)|0\rangle = \varepsilon_\mu \int du e^{iuq\cdot x}\varphi_\gamma(u) + O(x^2) \tag{4.1}$$

where $\varphi_\gamma(u)$ is the photon light-cone distribution function which encodes the non-perturbative effects included in the corresponding state's distribution amplitudes.

In [40], $D^*_{sJ}(2317) \to D^*_s\gamma$, $D_{sJ}(2460) \to D_s\gamma$, $D_{sJ}(2460) \to D^*_s\gamma$ and $D_{sJ}(2460) \to D^*_{sJ}(2317)\gamma$ decay are studied by using LCSR and are compared with other methods, namely Vector Meson Dominance(VMD) in the heavy quark limit, i.e. $m_c \to \infty$ and constituent quark model.

For the radiative transitions of $D_{sJ}(2317)$ the decay $D^*_{sJ}(2317) \to D^*_s\gamma$ is studied in LCSR and also in the heavy quark limit. The currents are chosen as the correlation function in LCSR for this decay is chosen as [40]:

$$F_\mu(p,q) = i\int d^4x e^{ip\cdot x}\langle \gamma(q,\lambda)|T[J^\dagger_\mu(x)J_0(0)]|0\rangle \tag{4.2}$$

where $J_0 = \bar{c}s$ and $J_\mu = \bar{c}\gamma_\mu s$ and $\gamma(q,\lambda)$ is an external photon state of momentum q and helicity $\lambda$. In Eq. (4.2), the products of the currents are performed in light-cone $x^2 \to 0$. From this correlation function, the coupling constant for the transition is predicted to be $-0.35GeV^{-1} \le g \le -0.28GeV^{-1}$. The calculated decay width using this coupling constant is 4-5 times larger than that one obtained by VMD. This difference might be explained by arguing the use of the heavy quark limit of VMD . In order to eliminate this choice, in [40], heavy quark limit is also considered. Finally, it is understood that finite mass effects are explained the difference between the calculations using VMD and LCSR.

For the radiative transitions of $D_{sJ}(2460)$, similar calculations are made. For analyzing the decay $D_{sJ}(2460) \to D_s\gamma$ in LCSR, the following correlation function is considered [40]:

$$T_\mu = i\int d^4x e^{ip\cdot x}\langle \gamma(q,\lambda)|T[J^\dagger_5(x)J^A_\mu(0)]|0\rangle \tag{4.3}$$

where $J_5 = \bar{c}i\gamma_5 s$ and $J^A_\mu = \bar{c}\gamma_\mu\gamma_5 s$.

Table 4.1: The results of the calculation using LCSR

| Initial state | Final state | LCSR | VMD | QM |
|---|---|---|---|---|
| $D_{sJ}^{+}(2317)$ | $D_s^{+}\gamma$ | 4-6 | 0.85 | 1.9 |
| $D_{sJ}(2460)$ | $D_s\gamma$ | 19-29 | 3.3 | 6.2 |
| | $D_s^{+}\gamma$ | 0.6-1.1 | 1.5 | 5.5 |
| | $D_{sJ}^{+}(2317)\gamma$ | 0.5-0.8 | - | 0.012 |

And the following correlation functions are chosen for the transitions $D_{sJ}(2460) \to D_s^*\gamma$ and $D_{sJ}(2460) \to D_{sJ}^*(2317)\gamma$ as:

$$T_{\mu\nu}(p,q) = i\int d^4x e^{ip\cdot x}\langle\gamma(q,\lambda)|T[J_\mu^\dagger(x)J_\nu^A(0)]|0\rangle \tag{4.4}$$

$$W_\mu(p,q) = i\int d^4x e^{ip\cdot x}\langle\gamma(q,\lambda)|T[J_0^\dagger(x)J_\mu^A(0)]|0\rangle \tag{4.5}$$

respectively. And the same calculations are carried out to get the decay width of the transitions.

In these calculations, it is understood that in charmed meson case, there are two main contributions. One of them corresponds to the perturbative photon emission from the heavy and light quarks and the other corresponds to the photon emission from the soft light quark [40]. From the table, it is seen that the radiative decay width of $D_{sJ}(2317) \to D_s^{+}\gamma$ is predicted to be more than 3 times the predictions of the VMD and quark models. The width of the decays $D_{sJ}(2460) \to D_s^{+}\gamma$ and $D_{sJ}(2460) \to D_{sJ}(2317)\gamma$ is in general predicted to be much smaller than other predictions. The largest difference between the predictions is in the decay $D_{sJ} \to D_s\gamma$ whose decay width is predicted to be more than 4 times larger than the predictions of the other methods. Note also that, other methods predict the decay width of the $D_{sJ} \to D_s\gamma$ decay to be more or less equal to the widths of the other channels, whereas LCSR prediction for $D_{sJ} \to D_s\gamma$ width is more than 4 times larger than the widths of the other ones. This result of LCSR can naturally explain why the only observed mode is $D_{sJ}(2460) \to D_s\gamma$. And from these results, it is suggested that other interpretations for these mesons are unnecessary to explain present experimental data [40].

Another study on $D_{sJ}(2317)$ and $D_{sJ}(2460)$ by using QCD sum rules is about strong decays of these mesons [48]. In this study again the light-cone QCD sum rules method is used by assuming the $c\overline{s}$ quark content. The strong decays $D_{sJ}(2317) \to D_s\pi^0$ and $D_{sJ}(2460) \to D_s^*\pi^0$ are studied in [48]. These decays violate isospin. Hence, in [48], these decays are modeled as two strange decays. First, $D_{sJ} \to D_s\eta$ which conserved isospin followed by a conversion of $\eta$ into $\pi^0$ due to isospin violation. The interpolating currents are chosen as
$J_0(x) = \overline{c}(x)s(x)$, $J_5(x) = \overline{c}(x)i\gamma_5 s(x)$, $J_\mu(x) = \overline{c}(x)\gamma_\mu s(x)$ and $J_\nu^A = \overline{c}(x)\gamma_\nu\gamma_5 s(x)$. And the correlation functions

$$F(p^2,(p+q)^2) = i\int d^4x e^{ip\cdot x}\langle\eta(q)|T[J_5^\dagger(x)J_0(0)]|0\rangle \tag{4.6}$$

for the $D_{sJ} \to D_s\eta$ coupling constant, and

$$F_{\mu\nu}(p^2,(p+q)^2) = i\int d^4x e^{ip\cdot x}\langle\eta(q)|T[J_\mu^\dagger(x)J_\nu^A(0)]|0\rangle \tag{4.7}$$

for the $D_{sJ}(2460) \to D_s^*\eta$ coupling constant are chosen. Following similar calculations, one can obtain the coupling constants which are used in the decay width calculations. In [48], the obtained results by using LCSR is compared with the experimental results. And it is observed that these results are consistent with the experimental ones. This consistency also supports the quark content $c\overline{s}$ for these mesons.

## 4.2 The Semileptonic $\mathbf{D_{sJ}(2317) \to D_0\ell\nu}$ Decay

In this thesis, the steps of the calculation with three-point QCD sum rules of semileptonic $D_{sJ}(2317) \to D_0\ell\nu$ decay which has not been observed yet, is investigated. This decay, when observed, can give insight into the structure of $D_{sJ}(2317)$. In the calculation of the form factors of this decay, the quark structure of the meson is assumed to be $c\overline{s}$. As it can be seen in Fig.4.1, the decay $D_{sJ}(2317) \to D_0\ell\nu$ is described by the quark level transition of an s quark into an u quark with emission of a virtual W. At low energies, since $p^2 \ll M_W^2$ where p is the momentum of the W boson, the momentum term in the W boson propagator ($\propto 1/(p^2 - M_W^2)$) can be neglected. When it is neglected, the effective hamiltonian takes the following form:

$$H_{eff} = \frac{G_f}{2^{1/2}} V_{us}\overline{s}\gamma_\mu(1-\gamma_5)u\overline{\ell}\gamma^\mu(1-\gamma_5)\nu \tag{4.8}$$

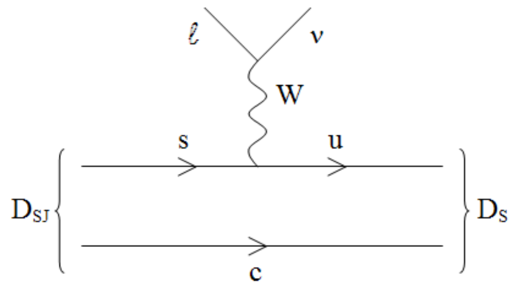


Figure 4.1: The Feynman diagram for $D_{sJ}(2317) \to D_0 \ell \nu$

and the matrix element is:

$$\begin{aligned} M \quad &= \langle final\ state | H_{eff} | initial\ state \rangle \\ &= \langle \ell \nu | \bar{\ell} \gamma_\mu (1-\gamma_5) \nu | 0 \rangle \langle D_0 | \bar{s} \gamma_\mu (1-\gamma_5) u | D_{sJ} \rangle \end{aligned} \tag{4.9}$$

where the necessary matrix element to investigate $D_{sJ}(2317) \to D_0 \ell \nu$ is:

$$\langle D_0 | \bar{s} \gamma_\mu (1-\gamma_5) u | D_{sJ} \rangle = f_1(q^2) p_\mu + f_2(q^2) q_\mu \tag{4.10}$$

So one needs to find the formfactors $f_1$ and $f_2$, for which a non-perturbative calculation is needed.

While studying the steps of the decay of concern, three-point QCD sum rules can be used. The methodology of the three point sum rules is very similar to two-point ones. In this method, the correlation function contains double integral unlike the two-point one, because of the consideration of 3 space-time points. The correlation function is given by:

$$\Pi(p^2, p^{'2}) = -\int\int d^4x d^4y e^{ip\cdot y} e^{ip^{'}\cdot x} \langle 0 | [J_1(x) J_2(y) J_3(0)] | 0 \rangle \tag{4.11}$$

and the dispersion relation for the three-point QCD sum rules is:

$$\Pi(p^2, p^{'2}) = \int\int ds_1 ds_2 \frac{\rho(s_1, s_2)}{(s_1 - p^2)(s_2 - p^{'2})} + polynomials\ in\ p^2\ and\, p^{'2} \tag{4.12}$$

where

$$\rho(s_1, s_2) \propto Im_{p^2} Im_{p^{'2}} \Pi(p^2, p^{'2}) \tag{4.13}$$

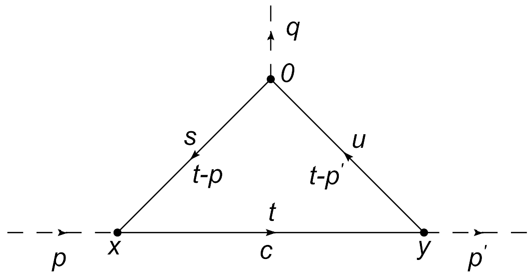


Figure 4.2: The Feynman diagram for the perturbative part

In the calculation of the coupling constant of the decay under consideration, firstly the following correlation function is studied:

$$\Pi_\mu(p^2,p^{'2}) = -\int\int d^4x d^4y e^{ip\cdot y} e^{-ip'\cdot x} \langle 0|\{\overline{u}(y)c(y)\overline{s}(0)\gamma_\mu(1-\gamma_5)u(0)\overline{c}(x)s(x)\}|0\rangle \quad (4.14)$$

After choosing the correlation function, firstly the OPE side is calculated. In this part, as it is explained in chapter 3, the perturbative and non-perturbative effects are calculated separately.

### 4.2.1 Perturbative Part of OPE

Figure 4.2 is the Feynman diagram of the perturbative part, where $q = p - p'$. It is expected that the dominant contribution comes from this perturbative part.The perturbative part is given by the following expression:

$$\Pi_\mu(p^2,p^{'2}) = -\int\int d^4x d^4y e^{ip\cdot y} e^{-ip'\cdot x} \langle 0|[\gamma_\mu(1-\gamma_5)]_{\gamma\beta}[S_u(0,y)]^{ab}_{\beta\alpha}[S_c(y,x)]^{ac}_{\alpha\lambda}[S_s(x,0)]^{bc}_{\lambda\gamma} : 1 : |0\rangle \quad (4.15)$$

where $S_u$, $S_c$ and $S_s$ are the free propagators, whose explicit forms are given in the Appendix B. The other terms go to zero, some of them are equal to zero after applying Borel transformation with respect to $p^2$ and $p^{'2}$ and some of them are zero because of the vacuum structure of QCD. And the final correlation function for perturbative part is:

$$\Pi_\mu = -i^3 \int \frac{d^4l}{(2\pi)^4} Tr\left[\gamma_\mu(1-\gamma_5)\frac{\not{l}-\not{p}+m_u}{((l-p)^2-m_u^2)}\frac{\not{l}+m_c}{(l^2-m_c^2)}\frac{\not{l}-\not{p}'+m_s}{((l-p')^2-m_s^2)}\right]$$

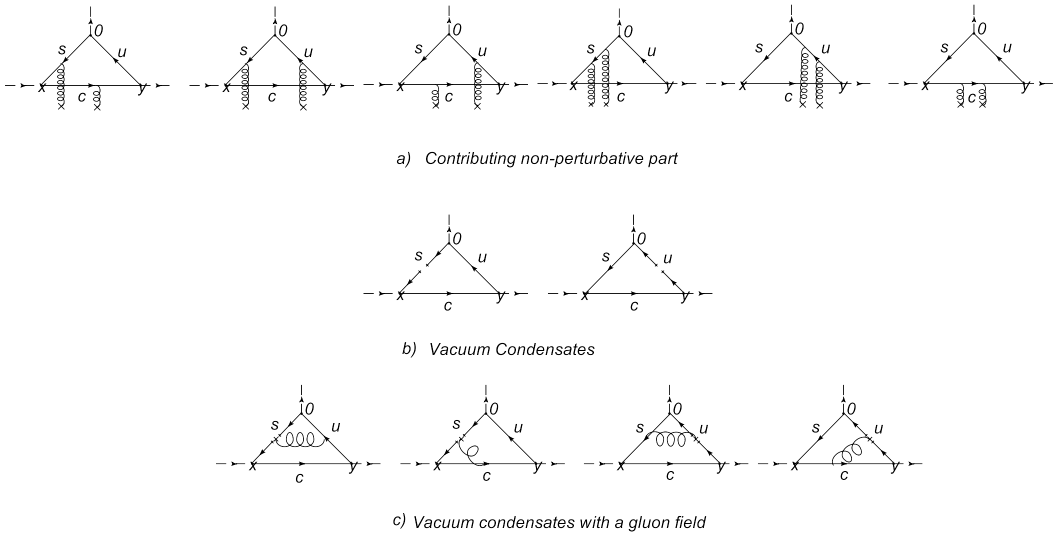


Figure 4.3: The Feynman diagrams of the non-perturbative part

(4.16)

The integrals are taken with the help of Feynman Parametrization, which is presented in Appendix A. Carrying out the trace and applying double Borel transformation with respect to $p^2$ and $p'^2$, the correlation function takes the following form:

$$BB(\Pi(p^2, p'^2)) = \int\int ds_1 ds_2 \rho(s_1, s_2) e^{-\frac{s_1}{M_1^2}} e^{-\frac{s_2}{M_2^2}} \tag{4.17}$$

and the spectral density of perturbative part can be found.

### 4.2.2 Non-Perturbative Part of OPE

As mentioned in Chapter 3, when one moves from the short distances to long distances, non-perturbative effects become important. It is expected that higher dimensional operators give small but non-negligible contributions to the calculation of the coupling constant. The diagrams that contribute up to dimension of 6 are shown in the Figure (4.3).

The diagrams in b and c do not contribute to the result, because after applying Borel transformation, they are equal to zero since they depend on $p^2$ only or $p'^2$ only.

For example, for the second figure, the form of the correlation function is as follows:

$$
\begin{aligned}
\Pi_\mu = \quad & -\int\int d^4x d^4y e^{ip\cdot y} e^{-ip'\cdot x} \\
& \langle 0|[\gamma_\mu(1-\gamma_5)]_{\gamma\beta}[S_u^G(0,y)]^{ab}_{\beta\alpha}[S_c^G(y,x)]^{ac}_{\alpha\lambda}[S_s^{free}(x,0)]^{bc}_{\lambda\gamma}|0\rangle \quad (4.18)
\end{aligned}
$$

where $S_s^{free}$ is free propagator, which is the same with the one in perturbative part and $S_u^G$ and $S_c^G$ are propagators of the quarks in a one-gluon field represented in Appendix B.

Like in the perturbative part, the integrations over loop momentum are performed using Feynman Parametrization. Afterwards, Borel transformation is applied for $p^2$ and $p'^2$. Then, the non-perturbative part of the spectral density is found. The correlation function calculated using OPE is an expression involving only the parameters of QCD such as the condensates, masses, etc.

### 4.2.3 Phenomenological Part

By inserting two complete sets of hadronic states into Eq.(4.10), the correlation function can be written as:

$$
\Pi = \frac{\langle 0|\overline{u}(y)c(y)|D_0\rangle\langle D_0|\overline{s}\gamma_\mu(1-\gamma_5)u|D_{sJ}\rangle\langle D_{sJ}|\overline{c}(x)s(x)|0\rangle}{(p^2_{D_{sJ}}-m^2_{D_{sJ}})(p^2_{D_0}-m^2_{D_0})} + h.s. \quad (4.19)
$$

where $\overline{u}(y)c(y) = J_{D_0}$ and $\overline{c}(x)s(x) = J_{D_{sJ}}$. And

$$
\begin{aligned}
\langle D_{sJ}|J_{D_{sJ}}|0\rangle &= f_{D_{sJ}} \\
\langle 0|J_{D_0}|D_0\rangle &= f_{D_0}\frac{m^2_{D_0}}{m_c+m_u} \\
\langle D_0|\overline{s}\gamma_\mu(1-\gamma_5)u|D_{sJ}\rangle &= f_1(q^2)P_\mu + f_2(q^2)q_\mu \quad (4.20)
\end{aligned}
$$

where

$$
\begin{aligned}
P &= p_{D_{sJ}} + p_{D_0} \\
q &= p_{D_{sJ}} - p_{D_0} \quad (4.21)
\end{aligned}
$$

And the correlation function of the phenomenological part becomes:

$$
\Pi_\mu = \Pi_1 P_\mu + \Pi_2 q_\mu \quad (4.22)
$$

where

$$\begin{aligned}
\Pi_1 &= f_{D_{sJ}} f_{D_0} \frac{m_{D_0}^2}{m_c + m_u} f_1(q^2) \frac{1}{(p_{D_{sJ}}^2 - m_{D_{sJ}}^2)} \\
\Pi_2 &= f_{D_{sJ}} f_{D_0} \frac{m_{D_0}^2}{m_c + m_u} f_2(q^2) \frac{1}{(p_{D_0}^2 - m_{D_0}^2)}
\end{aligned} \tag{4.23}$$

Eq. (4.19) and Eq. (4.23) give the phenomenological represantation of the correlation function Eq. (4.11). Note that the functions in Eq. (4.23)involve hadronic parameters.

By equating the OPE expression of the correlation function to the phenmenological expression, one can express the hadronic parameters in terms of the QCD parameters and vacuum condensates. After subtracting the contributions of higher states and continuum using quark-hadron duality, the sum rules is obtained by evaluating

$$\int_0^{s_0} ds_1 ds_2 \rho_i^{Phen}(s_1, s_2) e^{-\frac{s_1}{M_1^2}} e^{-\frac{s_2}{M_2^2}} = \int_0^{s_0} ds_1 ds_2 \rho_i^{OPE}(s_1, s_2) e^{-\frac{s_1}{M_1^2}} e^{-\frac{s_2}{M_2^2}} \tag{4.24}$$

where $i = 1, 2$ and $\rho_i$ corresponds to the spectral density of $\Pi_i (i = 1, 2)$.

### 4.3 Discussion

The semileptonic decay $D_{sJ}(2317) \to D_0 \ell \nu$ has not been observes yet.When it is observed this investigation may give some important clues for the structure of the meson. Hence, the study of the $D_{sJ}(2317) \to D_0 \ell \nu$ transition will be important in understanding the structure of this meson.

# CHAPTER 5

# CONCLUSIONS

$D_{sJ}(2317)$ and $D_{sJ}(2460)$ are recently discovered mesons by BaBar and CLEO Collaborations. The difference between the expected and the measured masses and the narrow decay widths stimulated studies of $D_{sJ}(2317)$ and $D_{sJ}(2460)$. The discrepancies between the predicted properties within the quark model and experiment caused speculations about the structure of these mesons. Various models are suggested for the structure of these mesons.

In this thesis, firstly, the Godfrey-Isgur potential-based quark model is investigated and it is shown that this model has been successfully applied to the $c\bar{q}$ $(q = u, d)$ mesons discovered previously. The predictions of this model on the $0^+$ and $1^+$ states of $c\bar{s}$ and the observed candidates $D_{sJ}(2317)$ and $D_{sJ}(2460)$ are shown to be inconsistent. Following this model, various models suggested for the structure of these mesons besides the quark model are investigated. These are four-quark structure, DK and $D^*K$ molecules and $D\pi$ atom. Afterwards, the model-independent QCD sum rules method, which is one of the non-perturbative methods to calculate the hadronic parameters. QCD sum rules method can also be used to study four-quark structure, etc. QCD sum rules predictions, assuming a $c\bar{s}$ structure for the $D_{sJ}$ mesons, are consistent with experimental predictions.

By using QCD sum rules method and accepting the quark structure of $D_{sJ}(2317)$ meson as $c\bar{s}$, we outlined how to study the decay $D_{sJ}(2317) \rightarrow D_0\ell\nu$ within QCD sum rules. Although this semileptonic decay has not been observed yet, the study of this decay will help to understand the structure of $D_{sJ}(2317)$ meson.

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# APPENDIX A

# FEYNMAN PARAMETRIZATION

In QCD sum rules method, the following types of integrals are frequently encountered.

$$I_1 = \int \frac{d^4k}{(2\pi)^4} \frac{1}{[k^2 - m_1^2]^n[(k-q)^2 - m_2^2]^m[(k-q')^2 - m_3^2]^l} \tag{A.1}$$

$$I_{2\mu} = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{[k^2 - m_1^2]^n[(k-q)^2 - m_2^2]^m[(k-q')^2 - m_3^2]^l} \tag{A.2}$$

In order to calculate these integrals, Wick rotation is used:

$$\begin{aligned} k_0 &\rightarrow ik_0^E \\ \overrightarrow{k} &\rightarrow \overrightarrow{k}^E \end{aligned} \tag{A.3}$$

After this step, Feynman parametrization is used. [49]

$$\begin{aligned} \frac{1}{a^n b^m} &= \frac{\Gamma(n+m)}{\Gamma(n)\Gamma(m)} \int_0^1 dx \frac{x^{n-1}\overline{x}^{m-1}}{[ax + b\overline{x}]^{n+m}} \\ \frac{1}{abc} &= 2\int_0^1 xdx \int_0^1 dy \frac{1}{[a\overline{x} + bxy + cx\overline{y}]^3} \end{aligned} \tag{A.4}$$

The integrals in Eq. (A.1) and Eq. (A.2) are evaluated as:

$$\begin{aligned} I_1 &= \frac{(-1)^{n+m+l} i}{(4\pi)^2} \frac{\Gamma(n+m+l-2)}{\Gamma(n)\Gamma(m)\Gamma(l)} \int_0^1 dxx \int_0^1 dy \frac{(xy)^{n-1}(x\overline{y}^{m-1}\overline{x}^{l-1})}{\Delta^{n+m+l-2}} \\ I_{2\mu} &= \frac{(-1)^{n+m+l} i}{(4\pi)^2} \frac{\Gamma(n+m+l-2)}{\Gamma(n)\Gamma(m)\Gamma(l)} \left[ q_\mu \left( \int_0^1 dx \int_0^1 dy \frac{(x^{n+m}y^{n-1}\overline{y}^m\overline{x}^{l-1})}{\Delta^{n+m+l-2}} \right) \right. \\ &\qquad \left. + q'_\mu \left( \int_0^1 dx \int_0^1 dy \frac{(x^{n+m-1}y^{n-1}\overline{y}^{m-1}\overline{x}^l)}{\Delta^{n+m+l-2}} \right) \right] \end{aligned} \tag{A.5}$$

where

$$
\begin{aligned}
\overline{x} &= 1 - x \\
\overline{y} &= 1 - y \\
\Delta &= p^2 x\overline{x}y + p'^2 x\overline{x}\overline{y} + q^2 x^2 y\overline{y} + m_3^2 x\overline{y} + m_2^2 x\overline{y} + m_1^2 \overline{x} \qquad \text{(A.6)}
\end{aligned}
$$

# APPENDIX B

# PROPAGATORS

The full propagator in the presence of an external gluon field can be written as:

$$S_q = S_q^{free} + S_q^G, \tag{B.1}$$

where

$$S_i^{free}(y,x) = -i \int \frac{d^4k}{(2\pi)^4} \frac{\not{k} + m_i}{k^2 - m_i^2} e^{-ik(y-x)} \tag{B.2}$$

where $i = u, d, s, c, b, t$ and

$$S_q^G(y,x) = -ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ik(y-x)}$$
$$\int_0^1 dv \left[ \frac{\not{k} + m_c}{(m_c^2 - k^2)^2} (G^{\beta\alpha})(ux + \overline{u}y)\sigma_{\beta\alpha} + \frac{1}{(m_q^2 - k^2)} (ux + \overline{u}y)_\beta (G^{\beta\alpha})\gamma_\alpha \right] \tag{B.3}$$