MULTI-CRITERIA DECISION MAKING WITH INTERDEPENDENT CRITERIA USING PROSPECT THEORY

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ABSTRACT

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In this study, an integrated solution methodology for a general discrete multi-criteria decision making problem is developed based on the well-known outranking method Promethee II. While the methodology handles the existence of interdependency between the criteria, it can also incorporate the prospect theory in order to correctly reflect the decision behavior of the decision maker. A software is also developed for the application of the methodology and some applications are performed and presented.

Keywords: Multiple Criteria Decision Making, Interdependency among Criteria, Prospect Theory

ÖZ

TERCİH TEORİSİ KULLANARAK BAĞIMLI KRİTERLERİN OLDUĞU DURUMDA ÇOK KRİTERLİ KARAR VERME

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Bu çalışmada en çok bilinen çok kriterli karar verme yöntemlerinden biri olan Promethee yöntemi baz alınarak melez bir sıralama metodolojisi geliştirilmiştir. Geliştirilen yöntem kriter ağırlıklarının hesaplanmasında kriterler arasındaki etkileşimin etkisini yansıtabilmesinin yanısıra karar vericinin karar vermede gösterdiği yaklaşımın sonuçlara etkisini de hesaplamada kullanabilmektedir. Ayrıca geliştirilen yöntemin uygulanmasında kullanılmak üzere bir yazılım da geliştirilmiştir. Bu yazılım kullanılarak yapılan örnek uygulamalar da tez kapsamında sunulmuştur.

Anahtar Kelimeler: Çok Kriterli Karar Verme, Kriterler Arası Bağımlılık, Tercih Teorisi

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CHAPTER 1

INTRODUCTION

Motivation and Scope

Discrete multi-criteria decision making (MCDM) problems are encountered very often within organizations. Some examples are R & D project selection, construction site selection, investment decisions, information system project selection problems etc. For decades various methodologies have been developed to systematically solve such problems. Since the topic covers a wide range of problems, most of the time, the methodologies developed concentrated on a specific property of the problem, that is, the methodologies are problem specific and it is very difficult to handle different scenarios. Also, some methodologies require important assumptions, for example ignoring the interdependency among criteria. With such assumptions, the methodology may end up with undesirable solutions when the impacts of such assumptions are not forecasted. For this reason, there is a clear necessity for a general methodology, which shall be applicable for all kinds of problems (or at least a high portion of them) with slight modifications. This master thesis study is conducted to develop such a methodology and the study of Karasakal et al. (2005) was the main inspiration point.

Since the main purpose is to rank the alternatives, the methodology is constructed upon the basis of the well-known outranking procedure called Promethee II. Promethee family methods do not suggest any specific technique to specify the weights of the criteria, which have a crucial influence towards the final ranking in a MCDM. In this study for the determination of the criteria weights, three alternative techniques are proposed namely, Analytical Hierarchy Process (AHP), Matrix Multiplication Technique and Analytical Network Process (ANP). The last two are suggested for the case of interdependency and feedback among the elements of the problem, which are important issues for MCDM problems. In literature, generally the criteria are assumed to be independent of each other, which is not the case most often however. For example, while deciding upon choosing which projects to follow, time duration that will elapse and the overall cost of the project can not be treated independently.

On the other hand, though it is proved with studies that prospect theory models the choice behavior of the decision maker more accurately, not many studies have been conducted to implement it into MCDM problems. Besides the classical preference functions proposed with Promethee family methods, functions representing the decision behavior of the decision maker in accordance with the prospect theory are also proposed in the methodology developed in this study.

Organization of the rest of the thesis is as follows: In the second chapter of this study, the literature survey about the topic and the related studies are presented. In chapter three, theoretical background about the techniques and methods that are utilized throughout the development of the methodology is given. Afterwards in chapter four the development of the methodology is described in detail. Chapter five introduces the software developed by explaining the user interfaces with an example application. A study conducted for the comparison of the weight determination techniques is presented in chapter six whereas in chapter seven and eight sample problems and their variations are solved and the final rankings are compared.

CHAPTER 2

LITERATURE SURVEY

In the context of discrete MCDM problems, many methodologies have been developed and proposed, utilizing numerous numerical and empirical methods. Analytical Hierarchy Process (AHP), developed by Thomas L. Saaty (1980), is one of the most popular techniques used by the researchers and practitioners. It is a pairwise comparison technique, which can model the complex problems in a unidirectional hierarchal structure assuming that there are no interdependencies between or within the levels. It is a relatively simple and intuitive tool, which allows the conversion of qualitative values into the quantitative ones. Some examples of the AHP applications in literature are as follows: Jain et al. (1996) uses simply AHP for a new venture selection problem, where both quantitative and qualitative values are easily handled. Khalil et al. (2002) used AHP to select the appropriate project delivery method. AHP is again used to assign proper weights in a 0-1 goal programming application by Kwak et al. (1997). Moreover, Gabriel et al. (2005) utilized AHP and Monte Carlo simulation when the data is uncertain. Tavana (2003) incorporated group decision making with AHP for evaluating and prioritizing advanced technology projects at NASA. In their study, Ramachandran et al. (2004) searched the applicability of AHP for sustainable energy policy decisions.

Saaty et al. (1986) revised AHP in their study so that it can handle the non-linear hierarchies. Saaty (1996) then developed the Analytical Network Process (ANP), which is a general form of AHP. Upon the beneficial properties of AHP, ANP can handle interdependence and feedback and reveals the composite weights through the calculations using the supermatrix phenomena. Ulutaş (2005) applied ANP to evaluate the alternative energy sources for the country. ANP is utilized for R & D project selection in a study by Meade et al. (2002). Cheng et al. (2007) re-solved the project selection problem, stating the errors made by Meade et al. They again studied the application of ANP in process models with an example on making decisions regarding strategic partnering. Shyur et al. (2005) used ANP during the development of a hybrid method where interdependency among criteria exists. Jarkharia et al. (2005) solved the problem of logistics service provider with an ANP application. Wey et al. (2007) used 0-1 goal programming together with ANP for a resource allocation problem in transportation infrastructure.

While ANP is a practical tool for handling interdependencies among the criteria, which is the main concern of this study, some other techniques are also developed for this purpose. For example, Karasakal et al. (2005) proposed a technique utilizing the "impact matrix" concept and the matrix multiplication operation in order to obtain the composite weights of the criteria. Carlsson et al. (1994) introduced interdependency concept into MCDM. Their suggestions are built upon three types of relationships between criteria, which are support, conflict and independency. They illustrated the technique with a numerical example. Later on, Östermark (1996) improved their technique. Preemptive Goal Programming technique is used in the study of Santhanam et al. (1994). They formed a multi-criteria model for solving an Information System project selection problem with interdependencies.

Another useful tool for solving discrete MCDM problems is Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE), a new class of outranking methods developed by Brans et. al. (1985). Using these methods, either a partial preorder (Promethee I) or a complete preorder (Promethee II) of all the alternatives can be proposed to the decision maker(s). Only a few parameters are asked to the decision maker(s), and they are easy to understand since they have an economic signification. Another Promethee Family method is Promethee V, introduced by Brans et al. (1992). With this method, a number of constraints are incorporated to the alternatives and the problem is converted to a 0-1 goal programming problem. This method is useful for resource allocation and project ranking-selection type of problems. Abu-Taleb et al. (1995) used Promethee V for a water resources planning problem in Middle East. Later Mavrotas et al. (2006) improved the Promethee V application of Abu-Taleb et al. (1995) with a project prioritization application.

AHP and PROMETHEE methods are analyzed and discussed together thoroughly in the study of Macharis et al. (2004). They state that operational synergies could be achieved by integrating Promethee and a number of elements associated with AHP. More specifically, they argue that AHP could be used during the weight determination stage of Promethee method, in which no particular weighing approach was suggested. Similarly, Wang et al. (2006) combined AHP and Promethee II to form a hybrid method to rank alternatives. They used AHP for determination of the weights of the criteria and to understand the structure of the problem whereas Promethee II for the final ranking. Similarly, Babic et al. (1996) used Promethee II together with AHP to specify the priorities of the criteria in a MCDM problem.

Besides using AHP-Promethee pair, there are also some other studies to develop hybrid methodologies which are the combinations of the unique and specific tools. For example, Lee et al. (2001) proposes an integrated approach for solving interdependent multi-criteria IS project selection problems using Delphi, ANP and 0-1 GP. In their study, ANP is used for weight determination of the criteria, since interdependencies exist between criteria.

Choice behavior of the decision maker is another issue that is concerned in this study. Keeney and Raiffa (1976) used Multi-Attribute Utility Theory (MAUT) to model the choice behavior of the DM for each criterion and evaluated the overall utility of each alternative for the DM by either additive or multiplicative utility function. The alternatives are then ranked according to the final utilities. However, it is commonly accepted that MAUT fails to reflect the actual choice behavior of the decision maker. Kahneman and Tversky (1979) came up with a new theory called Prospect theory, stating that the outcomes are expressed as positive or negative deviations (gains or losses) from a reference alternative or aspiration level and losses have higher impact than the gains. Currim et al. (1989) showed in their study that prospect theory “outperforms” utility theory for paradoxical choices.

Although researches, experiments and empirical studies prove that Prospect Theory better models the choice behavior, there are few studies in literature about applications within the context of MCDM. Though it was originally developed for single criterion problems, the ideas have been extended to MCDM problems as well by Korhonen et al. (1990). They conducted an experimental study to observe the choice behavior and their results were persistent with Prospect Theory. Salminen (1994) also incorporated the prospect theory to MCDM. In his study, piecewise linear marginal value functions are assumed to approximate the S-shaped value functions of prospect theory. Another study conducted by Karasakal et al. (2005) integrated the Prospect Theory into Promethee. They have adopted the weights associated with the criteria for the imprecise information situation through an interactive procedure with the decision maker.

This study incorporates the basic tools of MCDM to develop a ranking methodology. These tools are namely Promethee II method, AHP & ANP and Prospect theory.

CHAPTER 3

THEORITICAL BACKGROUNG

3.1 OUTRANKING METHODS

Ranking of alternatives is encountered very often in the MCDM type of problems. Outranking methods are utilized in order to derive a solution to such problems. They provide either weaker or poorer models than the utility function method; however, they are built upon fewer assumptions and require less effort, which makes these methods very popular and easily applicable indeed.

Outranking methods are based on pairwise comparison of possible alternatives along each criterion. The preference relations used for the pairwise comparisons are given in Table 3.1.

Table 3.1 Preference relations used in outranking methods

Preference relation	Explanation
a I b	There is indifference between a & b.
a P b	a is strictly preferred to b
a Q b	a is weakly preferred to b
a R b	a is incomparable with b
a S b	a is at least as good as b

The criterion of the concern could be of three types:

I. If the criterion is ***real (true) criterion*** the preference relation could be defined as

$$a\,P_j\,b \quad \Leftrightarrow g_j(a) > g_j(b)$$

where $g_j(a)$ is the j^{th} criterion value of alternative a.

II. The criterion might be a ***quasi-criterion***, which is the evaluation of the alternatives' performance in terms of that criterion is often uncertain and imprecise. In that case one way to take this into account is to introduce an indifference threshold, $q_j \geq 0$, such that if the performances of the two alternatives on criterion *j* differ by less than q_j, then there is indifference relation I_j such as,

$$a\,P_j\,b \quad \Leftrightarrow g_j(a) - g_j(b) > q_j$$

$$a\,I_j\,b \quad \Leftrightarrow \left|g_j(a) - g_j(b)\right| \leq q_j$$

III. If the criterion is ***Pseudo-criterion****,* in addition to the indifference threshold, there might be a strict preference threshold for the criterion *j,* $p_j \geq 0$, to distinguish between strict preference and weak preference. Hence,

$$a\,P_j\,b \quad \Leftrightarrow g_j(a) - g_j(b) > p_j$$

$$a\,Q_j\,b \quad \Leftrightarrow q_j \leq g_j(a) - g_j(b) \leq p_j$$

$$a\,I_j\,b \quad \Leftrightarrow \left|g_j(a) - g_j(b)\right| \leq q_j$$

The Promethee family methods (Brans, Vicke and Mareschal, 1986) are the most well known and applied outranking method. Their main features are simplicity, clearness and stability. These methods are suitable to solve the multi-criteria problem of the type

$$Max\{f_1(a),...,f_n(a) | a \in K\}$$

where K is a finite set of alternatives and f_i, $i = 1,...,n$, are n criteria to be maximized.

The method is based on the pairwise comparison of alternatives with respect to each criterion according to a valued outranking relation which belongs to that criterion. There is a weak assumption of preferential independence among the criteria in the Promethee methods, i.e., it is assumed that the criterion values has no influence on the preference function of another criterion. First the type of each criterion (preference function) and -if necessary- the corresponding parameter(s) are defined at the beginning of the decision process with the decision maker(s). The decision maker is asked only for a few parameters, which all have an economic significance so that the decision maker is able to determine their values intuitively.

The weights can be determined using various methods, and the overview of these methods are listed and analyzed in the study of Nijkamp et al. (1990). Promethee family methods do not provide specific guidelines for determining these weights, but assumes that the decision maker is able to assign weight to each criterion appropriately, at least when the number of the criteria is not large. In this study, the determination of the weights of the criteria covered extensively and alternative methods are presented in chapter 4.

The preference function, each of which is specific to the criterion assigned, translates the difference between the evaluations obtained by two alternatives in terms of a particular criterion n, into a preference degree ranging from 0 to1. Let $P_n(a,b)$ be the preference function associated to the criterion n, where

$$P_n(a_i, a_j) = G_n \lfloor f_n(a_i) - f_n(a_j) \rfloor$$

$$0 \leq P_n(a_i, a_j) \leq 1$$

Here G_n is a non-decreasing function of the observed deviation between $f_n(a_i)$ and $f_n(a_j)$. Six basic types of preference functions are proposed for the selection (Brans et al. 1986).

After a weight, w_n, is assigned to each criterion and the preference values are calculated, the outranking degree (or overall preference index), $\pi(a_i,a_j)$ is obtained for each alternative pair as follows:

$$\pi(a_i, a_j) = \sum_n P_n(a_i, a_j) \cdot w_n$$

Using the outranking degrees, the *entering flow* and the *leaving flow* indices for each alternative are computed as follows:

$$\phi^-(a_i) = \sum_j \pi(a_j, a_i) \quad \forall a_i \in K \quad \text{(Entering Flow)}$$

$$\phi^+(a_i) = \sum_j \pi(a_i, a_j) \quad \forall a_i \in K \quad \text{(Leaving Flow)}$$

Until this point, all the steps are same for all methods of the Promethee family.

Intuitively, higher the leaving flow and lower the entering flow, the more preferable the alternative is. *Promethee I* uses the entering flow and the leaving flow values to first obtain two separate rankings and finally a partial preorder of the alternatives by taking the intersection of these two rankings.

In Promethee II, the *net flow*, $\Phi(a_i)$, is calculated for each alternative as follows:

$$\Phi\ (a_i) = \phi^+(a_i) - \phi^-(a_i) \quad \forall a_i \in K$$

According to the net flow value of each alternative, a complete preorder on the set of possible alternatives is proposed to the decision maker.

3.2 ANALYTICAL HIERARCHY PROCESS (AHP)

The analytical hierarchy process (AHP) (Saaty, 1982, 1988, 1995) is one of the best known and most widely used MCDM approach. In AHP, the top element of the hierarchy is the overall goal for the decision model. The hierarchy decomposes from the general to a more specific criterion until a manageable decision criteria is met. On the other hand it can incorporate both quantitative and qualitative components in a complex decision making problem.

Overall, AHP is based on three principles, namely construction of the hierarchy, priority setting and logical consistency.

3.2.1 Construction of the Hierarchy

A decision problem centered around measuring contributions to an overall goal, is structured and decomposed into its constituent parts (i.e. criteria, sub-criteria alternatives, etc.), using a hierarchy.

3.2.2 Priority Setting

The relative “priority” given to each element in the hierarchy is determined by comparing pairwise the contribution of each element at a lower level in terms of the criteria (or elements) with which a causal relationship exists. The decision maker uses a pairwise comparison mechanism, the square matrix as shown in Figure 2.1.

$$
\begin{array}{c|ccccc}
C & x_1 & \cdots & x_j & \cdots & x_n \\
\hline
x_1 & & & & & \\
\vdots & & & & & \\
x_i & & & P_c(x_i, x_j) & & \\
\vdots & & & & & \\
x_n & & & & &
\end{array}
$$

Figure 3.1 Square matrix

The following statements hold for a pairwise comparison square matrix:

$P_c(x_i, x_j) = 1$ if $i = j$.

$P_c(x_i, x_j) = 1 / P_c(x_j, x_i)$ $\forall$ (i,j) pairs

The pairwise comparison is based on a scale of *1 – 9* as the interpretations are given in Table 3.2.

Table 3.2 Interpretation of Saaty`s 1 – 9 scale

Scale	**Interpretation**
1	Equal Importance
3	Weak importance of one over another
5	Essential or strong importance
7	Very strong importance
9	Absolute importance
2,4,6,8	Intermediate values between two adjacent values
Reciprocals	If factor *i* has one of the above numbers assigned to it when compared with *j*, then *j* has the reciprocal value when compared with *i*.

Formally the relative priorities (or weights) of each element with respect to a higher level element in the hierarchy are given by the right eigenvector (W) corresponding to the highest eigenvalue ($\lambda_{\max}$) as follows:

$$A \cdot W = \lambda_{\max} \cdot W$$

The pairwise comparison matrix shown in Figure 3.1 is represented by letter A. Its standard element is $P_c(x_i,x_j)$, that is the intensity of the preference (in terms of contribution to a specific criterion (c) of the row element (x_i) over the column element (x_j).

Since the decision maker makes multiple pairwise comparisons among a set of elements, the problem of "consistency" arises. The consistency check procedure for the pairwise comparisons is explained in the next section in this chapter.

In case the pairwise comparisons are completely consistent, the matrix A has rank 1 and $\lambda_{max} = n$. In that case weights can be obtained by normalizing any of the columns of A. Else if the consistency check reveals that the comparisons are not consistent enough, the data entered should be revised and updated by the decision maker and afterwards the weights could be calculated.

The procedure described above is repeated for all subsystems in the hierarchy. In order to synthesize various priority vectors, these priorities are weighted with global priorities of the parent criteria and synthesized. This process starts at the top of the hierarchy. As a result, the overall relative priority to be given to the lowest elements is obtained. These overall relative priorities indicate the degree to which the elements contribute to the top of the hierarchy (goal).

3.2.3 Consistency Check

As mentioned in section 3.2.2, in case the pairwise comparisons are completely consistent, the matrix A has rank 1 and $\lambda_{\max}$ = n. In that case weights can be obtained by normalizing any of the columns of A. In fact in case of complete consistency, the following preference relation holds for any subset (a_i, a_j, a_k):

$$P_c(a_i, a_j) = P_c(a_i, a_k)P_c(a_k, a_j) \qquad \forall i, j, k.$$

For example, let there be three elements, *x*, *y*, *z* to be compared. If *x* is preferred to *y* and *y* is preferred to *z*, then by transitivity property *x* should be preferred to *z*. If this property holds for all the comparisons of the decision maker for some degree, then the pairwise comparisons are said to be consistent (or consistent enough).

However it is very unlikely for a decision maker to make the pairwise comparisons through a perfect consistent manner. In case the inconsistency of the pairwise comparison matrices is limited, slightly the highest eigenvalue (λ_{max}) deviates from *n*. This deviation (λ_{max}-*n*) is used as the measure for inconsistency. This measure is divided by *n-1* to obtain the "*consistency index*" (*CI*) as follows:

$$CI = \frac{\lambda_{\max} - n}{n - 1}$$

The final "consistency ratio" (CR), on the basis of which one can conclude whether the evaluations are sufficiently consistent, is calculated as follows:

$$CR = \frac{CI}{CI *}$$

Where *CI* is the consistency index and *CI** is the random consistency index. The random consistency indices (*CI**`s given in Table 3.3) are the experimental results of studies conducted by the scientists and correspond to the degree of consistency that arises when random pairwise comparison matrices are generated with values on the 1-9 scale.

Table 3.3 Random consistency indices

Random consistency indices											
n	1	2	3	4	5	6	7	8	9	10	11
*CI**	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51

Saaty (1982) claimed that the inconsistency ratio (*CR*) should not be higher than 10% ($CR \leq 0.10$). An inconsistency level higher than 10% means that the consistency of the pairwise comparisons is insufficient and the pairwise comparison matrix is said to be "*not consistent enough*" so the decision maker must review his/her judgments he\she made during the pairwise comparison stage.

3.3 ANALYTICAL NETWORK PROCESS (ANP)

ANP is a general form of AHP, developed by Saaty (1996). While AHP models a decision making framework that assumes a unidirectional hierarchical relationship among clusters, ANP allows more complex interrelationships among the clusters and elements within the clusters. As illustrated in Figure 3.2 (3 node situation: Linear Hierarchy and Non-linear Network) and with a sample network in Figure 3.3 (two clusters, Main Goal and Criteria) ANP does not require the strict unidirectional hierarchy of the AHP. There are also arcs between the elements of a cluster and towards the top of the hierarchy. Two-way arcs in the network structure represent interdependencies and feedbacks among clusters and elements. In case of linear hierarchy matrix manipulation (multiplication) technique is proposed whereas supermatrix method is proposed for non-linear networks (Saaty et al. 1986). Matrix manipulation is utilized in the studies of Karasakal et al (2005), Wey et al (2007) and Shyur (2006).

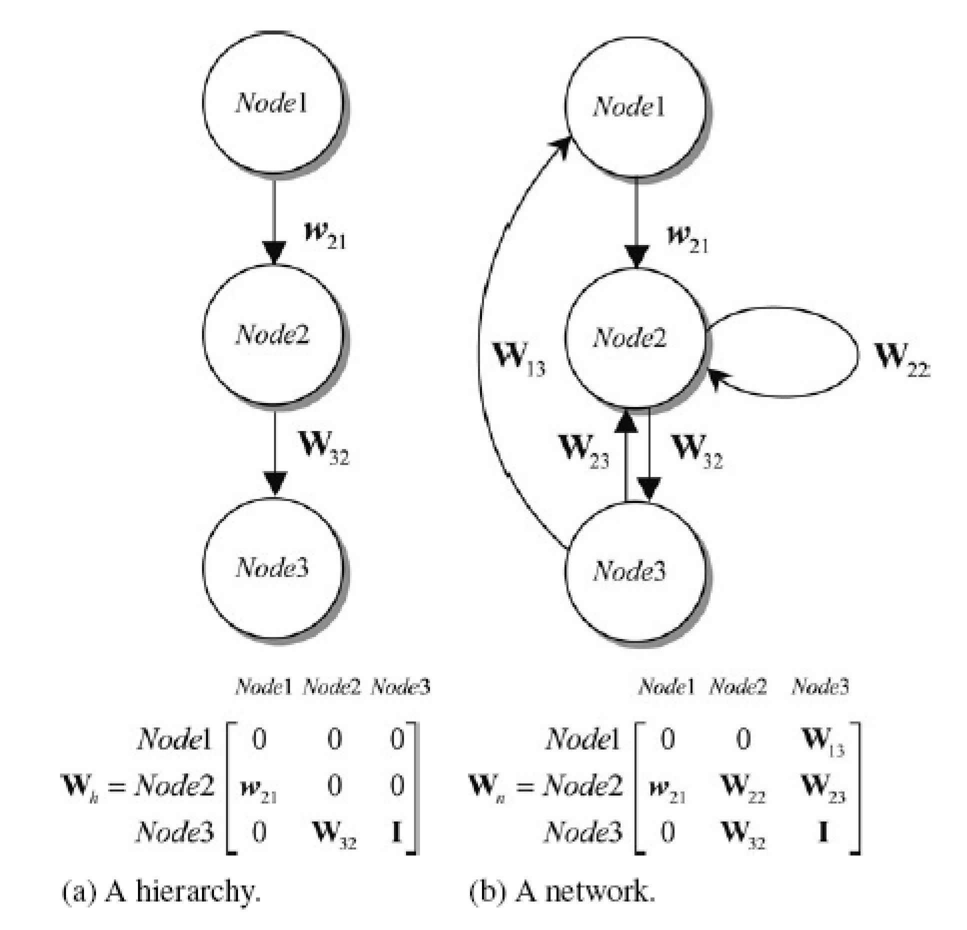


Figure 3.2 (a) Linear hierarchy and (b) nonlinear network. W_{ij} refers to influence matrix of cluster *i* on cluster *j*. (Wey et al., 2006)

Although all the arcs in a network have the same meaning mathematically, the interpretations differ according to whether they are between the clusters or within a cluster. Arcs emanating from an element indicate relative importance, influence or feedback. For example, the blue arcs in Figure 3.3 refer to relative priorities of the criteria with respect to the main goal while the red ones refer to the influences between the criteria and the black ones are the feedbacks from criteria to main goal. The corresponding values of the arcs are measured on a ratio scale similar to AHP.

ANP approach is capable of handling interdependency among elements by obtaining the composite weights through the development of a supermatrix. The supermatrix is the combination of individual square matrices that correspond to each cluster. They represent the network within each cluster. All in all, the supermatrix is a single matrix showing all the elements in all clusters.

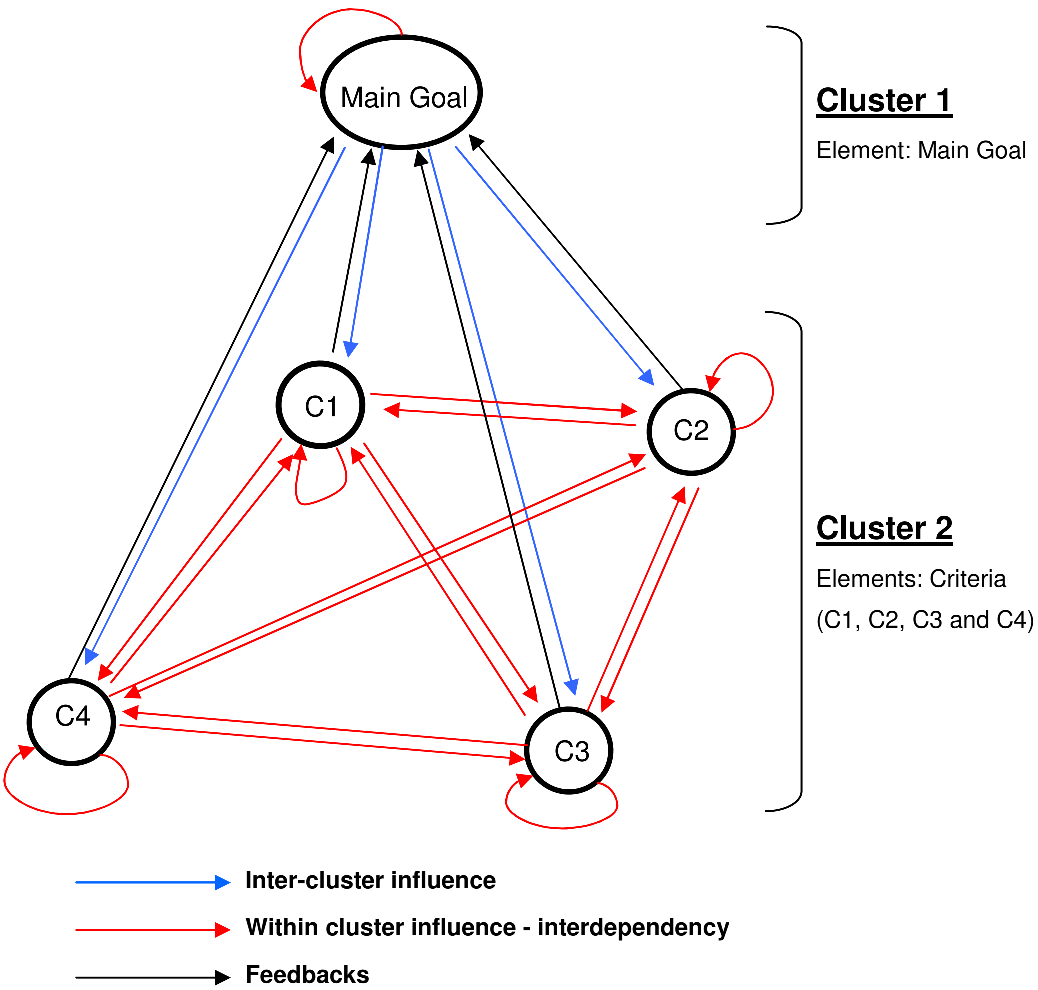


Figure 3.3 Illustration of a sample ANP network

Saaty explains the supermatrix concept parallel to the Markov Chain Processes. By incorporating interdependencies (i.e. addition of the feedback arcs in the model), the supermatrix is created. Feedback arcs are important because there should exist complete loops for supermatrix application. In other words, from the Markov Chains point of view, all the elements (nodes) should be recurrent instead of being transient so that the effects of the influences on the final results do not vanish for some elements (nodes) during the application.

Assume that there is a system of N clusters where the elements in each cluster have impact on or are influenced by some or all of the elements of that cluster or of other clusters with respect to a property governing the interactions of the entire system. Assume that cluster h, denoted by C_h, $h=1,\ldots,N$, has n elements denoted by e_{h1}, $e_{h2},\ldots,e_{hn}$. The structure of the corresponding supermatrix is illustrated in Figure 3.4.

During building up the supermatrix, it is extremely important to be consistent about the question asked to the decision maker. Saaty (1999) proposes two types of questions formulated in terms of dominance or influence. Given a parent element, which of two elements being compared with respect to it has greater influence (is more dominant) on that parent element? Or, which is influenced more with respect to that parent element?

For example, in comparing A to B (elements in a cluster) with respect to a criterion, the question asked is whether the criterion influences A or B more. Then if for the next comparison involving A and C the question asked is whether A or C influences the criterion more, this would be a change in perspective that would undermine the whole process. One must keep in mind whether the influence is flowing from the parent element to the elements being compared, or the other way around. Considering this, it is crucial to stick to the perspective during the pairwise comparisons

$$
\begin{array}{cc|cccc}
 & & C_1 & C_2 & \cdots & C_N \\
 & & e_{11}e_{12}\cdots e_{1n_1} & e_{21}e_{22}\cdots e_{2n_2} & \cdots & e_{N1}e_{N2}\cdots e_{Nn_N} \\
\hline
 & e_{11} & & & & \\
 & e_{12} & & & & \\
C_1 & \vdots & W_{11} & W_{12} & \cdots & W_{1N} \\
 & e_{1n_1} & & & & \\
 & e_{21} & & & & \\
 & e_{22} & & & & \\
C_2 & \vdots & W_{21} & W_{22} & \cdots & W_{2N} \\
 & e_{2n_2} & & & & \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
 & e_{N1} & & & & \\
C_N & e_{N2} & W_{N1} & W_{N2} & \cdots & W_{NN} \\
 & \vdots & & & & \\
 & e_{Nn_N} & & & &
\end{array}
$$

Figure 3.4 The supermatrix

Saaty suggests one of the following two questions throughout a process:

1. *Given a parent element and comparing elements A and B under it, which element has greater influence on the parent element?*
   (The direction of the arrow is to the parent element)

2. *Given a parent element and comparing elements A and B, which element is influenced more by the parent element?*
   (The direction of the arrow is from the parent element)

To be consisted throughout this study, the first question is posed during the pairwise comparisons. That is, the eigenvector in the column of an element (either the main goal or any of the criteria) in the supermatrix indicates the relative influences of the row elements on the column element. In other words, the numbers in a column are the relative priorities of the elements with respect to the element corresponding to that column.

After the pairwise comparisons, each eigenvector is obtained and introduced in the appropriate position as a column vector as shown in Figure 3.4. While building up the supermatrix, the eigenvectors in the individual matrices are adjusted by normalization with respect to the relative weights of the clusters they belong. When this is done, the supermatrix becomes column stochastic and from this point on it is called "weighted supermatrix". This should be performed before any operation on the supermatrix in order to derive meaningful limiting priorities. From the network perspective, this operation makes the sums of the arrows emanating from an element equal to unity, which is essential from the Markov Chains point of view before any limiting operations on the supermatrix. In general the supermatrix is rarely stochastic because, in each column, it consists of several eigenvectors each sums up to one, and hence the entire column of the matrix may sum up to an integer greater than one. Normalization would be meaningless and such weighting does not call for normalization.

When the matrix is column stochastic, the limiting priorities depend on its reducibility and periodicity properties (Kulkarni 1999). (Analogy: Here the term "limiting priorities" could be perceived as the "limiting probabilities" concept of the Discrete Time Markov Chains (DTMC). Mathematically, both terms refer to exactly the same value). Because of the existence of the feedback arcs, the elements of the supermatrix become recurrent, that is the supermatrix is irreducible form the DTMC point of view.

However, for limiting operations, it is important whether the supermatrix is periodic or aperiodic. According to the definition given below within the context of DTMC the supermatrix has to be aperiodic before being raised up to powers. (Here again there is analogy between the supermatrix and the "one step transition matrix" of DTMC):

Definition (Periodicity), (Kulkarni, 1999). Let $\{X_n, n \geq 0\}$ be an irreducible DTMC on state space S = {1,2,…,N}, and let *d* be largest integer such that

$$P(X_n = i | X_0 = i) > 0 \Rightarrow \quad n \text{ is an integer multiple } d,$$

for all $i \in S$. The DTMC is said to be periodic with period *d* if $d > 1$ and aperiodic if $d = 1$.

A DTMC with period *d* can return to its starting state only at times *d*, 2*d*, 3*d*,…. In particular if $P(X_1 = i | X_0 = i) > 0$ for any $i \in S$ for the irreducible DTMC, than *d* must be 1, and the DTMC must be aperiodic. The interpretation of this fact from the ANP (Supermatrix) point of view is as follows: If at least there is one element that has a self-influence (in the network the arrow emanates from and ends at the same element), then the supermatrix is said to be aperiodic. Since, in the methodology developed, all the criteria have self-influences, there is no risk for the weighted supermatrix to be periodic.

The weighted supermatrix is raised to a sufficiently large power until the priorities converges to stable values. An irreducible and aperiodic weighted supermatrix yields a limiting matrix with all the columns equal to each other. After the limiting operation, the values corresponding to the elements of the cluster under consideration are normalized among themselves to obtain the relative weights.

3.4. PROSPECT THEORY

Decision maker(s)' choice behavior is an important issue for the modeling of decision making problems. For decades, the classical expected utility theory developed by Keeney and Raiffa (1976) had been accepted as the dominant paradigm. However, there has been a general agreement that the theory does not represent the actual decision behavior of the decision maker(s). Many empirical studies proved that decision maker(s) systematically violate this theory's basic tenets. Kahneman and Tversky (1979) came up with a new theory called the Prospect Theory, which explains the major violations of utility theory and the intransitive behavior represented by the decision maker.

According to prospect theory, the outcomes are expressed as positive or negative deviations (gains or losses) from a reference alternative or aspiration level. Although value functions differ among individuals, Kahneman and Tversky propose that they are commonly S-shaped: concave above the reference point, and convex below it. Furthermore preference functions are commonly assumed steeper for losses than that of gains as shown on Figure 3.5. This can be interpreted as follows: Loosing has a higher impact (in magnitude) than winning the same amount.

Hence

$$-f(-x) \geq f(x)$$

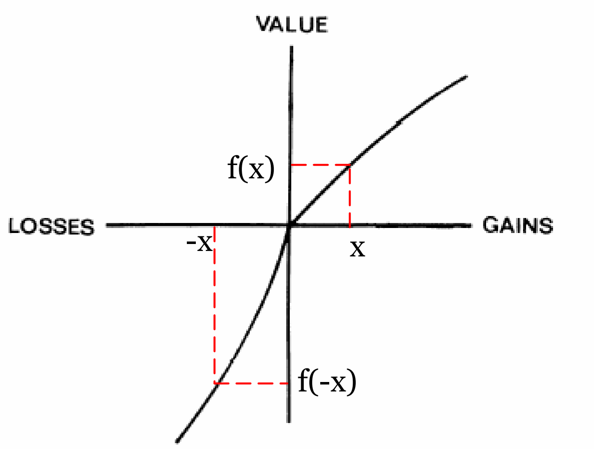


Figure 3.5 General preference function according to Prospect Theory

Prospect theory was originally developed for single criterion problems with uncertainty, but the ideas have been extended to multiple criteria decision making problems as well by Korhonen, et al. (1990). Salminen (1994), Karasakal et al. (2005) and Karasakal & Özerol (2006) also incorporated the Prospect Theory into MCDM. Apart from these studies, application of prospect theory in MCDM is not studied much and there are very limited resources in the literature.

CHAPTER 4

THE PROPOSED METHODOLOGY

The methodology proposed in this study is a combination of different methodologies that are individually very common, easy to understand and popular among the researchers in the field of multi-criteria decision making problems. Each methodology has a specific advantage with respect to different perspectives, and they are applicable for a problem only if some specific criteria corresponding to the methodology are met. Bringing up the useful properties of some methods and combining them to develop a hybrid methodology has been accepted as an important progress for some years in the field of MCDM and studied by many researchers. For example, Lee et al. (2001) proposed an integrated approach for solving interdependent multi-criteria IS project selection problems using Delphi, ANP and 0-1 GP. On the other hand, in the study of Macharis et al. (2004), AHP and PROMETHEE family methods are analyzed and discussed thoroughly and they state that operational synergies could be achieved by integrating Promethee and a number of elements associated with AHP. They proposed that AHP could be used during the weight determination stage of PROMETHEE method, in which no particular weighing approach was suggested. Similarly, Wang et al. (2006) combined AHP and PROMETHEE to form a hybrid method to rank alternatives. They used AHP for the determination of the weights of the criteria and to understand the structure of the problem, on the other hand PROMETHEE for the final ranking.

The developed methodology is explained in detail in this chapter. In the first part, alternative procedures proposed for the determination of the weight of the criteria are described. And in the second part Promethee II application is explained where new preference functions reflecting decision behavior of the decision maker are introduced.

4.1 DETERMINATION OF THE CRITERION WEIGHTS

The most crucial part of the multi-criteria decision making problem is to specify the weights of the criteria, which in essence, is the most determining stage for the final solution. Therefore, an intense care is spent throughout the study. Three alternative techniques, which are proposed to the decision maker, are presented in this part for weight determination.

4.1.1 *Technique 1*: AHP (Independent criteria)

In most cases to simplify the procedure in MCDM problems, the criteria are assumed to be independent of each other, that is, there are no interrelationships between the criteria. In other words, the existence or non-existence of a criterion has no effect on others. Simply AHP is utilized in such cases for the determination of the weights of the criteria. The question posed to the decision maker during the pairwise comparisons is as follows:

> *"Which criterion is more important with respect to the main goal and how much?"*

In this technique the decision maker makes $\frac{1}{2}*(n-1)*(n-2)$ pairwise comparisons (should be consistent enough), where n is the total number of criteria, and the eigenvector corresponding to the highest eigenvalue yields the weights for criteria W_o.

4.1.2 *Technique 2*: Matrix Multiplication Method

Matrix multiplication is composed of three phases. In the first phase just like in *technique 1,* the original weights, W_o of the criteria are calculated using AHP ignoring the interdependencies among them.

In the second phase the relative influences on each other is obtained again using AHP. Considering each criterion, a pairwise comparison is made among all criteria in terms of the magnitude of the impact inflicted on the criterion under consideration. The comparisons are made according to the Saaty's 1 - 9 scale (Table 3.2). During the pairwise comparisons, the decision maker is supposed to answer the following question:

> *"Given a reference criterion, which criterion influences the criterion under consideration more and how much?"*

Again here the consistency plays an important role, that is, the pairwise comparisons of the decision maker must be consistent enough among themselves. Suppose there are n criteria and the comparison matrix is formed as shown in Figure 4.1.

$$
\begin{array}{c|ccccc}
C_i & C_1 & \cdots & C_k & \cdots & C_n \\
\hline
C_1 & 1 & \cdots & x_{i1k} & \cdots & x_{i1n} \\
\vdots & \vdots & & \vdots & & \vdots \\
C_j & x_{ij1} & \cdots & x_{ijk} & \cdots & x_{ijn} \\
\vdots & \vdots & & \vdots & & \vdots \\
C_n & x_{in1} & \cdots & x_{ink} & \cdots & 1
\end{array}
$$

Figure 4.1 The criteria comparison matrix w.r.t. criteria *i*

In the comparison matrix x_{ijk} refers to the factor that how many times more the criterion j influences the parent (reference) criterion i than the criterion k.

After the comparison matrix is formed, the eigenvector corresponding to the highest eigenvalue is calculated, yielding the relative influences of the criteria on the criterion *i*:

$$E_i^T = [x_{1i}, \quad \cdots \quad x_{ji}, \quad \cdots \quad x_{ni}]$$

At this point, one must notice that these numbers are not the weights of the criteria with respect to the main goal; instead they can be interpreted as the relative influences on the criterion under consideration.

Similarly all the eigenvectors for each criterion are obtained and brought together to form the *impact matrix, E,* as shown in Figure 4.2. In the impact matrix, x_{ji} refers to the relative influence of criterion *j* on criterion *i*.

$$E = \begin{array}{c|ccccc} & C_1 & \cdots & C_i & \cdots & C_n \\ \hline C_1 & x_{11} & \cdots & x_{1i} & \cdots & x_{1n} \\ \vdots & \vdots & & \vdots & & \vdots \\ C_j & x_{j1} & \cdots & x_{ji} & \cdots & x_{jn} \\ \vdots & \vdots & & \vdots & & \vdots \\ C_n & x_{n1} & \cdots & x_{ni} & \cdots & x_{nn} \end{array}$$

Figure 4.2 The impact matrix

Lastly, the final weights *W*, of the criteria are obtained as follows:

$$W = E \times W_o$$

Where

E : Impact matrix

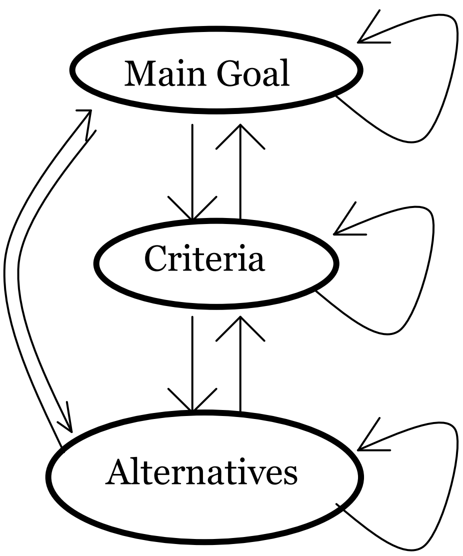
W_o : Weights obtained assuming there is no interdependency

W : Final weights of the criteria

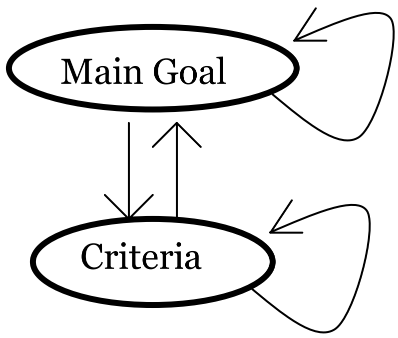
4.1.3 *Technique 3*: ANP - The Supermatrix Method

This technique is proposed if there exist interdependencies among criteria. It utilizes the supermatrix phenomena introduced by Saaty (1996) as the tool for the ANP concept. Since ANP can handle the interrelationships among the clusters and the elements, it can be used as the weight determination tool where there are interdependencies.

Thanks to the conventional problem structure shown in Figure 4.3.a, ANP can be used for discrete MCDM problems, where there are feedbacks and interrelationships. However, in this study, the main concern is to determine weights of the criteria with respect to the main goal. For this reason, unlike the classical structure, the alternatives cluster can be eliminated together with the arcs arriving at and leaving from it (Figure 3.3.b). In other words, while building up the supermatrix, ignoring the alternatives and leaving only the main goal and the criteria is sufficient to determine the weights of the criteria with respect to the main goal.



(a) The conventional structure



(b) The reduced structure

Figure 4.3 (a) The conventional and (b) the reduced problem structure

On the other hand, this reduction in the network structure also reduces the size of the supermatrix, which directly simplifies the operations to be done. As shown in Figure 4.4 all the rows and the columns corresponding to alternatives are deleted

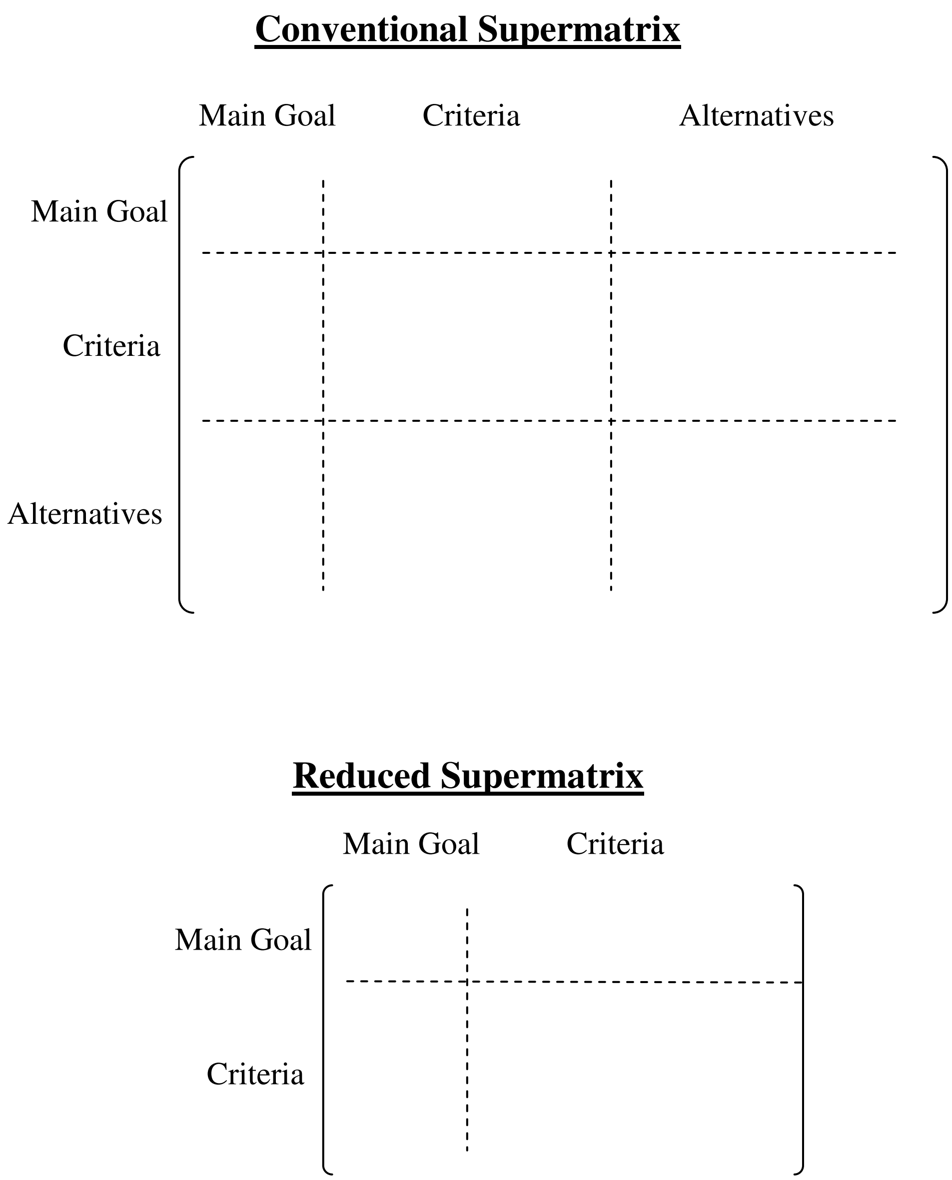


Figure 4.4 The conventional and the reduced supermatrix

Filling in the reduced supermatrix is another important step that requires extra care. Keeping in mind that Saaty (1996) developed the supermatrix tool parallel to the

"Markov Chains" concept, it is useful to construct the analogy. That is, considering every criteria and the main goal as the nodes or states, which are the elements of the set of possible outcomes, introduces meaning and significance to the technique applied

At this point, there is an important question that the decision maker has to answer:

> *"By how much amount do you want the interdependencies among the criteria to take effect on the final weights of the criteria?"*

Answering this question, the decision maker decides on the weights of the two clusters. They are the Main Goal cluster, which has only a single element the main goal, and the criteria cluster whose elements are the criteria. The weights are specified in a way that the sum equals to 1, that is

$$W_m + W_c = 1$$

where

W_m: weight of the main goal cluster (influence of the original weights)

W_c: weight of the criteria cluster (influence of the interdependency)

And the notation for the ANP application is ANP(w_1, w_2), where

$$w_1 = W_m \quad \text{and} \quad w_2 = W_c$$

For example, let's say in case 1, the decision maker sets W_c=0.1, W_m automatically becomes 0.9, which means that original weights have 9 times more impact on the final weights than the interdependencies. And in case 2 it is vice versa. According to these numbers, in case 1, the technique will yield closer solution to the original weights than in case 2.

After determining the weights of the two clusters, the supermatrix can be filled. First the original weights of the criteria with respect to main goal are determined as if there is no interdependency among them just like in *technique 1* and the numbers obtained are inserted into the first column to the corresponding rows, in the *region II* as indicated in Figure 4.5. Second the *impact matrix,* E, whose columns sum to unity, is obtained as described in technique 2. Before inserting the impact matrix into *region III*, it is updated by multiplying with W_c, weight of the criteria cluster, hence column sums of *region III* equals W_c.

$$S = \begin{array}{c|c:c} & MainGoal & Criteria \\ \hline MainGoal & I & IV \\ \hdashline Criteria & II & III \end{array}$$

Region I: Influence of **main goal** on **main goal**

Region II: Relative influences of **criteria** on **main goal**

Region III: Relative influences of **criteria** on **criteria**

Region IV: Influence of **main goal** on **criteria**

Figure 4.5 Regions of the reduced supermatrix and interpretations

Finally every element of the *region IV* is set to W_m, indicating the objective position of the main goal with respect to the interdependencies within the criteria cluster. The single node in *region I* is set to zero, because it has no effect in the final results even when set to a positive number, but only reduces the convergence speed during the limiting operations.

All in all, one must spent extra care while computing the eigenvectors in the regions II and III. For convenience, the question asked during the pairwise comparisons for these regions are given below (one must note that no question is asked for region I since its value is entered zero and the value for region IV is manipulated according the cluster weights):

> "*With respect to the main goal, which criterion has more influence and how much?*" (Region II)

> "*Given the reference criterion, which criterion has more influence on the criterion under consideration and how much?*" (Region III).

Finally the supermatrix is constructed as shown below. Before proceeding with the operations on the supermatrix, one must note that it is important to check that the supermatrix is column stochastic, i.e. all the column sums up to unity.

$$S = \begin{array}{c|cc} & MainGoal & Criteria \\ \hline MainGoal & 0 & W_m \\ Criteria & W_o & W_c \cdot E \end{array}$$

$$S = \begin{array}{c|c|cccc} & MainGoal & C_1 & C_2 & \cdots & C_n \\ \hline MainGoal & 0 & W_m & W_m & \cdots & W_m \\ \hline C_1 & W_{o1} & W_c \cdot x_{11} & W_c \cdot x_{12} & \cdots & W_c \cdot x_{1n} \\ C_2 & W_{o2} & W_c \cdot x_{21} & W_c \cdot x_{22} & \cdots & W_c \cdot x_{2n} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n & W_{on} & W_c \cdot x_{n1} & W_c \cdot x_{n2} & \cdots & W_c \cdot x_{nn} \end{array}$$

where

$$x_{ij} \in E \qquad i, j : 1,2,...,n$$

and

$$\sum_{i}^{N} x_{ij} = 1 \quad \forall \quad j : 1,2,...,n$$

At this point the analogy with the Markov Chains (MC) concept could be set as follows: All the elements within the clusters (i.e. the main goal and the criteria) could be perceived as states of a Discrete Time Markov Chain (DTMC) and the set of possible outcomes (states) conforms an irreducible closed set in which all the states are recurrent. From DTMC point of view, the supermatrix S obtained is the one-step transition matrix. Intuitively the limiting probabilities to be obtained will refer to the priorities of the elements. Since the one-step transition matrix (supermatrix, S) is irreducible and aperiodic as explained in section 3.3, the *Limiting Probabilities*, π could be obtained as follows:

$$\pi = \lim_{n\to\infty} S^n(i, j)$$

Not necessarily it is required to raise the supermatrix to a high degree power, instead in most cases it is sufficient to raise it to the power in the order of 100`s, because the system rapidly converges, and an ordinary CPU performs this operation within seconds for a 10x10 supermatrix (9 criteria and the main goal for example). During the operation, one must spend special care that the supermatrix is column stochastic, otherwise the matrix does not converge.

The columns of the obtained limiting matrix are exactly same and it sums up to unity. The numbers corresponding to the criteria in region III must be re-normalized since the number corresponding to the main goal element does not have any significant role. The re-normalized numbers are the weights of the criteria with respect to the main goal in the case of existence of interrelationships among criteria.

All in all, the criteria weights are computed considering that all the criteria are at the same level (single cluster). However, when the number of the criteria is large, constructing the impact matrix is not an easy task in case of existence of interdependency. For such situations the criteria could be rearranged such that sub-clusters are formed. Also in some problem, there must be some sub-clusters due to the nature of the problem. For such cases, instead of handling all the criteria at a single step simultaneously, simple operations could be performed on the sub-clusters using the matrix manipulation technique (Saaty et al. 1986) and the composite weights of the criteria could be obtained.

4.2 PROMETHEE II APPLICATION

After the determination of the weights of the criteria, alternatives could be ranked with Promethee II method. Like the weight determination stage, this part also requires interaction with the decision maker, in order to understand his/her perception of each criterion one by one.

At this stage, through an interactive procedure, the proposed methodology tries to grab three important aspects of the problem, which are crucial all the way towards the solution:

1. For a specific criterion, does the decision maker have a preference function that is parallel to prospect theory?

2. For a specific criterion, which preference function among the presented types, best suit and represent nature of that criterion?

3. What are the values of the parameters which are specific for the type of the preference function determined?

In the beginning, during the construction of the problem, while introducing the criteria, the decision maker is asked the following question for each criterion:

> *"Considering the criterion under consideration, minimum how many unit(s) of gain can satisfy you upon one unit of loss?"*

From the utility theory point of view, the answer should be always "*one*", which makes sense mathematically. From the prospect theory point of view, this is not always the case; generally "*more than one*" unit of gain is necessary for satisfaction. For this reason, according to the answer given to the question above, two groups of preference functions are proposed to the decision maker.

I. If the answer is "one", the six basic types of preference functions defined in the study of Brans et al. (1986) (I, II, .., VI) are proposed. These functions and the parameters required for each function are summarized in Figure 4.6. What is significant here is that these functions are symmetrical with respect to the vertical axis.

II. If the answer is "more than one", two new preference functions, VII and VIII, in accordance with prospect theory are proposed because this answer is perceived as the indication of prospect theory in the choice behavior of the decision maker for the criterion under consideration. These two preference functions are illustrated in Figure 4.7 and explained in detail in section 4.3. One must note that symmetrical property of the previous set of functions does not exist in this new set of functions.

After decision maker specifies all the preference functions and corresponding parameters for each criterion, the Promethee II method can be applied for the complete ranking.

In the methodology developed, the crucial part of the Promethee II application is the incorporation of the prospect theory. As explained in the previous chapter, the overall preference index of an alternative pair is calculated as

$d \geq 0 \Rightarrow P(d) = f(a,b)$, $d \leq 0 \Rightarrow P(d) = f(b,a)$

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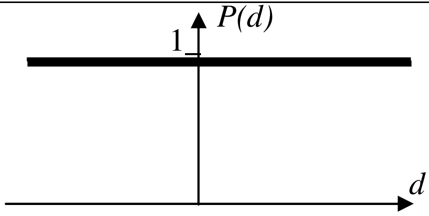
<tr><td>I. Usual Criterion</td><td>
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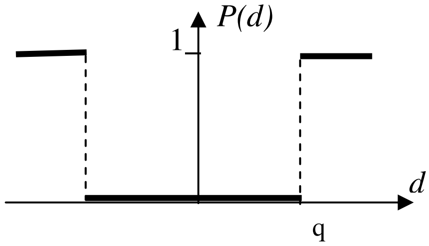
<tr><td>II. Quasi-Criterion</td><td>
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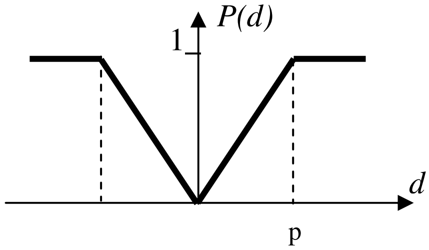
<tr><td>III. Criterion with Linear Preference</td><td>
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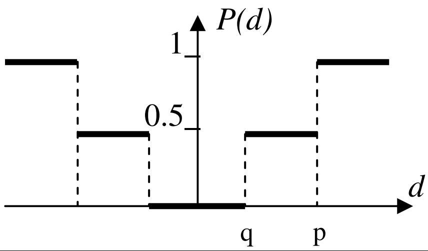
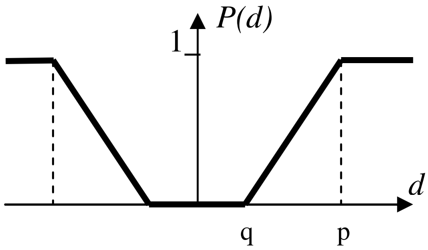
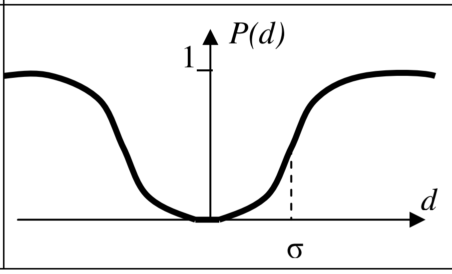
<tr><td>IV. Level Criterion</td><td>
</td><td><u>Parameters to be defined:</u>
q , p</td></tr>
<tr><td>V. Criterion with Linear Preference and Indifference Area</td><td>
</td><td><u>Parameters to be defined:</u>
q , p</td></tr>
<tr><td>VI. Gaussian Criterion</td><td>
</td><td><u>Parameters to be defined:</u>
σ</td></tr>
</table>

Figure 3.6 Preference Functions, Brans et al. (1986)

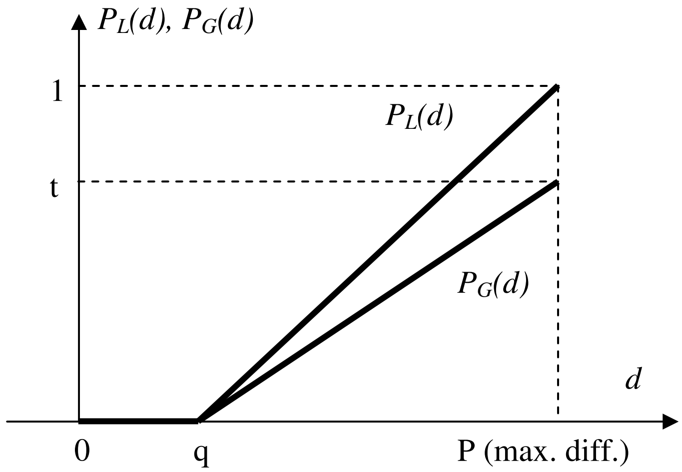
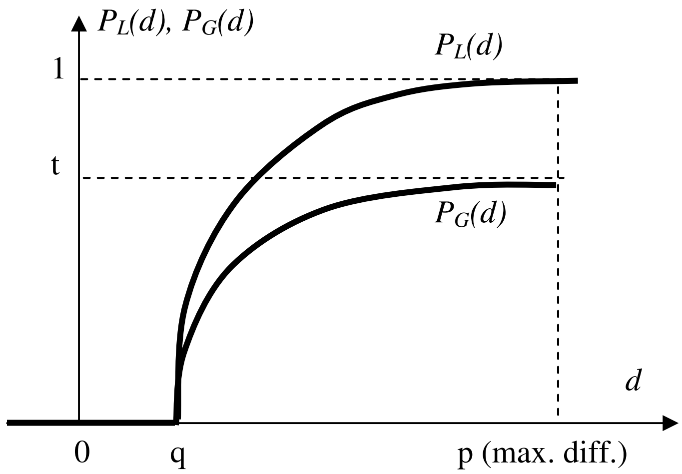
VII. Linear Criterion (Prospect Theory)		Parameters to be defined: q
VIII. Exponential Criterion (Prospect Theory)		Parameters to be defined: q

Figure 3.7 The introduced preference functions reflecting the Prospect Theory

$$\pi(a_i, a_j) = \sum_n P_n(a_i, a_j) \cdot w_n$$

where $P_n(a_i, a_j)$ is the preference function associated to the criterion n and w_n is the weight of the criterion n.

If the criterion n is associated with a preference function that is reflecting prospect theory (*gain/loss* > 1), $P_n(a_i, a_j)$ yields different results when either a_i or a_j is set as the reference alternative respectively. That is because when a_i is set as the reference alternative, $f_n(a_i) - f_n(a_j)$ difference -if positive- has "*gain*" property and whereas if a_j is set as the reference alternative, $f_n(a_i) - f_n(a_j)$ difference -if positive- has "*loss*" property and it was explained in section 3.4 that according to prospect theory gains have less impact than losses on outranking degree. This situation is illustrated in Figure 4.7 and the preference functions are presented in section 4.3.1 and 4.3.2.

In the methodology developed in this study, each alternative in the pair is set as the reference alternative separately, since the reference point is important for the prospect theory application. Hence for every alternative pair, two different preference indices are calculated respectively, and finally two separate preference index table is obtained (Π_1 and Π_2) as shown in Figures 4.8 and 4.9.

The "*leaving flow*" value could be interpreted as the overall dominance of the alternative on others and it is calculated by summing the preference indices that flow from the alternative under consideration to the others. In other words it is the total flow from one alternative to the rest. Therefore the alternative under consideration (row element) is the reference alternative, so, Π_1 table is used for calculating the "*leaving flow*" values as follows:

$$\phi^+(a_i) = \sum_j \pi_1(a_i, a_j) \quad \forall a_i \in K \qquad [\textit{Leaving Flow}]$$

$$\pi_1(a_i, a_j) = \sum_n P_n(a_i, a_j) \cdot w_n$$

where $P_n(a_i, a_j) = P_{nG}(a_i, a_j)$ if $gain/loss > 1$ for criterion n (Prospect Theory) and w_n is the weight associated with criterion n.

Π_1	a_1	a_2	$\cdots$	a_j	$\cdots$	a_k	$\phi^+(a_i)$
a_1	$\pi_1(a_1,a_1)$	$\pi_1(a_1,a_2)$	$\cdots$	$\pi_1(a_1,a_j)$	$\cdots$	$\pi_1(a_1,a_k)$	$\sum_j \pi_1(a_1,a_j)$
a_2	$\pi_1(a_2,a_1)$	$\pi_1(a_2,a_2)$	$\cdots$	$\pi_1(a_2,a_j)$	$\cdots$	$\pi_1(a_2,a_k)$	$\sum_j \pi_1(a_2,a_j)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
a_i	$\pi_1(a_i,a_1)$	$\pi_1(a_i,a_2)$	$\cdots$	$\pi_1(a_i,a_j)$	$\cdots$	$\pi_1(a_i,a_k)$	$\sum_j \pi_1(a_i,a_j)$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
a_k	$\pi_1(a_k,a_1)$	$\pi_1(a_k,a_2)$	$\cdots$	$\pi_1(a_k,a_j)$	$\cdots$	$\pi_1(a_k,a_k)$	$\sum_j \pi_1(a_k,a_j)$

Figure 4.8 Preference indices table (Π1) and calculation of leaving flows, $\phi^+(a_i)$ (first elements of the alternative pairs are the reference alternatives)

Π_2	a_1	a_2	$\cdots$	a_j	$\cdots$	a_k
a_1	$\pi_2(a_1,a_1)$	$\pi_2(a_1,a_2)$	$\cdots$	$\pi_2(a_1,a_j)$	$\cdots$	$\pi_2(a_1,a_k)$
a_2	$\pi_2(a_2,a_1)$	$\pi_2(a_2,a_2)$	$\cdots$	$\pi_2(a_2,a_j)$	$\cdots$	$\pi_2(a_2,a_k)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$
a_i	$\pi_2(a_i,a_1)$	$\pi_2(a_i,a_2)$	$\cdots$	$\pi_2(a_i,a_j)$	$\cdots$	$\pi_2(a_i,a_k)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
a_k	$\pi_2(a_k,a_1)$	$\pi_2(a_k,a_2)$	$\cdots$	$\pi_2(a_k,a_j)$	$\cdots$	$\pi_2(a_k,a_k)$
$\phi^-(a_j)$	$\sum_i \pi_2(a_i,a_1)$	$\sum_i \pi_2(a_i,a_2)$	$\cdots$	$\sum_i \pi_2(a_i,a_j)$	$\cdots$	$\sum_i \pi_2(a_i,a_k)$

Figure 4.9 Preference indices table (Π2) and calculation of entering flows, $\phi^-(a_j)$ (second elements of the alternative pairs are the reference alternatives)

On the other hand, the "*entering flow*" value could be interpreted as the overall dominance of the other alternatives on the alternative under consideration and it is calculated by summing the preference indices that flow to that alternative from the rest. In other words it is the total flow to one alternative from others. Therefore, the alternative under consideration is not the reference alternative, but the others are (column elements), so Π_2 table is used for calculating the "*entering flow*" values as follows:

$$\phi^{-}(a_j) = \sum_i \pi_2(a_i, a_j) \quad \forall a_j \in K \quad (\textit{Entering Flow})$$

$$\pi_2(a_i, a_j) = \sum_n P_n(a_i, a_j) \cdot w_n$$

where $P_n(a_i, a_j) = P_{nL}(a_i, a_j)$ if $gain/loss > 1$ for criterion n (Prospect Theory) and w_n is the weight associated with criterion n.

All in all, two different preference indices tables are obtained for the calculation of flow values. In the first one (Π_1), the first elements in the alternative pairs (row elements) are set as the reference alternatives, i.e. the criteria value differences have "gain" property. Whereas in the second table (Π_2), the second elements in the alternative pairs (the column elements) are set as the reference alternatives, i.e. the criteria value differences have "loss" property. The "net flow" values are calculated using the "leaving flow" and the "entering flow" values and the final ranking of the alternatives are obtained.

If in the beginning of the problem, all the answers to the "*gain/loss*" ratio question is given as "1" by the decision maker, the two preference indices tables become equal (Π_1 = Π_2), which makes sense and the problem returns to an ordinary Promethee II application.

4.3 THE INTRODUCED PREFERENCE FUNCTIONS

Two new preference functions, which are suitable for the Prospect Theory applications are introduced in this study and proposed to the decision maker in the methodology. The first one is a variation of the preference function V, criterion with linear preference and indifference area, proposed by Brans et al. (1986). The second one is a variation of the preference function proposed by Karasakal et al. (2005) and it is based on exponential function. Both are explained in detail below.

The decision maker must choose either one of these two in case his/her perception of the criteria under consideration best suits to the prospect theory. The most significant difference of the two is that one is linear; the other is concave, whereas both have an indifference area, which are specified by defining the corresponding indifference threshold value. On the other hand, if the contribution of small differences of the criterion value beyond the indifference area is significant, then it would be more appropriate for the decision maker to choose the preference function VIII (exponential function) because this function has a steeper slope just after the indifference area.

4.3.1 Preference Function VII (Linear criterion with indifference threshold area)

Let $d = f_n(a_i) - f_n(a_j)$ for criterion n.

If a_j is reference alternative (d has **loss** property),

$$P_n(a_i, a_j) = P_{nL}(d)$$

$$d \leq q \Rightarrow P_{nL}(d) = 0$$

$$d > q \Rightarrow P_{nL}(d) = \frac{(d-q)}{p-q}$$

Else if a_i is reference alternative (d has **gain** property),

$$P_n(a_i, a_j) = P_{nG}(d)$$

$$d \leq q \Rightarrow P_{nG}(d) = t \cdot P_{nL}(d) = 0$$

$$d > q \Rightarrow P_{nG}(d) = t \cdot P_{nL}(d) = t \cdot \frac{(d-q)}{p-q}$$

Where

t = (gain/loss)$^{-1}$ (to be defined by the decision maker)

q = indifference threshold (to be defined by the decision maker)

p = the max. absolute difference among the criterion values

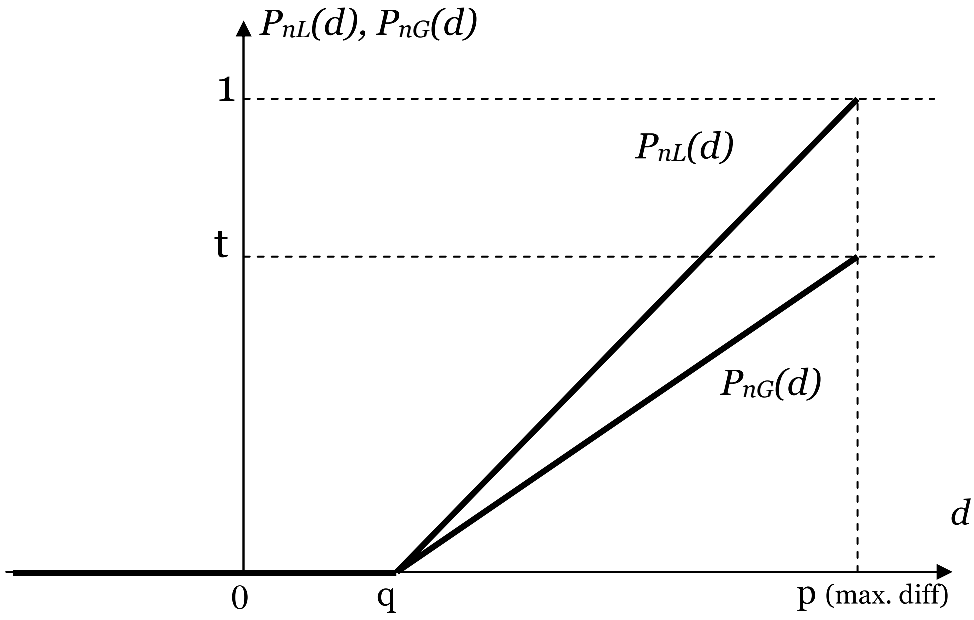


Figure 4.10 Linear criterion with indifference threshold area (Prospect Theory)

4.3.2 Preference Function VIII (Exponential Function with Indifference Area)

Let $d = f_n(a_i) - f_n(a_j)$ for criterion n.

If a_j is reference alternative (d has **loss** property),

$$P_n(a_i, a_j) = P_{nL}(d)$$

$$d \leq q \Rightarrow P_{nL}(d) = 0$$

$$d > q \Rightarrow P_{nL}(d) = 1 - e^{-\lambda(d-q)}$$

Where

$$\lambda = \frac{\ln(\varepsilon)}{p-q}$$

$$\varepsilon = 0.01$$

q = indifference threshold (to be defined by the decision maker)

p = the max. absolute difference among the criterion values

Else if a_i is reference alternative (d has **gain** property),

$$P_n(a_i, a_j) = P_{nG}(d)$$

$$d \le q \Rightarrow P_{nG}(d) = t \cdot P_{nL}(d) = 0$$

$$d > q \Rightarrow P_{nG}(d) = t \cdot P_{nL}(d) = t - t \cdot e^{-\lambda(d-q)}$$

Where

$$\lambda = \frac{\ln(\varepsilon/t)}{p-q}$$

$$\varepsilon = 0.01$$

$t = (\text{gain/loss})^{-1}$

q = indifference threshold (to be defined by the decision maker)

p = the max. absolute difference among the criterion values

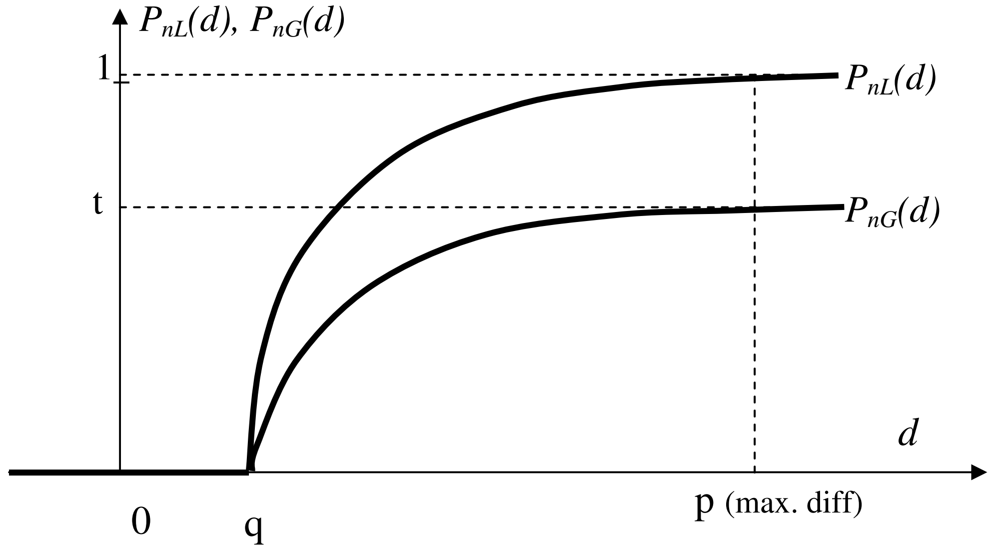


Figure 4.11 Exponential function with indifference area (Prospect Theory)

CHAPTER 5

THE SOFTWARE

In this chapter, the developed software is introduced thoroughly together with demonstrations of the windows of the user interface. Also the algorithm is summarized with the flowchart provided at the end of this section. For the development of the software Microsoft Visual C# 2005 programming language is used.

In the opening window of the software, the problem structure is built by the decision maker. Each criteria and the corresponding gain/loss ratio is entered in the first column, whereas the alternatives are introduced into the second as illustrated in Figure 5.1.

After this initial interaction, the decision maker now has to decide on how to define the weights of the criteria. At this stage, alternative procedures defined in chapter 3 are proposed to the decision maker. The decision maker chooses either “manual”, “independent” or “interdependent” in the weight determination section given in the lower left corner of the first window, below the problem definition columns. If the decision maker chooses “manual”, he/she specifies and enters the weights in the next window, where all other data is entered. Else if the “independent” is chosen, a new window opens and the decision maker makes pairwise comparisons for the AHP application as shown in Figure 5.2. For the consistency check, the consistency ratio is calculated instantaneously as the decision maker enters the pairwise comparison values and displayed in the lower left corner of the present window. Also a message indicating whether the comparisons are “consistent enough” is displayed below the consistency ratio. If the comparisons are not consistent, the decision maker shall go over the input values and revise the judgments he/she made where necessary.

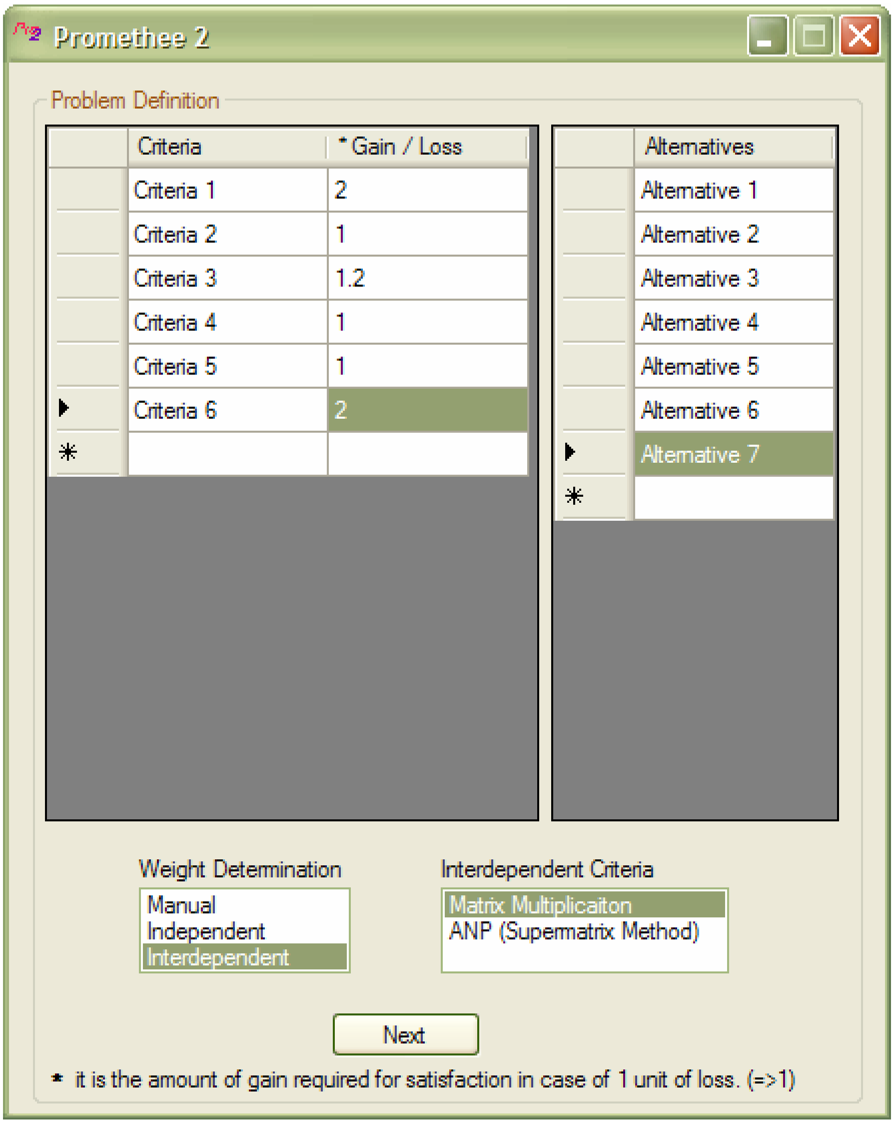


Figure 5.1 The opening window: Problem definition stage

When the comparisons are consistent enough, the decision maker clicks the "done" button and progresses to the evaluation window next.

If in the first window the decision maker chooses the "interdependent", meaning interrelationships between the criteria, two new alternative courses of actions are displayed on the right, which are “Matrix Multiplication” and “ANP (Supermatrix Method)” as shown in Figure 5.1. In both cases, sequential pairwise comparisons of criteria are performed with respect to first the main goal and then each criterion. In the “Matrix Multiplication” case, after the pairwise comparisons, the software manipulates the inputs to obtain the weights and the progresses to the evaluation window. In the “ANP (Supermatrix Method)” case, besides these pairwise comparisons, the decision maker is asked for the cluster weights (main goal and criteria clusters) before the calculation of the criteria weights (Figure 5.3). By specifying the cluster weights, the decision maker decides on the influence of the interrelationships among the criteria on the final weights.

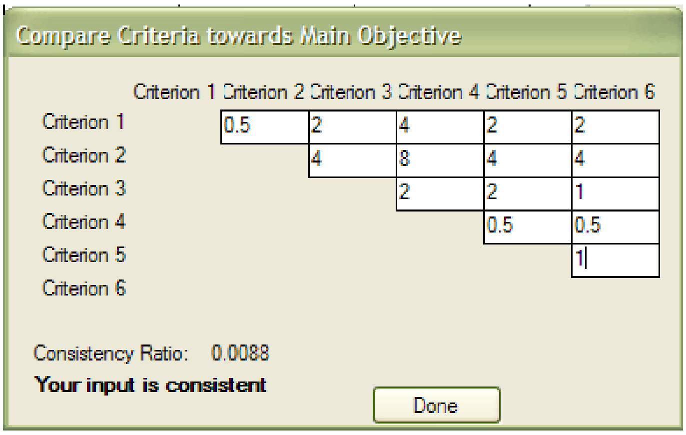


Figure 5.2 Pairwise comparison window

Figure 5.3 Specifying the cluster weights

After the weight determination stage is completed, the window illustrated in Figure 5.4 (evaluation window) appears. Here the rest of the necessary data is entered by the decision maker. The criteria values for each alternative, the optimization type, the preference function and the corresponding parameters for each criterion are set by the decision maker. Now the software is ready to reveal the result, which is the preference ranking of the alternatives.

When the "next" button is clicked, a new window appears with a table on the left showing a square matrix of alternatives with the outranking degrees (Π_1 / Π_2) and the Promethee flows (entering and leaving) (Figure 5.5). On the right, the final ranking of the alternatives is given in the order of descending net flow values. The net flow values, which are calculated using the entering and the leaving flow values are also listed in the final ranking.

The algorithm of the software is illustrated with the flowchart in Figure 5.6.

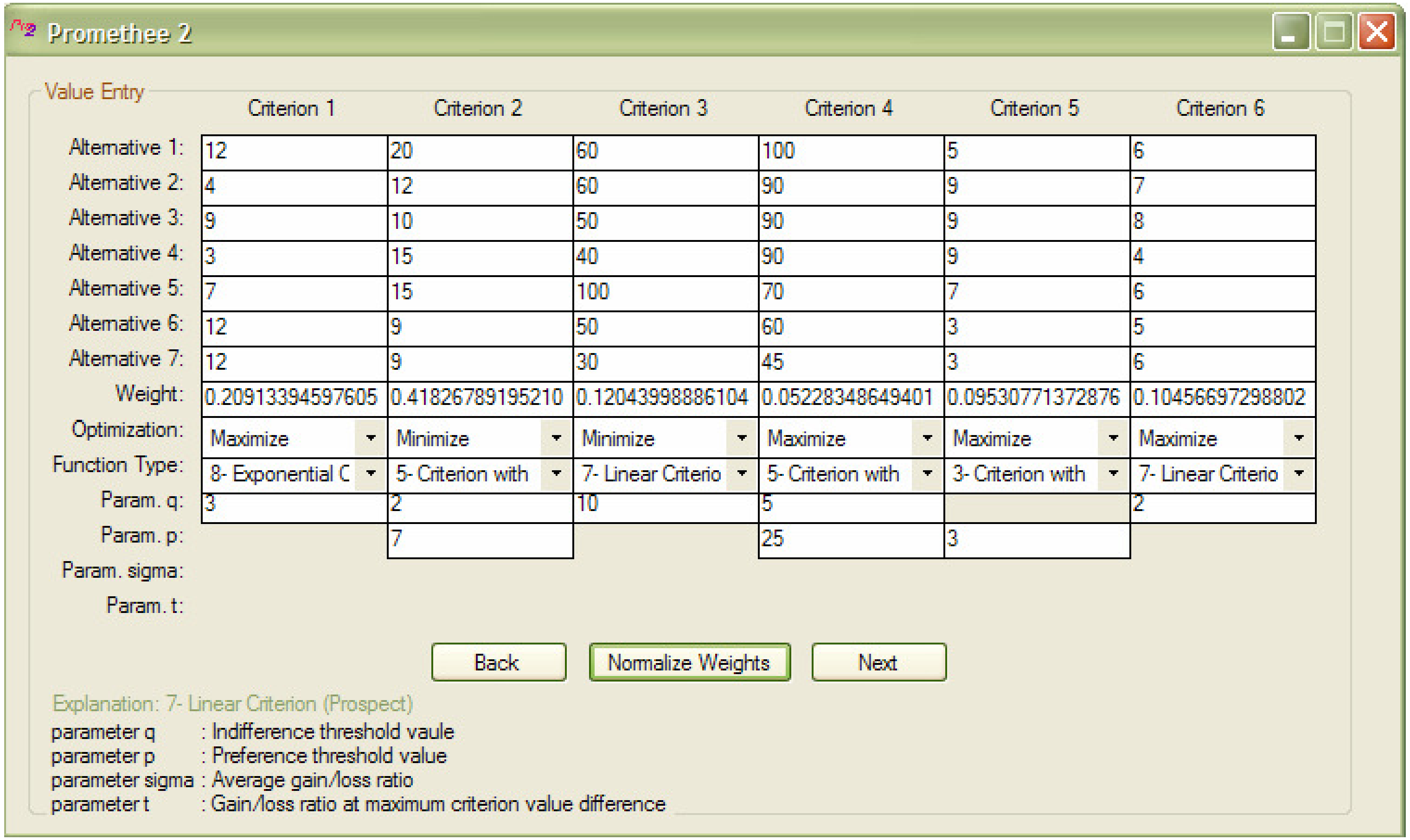
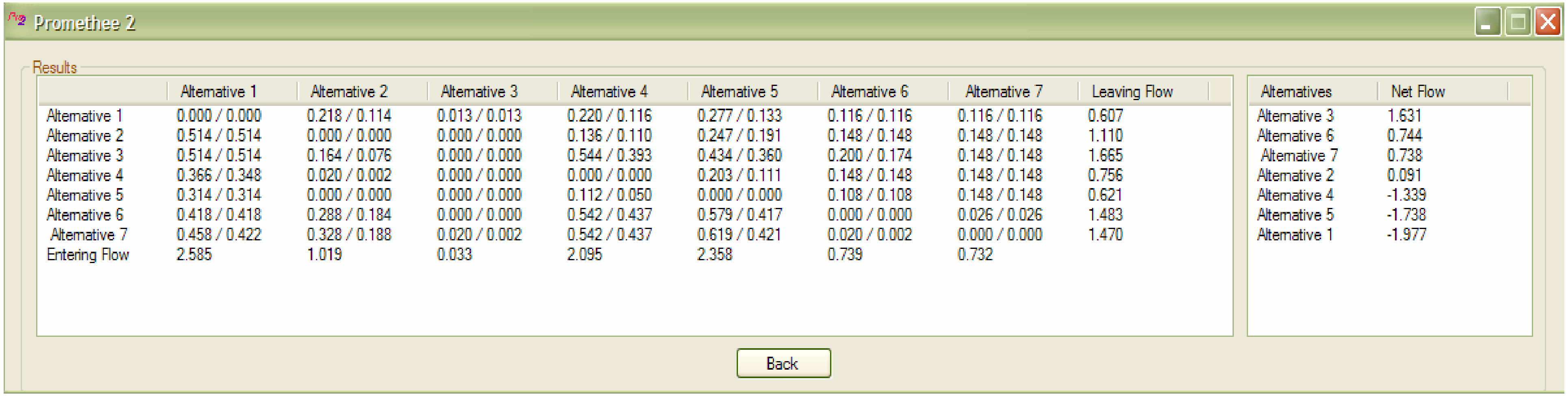


Figure 5.4 Evaluation window



Promethee 2

Results

	Alternative 1	Alternative 2	Alternative 3	Alternative 4	Alternative 5	Alternative 6	Alternative 7	Leaving Flow
Alternative 1	0.000 / 0.000	0.218 / 0.114	0.013 / 0.013	0.220 / 0.116	0.277 / 0.133	0.116 / 0.116	0.116 / 0.116	0.607
Alternative 2	0.514 / 0.514	0.000 / 0.000	0.000 / 0.000	0.136 / 0.110	0.247 / 0.191	0.148 / 0.148	0.148 / 0.148	1.110
Alternative 3	0.514 / 0.514	0.164 / 0.076	0.000 / 0.000	0.544 / 0.393	0.434 / 0.360	0.200 / 0.174	0.148 / 0.148	1.665
Alternative 4	0.366 / 0.348	0.020 / 0.002	0.000 / 0.000	0.000 / 0.000	0.203 / 0.111	0.148 / 0.148	0.148 / 0.148	0.756
Alternative 5	0.314 / 0.314	0.000 / 0.000	0.000 / 0.000	0.112 / 0.050	0.000 / 0.000	0.108 / 0.108	0.148 / 0.148	0.621
Alternative 6	0.418 / 0.418	0.288 / 0.184	0.000 / 0.000	0.542 / 0.437	0.579 / 0.417	0.000 / 0.000	0.026 / 0.026	1.483
Alternative 7	0.458 / 0.422	0.328 / 0.188	0.020 / 0.002	0.542 / 0.437	0.619 / 0.421	0.020 / 0.002	0.000 / 0.000	1.470
Entering Flow	2.585	1.019	0.033	2.095	2.358	0.739	0.732	

Alternatives	Net Flow
Alternative 3	1.631
Alternative 6	0.744
Alternative 7	0.738
Alternative 2	0.091
Alternative 4	-1.339
Alternative 5	-1.738
Alternative 1	-1.977

Back

Figure 5.5 The final window: Outranking degrees, Promethee flows & final ranking

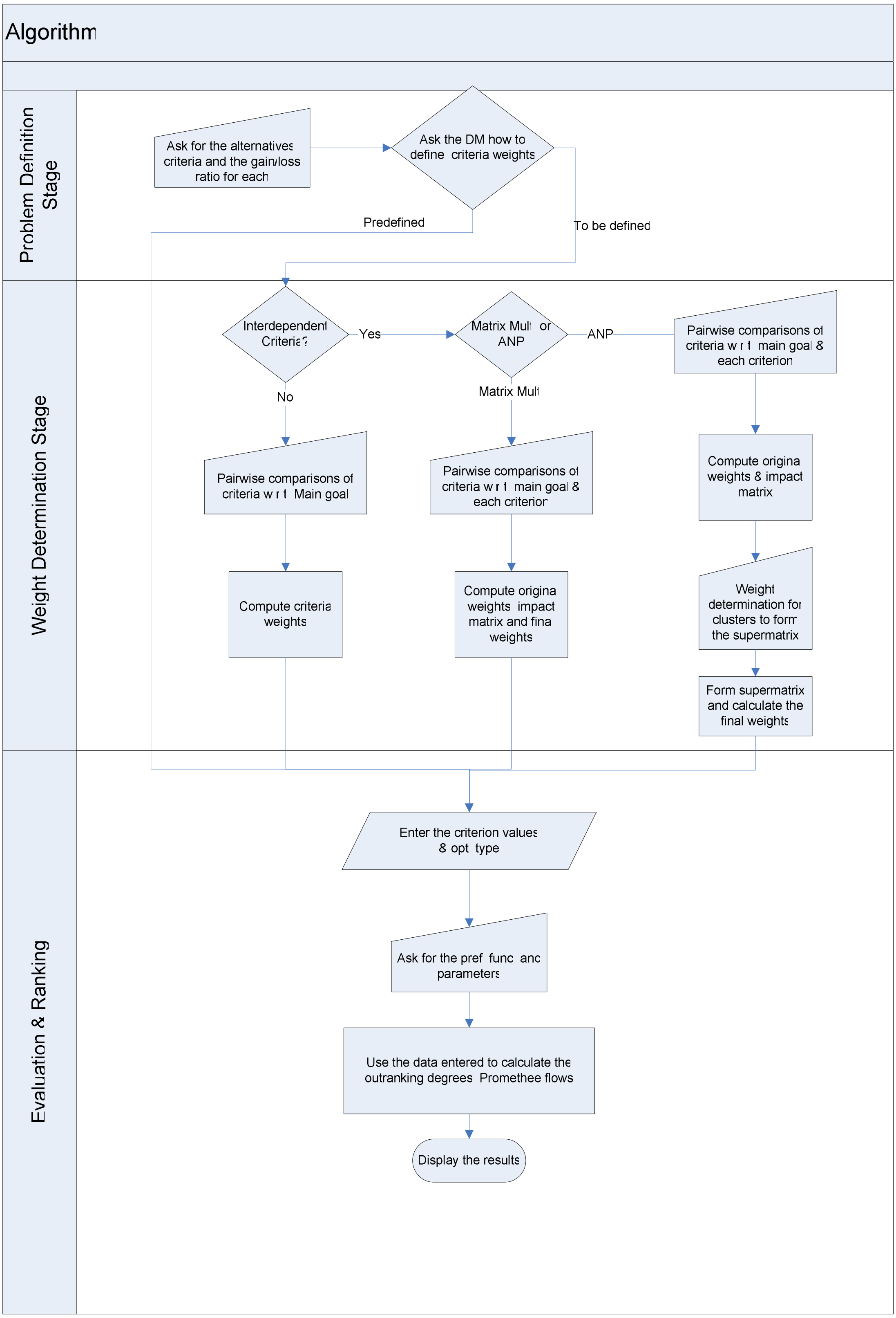


Figure 5.6 The algorithm of the software

CHAPTER 6

COMPARISON OF THE WEIGHT DETERMINATION TECHNIQUES

The weights of the criteria have a significant effect on the solution of a MCDM problem. In the methodology proposed in this study, alternative techniques to specify the weights are presented and explained in detail in chapter 4. The purpose of this chapter is to observe and analyze the differences and the variations in the final results when alternative techniques are applied.

To observe differences on the revealed solutions of the techniques proposed in section 4.1, they are applied on the same set of data for different cases. The techniques are applied for 3, 6 and 9 criteria cases respectively. For each case the weights are calculated for 5 different data sets obtained throughout a random number generation procedure. For clearness and simplicity, eigenvectors are generated instead of pairwise comparisons to form the comparison matrices assuming perfect consistency.

For all the data sets, technique 1 (AHP), technique 2 (Matrix Multiplication Method) and technique 3 (ANP – Supermatrix) are applied respectively. Since there is the flexibility of determining the cluster weights in ANP method, ANP(w_1, w_2) notation is used to indicate the cluster weights used in technique 3, where

w_1: weight of the main goal cluster
w_2: weight of the criteria cluster

ANP is applied with four different cluster weight set, that is ANP(1.0, 0.0), ANP(0.5, 0.5), ANP(0.9, 0.1) and ANP(0.1, 0.9). In order to observe the impact of the variation in the weights of the clusters on the final weights, cluster weights are specified in such a manner.

Weights obtained from one of the 3 criteria data sets is given in Table 6.1 and illustrated with a chart in Figure 6.1 below.

Table 6.1 Sample results from a 3 criteria case

Weights obtained with alternative techniques (3 criteria case)

	AHP	Matrix Mult.	ANP(1.0, 0.0)	ANP(0.9, 0.1)	ANP(0.5, 0.5)	ANP(0.1, 0.9)
C1	0.114	0.260	0.114	0.128	0.198	0.271
C2	0.797	0.578	0.797	0.774	0.648	0.404
C3	0.089	0.163	0.089	0.098	0.154	0.326

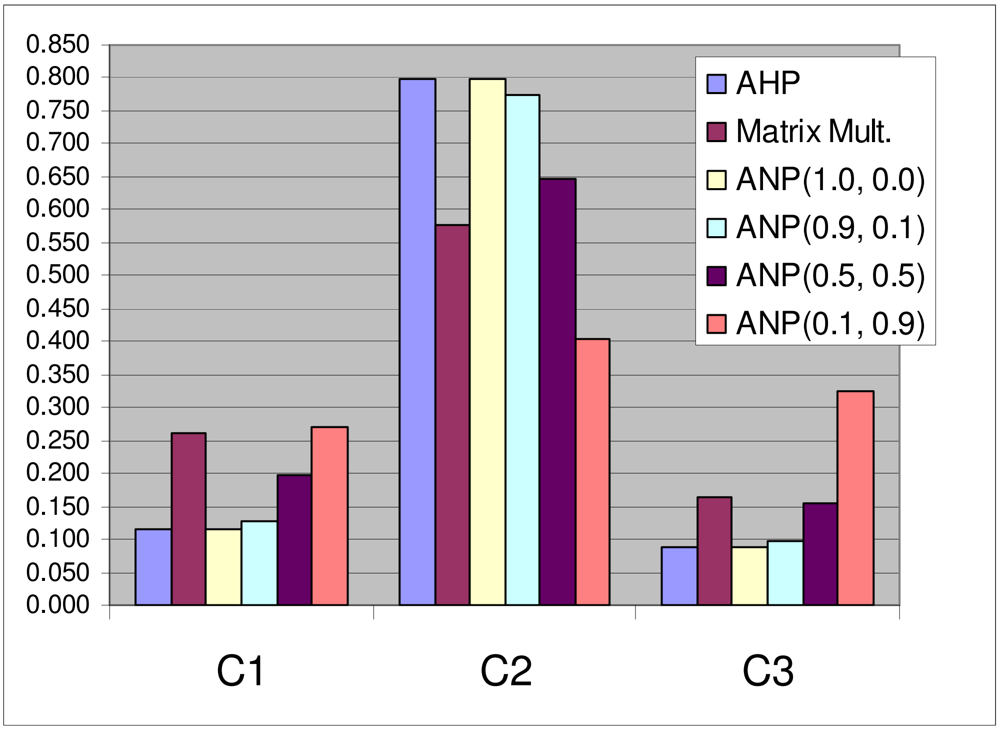


Figure 6.1 Weights obtained for a 3-Criteria case

The first observation is that, when the results of all three techniques are analyzed, it is obvious that the techniques 2 and 3 deviate the criteria weights in the same direction with respect to the original weights obtained with the technique 1 (AHP). However, naturally the degree of change differs. To analyze the degree of the deviations of the techniques 2 & 3, the average absolute deviations from the original weights (technique 1, AHP) are listed in Table 6.2 for problems with 3, 6 & 9 criteria. In the list, the deviations of the ANP – Supermatrix technique applied for four different cluster weight set are presented.

Table 6.2 Average deviations from the original weights

Average deviations (reference value: original weights obtained with technique 1: AHP)					
	Matrix Mult.	ANP(1.0, 0.0)	ANP(0.9, 0.1)	ANP(0.5, 0.5)	ANP(0.1, 0.9)
3 Criteria	0.089	0.000	0.012	0.056	0.134
6 Criteria	0.081	0.000	0.008	0.049	0.109
9 Criteria	0.066	0.000	0.007	0.037	0.073

The average absolute deviations indicate that due to the cluster weights, ANP results lies within a range, whose boundaries are two points in the weight space. The first point is the point of no interdependency, which is obtained by ANP(1.0, 0.0) and the second one is the point of pure interdependency, which is obtained by ANP(0.0, 1.0). Also the results show that the weights at the point of no interdependency are equal to the original weights obtained by technique 1 (AHP) with respect to main goal. This makes sense because ANP(1.0, 0.0) yields results, in which the interdependencies among the criteria have zero impact on the final weights.

Another observation is that the results of the Matrix Multiplication technique generally lies between the results of ANP(0.5, 0.5) and ANP(0.0, 1.0). This situation is observed in almost all solutions obtained. This indicates that Matrix multiplication technique deviates the original weights relatively more than the average ANP(0.5,0.5) technique. Naturally, this observation may not hold for some extreme cases of interdependencies between criteria.

The observations given above are re-illustrated with an example below. In this example there are 3 criteria (C1, C2 and C3) and the weight of the criteria are calculated using different cluster weights sets. For simplicity and clearness, the original weights with respect to the main goal are specified as follows:

$$W_o = [0.333, 0.333, 0.333]$$

The interrelationships among the criteria are given in the weighted supermatrix in Table 6.3 and the corresponding network is illustrated in Figure 6.2. In this example, the criteria C1 & C2 are twins with respect to the original weights and interrelationships so finally their weights will be equal. On the other hand C3 has different interrelationship eigenvector that is C3 is more conservative than C1 & C2. It is because it has a higher self-impact than C1 & C2 (0.8 > 0.4). Moreover C1 & C2 have a higher influence on C3 than C3 has on them. According to the interrelationships, it is easy to say that the final weight of C3 will be higher than that of C1 & C2 after the supermatrix operations.

Table 6.3 The weighted supermatrix for ANP(w_1, w_2) application

Supermatrix for ANP(w1, w2) application

		Main Goal	C1	C2	C3
Main Goal		0	w_1	w_1	w_1
Criteria	C1	0.333	w_2*0.4	w_2*0.3	w_2*0.1
	C2	0.333	w_2*0.3	w_2*0.4	w_2*0.1
	C3	0.333	w_2*0.3	w_2*0.3	w_2*0.8

After performing the limiting operations on the supermatrix for five cases of cluster weights, the final weights obtained are given in Table 5.4 and illustrated with a chart in Figure 6.3.

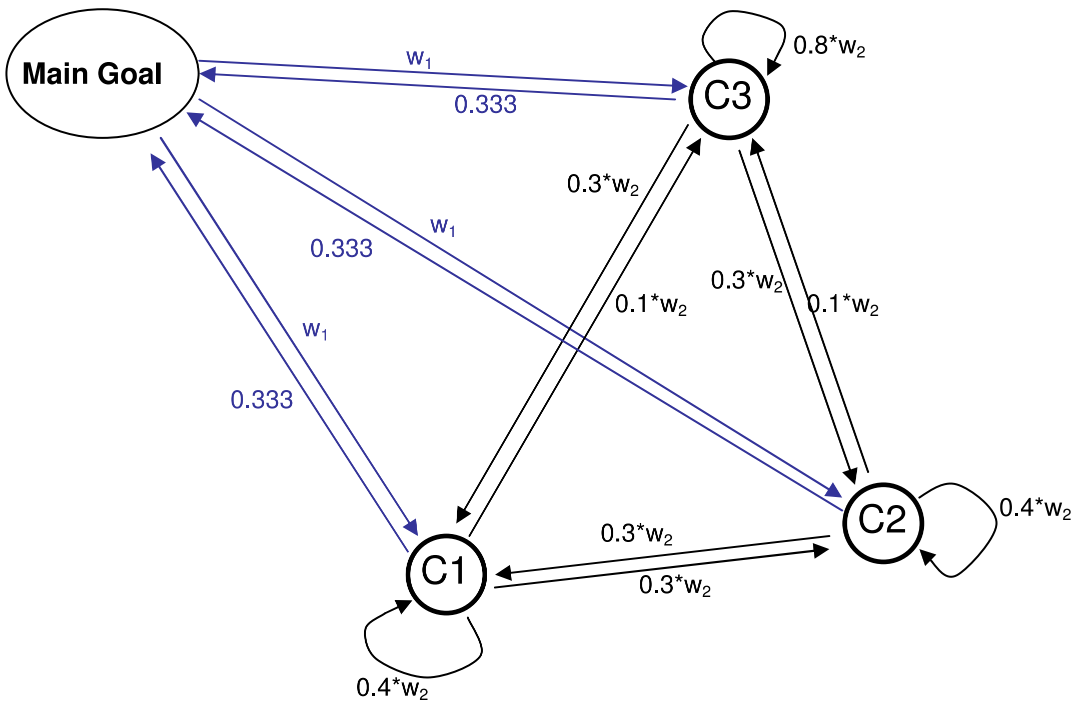


Figure 6.2 Network for the ANP(w_1, w_2) application

Since there is the flexibility to specify the cluster weights in ANP, the decision maker is able set the overall impact of the interrelationships among the criteria on the final weights; he/she can either increase or decrease it by a simple weight specification.

Table 6.4 Solutions with AHP, Matrix Mult. & ANP techniques

Final weights with alternative techniques

	Technique 1: AHP	Technique 2: Matrix Mult.	Technique 3: ANP (Supermatrix)					
			w_1:	1.0	0.9	0.5	0.1	0.0
			w_2:	0.0	0.1	0.5	0.9	1.0
C1	0.333	0.267		0.333	0.326	0.289	0.224	0.200
C2	0.333	0.267		0.333	0.326	0.289	0.224	0.200
C3	0.333	0.466		0.333	0.348	0.423	0.552	0.600

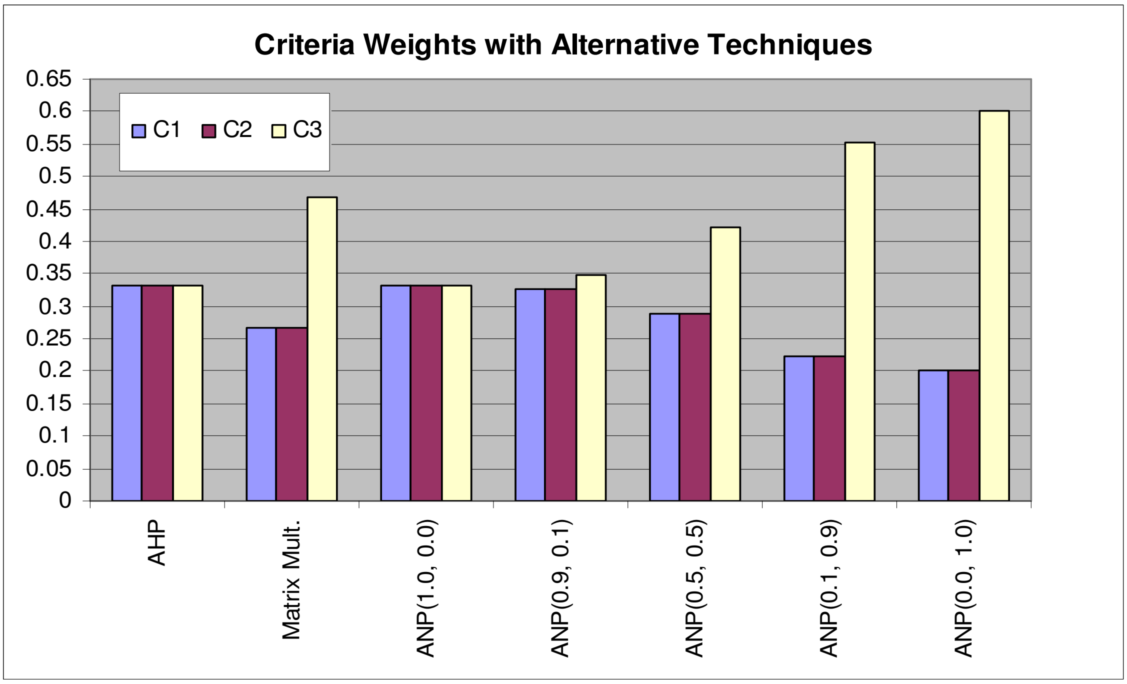


Figure 6.3 Variation of the weights with alt. techniques

CHAPTER 7

SAMPLE SOLUTIONS

In this chapter, sample problems from different scenarios are solved using the methodology developed. The scenarios are generated using the problem solved in a study conducted by Wang et al. (2006). The problem is ranking of outsourcing decisions of parts of IS functions. They treat the problem through a hybrid method composed of AHP and Promethee II. In their hybrid method, similar to the methodology developed in this study, AHP is used to analyze the structure of the outsourcing problem and to determine the weights of the criteria (independent criteria) and Promethee II is used for final ranking.

In order to observe the changes in the final ranking, each time a slight change is made on the original problem structure. More specifically, since the new methodology introduces techniques to handle the interdependency among criteria and new preference functions to reflect the decision behavior of the decision maker, modifications in accordance with these two important facts of the MCDM problems are introduced into the sample problem.

Firstly, interdependency among criteria is introduced. The proposed alternative techniques (*Matrix Multiplication* and *ANP*) are utilized separately to obtain two distinct final rankings. Secondly, it is assumed that *Prospect Theory* best suits the decision maker's choice behavior for some criteria and new preference functions are introduced and another new final ranking is obtained. Lastly, combinations of these two situations are applied so that the problem structure became more realistic.

Towards the solutions, the author was set as the decision maker and contributed with his judgments and evaluations where required. However, the criteria values (quantitative and qualitative) kept same as the original problem, in order to observe the differences between the methods applied. The obtained solutions for the generated scenarios are given in Table 7.17 at the end of this chapter.

In the first section, the original problem and its solution is presented in detail whereas in the second section the generated scenarios and their differentiations are briefly explained. Finally in the last section the final rankings of all the solutions are summarized with comments and discussions.

7.1 ORIGINAL PROBLEM

As mentioned before, the original problem is the ranking of outsourcing decisions of parts of IS functions. Six criteria are defined in agreement with a group of experts and managers and there are 5 candidate systems for outsourcing. The criteria and the alternatives are listed in Table 7.1 and Table 7.2 respectively. The descriptions of the criteria are as follows:

Economics

For economics (C1), the major consideration of a firm is to reduce costs of information systems. Because the vendors have a better management skill as well as higher productivity per employee, the costs can be reduced. Meanwhile, because of the scale of economics vendors have invested in the hardware, software and human resources, the cost can be reduced. Another consideration of economics is financial flexibility. Because of outsourcing, the facilities and employee would be transferred to the vendor side, which transform fixed costs into variable costs, resulting in increasing financial flexibility

Resource

For resource (C2), resources include new technologies and professional workers. The fastest and most effective way to get qualifiedworkers of IT is to outsource.

Outsourcing also can provide immediate access to the latest technologies with the lead time customary in in-house development. In-house workers can learn new technology of software management and development from the vendor [7,13,28].

Strategy

For strategy (C3), firms need to focus on their core activities and outsource non core activities. IS outsourcing allows management to focus available IS talent on important and strategic IT applications rather than the mundane and routine activities. The internal operations and outsourced operations should then work in union striving to optimize flexibility and responsiveness to customer and internal needs, and minimize unnecessary paperwork and bureaucracy. In addition, the firms can make strategic alliance with vendors to make up the shortage of resources. From strategic alliances, the firm even can develop and market new products. Other strategic consideration includes sharing risks and accelerating the time of product to market

Risk

For risk (C4), it is rare to experience opportunities in organizational life where the managerial actions taken to produce benefits are not associated with potential risks either. This is most certainly the case with IS outsourcing. The risks that have to be dealt with include: loss of core competence, loss of internal technical knowledge, loss of flexibility, damaging the firm’s innovative capability, increasing information services management complexity, etc. As being the factors with benefits, these risks factors should not be ignored in outsourcing activities.

Management

For management (C5), the problems that have to be dealt with include: improving communication problems and selfishness between IS department and operational department, stimulating IS department to improve their performance and enhance morale, increasing the ability of management and control of IS department, solving the floating and scarcity of employee, keeping the flexibility to adjust department, etc.

Quality

For quality (C6), because vendors may have access to more technological environments, have more qualified or more motivated personnel, provide a greater breadth of services, and simply be more committed than internal staff to making the alliance with the customer work well, outsourcing can improve the quality and services of the internal IS department. Therefore, good quality of service and good relationship are the significant success factors of outsourcing.

Table 7.1 Criteria of the sample problem

Criteria	
C1	Economics
C2	Resources
C3	Strategy
C4	Risk
C5	Management
C6	Quality

In the original problem it is assumed that there exists no interrelationships among the criteria and the weights of the criteria are calculated using AHP. The square matrix (pairwise comparisons) of this step, which is obtained through an interactive procedure carried out among experts and managers, is shown in Table 6.3. In this matrix, the numbers could be interpreted as the relative importance of the row element with respect to the column element.

Table 7.2 Alternatives of the original problem (Wang et al.)

Alternatives	
P1	Facilities management
P2	Development of internet homepage
P3	Maintenance of the customer relationship management information system
P4	Development of the supplier relationship management information system
P5	Development and maintenance of the online transaction processing system

Table 7.3 The square matrix (AHP) for weight determination (Wang et al.)

The square matrix

	Economics	Resources	Strategy	Risk	Management	Quality
Economics	1	1	1/2	1/2	2	1/2
Resources	1	1	1/2	1	2	1
Strategy	2	2	1	3	3	3
Risk	2	1	1/3	1	3	2
Management	1/2	1/2	1/3	1/3	1	1/2
Quality	2	1	1/3	1/2	2	1

AHP is also used for the decomposition of the problem into a multi-level hierarchy showing the overall goal of the decision process, each decision criterion to be used and the decision alternatives to be considered as candidates for selection. The hierarchy for this problem is illustrated in Figure 7.1.

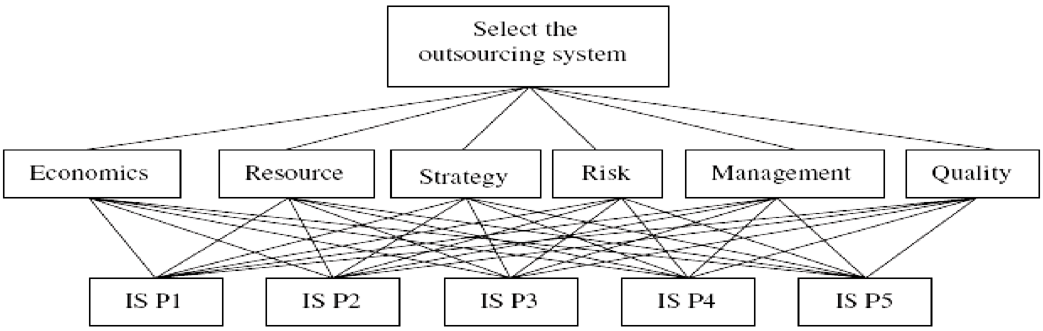


Figure 7.1 The hierarchy of the sample problem (Wang et al.)

After the necessary computations on the AHP comparison matrix, for consistency check, the following values are obtained in the given order according to the formulations given in section 3.2.3:

$\lambda_{\max} = 6.2218$, {highest eigenvalue where $n = 6$}

$CI = 0.04436$

$CI^* = 1.24$ {for $n = 6$}

$CR = 0.0358 < 0.1$

Since CR (consistency ratio) is less than 10 %, the evaluations made for the square matrix are said to be consistent enough. Hence, the eigenvector corresponding to the highest eigenvalue, i.e. the weight vector is

W_o = [0.12, 0.15, 0.33, 0.19, 0.07, 0.14]

Evaluations of the alternatives according to the criteria described above are provided in the evaluation matrix given in Table 7.4.

For the criteria 2 to 6, a qualitative impact value was used, expressed on a qualitative scale that is used for calculations (Judgments on a series of ordered semantic values, which are the elements of the set {very weak, weak, common, good, common good} is associated with a numerical value such as ranking from 1 to 5).

For the application of the Promethee II method to rank the candidate systems, specific preference functions with the corresponding necessary parameters defined for each criterion are listed in Table 7.5.

The problem is now ready for the implementation of the Promethee II method.

Table 7.4 The evaluation matrix of the original problem (Wang et al.)

Evaluation Matrix

Criteria	**Economics**	**Resources**	**Strategy**	**Risk**	**Management**	**Quality**
Max. / Min.	Max.	Max.	Max.	Min.	Max.	Max.
weight	0.12	0.15	0.33	0.19	0.07	0.14
P1	10	4	5	1	5	5
P2	25	5	4	2	4	4
P3	40	3	1	4	2	2
P4	20	5	3	2	2	4
P5	50	5	4	3	5	3

Table 7.5 Preference functions and the corresponding parameters

Preference Functions

		Parameters (Thresholds)		
Criteria	**Pref. Func.**	**q**	**p**	**σ**
Economics	Gaussian	-	-	15
Resources	Quasi (U-shape)	1.5	-	-
Strategy	Linear with indiff.	1	2	-
Risk	Level	0.5	1.5	-
Management	Level	0.5	1.5	-
Quality	Level	1	2	-

Performing the calculations of the preference indices with the weights obtained by the AHP method leads to the final values of leaving, entering and net flows given in Table 7.6 and the complete ranking of alternatives is obtained as illustrated with their net flows in Figure 7.2.

Table 7.6 Promethee flows of the original problem

Promethee Flows			
	Leaving Flow	Entering Flow	Net Flow
Alternative	φ^+	φ^-	Θ
P1	1.615	0.291	1.324
P2	1.029	0.302	0.727
P3	0.222	2.949	-2.727
P4	0.859	0.816	0.043
P5	1.084	0.450	0.634

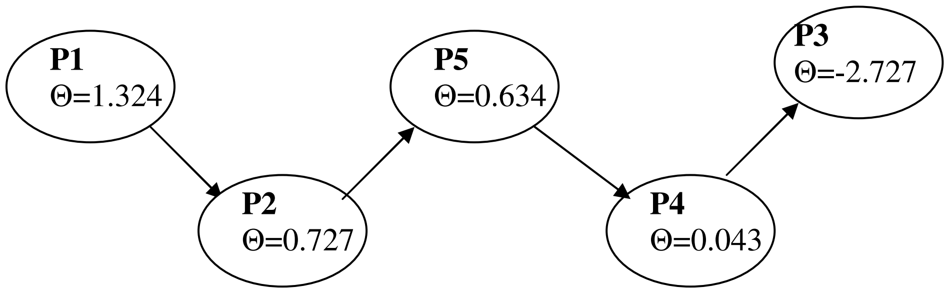


Figure 7.2 Complete ranking of alternatives (Original solution, Wang et al.)

7.2 GENERATED SCENARIOS FOR SAMPLE SOLUTIONS

7.2.1 Scenario I

Here, unlike the original solution, interrelationships between the criteria are not ignored any more, that is, the weight of a criterion has more or less has influence on the weight of another. In this scenario technique 2, *Matrix Multiplication* is used to handle the interrelationships. During the application, the original solution's weights are used for the initial weights, W_o. Moreover, to form the impact matrix E, it is necessary to do the pairwise comparisons of the criteria for each criterion. Here the author is set as the decision maker to do the comparisons. The impact matrix obtained as a result of the pairwise comparisons is given in Table 7.7. A column of the impact matrix gives the relative influences of the criteria on the criterion corresponding to that column. The highlighted values on the diagonal line indicate the self-influence of the criteria. These values are a little higher than the others in the same column, which is a natural outcome when the fact that each criterion influences itself more than the others is considered. The initial and the final weights and the deviations calculated after the implementation of Matrix Multiplication technique are presented in Table 7.8.

Table 7.7 Impact Matrix (Interrelationships among criteria)

Impact matrix

		Criteria that are impacted					
	Criteria	Economics	Resources	Strategy	Risk	Management	Quality
Criteria that impact	Economics	**0.41**	0.21	0.15	0.12	0.08	0.05
	Resources	0.10	**0.42**	0.15	0.12	0.13	0.38
	Strategy	0.25	0.12	**0.52**	0.26	0.16	0.09
	Risk	0.05	0.05	0.04	**0.34**	0.23	0.05
	Management	0.11	0.10	0.07	0.13	**0.36**	0.05
	Quality	0.08	0.10	0.07	0.03	0.04	**0.38**

Table 7.8 Weights deviations (Technique 2, Matrix Mult.)

Initial & final weights and the deviations

Criteria	Initial Weights	Final Weights	Change in Magnitude	% Change
Economics	0.12	0.17	0.05	38.0
Resources	0.15	0.21	0.06	39.7
Strategy	0.33	0.29	-0.04	-11.3
Risk	0.19	0.11	-0.08	-39.8
Management	0.07	0.11	0.04	54.6
Quality	0.14	0.11	-0.03	-21.9
Sums	**1.00**	**1.00**		

The first observation regarding the new weight set is that the dominance of the *Strategy* has decreased significantly in magnitude. On the other hand the influence of *Management* increased by 54.6 % with respect to its original weight.

The Promethee II application is performed exactly similar to the original solution, having the same preference functions and the threshold values so that the pure effect of the interdependency could be observed on the final ranking.

Table 7.9 Promethee flows for the scenario I

Promethee Flows			
	Leaving Flow	Entering Flow	Net Flow
Alternative	φ^+	φ^-	Θ
P1	1.350	0.413	0.937
P2	1.016	0.359	0.657
P3	0.314	2.759	-2.445
P4	0.754	0.931	-0.177
P5	1.304	0.275	1.029

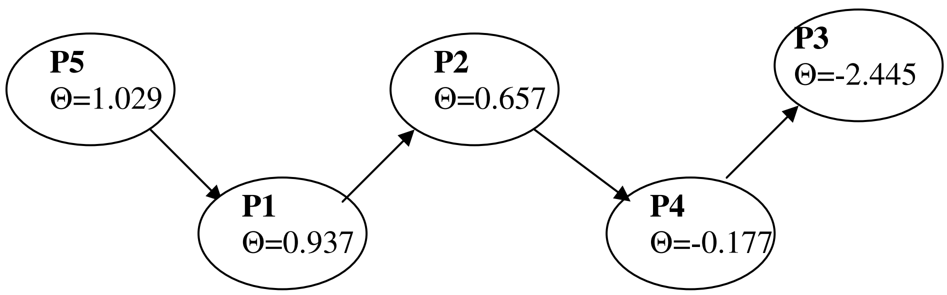


Figure 7.3 Complete ranking of alternatives (Scenario I)

7.2.2 Scenario II

This scenario is exactly similar to the previous one except the technique utilized to handle the interdependency. Using the same impact matrix and initial weights the given in Table 7.7 and 7.8 respectively, ANP and the supermatrix phenomena (technique 3) is used to calculate the final weights. Besides these the decision maker should also decide on another parameter, the weights of the clusters in the network. These are the main goal cluster weight and the criteria cluster weight, denoted by W_m and W_c respectively. By specifying these, the overall influence of the interrelationships among the criteria on the final weights is determined.

For this scenario, the cluster weights are set equal, that is

$$W_m = W_c = 0.5$$

The weighted supermatrix obtained using the cluster weights is given in Table 7.10. Since it is column stochastic matrix, the limiting operation yields the final weights, which are listed together with the initial weights and deviations in Table 7.11.

Finally Promethee II is applied same way (with the same preference functions and threshold values given in Table 7.5) to obtain the final ranking.

Table 7.10 The weighted supermatrix for ANP application (Scenario II)

Weighted supermatrix

	Main Goal	Economics	Resources	Strategy	Risk	Management	Quality
Main Goal	**0.000**	0.500	0.500	0.500	0.500	0.500	0.500
Economics	0.120	**0.205**	0.105	0.075	0.060	0.040	0.025
Resources	0.150	0.050	**0.210**	0.075	0.060	0.065	0.190
Strategy	0.330	0.125	0.060	**0.260**	0.130	0.080	0.045
Risk	0.190	0.025	0.025	0.020	**0.170**	0.115	0.025
Management	0.070	0.055	0.050	0.035	0.065	**0.180**	0.025
Quality	0.140	0.040	0.050	0.035	0.015	0.020	**0.190**

Table 7.11 Weights deviations (Technique 3, ANP(0.5,0.5))

Initial & Final Weights and the Deviations

Criteria	Initial Weights	Final Weights	Change in Magnitude	% Change
Economics	0.12	0.15	0.03	23.4
Resources	0.15	0.18	0.03	21.2
Strategy	0.33	0.31	-0.02	-7.2
Risk	0.19	0.15	-0.04	-22.0
Management	0.07	0.09	0.02	31.8
Quality	0.14	0.12	-0.02	-12.0

Table 7.12 Promethee flows for the scenario II

Promethee Flows

	Leaving Flow	Entering Flow	Net Flow
Alternative	φ^+	φ^-	Θ
P1	1.475	0.364	1.111
P2	1.022	0.337	0.685
P3	0.277	2.845	-2.568
P4	0.805	0.881	-0.076
P5	1.208	0.360	0.848

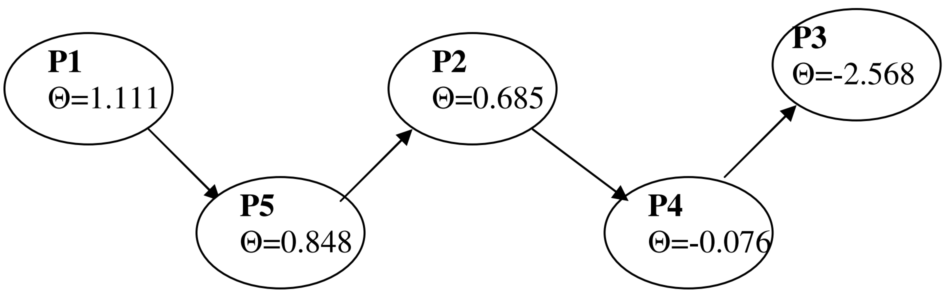


Figure 7.4 Complete ranking of alternatives (Scenario II)

7.2.3 Scenario III

In this scenario, it is aimed to see the effect of introduction of the prospect theory to the Promethee II application. For this reason, the original criteria weights are used. (Interdependency ignored). During the problem definition stage, the decision maker (author as in the previous cases) determines the criteria, which are more suitable for prospect theory application by entering the gain/loss ratio a value greater that 1. In this scenario, the gain/loss ratio for the criteria *Economics*, *Strategy* and *Management* are entered 2. With this modification on the original problem, gains and losses earn a defining meaning in a pairwise comparison of any alternative pairs.

The preference functions and the corresponding parameters defined by the decision maker for each criterion are listed in Table 7.13.

Table 7.13 Preference functions and the parameters (Scenario III)

Criteria Definitions					
	Gain / Loss	Preference Func.	Parameters		Opt. type
Economics	2	VIII- Exponential (Prospect Theory)		q=4	Max.
Resources	1	II- Quasi		q=1.5	Max.
Strategy	2	VII- Linear (Prospect Theory)		q=0.5	Max.
Risk	1	IV- Level	q=0.5	p=1.5	Min.
Management	2	VII- Linear (Prospect Theory)		q=0.5	Max.
Quality	1	IV- Level	q=1.0	p=2.0	Max.

The promethee flows and the final ranking obtained after the necessary calculations with the new preference functions for scenario III are given in Table 7.14 and Figure 7.5 respectively.

Table 7.14 Promethee flows for the scenario III

Promethee Flows			
	Leaving Flow	Entering Flow	Net Flow
Alternative	φ^+	φ^-	Θ
P1	1.14	0.389	0.751
P2	0.736	0.373	0.363
P3	0.148	2.584	-2.436
P4	0.604	0.747	-0.143
P5	0.661	0.497	0.164

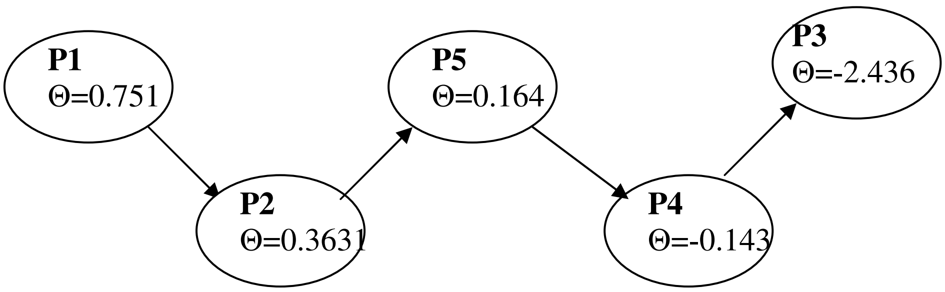


Figure 7.5 Complete ranking of alternatives (Scenario III)

7.2.4 Scenario IV

In this scenario a combination of the previous scenarios is applied, that is both the interdependency among criteria exists and preference functions parallel to Prospect Theory are introduced for some criteria.

In this solution the weights obtained through technique 2 in scenario I (Table 7.11) are used with the preference functions specified in scenario III (Table 7.13). The resulting Promethee flows are listed in Table 7.15 and the final ranking is illustrated in Figure 7.6.

Table 7.15 Promethee flows for the scenario IV

Promethee Flows			
	Leaving Flow	Entering Flow	Net Flow
Alternative	φ^+	φ^-	Θ
P1	0.865	0.552	0.313
P2	0.688	0.427	0.261
P3	0.209	2.441	-2.232
P4	0.533	0.88	-0.347
P5	0.791	0.316	0.475

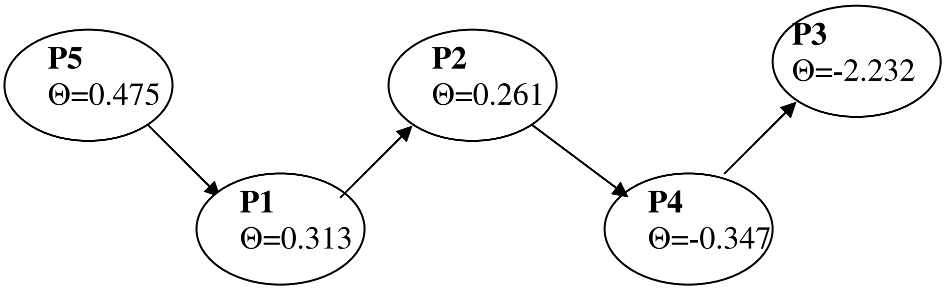


Figure 7.5 Complete ranking of alternatives (Scenario IV)

7.2.5 Scenario V

This scenario is exactly similar to the previous one but only the weights obtained in scenario 2 are used during the evaluation.

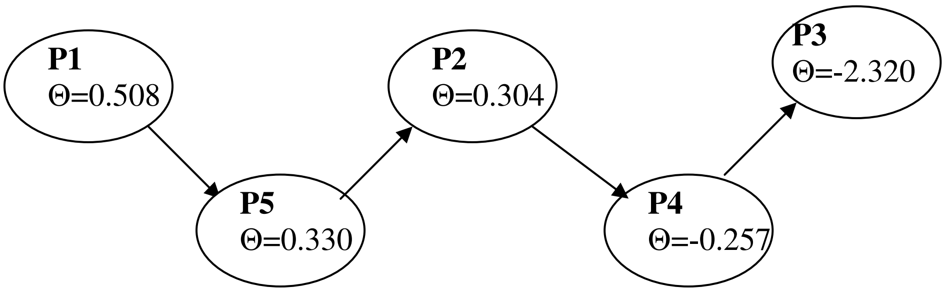


Figure 7.6 Complete ranking of alternatives (Scenario V)

Table 7.16 Promethee flows for the scenario V

Promethee Flows			
Alternative	Leaving Flow φ^+	Entering Flow φ^-	Net Flow Θ
P1	0.995	0.487	0.508
P2	0.712	0.408	0.304
P3	0.185	2.505	-2.320
P4	0.567	0.824	-0.257
P5	0.734	0.404	0.330

7.3 SUMMARY AND DISCUSSION

The criteria weights, final rankings and the corresponding net flow values for the original problem and the generated scenarios are summarized and illustrated in Table 7.17. Not surprisingly, newer approaches towards the solution of the problem slightly changed the final rankings.

For example, in the first scenario, interdependency among criteria is introduced and it is handled with technique 2, matrix multiplication. The final ranking deviated from the original solution which is because the technique employed updated the weights of the criteria significantly and hence the net flows changed, which lead to a slightly different final ranking. For example, the weights of the criteria economics Resources and Management has increased more than 30 % and *Risk* decreased from nearly 40% and these changes decreased the influences of the criteria. After re-calculating the net flows with the new set of weights, in the final ranking P1 & P2 moved down, while P5 moved up two ranks. This change in the rank makes sense when the criteria values for each alternative are considered.

Scenario II, where technique 3 (ANP - Supermatrix) is utilized with equal cluster weights (ANP(0.5, 0.5)), revealed a final ranking that is the intersection of the original solution and the scenario I. This outcome is reasonable because the deviations of the weights are less than scenario I and so the net flows deviated less.

For the scenarios II, IV and V, preference functions representing Prospect Theory properties are introduced for some criteria. The problem is solved in scenario II, IV and V with the criteria weights of the original solution, scenario I and scenario II respectively. Surprisingly, the introduction of Prospect Theory did not change the rankings due to the relatively small changes in the net flow values. Therefore it can be concluded that the deviation in the criteria weights contributes more to the final rankings rather than the introduction of Prospect Theory.

All in all, the sample solutions obtained above prove that the kind of approach to such discrete MCDM problems might be very much determining towards the solution, in other words if not thoroughly analyzed or digested, some simplifying assumptions, like ignoring the interdependencies among the criteria may obviously lead to undesired rankings.

Table 7.17 Summary: Weights Net Flows and Final Rankings

Summary: Final rankings

	Final Criteria Weights						Net flows				
	w1	**w2**	**w3**	**w4**	**w5**	**w6**	**P1**	**P2**	**P3**	**P4**	**P5**
Original Problem	0.12	0.15	0.33	0.19	0.07	0.14	**1.324**	**0.727**	**-2.727**	**0.043**	**0.634**
	P1 → P2 → P5 → P4 → P3										
Scenario I	0.17	0.21	0.29	0.11	0.11	0.11	**0.937**	**0.657**	**-2.445**	**-0.177**	**1.029**
	P5 → P1 → P2 → P4 → P3										
Scenario II	0.15	0.18	0.31	0.15	0.09	0.12	**1.111**	**0.685**	**-2.568**	**-0.076**	**0.848**
	P1 → P5 → P2 → P4 → P3										
Scenario III	0.12	0.15	0.33	0.19	0.07	0.14	**0.751**	**0.363**	**-2.436**	**-0.143**	**0.164**
	P1 → P2 → P5 → P4 → P3										
Scenario IV	0.17	0.21	0.29	0.11	0.11	0.11	**0.313**	**0.261**	**-2.232**	**-0.347**	**0.475**
	P5 → P1 → P2 → P4 → P3										
Scenario V	0.12	0.15	0.33	0.19	0.07	0.14	**0.508**	**0.304**	**-2.320**	**-0.257**	**0.330**
	P1 → P5 → P2 → P4 → P3										

CHAPTER 8

SAMPLE APPLICATION: RANKING THE UNIVERSITIES

In this chapter, a sample application of the methodology developed in this study is performed. The applied problem is the ranking of top 101 universities around the world within the context of 6 criteria. The problem data is obtained from the ranking study performed in the Institute of Higher Education, Shanghai Jiao Tong University. Using the alternative techniques, three different criteria weight set is obtained and rankings are obtained using the software developed. At the end of this chapter, a comparison study conducted among the original ranking and the new rankings is presented. All the rankings obtained are given in Appendix A.2.

8.1 THE ORIGINAL RANKING STUDY

The universities are ranked by several indicators of academic or research performance, including alumni and staff winning Nobel Prizes and Fields Medals, highly cited researchers, articles published in Nature and Science, articles indexed in major citation indices, and the per capita academic performance of an institution.

The data used during ranking and the sources for these data are provided in the Appendix A.1 and A.2 respectively. For each criterion, the highest scoring institution is assigned a score of 100, and other institutions are calculated as a percentage of the top score. The distribution of data for each criterion is examined for any significant distorting effect; standard statistical techniques are used to adjust the criteria if necessary.

Scores for every criteria are weighted as given in Table 8.1 to arrive at a final overall score for an institution. The highest scoring institution is assigned a score of 100, and other institutions are calculated as a percentage of the top score. An institution's rank reflects the number of institutions that sit above it.

The definitions of the 6 criteria that are used in the problem are as follows:

1. *Alumni*

The total number of the alumni of an institution winning Nobel Prizes and Fields Medals. Alumni are defined as those who obtain bachelor, Master's or doctoral degrees from the institution. Different weights are set according to the periods of obtaining degrees. The weight is 100% for alumni obtaining degrees in 1991-2000, 90% for alumni obtaining degrees in 1981-1990, 80% for alumni obtaining degrees in 1971-1980, and so on, and finally 10% for alumni obtaining degrees in 1901-1910. If a person obtains more than one degrees from an institution, the institution is considered once only.

2. *Awards*

The total number of the staff of an institution winning Nobel prizes in physics, chemistry, medicine and economics and Fields Medal in mathematics. Staff is defined as those who work at an institution at the time of winning the prize. Different weights are set according to the periods of winning the prizes. The weight is 100% for winners in 2001-2005, 90% for winners in 1991-2000, 80% for winners in 1981-1990, 70% for winners in 1971-1980, and so on, and finally 10% for winners in 1911-1920. If a winner is affiliated with more than one institution, each institution is assigned the reciprocal of the number of institutions. For Nobel prizes, if a prize is shared by more than one person, weights are set for winners according to their proportion of the prize.

3. HiCi

The number of highly cited researchers in broad subject categories in life sciences, medicine, physical sciences, engineering and social sciences. These individuals are the most highly cited within each category. The definition of categories and detailed procedures can be found at the website of Institute of Scientific information.

4. N&S

The number of articles published in Nature and Science between 2001 and 2005. To distinguish the order of author affiliation, a weight of 100% is assigned for corresponding author affiliation, 50% for first author affiliation (second author affiliation if the first author affiliation is the same as corresponding author affiliation), 25% for the next author affiliation, and 10% for other author affiliations. Only publications of article type are considered.

5. SCI

Total number of articles indexed in Science Citation Index-expanded and Social Science Citation Index in 2005. Only publications of article type are considered. When calculating the total number of articles of an institution, a special weight of two was introduced for articles indexed in Social Science Citation Index.

6. *Size*

The weighted scores of the above five indicators divided by the number of full-time equivalent academic staff. If the number of academic staff for institutions of a country cannot be obtained, the weighted scores of the above five indicators is used. For ranking 2006, the numbers of full-time equivalent academic staff are obtained for institutions in USA, UK, Japan, South Korea, Czech, China, Italy, Australia, Netherlands, Sweden, Switzerland, Belgium, Slovenia, New Zealand etc.

The criteria definitions and the weights utilized in the original study are summarized in Table 8.1.

Table 8.1 Criteria definitions & weights of the original study

Criteria	Indicator	Code	Weight
Quality of Education	Alumni of an institution winning Nobel Prizes and Fields Medals	Alumni	0.10
Quality of Faculty	Staff of an institution winning Nobel Prizes and Fields Medals	Award	0.20
	Highly cited researchers in 21 broad subject categories	HiCi	0.20
Research Output	Articles published in Nature and Science	N&S	0.20
	Articles in Science Citation Index-expanded, Social Science Citation Index	SCI	0.20
Size of Institution	Academic performance with respect to the size of an institution	Size	0.10
		Total:	**1.00**

8.2 RANKING USING THE METHODOLOGY

The data given in Appendix A.1 is used to rank the alternatives with the methodology developed. To observe the differences, first the weights used in the original study and then the weights updated with techniques 2 and 3 are used in the given order under three scenarios and finally three distinct rankings are obtained respectively. Wherever necessary the author is set as the decision maker (pairwise comparisons, preference function determination, etc.).

8.2.1 Scenario I

The weights used in the original study (Table 8.1) are used in this scenario. Preference functions and the corresponding parameter definitions given in Table 8.2 are used for the Promethee II application.

Table 8.2 Preference functions and the parameters (Scenario II & III)

Criteria Definitions					
	Gain / Loss	Preference Func.	Parameters		Opt. type
Alumni	2	VII- Linear (Prospect Theory)		q=4	Max.
Awards	1.5	VII- Linear (Prospect Theory)		q=3	Max.
HiCi	1	III- Linear		p=6	Max.
N & S	1	V- Linear with indiff.	q=2	p=6	Max.
SCI	1.5	VIII- Exponential (Prospect Theory)		q=3	Max.
Size	1	III- Linear		p=6	Max.

8.2.2 Scenario II

The final ranking is obtained considering the interdependencies among the criteria. The original weights are updated with matrix multiplication technique, where the initial weights (W_o) and the impact matrix (E) are as follows:

$$W_o = \begin{bmatrix} w_o^{Alumni} \\ w_o^{Award} \\ w_o^{HiCi} \\ w_o^{N\&S} \\ w_o^{SCI} \\ w_o^{Size} \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \\ 0.1 \end{bmatrix}$$

$$E = \begin{array}{r|cccccc} & Alumni & Awards & HiCi & N\,\&\,S & SCI & Size \\ \hline Alumni & 0.33 & 0.15 & 0.09 & 0.10 & 0.10 & 0.14 \\ Awards & 0.08 & 0.31 & 0.09 & 0.10 & 0.10 & 0.14 \\ HiCi & 0.17 & 0.15 & 0.36 & 0.10 & 0.10 & 0.14 \\ N\,\&\,S & 0.08 & 0.15 & 0.09 & 0.40 & 0.10 & 0.14 \\ SCI & 0.17 & 0.15 & 0.18 & 0.10 & 0.40 & 0.14 \\ Size & 0.17 & 0.08 & 0.18 & 0.20 & 0.20 & 0.29 \end{array}$$

And the weights obtained using technique 2 (Matrix Multiplication) are

$$W = \begin{bmatrix} w^{Alumni} \\ w^{Award} \\ w^{HiCi} \\ w^{N\&S} \\ w^{SCI} \\ w^{Size} \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.14 \\ 0.17 \\ 0.17 \\ 0.20 \\ 0.18 \end{bmatrix}.$$

The preference function definitions given in scenario I are used for the Promethee II application.

8.2.3 Scenario III

Again in this scenario interdependencies are not ignored and handled via technique 3, ANP. The cluster weighing is done as follows:

$$W_m = W_c = 0.5$$

The supermatrix constructed using the above cluster weights is given in Table 8.3.

The final rankings obtained for scenario I, II and III are provided in Appendix A.3 together with the final ranking of the original study.

Table 8.3 The weighted supermatrix for ANP application (Scenario II)

Weighted supermatrix

	Main Goal	Alumni	Award	HiCi	N&S	SCI	Size
Main Goal	0.00	0.50	0.50	0.50	0.50	0.50	0.50
Alumni	0.10	**0.18**	0.08	0.05	0.05	0.05	0.07
Award	0.20	0.04	**0.14**	0.05	0.05	0.05	0.07
HiCi	0.20	0.08	0.08	**0.17**	0.05	0.05	0.07
N&S	0.20	0.04	0.08	0.05	**0.20**	0.05	0.07
SCI	0.20	0.08	0.08	0.09	0.05	**0.20**	0.07
Size	0.10	0.08	0.04	0.09	0.10	0.10	**0.15**

And the weights obtained using technique 3 (ANP-supermatrix) are

$$W = \begin{bmatrix} w^{Alumni} \\ w^{Award} \\ w^{HiCi} \\ w^{N\&S} \\ w^{SCI} \\ w^{Size} \end{bmatrix} = \begin{bmatrix} 0.120 \\ 0.170 \\ 0.185 \\ 0.185 \\ 0.200 \\ 0.140 \end{bmatrix}.$$

The preference function definitions given in scenario I are used for the Promethee II application.

8.3 COMPARISONS

The final rankings obtained for scenario I, II, III and the ranking of the original study is given in Appendix A.3. The most obvious outcome of the study is that the application of the developed methodology yields rankings that have significant deviations from the original study. Ranking deviation calculations are presented in Table 8.4, where the original study is expressed as scenario 0. Application of a new methodology together with considering the interdependencies among the criteria and choice behavior of the decision maker is the reason for the deviation. Here again it is proved that the kind of approach and the assumptions are very much determining towards the solution in multiple criteria problems.

On the other hand when the rankings of the scenarios II and III are compared with respect to scenario I, a result that is consistent with the previous applications is easily derived. This is because scenario II deviates the results more that III since the matrix multiplication technique updated the weights more than the ANP(0.5, 0.5).

Table 8.4 Ranking deviations among the scenarios

Comparison of rankings

	Average Abs. Diff.*	Std. Deviation**	Max.***	# of dev. > 5****
| scen. 0 - scen. I |	7.52	7.11	30	48
| scen. 0 - scen. II |	7.43	6.63	30	50
| scen. 0 - scen. III |	7.31	6.78	29	50
| scen. I - scen. II |	2.97	2.84	11	19
| scen. I - scen. III |	1.60	1.55	6	1
| scen. II - scen. III |	1.62	1.64	8	1

(*): Average deviation of the rank of any alternative

(**): Standard deviation of the rank deviations

(***): Max. rank deviation

(****): # of alt.`s whose rank changed at least 6 places.

CONCLUSION

In this study, a methodology aiming to rank the alternatives in a discrete MCDM problem is developed and proposed. It is constructed upon the basis of the well known outranking method Promethee II. Alternative techniques are proposed for the determination of the criteria weights and new preference functions that incorporate the Prospect Theory into Promethee are suggested.

In discrete MCDM, there have been various methodologies developed based on the Promethee family methods as given in chapter one. This study differs with its two aspects. First is the way it handles the determination of the criteria weights and the second is its approach to incorporate the choice behavior of the decision maker.

Due to the alternative techniques to specify the criteria weights, decision maker is not restricted to a single method. He/She can either choose to incorporate the interdependencies between the criteria and perform the calculations according to that or ignore them. However, the two courses of actions, ignoring and not ignoring the interdependencies among the criteria, both have some disadvantages. When the calculations are performed without considering the interdependencies, the resulting weight set does not reflect the reality, in other words, the contribution of the interdependencies is totally disregarded and the existence of a criterion has no influence on the weights of the others. For this reason, the independent criteria assumption, which has been the dominant approach in MCDM applications and researches, has a negative influence while modeling the problem. On the other hand, it is not an easy process to calculate the weights with interdependency. In the study, two alternative techniques are proposed to handle the interdependency, which are the Matrix Multiplication technique and the ANP (supermatrix) technique. In both there is the necessity to make the pairwise comparisons of all the criteria pairs with respect to each criterion to obtain the relative influences between the criteria, which is represented with impact matrix. Suppose there are *n* criteria, then the impact matrix is an *n* x *n* matrix. The eigenvector in each column of the impact matrix represent the relative influence on the parent criterion (the one corresponding to that column) of the criteria. The eigenvectors are obtained through a pairwise comparison process of AHP according to Saaty`s 1-9 scale. For each eigenvector, (n-1)*(n-2)/2 comparisons are necessary. For the whole problem, the total number of comparisons (only for interdependency) is n*(n-1)*(n-2)/2, which is in the order of n^3. In other words, the number of comparisons increases rapidly with the criteria number. Moreover, it is a more difficult task to be consistent during the pairwise comparisons when n is large. The vast number of criteria in a problem could also be handled by grouping them into sub-clusters and the problem could be solved by Matrix Manipulation (Saaty, 1996), constructing multi-level hierarchy instead of single level.

One other important feature of the methodology developed is that it can manipulate the choice behavior of the decision maker with a simple interaction by asking a single question during the construction of the problem. This may seem to be a very insufficient way to model the decision behavior; however, one must note that there are yet no methods which are commonly accepted to model the decision behavior of the decision maker and there are still researches going on to obtain the preference function of the decision maker within the context of Decision Theory (Korhonen et al. 1990, Salminen 1994, Karasakal et al. 2005). Considering this, simply the method given in this study is proposed instead of a sophisticated one.

According to the answer given by the decision maker to the question asked in the beginning of the methodology, two different sets of preference functions are proposed as given in chapter 3. The first set is composed of conventional functions suggested with Promethee by Brans et al (1986). In the second set, there are two new preference functions that incorporate the Prospect Theory (Kahneman and Tversky, 1979). According to the definition of the prospect theory, the preference function is steeper for losses than for gains in the new preference functions. The preference function is updated according to the answer given by the decision maker, so that it yields less for the gains than losses.

All in all, the main characteristic of the methodology developed is its flexibility that there exist alternative courses of actions during the criteria weights determination stage and the specification of the preference functions with respect to each criterion.

The work presented in this study may be improved with respect to many different perspectives as listed below:

- Within organizations, important decisions are made by a board of executives instead of a single person. For this reason, the methodology may be extended by introducing the group decision making techniques.

- Besides the alternative techniques presented to specify the weights of the criteria, newer approaches may also be integrated to the methodology and proposed to the decision maker alternatively. Especially newer tools to handle the interdependency among criteria might be very useful since the proposed techniques are not easily applicable when the number of the criteria is large.

- The presented preference functions that incorporate Prospect Theory into Promethee could be extended with newer functions.

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APPENDIX A

APPLICATION: RANKING THE UNIVERSITIES

A.1 Problem Data (6 criteria, 101 alternatives)

Table A.1 Criteria values for the alternatives

Institution	Country	Score on Alumni	Score on Award	Score on HiCi	Score on N&S	Score on SCI	Score on Size	Total Score*
Harvard Univ	USA	100	100	100	100	100	73.6	100
Univ Cambridge	UK	96.3	91.5	53.8	59.5	67.1	66.5	72.6
Stanford Univ	USA	39.7	70.7	88.4	70	71.4	65.3	72.5
Univ California - Berkeley	USA	70.6	74.5	70.5	72.2	71.9	53.1	72.1
Massachusetts Inst Tech (MIT)	USA	72.9	80.6	66.6	66.4	62.2	53.6	69.7
California Inst Tech	USA	57.1	69.1	59.1	64.5	50.1	100	66
Columbia Univ	USA	78.2	59.4	56	53.6	69.8	45.8	61.8
Princeton Univ	USA	61.1	75.3	59.6	43.5	47.3	58	58.6
Univ Chicago	USA	72.9	80.2	49.9	43.7	54.1	41.8	58.6
Univ Oxford	UK	62	57.9	48	54.3	66	46	57.6
Yale Univ	USA	50.3	43.6	59.1	56.6	63	49.3	55.9
Cornell Univ	USA	44.9	51.3	56	48.4	65.2	40.1	54.1
Univ California - San Diego	USA	17.1	34	59.6	54.8	65.6	47.1	50.5
Univ California - Los Angeles	USA	26.4	32.1	57.6	47.5	77.3	34.9	50.4
Univ Pennsylvania	USA	34.2	34.4	57	41.7	73.6	40	50.1
Univ Wisconsin - Madison	USA	41.5	35.5	53.3	45.1	68.3	29.3	48.8
Univ Washington - Seattle	USA	27.7	31.8	53.3	47.6	75.5	27.8	48.5
Univ California - San Francisco	USA	0	36.8	55.5	54.8	61.1	48.2	47.7
Tokyo Univ	Japan	34.8	14.1	41.4	51.5	85.5	35.2	46.7
Johns Hopkins Univ	USA	49.5	27.8	40.7	52.2	68.8	25.3	46.6
Univ Michigan - Ann Arbor	USA	41.5	0	61.5	41.6	76.9	31.2	44.5
Kyoto Univ	Japan	38.3	33.4	36.9	36.2	72.4	31.7	43.9
Imperial Coll London	UK	20.1	37.4	40	39.7	64.2	40.2	43.4
Univ Toronto	Canada	27.1	19.3	38.5	36.5	78.3	44.8	42.8
Univ Illinois - Urbana Champaign	USA	40.1	36.6	45.5	33.6	57.7	26.3	42.5

(*)The ranking of the original solution is obtained referring to this value.

Table A.1 Criteria values for the alternatives **(continued)**

Institution	**Country**	**Score on Alumni**	**Score on Award**	**Score on HiCi**	**Score on N&S**	**Score on SCI**	**Score on Size**	**Total Score***
Univ Coll London	UK	29.6	32.2	38.5	43.2	60	33.4	42.2
Swiss Fed Inst Tech - Zurich	Switzerland	38.8	36.3	35.3	39.9	43.5	52.6	41.2
Washington Univ - St. Louis	USA	24.2	26	37.7	45.6	55.3	40.4	40.4
New York Univ	USA	36.8	24.5	42.8	34	54	26.4	38.4
Rockefeller Univ	USA	21.8	58.6	28.8	44.8	24.1	38.4	38.3
Duke Univ	USA	20.1	0	48	45.4	62.4	40.3	38.2
Univ Minnesota - Twin Cities	USA	34.8	0	50.4	34.1	69.7	24.3	37.8
Northwestern Univ	USA	21	18.9	44.9	33.6	57.1	36.7	37.6
Univ Colorado - Boulder	USA	16	30.8	40	37	46.4	30.1	36.4
Univ California - Santa Barbara	USA	0	35.3	42.1	37	43.7	35.7	36.1
Univ British Columbia	Canada	20.1	18.9	31.7	31.9	62.1	36.6	35.5
Univ Maryland - Coll Park	USA	25	20	40	32.7	53.8	26.4	35.4
Univ Texas Southwestern Med Center	USA	23.4	33.2	31.7	38.1	39.8	33.5	35.2
Univ Texas - Austin	USA	21	16.7	48	28.3	55.4	21.8	34.9
Univ Utrecht	Netherlands	29.6	20.9	28.8	27.5	57.3	26.9	33.4
Vanderbilt Univ	USA	12.1	29.6	32.6	24.7	50.6	36.2	33.2
Pennsylvania State Univ - Univ Park	USA	13.5	0	44.9	37.7	58	23.8	32.7
Univ California - Davis	USA	0	0	47.4	33.3	63.3	30.1	32.7
Univ California - Irvine	USA	0	29.4	35.3	28.9	49	32.4	32.6
Univ Paris 06	France	34.4	23.5	23.1	24.9	52.9	32.5	32.4
Rutgers State Univ - New Brunswick	USA	14.8	20	38.5	32.7	46.5	24.6	32.3
Univ Southern California	USA	0	26.8	37.7	24.1	54	26.6	32
Karolinska Inst Stockholm	Sweden	29.6	27.3	33.5	18	48.7	25.6	31.9
Univ Pittsburgh - Pittsburgh	USA	24.2	0	40	24	65	28.6	31.9
Univ Manchester	UK	26.4	18.9	24.3	24.9	58.7	28.7	31.7
Univ Munich	Germany	35.8	22.9	15.4	28	52.9	32.2	31.5
Univ Edinburgh	UK	21.8	16.7	25.5	35.4	49.3	30.3	31.4
Univ Florida	USA	21.8	0	36.1	25.1	65.6	26.7	31
Australian Natl Univ	Australia	17.1	12.6	37.7	30.1	44.4	32.8	30.8
Tech Univ Munich	Germany	41.5	23.6	24.3	19.5	46.2	30.7	30.8
Carnegie Mellon Univ	USA	33.7	32.8	32.6	12.7	37.5	31.8	30.5
Univ Copenhagen	Denmark	29.6	24.2	23.1	24.8	46.4	30	30.5

Table A.1 Criteria values for the alternatives **(continued)**

Institution	Country	Score on Alumni	Score on Award	Score on HiCi	Score on N&S	Score on SCI	Score on Size	Total Score
Univ Zurich	Switzerland	12.1	26.8	21.8	29.7	47.9	31.4	30.4
Univ North Carolina - Chapel Hill	USA	12.1	0	37.7	29.3	60.3	27.9	30.3
Hebrew Univ Jerusalem	Israel	32	20	25.5	25.2	44.7	29.5	30
Osaka Univ	Japan	12.1	0	25.5	30.7	67	29.9	29.6
McGill Univ	Canada	27.7	0	30.8	22.4	59.7	33.5	29.5
Univ Bristol	UK	10.5	17.9	29.8	26.3	47.8	33.2	29.5
Univ Paris 11	France	32	33.5	13.3	20.8	44.7	29.7	29.4
Uppsala Univ	Sweden	25	32.2	13.3	24.6	49.3	21.5	29.3
Ohio State Univ - Columbus	USA	17.1	0	40.7	20.6	61.3	19.7	29
Univ Heidelberg	Germany	19.1	27.2	18.8	21.5	49.5	29.5	29
Univ Oslo	Norway	25	33.4	18.8	17.7	42.7	28.5	28.6
Univ Sheffield	UK	22.6	14.1	23.1	29.2	45.8	30.2	28.5
Case Western Reserve Univ	USA	39.2	11.5	21.8	22	43.9	33.6	27.9
Moscow State Univ	Russia	49.5	34.2	0	5.6	54.3	33.4	27.9
Univ Leiden	Netherlands	24.2	15.5	28.8	18.9	46	28.5	27.8
Purdue Univ - West Lafayette	USA	18.2	16.7	27.7	20.7	50.6	19.9	27.7
Univ Helsinki	Finland	18.2	17.9	20.4	19.2	53.4	29.2	27.6
Univ Rochester	USA	32	8.9	26.6	21.6	43.3	35.6	27.6
Tohoku Univ	Japan	18.2	0	20.4	22.6	65.9	29.2	27.2
Univ Arizona	USA	0	0	28.8	36.7	54	25.6	27.2
Univ Melbourne	Australia	14.8	14.1	23.1	18.1	54.8	25.2	26.7
Univ Nottingham	UK	14.8	20	23.1	18.3	45	27.6	26.2
Michigan State Univ	USA	12.1	0	37.7	22.7	51.2	18.6	26.1
Boston Univ	USA	14.8	0	31.7	26.7	51.6	17.8	25.9
Univ Basel	Switzerland	25	17.1	20.4	22.4	36.2	35.4	25.9
King's Coll London	UK	16	23.1	20.4	16.7	43.9	26.7	25.8
Stockholm Univ	Sweden	28.4	29.6	15.4	18.5	36.9	19.7	25.6
Brown Univ	USA	0	13.6	28.8	26.7	40.5	28.4	25.4
Univ Goettingen	Germany	37.3	20	15.4	15.9	40.8	26	25.4
Rice Univ	USA	21	21.9	23.1	22	30.4	30.4	25.3
Texas A&M Univ - Coll Station	USA	0	0	31.7	24.4	55.7	20.8	25.1
Tokyo Inst Tech	Japan	16	0	23.1	23.3	51.2	32.5	25
Lund Univ	Sweden	28.4	0	24.3	20.2	52.2	18.8	24.7
McMaster Univ	Canada	16	18.9	21.8	14.2	44.6	25.6	24.7
Univ Birmingham	UK	24.2	10.9	21.8	15.2	46.6	27.6	24.7
Univ Freiburg	Germany	24.2	20.9	17.2	18.4	38.8	24.4	24.6
Univ Utah	USA	0	0	30.8	28.6	47.1	25.3	24.5
Univ Iowa	USA	0	0	33.5	22.4	51.6	21.8	24.3

Table A.1 Criteria values for the alternatives **(continued)**

Institution	**Country**	**Score on Alumni**	**Score on Award**	**Score on HiCi**	**Score on N&S**	**Score on SCI**	**Score on Size**	**Total Score**
Univ Strasbourg 1	France	28.4	22.5	18.8	16.7	33.6	23.6	24.2
Indiana Univ - Bloomington	USA	13.5	17.9	24.3	18.9	40.7	17.8	24.1
Nagoya Univ	Japan	0	14.1	15.4	21.6	52.9	25.8	24
Ecole Normale Super Paris	France	46.1	24.5	13.3	14.8	27.3	24.1	23.6
Arizona State Univ - Tempe	USA	0	14.1	21.8	27	42.6	18.1	23.5
Univ Roma - La Sapienza	Italy	16	15.5	10.9	19.4	53.3	14.8	23.5

A.2 Data Sources

- Nobel laureates. http://www.nobel.se.
- Fields Medals. http://www.mathunion.org/medals/.
- Highly cited researchers. http://www.isihighlycited.com.
- Articles published in Nature and Science. http://www.isiknowledge.com.
- Articles indexed in Science Citation Index-expanded, Social Science Citation Index, and Arts & Humanities Citation Index. http://www.isiknowledge.com

A.3 Rankings for the top 101 universities around the world

Table A.2 Rankings obtained

Institution	Original Study	Methodology Application: Scen. I Original Weights	Scen. II Matrix Mult.	Scen. III ANP (.5,.5)
Harvard Univ	1	1	1	1
Univ Cambridge	2	5	5	5
Stanford Univ	3	3	3	3
Univ California - Berkeley	4	2	2	2
Massachusetts Inst Tech (MIT)	5	4	4	4
California Inst Tech	6	6	6	6
Columbia Univ	7	7	7	7
Princeton Univ	8	13	11	11
Univ Chicago	8	16	15	15
Univ Oxford	10	10	10	10
Yale Univ	11	8	8	8
Cornell Univ	12	11	12	12
Univ California - San Diego	13	9	9	9
Univ California - Los Angeles	14	14	16	16
Univ Pennsylvania	15	15	14	14
Univ Wisconsin - Madison	16	19	23	20
Univ Washington - Seattle	17	21	24	21
Univ California - San Francisco	18	12	13	13
Tokyo Univ	19	17	18	17
Johns Hopkins Univ	20	24	29	28
Univ Michigan - Ann Arbor	21	18	19	19
Kyoto Univ	22	29	28	29
Imperial Coll London	23	22	20	22
Univ Toronto	24	25	21	24
Univ Illinois - Urbana Champaign	25	31	33	32
Univ Coll London	26	26	27	25
Swiss Fed Inst Tech - Zurich	27	28	25	27
Washington Univ - St. Louis	28	23	22	23
New York Univ	29	34	36	36
Rockefeller Univ	30	41	37	38
Duke Univ	31	20	17	18
Univ Minnesota - Twin Cities	32	33	35	33
Northwestern Univ	33	27	26	26
Univ Colorado - Boulder	34	35	34	35

Table A.2 Rankings obtained **(Continued)**

Institution	Original Study	Methodology Application: Scen. I Original Weights	Scen. II Matrix Mult.	Scen. III ANP (.5,.5)
Univ California - Santa Barbara	35	30	30	30
Univ British Columbia	36	37	31	34
Univ Maryland - Coll Park	37	38	41	40
Univ Texas Southwestern Med Center	38	39	38	39
Univ Texas - Austin	39	40	48	43
Univ Utrecht	40	56	57	57
Vanderbilt Univ	41	47	42	44
Pennsylvania State Univ - Univ Park	42	36	40	37
Univ California - Davis	42	32	32	31
Univ California - Irvine	44	43	43	42
Univ Paris 06	45	59	55	58
Rutgers State Univ - New Brunswick	46	44	51	46
Univ Southern California	47	52	58	56
Karolinska Inst Stockholm	48	70	74	71
Univ Pittsburgh - Pittsburgh	48	46	46	47
Univ Manchester	50	63	63	63
Univ Munich	51	65	60	62
Univ Edinburgh	52	48	47	48
Univ Florida	53	49	54	54
Australian Natl Univ	54	42	39	41
Tech Univ Munich	54	75	70	72
Carnegie Mellon Univ	56	77	72	76
Univ Copenhagen	56	68	66	64
Univ Zurich	58	58	56	59
Univ North Carolina - Chapel Hill	59	45	44	45
Hebrew Univ Jerusalem	60	50	53	51
Osaka Univ	61	53	52	52
McGill Univ	62	57	50	53
Univ Bristol	62	55	49	50
Univ Paris 11	64	86	83	83
Uppsala Univ	65	87	90	89
Ohio State Univ - Columbus	66	62	71	66
Univ Heidelberg	66	79	76	79
Univ Oslo	68	89	87	87
Univ Sheffield	69	61	61	60
Case Western Reserve Univ	70	73	65	69
Moscow State Univ	70	91	80	85

Table A.2 Rankings obtained **(Continued)**

Institution	Original Study	Methodology Application: Scen. I Original Weights	Scen. II Matrix Mult.	Scen. III ANP (.5,.5)
Univ Leiden	72	60	62	61
Purdue Univ - West Lafayette	73	80	86	82
Univ Helsinki	74	81	78	80
Univ Rochester	74	54	45	49
Tohoku Univ	76	76	73	74
Univ Arizona	76	51	59	55
Univ Melbourne	78	84	85	86
Univ Nottingham	79	85	84	84
Michigan State Univ	80	67	75	70
Boston Univ	81	69	77	73
Univ Basel	81	78	67	75
King's Coll London	83	93	91	93
Stockholm Univ	84	100	100	100
Brown Univ	85	66	68	68
Univ Goettingen	85	97	95	96
Rice Univ	87	82	82	81
Texas A&M Univ - Coll Station	88	72	79	77
Tokyo Inst Tech	89	71	64	67
Lund Univ	90	88	92	91
McMaster Univ	90	95	94	94
Univ Birmingham	90	92	88	90
Univ Freiburg	93	96	96	97
Univ Utah	94	64	69	65
Univ Iowa	95	74	81	78
Univ Strasbourg 1	96	99	98	99
Indiana Univ - Bloomington	97	94	97	95
Nagoya Univ	98	90	89	92
Ecole Normale Super Paris	99	101	101	101
Arizona State Univ - Tempe	100	83	93	88
Univ Roma - La Sapienza	100	98	99	98