LEPTON FLAVOR VIOLATION IN THE TWO HIGGS DOUBLET MODEL

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ABSTRACT

LEPTON FLAVOR VIOLATION IN THE TWO HIGGS DOUBLET MODEL

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The lepton flavor violating interactions are interesting in the sense that they are sensitive the physics beyond the standard model and they ensure considerable information about the restrictions of the free parameters, with the help of the possible accurate measurements. In this work, we investigate the lepton flavor violating $H^+ \to W^+ l_i^- l_j^+$ and the lepton flavor conserving $H^+ \to W^+ l_i^- l_i^+$ ($l_i = \tau, l_j = \mu$) decays in the general two Higgs doublet model and we estimate decay widths of these decays. After that, we analyze lepton flavor violating decay $\tau \to \mu \overline{\nu_i} \nu_i$ in the same model and calculate its branching ratio. We observe that the experimental results of the processes under consideration can give comprehensive information about the physics beyond the standard model and the existing free parameters.

Keywords: Lepton flavor violation, Lepton flavor conservation, Standard Model, Two Higgs doublet model, Decay width, Branching ratio.

ÖZ

İKİ HİGGS DUBLET MODELİNDE LEPTON ÇEŞNİ BOZULMASI

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Lepton çeşni bozan etkileşmeler, standart model ötesi fiziğe duyarlı olmaları ve teorik modellerdeki serbest değişkenlerin deneysel hassas ölçümler yardımıyla sınırlandırılmaları hakkında önemli bilgiler içermesi sebebiyle ilgi çekicidir. Bu çalışmada, lepton çeşni bozan $H^+ \to W^+ l_i^- l_j^+$ ve lepton çeşni koruyan $H^+ \to W^+ l_i^- l_i^+$ $(l_i = \tau, l_j = \mu)$ bozunumlarını genel iki Higgs dublet modeli çerçevesinde inceledik ve bu bozunumlar için bozunma genişliklerini hesapladık. Buna ek olarak, lepton çeşni bozan $\tau \to \mu \overline{\nu_i} \nu_i$ bozunumunu aynı modelde inceledik ve dallanma oranını hesapladık. Bu süreçlerin deneysel sonuçları kullanılarak, standart model ötesinde var olabilecek yeni fizik ve ortaya çıkabilecek serbest değişkenler hakkında kapsamlı bilgi edinilebilineceğini gözlemledik.

Anahtar Kelimeler: Lepton çeşni bozulması, Lepton çeşni korunması, Standart model, İki Higgs dublet modeli, Bozunma genişliği, Dallanma oranı.

...ANNEME

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CHAPTER 1

INTRODUCTION

The Standard Model (SM) is the simplest model which combines the electromagnetic and weak interactions. It is a comprehensive theory that explains all the hundreds of particles and complex interactions with only six quarks, six leptons and force carrier particles. The SM which is proposed by Glashow-Weinberg-Salam [1]-[2] has been a remarkable success story of modern theoretical and experimental high-energy physics, during the last decades. The discovery of neutral weak interactions and the production of intermediate vector bosons $W^{\pm}, Z^0$ with the expected properties increased our confidence in the model [3]. The SM contains the spin one-half quarks, leptons, the spin one gauge bosons and the spin zero Higgs field as fundamental degrees of freedom. For the details of the model construction see Chapter 2 and also textbooks [4]-[8] and review [9]-[11] existing in the literature.

The SM answers many of the questions about the structure and stability of matter with its six types of quarks, six types of leptons and three forces. But the SM is not complete and there are still many unsolved questions: What is the reason beyond the hierarchy of fundamental forces? What is the origin of the masses of fundamental particles and their mass hierarchies? Are quarks and leptons actually fundamental or made up of even more fundamental particles? Do neutrinos have finite masses? Does the Higgs boson exist? What is the origin of CP violation? Is it possible to unify the strong and electroweak interactions, as one has unified the electromagnetic and weak interactions? What is the unobserved dark matter which creates visible gravitational effects in the cosmology? Such questions stimulate the physicists to study the new physics beyond the SM in order to understand the problems under consideration . There are several candidates for the models beyond the SM. The multi Higgs doublet model [12, 13], the minimal supersymmetric model (MSSM) [14, 15, 16, 17], the Zee model [18], the see-saw model [19], left-right (super) symmetric model (LRSM) [20], tecnicolor models [21], extra dimensions [22, 23, 24] and Randrall-Sundrum model [25, 26] are some examples of these models.

The flavor changing neutral current (FCNC) processes are governed in the SM by the Glashow-Iliopoulos-Maiani (GIM) mechanism [27]. In this scenario such transitions are forbidden at tree level and leading contributions which can produce these processes only result from the one-loop diagrams known as the penguin and box diagrams [28]. The FCNC is referred to when the hadronic state changes its flavor composition without a change in charge. The experimental search of such procceses represents an important test of the validity of the SM, either by confirming its prediction or by indicating the need for physics beyond the SM if observed at larger probabilities [28, 29, 30]. In other words, FCNC processes have long served as a good thinking ground and a good experimental probe of new physics in each stage of the development of high energy physics [31].

The violation of flavor symmetry in the leptonic sector, known as lepton flavor violation (LFV), is of special interest to physicists. FCNC processes due to LFV are strictly forbidden in the SM with massless neutrinos. However, there is evidence for a very important new property of the neutrinos, i.e. they have masses and, as a result, mix with each other to lead to the phenomenon of neutrino oscilation [32]. Evidences that the neutrinos are massive particles come from three anomalous effects, the Liquid Scentilator Neutrino Detector (LSND) excess [33], the atmospheric anomaly [34] and solar neutrino deficit [35]. The atmospheric and the solar results are the most convincing one but the LSND has the small probability of mixing, compared to the atmospheric and solar anomalies [32]. Another clue that neutrinos have masses is the cosmological data. From cosmological data, the 2dF and SDSS galaxy redshift surveys and the WAMP measurement of the cosmic microwave background (CKM) temperature fluctuations, mass of the neutrino is restricted limits of $\sum m_\nu < (0.7 - 1.0)\, eV\, (95\%\, CL)$ [36]. The discovery in neutrino oscillations suggests that the lepton flavor is not strictly conserved in nature. In the SM, LFV decays are allowed by introducing the neutrino mixing with non zero neutrino masses. However, their branching ratios (BRs) are much below the experimental limits due to the smallness of neutrino masses. The conservation of the lepton flavor can be broken with the extension of the SM [37]. So, search for LFV in charged lepton processes is one of the promising way to look for physics beyond the SM [38].

The LFV interactions reach great interest since they are rich and clean theoretically. They are clean because they are free from the nonperturbative effects; they are rich because they ensure considerable information about the restrictions of the free parameters, appearing in the new models, with the help of the possible accurate measurements since the loop effects are necessary for their existences.

Among the LFV interactions, the flavor changing Z decay, such as $Z \to e\mu$, $Z \to e\tau$ and $Z \to \mu\tau$ are important for the search of neutrinos, their mixing and possible masses, and the physics beyond the Standard model (SM) (see [39] and references therein). With the Giga-Z option of the Tesla project, the production of Z bosons at resonance is expected to increase [40] and this forces to study on such Z decays more precisely. In order to describe the LFV Z decays, there is a need to extend the SM. One of the candidate model is so called νSM, by taking neutrinos massive and permitting the lepton mixing mechanism [41]. In this case the lepton sector is analogous to the quark sector. In this model, the theoretical predictions for BRs of the LFV Z decays are extremely small in the case of internal light neutrinos [42, 43] and far from the experimental limits obtained at LEP 1 [44].

To enhance the BRs of the corresponding LFV Z decays some other scenarios have been studied. The possible scenarios are the extension of νSM with one heavy ordinary Dirac neutrino [43], the extension of νSM with two heavy right-handed singlet Majorana neutrinos [43], the Zee model [18], the model III version of the two Higgs doublet model (2HDM), which is the minimal extension of the SM [39, 45], the supersymmetric models [46, 47], top-color assisted technicolor model [21].

Since quark mixing through the CKM matrix and neutrino oscillations are now established in the SM, the question may be asked why no mixing in the charged lepton has been observed [48]. In fact lepton flavor can change in the SM, mediated through a virtual W boson. However, the BR scales with the neutrino mass over the W mass by the fourth power in the case of LFV decays, the result at the order of the magnitude of 10^{-60} and, thus, immeasurably small [48]. So, the LFV processes are important and deserve to be analyzed experimentally and theoretically. Among them the radiative LFV $l_i \rightarrow l_j \gamma$ $(i \neq j; i, j = e, \mu, \tau)$ decays received great interest and there are various experimental and theoretical works done in the literature. In [49], $\mu \rightarrow e\gamma$ is searched by the MEGA experiment at the Los Alamos Meson Physics Facility (LAMPF) and the upper limit of the BR for this decay is found to be $< 1,2 \times 10^{-11}$ with 90% confidence. Furthermore, the MEG experiment searches for the LFV decay $\mu \rightarrow e\gamma$ will pull the BR down to the values of the order of 10^{-13} and it will go into data taking in 2007 [48]. Also, a new experiment at PSI has been described and aimed to reach to a sensitivity of $BR \sim 10^{-14}$ for $\mu \to e\gamma$ decay [50]. The LFV $\tau \to e\gamma$ decays have been searched in the Belle detector at the KEKB asymemetric e^+e^- collider and the upper limit of the BR is obtained as 3.9×10^{-7} 90% CL [51]. Another radiative LFV decay $\tau \to \mu\gamma$ has been searched at the BABAR detector at the PEP-II storage ring and the upper limit of the BR is obtained as 9.0×10^{-8} in the [52] and 6.8×10^{-8} in the [53] at 90% CL. From the theoretical point of view, there are extensive works on the radiative LFV decays in the literature. These decays were analyzed in the supersymmetric models in [15, 16, 17, 54]. [55, 56] were devoted to the radiative LFV decays in the framework of the 2HDM and in the [57] such decays were studied in a model independent way. In another work [58], they were analyzed in the framework of the 2HDM and in the supersymmetric model. Due to the extremely high suppression of the SM contribution to this decay, it is a very clean signature of physics beyond the SM.

Another motivation to search for new physics beyond the SM is electric dipole moments of fermions (EDMs). Elementary particles can possess EDM only if CP is violated. CP violation is carried by the complex CKM matrix elements in the quark sector and the possible lepton mixing matrix elements in the lepton sector, in the framework of the SM. However, the estimated fermion EDMs in the SM are negligibly small. So, this stimulates one to investigate these physical quantities in the framework of the new models beyond the SM. There are extensive theoretical and experimental works done on the EDMs of fermions. From the experimental point of view, the experimental results of the fermion EDMs are $d_e = (1.8 \pm 1.2 \pm 1.0) \times 10^{-27} e\,cm$ [59], $d_\mu = (3.7 \pm 3.4) \times 10^{-19} e\,cm$ [60] and $d_\tau = (3.1) \times 10^{-16} e\,cm$ [61] respectively. Also, the lepton EDMs have been predicted in various theoretical models beyond the SM. In [62], using the seesaw model the lepton electric dipole moments has been analyzed. [63] was devoted to the EDMs of the leptons in the model III version of the 2HDM and d_e has been predicted at the order of the magnitude of $10^{-32}\, e - cm$. The work [64] was related to the lepton EDM in the framework of the SM with the inclusion of non-commutative geometry. In [65], the effects of non-universal extra dimensions on the electric dipole moments of fermions in the 2HDM have been estimated. The EDMs of charged leptons were investigated in the split fermion scenario in the 2HDM in [66]. Furthermore, charged lepton EDMs were estimated in the framework of the 2HDM with the inclusion of two extra dimensions in [67].

As a summary, LFV interactions are important and give a comprehensive information about the new physics. Also, they have reached great interest with the improvement of experimental measurements. Such type of interactions are studied in different models beyond the SM. Among them, the simplest extention of the SM with one extra Higgs doublet is the 2HDM. The 2HDM, obtained by introducing another doublet, would automatically lead to FCNC problem in its Yukawa sector representing interactions between the Higgs fields and fermions [68].

This thesis is organized as follows. In Chapter 2 we give a brief review of the SM. Chapter 3 is devoted to the construction of the 2HDM and its different versions. In Chapter 4 we investigate lepton flavor violating (LFV) $H^+ \to W^+ l_i^- l_j^+$ and lepton flavor conserving (LFC) $H^+ \to W^+ l_i^- l_i^+$ ($l_i = \tau, l_j = \mu$) decays in the 2HDM, allowing the FCNC at tree level [69]. After giving the expression for the matrix element for this decay, the decay width of LFC and LFV decays are calculated. In Chapter 5, the LFV $\tau \to \mu \overline{\nu_i} \nu$ decay is investigated in the framework of the general 2HDM [70]. The BR is calculated and the limits of the BR's of the decay modes are discussed with respect to sensitivity of the model parameters. Chapter 6 represents our conclusions. In Appendix A, we present the local and global gauge invariance. Appendix B is devoted to the spontaneous symmetry breaking and Higgs mechanism. In Appendix C we present the propagators and vertices which we used in our calculations. Appendix D is related to Feynman parametrization and dimensional regularization.

CHAPTER 2

THE STANDARD MODEL

The SM is a quantum field theory which comprise all known particles and three out of the four known fundamental interactions. The elementary particles called fermions are leptons and quarks and the fundamental interactions are the strong, weak, electromagnetic and gravity (see for example [7], [71]). The gauge particles are $W^{\pm}$ and Z^0 bosons-photon-gluon for the weak-electromagnetic-strong interactions, respectively. The gravitational interaction is not described in the SM.

At this stage, we would like to give a brief information about the Fermi theory which describe the weak interaction phenomenology in the mid-1950.

2.1 Fermi Theory

In 1934, Fermi tried to describe the weak interactions existing in the $\beta-$decay $n \rightarrow p + e^- + \overline{\nu}_e$ in terms of quantum field theory [72] and he assumed that the emission of an electron-neutrino pair was analogous to the electromagnetic emission of a photon and theory was the first field theory in which the processes were described in terms of creation and annihilation of particles.

The form vector-axial (V-A) of the weak interactions was generalized by Feynman and Gell-Mann in 1958 through the current-current interactions [73]. This consists in writing the weak lagrangian in the form [4]

$$L_F = \frac{G_F}{\sqrt{2}} J^{\alpha}(x) J^{\dagger}_{\alpha}(x) \quad , \tag{2.1}$$

where G_F is the Fermi coupling and $J^{\alpha}(x) = \overline{\psi}_1 \gamma_{\alpha}(1-\gamma_5)\psi_2$, where ψ is the fermion field.

However, there are some problems in the Fermi theory. The main problem is its non-renormalizability due to the four-fermi interaction. When evaluating Feynman diagrams beyond the tree level, one encounters divergences due to the bad ultraviolet behaviour of the theory. Another problem of the Fermi theory is the unitarity. The unitarity requires that the scattering amplitudes are limited. However, the bad high-energy behaviour of the non-renormalizable theory leads to increasing amplitudes. As a result, the unitarity is violated in the Fermi theory. For example, if we consider $\nu_e + e \rightarrow \nu_e + e$ decay, the differential cross-section is obtained as

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 k^2}{\pi^2}, \tag{2.2}$$

then the cross-section is written as

$$\sigma = \frac{4 G_F^2 k^2}{\pi}, \;\; k^2 \gg m_e^2. \tag{2.3}$$

Since the interaction is point-like the scattering goes entirely via the s-wave, and partial wave unitarity then requires that

$$\sigma < \frac{\pi}{2k^2}. \tag{2.4}$$

Then, the energy k is obtained as

$$k^4 < \frac{\pi^2}{8G_F^2}. \tag{2.5}$$

So, we can say that the Fermi theory is an effective theory up to energies $k \simeq 300GeV/c$ and above this energy, the interaction Eq. (2.1) will violate unitarity. Furthermore, if one tries to evaluate higher order corrections, horrible divergences are found. Since this divergences cannot be eliminated by renormalization, the Fermi theory of weak interactions is non-renormalizable. To eliminate this

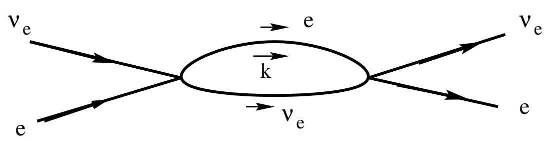

Figure 2.1: $\nu_e + e \longrightarrow \nu_e + e$ diagram for pointlike interaction .

problem, the idea is that the weak interaction are mediated by a vector boson similar to electromagnetic interactions where the mediating particle is photon. Since weak interactions are short range, we would rather need to exchange a massive particle. Therefore, Eq. (2.1) is replaced by a new weak Lagrangian given as

$$L = g_w(J_\mu W^\mu + h.c), \tag{2.6}$$

where $W_\alpha(x)$ is the field of the vector boson which is to be the analogue of the photon field $A_\alpha(x)$ and the coupling constant g_W is dimensionless. Then the differential cross section for the diagram $\nu_e + e \longrightarrow \nu_e + e$ is written as

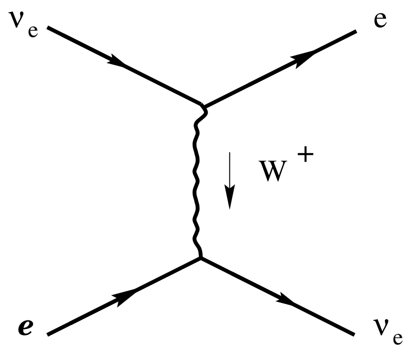

Figure 2.2: Diagram for $\nu_e + e \longrightarrow \nu_e + e$.

$$\frac{d\sigma}{d\Omega} = \frac{2g_W^4 k^2}{\pi^2(q^2 - m_W^2)^2};\ (k^2 \geq m_e^2) \quad , \tag{2.7}$$

where m_W is the mass of the $W^\pm$ boson and q is the momentum transfer vector defined as $q^2 \simeq -2k^2(1 - \cos\theta)$. Eq. (2.7) reduces to the Fermi result Eq. (2.2) as $q^2 \longrightarrow 0$ provided

$$\frac{g_W^2}{m_W^2} = \frac{G_F}{\sqrt{2}} \quad . \tag{2.8}$$

The introduction of the intermediate vector bosons in the theory is an improvement in the high-energy behaviour of the amplitudes. As a result, we see that the ingredients of electroweak interactions are intermediate massive vector bosons, conservation of the various leptonic numbers and renormalizability. As we shall see, in order to satisfy all these requirements new ideas in particle physics have been necessary, namely, local gauge invariance (see Appendix A for details), spontaneous symmetry breaking (SSB) and Higgs mechanism (see Appendix B).

2.2 The Standard Model

The SM of particle physics consists of the idea of local gauge invariance and SSB to implement a Higgs mechanism [1]. The local gauge symmetry under consideration is $SU(2)_L \times U(1)_Y$ and the SSB obeys the scheme $SU(2)_L \times U(1)_Y \to U(1)_Q$ where the subscript L means that $SU(2)$ only acts on left-handed doublets (in the case of fermions), Y is the generator of the original $U(1)$ group, and Q correspond to an unbroken generator (the electromagnetic charge). Specifically, the symmetry breaking is implemented by introducing a scalar doublet [1]

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \tag{2.9}$$

It transforms as an $SU(2)_L$ doublet, thus its weak hypercharge is one. In order to induce the SSB (see Appendix B) the doublet should acquire a VEV different from zero

$$\langle \Phi \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}. \tag{2.10}$$

The generators of the local gauge symmetry $SU(2)_L \times U(1)_Y$ are τ_i and Y defined as

$$\tau_i \equiv \frac{\sigma_i}{2} \tag{2.11}$$

where σ_i are the Pauli matrices. These generators obey the following lie algebra

$$[\tau_i, \tau_j] = i\varepsilon_{ij}^{k}\tau_k \;\; ; \;\; [\tau_i, Y] = 0 \tag{2.12}$$

When the symmetry is spontaneously broken, the doublet acquire a VEV and we can see easily that all generators of the $SU(2)_L \times U(1)_Y$ are broken [74]

$$\begin{aligned} \tau_1\langle\Phi\rangle &= \frac{1}{2}\begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \;\; ; \;\; \tau_2\langle\Phi\rangle = \frac{1}{2}\begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \\ \tau_3\langle\Phi\rangle &= \frac{1}{2}\begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \;\; ; \;\; Y\langle\Phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \end{aligned} \tag{2.13}$$

The unbroken generator is defined by Gellman-Nijishima relation [75] as

$$Q = \left(\tau_3 + \frac{Y}{2}\right) \;\; ; \;\; Q\langle\Phi\rangle = 0 \tag{2.14}$$

According to the Goldstone theorem, the number of Goldstone bosons generated after the symmetry breaking is equal to the number of broken generators. Therefore, instead of working with four broken generators we shall work with three broken generators and unbroken one, Q. This scheme ensures for the photon to remain massless, while the other three gauge bosons acquire masses [1].

2.2.1 The Weinberg-Salam Model

The Weinberg-Salam Model combine the weak and electromagnetic interactions [76, 77, 78]. From the phenomenology of the weak interactions, we know that, we require both charge-changing leptonic currents and neutral currents. Given that the weak interactions are to be mediated by our gauge vector bosons, we require three vector bosons $W_\mu^j (j = 1, 2, 3)$. The simplest group that contains the required three generators is SU(2). However, it is clear that this is not enough if we wish to include electromagnetic interactions as well. Thus we need one further gauge vector boson, called as B_μ, and correspondingly a group with one generator, U(1). The overall gauge group is then $SU(2)_L \otimes U(1)_Y$ with a total of four generators. The Weinberg-Salam Lagrangian reads

$$L_{WS} = L_G^{kin} + L_F^{kin} + L_H, \tag{2.15}$$

(see [8] for details), where G-F-H denote gauge-fermion-Higgs.

The Higgs part of Lagrangian contains three kinds of terms, namely,

$$L_H = L_{HG}^{kin} + L_Y + V(\Phi^+\Phi). \tag{2.16}$$

where the Higgs potential of the Lagrangian reads

$$V(\Phi^+\Phi) = \mu^2(\Phi^+\Phi) - \lambda(\Phi^+\Phi)^2 \tag{2.17}$$

with the free parameters μ^2 and λ and Φ is defined as in Eq. (2.9). For $\mu^2 > 0$, the scalar field Φ develops a non zero VEV given in Eq. (2.10) and the vacuum expectation value is $\upsilon = \mu/\sqrt{2\lambda}$ ($\simeq 246GeV$). Here the Higgs mass is to be yielded as a consequence of the Higgs mechanism (see Appendix B) and it reads $m_H = \sqrt{2\lambda}\upsilon$.

The L_{HG}^{kin} part of the lagrangian is responsible for the interaction of the gauge and Higgs particles and it is given by

$$L_{HG}^{kin} = D_\mu\Phi\ (D^\mu\Phi)^*, \tag{2.18}$$

with

$$D_\mu\Phi = (\partial_\mu + \frac{ig'}{2}YB_\mu + ig\tau_i W_\mu^i)\Phi, \tag{2.19}$$

and W_μ^i and B_μ are $SU_L(2)$ and $U_Y(1)$ gauge fields. The corresponding kinetic term for the gauge fields is

$$L_G^{kin} = -\frac{1}{4}F_i^{\mu\nu}F_{\mu\nu}^i - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}, \tag{2.20}$$

where $F_{\mu\nu}^i (i = 1, 2, 3)$ is the $SU(2)_L$ field strength,

$$F_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g\epsilon^{ijk}W_\mu^j W_\nu^k, \tag{2.21}$$

and $B_{\mu\nu}$ is the $U_Y(1)$ field strength,

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{2.22}$$

The gauge boson fields W_μ^1, W_μ^2, W_μ^3 couple to the weak isospin and B_μ couples to the weak hypercharge.

The L_Y part of the Lagrangian describes the interaction among the Higgs bosons and fermions, so called the Yukawa Lagrangian,

$$L_Y = \eta_{ij}^{D}\overline{Q}_{iL}\Phi D_{jR} + \eta_{ij}^{U}\overline{Q}_{iL}\widetilde{\Phi}U_{jR} + \eta_{ij}^{E}\overline{\ell}_{iL}\Phi E_{jR}, \tag{2.23}$$

where

$$\widetilde{\Phi} = i\tau_2\Phi^*, \tag{2.24}$$

$\overline{Q}_{i,L}$ are left-handed quark doublets, D_{jR} and U_{jR} are right-handed singlets of the up and down sectors of quarks, $\overline{\ell}_{i,L}$ are left-handed lepton doublets and E_{jR} are right-handed lepton singlets (see section (2.2.2) for quark-lepton doublets and singlets), the parameters $\eta_{ij}^{U,D,E}$'s are responsible for the masses of up-down quarks and leptons, with family indices $i\, and\, j$ (Here $\eta_{ij}^{U,D,E}$'s are flavor diagonal Yukawa couplings). In Eq. (2.23), there is not any right-handed neutrino term which is based on the assumption that the neutrinos are massless.

The fermionic sector of the Lagrangian which include both the left-handed and rigth-handed chiralities can be presented as

$$L_F^{kin} = \sum_{\psi_L} \overline{\psi_L}\, i\, \not{D}\psi_L + \sum_{\psi_R} \overline{\psi_R}\, i\, \not{D}\psi_R, \tag{2.25}$$

where $\not{D} = \gamma^\mu D_\mu$. ψ_L and ψ_R are left-handed weak isodoublets and right-handed weak isosinglets, respectively. Right-handed chiral fermions do not couple to weak isospin, so their covariant derivative has the simple form

$$D_\mu\psi_R = (\partial_\mu + i\frac{g'}{2}YB_\mu)\psi_R, \tag{2.26}$$

where g' is the coupling constant for $U(1)_Y$ group. The corresponding covariant derivative for the $SU(2)_L$ doublet ψ_L is given in terms of the $SU(2)_L$ gauge coupling g

$$D_\mu \psi_L = (\partial_\mu + \frac{ig'}{2} Y B_\mu + ig\tau_i W_\mu^i), \tag{2.27}$$

After a proper normalization, the gauge fields can be written in a form which corresponds to the physical photon and Z^0 boson fields [1]

$$\begin{aligned} A_\mu &= sin\theta_W W_\mu^3 + cos\theta_W B_\mu, \\ Z_\mu &= cos\theta_W W_\mu^3 - sin\theta_W B_\mu, \end{aligned} \tag{2.28}$$

where θ_W is the weak mixing angle or the Weinberg angle and defined as

$$sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}} \quad ; \quad cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}. \tag{2.29}$$

It is appropriate to redefine the other two components of the weak fields, which correspond to the physical charged weak bosons,

$$\begin{aligned} W_\mu^+ &= \sqrt{\frac{1}{2}}(W_\mu^1 - iW_\mu^2) \\ W_\mu^- &= \sqrt{\frac{1}{2}}(W_\mu^1 + iW_\mu^2) \end{aligned} \tag{2.30}$$

Finally, mass eigenstates of the gauge bosons are obtained [1]

$$M_{W^\pm}^2 = \frac{1}{4} g^2 \upsilon^2 \quad ; \quad M_Z^2 = \frac{1}{4} \upsilon^2 (g'^2 + g^2). \tag{2.31}$$

Notice that in general, the Yukawa Lagrangian is CP violating due to complex Yukawa couplings. The Hermiticity of the Yukawa Lagrangian implies that there

Table 2.1: The known quarks and leptons. Here and elsewhere we take $c = 1$.

Quarks				***Leptons***			
Charge 2/3		*Charge* −1/3		*Charge* −1		*Charge 0*	
	Mass(Gev)		Mass(Gev)		Mass(Gev)		Mass(Gev)
u	0.0015–0.003	d	0.003–0.007	e	0.000511	ν_e	$< 3 \times 10^{-9}$
c	1.25 ± 0.09	s	0.095 ± 0.025	μ	0.106	ν_μ	$< 190 \times 10^{-6}$
t	174.2 ± 3.3	b	4.2 ± 0.07	τ	1.777	ν_τ	$< 18.2 \times 10^{-3}$

are terms in pairs of the form:

$$\eta_{ij}\overline{\Psi}_{iL}\Phi\Psi_{jR} + \eta_{ij}^{*}\overline{\Psi}_{jR}\Phi^{\dagger}\Psi_{iL}. \tag{2.32}$$

However a transformation under CP gives

$$\overline{\Psi}_{iL}\Phi\Psi_{jR} \leftrightarrow \overline{\Psi}_{jR}\Phi^{\dagger}\Psi_{iL}, \tag{2.33}$$

and leaves the coefficients η_{ij} and η_{ij}^{*} unchanged. In fact CP is conserved in L_Y only if $\eta_{ij} = \eta_{ij}^{*}$. Moreover, the family indices $i\, and\, j$ are symmetric.

2.2.2 The Fermions and Bosons

The leptons are the fundamental fermions lacking strong interactions and the quarks are the fundamental building blocks of strongly interacting particles. The quarks are distinguished from the leptons by possesing a three fold charge known as color which enables them to interact strongly with another. All fermions are summarized in Table 2.1 [79].

In the SM, all fermions are placed in to left-handed doublets and right-handed singlets. The doublets and singlets of the quarks are (see for example [7])

$$\begin{pmatrix} u \\ d \end{pmatrix}_L, \qquad \begin{pmatrix} c \\ s \end{pmatrix}_L, \qquad \begin{pmatrix} t \\ b \end{pmatrix}_L,$$

$$d_R, \qquad u_R, \qquad s_R, \qquad c_R, \qquad b_R, \qquad t_R. \tag{2.34}$$

On the other hand, the lepton doublets and singlets are

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L, \qquad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L, \qquad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L,$$
$$e_R, \qquad \tau_R, \qquad \tau_R. \tag{2.35}$$

The charged weak interactions couple the upper members of the $SU(2)_L$ fermion doublets to the lower members (rotated quark doublets of the weak eigenstates in the case of quarks)

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L \tag{2.36}$$

where the weak eigenstates of the down-type quarks can be defined as linear combinations of the mass eigensates by using the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix V_{ij}

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{2.37}$$

Table 2.2: The elementary bosons according to SM .

Name	***Charge(e)***	***Spin***	***Mass (GeV)***	***Force mediated***
Photon	0	1	0	Electromagnetic
$W^{\pm}$	+1	1	80.403 ± 0.029	Weak nuclear
Z^0	0	1	91.1876 ± 0.0021	Weak nuclear
Gluon	0	1	0	Strong nuclear
Higgs	0	0	> 114.4	

The non-diagonal elements of the CKM matrix allow flavor transitions between families. The experimentally measured values of the matrix elements are [79]

$$V_{ij} = \begin{pmatrix} 0.97377 \pm 0.00027 & 0.2257 \pm 0.0021 & 0.00431 \pm 0.0003 \\ 0.230 \pm 0.011 & 0.957 \pm 0.017 \pm 0.093 & 0.0416 \pm 0.0006 \\ 0.0074 \pm 0.0008 & 0.0406 \pm 0.0027 & > 0.78 \end{pmatrix}. \quad (2.38)$$

The gauge bosons are mediating particles which are responsible for the electromagnetic, weak and strong forces. In the Table 2.2, the elementary bosons in the SM are summarized [79]. The $W^{\pm}$ and Z^0 bosons are the elementary particles that mediate the weak force. Their discovery at CERN in 1983 has been heralded as a major success for the SM of the particle physics. The Higgs boson is predicted by electroweak theory. It is the only SM particle not observed yet. Many physicists expect the Higgs to be discovered at the Large Hadron Collider (LHC) particle accelerator which starts operation in autumn 2007 at CERN.

CHAPTER 3

BEYOND THE STANDARD MODEL

The SM has been extremely successful in describing the experimentally observed particle physics phenomenology up to the electroweak energy scale of the order of 100 GeV. When going to higher energy scales, the SM becomes insufficient and unsatisfactory.

In the SM, the Higgs mechanism is presently only a hypothesis as no experimental evidence for it has been found. After the LHC experiment at CERN, the Higgs mechanism is expected to be proven right or wrong. Even if the Higgs mechanism of the SM is proven to be right, the SM gives no answers to many fundamental questions given in the introduction section, so a new physics beyond the SM is needed. There are many models beyond the SM such as multi Higgs doublet model (MHDM), supersymmetric model (SUSY), technicolor model, etc.. The work done in this thesis based on the most primitive candidate of MHDM, the 2HDM.

3.1 The Two Higgs Doublet Model

The simplest extension of the SM, compatible with the gauge invariance, is the so called 2HDM, which consists of adding a second Higgs doublet with the same quantum numbers as the first one. What is the motivation to introduce the second doublet? (see for example [74] and the references therein.) First, there is not any fundamental reason to assume that the Higgs sector must be minimal. Second motivation comes from the hierarchy of Yukawa couplings in the third generation of quarks, the ratio between the masses of the top and bottom quarks is of the order of $m_t/m_b \approx 174/5 \approx 35$. Since, in the SM, up and bottom quarks get masses from the same Higgs doublet, there is a non natural hierarchy between their corresponding Yukawa couplings. However, if one doublet (ϕ_1) gives masses to the up quarks and the other doublet (ϕ_2) gives masses to the down quarks, then the hierarchy of their Yukawa couplings could be more natural. Another motivation lies on the study of some rare processes called FCNC. It is well known that these kind of processes are severely suppressed by experimental data, despite they seem not to violate any fundamental law of nature. In the SM with massless neutrino, FCNC are absent in the lepton sector and in the quark sector they are prohibited at tree level and further supressed at one loop by the GIM mechanism [80]. Owing to the addition of the second doublet in the 2HDM, the Yukawa interactions lead naturally to tree level FCNC. Therefore, FCNC processes would imply the presence of new physics effects.

As explained above, we introduce a new Higgs doublet, so the Higgs sector includes two Higgs doublets with the same quantum numbers [81]

$$\phi_1 = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 0 \\ v_1 + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix} \right] , \tag{3.1}$$

$$\phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}H^+ \\ v_2 + H^1 + iH^2 \end{pmatrix} , \tag{3.2}$$

with the vacuum expectation values

$$< \phi_1 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix} , \quad < \phi_2 >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} , \tag{3.3}$$

where $v = (v_1^2 + v_2^2)^{1/2} = (\sqrt{2}G_F)^{-\frac{1}{2}} = 246\text{GeV}$. Here H^0 is the SM Higgs boson, H^1 and H^2 are the CP even and CP odd neutral Higgs bosons and H^+ is the charged Higgs boson.

In the 2HDM, the kinetic term of the SM Lagrangian Eq. (2.18) is extended to

$$L_{HG}^{kin} = (D_\mu \Phi_1)^+ (D^\mu \Phi_1) + (D_\mu \Phi_2)^+ (D^\mu \Phi_2) \tag{3.4}$$

where the covariant derivative D_μ is defined by Eq. (2.19). This Lagrangian endows the gauge bosons with mass and provides the interactions among gauge and Higgs bosons.

The most general renormalizable CP invariant Higgs potential subject to gauge invariance with discrete symmetry $\phi_1 \to -\phi_1$ [81]

$$\begin{aligned} V(\phi_1, \phi_2) &= \lambda_1(\phi_1^+\phi_1 - \upsilon_1^2)^2 + \lambda_2(\phi_2^+\phi_2 - \upsilon_2^2)^2 \\ &+ \lambda_3[(\phi_1^+\phi_1 - \upsilon_1^2) + (\phi_2^+\phi_2 - \upsilon_2^2)]^2 \\ &+ \lambda_4[(\phi_1^+\phi_1)(\phi_2^+\phi_2) - (\phi_1^+\phi_2)(\phi_2^+\phi_1)] \\ &+ \lambda_5[Re(\phi_1^+\phi_2) - \upsilon_1\upsilon_2]^2 \\ &+ \lambda_6[Im(\phi_1^+\phi_2)]^2 \ , \end{aligned} \tag{3.5}$$

where λ_i are real. Unlike the SM case, in the 2HDM the Higgs potential is not unique, and each potential leads to different Feynman rules.

3.1.1 The Yukawa Lagrangian in 2HDM

The most general gauge invariant Lagrangian that couples the Higgs fields to fermions reads

$$\begin{aligned} L_Y^{2HDM} &= \eta_{ij}^U\overline{Q}_{iL}\widetilde{\Phi}_1 U_{jR} + \eta_{ij}^D\overline{Q}_{iL}\Phi_1 D_{jR} + \xi_{ij}^U\overline{Q}_{iL}\widetilde{\Phi}_2 U_{jR} + \xi_{ij}^D\overline{Q}_{iL}\Phi_2 D_{jR} + \\ & \eta_{ij}^E\overline{l}_{iL}\Phi_1 E_{jR} + \xi_{ij}^E\overline{l}_{iL}\Phi_2 E_{jR} + h.c., \end{aligned} \tag{3.6}$$

where $\Phi_{1,2}$ represent the Higgs doublets, $\widetilde{\Phi}_{1,2} \equiv i\sigma_2\Phi_{1,2}$, η_{ij} and ξ_{ij} are non diagonal 3×3 matrices in general, so called Yukawa couplings, and i, j denote family indices. D_{jR} refers to the three down-type weak isospin quark singlets $D_{jR} \equiv (d_R, s_R, b_R)$, $U_{jR}j$ refers to the three up-type weak isospin quark singlets

$U_{jR} \equiv (u_R, c_R, t_R)$ and E_R to the three charged leptons. Finally, $Q_{iL},\ l_{iL}$ denote the quark and lepton weak isospin left-handed doublets respectively.

It is possible that 2HDM can be categorized into four types by explicitly imposing *ad-hoc* discrete symmetry.

- **The 2HDM type I**

  If we impose the following discrete symmetry sets,

$$\begin{aligned} \Phi_1 &\rightarrow -\Phi_1 \quad and \quad \Phi_2 \rightarrow \Phi_2 \\ D(E)_{jR} &\rightarrow -D(E)_{jR} \quad and \quad U_{jR} \rightarrow -U_{jR} \end{aligned} \tag{3.7}$$

  the Lagrangian is obtained as

$$\begin{aligned} L_Y^{2HDM}(type\ I) &= \eta_{ij}^U \overline{Q}_{iL} \widetilde{\Phi}_1 U_{jR} + \eta_{ij}^D \overline{Q}_{iL} \Phi_1 D_{jR} \\ &+ \eta_{ij}^E \overline{l}_{iL} \Phi_1 E_{jR} + h.c., \end{aligned} \tag{3.8}$$

  In the quark sector, Φ_2 decouples from the Yukawa sector and only Φ_1 couples and gives masses to the up and down sectors. This case is known as the 2HDM type I.

- **The 2HDM type II**

  When we use the following discrete symmetry sets,

$$\begin{aligned} \Phi_1 &\rightarrow -\Phi_1 \quad and \quad \Phi_2 \rightarrow \Phi_2 \\ D(E)_{jR} &\rightarrow -D(E)_{jR} \quad and \quad U_{jR} \rightarrow U_{jR} \end{aligned} \tag{3.9}$$

then the Lagrangian becomes

$$
\begin{aligned}
L_Y^{2HDM}(type\ II) &= \eta_{ij}^D \overline{Q}_{iL} \Phi_1 D_{jR} + \xi_{ij}^U \overline{Q}_{iL} \widetilde{\Phi}_2 U_{jR} \\
&\quad + \eta_{ij}^E \overline{l}_{iL} \Phi_1 E_{jR} + h.c., \qquad (3.10)
\end{aligned}
$$

Therefore, in the quark sector, Φ_1 couples and gives masses to the down sector while Φ_2 couples and gives to the up sector. In this case we call it, the 2HDM type II.

- **The 2HDM type III**

If there is no discrete symmetry, we obtain the model III in the 2HDM. The Yukawa Lagrangian for the model III is as in Eq. (3.6)

$$
\begin{aligned}
L_Y^{2HDM}(type\ III) &= \eta_{ij}^U \overline{Q}_{iL} \widetilde{\Phi}_1 U_{jR} + \eta_{ij}^D \overline{Q}_{iL} \Phi_1 D_{jR} + \xi_{ij}^U \overline{Q}_{iL} \widetilde{\Phi}_2 U_{jR} + \\
&\quad \xi_{ij}^D \overline{Q}_{iL} \Phi_2 D_{jR} + \eta_{ij}^E \overline{l}_{iL} \Phi_1 E_{jR} + \xi_{ij}^E \overline{l}_{iL} \Phi_2 E_{jR} + h.c.
\end{aligned}
\qquad (3.11)
$$

It is more convenient to choose the VEV of the doublets in the following way [82]

$$
< \Phi_1 > = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \; ; \; < \Phi_2 > = 0\,. \qquad (3.12)
$$

The two doublets in this case are of the form

$$
\Phi_1 = \begin{pmatrix} \chi^+ \\ \frac{v + H^0 + i\chi^0}{\sqrt{2}} \end{pmatrix} \; ; \; \Phi_2 = \begin{pmatrix} H^+ \\ \frac{H^1 + iH^2}{\sqrt{2}} \end{pmatrix}. \qquad (3.13)
$$

The Φ_1 and Φ_2 doublets correspond to the scalar doublet of the SM and the new scalar fields respectively. Although $H^\pm$ is the charged scalar mass eigenstate, H^0 and H^1 are not the neutral mass eigenstates. The mass eigenstates $\overline{H}^0, h^0, A^0$ are obtained as

$$\begin{aligned} \overline{H}^0 &= (H^0 cos\alpha - H^1 sin\alpha) \\ h^0 &= (-H^0 sin\alpha + H^1 cos\alpha) \\ A^0 &= H^2 \end{aligned} \tag{3.14}$$

where α is a mixing angle. If the mixing angle $\alpha = 0$, H^0 and H^1 coincide with the mass eigenstates. Notice that the most general CP invariant Higgs potential in $SU(2)_L \times U(1)$ gauge group is given in Eq. (3.5). In the model type III, FCNC is permitted in the tree level and the Flavor Changing (FC) part of the Yukawa Lagrangian looks like

$$L_{Y,FC}^{2HDM} = \xi_{ij}^U \overline{Q}_{iL} \widetilde{\Phi}_2 U_{jR} + \xi_{ij}^D \overline{Q}_{iL} \Phi_2 D_{jR} + \xi_{ij}^E \overline{l}_{iL} \Phi_2 E_{jR} + h.c., \tag{3.15}$$

- **The 2HDM type IV**

In the Model IV, the up-type quarks get masses from ϕ_2 and down-type quarks and leptons get masses from ϕ_1. In this model, VEV of the doublets are chosen in the following way [83]

$$< \Phi_1 >= \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \; ; < \Phi_2 >= \begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix} . \tag{3.16}$$

In this case the $\lambda_6(Im(\Phi_1^\dagger\Phi_2) - v_1\, v_2\, sin\xi)^2$ term in the Higgs potential (see Eq. (3.5)) is responsible for the CP violation in the Higgs sector.

CHAPTER 4

$H^+ \rightarrow W^+ l_i^- l_j^+$ DECAY

Although the SM of electroweak interactions [1] are very successful in explaining all experimental data available until now, the electroweak symmetry breaking mechanism still has to be established and the Higgs sector still remains untested. The main goals of future colliders such as LHC and International Linear Collider (ILC) are to study the scalar sector of the SM. Moreover, the scalar sector of the SM can be enlarged and some simple extensions such as the Minimal MSSM and 2HDM are intensively studied. In the 2HDM, the electroweak symmetry breaking is generated by two Higgs doublets field Φ_1 and Φ_2. After electroweak symmetry breaking we are left with five physical Higgs particles; two charged $H^{\pm}$, two CP-even H^0, h^0 and one CP-odd A^0.

The charged Higgs production has been studied in several theoretical and experimental works. A search for pair-produced charged Higgs bosons was analyzed with the L3 detector at LEP and its mass was obtained as $m_{H^+} > 76.5\,(GeV)$ [84]. The CDF and D0 collaborations have studied H^+ bosons at tevatron, in the case of $p\bar{p} \to t\bar{t}$, with at least one of the top quark decaying via $t \to H^+ b$ and they estimated the charged Higgs mass lower limits as $m_{H^+} > 77.4\,(GeV)$ [85]. In [86], a search for pair produced charged Higgs boson is performed using the data from the DELPHI detector at LEP II and the existence of this particle with mass lower than $76.7\,(74.4) GeV$ in the type I (II) 2HDM is excluded and the charged Higgs boson in the 2HDM model II has the following lower mass limits $m_{H^+} > 79.3\,(GeV)$ in [87] and $m_{H^+} \geq 1.71\, tan\beta\,(GeV)$ where $tan\beta$ is the ratio of vacuum expectation values of two Higgs doublets [88].

In the calculation of the BR of the charged Higgs boson, the dominant decays are $H^+ \to W^+ h^0$, $H^+ \to \tau^+ \nu$ and $H^+ \to t\bar{b}$ [89, 90, 91, 92]. The total decay width of the charged Higgs boson is approximated by

$$\Gamma_{tot}(H^+) = \Gamma(H^+ \to W^+ h^0) + \Gamma(H^+ \to t\bar{b}) + \Gamma(H^+ \to \tau^+ \nu) + \Gamma(H^+ \to c\bar{s}).$$

In [92], the various BRs of the charged Higgs boson decays have been obtained as:

$$\begin{aligned} BR(H^+ \to t\bar{b}) &< 1\,, \\ BR(H^+ \to \tau^+ \nu) &< 0.1\,, \\ BR(H^+ \to W^+ h^0) &< 0.01\,, \\ BR(H^+ \to \mu^+ \nu) &< 0.001\,, \\ BR(H^+ \to c\bar{s}) &< 0.0001\,, \end{aligned} \tag{4.1}$$

for $tan\beta \sim 10$ and $m_{H^+} \sim 400 GeV$, in the framework of the MSSM. These results are strongly sensitive to the choice of $tan\beta$, and increasing values of $tan\beta$ make $H^+ \to \tau^+\nu$ and $H^+ \to \mu^+\nu$ more dominant compared to the decay $H^+ \to W^+h^0$. In [93], $H^+ \to W^+h^0$ has been predicted to be of the order of $O(1)$, in the context of the effective lagrangian extension of the 2HDM.

Our task in this chapter is devoted to the analysis of the LFV $H^+ \to W^+l_i^-l_j^+$ and the LFC $H^+ \to W^+l_i^-l_i^+$ $(l_i = \tau, l_j = \mu)$ decays in the framework of the general 2HDM with massless neutrinos. The LFV interactions are interesting, since they do not exist in the SM. On the other hand, the high statistic results of the superkamiokande atmospheric neutrino experiment and the solar neutrino experiment have made one to believe that LFC is not exact [94]. In any case we expect that the contribution of possible lepton mixing mechanism to the BR's of such LFV processes are negligible small.

4.1 The Calculation of the $H^+ \to W^+l_i^-l_j^+$

In this section, we derive the basic steps for calculating the LFV $H^+ \to W^+l_i^-l_j^+$ and LFC $H^+ \to W^+l_i^-l_i^+$ $(l_i = \tau, l_j = \mu)$ decays in the general 2HDM. To this aim, let us first present part of the Lagrangian responsible for this LFV decay. From Eq. (3.11), the flavor changing part of the Yukawa Lagrangian in the leptonic sector is seen to be

$$L_{Y,FC} = \xi^{E}_{ij}\bar{l}_{iL}\Phi_2 E_{jR} + h.c., \tag{4.2}$$

where i, j are family indices of leptons, L and R denote chiral projections $L = 1/2(1-\gamma_5)$ and $R = 1/2(1+\gamma_5)$ respectively.

Fist of all, we consider LFV decay $H^+ \to W^+ l_i^- l_j^+$ which exists with the chain processes $H^+ \to W^+(h^{0*}, A^{0*}) \to W^+ l_i^- l_j^+$, given in figure Fig. (4.1), where h^0, A^0 are CP even and odd neutral Higgs bosons beyond the SM and l_i, l_j are different leptons flavors, e, μ, τ. This process is driven by the FC interaction in the leptonic sector and the strength of this interaction is carried by the Yukawa couplings ξ_{ij}^E [1], which are the free parameters of the general 2HDM. The Yukawa couplings can have complex entries in general and be restricted by using experimental measurements.

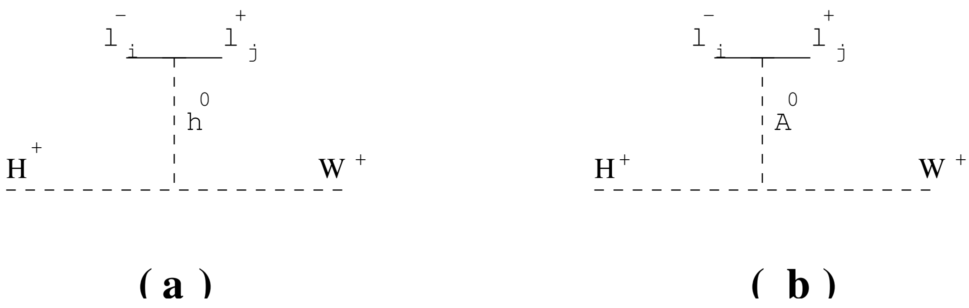

Figure 4.1: Tree level diagrams contribute to $\Gamma(H^+ \to W^+\, l_i^- l_j^+)$, $i = e, \mu, \tau$ decay in the general 2HDM.

Now, we would like to calculate amplitude $\boldsymbol{M}$ of LFV $H^+ \to W^+\, l_i^- l_j^+$ decay. It is evident that we have only four types of interaction vertices and the propagators for massive scalar particles which we present in Appendix C. The amplitude

[1] In the following we replace ξ^E with ξ_N^E where "N" denotes the word "neutral".

of this decay is defined as

$$M = M_{h^0} + M_{A^0} \,, \tag{4.3}$$

where M_{h^0} and M_{A^0} are the contributions of h^0 and A^0 bosons to the amplitude (see Fig. (4.1) (a) and (b)). At this stage we present the details of calculations of these amplitudes. The amplitude, M_{h^0} reads, (see Fig. (4.2)):

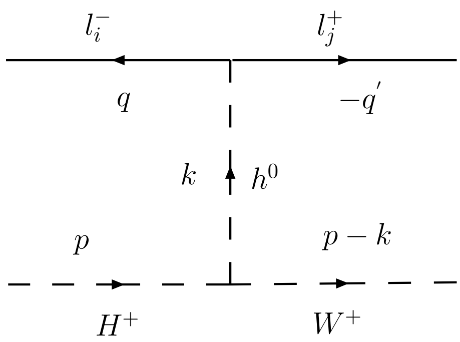

Figure 4.2: Tree level diagrams for h_0 .

$$\begin{aligned} M_{h^0} &= W^+[-\frac{ig}{2}(p+k)_\mu]H^- \frac{i}{k^2 - m_{h^0}^2 + i m_{h^0}\Gamma_{h^0}} \\ & \overline{l_i}^-[-\frac{i}{2\sqrt{2}}[(\xi^E_{N,ji} + \xi^{*E}_{N,ij}) + (\xi^E_{N,ji} - \xi^{*E}_{N,ij})\gamma_5]]l_j^- \,, \end{aligned} \tag{4.4}$$

and, $M^\dagger_{h_0}$ becomes:

$$\begin{aligned} M^\dagger_{h^0} &= H^+[\frac{ig}{2}(p+k)_\nu]W^- \frac{-i}{k^2 - m_{h^0}^2 - i m_{h^0}\Gamma_{h^0}} \\ & \overline{l_j}^-[\frac{i}{2\sqrt{2}}[(\xi^E_{N,ji} + \xi^{*E}_{N,ij}) + (\xi^E_{N,ji} - \xi^{*E}_{N,ij})\gamma_5]]l_i^- \,. \end{aligned} \tag{4.5}$$

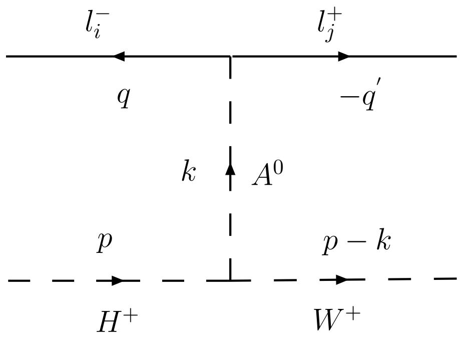

Figure 4.3: Tree level diagrams for A_0 .

The amplitude M_{A^0} can be written similar to the M_{h^0} (see Fig. (4.3)) and it reads :

$$\begin{aligned} M_{A^0} &= W^+[-\frac{g}{2}(p+k)^\mu]H^- \frac{i}{k^2 - m_{A^0}^2 + i m_{A^0}\Gamma_{A^0}} \\ &\overline{l_i}^-[\frac{1}{2\sqrt{2}}[(\xi^E_{N,ji} - \xi^{*E}_{N,ij}) + (\xi^E_{N,ji} + \xi^{*E}_{N,ij})\gamma_5]]l_j^-, \end{aligned} \tag{4.6}$$

and

$$\begin{aligned} M^\dagger_{A^0} &= H^+[-\frac{g}{2}(p+k)^\nu]W^- \frac{-i}{k^2 - m_{A^0}^2 - i m_{A^0}\Gamma_{A^0}} \\ &\overline{l_j}^-[-\frac{1}{2\sqrt{2}}[(\xi^E_{N,ji} - \xi^{*E}_{N,ij}) + (\xi^E_{N,ji} + \xi^{*E}_{N,ij})\gamma_5]]l_i^-. \end{aligned} \tag{4.7}$$

As a result, the square of the total amplitude is defined as

$$|M|^2 = |M_{h^0}|^2 + |M_{A^0}|^2 + 2\,Re[M^\dagger_{A^0}\, M_{h^0}], \tag{4.8}$$

where

$$\begin{aligned}
M^{\dagger}_{h^0}\, M_{h^0} &= \frac{g^2}{4}(p+k)_\mu (p+k)_\nu [-g_{\mu\nu} + \frac{(p-k)_\mu (p-k)_\nu}{m^2_{W^+}}] |p_{h^0}|^2 \\
& Tr[(A+B\gamma_5)(\not{q}' + m_{l_j})(A+B\gamma_5)(\not{q} + m_{l_i})], \\
M^{\dagger}_{A^0}\, M_{A^0} &= \frac{g^2}{4}(p+k)_\mu (p+k)_\nu [-g_{\mu\nu} + \frac{(p-k)_\mu (p-k)_\nu}{m^2_{W^+}}] |p_{A^0}|^2 \\
& Tr[(A'+B'\gamma_5)(\not{q}' + m_{l_j})(A'+B'\gamma_5)(\not{q} + m_{l_i})], \\
M^{\dagger}_{A^0}\, M_{h^0} &= \frac{ig^2}{4}(p+k)_\mu (p+k)_\nu [-g_{\mu\nu} + \frac{(p-k)_\mu (p-k)_\nu}{m^2_{W^+}}] p_{h^0} p^*_{A^0} \\
& Tr[(A+B\gamma_5)(\not{q}' + m_{l_j})(-A'-B'\gamma_5)(\not{q} + m_{l_i})], \qquad (4.9)
\end{aligned}$$

and the factors A, B, A', B' are

$$\begin{aligned}
A &= -\frac{i}{2\sqrt{2}}\,(\xi^E_{N,ji} + \xi^{*E}_{N,ij})\,, \\
A' &= \frac{1}{2\sqrt{2}}\,(\xi^E_{N,ji} - \xi^{*E}_{N,ij})\,, \\
B &= -\frac{i}{2\sqrt{2}}\,(\xi^E_{N,ji} - \xi^{*E}_{N,ij})\,, \\
B' &= \frac{1}{2\sqrt{2}}\,(\xi^E_{N,ji} + \xi^{*E}_{N,ij}). \qquad (4.10)
\end{aligned}$$

Here p_{h^0} and p_{A^0} are defined as

$$p_{h^0} = \frac{i}{k^2 - m^2_{h^0} + i m_{h^0}\,\Gamma_{h^0}}\ ;\ p_{A^0} = \frac{i}{k^2 - m^2_{A^0} + i m_{A^0}\,\Gamma_{A^0}}\,. \qquad (4.11)$$

In Eq. (4.9), the parameters p, q, q' and k are the four momentum vectors of the charged Higgs boson H^+, incoming lepton l_i, outgoing lepton l_j and the transfer four momentum respectively. The decay widths Γ_{h^0} and Γ_{A^0} are the total decay widths of the h^0 and A^0 respectively. For the matrix element square of the process

$H^+ \to W^+ l_i^- l_j^+$, we eventually get [69]

$$\begin{aligned}
|M|^2 &= \frac{g^2}{2}\, h\, \{ \Big(\big((m_{l_j}+m_{l_i})^2-k^2\big)|A|^2 + \big((m_{l_j}-m_{l_i})^2-k^2\big)|B|^2 \Big) |p_{h^0}|^2 \\
&+ \Big(\big((m_{l_j}+m_{l_i})^2-k^2\big)|A'|^2 + \big((m_{l_j}-m_{l_i})^2-k^2\big)|B'|^2 \Big) |p_{A^0}|^2 \\
&- 4 m_{l_j}\, m_{l_i} Im[(A\, A'^* - B\, B'^*)\, p_{h^0}\, p^*_{A^0}] \\
&- 2(m^2_{l_j}+m^2_{l_i}-k^2)\, Im[(A\, A'^* + B\, B'^*)\, p_{h^0}\, p^*_{A^0}]\} \,, \qquad (4.12)
\end{aligned}$$

where

$$h = \frac{k^4 + (m^2_{H^\pm} - m^2_W)^2 - 2\, k^2 (m^2_{H^\pm} + m^2_W)}{m^2_W} \,. \qquad (4.13)$$

Finally, the decay width $\Gamma(H^+ \to W^+ l_i^- l_j^+)$ is obtained in the $H^\pm$ boson rest frame by using the well known expression

$$d\Gamma = \frac{(2\,\pi)^4}{2 m_{H^\pm}}\, |M|^2\, \delta^4(p - \sum_{i=1}^{3} p_i) \prod_{i=1}^{3} \frac{d^3 p_i}{(2\pi)^3 2E_i} \,, \qquad (4.14)$$

where p (p_i, i=1,2,3) is four momentum vector of H^+ boson, (W^+ boson, incoming l_j, outgoing l_i leptons).

4.2 Numerical Analysis and Discussion

This section is devoted to the analysis of the charged Higgs decays $H^+ \to W^+ (\tau^- \mu^+ + \tau^+ \mu^-)$ and $H^+ \to W^+ \tau^- \tau^+$. The Yukawa couplings [2] $\bar{\xi}^E_{N,\tau\mu}$ and $\bar{\xi}^E_{N,\tau\tau}$ play the main role in the leptonic part of the LFV $H^+ \to W^+ (\tau^- \mu^+ + \tau^+ \mu^-)$ and

[2] The Yukawa couplings are complex in general and we use the parametrization $\xi^E_{N,ij} = \sqrt{\frac{4 G_F}{\sqrt{2}}}\, \bar{\xi}^E_{N,ij}$ where $G_F = 1.6637 \times 10^{-5} (GeV^{-2})$ is the fermi constant.

LFC $H^+ \to W^+ (l_\tau^- l_\tau^+)$ process respectively. These couplings are free parameters of the model used and they should be restricted by respecting the appropriate present and forthcoming experimental measurements. The upper limit of the coupling $\bar{\xi}^E_{N,\tau\mu}$ has been predicted as $< 30 \pm 5$ GeV, by using experimental result of anomalous magnetic moment of muon in [95]. However, the strength of the coupling $\bar{\xi}^E_{N,\tau\tau}$ is an open problem and waiting for new experimental results in the leptonic sector. Furthermore, the total decay widths of Γ_{h^0} and Γ_{A^0} are unknown parameters and we expect that they are at the same order of magnitude of $\Gamma_{H^0} \sim (0.1 - 1.0)\, GeV$, where H^0 is the SM Higgs boson. Now we start to analyze the three-body decay $H^+ \to W^+ (\tau^- \mu^+ + \tau^+ \mu^-)$.

In Fig.(4.4), we present $\bar{\xi}^E_{N,\tau\mu}$ dependence of the decay width Γ for the decay $H^+ \to W^+ (\tau^- \mu^+ + \tau^+ \mu^-)$, for the real coupling $\bar{\xi}^E_{N,\tau\mu}$, $\Gamma_{A^0} = \Gamma_{h^0} = 0.1\, GeV$, $m_{h^0} = 85\, GeV$ and $m_{A^0} = 90\, GeV$. Here solid, dashed and small dashed lines represent the cases for the Higgs mass $m_{H^\pm} = 200$, 300 and $400 GeV$ respectively. It can be easily seen from the Fig.(4.4) that the Γ is strongly sensitive to the coupling $\bar{\xi}^E_{N,\tau\mu}$, since it is proportional to its square . Moreover, this figure shows that the Γ enhances with the increasing values of the charged Higgs mass, as expected. The Γ is at the order of magnitude of $10^{-11}\, GeV$ for $m_{H^\pm} = 200\, GeV$ and it enhances to the values $10^{-5}\, GeV$ for $m_{H^\pm} = 400\, GeV$, for even the intermediate values of $\bar{\xi}^E_{N,\tau\mu}$.

Fig. (4.5) represents the $m_{H^\pm}$ dependence of the Γ for the fixed values of

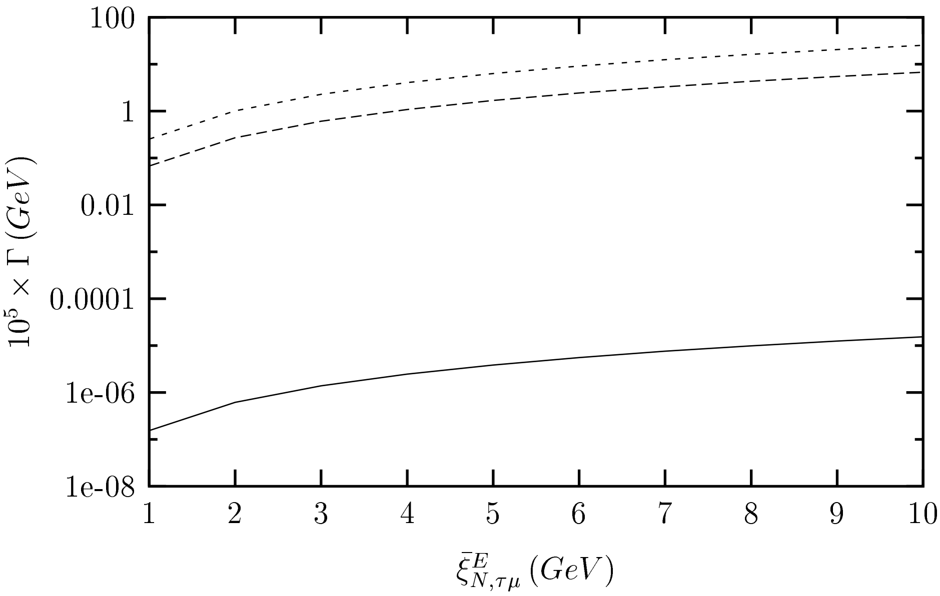

Figure 4.4: $\bar{\xi}^{E}_{N,\tau\mu}$ dependence of the decay width $\Gamma(H^+ \rightarrow W^+\,(\tau^-\mu^+ + \tau^+\mu^-))$, for the real coupling $\bar{\xi}^{E}_{N,\tau\mu}$, $\Gamma_{A^0} = \Gamma_{h^0} = 0.1\,GeV$ $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$. Here solid (dashed, small dashed) line represents the case for the mass value $m_{H^\pm} = 200\,(300, 400)\,GeV$.

$\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$, $\Gamma_{A^0} = \Gamma_{h^0} = 0.1\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$. It is observed that the Γ reaches large values at the order of magnitude of 10^{-5} even for the small coupling $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$. This is interesting in the determination of the upper limit for the charged Higgs mass $m_{H^\pm}$ and also the coupling $\bar{\xi}^{E}_{N,\tau\mu}$.

Fig. (4.6) denotes the total decay width Γ_{h^0} dependence of the decay width Γ for $\Gamma_{A^0} = \Gamma_{h^0}$, $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$, $m_{H^\pm} = 400\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$. It is evident from the Fig. (4.6) that Γ is sensitive to Γ_{h^0} and its value decreases with increasing value of the Γ_{h^0} .

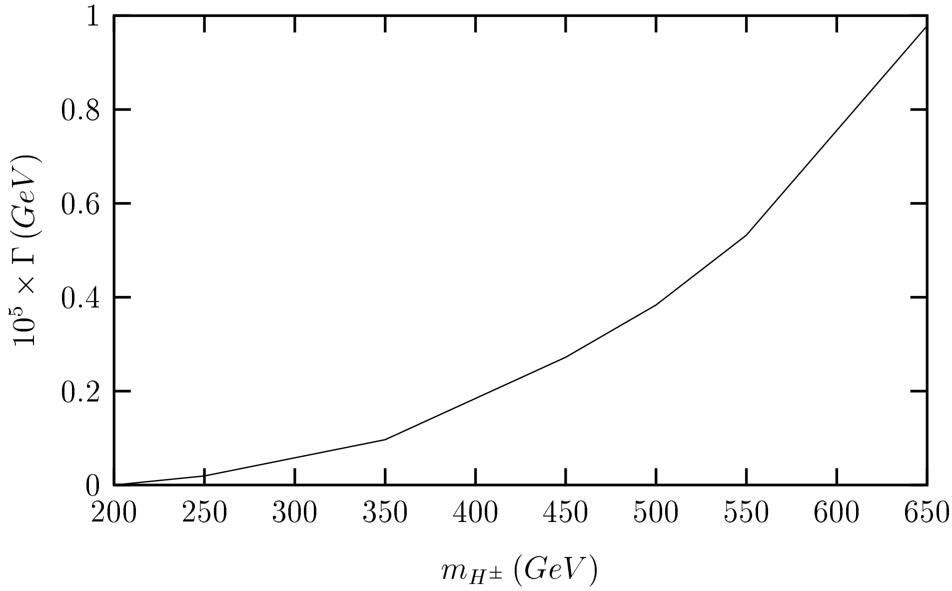

Figure 4.5: The $m_{H^\pm}$ dependence of the decay width $\Gamma(H^+ \rightarrow W^+(\tau^-\mu^+ + \tau^+\mu^-))$ for the fixed values of $\bar{\xi}^E_{N,\tau\mu} = 1\,GeV$, $\Gamma_{A^0} = \Gamma_{h^0} = 0.1\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$.

Now, we make the same analysis for the LFC process $H^+ \rightarrow W^+\tau^-\tau^+$. In Fig. (4.7) we present the $\bar{\xi}^E_{N,\tau\tau}$ dependence of the decay width Γ, for the real coupling, $\Gamma_{A^0} = \Gamma_{h^0} = 0.1\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$. Here solid, dashed and small dashed lines represent the cases for the mass value $m_{H^\pm} = 200$, 300 and $400\,GeV$, respectively. It is seen from the Fig. (4.7) that the Γ is strongly sensitive to the coupling $\bar{\xi}^E_{N,\tau\tau}$ and it enhances with the increasing values of the charged Higgs mass. The Γ is placed in the interval $10^{-9} - 10^{-4}\,(GeV)$ for $200(GeV) \leq m_{H^\pm} \leq 400(GeV)$, at the intermediate values of the coupling $\bar{\xi}^E_{N,\tau\tau}$.

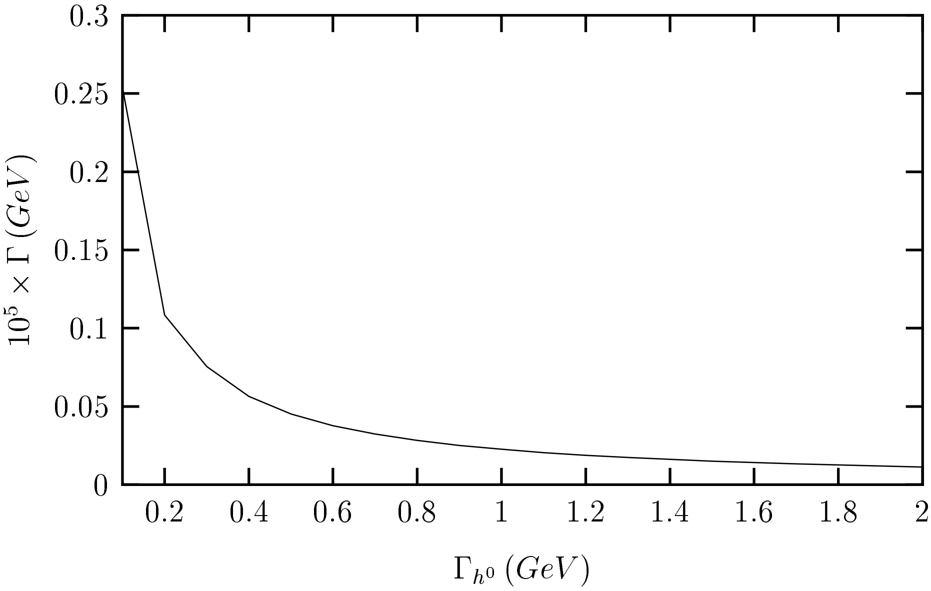

Figure 4.6: Γ_{h^0} dependence of the decay width $\Gamma(H^+ \to W^+\,(\tau^-\mu^+ + \tau^+\mu^-))$ for $\Gamma_{A^0} = \Gamma_{h^0}$, $\bar{\xi}^{E}_{N,\tau\mu} = 1\,GeV$, $m_{H^\pm} = 400\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$.

Fig. (4.8) denotes the $m_{H^\pm}$ dependence of the Γ for $\bar{\xi}^{E}_{N,\tau\tau} = 10\,GeV$, $\Gamma_{A^0} = \Gamma_{h^0} = 0.1\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$. From the figure it is observed that the Γ reaches large values of the order of magnitude of 10^{-4}, even for the small coupling $\bar{\xi}^{E}_{N,\tau\tau} = 10\,GeV$. The determination of the upper limit for the coupling $\bar{\xi}^{E}_{N,\tau\tau}$ would be possible with the measurement of the process under consideration.

Fig. (4.9) represent Γ_{h^0} dependence of the decay width Γ for $\Gamma_{A^0} = \Gamma_{h^0}$, for $\bar{\xi}^{E}_{N,\tau\tau} = 10\,GeV$, $m_{H^\pm} = 400\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$. It is observed that the Γ is sensitive to Γ_{h^0} and decreases with its increasing value, similar to the LFV process $H^+ \to W^+\,\tau^-\mu^+$.

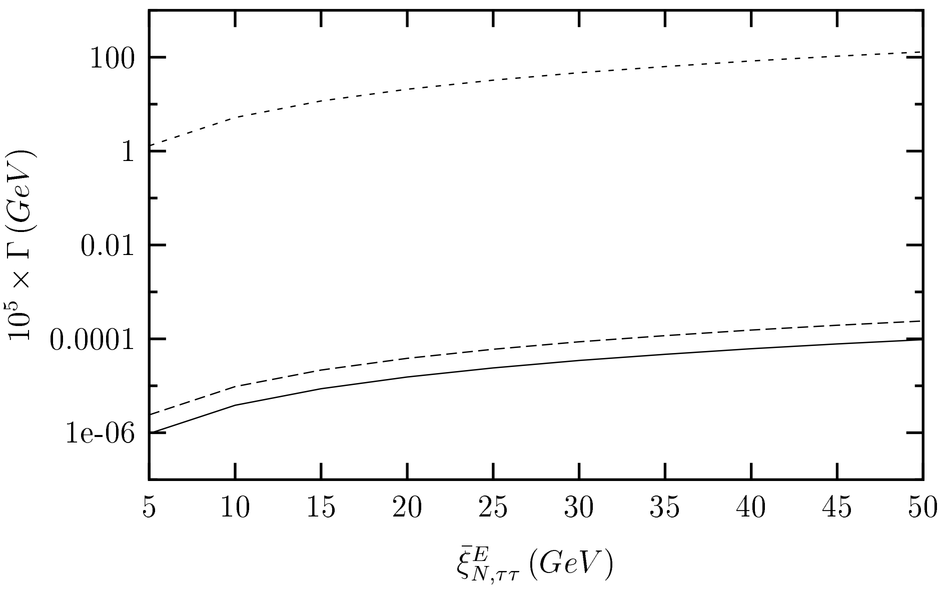

Figure 4.7: $\bar{\xi}^{E}_{N,\tau\tau}$ dependence of the decay width $\Gamma(H^+ \to W^+\,\tau^-\tau^+)$, for the real coupling $\bar{\xi}^{E}_{N,\tau\tau}$, $\Gamma_{A^0} = \Gamma_{h^0} = 0.1\,GeV$ $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$. Here solid (dashed, small dashed) line represents the case for the mass value $m_{H^\pm} = 200\,(300, 400)\,GeV$.

Finally, we consider the coupling $\bar{\xi}^{E}_{N,ij}$ as a complex matrix

$$\bar{\xi}^{E}_{N,ij} = |\bar{\xi}^{E}_{N,ij}|\, e^{i\theta_{ij}}\,, \tag{4.15}$$

and study the $\sin\theta_{ij}$ dependence of the decay width. We observe that the decay width is not sensitive to the complexity of the coupling $\bar{\xi}^{E}_{N,ij}$.

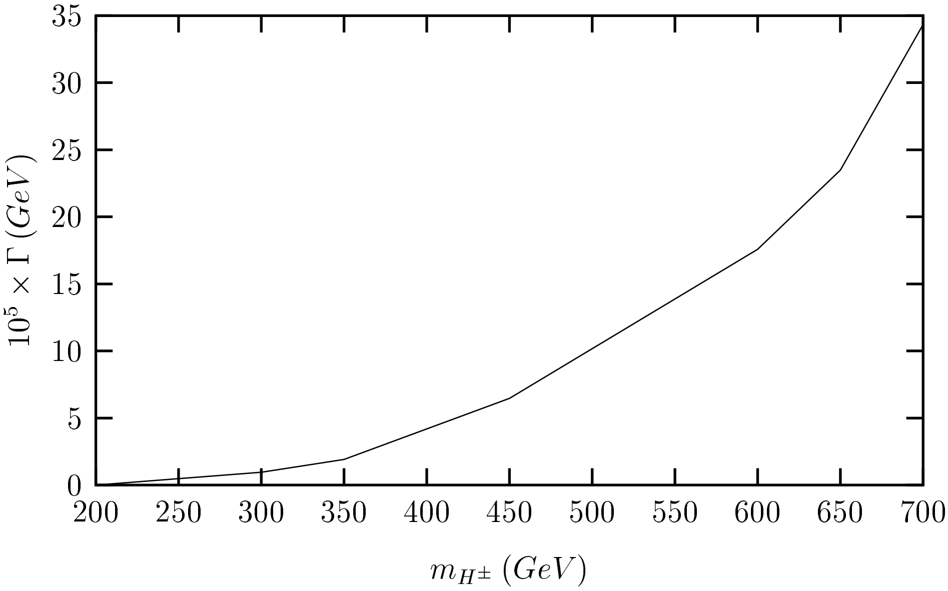

Figure 4.8: The $m_{H^\pm}$ dependence of the decay width $\Gamma(H^+ \to W^+\,\tau^-\tau^+)$ for the fixed values of $\bar{\xi}^E_{N,\tau\tau} = 10\,GeV$, $\Gamma_{A^0} = \Gamma_{h^0} = 0.1\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$.

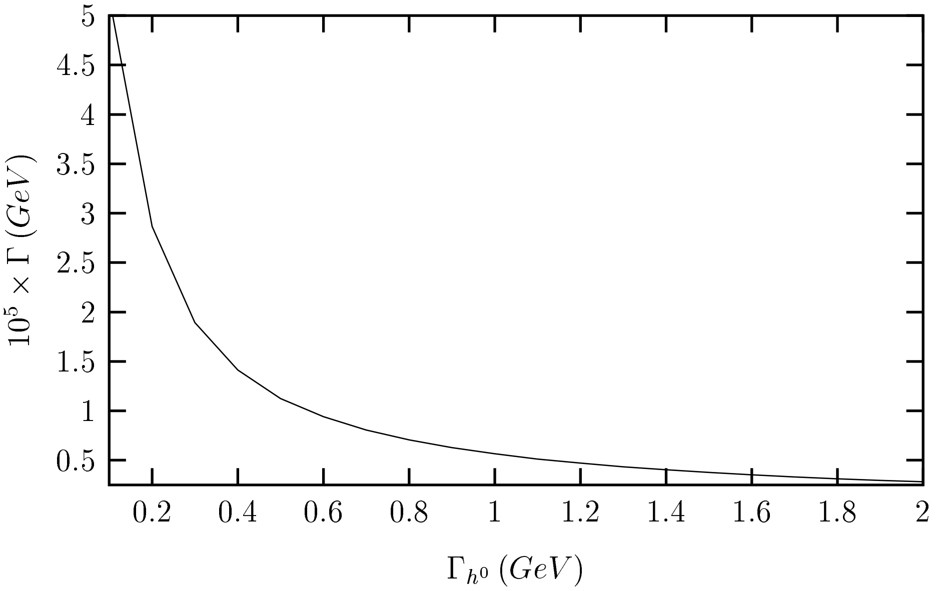

Figure 4.9: Γ_{h^0} dependence of the decay width $\Gamma(H^+ \to W^+\,\tau^-\tau^+)$ for $\Gamma_{A^0} = \Gamma_{h^0}$, $\bar{\xi}^E_{N,\tau\tau} = 10\,GeV$, $m_{H^\pm} = 400\,GeV$, $m_{h^0} = 85\,GeV$ and $m_{A^0} = 90\,GeV$.

CHAPTER 5

$\tau \to \mu \overline{\nu_i} \nu_i$ DECAY IN THE GENERAL TWO HIGGS DOUBLET MODEL

In the leptonic sector of the SM, the charged FC interactions with the massless neutrinos are forbidden due to the absence of a CKM type matrix. However, if neutrinos ν_i are massive particles and the lepton numbers L_i, which denote the leptons of i^{th} family, are not conserved, the lepton sector is an exact analogue of the quark sector and there exists a similar CKM type matrix, Maki-Nakagawa-Sakata (MNS) matrix $V_{l\nu}$ [96], and its elements are measured in neutrino oscillation experiments. It has been shown that the mixing between the muon neutrino and the heaviest mass eigenstate of the neutrino sector, the $V_{\mu 3}$ element, is nearly maximal [97, 98]. The experiments on solar neutrinoes [97], [99] (the reactor experiments such as CHOOZ [100]) predicted the mixing between electron neutrino and the second heaviest mass eigenstate of the neutrino sector, the V_{e2} element the heaviest mass eigenstate of the neutrino sector, the V_{e3} element). Notice that the corner element V_{e3} is much smaller than the others. On the other hand some off-diagonal elements of MNS matrix, such as $V_{\mu 3}$ are large and the MNS matrix is far from diagonal in contrast to the CKM matrix (see for example [101] for more discussion on lepton mixing). With the inclusion of MNS matrix to the SM, so called extended SM, the existence of the LFV processes, $\tau \to \mu$ transition would be possible. However, we expect that the tiny masses of the internal neutrinos bring small contribution even with the choice of maximal mixing in the leptonic sector. (See for example [102], for the discussion of the existence of the MNS matrix and its effects on a special LFV process). So, LFV interactions give strong signal about the new physics beyond and there are many models which appear in the literature as an extension of the SM to describe the LFV interaction. Such interactions are studied in a model independent way in [57], in the framework of the 2HDM [103], in supersymmetric models [15, 16, 17, 104, 105, 106, 107]. There are on-going and planned experiments for $\mu \to e\gamma$ ($\tau \to \mu\gamma$) and the current limits for their branching ratios (BR) are 1.2×10^{-11} [49] (1.1×10^{-6} [108]). The numerical estimates predict that the BR of the proceses $\tau \to e\bar{e}e$, $\tau \to e\bar{\mu}\mu$ are at the order of the magnitude of 10^{-6} [109], which is a measurable value in the LEP experiments and τ factories.

In this chapter, we analyze the BR of the LFV $\tau \to \mu\bar{\nu}_i\nu_i$, $i = e, \mu, \tau$ decay in the general 2HDM and study their dependencies on free parameters of the model used. This process exists with the help of internal scalar bosons h^0 and A^0 in order to obtain the flavor changing transition $\tau \to \mu$ and the internal Z boson in order to get the output $\bar{\nu}\nu$. There have been some experimental studies on this process in the literature [110]. The BR's of the LFV decay $\tau \to \mu\bar{\nu}_i\nu_i$ can be used for the prediction of the upper limit of the Yukawa couplings. These are all discussed in numerical analysis and discussion section.

5.1 The Calculation of the $\tau \to \mu\overline{\nu_i}\nu_i$

In this section, we outline the basic steps of the calculation the amplitude M and the BR for the one-loop LFV $\tau \to \mu\overline{\nu_i}\nu_i$ decay in the framework of the general 2HDM (see Eq. (4.2) for the definition of the FC part of the Yukawa Lagrangian in the leptonic sector).

The general effective vertex for the interaction of off-shell Z-boson with a fermionic current is obtained as

$$\Gamma_\mu^{(REN)}(\tau \to \mu Z^*) = f_1\,\gamma_\mu + f_2\,\gamma_\mu\gamma_5 + f_3\,\sigma_{\mu\nu}k^\nu + f_4\,\sigma_{\mu\nu}\gamma_5 k^\nu\,, \qquad (5.1)$$

where k is the momentum transfer, $k^2 = (p-p')^2$, p (p') are the four momentum vectors of incoming (outgoing) lepton (see Fig. (5.2 - 5.3 - 5.4)).

The LFV $\tau \to \mu\bar{\nu}_i\nu_i$ decay exists with the FC transition $\tau \to \mu$, which is carried by the internal scalar bosons h^0 and A^0. On the other hand, the internal Z boson is responsible for the output $\bar{\nu}\nu$ (see Fig. (5.1)).

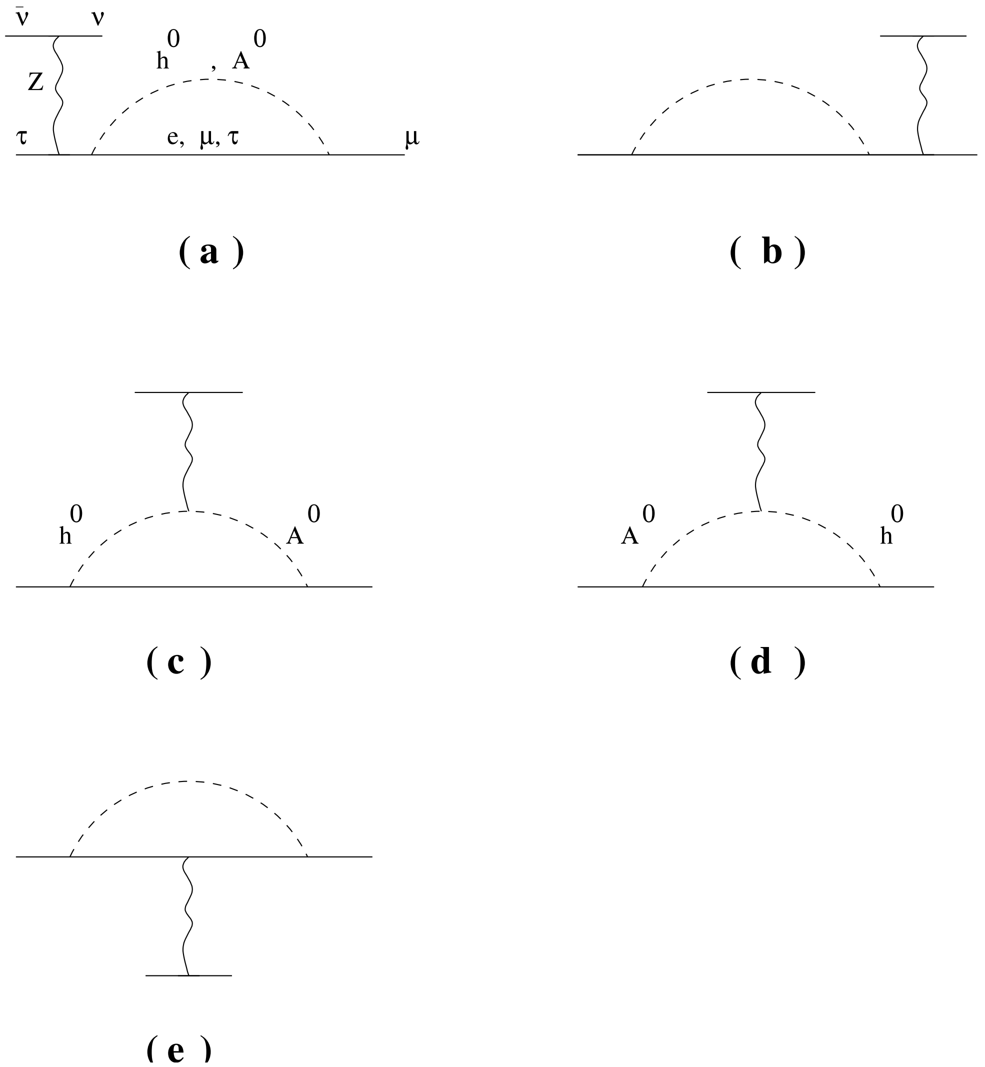

Figure 5.1: One loop diagrams contribute to $\tau \to \mu\,\bar{\nu}_i\,\nu_i$, $i = e, \mu, \tau$ decay due to the neutral Higgs bosons h_0 and A_0 in the general 2HDM. Solid lines represent leptons and neutrinos, curly (dashed) lines represent the virtual Z boson (h_0 and A_0 fields).

There are two types of diagrams in the Fig. (5.1), the self energy diagrams (Fig. (5.1) (a) (b)) and the vertex diagrams (Fig. (5.1) (c), (d), (e)). At this point, we present the details of the calculations for the self-energy and vertex diagrams. Notice that, the vertices and propagators used in the calculations are given in Appendix C.

Now, we consider the contribution of self-energy diagram to the effective vertex (see Fig. (5.2)):

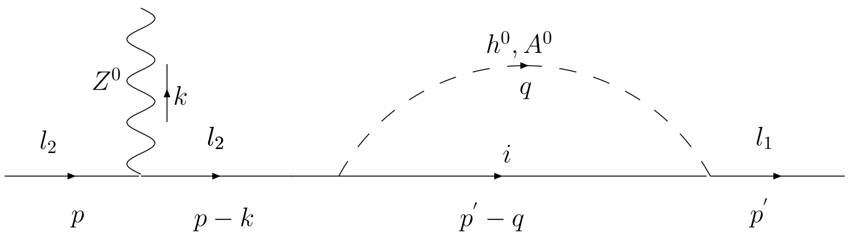

Figure 5.2: The self energy diagram for $l_2 \rightarrow l_1$ transition.

$$
\begin{aligned}
\Gamma^{\mu}_{a,h^0} &= \frac{4\,G_F}{\sqrt{2}} \int \frac{d^4q}{(2\pi)^4} \frac{-i}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_1 i} + \overline{\xi}^{E*}_{N,i l_1}) + (\overline{\xi}^{E}_{N,l_1 i} - \overline{\xi}^{E*}_{N,i l_1})\gamma_5] \frac{i}{\not{p}' - \not{q} - m_i} \\
& \frac{-i}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_2 i} + \overline{\xi}^{E*}_{N,i l_2}) + (\overline{\xi}^{E}_{N,l_2 i} - \overline{\xi}^{E*}_{N,i l_2})\gamma_5] \frac{i}{q^2 - m^2_{h^0}} \\
& \frac{i}{\not{p} - \not{k} - m_{l_2}} \Big(\frac{-i\,g}{\cos\theta_w}\Big) \gamma^{\mu} (c_L L + c_R R)\,, \\
\Gamma^{\mu}_{a,A^0} &= \frac{4\,G_F}{\sqrt{2}} \int \frac{d^4q}{(2\pi)^4} \frac{1}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_1 i} - \overline{\xi}^{E*}_{N,i l_1}) + (\overline{\xi}^{E}_{N,l_1 i} + \overline{\xi}^{E*}_{N,i l_1})\gamma_5] \frac{i}{\not{p}' - \not{q} - m_i} \\
& \frac{-1}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_2 i} - \overline{\xi}^{E*}_{N,i l_2}) + (\overline{\xi}^{E}_{N,l_2 i} + \overline{\xi}^{E*}_{N,i l_2})\gamma_5] \frac{i}{q^2 - m^2_{A^0}} \\
& \frac{i}{\not{p} - \not{k} - m_{l_2}} \Big(\frac{-i\,g}{\cos\theta_w}\Big) \gamma^{\mu} (c_L L + c_R R)\,, \qquad (5.2)
\end{aligned}
$$

On the other hand, vertex diagrams have two different types, Fig. (5.1) (c), (d), and Fig. (5.1) (e). Now, first, we present the details of the calculation for the effective vertex due to (c) part of Fig. (5.1) and it reads

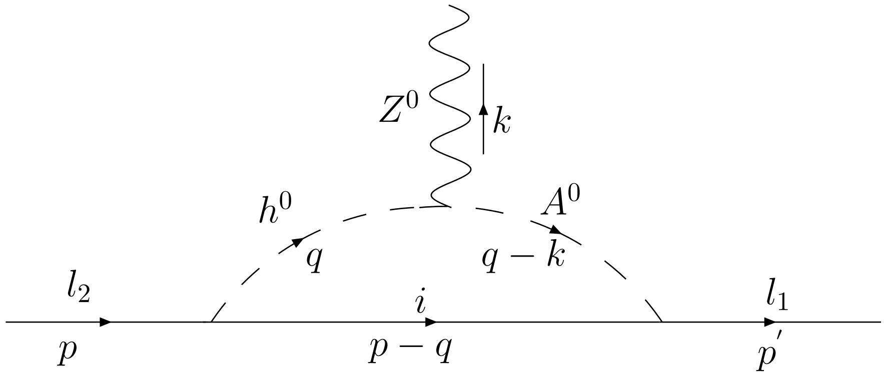

Figure 5.3: The one loop of the vertex diagram for (c) in the Fig. (5.1).

$$
\begin{aligned}
\Gamma_c^{\mu} \;=\; & \frac{4\,G_F}{\sqrt{2}} \int \frac{d^4q}{(2\pi)^4} \frac{1}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_1 i} - \overline{\xi}^{E*}_{N,i l_1}) + (\overline{\xi}^{E}_{N,l_1 i} + \overline{\xi}^{E*}_{N,i l_1})\gamma_5] \frac{i}{\not p - \not q - m_i} \\
& \frac{-i}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_2 i} + \overline{\xi}^{E*}_{N,i l_2}) + (\overline{\xi}^{E}_{N,l_2 i} - \overline{\xi}^{E*}_{N,i l_2})\gamma_5] \frac{i}{q^2 - m^2_{h^0}} \\
& \frac{g}{2\cos\theta_w} (2q-k)^{\mu} \frac{i}{(q-k)^2 - m^2_{A^0}} . \qquad (5.3)
\end{aligned}
$$

Furthermore, the effective vertex due to (e) part of Fig. (5.1) is calculated as

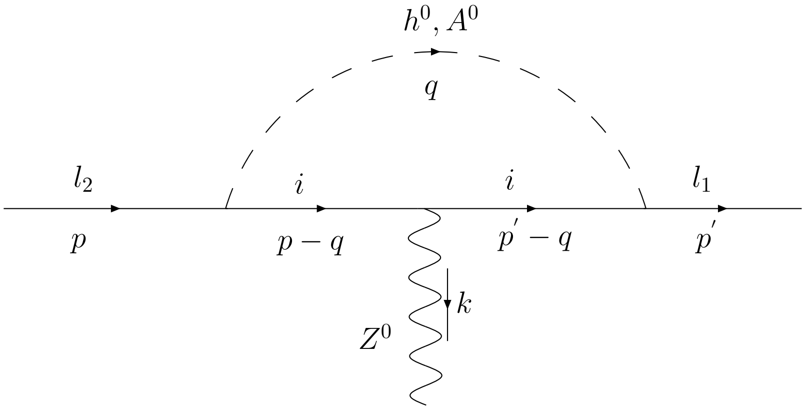

Figure 5.4: The one loop of the vertex diagram for (e) in the Fig. (5.1).

$$\begin{aligned}
\Gamma^{\mu}_{e,h^0} &= \frac{4\,G_F}{\sqrt{2}} \int \frac{d^4q}{(2\pi)^4} \frac{-i}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_1 i} + \overline{\xi}^{E*}_{N,il_1}) + (\overline{\xi}^{E}_{N,l_1 i} - \overline{\xi}^{E*}_{N,il_1})\gamma_5] \frac{i}{\not p' - \not q - m_i} \\
& \frac{ig}{\cos\theta_w} \gamma_\mu (c_V + c_A \gamma_5) \frac{i}{\not p - \not q - m_i} \\
& \frac{-i}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_2 i} + \overline{\xi}^{E*}_{N,il_2}) + (\overline{\xi}^{E}_{N,l_2 i} - \overline{\xi}^{E*}_{N,il_2})\gamma_5] \frac{i}{q^2 - m^2_{h^0}}, \\
\Gamma^{\mu}_{e,A^0} &= \frac{4\,G_F}{\sqrt{2}} \int \frac{d^4q}{(2\pi)^4} \frac{1}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_1 i} - \overline{\xi}^{E*}_{N,il_1}) + (\overline{\xi}^{E}_{N,l_1 i} + \overline{\xi}^{E*}_{N,il_1})\gamma_5] \frac{i}{\not p' - \not q - m_i} \\
& \frac{ig}{\cos\theta_w} \gamma_\mu (c_V + c_A \gamma_5) \frac{i}{\not p - \not q - m_i} \\
& \frac{-1}{2\sqrt{2}} [(\overline{\xi}^{E}_{N,l_2 i} - \overline{\xi}^{E*}_{N,il_2}) + (\overline{\xi}^{E}_{N,l_2 i} + \overline{\xi}^{E*}_{N,il_2})\gamma_5] \frac{i}{q^2 - m^2_{A^0}}. \qquad (5.4)
\end{aligned}$$

In the calculation of the momentum integrals, we used the Feynman parametrization in order to put the loop integrals into quadratic forms (see the Appendix D for details of Feynman parametrization). By using the dimensional regularization method (see Appendix D), the divergent terms are extracted in each diagrams and they cancel each other in the sum. Since Γ^{μ} is sandwiched between $\overline{l_1}(p')$ and $l_2(p)$, we used the following on shell mass conditions,

$$\overline{l_1}(p')\ \not p' = m_{l_1}\overline{l_1}(p'), \quad \not p l_2(p) = m_2 l_2(p), \qquad (5.5)$$

and

$$\begin{aligned}
\not p'^2 &= p'^2 = m^2_{l_1}, \\
\not p^2 &= p^2 = m^2_{l_2}. \qquad (5.6)
\end{aligned}$$

Taking into account all the masses of internal and external leptons (anti-leptons) and, following the procedure as stated, the explicit expressions for f_1, f_2, f_3 and

f_4 can be obtained as [70]:

$$
\begin{aligned}
f_1 &= \frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\,\frac{1}{m_{l_2}^2-m_{l_1}^2}\Bigg\{c_V\,(m_{l_2}+m_{l_1})\\
\Bigg(& (-m_i\,\eta_i^+ + m_{l_1}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2}\\
+ & (m_i\,\eta_i^+ - m_{l_2}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2}\\
+ & (m_i\,\eta_i^+ + m_{l_1}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2}\\
- & (m_i\,\eta_i^+ + m_{l_2}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}\Bigg)\\
+ & c_A\,(m_{l_2}-m_{l_1})\Bigg((-m_i\,\eta_i^- + m_{l_1}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2}\\
+ & (m_i\,\eta_i^- + m_{l_2}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2}\\
+ & (m_i\,\eta_i^- + m_{l_1}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2}\\
+ & (-m_i\,\eta_i^- + m_{l_2}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2}\Bigg)\Bigg\}\\
- & \frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\int_0^{1-x} dy\,\Bigg\{m_i^2\,(c_A\,\eta_i^A - c_V\,\eta_i^V)\,(\frac{1}{L^{ver}_{A^0}}+\frac{1}{L^{ver}_{h^0}})\\
- & (1-x-y)\,m_i\Bigg(c_A\,(m_{l_2}-m_{l_1})\,\eta_i^-\,(\frac{1}{L^{ver}_{h^0}}-\frac{1}{L^{ver}_{A^0}})\\
+ & c_V\,(m_{l_2}+m_{l_1})\,\eta_i^+\,(\frac{1}{L^{ver}_{h^0}}+\frac{1}{L^{ver}_{A^0}})\Bigg) - (c_A\,\eta_i^A + c_V\,\eta_i^V)\\
\Bigg(& -2+(k^2\,x\,y+m_{l_1}\,m_{l_2}\,(-1+x+y)^2)\,(\frac{1}{L^{ver}_{h^0}}+\frac{1}{L^{ver}_{A^0}}) - ln\,\frac{L^{ver}_{h^0}}{\mu^2}\,\frac{L^{ver}_{A^0}}{\mu^2}\Bigg)\\
- & (m_{l_2}+m_{l_1})\,(1-x-y)\Bigg(\frac{\eta_i^A\,(x\,m_{l_1}+y\,m_{l_2})+m_i\,\eta_i^-}{2\,L^{ver}_{A^0\,h^0}}\\
+ & \frac{\eta_i^A\,(x\,m_{l_1}+y\,m_{l_2})-m_i\,\eta_i^-}{2\,L^{ver}_{h^0\,A^0}}\Bigg)+\frac{1}{2}\,\eta_i^A\,ln\,\frac{L^{ver}_{A^0\,h^0}}{\mu^2}\,ln\,\frac{L^{ver}_{h^0\,A^0}}{\mu^2}\Bigg\}\,,\\
f_2 &= \frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\,\frac{1}{m_{l_2}^2-m_{l_1}^2}\Bigg\{c_V\,(m_{l_2}-m_{l_1})
\end{aligned}
$$

$$
\begin{aligned}
&\Bigg(&& (m_i\,\eta_i^- + m_{l_1}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} \\
&+ && (-m_i\,\eta_i^- + m_{l_2}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2} \\
&+ && (-m_i\,\eta_i^- + m_{l_1}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} \\
&+ && (m_i\,\eta_i^- + m_{l_2}(-1+x)\,\eta_i^A)\,ln\,\frac{L^{self}_{2,\,h^0}}{\mu^2}\Bigg) \\
&+ && c_A\,(m_{l_2}+m_{l_1})\Bigg((m_i\,\eta_i^+ + m_{l_1}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,A^0}}{\mu^2} \\
&- && (m_i\,\eta_i^+ + m_{l_2}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{2,\,A^0}}{\mu^2} \\
&+ && (-m_i\,\eta_i^+ + m_{l_1}(-1+x)\,\eta_i^V)\,ln\,\frac{L^{self}_{1,\,h^0}}{\mu^2} \\
&+ && (m_i\,\eta_i^+ - m_{l_2}(-1+x)\,\eta_i^V)\,\frac{ln\,L^{self}_{2,\,h^0}}{\mu^2}\Bigg)\Bigg\} \\
&+ && \frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\,\int_0^{1-x} dy\,\Bigg\{m_i^2\,(c_V\,\eta_i^A - c_A\,\eta_i^V)\,(\frac{1}{L^{ver}_{A^0}}+\frac{1}{L^{ver}_{h^0}}) \\
&- && m_i\,(1-x-y)\,\Bigg(c_V\,(m_{l_2}-m_{l_1})\,\eta_i^- + c_A\,(m_{l_2}+m_{l_1})\,\eta_i^+\Bigg)\,(\frac{1}{L^{ver}_{h^0}}-\frac{1}{L^{ver}_{A^0}}) \\
&+ && (c_V\,\eta_i^A + c_A\,\eta_i^V)\Bigg(-2+(k^2\,x\,y - m_{l_1}\,m_{l_2}\,(-1+x+y)^2)(\frac{1}{L^{ver}_{h^0}}+\frac{1}{L^{ver}_{A^0}}) \\
&- && ln\,\frac{L^{ver}_{h^0}}{\mu^2}\,\frac{L^{ver}_{A^0}}{\mu^2}\Bigg) - (m_{l_2}-m_{l_1})\,(1-x-y)\,\Bigg(\frac{\eta_i^V\,(x\,m_{l_1}-y\,m_{l_2}) + m_i\,\eta_i^+}{2\,L^{ver}_{A^0\,h^0}} \\
&+ && \frac{\eta_i^V\,(x\,m_{l_1}-y\,m_{l_2}) - m_i\,\eta_i^+}{2\,L^{ver}_{h^0\,A^0}}\Bigg) - \frac{1}{2}\,\eta_i^V\,ln\,\frac{L^{ver}_{A^0\,h^0}}{\mu^2}\,ln\,\frac{L^{ver}_{h^0\,A^0}}{\mu^2}\Bigg\}, \\
f_3 &= && -i\frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\,\int_0^{1-x} dy\,\Bigg\{\Bigg((1-x-y)\,(c_V\,\eta_i^V + c_A\,\eta_i^A) \\
&(&& x\,m_{l_1}+y\,m_{l_2}) + m_i\,(c_A\,(x-y)\,\eta_i^- + c_V\,\eta_i^+\,(x+y))\Bigg)\,\frac{1}{L^{ver}_{h^0}} \\
&+ && \Bigg((1-x-y)\,(c_V\,\eta_i^V + c_A\,\eta_i^A)\,(x\,m_{l_1}+y\,m_{l_2}) \\
&- && m_i\,(c_A\,(x-y)\,\eta_i^- + c_V\,\eta_i^+\,(x+y))\Bigg)\,\frac{1}{L^{ver}_{A^0}}
\end{aligned}
$$

$$
\begin{aligned}
& - \quad (1-x-y)\Big(\frac{\eta_i^A\,(x\,m_{l_1}+y\,m_{l_2})}{2}\,\big(\frac{1}{L^{ver}_{A^0\,h^0}}+\frac{1}{L^{ver}_{h^0\,A^0}}\big) \\
& + \quad \frac{m_i\,\eta_i^-}{2}\,\big(\frac{1}{L^{ver}_{h^0\,A^0}}-\frac{1}{L^{ver}_{A^0\,h^0}}\big)\Big)\Big\}\,, \\
f_4 \quad & = \quad -i\frac{g}{64\,\pi^2\,cos\,\theta_W}\int_0^1 dx\,\int_0^{1-x} dy\Big\{\Big((1-x-y)\,\Big(-(c_V\,\eta_i^A+c_A\,\eta_i^V) \\
& (\quad x\,m_{l_1}-y\,m_{l_2})\Big)-m_i\,(c_A\,(x-y)\,\eta_i^+ + c_V\,\eta_i^-\,(x+y))\Big)\,\frac{1}{L^{ver}_{h^0}} \\
& + \quad \Big((1-x-y)\,\Big(-(c_V\,\eta_i^A+c_A\,\eta_i^V)\,(x\,m_{l_1}-y\,m_{l_2})\Big) \\
& + \quad m_i\,(c_A\,(x-y)\,\eta_i^+ + c_V\,\eta_i^-\,(x+y))\Big)\,\frac{1}{L^{ver}_{A^0}} \\
& + \quad (1-x-y)\,\Big(\frac{\eta_i^V}{2}\,(m_{l_1}\,x-m_{l_2}\,y)\,\big(\frac{1}{L^{ver}_{A^0\,h^0}}+\frac{1}{L^{ver}_{h^0\,A^0}}\big) \\
& + \quad \frac{m_i\,\eta_i^+}{2}\,\big(\frac{1}{L^{ver}_{A^0\,h^0}}-\frac{1}{L^{ver}_{h^0\,A^0}}\big)\Big)\Big\}\,, \qquad (5.7)
\end{aligned}
$$

where

$$
\begin{aligned}
L^{self}_{1\,\{2\},\,h^0\,(A^0)} &= m^2_{h^0\,(A^0)}\,(1-x)+(m_i^2-m^2_{l_{1\,\{2\}}}\,(1-x))\,x\,, \\
L^{ver}_{h^0\,(A^0)} &= m^2_{h^0\,(A^0)}\,(1-x-y)+m_i^2\,(x+y)-k^2\,x\,y\,, \\
L^{ver}_{h^0\,A^0\,(A^0\,h^0)} &= m^2_{A^0\,(h^0)}\,x+m_i^2\,(1-x-y)+(m^2_{h^0\,(A^0)}-k^2\,x)\,y\,, \qquad (5.8)
\end{aligned}
$$

and

$$
\begin{aligned}
\eta_i^V &= \frac{4\,G_F}{\sqrt{2}}\,\big(\overline{\xi}^E_{N,l_1 i}\overline{\xi}^{E\,*}_{N,i l_2}+\overline{\xi}^{E\,*}_{N,i l_1}\overline{\xi}^E_{N,l_2 i}\big)\,, \\
\eta_i^A &= \frac{4\,G_F}{\sqrt{2}}\,\big(\overline{\xi}^E_{N,l_1 i}\overline{\xi}^{E\,*}_{N,i l_2}-\overline{\xi}^{E\,*}_{N,i l_1}\overline{\xi}^E_{N,l_2 i}\big)\,, \\
\eta_i^+ &= \frac{4\,G_F}{\sqrt{2}}\,\big(\overline{\xi}^{E\,*}_{N,i l_1}\overline{\xi}^{E\,*}_{N,i l_2}+\overline{\xi}^E_{N,l_1 i}\overline{\xi}^E_{N,l_2 i}\big)\,, \\
\eta_i^- &= \frac{4\,G_F}{\sqrt{2}}\,\big(\overline{\xi}^{E\,*}_{N,i l_1}\overline{\xi}^{E\,*}_{N,i l_2}-\overline{\xi}^E_{N,l_1 i}\overline{\xi}^E_{N,l_2 i}\big)\,. \qquad (5.9)
\end{aligned}
$$

The parameters c_V and c_A are $c_A = -\frac{1}{4}$ and $c_V = \frac{1}{4} - sin^2\,\theta_W$. In Eq. (5.9) the flavor changing couplings $\bar{\xi}^{E}_{N,l_j i}$ represent the effective interaction between the internal lepton i, $(i = e, \mu, \tau)$ and outgoing (incoming) $j = 1\,(j = 2)$ one. The parameter μ in Eq. (5.7) is the renormalization scale, the functions f_1, f_2, f_3, f_4 are finite and independent of μ. During the calculation, we take only the τ lepton in the internal line and we neglect all the Yukawa couplings except $\bar{\xi}^{E}_{N,\tau\tau}$ and $\bar{\xi}^{E}_{N,\tau\mu}$ in the loop calculation. Details of this choice is discussed in discussion section.

After calculating the f_1, f_2, f_3 and f_4, the matrix element M for the process $\tau \to \mu\, \bar{\nu}_i\, \nu_i$, $i = e, \mu, \tau$ is calculated in the general 2HDM, by connecting the $\tau \to \mu$ transition and the $\bar{\nu}_i \nu_i$ output with the help of the internal Z boson.

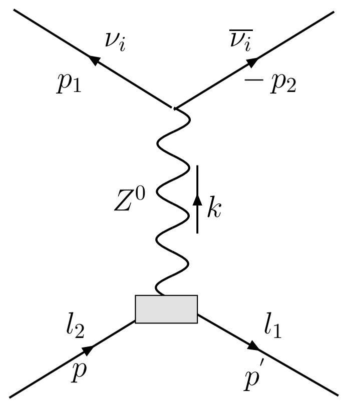

Figure 5.5: The diagram for $\tau \to \mu\, \bar{\nu}_i\, \nu_i$ decay.

The box in the Fig. (5.5) includes all the possible diagrams in Fig. (5.1). l_1 and l_2 are μ and τ lepton respectively. The Matrix element and its hermition conjugate are obtained as

$$\begin{aligned}
M &= [\overline{l_1}(f_1\,\gamma_\mu + f_2\,\gamma_\mu\gamma_5 + f_3\,\sigma_{\mu\nu}k^\nu + f_4\,\sigma_{\mu\nu}\gamma_5 k^\nu) l_2] \frac{-ig_{\mu\lambda}}{k^2 - m^2_{Z^0}} \\
& [\quad \overline{\nu_i}\frac{-ig}{4\cos\theta_w}\gamma_\lambda(1-\gamma_5)\nu_i], \\
M^\dagger &= [\overline{l_2}(f_1^\dagger\,\gamma_{\mu'} + f_2^\dagger\,\gamma_{\mu'}\gamma_5 + f_3^\dagger\,\sigma_{\mu'\nu'}k^{\nu'} - f_4^\dagger\,\sigma_{\mu'\nu'}\gamma_5 k^{\nu'}) l_1] \frac{ig_{\mu'\lambda'}}{k^2 - m^2_{Z^0}} \\
& [\quad \overline{\nu_i}\frac{ig}{4\cos\theta_w}\gamma_{\lambda'}(1-\gamma_5)\nu_i]. \qquad (5.10)
\end{aligned}$$

After a little algebra, absolute square of the amplitude becomes

$$\begin{aligned}
|M|^2 &= \frac{g^2}{\cos^2\theta_w}\frac{1}{(k^2-m^2_{Z^0})^2}[-k^4(|f_1|^2 + |f_2|^2 + |f_3|^2(m^2_{l_1} + 4E_1 m_{l_2} + 2m_{l_1}m_{l_2} \\
&- m^2_{l_2}) + |f_4|^2(m^2_{l_1} + 4E_1 m_{l_2} - 2m_{l_1}m_{l_2} - m^2_{l_2}) - 2Im|f_1^\dagger f_3|(m_{l_1} + m_{l_2}) \\
&+ 2Im|f_2^\dagger f_3|(m_{l_1} + m_{l_2}) + 2Im|f_1^\dagger f_4|(m_{l_1} - m_{l_2}) + 2Im|f_2^\dagger f_4|(-m_{l_1} + m_{l_2}) \\
&- 2Re|f_1^\dagger f_2| - 2Re|f_3^\dagger f_4|(m^2_{l_1} - m^2_{l_2})) \\
&- k^2(|f_1|^2(-m^2_{l_1} + m^2_{l_2} - 4E_1 m_{l_2} + 2m_{l_1}m_{l_2}) + |f_2|^2(-m^2_{l_1} + m^2_{l_2} \\
&- 4E_1 m_{l_2} - 2m_{l_1}m_{l_2}) + (|f_3|^2 + |f_4|^2)(-m^4_{l_1} - m^4_{l_2} - 4E_1 m^2_{l_1}m_{l_2} \\
&- 8E_1^2 m^2_{l_2} + 2m^2_{l_1}m^2_{l_2} + 4E_1 m^3_{l_2}) + 2Im|f_1^\dagger f_3|(m^3_{l_1} - m^2_{l_1}m_{l_2} \\
&- m_{l_1}m^2_{l_2} + m^3_{l_2}) + 2Im|f_2^\dagger f_3|(-m^3_{l_1} - 4E_1 m_{l_2}(m_{l_1} + m_{l_2}) \\
&- m_{l_1}m_{l_2}(m_{l_1} - m_{l_2}) + m^3_{l_2}) + 2Im|f_1^\dagger f_4|(-m^3_{l_1} - 4E_1 m_{l_2}(m_{l_1} - m_{l_2}) \\
&+ m_{l_1}m_{l_2}(m_{l_1} + m_{l_2}) - m^3_{l_2}) + 2Im|f_2^\dagger f_4|(m^3_{l_1} - m^3_{l_2} + m^2_{l_1}m_{l_2} - m_{l_1}m^2_{l_2}) \\
&+ 2Re|f_1^\dagger f_2|(m^2_{l_1} - m^2_{l_2} + 4E_1 m_{l_2}) + 2Re|f_3^\dagger f_4|(m^4_{l_1} + m^4_{l_1} - 2m^2_{l_1}m^2_{l_2} \\
&+ 4E_1 m_{l_2}(m^2_{l_1} - m^2_{l_2}))) \\
&- 4E_1 m_{l_2}(m^2_{l_1} + m_{l_2}(2E_1 - m_{l_2}))(|f_1|^2 + |f_2|^2)]\,, \qquad (5.11)
\end{aligned}$$

where m_{l_1} and m_{l_2} are masses of the μ and τ leptons, respectively and E_1 is the energy of the outgoing neutrino.

Finally, the decay width Γ is obtained in the τ lepton rest frame using the well known expression Eq. (4.14) and we get

$$d\Gamma = \frac{|M|^2}{128\pi^3 m_\tau} dE_1 dE_3 \tag{5.12}$$

where E_1 and E_3 are the muon and outgoing neutrino energies, respectively.

5.2 Numerical Analysis and Discussion

In the 2HDM the $\tau \to \mu Z^*$ transition can be switched on with the internal neutral Higgs bosons h^0 and A^0, and internal leptons e, μ, τ. This brings unknown free parameters $\bar{\xi}^E_{N,\mu j}$ and $\bar{\xi}^E_{N,\tau j}$, $i, j = e, \mu, \tau$ and since these couplings are free parameters of the theory, they should be restricted by using the present and forthcoming experimental limits of physical quantities, such as BR of various leptonic decays and electric dipole moments (EDM), anomalous magnetic moments (AMM) of leptons. In general, these Yukawa couplings are complex and they ensure non-zero lepton EDM. By assuming that only the internal τ lepton contribution is large, the Yukawa couplings which do not contain τ index can be neglected. Such a choice respects the statement that the strength of the couplings are related with the masses of leptons denoted by the indices of them, similar to the Cheng-Sher scenario [111]. Furthermore, we take $\bar{\xi}^E_{N,ij}$ symmetric with respect to the indices i and j. Finally, we are left with the couplings $\bar{\xi}^{E}_{N,\tau\tau}$ and $\bar{\xi}^{E}_{N,\tau\mu}$, which are complex in general.

The measurements of the BRs of $\tau \rightarrow \mu\,\bar{\nu_i}\,\nu_i$, $i = e, \mu, \tau$ decays [110] are based on counting the number of candidate jets and correcting for efficiency and event selection. In addition to this, the backgrounds coming from tau decaying to hadrons or cosmic rays should be detected. In the process, we study the output containing $\bar{\nu_i}\,\nu_i$, $i = e, \mu, \tau$ and the extraction of this output from the most probable one $\bar{\nu_\mu}\,\nu_\tau$ (BR $\sim$ 17.37 $\pm$ 0.09% [112]), which exist in the SM theoretically, is difficult from experimental point of view.

In the present work, we studied the BR of the process $\tau \rightarrow \mu\,\bar{\nu_i}\,\nu_i$, $i = e, \mu, \tau$ and we used the numerical value $\bar{\xi}^{E}_{N,\tau\mu}$ in the interval $5\,GeV < |\bar{\xi}^{E}_{N,\tau\mu}| < 25\,GeV$. Here, the upper limit for the coupling $|\bar{\xi}^{E}_{N,\tau\mu}|$ has been estimated in [95] as $\sim 30\,GeV$. In this work, it is assumed that the new physics effects are of the order of the experimental uncertainty of muon AMM, namely 10^{-9} by respecting the new experimental world average announced at BNL [113]

$$a_\mu = 11\,659\,203\,(8) \times 10^{-10} \;, \tag{5.13}$$

which has about half of the uncertainty of previous measurements. Here we have not used any restriction for the coupling $\bar{\xi}^{E}_{N,\tau\tau}$ except that we choose its numerical value larger compared to $\bar{\xi}^{E}_{N,\tau\mu}$. In addition to this, we expect the upper limit of $\bar{\xi}^{E}_{N,\tau\tau}$ by taking the upper limit of $\bar{\xi}^{E}_{N,\tau\mu}$ and the BR of the process as 10^{-6}.

Table 5.1: The numerical values of the physical parameters used in the numerical calculations.

m_τ	1.78 (GeV)
m_Z	91 (GeV)
m_W	80 (GeV)
s_w	$\sqrt{0}.23$
G_F	$1.16637 \times 10^{-5} (GeV^{-2})$
Γ_Z	2.49 (GeV)
Γ_τ	$2.27 \times 10^{-12}\,(GeV)$

Throughout our calculations we used the input values given in Table (5.1)

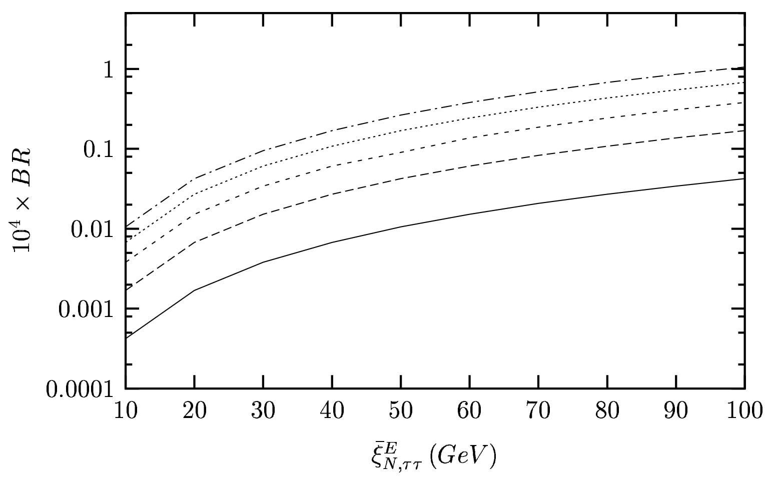

Figure 5.6: $\bar{\xi}^{E}_{N,\tau\tau}$ dependence of the BR for real couplings and $m_{h^0} = 85\,GeV$, $m_{A^0} = 90\,GeV$. Here solid (dashed, small dashed, dotted, dash-dotted) line represents the case for $\bar{\xi}^{E}_{N,\tau\mu} = 5\,GeV$ (10, 15, 20, 25) GeV.

The Fig. 5.6 represents $\bar{\xi}^{E}_{N,\tau\tau}$ dependence of the BR for real couplings. Here solid, dashed, small dashed, dotted, dash-dotted lines represent the cases for $\bar{\xi}^{E}_{N,\tau\mu} = 5, 10, 15, 20, 25\ GeV$, respectively. From this figure, we say that the BR enhances with the increasing values of both couplings and it reaches to values at order of magnitude of 10^{-4}.

Fig. 5.7 shows the possible values of $\bar{\xi}^{E}_{N,\tau\tau}$ and the ratio $\frac{\bar{\xi}^{E}_{N,\tau\mu}}{\bar{\xi}^{E}_{N,\tau\tau}}$ for the fixed values of the BR, $BR=10^{-4}$ (solid line) and $BR=10^{-6}$ (dashed line). For $\frac{\bar{\xi}^{E}_{N,\tau\mu}}{\bar{\xi}^{E}_{N,\tau\tau}}=0.1$, the $BR=10^{-4}$ is obtained if the coupling is $\bar{\xi}^{E}_{N,\tau\tau}\sim 150\,GeV$ and the $BR=10^{-6}$ is obtained if the coupling is $\bar{\xi}^{E}_{N,\tau\tau}\sim 50\,GeV$. The possible experimental search of the process $\tau\rightarrow\mu\,\bar{\nu}_i\,\nu_i$, $i=e,\mu,\tau$ would ensure a strong clue in the prediction of the upper limit of the coupling $\bar{\xi}^{E}_{N,\tau\tau}$.

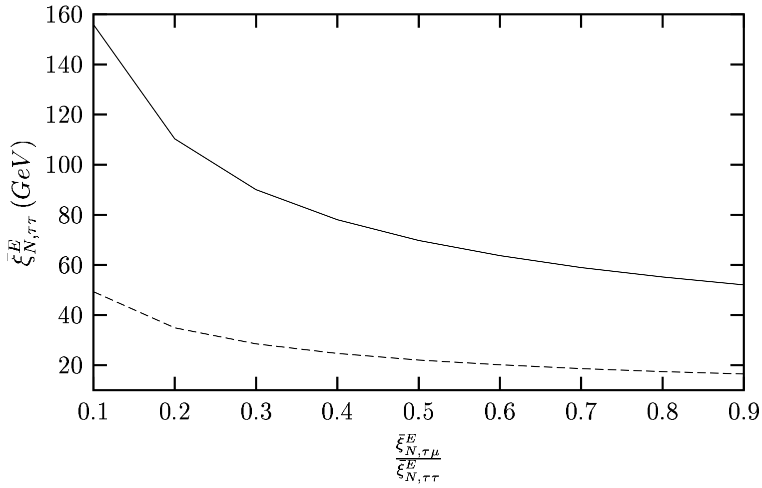

Figure 5.7: $\frac{\bar{\xi}^{E}_{N,\tau\mu}}{\bar{\xi}^{E}_{N,\tau\tau}}$ for $m_{h^0}=85\,GeV$, $m_{A^0}=90\,GeV$ and the fixed values of the BR, $BR=10^{-4}$ (solid line) and $BR=10^{-6}$ (dashed line).

In Fig. 5.8, we present the m_{h^0} dependence of the BR for the fixed values of $\bar{\xi}^{E}_{N,\tau\mu}$ and $\bar{\xi}^{E}_{N,\tau\tau}$, $\bar{\xi}^{E}_{N,\tau\mu}=10\,GeV$, $\bar{\xi}^{E}_{N,\tau\tau}=100\,GeV$. In this figure, we say that the BR is sensitive to m_{h^0} and it decreases with the increasing values of m_{h^0}.

In Fig. 5.9, we take the coupling $\bar{\xi}^{E}_{N,\tau\tau}$ complex $\bar{\xi}^{E}_{N,\tau\tau}=|\bar{\xi}^{E}_{N,\tau\tau}|\,e^{i\theta_{\tau\tau}}$. This figure represents the $\sin\theta_{\tau\tau}$ dependence of the BR for $\bar{\xi}^{E}_{N,\tau\tau}=100\,GeV$ for four different values of $\bar{\xi}^{E}_{N,\tau\mu}$, namely $\bar{\xi}^{E}_{N,\tau\mu}=5,10,15,20\,GeV$ (solid, dashed, small

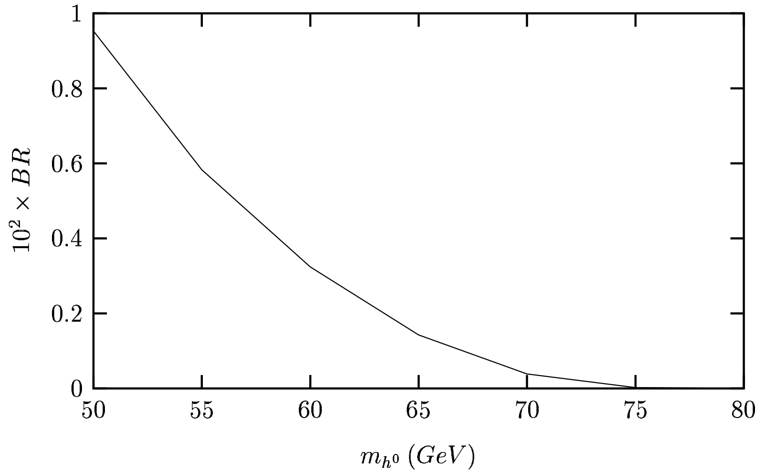

Figure 5.8: m_{h^0} dependence of the BR for the fixed values of $\bar{\xi}^{E}_{N,\tau\mu} = 10\,GeV$, $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$ and $m_{A^0} = 90\,GeV$.

dashed, dotted lines) . From this figure it can be shown that the BR is not sensitive to the complexity of the coupling $\bar{\xi}^{E}_{N,\tau\tau}$.

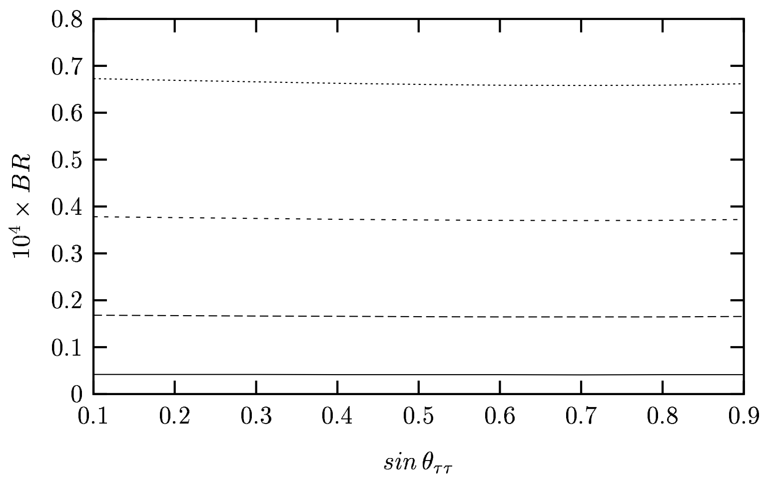

Figure 5.9: The sin $\theta_{\tau\tau}$ dependence of the BR for $m_{h^0} = 85\,GeV$, $m_{A^0} = 90\,GeV$, $\bar{\xi}^{E}_{N,\tau\tau} = 100\,GeV$ and three different values of $\bar{\xi}^{E}_{N,\tau\mu}$, $\bar{\xi}^{E}_{N,\tau\mu} = 5, 10, 15, 20\,GeV$ (dashed, small dashed, dotted lines).

CHAPTER 6

CONCLUSIONS

Although, the SM has passed many experimental tests and has been confirmed even in precision measurements, many physicists believe that it does not represent the final theory. Currently, there are many questions still open in the SM. For instance, within the concepts of the SM, there are no complete understanding about the hierarchy of charged fermion mass spectra and the smallness of neutrino masses, also little is known on the origin of flavor mixing and CP violation. Recently experiments on neutrino physics has suggested that neutrinos should be massive and lepton mixing should be exist. But the theoretical values of the BRs of LFV interactions calculated in the framework of the SM within the massive neutrino are extremely small. So, new physics beyond the SM becomes inevitable for investigation of fermion masses and flavor mixing problems. The LFV interactions and their BRs are being still investigated at many experiments such as MEG experiment [48], MEGA experiment [49], in the Belle detector at the KEKB [51] and BABAR detector at the PEP-II [52, 53]. Furthermore, there are numerous new experiments, planned for the next decade, to investigate electroweak physics and Higgs potential, at LHC and ILC [114]. For instance, at LHC, which will be operational by 2007, a proton-proton collision experiment is already scheduled. Because of the high collision energy that will be available at LHC, it will be possible to have large mass reach for direct discoveries. Also, the ILC, which will bring the electron to collision with positron, will start to take data in the middle of the next decade. Striking features of the ILC are its clean experimental environment, polarized beams, and known collision energy, enabling precision measurement and therefore detailed studies of directly accesible new particles as well as a high sensitivity to indirect effects of new physic. Therefore, the experimental works stimulate the theoretical studies on various decays and the LFV decays beyond the SM are among the important candidates. The LFV interactions are rich from the theoretical point of view since they exist at the loop level and the measurable quantities of these decays carry considerable information about the free parameters of the model used.

Within the general 2HDM, we have analyzed the FCNC processes in the leptonic sector. In the 2HDM, five types of Higgs particles exist, two CP-even boson h^0 and H^0, one CP-odd A^0, and two charged particles $H^{\pm}$. Experimental discoveries of these particles would become a clear signature for the presence of a non-minimal Higgs sector and the physics beyond the SM.

In the present work, we have investigated LFV interactions $H^+ \rightarrow W^+ l_i^- l_j^+$

and $\tau \to \mu \overline{\nu_i} \nu$ decays within the general 2HDM. Under the assumption that CKM type matrix in the leptonic sector does not exist, LFV interactions arise with the help of the neutral Higgs bosons h^0 and A^0, in the general 2HDM. In this model, there are many free parameters, namely, complex Yukawa couplings, $\xi_{ij}^{U,D}$ where i,j are flavor indices, masses of charged and neutral Higgs bosons, $m_{H^\pm}$, m_{h^0}, m_{A^0}. These free parameters should be restricted using the experimental measurements.

First, we would like to summarize our results for the $H^+ \to W^+ l_i^- l_j^+$ processes:

- The decay widths of the $\Gamma(H^+ \to W^+ (\tau^- \mu^+ + \tau^+ \mu^-))$ and $\Gamma(H^+ \to W^+ \tau^- \tau^+$ are in the interval $(10^{-11} - 10^{-5}))\, GeV$ and $(10^{-9} - 10^{-4})\, GeV$, respectively, for $200(GeV) \leq m_{H^\pm} \leq 400(GeV)$, and for the intermediate values of the couplings $\bar{\xi}^{E}_{N,\tau\mu} \sim 5\, GeV$ and $\bar{\xi}^{E}_{N,\tau\tau} \sim 30\, GeV$. With the possible experimental measurement of the processes under consideration, strong clues would be obtained in the prediction of the upper limit of the coupling $\bar{\xi}^{E}_{N,\tau\mu}$ ($\bar{\xi}^{E}_{N,\tau\tau}$). This result is also informative in the determination of the charged Higgs mass, $m_{H^\pm}$.

- We observe that the decay widths $\Gamma(H^+ \to W^+ (\tau^- \mu^+ + \tau^+ \mu^-))$ and $\Gamma(H^+ \to W^+ \tau^- \tau^+)$ are strongly sensitive to the charged Higgs mass, $m_{H^\pm}$.

- We observe that the decay widths $\Gamma(H^+ \to W^+ (\tau^- \mu^+ + \tau^+ \mu^-))$ and $\Gamma(H^+ \to W^+ (\tau^- \tau^+)$ are not sensitive to the possible complexity of the Yukawa couplings.

A possible discovery of charged Higgs bosons would be a solid proof for physics beyond the SM. The discovery of these particles and measurement of their properties would also provide useful information about the structure of the more general theory. Therefore, the experimental and theoretical investigations of these decays of the charged Higgs boson would ensure strong clues in the determination of the physics beyond the SM and the existing free parameters.

Next, we have analyzed the LFV $\tau \rightarrow \mu \overline{\nu_i} \nu_i$ decay in the framework of the general 2HDM. In this model, the $\tau \rightarrow \mu Z^*$ transition can be switched on with the internal neutral Higgs bosons h^0 and A^0, and internal leptons e, μ, τ. This brings unknown free parameters $\xi^E_{N,\mu j}$ and $\xi^E_{N,\tau j}$, $i, j = e, \mu, \tau$ which can be restricted using the experimental measurements. Now, we present the summary of our results:

- We predict the BR at the order of the magnitude of $10^{-6} - 10^{-5}$ for the range of the couplings, $\bar{\xi}^E_{N,\tau\tau} \sim 30 - 100\, GeV$ and $\bar{\xi}^E_{N,\tau\mu} \sim 10 - 25\, GeV$. We predict the upper limit of the coupling for the $h^0(A^0) - \tau - \tau$ vertex as ~ 0.3 in the case that the BR is $\sim 10^{-6}$.

- We observe that the BR is sensitive to the neutral Higgs masses m_{h^0} and m_{A^0}, but it is not sensitive to the possible complexity of the Yukawa couplings.

With the possible experimental measurement of the process $\tau \rightarrow \mu\, \bar{\nu}_i\, \nu_i$, $i =$

e, μ, τ, a considerable information for the upper limit of the coupling $\bar{\xi}^{E}_{N,\tau\tau}$ would be obtained. Notice that, in [95], the upper limit of the coupling $\bar{\xi}^{E}_{N,\tau\mu}$ was predicted by using experimental result of anomalous magnetic moment of muon. As a result, the future theoretical and experimental investigations of the process $\tau \rightarrow \mu\, \bar{\nu}_i \nu_i$ would be informative in the determination of the physics beyond the SM and the free parameters existing in this model.

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APPENDIX A

GLOBAL AND LOCAL GAUGE INVARIANCE

In the Lagrangian field theory formalism, the equations of motion describing the time evolution of a free fermions are defined by the fermion Lagrangian (see for example [115]),

$$L_{free} = \overline{\Psi}(i\gamma^{\mu}\partial_{\mu} - m)\Psi \tag{A.1}$$

where Ψ is the fermion spinor. We can see that such Lagrangian is invariant under global gauge transformation defined as

$$\Psi \rightarrow \Psi' = e^{i\theta}\Psi. \tag{A.2}$$

where θ is space-time invariant parameter. Nevertheless, such Lagrangian is not invariant under local gauge transformation defined by

$$\Psi \rightarrow \Psi' = e^{iq\theta(x)}\Psi. \tag{A.3}$$

Since the physical measurable quantities should not change in local gauge transformations, the equation of motion must be unchanged. As the equation of motion are derived from the Lagrangian, their invariance can be ensured if we require that the Lagrangian of the theory must be invariant under local gauge transformations. We can modify our Lagrangian and make it gauge invariant by replacing the *normal* derivative ∂_μ by the *covariant* derivative $D_\mu \equiv \partial_\mu + iqA_\mu$ where A_μ is a four vector field that transforms as $A_\mu \rightarrow A_\mu + \partial_\mu \theta(x)$. So, the Lagrangian Eq. (A.1) becomes

$$L = \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi = \overline{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - qA_\mu \overline{\Psi}\gamma^\mu \Psi = L_{free} - J^\mu A_\mu. \quad (A.4)$$

It is easy to see that this new Lagrangian is invariant under the combined transformations $\Psi \rightarrow e^{iq\theta(x)}\Psi$ and $A_\mu \rightarrow A_\mu + \partial_\mu \theta(x)$. If A_μ is defined as the four vector electromagnetic potential, then $J^\mu = q\,\overline{\Psi}\gamma^\mu \Psi$ is the four-vector electromagnetic current. With the addition of the local gauge invariant kinetic term that describe the propagation of free photons in the Lagrangian, the Lagrangian of Quantum Electrodynamics (QED) is obtained as

$$L_{QED} = L_{free} - J^\mu A_\mu - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \quad (A.5)$$

where the electromagnetic field tensor $F_{\mu\nu}$ reads

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (A.6)$$

APPENDIX B

SPONTANEOUS SYMMETRY BREAKING AND THE HIGGS MECHANISM

Using the local gauge invariance (see Appendix A for the details) is not enough to predict particle physics phenomenology since it leads to massless gauge bosons that do not correspond to physical reality. If the vacuum of a system (minimum of the potential) is degenerate, this minimum is not invariant under the symmetry of the Lagrangian. When the vacuum does not have the symmetry of the Lagrangian, we can say that the symmetry has been spontaneously broken. After this phenomenon, some other massless particles, called Goldstone bosons, appear in the spectrum. However, if the Lagrangian possesses a local gauge symmetry an interrelation among gauge and Goldstone bosons endows the former with a physical mass, while the latter disappear from the spectrum, this phenomenon is called the *Higgs mechanism* [116]. To explain the mechanism we shall use a toy model describing a couple of self interacting complex scalar fields ϕ and ϕ^*. The local gauge invariant Lagrangian is defined as

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\left|D^{\mu}\phi\right|^2 - V\left(\phi^*\phi\right) \,, \tag{B.1}$$

where $\phi = \phi_1 + i\phi_2$ is a complex field and

$$D_\mu \equiv \partial_\mu + iqA_\mu \quad ; \quad F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \,. \tag{B.2}$$

This Lagrangian has already the local gauge invariance described by simultaneous transformations

$$\phi(x) \to e^{-iq\theta(x)}\phi(x) \quad ; \quad A_\mu(x) \to A_\mu(x) + \partial_\mu\theta(x) \,. \tag{B.3}$$

The potential term of the Lagrangian is defined as

$$V\left(\phi\right) \equiv -\frac{1}{2}\mu^2\left|\phi\right|^2 + \frac{1}{4}\lambda^2(\phi^*\phi)^2 \,, \tag{B.4}$$

where the values of λ^2 must be positive to keep the energy of the vacuum bounded from below, but the value of μ^2 can be either positive or negative [7]. If $\mu^2 < 0$, the potential $V\left(\phi\right)$ posseses a unique minimum at $\phi = 0$ which preserves the symmetry of the Lagrangian. However, if $\mu^2 > 0$, the Lagrangian has a continuum degenerate set of vacuum lying on a circle of radius μ/λ

$$\langle\left|\phi\right|^2\rangle = \langle\phi_1\rangle^2 + \langle\phi_2\rangle^2 = \frac{\mu^2}{\lambda^2} \equiv \nu^2 \,, \tag{B.5}$$

any of them might be chosen as the fundamental state, but no one of them is invariant under local phase rotation. According to the definition made above, the symmetry of the Lagrangian has been spontaneously broken. Choosing a particular minimum:

$$\langle \phi_1 \rangle = \frac{\mu}{\lambda} \equiv \nu \;\; ; \;\; \langle \phi_2 \rangle = 0\,, \tag{B.6}$$

$\langle \phi_1 \rangle$ has acquired a Vacuum Expectation Value (VEV) . It is convenient to introduce new fields [74]

$$\eta \equiv \phi_1 - v \;\; ; \;\; \xi \equiv \phi_2\,, \tag{B.7}$$

and expanding the Lagrangian in terms of these new fields we obtain:

$$\begin{aligned} L \;&=\; \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2 \eta^2\right] + \frac{1}{2}\left[(\partial_\mu \xi)(\partial^\mu \xi)\right] + \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2 v^2}{2}A_\mu A^\mu\right] \\ &\quad -2iqv\left(\partial_\mu \xi\right)A^\mu + \Big\{ q\left[\eta\left(\partial_\mu \xi\right) - \xi\left(\partial_\mu \eta\right)\right]A^\mu + vq^2\left(\eta A_\mu A^\mu\right) \\ &\quad + \frac{q^2}{2}\left(\xi^2 + \eta^2\right)A_\mu A^\mu - \lambda\mu\left(\eta^3 + \eta\xi^2\right) - \frac{\lambda^2}{4}\left(\eta^4 + 2\eta^2\xi^2 + \xi^4\right)\Big\} \\ &+\; \frac{\mu^2 v^2}{4}\,. \end{aligned} \tag{B.8}$$

In this Lagrangian we can say that the particle spectrum consists of three fields: a vector boson A_μ with mass qv , a field η with mass $\sqrt{2}\mu$ and a massless field ξ called a Goldstone boson. However, this Lagrangian obtain some non-physical term such that $(\partial_\mu \xi)A^\mu$ which does not have an interpretation in the Feynman formalism. In order to remove the unwanted terms, Eq. (B.3) can be redefined in terms of ϕ_1 and ϕ_2

$$\phi \rightarrow e^{i\theta(x)}\phi = \left[\phi_1 \cos\theta\left(x\right) - \phi_2 \sin\theta\left(x\right)\right] + i\left[\phi_1 \sin\theta\left(x\right) + \phi_2 \cos\theta\left(x\right)\right] \tag{B.9}$$

where $\theta\,(x) \equiv -q\lambda\,(x)$, and

$$\theta\,(x) = -\arctan\left(\frac{\phi_2\,(x)}{\phi_1\,(x)}\right) . \tag{B.10}$$

Using the Eqs. (B.3), (B.9) and (B.10), the Lagrangian is obtained as,

$$\begin{aligned} L &= \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2\right] + \left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{q^2v^2}{2}A_\mu A^\mu\right] \\ &\quad + \left\{q^2 v\,(\eta A_\mu A^\mu) + \frac{q^2}{2}\eta^2 A_\mu A^\mu - \lambda\mu\eta^3 - \frac{\lambda^2}{4}\eta^4\right\} + \frac{\mu^2 v^2}{4}. \end{aligned} \tag{B.11}$$

So, we have got rid of the massless field ξ and all its interactions in the Lagrangian. On the other hand, we are left with a massive scalar field η which is a Higgs particle and a massive four vector field A_μ .

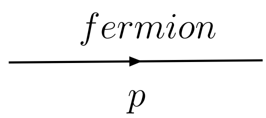
Notice that, in the SM, the Higgs mechanism creates three massive vector bosons $(W^\pm, Z)$ and one massless vector boson (the photon).

APPENDIX C

THE PROPOGATORS AND THE VERTICES

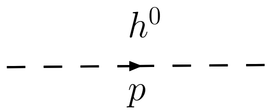
In order to calculate amplitude of decays, we used basic propagators and some vertex factors. The propagators which we used in the text are defined as follows:

- the fermion propagator is



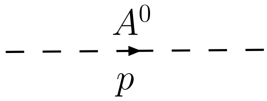
$$\frac{i}{\not{p} - m}$$

- h^0 Higgs boson propagator is



$$\frac{i}{p^2 - m_{h^0}^2}$$

- A^0 Higgs boson propagator is

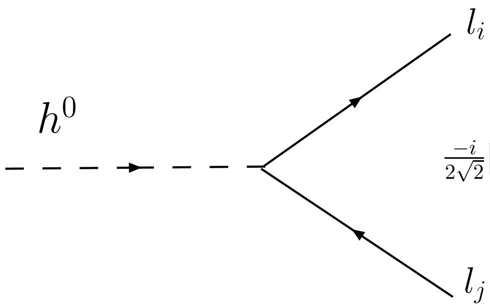


$$\frac{i}{p^2-m_{A^0}^2}$$

Figure C.1: Propogators.

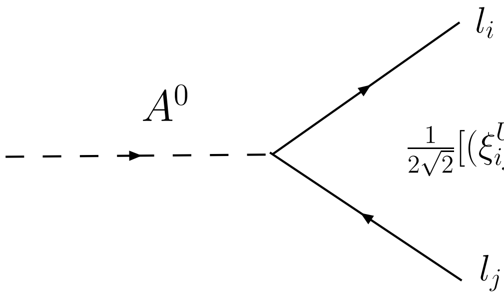
The vertices used in the text read:

- h^0 - l_j - $\bar{l}_i$ interaction



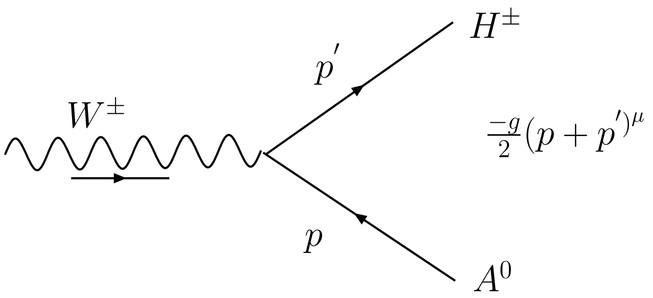
$$\frac{-i}{2\sqrt{2}}[(\xi_{ij}^{U,D}+\xi_{ji}^{U,D*})+(\xi_{ij}^{U,D}-\xi_{ji}^{U,D*})\gamma_5]$$

- A^0 - l_j - $\bar{l}_i$ interaction

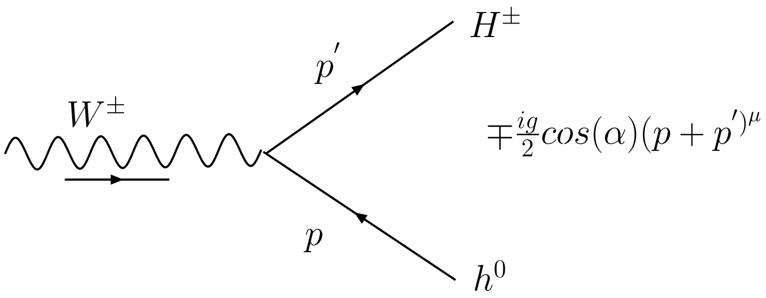


$$\frac{1}{2\sqrt{2}}[(\xi_{ij}^{U,D}-\xi_{ji}^{U,D*})+(\xi_{ij}^{U,D}+\xi_{ji}^{U,D*})\gamma_5]$$

- $W^{\pm}$ - $H^{\pm}$ - A^0 interaction

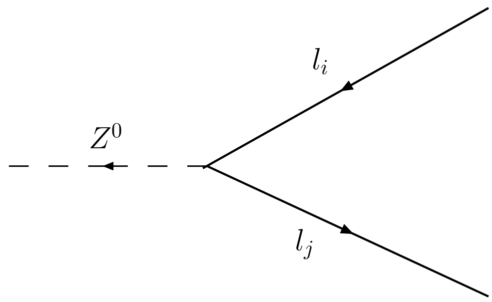


- $W^{\pm}$ - $H^{\pm}$ - h^0 interaction



Notice that we take that there is no mixing between CP-even Higgs bosons H^0 and h^0 and we choose the corresponding mixing angle $\alpha = 0$ in our work.

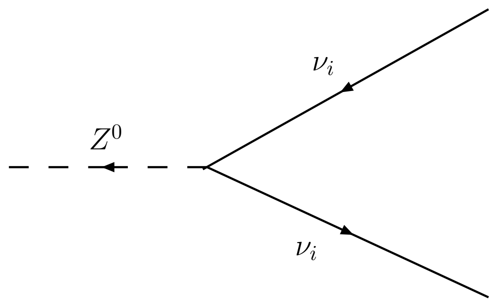
- Z^0 - l_i - l_j interaction



$\frac{-ig}{\cos\theta_w}\gamma^\mu(c_L L + c_R R)$

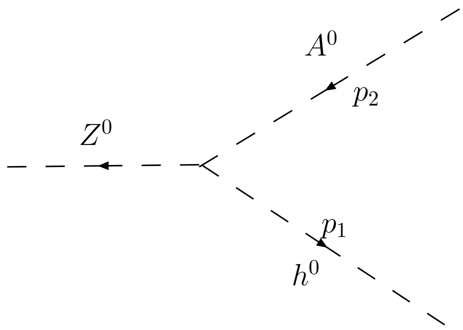
where $g = \frac{e}{\sin\theta_w}$, $c_L = \frac{-1}{2} + \sin^2\theta_w$, $c_R = \sin^2\theta_w$.

- Z^0 - ν_i - ν_i interaction



$\frac{-ig}{2\,\cos\theta_w}\gamma^\mu\, L$

- Z^0 - A^0 - h^0 interaction



$\frac{g}{2\cos\theta_w}(p_1 + p_2)^\mu$

Figure C.2: Vertices.

APPENDIX D

THE FEYNMAN PARAMETRIZATION AND DIMENSIONAL REGULARIZATION

D.1 The Feynman Parametrization

The Feynman Parametrization is very useful to calculate the loop integrals and the idea behind is that the different denominators are combined into a single denominator and the combined denominator is reduced to standard form by translating the internal loop momentum. The most general form of the Feynman Parametrization is

$$\Pi_{i=1}^{n} \frac{1}{A_i^{\alpha_j}} = \frac{\Gamma(\alpha)}{\Pi_{i=1}^{n}\Gamma(\alpha_i)} \int_0^1 (\Pi_{i=1}^{n} dx_i x_i^{\alpha_j - 1}) \frac{\delta(1-x)}{(\sum_{i=1}^{n} x_i A_i)^{\alpha}}, \tag{D.1}$$

where $\alpha_i (i = 1, 2, ..., n)$ are arbitrary complex numbers and $\Gamma(\alpha_i)$ is the gamma function. $\alpha \equiv \sum_{i=1}^{n} \alpha_i$ and $x \equiv \sum_{i=1}^{n} x_i$. With the above given formula, multiplication of propagators appearing during the calculation of the loop integrals are parametrized in the most appropriate form i.e., in case of two and three multiplication we have

$$\begin{aligned}
\frac{1}{AB} &= \int_0^1 \frac{dx}{[Ax + B(1-x)]^2}, \\
\frac{1}{ABC} &= 2\int_0^1 dx \int_0^{1-x} \frac{dy}{[Ay + B(1-x-y) + Cx]^3}.
\end{aligned} \tag{D.2}$$

D.2 The Dimensional Regularization

The regularization is a method of dealing with infinite and divergent expressions by introducing an auxiliary concept of a regulator. The correct physical result is obtained in the limit in which the regulator goes away, but the virtue of the regulator is that for finite value, the result is finite. Regularization is only a mathematical method and has no physical consequences. We have several regularization schemes. In our calculation, we used the dimensional regularization. The dimensional regularization is a particular way to get rid of infinities that occur when one evaluates Feynman diagrams. The basic idea of the dimensional regularization is that the spacetime dimension d is lower than four and any loop-momentum integral will converge for sufficiently small d. So momentum integral become

$$\int \frac{d^4q}{(2\pi)^4} \Rightarrow \mu^{4-d} \int \frac{d^dq}{(2\pi)^d}, \tag{D.3}$$

where μ is an arbitrary mass parameter.

Some Dirac algebra in d-dimension are given as:

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu},$$

$$\begin{aligned}
\gamma^{\mu}\gamma_{\mu} &= d, \\
\gamma_{\mu}\gamma_{\nu}\gamma^{\mu} &= (2-d)\gamma_{\nu}, \\
g^{\mu}_{\mu} &= g_{\mu\nu}g^{\mu\nu} = d.
\end{aligned} \tag{D.4}$$

After that, the one loop momentum integrals can be calculated with the help of the Feynman Parametrization and the dimensional regularization from the references [117]-[118]:

$$\begin{aligned}
\int \frac{d^d l}{(2\pi)^d}\frac{1}{(l^2-\Delta)^n} &= \frac{(-1)^n i}{(4\,\pi)^{d/2}}\frac{\Gamma(n-d/2)}{\Gamma(n)}(\frac{1}{\Delta})^{n-d/2}, \\
\int \frac{d^d l}{(2\pi)^d}\frac{l^2}{(l^2-\Delta)^n} &= \frac{(-1)^{n-1} i}{(4\,\pi)^{d/2}}\frac{d}{2}\frac{\Gamma(n-d/2-1)}{\Gamma(n)}(\frac{1}{\Delta})^{n-d/2-1}, \\
\int \frac{d^d l}{(2\pi)^d}\frac{l_\mu l_\nu}{(l^2-\Delta)^n} &= \frac{(-1)^{n-1} i}{(4\,\pi)^{d/2}}\frac{g_{\mu\nu}}{2}\frac{\Gamma(n-d/2-1)}{\Gamma(n)}(\frac{1}{\Delta})^{n-d/2-1}, \\
\int \frac{d^d l}{(2\pi)^d}\frac{(l^2)^2}{(l^2-\Delta)^n} &= \frac{(-1)^n i}{(4\pi)^d/2}\frac{d(d+2)}{4}\frac{\Gamma(n-d/2-1)}{\Gamma(n)}\Delta^{-n+\frac{d}{2}+2}, \\
\int \frac{d^d l}{(2\pi)^d}\frac{l_\mu l_\nu l_\rho l_\sigma}{(l^2-\Delta)^n} &= \frac{(-1)^n i}{(4\pi)^d/2}\frac{\Gamma(n-d/2-2)}{\Gamma(n)}\Delta^{-n+\frac{d}{2}+2} \\
&\times \frac{1}{4}(g_{\mu\nu}g_{\rho\sigma}+g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}).
\end{aligned} \tag{D.5}$$

where $d = 4-\epsilon$ in the $\epsilon \to 0$ limit. At this stage, we consider special case $n = 2:$

$$\begin{aligned}
(\frac{1}{\Delta})^{2-\frac{d}{2}} &= 1-\frac{\epsilon}{2}lnL+...., \\
\Gamma(2-\frac{d}{2}) &= \frac{2}{\epsilon}-\gamma+O(\epsilon).
\end{aligned} \tag{D.6}$$

Finally, we get

$$\int \frac{d^d l}{2\pi^d}\frac{1}{(l^2-\Delta)^n} = i\Big(\frac{2}{\epsilon}-(ln\Delta+\gamma-ln4\pi)+O(\epsilon)\Big) \tag{D.7}$$

where $\gamma = 0.5772$ is the Euler-Mascheroni constant. $\frac{2}{\epsilon}$ term is infinite when $\epsilon \to 0$. We can get rid off infinities by a renormalization procedure. However, sum of the all possible diagrams contributing the process considered make the coefficient of $\frac{1}{\epsilon}$ zero.

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