TERM STRUCTURE OF GOVERNMENT BOND YIELDS: A MACRO-FINANCE APPROACH

HALİL ARTAM

SEPTEMBER 2006

TERM STRUCTURE OF GOVERNMENT BOND YIELDS: A MACRO-FINANCE APPROACH

A THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF APPLIED MATHEMATICS

OF

THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

HALİL ARTAM

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN

THE DEPARTMENT OF FINANCIAL MATHEMATICS

SEPTEMBER 2006

Approval of the Graduate School of Applied Mathematics

Prof. Dr. Ersan AKYILDIZ
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Hayri KÖREZLİOĞLU
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. Kasırga YILDIRAK
Supervisor

Examining Committee Members

Assoc. Prof. Dr. Azize HAYFAVİ

Assoc. Prof. Dr. Gül ERGÜN

Assist. Prof. Dr. Kasırga YILDIRAK

Assist. Prof. Dr. Hakan ÖKTEM

Dr. Coşkun KÜÇÜKÖZMEN

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Halil ARTAM

Signature :

Abstract

Term Structure Of Government Bond Yields: A Macro-Finance Approach

Halil Artam
M.Sc., Department of Financial Mathematics
Supervisor: Assist. Prof. Dr. Kasırga Yıldırak

SEPTEMBER 2006, 74 pages

Interactions between macroeconomic fundamentals and term structure of interest rates be stronger according to the way of changes in structure of world-wide economy. Combined macro-finance analysis determines the joint dynamics of term structure of interest rates and macroeconomic fundamentals. This thesis provides analysis of two existing macro-finance models and an original one. Parameter estimations for these three macro-finance term structure models are done for monthly Turkish data by use of an efficient recursive estimator Kalman filter. In spite of the small scale application the results are satisfactory except first model but with longer sets of macroeconomic variables and interest rate data models provide more encouraging results.

Keywords: Term structure, interest rates, macroeconomic fundamentals, Kalman filter.

Öz

DEVLET TAHVİLİ GETİRİLERİ VADE YAPISI: MAKRO-FİNANS YAKLAŞIM

Halil Artam

Yüksek Lisans, Finansal Matematik Bölümü

Tez Yöneticisi: Yrd. Doç. Dr. Kasırga Yıldırak

Eylül 2006, 74 sayfa

Dünya ekonomisindeki yapısal değişiklikler doğrultusunda, temel makroekonomik göstergeler ve faiz haddi vade yapıları birbirlerine daha güçlü tesir etmeye başlamıştır. Birleşmiş makro-finans analizi faiz haddi vade yapıları ile temel makroekonomik değişkenlerin birleşmiş dinamiklerini açıklamaktadır. Bu çalışma varolan iki makro-finans modelini ve yeni önerdiğim modelin analizini ortaya koymaktadır. Bahsi geçen üç makro-finans vade yapısı modellerinin parametre tahminleri Kalman filtrelemesi kullanılarak aylık Türkiye verisiyle geçekleştirilmiştir. Küçük ölçekli veriler kullanılmasına rağmen sonuçların birinci model dışında gözlenilen getiri eğrisini iyi ifa etmesi, daha uzun makroekonomik değişkenler ve faiz veri kümeleri kullanılarak yapılacak olan çalışmalarda sonuçların daha iyi ifa eder şekilde çıkacağı beklentisini artırmıştır.

Anahtar Kelimeler: Faiz haddi, vade yapısı, temel makroekonomik göstergeler, Kalman filtresi.

To my family

Acknowledgments

I am grateful to my parents for always supporting, encouraging and motivating me throughout my life.

I acknowledge financial support for TUBÍTAK-BAYG through my graduate academic life.

I want to thank Assist. Prof. Dr. Kasırga Yıldırak who shows me the way and guide me to end up this study.

I am grateful to Prof. Dr. Hayri Körezlioğlu, Assoc. Prof. Dr. Azize Hayfavi sharing out their experience and knowledge with me.

Special thanks for Dr. Coşkun Küçüközmen and Prof. Dr. Gerhard Wilhelm Weber for helpful suggestions during the correction of this study.

I want to thank Írge Bulunç for patiently supporting and motivating me throughout this study.

Finally I want to thank to the members of the Institute of Applied Mathematics and all of my friends specially Tolga Aktük, Şirzat Çetinkaya, Korhan Nazlıben and Sühan Altay for their support.

Table of Contents

List of Figures

List of Tables

Chapter 1

INTRODUCTION

The term structure of interest rates is commonly cited in literature as an indicator of monetary policy especially for economic activity and inflation. Having a bidirectional characterization, term structure of interest rates is sensitive and responds to macroeconomic shocks. Hence, understanding the stochastic behaviour of term structure of interest rates in the light of macroeconomic evolution is very important to decide about monetary policy transmission.

The monetary authority to decide how to conduct the monetary policy monetary policy to achive the goals of economic stabilization is the central bank and short term interest rates are a key instrument to realize a powerful monetary policy under the control of central banks. The most cited theory to explain the determinants of the shape of the term structure of interest rates is expectation hypothesis theory. This theory argues that yields with longer term interest rates are equal to the average of expected future short term interest rates. According to the expectation hypothesis short term interest rates are a building block for longer term rates. These two properties imply that the short term interest rates is a critical intersection point between finance and macroeconomy.

The financing of public debt, expectations of real economic activity and inflation, interest rate risk management of a portfolio including interest rate sensitive instruments and valuation of interest rate sensitive derivatives are other reasons which make the importance of understanding term structure greater. These importances of understanding the behaviour of the term structure of interest rates force someone modeling and describing the term structure of interest rates. By fitting a model to an available interest rate data, both financial economists and macroeconomists would like to discover the dynamics of interest rates.

Financial economists try to develop models for pricing and forecasting interest rate related securities and these models are based on the assumption of no arbitrage. They are also called traditional models and focusses on jointly modeling the entire yield curve and propose the existence of unobserved latent risk factors or a few linear combination of interest rates with different maturities. While fitting these models is rather good since they could not provide relationships between term structure and macroeconomic fundamentals. The pioneers of this literature are Vasicek (1977), Cox Ingersoll and Ross (1985), Duffie and Kan (1996), Duffie and Singleton (1997), Dai and Singleton (2000, 2002), Duffee(2002), Knez Litterman and Scheinkmann (1994). [43] and [10] used dynamics of short term interest rate as the underlying state factor. Rest of papers commonly study with the principal components of yield curve as state factors and try to determine the shape of the yield curve by means of these principal components. [34] use principal component analysis to present factors affecting the yield curve in terms of the general level of interest rate, the slope of the yield curve and its curvature. These three latent factors are often interpreted as level, slope and curvature according to their effects on the yield curve following [34].

On the other hand macroeconomists try to draw the triangle between interest rates, monetary policy and macroeconomic fundamentals without any restrictions about absence of arbitrage and based on expectation hypothesis.

As an alternative approach, statistical techniques use to determine the shape of the yield curve usually do not consider the factors driving it. The pioneer of this approach, Nelson and Siegel (1987), have relatively few parameters and succeed fairly well in capturing the overall shape of the yield curve. Although they can mimic hump-shaped yield curves, they could not do so for spoon-shaped ones. Thus, in general, it is not possible to match arbitrary term structures with sufficient precision. Spline techniques, kernel methods and parametric classes such as Nelson-Siegel family of curves are most widely used estimation methods in statistical techniques approach.

In the beginning of the second millenium, the need to bridge the gap between macroeconomic and term structure models, a joint characterization of macroeconomy and term structure by adding observable macroeconomic variables to the latent factors which is presented by traditional term structure modelling literature became necessary. Macro-finance models present jointly developing dynamics of interest rates and macroeconomic variables which reciprocally interact between each other. The joint macro-finance modelling strategy provides the most comprehensive understanding of the term structure as Diebold Rudebusch and Aruoba (2006) argued.

This thesis provides detailed explanations about two existing joint macro-finance models and give empirical results by use of zero coupon Turkish government bond yields and related macroeconomic variables. One of the pioneering papers of this joint macro-finance modeling strategy literature is Ang and Piazzesi (2003). It offers a no arbitrage vector autoregression framework. [17] can be given as a second paper which offers a Nelson Siegel based representation of macro finance modeling in state space form. Comments about advantages and disadvantages of these two models and the empirical results for Turkish data will appear in following chapters. In the final an original macro-finance model based on Vasicek's term structure solution and forward-looking Taylor Rule has been proposed.

The thesis proceed as follows. In Chapter 2 preliminaries about interest rates and yield curves are provided and the most cited term structure theory, expectations hypothesis is discussed. Chapter 3 provides the most popular approaches for term structure modelling, equilibrium and no-arbitrage approaches. It presents the types of these two approaches and give explanations about them. A detailed description about the framework of two existing and the original macro-finance models is provided in Chapter 4. The data description and empirical analysis of these three models appear in Chapter 5 by use of zero coupon Turkish government bond yields and related macroeconomic variables. A brief explanation about the Kalman filter is also included in this chapter since it is used to estimate the models. In the final chapter conclusion and further suggestions are provided.

Chapter 2

PRELIMINARIES

2.1 ESSENTIALS ABOUT YIELD CURVE

Brief explanation about some key relationships between interest rates and term structure needs to be given since they are frequently used throughout the thesis.

A risk free zero coupon bond, a bond with no coupon payments, is a building block in fixed-income analysis. The price at time t for a bond mature at time T will be denoted by $P(t, T)$ for any $t < T$ as a function of the current time t and maturity time T where $P(T, T) = 1$. A zero coupon bond of maturity T pays to its holder one unit of cash at a pre-specified date T in the future. The term structure of interest rate estimation from bond market data is just a simple set of calculations.

The yield to maturity (YTM), is the continuously compounded rate of return that causes the market price of bond $P(t, T)$ to be equal to the present value of the future cash flows. The yield to maturity is denoted by $Y(t, T)$ and the price of bond at time t is given in terms of yield to maturity as

$$P(t, T) = e^{-(T-t)Y(t,T)}, \quad \forall t \in [t, T]. \tag{2.1.1}$$

The term structure of interest rates or the yield curve is the function that presents the relationship between yields $Y(t,T)$ and their maturities T. It is obvious that for arbitrary fixed maturity date T, there is a one-to-one correspondence between the bond prices process $P(t,T)$ and its yield to maturity process $Y(t,T)$ from (2.1.1). Given the yield to maturity process $Y(t,T)$, the corresponding bond price process $P(t,T)$ is uniquely determined by following formula

$$Y(t,T) = -\frac{ln(P(t,T))}{T-t}. \tag{2.1.2}$$

The initial term structure of interest rates may be represented either by the current bond prices with different time to maturity or by the initial yield curve $Y(0,T)$ as

$$P(0,T) = e^{-T\times Y(0,T)} \quad \forall T \in [0,T]. \tag{2.1.3}$$

Instantaneous interest rate is an important and useful instrument in modelling interest rates and denoted by $r(t)$. The limit of yield to maturity as $T \to t$ gives the instantaneous short rate as follows

$$r(t) = \lim_{T\to t} Y(t,T). \tag{2.1.4}$$

In reality, the instantaneous short term interest rate does not exist, it is just a theoretical construct used to make the modelling process easier since the most cited traditional stochastic interest rate models are based on the specification of a short term rate of interest.

The rate that can be agreed upon at time t for a risk-free loan starting at time T_1 and finishing at time T_2 is called the forward rate and it is denoted by $f(t,T_1,T_2)$

$$f(t,T_1,T_2) = \frac{\ln P(t,T_1) - \ln P(t,T_2)}{T_2 - T_1} \tag{2.1.5}$$

and the instantaneous forward rate is

$$f(t,T) = f(t,T,T). \tag{2.1.6}$$

It is the rate that one contracts at time t for a loan starting at time T for an instantaneous period of time. We have

$$f(t,T) = -\frac{\partial P(t,\tau)}{\partial \tau}|_{\tau=T} = -\frac{1}{P(t,T)}\frac{\partial P(t,T)}{\partial T} \tag{2.1.7}$$

under the assumption that the bond prices are differentiable. The bond price can be obtained in terms of forward rates as

$$P(t,T) = e^{-\int_t^T f(t,s)ds} \tag{2.1.8}$$

therefore we can write $r(t) = f(t,t)$.

The yield curve for any given day can be obtained from the daily prices of interest rate instruments. Yields using throughout this study are obtained by using zero coupon bond prices with different time to maturities. The relationship between yield on zero coupon bond and maturity is referred as the term structure of interest rates. Since there is a one-to-one correspondence between yields and bond prices, studying yield curve is equivalent with studying bond valuation.

Examination of the term structure of interest rate is crucial in the analysis of all interest rate sensitive securities, forecasting the future interest rate, pricing fix payment contracts, hedging for portfolios include interest rate sensitive instruments, having arbitrage between bonds with different maturities and figuring expectations about the future path of the economy.

The shape of the yield curve can be used when analyzing the evolution of the term structure of interest rate over time. The shape of the yield curve may be a flat one which means longer term rates are almost the same as shorter ones or upward sloping which means longer term rates are higher than shorter ones or downward sloping which says short term interest rate fluctuate more than long term rates.

An understanding of the stochastic behavior of yields is important for four reasons. The first one is the conduct of monetary policy since central banks seem to be able to move the short end of the yield curve to achieve their economic stabilization target and the long term rates are the average of expected future short rates at least after an adjustment of risk. A powerful model of the yield curve helps understanding the relationship between long and short term rates and deciding adjustment of short rate to manage a powerful monetary policy transmission. Forecasting is the second important aspect of understanding the yield curve. The current term structure contains information about the future path of the economy since long term interest rates are conditional expected values of future short rates. Thirdly yield curve can be given as constituting a debt strategy. Governments need to decide about the maturity of bonds when they issuing new debts. In this manner, it is important to consider how various issuance strategies perform under different interest rate outcomes. The last but not the least understanding the yield curve helps better risk management and derivative pricing. To manage risks like paying short term interest rates on deposits while receiving long term interest rates on loans, banks need to decide and compute hedging strategies with respect to changing state of the economy. The changing state of the economy affects the pricing processes for interest rate sensitive instruments.

2.2 EXPECTATIONS HYPOTHESIS

A number of theories have developed to understand the behavior of the term structure of interest rates. The most cited theories are based on market expectations hypothesis and generally the theory of term structure is called expectations hypothesis. Expectations hypothesis has an important role in the analysis of the term structure of interest rates. The hypothesis suggest that the continuously compounded zero coupon bonds with maturity T, $Y(t,T)$ equals the average of the current and expected future short rates $r(t+k)$ for $0 < k <$

$T - t$ plus a maturity specific constant:

$$Y(t, T) = \frac{1}{T} \sum_{k=0}^{T-t-1} E_t[r(t+k)] + \alpha_T. \tag{2.2.9}$$

The expectation hypothesis argue that the current term structure says something about investors expectations of future interest rates and by use of these information someone guess what actual future rates might be. The expectations hypothesis predict that when the expected future short rates are falling, the yield curve will slope downward and when we expect them to rise, the curve will be slope upward. Another alternative scenario concludes that the upward sloping yield curve can be given as uncertainty about long term bond yields which makes them systematically less attractive to lenders than the short term bonds. If it is certain that the short term rate will remain constant, lenders should be indifferent between lending on short or long term bonds with respect to expectation hypothesis. These two distinct scenarios give the same result shows the excellence flexibility of expectations hypothesis. The alternative scenario for the upward sloping of the yield curve based on uncertainty about future interest rates and this says that the expectation hypothesis is exactly true in a certain world by the force of no-arbitrage but in a stochastic world uncertainty causes systematic distortion of expectation hypothesis.

Chapter 3

TERM STRUCTURE MODELLING

An interest rate model characterizes the uncertainty on future interest rates based on today's information. The last three decades witnessed significant developments in term structure modelling. Conducting of monetary policy, forecasting for the future path of the economy, public debt policy and the risk management of a portfolio of interest rate sensitive securities make understandable the term structure of interest rate crucial. There are various types of approaches trying to model the term structure of interest rates. The most popular approaches are equilibrium, no-arbitrage models and functional form models.

Most of the one-factor interest rate models take the short rate as the basis for modelling the term structure of interest rate. The first interest rate models, equilibrium models, were not offered to fit an arbitrary initial term structure. The equilibrium approach focus on posit an endogenously specified term structure of interest rate under various assumptions about economic equilibrium and absence of arbitrage are based on a given market price of risk and other parameters governing collective expectations by using typically affine models. The equilibrium approach derives a stochastic process from the short term rate in a risk neutral world, typically using affine models, and finally contributes and decides about effects of these process on interest rate claims. A detailed information about affine models is given in [39]. This approach derive the term structure in models with consumer maximization and, occasionally, production functions.

In contrast with equilibrium approach, no arbitrage framework focuses on perfectly fitting the term structure at a given point in time to ensure that no arbitrage possibilities exist and try to choose parameters to determine the behavior of the term structure of interest rate in future under assumption of risk neutral scenario. Figure 3.1 shows the difference between these two term structure modelling approach. The idea is to write a plausible mathematical description of the term structure which is numerically tractable. Construction of no-arbitrage models imply that they are exactly consistent with the current term structure.

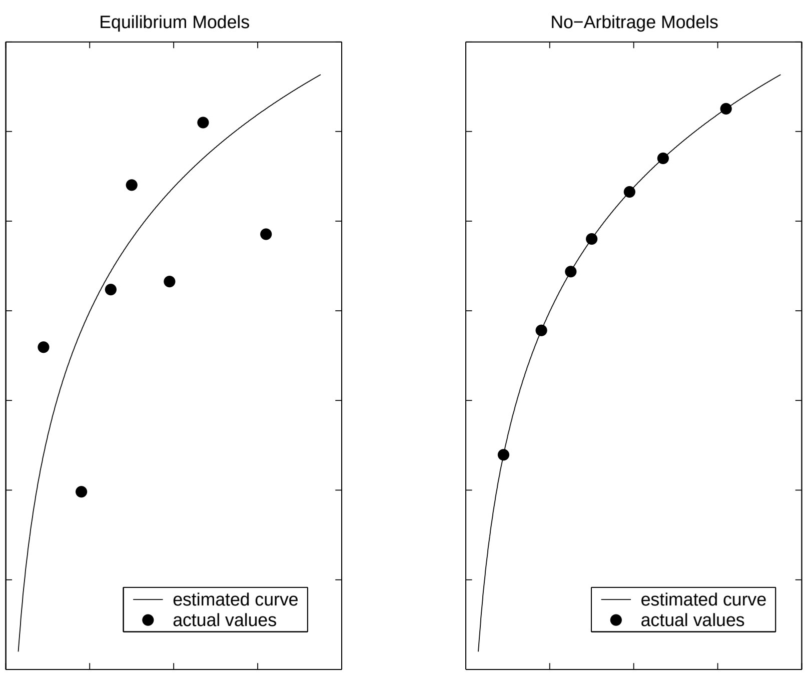

Figure 3.1: Estimation of Short Rate with Equilibrium and No-Arbitrage Models

3.1 EQUILIBRIUM MODELS

A general representation for one factor models is given by

$$dr(t) = \mu(t, r)dt + \nu(t, r)dW_t, \tag{3.1.1}$$

where r is the short rate, $\mu(t, r)$ and $\nu(t, r)$ represent the average rate of change or instantaneous drift and variance of the short rate process respectively. Drift term is the deterministic part and variance is the stochastic part of the short rate process. Furthermore dW_t represents an increment in a Wiener process over a small time interval dt. The value of a discount bond with maturity T for $t < T$ can be given as the expectation of the payoff discounted at the future levels of the short rate in following form

$$P(t, T) = E_t[e^{-\int_t^T r(s)ds}], \tag{3.1.2}$$

where E_t imply the conditional expected value in the risk neutral world on information available up to time t. The behavior of the term structure of interest rates can be determined by the short rate process under the assumption of no-arbitrage.

Although classical models are motivated by their analytical and mathematical tractability, if a so called market risk premium is not included, they have a weak ability to describe the real data. Absence of arbitrage restriction is the main reason creating this drawback. The equilibrium models do not tell us anything useful about the real world dynamics of interest rates since they assumed a risk neutral scenario.

3.1.1 Vasiček Model

[43] presented one of the first term structure model based on no-arbitrage considerations. This model assumes a risk neutral scenario and offer a model to determine the instantaneous short rate dynamics as an Ornstein-Uhlenbeck

process in following form:

$$dr(t) = \kappa(\theta - r(t))dt + \sigma dW_t \tag{3.1.3}$$

where κ, θ, σ are strictly positive constants W_t is a standard Wiener process and movements of the instantaneous short rate follow a Brownian motion, which indicates that the spot interest rate is a continuous stochastic process and the difference of two interest rates with different maturities has mean zero and variance exactly equal to the maturity gap κ. The Vasiček model says that the instantaneous short rate follows a trend with a mean reverting characteristic. If $r(t)$ goes over θ, then $r(t)$ tends to adjust with a speed of κ to its average long term level θ.

Vasiček model is popular because of its analytical and mathematical tractability. Integrating the stochastic differential equation (3.1.3) we get the expression for the instantaneous interest rate as

$$r(t) = r(s)e^{-\kappa(t-s)} + \theta(1 - e^{-\kappa(t-s)}) + \sigma \int_s^t e^{-\kappa(t-u)} dW(u) \tag{3.1.4}$$

for $s < t$. The instantaneous short rate $r(t)$ is normally distributed for the given set of information at time s:

$$r(t) \mid \mathcal{F}_s \sim N(\theta + (r(s) - \theta)e^{-\kappa(t-s)}, \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa(t-s)})). \tag{3.1.5}$$

The prices of a discount bond solves the following partial differential equation under all of the above assumptions

$$\frac{\partial P}{\partial t} + (\kappa(\theta - r(t)) - \lambda\sigma)\frac{\partial P}{\partial r} + \frac{1}{2}\sigma^2\frac{\partial^2 P}{\partial r^2} - rP = 0 \tag{3.1.6}$$

where $P(t,T)$ is the price at time t of a discount bond maturing at time T, with unit maturity value $P(T,T) = 1$. Furthermore $\kappa(\theta - r(t))$ the instantaneous drift of the process of r and $\lambda = \frac{\mu - r}{\sigma}$ is usually called the market price of risk. If r and λ are specified, the bond prices obtained by solving (3.1.6) subject to the boundary condition $P(T,T) = 1$. When the expected instantaneous rates of return on bonds of all maturities are the same, the bond price can be given as

$$P(t,T) = E_t(e^{-\int_t^T r(\tau)d\tau}), \tag{3.1.7}$$

and the analytical solution is

$$P(t,T) = e^{A(t,T)r(t)+B(t,T)}, \tag{3.1.8}$$

where

$$A(t,T) = \frac{1}{\kappa}(e^{-(T-t)\kappa} - 1), \tag{3.1.9}$$

$$B(t,T) = \frac{\sigma^2}{4\kappa^3}(1-e^{-2(T-t)\kappa})+\frac{1}{\kappa}(\theta-\frac{\lambda\sigma}{\kappa}-\frac{\sigma^2}{\kappa^2})(1-e^{-(T-t)\kappa})-(\theta-\frac{\lambda\sigma}{\kappa}-\frac{\sigma^2}{2\kappa^2})(T-t). \tag{3.1.10}$$

By use of (2.1.2) the yield to maturity can be written as

$$\begin{aligned} Y(t,T) &= -\frac{1}{T-t}[\frac{1}{\kappa}(e^{-(T-t)\kappa} - 1)r(t) + \frac{\sigma^2}{4\kappa^3}(1 - e^{-2(T-t)\kappa}) \\ &+ \frac{1}{\kappa}(\theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{\kappa^2})(1 - e^{-(T-t)\kappa}) - (\theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{2\kappa^2})(T - t)] \end{aligned} \tag{3.1.11}$$

The yield to maturity $Y(t,T)$ with respect to given set of information at $s \leq t$ is normally distributed

$$Y(t,T) \mid \mathcal{F}_s \sim N(\mu_R, \sigma_R^2) \tag{3.1.12}$$

with

$$\begin{aligned} \mu_R &= (1 - e^{-\kappa(t-s)})(R(t,\infty) + \frac{1 - e^{-\kappa T}}{\kappa T}(\theta - R(t,\infty)) + \frac{\sigma^2(1 - e^{-\kappa T})^2}{4\kappa^3 T}) \\ &+ e^{-\kappa(t-s)}R(s,t), \end{aligned} \tag{3.1.13}$$

$$\sigma_R = (\frac{1 - e^{-\kappa T}}{\kappa T})^2(1 - e^{-2\kappa(t-s)})\frac{\sigma^2}{2\kappa}, \tag{3.1.14}$$

where $R(t,\infty)$ is

$$R(t,\infty) = \lim_{t\to\infty} R(t,T) = \theta - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{2\kappa^2} \tag{3.1.15}$$

as time to maturity tends to infinity, volatility of the term structure tends to zero.

The Vasiček model has some drawbacks based on its assumptions. Since the movements of instantaneous short rate obeys Brownian motion, the spot rate can be negative. Another drawback of this model is the perfect correlation for yield curve from (3.1.4). Perfect correlation means that the correlation of two random interest rates with different maturities is always one, which means rejected. And as a common problem of equilibrium models, the Vasiček model also does not fit the initial term structure.

3.1.2 Cox-Ingersoll-Ross Model

[10] proposed an alternative model based on Vasiček to prevent the occurrence of negative short-term interest rate $r(t)$ faced in Vasiček model by introducing a $\sqrt{r(t)}$ in the variance term of the short rate process. The CIR model is

$$dr(t) = \kappa(\theta - r(t))dt + \sigma\sqrt{r(t)}dW_t \tag{3.1.16}$$

CIR has the same assumptions and parameters as the Vasiček model except that the standard deviation is proportional to $\sqrt{r(t)}$, which guarantees the nonnegativeness of the spot rate. The CIR model gives a detailed prediction about the response of the term structure into changes in underlying variables that effect the behavior of term structure of interest rate. This approach is a result of the CIR point of view since consider the problem of understanding the term structure as a problem in general equilibrium theory.

CIR model drawbacks show a similarity with the Vasiček model except avoiding the occurrence of negative spot rate. Both the Vasiček and CIR models are affine models since the yield to maturity $Y(t,T)$ is an affine function of the instantaneous short rate $r(t)$ as given in (3.1.11).

3.2 NO-ARBITRAGE MODELS

The no arbitrage approach provides an exact fit to the initial term structure of interest rates and specifies the stochastic evolution of the term structure. In order to achieve that there are no arbitrage possibilities.

3.2.1 Ho and Lee Model

The first no arbitrage model of the term structure of the interest rate is [29]. The short rate process under the no arbitrage assumption can be given as

$$dr(t) = \theta(t)dt + \sigma dz, \tag{3.2.17}$$

where $r(t)$ is the short rate at time t, σ is constant and $\theta(t)$ is a function of time, provides an exact fit to the initial term structure and given as

$$\theta(t) = f(0,t) + \sigma^2 t. \tag{3.2.18}$$

Here $f(0,t)$ is the instantaneous forward rate at $t = 0$. The value of a discount bond can be given analytically as

$$P(t,T) = A(t,T)e^{-r(T-t)}, \tag{3.2.19}$$

where

$$\ln(\frac{P(0,T)}{P(0,t)}) - (T-t)\frac{\partial \ln(P(0,T))}{\partial t} - \frac{1}{2}\sigma^2 t(T-t)^2 = 0, \tag{3.2.20}$$

The advantages of Ho and Lee model can be given as analytical tractability and exact fitting to the initial term structure of interest rates. On the other hand containing no mean reversion and having all spot and forward rates the same volatility can be given as drawbacks of this model.

3.2.2 Heath-Jarrow-Morton Model

Heath-Jarrow-Morton approach provide the stochastic evolution of dynamics of the entire yield curve by the use of instantaneous forward rates under no arbitrage considerations. They perform to equate the underlying securities with the entire yield curve by use the spot curve observed in the market. The absence of arbitrage implies that the discounted bond prices of such products are martingale under a risk neutral probability. The Heath-Jarrow-Morton process for forward rates can be given as

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW(t), \tag{3.2.21}$$

where $f(t,T)$ is the instantaneous forward rate of maturity $T-t$, mature at T and $\alpha(t,T)$ is the instantaneous drift as follows

$$\alpha(t,T) = \sigma(t,T)\int_t^T \sigma(t,\tau)d\tau.$$

Here, $\sigma(t,T)$ is the standard deviation of $f(t,T)$ and $W(t)$ is a standard Wiener process. The single factor Heath-Jarrow-Morton approach could be extended to a model with multi-factors

$$df(t,T) = \alpha(t,T)dt + \sum_k \sigma_k(t,T)dW(t), \tag{3.2.22}$$

where the instantaneous drift for the multi-factors Heath-Jarrow-Morton model is given as

$$\alpha(t,T) = \sum_k \sigma_k(t,T)\int_t^T \sigma_k(t,\tau)d\tau. \tag{3.2.23}$$

As a characteristic property of no arbitrage models, the Heath-Jarrow-Morton approach also consistent with the initial term structure data. On the other hand, it is difficult to obtain closed form solutions for the values of bonds and interest rate derivatives. Although classical models are motivated by their mathematical tractability, they have a weak ability to describe the real data unless a so-called market-risk premium is included, this is also true for the popular Heath-Jarrow-Morton model.

3.2.3 Hull White Model

Hull and White (1990) provide a class of models that both incorporate the properties of no arbitrage and equilibrium models. Ho-Lee model do not impose mean reversion property, however, Hull-White model incorporates deterministically the mean reversion property and exact fit of the initial yield curve. Hull-White extend the equilibrium models Vasiček and CIR with settling a time varying parameter to perform an exact fit to the currently observed yield curve. The extended Vasiček model provides a closed form solution for the instantaneous short rate. The short rate process for this model can be written as

$$dr(t) = (\theta(t) + \alpha(t)r(t))dt + \sigma(t)dW(t), \tag{3.2.24}$$

where $\theta(t), \sigma(t)$ and $\alpha(t)$ are functions of t. If $\sigma(t)$ and $\alpha(t)$ are assumed to be constants, the analytic solution for the short rate can be given as

$$\begin{aligned} r(t) &= e^{-\alpha t}r_0 + \int_0^t e^{-\alpha(t-s)}\theta(s)ds + \int_0^t e^{-\alpha(t-s)}\sigma dW(s) \\ &= e^{-\alpha(t-u)}r_u + \int_u^t e^{-\alpha(t-s)}\theta(s)ds + \int_u^t e^{-\alpha(t-s)}\sigma dW(s). \end{aligned} \tag{3.2.25}$$

To summarize the advantages and disadvantages of each approach can be given as follows in pricing interest rate derivatives. The advantage of the equilibrium models is that all interest rate derivatives are valued on a common basis. However, the equilibrium models have some disadvantages. First, they do not correctly price actual bonds and derivatives. Second, they have not sufficiently incorporated empirical realism, i.e., the model term structure does not fit the initial term structure and because of these reasons, they may admit arbitrage. On the other hand, no-arbitrage models have the advantage that the model term structure can fit the initial term structure. The disadvantages of the no-arbitrage models are as follows: First, there is no guarantee that the estimated function for the term structure of interest rate will be consistent with the previously estimated function. Second, it is difficult to obtain closed-form solutions for the value of bond and interest rate derivatives.

Chapter 4

MACRO-FINANCE MODELLING

4.1 Connecting The Edges

Understanding the term structure of interest rate is important from both finance and macroeconomic perspectives. On finance side, forecasting and pricing interest rate sensitive instruments in several fixed-income markets is crucial to manage the interest rate risk. Another reason is the importance of the short rate since the long term yields are risk adjusted averages of expected future short rates after at least an adjustment of risk, this says that the short rate is a fundamental building block for rates of other maturities. On the other hand, the term structure of interest rates provides useful information about underlying expectations of inflation and real activity from macroeconomic point of view. Future expectations about macroeconomic fundamentals open the way for central banks deciding monetary policy transmission perfectly by adjusting the most important key instrument short rate to achieve their economic stabilization goals.

The term structure models constructed in these two distinct branches have different aims. Financial economists develop models based on the absence of arbitrage and calibrating to the initial spot rate curve since their necessities are forecasting and pricing. Their models could not specify the relationship between the term structure and other economic variables. These traditional models of the term structure of interest rates determine the behavior of the yield curve by means of a limited number of unobservable latent factors where they are obtained by decomposing the yield curve using one of statistical methods. They fail to provide an economic interpretation of these factors. On the other hand, macroeconomists have developed term structure models to determine the relation between interest rates, monetary policy and macroeconomic fundamentals. These macroeconomic models are based on the expectation hypothesis and could not provide the interactions between macroeconomy and term structure dynamics.

It is very important to understand how the behavior of the yield curve reacts to macroeconomic shocks not only for traders but also for central banks and government agencies since yield curves provide a useful information about underlying expectations of inflation and output over a number of different horizons.

According to the way of changes in the worldwide economy, stronger links appear between macroeconomic fundamentals and asset pricing models of the yield curve. A key aspect of this change is the sharp decline in overall macroeconomic volatility depending on several factors. One of these factors is the stabilization of economies by use of a better economic policy transmissions which turn out less output volatility and a lower, stable inflation. Another factor is the development of financial markets and the beginning use of new financial instruments. These changes in nature of economy reflect the behavior of the term structure of interest rates and someone needs to determine how information about macroeconomy feeds into bond prices or yield curve. This can be done by providing an explanatory role to macro factors on term structure of interest rate. As [22] argued "... the factors moving the interest rates should be linked to monetary policy and to fluctuations in real economic activity ...".

In the beginnings of the second millennium, someone started to connect two edges, the macroeconomic and traditional term structure models and offered a joint characterization for these two literatures. The combined macro-finance analysis determines the behavior of yield curve in the light of macroeconomic issues since the addition of term structure information to a macroeconomic model provides sharpened inference. There have appeared various papers trying to determine the joint macro-finance characterization of the term structure of interest rates and macroeconomic fundamentals. Although papers based on different frameworks they have a number of common properties. First, the short end of the yield curve plays the most important role to explain the behavior of the entire yield curve. Considering the yield curve as a function, that is obviously contrary to vector autoregressive models, is another common property. These models could not provide an explicit relation between the determinants of the yield curve shape and macroeconomic factors.

The pioneers of macro-finance literature, [3] offer a vector autoregressive framework to capture the joint dynamics of macro factors and additional latent factors based on no arbitrage restrictions where macroeconomic factors are measures of inflation and real activity. This approach provides a unidirectional characterization between the term structure of interest rates and macroeconomic fundamentals. This means that shocks on interest rates do not effect macroeconomic fundamentals.

[17] offer a macroeconomic interpretation of Nelson-Siegel representation which allows that the latent variables and macroeconomic factors can be correlated and find strong effects of macroeconomic variables on the future movements of yield curve. This framework occurs under a standard macroeconomic vector autoregressive, but they do not preclude no-arbitrage movements of the yield curve. This paper provides a bidirectional characterization of macroeconomic factors real activity, inflation and a monetary policy instrument federal funds rate and term structure of interest rates. According to the paper, the bidirectional characterization shows that the causality from macroeconomy to yields is stronger than in reverse direction.

Further examples of recent papers are; [30] follows a framework like [3], but they remove the assumption that inflation and output macroeconomic variables are independent of the policy interest rate by building a structural macroeconomic model with both forward and backward looking expectations, rather than employing a bivariate vector autoregressive of inflation and output. They show that the out-of-sample forecasting performance is comparable to the best available affine term structure models, apart from long-term yields. [41] proposed a more macroeconomic structure which combine the affine dynamics for yields with a macroeconomic model which typically consists of a monetary policy reaction function, an output equation and an inflation equation under the no arbitrage restrictions. They allow for a bidirectional feedback between the term structure of interest rates, latent factors and macroeconomic variables which says latent factors are affected by macroeconomic variables through inflation targeting and monetary policy inertia. This paper argues that current and future yield curves show a significant response to the expectations of forward looking agents about the future dynamics of the economy. However, current observed yield curves and macroeconomic information include these expectations indirectly. [1] offer a bidirectional characterization of the macroeconomic fundamentals and the yields by use of the Markov Chain Monte Carlo method under no arbitrage restrictions and estimate forward looking Taylor rules using the short rate equation. The key observable factor is the federal reserves interest rate target. The short rate is modelled as the sum of the target and short lived deviations from target. This paper shows that lower pricing errors occur when using macroeconomic information rather than a traditional latent factor characterization. [44], [19], [4], [14] and [15] are the remaining papers in macro-finance literature that are study the joint dynamics of bond yields and macroeconomic variables.

All these studies offering distinct joint macro-finance models concluding that macroeconomic variables are useful for understanding and forecasting government bond yields. Although these models have different methodologies, the use of a small macroeconomic information for the analysis can be given a common feature for them. The most commonly used factors in these macro-finance models include a measure of inflation, real activity, output gap and at most two other macroeconomic factors and latent yield curve factors that relates yields of different maturities. Besides providing a useful compression of information, factor structure must also agree with the parsimony principle.

4.1.1 Advantages of Macro-Finance Modelling

The joint macro-finance modelling brings up some additional advantages to understand the behavior of the yield curve over traditional term structure modelling and pure macroeconomic modelling.

Traditional term structure models determine the yield curve by use of historical interest rates whereas the macro-finance models offer to characterize the bidirectional interactions between interest rates and macroeconomic variables since these two are jointly developing over time. A macro-finance model allows us to determine the behavior of risk premiums explicitly depending on macroeconomic conditions. This is an extra advantage of macro-finance models over standard consumption based models of asset returns since they determine the risk premium by the covariance of asset returns with the marginal utility consumption. This advantage was shown in empirical study [7] that macro-finance models find a strong relationship between economic activity and excess return in bond markets or in other words risk premiums. A third extra advantage of macro finance model over standard macroeconomic models is that the macro-finance model allows a substantial component of historical bond yields effecting the evolution of time varying term of risk premiums. In traditional finance modelling the same effect on time varying term and risk premium according to the estimated dynamics of model is captured by the absence of arbitrage restriction.

The rest of this chapter includes a description of three distinct macro-finance modelling frameworks. The first two are existing macro-finance models that were offered by [3] and [17]. The last but not least important one is as original alternative macro-finance model that we offer for the first time in this study under absence of arbitrage opportunities in a stochastic manner. From now on, Ang and Piazzesi, Diebold, Rudebusch and Aruoba and the alternative framework are going to be interpreted as AP, DRA and original macro-finance approaches respectively.

4.2 AP Approach

A no arbitrage based financial term structure model could determine movements of the yields of all maturities respond to the movements to the underlying state factors but could not identify the sources of those movements in state factors. On the other hand, a macroeconomic empirical vector autoregressive model could explain the economic sources of movements in state variables for given yields, but could not say anything about the response of the entire yield curve against those movements.

AP combine these two frameworks to derive the movements of yield curve and present a no arbitrage vector autoregressive model. This joint vector autoregressive framework captures the joint dynamics of the macroeconomic and bond yield factors based on the absence of arbitrage opportunities. Three latent factors and two measures of macroeconomic factors are included in the model as state variables. The macroeconomic factors are derived by using the first principal component of a set of large collection of candidate macroeconomic series for inflation, real activity and latent factors ending up measuring from yields.

In AP there are two maintained assumptions. The first can be given as independence of latent and macroeconomic factors and the second is that both latent and macroeconomic factors follow vector autoregressive processes. Throughout this study latent and macroeconomic factors for AP approach are denoted by un_t^i , m_t^j for $i = 1, 2, 3$, $j = 1, 2$, respectively. The vector autoregressive process is adjusted according to data available for Turkey and present VAR(1) and a bivariate VAR(3) process for latent and macroeconomic factors respectively. The forward looking rule for macroeconomic variables joined the ability of lagged macroeconomic variables to forecast the future. The dynamics of latent and lagged macro variables can be written as

$$X_t = \mu + \phi X_{t-1} + \Sigma\varepsilon_t, \tag{4.2.1}$$

where

$$X_t = \begin{pmatrix} m_t^1 & m_t^2 & m_{t-1}^1 & m_{t-1}^2 & m_{t-2}^1 & m_{t-2}^2 & un_t^1 & un_t^2 & un_t^3 \end{pmatrix}$$

and

$$\varepsilon_t = \begin{pmatrix} u_t^{m_1} & u_t^{m_2} & 0 & 0 & 0 & 0 & u_t^{un_1} & u_t^{un_2} & u_t^{un_3}. \end{pmatrix}$$

Here, $u_t^{m_i}$ and $u_t^{un_j}$ are the shocks to the macro and latent factors, respectively, for i=1,2 and j=1,2,3. The reduced form (4.2.1) is a VAR process of order 1 with nonlinear parameters.

It is assumed that the short term rate is affected from both macroeconomic variables as in the literature on simple monetary policy and unobservable factors as in the affine term structure literature. This means that is the one period short rate r_t is assumed to be affine functions of all state variables

$$r_t = \delta_0 + \delta_1 X_t, \tag{4.2.2}$$

where r_t is the two month yield in our framework according to Turkey data.

No arbitrage restriction guarantee the existence of a risk neutral measure Q or, in other words, existence of Radon-Nikodyn derivative, ξ_t, which is used to convert the risk neutral measure to data generating measure, such that the price of any asset V_t without dividend payments at time t+1 satisfies the following equation

$$V_t = E_t^Q(e^{-r_t}V_{t+1}). \tag{4.2.3}$$

This means that the current price of any asset is the expectations of one step ahead price discounted today under a risk neutral measure. By using Radon-Nikodyn derivative for any random variable Z_{t+1} at time $t+1$, the change of measure occurs as follows:

$$E_t^Q(Z_{t+1}) = E_t(\xi_{t+1} Z_{t+1} \mid \xi_t). \tag{4.2.4}$$

Assume that the Radon-Nikodyn derivative ξ_{t+1} follows the log-normal process, then one can write

$$\xi_{t+1} = \xi_t e^{\frac{-1}{2}\lambda_t' \lambda_t - \lambda_t \varepsilon_{t+1}}, \tag{4.2.5}$$

where λ_t are the time varying market prices of risk associated with the source of uncertainty ε_{t+1} in the economy. The market price of risk parameter is commonly assumed to be constant in a Gaussian models or proportional to the factor volatilities. The model combines a vector autoregressive framework for the unobservable and macroeconomic variables with an exponential affine pricing kernel. As a result, the implied risk premia are affine in the unobservable and macroeconomic variables as following

$$\lambda_t = \lambda_0 + \lambda_1 X_t. \tag{4.2.6}$$

Here, X_t is defined by (4.2.1). The equation (4.2.6) relates the shock in the underlying state dynamics to ξ_{t+1}. The constant risk premium parameter λ_0 is a 9×1 vector column, while time varying risk premium parameter λ_1 is a 9×9 matrix, but parameters in λ_0 and λ_1 that correspond to lagged macro variables are set to zero to reduce the number of parameters to be estimated. Since latent and macro variables are assumed to be orthogonal, let us set any λ_1 parameters corresponding to the latent variables to zero in estimations of models with macro variables. The matrix λ_1 is specified to be block diagonal, with zero restrictions on the upper -right and lower-left corner blocks. All of these settings leaves a 2×2 matrix for current macro variables and a 3×3 matrix for the latent variables different than zero in λ_1 matrix.

AP follows the standard dynamic arbitrage-free term structure literature and defines the nominal pricing kernel, pricing kernel prices all assets in the economy, as

$$m_{t+1} = e^{-r_t}(\frac{\xi_{t+1}}{\xi_t}). \tag{4.2.7}$$

Substituting (4.2.5) and (4.2.2) in (4.2.7), the expressions turns to be

$$\begin{aligned} m_{t+1} &= e^{-\delta_0-\delta_1' X_t} e^{\frac{-1}{2}\lambda_t'\lambda_t-\lambda_t'\varepsilon_{t+1}} \\ &= e^{\frac{-1}{2}\lambda_t'\lambda_t-\delta_0-\delta_1' X_t-\lambda_t'\varepsilon_{t+1}}. \end{aligned} \tag{4.2.8}$$

Let us denote $P(t, t+n+1)$ as P_t^{n+1} for tractability which represents the price at t of an $n+1$ period zero coupon bond, then the bond price can be computed recursively by use of pricing kernel as

$$P_t^{n+1} = E_t(m_{t+1}P_{t+1}^n)), \tag{4.2.9}$$

with

$$P_t^n = e^{\overline{A}_n+\overline{B'}_n X_t}. \tag{4.2.10}$$

To derive $\overline{A}_n$ and $\overline{B}_n$ in terms of market prices of risk and parameters in (4.2.1) and (4.2.2), first note that for a one period bond, $n = 1$, we have

$$\begin{aligned} P_t^1 &= E_t[m_{t+1}] \\ &= e^{-r_t} \\ &= e^{-\delta_0-\delta_1 X_t}. \end{aligned} \tag{4.2.11}$$

This adjustment turns the coefficients to be $\overline{A}_1 = -\delta_0$ and $\overline{B}_1 = -\delta_1$. From (4.2.9) and (4.2.10) we can write

$$
\begin{aligned}
P_t^{n+1} &= E_t[m_{t+1}P_{t+1}^n] \\
&= E_t[e^{(-r_t-\frac{-1}{2}\lambda_t^{'}\lambda_t-\lambda_t^{'}\varepsilon_{t+1}+\overline{A}_n+\overline{B^{'}}_n X_{t+1})}] \\
&= e^{(-r_t-\frac{1}{2}\lambda_t^{'}\lambda_t+\overline{A}_n)}E_t[e^{-\lambda_t^{'}\varepsilon_{t+1}\overline{B^{'}}_n X_{t+1}}] \\
&= e^{(-r_t-\frac{1}{2}\lambda_t^{'}\lambda_t+\overline{A}_n)}E_t[e^{-\lambda_t^{'}\varepsilon_{t+1}\overline{B^{'}}_n(\mu+\phi X_t+\Sigma\varepsilon_{t+1})}] \\
&= e^{(-\delta_0-\delta_1 X_t-\frac{-1}{2}\lambda_t^{'}\lambda_t+\overline{A}_n)}E_t[e^{-\lambda_t^{'}\varepsilon_{t+1}\overline{B^{'}}_n(\mu+\phi X_t+\Sigma\varepsilon_{t+1})}] \\
&= e^{(-\delta_0+\overline{A}_n+\overline{B^{'}}_n\mu+(\overline{B^{'}}_n\phi-\delta_1)X_t-\frac{1}{2}\lambda_t^{'}\lambda_t)}E_t[e^{-(\lambda_t^{'}+\overline{B^{'}}_n\Sigma)\varepsilon_{t+1}}] \\
&= e^{-\delta_0+\overline{A}_n+\overline{B^{'}}_n(\mu-\Sigma\lambda_0)+\frac{1}{2}\overline{B^{'}}_n\Sigma\Sigma^{'}\overline{B}_n-\delta_1^{'}X_t+\overline{B^{'}}_n\phi X_t-\overline{B^{'}}_n\Sigma\lambda_1 X_t},
\end{aligned}
\quad (4.2.12)
$$

and the coefficients $\overline{A}_n$ and $\overline{B}_n$ follows the difference equations

$$
\overline{A}_{n+1} = \overline{A}_n + \overline{B^{'}}_n(\mu - \Sigma\lambda_0) + \frac{1}{2}\overline{B^{'}}_n\Sigma\Sigma^{'}\overline{B}_n - \delta_0 \quad (4.2.13)
$$

and

$$
\overline{B^{'}}_{n+1} = \overline{B^{'}}_n(\phi - \Sigma\lambda_1) - \delta_1^{'} \quad (4.2.14)
$$

with $\overline{A}_1 = -\delta_0$ and $\overline{B}_1 = -\delta_1$.

By use of (2.1.2) bond yields can be written as an affine functions of the state variables in following form

$$
y_t^n = A_n + B_n^{'}X_t, \quad (4.2.15)
$$

where $A_n = -\frac{\overline{A_n}}{n}$ and $B_n = -\frac{\overline{B_n^{'}}}{n}$.

4.3 DRA Approach

DRA provides a simple way of adding macroeconomic variables in a finance specification of the yield curve. They provide a macroeconomic interpretation of the Nelson-Siegel representation by combining it with VAR dynamics for macroeconomic variables. The Nelson-Siegel framework drives out the principal components of the entire yield curve, period by period, into a three dimensional parameter that evolves dynamically. The Nelson-Siegel representation is

$$y_t(\tau) = \beta_{1t} + \beta_{2t}[\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau}] + \beta_{3t}[\frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau}], \tag{4.3.16}$$

where β_{1t} , β_{2t} , β_{3t} and λ_t are parameters and τ denotes maturity. The parameter λ_t governs with the exponential decay rate which satisfies better fit for the yield curve at long maturities with large values and short maturities with small values of it. Throughout this thesis, the exponential decay rate are assumed as constant for estimation tractability.

Three factors β_{1t}, β_{2t} and β_{3t} in Nelson-Siegel representation (4.3.16) can be interpreted in terms of time varying level slope and curvature factors. Limit of (4.3.16) is taken as time to maturity τ goes to infinity the result as longest term yield will be obtained

$$y_t(\infty) = \lim_{\tau \to \infty} y_t(\tau) = \beta_{1t}. \tag{4.3.17}$$

Since the coefficient of β_{1t} is one, it affects all maturities identically, that means changing the level of the yield curve.

The slope of the yield curve can be defined as the difference between long term and short term rates as $y_t(\infty) - y_t(0)$ which exactly equals to β_{2t} from (4.3.16). That is an increase in β_{2t} increase short term yields more than longer term yields because the short rates load on β_{2t} more heavily thereby this factor changing the slope of the yield curve.

Finally the last factor β_{3t} is closely related to the yield curve curvature that an increase in β_{3t} will have little effect on very short or very long yields which load minimally on it, but will increase medium term yields. Therefore one can say that β_{3t} increase the yield curve curvature.

$$y_t(\tau) = L_t + S_t(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}) + C_t(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}). \tag{4.3.18}$$

DRA wish to characterize the joint dynamics of L_t, S_t, C_t and the macroeconomic fundamentals. Capacity utilization is used to represent the level of real economic activity relative to potential. Federal fund rates and monthly CPI include in macroeconomic factor set to represent the monetary policy instrument and inflation rate respectively. These three macroeconomic factors are widely considered to be the minimum set of fundamentals to interpret the basic macroeconomic dynamics. The addition of macroeconomic factors to the Nelson-Siegel representation is easy to do. Nelson-Siegel representation can be extended after including the macroeconomic factors and turns to be as a measurement equation in the form

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ y_t(\tau_3) \\ y_t(\tau_4) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} & \frac{1-e^{-\lambda\tau_1}}{\lambda\tau_1} - e^{-\lambda\tau_1} & a_{14} & a_{15} & a_{16} \\ 1 & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} & \frac{1-e^{-\lambda\tau_2}}{\lambda\tau_2} - e^{-\lambda\tau_2} & a_{24} & a_{25} & a_{26} \\ 1 & \frac{1-e^{-\lambda\tau_3}}{\lambda\tau_3} & \frac{1-e^{-\lambda\tau_3}}{\lambda\tau_3} - e^{-\lambda\tau_3} & a_{34} & a_{35} & a_{36} \\ 1 & \frac{1-e^{-\lambda\tau_4}}{\lambda\tau_4} & \frac{1-e^{-\lambda\tau_4}}{\lambda\tau_4} - e^{-\lambda\tau_4} & a_{44} & a_{45} & a_{46} \end{pmatrix} \begin{pmatrix} L_t \\ S_t \\ C_t \\ RA_t \\ INFL_t \\ FFRs_t \end{pmatrix} + \begin{pmatrix} \varepsilon_t(\tau_1) \\ \varepsilon_t(\tau_2) \\ \varepsilon_t(\tau_3) \\ \varepsilon_t(\tau_4) \end{pmatrix}.$$

The system is constructed under several assumptions. First it is assumed that the factor dynamics are unconstrained and follows a VAR(1) process. This assumption gives the transition equation in following form:

$$\begin{pmatrix} L_t \\ S_t \\ C_t \\ RA_t \\ INFL_t \\ FFRs_t \end{pmatrix} = \begin{pmatrix} f_{11} & f_{12} & f_{13} & f_{14} & f_{15} & f_{16} \\ f_{21} & f_{22} & f_{23} & f_{24} & f_{25} & f_{26} \\ f_{31} & f_{32} & f_{33} & f_{34} & f_{35} & f_{36} \\ f_{41} & f_{42} & f_{43} & f_{44} & f_{45} & f_{46} \\ f_{51} & f_{52} & f_{53} & f_{54} & f_{55} & f_{56} \\ f_{61} & f_{62} & f_{63} & f_{64} & f_{65} & f_{66} \end{pmatrix} \begin{pmatrix} L_{t-1} \\ S_{t-1} \\ C_{t-1} \\ RA_{t-1} \\ INFL_{t-1} \\ FFRs_{t-1} \end{pmatrix} + \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{4,t} \\ \eta_{5,t} \\ \eta_{6,t} \end{pmatrix}.$$

Compact forms for the measurement and transition equations are

$$y_t = Af_t + \varepsilon_t, \tag{4.3.19}$$

$$f_t = Ff_{t-1} + \eta_t. \tag{4.3.20}$$

In addition to these assumptions, it is assumed that there is no relation between factor loadings and factor dynamics and for linear least square optimality of the Kalman filter which will be discussed in latter chapter, white noise transition and measurement disturbances are orthogonal to each other and to the initial state.

$$\begin{pmatrix} \eta_t \\ \varepsilon_t \end{pmatrix} \sim WN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & H \end{pmatrix}\right]. \tag{4.3.21}$$

It is assumed for estimation tractability that H is a diagonal matrix, implies that the deviation of yields of various maturities from the yield curve are uncorrelated, and Q is an upper triangular matrix, allows the shocks to the three term structure latent factors to be correlated.

Unlike the AP the model offer a bidirectional characterization between the term structure of interest rates and the macroeconomic variables. Hence, it is determined whether the relation flows from the term structure to the macroeconomic factors or vice-versa. The model present that the causality from the macroeconomy to yields is indeed significantly stronger than in the reverse direction for US data in original paper but that interactions in both directions can be important.

4.4 An Original Macro-Finance Model

The starting point of this thesis is to present a term structure model that adequately captures the dynamics of the Turkish term structure of interest rates.

The model must provide an analytic representation for the relationship between the state variables and the term structure of interest rates with having a relatively easy to estimate and interpret parameter set. Observable macroeconomic factors with unobservable long run expectations of these macroeconomic factors and two latent yield curve factors which one of them is the long run expected short term interest rate target are used to present that relationship. A measure of inflation and real activity are represented by monthly change in consumer price index (CPI) and capacity utilization (CU) , respectively, and two latent factors included in the state factors after taking difference of macroeconomic variables with unobservable long run expectations as central tendencies are used. The dynamics of the observed and the unobserved factors that drive the long run interest rates are assumed as follows

$$d\pi_t = \beta_\pi(\pi^\star - \pi_t)dt + \sigma_\pi dW_t^\pi \tag{4.4.22}$$

$$d\gamma_t = \beta_\gamma(\gamma^\star - \gamma_t)dt + \sigma_\gamma dW_t^\gamma \tag{4.4.23}$$

$$df_t = \beta_f f_t dt + \sigma_f dW_t^f, \tag{4.4.24}$$

where $W^\pi(t)$, $W^\gamma(t)$ and $W^f(t)$ denote independent Wiener processes defined on the probability space $(\Omega, \mathcal{F}, P)$ with filtration $\mathcal{F}_t$. Furthermore $\pi(t), \gamma(t)$ and $f(t)$ are inflation real activity and latent factors, respectively, and π^* and γ^* are the long run expectations about inflation and real activity.

Bond prices and consequently yields are sensitive against fluctuations in expectation about the future path of monetary policy. Therefore central bank policy rule about instantaneous interest rate plays an important role in term structure modelling. The most cited policy rule for the dynamics of interest rates is proposed in [42]. Taylor's model based on which allow the macroeconomic forecasts of the first opportunity to explain variation in yields. We formalize the policy rule according to forward looking Taylor rule based on the assumption that the variables in that process converge to their respective central tendencies as

$$r_t^* = \zeta_0 + \zeta_1(E(\pi_{t+1}|\mathcal{F}_t) - \pi^*) + \zeta_2(E(\gamma_{t+1}|\mathcal{F}_t) - \gamma^*) + \zeta_3 f_t, \tag{4.4.25}$$

where r_t^* is the time varying interest rate target. After total differentiate (4.4.25) and taking the conditional expectations with respect to t under the assumptions

$$\pi_{t+1}|\mathcal{F}_t \simeq \pi_t|\mathcal{F}_t,$$

$$\gamma_{t+1}|\mathcal{F}_t \simeq \gamma_t|\mathcal{F}_t,$$

the expressions turn to be as follows

$$E_t(dr_t^*) = \zeta_1 E_t(d\pi_t) + \zeta_2 E_t(d\gamma_t) + \zeta_3 E_t(df_t)$$

Substituting (4.4.22), (4.4.23) and (4.4.24) into this expression under assumption $E_t(r_{t+1}^*) = r_{t+1}^*$ and using Euler method for discretization, the following expression appears

$$E_t(r_{t+1}^*) = r_t^* + \zeta_1\beta_\pi(\pi_t - \pi^*) + \zeta_2\beta_\gamma(\gamma_t - \gamma^*) + \zeta_3\beta_f f_t, \tag{4.4.26}$$

$$r_{t+1}^* = r_t^* + \zeta_1\beta_\pi(\pi_t - \pi^*) + \zeta_2\beta_\gamma(\gamma_t - \gamma^*) + \zeta_3\beta_f f_t. \tag{4.4.27}$$

If we denote $\zeta_i\beta_j = \alpha_i$ for $i = 1, 2, 3$ and $j = \pi, \gamma, f$ then the last equations turns to

$$r_{t+1}^* = r_t^* + \alpha_1(\pi_t - \pi^*) + \alpha_2(\gamma_t - \gamma^*) + \alpha_3 f_t \tag{4.4.28}$$

The long run expectations of instantaneous interest rate r_t^* can be written as the average long term level parameter θ in Vasiček model (3.1.3) and obtain the process for instantaneous short rate as

$$dr_t = \kappa(r_t^* - r_t)dt + \sigma_r dW_t. \tag{4.4.29}$$

The no arbitrage restriction allows us to determine the relationship between the term structure of interest rates and the state variables by use of Vasiček model which permits analytic solutions for the term structure (3.1.11)

By use of Euler method, one can write the discrete time analogy for equations (4.4.22), (4.4.23), (4.4.24) as folows

$$\begin{aligned} \pi_{t+1} &= (1-\beta_\pi)\Pi_t + \beta_\pi \pi^* + \sigma_\pi \varepsilon_{1t} \\ \pi_{t+1} - \pi^* &= (1-\beta_\pi)\pi_t + (1-\beta_\pi)\pi^* + \sigma_\pi \varepsilon_{1t} \\ \pi_{t+1} - \pi^* &= (1-\beta_\pi)(\pi_t - \pi^*) + \sigma_\pi \varepsilon_{1t}, \end{aligned} \tag{4.4.30}$$

$$\begin{aligned} \gamma_{t+1} &= (1-\beta_\gamma)\gamma_t + \beta_\gamma \gamma^* + \sigma_\gamma \varepsilon_{2t} \\ \gamma_{t+1} - \gamma^* &= (1-\beta_\gamma)\gamma_t + (1-\beta_\gamma)\gamma^* + \sigma_\gamma \varepsilon_{2t} \\ \gamma_{t+1} - \gamma^* &= (1-\beta_\gamma)(\gamma_t - \gamma^*) + \sigma_\gamma \varepsilon_{2t}, \end{aligned} \tag{4.4.31}$$

$$\begin{aligned} f_{t+1} &= \beta_f f_t + f_t + \sigma_f \varepsilon_{3t} \\ f_{t+1} &= (1+\beta_f) f_t + \sigma_f \varepsilon_{3t}. \end{aligned} \tag{4.4.32}$$

The term structure solution of the Vasiček model (3.1.11) turns to be as follows after setting (4.4.28) as θ in (3.1.3):

$$\begin{aligned} Y(t,T) = Y(\tau) &= -\frac{1}{T-t}\Big[\frac{1}{\kappa}(e^{-(T-t)\kappa} - 1)r(t) + \frac{\sigma^2}{4\kappa^3}(1 - e^{-2(T-t)\kappa}) \\ &+ \frac{1}{\kappa}(r_t^* - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{\kappa^2})(1 - e^{-(T-t)\kappa}) \\ &- (r_t^* - \frac{\lambda\sigma}{\kappa} - \frac{\sigma^2}{2\kappa^2})(T-t)\Big], \end{aligned} \tag{4.4.33}$$

where r_t^* is given in (4.4.28) and λ is constant risk premium.

Kalman filter that exploits the affine relationships between bond prices and the state variables will be explained in the following chapter to subsequently estimate the parameter set. So far we provide the state variables processes

(4.4.30), (4.4.31), (4.4.32) and (4.4.28) and the term structure equation (4.4.33) and from now on formulation of these equations in state-space form involves the specification of the model's measurement and transition equations are given. Equation (4.4.33) presents the measurement system by using a sequence of four zero coupon bond yield series with distinct time to maturities as

$$\begin{pmatrix} y_t(\tau_1) \\ y_t(\tau_2) \\ y_t(\tau_3) \\ y_t(\tau_4) \end{pmatrix} = \begin{pmatrix} A_t(\tau_1) \\ A_t(\tau_2) \\ A_t(\tau_3) \\ A_t(\tau_4) \end{pmatrix} + \begin{pmatrix} B(\tau_1) \\ B(\tau_2) \\ B(\tau_3) \\ B(\tau_4) \end{pmatrix} \begin{pmatrix} r^*_{t-1} \\ \pi_t - \pi^* \\ \gamma_t - \gamma^* \\ f_t \end{pmatrix} + \begin{pmatrix} w_t(\tau_1) \\ w_t(\tau_2) \\ w_t(\tau_3) \\ w_t(\tau_4) \end{pmatrix}; \quad (4.4.34)$$

in compact form:

$$y(\tau) = A + H'X_t + w_t. \quad (4.4.35)$$

Here, τ_i denotes the time to maturity and

$$\begin{aligned} A_t(\tau_i) &= -\frac{1}{\tau_i}(\frac{(e^{-\tau_i\kappa-1})r_t}{\kappa} + \frac{\sigma_r^2(1-e^{-2\tau_i\kappa})}{4\kappa^3} - \frac{\sigma_r^2(1-e^{-\tau_i\kappa})}{\kappa^3} - \frac{\sigma_r^2\tau_i}{2\kappa^2} \\ &+ (-\frac{\lambda\sigma_r}{\kappa^2})(1-e^{-\tau_i\kappa}) + \frac{\lambda\sigma_r\tau_i}{\kappa}) \end{aligned} \quad (4.4.36)$$

$$B(\tau_i) = \begin{bmatrix} \frac{e^{-\tau_i\kappa}-1}{\tau_i\kappa} & \frac{e^{-\tau_i\kappa}-1}{\tau_i\kappa}\alpha_1 & \frac{e^{-\tau_i\kappa}-1}{\tau_i\kappa}\alpha_2 & \frac{e^{-\tau_i\kappa}-1}{\tau_i\kappa}\alpha_3 \end{bmatrix} \quad (4.4.37)$$

and the transition system can be written by use of (4.4.28), (4.4.30), (4.4.31) and (4.4.32) as

$$\begin{pmatrix} r^*_t \\ \pi_t - \pi^* \\ \gamma_t - \gamma^* \\ f_t \end{pmatrix} = \begin{pmatrix} 1 & \alpha_1 & \alpha_2 & \alpha_3 \\ 0 & (1-\beta_\pi) & 0 & 0 \\ 0 & 0 & (1-\beta_\gamma) & 0 \\ 0 & 0 & 0 & (1+\beta_f) \end{pmatrix} \begin{pmatrix} r^*_{t-1} \\ \pi_{t-1} - \pi^* \\ \gamma_{t-1} - \gamma^* \\ f_{t-1} \end{pmatrix} + \begin{pmatrix} v_t(\tau_1) \\ v_t(\tau_2) \\ v_t(\tau_3) \\ v_t(\tau_4) \end{pmatrix}.$$

In compact form:

$$X_t = FX_{t-1} + v_t. \quad (4.4.38)$$

The expressions (4.4.35) and (4.4.38) together represent the state-space form of our term structure model. The estimation results provided by using Kalman filter to this state space form will appear in the next chapter.

Our original model is being given the very definition of the yield curve as a function of maturity and other economic factors which gives the strong interaction between yields with different maturities make up the curve and macroeconomic factors. The Kalman filter estimation method allows dependence among the entire yield curve and macro economic factors since the filter procedure follow an update process when new information appears.

The rest of this thesis includes detailed explanations for the Kalman filter method and estimation results for three macro-finance models.

Chapter 5

Empirical Analysis of Macro-Finance Models

5.1 Data

There are various approaches to estimate the yield curves and forward curves from observed bond prices since in practice they are not observed. The yield data used throughout this paper are produced from Turkish government zero coupon bond prices using the Nelson-Siegel functional form. A limited sample of end of month zero coupon constant maturity Turkish government bond yields ranging from February 2003 to March 2006 with maturities 2, 6, 12 and 24 month are used throughout this study. Table 5.1 displays descriptive statistics and up to three lags autocorrelations. Yields with longer maturity bonds are not used since they are sparse and not liquid enough to be used in term structure analysis. Figure 5.1 plot a figure of yield data.

	mean	stdev	skew	kurt	lag1	lag2	lag3
2 month	20.2988	7.8855	1.1477	3.4074	0.8931	0.7579	0.6602
6 month	24.0012	10.6050	1.1371	3.3946	0.8964	0.7537	0.6575
12 month	27.0877	13.4642	1.2313	3.6206	0.8957	0.7458	0.6450
24 month	29.1636	15.5512	1.3137	3.8793	0.8901	0.7342	0.6309

Table 5.1: Descriptive Statistics for Yields

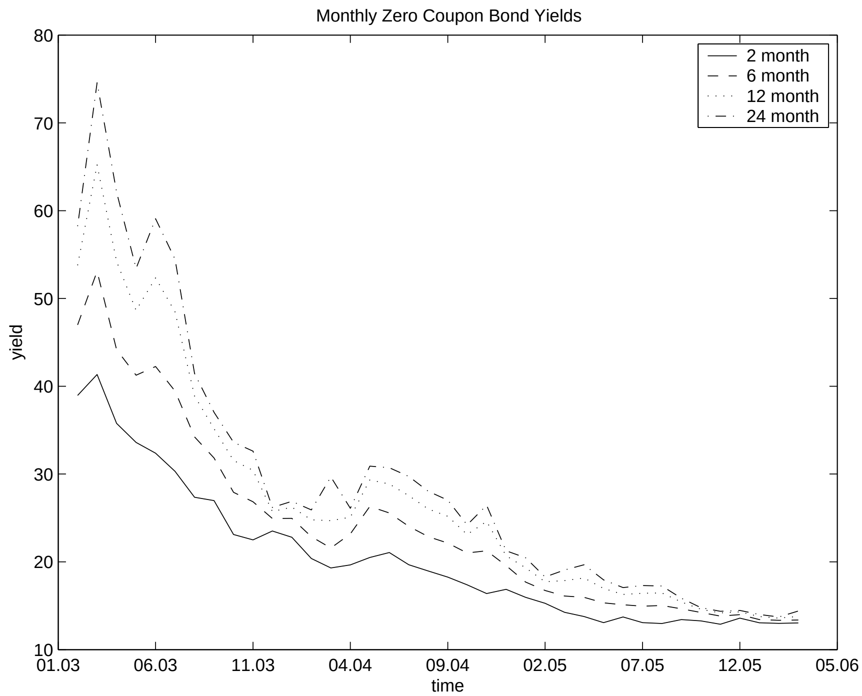

Figure 5.1: Monthly Zero Coupon Bond Yields

Different sets of macroeconomic variables are used in each model. All macroeconomic series used in this chapter obtained from the web-site of the Central Bank of the Republic of Turkey as follows

PPI	Producer Price Index
CPI	Consumer Price Index
WPI	Wholesale Price Index
CU	Capacity Utilization
IP	Industrial Production
O/N	Overnight Lending Interest Rate

Table 5.2: Macroeconomic Variables

Monthly returns of PPI, CPI, WPI, CU and IP series are used throughout this study after normalizing to zero mean and unit variance to achieve econometric identification. We have to achive this econometric identification since latent factors are also included in term structure model settings and give observationally equivalent systems with macroeconomic variables. Table 5.3 provides the descriptive statistics and up to three lags autocorrelations for these macroeconomic series as follows.

	mean	stdev	skew	kurt	lag1	lag2	lag3
PPI	0.0067	0.0125	1.0069	4.0472	0.2070	-0.1742	-0.1174
CPI	0.0071	0.0060	0.3992	3.0344	0.4113	-0.0399	-0.3673
WPI	0.0072	0.0076	-0.4174	4.3938	0.2856	-0.1138	-0.1074
CU	0.0041	0.0436	0.5163	3.5965	-0.3864	0.0724	0.0399
IP	0.0074	0.0950	0.0791	3.2247	-0.4327	-0.1107	0.2843

Table 5.3: Descriptive statistics for macroeconomic variables

In AP model, macroeconomic factors are derived by using the first principal component of set forms from candidate macroeconomic series for inflation and real activity. The first group consist of CPI, PPI and WPI as measures of inflation variables and the second group includes CU and IP to capture the dynamics of real activity. The loadings of the first three principle components for inflation and first two principle components for real activity and the variance explained of each principal components are given in Table 5.4 and Table 5.5.

	1st	2nd	3rd
PPI	-0.5571	0.7046	0.4395
CPI	-0.5562	-0.7096	0.4225
WPI	-0.6166	0.0035	-0.787
% variance explained	0.5545	0.8023	1

Table 5.4: Principal Component Analysis for Inflation

	1st	2nd
CU	-0.7071	-0.7071
IP	-0.7071	0.7071
% variance explained	0.7229	1

Table 5.5: Principal Component Analysis for Real Activity

As Table 5.4 and Table 5.5 display, over 50% of the variance of inflation group is explained by the first principal component. The first principal component of the inflation factors group loads negatively on PPI, CPI and WPI that says a positive shock on this variable loads negative shocks to inflation factors. From now on the first principle component of these macroeconomic factors are interpreted as measure of inflation. On the other hand, over 70% of the variance of real activity group are explained by the first principal component and this component has negative loadings on associated macroeconomic factors. Hence, we interpret the first principal component of the real activity factors as a measure of real activity. The measures of inflation and real activity are plotted in Figure 5.2.

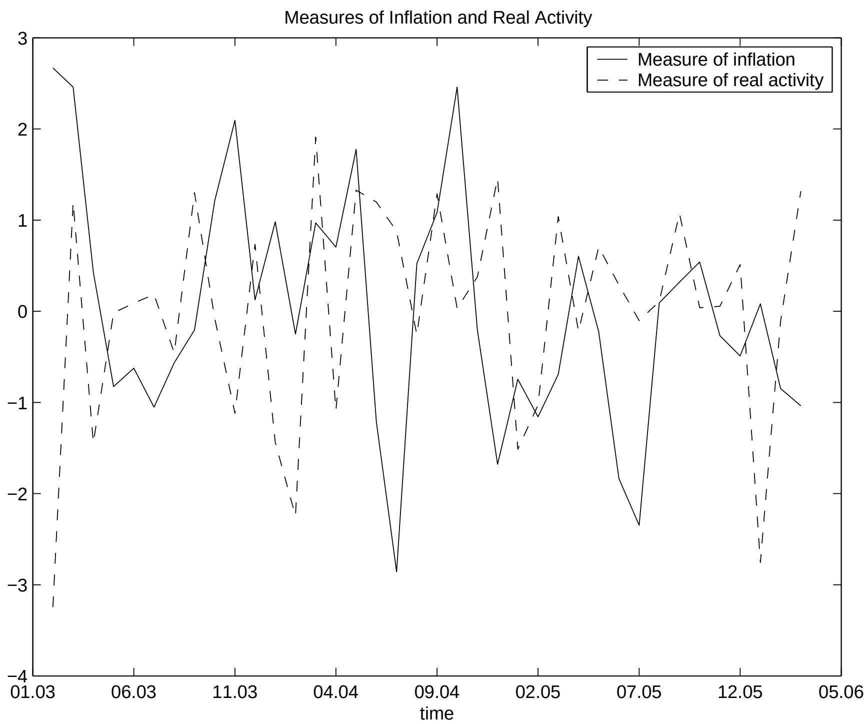

Figure 5.2: Measures of Inflation and Real Activity

Figure 5.3 and Figure 5.4 display the relationship between measures of inflation and normalized inflation factors and measure of real activity and normalized real activity factors, respectively.

The measure of inflation, which are displayed by circles on solid lines in Figure 5.3, move together with three normalized inflation factors. All of these

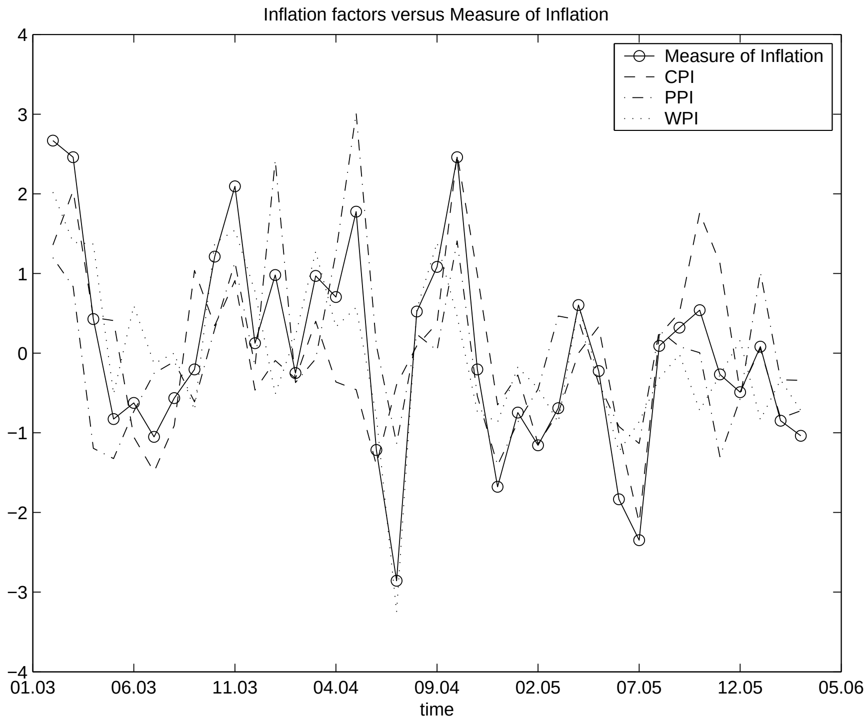

Figure 5.3: Measure of Inflation versus Inflation Factors

variables have roughly the same cycles. Measure of real activity in Figure 5.4 follows a closely relative path with two factors of real activity. The measure of real activity shows more close movements to its factors rather than inflation measures show to inflation factors, this can also be seen from the correlations between measures of macroeconomic variables and real macroeconomic factors are given in Table 5.6. This table also provides an intuition about the relationship between measures of inflation and real activity and yield curve.

Yields with 2, 6, 12 and 24 months to mature are used to estimate the rest of two macro-finance models. However, different macroeconomic series are used to capture the dynamics macroeconomic fundamentals. For DRA approach WPI series is used as measure of inflation and CU series is used as measure of real activity. As an additional macroeconomic factor, overnight short rate series are used as measure of monetary policy transmission instrument. For the original macro-finance model CPI and CU series are used as measures of inflation and

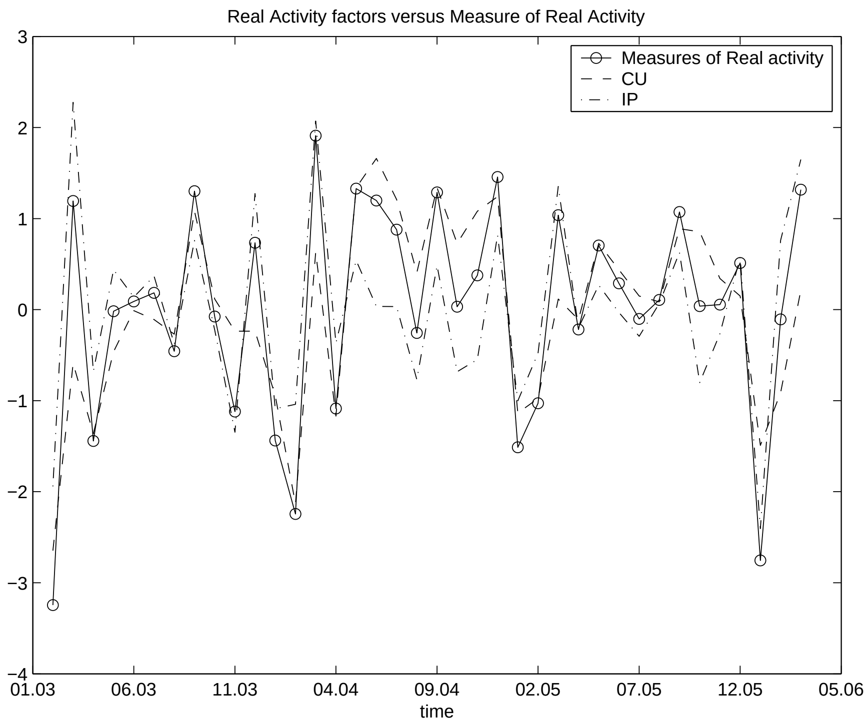

Figure 5.4: Measure of Real Activity versus Real Activity Factors

real activity respectively.

5.2 Kalman Filter

Parameter estimations for three macro-finance models are made by using the maximum likelihood estimation method via Kalman filter. The Kalman filter concept was arised to describe a recursive solution to the discrete-data linear filtering problem in engineering control literature. The idea is that one observes a stream of data over time which is subject to noise. This noise generally stems from measurement error arising in the devices used to measure the signal. In the context of the thesis, the noise in zero-coupon bond yields might relate to macroeconomic and latent yield factors. The Kalman filter is the method for filtering out the true signal and the unobserved components from this unwanted noise. More recently, it has also been used in some non-engineering applications

	PPI	CPI	WPI	
INFL	0.7186	0.7174	0.7953	
	CU	IP		
RA	0.8503	0.8503		
	INFL	RA	y_2	y_6
RA	-0.2066			
y_2	0.3524	-0.1814		
y_{12}	0.3222	-0.1229	0.9838	
y_{24}	0.3168	-0.1054	0.9762	0.9982

Table 5.6: Selected Correlations

such as short-term forecasting.

The Kalman filter is a set of mathematical equations that allow an estimator to be updated once a new observation becomes available. The Kalman filter provides an efficient recursive estimation by minimizing the mean of the squared error. The Kalman filter approach used in term structure literature is dealing with the estimation of affine term structure model. It is a linear estimation method and makes use of the assumption of an affine relationship between bond yields and state variables, which allows to be unobserved, to estimate the parameter set in a state-space framework. State-space representation consist of a measurement and a transition equation. The transition equation describe the dynamics of unobservable state variables assumed to follow a Markov process. The measurement equation describing how the observed data are generated from the state variables, that is provide the affine relation between observable bond yields and state variables. These state variables are unobservable factors but indeed observable factors can also be used as in our framework. Both observable macroeconomic variables and latent factors are included in state vector throughout this study. The Kalman filter use this type of formulation to recursively predict the unobservable values of the state variables based on the observation available up to the current time from observed market zero coupon yields. This property allows the state-space models to capture the dynamics of the system. As an example, when examining the relationship between term structure and macroeconomic factors, one should take into account the regime of the economy since economic recessions and expansions at a particular date is appropriately reflected in the estimate of the state variable for that date and the prediction for the next period will be more accurate using a state space model rather than a fixed coefficients methodology.

The general state-space models are in following form

$$y_t = A'z_t + H'X_t + w_t \tag{5.2.1}$$

$$X_{t+1} = FX_t + v_{t+1}, \tag{5.2.2}$$

where (5.2.1) is the measurement and (5.2.2) is the transition equations. y_t is a vector of observation and X_t is a vector of state factors. Furthermore, w_t and v_t are disturbance vectors for measurement and transition equations respectively. F is a transition matrix with appropriate number of rows and columns that relates the state vector at the previous time step $t-1$ to the state at current step t, in the absence of either a driving function or process noise. The matrix H' in the measurement equation relates the state to the measurement y_t. In practice, the matrices H' and F might be change with each time step or measurement, but here we assume coefficients in them as constants.

$$\begin{pmatrix} v_t \\ w_t \end{pmatrix} \sim WN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q & 0 \\ 0 & R \end{pmatrix}\right]. \tag{5.2.3}$$

The Kalman filter begins with a guess about the initial state vector since it is a recursive algorithm. This is the first and most difficult step to decide the appropriate initial values for the recursive filtering. The unconditional mean and variance used to forecast of X_1 are based on no observation of y_1 given as

$$\tilde{X}_{1|0} = E(X_1) = E(X_1|\mathcal{F}_0) = 0 \tag{5.2.4}$$

$$P_{1|0} = var(X_1) = var(X_1|\mathcal{F}_0). \tag{5.2.5}$$

For given $\tilde{X}_{1|0}$ and $P_{1|0}$ the next step is to calculate the conditional forecast of the measurement equation for the following day iterating on

$$\tilde{X}_{t|t} = \tilde{X}_{t|t-1} + P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'z_t - H'\tilde{X}_{t|t-1})) \qquad (5.2.6)$$

the MSE associated with this updated projection is given as

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1}. \qquad (5.2.7)$$

A sense of the error in our conditional prediction can be obtained since the true value of the measurement system y_t is now observed. The error is given as

$$err_t = y_t - A'z_t - H'\tilde{X}_{t|t-1}. \qquad (5.2.8)$$

Now the transition equation (5.2.2) is used to forecast X_t by using the Kalman gain matrix

$$K_t = FP_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}. \qquad (5.2.9)$$

The presence of the Kalman gain is what takes into account previous values of the explanatory variables and appropriately weights the previous prediction error and factors into the updated estimation of the coefficients. In other words, the Kalman gain allows the Kalman filter to adopt more quickly to structural change than would be the case under standard regression techniques.

In the next step, the unknown values of our state system for the next time period conditioning on the updated values for the previous period. The conditional expectations and conditional variance appears as

$$\begin{aligned}
\tilde{X}_{t+1|t} &= F\tilde{X}_{t|(t-1)} + FP_{t|(t-1)}H(H'P_{t|t-1}H + R)^{-1}(y_t - A'z_t - H'\tilde{X}_{t|t-1}) \\
&= F\tilde{X}_{t|(t-1)} + K_t(y_t - A'z_t - H'\tilde{X}_{t|t-1}) \\
&= F\tilde{X}_{t|t}, \qquad (5.2.10)
\end{aligned}$$

$$
\begin{aligned}
P_{t+1|t} &= F(P_{t|t-1} - P_{t|t-1}H(H'P_{t|t-1}H + R)^{-1}H'P_{t|t-1})F' + Q \\
&= FP_{t|t}F' + Q.
\end{aligned}
\tag{5.2.11}
$$

Those given steps must be iterated recursively for each discrete time step. At each step, a measurement-system prediction error (5.2.8) and a prediction error covariance matrix (5.2.11) appear. Under the assumption that measurement-system prediction errors are Gaussian, the likelihood function can be constructed as

$$
\begin{aligned}
L(\theta) &= \sum_{i=1}^{N} \ln[(2\pi)^{-\frac{n}{2}}(\det(H'P_{t|t-1}H + R))^{-\frac{1}{2}} e^{-\frac{1}{2}(err_t^{'}(H'P_{t|t-1}H)^{-1}err_t)}] \\
&= -\frac{nN\ln(2\pi)}{2} - \frac{1}{2}\sum_{i=1}^{N}[\ln(\det(H'P_{t|t-1}H + R)) + err_t^{'}(H'P_{t|t-1}H + R)^{-1}err_t].
\end{aligned}
\tag{5.2.12}
$$

The log-likelihood function (5.2.12) can then be maximized numerically with respect to the unknown parameters in matrix F, Q, A, H and R.

5.2.1 Impulse Response Function

We define the impulse response functions of the systems as the responses of the endogenous variables to one unit shock in the residuals of state vectors. One would often see responses to a "one unit" shock instead of the "one standard deviation" shock that we use.

The rest of this chapter includes empirical results for parameter estimations of three distinct macro-finance models for related Turkish government bond yields and macroeconomic data that were mentioned in the previous section.

5.3 Empirical Results for AP Approach

The orthogonality assumption between macro and latent factors brings on the upper right 6×3 corner and the lower left 3×6 corner of θ in (4.2.1) contains only zeros. This framework allows to focus on the impact of pure macroeconomic variables on yields since orthogonality vanishes uncertainties appear in the latent factors. This approach make it impossible to make a bidirectional characterization that is responses of macro factors on yield factors could not be explained.

5.3.1 Estimation Method

Estimation occurs by use of the maximum likelihood method via Kalman filter. A two step estimation procedure is constructed to reduce difficulties related with estimating a large number of factors in one step with maximum likelihood estimation. Firstly parameters in the state dynamics equation (4.2.1) and the coefficients of δ_0 and δ_1 corresponding to macro factors in the short rate dynamic equation (4.2.2) are estimated by use of ordinary least square method and the results appear in Table 5.7 and Table 5.8.

	I_t	RA_t	I_{t-1}	RA_{t-1}	I_{t-2}	$RAt-2$
I_{t-1}	0.3799	-0.0859	1.0000	0.0000	-0.0000	0.0000
	(0.0323)	(0.0267)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
RA_{t-1}	0.0772	-0.2431	-0.0000	1.0000	-0.0000	-0.0000
	(0.0310)	(0.0256)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
I_{t-2}	-0.2370	0.2686	0.0000	-0.0000	1.0000	0.0000
	0.0335	(0.0276)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
RA_{t-2}	-0.0702	-0.0615	-0.0000	0.0000	0.0000	1.0000
	(0.0330)	(0.0272)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
I_{t-3}	-0.1986	-0.3526	-0.0000	-0.0000	0.0000	-0.0000
	(0.0275)	(0.0227)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
RA_{t-3}	-0.1323	0.2022	-0.0000	-0.0000	-0.0000	0.0000
	0.0310	(0.0256)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Table 5.7: Reduced Form Θ Corresponding to Macro variables

	I_t	RA_t	I_{t-1}	RA_{t-1}	I_{t-2}	$RAt-2$
r_t	0.0001	0.0001	-0.0000	-0.0000	0.0001	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Table 5.8: Coefficients of δ_1 Corresponding to Macro variables

Both the state dynamics and short rate dynamics corresponding to latent factors are estimated by maximizing likelihood as a second step of the estimation procedure and results appearing in Table 5.9, Table 5.10 As appear in

	un_t^1	un_t^2	un_t^3
un_{t+1}^1	0.8945	0.0547	-0.0027
	(0.0310)	(0.1867)	(0.1822)
un_{t+1}^2	0.1387	0.3802	-0.0898
	(0.0274)	(0.1649)	(0.1610)
un_{t+1}^3	0.3106	-0.2843	0.2784
	(0.0253)	(0.1525)	(0.1489)

Table 5.9: Reduced Form Θ for Latent variables

Unobs1	Unobs2	Unobs3
2.93	-0.1900	-0.0900
(0.0015)	(0.0013)	(0.0016)

Table 5.10: Short Rate Equation Parameters δ_1 For Latent Factors ($\times 100$)

Table 5.10 the first unobservable factor is the most persistent factor determine the short rate dynamics. Table 5.12 displays estimation results for the risk premia parameters.

Table 5.11 provides a necessary prior knowledge about the strength of the AP model by use of its estimated yields. This table answer the question "How are the estimated yields fit to actual yield series?". As clearly seen, the AP approach provides a misfit estimated yield series. This table says that parameter estimates of AP approach could not provide reliable results to make comment on them but we present them for just giving information.

		mean		
	y_2	y_6	y_{12}	y_{24}
Actual	20.2988	24.0012	27.0897	29.1636
AP	13.9500	12.7009	12.4772	12.3786

Table 5.11: Comparing Estimated Yields For AP Model With Actual Data

	λ_0			λ_1		
		INF	RA	Unobs1	Unobs2	Unobs3
INF	0.0106	0.0136	0.0114	0.0000	0.0000	0.0000
RA	0.0135	0.0092	0.0159	0.0000	0.0000	0.0000
Unobs1	0.0042	0.0000	0.0000	0.0012	0.0083	0.0003
Unobs2	0.0076	0.0000	0.0000	0.0121	0.0061	0.0154
Unobs3	0.0157	0.0000	0.0000	0.0010	0.0175	0.0194

Table 5.12: Prices of Risk λ_0 and λ_1

The effects of each state factor on yield curve can be determined by the weights B_n' in (4.2.15). The loadings are as in Table 5.13

As Table 5.13 shows, the loadings on macroeconomic factors representing yields are very small and this means according to this model, macroeconomic factors have no dominant characteristics that determine the variation in the yield curve. The most powerful factor that represent yields is $Unobs_1$ factor, which has almost identical coefficients corresponding to all yields. The second powerful factor are $Unobs_2$ factors in determining yields and the $Unobs_3$ factor has small loadings as macroeconomic factors. These loadings say that the model with a large number of parameters to be estimated with limited Turkish data sample could not provide persistent results. This is why the state factors and the term structure of interest rates are unrelated. In addition, a series of regressions are run to provide the relation between macro factors and the latent yield factors estimated from the corresponding model. These regressions display that macroeconomic variables and latent factors are unrelated. The results appears in Table 5.14.

These results say that the AP approach is not suitable for Turkish data since it needs a large number of parameters to be estimated with a small number of

	y_2	y_6	y_{12}	y_{24}
Inf	0.0002	0.0002	0.0002	0.0002
RA	0.0001	0.0001	0.0001	0.0001
$Unobs_1$	-0.3078	-0.3082	-0.3083	-0.3083
$Unobs_2$	-0.0329	-0.0319	-0.0320	-0.0320
$Unobs_3$	0.0040	0.0040	0.0040	0.0040

Table 5.13: Weights of State Factors on Yields

	Inf	RA	$Unobs1$	$Unobs2$	$Unobs3$	Adj R^2
Unobs1	0.0000 (0.0000)	0.0000 (0.0000)	1.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.9999
Unobs2	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	1.0000 (0.0000)	0.0000 (0.0000)	0.9999
Unobs3	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	0.0000 (0.0000)	1.0000 (0.0000)	0.9999

Table 5.14: Regressions Unobservable Factors on State Factors

data set.

5.3.2 Impulse Response Analysis

We examine the orthogonal impulse response functions of yields with maturities 2, 12, 24 month against a one standard deviation shock to macroeconomic factors providing the relationship between macroeconomic variables and yields where Figure 5.5 presents the results, however we could not verify meaningful results by use of estimated parameters related with the term structure. Initial response of a 2 months yield appears as 3.41 basis points where yields with 12 month and 24 month to mature have negative initial responses with -25 and -2.6 basis points against a shock to inflation factor. These initial responses implies that inflation factor effect the yield with 12 month to mature stronger than yields with 2 and 24 month to mature.

Impulse response functions of 2, 12 and 24 month yields against one standard deviation shock to real activity factor appear in Figure5.6. Long term 24 month

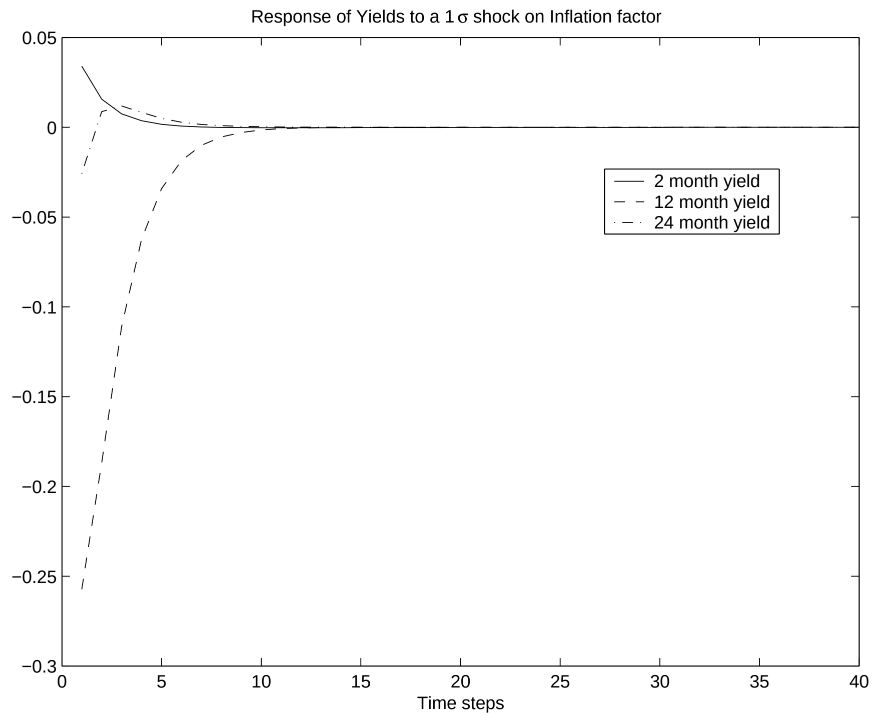

Figure 5.5: Orthogonal IRF : 1σ shock to Inflation

yield shows the highest initial response in magnitude with -36 basis points and medium term 12 month yield follows with a 27 basis points initial response. This says that a change in real activity factor effect yields with long term yields rather than short terms yields according to this model.

5.4 Empirical Results for DRA Approach

In previous chapters it is mentioned that three key macroeconomic variables CU, WPI and overnight interest rates additional to latent factors are used as the minimum set to capture the dynamics of economy and term structure in DRA approach.

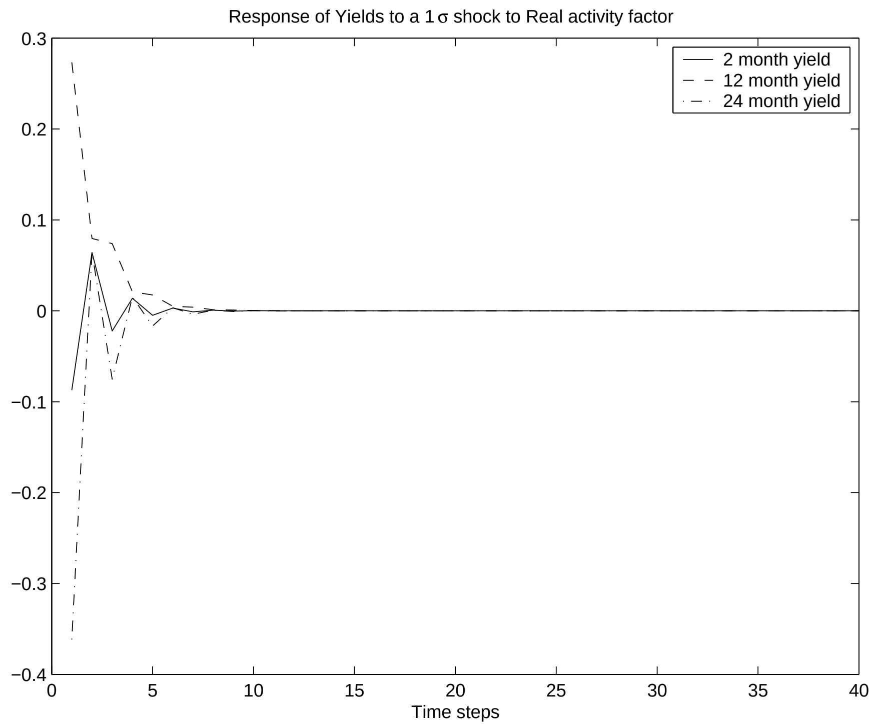

Figure 5.6: Orthogonal IRF: 1 σ shock to Real Activity

5.4.1 Estimation Method

The term structure model according to DRA approach in state space form is appeared in (4.3.20) and (4.3.19). Maximum likelihood method via Kalman filter is used to estimate parameters and the unobservable yield curve factors which we called as level, slope and curvature on that state-space form. Table 5.15 presents the estimate of parameters in transition equation which provides the interaction between macroeconomic and term structure dynamics. This table provides that the overnight lending interest rate which include in the model as monetary policy instrument factor have negligible loadings on other state factors. On the other hand real activity factor plays an important role in determining all of the latent factors which means any changes on real activity level effects all of the latent yield curve factors in one step ahead. The slope factor is a significant characteristic factor for one step ahead real activity with respect to the Table 5.15. The loading of slope factor on inflation is also significant coefficient in transition matrix of the state equation. Estimated Q matrix coefficients are displayed in Table 5.16. Covariances between real activity-slope and real activity-level factors are asserted coefficients.

	L_{t-1}	S_{t-1}	C_{t-1}	RA_{t-1}	INF_{t-1}	O/N_{t-1}
L_t	0.0989 (0.0198)	-0.3674 (0.0189)	-2.7616 (0.0210)	-0.2560 (0.0208)	0.3764 (0.0206)	0.0197 (0.0198)
S_t	-0.0160 (0.0193)	0.3150 (0.0204)	-0.4846 (0.0197)	-0.3752 (0.0183)	0.1917 (0.0211)	0.0035 (0.0175)
C_t	0.0591 (0.0232)	-0.0790 (0.0240)	0.3765 (0.0181)	0.3608 (0.0179)	-0.0357 (0.0204)	-0.0034 (0.0145)
RA_t	-0.0593 (0.0204)	0.2385 (0.0175)	0.4642 (0.0206)	0.0779 (0.0197)	-0.4171 (0.0175)	0.0014 (0.0222)
INF_t	-0.0052 (0.0233)	0.2923 (0.0196)	-0.0201 (0.0201)	0.0948 (0.0198)	0.1277 (0.0237)	0.0048 (0.0177)
O/N_t	19.9156 (0.0225)	2.9412 (0.0195)	19.8886 (0.0211)	13.5777 (0.0222)	-5.7744 (0.0251)	-0.8520 (0.0208)

Table 5.15: Transition Matrix Parameter Estimates for DRA approach

	L_t	S_t	C_t	RA_t	INF_t	O/N_t
L_t	0.0297 (0.0192)	-0.1039 (0.0214)	0.0124 (0.0211)	0.0248 (0.0197)	0.0101 (0.0206)	0.0002 (0.0191)
S_t		0.5753 (0.0213)	0.0264 (0.0241)	-0.1257 (0.0192)	-0.0812 (0.0198)	-0.0010 (0.0179)
C_t			0.0797 (0.0205)	0.0380 (0.0193)	-0.0341 (0.0223)	-0.0015 (0.0194)
RA_t				0.4399 (0.0213)	-0.0109 (0.0192)	0.0010 (0.0208)
INF_t					0.9649 (0.0231)	0.0018 (0.0237)
O/N_t						0 (0.0203)

Table 5.16: Estimated Q matrix for DRA approach

By use of estimated parameters according to DRA approach we present estimated yield curve mean and actual data mean in Table 5.17 to provide a prior knowledge about strength of DRA model. It can be easily seen that estimated yields corresponding to DRA provides almost an exact fit for yields with 2, 6 and 12 month to mature. This says estimation results are reliable about DRA models by use of Turkey data.

	mean y_2	y_6	y_{12}	y_{24}
$Actual$	20.2988	24.0012	27.0897	29.1636
DRA	20.0898	25.0626	27.0117	27.7131

Table 5.17: Comparing Estimated Yields For DRA Model with Actual Data

The loadings of state factors on four different yields in equation (4.3.19) are appeared in Table 5.18. Like AP approach the loadings on yields corresponding to macroeconomic factors are very small. This says level and slope factors determine the yield curve. Loadings on yields corresponding to slope factor is more persistent on short term yields with 2 months to mature and levels off as time to maturity increase on the other hand level factor has identical effect on each yield with different maturities. These results are presented as characteristics of coefficients in Nelson-Siegel representation.

5.4.2 Impulse Response Analysis

Interrelations between yields macroeconomic factors are provided via orthogonal impulse response functions. We consider responses of macroeconomic factors against shocks to latent factor, latent factor responses against shock to macro variables, yield curve responses against shock to macro variables and latent variables to understand the dynamics between macroeconomic fundamentals and yields according to DRA approach in following analysis.

	L_t	S_t	C_t	RA_t	INF_t	O/N_t
y_2	1	0.0769	0.0017	0.0000	0.0003	-0.0001
y_6	1	0.0256	0.0052	0.0012	-0.0006	-0.0014
y_{12}	1	0.0128	0.0102	0.0007	-0.0036	-0.0006
y_{24}	1	0.0063	0.0202	0.0011	-0.0006	-0.0005

Table 5.18: State Factors Loadings on Yields for DRA approach

Macroeconomic variables have negligible responses against shocks on level factor. Real activity, inflation and overnight interest rate factors shows -0.25, -0.1 and 0.89 basis points initial response to a one standard deviation shocks on level factor, respectively.

Initial response of real activity is -25 basis points due to shocks on slope factor but levels off corresponding to magnitude in continuing steps. Inflation factor shows 4.5 basis points response and overnight interest rates has a -13 basis point initial response against shocks on slope factor according to DRA model. These results show that slope factor is related closer with real activity rather than inflation and overnight interest rates as Figure 5.7 presents.

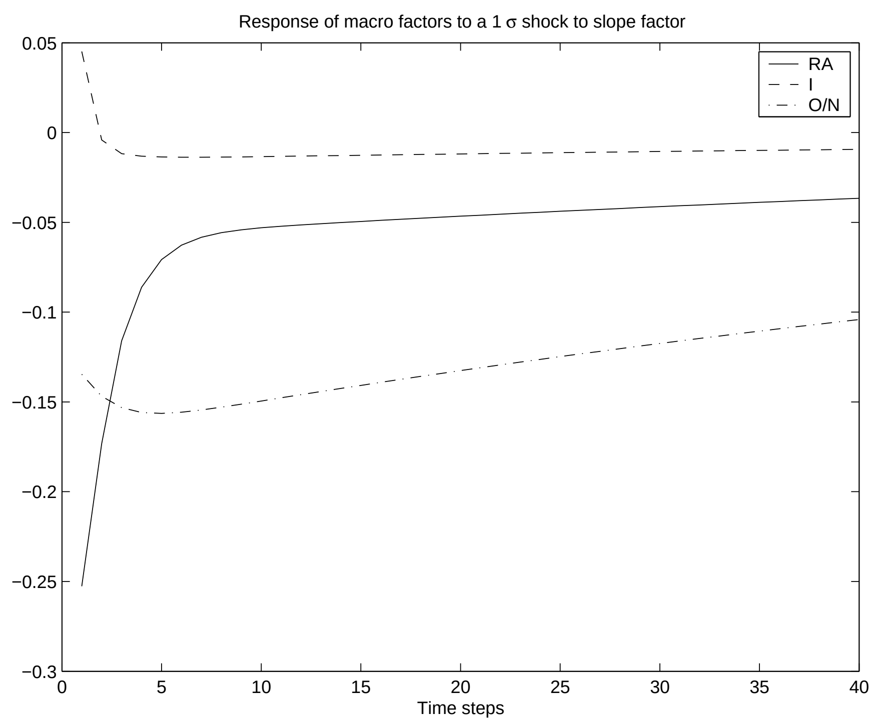

Figure 5.7: Orthogonal IRF : 1 σ Shocks to Slope Factor

A one standard deviation shock on curvature factor arise an initial response for real activity of 27.74 basis points. On the other hand inflation and overnight interest rates initial responses are -0.9 and -5.17 basis points respectively. Figure 5.8 presents these impulse response functions. The overall macroeconomic factors responses to a shock on curvature factor show that curvature factor is related with real activity stronger than other factors as occurs when a shock apply on slope factor. These two results provide that real activity factor is closely related with latent factors.

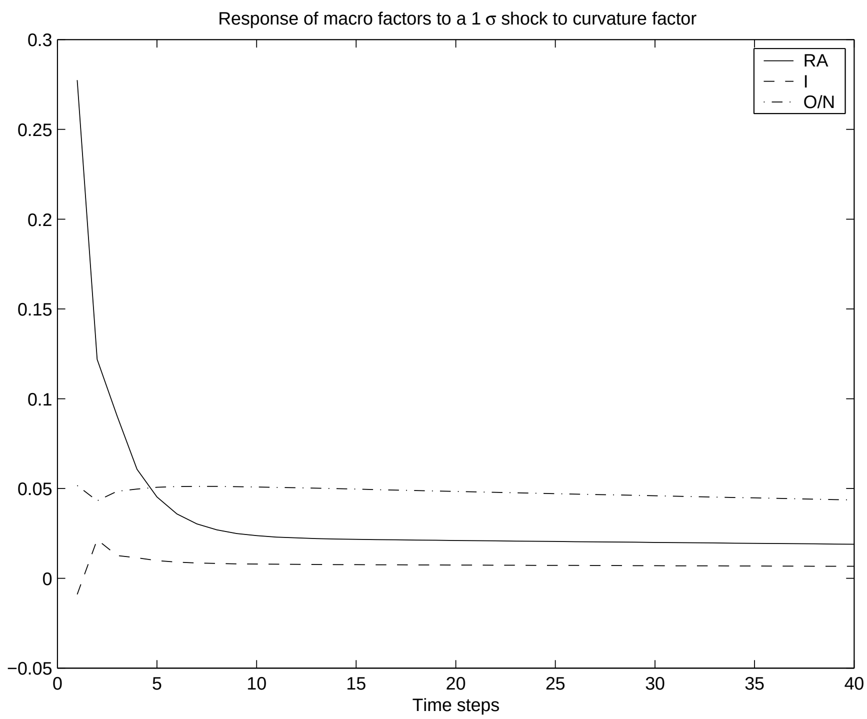

Figure 5.8: Orthogonal IRF for Macro Factors: 1 σ Shocks to Curvature Factor

As Figure 5.9 presents, one standard deviation shock to inflation factos represents identical initial impulse responses on latent factors as 8 basis points. Figure 5.10 displays the impulse response functions of latent factors against shock on RA. Level factor has an initial response with -5.35 basis points against a one standard deviation shock to real activity factor. On the other hand slope and curvature factors have -1.7 and 6.95 basis points initial responses respectively. Initial response of one standard deviation shocks on overnight rate are negligible.

Impulse response functions of yields with maturities 2, 12 and 24 months against shocks to macroeconomic variables are examined. As Figure 5.11 presents

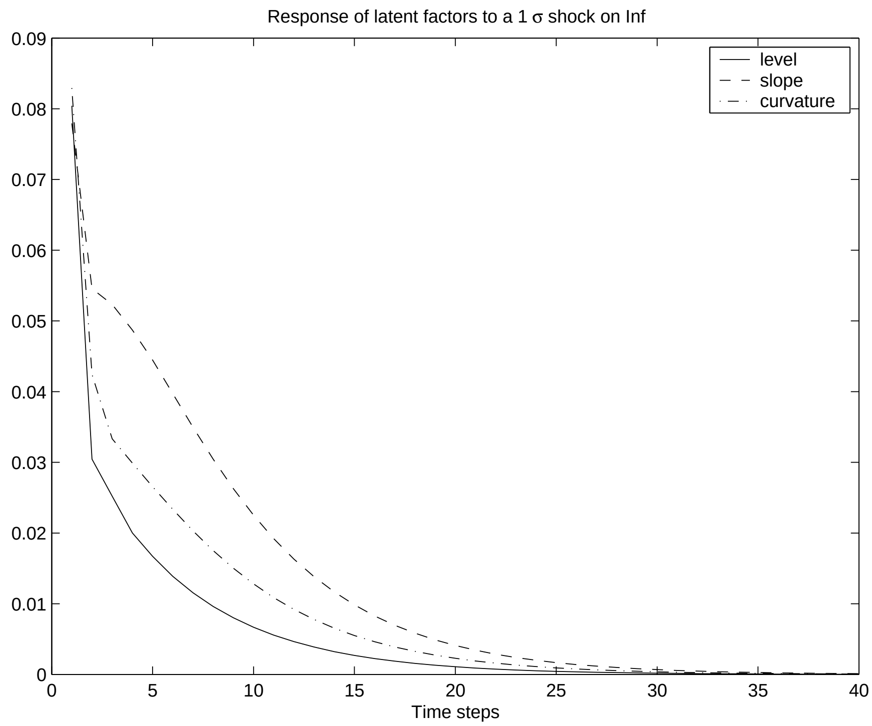

Figure 5.9: Orthogonal IRF for Latent Factors: 1 σ Shocks to Inflation

the 12 month yield has -11 basis points initial response where 2 month and 24 month yield have 2.85 and 4.81 basis points initial responses respectively against a one standard deviation shock to inflation. These results say that a shock on inflation effect the medium term rather than short and long term yields. Short term yield with 2 month to mature has a small initial response against shocks on real activity and overnight interest rates but 12 month and 24 month yields have considerable initial response to shocks on real activity which are 29 and -36 basis points respectively.

Impulse response of term structure against shocks on latent variables to check the model for Turkish data are also examined since we have knowledge about the effects of latent factors from Nelson-Siegel on term structure of interest rates. The initial impulse responses against a one standard deviation to slope factor for 2, 12 and 24 month yields are 3.93, 10.28 and 1.91 basis points respectively. These results coincident with the loadings of slope factor

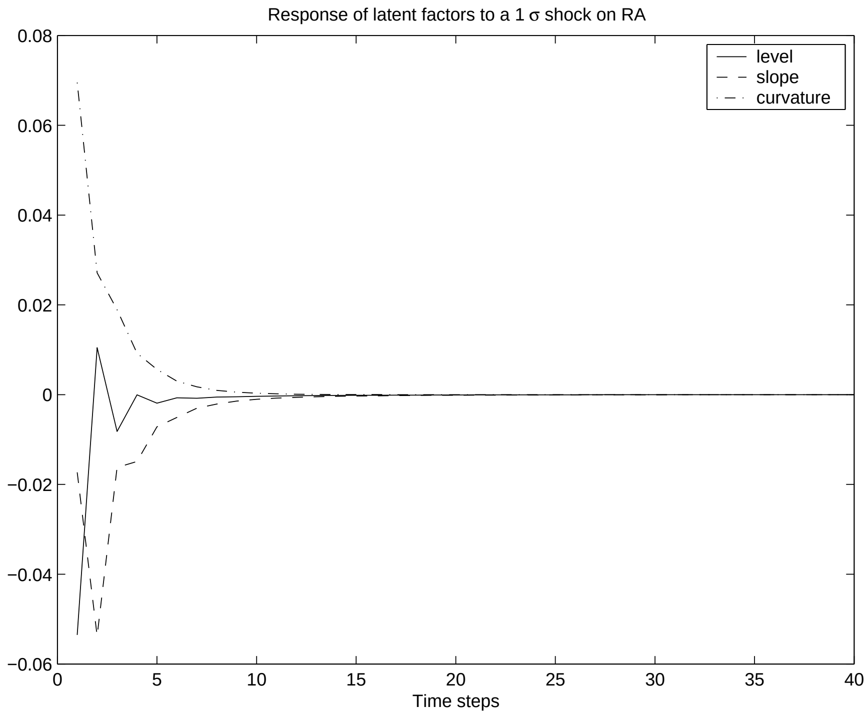

Figure 5.10: Orthogonal IRF for Latent Factors: 1 σ Shocks to Real Activity

in Nelson-Siegel representation since the loading says a change in slope factor effect short term yields more than long term yields. This impulse response functions appear in Figure 5.12 It was mentioned that curvature factor have little effect on very short and long yields and higher effect on medium term yields with respect to loadings comes from Nelson-Siegel representation. According to DRA model for Turkish data the impulse response of yields comes up with the same aspect as Figure 5.13 presents. The initial impulse response of 2 month and 24 month yields are -1.25 and 2.09 basis points respectively on the other hand the medium term yield 12 month has an initial response against shocks to curvature factor 8.62 basis points. Therefore we can say that DRA approach is an applicable method for Turkish data since presents expected results and relations.

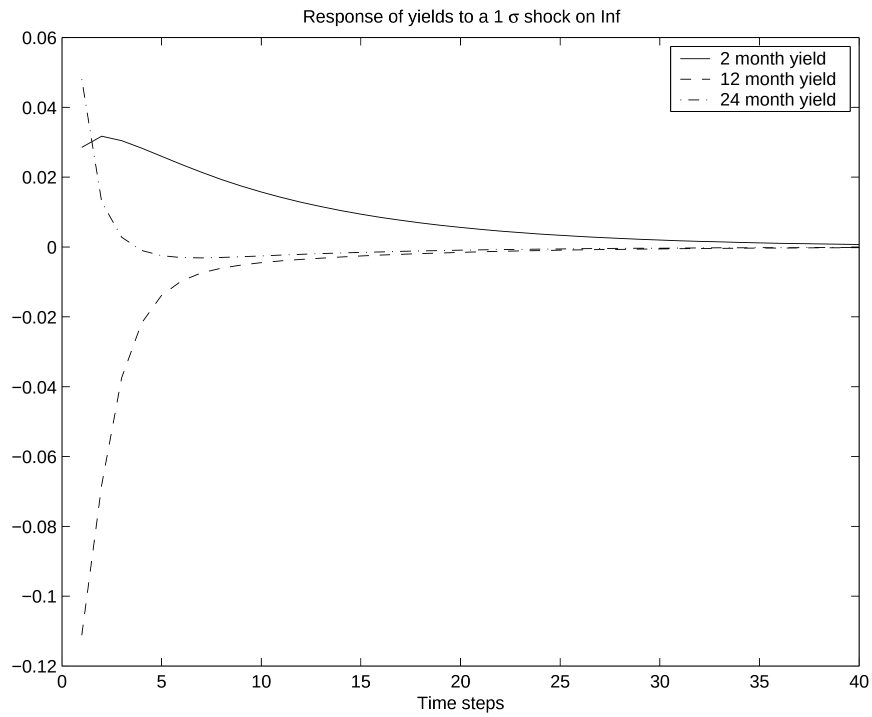

Figure 5.11: Orthogonal IRF for Yields : 1 σ Shocks to Inflation

5.5 Empirical Results for the Original Macro-Finance Model

5.5.1 Estimation Method

The term structure model is constructed by given processes for macroeconomic variables and the short term rates in previous chapter. The state-space system (4.4.35) and (4.4.38) is estimated by use of maximum likelihood estimation method via Kalman filter. The model is tested on a data set containing four yields with different maturities and two macroeconomic variables as a measure of inflation and real activity. Estimation results for coefficients in transition and measurement system appear in Table 5.19 with standard deviations in paranthesis.

Table 5.20 presents the estimation results for the transition equation (4.4.38)

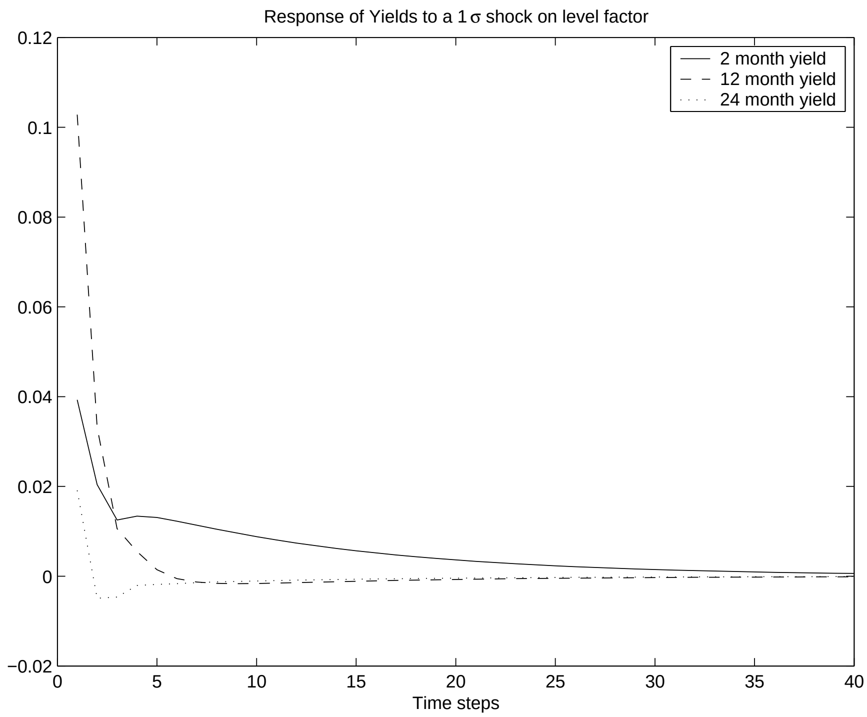

Figure 5.12: Orthogonal IRF for Yields : 1 σ Shocks to Slope Factor

α_1	α_2	α_3	β_π	β_γ	β_f
0.0030	-0.0034	0.1524	1.5040	-0.0631	-0.1293
(0.0394)	(0.0440)	(0.0332)	(0.0308)	(0.0068)	(0.0314)

Table 5.19: Estimated Coefficients

and the estimation results for measurement equation (4.4.34) is presented in Table 5.21.

Coefficients of latent factor f_{t-1} is more effective on determining the long run expectation of future short rates rather than macroeconomic factors as Table 5.20 presents. On the other hand measurement equations coefficients implies that the long term expectations of short term rates and the latent factors f_t undertake a dominant role in determining the yields rather than macroeconomic factors with respect to these results. According to the results, macroeconomic factors effect term structure of interest rates identically for all terms. That is a shock on macroeconomic factors affect all yields with different maturities iden-

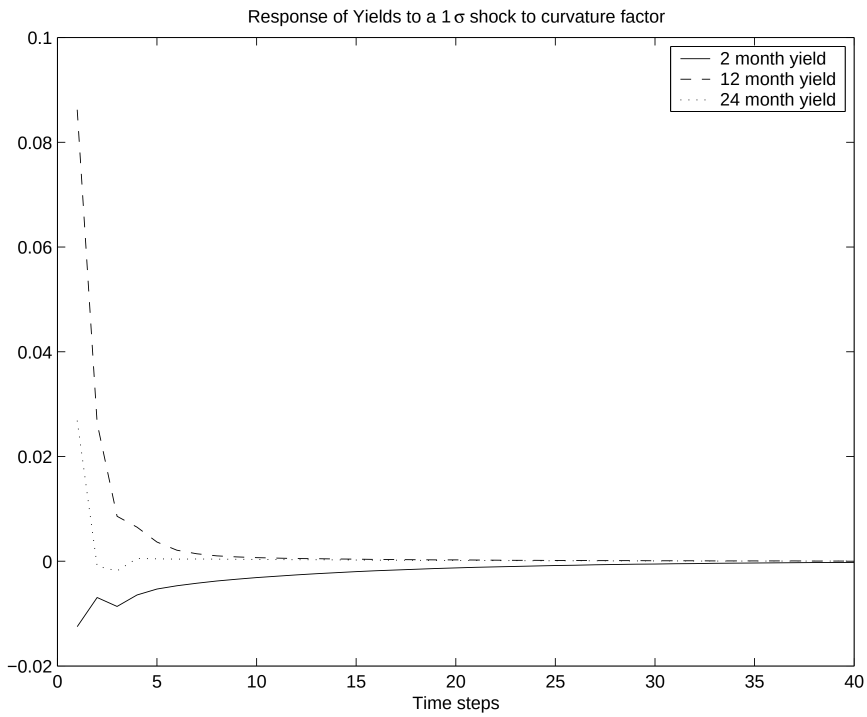

Figure 5.13: Orthogonal IRF for Yields : 1 σ Shocks to Curvature Factor

	r^*_{t-1}	$\pi_{t-1} - \pi^*$	$\gamma_{t-1} - \gamma^*$	f_{t-1}
r^*_t	1	0.0030	-0.0034	0.1524
$\pi_t - \pi^*$	0	-0.5040	0	0
$\gamma_t - \gamma^*$	0	0	1.0631	0
f_t	0	0	0	0.8707

Table 5.20: Estimated F Matrix for the Original Macro-Finance Approach

tically. Another approach examining the dynamic interaction between macro factors and yield curve present by use of orthogonal impulse response functions.

By using the estimated parameters according to the original macro-finance approach we present estimated yield curve mean and actual data mean in Table 5.22 to provide a prior knowledge about strength of the original macro-finance model. It can be easily seen that estimated yields corresponding to original model provides almost an exact fit for yields with 6, 12 and 24 month to mature. This says estimation results are reliable for the original macro-finance model by use of Turkey data.

	r_t^*	$\pi_t - \pi^*$	$\gamma_t - \gamma^*$	f_t
y_2	0.9889	0.0030	-0.0034	0.1520
y_6	0.9944	0.0030	-0.0034	0.1522
y_{12}	0.9963	0.0030	-0.0034	0.1723
y_{24}	0.9972	0.0030	-0.0034	0.1723

Table 5.21: Estimated H' Matrix for The Original Macro-Finance Approach

		mean		
	y_2	y_6	y_{12}	y_{24}
Actual	20.2988	24.0012	27.0897	29.1636
DRA	18.5100	24.0028	26.7492	28.9472

Table 5.22: Comparing Estimated Yields For Original Macro-Finance Model With Actual Data

5.5.2 Impulse Response Analysis

This section provide the impulse response functions of yields against a one standard deviation shock to state factors. Figure 5.14 presents the impulse response of yields against a one standard deviation shock to r_t^* and easily seen from the figure that 2 month and 12 month yields initial responses are 42 basis points and 48 basis points respectively. On the other hand the long term yield 24 month to mature has a 5 basis points initial response. These results imply that the latent factor r_t^* effect yields with shorter time to maturities. The impulse response of yields against a one standard deviation shock to central tendencies of inflation factor is presented in Figure 5.15 and shows longer term yields with 12 and 24 months to mature have negative initial responses with -9 and -11.5 basis points respectively and the short term yield initial response appear as 2.2 basis points. Figure 5.16 presents the impulse response function of yields against a shock to central tendencies of real activity factor. It shows initial response of short term yield is negligible. Yields with 12 and 24 months to mature show initial responses almost same in magnitude but different in signs with 34 and -37 basis points. Initial responses of yields against a shock to the latent factor f_t, as Figure 5.17 presents, are strong for 2 and 12 month

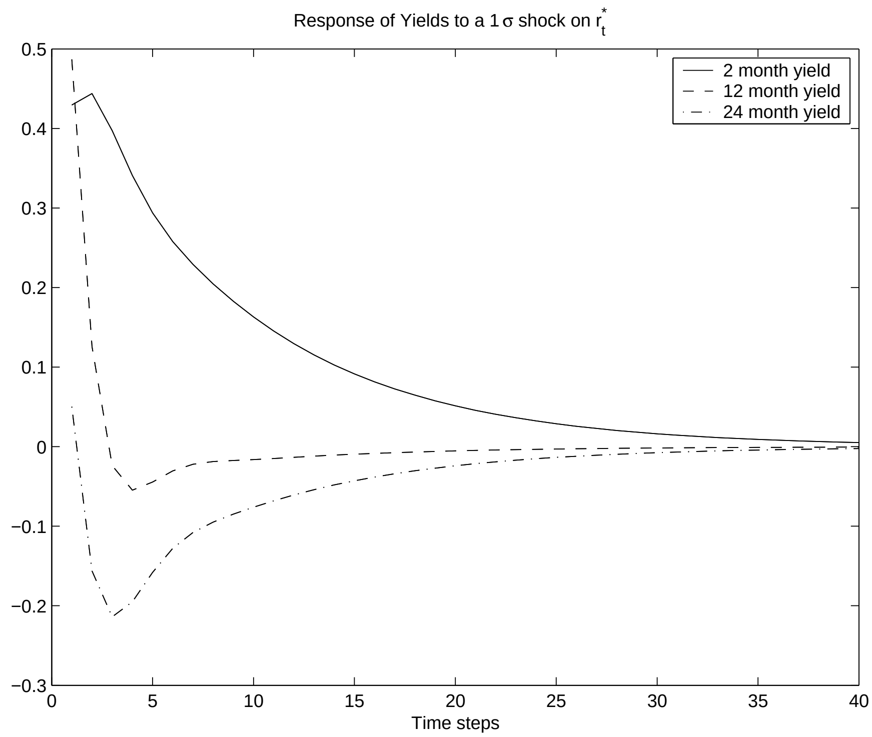

Figure 5.14: Orthogonal IRF for Yields : 1 σ Shocks to r_t^*

yields which are -240 and -280 basis points respectively. 2 month yield response functions has an upward sloping shape. Initial response for 24 month yield is 30 basis points and incline to 110 basis points at fourth step.

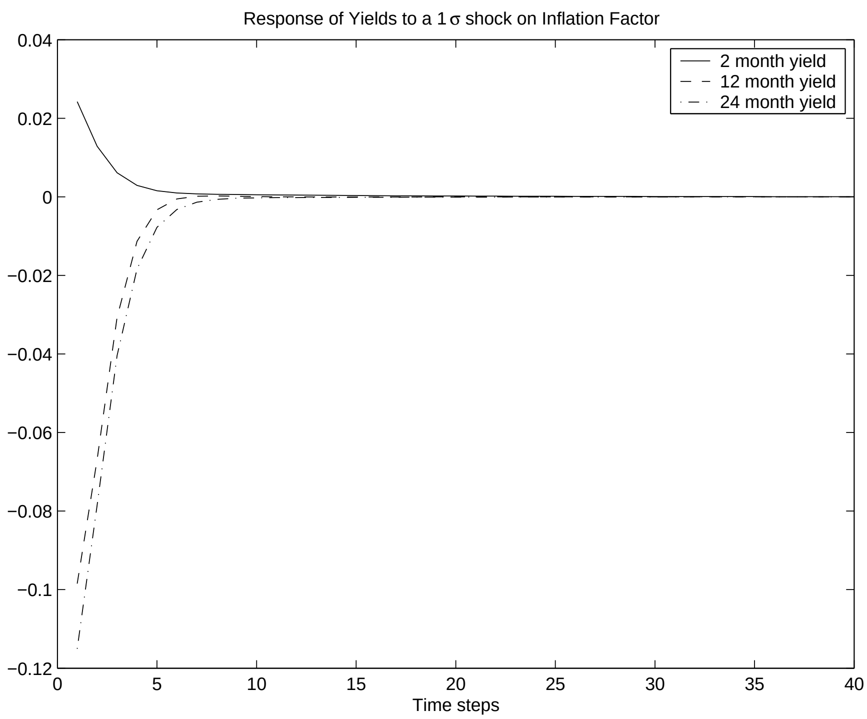

Figure 5.15: Orthogonal IRF for Yields : 1 σ Shocks to $\pi_t - \pi^*$

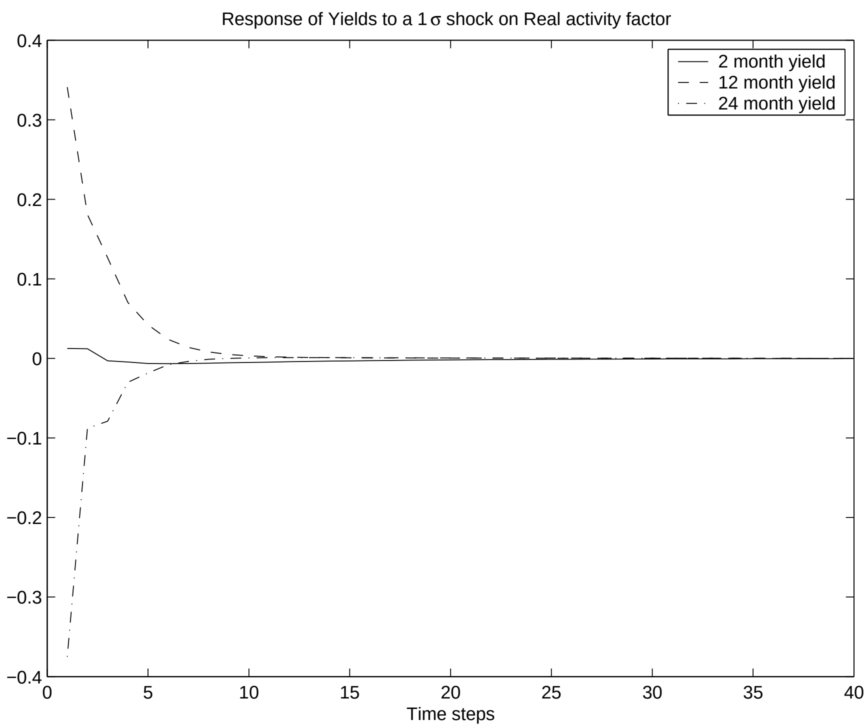

Figure 5.16: Orthogonal IRF for Yields : 1 σ Shocks to $\gamma_t - \gamma^*$

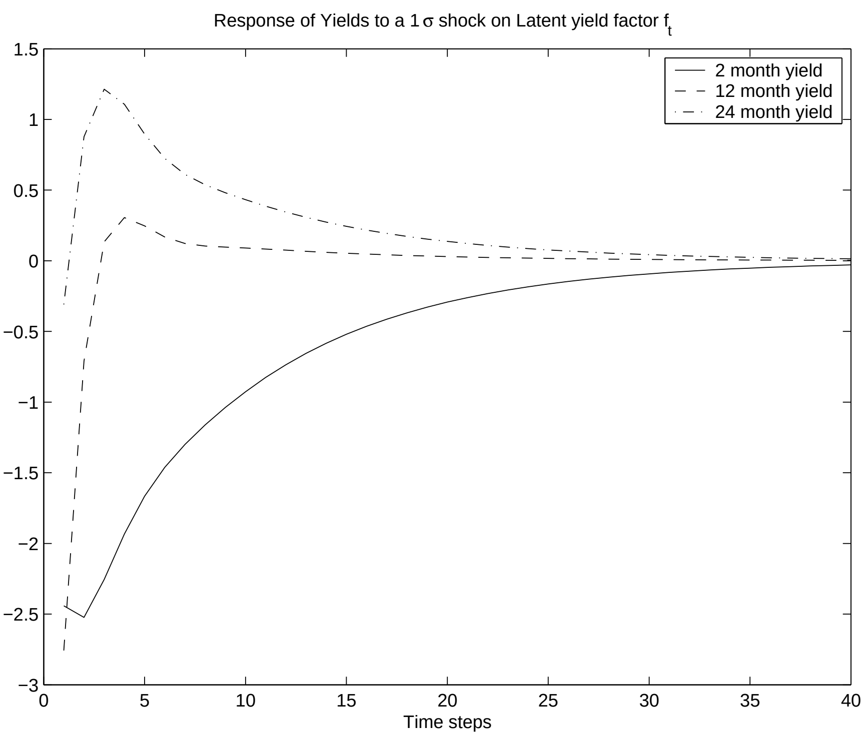

Figure 5.17: Orthogonal IRF for Yields : 1 σ Shocks to f_t factor

Chapter 6

CONCLUSION

In first part of this thesis the traditional model literature about term structure modelling has been reviewed and macro-finance models started by the seminal article Ang and Piazzesi(2003) has been discussed. Three macro-finance term structure models are explained and estimated throughout this paper after giving the importance and advantages of macro-finance modelling against traditional models. These macro-finance approaches estimate the joint dynamics of bond yields and macroeconomic variables together with affine term structure models. For AP approach, a well performing estimated yield curve that coincide with the historically observed term structure of interest rate by using estimated parameters could not be constructed. This may be a result of large number of parameters to be estimated with a small scale data sets. For DRA approach a better performing estimated yield curve had been constructed despite not indeed no-arbitrage restriction. Although macroeconomic variables have less impact on term structure rather than latent factors, macroeconomic variables have related with yield curve as following. RA factor effect medium and long term yields rather than short term yields and inflation factor effect medium term yields at most. On the other hand overnight lending rate has almost negligible effect on yield curve.

Estimation results show that the original macro-finance model better perform the yield curve rather than DRA approach. As in two macro finance models the macroeconomic variables have smaller impact on yield curve than latent factors r_t^* and f_t. r_t^* factor is dominant to determine the short term yields since it is constructed as the policy rule according to forward looking Taylor rule. Corresponding to estimation results inflation factor effects the long term yields rather than short terms, as real activity factor do.

Despite the small-scale application the results are quite encouraging and by a larger set of macroeconomic variables and interest rate data the models expected to be even more convincing.

References

[1] Andrew, A., Piazzesi, M., Dong, S., *No Arbitrage Taylor Rules*, Working Paper , University of Chicago, Chicago, (2005).

[2] Andrew, A., Bekaert, G., *The term structure of real rates and expected returns*, Working Paper, Columbia University, (2005).

[3] Andrew, A., Piazzesi, M., *A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables*, Journal of Monetary Economics, 50, 745-787 (2003).

[4] Andrew A., Piazzesi, M., Wei, M., *What does the yield curve tell us about GDP growth?*, Journal of Econometrics, 131, 359-403, (2006).

[5] Bekaert, G., Hodrick, R., *Expectations hypothesis tests*, Journal of Finance, 56, 115-38, (2001).

[6] Brigo D., Mercurio F., *Interest Rate Models Theory and Practice*, Springer, (2001).

[7] Cochrane, J. H., Piazzesi, M., *Bond Risk Premia*, American Economic Review, 95, 138-160, (2005).

[8] Cochrane, J. H., Piazzesi, M., *The Fed and Interest Rates: A High-frequency Identification*, American Economic Review, 92, 90-95, (2002).

[9] Constantinides, G., *A theory of the nominal term structure of interest rates*, Review of Financial Studies, 5, 531-552, (1992).

[10] Cox, J. C., Ingersoll, J. E., Ross, S. A., *A theory of the term structure of interest rates*, Econometrica, 53, 385-407, (1985).

[11] Dai, Q., Singleton, K., *Term structure dynamics in theory and reality*, Review of Financial Studies, 16, 631-678, (2003).

[12] Dai, Q., Singleton, K., *Specification analysis of affine term structure models*, Journal of Finance, 55, 1943-1978, (2000).

[13] Dai, Q., Singleton, K., *Expectation puzzles, time-varying risk premia, and affine models of the term structure*, Journal of Financial Economics, 63, 415-441, (2002).

[14] Dewachter, H., Lyrio, M., *Macro Factors and the Term Structure of Interest Rates*, Journal of Money Credit and Banking, 38(1), 119-140, (2006).

[15] Dewachter, H., Lyrio, M., Maes, K., *A joint model for the the term structure of interest rates and the macroeconomy*, Journal of Applied Econometrics, 21(4), 439-462, (2006).

[16] Diebold., F.X., Piazzesi, M., Rudebusch, G.D., *Modeling Bond Yields in Finance and Macroeconomics*, American Economic Review, 95, 415-430, (2005).

[17] Diebold, F.X., Rudebusch, G.D., Aruoba, B., *The Macroeconomy and the Yield Curve: A Dynamic Latent Factor Approach*, Journal of Econometrics, 131, 309-338, (2006).

[18] Diebold, F.X., Li, C., *Forecasting the Term Structure of Government Bond Yields*, Journal of Econometrics, 130, 337-364, (2006).

[19] Duffee, G.R., *Term premia and interest rate forecasts in affine models*, Journal of Finance, 57, 405-443, (2002).

[20] Duffie, D., Kan, R., *A yield-factor model of interest rates*, Mathematical Finance, 6, 379-406, (1996).

[21] Durbin, J., Koopman, S.J., *Time Series Analysis by State Space Methods*, Oxford University Press, Oxford, (2001).

[22] Evans, C.L., Marshall, D.A., *Monetary policy and the term structure of nominal interest rates: Evidence and theory*, Carnegie-Rochester Conference Series on Public Policy 49, 53-111, (1998).

[23] Evans, C.L., Marshall, D.A., *Economic determinants of the term structure of nominal interest rates*, Working paper, Federal Reserve Bank of Chicago, Chicago, IL, (2001).

[24] Fisher, M., Gilles, C., *Estimating exponential-affine models of the term structure*, Working paper, Federal Reserve Atlanta, (1996).

[25] Fisher, M., Gilles,C., *Around and around: The expectations hypothesis*, Journal of Finance, 53, 365-83, (1998).

[26] Hamilton, J.D., *Time series analysis*, Princeton University Press, Princeton, New Jersey, (1994).

[27] Harvey, A.C., *Time Series Models*, MIT Press, Cambridge, (1981).

[28] Heat, D., Jarrow, H., Morton, A., *Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claims Valuation*, Econometrica, 60, 77-105, (1992).

[29] Ho S.Y., Lee S., *Term structure movements and pricing interest rate contingent claims*, Journal of Finance, 41, 1011-1029, (1986).

[30] Hördahl, P., Tristani,O., Vestin, D., *A Joint Econometric Model of Macroeconomic and Term Structure Dynamics*, Journal Econometrics, 131, 405-444, (2006).

[31] Hull, J., *Options, futures, and other derivatives* (4th edition), Englewood Cliffs, NJ: Prentice Hall, (2000).

[32] Hull J., White A., *Pricing Interest Rate Derivative Securities*, The Review of Financial Studies, 3, 573-592, (1990).

[33] James J., Webber N., *Interest Rate Modelling*, Wiley (2002).

[34] Knez, P.J., Litterman, R., Scheinkman,J., *Explorations Into Factors Explaining Money Market Returns*, Journal of Finance, 49(5), 1861-1882, (1994).

[35] Litterman, R., Scheinkman, J., *Common factors affecting bond returns*, Journal of Fixed Income, 1, 54-61, (1991).

[36] Nelson, C.R., Siegel, A.F., *Parsimonious modelling of yield curves*, Journal of Business, 60, 473-489, (1987).

[37] Piazzesi, M., *An econometric model of the yield curve with macroeconomic jump effects*, NBER Working paper, no:8246, (2001).

[38] Piazzesi, M., *Bond Yields and the Federal Reserve*, Journal of Political Economy, 113(2),311-344, (2005).

[39] Piazzesi, M., *Affine Term Structure Models*, in: Ait-Sahalia, Y., Hansen, L.P., Handbook of Financial Econometrics, (2004).

[40] Rudebusch, G. D., Wu, T., *A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy*, Federal Reserve Bank of San Francisco working paper, 2003-17, (2004).

[41] Rudebusch, G. D., Wu, T., *The Recent Shift in Term Structure Behavior from a No -Arbitrage Macro -Finance Perspective*, Federal Reserve Bank of San Francisco working paper, 2004-25, (2004).

[42] Taylor, J.B., *Discretion versus policy rules in practice*, Carnegie-Rochester Conference Series on Public Policy, 39, 195-214, (1993).

[43] Vasicek, O., *An equilibrium characterization of the term structure*, Journal of Financial Economics, 5, 177-188, (1977).

[44] Wu, T., *Macro factors and the affine term structure of interest rates*, Federal Reserve Bank of San Francisco Working Paper, 2002-06, (2005).

[45] Wu, T., *Stylized Facts on Nominal Term Structure and Business Cycles: An Empirical VAR study*, Applied Economics, 35, 901-906, 2003.

[46] Wu, T., *Macro factors and the affine term structure of interest rates*, Working Paper, Federal Reserve Bank of San Francisco, (2001).

[47] Wu, T., *Monetary policy and the slope factor in empirical term structure estimations*, Working Paper, Federal Reserve Bank of San Francisco, (2001).

[48] Hull, J., White, A., *Pricing interest rate derivative securities*, Review of Financial Studies 3, 4, 573-92, (1990).