NEW DIRECTIONS IN THE DIRECTION OF TIME

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ABSTRACT

NEW DIRECTIONS IN THE DIRECTION OF TIME

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This thesis analyzes the direction of time problem in the framework of philosophy of science. The radiation arrow, Newtonian arrow, thermodynamic arrow and quantum mechanical arrow have been studied in detail. The importance of the structure of space-time concerning direction of time is emphasized.

Keywords: Direction of Time, Arrow of Time, Past Hypothesis

ÖZ

ZAMANIN YÖNÜNDE YENİ YÖNELİMLER

Bağcı, Gökhan Barış
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Bu tez zamanın yönü problemini bilim felsefesi çerçevesinde çözümlemektedir. Işıma oku, Newton oku, termodinamik ok ve kuvantum oku ayrıntılı bir şekilde çalışılmıştır. Uzay-zaman yapısının zamanın oku açısından önemi de vurgulanmıştır.

Anahtar Kelimeler: Zamanın Yönü, Zaman Oku, Geçmiş Hipotezi

To My Family

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

The concept of time has always been an interest in many a philosopher's mind. Many different forms of analysis have been tried to provide a deeper understanding of this concept. Some tried their hands with grandiose metaphysical questions whereas for some others it was a matter of psychological experience only. Some others tried to understand it better with scientific analysis.

Concerning its nature, the first point which is considered is whether time can be defined as an independent reality or something relational (Sol, p. 40). Aristotle thought that time is not independent of change (Aristotle, *Physics*, Bk. IV, pp. 20-22). On the other hand, for Hume, it is impossible to conceive a time when there is no change in any real existence (Hume, 1888, p. 40). Van Fraassen (1970, p. 15) considers the change as the means through which we become conscious of passing time.

The observation that time may be associated with change nevertheless does not entail that it does not have an existence of its own. The philosopher Sydney Shoemaker (1969, p. 64) reaches this conclusion with the help of a Gedanken experiment: he thinks of a time interval in which no change occurs and therefore thinks that Aristotelian argument is far from establishing the independent reality of time.

N. Rotenstreich takes a rather Kantian approach in this subject matter. He considers time as a form of relation of succession (Rotenstreich, 1958, p. 51). He then distinguishes pure time from empirical time which is not identical with the changes that take place in time. He asserts that pure time does not flow since it is only a form used for the cognition of reality echoing the Kantian standpoint which states time is a form of flow free from flow (ibid, pp. 60-61).

McTaggart distinguished two views of time and called them A series and B series. Classifying each position as Past, Present and Future is the so- called A series whereas labeling positions in time on the basis of earlier/later relations is called B series (McTaggart, 1908, p. 24). A series emphasizes the transient relation. On the other hand, B series emphasizes the permanence of events in time. Earlier/later relations are fixed once an event E_1 is earlier/later than a second event E_2. A nice example for A series is the death of Atif Yilmaz. His death was once in the future. Then, it became present. Finally, it has been past now.

McTaggart first showed that change in time requires the existence of the A series which then led to contradiction. He then rejected reality of time due to this contradiction. He later rejected the reality of B series, too. This is tantamount to saying the following

> Nothing is really present, past, or future. Nothing is really earlier or later than anything else or temporarily simultaneous with it. Nothing really changes. And nothing is really in time. (ibid. p. 34)

This was the view adopted by Parmenides already. Hugh Mellor (Mellor, 1981, p.92) criticized McTaggart's view stating that A series language (i.e., tensed sentences) can easily be translated into B series language (tenseless sentences). A past event can be identified with "earlier than", or present event can be with "simultaneous with". Since one can be translated into another, the ontology is all that matters. Denbigh writes

> Neither the A-theory nor the B-theory … is properly speaking a *scientific* theory-not at least in Popper's sense. There appear to be no empirical means by which either of them might be refuted. (1981, p. 54)

However, whichever theory we are tempted to pick, the time experienced by us is certainly unidirectional. We perceive it as flowing from past towards future, touching upon present. This is what is called direction of time or arrow of time in the related literature.

We will begin our investigation of time direction from a philosophy of science point of view. Along this line, the beginning has been marked by the seminal work of Hans Reichenbach i.e., *The Direction of Time* (Reichenbach, 1956). He first considered points on a straight line. He quickly saw that these points possess an asymmetric and transitive order under "to the left of/to the right of" relations and deduced that this order is not directed at all. He then investigated the directionality in the case of real numbers (ibid, p. 26). Reichenbach considered them to have a serial order as in the case of points on a straight line but found them directed as opposed to points on a straight line being not directed. He asserts

> The square of a positive number is positive, and the square of a negative number is also positive. We therefore can make this statement for the class of [negative] real numbers: Any number which is the square of another number is larger than any number which is not the square of another number. (ibid, p. 26)

Then, he considers the relation between the real numbers and time, deeming both as having direction and order.

Denbigh too thinks that the points on a straight line have no direction but real numbers do have direction. However, according to Denbigh, there is a major difference between the relation "greater than" and "later than". He states this as:

> There is no logical necessity that all change in the universe, including the ongoing of clocks, will not suddenly cease. (ibid, p. 63)

One important criticism along these lines against Reichenbach has been given by Mehlberg. He argued that not only "greater than" has the properties listed by Reichenbach in order to label the relation possessing serial order, but "smaller than” too has these properties i.e., asymmetry, transitivity and connectedness. The same holds for the relations “after” and “before”. Another issue raised by Mehlberg is that even if Reichenbach was to be found successful in his analysis, he cannot be taken to have proved that one is more privileged than another. All he could show was that one direction is just different than the other.

Adolf Grünbaum’s objection (1967, 1974) was straightly to the bull’s eye: Asymmetry in the order was already bringing unidirectional nature in both points on a straight line and case of real numbers independent of whether this order has intrinsic or extrinsic basis i.e., whether it is based on reference to an external viewer or not (ibid, pp. 214-215).

For our purposes, what we must learn from all these is that mere logical analysis of the subject is not enough in order to comprehend the privileged status of one direction over the other one. We can, by means of a logical analysis, see why there will be two directions, but that is all!

We take this to be an impetus enough to consider the direction of time from a philosophy of science point of view. And as far as science is considered, physics will be the science we are talking about. There are two reasons for this: firstly, as all hands agree upon, it is the developments in physics which have the most important bearings on our subject matter i.e., directionality of time. Another candidate would be to consider psychology but that field has registered developments in no way near the ones in physics. Second, it is the usual way to take for someone who is studying the direction of time. In fact, it is possible nowadays to see philosophers of science who distinguish themselves from the other philosophers of science calling themselves philosophers of physics. There are even some departments which offer philosophy of physics as a separate program such as University of Pittsburgh and University of Oxford. Naturally, the distinction between fundamental science and philosophy of science then blurs. It is usual to see some philosophers of physics to publish in journals of physics as well as journals of philosophy. I hope this explanation will be taken as a frank and genuine excuse for many equations found in the text of this dissertation although it is written for the sole purpose of being a philosophy of science dissertation.

The dissertation is organized as follows: First, in Chapter 2, we ponder about the arrow of time in Newtonian mechanics. Usually, this theory is taken to be completely reversible because of Newton' second law, but this stand has been challenged by some physicists and philosophers of science. We will consider the recent developments headed by Keith Hutchison (1993) as the core of this Chapter but it must be noted that similar ideas have been set forth before. For example, as early as 1956, Schlegel (1956) noted that classical mechanics must be assumed to provide time asymmetric solutions if the forces have explicit dependence on time (ibid, p. 382). Mehlberg (1961) too agreed on this issue and took side with Schlegel. Karl Popper (1956), on the other hand, argued against Schlegel's considering only one, single point particle. According to Popper, we must consider the whole universe. He states this as follows:

> If we reverse the velocity of one of the planets, at the time t_f and at the position x_f , the planet will clearly not reverse its path precisely… If, however, we reverse the motions of all the planets in the system, then the force will be the same; the system is reversible. (ibid, p. 382)

We will not repeat these historical remarks in Chapter 2 since in one way or another, they echo in the present debates on direction of time. What is important though is to be able to see these problems without prejudice and this is what we will attempt in Chapter 2. As we will see, uncertainties in measurement will also play an important role in our argumentation. One conclusion which is inevitable is related to how we assess the theories' ontological structure and ontological commitments.

Chapter 3 will deal with arrow of time in classical electrodynamics. This is the so called arrow of radiation which puzzled many. A point must be made here in order to explain that whatever is covered in Chapter 3 can be expanded to include any wave related phenomenon since classical electromagnetism is founded on wave equation too. Therefore, we will not specify whether we are dealing with electromagnetic waves or water waves for that matter.

The starting point of Chapter 3 is Maxwell equations since these four (two in the covariant formulation) equations give us all we need (not exactly, but more on his later) in order to solve any problem in electromagnetic phenomena. They are assumed to be time reversal invariant but the way it is done is subject to objection. This brings us to the very question about how time reversal invariance must be defined. Is there a general definition we can use or must we consider each case as a particular case? But, if the latter is the case, then how can we justify our use of different definitions in different cases? Another surprising result of this Chapter is to take us back to Zeno Paradox and teach us more about it as well as time reversal of the states. As we will see, even the philosophy of mathematics in the form of calculus and non-standard analysis will be invited to the court in order to testify for/against time reversal invariance and direction of time in physics as far as the radiation phenomena are considered. The next step in investigating Maxwell equations will be investigating them in a relativistic manner to shed some more light on the issue. As we will see, what is problematic in the non-relativistic case can be easily answered within a relativistic scheme.

The textbook answers to the main riddle of the arrow of radiation have almost always been based on causality. This is our aim in Section 3 in Chapter 3. We will see that a straightforward answer is not easy to be provided.

Then, we turn our attention to solutions of Maxwell equations instead of equations themselves. The main issue, almost a riddle from the Delphi temple, is that Maxwell equations provide us two kinds of solutions, namely retarded and advanced, but we observe only the retarded one in nature. This apparent asymmetry is taken to be providing a direction of time. Whether this is so will be discussed in Section 4 of Chapter 3.

The physicists John Archibald Wheeler and Richard Feynman (1945) have shown that one can formulate *classical* electrodynamics in a symmetric manner. Their symmetric treatment of the subject is now called “Absorber Theory of Radiation” and attracts attention of many philosophers of science. Their final result is embraced fully now by almost all philosophers of science. This is the so called origin of arrow of radiation being of thermodynamic nature. Indeed, this has been the view shared by distinguished scientists such as Einstein and Feynman. The current debates surrounding this issue bring the end of Chapter 3.

Chapter 4 is devoted to the study of thermodynamic arrow. The origin of this asymmetry is found in H theorem, or in other words Second Law of thermodynamics. We first discuss H theorem and see that it leads to some paradoxes. Then, we turn our attention to generalized H theorem and show that it is free of the paradoxes which ordinary H theorem faces. Indeed, this is due to the transition in perspective from the single particle point of view to Gibbs ensemble view. Generalized H theorem forms Section 2 of Chapter 4 while objections raised against H theorem are investigated fully in Section 3. Section 4 is about Reichenbach’s seminal work *The Direction of Time* and his branch structures. Furthermore, important critiques by Sklar and Earman have been explained and argued.

One important critique against Reichenbach is that he did not take the gravitation into account, be it Einsteinian or Newtonian gravity. In this sense, temporal orientability is explained and its relation to arrow of time in general has been discovered. In fact, the very idea of gravity, let aside the form of space-time we are in makes it impossible to talk about “isolation” as far as thermodynamic systems are considered. Since a lot of thermodynamic arguments (even the famous Second Law) include the idea of isolation at the core, the gravitational effects have to be taken into consideration for a full understanding of the subject.

All these considerations lead to the Past Hypothesis, i.e. the hypothesis that the universe has a low entropy initial state. We will see that this in itself forms the explanation needed to fulfill many a philosopher's needs. Of course, then we face with the dilemma of accepting an initial condition as lawlike. The two main problems for a deeper study of the Past Hypothesis are that the universe has been come into existence only once which means that a repeated experiment is impossible. Second, it forms a singularity and going beyond it is impossible. Any explanation which will explicate the Past Hypothesis must explain something beyond this singularity and this does not make sense as we argue in Chapter 4. Leaving Past Hypothesis unexplained is another problem since this is not an ordinary initial condition. It is an event with low probability. Although not all low probability events require some explanation, as Callender puts it, this is a rather bizarre result to digest. Callender (1996) puts it as

> Empiricists who think the sole goal of scientific inquiry is empirical adequacy will not find any epistemic reason to prefer dynamical explanations to special initial condition explanations if the two candidates are both empirically adequate. The models used to describe the phenomena are what count. Whether one chooses to pick out the class of relevant models with laws alone or with laws plus boundary conditions does not matter, and, indeed, may be viewed as merely a difference in language. Scientific realists, by contrast, are not solely constrained by empirical adequacy in their search to find epistemic reasons to prefer a theory, and therefore, they may have reasons to prefer dynamical explanations to non-dynamical ones. (ibid, pp. 232-233)

According to the quote above, the empiricist view can be seen to be the way out, but we must first understand how *special* this initial condition is. Again, a quote by Callender (2003), although it is relatively long, will explain the situation:

> Suppose that God or a demon informs you of the following future fact: despite recent cosmological evidence, the universe is indeed closed and it will have a 'final' instant of time; moreover, at that final moment, all 49 of the world's Imperial Faberge eggs will be in your bedroom bureau's sock drawer. You are absolutely certain that this information is true. All of your other dealings with supernatural powers have demonstrated that they are a trustworthy lot. After getting this information, you immediately run up to your bedroom and check the drawer mentioned. Just as you open the drawer, a Faberge egg flies in

> through the window, landing in the drawer. A burglar running from the museum up the street slipped on a banana peel, causing him to toss the egg up in the air just as you opened the drawer. After a quick check of the drawer, you close it. Reflecting on what just happened, you push your bed against the drawer. You quit your job, research Faberge eggs, and manage to convince each owner to place a transmitter on his egg, so that you can know the eggs whereabouts from the radar station in your bedroom. Over time you notice that, through an improbable set of coincidences, they are getting closer to your house. You decide to act, for the eggs are closing in and the news from astronomers about an approaching rapid contraction phase of the universe is gloomy. If-somehow-you can keep the eggs from getting into the drawer, perhaps you can prevent the world's demise. (Already eight eggs are in the drawer, thanks to your desire to peek and your need for socks.) Looking out your window, you can actually see eggs moving your way: none of them breaking laws of nature, but each exploiting strange coincidences time and again. Going outside, you try to stop them. You grab them and throw them away as far as you can, but always something –a bird, a strange gust of wind-brings the egg back. Breaking the eggs has proved impossible for the same kinds of reasons. You decide to steal all the eggs, seal them in a titanium box and bury it in Antarctica. That, at least, should buy some time, you think. Gathering all the eggs from outside, you go upstairs to get the ones from the drawer. The phone rings. It is a telemarketer selling life insurance. You decide to tell the telemarketer that their call their call is particularly ill-timed and absurd, given that the universe is about to end. Absent-mindedly, you sit down, start speaking, put the eggs down in the open bureau drawer… and the universe ends. (ibid, p. 1)

As Callender notes, the Past Hypothesis is pretty much like this. As he sates, Past Hypothesis is like trillions of eggs to be in your bedroom miraculously. This is why the initial condition is so special: because it has a very low probability.

Chapter 5 is about quantum theory and emergence of arrow of time in that specific theory. The presentation is independent of interpretations such as Bohmian or Copenhagen as much as possible. A brief investigation of the arrow of time in quantum electrodynamics is given, too.

CHAPTER 2

THE ARROW OF TIME IN CLASSICAL MECHANICS

2.1 Rudiments

In physics, the title "classical mechanics" represents Newtonian mechanics together with all its versions, i.e., the one due to Lagrange or Hamilton. Newtonian mechanics rests on three laws:

1) Law of Inertia: If there is no external force acting on a body, it will stay at rest if it is initially at rest or it will remain in motion with constant velocity if it is initially in motion.

2) Second Law: Newton's second law simply states that mass times acceleration is equal to net force exerted on the body or to write it as an equation it reads

$$\vec{F} = m\vec{a} \tag{2.1}$$

3) Third Law: For every action, there is a reaction which is of equal magnitude with the action but in opposite direction to it.

These three laws form the main skeleton of the Newtonian dynamics. Through these laws, together with the initial conditions, whole dynamics of a physical system can be computed for all times.

Classical mechanics has alternative formulations (Goldstein, 1950). We can cite, among some others, the one by Lagrange and Hamilton in particular. The version due to Lagrange is based on solving Lagrange equations which is given by

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0, i = 1,2,\ldots n \tag{2.2}$$

where the Lagrangian is defined as the difference of kinetic energy and potential energy and it is a function of position, velocity and time.

A second formulation is given by Hamilton and is based upon Hamiltonian instead of Lagrangian. Hamiltonian reads

$$H = \sum_i \dot{q}_i p_i - L(q, \dot{q}, t). \tag{2.3}$$

Hamiltonian is a function of position, momenta and time. The differential of H is given by

$$dH = \sum_i \frac{\partial H}{\partial q_i} dq_i + \sum_i \frac{\partial H}{\partial p_i} dp_i + \frac{\partial H}{\partial t} dt, \tag{2.4}$$

And from the definition of Hamiltonian in Eq. (2.3), we obtain

$$dH = \sum_i \dot{q}_i dp_i - \sum_i \dot{p}_i dq_i - \frac{\partial L}{\partial t} dt, \tag{2.5}$$

Also, making use of the following equation

$$\frac{\partial L}{\partial q_i} = \dot{p}_i. \tag{2.6}$$

Comparison of Eqs. (2.4) and (2.5) gives us the following set of equations, which is called canonical equations of Hamilton

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \dot{p}_i = -\frac{\partial H}{\partial q_i}, \frac{\partial L}{\partial t} = -\frac{\partial H}{\partial t}. \tag{2.7}$$

They constitute a set of 2n first order equations of motion instead of n second order equations of Lagrangian formalism.

Since we will deal with some physical systems in classical framework in this chapter, it is appropriate to study them using Newtonian equations of motion. We will base our discussion on Eq. (2.1), which is Newton’s second law since this is the form used in the literature of philosophy of science debates in general but also make some remarks relevant to Lagrangian formalism in subsequent pages.

As an application of classical mechanics, let us solve the problem of simple harmonic oscillator applying Eq. (2.1) to a mass-spring system. Imagining that the motion is taking place along the y axis, we can write the force acting as

$$\vec{F} = -k\vec{y}\,, \tag{2.8}$$

which is called Hooke's law and y denotes the vertical displacement. If we substitute the equation (2.8) into Eq. (2.1), we obtain

$$\ddot{y} + \frac{k}{m} y = 0\,, \tag{2.9}$$

where k is spring constant, m is the mass and double dots in the superscript denotes second time derivative. The term $\sqrt{k/m}$ is called angular frequency and will be denoted by ω_0. If the initial conditions are given by

$$y\,(0) = A, \qquad \dot{y}(0) = 0\,, \tag{2.10}$$

in which reference time is taken to be zero, the general solution to Eq. (2.9) will be given by

$$y(t) = ACos(\,\omega_0 t). \tag{2.11}$$

Now, we will solve Eq. (2.1) again but with damping. Damping can be caused in many ways. Even the twisting of the wire in the spring itself causes some damping. Other sources of damping might be due to a viscous medium in which mass-spring system is set in motion. The damping force is taken to be proportional to velocity and if we take the proportionality constant to be positive, Eq. (2.9) now becomes

$$m\ddot{y} + c\dot{y} + ky = 0\,, \tag{2.12}$$

where c is called damping constant. The solution consists of three cases. The first case applies when c^2-4km > 0. This case is called *overdamping* and its solution is given by

$$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}\,. \tag{2.13}$$

Since both r_1 and r_2 are negative numbers for overdamped case, we can safely say that the motion dies out with time i.e.,

$$\lim_{t\to\infty} y(t) = 0 . \tag{2.14}$$

The second case is called *critical damping* and happens to be the case whenever c^2-4km = 0. Since the characteristic equation now has a single root, the general solution reads

$$y(t) = (c_1 + c_2 t)e^{-rt} , \tag{2.15}$$

Where the single root r is equal to (-c/2m). Again, we have

$$\lim_{t\to\infty} y(t) = 0 . \tag{2.16}$$

The last case is the case for c^2-4km < 0. This is called *underdamping*. The general solution to Eq. (2.12) now becomes

$$y(t) = e^{-ct/2m}[c_1 Cos(\beta t) + c_2 Sin(\beta t)] , \tag{2.17}$$

where β is given by

$$\beta = \frac{\sqrt{4km - c^2}}{2m} . \tag{2.18}$$

Since c and m are both positive, we have again the condition

$$\lim_{t\to\infty} y(t) = 0 . \tag{2.19}$$

In all three cases in which there is damping, the motion dies out eventually. In this case of underdamping though, the motion is oscillatory, because of the sine and cosine terms but is not periodic due to exponential term.

Generally, scientists and philosophers of science alike considered that classical mechanics is time reversal invariant. For example, the Nobel laureate Anthony Leggett (1987) puts it as follows:

> Consider, first classical Newtonian mechanics. Newton’s first and third laws clearly do not refer to the sense of time, and would have identical forms in a time-reversed system. As to his second law, the acceleration which appears in it is the second derivative of position with time; so, if we reverse the sense of time, the velocity (and momentum) is reversed, but the acceleration is

> unchanged, and thus the second law also is the same in the time-reversed system. (Leggett, 1987, p. 149)

At least, this has been the case until it has been challenged by Keith Hutchison in a series of papers published in the *British Journal for the Philosophy of Science*. His ideas are explained and criticized below.

2.2 Hutchison's Defense

What happens to an insulated bar of iron , warm at one end, and cold at the other? Left to itself, heat will be transferred in such a way that there will be a common temperature throughout the iron bar. In other words, heat will be transferred from hot to cold parts of the iron bar. Now, one can never *witness* reverse change to occur spontaneously to isolated bars. Another example of same kind is our model of simple harmonic oscillator studied above through Eqs. (2.8)-(2.11). The solution includes a cosine term which is time symmetric. In short, the simple harmonic oscillator model is time symmetric.

Armed with the solutions to the differential equations in each of the three cases regarding the simple harmonic oscillator with friction term, we see that they are time asymmetric. This claim can be approached in two ways: first, intuitively, it is clear that the reverse motion cannot be witnessed. One cannot get almost oscillatory behavior out of an equation of motion such as $\lim_{t\to\infty} y(t) = 0$. Second, the solutions to the model with friction either contains sine terms which are not symmetric under the mapping $t \to -t$ or exponential terms which describe a very fast decay. The reverse motion would represent a motion with exponential increase in velocity which is impossible in a medium with friction. Concerning this case, Hutchison (1993) remarks

> Classical mechanics acts as a sort of algorithm, enabling an intelligent creature capable of solving differential equations, to calculate the full motions in terms of the initial conditions and the forces acting. *Whether the resulting motion is reversible or not depends on the latter, the forces*, part of the specifications of the system, setting out the details of how its components interact. The algorithm, the mechanics, is quite neutral on reversibility it is just as compatible with the forces that produce irreversible behavior as with those that produce reversibility. As a rough rule-of-thumb: The motions will be reversible if the forces depend only on geometric configurations; but when the forces vary with time, or the velocities of the interacting components, then irreversible motion results if the dependence is asymmetric (that is, if replacement of t by –t in the function specifying the dependence changes the force acting). (Hutchison 1993, p. 311)

For many physicists, engineering calculations are too mundane to ponder about since it is somehow more favorable to physicists to practice what is called *fundamental physics*. According to their view, what is seemingly a force in the Newtonian universe does not exist at the fundamental level but are only phenomenal. This view in itself is a reductionist view and open to attacks of the kind of Loschmidt paradox: how can we have macroscopic irreversibility in nature if we have only reversible constituents on the micro scale?

Concerning the same example above, i.e. the motion in a viscous medium, the physicist P. C. W. Davies (1977) seeks the way out in a more detailed account of motion in terms of the environment. Davies states that irreversibility will fade away once we take *the whole system* into account. He says:

> … The motion of the body is slowed by the communication of kinetic energy to the medium atoms in the form of *heat*. It follows that if the motions of the individual atoms are also reversed then, because of the invariance of the laws

> of physics governing the atomic interactions, each collision will be reversed, causing a cooperative transfer of momentum to the large body, which would then become exponentially accelerated. (P. C. W. Davies, 1977, p. 26)

What Davies means by this reasoning is that we need to take into account all degrees of freedom i.e., whole system, instead of taking only single degree of freedom (the body or particle in motion) into account. If we can reverse the motion of all degrees of freedom, then the reversibility will be obtained. Of course, in practice, this is utterly impossible which means that we have irreversibility due to our own limitations in one way or another.

Hutchison accepts this as a serious objection but seeks the solution in terms of idealizations and simulations of science. In many cases in physics, we simply ignore the effect of other molecules surrounding our particle of interest. Then, we get reversible equation out of this condition, and nobody would object to this since this is merely an idealization, our own simulation, and this fact alone cannot be used to invalidate classical mechanics. When we consider friction or air resistance, something similar happens indeed: We simply replace real air or real viscous medium by some terms which will somehow simulate the air resistance or friction. Hutchison remarks that once we simulate the motion as such, we are free of the obligation of thinking what the real air is doing. Saying so, he insists that irreversible simulation too is a part of classical mechanics.

Another problem with the explanation made by Davies is that it explains away all non-mechanical but irreversible processes, too. In the conduction of heat along an iron bar for example, if we were to reverse the motions of all the particles, then we would be able to observe a transfer of heat from cold to hot end. But, we never observe this kind of motion. Therefore, if one buys Davies' explanation, we do not have even non-mechanical irreversibility.

At this point, it is useful to pose the question: Why, then, is one assumed to believe in the reversibility of classical mechanics? The answer lies in the distinction of conservative / nonconservative forces. In Eq. (2.2), we wrote down the Lagrangian to be defined as the difference of kinetic energy and potential energy as Tolman (1938) did in his "The Principles of Statistical Mechanics". This consideration simply assumes that the forces are independent of time and velocity so that one can talk about reversibility of the Lagrangian formalism. Indeed, Tolman states it explicitly

> It is possible to look at any system from a point of view that would make this [presumption] true. (Tolman, 1938, p. 102)

This is a personal belief on the part of Tolman and many others like him. This is the belief that the fundamental forces governing the dynamics of the universe are fundamentally conservative. One can see a similar statement in the works of Feynman (1963), too. In his famous *Lectures*, he states that there are no non-conservative forces. The main reason for physicists to insist on conservative forces is because of the well-known results of the Noether theorem. According to Noether theorem, there is a relation for a conserved quantity and a law of invariance. In our case, whenever time reversal invariance holds, the conservation of energy is implied. In a sense, to consider the possibility of the *real* existence of the non-conservative forces in general, is tantamount to saying that the principle of the conservation of energy fails to be valid. One delicate point is to understand the fact that classical mechanics would not be affected by all these issues since it is valid whether Noether theorem holds or not. In other words, one can still have a non-reversible mechanics and embrace this view point without caring about Noether's theorem. This is an ontological position to be taken by the physicists, and Newton himself is among the physicists who accepted this position happily though due to other reasons.

Hutchison defended his case with a second paper in 1995. He made use of concepts like stability and uncertainty in order to show that classical mechanics can lead to irreversibility. His argument of stability can be understood if one is familiar with nonlinear physics and chaos. He first formulates the usual reversibility for a conservative and deterministic system.

> We imagine a (conservative and deterministic) mechanical system evolving from some initial state A to some final state A_T in time T. Consider now the same system (i.e. precisely the same collection of material objects interacting in precisely the same manner) evolving from A^*_T, the precise 'time-reversal' of the state A_T, viz. the state of the system in which all positions are left unchanged but all velocities reversed. Will the state of the system at time T later, viz. $(A^*_T)_T$ be just the state A*, time reversal of A? (Hutchison, 1995a,p. 223)

If the answer to the question above is YES, then we can say that the motion of the system is time reversal invariant, otherwise it is not time reversal invariant. Now, let us assume that the answer we provide to the question above is YES. Then, we can ask a similar question: Can we say YES if we apply the same reasoning to a point B in the neighborhood of A? In other words, when we reverse the motion of the system, will it evolve to a point near A*? Of course, if all the initial configurations are on equal footing as far as our equations are concerned, then we must be able to answer Yes to this question, too. But, the sole fact that the system is conservative does not ensure this, since as we know now very well, even the conservative systems can exhibit chaotic systems. What we understand by the word 'chaotic' in this context is related to the response of the system to a change in its initial position. This simple observation is indeed enough to see that conservative, deterministic but chaotic systems exhibit a genuine irreversibility.

The argument above also corrects one misunderstanding about the arrow of time in the literature (see, for example, Denbigh, 1981,p. 99): It is usually said that the source of irreversibility in some macroscopic systems is due to our or nature's failure in producing exact initial conditions to occur needed for the reverse motion. Then, it is deduced that the irreversibility is not in the laws of motion but simply reflects the human incapacity, or contingent features of the universe. This idea is partially true in stating that incapacity of the humans and nature does exist, but still this does not ensure the irrelevance of the laws of motion to irreversibility since not all systems which fail in achieving *exact* initial conditions will produce irreversibility. This only happens when the system is chaotic, i.e., the equations of motion of the dynamical system are not stable at all under small perturbations. Only then, small perturbations will cause bigger shifts away off the initial configuration which makes the system irreversible. Otherwise, if the system is dynamically stable, small perturbations will just cause a return to the initial configuration which will label the system as reversible. In this sense, the irreversibility is a part of the system and not simply a result of our ignorance or incapacity. To be able to see this in detail, one can consider the solutions to one of the simple examples of dynamical systems, so called harmonic oscillator. This system has been already studied above as Eq. (2.9). Its solutions are given by Eq. (2.11), and depending on initial conditions, they are either of the form of Cosine or Sine and can easily be represented in phase space. Each solution of the equation is represented by a point in phase space corresponding to the coordinates (x,v). Time reversal is represented by reflection in the x-axis since we require $v \rightarrow -v$ in time reversed state. Inspecting Fig. 1 below, one sees that also the uncertainties regarding two states i.e., any usual state and its time reversal are similar.

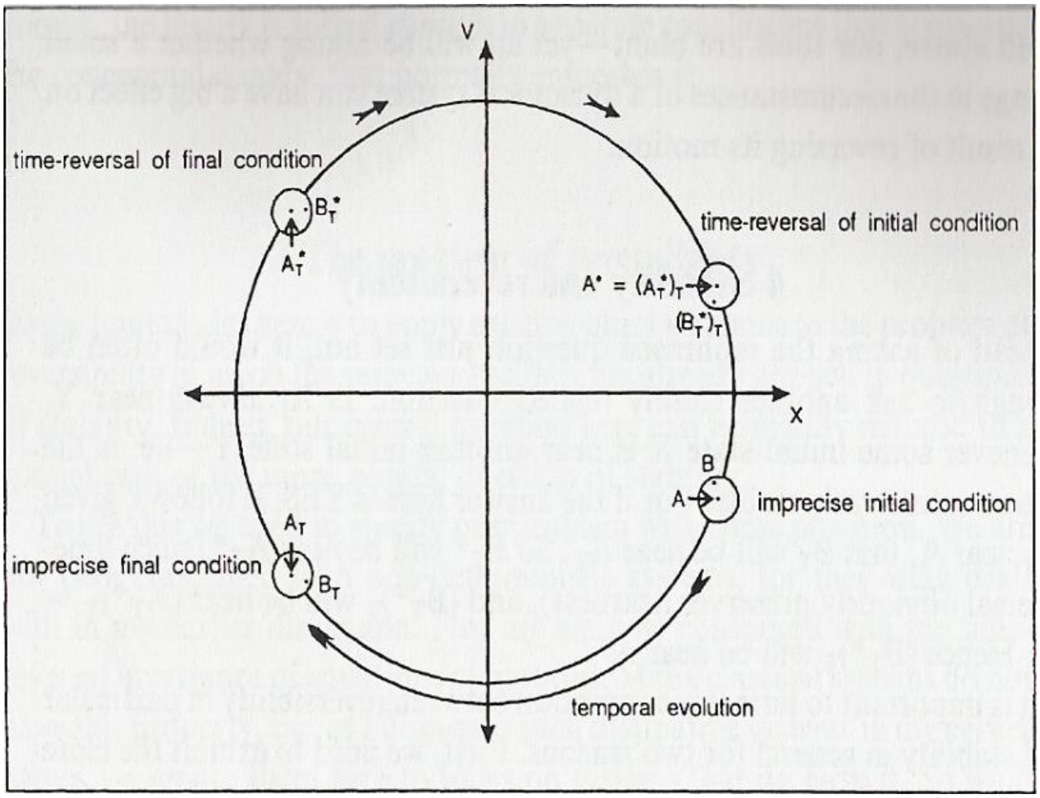

Figure 1.1: Harmonic Oscillator

Now, let us consider another mechanical example due to Keith Hutchison (1995a,p. 227): Imagine a free point-particle is projected at origin along the s-axis with a constant velocity V. After T seconds elapses, this particle will have position s = VT and velocity V since it is constant. If its velocity is reversed in order to obtain time reversal of the previous motion, the free particle will retrace its history. This case is shown in Fig. 2 below by solid lines.

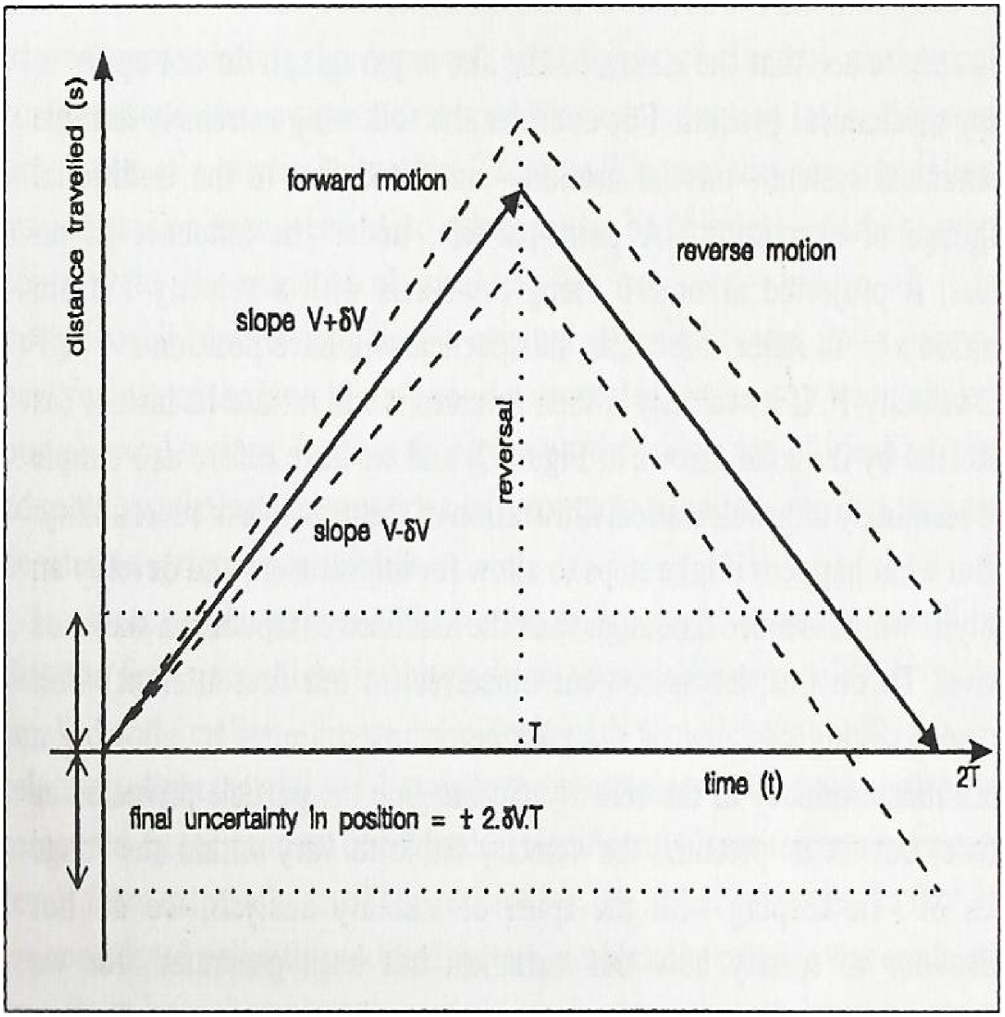

Figure 1.2: Uncertainty and Time Reversal

This motion, of course explained in the language of exactitude, does not show any sign of irreversibility and so can be another example to prove reversibility in classical mechanics as harmonic oscillator. Indeed, as Hutchison noticed one can rightfully state

> Our whole mathematical tradition is constructed on the notion of exactness, and we have a few mathematical or conceptual tools at our disposal for the systematic handling of vagueness, and such tools as we have seen extremely

> crude. We still teach theoretical physics via the ontology of exact values, for instance, then later teach advanced to make various *ad hoc* allowances when some special reason requires them temporarily to shun this ontology. (Hutchison 1995a, p. 222)

But, let us look closer, and scrutinize a little bit more. Let us allow a little bit of imprecision by imagining that velocity will be allowed to change in an interval as $V \pm \delta V$. This uncertainty in velocity can be thought in many ways e.g., because of the measurement. Then, Hutchison remarks

> After time T, the particle's position will then be somewhere in the range $VT \pm \delta V.T$, and its velocity will still be in the range $V \pm \delta V$. We now reverse the motion of the particle: i.e. we follow its motion given an initial position in the range $VT \pm \delta V.T$, and an initial velocity in the range $- V \pm \delta V$. Will classical mechanics show that the particle returns to the time-reversal of its original state another T seconds later? NO! For all we can predict about the position then is that it will be somewhere in the range $(VT \pm \delta V.T)-(V \pm \delta V)T = \pm 2\delta V.T$… *The uncertainties do not reverse themselves, only the precise values.* (Hutchison, 1995b, p. 227)

This quote can be interpreted in many ways: firstly, it shows that even classical measurement induces irreversibility in the dynamical system. This puts the classical mechanics on equal footing as the quantum mechanics as far as the arrow of time is considered. In my opinion, only this scheme of unification suffices to present Hutchison's ideas compelling. Why this simple observation has been concealed for many physicists and philosophers of science will be explained when we think about the cases against Hutchison. For the present, we just note it in passing. Second, it shows us that the apparent reversibility of classical mechanics was only a result of our modeling, the way we see things. Since when one takes inexactitudes into account, if one models classical dynamics as such, the apparent reversibility is lost immediately.

One immediate objection could be that the uncertainty introduced in the above example is small since it depends on δV, and therefore can be neglected.

However, this argument is misleading since what matters at the end is given by δV.T which means that uncertainty depends on amount of time T. Of course, there is no limit on the values of T. It can be as big as we want, so this renders it impossible to label the uncertainty introduced above as neglible.

Third point worth of remark concerning the Hutchison's case is relevant to famous Loschmidt (1876a, 1876b, 1877) paradox. According to this paradox, H-theorem, or in other words statistical mechanics, looks irreversible although the very dynamics i.e., Newtonian mechanics on which it is founded is time reversible. How can, Loschmidt questions, a foundationally reversible dynamics causes an irreversible theory such as statistical theory of mechanics? Now, we can think of a way out of this paradox as Hutchison remarks: If we model the universe by allowing uncertainties, even classical mechanics exhibits irreversibility. This means that there is nothing surprising in observing statistical mechanics to be irreversible since its very foundations are so. So is the end of the Loschmidt paradox.

In a paper entitled "Is classical mechanics time reversal invariant?", Steven Savitt (1994) objected to Hutchison's central premise that classical mechanics is not time reversal invariant. According to Savitt (1994, p. 910), how the time-reversed state is to be understood depends on the theory T under consideration. Savitt's objection that factors "outside" of Newtonian mechanics are invoked as the origin of the arrow of time is not on point in this discussion since it is exactly the same issue which makes important the debates about the famous entropic arrow of time that arise in both a scientific and philosophical sense; in thermodynamic irreversibility too, we must have some instances which will make entropy decrease (or H function increase) according to the statistical nature of the Second Law of Thermodynamics, yet we never observe (Savitt and Hutchison use the word "witness" rather than "observe") these instances. Formulated as such, Hutchison's contribution—as objected to by Savitt—is an attempt to set the problem of Newtonian and entropic time arrows on same footing. This, however, should not be taken as a fault. First, Hutchison seems to show that the two time arrows cited above can be understood as the same problem. Second, he proposes some solutions to solve this problem using the notion of uncertainty in experiments which further connects the classical case to the quantum one.

A second point of Savitt's objections to Hutchison's view is not well founded. As we have already indicated, Savitt considers how the time-reversed state is to be understood depends on the theory T under consideration. In other words, the measurement problem *per se* cannot be counted among the reasons one can classify classical mechanics as time reversal non-invariant since measurement does not form the core of the Newtonian theory. Savitt maintains that any discussion related to classical mechanics must be centered on Newton's equations, not on how we practice them, nor on how we simulate them.

This view is deleterious to the clarification of the issues revolving around arrows of time since it suggests an artificial richness of matters: one can look at classical mechanics, and say alternately that there is irreversibility due to friction or reversibility due to Newton's equations. We contend in reply that one must be able to have certain well-defined criteria that extend throughout the scientific and philosophical literature to form valid categories. If we are able to deem quantum mechanics to be irreversible due to the problems of measurement, we must be able to do so for the classical mechanics as well.

As another philosopher who participated in this discussion in the columns of the *British Journal for the Philosophy of Science*, Craig Callender (1995) dwells on the point of ignoring non-conservative forces. He states:

> … We make an ontological assumption… It is simply the following: classically, there are really only particles in motion and interparticulate (distance-dependent) forces… We ignore nonconservative forces simply because we are confident they do not exist. (Callender 1995, p. 333)

Callender, moreover, quotes Richard Feynman on this issue: in his famous lectures, Feynman says that there are *no* non-conservative forces (1963, section 14.6). Callender then goes on to ask the reasons underlying this belief; what he finds is another one, belief in so-called global conservation of energy. The connection between conservative forces and conservation of energy is construed through the well-known Noether's theorem. According to this theorem for every continuous symmetry of a dynamical system there must be a conserved quantity. In our example, this means that time translational invariance results in a conserved Hamiltonian, i.e., conserved energy if kinetic energy is homogeneous and quadratic in velocities. Callender further argues that if the only extant forces are the conservative ones, then we can show that the classical mechanics is time reversal invariant, and hence energy is conserved.

Callender is correct when he states that the belief in time reversal invariance is investigated by the conviction that energy is globally conserved. But, again, this shows that the ontology of a theory has a direct bearing on its acceptance. What is decisive in whether we apprehend a theory as time-reversal invariant or not is our beliefs and our prior ontological commitments. In fact, Callender fully embraces this point in the subsequent pages:

> Laws are either TRI (Time reversal invariant) or not, regardless of the ontology. However, when asking whether a theory is TRI, we need to know which laws to look at to make this judgment, for as Hutchison ever reminds us, there are TRI and non-TRI laws in classical mechanic. We have said we want the fundamental ones. This is where ontology enters the picture, since metaphysics determines which laws are fundamental. (Callender 1995, p. 336)

We argue that all forces are conservative only if an accurate account of all the energy of all the constituents of the system is kept. If any degree of freedom is left unspecified or incorrectly audited, then the subsystem will not be conservative. Thus, one is brought again to the place where knowledge and ignorance are the fundamental source of non-conservative forces and consequently a temporal arrow. This is where ontology enters, indeed.

Another issue raised by the ontological aspects of classical mechanics concerning the Hutchison's defense is the measurement problem: as we have already remarked, Hutchison centralizes the problematic of the arrow of time around the notion of measurement due to his interpretation. Once one begins to see the classical dynamics in terms of inexact variables due to measurement, i.e. once when one begins to take uncertainties due to measurement into consideration, the arrow of time emerges to appear exactly as it would be in the case of quantum mechanics since in that theory too, the source of the arrow of time is accepted to be due to the measurement problem as will be seen in a related chapter in this dissertation.

Although the observation that the measurement problem is at the heart of quantum mechanical arrow of time has been made almost right from the beginning, no one thought the same thing would happen with classical mechanics. Why? I believe that the answer to this question lies in the fact that quantum mechanical ontology was construed on uncertainty right from the start whereas the ontological framework of Newtonian mechanics was based on certainty. In this sense, when one talks about the measurement problem in the domain of quantum theory, we already know that it is important and forms the core of the body of the theory. On the contrary, we tend to neglect uncertainties arising in Newtonian mechanics since we believe that the theory at hand is one of certainty.

CHAPTER 3

THE ARROW OF TIME IN CLASSICAL ELECTRODYNAMICS

3.1 On Maxwell Equations

The classical electromagnetism is founded on Maxwell equations (Jackson, 1975), which can be given as

$$\nabla.\vec{E} = \rho \,, \tag{3.1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \,, \tag{3.2}$$

$$\nabla.\vec{B} = 0 \,, \tag{3.3}$$

$$\nabla \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{j} \,. \tag{3.4}$$

These equations are time reversal invariant if we define the time reversal as the following mapping

$$^{T}\rho \rightarrow \rho \,, \tag{3.5}$$

$$^{T}\vec{j} \rightarrow -\vec{j} \,, \tag{3.6}$$

$$^{T}\vec{E} \rightarrow \vec{E} \,, \tag{3.7}$$

$$^{T}\vec{B} \rightarrow -\vec{B} \,. \tag{3.8}$$

Indeed, this is how the time reversal is defined in many textbooks. But, if one just looks closely at the way how his transformation is made, i.e., leaving $\vec{E}$ invariant but changing the sign of $\vec{B}$, one begins to think that this is an ad hoc maneuver to save the electromagnetism from time reversal non-invariance since the fact that $\vec{E}$ and $\vec{B}$ is being treated in a different way is not justified at all. This problem can be traced back to the fundamental problem of how we define the inverse of a process: a process P is said to be irreversible if and only if R (P), the temporal inverse of the process P, is incompatible with the laws of nature. But, in many cases, it is not clear how we must define the temporal inverse of an arbitrarily given process. In other words, irreversibility is directly linked to the definition of the operator R and we must have an explicit form for it. It is important to quote Paul Horwich at this point. He argues as follows:

> A natural first thought will be that if process P is made up of the sequence of states, ABCD, then R (P) is the sequence, DCBA. In general, one is tempted to suppose that R (P) contains just the same events and states as P, but occurring in the opposite temporal order. However, this characterization must be rejected, for, on reflection, it clearly fails to capture what we have in mind by the inverse of a process. To illustrate, let P be the sequence, A (meteorite comes flying toward the Earth), B (hits the ground), C (bounces around) and D (stops). Surely, we don't suppose that the inverse of this type of process is DCBA-one in which a meteorite first stops, then bounces around, then hits the ground, and, finally, comes flying toward the Earth… The moral here is that if state A occurs in process P, then R (P) contains, not A itself but rather R (A), the temporal inverse of A. It is plausible to suppose that when A is a state involving a specific velocity, the temporal inverse of A will involve the opposite velocity. However, we yet have no *general* account of how to construct the temporal inverse of an arbitrarily given state. (Horwich, 1987)

The problem of defining the time reversal operator is also at the heart of matter when it comes to classical electrodynamics since we do not have a recipe cooked for each occasion. This problem is not only philosophical since the transformation of electric and magnetic fields through the time reversal operator R would lead to a new Lorentz force in the temporally inverted universe i.e.,

$$^{T}F_{L} = q\,^{T}\vec{E} + q\,^{T}\vec{v} \times {}^{T}\vec{B}\,, \tag{3.9}$$

and force, according to Newtonian picture, is just one of the most essential ingredients in calculating the future state of the universe once the initial conditions are given. The possibility of a new force in this temporal universe is not only a philosophical issue but also a challenge for the physicists. Another philosopher of science, Lawrence Sklar (Sklar, 1974) illustrates this point with an easy-to-understand example: let us imagine, as in figure 3, a current carrying wire and a magnet right below it with its north pole being closer to the wire itself. The current is moving from right to left. According to the right hand rule, the force acting on this wire will be into the page. When we consider the temporal inverse of this process, one can try to do it only by inverting the direction of the current passing through the wire and keeping the poles of the magnet fixed. This of course would provide us with an out-of-page force. Since we are still working in Newtonian universe, this simple observation, i.e. the observation that we would have a different force in the temporally inverted universe, leads us to conclude that classical electromagnetism is irreversible or in other words not time reversal invariant. But, if we pay more attention to inner workings of a magnet in detail, we see that its magnetism results from internal currents formed by electrons. This in turn means that the direction of these internal currents within the magnet must also be changed. This is tantamount to changing the orientation of the poles of the magnet as a whole. Only then, only when we change the orientation of the poles of the bar magnet and the direction of the current simultaneously, we obtain a force in the same direction (and also with the same magnitude). Understanding the inverse of a given process as this example shows requires an understanding of the *whole* mechanism.

Recently, David Albert (Albert, 2000) provided a fresh way of seeing the same kind of problematic explained above. First, he reminds us of the fact that the only dynamical variable in Newtonian universe-the parameters changing with time- is the position and then considers it in connection with what it means to have a complete description of the physical situation of the world at an instant. This issue is already important in the context of quantum mechanics due to the famous Einstein-Podolsky-Rosen paradox. He lists two criteria for completeness:

i. That it is genuinely instantaneous i.e., logical conceptual or metaphysical independence among the descriptions of the world at different times.

ii. That it be complete.

Albert calls any state satisfying these two criteria above an instantaneous physical state of the world. In Newtonian picture, the physical state of the world is, according to these criteria cited above, given by the positions of all the particles in the world at any one time.

Albert's first point of attack is very simple indeed: In most of the books written in this area, what is called the instantaneous state of the world consists of both positions and velocities of the particles at one particular time. But, when one defines the instantaneous state as such, then one immediately faces a serious problem: This definition breaks the independence postulate mentioned above since specifications of the position and velocity both result not in determination of the state of the world at that instant *alone*, but also for some interval of time which can be judged by the tools of calculus.

Having made this criticism, Albert considers a general outline of what the time reversal means. In his opinion, once the instantaneous states are determined, what is left is just to juxtapose them in the inverse order. For example, let us suppose that the instantaneous states are ordered as the sequence $S_I...S_F$ with respect to a theory T. Then, according to one account of time reversal, time reversal of this process is the sequence $S_F...S_I$. Now, according to the classical theory of electromagnetism, the instantaneous states are made of positions, magnitude and directions of magnetic and electric fields. One can easily see that this theory is not time reversal invariant with this adoption of time reversal invariance which itself is based on the specification of instantaneous states for a complete description of the state of the world at one instant. In other words, we would expect position, velocity (since this is nothing but the change of position with respect to time), electric field and magnetic field all to be inverted i.e., multiplied by a minus sign. Of course, in practice (this is tantamount to saying what is written in the textbooks in general), we invert the velocity and magnetic field, but not electric field. This is Albert's main objection. He thinks that time reversal invariance is forced on set of equations. This does not cause any problem for Newtonian mechanics since inverting the position and velocity is the one and same thing. In the end, velocity is nothing but the rate of change of position with respect to time as indicated before.

It is extremely instructive to look at how dressing the equations so that they will be time reversal invariant works. What is done at this stage of things is simply to operate on these states or more correctly first inverting them and then operating on them through some operator of which the explicit structure varies from one fundamental theory to another. In fact, what are accepted to be the description of the physical state in general by the scientists and philosophers of science alike are the *dynamical conditions* since for example, in Newtonian physics, these are position and velocity and give the theory its full predictive power. But the price one has to pay in return is to sacrifice the independence postulate cited above.

Concerning the time reversal problem in terms of dynamical conditions instead of instantaneous states is a difficult one as an example by David Albert shows: If $D_I...D_F$ is a sequence of dynamical conditions concerning a single free particle moving to the right, then $D_F...D_I$ will not correspond to a particle like that moving to the left but to a particle whose position is constantly being displaced toward the left, and whose velocity is constantly pointing to the right.

This example shows that if you would like to define the physical state of a system in terms of its dynamical conditions, then you must have to do something more than merely inverting the sequence i.e., you must have an operator to act upon these dynamical conditions as mentioned before. For each D, one must have some unique condition D* which is D's time reversal. The first flaw in this kind of reasoning makes it explicit even at this very beginning since one cannot be sure of what it means to talk about the reversal of one instantaneous physical situation. Inversion as a sequence can be understood easily but not one instant of it.

Proceeding with the idea of assuming the physical situations are nothing but the dynamical conditions, we now have a ready-to-cook recipe for any kind of time-reversal process: One starts with $D_I...D_F$ and ends up with $D^*_F...D^*_I$. This simply means that we must first invert each dynamical condition, and then apply the operator * onto them whose explicit structure is left unexplained since its form varies from one theory to the other. For example, in the case of Newtonian mechanics, we have to define * operator as an operator which reverses all the velocities but leave everything else, including position untouched.

If you would try to do the same in the language of *instantaneous states*, then what you have to do is simple: Let us imagine that we do not have access to the states but only to the dynamical conditions $D_I...D_F$. We then translate this sequence into a sequence of instantaneous states i.e., $S_I...S_F$, and invert it, writing it as $S_F...S_I$. Finally, we translate this sequence back into the language of the dynamical conditions and then call it $D^*_F...D^*_I$.

When we interest ourselves with Newtonian mechanics, these considerations do not cause any trouble. The velocities of the particles are rates of changes of positions. Therefore, transition from $S_I...S_F$ to $D^*_F...D^*_I$ and back to $S_I...S_F$ does not lead to any inconsistencies if we define the operator D as the operator which reverses the velocities only. But, mixing instantaneous states with dynamical conditions leads to confusions in some other fundamental theories. One immediate example, says Albert, is the classical electrodynamics. He says:

> What counts as an instantaneous state of the world according to classical electrodynamics is … a specification of the positions of all the particles and of the magnitudes and directions of the electric and magnetic fields at very point in space. And it isn't the case that for any sequence of such states $S_I...S_F$ which is in accord with the dynamical laws of classical electrodynamics, $S_F...S_I$ is too. And so classical electrodynamics is not invariant under time reversal. (Albert, 2000)

According to textbooks though, classical electrodynamics is as much time reversal invariant as classical mechanics. These books take the dynamical conditions to be defining the physical state of the system, and proceed by defining the transformation (3.5)-(3.8). With this definition of time reversal operator D, one recovers the time reversal character of the classical electrodynamics. The problem is, Albert emphasizes

> That this identification is *wrong*. Magnetic fields are not the sorts of things that any proper time reversal operation can possibly turn around. Magnetic fields are not-either logically or conceptually- the rates of change of anything. If $S_I...S_F$ is a sequence of instantaneous states of a classical electro dynamical world, and if the sequence of dynamical conditions corresponding to $S_I...S_F$ is $D_I...D_F$, and if we write the sequence dynamical conditions corresponding to $S_F...S_I$ as $D^*_F...D^*_I$, then the transformation from D to D* can involve nothing whatsoever other than reversing *the velocities of the particles*. And if that's the case, and if $D_I...D_F$ is in accord with the classical electrodynamical laws of motion, then , in general, $D^*_F...D^*_I$ will not be. (Albert, 2000)

In summary, the issue here is that there is no justification for the transformation (3.5)-(3.8). The fact that one is using the dynamical conditions hand one the freedom to choose the explicit form of the operator D. In other words, this D is chosen in such a way that classical electrodynamical theory has no other option than being time reversal invariant. This is nothing but an ad-hoc movement according to Albert since for each fundamental theory, one has to define a distinct operator D which will reverse the states. This simply does not make sense at all in his view.

The treatment of the subject of time reversal invariance by Albert led to the discussion of the topic in many aspects. One main discussion was related to the idea of instantaneous states. The writings on the existence of the instantaneous states first emerged in relation to Zeno’s arrow argument. Zeno of Elea argued that the motion of an arrow is impossible since it does not change its location at any instant. There are three general stands that one can take philosophically in view of Zeno’s paradox. First one is called “at-at” theory: According to this view, there is no such thing as instantaneous velocity, while motion is possible. Here, the term motion must be understood as the occupation of different locations at different times (Arntzenius, 2000).

Aristotle responded to Zeno’s argument by rejecting the sensibility of the notion of instantaneous velocity. He then described the situation in terms of average velocity. Any motion takes place over a period of time. Thus, the only notion which makes sense is average velocity over a time interval. Average velocity then is defined as the distance taken by the time of travel (Aristotle, Physics VI). His first step was to reject atomic units of time i.e., “instants”. According to Aristotle, there are no smallest time intervals. He not only rejects instantaneous velocity but also instantaneous position, too. This view is called “no-instant” view.

The second idea one can read in Aristotle’s writings which is contradictory to his former one is that there are instants and instantaneous positions but not instantaneous velocities. The reason for the exclusion of instantaneous velocity is the same as Zeno’s, that there are no changes of position in an instant. Later, this idea has evolved into what is called “at-at” theory. Another issue that one learns from these considerations is that motion is an entity defined in relation to some other fundamental quantities such as position and time.

The “at-at” theory resolves Zeno’s paradox but it is not that comforting at all since what it tells us is that there is no difference between a car moving to the right and the one moving to the left. As Frank Arntzenius (2000) summarizes in his paper in the Monist,

> Is there really no sense to be made of the claim that this car is moving right now, at this instant? Doesn't the complete state of the car at an instant not include the fact that it is moving? Aren't cars that are moving in different directions in different instantaneous states? (Arntzenius, 2000)

If we do not include the existence of instantaneous velocities into our definition of a physical state, we cannot even talk about why one ball moves to the right and other to the left.

Zeno's argument (and its so called solutions) understood as such is linked to the core of classical physics. As is explained in Chapter 2 in detail, the equations of motion in Newtonian physics are of second order. This requires the use of velocities which are calculated at one instant in order to give a full account of a physical state. The specification of a physical state requires the specification of both positions and velocities. Of course, all this boils down to is determinism in classical physics. So, the determinism in Newtonian universe requires the existence of both instantaneous positions and velocities. It is of course true that determinism is not something which must be guarded against all other arguments per se but nobody would like to lose determinism just because of Zeno's arguments.

A different way to see this problem is as follows: Even though determinism would fail, the Markovian nature (the feature of later evolutionary states to depend on the former ones in terms of conditional probabilities) of the world does not have to fail. We would still believe that the states at a time would fix the probabilities of future developments of states as is the case with quantum physics (consider the solution to Schroedinger equation). Since most of the theories of physics we now have are of Markovian nature, we would not think it to fail so easily.

One way out for the "at-at" theory is through Calculus developed by Newton and Leibniz. According to Calculus, we define the instantaneous velocity as

$$V(t) = \lim_{h \to 0} \frac{x(t+h) - x(t)}{h}, \qquad (3.10)$$

The above equation indicates that velocity at time t is defined in terms of finite or infinitesimal neighborhoods of that time t. This approach saves “at-at” theory in the sense that one now is not forced to buy the idea of the instantaneous state. It is enough to buy the ideas of instants and instantaneous position so that the left hand side of Eq. (3.10) is justified. Therefore, the left hand side which is instantaneous velocity is explained away with the help of instantaneous position and the concept of instant alone.

However, this explanation still does not explain why the balls continue to move in the directions that they do. The example given by Arntzenius is very helpful in understanding this point.

> Suppose that one defined an object to have the property X at time t iff it is blue at time t + 1. Suppose one sees a ball that turns from red to blue between t and t + 1, and one asks: “why did it turn blue during that period?” It seems clear that the answer “because it had property X at t,” is not to be regarded as a satisfactory answer. Property X is not the kind of intrinsic property that could cause it to turn blue. (Arntzenius, 2000)

We consider Eq. (3.10) to be different than the property X cited above, that much is sure but why? What are the differences between ordinary properties X and so called neighborhood properties even though both of them are not to be intrinsic? There are two main differences between Eq. (3.10) and property X mentioned above in the example given by Frank Arntzenius in his paper. First, property X is only approachable from the right i.e., from times greater than t. But, we know that in order to be able to speak about the existence of a limit, one must approach t from right as well as left. Therefore, the neighboring property V(t) stands in relation to not only t + 1 but also to t-1 as well. Second, contrary to X, we can define V(t) in terms of intrinsic quantities since position and time are intrinsic quantities according to anybody's theory of physical state. In my opinion, these two main differences show the sharp line of demarcation drawn between property X and V(t).

Next issue is how we must understand determinism concerning the neighborhood properties. Since we can now include velocity in this picture without invoking any paradox, the determinism is saved. Position and velocity both can be used in order to understand what is going to happen in the future. This is well known from the Newtonian equations of motion. This much is clear. The question is whether we do have determinism or not based on these quantities alone (logic and definition alone, in the words of David Albert) without invoking physics, too.

Neighborhood properties are rather different than intrinsic and non-intrinsic quantities. We can appreciate this fact by thinking in terms of an example: Imagine that the limiting value of a position at time t is equal to x(t). This is possible only when right hand and left hand limits approach this definite value which is x(t). But, this way approaching a number does not say anything about what the position will be, for example, at a time equal to t + 1. All one can entail from neighborhood properties is the position at time t even though we *know* more than this seemingly in the overall limiting process. We can infer the tendency around t but we cannot assign any particular definite value to any position after or before t. This simply shows that the determinism acquired by the use of neighborhood properties will not be a trivial one. We do have to rely on the equations of development of physical states which must be taken to be the physical laws governing the particular interactions in each case.

The second response to Zeno is called "impetus theory". Impetus theory claims that the reason that any object at any instant keeps moving in a definite direction is that it has an impetus of a certain magnitude and direction at each instant. In the words of Frank Arntzenius:

> On this view there is a kinematic quantity in addition to position, which one could call “intrinsic velocity,” which equals impetus divided by mass, which is part of the intrinsic state of an object at time t. This quantity is not defined in terms of position developments, but it is a law of nature that “intrinsic velocities” always equal the temporal derivatives of position developments. (Arntzenius, 2000)

So, in this view, we have an additional state variable called “intrinsic velocity” and an additional law which disables the possibility that this intrinsic quantity does not correspond to position developments in a neighborhood of time. Another version of impetus view could be formulated in terms of Hamiltonian dynamics. The full physical state, according to Hamilton formalism of Newtonian mechanics, is determined by canonical position and momentum. Then, Hamilton’s equations of motion will determine what these variables are and the relation between the canonical momentum and position. Since this can change from one case to the other, this has to be understood as a law instead of identification of canonical momentum with the kinematical momentum all the time.

One problem with this view is in the context of Ockham’s razor. All other things being equal, one would really like to get rid of this “additional” state variable called impetus.

The other issue which impetus theory has to face with is that the existence of intrinsic velocities breaks the time reversal invariance of theories which have been accepted to be time reversal invariant until now. The time reversal of any state is formed by reversing the order of physical states. According to this recipe, one has to reverse the order of positions and also intrinsic velocities but intrinsic velocities, since they are intrinsic, will be pointing in the wrong direction. This contradicts with the fact that there is nothing in Newtonian physics which would suggest an objective temporal direction. So, at this point, one has two ways out of this dilemma: one either accepts the view that classical mechanics is time reversal invariant and therefore impetus theory is wrong or we must include time reversal operations on physical states as is explained above.

Frank Arntzenius opts for the second one and argues favorably for time reversal operations. If we rule out such transformations on states, then we will lose the possibility of having non-trivial, deterministic and time reversible theories since

> It is impossible to have any non-trivial theory which both implies that the state at a time fully determines all future and past states, and implies that any reverse of any allowed sequence of states at times is also allowed. This would imply mirror symmetry of developments of states in both directions of time around any point in time. And that is impossible unless there is no state of change ever. Surely, theories can be deterministic, time reversible and non-trivial (Arntzenius, 2000)

Finally, we turn to "no-instants" view. According to this view, there are neither instants nor instantaneous velocity. This is tantamount to saying that time is *atomless*. Let us see how this can be done in the case of a *pointless geometry* (Skyrms, 1993): We begin with the collection of open intervals of the real line and then form Borel algebra by closing this collection up under complementation and countable intersection and union. One can form an *atomless* algebra by identification of regions that differ by Lebesque measure 0. We then identify these regions of 0 measure with the null element of the algebra. The remaining algebra of regions can be handled exactly as Caratheodory (Caratheodory, 1963) wished. There are no regions of zero measure anymore. Borel algebra, in this sense, represents a solution to Zeno's paradox by removing the possibility of making finite sized regions out of 0 sized points since we do not have any 0 sized points to begin with in this new algebra.

Now, the question arises: How can one make sense out of functions of space or functions of time if they are atomless? They cannot be thought as being formed from point values to point values. This point is summarized very well by Arntzenius himself as

> … One can still have maps from non-atomic regions to non-atomic regions… In other words, if one supposes that space and time (and perhaps other physical quantities) do not consist of points (do not have point values), but form atomless algebras as outlined above, it is just as if one is working with equivalence classes of point functions from the reals to the reals that differ at most on sets of points of measure 0. (Arntzenius, 2000)

What about time reversibility if we adopt a "no-instant" view? To be able to talk about time reversal, we must talk in terms of reversing the history of states at times. In other words, we have to talk in terms of spaces occupied at an instant. But, now, since we do not have the language of instants accessible to us, all we can hope for is using the languages of mapping explained above in the quote. The time reverse of a mapping is simply the mapping that corresponds to the time reverse of the equivalence class of point functions that corresponds to the original mapping. The problem is that we do not have a simpler picture with this approach. This can be compared to the case of point particles versus extended objects. In particular, consider how one can understand the motion of extended objects over a period of time adopting the view that there are only point particles. One might consider a mapping in terms of points and space that they occupy and try to see the evolution of this mapping but of course this will have some difficulties. One counter-example can be given at once: What if one has a homogeneous rotating disk about its own axis? Now, all point particles will occupy the same position for all time during the interval of motion but we cannot deny the fact that disk is rotating i.e., it is not at rest at all. In other words, the difference between rotational and translational motion is easily lost when one adopts only the point particle view.

It seems that adoption of pointless views is not natural (or simple) enough in the sense that one would expect from a scientific theory.

Recently, Sheldon Smith (Smith, 2003) opposed the views expressed by Frank Arntzenius by affirming that the instantaneous velocities are real. His first vantage point was to link Arntzenius to Bertrand Russell: Russell, considering that there were some problems concerning the calculus definition of instantaneous velocity in regard to Zeno's paradox, adopted the view that the concept of motion only involves being at different locations at different times. Before proceeding further, it is wiser to write down the formulation of Zeno's paradox as stated in Sheldon Smith's paper concerning how Russell understood it. Smith considers Zeno's paradox to be formed as follows:

1) At each instant of its "flight", an arrow occupies only one position.
2) If something only occupies one position, then it is not in a state of motion.
3) Therefore, at each instant, an arrow is not in a state of motion.
4) If at each instant it is not in a state of motion, then it has not moved over the entire time interval of its "flight".
5) Therefore, motion-even over non-zero time intervals-does not take place.

Obviously, the way Russell got rid of this paradox depended on granting 1, 2 and 3 but blocking 4 so that 5 does not follow. In his "Mathematics and the Metaphysicians", he wrote,

> People used to think that when a thing changes, it must be in a state of change, and when a thing moves, it is in a state of motion. This is now known to be a mistake. When a body moves, all that can be said is that it is in one place at one time and in another at another. We must not say that it will be in a neighboring place at the next instant. Philosophers often tell us that when a

> body is in motion, it changes its position within the instant. To this view Zeno long ago made the fatal retort that every body always is where it is; … It was only recently that it became possible to explain motion in detail in accordance with Zeno's platitude, and in opposition to the philosopher's paradox. We may now at last indulge the comfortable belief that a body in motion is just as truly where it is as a body at rest. (Russell, 1929)

Russell thought that the only way 1 can be wrong is if the infinitesimals were coherent. He dismissed this idea very quickly so the only way out, according to him, was to block 4 so that the conclusion 5 does not follow at all.

Bertrand Russell thought that Weierstrass, by his efforts to "arithmetize", founded mathematical analysis on the basis of numbers alone. He replaced the continuous by the discrete so that he banished infinity from the realm of mathematics. One major difference of opinion between Russell's view and Arntzenius' view is that the former denied the existence of instantaneous velocity due to calculus without infinitesimals whereas the latter tries to save the "at-at" theory by calculus with infinitesimals.

Russell thought, due to the contributions of Weierstrass, that there were no infinitesimals, so there were no instantaneous velocities. A response to this view can now be given, Smith remarks. He appeals to the recent construction of so called "smooth world" account of infinitesimals pioneered by Lawvere (Bell, 1998). Smooth world account is also called "smooth infinitesimal analysis" (SIA). According to this account, infinitesimals are rather *fuzzy* (italics are Smith's) things, and the continuous is not explicable in terms of the discrete. The use of limits is replaced by the use of nilpotent infinitesimals, quantities which are nonzero but small and whose squares vanish. In order to understand SIA, let us begin with the ordinary calculus definitions. Let y = f(x) be a differentiable function on the real line $\Re$. Then, the increment δy is given by

$$\delta y = f(x+\delta x) - f(x) . \qquad (3.11)$$

Using Taylor's theorem, we can write it as

$$\delta y = f'(x)\delta x + A(\delta x)^2, \qquad (3.12)$$

Where $f'(x)$ is the derivative of the function f with respect to x, and the value of A depends on both x and δx. If we could assume δx to be so small but nonzero that we could equate $(\delta x)^2 = 0$, then Eq. (3.12) would take the form

$$f(x+\delta x) - f(x) = f'(x)\delta x. \qquad (3.13)$$

A quantity whose square is zero is called nil square infinitesimal or micro quantity. Now, the equation (3.13) holds trivially in standard analysis since zero is the only micro quantity. In SIA, there are enough micro quantities which ensures Eq. (3.13) to hold non-trivially because we can replace δx by any ε, i.e. for any micro quantities. Then, the derivative may be defined to be a unique quantity D which holds for all micro quantities as follows

$$f(x+\varepsilon) - f(x) = \varepsilon D. \qquad (3.14)$$

Setting x equal to zero above, we get

$$f(\varepsilon) = f(x) + \varepsilon D, \qquad (3.15)$$

for $\forall\varepsilon$. The Eq. (3.15) is the axiom of SIA together with the above mentioned definition of micro quantities,

$$\Delta = \{x : x \in \Re \wedge x^2 = 0\}. \qquad (3.16)$$

Then, it is postulated that, for any f:$\Delta \to \Re$, there is a unique D$\in \Re$ such that the Eq. (3.15) holds for all ε. This postulate is called the principle of microaffineness since any function on Δ is affine due to the reason that the Eq. (3.15) represents a line with slope Δ. Δ is not a point which would be the case within the framework of standard analysis but it can be rather thought to be of as an entity possessing position but without any extension.

Now, if we think of a function y = f (x) as representing a curve, then the image of Δ+a under the mapping f is obtained by translating Δ to a. This image will coincide with the tangent to the curve at x = a i.e., each curve is infinitesimally straight. Another point of interest is called the principle of micro cancellation which rests upon the principle of microaffineness i.e., Eq. (3.15) above. It reads,

If $a = \varepsilon b$ for all ε, then a = b. (3.17)

Again from Eq. (3.15), it follows that all functions on $\Re$ are continuous. In SIA, the fact that a function is continuous on $\Re$ simply means the following: let x and y be two points on $\Re$. They are said to be neighbors if x-y is in Δ i.e., if x and y differs by a micro quantity. Continuity then simply means a mapping from neighboring points to neighboring points. In order to see this, imagine a function f from $\Re$ to $\Re$ and two neighboring points x and y which is tantamount to writing $y = x + \varepsilon$ with ε in Δ. Then, right after the mapping under the function f, we have

$$f(y) - f(x) = f(x+\varepsilon) - f(x) = \varepsilon f'(x). \quad (3.18)$$

Since any multiple of a micro quantity is also a micro quantity, so $\varepsilon f'(x)$ is a micro quantity, too. Therefore, all functions on $\Re$ are continuous. Since the equation above is valid for all functions f, it follows that all functions are differentiable arbitrarily many times which explains the word “smooth” in SIA.

One interesting observation is that SIA is incompatible with the law of excluded middle or in other words principle of tertium non datur. This principle can easily be written in terms of classical logic as

$$p \vee \neg p \Re \varepsilon \ . \quad (3.19)$$

An example in sentences can be, for example, “ I am going to have my Ph. D. in philosophy or I will not have my Ph. D in philosophy.”. The truth value of this proposition is always true independent of whether I will have my Ph. D. or not. There is no other possibility anyway.

There are two ways to assess the situation regarding the law of excluded middle in the framework of SIA. First one can be put like this: Consider the function defined for real numbers x by f(x) = 1 if x = 0 or f(x) = 0 whenever $x \neq 0$. According to law of excluded middle, each real number would either be equal to zero or unequal to zero. But considered as a function with domain $\Re$, f is clearly discontinuous. Since every function on $\Re$ is continuous in SIA, f cannot have domain $\Re$. In other words, universal continuity is the source of the failure of the law of excluded middle.

A more rigorous argument can be given as follows: If $x \neq 0$, then $x^2 \neq 0$, so that, if $x^2 = 0$, then necessarily not $x \neq 0$. Now, instead of x, substitute ε. This means that, combined with the fact that $\varepsilon^2 = 0$, i.e., Eq. (3.16),

$$\text{For all infinitesimal } \varepsilon, \text{ not } \varepsilon \neq 0. \qquad (3.20)$$

If the law of excluded middle held, then for any ε, we would have either $\varepsilon = 0$ or $\varepsilon \neq 0$. But due to Eq. (3.20) above, second possibility is excluded, leaving us with $\varepsilon = 0$. This can be written as

$$\text{For all } \varepsilon,\ \varepsilon.1 = \varepsilon.0, \qquad (3.21)$$

From which one obtains, by micro cancellation, the falsehood

$$1 = 0. \qquad (3.23)$$

Therefore, the way out is the law of excluded middle to fail. The logic of SIA is not completely classical as one can see from the arguments above. One does not see this difference if one is only willing to give his strength to computational aspects within this framework since logic veils itself there.

Sheldon Smith, in the light of all these developments in SIA, states,

> The instant t, consists of indistinguishable points (the set of points not not equal to equal to t) but whose identity does not follow from their indistinguishability. (This is possible because of the denial of the excluded middle within smooth world account.) So, at an instant, it cannot be said that the arrow only occupies one *fixed* position, t. Rather, it occupies some vague smear. Thus, with infinitesimals, like Russell suspected, we can reject premise 1 of the arrow argument as giving an improper picture of states, so we never get to the denial of states of motion claim. (Smith, 2003)

On the contrary to what is believed by Berkeley and Russell, we have a coherent picture of infinitesimals thanks to SIA. But, what about premise 2 in the argument above? According to Smith, premise 2 is about the time intervals not instants. It is true that a thing does not move at all if it occupies only one position in time interval. One can grant premise 1 but can deny premise 2 i.e., one can grant that a thing occupies only one position at an instant, but deny that this indicates that it is not moving since its instantaneous velocity can be nonzero. Smith gives the following example,

> There is nothing incoherent about saying that a particle in an instantaneous state of motion of 1000 miles per hour travels no farther in the instant than a particle going 5 miles per hour, but nonetheless they are in different states of motion. (Smith, 2003)

In short, one is misled by the idea of average velocity applied to the case of the instant. Russell was misled in deducing that the instantaneous velocity cannot be attributed to instants. All he could deduce is that motion, in the sense of change of place which requires two places, cannot be attributed to instants.

Smith also attacks another view expressed by Arntzenius. Arntzenius was complaining about the fact that “even a well-defined velocity cannot account for why the object after time t moved in the direction that it did”. Smith, accepting this to be true, finds it irrelevant to the existence of instantaneous velocity since it is not a task it has to undertake. The laws of motion do exist what velocity an object will have at some later time t. It is not easy to understand why Frank Arntzenius would like to have this feature as a source of complaint. In historical impetus theory, this could have been an issue to worry but certainly not in Newtonian physics. This is a problematic only in the Aristotelian physics which were unable to explain constant velocity motion when there were no forces exerted on the object. Even, a modern day version of the impetus view which would consider mass times velocity as the modern impetus would fail since momentum by itself does not tell us anything abut the future developments of states. One still needs Newtonian equations of motion.

According to Arntzenius, the velocity is not a property of instants even though it gives us values at instants. Smith understands this as the following

> … If, however, the claim is that the value of the derivative depends upon the behavior of the function *throughout* a certain finite neighborhood and thus can only be considered a property of that neighborhood, then it is not true. If we have any neighborhood (t-δ, t + δ) around t, there is always a smaller one (t-ε, t + ε) where ε< δ in which the derivative is still determined. (Smith, 2003)

Therefore, there is no finite neighborhood within which the values of X (t) are all required for the value of the derivative. No "special: neighborhood is needed. Any will do! This again makes the independence of the velocity and position values explicit to us.

A related issue raised by David Albert is the use of the word *temporal vicinity*. Albert states that the instantaneous velocity at t = 7 seconds is nothing but the rate of change of the position of the particle in the immediate temporal vicinity of t = 7 seconds. Against this, Smith remarks that there is no "immediate vicinity" of any point in a standard continuum i.e., one without infinitesimals. In Smith's words, there is no *smallest* finite interval around a point t that can be considered "immediate."

The source of this debate indeed lies in the definition given by Albert in his book "Time and Chance". He has different definition of genuinely instantaneous states. According to him, knowledge of the state at all points other than t ought to have no logical implication for the instantaneous state at t. But, when we reflect on velocity, knowing the position at certain points in the neighborhood of t has implications about the velocity. Smith illustrates this point with the following example

> … Suppose at all temporal points of an interval (t-δ, t + δ) around t other than t, the position of a particle is just zero with the behavior at t not being stipulated. As a matter of logic (or conceptual necessity), the *position* at t can

> still be *whatever you like*. So, the state is not completely constrained by the non-t knowledge. In fact, the position is not constrained at all. However, logic-at least classical logic-does lay down that either the position is zero at t (and, thus, by definition that the velocity is zero as well given the non-t behavior) or that the position is not zero at t (and, thus, that the velocity is undefined at t according to the standard definition since that would make the function discontinuous there). We can deduce *something* about what the state is , at least more than we could *without* this information of what is going on around t. That is, we can deduce things like given the above behavior of the particle around t, its velocity is either zero or undefined at t. We have done this without any knowledge whatsoever of the evolutionary differential equation that might govern the process; we only know the state at certain non-t times. (Smith, 2003)

Therefore, for Albert, this is about the doubtful nature of velocity at time t. If it were a property of t alone, we could have been unable to deduce anything about its value at t by non-t behavior**.**

Sheldon Smith opposes to this view since one cannot pinpoint what other than t the velocity is a property of since whenever we attempt to do this, we face with the dilemma that there is no minimal neighborhood of t. In other words, one can always choose a different neighborhood of t but what will be common to all these choices would be to choose the intervals around t alone. This point forms the contra-move of Smith (Smith, 2003) against what becomes the intuition of Arntzenius and Albert.

Arntzenius further notices that even though one considers only velocities to form the fundamental instantaneous properties, the problem will stay unresolved since there happens to exist some velocity developments which are incompatible with the calculus. He cites the following example, due to Hartry Field: Let velocity be equal to 1 at rational times and equal to zero at irrational times. This cannot be since the relevant limits converge to 1 at rational times and 0 at irrational times according to calculus. So, even if one discards the positions from being the fundamental instantaneous property, *logic and definition alone* would imply constraints between instantaneous states at different times (Arntzenius, 2003).

Another complaint about the instantaneous velocity mentioned by Frank Arntzenius is that it is relationally defined. He claims that velocity is constructed from more basic ingredients of position and time. In fact, this is why he feels attracted to impetus view for a while before he also begins to criticize that view being guilty under Ockham's razor. Since velocity is related to being at different places at different times, it is indeed natural that it is defined by the notions of position and time. Of course, then, it cannot be counted among the additional properties of the particle. As Sheldon Smith puts it: "… Once we are given position information throughout an interval, velocity comes along with it for free." This causes, according to David Albert and Frank Arntzenius at least, nothing but a reduction of the kinematical state of the particle into two, one of position and other being time. Of course, the way out is the impetus view which requires ontologically added ingredient. Therefore, if we accept the standard view of the instantaneous velocity, it does not do us any good since it is an additional property other than position developments. If we accept impetus view, then we have to have another additional property which means an enlarged ontological kinematic state. Smith remarks that position and velocity works well together with the laws of motion and in this important sense, velocity is not additional. The adoption of impetus view can be a remedy only if we do not have laws of motion which are due to Sir Isaac Newton.

3.2 On Maxwell Equations Again

One can write Maxwell equations in a way which will conform to the theory of relativity. In order to this, we assume that space-time continuum is defined in terms of a four-dimensional space with coordinates x^α where index α ranges from 0 to 3. We suppose that there is a well-defined transformation that yields new coordinates x'^α according to some rule unspecified for now.

Tensors of rank k associated with the space time point x is defined by their transformation properties under the transformation $x \rightarrow x'$. A scalar (tensor of rank zero) is a single quantity whose value is not changed by the transformation. For tensors of rank one (i.e. vectors), we have two kinds, contravariant tensor A^{α} and covariant tensor A_{α}. A contravariant tensor is transformed according to the following rule

$$A'^{\alpha} = \frac{\partial x'^{\alpha}}{\partial x^{\beta}} A^{\beta} , \qquad (3.24)$$

whereas a covariant tensor transforms according to the rule below

$$B'_{\alpha} = \frac{\partial x^{\beta}}{\partial x'^{\alpha}} B_{\beta} , \qquad (3.25)$$

where both α and β runs from 0 to 3. Of course, we employ Einstein summation convention for repeated indices.

The inner or scalar product is defined as the product of the components of a covariant and a contravariant vector.

$$A.B \equiv A^{\alpha} B_{\alpha} . \qquad (3.26)$$

The metric is written as

$$(ds)^2 = g_{\alpha\beta} dx^{\alpha} dx^{\beta} , \qquad (3.27)$$

where g is called metric tensor. For flat space-time of special relativity, we have

$$g_{00} = 1, g_{11} = g_{22} = g_{33} = -1 . \qquad (3.28)$$

All off-diagonal elements are zero. We also have

$$x^{\alpha} = g^{\alpha\beta} x_{\beta} , \qquad (3.29)$$

and its inverse

$$x_{\alpha} = g_{\alpha\beta} x^{\beta} . \qquad (3.30)$$

With the choice of the metric tensor given by Eq. (3.28), we see that if we have a contravariant 4 vector with components (A^0, A^1 , A^2, A^3), we will have a covariant vector with components (A^0, $-A^1$, $-A^2$, $-A^3$). We write this as

$$A^{\alpha} = (A^0, \vec{A}), A_{\alpha} = (A^0, -\vec{A}) . \tag{3.31}$$

What about partial derivative operators? We can write them as follows

$$\partial_{\alpha} \equiv \frac{\partial}{\partial x^{\alpha}} = (\frac{\partial}{\partial x^0}, \vec{\nabla})$$
$$\partial^{\alpha} \equiv \frac{\partial}{\partial x_{\alpha}} = (\frac{\partial}{\partial x^0}, -\vec{\nabla}) \tag{3.32}$$

Therefore, the divergence of a four vector A can be written as

$$\partial^{\alpha} A_{\alpha} = (\frac{\partial A^0}{\partial x^0} + \vec{\nabla}.\vec{A}) . \tag{3.33}$$

The four dimensional Laplacian is given by

$$\Box \equiv \partial^{\alpha} \partial_{\alpha} = (\frac{\partial^2}{\partial x^{02}} - \vec{\nabla}^2) . \tag{3.34}$$

The electric and magnetic fields can be written in terms of scalar and vector potentials

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\Phi$$
$$\vec{B} = \vec{\nabla} \times \vec{A} \tag{3.35}$$

These equations imply that the electric and magnetic fields are the elements of a second rank, antisymmetric field-strength tensor,

$$F^{\alpha\beta} = \partial^{\alpha} A^{\beta} - \partial^{\beta} A^{\alpha} . \tag{3.36}$$

Or, in explicit form,

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} . \tag{3.37}$$

The elements of $F_{\alpha\beta}$ are obtained from $F^{\alpha\beta}$ by putting $\vec{E} \to -\vec{E}$ since the dual $F^{\alpha\beta}$ is defined by ½ $\varepsilon^{\alpha\beta\beta\gamma\delta}$ $F_{\gamma\delta}$ .The dual field-strength tensor is defined as

$$F_{dual}{}^{\alpha\beta} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}. \tag{3.38}$$

The inhomogeneous Maxwell equations read

$$\begin{aligned} &\vec{\nabla} \bullet \vec{E} = 4\pi\rho \\ &\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{j} \end{aligned}. \tag{3.39}$$

They can be written in a covariant form as follows

$$\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} j^\beta . \tag{3.40}$$

The homogeneous Maxwell equations read

$$\begin{aligned} &\vec{\nabla} \bullet \vec{B} = 0 \\ &\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0 \end{aligned}. \tag{3.41}$$

These two equations can be written in a covariant form as

$$\partial_\alpha F_{dual}{}^{\alpha\beta} = 0 . \tag{3.42}$$

The equations (3.47) and (3.49) form the relativistic Maxwell equations in flat space.

The continuity equation which follows from Eq. (3.40) reads

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.\vec{J} = 0 , \tag{3.43}$$

where ρ(x, t) is charge density and J(x, t) is current density. If we postulate that they form a 4 vector J^α together as

$$J^\alpha = (c\rho, \vec{J}) , \tag{3.44}$$

Then the continuity equation (3.35) takes the form

$$\partial_\alpha J^\alpha = 0 . \tag{3.45}$$

Within the scheme of Lorentz gauge, The equations for the vector potential $\vec{A}$ and the scalar potential Φ are

$$\begin{aligned} &\frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \nabla^2 \vec{A} = \frac{4\pi}{c}\vec{J} \\ &\frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t^2} - \nabla^2 \Phi = 4\pi\rho \end{aligned} \tag{3.46}$$

together with the Lorentz condition

$$\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla.\vec{A} = 0\,. \tag{3.47}$$

Defining the vector potential

$$A^{\alpha} = (\Phi, \vec{A})\,, \tag{3.48}$$

We see that Eqs. (3.38) and (3.39) reads

$$\begin{aligned} &\Box A^{\alpha} = \frac{4\pi}{c} J^{\alpha} \\ &\partial_{\alpha} A^{\alpha} = 0 \end{aligned} \tag{3.49}$$

At this point, we claim that time reversal invariance is to be given as

$${}^{T}F_{\alpha\beta} = -F^{\alpha\beta},\;\; {}^{T}J^{\alpha} = -J_{\alpha}. \tag{3.50}$$

It is easy to see that the electric, magnetic fields and charge are all on equal footing in the above equation (for details, see Malament 2004). With the time reversal given in Eq. (3.50), one can easily see that Eqs. (3.47) and (3.49) are time reversal invariant. The property of Maxwell equations being time reversal invariant is nontrivial. In order to emphasize this point, David Malament chooses the following arbitrary Maxwell-like equation

$$\partial_{\gamma}(F_{\alpha\beta}F^{\alpha\beta}) = \frac{4\pi}{c} j_{\gamma}\,. \tag{3.51}$$

The left hand side of the equation above is time reversal invariant, whereas the right hand side is not. The Eq. (3.51) too is written in a covariant way but it is not time reversal invariant. Therefore, the fact that we have written Maxwell equations in covariant form does not ensure that it is time reversal invariant. These are two different issues not to be confused with one another. The fact that Maxwell equations are time reversal invariant is independent of the definitions of $F^{\alpha\beta}$ or j^{α} . It is only easier to see it this way!

In order to understand David Albert's claim, it is better even to get deeper in relativistic approach. For this purpose, let us adopt a general relativistic formulation due to Malament. To this end, let (M, $g_{\alpha\beta}$) be a relativistic space time, in other words, let M be a smooth, connected, four-dimensional manifold and g_{ab} be the metric with signature (1, 3) associated with this manifold (Wald, 1984). With this signature, a vector is timelike if its norm is greater than zero, spacelike if the norm is less than zero and null if it is zero. Let τ^{α} be a continuous timelike vector field on M. This is tantamount to assume that (M, g_{ab}) is temporally orientable. All timelike vectors at all points qualify as either future-directed or past-directed relative to τ^{α} since it is impossible to have two timelike vectors orthogonal to one another. A timelike vector ξ^{α} is future-directed relative to τ^{α} if $\tau^{\alpha}\xi_{\alpha} > 0$ and past-directed relative to τ^{α} if $\tau^{\alpha}\xi_{\alpha} < 0$. A four-dimensional volume element is a smooth tensor field $\varepsilon_{\alpha\beta\gamma\delta}$ on M that is completely anti-symmetric and satisfies the normalization condition $\varepsilon_{\alpha\beta\gamma\delta}\,\varepsilon^{\alpha\beta\gamma\delta} = -24$. If there exists such a volume element in M, then (M, g_{ab}) is said to be orientable. Then, there are two volume elements in M, one $\varepsilon_{\alpha\beta\gamma\delta}$ and the other being - $\varepsilon_{\alpha\beta\gamma\delta}$. So, we assume from now on that (M, g_{ab}) is orientable as well as temporally orientable and $\varepsilon_{\alpha\beta\gamma\delta}$ is volume element. Let η^{α} be a frame of reference on (M, g_{ab}). Let $\varepsilon_{\alpha\beta\gamma}$ be the spatial volume element relative to η^{α}. Now, let us consider the effects of three operators on these objects:

Time reversal operator T does not act on $\varepsilon_{\alpha\beta\gamma}$ but affects τ^{α} and $\varepsilon_{\alpha\beta\gamma\delta}$ by multiplying the latter two with a minus sign. Spatial parity reversal P does not act on τ^{α} but affects $\varepsilon_{\alpha\beta\gamma}$ and $\varepsilon_{\alpha\beta\gamma\delta}$ by multiplying the latter two with a minus sign. Together, they do not change $\varepsilon_{\alpha\beta\gamma\delta}$ but multiply τ^{α} and $\varepsilon_{\alpha\beta\gamma}$ by a minus sign. These important fundamental properties are summarized in the following table.

Table 3.1: Fundamental Reversals

Fundamentals	Time Reversal T	Spatial Parity Reversal P	TP
τ^{α}	$-\tau^{\alpha}$	τ^{α}	$-\tau^{\alpha}$
$\varepsilon_{\alpha\beta\gamma\delta}$	$-\varepsilon_{\alpha\beta\gamma\delta}$	$-\varepsilon_{\alpha\beta\gamma\delta}$	$\varepsilon_{\alpha\beta\gamma\delta}$
$\varepsilon_{\alpha\beta\gamma}$	$\varepsilon_{\alpha\beta\gamma}$	$-\varepsilon_{\alpha\beta\gamma}$	$-\varepsilon_{\alpha\beta\gamma}$

Another table can be prepared in order to explain what will happen to electric, magnetic fields and current densities in terms of these transformations.

Table 3.2: Particular Reversals

Time Reversal	**Parity Reversal**	**TP Reversal**
${}^{T}\rho = \rho$	${}^{P}\rho = \rho$	${}^{TP}\rho = \rho$
${}^{T}j^{\alpha} = -j^{\alpha}$	${}^{P}j^{\alpha} = j^{\alpha}$	${}^{TP}j^{\alpha} = -j^{\alpha}$
${}^{T}E^{\alpha} = E^{\alpha}$	${}^{P}E^{\alpha} = E^{\alpha}$	${}^{TP}E^{\alpha} = E^{\alpha}$
${}^{T}B^{\alpha} = -B^{\alpha}$	${}^{P}B^{\alpha} = -B^{\alpha}$	${}^{TP}B^{\alpha} = B^{\alpha}$

Let us try to understand how these entries in Table 3.2 is formed. The definitions that we will need are given below:

$$\begin{aligned}
\rho &= J^{\alpha}\eta_{\alpha} \\
J^{\alpha} &= (g^{\alpha}_{\beta} - \eta^{\alpha}\eta_{\beta})J^{\beta} \\
E^{\alpha} &= F^{\alpha}_{\beta}\eta^{\beta} \\
B^{\alpha} &= \frac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}\eta_{\beta}F_{\gamma\delta}
\end{aligned} \tag{3.52}$$

According to these definitions above (see Malament 2004 for details), time reversal of charge is given by

$$^{T}\rho = {}^{T}J^{\alpha}\, {}^{T}\eta_{\alpha} = (-J^{\alpha})(-\eta_{\alpha}) = J^{\alpha}\eta_{\alpha}\,. \tag{3.53}$$

The time reversal of J was already given in Eq. (3.50). It makes future-directed timelike vectors into past-directed ones i.e., η^{α} = - η^{α}. Therefore, Eq. (3.53) follows. For the other entries in the first column, we have

$$\begin{aligned}
{}^{T}J^{\alpha} &= ({}^{T}g^{\alpha}_{\beta} - {}^{T}\eta^{\alpha}\, {}^{T}\eta_{\alpha})\, {}^{T}J^{\beta} = (g^{\alpha}_{\beta} - \eta^{\alpha}\eta_{\alpha})(-J^{\beta}) = -J^{\alpha} \\
{}^{T}E^{\alpha} &= {}^{T}F^{\alpha}_{\beta}\, {}^{T}\eta^{\beta} = (-F^{\alpha}_{\beta})(-\eta^{\beta}) = F^{\alpha}_{\beta}\eta^{\beta} = E^{\alpha} \\
{}^{T}B^{\alpha} &= \frac{1}{2}\, {}^{T}\varepsilon^{\alpha\beta\gamma\delta}\, {}^{T}\eta_{\beta}\, {}^{T}F_{\gamma\delta} = -B^{\alpha}
\end{aligned} \tag{3.54}$$

The table 3.2 shows that what Albert's proposed as the genuine transformation for electric and magnetic fields corresponds to the last column which is a combination of spatial parity and time reversal operations. Magnetic fields do not just lie there as Albert puts it in his book "Time and Chance" but are left intact under TP because the actions of the two operations cancel one another. The transformation properties of magnetic field are exactly the same as of angular velocity. As David Malament puts it, concerning the last entry in the last column,

> If we make a movie of a fluid whirling in a clockwise direction, and then play the movie backwards, we see the fluid whirling in a counterclockwise direction. The angular velocity of the fluid is reversed. On the other hand, if we play it backwards, project the image onto a mirror, and then watch the reflected image, we see the fluid whirling in a clockwise direction again, as in the original. In this case, angular velocity is not reversed. (Malament, 2004)

One point of debate might be the claim that Maxwell equations are invariant under Albert's proposed time reversal. Let us make this point clearer: we have observed that Albert's proposal corresponds to TP invariance. In other words, why not use Albert's definition, why not buy his transformation rules since Maxwell equations are also invariant under those operations. This somehow looks like providing support in Albert's claim. This apparent conclusion is deceiving since TP invariance does not ensure T invariance. Take, for example, Eq. (3.48). This equation is TP invariant as we already stated but it is not time reversal invariant if we understand Albert's version as a definition of time reversal operation. The reason for this is that although electric and magnetic fields stay invariant under this transformation, t has to be replaced by –t, thereby breaking the time reversal invariance of the equation. If we consider Albert's transformation as TP though, there comes another minus sign to cross product (curl) which makes both sides of the equation even. This simply shows us that although Maxwell equations are TP invariant, the same set of operations do not ensure the time reversal invariance of the Maxwell equations.

This relativistic approach we considered above is founded on the idea of temporal orientability. The textbook approach assumes the background temporal orientation fixed, but inverts dynamical histories under the action of symmetries.

3.3 Self-interaction and Causality

Every classical object has a self-field which affects its motion. It is generated by the moving object and acts back on it. Therefore, this self-force must be added into the equations of motion, in other words, it must be taken into account in all cases. One must note that only part of the self force can be identified as due to the reaction of the emitted radiation. But, the remaining part is due to the nonlocality of this self-interaction term (Rohrlich, 2000).

In Newtonian physics, particles must be classical i.e., they must be macroscopic. So, from now on, what we understand by the word “particle” will be only extended objects, not point particles.

Another important distinction must be made of the use of the word causality in this section: causality here must be taken to be the claim that a cause cannot be later in time than the effect it causes. It is not meant to be the same as predictability nor determinism. We also assume that integrability holds. Integrability in this sense means that small changes in initial conditions only yield small changes in the prediction of a later state. Chaotic motions are classified as non-integrable ones.

Historically, it was first Lorentz who calculated the self-field of an extended object non-relativistically. He found that this additional term is

$$F_{self} = \frac{2}{3}\frac{e^2}{c^3}\ddot{v}, \tag{3.55}$$

where e is the total charge, v the velocity of the particle, c speed of light. Double dot above the velocity term indicated double differentiation with respect to time. The equation above is called Lorentz equation in the literature (Lorentz, 1892). This additional term above explicitly contradicts with the Newtonian law that an equation must be first order in time derivative of the velocity. The double derivative simply means that we must specify not only the initial position and initial velocity but also initial acceleration of the particle.

Later, due to the discovery of the electron, the same problem have been handled relativistically. The first two scientists working on this problem were Thomson (Thomson, 1897) and Abraham (1904, 1905). Abraham did not know about Einstein’s theory of special relativity since it was still unpublished then. His equation took into account the action of the self-field on the motion of the particle. The same term given by Eq. (3.55) is obtained also in the relativistic framework and given the name “Schott term”. Later on, Dirac took over this problem and he also obtained the same term and especially because of the prominence of Dirac, this equation became highly respected and called Lorentz-Abraham-Dirac (LAD) equation. Together with the four-vector force of radiation reaction, LAD equation reads

$$F^{\mu}{}_{self} = \frac{2}{3}\frac{e^2}{c^3}(\ddot{v}^{\mu} - \dot{v}^{\alpha}\dot{v}_{\alpha}v^{\mu}/c^2)\,. \qquad (3.56)$$

Forcing classical physics to provide solutions outside its own domain of applicability resulted in two main "pathologies": one is the self-accelerating solutions of LAD equation and the other is acausal solutions. One obtains self-accelerating solutions out of LAD equation when there are no external forces. According to Newton's laws of dynamics, this case must result in a constant velocity motion. Instead, as already stated above, one has a particle which constantly accelerates without any force exerted on the particle. The second pathology indicates the existence of solutions which show acceleration due to the future action of a force which breaks the causality. Fritz Rohrlich summarizes what happened next in a way which must be very pedagogical to all scientists:

> Unfortunately, during much of the half-century following Dirac's work, some physicists tried to 'repair' the LAD equations instead of recognizing that its pathologies are symptoms of the inapplicability of classical physics to point particles. Such particles must be treated by quantum mechanics and are outside the validity limits of any classical theory. Therefore, this 'repair work' led to a useless literature but was unfortunately quite voluminous. (Rohrlich, 2000)

The irony is that these scientists working on LAD had completely overlooked a work done by Sommerfeld in 1904 (Sommerfeld, 1904). Sommerfeld calculated for a surface-charged sphere of total electric charge e moving with non-relativistic velocity. Later in 1918, this calculation has been repeated by Page. This equation reads

$$F_{self} = m_e[v(t-\tau_a) - v(t)]/\tau_a\,, \qquad (3.57)$$

where m_e is mass, v is the velocity, a is the radius of the sphere, and τ_a is the time it takes a light ray to traverse the diameter of the sphere. The Sommerfeld-Page (SP) equation is not a differential equation of motion but a differential-difference equation. Moreover, it has no third order derivative term. SP equation also has no pathological solutions as LAD equation since it does deal with finite size particle. The SP equation came on stage for brief period of time in 1977 due to papers written by Levine, Moniz ad Sharp but remain almost forgotten until the works of Yaghjian in 1992.

Yaghjian considered a sphere with radius a and a uniform charge distribution on the surface. He then showed that the self-force due to self-field is proportional to the earlier velocity v(t- τ_a) at a time t, observed in its own rest frame. Yaghjian did not make any non-relativistic assumption so it has been easy to generalize it into relativistic reference frame. When this equation is inserted into the equation of motion, one obtains an equation first conjectured by Caldirola in 1956 (Caldirola, 1956). This equation is now called Caldirola-Yaghjian (CY) equation and reads

$$\begin{aligned} m\dot{v}^{\mu}(\tau) &= F^{\mu}{}_{in}(\tau) + \\ & m_e[v^{\mu}(t-\tau_a) - v^{\mu}(\tau)v^{\alpha}(\tau)v_{\alpha}(\tau-\tau_a)]/\tau_a \end{aligned} \tag{3.58}$$

The Eq. (3.58) replaces Eq. (3.57). CY equation reduces to SP equation in the non-relativistic limit. It reduces to LAD equation as the radius a goes to zero. And finally, it has no pathological solutions. CY is a relativistic equation which takes into account self-field for a finite size charged particle. The SP and CY equations are the only classical equations for an extended charged particle which include electromagnetic self-interaction. Both CY and SP are not time reversal invariant due to the explicit occurrence of t and t- τ_a in Eq. (3.57) and τ and τ - τ_a in Eq. (3.58). Rohrlich comments on this asymmetry as follows

> And it is physically intuitive because self-interaction involves the interaction of one element of charge on the particle wit another such element That

> interaction takes place by the first element emitting an electromagnetic field, propagating it along the future light cone, and then interacting with the other element of charge. The *future light cone* (rather than the past light cone) was selected by using the *retarded* fields (rather than the advanced fields). An asymmetry in time was thus introduced according to the causal structure of this process. What is at first somewhat surprising, however, is that LAD equation is invariant under time reversal (Rohrlich, 1965). But this, too, is now easily understood: that equation describes a point charge; therefore if that point charge is thought of as a charged sphere that shrank to a point, the light cones that send the self-field from one element of charge to another also shrink to a point. In that limit, therefore, there is no difference between past and future light cones. (Rohrlich, 2000)

The main result of Rohrlich's paper is to state that classical physics is not time reversal invariant if one includes the self-interaction in the picture. This case is reminiscent of Hutchison's paper which was defending the case that Newtonian mechanics is not time reversal invariant. Together with Hutchison, Rohrlich tries to show that time reversal invariance is not broken only by Second Law but also lacking in the case of classical physics under certain conditions.

An objection has been made against Rohrlich's arguments by Carlo Rovelli (Rovelli, 2004). First, let us see how Rovelli understands Rohrlich's argument. This will prove to be important in order to understand his vantage point. Rovelli thinks that Rohrlich founds his case on an equation of the form

$$F(t) = F_{ext}(t) + F_{self}{}^{earlier}, \qquad (3.59)$$

where external force is applied for a short finite time interval to accelerate the particle. This acceleration generates a radiation field which in turn acts back on the extended particle causing it to feel a self-force (the second term on the right hand side). This process of course takes some time, there will indeed be delay. Then, he takes what the time reversal of Eq. (3.59) is supposed to be as

$$F(t) = F_{ext}(t) + F_{self}{}^{later}. \qquad (3.60)$$

This time, instead of an earlier time, there is a later time involved in the time reversed of the Eq. (3.59). Rohrlich immediately judges this case to be in complete violation of causality since Eq. (3.60) requires the specification of initial points in the future. This cannot be, in view of causality so Rohrlich argues, we can discard Eq. (3.60). So, we do not have a time reversed equation for Eq. (3.59), and we do not have time reversibility in classical physics.

First of all, Carlo Rovelli does not agree with Rohrlich on using causality in this manner. He expresses his criticism in the following words

> … We can always write an equation that connects a force at time t_2 with some events that happened at an earlier time t_1. We can also argue that the event at time t_1 was the "cause" of the force acting at time t_2, if we like to think in terms of "causes". But, in the time-reversed process, we cannot keep the same causal connections. If we want to think in terms of causes, causal connections must be reversed. If in the "forward" tennis game a bounce A happens first and a bounce B happens later, then we can say that the bounce A is the "cause" of the later force at the bounce B. But, in the time-reversed process, it is the bounce B that happens first. Therefore, we cannot say anymore that A causes the force at B. This does not contradict the fact that there exists an equation connecting the force at B with the (later in the time-reversed process) bounce at A. (Rovelli, 2004)

In other words, Rovelli does not allow causality to play a role in distinguishing the understanding of time reversal of these equations. He sees it merely as a matter of words such as "earlier" and "later" to be replaced with one another.

Secondly, Rovelli notices one important point: right from the beginning, Rohrlich decomposes the overall field into two i.e.,

$$F_{\mu\upsilon}(x,t) = F_{\mu\upsilon}^{ext}(x,t) + F_{\mu\upsilon}^{self}(x,t)\,. \qquad (3.61)$$

The first component is the external field, the second is the self-field generated by the acceleration of the particle itself as is already indicated above. Now, the crucial point is that this decomposition is already non-time-reversible, since external field is present even when the particle is not there by very definition. In other words, this field is same in the past as it is in future. Of course, one can decompose the overall field into two, this is permissible. What is wrong here is to insist to obtain the time reversal also in this decomposed form in terms of causality. If one really needs to do his, one must give a new meaning to external force.

Another important point made by Rovelli is as follows: Rohrlich must specify everything in terms of particles and fields. But, at some point in his paper, he drops the field in explaining the dynamics. The key assumption of Rohrlich is his specific choice of initial conditions. The field generated by the accelerating particle does not only act back on the particle itself in the form of self-force but also radiates away in to the future. Therefore, the main assumption in Rohrlich's case is that he allows the outgoing radiation but not incoming radiation when he wants to treat the time reversal of the same problem. It is especially this assumption that breaks time reversal invariance. If there is no incoming radiation, which is the case with the Rohrlich's assumption, then we can use retarded potentials.

Fritz Rohrlich responded these criticisms in an online paper in Phi. Sci. Archive (Rohrlich, 2004). He first stated that he also took into account the incoming radiation (Rohrlich, 1999). He finds Rovelli guilty of suppressing all the relevant indices. He presents the matter more cautiously as follows: He first gives a clearer explanation for LAD equation by decomposing it into components but this time also taking into account incoming field:

$$F^{\mu\upsilon}(x,t) = F^{\mu\upsilon}_{in}(x,t) + F^{\mu\upsilon}_{ret}(x,t) = F^{\mu\upsilon}_{out}(x,t) + F^{\mu\upsilon}_{adv}(x,t)\,. \qquad (3.62)$$

He then defines the symmetric and asymmetric combinations as

$$2F_{+}^{\ \mu\upsilon}(x,t) = F^{\mu\upsilon}_{ret}(x,t) + F^{\mu\upsilon}_{adv}(x,t) \qquad (3.63)$$

and

$$2F_{-}^{\ \mu\upsilon}(x,t) = F^{\mu\upsilon}_{ret}(x,t) - F^{\mu\upsilon}_{adv}(x,t)\,. \qquad (3.64)$$

When the Lorentz force on a point charge is evaluated (Rohrlich, 1990), one obtains

$$\begin{aligned} F_{+}^{\ \mu} &= eF_{+}^{\ \mu\upsilon}v_{\upsilon} = -m_e\dot{v}^{\mu} \\ F_{-}^{\ \mu} &= eF_{-}^{\ \mu\upsilon}v_{\upsilon} = \Gamma^{\mu} \end{aligned}\,. \qquad (3.65)$$

where the dot indicated differentiation with respect to proper time τ, m_e is the electrostatic mass of a surface charged sphere of radius a and equal to $e^2/2a$.

$$\Gamma^{\mu} = \frac{2e^2}{3}(\ddot{v}^{\mu} - \dot{v}^{\alpha}\dot{v}_{\alpha}v^{\mu}) \tag{3.66}$$

is the radiation reaction written in Gaussian units with c = 1. Since we are trying to treat a point particle, the radius a must approach to zero giving us a divergent m_e. After normalization, we are led to following equation

$$m\dot{v}^{\mu} = F_{in}^{\mu} + \Gamma^{\mu}, \tag{3.67}$$

where mass term m is the difference between the renormalized (observable) mass and m_e. F_{in}^{μ} is the Lorentz force due to the incoming field. The time reversal state is described by

$$\begin{aligned} &x^{\mu} \to x_{\mu} \\ &v^{\mu}(\tau) \to -v_{\mu}(\tau) \\ &F_{ret}^{\mu\upsilon}(x) \to -F_{\mu\upsilon}^{adv}(x) \\ &F_{adv}^{\mu\upsilon}(x) \to -F_{\mu\upsilon}^{ret}(x) \quad . \\ &F_{in}^{\mu\upsilon}(x) \to -F_{\mu\upsilon}^{out}(x) \\ &F_{out}^{\mu\upsilon}(x) \to -F_{\mu\upsilon}^{in}(x) \\ &\Gamma^{\mu} \to -\Gamma_{\mu} \end{aligned} \tag{3.68}$$

The last term is obtained by inspecting Eq. (3.66). This set of transformations show us that the time reversed form of the LAD equation written above as Eq. (3.67) is given by

$$m\dot{v}_{\mu} = F_{\mu}^{out} - \Gamma_{\mu}. \tag{3.69}$$

This shows us that LAD equation is apparently not time reversal invariant. However, note that one can get, from the Eqs. (3.62), (3.63), (3.64) and (3.65)

$$F_{in}^{\mu} + \Gamma^{\mu} = F_{out}^{\mu} - \Gamma^{\mu}. \tag{3.70}$$

This means that LAD equation, in opposition to the apparent asymmetry in Eq. (3.69), is indeed time reversal invariant. It is so even though there is reference to retarded fields. In Rohrlich's words, what matters is only the total field given by Eq. (3.62) and it can be expressed in either way.

Then, we have the Caldirola-Yaghjian equation for a sphere with radius a. This equation already written above as Eq. (3.58) and its properties are well listed. The CY equation is not time reversal invariant as mentioned before. Another way to see this is to know that it can be written, by expanding in powers of 2a, in the following form

$$m\dot{v}^{\mu} = F_{in}^{\mu} + \Gamma^{\mu} + R^{\mu}(\tau, a)\,, \qquad (3.71)$$

Where the last term is a remainder term O^{r} which contains third and higher derivatives of velocity four vector and it vanishes for a = 0. Since this last term is not time reversal invariant, CY equation overall is not time reversal invariant.

Rohrlich warns that one must not think T violating term in the equation above are relatively small. Though small, that term is responsible for establishing the agreement between the theory and the observation. In that sense, they are essential.

3.4 On Solutions of Maxwell Equations

We will, for the sake of simplicity, assume the medium to be non-dispersive now so that wave velocity c is constant. From Maxwell equations, one can obtain the following wave equation (Jackson, 1975)

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\Phi(\vec{r},t) = 4\pi\rho(\vec{r},t)\,, \qquad (3.\ 72)$$

where $\Phi(\vec{r},t)$ is the amplitude of the wave and $\rho(\vec{r},t)$ is the corresponding source density. Eq. (5.72) has two solutions. First one reads

$$\Phi_{ret}(\vec{r},t) = \int \left[\frac{\rho(\vec{r}\,',t')}{|\vec{r}-\vec{r}\,'|}\right]_{ret} d^3r'\,, \qquad (3.73)$$

where the integral is taken over all space. The subscript “ret” denotes that time t' is evaluated as the retarded time $t-\frac{|\vec{r}-\vec{r}'|}{c}$. This “retarded” solution expresses the fact that the disturbance at position $\vec{r}$ at time t is caused by the source at another point $\vec{r}'$, but not at a simultaneous time t; instead at an earlier time t',the difference being due to the delay in propagation of the disturbance. The total amplitude $\Phi_{ret}(\vec{r},t)$ is the linear superposition of all these earlier sources. Another solution to wave equation can be written as

$$\Phi_{adv}(\vec{r},t)=\int[\frac{\rho(\vec{r}',t')}{|\vec{r}-\vec{r}'|}]_{adv}\,d^3r' , \qquad (3.74)$$

where the integral is taken over all space. The subscript “adv” denotes that time t' is evaluated as the advanced time $t+\frac{|\vec{r}-\vec{r}'|}{c}$. This “advanced” solution expresses the fact that the disturbance at position $\vec{r}$ at time t is caused by the source at another point $\vec{r}'$, but not at a simultaneous time t; instead at a later time t'. The total amplitude $\Phi_{ret}(\vec{r},t)$ is the linear superposition of all these later sources.

One point is worth making: The Eqs. (3.73) and (3.74) are not time reverses of one another. In order to obtain Eq. (3.73) from Eq. (3.74), it does not suffice only to reverse the time but is necessary to invert source, too. What we observe in nature though as a solution of wave equation is not advanced solutions but the retarded ones. The retarded solutions correspond to a radio wave coming from infinity and converging onto a radio transmitter for example.

Following the explanation given by Davies and Jackson, let us first try to understand the relevance of boundary conditions related to advanced and retarded solutions. The wave equation is a second order inhomogeneous hyperbolic partial differential equation. In order to have a unique solution, one must specify Dirichlet and Neumann boundary conditions together i.e. both Φ and $\frac{\partial\Phi}{\partial t}$ throughout all space at one time t. For example, a possibility would be the following

$$\Phi = \frac{\partial\Phi}{\partial t} = 0\,, \tag{3.75}$$

For t < 0 in the case of retarded solutions and

$$\Phi = \frac{\partial\Phi}{\partial t} = 0\,, \tag{3.76}$$

For t > 0 in the case of advanced solutions. From any solution, another may always be obtained by adding a solution to the homogeneous (source free) equation. Also the difference between retarded and advanced solution forms a solution to the homogeneous equation.

In order to have a deeper understanding, we can write the wave equation in integral representation as

$$\begin{aligned}\Phi(\vec{r},t) = &\int_V \frac{[\rho]_{ret}}{R}\,dV + \\ &\frac{1}{4\pi}\int_S \{[\Phi]_{ret}\,\frac{\partial}{\partial n}(\frac{1}{R}) - \frac{1}{R}\frac{\partial R}{\partial n}[\frac{\partial\Phi}{\partial t}]_{ret} - \frac{1}{R}[\frac{\partial\Phi}{\partial n}]_{ret}\}dS\end{aligned}\;. \tag{3.77}$$

In the equation above, we have taken c equal to 1, and R = $|\vec{r} - \vec{r}'|$. The meaning of the terms are illustrates in Fig. (3.1) below.

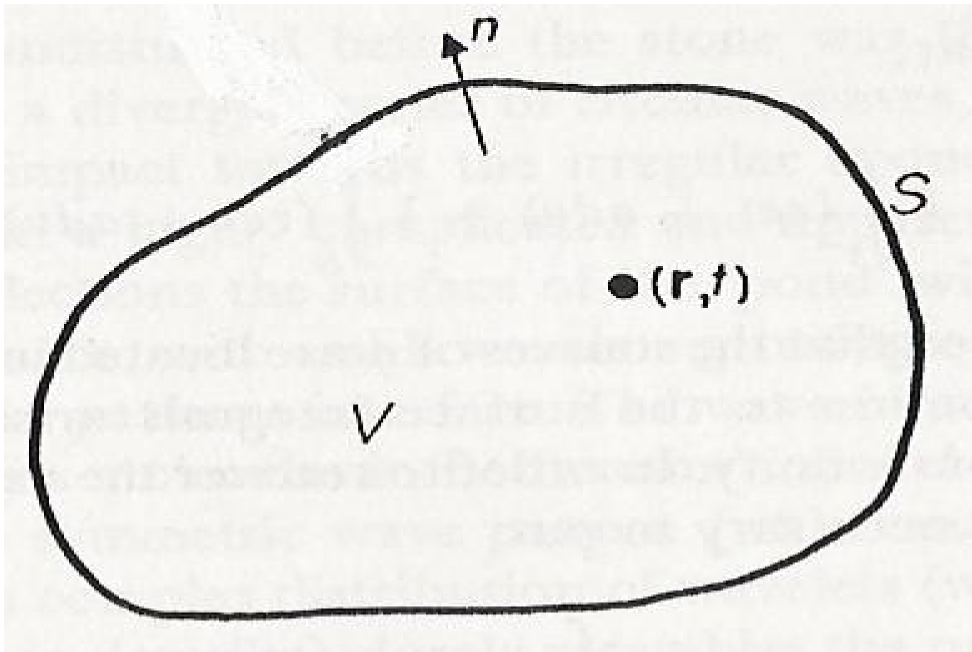

Figure 3.1: Surface and Volume Integrals

In Fig. (3.1), we have a smooth closed surface S which bounds a volume V. The surface S has an outward normal $\vec{n}$. The position vector $\vec{r}$ refers to a point inside V. Keeping this figure in mind, one can find three sources that contribute the total field $\Phi(\vec{r},t)$ as:

I. The first volume integral in Eq. (3.77) corresponds to the sources inside the volume V.
II. The sources outside V which is taken care of by surface integral.
III. Source free disturbances coming from infinity which is still represented by some parts in surface integral.

The volume integral satisfies the inhomogeneous equation whereas the surface integral satisfies the homogeneous equation, inside V.

There is another integral representation which will be very useful in further discussions in this Section. The total field can be written as

$$\Phi(\vec{r},t) = \int_V adv + \int_S adv \,. \tag{3.78}$$

Or,

$$\Phi(\vec{r},t) = \int_V ret + \int_S ret \,. \tag{3.79}$$

The abbreviations “ret” and “adv” correspond to “retarded” and “advanced” respectively. Because of the linearity of the wave equation, any linear combination of ret and adv may be taken as

$$\Phi(\vec{r},t) = k\int_V ret + (1-k)\int_V adv + k\int_S ret + (1-k)\int_S adv \,, \tag{3.80}$$

Where k < 1. In particular, we can equate k = ½ and obtain the following expression:

$$\Phi(\vec{r},t) = \frac{1}{2}\int_V (ret + adv) + \frac{1}{2}\int_S (ret + adv) \,. \tag{3.81}$$

If we now suppose that the sources creating the disturbance are located in a small region within V, then the contribution to the surface integral is only due to the source free disturbances coming from infinity. In order to have Eq. (3.73) from Eq. (3.79), we have to have

$$\int_S ret = 0 \,, \tag{3.82}$$

i.e., source free radiation in the retarded case should vanish, too. But, in order to recover Eq. (3.74) from Eq. (3.78), we cannot say that

$$\int_S adv = 0 \,. \tag{3.83}$$

In fact, we have

$$\int_S adv = \int_V ret - \int_V adv \,., \tag{3.84}$$

which in general does not vanish. This shows us that although we are allowed to use both retarded and advanced formulation equivalently, the boundary conditions must be chosen in a different way. The interpretation of Eqs. (3.82) and (3.84) is odd enough to be written: one must not allow for source free radiations coming into region of interest from remote *past*, but must allow for disturbances propagating outwards from the region of interest into the remote *future*.

Since the boundary conditions play a very important role as is seen above, let us try to have a better understanding of them. In order to do this, let us use the following notation due to Dirac (Dirac, 1938) and Davies (Davies, 1974):

$$\begin{aligned} F_{tot} &= F_{ret} + F_{in} \\ F_{tot} &= F_{adv} + F_{out} \end{aligned} \tag{3.85}$$

Where the total field is either decomposed into the incident field from outside the volume V plus the retarded contribution or outgoing field plus the advanced contribution. The incident field F_{in} satisfies the homogeneous equation for $t \to -\infty$. Similarly, The outgoing field F_{out} satisfies the homogeneous equation for $t \to +\infty$. Dirac then defined the radiation field as

$$F_{rad} = F_{out} - F_{in} \,. \tag{3.86}$$

The equation above can be written, due to Eq. (3.85), as

$$F_{rad} = F_{ret} - F_{adv} \,. \tag{3.87}$$

The corresponding potential $(A_{ret} - A_{adv})$ is a solution of the homogeneous equation

$$\Box^2 A_\mu = 0 \,. \tag{3.88}$$

The solutions of Eq. (3.88) have the property that if they vanish on the surface S at all times, they vanish everywhere. Consequently, $F_{rad} = 0$ everywhere when the particle acceleration is zero.

Let us also introduce the following fields:

$$\begin{aligned} \bar{F} &= \frac{1}{2}(F_{ret} + F_{adv}) \\ \bar{A} &= \frac{1}{2}(A_{ret} + A_{adv}) \end{aligned} \tag{3.89}$$

That they are time symmetric can easily be seen now. These fields are solutions of the inhomogeneous equation. The appropriate solutions of the corresponding homogeneous equation can always be added to these equation above, so that we can, for example, write

$$F_{ret} = \bar{F} + \frac{1}{2}F_{rad}$$
$$F_{adv} = \bar{F} - \frac{1}{2}F_{rad} \qquad (3.90)$$

Rewriting the total radiation as,

$$F_{tot} = \bar{F} + \frac{1}{2}F_{rad} + F_{in}, \qquad (3.91)$$

Dirac evaluated the effect of this total field when surface S encloses a single charge. According to Dirac's calculations, The first two terms represent the particle's self fields, while the third term represents the incoming field due to all other particles in the world outside V and any radiation coming from past infinity. Dirac then showed that self field results in divergence for a point source.

We did not specify anything particular about F_{in}. Inspecting Eq. (3.91), one sees how the retarded field F_{in} of a single charge can be decomposed into a source free part $1/2(F_{ret} - F_{adv})$ which causes the observable, finite radiation damping force and $1/2(F_{ret} + F_{adv})$ which leads to a part of self energy.

Let us now consider a collection of charged particles in a volume V bounded by a smooth surface S in an otherwise empty world (Davies, 1974). The total force acting on particle i is due to the field

$$\sum_{j \neq i} F_{(j)ret} + \frac{1}{2}(F_{(i)ret} - F_{(i)adv}) + F_{in}. \qquad (3.92)$$

F_{in} now includes only the source free fields coming from infinity since there are supposedly no charges outside the volume V. But, there is another situation which is similar to the equation above. In Eq. (3.92), we did not take boundary conditions into account. Therefore, this equation must still preserve its time symmetrical nature due to the Maxwell equations. In fact, we could have started right from the beginning with the following equation

$$\sum_{j\neq i} F_{(j)adv} - \frac{1}{2}(F_{(i)ret} - F_{(i)adv}) + F_{out} . \tag{3.93}$$

Moreover, summing the last two equations, we obtain

$$\frac{1}{2}\sum_{j\neq i}(F_{(j)ret} + F_{(j)adv}) + \frac{1}{2}(F_{in} + F_{out}) . \tag{3.94}$$

Obviously, the Eq. (3.94) is time symmetric. Now, the issue is that we only have retarded waves in nature. If we would like to simulate this case with the equation above, we have to set the boundary condition as follows

$$F_{in} = 0 . \tag{3.95}$$

Thos boundary condition is called "Sommerfeld radiation condition". Inspection of Eq. (3.92) shows us that, once we set this condition, the fields acting on particle i is only retarded fields of the other particles plus the self field (finite part). We also have, from Eqs. (3.86) and (3.87), we obtain

$$F_{out} = \sum_{all j}(F_{(j)ret} - F_{(j)adv}) . \tag{3.96}$$

This means that Eq. (3.93) reduces to Eq. (3.92). This simply dictates that apparent time symmetry is lost!

One can defend the time symmetrical view still by stating that one could have equally started by the boundary condition

$$F_{out} = 0 . \tag{3.97}$$

Then, we would have advanced fields and a radiation which converges onto particles and accelerates them. In short, the damping force would change sign. Now, we are ready to discuss the last Section of this Chapter which is about the absorber theory of radiation.

3.5 The Absorber Theory of Radiation

The original article written by Wheeler and Feynman (Wheeler and Feynman, 1945) almost begins with a simple statement: they write that they accept the proposal made by Tetrode (Tetrode, 1922) which is the following:

> The sun would not radiate if it were alone in space and no other bodies could absorb its radiation… If for example I observed through my telescope yesterday evening that star which let us say is 100 light years away, then not only did I know that the light which it allowed to reach my eye was emitted 100 years ago, but also the star or individual atoms of it knew already 100 years ago that I, who then did not exist, would view it yesterday evening at such and such a time… One might accordingly adopt the opinion that the amount of material in the universe determines the rate of emission. Still, this is not necessarily so, for two competing absorption centers will not collaborate but will presumably interfere with each other. If only the amount of matter is great enough and is distributed to some extent in all directions, further additions to it may well be without influence. (Tetrode, 1922)

Wheeler and Feynman accepted the proposal of Tetrode and agreed upon treating the radiation not as an elementary process but as a consequence of the interaction between a source and an absorber. They present four different derivations. We will follow the fourth one since this has been the most general derivation and followed by many philosophers of science working in the field.

According to this fourth derivation, we do not take the refractive index nor density of the absorber into account. The only assumption to the medium to be a complete absorber. This simply means that any charged particle outside the absorber will experience no disturbance. Then, Wheeler and Feynman continues to write

$$\sum_k \frac{1}{2}(F^{(k)}{}_{ret} + F^{(k)}{}_{adv}) = 0 \text{ (outside the absorber).} \qquad (3.98)$$

Since this sum vanishes outside the absorber everywhere and at all times, we must have

$$\sum_k F^{(k)}{}_{ret} = 0 \text{ (outside).} \tag{3.99}$$

and

$$\sum_k F^{(k)}{}_{adv} = 0 \text{ (outside).} \tag{3.100}$$

Retarded waves represent the outgoing waves, and advanced fields represent the incoming (converging) waves. But, complete destructive interference between these two is impossible. Therefore, the fact that their sum is equal to zero, simply means that they have to be equal to zero independently. From Eqs. (3.99) and (3.100), we can write

$$\sum_k \frac{1}{2}(F^{(k)}{}_{ret} - F^{(k)}{}_{adv}) = 0 \text{ (outside).} \tag{3.101}$$

This field is a solution of Maxwell's equations for free space. Since it vanishes everywhere outside, it must be equal to zero inside, too. Therefore,

$$\sum_k (F^{(k)}{}_{ret} - F^{(k)}{}_{adv}) = 0 \text{ (everywhere).} \tag{3.102}$$

According to the theory of action at distance, the entire field , on the ath charge is given by

$$\sum_{k \neq a} \frac{1}{2}(F^{(k)}{}_{ret} + F^{(k)}{}_{adv}) . \tag{3.103}$$

The expression above can be broken into three different parts:

$$\sum_{k \neq a} F^{(k)}{}_{ret} + \frac{1}{2}(F^{(a)}{}_{ret} - F^{(a)}{}_{adv}) - \sum_{allk} \frac{1}{2}(F^{(k)}{}_{ret} - F^{(k)}{}_{adv}) . \tag{3.104}$$

Third term vanishes for an absorber as shown previously. The second term gives rise to radiation damping.

The results of Wheeler-Feynman theory is very important since it gives us the classical time-asymmetric solutions of electrodynamics in a time-symmetric way since they presuppose the existence of both the retarded and advanced fields at the beginning. One also notes the fact that Eq. (3.102) is time symmetrical. We can easily change the subscript advanced to retarded and vice versa. However, we will have a radiation term with opposite sign in this case, suggesting that the charged particle will gain energy instead of losing energy while it accelerates. This point is interpreted by Wheeler and Feynman as follows:

> …Evidently the explanation of the one sidedness of radiation is not purely a matter of electrodynamics… We have to conclude with Einstein (W. Ritz and A. Einstein,1909) that the irreversibility of the emission process is a phenomenon of statistical mechanics connected with the asymmetry of the initial conditions with respect to time. In our example the particles of the absorber were either at rest or in random motion before the time at which the impulse was given to the source. It follows that in the equation of motion, the sum, $\sum_{k\neq a} F_{n\alpha}{}^{(k)}{}_{ret}$, of the retarded fields of the absorber particles had no particular effect on the acceleration of the source. Consequently, the normal term of radiative damping dominates the picture. In the reverse formulation of these equations of motion, the sum of the advanced fields of the absorber particles is not at all neglible, for they are put into motion by the source at just the right time to contribute to the sum $\sum_{k\neq a} F_{n\alpha}{}^{(k)}{}_{ret}$. This contribution, apart from the natural random effects of the changes of the absorber, has twice the magnitude of the usual damping term. The negative reactive force of the reversed equation of motion is therefore cancelled out, and a force of the expected sign and magnitude remains. (Wheeler and Feynman, 1945)

After this last remark, one can take the work of Wheeler and Feynman not proving the classical electrodynamics to be time symmetric but rather showing that the time asymmetry in classical theory is due to the special role played by initial conditions. In other words, according to Wheeler and Feynman, it is the initial conditions which create the time asymmetry in classical electrodynamics.

The works of Wheeler and Feynman regarding the absorber theory of radiation had great popularity among the philosophers of science such as Zeh (Zeh, 1999), Price (Price 1991a, 1991b, 1994, 1996), Ridderbos (Ridderbos, 1997), Leeds (1994, 1995), Frisch (Frisch, 2000) working in the field of time arrow in electromagnetic radiation. One interesting point made by Huw Price has been to claim that he himself has reinterpreted the "core" of their theory in order to show that electromagnetic radiation is time symmetric on the micro level. Zeh or Jackson considers radiative asymmetry described as

(3.0) All accelerated charges or sources can be associated with fully retarded (but not fully advanced) radiation fields.

They also agree that the microscopic fields associated with the individual charges exhibit time asymmetry. But, Price believes that the apparent asymmetry of radiation arises only in the macroscopic case, and agues that the asymmetry can be characterized by

(4.0) Organized waves get emitted, but only disorganized waves get absorbed.

According to Price, an emitter is a charge or a distribution of charges that emits electromagnetic energy, while an absorber is a charge that absorbs energy. Price further thinks that only emitters are associated with retarded waves, he then proceeds to write

(5.0) All emitters produce retarded rather than advanced wave fronts.

The difference between (3.0) and (5.0) is that while retarded fields are associated with all kind of charges in (3.0), it is related only to emitters in (5.0). Price first dismisses (5.0) on the basis that it gives us a symmetric picture of radiation. Then, he discards (4.0) on the basis of the fact that radiation is time symmetric at the micro level. Doing so, he takes full support from Wheeler-Feynman theory. Based on their theory, he finally proposes the following

(6.0) Both emitters and absorbers are centered on coherent wave fronts (these being outgoing in the first case and incoming in the second)

Therefore, he believes that there is no riddle to solve at micro level since radiation is time symmetric in that domain whereas this is not true for macroscopic case. So, the only riddle to be solved is the macroscopic time asymmetry. Price argues that the solution to this riddle is because of the cosmological initial conditions. There are large macroscopic coherent emitters but no macroscopic coherent absorbers. Leeds, Ridderbos and Frisch all argued against the reinterpretation of Huw Price.

Here, I will adopt the version due to Frisch since I believe his arguments are the strongest and more general than the others.

The first objection by Price and Frisch against Wheeler and Feynman is about the temporal double standard: Wheeler and Feynman begins their paper with a time symmetric equation, i.e. half retarded and half advanced. Then, when they consider the time symmetric case, they appeal to the statistical argument. But, this does not mean one is able to explain away the macroscopic existence of retarded fields only. This is what is called double temporal standard and will be explained in detail in next Chapter.

The second objection by Frisch against Price is also the one mentioned by Leeds and Ridderbos. This objection is based on the reinterpretation of Price based on Wheeler-Feynman theory. Price says that

> The real lesson of the Wheeler-Feynman argument is that the same radiation field may be described equivalently either as a coherent front or diverging from [the charge a], or as the sum of coherent wave fronts converging on the absorber particles. (Price, 1996)

Where the diverging wave is a fully retarded wave and the converging waves are fully advanced. Therefore, according to him,

$$F^{(a)}{}_{ret} = \sum_{k \neq a} F^{(k)}{}_{adv} \ . \qquad (3.105)$$

Since Price is associating only retarded fields with a point charge, he will face with the infinities related to self interaction which Wheeler and Feynman was trying to avoid. Another possibility is that he is on the same page with Wheeler and Feynman in thinking that the force on a charge is due to fields of other charges only. But it is not obvious how he can get the radiation term in his theory. The radiation term arises in Wheeler-Feynman theory only because their time symmetric fields of the source interact with the time symmetric field of the absorber. To see the impossibility of this option in Price's reinterpretation, let us use an example we borrow from Frisch: we can calculate the field of a second charge b some distance away from a. If b is one of the charges on the absorber, then we would have, according to Price,

$$F^{(a)}{}_{ret} = \sum_{k \neq a,b} F^{(k)}{}_{adv} + F^{(b)}{}_{adv} \,. \tag{3.106}$$

As one can easily see, there occurs no radiation term above related to the charge b. Another simple algebra shows us that we can obtain, from Eq. (3.105), the following

$$(F^{(a)}{}_{ret} + F^{(a)}{}_{adv}) = \sum_{all k} F^{(k)}{}_{adv} \,. \tag{3.107}$$

The left hand side of this equation will in general not be equal to zero far away from the charge. This must be so then also for the right hand side. But, this violates the fundamentals on which Wheeler-Feynman theory is built on. This shows us that Price's proposal conflicts with the absorber theory and it cannot merely be its reinterpretation.

Even though we can reject Price's theory on the grounds that it fails to be a reinterpretation of absorber theory, we cannot reject it wholly on these grounds. Whether it is a theory on its own rights requires more study.

Previously, we have seen that Maxwell equations reduce to two. One of these equations, i.e. Eq. (3.47) shows us that the four dimensional divergence is related to four current. If we look at the region surrounding the charge a, the retarded field of the charge in this region has a source but advanced field of the absorber particles does not. This retarded field due to charge a is a solution to Maxwell equations if a is the only charge in the world. Likewise, the advanced field of the absorber particles is a solution if the absorber particles are the only charges. Therefore, the divergence of the absorber field is equal to zero whereas that of the charge is not. Therefore, if they are equal, then one of then cannot satisfy Maxwell's equations which is a contradiction. This shows us that they cannot be equal to one another. Price's theory cannot be true in the fundamental sense.

At this point, something similar to what Craig Callender defended in the Hutchison case in Chapter 1 is mentioned by Frisch. Trying to answer the question why the arrow of radiation is a genuine problem, he answers by the ontological value of the Maxwell equations in electromagnetic theory. Since the solutions to Maxwell equations include both advanced and retarded solutions, we deem both of them to be actual. In this sense, we take it to be the same thing if something is physically possible or actual. One can easily read Maxwell equations giving us all the possibilities but not all actualities. One can then easily define *retardation* as a law. This is in a way similar to the solutions of quadratic equations. In general, we solve for the unknowns and obtain two unknowns. Then, the choice of the particular solution depends on the physical problem at hand. Accordingly, one can easily discard negative solution deeming it to be unphysical. This does not mean it is not a solution to the quadratic equation we are trying to solve. It is just possible but not in this *actual* case. That is all!

Another way of looking at this is related to inspecting the content of a theory and actual data. If the content of a theory exceeds of what is actual i.e., it is able to explain what really happens and more, then we do not have to discard it straight away. Even the possibility of mathematics going beyond the actual can be thought enough of a reason for not to see arrow of radiation as a genuine problem. It is true that retardation as a law will not be as profound as the Maxwell laws but still this is not a criteria for lawhood anyway.

This view has been criticized by Jill North (North, 2003). She begun her analysis by noting the importance of free field in the description of any electromagnetic phenomenon. As we have noted before, any field can be written in the form of retarded plus incoming or advanced plus outgoing fields. In this sense, North redefines the problem of asymmetry of radiation. Even though one can describe any radiation field in the way explained above, why do we perceive that it is in the form of retarded fields after all? She moreover notices that the free field in the universe do exist even though it is weak i.e., so called background radiation. According to her, we might be perceiving the advanced fields anyway since these fields might be coupled to some free fields and give us the impression that they are nothing but retarded fields.

She thinks that most of the philosophers of science appeals to the simplicity of equations when one chooses the retarded case. Since only then, the free field can be chosen small, and this seems reasonable. But, this does not alter the fact that the same situation can easily be written with a superposition of advanced field together with a source free field chosen appropriately.

There is still an apparent asymmetry in radiation though. North is against explaining this away by the retardation condition proposed above by Frisch. According to North, the existence of free fields is enough to show that not all fields comply with the retardation condition. Moreover, it cannot be derived from initial conditions plus deterministic Maxwell laws which further shadows its status as a scientific law.

Jill North offers the thermodynamic arrow (in its connexion to cosmological arrow) as a solution to the arrow of radiation. This is usually called Past Hypothesis (more in next Chapter). In the case of arrow of radiation, it can be explained as follows: let us imagine the situation in the universe right after the Big Bang. The state would be one of extremely low gravitational entropy. Since everything is in one uniformly hot soup, the universe was in thermal equilibrium. Therefore, the lowness of the entropy in this stage is not due to thermal gradients but due to gravitational entropy. Then the process of clumping up begins and stars start to form. This forms the continual change towards gravitationally higher gravitational entropy states. Of course, the universe now moves away off thermal equilibrium. Then, as a tendency to go back to the state of thermal equilibrium, accelerating charges will begin to radiate energy into the surrounding place. Since we are following the footsteps of Boltzmann, we must assert that it is quite possible for radiation to take place in both ways i.e., through retarded or advanced fields. But, note that it will be more in retarded form since advanced means radiation towards past cone. It is still probable but the probability of this event will be less since there happened to be thermal equilibrium in the past anyway.

What North is trying to do mainly is to try to explain radiative asymmetry based on existent laws plus initial conditions (Past Hypothesis). In fact, both Frisch's and North's account were predicted by Callender (Callender, 2002): he states that there are two viable stands to thermodynamic entropy. First, one can assume asymmetrical boundary conditions as North did above. Second, one can posit an additional time asymmetric law. This is what Mathias Frisch tried to do by elevating retardation condition to the privileged status of a law.

One can easily note that even though there are seemingly two viable posits, there is a major similarity between them. The dissimilarity between them is founded on their acceptance or refusal, for that matter, of the basic asymmetrical law, e.g., thermodynamical asymmetry. Frisch does not accept the thermodynamic asymmetry to be the solution to radiative phenomena as long as time arrow is considered. Of course, this is so in the first place since he does not consider the arrow of radiation to be a genuine problem. But, leaving this aside, we can easily observe that he does not choose to consider the thermodynamic asymmetry as a solution to radiative phenomena. What he reaches instead is retardation condition as a law.

Of course, thinking retardation condition in the status of law is not satisfying to many including Jill North. On the other hand, North and other philosophers of science like her take the other way out i.e., believing the Past Hypothesis to be the cure to the case under study. At this point, one can easily catch a similarity between these two approaches even though they look different. As much as retardation condition is in need of explanation, so is Past Hypothesis. Both of them looks simple enough. Both of them gives us a plausible solution. Both of them represents what is in our belt as scientists and philosophers of science; the retardation condition is already explicit in the solutions of Maxwell equations. If one accepts the general view that the theory just provides the physical solutions not the actual ones, then there occurs to be no problem as Frisch states. On the other hand, Past Hypothesis is also nothing but an initial condition which one requires in order to solve some differential equations. In fact, both of these approaches rely on the struggle of getting actual out of what is physical. One chooses to delimit the theory via initial conditions and the other through retardation condition.

Note that both approaches mentioned above will have the same predictive power as far as natural phenomena is considered.

CHAPTER 4

THE ARROW OF TIME IN STATISTICAL MECHANICS

4.1 H Theorem and the Second Law of Thermodynamics

There are many phenomena in nature which exhibit thermodynamic time asymmetric behavior. Although placing an ice cube in war water and observe it to melt is an ordinary phenomenon one can observe in daily life, the time reverse of this process i.e., the spontaneous freezing of a small part of warm water is never to be seen. Examples of this kind are many and they have one thing in common: they exhibit the so called time asymmetry in thermodynamics which is summarized in the so called "second law of thermodynamics".

One version of the second law owes itself to Lord Kelvin and Clausius. They state that heat does not, of its own accord, move from cold to hot bodies (Davies, 1974). In other words, all isolated systems tend to approach equilibrium and not to leave it again. In the language of physics, this fact is stated as

$$\Delta S \geq \int \frac{dQ}{T}, \tag{4.1}$$

where ΔS is the change in a quantity called entropy, Q is heat and T is temperature. The equality sign is applicable only when the process is reversible. For an adiabatic enclosure, we have dQ = 0, so that

$$\Delta S > 0. \tag{4.2}$$

Eq. (4.2) is valid for any change which occurs in the "real" world. It says that the entropy of an isolated system never decreases. Since all the natural changes increase the entropy of an isolated system, a condition for no change to occur must be maximum entropy condition. Such a maximum entropy state is called equilibrium state.

In order to illustrate second law, let us give the example of heat reservoirs: suppose that we have two heat reservoirs at constant temperatures T_1 and T_2 respectively. Then, we let them interact with one another thermally. If $T_1 > T_2$, then we expect that an amount of heat Q will flow from the first reservoir to the second one. The entropy change for the first reservoir will be $-Q/T_1$ and the entropy change for the second reservoir will be $-Q/T_2$. The total entropy change will therefore be equal to $Q(1/T_2 - 1/T_1)$ which is a positive quantity. This indicates that the overall entropy increased. This example shows us that the heat will spontaneously move only from hot bodies to cold bodies and not vice versa. The inverse is not to be observed since this will violate the second law of thermodynamics. Only when the two temperatures become equal, the entropy change will be zero indicating the reversibility of the situation in which case there is no temperature difference to be seen.

As a second example, one can consider an ideal gas enclosed in a cylinder composed of N number of single atoms thereby making it possible for us to neglect the intermolecular interactions. Let us assume the cylinder to be an adiabatic enclosure. The entropy of this kind of gas is given by

$$S = Nk\log(VT^{3/2}), \tag{4.3}$$

V being the volume of the cylinder, and k being a constant called Boltzmann constant . Now, let us imagine that we are expanding this gas with the help of a removable piston very quickly. Then, the gas will not do any work on the piston but will only fill the vacuum very quickly. The temperature of the gas will be constant due to the first law of thermodynamics which states

$$dE = dQ - dW \; . \tag{4.4}$$

Due to the adiabaticity of the enclosure, dQ has to be equal to zero. Since dW too is zero, we must have dE = 0, which means that energy is constant. Since

$E = E(T)$ for an ideal gas, the energy being constant implies the temperature being constant. This will require that the change in entropy be given by

$$\Delta S = Nk \log(V_2 / V_1) , \qquad (4.5)$$

where V_2 and V_1 are the final and initial volumes respectively. The change in entropy in this particular case will be positive indicating the irreversible nature of the process.

Another way to expand the gas would be to do it as we withdraw the piston infinitesimally slowly. This will ensure the states of the gas to be at equilibrium at each instant. Also, we must note that the state of an ideal gas is given by

$$PV = NkT. \qquad (4.6)$$

Since we still do have adiabatic enclosure, we deduce, from the first law of thermodynamics, that

$$dE = \frac{3}{2} NkdT = \frac{NkTdV}{V} . \qquad (4.7)$$

From the equation above, it is easy to find that

$$\log(VT^{3/2}) = cons\tan t . \qquad (4.8)$$

Inspection of Eq. (4.3) then tells us that the change in entropy is equal to zero. This simply means that the quasistatic expansion is reversible.

From the considerations above, it is explicit that the macroscopic time asymmetry is mainly founded on the second law of thermodynamics. Historically, this law was stated as H theorem by Ludwig Boltzmann. In order to proceed further, it is important to see the derivation of this theorem.

A simple model for an ideal gas consists of N identical spherical particles in a box with volume V. The number of particles must be large enough in order to allow for a statistical treatment of the gas. The container must have perfectly rigid walls so that particles will collide elastically with the walls of the container. Let us also assume the box to be adiabatic i.e., no heat transfer occurs. Assuming that the gas is dilute enough, we will neglect long-range interactions and only focus on binary interactions. We will, in other words, allow the particles to collide with one another. We will treat the subject matter classically (and non-relativistically) since quantum mechanics does not alter the picture which classical physics provides as far as temporal asymmetry is considered.

In order to describe the physical state of the ideal gas explained above, we need to determine all position and momentum coordinates of each particle. The space consisting of all these position and momentum coordinates is called μ space and its dimension is given by 6 N. The position or momentum (or velocity for that matter) of a particle at any time is associated with a point in μ space. Therefore, the entire state of the ideal gas can be traced as observing N points in this space. As the microscopic state of the gas evolves, these N points will move, too. Now, we divide this space into small cells in such a manner that the volume of the cells is large enough to contain many particles, but still small enough to be considered as infinitesimal compared to macroscopic dimensions. The size of these cells is also determined by the limits of resolution of macroscopic observation. Then, each cell will have a volume (d^3qd^3p). The total number of particles in each cell is given by f (q, p, t) d^3qd^3p where f is the density of points and is called distribution function. Integrating over all the cells in μ space, we obtain

$$N = \iint \mathrm{f}\,(\mathrm{q},\, \mathrm{p},\, \mathrm{t})\; \mathrm{d}^3\mathrm{q}\mathrm{d}^3\mathrm{p}\,. \tag{4.9}$$

In fact, knowing the explicit form of distribution function, one can calculate any macroscopic variable of interest by a suitable averaging procedure.

For many purposes, we will need the distribution function and how it evolves with time. Excluding any possibility of collision for now, we can imagine the behavior of this function to be like a fluid in phase space. Then, we can write the usual conservation equation given by

$$\frac{\partial f}{\partial t} + \vec{\nabla} \bullet (\vec{u} f) = 0\,. \tag{4.10}$$

In the equation above, f is the distribution function, $\vec{\nabla}$ is the six dimensional divergence being equal to $(\frac{\partial}{\partial q_i}, \frac{\partial}{\partial p_i})$ where the index i runs from 1 to 3. $\vec{u}$ the six dimensional velocity vector given by $(\dot{q}_i, \dot{p}_i)$. Since there are no collisions i.e., since we neglect the interaction between the particles, their energy will be constant or in other words, their momentum will be conserved. This reduces Eq. (4.10) into the following equation

$$\frac{\partial f}{\partial t} + (\vec{v} \bullet \vec{\nabla}_3) f = 0 , \tag{4.11}$$

where $\vec{v}$ is the three dimensional velocity vector and $\vec{\nabla}_3$ is the gradient operator acting only on position. If we also include an external force in Eq. (4.11), we obtain

$$(\frac{\partial}{\partial t} + \vec{v} \bullet \vec{\nabla}_3 + \frac{\vec{F}}{m} \bullet \vec{\nabla}_v) f = 0 . \tag{4.12}$$

In the equation above, the last term within the parentheses is gradient operator acting on velocity v. If we include the collisions between the particles, we must add another term on the right hand side giving us

$$(\frac{\partial}{\partial t} + \vec{v} \bullet \vec{\nabla}_3 + \frac{\vec{F}}{m} \bullet \vec{\nabla}_v) f = (\frac{\partial f}{\partial t})_{collision} . \tag{4.13}$$

This equation is the famous Boltzmann's equation. The collision term on the right hand side will ensure the sudden disappearances and appearances of cell points due to abrupt collisions taken into account. Because of the collisions, the points in μ space will be reshuffled at random.

If we throw the N points into μ space randomly, the only constraint being constant energy, each throw will produce a different microscopic arrangement of points. Many throws will essentially lead to same distribution function since we cannot see beyond the resolution scale anyway. For fixed number of particles and fixed energy, this gives us the famous Maxwell distribution i.e.,

$$f(\vec{v}) \propto e^{-\beta v^2} . \quad (4.14)$$

Due to each collision, the system will be reset. But, at one point, the state will be the most probable one i.e., the Maxwell distribution. Once Maxwell distribution is obtained, it is less likely that subsequent reshuffling will remove it. The Maxwell distribution is therefore regarded as the equilibrium distribution. The time required to reach equilibrium is called relaxation time. For N of the order of Avogadro number, deviations from Eq. (4.14), once attained, are exceedingly small. But, in order to have this picture which is consistent with the empirical results, we need to make one statistical assumption which states that the points in μ space are reshuffled at random after each collision. Let us also call this assumption as assumption A following the convention of Paul Davies (Davies, 1974). In order to move from Eq. (4.14) to H theorem, one needs a second assumption which is independent of assumption A. This assumption has been called assumption of molecular chaos (or Stosszahlansatz) by Boltzmann. According to this assumption, the positions and velocities of the particles are uncorrelated before they collide but not after the collision. In order to understand what the mathematical meaning of this assumption can be, we can try to think in terms of the following simple picture now: let us imagine two particles (also generalize them as Type 1 and Type 2 particles concerning whole distribution) moving towards one another. Let one of them have the velocity $\vec{v}_1$ and the other particle have the velocity $\vec{v}_2$. After the collision, the velocities will be changed to $\vec{v}_1'$ and $\vec{v}_2'$ respectively. The molecular chaos assumption states that the distribution functions f_1 and f_2 are the same function. In other words, the distribution functions of two types of particles are same. Of course, this cannot be said for f'_1 and f'_2 where the prime denotes the new distribution functions being changed after the collision. We can further specify the collision as a mapping from $\{\vec{v}_1, \vec{v}_2 \rightarrow \vec{v}_1', \vec{v}_2'\}$. Assuming that the forces of interaction between the particles forming the gas during the collision are time symmetric, we can see that the inverse collision is understood to be $\{\vec{v}_1{}'$, $\vec{v}_2{}' \to \vec{v}_1, \vec{v}_2\}$. The reverse collision is given by $\{-\vec{v}_1{}', -\vec{v}_2{}' \to -\vec{v}_1, -\vec{v}_2\}$. The fact that the cross section of ordinary collision and the reverse collision is equal to one another is called the classical principle of microreversibility.

If the external force is taken to be zero, the condition for equilibrium i.e., $\frac{\partial f_1}{\partial t} = 0$, given also the Boltzmann's equation in terms of collision cross sections, provides us the following equation

$$\int d^3\vec{v}_2 \int d\Omega \sigma(\Omega) |\vec{v}_1 - \vec{v}_2| (f_2' f_1' - f_2 f_1) = 0 . \qquad (4.15)$$

A necessary and sufficient condition for this to happen is

$$f_2' f_1' = f_2 f_1 . \qquad (4.16)$$

The Eq. (4.16) states that all types of collisions are exactly balanced by their inverses in equilibrium. This is an example of detailed balance. From the Eq. (4.16), we have

$$\log(f_2') + \log(f_1') = \log(f_2) + \log(f_1) . \qquad (4.17)$$

The equation above shows us that left hand side is unchanged due to the collision. In other words, the distribution function must be chosen in terms of the kinematic quantities which are conserved during the collision. These kinematic quantities are total momentum and energy. Therefore, the most general form of the distribution function will be

$$\log(f) = -\beta(\vec{v} - \vec{v}_0)^2 + \log C . \qquad (4.18)$$

The constant $\vec{v}_0$ represents the velocity of the gas as a whole. If the container is at rest, then this term is equal to zero leading to Maxwell distribution in Eq. (4.14).

In order to show that Maxwell distribution is indeed the equilibrium distribution for an arbitrary initial state, Boltzmann introduced his famous H function defined as

$$H = \int d^3 v f(\vec{v},t) \log f(\vec{v},t)\,. \qquad (4.19)$$

Differentiation of above equation with respect to time gives us

$$\frac{dH}{dt} = \int d^3 v \frac{\partial f}{\partial t}(1 + \log f)\,. \qquad (4.20)$$

We then substitute $\frac{\partial f}{\partial t}$ from Boltzmann's equation, we finally obtain

$$\frac{dH}{dt} \leq 0\,. \qquad (4.21)$$

The equation (4.21) tells us that when a gas is in a condition of molecular chaos, H will decrease. H will attain a minimum value at equilibrium. H is of course minimized by the Maxwell distribution.

The H function for an ideal monatomic gas is given by

$$H = -\frac{N}{V}\log(VT^{3/2}) + cons\tan t\,. \qquad (4.22)$$

The entropy in this case reads

$$S = -kVH\,. \qquad (4.23)$$

The negative sign in the Eq. (4.23) is important and shows us that while H function decreases towards a minimum, entropy increases towards a maximum. The equilibrium state is characterized by either minimum H or maximum S.

Concerning entropy, we also have another important relation which is engraved in the tombstone of Ludwig Boltzmann. It reads

$$S = k\log(\Omega) + cons\tan t\,, \qquad (4.24)$$

where the letter Ω is reserved for the total number of microstates compatible with the given macrostate which will lead to entropy S. This relation above indicates that the maximum entropy state will be the most shuffled state. In other words, disordered states are more probable than the ordered ones. In order to support this interpretation of entropy, we can cite the example of two different gases in two different compartment separated by a partition in between. When we remove the partition, the gases will mix evenly with each other in a short time. This example shows the natural tendency for the transition from order to disorder.

The interpretation based on Eq. (4.24) above also suggests us some uses of the concept of entropy beyond thermodynamic systems. A less (more) ordered state clearly requires (consider the example above) more (less) information for a full specification of the macrostate of the system. This picture suggests us that negative entropy (negentropy hereafter) must be associated with information.

A more quantitative picture can be provided (Davies, 1974): Consider having a discrete number q of possible outcomes. If we do not have any additional information about the situation, each outcome is equally likely. But, it may happen that we have some additional information which will enable us to reduce the number of choices to p where p < q. Then, the amount of information is defined as

$$\Delta I = k \log(q / p) = -k \log(p) + constant \, . \qquad (4.25)$$

If we compare the last two equations above, we see that

$$\Delta I = -\Delta S \, . \qquad (4.26)$$

In principle, it is impossible to make experiment on a physical system without perturbing it. Finding the temperature of a room requires one to make measurements by the use of a thermometer, which in turn interacts with the room temperature and perturb the very quantity i.e., thermal equilibrium it was supposed to measure. The acquisition of information is always associated with some negentropy which is “negentropy principle of information” by Brillouin which is mathematically stated as

$$\Delta I \leq \Delta S \, . \qquad (4.27)$$

One point worth remark is the following: everyday experience suggests us that information only increases with time. Does this not point to a simultaneous increase of both information and entropy? There is none if we understand that traces we leave behind increase information content (and hence decrease entropy) at the expense of an entropy increase in the universe. In other words, entropy increases in a closed universe but information increases in an open one considering the local environment as a universe.

One very unorthodox view has recently been advocated by Jos Uffink (Uffink, 2001) and Harvey Brown (Brown and Uffink, 2001). According to these philosophers, the origin of time asymmetry in thermodynamics lies in anything but in thermodynamics itself. One important point which is usually forgotten to be discussed by many authors is apparent for example in Kelvin's formulation of Second Law: what this law states then refers to the irreversibility of cyclic processes. It is almost a common mistake to use this formulation anywhere one wants and deduce the irreversibility of system as such. But all that Kelvin formulations states, for example in the case of adiabatic expansion of a gas, in the words of Brown and Uffink, is

> If the gas spontaneously expands to a new equilibrium state, and if certain other processes are available, then the converse transition is impossible. But that this expansion occurs spontaneously, I not part of the content of the Law. (ibid, p. 527)

The problem is that the spontaneous approach to equilibrium is taken as a definition in thermodynamics and this definition is time symmetric. We define a thermodynamical equilibrium a one-way road. One state can evolve into an equilibrium state but not vice versa. This state is supposed to be unique too. Thermodynamic equilibrium once attained cannot be disturbed unless an intervention from the environment has been made. This situation of equilibrium principle overwriting Second Law is called "Minus First Law" by Brown and Uffink (ibid, p. 529).

This is not the same for statistical mechanics though. In statistical approach, we have a time-symmetric formulation (apart from Gibbsian approach). Fluctuations out of equilibrium occur spontaneously for almost all microstates (ibid, p. 530). Of course, statistical approach has its own problems with it as we will see later in this Chapter. But, it is safe at least that its time-symmetric formulation is possible and we know this if we keep our eyes on the ball.

Another way to see the difference between the thermodynamic and statistical cases is to look at one possible (arguably the best candidate we have so far) solution of the Past Hypothesis. In order for Bolztmann account to hold firm, one has to posit that the early state of the universe must be a low entropy initial state. This is called Past Hypothesis. If this is really correct, then the origin of statistical asymmetry is totally different than thermodynamic asymmetry since the former lies in some cosmological initial value issue whereas the latter is already written in the theory by definition alone.

The next step of course is to wonder whether we can have a time symmetric formulation of Second Law, following the line of development of statistical mechanics. This has been shown to be possible recently by Lieb and Yngvason (1999). Its details however are out of scope of this dissertation. As is seen clear already from this Section, we will take the statistical arguments serious and move on from there.

4.2 Generalized H Theorem

The domain of ordinary H theorem is a system of interest. If one has to use H theorem for an ensemble of systems, then the generalized H theorem must be used. The generalized H theorem is due to Gibbs who took over H theorem from Boltzmann and handled it in a way which will be suitable for ensembles. The presentation of generalized theorem in this Section is due to Richard Tolman (1979).

In order to begin our investigation, we must define two kinds of density functions i.e., fine-grained and coarse-grained density functions. From equation (4.9), we can define a fine–grained density ρ (q, p, t) normalized to unity as

$$\rho = \frac{1}{N}\ f\,(q, p, t)\ . \tag{4.28}$$

It is obvious now that

$$\iint \rho\,(q, p, t)\ d^3qd^3p = 1\,. \tag{4.29}$$

However, when we make a real measurement of momenta and coordinates of a system, there occurs some uncertainty in measurements. Because of this uncertainty in measurement, we will define another distribution function which will tell us about the probability of finding members of an ensemble within small but finite regions having extensions δq and δp which are related to the accuracy of the measurement. For this, we define another distribution function i.e., coarse-grained density whose notation will be P. It will be given by

$$P = \frac{\iint \rho\,(q, p, t)\ d^3qd^3p}{\delta\, q_1 ... \delta\, p_3}\,. \tag{4.30}$$

The coarse-grained density, too is normalized to unity. Now, we are ready to define the generalized H function as

$$\bar{H} = \int d^3qd^3p P \log P\ . \tag{4.31}$$

We can also write the equation above as

$$\bar{H} = \int d^3qd^3p \rho \log P\,, \tag{4.32}$$

Since logP will be constant over each one of the small regions δq…δp and the integration of fine-grained density is equal to (P δq…δp) due to the relation in Eq. (4.30). Eq. (4.32) shows us that such a defined coarse-grained density function is indeed nothing but the mean of logP over the ensemble. This observation also justifies the bar over H in the notation.

One useful remark about the fine-grained density would be to indicate that it must obey Liouville's theorem which states that

$$\frac{d\rho}{dt} = 0\ . \tag{4.33}$$

Liouville's theorem allows us to write

$$\frac{d}{dt}\int d^3q d^3p \rho \log \rho = 0 \,. \tag{4.34}$$

This observation will be proven to be useful in understanding the evolution of generalized density function.

In order to understand how the generalized density function will evolve in time, let us consider what is going to happen at an initial time t_1: According to the fundamental postulate of statistical mechanics, we assume equal a priori probabilities for the fine-grained density function. But, this simply means that this fine-grained density function will be constant inside each related phase space volume. It will also be equal to coarse-grained density since coarse grained density is nothing but mean value of fine grained density over δq…δp by very definition. Therefore, we can write, initially, the following equality

$$\rho_1 = P_1 \,. \tag{4.35}$$

This equality is valid at all points in phase space. We can then write, for this initial case,

$$\bar{H}_1 = \int d^3q d^3p \rho_1 \log\rho_1 \,. \tag{4.36}$$

For a later time t_2

$$\bar{H}_2 = \int d^3q d^3p P_2 \log P_2 \,. \tag{4.37}$$

To be able to compare the values of generalized H function, we subtract these equations above from one another to get

$$\bar{H}_1 - \bar{H}_2 = \int d^3q d^3p (\rho_1 \log \rho_1 - P_2 \log P_2) \,. \tag{4.38}$$

Now, we can write the equation above in the following form

$$\bar{H}_1 - \bar{H}_2 = \int d^3q d^3p (\rho_2 \log \rho_2 - \rho_2 \log P_2 - \rho_2 + P_2) \,. \tag{4.39}$$

The change in the first term is due to Eq. (4.34) whose existence is due to the Liouville's theorem. Second term owes itself to the observation (4.32). The last two terms cancel each other after the integration since both fine-grained and coarse-grained density functions are normalized to unity.

At this point, we make use of the following lemma: For any two quantities ρ and P, we have

$$\rho \log \rho - \rho \log \mathrm{P} - \rho + \mathrm{P} \geq 0 \,. \tag{4.40}$$

The only requirement for Eq. (4.40) to hold is that ρ and P to be assuming nonnegative values. This is of course true for ρ and P by definition. With the help of this lemma, we obtain, from Eq. (4.39),

$$\begin{aligned} &\bar{H}_1 - \bar{H}_2 \geq 0 \\ &\bar{H}_1 \geq \bar{H}_2 \end{aligned} \,. \tag{4.41}$$

Since the equality holds only when ρ and P equal to one another, and we are interested in cases which are different than this, we finally write

$$\bar{H}_1 > \bar{H}_2 \,. \tag{4.42}$$

This final result shows us that also generalized form of H functional decreases by time. This result is called generalized H theorem.

The relation between the $\bar{H}$ for an ensemble and H for a system is given as follows (Tolman, 1979)

$$\bar{H} = \bar{H}_{system} + \sum_k P_k \log P_k + cons\tan t \,. \tag{4.43}$$

In the above formula, P_k is the total probability of finding a member of our ensemble in the condition k. The first term on the right hand side is given by

$$\bar{H}_{system} = \sum_k P_k H_k \,. \tag{4.44}$$

In other words, this term is the average for all system values. If the probability P_k = 1, then we have these two different function equal to one another apart from a constant. After this relatively brief account, we then proceed to the objections raised against H theorem and see what kind of relation it bears to the direction of time.

4.3 Some Remarks on Reversibility Objections

Historically, there have been two main attacks against the Boltzmann's famous H theorem. The first of them is due to Loschmidt and Zermelo and calls for a reversibility paradox inherent in H theorem itself. According to Loschmidt, H theorem is based on classical mechanics, or in other words, microreversibility of the collision processes. Since these kind of processes are known to be time reversal invariant, how come one obtains a result such as H theorem which indicates a behavior in single time direction only i.e., towards future. This apparent conflict between the underlying time reversal invariance of classical mechanics and non-invariance of H theorem forms the paradox itself. Of course, the time reversal non-invariance of H theorem must be understood in that it only specifies a certain function (H function) to decrease as we move towards a future state.

I believe the resolution of this first paradox (we will see the second one in a while) is closely associated with some misunderstandings on part of the structure of H theorem. In order to see this, one needs to look closer to what it assumes to be able to say what it says. As we have seen in the previous Section, we needed two assumptions in order to obtain H theorem. These were called equal a priori postulate and assumption of molecular chaos. The latter has been time asymmetric right from the start since it stated a certain property which was inherent in the picture before collision and not thereafter. This indeed allowed us to write number of collisions as the multiplication of two distribution functions. The temporal distinction of before/after was already there in the assumption of molecular chaos. This point is important in order to show us that the time reversal invariance of classical mechanics is not broken at all. It is the assumption which we make. It is this assumption as a seed for time reversal non-invariance. Concerning the remark that H theorem is not immune to reversibility paradox, let us try to formulate it in a better way. This solution will be called textbook solution since it is the one presented in the textbooks like Huang (Huang, 1987) and the one that I believe to be the true account of the theorem concerning the objections related to the time reversal.

First of all, let us understand how this paradox emerges with the following example: as we have already mentioned, the molecular chaos assumption i.e., Stosszahlansatz, is responsible for the emergence of time reversal non-invariance. This assumption does not say anything about the explicit form of the distribution functions which we multiply in order to get the number of collisions. Since this is the case, in order to see how the paradox emerges easily, let us assume that we have a particular kind of distribution function, $f = f(|\vec{v}|)$. This simply means that we have a distribution function which only depends on the magnitude of the velocity vector explicitly. Let us also assume that our gas is in a state of molecular chaos and not be Maxwell-Boltzmannian at time t = 0. According to H theorem then, we must have a decreasing H at time $t = 0^+$. Now, let us consider another gas which has exactly the same properties as the previous one but the velocities reversed this time. This gas will have the same distribution function of course since the distribution function only depends on the magnitude of velocities and it is left unchanged when we reverse the velocities since this is tantamount to the mapping $v \rightarrow -v$. Since it has the same distribution function, it will also have same H and must also be in the same state of molecular chaos. So, also for this new gas, we must have a decreasing H for $t = 0^+$. But, there must be something wrong with this picture since the future of the new gas is the past of the old one. Therefore, for the original gas, we must have a decreasing H for $t = 0^+$ and an increasing H for $t = 0^-$. Therefore, H (or S for that matter) must be at a local peak (minimum) as shown in the following figure.

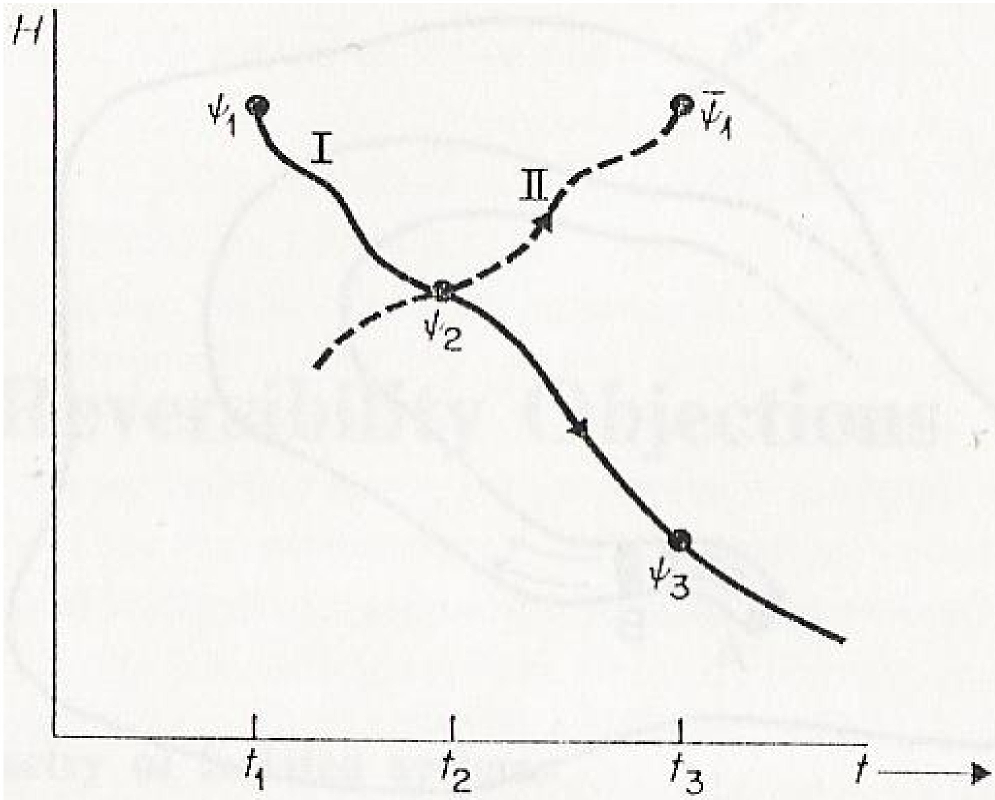

Figure 4.1: H Function versus Time

When H is not at a local peak, then it is not in a state of molecular chaos. Molecular collisions can create molecular chaos when there is none and can destroy molecular chaos when there is one. dH/dt cannot of course be continuous function of time since it can undergo drastic changes abruptly due to the collisions. The more improbable the state, the sharper the peak. In general, the value of H fluctuates a little bit above the minimum. This range is called “noise range”. It is very improbable for H to have a value more than one which lies within the noise range. Following Huang (Huang, 1987), these results can be summed up nicely in three items as follows:

I. For all practical purposes, H never fluctuates spontaneously above the noise range. This corresponds to the observed fact that a system in thermodynamic equilibrium never spontaneously goes out of equilibrium.

II. If at an instant H has a value above the noise range, then, for all practical purposes, H always decreases after that instant. In a few collision times, it will within the noise range. This corresponds to the observed fact that if a system is initially not in equilibrium, it always tends to equilibrium. Together with item I, this forms the second law of thermodynamics.

III. Most of the time, the value of H fluctuates in the noise range, in which dH/dt is positive as frequently as negative. These small fluctuations produce no observable change in the equation of state and other thermodynamics quantities. When H is in the noise range, the system is, for all practical purposes, in thermodynamic equilibrium. However, they lead to some observable effects such as blue sky since it is due to nothing but the fluctuations scattering of light.

All these moves above are indeed directed in explaining the statistical nature of the H theorem. H theorem cannot be thought as the Newton's laws. There is a difference in nature between a fully deterministic theory and the one of statistical nature and this is what caused trouble for a lot of scientists and philosophers of science in this field. These explanations above are summarized in the two figures below.

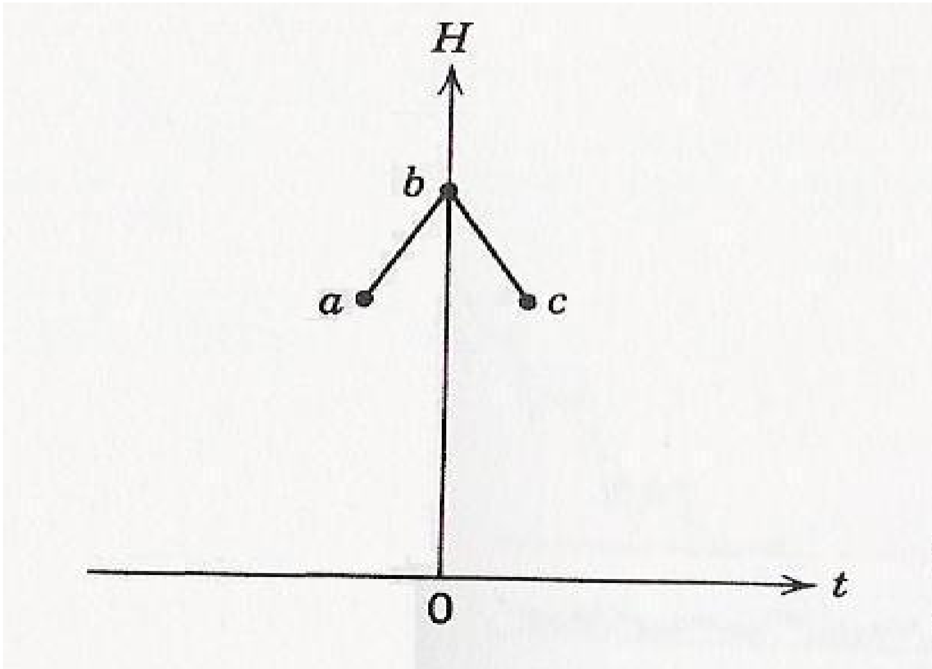

Figure 4.2: H Function and Local Peak

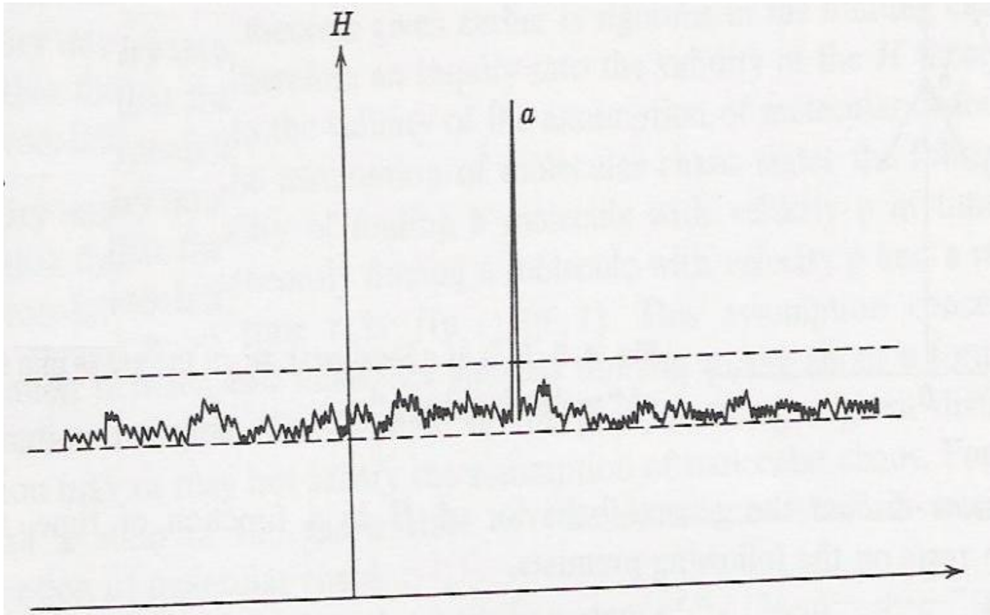

Figure 4.3: H Curve

The second objection is due to the works of Poincaré. Poincaré's "recurrence" theorem states that any state will be revisited to arbitrary closeness an infinite number of times in an isolated system. When accepted, it is obvious that eventually very low entropy states (peaks in H above the noise range) must occur. The textbook version we have just provided already is independent of such flaws since it is already inherent in that picture that from time to time we will have those peaks which lie way above the noise range. They are highly improbable but yet do happen. We must again emphasize the statistical nature of H theorem. H theorem does not say that $dH/dt < 0$ all the time.

The time required for a second fluctuation which would lie above the noise range is called a Poincaré cycle. A crude estimate (see Huang, page 90) shows us that this cycle is of the order of e^N where N is the number of the molecules in the system. Especially concerning the fact that the age of the universe is merely 10^{10} years, one comes to realize that these cycles are too long.

Now, it is time to see how the literature about statistical direction of time has been built and in order to so, there is no better place than starting from the classical treatise of Hans Reichenbach called “the direction of time”.

4.4 Reichenbach on Direction of Time

Hans Reichenbach (1956) was the first philosopher of science who analyzed the philosophical implications of the direction of time. His analysis begins with a causal definition of time order. According to Reichenbach, the relationship defined by “between” is an order relation and can be used to shed more light on reversible processes since whatever is in between will stay invariant under the time reversal.

Then, he observes the difficulty of noting the arrow of time in physical processes, stating

> Neither the laws of mechanics nor mechanical observables give us a direction of time, unless such a direction has been defined previously by reference to some irreversible process. For instance, if the velocity of a body is regarded as an observable, its direction must be ascertained by comparison with some temporally directed process, such as the time of psychological experience, which is derived from the irreversible processes of the human organism. But if no such standard is used, we cannot regard a velocity as an observable. We can merely derive its value from other observables, which, however, leave the sign of the velocity undetermined. (ibid, pp. 35-36)

But, he argues, one can obtain what is in between out of mechanical arguments and therefore can form a *causal net* which has a lineal (e.g., the assignment of one direction to one line fixes the direction for al the other lines) order. Only the knowledge of “between” will suffice to form a causal net and moreover, this is something we can achieve without irreversible processes. Below, one can find an example of a causal net. If several arrows depart from one point, we choose any one we like. According to Reichenbach (ibid, p. 36), sum of these lines may be called a *causal chain*. Once we begin our journey on a causal chain, we can never return to the starting point i.e., there are no *closed causal chains*. Openness of the net is an empirical fact not a logical necessity. However, the existence of a time order for our universe is founded on the openness of the causal net. In other words, one can easily deduce whether an event is before or after another event. A closed causal chain would violate this property since then an event could be before and after another event at the same time. This time order we mention here still does not give us any direction. Moreover, we must note that the reversing the direction would not change anything in the causal chain as far as we are interested in a picture with ever repeating past.

Reichenbach's next important step is his treatment of probability lattice (ibid, 96). Using the now usual frequency interpretation he makes distinction between the time ensemble and space (what physicists today call Gibbs ensemble) ensemble. To be able to understand how he does this, let us imagine a mixing process where we have nitrogen molecules on one side of the partition which we can label as B, and on the other side, we have oxygen i.e., on part $\bar{B}$. When we remove the partition, we know that these two gases will ix with one another. After enough time, there will be equal amount of nitrogen and oxygen molecules in B and $\bar{B}$. Once a molecule is in B, it will stay there for short time later. The same applies also to molecules in $\bar{B}$. Reichenbach calls this "aftereffect". If we form a sequence for all the molecules in the enclosure, we obtain what is called a probability lattice for this diffusion process in the following form

$x_{11}\ x_{12}\ \ldots x_{1i}\ \ldots \rightarrow\ p$

$x_{21}\ x_{22}\ \ldots x_{2i}\ \ldots \rightarrow\ p$

……………….

x_{k1} x_{k2} …x_{ki} … → p

……………….

p_1 p_2…p_i……. → p

Each horizontal row represents the history of a molecule whereas the vertical column represents the space ensemble.

Reichenbach (ibid, pp. 99-101) considered two limitations concerning the lattice structure. Firstly, he assumed the independence of the rows. Secondly he assumed lattice invariance holds. It means that the horizontal probability of finding a B in the kth row reappears as a vertical probability. It measures the number of B terms selected from the ith column by the condition that the preceding column has a B at the same place. Then, he proves that the inference from the time ensemble to space ensemble is valid for every lattice of mixture.

Having defined time order in terms of causality, Reichenbach moves on to analyze the unidirectionality of time. There are two main questions to be answered, he adds (ibid, pp. 114-116): the first one is a possibility of inference from time to entropy and the latter being the possibility of inference from entropy to time.

The first of these cases can be put as follows(ibid, p. 114): Given the time direction and an initial nonequilibrium macrostate A of entropy S(A), will the entropy S(B) of a macrostate B be higher if B is later than A? The answer by Reichenbach (and Boltzmann) is that $S(B) > S(A)$ is more probable than $S(B) < S(A)$ if A has a low entropy. The problem is that this is also true if B precedes A. In other words, It is more probable that $S(B) > S(A)$ than $S(B) < S(A)$ even though A is later than B in time. This simply expresses the symmetry of time direction for the entropy curve (ibid, p. 115). In sum, we cannot use the entropy curve to define a time direction since it is symmetric in the last analysis.

To find a way out of this dilemma, Reichenbach, following Gibbs' analysis, turns his attention to space (in other words, Gibbs ensemble) ensembles. In order to do this, he introduces the issue of interaction:

> For example, if one observer tells us he saw the gases I a container rather well separated, though they were not divided by a partition, and another observer informs us that he saw the well mixed, we shall conclude that the second observation was made later than the first. We shall add the further conclusion that originally the gases were separated by a partition, and that someone must have removed the partition shortly before the first observation was made. This means that, rather than proceeding on the assumption that the gas system was closed all the time, we assume that it was originally in interaction with its environment; and we conclude that that improbable state is the product of this interaction rather than the result of a separation process produced by mere chance in the history of closed system (ibid, p. 117)

Taking this example further, he arrives at the concept of *branch systems*. These are subsystems which branch off from another system and remain isolated for some time thereafter. They state with a low entropy and progress towards relatively high entropy. Once we pose the problem of direction of time in terms of these branch systems with same initial conditions i.e., being all low entropy states and adopt Gibbsian point of view, i.e. space ensembles, it is possible to show that the probability that a low entropy state is followed by a high entropy state is greater than the probability that a high entropy state is followed by a low entropy state. The figure related to these branch systems in an entropy upgrade given below. The reversibility objection does not apply here since we are dealing with many-system probability rather than one-system probability (ibid, p. 121).

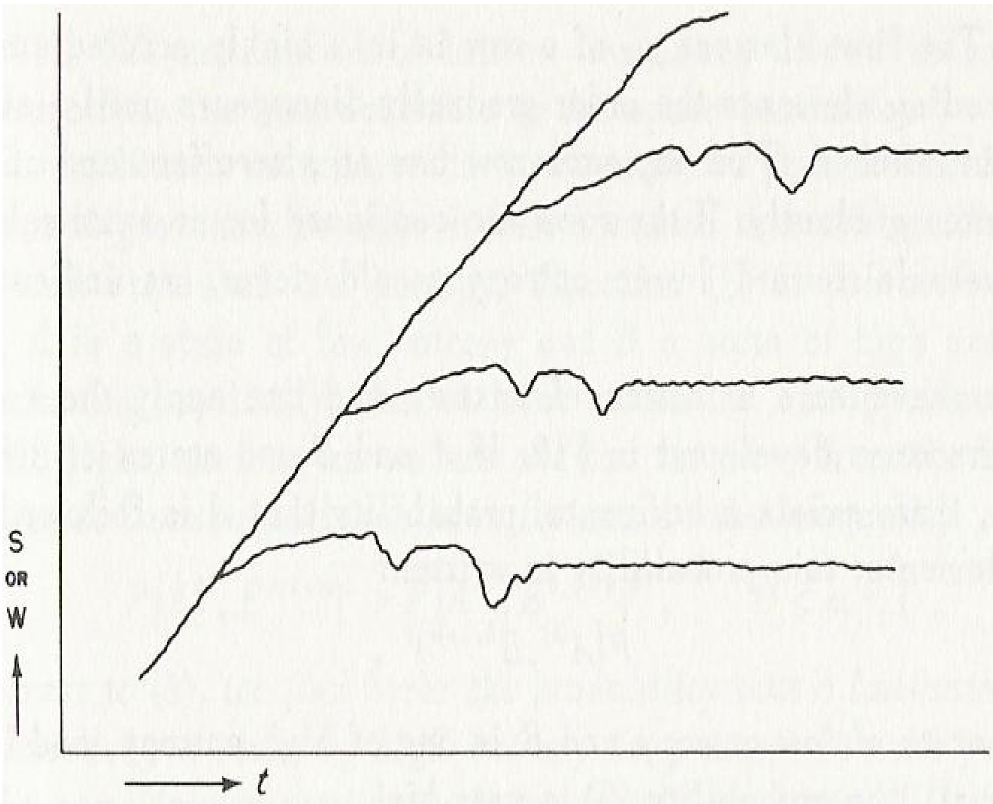

Figure 4.4: Branch Systems

According to Hans Reichenbach, the approach of moving from time ensemble to space ensemble is imperative in the probabilistic sense since probability, in the frequency interpretation at least, requires many-systems anyway. He adds that probability statement concerning a single event does not have any meaning. In short, even probabilistic concepts require us to consider space ensembles rather than time ensembles.

There are two things about the figure above which is not realistic: first, there is only one upgrade in the figure above, but we know that there must be random upgrades and downgrades and some horizontal sections. Second, the branch systems above extend to infinity in time. This is also not realistic since we know that the branch systems are isolated for a limited time and turn back to environment after that time. If we put coffee and cream into a thermos bottle, the resulting mixture will not stay in the bottle for infinitely long time(ibid, p. 126). Having considered these two realistic corrections, we have the figure below.

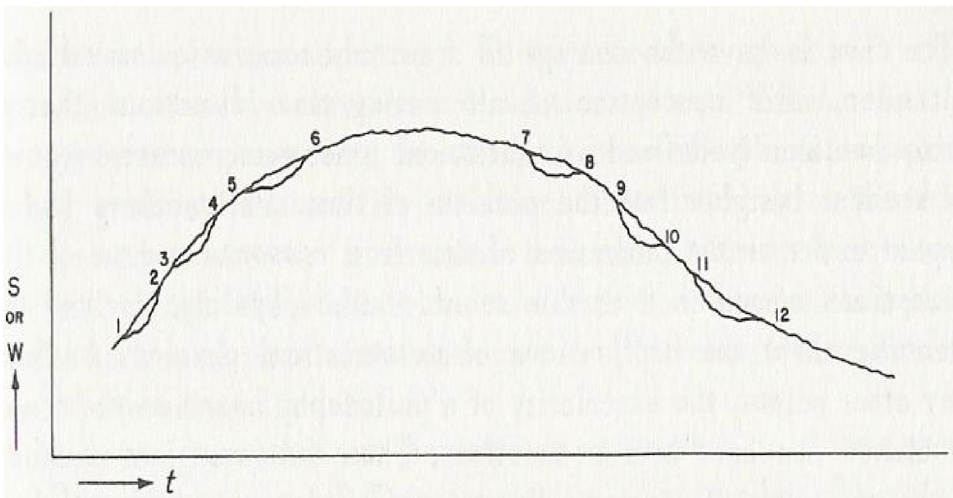

Figure 4.5: S or W versus Time

This more realistic picture however shows us some other features which were unknown to us before. The branch systems starting with initial low entropy are possible only for the entropy upgrade such as number 1-2 or 5-6. If we inspect the section given by 7-8 for example, we see that this branch system begins with high entropy and ends up in low entropy opposite to what is being stated before. This simply shows the dependence of direction of time on different sections. As Reichenbach puts it; a time direction can be defined only for sections of the total entropy curve since the frequencies of overall upgrades and downgrades will be equal. If the universe is on an entropy upgrade, then the statistics accessible to us will give a direction of time. In other words, main ingredient in Reichenbach or Boltzmann recipe is to assess whether we i.e., the universe, are on an entropy upgrade or downgrade.

Thinking in terms of the problem of direction of time in the context of different sections of the universe, leads us to the understanding that one can easily mention opposite direction of times related to different sections. The upgrade and downgrade parts will provide different (in fact, opposite) directions of time. Reichenbach then notes that

> Philosophers had attempted to derive the properties of time from *reason*; but none of their conceptions compares with this result that a physicist derived from *reasoning* about the implications of mathematical physics. As in so many other points, the superiority of a philosophy based on the results of science has here become manifest. There is no logical necessity for the existence of a unique direction of total time; whether there is only one time direction, or whether time directions alternate, depends on the shape of the entropy curve plotted by the universe. (Reichenbach, p. 128)

There happens to be a serious possibility of us being in entropy downgrade section of the universe without our awareness of the situation.

Reichenbach, at this point, introduces the notion of *supertime*. It has no direction but an order. This supertime works even when there is a horizontal plateau on the entropy curve. When this is the case, one cannot talk in terms of entropy since there occurs neither entropy increase nor decrease.

How can we know then that our universe is at present on an upgrade? As is argued before, one system solutions do not work. We need to consider space ensembles to be able to answer the question posed about. Reichenbach (ibid, pp. 130-131) notices that these kinds of inferences are already in use. For example, the crust of Earth is regarded as the product of cooling process in geology (ibid, p.

130). The plurality of branch systems enables us to deduce a definite time direction. This shows how a powerful tool the concept of branch system is. He also states

> The existence of a long upgrade of entropy, though a necessary condition for the phenomenon of time direction, is therefore not a sufficient condition. Time direction becomes apparent to us only because the upgrade contains a large number of situations in which subsystems branch off, disclosing in their further development the universal growth of entropy (Reichenbach, p. 131).

One must make a difference between the time direction perceived by us and the direction of time as a whole.

At this point, it is very important to understand the assumptions we made so far in order the branch structure to work in shedding light on the problem of arrow of time. The first assumption we made is that the entropy of the universe at present is low and is on a slope of the entropy curve (ibid, p. 136). Second assumption is related to the existence of many branch systems, which are isolated from the main system for a certain period of time (but not infinite) but connected with the main system at two ends. This assumption is very plausible in the sense that it requires nothing but the existence of Gibbs ensembles which are being used more often than the 1950s in which Hans Reichenbach had written his treatise "The Direction of Time". It has also been assumed that majority of branch systems, one end is a low point and the other end a high-point. The last assumption is that the directions toward higher entropy are parallel to one another and to that of the main system in majority of the branch systems. This last assumption is called "the principle of the parallelism of entropy increase" in Reichenbach's terminology. The principle of parallelism cannot be derived from the assumption that the entropy of the universe at present is low and is on a slope of the entropy curve i.e., our first assumption. If the entropy gradient of a branch system or all of them for that matter is counterdirected to that of the main system, there is nothing violated as far as laws of mechanics is considered. This also means that it would be consistent with the causal laws, too. This is why it has to be postulated on its own merit as a separate assumption. In short, the principle of the parallelism of entropy increase ensures that the time direction from point 1 to point 2 (see Fig. 4.5 above)is the same for the main system as for the branch system (ibid, p. 137).

4.5 After Reichenbach

Even though Reichenbach wrote his treatise in 1956, his "Direction of Time" has been accepted to be the most thorough discussion in the literature of time arrow (Sklar, 1993). But, the ideas on the arrow of time did not cease to emerge and even Reichenbach had taken his share from these developments in the form of critiques. In fact, anyone who decides to work on temporal asymmetry related to entropy and universe in general has to pay homage to him one way or another.

The first critique of Reichenbach, at least to my knowledge, has been done by Stein (1967, 1968) and Earman (1974). A similar route has also been adopted by late Robert Weingrad (1977). According to these philosophers, the main issue which has been overseen by Reichenbach is the *temporal orientability* and gravitational issues related to the direction of time. According to Earman, a relativistic space-time is temporally orientable if there exists a continuous nonvanishing vector field on the differential manifold which is time like with respect to the metric. The temporal orientability has to be taken into account since the very space-time we are embedded in is relativistic. Having defined the temporal orientability, he goes on to enumerate the following three research programmes which has to be dealt with of one would like to have a concise view on the direction of time (ibid, pp. 18-19).

I. Can any nontemporally orientable space-time be ruled out a *priori* as an arena for physics?

II. Is the actual world temporally orientable?

III. By means of what kind of evidence could we come to know the answer to second item above?

IV. Does the world come equipped with a time orientation?

V. If the answer to fourth item is affirmative, where does it come from? If the answer is negative, what explains our psychological feeling of a direction for time?

VI. If the answer to the fourth item is affirmative, how do we know which of the two possible orientations is the actual one?

Reichenbach had not worried about the first three of these items which is based on the general relativity. He just took it for granted and moved from therein. As is explained in the previous Section, he just believed that the direction associated with the entropy increase can be “labeled” as future. In the end, he discovered, together with Boltzmann, that a global direction if time is unattainable. Of course, as Earman noticed, it is not easy to make sense out of usages like “region of space time” (ibid, p. 21) or the space-time sections with different time directions he has offered since it is almost impossible to partition space-time regions in terms of the issues of temporality. Therefore, this is an assumption to be mentioned. John Earman then proceeds to define a new and more powerful criterion to be able to understand the time sense of a temporally orientable space-time. This is called “Principle of Precedence” (PP hereafter) and it reads

> Assuming that space-time is temporally orientable, continuous timelike transport takes precedence over any method (based on entropy or the like) of fixing time direction; that is, if the time senses fixed by a given method in two regions of space-time (on whatever interpretation of ‘region’ you like) disagree when compared by means of transport which is continuous and which keep timelike vectors timelike, then if one sense is right, the other is wrong. (ibid, p.22)

Now, due to this principle and the statistical analysis of Reichenbach, Earman concludes

> With Reichenbach's entropy method it is always physically possible and in many cases highly likely (according to statistical mechanics) that there will be disagreement. (ibid, p. 22)

One can simply consider a combined use of Reichenbach's entropy analysis and PP i.e., one can take a piece of space-time region and apply the statistical method in order to determine the time sense and then use the timelike vector transport beginning from there. But, Earman finds it problematic since one cannot determine which region will provide us with the correct sense of time since believes in a unique global time sense.

Another important issue raised by Earman is about whether we can take "isolation" of the systems of interest for granted. Reichenbach apparently did to certain extent but Earman is against this very idea by defending his case through the effects of gravitational field since there is no way to shield a system from its effects. In fact, Earman (ibid, p. 38) quotes Morrison on this issue who states

> … a gravitational force exerted by a falling apple a kilometer away over an arc of ten centimeters is ample to mix up the trajectories of a mole of normal gas in a time of milliseconds. (Morrison, p. 350)

This is a very important critique but one must also keep in mind that the intricate relationship between the entropy and the gravitation is not clear even today. Therefore, even though Reichenbach would like to study the gravitational effects on the direction of time, he would have no means for it, let alone a result coming out of his study. Moreover, Reichenbach (p. 113), as noted by Earman, noted the instability of the microstates which lead to order under small perturbations.

Another well known fact about the physical systems of statistical nature is the observation time. If one observes the system shorter than the relaxation time, one will certainly misinterpret the situation due to the lack of full knowledge of the system. Indeed, Earman (ibid, p. 38) cites Chandrasekhar who states

> … An isolated system appears irreversible (or reversible) according as whether the initial state is characterized by a long (or short) time of recurrence compared to the times during which the system is under observation (Chandrasekhar, p. 56)

Reichenbach considers this point in favor of his and Boltzmann's argument by stating that it is this feature of the processes which allow a statistically solid argument of symmetrical treatment of time.

The importance of temporal orientability has also been emphasized by Robert Weingrad (Weingrad, 1977) in a paper entitled "Space-time and the direction of time" published in Noûs. What Weingrad argued in his paper was the existence of space-time constructs with past/future distinction but without asymmetric earlier/later relation. One example to this, says Weingrad, is a Minkowski space-time with closed time-like world lines. This is very obtain to simple indeed, it is just enough to roll up a two dimensional Minkowski space-time into a cylinder along its time-like worlds lines. Then, one has the following figure which has been adopted from Weingrad (1977, p. 122).

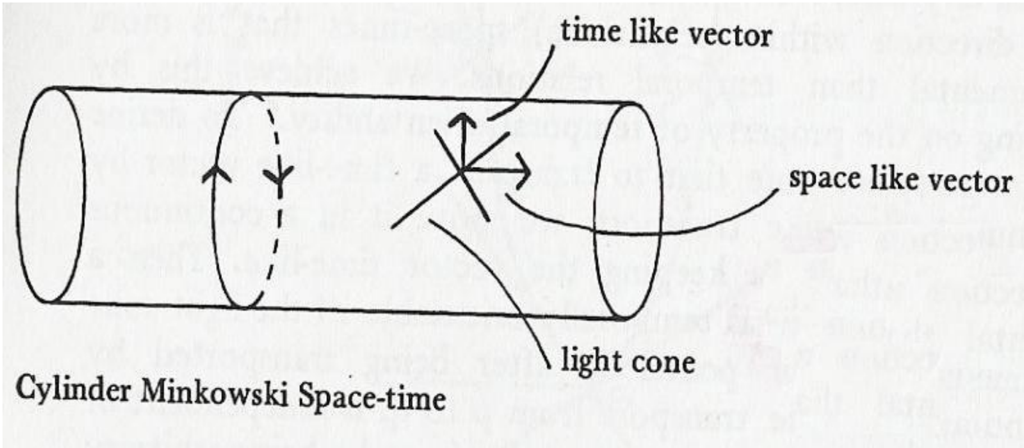

Figure 4.6: Cylinder Minkowski Space-time

As can easily be seen from the figure above, there will be closed time-like curves on this space which will allow the past/future distinction as far as PP holds but an earlier/later distinction will not make any sense at all. In fact, Weingrad uses this simple example to justify the superlative use of PP since he then goes on to state

> In any case, it seems desirable then, to develop a notion of time direction within (relativistic) spacetimes that is more fundamental than temporal relations. We achieve this by focusing on the property of temporal orientability. (ibid, p. 123)

In order to understand the concept of temporal orientability better, we adopt another figure from the same paper of Weingrad (p. 124) below:

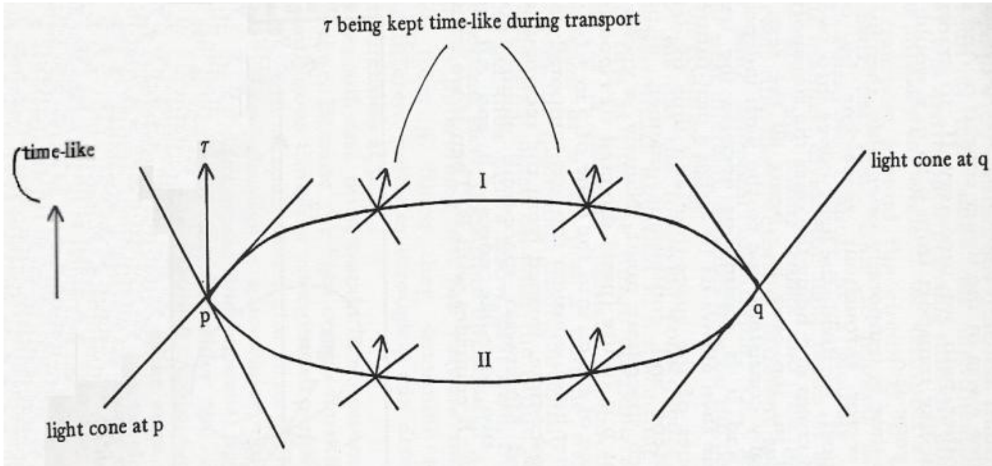

Figure 4.7: Parallel Transport

If we apply PP and transport a timelike vector from p to q along the path I in the figure above in a continuous and timelike manner, then the vector τ will point into the upper light cone at q. This result is independent of the path. PP simply states that a continuous transport of a timelike vector divides all timecone into two classes. One of them can be labeled as + and the other -. It must be noted that this is also global i.e., once fixed, it determined the temporality if whole space-time manifold (Weingrad, p. 125).

The last example of this sort is a Moebius strip space-time. In the figure 4. 8, we have this structure which is formed by the coincidence of A with $\overline{A}$, and B with $\overline{B}$. This time, the transport of the timelike vector τ around I will twist it to be in the opposite direction compared to its initial orientation whereas its transportation around II will cause it to point in the same direction. PP states that we cannot define a global time sense on this space-time and we cannot talk about a future/past division globally.

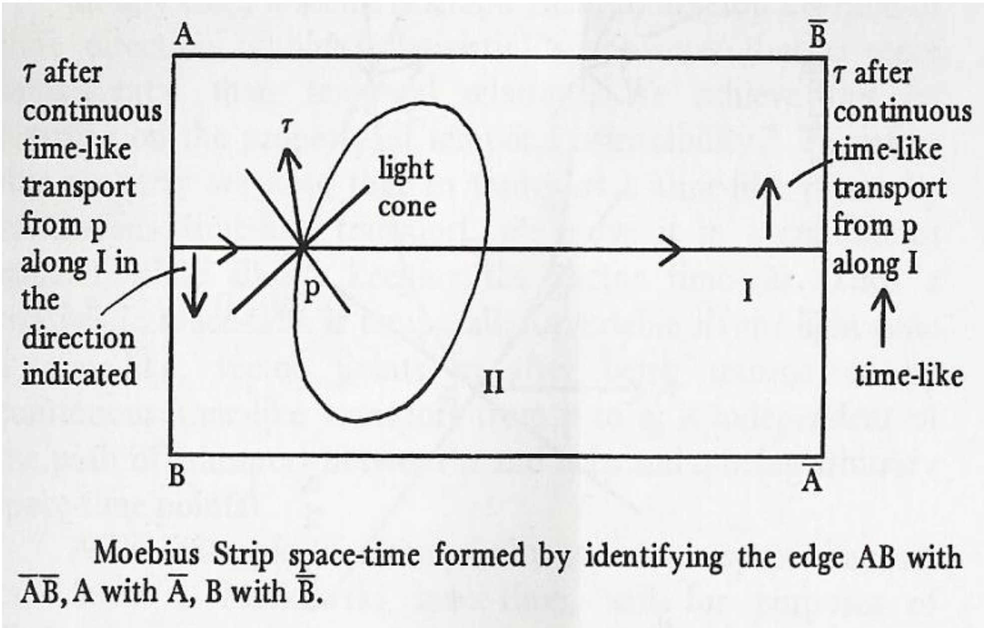

Figure 4.8: Moebius Strip Space-time

Another philosopher of science who criticized Reichanbach's treatment of the direction of time was Lawrence Sklar (Sklar, 1993) but his critiques were more towards Reichanbach's assumptions instead. On other words, He as criticizing Reichenbach not because of what he did not argue but because of his assumptions in his arguments. One important point he made in his case against Reichanbach is his assumption on parallelism. He considers this to be a circular argument overall since assuming this parallelism brings out the desired result. Instead of explaining the underlying symmetry of H theorem, what Reichenbach does, according to Sklar (ibid, p. 325), is to reduce problem into the parallelism of branch systems. Now, we are in a position of explaining why branch systems behave the way they behave. But, this is not an explanation. It is merely a shift in the explanandum, that is all! According to Sklar then, Reichenbach's analysis does not *explain* anything at all.

Craig Callender does agree with Sklar in not buying Reichenbach's analysis as an explanation. He first notes that the Gibbs perspective and Boltzmann perspective conflicts with one another (Callender, 1997). We do not have an entropy definition which can decrease with time when we adopt the Gibbsian point of view whereas Boltzmann entropy is entitled to such changes from time to time. What Callender does not accept is that the generalized H theorem is a success in its own right removing the conflict with the observation. He writes

> The S (ρ) associated with an ice cube on the floor might increase even if (or when) the individual ice cube suddenly starts to freeze!... The switch from S (X) to S (ρ) hardly appears to be a harmless case of concept extension. (ibid, p. 227)

But, what Craig Callender (2004) offers instead is a novel approach based on Hume and what is called best system-analysis; in Reichenbach's analysis, the first assumption was the universe to have a low entropy at present state and evolving towards a higher entropy condition. Therefore, the initial state of the universe must have been an even lower entropy state. But, Boltzmann's view is time symmetric so it must work in both directions. This states that the initial state can also be a higher entropy increase (more on this issue later). Therefore, according to philosophers of science such as Callender and Price, the question to be posed is why the initial state of the universe has his property of being low in entropy. Now, at his junction, Callender needs help from David Hume's argument against the classical cosmological argument for the existence of God. Hume (Hume, 1980) , to begin with, assumes that every effect in the universe must have a cause. He thinks that there would be no sufficient reason for the effect otherwise. Then, we are given two options if we follow the reasoning of Hume: either there is an infinite chain of causes or there was an Uncaused Cause. We choose the latter.

The same argument by Hume is used by Callender in order to shed light on initial value problem concerning the direction of time. This initial condition is called as Past Hypothesis after David Albert (Albert, 2000). Callender thinks that we must be done with this problem by positing the Past Hypothesis and move on. What would, asks Callender, explain the Past Hypothesis anyway? This initial condition forms the boundary of the known facts. If we also consider that the universe has come to existence only once and there is no way of observing this, he insists that Past Hypothesis solves all the problems related to thermodynamic asymmetry of time as distinguished physicists such as Schroedinger and Feynman believed. In Hume's "Dialogues Concerning Natural Religion", Philo argues about the cosmos' coming into existence in the following way:

> The subject in which you [Cleanthes] are engaged exceeds all human reason and inquiry. Can you pretend to show any such similarity between the fabric of a house and the generation of a universe? Have you ever seen Nature in any situation as resembles the first arrangement of the elements? Have worlds ever been formed under your eye[…]? If [so] […] then cite your experience and deliver your theory. (ibid, p. 22)

Then, Callender contrasts the case of Past Hypothesis to a historical example:

> … Consider an old chestnut in the history and philosophy of science, namely the example of scientists rejecting Newton's gravitational theory because it posited an action-at-a-distance force. Such a force could not be basic because it was judged to be not explanatory. But a priori, why are non-local forces not explanatory and yet contact forces explanatory?... Furthermore, note that believing Newton's action-at-a-distance problematic simulated scientists to posit all manner of mechanisms that would restore contact forces. Not only were these efforts ultimately in vain, but many of these posits came at a price of their mechanisms not being independently testable. (ibid. p. 205)

So, there is no problem of justification of Past Hypothesis for Callender.

Another problem with the Past Hypothesis is that it is a low probability event. But, Callender argues that there are lots of low probability events and they do not require explanation at all. The probability of an asteroid to strike Earth is also low but when it happens, it does not require explanation at all. As Callender notes, these low probability events even serve as the explananda, not merely the explanans. An asteroid strike is a low probability event in itself but it might be arguably used in explaining the death of dinosaurs. In short, there do not need to be a close relation between the probability and explanation.

This issue is also related to one's definition of scientific explanation and scientific knowledge. Without any new evidence, we had better stick to the preexisting explanation under the heavy empirical data which is the whole universe in this particular case under study.

Callender also appeals to "Best-System" argument advocated by Ramsey and Lewis (Lewis, 1994). Lewis states this as:

> Take all deductive systems whose theorems are true. Some are simpler, better systematized than others. Some are stronger, more informative than others. These virtues compete: An uninformative system can be very simple, and an unsystematized compendium of miscellaneous information can be very informative. The best system is the one that strikes as good a balance as truth will allow between simplicity and strength. How good a balance that is will depend on how kind nature is. A regularity is a law IFF it is a (contingent) theorem of the best system. (ibid, p. 478)

Basing his argument on the quotation above, Callender rests his case by saying that the laws of nature are the axioms of those true deductive systems with the greatest balance of simplicity and strength and arguing that Past Hypothesis satisfies these criteria (Callender, 2004, p. 207). We always use Past Hypothesis to explain thermodynamic behavior of ordinary mixing processes anyway in our daily life. Why not making the Past Hypothesis a law? It is non-dynamical, yes, but, there is no explicit statement about what a scientific law can be. We also do not need to explain it since we can state easily that it is impossible a violation of Past Hypothesis to occur.

It must be noted that the same kind of approach has been made by Frisch in the case of electromagnetic arrow of radiation. He was offering a way out by accepting retardation condition as a law. His main argument was that not all nomologically possible situations happen in the universe. One can also classify advanced solutions as such. They can be *physically* possible but are not *actually* possible. What happens in actuality can be differentiated as a scientific law. This has been Frisch's stand before about the electromagnetic arrow of time. Callender's point is not the same exactly since Past Hypothesis is about something which happened once and only once, i.e. the universe coming into existence. Contrary to this, electromagnetic radiations happen all the time and might be checked better later on about the retardation condition, therefore, this is a plus on Callender's case. So, we can be inclined to assume that Callender is right in his own case more than Frisch.

The philosopher Huw Price, too defends the atemporal view. He defines the problem exactly as Callender does. The fact that entropy is increasing is not a matter of explanation since this corresponds to an approach to equilibrium. What is in need of explanation is the fact that it has been low to begin with. Price finds Boltzmann/Reichenbach view as a great advance (Price, 1996, p. 35) since one can be convinced that the direction of time is a subjective matter. But, this does not mean he endorses it completely since according to Price, Boltzmann/Reichenbach view misses the real point about the relation between entropy and probability. Price states:

> If the choice is between (1) fluctuations which create the very low-entropy conditions from which we take our world to have evolved, and (2) fluctuations which simply create it from scratch with its current macroscopic configuration, then choice (2) is overwhelmingly the more probable. Why? Simply by definition, once entropy is defined in terms of probabilities of microstates for

> given macrostates. So the most plausible hypothesis-overwhelmingly so- is that the historical evidence we take to support the former view is simply misleading, having itself been produced by the random fluctuation which produced our world in something very close to its current condition. (It is no use objecting that such a fluctuation would have to involve all kinds of "miraculous" correlated behavior. It would indeed, but not half as miraculous as that required by option [1]!) (ibid, p. 35)

This simply means the negation of all historical evidence. According to Boltzmann's view, the universe could have come into existence just some minutes ago as well.

The second main objection uttered by Huw Price is that we should avoid fluctuations which extend the low entropy region. In other words, we should not expect more of these kinds of region like ours to be wide spread. This is also a problem which conflicts with the recent cosmological data since we continue to discover order as much as we go on with our research.

The main difference between the views of Callender and Price happens to be the one related to Past Hypothesis. According to Price, Past Hypothesis itself can be taken to be a scientific law whereas It is still an enigma, a riddle waiting to be solved for Price. For Price, the solution to the Past Hypothesis is of cosmological kind since the question is seemingly one of large structure. The cosmological help one can get for the case of time asymmetry will be inquired in next Section.

4.6 Cosmology and Thermodynamical Time Asymmetry

We begin this Section by trying to understand what the problems related to Past Hypothesis are. In order to do this, we must first understand the early smoothness of the universe and its implications. Smoothness simply means, at least in this context, that matter was distributed with same density everywhere. But, why does this call for an explanation? Why do we have to call it special state even though the smoothness looks like a property of equilibrium? The key to understanding this is to consider the gravitational forces. If we look at a gas in a container, we consider it to be uniformly spread throughout the container. But, in that case, we have the interaction among the particles forming the gas which is in the form of repulsion. Gravitation is a force which exists in the form of attraction. This means that the equilibrium configuration for a thermodynamical system under the effect of gravitation will be in a clumpy one as has been emphasized very nicely by Roger Penrose (Penrose, 1989). This situation is nicely illustrated in the following figure adapted from Penrose (1989).

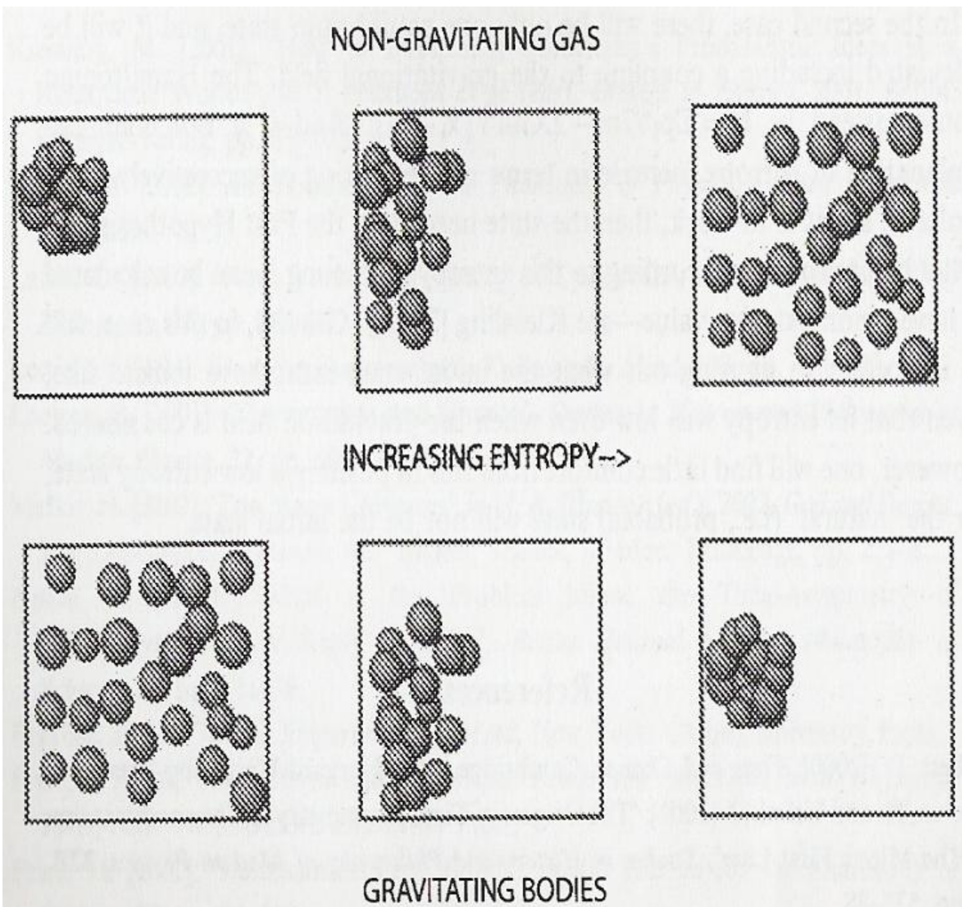

Figure 4.9: Entropy with and without Gravity

The smoothness of the earlier universe then simply means that it was out of equilibrium to begin with. This out-of-equilibrium behavior is in need of explanation. This initial low entropy state is what needs to be explained because it is simply a low probability event. In fact, according to Roger Penrose, its probability is given by $10^{10^{123}}$. So, our universe is very "special" in this probabilistic sense. The smoothness at the beginning of the universe is a must since only then our universe could have been evolved into the state it is in today. Today, we even have some indirect proofs for smoothness of the universe and it is widely accepted among physicists.

Any discussion which will connect the thermodynamic asymmetry and cosmology is centered around the Gold Universe (Gold, 1962). It is a scientific fact that the force of gravity will be sufficient to overcome the expansion of the present universe if the gravitational force is strong enough. Then, instead of expansion, we will have a contracting universe towards an end called big crunch. Will it be a kind of mirror image of the big bang? The Gold universe is this kind of universe model which has been set forth by Thomas Gold in 1962. Gold's hypothesis has not been taken very seriously since the physicists were also committing what Price (ibid, p. 82) called a "temporal double standard". But as Price puts it nicely

> People argue that if Gold were right, matter would have to behave in extremely unlikely ways as entropy decreased. They fail to appreciate that what Gold's view requires towards the future is just what the standard view requires towards the past. (ibid, p. 82)

Therefore, if we do assume that the laws of physics are time symmetrical, then we will have two options: both ends can be smooth which is Gold's universe, or neither ends are. If the second option is adopted, then the Past Hypothesis remains unexplained of course.

According to inflation model, one assumes that the force of gravity is repulsive in the early states of the universe which lead the universe to expand. It cools down as it grows, and a phase transition occurs at some point. Then, the gravitational force becomes attractive, and the ordinary big bang scenario begins. If we look at it from the atemporal view as Price suggests, we must have a collapse with deflation at the other end. If the second law of thermodynamics changes direction when the universe recon tracts, the universe would witness many miracles such as converging radiation, growing younger etc. (Price, 1996, p. 100). But, this does not constitute any argument against the symmetric model of the universe. Paul Davies (Davies, 1977) states this fact with the following words:

> It is curious that this seems so laughable, because it is simply a description of our present world given in time-reversed language. Its occurrences *are no more remarkable* than what we at present experience-indeed it *is* what we actually experience-the difference in description being purely semantic and not physical. (ibid, p. 196)

According to Huw Price then, there is no objective reason to discard Gold's hypothesis until a better theory comes up.

CHAPTER 5

THE ARROW OF TIME IN QUANTUM MECHANICS

The dynamical equation governing the quantum realm, which corresponds to Newton's second equation in classical mechanics is the so called Schroedinger equation, and it reads:

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \,, \tag{5.1}$$

Where $\hbar$ is a constant called Planck's constant, H is the Hamiltonian, i is the complex number and finally Ψ is the wave function. According to the orthodox view (i.e., Copenhagen interpretation) at least, the probability of a particle to be found at a specific space-time point is given by the Born Postulate which states that the modulus squared of the wave function provides this link between the unobservable wave function and the observable particle. Born postulate can be written as

$$P(x,t) \equiv \left|\Psi(x,t)\right|^2 . \tag{5.2}$$

The Schroedinger equation is accepted independent of the interpretations of quantum theory, be it many-world, Bohm or Copenhagen. It provides the deterministic evolution of the wave function once the initial data is given.

The second fundamental postulate is the measurement postulate. Assuming that the state (or wave function) can be decomposed into any orthonormal basis as

$$\Psi = \sum_i c_i \Psi_i \,, \tag{5.3}$$

where Ψ_i's are eigenvectors of the relevant observable and c_i's are the relevant coefficients, the modulus squared of these coefficients gives the probability of finding that particular value.

First, let us look closer to the measurement postulate. In order to assess the situation, it is instructive to adopt an example due to Roger Penrose (Penrose, 1989, p. 358) in the form presented by Craig Callender (Callender, 2000). It reads

> At L we place a source of photons- a lamp- that we direct precisely at a photon detector- a photocell- located at P. Midway between L and P is a half-silvered mirror tilted at 45 degrees from the line between L and P. Speaking loosely, when a photon's wave function hits the mirror it will split into two components, one continuing to P and the other to a perpendicular point A on the laboratory wall. Since the wave function determines the quantum probabilities, and by assumption it weights both possibilities equally, we should expect one-half of the photons aimed from L to make it to P and one-half to be reflected to A. Each photon has a one-half chance of either being reflected to A or passing through to P. (Callender, 2000, p. 7)

Penrose, and together with him Callender, ask what the conditional probability of L registering will be given that P registers i.e., P(L, P) and vice versa. Penrose then concludes that P(P, L) is ½ whereas P(L, P) is equal to unity since there is a certainty that the photon came from the lamp and not from the laboratory wall (i.e., A) if the photocell indeed registers. Then, Penrose argues that there is the time asymmetry in quantum mechanics due to the fact that these two processes are temporal inverses of one another (ibid, p. 8). Of course, there are some gaps to be filled in this Gedanken experiment such as the issue of extra information. But the most important issue which can be objected is the use of comparison of $P(S_i \rightarrow S_f)$ with $P(S_f \rightarrow S_i)$ but not with $P(S_f^T \rightarrow S_i^T)$. Penrose should have treated emitter as the absorber and the absorber as the emitter in this time symmetric case. According to Callender, he should have asked instead what the probability that the time reversed photocell will emit a photon to the time reversed lamp would be.

Even though there are some objections which can be raised against Penrose's example, it reminds us one important fact about quantum theory: It is predictive but not retrodictive. In other words, forward transition probabilities (FOR) are not equal to backward transition probabilities (BAC) in general. This shows that there is temporal asymmetry in the measurement process. It is not time reversal invariant since the theory does not tell us the same thing in two different directions (ibid, p. 11).

One important step to take is about the law-likeness or fact-likeness of FOR. In Chapter 2, it has been observed that the ontology of the theory at hand plays a very decisive role in judging whether that particular theory is time reversal invariant or not. The same issue is being raised here by asking the role of FOR in different interpretations of quantum theory. If one adopts the Ghirardi-Rimini-Weber (1980) collapse interpretation (GRW in short), FOR is a fundamental law, therefore deeming quantum theory to be not time reversal invariant. In Bohmian (Bohm, 1952) mechanics though, the motion of the particles is governed by the so called guidance principle

$$\vec{v} = \mathrm{Im}\,\nabla\Psi / \Psi\,. \qquad (5.4)$$

Therefore, the time reversal invariance of Bohm's interpretation is based on first postulate and has nothing to do with the measurement postulate. Then, the observed asymmetry is explained away with the help of initial conditions (Arntzenius, 1997): according to Bohmian kind of noncollapse interpretation, in addition to their quantum state, each particle has to obey the guidance principle given in Eq. (5.4) above. Looking at Fig. (5.1) below which is modified from Arntzenius (1197, p. 214), we deduce that

> If the photon comes from A, then the initial quantum state will be in a wave packet Φ_A concentrated around A. Since there is no collapse in Bohm's theory, after encountering the mirror the quantum state will become a superposition $\Phi_C + \Phi_D$ of wave packets centered around C and D. But, each

particle also has a definite position, and it is that position which will determine which way it will go. Bohm's theory says that half the initial positions will pass through the mirror and the other half will bounce off the mirror. Thus one will get the desired invariant forward transition chances for particles starting from A. (ibid, p. 219)

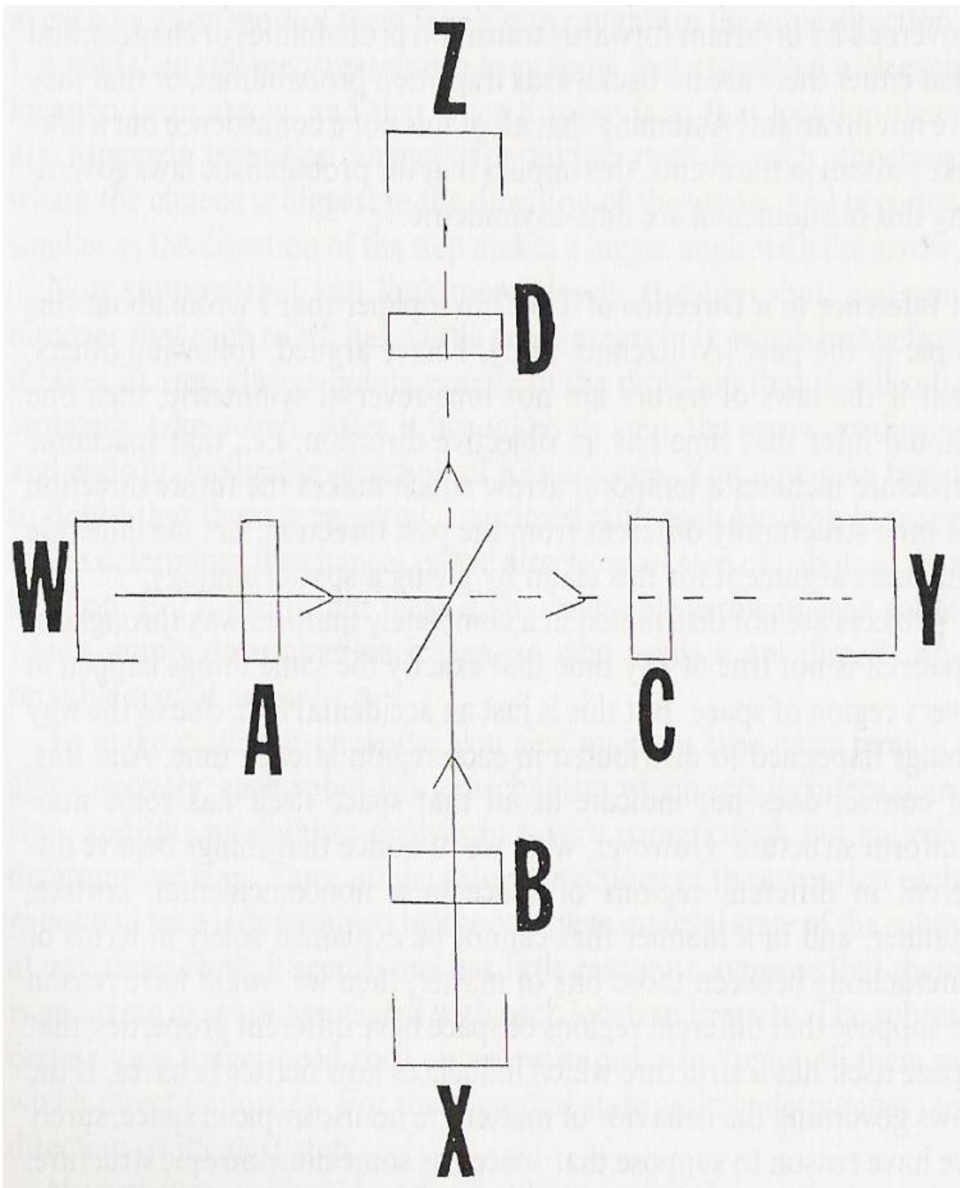

Figure 5.1: Arntzenius Gedanken Experiment

If the particles would begin in a quantum state Φ_B centered around B, they would develop into $\Phi_C - \Phi_D$. Now, if we think in terms of backward transition probabilities, it is obvious that we must consider the final states as a criterion to determine what is going to happen. If the final state is $\Phi_C - \Phi_D$, then it is obvious that all the particles came from B. On the other hand, if the final state is equal to $\Phi_C + \Phi_D$, then we will be sure of the fact that all the particles came from A. This simply shows us that backward transition chances depend on the *final quantum states* and not on the *final positions*. Since the final state depends on the initial state (whether we started from A or B), we have the temporal asymmetry. The source of this asymmetry hence lies in the initial states. After all, this is not surprising at all since Bohmian interpretation is a deterministic one. In this sense, i.e., relying on initial conditions to account for time asymmetry, Bohmian mechanics reflects the thermodynamic account of time asymmetry inherently.

Now, let us focus our attention on the first postulate which is Schroedinger equation plus Born interpretation. This is same as investigating whether Schroedinger equation is time reversal invariant or not. It is very simple to notice that it is not indeed since it is a first order equation in time derivative and its temporal inverse will give us

$$-i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \,. \qquad (5.5)$$

This equation is certainly not equal to the ordinary Scroedinger equation. As Callender (2000, p. 13) notices, this is exactly the same case one would have if Newton's second law would be written as F = mv.

The way out of this non-invariance in textbooks is explained by referring to what is called Wigner (1936) reversal. According to Wigner reversal, we must not only reverse the time order but also apply the complex conjugate operator on the state. Then, we will have

$$i\hbar \frac{\partial \Psi^*}{\partial t} = H\Psi^* , \qquad (5.6)$$

Which has the same form as Eq. (5.1) i.e., ordinary Schroedinger equation. If Ψ(x,t) is a solution to Schroedinger equation, then so is Ψ^*(x,-t). Let us, following Callender (2000), call this symmetry as WRI instead of ordinary time reversal invariant, TRI. Since what is important for all practical purposes is the probabilities (at the end, this is observable!), WRI restores the Born postulate which is given by Eq. (5.2).

Callender argues that symmetries must be applied o states and this must suffice for us to deduce whether a theory is time reversal invariant or not. Explicitly, Eq. (5.5) is not. In other words, WRI and TRI are two different symmetries. If TRI fails, it will tell us something abut time's handedness in quantum mechanics. On the other hand, the failure of WRI can be either due to TRI or complex conjugate operation. One cannot infer time is handed in quantum theory just by looking at the failure of WRI. It must be noted that all interpretations of quantum theory is time reversal non-invariant since they all embody Schroedinger equation, be it collapse or no-collapse theories.

However, it is easy to see why physicists insist on using WRI. It rests on a principle called Correspondence Principle (CP). In order to see this, we can use Ehrenfest theorem. According to Ehrenfest theorem, we have

$$\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m} . \qquad (5.7)$$

This equation shows the classical correspondence of the quantum mechanical operators when the position and momentum operators are averaged in terms of wave function. Now, looking at equation (5.7), we can apply TRI to get

$$-\frac{d\langle x\rangle}{dt} = \frac{\langle p\rangle}{m} . \qquad (5.8)$$

The right hand side of the equation must also have a minus sign if we want $\langle x(-t) \rangle$ to follow lawfully. But, this is not possible since momentum operator is defined as

$$\langle p \rangle = \int \Psi^*(x,t)(\frac{\hbar}{i})\frac{\partial \Psi(x,t)}{\partial x} dx \,. \qquad (5.9)$$

The equation above makes it explicit that the expectation value of the momentum operator does not change sign under the time reversal invariance TRI. Choosing to apply a second operation which will turn i to (-i), i.e., conjugation, will do the work however. In other words, the CP commands us to adopt WRI instead of TRI. This means that even Bohmian mechanics is not time reversal invariant if we adopt TRI instead of WRI. Indeed, this result is independent of any interpretation one can adopt since Schroedinger equation is fundamental to each one of them.

In GRW, it must be noted that even without Schroedinger equation, there is a preferred orientation of time. In all collapse theories, certain feature of the system such as particle number or mass will ignite a non-unitary indeterministic collapse to one of the eigenfunctions of the state. Therefore, there is certainly a temporal preference in these theories.

The experimental determination of temporal asymmetry in Bohmian interpretation is not trivial since there is no possible experiment which can tell us the difference whereas it is possible to do with the collapse theories (Callender, 2000, p. 13).

Reichenbach too accepts TRI as the correct time reversal invariant but thinks that the ordinary wave function and the time reversed one is indistinguishable. He then concludes that quantum mechanics is time reversible. He states this explicitly in following words

> There remains the problem of distinguishing between Ψ(q,t) and Ψ(q,-t). In order to discriminate between these functions, we should first have to know whether [$E\Psi = i\hbar\partial\Psi/\partial t$ or: $E\Psi = -i\hbar\partial\Psi/\partial t$] is the correct equation. But the

> sign on the right in Schroedinger's equation can be tested observationally only if a direction of time has been previously defined. We use here the time direction of the macroscopic systems by the help of which we compare the mathematical consequences of Schroedinger's equation with observation. Therefore, to attempt a definition of time direction through Schroedinger's equation would be reasoning in a circle; this equation merely presents us with the time direction we introduced previously in terms of macroscopic processes. (Reichenbach, 1956, pp. 209-210)

Andrew Holster (Holster, 2003, pp. 18-19) considers Reichenbach's view confusing since according to Reichanbach, we can equally go with TRI which will give us a time reversal non-invariant macroscopic picture due to CP.

So far, we considered the time reversal quantum states being formed by the action of an operator. In Chapter 3, it has been discussed that whether the actions of operators on states make sense. The time reverse of any sequence of states can simply be the inverse of the same sequence. But, in general, we do not be content with just inverting the sequence of states. We also apply an operator to this inverted sequence. Arntzenius (2004) considers this aspect from a quantum theoretical point of view. But, first he makes the point that he considers the use of operators is necessary in order to have non-trivial time asymmetries. Since one cannot have both deterministic and trivially time reversal non-invariant theories if we allow any kind of time reversal operator to act on the quantum states, we must either allow the states to be inverted without reversal operations acted upon at all or some certain operations to be acted. Why this is so is worth some pause: Let us imagine a history which can evolve into some other state in time as S(t) following Frank Arntzenius (ibid, p. 32) in a deterministic theory. Let us take the time reverse of the state $S(t_0+\delta t)$ to be $S(t_0-\delta t)$ where t_0 is a fixed time. In other words, we assume

$$(S(t_0+\delta t))^T = S(t_0-\delta t). \qquad (5.10)$$

Of course, this definition itself is compatible with $S^{TT} = S$. Now, let us suppose that S(t) develops into the state S(t+dt). Now, if we define $s \equiv t_0 - t$, we get

$$(S(t+dt))^T = (S(t_0-s+dt))^T = S(t_0+s-dt). \quad (5.11)$$

Now, according to rules of evolution for the states, it is obvious that $S(t_0+s-dt)$ will evolve into $S(t_0+s)$. Now, note that

$$(S(t))^T = (S(t_0-s))^T = S(t_0+s). \quad (5.12)$$

This proves that $(S(t+dt))^T$ evolves into $(S(t))^T$ if S(t) develops into the state S(t+dt). These considerations are also unique since the theory is assumed to be deterministic. In other words, if we choose just inverting the order of sequences, every deterministic theory is time reversal invariant. Since we do not want to have such a restriction, we now turn to understand more and use quantum mechanical arguments in order to understand what kind of time reversal operations must be used. Note that standard textbooks solve it in terms of four-potentials but reflect the act of time reversal operation as $A^0 \rightarrow A^0$ and $A^i \rightarrow -A^i$ for all components other than zero. This is not a four-vector transforms, states Arntzenius and look for another explanation to be able to justify the use of operators on states.

In quantum field theory, creation fields and annihilation fields can be written as

$$\begin{aligned} \Phi^+(x,t) &= k\sum_s \int u(p,s,x,t)a^+_{p,s}d^3p \\ \Phi^-(x,t) &= k\sum_s \int v(p,s,x,t)a_{p,s}d^3p \end{aligned} \quad (5.13)$$

Where k is normalization constant, u and v are coefficients, p is momentum, s is spin, and a's are operators which create and annihilate the single particle states with definite momentum and spin. It is possible to form scalar Lagrangians out of these quantum fields if u and v transforms like irreducible representations of the proper Lorentz group. In assessing the transformation properties, be it parity or time reversal, one assumes the energy to be positive (ibid, p. 39). This is necessary in order to avoid the possibility that of extracting unlimited amounts of energy through decays into deeper and deeper negative energy states. As an example for transformation properties, we consider, following Arntzenius, chargeless massive spin-0 particle. Under parity, an eigenstate $|p\rangle$ of three-momentum must transform to $\eta|-p\rangle$ where η is a phase factor. Then, the next step is to see that a_p^+ transforms to ηa_{-p}^+ and annihilation operator to $\eta^* a_{-p}$. Also, using p $\rightarrow$ -p, we find that under parity Φ(x, t) $\rightarrow$ η Φ(-x, t) if we assume η = η*. This assumption means that η is equal to 1 or -1. If it is equal to 1, then Φ(x, t) is invariant under parity and is called a true scalar. If it is equal to (-1), then Φ changes its sign and becomes pseudo-scalar.

In order to do the same kind of analysis for the case of time reversal, T must be anti-unitary i.e., it must transform ca_p to c^*a_{-p}. Only then, together with the assumptions that energy is always positive and transition probabilities are to remain invariant, we obtain Φ(x, t) $\rightarrow$ α Φ(x, -t) since we also change the sign of complex numbers in the exponentials. Of course, we still have the possibility of α being equal to +1 or -1.

Let us assume that we start with 0-memntum eigenstate $|0\rangle$ which by assumption changes its sign under time reversal (ibid, p. 40) i.e., $T|0\rangle = -|0\rangle$. We can then define a new zero-momentum eigenstate as $|0'\rangle = i|0\rangle$. When we apply T to this new state, we see that it is given by $T|0'\rangle = |0'\rangle$. Its phase factor is equal to 1 now instead of -1. Then, the other states can be defined by Lorentz boosting $|0'\rangle$. They will all have the same phase factors. This cannot be done for parity operator since it does not affect the complex numbers at all. Therefore, we can choose a new state and always have 1 as phase factor. Lorentz group properties suffice in order to explain the intrinsic structure of these transformation operators.

One interesting unorthodox proposal to connect quantum theory with the thermodynamic asymmetry of time has been offered by David Albert (Albert, 1994). Albert begins his discussion by considering two bodies in thermal contact with a temperature gradient (ibid, p. 671). Then we know that we can talk about two kinds of microstates, normal and abnormal, of the system which are compatible with its initial macrostate. The normal microstates are the ones which will decrease the temperature gradient whereas the abnormal microstates are the ones which will increase the temperature gradient. Of course, we know from statistical considerations that normal microstates will outnumber the abnormal ones. Moreover, the normal microstates will be stable under small perturbations while abnormal states will be unstable. Then, David Albert states

> Therefore, if the two bodies we have been talking about here *were*, in fact, somehow being frequently and microscopically and randomly perturbed, then the fact that their temperatures approached one another could be explained *objectively*, it could be explained (that is) *without reference to anything about what anybody happens to have known*. (ibid, p. 672)

These perturbations must be genuinely random but must be proven to be useful in their connection with the physical chances (ibid, p. 672). These chances must be such that they must have nothing to do with measurement problem since the tendency of temperature equalization of these two bodies is a fact independent of measurement.

What is Ghirardi-Rimini-Weber theory of collapse (GRW, 1980) then? According to GWR, the wave function *usually* evolves with respect to deterministic laws i.e., Schroedinger equation. But, from time to time, in a random way, the wave function of N particles is multiplied by a Gaussian of the form below

$$\Psi = K\exp[-(r - r_k)^2 / 2]. \qquad (5.14)$$

where K is of course normalization constant. r_k is chosen at random from the arguments of the N particle wave function. It is in general of the order of 10^{-5} cm. The probability of such jumps per particle per second and the width of the multiplying Gaussian are new constants of nature (ibid, p. 675). According to

Albert then, every single one of the microstates (not majority of them) will be overwhelmingly likely will evolve into states which the temperature difference gets smaller. In other words, these jumps are playing the role of perturbations we are looking for and in need of (ibid, p. 676).

Craig Callender (Callender, 1998) notes that Albert's proposal has everything one expects from a dynamical theory and can stand as an explanation but only up to a certain point: GRW theory cannot explain why we have initial low entropy state to to begin with. In other words, Albert's proposal is unable to explain Past Hypothesis. But, still, it is a good proposal in that one gets a dynamical explanation out of it (ibid, p.148).

Jos Uffink (2002) also hailed Albert's proposal a new approach but there is one big flaw in all this according to him: GRW only applies to solids not gases. Therefore, an ideal gas initially in a product state will not evolve into quasiclassical state, in which the center of mass is sharply localized (ibid, p. 562).

When the classical treatment of Boltzmann is applied to radiation which is modeled as a group of harmonic oscillators, it leads to Rayleigh-Jeans distribution whose output is that the total energy of any radiation is infinite. This is the so called UV catastrophe. The way out is the quantum mechanical treatment which then leads to the correct Planck distribution. The final answer lies in quantum electrodynamics (QED from now on).

As we have seen earlier in Chapter 3, it has been claimed in the literature that pure emission of light without absorption is possible but pure absorption without emission is impossible. Then, it is simple matter to label this as temporal asymmetry. Is this indeed so? The interaction of electrons and photons is given by QED. According to QED, these processes are called Compton scattering: an electron of momentum p and spin s absorbs a photon of momentum q and polarization λ. The intermediate electron will have momentum p+q due to the conservation of momentum at each vertex. The final state then consists of an electron with momentum p' and spin s', and a photon with momentum q' and polarization λ'. This case is plotted in Fig. 5.2 below adopted by Atkinson (ibid, p. 4)

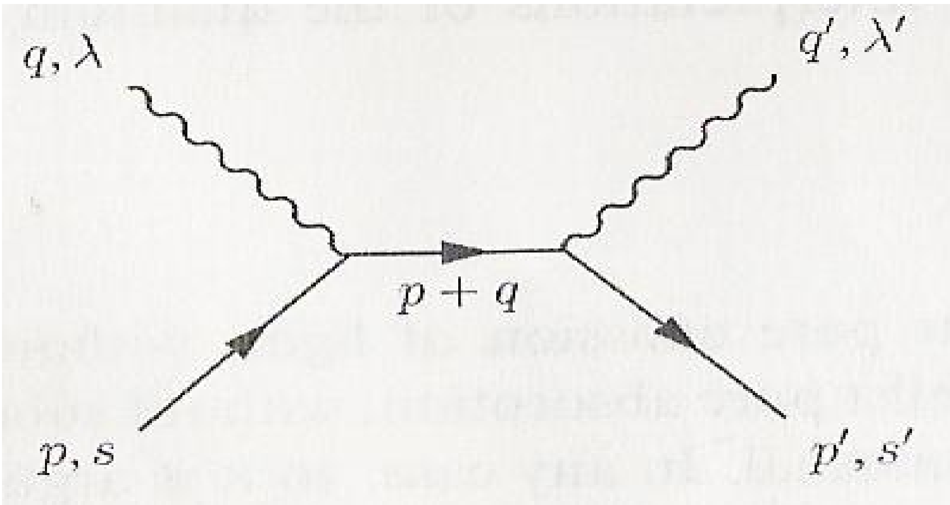

Figure 5.2: Compton Scattering

When q'=0, the energy of the outgoing photon too is zero. This simply is tantamount to say that the outgoing radiation is zero. But, this is kinematically impossible due to energy and momentum conservation and mass-shell condition (Atkinson, 2006, p. 4). Mass-shell condition applies to initial and final states not the intermediate ones and command the square of the energy minus momentum squared is equal to mass squared in such units that speed of light is taken to be equal to one. However, we also cannot have q = 0 exactly due to same reasons. In other words, the emission without absorption is also impossible due to kinematic constraints.

Although pure emission is not possible for a free electron, this is not the case for a bound electron. The Feynman diagram for this case is given blow as Fig. 5.3 adopted by Atkinson (ibid, p. 5).

If the atom is in excited state, it can undergo a transition to the ground state, with the emission of photon. The interaction will be between the electron (e) and an up quark (u) in the proton for example. This interaction will be accounted for by a virtual photon (ibid, p. 5). The energy of the photon will be equal to the energy of the excitation of the atom. Therefore, pure emission is possible for a bound state electron. But, its inverse is also possible which is simply the absorption of a photon in an atom so that the atom ends up being in excited state. The Feynman diagram for this pure absorption is also given in Fig. 5.3.

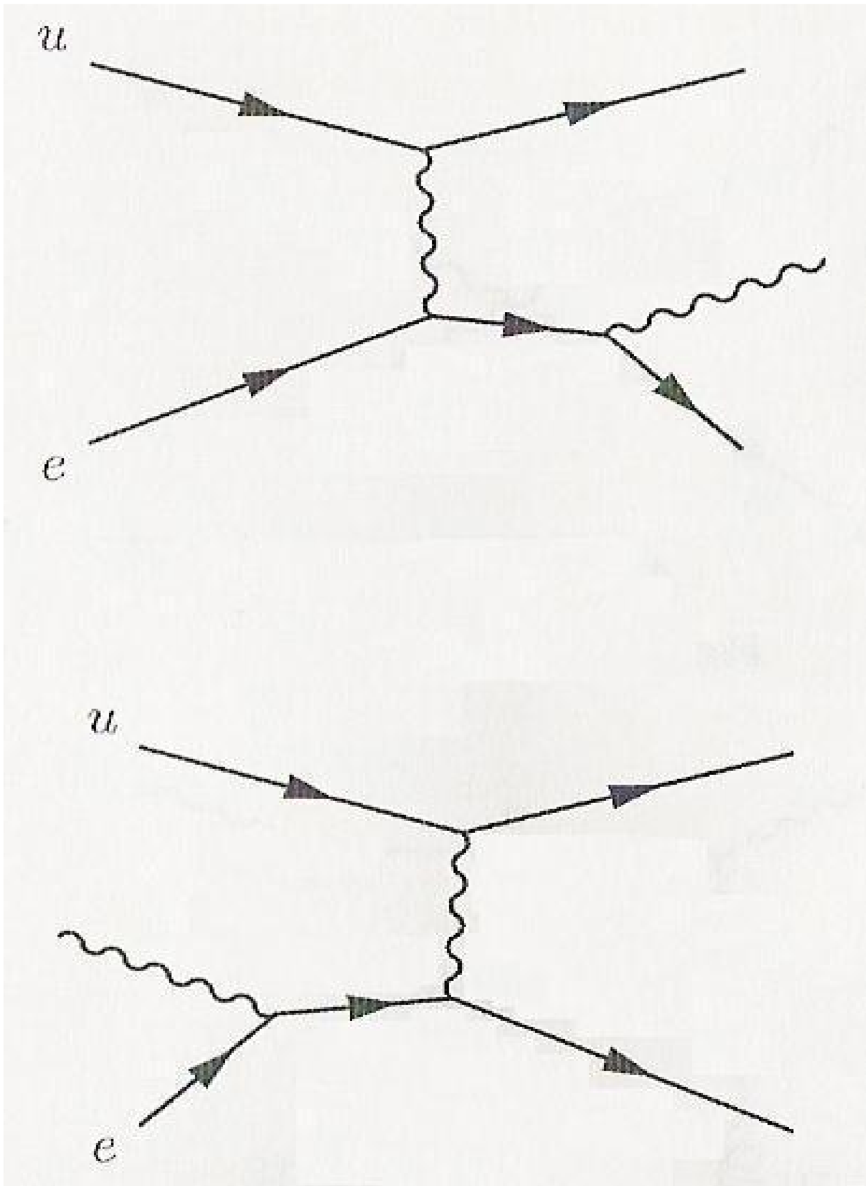

Figure 5.3: Pure Absorption and Pure Emission

In short, as David Atkinson remarks (ibid, p. 5), no arrow of time is obtained by the phenomena of emission and absorption of photons.

One important issue is to understand that there are other contributions to Compton scattering. For example, one needs to add another amplitude corresponding to Fig. 5.4 adopted from Atkinson (2006, p. 6) to Fig. 5.2 at two-vertex model in order to have a complete description of the overall process. What is happening in this figure? According to this figure, the emission of an outgoing photon happens before the absorption of the incoming photon, which causes the emission somehow, although this has already happened (ibid, p.5). Then, but only then, the sum of these two contributions, both being two-vertex contributions, is time symmetric. As Atkinson reminds us, this is the case for all perturbative levels of QED. Atkinson states that

> The Green's function that is used to calculate scattering amplitudes can be written as the sum of three parts (see Atkinson, 2000, p. 48): a retarded Green's function, an advanced Green's function, each with the same strength, and a self-interaction term, reflecting the fact that an electron interacts with the electromagnetic field that it produces itself. (Atkinson, 2006, p.6)

QED is a time symmetric theory which describes the interaction of photons with electrons.

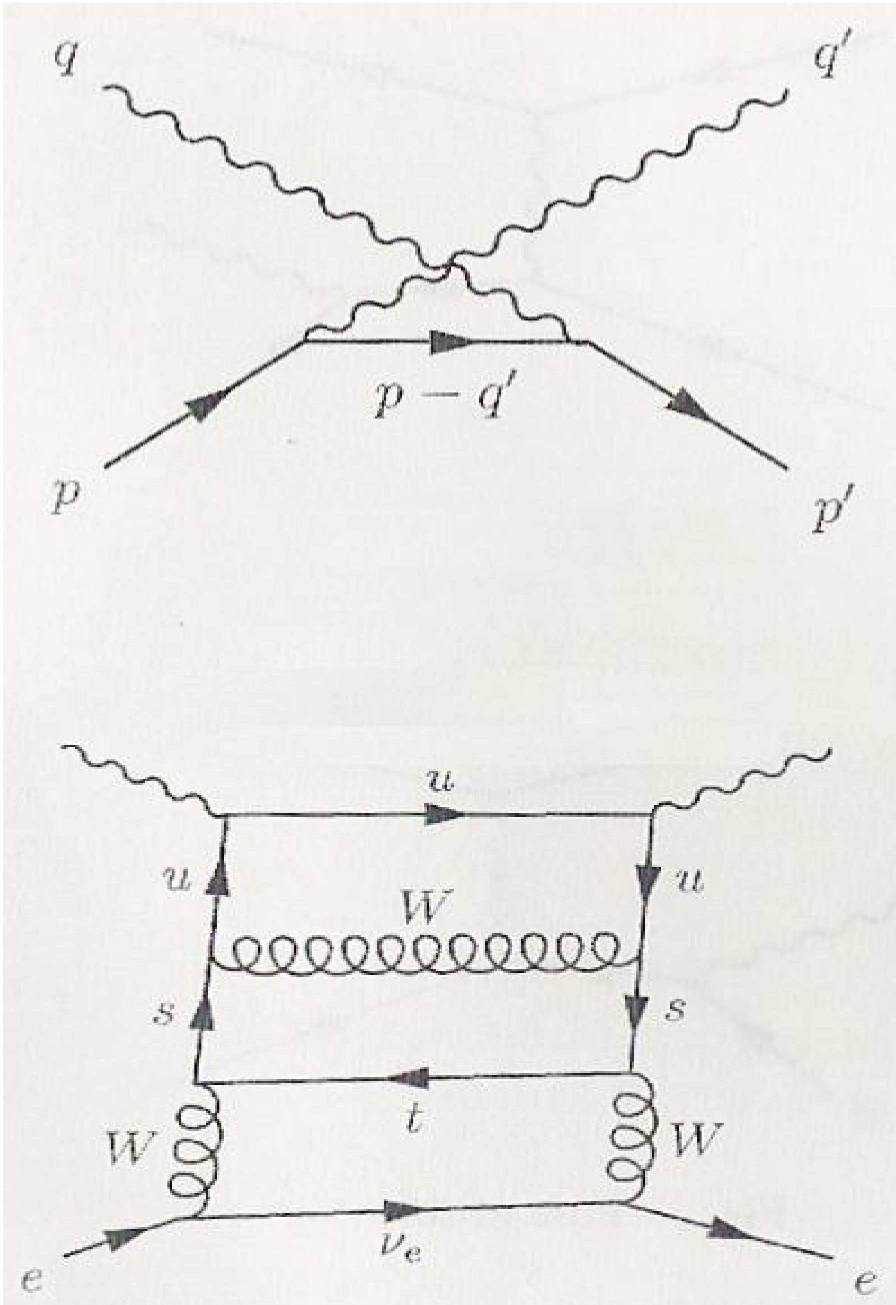

Figure 5.4: Symmetry in QED

Electrons also undergo weak interactions and weak interaction is not time symmetric. They violate this symmetry by about one part in thousand. Quarks, having charges themselves, couple to photons and violate T-invariance. The theory which unifies QED and weak interactions is called electroweak theory. Even though this is a small violation, it is a violation nevertheless. This violation in itself would not explain Sommerfeld radiation condition. It would not be able explain the arrow of radiation in the classical case since there are differences in magnitudes in non-invariance. Yet, one can deduce that there is a microscopic arrow of time.

CHAPTER 6

CONCLUSION

One important conclusion which can be drawn from this dissertation is that there is no master arrow or at least we are far from perceiving such an arrow in philosophy of science or science itself. The temporal invariance we have faced in Newtonian mechanics in Chapter 2 is certainly different than the thermodynamical arrow. One can of course argue that both of them might be interpreted as a problem of initial condition since we also emphasized the effect of initial conditions on mechanical systems as we did in the case of thermodynamic arrow by arguing the Past Hypothesis. However plausible this can seem, it is misleading: The initial value enters into our discussion related to chaotic behavior of mechanical systems whereas the Past Hypothesis has got nothing to do with mechanical systems. Another difference can be traced back in the observation that the Past Hypothesis is needed to explain something else i.e., the present high entropy state and in need of explanation while initial conditions in chaotic systems do not require further explanation. This is apparent since we would not try to consider initial conditions in mechanical systems lawlike. However, as we have seen, this is not the case for the Past Hypothesis.

Many scientists and philosophers now agree that the origin of the thermodynamic arrow is cosmological. However, the science which will explain this further is not adequate enough to provide the ultimate solution to the cosmological arrow of time. There are many debates surrounding the issues of big bang or big crunch.

I do however believe that certain parallels can be drawn between some issues. The problem of measurement in Newtonian mechanics and quantum mechanics both share the same feature in playing the role in making the theory time reversal non-invariant. The reason that we consider the measurement process in quantum theory by itself as opposed to our treatment in Newtonian mechanics is due to the difference in ontologies of these theories. Newtonian universe is led by the certainty written all over it whereas uncertainty is a common feature of quantum world. Nevertheless, this does not make Newtonian mechanics immune to the critiques mentioned by Keith Hutchison as we had the opportunity to see in Chapter 2.

One of the important lessons one can learn from Chapter 3 is the distinction between the definitions of time reversals. One can choose to do it in terms of the instantaneous states or dynamical conditions. If one adopts the latter view, one is forced to choose to apply a definite time reversal operator on dynamical states. Just inverting the dynamical conditions is not enough. This brings us to the problem of defining time reversal operators explicitly. In many cases, this is a difficult procedure and there is no ready-to-cook recipe. Choosing to use the instantaneous states in description of the universe on the other hand does not require such a time reversal operator to act. The problem is that state description gives us more than we bargain for: according to this view, even Maxwell equations become time reversal non-invariant.

As we have seen, one way out of this dilemma lies in covariant formulation of classical electrodynamics. Once we increase the dimension from three to four, what has seemingly been a problem appears to be a normal case in which one can defend time reversal invariance easily. This example is important from another aspect, too. It shows us how important developments in physics are in order to shed some light on topics which philosophers (remembering the connection of the problem stated above to the famous Zeno paradox) and philosophers of science argue about.

Another interesting topic which has been covered within Chapter 3 is the famous Wheeler-Feynman theory of radiation. This theory gives a successful treatment of electrodynamics in a time symmetric way. The down side of the theory is that it ends up in an asymmetrical conclusion and is forced to accept the thermodynamic arrow i.e., initial conditions as the adequate explanation. The Wheeler-Feynman theory is important in showing us that there might be alternative formulations, different points of view. Recently, philosophers of science got into a very hot debate to discuss Wheeler-Feynman theory, its implications and alternatives. All these topics have been covered in Chapter 3.

The relation of causality on time asymmetry is discussed in the framework of LAD equation. This topic has brought us to whether we must allow the use of point particles in classical theories. As we all know, the limit of classical theory is De Broglie wavelength. For sizes smaller than this wavelength, one has to adopt a quantum mechanical perspective. As we have observed, confusing the domain of use of a theory can lead to some strange behaviors which can be mistakenly interpreted in terms of temporal symmetry/asymmetry although it has nothing to do whatsoever with it. In fact, this allows us to have two different readings of Chapter 2: In Chapter 2, we discovered the asymmetry hidden in Newtonian mechanics. It was hidden because we underestimated the domain of use of the theory. In the case of Chapter 3 though, we overestimated the use of theory and used it in a way we were not allowed to. The Classical mechanics is not the proper domain to talk in terms of point particles.

Chapter 4 had its starting point in Hans Reichenbach’s work the *Direction of Time*. Reichenbach’s main analysis was based on the works by Boltzmann and in this regard historical. But, he went beyond a simple historical account by providing all the philosophical and logical background. His emphasis on branch structures were based on a transition from time ensemble to space ensemble i.e., Gibbs ensemble. The problem is that there are many philosophical problems with

Gibbs ensemble, and Reichenbach has just inherited these problems since his solution included the use of Gibbs ensemble.

The post-Reichenbach period is marked by the emphasis on two issues: Firstly, its emphasis on gravitational effects. Second, Reichenbach's assumptions in order to have a consistent formulation of the problem. The first critique of Reichenbach has been anticipated by Reichenbach himself but he did not pursue his route. One reason might be that one cannot talk about any thermodynamic isolation if one does not neglect gravitation. Even today, we do not know how to consider gravitational cases in a consistent manner. The Boltzmannian approach, right from the beginning, assumed short range interactions. Therefore, it is not adequate to handle gravitational forces which are long range. In short, I believe that Reichanbach was right in neglecting gravity in this sense. But, there is a second sense in which gravitation plays a very important role. It is the very structure of space-time itself. After general theory of relativity, the structure of space-time in which we are living has been extremely important. Therefore, the form of space-time has to be taken into account in order to be able to talk about any direction of time in a consistent way. Even Boltzmann (and later Reichenbach for that matter) wrote about different space-time points having different temporal orientabilities but that was it. Neither Boltzmann (he could not know about Einsteinian theory of gravity then anyway) nor Reichenbach (he excluded all issues related to gravity somehow when he was discussing the direction of time) had a consistent study of the direction of time as far as the structure of space-time is considered. This is surprising since Reichenbach was a philosopher of science who also studied Einsteinian theory of gravitation and knew it well enough. One explanation for this might be that Reichenbach's *The Direction of Time* was left unfinished and published posthumously.

One critique which can be made about this dissertation which is also applicable to Reichenbach himself as far as general theory of gravity is considered is that we also did not discuss some important aspects of general relativity on the direction of time. Let us make this point clearer: any kind of closed causal net will have important bearance on our topic since a direction of time cannot be specified then. This point has been emphasized in this dissertation. The problem is that we are beginning to take the possibility of such a space-time serious as some developments in physics are emerging. The point is the famous Gödel universe. When Gödel was working on Einstein's theory of gravity, he quickly discovered that Einstein's theory allows a model of universe in which closed timelike curves are possible. This simply means that the direction of time has no ordinary sense since it forms a closed causal net as Reichenbach calls it. The possibility of time travel and other implications of Gödel universe are studied in detail now by physicists and philosophers of science. We omitted this part in this dissertation since the findings are mixed with speculations yet.

Chapter 5 has aimed to give a concise description of what the quantum mechanical arrow of time is all about. The main results is that quantum mechanical world is suspected to be time asymmetric but gives rise to a time symmetric view of universe at macro level (arguably). As is well known, there are many interpretations of quantum theory and each of them may provide a new insight into the quantum mechanical arrow of time. We excluded many-world interpretation (Everett, 1957), modal-interpretation (van Fraasen, 1974) and transactional interpretation of quantum mechanics (Cramer, 1986) since these topics by themselves form a dissertation topic. Nevertheless, we believe that the almost interpretation-independent view in Chapter 5 will form the next step in assessing the true meanings of the quantum mechanical arrows of time when one would like to consider the arrow of time emerging in a particular interpretation of quantum theory.

One important topic which has not been considered in this dissertation is the arrow of time in quantum gravity. The physicist/philosopher Julian Barbour (Barbour,2001) has been the first person, as far as I know, who drew attention to the outcomes of this theory concerning the arrow of time. Indeed, the main result is simple and very direct: there is no such thing as time. Why is this result important? The answer to this question lies in understanding what quantum gravity is: it is supposed to be the holy grail of physics. In other words, many physicists believe that it is either the ultimate theory or an important part of it. Therefore, a theory of quantum gravity is important. The theory of quantum gravity developed so far has a main formula (main in the same sense of second law being the main equation in Newtonian mechanics) which does not have anything to do with time. This is a timeless equation which suggests the possibility of a timeless existence.

We believe that developments in physics and the works done by physicists and philosophers related to these developments will shed new light on the famous arrow of time problem in the future.

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TURKISH SUMMARY

Zamanın oku problemi, genel olarak felsefe, özeldeyse bilim felsefesinin önemli konuları arasındadır. Önemi sadece kapsadığı konular açısından değil, farklı disiplinler ve felsefe sistemleri arasında kurduğu bağlantı dolayısıyladır. Zamanın yönünün çalışılması felsefenin herhangi bir alt bölümü esas alınarak yapılabileceği gibi (metafizik ya da ontoloji) farklı bilim dallarından da yararlanılarak yapılabilir. Psikolojik zaman ile fiziğin bize sunduğu zaman anlayışı birbirinden farklı zaman tanımlarına yaslandığından, her bir bilim dalının seçimi kendi öz felsefesini de beraberinde getirecektir. Bu tezde yararlanılan bilim fizik olduğu için bu tezin fizik felsefesi alanında olduğunu saptamak yanlış olmayacaktır.

Zaman oku sözü ilk defa Sir Arthur Eddington tarafından edilmiş olup, zamanın hep ileriye doğru akmasını, yani tek yönlülüğünü belirtmek için kullanılmaktadır. Devamlı yaşlanmamız, sütle kahvenin kendiliğinden birbirine karışması (ayrışmaması) hep zamanın tek yönlülüğüne işaret eder.

Fizik felsefesindeyse, zaman oku yolundan yapılacak herhangi bir araştırma, zamanın tersinirliği işlemi cinsinden yapılır. Denklemlerimizdeki zaman parametresi negatifiyle değiştirildiğinde denklemin aynı kalması bize bu denklemin belirli bir zaman yönünü ayrıcalıklı saymadığını gösterir. Bunun tersi bir durumsa, o denklemin zamanın yönü açısından belirli bir yönü ayrıcalıklı saydığını düşündürür.

Denklemlerin zamanla ilişkisini teorinin zamanla ilişkisine dönüştürmekse kolay değildir. Bir teori yapısı itibariyle birçok denklemi içerebilir. Bunların herhangi birini temel denklem saymak her zaman kolay bir iş olmayacaktır. Bu kişinin öznel kıstaslarına göre değişen bir uğraş olacağından doğrudan o kişinin ontolojik seçimlerine bağlıdır. Buna örnek olarak Newton mekaniğini verebiliriz: Newton mekaniğinde genellikle en temel olarak alınan yasa ikincisidir. Bu yasa, kuvvet, kütle ve ivme arasındaki ilişkiyi verir. Newton'un ikinci yasasında belirtilen kuvvet herhangi bir kuvvet olabilir. Diğer bir deyişle geneldir. Bu kuvvet ifadesine sürtünme kuvvetini eklediğimiz anda Newton mekaniğinin zaman oku açısından simetriye sahip olmadığını görürüz. Genel ifadenin kendisiyse zaman açısından simetriktir. Eğer sürtünme kuvvetini doğada bulunan bir kuvvet olarak düşünürsek, Newton mekaniğinin zaman simetrik olmadığı sonucuna varırız ki, bu da bizi Newton denklemlerinin geçmiş ile gelecek arasında ayrım yapabildiğini düşünmeye sevk eder. Eğer denklemin genel halini düşünürsek, zaman simetrik olduğundan çıkan sonuç tam aksi olacaktır. Bunun hangisi doğrudur? Cevap kişinin ontolojik seçimlerine bağlıdır. Eğer ünlü fizikçi Richard Feynman gibi doğadaki bütün kuvvetlerin korunumlu olarak ifade edilebileceğini düşünüyorsak, denklemlerin geçmiş ve gelecek arasında ayrım yapmadığını düşüneceğiz.

Klasik mekanikte zaman oku problemi açısından önemli olan bir diğer konu da kaos teorisidir. Bu teoriye göre, belirli bir noktadan başlayan hareket, klasik olsa bile, tekrar o noktaya dönemez. Bunun nedeni, sistemin bizzat kendisinin küçük değişimlere aşırı hassasiyetidir. Bu açıdan bakıldığında bir sistemin klasik olması onun zaman oku simetrik olmasını gerektirmez.

Tezin ilgili kısmında bahsi geçen son konu klasik fizikte ölçmenin zaman oku problemine etkisidir. Eğer ölçümlerde, ölçümün bizzat kendisinden doğan hatalar da düşünülürse, o zaman sistemin zaman simetrik olmayacağı açıktır. Çünkü her ne kadar ölçüm sonucu bulunan değerin kesinlik içeren kısmı zaman simetrik olacaksa da, aynısını kesinliği az olan kısım için söylemek mümkün değildir. Bütün bu yukarıda özeti yapılmaya çalışıla konular tezin ikinci kısmını oluşturmaktadır.

Tezin üçüncü bölümü klasik elektrodinamikte zamanın oku konusunu incelemektedir. Klasik elektrodinamik Maxwell'in dört denklemi yardımıyla incelenir. Genellikle bu denklemler zaman açısından simetrik olarak Kabul edilirler. Bunu görebilmek için, elektrik alan aynı kalmasına rağmen, manyetik alanın negatifinin alınması gerekir. Bu tür bir zaman tersiniri işleyişi anlamlı gözükmemektedir. Çünkü biz her şeyden önce elektrik ve manyetik alanın aynı şekilde düşünülmesi gerektiğini biliyoruz. Nasıl oluyor da elektrik alan aynı kalırken manyetik alanın negatifinin alınması haklılık kazanmış oluyor o zaman?

Bu konuda yapılabilecek ciddi bir çözümleme bizi zaman oku probleminin yeniden tanımlanmasına götürür. Bir eylemler dizininin tersi sadece bu eylemlerin tersyüz edilmesiyle mi elde edilir yoksa her eylemin tersine bir başka işlem daha uygulamak mı gerekir? Bu soru sadece felsefi açıdan önemli olmayıp, fizik bilimi açısından da oldukça önemlidir. Zamansal olarak tersten işleyen bir dünyada yeni kuvvetlerin olup olmaması olasılığı da buna bağlıdır. Örneğin, eğer elektrik ve manyetik alanın zaman oku karşısındaki değişimleri yeni bir Lorentz kuvvetine yol açacak cinstense, bunun fiziki önemi aşikardır.

Bir olaylar dizininin zamansal açıdan tersinin tanımlanması iki şekilde yapılabilir: Anlık değerlerin tersine çevrilmesi ya da dinamik koşulların tersine çevrilmesi. Eğer dinamik koşullar diliyle konuşacak olursak, sadece bu koşulların tersine çevrilmesinin bizim amaçlarımız için yeterli olmayacağı açıktır. O halde, bu dinamik koşullar ek bir işleme tutulmalıdırlar.

Anlık değerler ya da dinamik koşullar cinsinden konuşmak klasik mekanik söz konusu olduğunda sorun teşkil etmez. Çünkü bu teori açısından bakıldığında anlık değer olan konum vektörüyle dinamik koşul olan hız aynı anda ve tutarlı bir biçimde ters çevrilebilirler. Aynı şeyi klasik elektromanyetik alanında yapmaya kalktığımızdaysa sorun çıkar çünkü elektrik ve manyetik alanlar arasında hız ile konum arasındaki bağdan farklı bir bağ vardır. Bu bize aslında zamanın oku probleminin tanımlamasının bile ne kadar çetrefil bir iş olduğunu göstermektedir.

Aslında olası bir çözüm boyutun arttırılması ile mümkün gözükmektedir. Maxwell denklemlerini dört boyutta yazdığımızda elektrik ve manyetik alanları bir alan tensörü içinde yazmak mümkün olduğundan dinamik koşulların ya da anlık koşulların önemi kalmamaktadır. Bu örnek aslında bize çok daha genel bir oluşumu işaret etmektedir: Boyut arttırımı bazen basit bir matematiksel işlem olmaktan çok aynı olayın farklı görünmesini sağlamaktadır.

Dikkat çekilmesi gereken konulardan biri yukarıda anılan çözümün sadece kovaryant yazımdan kaynaklanmadığıdır. Her kovaryant yazım bize istenilen sonucu vermemektedir. Bu yüzden özel görecelik kuramının yeri ayrıdır.

Üçüncü kısımda ele alınan konulardan biri de fiziksel sistemlerin kendi üzerine etkilerinin ele alınmasıdır. Tarihsel olarak ilk defa Lorentz tarafından ele alınan bu konu hızın zamana göre ikinci dereceden türevi ile verilen bir terime yol açmıştır. Daha sonar aynı terim, Dirac ve Abraham tarafından da elde edilmiştir. Bu terim temelde iki sorunu da beraberinde getirmiştir. İlk olarak kendi kendine hızlanan bir sistemi betimlemesi Newton yasaları açısından tutarsızdır. Çünkü Newton yasaları uyarınca, üzerine hiç bir kuvvet etki etmeyen bir nesne ya sabit hızla hareketine devam etmelidir ya da durmalıdır. Diğer bir sorunda nedensellik ilkesinin bu tür durumlarda geçersiz kılınmasıdır.

Lorentz-Abraham-Dirac denkleminin bu iki sorununun çözümü aslında çok önceleri Somerfeld tarafından verilmiş olmasına rağmen bilim tarihinin karanlık sayfalarının arasında kalmıştı. Sommerfeld'in çözümü klasik elektrodinamiğin uzanımlı bir parçacığa uygulanışı ile ilgiliydi. Diğer bir deyişle Sommerfeld nokta parçacıklar yerine, yarıçapı sıfırdan farklı olan parçacıkları kullanarak hesap yapmış ve az önce bahsedilen iki sorun da ortadan kalkmıştı. Bu örnek bize klasik elektrodinamiğin aslında zaman oku yönünden simetrik olmadığını gösterir. Çünkü nokta parçacıklar sadece kuvantum fiziği ile anlaşılabilir. Eğer parçacığın uzanımı varsa, klasik dinamik kullanılabilir ve o zamanda asimetrik bir durumdan söz edilebilir. Aslında bu örneğin bize gösterdiği, bir teorinin tanım kümesinde kullanılması gerekliliğidir. Bohr dalga boyundan az olan bir yarıçapa sahip parçacık her zaman kuvantum teorisi ile anlaşılmaya çalışılmalıdır ve bu tür durumlarda klasik teori kullanılmamalıdır.

Daha önce Maxwell denklemlerinin kendisinden bahsetmekle birlikte, bunların çözümü üzerinde durulmadı. Bu tezin üçüncü bölümünün bir kısmı buna ayrılmıştır. Burada bizim açımızdan en önemli sorun şudur: Maxwell denklemlerinin iki tür çözümü vardır. Bunlardan birisi nedensellik ilkesi ile bağdaşmakta, diğeriyse bağdaşmamaktır. Nedensellik her ne kadar fiziğin içinde fiziksel yasa olarak kabul edilmese de, doğada her zaman nedensellik ilkesiyle bağdaşan çözüm görülmektedir.

Maxwell denklemlerinin her iki tür çözümü de sunmalarına rağmen bizim sadece bunlardan birini gözlemlememiz aslında çok da anlaşılmaz bulunmamalıdır. Maxwell denklemleri bütün olasılıkları verecek güçtedir. Her iki çözümün de aynı anda evrende bulunmasını beklemek safdillik olacaktır. Bunun bir benzeri ikinci dereceden denklem çözümlerinde görülür. Bu tür denklemler biri pozitif diğeri negatif olmak üzere iki çözüm önermelerine rağmen, hesaplanılan fiziksel niceliğe göre uygun olan çözüm seçilmektedir. Aynısı Maxwell denklemlerinin çözümü için de düşünülebilir o halde. Tabii ki bu nedensellik ilkesinin fiziksel bir yasa olarak Kabul edilmesiyle eş değerli düşünülmelidir.

Maxwell denklemlerinin bu iki tür çözümleri düşünüldüğünde, çözüm en sonunda istatiksel okun kendisine indirgenir. Wheeler-Feynman teorisinin de önemi bu noktada ortaya çıkmaktadır. Bu teoriye göre aslında bütün klasik elektrodinamiği her iki çözümle de elde etmek mümkündür. Ama yine de bu teorinin vardığı sonuç, zaman okunun tersine çevrilmesinde karşılaşılan sorunun istatiksel olduğudur. Diğer bir deyişle sistemin ilk durumu çok büyük önem kazanmaktadır.

Bazı felsefeciler bu noktadan hareketle elektrodinamik okun temelini Geçmiş Hipotezinde ararlar. Buna göre, evren ilk aşamasında çok düşük bir çekimsel entropiye sahipti. Daha sonra bu entropi artmaya başladı. Söz konusu entropi artışına neden olan şey çekim kuvveti olduğundan, görünümü cisimlerin bir araya gelmesiyle olmuştur. Bütün bunlar olurken evren başlamış olduğu termal dengeden de uzaklaşmaya başlamıştır. Termal dengenin tekrar sağlanabilmesi için ivme kazanmış olan parçacıklar ışımaya başlayacaklardır. İstatiksel teoriye göre bu ışımalar her iki yönde de, yani hem geçmişe doğru, hem de geleceğe doğru olmalıdır. Ama geçmişte zaten termal denge söz konusu olduğundan bu yöndeki ışımalar az olacaktır. Bu da istatiksel olarak klasik elektrodinamik zaman okunun açıklanışıdır.

Tezin dördüncü bölümü az önce bahsettiğimiz istatiksel zaman okunun anlaşılmasına yöneliktir. Burada en önemli konu H teoremdir. Boltzmann tarafından bulunan H teorem istatiksel olarak her iki yönde de zaman okuna açıktır. Bu yapısındaki asimetriden kaynaklanmaktadır. Söz konusu asimetri moleküler kaos denilen bir varsayımdan kaynaklanmaktadır. Bu varsayım olmadan H teorem olamayacağından, H teoremin asimetrisi aslında moleküler kaosun kendisidir. Çözüm genelleştirilmiş H teoremidir. Bu teoremde artık moleküler kaos varsayımına gerek duyulmamaktadır. Önemli olan birbirinin aynı olan gruplar cinsinden istenilen niceliklerin ortalamasının bulunmasıdır.

Genelleştirilmiş H teoreminin temelinde ölçüme dair belirsizliklerin hesaba katılması rol oynamaktadır. Belirsizlikler hesaba katıldığında zaman oku problemi de çözülmüş olmaktadır. Bu durum aslında daha önceden de konu ettiğimiz klasik mekanikteki duruma çok benzemektedir. Klasik mekanikte de ölçüme dair belirsizlikler hesaba katıldığında sistemin zaman oku yönünden tersinmez olduğu görülür. Klasik mekaniğin istatistiksel mekaniğin de temelinde olduğu düşünülürse bu tür bir düşünüşün ünlü Loschmidt paradoksunun çözümünü de beraberinde getirdiği kolaylıkla anlaşılacaktır.

Bilim felsefesi alanında bu doğrultuda en önemli çalışmayı Hans Reichenbach yapmıştır. Onun temel katkısı Boltzmann'ın düşüncelerini büyük bir berraklıkla açımlayabilmesidir. Tanımladığı branş sistemleri aracılığıyla entropinin genel artışını bu branş sistemlerinin davranışına bağlamıştır. Bütün bunlardan çıkan en önemli sonuçsa, zamanın okunun yönünün evrende bir alandan diğer alana değişiklik gösterebileceğinin anlaşılmasıdır. Diğer bir deyişle zaman oku bizim nerede olduğumuza bağlıdır. Bu bizim algıladığımız zaman ile evrensel zaman arasındaki bağlantıyı gözler önüne serer.

Reichenbach sonrası yapılan her çalışma bir şekilde ondan yola çıkmıştır. Onu eleştirel gözle okuyanların başında felsefeci John Earman gelir. Earman'a göre Reichenbach Einstein'ın genel görelilik yasasından tam anlamıyla faydalanamamış, çıkarması gereken dersi çıkarmamıştır. Earman'a göre, zamanın yönü problemi global özelliğe sahip olmakta yani evrenin her yerinde geçerli bir zaman okundan söz etmektedir. Bu zaman oku bir kere belirlendiğinde değişmesi söz konusu olamaz. Buna göre, evrende, herhangi bir alanında zamanın yönünün belirlenmesi için iki temel yöntem vardır. Ya Boltzmann'ın entropi metodu kullanılabilir ya da Earman'ın paralel taşıma metodu. İşte sorun da tam bu noktada belirmektedir. Diyelim ki Earman'ın metodu herhangi bir bölge için gelecek yönünü gösterdi. Boltzmann'ın metodu da bunu doğrular nitelikte diyelim. Boltzmann'ın metodu doğası gereği istatiksel olduğundan bir sure sonra aynı yönü geçmiş olarak göstermesi kaçınılmazdır. Buna rağmen Earman'ın metodu hala bu nokta için geleceği işaret edecektir. Bu iki yöntemin çelişmesi kaçınılmazdır ve çeliştiklerinde de hangisine güvenilebileceği açık değildir.

Tezin beşinci bölümü kuvantum mekaniğindeki zaman oku üzerinedir. Bu teoride iki ana nokta vardır. Birincisi Schrödinger denklemine ilişkindir. Bu denklem zaman oku yönünden asimetriktir. Buna rağmen bu durum engel teşkil etmez. Çünkü kuvantum mekaniği uyarınca önemli olan dalga fonksiyonunun kendisi değil, bundan elde edilen olasılıktır. Bu olasılık ta aynı kalmaktadır.

Diğer sorunda kuvantum mekaniğindeki ölçme sorunudur. Ölçmeden hemen sonra elde kalan dalga fonksiyonu tersinmez niteliktedir. Zaman oku yön değiştirdiğinde aynı sonuç elde edilemeyeceğinden ölçmenin kendisi dolayısıyla kuvantum mekaniğinin zaman oku açısından asimetrisine hükmedebiliriz. Bu açıdan bakıldığında David Albert'in kuvantum mekaniğiyle istatistik mekanik arasında kurduğu ilişki çok önem kazanmaktadır. Böylece artık kuvantum mekaniğinin zaman okuyla istatistiksel zaman okunun kaynağının bir ve tek olduğu düşünülebilir.

Son olarak beşinci bölümde analize tutulan konu kuvantum elektrodinamiğinde zaman okudur. Bütün derecelerden perturbasyonlar dikkate alındığında bu teori de zaman oku açısından nötrdür.

VITA

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