ESTIMATING THE NEUTRAL REAL INTEREST RATE FOR TURKEY BY USING AN UNOBSERVED COMPONENTS MODEL

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ABSTRACT

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In this study, neutral real interest rate gap and output gap are estimated jointly under two different multivariate unobserved components models with the motivation to provide empirical measures that can be used to analyze the amount of stimulus that monetary policy is passing on to the economy, and to understand historical macroeconomic developments. In the analyses, Kalman filter technique is applied to a small-scale macroeconomic model of the Turkish economy to estimate the unobserved variables for the period 1989-2005. In addition, two alternative specifications for neutral real interest rate are used in the analyses. The first model uses a random walk model for the neutral real interest rate, whereas the second one employs more structural specification, which specifically links the neutral real rate with the trend growth rate and the long-term course of the risk premium. Comparison of the models developed by using various performance criteria clearly indicates the use of more structural specification against random walk specification. Results suggest that though there is relatively high uncertainty surrounding the neutral real interest rate estimates to use them directly in the policy-making process, estimates appear to be very useful for ex-post monetary policy evaluations.

Keywords: Kalman filter, Random walk model, State space models, Output gap, Real interest rate gap

ÖZ

GÖZLENEMEYEN BİLEŞENLER MODELİ İLE NÖTR REEL FAİZ ORANININ TÜRKİYE İÇİN TAHMİNLENMESİ

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Bu araştırmada, Türkiye'deki tarihsel makroekonomik gelişmeleri anlayabilmek ve para politikasının Türkiye ekonomisi üzerindeki uyarıcı etkisini analiz etmede kullanılabilecek gözlemlere dayalı (ampirik) ölçümlerin elde edilmesi amacı ile çoklu gözlenemeyen bileşenler modelleri kullanılarak nötr reel faiz oranı ve üretim açığı birlikte tahminlenmeye çalışılmıştır. Analizlerde gözlenemeyen değişkenlerin 1989-2005 yılları arasındaki tahminleri Türkiye ekonomisinin küçük-ölçekli bir makroekonomik modeline Kalman filter tekniği uygulanarak elde edilmiştir. Bunun yanı sıra nötr reel faiz oranı iki yaklaşımla modellenmiştir. Birinci yaklaşımda rasgele yürüyüş modeli kullanılırken; ikinci yaklaşımda nötr faiz oranını potansiyel üretimin büyüme oranı ile risk piriminin uzun dönemli seyrine ilişkilendiren daha yapısal bir modelden yararlanılmıştır. Geliştirilen modeller çeşitli başarım kıstasları ile karşılaştırıldığında yapısal modelinin rasgele yürüyüş modeline göre üstünlüğü açıkça görülmektedir. Araştırma sonuçlarına dayanarak nötr reel faiz oranı tahminlerinin oldukça yüksek belirsizlik içermesi nedeni ile doğrudan politika yapma sürecinde kullanılmasının sakıncalı olabileceği, buna karşın para politikasının geçmişe yönelik bir değerlendirmesini yapmada yararlı olabileceği söylenebilir.

Anahtar Kelimeler: Kalman filtresi, Rasgele yürüyüş modeli, Durum uzay modelleri, Üretim açığı, Reel faiz oranı açığı

To My Family,

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TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

One of the central issues in the monetary policy decision-making has been to evaluate the monetary policy stance. Monetary policy authorities, in setting the interest rates, commonly need a benchmark to assess whether level of the interest rate is stimulative or contractionary for the economic activity. This benchmark is generally called as "neutral real interest rate[1]". Archibald and Hunter (2001) define the neutral rate as the level of the real interest rate that would prevail if there were no inflationary or deflationary pressure requiring the central bank to lean in either direction. In other words, neutral real interest rate is the real rate consistent with the real gross domestic product (GDP) at its potential level and stable inflation, which corresponds to a neutral monetary policy setting. The real interest rates above (below) the neutral rate -for a prolonged period of time- would tend to contract (stimulate) the economic growth through lowered consumption and investment, and inflation would eventually fall (rise) in the absence of shocks to the economy or other imbalances. Thus, it is vital to quantify the level of the neutral real interest rate for any monetary authority whose primary goal is to achieve and maintain price stability.

[1] Neutral real interest rate is sometimes called as natural rate of interest or equilibrium real interest rate. While the former can be used with neutral real interest rate interchangeably, the latter is frequently referred to "the long-run equilibrium level". Regarding the conceptual difference between neutral (or natural) real interest rate and the long-run equilibrium real interest rate, see Chapter 2.

The concept of neutral real interest rate is commonly ascribed to the Swedish economist Knut Wicksell (1907)[2]. Wicksell provided an answer to the question of how changes in the real interest rate affect the general price level in the economy. According to his view, the price level is determined by the difference between the actual and the neutral real interest rate, which is named as real interest rate gap in general terminology. Jonsson (2002) briefly explains, in his own words, the Wicksell's framework as follows:

An increase in the natural interest rate (given the actual interest rate) implies that the firms' profits increase due to an increase in the return on their capital. This leads to an increased demand for labor and thus, eventually, increased wages. The increased wages increases households' demand for consumer goods, which pushes up prices. That is, if the natural interest rate exceeds the actual interest rate, the price level will increase.

The modern explanation of Wicksell's idea is based on the real interest rate that stabilizes inflation rather than the price level. In order to overcome the inflationary pressures and to prevent the occurrence of an interest rate gap, central banks, including Central Bank of the Republic of Turkey (CBRT), control their short-term nominal interest rate as policy instrument. Sometimes, policy rates will be increased or unchanged for a long period of time against the possibility of inflation rising too much, and sometimes, lowered to avoid the possibility of inflation falling too much. At this point the question is what the balance level should be. Hence, in order to provide an answer to this question, this thesis aims to estimate the neutral real interest rate for the Turkish economy. It would seem that this research is the first attempt to model neutral real interest rate in Turkey, apart from the simple statistical filter estimates. With this thesis, we hope to contribute to the better understanding of the monetary policy (and thereby inflation dynamics) conducted in Turkey as a recent inflation-targeting country and provide some insight to the policymakers in their analysis and decision-making process. We think that taking account of the information content of

[2] On the other hand, there are some arguments that the idea dates back to 70 years earlier to the economists Thornton and Joplin.

neutral real interest rate, or alternatively, interest rate gap[3] could probably help the policymakers in the formulation of monetary policy.

Turkey implemented implicit inflation-targeting regime after being hit by the severe 2001 crisis. This new policy regime in addition to increased central bank independence and enhanced price stability objective has changed the monetary policy setting of the CBRT. The new policy framework has put the "inflation target" at the core and let the central bank control its short-term interest rate to restrain the inflation. Accordingly, the importance of the quantification of unobserved variables (to know the current state of the economy) such as the neutral real interest rate and the output gap, which are crucial for both the near-term and long-term inflation forecasts, has increased with this "inflation forecast targeting" regime. To this end, the most important motivation behind estimating the neutral real interest rate for the Turkish economy in this study is the absence of a certain measure for neutral real interest rate or real interest rate gap that can be used within this framework.

The question of how the neutral real interest rate became so popular in recent years is related to growing monetary policy rule literature, in which the pioneer work of Taylor (1993) is influential. Taylor proposed a simple interest rate rule, where neutral rate is used as the intercept term and so the success of the rule has increased the interest to the estimation of the neutral interest rate (Wintr et al., 2005). The work of Woodford (2003)[4] is remarkably influential in the recent resurgence of the concept. After the introduction of the Kalman filter into the estimation of the neutral rate by Laubach and Williams (2003), number of studies for estimating the neutral rate of interest has been increased noticeably.

Although the neutral real interest rate is a practical guide to policymakers, it cannot be directly observed and thus has to be estimated. The concept of the neutral real interest rate is useful from a theoretical perspective, but it is problematic from a practical point of view due to uncertainty related with the

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[3] Unless otherwise stated, interest rate refers to the real interest rate.

[4] Indeed, many of the studies refer to various older manuscript versions of the Woodford.

measurement issue. Unobservable nature of the variable makes it model specific, which in turn, is subject to significant revisions as time passes. Thus, there is adequate uncertainty about determining the true level of the neutral rate at a point in time (they may not be reliable enough to aid the policymakers). Nevertheless, as stated by Björksten and Karagedikli (2003), since the policy-makers usually deal with a great deal of uncertainty in determining the current state of the economy, sensible judgment based on experience, plus a sense that monetary policy is either broadly stimulatory or contractionary may be enough[5], unless the neutral real interest rate is exploited for modeling purposes[6]. Furthermore, the growing literature on estimating the neutral rate indicates that, despite their measurement uncertainty, estimates are still useful for ex-post monetary policy evaluations.

Multivariate unobserved components model has preferred against its alternatives in this thesis, in view of the fact that it provides a compromise between univariate statistical approaches and theoretically well-defined dynamic stochastic general equilibrium (DSGE) models, as highlighted by Larsen and McKeown (2003). One main advantage of this model is that it combines the time series techniques with the economic theory; hence have the ability to provide economic interpretation for both unobserved variables and model parameters. Multivariate unobserved component models are mostly tractable and allow various structural specifications. The most importantly, since they can give an explanation for large changes in structural variables, they may possibly perform better in emerging market economies that are exposed to large shocks and/or many structural changes like Turkey. As a result, unobserved components model (thereby Kalman filter) emerges as the appropriate estimation technique for the estimation of the neutral rate due to its advantages. It should also be noted that

[5] In this respect, it is more common to define a given interest rate level as being “broadly” rather than “exactly” neutral (Archibald and Hunter, 2001).

[6] Modeling requires the point estimate of the neutral rate, so a great deal of caution is needed given the high uncertainty surrounding the estimates. Regarding the consequences of mismeasurement of the neutral rate on output and inflation stabilization see Laubach and Williams (2003) and Orphanides and Williams (2002).

some of the alternative techniques, such as that of Bomfim (2001), could not be applied to Turkish case due to the lack of data[7].

This thesis presents two different multivariate unobserved components models to estimate jointly the neutral real interest rate or the real interest rate gap and the output gap using the recursive algorithm Kalman filter. Both models are built consistent with the output gap literature; however they differ in terms of the specification of the neutral real interest rate. While the first specification uses a random walk model for the neutral real interest rate, second one employs more structural specification, which specifically links the neutral real rate with the trend growth rate and the risk premium and principally relies on the study of Laubach and Williams (2003).

Model results indicate similar output gap estimates, but they differ in terms of neutral real interest rate estimates. The economic relevance of the estimated figures generally seems to be satisfactory and estimates confirm that using of variables in the form of deviation from their equilibrium undoubtedly has stronger explanatory power for the Turkish economy dynamics. Especially, deviation of the actual real interest rate from neutral rate appears to be very meaningful in explaining the output gap dynamics. Model comparisons evidently recommend the use of more structural specification against random walk specification. Lastly, it can be said that the general shortcoming of high level of uncertainty surrounding the estimates of neutral real interest rate also applies to our study.

The thesis is organized as follows. Chapter 2 reviews the concept of the neutral real interest rate and explains the determinants of the neutral real interest rate in order to better understand the model dynamics. Chapter 3 covers brief information and literature review on the alternative estimation techniques of the neutral real interest rate. Chapter 4 summarizes the methodology used throughout the thesis, namely theory of discrete-time Kalman filter, state space form and the

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[7] For the details regarding the alternative estimation techniques, see Chapter 3.

prediction error decomposition. Chapter 5 introduces the empirical models developed with a special focus on the historical evolution of the neutral real interest rate models and explains the dynamics of models developed. Chapter 6 presents the estimation results. Chapter 7 discusses model comparisons and empirical findings through drawing some conclusions about the monetary policy stance and dynamics of the Turkish economy. Finally, Chapter 8 derives the main conclusions.

CHAPTER 2

THE CONCEPT OF NEUTRAL REAL INTEREST RATE

In the literature, various expressions have been used for the neutral real interest rate including natural real interest rate, neutral rate of interest, and equilibrium real interest rate depending on the definition or the time horizon to which the concept refers. On the other hand, as mentioned by Bernhardsen[8] (2005), it is possible to deliberate at least two definitions; namely medium-run concept of the neutral real interest rate and the long-run equilibrium real interest rate.

Medium-run concept of the neutral real interest rate is defined as real interest rate consistent with stable inflation and the real GDP at its potential level over a horizon relevant to monetary policy decisions[9]. This definition of the neutral rate does not respond to temporary shocks, but instead evolves over time in response to structural factors. In this respect, in this thesis the term neutral real interest rate is used to refer to the medium-run, which is more relevant for the period of setting a monetary policy. The equilibrium real interest rate concept, on

[8] For a more detailed scrutiny about the theory of the neutral real interest rate, see Woodford (2003), Bernhardsen (2005), and Archibald and Hunter (2001) from which this chapter emerges. For the role of neutral real interest rate in the conduct of monetary policy, one can refer to Amato (2005).

[9] This definition is widely cited in the literature, see Laubach and Williams (2003), Cuaresma et al. (2003), Basdevant el at. (2004), Bernhardsen (2005), Wintr et al. (2005).

the other hand, is a more abstract[10] concept and applies to very long-run. Archibald and Hunter (2001) describe this concept as "all markets are in equilibrium and there is therefore no pressure for any resources to be redistributed or the growth rates for any variables to change". Thus, they explain this period as a period that is sufficiently long enough to enable all markets to be in equilibrium meaning that the real interest rate is equal to the neutral rate. For a more complete study of the long-run equilibrium real interest rate, readers can refer to Bernhardsen (2005).

According to Bernhardsen (2005), the time horizon over which these two concepts are related is an important difference between the neutral real interest rate and the equilibrium (long-run) real interest rate. This means that (in the medium-run) the real interest rate may be neutral with output gap equal to zero and stable inflation but the economy may not be settled at its long-run values, that is, growth rate of the economy may depart from its long-run level. In such a situation, though some of the factors may not be compatible with the long-run, they might not have any significant impact on output gap and inflation, so the monetary policy over the medium-run might not be affected significantly. From this point of view, the neutral real interest rate is the concept that is more relevant -compared to long-run counterpart- for monetary policy analysis. In the following paragraphs, we will briefly explain determinants of the neutral real interest rate.

In order to understand the determinants of the neutral real interest rate, it is a good way to follow the decomposition of the short-term nominal interest rate. Table 1 illustrates the decomposition of the nominal interest rate into its components according to Archibald and Hunter (2001). As it is known, the nominal interest rate that we observe in our daily life is the sum of real interest rate and the inflation[11]. Real interest rate, then again, is the sum of neutral real interest rate (r*) and cyclical factors that stand for real interest rate gap. Real

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[10] Though its abstractness, most of the economists find this concept very helpful to understand the variations in the neutral rate of interest over the medium-run.

[11] So, the real interest rate is the one adjusted by the amount with which prices change over the time rate refers to. For ex-post measure of the real interest rate, one should simply replace the expected inflation with the observed inflation.

interest rate gap is related to business cycle and it gives us an idea about the effort of the monetary authority in fighting with the inflationary pressures over the business cycle -in cyclical up and downs.

Table 1: Decomposition of the Short-term Nominal Interest Rate

<table>
<tr><td colspan="9">Nominal interest rate (i)</td></tr>
<tr><td colspan="7">Ex-ante real interest rate (r)</td><td>+</td><td>Expected Inflation (π^e)</td></tr>
<tr><td colspan="5">Neutral real interest rate (r*)</td><td>+ / -</td><td>Cyclical factors (Monetary policy leaning against inflationary pressure)</td><td colspan="2"></td></tr>
<tr><td>Fundamentals affecting saving and investment decisions, hence the (risk-free) long-run equilibrium real interest rate</td><td>+ / -</td><td>Impediments to international capital flows</td><td>+</td><td>Country specific risk premia (rp)</td><td colspan="4"></td></tr>
</table>

Source: Archibald and Hunter (2001), page 20.

The neutral real interest rate is obtained by adding country specific risk premium and impediments to capital flow to the risk-free long-run equilibrium real interest rate. Hence, the neutral real interest rate changes to the extent that these factors (if exist) change or are influenced. Since our concept of the neutral real interest rate is a function of the long-run equilibrium real rate, the determinants of the long-run equilibrium real rate are also the determinants of the neutral rate. Subsequently, it can be thought that the neutral rate fluctuates around the equilibrium rate and in the absence of shocks such as risk premium, neutral rate moves towards the long-run equilibrium rate.

The real interest rate can be thought of the price which equates saving (supply of loanable funds) and investment (demand for loanable funds) and the long-run equilibrium real interest rate -in a risk-free world - is affected by the fundamentals affecting saving and investment decisions (Archibald and Hunter, 2001). Therefore, all of the structural factors that tend to have an effect on saving and investment will also change the neutral real interest rate. All factors that decrease (increase) investment, or increase (decrease) saving will result in a lower (higher) neutral real interest rate. Such structural factors include the time preference of consumers (when to consume), shocks that affect households' saving decisions, population growth and factors related to productivity of capital (technological improvements, level of the capital stock etc.)[12]. Another factor that may have an influence on the change of the neutral rate is the impediments to international capital flows. Archibald and Hunter (2001) argues that in most cases, there are some impediments such as capital controls, taxes to the flow of capital across countries and these impediments might be one source of cross-country differences in neutral real interest rate. Final component that constitutes the neutral real interest rate is the country specific risk premium. Due to uncertainty associated with the ex-post real return at the time of investment, investors demand some premium according to their perception of risk. For this reason, countries that are assessed to be riskier should pay more to catch the attention of investors. This risk premium is governed by the countries' domestic fundamentals depending on wide range of considerations. Fiscal policy, liquidity risk, exchange rate considerations, concerns on debt sustainability and perception of political uncertainty are just some of them.

Briefly, all economic factors that influence saving and investment decisions and the risk premium attributable to domestic fundamentals of the economy are the main factors affecting the neutral real interest rate. Note that

[12] Global structural factors may also influence domestic neutral rate of the interest. For example, if the global neutral real interest rate increases due to advances in the technology, then domestic neutral rate might respond the global one and have a tendency to increase. Since high global real interest rates attract the global liquidity and create a tendency for the domestic currency to depreciate ensuing an inflationary pressure, so this possibly will end in a situation where domestic interest rates tend to increase.

most of these structural factors affecting the neutral rate are also not easy to measure, which indeed constitutes one source of the uncertainty associated with the estimation of the neutral rate. Furthermore, it is worth emphasizing that all these structural factors are expected to evolve gradually through time meaning that the neutral rate evolves gradually throughout time as well (Archibald and Hunter, 2001).

CHAPTER 3

LITERATURE REVIEW ON THE ESTIMATION OF NEUTRAL REAL INTEREST RATE

The neutral real interest rate has received growing attention in recent years. When we look at the empirical literature on the estimation of neutral real interest rate, it is clearly seen that the central bankers do much of the current research. Number of studies on monetary policy rules has increased considerably after the success of the Taylor (1993) rule, where neutral rate is used as the intercept term. Woodford (2003) allowed the neutral rate (in the Taylor rule) to evolve throughout the time in response to various shocks and played a central role in the recent resurgence of the concept. To the extent that such rules are used by central banks with the aim of stabilizing inflation and output, the estimation of the neutral rate has gained importance. This section concentrates on this growing literature associated with the estimation of the neutral real interest rate with the focus on the estimation methodology[13]. Giammarioli and Valla (2004) present a comprehensive discussion of the alternative estimation methods and argue that there is not a single consensual concept behind the neutral rate, and therefore, the comparison of alternative measures can be insightful. They classify the alternative

[13] For a comprehensive discussion of the alternative estimation methods, see Giammarioli and Valla (2004).

measures of the neutral real interest into three main categories, namely, empirical measures based on time-series, measures based on the yield curve and asset pricing models and measures derived from DSGE models.

The simplest approach to estimate the neutral real interest is to make use of simple historical average of the real interest rate over a long period of time if it is believed that the neutral real interest rate is constant over time. The logic behind this undemanding approach is that over a long period of time positive and negative shocks affecting the neutral rate would offset each other, so that their net effect is neutral. However, if the neutral rate has been changing through time in response to factors affecting demand and supply, the long-term averaging will not be a good indicator of the neutral real interest rate[14]. When inflation and output are fairly stable, such approach could provide information about the neutral rate (Williams, 2003). However, we expect that estimates based on this approach do not work well in practice for developing countries like Turkey due to deviation of the real interest rate from its neutral rate for long periods as a result of unstable inflation and the unsustainable growth path that the Turkish economy followed in the last decade[15].

The neutral real interest rate also might be estimated by applying univariate filters such as linear trending or Hodrick-Prescott (1997) filter[16]. These

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[14] An alternative way of such simple averaging is to allow more changes in the neutral real interest rate by giving more weight to recent data compared to past data.

[15] For the last couple of years where the Turkish economy followed stabilization programs and gained significant improvement in macroeconomic conditions, such a simple averaging results in implausibly high neutral rate estimates due to the dominance of the previous regimes.

[16] There are some shortcomings regarding the use of Hodrick-Prescott (HP) filter in the literature. Specifically, one of the serious drawbacks is the poor performance of the filter at the end periods. Since the HP filter is a two-sided optimization procedure, which uses both lead and lagged information, the accuracy of the filter diminishes at the end of sample due to missing lead information. In fact, this criticism is also applicable to other filters like the univariate Kalman filter. However, the use of more structural models (more economic information) and forecasted values for the end-sample observations reduces this poor performance of the filters. Moreover, the cyclical component always sums up to zero in HP filter, which needs not necessarily hold, especially, when the sample size is small. This caveat can particularly be important when the HP filter is used to estimate the output gap, since the output gap might be more negative or positive. The criticisms also focus on the view that, the values for smoothing parameter suggested by Hodrick and Prescott are specific to the US data and may lead to misspecification of the underlying economic structure of other economies. However, as shown by Harvey and Jaeger (1993) and King and Rebelo (1993), HP filter can also be obtained in the unobserved components

kinds of filters are completely statistical approaches (solely reliance on the actual data), and consequently, suffer from the shortcoming that they disregard the information from macroeconomic relationships such as inflation, output and the real interest rates, so no structural interpretation of the estimates like the simple averaging are probable.

Another method to derive the neutral real interest rate is to utilize the information in the market data that is contained in the yield curve. Bomfim (2001) estimates the equilibrium real rate using data from the yield curve for inflation-indexed bonds for the US. This approach has the shortcoming that estimates may be distorted by the estimates of term and liquidity premiums and the assumption of the constancy of expected inflations. Moreover, such an approach requires the issue of inflation-indexed bonds; however the market for inflation-indexed bonds simply does not exist for our economy.

Christensen (2002) proposes a non-model based approach where the real interest rate gap is approximated by a simple transformation of the difference between two nominal interest rates, namely the central bank's interest rate and the interest rate on long-term government bonds. Though this approach is simple and intuitive, it makes the assumptions of stable inflation and mean-reversion of the term premium during the studied time period. This yield curve based approach is employed by Basdevant et al. (2004) along with the Kalman filter approach. In this study, they clearly state the shortcomings associated with the above approach to assess monetary policy stance and adopt a slightly more structural approach within a multivariate HP filter context to determine the neutral real interest rate[17].

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framework. Furthermore, the smoothing parameter, which determines the smoothness of the estimated trend component, can be estimated by prediction error decomposition via the Kalman filter (Boone et al., 2002).

[17] Multivariate HP filter is an alternative way of estimating unobserved variables. This method extends the standard HP filter by using relevant economic information (see Laxton and Tetlow (1992) for more information). In their paper, Basdevant et.al (2004) reproduce the multivariate HP filter with Kalman filter to use the further advantages of Kalman filter such as the estimation of hyper-parameters and reduction of the steps in the estimation procedure .

Another aspect of the literature that grows in the estimation of the neutral rate deals with the DSGE models[18]. On this view, the neutral interest rate could be defined as the real interest rate that would prevail in an economy where prices are fully flexible (Giammarioli and Valla, 2004). These kinds of theoretical models enable somewhat complicated structures and have structural interpretation; yet construction of such models necessitates quite modeling experience, and is not a straightforward task. DSGE models are mostly applicable to stable or developed economies, but not the emerging market economies like Turkey, which is subject to intensive structural changes.

The complication of large structural models has encouraged the development of small-scale macroeconomic models that are easier to apply compared to DSGE models, but more structured than simple statistical approaches. The reference paper for this sort of method is Laubach and Williams (2003), where the authors apply the Kalman filter to a simple small-scale macroeconomic model to jointly estimate the neutral real interest rate, potential output and its growth rate. In this model, in addition to existent output gap model literature, authors relate the neutral real interest rate to the potential growth rate of the economy in line with the economic theory. Though this method has less structural interpretation, it is more tractable and for this reason, more commonly applied. Recent examples for the application of this method include Clark and Kozicki (2004) for the US, Cuaresma et al. (2003), Mesonnier and Renne (2004), and Garnier and Wilhelmsen (2005) for Euro area, Wintr et al. (2005) for Euro area and Luxembourg, and Larsen and McKeown (2003) for the UK.

In practice, multivariate unobserved component models[19] are widely preferred to estimate the neutral real interest rate due to their distinctive advantages. The general form of the system permits for imposing various economic structures and encompasses a wide range of possibilities from simple

[18] Rotemberg and Woodford (1997), Neiss and Nelson (2003), Smets and Wouters (2002), Giammarioli and Valla (2003) are among the well-known examples that considered DSGE models to estimate the neutral rate.

[19] State space models are analyzed using recursive algorithm known as Kalman filter.

statistical filters such as the famous HP filter to multivariate semi-structural macroeconomic models[20]. They have the ability to provide economic interpretation for both unobserved variables and model parameters. They are mostly tractable and robust to particular structural assumptions. Moreover, since they enable large changes in unobserved variables, they can manage and reasonably account for the large shocks and many structural changes (Mesonnier and Renne, 2004). In this respect, these type of models may possibly perform better to estimate the "true" level of the neutral rate in emerging economies that are exposed to large shocks and/or many structural changes like Turkey, in which the neutral rate may likely to be more often changing through time.

[20] As regards the use of unobserved components models, Larsen and McKeown (2003) state that "by pursuing an approach that entails less structure than a dynamic general equilibrium model, we, loosely speaking, lose the ability to provide a structural interpretation of the data, but gain a better fitting explanation."

CHAPTER 4

METHODOLOGY

This thesis aims to use unobserved components model for the purpose of estimating the neutral real interest rate in an emerging market economy. The unobserved components model is a method to estimate the unobserved variables such as neutral real interest rate, potential output and potential output growth rate using the information from observed variables. Once the model is formulated and specified in the state space form and the initial values are given for the unobserved states, the unobserved components can be estimated by an algorithm known as Kalman filter[21]. Kalman filter is a recursive procedure, which computes the estimate of the unobserved state vector at time point t, given available information about observed variables at time t. Hence, state space form and the Kalman filter allow us to make statistical inferences about unobserved variables in the time series context.

Kalman filter uses the initial values for the unobserved state vector in order to predict the unobserved variables, and then, updates the predictions based on the prediction errors. When all of the observations have been processed, the

[21] The name of the filter comes from its inventor, Rudolf E. Kalman. The Kalman filter was extended in several papers by Swerling (1958), Kalman (1960) and Kalman and Bucy (1961). Various other filters have been formulated in the course of time after Kalman's original formulation. These filters include Schmidt's extended filter, the information filter and a range of square-root filters developed by Bierman, Thornton and many others (Wikipedia contributors, 2006).

smoothing equations give the best estimates of the unobserved variables based on all of the information.

4.1. State Space Form

A state space form is a mathematical model of a dynamic system as a set of observed and state variables linked by first-order difference equation. State space representation provides a convenient and compact way to model dynamic time series models that involve unobserved variables. The form is composed of two equations namely transition and measurement equations. The transition equation (4.1.1) describes the evolution of unobserved state variables and assumes that unobserved state variables evolve according to a first-order Markov process. Following Harvey (1990), consider the following discrete time-invariant linear representation[22]:

$$\alpha_t = T\alpha_{t-1} + Cw_t + R\eta_t, \text{ t = 1,…,T,} \tag{4.1.1}$$

where α_t is a m×1 vector of unobserved state variables, T is an m×m matrix, C is an m×k matrix, w_t is k×1 vector of exogenous (predetermined) variables, R is an m×g matrix and η_t is a g×1 vector of serially uncorrelated disturbances with mean zero and covariance matrix Q, that is to say, $E(\eta_t) = 0$ and $Var(\eta_t) = Q$. Here, m, k and g represent the number of unobserved states, exogenous variables, and disturbance terms correspondingly. Note also that the initial state vector α_0 has a mean of a_0 and a covariance matrix of P_0.

$$y_t = Z\alpha_t + Dw_t + \varepsilon_t, \text{ t = 1,…,T.} \tag{4.1.2}$$

The equation (4.1.2) is the measurement equation, which defines the evolution of N×1 observed variables y_t (N denotes the number of observed variables in y_t) as a function of the unobserved state vector. Here, Z is an N×m

[22] For a detailed description of the state space models and the Kalman filter, interested readers are suggested to refer to Anderson and Moore (1979), Harvey (1990) and Kim and Nelson (2000) from which this chapter mostly emerges.

matrix that relates the observed y_t vector and the unobserved state vector, D is an N×k matrix, and finally ε_t is an N×1 vector of serially uncorrelated disturbances with mean zero and covariance matrix H, namely $E(\varepsilon_t) = 0$ and $Var(\varepsilon_t) = H$. The system further assumes that the disturbances η_t and ε_t are uncorrelated with each other and uncorrelated with the initial state. On the other hand, when measurement and transition equations are correlated in a known form, then it is possible to modify the Kalman filter, accordingly[23].

The equations (4.1.1) and (4.1.2) together form the general state space representation. Note that the system matrices T, C, R, Q, Z, D and H might also change with each time step; however, in this thesis, they are assumed to be time-invariant. If the elements of these matrices are allowed to change in time together with unobserved states (both state and parameters are estimated simultaneously), the extended Kalman filter emerges as the appropriate estimation procedure. For a recent application of the extended Kalman filter to the Turkish economy, readers can refer to Sarıkaya et al. (2005).

Most of the statistical models, including autoregressive integrated moving-average processes, can be written in a state space form and the parameters of the model could be easily estimated via the prediction error decomposition. Note also that in the maximum likelihood estimation of the unknown system parameters, expectation maximization (EM) algorithm can also be used as an alternative method for maximizing the likelihood function[24].

4.2. Discrete-time Kalman Filter

The Kalman filter is a set of recursive mathematical equations that provides an estimate of the unobserved state α_t (based on the information set ψ_t) by means of minimizing the mean of the squared error. Hence, Kalman filter is a

[23] See section 3.2.4 in Harvey (1990) for further details.

[24] For details, see Digalakis et al. (1993).

minimum mean-square error estimator. The filter recursively updates a linear projection for the system. It provides two different estimates, namely filtered and smoothed, of the state variables based on the information set used in the filter. The filtered estimate at time t is also named as one-sided and uses the information available up to time t ($y_{t|t}$) to estimate the state α_t. One-sided estimate generally corresponds to a real-time estimate. On the other hand, a smoothed estimate is two-sided and uses all the available information in the sample up to time T ($y_{t|T}$ where $0 \leq t \leq T$), in other words, it also uses the future information to compute the state at time t. Therefore, it is worth to note that the filtered and smoothed values are identical for the last observation of the sample due to lack of future information.

The filter assumes that the system matrices (T, C, R, Q, Z, D and H), initial state and covariance matrices are known. When the disturbances and the initial state vector are normally distributed, the filter enables the estimation of unknown parameters (elements of the system matrices, sometimes named as hyper-parameters) in the model via the prediction error decomposition (Harvey, 1990). In this context, it opens a way to apply statistical testing of model parameters as well.

Since the filter is a recursive estimator, it needs both the estimated state from the previous time step and the current measurement to compute the estimate for the current state. To this end, the algorithm basically consists of two steps or sets of equations, namely time update (prediction) and measurement update (updating). Welch and Bishop (2004) define these equations in the following way: “the time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the *a priori* estimates for the next time step. The measurement update equations are responsible for the feedback- i.e. for incorporating a new measurement into the *a priori* estimate to obtain an improved *a posteriori* estimate”. To be more explicit, the general form of the time and measurement update equations for a time-invariant model is as follows:

Time Update

$$a_{t|t-1} = Ta_{t-1} + Cw_t \text{ (Predicted state)} \quad (4.2.1)$$

$$P_{t|t-1} = TP_{t-1}T' + RQR' \text{ (Predicted estimate covariance)} \quad (4.2.2)$$

Measurement Update

$$v_t = y_t - \tilde{y}_{t|t-1} = y_t - Za_{t|t-1} - Dw_t \text{ (Prediction error)} \quad (4.2.3)$$

$$F_t = ZP_{t|t-1}Z' + H \text{ (Prediction error covariance)} \quad (4.2.4)$$

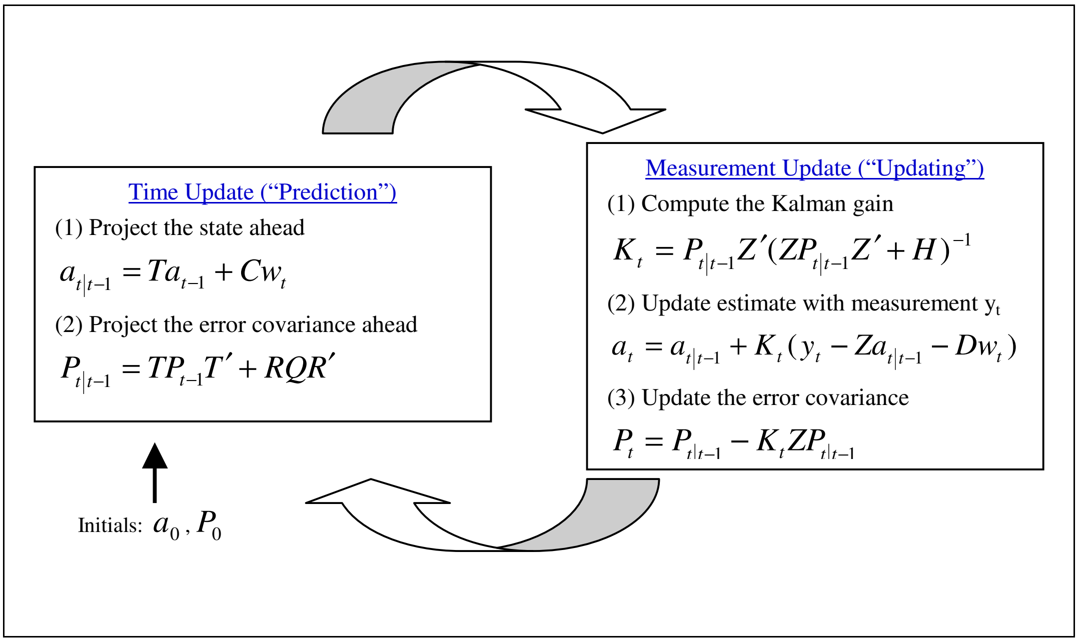
$$K_t = P_{t|t-1}Z'F_t^{-1} \text{ (Kalman gain)} \quad (4.2.5)$$

$$a_t = a_{t|t-1} + K_t v_t \text{ (Updated state estimate)} \quad (4.2.6)$$

$$P_t = P_{t|t-1} - K_t ZP_{t|t-1} \text{ (Updated state covariance),} \quad (4.2.7)$$

where $a_{t|t-1}$ denotes the predicted state that is the estimator of α_t conditional on the information up to t-1 (ψ_{t-1}), whereas $P_{t|t-1}$ stands for the covariance matrix of α_t conditional on the information up to t-1. In the time update phase, we make a prediction of y_t, that is $\tilde{y}_{t|t-1}$, based on the information set ψ_{t-1}. When a new observation y_t is realized, then the prediction error (4.2.3), which includes new information about α_t further than contained in $\alpha_{t|t-1}$, is calculated. With the equation (4.2.6), a more accurate inference can be made about the state vector α_t and in this case, Kalman gain (K_t) determines the weight assigned to new information about α_t contained in the prediction error (Kim and Nelson, 2000).

Given the initial values a_0 and P_0, filter operates through t=1,…,T and filtered estimates can be obtained (See Figure 1). The resultant estimate of α_t, namely a_t, is the minimum mean squared error estimate based on the information used up to t-1 or t.



Source: Welch and Bishop (2004)

Figure 1: Kalman Filter Recursive Algorithm–Time and Measurement Updates

4.3. Derivation of the Kalman filter

This section is devoted to the derivation of the Kalman filter equations. In the Kalman's original seminal paper (1960), filter is derived based on the orthogonality properties associated with linear minimum variance estimation and orthogonal projection arguments are used to provide evidence for the optimality of the state estimates. Kalman also recognized that when the disturbances and initial state vector are normally distributed, the quantities computed by his recursive equations are mean and covariance of a conditioned normal random variable, and accordingly describe recursively a conditional probability density (Anderson and Moore, 1979). Thus, it is seen that, in the normality situation the filter produces mean and covariance of the unknown state and encloses all information contained in the conditional probability density.

Numerous approaches have been formulated in the course of time to derive the algorithm of the Kalman filter ranging from the least square to Bayesian point of view. Chui and Chen (1991) demonstrate the derivation of linear, unbiased, minimum variance least-square estimate a_t of α_t using the information from y_t. They also illustrate the optimality in the sense of choosing the optimal weight matrix that gives a minimum variance estimate. Duncan and Horn (1972) formulated the recursive estimation from the regression point of view, whereas Meinhold and Singpurwalla (1983) present the Bayesian derivation and interpretation of the filter. Readers may also be referred to Özbek (2000) for the different derivations of the filter.

Following derivation of the filter is taken from Harvey (1990) and depends on the assumption that the disturbances η_t, ε_t and initial state vector α_0 are normally distributed. In this way, a standard result on multivariate normal distribution is used to derive the equations for a discrete time-invariant linear filter. To this end, we go over the assumptions presented in section 4.1 and add a further assumption of multivariate normality:

(1) Initial state vector has a multivariate normal distribution, specifically $\alpha_0 \sim MVN(a_0, P_0)$.

(2) The disturbance terms also have a multivariate normal distribution, that is $\eta_t \sim MVN(0, Q)$ and $\varepsilon_t \sim MVN(0, H)$.

(3) The disturbances η_t and ε_t are uncorrelated with each other and uncorrelated with the initial state.

Based on (4.1.1), for t=1, the state vector is written as:

$$\alpha_1 = T\alpha_0 + Cw_1 + R\eta_1 . \tag{4.3.1}$$

Note that α_1 is a linear combination of two normal random variables (assumptions (1) and (2)) and thereby distributed as multivariate normal, that is $\alpha_1 \sim MVN(a_{1|0}, P_{1|0})$ where $a_{1|0}$ denotes the mean of the distribution of α_1 conditional on the information at time t=0.

$$a_{1|0} = Ta_0 + Cw_1, \tag{4.3.2}$$

and

$$P_{1|0} = TP_0T' + RQR'. \tag{4.3.3}$$

Note that (4.3.2) and (4.3.3) are simply the time update equations. Hence, it can be said that the time update equations are just the distribution of α_t conditional on the information ψ_{t-1}. When a new observation y_1 is realized, this new information is used in the measurement update part, thus the measurement update equations are in simple terms of the conditional distribution of α_1 on y_1. Before we proceed further to derive this conditional distribution, the following rearrangements on α_1 and (4.1.2) are made:

$$\alpha_1 = \alpha_1 + a_{1|0} - a_{1|0} = a_{1|0} + (\alpha_1 - a_{1|0}), \tag{4.3.4}$$

$$y_1 = Z\alpha_1 + Dw_1 + \varepsilon_1 + Za_{1|0} - Za_{1|0},$$

$$= Za_{1|0} + Dw_1 + Z(\alpha_1 - a_{1|0}) + \varepsilon_1. \tag{4.3.5}$$

From (4.3.4) and (4.3.5), it can be seen that

$$\begin{bmatrix} \alpha_1 \\ y_1 \end{bmatrix} \sim MVN\left(\begin{pmatrix} a_{1|0} \\ Za_{1|0} + Dw_1 \end{pmatrix}, \begin{pmatrix} P_{1|0} & P_{1|0}Z' \\ ZP_{1|0} & ZP_{1|0}Z' + H \end{pmatrix}\right).$$

To write the distribution of α_1 conditional on a particular value of y_1, the following well-known lemma on multivariate normal distribution is exploited.

Lemma 4.3.1: Let the pair of vectors x_1 and x_2 be jointly multivariate normal and mean and covariance matrix are partitioned as follows:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$$

then the distribution of x_1 conditional on x_2 is multivariate normal $x_1 \mid x_2 \sim MVN(\mu_{1|2}, \Sigma_{11|2})$ where

$$\mu_{1|2} = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2), \quad (4.3.6)$$

$$\Sigma_{11|2} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}. \quad (4.3.7)$$

By using lemma 4.3.1, we can write the distribution of α_1 conditional on y_1 using the information ψ_t, measurement update equations, that is $\alpha_1 \mid y_1 \sim MVN(a_1, P_1)$ with

$$a_1 = a_{1|0} + P_{1|0}Z'(ZP_{1|0}Z' + H)^{-1}(y_1 - Za_{1|0} - Dw_1), \quad (4.3.8)$$

$$P_1 = P_{1|0} - P_{1|0}Z'(ZP_{1|0}Z' + H)^{-1}ZP_{1|0}, \quad (4.3.9)$$

where the term $ZP_{1|0}Z' + H$ is the prediction error covariance. Repeating the step for t=2,…T produces the Kalman filter equations given in Figure 1.

Since the above derivation depends on the assumption of normality, when the disturbances (η_t, ε_t) are not normally distributed, there is no longer any guarantee that the Kalman filter will yield the conditional mean of the state vector. Nevertheless, among the class of all linear estimators, the Kalman filter produces the smallest minimum mean square error (MSE) whether or not the disturbance and the initial state vector are normally distributed. So that a_t is an optimal estimator in the sense of minimizing MSE (Harvey, 1990). Proof of the best linear estimator property of the Kalman filter can be found in Anderson and Moore (1979, p.46-49). Briefly, as stated by Harvey (1990), a_t is *the minimum mean square linear estimator* of the unobserved state vector based on observations up to and including time t, it is unconditionally unbiased and unconditional error covariance matrix is P_t as defined in (4.2.7).

4.4. MLE and Prediction Error Decomposition

When observations are independent and identically distributed, it is straightforward to derive the likelihood function. However, for dependent observations like in the time series models, it is not that simple and in these cases the product of conditional probability densities is used to obtain the joint density function[25]:

$$L(\theta \mid \tilde{y}_T) = \prod_{t=2}^{T} p(y_t \mid \tilde{y}_{t-1}, \theta) p(y_1 \mid \theta), \tag{4.4.1}$$

where $p(y_t \mid \tilde{y}_{t-1}, \theta)$ is the conditional density with $\tilde{y}_{t-1} = [y_1, y_2, ..., y_{t-1}]$, $p(y_1 \mid \theta)$ is the marginal density of y_1 and θ is the vector of parameters to be estimated. Under the normality assumption of disturbances, the vector of observations $\tilde{y}_T$ has a multivariate normal distribution with μ and variance-covariance matrix Σ (positive definite and symmetric) and has the following likelihood function:

$$L(\theta \mid \tilde{y}_T) = (2\pi)^{-T/2} |\Sigma|^{-1/2} \exp\left[-\frac{1}{2} (\tilde{y}_T - \mu)' \Sigma^{-1} (\tilde{y}_T - \mu) \right], \tag{4.4.2}$$

where mean and variance-covariance matrix depend on a set of the unknown hyper-parameters (parameters to be estimated). Harvey (1981) proposes the following triangular factorization of the variance-covariance matrix Σ to solve the possible difficulties regarding (4.4.2):

$$\Sigma = AFA' \tag{4.4.3}$$

with the diagonal matrix $F = \begin{bmatrix} F_1 & 0 & \cdots & 0 \\ 0 & F_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & F_T \end{bmatrix}$, $F_t>0$ for all t and lower triangular matrix $A = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ A_{21} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ A_{T1} & A_{T2} & \cdots & 1 \end{bmatrix}$.

Using this triangular factorization in the likelihood function (4.4.2) yields:

$$L(\theta \mid \tilde{y}_T) = (2\pi)^{-T/2} |F|^{-1/2} \exp\left[-\frac{1}{2} v' F^{-1} v \right], \tag{4.4.4}$$

[25] Comprehensive treatment of MLE and prediction error decomposition can be found in Harvey (1990) and Kim and Nelson (2000). This chapter mainly emerges from Kim and Nelson (2000).

$$L(\theta \mid \tilde{y}_T) = (2\pi)^{-T/2} \left[\prod_{t=1}^{T} F_t \right]^{-1/2} \exp\left[-\frac{1}{2} \sum_{t=1}^{T} v_t' F_t^{-1} v_t \right], \qquad (4.4.5)$$

where $v = A^{-1}(\tilde{y}_T - \mu)$ and it can be seen directly that t[th] element of v is the prediction error, that is $v_t = y_t - \tilde{y}_{t|t-1}$, t=1,…T. Note that (4.4.5) is the likelihood of the normal density function of y_t conditional on the information $\tilde{y}_{t-1}$, that is $y_t \mid \tilde{y}_{t-1} \sim N(Za_{t|t-1} + Dw_t, F_t)$. For a multivariate case, the log-likelihood function can be easily written as:

$$\log L = -\frac{NT}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \log|F_T| - \frac{1}{2} \sum_{t=1}^{T} v_t' F_t^{-1} v_t \ , \qquad (4.4.6)$$

where N is the dimension of y_t, and F_t is the variance of the prediction error, which is defined by (4.2.4). The above decomposition of the likelihood is also known as the prediction error decomposition form. If the normality of disturbances and the initial state is guaranteed, then the conditional distribution of y_t on $\tilde{y}_{t-1}$ is also normal, and the mean and covariance of this conditional distribution are provided directly by the Kalman filter (Harvey, 1990). Hence, one can easily compute the log-likelihood and estimate the unknown parameters of the model.

4.5. Smoothing

Unobserved variables can be estimated in two different ways depending on what information is being used. As stated before, the filtered estimate uses the information available up to time t ($y_{t|t}$), yet we have more information available (information made available after time t, namely future information) to estimate the state at a given time t. To this end, the smoothed estimate uses all available information in the sample up to time T ($y_{t|T}$ where $0 \leq t \leq T$). It supplies a more accurate estimate of the state given that it uses more information than the filtering, and so, the smoother naturally provides a smaller MSE compared to the filtered estimator. Once the basic filtering equations run and filtered estimates are achieved, then the smoothed estimates are obtained in backward-iteration, using the filtered estimates at time T as the initial values for the smoothing process and proceeding from T to 1. Although there are different types of smoothing methods (fixed-point, fixed-lag and fixed-interval), the fixed-interval smoothing will be used in this thesis, in view of the fact that, it is the most widely used algorithm for the economic and social data (Harvey, 1990). The smoothed estimates can be achieved through the backward recursion for t=T-1,…,1:

$$a_{t|T} = a_t + P_t T' P_{t+1|t}^{-1} (a_{t+1|T} - T a_t) , \tag{4.5.1}$$

$$P_{t|T} = P_t + P_t T' P_{t+1|t}^{-1} (P_{t+1|T} - P_{t+1|t}) P_{t+1|t}^{-1 \, \prime} T P_t' , \tag{4.5.2}$$

with $a_{T|T} = a_T$ and $P_{T|T} = P_T$. Readers can refer to Anderson and Moore (1979) for derivation of the above equations and the comprehensive treatment of the smoothing issue.

CHAPTER 5

MODELS USED IN THE STUDY

This chapter presents the general form of the models that are used throughout the thesis: specifically, two different multivariate unobserved components models to estimate jointly the neutral real interest rate and the output gap are introduced. Both models are built consistent with the output gap literature; decompose actual output into potential output that follows a random walk with a time-varying potential growth rate and a stationary output gap. However, models differ in terms of the specification of the behavior of the neutral real interest rate. While the first specification uses a random walk model for the neutral real interest rate based on Orphanides and Williams (2002), Larsen and McKeown (2003) and Kozicki (2004), the second one employs more structural specification, which links the neutral real rate with the trend growth rate and the risk premium and principally relies on the study of Laubach and Williams (2003).

Initially, the data used in the estimations is clarified. In the second part of the chapter, fundamentals of the models used in the thesis are discussed and lastly, state space representations of the models are presented.

5.1 Data

This section gives the description of data used in this study. Data exploited in the estimations are quarterly ranging from the first quarter of 1988 to last quarter of the year 2005. Logarithmic seasonally adjusted real GDP at 1987 constant prices is used as output data (Figure 2). In the seasonal adjustment, we utilized commonly used program TRAMO/SEATS (Gomez and Maravall, 1998), which is developed by the Bank of Spain and promoted by Eurostat[26].

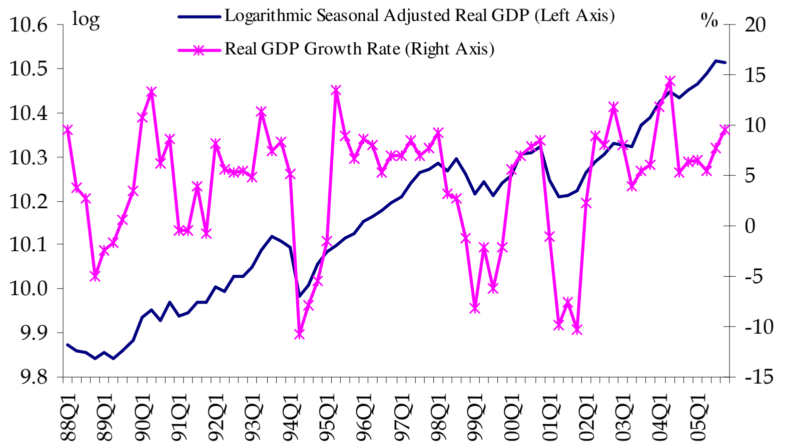


Figure 2: Seasonal Adjusted Real GDP and Quarterly Real GDP Growth Rate

In the calculation of the real interest rates, actual consumer inflation is employed. In this respect, our definition of the real interest rate is the ex-post real interest rate on government securities. Though the ex-ante real interest rate, calculated by using the inflation expectations, is preferable in the estimations, the absence of the data for inflation expectations forces us to employ the ex-post real interest rates. However, it is thought that our ex-post real interest rate measure should be comparable to any ex-ante measure due to dominance of the backward-

[26] In the applications, we used DEMETRA package program developed by Eurostat. It includes the TRAMO/SEATS method in addition to the X-12 ARIMA. Interested readers can download the program from the web page of Eurostat.

looking nature in the inflation expectations of Turkish economy. In computations, we make use of the compound interest rates for discounted Treasury auctions, which is weighted by the share of respective amounts sold, as the nominal interest rate. Figure 3 abridges the general juncture for consumer inflation, real and nominal interest rates for 1988-2005 periods.

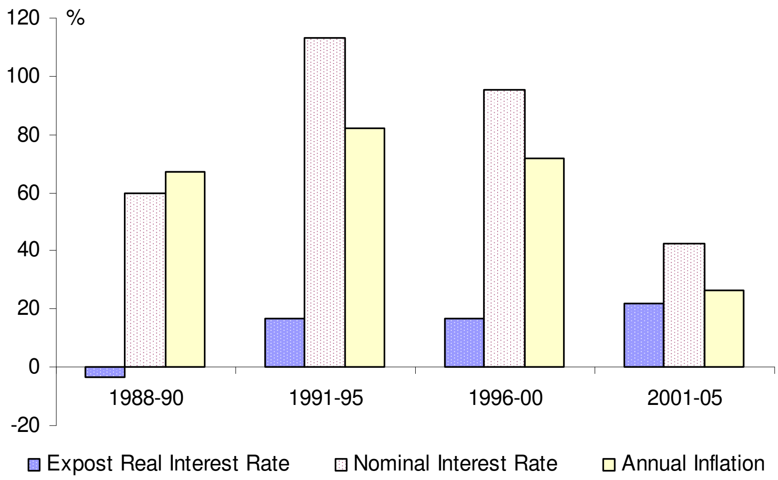


Figure 3: Real and Nominal Interest Rates and Inflation (Average Percentage Change)

In model equations, inflation rate is defined as the quarterly percentage change of seasonally adjusted consumer price index (CPI) with different base years. Our real effective exchange rate (REER) definition is the CPI-based real effective exchange rate with the base year of 1995[27]. An increase in the REER denotes a real appreciation of the Turkish Lira, whereas a decrease denotes a real depreciation. For the risk premium variable, we use the average borrowing maturity (months) in Treasury auctions, which is calculated by the Turkish

[27] The index uses the IMF weights for 19 countries, namely Germany, USA, Italy, France, the United Kingdom, Japan, Netherlands, Belgium, Switzerland, Austria, Spain, Canada, Korea, Sweden, Taiwan, Iran, Brazil, China and Greece.

Treasury. For output and the inflation figures, main data source is the CBRT Electronic Data Delivery System, the source for the nominal interest rate is the Turkish Treasury and lastly the real effective exchange rate data are obtained from the calculations of the Central Bank of the Republic of Turkey. Note also that in this thesis, the symbol Δ is used to denote the percentage change of the related variable. The following table describes the variables and the notation used in the thesis for the sake of simplicity.

Table 2: Notation Used and the Variable Descriptions

π_t	Quarterly percentage change of seasonally adjusted CPI
y_t	Logarithmic seasonally real gross domestic product at 1987 constant prices
$\tilde{y}_t$	Output gap (unobserved), $y_t - y_t^*$
$\Delta reer_t$	Quarterly percentage change of CPI-based real effective exchange rate with the base year of 1995
r_t	Quarterly average of real interest rate, which is calculated by compound interest rates for discounted Treasury auctions, weighted by the share of respective amounts sold
y_t^*	Potential output (unobserved)
r_t^*	Neutral real interest rate (unobserved)
$\tilde{r}_t$	Real interest rate gap (unobserved)
g_t	Potential output growth rate (unobserved)
rp_t	Quarterly average borrowing maturity (months) in Treasury auctions
$I9402$	Indicator dummy variable for second quarter of 1994 currency crisis

5.2 Model Specifications

In this section, the general forms of the models utilized in the thesis are presented. Before stating the model specifications, it is thought that a brief historical background information on the output gap literature can be valuable in order to better comprehend the model dynamics.

Early specification of the trend component of the output was a linear trend, which is based on a strong assumption that the supply side of the economy is deterministic and economic fluctuations usually depend on the changes in the demand-side. Later, Nelson and Plosser (1982) suggested that the nonstationarity in economic activity should be removed by first differencing, which means the trend component is a random walk with drift rather than a straight line. It was followed up by Watson's (1986) specification, which characterized potential output and output gap as random walk with drift and AR (2), respectively. However, in all of these settings, economy's trend growth rate was assumed to be constant and there was no reason for these components to be constant over time, especially when an economy is experiencing considerable structural change. Hence, Clark (1987) and afterward Kuttner (1999) constructed a variable growth rate model given the decline of the US productivity growth in the 1970s, reduction of labor force growth in the 1980s and the apparent increase in the trend growth in mid-1990s.

Kuttner (1994) enhanced the model by including the output gap and inflation relationship, namely the Phillips curve to the system. This specification brought in more structural interpretation to the model, and with this study, the strengths of the statistical models were integrated with the structural ones[28]. Gordon (1997), Laubach (2001) and Fabiani and Mestre (2004) used a similar specification, but replaced the output gap with the unemployment gap, thereby estimated the non-accelerating inflation rate of unemployment (NAIRU)[29]. Laubach and Williams (2003) contributed to the literature by incorporating the neutral real interest rate into this model and estimating the neutral real interest rate and output gap simultaneously. This specification partly follows the lines of

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[28] Following the Kuttner's (1994) specification, Öğünç and Ece (2004) estimated the output gap for the Turkish economy. Afterwards, Sarıkaya et al. (2005) employed a non-linear time series framework along with extended Kalman filter to estimate the output gap, in which the parameters of the system are allowed to be time varying. In the first study, a simple first order autoregressive model was used to model output gap dynamics, whereas Sarıkaya et al. proposed more structural equation for the output gap dynamics.

[29] In terms of output, the NAIRU corresponds to potential output, the highest level of real GDP that can be sustained at any time.

Rudebusch and Svensson (1998), where the Phillips curve is combined with a reduced form of IS curve in a small-scale backward-looking macroeconomic model[30]. In this respect, this study generalized the above model by means of linking the neutral real interest rate to the existing abounding output gap literature. Following the study of Laubach and Williams, the number of studies estimating the neutral rate has increased considerably.

In the light of the literature, in this study, we propose two alternative models, namely Model 1 and Model 2, for the Turkish economy case. Models have a neo-Keynesian motivation, uses deviation of the variables from their equilibrium levels, and describes the behavior of inflation and the output gap simultaneously through IS and Phillips curves, as expressed by Garnier and Wilhelmsen (2005). The proposed models acquire different characters on the basis of the neutral real interest rate dynamics. The first model is composed of the following equations:

Model 1:

$$\pi_t = b_0 + b_1\pi_{t-1} + b_2\pi_{t-2} + b_3\tilde{y}_{t-1} + b_4\Delta reer_t + b_5 I9402 + \varepsilon_t^{\pi} \quad (5.2.1)$$

$$\sigma_{\pi} \equiv st.dev(\varepsilon_t^{\pi})$$

$$y_t = y_t^* + \tilde{y}_t \quad (5.2.2)$$

$$r_t = r_t^* + \tilde{r}_t \quad (5.2.3)$$

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*}, \sigma_{y^*} \equiv st.dev(\varepsilon_t^{y^*}) \quad (5.2.4)$$

$$g_t = \rho g_{t-1} + (1-\rho) g_0 + \varepsilon_t^{g}, \sigma_g \equiv st.dev(\varepsilon_t^{g}) \quad (5.2.5)$$

$$r_t^* = r_{t-1}^* + \varepsilon_t^{r^*} \quad (5.2.6)$$

$$\tilde{r}_t = \tau_1\tilde{r}_{t-1} + \tau_2 I9402 + \varepsilon_t^{\tilde{r}}, \sigma_{\tilde{r}} \equiv st.dev(\varepsilon_t^{\tilde{r}}) \quad (5.2.7)$$

[30] A similar model is used by Gerlach and Smets (1999) as well.

$$\tilde{y}_t = a_1 \tilde{y}_{t-1} + a_2 \tilde{r}_t + a_3 \Delta reer_t + \varepsilon_t^{\tilde{y}}, \sigma_{\tilde{y}} \equiv st.dev(\varepsilon_t^{\tilde{y}}). \quad (5.2.8)$$

There are five shocks in the model: ε_t^{π} is the shock to the inflation (supply side shock), $\varepsilon_t^{y^*}$ is the level shock, ε_t^{g} is the growth rate shock, $\varepsilon_t^{\tilde{r}}$ is the real interest rate gap shock, and $\varepsilon_t^{\tilde{y}}$ is the demand shock; the standard deviations of the shocks are σ_{π}, σ_{y^*}, σ_g, $\sigma_{\tilde{r}}$ and $\sigma_{\tilde{y}}$ respectively.

First three equations above are the measurement equations, defining the relationship between observed variables (output, inflation and real interest rate) and unobserved state variables (potential output, potential output growth, output gap, neutral real interest rate and real interest rate gap), whereas the equations through (5.2.4) to (5.2.8) are the transition equations.

Equation (5.2.1) can be read as an aggregate supply equation, or a standard reduced form backward-looking Phillips curve. Inflation is explained by its own lags, representing the inertia in the inflation, lagged output gap, the change in the real effective exchange rate and a dummy variable for the 1994 currency crisis in Turkey. Two lags of inflation indicate that though the recent developments; backward-looking behavior dominates the price expectations of the agents due to past high inflationary environment. Note also that effect of real exchange rate on inflation is immediate[31] and the influence of excess demand is captured through the first lag of the output gap. As stated by Sarıkaya et al. (2005), the real effective exchange rate has an effect on the inflation dynamics both through the cost of production channel and through the prices of imported final goods.

The second equation (5.2.2) is an identity, demonstrating that the actual output is the sum of potential or trend output and the output gap. Equation (5.2.3) is also an identity, defining real interest rate as the sum of the neutral real interest rate (trend component) and the real interest rate gap (gap component).

[31] Regarding the effect of exchange rate on inflation dynamics, see Kara et al. (2005) and Kara and Öğünç (2005).

As shown in the equation (5.2.4) and (5.2.5), in parallel with the studies of Nelson and Plosser (1982) and Kuttner (1999), potential output follows a random walk with drift model and trend growth rate can be shaped with respect to different values of ρ parameter, representing the persistence in trend growth rate. In the model, ρ is calibrated to 0.8[32] based on the finding of Sarıkaya et al. (2005) and in so doing a serial correlation is assumed in the trend growth rate. Furthermore, g_0 parameter in the model, the sustainable steady-state real growth rate[33] for our economy, is calibrated to 5 percent on annual basis, which corresponds to 1.23 percent per quarter.

In equation (5.2.6), it is assumed that neutral real interest rate r_t^* follows a random walk model based on the specifications of Orphanides and Williams (2002) and Kozicki (2004) for the US and Larsen and McKeown (2003) for the UK. As mentioned before, in the first model, we plainly do not impose any theoretical priors on the movements of the neutral real interest rate, but rather we model it as a statistical process. In equation (5.2.7), real interest rate gap is assumed to be evolving according to an autoregressive process.

One of the fundamental equations in our modeling is the equation (5.2.8). It is a reduced form aggregate demand equation, or the IS equation, and relates the dynamic of the output gap to lagged output gap, difference between the real interest rate and its neutral level that is real interest rate gap and the change in the real effective exchange rate. Since Turkey is a small open economy, the real effective exchange rate developments are one of the determinants of the aggregate demand. The role of the exchange rate on output gap is clearly explained in Sarıkaya et al. (2005). The striking point of our aggregate demand equation compared to earlier studies is the use of real interest rate gap, rather than the real interest rate itself. Indeed, this is one of the important questions that this thesis

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[32] It means that, in the absence of shocks, output growth would converge to the within one percent of the steady-state rate in just about 5 years.

[33] Generally, it is thought that the potential output corresponds to the ideal equilibrium position for all output variables; this particular position corresponds to the so-called “steady state”. Here, this steady-state value is calibrated by using expert judgment.

aims to enlighten; to be precise whether the deviation of the real interest rate from its neutral level improves our understanding (or explanation) of the aggregate demand dynamics. According to our specification, in the case under which there are no demand and exchange rate shocks, and the real interest rate is at its neutral level, the output gap turns out to be close to zero.

The second model applies the model suggested by Laubach and Williams (2003). Some recent studies employed models, which are similar to Laubach and Williams' include Clark and Kozicki (2004) for the US, Mesonnier and Renne (2004), and Garnier and Wilhelmsen (2005) for Euro area, Wintr et al. (2005) for Euro area and Luxembourg. As implied by the economic theory, Laubach and Williams model the neutral real interest rate as function of potential (trend) output growth rate (g_t) and a random factor (z_t), which captures other determinants of the neutral rate such as population growth, households' rate of time preference, technological progress. Their model further assumes that z_t follows an autoregressive process. Specifically,

$$r_t^* = cg_t + z_t , \tag{5.2.9}$$

and

$$z_t = D_z(L)z_{t-1} + \varepsilon_t^z . \tag{5.2.10}$$

where, $D_z(L)$ is the lag operator. However, our model partly deviates from the Laubach and Williams' model in terms of the potential output growth and the neutral real interest rate specification. First of all, in our model potential output growth follows a stationary autoregressive process instead of the random walk specification, however it is thought to be persistent, hence ρ is calibrated to 0.8, as stated above. Secondly and the most importantly, as expressed by Björksten and Karagedikli (2003), the approach applied by Laubach and Williams is better suited for a large and relatively closed economy (such as the US) than for a small open one like Turkey. Laubach and Williams (2003) found that the variation in the trend growth rate is an important determinant of the neutral real interest rate in the United States. But, this finding may not hold for emerging market economies, which are subject to intense structural changes and severe shocks; moreover, these economies have high and volatile real interest rates compared to developed economies. Thus, there might be some other factors affecting the neutral real interest rate notably along with trend growth rate. Among those factors, long term expectations regarding fiscal soundness, a reliable monetary policy framework, robustness to exchange rate shocks or sustainable external debt emerges as the relevant ones for the emerging market economies. Contrary to close economy case, each of these factors has also an influence on the accessibility of the small-open economies to the international funds; thus, they affect the neutral interest rates through the amount of the funds available for the domestic use. Hence, it is important to note that, the neutral real interest rate specification of the Laubach and Williams needs to be modified in order to take small open economy consideration into account, by comprising the dynamics mentioned above, accordingly.

Along with this, Laubach and Williams models the neutral real interest rate (an unobserved variable) as linear functions of two unobserved variables (g_t and z_t), which creates an extra uncertainty in the estimation of the neutral rate. This problem has been drawn attention by Mesonnier and Renne (2004). This is actually one of the caveats related with interpreting the neutral real interest rate estimates. It can be clearly observed from the literature that estimation of the neutral interest rate contains more ambiguity compared to estimation of the output gap. This is mainly due to the fact that some of structural factors, like household time preferences, technological progress affecting the neutral rate are also not directly observed or not easy to measure, whereas the relationship between the output gap and its determinants can be clearly formulated. But, it is considered that a more structural specification for the neutral real interest rate by means of including the country specific dynamics, instead of just using two unobserved variables, could improve the estimates and as a result might reduce the measurement uncertainty to some extent. In this respect, the model suggested by Laubach and Williams should be modified so as to include some structural factors, which are directly observed, to be able to reduce the uncertainty in the modeling of the neutral interest rates.

Taking these two different but related considerations into account, the behavior of the risk premium emerges as a candidate for the basic determinant of the neutral interest rate, especially in emerging markets experiencing economic transformation process. Besides the direct effect, the risk perceptions of the economic agents regarding the future course of the economy affect their saving and investment decisions through expectations channel and hence have an influence on the behavior of the neutral interest rates. In addition, the course of the risk premium reflects the economy's ability to access international sources, hence, affects the supply of funds available for domestic use and thereby neutral interest rates.

At this point, it would be useful to analyze the determinants of the risk premium in detail since by putting the short-run fluctuations aside, the long-run behavior of the risk premium is thought to be almost the single most important determinant for the neutral interest rate of Turkey.

The risk premium has basically three components: the default risk, the liquidity risk and the inflation risk premium. The Eurobonds issued by the Turkish government in the international markets are almost equally liquid compared to their foreign counterparts; hence, the effect of liquidity premium is negligible in the current analysis. As to inflation risk, although declining in the last four years, the chronically high and volatile nature of the inflation in Turkey has led to higher inflation risk premium, which in turn, caused relatively higher neutral interest rate. However, it should be emphasized that inflation risk has declined along with the decisive implementation of the macroeconomic policies, where the price stability is the primary objective of the monetary policy in the last four years.

Default risk, the most important determinant of the risk premium, is basically stemming from the risk factors related to the structural deficiencies of the countries such as concerns of the economic agents regarding debt and current account sustainability, the continuity of the prudent economic policies and ongoing structural reforms that reinforce the structural transformation process of the economy towards developed economies. For example, in an environment where the domestic public debt is high and the average maturity is short, concerns on debt sustainability increase the default risk and hence risk premium significantly. However, it should be emphasized that Turkish economy has been experiencing structural transformation in the last four years, during which default risk has decreased considerably reflecting the fact that economic policies implemented since 2001 crisis is considered more and more credible. As the economic agents become more and more confident for the future, especially as regards to debt sustainability and continuity of the economic policies decisively implemented, the default risk, hence the risk premium decreases.

As it is mentioned before, the long-term course of the risk premium is considered to be one of the most basic determinants of neutral interest rate. In this respect, the following model is proposed for the neutral real interest dynamics in the second model:

$$r_t^* = 4c_1 g_t + c_2 rp_t^* + z_t \,, \qquad (5.2.11)$$

and

$$z_t = \phi z_{t-1} + \varepsilon_t^z \,, \qquad (5.2.12)$$

where rp_t^* denotes the trend component of the risk premium measure[34]. Note that, growth rate in the neutral rate specification is scaled by four to annualize the growth rate. In Laubach and Williams (2003), two different specifications, namely a stationary AR process and a random walk process, are considered for z in the estimations. However, it is argued that the probable components composing z like households' rate of time preferences are supposed to be stationary, thereby it is

[34] Trend component of the risk premium measure is obtained from the HP filter. Two quarters of forecasted values are exploited, so that the influence of the end-sample uncertainty is reduced. Rather than using the smoothing parameter values suggested by Hodrick and Prescott, filter is formulated in the unobserved components framework, and the variances are estimated by prediction error decomposition via the Kalman filter. See appendix for the state space representation of the HP filter.

assumed that z follows a stationary AR process like in the studies of Mesonnier and Renne (2004) and Garnier and Wilhelmsen (2005).

Our second model is the same as our Model 1, excluding the neutral real interest rate specification. Explicitly, Model 2 exploits equations (5.2.11) and (5.2.12) for r_t^*, instead of equation (5.2.6) in Model 1 and can be represented as follows:

Model 2:

$$\pi_t = b_0 + b_1\pi_{t-1} + b_2\pi_{t-2} + b_3\tilde{y}_{t-1} + b_4\Delta reer_t + b_5 I9402 + \varepsilon_t^{\pi}\ ,\ \sigma_{\pi} \equiv st.dev(\varepsilon_t^{\pi})$$

$$y_t = y_t^* + \tilde{y}_t$$

$$r_t = r_t^* + \tilde{r}_t$$

$$y_t^* = y_{t-1}^* + g_{t-1} + \varepsilon_t^{y^*}\ ,\sigma_{y^*} \equiv st.dev(\varepsilon_t^{y^*})$$

$$g_t = \rho g_{t-1} + (1-\rho) g_0 + \varepsilon_t^{g}\ ,\sigma_g \equiv st.dev(\varepsilon_t^{g})$$

$$r_t^* = 4c_1 g_t + c_2 rp_t^* + z_t$$

$$z_t = \phi z_{t-1} + \varepsilon_t^{z}\ ,\ \sigma_z \equiv st.dev(\varepsilon_t^{z})$$

$$\tilde{r}_t = \tau_1\tilde{r}_{t-1} + \tau_2 I9402 + \varepsilon_t^{\tilde{r}}\ ,\sigma_{\tilde{r}} \equiv st.dev(\varepsilon_t^{\tilde{r}})$$

$$\tilde{y}_t = a_1\tilde{y}_{t-1} + a_2\tilde{r}_t + a_3\Delta reer_t + \varepsilon_t^{\tilde{y}}\ ,\sigma_{\tilde{y}} \equiv st.dev(\varepsilon_t^{\tilde{y}})\,. \qquad (5.2.13)$$

5.3 State Space Representations of the Models

In order to estimate unobserved state variables by Kalman filter, the models should be written in the state space representation (form). State space representation is a matrix-based formulation of the dynamic model. All dynamic models can be put into their corresponding state space representation and there is generally more than one way of formulating a dynamic system in state space representation. In this context, it is straightforward to represent Model 1 as in equations (4.1.1) and (4.1.2):

$$\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ g_t \\ r_t^* \\ \tilde{r}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \tau_1 \end{bmatrix} \begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ g_{t-1} \\ r_{t-1}^* \\ \tilde{r}_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\rho)g_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tau_2 \end{bmatrix} \begin{bmatrix} 1_{T\times 1} \\ y_{t-1} \\ \Delta reer_t \\ \pi_{t-1} \\ \pi_{t-2} \\ I9402 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{y^*} \\ 0 \\ \varepsilon_t^{g} \\ \varepsilon_t^{r^*} \\ \varepsilon_t^{\tilde{r}} \end{bmatrix}$$

$$\begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = \begin{bmatrix} 1 & -a_1 & 0 & 0 & a_2 \\ 0 & -b_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_t^* \\ y_{t-1}^* \\ g_t \\ r_t^* \\ \tilde{r}_t \end{bmatrix} + \begin{bmatrix} 0 & a_1 & a_3 & 0 & 0 & 0 \\ b_0 & b_3 & b_4 & b_1 & b_2 & b_5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1_{T\times 1} \\ y_{t-1} \\ \Delta reer_t \\ \pi_{t-1} \\ \pi_{t-2} \\ I9402 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ 0 \end{bmatrix} \quad . \qquad (5.3.1)$$

The first and the second equations above are the measurement and the transition equations, respectively, for Model 1. Note that the system matrices T, C, Z, D, Q and H depend on a set of unknown parameters that need to estimated or calibrated. Likewise, the second model can be put into state space from as follows:

$$\begin{bmatrix} y_t^* \\ y_{t-1}^* \\ r_t^* \\ z_t \\ g_t \\ \tilde{r}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi & 4c_1\rho & 0 \\ 0 & 0 & 0 & \phi & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & 0 & 0 & \tau_1 \end{bmatrix} \begin{bmatrix} y_{t-1}^* \\ y_{t-2}^* \\ r_{t-1}^* \\ z_{t-1} \\ g_{t-1} \\ \tilde{r}_{t-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4c_1(1-\rho)g_0 & 0 & 0 & 0 & 0 & 0 & c_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\rho)g_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tau_2 & 0 \end{bmatrix} \begin{bmatrix} 1_{T\times 1} \\ y_{t-1} \\ \pi_{t-1} \\ \pi_{t-2} \\ \Delta reer_t \\ I9402 \\ rp_t^* \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 4c_1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^{y^*} \\ 0 \\ 0 \\ \varepsilon_t^{z} \\ \varepsilon_t^{g} \\ \varepsilon_t^{\tilde{r}} \end{bmatrix}$$

$$\begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = \begin{bmatrix} 1 & -a_1 & 0 & 0 & 0 & a_2 \\ 0 & -b_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_t^* \\ y_{t-1}^* \\ r_t^* \\ z_t \\ g_t \\ \tilde{r}_t \end{bmatrix} + \begin{bmatrix} 0 & a_1 & 0 & 0 & a_3 & 0 & 0 \\ b_0 & b_3 & b_1 & b_2 & b_4 & b_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1_{T\times 1} \\ y_{t-1} \\ \pi_{t-1} \\ \pi_{t-2} \\ \Delta reer_t \\ I9402 \\ rp_t^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\tilde{y}} \\ \varepsilon_t^{\pi} \\ 0 \end{bmatrix} \quad . \qquad (5.3.2)$$

CHAPTER 6

ESTIMATION RESULTS

This chapter of the study consists of two parts. The first part of chapter explains how we deal with the initial values for the state and its covariance matrix. The second part presents the estimation results together with the model diagnostics.

6.1 Setting of Initial Values

Estimation of the unobserved state vector requires the setting of initial values for both the state vector and its covariance matrix (a_0 and P_0) and the unknown system parameters. As explained in Kim and Nelson (2000), for a stationary state vector, the unconditional mean and covariance matrix of the state vector can be used as the initial values of a_0 and P_0. However for a non-stationary state vector, the unconditional mean and the covariance matrix of the state vector do not exist, hence a_0 may be determined based on the past knowledge or the economic intuition. Kim and Nelson (2000) asserts that if the uncertainty about this initial guess is large, one can assign sufficiently large values to the diagonal elements of P_0, which measures the confidence on the priors. If $P_{t|t-1}$ is a large positive definite matrix, then as seen in equations (4.2.5) and (4.2.6), the most of the weight in the state update equation is given to the new information contained in the forecast error for y_t and in this case the information content of $a_{t|t-1}$ decreases, since increased uncertainty in $a_{t|t-1}$ could intuitively be interpreted as a worsening of the information content of $a_{t|t-1}$ relative to that of the prediction error (Kim and Nelson, 2000). In light of this information, a common practice exploited in the literature is resorted in the estimations. Specifically, for the initial values of potential output and trend growth rate and their variances, the smoothed estimates from a previous study - Sarıkaya et al. (2005) - are employed. On the other hand, the HP filter estimate is used to get the prior estimate of the neutral real interest rate and initial condition for z_t variable is set to -0.2 based on other initial conditions through the equation (5.2.11). Furthermore, compared to y_t^* and g_t, relatively higher values for the variance of r_t^* and z_t are assumed in the initial covariance matrix P_0 in order to reflect our relative uncertainty about these two unobserved states.

The second step is to find the initial values for the parameters to be estimated. For this purpose, the proposed equations for the IS, Phillips curve, real interest rate gap and neutral real interest rate are estimated based on single-equation least square (LS) regressions by using proxy variables for the unobserved states and the parameters in the prediction error decomposition are initialized to these simple LS coefficients. The output gap estimate from the study of Sarıkaya et al. (2005) is used as a preliminary estimate for the output gap in the LS estimates, while HP filtered series are utilized to initialize the state vector of the neutral real interest rate and the real interest rate gap.

6.2 Empirical Models

Given the state space representation and the above initial values for the state vector and its covariance matrix, the unobserved state variables, their covariance matrices and the unknown system parameters are estimated through the Kalman filter.

The first model is estimated using data from the first quarter (q1) of 1988 to the last quarter (q4) of 2005; on the other hand the second model's sample period covers the data from 1989q2 to 2005q4 due to the lack of data for the average maturity prior to 1989. For the numerical optimization procedure BFGS provided by the GAUSS package program (Gauss 7.0 User Guide, 2005) is applied. In Table 3, estimation results for both models are reported. First and second columns are the results from the Kalman filter through the prediction error decomposition, whereas the third and fourth columns are estimated by ordinary least squares in order to make a simple comparison. In LS estimations, the estimates of the unobserved variables like output gap, real interest rate gap from Model 1 and 2 are used as explanatory variables to estimate the Phillips and the IS curves.

Before proceeding to the discussion of estimation results, the underlying assumptions of the models are tested in the following lines. Almost all statistical methods make assumptions about the model formulation and the shape of the population distribution. As it is mentioned in Chapter 4, one of the fundamental assumptions of the model is the normality of disturbances, which is essential for estimation of the unknown system parameters through the prediction error decomposition. An informal test of normality can be based on a normal probability plot (sometimes called as the Q-Q plot), which plots the data against the normal scores, or expected normal order statistics for a sample with the given number of observations. To this end, Ryan-Joiner test, which is similar to the Shapiro-Wilk test, is conducted and results are presented in Figures 4 and 5. The normal probability plots are fairly linear and the approximate p-values from the Ryan-Joiner test are greater than 0.1 indicating that the normality assumption is plausible for our models. In addition to normal probability and Ryan-Joiner test results, Table 4 summarizes the results of two further commonly used normality tests namely Jarque-Bera and the Anderson-Darling that support the normality finding of the Ryan-Joiner test at 0.05 significance level.

Table 3: Parameter Estimates for the Model 1 and 2

	Model 1	**Model 2**	**LS Estimates of Model 1**	**LS Estimates of Model 2**
Sample:	1988q1-2005q4	1989q2-2005q4	1989q4-2005q4	1989q4-2005q4
Phillips curve				
b_1	0.337 (0.000)	0.303 (0.004)	0.304 (0.000)	0.310 (0.000)
b_2	0.396 (0.000)	0.432 (0.000)	0.436 (0.000)	0.441 (0.000)
b_3	0.211 (0.052)	0.261 (0.115)	0.245 (0.007)	0.308 (0.005)
b_4	-0.129 (0.017)	-0.130 (0.013)	-0.137 (0.025)	-0.128 (0.037)
b_5	0.249 (0.000)	0.247 (0.000)	0.246 (0.000)	0.246 (0.000)
σ_{π}	0.030 (0.000)	0.029 (0.000)	0.029	0.029
IS curve				
a_1	0.758 (0.000)	0.723 (0.000)	0.735 (0.000)	0.694 (0.000)
a_2	-0.084 (0.001)	-0.082 (0.000)	-0.092 (0.000)	-0.085 (0.000)
a_3	0.120 (0.012)	0.120 (0.012)	0.115 (0.001)	0.118 (0.000)
$\sigma_{\tilde{y}}$	0.018 (0.000)	0.017 (0.000)	0.017	0.014
Neutral real interest rate equation				
c_1		0.151 (0.098)		
c_2		1.153 (0.030)		
ϕ		0.929 (0.000)		
Real interest rate gap equation				
τ_1	0.551 (0.000)	0.564 (0.000)		
τ_2	0.876 (0.000)	0.857 (0.000)		
$\sigma_{\tilde{r}}$	0.106 (0.000)	0.113 (0.000)		

Note: Results from the Model 1 (random walk specification for the neutral real interest rate) and the Model 2 (model similar to Laubach and Williams) are presented in columns 1 and 2, respectively. Associated p-values are given in brackets.

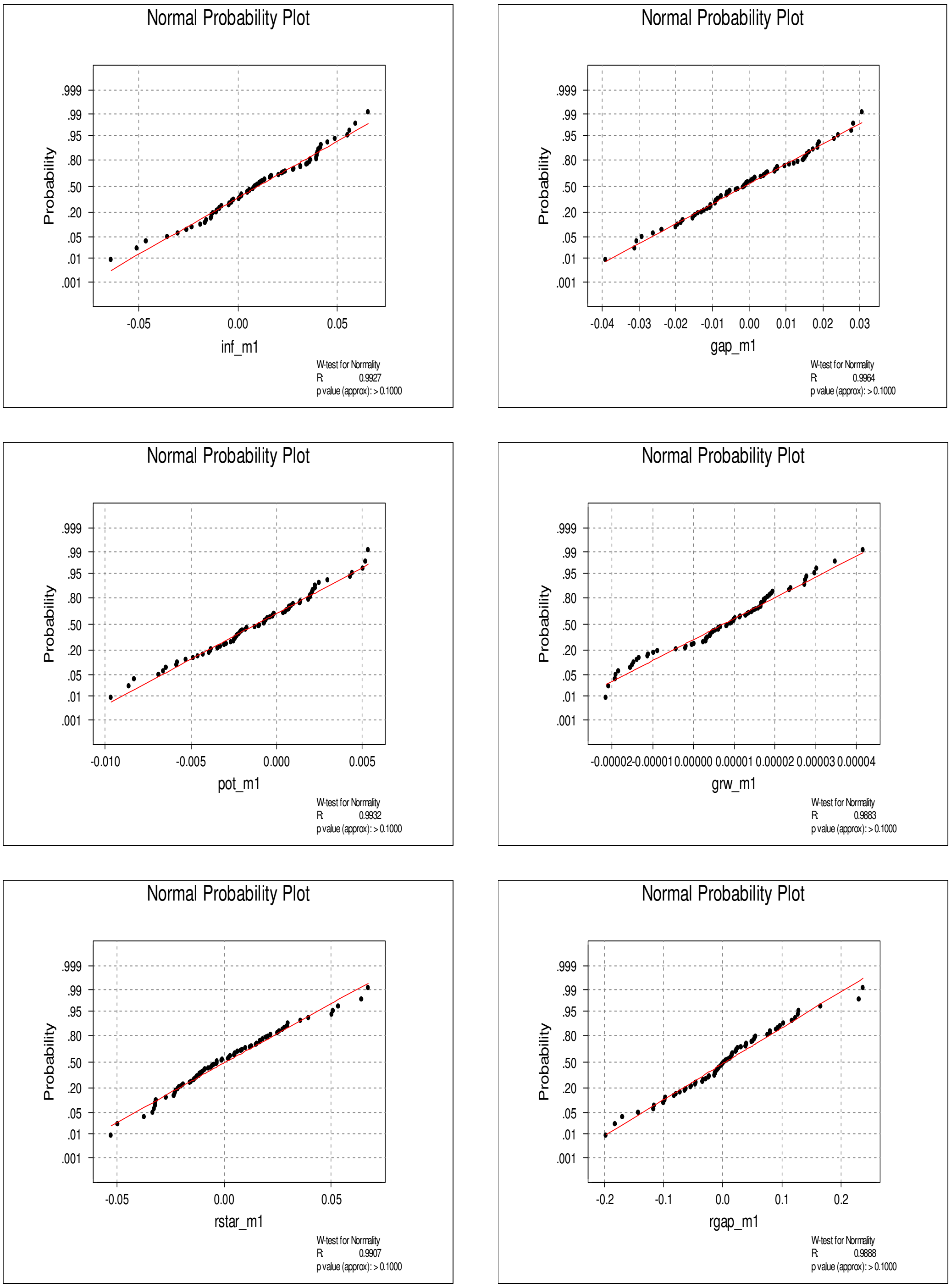

Figure 4: Normal Probability Plots for the Model 1

Note: The abbreviations inf, gap, pot, grw, rstar and rgap are used to symbolize the residuals from inflation, output gap, potential output, potential output growth, neutral real interest rate and real interest rate gap equations, respectively.

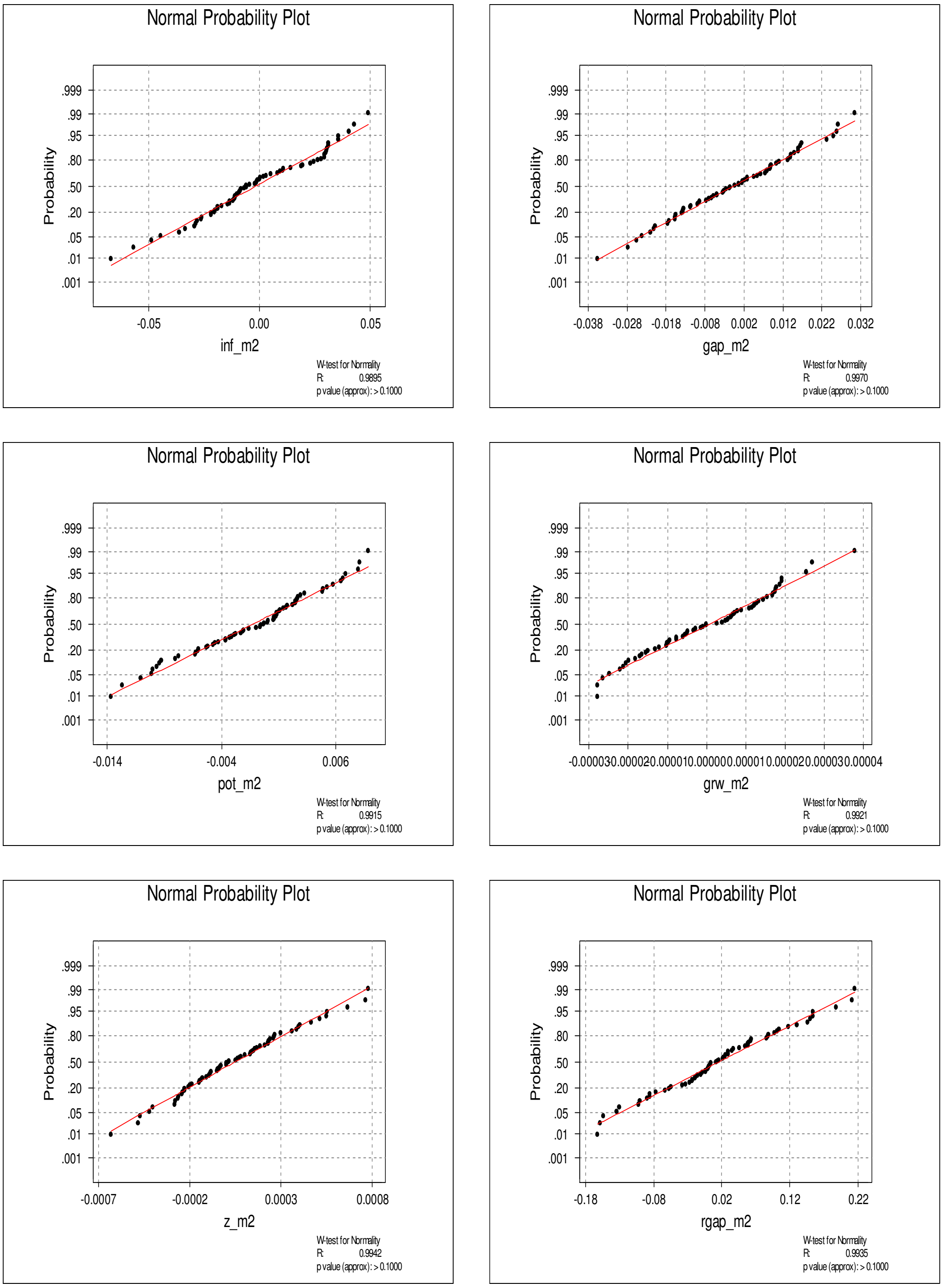

Figure 5: Normal Probability Plots for the Model 2

Note: The abbreviations inf, gap, pot, grw, z and rgap are used to symbolize the residuals from inflation, output gap, potential output, potential output growth, unobserved z variable and real interest rate gap equations, respectively.

Table 4: Jarque-Bera and the Anderson-Darling Test Results

	Jarque-Bera Statistics	p-value	Anderson-Darling (A2) Statistics	p-value
Model 1:				
e_t^{π} (inf_m1)	0.803	0.670	0.329	0.511
$e_t^{\tilde{y}}$ (gap_m1)	0.461	0.794	0.208	0.861
$e_t^{y^*}$ (pot_m1)	1.189	0.552	0.308	0.553
e_t^{g} (grw_m1)	0.966	0.617	0.683	0.071
$e_t^{r^*}$ (rstar_m1)	2.411	0.300	0.395	0.364
$e_t^{\tilde{r}}$ (rgap_m1)	1.723	0.423	0.536	0.164
Model 2:				
e_t^{π} (inf_m2)	0.380	0.827	0.596	0.117
$e_t^{\tilde{y}}$ (gap_m2)	0.762	0.683	0.164	0.940
$e_t^{y^*}$ (pot_m2)	1.634	0.442	0.450	0.269
e_t^{g} (grw_m2)	1.441	0.486	0.404	0.345
e_t^{z} (z_m2)	1.010	0.603	0.261	0.699
$e_t^{\tilde{r}}$ (rgap_m2)	0.197	0.906	0.352	0.459

Various specifications are tried to achieve the final models presented. Among these efforts, the models, which are not satisfying the model diagnostics and/or insignificant ones, are discarded. In the inflation and the real interest rate gap equations, an indicator dummy variable for the second quarter of 1994 is employed to represent the negative effects of 1994 currency crisis, since, the normality assumption is not satisfied otherwise. For the Phillips curve equation, various nominal exchange rate definitions, YTL based import price inflation, some oil price and cost measures (such as average unit cost expectations from the Business Tendency Survey of CBRT, energy prices) are tested as the explanatory variables. Exchange rate based measures are mostly significant; hence, the real effective exchange rate is chosen among the alternatives. But the remaining variables such as energy prices are found to be insignificant, hence not presented in the thesis. Moreover, different lag structures, such as inclusion of the second lag of output gap into the inflation equation, the first and second lags of real interest rate gap into the aggregate demand equation and an AR (2) specification for the real interest rate gap equation are tested but also found to be insignificant. Different representations that appeared in the literature, for instance the random walk specification for the potential output growth, are tested as well. But, the related mechanisms are far from being satisfactory compared to the models finally presented.

Another assumption of the system is that the disturbance terms of measurement and transition equations are uncorrelated. Indeed, as mentioned before, if they are correlated in a known form, then the filter equations can be modified to take into account that information. Table 5 displays the correlation matrices between measurement and transition equation residuals. Results indicate that uncorrelated measurement and transition disturbances assumption seems to be reasonable.

Table 5: Pair-wise Correlation Matrices

Model 1:

$$\begin{array}{c} \\ e_t^{y^*} \\ e_t^{g} \\ e_t^{r^*} \\ e_t^{\tilde{r}} \end{array} \begin{array}{c} \begin{array}{cc} e_t^{\pi} & e_t^{\tilde{y}} \end{array} \\ \begin{bmatrix} -0.048 & -0.246 \\ 0.048 & -0.132 \\ 0.079 & -0.282 \\ 0.085 & -0.205 \end{bmatrix} \end{array}$$

Model 2:

$$\begin{array}{c} \\ e_t^{y^*} \\ e_t^{g} \\ e_t^{z} \\ e_t^{\tilde{r}} \end{array} \begin{array}{c} \begin{array}{cc} e_t^{\pi} & e_t^{\tilde{y}} \end{array} \\ \begin{bmatrix} 0.044 & -0.231 \\ 0.021 & -0.264 \\ -0.038 & -0.092 \\ 0.089 & -0.066 \end{bmatrix} \end{array}$$

Lastly, to check the higher-order serial correlation among the residuals, Ljung-Box Q-statistics (Ljung and Box, 1979) are computed and the results are reported in Table 6. The Q-statistic at lag k is a test statistic for the null hypothesis that there is no autocorrelation up to order k and is calculated as:

$$Q = T(T+2)\sum_{i=1}^{k} \frac{\kappa_i^2}{T-i}, \qquad (6.2.3)$$

where κ_i is the i[th] autocorrelation. Under the null hypothesis, Q is asymptotically distributed as chi-square with degrees of freedom equal to the number of autocorrelations. If there is no serial correlation in the residuals, then Q-statistics should be insignificant with large p-values. In this respect, results indicate that there is no significant serial correlation in the residuals up to order five according to 0.05 significance level[35]. Though the p-values for residuals from the potential growth rate equation of first model are close to five percent at second and third lags, Q-statistics at lag 5 indicates that there is no autocorrelation up to order five.

Table 6: Ljung-Box Q-Statistics

Model 1:

	e_t^{π}		$e_t^{\tilde{y}}$		$e_t^{y^*}$		e_t^{g}		$e_t^{r^*}$		$e_t^{\tilde{r}}$	
	Q-Stat	p-value	Q-Stat	p-value	Q-Stat	p-value	Q-Stat	p-value	Q-Stat	p-value	Q-Stat	p-value
1	0.33	0.57	1.07	0.30	0.87	0.35	0.37	0.54	2.32	0.13	3.25	0.07
2	0.60	0.74	3.22	0.20	1.58	0.45	5.99	0.05	2.41	0.30	3.74	0.15
3	1.41	0.70	3.78	0.29	1.77	0.62	7.37	0.06	3.36	0.34	4.95	0.18
4	5.84	0.21	4.36	0.36	4.89	0.30	8.40	0.08	3.39	0.50	5.32	0.26
5	5.84	0.32	6.70	0.24	4.91	0.43	9.03	0.11	4.07	0.54	6.86	0.23

Model 2:

	e_t^{π}		$e_t^{\tilde{y}}$		$e_t^{y^*}$		e_t^{g}		e_t^{z}		$e_t^{\tilde{r}}$	
	Q-Stat	p-value	Q-Stat	p-value	Q-Stat	p-value	Q-Stat	p-value	Q-Stat	p-value	Q-Stat	p-value
1	1.75	0.19	0.15	0.70	0.36	0.55	1.40	0.24	2.57	0.11	2.70	0.10
2	1.78	0.41	2.23	0.33	0.80	0.67	4.09	0.13	2.96	0.23	3.89	0.14
3	2.45	0.48	2.55	0.47	0.84	0.84	4.13	0.25	6.88	0.08	5.03	0.17
4	6.38	0.17	2.66	0.62	4.30	0.37	5.22	0.27	6.93	0.14	5.53	0.24
5	6.38	0.27	4.41	0.49	5.05	0.41	5.69	0.34	6.94	0.23	8.03	0.15

[35] Choosing the order of lag to use for the test is a practical problem. Here, we follow the general practice and choose 5 lags, since our model is a quarterly model. But there is trade-off for this kind of tests; if a too small lag is chosen, the test may not detect serial correlation at high-order lags, whereas if too large lag is chosen, the test may have low power since the significant correlation at one lag may be diluted by insignificant correlations at other lags.

CHAPTER 7

MODEL COMPARISONS AND EVALUATION OF THE FINDINGS

In this chapter, estimation results are evaluated by comparing the two model results developed with respect to various performance criteria. Moreover, the empirical findings are discussed with special emphasize on the Turkish economy. In the first part of chapter, a short overview of the macroeconomic environment of the Turkish economy is presented. In the following parts of chapter, we compare the models from different perspectives such as evaluation of the estimated figures from economical point of view, uncertainty surrounding the estimates, significance and interpretation of the model parameter estimates, indicator and real-time properties of the model estimates.

7.1 Brief Overview of Macroeconomic Environment in Turkey

Before evaluating the estimation results, it would be useful to give brief historical background regarding the macroeconomic environment of the Turkish economy during the sample period. Actually, this is also important to distinguish the periods over which the behavior of output gap and neutral interest rate estimates changes dramatically.

The chronically high and volatile inflation environment, the unstable growth performance, the volatile nature of the short term capital inflows, the political instability, fast growing public debt stock and high real interest rates (especially in post-1994 period) were among the factors characterizing the macroeconomic environment of the Turkish economy until 2001. This environment of economic and political instability led to a series economic crisis in this period; namely, the currency crisis of 1994, contagion effect of both Asian and Russian crises in 1997 and 1998, finally the deepest crisis of the Turkish economic history in November 2000 and February 2001. These developments resulted in further increase in the real rates, which caused deepest problems in public debt sustainability, and hence, inflation. Therefore, monetary policy implementations of the CBRT focused on achieving stability in interest rates and exchange rates in order to cope with the deterioration in the stability of the financial markets and support sustainability of public debt. Last but not least aim of the CBRT in 1990s was to prevent sharp fluctuation in inflation by controlling the growth rate of its own balance sheet.

Due to the unstable nature of the money demand together with the basic premises of the economic literature saying the link between monetary aggregates, such as reserve money or monetary base and inflation is neither tight nor stable, monetary targeting regime was replaced with the exchange rate based stabilization program in 2000. Following the February 2001 crisis, with which the exchange rate base stabilization program ended, Turkey has adopted implicit inflation targeting monetary policy strategy where short-term interest rates are used as the main monetary policy instrument under floating exchange rate regime. The new monetary policy regime is different from its predecessors in the sense that more sound regulatory arrangements and structural reforms has put into practice together with the fact that the new regime has put strong emphasize on transparency and accountability in the macroeconomic policy implementations. Moreover, the control of CBRT on the short-term interest rate was strengthened.

With decisively implemented macroeconomic policies, the inflation targets have been met for four years in a row; resulting in single digit inflation rates today. Moreover, high and stable rates of economic growth have been achieved in the last four years. Deepening of financial markets has been experienced; in line with these, both nominal and real interest rates have declined considerably. The average maturity of the Treasury's domestic borrowing has increased together with the dramatic decline in the Turkish Eurobond spreads, which is used as proxy for the country specific risk premium.

With this very brief macroeconomic environment, it is obvious that Turkish economy has experienced a very intense structural transformation process in the last one and half decade. Therefore, while evaluating the findings of the models developed in this study, it is important to take the above-mentioned characteristics of the Turkish economy into consideration.

7.2 Evaluating the Output Gap and Neutral Real Interest Rate Estimates

This section analyzes the estimated output gap and neutral real interest rate (or real interest rate gap) by comparing the results across models. At first, we begin by focusing on the estimated output gap figures, which are portrayed in Figure 6. Results points out that estimate of the output gap from Model 1 is similar to one obtained from Model 2 to a great extend. In other words, the difference between the two estimates is not striking. This may be due to the fact that across the models, the underlying mechanisms are the same for both output gap and potential output dynamics. Hence, the estimated figures differ to the extent that neutral real interest rates (in other words the real interest rate gaps) behave differently.

Model 1 estimate of the output gap indicates a higher expansion and recession in the economic activity in 1998 and 2001-2002 correspondingly. But more or less, output gap estimates point out similar expansion and recession periods and are close to earlier studies such as Sarıkaya et al. (2005). Our estimated figures confirm the finding of Sarıkaya et al. (2005) that a) our economy has been exposed to rigorous shocks for several times during the sample period, b) period of expansion usually ends with economic crises, and c) three separate periods of recession are noted in the last decade. The first period is the

1994 currency crises, in which negative output gap reaches to 9 percent according to our estimates. After that, economy recovers and the period roughly between 1997 and 1998q4 emerges as an expansionary period. Then, with two major shocks, namely the Russian crisis at 1998q3 and the earthquake at 1999q3, Turkish economy encounters second recession phase. After the three quarters of positive output gap, Turkish economy has experienced the deepest recession of its history with collapse of the crawling peg regime at February 2001[36]. For this period, Model 1 and Model 2 estimates indicate a negative output gap of 9 and 11.5 percent, respectively. Estimates verify that 2001 economic crises have a long-lasting devastating effect on the economic activity compared to the 1994 crises. In addition, estimates point out that the gradual convergence of the actual output to its potential after 2001 economic crises has noticeably contributed to the disinflation process. Most recent estimates also point out that actual output is still below its potential level implying a limited degree of demand side pressure on the inflation. However, the negative gap seems to be closing, indicating the contribution of the output gap to disinflation process has been diminishing compared to the previous years.

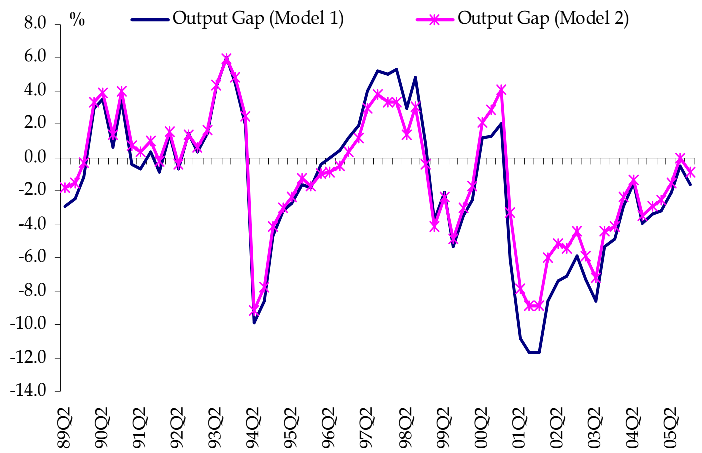

Figure 6: Smoothed (2-sided) Estimates of the Output Gap

[36] For a detailed discussion of historical evaluation of the output gap estimate, readers can refer to Sarıkaya et.al (2005).

The unobserved components method has the advantage that it allows the construction of confidence bands for the estimated unobserved variables. The estimates of the error covariance matrix of the state variables can be obtained by Kalman recursion. Figure 7 and 8 demonstrate the two-sided output gap estimates with 90 percent confidence bands[37].

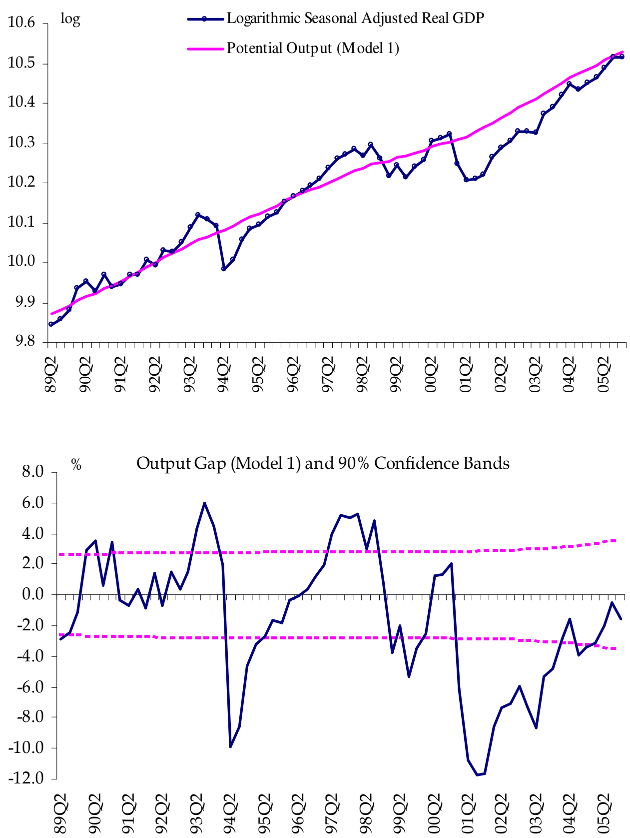

Figure 7: Smoothed Estimates of the Potential and Output Gap (Model 1)

[37] Since expected value of the output gap in the long-run is theoretically zero, the error bands to specify the recession and expansion dates are calculated based on the following formula; 0±1.645*SE.

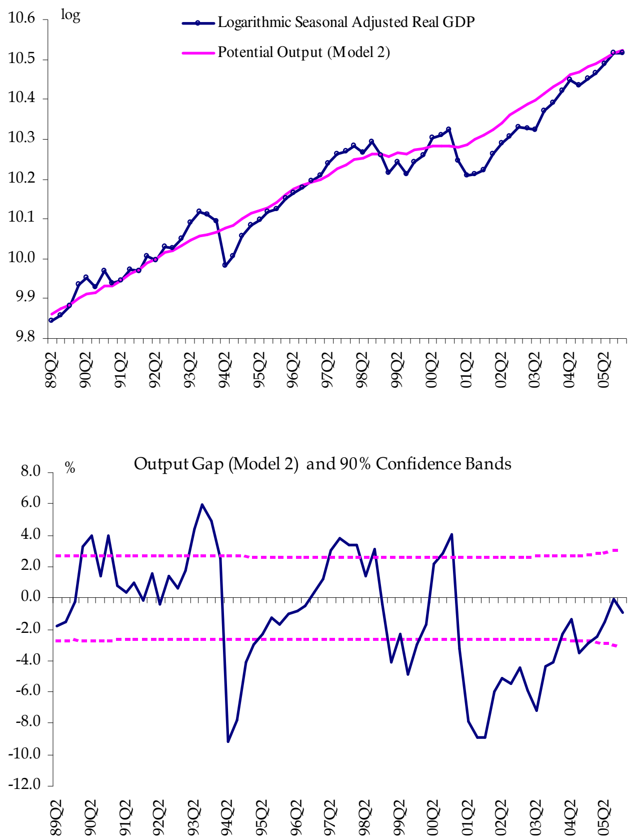

Figure 8: Smoothed Estimates of the Potential and Output Gap (Model 2)

Average standard error of the first model's output gap estimate is slightly larger than the second one, implying a little higher uncertainty around the first model output gap estimate. As can be seen from the figures, standard error bands widens at the end of the sample period. Error bands roughly indicate that 1990, 1993 and 1997-1998 periods are the expansions, whereas 1994, 1999 and 2001-2003 periods are the recessions.

In general, it can be concluded that comparison of output gap estimates across the two models does not give a clear distinction in favor of any model. On the other hand, the sample average standard error of the output gap for the random walk and the structural specification models are 1.74 and 1.62, correspondingly. Hence, results suggest slightly higher uncertainty regarding the first model estimate.

The smoothed estimates of the neutral real interest rates obtained from the models are plotted in Figure 9 together with the actual real interest rate. The distinction between the estimated neutral real interest rates of two models is considerable, compared to output gap estimates. The random walk specification yields a more volatile estimate suggesting considerable variation over time in the neutral real interest rate, whereas the more structural specification of the neutral rate produces a smoother estimate. On the other hand, sample means of both model specification estimates are almost the same.

Figure 9: Smoothed Estimates of Neutral Real Interest Rates

Both models follow a similar pattern in two different periods; namely until 1996 and after 2003. Accordingly, both models point out that neutral real interest rate is rising until 1996 and then declining after 2003. However, models generate quite different estimates between the years 1997 and 2003; while random walk specification points out first a rising, then declining and finally again a rising neutral real interest rate, the second specification indicates a more stable neutral real rate. The other striking difference between the models is that although the neutral rate obtained from the second model starts to decrease right after the financial crisis at 2001, consistent with the declining trend of the risk premium as mentioned above, the neutral rate estimates of the first model decreases only after two years following the crises period. However, according to both measures, the neutral real interest rate declines in the last years in the light of the favorable developments in the economy.

Archibald and Hunter (2001) state that main drivers of the neutral real interest rate are the structural factors affecting saving and investment decisions and country-specific risk premium. Hence, they expect these factors to change slowly through time, meaning that the neutral rate also changes slowly rather than varying significantly over the business cycle. If the estimates are evaluated in this respect, the second model turns out to be more sensible estimate of the neutral rate.

For more detailed analysis, it is important to evaluate estimated neutral interest rates together with output gap developments. Expressing both the real interest rate and the actual output as the deviation from neutral rate and potential output respectively, Figure 10 plots the estimated real interest rate gaps together with the estimated output gaps. As it is obvious from this figure, there is a clear relationship between the estimated real interest rate gap and the output gap for both models for the entire sample period. Whenever the real interest rates increase (decline) above (below) its neutral level, economic activity declines (increases).

For the detailed analysis, the whole sample can be divided into three sub-periods based on the macroeconomic environment explained above. The first sub-sample covers the first five years of the sample, hence ranges from 1989q2 until the post-currency crises in 1994q2. The major characteristic of this period is the high and volatile inflation rates together with emergence of "fiscal dominance" phenomenon on the monetary policy implementations. Excluding the crises period in 1994, during which both nominal and real interest rates increased considerably, the observed real interest rate is generally below the estimated neutral rate for this period. As the rise in inflation was not followed by the similar rise in the nominal interest rates, the real interest rates not only decreased but also became negative during some part of this sub-period. This factor points out that monetary policy was too loose in the first half of 1990s to control the high inflation rates. As it is observed from Figure 10, the output gap estimates confirms the findings that loose monetary policy implied by negative real interest rate gap resulted in output above its potential level up to currency crises in April 1994; which puts upward pressure on inflation through aggregate demand channel.

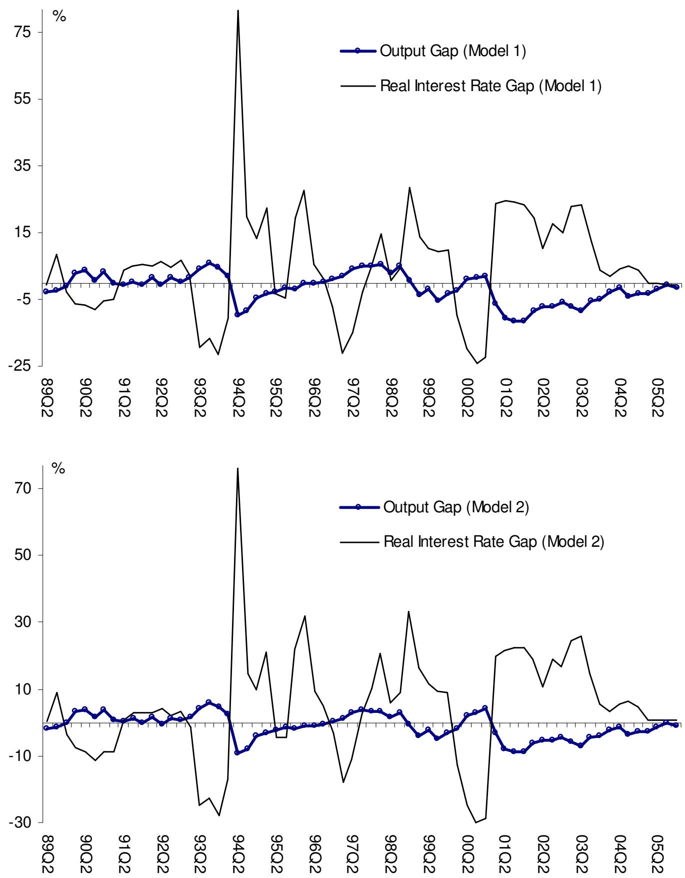

Figure 10: Smoothed Estimates of the Output Gap and Real Interest Rate Gap

The second sub-period ends with the deepest financial crisis that Turkey experiences in its economic history. More clearly, the sub-sample covers the period between 1994q3 and 2001q1. Besides chronically high inflation rates, which is mainly resulting from high budget deficits together with structural fiscal and financial side problems, the external shocks originated from international markets such as Asian and Russian crises, was the most striking period-specific characteristics of the Turkish economy in this sub-period. Actually, it was high and volatile risk premium resulting from these structural problems and contagion effects of the abovementioned crises caused considerably high levels of nominal and real interest rates. When we exclude the period of 1996q4-1997q2, the observed real interest rate was realized well above the neutral rate until the adoption of exchange rate base stabilization program. In other words, during the mentioned period, the real interest rate gap turned out to be positive. However, it was not possible for the CBRT to decrease inflation rates considerably in those years due to some structural problems in the economy as mentioned above. On the other hand, short-lived negative real interest rate gap observed in the period of 1996q4-1997q2 seems to be consistent with the speed up of the economic activity that ended with the Russian crises.

Immediately after the implementation of exchange rate based stabilization program in 2000, real interest rates dropped significantly and realized below the neutral rate until the end of this sub-period. Since the exchange rate was main nominal anchor of the monetary policy implementations, the CBRT had no control over the interest rates. Throughout this period, as it is also consistent with the negative real interest rate gap, economic activity rebounded and positive output gap was observed.

The last period from the 2001q2-2005q4 was characterized by declining inflation together with high growth rates, improved fiscal discipline, positive expectations for the future course of the economy on the way of convergence with the European Union, and hence declining country specific risk premium, relatively stable exchange rates and declining real interest rates.[38] Although declining, the actual real interest rates were above the neutral rate, implying that the CBRT adopted restrictive monetary policy to reduce inflation to single digit levels. Only very recently, the real interest rates realized very close to neutral interest rate. Actually, an important feature of this period is that CBRT started to use the nominal interest rates as a policy instrument to achieve the inflation target after the floating exchange rate regime.

When we focus on Figure 10 for this sub-sample, it is evident from the real interest rate gap that the CBRT implemented tight monetary policy to cope with high inflation rates. Actually, the real rate was above its neutral rate for a long period of time. As a result of this, in spite of high growth rates in the last four years, the output gap closes very gradually, which is another factor supporting the decline in the inflation rates.

In general, the recent Turkish experience seems to be very much consistent with the Bernhardsen's (2005) statement about the global neutral real interest rate that going through a transformation from high and volatile inflation to an environment of low and stable inflation has probably contributed to a fall in the inflation risk premium and hence neutral real interest rates.

After detailed discussion of the estimated figures, it would be helpful to evaluate the uncertainty surrounding the estimated neutral real rates across the models. As shown in Figure 9, the differences between the estimates of the neutral rate across models can be substantial at any point in time, signifying the prominent degree of "specification uncertainty". Orphanides and Williams (2002) highlight that another source of the uncertainty is to allow neutral rate to change unpredictably over time. To exemplify this point, the estimated neutral real interest rates are plotted with 90 percent confidence bands in Figure 11, in which the width of bands are quite large. To illustrate the extent of uncertainty, it would be useful to look at the upper and lower bounds for the last point estimate, that is

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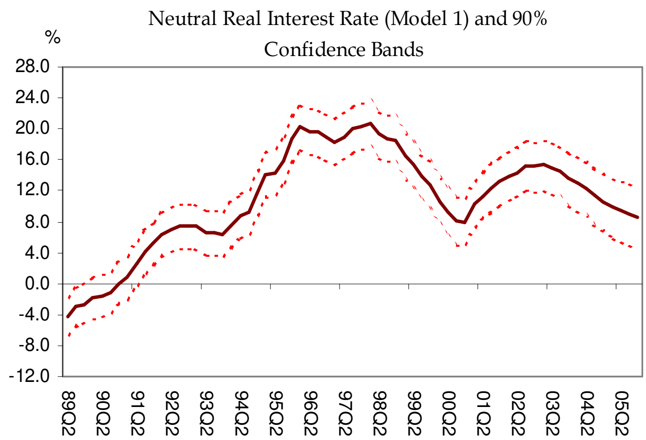
[38] The halt in the decrease of the real interest rate in 2003 is mainly resulting from concerns regarding Iraqi war in the first quarter of 2003.

fourth quarter of 2005. For the first model, upper and lower bands are 4.55 and 12.53 (band width is 7.98), whereas they are 4.51 and 9.77 (band width is 5.26) for the second model. In addition, the sample average standard error of the neutral real interest rates for the random walk and the structural specification models are 1.85 and 1.42, respectively[39]. Note that the degree of uncertainty surrounding estimates in Model 2 is relatively smaller than the one in Model 1.



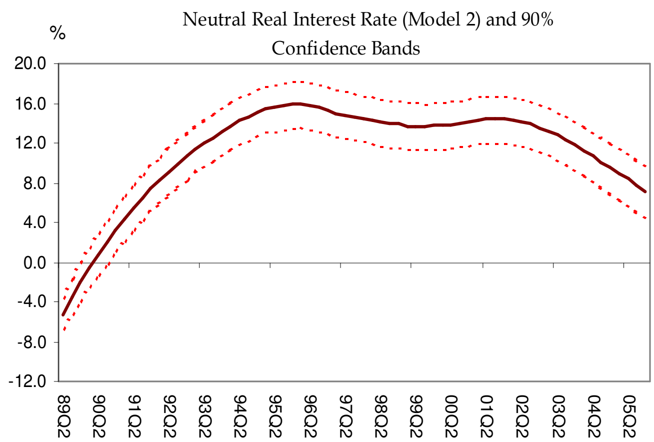


Figure 11: Smoothed Estimates of Neutral Real Interest Rate with 90 Percent Confidence Band

[39] As noted in Chapter 4, Kalman filter (smoother) computes the variance of the state vector conditional on the data available up to the current quarter (conditional on the data through the whole sample).

All these results suggest that it is quite difficult to come up with an accurate point estimate, since the estimates of the neutral rate are highly imprecise. For that reason, one has to be rather cautious about the point estimate of the neutral rate at a particular point in time. Indeed, this point is stressed in almost all studies in the literature that attempt to estimate the neutral rate. By attributing this imprecise nature of the estimates, Orphanides and Williams (2002) declares that "even with the benefit of hindsight and "best practice" techniques, our knowledge about the natural rates remains cloudy, and this situation is unlikely to improve in the foreseeable future". Despite the uncertainty surrounding them, the number of studies estimating the neutral rates is growing considerably in the view of their usefulness in understanding and explaining past macroeconomic developments.

7.3 Evaluating the Parameter Estimates

For evaluating the parameter estimates, the following issues are worth to be mention. First of all, as shown in Table 3, parameter estimates of Phillips and IS curves together with the neutral real interest rate equation are generally sensible. Moreover, they are very similar across the model specifications and all of them have the expected signs.

As regards the Phillips curve, the first two lags of inflation are highly significant and the sum of coefficients of the lagged inflation terms ($b_1 + b_2$) is around 0.74, indicating high level of inflation inertia. Significance of two lags of quarterly inflation reveal that backward-looking behavior dominates the pricing behavior of the agents, stemming from past high and persistent inflationary environment. On the other hand, with the implementation of the inflation-targeting regime, it is thought that the percentage of the agents forming their expectations and thereby determining their pricing policy in a forward-looking manner has been increasing in line with the credibility of the implemented policies.

The estimated output gap coefficients on inflation equation (b_3) in Model 1 and 2 are 0.21 and 0.26, respectively. There is a positive relationship between inflation and the output gap as expected. Note that, parameter estimate of the Model 2 is not significant at 5 percent significance level (p-value for the first model is 0.052), whereas the least square estimates point out to a significant effect of output gap on inflation. An important reason for the limited information content of the level of output gap for inflation is that exchange rate pass-through and inflation inertia had been significant in Turkey for a long period of time. However, with the implementation of the floating exchange rate and the inflation-targeting regime, the functioning of transmission mechanism has been clearer. In line with this, it is though that the expected relation between output gap and inflation has begun to be more evident. The estimated coefficient of the output gap indicates that a one-percentage point increase in the output over its potential raises the inflation about 0.21 (0.26) percent in the next quarter for Model 1 (Model 2). Note also, due to the highly persistent output gap, we also expect that the future inflation rates will be influenced.

The parameter estimate of real effective exchange rate in the inflation equation (b_4) indicates that depreciation of the domestic currency leads to a higher level of inflation, as expected. The estimated coefficients are almost the same for both models including the LS estimates. Depreciation of the Turkish Lira results in the price increases for both imported consumption goods and intermediate goods, which in turn increases inflation. While the price increase of imported consumption goods is directly reflected into inflation, intensive use of imported inputs in production processes causes higher consumer inflation through higher production costs.

As regards to IS equation, coefficient of the lagged output gap (a_1) is around 0.74, signifying a persistent behavior of the output gap. In terms of the monetary transmission mechanism, one of the most important parameters is the coefficient of the real interest rate gap on the IS curve. Consistent with the theory, the relationship between real interest rate gap and output gap is found to be negative. Two lags of real interest rate gap are tested in the models but they are found to be insignificant. Only the contemporaneous effect of real interest rate gap turns out to be significant, implying that the pass-through effect is immediate. Most importantly, although the neutral rate estimates differ considerably, the coefficients of the real interest rate gap in the IS equation (a_2) are estimated as -0.084 and -0.082, which are almost implying the same effect. This finding suggests that actual real interest rate developments dominate the real interest rate gap compared to the changes in the neutral real interest rate. Indeed, this seems to be reasonable due the fact that in general real interest rates in our economy remained at markedly high levels, particularly through 1990s as a result of unstable economic environment with high and persistent inflation and high degree of uncertainty.

Another important result is the significant effect of real effective exchange rate on the output gap. The coefficient (a_3) is estimated as 0.12 across models, which means appreciation of the Turkish Lira leads to a higher positive output gap. Such a finding suggests the view that appreciation of the domestic currency induces a higher consumption and investment demand due to decline in the prices of both imported and domestically produced goods, which offset the contractionary effect of the export channel.

When the parameter estimates of the neutral rate equation are considered, trend growth rate and trend risk premium have the expected signs in explaining the neutral real interest rate dynamics. However, the effect of a change in trend growth rate on the neutral rate seems to be weak due to the insignificance of the parameter at 0.05 significance level. Note also that it is significant at 0.1 significance level. On the other hand, estimates point out a close association between the trend component of the risk premium and the neutral rate. Indeed, these findings are consistent with our intuition that apart from short-run fluctuations, the long-run behavior of the risk premium is one of the most important determinants of the neutral interest rate in Turkey.[40]

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[40] See section 5.2 for a detailed explanation.

7.4 Statistical Properties of the Model Estimates

In this section, some properties of our model estimates, concentrating on statistics describing the relationship among real interest rate gap, aggregate demand and inflation are examined. By this way, we investigate the indicator properties of the gap estimates. Table 7 summarizes standard deviation and correlation of some selected variables and estimates. It is thought that these simple statistics provide some important information concerning the transmission mechanism of the Turkish economy.

Table 7: Model Statistics (Sample: 1989q2-2005q4)

		Model 1	**Model 2**
Standard Deviations			
y_t		0.173	0.173
y_t^*		0.186	0.184
$\tilde{y}_t$		4.384	3.684
r_t		18.934	18.934
r_t^*		6.524	5.007
$\tilde{r}_t$		16.261	17.130
Cross Correlations			
	k		
Corr($r_t, \tilde{r}_{t-k}$)	0	0.942	0.966
Corr($\Delta y_t, r_{t-k}$)	0	-0.384	-0.384
Corr($\tilde{y}_t, \tilde{r}_{t-k}$)	0	*-0.662*	*-0.707*
	1	-0.582	-0.613
	2	-0.310	-0.308
	3	-0.131	-0.099
Corr($\pi_t, \tilde{r}_{t-k}$)	1	-0.098	-0.132
	2	-0.171	-0.212
	3	-0.283	-0.321
	4	*-0.331*	*-0.364*
	5	-0.195	-0.227
	6	-0.167	-0.194
	7	-0.105	-0.130

Note: In the table, k denotes the number of lags

At a first glance, estimates of Model 2 have less variability compared to that of Model 1, except for the real interest rate gap. Similar to the above discussion, the correlation between actual real interest rate and the estimated real interest rate gaps indicates that the behavior of the real interest rate seems to be a reasonable approximation for the behavior of the real interest rate gap. The correlation between these two series is relatively high, especially for Model 2. The variability of the gap measures is close to that of actual real rate. In parallel to this, the variability of the neutral rate is much less than the actual rate. As shown in the figures, the estimate of the neutral real rate from Model 1 fluctuates more than the estimate from second one. All these imply that the behavior of the real interest rate gap is dominated by the behavior of the real interest rate itself, rather than the neutral real interest rate component. This finding is similar to other studies, such as Garnier and Wilhelmsen (2005) for Euro area (unobserved component based model) and Neiss and Nelson (2003) for the UK (DSGE based model).

Consistent with the theory, output gap and real interest rate gap, as also shown in Figure 10, have a strong negative relationship at all lags for both models. Most of the effect is felt in the contemporaneous quarter. The effect of real interest rate gap on the aggregate demand gradually diminishes and it is completed almost within one year. Table 7 indicates that the relationship is stronger in Model 2 estimates than in Model 1's. Note that the contemporaneous correlation between actual output and real interest rate is also negative, but it is much lower than the gap estimates, which indicates that the use of measures in the form of deviation from its equilibrium clearly has a stronger explanatory power regarding dynamics of the Turkish economy compared to the use of directly actual variables.

Regarding the cross correlation between real interest rate gap and inflation, the real interest rate gap seems to have a weak negative correlation with quarterly inflation. Roughly, it takes up to one and half year to affect the inflation by the interest rate policy. This finding is consistent with the statements of the Central Bank of Turkey. The long lags, despite their low magnitude, seem in agreement with the conventional view that it takes a considerable time for monetary policy to affect the inflation. But as we have mentioned, extent of the association is not strong. This indicates that developments in inflation in the sample period at hand are not that much related to the evolution of the real interest rate gap. There are some other factors such as the exchange rate developments and the inertia dominating the inflationary behavior.

According to the cross correlations, Model 2 estimates reveal better indicator properties than Model 1 estimates in terms of the both inflation and output gap correlations.

7.5 Real-time Properties of the Estimates (Filtered vs. Smoothed)

As mentioned in Chapter 4, two different estimates, namely filtered and smoothed estimates (occasionally called as 1 and 2-sided estimates respectively), can be obtained depending on information used. At the end of the sample period, both the filtered and smoothed estimates are the same since there is no any future data. In fact, this is unfortunate given that the last observations are often the most important ones for monetary policy analysis. Therefore, policy makers must act according to filtered estimates; this is the only data at hand at time of decision. For that reason, filtered and smoothed estimates can sometimes also be referred as the real-time[41] and retrospective estimates correspondingly. For a given model, if the difference between the real-time and the retrospective estimates is substantial, then policy makers will probably give a wrong decision. In this respect, the difference between the filtered and smoothed estimates can provide a measure of misperception and can be utilized to compare the real-time performances of the models. Consequently, a minor difference between the filtered and smoothed estimates is a desirable feature. To this end, Figure 12 and 13 are presented to compare filtered and smoothed estimates for both models and the Table 8 summarizes the main statistics regarding the misperception between models.

[41] Since the model parameters are estimated based on the whole sample, the equivalence is not exact.

Table 8: Differences between the Filtered and Smoothed Estimates (Sample: 1989q2-2005q4)

	Mean Absolute Deviation (MAD)		**Standard Deviation of the Difference**	
	Model 1	**Model 2**	**Model 1**	**Model 2**
y_t^*	0.024	0.011	0.019	0.012
g_t	0.089	0.042	0.069	0.049
r_t^*	5.019	1.324	4.891	1.769
$\tilde{y}_t$	2.391	1.081	1.881	1.240

A comparison of the filtered and smoothed estimates points out that filtered estimates are highly imprecise, especially for Model 1, and may lead to incorrect policy decisions. This problem is emphasized in Orphanides and van Norden (2002) for the estimation of output gap measures, Orphanides and Williams (2002) and Laubach and Williams (2003) for the neutral real rate. These studies point out that filtered estimates are important source of data revisions. To illustrate the extent of this issue, following example can be given. The filtered estimates of Model 1 produces a negative neutral rate for the last quarter of 2000; hence, the output declines below its potential. However, retrospective estimates for both models indicate a positive output gap for that period. In summary, as shown in Table 8 and Figures 12 and 13, Model 2 produces smaller differences between retrospective and real-time estimates for all unobserved variables, so has better real-time properties.

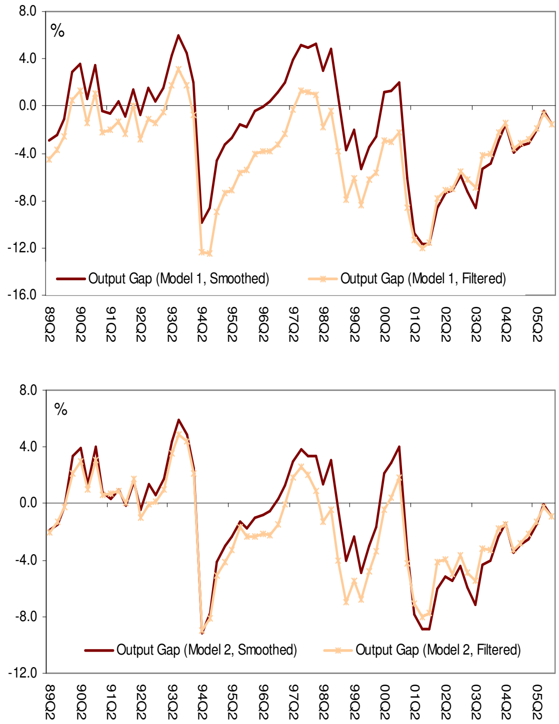

Figure 12: Smoothed and Filtered Estimates of Output Gap

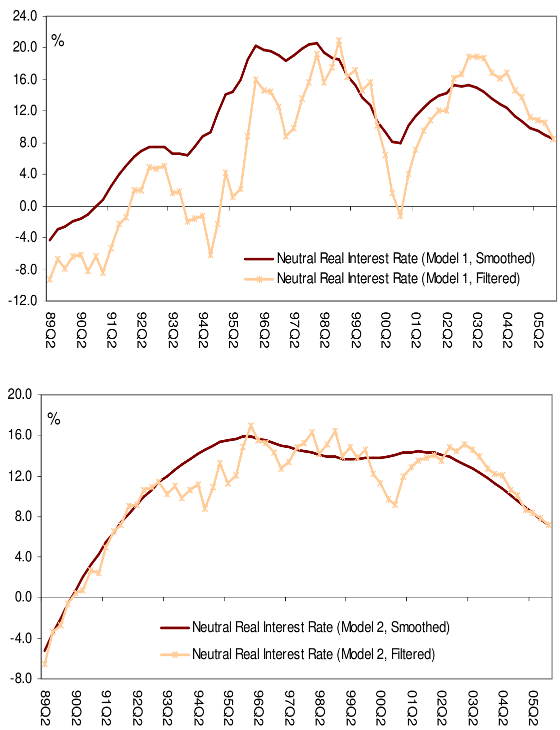

Figure 13: Smoothed and Filtered Estimates of Neutral Real Interest Rate

CHAPTER 8

CONCLUSION

Neutral real interest rate can simply be defined as the real interest rate consistent with the real GDP at its potential level and stable inflation. The real interest rates above or below the neutral rate for a prolonged period of time would have a tendency to contract or stimulate the economic activity. Therefore, the deviation of actual real interest rate from the neutral rate, namely real interest rate gap, is an important variable not only to evaluate the amount of stimulus that monetary policy is passing on to the economy but also to understand historical macroeconomic developments. To this end, this thesis estimates the neutral real interest rate for the Turkish economy. The most important reason behind estimating the neutral rate for the Turkish economy in this study is the lack of a certain measure for the neutral rate or real interest rate gap.

This thesis derives estimates of a time-varying neutral real interest rate under two different multivariate unobserved components models. Both models are built on the output gap literature. However they differ in terms of neutral rate specification. The first model uses a random walk specification for the neutral rate based on Orphanides and Williams (2002), Larsen and McKeown (2003) and Kozicki (2004), whereas the second one exploits a more structural specification, specifically links the neutral real rate with the trend growth rate and the long-term course of the risk premium and principally relies on the study of Laubach and Williams (2003).

In general, the economic relevance of the estimated figures is found to be satisfactory. Both models yield similar results in terms of the output gap estimates, which reveal similar expansion and recession periods and are in line with the results of Sarıkaya et al. (2005). Estimated output gap figures confirm the fact that our economy has been exposed to rigorous shocks for several times in the sample period. They also point out that the gradual convergence of the actual output to its potential after 2001 economic crises has noticeably contributed to the disinflation process. Most recent estimates indicate that actual output is still below its potential, implying a limited degree of demand side pressure on the inflation; however, the negative gap seems to be closing, indicating the contribution of the output gap to disinflation process has been diminishing compared to the previous years.

The difference between the two estimated neutral real interest rates is considerable compared to output gap estimates. The extent of the variation in neutral real interest rate significantly differs across model estimates; specifically, random walk specification suggests considerable variation over time in the neutral real interest rate. However, recent estimates in both models confirm that neutral real interest rate has been declining in the last couple of years in the light of favorable developments in the economy. Estimation results suggest that there is a clear negative relationship between the estimated real interest rate gap and the output gap for both models. For the last couple of years, neutral rate estimates indicate that though declining, the actual real interest rates are above the neutral rate, implying that the CBRT has adopted tight monetary policy to reduce inflation to single digit levels. Actually, the real rate is above its neutral rate for a long period of time. Only very recently, the real interest rates realized very close to neutral interest rate. As a result of this, in spite of high growth rates in the last four years, the output gap closes very gradually, which is one of the factors supporting the decline in the inflation rates.

The parameter estimates of the models have the expected signs and are consistent with Turkish economy dynamics. The effect of a change in potential growth rate on the neutral rate seems to be weak, whereas estimates show a closer relationship between the trend component of the risk premium and the neutral rate. Indeed, these findings are consistent with our intuition that apart from short-run fluctuations, the long-run behavior of the risk premium is one of the most important determinants of the neutral interest rate in Turkey.

The comparison of real interest rate gap with the output gap reveals the fact that there is significant negative contemporaneous correlation between the real interest rate gap and the output gap. According to cross correlations, the effect of real interest rate gap on the output gap gradually diminishes and it is completed almost within one year. On the contrary, it takes more time (up to one and half year) to affect the inflation by the interest rate policy and additionally weak negative cross correlations between inflation and real interest rate gap imply that the real interest rate gap is not a strong indicator of future inflationary pressure.

Both the estimated parameters of real interest rate gaps on the IS equations and model statistics imply that the variation in the real interest rate gap is dominated by the variation in the real interest rate itself, rather than the neutral real interest rate component. In this respect, it can be considered that the behavior of the real interest rate may be a reasonable approximation for the behavior of the real interest rate gap. However, estimates also indicate that the use of measures in the form of deviation from their equilibrium has stronger explanatory power for Turkish economy dynamics compared to the use of actual variables.

As indicated by the cross correlations, Model 2 estimates show a better indicator properties compared to Model 1 estimates in terms of both inflation and output gap correlations. Furthermore, Model 2 produces smaller differences between retrospective and real-time estimates for all estimates, so has better real-time performance. Note that degree of uncertainty surrounding the estimates in the second model is relatively smaller than the first one and on the whole average size of the error attributable to the second model seems to be lower. All these model comparisons evidently point out the use of more structural specification against purely statistical (random walk) specification.

Our practice suggests that there is a lot of uncertainty related with the measurement issue of the neutral real interest rate. Substantial differences between the estimates of the neutral rate across models at some points imply that model specification used can place an important role in the decomposition. Diverse assumptions regarding the specification may bring about a range of possible estimates that may differ in their policy implications. All these issues emphasize the importance of a careful analysis. It is unlikely to come up with one formulation of the neutral real interest rate that would be accurate and cover the needs of policy makers. In fact, it seems to be practical to estimate a range of measures on the basis of a range of alternative considerations. Moreover, confidence bands for the estimated neutral rates suggest that uncertainty surrounding the estimates is considerable and it is quite difficult to find an accurate point estimate. Therefore, one has to assign little weight to the point estimate of the neutral rate at a particular point in time, especially in the formulation of the monetary policy decisions. Real-time properties of neutral rate estimates also support this finding. Our results propose that although the specification uncertainty is inherent at all times, filter uncertainty could be reduced by using more structural specifications for the neutral real interest rate. Notwithstanding their uncertainty, the estimates appear to be very helpful in explaining historical macroeconomic developments in the Turkish economy. It would not be incorrect to state that estimates are notably contributed to our understanding of the monetary policy (and the transmission mechanism) conducted in Turkey. Especially, deviation of the actual real interest rate from the neutral rate seems to be very meaningful in explaining the output gap dynamics.

As far as we know, this study is the first attempt to estimate the neutral rate for Turkish economy. From this point of view, this thesis can be seen as a basis for other attempts to estimate the neutral rate for the Turkish economy. An important issue for further research may be to expand our small-scale macroeconometric model by incorporating equilibrium real exchange rate dynamics. Though it is quite difficult to formulate a model that yield the joint estimate of twin gaps, this kind of a research could possibly give an idea to the policy makers about real monetary conditions.

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APPENDIX

STATE SPACE REPRESENTATION OF THE HODRICK PRESCOTT FILTER

Decomposition of a series into trend and cycle components is widespread in modern macroeconomics. The Hodrick-Prescott filter is commonly used to get a smooth estimate of the long-term trend component of a series. Hodrick and Prescott (1997) proposed to the following minimization problem to extract the stochastic long-term trend and cycle:

$$\min_{(rp_t^*)} \sum_{t=1}^{T} \left(rp_t - rp_t^* \right)^2 + \lambda \sum_{t=2}^{T-1} \left(\left(rp_{t+1}^* - rp_t^* \right) - \left(rp_t^* - rp_{t-1}^* \right) \right)^2 \tag{A.1}$$

where, in our case, rp denotes the risk premium, which is defined in Table 2. The filter is a two-sided linear filter that computes the trend series of rp^* by minimizing the variance of rp around rp^*, subject to a penalty that constrains the second difference of rp^* (Eviews 5.1 User's Guide, 2005). Here, the smoothing parameter (λ) controls the smoothness of the economic series of interest, that is, as lambda goes to zero, the trend moves towards to the actual series, and as lambda goes to infinity, it approaches a linear trend. Economic theory provides little information regarding the value of this smoothing parameter. In applications, most studies use the value originally suggested by Hodrick and Prescott, i.e. λ=1600 for quarterly data. Filter can be also obtained in the context of an unobserved component model, as noted by Harvey and Jaeger (1993) and King and Rebelo (1993). In this case, the smoothing parameter can be estimated by prediction error decomposition via the Kalman filter. Based on our model notation, general form of the unobserved component model that encompasses (including the HP filter) a number of popular decompositions can be written as the following:

$$rp_t = rp_t^* + \widetilde{rp}_t \tag{A.2}$$

$$rp_t^* = rp_{t-1}^* + \mu_{t-1} + \varepsilon_t^{rp^*}, \sigma_{rp^*} \equiv st.dev(\varepsilon_t^{rp^*}) \tag{A.3}$$

$$\mu_t = \rho\mu_{t-1} + (1-\rho)\mu_0 + \varepsilon_t^{\mu}, \sigma_{\mu} \equiv st.dev(\varepsilon_t^{\mu}) \tag{A.4}$$

$$\phi(L)\widetilde{rp}_t = \varepsilon_t^{\widetilde{rp}}, \sigma_{\widetilde{rp}} \equiv st.dev(\varepsilon_t^{\widetilde{rp}}) \tag{A.5}$$

where $\widetilde{rp}$ and μ_t refer to cycle or gap component of risk premium and growth rate of trend component. HP filter is a special case of above model. Smoothing parameter in HP filter in our notation corresponds to $\lambda = \sigma_{\widetilde{rp}}^2 / \sigma_{\mu}^2$. If shocks to the trend component is assumed to be zero ($\sigma_{rp^*} = 0$) and growth rate of trend component is assumed to be random walk ($\rho = 1$) together with the assumption of a serially uncorrelated gap ($\phi(L) = 1$), the above system is reduced into the HP filter. Note also that the lambda value of 1600 suggested by Hodrick and Prescott corresponds to the assumption of $\sigma_{\mu} = 0.025\sigma_{\widetilde{rp}}$ and the μ_0 parameter disappears since ρ equals to 1 (Kuttner, 2002). In this context, the state space representation of HP filter can defined as in (A.6) and (A.7).

$$\left[rp_t\right] = \left[1 \quad 0 \quad 1\right]\begin{bmatrix} rp_t^* \\ \mu_t \\ \widetilde{rp}_t \end{bmatrix} \tag{A.6}$$

$$\begin{bmatrix} rp_t^* \\ \mu_t \\ \widetilde{rp}_t \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} rp_{t-1}^* \\ \mu_{t-1} \\ \widetilde{rp}_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^{\mu} \\ \varepsilon_t^{\widetilde{rp}} \end{bmatrix} \tag{A.7}$$