THE INFLUENCE OF DIGITAL TECHNOLOGIES ON THE INTERACTION OF DESIGN AND MANUFACTURING PROCESSES

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Güney Çıngı

ABSTRACT

THE INFLUENCE OF DIGITAL TECHNOLOGIES ON THE INTERACTION OF DESIGN AND MANUFACTURING PROCESSES

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This study aims to analyze and evaluate the influence of digital technologies on the interaction of design and manufacturing processes by representing an outlook of digital technologies through developments in modeling capabilities, manufacturing techniques, material science, and design strategies.

The digital era reached by the technological developments in different fields of science influenced the field of architecture, just like the others. Thus, a new kind of spatial and tectonic quality in architecture is emerging with the lately introduced design tools and materials that are novel to the building industry, while redefining the role of architect in this contemporary medium.

The evolutionary process of Frank O. Gehry and his office, being a pioneer in using digital design and manufacturing tools in architecture, is represented with realized examples that point out the formerly discussed developments in the realm of architecture and visualize the tectonics of the digitally designed and produced buildings; culminating with the case study of Guggenheim Museum, Bilbao.

Keywords: computer aided design (CAD), computer aided manufacturing (CAM), geometric modeling, digital design strategies

ÖZ

SAYISAL TEKNOLOJİLERİN TASARIM VE ÜRETİM SÜREÇLERİNİN ETKİLEŞİMİ ÜZERİNDEKİ ETKİLERİ

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Bu çalışma, sayısal teknolojilerin genel durumunu, modelleme becerileri, üretim teknikleri, malzeme bilimi, ve tasarım stratejilerindeki ilerlemeler yoluyla ifade ederek; bunların tasarım ve üretim süreçlerinin etkileşimi üzerindeki etkilerini incelemeyi ve değerlendirmeyi amaçlamaktadır.

Bilimin çeşitli alanlarındaki teknolojik gelişmelerle ulaşılan sayısal çağ, diğer bütün alanları olduğu gibi, mimarlığı da etkilemiştir. Böylece, bu günümüz ortamında mimarın rolü yeniden tanımlanırken; yeni sunulan tasarım araçları ve yapım endüstrisine yeni olan malzemeler, mimarlıkta yeni bir mekansal ve tektonik anlayış ortaya çıkarmıştır.

Sayısal tasarım ve üretim teknolojilerinin mimarlıkta kullanımında öncü olan Frank O. Gehry ve ofisinin evrimsel süreci, mimarlık alanındaki daha önce incelenmiş gelişmeleri vurgulayan ve sayısal olarak tasarlanan ve üretilen yapıların tektonik-lerini imgeleyen örnekler yoluyla anlatılmıştır.

Anahtar Kelimeler: Bigisayar destekli tasarım (CAD), bilgisayar destekli üretim (CAM), geometrik modelleme, sayısal tasarım stratejileri

To my wife,

Tuba Çıngı

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2D	Two-Dimensional
3D	Three-Dimensional
B-Rep	Boundary Representation
C-Rep	Constructive Representation
CAD	Computer Aided Design
CAE	Computer Aided Engineering
CAM	Computer Aided Manufacturing
CATIA	Computer Aided Three-Dimensional Interactive Application
CMM	Coordinate Measuring Machine
CNC	Computer Numerically Controlled
CSG	Constructive Solid Geometry
FEA	Finite Element Analysis
LOM	Laminated Object Manufacturing
MJM	Multi Jet Manufacturing
NC	Numerically Controlled
NURBS	Non-Uniform Rational B-Spline
SLS	Selective Laser Sintering

CHAPTER 1

INTRODUCTION

> Having abandoned the discourse of style, the architecture of modern times is characterized by its capacity to take advantage of the specific achievements of that same modernity: the innovations offered it by present-day science and technology. The relationship between new technology and new architecture even comprises a fundamental datum of what are referred to as avant-garde architectures, so fundamental as to constitute a dominant albeit diffuse motif in the figuration of new architectures.[1]

With the aid of developments in various fields of science, design and construction abilities of contemporary architects have gone far beyond of the Industrial Age's. This new potential of design merits are being used since the digital information revolution, which took place in the field of architecture parallel to the widespread expansion of computer use in design studios. Computer use in the studios extended from being limited to representational tool to the single source of both design and manufacturing processes. This period, with the advances in the material and construction industries, has drawn attention to complex-shaped forms and highly curvilinear surfaces, which were, until recently, very difficult and expensive to design and produce using traditional methods.

The challenges of constructability forced the architects to become closely involved with the fabrication and construction process, if they were to see their projects realized. However, building contractors that are used to analog forms of practice were reluctant to take on projects they saw as apparently unbuildable. The architects had

[1] Morales, I. S. *Differences: Topographies of contemporary architecture.* MIT Press, Cambridge. 1997, cited in: Kolarevic, B. Introduction, in: *Architecture in the digital age: Design and manufacturing*. (pp. 1-10). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 3.

to find contractors and fabricators that are capable of digitally driven production, which were often not in building but in shipbuilding. They had to provide digital design information needed to fabricate buildings. Thus they became involved with the digitally generation of the building. They found out that the digital information they generated could be used directly in the fabrication to directly drive the computer-controlled machinery that saves time, makes error-prone drawing unnecessary. Numerically controlled machines could enable the production of unique parts in fabrication process. Thus, new formal approaches emerged that are not necessarily standardized. In addition, new medium enabled them to produce scale models of their designs through the process using the same techniques of those used in the industry.[2] Several new opportunities revolutionized the design approaches. These unintended outcomes of the progress could potentially reemerge the close relationship between the architecture and construction as once existed. That would place more power into the hands of architects, hence more responsibility.

1.1 Brief History of CAD / CAM

Computer aided design (CAD) and computer aided manufacturing (CAM), as its name implies, involves with the use of computer in the design and manufacturing processes. CAD is essentially based on computer graphics, which means the creation and manipulation of pictures on a display device with the aid of a computer. CAM, on the other hand, has originally started from numerically control (NC). A method of controlling machines was invented by John T. Parsons in the late 1940s. His method involved using punched cards that contains the coordinate data to direct the machine to move various positions that defined the desired surface to be machined.[3]

Since 1950s, the applications of computers have dramatically increased with the rapid development of computer hardware and software. “In the nineteenth century, the industrial revolution considerably enhanced man’s physical power. In the present

[2] Kolarevic, B. Information master builders, in: *Architecture in the digital age: Design and manufacturing*. (pp. 55-62). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 57.

[3] Besant, C. B., C. W. K. Lui. *Computer-aided design and manufacture.* Ellis Horwood Limited, England. 1986. p. 15.

century, a second industrial revolution is taking place, with computers offering an enhancement of man's mental capabilities."[4]

"The major businesses that use CAD / CAM are aerospace, automotive and architectural businesses, but they are by no means the exclusive users. It could be said that they have performed a pioneering role in its development."[5] Just like the effects of Industrial Age on architecture before, the Information Age had its most profound challenges not only in design stage, but also in manufacturing and construction processes.

> During the 1970s, the few architects who worked with a computer were seen as a group of strange, determined Utopians. During the 1980s, they were looked upon as specialists who spoke language incomprehensible to most but during the 1990s –parallel to the widespread expansion of computer use in design studios– the understanding also grew that those Utopian or specialist architects were following lines of research that would become fertile ground for new developments for everyone.[6]

Architecture is highly influenced by the information system for designing and planning, construction materials and even by the methods of designing. More and more multifunctional spaces are created using complex geometries. The construction process is turned into a computerized craftsmanship. However, it is the information above all that is becoming an essential component of the new architecture. Forward-looking architects are attempting to create buildings and spaces that are *conscious* of the changes in the operational and social framework caused by information technology.[7]

New digital architectures, which have found their expressions in highly complex, curvilinear forms, are emerging from the digital revolution. "What unites the digital

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[4] Ibid. p. 13.

[5] Beeby, W. D., P. K. Collier. *New directions through CAD / CAM.* Society of Manufacturing Engineers publications Development Department, Michigan. 1986. p. 2.

[6] Saggio, A. How, foreword by Antonino Saggio, in: *Behind the scenes: Avant-garde techniques in contemporary design*. (pp. 5-7). Luca, F. D., M. Nardini. Birkhäuser, Berlin. 2002. p. 5.

[7] Ibid. p. 7.

architects, designers and the thinkers is not a desire to *blobify* all and everything, but the use of digital technology as an enabling apparatus that directly integrates conception and the production in ways that are unprecedented since the medieval times of master builders."[8]

1.2 Object and Scope of the Study

This study intends to explore the new fields of architecture introduced by the technological innovations in all phases of construction process. Integration of architectural design and manufacturing process with digital technologies, which are essentially developed for other disciplines, is examined via the evolution of an architectural office. Investigation of the outcomes of this integration in design and construction processes is within the scope of the study.

Digital design tools of the digital era are explored through chapter 2. Being the starting point of the digital design and manufacturing process, modeling techniques are explored in accordance to their creation techniques and purpose of use. Curves and their mathematical background are represented as a prelude to surface creation methods.

In chapter 3, methods of manufacturing and construction that are introduced to architecture from other industries are presented. Many of these methods are adopted from digital design and production techniques invented for automotive, aerospace and shipbuilding industries. It is aimed to establish a basic understanding for the potential contributions of these methods to the realm of architecture.

In chapter 4, a departure from the traditional way of designing to the new design approaches is aimed to be represented. Innovative design tools that are discovered by the introduction of the fourth variable –time– to the three-dimensional space are explored. Basic operations or considerations for perpetuating more feasible and economic designs are represented in order to emphasize the rationality of the new ap-

[8] Kolarevic, B. Introduction, in: *Architecture in the digital age: Design and manufacturing*. (pp. 1-10). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 4.

proaches. Effects of new materials to new design concepts will shortly be demonstrated by briefly introducing some innovative materials of the era.

Chapter 5 focuses on Frank Gehry's evolving design process and how digital tools and processes have been adopted to a collaborative practice. It will describe the organization of Gehry's office and its associated teams, and it will outline the role of digital tools in an emerging design and construction process where a renewed collaboration is resulting in a new architecture and the reestablishment of the architect as master builder. Guggenheim Museum, Bilbao will be studied as a case study, as being "the first Crystal Palaces and Eiffel Towers of the new Information Age"[9]. Background of the design survey will be explored in accordance with the realization process in order to clearly represent the effects of digital tools to the design and manufacturing processes.

[9] Branko Kolarevic referred Frank Gehry's *Guggenheim Museum* in Bilbao in Introduction, in: *Architecture in the digital age: Design and manufacturing*. (pp. 1-10). p. 3.

CHAPTER 2

DIGITAL DESIGN TOOLS

"A model is a mathematical representation of a geometric form that is stored in the computer memory of CAD / CAM system."[1] Entries of the two-dimensional (2D) systems in computer memory are recognized as flat frames and they can be defined by X/Y coordinates system. For three-dimensional (3D) systems, a third coordinate, Z is added for the definition of the entries. Typically, a right-handed Cartesian system is utilized. Most computer software utilizes two separate coordinate systems. First is a fixed coordinate system that is used for the overall definition of the model, which is called global coordinate system. Second is a moveable, user adjustable coordinate system to aid the creation of the model. By adjusting the coordinate system, the user changes the drawing plane, thus can easily adopt the drawing tools according to the position of the surface. This system is called local coordinate system. Efficient use of the two coordinate systems enables the user to define complex 2D and 3D models in computers memory.

Definition of the shape and other geometric characteristics of a model in terms of a collection of methods is the starting point of the digital design and manufacturing. This is often referred as geometric modeling, a term that is first came into use in the early 1970s with the rapidly developing computer graphics and computer aided design and manufacturing technologies.[2] The model that is going to be acquired at the end of this process is going to be used in several ways such as engineering purposes, manufacturing data, or representation medium. "For many applications, the geometric model of a physical object may require the complete description of surface reflec-

[1] Hawkes, B. *The CADCAM process.* Pitman Publishing, London. 1994. p. 69.

[2] Mortenson M. E. *Geometric Modeling.* John Wiley & Sons, Inc., Canada. 1985. p. 1.

tance properties, texture and color; or it may include only information on the elastic properties of the object's material."[3] The way that the model is structured and stored in computer's memory has a crucial importance in this manner, since the model has to serve its specific purpose and has to be modeled accordingly.

There are many applications software in the market for modeling purposes. Different software producers solve different modeling problems of specific industries. They try to satisfy the customer with easy-to-use interfaces, ready-to-use analyzes tools, and graphic enhancements. They provide additional developments every term to extend their end-user profile. There is a feedback mechanism between the producer and the customer to improve the application continuously. In some cases such as Frank Gehry's office, software called CATIA (Computer Aided Three-Dimensional Interactive Application), which is developed for aerospace industry by a French software company Dassault, is used for the generation and production of surfaces. Gehry's colleagues later improved the software in collaboration with the developer company for architectural use.

"Geometric modeling is often performed with the assistance of graphics systems because interactive graphics enables user easily to enter, manipulate and modify the data for the construction of geometric models, although it is not absolutely necessary."[4] Computer graphics is a tool of communication, thus productivity, between the human and the computer. Graphical interpretation of the numerical data that the computer generates enables the human to give quick decisions. Otherwise, interpretation of the numerical data would be time-consuming, hence would cause ineffective use of human resources.

Software would also serve ease of communication. Therefore, they are equipped with some basic tools such as zooming and panning. These tools simply provide the user the ability to magnify and pan individual localities of the drawing for the purpose of

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[3] Ibid.

[4] Besant, C. B., C. W. K. Lui. *Computer-aided design and manufacture.* Ellis Horwood Limited, England. 1986. p. 152.

observing, adding or editing intricate details.[5] There are other facilities that the software provides to the user. Layering, for example, is a common facility to many applications software. "The principle of layering may be understood by considering a number of drawings, each done on a separate transparent sheet. Each sheet could be viewed separately or, alternatively, placed against each other as layers in a stack and viewed collectively."[6] This basic understanding, clarifies the use of layering in CAD. Every different aspect of the drawing is placed on a separate layer; hence, the user can work in any layer separately or together with others. Number of the layers needed depends on the complexity level of the drawing. For further clarification of complex drawings, layers accommodate the information of pen color and thickness. This would be helpful in hard copy plots, as well as the drawing creation process.

A practical geometric model should be easy to manipulate. The manipulation of the geometry can be accomplished using basic transformation techniques using matrix operations[7]. "Transforming an object implies changing it in either position, orientation, or shape."[8] We may list three main operations: translation, rotation and scaling.[9] "These may be described in terms of vector subtraction (C* = C - t) for translation, where **t** is the translation vector, and matrix multiplications (C* = RC) for the other operations, where **C** is an initial x, y, z position vector, **C*** a transformed position, and **R** a 3x3 matrix to describe rotation or scaling."[10]

There are several techniques for geometric modeling purposes. In general, 2D systems utilize a method that is almost parallel to the conventional drawing habits is followed. The complete perception of the model is assured by the interpretation of multiple 2D views of the model. On the other hand, 3D systems can be modeled in three

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5 Hawkes, B. *The CADCAM process.* p. 58.

6 Ibid. p. 60.

7 Basic matrix operations will be discussed in Appendix A.

8 Mortenson, M. E. *Geometric Modeling.* p. 345.

9 Basic transformations and their mathematical basis will be discussed in Appendix B.

10 McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* Addison Wesley Longman Limited, England. 1998. p. 131.

different ways, according to the model's purpose of use: wireframe modeling, surface modeling and solid modeling.

2.1 Multiple-View Two-Dimensional Drafting

Just like the traditional way of drawing on paper, as a way of communicating the ideas, a designer can sketch various views of a model on a computer, using input devices like a tablet or a mouse. The first step may be the positioning of the vertices of the shape in pair coordinates x,y for the plan view and x,z or y,z for the elevation views. Next, the designer may connect the vertices with line segments. The result is the most elementary representation of a model composed of a plan view and two or more side views as necessary. However, if the designer wants to query the represented model, there is no straightforward information about the three coordinates of any vertex. Because the computer database stored the input coordinates in doubles, there is no definition of the relationships among the separately stored views. Thus, the model does not exist as a 3D model, but as multiple 2D views. There are some limitations regarding this character. "Firstly, skill is required in the construction and interpretation of drawings."[11] From the technical director to the machinist in the shop level, everyone involved throughout the process should be able to understand and interpret the complex drawings and established syntaxes. "Secondly, it is possible to have conflicting or erroneous models –perhaps views on a drawing that do not correspond, or diagrams with unmatched connections on symbols."[12] Interpretation of the model turns out to be a difficult and time-consuming task for complex geometries. The practice of drawing and interpretation of it becomes prone to inaccuracies. "Finally, complexity in the product may stretch the techniques to their limits. For example, certain geometries may be very difficult to represent using drawings – particularly where there are complex, doubly curved surfaces such as on automobile or aircraft bodies."[13]

[11] Ibid. p. 27.

[12] Ibid.

[13] Ibid.

2.2 Wireframe Modeling

Wireframe modeling technique is used by many CAD systems for defining simple geometries. The construction is only made up of points and lines. The user enters the x, y and z coordinates of vertices of the object and combines them with lines (usually straight lines). This lowest level of modeling requires very little computer memory compared to the other techniques however, it has serious limitations regarding face data of the model. Surface geometries that are very important for manufacturing and analyze procedures are not present.

Another disadvantage of this technique is the confusion caused by the imperceptibility of the geometry without removing the hidden lines. Since there is no perception of surface alterations or solid shape, there is no distinguish between visible and hidden edges. Though hidden line removal can be done manually for a view, this would be self-defeating, since a part of line, which is removed for that specific view, may need to be shown on other views.

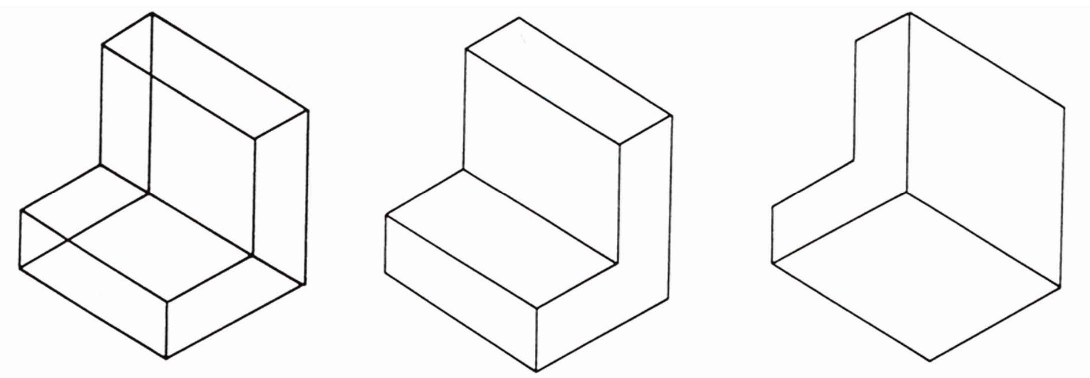

Figure 1. Ambiguities of wireframe modeling

(Hawkes, B. *The CADCAM process.* Pitman Publishing, London. 1994.)

Wireframe models cannot recognize curved profiles. Longitudinal profiles of cylindrical shapes are not fixed edges between defined points in space; they are seen as the silhouette of component face that could vary in position depending on the direction of view. Hence, they are not recognized as wireframe elements and are omitted.[14]

Shade line at regular angular intervals may be used to indicate the curved profile; however, these non-existing lines may lead further confusions on a drawing, which is already imperceptible. Another consequence of the lack of surface data is the difficulty in calculating physical properties. Therefore, accurate calculation of volume, mass, surface area of anything other than very basic shapes is not possible. Tolls like shading and color tone variation, used by the artists effectively in 3D modeling, cannot be used since they are applicable to the surfaces not the edges.

2.3 Surface Modeling

Surface modeling technique adds information about the surfaces in order to solve the drawbacks of the wireframe modeling. The definition of the model is constructed in terms of points, lines and faces. By providing the information of surface to the model, some basic problems of wireframe modeling caused by the lack of surface data is overcome. Surface models may (or may not) define a closed volume, but it has no concept of inside/outside. The model is not a "solid" in essence, but it is a shell-like definition either enclosing a volume or not. Since we cannot talk about "mass", the modeler cannot calculate mass properties like moments of inertia and principal axes.

> In fact, a surface modeler cannot guarantee that the designer has described a realizable object. It may be a collection of surfaces, which do not define a physical part, as in the case where the surfaces may not be connected. A complete part description would have sufficient information to answer any question one could ask about a physical solid object, including a guarantee of solidity.[15]

[14] Hawkes, B. *The CADCAM process.* p. 73.

[15] Bedworth, D. D., M. R. Henderson, P. M. Wolfe. *Computer-integrated design and manufacturing.* McGraw-Hill, New York. 1991. p. 30.

Although less advanced compared to the solid modeling, surface modeling is the most suitable choice for handling the design and manufacturing process of complex curved surfaces, such as car bodies.[16]

“The most elementary of surface types is the flat plane, which may be defined in a number of ways including between two parallel lines, through three points or through a line and a point.”[17] Other surface definitions, in general, fall, into three main categories. In the first category, surfaces are fitted to data points, called control points, and the surface is generated either to pass through approximately or to interpolate the points. The second category comprises surfaces that are based on curves. One or more curves define the surface by means of some operations. For instance, a tabulated cylinder is defined by projecting a generating curve along a vector. A ruled surface in produced by linearly interpolation of two different generating curves. A translational sweep surface is defined by the path of a generating curve swept along a directing curve. A rotational sweep surface, also known as surface of revolution, is produced by revolving a generating curve about a vector. It is particularly useful when modeling objects that possess axial symmetry.

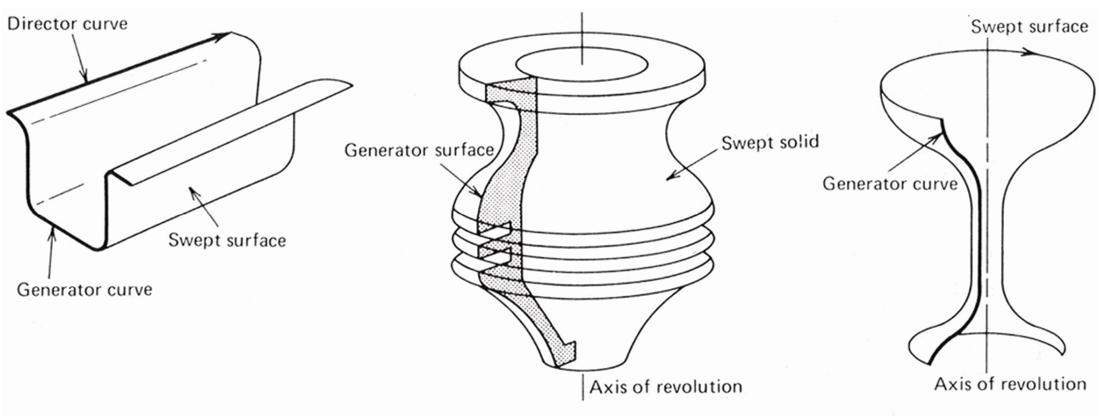

Figure 2. Examples of sweep representations

(Mortenson M. E. *Geometric Modeling*. John Wiley & Sons, Inc., Canada. 1985.)

[16] Hawkes, B. *The CADCAM process.* p. 74.

[17] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 37.

In the third category, surfaces are defined to interpolate between other surfaces. Chamfer and fillet surfaces are common examples of this category. Each of the three categories describes the way that the surface is defined. The mathematical basis or the way that is stored by the system may be the same.

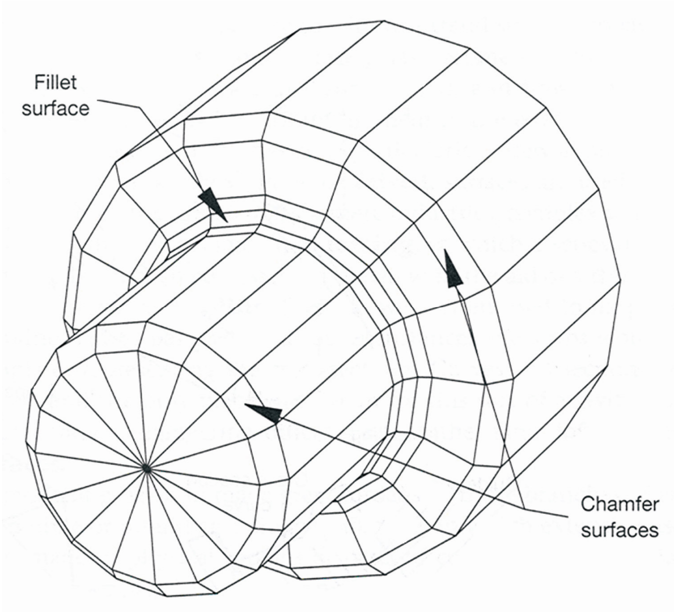

Figure 3. Examples of surfaces defined from other surfaces

(McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* Addison Wesley Longman Limited, England. 1998.)

2.3.1 Curve Representation

Continuous objects are best presented with the help of curves and surfaces. A mathematical tool is needed for the accurate representation that appears as real as possible.[18] Following are the familiar representations of some planar curves:

$$\text{Line:} \quad y = mx + c \qquad \text{Parabola:} \quad y = bx^2 + c$$
$$\text{Circle:} \quad (x-a)^2 + (y-b)^2 = r^2 \qquad \text{Hyperbola:} \quad xy = k$$
$$\text{Ellipse:} \quad x^2 / a^2 + y^2 / b^2 = 1$$

[18] Amirouche, F. M. L. *Computer-aided design and manufacturing.* Prentice-Hall, Inc., New Jersey. 1993. p. 142.

These equations are known as analytical or nonparametric representations of curves. For plane curves these equations take the form f(x, y) = 0 or y = f(x), the first being known as the implicit and the second the explicit nonparametric form.[19] For several reasons, the implicit and the explicit form do not completely fulfill the needs of CAD. Firstly, they represent unbounded geometry without additional constraints. In CAD, normally bounded geometry is used so that lines or curves do not extend to infinity. Secondly, for a chosen value of x there may be two solutions for y; ideally, there should be unique points on the curve defined by a single value of a variable. Thirdly, it is difficult to perform geometric transformations on the curve. Lastly, the equation is dependent on the choice of the coordinate system. The parametric form of a curve equation overcomes these problems. The equations are decoupled in this form, so that there are separate equations of each of the coordinates expressed in terms of additional variable u, were u called parametric variable. It varies in a convenient range, usually from 0 to 1.[20] Additional parameters are used for 2D and 3D systems, v and w, respectively.

$$x = x(u) \qquad y = y(u) \qquad z = z(u)$$

“In 3D modeling, a geometric description is required to describe non-planar curves, but which will also avoid computational difficulties and unwanted undulations that might be introduced by high-order polynomial curves.”[21]

Parametric cubic polynomial curves, being the lowest-order polynomial that can describe a non-planar curve, has become very popular as a basis of computational geometry. Four points in the space provides boundary conditions for cubic polynomials. The fitting of a curve through these points is known as *Lagrange interpolation*. A cubic curve may also be fitted to two points and two slope conditions, which is know as *Hermite interpolation*. It has some advantages where close control of curve

[19] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* John Wiley & Sons, Inc., Canada. 1995. p. 29.

[20] Ibid. p. 30.

[21] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 59.

slope is desired.[22] They are classified as interpolating curves as the curve passes through all the points exactly.

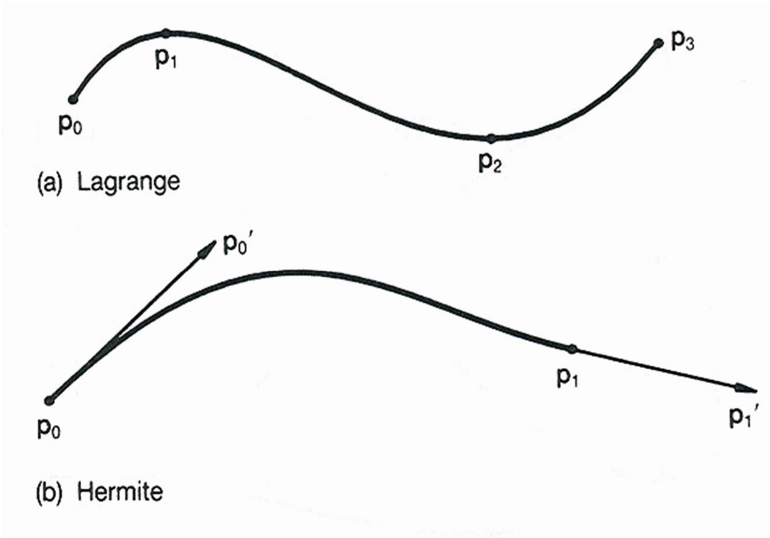

Figure 4. Lagrange and Hermite Interpolation

(McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* Addison Wesley Longman Limited, England. 1998.)

Providing boundary values for curves in terms of points and tangent vectors is not sufficient for interactive design process, as numerical values of slopes may not mean much to the user. This difficulty was resolved by Pierre Bézier, who pioneered the use of computer modeling of surfaces in design. ***Bézier curves***[23] utilizes control polygon, instead of points and tangent vectors. This polygon is approximated by a polynomial curve whose degree is one less than the number of vertices of the polygon, which are also known as control points. Unlike cubic polynomial curves, these curves are classified as approximated curves, because the curve does not pass through the control points except the first and the last points. The cubic polynomial is an example of more general curves that may be fitted to control polygons with arbitrary numbers of control points. In any condition, the curve passes through the first

[22] Ibid.

[23] McMahon (1998) states that Bézier curve is also known as a Bézier-Bernstein polynomial, because the Bézier technique applies a vector formulation of a method of approximation developed by Bernstein earlier this century.

and the last control points of the polygon, and is tangential to the polygon at these end points to the vectors formed between the first and last pairs of control points respectively. Moreover, the interpolating curve does not have more intersections with any plane than the control polygon, and does not change direction more frequently than the control polygon.[24] Therefore, the Bézier curve will not introduce any unexpected behavior. The curve can be considered as a combination of blending functions that represents the influence of each control point on the curve. Since the sum of blending functions is equal to one for any value of u, the curve stands within the convex hull[25] of the defining points. As influence coefficient of any blending function is non-zero in the whole range, moving one control point in the polygon affects every position in the curve. Thus, Bézier curves are *globally modified*. There are other curve formulations that achieve *local modifications*, in which the movement of one control point only affects the curve in the vicinity of the point.[26]

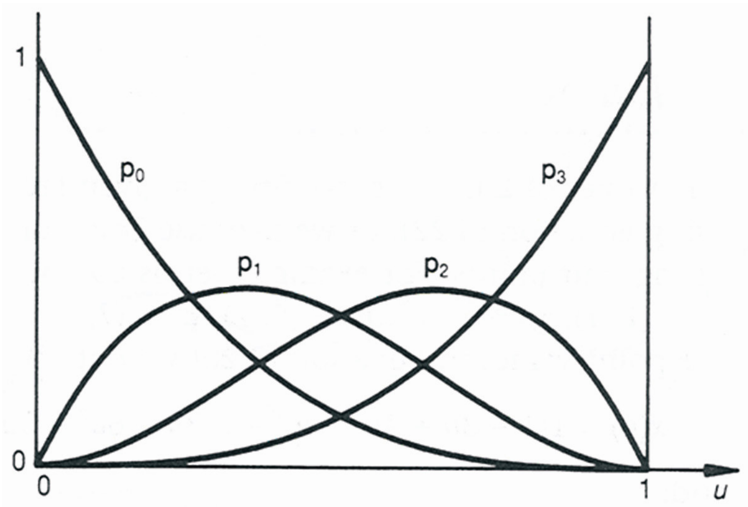

Figure 5. Blending functions for a cubic Bézier curve

(McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* Addison Wesley Longman Limited, England. 1998.)

[24] This property is known as variation diminishing property.

[25] Convex hull is the minimal convex region enclosing the control points.

[26] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* pp. 62-64.

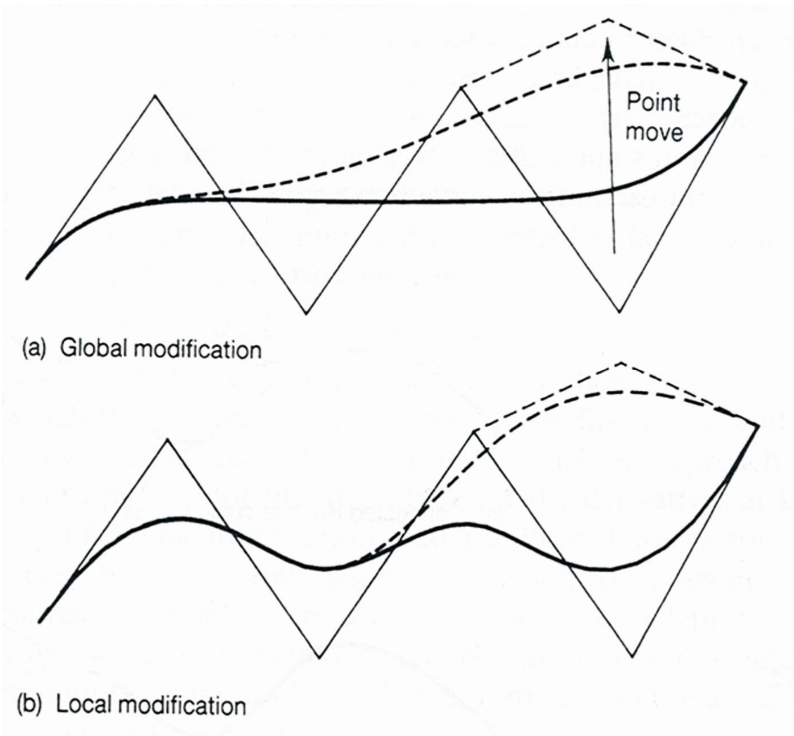

Figure 6. Global and local modification of curves

(McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* Addison Wesley Longman Limited, England. 1998.)

It is often necessary to synthesize a curve from several separate segments in the geometric modeling. When these segments are joined, there are different continuity conditions between them. Parametric continuity is characterized as C^o, C^1, C^2,..., C^n, where the *n*th derivative of its parametric form is continuous. C^o continuity, or point continuity, implies that two curves are joined end to end with no restrictions on end slopes or curvature. C^1 continuity, or tangent continuity, implies that two curves have the same slope (direction and the magnitude of the tangent vector is same) at the common meeting point. C^2 continuity, or curvature continuity, implies that not only the slope but also the curvature is same at the meeting point of the curves.[27]

B-spline curve (basis spline curve) is a generalization of Bézier curves. Just like Bézier curves, they pass through the first and last points, and they are tangent to the first and last segments. They are controlled by a set of point lying on an open polygon.

[27] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* pp. 34-35.

Unlike Bézier curves, B-spline curves can be modified locally, and the degree of the curve is independent of the control points. This makes the control of composite curves possible. Moreover, they can be implemented either approximated or interpolated form. For all of these reasons they are very popular for geometry definition in CAD.[28] If an order k polynomial is defined with k knot points, the lending functions are identical to those for a Bézier curve. As the polynomial order is reduced, the influence of each track point becomes more marked locally. The influence of track point may further be increased by repeating points, which has the effect of first pulling the curve and than causing the curve to pass through the curve. If the spacing value of knot is equal, it is called to be uniform. Conversely, if the not points are at arbitrary ascending numerical values, the knot vector is called to be non-uniform knot vector. A B-spline defined on a non-uniform knot vector is called a non-uniform B-spline.[29]

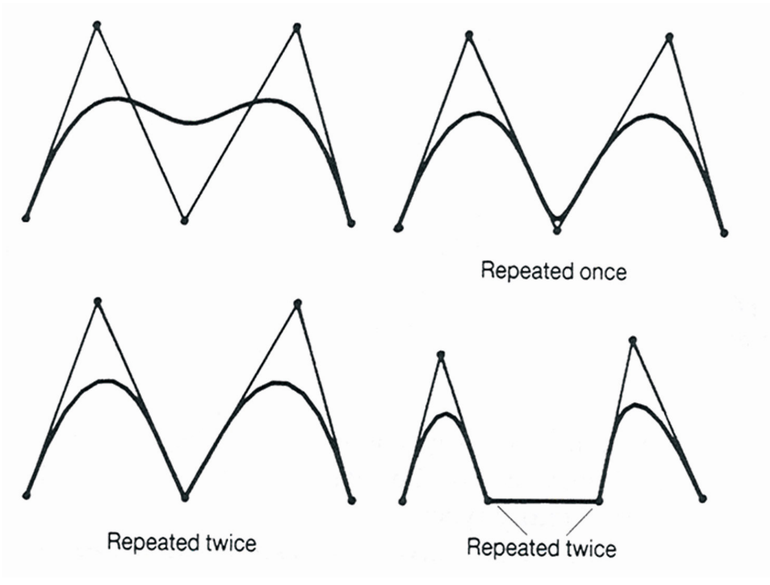

Figure 7. The effect of repeated points on a B-spline curve

(McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* Addison Wesley Longman Limited, England. 1998.)

[28] Ibid. p. 39.

[29] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* pp. 70-71.

The entire curve forms so far, used in CAD for the representation of free form curves and data. In engineering design, however, standard analytic shapes such as arc, cylinders, cones, lines and planes predominate. Consequence is the need for systems that involve both free form and analytic geometry. An ideal modeling method should allow the representation of both analytic and free form geometries in a single unified form. Furthermore, the method should have the advantage of reducing the database complexity and the number of procedures required in a CAD system for the display and manipulation of geometric entities. Theses issues are resolved by ***rational polynomials***. They are capable of exactly representing conic and general quadric functions as well as representing various polynomial types.[30]

A very wide used form is the ***non-uniform rational B-spline***, or ***NURBS***, so called because it is a rational basis spline function allowing a non-uniform knot vector. NURBS are capable of representing in a single unified form non-rational curves, as well as analytic curves, and may be used in approximated and interpolated mode.[31]

Modification of the curve can be easily managed after the creation, in terms of control vertices, weights, knots and the degree of the curve itself. Each control point has an associated weight that determines the extent of its influence over the curve. Increasing the weight of a control point pulls the curve towards that control point and vice versa.[32]

2.3.2 Surface Representation

Parametric methods for representing a curve may be generalized for the parametric surface representations. Portions of non-planar surfaces are referred as surface patches, and they involve two parameters, u and v. The general form of parametric representation of a surface is as follows:

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[30] Ibid. pp. 71-72.

[31] Ibid. p. 72.

[32] Kolarevic, B. Digital Morphogenesis, in: *Architecture in the digital age: Design and manufacturing*. (pp. 11-28). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 16.

$$x = x(u, v) \qquad y = y(u, v) \qquad z = z(u, v)$$

Just as the curve segment is the fundamental building block for the curve entities, so patches are the fundamental building blocks for surfaces. In addition, just as the parametric variable *u* varies monotonically along the segment, two variables *u* and *v* vary across the patch. Parametric values often lie in the range 0 to 1, although other parametric intervals may be used if appropriate. Fixing the value of one of the parameters, results in a curve on the patch in terms of other variable.[33] If the procedure is repeated for a variety of values of both parameters, the result is an intersecting mesh of curves on the patch.[34] Four bounding curve of the patch may be found by substituting, in turn, the upper and lower limits of one parameter and keeping the other parameter as variable.[35]

The parametric cubic curve, which is widely used in the representation of curves, is commonly utilized in surface modeling as an edge curve defined in terms of point and tangent vector information. The equivalent surface form, the ***bicubic patch***, is an important entity for surface descriptions.

In the same way that the Bézier curve uses a more tractable control polygon instead of interpolations of points and tangent vectors, so does the ***Bézier surface*** formulation use a *characteristic polyhedron*.[36] The behavior of the representation is analogous to curves; the surface approximately passes through the control points, except for the four corner points through which the surface passes exactly. The surface can be pulled closer to the characteristic polyhedron by using coincident control points.[37]

[33] This is defined as isoparametric curve in: Alias. *Maya Personal Learning Edition software help contents*. 2005.

[34] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 73.

[35] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* p. 46.

[36] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 75.

[37] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* p. 48.

Further properties of Bézier curves are valid for Bézier surfaces such as the variation diminishing property, convex hull property and global modification restriction.

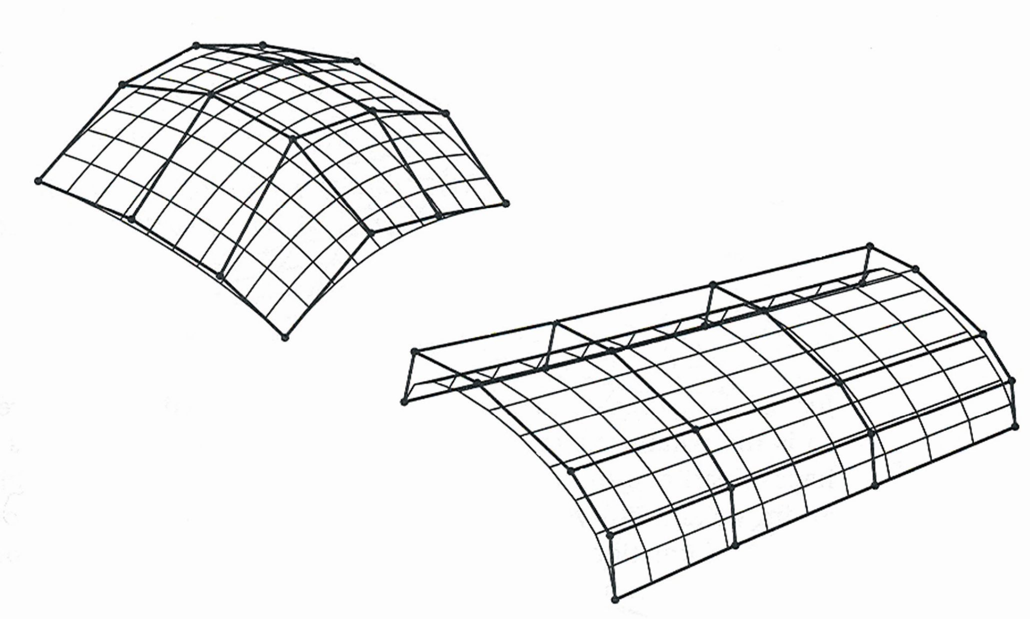

Figure 8. Examples of Bézier surfaces

(McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* Addison Wesley Longman Limited, England. 1998.)

Some limitations of the Bézier scheme are overcome by ***B-spline surface*** formulation. These kinds of surfaces may be locally modified and the degree of approximation may be varied by varying the order of the B-spline blending curves employed.[38] They may also be interpreted in interpolated form.

Rational parametric surfaces, which are defined analogously to the rational parametric curves, gives a surface formulation that can record various commonly used surfaces such as spheres, cylinders, and cones in addition to the forms captured by the conventional non-rational forms.[39]

[38] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 77.

[39] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* p. 50.

2.4 Solid Modeling

"A solid model is described in terms of the volumetric shape which it occupies. Solid modeling is thus the only technique which provides a full, unambiguous description of a 3D shape."[40] Solid modeling technique offers complete definition of volumetric shape that provides the ability to distinguish the inside and the outside of the modeled object. Easy determination of mass properties is another advantage of this method in terms of engineering applications. Hidden line removal and color shading are some basic tools serve the easiness of perception of the model.

In the design of a solid modeler, there are two major issues: integrity and complexity. A solid modeler, in this manner, should have the capability of enforcing the model to be correct, either with integrity-checking algorithm or by providing only integrity-preserving operations. High level modeling tools should be provided in order to overcome complexity issues that may be faced in large models with hundreds of thousands of modeling primitives.[41]

Many methods have been proposed for solid modeling, but none of them is entirely satisfactory. Two of them have been partially successful and have dominated the development of the practical systems. These are the ***constructive solid geometry*** (CSG) method (C-Rep for short, set-theoretic, or Boolean method) which achieved early prominence in CAD, and the ***boundary representation*** method (B-Rep for short or graphic-based method) which dominates in today's applications.[42] There are other methods used in specific fields of science and engineering such as ***decomposition models*** and ***non-manifold models***. Non-manifold models may be regarded as a hybrid of decomposition model and boundary model, and they are utilized in order to overcome some restrictions of both.

[40] Hawkes, B. *The CADCAM process.* p. 80.

[41] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* p. 52.

[42] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 43.

2.4.1 Constructive Models

"Constructive models follow a set-theoretic approach to solid modeling where models are defined as combinations of primitive sets by Boolean operations."[43] The principle that this technique is based on argues that any part, no matter how complex can be modeled by adding or subtracting elementary shapes and putting them in the appropriate position. Each of the elementary shapes structuring the finished solid model may be considered as a solid model with associated mass properties.[44]

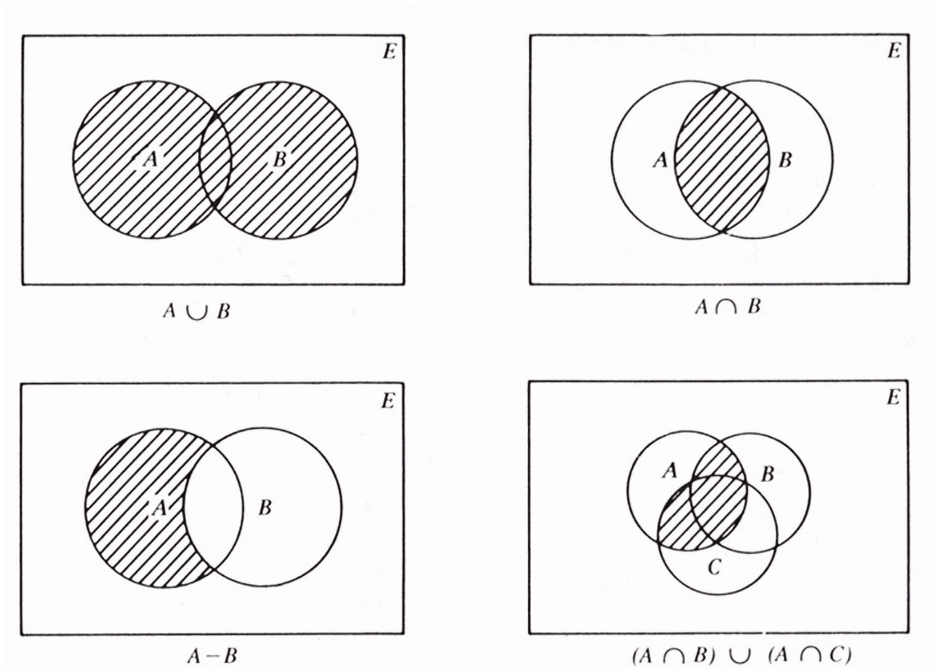

Figure 9. Venn diagrams and set theory

(Mortenson M. E. *Geometric Modeling.* John Wiley & Sons, Inc., Canada. 1985.)

Constructive solid modeling systems use basic primitives such as plane, cuboids, cylinder, sphere, cone, wedge, torus etc. "The primitives themselves may be defined in a number of different ways."[45] In some systems, instances of bounded solid primitives

[43] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* p. 56.

[44] Besant, C. B., C. W. K. Lui. *Computer-aided design and manufacture.* p. 158.

[45] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 81.

are used, known as constructive solid geometry (CSG), whereas in other cases they may be defined by intersections of simpler primitives, which are known as half-space models. In half-space models, logical operators are applied to a number of sets of infinite points in the space, defined by analytical functions, in order to define the desired model. They are not commonly used in modeling.

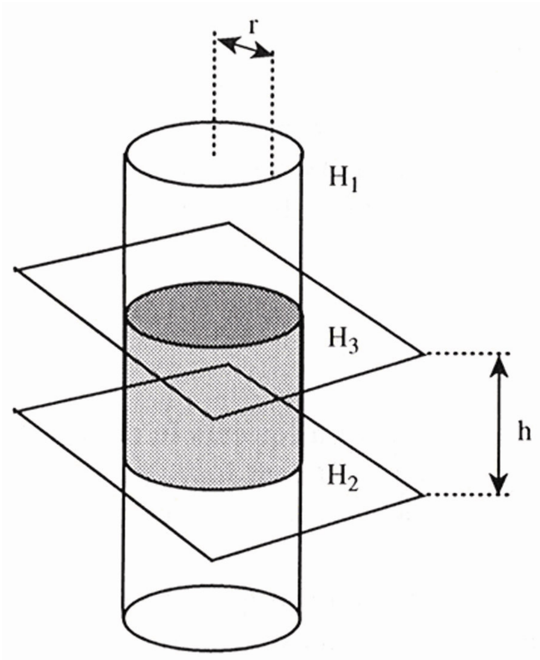

Figure 10. Half-space model of a finite cylinder

(Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* John Wiley & Sons, Inc., Canada. 1995.)

In constructive solid geometry, primitives are inserted for the Boolean operations. Software provides a number of parameterized instances of solid primitives. By determining the independent parameters, which may include orientation, position, overall scale factor, independent scale and size parameters, and feature form parameters, the primitive is recalled from the library. This technique is known as instantiation. "Usefulness of this technique depends on the range of the primitives that is available, and the number and types of the values that can be specified at the time of instantiation."[46] By properly positioning primitives and applying the required logical

[46] Amirouche, F. M. L. *Computer-aided design and manufacturing.* p. 173.

operator, such as union, intersection and difference, even complex sculptured surfaces may be synthesized; however, they work best on models that do not have complex surfaces. The main problem with the generation of complex surfaces is the large quantity of possible intersecting curves for such surfaces. As a result, a great deal of computer time is needed to produce the desired shape.[47]

"Algorithms for the model work by inspecting the Boolean combination typically recorded in a tree-type data structure."[48] In the leaves, there are the primitives and the nodes resemble the logical operator performed on the solids or the rigid transformation. One node, that is an operator, may only connect two children to one parent. The root, that is finished design, has no parent, and the leaves have no children. This is known as binary tree.[49] Since the set operations and the transformations do not destroy the solid nature of the primitives, the finished design is guaranteed to preserve its solidity. The way that the model is stored in computer memory requires less storage area but more computation to reproduce the model and its image.[50] Calculation of intersections between bounded solids is computationally intensive when a large number of primitives are involved in the model. "The intensity of this task may be reduced by such means as spatial division of the model such that the intersections are only tested for primitives in proximity to each other."[51]

2.4.2 Boundary Models

"Boundary representation is scheme wherein the objects are defined by their enclosing surfaces or boundaries. This technique consists of listing all faces, vertices, and edges of an object."[52] Faces of the modeled object must be recorded in boundary data

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[47] Besant, C. B., C. W. K. Lui. *Computer-aided design and manufacture.* p. 158.

[48] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* p. 56.

[49] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 81.

[50] Groover, M. P., E. Zimmers. *CAD / CAM: Computer-aided design and manufacturing.* p. 127.

[51] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 81.

[52] Amirouche, F. M. L. *Computer-aided design and manufacturing.* p. 184.

structure. Normally a hierarchical approach is used for this purpose. Faces are defined in terms of their bounding edges, and edges are defined similarly in terms of their bounding vertices.

The principle of the technique is that part geometry is different from part topology and that they can be defined separately. The topology of an object is the description of how its vertices, edges, and faces are connected. For instance, topology of an object provides the information of which two vertices are connected to form an edge, and which edges are connected to form a face of an object. Conversely, topology of an object provides such information as what faces share what edges, how many faces meet at a given point, and so on. The geometry of an object, on the other hand, defines the positions and dimensions of its vertices, edges, and faces. Without the geometrical information, in general terms, cube and other parallelepipeds are identical topologically.[53] Topological and geometrical information together defines the model completely and uniquely in the space. "In boundary modeling, once particular topology has been defined, many different operations can be performed on the part to adjust geometry without changing basic topology."[54]

One of the central problems of boundary modeling is how to ensure the models defined by the system will always be topologically valid, even during interactive modifications. This problem is managed in two ways: by appropriate choice of the data structure and by ensuring that the model conforms to a set of mathematical rules that control the topology. Topological[55] and geometrical[56] consistency can be determined by a set of rules throughout the process.

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[53] Besant, C. B., C. W. K. Lui. *Computer-aided design and manufacture.* p. 160.

[54] Ibid.

[55] Mortenson (1985) noted these rules as, faces should be bounded by a single ring or loop of edges; each edge should adjoin exactly two faces and have a vertex at each end; at least three edges should meet at each vertex; Euler's rule should apply. Euler's rule implies, V (vertices) + F (faces) – E (edges) = 2; when the bodies contains holes and passages V + F – E – H (holes) – 2B (bodies) – 2P (passages) = 0

[56] Mäntylä (1988) noted these rules as, faces of the model do not intersect each other except at common vertices and edges; the boundaries of the faces are simple loops of edges that do not intersect themselves; the set of faces of the model close to form the complete skin of the model with no missing parts.

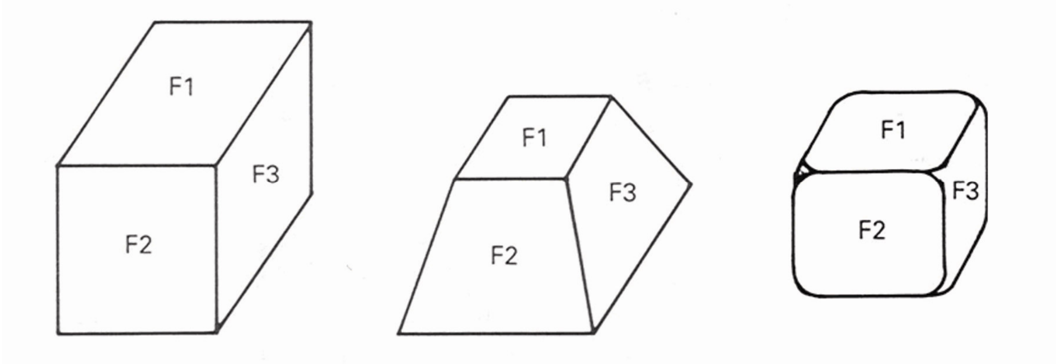

Figure 11. Topologically identical, but geometrically different solid models

(Besant, C. B., C. W. K. Lui. *Computer-aided design and manufacture.* Ellis Horwood Limited, England. 1986.)

2.4.3 Decomposition Models

Decomposition models represent solid objects as non-overlapping basic blocks of material that are pasted together. Various alternative schemes are further characterized by the types of blocks available and the way the collection of blocks constituting a solid. In cellular decomposition method, solids are represented as a combination of irregular cells that are pasted together over common faces.[57] "Although this technique is not used widely in geometric modeling, it is the basis of finite element analysis (FEA), in which the analysis of a complex shaped is approximated buy the analysis of an assembly of simple elements representing the shape."[58] In exhaustive enumeration method, regular cellblocks are used, and they denote either solid material of several types or empty space. They are primarily used in the generation of information directly from solid models.

2.4.4 Non-Manifold Models

Non-manifold models relax some of the topological restrictions of the boundary models and decomposition models. With this kind of modeling system, it is possible

[57] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* p. 52.

[58] McMahon, C., J. Browne. *CADCAM principles, practice and manufacturing management.* p. 82.

to draw different types of point neighborhoods, which boundary systems do not allow. For instance, this technique may be used to compute the crack propagation of a model under a load, as it requires one- and two-dimensional geometric elements within the model.[59]

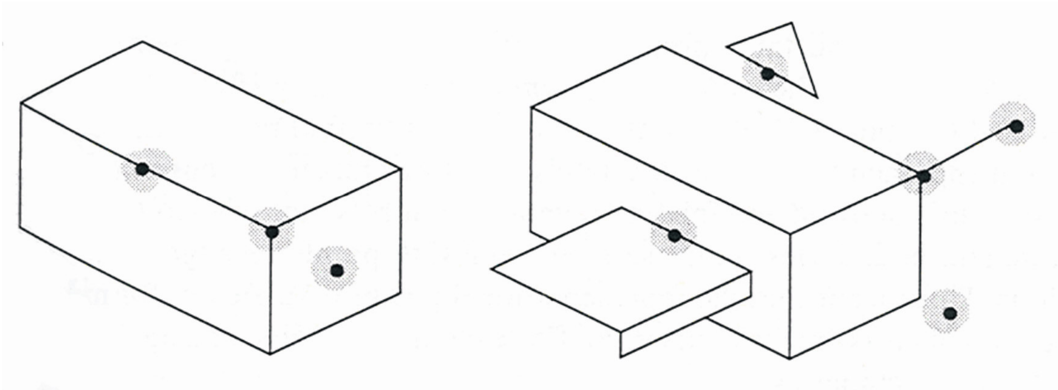

Figure 12. Manifold and non-manifold neighborhood configurations

(Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* John Wiley & Sons, Inc., Canada. 1995.)

[59] Mäntylä, M., J. J. Shah. *Parametric and feature-based CAD / CAM: Concepts, techniques, and applications.* p. 74.

CHAPTER 3

DIGITAL PRODUCTION TECHNOLOGIES

Introduction of digital production techniques into the architecture has radically changed the ideas of conception and production heralding a new kind of architecture that born out and constructed digitally. Emergence of complex *blobby* forms, has forced architects to be involved with the constructability. Difficulties on structural realization and construction of highly curvilinear surfaces of such structures were the major challenge of reliability of their spatial and tectonic qualifications. These challenges have played an important role on the innovative developments in building industry. With developments and technologies borrowed from other industries, it has become the question of computability rather than constructability for the designer.[1]

Computer Aided Design (CAD), Computer Aided Manufacturing (CAM) and Computer Aided Engineering (CAE) systems have long been used by aeroplane, shipbuilding and automobile industries both in design and production phase. Use of computer in architecture design and production phase happened to be in 1980's by some architectural offices for realization of some specific designs. Past two decades, there had been unusual developments in computer technology. Spread of computer related technologies in terms of hardware and software has resulted in evolutionary changes in designer's approach to computer. Today, CAD is not apart from the design process itself, and obviously, not just a tool for elaborately visualizing images of the end product, but rather a continuous, multidirectional feedback mechanism that connects various part of the design and construction phases.[2] Extensive use of parametric

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[1] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 31.

[2] Luca, F. D., M. Nardini. *Behind the scenes: Avant-garde techniques in contemporary design*. Birkhäuser, Berlin. 2002. p. 40.

technologies in CAD systems has created close relationship between various design phases and transformed digital design via modeling into a highly interactive process. The contact between digital and physical model developed by CAM and CAE systems, has opened up new potentials in design field. In other words, a reciprocal interaction between digital and physical environment has been constructed: any physical model, even the most complex forms, can be digitally realized on computers; and any digital model can be reproduced physically.

Effective use of CAD / CAM systems in design and production processes, resulted in developments in prototyping techniques. "Prototype is an approximation of a product (or system) or its components, in some form, for a definite purpose in its implementation."[3] The word prototype used in design, covers all kind of pencil sketches, mathematical models and of course functional physical models used in the product development process. Prototyping is the process of realizing these. There are three aspects of interest in the definition of prototype: ***implementation***, ***form***, ***degree of approximation***.

Implementation aspect of the prototype covers the range from prototyping fully functional complete product to prototyping a part or sub-assembly of a product.[4] A good example of the complete product prototyping is the one, which is tested by a group of specialist in order to identify outstanding problems before the design is finalized. Sub-assembly or partial product prototyping can be used to test the product for a specific purpose associated with one component. The second aspect, *form*, covers digital prototype and physical prototype.[5] Digital prototypes are mathematical models of the intended design and they base on the assumed principles. Thus, they will not be able to predict any unexpected phenomenon. Physical prototype, on the other hand, may even lead to aesthetical or human factor evaluation. The third aspect, *degree of approximation*, covers from a very rough representation to the exact replication of a

[3] Chua, C. K., K. F. Leong, C. S. Lim. *Rapid Prototyping: Principles and applications.* World Scientific, New Jersey. 2003. p. 2.

[4] Ibid. p. 3.

[5] Ibid.

product.[6] Rough prototypes, such as foam model, can be used for overall formal considerations and dimensional studies in the initial stages of the design. On the other hand, exact replications that models every aspect of the product becomes important at the end-stage of the product since it may even address manufacturing issues and concerns.

The roles that prototypes play in the product development process are several. They include the following: *experimentation and learning*, *testing and proofing*, *communication and interaction*, *synthesis and integration*, *scheduling and markers*.[7] To the design team, most appropriate and reliable way of testing various properties is building and studying prototypes. For instance, physical prototypes can be built in order to test the comfort of a chair when performing typical tasks. Alternatively, some rough prototypes can be used in order to proof the concepts of folding mechanism of a foldable chair. At a later stage of the design, a more accurate prototype can be built in order to communicate with the management or the client. There is nothing clearer than the precise physical prototype to communicate design ideas, since the management / client may have the full experience of the physical and tactile impression of the product. This interaction between the management / client and the product will provide valuable information for the improvement of the design. Talking about a foldable chair –or a more complex design product that requires collaboration of various disciplines–, prototyping is a very important design stage in order to integrate different aspects of design. The successful integration of the mechanical folding system into the aesthetically designed chair components is vital for the design process. Different aspects of design come together from different design department in the prototyping phase and a more interactive design platform is achieved to satisfy a variety of design problems. "Along the process, each prototype usually marks a completion of a particular development phase, and with proper planning, the development schedule can be enforced."[8]

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[6] Ibid. p. 4.

[7] Ibid. p. 5.

[8] Ibid. p. 7.

Independent of the prototype being complete or a part, physical or virtual, rough or accurate, prototypes do not necessarily serve all of these roles concurrently for a design team, but they are certainly a necessity in any product development project. Technological contribution of CAD / CAM / CAE to the prototyping –and inevitably to the design process– realized via different techniques. It is possible to group them in two directions according to the disposition of the final model –virtual or physical: from physical model to digital model and from digital model to physical model.

3.1 From Physical Model to Digital Model

"For some designers, such as Frank Gehry, the direct tactility of a physical model is a much preferred way of designing than a "flat" digital manipulation of surfaces on a computer screen."[9] However, the physical model has to be digitized for optimizing or analyzing the geometry. Consecutively, translation from physical model to digital medium becomes an inevitable part of design.

> In Gehry's case, the digital technologies are not used as a medium of conception but as a medium of *translation* in a process that takes as its input the geometry of the physical model and produces as its output the digitally-encoded control information which is used to drive various fabrication machines.[10]

The process of gathering surface data from a handmade physical model by sophisticated scanning techniques –which is often referred to as 'reverse engineering'– results in a *point cloud*, a pattern of points. A point cloud has the surface data of the scanned object, and it is then interpreted by the conversion software to produce a close approximation of the model's geometry. Acquiring the point cloud and its interpretation is referred as ***digitalization*** and ***mathematization*** respectively.

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[9] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 31.

[10] Ibid.

Digitalization is the process of recording the coordinates of series of point on the surface via position sensors. This is done by two techniques: *contact* and *non-contact*. The *contact technique* involves the use of digitizing position probe that traces the surface of the physical model and transmits the coordinates of the surface points to the computer. It can be derived manually by digitizing arms or automatically by Coordinate Measuring Machine (CMM) that is mechanically kept in contact with the surface to be scanned. Frank Gehry's office transmits the surface coordinates of the physical models by contact technique utilizing manually driven digitizing arms.

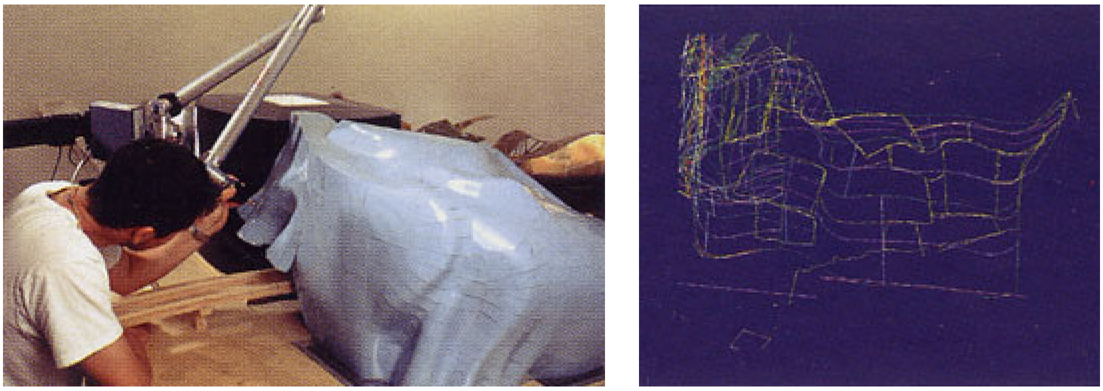

Figure 13. Images of the digitizing of the model for the design of the EMP by Frank. O. Gehry

(Luca, F. D., M. Nardini. *Behind the scenes: Avant-garde techniques in contemporary design*. Birkhäuser, Berlin. 2002.)

The *non-contact technique* is performed by using laser light that illuminates the surface of the scanned object, producing a pattern of bright dots, lines or grids, captured by optical sensors, which are later processed to construct the 3D photographic relief. Optical sensors capture four pieces of information from the reflected laser beam: X, Y, Z coordinates and its intensity. The complete digitization of the object is obtained by reassembling a series of photographic relief from various points of view.

Transformation of the point cloud obtained at the end of the digitalization process into a 3D digital model is known as ***mathematization***. Software that perform mathematization process, allows a certain level of editing to the model, such as correcting irregularities, diminishing some points in low level details, surface joining etc. After various operations, the point cloud is transformed into either a mesh model, which is defined by adjacent polygons, or a patch model, which is defined by continuous surfaces, Non-Uniform Rational B-Splines (NURBS).

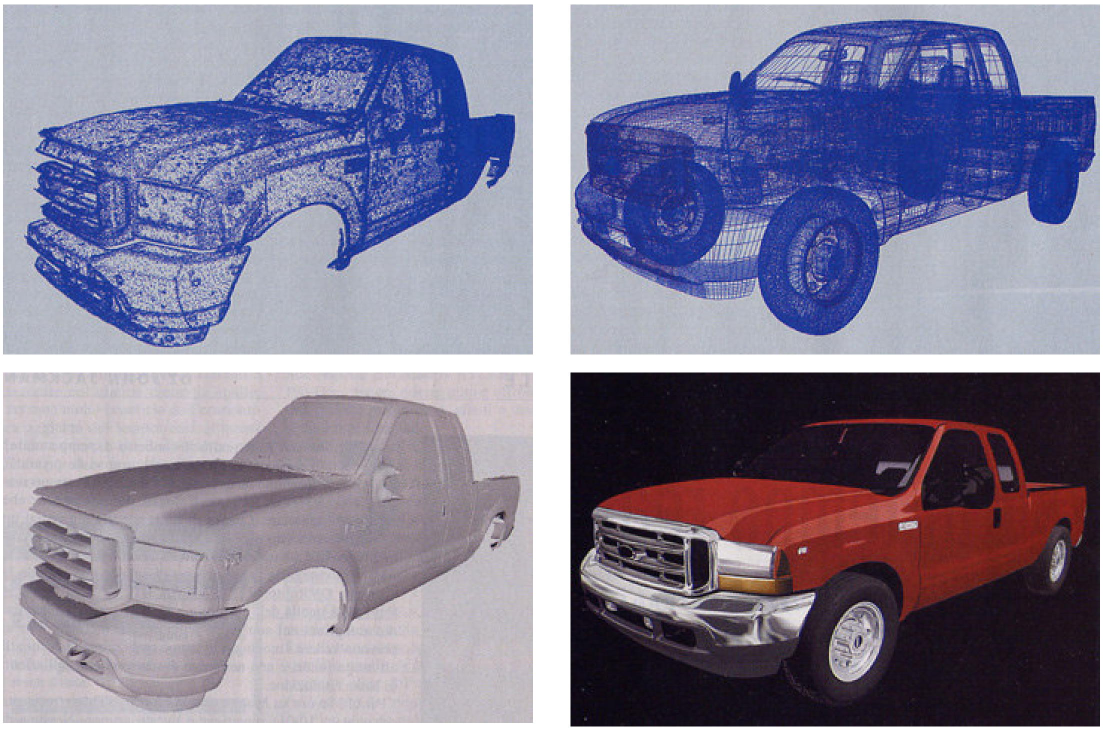

Figure 14. Digital reconstruction of a model, point cloud derived from 3D scanning, NURBS model, shading and texturing.

(Luca, F. D., M. Nardini. *Behind the scenes: Avant-garde techniques in contemporary design*. Birkhäuser, Berlin. 2002.)

The final digital prototype derived from 3D scanning has different applications. First, it constitutes the initial design phase and feeds it with a series of topological information through CAE. In the rest of the procedure, there is a continuous interaction between physical and the digital model. Another application is the ability to reuse the document repeatedly without the risk of deterioration as in the case of physical model.

Three-dimensional (3D) scanning techniques may also be used to capture the existing conditions or even the landscapes digitally. They are being used increasingly on construction sites in place of conventional measure devices for quickly measuring distances and precisely determining the locations for installations of various building components.

3.2 From Digital Model to Physical Model

Architects have been using CAD systems for the last two decades. It is only in the last few years however that the CAM functionalities that we see in industrial production are being utilized significantly in architectural design.

> Knowing the production capabilities and availability of particular digitally driven fabrication equipment enables architects to design specifically for the capabilities of those machines. The consequence is that architects are becoming much more directly involved in the fabrication processes, as they create the information that is translated by fabricators directly into the control data that drives the digital fabrication equipment. For instance, the irregularly shaped glass panels on Frank Gehry's Nationale-Nederlanden Building in Prague, Czech Republic, were cut using digitally driven cutting machines from the geometric information extracted directly from the digital model.[11]

Physical reproduction of the digital model can be done by different techniques, according to its characteristic and topological properties. ***Two-dimensional fabrication***, ***formative fabrication***, ***subtractive fabrication*** and ***additive fabrication*** are the four fundamental fabrication processes.

3.2.1 Two-Dimensional Fabrication

Two dimensional (2D) cutting, or CNC cutting, is the most common way of digital manufacturing. Among various cutting technologies, e.g. laser beam, water jet, and plasma-arc, the cutting principle depends on the relative motion of the cutting head to the sheet material on two axes. Choice of the technique is related to the physical and chemical properties of the sheet material.

[11] Ibid. pp. 32-33.

Laser beam cutting is usually preferred with materials that can absorb light energy with thicknesses up to 16 mm. A high-intensity focused beam of infrared light and a jet of pressurized gas (carbon dioxide) is used to melt or burn the material sheet. Water jet cutting involves highly pressurized water mixed with solid abrasive particles, forced through a nozzle, producing very clean and accurate cut with the rapid erosion of the material in its path. Almost any material can be cut with this technique. Plasma-arc cutting is achieved by passing an electric arc through compressed gas jet, heating the gas into plasma state with very high temperatures (14000°C), which converts back into gas when it transfers the heat to the cutting zone.[12]

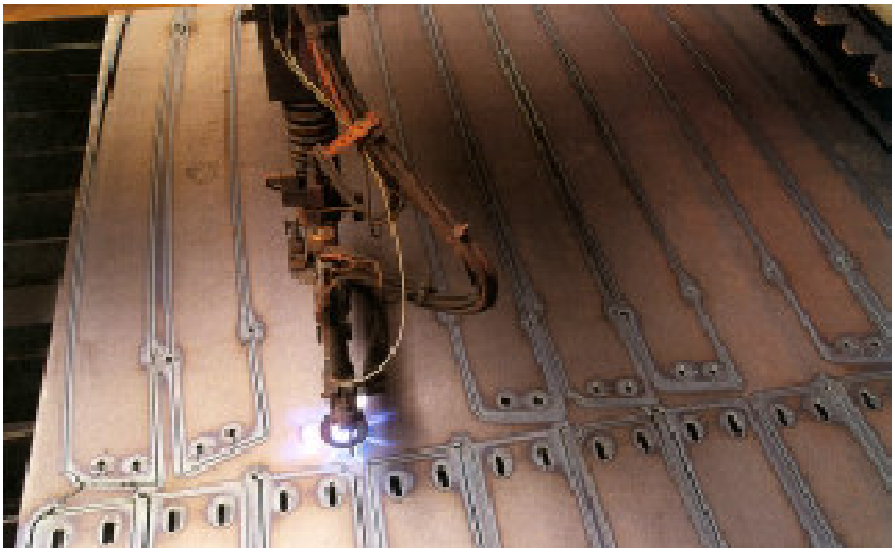

Figure 15. CNC cutting of steel supports for masonry walls in Zollhof Towers

(Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). Branko Kolarevic, ed. Spon Press, New York. 2003.)

This kind of fabrication has many applications in architecture. Specific surfaces that will be introduced in next chapter (such as developable or ruled surfaces) can be produced by means of cutting 2D flat pieces of metal. Proper positioning and combining of these flat sheets achieves 3D curvilinear shape. In other words, curvilinear surfaces and even their supporting frames can be cut from 2D planes. The aluminum

[12] Ibid. p. 34.

frame of BMW Pavilion Frankfurt, Germany is cut directly from the digital data using CNC water-jet technology.[13]

3.2.2 Formative Fabrication

In formative fabrication, material is given shape by mechanical forces, restricting forms, heat of steam by deforming it, which can be axially or surface constrained. Deformations can be done permanently by different processes such as stressing the metal past its elastic limit, heating and bending it when it is in softened state, steam bending boards etc. Approximation of double-curved surfaces can be done by arrays of height adjustable numerically controlled pins.[14]

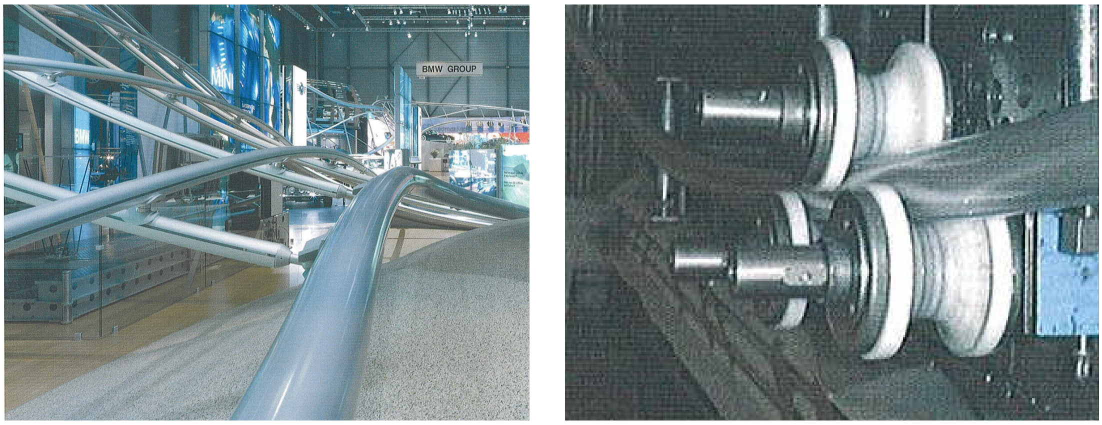

Figure 16. CNC bending of the aluminum profiles for the BMW Pavilion Geneva, Switzerland

(Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). Branko Kolarevic, ed. Spon Press, New York. 2003.)

3.2.3 Subtractive Fabrication

Subtractive fabrication involves the principle of removing specified part of a solid using electro-, chemically- or mechanically-reductive milling processes. The milling can be axially, surface or volume constrained.

[13] Ibid.

[14] Ibid. p. 38.

In axially constrained (one axis) milling the material that is going to be milled has one axis of rotational motions, and the milling head has two transformational motions. Surface constrained (two axes) milling machines, has the ability two move the milling head along X and Y-axes to remove 2D patterns of material. By adding the milling head the ability to move in the third axis, Z, volume constrained (three axes) milling of a solid becomes possible. However, with 3-axes milling machines undercuts cannot be accomplished, and the range of producible shapes with such machines is limited. If the 4^{th} and 5^{th} axes rotation abilities are added to the milling head, the range of shapes that can be produced is increased tremendously.[15]

Drill bits used in the milling can be of different dimensions. Whereas large diameter bits are used for course removals, smaller bits are necessary for the finishing. Rotational speed of the bits may also vary according to the properties of the material that is milled.[16]

Subtractive fabrication technique can be used in different scales. In early 1970s, experiments of CNC milling machines in the field of architectural modeling were carried out in United Kingdom. “Large architectural firms in the United States, such as Skidmore, Owings and Merrill’s (SOM) office in Chicago, have used CNC milling machines and laser cutters extensively in the production of architectural models and studies of construction assemblies.”[17] Afterward, automated milling machines of 1980s were used to produce construction components, such as columns for the Sagrada Familia Church in Barcelona, in 1990s. In Gehry’s project in Düsseldorf, Germany (Zollhof Towers), this fabrication technique was used to produce load-bearing external wall panels, made of double-curved reinforced concrete. Blocks of lightweight polystyrene were CNC milled to produce 355 different curved molds for the casting of the concrete.[18]

[15] Ibid. p. 34.

[16] Ibid.

[17] Ibid. p. 35.

[18] Ibid. p. 36.

Figure 17. Fabrication of concrete panels for Zollhof Towers

(Raghep, J. F., K. W. Weg. *Frank Gehry, architect.* Guggenheim Museum Publications, New York. 2001.)

3.2.4 Additive Fabrication

Additive fabrication is a process of forming the model by adding material, opposite to the milling, layer by layer. There is the same principle behind all various kinds of additive fabrication: the digital model is sliced into 2D layers. After the transfer of 2D layer data, physical product is generated incrementally with different base material and different techniques of solidifying according to the base material.

Classification of different techniques is broadly done according to the initial form of its material. In this manner, they can be easily categorized into liquid-based, solid-based and powder-based.[19] In liquid-based systems, the liquid material is solidified through a process known as curing. Curing can be achieved by Single Laser Beam,

[19] Chua, C. K., K. F. Leong, C. S. Lim. *Rapid Prototyping: Principles and applications.* p. 19.

Two Laser Beam or Masked Lamp Method. They are all concluded as Photo-curing Methods.[20] In solid-based systems, solid material –in one form or another– is utilized as the primary medium to create the prototype. The form of the solid can be a wire, a roll, laminates and pellets. Two possible methods for solid-based systems are Cutting and Gluing / Joining Method and Melting and Solidifying / Fusing Method.[21] Powder in grain-like form is created as another category outside the solid based systems: powder-based systems. These systems use materials in the form of powder that all employs Joining/Binding Method.[22]

Stereolitrography is based on a liquid polymer that solidifies when exposed to laser light. Laser beam traces the cross-section of the model into the light sensitive polymer. That thin layer of solidified polymer is than lowered by a small increment depending on the thicknesses of the sections and more material is emitted to fill the missing parts. The process continues until the entire model is 'printed', submerged into the base material. After the cure of the model to remove waste liquid and to give extra rigidity, the prototype is completed.[23]

Choice of base material and solidifier may change according to the purpose of use. For instance, in Selective Laser Sintering (SLS), metal powder is solidified by laser beam, that may turn into a 1:1 scale solid object. Plaster or ceramic powder and glue in 3D Printing, sheets of paper or plastic and glue in Laminated Object Manufacture (LOM), are examples of base material and solidifier. Loss of heat is also used a solidifier in some techniques such as Multi Jet Manufacture (MJM) technique in which a print head deposits melted thermoplastic wax material is used. By adding thin layers of material each time, a 3D solid is created and the solidification occurs upon cooling.

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[20] See The Rapid Prototyping Wheel in: Chua, C. K., K. F. Leong, C. S. Lim. *Rapid Prototyping: Principles and applications.* World Scientific, New Jersey. 2003. p. 12.

[21] Ibid.

[22] Ibid.

[23] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 36.

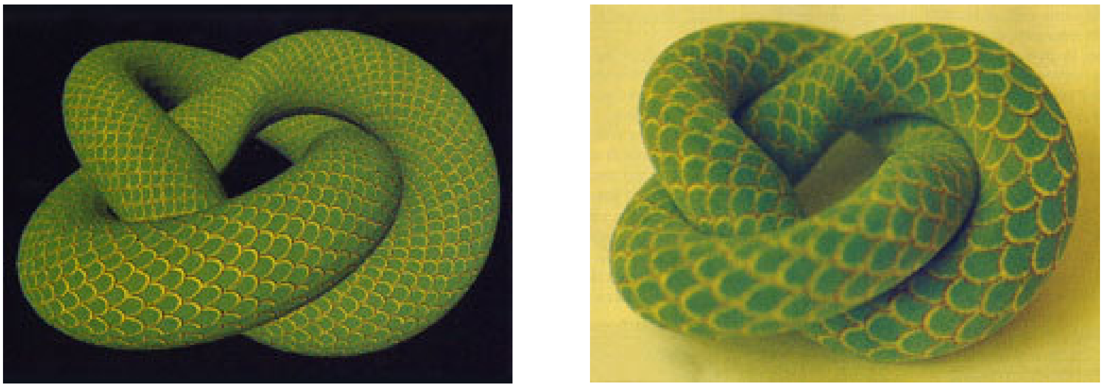

Figure 18. *Trefoil Umbilical Torus,* a mathematical sculpture by S. Dickson, built with Stereolitrography

(Luca, F. D., M. Nardini. *Behind the scenes: Avant-garde techniques in contemporary design*. Birkhäuser, Berlin. 2002.)

Despite the unlimited range of typologies that may be produced as such, limited size of the objects and costly equipments of the fabrication process, this technique becomes rather unusable in the field of building design and manufacturing. In design phase, they are generally used for producing scaled mass models of complex geometries. The scale model can be utilized for either visualizing massive qualities or checking the integrity of the digital model. In construction, there are examples that additive fabrication technique is used for manufacturing truss elements.

CHAPTER 4

EVOLVING DESIGN PROCESS

Design is practiced by a large group of professionals. A design is usually produced to satisfy the need of a particular person or group. Therefore, to design something, a designer needs to know the problem constraints and then propose a solution that will operate within the limits of these constraints. Along the process, it will be apparent that there exists merely one solution or perhaps none does. Optimum design solution takes shape with further information in terms of social and economic variables pertaining to the use of the product being designed. On the factors that the constraints are unknown, the designer faces decisions, which requires experience and professional judgment.[1]

Conventional engineering design phases are defined in four steps by Amirouche as follows: *Problem definition* is the first step in the design process. A well-defined problem leads the designer to a successful solution. It should be clearly stated in order to avoid unnecessary constraints and to focus on what has to be done. *Conceptualizing* is the second step of the design process, which requires ingenuity, experience, and knowledge. This stage may involve making of concept models to work out what the design should look like. *Synthesis* is the third stage of design process that involves taking elements of the concept and arranging them in the proper order. A successful and effective design relies a great deal on the synthesis aspect of the problem. The information required for the proposed conceptualization is organized and a plan devised for achieving that particular design. *Analysis* is concerned with the mathe-

[1] Amirouche, F. M. L. *Computer-aided design and manufacturing*. Prentice-Hall, Inc., New Jersey. 1993. pp. 45-46.

matical or experimental testing to make sure it meets the criteria set forth in the problem definition. The engineer must test all possible factors important to the design.[2]

This process is relatively applicable to any field of design; however, design tools and constraints may vary from one discipline to the other. Design tools and constraints also diverge relative to the geographic location, cultural / professional habits and practices, and apparently, time. "The landscape of every architect's office has changed over the past 20 years. Gone is the gentle squeak of Rotring pen on Mylar or tracing paper to be replaced by the hum of computers and the intense clicking of mice."[3] The technological developments in the field of CAD and CAM have revolutionized the procedures used in conceptualizing and designing. The computer, being a representation tool for the offices, has become involved in any stage of the realization process from design to manufacture. No other medium is required between design and manufacture processes, since the digital design data is directly transferred and used in the manufacturing systems. The compatibility of the data decreases the possibilities of erroneous information transfers and communication problems between the designer and the manufacturer.

Capabilities of the designers increased tremendously allowing them to discover geometries other than the Euclidian geometry. New forms that were once very difficult to conceive, develop and represent –and almost impossible to manufacture– are introduced into the designers' terminology[4]. Obviously, the designer has to reveal the fundamental understanding of the digitally driven design and production technologies without being possessed by the formal aesthetics of blobby forms.

> The "blobby" aesthetics, which seem to be pervasive, in the projects of the avant-garde, are often sidetracking the critical discourse into the more immediate territory of formal expression and away

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[2] Ibid. pp. 47-48.

[3] Stacey, M., P. Beesley, V. Hui. *Digital fabricators*. University of Waterloo School of Architecture Press, Cambridge. 2004. p. 6.

[4] *Möbius House*, 1995 by UN Studio and *Torus House*, 2001 by P. S. Cohen are examples of project that are inspired –and named– by topological forms of non-Euclidian geometry: Möbius strip and torus.

> from more fundamental possibilities that are opening up, such as the opportunity for architects to reclaim the lost ground and once again become fully engaged in the act of building (as information master-builders). This is not to say that the profession should not maintain a critical attitude towards the potentiality of the digital, but that it should attempt to see beyond the issues of the formal aesthetics.[5]

Beyond modeling tools that permit flexible and expressive forms, software provides the user, time dependent simulations, which are utilized by different architectures. Interaction with other industries –aerospace, automotive and shipbuilding in the case of modeling and film industry in the case of animation– has been at the core of many successful advances.[6]

Figure 19. Torus House

(Cohen, P. S. 100 architects, in: *10x10*. (pp. 109-111). Iona Baird ed. Phaidon Press Ltd., London. 2000.)

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[5] Kolarevic, B. Digital Morphogenesis, in: *Architecture in the digital age: Design and manufacturing*. (pp. 11-28). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 27.

[6] Kolarevic, B. Introduction, in: *Architecture in the digital age: Design and manufacturing*. (pp. 1-10). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 10.

4.1 Digital Design Strategies

Computer use was limited to the representational purposes in architecture; however, with the progress of applications, today, it is not only a part of design process but also a part of manufacturing process. Architecture is passing through pioneer phase of CAD applications and moving toward a more mature phase. On one hand, real possibilities of digital era are being explored and, on the other, the results of these possibilities have begun to be translated into material.[7]

Everyday architects are experimenting formal and conceptual designs to attain the essence of the new architectures. Design tools of other industries are being explored by this projects and a future conception of architecture is being experienced.

4.1.1 Interactive Membranes: Polysurfaces

Exterior aspect of the architectural works has long been important, if not predominant. As early as the Baroque era, masterpieces of architecture were produced in which the value of the façade was perceived as not only a celebrative instrument but also a membrane, capable of spatially creating a relationship between interior and exterior environments.[8]

“The term ***polysurfaces*** attempts to explain a topological surface characterized by a high complexity that can be represented only in the 3D environment of a modeling program since it is the expression of particularly complex mathematical equations and not parameters.”[9]

> Greg Lynn in his book Animate Form points out that it is due to animation software that architects are able to “sketch with calculus” for the first time. This has had a liberating effect on the design

[7] Luca, F. D., M. Nardini. *Behind the scenes: Avant-garde techniques in contemporary design*. Birkhäuser, Berlin. 2002. p. 21.

[8] Ibid. pp. 22-23.

[9] Ibid. p. 25.

process, allowing architects to work on topological surfaces with increasing levels of complexity and a smooth connection between landscape and building.[10]

Among various techniques for generating polysurfaces, the most flexible method is the creation of surfaces through the use of NURBS (non-uniform rational B-splines). NURBS have long been used by many CAD systems as the mathematical expression of surfaces.[11] The potential of NURBS tool lies in the modifiability of the surfaces. After the surface is created, it can easily be edited via control vertices, weights and knots of the individual curves. With this technique, it is possible to generate architectural coverings as topological surfaces, making it easy to produce variations and deformations depending on the interior characteristics or exterior situations.[12]

4.1.2 Spatial-Temporal Alterations: Keyframing, Morphing, Inverse Kinematics

The techniques of keyframing, morphing, and inverse kinematics are basic introductions to the modeling software that add a fourth variable to the three traditional spatial variables: *time*. The concept of time in the design process was only used for representational purposes at the beginning. With the advancement of systems, animation increased the level of complexity in designs with ever-growing heterogeneity of functional and spatial programs. Moreover, it triggered a series of new relationship between the architectural organism and several design factors such as public use, the site and the program. Use of animation in the design process allows us to observe modifications of various characteristics of architectural organism over time, due to several factors that can be inserted with their relative values depending on the time factor. Once different configurations of the design at different times are obtained, they can be utilized as elements of a new design perceptive. These still images of the design process imply that, animation does not necessarily mean movement. “Animation is a term that differs from, but is often confused with, motion. While motion im-

[10] Imperiale, A. *New Flatness: Surface tension in digital architecture*. Birkhäuser, Berlin. 2000. p. 42.

[11] Chiyokura, H., T. Takamura, K. Konno, T. Harada. G^1 surface interpolation over irregular meshes with rational curves, in: *NURBS for curve and surface design.* (pp. 15-34). Gerald Farin, ed. Society for Industrial and Applied Mathematics, Philadelphia. 1991. p. 15

[12] Luca, F. D., M. Nardini. *Behind the scenes: Avant-garde techniques in contemporary design*. p. 25.

plies movement and action, animation implies the evolution of a form and its shaping forces; it suggests animalism, animism, growth, actuation, vitality, virtuality."[13]

Animation is based on human perception. A series of related still images are perceived as continuous motion by the human brain if viewed in a quick succession. Each image is called *frame*.

Historically, the major difficulty in creating an animation has been that the animator had to produce large amount of frames. Depending on the quality, one minute of animation might require 720 to 1800 still images. However, most of the frames are routine, incremental changes from the previous one. Thus early animation studios realized that they could increase the productivity of the master artists by having them draw only the important frames, called *keyframes.* Assistants could then figure out the frames that were required in between the keyframes. These frames are called *tweens*.[14]

The historical background makes the ***keyframing*** method in the digital modeling software explicable. To create an animation, software produces a sequential series of transformations, corresponding to single frames that give the illusion of fluid transitions when reproduced at high speed. Keyframes represent particularly important moments. Data related to the design models is input by the user, describing the properties at various states. The software then creates an interpolation of values, generating the transitions between the keyframes. The fundamental issue is to input the parameters of the keyframes.[15]

Morphing is a term derived from metamorphosis, which means to change physical form. This technique transforms one form into other making a fluid transition between them. By defining several destinations, the key passages that the form must to through in its transformation cab be established. These destinations are obtained by

[13] Lynn, G. *Animate Form*. Princeton Architectural Press, New York. 1999. p. 9.

[14] See Autodesk *3ds Max software help contents*. 2005.

[15] Luca, F. D., M. Nardini. *Behind the scenes: Avant-garde techniques in contemporary design*. p. 27.

creating copies of the initial form and deforming them via parametric deformers or directly by controlling the mesh.[16]

Inverse kinematics is used to transmit the movement of an object, called *children*, to the principal object, called *parents.* As opposed to the *forward kinematics*, which transmits the movements of parents to the children and so can be utilized for simulating even complex mechanical devices, inverse kinematics is more adapted to reproducing the behavior of skeletal structures where the actions of a terminal part, the hand for example, determine the movement of the rest of the structure up to the principal still body.[17]

4.1.3 Dynamic Physical Interactions: Metaballs, Particle Systems / Space Warps, Dynamic

Metaball techniques, particle systems and dynamics are great innovations introduced by new animation software. These techniques fully exploit the true potential of the calculation engines. Computer unifies the simple functions of animation with more complex ones, dealing with relations between the elements. The intermediate or final configurations created through the interaction of elements in the scene, can be an extraordinarily important contribution especially in the creation and conceptualization phases of the design process.[18]

> Interactivity is key for these tools. If we consider an architectural organism as an aggregation of functions and a collection of relationships, then it would be interesting to reorder those mental processes we use to establish the relationship between these functions and the entity of the relations, from the overall view of an evolutionary system subject to modifications and alterations found in an apparently calm state. This occurs using a map sensitive to the interactions. Each change in the individual parts of this map influences the relationships of the whole. So the system is evolutive to

[16] Ibid. p. 30.

[17] Ibid. p. 31.

[18] Ibid. p. 33.

meet the needs of the moment. As it interacts, it adapt to environmental conditions just like real organism.[19]

Metaballs help control the simulation by making the elements react among themselves and generate fluid forms capable of simulating liquids, viscous substances, molten metals, etc.

Figure 20. The Bubble, BMW's exhibition pavilion by Bernard Franken and ABB Architekten

(Kolarevic, B. Digital Morphogenesis, in: *Architecture in the digital age: Design and manufacturing.* (pp. 11-28). Branko Kolarevic, ed. Spon Press, New York. 2003.)

Particle systems can either simulate elements such as snow, dust, spray and others or represent geometric primitives, blobs in automatic fusion and complex objects. The particle system icon is represented by a reference plane and a directional vector. It is managed by a large number of parameters such as quantity, size, motion, type, time intervals, etc. Flows can be generated and their behavior studied in relation to the application of the ***space warp***. Space warps allow a set entity or region of space to be assigned the characteristic of inducing particular deformations and effects in the object found, either permanently or temporarily, within their range of action. A large quantity of space warps exists such as gravity, wind, waves, and collisions.[20]

[19] Ibid.

[20] Ibid. pp. 35-38.

Dynamics is defined as the science of movement that takes into account the mass, elasticity, roughness, etc. of an object and the forces applied to it. This technique allows creating realistic motion by using the rules of physics to simulate natural forces.[21] Used especially in other industries for studying aerodynamic properties, automobile crash tests, just to name few, this tool is being used by architects still in the early experimentation phases.

4.2 Surface Strategies

"Architects today digitally create and manipulate NURBS surfaces, producing building skins that result not only in new expressive and aesthetic qualities, but also in new tectonic and geometric complexities."[22] In the exploration of new territories of the new digital avant-garde, the exterior surface of the building becomes emphasized due to the logics of the formal conception inherent in the NURBS-based software.

A new understanding of surface tectonics is established as a result of examinations in constructability of complex envelopes. "The building envelope is increasingly being explored for its potential to reunify the skin and the structure in opposition to the binary logics of the Modernist tectonic thinking."[23] The structure becomes embedded into the skin, creating self-supporting units that require no armature. The consequences are not only the search of new materials but also new geometries such as curves and folds that would enable the continuous skin to act structurally.

> In some ways the search for a material and form that unifies structure and skin is a counterrevolution to Le Corbusier' Domino House, in which the master separated structure from skin. The new conflation is a return to the bearing wall, but one with freedoms that Corb never imagined possible. Architects could build many more exciting buildings on the Statue of Liberty paradigm, but

[21] See Alias. *Maya Personal Learning Edition software help contents*. 2005.

[22] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 39.

[23] Ibid.

> complex surfaces with integrated structures promise a quantum leap of engineering elegance and intellectual satisfaction.[24]

The technique of unifying, which is a radical departure from the Modernism's ideals, did not only provide enclosure and structural support but also contained other systems typically placed into ceilings and floors.

Other less radical strategies involve the offsetting of the structure from the skin, which is a clear departure from the *primacy of structure* logics of the Modernism, a distinct separation of the structure, where juxtaposition can produce potent visual interplays, and a more conventional approach, where the sinuous skin is attached to conventionally conceived structural grid. Each of these approaches to the concept of skin and structure are perfectly valid and each has different repercussions of the development of the project relative to its overall cost and desired spatial qualities. The digital technologies enable architects to achieve exact control over the process by precisely controlling the geometry, thus the budget.[25]

4.3 Production Strategies

"The production strategies used for two-dimensional fabrication often include contouring, triangulation (or polygonal tessellation), use of ruled, developable surfaces and unfolding."[26] Each of these techniques involves the extraction of 2D planar parts from complex shaped building forms. The challenge in this process is choosing the appropriate geometric approximation method that will preserve the essential qualities of the initial form.

Contouring can be used to articulate the structural system of complex forms. A parallel sequence of planar sections that are placed at regular intervals defines the con-

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[24] As quoted from Giovannini J. (2000), by Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 41.

[25] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). pp. 42-43.

[26] Ibid. p. 43.

tours of the form. Conceptually, contouring is identical to a process called *lofting* in shipbuilding. Ships hulls are constructed in the lofts with the help of a sequence of planar lateral cross-sections that become ribs mounted on a spine that runs lengthwise.

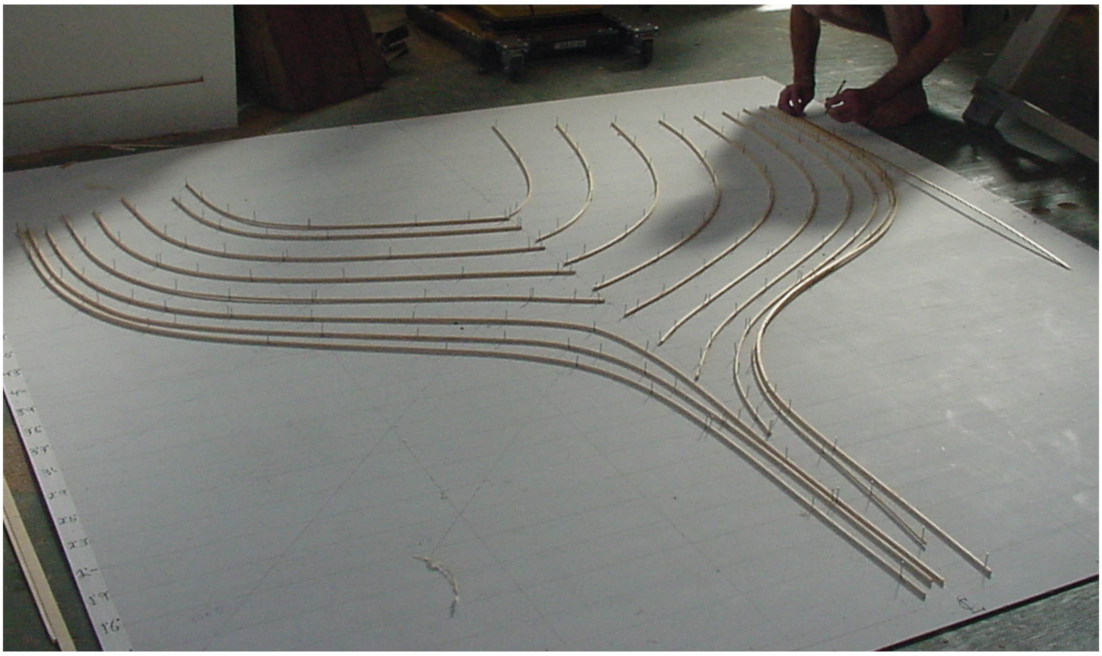

Figure 21. Contours of a ship hull

(IYRS. *International Yacht Restoration School albums*. http://www.iyrs.org/albums/ 17.jpg. Last accessed in December 2005.)

The wireframe cross-sections can be further manipulated to create complete abstraction of the building's structural framework. This information can easily be used by structural analysis software to produce the precise definition of every structural member. Furthermore, same software may produce the fabrication drawings, or CNC data to cut various components.

Extracting the isoparametric curves that are used to aid in visualizing the NURBS surfaces is another technique of contouring. However, due to the budgetary or other production-related restrictions, complex geometry of NURBS curves can be approximated with circular, radial geometry, which can be inexpensively manufactured using rolling machines.[27]

"Complex, curvilinear surface envelopes are often produced by either triangulation or some other planar tessellation, or by the conversion of double-curved into ruled surface, which are generated by linear interpolation between two curves."[28] The acquired surfaces are then unfolded into planar strips that are laid out as 2D shapes on a metal sheet to be cut by one of the CNC cutting techniques introduced in chapter 3.

Triangulation or other ***planar tessellation*** techniques involve the representation of a complex surface in terms of patches of different geometries and size. Sophisticated modeling programs offer the user rich options of tessellations, allowing them to control not only the geometry of the patches but also their minimum and maximum size. Small sized patches result in smoother surfaces and probably higher costs and vice versa. Through the use of different tessellation parameters, the designer may explore various approximation strategies interactively to match cost and production constraints. Sydney Opera House (1973) by Jørn Utzon and Great Court in the British Museum by Foster and Partners are well known examples of the polygonal tessellation.

Conversion of the complex double-curved surfaces to ***ruled surfaces*** is another method of rationalizing the production process. A wide variety of surfaces, including cones, cylinders as the simplest ones, can be generated by linear interpolation of two distinct curves. The fact that makes ruled surfaces to be used extensively in contemporary architecture is that they can be *developed*, which means they can be digitally fabricated out of flat sheets without deformation.

Whether a particular building envelope is produced as a developable or doubly curved surface can be determined by applying the ***Gaussian analysis*** to the surface model. Use of Gaussian analysis is particularly significant if the digital model is ac-

[27] Ibid.

[28] Ibid. p. 44.

quired by digitalization of a physical model –as in the case of Guggenheim Museum Bilbao, or other projects by Frank Gehry– since determination of the degree of the curvature is not possible in the physical models. The Gaussian analysis evaluates the degree of curvature in the surface and produces colored images that indicate the extent of the surface curvature through various colors. This gives feedback to the designer for changing the curvature of the highly curved parts without disturbing the overall design. Developable surfaces, for example, have zero Gaussian curvature at every point on the surface, because they are linear in one direction.[29]

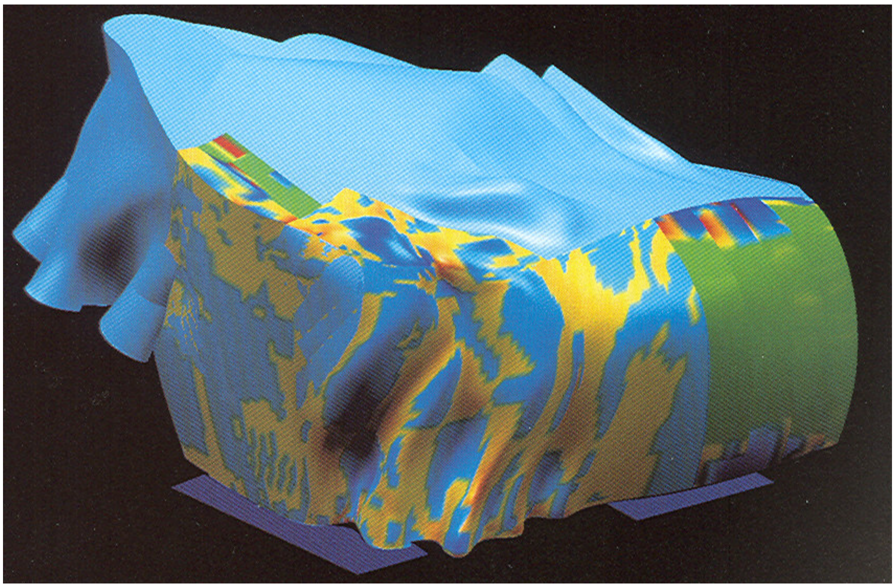

Figure 22. Gaussian analysis of EMP project designed by Frank O. Gehry

(Raghep, J. F., K. W. Weg. *Frank Gehry, architect.* Guggenheim Museum Publications, New York. 2001.)

> The sparse geometries of the twentieth century Modernism were, in large part, driven by Fordian paradigms of industrial manufacturing, imbuing the building production with the logics of standardization, prefabrication and on-site installation. The rationalities of manufacturing dictated geometric simplicity over complexity and the repetitive use of low-cost-mass-produced components. But these rigidities of production are no longer necessary, as digitally controlled machinery can fabricate unique, complexly shaped components at a cost that is no longer prohibitively expensive. Variety,

[29] Ibid. p. 47.

> in other words, no longer compromises the efficiency and economy of production.[30]

The concept of ***mass customization*** is introduced by the ability to produce highly differentiated building components with the same facility as standardized parts. Producing a thousand unique components is as easy and cost effective as producing a thousand identical components with the CNC technologies introduced in chapter 3. The concept of mass customization affected almost every segment of the industrial production. World famous brands offer their clients *customized product* at a cost slightly more than standard products. “The notion that uniqueness is now as economic and easy to achieve as repetition challenges the simplifying assumptions of Modernism and suggests the potential of a new, post-industrial paradigm based on the enhanced, creative capabilities of electronics rather than mechanics.”[31]

4.4 New Materiality

The problem of constructability of the geometrically complex envelopes has led to the rethinking of the surface tectonics. New design approaches that the principle idea was to conflate the skin and the structure in single element, which do not require additional supporting elements emerged. That prompted a search for new building materials, such as high temperature foams, rubbers, plastics and composites, which were, until recently, rarely used in the building industry.[32] The emphasis on the new surface articulations is depending on the possibilities and the resistance offered by the material technology. New found or adopted materials are offering new surface effects, interactive and dynamic reactions, and unprecedented thinness. For instance, the titanium skin that covers the Guggenheim Museum in Bilbao has the thickness of only 0.38 mm. and has the most impressing daylight effects throughout the day. It is not only the thinness that derives the increasing interest on the building materials.

[30] Ibid. p. 52.

[31] As quoted from Slessor, C. (2000), by Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 53.

[32] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 39.

The mechanic and electronic systems that are embedded in the composite building skins are able to create dynamic behavior as a response of outside stimuli. As a consequence of the new understanding regarding the concept of the surface, architects have to tackle with the material and construction aspects besides formal concerns.

> The problem today is that you can build everything in the computer, but if you want to construct these things in reality, the gap you have to bridge is very complicated. In general, I think that projects become much more interesting if they take the geometrical qualities of materials into account. If architects do not try to feed material constraints into software, they become moviemakers or image manipulators instead of designers who actually construct things.[33]

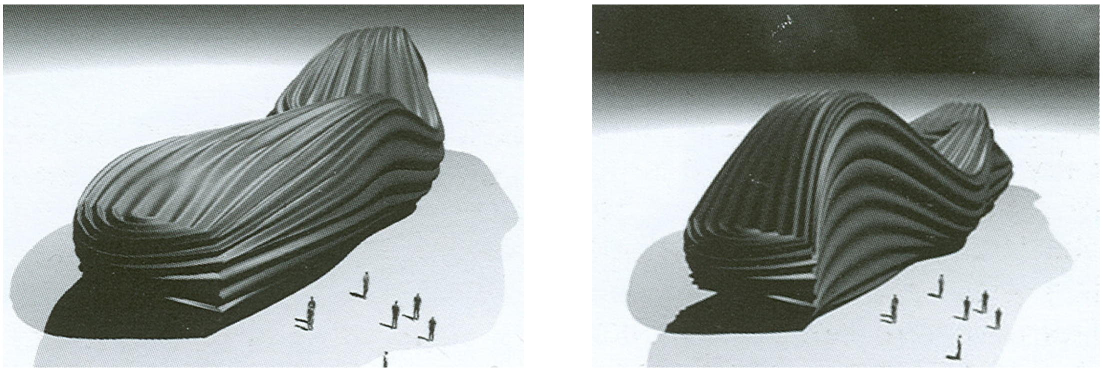

Figure 23. The Trans-Ports 2001 designed by Oosterhuis Associates, is modified via sensors and automatic mechanisms in the building.

(Luca, F. D., M. Nardini. *Behind the scenes: Avant-garde techniques in contemporary design*. Birkhäuser, Berlin. 2002.)

Construction of complex shaped forms aroused particular interest for liquid materials, such as composites, among architects. Composites[34] that have two primary components –reinforcement and matrix– offer improvement in performance that is supe-

[33] Zaera-Polo, A. Knowledge of reality: Interview with Alejandro Zaera-Polo, by Olv Klijn, in: *Concrete design book on robustness.* (pp. 96-99). Siebe Bakker ed. ENCI Media, the Netherlands. 2004. p. 98.

[34] Composites are defined as “materials consisting of two or more distinct phases on a macroscopic scale, which were produced by the combinations of materials put together to accomplish a specific purpose”, in: Yun, Y. G., D. L. Schodek. Development of boundary structures for complex-shaped buildings, in: *Journal of Architectural Engineering.* (pp. 18-25). 2003, March 3. p. 20.

rior to those of the original components. Matrix is typically, metal, ceramic or polymer, whereas reinforcement is fiber, particle or whisker. Most familiar reinforcements are glass, carbon, and polyethylene. Various chemical components may as well be added in order to attain desired color or to improve fire or thermal performance.[35]

Composites, in general, manifest high strength-to-weight ratios.[36] Flexibility of surface and color variations provides superior material potentials for the designers. They require minimum maintenance and relatively low cost.

"It is the *functionally gradient polymer* composite materials that offer the promise of enclosures in which structure, glazing and mechanical and electrical systems are synthesized in to a single material entity."[37] Even the variation of quantity and the pattern of the reinforcement exhibit different material properties.

New skins responses various environmental influences such as light, heat or movements. They are equipped with sensor and digitally controlled pistons that provide a real-time intelligent behavior. Intelligence of the skin, in other words, is achieved through the use of electronic ad mechanic parts embedded into the layers of the skin. In material science, *intelligent*, *smart* and other terms are used to describe a higher form of composite materials that have sensing, actuation, control and intelligence capabilities. These materials have their own sensors, actuators, and computational and control firmware built into their layers. According to another definition, intelligent materials are those materials that possess adaptive capabilities to external stimuli through built-in *intelligence* that can be programmed through its composition, its mi-

[35] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 50.

[36] Specific tensile strength (ratio of material strength to density) is approximately four or six times greater, and the specific modulus (ratio of material stiffness to density) is three or five times greater than those of steel or aluminum; as stated in: Yun, Y. G., D. L. Schodek. Development of boundary structures for complex-shaped buildings, in: *Journal of Architectural Engineering.* (pp. 18-25). 2003, March 3. p. 21.

[37] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 50.

crostructure, or by conditioning to adapt in a certain manner to different levels of stimuli.[38]

> The intelligence of the material can be limited to sensing or actuation only. For example, a sensory material is capable of determining particular material states or characteristics and sending an appropriate signal; an adaptive material is capable of altering its properties, such as volume, opacity, color, resistance, etc. in response to external stimuli. An active material, however, contains both sensor and actuators, with a feed back loop between the two, and is capable of complex behavior –it can not only sense a new condition, but can also respond to it.[39]

Some of the intelligent materials are capable of sensing the temperature and stress change through the embedded sensors, which can be utilized to monitor the stresses and detect potential damage of the concrete structures. By producing materials in a layer-by-layer fashion, as in additive fabrication, it is possible to embed various functional components, thus making them an integral part of a single, composite material.[40]

> Other potential applications of smart materials that would be enabled by 2015 include: clothes that respond to weather, interface with information systems, monitor vital signs, deliver medicines, and automatically protect wounds; airfoils that respond to airflow; buildings that adjust to the weather; bridges and roads that sense and repair cracks; kitchens that cook with wireless instructions; virtual reality telephones and entertainment centers; and personal medical diagnostics (perhaps interfaced directly with medical care centers). The level of development and integration of these technologies into everyday life will probably depend more on consumer attitudes than on technical developments.[41]

All these developments will radically redefine the relation between the architecture and the material reality in the 21st century. Designers will respond dynamically to the

[38] Ibid. p. 51.

[39] Ibid.

[40] Ibid.

[41] Antón, P. S., R. Silberglitt, J. Schneider. *The global technology revolution: Bio / nano / materials trends and their synergies with information technology by 2015*. RAND, California. 2001. p.20

internal logics and external influences of the environment. “Designs are already *alive* –the buildings will soon be as well.”[42]

Nanomaterials, produced by nanotechnology[43] are referred as the future of material technology. By manipulating materials in molecular scale, it is possible to produce materials with a strength-to-weight ratio about a hundred times that of mediocre steel, and tens of times better than the best steel.[44]

> Today, we make most things from big chunks of metal, wood, plastic, and the like, or from tangles of fibers. Objects made with molecular manufacturing can contain trillions of microscopic motors and computers, forming parts that work together to do something useful…Walls and furniture can be made to repair themselves, instead of passively deteriorating. On a mundane level, this sort of flexibility will increase reliability and durability. Beyond this, it will make possible new products with abilities we never imagined we needed so badly. And beyond even this, it will open new possibilities for art.[45]

Nowadays, first examples of nanotechnologic materials are seen in various products such as paints that are more durable and self-cleaning. However, in a decade or so, we will probably start to see some incredibly thin, but exceptionally strong, beams and walls. Apparently, it is not only the building industry, but also any field of science and production that will be revolutionized with the nanoscale products.

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[42] Kolarevic, B. Digital Production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 51.

[43] The term nanotechnology is first used by Eric Drexler in *Engines of creation: The coming era of nanotechnology.* Anchor Book, New York. 1986. The term is defined by Drexler as, “technology based on the manipulation of individual atoms and molecules to build structures to complex, atomic specifications.”

[44] Drexler, E., C. Peterson, G. Pergamit. *Unbounding the future: The nanotechnology revolution*. William Morrow and Company, Inc., New York. 1991. p. 80.

[45] Ibid. pp. 75-76.

CHAPTER 5

A CONTEMPORARY MASTER BUILDER[1]: FRANK O. GEHRY

Frank Gehry was born in Toronto, Ontario, Canada in 1929. He studied at the Universities of Southern California and Harvard, before he established his first practice, Frank O. Gehry and Associates in 1962. In 1979, this practice was succeeded by the firm Gehry and Krueger Inc.

Over the years, Gehry has moved away from a conventional style of architecture to an artistically directed atelier. His deconstructed architectural style began to emerge in the late 1970s when Gehry, created collage-like compositions out of found materials as a result of his personal vision of architecture. Instead of creating buildings, Gehry creates pieces of functional sculpture. In the large-scale public commissions, he melds formal compositions with an exploded aesthetic. Most recently, Gehry has combined sensuous curving forms with complex deconstructive massing, achieving significant new results.[2]

> More than any architect of his generation, Frank Gehry is an innovator whose vision reaches beyond the accepted aesthetic and technical constraints of twentieth-century architecture. His singular formal / philosophical stance developed slowly. In late 1950s and 1960s –the earliest years of his practice–, his work was well

[1] Kolarevic, B. defined "master builder" in, Information master builders in: *Architecture in the digital age: Design and manufacturing*. (pp. 55-62). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 57. as: "The master builders, from the Greek *tekton* (builder), to the master masons of the Middle Ages were in charge of all aspects of buildings, from their form to the production techniques used in their construction. They had the central, most powerful position in the production of buildings, stemming from their mastery of the material and its means of production. As the palette of materials broadened and the construction techniques became more elaborate, the medieval master masons evolved into master builders (or architects) who would integrate increasingly multiplying trades into an increasingly more complex production process."

[2] Stern, R. A. M. *Modern classicism.* Thames and Hudson Ltd., London. 1988. pp. 90-92.

> planned and handsome, and those who knew it regarded him as a genuine talent. But it was not until 1970s that the box began to break apart, and by the end of that decade, he had ventured into absolutely unknown territory with his own *dumb little house* –a small, pink Santa Monica bungalow. It became a laboratory in which it was possible to try anything, and he did. Since then, many barriers to self-expression have come down at his bidding.[3]

5.1 Gehry and His Office

Frank Gehry established the architecture firm of Frank O. Gehry and Associates in 1962. Before he established the office, he had trained in architectural rendering, and practicing early in his career for many architects, Gehry was a very capable draftsman. Thus, it was inevitable that sketching is the key point where Gehry starts thinking on a project. “As soon as I understand the scale of the building and the relationship to the site and the relationship to the client, as it becomes more and more clear to me, I start doing sketches.”[4] A Gehry building begins with a sketch, which are characterized by a sense of off-hand improvisation, of intuitive spontaneity. The fine line in his sketches is fluid. “The drawings convey no architectural mass or weight, only loose directions and shifting spatial relationships.”[5]

Another way for Gehry to focus on the building that he is designing is translating the drawing into a physical model. According to him, the model, existing in space is less abstract then drawing and is made of actual materials. The behavior of material, although not the very same as the actual construction, provides a feedback for the design process. The material matter is missing when in the case of building virtual models in computer. This will be the basis of Gehry’s suspicions about the adaptation of digital tools.[6]

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[3] Friedman, M. S. Gehry talks: Architecture + process. Rizzoli, New York. 1999. p. 8.

[4] Gehry stated in the conversation that took place in December 1998 with Robert Ivy, editor-in-chief of Architectural Record, in: Arcspace. *Frank O. Gehry: The architect's studio*. http://www.arcspace.com/gehry_new/. Last accessed in December 2005.

[5] Knight, C. Full of generosity, in: Arcspace. *Frank O. Gehry: The architect's studio*. http://www.arcspace. com/gehry_new/. Last accessed in December 2005.

[6] Lindsey, B. *Digital Gehry: Material resistance / digital construction.* Birkhäuser, Berlin. 2001. pp. 23-24.

In 1989, there were about twenty people in Gehry's office. There were two computers –one word processor and one in accounting. In those years, Gehry worked with outside executive architects on major projects. That relationship can still be seen in many firms today, which often leads to misunderstandings, errors in construction, and increased costs. For Gehry's office, the most remarkable example of the way in which the executive architect system can go wrong is found in the Walt Disney Concert Hall. Since Gehry's design in 1988 for the hall was complex and not thoroughly understood by the executive architect, the estimates generated from their work were astronomical and the project was nearly to stop. Gehry realized that in-house technical expertise was essential to his growing practice. Therefore, radical changes in Gehry's office structure were underway. He turned to Jim Glymph, who joined the office on the condition that they would no longer split the work with the outside executive architects, but would develop the essential in-house technical expertise that would permit them to develop projects from beginning to end. Finally, by the summer of 1998, after ten years, the realization of Disney Hall –designed and controlled in-house– was assured.[7]

5.2 On the Use of Computer

The computer was introduced to Gehry's office in a way that would not interfere the process, which have been continuing for over thirty years. The revolution is precipitated by three things. To start with, the speed of technological development as described in Moore's Law[8]; the transmissibility of digital information breaking down traditional boundaries of time and space; and the seemingly infinite forms that information can take. Secondly, buildings of the traditional process generally seemed to be slow, immovable, site- and time-specific and singular in form. However, Gehry's structures are fast, move and are very recognizable wherever they are. This leads Gehry to an evolutionary process. As a consequence, the introduction of digital

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[7] Friedman, M. S. *Gehry talks: Architecture + process.* p. 16.

[8] Intel co-founder Gordon E. Moore made a prediction on 19 April 1965 that states the growth of computing power follows an empirical exponential law in his publication *Cramming more components onto integrated circuits,* in Electronics Magazine. (ftp://download.intel.com/museum/Moores_Law/Articles-Press_Releases/Gordon_Moore_1965_Article.pdf)

tools did change Gehry's process; they also changed his architecture. Gehry has already won the Pritzker Prize in 1989 without the help of computers before Jim Glymph and Rick Smith, who were talented about computer technology, joined the office. After they joined him, they designed and constructed the Guggenheim Museum in Bilbao, and it would probably be a different building without the use of digital tools. "The *smart machines*, and the people who operate them, have given him the long-hoped-for freedom to create ever more inventive ways to enclose space."[9]

Another reason for Gehry's adaptation of digital tools was the difficult task of describing the innovative new designs to the contractor. It was hard to explain his complex three-dimensional forms, when represented in traditional two-dimensional plans, sections and details.

Initially Gehry was resistant to using computer in his design process. The program seemed to limit his architecture to symmetries, mirror imagery and simple Euclidian geometries. However, questions of how to visualize Gehry's gestural moves resulting in sculptural three-dimensional forms –his sketches–, or how to translate them into a very large scale, were unresolved. "I just didn't like the images of the computer", Gehry said, "but as soon as I found a way to use it to build, then I connected."[10]

> …we developed a process through digitizing and visualization on the screen, and a number of other things, where we started to capture the physical mode. And unlike everybody else, we always went back to physical model.[11]

Until then, Gehry had been having problems with contractors or manufacturers. They generally claimed Gehry that his sculptural shapes could not be built or were not economical. He began to lean more and more Frank Lloyd Wright's theory that an architect has to be a *master builder* as well. A change had taken place in the office, first with the arrival of Glymph, and then the architects Randy Jefferson and Vano

[9] Friedman, M. S. *Gehry talks: Architecture + process.* p. 8.

[10] Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* Guggenheim Museum Publications, New York. 1998. p. 136.

[11] As Glymph J. stated in: Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* p. 136.

Haritunians, as respectively Principal and Associate Principal, to manage projects and break the pattern of having to rely on outsiders. By starting to use the new computer program, sculptural shapes could be computed, a more time-saving, economic way of building was devised, affecting, for instance, the structuring of a steel frame, or figuring out what it takes to fit panels on a wall. The new process is suitable for both high technology in terms of construction, such as numerically controlled machines, and traditional craft well.[12]

In order to allow Gehry more freedom in his designing of sculptural shapes, Glymph and his team started to analyze how realizable the forms were while also comparing cost efficiency in terms of volume, surface and structure.

> We could refine a little bit based on those kinds of criteria, put them back in front of Gehry as a physical model, so he could again deal with the slightly altered shapes as a thought…and as a gesture.[13]

While the designs were brought closer to their immediate realization, Gehry became aware of the computer's power to generate form.

> Many of the forms he is developing now are only possible through the computer. Bilbao is a perfect example. Prior to the development of the computer applications in the office, they would have been considered something to move away from. It might have been a sketch idea, but we would never be able to build it. Bilbao could have been drawn with a pencil and straightedge, but it would take us decades.[14]

These ideas have contributed to an inevitable change in his way of practicing architecture.

> Jim developed the computer thing slowly, and that was expensive. But he does make it work for us. That is how we controlled the

[12] Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* pp. 136-138.

[13] As Glymph J. stated in: Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* p. 138.

[14] Zaera-Polo, A. Interview with Frank O. Gehry, in: *El Croquis*. (pp. 6-37). 1995, vol. 74/75.

> costs of Bilbao and how we can do those curves now. Consequently, we have a lot of freedom, I can play with shapes. When I create the curved shapes on all the little models, we have a gadget that digitizes them. It is becoming quicker and quicker. With our new equipment, shapes can be transferred to the computer in fifteen minutes, and now we know how much it is going to cost per square foot to build to those shapes, because we have had the necessary experience. Now we can budget jobs in the earliest design phases. And we know if we use flat materials, it is relatively cheap, when we use single curved materials it is a little more expensive, and it is most expensive when we warp materials. So we can rationalize all these shapes in the computer and make a judgment about the quantity of each shape to be used. It is not possible to know this by looking at he completed building. The most important thing is that the computer gives us a tool we can use to communicate with the contractors.[15]

Gehry thinks that the new process leads the architect to the master builder position, which he is looking for in his architectural practice to be able to construct his buildings as his models and drawings.

> The new computer and management system allows us to unite all the players –the contractor, the engineer, the architect– with one modeling system. It is the master builder principle. I think it makes the architect more the parent, and the contractor more the child-the reverse of the twentieth century system.[16]

The question is how was Gehry fit into new process? His working method has not change because of the computer. He still develops his ideas slowly, from sketches through a long series of physical models. Gehry and his studio generate dozens and dozens of physical models. He creates sketches, his drafts shape the material, and by his models, Gehry tests the spaces, the three-dimensional affects, the play of hollows and solids.[17] “I sit and I watch and I move things. I move a wall, I move a piece of paper, I move something, and I look at it –and it evolves.”[18] The new process for Gehry starts from this point. Once a satisfying physical model has been built, it can be

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[15] Friedman, M. S. *Gehry talks: Architecture + process.* pp. 50-52.

[16] Ibid. p. 52.

[17] Lindsey, B. *Digital Gehry: Material resistance / digital construction.* p. 6.

[18] As Gehry stated in: Friedman, M. S. *Gehry talks: Architecture + process.* p. 19.

digitized by means of digitizing arms and a new model built, this time digital, that will then become the basis for thousands of other verifications and modifications. Gehry's office builds a virtual building in the computer utilizing three-dimensional scanning techniques as mentioned in chapter 3.

The first opportunity for Glymph to take the responsibility of the whole process came along with the *Barcelona Fish* for the Villa Olimpica, a hotel / commercial development commissioned for the 1992 summer Olympic Games, a steel sculptural element of 54 meter long and 35 meter high. The client was in a hurry, and the Gehry office had less than a year to finish the project until the games.

After looking at a variety of systems that was not suitable for the job, Glymph found a software created by Dassault Systems for the French aerospace industry. The answer was the CATIA program, designed to represent complex three-dimensional objects. "These programs array points in space and then there are massive holes in between…but CATIA, a program that deals with polynomial equations instead of polygons, is pretty much capable of defining any surface as an equation, which means that if you query the computer for any point on that surface, it knows it."[19] Gehry's office made the Barcelona Fish happen with the help of CATIA with a collaborating contractor from Italy.

> The Barcelona "Fish" project was done very early as a paperless project using CATIA. It was relatively simple cladding on the structural steel. It was all coordinated through the computer model. There were about six drawings by hand and a computer model for the entire process. The computer model was used for tracking parts, design and the layout of the system. Through that project, we became familiar with downloading information to laser cutters from a three-dimensional model and working with a contractor on the layout in fabrication from a single database that we all shared. We also did a project in Prague in the same timeframe soon after the concert hall was stopped. It was a complex metal and glass construction, modeled in the computer and then templated for what looks like a nineteenth-century construction of the steel but is, in fact, all computer-templated. We also had this notion that we could mass-produce, using a CAD / CAM process, individual precast concrete

[19] As Glymph J. stated in: Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* p. 135.

> wall panels that were all slightly different, but the Czech labor rates were so low that we wound up simply templating directly from the computer to craftsman, who would build wood forms and then ultimately complete the wall.[20]

Figure 24. Barcelona Fish for the Villa Olimpica

(Lindsey, B. *Digital Gehry: Material resistance / digital construction.* Birkhäuser, Berlin. 2001.)

In the beginning, Dassault gave the architects a great deal of support, but when Boeing ordered a thousand stations, it disappeared for a while, because repetitive, mass-produced products, such as cars and planes, become more appealing for them. However, Bilbao changed everything and Dassault now sees the potential in the development of new programs for the architects. Nevertheless, architects eventually will develop their own software, and the Gehry team is already well on the way to doing just that.[21]

A *bus stop* in Hanover, Germany was the next project that allowed Gehry to push the process begun with the Fish in Barcelona. The project was to design bus stops for Expo 2000. Gehry designed a shallow arching vault of silver and green stainless steel

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[20] Glymph, J. Evolution of the digital design process, in: *Architecture in the digital age: Design and manufacturing*. (pp. 101-120). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 108.

[21] Friedman, M. S. *Gehry talks: Architecture + process.* p. 18.

ribbons. The small project was completely developed from the CATIA model without any other construction documentation. This paperless experiment on a manageable project provided valuable lessons for future larger-scale uses of these digitally adapted processes.[22]

Figure 25. Bus stop in Hanover

(Lindsey, B. *Digital Gehry: Material resistance / digital construction.* Birkhäuser, Berlin. 2001.)

In 1991, when Gehry won the competition for the *Guggenheim Museum* in Bilbao, his process was in the middle of changing from a traditional practice to a digitally adapted one.

> And then, after the concert hall stopped, we had the miracle in Bilbao. It was a miracle, I think, because we were very lucky. At that point, we were committed to trying to prove that what we wanted to have done on Disney could be done. We began a process with the same kind of exchange between physical and computer models,

[22] Lindsey, B. *Digital Gehry: Material resistance / digital construction.* pp. 41-42.

> with the notion that we build a CATIA model that would be able to control all the major elements of construction.[23]

"The project in Seattle, the *Experience Music Project* (EMP), was an exploration of what we could do with sheet metal and how it performs on various surfaces."[24] In this project, CNC guided plasma cutters were used to cut the flanges of the curving structural steel members. Computer controlled rolling machines were used to bend the flanges and an automated trolley, which ran along the flange, welded the assembly together. As a result of this process, none of the steel ribs, which are curves of 11th order, became alike each other.[25] The project leads Gehry's office to a completely digitally adapted process.

Figure 26. Experience Music Project (EMP), Seattle

(JBL. *Press room*. http://www.jblpro.com/pressroom/images/aeriel_view_72.jpg. Last accessed in December 2005.)

[23] Glymph, J. Evolution of the digital design process, in: *Architecture in the digital age: Design and manufacturing*. (pp. 101-120). p. 109.

[24] Ibid. p. 110.

[25] Lindsey, B. *Digital Gehry: Material resistance / digital construction.* pp. 84-86.

> There was a major cultural transition that everybody had to go through, to get used to the notion that one no longer measures things with a ruler and a tape, and then runs offsets from specific points, and that entire building actually works from one zero point.[26]

On the *DG Bank* project in Berlin, there is an interesting contrast between the irrational, free-form steel on the "Horse's Head" conference room and a much-rationalized form of the skylight above it. In the skylight, there is the rigorous geometry and, in the Horse's Head, the free-form geometry done in a quarter inch stainless steel.[27] "Those are two different ways of approaching the problem of geometry and tectonics, and both are equally valid. One does not need arcs and rationalized geometry to build those shapes."[28]

The *Walt Disney Concert Hall* also makes extensive use of computer technology. When Gehry and his office won the competition for the Walt Disney Concert Hall, Los Angeles, the owner and users sat down to talk about what they wanted to do, and what that might cost, and they realized the project would not be able to stay within the budget. Some iterations are proposed for the design. Passing through a period of developing a new design, Gehry and his office figured out that they are in need of emerging computer modeling and CAD / CAM technologies to be able to build the hall within the budget and time. Gehry's firm decided to use CATIA to render the plans. After a physical model is built, the model is scanned by a laser device that transmits coordinates to the CATIA program. CATIA then shows a 3D section of the model. These paperless plans are more easily understood by a contractor and construction crew and allowed the project to take shape.

> Using a digitizer arm, we digitized the previously built model, rationalized it to a degree in the computer, and then rebuilt it from templates to look very specifically at the surfaces that were to hold stone. The stone surfaces were modeled with rational breakpoints

[26] Glymph, J. Evolution of the digital design process, in: *Architecture in the digital age: Design and manufacturing*. (pp. 101-120). p. 110.

[27] Ibid. p. 115.

[28] Ibid. p. 115.

> to create curves and arcs in some cases, and in other cases, we left the natural form since we knew we were going to be milling.[29]

CATIA can also give a schedule of the construction process to ensure that all components of the structure are completed on track. In the Walt Disney Concert Hall case, the general contracting company decided to integrate the fourth dimension of computer modeling and marry the building's curvilinear geometry to the planned schedule. The output is an animated four-dimensional CAD visualization tool that plays out the construction sequence.[30]

Figure 27. Walt Disney Concert Hall, Los Angeles

(Raghep, J. F., K. W. Weg. *Frank Gehry, architect.* Guggenheim Museum Publications, New York. 2001.)

5.3 Guggenheim Museum in Bilbao, Spain

Designed by the North American architect Frank O. Gehry, Guggenheim Museum is built on a 32500 square meter site in the center of Bilbao. One side it runs down to

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[29] Ibid. pp. 105-106.

[30] Illumin. *Curves of steel: CATIA and the Walt Disney Concert Hall.* http://illumin.usc.edu/article.php?articleID=120 &page=1. Last accessed in December 2005.

the waterside of the Nervión River. One end is pierced through by the huge Puente de La Salve, one of the main access routes into the city.

In Guggenheim Museum Bilbao, Gehry displays his tremendous ability to marry the disciplines of art and history to architecture and new technologies. The building itself is an extraordinary combination of interconnecting shapes. As a whole, Gehry's design creates a spectacular, remarkable structure that has the presence of a huge sculpture set against the backdrop of the city.

The Guggenheim Bilbao has received extraordinary amounts of attention, not only in print, but also in the realm of media-induced popular culture. Since the importance of the building's design and construction has been established, authors have subsequently turned their attention to the long-term effects it will have on the architectural world. Either negative or positive, most critics feel the *Bilbao Effect* is a valid architectural issue.

5.3.1 The Origins of the Museum

Since the latter part of the nineteenth century, Bilbao had been an industrial and mercantile city, but in 1980's, in the time of recession, the city had some difficulties in making a transition to high-service industries. The Basque Administration devoted major resources to an urban renewal: a new terminal and a control tower by Santiago Calatrava, a suspension pedestrian bridge by the Nervion River by the same architect, and the first phase of the subway by the English architect, Sir Norman Foster.[31] The Guggenheim Museum Bilbao is one of the most important ingredients in the plan to redevelop the city of Bilbao. These initiatives were also seen as a means of increasing the chance of the city's metropolitan area becoming the major reference point for European regions of the Atlantic seaboard.[32]

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[31] Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* pp. 17-18.

[32] Museo Guggenheim Bilbao. *Guggenheim*. http://www.guggenheim-bilbao.es/ingles/edificio/el_edificio.htm. Last accessed in December 2005.

In early 1991, the Basque administration was also planning to convert a former wine storage warehouse called the Alhongida into a cultural facility, a museum. Nevertheless, the Basques did not have an internationally renowned collection to put on view in the museum, nor did they have the expertise to run it. Therefore, they demand assistance of Thomas Krens, the director of Solomon R. Guggenheim Foundation. Asked by the Basque Administration what recommendations he would make for the architectural development for the Alhongida, he realized the fact that the interior space of Alhongida, with intervals no more than three meters and low ceilings of five meters, would present a problem. Krens decided to get the opinion of Frank O. Gehry and invited him to Bilbao.[33]

When Gehry came to Bilbao, his immediate reaction, along lines similar to Krens's, was that the Alhongida was an unworkable proposition for a museum. Gehry and Krens's idea was to build a new museum somewhere else. When questioned by the Basques as to which location he would pick, Gehry responded, "By the river – because they had been telling me all day that the river is being redeveloped–I like the site because it went under the bridge." Krens agreed to Gehry that the waterfront was in the middle of three major cultural facilities: the Bellas Artes Museum, the university and the Opera House what he called the geocultural triangle of Bilbao.[34]

The Guggenheim Museum Bilbao is the result of a unique process of collaboration, based largely on the complementary nature of their resources, between the Basque authorities and the Solomon R. Guggenheim Foundation. In December 1991, the Basque Administration and the Solomon R. Guggenheim Foundation signed the Development and Programming Services Agreement for the Guggenheim Museum Bilbao at the Provincial Council's headquarters in Bilbao.[35]

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[33] Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* p. 17.

[34] Ibid. p. 21.

[35] Museo Guggenheim Bilbao. *Guggenheim*. http://www.guggenheim-bilbao.es/ingles/edificio/el_edificio.htm. Last accessed in December 2005.

After the Basque Administration cleared the purchase of the new river location, the next step was a brief competition, involving an American, a European, and an Asian architect, all of whom Krens could recommend, was agreed upon. Isozaki was selected from Japan, Gehry was selected as the American, and the European participant became the Coop Himmelblau.[36]

The selection committee was not interested in technicalities and details; they set no rules for presentation, because they were interested in an impression of the overall vision of each architect. The aim of the selection committee was to choose a building that would be greater than the sum of its parts and they were looking for a strong iconic identity for the museum so that people would want to visit the building for itself. In the end, it was Gehry's proposal for the Guggenheim Museum Bilbao that was selected during the course of meetings on July 20 and 21.[37]

Architect Frank O. Gehry presented his model for the Museum building in February 1993, and the foundation stone was laid on October 23 of that year. In October 1994, work began on the structure of what was to become the Guggenheim Museum Bilbao.

5.3.2 Design and Realization of the Project

Frank Gehry and Associates won the competition for Guggenheim Museum, Bilbao in 1991. The design team's winning competition model was accompanied by beautiful watercolor rendered plan, section, and elevation drawings, as if opposing to the arising digital invasion in the office. These drawings were done using traditional methods as early design development for the project that lasted almost three years. At that time, there was CATIA in only one of the office's three workstations and the work on the Nationale-Nederlanden Building in Prague, and the Team Disneyland Administration Building, in Anaheim, California, which were occurring at the same time with Guggenheim, Bilbao, were executed on CATIA. Therefore, initially tradi-

[36] Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* p. 27.

[37] Ibid. pp. 27-29.

tional methods were preferred for Bilbao. Using the pre-digital methods of the office, most of the early development of the museum's complex three-dimensional form was drawn by hand –descriptive geometry[38] replacing the computer screen. Until the final design model was complete, the computer was not extensively used. After the model was finished, a digital model was produced in CATIA by digitizing the design model. Every volume has been shaped in three-dimensions, tested and modified by computer plotting, just as every part of its physical assembly –steel frame, cladding etc.– was fabricated on the basis of computer-generated construction documents.[39] The office was still unsure about the accuracy of the translation from physical model to digital model, so, a three-dimensional check model was produced using an automated milling machine. The milling machine took its information from the digital model. Verifying that the digital information was correct, the digital model became the dimensional reference that allowed the working drawings to be developed, and later helped to coordinate the construction of the project.[40]

The digital process of the Guggenheim museum can briefly be summarized in several steps. First step is digitizing the physical model. In digitizing a series of points on the computer screen is produced, which together create a shape, which roughly resembles the shape of the physical model. The points are then manipulated, cleaned up and smoothed out, which may be recalled as mathematization of the point cloud as mentioned in chapter 3. Then a surface model is created, which is the outline of the physical model as a wireframe model. The shaded surface model is derived from the surface model. The lines represent the cladding pattern of the titanium panels. The fourth step involves creating the primary structure of the building. For Bilbao, this step represents a sort of map of the steel skeleton of the building. Then the secondary structure is analyzed. In the case of Bilbao, it is part of the structure, which supports the galvanized steel under-layer of the cladding system. Afterwards, curvature

[38] Descriptive geometry is the branch of geometry that is concerned with the two-dimensional representation of three-dimensional objects; it was introduced in 1795 by Gaspard Monge. Traditional architectural drawings are based on the principles of descriptive geometry.

[39] Dal Co, F., K. W. Forster. *Frank O. Gehry: The complete works*. The Monacelli Press, New York. 1998. p. 31.

[40] Lindsey, B. *Digital Gehry: Material resistance / digital construction.* pp. 43-44.

(Gaussian) analysis is done. The curvature analysis is about budget. It is done to determine whether the metal panels will naturally adhere to a certain curve. Otherwise, they would need to be pressed to adhere to the curve. Obviously, pressed metal panels are more expensive. This analysis may be conducted at several stages, or may not be conducted at all. Consequently, the CATIA computer model is used to generate the steel shop drawings as a last step.[41]

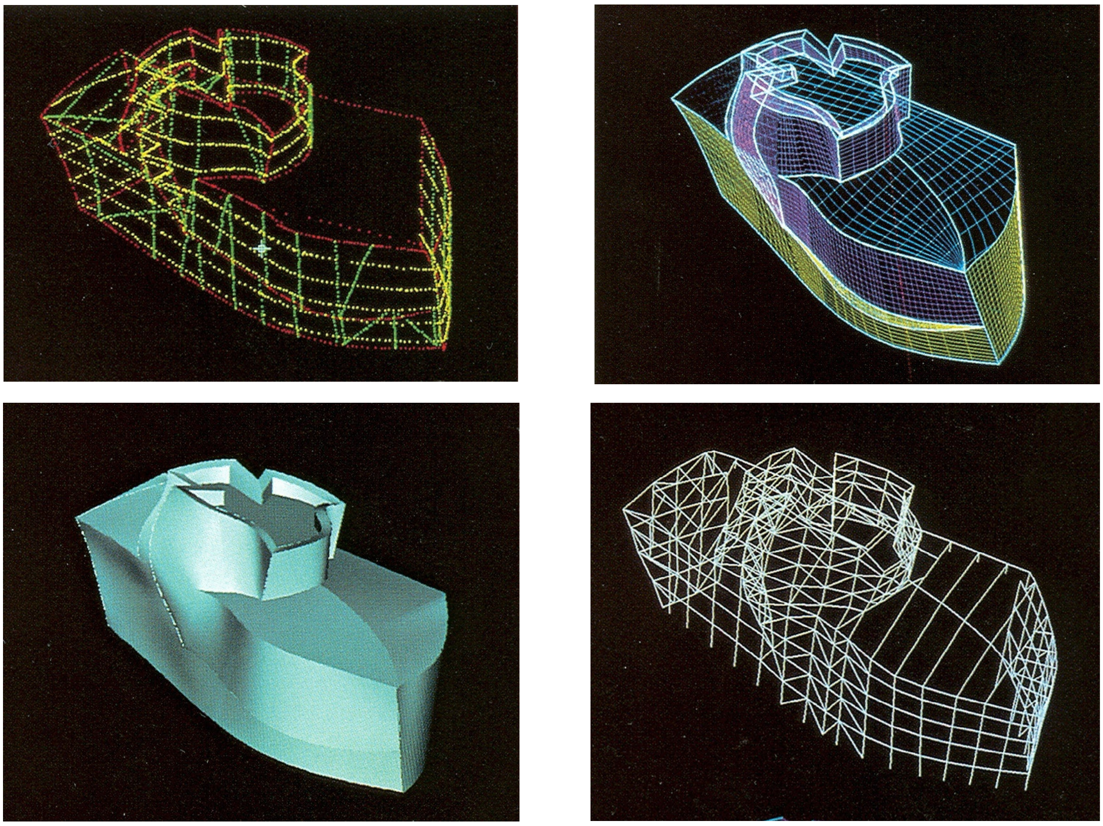

Figure 28. Digitizing sequence of Guggenheim Museum developed in CATIA

(Raghep, J. F., K. W. Weg. *Frank Gehry, architect.* Guggenheim Museum Publications, New York. 2001.)

The winning project of Gehry caused many speculations at that time. Many local and international architects described the building as *not buildable*. The exterior surface wrapping the building has an extraordinary shape, which cannot be defined any

[41] Arcspace. *Frank O. Gehry: The architect's studio.* http://www.arcspace.com/ge-hry_new/. Last accessed in December 2005.

mathematical formulas. Due to its complexity, a new approach had to be emerged to represent this challenging design.[42] Gehry with his project designers Edwin Chan, Jefferson and the design team won the confidence of the engineering team IDOM, put in place to manage the Guggenheim project for Krens and the Basque Government, and the design team gave the Basque Government proof that they could work with budget, and time. Groundbreaking occurred on October 22, 1993 while the design development was still in progress. The building was completed in March of 1997.

In Guggenheim museum, the primary load-bearing structure is quite straightforward, which means that most of the steel structural members are regular and straight (except in the long gallery and the tower). Therefore, the length and the connecting angles of the structural members are varied.[43] Bolted one-to-the-other, the form of the building emerged as the pre-fabricated steel frames were assembled. The frames required no additional shoring to be erected. Splines of 60 mm diameter steel tubes established the horizontal curvature by off setting from the main frame. Light gauge steel studs at right angles to the splines describe the vertical curvature. The splines were connected to the frame with a uni-strut adjustable joint that Matt Fineout, project team member, describes as, “the secret of the construction of Bilbao”. The joint allowed for the tuning of the splines to support the titanium skin. All of the titanium cladding panels was supplied flat and four panel sizes were used for cladding 80 percent of the surface.[44]

None of the elements of the structure for the 24000 square meter building is the same. 50000 drawings and 60000 hours of computing time were needed to produce the elements of the building façade; this describes the complexity and impossibility of organizing the project without the use of the CATIA digital model. By the whole construction process, 18 CATIA stations leased from aerospace contractors were be-

[42] Yun, Y. G., D. L. Schodek. Development of boundary structures for complex-shaped buildings, in: *Journal of Architectural Engineering* (pp. 18-25). 2003, March 3.

[43] Ibid.

[44] Lindsey, B. *Digital Gehry: Material resistance / digital construction.* p. 45.

ing used to detail the glazing package for the museum.[45] In Gehry's Bilbao project, the contractor –Spanish company Urssa– used a software program from Germany called BOCAD to automatically generate a comprehensive digital model of the structural steel, including the brace-framed and secondary steel structures for the museum and this work logged much of the computing time.[46] More importantly, that same program was used to automatically produce the fabrication drawings, or CNC data, to precisely cut and pre-assemble the various components.[47] Consequently, the steel bids for Bilbao came in at 18 percent under budget.

After the components are digitally fabricated, their assembly on site can also be established with digital technology. Digital three-dimensional models can be used to determine the location of each component, afterwards to move each component to its location, and finally, to fix each component in its right place.

> Frank Gehry's Guggenheim Museum in Bilbao was built without any tape measures. During fabrication, each structural component was bar coded and marked with the nodes of intersection with adjacent layers of structure. On site, bar codes were swiped to reveal the coordinates of each piece in the CATIA model. Laser surveying equipment linked to CATIA enabled each piece to be precisely placed in its position as defined by the computer model.[48]

While designing the Guggenheim Museum in Bilbao, Gehry's office used the Gaussian analysis to determine the areas of excessive curvature, as there are limits as to how much the sheets of metal could be bent in two directions. This method also let to determine which of the double-curved surface patches can be converted into developable ones and which ones need to be complexly shaped. This provided Gehry's office an important ability to determine and control the overall cost of manufacturing elements of a particularly complex envelope.[49]

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[45] Ibid. pp. 44-45.

[46] Stephens, S. The Bilbao effect, in: *Architectural Record*. (pp. 168-173). 1999, May.

[47] LeCuyer, A. Building Bilbao, in: *Architectural Review*. (pp. 43-45). 1997, December, vol. 102.

[48] Ibid.

[49] Kolarevic, B. Digital production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). p. 48.

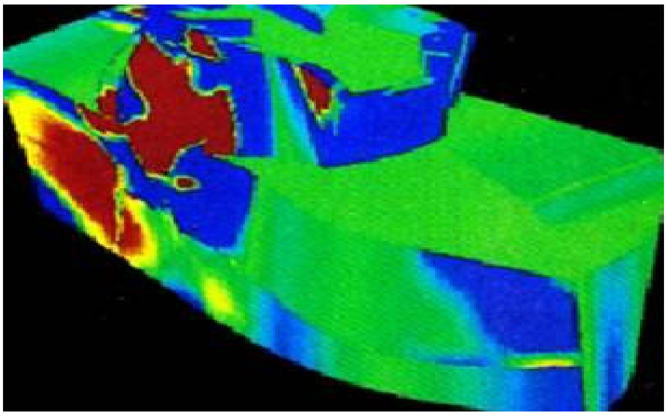

Figure 29. Gaussian analysis of Guggenheim Museum, Bilbao

(Kolarevic, B. Digital production, in: *Architecture in the digital age: Design and manufacturing*. (pp. 29-54). Branko Kolarevic, ed. Spon Press, New York. 2003.)

In the assembling of the building, stone fabrication for the project also utilized computer aided manufacturing techniques. The Spanish limestone for the museum was cut using computer numerically controlled routers with the help of the CATIA model –named as a *digital prosthesis* by the design team– providing all dimensional information.[50]

"The exteriors of the Guggenheim museum take the liberty of the manipulation of the facings, which we have seen in so many of Gehry's works, to its extreme consequences."[51] New materiality is one of the basic concepts that encourage this manipulation. Gehry and his office searched for a material that would overlap with the character of the unusual form of the museum. The choice of titanium as the facing material was a consequence of these researches.

Originally, lead copper was planned to use but it was outlawed as a toxic material. The design team started to look for another material that could play with the light like the lead copper, since the interaction of the light and the building is an important criterion for the design of the Guggenheim Museum. Afterwards, stainless steel was analyzed, but the design team had suspicions because of the cold industrial look of

[50] LeCuyer, A. Building Bilbao, in: *Architectural Review*. (pp. 43-45). 1997, December, vol. 102.

[51] Dal Co, F., K. W. Forster. *Frank O. Gehry: The complete works*. p. 58.

the material. During that frustration, Gehry's office found some samples of titanium. Looking at the material in the light, they realized that there was some potential for a metal that had warmth and character. Nevertheless, titanium was much more expensive than the steel, so the team worked in two directions in case the titanium could not be financially viable. Titanium at this point had rarely been used as an exterior material for buildings. It was used as castings for airplane parts, golf clubs, and many other things where strength is a factor. The rolling of the material was very delicate. It can lead to a dead surface or a wonderful light-receptive one. The team and the titanium fabricator searched for the right mix of oil, acids, rollers, and heat to arrive at the material they want. The titanium is thinner than stainless steel would have been; it is a third of a millimeter thick and it is pillowy, it does not lie flat and a strong wind makes it surface flutter, moreover a third of a millimeter of titanium is a hundred-year guarantee against city pollution. These are all characteristics that the team was looking for the material on the building.[52]

Figure 30. Light effects on the exterior cladding of Guggenheim Museum, Bilbao

(Çıngı, G. 2004.)

[52] Bruggen, C. V. *Frank O'Gehry Guggenheim Museum Bilbao.* pp. 141.

As a consequence, Gehry and his office, agreed on titanium as the facing for the curvilinear volumes. Probably, this was one of the most critical decisions on Guggenheim Museum's design process that gives the museum its character.

> Different types of facings or claddings are used for the different volumes, from titanium sheets to the most ordinary stucco. In each situation, the materials are used to produce the effect that is appropriate for particular surfaces….the titanium panel cladding grants homogeneity to even the most irregular forms, reminding the observer of a sort of continuous elastic sheath composed of a material that permits the hand of the architect to model the forms.[53]

Figure 31. Light effects on the exterior cladding of Guggenheim Museum, Bilbao

(Çıngı, G. 2004.)

The Bilbao museum plays an important role on the evolutionary process of Gehry and his office. Utilizing computer-aided design revolutionized their understanding of

[53] Dal Co, F., K. W. Forster. *Frank O. Gehry: The complete works*. pp. 58-59.

the interaction of designing a building and making the design *buildable*. This mentality of the office is also reflected to the construction processes of buildings. The ability to digitally generate and analyze the design information, and then use it directly to manufacture and construct buildings, redefined the relationship between conception and production.[54] The boundaries between architecture, engineering and construction blur and the role of *master builder* was reinvented in Gehry's office.

> The age-old split between the hands that design and the instruments execute has been overcome; the separate phases and techniques of conceiving and executing a building were woven into an unbroken loop…Only in this way can the inaccurate fit among the conventionally separate phases of invention, transcription, and execution be perfected, and the exponential degree of geometric complexity of such a structure be realized without costly trial and error.[55]

5.3.3 The Bilbao Effect

> Architecture is a marketing tool. The *Bilbao Effect* named after the incredible success of the Frank Gehry-designed Guggenheim Museum in Bilbao, whose architecture manages to draw over a million visitors a year to this former industrial backwater without much of a collection to house, had led to 'signature' structures around the globe. These are not so much buildings as they are sites of attraction. However much their designers might argue that they create magnificent spaces or critiques of dwelling, it is their newness, the oddness of their imagery, and their scale that brings in the crowds.[56]

As one might imagine, when idea is revolutionary like a new technological breakthrough in discipline, it is inevitable that there is great discussion on that innovation. As in Guggenheim Museum, Bilbao case, after it was completed, the ideas and methods of the Guggenheim Bilbao were discussed so much. Critics began using the term *Bilbao Effect* to discuss the potential ramifications of the building. Suzanne

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[54] Kolarevic, B. Information master builders, in: *Architecture in the digital age: Design and manufacturing*. (pp. 55-62). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 59.

[55] Dal Co, F., K. W. Forster. *Frank O. Gehry: The complete works*. p. 31.

[56] Betsky, A. Ten modernist architects in: *10x10.* (p. 410). Iona Baird ed. Phaidon Press Ltd., London. 2000. p. 410.

Stephens was one of the firsts to use the term. She wrote an article called "The Bilbao Effect" appearing in *Architectural Record,* May 1999. In the article, she proposed that the unusual building would create *shockwaves* in the architectural world.[57] She was correct.

Focusing on the design component of the building, some writers were on the side of the museum's design and others in opposition to it. The negative critics go on two primary groups. In one group, critics state that the Guggenheim Bilbao is too expressive and theatrical that it upstages the art it houses; and in the other, there are those who fear its continual replication. Some critics are suspicious about the possible ongoing repetition of the Guggenheim's curves, double curves etc., made easily replicable by the computer. They state that Gehry can repeat the Guggenheim Museum in any site he wants by the help of the computer and database, and he can reproduce Guggenheim only by increase and decrease the scale according to the specifics of another site. Some architects defend that engineers should use computers but architects should maintain the aesthetic of a project. Another point that some authors criticize about Guggenheim Bilbao is that the museum represents an architecture for tourist destination rather than a home for art.[58] There is also a positive understanding for the *Bilbao Effect*. Some critics state that Guggenheim has an important role for the revitalization of Bilbao due to its iconic architectural image. The economical and cultural contribution of the museum shows that Basque Government reached one of their purposes on the museum. The museum takes the attention of the entire world and the government is delighted with the economic turn-around. Although there are many critics –positive or negative– on Guggenheim Museum, Bilbao, no one disagrees with the rising economic status of the city of Bilbao, nor do they diminish the Guggenheim's role in that turn-around and today the *Bilbao Effect* still serves as a catalyst for new architectural dialogue.

[57] Stephens, S. The Bilbao effect, in: *Architectural Record*. (pp. 168-173). 1999, May.

[58] Tradarch. *Archives, October 2002 (213).* http://listserv.miami.edu/scripts/wa.exe?A2=ind0210&L=tradarch&F=&S=&P=24439. Last accessed in December 2005.

CHAPTER 6

CONCLUSIONS

In this study, a survey about the digital technologies influencing the architectural practice is carried out. Computer aided design and manufacturing techniques are introduced in order to represent the interaction and progression between them. The digital continuum from conception to construction is explored via Frank O. Gehry's office in order to understand the essence of the process. Obviously, it is not the rediscovery of the complex curving forms, but the newfound ability to generate construction information directly from design information through new processes and techniques of digital design and production.

For centuries, being an architect meant being a builder. Architects was not only the masters of spatial organization, but were also closely involved in the construction of the buildings. They had to most powerful position in the production of building. With the disassociation of architecture from building –through the use of perspective representation and orthographic drawings as a medium of communicating the information in the late Renaissance– architect no longer had to be present on site to supervise the construction process. The rift between the architecture and construction, started with the externalization of design information via drawings, dramatically widened when *drawings* became *contract documents* in the nineteenth century. Disassociation in the twentieth century is brought by the invention of numerous new materials, technologies and processes. Increasing complexity in techniques resulted in increased specialization in different building systems in both design and construction.

> The outcome of this progressive disassociation of architecture from the rest of the building industry is a profession unsure of its role in contemporary society and its economy, and a profession unable to

respond to the challenges and opportunities of the Information Age.[1]

Innovations in computer technology emerged new modeling techniques and highly accelerated computing capabilities, which, in turn, offer architects more freedom in formal aspects. Moreover, these innovations offer the architect the ability to build what they draw in a collaborative medium in which the construction information is directly derived from design information. To be able to lead this collaborative medium of various disciplines, architect has to be involved with both construction and material aspects besides the design aspect. The historical relationship between the architect and the builder could reemerge as an outcome of the new digital design process.

Guggenheim Museum in Bilbao has one of the most important roles in the realm of digitally realized projects that would be impossible to build without the aid of collaborative medium. Although, it is subject to many critiques for its expressive quality that upstages the art it houses, easily replicable curved and shiny character, and structural system that does not follow the elegant exterior surface, the museum building takes its place in the scene, being the outstanding realization process of the time. The building is a pioneer not only in terms of implementation but also in the utilization of the digital data as a contract document. By means of architectural practice, as a remarkable departure from the current norms, the three-dimensional digital model of the Guggenheim Museum, which is created in CATIA, is a key part in the contract documents. Legally and in practice, the digital model takes precedence over any other construction document. According to Kolarevic, this is a radical, revolutionary change in building practice, for which Gehry's office will probably be remembered in future history books –and not only for the sinuous, curving geometries of the Guggenheim Museum in Bilbao, Spain.[2]

[1] Kolarevic, B. Information master builders, in: *Architecture in the digital age: Design and manufacturing*. (pp. 55-62). Branko Kolarevic, ed. Spon Press, New York. 2003. p. 58.

[2] Ibid. p. 59.

With the ability to unite all phases of realization in one single source –digital data– has numerous advantages compared to the building with traditional methods. It not only saves time by eliminating externalization of design in terms of drawings for the manufacturers, but also is cost effective due to the optimized construction procedure. The digital data can be modified through analysis tools to derive more economical and feasible design solutions. Moreover, the collaborative digital medium minimizes the misunderstandings between the designer and the contractor. Three-dimensional modeling provides visualizations for the customer and the contractor that was very difficult to represent with the traditional two-dimensional drawings. Use of numerically controlled machines in the manufacturing process, renders the need for standardization invalid. The principle of producing unique parts at the same cost of identical parts enables the realization of buildings such as Guggenheim Museum.

Undeniable contribution of utilization of digital medium to the field of architecture broadens the limits further and new projects are accomplished. Conceptual studies of unknown territories of the digital realm are being carried. Realizable or not, future-looking projects have the potential to draw paths for the future of architecture. Today affordability of the technology is increasing every minute and information is so accessible as it has never been which directly contributes emergence of experimental studies. Referring the era we live in, Information Age, developments should not stay unnoticed in architecture. Having witnessed the fascinating effects of Gehry's architecture to the city of Bilbao, there is much to learn from the progress; and there is no need to abstain from the practice and education of digital continuum.

Digital tools compromise neither creativity and ingenuity nor economy for architects. Understanding the essence of the digital era emancipates the architect in several ways. Design and production territories are extended exceptionally. The potential beyond the formal issues should be captured and the digital era should be perceived as a medium to reunify the capabilities of the master builders in the act of building.

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APPENDIX A

MATRICES[1]

A.1 Definition

A matrix consists of a set of numbers or elements arranged in rows and columns as in double-entry tabular form. A typical matrix is,

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

Where *m* indicates the number of rows, and *n* the number of columns.

A.2 Notation and Principal Types of Matrices

A.2.1 Order of a Matrix

A matrix having *m* number of rows and *n* number of columns is said to be of the order *m* by *n*, conveniently written as *m x n*. however, *m x n* can also be called the dimension or size of a matrix. For example,

$$A = \begin{bmatrix} 3 & 2 & 4 & 5 \\ 5 & 1 & 2 & -7 \\ 6 & 4 & -1 & -8 \end{bmatrix}$$

[1] This appendix consists of a reprint of an appendix written by Amirouche, F. M. L. Computer-aided design and manufacturing. Prentice-Hall, Inc., New Jersey. 1993. pp. 508-521.

Matrix A is said to be of the order 3 x 4, where $m = 3$ indicates the number of rows, and $n = 4$ the number of columns.

A.2.2 Row Matrix

In an $m \times n$ matrix, if $m = 1$, it is called a row matrix. For example,

$$B = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Matrix B is a row matrix.

A.2.3 Column Matrix

In an $m \times n$ matrix, if $n = 1$, it is called a row matrix. For example,

$$C = \begin{bmatrix} 4 \\ -5 \\ 7 \end{bmatrix}$$

Matrix C is a row matrix.

A.2.4 Rectangular and Square Matrices

In an $m \times n$ matrix, if the number of rows is not equal to the number of columns, that is, $m \neq n$, then it is a rectangular matrix. On the other hand, if the number of rows is equal to the number of columns, that is m = n, it is a square matrix. For example,

$$D = \begin{bmatrix} 4 & 3 & 5 & 6 \\ -9 & 6 & -8 & 4 \\ 4 & -1 & 0 & -5 \end{bmatrix} \quad \text{and} \quad E = \begin{bmatrix} 4 & -3 & -1 \\ -2 & 0 & 6 \\ 0 & 4 & 4 \end{bmatrix}$$

Matrix D is a rectangular matrix and matrix E is a square matrix.

A.2.5 Identity Matrix

The unity or identity matrix is a square matrix in which the diagonal elements are ones and all other elements are zeros. For example,

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrix I is identity matrix.

A.2.6 Null Matrix

The null or zero matrix is a matrix in which all the elements are zero. The dimension of the null matrix is defined according to the dimension of the adjacent matrices. The notation for the null matrix is G = 0.

A.2.7 Transpose of a Matrix

Let A be a matrix of the order *m x n.* When the rows and columns are interchanged, the resultant matrix is called the transpose matrix of A. the transpose matrix of A is denoted by A^T. For example,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{and} \quad A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

A.2.8 Inverse of a Matrix

The inverse of the matrix A is written as A^{-1}, satisfying the following relationship:

$$A = A^{-1}A = I$$

This relationship means that the product of A by its inverse yields the identity matrix I.

A.2.9 Orthogonal Matrix

The matrix is said to be orthogonal when its transpose is equal to its inverse. A matrix A is said to be orthogonal is the following relationship is satisfied:

$$A\,A^{-1} = A\,A^{T} = I$$

Where I is identity matrix.

A.2.10 Minors, Cofactors and Adjoints

Consider the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The determinant obtained by deleting the *i*th row and the *j*th column of the matrix A is called the minor of element a_{ij} and is represented as M_{ij}.

The cofactor of element a_{ij} is denoted as A_{ij}. The cofactor A_{ij} is defined as

$$A_{ij} = (-1)^{i+j} M_{ij}$$

The adjoint matrix of A is the transpose of the matrix formed by the cofactors of all elements a_{ij} of matrix A. the adjoint of A is represented by $\tilde{A}$.

$$\tilde{A} = \left[\tilde{A}_{ij}\right] = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \tilde{A}_{13} \\ \tilde{A}_{21} & \tilde{A}_{22} & \tilde{A}_{23} \\ \tilde{A}_{31} & \tilde{A}_{32} & \tilde{A}_{33} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

From this equation, it is apparent that the adjoint of A is the transpose of the matrix of cofactors of A. For example, for the given matrix A, find its adjoint matrix.

A.2.11 Symmetric Matrix

Matrix A is said to be symmetric when it is equal to its transpose matrix, that is A = A^T. For example,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 5 \\ 3 & 5 & 6 \end{bmatrix} \Rightarrow A^T = A$$

A.2.12 Skew Symmetric Matrix

The skew symmetric matrix is a matrix in which the diagonal elements are zeros and the rest are such that $a_{ij} = - a_{ij}$. For example,

$$A = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

A.2.13 Trace of a Matrix

The trace of a square matrix A is the sum of all the elements in the main diagonal. For example,

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 8 & 9 & 0 \end{bmatrix}$$

Then the trace of the matrix is defined as *tr*(A) = 2 + 6 + 0 = 8.

A.3 Determinants

The determinant of matrix A is denoted by the following symbols: $|A|$, $|a_{ij}|$, or det A.

The determinant can be evaluated using Laplace's expansion:

$$\det A = \sum_j a_{ij}\gamma_{ij} \qquad (i = 1, 2, ..., n)$$

Where γ_{ij} denotes the cofactor corresponding to a_{ij}. Then the determinant for a 2 x 2 matrix is $|A| = a_{11}a_{22} - a_{12}a_{21}$. The following illustrates the application of Laplace's expansion to a 2 x 2 matrix.

By applying Laplace's expansion, the determinant for a 3 x 3 matrix is found as follows: Let

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Then

$$\det A = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{31}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

A.3.1 Properties of Determinants

1. The value of det A remains the same regardless of the row or column chosen to find it.

2. If any two rows or columns are the same, the relevant determinant obtained is equal to zero.

3. If two parallel lines of the matrix are interchanged, the sign of the determinant of that matrix changes but mot its magnitude.

4. If any row or column of a matrix is zero, the value of the determinant is zero.

A.3.2 Singularity of a Matrix

A matrix is said to be singular if the value of its determinant is equal to zero.

A.4 Matrix Partitioning

If some rows and/or columns of a matrix are deleted, then the remaining array is called a submatrix of the original matrix. A matrix can be considered a submatrix of itself. For example,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 6 & 9 \\ 2 & 5 & 9 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$$

Where

$$A_{11} = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \qquad A_{12} = \begin{bmatrix} 1 \\ 9 \end{bmatrix}$$
$$A_{21} = \begin{bmatrix} 2 & 5 \end{bmatrix} \qquad A_{22} = \begin{bmatrix} 9 \end{bmatrix}$$

A.5 Matrix Operations

A.5.1 Addition

Given two matrices, A = $\left[a_{ij}\right]$ and B = $\left[b_{ij}\right]$, the sum is defined as

$$A + B = \left[a_{ij} + b_{ij}\right]$$

For example,

$$A = \begin{bmatrix} 2 & 2 \\ 6 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2+3 & 2+1 \\ 6+1 & 9+0 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 7 & 9 \end{bmatrix}$$

A.5.2 Subtraction

Given two matrices, C = $\left[c_{ij}\right]$ and D = $\left[d_{ij}\right]$, then the difference matrix E is given by

$$E = C - D = \left[c_{ij} - d_{ij}\right]$$

For example,

$$C = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 5 & 6 \\ 4 & 5 \end{bmatrix}$$

$$E = C - D = \begin{bmatrix} 4-5 & 3-6 \\ 2-4 & 1-5 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ -2 & -4 \end{bmatrix}$$

A.5.3 Multiplication

Scalar Multiplication. Given matrix A = $\left[a_{ij}\right]$, and a scalar α, then

$$\alpha \text{A} = \alpha \left[a_{ij}\right]$$

For example,

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \qquad \text{and} \qquad \alpha=3$$

$$\alpha A = \begin{bmatrix} 6 & 12 \\ 15 & 18 \end{bmatrix}$$

Matrix Multiplication. Let A be a *m x p* matrix and B a *p x n* matrix. The product C = AB is a *m x n* matrix and each element C_{ij} of matrix C is obtained by multiplying the correspondent elements of the *i*th row of A by those of the *j*th column of matrix B, adding products. The multiplication of any two matrices exists only if the number of columns of the first matrix is equal to the number of rows of the second matrix. For example,

$$A = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix}_{2x2} \qquad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}_{2x1}$$

Thus,

$$C = A.B = \begin{bmatrix} 2 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} (2.3)+(4.1) \\ (4.3)+(3.1) \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix}_{2x1}$$

A.6 Commutative, Distributive, and Associative Laws

A.6.1 Commutative Law

A + B = B + A

α A = A α (for any scalar α)

A.6.2 Distributive Law

α (A + B) = α A + α B

A(B + C) = AB + AC

(A + B)C = AC + BC

A.6.3 Associative Law

(A + B) + C = A + (B + C)

(AB)C = A(BC)

A.7 Method to Find the Inverse of a Matrix

A.7.1 Inverse of a 2 x 2 Matrix

Given the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The determinant of the matrix A is given by

$$|A| = a_{11}a_{22} - a_{21}a_{12}$$

By interchanging the positions of the element in the main diagonal and changing the algebraic sign of the remaining elements, the resultant matrix is

$$B = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

The inverse matrix of A is obtained by dividing all the elements of matrix B by the determinant value of matrix A, which yields

$$A^{-1} = \frac{1}{a_{11}a_{22} - a_{12}a_{21}} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

A.7.2 Inverse of a 3 x 3 Matrix

Given the matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

The determinant of the matrix A is defined by

$$|A| = \left| a_{11}\left(a_{22}a_{33} - a_{32}a_{23}\right) - a_{12}\left(a_{21}a_{33} - a_{31}a_{23}\right) + a_{13}\left(a_{21}a_{32} - a_{31}a_{22}\right) \right|$$

Find the adjoint matrix of A, which is denoted by B.

The inverse matrix of A is obtained by dividing all the elements of matrix B by $|A|$:

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

A.8 Solution of Simultaneous Linear Equations

There are several methods in solving simultaneous equations. Cramer's rule is one of the simplest approaches to solve a set of *n* equations with *n* unknowns, especially if *n* is small. Thus, consider two equations having two unknowns *x* and *y*:

$$2x - 3y = 5$$
$$x + y = 5$$

These equations rewritten in matrix form yield

$$\underset{A}{\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix} = \underset{B}{\begin{bmatrix} 5 \\ 5 \end{bmatrix}}$$

Using Cramer's rule, we solve for *x* and *y* as follows:

$$x = \frac{\begin{vmatrix} 5 & -3 \\ 5 & 1 \end{vmatrix}}{\det A} \qquad x = \frac{\begin{vmatrix} 5 & -3 \\ 5 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}} = \frac{20}{5} = 4$$

The matrix in the numerator is obtained by deleting the first column of A and replacing it with the vector matrix B. Similarly, in solving for *y*, the second column of A is replaced with vector B. Therefore,

$$y = \frac{\begin{vmatrix} 2 & 5 \\ 1 & 5 \end{vmatrix}}{\det A} \qquad y = \frac{\begin{vmatrix} 5 & -3 \\ 5 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}} = \frac{5}{5} = 1$$

Then the solution is $x = 4$ and $y = 1$.

An example for the solution of three equations with three unknowns using Cramer's rule is shown next.

Given: 2x + 2y + z = 1 x + 3y + 3z = 4 2x – y – 2z = -2

These equations can be rewritten in matrix form as

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 3 \\ 2 & -1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$$

Then, applying Cramer's rule,

$$x = \frac{\begin{vmatrix} 1 & 2 & 1 \\ 4 & 3 & 3 \\ -2 & -1 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 3 \\ 2 & -1 & -2 \end{vmatrix}} = \frac{(1)\begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix} - (2)\begin{vmatrix} 4 & 3 \\ -2 & -2 \end{vmatrix} + (1)\begin{vmatrix} 4 & 3 \\ -2 & -1 \end{vmatrix}}{(2)\begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix} - (2)\begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} + (1)\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}} = 1$$

$$y = \frac{\begin{vmatrix} 2 & 1 & 1 \\ 1 & 4 & 3 \\ 2 & -2 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 3 \\ 2 & -1 & -2 \end{vmatrix}} = \frac{(2)\begin{vmatrix} 4 & 3 \\ -2 & -2 \end{vmatrix} - (1)\begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} + (1)\begin{vmatrix} 1 & 4 \\ 2 & -2 \end{vmatrix}}{(2)\begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix} - (2)\begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} + (1)\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}} = -2$$

$$z = \frac{\begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 4 \\ 2 & -1 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 3 \\ 2 & -1 & -2 \end{vmatrix}} = \frac{(2)\begin{vmatrix} 3 & 4 \\ -1 & -2 \end{vmatrix} - (2)\begin{vmatrix} 1 & 4 \\ 2 & -2 \end{vmatrix} + (1)\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}}{(2)\begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix} - (2)\begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} + (1)\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}} = 3$$

Therefore, x = 1, y = -2, and z = 3.

APPENDIX B

TRANSFORMATIONS[1]

To be of any practical value, the geometric model of an object must be easy to manipulate or transform. Transforming an object implies changing it in either position, orientation, or shape. Since a geometric model must be stored in a computer as numerical data in terms of some coordinate system, we must devise ways of transforming the data to represent changing an object's position and orientation.

The simplest changes are the so-called rigid-body transformations, such as translation and rotation. We can operate directly on the parametric representation of geometric objects, such as points, vectors, curves, and surfaces, to effect these changes. We will see how this is done by the simple expediency of matrix multiplication and addition.

B.1 Translation

The rigid-body translation of a geometric object implies that every point on the object is moved equally a given distance in a given direction. We may specify translation by a vector, say, t; Figure 32 shows a curve translated by t. Every point p on the curve is translated by an amount t, so that a point p* on the fully translated curve is given by

$$p^* = p + t \tag{B.1}$$

[1] This appendix consists of a reprint of a text written by Mortenson, M. E. *Geometric Modeling*. John Wiley & Sons, Inc., Canada. 1985. pp. 345-349

Notice that an asterisk denotes a transformed vector or matrix quantity.

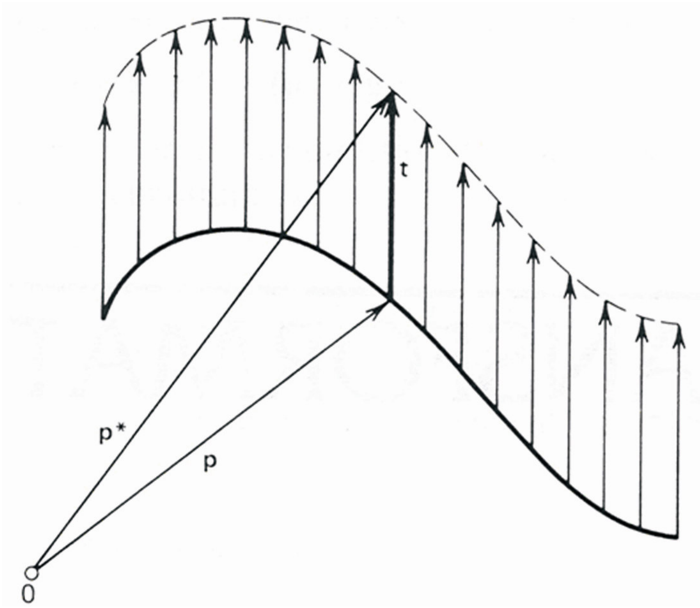

Figure 32. Translation

Equation B.1 represents the direct point wise transformation of p and requires that we retain a definition of the curve through either the B matrix of geometric coefficients or the A matrix of algebraic coefficients. In most applications, it is preferable to transform the A or B matrices and then discard the old definition.

The B matrix for a parametric cubic curve is transformed as follows:

$$B^* = B + T \qquad \text{where } T = \begin{bmatrix} t & t & 0 & 0 \end{bmatrix}^T$$

For a bicubic patch,

$$T = \begin{bmatrix} t & t & 0 & 0 \\ t & t & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

B.2 Rotation

There are several ways of rotating an object. We will investigate rotation around the origin (that is, the principal axes) and an arbitrary line in space. The rigid-body rotation R of a geometric object must meet the following conditions:

1. Relative distances and angles between points and slopes of an object are maintained.

2. A right-hand convention applies to signs of rotation angles.

3. The embedding coordinate system is a right-hand system.

4. For the case of rotation around the origin, a rotation has three possible components: γ, β, θ; where γ is the angle of rotation around the x-axis, β the angle around the y-axis, and θ the angle around the z-axis. In keeping with the right-hand convention, θ is positive in a counterclockwise sense when viewed from a point on the +z-axis and toward the origin; β is positive in a counterclockwise sense when viewed from a point on the +y-axis; and γ is positive in a counterclockwise sense when viewed from a point on the +x-axis.

5. When the rotation of an object is specified by all three components, the order is important. In the absence of other constraints, the following convention is useful.
(a) First, rotate around the z-axis if $\theta \neq 0$.
(b) Next, rotate around the y-axis if $\beta \neq 0$.
(c) Finally, rotate around the x-axis if $\gamma \neq 0$

Figure 33 shows a curve rotated through the angle θ in the positive direction around the z-axis. Notice that every point on the curve undergoes a rotation θ around the z-axis and that its trajectory lies in planes perpendicular to the z-axis and parallel to the x, y plane. For a parametric curve, the rotation changes (transforms) the functions x(u) and y(u), but not z(u). θ is positive when in the same direction as that assumed for the vector product of unit vectors that produces i x j = k; that is, rotating the positive x-axis into the y-axis when rotation is around the z-axis.

Conversely, Figure 33 shows a rotation sequence applied to the coordinate system. Here, θ and β are nonzero and γ=0. Rotation through the angle θ is executed first and then β. As shown, both rotational components are negative, in accordance with the preceding conventions, with respect to object that we assume to remain fixed relative to the initial, unrotated system.

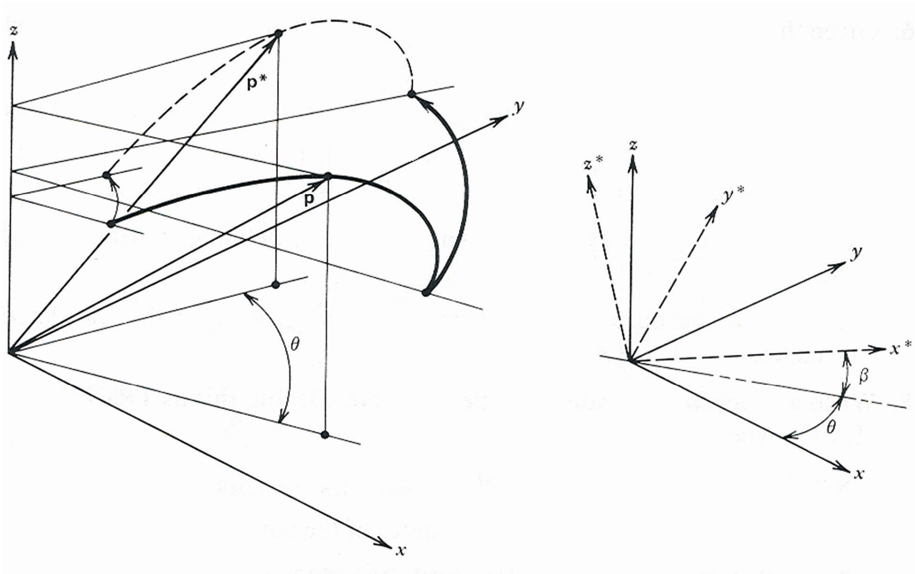

Figure 33. Rotation of a curve

We will now develop rigid-body rotation of points and vectors within a fixed coordinate system and show how they are related. Points can be rotated and translated with respect to the origin, and vectors can be rotated. A curve or other geometric object defined in terms of points, or points and tangent vectors, is clearly subject to general rigid-body transformations consisting of both translation and rotation, as expressed by

$$p^* = pR + t \tag{B.2}$$

where

p* = the transformed point.	p = the initial point.
R = a 3 x 3 rotation matrix.	t = a translation vector.

Notice that Eq. B.2 implies that the point is first rotated –in other words, we first computed pR– and then translated. It is possible to perform the transformation in the reverse order; that is, translate first and then rotate.

$$p^* = [p + t]R$$

However, in general, $pR + t \neq [p + t]R$.

Now, let us derive the elements of the R matrix. Figure 34 shows the geometry describing the rotation of point p through an angle θ around the z-axis, so that given p and θ, we can find p*, the transformed point. The relationship between the components of p and p* is

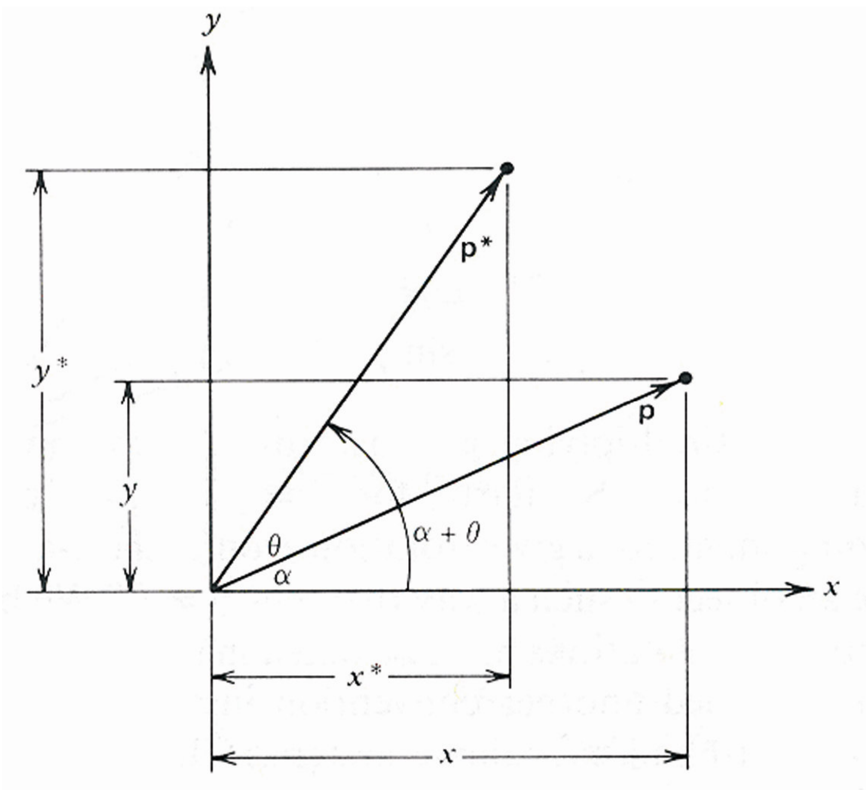

Figure 34. Rotating a point or vector about the z-axis

$$x^* = |p|\cos(\alpha + \theta) \qquad \text{and} \qquad y^* = |p|\sin(\alpha + \theta) \tag{B.3}$$

where $|p| = |p^*| = \sqrt{x^2 + y^2}$

From elementary trigonometry,

$$\cos(\alpha + \theta) = \cos\alpha\cos\theta - \sin\alpha\sin\theta$$
$$\sin(\alpha + \theta) = \sin\alpha\cos\theta + \cos\alpha\sin\theta$$

Furthermore,

$$\cos\alpha = \frac{x}{|p|} \qquad \text{and} \qquad \sin\alpha = \frac{y}{|p|}$$

Substituting appropriately, we obtain

$$\begin{aligned} \cos(\alpha + \theta) &= \left(\frac{z}{|p|}\right)\cos\theta - \left(\frac{y}{|p|}\right)\sin\theta \\ \sin(\alpha + \theta) &= \left(\frac{x}{|p|}\right)\sin\theta - \left(\frac{y}{|p|}\right)\cos\theta \end{aligned} \tag{B.4}$$

Substitute Eq. B.4 into Eq. B.3 to yield

$$x^* = x\cos\theta - y\sin\theta \qquad y^* = x\sin\theta + y\cos\theta$$

Since this rotation is around the z-axis, z* = z. the transformation matrix for these rotations is

$$R_\theta = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So that $p^* = pR_\theta$. This formulation of R assumes that both p and p* are 1 x 3 matrices.

Generalize this procedure to obtain the rotation transformation matrices for rotation around the y- and x-axes R_β and R_γ.

$$R_\beta = \begin{bmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{bmatrix} \quad \text{and} \quad R_\gamma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & \sin\gamma \\ 0 & -\sin\gamma & \cos\gamma \end{bmatrix}$$

For $\theta = \gamma = 0$, R_β post multiplying p transforms the component for a given rotation around the y-axis. Similarly, for $\theta = \beta = 0$, R_γ post multiplying p transforms the component for a given rotation around the x-axis. What happens if we must rotate an object in such a way that θ, β, $\gamma \neq 0$? We have already said that order is important –rotations must be taken in the order θ, β, γ. (Of course, we could have established another convention just as easily, but this one is consistent with a right-hand coordinate system.) this means that for general rotations with three components,

$$p^* = pR_\theta R_\beta R_\gamma = pR$$

Find R by simply performing the indicated matrix multiplications.

$$R = \begin{bmatrix} \cos\theta\cos\beta & \sin\theta\cos\gamma + \cos\theta\sin\beta\sin\gamma & \sin\theta\sin\gamma - \cos\theta\sin\beta\cos\gamma \\ -\sin\theta\cos\beta & \cos\theta\cos\gamma - \sin\theta\sin\beta\sin\gamma & \cos\theta\sin\gamma + \sin\theta\sin\beta\cos\gamma \\ \sin\beta & -\cos\beta\sin\gamma & \cos\beta\cos\gamma \end{bmatrix}$$

This rotation matrix is applicable for any combination of component rotations R_θ, R_β, R_γ so long as they are taken in that order. Thus, if β=0, merely substitute zero for β in the appropriate terms of the matrix elements, and the result applies to rotations around the x- and z-axes.

It is easy to show that the rotation matrix applies equally well to the algebraic and geometric coefficient matrices of curves and surfaces. Thus,

$$A^* = AR \qquad \text{and} \qquad B^* = BR$$

With the techniques we have just developed, we can now derive transformation matrices for more general rigid-body movements. One of the most useful ways of rotation an object is around some given axis in space. This is illustrated in Figure 35 for a curve. The problem is to find the transformation matrices and operation that will yield the points of a curve rotated from its initial position to its final position through an angle ϕ around an axis whose end points are given by vectors r_1 and r_2.

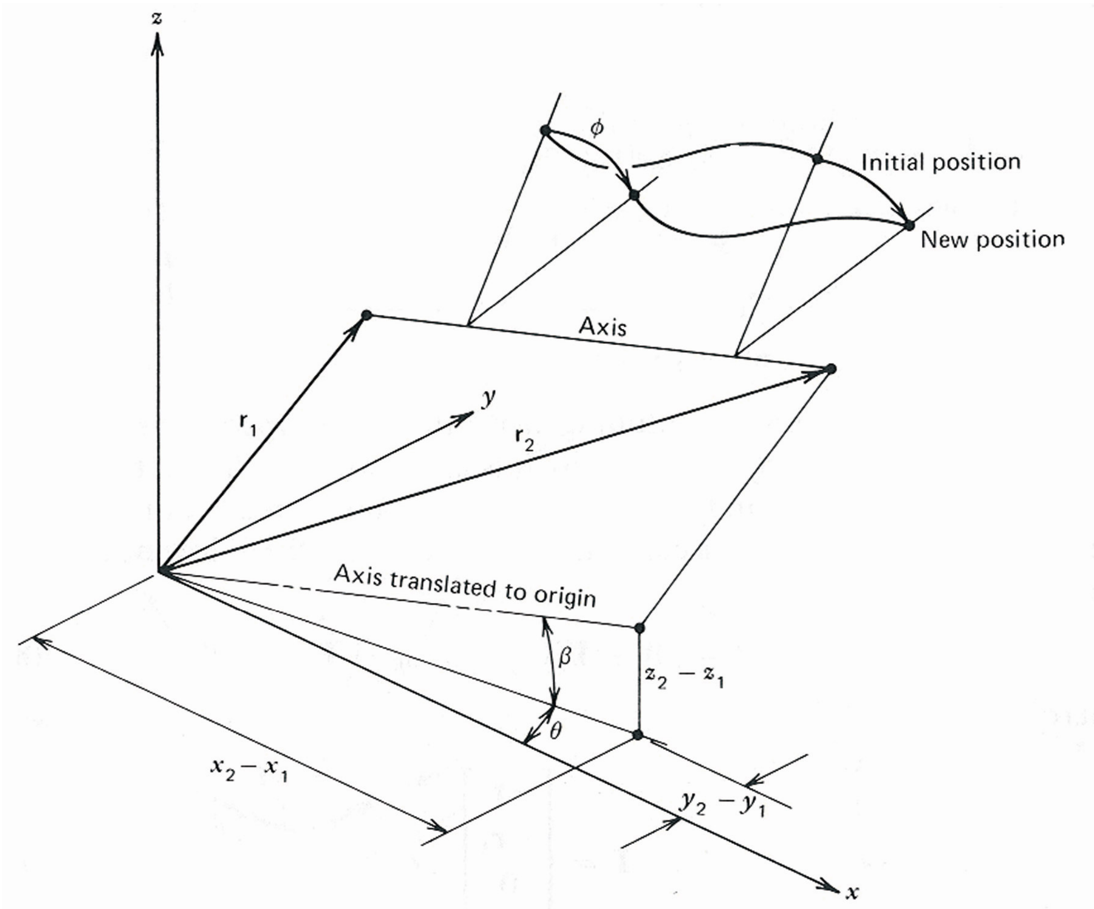

Figure 35. Rotating a curve about an arbitrary axis in space

An algorithm for doing this follows.

Step 1. Translate the curve and axis so that one end of the axis, say, r_1, is at the origin of the coordinate system. Thus, t = - r_1, and if p is any point on the curve, then

$$p^* = p - r_1$$

Step 2. Rotate the curve and the axis system so that the axis is collinear with the x-axis. Do this by rotating -θ degrees around the z-axis and then -β degrees around the y-axis, where

$$\theta = \tan^{-1}\frac{y_2 - y_1}{x_2 - x_1} \qquad \text{and} \qquad \beta = \sin^{-1}\frac{z_2 - z_1}{|r_2 - r_1|}$$

Thus, p* now becomes

$$p^* = [p - r_1]R_{\theta\beta}$$

Step 3. Rotate the curve around the x-axis. After the preceding two steps, $\gamma = \phi$, to yield

$$p^* = [p - r_1]R_{\theta\beta}R_{\gamma}$$

Step 4. Reverse step 2. Notice that there is a sign change in the rotation angles.

$$p^* = [p - r_1]R_{\theta\beta}R_{\gamma}R_{-\theta\beta}$$

Step 5. Reverse step 1 to obtain the final transformation.

$$p^* = [p - r_1]R_{\theta\beta}R_{\gamma}R_{-\theta\beta} + r_1$$

The set of components resulting from these five operations defines the curve point in the new position. Again, it is usually more efficient to transform the B matrix of the object and then proceed to compute points or other properties in the transformed state. For a curve, these steps result in the following analogous transformation sequence:

$$B^* = \left[B + T\right] R_{\theta\beta} R_{\gamma} R_{-\theta\beta} - T \qquad \text{Where } T = \begin{bmatrix} -r_1 \\ -r_1 \\ 0 \\ 0 \end{bmatrix}$$

Now, let $R_{\theta\beta} R_{\gamma} R_{-\theta\beta} = R_a$. Then

$$B^* = \left[B + T\right] R_a - T$$

Which expands to

$$B^* = BR_a + TR_a - T \tag{B.5}$$

Then let $TR_a - T = T_a$. Substitute into Eq. B.5 to obtain

$$B^* = BR_a + T_a$$

Thus, the rigid-body transformations for a curve or any geometric object rotated around an arbitrary axis in space mathematically reduced to a rotation around the coordinate system axes and a subsequent translation.

As an interesting exercise, investigate other, perhaps more exotic, transformation situations, such as rotating a curve in space around a line joining its end points or around one of its tangent vectors. What is the effect on the transformed B matrix in either case?

There is one final transformation algorithm to explore. It, too, combines rotations and translations and is of considerable practical value to geometric modeling in CAD systems. It is called the three-point-to-three-point transformation, or the three-point transformation. The problem is as follows: Given a geometric model that contains points p_1, p_2 and p_3 and given three other points q_1, q_2 and q_3, find the total rigid-body transformation that (1) transforms p_1 into q_1; (2) transforms the vector (p_2 - p_1) into the vector (q_2 - q_1) (direction only); and (3) transforms the plane containing the three points p_1, p_2 and p_3 into the plane containing q_1, q_2 and q_3.

Note: p_1, p_2 and p_3 can be reference points in the model system. It is their relationship to q_1, q_2 and q_3 that determines the total rigid-body transformation applied to the geometric model. Figure 36 illustrates these conditions and relevant parameters.

The following algorithm produces the desired transformation.

Step 1. Construct vectors (p_2 - p_1), (p_3 - p_1), (q_2 - q_1), and (q_3 - q_1).

Step 2. Let $V_1 = p_2 - p_1$, $W_1 = q_2 - q_1$.

Step 3. Construct V_3 and W_3 by

$$V_3 = V_1 \text{ x } (p_3 - p_1) \qquad W_3 = W_1 \text{ x } (q_3 - q_1)$$

Step 4. Construct V_2 and W_2 by

$$V_2 = V_3 \text{ x } V_1 \qquad W_2 = W_3 \text{ x } W_1$$

Clearly the vectors V_1, V_2, and V_3 form a right-hand orthogonal system, as do W_1, W_2, and W_3.

Step 5. Construct the unit vectors.

$$v_1 = \frac{V_1}{|V_1|} \qquad w_1 = \frac{W_1}{|W_1|}$$

$$v_2 = \frac{V_2}{|V_2|} \quad \text{and} \quad w_2 = \frac{W_2}{|W_2|}$$

$$v_3 = \frac{V_3}{|V_3|} \qquad w_3 = \frac{W_3}{|W_3|}$$

Step 6. To transform any point p in the v system into the w system, use the transformation relationship

$$p^* = pR + T \tag{B.6}$$

Step 7. $\begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} R$, since $[w]$ and $[v]$ are the unit vector matrices. Then the required rotation matrix with respect to the w system is simply

$$R = [v]^{-1}[w] \tag{B.7}$$

Step 8. Obtain the required translation matrix by substituting Eq. B.7 for Eq. B.6 and solving for T with p* = q_1 and p = p_1

$$T = q_1 - p_1 [v]^{-1}[w]$$

Step 9. Rewrite Eq. B.6 as

$$p^* = p[v]^{-1}[w] - p_1[v]^{-1}[w] + q_1$$

This transformation is extremely useful for moving two solids into coincidence with each other and for simply repositioning element in a geometric model.

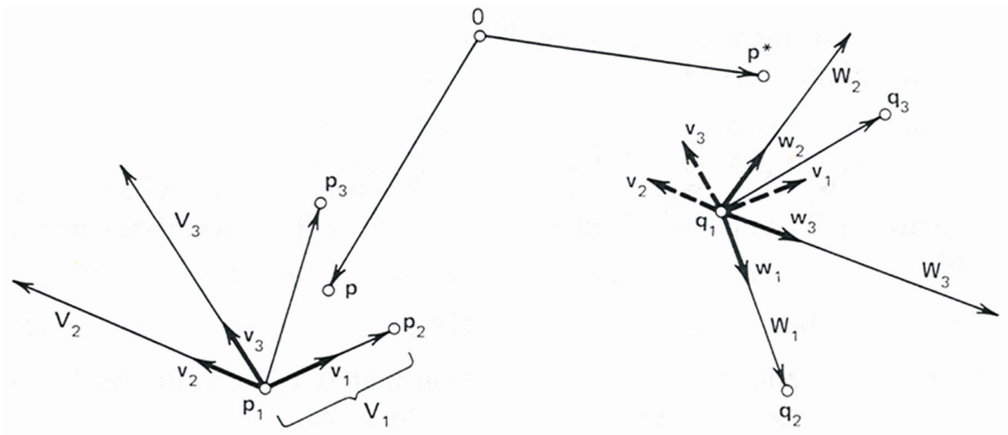

Figure 36. Three-point to Three-point transformation

B.3 Scaling

The absolute size of a geometric element, such as a curve or a surface, may be changed by multiplying its geometric coefficients by a scale factor. If we apply the same scale factor to each coordinate component, then the element will change in size, but not in shape. If we apply different scale factors to each of the components, then both size and shape will change. We will consider each of these cases.

The simplest kind of scaling occurs by applying a positive scalar multiplier to each of the geometric coefficients

$$B^* = sB \tag{B.8}$$

The scalar is always positive. (If it is negative, we are then dealing with reflection.) In Eq. B.8, B is the matrix of geometric coefficients before scaling and B* the scaled or transformed coefficients. According to matrix algebra, each element is multiplied by s. thus, for a parametric cubic curve,

$$B^* = \begin{bmatrix} sp_0 \\ sp_1 \\ sp_0^u \\ sp_1^u \end{bmatrix} \tag{B.9}$$

Figure 37 graphically shows the effect of this scaling transformation. The shapes if the curves p(u) and p*(u) are identical, but curve p*(u) is in this case larger by a factor of s. It is s times longer, and it occupies a different position in space, since the components of each point p* are s times larger than the corresponding components of p. The tangent vectors p_0^{*u} and p_1^{*u} are likewise proportionally greater than p_0^u and p_1^u. The direction cosines at any point are unchanged, however. The point wise transformation of a curve is clearly

$$p^* = sp \tag{B.10}$$

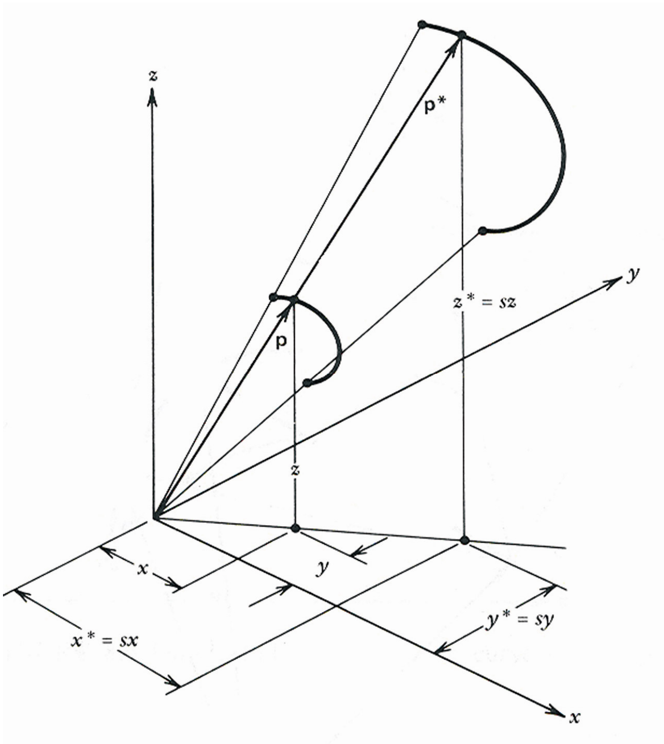

Figure 37. Scaling a curve with respect to the origin

This transformation causes scaling, expansion or contraction, around the system's origin. We can adjust the equation, however, to cause scaling around (with respect to) any point q. Figure 38 illustrates the effect of a curve on a point p when the curve is scaled around point q, where s is the scalar multiplier.

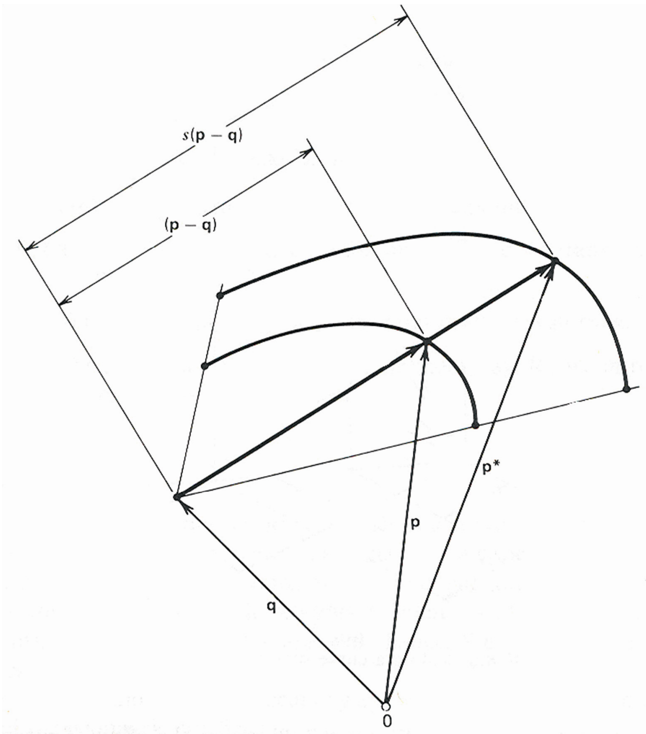

Figure 38. Scaling a curve with respect to an arbitrary point

Vector arithmetic simplifies the derivation of the relationship between point p and its transformed counterpart p*

$$p^* = q + s(p - q) \qquad \text{(B.11)}$$

Rewrite Eq. B.11 as

$$p^* = sp - (s - 1)q$$

Figure 39 illustrates the effect of this type of scaling on the tangent vectors. From the properties of similar triangles,

$$p^{*u} = sp^u$$

By assembling these results, we obtain the transformation equation for scaling a parametric cubic curve around any point in space.

$$B^* = sB + T_s \qquad \text{Where } T_s = \begin{bmatrix} -q(s-1) \\ -q(s-1) \\ 0 \\ 0 \end{bmatrix}$$

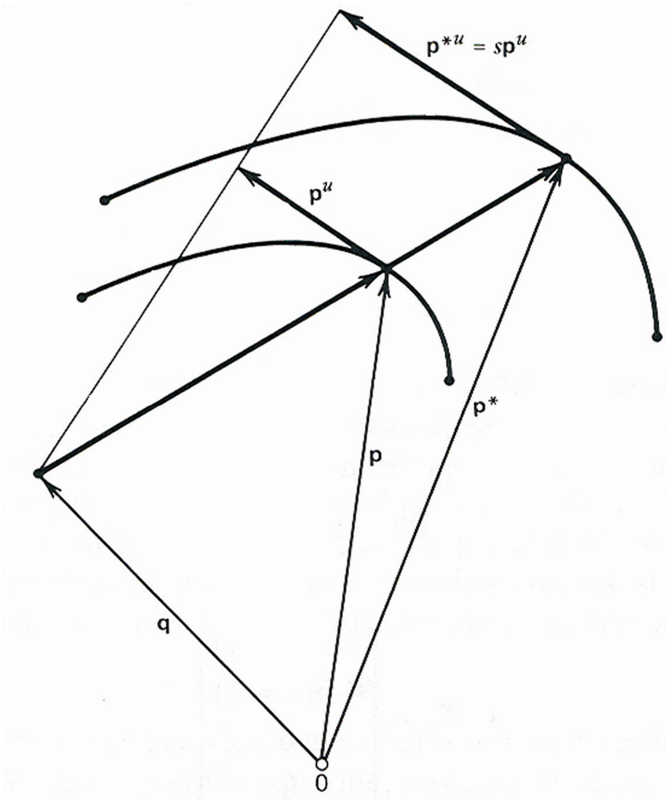

Figure 39. The effect of scaling on tangent vectors

As in Eqs. B.9 and B.10, apply the same scale factor to each component p_x, p_y, and p_z. We are not constrained to do this in all cases. There are often situations in geometric modeling where it is necessary to scale (stretch or shrink) each component by a different factor. We denote this scale factors by s_x, s_y, and s_z. The operation of applying them is called differential scaling. Figure 40 shows an example of this for a curve in the x, y plane. Eq. B.12 shows how to use these scale factors to construct a general, differential scaling transformation matrix for a parametric cubic curve.

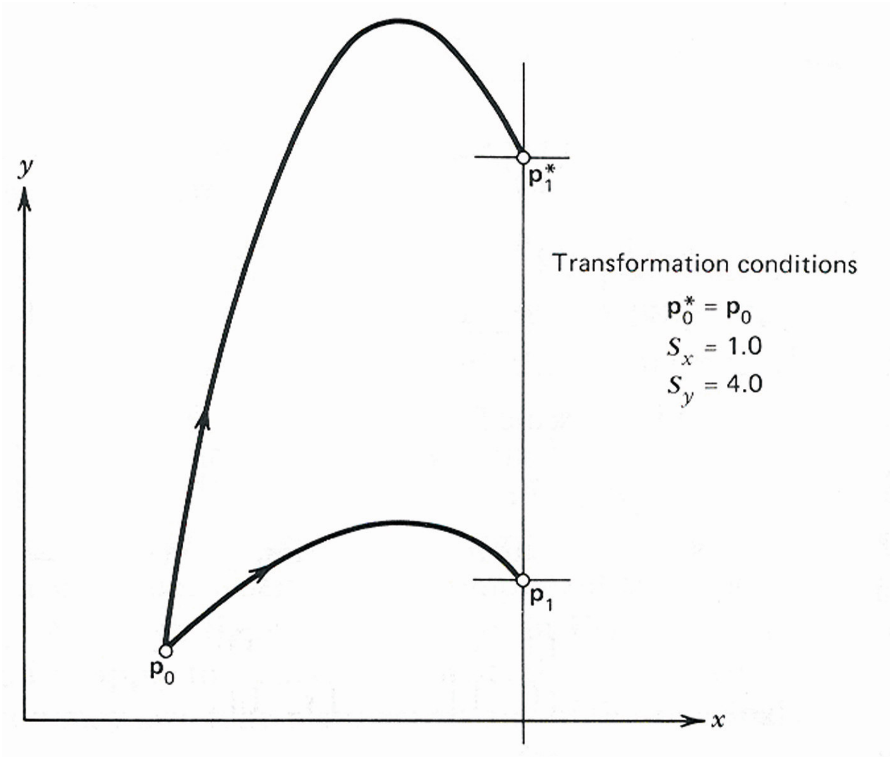

Figure 40. Differential scaling of a parametric cubic curve with respect to p_0

$$B^* = \begin{bmatrix} s_x x_0 - q_x(s_x - 1) & s_y y_0 - q_y(s_y - 1) & s_z z_0 - q_z(s_z - 1) \\ s_x x_1 - q_x(s_x - 1) & s_y y_1 - q_y(s_y - 1) & s_z z_1 - q_z(s_z - 1) \\ s_x x_0^u & s_y y_0^u & s_z z_0^u \\ s_x x_1^u & s_y y_1^u & s_z z_1^u \end{bmatrix} \qquad \text{(B.12)}$$