THE EFFECT OF INSTRUCTION WITH PROBLEM POSING ON TENTH GRADE STUDENTS' PROBABILITY ACHIEVEMENT AND ATTITUDES TOWARD PROBABILITY

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ABSTRACT

THE EFFECT OF INSTRUCTION WITH PROBLEM POSING ON TENTH GRADE STUDENTS' PROBABILITY ACHIEVEMENT AND ATTITUDES TOWARD PROBABILITY

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The purpose of the study was to investigate the effects of instruction with problem posing on tenth grade students' probability achievement and attitudes towards probability. The study was conducted in Nallıhan-Ankara with a total of 82 tenth grade students who were enrolled in one Public High School and one Anatolian High School. Twenty-seven of the subjects received instruction with Problem Posing (PPI), and fifty-five of the subjects received instruction with Traditional Method (TM).

The following measuring instruments were used to collect data: Probability Attitude Scale (PAS), Probability Achievement Test (PAT) and Mathematics Attitude Scale (MAS). The PAS and MAS were administered as both pre and post-tests. The PAT was administered as post-test. In addition, students' overall academic year of

2004-2005 Mathematics and Turkish course grades were collected from the school administration in order to interpret the effects of those grades on students' probability achievement.

The results of the study indicated that: There was a statistically significant difference between the mean scores of students received instruction with problem posing and those received instruction with traditional method in terms of probability achievement, attitudes toward probability and mathematics in the favor of PPI.

Key Words: Mathematics, Probability, Achievement, Attitude, Problem Posing

ÖZ

PROBLEM KURARAK DERS İŞLENİŞ YÖNTEMİNİN ÖĞRENCİNİN OLASILIK BAŞARISINA ETKİSİ VE OLASILIĞA YÖNELİK TUTUMUNA ETKİSİ

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Bu çalışmanın amacı problem oluşturma öğretim yönteminin öğrencinin olasılık konularındaki başarısına, olasılığa ve matematiğe yönelik tutumuna etkisini araştırmaktır. Araştırma Nallıhan-Ankara daki bir genel ve bir de anadolu lisesinde toplam 82 onuncu sınıf öğrencisi ile yürütülmüştür. Çalışmanın 27 deneği Problem Kurma Öğretim Yöntemi (PKÖY) ile 55 deneği ise Geleneksel Öğretim Yöntemi (GÖY) ile öğretim almışlardır.

Bu araştırmada veri toplamak için şu ölçme araçları kullanılmıştır. Olasılık Tutum Ölçeği (OTÖ), Olasılık Başarı Testi (OBT) ve Matematik Tutum Ölçeği (MTÖ). MTÖ ve OTÖ ön ve son test olarak uygulandı, OBT ise son test olarak uygulandı. Ayrıca, öğrencilerin 2004–2005 eğitim-öğretim yılı Matematik ve Türkçe ders notları,

öğrencilerin olasılık başarılarına etkilerini yorumlamak için, okul idarelerinden temin edildi.

Bu araştırmanın sonuçları gösteriyor ki: Problem Kurma Öğretim Yöntemi grubundaki öğrenciler ile Geleneksel Öğretim Yöntemi grubundaki öğrenciler arasında olasılık başarı sonuçlarına, olasılığa ve matematiğe tutumlarına göre PKÖY lehine istatiksel olarak anlamlı bir fark görülmüştür.

Anahtar Kelimeler: Matematik, Permütasyon, Kombinasyon, Olasılık, Başarı, Tutum, Problem Kurma.

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LIST OF ABBREVIATIONS

PPI: Problem posing instruction

TM: Traditional Method

PAT: Probability Achievement Test

PAS: Probability Attitude Scale

MAS: Mathematics Attitude Scale

EG: Experimental Group

CG: Control Group

MP: Main Problem

SP: Sub problem

ATM: Attitudes towards mathematics

ATP: Attitudes towards probability

PAch: Probability Achievement

TG: Turkish Grade

MG: Mathematics Grade

ANCOVA: Analysis of Covariance

SD: Standard Deviation

CHAPTER 1

INTRODUCTION

Regardless of the topics researchers have studied, researches have focused on new instructional methods in order to improve students' problem solving achievements in mathematics. The need is to find the best way that meets students with the conceptualized abstract concepts of mathematics. Researchers argue that problem solving is a complex mental process that involves, for example, using background knowledge (concepts, facts, structures), making connections by associating ideas, reasoning, abstracting, self-monitoring, questioning, evaluating, and visualizing (Gonzales, 1999). The challenge is to translate this complex mental process into reality by understanding students' world of mathematics.

Winograd (1990) states that students typically think that:

1) Mathematics is computation and it means following rules and memorizing.
2) Mathematical problems should be solved quickly and in a few steps.
3) Only geniuses can create problems in mathematics.
4) What matter in a problem is NOT the meaning, but the size of the numbers.
5) Non-routine problems should only be for the "bonus."
6) The teacher should provide all the information to learn, requiring students to merely regurgitate it at the appropriate time.
7) To solve problems, students should look for one "key" word, resulting in a single operation for problem solution.

Such beliefs of students indicate that students' world of mathematics need diagnostic solutions. At this point of view, problem posing becomes the appropriate instructional method to teach mathematics as a specific alternative to traditional instructional methods. Since the nature of problem posing requires creativity, imagination and exploration, encouraging students to write their own words of problems result in their increasing participation in classroom environments. NCTM standards envision a mathematics education program that encourages students to appreciate the value and beauty of mathematics, to be able to understand and use quantitative information, and to engage in the real work of mathematics, including conjecture, reasoning and exploration (Geeslin, 1977; NCTM, 2000). Thus, understanding mathematics requires to get involve in building it from infrastructure to main edifice.

On the other hand, the selection of the topic probability has several reasons for the present study. First of all, rather than the other topic of mathematics, probability as a subject gives more opportunities to students in order to apply their real life experiences to mathematical situations. Moreover, the related literature indicates that students have some difficulties in learning probability concepts which are generally based on interpreting the problems, and highly abstract and formal ways to teach it. Students, in addition, are unaware of the relations of probability with real life situation. Therefore, applying problem posing instruction may give more effective and significant results than traditional instructional methods.

Considering the studies performed in our country, problem posing has long been under the shadow of problem solving in mathematics education. In fact, there is no related study about problem posing performed in our country, or there is some but not finished yet. Thus, in order to understand the results of studies applied in abroad, we need to apply similar studies in our country. Moreover, as reported by the Third International Mathematics and Science Study (TIMSS, 1999), students in Turkey are lagging behind other nations in the areas of mathematics. According to the mathematics results, Turkey has the rank of 31 among 38 countries. It was stated that, integration of new teaching methods to the current education standards in Turkey, especially student-centered ones, may not increase students' mathematics achievements. However, it was also stated that this does not require giving up student-centered activities. In fact, this was explained as (1) in what extents student-centered activities are implemented correctly by teachers and (2) in what extents student-centered activities are adapted to current ingredients of Turkish Education system (TIMMS, 1999). By considering these results, current education system should be modified with correct integration of new teaching-learning activities into the education system in our country. Another implication of TIMSS results for students' mathematics achievement is students' lack of motivation and self-confidence on solving mathematical problems. Since problem posing provides students discover and experience mathematics in their own world, it may foster their motivation and self-confidence on solving mathematical problems. Thus, as a being newly teaching activity in Turkey, problem posing needs to be studied in our country.

The purpose of the present study, then, was to investigate the effect of instruction with problem posing on tenth grade students' probability achievement and attitude towards probability and mathematics in general. Selection of the subjects of the study and other related information are given in following chapters.

CHAPTER 2

LITERATURE REVIEW

In this chapter, the literature related to the present study is reviewed and discussed. Based on the content and the main objectives of the study, the literature is classified as educational studies on probability and the problem posing instruction (PPI).

2.1 Educational Studies on Probability

There are numerous studies on probability in the domain of educational research. Researches indicate that patterns of thinking and ideas related to probability grow over time (Davies, 1965; Piaget & Inhelder, 1975; Fischbein, 1975; Moran & McCullars, 1979; Carpenter et al., 1981; Fischbein & Gazit, 1984; Hoemann & Ross, 1991).

According to Piaget and Inhelder (1975), the development of probabilistic thinking in children occurs in three stages. (i) Stage 1/Sensory Motor (up to 7 years old), (ii) Stage 2/ Concrete-operational (approximately age 7 through 10 years old), and (iii) Stage 3/ Formal-operational (beginning at approximately age 11). It is only during the third stage (after eleven or twelve years) that the judgement of probability becomes organized (Piaget & Inhelder, 1951). That is to say full understanding of probability is only achieved in adolescence, i.e. in the formal-operational stage.

Similar to Piaget and Inhelder findings, Engel (1966) found that students can learn probability theory in every secondary classroom through the study of simple examples and by carrying out many experiments. Carpenter and his colleagues (1981) pointed out that the percentage of correct responses on probability items increased with age (*i.e.* ages 13 and 17) but was still low. Although Engel (1966) stated that children entering secondary school usually have no intuitive notions of probability, Carpenter and his colleagues (1981) found that students have some intuitions but they do not know how to report probability. They have difficulty in developing an intuition about the fundamental ideas of probability even after instruction (Shanghnessy, 1977). Moreover, Garfield and Ahlegen (1988) stated that students' level of specific mathematics skill and students' mental maturity affect the learning of probability. Ford and Kuhns (1991) pointed out that language development of children is important for understanding probability concepts.

Nevertheless, some studies on probability contrary to those above mentioned. For instance, Yost and his colleagues (1962), Siegel and Andrews (1962), and Davies (1965) found that young children had some understanding of probability. More specifically, the studies involving actual teaching programs indicated that appropriate concepts of probability can be successfully taught in elementary and lower secondary school (Ojeman et al., 1965; and Romberg and Shepler, 1973).

Whatever the reasons are, it is obvious that students have difficulties in learning probability concepts. Especially, prerequisite knowledge has great impact on students' probability achievement. They have difficulty with prerequisite concepts including fractions, decimals, percents (Carpenter et al., 1981), or operations on sets (Baron and Or-Bach, 1988). This difficulty may raise insufficient interpretations of a given problem. Mosteller (1967) and Carpenter et al (1981) mentioned that they have difficulty in interpreting the problems. Moreover, abstract and formal teaching methods make students develop distaste for probability (Garfield and Ahlegen, 1988). There becomes a conflict between probability ideas and students' experiences and how they view the world (Hope and Kelly, 1983).

According to Myers and his colleagues (1983), it may be concluded that teaching subjects to solve probability problems by different instructional methods may produce different types of learning outcomes. For example, Cankoy (1989) studied the effects of a mathematics laboratory method on probability achievement of eighth grade students. He found that there was a significant mean difference in the favor of the mathematics laboratory group over those taught traditionally.

Some recommendations for probability education says that teaching probability through concrete experiments holds much more promise than teaching through traditional methods (Lappan and Winter, 1980). For instance, elementary grades students should experience concepts in probability by using an exploratory approach rather than by focusing on the theory of probability (Burns, 1983).

In conclusion, students' understandings of probability increase with age and the type of instructions used. However, learning of probabilistic concepts is limited by students' misconceptions with and unawareness of the relations between real-life situations and probability.

2.2 Problem Posing

In contrast to probability, problem posing has less literature, yet growing. Therefore, before giving the related literature of problem posing, it is more appropriate to first give the definition.

2.2.1 Definition

In the literature of problem posing, the definitions were given by the researchers from different perspectives: problem posing has been defined as a generation of a new problem or reformulation of a given problem (Duncker,1945), as a formulation of a sequence of mathematical problems from a given situation (Shukkwan,1993), as a resultant activity when a problem invites the generation of other problems (Mamona-Downs, 1993). Moreover, Silver defined problem posing as the creation of a new problem from a situation or experience, or reformulation of given problems (1993), and Dickerson said that problem posing allows students to write problems using their own language, syntax, grammar and context (1999). Similar to Dickerson (1999), it was stated that problem posing allows students to formulate problems, using their own language, vocabulary, grammar, sentence structure, context, and syntax for the problem situation (Brown, 1981a; Burns&Richards, 1981; Brown&Walter, 1990; Cromwell&Sasser, 1987; Frankenstein, 1987).

In the mathematics classrooms, problem posing can be viewed as a teaching activity where the teacher intentionally poses questions for students to solve, and can also be viewed as a learning activity, where the student poses questions in response to different circumstances; real-life situations, another mathematical problem, or the teacher (Stoyanova, 1998).

With those definitions of problem posing from different perspectives, the researcher also have defined categories for the problem-posing situations. For instance, Stoyanova and Ellerton introduced three main categories namely *free, semi-structured* and *structured* for the problem-posing situations (1996). In a *free problem-posing situation*, students simply asked to generate a completely new problem on a basis of contrived or naturalistic situations. Students' may be asked to write a new question about a specific topic of a given content. For instance, a teacher may ask to write a problem about "conditional probability" without giving any starting point or data. At this stage, students need to be ready to generate all of the major ingredients of a problem. A problem-posing situation where students are given an open situation and are invited to explore the structure or to finish it using knowledge, skills, concepts and relationship from their previous mathematical experiences is defined as *semi-structured situations*. This involves asking students to pose questions from a collection of data or from the given answer or a calculation: posing a class of problems related to a specific solution method, posing sequences of interconnected problems related to a specific concept(s), posing problems driven from pictures, equations, inequalities, etc. For example, a teacher may ask to students: "Consider that you have 2 coins to toes" and want them to complete the sentence in order to pose a problem. In a *structured problem-posing situation*, a well structured problem or problem solution is given, and the task is to construct new problems which relate, somehow, to the given problem or solution. An example of a tool for structured problem posing situation is 'what if not?' strategy (Brown&Walter, 1983,1993) that makes students examine each component of the problems' data and questions, and makes them manipulate the problem through the process of asking "what if not?". For example, students may be given a problem: "A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if there are no restrictions?" Then, they manipulate the data or conditions of the problem. For instance, one may manipulate the given problem as: "A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carried out if there must be an even number of women in the committee?"

Moreover, in *A Blueprint for Problem Posing* , which consists of five phases in order to teach problem posing to students, Gonzales (1998) suggests that the teacher should emphasize posing questions and ask students to pose a related problem after using Polya' s problem solving methods (1973).

2.2.2 Background

In mathematics education problem posing has long been under the shadow of problem solving (Stoyanova, 1998), yet researchers started to be aware of the potential of problem posing and there has been a growing recognition of the need to incorporate problem-posing activities into the mathematics classrooms. For example, it was stated that problem posing is a significant component of the mathematics curriculum and is considered to lie in the heart of mathematical activities (Brown & Walter, 1983, 1993; Kilpatrick, 1987; Moses, Bjork & Goldenberg, 1990; National Counsel of Teachers of Mathematics [NCTM], 1989, 1991; Silver, 1990, 1994). The inclusion of problem-posing activities in the curriculum can foster more diverse and flexible thinking, enhance students' problem-solving skills, broaden their perception of mathematics, and enrich and consolidate basic concepts (Brown & Walter, 1993; English, 1996, in press a; Silver & Burkett, 1993; Simon, 1993). Furthermore, problem posing activities can provide us with important insights into children's understanding of mathematical concepts and processes, as well as their perception of, and attitudes towards, problem solving and mathematics in general (Brown & Walter, 1993; English, 1996; Van den Heuvel-Panhuizen, Middleton, & Streefland, 1995). The importance of an ability to pose significant problems was recognized by Einstein and Infeld (1938), who wrote:

> The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skills. To raise new questions, new possibilities, to regard old questions from a new angle, require creative imagination and marks real advance in science. (Ellerton&Clarkson, p.1010)

One way to provide pupils with such opportunities that stimulate higher-order thinking is to let them carry out investigations, especially open-ended investigations, where pupils pose the problem to be investigated and design their own procedures to answer the question (Chin&Kayalvizhi, 2002). Since whole investigations can provide pupils with the opportunity to investigate problems of particular relevance to them, they encourage ownership while also engaging the integrated processes which are commonly found to be most difficult to learn (Arena, 1996).

Problem posing can also promote a spirit of curiosity and more diverse and flexible thinking (English,1997). Students who are engaged in problem posing activities become enterprising, creative and active learners. They have the opportunity of navigating the problems they pose to their domains of interest according to their cognitive abilities (Goldenberg, 1993; Mason, 2000; Moses, Bjork, & Goldenberg, 1990). Studies show that problem posing might reduce common fears and anxieties about mathematics (Brown&Walter, 1993; English, 1997; Moses et al., 1990; Silver, 1994). The inclusion of problem posing activities might help students develop improved attitudes towards mathematics, reduce erroneous views on the nature of mathematics and become more responsible for their learning (Brown & Walter, 1993; English, 1997; Silver, Mamona-Downs, Leung, & Kenney, 1996). Students read, examine data and think critically about problem formation and structured, they are actively engaged, minimizing inattention and off task behavior (Davidson&Pearce 1988). When students write their own problems, they become actively engaged, eager, involved in the art of thought, and motivated to solve the more realistic, meaningful problems that they wrote which reflect their own life (Rudnitsky 1995; Ford, 1990).

Moreover, constructivism and task theories provide the theoretical basis for problem posing. If educators accept the constructivist theory of learning as a “progressive organization and reorganization of ideas under the stimulus of a dynamic environment” (Connor&Howkins, 1936, p.20), then problem posing, which seeks to build meaning, relevance and logic through a child’s own language and experiences, seems a promising pedagogical approach (Dickerson,1999). According to constructivists, students actively build mathematical knowledge when they strive to make a sensible pattern out of the confusion of the world around them (Cobb, Yackel, & Wood, 1992; Battista, 1999). In addition, learning is firstly constructed in the social area between students, or between teachers and students. This is considered the interpsychological arena. Students learn and gain behaviors in this arena with reflections of the social interactions.

The social nature of problem posing, where students write and share problems together, facilitates learning and cognitive change (Winograd, 1990). Collaboration in learning is valued as students learn to “establish and defend their own positions while respecting the positions of others” (Pajares, 1998). As students listen to others' problems, they are able to clarify and refine their own concept understanding, promoting their own ability to go from solution of specific problems to production of generalized problem-solving strategies (Cappo&Osterman, 1991). Students feels more equal in status with their peers, than with teachers, so they may be more likely to ask questions, to seek classifications within their small groups (Dickerson,1999).

The writing aspect of problem posing maximizes learning as it forces student to take an idea, interact with it cognitive, engaging in reflection, analysis and synthesis. Writing problems improves students' cognitive and communication skills, at the same time clarifying their understanding of mathematical process and providing linkage across the curriculum (Burton, 1992; Richards, 1990; Matz&Leier, 1992). When students write their own problems, many of the linguistic and reading difficulties of solving textbook "word problems" may be reduced, if not eliminated as their familiarity with language maximizes success (Burns&Richards, 1981; Wirtz&Kahn, 1982; Wright&Stevens, 1983; Kilpatrick, 1987; Resnick&Resnick, 1996). When students write their own problems, they use their own ideas, giving them time to think and thus increasing comprehension and precision knowledge (Geeslin, 1997; Cappo&Osterman, 1991). Bell and Bell (1985) propose that writing problems may promote learning because the writing process itself is self-paced and review-oriented. When students write their own problems, they can gain an appreciation and understanding of the underlying structure of problems, developing their abilities to sense number relationships and generalize these concepts to the real world (Dickerson, 1999). When students use their own language, vocabulary, grammar, interests and contexts, the connections between the old and new are made strongly, quickly and easily (Lesh, 1981; Walter, 1992; Winograd, 1990).

The use of problem posing decreases dependence on the one right answer syndrome of most textbook problem-solving applications, fostering the natural curiosity of children (Brown, 1976, 1983; Walter&Brown, 1977; Burton, 1984; Cromwell&Sasser, 1987; Lerman, 1987; Silver & Mamona, 1989a; Silver & Mamona, 1989b; Silver et al., 1990). Instead, children are encouraged to develop their own abilities to generalize, specialize and analyze, at the same time improving their writing skills in terms of clarity, accuracy and organization (Polya, 1945; Wright&Stevens, 1980; Petersen&Jungck, 1988; Solorzano, 1989). Problem posing encourages students to “clarify, refine, and consolidate their thinking” (NCTM, 1989, p.9; Kliman&Richards, 1992).

According to Dickerson (1999), problem posing, which allows students to write problems using their own language, grammar, syntax, and context, may provide a viable alternative to traditional problem-solving instruction. It encourages students to use mathematics to make sense out of their world by building connections between previous and new knowledge through authentic, personally meaningful experiences. He found that problem-posing instruction appears to be an effective approach to increase the problem-solving achievement of success. He claims that problem posing offers a better instructional strategy that meets the goals of NCMT for reasoning, communication, connections within mathematics.

Furthermore, problem posing has constant roots through the Polya’s (1957) four stage model of problem solving. Polya’s model states that the problem solver must understand the problem, devise a plan, carry out the plan, and then look back at his action. The “looking back” stage involves checking for correctness and determines the best solution as well as asks the problem solvers to pose or formulate original problems that are in some way related to the problem just solved. Problem posing, along with problem solving, is central to the discipline of mathematics and the nature of mathematical thinking (Silver, 1994).

2.2.3 Research in Problem Posing

In mathematics education research, problem posing has been used both as an instruction method and an activity. For example, in her one-year study involved designing and implementing a problem posing program for fifth grade children’s number sense and novel problem solving skills, English (1999) found substantial developments in (a) children’s recognition and utilization of problem structures, (b) their perceptions of, and preferences for, different problem types, and (c) their development of diverse mathematical thinking in contrast to those who did not participate in problem posing program.

In addition, Lavy and Bershadsky (2003) observed the kinds of problems posed by pre-service teachers on the basis of complex solid geometry tasks using the “what if not?” strategy and the educational value of such an activity. Twenty-eight pre-service teachers participated in two workshops in which they had to pose problems on the basis of given problems. Analysis of the posed problems revealed a wide range of problems including those containing a change of one of the numerical data to another specific one, to a proof problem. Different kinds of posed problems enlightened some phenomena such as a bigger frequency of posed problems with another numerical value and a lack of posed problems including formal generalization. They also discussed the educational strengths of problem posing in solid geometry using the "what if not?" strategy (Brown&Walter, 1983,1993), which could make the learner rethink the geometrical concepts he uses while creating new problems, make connections between the given and the new concepts and as a result deepen his understanding of them.

The study, performed by Cai and Hwang (2002), examined US and Chinese 6th grade students' generalization skills in solving pattern-based problems, their generative thinking in problem posing, and the relationships between students' performance on problem solving and problem posing tasks. Across the problem solving tasks, Chinese students had higher success rates than US students. The disparities appear to be related to students' use of differing strategies. Chinese students tend to choose abstract strategies and symbolic representations while US students favor concrete strategies and drawing representations. If the analysis is limited to those students who used concrete strategies, the success rates between the two samples become almost identical. With regard to problem posing, the US and Chinese samples both produce problems of various types, though the types occur in differing sequences. There was a much stronger link between problem solving and problem posing for the Chinese sample than there was for the US sample.

Nicolaou and Philippou (2004) examined the relation among efficacy in problem posing, problem-posing activity, and mathematical achievement. They found a strong correlation between students' ability in problem posing and their general mathematical achievements, significant differences were also found in problem posing ability, between sixth and fifth grade students. In addition, prior efficacy beliefs in problem posing were a strong predictor of students' performance in problem posing and also in general mathematics achievement.

Chin and Kayalvizhi (2002) conducted a study to investigate students' answers for a given open-ended questions based on two type of investigations. The questions for the first investigation were generated individually, while the other was generated in groups. Among the questions that were posed individually, only 11,7%

could be answered by performing hands-on investigations. Most of the questions asked were based on general knowledge and covered a wide range of topics. However, when questions were generated in groups after examples were shown, there was a significant increase in the number of questions that were amenable to science investigations (71,4%) but they related to fewer topics.

The study, performed by Cai (2002), in which Singaporean fourth, fifth, and sixth grade students' mathematical thinking in problem solving and problem posing were explored showed that the majority of Singaporean fourth, fifth, and sixth graders are able to select appropriate solution strategies to solve these problems, and choose appropriate solution representations to clearly communicate their solution processes. Most Singaporean students are able to pose problems beyond the initial figures in the pattern. The results of this study also showed that across the four tasks, as the grade level advances, a higher percentage of students in that grade level show evidence of having correct answers. Surprisingly, the overall statistically significant differences across the three grade levels are mainly due to statistically significant differences between fourth and fifth grade students. Between fifth and sixth grade students, there are no statistically significant differences in most of the analyses. Compared to the findings concerning US and Chinese students' mathematical thinking, Singaporean students seem to be much more similar to Chinese students than to US students.

Stover (1982) investigated the consequences of having students make format changes to mathematics problems. He observed substantial improvement in students' ability to solve problems of the type they had learn to modify. In their study, Silver and Cai (1993) found a strong positive relationship between problem posing based on a brief story with an unstated question and the problem-solving performance of the middle school students on open-ended mathematical problems. In her study, Stoyanova (1998) investigated the effects of a range of problem posing situations on students' problem-solving and problem-posing mathematical performances. She reported that, on a range of predefined performing categories, students exposed to problem-posing and problem-solving activities outperformed students exposed only to problem-solving activities. Stressing self-perception, problem-posing education bases its philosophy on creativity and stimulates true reflection and action upon reality (Milner, H.R., 2003).

Owens (1999) found that problem posing instructed students performed significant achievement than traditionally instructed students in algebra. Overall, instructors using problem posing lessons should provide students showing abstract learning traits with extra assistance and guidance in problem posing lessons to maximize their achievement gains.

Another research conducted on preservice teacher performed by Crespo (2003), he claims that learning to pose mathematical tasks is one of the challenges of learning to teach mathematics. How and when preservice teachers may learn this essential practice, however, is not at all clear. She reports on a study that examined the changes in the problem posing strategies of a group of elementary preservice teachers as they posed problems to pupils. It reports that their later problem posing practices significantly differed from their earlier ones. Rather than posing traditional single steps and computational problems, these preservice teachers ventured into posing problems that had multiple approaches and solutions, were open-ended and exploratory, and were cognitively more complex. Their problem posing style also changed. Rather than making adaptations that made students' work easier or narrowed the mathematical scope of the problem, their adaptations became less leading and less focused on avoiding pupils' errors. Posing problems to an authentic audience, engaging in collaborative posing, and having access and opportunities to explore new kinds of problems are highlighted as important factors in promoting and supporting the reported changes.

In his study Craig (1999), found that preservice elementary teachers cannot be expected to teach differently than they are taught. Problem posing should be modeled and implemented in the university mathematics classroom as well in kindergarten through high school. Problem posing as a method of teaching a mathematical concept is difficult for preservice elementary teachers Craig (1999).

1.3 Summary

To sum up, the use of problem posing in mathematics classrooms provides opportunities for every child to link their own interest with all aspects of their mathematics education. Several aspects of problem posing are thought to play an important role for linking students' personal interests with their education. Mathematics is a way of thinking. Problem posing activities provide environments that seem to engage students in a natural way in reflective mathematical abstraction.

Such activities nurture students' attempts to explore problem and solution structures rather than to focus only on finding solutions. Therefore, developing students' ability to pose and explore problems, and to mathematise their everyday experiences, should be seen as a vital component of mathematical instruction at all levels.

The studies all above carried out abroad discussed specifically the effects of problem posing instruction or activities on number sense, solid geometry, and generally mathematical achievement and thinking of preservice teachers as well as elementary school students. In mathematics education problem posing has long been under the shadow of problem solving (Stoyanova, 1998). In fact, there is no study about problem posing in Turkey. Furthermore, when combining the literature of probability and problem posing, problem posing activities will have significant effects on students' probability achievements. Thus, I would like to open the door by applying problem posing activities in order to teach probability.

CHAPTER 3

METHOD OF THE STUDY

This chapter explains the main problem and the hypotheses of the present study, research design, and subjects of the study, definitions of terms used in the study, statement of the variables, measurement instruments, procedures followed, and tools used for analyzing the data.

3.1 Research Design of the Study

The purpose of the study was to investigate the effect of instruction with problem posing on tenth grade students' probability achievement and attitude towards probability and mathematics in general.

The Probability Achievement Test (PAT), the Probability Attitude Scale (PAS) and the Mathematics Attitude Scale (MAS) were administered in the present study.

This study utilized the matching-only pretest-posttest control group design as outlined in Table 3.1.

Table 3.1 Research Design of the Present Study

Group	**Pre-test**	**Treatment**	**Post-test**
EG	T1,T2,TG,MG	PPI	T1,T2,T3
CG	T1,T2,TG,MG	TM	T1,T2,T3

In Table 3.1, the abbreviations have the following meanings: EG represent experimental group, which received instruction with the “Problem Posing” (PPI); CG represent the control group, which received instruction with the "Traditional Method" (TM).

The measuring instruments in Table 3.1 are the following: T1—Probability Attitude Scale (PAS); T2—Mathematics Attitude Scale (MAS); T3—Probability Achievement Test (PAT); MG—Mathematics Grade; TG—Turkish Grade. The PAS and MAS were administered as pre-tests and post-tests. The PAT was administered as post-test. In addition, 2004-2005 mathematics and Turkish grades were taken from the schools’ administrations.

3.2 Main and Sub-problems and Associated Hypotheses

This section presents the main problem and related sub-problems of the thesis, and examines relevant hypotheses.

The main problem of the present study is the following:

- MP: What is the *effect of instruction with problem posing* on students' probability achievement and attitudes toward probability and mathematics in general?

The main problem has been divided into three sub-problems:

- *SP1: What is the effect of instruction with problem posing* on students' probability achievement?
- SP2: What is the *effect of instruction with problem posing* on students' attitudes toward probability?
- SP3: What is the *effect of instruction with problem posing* on students' attitudes toward mathematics?

Before studying the first sub-problem SP1, the following hypothesis was stated:

H1: There is no significant difference among the mean scores of students received instruction with problem posing and those received instruction with traditional method in terms of probability achievement (PAch).

To study the second sub-problem SP2, the following hypothesis was tested:

H2: There is no significant difference between the mean scores of the students received instruction with problem posing and those received instruction with traditional method in terms of attitudes toward probability (ATP).

To study the third sub-problem SP3, the following hypothesis was tested:

H3: There is no significant difference between the mean scores of the students received instruction with problem posing and those received instruction with traditional method in terms of attitudes toward mathematics (ATM).

As shown above, the hypotheses are defined in the null form. They were tested at the level of significance $\alpha=0.01$ after the treatment of subjects in the experimental and control groups.

3.3 Subjects of the Study

The subjects of the study were 82 tenth grade students, 49 enrolled in a public general high school, and 33 enrolled in an Anatolian high school. All the subjects learned the same mathematical content with the same textbook in the same period of time. The students were assigned to classes randomly by the schools administration when they started the tenth grade and the classes were heterogeneous. The study was carried out during the spring semester of 2004-2005 academic years. The distribution of the subjects is given in Table 3.2.

Table 3.2 Distributions of Subjects of the Present Study

	Groups		
School Types	Experimental (PPI)	Control (TM)	Total
Public High School	14	35	49
Anatolian High School	13	20	33
Total	27	55	82

3.4 Definition of Terms

In this section, some of terms that were used in this study are defined to prevent any misunderstandings.

1. Probability Achievement refers to subjects' achievement scores on permutation, combination and probability measured by PAT.

2. Attitude toward Probability refers to subjects' attitude scores on the "Probability Attitude Scale".

3. Attitude toward Mathematics refers to the subjects attitude scores on the "Mathematics Attitude Scale".

4. Problem Posing refers reformulation of a given problem, creation of a new problem, or generation of a problem.

5. Treatment refers to the method of instruction; either instruction given by Traditional Method (TM) or instruction with Problem Posing (PPI).

6. Control Group (CG) refers to the group who received instruction with the Traditional Method.

7. Experimental Group (EG) refers to the group received instruction with Problem Posing.

8. Math grade (MG) refers to subjects' 2004-2005 mathematics grades.

9. Turkish grade (TG) refers to subjects' 2004-2005 Turkish grades.

3.5 Procedure

In this section procedure of the study is explained.

3.5.1. Steps of the Study

1- The study began with the review of literature about various aspects and current state of questions researched in the current study.

2- The probability attitude scale (PAS) was developed by Bulut (1994). The researcher developed the probability achievement test (PAT). The mathematics attitude scale (MAS) was developed by Aşkar (1986).

3- The PAT was piloted with 100 10^{th} and 11^{th} grade students at a high school in İnegöl-Bursa February 2005. This pilot study allowed testing the reliability and validity of PAT. According to the results of this pilot study, the PAT was revised.

4- Activity sheets were prepared using appropriate problem posing statements as recommended by reports of research found in the literature.

5- Mathematics teachers administered the PAS and MAS to the students before and after the treatment during a mathematics lesson. Both schools provided one control and one experimental group.

6- Researcher taught groups in the public high school, other teacher taught g roups in Anatolian high school.

7- The study ran into a period of six weeks with 5 hours in each beginning in May 2005.

8- The data obtained from the PAS, PAT and MAS before and after the study, and students' 2004-2005 mathematics and Turkish grades were analyzed and used in reaching conclusions about the problem.

3.6 Development of the Measuring Instruments

In the present study, the following measuring instruments were used:

1. Probability Achievement Test

2. Probability Attitude Scale

3. Mathematics Attitude Scale

The development process of each measuring instruments is explained below.

3.6.1 Probability Achievement Test (PAT)

This test was developed by the researcher to determine students' permutation, combination and probability achievement (see Appendix A). The test and course content and objectives were determined according to high school curriculum of Ministry of National Education. The content of PAT included product rule, permutations with repetition, circular arrangements, combinations, fundamental concepts of probability, function of probability and types of probability. The test was developed as an open-ended. It was used as a post test.

Pilot study of the PAT was conducted in a high school in Inegöl-Bursa with 100 pupils of 10^{th} and 11^{th} grade in Spring 2005. The administration of the test held in two lesson hours. After the pilot study, no items were eliminated. Each item had different weight, and was graded by using an analytic approach. Hence, an answer key was prepared to eliminate subjectivity. Each item of PAT was given five points. Since there was 20 items in PAT, it was scored over 100. The rubric of PAT was given in Appendix G. The content related validity of the instrument was established by researcher and a mathematics teacher. It was suitably prepared according to the tenth grade mathematics program. Since the PAT did not contain objective test items, the researcher and a mathematics teacher scored the test administered in the pilot study. The correlation between the two scorings was determined to test the reliability of the test. The correlation coefficient was found as 0.99 for the rater reliability.

3.6.2 Probability Attitude Scale

Probability Attitude Scale (PAS) was developed by Bulut (1994) (see Appendix B). The 28-item PAS consisted of 15 positive items and 13 negative items and was scaled on a six-point Likert Type scale: Strongly Agree, Agree, Tend to Agree, Tend to Disagree, Disagree, Strongly Disagree. The positively worded items were scored starting from Strongly Agree as 6, to strongly disagree as 1, and negatively worded items were reversed to a positive direction for scoring purposes. This six-point scale was used to disallow the undecided response in five-point scales.

The PAS has one-dimension which was labeled "general attitude toward probability". Also, content validity of the PAS was checked by a mathematics education researcher. In the present study, the alpha reliability coefficient of the PAS with 28 items was found as 0.95. The total score of PAS was between 28 and 168.

3.6.3 Mathematics Attitude Scale

Mathematics Attitude Scale (MAS) was developed by Aşkar (1986) (see Appendix C). It consisted of 10 positive and 10 negative items about attitude toward mathematics. They were in five-point Likert-type scale: Strongly Agree, Agree, Undecided, Disagree, Strongly Disagree. Positive items were coded starting from Strongly Agree as 5 to Strongly Disagree as 1. Negative items were coded as from 1 to 5. She found the alpha reliability coefficient 0.96 with SPSS. It has one dimension called as “general attitude toward mathematics”. “The total score of MAS is between 20 and 100.

3.7 Activities based on Problem Posing

During the treatment of experimental groups, developed problem posing activities was applied about the topic of permutation, combination and probability. The framework of these activities was developed by researcher and the participant teacher by considering the "what if not" strategy, semi-structured problem posing situations, structured problem posing situations and free problem posing situations (Stoyanova & Ellerton, 1996). Activities were started to ask students to reformulate a given specific problem without changing the mathematical nature of the problem for the related content. For example, students were given the question: Find the different selections of 4 mathematics books among 9 mathematics books. One of the reformulation of this question were "How many different 4 mathematics books can be selected among 9 mathematics books?". In order to start individual discussions, problems with surplus or insufficient information were also used in these activities. For instance, students were given an open statement: "Consider that you have 3 railways from Ankara to Eskişehir". They were asked to complete the statement in order to pose a problem. In addition, activities that fostered students to analyze the sequences of problems with different verbal context but same solution, and activities requiring numerical changes rather than verbal changes were also applied. In the activities including "what if not" strategy (Brown and Walter, 1983, 1993), students listed all attributes of a given problem, and then they had to ask "what if not attribute k". If negation of an attribute spoiled the nature of a given problem, then they had to find an alternative attribute. By applying this strategy, they had already posed a new problem. Sample activity sheets and students' sample answers to these questions were given in the Appendix D and E respectively.

3.8 Treatments

Different treatments were administered to the control and the experimental groups, but both the experimental groups and the control groups received instruction from their own mathematics teacher (researcher is one of the mathematics teacher). The two groups were taught the same content to reach exactly the same objectives, which are presented in Appendices F. There were two control groups and two experimental groups, which received the treatments described below.

3.8.1 Treatment of the Control Group

The instruction given to the control group (CG) was called the Traditional Method (TM) because the teacher taught concepts and skills directly to the whole class. The subjects were taught in a teacher-centered way. The only interaction between students and the teacher occurred when students asked questions. This class received 30 hours instruction during six weeks. Students did not use problem posing in the control group. The teacher only taught the curriculum as a way of instruction by lecturing. Students worked individually during the class. The control groups were given PAS and MAS before and after the treatment, whereas PAT was administered at the end of the treatment. The teacher explained to the students the purpose of the attitude scales and achievement test.

3.8.2 Treatment of the Experimental Group

The experimental Group (EG) was instructed based on problem posing activities. The instruction of the EG groups lasted 30 hours during six weeks. One day before the treatment the students were explained the purpose of the treatment, procedure to be followed. They were also explained to behave collaboratively. The process of problem posing was discussed and modeled by the instructors at the beginning of the study.

In the experimental group, the related topics were lectured at the beginning of the lessons about in 10-20 minutes to students. Activities that required discussions were accomplished by assigning students to work in pairs. Simply, activities were started to ask students to reformulate a given specific problem without changing the mathematical nature of the problem for the related content. For example, students were given the question: Find the different arrangements of 8 different mathematics books. One of the reformulation of this question were "How many different ways can 8 mathematics books be arranged?". After allowing students rewrite the given problems in their own words, "what if not" strategy (Brown and Walter, 1983, 1993) were used to start individual discussions. In this strategy, students listed all attributes of a given problem, and then they had to ask "what if not attribute k". If negation of an attribute spoiled the nature of a given problem, then they had to find an alternative attribute. By applying this strategy, they had already posed a new problem. Accomplishment of "what if not" strategy activities were followed by problems with insufficient information. For instance, students were given an open statement: "Consider that you have 3 railways from Ankara to Eskişehir". They were asked to complete the statement in order to pose a problem. In addition, activities that fostered students to analyze the sequences of problems with different verbal context but same solution, and activities requiring numerical changes rather than verbal changes were also applied. Sample posed problems shared with each other and were put on the board to get solutions as well. By the completion of a specific topic, students were given structured situations such as "Write a question which has the answer C(8,3)". And finally, they were assigned to pose free situational problems such as "Write a problem related with conditional probability". The order of the activities was (1) rewriting given problems, (2) "what if not" strategy, (3) problems with insufficient information, (3) problems with different contexts but same solutions, (4) problems with same context but different solutions and (5) free situational problems. All these strategies were used with the current and appropriate instructional curriculum during regular class time under the study. By taking into account the Bloom's taxonomy (Bloom, 1984), students' posed problems represented all levels of abstractions of problems. Sample activity sheets and students' sample answers to these questions were given in the Appendix D and E respectively.

3.9 Variables

Four variables were considered in the present study. Three are dependent variables, and one is independent variable. The dependent variables are the following:

1. Probability Achievement,

2. Attitude toward Probability, and

3. Attitude toward Mathematics.

The independent variable of the present study is considered in a group:

1. Teaching Method, this includes

(i) Traditional Method (TM),

(ii) Instruction with Problem Posing (PPI), and

(iii) Students' 2004-2005 mathematics and Turkish grades

3.10 Data Analysis

Data of the present study were analyzed by descriptive and inferential statistics. Hypotheses of the study were analyzed by Analysis of Covariance (ANCOVA) and independent samples t-tests with the statistical package program SPSS.

3.11 Assumptions and Limitations

As in other studies there are several assumptions and limitations in the present study.

3.11.1 Assumptions

The main assumptions of the present study are the following:

1. There was no interaction between the experimental and control groups to affect the results of the present study.
2. No outside event occurred during the experimental study to affect the results.
3. The instructors were not biased during the treatment.
4. The instructors were considered as equal.
5. The administration of the tests, scales, and questionnaire were completed under standard conditions.
6. All subjects of the control and experimental groups answered the measurement instruments accurately and sincerely.

3.11.2 Limitations

The limitations of the present study are as listed below:

1. This study was limited to the 10^{th} -grade students in two kind of public schools in Nallıhan-Ankara during the spring semesters of 2004 and 2005 academic years.
2. Self-report techniques, which require the subject to respond truthfully and willingly, were applied.
3. The research was conducted in high schools classes, urban, public school, so results are limited in generalizability.

CHAPTER 4

RESULTS AND CONLUSIONS

Results and conclusions of the present study are explained in this chapter.

4.1 Results of Pre-treatment Measures

At the beginning of the treatment, the Probability Attitude Scale (PAS) and the Mathematic Attitude Scale (MAS) were administered as pre-tests. Moreover, students' 2004-2005 mathematics grade (MG) and Turkish grade (TG) were obtained from the administration in the high schools. Equivalency of the treatment groups were tested in terms of pre-treatment measures by using independent samples t- tests. The results were given in Table 4.1.

Table 4.1 Results of Independent Samples t-test for Pre-Treatment Measures

Variables				Levene's Test				
	Group	Mean	SD	F	Sig.	t	df	Sig
MG	PPI	3.89	0.892	1.506	0.223	1.801	80	0.076
	TM	3.45	1.086					
TG	PPI	4.41	0.572	0.354	0.553	2.072	80	0.041*
	TM	4.04	0.838					
Pre-PAS	PPI	125.52	25.521	1.315	0.255	0.029	80	0.977
	TM	125.36	21.819					
Pre-MAS	PPI	75.48	17.368	0.031	0.860	1.112	80	0.270
	TM	71.13	16.323					

*p<0.01

As seen in Table 4.1 there was a statistically significant difference between the mean scores of students taught by problem posing instruction (PPI) with those taught by traditional method (TM) with respect to TG ($p< 0.01$). The Table 4.1 also indicates that there were no significant mean differences between the treatment groups with respect to pre-PAS, pre-MAS scores and MG ($p>0.01$).

4.2 Common assumptions of the analyses

The hypotheses of the present study were tested by ANCOVA and independent samples t-test. Their common assumptions were explained below:

1. Assumption on independence of observations might not be satisfied because some of the situations stated by Pallant (2001) might be happened in the present study. Pallant(2001) stated that

> *"Studying teaching methods within a classroom and examining the impact on students' behavior and performance. In this situation all students could be influenced by the presence of a small number of trouble-makers, therefore individual behavioral or performance measurements are not independent"(p.171)*

We suspect some violation of assumption on the independence of observations. Stevens (1996, p241) recommends that you should set a more stringent alpha value (e.g. $p<0.01$). So, in the current study the alpha level was set as 0.01.

2. In Normal Q-Q Plots the observed values for all dependent variables were plotted against expected values from normal distribution. Reasonably straight lines were obtained. The value of skewness and curtosis is approximately between -2 and 2. They were given below for each group in Table 4.2

Table 4.2 Values of Skewness and Kurtosis of the dependent variables for each group

Groups	n	Skewness/Kurtosis	Post-PAS	Post-MAS	PAT
PPI	27	Skewness	-0.863	-0.471	-0.251
		Kurtosis	0.843	-0.505	-0.931
TM	55	Skewness	0.054	-0.036	0.409
		Kurtosis	-1.359	-1.190	-0.224

Pallant (2001) stated that "Fortunately, most of the techniques are reasonably robust or tolerant of violation of this assumption. With large enough sample sizes (e.g.30+) the violation of this assumption should not cause any major problem" (p.172). Consequently, assumption of normal distribution was approximately satisfied.

4.3 Results of Testing of the First Hypothesis of the Problem

The first hypothesis of the problem (H1) was "There is no significant difference between the mean scores of students received instruction with problem posing and those received instruction with traditional method in terms of probability achievement (PAch)". This hypothesis was tested by the ANCOVA.

To test the hypothesis in addition to assumption explained in section 4.1, special assumptions of ANCOVA were discussed below:

Homogeneity of variance: "Levene's test showed that error variance of the PAch Score is equal across groups ($p>0.01$; $F=1.54$, $df1= 1$, $df2=80$, $p=2.18$).

Correlations amongst the covariates: Covariates-OMG and OTG- were not strongly correlated because the Pearson product moment correlation coefficient was found as $r=0.52$.

Linearity: We generated scatter plots between dependent variables and each covariates for each group. It was found that there were a linear relationship between dependent variables-PAch- and the covariates for treatment groups.

The assumption was also tested statistically. The R squared values gave an indication of the strength of the relationship between dependent variables and covariates. In the PPI group, values were found as $R^2_{\text{PAch-OMG}}=0.99$; $R^2_{\text{PAch-OTG}}=0.95$. In the TM group, R squared values were found as $R^2_{\text{PAch-OMG}}=0.98$; $R^2_{\text{PAch-OTG}}=0.95$.

Homogeneity of Regression Slopes: As seen in Table 4.3 interactions between groups and covariates were not statistically significant at the level of significance 0.01 ($F_{\text{Group-OMG}}=0.96$, $p_{\text{Group-OMG}}=0.47$; $F_{\text{Group-OTG}}=2.87$, $p_{\text{Group-OTG}} = 0.09$).

Table 4.3 The results of ANCOVA for Testing Assumption on Homogeneity of Regression Slopes

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	18316.414	5	3663.283	95.323	.000
Intercept	359.553	1	359.553	9.356	.003
GROUP	146.621	1	146.621	3.815	.054
MG	9691.089	1	9691.089	252.173	.000
TG	36.964	1	36.964	.962	.330
GROUP * MG	20.093	1	20.093	.523	.472
GROUP * TG	110.210	1	110.210	2.868	.094
Error	2920.708	76	38.430		
Total	299410.000	82			
Corrected Total	21237.122	81			

After testing the assumptions of ANCOVA, the hypothesis was tested by ANCOVA with covariates- MG and TG- at the significance level 0.01. The results were given in Table 4.4

Table 4.4 The results of ANCOVA for PAT Scores

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Observed Power**
Corrected Model	18203.905	3	6.067.968	156.039	.000	.857	1.000
Intercept	376.666	1	376.666	9.686	.003	.110	.867
MG	11.318.691	1	11.318.691	291.063	.000*	.789	1.000
TG	1.930	1	1.930	.050	.824	.001	.056
GROUP	1.028.326	1	1.028.326	26.444	.000*	.253	.999
Error	3.033.217	78	38.887				
Total	299.410.000	82					
Corrected Total	21.237.122	81					

* $p<0.01$

** $\alpha=0.05$

As seen in Table 4.4 students' TG and MG were taken as covariates for PAT scores.

While the MG was statistically significant covariate for the PAch ($p<0.01$, $F_{MG}=291.06$, p =0.00), the TG was not statistically significant covariate ($p>0.01$, $F_{TG}=0.50$, $p_{OTG} = 0.82$).

As seen in Table 4.4, it was found that there was a statistically significant difference between the mean scores of students received instruction with problem posing and those received instruction with traditional method in terms of probability achievement (PAch) in the favor of PPI ($p<0.0$). The effect size was 0.25. This meant that it was large effect size because it was greater than 0.14 (Cohen, 1988). It also indicated that 25 percent of variance in PAch score was explained by independent variable-group. Mean and standard deviations of PAT scores were given in Table 4.5

Table 4.5 Means and Standard Deviations of PAT scores

GROUP	n	Mean	SD
PPI	27	67.30	13.179
TM	55	53.80	15.766

4.4 Results of Testing of the Second Hypothesis of the Problem

The second hypothesis of the problem (H2) was "There is no significant difference between the mean scores of the students received instruction with problem posing and those received instruction with traditional method in terms of attitudes toward probability (ATP)". This hypothesis is tested by the independent samples t-test. The results are given in Table 4.6.

Table 4.6 Results of Independent samples t-test for Post-PAS Scores

				Levene's Test				
Variable	Group	Mean	SD	F	Sig.	t	df	Sig.
ATP	PPI	137.67	17.994	20.488	0.000*	5.111	76.686	0.000*
	TM	110.44	30.033					

*p<0.01

As seen in Table 4.6 Levene's Test showed that there was no equality of variances. So, the values in equal variances not assumed were used to test the hypothesis. It was found that there was a statistically significant difference between the mean scores of the students received instruction with problem posing and those received instruction with traditional method in terms of ATP in the favor of PPI ($p<0.01$). To determine the effects size eta-squared was computed by using the following formula (Pallant, 2001, p.180):

$$\text{Effect size (eta squared)} = \frac{t^2}{t^2 + (n_1 + n_2 - 2)}$$

The effect size was computed as 0.25. This meant that it was large effect size (Cohen, 1988). It also indicated that 25 percent of variance in PAS score was explained by independent variable-group.

4.5 Results of Testing of the Third Hypothesis of the Problem

The third hypothesis of the problem (H3) was "There is no significant difference between the mean scores of the students received instruction with problem posing and those received instruction with traditional method in terms of attitudes toward mathematics (ATM)". This hypothesis was tested by the independent samples t- test. The results are given in Table 4.7.

Table 4.7 Results of Independent samples t-test for Post-MAS

Variable	Group	Mean	SD	Levene's Test F	Levene's Test Sig.	t	df	Sig.
ATM	PPI	82.67	11.446	9.558	0.003*	4.497	73.809	0.000*
	TM	68.07	17.676					

* $p<0.01$

As seen in Table 4.7 Levene's Test showed that there was no equality of variances. So, the values in equal variances not assumed were used to test the hypothesis. It was found that there was a statistically significant difference between the mean scores of the students received instruction with problem posing and those received instruction with traditional method in terms of ATM in favor of PPI ($p<0.01$).

The effect size was computed as 0.20. This meant that it was large effect size. It also indicated that 20 percent of variance in MAS score was explained by independent variable-group.

4.6 Conclusions

In the light of the above findings of the present study, the following conclusions can be stated for the present study:

1. There was a significant difference between the mean scores of students received instruction with problem posing and those received instruction with traditional method in terms of PAch. The students taught by PPI had significantly greater probability achievement than the students taught by TM.
2. There was a significant difference between the mean scores of the students received instruction with problem posing and those received instruction with traditional method in terms of ATP in the favor of PPI.
3. There was a significant difference between the mean scores of the students received instruction with problem posing and those received instruction with traditional method in terms of ATM in the favor of PPI.

CHAPTER 5

DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

This chapter restates the treatment, and interprets the results of the present study in discussion. Then, implications and recommendations are mentioned.

5.1 Discussion

The purpose of the study was to investigate the effects of problem posing based instructions on tenth grade students' probability achievement and attitudes towards probability and mathematics in general.

For the present study, conducted in Nallıhan-Ankara, 82 tenth grade students were totally selected in one public general and one Anatolian high school. 49 of the subjects enrolled in the public general high school, and 33 of the subjects enrolled in the Anatolian high school. 14 of the subjects enrolled in public high school and 13 of the subjects enrolled in Anatolian high school received instruction with Problem Posing (PPI), and 35 of the subjects enrolled in the public high school and 20 of the subjects enrolled in the Anatolian high school received instruction with Traditional Method (TM). Probability Attitude Scale (PAS), Probability Achievement Test (PAT) and Mathematics Attitude Scale (MAS) were used to collect the data. In addition, students' 2004-2005 Academic year Mathematics and Turkish grades were taken from the school administration.

After the data were analyzed to determine which treatment group had a significant mean difference, it was found that the students taught by PPI not only had significantly greater probability achievement than the students taught by TM but also had significantly greater attitudes towards probability and mathematics.

The results of attitudes differences between treatment groups confirm the findings of Brown and Walter (1993), English (1997), Moses et al.(1990), Silver (1994), Nicolaou and Philippou (2004) who stated that problem posing might reduce common fears and anxieties about mathematics and foster attitudes towards mathematics. The inclusion of problem posing activities might help students develop improved attitudes towards mathematics, reduce erroneous views on the nature of mathematics and become more responsible for their learning (Brown & Walter, 1993; English, 1997; Silver, Mamona-Downs, Leung, & Kenney, 1996). When students write their own problems, they become actively engaged, eager, involved in the art of thought, and motivated to solve the more realistic, meaningful problems that they wrote which reflect their own life (Rudnitsky 1995; Ford, 1990). However, the results cannot necessarily be generalized beyond the sample population of suburban Turkish students in regular tenth grade students and the subject of probability.

Although there is no research combining problem posing to probability, the findings of the present study indicate that similar to other topics, such as algebra in Owen's (1999) study, problem posing activities may develop the probability ability of students. Since the nature of probability problems is directly related to realife situations, the realm of problem posing satisfies the responsibility of conducting this relationship between the abstract world of probability and students' amplify prior knowledge and life experiences. Students are beginning to communicate mathematically as they pose their own problems.

Furthermore, the effectiveness of problem posing in current research provides further evidence for the validity of constructivist theory of learning for which students learn best when they engage in active sense-making activities that relates their present understandings to a new situation. When students interact with mathematical concepts and process, they may be in the process of forming internal structures of understanding that can be used to reconstruct and amplify prior knowledge (Silver, Kilpatrick, Schlesinger, 1990).

Problem posing activities give the chance of creating your problem with all aspects of your own language, grammar and experiences. Students actively communicate with each other, eagerly participate with their partners, thus, problem posing activities built in social interaction. Moreover, the student-centered aspect of problem posing promotes teacher to have the opportunity to get inside their students' heads, identifying potentially troublesome misconceptions before they become patterns of incorrect thought. Misconceptions with and unawareness of the relations between real-life situations and probability may be identified and remedy diagnostic actions can be taken with problem posing activities. In addition, Bruns' (1983) recommendations for teaching probability with an exploratory approach and student-centered activities may be provided by using problem posing activities.

Since the current study was limited to students of grade ten, it does not give any clues for the findings of Piaget and Inhelder (1975) that full understanding of probability is only achieved in adolescence, i.e. in the formal-operational stage. Likewise, the current study makes no sense for the effects of students' prerequisite knowledge on their probability achievement.

Contrary to traditional mathematics instruction which yields students understanding to rote application of memorized formulas, problem posing supports students with the opportunity to think and reason logically. Regardless of students' previous achievement, teachers may improve students' problem solving achievements, specifically in probability. The teacher in a problem posing environment becomes a facilitator rather than an answer bank. In addition, students' participation of created problems may reduce the differences in students' prerequisite knowledge, and make them be aware of multiple routes to a variety of solutions.

5.1.1 Internal Validity

Fraenkel and Wallen (1996) states that internal validity means that observed differences on the dependent variable are directly related to the independent variable, and not due to some unintended variable.

The possible threats on internal validity of the current study were subject characteristics, location, history, instrument decay, maturation, regression, data collector characteristics, data collector bias, confidentiality, implementation of the treatment and Hawthorne effect. The way of controlling these threats were discussed.

In the present study, subject characteristics could not be a problem for the internal validity. Subjects were at the same age. Hence, those characteristics did not affect research results unintentionally. Moreover, subjects' socioeconomic backgrounds were almost the same. Subjects were tenth grade students so that they were given the same courses through their entire education. Their differences on pre-treatment measures were taken by using ANCOVA. Therefore, their educational backgrounds could not be a threat.

Location and history threats were controlled by administering the pre and post-tests to all groups almost at the same time. The testing locations were not different in terms of physical conditions since they were at the same school even at the same floor. Although maturation was threat in many studies, they were not in the present study. Since all the subjects were at the same age and duration of the treatment was not long, the maturation threat was controlled.

Data collector characteristics and data collector bias should not be threats in this study because data collector followed the same procedure and there was one data collector. While scoring the instruments, the researcher scored an item for all students then passed to the next item in order to prevent instrument decay threat to internal validity.

Confidentiality was satisfied without taking account the names of the subjects. Regression should not be a threat for this study because the subjects were not from the gifted or remedial classrooms. On the other hand, treatment might be a threat to internal validity. The one of the groups in PPI and TM groups were instructed by the reseracher, but the other two groups had a different teacher. To control this threat, the instructions were observed. The observer noted that the teachers solved the same problems and did not favor any of the methods. Since the study took place in regular school settings, Hawthorne effect could be reduced.

5.1.2 External Validity

5.1.3 Population Validity

In the present study, convenience sampling was utilized. Therefore, generalizations of the findings of the study were limited. However, generalizations can be done on subjects having the same characteristics mentioned in chapter 3.

5.1.4 Ecological Validity

Fraenkel and Wallen (1996) states that the ecological validity refers to the degree to which results of a study can be extended to other settings or conditions. The treatments and the instruments were utilized in regular classroom conditions. The results of the present study can be generalized to classroom settings similar to this study.

5.2 Implications

The result of this research showed that problem posing instruction produces significantly positive results in students' probability achievements, attitudes toward probability and mathematics.

Successful mathematics instructions demand more than lecture and require active participation on the part of the learner. Problem posing satisfies such an active participation with an inexpensive and easy to use implement. Moreover, giving students the chance of making mathematics with their own way encourages their self-confidence. Thus, full participations occur in problem posing instruction classes. It also offers teachers insight into patterns of student thought. As students write their own problems, linguistic and reading difficulties may be reduced with diagnostic actions. Perception of problem solving tactics can be modified or improved by allowing students' this chance, namely chance to write their own problems.

On the other hand, there are difficulties in teaching probability among teachers. Hope and Kelly (1983) stated that adolescents also have difficulties in perception of some probability concepts. When combining this with the results of the studies on preservice teachers' problem posing in Lavy and Bershadsky (2003), Crespo (2003) and Craig (1999), it is open to discuss that problem posing instructions promote the view of teachers' probability concepts.

To sum up, rather than traditional instructions, problem posing instruction environment offers better strategies for teaching probability. The results of the present study reinforces the urgency of encouraging mathematics educators to employ problem posing as an instructional strategy to improve student probability achievement, attitudes towards mathematics and probability. The results, in fact, can be generalized to other topic of subjects. It seems that the way of searching the best instructional strategy for the mathematics achievements of students will end up with the understanding of the power of problem posing instructions. Yet, the development of problem posing environment need to be improved by means of its implementation by teachers.

5.3 Recommendations for Further Research

Following are some recommendations for further research on the effects of problem posing instruction on students' probability achievement and attitudes towards probability and mathematics.

- The sample size must be increased in further studies.
- The study can be conducted for different grade levels and school types.
- The duration of application of the treatment can be increased in further studies.
- More research can be done with students of different cultural and linguistic backgrounds.
- Specific research can be done with different problem posing strategies such as "what if not".
- The relationship between problem solving and problem posing can be investigated under the topic of probability.
- The effect of linguistic background of students on posing problems can be searched.

REFERENCES

Arena, P. (1996). The role of relevance in the acquisition of science process skills. *Australian Science Teachers Journal*, *42* (4), pp. 34–38.

Battista, M. (1999) The mathematical miseducation of America's youth: Ignoring research and scientific study in education. *Phi Delta Kaplan. 18* (1) , 425-432.

Bell, E., & Bell, R. (1985). Writing and mathematical problem solving: Arguments in favor of synthesis. *School Science and Mathematics, 85* (3), 210-221.

Bloom, B. J. (1984). *Taxonomy of educational objectives*: Pearson Education, Boston: Allyn and Bacon

Borasi, R., Sheedy, J., & Siegel, M. (1990). The power stories in learning mathematics. *Language Arts,67* (2), 174-184.

Brown, S. (1976). From golden rectangle and fibanocci to pedagogy and problem posing. *Mathematics Teacher, 69,* 180/8.

Brown, S. (1981) Ye shall be known by your generation. *For the Learning of Mathematics*, *3*, 27-36.

Brown, S. (1983). *The Logic of problem generation: From morality and solving to de posing and rebellion.* Canadian Mathematics Education Study Group: British Colombia.

Brown, S. I., & Walter, M. I. (1983a). Some natural links between problem posing and problem solving. In S. I. Brown, & M. I. Walter (Eds.), *The art of problem posing* (pp. 107–122). Philadelphia, PA: Franklin Institute Press.

Brown, S. I., & Walter, M. I. (1983b). The "What-if-not" strategy in action. In S. I. Brown, & M. I. Walter (Eds.), *The art of problem posing* (pp. 63–105). Philadelphia, PA: Franklin Institute Press.

Brown, S. I., &Walter, M. I. (1993). Problem posing in mathematics education. In S. I. Brown, & M. I.Walter (Eds.), *Problem posing: reflection and applications* (pp. 16–27). Hillsdale, NJ: Lawrence Erlbaum Associates.

Burns, M., & Richards, K. (1981). Making sense out of word problems. *Learning, 9* (6), 26-32

Burton, G. (1992). Using language arts to promote mathematical learning. *The Mathematics Teacher, 3* (2), 26-31

Burton, L. (1984). Mathematical thinking: The struggle for meaning. *Journal of Research in Mathematics Education, 15* (1), 35-49

Cai, J. (2003). Singaporean students' mathematical thinking in problem solving and problem posing: an exploratory study. *Int. J. Math. Educ. Sci. Technol.*, *34* (5), 719–737

Cai, J. & Hwang S., (2002) Generalized and generative thinking in US and Chinese students' mathematical problem solving and problem posing: *Journal of Mathematical Behavior 21* 401-421.

Cankoy, O. (1998). Determining and Overcoming Preservice Elemantary Teachers' Perceptions in Interpreting and Applying Decimals: Unpublished Doctoral Dissertation: METU

Cappo, M.,& Osterman, G. (1991). Teach students to communicate mathematically,
The Computing Teacher, 2, 34-39

Carpenter, T.P., Corbitt, M. K., Kepner, H.S. jr, Lindquist, M. M., Reys, R. E. (1981) What are the chances of your students knowing probability? *Mathematics Teacher,* 74, 342-344.

Chin, C. & Kayalvizhi, G. (2002). Posing Problems for open Investigations: What questions do pupils ask? *Research in Science & Technological Education, 20*, (2)

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist approach to the representational view of mind in mathematics education. *Journal of Research in Mathematics Education ,23*, (1), 2-34.

Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences*.

Hillsdale, NJ: Erlbaum.

Connor, W., & Hawkins, G. (1936). What materials are most useful to children in learning to solve problems. *Educational Method, 16* (1), 21-29

Craig, D.,S. (1999). Preservice Elementary Teachers' Problem Posing and its' relationship to mathematical knowledge and attitudes [online] http:e-archive.library.okstate.edu/dissertations/, last visited date: December 2005

Crespo, S. (2003). Learning To Pose Mathematical Problems: Exploring changes in preservice teachers' practices. *Educational Studies in Mathematics* 52**:** 243–270.

Cromwell, A., & Sasser, S. (1987). *Putting the shoe on the other foot:* A Jigsaw Lesson in Point of View. (Eric Document Reproduction No. Ed 317 045).

Davidson, D. & Pearce, D (1988) . Using writing activities to reinforce mathematics instruction. *Arithmetic Teacher*, *35*, (18) 42-45.

Davies, C. M. (1965). Development of the probability concept in children. *Child Development, 36*, 779-788

Dickerson, V.,M. (1999). The Impact of Problem-posing instruction on the mathematical Problem Solving achievement of seventh-graders. Doctoral Dissertation:Emory University:Umi Microform 9931793.

Duggan, S. & Gott, R. (1995) The place of investigations in practical work in the UK National Curriculum for Science, *International Journal of Science Education, 17* (2), 137–147.

Duncker, K. (1945). On problem solving. *Psychological Monographs, 58* (5), 270.

Einstein, A., & Infeld, L. (1938). *The evolution of physics*. New York: Simon & Schuster.

Ellerton,N., Clements,M.A., & Clarkson,P.C. (2000). Language factors in mathematics education. In K.Owens & J.Mousley (Eds.), *Research in mathematics education in Australasia 1996 – 1999* (pp. 29-96). Sydney: Mathematics Education Research Groups of Australasia.

.

Engel, A. (1966). Mathematical Research and Instruction in Probability Theory *International Mathematics Education, 59* (8): 771-782

English, L.D., (1997). The Development Of Fifth grade Children's Problem Posing Abilities. *Educational Studies in Mathematics, 34,* 183-217

Fischbein, E. (1975) The intuitive sources of probability thinking in children. D.Reidel, Dordrecht.

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? *Educational Studies in Mathematics, 15,* 1-24.

Fischbein, E., Nello, M.S., & Marino, M.S. (1991). Factors affecting probabilistic judgements in children and adolescents. *Educational Studies in Mathematics, 22,* 523-549

Ford, M. (1990) The writing Process: A strategy for problem solvers. *Arithmetic Teacher*, *38* (3), 35-38.

Fraenkel, J.R., & Wallen, N.E. (1996) *How to Design and Evaluate Research in Education*, New York: McGraw-Hill, Inc.

Frankenstein, M. (1987). Critical mathematics education: An application of Paulo Freire's epistemology. In Ira Shor (Ed.), *A sourcebook for Liberatory Teaching* (pp. 180-210). Porstmount: Boynton Cook.

Freire, P. (1998) *Pedagogy of the Oppressed.* New York: Continuum.

Frenn, M.,J. (2002). Benefits of algebraic proof and problem posing: A study into creative strategies for solving Linear Equation.[online] http://sunzi1.lib.hku.hk/ER/detail/2708120, last visited date: December 2005

Garfield, J., & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. *Journal for Research in Mathematics Education, 19,* 44-63.

Geeslin, W. (1977). Using writing about mathematics as a teaching technique. *Mathematics Teacher,70* (2), 112-115.

Gonzales, N. A. (1998) . A blueprint for problem posing. *School Science and Mathematics*, *94* (2), 78- 85.

Hoeman, H., & Ross, B. M. (1971) .Children's understanding of probability concepts. *Child Development, 42*. 221-236.

Hope J.A. & Kelly I.W. (1983). Common difficulties with Probabilistic Reasoning, Mathematics Teacher, 565-570.

Kilpatrick, J. (1987). Where do good problems come from? In A. Schoenfeld, (Ed.), *Cognitive Science and Mathematics* (pp.123-147). Hillsdale: Lawrance Erlbaum Associates.

Kliman, M., & Richards, J. (1992). Writing, sharing and discussing mathematics stories. *Arithmetic Teacher,40* (3), 138-141.

Lappan, G., dc Winter, M. J. (1980). Probability simulation in middle school. *Mathematics Teacher*, *73* (6), 446-449.

Lavy, I., & Bershadsky, I. (2003). Problem Posing via “waht if not” strategy in solid geometry- a case study: *Journal of Mathematical Behavior, 22*,369-387.

Lesh, R. (1981). Applied mathematical problem solving. *Educational Studies in Mathematics,12,* 235-264.

Lijnse, P.,& Klaasen, K. , (2004) . Didactical structures as an outcome of research on teaching–learning sequences*? Int. J. Sci. Educ., 26(5),* 537–554.

Mamona-Downs, J. (1993). On analysing problem posing. In I. Hirabayashi, N. Nohda, K. Shigematsu & F. L. Lin (Eds), *Proceedings of the Seventeenth International Conference for the Psychology of Mathematics Education Vol. III* (pp. 41–47). Tsukuba, Japan: International Group for the Psychology in Mathematics Education

Matz, K. & Leier, C. (1992). Word problems and the language connection. *Arithmetic Teacher,39,* 14-17.

Milner, H.,R., (2003) . Reflection, Racial Competence, and Critical Pedagogy: how do we prepare pre-service teachers to pose tough questions? *Race Ethnicity and Education, 6* (2)

Moses, B. E., Bjork E., & Goldenberg, P. E. (1990). Beyond problem solving: problem posing. In: T. J. Cooney, & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990's* (pp. 82–91). Reston, VA: National Council of Teachers of Mathematics.

Mosteller, F. (1967) What has happened to probability in the high school? *Mathematics Teacher, 55*, 824-831.

Myers, J. L., Hansen, R. S., Robson, R.C., & McCann, J. (1983) The role of explanation in learning elementary probability. *Journal of Educational Psychology, 75*, 374-381.

National Council of Teachers of Mathematics (2000). *Curriculum Standards for School Mathematics.* Reston, Va.: National Council of Teachers of Mathematics

Nicolaou, A. A., & Philippou, N. G. (2002). Efficacy Beliefs, Ability in Problem Posing, and Mathematics Achievement **,** *University of Cyprus.*[online] http://ning.ntctc.edu.tw/resource/PME%2025/content/VOLUME4/Philippou &_RR180.pdf, last visited date: December 2005

Ojeman, R. H., Maxey, E. J., & Snider, B. C. (1965) Effects of guided learning experiences in developing probability concepts at the fifth grade level. *Perceptual and Motor Skills,* 21, 415-427.

Or-Bach R. and Bar-On E. (1989). PROBIT- Developing a Diagnostic intelligent tutor. *Artificial Intelligence and Education. Proceedings of the 4th International Conference on AI and Education*, Amsterdam/Netherlands, Bierman D., Breuker J. and Sandberg J (Eds).

Owens, M.,E., (1999). The relationship of Cognitive learning styles, Mathematic Attitudes, and Achievement in a Problem Posing Classroom.

Pallant, J.(2001).*SPSS Survival Manual*. Open University Press:Buckingham

Polya, G. (1945). *How to solve it, a new aspect of mathematical method.* Princeton, N.J.:Princeton University Press.

Polya, G. (1973). How to solve it: A new aspect of mathematical method (2nd ed.). Princeton, NJ: PUP.

Resnick, D. & Resnick, L. (1996). Performance assessment and the multiple functions of educational measurement. In M.B. Kane and R. Mitchell (Eds.), *Implementing performance assessment: Promises, problems, and challenges* (pp. 23-29). New Jersey: Lawrence Erlbaum Associates.

Richard, L. (1990) Measuring things in words: Language for learning mathematics. *Language Arts*, *67* (1), 14-25.

Romberg, T. A., & Shepler J. (1973). Retention of Probability Concepts: A pilot study into the effects of mastery learning with sixth-grade students. *Journal for Research in Mathematics Education 4*, 26-32.

Rudnitsky, A., Etheredge, S., Freeman, S., & Gilbert, T. (1995). Learning to solve addition and subtraction word problems through a structure-plus-writing approach. *Journal for Research in Mathematics Education, 26* (5), 467-486.

Shaughnessy, M. (1977), "Misconceptions of Probability: An experiment with a small-group, activity-based, model building approach to introductory probability at the college level," *Educational Studies in Mathematics*, *8*, 295-316.

Siegel, S., & Andrews, J. M. (1962). Magnitude of reinforcement and choice behavior in children. *Journal of experimental Psychology, 63*, 337-340.

Silver, E. A. (1994). On mathematical problem posing. *For the Learning of Mathematics, 14*(1), 19–28.

Silver, E. A. and Burkett, M. L. 1993, *The Posing of Division Problems by Preservice Elementary School Teachers.* Unpublished manuscript, Learning Research and Development Center, University of Pittsburgh.

Silver, E. A. & Cai, J. (1993). Mathematical problem posing and problem solving by middle school students. In C. A. Maher, G. A. Goldin & R. B. Davis (Eds), *Proceedings of PME-NA 11, Vol. 1* (pp. 263–269). New Brunswick NJ: Rutgers University.

Silver, E. A., Mamona-Downs, J., Leung, S. S., & Kenney, P. A. (1996). Posing mathematical problems: an exploratory study. *Journal for Research in Mathematics Education*, *27*(3), 293–309.

Simon, M. A. (1993) Prospective elementary teachers' knowledge of division. *Journal for Research in Mathematics Education, 24* , 233–254.

Solorzano, D. (1989). Teaching and social change: Reflection on a Freire an approach in a college classroom. *Teaching Sociology, 17,* 218-225.

Stevens, J. (1996). *Applied Multivariate Statistics for the Social Sciences*. Mahway,NJ: Lawrence Erlbaum.

Stover, G. B. (1982). Structural Variables Affecting Mathematical Word Problem Difficulty in 6th-graders. *Unpublished doctoral dissertation, University of San Francisco.* (Dissertation Abstract International, 42, 5050A)

Stoyanova, E. (1998). Extending and Exploring Students' Problem Solving via Problem Posing: A Study of Years 8 and 9 Students involved in Mathematics Challenge and Enrichment Stages of Euler Enrichment Program for Young Australians.*Unpublished doctoral thesis, Edith Cowan University*.

Stoyanova, E. & Ellerton, N. F. (1996). A framework for research into students' problem posing. In P. Clarkson (Ed.), *Technology in Mathematics Education* (pp. 518–525). Melbourne: Mathematics Education Research Group of Australasia.

TIMSS Ulusal Rapor: [online] http://earged.meb.gov.tr/Projsb/TIMSS/TIMMSulusrap.pdf, last visited date: December 2005

Van den HeuvelPanhuizen, M., Middleton, J. A.&Streefland, L.(1995). Studentgenerated problems: Easy and difficult problems on percentage, *For the Learning of Mathematics, 15*(3), 21–27.

Walter, M. (1992). Mathematics from almost anything, part I. *Mathematics teaching, 120,* September 1987, 3-7; part II. *Mathematics Teaching,121,* December 1987, 2-6.

Walter, M., & Brown, S. (1977). Problem posing and problem solving: An illustration of their interdependence. *Mathematics Teacher, 70,* 4-14

Winograd, K. (1990). Writing, solving, and sharing original math story problems: Case studies of fifth grade children's cognitive behavior. *Unpublished doctoral dissertation . University of Northern Colorado.*

Wirtz, R., & Kahn, E. (1982). Another look at application in elementary school mathematics. *Arithmetic teacher, 30* (1), 21-25.

Wright, J. & Stevens, N. (1980). Improving verbal problem solving performance. *Arithmetic Teacher, 31* (2), 40-42

Yost, P. A., Siegel, A.E., & Andrews, J. M. (1962) .Nonverbal probability judgments by young children. *Child Development, 33,* 769-780

APPENDIX A

OLASILIK BAŞARI TESTİ

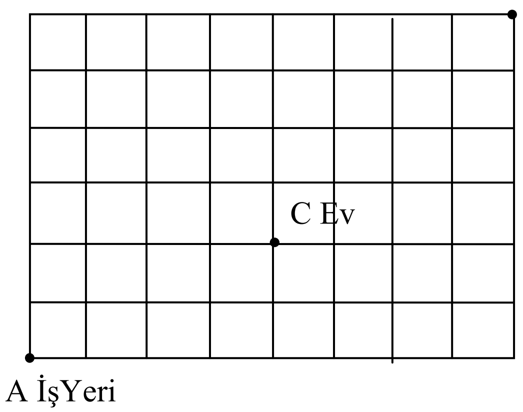
Açıklamalar

Bu testte 5 i permütasyon ve kombinasyon ile ilgili, 15 ise olasılık ile ilgili toplam 20 soru vardır. Sorulara verebileceğiniz en açık cevapları veriniz. İstediğiniz sorudan cevaplamaya başlayabilirsiniz. Her soru 5 puan değerindedir. Sınav 100 üzerinden değerlendirilecektir.

ADI SOYADI			
OKULU / SINIFI / NUMARASI			

1) Ankara'dan Bursa'ya giderken Eskişehir'den geçmek gerekir. Ankara'dan Eskişehir'e 5 farklı yoldan, Eskişehir'den Bursa'ya ise 3 farklı yoldan gidilebilmektedir. Buna göre, Ankara'dan Bursa'ya kaç farklı yoldan gidilebilir?

2)



Şekildeki çizgiler bir kentin birbirini dik kesen sokaklarını göstermektedir. İş yerinden çıkan bir kişi, eve uğramadan, alışveriş merkezine en kısa yoldan gitmek istiyor. Kaç değişik yol izleyebilir?

3) Mehmet ve 5 arkadaşı bir pastaneye gittiklerinde yuvarlak bir masa etrafında kaç farklı şekilde oturabilirler?

4) Dünya Sağlık Örgütü (WHO); 3 göz, 2 diş doktoru ve 4 cerrahtan oluşacak bir sağlık ekibini Endonezya'ya tsunami felaketi için göndermek istiyor. Bu ekip, 5 göz, 4 diş ve 6 cerrah arasından kaç farklı şekilde seçilebilir?

5) Bir pizzacı, sade pizzanın üzerine **istendiğinde** sucuk, salam, sosis, mantar ve zeytin çeşitlerinde bir veya birkaçını ekleyerek servis yapmaktadır. Pizzalar küçük, orta ve büyük boy olmak üzere üç farklı büyüklükte servis yapıldığına göre, pizza siparişi verecek olan bir kişinin kaç değişik seçeneği vardır?

6) "Farklı 2 tane hilesiz madeni paranın atılması sonucunda, paraların en az bir tanesinin yazı gelme olasılığı nedir?" sorusunu göz önüne alarak, sorunun :

- Deneyini
- Örnek uzayını ve eleman sayısını
- İstenen olayı ve istenen olayın olma sayısını yazınız.

7) Yaren, Bahar ve Ezgi arasında bir okçuluk yarışması düzenleniyor. Bu yarışmayı Yaren'in kazanma olasılığı Bahar'ın kazanma olasılığının 4 katı, Ezgi'nin kazanma olasılığı ise Bahar'ın kazanma olasılığının 3 katıdır. Yarışmayı sadece biri kazanacağına göre, Ezgi'nin kazanma olasılığı kaçtır?

8) Aşağıda verilen olasılık sorularında olay çeşitlerini, yazılan yedi çeşit ile eşleştiriniz.

(1) Kesin olay (2) İmkansız Olay (3) Ayrık Olay
(4) Ayrık Olmayan Olay (5) Bağımlı Olay (6) Bağımsız Olay
(7) Koşullu Olay

a) Aynı gün içinde seçilen iki kişinin aynı televizyon programını izleme olasılığı

Olay Çeşidi (……)

b) 10 A Rh (+), 2 B Rh (-), 23 0 Rh (+) ve 13 AB Rh (+) kan grubundan 48 kişinin bulunduğu bir topluluktan seçilen bir kişinin kan grubunun 0 Rh (-) olma olasılığı

Olay Çeşidi (……)

c)

	Kanser Olan	**Kanser Olmayan**
Sigara İçen	15	1
Sigara İçmeyen	2	10

Şekildeki tablo küçük bir köyde 28 kişi arasında yapılan bir araştırmayı gösteriyor. Bu kişiler arasından seçilen bir kişinin sigara içen veya kanser olmayan bir kişi olma olasılığı

Olay Çeşidi (……)

d) İçinde sadece 5 tane mavi top bulunan bir torbadan, mavi top çekme olasılığı

Olay Çeşidi (……)

e) Bir torbada aynı özelliklere sahip 2 yeşil, 3 mavi ve 4 kırmızı top arasından sırayla üç top çekildiğinde, birincinin sarı, ikincinin yeşil ve üçüncünün mavi olma olasılığı

Olay Çeşidi (……)

	Yetişkin Kadın	**Yetişkin Erkek**	**Kız Çocuğu**	**Erkek Çocuğu**
Hayatta Kalan	332	318	29	27
Ölen	1360	104	35	18

Tablo, Titanik mürettebatında hayatta kalan ve ölenlerin sayısı göstermektedir. Buna göre;

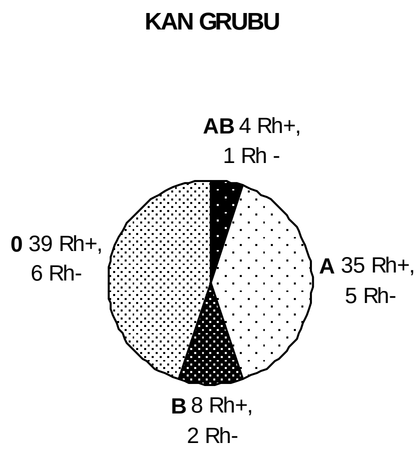
f) Seçilen bir kişinin erkek veya hayatta kalan bir kız çocuğu olma olasılığı

Olay Çeşidi (……)

g) Seçilen bir erkeğin hayatta kalan bir çocuk olma olasılığı

Olay Çeşidi (……)

9–14. sorularını bu figürden yararlanarak çözünüz.



Şekildeki figür, Kızılay kurumuna kan vermiş 100 kişinin kan gruplarını ve Rh çeşitlerini göstermektedir.

9) Seçilen bir kişinin A kan grubundan **olmama** olasılığı kaçtır?

10) Seçilen bir kişinin Rh – çeşit kan grubundan olma olasılığı kaçtır?

11) Seçilen bir kişinin A grubu veya Rh – çeşit kan grubundan olma olasılığı kaçtır?

12) Seçilen bir kişinin A grubu veya B grubu olma olasılığı kaçtır?

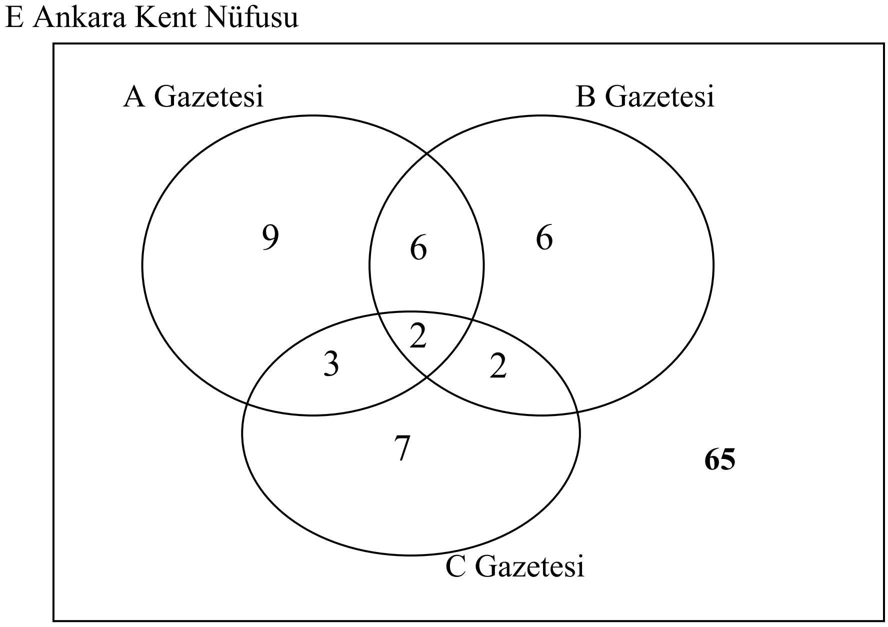
13) AB grubundan seçilen bir kişinin Rh + çeşit kan grubundan olma olasılığı kaçtır?

14) Seçilen bir kişinin AB grubu veya 0 grubu veya Rh + çeşit kan grubundan olma olasılığı kaçtır?

15) Matematik sınavına girmeyen 4 öğrenci mazeret olarak sınava gelirken kullandıkları arabanın tekerinin patladığını söylemişlerdir. Mazeret sınavı yapmayı kabul eden ders öğretmeni, sınavda öğrencilere “Arabanın hangi tekeri patladı?” sorusunu sormuştur. Öğrencilerin mazeretinin doğru olma olasılığı kaçtır?

16) 8 kişilik bir afet kurtarma ekibinin başkan ve başkan yardımcısını seçmek için kişilerin isimleri özdeş kartlara yazılarak bir torbaya konuluyor. Birinci seçilecek karttaki kişi başkan, ikinci seçilecek karttaki kişi ise başkan yardımcısı olacağına göre, bu ekipte bulunan Ayşe’nin başkan, Ali’nin ise başkan yardımcısı olma olasılığı kaçtır?

17–19. soruları aşağıdaki şemaya göre cevaplayınız.



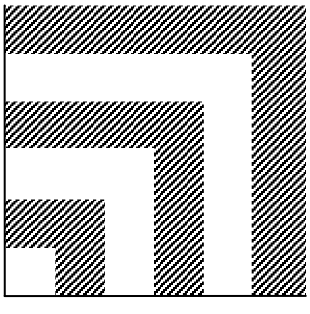
Şekildeki figür, Ankara’da basılan A, B ve C gazetelerinin okunma yüzdelerini gösteriyor. Örneğin sadece A gazetesi okuyan kişi sayısı %9 dur. A, B ve C gazetelerinin her üçünü de okuyan kişi sayısı %2 dir.

17) Ankara’dan seçilecek bir kişinin bu gazetelerden hiçbirini **<u>okumama</u>** olasılığı kaçtır?

18) Ankara’dan seçilecek bir kişinin bu gazetelerden sadece birini okuma olasılığı kaçtır?

19) Seçilen bir kişinin en az bir gazete okuduğu bilindiğine göre, A ve B gazetelerinden her ikisini de okuma olasılığı kaçtır?

20)



Şekildeki şeritte taralı ve beyaz şeritler eşit genişliktedir. Atılan iğne uçlu bir okun kareye isabet ettiği bilindiğine göre, okun taralı şeritte olma olasılığı kaçtır?

APPENDIX B

1. Olasılık konularını severim.	O	O	O	O	O	O
2. Olasılık konuları sevimsizdir.	O	O	O	O	O	O
3. Olasılıkla ilgili konuları tartışmaktan hoşlanırım.	O	O	O	O	O	O
4. Olasılıkla ilgili bilgiler can sıkıcıdır.	O	O	O	O	O	O
5. Olasılıkla ilgili bilgiler zihin gelişmesine yardımcı olur.	O	O	O	O	O	O
6. Olasılık konusu beni huzursuz eder.	O	O	O	O	O	O
7. Olasılıkla ilgili ders saatlerinin daha çok olmasını isterim.	O	O	O	O	O	O
8. Olasılık konuları rahatlıkla/kolaylıkla öğrenilebilir.	O	O	O	O	O	O
9. Olasılıkla ilgili sınavlardan korkarım.	O	O	O	O	O	O
10. Olasılık konuları ilgimi çeker.	O	O	O	O	O	O
11. Olasılığın doğru karar vermemizde önemli rolü vardır.	O	O	O	O	O	O
12. Olasılık konuları aklımı karıştırır.	O	O	O	O	O	O
13. Olasılık konusunu severek çalışırım.	O	O	O	O	O	O
14. Olasılık konusunu elimde olsa öğrenmek istemezdim.	O	O	O	O	O	O
15. Olasılık ilginç bir konu değildir.	O	O	O	O	O	O
16. Olasılıkla ilgili ileri düzeyde bilgi edinmek isterim.	O	O	O	O	O	O
17. Olasılık hemen hemen her iş alanında kullanılmaktadır.	O	O	O	O	O	O
18. Olasılık konusunu çalışırken canım sıkılır.	O	O	O	O	O	O
19. Olasılık kişiye düşünmesini öğretir.	O	O	O	O	O	O
20. Olasılığın adını bile duymak sinirlerimi bozuyor.	O	O	O	O	O	O
21. Olasılık konusundan korkarım.	O	O	O	O	O	O
22. Olasılık herkesin öğrenmesi gereken bir konudur.	O	O	O	O	O	O
23. Olasılık konusundan hoşlanmam.	O	O	O	O	O	O
24. Olasılıkla ilgili bilgiler kişinin tahmin etme yeteneğini artırır.	O	O	O	O	O	O
25. Olasılık konusu anlatılırken sıkılırım.	O	O	O	O	O	O
26. Olasılıkla ilgili bilgilerin günlük yaşamda önemli bir yeri vardır.	O	O	O	O	O	O
27. Olasılık konusu okullarda öğretilmese daha iyi olur.	O	O	O	O	O	O
28. Olasılık konuları eğlencelidir.	O	O	O	O	O	O

APPENDIX C

Adınız Soyadınız:……………………………………….. Cinsiyetiniz:…………..

Okulunuzun İsmi:………………………………………. Sınıfınız:………………

MATEMATİK DERSİNE KARŞI TUTUM ÖLÇEĞİ

Genel Açıklama: Aşağıda öğrencilerin matematik dersine ilişkin tutum cümleleri ile her cümlenin karşısında "Tamamen Uygundur", "Uygundur", "Kararsızım", "Uygun Değildir" ve "Hiç Uygun Değildir" olmak üzere beş seçenek verilmiştir. Lütfen cümleleri dikkatli okuduktan sonra her cümle için kendinize uygun olan seçeneklerden birini işaretleyiniz.

	Tamamen Uygundur	Uygundur	Kararsızım	Uygun Değildir	Hiç Uygun Değildir
1. Matematik sevdiğim bir derstir.	O	O	O	O	O
2. Matematik dersine girerken büyük sıkıntı duyarım.	O	O	O	O	O
3. Matematik dersi olmasa öğrencilik hayatı daha zevkli olur.	O	O	O	O	O
4. Arkadaşlarımla matematik tartışmaktan zevk alırım.	O	O	O	O	O
5. Matematiğe ayrılan ders saatlerinin fazla olmasını dilerim.	O	O	O	O	O
6. Matematik dersi çalışırken canım sıkılır.	O	O	O	O	O
7. Matematik dersi benim için angaryadır.	O	O	O	O	O
8. Matematikten hoşlanırım.	O	O	O	O	O
9. Matematik dersinde zaman geçmez.	O	O	O	O	O
10. Matematik dersi sınavından çekinirim.	O	O	O	O	O
11. Matematik benim için ilgi çekicidir.	O	O	O	O	O
12. Matematik bütün dersler içinde en korktuğum derstir.	O	O	O	O	O
13. Yıllarca matematik okusam bıkmam.	O	O	O	O	O
14. Diğer derslere göre matematiği daha çok severek çalışırım.	O	O	O	O	O
15. Matematik beni huzursuz eder.	O	O	O	O	O
16. Matematik beni ürkütür.	O	O	O	O	O
17. Matematik dersi eğlenceli bir derstir.	O	O	O	O	O
18. Matematik dersinde neşe duyarım.	O	O	O	O	O
19. Derslerin içinde en sevimsizi matematiktir.	O	O	O	O	O
20. Çalışma zamanımın çoğunu matematiğe ayırmak isterim.	O	O	O	O	O

APPENDIX D

SAMPLE ACTIVITY SHEETS

Aktivite 1 ("eğer olmazsa ne olur" yöntemi)

Konu: *Çarpım Kuralı*

Problem1: İzmir den Eskişehir e 4 farklı yol, Eskişehir den Ankara ya ise 5 farklı yol olduğunu düşünürsek, İzmir den Ankara ya Eskişehir den geçmek şartıyla kaç farklı şekilde gidilebilir?

Sayısal değerleri ya da bilgilerin cinsini değiştirerek soru oluşturunuz.

Aktivite 2 (yarı-yapısal cümleler)

Konu: *Olasılık*

Verilen cümle: 5 tane hilesiz madeni paranın aynı anda atıldığını düşünelim.

Verilen bu cümleyi tamamlayarak soru oluşturunuz.

Aktivite 3 (yapısal cümleler)

Konu: *Kombinasyon*

Verilmiş cevap: C(8,3).

Cevabı yukarıdaki cevap olan sorular oluşturunuz.

Aktivite 4 (Serbest Yöntem)

Konu: *Olasılık*

1) Bağımsız olasılıkla ilgili bir problem yazınız.
2) Bağımlı olasılıkla ilgili bir problem yazınız.
3) Koşullu olasılıkla ilgili bir problem yazınız.
4) Geometrik olasılıkla ilgili bir problem yazınız.

APPENDIX E

SAMPLE ANSWERS OF STUDENTS TO THE ACTIVITY SHEETS

Aktivite 1

1) İzmir den Eskişehir e 5 farklı yol, Eskişehir den Ankara ya ise 6 farklı yol olduğunu düşünürsek, İzmir den Ankara ya Eskişehir den geçmek şartıyla kaç farklı şekilde gidilebilir?
2) İzmir den Eskişehir e 4 farklı yol, Eskişehir den Ankara ya 5 farklı yol ve Ankara dan Kayseri ye 3 farklı yol olduğunu düşünürsek, İzmir den Kayseri ye Eskişehir den ve Ankara dan geçmek şartıyla kaç farklı şekilde gidilebilir?
3) 5 farklı renkte pantolonu ve 6 farklı gömleği olan bir kişi bir gömlek ve bir pantolon giyecektir. Kaç farklı giyim yapabilir?

Aktivite 2

1) …Hepsinin yazı gelme olasılığı kaçtır?
2) …En az iki yazı gelme olasılığı kaçtır?
3) …2 tanesinin yazı geldiği bilindiğine göre diğer üçünden birinin yazı diğer ikisinin tura gelme olasılığı kaçtır?

Aktivite 3

1) 8 göz doktoru arasından 3 doktor kaç farklı şekilde seçilebilir?
2) 8 elemanlı bir kümenin 3 elemanlı kaç tane alt kümesi vardır?
3) Edebiyat yazılısında öğrenciye 8 soru verilmiştir. Bunlardan herhangi 3 ünü seçip cevaplaması istendiğine göre kaç farklı seçim yapabilir?

Aktivite 4

1) Bir çocuğun sağ cebinde 2 tane madeni 1 YTL ve 3 tane madeni 50 YK, sol cebinde ise 4 tane madeni 25 YK ve 3 tane madeni 10 YK vardır. Her iki cebinden de bir madeni para çeken çocuğun elinde toplam 60 YK madeni para olma olasılığı kaçtır?
2) İçinde 2 kırmızı ve 4 beyaz bilye bulunan bir torbadan rasgele bir bilye seçiliyor. Çekilen bilye torbaya geri konulduktan sonra torbadan bir bilye daha çekiliyor. Çekilen bilyelerin aynı renkte olma olasılığı kaçtır?
3) İki hilesiz zar aynı anda atılıyor. Zarlarda gelen sayılardan birinin çift olduğu bilindiğine göre, zarlarda gelen sayıların toplamının asal olma olasılığı kaçtır?
4) n uzunluğundaki bir ip rasgele bir yerinden kesiliyor. Oluşan parçalardan birinin, diğerinin en az iki katı uzunlukta olma olasılığı kaçtır?

Table F Unit Plan for Permutation, Combination and Probability

			ÜNİTE: PERMÜTASYON, KOMBİNASYON, OLASILIK			
MAYIS	1	2 2 1	Faktöriyeli kavrayabilme Permütasyonu kavrayabilme	**PERMÜTASYON, KOMBİNASYON, OLASILIK** Faktöriyel Kavramı Sayma Kuralları Permütasyon Tanımı ve Özellikleri	Anlatım, Soru - Cevap, Problem Çözme, Mukayese Etme, Analiz Etme	MEB Tavsiyeli Kitaplar, Ders Kitabı, Akademia CD Seti,ÖSS Test Kitapları
	2	2 2 1	Permütasyon ile ilgili uygulama yapabilme Ölçme ve değerlendirme	Dönel Permütasyon , Tekrarlı Permütasyon Alıştırma Çözümü 1.Yazılı Yoklama	Anlatım, Soru - Cevap, Problem Çözme, Mukayese Etme, Analiz Etme	MEB Tavsiyeli Kitaplar, Ders Kitabı, Akademia CD Seti,ÖSS Test Kitapları
	3	2 2 1	Kombinasyonu kavrayabilme Kombinasyon ile ilgili uygulama yapabilme	Kombinasyon Tanımı ve Özellikleri Alıştırma Çözümü	Anlatım, Soru - Cevap, Problem Çözme, Mukayese Etme, Analiz Etme	MEB Tavsiyeli Kitaplar, Ders Kitabı, Akademia CD Seti,ÖSS Test Kitapları
	4	2 2 1	Olasılığı kavrayabilme Olasılık İle İlgili Uygulama Yapabilme	Olasılık Tanımı Olasılık Fonksiyonu, Eş olumlu örneklem uzay Alıştırma çözümü	Anlatım, Soru - Cevap, Problem Çözme, Mukayese Etme, Analiz Etme	MEB Tavsiyeli Kitaplar, Ders Kitabı, Akademia CD Seti,ÖSS Test Kitapları
	5	2 2 1	Koşullu olasılığı kavrama Bağımsız ve bağımlı olayları kavrama	Koşullu Olasılık Bağımsız ve Bağımlı Olaylar	Anlatım, Soru - Cevap, Problem Çözme, Mukayese Etme, Analiz Etme	MEB Tavsiyeli Kitaplar, Ders Kitabı, Akademia CD Seti,ÖSS Test Kitapları
NİSAN	1	2 2 1	PERMÜTASYON, KOMBİNASYON, OLASILIK ile ilgili uygulumu yapabilme	Alıştırma Çözümü Bölüm ile ilgili Test	Anlatım, Soru - Cevap, Problem Çözme, Mukayese Etme, Analiz Etme	MEB Tavsiyeli Kitaplar, Ders Kitabı, Akademia CD Seti,ÖSS Test Kitapları

APPENDIX F

APPENDIX G

Olasılık Başarı Testini Değerlendirme Kriterleri

5 Puan: Soru tam ve doğru cevaplanırsa

4 Puan: Soru doğru yolla çözülmüş fakat işlem hatasından kaynaklı yanlış cevap verilmişse

3 Puan: Çözümde birden fazla işlem hatası yapılmışsa veya çözüm yolu yarım bırakılmışsa

2 Puan: Çözümde sorunun yarısından azı cevaplanmışsa

1 Puan: Birden fazla seçeneği olan soruların sadece 1 seçeneği cevaplanmışsa veya bir soruyu yanlışta olsa çözmeye çalışılmışsa

0 Puan: Herhangi bir çözüm yapılmamışsa

Table H Table of Specification

Topic / Levels	Product Rule	Repeating Arrangements	Circular Arrangements	Combination	Fundamental Concepts of Probability	Probability Function	Events Type
Knowledge					6		
Comprehension			3				8
Application	1			4		7	16
Analysis				5			
Synthesis		2					9-15,17-20