THE INVENTORY ROUTING PROBLEM
WITH DETERMINISTIC ORDER-UP-TO LEVEL INVENTORY POLICIES

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Prof. Dr. Canan Özgen
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Çağlar Güven
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Asst. Prof. Dr. Haldun Süral
Supervisor

Examining Committee Members

Prof. Dr. Nur Evin Özdemirel (METU,IE)

Asst. Prof. Dr. Haldun Süral (METU,IE)

Asst. Prof. Dr. Sedef Meral (METU,IE)

Dr. Seçil Savaşaneril (METU,IE)

Prof. Dr. Refik Güllü (Boğaziçi Unv.,IE)

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Özlem Pınar

Signature :

ABSTRACT

THE INVENTORY ROUTING PROBLEM
WITH DETERMINISTIC ORDER-UP-TO LEVEL INVENTORY POLICIES

Pınar, Özlem
M.Sc., Department of Industrial Engineering
Supervisor: Asst. Prof. Dr. Haldun Süral

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This study is concerned with the inventory routing problem with deterministic, dynamic demand and order-up-to level inventory policy. The problem mainly arises in the supply chain management context. It incorporates simultaneous decision making on inventory management and vehicle routing with the purpose of gaining advantage from coordinated decisions.

An integrated mathematical model that represents the features of the problem is presented. Due to the magnitude of the model, lagrangean relaxation solution procedures that identify upper bounds and lower bounds for the problem are developed. Satisfactory computational results are obtained with the solution procedures suggested on the test instances taken from the literature.

Keywords: Inventory Routing Problem, Deterministic Order-Up-To Level Policy, Lagrangean Relaxation

ÖZ

DETERMİNİSTİK TALEPLİ, BELİRLİ BİR SEVİYEYE KADAR ISMARLAMALI ENVANTER YÖNETİMİ VE GÜZERGAH BELİRLEME PROBLEMİ

Pınar, Özlem
Yüksek Lisans, Endüstri Mühendisliği Bölümü
Tez Yöneticisi: Y. Doç. Dr. Haldun Süral

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Bu çalışmada deterministik, dinamik talepler ve belirli bir seviyeye kadar ısmarlamalı envanter politikası içeren bütünleşik envanter yönetimi ve güzergah belirleme problemi incelenmektedir. Tedarik zinciri yönetimi kapsamında sıklıkla ortaya çıkan bu problemde, envanter yönetimi ve güzergah belirleme kararlarının koordineli olarak ele alınmasıyla avantajlar elde edilmesi hedeflenir.

Ele alınan problemin özelliklerini ortaya koyan bütünleşik bir model oluşturulmuştur. Bu modelin geniş kapsamlı olmasından dolayı, probleme üst ve alt sınırlar belirlemek amacıyla Lagrange gevşetim esasına dayalı çözüm yöntemleri geliştirilmiştir. Literatürden alınan test problemleriyle yapılan sayısal deneylerle elde edilen sonuçlar, geliştirilen yöntemlerin başarılı olduğunu göstermektedir.

Anahtar Kelimeler: Bütünleşik Envanter Yönetimi ve Güzergah Belirleme Problemi, Belirli Bir Seviyeye kadar Ismarlamalı Envanter Politikası, Lagrange Gevşetimi

To my family

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CHAPTER 1

INTRODUCTION

The main motivations for carrying inventory, as mentioned in Nahmias (1997), are gaining from economies of scale; dealing with uncertainties, speculations, and changes in demand patterns; complying with the restrictions on the amount that can be purchased or distributed (such as the restrictions on the minimum amounts that must be purchased); and reducing control costs (as the inventory levels are reduced, the inventory controls should be increased to ensure that stockouts do not occur.)

Due to these properties, keeping inventories may become inevitable and the inventory management problem arises since there are costs related with inventory policies. Following Nahmias (1997), the related costs can be classified as inventory holding costs, ordering costs, and penalty costs. The inventory holding cost is incurred related with the inventory on hand, i.e., the opportunity cost, the cost of the physical space used for inventories, and so on. The ordering cost, as its name implies, is incurred according to the amount of inventory ordered or produced (It usually has a fixed and a variable component.), whereas the penalty cost is incurred, when the inventory on hand is insufficient to meet the demands. Thus, the inventory management problem is based on identifying inventory holding policies according to the characteristics of the system (the constraints that are imposed), while minimizing the related costs.

The vehicle routing problem, on the other hand, as stated in Fisher (1995), arises in the distribution systems with the purpose of using a fleet of vehicles efficiently, which either distribute or collect products by making a number of stops. Costs can be incurred related with the distances traveled, the amount of product distributed, the number of stops made, and the number of routes executed. Thus, the vehicle routing problem is based on determining the visits to be made by each vehicle and the order of the visits under restrictions imposed such as vehicle capacity, delivery times, and so on, while minimizing the related costs.

The inventory management problem and the vehicle routing problem have frequently been studied for years, whereas coordinated decision making for these problems, i.e., the inventory routing problem (IRP), has been of interest mostly in the last two decades. The main inspiration for studying IRP is the logistics systems, in which either vendor-managed replenishment systems are in use or when the supplier and the multiple retailers represent different echelons in the supply chain of a single firm. As generally cited in the literature, vendor-managed replenishment systems can be seen in the grocery industry, for instance, when the producer (supplier) of the goods on the shelves of the supermarkets (retailers) has the responsibility of monitoring and replenishing the products. In these types of distribution systems, the decision of how much inventory to maintain at the supplier and the retailers is affected by delivery times and amounts for the retailers, which in turn is affected by the capacity of the vehicles used for the deliveries. Thus, simultaneous decision making is important in these systems to obtain significant cost savings.

In this study, an inventory routing problem with deterministic order-up-to level inventory policy (IRDOP) is considered. The problem consists of coordinated decision making for inventory management and vehicle routing in a distribution system composed of a supplier and multiple retailers. The demands at the retailers are deterministic, dynamic, and the inventory that can be kept at each retailer is bounded from below and above by predetermined levels, which give rise to the so called deterministic order-up-to level policy. This problem is introduced by Bertazzi, Paletta and Speranza (2002).

Deterministic demand assumption can be considered as too restrictive, when modeling a real life problem since the uncertainties inherent in real life are ignored.

However, with the advanced technology, it is possible to retrieve point of sales data by electronic data interchange, if strategic alliances are formed between the retailers and the supplier. Thus, if advanced technologies are in use, the supplier can rapidly get information about the inventory status at the retailers. By this means, the uncertainties inherent in the system can be reduced to some extent.

Most of the earlier works on the IRP are based on comparing the solutions obtained with methods that incorporate a coordinated approach and a decoupled approach. To solve the coordinated and the decoupled problems, both mathematical models and intuitive heuristics have been developed. Even though mathematical formulations have been presented for the IRP, the solution procedures provided are generally based on sequentially solving the inventory management problem and the vehicle routing problem and resorting to improvement steps later on. Another approach taken is assuming that the replenishments are carried out by direct deliveries from the supplier to the retailers, who need to be resupplied. This assumption simplifies the model by eliminating the routing component. Thus, the studies that suggest approaches which consolidate finding routes and satisfying inventory policies are rare in the literature.

The IRDOP, on the other hand, has not been modeled mathematically so far, which forms one of the main motivations for this study. Besides, although a heuristic method has been developed to obtain an upper bound to the IRDOP by Bertazzi et al. (2002), a lower bound has not been identified, yet. Since the problem is difficult to solve, lower bounds are needed to evaluate the performances of the solution procedures suggested. Therefore, developing lower bounds is our secondary motivation for this study.

We study a distribution system consisting of a supplier of a product and several retailers. The retailers encounter retailer dependent, deterministic, and dynamic demands for the product that need to be met without backlogs.

A capacitated vehicle is available at the supplier to distribute the product to the retailers. It is assumed that the deliveries are made on a route that starts from the supplier, serves the retailers to be visited, and ends at the supplier. Upon deliveries, a transportation cost related with the distance traveled is incurred.

Both the retailers and the supplier can hold inventory to be able to meet the demand on time. Each retailer has a retailer dependent predetermined minimum and maximum inventory levels. The retailers must be visited before their inventory falls below the minimum level. Whenever a retailer is visited, its inventory is filled up to the maximum level. A predetermined amount of product becomes available at the supplier in each period, which is used in the deliveries to the retailers. Backlogging is not allowed at the retailers and the supplier. Inventory holding costs are incurred at the retailers and the supplier according to amount of product carried in the inventory.

In short, the problem determines the retailers to be visited in each period, the amount of product to be delivered, and the route of a single vehicle, while minimizing the inventory holding cost at the retailers and the supplier and the transportation cost.

An integrated mathematical model is developed for the IRDOP. Due to difficulties inherent in the mathematical formulation provided for the IRDOP, it is not expected to solve this model optimally even for moderate-size instances. Thus, methods that provide upper and lower bounds for the IRDOP are developed. The focus in this study is the difficulties caused by the subtour elimination constraints and so, methods to identify upper and lower bounds are developed with the purpose of removing these constraints from the formulation of the problem. The first method developed is based on relaxing the subtour elimination constraints by lagrangean relaxation. Both an upper bound and a lower bound on IRDOP are identified by this method. The second method starts with identifying the minimum cost tour that visits all retailers. The precedence relationships of the visits on this a priori tour are substituted for the subtour elimination constraints. Computational studies are conducted to evaluate the performances of the methods developed.

The chapters in this thesis are organized as follows.

In Chapter 2, we discuss the related studies on inventory routing problems in the literature. A scheme to classify the inventory routing problems, based on Baita, Ukovich, Pesenti and Favaretto (1998), is described in Section 2.1. In Section 2.2, the features of the problem under consideration are presented for the study reviewed according to this classification scheme. The solution methods employed in the related study, the bounds obtained, and the qualities of the bounds are discussed afterwards.

In Chapter 3, the mixed integer programming (MIP) model developed for the IRDOP is presented. Section 3.1 starts with a description of the problem environment. Then, the features of our problem are summarized according to the classification scheme given in Section 2.1 and the basic assumptions made. In Section 3.2, the mathematical formulation of the IRDOP is presented and the constraints of this model are described in detail. The difficulties related with this model are explained afterwards.

Due to difficulties faced with the solution to the mathematical formulation of the IRDOP, methods for identifying upper and lower bounds for the IRDOP are suggested in Chapter 4. The formulation of the lagrangean relaxation model that is based on relaxing the subtour elimination constraints is provided in Section 4.1. A solution for the subproblem obtained by this relaxation identifies the retailers to be visited and the amount of product to be delivered to the visited retailers. This solution is not feasible for the integrated model since the deliveries may be made with subtours. Thus, it provides a lower bound on the IRDOP. An upper bound for the IRDOP is identified by converting the subtours in each period into a single tour for that period. In Section 4.2, another method is described, which is based on obtaining an a priori tour that visits all retailers and fixing the precedence relationships of the visits on this a priori tour. The simplification provided by this approach results from separating the inventory management and vehicle routing problems to some extent. This method provides an upper bound for the IRDOP since its formulation is more restrictive than the integrated model.

Numerical experiments are carried out both with the integrated model given in Chapter 3 and with the methods of Chapter 4 that are used to obtain upper and lower bounds for the IRDOP. Chapter 5 includes all results obtained with these methods in two sets of randomly generated numerical tests and a set of test instances taken from the literature. In Section 5.1, the properties of the test problems are discussed. Section 5.2 starts with a description of the statistics monitored to evaluate the results of the experiments. Then, the results obtained for the test instances are presented for each set of experiment in sequence.

In Chapter 6, main stages and contributions of our study are summarized. The performances of the upper and lower bounds that are identified with the methods developed are presented. Then, two basic directions for future research are discussed. The first stream of possible future research areas is based on improving the results that are obtained by either enhancing the methods we developed or developing new methods. The second area of further studies is extending the study to more general problems.

CHAPTER 2

LITERATURE REVIEW FOR THE INVENTORY ROUTING PROBLEM

Both inventory management problems and vehicle routing problems have independently been analyzed in numerous studies for years. On the other hand, combined analysis of inventory management and vehicle routing problems, i.e., inventory routing problems, has mostly arisen in the last two decades. Although the contexts of the studies show differences, the underlying idea of gaining advantage from simultaneous decision making is the same.

In this chapter, we describe the classification scheme used for reviewing inventory routing problems studied in the literature and we present review of the related literature according to this scheme.

2.1 Classification Scheme

To classify the literature on inventory routing problems, a system similar to the one provided in Baita, Ukovich, Pesenti and Favaretto (1998) is used. The elements of this classification scheme are explained below.

Number of items

- One: The items to be distributed are of one type.
- Many: The items to be distributed are of multiple types.

Decision domain

- Time: The decisions related to inventory and distribution problems are carried out over time periods.
- Frequency: The decisions are executed over delivery frequencies. The studies, whose decision domains are frequency, consider infinite-horizon problems.

Demand

- Deterministic: The demands at the retailers are assumed to be known.
- Stochastic: The demands at the retailers are uncertain.

Time behavior of the demand

- Constant: The demand at each retailer is constant over time.
- Dynamic: The demands at the retailers vary over time.

Note that constant demand over time brings changes in the solution procedure adapted particularly for the studies whose decision domain is frequency. Assuming that demand is constant over time at the retailers makes it possible to determine fixed visit frequencies for the retailers.

Number of vehicles

- Given: The distribution is performed with a given number of vehicles.
- Not constraining: It is assumed that there are enough number of vehicles to make the required deliveries.

Vehicle capacity

- Equal: In case of multiple vehicles, the capacity of each vehicle is the same.
- Different: In case of multiple vehicles, the vehicles have different capacities.
- NA: It is used when single vehicle case is considered.

Note that vehicles with unequal capacities do not generally result in significant changes in the solution methods developed for the studies whose decision domain is time. For the problems, in which the decision domain is frequency, vehicles with unequal capacities give rise to different visit frequency bounds for the regions that are visited with different vehicles.

Stock capacity constraints

- Yes: There are limits on the amount of inventory carried at the retailers.
- No: The amount of inventory carried at the retailers is not restricted.

Supply capacity constraints

- Yes: There exists a limit on the amount of product that is supplied. Limits on the production capacity (i.e., time availability, resource availability, and so on) are considered in this class, as well.
- No: There is no limit on the product supply.

Inventory parameters

- H1: Inventory holding cost is incurred at the retailers.
- H2: Inventory holding cost is incurred at the supplier.
- Penalty: Inventory stockout or other penalty costs are taken into account.
- Order: Product ordering costs are considered.
- Revenue: Revenue is earned in proportion to the amount of product distributed.
- Setup: The cost incurred when setting up the facility for production.
- No: Inventory related costs are not taken into account.

Transportation costs

- Fixed: A fixed transportation cost is incurred at each trip.
- Distance: Transportation cost is incurred according to the distance traveled.
- Amount: Transportation cost is incurred in proportion to the amount of product carried or unloaded.

2.2 Literature review

Due to the characteristics of the problem we studied, the literature examined mostly consists of works, in which retailers with deterministic demands are taken into account and the amount of production at the supplier is not a consideration. The studies reviewed are infinite-horizon (i.e., frequency is the decision domain) and multi-period (i.e., time is the decision domain) inventory routing problems. The reader is referred to Baita et al. (1998) and Federgruen and Simchi-Levi (1995) for more comprehensive reviews of inventory routing problems.

Our review of each study starts with a table consisting of the features of the problem under consideration according to the classification elements that are described in Section 2.1 and the solution methodologies used in the study are discussed afterwards.

Table 2.1 Campbell, Clarke and Savelsbergh (2002)

Element	Feature
Number of items	One
Decision domain	Time
Demand	Deterministic
Demand time behavior	Constant
Number of vehicles	Given
Vehicle capacity	Equal
Stock capacity	Yes
Supply capacity	No
Inventory parameters	No
Transportation costs	Distance

In Campbell et al. (2002), a real life inventory routing problem of a firm which negotiated with its customers about initiating a vendor-managed replenishment policy is studied. The environment of the problem is as seen in Table 2.1. Although meeting the demands at the customers on time is required, inventory related costs are not considered when making decisions.

The solution approach suggested is composed of two phases. In Phase I, the customers to be visited in each day and the delivery amounts are determined, whereas in Phase-II, the actual delivery routes and schedules for each day are determined.

For the first phase, an integer programming model is constructed. Upon this basic model, three variations are considered: (1) handling the stop times at the customers together with the vehicle reloading time at the supplier, (2) the start and end times of customer usage, and (3) the time windows that restrict the delivery times.

For solving the integer programming model constructed in phase I, the customers are clustered at the beginning and it is assumed that only the customers that are in the same cluster can be on the same route. After identifying the clusters, the integer programming model is solved by replacing the daily variables with weekly variables for the days after a specific day (k days) and removing integrality for these weekly variables. Thus, the volume of product that will be delivered to each customer in the next k days is determined in phase I. In phase II, vehicle routing problems with time windows are solved to obtain daily vehicle routes and schedules.

For the computational experiments, the actual data of two production facilities of the company is used. The solutions obtained are compared with another heuristic, which is composed of the rules that constitute an approximation of the methods used in the industry. It is seen that the two-phase approach outperforms the industry approximation approach for both facilities considered on the important statistics.

Table 2.2 Bell, Dalberto, Fisher, Greenfield, Jaikumar, Kedia, Mack and Prutzman (1983)

Element	Feature
Number of items	One
Decision domain	Time
Demand	Deterministic
Demand time behavior	Dynamic
Number of vehicles	Given
Vehicle capacity	Different
Stock capacity	Yes
Supply capacity	Yes (Resource availability)
Inventory parameters	Revenue
Transportation costs	Fixed + Amount

Bell et al. (1983) also consider a real life problem, whose characteristics are given in Table 2.2. Although costs related with inventories are not taken into account in this study as in Campbell et al. (2002), it is assumed that revenue related with the amount of product delivered is earned. A fixed cost and cost related with the amount of product unloaded form the transportation costs.

Customer data (i.e., tank capacity, historical product usage), resource data (i.e., truck capacities, product availabilities), cost data, time and distance data, and schedule data of the firm are incorporated into a MIP formulation as problem parameters. To formulate the model, demand forecasts are used to compute minimum and maximum inventory levels.

To formulate the problem as a MIP, a set of vehicle routes, each composed of a set of customers to be visited at a trip, is generated by a program, which produces feasible and efficient routes. The least-cost order of the customers on the route is decided by complete enumeration. A subset of routes that are to be driven are selected from the set of routes generated and the starting time of each route, the vehicle to be used at each route, and the delivery amount to each customer on the route are determined with the aim of maximizing the value of deliveries minus the cost of deliveries by using the MIP model. Due to large size of the model, lagrangean relaxation is applied, after which the problem is decomposed into subproblems (one for each vehicle) and an upper bound is obtained for the problem.

A heuristic based on the lagrangean relaxation approach is used to find a feasible solution (a lower bound) to the problem.

It is seen that the gap between the upper and the lower bounds is at most 2% in the computational experiments performed and it is stated that the approach resulted in important savings for the firm.

Table 2.3 Anily and Federgruen (1990)

Element	Feature
Number of items	One
Decision domain	Frequency
Demand	Deterministic
Demand time behavior	Constant
Number of vehicles	Not constraining
Vehicle capacity	Equal & Different
Stock capacity	No
Supply capacity	No
Inventory parameters	H1
Transportation costs	Fixed + Distance

A stream of studies dealing with fixed-partition policies are started with Anily and Federgruen (1990). The characteristics of this study are given in Table 2.3. In this study, it is assumed that inventory is not kept at the warehouse and an upper and a lower bound to the long run average transportation and retailer inventory holding costs are determined.

The routing schemes are identified by a modified circular partitioning scheme. Following the partitioning of the customers, the customers in each partition are separated into regions. Whenever a customer in a region receives a delivery with a vehicle, all other customers in the related region are also visited by the same vehicle. Under this strategy, partial fulfillment is possible since it is probable to assign a customer to multiple regions.

It is seen that the average difference between the upper and lower bounds ranges from 1% to 19% for different problem settings.

Table 2.4 Anily (1994)

Element	Feature
Number of items	One
Decision domain	Frequency
Demand	Deterministic
Demand time behavior	Constant
Number of vehicles	Not constraining
Vehicle capacity	Equal
Stock capacity	No
Supply capacity	No
Inventory parameters	H1 (Retailer dependent)
Transportation costs	Fixed + Distance

The author studies the same problem, whose characteristics are given in Table 2.4, and generalizes the results of Anily and Federgruen (1990) for the case, in which holding costs at the retailers are not identical. Due to this property, a difference of the proposed solution from that of the previous work is that the partitioning of the retailers to the regions is performed taking the holding costs at the retailers into account.

In the experiments, it is seen that the gaps between the lower and upper bounds are less than 10% for all instances and the computational time is at most a few seconds. However, the author recommends use of the algorithm presented by Anily and Federgruen (1990), if the holding costs at the retailers are identical since it provides better quality policies.

Table 2.5 Anily and Federgruen (1993)

Element	Feature
Number of items	One
Decision domain	Frequency
Demand	Deterministic
Demand time behavior	Constant
Number of vehicles	Not constraining
Vehicle capacity	Equal
Stock capacity	No
Supply capacity	No
Inventory parameters	H1 + H2
Transportation costs	Fixed + Distance

Table 2.5 presents the environment of the problem studied by Anily and Federgruen (1993). In this article, the authors consider an extension to their previous work, i.e., Anily and Federgruen (1990), so that keeping inventory at the depot is allowed, i.e., central inventories are possible. For obtaining a solution to this problem, a similar strategy to the one used in the preceding work is utilized. It is observed that the average gap between the lower and the upper bounds that ranges between 6% and 12% is usually better than the gap seen in the system without central inventories (Anily and Federgruen (1990)).

Table 2.6 Gallego and Simchi-Levi (1990)

Element	Feature
Number of items	One
Decision domain	Frequency
Demand	Deterministic
Demand time behavior	Constant
Number of vehicles	Not constraining
Vehicle capacity	Equal
Stock capacity	No
Supply capacity	No
Inventory parameters	H1 (Retailer dependent) + Order
Transportation costs	Fixed + Distance

The characteristics of the problem considered by Gallego and Simchi-Levi (1990) are given in Table 2.6. In this study, a lower bound on the long run average retailer ordering, retailer inventory holding, and transportation costs is obtained for all inventory-routing strategies. Effectiveness of direct shipping, i.e., making delivery to each retailer by a separate shipment, is evaluated as a function of the truck capacity. An upper bound is identified for the case in which direct shipments are carried out by fully loaded trucks. Using the lower and the upper bounds obtained, it is shown that when the economic lot size of each retailer amounts to more than 71% of the capacity of the truck, direct shipping is at least 94% effective. Moreover, it is stated that the error of direct shipping increases as the lot sizes decrease.

Table 2.7 Chan, Federgruen and Simchi-Levi (1998)

Element	Feature
Number of items	One
Decision domain	Frequency
Demand	Deterministic
Demand time behavior	Constant
Number of vehicles	Not constraining
Vehicle capacity	Equal
Stock capacity	No
Supply capacity	No
Inventory parameters	H1
Transportation costs	Fixed + Distance

The environment of the problem studied by Chan et al. (1998) is seen in Table 2.7. In this study, the effectiveness of fixed partition policies (i.e., partitioning the retailers into regions and serving each region independently) and zero inventory ordering policies (i.e., a retailer receives a delivery when its inventory level reaches zero) are examined and the worst case analysis together with the probabilistic analysis are provided. Two lower bounds on the total cost are presented.

Besides, a heuristic algorithm is developed for partitioning the retailers into regions so that each region is assigned a vehicle that visits all retailers in that region at equidistant epochs. Numerical results are reported for randomly generated instances and it is seen that the gap between the heuristic solution (the upper bound) and the lower bound is less than 19%.

Table 2.8 Dror and Ball (1987)

Element	Feature
Number of items	One
Decision domain	Time
Demand	Deterministic
Demand time behavior	Constant
Number of vehicles	Given
Vehicle capacity	Equal
Stock capacity	Yes
Supply capacity	No
Inventory parameters	Penalty (& Incentive)
Transportation costs	Distance

In the study of Dror and Ball (1987) given in Table 2.8, a long term inventory routing problem is reduced to a short term problem. It is assumed that the customers keep inventory in tanks with predetermined sizes and whenever a customer receives a delivery, its tank is filled up.

The relationship between the annual distribution cost, the fixed delivery cost, and the amount delivered to the customers are examined and the customers to be visited on a given day are selected according to these costs. The authors formulate a mathematical model that takes into account the relationships between the costs mentioned. Single-customer deterministic, single-customer stochastic, and multiple-customers cases are considered in the paper.

In the single-customer deterministic problem, to reduce the annual problem to a single period problem, a penalty cost for the long term effects of the decisions that are carried out in the short term is utilized and a continuous time deterministic model is formulated. For this purpose, an m-day period, in which all costs incurred are considered explicitly, and the following n-day period, in which the effects of the decisions made for the m-day period are considered, are defined. The changes in the costs over the succeeding n days are investigated depending on whether the customer requires replenishment during m days or not. The increase in the costs over the succeeding n days, if a customer who needs replenishment on day t (t<m) not to stock out is resupplied before day t and the decrease in the costs over n days, if a customer who does not need replenishment in m-day period is replenished within m days, are calculated.

In the single-customer stochastic problem, it is decided whether or not to visit a customer at the beginning of the planning period. In case a customer that is not visited runs out of the stock, a penalty cost is incurred and it is assumed that the customer is automatically replenished. Since the customer runs out of stock, if it is not resupplied, the expected stock out payment, the expected future cost penalty, and a safety stock level are calculated.

Single-customer optimal replenishment policies are identified for both deterministic and stochastic demand cases.

In the multiple-customer case, a mathematical model for the inventory routing problem is constructed using a generalized assignment VRP formulation that includes transportation costs and costs and incentives related with early deliveries. Although stochastic demands are used in safety stock calculations, the formulation of the model is based on deterministic demands. In this formulation, the amount of product to be delivered to a customer on a visit is determined by the day of the visit. Therefore it is not considered as a decision variable. This problem is solved by a modified generalized assignment algorithm and it is seen that the algorithm provides more than 50% increase in the performance over a manual system that is used at the time of the study and more than 25% increase in the performance over another existing system.

Table 2.9 Chien, Balakrishnan and Wong (1989)

Element	Feature
Number of items	One
Decision domain	Time
Demand	Deterministic
Demand time behavior	Constant
Number of vehicles	Given
Vehicle capacity	Different
Stock capacity	Yes
Supply capacity	Yes
Inventory parameters	Penalty + Revenue
Transportation costs	Fixed + Amount

In Chien et al. (1989), a series of single period inventory allocation and vehicle routing problems is solved with the aim of maximizing revenues minus costs to obtain an approximate solution to the multi-period inventory routing problem. Table 2.9 presents the characteristics of the problem.

As a first step for solving this problem, a multi-commodity flow-based mixed integer programming model is formulated. Since it is difficult to solve this problem optimally, a lagrangean relaxation based approach is developed to obtain upper and lower bounds for the problem.

By applying lagrangean relaxation to four constraints of the model, two subproblems are obtained. The first subproblem is an inventory allocation problem and the second subproblem is a customer assignment/vehicle utilization problem, which can further be decomposed into customers and vehicles. Subproblems are solved by greedy procedures to identify an upper bound to the original problem.

To find a lower bound for the problem, a heuristic composed of two phases is applied. In phase I, an initial set of vehicle routes is obtained using the solutions of the inventory allocation and customer assignment/vehicle utilization subproblems. The first step of Phase II is to check for feasibility. In case the solution is not feasible, a feasible solution is found. Then, it is checked whether the amount supplied to the customers on the routes can be increased. If the customers that are currently on a route are fully supplied, new customers with unsatisfied demands are inserted to the related route.

Experiments that are performed on randomly generated test problems show that the solutions are within 1-3% of optimality.

Table 2.10 Chandra (1993)

Element	Feature
Number of items	Many
Decision domain	Time
Demand	Deterministic
Demand time behavior	Dynamic
Number of vehicles	Not constraining
Vehicle capacity	Equal
Stock capacity	No
Supply capacity	No
Inventory parameters	H1 + H2 + Order
Transportation costs	Fixed + Distance

In the study of Chandra (1993), whose characteristics are given in Table 2.10, the decisions related with the inventory policies of the supplier (warehouse) are also taken into account. The aim is to determine replenishment quantities for both the warehouse and the customers together with the delivery routes.

The author provides a MIP model, which is decomposed into two subproblems by separating the constraints related with the warehouse and the customer replenishments. The first subproblem is a single facility, uncapacitated, multi-product, multi-period warehouse ordering problem (WOP), which is NP-hard. The second subproblem is the distribution planning problem (DP), which determines delivery routes in every period and amount of product to be delivered to each customer upon delivery.

An approximate solution algorithm to the integrated problem is defined and the solutions obtained by this approach are compared to the case in which the two subproblems mentioned above are solved separately and sequentially. The approximation algorithm finds an initial feasible solution to the WOP. Based on the distribution lots obtained from the solution of WOP, DP is solved. Then possible cost reductions are searched, when the distribution patterns are changed. This iterative procedure is applied until further gains are not realized.

Randomly generated test problems are used in the experiments and it is seen that the integrated approach provides decrease in the costs ranging from 3% to 11% on average over the decoupled approach.

Table 2.11 Chandra and Fisher (1994)

Element	Feature
Number of items	Many
Decision domain	Time
Demand	Deterministic
Demand time behavior	Dynamic
Number of vehicles	Not constraining
Vehicle capacity	Equal
Stock capacity	No
Supply capacity	Yes (Production time availability)
Inventory parameters	H1 (Retailer dependent) + H2 + Setup
Transportation costs	Fixed + Distance

The characteristics of the problem studied by Chandra and Fisher (1994) are given in Table 2.11 above. In this study, production, inventory, and routing decisions are considered together. The authors compare the two approaches in which production and distribution decisions are taken separately and in coordination with each other.

In the decoupled approach, the first step is determining a production schedule so as to minimize the setup and inventory holding costs, while meeting demands on time. For this subproblem, a MIP formulation is provided and it is solved optimally.

Using the available amounts obtained from the production scheduling problem, vehicle routing problems are solved by a heuristic approach, and improvement steps are applied later on without changing the production schedule. In the coordinated approach, the production and distribution decisions are taken into consideration within a single model, which is solved by a heuristic method. The method is composed of solving the two subproblems as in the decoupled approach and resorting to more comprehensive improvement steps that allow changes in the production schedule, as well.

In the numerical results, it is seen that the total cost reduction obtained by coordinating the production and distribution decisions ranges from 3% to 20%.

Table 2.12 Fumero and Vercellis (1999)

Element	Feature
Number of items	Many
Decision domain	Time
Demand	Deterministic
Demand time behavior	Dynamic
Number of vehicles	Given
Vehicle capacity	Equal
Stock capacity	No
Supply capacity	Yes (Resource availability)
Inventory parameters	H1 (Retailer dependent) + H2 + Setup
Transportation costs	Fixed + Amount + Distance

As in Chandra and Fisher (1994), the production decisions are incorporated in this study, as well. The features of the problem are presented in Table 2.12.

The production and distribution decisions are isolated by relaxing the constraints that link the two decisions to obtain solvable subproblems. After these constraints are relaxed, it is possible to decompose the problem into four subproblems that are used to make decisions on production, inventory, distribution, and routing, separately. The solution of the subproblems gives a lower bound on the original problem.

Using the solutions of the subproblems, a feasible solution (i.e., an upper bound) for the original problem is identified by a heuristic procedure.

Randomly generated test instances are used in the experiments. The computational times range from a few minutes to 1 hour, the gap between the upper and lower bounds that is calculated as $100 * \frac{(UB - LB)}{UB}$ assumes an average of 5.5%. It needs to be mentioned that the statistic generally used in the literature to measure this gap is $100 * \frac{(UB - LB)}{LB}$, which will give a greater gap value than the one obtained in this study. The average percentage improvement lagrangean relaxation provides over the continuous relaxation of the original problem is about 15%.

The feasible solutions obtained by the integrated approach described above are compared to a decoupled approach. It is seen that the percentage of improvement gained by the integrated approach over the decoupled approach averages to slightly above 10%.

Table 2.13 Bertazzi, Paletta and Speranza (2002)

Element	Feature
Number of items	Many / One
Decision domain	Time
Demand	Deterministic
Demand time behavior	Dynamic
Number of vehicles	Given (One)
Vehicle capacity	NA
Stock capacity	Yes
Supply capacity	Yes
Inventory parameters	H1 (Retailer dependent) + H2
Transportation costs	Distance

The characteristics of the problem considered in Bertazzi et al. (2002) are given in Table 2.13. Although the problem environment is described as distributing a set of products to several retailers by a vehicle, the solution algorithm is provided for the single product case. Each retailer has a minimum and a maximum level of inventory. The retailers must receive a delivery before their inventory falls below the minimum level and the amount of product delivered must fill the inventory up to the maximum level. Thus, the problem introduced is called as deterministic order-up-to level policies in an inventory routing problem.

The authors suggest a two-step heuristic method for solving the problem. Firstly, a set of delivery times are obtained for the retailers. For this purpose, an acyclic network is formed for each retailer. The nodes on these networks correspond to time instants. The weight of an arc between any two nodes, say node k and t, is the estimated change in the total cost, when the last visit to the retailer occurred at time k (node k) and the next visit to the retailer is to occur at time t (node t). Using the estimated costs associated with each arc on the network of a retailer, the shortest path between the time (node) 0 and T+1, where T is the number of periods in the planning horizon, is identified and a set of delivery times are obtained for each retailer. Then, for these selected delivery times, the retailers are inserted into the routes by using insertion at cheapest cost method. These steps form Phase-I of the two-phase heuristic developed. In the second phase, iterative improvements are made in the following way. A pair of retailers is removed from the routes and possible improvements are searched by identifying new sets of delivery time instants for these retailers and repeating the cheapest insertion method.

The upper bounds obtained with the two-step heuristic described above are compared to two trivial heuristics, i.e., 'every' and 'latest'. 'Every' is based on visiting all retailers in each period with the minimum cost tour, whereas in 'latest' the retailers that are visited in a period are the ones that will stockout in the next period if a delivery is not made in the current period. It is seen that the two-step heuristic outperforms the trivial heuristics. The gap between the solutions obtained by the two-step heuristic and ‘every’ averages to 14%, whereas the related gap with ‘latest’ averages to 5%. Furthermore, the authors investigate the changes in the solutions under different objectives. Since a lower bound is not provided in this study, the performance of the heuristic developed can not be evaluated objectively.

Regarding the studies reviewed, it is seen that optimal solutions are not identified for the problems since the inventory routing problem is a hard problem. The method that is frequently used is decomposing the integrated inventory routing problem into subproblems, solving the subproblems by heuristic methods, and identifying upper and lower bounds to the integrated problem. Some of the studies, on the other hand, are based on providing a coordinated and a decoupled approach and evaluating the gains of the coordinated approach.

Although most of the studies take into account the inventory holding policies at the retailers, the studies that consider the inventory policies of the supplier (taking into consideration either production or inventory related costs) in addition to the inventory policies of the retailers are rare.

Similar to the previous works, we do not deal with identifying optimal solutions for the hard inventory routing problem. However, when compared to the previous studies, we provide an integrated mathematical model for the deterministic inventory routing problem with order-up-to level inventory policy that captures all decisions related with inventory management both at the retailers and at the supplier and routing. Furthermore, we propose methods to identify a lower bound on this problem, which is not provided in the literature so far.

CHAPTER 3

MIP FORMULATION FOR THE IRDOP

In this chapter, firstly, the problem environment is introduced, next basic assumptions made to model the problem are listed, and then an integrated model to formulate the problem is described. A discussion of difficulties faced upon identifying a solution to IRDOP using the integrated model is given afterwards.

3.1 Environment

IRP incorporates simultaneous decision making on inventory management and vehicle routing problems for a distribution system. The motivation for integrating inventory management and routing decisions mainly arises in real life either when the supplier and the retailers represent different echelons in a supply chain or when vendor-managed inventory systems are present. In IRP, the aim is to determine the route to distribute the product(s) under concern and the amount of inventory to keep at the stocks over a planning horizon.

In this study, a distribution system consisting of a supplier and several retailers is considered. The retailers face retailer dependent, deterministic, and dynamic demands for a product that have to be met without backlogs. The supplier distributes the product to the retailers with a capacitated vehicle and a transportation cost is incurred (vehicle routing problem). Both the supplier and the retailers hold inventory to be able to meet the demand on time (inventory management problem). Each retailer has a predetermined retailer dependent minimum and maximum inventory level and must be visited before its inventory falls below the minimum level. During a visit to the retailers, the amount for the difference between the on-hand inventory and the maximum level is delivered, which gives rise to the so called deterministic order-up-to level policy.

The problem is to determine which retailers to visit in each period, the amount of product to be delivered to the retailers, and the route of the vehicle, while minimizing the inventory holding costs at the retailers and the supplier and the transportation costs.

Thus, we study a deterministic IRP with order-up-to level inventory policies. The maximum inventory level in this problem can be considered as the shelf space available for a product in a supermarket or the size of a tank that is used to keep inventory of a product such as heating oil. The minimum inventory level, on the other hand, can be considered as safety stock level required not to stock out since backlogging is not allowed. If maximum and minimum inventory levels are removed, the problem corresponds to the classical inventory routing problem in the literature.

In fact, the property of filling the inventory at the retailers up to the maximum levels at each delivery is one of the distinguishing features handled in our mathematical formulation for IRDOP, when compared with earlier works on IRP. On the other hand, it can be argued that a minimum inventory level is not essential in systems with deterministic demands. Nevertheless, taking this feature into account can be considered as an initial step for modeling systems with stochastic demands, where the minimum inventory level will be an estimated safety stock level. Moreover, as it will be declared later on in detail, the minimum inventory level can simply be dropped from the formulation.

The features of the problem we studied are summarized in Table 3.1 below according to the elements of the classification scheme described in Section 2.1.

Table 3.1 The characteristics of the problem we studied

Element	Feature
Number of items	One
Decision domain	Time
Demand	Deterministic
Demand time behavior	Variable
Number of vehicles	Given (One)
Vehicle capacity	NA (Single vehicle)
Stock capacity	Yes
Supply capacity	Yes
Inventory parameters	H1 (Retailer dependent) + H2
Transportation costs	Distance

Basic Assumptions

- A deterministic and dynamic amount of product becomes available at the supplier in each period.
- The supplier can keep inventory and incurs a time independent unit inventory holding cost per period.
- The product is distributed by a capacitated vehicle on a tour that starts from the supplier, visits the retailers, and ends at the supplier.
- A distance dependent transportation cost is incurred upon deliveries. The formulation developed for IRDOP is valid for symmetrical and unsymmetrical distances and the triangle inequality is not necessarily satisfied.
- Retailers face retailer dependent, deterministic, and dynamic demands for the product.
- Retailers can keep inventory and incur a unit inventory holding cost per period that is retailer dependent and constant over time.
- Inventory holding costs are incurred at the beginning of each period. For this reason, to take into account the effects of the decisions made in the last period of the planning horizon, the inventory levels at the beginning of the period that succeeds the last period are considered in inventory holding cost calculations, as well.

- Retailers have retailer dependent minimum inventory levels and the inventories at the retailers are not allowed to be less than these levels.
- Retailers have retailer dependent maximum inventory levels and the replenishments must fill the inventories up to these levels.
- A lead time on using the amount of product available at the supplier is included by assuming that the amount that becomes available at the supplier in a period is not used in the deliveries in that period.
- The sequence of the events for any time period t is as follows. Inventory holding costs are incurred according to the on hand inventory at the retailers and the supplier at the beginning of the period. The product is distributed from the supplier to the retailers and transportation costs are incurred according to the distance traveled. A given amount of product becomes available at the supplier and the retailers meet the demands that they face. The events occurring in any period t are as seen in Figure 3.1.

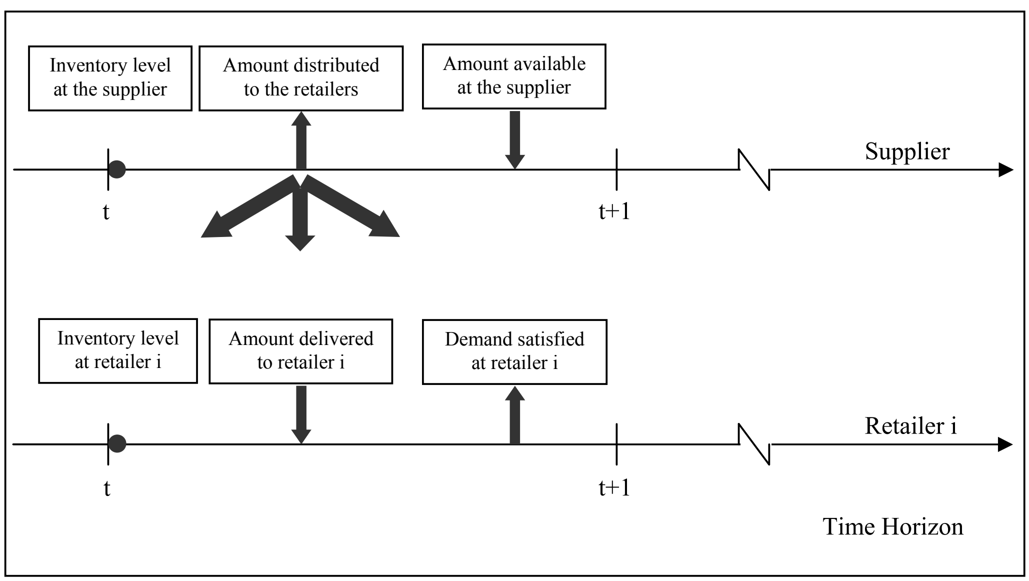

Figure 3.1 The order of the events in any period t

3.2 Integrated Model (IM) Formulation

The mathematical formulation for the IRDOP is given below. In this formulation, R represents the set of retailers. The supplier is associated with {0}, and set N, where $N = R \cup \{0\}$, includes both the supplier and the retailers. T stands for the set of periods. Also, set H is included to represent the T period planning horizon plus the period succeeding the planning horizon, i.e., $H = T \cup T+1$, basically for inventory holding cost computation purposes. Note that T denotes both the set of periods and the last period in the planning horizon, which can be distinguished from the context.

Sets

R: Set of retailers, R = {1, 2, 3, …, N}

T: Set of periods in the planning horizon, T = {1, 2, 3, …, T}

R_0: $R \cup \{0\}$

H: $T \cup \{T+1\}$

Parameters

h^i : Unit inventory holding cost at retailer i in each period

hs : Unit inventory holding cost at the supplier in each period

c^{ij} : Transportation cost incurred whenever j is visited after i

d_t^i : Demand at retailer i in period t

$\underline{S^i}$: Minimum inventory level at retailer i

$\overline{S^i}$: Maximum inventory level at retailer i

C : Capacity of the vehicle

a_t : Amount of product that becomes available at the supplier in period t

I_1^i : Initial inventory level at retailer i at the beginning of period 1

Is_1 : Initial inventory level at the supplier at the beginning of period 1

Decision Variables

X_t^i : Amount of product delivered to retailer i in period t

I_t^i : Inventory kept at retailer i at the beginning of period t

Is_t : Inventory kept at the supplier at the beginning of period t

$$Y_t^i : \begin{cases} 1 & \text{if retailer i is visited in period t} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_t^{ij} : \begin{cases} 1 & \text{if i is visited after j in period t} \\ 0 & \text{otherwise} \end{cases}$$

U_t^i : Total amount of product delivered up to and including retailer i in period t

Using these variables and parameters, the order of the events that occur in any period t is given in Figure 3.2.

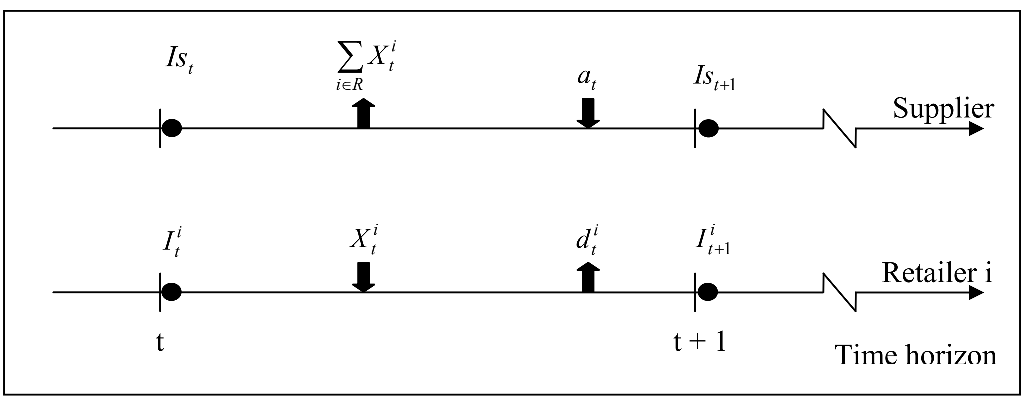

Figure 3.2 Events that occur in any time period t

[IM]

Minimize $\sum_{i \in R} \sum_{t \in H} h^i I_t^i + \sum_{t \in H} hsIs_t + \sum_{i \in R_0} \sum_{j \in R_0} \sum_{t \in T} c^{ij} Z_t^{ij}$ (3.1)

Subject to

$$I_{t-1}^i + X_{t-1}^i - I_t^i = d_{t-1}^i \qquad \forall i \in R, t = 2,...,T+1 \qquad (3.2)$$

$$X_t^i \le MY_t^i \qquad \forall i \in R, \forall t \in T \qquad (3.3)$$

$$\sum_{i \in R} X_t^i \le C \qquad \forall t \in T \qquad (3.4)$$

$$\sum_{i \in R} X_t^i \le Is_t \qquad \forall t \in T \qquad (3.5)$$

$$I_t^i \ge \underline{S^i} \qquad \forall i \in R, \forall t \in H \qquad (3.6)$$

$$I_t^i + X_t^i \le \overline{S^i} \qquad \forall i \in R, \forall t \in T \qquad (3.7)$$

$$\overline{S^i} - X_t^i - I_t^i \le M\left(1 - Y_t^i\right) \qquad \forall i \in R, \forall t \in T \qquad (3.8)$$

$$Is_t + \sum_{i \in R} X_{t-1}^i - Is_{t-1} = a_{t-1} \qquad t = 2,...T+1 \qquad (3.9)$$

$$\sum_{j \in R_0} Z_t^{ij} = Y_t^i \qquad \forall i \in R, \forall t \in T \qquad (3.10)$$

$$\sum_{j \in R_0} Z_t^{ji} = Y_t^i \qquad \forall i \in R, \forall t \in T \qquad (3.11)$$

$$U_t^i - U_t^j + C * Z_t^{ij} \le C - X_t^j \qquad \forall i \in R, \forall j \in R, \forall t \in T \qquad (3.12)$$

$$U_t^i \ge X_t^i \qquad \forall i \in R, \forall t \in T \qquad (3.13)$$

$$U_t^i \le C \qquad \forall i \in R, \forall t \in T \qquad (3.14)$$

$$X_t^i \ge 0 \qquad \forall i \in R, \forall t \in T \qquad (3.15)$$

$$Is_t \ge 0 \qquad \forall t \in H \qquad (3.16)$$

$$Y_t^i \in (0, 1) \qquad \forall i \in R, \forall t \in T \qquad (3.17)$$

$$Z_t^{ij} \in (0, 1) \qquad \forall i \in R_0, \forall j \in R_0; i \ne j, \forall t \in T \qquad (3.18)$$

where $M = \overline{S^i} - \underline{S^i}$ $\quad \forall i \in R$ (3.19)

In the above formulation, the objective function (3.1) minimizes the total of inventory holding cost at the retailers, inventory holding cost at the supplier, and transportation cost.

Constraint (3.2) is the flow balance restriction that ensures that demand in a period is met from the inventory at the beginning of that period and the delivery in that period without backlogs. In the flow balance equation for the first period, the parameter I_1^i that denotes the initial inventory level at a retailer is substituted for the term I_{t-1}^i.

Constraint (3.3) enforces that a delivery can be made to a retailer in a period only if the retailer is visited in that period. The parameter M used in this constraint is an appropriately large number. In fact, the value of M given by (3.19), i.e., the difference between the maximum and the minimum inventory levels, is the maximum possible value that the variable X_t^i, i.e., the amount of product delivered to retailer i in period t, can take on since inventory of a retailer is at least at the minimum level and must be filled up to the maximum level.

Constraints (3.4) and (3.5) guarantee that the total amount of product distributed to the retailers in a period does not exceed the vehicle capacity and the inventory available at the supplier at the beginning of the period, respectively. By this means, constraint (3.5) imposes that the deliveries to the retailers in a period are restricted with the inventory level at the supplier at the beginning of the period. Thus, the amount of product that becomes available at the supplier in a period is not used up for the deliveries in that period, which can be considered as inclusion of an implicit lead time.

Constraint (3.6) is for ensuring that the inventory level at each retailer at the beginning of each time period is above the minimum level.

Constraint (3.7) makes sure that the inventory level at a retailer at the beginning of a period and the amount of product delivered in that period is below the maximum level. Therefore, the inventory level at a retailer is below the maximum level at any instant.

Constraint (3.7) and constraint (3.8) ensure that whenever a delivery is made to a retailer, the inventory at the retailer reaches the maximum level. The value assumed for the parameter M is as given in (3.19). To give a brief explanation on how constraints (3.7) and (3.8) work, assuming that retailer i is visited in period t (i.e., $Y_t^i = 1$), the term on the right hand side of the constraint (3.8) is equal to 0 and the constraint can be restated as follows.

$$-X_t^i + \overline{S^i} - I_t^i \leq 0 \qquad \forall i \in R, \forall t \in T \qquad (3.8')$$

Arranging (3.8') as $X_t^i + I_t^i \geq \overline{S^i}$, and taking constraint (3.7) into account, i.e., $I_t^i + X_t^i \leq \overline{S^i}$, it is seen that the delivery amount X_t^i is forced to be equal to the difference between the maximum inventory level for the related retailer and the inventory on hand at the beginning of the period, whenever the retailer is visited.

The coordinated use of constraint (3.7) and (3.8) is explained above for the case, where $Y_t^i = 1$. Now, assuming that $Y_t^i = 0$ and substituting the value of M as given in (3.19), constraint (3.8) can be expressed as follows.

$$-X_t^i + \overline{S^i} - I_t^i \leq \overline{S^i} - \underline{S^i} \qquad \forall i \in R, \forall t \in T \qquad (3.8'')$$

(3.8'') can be arranged as $X_t^i + I_t^i \geq \underline{S^i}$, which does not impose additional restrictions on the formulation since the inventory on its own is always above the minimum level by means of constraint (3.6).

Constraint (3.9) establishes the flow balance for the supplier. It balances the inventory, the total amount delivered to the retailers and the amount that becomes available at the supplier in each period. Note that although the amount that becomes available at the supplier in period t (i.e., a_t) is not allowed to be used in the deliveries during period t, it is present in this constraint for the purpose of on hand inventory balancing. As stated before, its not being used in the deliveries during period t is assured with constraint (3.5).

Constraints (3.10) and (3.11) ensure that if a retailer receives a delivery in a period, that retailer is visited before and after the supplier or a retailer on the route followed in that period, respectively.

Constraint (3.12) is the subtour elimination constraint, developed for the Traveling Salesman Problem (TSP) by Miller, Tucker and Zemlin (1960) and adapted to the Vehicle Routing Problem by Kulkarni and Bhave (1985), to find a feasible tour.

Constraints (3.13) and (3.14) define the minimum and the maximum value that the total amount of product delivered to the retailers up to and including retailer i, i.e., U_t^i, can take on, respectively. The total amount distributed up to and including retailer i can not be less than the amount delivered to retailer i (i.e., X_t^i) and can not be more than the capacity of the vehicle (i.e., C).

Constraints (3.15) and (3.16) are nonnegativity and constraints (3.17) and (3.18) are integrality restrictions on the related decision variables. Note that a nonnegativity restriction is not required for the decision variable I_t^i since it is already forced to be above the minimum inventory level.

It must be remarked that since constraint (3.6) forces the inventory level of each retailer to be over the minimum level, an amount that corresponds to this minimum level is always kept at the stocks of the retailers. It is possible to drop out the minimum inventory level from the formulation and use the usual restriction of the policies without backlogging which ensures that the inventory level is not allowed to be negative. This can be handled as follows: Set the minimum inventory level at zero and reduce the maximum inventory level by the minimum inventory level. Therefore, to simplify the formulation, the minimum inventory level is eliminated from the model by modifying constraints (3.6), (3.7), and (3.8) as below.

$$I_t^i \geq 0 \qquad \forall i \in R, \forall t \in H \qquad (3.6')$$

$$I_t^i + X_t^i \leq \overline{S^i} - \underline{S^i} \qquad \forall i \in R, \forall t \in T \qquad (3.7')$$

$$-X_t^i + \overline{S^i} - \underline{S^i} - I_t^i \leq M\left(1 - Y_t^i\right) \qquad \forall i \in R, \forall t \in T \qquad (3.8''')$$

It must be noticed that the adjustment replaces the term $\underline{S^i}$ with 0 in (3.6) and the term $\overline{S^i}$ with $\overline{S^i} - \underline{S^i}$ in (3.7) and (3.8). Although the formulation is expressed including the minimum inventory levels throughout the rest of the text in accordance with the definition of the problem, it must be kept in mind that the experiments are performed using the modified version above.

A solution for IM will identify the following basic issues, while minimizing the total cost.

- The retailers that receive a delivery in each period.
- The amount of product delivered to the retailers that are visited in each period.
- The route followed by the vehicle when distributing the product.

Putting aside the restrictions related with the inventory policies and the costs associated with them, and assuming that the vehicle capacity is large enough, what remains to be solved is a TSP for a period, if some retailers receive deliveries in that period, i.e. for the periods in which there exist some $Y_t^i = 1$.

For this setting, let T' be the set of periods in which deliveries exist, R' be the set of retailers that are visited in period $t \in$ T', and let $R_0' = R' \cup \{0\}$.

With these modifications, the resulting formulation to be solved can be expressed as below.

Minimize $\sum_{i \in R_0'} \sum_{j \in R_0'} \sum_{t \in T'} c^{ij} Z_t^{ij}$ (3.1')

Subject to

$\sum_{j \in R_0'} Z_t^{ij} = 1$ $\quad \forall i \in R', \forall t \in T'$ (3.10')

$\sum_{j \in R_0'} Z_t^{ji} = 1$ $\quad \forall i \in R', \forall t \in T'$ (3.11')

$U_t^i - U_t^j + (n-1) * Z_t^{ij} \leq n-2$ $\quad \forall i \in R', \forall j \in R', \forall t \in T'$ (3.12')

$U_t^i \geq 1$ $\quad \forall i \in R', \forall t \in T'$ (3.13')

$U_t^i \leq n-1$ $\quad \forall i \in R', \forall t \in T'$ (3.14')

$Z_t^{ij} \in (0, 1)$ $\quad \forall i \in R_0', \forall j \in R_0'; i \neq j, \forall t \in T'$ (3.15)

The objective function (3.1') is a simplified version of the objective function of IM, which includes only transportation related costs.

The right hand sides of constraints (3.10') and (3.11') are 1 because the model is formulated for only the retailers to be visited.

Constraints (3.12'), (3.13'), and (3.14') are the Miller-Tucker-Zemlin subtour elimination constraints and constraint (3.15) ensures integrality.

The model given above turns out to be a TSP. Thus, even if the inventory related decisions are excluded from the formulation, it is required to identify solutions to several TSPs to solve our model. The TSP is known to be NP-hard, so it is hard to come up with a polynomial time algorithm to solve it. This implies that IM is also difficult to solve. In addition to the routing difficulty, the complexity caused by other decisions included in IM motivates us to develop heuristic procedures to solve the IRDOP. In the following chapter, our approaches to find lower and upper bounds on the solution are discussed.

CHAPTER 4

SOLVING THE IRDOP

In this chapter, the methods used to obtain lower and upper bounds for the IRDOP will be discussed. Identifying a lower bound for the IRDOP is a significant concern in this study since a lower bound has not been identified for this problem so far.

To avoid dealing with the NP-hard TSPs, the approach taken in this study is the exclusion of the subtour elimination constraints from the model. Removal of these constraints is carried out in two basic ways: using lagrangean relaxation approach on IM or incorporating an a priori tour into IM.

4.1 A Lagrangean Relaxation Based Approach

A lagrangean relaxation based approach is used to obtain lower and upper bounds for the IRDOP. In this procedure, the subtour elimination constraints in IM are relaxed. Thus, a solution identified by the relaxed problem is enforced to satisfy all restrictions imposed in IM except the subtour elimination constraints. So, this solution provides a lower bound to the IRDOP.

The solution to the relaxed problem may result in visiting the retailers by subtours because of ignoring subtour elimination. Except for this violation, however, remaining restrictions on IM hold for the solution obtained by lagrangean relaxation problem. As a result, to identify a feasible solution to IM, the subtours that exist in the solution for the relaxed problem must be turned into tours. For this purpose, a heuristic method is developed to convert the subtours into a tour. When a tour for each period that contains all retailers visited in the related period is determined, a feasible solution to the original problem (i.e., an upper bound) is identified.

4.1.1 Lagrangean Relaxation Problem

The subtour elimination constraints are multiplied by the lagrangean multipliers (i.e., the dual variables associated with the constraints) and added into the objective function. The relaxed problem (LRP) is formulated below.

Sets

R: Set of retailers, R = {1, 2, 3, …, N}

T: Set of periods in the planning horizon, T = {1, 2, 3, …, T}

R_0: $R \cup \{0\}$

H: $T \cup \{T+1\}$

Parameters

h^i : Unit inventory holding cost at retailer i in each period

hs : Unit inventory holding cost at the supplier in each period

c^{ij} : Transportation cost incurred whenever j is visited after i

d_t^i : Demand at retailer i in period t

$\underline{S^i}$: Minimum inventory level at retailer i

$\overline{S^i}$: Maximum inventory level at retailer i

C : Capacity of the vehicle.

a_t : Amount of product that becomes available at the supplier in period t

I_1^i : Initial inventory level at retailer i at the beginning of period 1

Is_1 : Initial inventory level at the supplier at the beginning of period 1

λ_t^{ij} : Lagrangean multiplier for the subtour elimination constraint over i, j, t

Decision Variables

X_t^i : Amount of product delivered to retailer i in period t

I_t^i : Inventory kept at retailer i at the beginning of period t

Is_t : Inventory kept at the supplier at the beginning of period t

$$Y_t^i : \begin{cases} 1 & \text{if retailer i is visited in period t} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_t^{ij} : \begin{cases} 1 & \text{if i is visited after j in period t} \\ 0 & \text{otherwise} \end{cases}$$

U_t^i : Total amount of product delivered up to and including retailer i in period t

The initial formulation of LRP is the same as IM formulation with only exception being the relaxed subtour elimination constraints that are included in the objective function, i.e., the last term in the objective function of LRP given with (4.1) below.

[LRP]

$$\text{Minimize} \sum_{i \in R} \sum_{t \in H} h^i I_t^i + \sum_{t \in H} hsIs_t + \sum_{i \in R_0} \sum_{j \in R_0} \sum_{t \in T} c^{ij} Z_t^{ij} + \sum_{\substack{i \in R \\ i \neq j}} \sum_{j \in R} \sum_{t \in T} \lambda_t^{ij} (U_t^i - U_t^j + C * Z_t^{ij} + X_t^j - C) \quad (4.1)$$

Subject to

$$I_{t-1}^i + X_{t-1}^i - I_t^i = d_{t-1}^i \qquad \forall i \in R, t = 2,...,T+1 \qquad (4.2)$$

$$X_t^i \leq MY_t^i \qquad \forall i \in R, \forall t \in T \qquad (4.3)$$

$$\sum_{i \in R} X_t^i \leq C \qquad \forall t \in T \qquad (4.4)$$

$$\sum_{i \in R} X_t^i \leq Is_t \qquad \forall t \in T \qquad (4.5)$$

$$I_t^i + X_t^i \leq \overline{S^i} \qquad \forall i \in R, \forall t \in T \qquad (4.6)$$

$$I_t^i \geq \underline{S^i} \qquad \forall i \in R, \forall t \in H \tag{4.7}$$

$$\overline{S^i} - X_t^i - I_t^i \leq M\left(1 - Y_t^i\right) \qquad \forall i \in R, \forall t \in T \tag{4.8}$$

$$Is_t + \sum_{i \in R} X_{t-1}^i - Is_{t-1} = a_{t-1} \qquad t = 2, \ldots T+1 \tag{4.9}$$

$$\sum_{j \in R_0} Z_t^{ij} = Y_t^i \qquad \forall i \in R, \forall t \in T \tag{4.10}$$

$$\sum_{j \in R_0} Z_t^{ji} = Y_t^i \qquad \forall i \in R, \forall t \in T \tag{4.11}$$

$$U_t^i \geq X_t^i \qquad \forall i \in R, \forall t \in T \tag{4.12}$$

$$U_t^i \leq C \qquad \forall i \in R, \forall t \in T \tag{4.13}$$

$$X_t^i \geq 0 \qquad \forall i \in R, \forall t \in T \tag{4.14}$$

$$Is_t \geq 0 \qquad \forall t \in H \tag{4.15}$$

$$Y_t^i \in (0, 1) \qquad \forall i \in R, \forall t \in T \tag{4.16}$$

$$Z_t^{ij} \in (0, 1) \qquad \forall i \in R_0, \forall j \in R_0; i \neq j, \forall t \in T \tag{4.17}$$

where $M = \overline{S^i} - \underline{S^i}$ $\qquad \forall i \in R$ (4.18)

A solution for LRP identifies the retailers that are visited in each period, the amount of product delivered at each visit, and the (sub) routes executed to perform the visits. The difference of the solution obtained for LRP from that of IM is that the deliveries can be made to the retailers by several subtours in each period since the subtour elimination constraints are relaxed into the objective function.

4.1.2 Enhancements

After the relaxation, there may be cases in which the supplier is included in more than one subtour in a period. This situation, however, can be prevented to obtain solutions for LRP in line with those of IM. If the supplier is included in more than one subtour in a period, when the subtours in the related period are merged into a tour, there will be multiple visits to the supplier. To overcome this unrealistic outcome, the decision variable that identifies whether a retailer is visited in a period or not is extended to include the supplier as shown below.

$$Y_t^0 : \begin{cases} 1 & \text{if the supplier is included in a tour in period t} \\ 0 & \text{otherwise} \end{cases}$$

Using Y_t^0, constraints (4.10) and (4.11) in LRP are extended with the insertion of constraints (4.19) and (4.20) given below to include the entries to and the exits from the supplier, as well.

$$\sum_{j \in R} Z_t^{0j} = Y_t^0 \qquad \forall t \in T \qquad (4.19)$$

$$\sum_{j \in R} Z_t^{j0} = Y_t^0 \qquad \forall t \in T \qquad (4.20)$$

Constraints (4.19) and (4.20) guarantee that there is no more than one entrance to and one exit from the supplier in a period, respectively. Thus, to include (4.19) and (4.20) in LRP formulation, constraints (4.10), (4.11), and (4.16) are modified as follows.

$$\sum_{j \in R_0} Z_t^{ij} = Y_t^i \qquad \forall i \in R_0, \forall t \in T \qquad (4.10')$$

$$\sum_{j \in R_0} Z_t^{ji} = Y_t^i \qquad \forall i \in R_0, \forall t \in T \qquad (4.11')$$

$$Y_t^i \in (0, 1) \qquad \forall i \in R_0, \forall t \in T \qquad (4.16')$$

Another impractical solution that may be faced with is that deliveries may be made to the retailers with several subtours in a period without including the supplier in any of these subtours in that period. To preclude this outcome, constraints (4.21) and (4.22) given below, which ensure that whenever a retailer is visited in a period, there must be an entrance to and an exit from the supplier in that period, are inserted into the model.

$$\sum_{i \in R} Y_t^i \leq N \sum_{i \in R} Z_t^{i0} \qquad \forall t \in T \qquad (4.21)$$

$$\sum_{i \in R} Y_t^i \leq N \sum_{i \in R} Z_t^{0i} \qquad \forall t \in T \qquad (4.22)$$

where N is the number of retailers in the distribution system.

Note that the enhancements given above are already satisfied in IM and therefore they are redundant for IM.

Moreover, to strengthen the solutions obtained from the relaxed problem, the cuts proposed in Barany, Van Roy and Wolsey (1984) and improved in Denizel and Süral (2005) are adapted to our problem as follows.

$$\sum_{k=1}^{t-1} X_k^i + d_t^i Y_t^i \geq \max\left(0, \sum_{k=1}^{t} d_k^i - I_1^i\right) \qquad \forall i \in R, t = 2,3,\ldots,T \qquad (4.23)$$

Constraint (4.23) makes the model stronger by forcing that if a retailer is not visited in period t, demand at that retailer up to and including period t must be met by the amount of product delivered to the retailer up to and including period t-1 and the retailer's initial inventory.

Another issue that needs to be remarked is that the LP relaxation of the strengthened LRP that is obtained by the enhancements above does not necessarily provide an integral solution. Therefore, as discussed in Geoffrion (1974), and restated in several succeeding studies as Fisher (1981), Fisher (1985), and Beasley (1993), we expect to get a lower bound with LRP that is not of inferior quality than the lower bound obtained by the LP relaxation of IM.

Whenever LRP is referred in the text without a notification, the formulation with the enhancement should be considered.

4.1.3 Lower Bounding Procedure

Any solution to LRP identifies a lower bound for IM. However, the quality of the lower bound obtained depends on several issues, which are discussed in the sequel.

One of the fundamental issues that affect the quality of a solution for LRP is the way the lagrangean multipliers are updated. From the two possible multiplier updating methods used in the literature, namely, subgradient optimization and multiplier adjustment, subgradient optimization is selected and employed in this study. As stated in Beasley (1993), although multiplier adjustment usually requires less computational effort, the lower bound obtained can be weaker than that of subgradient optimization. Furthermore, different procedures are required for applying multiplier adjustment heuristic to different problems, whereas subgradient optimization is a robust procedure that can be applied to any kind of problem in the same way. Nevertheless, a multiplier adjustment heuristic that is specific to the problem on hand may be considered in a more comprehensive study. For gaining more insight into lagrangean relaxation based approaches, Held, Wolf and Crowder (1974); Geoffrion (1974); Fisher (1981); Fisher (1985); and Beasley (1993) are some basic references.

The procedure utilized to obtain a lower bound for the IRDOP using LRP is described below.

[Lower Bound Identifying Procedure]

Initialization. Initialize the parameters.

$\pi = 2$ (Step size parameter)

$\lambda_t^{ij} = 0 \;\; \forall i \in R, \forall j \in R, \forall t \in T$ (Lagrangean multipliers)

Z_{UB} = A heuristic method is used (Upper bound). The reader is referred to Appendix A for a definition of the heuristic 'every'.

Z_{max} = A small, possibly negative, number (Maximum lower bound).

I_{max} = Used if a limit is required for the number of iterations (Maximum iterations).

INo = 1 (Number of iterations)

Step 1. Solve LRP with current set of multipliers and obtain Z_{LB}.

Let, Z_{max} = max (Z_{max}, Z_{LB})

Step 2. Calculate subgradients for the relaxed subtour elimination constraints as below.

$$s_t^{ij} = U_t^i - U_t^j + C * Z_t^{ij} + X_t^j - C \qquad \forall i \in R, \forall j \in R, \forall t \in T$$

Step 3. Identify the step size as follows.

$$T = \frac{\pi\left(Z_{UB} - Z_{LB}\right)}{\sum_{t\in T}\sum_{j\in R}\sum_{i\in R}\left(s_t^{ij}\right)^2}$$

Step 4. Update the lagrangean multipliers as seen below.

$$\lambda_t^{ij} = \max\left(0, \lambda_t^{ij} + T\ s_t^{ij}\right) \qquad \forall i \in R, \forall j \in R, \forall t \in T$$

Step 5. Let, INo = INo + 1. Update π according to the scheme used.

Step 6. Go to step 1 until $(\pi < 0.005)$ or (INo = I_{max}).

In the procedure above, a lower bound for IM is obtained each time step 1 is executed. The maximum lower bound is kept as Z_{max} to be used as the final lower bound obtained with this method.

In this procedure, multiplier initialization is a concern that needs to be clarified. We initialize lagrangean multipliers as zero since this is the most natural choice frequently used in the literature. Also, it is stated in Beasley (1993) that the initialization of the lagrangean multipliers does not have a high influence on the quality of the lower bound identified.

The parameter π used in the step size calculation can assume values between 0 and 2. The reader is referred to Held et al. (1974) for a discussion on the step size calculation. A common approach used in the literature is initialing π at 2 and halving it if the lower bound does not improve for a certain number of iterations (see Fisher (1981), Fisher (1985), or Beasley (1993)). In addition to this usual approach, we made use of another method, in which π is initialized at 2 and divided by 1.005 at each iteration. Thus two different schemes are employed for updating π to assess its effect on the lower bound.

Method 1: Halving π, if Z_{max} does not alter for a certain number of iterations.
Method 2: Dividing π by 1.005 at each iteration.

The quality of the solutions obtained by different π updating schemes will be discussed in the computational experiments.

4.1.4 Upper Bounding Procedure

Note that the solution obtained for LRP may not be feasible for IM since the deliveries in a period may be made to the retailers by several subtours. To obtain a feasible solution for the IRDOP, the nearest merger TSP algorithm (NM) as defined in Johnson and Papadimitriou (1985) for the problems with symmetric distances is used. The run time of the algorithm is $O(n^2)$ and its worst case bound approaches twice the length of the optimal tour.

Previously, it was mentioned that IM is applicable whether the distance between any two retailers is assumed as symmetric or not. However, NM requires the distances between the retailers to be symmetrical.
The first step of NM is identifying the subtours in each period. If there are two or more subtours in any period, the algorithm is used to merge these subtours into a

single tour. When the subtours in each period are merged into a single tour for the related period, a feasible solution for the IRDOP is identified.

NM algorithm works as follows. For each subtour pair in each period, cost effects of removing one of the arcs from each of the two subtours and merging the two subtours into a tour is assessed and the merger with the minimum cost is selected out of possible merging schemes for the related period.

For instance, if the solution of LRP contains two subtours in a period as seen in Figure 4.1, the effects of removing the arc connecting the supplier and retailer 1 from the first subtour and the arc connecting retailer 2 and retailer 3 from the second subtour are evaluated as seen in Figure 4.2. It must be noticed that the grey-colored arcs are the ones, effects of whose removal are assessed in Figure 4.2, whereas the thick arcs are the ones that are considered to be inserted to obtain a tour. All arcs in the two figures below are undirected since the algorithm is applicable for the cases with symmetric distances.

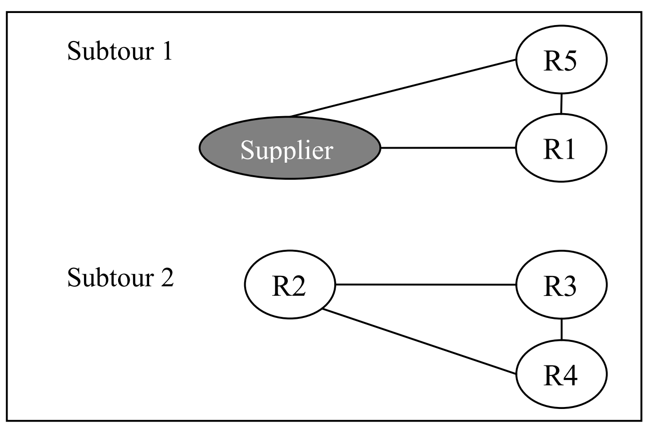

Figure 4.1 An illustrative solution in a period for LRP

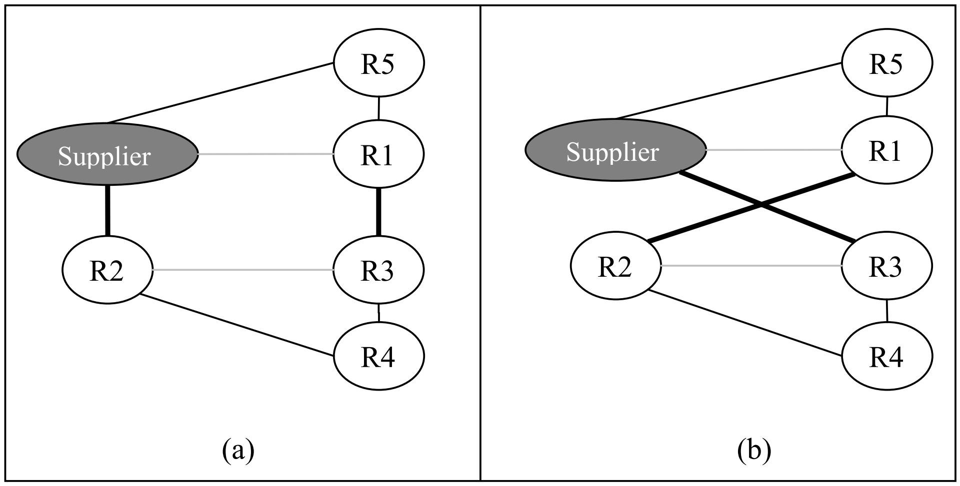

Figure 4.2 Possible merging schemes when the grey-colored arcs are removed

The tour obtained by the merging scheme that is demonstrated in Figure 4.2 (a), read clockwise, starts from the supplier, visits the retailers 5, 1, 3, 4, 2, and ends at the supplier.

Letting c^{ij} denote the cost of traveling from node i to node j, where i = 0 stands for the supplier and i = 1, 2, 3, 4, and 5 stand for respective retailers, the cost of the merging scheme in Figure 4.2 (a) i.e., CM_1, is calculated as follows:

$$CM_1 = c^{02} + c^{13} - c^{01} - c^{23}$$

The tour obtained with the merging scheme in Figure 4.2 (b), read clockwise, starts from the supplier, visits the retailers 5, 1, 2, 4, 3, and ends at the supplier.

Similarly, the cost of the merging scheme in Figure 4.2 (b), i.e., CM_2, is calculated as follows:

$$CM_2 = c^{03} + c^{12} - c^{01} - c^{23}$$

The effects of excluding two of the arcs on transportation costs, where each arc comes from different subtours, are assessed as in the example above. In our NM algorithm, the merging scheme with the minimum cost among all subtour pairs is selected and the related subtours are merged into a tour. This procedure is repeated for each period until a single tour is obtained in each period.

Figure 4.3 shows any iteration of NM for the nodes being considered. It will be used to explain the steps of the entire procedure.

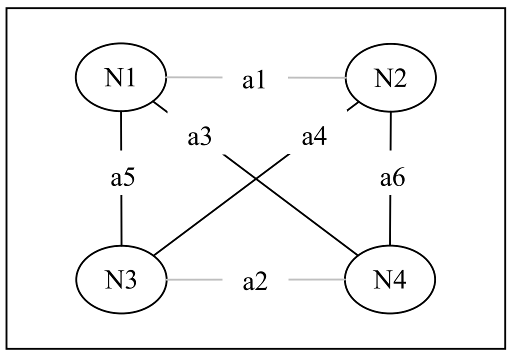

Figure 4.3 The nodes and arcs under consideration at any iteration of the nearest merger heuristic

In Figure 4.3, N1 and N2 are any two nodes on one of the subtours (s1) and N3 and N4 are any two nodes on another subtour (s2). Let the arc between N1 and N2 on s1 and the arc between N3 and N4 on s2 be the arcs (the grey colored arcs in Figure 4.3), whose removals are evaluated. Let the costs of traveling along the arcs a1, a2, a3, a4, a5, and a6 be c1, c2, c3, c4, c5, and c6, respectively, for the nodes under consideration. According to the arrangement in Figure 4.3, NM can be summarized as follows.

[Nearest Merger Procedure]

Initialization.

t = 0

CM_{min} = A large number (Minimum merging cost).

Step 1. Let t = t+1

Step 2. Identify the subtours in period t.

Create an ordered list of the subtours by associating each subtour in each period with a number starting with 1 and ending with the number of subtours in the related period in the following way.

List of subtours: 1, 2, 3, …, n_t $\forall t$, where n_t is the number of subtours in period t.

Create an ordered list of arcs for each subtour in each period by associating each arc with a number starting with 1 and ending with the number of the arcs included in the related subtour as follows.

List of arcs: 1, 2, 3, …, m_{ts} $\forall t, s$, where m_{ts} is the number of arcs included in subtour s in period t.

Step 3. Let s1 = 0

Let s2 = 0

Let a1 = 0

Let a2 = 0

Step 4. Let s1 = s1+1

Let s2 = s1

Step 5. Let s2 = s2+1

Step 6. Let a1 = a1+1

Step 7. Let a2 = a2+1

Step 8.

i. Calculate the cost of removing the arcs a1 and a2 from the subtours s1 and s2, respectively and including the arcs a3 and a4 in the tour.

$CM_{t-s1s2}^{a3a4-a1a2} = c^3 + c^4 - c^1 - c^2$

ii. Calculate the cost of removing the arcs a1 and a2 from the subtours s1 and s2, respectively and including the arcs a5 and a6 in the tour.

$CM_{s1s2}^{a5a6-a1a2} = c^5 + c^6 - c^1 - c^2$

* Let CM^t_{min} = Min (CM^t_{min}, $CM_{s1s2}^{a3a4-a1a2}$, $CM_{s1s2}^{a5a6-a1a2}$)

* If $CM_{s1s2}^{a3a4-a1a2}$ and $CM_{s1s2}^{a5a6-a1a2}$ are equal for s1 and s2 that are currently under consideration, the value to keep as the minimum cost to connect s1 and s2 (i.e., CM^t_{min}) is selected randomly out of the two possible schemes.

Step 9. If a2 $\neq$ m_{ts2} go to Step 7, else a2 = 0.

Step 10. If a1 $\neq$ m_{ts1} go to Step 6, else a1, a2 = 0.

Step 11. If s2 $\neq$ n_t go to Step 5, else a1, a2 = 0.

Step 12. If s1 $\neq$ n_{t-1} go to Step 4, else a1, a2 = 0.

Step 13. Merge the subtour pair with the least CM^t_{min} and go to Step 2 until a single tour is obtained for period t.

Step 14. Calculate the transportation cost incurred in period t. Go to Step 1 until t is the last period in the problem.

Step 15. Calculate the total transportation cost for the solution obtained.

4.1.5 Lagrangean Relaxation Solution Procedure

By using LRP in cooperation with the nearest merger heuristic (LRP-NM), a lower bound and an upper bound for the IRDOP are obtained. The procedure is based on making successive iterations for LRP by updating the lagrangean multipliers to obtain a lower bound and calling NM at a predetermined frequency to obtain an upper bound.

[LRP-NM]

Initialization.

I1. Initialize the parameters.

$\pi = 2$ (Step size parameter)

$\lambda_t^{ij} = 0 \ \forall i \in R, \forall j \in R, \forall t \in T$ (Lagrangean multipliers)

Z_{UB} = Heuristic method 'every' is used (Initial upper bound).

Z_{max} = A small, possibly negative, number (Maximum lower bound).

I_{max} = Used if a limit is required for the number of iterations (Maximum iterations).

I_{No} = 1 (Number of iterations)

m = Number of successive LRP-MIP iterations to be performed before finding an upper bound with NM (Frequency).

I2. Solve LRP with initial set of multipliers and obtain a lower bound.

Let, $Z_{max} = \max(Z_{max}, Z_{LB})$

Step 1. Calculate subgradients for the relaxed subtour elimination constraints as below.

$$s_t^{ij} = U_t^i - U_t^j + C * Z_t^{ij} + X_t^j - C \qquad \forall (i,j) \in R, \forall t \in T$$

Step 2. Identify the step size as follows.

$$T = \frac{\pi\left(Z_{UB} - Z_{LB}\right)}{\sum_{t \in T} \sum_{j \in R} \sum_{i \in R} \left(s_t^{ij}\right)^2}$$

Step 3. Update the lagrangean multipliers as seen below.

$$\lambda_t^{ij} = \max\left(0, \lambda_t^{ij} + Ts_t^{ij}\right) \qquad \forall i \in R, \forall j \in R, \forall t \in T$$

Step 4. Let, $I_{No} = I_{No} + 1$

Step 5. Solve LRP with current set of multipliers, obtain the total cost incurred (Z_{LB}), and update the lower bound as below.

Let, $Z_{max} = \max(Z_{max}, Z_{LB})$ and update π according to the scheme used.

Step 6. At every m iterations, identify a feasible solution for the IRDOP by converting the subtours into a tour for each period with NM and obtain an upper bound. Compute the total cost (Z_{UB}) using the transportation cost given by NM and the inventory holding cost given by the last solution obtained for LRP at Step 5 and update the upper bound as below.

Let, $Z_{min} = \min(Z_{min}, Z_{UB})$

Step 7. Go to step 1 until $(\pi < 0.005)$ or ($Z_{max} = Z_{min}$) or (INo = I_{max}).

When the procedure terminates, the final value of Z_{max} defines the maximum (best) lower bound obtained for the IRDOP. Likewise, the final value of Z_{min} is the minimum (best) upper bound obtained for the IRDOP.

The lower and upper bound updating scheme mentioned above is used to obtain lower and upper bounds for small-sized problems. However, solving LRP optimally is not possible for large-scale problems because too much time is needed to solve it. Thus, another solution methodology is required for large-scale problems, which is described in the next section.

4.1.6 Lagrangean Relaxation Solution Procedure for Large Problems

Since solving LRP optimally may not be affordable for large-size problems, LP relaxation of LRP (LRP-LP) is solved to obtain lower bounds for these problems. However, the decision variables that identify the routes executed will most probably be noninteger in the solution provided by LRP-LP. Thus, it is not possible to apply NM over the solution of LRP-LP. For this reason, to obtain upper bounds for large-scale problems, LRP is run until the first integer feasible solution is identified with CPLEX and NM is applied over this solution (NM-1).

It is apparent that LP relaxation of a lagrangean relaxation formulation can not provide a better quality lower bound than the solution provided by LP relaxation of the original problem (without lagrangean relaxation). However, due to enhancements included in LRP with the usages of (4.19), (4.20), (4.21), (4.22), and (4.23); it may be possible to obtain better bounds by LRP-LP than the LP relaxation of IM. In fact, as it will be mentioned in the computational results, it is observed during the numerical experiments that these constraints happen to be more restrictive than the subtour elimination constraints in the LP relaxation of the formulation.

The lower bound and upper bound updating procedure is modified as follows for large scale problems (LRP-NM-1).

[LRP-NM-1]

Step 1. Solve LRP-LP with zero lagrangean multipliers ($\lambda_t^{ij} = 0,\ \forall i \in R,\ \forall j \in R,\ \forall t \in T$) and obtain a lower bound for the IRDOP.

Step 2. Solve LRP with zero lagrangean multipliers ($\lambda_t^{ij} = 0,\ \forall i \in R, \forall j \in R, \forall t \in T$) until the first integer solution is obtained.

Step 3. Using the first integer solution obtained at Step 2, identify a feasible solution for the problem by NM.

Step 4. Determine an upper bound for the IRDOP by computing the total cost using the transportation cost given by NM and the inventory holding cost given by the first integer feasible solution of LRP at Step 3.

The reason why the lagrangean multipliers are set at zero in this procedure needs to be clarified. It is seen during the first set of preliminary experiments that the best lower bound with LRP-LP is identified in the first iteration when all of the lagrangean multipliers are equal to zero. When the subtour elimination constraints are removed from IM and the enhancements given in Section 4.1.2 are included in IM, an equivalent formulation to LRP with zero lagrangean multipliers is obtained. Accordingly, LRP-LP with zero lagrangean multipliers corresponds to the LP relaxation of IM, when the enhancements are included in IM and the subtour elimination constraints are removed from IM. It is known that removing a constraint entirely from a formulation causes the feasible region of the problem to enlarge if the detached constraints are binding, or to remain the same if they are nonbinding. Therefore, the LP relaxation of IM without the subtour elimination constraints will provide a solution that is at least as good as the formulation including them. As a result, it is expected to obtain better solutions when the lagrangean multipliers are zero.

However, what needs to be discovered is the circumstance that makes it possible to obtain zero multipliers. To see why this is the case, the enhancements presented in Section 4.1.2 are inserted into IM and sensitivity analysis is performed for the LP relaxation of IM. It is observed in the solution to the LP relaxation of IM that the subtour elimination constraints happen to be nonbinding for 25 instances out of 30 tested (the instances that are used in the preliminary experiments), which means that the optimal dual variables (i.e., the lagrangean multipliers) associated with them are zero. In each of the remaining 5 instances, only two of the subtour elimination constraints are binding (i.e., only two of the lagrangean multipliers are nonzero). So, setting the lagrangean multipliers at zero in LRP-LP appear to produce fine lower bounds.

Considering that it may not be affordable to perform numerous iterations for large-scale problems and that the best lower bounds are identified mostly when the lagrangean multipliers are zero, the multipliers are set at zero and only one iteration is performed for LRP-LP in the procedure above. Since it is not possible to determine initial multipliers when solving LRP in a simple manner, they are set at zero, as well.

4.2 MIP Formulation with a Priori Tour

IM that is developed to obtain a solution for the IRDOP contains several NP-hard TSPs to be solved. Basic motivation for formulating the problem with a priori tour is diminishing the number of inherent TSPs that are solved. In IM, a TSP for each period, in which some of the retailers are visited, needs to be solved, whereas in the formulation with a priori tour, only one TSP is solved.

This approach is based on an a priori minimum cost tour that is identified once as a preprocessing step. An a priori tour that starts at the supplier, visits all retailers, and ends at the supplier is determined by solving a TSP optimally for small-sized problems and using software, called Concorde, to obtain the optimal tour for large-scale problems (*http://www.tsp.gatech.edu/concorde*).

A priori tour will be executed in each period, in which deliveries exist, with some modifications depending on the retailers that are visited in the related period. For this purpose, the precedence relationships of the visits to the retailers in a priori tour are fixed. For instance, let a priori least cost tour in a distribution system including five retailers be as in Figure 4.4 below.

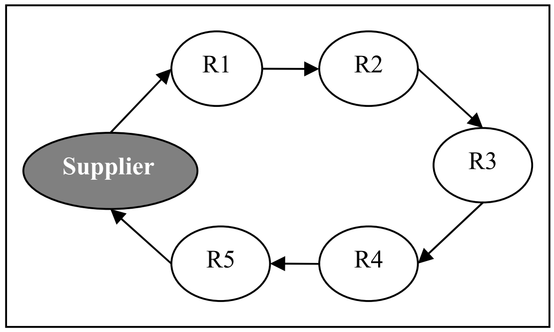

Figure 4.4 A priori tour for an illustrative distribution system

Letting P(X) denote the set of retailers that can take place prior to a visit to retailer X, the precedence relations are set as follows.

P(R2) = {R1}
P(R3) = {R1, R2}
P(R4) = {R1, R2, R3}
P(R5) = {R1, R2, R3, R4}

The precedence relationships above reveal the set of successors of a visit to retailer X, i.e., S(X), as given below, which are to be used in the formulation, as well.

S(R1) = {R2, R3, R4, R5}
S(R2) = {R3, R4, R5}
S(R3) = {R4, R5}
S(R4) = {R5}

In addition to the relations given above, it must be remarked that a direct visit is possible from the supplier to each retailer and from each retailer to the supplier. To give an example for the usage of these relationships, suppose that retailer 2 is to be visited in a period. The visit that can take place before a visit to retailer 2 is a visit to retailer 1, i.e., P(R2) = {R1}, or else a direct trip can be executed from the supplier to the retailer 2. Likewise, the visits that can take place following a visit to retailer 2 are visits to retailer 3, 4, or 5, i.e., S(R2) = {R3, R4, R5}, or else a direct trip from retailer 2 to the supplier can be carried out.

After the predecessor and successor relationships are fixed as above, the retailers to be visited in each period and the amount of product to be delivered during the visits are determined considering these relations. According to the relationships expressed above for the illustrative distribution system in Figure 4.4 and assuming that retailers 2, 4, and 5 are visited in a period in the planning horizon, the delivery route in that period will be as seen in Figure 4.5.

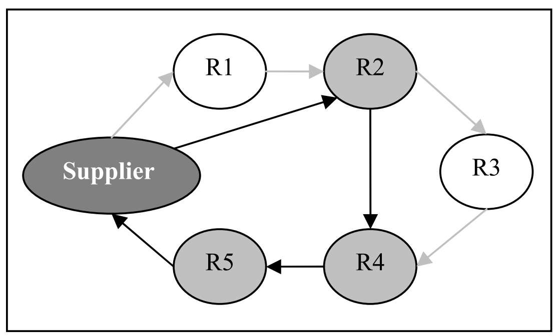

Figure 4.5 The tour to be executed according to the illustrative distribution system when retailers 2, 4, and 5 are visited in a period

An advantage of this approach is that the decisions related with obtaining the least cost tour and the least cost inventory policy in each period are separated to some extent.

4.2.1 Basic Formulation

The problem with a priori tour can be formulated as Traveling Salesman Problem with Profits as in Feillet, Dejax and Gendreau (2002), where there exist revenue and cost associated with visiting each retailer and the retailers to be visited are selected so as to maximize the revenue minus the cost, i.e., the profit, on a given a priori tour τ containing all the retailers.

Sets

R: Set of retailers, R = {1, 2, 3, …, N}

T: Set of periods in the planning horizon, T = {1, 2, 3, …, T}

R_0: $R \cup \{0\}$

S(i): Set of retailers that are allowed to be visited after retailer i $(i \in R)$ on route τ. Direct visit to the supplier is also included in the set, i.e., $(S(i) \subset R_0)$

P(i): Set of retailers that are allowed to be visited before retailer i $(i \in R)$ on route τ. Direct visit from the supplier is also included in the set, i.e., $(P(i) \subset R_0)$

Parameters

r_t^i: Revenue associated with visiting retailer i in period t

c^{ij}: Transportation cost incurred whenever j is visited after i

Decision Variables

$$Y_t^i : \begin{cases} 1 & \text{if retailer i is visited in period t} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_t^{ij} : \begin{cases} 1 & \text{if i is visited after j in period t} \\ 0 & \text{otherwise} \end{cases}$$

[Traveling Salesman Problem with Profits]

$$\text{Maximize } \sum_{t \in T} \sum_{i \in R} r_t^i Y_t^i - \sum_{t \in T} \sum_{j \in R_0} \sum_{i \in R_0} c^{ij} Z_t^{ij} \tag{4.24}$$

Subject to

$$\sum_{j \in S(i)} Z_t^{ij} = Y_t^i \qquad \forall i \in R, \forall t \in T \qquad (4.25)$$

$$\sum_{j \in P(i)} Z_t^{ji} = Y_t^i \qquad \forall i \in R, \forall t \in T \qquad (4.26)$$

$$\sum_{i \in R} Z_t^{i0} \leq 1 \qquad \forall t \in T \qquad (4.27)$$

$$\sum_{i \in R} Z_t^{0i} \leq 1 \qquad \forall t \in T \qquad (4.28)$$

$$Y_t^i \in (0, 1) \qquad \forall i \in R, \forall t \in T \qquad (4.29)$$

$$Z_t^{ij} \in (0, 1) \qquad \forall i \in R_0, \forall j \in R_0; i \neq j, \forall t \in T \qquad (4.30)$$

In the formulation above, the first term in the objective function (4.24) is the revenue earned when a retailer is visited, whereas the second term is the cost incurred depending on the route followed. Thus, the objective is maximizing the profit associated with visits to the retailers.

Constraints (4.25) and (4.26) ensure that if a retailer is visited in a period, it is visited before and after the predetermined successors and predecessors of it on a priori tour τ, respectively.

Constraints (4.27) and (4.28) make sure that there is at most one entrance to and one exit from the supplier in each period, respectively. These constraint sets are included in the formulation to eliminate possible subtours. Unless these restrictions are imposed, there may be several subtours in any period, each starting from the supplier, visiting some of the retailers, and ending at the supplier.

Constraints (4.29) and (4.30) are the integrality restrictions on the related decision variables.

The difference of the formulation we provided from the formulation in Feillet et al. (2002) is that our formulation is based on the precedence relationships that are determined according to the least cost a priori tour obtained. We use these precedence relationships in constraints (4.25) and (4.26) to restrict the tours that can be executed. Feillet et al. (2002), on the other hand, use no advance information about the tour and seek the best possible tour. In the formulation provided by Feillet et al. (2002), the visit that precedes or succeeds a visit to a customer can be made to any customer, whereas we require the precedence relationships given by a priori tour to be satisfied.

4.2.2 The Formulation with a Priori Tour (APT) for the IRDOP

Sets

R: Set of retailers, R = {1, 2, 3, …, N}

T: Set of periods in the planning horizon, T = {1, 2, 3, …, T}

R_0: $R \cup \{0\}$

H: $T \cup \{T+1\}$

S(i): Set of retailers that are allowed to be visited after retailer i $(i \in R)$ on the route τ. Direct visit to the supplier is also included in the set, i.e., $(S(i) \subset R_0)$

P(i): Set of retailers that are allowed to be visited before retailer i $(i \in R)$ on the route τ. Direct visit from the supplier is also included in the set, i.e., $(P(i) \subset R_0)$

Parameters

h^i : Unit inventory holding cost at retailer i in each period

hs : Unit inventory holding cost at the supplier in each period

c^{ij} : Transportation cost incurred whenever j is visited after i

d_t^i : Demand at retailer i in period t

$\underline{S}^i$: Minimum inventory level at retailer i

$\overline{S}^i$: Maximum inventory level at retailer i

C : Capacity of the vehicle

a_t : Amount of product that becomes available at the supplier in period t

I_1^i : Initial inventory level at retailer i at the beginning of period 1

Is_1 : Initial inventory level at retailer i at the beginning of period 1

Decision Variables

X_t^i : Amount of product delivered to retailer i in period t

I_t^i : Inventory kept at retailer i at the beginning of period t

Is_t : Inventory kept at the supplier at the beginning of period t

$$Y_t^i : \begin{cases} 1 & \text{if retailer i is visited in period t} \\ 0 & \text{otherwise} \end{cases}$$

$$Z_t^{ij} : \begin{cases} 1 & \text{if i is visited after j in period t} \\ 0 & \text{otherwise} \end{cases}$$

[APT]

$$\text{Minimize } \sum_{i \in R} \sum_{t \in H} h^i I_t^i + \sum_{t \in H} hs Is_t + \sum_{i \in R_0} \sum_{j \in R_0} \sum_{t \in T} c^{ij} Z_t^{ij} \tag{4.31}$$

Subject to

$$I_{t-1}^i + X_{t-1}^i - I_t^i = d_{t-1}^i \qquad \forall i \in R, t = 2,...,T+1 \tag{4.32}$$

$$X_t^i \le MY_t^i \qquad \forall i \in R, \forall t \in T \tag{4.33}$$

$$\sum_{i \in R} X_t^i \le C \qquad \forall t \in T \tag{4.34}$$

$$\sum_{i \in R} X_t^i \le Is_t \qquad \forall t \in T \tag{4.35}$$

$$I_t^i \ge \underline{S^i} \qquad \forall i \in R, \forall t \in H \tag{4.36}$$

$$I_t^i + X_t^i \le \overline{S^i} \qquad \forall i \in R, \forall t \in T \tag{4.37}$$

$$\overline{S^i} - X_t^i - I_t^i \le M\left(1 - Y_t^i\right) \qquad \forall i \in R, \forall t \in T \tag{4.38}$$

$$Is_t + \sum_{i \in R} X_{t-1}^i - Is_{t-1} = a_{t-1} \qquad t = 2,...,T+1 \qquad (4.39)$$

$$\sum_{j \in S(i)} Z_t^{ij} = Y_t^i \qquad \forall i \in R, \forall t \in T \qquad (4.40)$$

$$\sum_{j \in P(i)} Z_t^{ji} = Y_t^i \qquad \forall i \in R, \forall t \in T \qquad (4.41)$$

$$\sum_{i \in R} Z_t^{i0} \leq 1 \qquad \forall t \in T \qquad (4.42)$$

$$\sum_{i \in R} Z_t^{0i} \leq 1 \qquad \forall t \in T \qquad (4.43)$$

$$X_t^i \geq 0 \qquad \forall i \in R, \forall t \in T \qquad (4.44)$$

$$Is_t \geq 0 \qquad \forall t \in H \qquad (4.45)$$

$$Y_t^i \in (0, 1) \qquad \forall i \in R, \forall t \in T \qquad (4.46)$$

$$Z_t^{ij} \in (0, 1) \qquad \forall i \in R_0, \forall j \in R_0; i \neq j, \forall t \in T \qquad (4.47)$$

The formulation above includes the same objective function and restrictions with IM except for the removal of the subtour elimination constraints given by (3.12), (3.13), and (3.14). Instead of these three constraints, constraints (4.40), (4.41), (4.42), and (4.43) are included in the formulation.

The restrictions on the precedence relations are satisfied with the inclusion of constraints (4.40) and (4.41). Constraint (4.40) is for ensuring that if a retailer is visited in a period, it is visited before the nodes that are successors of it on a priori tour τ. Similarly, constraint (4.41) enforces that if a retailer is visited in a period, it is visited after the nodes that are predecessors of it on a priori tour τ.

Constraints (4.42), and (4.43) help prevent occurrence of subtours. Constraint (4.42) guarantees that there is at most one entrance to the supplier in each period. Likewise, constraint (4.43) enforces that there is at most one exit from the supplier in each period. If these constraints are not included in the formulation, the visits to the retailers may be split into subtours, each of which start from the supplier, visits some of the retailers, and ends at the supplier.

Solution of APT identifies the retailers to be visited, the amount of product to be delivered to these retailers, and the route to be executed to make the deliveries in each period according to the precedence relationships determined with a priori tour τ. Since APT formulation is more restrictive than IM, a solution for APT provides an upper bound for IM.

It is for sure that if the difference between the number of retailers that are visited in a period and the total number of retailers is large, the order given by a priori tour to visit the retailers may be considerably different from the minimum cost tour to visit these retailers. Although the formulation of APT is valid under any kind of distance assumption, i.e., symmetrical, unsymmetrical, Euclidean, and so on, the distances are symmetrical and the triangle inequality holds in the numerical experiments. The triangle inequality assumption may give rise to obtaining satisfactory results with the order given by a priori tour since skipping retailer i that is not visited in a period results in a direct visit from the retailer that precedes i to the retailer that succeeds i, which is less expensive than the case that retailer i is visited between them.

4.2.3 Improvements

Although a priori tour determined is the minimum cost tour that visits all the retailers, the route executed in each period according to the precedence relationships need not be the least cost tour to make the deliveries to a subset of retailers that are visited in the related period. Therefore, an improvement step is inserted into the procedure which identifies the minimum cost tour for each period that contains the retailers that are visited according to the solution of APT. The steps of the entire procedure including the improvement stage are summarized below.

[The Procedure with APT]

Step1. Solve a TSP to obtain a priori minimum cost tour starting from the supplier, visiting all of the retailers, and ending at the supplier.

Step 2. Fix the precedence relationships given by a priori tour.

Step 3. Formulate and solve the model with a priori tour according to the precedence relations determined in Step 2.

Step 4. Solve a TSP for each period to identify the minimum cost tour including the retailers that are visited in that period.

Step 5. If any tour that is identified in Step 3 changes in Step 4, calculate the total cost which is composed of inventory holding costs incurred according to the deliveries determined in Step 3 and transportation costs incurred according to the tours obtained in Step 4. If none of the tours has changed, the total cost incurred is equal to the cost of the solution provided by APT in Step 3.

The improvement step causes an increase in the number of TSPs to be solved. However, a separate TSP is solved for each period in this improvement step, while all inherent TSPs are solved at the same time in IM. Furthermore, another concern for using APT has been stated as separating the decisions related with obtaining the least cost tour and the least cost inventory policy to some extent to simplify the problem to be solved. This purpose is reinforced by the usage of the improvement step.

CHAPTER 5

COMPUTATIONAL RESULTS

In this chapter, we give a brief explanation about our computational experiments, our purposes of performing them, and the environment we work in. In Section 5.1, we present the data generation scheme used for the test problems in three parts of experiments. The results obtained in each experiment are provided in Section 5.2.

The experiments start with a preliminary run set, in which a distribution system composed of 6 retailers and 6 periods is considered. The purpose of these runs is to obtain optimum solutions for IM and to examine the quality of the lower and upper bounds obtained with the methods we developed under different parameter settings.

The second part of preliminary experiments has been carried out considering a distribution system involving 8 retailers and 8 periods. Since it is not expected to identify optimal solution to IM in a reasonable time in this experiment, a time limit of 3600 CPU seconds is set when solving IM. Other methods are tested without time limitation. The aim of the runs performed with this problem set is to gain an insight into the changes in the performances of the lower and upper bound identifying procedures as the problem size slightly enlarges.

The last part of experiments (main experiment) performed involves test problems with 50 retailers and 30 time periods. These runs are carried out to test the performances of the lower and upper bound obtaining procedures in large-scale problems.

All models and the interactions within them are coded with Turbo Pascal 7.0 and solved by CPLEX 8.1.0. All runs are carried out on Pentium IV 1.6 GHz. PC's with 256 MB RAM that run Windows NT Workstation 4.0.

5.1 Test Problems

The preliminary test problems are produced according to the generation scheme used in Bertazzi et al. (2002) with the exceptions of the number of retailers, the number of periods, and the parameter (g^i) that depends on the number of periods considered in our instances.

For conducting the main experiments with 50 retailers and 30 periods, a selection from the data used in Bertazzi et al. (2002), which is provided by the authors, is utilized.

Ranges of the parameters used in different experiments are as given in Table 5.1.

Table 5.1 The distinctive parameters used in the experiments

	Preliminary Experiment I	Preliminary Experiment II	Main Experiment
Number of retailers	6	8	50
Time horizon	6	8	30
Retailer holding cost (h^i)	[0.6, 1]	[0.6, 1]	[0.1, 0.5]; [0.6, 1]
Supplier holding cost (hs)	0.3	0.3	0.3; 0.8
Coordinates (x_i, y_i)	[0, 500]	[0, 500]	[0, 1000]
No. of periods to consume $\overline{S^i} - \underline{S^i}$ (g_i)	{2, 3, 5, 6}	{2, 3, 5, 6}	{2, 3, 5, 6, 10, 15, 30}
Vehicle capacity (C)	$\sum_{i \in R} d^i$	$\sum_{i \in R} d^i$	$\sum_{i \in R} d^i$; $3\sum_{i \in R} d^i$

The parameters of the test instances are generated as follows:

h^i : Randomly generated in the intervals seen in Table 5.1 according to uniform distribution.

c^{ij} : Calculated as $\left\lfloor \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \right\rfloor$, where (x_i, y_i) and (x_j, y_j) are randomly generated in the intervals seen in Table 5.1 according to uniform distribution.

d_t^i : Randomly generated as an integer number in the interval [10, 100] according to uniform distribution, and assumed to be constant over time in the experiments (i.e., $d_t^i = d^i$).

$\underline{S}^i$: Randomly generated as an integer number in the interval [50, 150] according to uniform distribution. Recall that this parameter is dropped from the formulation during the experiment phases (see Section 3.2).

$\overline{S}^i$: Calculated as $\underline{S}^i + d^i g^i$, where the parameter g^i, which represents the number of periods needed for retailer i to consume the amount $\overline{S}^i - \underline{S}^i$, is randomly selected from the sets given in Table 5.1. As mentioned in Section 3.2, this parameter is modified since the minimum inventory level is dropped from the formulation during the experiments.

a_t : Calculated as $\sum_{i \in R} d^i$ and assumed to be constant over time in the experiments.

I_1^i : Calculated as $\overline{S}^i - d^i$.

Is_1 : Calculated as $\sum_{i \in R} d^i$.

5.2 Results

In this section, the results obtained throughout the three parts of experiments are presented sequentially.

The abbreviations used in presenting the results are given below:

- CPU: It denotes the solution time (in CPU seconds) of the method under consideration.
- CPU-1: This denotes the average solution time (in CPU seconds), whenever more than one iteration is performed with the method under consideration.
- TC: It is the total cost incurred in the solution obtained with the method under consideration.
- LB: It is the lower bound value obtained with the method being discussed.
- UB: It is the upper bound value obtained by the method under consideration.
- IGap % = 100*(opt-LP)/opt: It is used, whenever the gap between the optimal solution value of a MIP formulation (denoted by opt) and its LP relaxation value, i.e., the integrality gap, is measured.
- LGap % = 100*(opt-LB)/opt: It measures the gap between the optimum solution value obtained with IM and the lower bound obtained by the method under study.
- UGap % = 100*(UB-opt)/opt: This statistic measures the gap between the optimum solution value obtained by IM and the upper bound obtained by the method used.
- LUB % = 100*(UB-LB)/LB: It is the percent difference between the upper bound and the lower bound that are obtained with the methods being studied.
- UDiff % = 100*(UB1-UB2)/UB2: It is used to compare the differences between the costs obtained by different upper bounding procedures.
- LDiff % = 100*(LB1-LB2)/LB1: It is used, whenever the differences between the costs obtained by different lower bounding procedures are compared.

Although the minimum inventory level is eliminated from the formulations during the experiments; the total costs, lower bounds, and the upper bounds given in the results include the inventory holding cost incurred on the minimum inventory level, as well. This is accomplished by adding the holding cost that results from the minimum inventory level ($\sum_{t \in H} \sum_{i \in R} h^i \underline{S}^i$) over the cost obtained from the solution of the problem with the method being discussed.

5.2.1 Results of Preliminary Experiment I (PE-I)

15 test instances are generated according to the scheme presented in Section 5.1.

5.2.1.1 Integrated Model

The instances are solved with the integrated model (IM) and optimal solutions are obtained for 14 instances out of 15 instances that are generated. The minimum total cost obtained by solving IM, the minimum total cost obtained by solving LP relaxation of IM, the times (in CPU seconds) required to solve the related models, and the integrality gap between the optimal solution of IM and its LP relaxation value are given in Table 5.2 below.

Table 5.2 Results for IM in PE-I

	IM		LP relaxation of IM		
Instance	TC	CPU	TC	CPU	IGap%
1	10588	288	7584	0.17	28.37
2	12633	35	9673	0.02	23.44
3	9831	3165	7167	0.02	27.09
4	11718	524	9094	0.02	22.40
5	8641	814	6177	0.02	28.52
*6	12664	31638	8337	0.02	-
7	12958	4958	9232	0.02	28.76
8	11947	53	8626	0.02	27.80
9	10021	107	7500	0.02	25.16
10	11345	59984	7849	0.02	30.82
11	11057	53	8778	0.02	20.61
12	9566	110	7459	0.02	22.03
13	10839	2213	7705	0.02	28.91
14	11646	1428	8415	0.02	27.74
15	9619	4580	7000	0.02	27.23
Average		7330		0.03	26.35

* Instance 6 could not be solved optimally due to memory limitations.

Instance 6 could not be solved optimally with the integrated model due to memory limitation of the PC used. In fact, the features of the data used in instance 6 require frequent visits to all retailers, when compared to remaining 14 instances, which make it hard to obtain feasible solution for this instance due to capacity limitations.

The solution times that average to 7330 CPU seconds for the MIP formulation decrease to less than 1 CPU second in all instances with the LP relaxation. The average solution time is more than 2 CPU hours for IM, which is a sign of the fact that we may not be able obtain optimal solutions for IM in reasonable times for larger problems.

The LP relaxation of IM provides solutions with integrality gaps that range between 21% and 31%, and of 26% on average.

5.2.1.2 Lagrangean Relaxation Solution Procedure

As mentioned in Section 4.1.2, LRP does not satisfy the integrality property. Thus, we expect the quality of the lower bounds provided with LRP to be no worse than the lower bounds obtained by the LP relaxation of IM.

During the first part of preliminary runs conducted with the lagrangean relaxation based approach (LRP-NM), effects of changing the following parameters on the quality of the bounds obtained are observed.

Let u be the parameter that denotes the update scheme for parameter π. Two basic schemes are employed for updating π as mentioned in Section 4.1.3. The first method is halving π, if Z_{max} does not alter for a certain number of iterations. The alternative values used for the number of iterations to halve π are u = 10, 15, and, 30. The second method is dividing π by 1.005 at each iteration. This setting is shown in the results as u = 1. However, it must be kept in mind that it does not indicate that π will be halved, if Z_{max} does not alter for one iteration.

Let m be the parameter that sets the number of iterations to be performed by LRP before identifying a feasible solution for the problem with nearest merger heuristic (NM). The alternative values that are tested for m are 5 and 10, which are given in the following results as m = 5 and m = 10, respectively.

Note that in the tables (Table 5.3 through Table 5.7) below, the solution times given in terms of CPU seconds denote the average total time to execute the lower and the upper bounding procedures (LRP-NM).

Instance 6 is given in the results that follow, as well. However, it must be kept in mind that it could not be solved optimally with IM and so, the total cost of the feasible solution provided by IM is greater than the optimum cost for this instance. Thus, the actual UGap% values are more than the figures given below, while actual LGap% values are less than the given figures for this instance.

Table 5.3 Results obtained with LRP-NM for u = 10, m = 5 and u = 10, m = 10 in PE-I

	u=10, m=5						u=10, m=10					
Instance	UB	LB	CPU-1	UGap%	LGap%	LUB%	UB	LB	CPU-1	UGap%	LGap%	LUB%
1	10722	9588	4.25	1.26	9.44	11.82	11071	9588	3.52	4.56	9.44	15.46
2	12857	12411	3.06	1.77	1.76	3.60	12857	12411	3.94	1.77	1.76	3.60
3	9964	9042	4.00	1.35	8.03	10.20	10087	9042	4.92	2.61	8.03	11.56
4	11855	10928	6.54	1.16	6.74	8.48	12179	10805	6.59	3.93	7.79	12.72
5	8669	8388	1.23	0.33	2.92	3.35	9228	8388	0.98	6.80	2.92	10.01
6	12785	11752	4.72	0.96	7.20	8.78	12714	11752	4.08	0.40	7.20	8.19
7	13051	12290	2.10	0.72	5.15	6.19	13051	12290	2.75	0.72	5.15	6.19
8	12335	10893	2.50	3.25	8.82	13.24	12489	10893	3.88	4.53	8.82	14.65
9	10269	9092	3.37	2.48	9.27	12.94	10166	9092	1.90	1.45	9.27	11.81
10	12302	10464	3.84	8.44	7.76	17.56	12370	10464	2.60	9.04	7.76	18.22
11	11141	10737	0.77	0.76	2.89	3.75	11487	10737	1.19	3.89	2.89	6.98
12	9673	8981	1.09	1.12	6.12	7.71	9920	8981	1.47	3.70	6.12	10.46
13	11319	9468	4.16	4.43	12.65	19.55	11328	9468	2.47	4.51	12.65	19.65
14	11646	10502	2.03	0.00	9.82	10.89	11919	10502	1.74	2.35	9.82	13.49
15	9731	9162	2.41	1.16	4.75	6.21	9661	9162	3.78	0.44	4.75	5.45
Average			3.07	1.95	6.89	9.62			3.05	3.38	6.96	11.23

Table 5.4 Results obtained with LRP-NM for u = 15, m = 5 and u = 15, m = 10 in PE-I

	u=15, m=5						u=15, m=10					
Instance	UB	LB	CPU-1	UGap%	LGap%	LUB%	UB	LB	CPU-1	UGap%	LGap%	LUB%
1	10794	9752	3.54	1.95	7.89	10.69	10794	9588	4.33	1.95	9.44	12.58
2	12857	12411	3.53	1.77	1.76	3.60	12857	12411	3.54	1.77	1.76	3.60
3	9964	9042	4.25	1.35	8.03	10.20	10358	9042	5.35	5.36	8.03	14.55
4	11855	10941	6.92	1.16	6.63	8.35	12179	10805	5.91	3.93	7.79	12.72
5	8669	8388	1.27	0.33	2.92	3.35	8680	8388	1.15	0.46	2.92	3.48
6	12768	11898	5.14	0.83	6.04	7.31	12714	11752	5.34	0.40	7.20	8.19
7	13051	12290	2.24	0.72	5.15	6.19	13051	12290	2.60	0.72	5.15	6.19
8	11976	10893	2.72	0.24	8.82	9.94	11976	10893	4.05	0.24	8.82	9.94
9	10193	9092	2.70	1.72	9.27	12.11	10166	9092	2.57	1.45	9.27	11.81
10	12302	10464	3.97	8.44	7.76	17.56	12302	10545	3.97	8.44	7.05	16.67
11	11141	10755	0.77	0.76	2.73	3.59	11340	10737	1.26	2.57	2.89	5.62
12	9673	8981	1.17	1.12	6.12	7.71	9756	8981	1.70	1.99	6.12	8.64
13	11319	9521	3.57	4.43	12.16	18.89	11319	9468	3.11	4.43	12.65	19.55
14	11646	10565	1.70	0.00	9.28	10.23	11646	10502	1.79	0.00	9.82	10.89
15	9731	9162	2.78	1.16	4.75	6.21	9661	9162	3.53	0.44	4.75	5.45
Average			3.08	1.73	6.62	9.06			3.35	2.28	6.91	9.99

Table 5.5 Results obtained with LRP-NM for u = 30, m = 5 and u = 30, m = 10 in PE-I

	u=30, m=5						u=30, m=10					
Instance	UB	LB	CPU-1	UGap%	LGap%	LUB%	UB	LB	CPU-1	UGap%	LGap%	LUB%
1	10794	9755	3.19	1.95	7.87	10.66	10794	9752	3.15	1.95	7.90	10.69
2	12857	12411	3.53	1.77	1.76	3.60	12937	12418	3.30	2.40	1.71	4.18
3	9964	9046	4.52	1.35	7.98	10.15	10317	9046	4.45	4.94	7.99	14.05
4	11855	10942	6.53	1.16	6.62	8.34	12179	10937	6.33	3.93	6.67	11.35
5	8669	8500	1.49	0.33	1.63	2.00	8680	8388	1.40	0.46	2.92	3.48
6	12768	11898	5.15	0.83	6.04	7.31	12714	11897	5.14	0.40	6.05	6.87
7	13051	12314	2.92	0.72	4.97	5.98	13051	12315	2.83	0.72	4.96	5.97
8	11976	10954	2.19	0.24	8.31	9.33	11976	10953	2.40	0.24	8.32	9.33
9	10193	9125	2.45	1.72	8.95	11.71	10166	9102	2.45	1.45	9.17	11.69
10	12302	10543	4.15	8.44	7.07	16.69	12302	10544	4.14	8.44	7.06	16.67
11	11141	10757	0.79	0.76	2.71	3.57	11141	10757	0.91	0.76	2.71	3.56
12	9673	8987	1.34	1.12	6.05	7.63	9673	8988	1.51	1.12	6.04	7.62
13	11319	9518	3.52	4.43	12.19	18.93	11319	9517	3.43	4.43	12.19	18.93
14	11646	10579	1.59	0.00	9.16	10.09	11646	10581	1.41	0.00	9.14	10.06
15	9852	9193	3.55	2.42	4.43	7.16	9661	9162	3.38	0.44	4.75	5.45
Average			3.13	1.82	6.38	8.88			3.08	2.11	6.50	9.33

Table 5.6 Results obtained with LRP-NM for u = 1, m = 5 and u = 1, m = 10 in PE-I

	u=1, m=5						u=1, m=10					
Instance	UB	LB	CPU-1	UGap%	LGap%	LUB%	UB	LB	CPU-1	UGap%	LGap%	LUB%
1	10649	9744	3.23	0.58	7.97	9.29	10794	9740	3.06	1.95	8.01	10.82
2	12857	12413	3.26	1.77	1.74	3.57	12937	12413	3.26	2.40	1.74	4.21
3	9964	9042	4.13	1.35	8.03	10.20	10100	9042	4.58	2.74	8.03	11.70
4	11855	10940	6.62	1.16	6.64	8.36	12179	10941	6.39	3.93	6.64	11.32
5	8669	8499	1.48	0.33	1.64	2.01	8669	8497	1.53	0.33	1.66	2.02
6	12714	11882	4.99	0.40	6.17	7.01	12743	11880	5.02	0.63	6.19	7.26
7	13051	12309	2.78	0.72	5.01	6.03	13051	12309	2.74	0.72	5.01	6.03
8	11976	10944	2.17	0.24	8.39	9.42	11976	10945	2.34	0.24	8.39	9.42
9	10269	9110	2.41	2.48	9.09	12.73	10166	9112	2.30	1.45	9.07	11.57
10	12302	10525	3.88	8.44	7.23	16.88	12302	10528	3.96	8.44	7.20	16.85
11	11141	10753	0.83	0.76	2.75	3.60	11141	10754	0.90	0.76	2.74	3.59
12	9673	8981	1.30	1.12	6.12	7.71	9673	8981	1.34	1.12	6.12	7.71
13	11319	9488	3.42	4.43	12.46	19.30	11319	9489	3.29	4.43	12.45	19.29
14	11646	10573	1.56	0.00	9.21	10.15	11646	10573	1.44	0.00	9.21	10.14
15	9852	9186	3.48	2.42	4.50	7.24	9661	9189	3.39	0.44	4.47	5.14
Average			3.04	1.75	6.46	8.90			3.04	1.97	6.46	9.14

It can be seen in the tables above that the upper bounds obtained for instance 14 are indeed the optimal solutions under all parameter settings except for the setting u=10 and m=10.

Table 5.7 Summary table for LRP-NM in PE-I

		u=10 m=5	u=10 m=10	u=15 m=5	u=15 m=10	u=30 m=5	u=30 m=10	u=1 m=5	u=1 m=10
CPU-1	Avg.	3.07	3.05	3.08	3.35	3.13	3.08	3.04	3.04
	Max.	6.54	6.59	6.92	5.91	6.53	6.33	6.62	6.39
	Min.	0.77	0.98	0.77	1.15	0.79	0.91	0.83	0.90
UGap %	Avg.	1.95	3.38	1.73	2.28	1.82	2.11	1.75	1.97
	Max.	8.44	9.04	8.44	8.44	8.44	8.44	8.44	8.44
	Min.	0.00	0.40	0.00	0.00	0.00	0.00	0.00	0.00
LGap %	Avg.	6.89	6.96	6.62	6.91	6.38	6.50	6.46	6.46
	Max.	12.65	12.65	12.16	12.65	12.19	12.19	12.46	12.45
	Min.	1.76	1.76	1.76	1.76	1.63	1.71	1.64	1.66
LUB %	Avg.	9.62	11.23	9.06	9.99	8.88	9.33	8.90	9.14
	Max.	19.55	19.65	18.89	19.55	18.93	18.93	19.30	19.29
	Min.	3.35	3.60	3.35	3.48	2.00	3.48	2.01	2.02

It can be seen that the average gap between the optimal solution and the upper bound ranges from 1.73% to 3.38%, whereas the average gap between the optimal solution and the lower bound ranges from 6.38% to 6.96%. Regarding the solution times that average to only about 3 CPU seconds, the performances of the upper bounding and the lower bounding procedures are satisfactory.

Table 5.7 shows that the quality of the upper bound improves when NM is applied at every 5 iterations (m=5) of LRP over the case in which it is applied at every 10 iterations (m=10). Thus, calling NM and identifying an upper bound more frequently results in getting better upper bounds, as it can be anticipated.

The frequency of identifying an upper bound also affects the lower bound obtained. Particularly, identifying an upper bound more frequently (m=5 rather than m=10) gives better lower bounds. In fact, this is also an expected outcome since the upper bound is utilized in the subgradient optimization step of the lower bounding procedure, which in turn affects the lower bound obtained.

Thus, it is observed that identifying an upper bound with NM at every 5 iterations (m=5) of LRP performs better than identifying an upper bound at every 10 iterations (m=10) of LRP in terms of both lower bound and upper bound qualities. Besides, since the CPU times do not seem to be affected much by the parameter settings, the frequency parameter m is decided to be set at 5.

It can be seen that the results obtained by halving π if the maximum lower bound does not change for 10 iterations (u = 10) are worse than the results of the other three update schemes in terms of the qualities of the lower and the upper bounds under a given setting of m.

The results show that the average gap between the lower bound and the optimal solution depends on updating schedule of the step size parameter π. The cases, in which π is halved at every 30 iterations (u=30) and π is divided by 1.005 at each iteration (u=1), give better results in terms of the average gap between the lower bound and the optimal solution. Updating π if the maximum lower bound does not alter for 30 iterations makes sense if numerous (at least more than 30) iterations are performed. However, for large test instances, it may not be affordable to perform even 30 iterations.

Moreover, u=15 and u=1 provide better upper bounds than u=10 and u=30. Although setting u at 30 is still meaningful, u will be set at two values, 15 and 1, in the succeeding experiments on LRP-NM method because of time concern.

Thus, to summarize, the levels to be used in solving larger scale test problems are selected as 15 and 1 for the step size parameter u and 5 for the frequency parameter m.

To get a better understanding, the average LGap%, UGap%, and LUB% are given graphically in Figure B.1 in Appendix B. The individual results can be seen in the figures given in Appendix C. Figure C.1 visualizes the gap between the upper bound and the lower bound for each instance. Figure C.2 (a) illustrates the gap between the optimal solution and the lower bound, whereas Figure C.2 (b) illustrates the gap between the optimal solution and the upper bound. It is seen in Figure C.2 (a) that the gap between the optimal solution and lower bound is highly dependent on the problem instance. However, the variation in lower bound for a specific problem is not too high under different parameter settings. Likewise, Figure C.2 (b) shows that the gap between the optimal solution and the upper bound depends on the instance examined. Moreover, there exists an obvious saw-tooth pattern in the results for some of the instances in Figure C.2 (b), which is an indication of the performance difference caused by the frequency that NM is utilized. It can be seen that Figure C.1 also has the saw-tooth pattern that results from the frequency of calling NM.

In the experimentations with LRP-NM, the stopping condition for the procedure is as stated in Section 4.1.5 and the maximum number of iterations is set at 200. The changes in lower and upper bounds are observed according to number of iterations performed to decide on maximum number of iterations to perform when larger problems are solved. To gain a better understanding on the changes in maximum lower bounds (Z_{max}) and minimum upper bounds (Z_{min}) that arise from the number of iterations performed, the costs in these figures do not involve the inventory holding costs that result from the minimum inventory levels. The related figures are provided for the selected parameter combinations below in Figure 5.1 and Figure 5.2, and for the other parameter settings in Appendix D.

Note that the number of iterations performed for the upper bounding heuristic NM depends on the number of iterations performed for LRP in addition to the frequency of identifying an upper bound, i.e., the parameter m. For instance, if m is set at 5 (NM will be called at every 5 iterations of LRP) and the maximum number of iterations to execute LRP is determined to be 200 as in the two figures above, then NM will be applied 40 times.

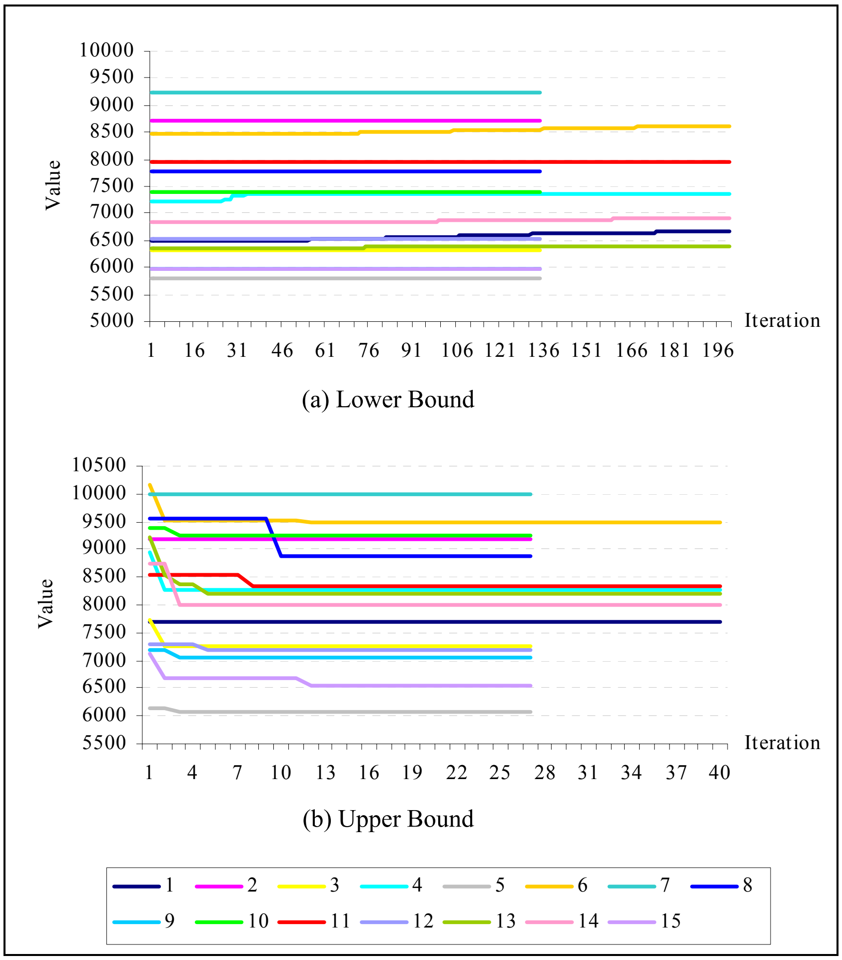

Figure 5.1 Lower and upper bounds for u = 15, m = 5 through iterations in PE-I for 15 instances

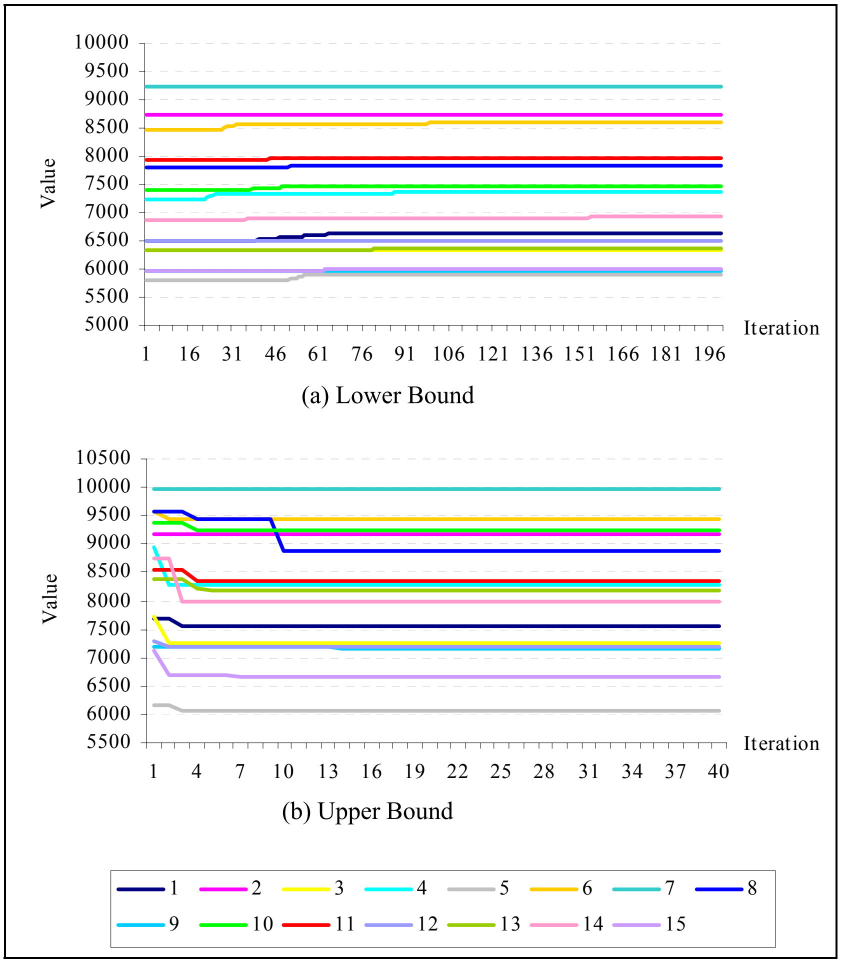

Figure 5.2 Lower and upper bounds for u = 1, m = 5 through iterations in PE-I for 15 instances

The figures above demonstrate that the lower bounds are getting only slightly better after several iterations. In addition to this, the upper bounds are almost fixed after several iterations. Taking into account that making numerous iterations is not affordable for large-size problems, it is decided to keep the maximum number of iterations at 30 when using LRP-NM for larger problems. This means that NM will be applied 6 times, because m is decided to be set at 5. It can be seen in Figure 5.1 and Figure 5.2 that 6 iterations of NM capture the changes in the upper bounds for most of the instances for the parameter settings given in these figures.

5.2.1.3 Lagrangean Relaxation Solution Procedure for Large Problems

Since LRP is expected to require a long solution time for large problems, lower bounds may be obtained by solving the LP relaxation of LRP (LRP-LP). Although we obtained lower bounds for this set of runs using LRP as given in Section 5.2.1.2, to have an idea about the quality of the lower bounds provided by LRP-LP, all runs are repeated with LRP-LP. Recall that LRP includes extra variables and constraints that are not present in IM. Thus, it may be possible to obtain solutions that are of better quality with the LP relaxation of LRP than that of IM.

As stated in Section 4.1.6, the lagrangean relaxation solution procedure for large problems (LRP-NM-1) is based on running LRP until the first integer feasible solution is identified by CPLEX and applying NM over this solution. Although upper bounds for this set of experiments are already obtained with LRP-NM as given in Section 5.2.1.2, new runs are performed to get an insight about the quality of the upper bounds obtained by LRP-NM-1. The results obtained with LRP-NM-1 are given in Table 5.8 below.

Table 5.8 Results obtained with LRP-NM-1 in PE-I

Instance	UB	LB	CPU	UGap%	LGap%	LUB%
1	13034	7922	0.24	23.10	25.18	64.52
2	12860	10113	0.31	1.79	19.95	27.15
3	10107	7524	0.24	2.81	23.46	34.32
4	13800	9500	0.24	17.77	18.93	45.27
5	9211	6708	0.07	6.60	22.37	37.32
*6	13876	9427	0.49	9.58	25.55	47.19
7	14208	9893	0.31	9.65	23.65	43.62
8	12222	9000	0.10	2.30	24.67	35.80
9	10913	7996	0.06	8.90	20.21	36.49
10	13295	8462	0.33	17.19	25.41	57.11
11	11467	9273	0.15	3.71	16.13	23.66
12	9894	7796	0.12	3.43	18.50	26.91
13	11936	8121	0.11	10.12	25.07	46.96
14	13597	8777	0.38	16.76	24.63	54.92
15	10315	7635	0.06	7.24	20.62	35.10
Average			0.21	9.40	22.29	41.09

* Instance 6 could not be solved optimally with the integrated model

It can be seen in Table 5.8 that the gap between the optimal solution and the lower bound ranges between 16% and 26%, while the gap between the optimal solution and the upper bound ranges between 2% and 23%. The solution times that involve the times to identify lower and upper bounds are less than 1 CPU second. Note that although the solution times are provided for 1 iteration, the statistic is denoted as CPU rather than CPU-1 in the table. The reason for this is that LRP-NM-1 procedure requires 1 iteration of LRP-LP (to obtain lower bound) and 1 iteration of LRP (to obtain upper bound).

LRP-NM method (with the selected parameter settings) has provided an average gap less than 7% between the optimum solution and the lower bound and an average gap less than 2% between the optimum solution and the upper bound. The solution times of LRP-NM were slightly above 3 CPU seconds on the average. As expected, a quality loss is experienced for the sake of decreasing the solution times when using LRP-NM-1 instead of LRP-NM.

5.2.1.4 The Procedure with a Priori Tour

Table 5.9 Results obtained with APT in PE-I

	APT			IM
Instance	UB	UGap%	CPU	CPU
1	10588	0.00	1.94	288
2	12633	0.00	1.54	35
3	9834	0.03	3.93	3165
4	11719	0.01	1.52	524
5	8641	0.00	0.69	814
*6	12657	-0.05	4.38	31638
7	12958	0.00	1.81	4958
8	11947	0.00	1.59	53
9	10021	0.00	0.90	107
10	11345	0.00	2.07	59984
11	11057	0.00	0.69	53
12	9567	0.01	1.09	110
13	10839	0.00	2.47	2213
14	11646	0.00	1.09	1428
15	9619	0.00	2.15	4580
Average			1.86	7330

* Instance 6 could not be solved optimally with the integrated model

The results obtained by solving the model with a priori tour (APT) are seen in Table 5.9 above.

The solution times of APT given in CPU seconds include the times consumed for all the steps of the procedure given in Section 4.2.3, i.e., solving a TSP to obtain a priori tour, solving APT, and solving a TSP for each period, if necessary. The CPU seconds required to solve the instances using IM are repeated in the table above to draw attention to the decrease in the solution times with the usage of APT.

Table 5.9 shows that the model with a priori tour identifies the optimum solution for eleven instances out of fifteen. While in three instances the gaps between the solutions of IM and the solutions given by APT are less than 0.03%, APT identifies a less cost solution for instance 6 in only about 4 CPU seconds, which can not be solved optimally with IM in almost 9 CPU hours.

Regarding the quality of the solutions provided by APT together with the solution times that range between 0.69 and 4.38 CPU seconds, solving APT can be preferred over solving IM, which requires more than 2 CPU hours on the average.

5.2.1.5 Benchmarking

It may not be possible to obtain optimal solutions of IM for larger problems. Following Bertazzi et al. (2002), the upper bounds identified for larger problems with the methods used in this study will be compared to the results provided by the heuristic ‘every’ (Refer to Appendix A).

For this purpose, the average gap between the solution given by IM and the solution obtained by ‘every’ is computed to have an idea on the performance of the upper bound provided by ‘every’.

Table 5.10 Gap between the solution of IM and the solution provided by 'every'

Instance	Every	UGap%
1	16858	59.22
2	19156	51.63
3	14288	45.33
4	17133	46.21
5	13655	58.03
*6	16035	26.62
7	19155	47.82
8	18814	57.48
9	14251	42.21
10	15735	38.70
11	17024	53.98
12	14842	55.16
13	16068	48.25
14	17341	48.90
15	14752	53.36
Average		48.86

* Instance 6 could not be solved optimally with the integrated model

The gap between the solutions of the integrated model and 'every' depends on the combination of the inventory holding costs at the retailers, the inventory holding cost at the supplier, the transportation costs, and the parameter g^i (i.e., the number of periods needed for retailer i to consume the amount for the difference between the maximum and minimum inventory levels). In general, if the holding cost at the supplier is high with respect to the holding costs at the retailers, the transportation cost incurred when executing the least cost tour that visits all retailers is low, and the parameter g^i is small for all retailers (i.e., frequent visits are required to the retailers), 'every' provides better solutions since it is based on visiting all retailers in each period with the least cost tour.

The gap between the solutions of IM and 'every' ranges between 27% and 59%. The least gap of 27% is obtained for instance 6. The one reason could be features of data of the instance 6, which require frequent visits to all retailers. Thus, the best solution would be close to the solution of 'every' (i.e., visiting all retailers in every period) in this respect. The other reason might be related with the fact that IM could not be solved at optimality for this instance. If the instance could have been solved optimally, the difference between the solutions of the integrated model and 'every' would have increased to some extent.

Although comparisons with optimal solutions of IM have been performed to assess the quality of the upper bounds for this part of experiments, the best upper bound obtained with our methods is compared with the solution given by the heuristic 'every' in the table below for convenience.

Table 5.11 Comparison of the best upper bounds obtained in PE-I with the solutions of 'every'

Instance	Every (UB1)	Best UB (UB2)	UDiff %
1	16858	10588	59.22
2	19156	12633	51.63
3	14288	9834	45.29
4	17133	11719	46.20
5	13655	8641	58.03
6	16035	12657	26.68
7	19155	12958	47.82
8	18814	11947	57.48
9	14251	10021	42.21
10	15735	11345	38.70
11	17024	11057	53.98
12	14842	9567	55.14
13	16068	10839	48.25
14	17341	11646	48.90
15	14752	9619	53.36
Average			48.86

The values given under the column 'best UB' in Table 5.11 are the minimum cost solutions among the solutions provided by LRP-NM with the selected parameter settings (u=15, m=5 and u=1, m=5) and by APT. A note to be made is that the minimum cost solutions happen to be the solutions of APT for all instances except instance 14, for which all methods provide the same solution that is optimal for IM. The difference between our upper bound and upper bound provided by 'every' ranges from 27% to 59% and our methods outperform 'every'.

5.2.2 Results of Preliminary Experiment II (PE-II)

5.2.2.1 Integrated Model

Solution times of IM have averaged to more than 2 CPU hours in PE-I. Thus, we do not expect to get optimal solutions for IM in reasonable times in PE-II due to the increase in the problem size. For this reason, a time limit of 3600 CPU seconds is used in CPLEX for the runs with IM in PE-II. The best integer feasible solutions identified for the instances by IM in the specified time limit are given in Table 5.12 below. Solutions of the LP relaxation of IM, the solution times and the integrality gaps are also presented in the table below.

Table 5.12 Results for IM in PE-II

	IM		LP relaxation of IM		
Instance	TC	CPU	TC	CPU	LUB%
1	30129	3600	19494	0.25	35.30
2	26686	3600	16886	0.04	36.72
3	23188	3600	13170	0.07	43.20
4	25185	3600	13543	0.06	46.22
5	25398	3600	14854	0.06	41.51
6	24486	3600	13616	0.06	44.39
7	18920	3600	12373	0.05	34.60
8	28988	3600	17407	0.07	39.95
9	29278	3600	17991	0.05	38.55
10	23462	3600	15717	0.04	33.01
11	25785	2593	19042	0.05	26.15
12	21855	3600	13371	0.05	38.82
13	28531	3600	17414	0.05	38.97
14	28348	3600	16065	0.06	43.33
15	19629	3600	11324	0.05	42.31
Average		3533		0.07	38.87

The optimum solution is identified only for instance 11 within the time limit. The number of instances, for which an optimal solution can not be determined in 3600 CPU seconds, has been four in PE-I, whereas it is fourteen for this experiment set.

Thus, it is possible to say that the solution times for IM increase with the sizes of the problems.

The statistic is denoted as LUB% in Table 5.12 since we do not have optimal solutions for 14 instances and thus the total costs for IM provide upper bounds rather than optimal solutions for these instances. The true integrality gap can only be examined for instance 11, which is solved optimally.

The gap between the total costs given by IM and its LP relaxation ranges from 26% to 46% with an average of 39%, whereas it has been 26% on average in PE-I. However, this is not necessarily an indication of wide integrality gaps since optimal solutions can not be identified for 14 instances in PE-II.

5.2.2.2 Lagrangean Relaxation Solution Procedure

During PE-I, it has been decided to perform 30 iterations of LRP for the problems that are of larger sizes and to use the parameter combinations u=15, m=5 and u=1, m=5 (see Section 5.2.1.2). Results for these parameters are presented in Table 5.13.

Table 5.13 Results obtained with LRP-NM for u = 15, m = 5 and u = 1, m = 5 in PE-II

	u=15, m=5				u=1, m=5			
Instance	UB	LB	CPU-1	LUB%	UB	LB	CPU-1	LUB%
1	28589	27276	213	4.81	28589	27270	184	4.84
2	27313	23804	157	14.74	27221	23808	171	14.34
3	22798	19534	234	16.71	22798	19535	230	16.70
4	25371	18291	407	38.71	25371	18291	339	38.71
5	23095	19479	1096	18.56	22996	19479	1117	18.06
6	22300	20846	174	6.98	22352	20844	153	7.24
7	19039	16935	87	12.42	18402	16937	81	8.65
8	29771	24284	612	22.59	29771	24274	589	22.64
9	27316	24744	520	10.39	27316	24709	506	10.55
10	23050	19810	111	16.35	23050	19809	109	16.36
11	26424	25291	124	4.48	26424	25286	116	4.50
12	21208	18130	217	16.98	21208	18130	196	16.98
13	28269	24469	1049	15.53	28269	24514	951	15.32
14	25776	21927	1020	17.55	25776	21916	906	17.61
15	19355	16129	38	20.00	19102	16103	40	18.63
Average			404	15.79			379	15.41

UGap% and LGap% statistics are not presented in the table above since 14 instances can not be solved optimally with IM. Besides, LRP-NM with parameter settings u=15 and u=1 provide solutions that are better than the solutions of IM for ten and eleven instances, respectively.

The gap between the upper bound and the lower bound averages to 16% and 15% when u=15 and u=1, respectively for this set of runs, while the solution times are close to 400 CPU seconds. They have been averaging to almost 9% with an average solution time of nearly 3 CPU seconds in PE-I. So, we can conclude that the qualities of the bounds deteriorate and that the solution times increase, as the problems are enlarged.

The stopping condition for the procedure is as stated in Section 4.1.5 and the maximum number of iterations is set at 30 as mentioned. The changes in maximum lower bounds (Z_{max}) and minimum upper bounds (Z_{min}) are observed according to number of iterations performed in the figures in Appendix E. As in PE-I, to gain a better understanding on the changes in upper and lower bounds that result from the number of iterations performed, the costs in these figures do not involve the inventory holding costs that are caused by the minimum inventory levels.

5.2.2.3 The Procedure with a Priori Tour

In PE-I, the TSPs are solved by Concorde. When the total costs obtained by IM and APT are compared, it is seen that APT identifies solutions with less costs than IM in all of the instances except 11, for which IM and APT identify the same optimal solution. Although the same solution is obtained for instance 11 with IM and APT, the solution times of the models are extremely different. While it takes 2593 CPU seconds for IM, APT achieves the same solution in only 30 CPU seconds. The results obtained by APT are seen in Table 5.14 .

Table 5.14 Results obtained with APT in PE-II

Instance	UB	CPU
1	28398	71.65
2	25781	18.89
3	21801	36.81
4	23802	287.65
5	22639	703.64
6	22188	54.49
7	18402	24.81
8	27455	59.18
9	27005	349.59
10	22332	22.49
11	25785	29.50
12	20825	22.26
13	26766	116.52
14	25085	146.72
15	18550	19.81
Average		130.93

The solution times of APT range between 19 and 704 CPU seconds. Since the solutions of IM are obtained with 3600 CPU seconds of time limit, except for instance 11 (solved in 2593 CPU seconds), the remark made for APT in PE-I can be restated in a more powerful way that APT is preferred to IM due to qualities of the solutions it provides in reasonable times.

5.2.2.4 Comparison of the Upper Bounds Obtained in PE-II

Different from PE-I, comparison of the upper bounds obtained by LRP-NM and APT is provided in Table 5.15 below since optimal solutions of IM can not be identified for most of the instances in PE-II.

The figures provided for LRP-NM in Table 5.15 are the upper bounds obtained when either u=1 or u=15, whichever is better.

Table 5.15 Comparison of the upper bounds obtained in PE-II

	LRP-NM		APT		
Instance	(UB1)	CPU	(UB2)	CPU	UDiff %
1	28589	184	28398	72	0.67
2	27221	171	25781	19	5.58
3	22798	230	21801	37	4.57
4	25371	339	23802	288	6.59
5	22996	1117	22639	704	1.58
6	22300	174	22188	54	0.51
7	18402	81	18402	25	0.00
8	29771	589	27455	59	8.43
9	27316	506	27005	350	1.15
10	23050	109	22332	22	3.21
11	26424	116	25785	30	2.48
12	21208	196	20825	22	1.84
13	28269	951	26766	117	5.62
14	25776	906	25085	147	2.76
15	19102	40	18550	20	2.98
Average		381		131	3.20

APT outperforms LRP-NM in terms of the quality of the upper bounds it provides and the solution times as in PE-I.

5.2.2.5 Benchmarking

The best upper bound obtained for each instance with the methods we developed is compared to the solution provided by ‘every’ in Table 5.16 below.

The values given under the column ‘best UB’ in the table below are the minimum cost solutions among the solutions provided by LRP-NM and APT. The minimum cost solutions are provided by APT for all instances except instance 7, for which both APT and LRP-NM with u=1 and m=5 provide the same solution. The gap between the solution of ‘every’ and the best solution obtained by our methods ranges from 37% to 103% and our methods clearly outperform ‘every’.

Table 5.16 Comparison of the best upper bounds obtained in PE-II with the solutions of 'every'

Instance	Every (UB1)	Best UB (UB2)	UDiff %
1	46236	28398	62.81
2	39775	25781	54.28
3	33077	21801	51.72
4	33934	23802	42.57
5	32254	22639	42.47
6	30328	22188	36.68
7	34648	18402	88.29
8	43186	27455	57.30
9	39855	27005	47.58
10	36886	22332	65.17
11	52244	25785	102.62
12	35585	20825	70.88
13	43003	26766	60.66
14	38120	25085	51.97
15	35207	18550	89.80
Average			61.65

5.2.3 Results of Main Experiment (ME)

In the main experiments, 80 out of the 240 instances used in Bertazzi et al. (2002) are tested. The index numbers associated with the instances are the numbers used by Bertazzi et al. (2002) for the related instances.

The instances that are tested in the main experiments differ in three parameters, namely, the holdings cost at retailers, the holding cost at supplier, and the vehicle capacity. Since each of these three parameters may assume two different levels (see Section 5.1), there are eight possible combinations of these parameters as given in Table 5.17 below. Note that H (for high) denotes the higher level value and L (for low) denotes the smaller one. For instance, H, L, and H in the column numbered 141-150 show that the retailer holding costs are in the range [0.6,1], the supplier holding cost is 0.3, and the vehicle capacity is $3\sum_{i\in R} d^i$. Similarly, L, H, and L in the column numbered 211-220 means that the retailer holding costs are in the range [0.1,0.5], the supplier holding cost is 0.8, and the vehicle capacity is $\sum_{i\in R} d^i$.

Note that the frequency of visiting retailer i is bounded below by a number that depends on the parameter g^i (i.e., the number of periods required for retailer i to consume the amount for the difference between the maximum and the minimum inventory levels). To be precise, retailer i should be visited at least once in every g^i periods for its inventory not to fall below the minimum level. Thus, as g^i gets smaller in value, retailer i must be visited more frequently. Another issue that has an effect on the frequency of visiting a retailer is the relationship between the holding costs at the retailer and the supplier. If the inventory holding cost at the supplier is high compared with the holding cost at a retailer, it may be advantageous to visit the related retailer more frequently.

Table 5.17 The characteristics of the instances used in ME

	121-130	141-150	151-160	171-180	181-190	201-210	211-220	231-240
Retailer Holding Cost	H	H	H	H	L	L	L	L
Supplier Holding Cost	L	L	H	H	L	L	H	H
Vehicle Capacity	L	H	L	H	L	H	L	H

Since the main experiments are performed for large instances, we do not expect to identify optimal solutions with the MIP formulations of the models. For this reason, when solving MIP formulations, finding feasible solutions is emphasized in CPLEX. On the other hand, LP relaxations of the models are run as usual with the default option of balancing optimality with feasibility.

Additional abbreviations used in the main experiments are given below:

- CPU w: It is the solution time (in CPU seconds) of the method under study with modified Barany et al. cuts.
- CPU w/o: It denotes the solution time (in CPU seconds) of the method considered without modified Barany et al. cuts.

- Imp% = 100*(TC with-TC without) / TC without: This statistic measures the improvement obtained in the lower bound with the method studied, when modified Barany et al. cuts are present in its formulation. TC with (TC without) cuts is the total cost obtained by the method under study with (without) Barany et al. cuts.

5.2.3.1 Integrated Model

Considering the observations made during the preliminary experiments, it is expected that solving IM optimally will require a long solution time. Therefore, IM is run with a time limit of 3600 CPU seconds. 6 instances out of 80, for which integer feasible solutions can be obtained, are given in Table 5.18 below.

Table 5.18 Feasible solutions obtained by IM in 3600 CPU seconds in ME

Instance	TC
141	954968
142	967293
145	789537
146	1020641
147	848527
149	962884

The LP relaxation of IM is solved as in the preliminary experiments. Additionally, modified Barany et al. cuts given with the relation (4.23) are inserted into IM and its LP relaxation is solved again to see whether we can obtain better lower bounds when these cuts are used. The individual results can be seen in Appendix F and the average results are given in Table 5.19 below.

Table 5.19 Results of the LP relaxation of IM without and with (w) modified Barany et al. cuts

Instance	CPU	CPU w	Imp%
121-130	10.64	11.84	0.068
141-150	10.80	11.94	0.068
151-160	23.88	26.25	0.005
171-180	24.39	25.59	0.005
181-190	23.38	25.95	0.013
201-210	24.30	27.08	0.013
211-220	71.87	80.13	0.000
231-240	73.05	77.87	0.000
Overall	32.79	35.83	0.021

It can be seen that equal improvements are obtained for each consequent two rows, i.e., 121-130 and 141-150; 151-160 and 171-180; and so on. Considering the couple 121-130 and 141-150, when the tables provided in Appendix F are examined, it is seen that identical total costs are obtained for 121 and 141; 122 and 142; through 130 and 150. The differences between the data of these couples arise from the vehicle capacities. This means that modifying the vehicle capacity between the two possible levels do not bring a change in the solutions obtained by the LP relaxation of IM.

It is seen in Table 5.19 that the LP relaxation of IM with the cuts give only slightly better results than the original formulation with almost 3 CPU seconds increase in the solution time. If these cuts had been used when solving the integrated model rather than its LP relaxation, it would have been expected to obtain more significant improvements in the solutions. However, the solution times, which are already not acceptable, would have increased, as well.

5.2.3.2 Lagrangean Relaxation Solution Procedure for Large Problems

LRP-LP is solved as in the preliminary experiments. In addition to this, to compare the performance of LRP-LP with the LP relaxation of IM, modified Barany et al. cuts (see relation 4.23) are removed from LRP-LP and it is solved again. The individual results for LRP-LP can be seen in Appendix G and the average results are given in Table 5.20 below.

Table 5.20 Results of LRP-LP without (w/o) and with modified Barany et al. cuts

Instance	CPU	CPU w/o	Imp %
121-130	11.54	2.55	3.846
141-150	8.80	2.62	3.818
151-160	11.57	5.00	0.229
171-180	8.40	4.64	0.229
181-190	11.13	4.87	0.613
201-210	8.43	4.72	0.613
211-220	10.62	6.98	0.004
231-240	9.57	7.26	0.004
Overall	10.01	4.83	1.169

Regarding the figures given in the table above, the cuts bring about 1% improvement with almost 5 CPU seconds increase in the average solution time. Overall improvement provided by the cuts in LRP-LP is greater than the overall improvement observed in the LP relaxation of IM with cuts.

Average results obtained with LP relaxation of IM and LRP with and without modified Barany et al. cuts are compared in Table 5.21 below. The results are provided in Appendix H for each instance. It is worthwhile to remind that LRP-LP can provide better results than the LP relaxation of IM due to enhancements (extra variables and constraints) included in its formulation (Refer to Section 4.1.2).

Table 5.21 Comparison of the LP relaxation results of IM and LRP

	Results without the cuts			Results with the cuts		
	CPU			CPU		
Instance	LRP-LP (LB1)	LP relaxation of IM (LB2)	LDiff %	LRP-LP (LB1)	LP relaxation of IM (LB2)	LDiff %
121-130	2.55	10.64	0.113	11.54	11.84	3.741
141-150	2.62	10.80	0.113	8.80	11.94	3.714
151-160	5.00	23.88	0.088	11.57	26.25	0.311
171-180	4.64	24.39	0.088	8.40	25.59	0.311
181-190	4.87	23.38	0.236	11.13	25.95	0.831
201-210	4.72	24.30	0.236	8.43	27.08	0.831
211-220	6.98	71.87	0.190	10.62	80.13	0.195
231-240	7.26	73.05	0.190	9.57	77.87	0.195
Average	4.83	32.79	0.157	10.01	35.83	1.266

It can be seen that LRP-LP provides better lower bounds than the LP relaxation of IM in overall with a reduction in solution time. This shows that the enhancements made in LRP formulation in Section 4.1.2 to preclude occurrences of unrealistic solutions, result in obtaining better lower bounds with LRP-LP. These enhancements have been needed in LRP formulation due to removal of the subtour elimination constraints. However, the results above show that these enhancements happen to be more restricting than the subtour elimination constraints in the LP relaxation of the formulations.

Although LRP has been extended to include modified Barany et al. cuts with relation (4.23), they are removed from the formulation of LRP since their inclusions increase the solution time extremely. Apart from this difference in the formulation of LRP, the procedure for LRP-NM-1 given in Section 4.1.6 is followed (i.e., LRP is run until the first integer solution is found and NM is applied over this solution).

To see whether improvements can be obtained by increasing the solution times for the instances, whose first integer feasible solutions are obtained in short times (less than 600 CPU seconds), the termination rule of finding the first integer feasible solution (see Section 4.1.6) is removed and a time limit of 600 CPU seconds is set. The results obtained by applying NM over the solutions got within the time limit of

600 CPU seconds for these instances are compared to the results got by applying NM over the first integer feasible solutions of the instances in Appendix I.

The gap between the lower bound obtained with LRP-LP (using the modified Barany et al. cuts) and the best upper bound obtained with LRP-NM-1 (either the first integer feasible solution obtained or the solution got in 600 CPU seconds, whichever is better) are given for each instance in Appendix J. The average figures are given in Table 5.22. It can be checked from Appendix J that there are instances, for which feasible integer solutions can not be identified with LRP-NM-1. These instances were stopped either because of memory limitations of the PC's used or the solution times over 120,000 CPU seconds.

Table 5.22 Results obtained with LRP-NM-1 in ME

	CPU		
Instance	LRP-NM-1	LRP-LP	LUB%
121-130	28684	12	28.04
141-150	600	9	25.59
151-160	26580	12	9.89
171-180	456	8	9.99
181-190	28799	11	27.30
201-210	501	8	25.62
211-220	41699	11	33.21
231-240	600	10	31.91
Overall	12452	10	23.52

Average gap between the lower and upper bounds for 8 problem classes range from 10% to 33% and the average solution time over the instances, for which integer feasible solutions are identified, is nearly 3.5 CPU hours. Considering the characteristics of the problem classes given in Table 5.17, it is seen that an increase in the vehicle capacity reduces the solution time remarkably (the classes 141-150, 171-180, 201-210, and 231-240), whereas it does not have a significant effect on the quality of the bounds obtained. In fact, when the individual results that are provided in Appendix J are examined, it is observed that the lower bounds are only slightly affected by the vehicle capacity. Therefore, the differences among the gaps between the lower and upper bounds, i.e., LUB% statistics, mostly result from the differences in the upper bounds obtained. For instance, data in classes 121-130 and 141-150 differ only in terms of the vehicle capacity. It can be checked from the table given in Appendix J that the lower bounds for instance 121 and 141 are 615,460 and 615,297, respectively, while the related upper bounds are 775,097 and 749,299. Thus, the lower bounds are not noticeably affected by the vehicle capacity and the differences between LUB% statistics for these instances mainly result from the differences in the upper bounds.

The bounds seem to perform better in the problem classes, in which the unit inventory holding costs of both the supplier and the retailers are at the high levels, i.e., classes 151-160 and 171-180. Since the unit holding costs are high in these instances, the effects of the inventory holding costs (and so the inventory holding policies) on the results are high. For this reason, it makes sense to obtain better bounds for these problems since all the restrictions on the inventory policies in IM are included in LRP-NM-1 method, while the restrictions on routing decisions are not fully included due to removal of subtour elimination constraints.

Another point to mention is that while LRP-NM-1 identifies integer feasible solutions for 45 instances out of 80 in 3600 CPU seconds (see Appendix J), IM can find integer feasible solutions only for 6 instances in 3600 CPU seconds, which are clearly worse than the solutions provided by LRP-NM-1.

5.2.3.3 The Procedure with a Priori Tour

In ME, LP relaxation of APT (APT-LP) is solved to get an idea about the quality of the solutions provided by APT. To see whether we can get improvements in APT-LP, the modified Barany et al. cuts given with the relation (4.23) are inserted into APT formulation and its LP relaxation is solved. The individual results obtained with and without the cuts are compared to each other in Appendix K and the averages are provided in Table 5.23 below.

It must be remarked that the procedure given in Section 4.2.3 is modified when solving APT-LP as follows. First a TSP is solved with Concorde and a priori tour is obtained, next APT-LP is solved. Since the decision variables that indicate whether the retailers are visited or not in a period are possibly noninteger in the solutions of APT-LP, the improvement step of the procedure, in which a TSP is solved for the retailers visited in each period, can not be executed. Thus, the total cost figures are the costs provided by the solutions of APT-LP.

Table 5.23 Results of APT-LP without and with (w) modified Barany et al. cuts

Instance	CPU	CPU w	Imp %
121-130	2.79	3.36	0.067
141-150	2.85	3.43	0.067
151-160	9.71	11.31	0.005
171-180	8.70	10.29	0.005
181-190	10.00	11.95	0.012
201-210	8.88	10.33	0.012
211-220	13.04	16.19	0.000
231-240	12.13	16.56	0.000
Overall	8.51	10.43	0.021

Different from the preliminary experiments, modified Barany et al. cuts are inserted into APT to examine whether improvements can be obtained. However, it is observed that these cuts increase the solution time extremely. Thus, the original formulation of APT that is used in the preliminary experiments, i.e., the formulation without the cuts, is utilized in the main runs, as well.

APT is run for each of the 80 instances until the first integer feasible solution is found. The instances, for which the first integer solution is obtained in less than 3600 CPU seconds, are run with a time limit of 3600 CPU seconds without the termination rule of identifying the first integer feasible solution. The results got by using the two termination rules are compared in Appendix L.

The gap between the lower bound obtained for APT using APT-LP (with the modified Barany et al. cuts) and the best upper bound obtained with APT (either the first integer feasible solution obtained or the solution got in 3600 CPU seconds, whichever is better) are given for each instance in Appendix M and presented in averages in Table 5.24 below. It can be seen in Appendix M that there exist 3 instances out of 80, for which feasible integer solutions can not be identified by APT. Although the solution times were less than 120,000 CPU seconds (the time limit used for LRP-NM-1), they were stopped due to memory limitations of the PC's used.

Table 5.24 Results obtained with APT in ME

	CPU		
Instance	APT	APT-LP	LUB%
121-130	20062	3.36	26.52
141-150	3600	3.43	22.55
151-160	14144	11.31	6.36
171-180	3600	10.29	3.59
181-190	14312	11.95	16.33
201-210	3600	10.33	8.81
211-220	11432	16.19	21.52
231-240	3600	16.56	15.95
Overall	9098	10.43	14.89

All TSPs are solved by Concorde in ME. The results above show that the gap between the lower and upper bounds ranges between 4% and 27% with an average solution time (excluding the 3 instances, for which integer feasible solutions can not be identified) of almost 2.5 CPU hours. It needs to be pointed out that the solutions of APT-LP provide lower bounds for APT but not for IM since it is based on a different formulation (fixing precedence relationships of the visits) that is more restricting, i.e., probably causing a greater minimum cost, than the formulation of IM.

5.2.3.4 Comparison of the Upper Bounds Obtained in ME

Comparison of the best upper bounds obtained with LRP-NM-1 and APT are provided in Table 5.25 below in averages and in Appendix N for each instance, for which both methods provide integer feasible solutions. The CPU seconds given for some of the classes and in overall are different from the figures given in Table 5.22 and Table 5.24 since the figures given below include only the instances, for which both LRP-NM-1 and APT provide integer feasible solutions.

Table 5.25 Comparison of the upper bounds obtained in ME

	CPU		
Instance	LRP-NM-1 (UB1)	APT (UB2)	UDiff %
121-130	28684	16436	1.21
141-150	600	3600	2.00
151-160	26580	8235	1.23
171-180	456	3600	3.37
181-190	28799	9570	3.25
201-210	501	3600	7.60
211-220	41674	5806	3.28
231-240	600	3600	6.93
Overall	11552	6482	3.84

As in the preliminary experiments, APT provides better upper bounds than LRP-NM-1 and for more instances. Moreover, in ME, the solution times required for APT are also less than that of LRP-NM-1.

5.2.3.5 Benchmarking

In this section, the best upper bounds obtained with our methods are compared to the results of the heuristic 'every' and the results of Bertazzi et al. (2002). The statistics used for benchmarking are described below.

- BestUB: It denotes the best upper bound obtained by our methods.
- UBe: It is the upper bound provided by 'every'.
- UBb: This denotes the upper bounds provided by Bertazzi et al. (2002).
- UDiff E% = 100*(UBe-BestUB)/BestUB: This statistic denotes differences between our upper bounds and the upper bounds given by 'every'.
- UDiff B% = 100*(BestUB-UBb)/UBb: It gives the differences between our upper bounds and the upper bounds given by Bertazzi et al. (2002).

The comparison for each instance is given in Appendix O and the average results are provided in Table 5.26 below.

Table 5.26 Comparison of the best upper bounds obtained in ME with the solutions of 'every' and Bertazzi et al. (2002)

Instance	UDiff E %	UDiff B %
121-130	33.19	7.52
141-150	37.75	5.46
151-160	10.07	2.41
171-180	13.02	1.32
181-190	23.50	5.95
201-210	32.04	2.96
211-220	6.17	2.82
231-240	12.08	3.42
Overall	20.82	3.94

As in the preliminary experiments, the upper bounds we obtain always outperform the upper bounds provided by 'every'. In this case, a remarkable outcome is that the upper bounds of ‘every’ are closer to our upper bounds when the unit inventory holding cost at the supplier is at the high level. This is an anticipated result since ‘every’ requires the retailers to be visited in every period and so, the inventory is preferred to be kept at the retailers as much as possible rather than keeping it at the supplier. In addition to this, if the unit inventory holding costs at the retailers are at the low level, solutions of ‘every’ get closer to our solutions under a given capacity level, i.e., solutions of 211-220 (231-240) are closer to our solutions than 151-160 (171-180).

The upper bounds provided by Bertazzi et al. (2002) are of better quality than the upper bounds we obtained with an average difference of almost 4%.

CHAPTER 6

CONCLUSION

In this study, an inventory routing problem with deterministic order-up-to level inventory policy (IRDOP) has been analyzed and an integrated mathematical formulation for the IRDOP has been developed. Optimal solutions could be identified with the integrated model for small problem instances.

The integrated model provided for the IRDOP turned out to be a complex model due to several inherent traveling salesman problems that needed to be solved and large number of integer variables included in its formulation. Considering difficulties caused by subtour elimination constraints, we developed methods to obtain upper and lower bounds for the IRDOP and examined their performances in a set of experiments. The test instances were either generated according to a scheme provided in the literature or directly taken from the literature. A secondary purpose of the first preliminary experiment that includes instances with known optimal solutions was to decide on the settings of the parameters in our proposed methods.

A lagrangean relaxation based approach, founded on relaxing the subtour elimination constraints and incorporating them into the objective function, was utilized to identify upper and lower bounds for the IRDOP. Upper and lower bounds were almost 7% and 2% within the optimality on average, respectively. For larger instances with unknown optimal solutions, the gap between the upper and lower bounds was almost 16% on average. For the largest instances that were tested, the procedure was simplified to be able to obtain bounds in reasonable times and the average gap between the upper and lower bounds increased up to 24%.

Another method for obtaining upper bounds for the IRDOP, which is based on identifying a priori tour and fixing the precedence relationships of the visits on this tour, is developed. The results demonstrated that this method provided better upper bounds than the lagrangean relaxation based approaches most of the time. The model with a priori tour happened to be more advantageous than the lagrangean relaxation problem in terms of the solution times, as well.

The upper bounds obtained by the methods developed in this study were compared to the solutions provided by a trivial heuristic approach 'every' (see Appendix A) and to the solutions of Bertazzi et al. (2002). Our methods outperformed 'every' all the time with average differences between them ranging from 21% to 62%. Although the upper bounding procedure given by the Bertazzi et al. (2002) provided slightly better values than ours, it must be pointed out that we do not conduct comprehensive improvement steps and there is no basis to compare the execution times because the solution times of Bertazzi et al. (2002) heuristics are unknown.

The main contributions of this thesis are to develop a mathematical model for the IRDOP and identify a lower bound for the problem, both of which have not been accomplished so far. Besides, to the best of our knowledge, this is the first time a priori tour is used to fix the precedence relationships of the visits and substitute these precedence relationships for the subtour elimination constraints in TSP context.

The methods we developed can be considered as an initial attempt to solve the IRDOP. Further improvements may be possible in two basic ways: making modifications in the procedures that are used currently and developing new procedures.

The first possible way to modify the procedures developed is enhancing the formulations of the models using relevant cuts. It was observed that the LP relaxation of the lagrangean relaxation problem was providing better solutions than

LP relaxation of the integrated model due to enhancements included in its formulation. It may be possible to obtain better results for both the lagrangean relaxation problem and the model with a priori tour by including further cuts.

The second way to improve the solutions we obtained is to build post improvement steps that will be executed following the usual stages of the procedures.

While executing iterations for the lagrangean relaxation problem, the lagrangean multipliers were updated by general subgradient optimization. Thus, as a third modification, different methods can be identified to improve the performance of the subgradient optimization. Besides, different values for initializing the lagrangean multipliers can be tested and the value that provides the best bounds for the instances can be used as the initial multiplier.

Considering new procedures that can be developed, it was seen that the upper and lower bounds provided by the lagrangean relaxation based approach were satisfactory particularly for small instances. However, the lagrangean relaxation problem could not be solved optimally as MIP for large instances. Thus, an apparent extension of this study is identifying a procedure to solve the MIP formulation of the lagrangean relaxation problem optimally.

Besides, the solution approach with a priori tour provided quite good upper bounds in reasonable times for small problems. The upper bounds that were obtained at the first integer feasible solutions for large problems were also satisfactory. It is certain that if this model could have been solved to optimality, better upper bounds would have been obtained for large instances, as well. Thus, identifying a method that is based on either lagrangean relaxation approach or other means to solve the model with a priori tour is a possible extension of this study. Moreover, it can be analyzed how the performance of the model with a priori tour is affected by the distance structure.

Regarding the assumptions made to model the system, the minimum and the maximum inventory levels at the retailers can be decision variables, as well, i.e., these levels can be determined by the model. In addition to this, rather than using a deterministic value for the amount of product that becomes available at the supplier in each period, a decision variable may be employed to get optimal value for this amount. Furthermore, allowing transshipments between retailer pairs and backlogs, considering multiple vehicles and multiple products are also possible extensions that can be studied.

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APPENDIX A

A TRIVIAL HEURISTIC METHOD 'EVERY'

A heuristic method named as 'every' is used by Bertazzi et al. (2002) to compare the results obtained in their study. This heuristic method is employed in our study either to obtain an initial upper bound or to make comparisons with the results we obtained. Below is a description of this heuristic.

The method is based on visiting all retailers in each period with the least cost tour. Except for this, the remaining assumptions on the original problem such as the delivery amounts are taken into account in this method, as well.

Due to some properties of the instances that are used in the numerical experiments, 'every' always provides a feasible solution for the IRDOP. These properties are discussed in the sequel.

A restriction that should be taken into account to obtain feasible solutions is the capacity of the vehicle. Data generation scheme used in the experiments guarantees satisfaction of the vehicle capacity restriction as follows. The initial inventory level of a retailer (at the beginning of period 1) is set at the difference between the maximum inventory level and the demand at the retailer ($I_1^i = \overline{S^i} - d^i$). Therefore, during a visit to a retailer in period 1, the amount of product delivered to the retailer that fills the inventory up to the maximum level is equal to the demand that needs to be satisfied by the respective retailer. After this amount is delivered to the retailer, the retailer meets the demand incurred in period 1. As a result of these events, the inventory level of retailer i at the beginning of period 2 is also equal to $\overline{S^i} - d^i$ and the same events are observed recurrently. This indicates that in every period each retailer receives a delivery that corresponds to the demand at it and so, the total amount of product distributed to the retailers in each period equals the sum of the demands at the retailers ($\sum_i d^i$). Thus, the total product distributed to the retailers in each period is always less than (if the vehicle capacity is $3\sum_i d^i$) or equal to (if the vehicle capacity is $\sum_i d^i$) the capacity of the vehicle.

Another concern is the inventory available at the supplier. Since the initial inventory level at the supplier is $\sum_i d^i$, the first period requirements of the retailers ($\sum_i d^i$) can be satisfied out of this amount. Also, an amount equal to $\sum_i d^i$ becomes available at the supplier in each period. Assuming that the first period requirements of the retailers are satisfied from the initial inventory of the supplier, the amount that becomes available in each period ($\sum_i d^i$) can be used to meet the requirements of the retailers in the next period.

Thus, the capacity of the vehicle and the amount available at the supplier can be used to meet the demands at the retailers on time. Considering the features of the data set given above, the costs are computed as follows.

Transportation cost:

The tour with the minimum cost that visits all retailers is identified. Because this tour is executed in each period, the cost of executing this tour is multiplied by the number of periods in the planning horizon (T).

Inventory holding cost at the retailers:

Knowing that every retailer receives a delivery that equals to the demand in each period, the flow balance constraint, i.e., constraint (3.2), can be expressed as $I^i_{t-1} + d^i_{t-1} - I^i_t = d^i_{t-1}$, i.e., $I^i_{t-1} = I^i_t$. This equality shows that the inventory level at each retailer at the beginning of each period equals the starting inventory level of the related retailer, i.e., $\overline{S^i} - d^i$. It must be reminded that the inventory holding cost at the retailers is computed according to the inventory available at the beginning of each period in the planning horizon (i.e., T periods) and at the beginning of the period T+1. Thus, to obtain the inventory holding cost at a retailer, the inventory level given above is multiplied by the unit holding cost at the relevant retailer and T+1, which is then summed up over all retailers.

Inventory holding cost at the supplier:

The flow balance constraint at the supplier, i.e., constraint (3.9), can be restated as $Is_t + \sum_i d^i - Is_{t-1} = \sum_i d^i$, i.e., $Is_t = Is_{t-1}$ since both the amount that becomes available at the supplier and the amount distributed to the retailers from the supplier in each period is $\sum_i d^i$. This equality demonstrates that the inventory level at the supplier in each period is equal to the starting inventory level of the supplier, i.e., $\sum_i d^i$. Therefore, in the same way as the retailers, the inventory holding cost at the supplier is calculated by multiplying this inventory level with the unit holding cost at the supplier and T+1.

The total cost incurred is computed by summing up the total transportation and inventory holding costs that are discussed above. Thus, a feasible solution for the IRDOP with the total cost that is calculated as above can be obtained using ‘every’.

APPENDIX B

AVERAGE GAPS FOR THE BOUNDS OBTAINED WITH LRP-NM IN PE-I

The figure below demonstrates the average percent gaps for the bounds obtained with lagrangean relaxation solution procedure (LRP-NM) over 15 instances that are tested in preliminary experiment I (PE-I).

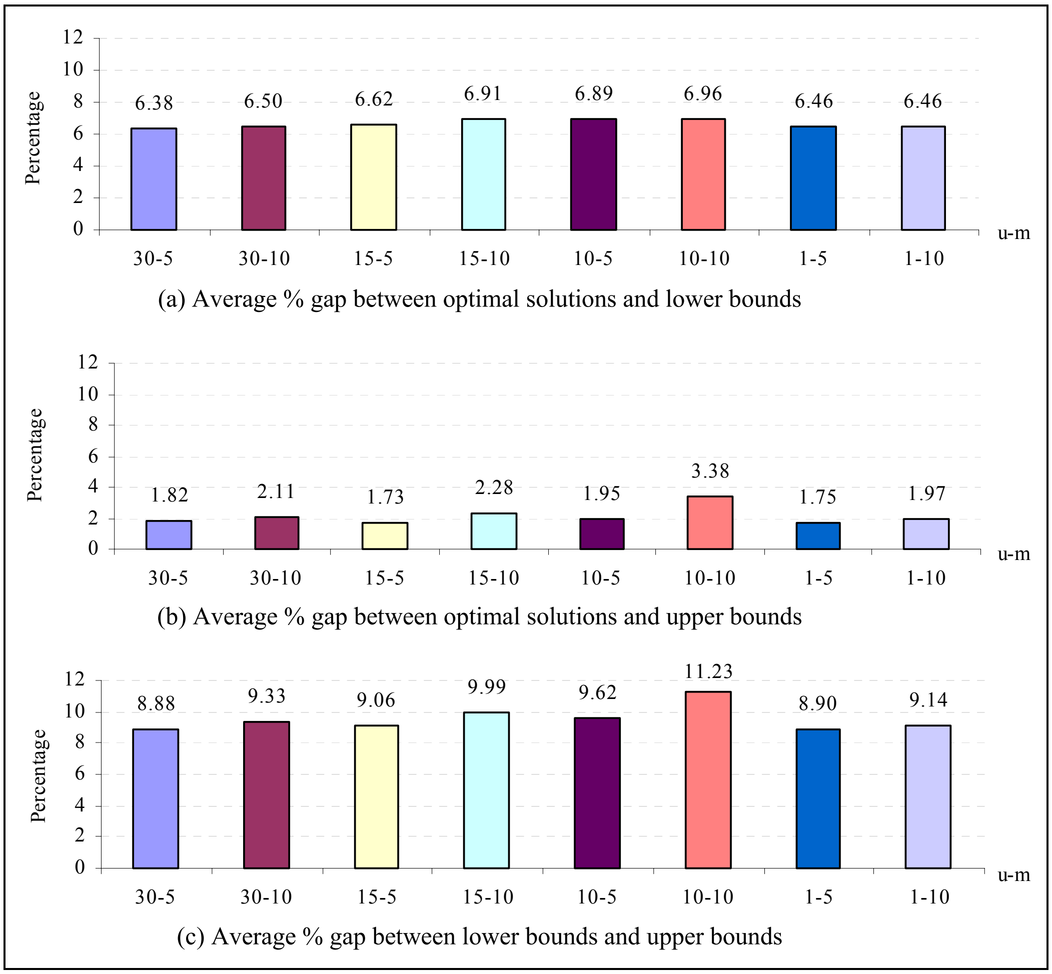

Figure B.1 Average % gaps over the instances tested in PE-I

APPENDIX C

INDIVIDUAL GAPS FOR THE BOUNDS OBTAINED WITH LRP-NM IN PE-I

Figures in this part illustrate percent gaps for the bounds obtained by the lagrangean relaxation solution procedure (LRP-NM) for each instance tested in preliminary experiment I (PE-I) individually.

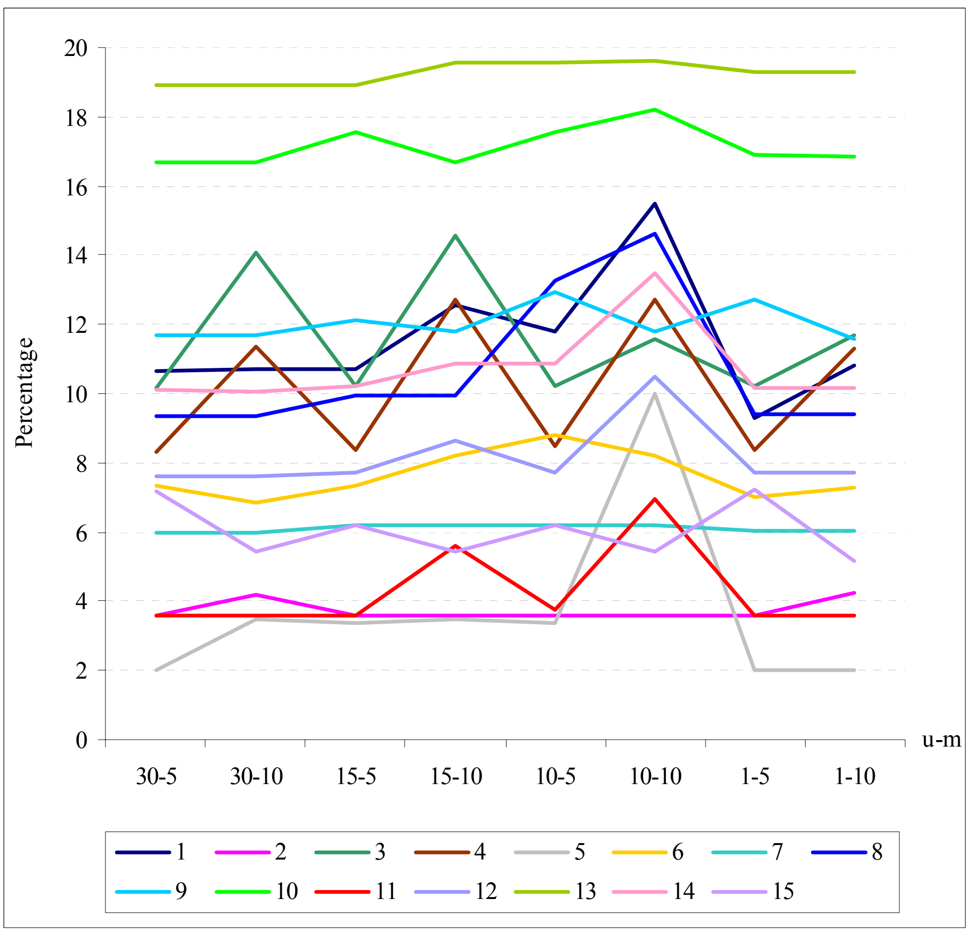

Figure C.1 % Gaps between upper and lower bounds for the instances tested in PE-I

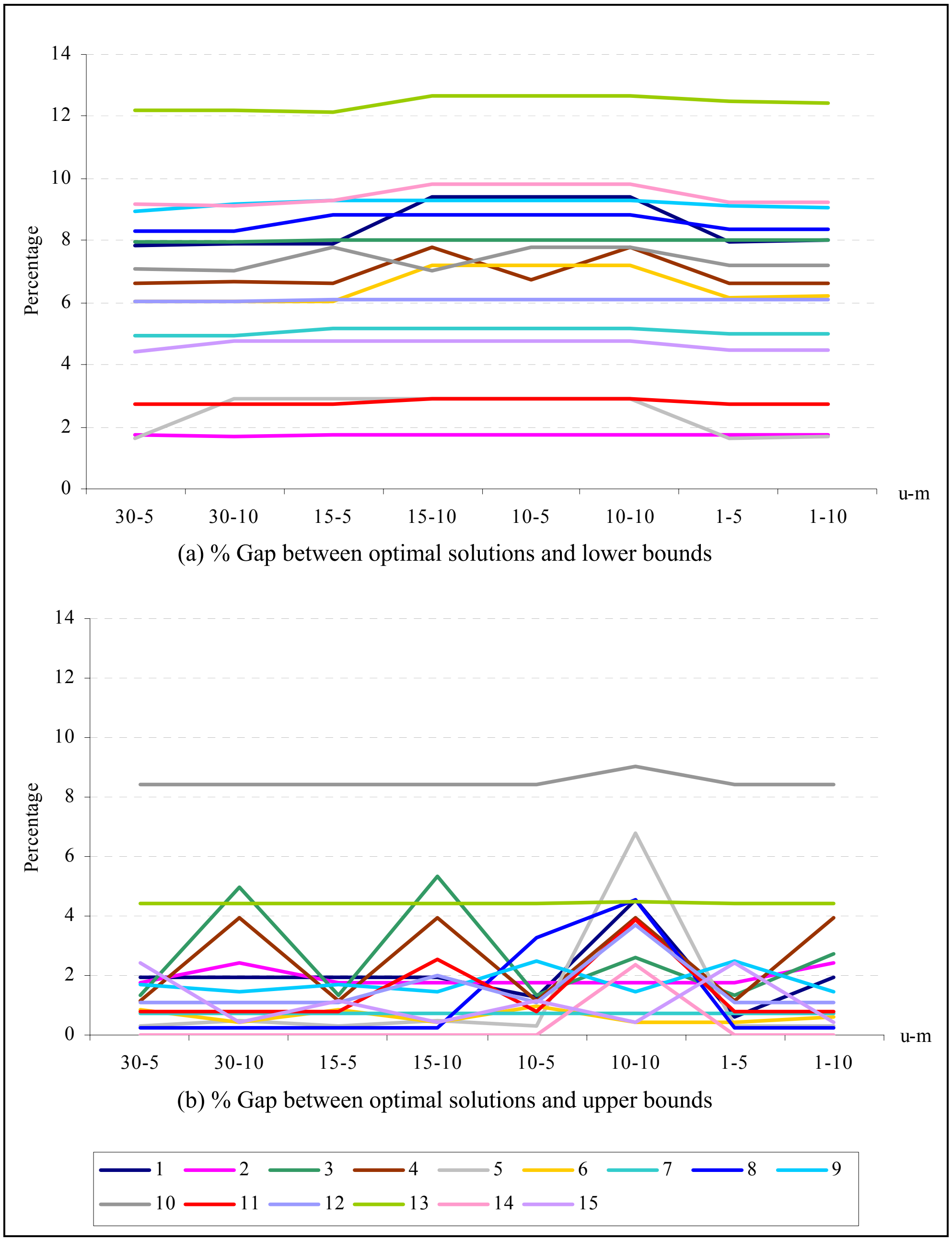

Figure C.2 % Gaps for the instances tested in PE-I

APPENDIX D

BOUNDS OBTAINED WITH LRP-NM IN PE-I THROUGH ITERATIONS

Figures in this part show the changes in bounds obtained with lagrangean relaxation solution procedure (LRP-NM) for each instance tested in preliminary experiment I (PE-I) according to number of iterations performed.

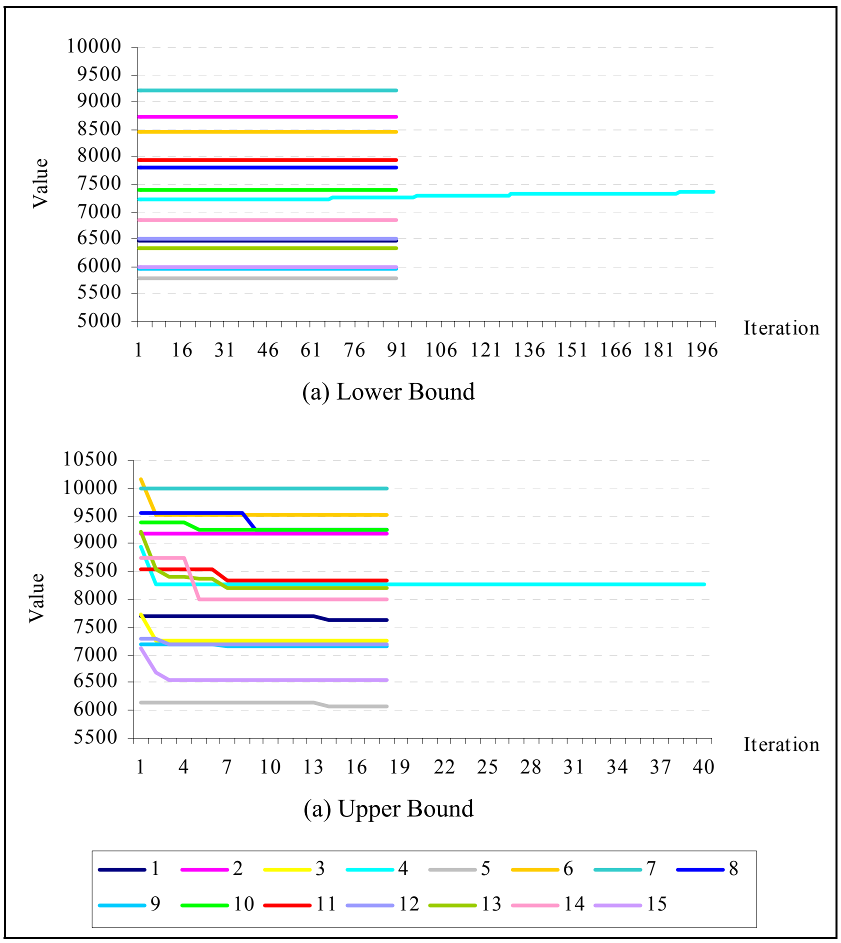

Figure D.1 Lower and upper bounds for u = 10, m = 5 through iterations in PE-I for 15 instances

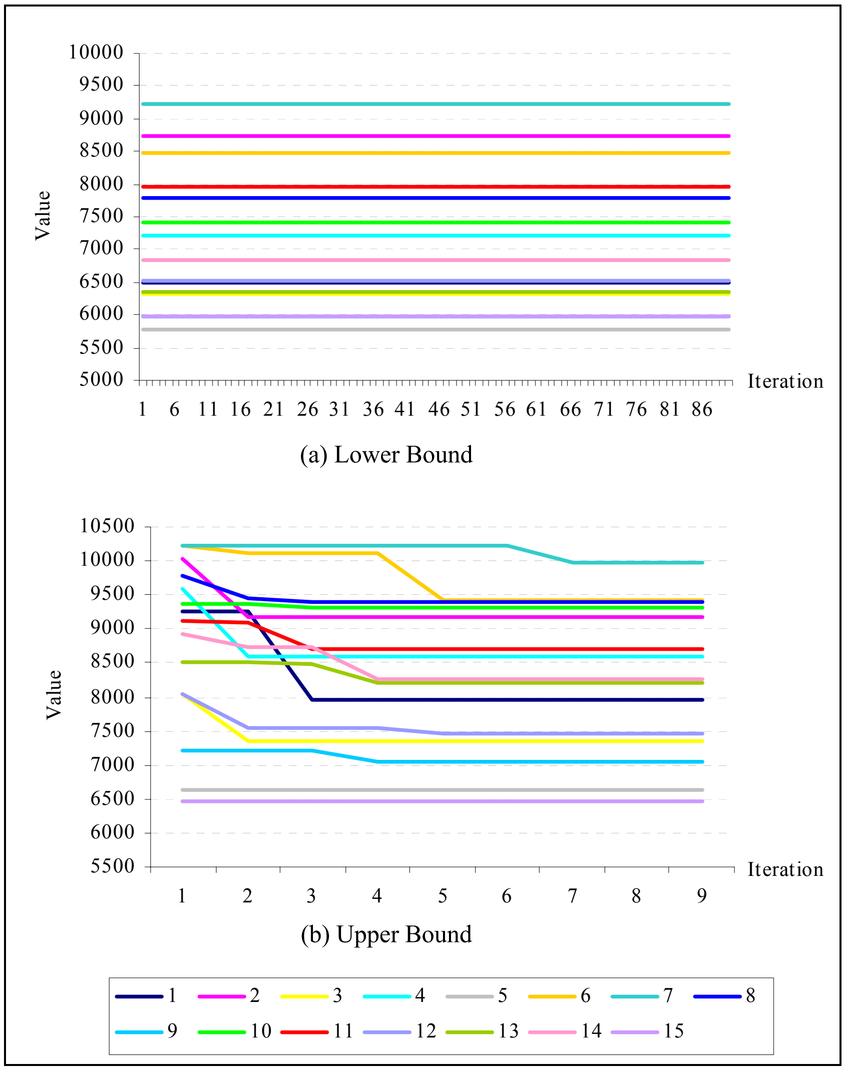

Figure D.2 Lower and upper bounds for u = 10, m = 10 through iterations in PE-I for 15 instances

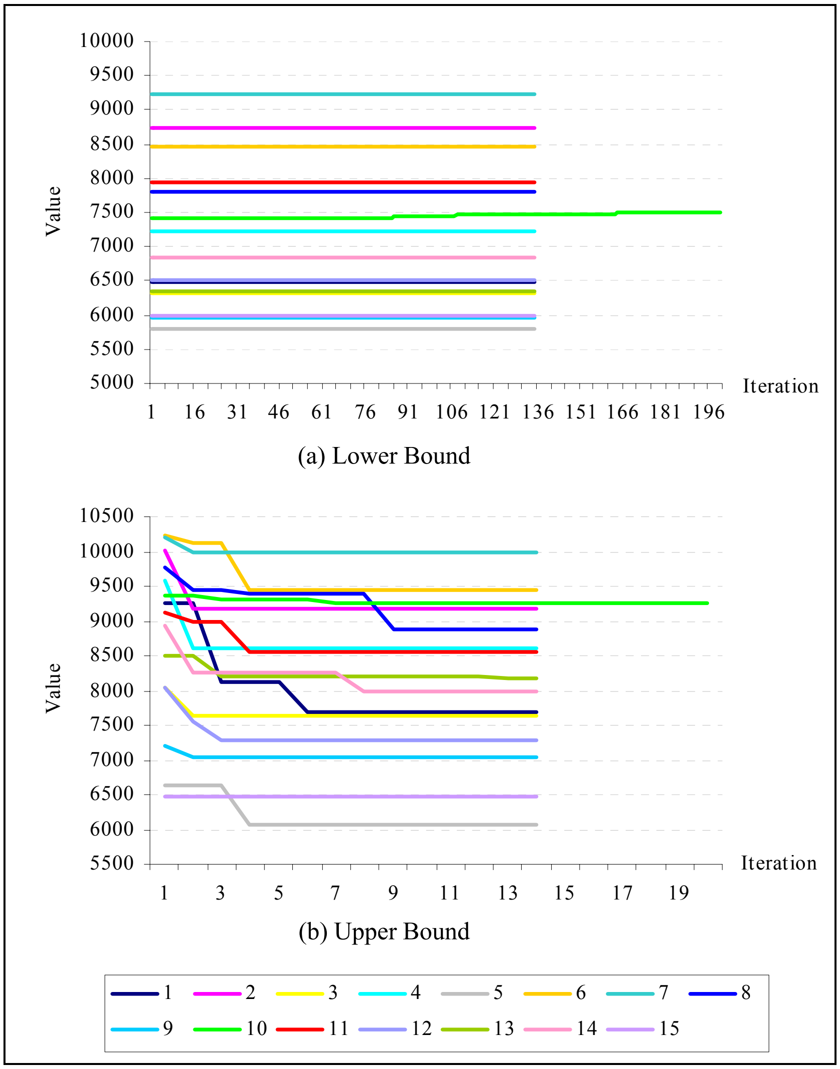

Figure D.3 Lower and upper bounds for u = 15, m = 10 through iterations in PE-I for 15 instances

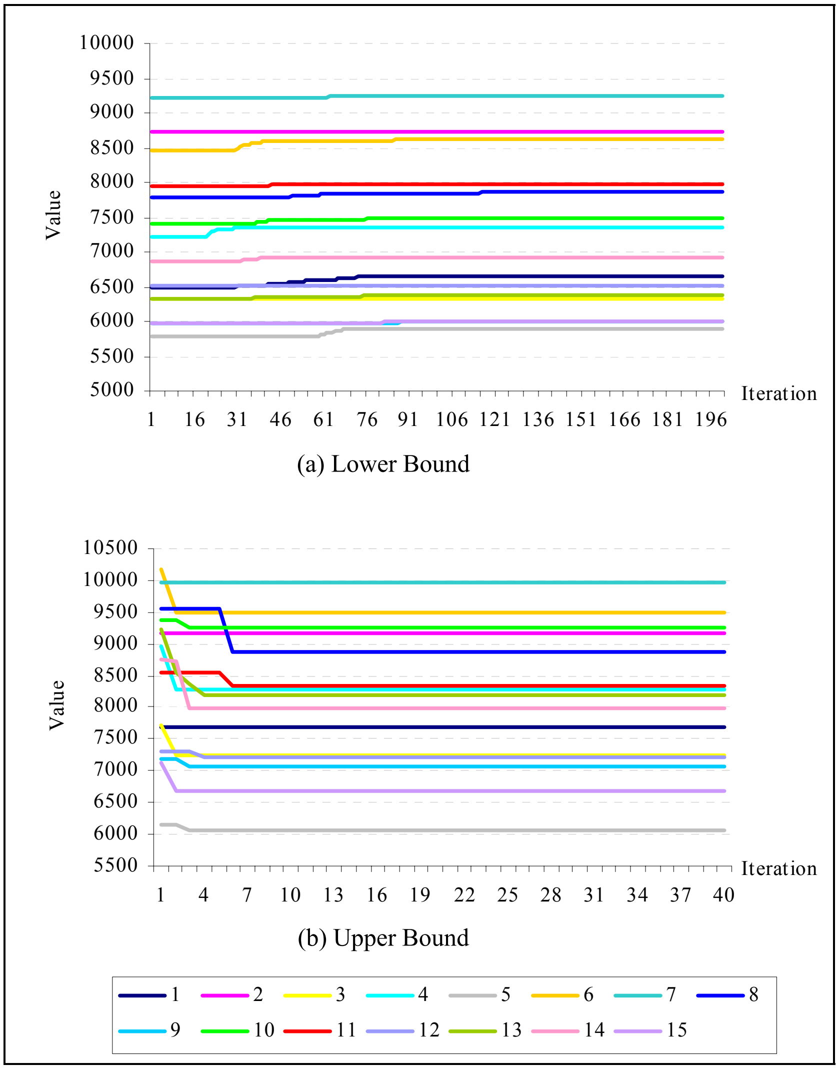

Figure D.4 Lower and upper bounds for u = 30, m = 5 through iterations in PE-I for 15 instances

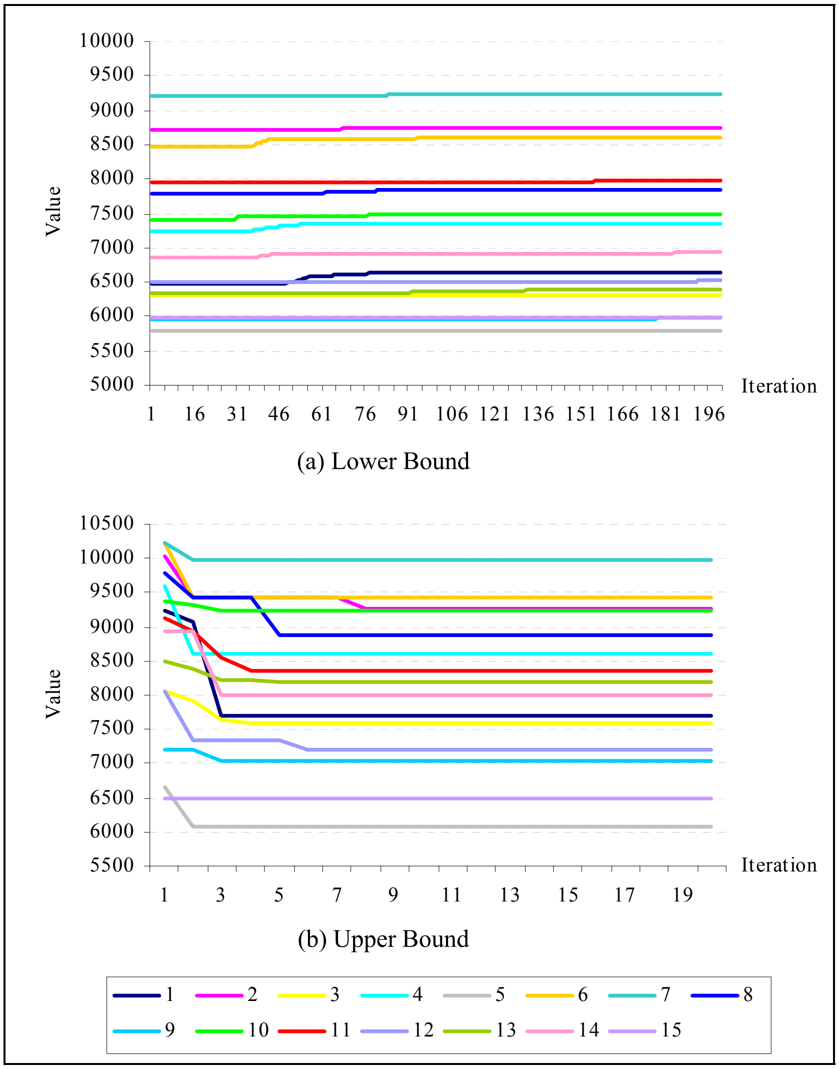

Figure D.5 Lower and upper bounds for u = 30, m = 10 through iterations in PE-I for 15 instances

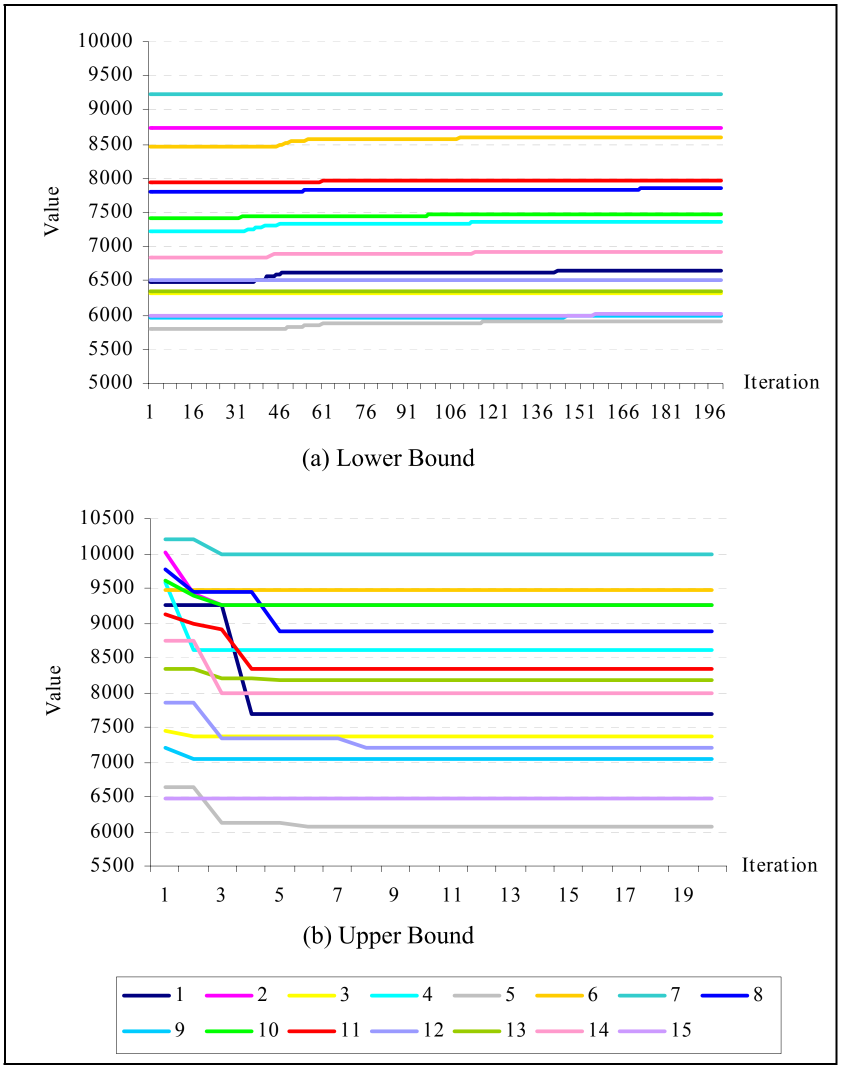

Figure D.6 Lower and upper bounds for u = 1, m = 10 through iterations in PE-I for 15 instances

APPENDIX E

BOUNDS OBTAINED WITH LRP-NM IN PE-II THROUGH ITERATIONS

Figures below show the changes in bounds obtained with lagrangean relaxation solution procedure (LRP-NM) for each instance tested in preliminary experiment II (PE-II) according to number of iterations performed.

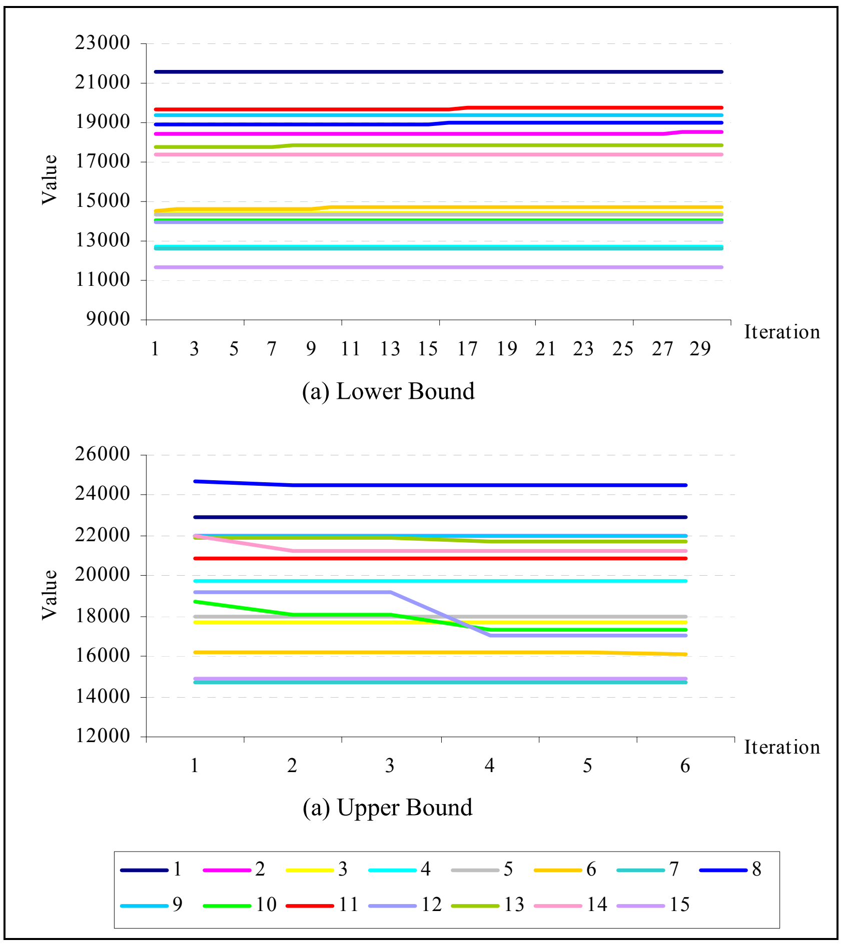

Figure E.1 Lower and upper bounds for u = 15, m = 5 through iterations in PE-II for 15 instances

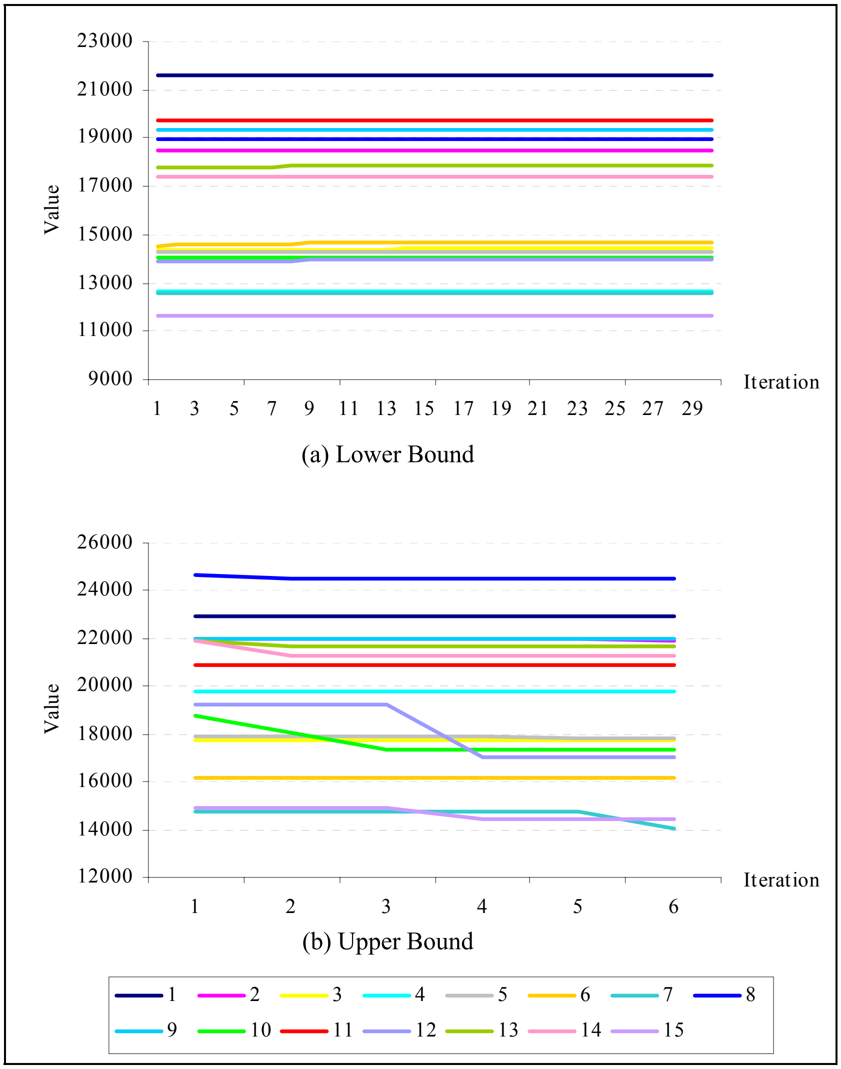

Figure E.2 Lower and upper bounds for u = 1, m = 5 through iterations in PE-II for 15 instances

APPENDIX F

RESULTS FOR LP RELAXATION OF IM IN ME

Results obtained with LP relaxation of the integrated model (IM-LP) during main experiments (ME) are provided in this part, together with the results obtained when modified Barany et al. cuts are included in this formulation.

Table F.1 Results of IM-LP without and with modified Barany et al. cuts in ME *

	IM-LP		IM-LP with cuts		
Instance	TC	CPU	TC	CPU	Imp %
121	596221	9.31	596221	9.84	0.0000
122	636839	9.80	637577	10.90	0.1158
123	482182	9.87	482777	10.94	0.1235
124	571207	12.03	571570	12.70	0.0635
125	387533	9.87	387827	11.76	0.0760
126	574754	10.72	575314	12.31	0.0975
127	515018	10.53	515656	12.02	0.1240
128	562478	11.54	562498	11.99	0.0034
129	513177	11.60	513259	13.09	0.0160
130	552702	11.14	553048	12.85	0.0626
141	596221	9.53	596221	10.09	0.0000
142	636839	9.89	637577	10.72	0.1158
143	482182	10.25	482777	11.21	0.1235
144	571207	11.90	571570	12.35	0.0635
145	387533	10.14	387827	11.94	0.0760
146	574754	10.82	575314	12.48	0.0975
147	515018	10.58	515656	12.28	0.1240
148	562478	12.06	562498	12.02	0.0035
149	513177	11.61	513259	13.08	0.0160
150	552702	11.24	553048	13.22	0.0626
151	893385	16.84	893385	20.97	0.0000
152	971751	22.42	971840	23.34	0.0092
153	747551	30.90	747659	38.89	0.0145
154	848551	25.67	848628	28.38	0.0091
155	611137	29.73	611137	25.78	0.0000
156	880594	29.02	880609	26.64	0.0017
157	796425	20.90	796517	26.14	0.0115
158	829048	19.99	829050	27.91	0.0003
159	802735	23.37	802735	22.81	0.0000
160	843241	19.92	843248	21.63	0.0008
171	893385	19.57	893385	21.32	0.0000
172	971751	23.62	971840	26.18	0.0092
173	747551	29.14	747659	27.32	0.0145
174	848551	25.77	848628	26.44	0.0091
175	611137	28.72	611137	28.75	0.0000
176	880594	31.22	880609	29.00	0.0017
177	796425	21.04	796517	21.11	0.0115
178	829048	20.40	829050	26.64	0.0003
179	802735	25.40	802735	27.50	0.0000
180	843241	18.97	843248	21.67	0.0008

Table F.1 (continued)

	IM-LP		IM-LP with cuts		
Instance	TC	CPU	TC	CPU	Imp %
181	336935	18.68	336935	19.58	0.0000
182	340932	20.40	341021	23.46	0.0261
183	267159	27.89	267268	36.62	0.0407
184	308671	24.10	308747	28.97	0.0249
185	227001	28.13	227001	34.39	0.0000
186	316099	28.81	316115	26.69	0.0048
187	305726	22.78	305818	22.50	0.0300
188	308604	21.05	308607	24.98	0.0007
189	315462	19.59	315462	19.66	0.0000
190	326781	22.34	326788	22.68	0.0021
201	336935	19.53	336935	20.01	0.0000
202	340932	24.76	341021	26.96	0.0261
203	267159	23.06	267268	33.82	0.0407
204	308671	27.51	308747	30.83	0.0249
205	227001	26.88	227001	31.94	0.0000
206	316099	28.83	316115	30.24	0.0048
207	305726	21.93	305818	20.50	0.0300
208	308604	24.74	308607	35.04	0.0007
209	315462	26.06	315462	21.66	0.0000
210	326781	19.67	326788	19.76	0.0021
211	407940	65.26	407940	63.75	0.0000
212	413537	78.65	413537	78.31	0.0000
213	329658	73.30	329658	80.71	0.0000
214	376602	73.21	376602	88.23	0.0000
215	286537	75.94	286537	91.90	0.0000
216	382274	70.94	382274	72.91	0.0000
217	377482	79.18	377482	103.20	0.0000
218	375081	63.32	375081	68.19	0.0000
219	393506	84.50	393506	87.70	0.0000
220	392017	54.44	392017	66.37	0.0000
231	407940	70.76	407940	62.54	0.0000
232	413537	69.60	413537	80.87	0.0000
233	329658	67.35	329658	79.33	0.0000
234	376602	90.00	376602	89.02	0.0000
235	286537	87.97	286537	83.58	0.0000
236	382274	57.25	382274	68.39	0.0000
237	377482	69.50	377482	91.22	0.0000
238	375081	71.02	375081	66.60	0.0000
239	393506	82.23	393506	95.41	0.0000
240	392017	64.85	392017	61.71	0.0000
Average		32.79		35.83	0.0215

* TC is the total cost incurred in the solution with the related formulation.
CPU is the solution time for the related formulation.
Imp %, given by % (TC with cuts-TC without cuts)/TC without cuts, denotes the improvement obtained with modified Barany et al. cuts over the formulation without the cuts.

APPENDIX G

RESULTS FOR LP RELAXATION OF LRP IN ME

Results obtained in main experiment (ME) with LP relaxation of the lagrangean relaxation problem (LRP-LP) are provided in this part, together with the results obtained when modified Barany et al. cuts are excluded from its formulation.

Table G.1 Results of LRP-LP with and without modified Barany et al. cuts in ME *

	LRP-LP		LRP-LP w/o cuts		
Instance	TC	CPU	TC	CPU	Imp %
121	615460	10.02	596877	2.62	3.0194
122	657499	15.24	637585	2.42	3.0287
123	503206	9.70	482682	2.82	4.0786
124	592218	13.22	571214	2.31	3.5468
125	411641	12.33	389722	2.44	5.3247
126	595706	7.80	575173	2.81	3.4469
127	536452	9.24	515433	3.27	3.9183
128	579296	10.33	562492	2.07	2.9008
129	537542	12.21	513491	2.20	4.4743
130	571351	15.27	552809	2.55	3.2452
141	615297	9.00	596877	2.65	2.9938
142	657368	11.58	637585	2.54	3.0094
143	503032	6.27	482682	2.93	4.0454
144	592077	10.25	571214	2.38	3.5237
145	411432	10.47	389722	2.52	5.2766
146	595624	6.39	575173	2.81	3.4336
147	536299	5.94	515433	3.32	3.8908
148	579199	9.80	562491	2.14	2.8846
149	537344	8.29	513491	2.27	4.4391
150	571205	10.04	552809	2.64	3.2205
151	895565	16.97	894097	5.35	0.1639
152	974395	11.92	972888	4.98	0.1546
153	750190	9.96	748177	6.70	0.2684
154	851075	10.32	848582	4.35	0.2928
155	615686	14.70	613839	6.19	0.3000
156	882491	10.75	880918	4.54	0.1782
157	798876	11.74	796787	4.61	0.2614
158	830352	6.57	829048	4.51	0.1570
159	805951	10.82	803148	3.86	0.3478
160	844720	11.97	843404	4.91	0.1557
171	895565	11.18	894097	5.08	0.1639
172	974395	6.46	972888	4.04	0.1546
173	750190	8.87	748177	5.45	0.2684
174	851072	7.77	848582	3.96	0.2925
175	615686	10.58	613839	6.19	0.3000
176	882491	6.32	880918	4.01	0.1782
177	798875	7.37	796787	4.45	0.2614
178	830352	6.43	829048	4.60	0.1570
179	805951	8.56	803148	4.12	0.3478
180	844720	10.46	843404	4.48	0.1557

Table G.1 (continued)

	LRP-LP		LRP-LP w/o cuts		
Instance	TC	CPU	TC	CPU	Imp %
181	339115	12.30	337647	5.04	0.4346
182	343576	9.38	342069	4.55	0.4405
183	269798	11.69	267785	6.30	0.7518
184	311194	11.97	308702	4.42	0.8073
185	231549	16.59	229702	5.86	0.8042
186	317996	13.20	316423	4.66	0.4971
187	308177	12.23	306088	4.58	0.6823
188	309908	5.93	308605	4.62	0.4224
189	318678	8.64	315874	4.01	0.8875
190	328260	9.35	326944	4.70	0.4024
201	339115	12.01	337647	5.23	0.4346
202	343576	5.99	342069	3.89	0.4405
203	269798	8.22	267785	5.53	0.7518
204	311191	11.02	308702	4.15	0.8063
205	231549	9.86	229702	6.46	0.8042
206	317996	6.27	316423	4.16	0.4971
207	308176	7.65	306088	4.19	0.6823
208	309908	6.52	308605	4.72	0.4224
209	318678	8.58	315874	4.25	0.8875
210	328260	8.14	326944	4.60	0.4024
211	408761	10.32	408749	6.86	0.0030
212	414758	9.45	414746	5.96	0.0029
213	330163	11.26	330162	8.74	0.0004
214	376663	11.24	376636	7.83	0.0070
215	289414	11.72	289412	7.55	0.0007
216	382550	12.02	382505	5.63	0.0117
217	377617	11.74	377599	7.05	0.0046
218	375084	7.85	375081	6.17	0.0010
219	393959	12.47	393946	7.64	0.0032
220	392269	8.09	392234	6.39	0.0087
231	408761	9.59	408749	6.90	0.0030
232	414758	8.82	414746	6.02	0.0029
233	330163	11.24	330162	8.49	0.0004
234	376663	11.15	376636	8.82	0.0070
235	289414	11.25	289412	9.45	0.0007
236	382550	8.56	382505	6.36	0.0117
237	377617	10.77	377599	7.87	0.0046
238	375084	7.17	375081	5.82	0.0010
239	393959	9.50	393946	6.98	0.0032
240	392269	7.60	392234	5.86	0.0087
Average	516710	10.01	510732	4.83	1.1326

* TC is the total cost incurred in the solution with the related formulation.
CPU is the solution time for the related formulation.
Imp %, given by % (TC with cuts-TC without cuts)/TC without cuts, denotes the improvement obtained with modified Barany et al. cuts over the formulation without the cuts.

APPENDIX H

COMPARISON OF LP RELAXATION RESULTS OF IM AND LRP IN ME

Below, the results obtained in main experiment (ME) with LP relaxations of the lagrangean relaxation problem (LRP-LP) and integrated model (IM-LP) are compared under the formulations with and without modified Barany et al. cuts.

Table H.1 Comparison of LRP-LP and IM-LP without and with modified Barany et al. cuts in ME *

	Results without the cuts					Results with the cuts				
	LRP-LP		IM-LP		LDiff %	LRP-LP		IM-LP		LDiff %
Instance	LB1	CPU	LB2	CPU		LB1	CPU	LB2	CPU	
121	596877	2.62	596221	9.31	0.1098	615460	10.02	596221	9.84	3.1259
122	637585	2.42	636839	9.80	0.1171	657499	15.24	637577	10.90	3.0301
123	482682	2.82	482182	9.87	0.1037	503206	9.70	482777	10.94	4.0598
124	571214	2.31	571207	12.03	0.0011	592218	13.22	571570	12.70	3.4867
125	389722	2.44	387533	9.87	0.5618	411641	12.33	387827	11.76	5.7850
126	575173	2.81	574754	10.72	0.0729	595706	7.80	575314	12.31	3.4232
127	515433	3.27	515018	10.53	0.0805	536452	9.24	515656	12.02	3.8766
128	562492	2.07	562478	11.54	0.0024	579296	10.33	562498	11.99	2.8998
129	513491	2.20	513177	11.60	0.0612	537542	12.21	513259	13.09	4.5176
130	552809	2.55	552702	11.14	0.0195	571351	15.27	553048	12.85	3.2035
141	596877	2.65	596221	9.53	0.1098	615297	9.00	596221	10.09	3.1003
142	637585	2.54	636839	9.89	0.1171	657368	11.58	637577	10.72	3.0107
143	482682	2.93	482182	10.25	0.1037	503032	6.27	482777	11.21	4.0266
144	571214	2.38	571207	11.90	0.0011	592077	10.25	571570	12.35	3.4635
145	389722	2.52	387533	10.14	0.5618	411432	10.47	387827	11.94	5.7371
146	575173	2.81	574754	10.82	0.0729	595624	6.39	575314	12.48	3.4099
147	515433	3.32	515018	10.58	0.0805	536299	5.94	515656	12.28	3.8490
148	562491	2.14	562478	12.06	0.0024	579199	9.80	562498	12.02	2.8834
149	513491	2.27	513177	11.61	0.0612	537344	8.29	513259	13.08	4.4824
150	552809	2.64	552702	11.24	0.0195	571205	10.04	553048	13.22	3.1788
151	894097	5.35	893385	16.84	0.0796	895565	16.97	893385	20.97	0.2433
152	972888	4.98	971751	22.42	0.1169	974395	11.92	971840	23.34	0.2622
153	748177	6.70	747551	30.90	0.0837	750190	9.96	747659	38.89	0.3373
154	848582	4.35	848551	25.67	0.0037	851075	10.32	848628	28.38	0.2875
155	613839	6.19	611137	29.73	0.4401	615686	14.70	611137	25.78	0.7388
156	880918	4.54	880594	29.02	0.0368	882491	10.75	880609	26.64	0.2132
157	796787	4.61	796425	20.90	0.0454	798876	11.74	796517	26.14	0.2952
158	829048	4.51	829048	19.99	0.0000	830352	6.57	829050	27.91	0.1568
159	803148	3.86	802735	23.37	0.0513	805951	10.82	802735	22.81	0.3990
160	843404	4.91	843241	19.92	0.0193	844720	11.97	843248	21.63	0.1742
171	894097	5.08	893385	19.57	0.0796	895565	11.18	893385	21.32	0.2433
172	972888	4.04	971751	23.62	0.1169	974395	6.46	971840	26.18	0.2622
173	748177	5.45	747551	29.14	0.0837	750190	8.87	747659	27.32	0.3373
174	848582	3.96	848551	25.77	0.0037	851072	7.77	848628	26.44	0.2871
175	613839	6.19	611137	28.72	0.4401	615686	10.58	611137	28.75	0.7388
176	880918	4.01	880594	31.22	0.0368	882491	6.32	880609	29.00	0.2132
177	796787	4.45	796425	21.04	0.0454	798875	7.37	796517	21.11	0.2952
178	829048	4.60	829048	20.40	0.0000	830352	6.43	829050	26.64	0.1568
179	803148	4.12	802735	25.40	0.0513	805951	8.56	802735	27.50	0.3990
180	843404	4.48	843241	18.97	0.0193	844720	10.46	843248	21.67	0.1742

Table H.1 (continued)

	Results without the cuts					Results with the cuts				
	LRP-LP		IM-LP		LDiff %	LRP-LP		IM-LP		LDiff %
Instance	LB1	CPU	LB2	CPU		LB1	CPU	LB2	CPU	
181	337647	5.04	336935	18.68	0.2108	339115	12.30	336935	19.58	0.6427
182	342069	4.55	340932	20.40	0.3325	343576	9.38	341021	23.46	0.7437
183	267785	6.30	267159	27.89	0.2337	269798	11.69	267268	36.62	0.9379
184	308702	4.42	308671	24.10	0.0101	311194	11.97	308747	28.97	0.7862
185	229702	5.86	227001	28.13	1.1762	231549	16.59	227001	34.39	1.9645
186	316423	4.66	316099	28.81	0.1024	317996	13.20	316115	26.69	0.5918
187	306088	4.58	305726	22.78	0.1181	308177	12.23	305818	22.50	0.7653
188	308605	4.62	308604	21.05	0.0001	309908	5.93	308607	24.98	0.4200
189	315874	4.01	315462	19.59	0.1305	318678	8.64	315462	19.66	1.0091
190	326944	4.70	326781	22.34	0.0498	328260	9.35	326788	22.68	0.4483
201	337647	5.23	336935	19.53	0.2108	339115	12.01	336935	20.01	0.6427
202	342069	3.89	340932	24.76	0.3325	343576	5.99	341021	26.96	0.7437
203	267785	5.53	267159	23.06	0.2337	269798	8.22	267268	33.82	0.9379
204	308702	4.15	308671	27.51	0.0101	311191	11.02	308747	30.83	0.7853
205	229702	6.46	227001	26.88	1.1762	231549	9.86	227001	31.94	1.9645
206	316423	4.16	316099	28.83	0.1024	317996	6.27	316115	30.24	0.5917
207	306088	4.19	305726	21.93	0.1181	308176	7.65	305818	20.50	0.7653
208	308605	4.72	308604	24.74	0.0001	309908	6.52	308607	35.04	0.4200
209	315874	4.25	315462	26.06	0.1305	318678	8.58	315462	21.66	1.0091
210	326944	4.60	326781	19.67	0.0498	328260	8.14	326788	19.76	0.4483
211	408749	6.86	407940	65.26	0.1978	408761	10.32	407940	63.75	0.2008
212	414746	5.96	413537	78.65	0.2915	414758	9.45	413537	78.31	0.2943
213	330162	8.74	329658	73.30	0.1527	330163	11.26	329658	80.71	0.1531
214	376636	7.83	376602	73.21	0.0091	376663	11.24	376602	88.23	0.0161
215	289412	7.55	286537	75.94	0.9934	289414	11.72	286537	91.90	0.9940
216	382505	5.63	382274	70.94	0.0602	382550	12.02	382274	72.91	0.0720
217	377599	7.05	377482	79.18	0.0310	377617	11.74	377482	103.20	0.0356
218	375081	6.17	375081	63.32	0.0000	375084	7.85	375081	68.19	0.0010
219	393946	7.64	393506	84.50	0.1117	393959	12.47	393506	87.70	0.1149
220	392234	6.39	392017	54.44	0.0554	392269	8.09	392017	66.37	0.0641
231	408749	6.90	407940	70.76	0.1978	408761	9.59	407940	62.54	0.2008
232	414746	6.02	413537	69.60	0.2915	414758	8.82	413537	80.87	0.2943
233	330162	8.49	329658	67.35	0.1527	330163	11.24	329658	79.33	0.1531
234	376636	8.82	376602	90.00	0.0091	376663	11.15	376602	89.02	0.0161
235	289412	9.45	286537	87.97	0.9934	289414	11.25	286537	83.58	0.9940
236	382505	6.36	382274	57.25	0.0602	382550	8.56	382274	68.39	0.0720
237	377599	7.87	377482	69.50	0.0310	377617	10.77	377482	91.22	0.0356
238	375081	5.82	375081	71.02	0.0000	375084	7.17	375081	66.60	0.0010
239	393946	6.98	393506	82.23	0.1117	393959	9.50	393506	95.41	0.1149
240	392234	5.86	392017	64.85	0.0554	392269	7.60	392017	61.71	0.0641
Average		4.83		32.79	0.1568		10.01		35.83	1.2659

* CPU is the solution time for the related formulation.
LDiff %, given by % (LB1-LB2)/LB1, denotes the difference between the costs obtained by different lower bounding procedures, where LB1 and LB2 are the total costs provided by LP relaxation of LRP and IM, respectively.

APPENDIX I

UPPER BOUNDS OBTAINED WITH LRP-NM-1 IN ME

Below are the upper bounds given by LRP-NM-1 in main experiment (ME). If the first integer feasible solution (FIS) of LRP-MIP is identified in less than 600 CPU seconds, another upper bound is identified over the solution of LRP-MIP in 600 CPU seconds.

Table I.1 Upper bounds with FIS and the solution in the time limit (TL) by LRP-NM-1 in ME *

	Over FIS		Over TL	
Instance	UB1	CPU	UB2	UDiff %
121	775097	8267		
122	869551	63171		
123	667805	119510		
124	738676	1344		
125	NFS			
126	755668	2389		
127	704162	31939		
128	715858	418	709227	0.94
129	NFS			
130	732178	2248		
141	775063	455	749299	3.44
142	823613	390	806879	2.07
143	652917	396	645251	1.19
144	744101	447	723136	2.90
145	553825	386	552475	0.24
146	777148	306	744206	4.43
147	702620	472	682210	2.99
148	733367	284	711090	3.13
149	688400	419	681333	1.04
150	727103	268	713569	1.90
151	975559	6608		
152	1060872	53808		
153	NFS			
154	939589	58212		
155	NFS			
156	972511	22724		
157	NFS			
158	921732	7628		
159	882992	28536		
160	931119	8546		
171	975408	336	975588	-0.02
172	1062675	290	1062908	-0.02
173	830908	256	831300	-0.05
174	934672	404	937365	-0.29
175	696562	187	687883	1.26
176	966307	163	964721	0.16
177	883478	278	883725	-0.03
178	920210	167	917740	0.27
179	880123	259	879953	0.02
180	929321	250	929196	0.01

Table I.1 (continued)

	Over FIS		Over TL	
Instance	UB1	CPU	UB2	UDiff %
181	423001	1744		
182	433412	24193		
183	NFS			
184	399841	70424		
185	NFS			
186	405665	18587		
187	402699	50017		
188	400069	10342		
189	398547	22786		
190	415812	32300		
201	417941	396	415082	0.69
202	430330	272	426712	0.85
203	350464	247	347383	0.89
204	390509	304	393460	-0.75
205	310743	198	297591	4.42
206	400855	164	399863	0.25
207	390729	291	391105	-0.10
208	395507	144	395345	0.04
209	389140	219	391487	-0.60
210	410946	256	408935	0.49
211	NFS			
212	546198	30406		
213	450645	58818		
214	NFS			
215	NFS			
216	NFS			
217	503127	52943		
218	NFS			
219	NFS			
220	515539	24631		
231	540646	140	526523	2.68
232	549286	110	539085	1.89
233	460450	117	445255	3.41
234	511755	144	493294	3.74
235	413327	106	395780	4.43
236	523805	108	510968	2.51
237	517902	117	502322	3.10
238	513609	65	502487	2.21
239	513516	99	505284	1.63
240	521169	129	505828	3.03
Average		12277		1.47

* UB1 is the upper bound obtained over the first integer feasible solution of LRP-MIP.
UB2 is the upper bound obtained over the solution identified by LRP-MIP in 600 CPU seconds.
UDiff %, given by % (UB1-UB2)/UB2, denotes the difference between the costs obtained by LRP-NM-1 over the first integer solution of LRP-MIP and over the solution LRP-MIP provides in 600 CPU seconds.
NFS denotes that an integer feasible solution can not be identified for the related instance.
The empty cells under the column 'over TL' are the instances, for which the first integer feasible solutions are identified in more than 600 CPU seconds. Thus, they are not solved again with a time limit.
Shaded cells show the instances, for which the solutions in 600 CPU seconds, are worse than the solutions obtained with the other termination rule of finding the first integer solution.

APPENDIX J

UPPER AND LOWER BOUNDS OBTAINED WITH LRP-NM-1 IN ME

The upper bounds (UB) and lower bounds (LB) provided by the lagrangean relaxation solution procedure for large-scale problems (LRP-NM-1) in main experiment (ME), the solution times of the methods and the gaps between the bounds are provided below.

Table J.1 Upper and lower bounds obtained by the procedure LRP-NM-1 in ME *

	LRP-NM-1		LRP-LP		
Instance	UB	CPU	LB	CPU	LUB%
121	775097	8267	615460	10	25.94
122	869551	63171	657499	15	32.25
123	667805	119510	503206	10	32.71
124	738676	1344	592218	13	24.73
125	NFS		411641	12	
126	755668	2389	595706	8	26.85
127	704162	31939	536452	9	31.26
128	709227	600	579296	10	22.43
129	NFS		537542	12	
130	732178	2248	571351	15	28.15
141	749299	600	615297	9	21.78
142	806879	600	657368	12	22.74
143	645251	600	503032	6	28.27
144	723136	600	592077	10	22.14
145	552475	600	411432	10	34.28
146	744206	600	595624	6	24.95
147	682210	600	536299	6	27.21
148	711090	600	579199	10	22.77
149	681333	600	537344	8	26.80
150	713569	600	571205	10	24.92
151	975559	6608	895565	17	8.93
152	1060872	53808	974395	12	8.87
153	NFS		750190	10	
154	939589	58212	851075	10	10.40
155	NFS		615686	15	
156	972511	22724	882491	11	10.20
157	NFS		798876	12	
158	921732	7628	830352	7	11.00
159	882992	28536	805951	11	9.56
160	931119	8546	844720	12	10.23
171	975408	336	895565	11	8.92
172	1062675	290	974395	6	9.06
173	830908	256	750190	9	10.76
174	934672	404	851072	8	9.82
175	687883	600	615686	11	11.73
176	964721	600	882491	6	9.32
177	883478	278	798875	7	10.59
178	917740	600	830352	6	10.52
179	879953	600	805951	9	9.18
180	929196	600	844720	10	10.00

Table J.1 (continued)

	LRP-NM-1		LRP-LP		
Instance	UB	CPU	LB	CPU	LUB%
181	423001	1744	339115	12	24.74
182	433412	24193	343576	9	26.15
183	NFS		269798	12	
184	399841	70424	311194	12	28.49
185	NFS		231549	17	
186	405665	18587	317996	13	27.57
187	402699	50017	308177	12	30.67
188	400069	10342	309908	6	29.09
189	398547	22786	318678	9	25.06
190	415812	32300	328260	9	26.67
201	415082	600	339115	12	22.40
202	426712	600	343576	6	24.20
203	347383	600	269798	8	28.76
204	390509	304	311191	11	25.49
205	297591	600	231549	10	28.52
206	399863	600	317996	6	25.74
207	390729	291	308176	8	26.79
208	395345	600	309908	7	27.57
209	389140	219	318678	9	22.11
210	408935	600	328260	8	24.58
211	NFS		408761	10	
212	546198	30406	414758	9	31.69
213	450645	58818	330163	11	36.49
214	NFS		376663	11	
215	NFS		289414	12	
216	NFS		382550	12	
217	503127	52943	377617	12	33.24
218	NFS		375084	8	
219	NFS		393959	12	
220	515539	24631	392269	8	31.43
231	526523	600	408761	10	28.81
232	539085	600	414758	9	29.98
233	445255	600	330163	11	34.86
234	493294	600	376663	11	30.96
235	395780	600	289414	11	36.75
236	510968	600	382550	9	33.57
237	502322	600	377617	11	33.02
238	502487	600	375084	7	33.97
239	505284	600	393959	10	28.26
240	505828	600	392269	8	28.95
Average		12452		10	23.52

* CPU is the solution time for the related formulation.
LUB%, given by % (UB-LB)/LB, denotes the gap between the upper and lower bounds.
NFS denotes that an integer feasible solution can not be identified for the related instance.

APPENDIX K

RESULTS FOR LP RELAXATION OF APT IN ME

Results obtained with LP relaxation of the model with a priori tour (APT-LP) during main experiments (ME) are provided in this part, together with the results obtained when modified Barany et al. cuts are included in this formulation.

Table K.1 Results of APT-LP without and with modified Barany et al. cuts in ME *

	APT-LP		APT-LP with cuts		
Instance	TC	CPU	TC	CPU	Imp %
121	615927	2.13	615927	2.83	0.0000
122	660270	3.42	661071	4.35	0.1214
123	505049	2.66	505723	3.10	0.1334
124	594125	2.28	594467	2.64	0.0576
125	418344	4.14	418622	4.89	0.0665
126	601039	2.48	601604	3.07	0.0939
127	536794	3.70	537416	4.34	0.1159
128	586536	2.13	586556	2.40	0.0034
129	531148	2.58	531235	3.12	0.0163
130	571197	2.38	571532	2.86	0.0587
141	615927	2.27	615927	3.01	0.0000
142	660270	3.56	661071	4.30	0.1214
143	505049	2.86	505723	3.41	0.1334
144	594125	2.39	594467	2.57	0.0576
145	418344	3.96	418622	5.03	0.0665
146	601039	2.39	601604	3.21	0.0939
147	536794	3.87	537416	4.32	0.1159
148	586536	2.13	586556	2.47	0.0034
149	531148	2.65	531235	3.27	0.0163
150	571197	2.37	571532	2.71	0.0587
151	913533	9.93	913533	9.52	0.0000
152	996919	10.49	997025	12.21	0.0106
153	771974	9.08	772087	12.61	0.0146
154	873347	9.21	873421	10.17	0.0084
155	643216	14.13	643216	15.87	0.0000
156	908996	12.12	909008	14.84	0.0013
157	818829	10.24	818919	10.31	0.0110
158	855797	6.57	855799	9.34	0.0003
159	821867	6.48	821867	8.04	0.0000
160	864101	8.85	864109	10.19	0.0010
171	913533	7.76	913533	9.67	0.0000
172	996919	9.18	997025	10.84	0.0106
173	771974	9.33	772087	10.05	0.0146
174	873347	8.09	873421	9.38	0.0084
175	643216	11.72	643216	14.74	0.0000
176	908996	9.40	909008	10.54	0.0013
177	818829	8.69	818919	11.79	0.0110
178	855797	6.54	855799	7.79	0.0003
179	821867	7.00	821867	7.92	0.0000
180	864101	9.30	864109	10.16	0.0010

Table K.1 (continued)

	APT-LP		APT-LP with cuts		
Instance	TC	CPU	TC	CPU	Imp %
181	357083	7.35	357083	8.60	0.0000
182	366100	10.43	366206	12.48	0.0289
183	291583	10.09	291696	12.84	0.0387
184	333467	10.53	333540	11.04	0.0220
185	259080	14.31	259080	20.60	0.0000
186	344502	12.26	344513	12.70	0.0033
187	328130	8.88	328220	10.87	0.0274
188	335353	7.06	335356	8.76	0.0008
189	334594	8.87	334594	9.79	0.0000
190	347641	10.19	347649	11.83	0.0024
201	357083	8.08	357083	9.95	0.0000
202	366100	8.42	366206	9.94	0.0289
203	291583	9.33	291696	10.59	0.0387
204	333467	9.72	333540	10.56	0.0220
205	259080	11.61	259080	13.14	0.0000
206	344502	8.82	344513	11.16	0.0033
207	328130	8.53	328220	9.98	0.0274
208	335353	7.11	335356	9.66	0.0008
209	334594	7.28	334594	7.85	0.0000
210	347641	9.91	347649	10.50	0.0024
211	428301	12.30	428301	15.20	0.0000
212	438486	11.97	438486	15.26	0.0000
213	353432	13.21	353432	19.69	0.0000
214	400592	12.98	400592	16.19	0.0000
215	317942	20.16	317942	20.06	0.0000
216	410066	13.84	410066	13.18	0.0000
217	399512	14.49	399512	17.84	0.0000
218	401981	8.89	401981	12.19	0.0000
219	412850	11.73	412850	16.13	0.0000
220	412921	10.85	412921	16.13	0.0000
231	428301	8.72	428301	14.29	0.0000
232	438486	11.92	438486	14.30	0.0000
233	353431	14.22	353431	21.50	0.0000
234	400592	11.27	400592	18.07	0.0000
235	317942	17.19	317942	20.51	0.0000
236	410066	11.39	410066	13.54	0.0000
237	399512	12.80	399512	18.18	0.0000
238	401981	8.98	401981	14.35	0.0000
239	412850	13.02	412850	17.04	0.0000
240	412921	11.75	412921	13.79	0.0000
Average		8.51		10.43	0.0209

* TC is the total cost incurred in the solution with the related formulation.
CPU is the solution time for the related formulation.
Imp %, given by % (TC with cuts-TC without cuts)/TC without cuts, denotes the improvement obtained with modified Barany et al. cuts over the formulation without the cuts.

APPENDIX L

UPPER BOUNDS OBTAINED WITH APT IN ME

Below are the upper bounds given by APT-MIP in main experiment (ME). If the first integer feasible solution (FIS) for APT-MIP is identified in less than 3600 CPU seconds, another upper bound is obtained by solving APT-MIP for 3600 CPU seconds.

Table L.1 Upper bounds with FIS and the solution in the time limit (TL) by APT in ME *

	Over FIS		Over TL	
Instance	UB1	CPU	UB2	UDiff %
121	757152	1044	753208	0.52
122	828336	12467		
123	649998	37838		
124	744231	2483	737665	0.89
125	NFS			
126	758259	21620		
127	695488	27405		
128	725733	638	721371	0.60
129	699096	49067		
130	734132	21359		
141	773872	145	740360	4.53
142	812981	122	792939	2.53
143	644216	172	633805	1.64
144	731843	109	708182	3.34
145	552416	127	542050	1.91
146	748348	119	722384	3.59
147	684577	133	671880	1.89
148	724913	99	686127	5.65
149	678966	125	672151	1.01
150	723136	138	702057	3.00
151	968731	2311	968731	0.00
152	1047426	1533	NFS	
153	821935	31270		
154	931995	2511	926119	0.63
155	700466	28395		
156	962714	1696	962714	0.00
157	873949	24130		
158	904885	5618		
159	876827	37777		
160	916885	5111		
171	966246	216	942330	2.54
172	1038805	199	1027000	1.15
173	817652	322	804679	1.61
174	921090	189	901555	2.17
175	685521	213	669071	2.46
176	958729	225	935417	2.49
177	871791	269	853004	2.20
178	899721	156	886147	1.53
179	871472	190	855328	1.89
180	919875	205	894811	2.80

Table L.1 (continued)

	Over FIS		Over TL	
Instance	UB1	CPU	UB2	UDiff %
181	413090	4360		
182	419284	1409	413837	1.32
183	347424	46613		
184	388060	8167		
185	308502	19950		
186	397778	19120		
187	387015	5911		
188	386122	5519		
189	388506	23076		
190	401461	6806		
201	407036	222	391443	3.98
202	409555	203	396357	3.33
203	336366	277	321227	4.71
204	379174	205	356123	6.47
205	306901	254	284477	7.88
206	387018	210	368559	5.01
207	379550	291	358813	5.78
208	381676	171	365309	4.48
209	381811	232	366898	4.06
210	396148	233	377749	4.87
211	516660	29924		
212	528358	1445	528358	0.00
213	NFS			
214	483398	14593		
215	406625	2395	NFS	
216	496718	5880		
217	487628	10167		
218	478060	21073		
219	496816	5981		
220	NFS			
231	518150	214	496391	4.38
232	519143	138	498113	4.22
233	435499	223	416099	4.66
234	483286	131	455983	5.99
235	402284	161	376601	6.82
236	505799	161	477934	5.83
237	516145	176	469489	9.94
238	477249	144	458363	4.12
239	492444	188	482206	2.12
240	493761	181	475425	3.86
Average		7197		3.26

* UB1 is the upper bound given by the first integer feasible solution of APT-MIP.
UB2 is the upper bound given by the solution identified by APT-MIP in 3600 CPU seconds.
UDiff %, given by % (UB1-UB2)/UB2, denotes the difference between the costs of the first integer feasible solution of APT-MIP and the solution APT-MIP provides in 3600 CPU seconds.
CPU is the solution time for the related formulation.
NFS denotes that an integer feasible solution can not be identified for the related instance.
The empty cells under the column 'over TL' are the instances, for which the first integer feasible solutions are identified in more than 3600 CPU seconds. Thus, they are not solved again with a time limit.

APPENDIX M

UPPER AND LOWER BOUNDS OBTAINED FOR APT IN ME

Upper bounds (UB) for the model with a priori tour given by MIP solution of APT (APT-MIP), lower bounds (LB) given by LP relaxation of APT (APT-LP) in main experiment (ME), solution times and gaps between the bounds are provided below.

Table M.1 Upper and lower bounds obtained by APT in ME *

	APT-MIP		APT-LP		
Instance	UB	CPU	LB	CPU	LUB%
121	753208	3600	615927	2.83	22.29
122	828336	12467	661071	4.35	25.30
123	649998	37838	505723	3.10	28.53
124	737665	3600	594467	2.64	24.09
125	NFS		418622	4.89	
126	758259	21620	601604	3.07	26.04
127	695488	27405	537416	4.34	29.41
128	721371	3600	586556	2.40	22.98
129	699096	49067	531235	3.12	31.60
130	734132	21359	571532	2.86	28.45
141	740360	3600	615927	3.01	20.20
142	792939	3600	661071	4.30	19.95
143	633805	3600	505723	3.41	25.33
144	708182	3600	594467	2.57	19.13
145	542050	3600	418622	5.03	29.48
146	722384	3600	601604	3.21	20.08
147	671880	3600	537416	4.32	25.02
148	686127	3600	586556	2.47	16.98
149	672151	3600	531235	3.27	26.53
150	702057	3600	571532	2.71	22.84
151	968731	2311	913533	9.52	6.04
152	1047426	1533	997025	12.21	5.06
153	821935	31270	772087	12.61	6.46
154	926119	3600	873421	10.17	6.03
155	700466	28395	643216	15.87	8.90
156	962714	1696	909008	14.84	5.91
157	873949	24130	818919	10.31	6.72
158	904885	5618	855799	9.34	5.74
159	876827	37777	821867	8.04	6.69
160	916885	5111	864109	10.19	6.11
171	942330	3600	913533	9.67	3.15
172	1027000	3600	997025	10.84	3.01
173	804679	3600	772087	10.05	4.22
174	901555	3600	873421	9.38	3.22
175	669071	3600	643216	14.74	4.02
176	935417	3600	909008	10.54	2.91
177	853004	3600	818919	11.79	4.16
178	886147	3600	855799	7.79	3.55
179	855328	3600	821867	7.92	4.07
180	894811	3600	864109	10.16	3.55

Table M.1 (continued)

	APT-MIP		APT-LP		
Instance	UB	CPU	LB	CPU	LUB%
181	413090	4360	357083	8.60	15.68
182	413837	3600	366206	12.48	13.01
183	347424	46613	291696	12.84	19.11
184	388060	8167	333540	11.04	16.35
185	308502	19950	259080	20.60	19.08
186	397778	19120	344513	12.70	15.46
187	387015	5911	328220	10.87	17.91
188	386122	5519	335356	8.76	15.14
189	388506	23076	334594	9.79	16.11
190	401461	6806	347649	11.83	15.48
201	391443	3600	357083	9.95	9.62
202	396357	3600	366206	9.94	8.23
203	321227	3600	291696	10.59	10.12
204	356123	3600	333540	10.56	6.77
205	284477	3600	259080	13.14	9.80
206	368559	3600	344513	11.16	6.98
207	358813	3600	328220	9.98	9.32
208	365309	3600	335356	9.66	8.93
209	366898	3600	334594	7.85	9.65
210	377749	3600	347649	10.50	8.66
211	516660	29924	428301	15.20	20.63
212	528358	1445	438486	15.26	20.50
213	NFS		353432	19.69	
214	483398	14593	400592	16.19	20.67
215	406625	2395	317942	20.06	27.89
216	496718	5880	410066	13.18	21.13
217	487628	10167	399512	17.84	22.06
218	478060	21073	401981	12.19	18.93
219	496816	5981	412850	16.13	20.34
220	NFS		412921	16.13	
231	496391	3600	428301	14.29	15.90
232	498113	3600	438486	14.30	13.60
233	416099	3600	353431	21.50	17.73
234	455983	3600	400592	18.07	13.83
235	376601	3600	317942	20.51	18.45
236	477934	3600	410066	13.54	16.55
237	469489	3600	399512	18.18	17.52
238	458363	3600	401981	14.35	14.03
239	482206	3600	412850	17.04	16.80
240	475425	3600	412921	13.79	15.14
Average		9098		10.43	14.89

* CPU is the solution time for the related formulation.
LUB%, given by % (UB-LB)/LB, denotes the gap between the upper and lower bounds.
NFS denotes that an integer feasible solution can not be identified for the related instance.

APPENDIX N

COMPARISON OF THE UPPER BOUNDS OBTAINED IN ME

Comparisons of the best upper bounds obtained by lagrangean relaxation solution procedure for large-scale problems (LRP-NM-1) and by the model with a priori tour (APT-MIP) are presented below for the instances, for which both methods can identify integer feasible solutions.

Table N.1 Comparison of the upper bounds obtained with LRP-NM-1 and APT-MIP in ME *

	LRP-NM-1		APT-MIP		
Instance	UB1	CPU	UB2	CPU	UDiff %
121	775097	8267	753208	3600	2.91
122	869551	63171	828336	12467	4.98
123	667805	119510	649998	37838	2.74
124	738676	1344	737665	3600	0.14
126	755668	2389	758259	21620	-0.34
127	704162	31939	695488	27405	1.25
128	709227	600	721371	3600	-1.68
130	732178	2248	734132	21359	-0.27
141	749299	600	740360	3600	1.21
142	806879	600	792939	3600	1.76
143	645251	600	633805	3600	1.81
144	723136	600	708182	3600	2.11
145	552475	600	542050	3600	1.92
146	744206	600	722384	3600	3.02
147	682210	600	671880	3600	1.54
148	711090	600	686127	3600	3.64
149	681333	600	672151	3600	1.37
150	713569	600	702057	3600	1.64
151	975559	6608	968731	2311	0.70
152	1060872	53808	1047426	1533	1.28
154	939589	58212	926119	3600	1.45
156	972511	22724	962714	1696	1.02
158	921732	7628	904885	5618	1.86
159	882992	28536	876827	37777	0.70
160	931119	8546	916885	5111	1.55
171	975408	336	942330	3600	3.51
172	1062675	290	1027000	3600	3.47
173	830908	256	804679	3600	3.26
174	934672	404	901555	3600	3.67
175	687883	600	669071	3600	2.81
176	964721	600	935417	3600	3.13
177	883478	278	853004	3600	3.57
178	917740	600	886147	3600	3.57
179	879953	600	855328	3600	2.88
180	929196	600	894811	3600	3.84

Table N.1 (continued)

	LRP-NM-1		APT-MIP		
Instance	UB1	CPU	UB2	CPU	UDiff %
181	423001	1744	413090	4360	2.40
182	433412	24193	413837	3600	4.73
184	399841	70424	388060	8167	3.04
186	405665	18587	397778	19120	1.98
187	402699	50017	387015	5911	4.05
188	400069	10342	386122	5519	3.61
189	398547	22786	388506	23076	2.58
190	415812	32300	401461	6806	3.57
201	415082	600	391443	3600	6.04
202	426712	600	396357	3600	7.66
203	347383	600	321227	3600	8.14
204	390509	304	356123	3600	9.66
205	297591	600	284477	3600	4.61
206	399863	600	368559	3600	8.49
207	390729	291	358813	3600	8.89
208	395345	600	365309	3600	8.22
209	389140	219	366898	3600	6.06
210	408935	600	377749	3600	8.26
212	546198	30406	528358	1445	3.38
217	503127	52943	487628	10167	3.18
231	526523	600	496391	3600	6.07
232	539085	600	498113	3600	8.23
233	445255	600	416099	3600	7.01
234	493294	600	455983	3600	8.18
235	395780	600	376601	3600	5.09
236	510968	600	477934	3600	6.91
237	502322	600	469489	3600	6.99
238	502487	600	458363	3600	9.63
239	505284	600	482206	3600	4.79
240	505828	600	475425	3600	6.39
Average		11552		6482	3.86

* UB1 is the best upper bound provided by LRP-NM-1.
UB2 is the best upper bound provided by APT-MIP.
UDiff %, given by % (UB1-UB2)/UB2, denotes the difference between the costs of the solutions provided by LRP-NM-1 and APT-MIP.
CPU is the solution time for the related method.

APPENDIX O

UPPER BOUND BENCHMARKING FOR ME

The best upper bounds obtained during the main experiment (ME) are compared to the solutions provided by a trivial heuristic 'every' and the solutions of Bertazzi et al. (2002).

Table O.1 Comparison with the results of 'Every' and Bertazzi et al. (2002) in ME *

		Every		BPS	
Instance	Best UB	UBe	UDiff E %	UBb	UDiff B %
121	753208	1039269	37.98	712497	5.71
122	828336	1086268	31.14	766561	8.06
123	649998	869906	33.83	606824	7.11
124	737665	967435	31.15	681950	8.17
125	NFS	733774		518729	
126	755668	995206	31.70	700646	7.85
127	695488	935759	34.55	644218	7.96
128	709227	948758	33.77	664914	6.66
129	699096	937486	34.10	648914	7.73
130	732178	955601	30.51	675101	8.45
141	740360	1039269	40.37	700530	5.69
142	792939	1086268	36.99	757277	4.71
143	633805	869906	37.25	596325	6.29
144	708182	967435	36.61	671001	5.54
145	542050	733774	35.37	505109	7.31
146	722384	995206	37.77	691880	4.41
147	671880	935759	39.27	633749	6.02
148	686127	948758	38.28	657162	4.41
149	672151	937486	39.48	638864	5.21
150	702057	955601	36.11	668728	4.98
151	968731	1080390	11.53	942084	2.83
152	1047426	1133186	8.19	1026314	2.06
153	821935	912950	11.07	804518	2.16
154	926119	1012060	9.28	909074	1.87
155	700466	773128	10.37	674949	3.78
156	962714	1041753	8.21	945787	1.79
157	873949	975578	11.63	852227	2.55
158	904885	991600	9.58	886139	2.12
159	876827	981955	11.99	854803	2.58
160	916885	997606	8.80	895482	2.39
171	942330	1080390	14.65	931262	1.19
172	1027000	1133186	10.34	1018474	0.84
173	804679	912950	13.46	791250	1.70
174	901555	1012060	12.26	890032	1.29
175	669071	773128	15.55	657206	1.81
176	935417	1041753	11.37	926520	0.96
177	853004	975578	14.37	839862	1.56
178	886147	991600	11.90	875680	1.20
179	855328	981955	14.80	844427	1.29
180	894811	997606	11.49	882535	1.39

Table O.1 (continued)

		Every		BPS	
Instance	Best UB	UBe	UDiff E %	UBb	UDiff B %
181	413090	523940	26.83	387493	6.61
182	413837	502367	21.39	396061	4.49
183	347424	432558	24.50	320465	8.41
184	388060	472179	21.68	367308	5.65
185	308502	388992	26.09	290882	6.06
186	397778	477258	19.98	377025	5.50
187	387015	484879	25.29	363606	6.44
188	386122	471157	22.02	363856	6.12
189	388506	494682	27.33	368798	5.34
190	401461	481146	19.85	382672	4.91
201	391443	523940	33.85	375088	4.36
202	396357	502367	26.75	386512	2.55
203	321227	432558	34.66	310518	3.45
204	356123	472179	32.59	350133	1.71
205	284477	388992	36.74	272947	4.22
206	368559	477258	29.49	362484	1.68
207	358813	484879	35.13	349121	2.78
208	365309	471157	28.97	355165	2.86
209	366898	494682	34.83	357028	2.76
210	377749	481146	27.37	365892	3.24
211	516660	565062	9.37	505559	2.20
212	528358	549286	3.96	521664	1.28
213	450645	475602	5.54	429730	4.87
214	483398	516804	6.91	468411	3.20
215	406625	428346	5.34	383325	6.08
216	496718	523805	5.45	491056	1.15
217	487628	524699	7.60	478558	1.90
218	478060	513999	7.52	473181	1.03
219	496816	539151	8.52	488550	1.69
220	515539	523151	1.48	491793	4.83
231	496391	565062	13.83	477960	3.86
232	498113	549286	10.27	490768	1.50
233	416099	475602	14.30	401944	3.52
234	455983	516804	13.34	447875	1.81
235	376601	428346	13.74	359044	4.89
236	477934	523805	9.60	461669	3.52
237	469489	524699	11.76	451131	4.07
238	458363	513999	12.14	444994	3.00
239	482206	539151	11.81	460499	4.71
240	475425	523151	10.04	460330	3.28
Average			20.82		3.94

* Best UB is the best upper bound given by our methods.
UBe is the total cost provided by 'every'.
UDiff E %, given by % (UBe-BestUB)/BestUB, denotes the difference between our upper bounds and UBe
UBb is the total cost provided by Bertazzi et al. (2002)
UDiff B %, given by % (BestUB-UBb)/UBb, denotes the difference between our upper bounds and UBb
NFS denotes that an integer feasible solution can not be identified for the related instance.