INFORMATION THEORY, ENTROPY
AND
URBAN SPATIAL STRUCTURE

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ABSTRACT

INFORMATION THEORY,ENTROPY
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Urban planning has witnessed the profound changes in the methodologies of modelling during the last 50 years. Spatial interaction models have passed from social physics, statistical mechanics to non-spatial and spatial information processing stages of progress that can be designated as paradigm shifts.

This thesis traces the Maximum Entropy (MaxEnt) approach in urban planning as pioneered by Wilson (1967,1970) and Spatial Entropy concept by Batty (1974) based on the Information Theory and its developments by Shannon (1948), Jaynes (1957), Kullback (1959) and by Tribus (1962,1969).

Information-theoric methods have provided the theoretical foundation for challenging the uncertainty and incomplete information issues concerning the complex urban structure. MaxEnt, as a new logic, gives probabilities maximally noncommittal with regard to missing information. Wilson (1967,1970) has replaced the Newtonian analogy by the entropy concept from statistical mechanics to alleviate the mathematical inconsistency in the gravity model and developed a set of spatial interaction models consistent with the known information.

Population density distribution as one of the determinants of the urban structure has been regarded as an exemplar to show the paradigm changes from the analysis of density gradients to the probabilistic description of density distributions by information-theoric methods.

Spatial Entropy concept has introduced the spatial dimension to the Information Theory. Thesis applies Spatial Entropy measures to Ankara 1970 and 1990 census data by 34 zones and also obtains Kullback's Information Gain measures for population changes during the two decades.

Empirical findings for Spatial Entropy measures show that overall Ankara-1970 and 1990 density distributions are ''Uneven'' and the uniform distribution hypothesis is not confirmed. These measures also indicate a tendency towards "More Uniformity" for density distributions in comparison to 1970. Information Gain measure for population changes also deviates from zero and direct proportionality hypothesis between posterior 1990 and prior 1970 population distributions by zones is not confirmed.

Current research is focused on information processing with more engagement in the urban spatial structure and human behavior. This thesis aims to participate with these efforts and concludes that Information Theory has the potential to generate new profound changes in urban planning and modelling processes.

Keywords : Information Theory, Spatial Entropy, Maximum Entropy, Urban Modelling Process, Ankara Maxent

ÖZ

ENFORMASYON KURAMI, ENTROPİ
VE
KENTSEL MEKAN YAPISI

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Kent planlaması son 50 yılda önemli değişmelere tanık olmuştur. Mekansal etkileşim modelleri, birer paradigma değişimleri olarak tanımlanabilecek sosyal fizik,istatistiksel mekanikten mekansal-olmayan ve mekansal enformasyon işlem aşamalarına geçişle ilerlemiştir.

Tez, Shannon (1948), Jaynes (1957), Kullback (1959), ve Tribus (1962,1969) tarafından geliştirilmiş olan Enformasyon Kuramına dayalı olarak, kent planlamasında Wilson (1967,1970) öncülüğündeki Maksimum Entropi (MaxEnt) ve Batty'nin(1974) Mekansal Entropi yöntemlerinin ortaya çıkışını izler.

Enformasyon-Kuramsal yöntemleri, kentin karmaşık yapısına ilişkin belirsizlik ve eksik veri sorunları ile başedebilecek kuramsal temeli hazırlamıştır. Yeni bir mantık olarak MaxEnt , elde olmayan verilere olabildiğince bağlı kalmadan olasılık dağılımlarını hesaplamaktadır. Wilson (1967,1970), Çekim Modelinin matematiksel tutarsızlığını ortadan kaldırmak için modelin Newton'cu çekim analojisi yerine istatistiksel mekaniğin entropi kavramını yerleştirmiştir. Mekansal Entropi kavramı ise Enformasyon Kuramına mekansal boyut kazandırmıştır.

Nüfus yoğunluğunun dağılım konusu, kentsel yapının önemli bir belirleyicisi ve eğim analizlerinden yoğunluk dağılımlarının olasılıksal tanımına doğru bir paradigma değişim örneği olarak tezde ele alınmıştır.

Tez, Ankara-1970 ve 1990 nüfus sayım verilerine ve 34 alt-bölgeye göre nüfus yoğunluğunun Mekansal Entropi ölçülerini bulmakta ve nüfusun yirmi yıllık dönem içindeki değişimlerini Kullback'ın Enformasyon Kazanım ölçüsü ile hesaplamaktadır.

Mekansal Entropi ölçüsü üzerinden varılan bulgular, Ankara-1970 ve 1990 nüfus yoğunluğu dağılımlarının her alt-bölgede ''Tekdüze Olamadığını'' göstermiş ve ilişkin tekdüze dağılım varsayımı doğrulanmamıştır.Sonuçlara göre Ankara-1990 , Ankara-1970'den ''Daha Tekdüze '' dağılıma doğru bir geçiş yaşamıştır.Salt nüfus değişimlerinin Enformasyon Kazanım ölçüsü ile incelenmesinde ise , bulguların sıfır değerinden uzaklaştığı ve sonraki-1990 nüfus dağılımlarının önceki-1970 dağılımları ile doğrudan orantılı olduğuna ilişkin varsayımın desteklenmediği görülmüştür.

Varolan araştırmalar, kentsel mekan yapısı ve insan davranışları ile daha çok ilgilenen enformasyon işlem süreçlerine odaklanmıştır.Tez, bu yöndeki yoğun çabalara katılmak üzere, Enformasyon Kuramının kentsel planlama ve modelleme süreçlerinde yeni ve derin değişimler yaratacak bir potansiyel taşıdığını savunmaktadır.

Anahtar Kelimeler : Enformasyon Kuramı, Mekansal Entropi,Maksimum Entropi, Kentsel Modelleme, Ankara MaxEnt

ACKNOWLEDGEMENTS

This thesis tried to combine and unify different types of my personal academic interests and different approaches in the field of quantitative urban geography encountered during the researches and readings that took almost more than three decades, dating back to the early 1970s. from the vantage points of 2000s, years of the New Millennium, I have the opportunity to remember many colleagues from both national & international meetings that I participated, the additional courses that I took as a listener on Fortran-IV Programming from Dr. Hamit Fişek (1976); on statistics & probability from Dr. Çağlar Güven (1979/80) definitely provided stimuli in the evolution of my thoughts. French courses enabled me to follow such periodicals Urbanismé and to see the other side of the Channel Sea.

Such contributors would be not only too numerous to list, but it would be rather incomplete and even misleading. I hope these many unnamed contributors will not be offended by this short and general acknowledgement of their assistance. Hence, only the more recent contributions to the preparation of this thesis shall be acknowledged.

I should thank my supervisor Prof. Dr. Gönül Tankut for her tolerance and continuous interest in the field of "Information Theory" in the context of urban structure. Her explanations of the spatial developments in the "Ottoman City" by "Information Channels" and other related concepts of the Information Theory, should be the first of this type of analysis and evaluation during 1985s. perhaps it is still the only example in this field in Turkey.(*)

Prof. Dr. Murat Balamir provided me with the John Clark's seminal 1951 – paper "Urban Population Densities" many years ago.

Assoc. Prof. Dr. Serap Erdoğan Kayasü and Assoc. Prof. Dr. Gülden Berkman not only solved my procedural problems with the Graduate School but also did "Not Forget" my password during the registration days at each terms.

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Assoc. Prof. Dr. Baykan Günay and Assist. Prof. Dr. Adnan Barlas, with whom I shared the Urban Design Studios in recent years, encouraged me for thesis completion, as the reserve Committee members.

I am still keeping the book "Ankara: 1985'ten 2015'e" (1986) that I borrowed from Assoc. Dr. Murat Güvenç.

Assist. Prof. Dr. Yaşar Bahri Ergen and Assoc. Prof. Dr. Hülagü Kaplan, the colleagues that I met during my assignment at Gazi University, with respect to Law 2547/40-b, between years 1990-1992, who were in the Examining Committee list, in different periods, always presented their interests in the topic of the thesis on occasions we met each other.

My sincere thanks also should be extended to my formerly students who helped me on different cases. Ali Rıza Demirel, from the Regional Planning Graduate Program, not only typed the whole thesis full with difficult equations to follow, but also translated equations into Excel format for computations and developed the figures and maps by using his computer skills.I would be still calculating the ''Spatial Entropy '' measures for Ankara- 1970 & 1990 without his sincere help. Uğraş

Doyduk, Bülent Açıkgöz, Cihan Polat made connections to the State Institute of Statistics (SIS) and research assistants Fikret Zorlu, Tolga Levent, Olgu Çalışkan provided me the 1990 census data according to the zones and "mahalle" boundaries of the Ankara Metropolitan Area that they obtained from the Greater Municipality for the use of CP301-302: Planning Studio, in years 2002-2003 and other studios. I had the opportunity to compare and revise the census data that I collected previously with data they supplied me more recently. Definitely, all mistakes if made are mine.

My thanks are also due to İbrahim Dolanbay, at the Computer Room, for helping not only in typing but also producing the prints and solving the computer problems whenever they rose.

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Many thanks to my wife Bilge for her silent supports and patience that continued over years and also my mother born in 1919.

March 2005, METU

(*) Really sorry to write here that we lost Prof. Dr. Gönül Tankut on 27 April 2005.City Planners and the planning field shall miss her much.

PREFACE

This thesis represents only a small section of the different issue and topics that I was involved since 1970s. Yet, it shows one of the consequences of my engagements during the past decades. Over these years, I had the historical opportunity to read and witness the rise and the fall of the important theories that are cited in this study and my involvements in these topics became a part of my whole academic life. It is now my privilege to note here that the Conference on Urban Development Models, held in Cambridge University (UK) in 1974 was the first international meeting that I attended where I met the well-known researchers such as A. G. Wilson, M. Batty, Britton Harris, R. Baxter, J. March and other participants. After the Cambridge Conference, I ordered A. G. Wilson's (1974) book "Urban and Regional Models in Geography and Planning" and it was sent to me with his "compliments". Later I ordered his another pioneering 1970 book "Entropy in Urban and Regional Planning" to obtain more knowledge on the topic.

I'm glad to note here that Michael Batty, during his visit to our Department for a lecture on computer applications in urban planning, in March 1997, accepted my invitation to my course CP452 – Models In Urban Planning. The topic of the week was the relationship between the "Chaos Theory" and the urban models. He signed his 1976 book "Urban Modelling: Algorithms, Calibrations, Predictions" that I had (26 March 1997). On our trip to Ankara Citadel and the Museum of Anatolian Civilizations, we talked not only on the "Cambridge School" of thought but also on the "Spatial Entropy" concept that was originated by himself. As an architect, it was a surprise, i.e., "Information" for me to hear that he had also an architectural education background.

One year later, in the spring of 1998, Bernard Marchand from the Université de Paris VIII, lectured at the Department on the applications of the entropy concept in geographical studies. He also visited our CP402 – Istanbul Metropolitan Area Planning Studio during the preliminary jury of the student projects.

Myron Tribus, a contributor to the information theory, explained how he met with E. T. Jaynes at Stanford University in 1958 and how the MaxEnt Conferences became "International Workshops", in his paper "A Tribute to Edwin T. Jaynes" (1998) presented at the 18th International Workshop at the Max Planck Institute, Garching / Munich, during 27-31 July 1998 where I also attended and met M. Tribus.

I should also add Jay W. Forrester and his son Nathan Forrester, who chaired the session where I presented my " 'Carmen', Catastrophe Theory and System Dynamics: A Revitalization" in the International Conference on System Dynamics, Oct. 1986, Sevilla University / Spain. This paper drew the attention not only Javrier Aracil, the well-known Spanish dynamicist but also the Forresters. As indicated in my CV, included at the end, this paper was published in the Proceedings of the 12th Congress on Cybernetics, Namur / Belgique, 1990, based also on the "Fourth Cybernetics" that I proposed first in the previous 11th Congress in Namur, 1986.

The above lines cite some of the authors that I met in international meetings. I owe much also to the authors I could not meet personally. For example, if I would not have the chance of acquiring the book by Ian Masser (1972) "Analytical Models for Urban and Regional Planning", David & Charles Publishers, perhaps I would not fully understand the function of "K", the scaling factor in the calibration process of the Traditional Gravity Model in Equation I-8 and Equation III-42, and would not see how A. G. Wilson (1967, 1974) demonstrated the mathematical inconsistency in the "Gravity Model", as explained in Chapter-I.3.4.4

Although many articles mentioned about the "link" between the entropy concept in thermodynamics, statistical mechanics and the entropy in the Shannonian Information Theory; the "link" was not clear at all to me until I read James Pooler's (1983) article "Information Theoric Methods of Spatial Model Building", as explained in Chapter-III.7

Similarly, before my reading the now classical article by Jay W. Forrester and Peter M. Senge (1980) – "Tests For Building Confidence in System Dynamics Models", the terms like 'validation', 'verification', 'reliability', 'accuracy' or 'correctness', in relation to model evaluation or testing process did not mean much to me. Definitions of these and other similar terms were depending heavily on the textbooks on engineering and sometimes they were even contradictory to each other. The warning statements that

> We believe confidence is the proper criterion because there can be no proof of the absolute correctness with which a model represents reality. There is no method for providing a model to be correct. Einstein's theory of relativity has not been proven correct; it stands because it has not been disproved…(Forrester and Senge, 1980, pp.211)

were illuminating. Above authors maintained instead of single test, there should be more tests applied to building our "confidence" in a model. Hence, the normal engineering methods, depending on the measurements on the correspondence between the model outputs with the actual data, calculating the "Root Mean Square Error" (RMSE), or chi-squares for examples, do not provide us sufficient criteria to evaluate models, especially the complex and non-linear models, as 'valid' or 'correct'. I dealt with these important issues in my (Turkish) paper (2003) criticizing the existing methods of statistical test to evaluate the earthquake probability models.

Finally, I would like to note here that the pioneering 2 articles on "Information Theory & Statistical Mechanics" were published in "Physical Review" in May 1957 and October 1957 and these were cited in this thesis. However, the first article was sent to the periodical in Sept. 1956, i.e., in the foundation year of METU. Hence, Jaynesian "Information Theory" and METU have the same birthdates. In 2006, the next year, their 50^{th} Anniversary Years will be celebrated.

It took more than my 30 years, i.e., almost whole my academic lifetime; to read, to learn and follow the developments in the information theory with respect to urban planning. Moreover, 1957 is my entrance year to METU as a student of architecture.

All these plain facts should be sufficient to “justify” why I chose the Information Theory as the main topic of the thesis.

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CHAPTER I

INTRODUCTION

The history of urban modeling and planning demonstrate that profound transformations have occurred during the last 50 years and it is still continuing. In the quest for integration of both quantitative methods and quality in disciplines such as geography, sociology and urban studies have turned to modern physics or similar fields, in the hope that powerful analogies might exist and thus they may lead to more related theories and human behavior. The change in the urban studies and in the social sciences in general, which began in the late 1950s was founded on the belief that the progress in the knowledge could best be achieved by rigorous theory-building rather than by loose and not integrated speculations of the previous decades. This change in approach which has prevailed in every social science during the above two decades mainly has been referred in various ways, but the "Quantitative Revolution" and the "Systems Approach" are the two best-known terms summing up these developments. It should also be noted that these developments coincided and supported with the launching of large scale land use and transportation studies of the "First-Generation" urban models, mainly in the USA, and the use of computers that made these new approaches possible. The evaluations and relevance of these developments have been formally traced in the works of many authors, such as D.B. Lee (1973), Baxter (edit., et.al. 1974), M. Batty (1976, Chapter I: The Art of Urban Modeling, pp. 1-19 and 1979), Tocalis (1978) and including more recent developments by Wegener (1994).

T. Kuhn (1962, 1970) asserts that the scientific progress is achieved not by cumulative process or by an articulation or extension of the old paradigm (1979, p.84-85), but by successive periods of "Normal Science" interspersed by revolutionary "Paradigm Shifts". Kuhn used the notion of a "paradigm" to indicate existence of a coherent, unified viewpoint, a kind of "weltanschauung" which determines the way the scientists view the world and practice their craft. Kuhn (1970, pp.52-53) notes that discovery commences with the awareness of "anomaly" that somehow violated the paradigm-induced expectations that govern "normal science". "Normal science does not aim at novelties of fact or theory" and when successful, finds "no new sort of fact" (Kuhn, 1970, p.52 – 61).

In the traditional Kuhnian formulation above (1962, 1970) changes in science are caused by a "revolution" or a "Paradigm Shift" which can be represented briefly by a linkage of events as adapted from M. E. Harvey and B. P. Holly (1981, p.16):

Paradigm A → Normal Science → Anomalies → Crisis → Response to Crisis → Revolution → Paradigm B

If we take "Paradigm A" as the existing paradigm, there is generally a period of "Normal Science" when scientists practice within the dominant paradigm and accumulate knowledge. There is also a gradual accumulation of anomalies that cannot be solved by the existing Paradigm-A. As these increase, a crisis ends with the rejection of the old paradigm and accepting the new one. This results in a scientific revolution and hence the new Paradigm-B emerges.

This thesis and Chapter I maintain that the changes in the modeling approaches to the urban phenomena can be regarded as the "Paradigm Shifts" of the natural sciences in the above Kuhnian sense of the term. Chapter I aim to review the developments in the urban studies and to point out the problems aroused, are to be regarded as the "Anomalies" emerged during progress in the field. Such authors like Tocalis (1978), B. Berry (1978), E. Harvey and P. Holly (1981) draw a "Kuhnian Perspective" and accept that Wilson's introduction of "MaxEnt" method has caused a "Progress" in urban studies by changing the existing ways we look at the world. Similarly, contributions by Shannon (1948), E.T.Jaynes (1957), Kullback (1959) and Tribus (1962, 1969) are to be considered as the

‘’Paradigm Shifts’’, not solutions merely to the technical and or mathematical problems aroused in the information theory.

The structure of thesis is given in the following lines.

The aim of Chapter-I is to pose the nature of the description, explanation and prediction problems in complex urban structures in both spatial and socio-economic terms. Chapter starts with an overview of the study (Esmer,1979) on Ankara residential population density changes between years 1965-1970 and draws some general conclusions for the 3 decades between 1960-1990. Chapter compares 3 generation of urban models developed between 1960-1980s. To define the “Scope of the Thesis”, chapter explains some problematic issues in developing urban models. For example, First and Second Generation of Urban models depended on arbitrary assumptions leading to the unrelated and inconsistent results with respect to known data. Or, Gravity Models, as A. G. Wilson (1967, 1970) demonstrated, had also mathematical inconsistency in the formulation that cannot be solved by calibration methods. Chapter asserts that these and similar problems can be alleviated by using the MaxEnt method as an outcome of the scientific revolutions in the Information Theory.

Chapter-II introduces the “Kinds of Probability” and it compares the “Objective” and “Subjective” views of the probability concept. It asserts that “Long-Run Frequency” concept of probability of the “Objective” view cannot be achieved and introduces the Bayes Theorem of the Subjective view.

Chapter-III explains the concepts of “Uncertainty”, “Information” and “Entropy” in Shannon’s (1948), Kullback’s (1959) Information Theories and makes comparisons. Chapter summarizes the E. T. Jaynes’ “Maximum Entropy” (MaxEnt) Principle, to determine the distribution of probabilities with “incomplete information”. M. Tribus (1962, 1969) made an important contribution to the MaxEnt method by generalizing to any probability distribution. Chapter further introduced the MaxEnt method in urban transportation planning and how he extended the traditional “Gravity Model” to the 5 members of “Spatial Interaction

Models". The Chapter ends with the comparison of the now "Classical" and the recent "Quantum" information theories.

Chapter-IV aims to show the MaxEnt & Minimum Information Principle (MIP) give the same results. For the demonstration, a numerical example is used for a simple urban problem with 3-zone city.

Chapter-V is based on the concept of "Spatial Entropy" as originated by M. Batty (1974) where he introduced the areal sizes of zones as the spatial dimension in his formulations based on Kullback's Information Theory. Chapter also shows Adams & Storbeck (1983) used information theoric methods for the analysis of population changes.

Chapter-VI applies M. Batty's "Spatial Entropy" measures and computes spatial entropy values of population density distributions by 34 zones of Ankara Metropolitan Area, for census years 1970 and 1990. Spatial Entropy measures S(70), S(90) and Information Gain measures I(70), I(90) are computed. Population changes by zones are computed according to Adams & Storbeck's (1983) method with no zone sizes. Interpretation and comparison of methods are presented by 8 bar-charts, 13 maps and their 3-dimensional illustrations based on the results of the computations. Definition of the variables are given at the end of the Chapter.

Chapter-VII summarizes the main theoretical and empirical conclusions of the thesis. The thesis maintains that the existing methods of model building of the complex urban spatial structures are to be replaced by the information theoric approaches, including the MaxEnt method, as a new logic of inference, in order to be maximally consistent with the known data.

Chapter I starts with the analysis of the population density distribution in Ankara 1965 & 1970 by "density gradient" method within then existing paradigm. To demonstrate the "Paradigm Shift" occurred, Chapter VI "Revisits Ankara Ankara 1970 and 1990" with the new insights provided by the information theoric measures presented in the previous chapters.

I.1 Ankara Population Density Changes: 1965-1970

Paper "Spatial Dimensions Of Human Crowding" was inspired from the classic works such as by C. Clark (1951), Newling (1969) ,Mills (1970) B. Berry & Horton(1970,pp.276-305) , aimed to obtain the pattern of population density distributions and the density gradient changes between years 1965 and 1970 (Esmer, 1979). Depending on the "Population Density-Distance Graphs", paper pointed 3 main results in comparing 1965-1970 density data, according to the geographical sectors of the city.

(i) Densities declined with distance from Ulus Center; as the Clark's (1951) hypothesis that "Residential population densities decline exponentially from the city center" asserted.

(ii) Central densities increased in 3 sectors of the city, except the North-East sector (i.e. Ulus-Altındağ-Citadel area), where there was a population decrease. Furthermore, central densities of both NE & SE sectors are higher than the central densities of the other two (NW & SW) sectors.

(iii) Density gradients were all negative as expected from the negative exponential functions; although there was a slight increase in the SE (i.e. Kızılay-Cebeci area) sector.

(iv) Expansion of Ankara along the NW (Ulus-Yenimahalle) and SW (Kızılay-Bahçelievler) directions can be seen from their decreasing gradients with time. Their gradients were lower than the gradients of the Se & NE sectors in years 1965-1970, as the graphs and the table show. (Fig.I.1 & Table.I.1). Although it was indicated that Ankara was not very different from Berry-type non-western model with a high population concentration at the center and a sharp density gradient declining also by time; it was also added that a 5-year period was too short for such conclusions or temporal generalizations (Esmer, 1979).

TABLE.I.1-) THE DECLINE OF POPULATION DENSITY & GRADIENT – ANKARA, 1965-1970

	North-West (N=31)		North-East (N=100)		South-West (N=27)		South-East (N=37)	
	1965	1970	1965	1970	1965	1970	1965	1970
a	4,669	4,827	6,988	9,394	4,927	5,163	5,33	5,847
b	-0,36	-0,26	-1,1	-0,8	-0,44	-0,32	-0,51	-0,67
D_0	106	125	811	598	138	174	207	346

Source: Esmer (1979)-" "Spatial Dimensions of Human Crowding: An Analysis by the Density Gradients for Ankara, 1965-1970", M. Gürkaynak & W. Ayhan LeCompte (edits) – Human Consequences of Crowding, NATO Conference Series III, Human Factors Vol.10, (p.94)

N = Number of "Mahalle" units

a = Intercept of regression line with density axis measured in $\log_e$ units.

b= Density gradient measured in units of natural log / kilometer (i.e. the fall of natural log of density per km. of distance).

D_0 = Central Density in persons per hectare (pph)

X = Distance in kms. From Ulus Center. Thus, the first-degree density function for the NW sector in 1965 becomes $D_x = 106 e^{-0,36x}$

[It should be noted here that D_0 values in the Fig.I.1 & Table.I.1 represent the results from the regression equations, where the line intercepts the Y- axis. The actual central density values may be higher or lower than the computed D_0 values.]

I.2 Ankara Population Density Changes: 1970-1990

The availability of 1990 data gives the possibility of making comparisons with the past. Fig.I.2 & Fig.I.3 have been drawn to help such comparisons between the 1970 and 1990 gross densities by zones.

During the two decades, between 1970 & 1990, gross densities of old settlement zones of Ulus (-47,35 %), Samanpazarı-Eski Ankara (-35,48 %), Altındağ (-7,73 %) declined significantly, but densities in the peripheral newly developing zones such as Keçiören (+186,81 %), Sanatoryum (+126,23 %), Karaağaç (+100,56 %), Siteler-Ulubey (+44,84 %) increased sharply, in the NE section of the city. Densities of the zones in the SW section, generally increased and Söğütözü (+273,87 %), Balgat (+87,46 %), AOÇ-Fabrika (+79,85 %), Dikmen-Öveçler (+33,82 %), Bahçeli-Emek (+21,33 %). There was a slight decrease in the Maltepe-Anıttepe zone (-7,57 %) (TABLE.3.4&3.5 in Appendix-A).

Fig.I.4 has been drawn to give a combined representation of the SW & NE section. Thus, Fig.I.4 shows a continuous "Population Gross Density Profile" along these SW & NE directions. [Distances have been taken from the Ulus Center, assuming 1,5 kms within zone distance. Hence, on the X- axis, zero point is not indicated.]

It can be said that the centrally – located old zones within approximately 5 kms of radius, lost their higher densities during 1970s and, peripheral zones increased their populations and densities, during the two decades of period. It should also be stated here that the observations made for years 1965 & 1970 *that* ***"The Old Ankara and Altındağ area should be losing its population"***, with an indication of ***"deconcentration in the parts near Ulus",*** hold also true for the following two decades up to the year 1990, as the Fig.I.4 & Table.2.1 & 3.4 & 3.5 in Appendix-A demonstrate.(Esmer,1979)

The continuation of the above general trends in the population density changes has also been supported in other researches on Ankara. J. Clark (1970) compares the negative-exponential density functions developed for “planned & unplanned” parts as well as the “whole” Ankara city, for years 1961, 1965 & 1969 (As cited by Tekeli & Güvenç (1986)). It is argued that though the central densities (D_0) of the “planned” parts remained almost unchanged between 1961-1969; there was a continuously declining trend in the “unplanned” parts and the “whole” city. There was also a decline in the gradient (b) of the function in all cases. Authors compare the findings of J. Clark (1970) with their study for years 1970-1985 and point out that the trend in the changes during 1961-1969 has also continued in the next 15 years. Hence, during the almost 25 years between 1961-1985, densities of the central zones within 5 km from Sıhhıye Center have fallen and gradients have declined by time. Authors also “explain” the reasons for these changes in (D_0) and (b) values in relation to the multiple factors, such as developments in the public transportation system, increases in the private car ownership rates, changes in the urban land use types, changes in the zoning regulations, including the increasing location rents at the Ankara sub-centers.

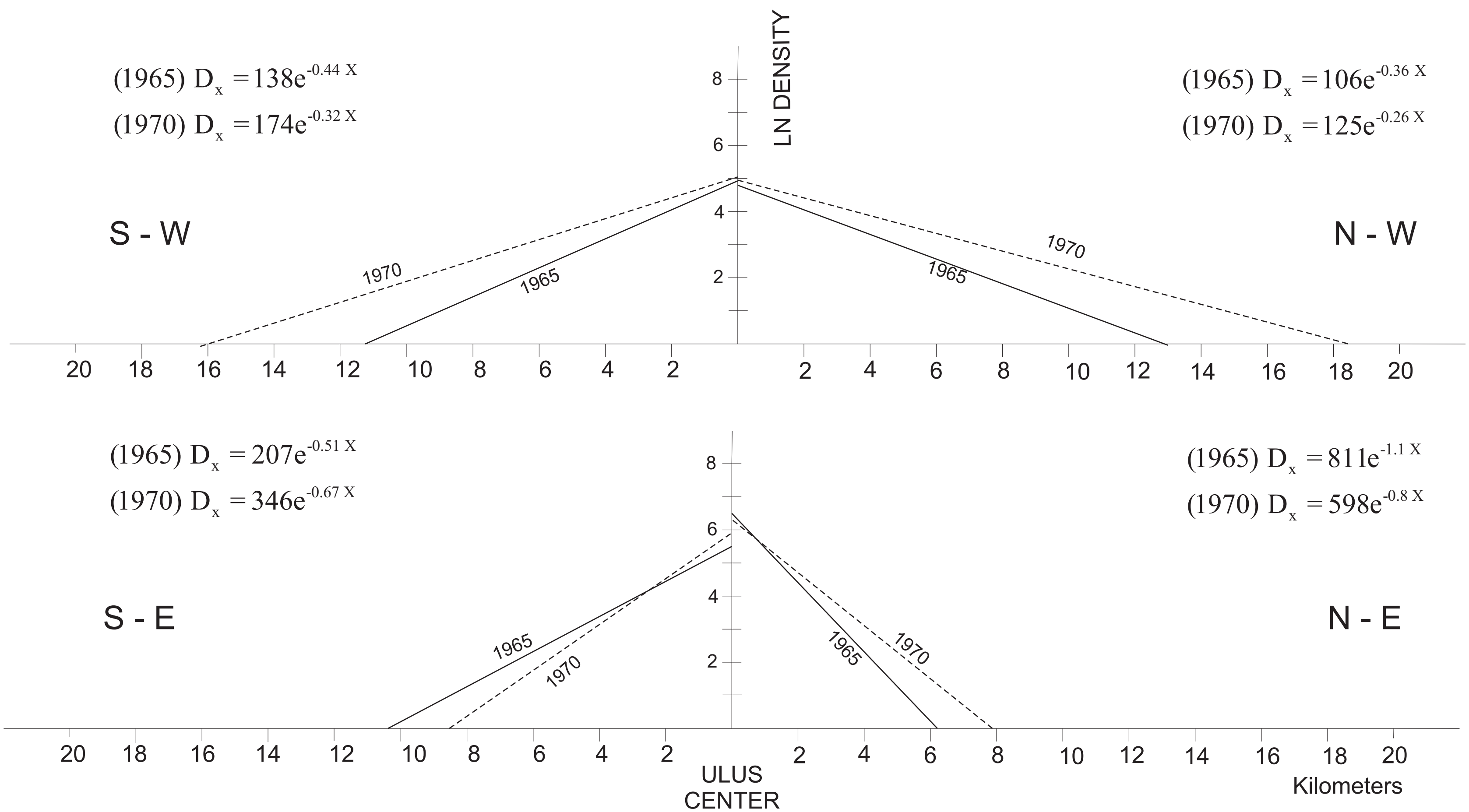


Figure I.1- Population Density Gradients For Ankara 1965-1970
Source: Esmer (1979, p.95)

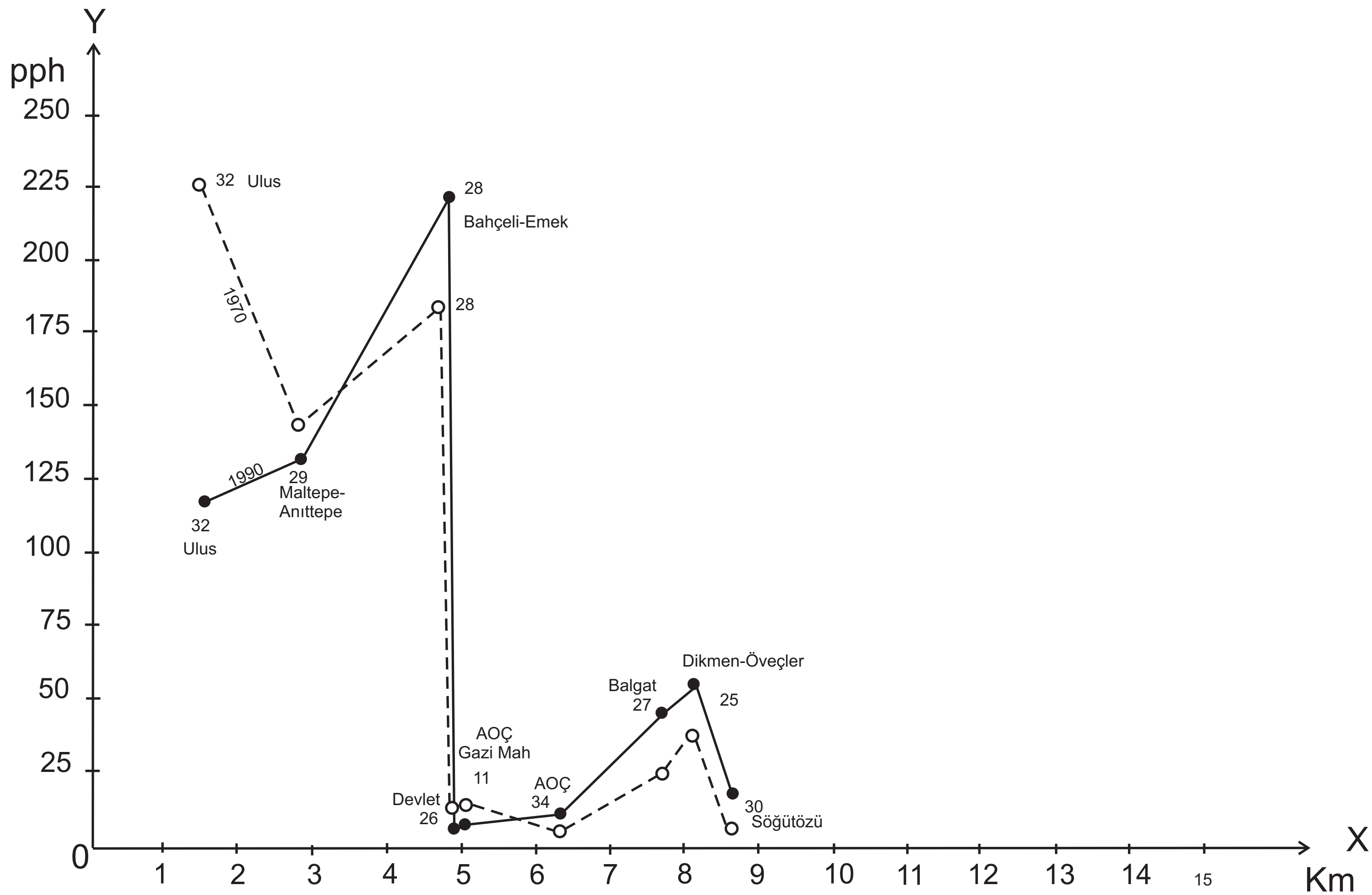


Figure I.2. Population Gross Density Changes in SW Section 1970-1990
Table 3.3 & 3.5 in Appendix-A

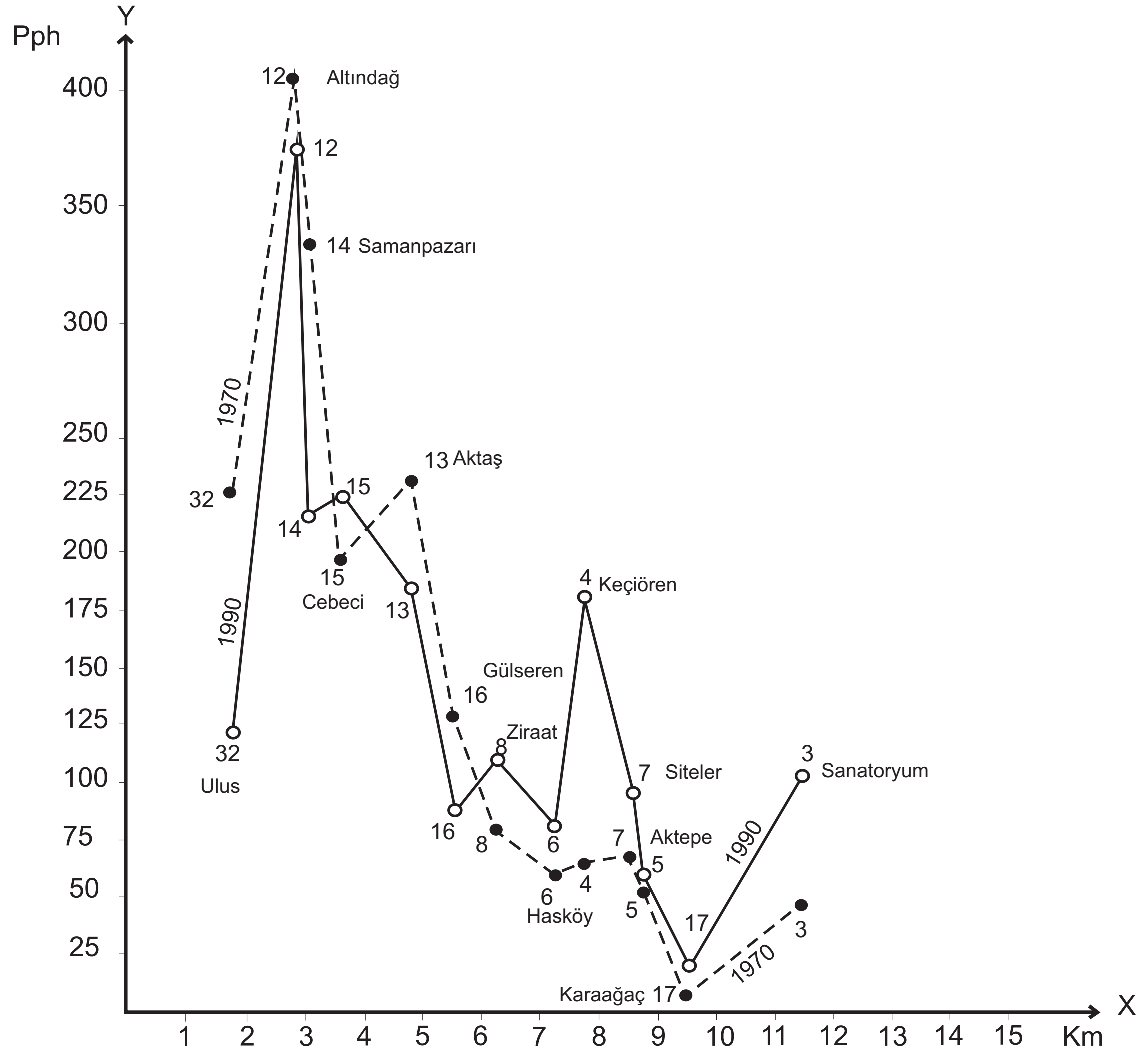
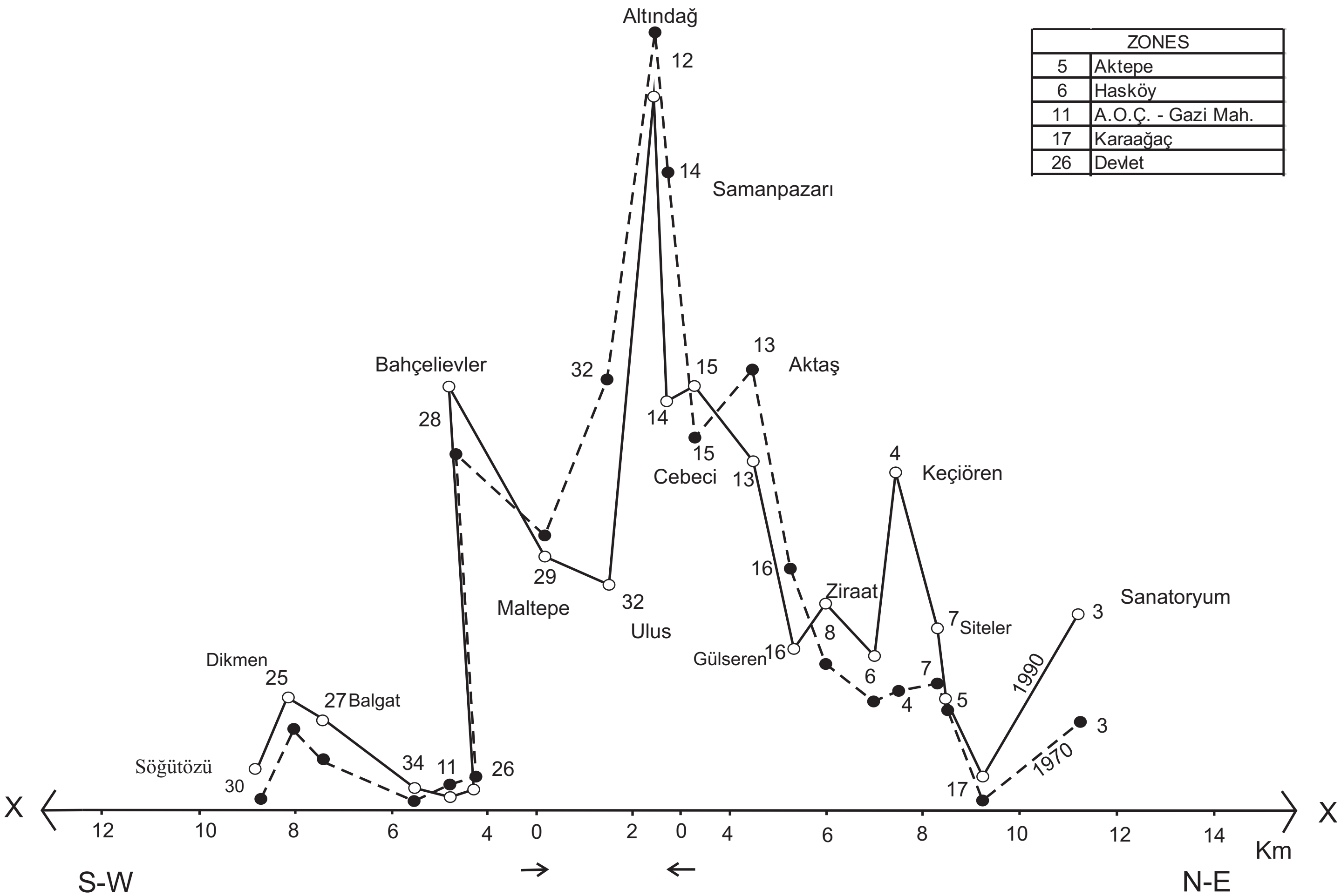


Figure I.3 Population Gross Density Changes in NE Section
Table 3.3 & 3.4 in Appendix-A



ZONES	
5	Aktepe
6	Hasköy
11	A.O.Ç. - Gazi Mah.
17	Karaağaç
26	Devlet

Figure I.4-) Population Gross Density Changes in SW & NE Sections, Ankara 1970-1990
Table 3.3 in Appendix-A

I.3 A New Look Into The Urban Structure

In this section, 3 generations of models aiming to "explain" the observed structure of an urban area shall be reviewed and compared.

I.3.1 First Generation Models: 1960s

Up to the late 1950s, urban economists, geographers, regional and urban planners had at their disposal two significant theories: (i) The location rent theory of Von Thünen (1826), developed within the context of the location of agricultural activity and, (ii) The central place theory of Lösch (1954). The 1960s witnessed an extension of the Von Thünen Model to the urban context. Wingo (1961), Alonso (1964) and Muth (1961, 1969) are the representative "triumvirate" of the "First Generation Models", as Anas & Dendrinos (1976) called the term (References are given in Appendix-B and Appendix-C).

I.3.2 Second Generation Models: 1970s

According to the evaluations by Anas & Dendrinos (1976), Landmarks in "Second Generation Models" were the three papers by Mills (1967), Solow (1972) and Mirrless (1972). With the exception of the 1967 Mills model, all of the "Second Generation Models" were developed during the 1970s. In 1974, the "Journal of Urban Economics" appeared in response to the growing interests in urban problems and uninvestigated theoretical issues by economists and other concerned researchers.(References are as cited by Anas & Dendrinos (1976))

There is no consensus about the origins of the new subfield, labeled the "New Urban Economics" (NUE). Some trace them back to the works of the "Triumvirate" of First Generation developments; but Richardson (1976) regarded them as the "heralds" rather than participants in the general equilibrium models of NUE. Anas and Dendrinos (1976), summarizes the First-Second Generation dichotomy as follows: 1-) Partial vs. general equilibrium models, 2-) Positive models (no public sector) vs. policy-oriented models (implicit public sector) and normative (explicit public sector), 3-) Utility maximization with flexible demands for space and the composite commodity vs. cost minimization and inelastic demand for space, 4-) Homogeneous vs. heterogeneous incomes and tastes among locators, 5-) Monocentric vs. polycentric urban form, 6-)Continuous vs. discrete representation of space, 7-) Absence vs. presence of externalities.

I.3.3 Third Generation Models: 1980s - ?

According to Anas & Dendrinos (1976), though there were some contributions on issues such as zoning policy, income and property taxation by various authors, these researches received isolated attention from the researchers within the field. These attempts can be characterized as extensions of either First or Second Generation Models, and in the view of Anas and Dendrinos (1976), their proximity to the main models does not justify the title "Third Generation". They feel that the title should be reserved for the developments of "Optimum Geography" and "The Optimum Town" by Mirrless (1972) and the "dynamic models". The outcome of "Optimum Geography" was the optimum number of urban areas in the country and their respective market area with their surrounding agricultural hinterland and population size.

Yet, Anas & Dendrinos (1976) did not explain what was meant by "dynamic models" in their survey. On the other hand, Richardson (1976) gives J. Forrester's (1969) "Urban Dynamics" as an example to alternatives to models of NUE. According to Richardson (1976), urban economics should be primarily a policy-oriented field, but NUE had not much to offer as a guide to policy makers. The alternative to NUE models fall within the classification of "simulation" models. The econometric models of NUE and "system dynamics" models are based on different paradigms and therefore they are regarded as "irreconcilable" views. Econometric models are firmly founded on past data and take a "correlative" view. In contrast, system dynamic models are based on "causal" view with feedback-loop structures. Meadows (1980) and Legasto & Maciarello (1980) compares these two different modeling paradigms and conclude that these two views can and will co-exist.

Similarly, we can assert that the Lowry Model (Lowry, 1964) and its descendants, Gravity Model (Isard, 1965,Chapter-11; Isard,et.al.,1998, pp.240-280) and "Spatial Interaction Models" originated by Wilson (1967, 1970, 1970a, 1974) using MaxEnt methodology, i.e., simulation models in general cannot be regarded as merely alternatives to NUE models but as "irreconcilable" views and therefore these simulation models should not be compared to the First & Second Generation NUE models.

As far as the urban simulation models are concerned, Wegener (1994) describes the "State-of-the-Art", for the 20 years after Lee's (1973) "Requiem For Large-Scale Models" and shows that the urban modeling, despite the opinions that their time has passed with late modernism, has increased steadily. Wegener (1994) reviews and compares 12 operational (i.e. implemented) urban models selected from 20 modeling centers in different countries. Those models tried to "integrate" the subsystems of the city, such as transportation networks, employment centers, residential areas in relation to policy issues. Wegener (1994) concluded that growing environmental awareness may accelerate the greening of urban models, and in particular, models should be made more sensitive to the issues of equity and environmental sustainability.

From the viewpoint of consideration of ecological and environmental sustainability issues, at all levels from urban to national and global, it seems that the models developed after 1980s deserve the title of "Third Generation", without making a distinction between the "econometric" and "simulation" models of the urban structure.

I.3.4 Statement Of Problems

My 1998a unpublished study "Some Critical Notes On Urban Density Models" consisted of two parts, i.e., the "Introduction" giving a summary of the 1977 paper and the second part over viewing the developments in the field during 1980s & 1990s (Esmer, 1998a & 1977; Appendix-B & C). It was indicated how the Maximization of Entropy (MaxEnt) paradigm, after the pioneering works of A. G. Wilson (1967, 1970, 1970a) has attracted many researchers from different fields. Although Cesario (1975) stated that "the precise meaning of entropy is all but lost to the average reader", there have been significant contributions to the study of the urban structure by using the MaxEnt methodology. Of course, the method is difficult to express in a non-mathematical manner and radically different from that employed by urban researchers whose models of behavior are rooted in the concepts from urban economics (Webber, 1977a). The previous section reviewed the generations of models developed within NUE.

In this section, some reasons why the MaxEnt has been introduced to the field of urban study and planning shall be outlined briefly.

I.3.4.1 The Need For Integration

In their paper, Batty and March (1976) asserted that, literature on urban spatial planning revealed different approaches to the subject matter which imply different objectives on designing such models. The particular approach adopted tended to dominate the presentation of model produced; yet there was a concern among different schools of model-builders that an "integration" was urgently required. For the purposes of their paper (1976), authors classified the approaches, developed to date, into 3 types.

i-) The first approach, which might be called "the planning approach", characterized by the design of theoretically-crude simulation models, in which the emphasis has been upon developing working models useful for predicting variables of interest to physical planners. These models were macro in emphasis and were postulated in an " a priori" manner, and were fitted to empirical data using numerical methods, rather than statistical theory.

ii-) The second approach, was that referred to as "new urban economics", starting with Alonso's (1964) theory of the urban land market, micro-economic theory such as the consumer theory of utility-maximizing, has been extensively used in deriving theoretical urban land use patterns.

It is clear that the second approach as defined by above authors, corresponds to the First & Second Generation of models as reviewed in sections 3.1, 3.2 & 3.3 of this Chapter.I.

iii-) A third approach, is different from the previous two. Whereas the "macro-planning" and "micro-economic" approaches are both characterized by a "a priori" model design; the third approach is more inductive in that the emphasis is upon searching for appropriate models which reflect the data in certain ways. These models are "statistical" in the general sense.

According to Batty & March (1976), any attempt to "integrate" these various streams was a long term endeavor. Yet, their paper introduced the "Information Minimizing" methodology for deriving models which were consistent with both the "planning" and "statistical" approaches and with their related urban models. The theory of "Information Minimizing" is concerned with deriving least prejudiced models which meet sets of known information. Of course, the theory is consistently and clearly related to the "Maximizing-Entropy", the MaxEnt theory, that this thesis introduced and some of its pioneering applications are reviewed.

I.3.4.2 Urban Models Should Be Based on the Known Information with Minimal Prejudices or Assumptions:

Webber (1977) explains that the facts known about the distributions of urban activities or land uses such as people, income, housing, transport are in an aggregate way. A detailed prediction is to be made about the distribution of probabilities of the related urban activities from the city center. The known information is not sufficient to predict the detail required and it is the research task to bridge this information gap. Economic and psychological models of urban consumer behavior between people in different areas fill this gap by making assumptions about individual behavior.

In his seminal work, “Location and Land Value”, Alonso (1964) places the rational households within an ideal city in which all jobs are concentrated at the city center, (CBD), and assumes that households (i) maximize their utility by choosing an optimal mix of quantity of residential land, of distance from the city center, as well as consumption goods, (ii) subject to the constraint that the sum of the expenditures does not exceed their income.

Beckmann (1969), in his “On the Distribution of Urban Rent and Residential Density”, also assumed that “Given the rent or value of land as a function of distance from the CBD, a household will choose to live at a distance which maximizes its utility”. Papageorgiou and Casetti (1971); Papageorgiou and Brummel (1975) extended the Alonso’s “single-center” model to the “multiple-center” urban framework. The main problem of NUE was to deduce urban spatial structure from the urban economic theory. Webber (1977) explains that the “Entropy Maximizer” has a different viewpoint than the above classical urban studies based on NUE:

> The entropy maximizing research task is to infer detailed patterns from known aggregate data directly without intermediary assumptions…Entropy maximizers do not construct models of choice; they try to draw inferences from data.(Webber,1977,p.260)

By “intermediary assumptions”, Webber (1977) meant the assumptions adopted from urban economics, including the “utility maximization”. Webber (1979) generalized and extended his viewpoints in his book “Information Theory and Urban Spatial Structure”. Therefore, the aim of a model or a research based on

MaxEnt formalism, or on information-theoric framework, is to rely solely on logical principles, nor on a prejudiced or biased model of urban economic behavior.

Appendix-B (2.2) shows how March (1972) derived Clark's empirically grounded hypothesis solely by MaxEnt principles. In regional science context, Anastassiadis (1986) derives the empirical Rank-Size Rule by MaxEnt methods and compares the real and estimated population of cities and their ranks.

I.3.4.3 Urban Spatial Interaction Structure is to be Redefined by the New Concepts of Macrostates, Microstates and the Most Probable State.

Wilson (1970, Chapter 1), pointed out that the question "What information do we need to be given to specify fully a system state?" is far from trivial question. In classical physics, the state of the gaseous system is fully specified by coordinates and velocities of each particle in the gas at any time. However, such "Microanalytic" techniques prove too difficult to handle when there are many particles involved. A branch of physics, known as "Statistical Mechanics", developed new methods which enabled the physicist to explain and predict certain "macroproperties" of the system without having to explain the behavior of each individual particle at the microlevel; by using the concept of "Entropy". In an "Origin-Destination" matrix, he shows how the O_i or D_j sets of numbers, i.e. a macrostate distribution can be obtained by many microstates, represented by T_{ij}, the total number of individuals who travel from origin (i) to destination (j), to be estimated by the interaction model. Finding the "most probable trip distribution" by calculating the number of microstates associated with each macrostate distribution subject to relevant locational constraints, required new insights into the urban spatial structure that Wilson (1967, 1970) introduced and pioneered in the field.

To illustrate the Wilson's model simply, Cesario's example shall be summarized below (Cesario, 1975).

Let there be N origins and M destinations of trips in an urban area divided by zones. Let O_i denote the total number of trips emanating from origin (i) and let D_j be the total number of trips terminating at destination (j) during some time period. The Origin-Destination matrix T_{ij} is assumed to be known. Assuming that O_i and D_j are given exogenously, i.e., are known, we have:

$$\sum_{j=1}^{M} T_{ij} = O_i \quad (\ i=1, 2,\ldots\ldots\ldots N\ ) \qquad (\ I\text{-}1\ )$$

$$\sum_{i=1}^{N} T_{ij} = D_j \quad (\ j=1, 2, \ldots\ldots\ldots N\ ) \qquad (\ I\text{-}2\ )$$

$$T=\sum_{i=1}^{N} O_i = \sum_{j=1}^{M} D_j = \sum_{i=1}^{N}\sum_{j=1}^{M} T_{ij} \qquad (\ I\text{-}3\ )$$

Letting C_{ij} be the generalized cost of travel between (i) and (j), the total amount of cost spent on travel for a given distribution T as:

$$C=\sum_{i=1}^{N}\sum_{j=1}^{M} T_{ij}c_{ij} \qquad (\ I\text{-}4\ )$$

We wish to find a “macrostate” distribution T according to the MaxEnt principle.

Suppose N=M=2 and we have $O_1 = 3$; $O_2 = 3$ and $D_1 = 4$; $D_2 = 2$. The trip distribution matrix can be structured as follows:

TABLE.I.2 TRIP DISTRIBUTION MATRIX

		Destinations (D_j)		$\sum_{j=1}^{N} T_{ij} = O_i$
		D_1	D_2	
Origins (O_i)	O_1	T_{11}	T_{12}	3
	O_2	T_{21}	T_{22}	3
$\sum_{i=1}^{N} T_{ij} = D_j$		4	2	T=6

The problem is to find the distribution (T_{11}, T_{12}, T_{21}, T_{22}) which is "maximally noncommittal and unbiased, consistent with the amount of information at our disposal". The distribution selected must be in accord with the constraint equations (I-1), (I-2), and (I-3).

A "microstate" is a complete specification of the system. If each of the (T) individuals can be identified, a "microstate" in this case is an enumeration of who travels where. A "macrostate", on the other hand, merely specifies how many people travel between (i) and (j), that is T_{ij} without regard for individual identification.

If we enumerate the possible macrostates in the system subject to (I-1) and (I-2) for the moment, there shall be only 3 possibilities as shown in Figure I.5.

$$T_{ij} = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} \qquad T_{ij} = \begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix} \qquad T_{ij} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$$

(a) (b) (c)

FIGURE.I.5. Macrostates

Any other arrangement of the 6 travelers would violate the given constraints. Now, let us examine the "microstates" associated with each "macrostate" in Figure.I.5. For particular distribution (T_{11}, T_{12}, T_{21}, T_{22}), where T = $T_{11}+T_{12}+T_{21}+T_{22}$, let us find the number of ways T_{11} trip makers can be selected from T:

$$\frac{T!}{T_{11}!(T\text{-}T_{11})!} \qquad \text{(I-5)}$$

Where (!) represents the factorial operation in the familiar combinatorial equation of statistics. Using this equation, we can select T_{11} in Figure.I.5 (a), (b), (c) in

$$\frac{6!}{3!3!}=20 \ ; \qquad \frac{6!}{2!4!}=15 \ ; \qquad \frac{6!}{1!5!}=6$$

ways respectively. Number of ways to select T_{12} out of the remaining $(T\text{-}T_{11})$ travelers is given by:

$$\frac{(T\text{-}T_{11})!}{T_{12}!(T - T_{11}\text{-}T_{12})!} \qquad \text{(I-6)}$$

Total number of ways of selecting T_{11} out of T and T_{12} out of $(T\text{-}T_{11})$ is given by the product of equations (I-5) and (I-6). The total number of ways (W) in which we can select a particular distribution T_{ij} from T is;

$$W(T_{ij}) = \frac{T!}{T_{11}!T_{12}!T_{21}!T_{22}!} = \frac{T!}{\prod_{ij} T_{ij}!} \qquad \text{(I-7)}$$

(where $\prod$ means multiplication of $T_{ij}!$). Using equation (I-7), we get W=60, W=180 and W=60 for Figure.I.5 (a), (b), (c) respectively. We now make a critical "Entropy" assumption that each microstate is equally probable. Hence, "the most probable" T_{ij} distribution is given in Figure.I.5 (b) since it can be achieved in W=180, i.e., the greatest number of ways.

For a complete treatment of Wilson's MaxEnt method, the definition of the concept of "entropy" is needed, as shall be given in Chapter III (Also in Appendix-C).

I.3.4.4 Deficiencies & Inconsistencies in the Traditional Gravity Model

In his pioneering paper Wilson (1967), demonstrates "the obvious deficiency" in the well known Gravity Model:

$$T_{ij} = KO_iD_jd_{ij}^2 \qquad \text{(I-8)}$$

where T_{ij} is the number of trips to be estimated, d_{ij} the distance between (i) and (j); O_i and D_j are the numbers of trip origins and destinations respectively. If a particular O_i and D_j are each doubled, then the number of trips between the zones would quadruple according to the Gravity Model, when it would be expected that they would double also. More precisely; the following constraint equations on T_{ij} should always be satisfied:

$$\sum_j T_{ij} = O_i \qquad \text{(I-9)}$$

$$\sum_{i} T_{ij} = D_j \qquad (\text{I-10})$$

That is, the row and column sums of the trip matrix should be equal to the trips generated in origin (i) and attracted by destination (j) zones. These constraint equations can be satisfied if sets of "balancing factors" A_i and B_j are introduced. Also, (d_{ij}) is replaced by the "generalized cost" function (c_{ij}), to emphasize that the measure of impedance or deterrence need not be (Newtonian) distance. The new "Spatial Interaction Model" is then

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \qquad (\text{I-11})$$

$$A_i = \left[\sum_{j} B_j D_j f(c_{ij}) \right]^{-1} \qquad (\text{I-12})$$

$$B_j = \left[\sum_{i} A_i O_i f(c_{ij}) \right]^{-1} \qquad (\text{I-13})$$

The above set of equations satisfy the constraints and alleviate the deficiency in the conventional Gravity Model.

Wilson (1974, p41-43) uses Reilly-type equation in the form

$$S_{ij} = K(e_i P_i) W_j^{\alpha} c_{ij}^{-n} \qquad (\text{I-14})$$

Where S_{ij} is sales in shops in zone (j) to residents of zone (i); P_i is the population of zone (i); e_i means expenditure on shopping per person in zone (i), $e_i P_i$ money spent by residents of (i), W_j size or attractiveness index of shopping center in (j), c_{ij} generalized cost of travel, (α) and (n) are powers of W_j and (c_{ij})

respectively. Then, S_{i*}, the total flow of cash out of residential zone (i) shall be calculated for N zones using equation:

$$S_{i*} = \sum_{j=1}^{N} S_{ij} = \sum_{j=1}^{N} K(e_i P_i) W_j^{\alpha} c_{ij}^{-n} \qquad \text{(I-15)}$$

However, since we know $S_{i*} = e_i P_i$, so we obtain by replacement:

$$K = \left[\sum_{j=1}^{N} W^{\alpha} c_{ij}^{-n} \right]^{-1} \qquad \text{(I-16)}$$

Hence, Wilson (1974, p.42) "unearth an inconsistency" in the Reilly model: The right-hand side of equation (I-16) is depended on (i) and so it is "impossible" to solve the equation for K, as there are N (i.e., one for each (i) possible values of the right-hand side. For consistency, Wilson (1974, p.42) replaces K by a set of K_i, depended on (i). By substitution of K_i, equation (I-14) is revised and it becomes "consistent" with the initial constraint represented by $S_{i*} = e_i P_i$. For a more general notation, K_i is replaced by A_i (Wilson, 1974, p. 65-66).

Above explanations should not be regarded as technical midcourse corrections. For the alleviation of inconsistencies and the Newtonian gravitational analogies existing in the traditional Gravity Model, Wilson (1967, 1970, 1974) developed new insights into the model that were inspired from statistical mechanics.

I.3.4.4.1 "Gravity Model" and "Spatial Interaction Models"

The derivation of the Gravity Model as given in Equation (I-8) is not included in this thesis, since they are available in textbooks (Lee, 1980) and generally they refer to W. Isard's 1960 book.(1960, Chapter-11 Gravity Models). Tocalis (1978) give an excellent overview in the developments of the Gravity Model and shows how the deterministic model has been changed into the probabilistic model by the efforts of Huff, Warntz and and Isard during 1955s and early 1960s.

Isard (1975) explains that he was among the first to lecture on this topic, starting in the late 1940s at Harvard University in his location theory course and he received sharp criticisms from his fellows not to provide any rationale for the model. Isard (1975, p.25) writes that he always have remained unhappy with the rationale presented for explaining travel behavior adding that "The current development of entropy maximization models by Alan Wilson… leaves me still unhappy". In his 1975 paper, Isard develops a rationale for travel behavior which is consistent with the gravity model trip patterns by using the utility concept expressed as a function of the discounted trip level leading to the exponential distance function. In his 1998 book, Chapter 6 is devoted to "Gravity and Spatial Interaction Models" where the basic derivation of the model remained the same as in his 1960 book (Isard, 1960, Chapter-11) and also in his 1975a book (Isard, 1975a, "The Gravity Model", pp.39-50). However, the chapter in 1998 book includes sections explaining the "Single-Constrained" and "Double-Constrained" models with numerical examples. According to him, the early development of exponential deterrence models was stimulated by "the pioneering work of Wilson (1970, 1974) … These models have often been designated as maximum entropy and/or information-minimization models" (Isard, 1998, p.257-258). He further notes that the presentation of the "basis of these designations is beyond the scope of this chapter" (Isard, 1998, ibid). Therefore, it can be argued here that Isard (1998) omits the two distinct "rationalities" of the theoretical foundations of "Gravity Model" and "Spatial Interaction Models" and treat them as if they are the same kind of models. As explained in Chapter I.3.4.4 and Chapter III.6, Wilson's contribution cannot be regarded as a simple mathematical treatment of a calibration problem or finding the exponential deterrence functions. In Kuhnian terms, given in Chapter I, the "Anomaly" in the Gravity Model, expressed as the ''mathematical inconsistency" in equations of Chapter I.3.4.4, was eliminated by a "Paradigm Shift" resulting in a change from "Gravity" analogy based on Newtonian physics to "MaxEnt'' method based on statistical mechanics and information theory.A numerical example to show the mathematical inconsistency and the role of constraint functions is given in Chapter III.6.1.

I.3.4.5 Complex Systems Require Theory of Large Numbers & Probability Concepts

Mogridge (1972), in his paper delivered to a conference celebrating the 10th anniversary of the foundation of "Centre de Recherche d'Urbanism", Paris, pointed out the importance of "entropy" as a tool in urban and regional model building. According to him, we must use theories based "on the laws of large numbers and on probabilities", where we deal with large, complex and interacting systems, such as in urban system and "Entropy Maximizing" methods provided much firmer theoretical and scientific base to our urban and regional models.

I.3.4.6 Statistical Tests Do Not Suffice to Determine the Best-Fit Curve

Casetti (1969), in his paper "Alternate Urban Population Density Models: An Analytical Comparison of Their Validity Range", discusses alternative procedures for determining which one of several families of functions is best suited to describe the relationship between population densities and distance from city centers for specific data. His investigations on 6 cities with 15 families of regression equations have shown that there was a decline of population densities with distance from the city centers, and that negative exponential functions of degree from 1 to 3 are better suited to represent the density decline in central urban areas. The statistical analysis was carried out using a stepwise computer program. Those steps were selected which gave regression equations with the highest correlation and with coefficients significantly different from zero on the basis of a t-test at the 5 per cent significance level.

However, Casetti's (1969) following general conclusion is extremely important:

> However, all the functions investigated give excellent fits, so that good results obtained with one particular

> family of functions is, in itself, not a good enough reason for preference. (Casetti,1969,p.111)

Papageorgio (1971), by referring to the Casetti's (1969) above conclusion, also claimed that "***In consequence, it seem unfeasible to select the best description of the density gradient by using only empirical evidence"*** (italics are mine). As an alternative approach, he proposed "to examine those functions, if possible, within the framework of a certain deductive system". By "***deductive system***", he meant "***a particular set of postulates***". Therefore, according to Papageorgio (1971), "The emphasis would be laid upon logical rather than empirical grounds". The selection criterion was also given :

> The function associated with the least restrictive assumptions should be selected as the best description of the density gradient. (Papageorgio, 1971,p.22)

Nevertheless, neither Casetti (1969) nor Papageorgio (1971) defined what they operationally meant by "**A particular set of postulates**", or "**Logical rather empirical grounds**". Their suggestion that "***The selection of the function associated with the least restrictive assumptions***" was right but there was no procedure presented to satisfy the above stated proposals. Actually, meeting these demands would not be possible anyway within their paradigm of research. The achievement of the objective "***selection with the least restriction***" would require the concept of entropy and the principle of MaxEnt as originally put forward by E. T. Jaynes (1957).

Similar complaints do exist also in the field of natural sciences. In a recent paper on the recurrence of earthquake probabilities, Ellsworth (et.al., 1999) asserted that a number of candidate statistical models have been proposed for the computation of conditional probabilities of future earthquakes, including the Double Exponential, Gaussian, Weibull, Lognormal and Gamma distributions by various researchers. Ellsworth (et.al., 1999) complained that,

> **All of these distributions have been widely discussed, ……. although none of them has any particular claim**

> **as a proper model for earthquake recurrence……**At present, ***it is not possible to discriminate between such candidate models***, given the limited and uncertain nature of earthquake recurrence data… (Ellsworth,et.al.,p.2)

It is clear that Casetti (1969) and Ellsworth (et.al. 1999) used elaborated statistical tests to identify the best-fit curve for a given set of empirical data, the conclusions were not satisfactory as the researchers pointed out. The issues related to the "**Verification**" and "**Validation**" particularly in system dynamics models and models in general, were investigated by J. Forrester & Senge (1980) in their classic paper. These authors claimed that "**There can be no proof of the absolute correctness with which a model represents reality**" and proposed 17 tests for building "confidence" in system dynamics models, instead of applying single test which serve to "validate" a given model (J. Forrester & Senge, 1980). The traditional treatment of "validation & verification" issues in engineering sciences were criticized and evaluated in my 2003 paper (Esmer, 2003).

Bratley (et.al. 1983, p.134-135) explains the defects of goodness-of-fit tests, such as chi-square, and points out the risk that theoretical distribution does not generate the data no matter what parameters are chosen: "**God does not usually tell us from what distribution the data come**".

I.4 Scope of the Thesis

In 1970s my interests were influenced mainly by the existing research and application methods of the First-Generation Models, though I was also attracted by the developments in cybernetics. My 1979 paper was one of the outcomes of my interests in the study of intra-urban population distributions by the use of density functions (Esmer, 1979).

Chapter I.1 & 2 shows that the trends characterized by decreases at the old central zones around Ulus and increases at the peripheral zones that were observed during 1960s have continued during the following 3 decades in Ankara Metropolitan Area, within the same 34 zonal boundaries.

My 1977 and 1998 unpublished reports reviewed the theoretical developments in models of the urban structure and showed that these were important contributions to the field by generating alternative lines with respect to the First & Second Generation Models. Developments were particularly in "Simulation Models", that were regarded as "alternatives" to the existing models and Wilson's (1967,1970) introduction of MaxEnt methods into the modeling process provided new insights into the field of urban studies and planning, giving rise to new applications. The reasons or rationale for the introduction of MaxEnt methodology and information-theoric concepts in general have been justified by various authors as follows:

(1) There was an urgent need to "integrate" the different modeling approaches (Batty & March, 1976).

(2) First & Second Generation Models of NUE depended on some "intermediary assumptions", such as "utility-maximization" by urban consumers. These "assumptions" were needed to fill the information gap between the urban aggregate data and the detailed distribution of data. MaxEnt methodology provided a new logic to make inferences or predictions directly from the known data, without making further assumptions in a prejudiced way (Webber, 1977).

(3) Urban spatial interaction structure needed new concepts of "macrostates" and "microstates" and the "most probable state" concepts, borrowed from statistical mechanics, to deal with the uncertainties and associated probabilities in relation to "entropy" and "information" concepts. (Wilson, 1970)

(4) There were obvious deficiencies and inconsistencies in the formulation of the traditional "Gravity Model". Wilson (1967, 1970) proposed the new statistical theory of spatial interaction distribution models based on the above concepts in (3) and the MaxEnt methodology to eliminate the Newtonian analogies and inconsistencies in the "Gravity Model"

(5) Complex and interacting systems, such as urban systems, require theories based on the laws of large numbers and on probabilities. MaxEnt methods

provided much firmer theoretical base to our urban and regional models. (Mogridge, 1972)

(6) Conventional statistical tests are not sufficient to select among the candidate family of best-fit curves. MaxEnt provides the procedures for the selection in with respect to the known data, in a minimally biased way.

Although the theoretical foundations were first laid by C. Shannon (1948) and E. T. Jaynes (1957), there were resistances by the statisticians, physicists, mathematicians; up to the 1980s and in the last decades it has attracted many scientists from various disciplines and its applications have brought new theoretical issues to the for. The first proceedings on MaxEnt and Bayesian methods was published in 1979, after the first meeting at MIT in 1978. the first international workshop was held in 1981 and proceedings are published yearly since then. (Tribus, 1999).

The special commemorative issue of the ***IEE Transactions On Information Theory*** (October 1998, Vol.44, No:6) celebrating the 50th anniversary of C.E. Shannon's (1948) "***A Mathematical Theory of Communication***" (Verdú, 1998). This special issue evaluates the great accomplishments of 5 decades of the "Information Theory" and introduces the recent progresses in the "Quantum Information Theory". It is exciting to compare the differences between these two theories and to read that "**information cannot be read or copied without disturbing it**" (Bennett & Shor, 1998). Next decades of the new millennium may witness the emergence of new lines of developments and paradigm changes in the field of urban modeling.

It is clear that information-theoric concepts have given new impetus to the foundations of all sciences and consequently to the rationale for the derivation of distribution models in the urban spatial interaction systems.

This thesis is an inquiry into Bayesian, Shannonian and Jaynesian revolutions in "Information Theory" and their implications for the urban studies and planning.

More specifically, information-theoric concepts and MaxEnt methodology are introduced and M. Batty's (1974a,b) "Spatial Entropy" concept is applied to the Ankara population distributions by zones between years 1970 and 1990 as a case study to acquire new kinds of interpretations.

I.5 Concluding Remarks

Chapter I aims to demonstrate that the historical evolution of urban NUE models needs a new impetus for the solution of the important theoretical issues that were identified by the authors in different periods.

(i) Chapter I.3.4: Statement of Problems lists these issues that led to the inconsistent results.

(ii) Urban structure is a complex system requiring probability concepts for its description and explanation. Incomplete data or uncertainties that exists cannot be omitted.

(iii) Existing methods are not consistent with the known data on urban structure and are making arbitrary and biased assumptions.

Chapter I.4: Scope of the Thesis states that the new framework for the alleviation of the above issues has been provided by the Maximum Entropy (MaxEnt) method.

CHAPTER II

KINDS OF PROBABILITY

Early work in mathematical probability was dominated by a consideration of “equally likely” cases. The greatest incentive for mathematical development in this field was the analysis of “games of chance”. Fermat, Pascal, Huygens, Jacob Bernoulli and others in the 17th century had examined how the mathematics of permutations and combinations could be used in probability theory to describe “favorable” cases. Mathematicians had been dealing with problems of a certain kind: Given the underlying properties of a game, experiments, etc, what can we say about the outcome or probability of an outcome? These theorems were brought together and synthesized in the work of Laplace (1814). According to Laplace, “all events are regulated by ***the great laws of nature***”, and it was necessary to develop some theory which could be applied to situations in which we were uncertain. In particular, Laplace assigned numerical values to probabilities that could be judged as “equally possible”. His “**The Principle of Insufficient Reason**” meant that equal probabilities could be assigned to the various outcomes provided there was no evidence to suggest otherwise. Thus, the assignment of probabilities of a six-faced die would be 1/6 to each face unless there was definite evidence or reason that the die was biased in some way. The application of Laplacian method of assigning probabilities on an “A Priori” basis raised a number of difficult problems. Attempts to be more “objective” about the assignment of probabilities to events, in the works of Pearson, Fisher and Neyman, led to the foundation of a school of “Statistical Orthodoxy”, based on the “Relative-Frequency” interpretation of probability.

Most writers on probability theory identify 3 major views within which there may be considerable variety of interpretations. Next sections review these 3 views. (Harvey, 1969, pp.230-259)

II.1 The Relative-Frequency View of Probability

There are a number of variants of the view, but essentially it rests on the belief that there is some ratio between the actual number of times, a particular outcome of an event and the total number of events. Given a total set of events, R, and a subset of R exhibiting a certain property, represented by subset A, then the frequency, (r), of A in R is given by:

$$r(A,R)=\frac{n(A)}{n(R)} \qquad \text{(II-1)}$$

which can be empirically determined, given any reasonable number of events R. The relative-frequency view goes on to postulate that (r) stabilizes as (n) is increased and the term "probability" can be defined by stating that the probability (p) is the value of this ratio at the limit. Thus:

$$p(A,R) = \lim_{n\to\infty}\frac{n(A)}{n(R)} \qquad \text{(II-2)}$$

It replaces the "A Priori" assignment of probabilities by an empirical method for determining those probabilities. Such a method minimizes individual judgment and this view is also known as the "Objective Probability". It assumes the existence of some hypothetical "infinite population" and it also assumes that the value of (p) in the limit can be estimated. The "Relative-Frequency" view is based on the notion of "Randomness" and thus "Random Experiment".

M. Tribus (1969, pp.59-60) asserts that the Equation (II-2) has been called the "Long-Run Frequency" interpretation of probability and many writers, principally Von Mises, have insisted that probability be "defined" on the basis of frequencies. To make this definition meaningful, the observer is supposed to imagine an experiment repeated many times. Tribus (1969, p.60) gives some counter examples to show the deficiencies in the "Frequency View" of probability. He considers a cube of sugar selected from a bowl of sugar cubes and supposes that ink dots on the 6 surfaces are marked to produce a die. If the die is to be tossed on a table and if the surface of the table is very wet with water; in these circumstances it shall be impossible to imagine a "Long-Run" experiment. As soon as the first toss is made, the sugar cube will begin to crumble.

McGee (1971, pp.304-305) and Tribus (1969, pp.421-422) give the example of an astronaut sitting in his capsule on the top of the rocket and asking himself, "What is the probability of success of this mission?". The astronaut knows about the previous successes and failures of the rocket, about the previous successes and failures of unmanned test flights. But he does not know whether his own flight will be a success or not. There is no such thing as "Long-Run Frequency" in connection with his own mission: It has never been run before; it will never be run again. Both sugar cube and the launching of a space rocket cases defy the use of the "Frequentist" interpretation of probability.

II.2 The Logical View of Probability

Harvey (1969, pp.239-240) writes that Carnap (1950,1952) is the most important philosopher to expound the "Logical View" of probability. According to Harvey (1969, idem) Carnap does not object to the frequency view but suggest that an alternative probability may be employed which explores the relationship between certain aspects of mathematical probability and inductive logic. The logical relation between a hypothesis and the confirming evidence is conceived by Carnap as being wholly analytic and therefore entirely independent of personal belief or judgment. Thus, probability statements are entirely formal and have no empirical content. However, there were serious difficulties in developing such an inductive logic and the exponents of the logical view, such as Nagel (1939), Carnap (1950, 1952), Keynes (1966) were actually aware. (Harvey, 1969, ibid.)

II.3 The Subjective View of Probability

The "Subjective View" of probability is similar to the logical view in that it accepts that probability represents a relation between a statement and a body of evidence. The main difference is that the "Subjectivist" denies that this is a purely logical relationship and conceives of the relationship as representing a "degree of belief". Clearly, this degree of belief may vary from person to person, and there is no unique relationship between a statement and the evidence for it. Proponents of the subjective view, such as Savage (1954), Ramsey (1960) and R.C. Jeffrey (1965) aimed to develop a normative theory of rational choice in the face of uncertainty.

The principal mathematical tool for the assignment of the "Prior Probability" is known as "Bayes Rule", named for the R. Thomas Bayes (1702-1761) who first proposed it in a paper. (Harvey, 1969, pp.240-242), (Tribus, 1969, pp.73-86).

II.4 The Bayesian View of Probability

Let us consider 2 mutually exclusive and exhaustive hypotheses, H_1 and H_2. An experimenter is conducting an experiment in which he believes that either H_1 or H_2 is true. We shall designate everything he knows about experiment, i.e., his past experience, hunches, etc., by the symbol X, and the data he obtains in his current experiment by D. Now consider the following stages in his reasoning (McGee, 1971, pp.293-294):

(a) His state of knowledge about H_1 before he obtains the new data, or the "Prior Probability" of H_1, given all that he knows is represented by:

$$p(H_1 \mid X) \qquad \text{(II-3)}$$

And be read "The Probability of the truth of H_1 conditional upon the truth of statements X", i.e., they specify the facts on which the probability (p) is based. The notation $p(H_1)$ is meaningless because the data on which the truth of the hypothesis H_1 is to be assessed has not been specified. In brief, the notation argues that "***there is no such thing as an unconditional probability***" and the symbol (p) represents an intermediate "mental construct" in a chain of inductive logic and does not necessarily relate to a physical property existing "out there", i.e., independent of the human mind. (Tribus, 1969, pp.24-25).

Since H_2 is the denial of H_1, we can write:

$$p(H_1 \mid X) + p(H_2 \mid X) = 1.0 \qquad \text{(II-4)}$$

$$p(H_2 \mid X) = 1.0 - p(H_1 \mid X) \qquad \text{(II-5)}$$

(b) The experimenter than obtains new data, D, which affects the plausibility of hypothesis of H_1. He reasons as follows: First, what are the chances of obtaining data, D, if hypothesis H_1 is true? Next, he wants to compare this probability $p(D \mid H_1X)$ with the probability of getting data, D, under any hypothesis whatsoever. The ratio of these two probabilities is called the "Likelihood Ratio":

$$\frac{p(D \mid H_1X)}{p(D \mid X)} \qquad \text{(II-6)}$$

Giving him some feeling for how likely hypothesis H_1 when new data such as D are obtained.

(c) Finally, having established his "Prior Probabilities" and the "Likelihood Ratio", how may the experimenter reason rationally to obtain the "Posterior Probability" of H_1 being true? The model for plausible reasoning requires the use of Bayes Theorem to produce the posterior probability $p(H_1 \mid DX)$:

$$p(H_1 \mid DX) = p(H_1 \mid X)\frac{p(D \mid H_1X)}{p(D \mid X)} \quad \text{(II-7)}$$

(McGee,1971, pp.293-294)

Verbally, Bayes Theorem says that:

> Probability (Hypothesis given Data) is proportional to Probability (Hypothesis) x Probability (Data given Hypothesis).

Or, shortly:

POSTERIOR is proportional to PRIOR times LIKELIHOOD

(http://www.keycollege.com/ws/Bayesian/primer/topic15.htm) (As of date 4 Oct 2003)

Bayes Theorem is important because it provides a means for incorporating previously known information and for updating beliefs as more information becomes available. (E.T. Jaynes, 1985). Illustrative examples to show how to convert priors into posteriors are available in McGee (1971, pp.325-326).

It also provides the method for the "Inverse" questions: Given the outcome of an experiment or observation, what can we say about the underlying situation? Evidently, "Inverse" problems are opposite of the problems trying to predict the outcome of an experiment with given properties. Shapiro (edit. in chief, 1990, pp.49-56) gives examples to develop retrospective supports for hypotheses H_i by the evidences actually observed.

Tribus (1971, pp.78-79) shows how to extend the Standard Bayesian Equation. If the prior assignment of probability is $p(A_i \mid X)$, and some new data D become available, the probabilities are adjusted according to:

$$p(A_i \mid DX) = \frac{p(A_i \mid X)\, p(D \mid A_i X)}{\sum_i p(A_i \mid X)\, p(D|A_i X)} \qquad \text{(II-8)}$$

in which denominator has been "extended" by the Extension Rule. (The derivation is available in Tribus (1971,ibid.)

Since the term on the right depends on the assignment of $p(A_i \mid X)$, there has been much controversy over the question of how to assign "prior probabilities" and the solution to this important problem has been provided by the concept of "entropy" and E.T. Jaynes' (1957) MaxEnt Methodology.

II.5 Concluding Remarks

Chapter II introduces that there are different "Kinds of Probability" concepts and shows that the Subjective / Bayesian view is more appropriate for the complex urban phenomena.

(i) Objective view defines probability as a "Long-Run Frequency".

(ii) Contrary to the above, Subjective view accepts that all probabilities are "Conditional".

(iii) The traditional statistical methods are very insufficient in the problem of assignment of "Prior" probability distributions.

(iv) Bayesian Theory that defines "Prior" and "Posterior" probabilities and the "Likelihood Ratio" values is related to the concept of "Entropy" and to E. T. Jaynes' (1957) MaxEnt method.

CHAPTER III

INFORMATION THEORY & ENTROPY

The purpose of this chapter is to clarify the concepts of "uncertainty", "information" and "entropy" which are also used in constructing the theory of spatial distribution of urban activity. Since Claude E. Shannon (1948) proposed his theory of information, there has been a fierce debate as to what constitutes an appropriate measure or statistic of information. In this debate, there are two schools; the first who believe that information must be an "absolute" characterization of the system of interest; the second who believe that information is a "relative" measure. The first school follows Shannon in that communication engineers regard "information" and degree of "uncertainty" as synonymous. Hence, an increase of information would "remove uncertainty"; while decrease of information would connote increase of uncertainty.

The alternative view of information, which considers the quantity as a "relative measure", is held by the school of "Probability Theorists" who include a number of eminent statisticians such as Fisher, Good, Lindley and Kullback among others. (Batty & March, 1976). In short, this school believes that information can only be measured by comparing one state of the system with another, and computing the information difference. The argument has quite profound philosophical implications for it relates the way in which the world is perceived. The idea that information is a measure of the "gain" in knowledge through "updating" a prior to a posterior probability distribution has an implicitly dynamic quality and has loosely been considered as related to the Bayesian Methodology.

This chapter provides reviews Shannon's and Kullback's information theories, representing the two differing schools and introduces the contributions by E.T. Jaynes (1957) and M. Tribus (1962, 1969) with respect to MaxEnt methodology.

III.1 Shannon's Information Theory

Claude E. Shannon's "A Mathematical Theory of Communication" (1948) is considered as the "*Magna Carta*" of the Information Age. Shannon's discovery of the fundamental laws of data comprehension and transmission marks the birth of "Information Theory". (Verdú, 1998).

Information theory started out as an engineering project. Shannon's simple goal was to find a way to clear up noisy telephone connections. Today, there would be no internet without Shannon's theory. Every new modem upgrade, every compressed file, which includes any in (.gif) or (.jpeg) format, owes something to information theory of Shannon. Even the everyday compact disc would not be possible without error connection based on information theory. (Gimon, 2002). To solve the "noise" problem in communications, Shannon developed a new concept, the "channel" and its associated concepts "the channel capacity" and the "redundancy". In the introduction of his paper (1948), he wrote that:

> Information was the outcome of a finite number of possibilities...The significant aspect is that the actual message is one *selected from a set* of possible messages. (Verdú, 1998,p.2058) (italics in the original).

Shannon and Weaver (1949, pp.48-51) suppose a set of possible events whose probabilities of occurrence are ($p_1, p_2, ..., p_n$). These probabilities are known but that is all we know concerning which event will occur. Then it is asked: "Can we find a measure of how much 'choice' is involved in the selection of the event or how uncertain we are of the outcome?" If there is such a measure, say H($p_1, p_2, ..., p_n$), it is reasonable to require of it the following properties:

(i) It should be continuous in the probabilities (p_i).

(ii) If all the (p_i) are equal, $p_i = \frac{1}{n}$, then H should be monotonic increasing function of (n). With equally likely events, there is more choice, or uncertainty, when there are more possible events.

(iii) If a choice be broken down into two successive choices, the original H should be the weighted sum of the individual values of H.

Shannon then states his "Theorem":

> Theorem 2: The only H satisfying the three above assumptions is the form :
>
> $$H = -K\sum_{i=1}^{n} p_i \log p_i$$
>
> where K is a positive constant. (Shannon & Weaver, 1949, pp.49-50)

There is no proof in the book, but it is indicated that the theorem is given to lend a plausibility to some later definitions and the real "justification" of these definitions will reside in their implications. H function was recognized as "Entropy" as in Boltzmann's famous H theorem in statistical mechanics. Some authors claim that the name was given on the advice of Von Neumann (Webber, 1979, p.29). But S. Verdú (1998), in his paper "Fifty Years of Shannon Theory", notes that full and sole credit is due Shannon for the introduction of "Entropy" in information theory.

E.T. Jaynes (1957, p.630) in his appendix and A.G. Wilson (1970, pp.131-132) also maintain that the function

$$H = -K\sum_{i=1}^{n} p_i \log p_i \qquad \text{(III-1)}$$

measures in a unique way the amount uncertainty presented by the probability distribution $H(p_1, p_2, \ldots\ldots\ldots, p_n)$. In other words, the only function which satisfies the above three conditions is the "Entropy" Equation (III-1). M.J. Webber (1979, pp.25-35) interprets the three conditions of Shannon and extends them to five desiderata.

III.2 Kullback's Information Theory

According to Shannon (1948), information is accumulated as probabilities assigned to propositions change. In the Equation (III-1), as the number of events increases, information H increases. Kullback's (1959) measure of "information gain" compares probabilities "before" and "after" an observation:

$$I = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \qquad \text{(III-2)}$$

Where;

I = Information measure
p_i = Posterior probabilities
q_i = Prior probabilities

Information gain or difference can be controlled by taking the uncertainty of the posterior probability (p_i) from the uncertainty of the prior probability (q_i) and calculating its "expected value" in terms of the posterior probability distributions. Uncertainty of an event, or information (I_i) obtained from an observation that (x_i) is true is :

$$I_i = \log \frac{1}{x_i} = -\log x_i \qquad \text{(III-3)}$$

as Hartley (1928) suggested. (cited in Hare, Van Court, Jr., 1967, p.509). Gigch (1974) also gives the same equation above. Verdú (1998) informs us that Shannon has read Hartley's (1928) paper. Taking the differences between prior (q_i) and posterior (p_i) probability distributions and obtaining the expected value in terms of posteriors we can write :

$$I = \sum_{i=1}^{n} p_i \left[-\log q_i - (-\log p_i \right] = \sum_{i=1}^{n} p_i (\log p_i - \log q_i) \quad \text{(III-4)}$$

$$I = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \quad \text{(III-5)}$$

where logarithms are to the base ***e***. (Batty & March, 1976).

III.3 Comparison of Shannon's and Kullback's Information Theories

Equation III-5 is known as the Kullback's (1959) "Information Gain" or "Expected Information" measure. Shannon's "uncertainty" and Kullback's "information gain" share many common properties. Nevertheless, they differ in one significant feature: Shannon's is a measure of uncertainty whereas Kullback's a measure of information gain. Webber (1979, pp.74-78) explores the implications of differences between Shannon's and Kullback's measures as follows:

In passing from a probability distribution which represents maximum uncertainty $P_0 = (\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})$ to one which represents maximum certainty (P_1), one acquires $I_K(P_1, P_0)$ of Kullback information (I_K). On the other hand, $I_K(P\ , P_0)$ is obtained in passing from P_0 to $P = (p_1, p_2, ..., p_n)$. The difference between these two information gains is the information which still must be acquired to pass from P to P_1. This quantity is defined as the Kullback uncertainty H_K associated with P and the function is defined by:

$$H_K[P] = I_K[P_1; P_0] - I_K[P; P_0] \quad \text{(III-6)}$$

Webber (1979, pp.75) introduces four measures or uncertainty depending on the above definitions:

Shannon's Uncertainty: $H[P] = -\sum_{i=1}^{n} p_i \log p_i$ (III-7)

Shannon's Information: $I[P;P_0] = H[P_0] - H[P]$ (III-8)

Kullback's Uncertainty: $H_K[P] = I_K[P_1;P_0] - I_K[P;P_0]$ (III-9)

Kullback's Information: $I_K[P;P_0] = \sum_{i=1}^{n} p_i \log \frac{p_i}{p_{0i}}$ (III-10)

The following points are also important in the comparisons made by Webber (1979, pp.75-78):

(i) If data or observations cause to change one's beliefs from prior $Q = (q_1, q_2, ..., q_n)$ to posterior $P = (p_1, p_2, ..., p_n)$ then Kullback information gained; no matter whether the data are correct or not. If beliefs are not changed, information is not acquired, as the Kullback's Equation III-10 demonstrates.

(ii) By Shannon's definition, the manners in which priorities are changed to posterior probabilities are irrelevant; which depends only on the initial and final probability distributions and not on the intermediate probabilities. By contrast, if we employ Kullback's measure of information gain, the two observers acquire different amounts of information, depending not only on the initial and final but also on the intermediate probability distributions.

(iii) In a case where observations cause the final probabilities to be changed back to the initial probabilities, so that one's initial and final estimates of the probabilities are the same, then by Shannon's measure one has acquired no information, whereas if Kullback's measure is used, one may have obtained a positive or negative quantity of information. This means that Shannon's information measure is a function of states, whereas Kullback's measure is a function of the path taken between states.

(iv) Before observing (A), one's uncertainty about (B) is $H\left[P(B|a_j)\right]$, depending on the conditional uncertainty about (B) given (a_j). When observations have been made, one is forced to change one's opinion about the probabilities of the propositions in (B) from P(B) to $P(B|a_j)$, so that uncertainty about (B) changes:

$$I\left[B|a_j\right] = H\left[P(B)\right] - H\left[P(B|a_j)\right] \qquad \text{(III-11)}$$

As the above Equation III-11 shows, the difference between the two levels of uncertainty measures the information acquired and this information may be "positive" or "negative", or even zero, if the observation is irrelevant to the scheme (B). Hence, Shannon's information received from a particular message, may be "negative", since information is accumulated as the probabilities attached to propositions change; though the expected Shannon information conveyed by a message is always positive. Shannon's uncertainty is maximized when all propositions are all equally probable, $P_0 = (\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})$. (Webber, 1979, p.46, p.60).

(v) Shannon's measure has the advantage over Kullback's measure because Shannon's measure does not depend on the order of data collection, whereas I_K is affected by the order in which adjustments are made.

(vi) Some authors suggested that Kullback's measure of information gain $I_K\left[P;Q\right]$ be used to define one's uncertainty about the state of the system. This suggestion includes the urban spatial systems and arises from several reasons.

Firstly, the obvious reason is that Shannon's H can always be obtained from I_K, but not vice versa. (Batty & March, 1976). Shannon's H is merely a special case of Kullback's I_K. (Webber, 1979, p.76, p.98).

Secondly, there exists some zoning and spatial modeling problems in which we should use I_K with non-uniform Q distributions. In the modeling of spatial systems, models ought to be dimensionally correct so that the properties of the system which distort the model should be filtered out. In this sense, measure of information used must reflect such properties. In this example, if land area is regarded as a measure of partitioning, the formula for information must be consistent for both discrete and continuous distributions. M. Batty (1974a), argues that Shannon's analogy between discrete and continuous entropy equations lacks rigor and this problem is disturbing where "interval size" is important

Thirdly, in urban spatial systems, there are several distributions which might be considered "prior", such as land areas, and I_K provides as a useful way of considering such information. (Batty & March, 1976).

Finally, there is the question of the method for comparing priors and posteriors. The choice of I_K, rather than H enables this kind of building dynamic models to be used in which new information changes the state of the system.

These important reasons also justify why this thesis used the "Spatial Entropy" concept based on I_K measure in the Ankara case study. Chapter V is devoted for the review of the "Spatial Entropy" concept as developed by M. Batty (1974a; 1974b).

III.4 E. T. Jaynes' Maximum Entropy Principle

Edwin T. Jaynes (1922-1998), published his first articles in information theory, "Information Theory and Statistical Mechanics" in 1957. In these two articles appeared in "Physical Review", Jaynes reformulated statistical mechanics in terms of probability distributions derived by the use of the "Principle of Maximum Entropy". This reformulation of the theory simplified the mathematics, allowed for fundamental extensions of the theory, and reinterpreted statistical mechanics as inference based on "incomplete information". (Bretthorst, G. L., 1999).

Jaynes (1957, pp.620-623) in his first part of the articles points out that there were some "unsolved" problems in statistical mechanics and there was no a satisfactory theory in the sense that there is no line of argument proceeding from the laws of microscopic mechanics to macroscopic phenomena. Jaynes (1957, idem) defined the general problem as follows:

> The quantity x is capable of assuming the discrete values $x_i (i=1,2,...,n)$. We are not given the corresponding probabilities p_i; all we know is the expectation value of the function f(x):
>
> $$< f(x) > = \sum_{i=1}^{n} p_i f(x_i)$$
>
> On the basis of this information, what is the expectation value of the function g(x)? (Jaynes, 1957, p.621)

Jaynes (1957, idem) adds that at the first glance, it seems "insoluble", because the given information is not sufficient to determine the probabilities (p_i), though $\sum_{i=1}^{n} p_i = 1$. Hence, the problem was the assignment of probabilities in cases where little or no information is available. Laplacian "Principle of Insufficient Reason", which asserts that two events are to be assigned equal probabilities if there is no reason to think otherwise, was an arbitrary assumption. He writes that our problem is that of finding a probability assignment which avoids bias, while agreeing with whatever information is given. (Jaynes, 1957, idem). Jaynes accepts that the great advance provided by Shannon's (1948) information theory lies in the discovery that there is a unique, unambiguous criterion for the "amount of uncertainty" represented by a probability distribution:

$$H(p_1, p_2, ..., p_n) = -K \sum_{i=1}^{n} p_i \ln p_i \qquad \text{(III-12)}$$

since, this is just the expression for "entropy" as found in statistical mechanics, it is called the "entropy of the probability distribution p_i", the terms "Entropy" and "Uncertainty" should be considered as synonymous. Thus the concept of "Entropy" supplies the missing criterion of choice and the Jaynes' principle of "Maximum Entropy", i.e., MaxEnt, is stated as follows:

> ***The minimally prejudiced probability distribution is that which maximizes the entropy subject to constraints supplied by the given information.*** (Tribus, 1969, p.120)

Therefore, the MaxEnt distribution is "**uniquely determined as one which is maximally noncommittal with regard to missing information"**. (Jaynes, 1957, p.623).

During his stay at St. College in Cambridge, England; he tried to find the tomb of Thomas Bayes and he succeeded at this. (Bretthorst, 1999).

Hestenes (1984), reviews the book of collected articles by E. T. Jaynes and asserts that the book is the evidence that widely claimed "***Bayesian Revolution***" in statistics has been superseded by a "***Jaynesian Revolution***" of far greater consequence.

III.5 Generalization by Myron Tribus

M. Tribus (1999), in his presentation of the paper "A Tribute to Edwin T. Jaynes" at the 18th Workshop on Maximum Entropy and Bayesian Methods, Max Planck Institute, in Garching,/Munich, explained how he met Ed Jaynes in 1958:

> …When I was examined for my doctoral degree, I was asked to explain the connection between the entropy

> defined by C. Shannon and the entropy defined by Clausius a century earlier… Neither I nor my committee knew the answer. I was not at all satisfied with the answer I gave. That was in 1948… I read everything… to explain the connection between the entropy of Clausius and the entropy of Shannon. I got nowhere… One day, in 1958, a student of mine,… replied to my question about the connection between Shannon and Clausius by saying "Oh, that's already been solved". He referred me to Ed Jaynes' famous paper in Physical Review… Here was my Rosetta stone!... I went home and worked with that paper for a week, almost without sleep… Ed Jaynes was then at Stanford. I took an overnight train and showed up in his office… Ed listened my presentation, encouraged me, and that started our friendship. (Tribus, 1999, pp.11-12)

Tribus published two now-classic books, "Thermostatics and Thermodynamics" (1961) and "Rational Descriptions, Decisions and Designs" (1969), among others.

Tribus has provided a more general and formal framework to the Jaynesian MaxEnt methodology. The formalism that Tribus has developed may be characterized as follows (Tribus, 1962; 1969, pp.119-123):

(i) There is some variable, (x), which can take on different possible values, but we don't know which value it has. We know the "***possibilities***"; we wish to assign "***probabilities***".

(ii) Certain functions of (x) have been measured but we do not have the individual measurements. All we have are averages, that is, we know the functions, say $g_j(x)$, but all we have is some mean value, $\overline{g}_j$, for each of the functions, $g_1(x)$, $g_2(x)$,…, $g_j(x)$.

(iii) We would like to generate probability distributions which agree with these averages but are "maximally non-committal" with respect to the missing information: Maximize the entropy by finding the probability distribution which makes $S = -\sum p_i \ln p_i$ a maximum while agreeing with the equations which represent the given information.

(iv) Use the resulting probability distribution in Bayes' equation when the new information becomes available. Mathematically we can define the Jaynes-Tribus formalism as follows:

Maximize:

$$S = -\sum_{i=1}^{n} p_i \ln p_i \qquad \text{(III-13)}$$

Subject to the constraints:

$$\sum_i p_i = 1 \qquad \text{(III-14)}$$

$$\sum_i p_i g_1(x_i) = \overline{g}_1 \qquad \text{(III-15)}$$

$$\vdots$$

$$\sum_i p_i g_j(x_i) = \overline{g}_j \qquad \text{(III-16)}$$

Where:

$p_i = p(x_i \mid \overline{g}_1, \overline{g}_2, \ldots \overline{g}_j \ldots X)$ is the probability of i.
x_i = The value of x is x_i
$g_i(x_i)$ = is the function of x_i which embodies information on the x's
$\overline{g}_1$ = The mean value of $g_1(x)$ is $\overline{g}_1$
$\overline{g}_j$ = The mean value of $g_j(x)$ is $\overline{g}_j$
("mean" values are also the "expected values)
X= All the other facts in the case

There will be an infinite number of possible probability assignments. The distribution which maximizes S in Equation III-13 is considered the "minimally prejudiced assignment" in that it makes the distribution maximally broad. To maximize the entropy in S, subject to the constraints in equations above, Lagrange's method of multipliers is used. If S is to be maximum, the ∂S must be zero, with respect to ∂p_i. Then

$$\partial(\text{-S}) = \sum_i (\ln p_i + 1)\partial p_i = 0 \qquad \text{(III-17)}$$

Since the mean values do not depend on p_i, their variations are zero. We find:

$$\sum \partial p_i = 0 \qquad \text{(III-18)}$$

$$\sum g_1(x_i)\partial p_i = 0 \qquad \text{(III-19)}$$

$$\vdots$$

$$\sum g_j(x_i)\partial p_i = 0 \qquad \text{(III-20)}$$

If we multiply the first equation by $(\lambda_0 - 1)$, the second by λ_1, etc, and add then all to the equation $\partial(\text{-S})$ above, we find:

$$\sum_i (\ln p_i + 1)\partial p_i + (\lambda_0 - 1)\sum_i \partial p_i + \lambda_1 \sum_i g_1(x_i)\partial p_i + \ldots$$
$$\ldots + \lambda_j \sum_i g_j(x_i)\partial p_i = 0 \qquad \text{(III-21)}$$

where $\lambda_0, \lambda_1, \ldots, \lambda_j$ are the Lagrangian multipliers. Then it follows that, by collecting the terms:

$$\sum_{i}\left[\ln p_i + 1 + \lambda_0 - 1 + \lambda_1 g_1(x_i) + \ldots + \lambda_j g_j(x_i)\right]\partial p_i = 0 \qquad \text{(III-22)}$$

For this equation to be satisfied, the term in the parenthesis must be zero. If we solve for $\ln p_i$:

$$\ln p_i = -\lambda_0 - \lambda_1 g_1(x_i) \ldots - \lambda_j g_j(x_i) \qquad \text{(III-23)}$$

$$p_i = \exp\left[-\lambda_0 - \lambda_1 g_1(x_i) \ldots - \lambda_j g_j(x_i)\right] \qquad \text{(III-24)}$$

There are as many Lagrangian multipliers as there are equations for constraint. This is the "**General Form**" for the least prejudiced probability distribution. It is only necessary to substitute the information available on the $g_1, g_2, \ldots$ in terms of equation p_i and to solve for the undetermined multipliers. By substituting back their values one obtains the least prejudiced distribution for a given set of constraints. Tribus (1962;1969, pp.124-179) show the Exponential, Gaussian, Gamma, Poisson and other distributions are obtained in various examples with the given constraints.

III.6 The Wilson Model For MaxEnt Method

Let us refer to the example in Chapter I.3.4.3, explaining the concepts of macro, micro and the most probable states, based on the Wilson (1967, 1970) Model. To find the most probable state, we need to find the matrix (T_{ij}) which maximizes entropy;

$$W(T_{ij}) = \frac{T!}{\prod_{ij} T_{ij}!} \qquad \text{(I-7) \& (III-25)}$$

subject to the macro level constraints:

$$\sum_j T_{ij} = O_i \qquad \text{(I-1) \& (III-26)}$$

$$\sum_i T_{ij} = D_j \qquad \text{(I-2) \& (III-27)}$$

$$\sum_i \sum_j T_{ij} c_{ij} = C \qquad \text{(I-3) \& (III-28)}$$

As defined before in Chapter I. Wilson and Kirkby (1980, pp.308-311) maximizes:

$$S = \log W(T_{ij}) \qquad \text{(III-29)}$$

instead of Equation (III-25) for convenience. The method is called an "**entropy maximizing method**", because when a function such as S in Equation (III-29) is formed in an equivalent analysis in physics, it forms the entropy of the system. If there are N zones in an urban area, S is a function of N^2 variables. Lagrangian *L* has to be maximized:

$$L = S + \sum_i \lambda_i^{(1)} (O_i - \sum_j T_{ij}) + \sum_j \lambda_j^{(2)} (D_j - \sum_i T_{ij}) + \beta(C - \sum_i \sum_j T_{ij} c_{ij}) \qquad \text{(III-30)}$$

where $\lambda_i^{(1)}$ and $\lambda_j^{(2)}$ are the Lagrangian multipliers associated with O_i and D_j constraint equations, and B with C constraint. We must get S into a more convenient form so that we can find $\frac{\partial S}{\partial T_{ij}}$.

$$S = \log W(T_{ij}) = \log \frac{T!}{\prod_{ij} T_{ij}!} = \log T! - \sum_i \sum_j \log T_{ij}! \qquad \text{(III-31)}$$

Using Stirling's approximation:

$$\log N! = N \log N - N \qquad \text{(III-32)}$$

$$\log T_{ij}! = T_{ij} \log T_{ij} - T_{ij} \qquad \text{(III-33)}$$

$$\frac{\partial \log N!}{\partial N} = \log N$$

We must find T_{ij} matrix to satisfy the following conditions:

$$\frac{\partial S}{\partial T_{ij}} = -\log T_{ij} - \lambda_i^{(1)} - \lambda_j^{(2)} - \beta c_{ij} = 0 \qquad \text{(III-34)}$$

$$O_i - \sum_j T_{ij} = 0 \qquad \text{(III-35)}$$

$$D_j - \sum_i T_{ij} = 0 \qquad \text{(III-36)}$$

$$C - \sum_i \sum_j T_{ij} c_{ij} = 0 \qquad \text{(III-37)}$$

From Equation (III-34), by taking anti-logs, we can obtain:

$$T_{ij} = e^{-\lambda_i^{(1)} - \lambda_j^{(2)} - \beta c_{ij}} \qquad \text{(III-38)}$$

We can find $\lambda_i^{(1)}$ and $\lambda_j^{(2)}$ by substitution in Equation (III-35) and (III-36) and we make transformations:

$$e^{-\lambda_i^{(1)}} = A_i O_i \qquad \text{(III-39)}$$

$$e^{-\lambda_j^{(2)}} = B_j D_j \qquad \text{(III-40)}$$

Then Equation (III-38) can be written as:

$$T_{ij} = A_i B_j O_i D_j e^{-\beta c_{ij}} \qquad \text{(III-41)}$$

Thus, this is the usual "Doubly Constrained Spatial Interaction Model", $T_{ij} = A_i B_j O_i D_j c_{ij}^{-\beta}$, except that deterrence function $c_{ij}^{-\beta}$ is replaced by the negative exponential function $e^{-\beta c_{ij}}$. The above equation therefore represents the "most probable" distribution of trips and its derivation constitutes a new theoretical base. (Wilson & Kirkby, 1980, pp.308-311; Wilson, 1970, pp.15-19; Wilson, 1967). More detailed explanations, illustrations are available in Webber (1977) and Gould (1972). (In Appendix-B (1977), Equations from (25) to (30) were also presented in a summary form).

By his pioneering MaxEnt method, A. G. Wilson (1967, 1970) "generalized" the traditional Gravity Model, $T_{ij} = KO_i D_j f(c_{ij})$, developed and applied during the decades before 1967, and derived 5 different cases of spatial interaction models.

Case (1): Unconstrained Flows: In this case, neither the set of totals O_i nor the set of totals D_j is known. Hence, O_i is replaced by W_i and D_j by W_j, i.e., by their "attractiveness" measures, and the factor of proportionality K suffices for the model:

$$T_{ij} = KW_i W_j f(c_{ij}) \qquad \text{(III-42)}$$

where $f(c_{ij})$ represents the "generalized cost" function, used instead of the distance deterrence function $f(d_{ij}^{-\alpha})$.

Case (2): Origin-Constrained Flows: In this case, O_i is known or given, while D_j is not, and so D_j in Equation (III-42) is replaced by W_j. Also, a proportionality factor (A_i) to ensure that $\sum_{j=1} T_{ij} = O_i$ is satisfied. K did not have a subscript for zones, which was an "inconsistency" in the model equation. More explanations about the mathematical inconsistencies and deficiencies were given in Equations I-14, 15 and 16 of Chapter I.3.4.4. Replacement of K by set of factors A_i alleviates the mathematical inconsistency and also satisfies the constraint equation:

$$T_{ij} = A_i O_i W_j f(c_{ij}) \qquad \text{(III-43)}$$

$$\sum_{j=1}^{N} T_{ij} = O_i \qquad \text{(III-44)}$$

where

$$A_i = \left[\sum_{j=1}^{N} W_j f(c_{ij}) \right]^{-1} \qquad \text{(III-45)}$$

Case (3): Destination-Constrained Flows: This case is very similar to case (2). We know D_j but not O_i:

$$T_{ij} = B_j D_j W_i f(c_{ij}) \qquad \text{(III-46)}$$

$$\sum_{i=1}^{N} T_{ij} = D_j \qquad \text{(III-47)}$$

where

$$B_j = \left[\sum_{i=1}^{N} W_i f(c_{ij}) \right]^{-1} \qquad \text{(III-48)}$$

Case (4): Origin-Destination Constrained Flows: The inconsistencies in this case can be resolved if K is replaced by a product of proportionality factors $A_i B_j$ which are then calculated to ensure that the constraint equations are satisfied simultaneously:

$$T_{ij} = A_i B_j O_i D_j f(c_{ij}) \qquad \text{(III-49)}$$

$$\sum_{j=1}^{N} T_{ij} = O_i \qquad \text{(III-50)}$$

$$\sum_{i=1}^{N} T_{ij} = D_j \qquad \text{(III-51)}$$

where

$$A_i = \left[\sum_{i=1}^{N} B_j D_j f(c_{ij}) \right]^{-1} \qquad \text{(III-52)}$$

$$B_j = \left[\sum_{i=1}^{N} A_i O_i f(c_{ij}) \right]^{-1} \qquad \text{(III-53)}$$

Although Wilson (1974, p.67) writes that "we have now derived the whole family of spatial interaction models to fit a variety of circumstances", there is also the fifth member of the "family".

Case (5): Total Cost Constrained Flows: the case (4) is also known as "Doubly-Constrained" case. In addition to the two constraints in case (4), the third total cost constraint:

$$\sum_{ij} T_{ij} c_{ij} = C \qquad \text{(I-4) \& (III-54)}$$

is ensured in this case, as given and illustrated in Equation (I-4). The full derivation of the Total Cost Constrained Flows Model;

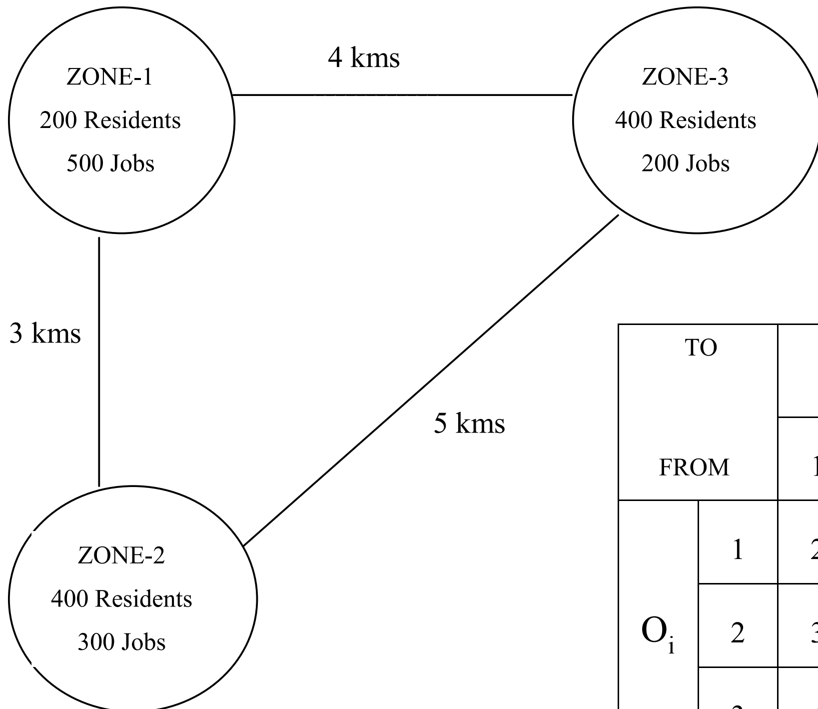
$$T_{ij} = A_i B_j O_i D_j e^{-\beta c_{ij}} \qquad \text{(III-41)}$$

was given in Equations (III-25) & (III-41). Since the A_i and B_j terms are solved to satisfy the O_i and D_j constraints, only the parameter (β) remains for modification to satisfy the Total Cost Constraint in Equation (III-54). In other terms, $f(c_{ij})$ is replaced by the negative exponential function $e^{-\beta c_{ij}}$ in this model (Wilson & Kirkby, 1990, p.310)

III.6.1 The Function of Scaling Factor K in Case (1) Model

This section illustrates the meaning and function of K used in Traditional Gravity Equation (I-8) $T_{ij} = KO_i D_j d_{ij}^{-2}$ and the Equation (III-42) for the Case (I): "Unconstrained Flows Model".

Let us summarize I. Masser's (1972, pp. 95-98) numerical example. Let the 3 zones of a city have the following given data in Fig.III-1.



FROM \ TO		D_j 1	D_j 2	D_j 3
O_i	1	2	3	4
	2	3	2	5
	3	4	5	2

Distance Matrix (kms)

Figure III-1 A 3-Zone City For Case (I)

In this example, the average distance traveled within each zone from home to work is assumed to be 2 kms and the distance deterrence function $f(c_{ij})$ is taken as the predetermined value of 2. The problem is to estimate, or to predict, the unknown amounts of interactions between zones given the total value of trips T = 1000.

However, in general, $\sum_i T_{ij} \neq D_j$ and $\sum_j T_{ij} \neq O_i$ (Baxter, 1972, p.2). The total interaction between all zones must be calculated before the value of the factor K can be calculated for this system. This derived by multiplying out the basic

Equation (I-8) for each pair of zones, setting K=1.0 at this first step. For instance, the interaction between origin O_1 and destination D_1 of Zone-1 will be

$$T_{11} = K O_1 D_1 d_{11}^{-2}$$

where, K=1.00 and O_1=200 residents, D_1=500 jobs, d_{11}=2 kms. Hence for the first cell of the matrix of interaction T_{11} shall be calculated as:

$$T_{11} = 1*200*500*(2*2)^{-1} = 25\,000$$

In this way, a matrix of interactions can be constructed to find the total amount of interaction in the city as a whole. (TABLE III-1)

TABLE III-1 MATRIX OF INTERACTIONS (K=1.0)

From \ To	Jobs (D_j)				TOTAL RESIDENTS (Estimated) $\sum_j T_{ij}$
Residents (O_i)	ZONE	1	2	3	
	1	25 000	6667	2500	34 167
	2	22 222	30 000	3200	55 422
	3	12 500	4 800	20 000	37 300
	TOTAL JOBS (Estimated) $\sum_i T_{ij}$	59 772	41 467	25 700	126 889

Source: Adapted from Masser, 1972 (pp. 96-98)

From Table III-1 it can be seen that the total interaction predicted by the model is 126889. This volume of interaction must now be adjusted so that it is made equivalent to the total number of trips given which refers to T=1000. Then, the value of the scaling factor K can be calculated:

$$K*126\ 886 = 1000 \text{ (trips)}$$

$$K = 1000 / 126\ 886 = 0.007881$$

Each of the estimates of interaction in Table III-1 are multiplied by this adjusted factor to obtain the new estimated trip distribution matrix. (Table III-2)

TABLE III-2 MATRIX OF INTERACTIONS (K=0.007881)

From \ To	Jobs (D_j)				TOTAL RESIDENTS (Estimated) $\sum_j T_{ij}$	TOTAL RESIDENTS (Given)
Residents (O_i)	ZONE	1	2	3		
	1	197	52	20	269	200
	2	175	236	25	436	400
	3	99	38	158	295	400
	TOTAL JOBS (Estimated) $\sum_i T_{ij}$	471	326	203	1000	1000
	TOTAL JOBS (Given)	500	300	200	1000	1000

Source: Adapted From: Masser, 1972 (pp. 96-98)

It is clear that the function of the scaling factor K is to ensure that $\sum_i \sum_j T_{ij} = \sum_j D_j = T$ and in this example the total interaction T=1000 work trips. As no further constraints were included in the model, there are discrepancies between the total number of residents and jobs "estimated" from the "given" O_i and D_j values as shown in Table III-2 above. (It should be noticed that O_i and D_j values are not "actual" or "known" amounts but they represent the "initial estimates" that generate the distribution in Table III-2). As explained in section III.7, Pooler (1994) designated this model as the "Total Interaction Constrained", contrary to the A. G. Wilson's "Unconstrained Model". Obviously, Case(I) is constrained to the total trip T by the scaling factor K. For the other members of the "Spatial Interaction Models", K is replaced by A_i and B_j proportionality factors. The process of calibration of A_i and B_j factors is long and outside of the scope of this thesis. Masser (1972, Chapter 4), Lee (1980, Chapter 5, pp.57-88) and Isard (1998, et.al., Chapter 6) give numerical examples to find A_i and B_j values.Ottensmann (1985) gives software programs for the constrained cases.

III.6.2 Extension of Spatial Interaction Models by J. Pooler

Pooler (1994), in his "An Extended Family of Spatial Interaction Models", introduces "Cost Constrained" cases to the "Singly-Constrained" cases, i.e., Origin-Constrained and Destination-Constrained cases, and to the Doubly-Constrained Case and thus obtains 8-member Wilson family of spatial interaction models. Pooler (1994) uses the term "Total Interaction Constrained", $\sum_i \sum_j T_{ij} = T$, instead of the term "Unconstrained". Obviously, case (1) is also a "constrained" model to the total interaction amount T.

Pooler (1994), further extends the 8-member Wilson models by considering the "relaxed" spatial interaction models. The marginals of the predicted matrix are not constrained to match exact totals in the Wilson's way, but are constrained instead to lie within a specified range of values, i.e., an upper (U) and lower (L) bounds specified a priori. For example:

$$O_i^L \le \sum_j T_{ij} \le O_i^U \qquad \text{(III-55)}$$

$$\sum_i T_{ij} = D_j \qquad \text{(III-56)}$$

$$\sum_i \sum_j T_{ij} c_{ij} = C \qquad \text{(III-57)}$$

is an "Origin Relaxed – Destination Constrained" and "Total Cost Constrained" case with Equation (III-49). Pooler (1994) shows that the basic Wilson family of spatial interaction models can be extended and generalized and they are all derived in the common framework of information theory, using MaxEnt and Information Minimizing methods.

III.6.3 General Criticisms and Recent Trends in Spatial Interaction Models

This section gives an overview of the general criticisms on the gravity model and recent contributions to the spatial interaction models as additional information to Chapter I.3.4.4.1.

One of the most influential and comprehensive criticisms against not only the gravity and spatial interaction models but to the whole urban modeling have been made by Sayer (1976; 1979). Some of the important issues Sayer (1976, Chapter 3: Critique of Gravity / Entropy Maximizing Models, pp.202-207; Chapter 4: Towards an Alternative Approach, pp.218-229) can be summed as follow:

1-) The term "Gravity Model" has become a misnomer and is sometimes replaced by "Spatial Interaction Model". The former term will be retained because its continued popularity and because the latter term is misleading as there are a large number of spatial interaction models which do not belong to the entropy maximizing model.

2-) Mathematical consistency is no guarantee of theoretical consistency, for the latter depends on the assumptions and the type of "abstraction" and the way in which constraints in Wilson's (1967, 1970) approach are specified.

3-) His "Critique" (1976), does not challenge to the mathematical consistency of MaxEnt models.

4-) both doubly and singly-constrained interaction models assume a fixed amount of expenditure, where Sayer (1976, p.205) means the total cost Equation (III-54), and therefore an inelastic demand for travel.

5-) The separation of "Trip Generation" and "Trip Distribution" is a long-standing problem of gravity models and the MaxEnt derivation has nothing to mitigate this artificial and arbitrary separation.

6-) Gravity Models fail to specify the essential links between the "reality" and its "abstraction". They abstract from the social and economic relations which determine the supply and demand for travel. Hence, Gravity Models use a posterior form of analysis. The origin and destination constraints from which the Spatial Interaction Models are derived are "market outputs" but not " market determinants". As such, these ex post constraints are nothing more than arithmetical truism which may make for mathematical consistency, but which hide the ex ante determinants of behavior.

7-) Spatial constraints are omitted relating to the effect of use of a bounded area, of spatial autocorrelation in origins and destinations.

8-) The trip distribution is time-dependent and not time independent as usually assumed. There is an implicit assumption of instantaneous equilibrium that demand for trip-ends create its own supply perfectly and instantly.

9-) The distance terms, as given in gravity models, represent an implicit assumption of "Absolute Space". They imply that trip frequency declines in an absolute, i.e., empty space as a function of distance and independently of objects located in space. The absolute and relative space concepts are quite incompatible and their incompatibility is probably the reason for many of the problems of gravity models.

10-) Both gravity and utility-maximizing models have the similar deficiencies: Trip distribution is cut off from its determining spatial and socio-economic relations in gravity models; in utility theory, ex post economic behavior is examined in abstraction from its determinants of human needs and supply conditions. As a result, both models can only be descriptive rather than explanatory and while they may be fitted with reasonable success, they do not offer worthwhile testable hypotheses (p.213).

11-) Although gravity and utility-maximizing models examine ex post market relations, the insights gained from such relations are very limited. However, such studies do not gain anything through the rationalization of utility theory.

12-) Although such models might refer to **travel costs** and **not distance**, and hence seem to escape the absolute space implications, their faults are structurally identical. Utility theory and gravity models share the same "defective" mode of abstraction.

Wilson (1978) reviews Sayer's (1976) book above and replies that "there was no clear presentation of the philosophical basis" of Sayer's attract on urban modeling. Wilson also complains about the "difficulty" of concepts such as "absolute" and "relative" space. Sayer (1979), in his "A Reply to Wilson" discusses the philosophical bases of the "Critique" and clarifies why positivism in social studies fail to understand the differences between social and natural facts.

Fotheringham (et. al., 2000, pp.213-235) points out that the major criticisms leveled at spatial interaction modeling stem from the first three phases of the evolution of the spatial interaction theory when urban geography borrowed heavily from concepts developed in other "aspatial" disciplines such as economics and physics. According to Fotheringham (et.al, 2000, ibid), four distinct phases of the spatial interaction can be identified in chronological order as follows: Spatial Interaction (a) as “social physics” (1860-1970); (b) as “statistical mechanics” (1970-1980); (c) as “aspatial information processing” (1980-1990) (d) as “spatial information processing” (1990 onwards).

Fotheringham (et.al, 2000, ibid), explain how Wilson’s (1967) entropy-maximizing derivation of a “Family of Spatial Interaction Models” provided a theoretical justification for what had been until that time only an empirical observation. Authors give the criticism which can be made in the derivation: (a) Replacement of one physical analogy, i.e, that of gravitational attraction with another, i.e., that of statistical mechanics. There is still sterile in terms of the processes by which individual make decisions. (b) While some of the constraint equations have behavioral interpretation, others, such as those on the populations, are more difficult to justify. (c) The use of Stirling’s approximation to derive the entropy formulation is highly suspect. When the T_{ij} values are small, the approximation is rather poor.

Authors above point out that Tribus (1969) and others eliminate the need to derive the entropy formulation from a discussion of microstates and macrostates and argue that entropy formulation satisfies the reasonable requirements of a measure of statistical uncertainty and that uncertainty about any outcome should be maximized subject to constraints. Otherwise, bias would be added into the model-building procedure. Despite this argument, considerable difficulties with entropy maximization exist as a framework for the development of human spatial behavior. Spatial interaction models as “aspatial” information processing (1980-1990) have a theoretical foundation based on human behavior and information processing related to the utility of choosing among the alternative destinations. However, it is a framework which has also been “borrowed” from economics, another “aspatial” discipline. Models developed during 1990s and onwards aimed to explain the information processing likely to take place in “spatial” choice among competing alternatives.

Consequently, the derivation of Spatial Interaction Models in the framework of discrete spatial choice represents an advance over the physical analogies from physics or statistical mechanics. In the discrete choice framework, the individual is assumed to process information on all the large number of alternatives. To overcome difficulty, Fotheringham developed a new form of spatial interaction model termed a "Competing Destination" model where individuals do not evaluate every spatial alternative and the choice is made from restricted set of spatial alternatives. (Fotheringham, et.al, 2000; Roy and Thill, 2004).

As seen above overview, Fortheringham (et.al., 2000) put the evolutionary stages of spatial interaction models into a chronological perspective. The conceptual changes during the four stages each of which has provided a "quantum leap" are in accordance strongly with Kuhn's (1970, 1962) concept of "Paradigm Shift" as stated in Chapter I. Introduction.

Roy & Thill (2004) in their article "Spatial Interaction Modeling" review also the progress of Spatial Interaction Models but authors put more emphasis to the entropy approach and its relations with the recent developments of spatial interaction models during 1990s. According to Roy and Thill (2004, p.339) "The key advances include the replacement of the gravity analogy by the more general concepts of entropy or information theory".

However, it seems that Sayer's "Critique" raised in 1976 & 1979 have not been dealt with yet. It can be asserted here that the developing Quantum Information Theory may influence deeply the new derivation methods based information processing in the near future.

The current research on space-oriented spatial interaction models that are linked to information-theoric methods can also be related to Batty's (1974a, b) "Spatial Entropy" concept as presented and applied in Chapter V and VI.

III.7 The Link Between the Two Entropies

Is there a link between the "entropy" in Shannonian information theory, and the "entropy" used in classical physics and statistical mechanics, or, is the resemblance between these two entropies is merely an analogy? Does MaxEnt methodology used in urban and regional researches borrowed the term "entropy" directly from statistical mechanics, developed by Boltzmann (1844-1906), Gibbs (1839-1903) and others?

This section aims to make some clarifications and to indicate that the methods used in both fields are the same and both deal with the probability distributions. The concept of "entropy" has had a long history of development. Dutta, (1968) in his article "A Hundred Years of Entropy" gives a summary of the contributions made to the entropy concept in different disciplines. The word was originally coined from the Greek analogue of the Latin term "evolution" by Classius in 1865 in his definition,

$$\Delta S = \Delta Q / T \qquad \text{(III-58)}$$

which means the change of entropy S equals the change of energy Q divided by the temperature T. Classius chose the term to mean "transformation" (**τροπη**) "+ en" to make it resemble <u>en</u>ergy. Mogridge (1972) points out several important properties of the term entropy:

(1) The term is defined as a "<u>difference</u>", not as an absolute amount. One cannot have an absolute entropy.

(2) The term is a pure number. According to Mogridge (1972, idem), "This is the most important and confusing property". Being a pure number, entropy has no physical embodiment; being a "<u>difference</u>", it has "<u>no beginning and no end</u>"; being a ratio of two energies, it can only be used with analogies of energy.

(3) It measures energy change ΔQ. In other words, only where energy values are changing, where there is interaction, we can have an entropy value. Classius' equation was a description of the "macroscopic" properties of interacting systems. Later the function:

$$S = kNH \qquad (III-59)$$

$$H = \sum_i f_i \ln f_i \qquad (III-60)$$

where

N = Total number of particles
N_i = Total number of particles in state (i)
$f_i = N_i / N$

was proposed by Boltzmann (1877). Similarly, he reasoned that Classius' equation would be expressed in statistical term:

$$S = k\ln W \qquad (III-61)$$

where S=entropy, k=constant, $W = N!/ N_1 ! N_2 ! ... N_m !$ and m= the total number of states in the system.

Boltzmann shows the number of microstates corresponding to macroscopic equilibrium is "overwhelmingly" large compared to other microstates and the "most probable state" is that for which S is a maximum. Shannon's (p_i) in Equation (III-1) is the same as the one used by Boltzmann to derive his H function above. Therefore, Shannon's equation is equivalent to Boltzmann H function, except that Shannon (1948) put a negative sign in the equation to get a positively valued measure. (Haynes, et.al, 1980). According to Tribus (1969), there was considerable debate whether Shannonian equation was the same or merely an analog to the entropy of statistical mechanics. Founders of statistical mechanics posed the fundamental problem as: “How can microscopically highly complex motions of atoms give rise to macroscopic phenomena with deterministic descriptions?” (Wightman, 1971). Jaynes in his 1957 paper and later publications demonstrated that the two entropies were in fact, examples of the same idea, and therefore they were not merely analogies. (Tribus, 1969, p.110; Tribus, 1999).

Yet, Webber (1976) points out that the philosophic basis of MaxEnt model seems to be misunderstood. Such misconceptions are primarily associated with the view that these models are merely analogies drawn from statistical mechanics; they regard entropy as a physical concept and they only describe systems at equilibrium state. Hence Webber (1976; 1979, pp.105-109) aims to demonstrate the falsity of such views in regional and urban research and to dispel them. Pooler (1983) very clearly asserts that:

> Regardless of whether we regard entropy as a measure of order, uncertainty of information; regardless of whether we are looking at the distribution of temperature, molecules, information or population; and regardless of whether we take Classius, Boltzmann, Shannon or Jaynes as our source, the reasoning behind the concept remains the same throughout: Entropy is simply a measure of orderliness of a distribution at a point in time.(Pooler,1983,p.155)

Above explanations should justify the conclusion that “entropy” has a deeper meaning than its assumed physical analogy and it is not a concept borrowed from physics, MaxEnt methodology provides a way of making maximally unbiased inferences consistent with the given amount of information. Contrary to the assertion that the “**controversy is vacuous**” by Rapoport (1972), this thesis considers that the “**controversy is not vacuous**”.

III.8 Quantum Information Theory

In Chapter I.5 explaining the “Scope of the Thesis” it was indicated that the special issue of ***IEEE Transactions On Information Theory*** (Vol.44, No:6, 1998), aimed to evaluate the developments in the "Information Theory" during the five decades since 1948. Bennett and Shor (1998), in their invited paper compares the "Classical" and "Quantum" information theories. Authors assert that an information theory based on quantum principles extends and completes classical information theory, somewhat as complex numbers extend and complete the real numbers. Hence, the "New Theory" includes quantum generalizations of classical notions such as sources, channels and codes as well as two complementary quantifiable kinds of information: Classical Information" and "Quantum Entanglement". Classical information can be copied freely, but can only be transmitted forward in time to a receiver. "Quantum Entanglement", by contrast, cannot be copied, but can connect any two points in space-time. Moreover, quantum information cannot be read or copied without "disturbing" it. Conventional data-processing operations destroy entanglement, but quantum operations can create it, preserve it and use it for various purposes, notably speeding up computations of classical data or "quantum teleportation" from a sender to a receiver.

In the classical information theory, the logarithm is to the base 2 and entropy is expressed in "Bits", as in the Shannonian entropy (Equation III-1). "Bit" is derived from "Binary Digit", i.e., binary choice, like "Yes-No", "0-1", and measures the amount of information. Hence, the entropy of a fair coin toss is 1, since we have two possible outcomes, i.e., "Head" or "Tail": $H = \log_2 2 = 1$ bit. Similarly, the information associated with an experiment to see whether the sun will rise between midnight and noon tomorrow shall be zero, since there is only one outcome possible: $H = \log_2 1 = 0$ (Raisbeck, 1963, pp.6-8). If the base of the logarithm is (e), then the entropy is measured in "nats" (Cover & Thomas, 1991, p.13).

A quantum "bit", or "Qubit", by contrast is typically a microscopic system, such as an atom or nuclear spin or polarized photon. A qubit can also exist in a two-dimensional "Hilbert Space". The quantum states identified with "rays" in Hilbert

Space, represent both (known) "Pure States" or "Mixed States". Since a mixed state represents incomplete information, the entropy of a mixed state is given by the von Neumann formula. In the "Pure States", there are conditions where the von Neumann entropy is equal to the Shannon entropy (Bennett & Shor, 1998).

Caticha (1998) argues that although probabilities play a most central role in quantum mechanics, the concept of entropy appears only later as an auxiliary quantity to be used only when a problem is sufficiently complicated that "clean" deductive methods have failed and indicates the relationships between the von Neumann and Shannon entropies with respect to the quantum entropy.

Meier (1962), in his book "***A Communications Theory of Urban Growth***", has attempted to build a systematic theory of urban growth based on the concept of information flows, inspired from Shannon's information theory and its related concepts such as "Channel", "Channel Capacity", "Sender" and "Receiver". He introduced a new term "Hubits", meaning a bit of "meaningful information received by a single human being". (Meier,1962. p.131). for example, information transmissions by reading, radio, TV, films etc. in a metropolitan area of 5,000,000 population is estimated as 100,000,000 hubits per year. In relation to his explanation of "Civic Bond", he suggested that the Gravity Model should be replaced by Quantum Theory, because in Quantum Theory the bonds between two different elementary units are created by the sharing of some units which are still simpler and smaller; just like protons and neutrons are bound to each other by the cloud of mesons which they share as satellites. Similarly, in the case of the transmissions of separate "messages" in a city, the "civic bond" shall be stronger and give rise to the urban growth, as the messages are shared and exchanged by individuals or groups. (Meier, 1962, pp.20-44).

D. Deutsch (1985), in his seminal paper "Quantum Theory, the Church-Turing Principle and the Universal Quantum Computer", has shown that the existing theory of digital computation, based on "Turing Machine", can be generalized and that universal quantum computation is possible. Hence, the universal quantum machine can describe any finitely realizable physical system, classical or quantum, discrete or stochastic. In Deutsch's Theory (1985), the Church-Turing Hypothesis that "***Every 'function which would naturally be regarded as computable' can be computed by the universal Turing Machine***" is replaced by a new and more exact physical principle. Turing computation that stands at the heart of present computer science is in the strict sense incorrect (Marcer, 1987). Classical physics is continuous and the Turing Machine is discrete, thus they don't obey the Church-Turing Hypothesis. Deutsch (1985) asserts that there are the motivations for seeking a truly Quantum Model, yet, "***The more urgent motivation is, of course, that classical physics is false***".

Smith (1995) gives reasons why the existing inductive methods are not applicable to the complex events and systems. Prior to the 1980s, "Chaos" was a word that indicated disorder, unpredictability and something to be either avoided or eliminated in the quest for certitude and safety. But since 1980, "Chaos" has ceased to be just a word, and has become instead a "New Science" especially after Gleick's (1988) book "***Chaos: Making A New Science***". Smith (1995) also explains that we have been dealing with a given mathematical model for some observable system and our task was to verify or refine it by use of predictions. These were derived from two sources: Deduction and Induction methods of thinking. Models are reached inductively until a satisfactory formula is obtained. It is argued that the Heisenberg Principle, Gödel's Incompleteness Theorem and recent developments in Chaos Theory introduce restrictions to our model building processes based on inductive or deductive methods. All chaotic functions are extremely sensitive to minor variations in the initial conditions to which the Galilean-inductive method is therefore "a priori" inapplicable. (Smith, 1995).

All these recent scientific developments such as in quantum and chaos theories imply that our descriptions of the urban structure and its modeling and simulation methods will be subject to profound changes in the next decades. Even now, since it is shown that small variations in the initial conditions generate very large deviations in the outputs, because of the sensitivity to the changes, assignment of not only prior but also the posterior probabilities has become a more critical issue in complex and nonlinear large systems such as in urban systems. Evidently, Bayesian, MaxEnt and MIP methods shall have new and significant roles in the future.

Chapter V shall introduce how the spatial dimension has been incorporated into the concept of entropy in urban studies as contributed by Batty (1974a, 1974b).

III.9 Concluding Remarks

(i) Chapter III reviews Shannon's (1948) & Kullback's (1959) Information Theories and shows that Kullback's "Information Gain" measure is more appropriate for urban studies.

(ii) E. T. Jaynes' MaxEnt Principle states that "Entropy" and "Uncertainty" in a probability distribution are synonymous.

(iii) MaxEnt distribution is "Maximally noncommittal with regard to missing information" (Jaynes, 1957, p.623).

(iv) M. Tribus (1962, 1969) generalized the Jaynesian MaxEnt.

(v) A. G. Wilson (1967, 1970), by using the MaxEnt approach, emancipated the Newtonian analogy in the traditional Gravity Model and derived the Family of Spatial Interaction Models with 5 members.

(vi) Sayer's (1976,1979) ''Critique '' on urban planning including the gravity and MaxEnt models are important. Four stages of progress in spatial interaction models can be identified.

(vii) "Entropy" is not a term borrowed from physics or statistical mechanics and the link between different definitions have deeper meaning than assumed physical analogies.

(viii) Recent developments in Quantum and Chaos theories definitely be influential in the MaxEnt method.

CHAPTER IV

MAXIMUM ENTROPY & MINIMUM INFORMATION PRINCIPLES

IV.1 The Minimum-Information Principle (MIP)

Chapter-III reviewed the concept of "entropy" in relation to the information theory and the maximum entropy (MaxEnt) methodology. This chapter deals with both MaxEnt and Minimum-Information Principle (MIP) and attempts to show similarities or differences between them in an simple urban numerical example.

The MIP implies that posterior $P = (p_1, p_2, ..., p_n)$ is obtained by "minimizing" I_K in Kullback's Equations III-2 & III-10 subject to the known information; and Jaynes' principle requires that entropy H be "maximized", in Equation III-1, again subject to the known information. Therefore, the problem of minimizing I_K is mathematically equivalent to that of maximizing $(-I_K)$. A detailed description of MIP is given in Webber (1979, pp.110-142) and relationships between the Jaynesian MaxEnt method are demonstrated.

Fisk's (1985) important article "Entropy and Information Theory: Are We Missing Something?" in Environment & Planning A was followed by five commentary papers and a response by Fisk. Her objective was to compare microstate approaches, aimed to find the "Most Probable State", with Shannon's entropy and Kullback's information gain procedures. According to Fisk (1985), there were some shortcomings of the MIP approach as currently applied in the derivation of urban spatial distribution models and her researches show that the "Most Probable State" and MIP frameworks do not always lead to the same model because certain properties relating to the microstructure of the urban system are not included as information. As the debate in commentary papers show, there are controversial views as to the interpretation and relative merits of each approach. Leaving aside the long-standing controversy between the "Most Probable State" approach and MIP or MaxEnt, in the following sections a simple urban problem shall be presented and solved by both the MaxEnt and MIP to help clarification of the procedures, as drawn from Pooler's (1983) article.

IV.2 A Simple Urban Problem

Let's imagine a simple linear city composed of 3 equal area zones. 4 people work in Zone-1, and the problem is to assign them to homes in Zone-2 and/or Zone-3 in an unbiased way. Given only the information that 4 workers in Zone-1 are to be assigned to the 2 residential destination zones, how should we proceed? (Figure IV-1)

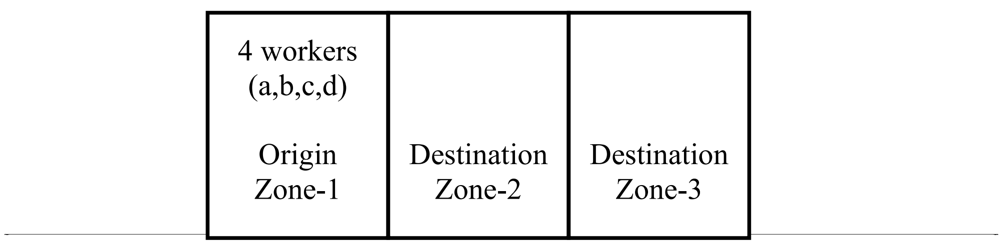


Figure IV-1 A 3-Zone City

TABLE IV-1 DISTANCE VECTOR

from \ to	Zone-2	Zone-3
Zone-1	1 km	2 kms

(Figure IV-1 & Table IV-1 are obtained according to descriptions by Pooler (1983))

IV.3 Solution By the Most Probable State Method

In the absence of any other information, it might seem that an intuitively unbiased assignment would be to locate 2 workers in each of the 2 residential zones, for this would give probabilities, as in the case of unbiased coin toss, of 0,5 for each zone. But this would be in the line of Laplace's "Principle of Insufficient Reason" and thus such an assignment would not be appropriate. To solve the problem, let us enumerate the "macrostates" and "microstates" with respect to the assignment of numbers of workers to Zone-2 and 3.

If the 4 individual workers are identified as (a, b, c, d), their possible distributions are listed in the second column of Table IV-2. The first column shows the "macrostates" and the second shows the possible "microstates" giving rise to these macrostates. From the third column of the Table IV-2, it is apparent that assignment of the 2 workers to each of the 2 residential zones can occur in **the greatest number of ways**. As it was explained in Chapter I.3.4.3, in the language of the statistical mechanics, the macrostate which has the greatest number of microstates associated with it represents the "**Most Probable State**".

The fourth column gives the probabilities of occurrences of these macrostates and it is clearly seen that the uniform distribution can occur 6 times in the total number of 16 macrostates with the highest probability of occurrence 6/16=0,375.

Of course, this method depended on making the a priori assumption that all "microstates are equally likely". This can be interpreted as a condition which is identical to the "Principle of Insufficient Reason". If the microstates are not a priori equally likely, the MIP allows such prior information to be explicitly incorporated into the analysis, as it shall be shown in Section IV-5 below.

TABLE IV-2 MACRO, MICRO & THE MOST PROBABLE STATES

MACROSTATES Distribution of workers to residential zones (2) & (3) from work Zone (1)		MICROSTATES All possible assignments of individual workers to zone (2) & (3)	Number of Ways of Macrostates Occurring (W_i)	Probability of Occurrence of Macrostates $\frac{W_i}{\sum W_i}$
$T_{ij}^{(1)}$	(1): – | (2): 4 | (3): 0	(1): – | (2): abcd | (3): 0	1	$\frac{1}{16} = 0,063$
$T_{ij}^{(2)}$	(1): – | (2): 0 | (3): 4	(1): – | (2): 0 | (3): abcd	1	$\frac{1}{16} = 0,063$
$T_{ij}^{(3)}$	(1): – | (2): 3 | (3): 1	abc d abd c adc b bcd a	4	$\frac{4}{16} = 0,250$
$T_{ij}^{(4)}$	(1): – | (2): 1 | (3): 3	d abc c abd b adc a bcd	4	$\frac{4}{16} = 0,250$
$T_{ij}^{(5)}$	(1): – | (2): 2 | (3): 2 (Uniform Distribution)	ab cd ac bd ad cb bc ad bd ac cd ab	6	$\frac{6}{16} = 0,375$ (The Most Probable State)
			$\sum_i W_i = 16$	$\approx 1,00$

Adapted from: Pooler (1983), Socio-Econ. Plan. Sci., Vol.17, No: 4, pp.153-164

The definition of $W(T_{ij})$, which is the number of microstates associated with each macrostate (T_{ij}), from statistical mechanics is:

$$W(T_{ij}) = \frac{T!}{\prod_{ij} T_{ij}!} \qquad (\text{IV-1})$$

as we used in Equations (I-7) & (III-25). If we apply the above equation, we have:

TABLE IV-3 CALCULATION OF $W(T_{ij})$

MACROSTATES	$W(T_{ij})$ = Number Of Ways Occurring for Each T_{ij} Macrostates
4 0	$\frac{4!}{0!4!} = \frac{4\times3\times2\times1}{1\times4\times3\times2\times1} = 1$
0 4	
3 1	$\frac{4!}{0!3!} = \frac{4\times3\times2\times1}{1\times3\times2\times1} = 4$
1 3	
2 2	$\frac{4!}{2!2!} = \frac{4\times3\times2\times1}{2\times1\times2\times1} = 6$

Adapted from: Pooler (1983), Socio-Econ. Plan. Sci., Vol.17, No:4, pp.153-164

The results in Table IV-3 agree exactly with those in Table IV-2; where 0!=1: The uniform distribution is the ''Most Probable State'' and it has the maximum number of ways of occurring.

Before maximization, let us apply the Shannonian Equation III-1 to the macrostates in our urban example.

Let T_i represent the number of trips to each zone (i) and T the total number of trips. Then

$$p_i = \frac{T_i}{T} \qquad \text{(IV-2)}$$

is the probability that a randomly selected worker will be assigned to a particular zone (i). Let's note that this simple problem does not involve a trip "matrix", since assignment of workers are made from work zone-1 to residential zones-2 and 3. This is analogous to a single row of a trip matrix. Since T=4 in our example, p_i probabilities and the corresponding Shannonian H entropies in relation to the macrostates can be calculated as in the Table IV-4.

TABLE IV-4 MACROSTATES AND THEIR ENTROPIES

MACROSTATES	Value for Shannonian Entropy $H = -\sum_i p_i \ln p_i$
4 0 0 4	$-\left[\left(\frac{4}{4}\ln\frac{4}{4}\right)+\left(\frac{0}{4}\ln\frac{0}{4}\right)\right] = 0,00$
3 1 1 3	$-\left[\left(\frac{3}{4}\ln\frac{3}{4}\right)+\left(\frac{1}{4}\ln\frac{1}{4}\right)\right] = 0,652$
2 2	$-\left[\left(\frac{2}{4}\ln\frac{2}{4}\right)+\left(\frac{2}{4}\ln\frac{2}{4}\right)\right] = 0,693$

Again, it is evident that the solution for the maximum value for H occurs in the case of uniform distribution.

IV.4 Solution By the MaxEnt Method

In the solution by the "Most Probable State" method above, the assignment of workers to the residential zones problem, we found the maximum by calculating every possible value for the number of microstates for each macrostates. Evidently, this approach will be impractical for large numbers of macrostates and therefore we require a method to find the maximum of the function by analytic means. The usual approach involves taking the first derivative of the function, setting it equal to zero and solving for the unknown. However, this usual method is applicable only to unconstrained functions. According to Jaynes' MaxEnt method, we should maximize the entropy function (H) subject to the constraint equation:

Maximize:

$$H(p_1, p_2, ..., p_n) = -K\sum_{i=1}^{n} p_i \ln p_i \qquad \text{(III-12) \& (IV-3)}$$

Subject to:

$$\sum_{i=1}^{n} p_i = 1,0 \qquad \text{(IV-4)}$$

As it was explained in section III.5, it is necessary to employ the method of Lagrange multipliers in order to maximize the constrained entropy function above. Hence we can write the Lagrangian function:

$$L = -\sum_{i} p_i \ln p_i + \left[(\lambda\text{-}1)\left(1 - \sum_{i} p_i\right)\right] \qquad \text{(IV-5)}$$

Taking the first derivative with respect to (p_i) and setting the result equal to zero, we have:

$$\frac{\partial L}{\partial p_i} = -\ln p_i - 1 - \lambda + 1 = 0 \qquad \text{(IV-6)}$$

$$\ln p_i + \lambda = 0 \qquad \text{(IV-7)}$$

By taking antilog, we find:

$$p_i = \exp(-\lambda) = e^{-\lambda} \qquad \text{(IV-8)}$$

This is the MaxEnt distribution, in other words, the macrostate probability distribution representing the "Most Probable State", with respect to the given information, i.e, constraint. **It can be also noted that MaxEnt method identifies a method for finding the maximum value of (H) without calculating every possible value for (H).** By substituting $p_i = \exp(-\lambda)$ into the constraint equation:

$$\lambda = \ln n \qquad \text{(IV-9)}$$

$$p_i = \frac{1}{n} \qquad \text{(IV-10)}$$

Tribus (1969, pp.124-128) explains how the above result is found for the uniform distribution and the Lagrange multiplier λ plays the role of a normalizing factor. In the context of our urban problem, since there are 2 zones for assignment:

$$p_i = \frac{1}{n} = \frac{1}{2} = 0,5 \qquad \text{(IV-11)}$$

$$H = -\sum_{i=1}^{n} \frac{1}{n} \ln \frac{1}{n} = -\left(\frac{n}{n} \ln \frac{1}{n} \right) = \ln n \qquad \text{(IV-12)}$$

$$H = \ln 2 = 0,693 \qquad \text{(IV-13)}$$

which agrees exactly with the result obtained using Shannonian Equation III-1 as shown in Table IV-4 above.

IV.5 Solution By The Minimum-Information Principle (MIP)

In the urban problem above, the assumption has been made that characteristics of the destination zones have no effect on the assignment of workers to zones, since all zones are of equal size, each contains the same number of homes, etc. In the language of information theory, this is the assumption of “a priori” equally likely microstates, that is, in the absence of such prior information, the probability that a worker will be assigned to any particular zone is equal all over zones. According to Pooler (1983), the assumption of “equally likely prior microstates” is a biased one. Suppose that some prior information is available in the form of the unequal areas of the destination zones. We would expect that such information to have an effect on the assignment of workers to zones in the sense that it would be expected that the assignments would be in direct proportion to the sizes of zones. The statistically most likely form of a probability distribution (p_i) which takes information concerning some prior probability distribution (q_i) into account, is one which “minimizes” the Kullback’s information measure:

$$I(p:q) = \sum_i p_i \ln \frac{p_i}{q_i} \qquad \text{(III-2) \& (IV-14)}$$

In the context of the urban example, the prior (q_i) can be defined with respect to the areas (a_i) of the destination zones such that:

$$q_i = \frac{a_i}{\sum_i a_i} \qquad \text{(IV-15)}$$

$$\sum_i q_i = 1,0 \qquad \text{(IV-16)}$$

we have also the normalization constraint:

$$\sum_i p_i = 1,0 \qquad \text{(IV- 17)}$$

to find the "Minimum" of the Kullback's information. Considering the above prior (q_i) and the normalization (p_i) constraints, we can write a Lagrangian function in order to "Maximize" $\left[-I(p:q)\right]$:

$$L = -\sum_i p_i \ln \frac{p_i}{q_i} + \left[(\lambda\text{-}1) \left(1 - \sum_i p_i \right) \right] \qquad \text{(IV-18)}$$

$$\frac{\partial L}{\partial p_i} = -\ln p_i - 1 + \ln q_i - \lambda + 1 = 0 \qquad \text{(IV-19)}$$

$$-\ln p_i + \ln q_i - \lambda = 0 \qquad \text{(IV-20)}$$

solving for p_i, we obtain:

$$p_i = q_i \exp(-\lambda) = q_i e^{-\lambda} \qquad \text{(IV-21)}$$

since, exp(-λ) is a constant, by substituting in Equation IV-15 we can write:

$$p_i = \frac{a_i}{\sum_i a_i} = q_i \qquad \text{(IV-22)}$$

The result means that, when the only information available is that concerning the prior and the normalization constraint, the MIP results in an assignment which is in direct proportion to the prior, in our example, the area (a_i) of the zones.

Pooler (1983) introduces the information concerning the mean distance such that:

$$\sum_i p_i r_i = \bar{r} \qquad \text{(IV-23)}$$

where (r_i) is the distance traveled and ($\bar{r}$) is the average or mean distance. The equation means that the sum of the probabilities times the distance equals the mean distance ($\bar{r}$). Now the Lagrangian function also includes the above mean distance constraint:

$$L = -I(p{:}q) + \left[(\lambda\text{-}1)(1 - \sum_i p_i) \right] + b\left[\bar{r} - \sum_i p_i r_i \right] \qquad \text{(IV-24)}$$

$$\frac{\partial L}{\partial p_i} = -\ln p_i + \ln q_i - \lambda\text{-}br_i = 0 \qquad \text{(IV-25)}$$

$$p_i = q_i \exp(-\lambda)\exp(-br_i) \qquad \text{(IV-26)}$$

where (b) is a Lagrangian multiplier which will ensure that constraint Equation IV-23 is satisfied.

The Equation IV-26 will assign workers to zones in direct proportion to the size of zones (a_i) and inverse proportion to an exponential function of distance (r_i). Pooler (1983) transforms the Equation IV-26 to a more suitable mathematical form:

$$p_i = \frac{q_i \exp(-br_i)}{\sum_i q_i \exp(-br_i)} \qquad \text{(IV-27)}$$

If we suppose that zone-2 contains 0,3 and zone-3 contains 0,7 of the total area of the two destination zones and zone-1 is only an origin, b = 0,8 where distances to destination zones $r_2 = 1\,\text{km}$ and $r_3 = 2$ kms, as given in Table IV-1, the following calculations can be made:

$$T_{12} = T.p_2 \qquad (\text{ IV-28 })$$

$$T_{13} = T.p_3 \qquad (\text{ IV-29 })$$

Since total trip T = 4, by substituting (p_i) values, we obtain:

$$T_{12} = \frac{4(0,3)\exp(-0,8.1)}{\left[0,3\exp(-0,8.1)\right]+\left[0,7\exp(-0,8.2)\right]} = 1,95 \ (\approx 2,0) \text{ workers}$$

$$T_{13} = \frac{4(0,7)\exp(-0,8.2)}{\left[0,3\exp(-0,8.1)\right]+\left[0,7\exp(-0,8.2)\right]} = 2,05 \ (\approx 2,0) \text{ workers}$$

Once again, 2 workers, in integer form, have been assigned to each of the two residential destination zones. Evidently, this result obtained by the method of MIP, agrees with the previous solutions by the "Most Probable State", Shannonian entropy H and the Jaynesian MaxEnt methods.

IV.6 Concluding Remarks

The minimum Information Principle (MIP), The Most Probable State Method and the MaxEnt method gives the same probability distributions as shown in a numerical example.

CHAPTER V

SPATIAL ENTROPY

V.1 Introduction

According to Sheppard (1976), broadly speaking two main uses of the entropy in urban and regional studies may be identified. The first group uses entropy as a "descriptive statistics"; the second group uses as the "maximization of the entropy of a distribution" in order to produce a maximally unbiased estimate of probabilities. The former approach is typified in works of Chapman (1970), Medvedkov (1971), Marchand (1972) and recently Adams and Storbeck (1983), among others, to describe the level of "uncertainty" present in an observed spatial distribution. The second approach was pioneered by Wilson (1967, 1970, 1970a) and elaborated by Batty (1976) and others. Although Sheppard regarded Batty's (1974a, 1974b) papers in the first group, Batty (1974a, 1974b) has developed and introduced the concept of "Spatial Entropy" to the descriptive urban studies. A third approach, aiming to integrate the NUE, as explained in Chapter I, and the urban structure within the information-theoric framework, as in the book of Webber (1979), may be added to Sheppard's (1976) analysis.

Chapter VI on the population and population density probability distributions in Ankara between years 1970 and 1990, based on the "Spatial Entropy" concept of Batty, can be associated with the first type of "descriptive" researches.

V.2 Spatial Entropy

The concept of "Spatial Entropy" has been developed first by Batty (1974a, 1974b) and used to test various hypotheses concerning the distributions of population and its density, in the New York, London & Los Angeles regions.

Batty takes the view of Kullback as a point of departure in the derivation of a "Generalized Information" or "Entropy Statistic". In defining information or uncertainty relative to two distributions, regarded as "prior" and "posterior" probabilities q_i and p_i respectively, information I_i is given as the difference between these information. This measure is called "Expected Information" or "Information Gain", then

$$I_i = \ln\frac{1}{q_i} - \ln\frac{1}{p_i} = \ln\frac{p_i}{q_i} \qquad \text{(V-1)}$$

information for the complete set of events characterizing the distributions prior q_i and posterior p_i is calculated by weighting the above equation by the posterior probabilities and summing over all events:

$$I = -\sum_i p_i \ln q_i + \sum_i p_i \ln q_i$$

$$I = \sum_i p_i \ln\frac{p_i}{q_i} \qquad \text{(V-2)}$$

The distributions q_i and p_i must be normalized to sum to unity for the equation to hold.

In a geographic context, where the number of events may comprise the number of zones, expected information can be used to compare systems with different numbers of zones. To explicate this point, Batty (1974a) defines the equivalents of probabilities q_i and p_i for the interval size Δx_i, over which the average point densities $q(x_i)$ and $p(x_i)$ are defined:

$$q(x_i)\Delta x_i = q_i \qquad \text{(V-3)}$$

$$p(x_i)\Delta x_i = p_i \qquad (\text{V-4})$$

Substituting these continuous equivalents into equation gives;

$$I = \sum_i p(x_i)\Delta x_i \ln \frac{p(x_i)\Delta x_i}{q(x_i)\Delta x_i} \qquad (\text{V-5})$$

$$I = \sum_i p(x_i) \ln \frac{p(x_i)\Delta x_i}{q(x_i)\Delta x_i} \qquad (\text{V-6})$$

In the limit where $\Delta x_i \to 0$, the continuous form is derived:

$$I(x) = \int p(x) \ln \frac{p(x)}{q(x)} dx$$

$$I(x) = -\int p(x) \ln q(x) dx + \int p(x) \ln p(x) dx \qquad (\text{V-7})$$

Batty (1974a, 1974b) demonstrates that the discrete equivalent of the first term of Equation (V-7) can be written as

$$\lim_{\Delta x_i \to 0} -\sum_i p_i \ln \frac{q_i}{\Delta x_i} = -\int p(x) \ln q(x) dx$$

which is referred as "Spatial Entropy".

By ignoring the posterior p_i in Kullback's equation (V-2) Shannon's formula is derived:

$$H = -\sum q_i \ln q_i \qquad (\text{V-9})$$

The "Spatial Entropy" is developed by the above arguments:

$$S = -\sum_i q_i \ln \frac{q_i}{\Delta x_i} \qquad \text{(V-10)}$$

where

$$\sum_i \Delta x_i = \text{Total Urban Area X}$$

$$\sum_i q_i = 1,0$$

V.3 Spatial Hypothesis Testing

To show how the expected information formula for testing can be used for the distribution and density of population, Batty (1974b) gives two different applications. In terms of the first application, the hypothesis that the distribution of is "uniform" in each zone. In this case, the hypothesis suggests that "There is an equal amount of population in each zone". The prior and posterior probabilities are defined as follows:

$$q_i = \frac{1}{N} = \frac{1}{\text{Number of Zones}}$$

$$p_i = \frac{Pop_i}{\sum_i Pop_i} = \frac{Pop_i}{Pop} = \frac{\text{Population in zone (i)}}{\text{Total City Population}}$$

substituting these probabilities in equation (V-2),

$$I_1 = \sum_i p_i \ \ln\ (p_i N) = \ln N + \sum_i p_i \ \ln p_i \qquad \text{(V-11)}$$

The second hypothesis involves the "Density of Population", i.e., relationship between the distribution of land area in each zone and the distribution of population. In this case, it is required to test the hypothesis that the "Density of

Population is uniform". The posterior p_i is above but the prior q_i is redefined to be:

$$q_i = \frac{\Delta x_i}{\sum_i \Delta x_i} = \frac{\Delta x_i}{X} = \frac{\text{Area of zone (i)}}{\text{Total City Area}} \qquad (\text{V-12})$$

Again, substituting the appropriate probabilities gives:

$$I_2 = \sum_i p_i \ln \frac{p_i X}{\Delta x_i} = \ln X + \sum_i p_i \ln \frac{p_i}{\Delta x_i} \qquad (\text{V-13})$$

$$I_2 = \ln X - S \qquad (\text{V-14})$$

If the "Spatial Entropy" S is a uniform distribution, it is a MaxEnt distribution and has a value of lnX. In such a case, I_2 is zero, (I_2=0), the distributions of population and land area in the same ratio and thus the hypothesis is confirmed. In a second interpretation of this hypothesis, Equation V-13 or V-14 can be written out explicitly as :

$$I_2 = \sum_i p_i \ln\left(\frac{Pop_i}{\Delta x_i} . \frac{X}{Pop}\right) = \sum_i p_i \ln \frac{Pop_i}{\Delta x_i} - \ln \frac{Pop}{X} \qquad (\text{V-15})$$

From Equation V-15, I_2 can be interpreted as the difference between the expected log of population density in the system and the actual log of density. When these two values are equal, the two distributions of population and land area coincide and I_2 is zero.

Table V-1 shows the results of "Entropy Measures" in spatial hypothesis testing (Batty, 1974b).

TABLE V.1 ENTROPY MEASURES IN SPATIAL HYPOTHESIS TESTING

	New York	**Los Angeles**	**London**	**Ankara (*)**
Number of Zones N	158	274	186	-
ln N	5,0626	5,6131	5,2257	-
Discrete Entropy H	4,8592	5,2234	5,0428	-
Expected Information I_1	0,2014	0,3897	0,1829	-
Area X in sq miles	3476	8966	940	70
ln X	8,1538	9,1012	6,8459	9,8129
Spatial Entropy S	7,2013	7,6118	6,4096	9,4418
Expected Information I_2	0,9525	1,4893	0,4363	0,3711

Source: Batty, 1974b; (Simplified Table). (*)Ankara measures are from this study. Ankara area is 70 sq miles within 1970 zone boundaries.(Appendix-A, Table 1.0)

[Ankara (1970) entropy measures S(70) and I_2(70) have been added to the above Table V.1 from Table VI.1. Ankara (1970) population density distribution is "more even" than London's. The first hypothesis with H and I_1 measures were not calculated for Ankara].

In Table V-1, each of the expected information, for the first hypothesis are different from zero, I_1 values for New York (0,2014), Los Angeles (0,3897) and London (0,1829) are far from the MaxEnt distribution. In comparison to the other two cities, London has the "most even", Los Angeles has the "most uneven" population distributions. Thus, in the London region the correlation between zone size and population is strong whereas New York and Los Angeles regions correlations are weak.

The second hypothesis, is also rejected as I_2 is significantly different from zero in all three cases. London has the "most even" and Los Angeles has the "most uneven" density of population. Evidently, these results are in accordance with the above I_1 results. (Figure V.1)

It should be noted have that Batty (1974b) did not interpret the results of "Discrete Entropy" H and "Spatial Entropy" S that appeared in the Table V-1. The only explanation was for the "difference between the log of system size and spatial entropy", where he meant the Equation (V-14).

V.4 Population Changes and Information Theory Measures

Adams & Storbeck (1983), developed "An Information Theoretic Approach" for the urban development strategies for USA's largest 49 central cities where there was a population decline by an average of over 6% from 1970 to 1980. Did this decline point up a failure of urban development policies undertaken during the seventies? To analyze the problem, they adopted an "Expected Information Measure" expressed in terms of "Prior" (q_i) and "Posterior" (p_i) distributions, which gives the amount of "Information Gained", by comparing the state of "ignorance" encoded in the prior distribution with the additional knowledge obtained from the posterior distribution. Thus, the mean value of "Expected Information" is defined as:

$$I(p:q) = \sum_{i=1}^{n} p_i \ln \frac{p_i}{q_i} \qquad \text{(V-16)}$$

with constraints:

$$\sum_{i=1}^{n} q_i = 1{,}0 \text{ and } \sum_{i=1}^{n} p_i = 1{,}0 \qquad \text{(V-17)}$$

The Equation (V-16) and Equation (V-17) are Kullback's "Information Gain" as given in Chapter III.2.

To identify population changes associated with the regional and city-suburban disparity components, authors used Theil's method of decomposition with the grouping of events. Hence, total expected information was decomposed into "within each set" and "between set" information. This decomposition is first performed on a regional basis and then on a disparity group basis. The significance of the "between" and "within set" information components for each decomposition is subsequently determined by comparing these measures to their respective maximum values. In the analysis, posterior probabilities denote the distribution of central city population among the 49 urban centers for 1980, where

$$p_i = \frac{\text{Population of City}}{\text{Total Population}}$$

Similarly, q_i values denote the prior probabilities for 1970,

$$q_i = \frac{\text{Population of City}}{\text{Total Population}}$$

By substituting these values of the above posterior and prior probabilities in Equation V-16 authors determined the amount of "Information Gain" for the entire urban system.

In a broad sense, they were "testing the hypothesis" that "**the distribution of 1980 central city populations is directly proportional to the distribution of 1970 central city populations**". In such a case, the value of resulting information gain measure is exactly zero. If this value deviates from zero, it can be identified the extent to which these two distributions are deviating from direct proportionality. An information gain, which approaches the maximum, would indicate an urban system experiencing "substantial" shifts in population structures.

Authors show how the 1970 and 1980 probabilities can be grouped on the basis of regions. It can be argued that there is no explicit equation of "Spatial Entropy", in

Batty's sense in this study but the total information gain is decomposed into inter-regional or intra-regional components. In this section, Theil's method of decomposition that the authors used is not summarized. But it is evident that Kullback's measures can be applicable for the evaluation of regional policies and planning decisions.

V.5 Concluding Remarks

The aim of this chapter is to introduce the Spatial Entropy concept and its measures that to be applied in Ankara Metropolitan Area for 1970-1990 data in the next Chapter VI.

(i) In urban and regional studies, there are mainly two types of researches using the entropy concept. The first type uses as a "descriptive statistics" and the second one as the "MaxEnt" method. Chapter asserts that there is a 3rd type aiming at "Integration" of the urban models.

(ii) The Spatial Entropy concept has been developed first by M. Batty (1974) and used to test various hypotheses of population and population density distributions in cities.

(iii) Spatial Entropy is based on the Kullback's Information Gain method.

(iv) Overall Spatial Entropy measures for population density indicate that Ankara (0.3711) has a "more even" distribution that London (0.4363) by 1970 data but not for (S) values.(TableV.1).

(v) Information Gain method can also be applied to population changes in city populations or in zones of a city.

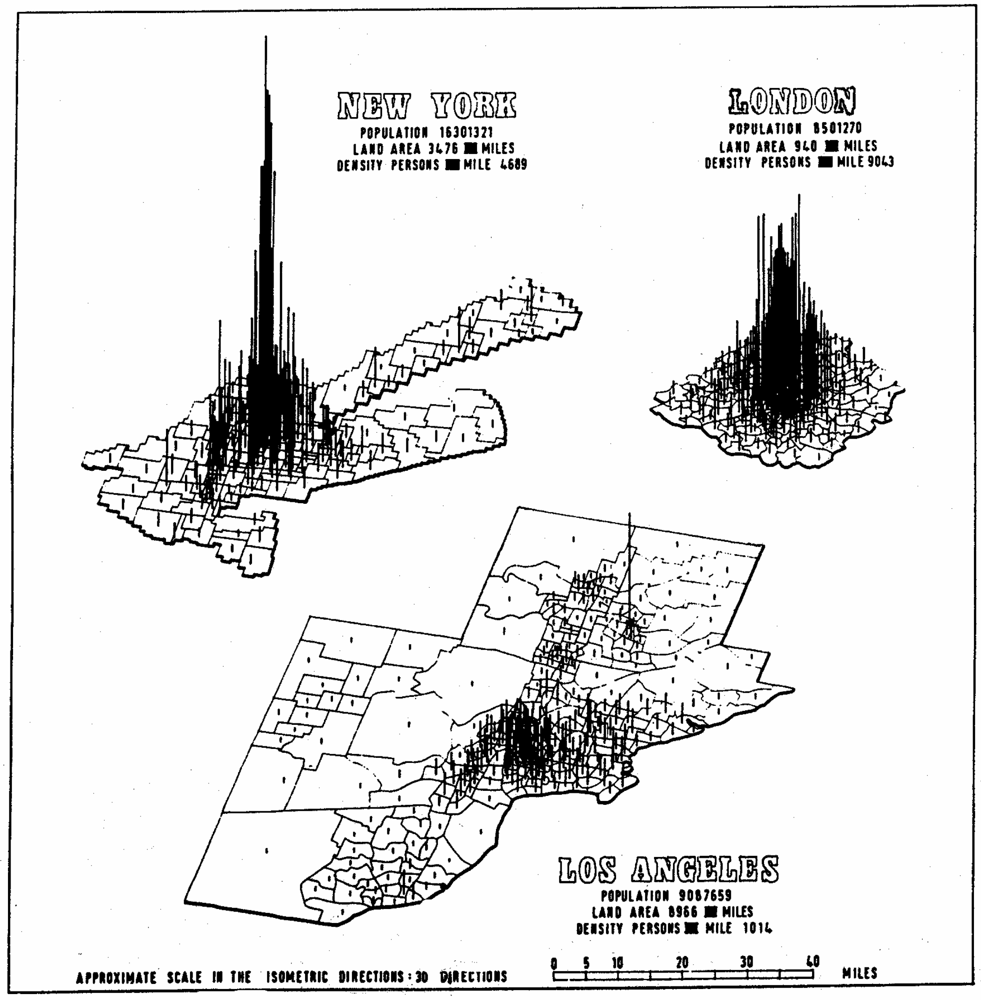


Figure V.1- Isometrics Of Population Density
Source : Batty (1974b, p.46)

CHAPTER VI

ANKARA REVISITED:
EVALUATION OF INFORMATION-THEORIC MEASURES (1970-1990)

VI.1 Introduction

This chapter explains the data and computation tables in Appendix-A and maps in Chapter VI and gives some preliminary evaluations of some of the results of application of the information-theoric methods to the available data of the Ankara Metropolitan Area for census years 1970 and 1990.

Before the following evaluations, it should be noted here that logarithms are natural logarithms to the base (e), therefore the unit of measurement is "Nat" in all tables developed for spatial entropy and information gain computations. However, as in Batty's papers (1974a&b) authors do not explicitly indicate the unit of measurement, except in communication theory where "Bits" are given to the base 2.

Like Mogridge (1972), as it was noted in Chapter I, Webber (1976a) also writes clearly that entropy is a **"**pure number", as it has no dimensions because it is computed from probabilities, which are themselves pure numbers. Information statistics suggest that the measure vary from zero to infinity and the measure would be additive between independent events. (Batty, 1974a). "Test of Hypothesis" considered the prior "Zone Area" and the posterior "Population Density" distributions, but not population distributions in the given 34 zones. The difference between these two applications are explained in Chapter V. However, these tests are not in the strict sense of the traditional statistics, since confidence limits are not specified for information-theoric measures. (Batty, 1974b; Adams & Storbeck, 1983).

Cover & Thomas (1991, p.19) consider Kullback's information I_K as "relative entropy" that measures the "distance" between two distributions. Hence, the relative entropy is a measure of the "inefficiency" in assuming that the prior distribution is (q) when the true posterior is (p), and is zero if p=q. In spatial Information Gain measures I(70) and I(90) formulations, prior (q_i) represents the area of zone (i) ratio, and posterior (p_i) represents the population ratio. (Chapter VI.4, Equation (VI-2) and (VI-4)). Therefore to think I(70) and I(90) values as measures of "distances" between prior and posterior distributions, not just deviations from zero, is highly useful.

In Batty's papers (1974a,b) there are no evaluations based on the zonal variations of spatial entropy measures. This thesis considers both the zonal measures and their changes in the two decades between years 1970 and 1990, and develops a series of bar-charts and maps in this chapter to visualize the distributions in the geographical dimension.

VI.2 Preliminary Evaluations

Table 1.0 (Appendix-A) gives 34 zones with their "mahalle" units in each zone and their corresponding population, area size in hectares, population density amounts for years 1970 and 1990, also shows the percentages of increases in population and density values. Table 1.0 (Appendix-A) has 11 pages and it includes the "mahalle" units in the 34 zones within the 1970 boundary of Ankara Metropolitan Area. All other data and the related computations are developed for 34 zones as listed in the tables. Some of the results are "sorted" from highest to lowest values to help comparisons.

Table 4.1 finds “Spatial Entropy”, S(70), based on Equation (V-10) but with modified definition of (q_i) values as zonal areas, to test the second hypothesis, using Equation (V-12). Table 4.1 also computes “Information Gain”, I(70), for population density distributions over 34 zones, according to year 1970, based on Equation (V-13). The overall S(70) is 9,44182 and I(70) is 0,37119 as computed in Table 4.1 (Appendix-A) for the whole Ankara Metropolitan Area, within 1970 boundary.

Table 4.2 (Appendix-A) gives the sorted S(70) and I(70) values from highest to lowest.

Table 5.1 and Table 5.2 (Appendix-A), as in the above tables, compute S(90) and I(90) values for the year 1990 and sort them. Depending on the tables given in the Appendix-A, a set of 8 graphs and 10 maps are developed as Figures VI.1-8 and Maps VI-(1-10). Additionally, Maps VI-(11-20) aims to give the 3-dimensional images of the values given or computed in Tables, Figures VI.1-8 and Maps VI-(1-10).

VI.2.1 Overall Evaluations For Spatial Entropy and Information Gain Measures For Population Densities

In Table VI.1, both Information Gain measures, I(70) and I(90), i.e., 0,3712 and 0,3044 respectively, deviate significantly from zero; the hypothesis that “**Density of population is uniform in each zone**” is not confirmed. Therefore, both in years 1970 and 1990, there were “uneven” distributions and Ankara-1970 was more “uneven” than Ankara-1990, since I(70)>I(90). Hence, there has been a tendency to “more even” or “homogeneous” population density distribution during the two decades.

TABLE VI.1 OVERALL SPATIAL ENTROPY & INFORMATION GAIN

S(70)	S(90)	S(90)-S(70)	I(70)	I(90)	I(90)-I(70)
9,4418	9,5277	0,0859	0,3712	0,3044	-0,0668

From Table 4.1 & 5.1 (Appendix-A)

Since total 1970 gross area of Ankara is 18270 hectares, the maximum entropy of the whole area is ln18270=9,8130. Both spatial entropy amounts, S(70) and S(90) are less than 9,8130; I(70) and I(90) distributions are not regarded as MaxEnt distributions as the Equation (V-14) implies.

Although the Table VI.1 shows that S(90) > S(70), meaning Ankara-1990 is "More Uneven" than Ankara-1970, the overall increase is 0,0859 / 9,4412 = 0,009 or 1,0% of the S(70) amount. On the other hand, the difference I(90) – I(70) = -0,0668 represents 0,0668 / 0,3712 = 19% decrease in the I(70) amount. Therefore, it can be asserted that since I(70) > I(90), Ankara-1990 has become "More Even" than Ankara-1970 in Kullback's Information Gain measures and the change has been "Negligible" in terms of Shannon's Spatial Entropy measure, though it is found as S(90) > S(70), in the context of population density distributions.

The following sections review the results at zone levels based on Shannon's and Kullback's spatial entropies and aim to make some general evaluations for their spatial distributions.

VI.2.2 Evaluations By Zones For Spatial Entropy Measures S(70) and S(90) of Population Densities

Table 7.2 Spatial Entropy Differences By Zones (in Appendix-A) gives the sorted values of S(70) and S(90) and S(90)-S(70) difference values. Figure VI.3 gives S(70) and S(90) values and Figure VI.7 gives the differences as bar charts.

Map VI-4, 5 and 6 are the respective maps for the same computation results and Map VI-14, 15 and 16 are the related 3-dimensional images of them. Table 7.2

(Appendix-A) lists the Akdere (0,5865), Gülseren (0,5635), Karşıyaka (0,4451), Aktaş (0,4255) and Bahçelievler (0,3995) as the top five zones for S(70) values. Keçiören (0,7593), Karşıyaka (0,6429), Etlik (0,6233), Kayaş (0,4587) and Ziraat Fk.-Aydınlık (0,4091) appear as the top five zones for S(90). Such high spatial entropy values indicate relative "Uneven" population distributions that are "Not Proportional" to the zone sizes.

In Spatial Entropy Equations (VI-1) and (VI-3), as defined in Chapter VI.4; (a_i) represents the area of zone (i) in hectares with no changes in 1970 and 1990 zone boundaries. Since (a_i) values are held constant, the variations in the spatial entropy values are determined by the rest of the terms of the spatial entropy Equation VI-1 and VI-3.

In other words, with their high S(90) values, Keçiören, Karşıyaka, Etlik and Kayaş contained high "Unexpected" information, i.e., they had more population growths than their zone sizes implied.

To compare the S(90) and S(70) spatial entropies, differences S(90)-S(70) are considered to find the relative tendencies at the zone level during years 1970 and 1990. Figure VI.7, Map VI-6 and its 3-dimensional image Map VI-16 show that, such zones as Keçiören, Etlik, Karşıyaka, Sanatoryum, Kayaş and Balgat, ranging from (0,4151) to (0,0925) difference values, have "More Uneven" population density distributions in relation to S(70) values. On the other hand, Akdere, Gülseren, Küçükesat, Ulus, Yenişehir, Aktaş, Samanpazarı and Yenimahalle zones ranging from (-0,2468) to (-0,0823) with their negative values, since S(70)>S(90) in these zones, have "More Even" or "Uniform" density distributions in comparison to their S(70) values.

According to the Figure VI.7, it can be asserted that 20 zones out of total 34, had a range of differences from Ziraat Fk.- Aydınlık (+0,0592) to Akköprü (-0,0636) that can be considered as "Negligible Differences". Hence, the spatial entropies

S(90) and S(70) of these 20 zones had a very similar spatial entropy distributions to each other (Table 7.2, Appendix-A).

VI.2.3 Evaluations By Zones For Information Gain Measures I(70) and I(90) of Population Densities

The Equations for Information Gain Measures I(70) and I(90) were developed from the basic Equation V-13 and given in Chapter VI.4.

$$I_2 = \sum_i p_i \ln \frac{p_i X}{\Delta x_i} = \sum_i p_i \ln \frac{p_i}{\frac{\Delta x_i}{X}} = \sum_i p_i \ln \frac{p_i}{\frac{a_i}{A}} \qquad (\text{V-13})$$

$$I(70) = \sum_i^{34} p_{i,70} \ln p_{i,70} \frac{A}{a_i} \qquad (\text{VI-2})$$

$$I(90) = \sum_i^{34} p_{i,90} \ln p_{i,90} \frac{A}{a_i} \qquad (\text{VI-4})$$

Where X and A represent the total land area and (Δx_i) and (a_i) the area of zone (i).

In Equation (V-13), it is clear that (p_i) are posterior probabilities representing the population percentages according to total population and (a_i/A) represents the prior probabilities as the percentages of zone sizes according to the total city area.

As it is explained in Chapter VI.1 Introduction, high Information Gain Values I(70) and I(90) mean that the “distance” between these two prior and posterior distributions with their above contents are also large (Cover and Thomas, 1991, p.19). In such cases, population and zone area distributions deviate from each other. On the contrary, for relatively low Information Gain Values of I(70) and/or I(90) indicate the similarity of the two distributions where the posterior population distributions are proportional to the prior zone area distributions.

Table 7.3 (Appendix-A) with sorted results and Figure VI.4 help to analyze the degree of influence of each zone on the overall results I(70) = 0,3712 and I(90)= 0,3044, as explained in Chapter VI.2.1. According to the Table 7.3, the top five zones for I(70) include Altındağ (0,0668), Aktaş (0,0611), İncesu (0,0545), Samanpazarı (0,0493) and Küçükesat (0,0492).

The top five zones for I(90) are given as Keçiören (0,0680), Bahçelievler (0,0482), İncesu (0,0460), Altındağ (0,0449) and Cebeci (0,0442). Map VI-17 for I(70) and Map VI-18 for I(90) provide 3-dimensional images of Map VI-7 and Map VI-8.

The presence of negative I(70) and I(90) in Table 7.3 (Appendix-A) can be argued that the posterior or actual probability distributions on zone populations produce little or no contribution to, or even worsen, the prior information on the zone size probability distributions.

Map VI-7 and Map VI-8 provide ranges for the I(70) and I(90) values changing from +0,0725 to -0,0400 amounts. Table VI-2 aims to identify the zones with MaxEnt distributions for I(70) and I(90) values as sorted in Table 7-3 (Appendix-A).

It is assumed that, a range of Information Gain values between + 0,0160 and -0,0080 fairly represent MaxEnt distributions, since the range does not deviate significantly from zero value.

TABLE VI.2 MAXENT DISTRIBUTIONS ZONES FOR I(70) & I(90)

Zone Number	Zones I(70)	Zone Number	Zones I(90)
19(*)	Mamak	19(*)	Mamak
8 (*)	Ziraat Fk.-Aydınlıkevler	1 (*)	Karşıyaka
10(*)	Yenimahalle-Demetevler	8 (*)	Ziraat Fk. – Aydınlıkevler
33(*)	Kültür Aksı	22	Küçükesat – Kavaklıdere
7 (*)	Siteler – Ulubey	5 (*)	Aktepe
4	Keçiören	29	Maltepe – Anıttepe
17(*)	Karaağaç	3 (*)	Sanatoryum
6 (*)	Hasköy	7 (*)	Siteler
5 (*)	Aktepe	32	Ulus
30	Söğütözü	31	Yenişehir
9	Akköprü	16	Gülseren – Gülveren
34	AOÇ – Fabrika	6 (*)	Hasköy
26(*)	Devlet	33(*)	Kültür Aksı
3 (*)	Sanatoryum	10(*)	Yenimahalle-Demetevler
1 (*)	Karşıyaka	26(*)	Devlet
		17(*)	Karaağaç

Source: Table 7.3 (Appendix-A)
(*) MaxEnt distribution zones in both I(70) and I(90) lists above.

Table VI.2 shows the 15 zones in 1970 and 16 zones in 1990 with MaxEnt distributions where I(70) and I(90) zone values are zero or "nearly zero" within the above range. These zones also represents where the second hypothesis "**Density of population is uniform in each zone**" is confirmed. Moreover, 11 zones are common in both I(70) and I(90) lists of MaxEnt distributions.(Map VI-21, Map VI-22).

Figure VI.8, Map VI-9 and the image Map VI-19 obtained for I(90)-I(70), shows clearly the zones that have "Uneven" and "Even" tendencies of population density distributions with respect to the zone sizes and also the zones with "Negligible" changes during the 1990-1970 period. Thus, Keçiören, Etlik, Karşıyaka, Aktepe, Kayaş, Sanatoryum with higher I(90) values than I(70) values, had trends towards "Unevenness"; such zones as Gülseren-Gülveren, Küçükesat-Kavaklıdere, Samanpazarı-Eski Ankara, Ulus, Akdere, Yenişehir, Yenimahalle towards "Evenness", since their I(70) values were higher in comparison to I(90) values.

The similarities in the general pattern in the order od the zones are clearly demonstrated in two bar-charts Figure VI.7 and Figure VI.8. Zone that tended to become "More Uneven" and "More Even" and zones that do not change relatively are almost the same in both Shannon's and Kullback's measures.

VI.2.4 Evaluations For Information Gain I(p90:q70) For Population Changes

As in the applications by Haynes & Storbeck (1978) and Adams & Storbeck (1983), reviewed in Chapter V.4, Kullback's information measure without "Spatial Entropy" concept has been applied to the Ankara population changes data by zones for years 1970 and 1990. Table 6.1 & 6.2 (Appendix-A), bar-chart Figure VI.6 and Map VI-10 with its image Map VI-20 are based on these computations.Map VI-23 shows the ''Minimum Information Gain '' zones.

In Table 6.1 & 6.2, contrary to the "Spatial Entropy" method, zones sizes and population densities are not included in the equations. In this sense, it is simpler than the previous "Spatial Entropy" method applied.

According to Table 6.2 (Appendix-A) the overall computed Information Gain I(p90:q70) is (0,10927) and since this result deviates from zero, the hypothesis that "**1990 population distributions in Ankara zones are directly proportional to the distribution of 1970 zonal populations**" is rejected. In other words, posterior 1990 population distributions by zones are not in direct proportionality to the prior 1970 distributions.

To find out the contributions of each individual 34 zones on the overall result of the hypothesis testing as stated above, Figure VI.6, Map VI-10 and the image Map VI-20 are developed based on the computations in Table 6.1 (Appendix-A).

According to the Table 6.2 (appendix-A), with sorted Information Gain measures; Keçiören (0,0746), Etlik (0,0522), Karşıyaka (0,0274), Sanatoryum (0,0258), Kayaş (0,0136) are the top five zones with higher values. There is a group of zones with almost zero values, such as Ziraat Fk.-Aydınlıkevler (0,0075), Ayrancı (0,0062), Hasköy (0,0049), Karaağaç (0,0013) and Bahçelievler-Emek (0,0011). Such zones as Gülseren-Gülveren (-0,0204), Akdere-Türközü (-0,0193), Ulus (-0,0108), Yenişehir (-0,0096), Altındağ (-0,0072) have negative values.

As it is given at Chapter VI.3, prior probability ($q_{i,70}$) equals the 1970 zone(i) population divided by the total 1970 Ankara population. Similarly, posterior probabilities denote the distribution of zone populations for 1990, where ($p_{i,90}$) equals the 1990 population of zone(i) divided by the total Ankara population in 1990. (Chapter VI.4 Definition of Variables)

In the above context of definitions, high values of Information Gain means that the posterior population distributions in 1970 by zones are not directly proportional to the prior 1970 distributions. In other words, relatively high values represent Information Gains "More Than Expected" values as given by the prior distributions. As it was argued for the negative I(70) and I(90) values; similar interpretations can be made for the negative Information Gain values in the Table 6.2 (Appendix-A). Hence, in such cases the posterior 1990 population probability distributions have "Less Than Expected" values.

Since the prior and posterior probability values are determined by the population changes in the zones according to the total populations in 1970 and 1990, I(p90:q70) values can be compared and related to the change of population percentages in zones.

Table VI.3, Figure VI.5 and Figure VI.6 clearly show that the top five zones with higher Information Gain values are followed by high population increases according to total growth between years 1990 and 1970, as Keçiören (35,38 %), Etlik (26,19%), Karşıyaka (18,96 %), Sanatoryum (14,30 %) and Kayaş (10,88 %). Zones with medium growth rates ranging from 8,0 % to 5,0 % have "Near Zero" I(p90:q70) values and out of 19 zones of negative I(p90:q70) values have also negative population growth, indicating the population loss, except Mamak (3,48 %), Çankaya-Yıldız (2,21 %), Cebeci (3,32 %) and İncesu (1,27 %) having population growths.

TABLE VI.3 COMPARISON OF I(p90:q70) WITH POPULATION CHANGES

Zone Numbers	Zones	I(p90:q70)	Population Increases in Zones According to Total Growth 1970-1990 (%)
4	Keçiören	0,074607624	35,38%
2	Etlik	0,052170462	26,19%
1	Karşıyaka	0,027443795	18,96%
3	Sanatoryum	0,025753539	14,30%
18	Kayaş	0,013623488	10,88%
27	Balgat - Çukurambar	0,012114273	7,92%
30	Söğütözü	0,008396091	3,45%
8	Ziraat Fk. - Aydınlıkevler	0,007515978	8,06%
7	Siteler - Ulubey	0,007219899	7,14%
23	Ayrancı	0,006166955	6,69%
6	Hasköy	0,004901029	5,50%
25	Dikmen - Öveçler	0,004708569	6,25%
34	A.O.Ç. Fabrikalar	0,001741504	1,19%
17	Karaağaç	0,001310785	0,80%
28	Bahçelievler - Emek	0,001109616	5,25%
19	Mamak	-0,000575756	3,48%
24	Çankaya - Yıldız	-0,000772653	2,21%
26	Devlet	-0,001617123	-1,07%
15	Cebeci	-0,001840274	3,32%
5	Aktepe	-0,002073348	0,57%
21	İncesu - Seyranbağları	-0,003188982	1,27%
11	A.O.Ç. - Gazi Mah.	-0,003611143	-1,63%
33	Kültür Aksı - Gençlik Parkı	-0,003787613	-1,97%
9	Akköprü - Varlık Mah.	-0,005604040	-2,28%
29	Maltepe - Anıttepe	-0,005905055	-1,26%
12	Altındağ	-0,007181118	-1,56%
10	Yenimahalle - Demetevler	-0,008009474	-2,40%
31	Yenişehir	-0,009649744	-7,23%

14	Samanpazarı - Eski Ankara	-0,010153901	-5,93%
32	Ulus	-0,010801241	-7,73%
22	Küçük Esat - Kavaklıdere	-0,012351800	-7,56%
13	Aktaş - Asrimezarlık	-0,012694474	-4,95%
20	Akdere - İmrahor - Türközü	-0,019300919	-11,24%
16	Gülseren - Gülveren	-0,020394910	-12,00%
Totals		0,10927004	100,00%

From (sorted) Table 6.2 and Table 6.3 (Appendix-A)

In cases where prior and posterior probabilities equal, i.e., $q_{i,70} = p_{i,90}$, Information Gain would be zero, since $p_i \ln 1 = 0$. (Webber, 1979, p.74)

In a similar way carried out for determining the zones of MaxEnt distributions for I(70) and I(90), Table VI.4 and Map VI-23 are obtained for the "Minimum Information Gain Zones". There are 11 zones out of total 34 within the range of 0,136 (Kayaş) and 0,0011 (Bahçelievler-Emek). Table VI.4 also shows the posterior and prior probability values approximately equal to each other.

TABLE VI.4 MINIMUM INFORMATION GAIN ZONES FOR I(p90:q70)

Zone Number	Zone Name	I(p90:q70)	$p_{i,90}$	$q_{i,70}$
18	Kayaş	0,0136	0,0448	0,0331
27	Balgat-Çukurambar	0,0121	0,0264	0,0166
30	Söğütözü	0,0084	0,0073	0,0023
8	Ziraat Fk.-Aydınlıkevler	0,0075	0,0431	0,0362
7	Siteler-Uluğbey	0,0072	0,0358	0,0292
23	Ayrancı	0,0062	0,0361	0,0304
6	Hasköy	0,0049	0,0303	0,0258
25	Dikmen-Öveçler	0,0047	0,0384	0,0340
34	AOÇ-Fabrikalar	0,0017	0,0042	0,0027
17	Karaağaç	0,0013	0,0025	0,0015
28	Bahçelievler-Emek	0,0011	0,0465	0,0454
19(*)	Mamak	-0,0006	00380	0,0386
16(*)	Gülseren-Gülveren	-0,0204	0,0333	0,0614

Source: Table 6.1 & Table 6.2 (Appendix-A)

(*) Zones with negative Information Gain values

It is also demonstrated that negative Information Gain values are followed by prior $q_{i,70}$ probabilities greater than posterior $p_{i,90}$ probabilities. Mamak (-0,0006) and Gülseren-Gülveren (-0,0204) are the two examples in the Table VI.4 with negative Information Gain values that have prior probabilities 0,0386 and 0,0614 greater than the posterior 0,0380 and 0,0333 respectively.

VI.3 Concluding Remarks

The Spatial Entropy and Information Gain measures have been applied to the Ankara Metropolitan Area by 34 zones with their 1970-1990 populations, area sizes in hectares and population densities. Chapter does not intent to make rigorous statistical test "per se" and there are some theoretical debates on the application of such standard tests for the information measures. Yet, it draws some preliminary conclusions for these measures and also try to make contributions to the method itself:

(i) Chapter VI applies the "Spatial Entropy" measures developed by Batty (1974a, b) for population density distributions and Information Gain measure for population changes as used by Adams & Storbeck (1983).

(ii) Definitions of Spatial Entropy measures S(70) and S(90) using zone sizes as the spatial variable and Information Gain measures I(70) and I(90) for population changes by zones in Ankara in years 1970 and 1990 are given fully in Chapter VI.4.

(iii) Results of the computations for entropy measures above are given as "dimensionless" pure numbers, since they are derived from probabilities that are also pure numbers.

(iv) Table VI.1 gives the overall results of the computations. Since the overall S(90) > S(70), Ankara-1990 is more "Uneven" than Ankara-1970, for the population density distributions. However, the difference S(90)-(70) =

9,5277-9,4412 = 0,0859 and therefore the increase rate is 0,0859 / 9,4412 = 0,009 or 1,0% of the S(70) amount. Evidently, 1,0% increase can be considered as "Negligible" and does not provide an evidence for the trend from "Uneven" to "More Even" distribution.

(v) Since the overall I(70) > I(90), it can be concluded that Ankara-1990 has tended to become "More Even" than Ankara-1970. The difference I(70)-I(90) = 0,3712-0,3044 = 0,0668 represents 0,0668 / 0,3712 = 19% decrease in the I(70) amount and this rate of change is to be regarded as significant. (Table VI.1).

(vi) Akdere, Gülseren, Karşıyaka, Aktaş and Bahçelievler are the top five zones for S(70) and Keçiören, Karşıyaka, Etlik, KAyaş and Ziraat Fk.-Aydınlıkevler are the top five zones for their S(90) values. (Table 7.2 Appendix-A). These relatively high Spatial Entropy values indicate relative "Uneven" population density distributions that are "not proportional" to the zone sizes.

(vii) In Spatial Entropy equations, as defined in Chapter VI.4, (a_i) represents the zone size in hectares and zone boundaries have not changed during 1970-1990 period. Hence, variations in the Spatial Entropy values for S(70) and S(90) are determined by the rest of the terms, by percentages of zone populations $(p_{i,70})$ or $(p_{i,90})$ and the logarithmic function.

(viii) Spatial Entropy differences [S(90) – S(70)] are useful to find the relative tendencies at the zone levels. Keçiören, Etlik, Karşıyaka, Sanatoryum, Kayaş and Balgat have "More Uneven" density distributions with (+) values; whereas Akdere, Gülseren, Küçükesat-Kavaklıdere, Ulus, Yenişehir, Aktaş, Samanpazarı and Yenimahalle zones have "More Even" density distribution tendencies with their (-) values during the period.

(ix) Results show that 20 zones out total 34 has “Negligible Differences” in their [S(90) –S(70)] values, indicating a similar Spatial Entropy distributions that do not deviate from each other.

(x) Information Gain measures I(70) and I(90) include population percentages (Pop_i / POP) as posterior (p_i) and zone size percentages (a_i / A) as prior (q_i) probabilities. Kullback’s Information Gain measures compare the posterior and prior probabilities to determine the “distance” between them. High values imply relatively larger “distances” and deviations from each other and low values indicate a similarity between the two distributions. It can be argued that negative Information Gain values imply that the posterior probability distributions do not contribute to, or even worsen the information encoded in prior distributions.

(xi) Overall Information Gain values I(70) = 0,3712 and I(90) = 0,3044 deviates significantly from zero. Hence, hypothesis that “**Ankara population density is uniform in each zone**” is not confirmed for years 1970 and 1990 in the overall evaluation. (Table VI.1).

(xii) In a MaxEnt distribution, Information Gain values are equal to zero. It is assumed that a range of values between (+) 0,0160 and (-) 0,0080 fairly represent “Near Zero” values and Table VI.2 gives the list of zones with MaxEnt distributions. Results show that 15 zones in 1970 and 16 zones in 1990 have MaxEnt distributions within the above range.

(xiii) In the zones of MaxEnt distributions of I(70) &I(90), the hypothesis “**Density of population is uniform in each zone**” is confirmed.

(xiv) It is found that 11 zones, out of 16 MaxEnt zones are common in the lists of I(70) and I(90).

(xv) The analysis of I(90)-I(70) differences show that Keçiören, Etlik, Karşıyaka, Aktepe, Kayaş, Sanatoryum had trends towards “Unevenness” with respect to their I(90) > I(70) values. Gülseren-Gülveren, Küçükesat-Kavaklıdere, Samanpazarı-Eski Ankara, Ulus, Akdere, Yenişehir, Yenimahalle and Maltepe tended towards "Evenness" or "Uniformity" since results are I(70) > I(90) for these zones. (Figure VI.8, Map VI-9 and Map VI-19).

(xvi) Kullback's Information Gain measure is also applied to the Ankara 1970-1990 census data for population changes but without the spatial variable, as represented by I(p90:q70). Definitions of variables are in Chapter VI.4

(xvii) The overall Information Gain for the whole Ankara Metropolitan Area is computed as I(p90:q70) = 0,10927 that deviates significantly from zero. Therefore, the hypothesis "**There is direct proportionality between 1990 and 1970 zone population distributions**" is not confirmed.

(xviii) Keçiören, Etlik, Karşıyaka, Sanatoryum and Kayaş are the top five zones with higher values. Ziraat Fk.-Aydınlıkevler, Ayrancı, Hasköy, Karaağaç and Bahçelievler-Emek have almost zero Information Gain values. Such zones as Gülseren-Gülveren, Akdere-Türközü, Ulus, Yenişehir, Altındağ have negative values. In cases where $q_{i,70} = p_{i,90}$, i.e, prior and posterior probabilities are equal, the Information Gain I(p90:q70) give zero results. These zones also represent the "Minimum Information Gain" conditions. Hence, higher Information Gain values indicate higher posterior probabilities than the prior probabilities. It is argued that in zones with negative values, posterior 1990 probabilities have "Less Than Expected" prior probability distributions.

(xix) Since the prior and posterior probability values are determined by the population changes in the zones according to the total 1970 and 1990 populations, I(p90:q70) values can be related to the change of population percentages in zones of Ankara. High population growth rates of Keçiören, Etlik, Karşıyaka, Sanatoryum and Kayaş are followed by the higher Information Gain values. Population losing 14 zones all have negative

I(p90:q70) values, including old settlement zones such as Ulus, Yenişehir, Samanpazarı-Eski Ankara, Altındağ, Maltepe (Table VI.3, Figure VI.5, Figure VI.6 and Map VI-10, Map VI-20).

(xx) Chapter VI develops a method of finding differences in Spatial Entropy and Information Gain measures for population density distributions by the introduction of "time" dimension to the analysis. Thus, difference equations S(90)-S(70) and I(90)-I(70) are not just arithmetic operations but a method of comparison to find out the tendencies for "Even" or "Uneven distributions in these information-theoric measures with respect to Ankara population census data 1970 and 1990.

VI.4 Definition of Variables in Spatial Entropy & Information Gain Equations

The definition of variables used in Spatial Entropy and Information Gain equations for the computations in Table 4.1, Table 5.1 and Table 6.1 (in Appendix-A) are given as follows. The related maps are also based on the same definitions.

Equations in Table 4.1 Spatial Entropy for Population Density & Information Gain Distributions (1970) (Appendix-A):

$$S(70) = -\sum_{i}^{n} p_{i,70} \ln \frac{p_{i,70}}{a_i} = -\sum_{i=1}^{34} \left[\frac{Pop(i,70)}{POP(70)} \right] LN \left[\frac{\frac{Pop(i,70)}{POP(70)}}{a_i} \right] \qquad (\text{VI-1})$$

Where:

S (70)	:	Spatial Entropy measure for population density for year 1970
$p_{i,70}$	:	Percentage of population in zone (i), in 1970 according to total population POP (70)
Pop(i,70)	:	Population of zone (i) in 1970
POP (70)	:	Total Ankara Population in 1970
a_i	:	Area of zone (i) in hectares in 1970. (Remains the same for year 1990).
ln and LN	:	Natural Logarithm (to base e=2,718…..)
n		Number of zones
$\sum_{i}^{n} p_i$	:	1.00

$$I(70) = \sum_{i}^{n} p_{i,70} \ln p_{i,70} \frac{A}{a_i} \qquad (\text{VI-2})$$

Where:

I(70)	:	Information Gain for Population Density in Year 1970
$p_{i,70}$	:	As given in S (70) equation above (VI-1)
A	:	Total Area of Ankara in hectares within 1970 boundaries.
a_i	:	As defined above

$\sum_{i}^{n} a_i$: A

Equations in Table 5.1 Spatial Entropy For Population Density & Information Gain Distributions (1990) (Appendix-A).

$$S(90) = -\sum_{i}^{n} p_{i,90} \ln \frac{p_{i,90}}{a_i} = -\sum_{i=1}^{34} \left[\frac{Pop(i,90)}{POP(90)} \right] LN \left[\frac{\frac{Pop(i,90)}{POP(90)}}{a_i} \right] \qquad (VI-3)$$

Where the definitions of variables are the same as given above for S (70), but the values are to be taken for year 1990.

$$I(90) = \sum_{i}^{n} p_{i,90} \ln p_{i,90} \frac{A}{a_i} \qquad (VI-4)$$

Where the definitions of the terms are the same as in I(70) except that 1990 values should be used.

Equations in Table 6.1 Information Gain Distributions I(p90:q70) For Population Changes (1970–1990) (Appendix-A).

$$I(p90{:}q70) = \sum_{i}^{n} p_{i,90} \ln \frac{p_{i,90}}{q_{i,70}} = \sum_{i=1}^{34} p_{i,90} \ln \frac{p_{i,90}}{q_{i,70}} \qquad (VI-5)$$

$$I(p90{:}q70) = \sum \frac{Pop(i,90)}{POP(90)} LN \frac{\frac{Pop(i,90)}{POP(90)}}{\frac{Pop(i,70)}{POP(70)}} \qquad (VI-6)$$

Where:

$p_{i,90}$	:	Posterior percentage of population of zone (i) in year 1990 according to total population POP (90)
POP (90)	:	Total Ankara population in 1990
Pop (i, 90)	:	Population of zone (i) in 1990
$q_{i,70}$	:	Prior percentage of population in zone (i) in 1990
Pop (i, 70)	:	Population of zone (i) in 1970
POP (70)	:	Total Ankara population in 1970

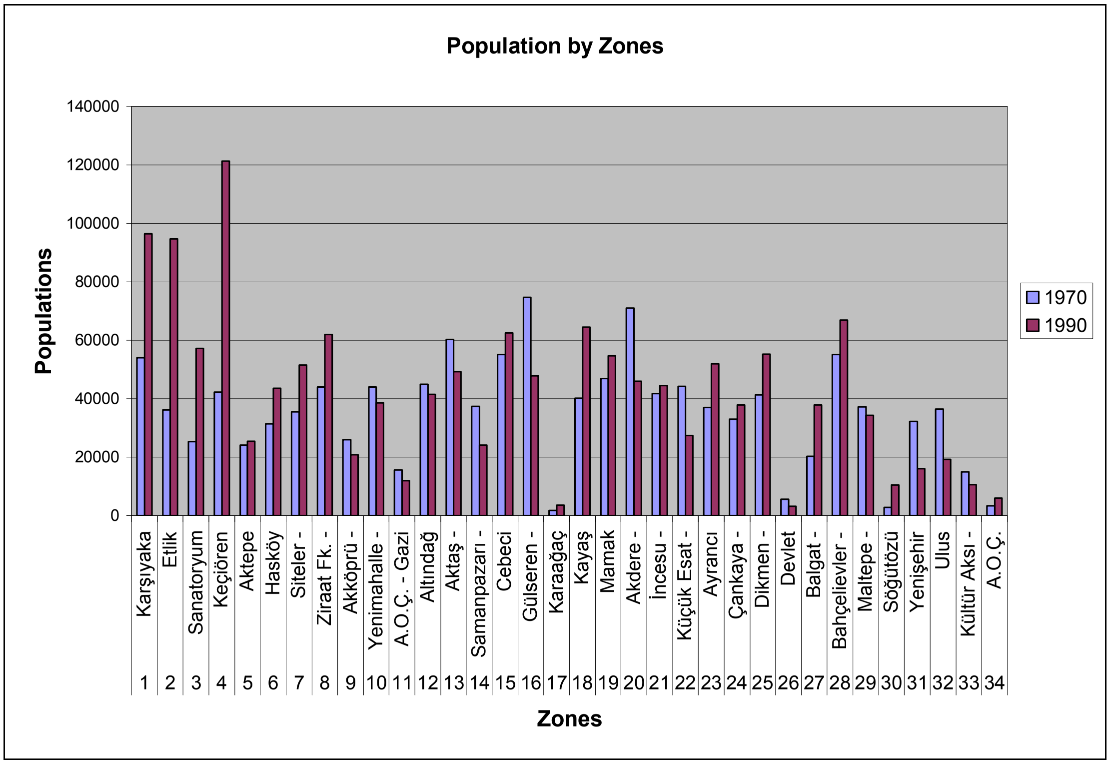


Figure VI.1-) Population by Zones

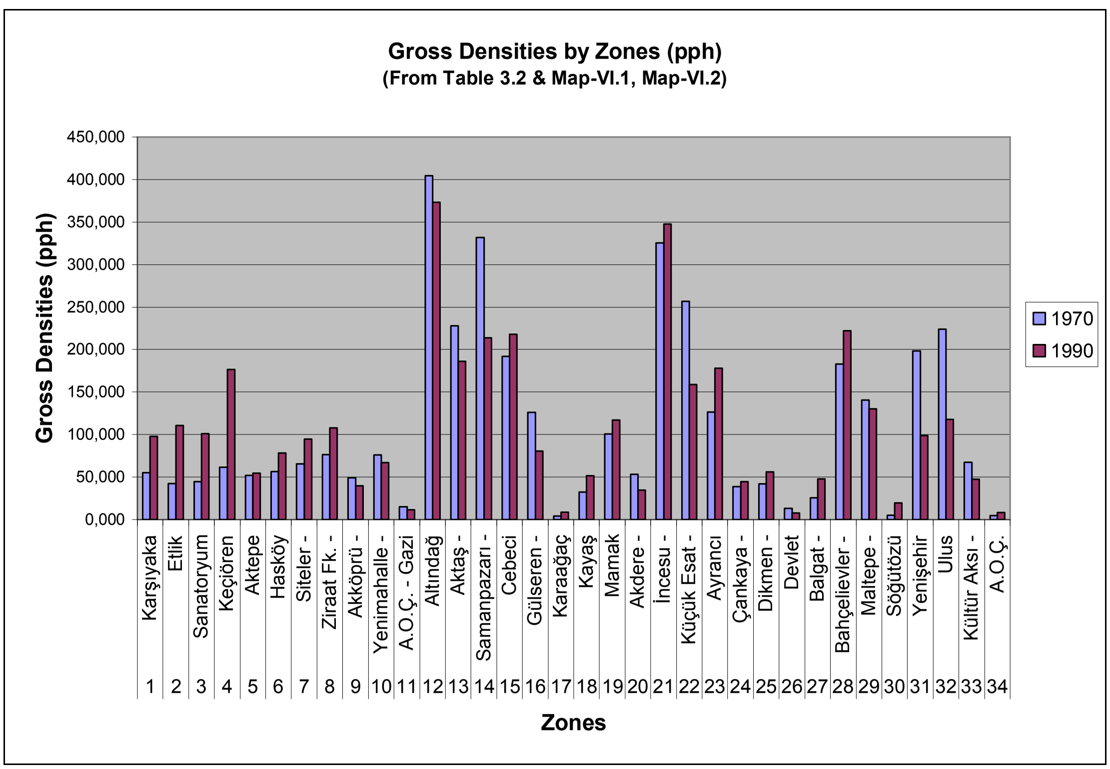


Figure VI.2-) Gross Densities by Zones (pph)

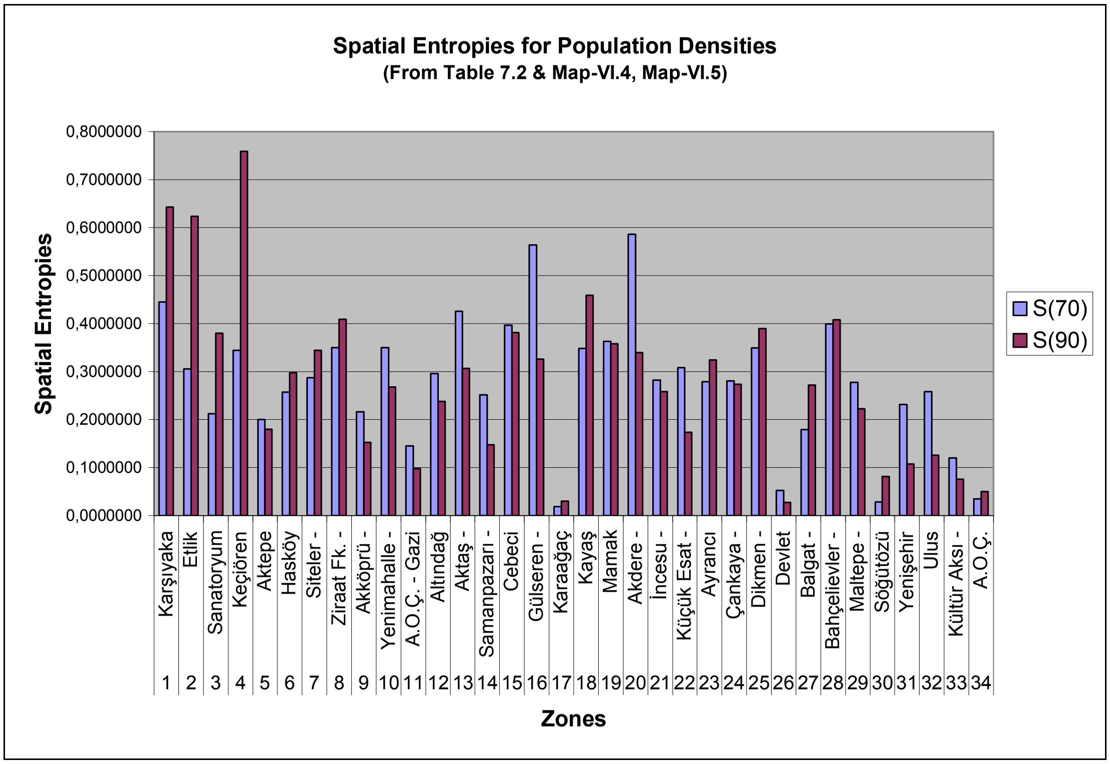


Figure VI.3-) Spatial Entropies for Population Densities

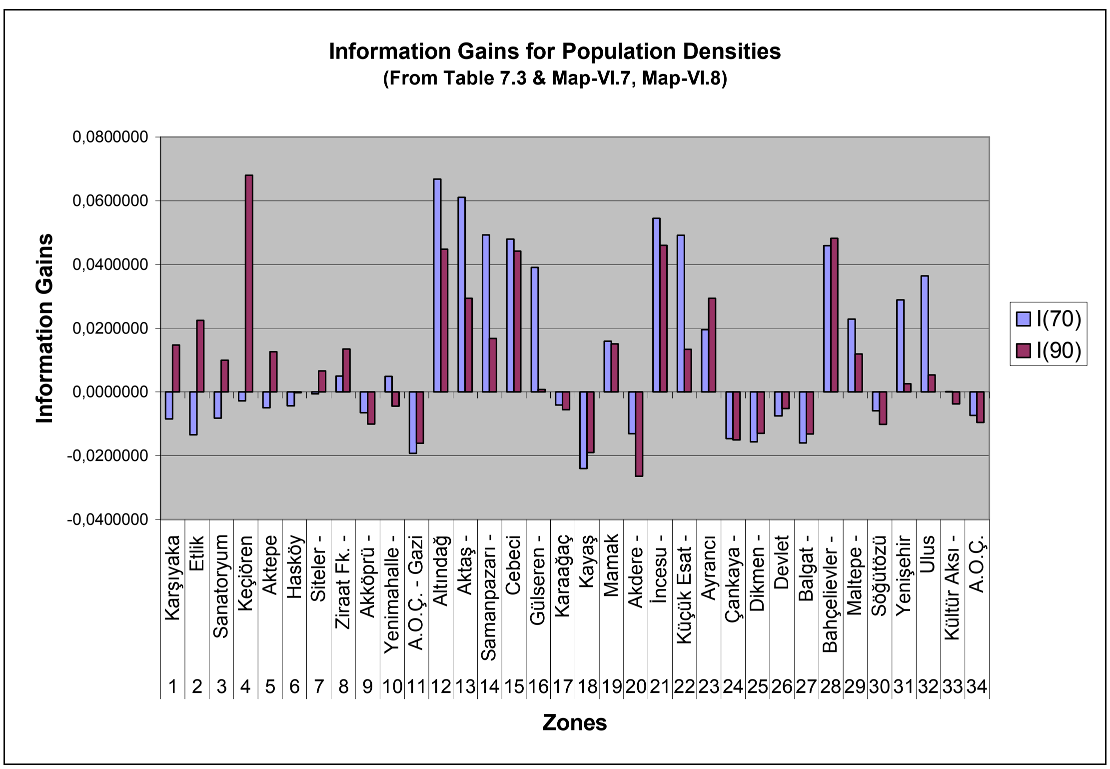


Figure VI.4-) Information Gain for Population Densities

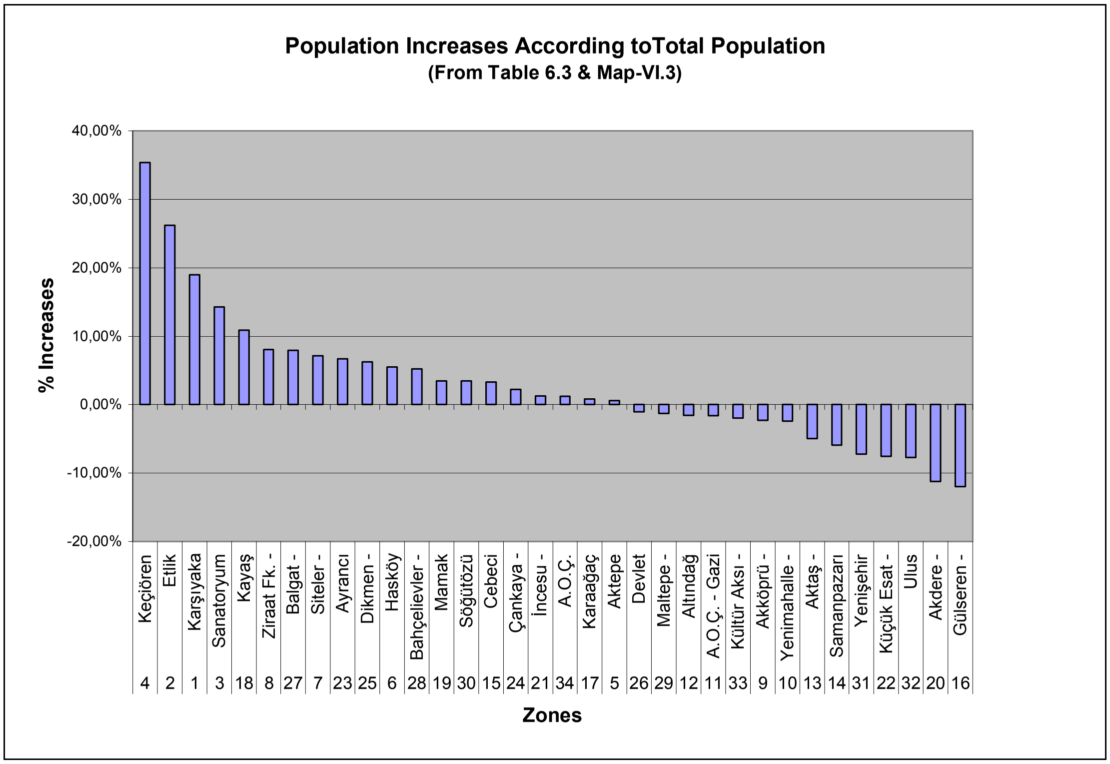


Figure VI.5-) Population Increases According to Total Population

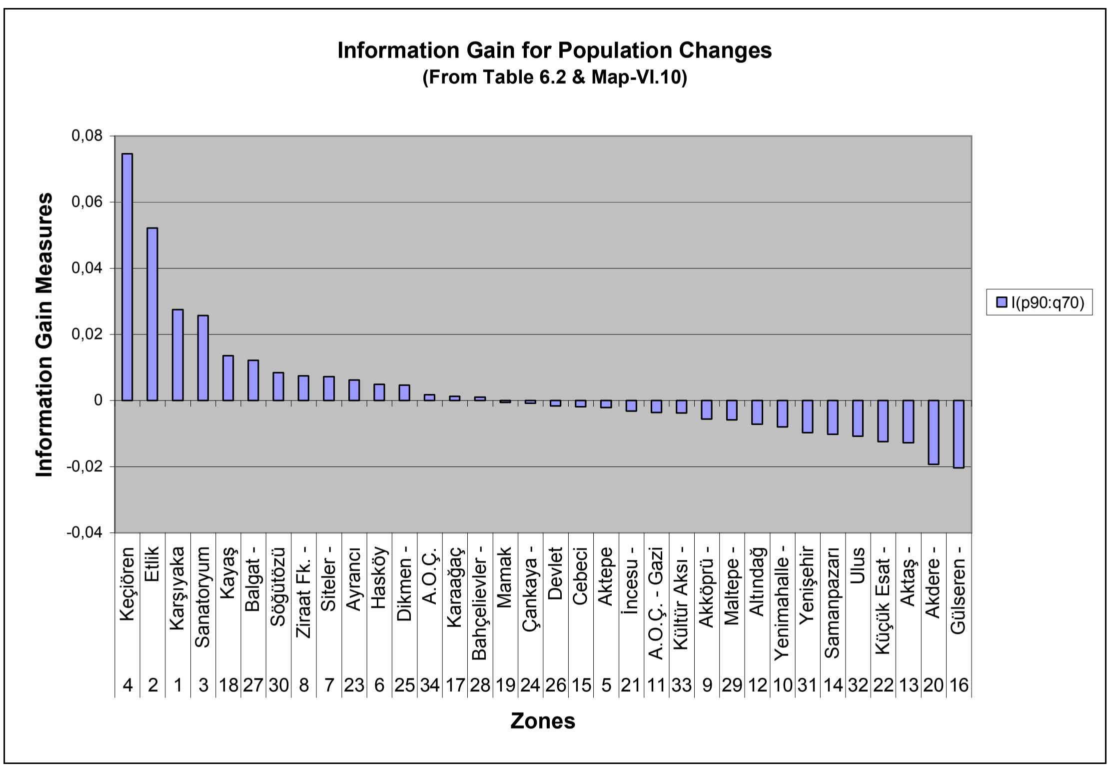


Figure VI.6-) Information Gain for Population Changes

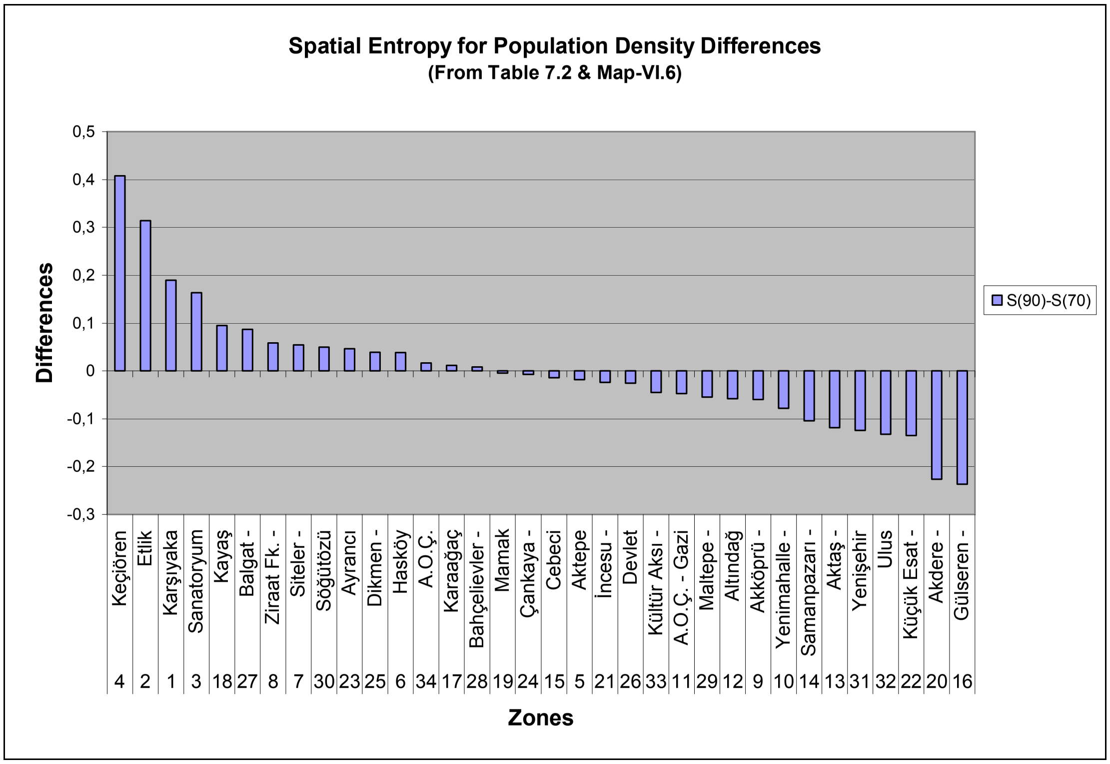


Figure VI.7-) Spatial Entropy for Population Density Differences

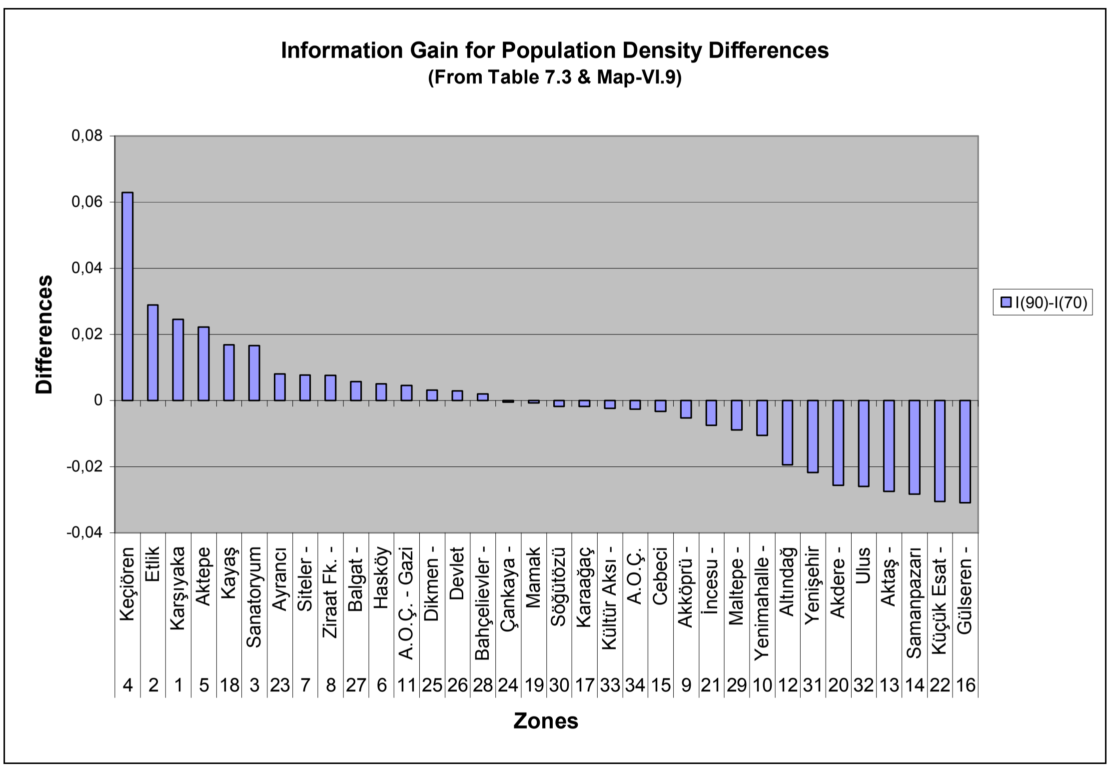
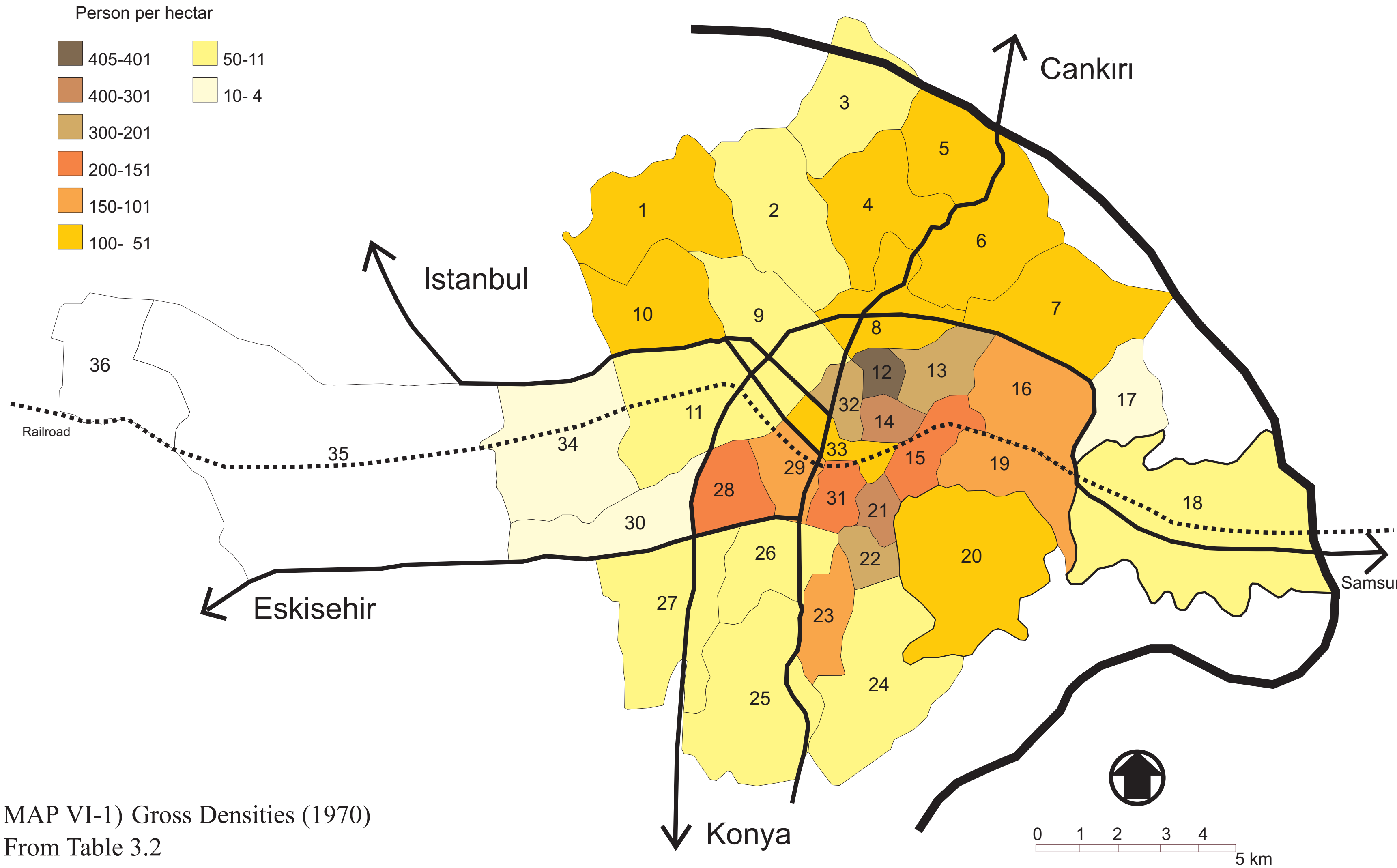
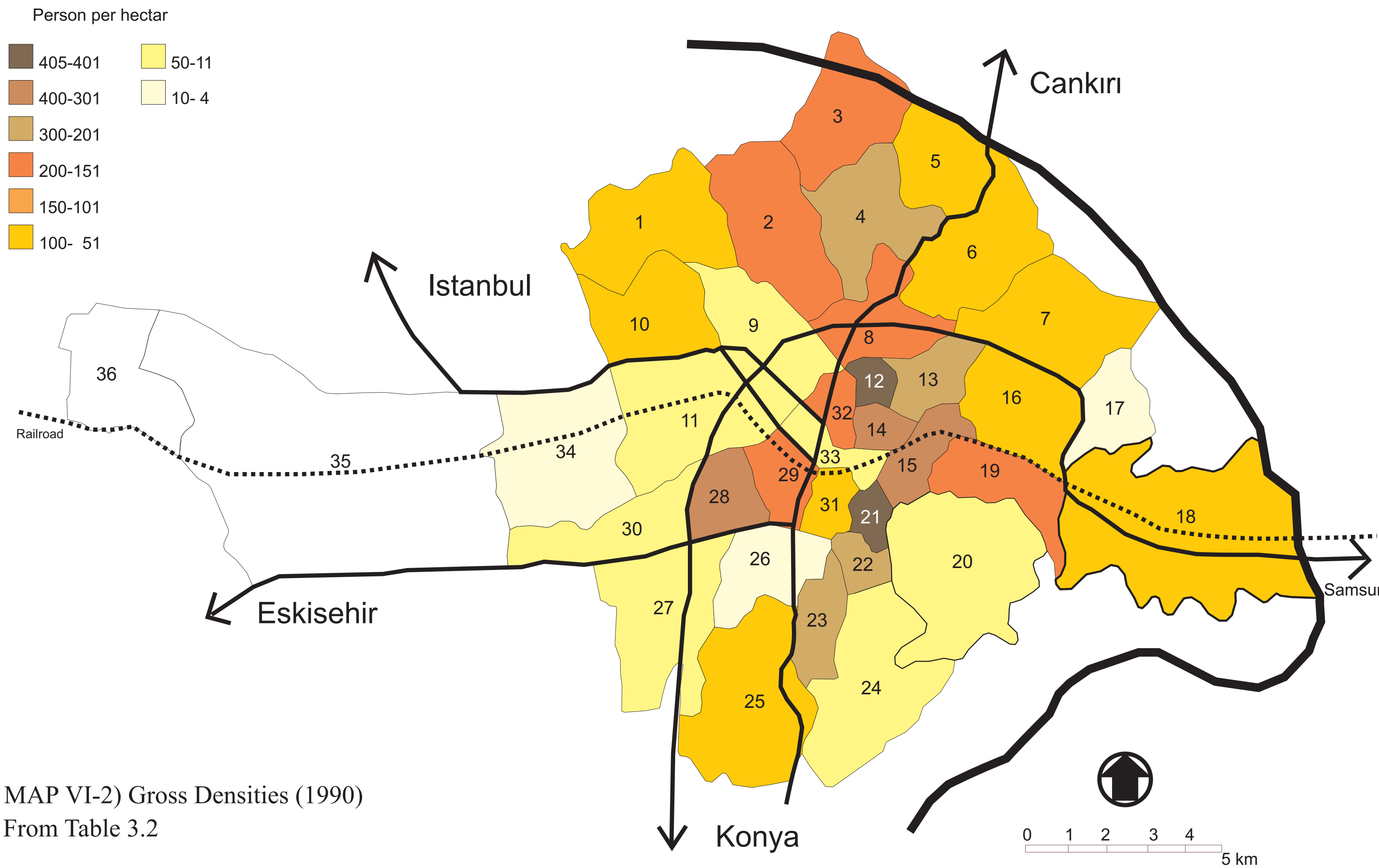


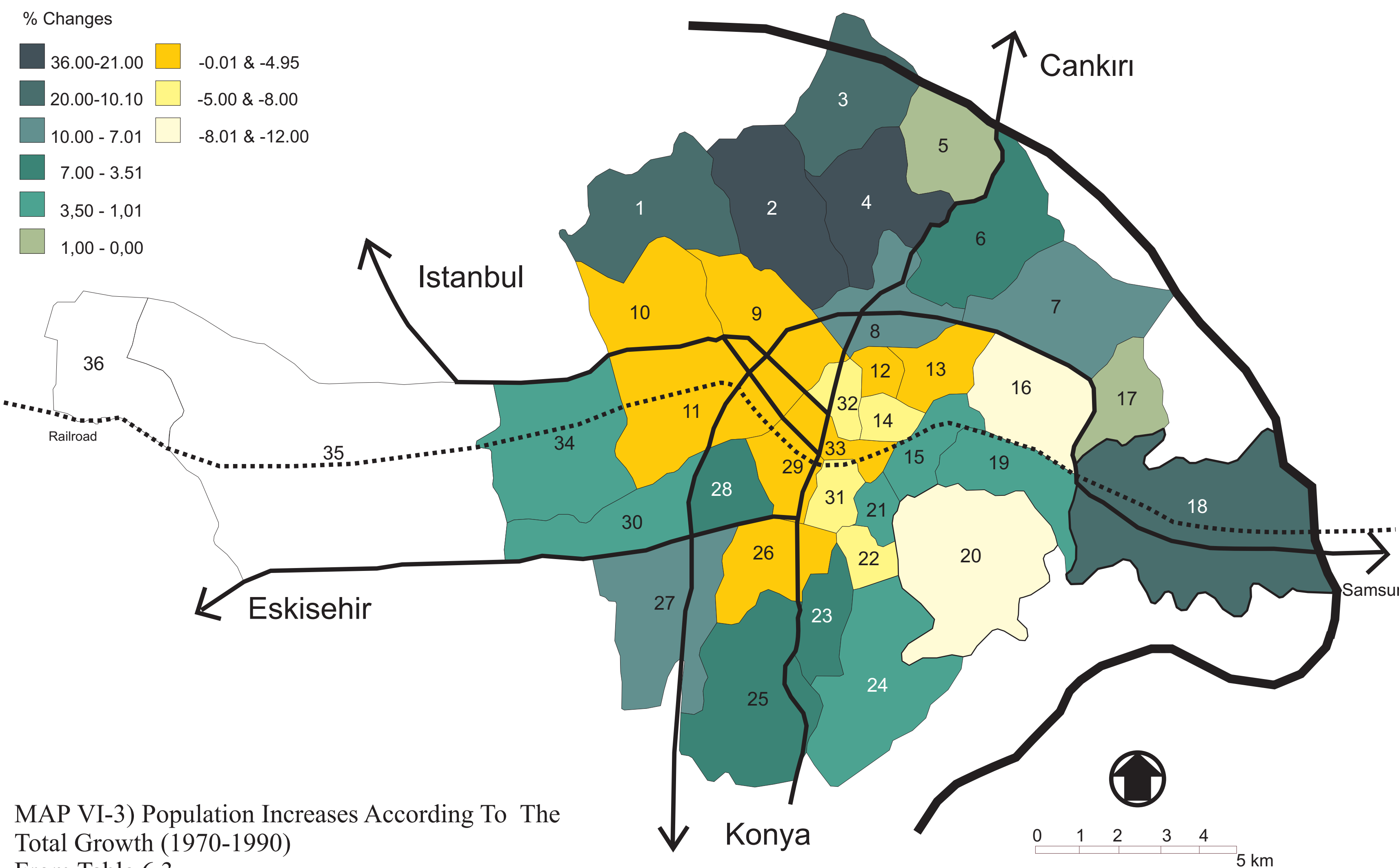
Figure VI.8-) Information Gain for Population Density Differences



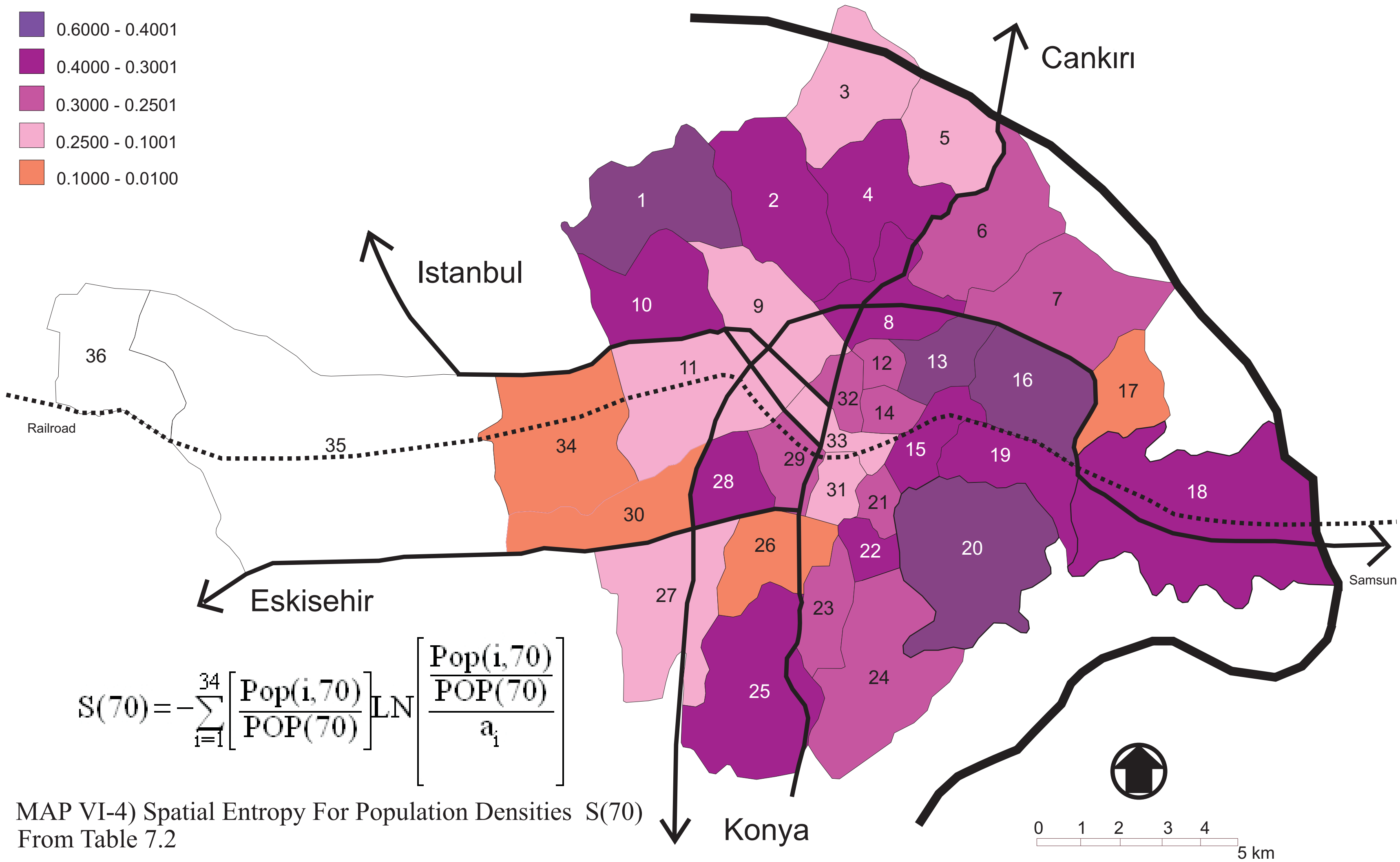
MAP VI-1) Gross Densities (1970)
From Table 3.2



MAP VI-2) Gross Densities (1990)
From Table 3.2

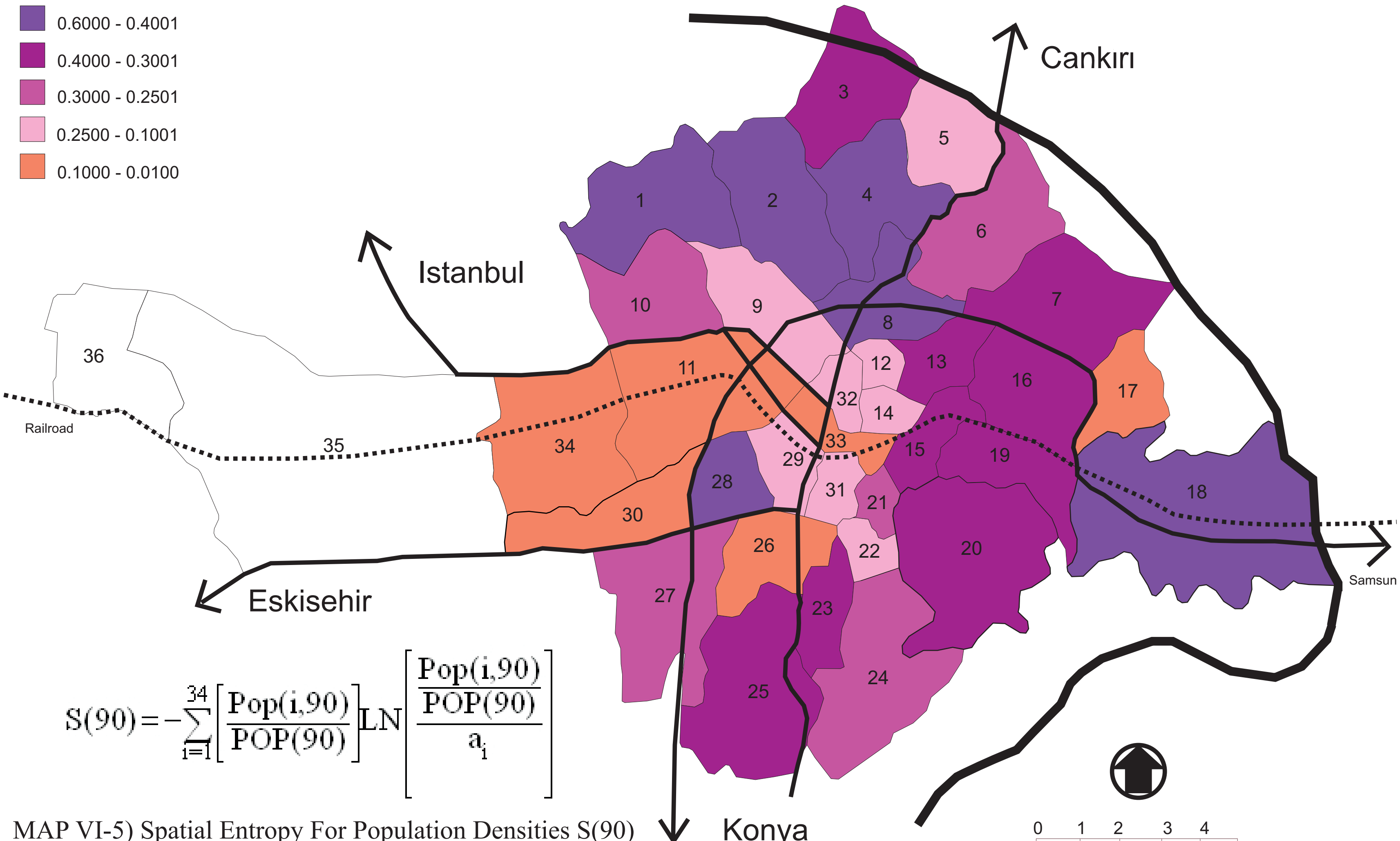


MAP VI-3) Population Increases According To The Total Growth (1970-1990)
From Table 6.3



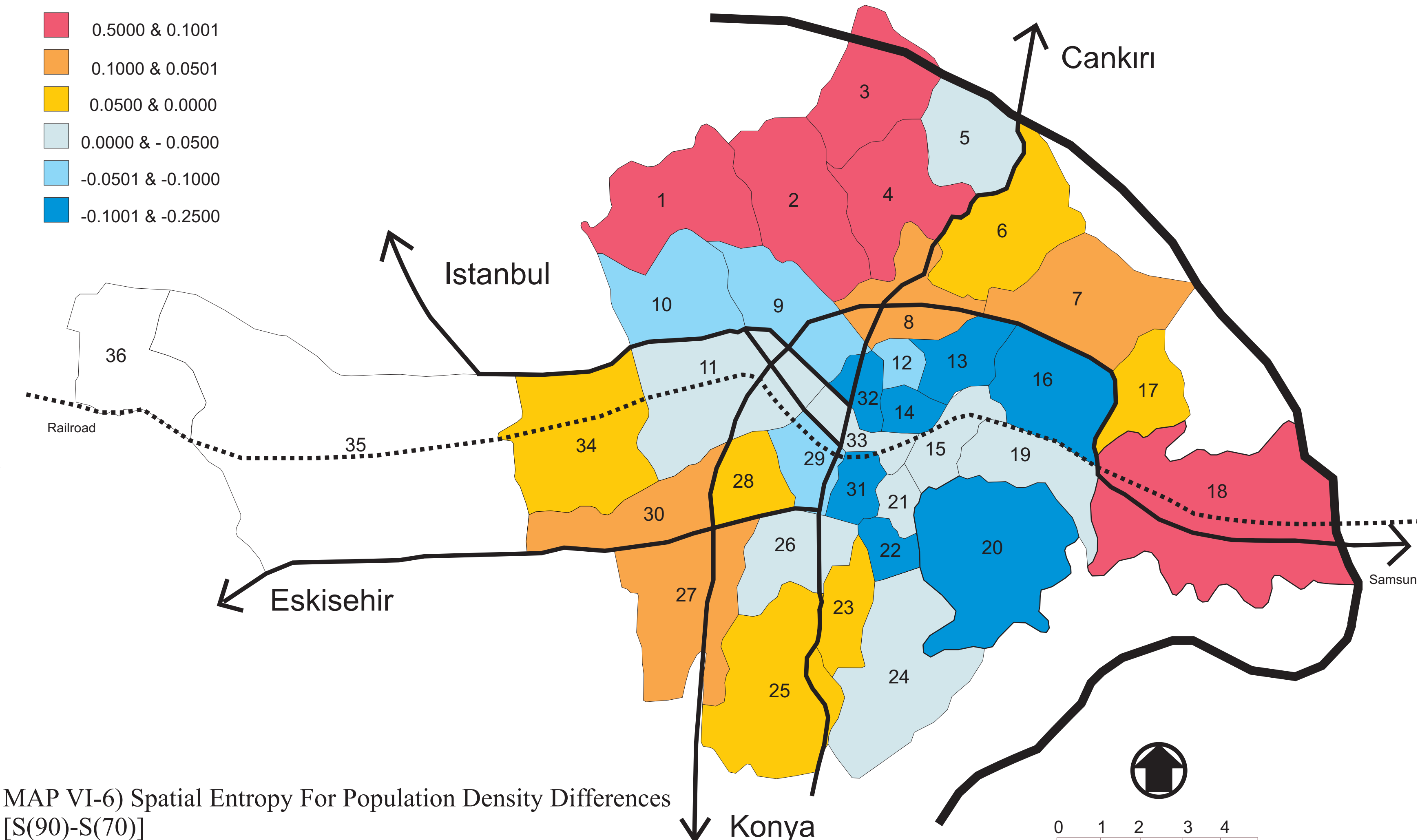
$$S(70) = -\sum_{i=1}^{34}\left[\frac{Pop(i,70)}{POP(70)}\right] LN\left[\frac{\frac{Pop(i,70)}{POP(70)}}{a_i}\right]$$

MAP VI-4) Spatial Entropy For Population Densities S(70)
From Table 7.2

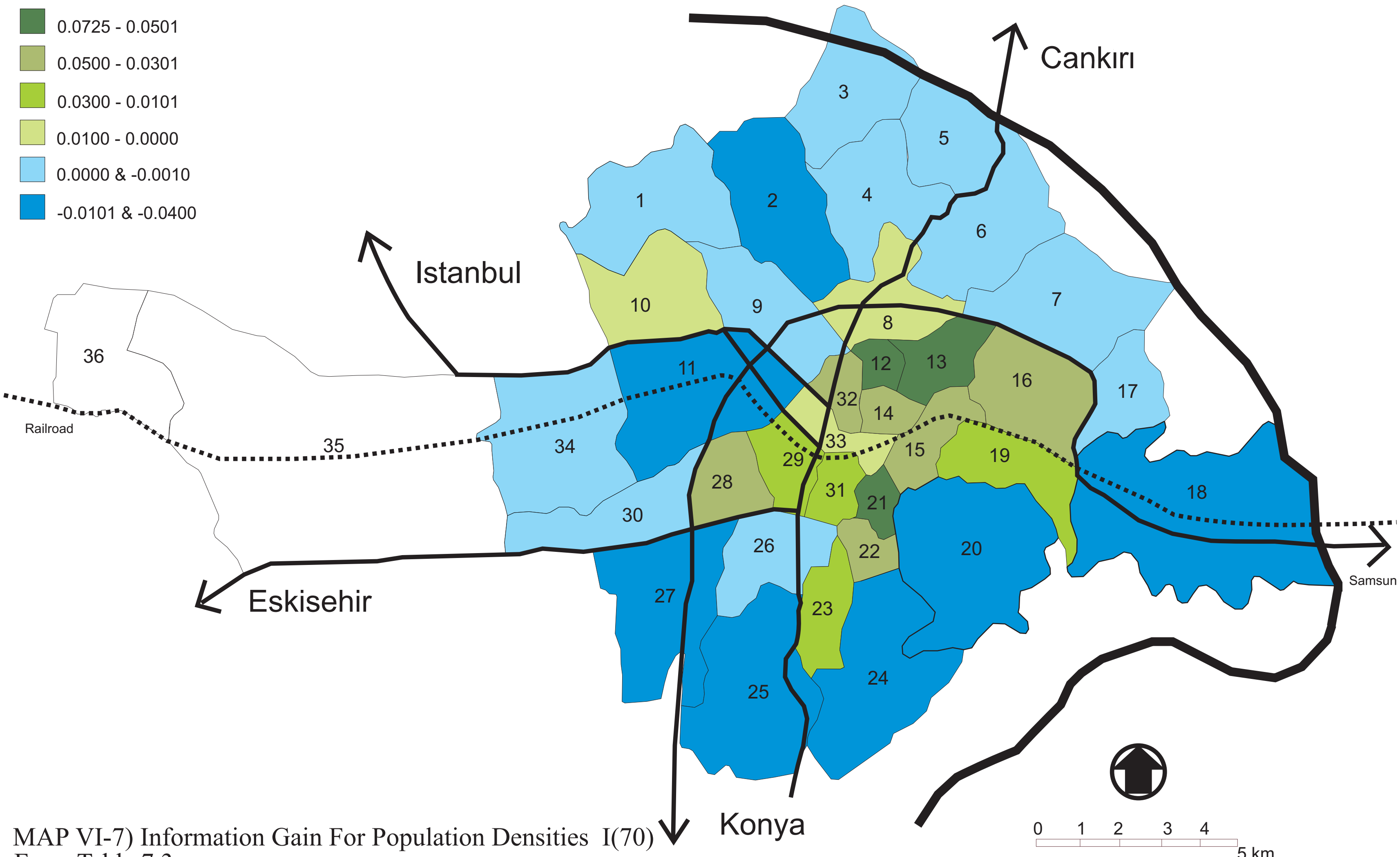


$$S(90) = -\sum_{i=1}^{34}\left[\frac{Pop(i,90)}{POP(90)}\right] LN\left[\frac{\frac{Pop(i,90)}{POP(90)}}{a_i}\right]$$

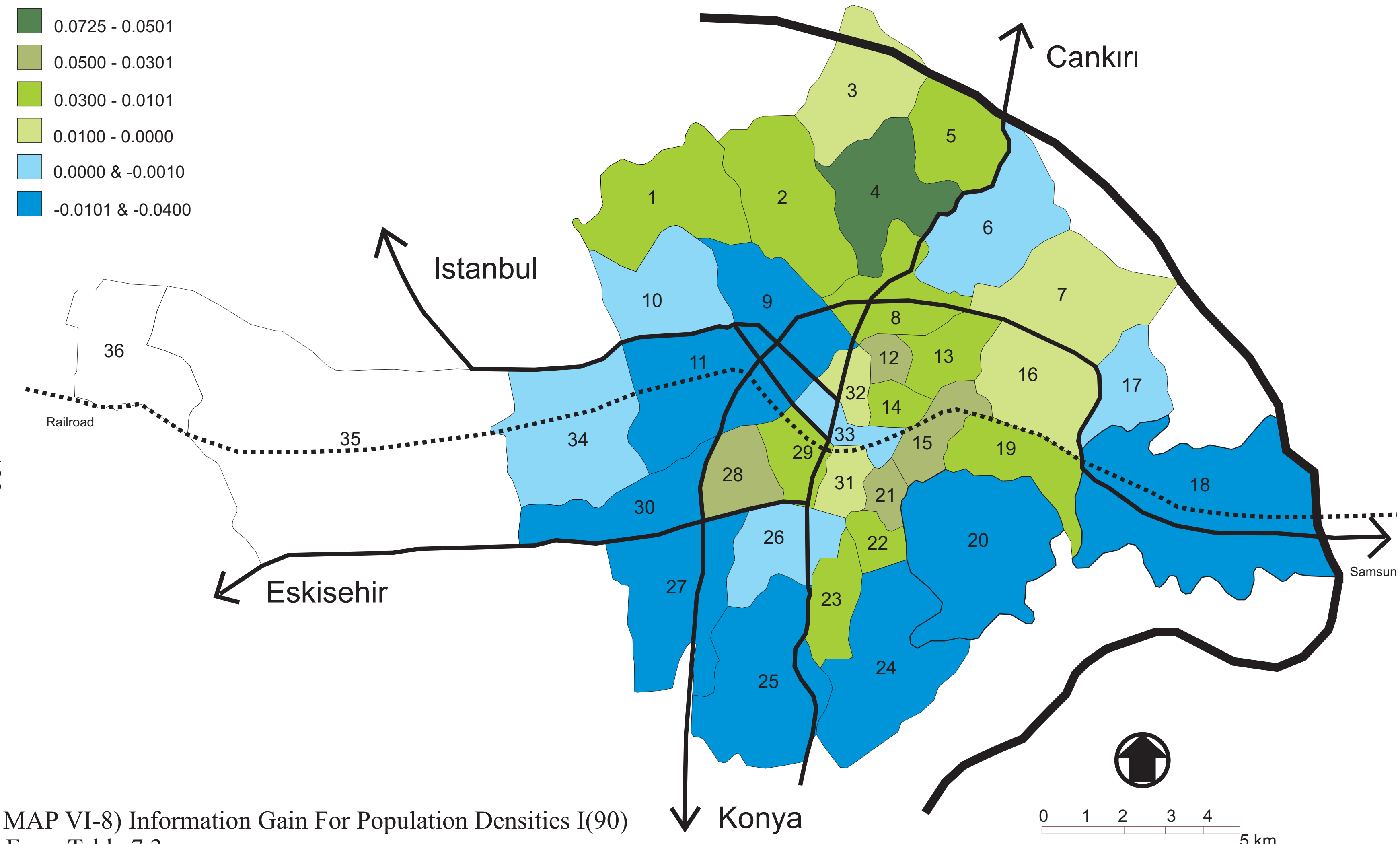
MAP VI-5) Spatial Entropy For Population Densities S(90)
From Table 7.2



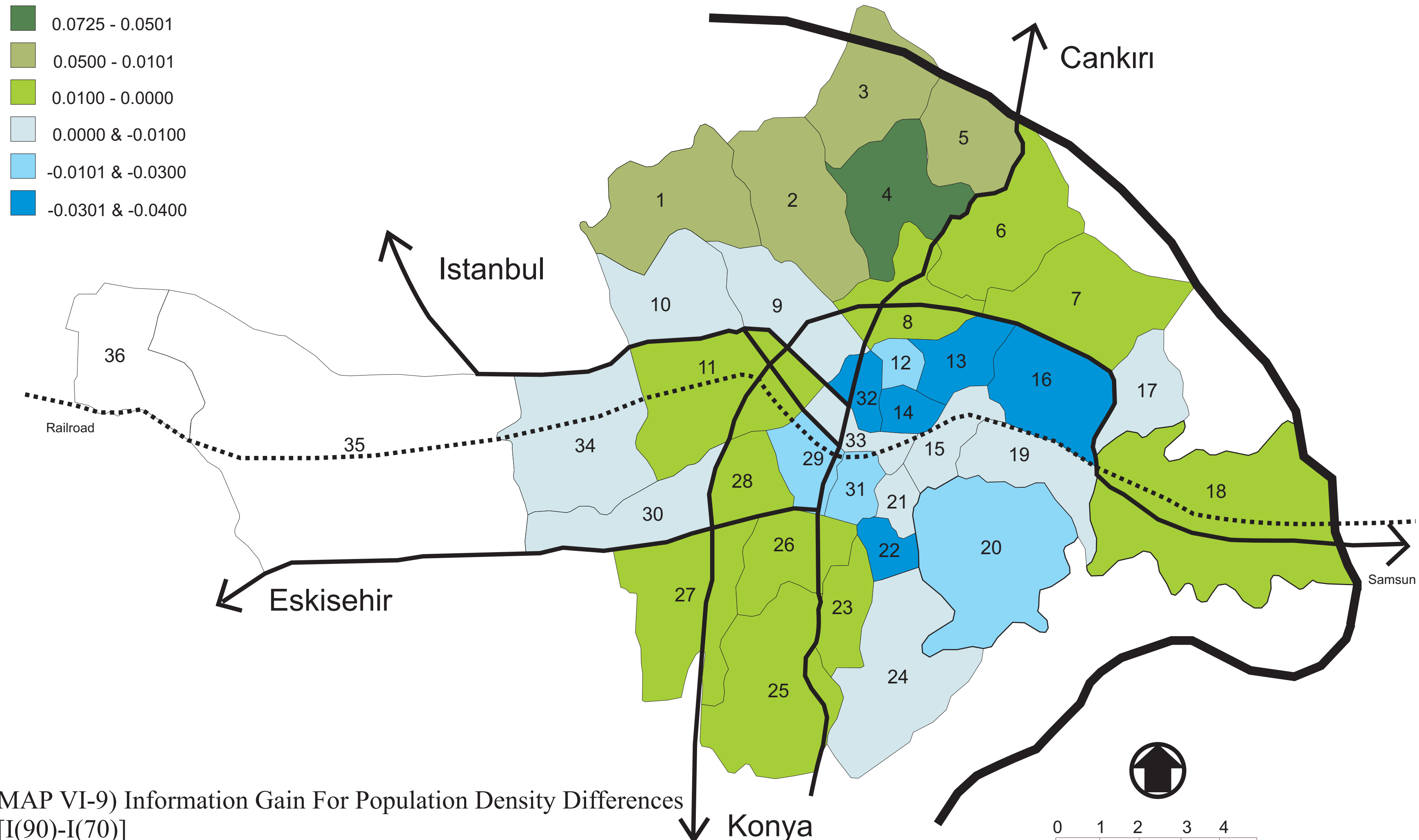
MAP VI-6) Spatial Entropy For Population Density Differences
[S(90)-S(70)]
From Table 7.2



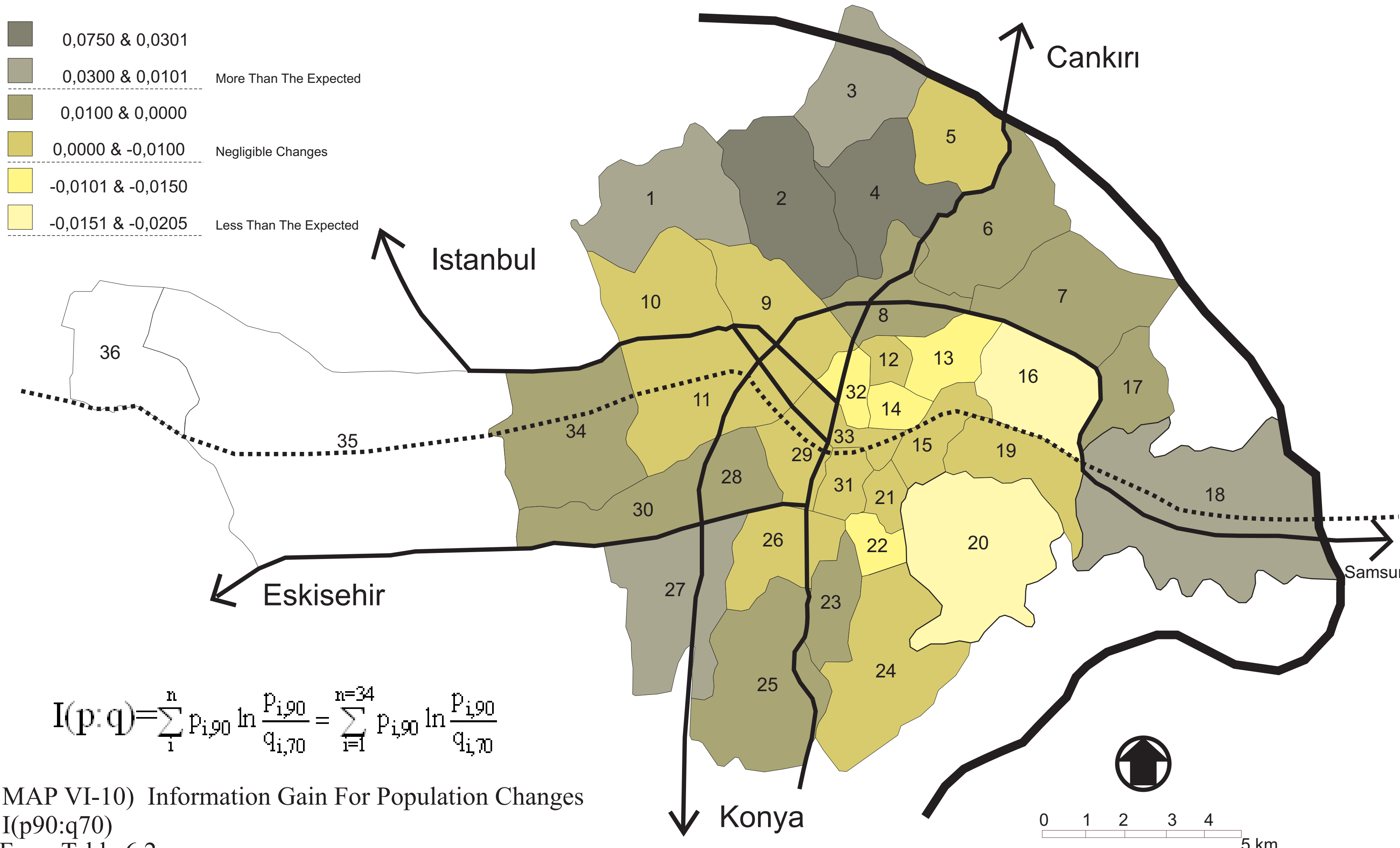
MAP VI-7) Information Gain For Population Densities I(70)
From Table 7.3



MAP VI-8) Information Gain For Population Densities I(90)
From Table 7.3

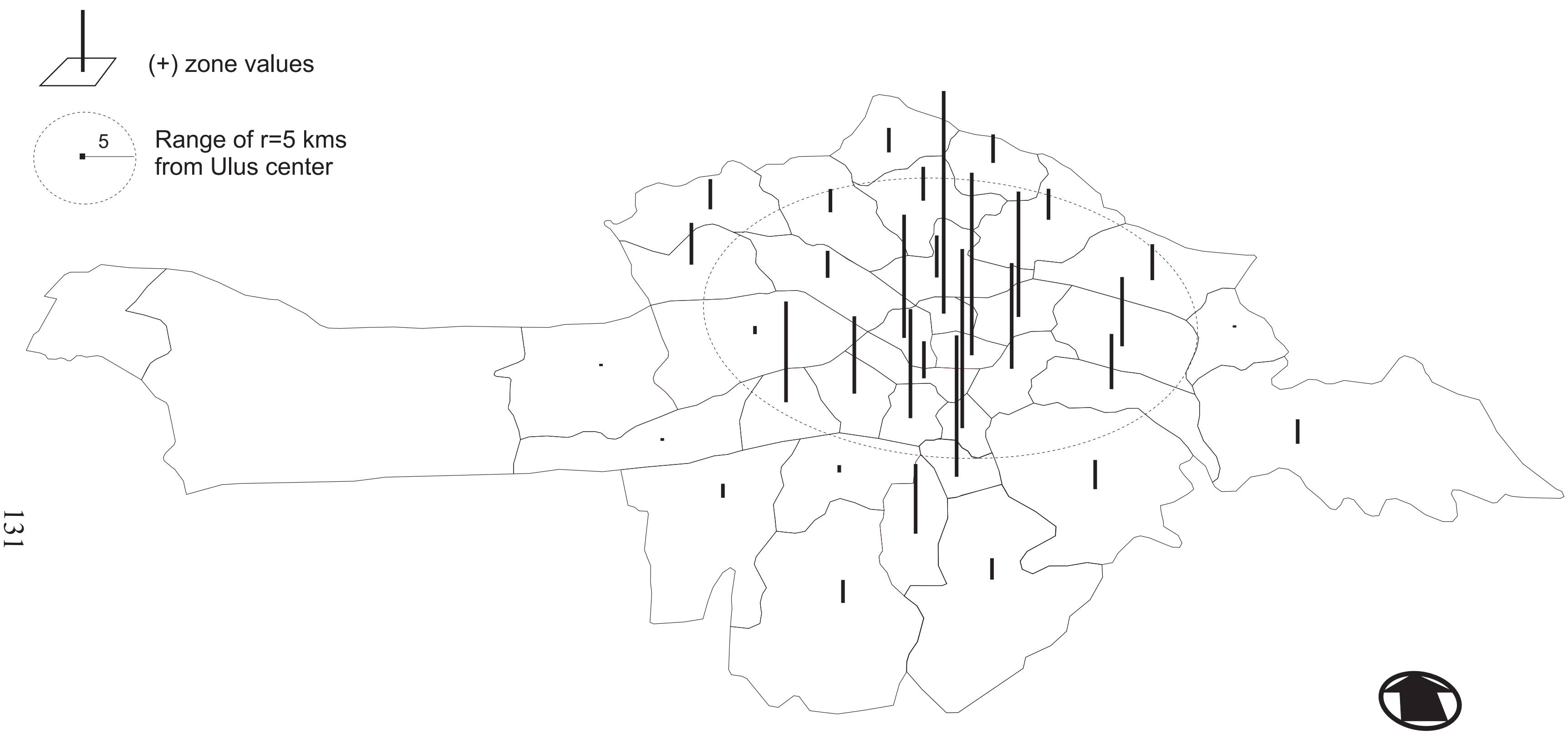


MAP VI-9) Information Gain For Population Density Differences
[I(90)-I(70)]
From Table 7.3

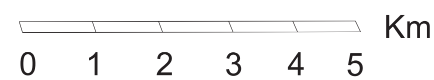


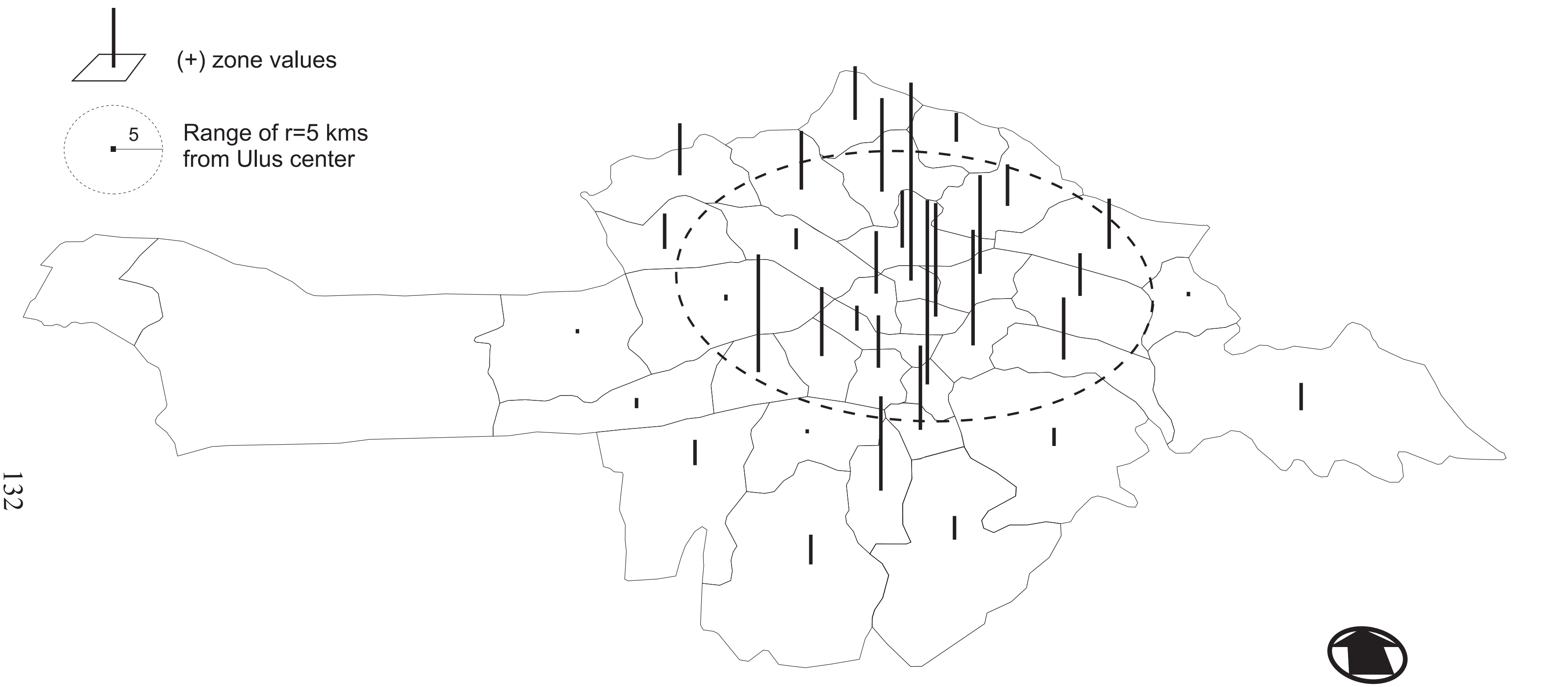
$$I(p:q)=\sum_{i}^{n} p_{i,90} \ln \frac{p_{i,90}}{q_{i,70}} = \sum_{i=1}^{n=34} p_{i,90} \ln \frac{p_{i,90}}{q_{i,70}}$$

MAP VI-10) Information Gain For Population Changes
I(p90:q70)
From Table 6.2



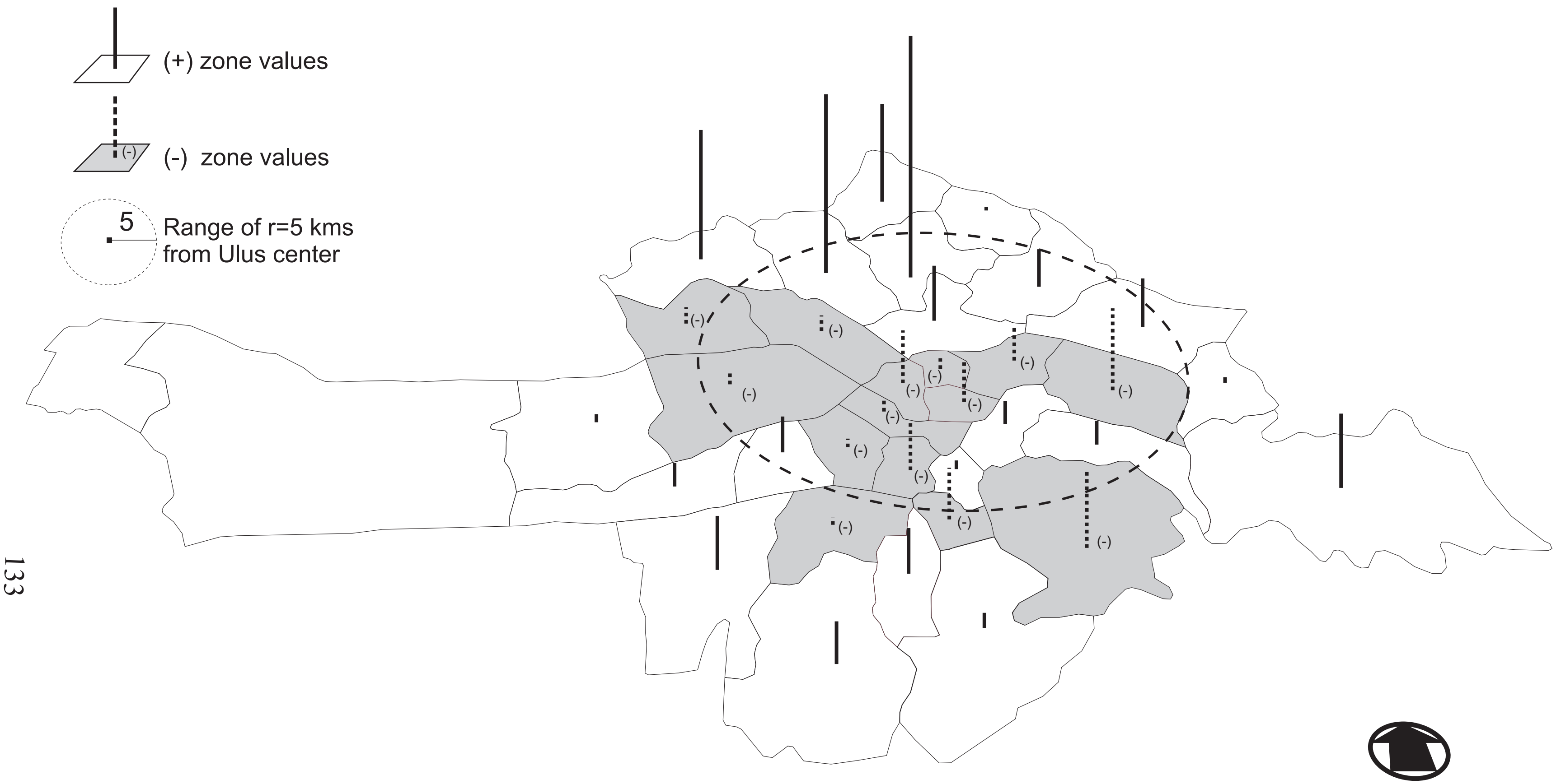
MAP VI-11) Gross Densities (1970)
(From Table 3.2 & Map VI-1) (Table values are divided by a factor F=12)

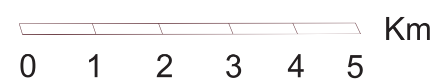




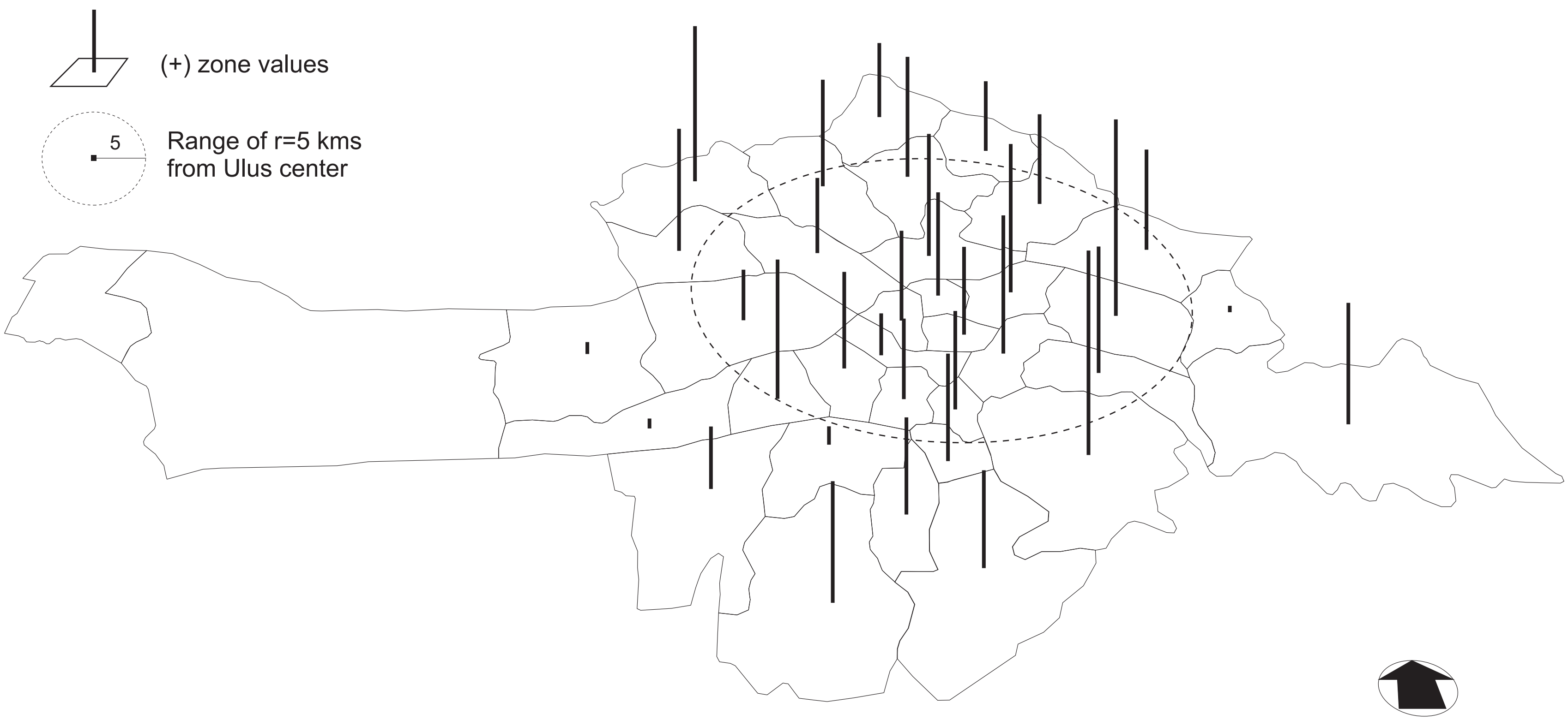
MAP VI-12) Gross Densities (1990)
(From Table 3.2 & Map VI-2) (Table values are divided by factor F=12)

Km
0 1 2 3 4 5

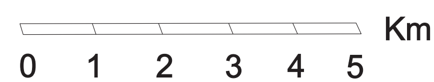


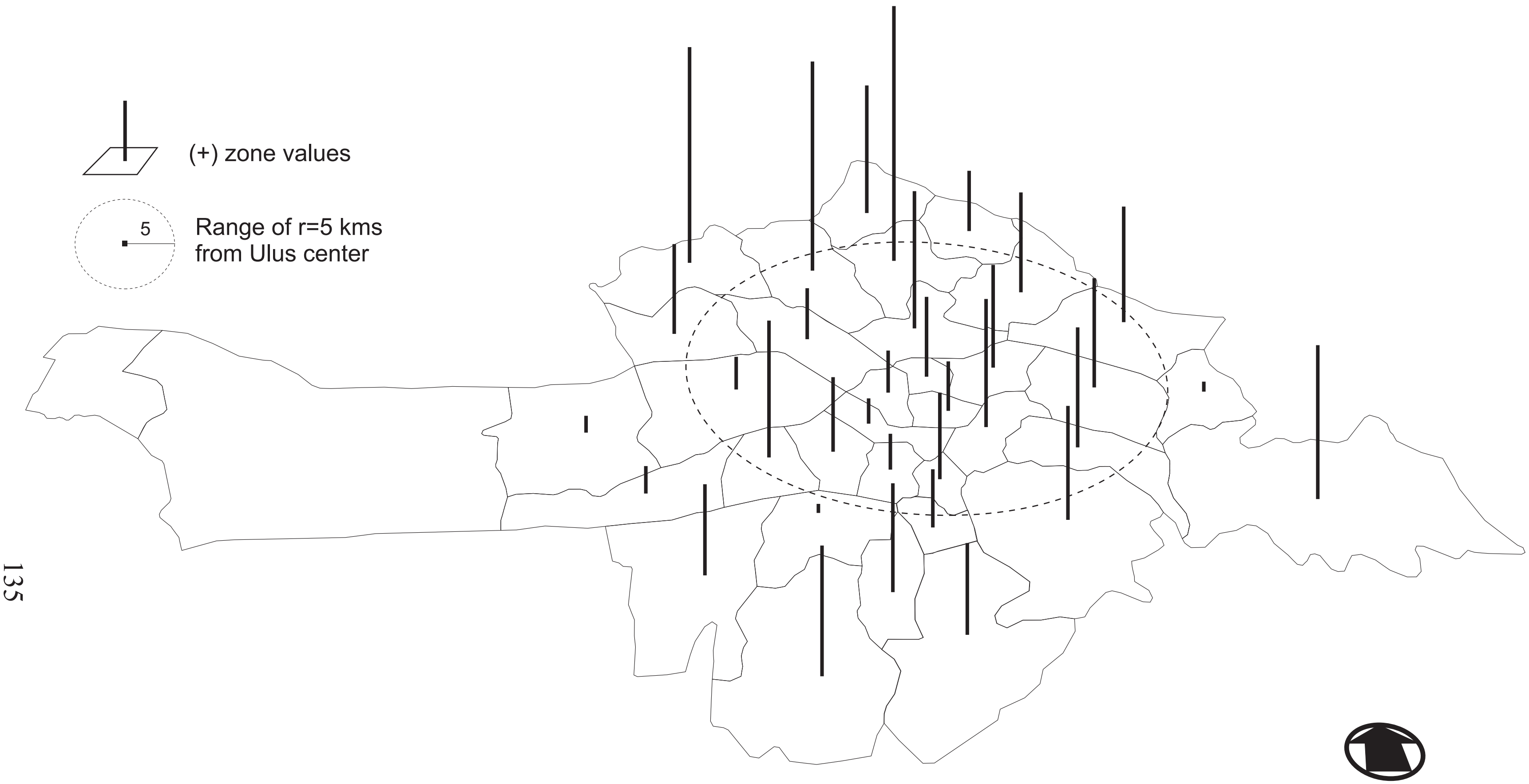


MAP VI-13) Population Increases According To Total Growth
(From Table 6.3 & Map VI-3)

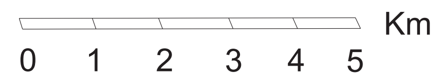


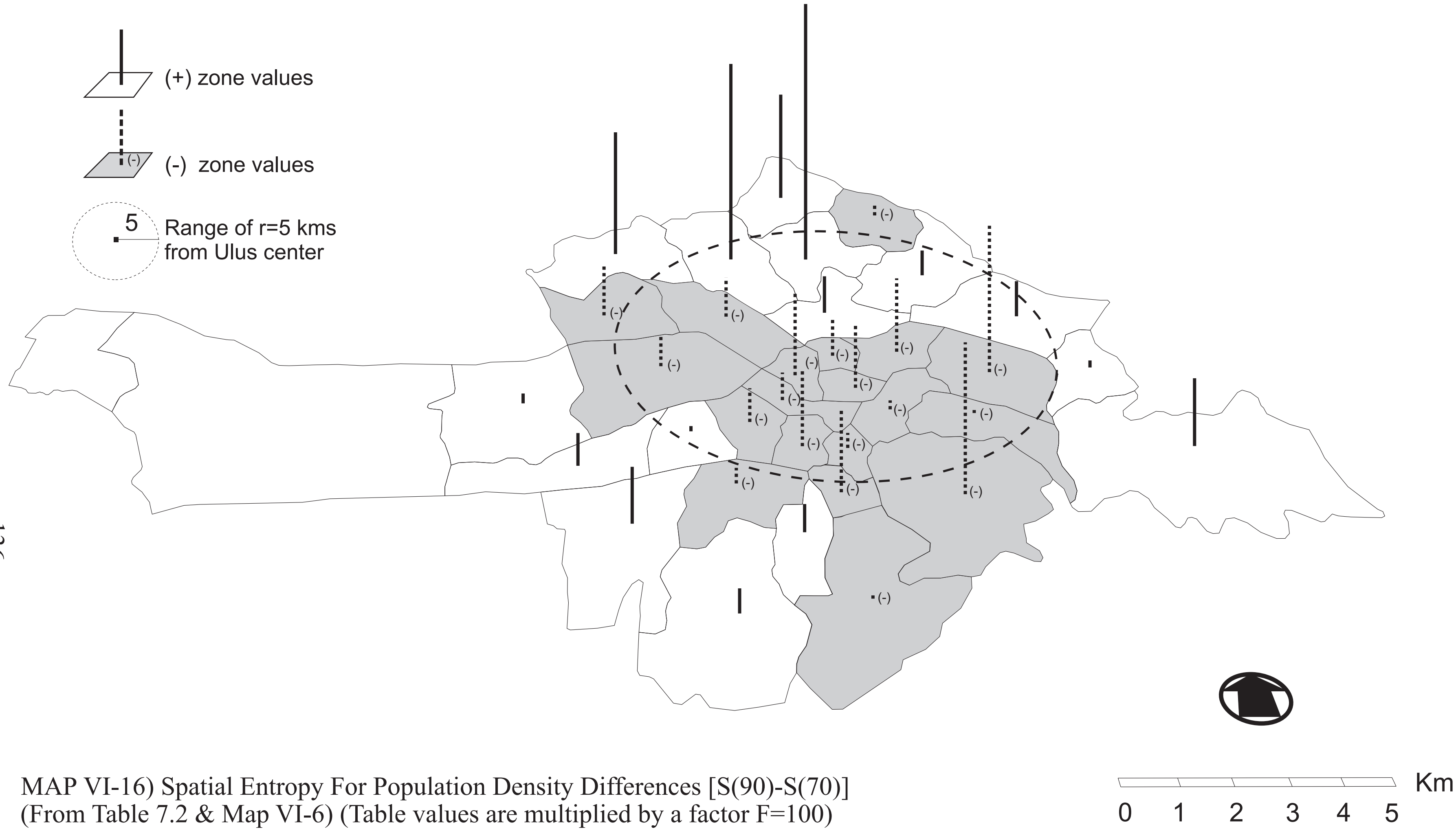
MAP VI-14) Spatial Entropy For Population Densities S(70)
(From Table 7.2 & Map VI-4) (Table values are multiplied by a factor F=50)



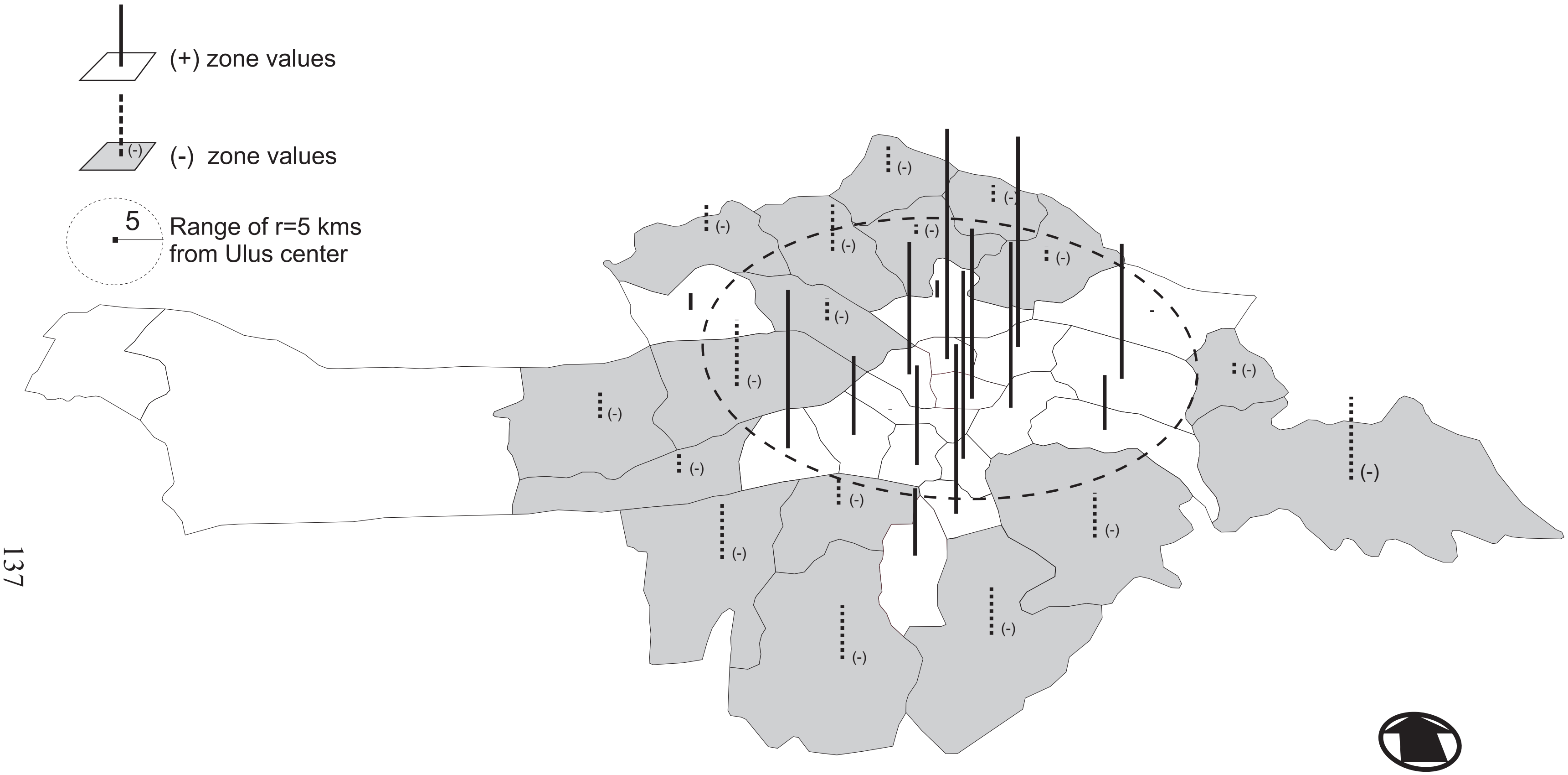


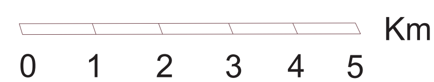
MAP VI-15) Spatial Entropy For Population Densities S(90)
(From Table 7.2 & Map VI-5) (Table values are multiplied by a factor F=50)



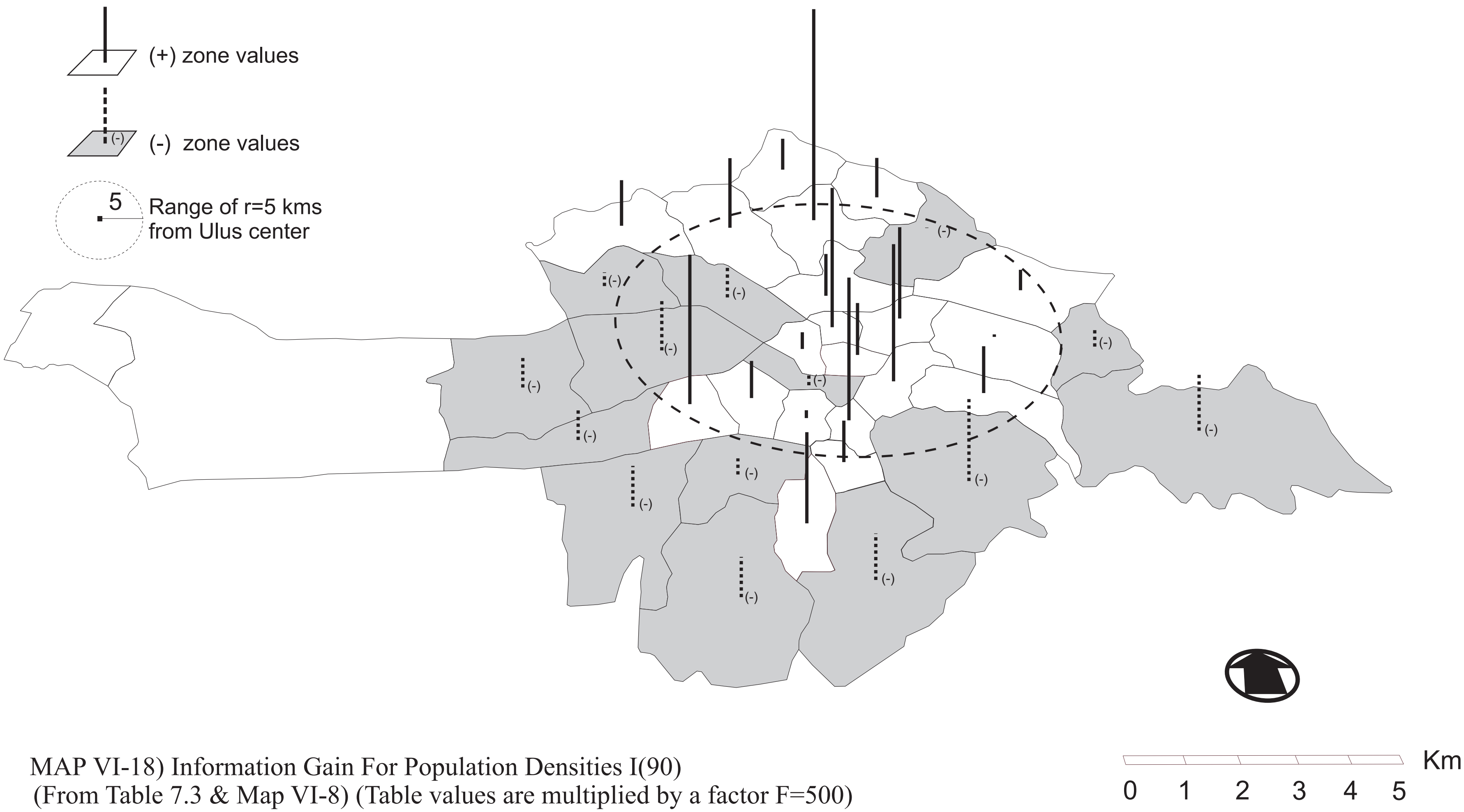


MAP VI-16) Spatial Entropy For Population Density Differences [S(90)-S(70)]
(From Table 7.2 & Map VI-6) (Table values are multiplied by a factor F=100)

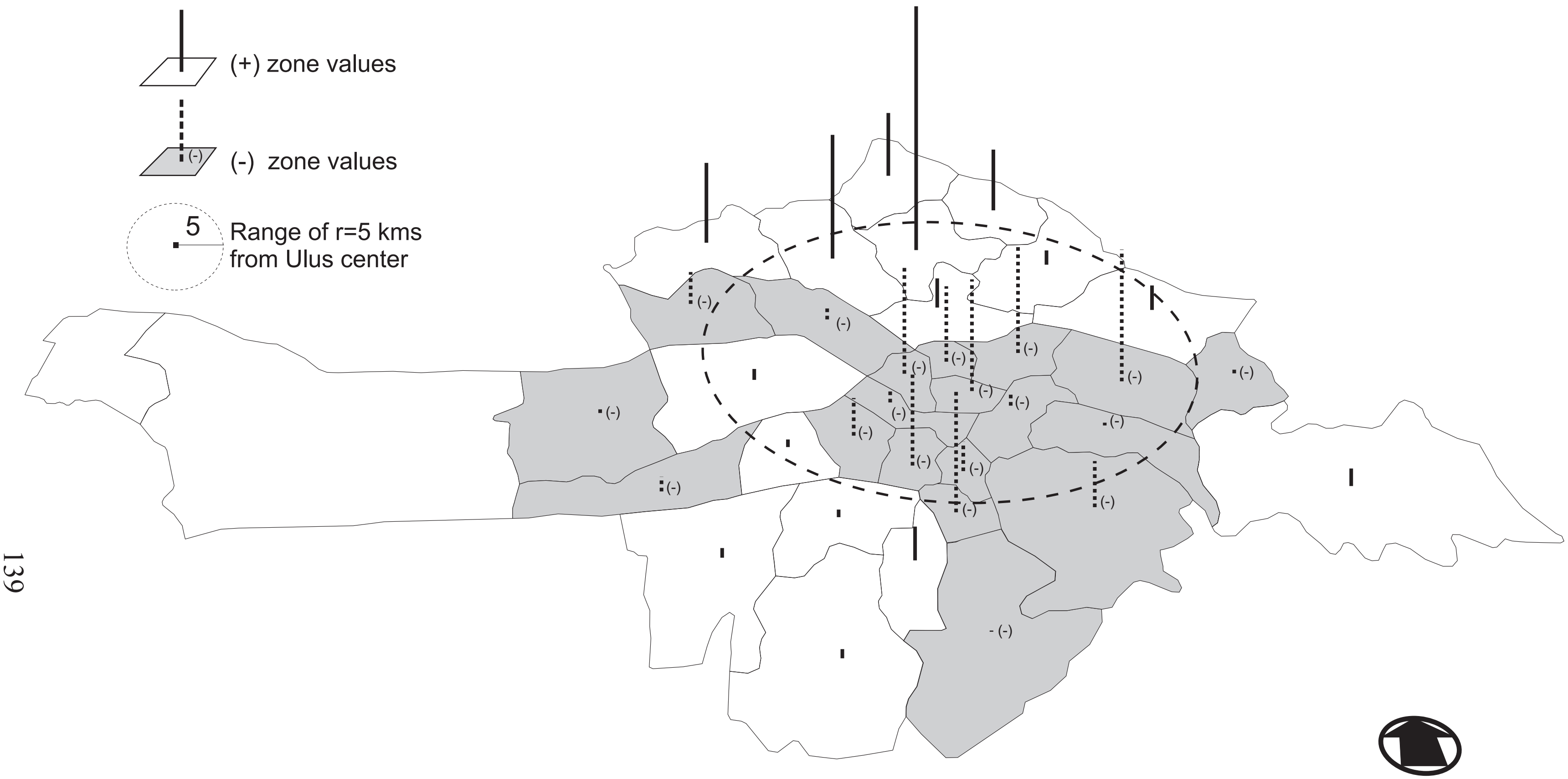


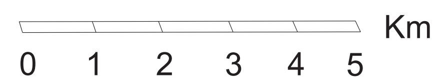


MAP VI-17) Information Gain For Population Densities I(70)
(From Table 7.3 & Map VI-7) (Table values are multiplied by a factor F=500)

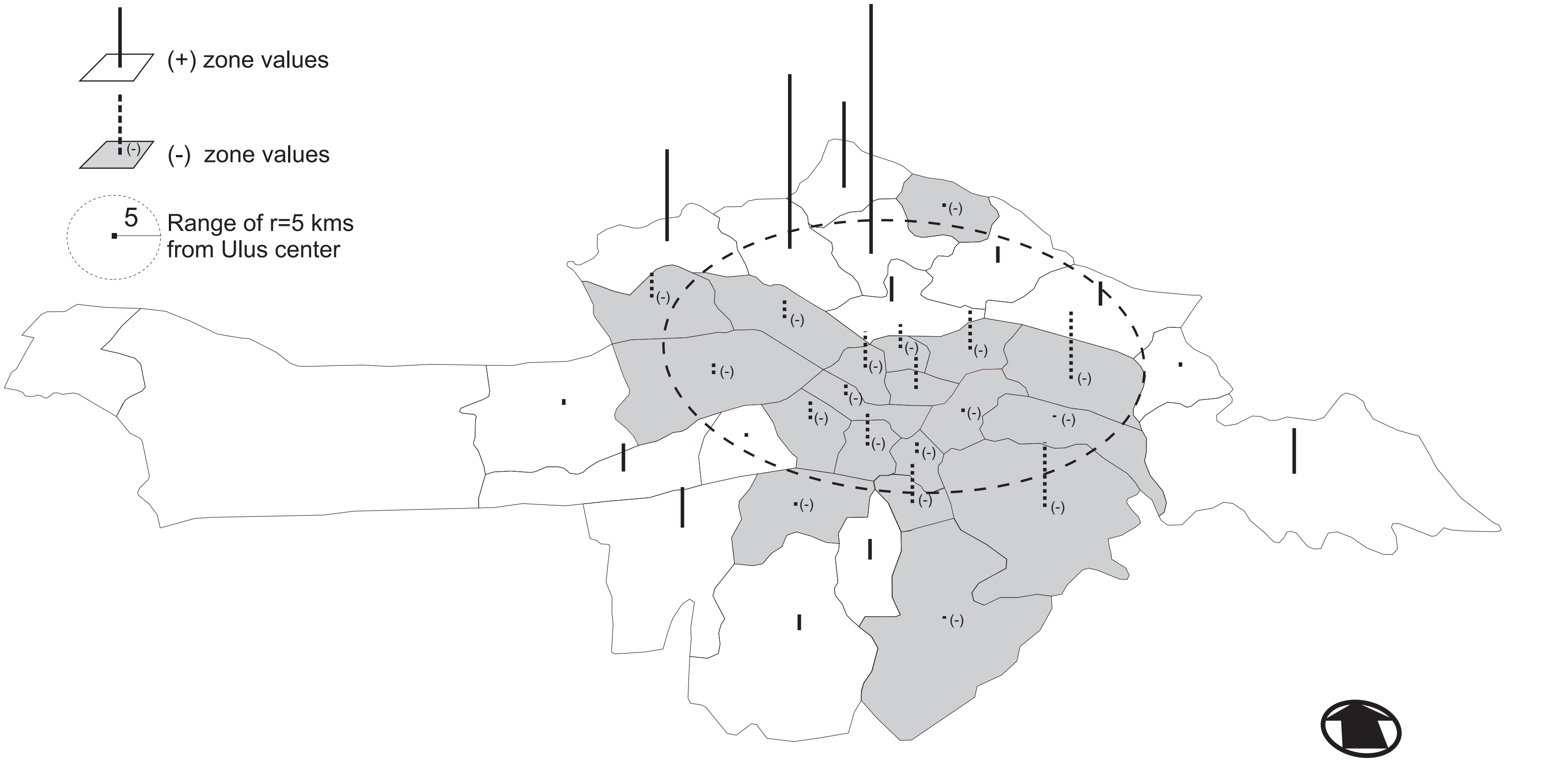


MAP VI-18) Information Gain For Population Densities I(90)
(From Table 7.3 & Map VI-8) (Table values are multiplied by a factor F=500)

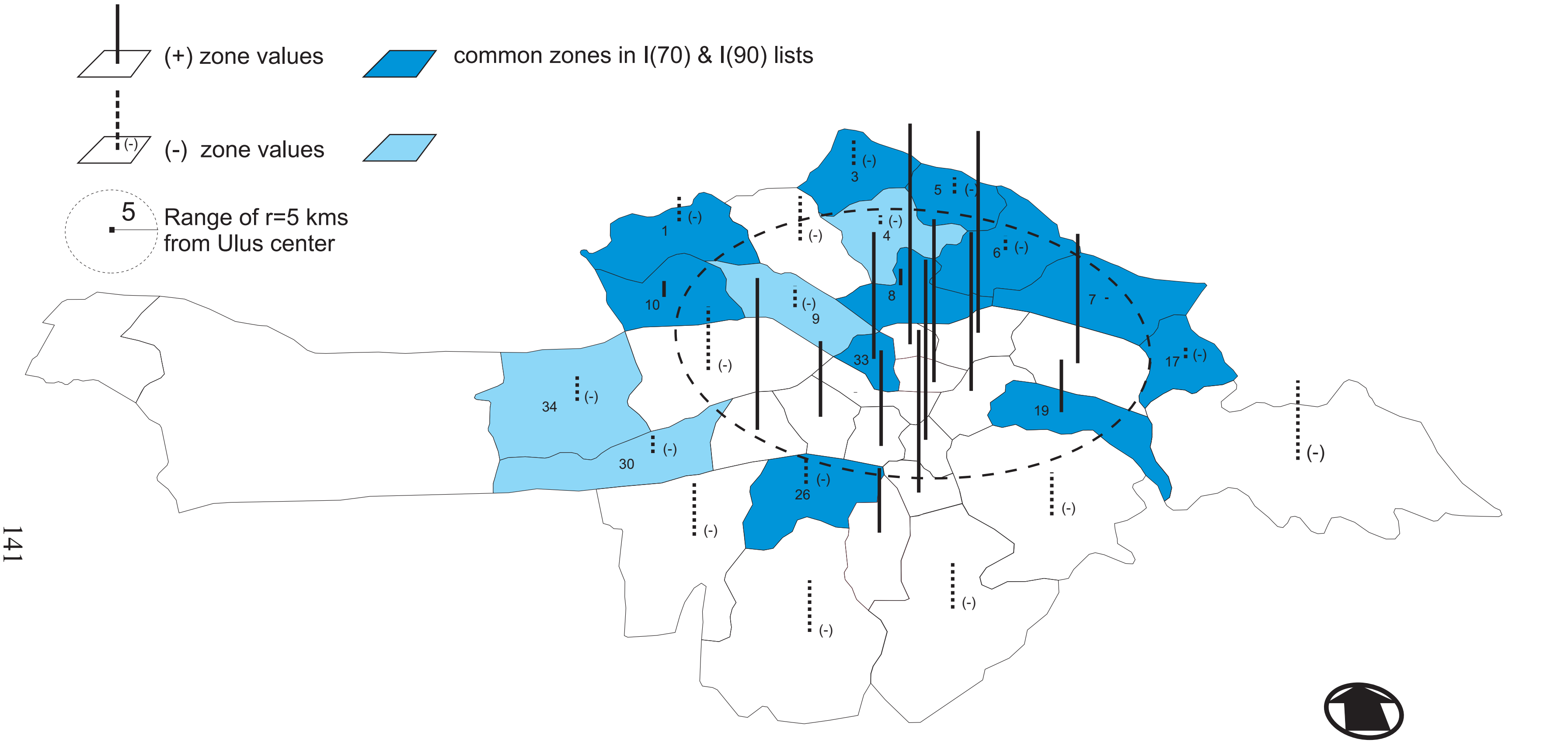


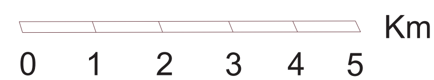


MAP VI-19) Information Gain For Population Density Differences [I(90)-I(70)]
(From Table 7.3 & Map VI-9) (Table values are multiplied by a factor F=500)

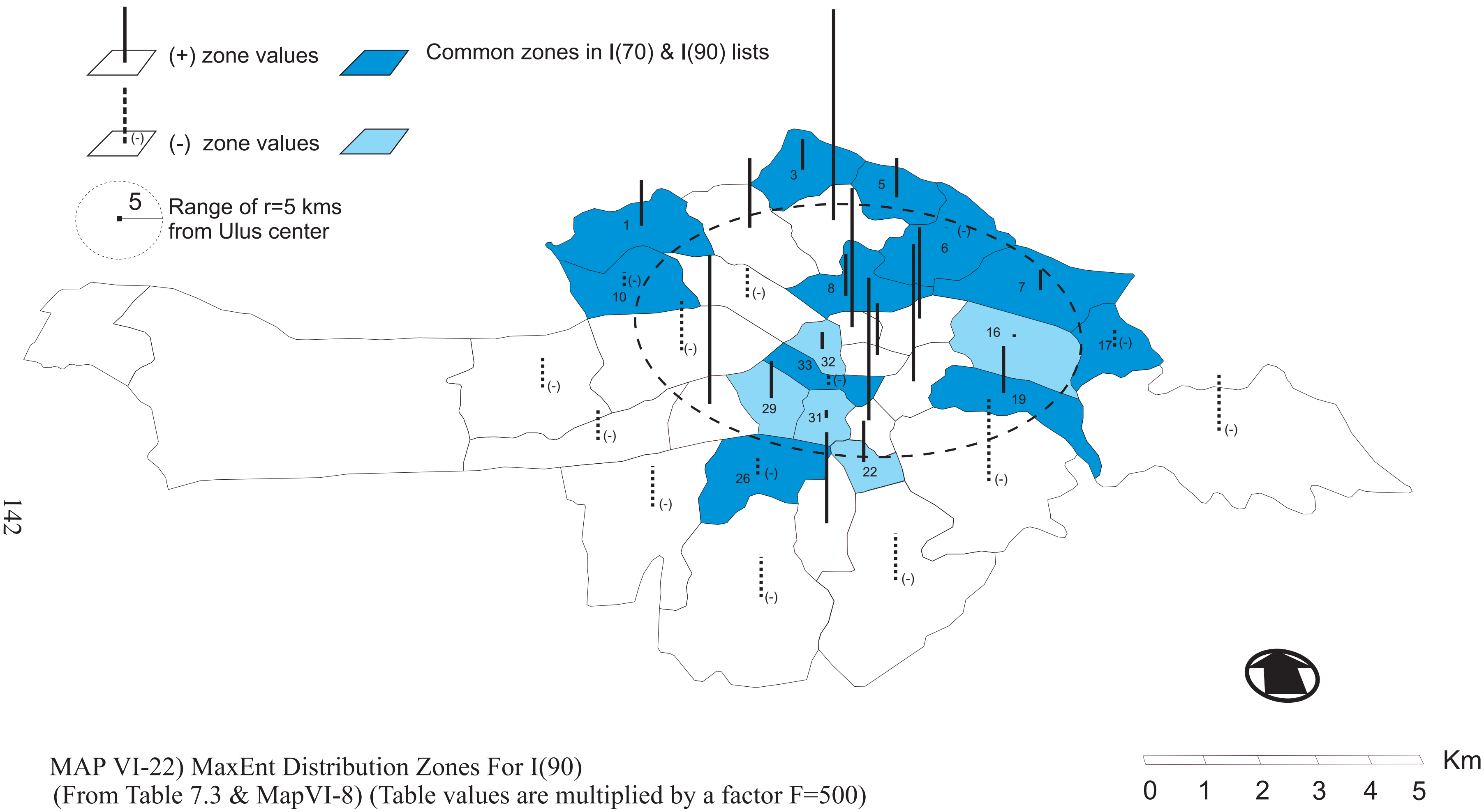


MAP VI-20) Information Gain For Population Changes: I(p90:q70)
(From Table 6.2 & Map VI-10) (Table values are multiplied by a factor F=500)

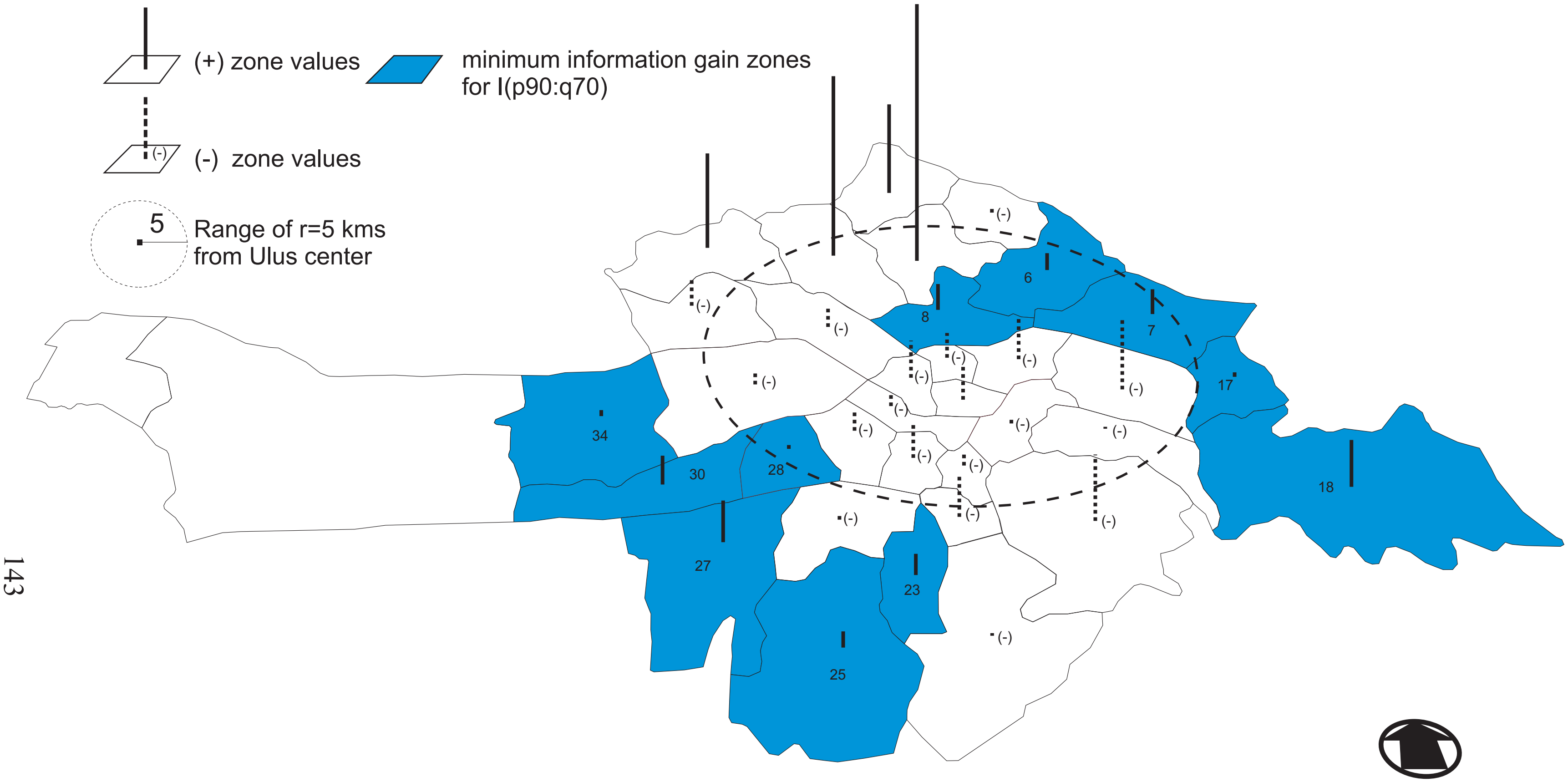


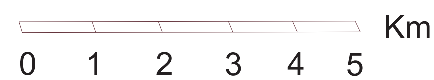


MAP VI-21) MaxEnt Distribution Zones For I(70)
(From Table 7.3 & Map-VI.7) (Table values are multiplied by a factor F=500)



MAP VI-22) MaxEnt Distribution Zones For I(90)
(From Table 7.3 & MapVI-8) (Table values are multiplied by a factor F=500)





MAP-VI.23) Minimum Information Gain Zones For I(p90:q70)
(From Table 6.2 & Map-VI.10) (Table values are multiplied by a factor F=500)

CHAPTER VII

CONCLUSIONS

In Chapter I.3.4 & I.4, defining the problems encountered and scope of the thesis, six basic reasons were given why the topic of inquiry was on the relations between Information Theory, Entropy and the urban spatial structure. The problems stated were not trivial: Each pointed out important issues that could not be possible to deal with or find solutions with respect to social and economic conditions had a long history of development and particularly the last fifty years witnessed the paradigm changes that Chapter I.3 reviewed the First, Second & Third Generation of urban models.

The year 1948 regarded as the date of birth of Information Theory, due to the publication of two papers by C.E. Shannon. His main was to eliminate "noise" in the communication channels but the origination of the Information Theory influenced other fields of research in various sciences. The special issue of the **IEEE Transactions On Information Theory** (1998, Vol.44, No:6) celebrated the 50th anniversary of the Shannon's theory (Verdú, 1998).

Similarly, the year 1967 can be regarded as the "birth-date" of the new urban research methods based on the "Entropy Maximization" (MaxEnt) by A.G. Wilson. As a student of mathematics at Cambridge University (UK), A. Wilson had a special option in statistical mechanics and saw the mathematical "inconsistency" in the formulation of "Gravity Model" used by the urban and regional planners, as explained in Chapter I.3.4.3. & I.3.4.4. **Progress in Human**

Geography (1991, Vol.15, No:4) published "Commentary 1" by M. Batty & "Commentary 2" by A. Hay & A. Wilson's "Response", in its series of "Classics in Human Geography Revisited". MaxEnt methodology that was introduced by A.G. Wilson (1967) and contributions by other authors in the following years represent a rise of a new paradigm in the urban research field, i.e., a new way looking at the urban structure; that attracted many new researchers to explore the new themes and problems in the field. Hence, MaxEnt should not be regarded merely as a "different" branch of modelling. As conclusions, the following issues, points and findings can be stated with respect to the use of MaxEnt and information-theoric methods in the field of urban research & planning with grouping under the theoretical & empirical headings.

VII.1 Theoretical Conclusions

1- The First & Second Generations of urban models that were developed during 1960s & 1970s and their extensions or modifications in the 1980s had the problem of "integration" of these different approaches. MaxEnt & MIP (Minimum Information Principle) provided a new methodology to integrate or to link the partial and disaggregated models such as land use, transportation, residential, location, retail, population density and urban rent models.

2-) As Webber (1977) asserted, the known information is not sufficient to explain and predict the urban structure and predict the urban structure and the "gap" was to be filled by the assumptions of the NUE (New Urban Economics) or the related sciences, leading to the "inconsistent & biased" urban models.

3-) The alleviation of the Newtonian analogy in the "Gravity Model" was not a simple problem of calibration to fit the trip generation model to the actual data: It required a new **"Weltanschauung"** to identify and to solve the problem. The theoretical ground needed has been already provided by the statistical mechanics and by the "Jaynesian Revolution" (1957). Chapter I.3.4.3 & 3.3.4 reviews how the Gravity Model has been transformed to a "consistent" model by the pioneering contributions of A.G. Wilson.

4-) There were complaints on the statistical tests for the selection of the “Best-Fit” curves to represent the empirical distributions of population density in an urban area (Casetti, 1969). Yet, the same complaints persistently exist, presently, for example, in the selection of candidate statistical models for the probabilities of earthquake recurrences (Ellsworth, 1999). The standard tests of traditional statistics are not adequate to tackle with the “validity” issue in model building.

5-) Chapter II introduces the traditional “Objective” and Bayesian “Subjective” concepts of probability and points out that all probabilities are to be regarded as “Conditional” and Bayesian Rule shows how to change “Prior” to “Posterior” Probability is not a property of things or events to be “discovered”, but a state of our information due to our observations (Tribus, 1969).

6-) Chapter III reviews and compares the Shannon’s Entropy (H) and Kullback’s Information (I_K) Theories and give reasons why Kullback’s information measure is more suitable for the urban spatial structure, in addition to other mathematical and conceptual reasons.

7-) Chapter III defines the Jaynesian (1957) MaxEnt distribution as **“One which is noncommittal with regard to missing information”** based on the Shannonian definition of “Entropy” or “Uncertainty”. Tribus (1962, 1969) generalized the Jaynesian “MaxEnt” method to obtain the **“Least prejudicial distribution with respect to the given or known information”**.

8-) A. G. Wilson (1967, 1970a, 1974), by discovering inconsistency and deficiency in the traditional Gravity Model, developed **“Spatial Interaction Family of Models”**, as a remedy to the situation by using the MaxEnt method. J. Pooler (1994) generalized and extended the five-member “Family of Models” to the multiple numbers of types.

9-) The deterministic formulations and the Newtonian analogies of the ‘’Gravity Model ‘’ was given a ’’Probabilistic ‘’ foundation by Huff, Warntz and Isard , inter alia, during the 1955s and early 1960s. However, A.G.Wilson (1967,1970) changed the base of the Gravity Model to a firm theoretical foundation by using the statistical mechanics and information theory concepts. (Chapter III.6)

10-) In Kuhnian terms, A.G.Wilson‘s contribution can be considered as a ‘’Paradigm Shift ‘’. Authors like Tocalis (1978), B.Berry (1978) and Harvey & Holly (1981) evaluate Wilson’s and some other researchers’ contributions to human geography from the ‘’Paradigm Shift ‘’ viewpoint.

11-) The three formulations of “Entropy” concept, i.e., the classical (Classius), the statistical mechanics (Boltzmann & Gibbs) and informational (Shannon), are not distinct; the **“link”** between all these definitions exist, because all deal with the probability distributions. The long debates that took decades were “vacuous” & futile (Chapter III.7).

12-) The recent development in the Quantum Information Theory and the “Quantum Computer” that D. Deutsch (1985) envisaged provide new opportunities for the developments in urban modeling (Chapter III.8).

13-) As it is shown in a simple urban problem, The Most Probable, MaxEnt and The Minimum Information Principle (MIP) methods are mathematically equivalent, i.e., they give the same probability distributions. Yet, there exists a debate on the merits and demerits of these methods in urban modeling (Chapter IV).

14-) Chapter V introduces the “Spatial Entropy” concept by M. Batty (1974 a, b) shows how it can be used to test various hypothesis concerning the population, population density and zone area distributions. The classical “Goodness of Fit” tests are not applicable to the information-theoric measures.

15-) In comparison to Shannonian measure, Kullback’s measure is more suitable for the evaluation of urban planning & policy decisions. Theil’s decomposition method, as described in Adams & Storbeck (1983), provides a technique todecompose a system into its constituent ‘’between ‘ and ‘’within set ‘’ parts for comparisons.

16-) Sayer’s (1976,1979) rigorous ‘’Critique ‘’ on urban planning and modeling as summarized in Chapter III.6.3 seems to be still valid.

17-) Fotheringham (et.al.,2000) identify four stages of progress of spatial interaction models.Authors indicate that in Wilson’s (1967,1970) MaxEnt method, gravitational attraction has been replaced by another concept of entropy borrowed from statistical mechanics.

18-) Roy (1990), Fotheringham (et.al.,2000) and Roy & Thill (2004) point out the ‘’information processing ‘’ aspects of spatial interaction models and discuss the future research possibilities.

19-)Yet, the relationships between the current information processing methods above and Bayesian and MaxEnt approaches need to be clarified.

20-) The conventional way of determining the ‘’validity ‘’ of a model is to compare the model results with obsevations or empirical data . The recent framework adopted takes a broader view of validation and makes distinctions among such concepts as verification, reliability, correctness, and adequacy. There is no single test which serve for ‘’validation ‘’. Rather, there should be more tests to contribute to build our ‘’confidence ‘’ in a model. (Forrester & Senge, 1980). In this broader view, Gravity Model and Spatial Interaction Models, with their distinct theoretical foundations, cannot be compared each other on empirical grounds, i.e., according to their degree of correspondences between the theoretical and actual data.The use of statistical techniques of correlations neither deny nor prove the existence of causal relations. (Sayer,1979). Moreover, Gravity Model aims to ‘’describe’’the interaction patterns but not to ‘’ explain ‘’ them. (C. Lee 1980, pp.66-67). MaxEnt approach provides the probalistic foundation for developing interactions based on the entropy concept, by using the ‘’Most Probable State ‘’ to ‘’explain ‘’ the interaction patterns

VII.2 Empirical Findings

1-) During the two decades, between 1970 & 1990, gross densities of old settlement zones, such as Ulus (-47,35%), Samanpazarı-Eski Ankara (-35.48%), Altındağ (-7,73%) declined significantly, according to within-zone populations, but densities in the peripheral newly developing Keçiören (+186.81%), Sanatoryum (+126,23%), Karaağaç (+100,56%), Siteler-Ulubey (+44,84%) zones increased sharply in the NE section of the city. Similar tendencies were also observed between years 1961-1970 . (Chapter I.2 & Figures I.1, 2,3, &4) (Map-1,2,3 ; Chapter VI)

2-) Table VI.1 gives the overall results of the computations for S(70), S(90) and I(70), I(90) of Spatial Entropy measures, including the zone area (a_i) as the spatial dimension. All these measures deviate significantly from zero and therefore Ankara-1970 and Ankara-1990 population density distributions by zones are found as "Uneven" according to both Spatial Entropy measures.

3-)Overall Information Gain values I(70) = 0,3712 and I(90) = 0,3044 deviate significantly from zero. Therefore, the "Uniformity" hypothesis that "**Density of population is uniform in each zone of Ankara**" is not confirmed for years 1970 and 1990. (Table VI.1).

4-)Since the overall S(90) > S(70), Ankara-1990 seems to be "More Uneven" than Ankara-1970. however, the difference S(90) – S(70) = 9,5277–9,4412 = 0,0859 represents only 0,0859 / 9,4412 =0,009 or approximately 1.0 % increase of the S(70) amount. Hence, it can be concluded that the increase from S(70) to S(90) is "Negligible" to indicate a tendency of the distributions.

5-) Since I(70) > I(90), Ankara-1970 is "More Uneven" than Ankara-1990 as an overall tendency, and it can be concluded that Ankara has tended to become

"More Even" during years 1970 & 1990, in the sense of the Information Gain measure. This conclusion is regarded as more plausible than the above finding that S(90) > S(70), since the overall difference I(70) – I(90) = 0,3712-0,3044 = 0,0668 indicates 0,0668 / 0,3712 = 19 % decrease of the I(70) amount during the period that should be considered as significant. (Table VI.1).

6-)The overall Information Gain values I(70) and I(90) do not represent "MaxEnt" population density distributions. In a MaxEnt distribution, Information Gain should yield zero value where the Spatial Entropy amounts of S(70) and S(90) should be maximum and equal to the logarithm of the total urban area where ln18270 = 9,8130. Evidently, since S(70) = 9,4418 and S(90) = 9,5277, both spatial entropies are less than the maximum 9,8130 amount. Therefore, on the overall basis, S(70) and S(90) do not represent MaxEnt distributions. (Table VI.1, Equation (V-14)).

Findings on the Zone Level

7-) Relatively high Spatial Entropy values indicate relative "Uneven" population density distributions that are "not proportional" to the zone sizes. Akdere, Gülseren, Karşıyaka, Aktaş and Bahçelievler are the top five zones for their S(70) values and Keçiören, Karşıyaka, Etlik, Kayaş and Ziraat Fk.-Aydınlıkevler are the top five zones for their S(90) values. (Table 7.2 Appendix-A)

8-) Spatial Entropy differences S(90)-S(70) indicate the relative tendencies towards "More Even" or "More Uneven" population density distributions at the zone level. Hence, between years 1970-1990, Keçiören, Etlik, Karşıyaka, Sanatoryum, Kayaş, Balgat and others tended to be "More Uneven"; whereas such zones as Gülseren, Akdere, Küçükesat-Kavaklıdere, Ulus, Yenişehir, Aktaş, Samanpazarı-Eski Ankara tended to be "More Even" in their population density distributions (Table 7.2 Appendix-A).

9-) Results show that 20 zones out of total 34 have “Negligible Differences” in their S(90)-S(70) values within the range of (+ 0,0592) to (-0,0636), indicating a similarity in the spatial entropy distributions. (FigureVI.7; Map VI-6, Map VI-16).

10-) Information Gain measures compare the posterior and prior probabilities to determine the “distance” between them. Hence, high values imply relatively larger “distances” and deviations from each other and low values indicate a similarity between the two distributions. Negative Information Gains in I(70) or I(90) values imply that the posterior probability distributions do not improve the information encoded in prior distributions.

11-) With their I(70) > I(90) values, generally old settlement and inner zones, Gülseren-Gülveren, Küçükesat-Kavaklıdere, Samanpazarı-Eski Ankara, Ulus, Akdere, Yenişehir, Yenimahalle and Maltepe tended towards “Evenness” or “Uniformity”. (Figure VI.8, Map VI-9, MapVI-19).

12-) The newly developing zones on the northern periphery, i.e., Keçiören, Etlik, Karşıyaka, Aktepe, Sanatoryum and Kayaş on the eastern periphery had trends towards “Unevenness” with in comparison to their I(90) > I(70) values.

13-) It is assumed that a range of values between (+0,0160) and (-0,0080) fairly represent “Near Zero” values and Table VI.2 gives the lists of 15 zones in 1970 and 16 zones in 1990 where I(70) and I(90) have MaxEnt distributions. (Table VI.2; MapVI-21, Map VI-22).

14-) The above zones of I(70) and I(90) MaxEnt distributions confirm the hypothesis that “**Density of population is uniform in each zone**”.

15-) Out of 16 MaxEnt zones in I(90) list 11 zones also exist in the I(70) list.

Findings on Information Gain Measure I(p90:q70)

16-) As different from the Spatial Entropy measures S(70), S(90) and I(70) and I(90); Information Gain measure I(p90:q70) does not contain a spatial dimension but compares the posterior 1990 and prior 1970 population change distributions by zones.

17-)The overall Information Gain for the Ankara Metropolitan Area is computed as I(p90:q70) = 0,10927 that deviates significantly from zero. Therefore, the "**direct proportionality"** hypothesis between the posterior 1990 and prior 1970 zone population distributions is not confirmed.

18-) Keçiören, Etlik, Karşıyaka, Sanatoryum located on the northern and Kayaş on the eastern periphery are the top five zones for their I(p90:q70) values. These and other zones within the range of (0,0750 and 0,0101) indicate the "More Than The Expected" Information Gain distributions. Similarly, there are Information Gain zones of "Negligible Changes" around zero and "Less Than The Expected" are given in Map VI-10, Map VI-20 and Figure VI.6.

19-) Such zones as Gülseren-Gülveren, Akdere-Türközü, Ulus, Yenişehir and Altındağ have negative I(p90:q70) values where $p_{i,90} < q_{i,70}$.

20-) There are 11 zones out of total 34, where the posterior $p_{i,90}$ and prior $q_{i,70}$ probabilities are equal. In such cases, Information Gain I(p90:q70) yields zero results representing the "Minimum Information Gain" or equivalently "MaxEnt Distributions" for I(p90:q70). To differentiate the MaxEnt distributions for I(p90:q70) from the previous MaxEnt distributions for I(70) and I(90), these are designated as "Minimum Information" zones in Map VI-23, where $p_{i,90} = q_{i,70}$. (Table VI.4).

21-) It is found that variations in Information Gain I(p90:q70) values can be related to the change of population percentages by zones. High population growth rates correspond also to the zones with high Information Gain values. Population losing 14 zones also have negative I(p90:q70) values. (Table VI.3, Figure VI.5, Figure VI.6 and Map VI-10, Map VI-20).

22-) Chapter VI introduces the "time dimension" to the information-theoric analyses. The difference equations S(90)-S(70) and I(90)-I(70) provide a method for finding out tendencies for "Even" or "Uneven" population density distributions between years 1970 and 1990 in the Ankara case study.

23-) Methods used in the study of population density distributions have also been subject to changes from the descriptive curve-fitting to more explanatory methods.(Esmer,1977 & 1998a , Appendix-B & C).Batty's (1974) Spatial Entropy concept has been applied to both population and population density distributions to show how the spatial dimension can be incorporated into the information-theoric methods in case studies.These efforts can be associated with the current researches in ''spatial '' information processing during 1990s and onwards. The new research trends as Fotheringham (et.al.2000) and Roy &Thill (2004) explained justify the reason of application of Spatial Entropy concept to Ankara in ChapterVI. The term ''Ankara Revisited '' implied the paradigm shift in the study of population density distributions .Definitely, Spatial Entropy and other information-theoric approaches can be applied to all problems dealing with probability distributions; not just to population distributions.

VII.3 General Conclusions

Without MaxEnt & MIP methods, we would be still using the "inconsistent & biased" urban structure & simulation models, with no "integration" between the micro & macro theories upon with these models were based, or with no link between the partial and the comprehensive models. Surely, the Bayesian, Shannonian, Jaynesian & Kullbackian "Information Theories" have influenced deeply urban research & planning methods; and the "Quantum Information Theory" is twinkling at the dawn of the New Millennium…,

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APPENDIX - A

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
1	**Karşıyaka**	Karşıyaka	15671	55096	212,2	73,8501	259,641847	39425	251,5793504
		Kurtini	2117		423,3	5,00118	0	-2117	-100
		Pamuklar		8896				8896	
		Demetevler	12287	25696	306,7	40,0619	83,7821976	13409	109,1316025
		Güventepe		6719				6719	
		ZONE TOTAL	**54078**	**96407**	**682,55**	79,2294	141,24533	42329	78,27397463
2	**Etlik**	Aşapı Eğlence	5848	29509	74	79,027	398,77027	23661	404,5998632
		Ayvalı	14882	18899	366	40,6612	51,636612	4017	26,99233974
		Emrah	8888	8824	202	44	43,6831683	-64	-0,720072007
		İncirli	15064	22941	268	56,209	85,6007463	7877	52,29022836
		Esertepe		14493				14493	
		ZONE TOTAL	**35195**	**94666**	**766,87**	45,8943	123,444652	59471	168,9757068
3	**Sanatoryum**	Pınarbaşı	21769	24808	482	45,1639	51,4688797	3039	13,96021866
		Kuşcağız	8188	13882	170	48,1647	81,6588235	5694	69,5407914
		Ufuktepe		9812				9812	
		Bademlik		8695				8695	
		ZONE TOTAL	**25283**	**57197**	**453,75**	55,7201	126,053994	31914	126,2271091
4	**Keçiören**	Çiçekli	16263	9473	201	80,9104	47,1293532	-6790	-41,75121441
		Kalaba	3987	13998	51	78,1765	274,470588	10011	251,0910459
		Şenlik	3848	32586	69	55,7681	472,26087	28738	746,8295218
		Şevkat	5526	10981	82	67,3902	133,914634	5455	98,71516468
		Şentepe	5809		143	40,6224	0	-5809	-100
		Yakacık	6851	15051	114	60,0965	132,026316	8200	119,6905561
		Tepebaşı		27432				27432	
		Kamilocak		11753				11753	
		ZONE TOTAL	**42284**	**121274**	**595**	71,0655	203,821849	78990	186,808249

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990) (continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*) 1970	POPULATION (*) 1990	AREA (hec.) (**)	POP. DENSITY (person/hec) 1970	POP. DENSITY (person/hec) 1990	POP. Increase 1970-1990	POP. & Density Increase (%) 1970-1990
5	Aktepe	Bağlarbaşı	18471	14132	624	29,601	22,6474359	-4339	-23,49087759
		Yeşilöz	3107	2461	143	21,7273	17,2097902	-646	-20,79176054
		Aktepe	7633	5285	27,9	273,584	189,426523	-2348	-30,76116861
		Köşk		3509				3509	
		ZONE TOTAL	**24121**	**25387**	**247**	97,6559	102,781377	1266	5,248538618
6	Hasköy	Hasköy	21544	1752	561	38,4029	3,12299465	-19792	-91,86780542
		Yeşilöz	3107	2461	286	10,8636	8,6048951	-646	-20,79176054
		Güneşevler	3255	9429	70,1	46,4337	134,507846	6174	189,6774194
		Yıldıztepe	3608	11290	57,6	62,6389	196,006944	7682	212,9157428
		Karakum		5797				5797	
		Gülpınar		8208				8208	
		Doğu		4676				4676	
		ZONE TOTAL	**31345**	**43613**	**336,5**	93,1501	129,607727	12268	39,1386186
7	Siteler- Uluğbey	Uluğbey	30217	11478	520	58,1096	22,0730769	-18739	-62,0147599
		Önder		12738				12738	
		Hacılar		12824				12824	
		Battalgazi		9639				9639	
		Bostancık	4310	4789	414,1	10,4081	11,5648394	479	11,1136891
		ZONE TOTAL	**35535**	**51468**	**365**	97,3562	141,008219	15933	44,83748417
8	Ziraat Fkl. - Aydınlıkevler	Aydınlıkevler	30053	15961	125	240,424	127,688	-14092	-46,89049346
		Gümüşdere	2820	1484	100	28,2	14,84	-1336	-47,37588652
		Kavacık Subayevleri	2820	10633	43	65,5814	247,27907	7813	277,0567376
		Ziraat	8264	5559	133	62,1353	41,7969925	-2705	-32,73233301
		Ahiler	17739	9695	36,3	488,678	267,07989	-8044	-45,34641186
		Örnek		18621				18621	
		ZONE TOTAL	**43957**	**61953**	**480,5**	91,4818	128,934443	17996	40,94000955

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990) (continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
9	Akköprü - Varlık Mah.	Akköprü	3167	260	133	23,812	1,95488722	-2907	-91,79033786
		Altınbaş	4465	403	19	235	21,2105263	-4062	-90,97424412
		Bozkurt	3217	990	17	189,235	58,2352941	-2227	-69,22598694
		Evliya Çelebi	4085	3133	16	255,313	195,8125	-952	-23,30477356
		Fevzi Paşa	3067	652	23	133,348	28,3478261	-2415	-78,74144115
		Yeni Turan	3242	2053	40	81,05	51,325	-1189	-36,67489204
		Zübeyde Hanım	7574	9844	112	67,625	87,8928571	2270	29,97095326
		Varlık	3387	3503	319	10,6176	10,9811912	116	3,424859758
		ZONE TOTAL	**25920**	**20838**	**304,4**	85,1511	68,455979	-5082	-19,60648148
10	Yenimahalle - Demetevler	Çarşı	3222	3618	15	214,8	241,2	396	12,29050279
		Çerçi Deresi	15084	7872	418	36,0861	18,8325359	-7212	-47,81225139
		Esentepe	3750	4712	21	178,571	224,380952	962	25,65333333
		Işınlar	9221	6708	61	151,164	109,967213	-2513	-27,25300943
		Orman Çiftliği	4382	4002	633	6,92259	6,32227488	-380	-8,671839343
		Ragıp Tüzün	6679	5066	16	417,438	316,625	-1613	-24,1503219
		Tepealtı	5235	3602	21	249,286	171,52381	-1633	-31,1938873
		Yeniçağ	4995	3018	22,5	222	134,133333	-1977	-39,57957958
		ZONE TOTAL	**43951**	**38598**	**374,5**	117,359	103,065421	-5353	-12,17947259
11	A.O.Ç - Gazi Mah.	Gazi	10418	6483	407	25,5971	15,9287469	-3935	-37,77116529
		Mebusevleri	1901	1445	46,75	40,6631	30,9090909	-456	-23,98737507
		Orman Çiftliği	4382	4002	633	6,92259	6,32227488	-380	-8,671839343
		ZONE TOTAL	**15578**	**11930**	**1050,25**	14,8327	11,3592002	-3648	-23,41764026

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)(continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*) 1970	POPULATION (*) 1990	AREA (hec.) (**)	POP. DENSITY (person/hec) 1970	POP. DENSITY (person/hec) 1990	POP. Increase 1970-1990	POP. & Density Increase (%) 1970-1990
12	Altındağ	Atıf Bey	1382	1176	4	345,5	294	-206	-14,90593343
		Çandarlı	2694	1530	5	538,8	306	-1164	-43,20712695
		Doğanşehir	4599	2658	6	766,5	443	-1941	-42,20482714
		Engürü	2877	1838	4	719,25	459,5	-1039	-36,11400765
		Enver Paşa	1293	1078	2	646,5	539	-215	-16,62799691
		Fatih	3261	2952	5	652,2	590,4	-309	-9,475620975
		Fazıl Ahmet Paşa	315	582	2	157,5	291	267	84,76190476
		Fehmi Yağcı	1287	2745	2,5	514,8	1098	1458	113,2867133
		Fermanlılar	971	1175	3,9	248,974	301,282051	204	21,0092688
		Gökçenefe	2832	1938	6,5	435,692	298,153846	-894	-31,56779661
		Hayri Artmanlar	2908	2486	3	969,333	828,666667	-422	-14,51169188
		Kartallar	2706	3388	6,5	416,308	521,230769	682	25,20325203
		Orhan Gazi	3779	2823	7,5	503,867	376,4	-956	-25,2976978
		Öncüler	2586	1609	1,9	1361,05	846,842105	-977	-37,78035576
		Sinan Paşa	1729	1418	4,5	384,222	315,111111	-311	-17,98727588
		Ulubatlı Hasan	1211	1251	4,5	269,111	278	40	3,303055326
		Yamaç	845	1069	5,5	153,636	194,363636	224	26,50887574
		Yavuz Selim	1426	937	4,5	316,889	208,222222	-489	-34,29172511
		Yıldırım Beyazıt	1123	1112	2,5	449,2	444,8	-11	-0,979519145
		Yılmazlar	1710	1263	5,5	310,909	229,636364	-447	-26,14035088
		Yiğitler	3419	6452	8,5	402,235	759,058824	3033	88,71014917
		ZONE TOTAL	**44953**	**41480**	**111,15**	404,435	373,189384	-3473	-7,725846996

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)(continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
13	Aktaş - Asrimezarlık	Aktaş	3085	2243	7	440,714	320,428571	-842	-27,29335494
		Attila	3943	3347	17	231,941	196,882353	-596	-15,11539437
		I. Murat	6895	9087	20	344,75	454,35	2192	31,79115301
		Cemal Bey	5103	4311	9	567	479	-792	-15,52028219
		Çalışkanlar	15752	8262	20	787,6	413,1	-7490	-47,54951752
		Gültepe	7923	5223	114	69,5	45,8157895	-2700	-34,07800076
		Hürriyet	3324	2431	6	554	405,166667	-893	-26,86522262
		Kemal Zeytinoğlu	1924	1913	98	19,6327	19,5204082	-11	-0,571725572
		Özgürlük	2763	1876	3	921	625,333333	-887	-32,10278683
		Plevne	5129	2825	14	366,357	201,785714	-2304	-44,92103724
		Servet Somuncuoğlu	1785	6062	28	63,75	216,5	4277	239,6078431
		Sokullu	2631	1635	7	375,857	233,571429	-996	-37,85632839
		ZONE TOTAL	**60257**	**49215**	**255,65**	235,701	192,50929	-11042	-18,32484193
14	Samanpazarı Eski Ankara	Akalar	4361	1329	3,5	1246	379,714286	-3032	-69,52533823
		Akbaş	1836	1031	2,5	734,4	412,4	-805	-43,8453159
		Alpaslan	1481	927	3,8	389,737	243,947368	-554	-37,40715733
		Başkır	2892	1329	6	482	221,5	-1563	-54,04564315
		Çeşme	2746	711	3,6	762,778	197,5	-2035	-74,10779315
		Çimentepe	1156	127	1,7	680	74,7058824	-1029	-89,01384083
		Demir Fırka	2316	1089	3,2	723,75	340,3125	-1227	-52,97927461
		İçkale	3622	1811	9	402,444	201,222222	-1811	-50
		Kaledibi	2633		5,4	487,593	0	-2633	-100
		Kılıçarslan	750	381	4,3	174,419	88,6046512	-369	-49,2
		Nazım Bey	872	827	2,5	348,8	330,8	-45	-5,160550459
		Pazar	872	807	3,5	249,143	230,571429	-65	-7,45412844
		Oğuz	451	273	3,1	145,484	88,0645161	-178	-39,46784922

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)(continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
14	Samanpazarı Eski Ankara	Özbekler	1557	469	4,3	362,093	109,069767	-1088	-69,87797046
		Sakarya	2337	2741	6,6	354,091	415,30303	404	17,28712024
		Sığınaklar	213	1732	5,3	40,1887	326,792453	1519	713,1455399
		Şükriye	2833	4119	20	141,65	205,95	1286	45,39357571
		Turan	1452	1469	6	242	244,833333	17	1,170798898
		Yalçınkaya	1757	2119	5,5	319,455	385,272727	362	20,60330108
		Yenihayat	1920	779	2	960	389,5	-1141	-59,42708333
		ZONE TOTAL	**37307**	**26060**	**112,5**	331,618	231,644444	-11247	-30,14715737
15	Cebeci	Cebeci	7536	7906	27,5	274,036	287,490909	370	4,909766454
		Çamlıtepe	7993	8781	24,5	326,245	358,408163	788	9,858626298
		Demirlibahçe	12286	11159	51,2	239,961	217,949219	-1127	-9,173042487
		Erzurum	3413	3726	14,2	240,352	262,394366	313	9,170817463
		Ertuğrul Gazi	5976	10191	21	284,571	485,285714	4215	70,53212851
		Fakülteler	8603	8061	38	226,395	212,131579	-542	-6,300127862
		Sümer	1302	1905	2,5	520,8	762	603	46,31336406
		Şafaktepe	6574	8112	54,5	120,624	148,844037	1538	23,39519319
		Topraklık	2668	2616	14,1	189,22	185,531915	-52	-1,949025487
		ZONE TOTAL	**55049**	**62457**	**288,65**	190,712	216,376234	7408	13,45710185
16	Gülseren Gülveren	Bahçeleriçi	7149	4361	31	230,613	140,677419	-2788	-38,99846132
		Bahçelerüstü	10028	6079	51	196,627	119,196078	-3949	-39,37973674
		Gülseren	19786	11673	129	153,38	90,4883721	-8113	-41,00374002
		Gülveren	11521	8359	39,5	291,671	211,620253	-3162	-27,44553424
		Harman	4961	6019	87	57,023	69,183908	1058	21,32634549
		Hüseyin Gazi	21188	4485	229	92,524	19,5851528	-16703	-78,83235794
		Yatıkmusluk	7418	6879	51,9	142,929	132,543353	-539	-7,266109463
		ZONE TOTAL	**74636**	**47855**	**570**	130,94	83,9561404	-26781	-35,88214803

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)(continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
17	Karaağaç	Karaağaç	6123	3576	712,5	8,59368	5,01894737	-2547	-41,59725625
		ZONE TOTAL	**1783**	**3576**	**423,45**	4,21065	8,44491676	1793	100,5608525
18	Kayaş	Araplar	3313	2971	145	22,8483	20,4896552	-342	-10,32297012
		Derbent	18214	11950	406	44,8621	29,4334975	-6264	-34,3911277
		Kayaş	5894	3663	331,5	17,7798	11,0497738	-2231	-37,85205294
		Köstence	8240	5955	125	65,92	47,64	-2285	-27,73058252
		Küçük Kayaş	4017	7559	362,5	11,0814	20,8524138	3542	88,17525517
		Üreğil	510	2321	175	2,91429	13,2628571	1811	355,0980392
		Boğaziçi	12344	12079	324,7	38,0166	37,2004928	-265	-2,146791964
		Yeşil Bayır	8116	11262	155	52,3613	72,6580645	3146	38,76293741
		Tepecik		6710				6710	
		ZONE TOTAL	**40188**	**64470**	**340,65**	117,974	189,255834	24282	60,4210212
19	Mamak	Abidinpaşa	8890	17636	96	92,6042	183,708333	8746	98,38020247
		Balkiraz	5952	12505	47,5	125,305	263,263158	6553	110,0974462
		Kartaltepe	9553	5911	56,5	169,08	104,619469	-3642	-38,12414948
		Saime Kadın	6967	10152	28	248,821	362,571429	3185	45,715516
		Tuzluçayır	15523	8459	188	82,5691	44,9946809	-7064	-45,50666753
		ZONE TOTAL	**46885**	**54663**	**385,5**	121,621	141,797665	7778	16,58952757
20	Akdere İmrahor Türközü	Akdere	9442	6874	37,9	249,129	181,372032	-2568	-27,19762762
		Şehit Cengiz Topel	7242	6792	26	278,538	261,230769	-450	-6,213753107
		Arka Topraklık	3261	3374	12,5	260,88	269,92	113	3,465194726
		Aşağı İmrahor	352	595	62	5,67742	9,59677419	243	69,03409091
		Bağcılar	15740	2576	431	36,5197	5,97679814	-13164	-83,63405337
		Dilekler	3149	3046	12,5	251,92	243,68	-103	-3,270879644
		Orta İmrahor		733	69,5	0	10,5467626	733	

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)(continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
20	Akdere İmrahor Türközü	Samanlık Bağları	17177		236	72,7839	0	-17177	-100
		Türközü	5237	10154	127	41,2362	79,9527559	4917	93,88963147
		Zafertepe	9799	11777	68	144,103	173,191176	1978	20,18573324
		ZONE TOTAL	**71017**	**47911**	**621,78**	114,216	77,0545852	-23106	-32,53587169
21	İncesu Seyranbağları	İncesu	12591	4992	23,5	535,787	212,425532	-7599	-60,35263283
		Kültür	8087	5739	16	505,438	358,6875	-2348	-29,0342525
		Öncebeci	6800	7291	17,5	388,571	416,628571	491	7,220588235
		Seyranbağları	14252	10683	49,5	287,919	215,818182	-3569	-25,04209935
		İleri		7224				7224	
		Tınaztepe	7841	8637	16,1	487,019	536,459627	796	10,15176636
		ZONE TOTAL	**41732**	**44566**	**128,25**	325,396	347,493177	2834	6,790951788
22	Küçükesat Kavaklıdere	Barbaros	10626	9150	30,5	348,393	300	-1476	-13,89045737
		Kavaklıdere	9204	8658	39,8	231,256	217,537688	-546	-5,93220339
		Küçükesat	20907	5208	40,7	513,686	127,960688	-15699	-75,08968288
		Remzi Oğuz Arık	3526	4368	29	121,586	150,62069	842	23,87975043
		ZONE TOTAL	**44265**	**27384**	**172,35**	256,832	158,885988	-16881	-38,13622501
23	Ayrancı	Ayrancı	10016	24534	59	169,763	415,830508	14518	144,9480831
		Güven	8574	13410	39,5	217,063	339,493671	4836	56,40307908
		Güzeltepe	20602	8118	450	45,7822	18,04	-12484	-60,59605864
		Kavaklıdere	1534	1443	6,7	228,955	215,373134	-91	-5,93220339
		Remzi Oğuz Arık	3527	4368	29	121,621	150,62069	841	23,84462716
		ZONE TOTAL	**36934**	**51873**	**291,82**	126,564	177,756836	14939	40,44782585
24	Çankaya Yıldız	Büyükesat	6709	11070	172,2	38,9605	64,2857143	4361	65,0022358
		Çankaya	8855	8811	162	54,6605	54,3888889	-44	-0,49689441
		Gazi Osman Paşa	8386	5936	35	239,6	169,6	-2450	-29,21535893
		Yıldızevler		12072				12072	
		ZONE TOTAL	**32962**	**37889**	**573,1**	57,5153	66,1123713	4927	14,94751532

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)(continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
25	Dikmen Öveçler	Dikmen	12321	15954	541	22,7745	29,4898336	3633	29,486243
		Öveçler	23763	23694	740	32,1122	32,0189189	-69	-0,290367378
		Keklik Pınarı	768	9518	126,2	6,08558	75,4199683	8750	1139,322917
		Ata	16291	6055	359,8	45,2779	16,8287938	-10236	-62,83223866
		ZONE TOTAL	**41266**	**55221**	**644,3**	64,0478	85,7069688	13955	33,81718606
26	Devlet	Devlet	5601	3217	304	18,4243	10,5822368	-2384	-42,56382789
		ZONE TOTAL	**5601**	**3217**	**414,92**	13,499	7,75330184	-2384	-42,56382789
27	Balgat Çukurambar	Balgat	8533	17954	273,5	31,1993	65,6453382	9421	110,4066565
		Kızılırmak	9307	5621	197,5	47,1241	28,4607595	-3686	-39,60459869
		Cevizlidere		14331				14331	
		ZONE TOTAL	**20221**	**37906**	**431,25**	46,8893	87,897971	17685	87,45858266
28	Bahçelievler Emek	Bahçelievler	20666	17711	56,5	365,77	313,469027	-2955	-14,29884835
		Emek	19857	28102	133	149,301	211,293233	8245	41,52188145
		Yukarı Bahçelievler	14637	21057	91,5	159,967	230,131148	6420	43,86144702
		ZONE TOTAL	**55160**	**66870**	**278,1**	198,346	240,453074	11710	21,22915156
29	Maltepe Anıttepe	Anıttepe	6932	7390	22,5	308,089	328,444444	458	6,607039815
		Eti	3612	2206	46	78,5217	47,9565217	-1406	-38,92580288
		Maltepe	14151	11536	42,5	332,965	271,435294	-2615	-18,47925942
		Mebusevleri	7606	5780	187	40,6738	30,9090909	-1826	-24,00736261
		Yücetepe	6756	7433	43,5	155,31	170,873563	677	10,02072232
		ZONE TOTAL	**37157**	**34345**	**264,15**	140,666	130,020822	-2812	-7,567887612
30	Söğütözü	Beştepeler	2809	10502	59,1	47,5296	177,698816	7693	273,8697045
		ZONE TOTAL	**2809**	**10502**	**236,05**	11,9	44,490574	7693	273,8697045
31	Yenişehir	Cumhuriyet	3958	532	9	439,778	59,1111111	-3426	-86,55886812
		Fidanlık	3894	2847	8,9	437,528	319,88764	-1047	-26,88751926
		Kızılay	5225	2685	23	227,174	116,73913	-2540	-48,61244019
		Kocatepe	2780	1839	8,5	327,059	216,352941	-941	-33,84892086

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)(continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
31	Yenişehir	Korkut Reis	6228	2299	16	389,25	143,6875	-3929	-63,08606294
		Meşrutiyet	3190	2025	15	212,667	135	-1165	-36,52037618
		Namık Kemal	2287	2105	14	163,357	150,357143	-182	-7,958023612
		Sağlık	4638	1717	25	185,52	68,68	-2921	-62,97973264
		ZONE TOTAL	**32200**	**18039**	**162,45**	198,215	111,043398	-14161	-43,97826087
32	Ulus	Altınbaş	3259	326	5	651,8	65,2	-2933	-89,99693157
		Anafartalar	1377	246	2,9	474,828	84,8275862	-1131	-82,13507625
		Bentderesi	3280	1084	6	546,667	180,666667	-2196	-66,95121951
		Bozkurt	3217	990	17,5	183,829	56,5714286	-2227	-69,22598694
		Doğanbey	2892	1291	38,5	75,1169	33,5324675	-1601	-55,35961272
		Fevzi Paşa	3067	652	23	133,348	28,3478261	-2415	-78,74144115
		İnkılap	497	714	4	124,25	178,5	217	43,66197183
		İstiklal	2249	894	3,3	681,515	270,909091	-1355	-60,24899956
		Kızılelma	801	302	2,1	381,429	143,809524	-499	-62,29712859
		Koyunpazarı	459	214	4,1	111,951	52,195122	-245	-53,37690632
		Köprübaşı	2049	3005	5	409,8	601	956	46,65690581
		Misakı Milli	1519	254	5,9	257,458	43,0508475	-1265	-83,27847268
		Necati Bey	1335	656	5,2	256,731	126,153846	-679	-50,86142322
		Özgen	1010	143	4,1	246,341	34,8780488	-867	-85,84158416
		Öztürk	1202	903	5,8	207,241	155,689655	-299	-24,87520799
		Sakalar	918	829	3,5	262,286	236,857143	-89	-9,694989107
		Sutepe	342	35	3,3	103,636	10,6060606	-307	-89,76608187
		Şenyurt	872	318	3	290,667	106	-554	-63,53211009
		Tabakhane	1047	249	2,5	418,8	99,6	-798	-76,21776504
		Turgut Reis	2996	3752	9,9	302,626	378,989899	756	25,23364486
		Yeğen Bey	505	306	3	168,333	102	-199	-39,40594059

TABLE 1.0-) ZONE & MAHALLE POPULATION & AREA DATA (1970 - 1990)(continued)

Zone Number 1970	Zone Name	Name of the Mahalle	POPULATION (*)		AREA (hec.) (**)	POP. DENSITY (person/hec)		POP. Increase	POP. & Density Increase (%)
			1970	1990		1970	1990	1970-1990	1970-1990
32	Ulus	Yenice	1536	2018	8	192	252,25	482	31,38020833
		ZONE TOTAL	**36429**	**21171**	**162,67**	223,944	130,146923	-15258	-41,88421313
33	Kültür Aksı Gençlik Parkı	Akalar	4361	1329	3,6	1211,39	369,166667	-3032	-69,52533823
		Alpaslan	1481	927	3,8	389,737	243,947368	-554	-37,40715733
		Demirtaş	2767	1233	1,8	1537,22	685	-1534	-55,43910372
		Gündoğdu	2404	1946	1,6	1502,5	1216,25	-458	-19,0515807
		Meydan	851	529	1,9	447,895	278,421053	-322	-37,83783784
		Sümer	1302	1905	3,6	361,667	529,166667	603	46,31336406
		Ülkü	2854	2664	62	46,0323	42,9677419	-190	-6,657323055
		ZONE TOTAL	**14927**	**10533**	**221,85**	67,2842	47,4780257	-4394	-29,43659141
34	A.O.Ç Fabrikalar	Macun	1139	3988	432	2,63657	9,23148148	2849	250,1316945
		Orman Çiftliği	2191	2001	316,5	6,92259	6,32227488	-190	-8,671839343
		ZONE TOTAL	**3330**	**5989**	**748,5**	4,4489	8,00133601	2659	79,84984985

NOTES:

Underlined figures: from 2002/2003 Studio Projects at the Department / METU. (Not included in 1990 zone totals.)

Blank cells represents the incomplete data.

(*) 1970 Populations are taken from AMPB (1977)

1990 "Mahalle" populations are from SIS (DIE). Zone totals are taken as the sum of "Mahalle" populations.

(**) Area zone totals are built-up areas (AMPB, 1977). "Mahalle" areas do not add up to zone totals.

TABLE 2.1) ZONE POPULATION & AREA DATA (1970 - 1990)

ZONE NUMBER	ZONE NAME	POPULATION		GROSS AREA (hec.)	POP. DENSITY (person/hec)		POP. INCREASE	POP. & DENSITY INCREASE (%)
		1970	1990		1970	1990	1970-1990	1970-1990
1	Karşıyaka	54078	96407	983,25	55,00	98,05	42329	78,27%
2	Etlik	36195	94666	855,00	42,33	110,72	58471	161,54%
3	Sanatoryum	25283	57197	565,50	44,71	101,14	31914	126,23%
4	Keçiören	42284	121274	687,50	61,50	176,40	78990	186,81%
5	Aktepe	24121	25387	464,50	51,93	54,65	1266	5,25%
6	Hasköy	31345	43613	556,50	56,33	78,37	12268	39,14%
7	Siteler - Ulubey	35535	51468	543,75	65,35	94,65	15933	44,84%
8	Ziraat Fk. - Aydınlıkevler	43957	61953	575,50	76,38	107,65	17996	40,94%
9	Akköprü - Varlık Mah.	25920	20838	527,40	49,15	39,51	-5082	-19,61%
10	Yenimahalle - Demetevler	43951	38598	578,00	76,04	66,78	-5353	-12,18%
11	A.O.Ç. - Gazi Mah.	15578	11930	1050,25	14,83	11,36	-3648	-23,42%
12	Altındağ	44953	41480	111,15	404,44	373,19	-3473	-7,73%
13	Aktaş - Asrimezarlık	60257	49215	264,40	227,90	186,14	-11042	-18,32%
14	Samanpazarı - Eski Ankara	37307	24070	112,50	331,62	213,96	-13237	-35,48%
15	Cebeci	55049	62457	286,65	192,04	217,89	7408	13,46%
16	Gülseren - Gülveren	74636	47855	593,10	125,84	80,69	-26781	-35,88%
17	Karaağaç	1783	3576	423,45	4,21	8,44	1793	100,56%
18	Kayaş	40188	64470	1250,00	32,15	51,58	24282	60,42%
19	Mamak	46885	54663	466,75	100,45	117,11	7778	16,59%
20	Akdere - İmrahor - Türközü	71017	45921	1334,25	53,23	34,42	-25096	-35,34%
21	İncesu - Seyranbağları	41732	44566	128,25	325,40	347,49	2834	6,79%
22	Küçük Esat - Kavaklıdere	44265	27384	172,35	256,83	158,89	-16881	-38,14%
23	Ayrancı	36934	51873	291,82	126,56	177,76	14939	40,45%
24	Çankaya - Yıldız	32962	37889	850,00	38,78	44,58	4927	14,95%
25	Dikmen - Öveçler	41266	55221	983,75	41,95	56,13	13955	33,82%
26	Devlet	5601	3217	421,42	13,29	7,63	-2384	-42,56%
27	Balgat - Çukurambar	20221	37906	792,50	25,52	47,83	17685	87,46%
28	Bahçelievler - Emek	55160	66870	301,40	183,01	221,86	11710	21,23%
29	Maltepe - Anıttepe	37157	34345	264,15	140,67	130,02	-2812	-7,57%
30	Söğütözü	2809	10502	539,35	5,21	19,47	7693	273,87%
31	Yenişehir	32200	16049	162,45	198,21	98,79	-16151	-50,16%
32	Ulus	36429	19181	162,67	223,94	117,91	-17248	-47,35%
33	Kültür Aksı - Gençlik Parkı	14927	10533	221,85	67,28	47,48	-4394	-29,44%
34	A.O.Ç. Fabrikalar	3330	5989	748,50	4,45	8,00	2659	79,85%
ZONES TOTAL		**1215315**	**1438563**	**18269,86**	**66,5202**	**78,7397**	**223248**	**18,37%**
ANKARA TOTAL		**1467304**	**2584594**					

TABLE 2.2-) DENSITIES BY BUILT-UP ZONE AREAS (1970)

Zones		Area (built-up) (hec.)	1970 Population	Density	LN of Densities
12	Altındağ	111,15	44953	404,44	6,002
14	Samanpazarı - Eski Ankara	112,5	37307	331,62	5,804
21	İncesu - Seyranbağları	128,25	41732	325,40	5,785
22	Küçük Esat - Kavaklıdere	172,35	44265	256,83	5,548
13	Aktaş - Asrimezarlık	255,65	60257	235,70	5,463
32	Ulus	162,67	36429	223,94	5,411
28	Bahçelievler - Emek	278,1	55160	198,35	5,290
31	Yenişehir	162,45	32200	198,21	5,289
15	Cebeci	288,65	55049	190,71	5,251
29	Maltepe - Anıttepe	264,15	37157	140,67	4,946
16	Gülseren - Gülveren	570	74636	130,94	4,875
23	Ayrancı	291,82	36934	126,56	4,841
19	Mamak	385,5	46885	121,62	4,801
18	Kayaş	340,65	40188	117,97	4,770
10	Yenimahalle - Demetevler	374,5	43951	117,36	4,765
20	Akdere - İmrahor - Türközü	621,78	71017	114,22	4,738
5	Aktepe	247	24121	97,66	4,581
7	Siteler - Ulubey	365	35535	97,36	4,578
6	Hasköy	336,5	31345	93,15	4,534
8	Ziraat Fk. - Aydınlıkevler	480,5	43957	91,48	4,516
9	Akköprü - Varlık Mah.	304,4	25920	85,15	4,444
1	Karşıyaka	682,55	54078	79,23	4,372
4	Keçiören	595	42284	71,07	4,264
33	Kültür Aksı - Gençlik Parkı	221,85	14927	67,28	4,209
25	Dikmen - Öveçler	644,3	41266	64,05	4,160
24	Çankaya - Yıldız	573,1	32962	57,52	4,052
3	Sanatoryum	453,75	25283	55,72	4,020
2	Etlik	766,87	36195	47,20	3,854
27	Balgat - Çukurambar	431,25	20221	46,89	3,848
11	A.O.Ç. - Gazi Mah.	1050,25	15578	14,83	2,697
26	Devlet	414,92	5601	13,50	2,603
30	Söğütözü	236,05	2809	11,90	2,477
34	A.O.Ç. Fabrikalar	748,5	3330	4,45	1,493
17	Karaağaç	423,45	1783	4,21	1,438
TOTAL AREA		13495,41	1215315		
LN of Total Area		9,510			

Sorted Density Data From Table 1.0

TABLE 2.3-) DENSITIES BY BUILT-UP ZONE AREAS (1990)

Zones		Area (built-up) (hec)	1990 Population	Density	LN of Densities
1	Karşıyaka	682,55	96407	141,25	4,9505
2	Etlik	766,87	94666	123,44	4,8158
12	Altındağ	111,15	41480	373,19	5,9221
21	İncesu - Seyranbağları	128,25	44566	347,49	5,8507
28	Bahçelievler - Emek	278,10	66870	240,45	5,4825
15	Cebeci	288,65	62457	216,38	5,3770
14	Samanpazarı - Eski Ankara	112,50	24070	213,96	5,3658
4	Keçiören	595,00	121274	203,82	5,3172
13	Aktaş - Asrimezarlık	255,65	49215	192,51	5,2601
18	Kayaş	340,65	64470	189,26	5,2431
23	Ayrancı	291,82	51873	177,76	5,1804
22	Küçük Esat - Kavaklıdere	172,35	27384	158,89	5,0682
19	Mamak	385,50	54663	141,80	4,9544
7	Siteler - Ulubey	365,00	51468	141,01	4,9488
29	Maltepe - Anıttepe	264,15	34345	130,02	4,8677
6	Hasköy	336,50	43613	129,61	4,8645
8	Ziraat Fk. - Aydınlıkevler	480,50	61953	128,93	4,8593
3	Sanatoryum	453,75	57197	126,05	4,8367
32	Ulus	162,67	19181	117,91	4,7700
10	Yenimahalle - Demetevler	374,50	38598	103,07	4,6354
5	Aktepe	247,00	25387	102,78	4,6326
31	Yenişehir	162,45	16049	98,79	4,5930
27	Balgat - Çukurambar	431,25	37906	87,90	4,4762
25	Dikmen - Öveçler	644,30	55221	85,71	4,4509
16	Gülseren - Gülveren	570,00	47855	83,96	4,4303
20	Akdere - İmrahor - Türközü	621,78	45921	73,85	4,3021
9	Akköprü - Varlık Mah.	304,40	20838	68,46	4,2262
24	Çankaya - Yıldız	573,10	37889	66,11	4,1914
33	Kültür Aksı - Gençlik Parkı	221,85	10533	47,48	3,8603
30	Söğütözü	236,05	10502	44,49	3,7953
11	A.O.Ç. - Gazi Mah.	1050,25	11930	11,36	2,4300
17	Karaağaç	423,45	3576	8,44	2,1336
34	A.O.Ç. Fabrikalar	748,50	5989	8,00	2,0796
26	Devlet	414,92	3217	7,75	2,0481
TOTAL AREA		13495,41			
LN of Total Area		9,510			

Sorted Density Data From Table 1.0

TABLE 3.1-) GROSS DENSITY CHANGES BY ZONES (1970-1990)

Zones		Zone Gross Areas (hec.)	Population		Gross Density	
			1970	1990	1970	1990
1	Karşıyaka	983,25	54078	96407	54,999	98,049
2	Etlik	855,00	36195	94666	42,333	110,720
3	Sanatoryum	565,50	25283	57197	44,709	101,144
4	Keçiören	687,50	42284	121274	61,504	176,399
5	Aktepe	464,50	24121	25387	51,929	54,654
6	Hasköy	556,50	31345	43613	56,325	78,370
7	Siteler - Ulubey	543,75	35535	51468	65,352	94,654
8	Ziraat Fk. - Aydınlıkevler	575,50	43957	61953	76,381	107,651
9	Akköprü - Varlık Mah.	527,40	25920	20838	49,147	39,511
10	Yenimahalle - Demetevler	578,00	43951	38598	76,040	66,779
11	A.O.Ç. - Gazi Mah.	1050,25	15578	11930	14,833	11,359
12	Altındağ	111,15	44953	41480	404,435	373,189
13	Aktaş - Asrimezarlık	264,40	60257	49215	227,901	186,138
14	Samanpazarı - Eski Ankara	112,50	37307	24070	331,618	213,956
15	Cebeci	286,65	55049	62457	192,043	217,886
16	Gülseren - Gülveren	593,10	74636	47855	125,840	80,686
17	Karaağaç	423,45	1783	3576	4,211	8,445
18	Kayaş	1250,00	40188	64470	32,150	51,576
19	Mamak	466,75	46885	54663	100,450	117,114
20	Akdere - İmrahor - Türközü	1334,25	71017	45921	53,226	34,417
21	İncesu - Seyranbağları	128,25	41732	44566	325,396	347,493
22	Küçük Esat - Kavaklıdere	172,35	44265	27384	256,832	158,886
23	Ayrancı	291,82	36934	51873	126,564	177,757
24	Çankaya - Yıldız	850,00	32962	37889	38,779	44,575
25	Dikmen - Öveçler	983,75	41266	55221	41,948	56,133
26	Devlet	421,42	5601	3217	13,291	7,634
27	Balgat - Çukurambar	792,50	20221	37906	25,515	47,831
28	Bahçelievler - Emek	301,40	55160	66870	183,013	221,865
29	Maltepe - Anıttepe	264,15	37157	34345	140,666	130,021
30	Söğütözü	539,35	2809	10502	5,208	19,472
31	Yenişehir	162,45	32200	16049	198,215	98,793
32	Ulus	162,67	36429	19181	223,944	117,914
33	Kültür Aksı - Gençlik Parkı	221,85	14927	10533	67,284	47,478
34	A.O.Ç. Fabrikalar	748,50	3330	5989	4,449	8,001
Totals		18269,86	1215315	1438563	66,52021417	78,73968383

TABLE 3.2-) GROSS DENSITIES BY ZONES 1970 & 1990

1970				1990			
Zones		Gross Density (pphect.)	Natural Log (LN)	Zones		Gross Density (pphect.)	Natural Log (LN)
12	Altındağ	404,44	6,00249	12	Altındağ	373,19	5,9221
14	Samanpazarı - Eski Ankara	331,62	5,80398	21	İncesu - Seyranbağları	347,49	5,8507
21	İncesu - Seyranbağları	325,40	5,78504	28	Bahçelievler - Emek	221,86	5,4021
22	Küçük Esat - Kavaklıdere	256,83	5,54842	15	Cebeci	217,89	5,3840
13	Aktaş - Asrimezarlık	227,90	5,42891	14	Samanpazarı - Eski Ankara	213,96	5,3658
32	Ulus	223,94	5,41140	13	Aktaş - Asrimezarlık	186,14	5,2265
31	Yenişehir	198,21	5,28935	23	Ayrancı	177,76	5,1804
15	Cebeci	192,04	5,25772	4	Keçiören	176,40	5,1727
28	Bahçelievler - Emek	183,01	5,20956	22	Küçük Esat - Kavaklıdere	158,89	5,0682
29	Maltepe - Anıttepe	140,67	4,94639	29	Maltepe - Anıttepe	130,02	4,8677
23	Ayrancı	126,56	4,84075	32	Ulus	117,91	4,7700
16	Gülseren - Gülveren	125,84	4,83502	19	Mamak	117,11	4,7631
19	Mamak	100,45	4,60966	2	Etlik	110,72	4,7070
8	Ziraat Fk. - Aydınlıkevler	76,38	4,33573	8	Ziraat Fk. - Aydınlıkevler	107,65	4,6789
10	Yenimahalle - Demetevler	76,04	4,33126	3	Sanatoryum	101,14	4,6165
33	Kültür Aksı - Gençlik Parkı	67,28	4,20893	31	Yenişehir	98,79	4,5930
7	Siteler - Ulubey	65,35	4,17978	1	Karşıyaka	98,05	4,5855
4	Keçiören	61,50	4,11910	7	Siteler - Ulubey	94,65	4,5502
6	Hasköy	56,33	4,03114	16	Gülseren - Gülveren	80,69	4,3906
1	Karşıyaka	55,00	4,00732	6	Hasköy	78,37	4,3614
20	Akdere - İmrahor - Türközü	53,23	3,97455	10	Yenimahalle - Demetevler	66,78	4,2014
5	Aktepe	51,93	3,94988	25	Dikmen - Öveçler	56,13	4,0277
9	Akköprü - Varlık Mah.	49,15	3,89481	5	Aktepe	54,65	4,0010
3	Sanatoryum	44,71	3,80018	18	Kayaş	51,58	3,9431
2	Etlik	42,33	3,74557	27	Balgat - Çukurambar	47,83	3,8677
25	Dikmen - Öveçler	41,95	3,73642	33	Kültür Aksı - Gençlik Parkı	47,48	3,8603
24	Çankaya - Yıldız	38,78	3,65787	24	Çankaya - Yıldız	44,58	3,7972
18	Kayaş	32,15	3,47042	9	Akköprü - Varlık Mah.	39,51	3,6766
27	Balgat - Çukurambar	25,52	3,23928	20	Akdere - İmrahor - Türközü	34,42	3,5386
11	A.O.Ç. - Gazi Mah.	14,83	2,69683	30	Söğütözü	19,47	2,9690
26	Devlet	13,29	2,58707	11	A.O.Ç. - Gazi Mah.	11,36	2,4300
30	Söğütözü	5,21	1,65022	17	Karaağaç	8,44	2,1336
34	A.O.Ç. Fabrikalar	4,45	1,49266	34	A.O.Ç. Fabrikalar	8,00	2,0796
17	Karaağaç	4,21	1,43762	26	Devlet	7,63	2,0326

(Sorted Data From Table 3.1)

TABLE 3.3-) GROSS DENSITIES (1970 & 1990) AND DISTANCES FROM ULUS CENTER

Zones		Distances (meters)	Gross Densities (1970)	LN(70)	Gross Densities (1990)	LN(90)
1	Karşıyaka	9000	55,00	4,007319	98,05	4,585471
2	Etlik	6700	42,33	3,745575	110,72	4,707009
3	Sanatoryum	11200	44,71	3,800177	101,14	4,616546
4	Keçiören	7600	61,50	4,119102	176,40	5,172746
5	Aktepe	8500	51,93	3,949877	54,65	4,001031
6	Hasköy	7000	56,33	4,031143	78,37	4,361443
7	Siteler - Ulubey	8300	65,35	4,179784	94,65	4,550226
8	Ziraat Fk. - Aydınlıkevler	6000	76,38	4,335728	107,65	4,678892
9	Akköprü - Varlık Mah.	5000	49,15	3,894811	39,51	3,676574
10	Yenimahalle - Demetevler	6500	76,04	4,331257	66,78	4,201382
11	A.O.Ç. - Gazi Mah.	5000	14,83	2,696831	11,36	2,430028
12	Altındağ	2500	404,44	6,002492	373,19	5,922086
13	Aktaş - Asrimezarlık	4500	227,90	5,428911	186,14	5,226491
14	Samanpazarı - Eski Ankara	2750	331,62	5,803983	213,96	5,365768
15	Cebeci	3200	192,04	5,257717	217,89	5,383972
16	Gülseren - Gülveren	5250	125,84	4,835015	80,69	4,390568
17	Karaağaç	9250	4,21	1,437617	8,44	2,133565
18	Kayaş	11500	32,15	3,470425	51,58	3,943056
19	Mamak	6100	100,45	4,609659	117,11	4,763149
20	Akdere - İmrahor - Türközü	5250	53,23	3,97455	34,42	3,538553
21	İncesu - Seyranbağları	3300	325,40	5,785042	347,49	5,850745
22	Küçük Esat - Kavaklıdere	3350	256,83	5,548422	158,89	5,068187
23	Ayrancı	5500	126,56	4,840751	177,76	5,180417
24	Çankaya - Yıldız	6500	38,78	3,657874	44,58	3,797180
25	Dikmen - Öveçler	8250	41,95	3,736422	56,13	4,027727
26	Devlet	4850	13,29	2,58707	7,63	2,032575
27	Balgat - Çukurambar	7700	25,52	3,239284	47,83	3,867672
28	Bahçelievler - Emek	4750	183,01	5,209555	221,86	5,402067
29	Maltepe - Anıttepe	2750	140,67	4,94639	130,02	4,867695
30	Söğütözü	8600	5,21	1,650219	19,47	2,968956
31	Yenişehir	2250	198,21	5,289351	98,79	4,593032
32	Ulus	1500	223,94	5,411397	117,91	4,769952
33	Kültür Aksı - Gençlik Parkı	1750	67,28	4,208925	47,48	3,860267
34	A.O.Ç. Fabrikalar	6250	4,45	1,492656	8,00	2,079609

TABLE 3.4-) POPULATION DENSITIES IN THE NE SECTION: 1970-1990

Zone No	Zone Name	Distances from Ulus Center (meters)	1970 Pop. Densities (pph)	1990 Pop. Densities (pph)	Changes in Densities (%)
32	Ulus	1500	223	118	-47,35
12	Altındağ	2500	404	373	-7,73
14	Samanpazarı - Eski Ankara	2750	331	213	-35,48
15	Cebeci	3200	192	217	13,46
13	Aktaş - Asri Mezarlık	4500	228	186	-18,32
16	Gülseren - Gülveren	5250	126	80	-35,88
8	Ziraat F. - Aydınlıkevler	6000	76	108	40,94
6	Hasköy	7000	56	78	39,14
4	Keçiören	7600	62	176	186,81
7	Siteler - Uluğbey	8300	65	95	44,84
5	Aktepe	8500	52	55	5,25
17	Karaağaç	9250	4*	8*	100,56*
3	Sanatoryum	11200	45	101	126,23

Sorted From Table 2.1 & 3.3
(*) Density figures rounded but % change not rounded

TABLE 3.5-) POPULATION DENSITIES IN THE SW SECTION: 1970 - 1990

Zone No	Zone Name	Distances from Ulus Center (meters)	1970 Pop. Densities (pph)	1990 Pop. Densities (pph)	Changes in Densities (%)
32	Ulus	1500	224	118	-47,35
29	Maltepe - Anıttepe	2750	141	130	-7,57
28	Bahçeli - Emek	4750	183	222	21,23
26	Devlet	4850	13	8	-42,56
11	AOÇ - Gazi	5000	15	11	-23,42
34	AOÇ - Fab.	6250	4*	8*	79,85*
27	Balgat	7700	26	48	87,46
25	Dikmen - Öveçler	8250	42	56	33,82
30	Söğütözü	8600	5	19	273,87

Sorted From Table 2.1 & 3.3
(*) Density figures rounded but % changes not rounded

TABLE 4.1-) SPATIAL ENTROPY FOR POPULATION DENSITY & INFORMATION GAIN DISTRIBUTIONS (1970)

Zones		Gross Area (hec.)	Population (Pop(i))	p(i)= Pop(i) / POP(70)	p(i)/a(i)	ln[p(i)/a(i)]	Spatial Entropy for Pop. Density S(70)	Information Gain for Pop. Density I(70)
1	Karşıyaka	983,25	54078	0,044497106	0,000045255	-10,003195	0,445113205	-0,008462751
2	Etlik	855,00	36195	0,029782402	0,000034833	-10,264939	0,305714543	-0,013459593
3	Sanatoryum	565,50	25283	0,020803660	0,000036788	-10,210337	0,212412372	-0,008265890
4	Keçiören	687,50	42284	0,034792626	0,000050607	-9,891412	0,344148184	-0,002727869
5	Aktepe	464,50	24121	0,019847529	0,000042729	-10,060637	0,199678793	-0,004914830
6	Hasköy	556,50	31345	0,025791667	0,000046346	-9,979371	0,257384615	-0,004290779
7	Siteler - Ulubey	543,75	35535	0,029239333	0,000053773	-9,830730	0,287443989	-0,000518181
8	Ziraat Fk. - Aydınlıkevler	575,50	43957	0,036169224	0,000062848	-9,674786	0,349929496	0,004999385
9	Akköprü - Varlık Mah.	527,40	25920	0,021327804	0,000040440	-10,115703	0,215745730	-0,006455820
10	Yenimahalle - Demetevler	578,00	43951	0,036164287	0,000062568	-9,679257	0,350043427	0,004837006
11	A.O.Ç. - Gazi Mah.	1050,25	15578	0,012818076	0,000012205	-11,313682	0,145019641	-0,019235759
12	Altındağ	111,15	44953	0,036988764	0,000332782	-8,008022	0,296206827	0,066764211
13	Aktaş - Asrimezarlık	264,40	60257	0,049581384	0,000187524	-8,581603	0,425487753	0,061054767
14	Samanpazarı - Eski Ankara	112,50	37307	0,030697391	0,000272866	-8,206531	0,251919087	0,049314658
15	Cebeci	286,65	55049	0,045296076	0,000158019	-8,752797	0,396467347	0,048023404
16	Gülseren - Gülveren	593,10	74636	0,061412885	0,000103546	-9,175499	0,563493840	0,039151288
17	Karaağaç	423,45	1783	0,001467109	0,000003465	-12,572897	0,018445814	-0,004049059
18	Kayaş	1250,00	40188	0,033067970	0,000026454	-10,540089	0,348539346	-0,024043092
19	Mamak	466,75	46885	0,038578476	0,000082653	-9,400855	0,362670638	0,015900251
20	Akdere - İmrahor - Türközü	1334,25	71017	0,058435056	0,000043796	-10,035964	0,586452113	-0,013028442
21	İncesu - Seyranbağları	128,25	41732	0,034338423	0,000267746	-8,225472	0,282449728	0,054513487
22	Küçük Esat - Kavaklıdere	172,35	44265	0,036422656	0,000211330	-8,462092	0,308211849	0,049203963
23	Ayrancı	291,82	36934	0,030390475	0,000104141	-9,169763	0,278673459	0,019548514
24	Çankaya - Yıldız	850,00	32962	0,027122186	0,000031908	-10,352640	0,280786220	-0,014635988
25	Dikmen - Öveçler	983,75	41266	0,033954983	0,000034516	-10,274091	0,348856600	-0,015656082
26	Devlet	421,42	5601	0,004608682	0,000010936	-11,423443	0,052647014	-0,007421984
27	Balgat - Çukurambar	792,50	20221	0,016638485	0,000020995	-10,771229	0,179216936	-0,015943353
28	Bahçelievler - Emek	301,40	55160	0,045387410	0,000150589	-8,800959	0,399452725	0,045934291
29	Maltepe - Anıttepe	264,15	37157	0,030573966	0,000115745	-9,064124	0,277126208	0,022896368
30	Söğütözü	539,35	2809	0,002311335	0,000004285	-12,360295	0,028568781	-0,005887633
31	Yenişehir	162,45	32200	0,026495188	0,000163097	-8,721162	0,231068841	0,028928655
32	Ulus	162,67	36429	0,029974945	0,000184268	-8,599117	0,257758058	0,036386314
33	Kültür Aksı - Gençlik Parkı	221,85	14927	0,012282412	0,000055364	-9,801588	0,120387151	0,000140260
34	A.O.Ç. Fabrikalar	748,50	3330	0,002740030	0,000003661	-12,517857	0,034299310	-0,007411370
TOTAL		**18269,86**	**1215315**	**1,00**			**9,441819639**	**0,371188348**

$$S(70) = -\sum_{i=1}^{34}\left[\frac{\mathrm{Pop}(i,70)}{\mathrm{POP}(70)}\right] LN \left[\frac{\frac{\mathrm{Pop}(i,70)}{\mathrm{POP}(70)}}{a_i}\right] \qquad I(70) = \sum_i p_{i,70} \ln p_{i,70} \frac{A}{a_i}$$

* Definitions of variables: Chapter VI

TABLE 4.2-) SPATIAL ENTROPY FOR POPULATION DENSITY & INFORMATION GAIN DISTRIBUTIONS (1970)

Zones		S(70)	Zones		I(70)
20	Akdere - İmrahor - Türközü	0,58645211	12	Altındağ	0,066764
16	Gülseren - Gülveren	0,56349384	13	Aktaş - Asrimezarlık	0,061055
1	Karşıyaka	0,44511320	21	İncesu - Seyranbağları	0,054513
13	Aktaş - Asrimezarlık	0,42548775	14	Samanpazarı - Eski Ankara	0,049315
28	Bahçelievler - Emek	0,39945272	22	Küçük Esat - Kavaklıdere	0,049204
15	Cebeci	0,39646735	15	Cebeci	0,048023
19	Mamak	0,36267064	28	Bahçelievler - Emek	0,045934
10	Yenimahalle - Demetevler	0,35004343	16	Gülseren - Gülveren	0,039151
8	Ziraat Fk. - Aydınlıkevler	0,34992950	32	Ulus	0,036386
25	Dikmen - Öveçler	0,34885660	31	Yenişehir	0,028929
18	Kayaş	0,34853935	29	Maltepe - Anıttepe	0,022896
4	Keçiören	0,34414818	23	Ayrancı	0,019549
22	Küçük Esat - Kavaklıdere	0,30821185	19	Mamak	0,015900
2	Etlik	0,30571454	8	Ziraat Fk. - Aydınlıkevler	0,004999
12	Altındağ	0,29620683	10	Yenimahalle - Demetevler	0,004837
7	Siteler - Ulubey	0,28744399	33	Kültür Aksı - Gençlik Parkı	0,000140
21	İncesu - Seyranbağları	0,28244973	7	Siteler - Ulubey	-0,000518
24	Çankaya - Yıldız	0,28078622	4	Keçiören	-0,002728
23	Ayrancı	0,27867346	17	Karaağaç	-0,004049
29	Maltepe - Anıttepe	0,27712621	6	Hasköy	-0,004291
32	Ulus	0,25775806	5	Aktepe	-0,004915
6	Hasköy	0,25738462	30	Söğütözü	-0,005888
14	Samanpazarı - Eski Ankara	0,25191909	9	Akköprü - Varlık Mah.	-0,006456
31	Yenişehir	0,23106884	34	A.O.Ç. Fabrikalar	-0,007411
9	Akköprü - Varlık Mah.	0,21574573	26	Devlet	-0,007422
3	Sanatoryum	0,21241237	3	Sanatoryum	-0,008266
5	Aktepe	0,19967879	1	Karşıyaka	-0,008463
27	Balgat - Çukurambar	0,17921694	20	Akdere - İmrahor - Türközü	-0,013028
11	A.O.Ç. - Gazi Mah.	0,14501964	2	Etlik	-0,013460
33	Kültür Aksı - Gençlik Parkı	0,12038715	24	Çankaya - Yıldız	-0,014636
26	Devlet	0,05264701	25	Dikmen - Öveçler	-0,015656
34	A.O.Ç. Fabrikalar	0,03429931	27	Balgat - Çukurambar	-0,015943
30	Söğütözü	0,02856878	11	A.O.Ç. - Gazi Mah.	-0,019236
17	Karaağaç	0,01844581	18	Kayaş	-0,024043

(Sorted Data From Table 4.1)

TABLE 5.1-) SPATIAL ENTROPY FOR POPULATION DENSITY & INFORMATION GAIN DISTRIBUTIONS (1990)

	Zones	Gross Area (hec.)	Population Pop(i)	p(i)= Pop(i) / POP(90)	p(i)/a(i)	ln[p(i)/a(i)]	Spatial Entropy for Pop. Density S(90)	Information Gain for Pop. Density I(90)
1	**Karşıyaka**	983,25	96407	0,067016182	0,000068158	-9,59368	0,642932113	0,014698218
2	**Etlik**	855,00	94666	0,065805947	0,000076966	-9,47215	0,623323570	0,022430710
3	**Sanatoryum**	565,50	57197	0,039759816	0,000070309	-9,56261	0,380207566	0,009955825
4	**Keçiören**	687,50	121274	0,084302182	0,000122621	-9,00641	0,759259962	0,067998024
5	**Aktepe**	464,50	25387	0,017647472	0,000037992	-10,17812	0,179618162	0,012707211
6	**Hasköy**	556,50	43613	0,030317059	0,000054478	-9,81771	0,297644155	-0,000142608
7	**Siteler - Ulubey**	543,75	51468	0,035777369	0,000065797	-9,62893	0,344497762	0,006585851
8	**Ziraat Fk. - Aydınlıkevler**	575,50	61953	0,043065893	0,000074832	-9,50026	0,409137316	0,013468635
9	**Akköprü - Varlık Mah.**	527,40	20838	0,014485288	0,000027465	-10,50258	0,152132915	-0,009988664
10	**Yenimahalle - Demetevler**	578,00	38598	0,026830942	0,000046420	-9,97777	0,267713056	-0,004420811
11	**A.O.Ç. - Gazi Mah.**	1050,25	11930	0,008292998	0,000007896	-11,74913	0,097435488	-0,016056233
12	**Altındağ**	111,15	41480	0,028834330	0,000259418	-8,25707	0,238087058	0,044864451
13	**Aktaş - Asrimezarlık**	264,40	49215	0,034211223	0,000129392	-8,95266	0,306281609	0,029433399
14	**Samanpazarı - Eski Ankara**	112,50	24070	0,016731975	0,000148729	-8,81339	0,147465369	0,016725634
15	**Cebeci**	286,65	62457	0,043416242	0,000151461	-8,79518	0,381853824	0,044190110
16	**Gülseren - Gülveren**	593,10	47855	0,033265835	0,000056088	-9,78859	0,325625537	0,000812372
17	**Karaağaç**	423,45	3576	0,002485814	0,000005870	-12,04559	0,029943097	-0,005549785
18	**Kayaş**	1250,00	64470	0,044815556	0,000035852	-10,23610	0,458736454	-0,018961050
19	**Mamak**	466,75	54663	0,037998336	0,000081410	-9,41601	0,357792585	0,015085388
20	**Akdere - İmrahor - Türközü**	1334,25	45921	0,031921438	0,000023925	-10,64060	0,339663322	-0,026417993
21	**İncesu - Seyranbağları**	128,25	44566	0,030979526	0,000241556	-8,32841	0,258010202	0,045992135
22	**Küçük Esat - Kavaklıdere**	172,35	27384	0,019035663	0,000110448	-9,11097	0,173433320	0,013363789
23	**Ayrancı**	291,82	51873	0,036058900	0,000123566	-8,99874	0,324484624	0,029361654
24	**Çankaya - Yıldız**	850,00	37889	0,026338089	0,000030986	-10,38198	0,273441392	-0,014985517
25	**Dikmen - Öveçler**	983,75	55221	0,038386223	0,000039020	-10,15143	0,389674996	-0,012990684
26	**Devlet**	421,42	3217	0,002236259	0,000005306	-12,14658	0,027162905	-0,005218474
27	**Balgat - Çukurambar**	792,50	37906	0,026349906	0,000033249	-10,31148	0,271706611	-0,013134772
28	**Bahçelievler - Emek**	301,40	66870	0,046483887	0,000154227	-8,77709	0,407993159	0,048153596
29	**Maltepe - Anıttepe**	264,15	34345	0,023874519	0,000090382	-9,31146	0,222306646	0,011974201
30	**Söğütözü**	539,35	10502	0,007300341	0,000013535	-11,21020	0,081838272	-0,010199970
31	**Yenişehir**	162,45	16049	0,011156272	0,000068675	-9,58612	0,106945403	0,002531183
32	**Ulus**	162,67	19181	0,013333445	0,000081966	-9,40920	0,125457092	0,005384106
33	**Kültür Aksı - Gençlik Parkı**	221,85	10533	0,007321890	0,000033004	-10,31889	0,075553765	-0,003704000
34	**A.O.Ç. Fabrikalar**	748,50	5989	0,004163182	0,000005562	-12,09955	0,050372619	-0,009519278
	TOTAL	**18269,86**	**1438563**	**1,00**			**9,527731924**	**0,304426653**

$$S(90) = -\sum_{i=1}^{34}\left[\frac{Pop(i,90)}{POP(90)}\right] LN\left[\frac{\frac{Pop(i,90)}{POP(90)}}{a_i}\right] \qquad I(90) = \sum_i p_{i,90} \ln p_{i,90} \frac{A}{a_i}$$

* Definitions of variables: Chapter VI

TABLE 5.2-) SPATIAL ENTROPY FOR POPULATION DENSITY & INFORMATION GAIN DISTRIBUTIONS (1990)

Zones		S(90)	Zones		I(90)
4	Keçiören	0,759259962	4	Keçiören	0,0679980
1	Karşıyaka	0,642932113	28	Bahçelievler - Emek	0,0481536
2	Etlik	0,623323570	21	İncesu - Seyranbağları	0,0459921
18	Kayaş	0,458736454	12	Altındağ	0,0448645
8	Ziraat Fk. - Aydınlıkevler	0,409137316	15	Cebeci	0,0441901
28	Bahçelievler - Emek	0,407993159	13	Aktaş - Asrimezarlık	0,0294334
25	Dikmen - Öveçler	0,389674996	23	Ayrancı	0,0293617
15	Cebeci	0,381853824	2	Etlik	0,0224307
3	Sanatoryum	0,380207566	14	Samanpazarı - Eski Ankara	0,0167256
19	Mamak	0,357792585	19	Mamak	0,0150854
7	Siteler - Ulubey	0,344497762	1	Karşıyaka	0,0146982
20	Akdere - İmrahor - Türközü	0,339663322	8	Ziraat Fk. - Aydınlıkevler	0,0134686
16	Gülseren - Gülveren	0,325625537	22	Küçük Esat - Kavaklıdere	0,0133638
23	Ayrancı	0,324484624	5	Aktepe	0,0127072
13	Aktaş - Asrimezarlık	0,306281609	29	Maltepe - Anıttepe	0,0119742
6	Hasköy	0,297644155	3	Sanatoryum	0,0099558
24	Çankaya - Yıldız	0,273441392	7	Siteler - Ulubey	0,0065859
27	Balgat - Çukurambar	0,271706611	32	Ulus	0,0053841
10	Yenimahalle - Demetevler	0,267713056	31	Yenişehir	0,0025312
21	İncesu - Seyranbağları	0,258010202	16	Gülseren - Gülveren	0,0008124
12	Altındağ	0,238087058	6	Hasköy	-0,0001426
29	Maltepe - Anıttepe	0,222306646	33	Kültür Aksı - Gençlik Parkı	-0,0037040
5	Aktepe	0,179618162	10	Yenimahalle - Demetevler	-0,0044208
22	Küçük Esat - Kavaklıdere	0,173433320	26	Devlet	-0,0052185
9	Akköprü - Varlık Mah.	0,152132915	17	Karaağaç	-0,0055498
14	Samanpazarı - Eski Ankara	0,147465369	34	A.O.Ç. Fabrikalar	-0,0095193
32	Ulus	0,125457092	9	Akköprü - Varlık Mah.	-0,0099887
31	Yenişehir	0,106945403	30	Söğütözü	-0,0102000
11	A.O.Ç. - Gazi Mah.	0,097435488	25	Dikmen - Öveçler	-0,0129907
30	Söğütözü	0,081838272	27	Balgat - Çukurambar	-0,0131348
33	Kültür Aksı - Gençlik Parkı	0,075553765	24	Çankaya - Yıldız	-0,0149855
34	A.O.Ç. Fabrikalar	0,050372619	11	A.O.Ç. - Gazi Mah.	-0,0160562
17	Karaağaç	0,029943097	18	Kayaş	-0,0189611
26	Devlet	0,027162905	20	Akdere - İmrahor - Türközü	-0,0264180

(Sorted Data From Table 5.1)

TABLE 6.1-) INFORMATION GAIN DISTRIBUTIONS FOR POPULATION CHANGES (1970 - 1990)

	ZONES	POP(i,70)	POP(i,90)	POP Increase [Pop(i,90)-Pop(i,70)]	% Increase In Zone Pops	% Increases Acc. To Total Pop Changes	p(i,90)	q(i,70)	p(i,90) / q(i,70)	LN[p(i,90) / q(i,70)]	I(p90:q70)
1	Karşıyaka	54078	96407	42329	78,27%	18,96%	0,06702	0,04450	1,50608	0,409509968	0,027443795
2	Etlik	36195	94666	58471	161,54%	26,19%	0,06581	0,02978	2,20956	0,792792523	0,052170462
3	Sanatoryum	25283	57197	31914	126,23%	14,30%	0,03976	0,02080	1,91119	0,647727820	0,025753539
4	Keçiören	42284	121274	78990	186,81%	35,38%	0,08430	0,03479	2,42299	0,885002288	0,074607624
5	Aktepe	24121	25387	1266	5,25%	0,57%	0,01765	0,01985	0,88915	-0,117486995	-0,002073348
6	Hasköy	31345	43613	12268	39,14%	5,50%	0,03032	0,02579	1,17546	0,161659110	0,004901029
7	Siteler - Ulubey	35535	51468	15933	44,84%	7,14%	0,03578	0,02924	1,22360	0,201800733	0,007219899
8	Ziraat Fk. - Aydınlıkevler	43957	61953	17996	40,94%	8,06%	0,04307	0,03617	1,19068	0,174522753	0,007515978
9	Akköprü - Varlık Mah.	25920	20838	-5082	-19,61%	-2,28%	0,01449	0,02133	0,67917	-0,386878025	-0,005604040
10	Yenimahalle - Demetevler	43951	38598	-5353	-12,18%	-2,40%	0,02683	0,03616	0,74192	-0,298516312	-0,008009474
11	A.O.Ç. - Gazi Mah.	15578	11930	-3648	-23,42%	-1,63%	0,00829	0,01282	0,64698	-0,435444823	-0,003611143
12	Altındağ	44953	41480	-3473	-7,73%	-1,56%	0,02883	0,03699	0,77954	-0,249047512	-0,007181118
13	Aktaş - Asrimezarlık	60257	49215	-11042	-18,32%	-4,95%	0,03421	0,04958	0,69000	-0,371061689	-0,012694474
14	Samanpazarı - Eski Ankara	37307	24070	-13237	-35,48%	-5,93%	0,01673	0,03070	0,54506	-0,606856121	-0,010153901
15	Cebeci	55049	62457	7408	13,46%	3,32%	0,04342	0,04530	0,95850	-0,042386774	-0,001840274
16	Gülseren - Gülveren	74636	47855	-26781	-35,88%	-12,00%	0,03327	0,06141	0,54168	-0,613088755	-0,020394910
17	Karaağaç	1783	3576	1793	100,56%	0,80%	0,00249	0,00147	1,69436	0,527306122	0,001310785
18	Kayaş	40188	64470	24282	60,42%	10,88%	0,04482	0,03307	1,35526	0,303990160	0,013623488
19	Mamak	46885	54663	7778	16,59%	3,48%	0,03800	0,03858	0,98496	-0,015152127	-0,000575756
20	Akdere - İmrahor - Türközü	71017	45921	-25096	-35,34%	-11,24%	0,03192	0,05844	0,54627	-0,604638152	-0,019300919
21	İncesu - Seyranbağları	41732	44566	2834	6,79%	1,27%	0,03098	0,03434	0,90218	-0,102938380	-0,003188982
22	Küçük Esat - Kavaklıdere	44265	27384	-16881	-38,14%	-7,56%	0,01904	0,03642	0,52263	-0,648876792	-0,012351800
23	Ayrancı	36934	51873	14939	40,45%	6,69%	0,03606	0,03039	1,18652	0,171024491	0,006166955
24	Çankaya - Yıldız	32962	37889	4927	14,95%	2,21%	0,02634	0,02712	0,97109	-0,029335947	-0,000772653
25	Dikmen - Öveçler	41266	55221	13955	33,82%	6,25%	0,03839	0,03395	1,13050	0,122663003	0,004708569
26	Devlet	5601	3217	-2384	-42,56%	-1,07%	0,00224	0,00461	0,48523	-0,723137301	-0,001617123
27	Balgat - Çukurambar	20221	37906	17685	87,46%	7,92%	0,02635	0,01664	1,58367	0,459746346	0,012114273
28	Bahçelievler - Emek	55160	66870	11710	21,23%	5,25%	0,04648	0,04539	1,02416	0,023870987	0,001109616
29	Maltepe - Anıttepe	37157	34345	-2812	-7,57%	-1,26%	0,02387	0,03057	0,78088	-0,247337127	-0,005905055
30	Söğütözü	2809	10502	7693	273,87%	3,45%	0,00730	0,00231	3,15850	1,150095771	0,008396091
31	Yenişehir	32200	16049	-16151	-50,16%	-7,23%	0,01116	0,02650	0,42107	-0,864961306	-0,009649744
32	Ulus	36429	19181	-17248	-47,35%	-7,73%	0,01333	0,02997	0,44482	-0,810086351	-0,010801241
33	Kültür Aksı - Gençlik Parkı	14927	10533	-4394	-29,44%	-1,97%	0,00732	0,01228	0,59613	-0,517299864	-0,003787613
34	A.O.Ç. Fabrikalar	3330	5989	2659	79,85%	1,19%	0,00416	0,00274	1,51939	0,418310753	0,001741504
	Totals	**1215315**	**1438563**	**223248**	**18,37%**	**100,00%**	**1,00**	**1,00**			**0,10927004**

$$I(p:q) = \sum_{i}^{n} p_{i,90} \ln \frac{p_{i,90}}{q_{i,70}} = \sum_{i=1}^{n=34} p_{i,90} \ln \frac{p_{i,90}}{q_{i,70}}$$

* Definitions of variables: Chapter VI

TABLE 6.2-) INFORMATION GAIN FOR POPULATION CHANGES: I(p90:q70)

Zones		I(p90:q70)
4	Keçiören	0,0746
2	Etlik	0,0522
1	Karşıyaka	0,0274
3	Sanatoryum	0,0258
18	Kayaş	0,0136
27	Balgat - Çukurambar	0,0121
30	Söğütözü	0,0084
8	Ziraat Fk. - Aydınlıkevler	0,0075
7	Siteler - Ulubey	0,0072
23	Ayrancı	0,0062
6	Hasköy	0,0049
25	Dikmen - Öveçler	0,0047
34	A.O.Ç. Fabrikalar	0,0017
17	Karaağaç	0,0013
28	Bahçelievler - Emek	0,0011
19	Mamak	-0,0006
24	Çankaya - Yıldız	-0,0008
26	Devlet	-0,0016
15	Cebeci	-0,0018
5	Aktepe	-0,0021
21	İncesu - Seyranbağları	-0,0032
11	A.O.Ç. - Gazi Mah.	-0,0036
33	Kültür Aksı - Gençlik Parkı	-0,0038
9	Akköprü - Varlık Mah.	-0,0056
29	Maltepe - Anıttepe	-0,0059
12	Altındağ	-0,0072
10	Yenimahalle - Demetevler	-0,0080
31	Yenişehir	-0,0096
14	Samanpazarı - Eski Ankara	-0,0102
32	Ulus	-0,0108
22	Küçük Esat - Kavaklıdere	-0,0124
13	Aktaş - Asrimezarlık	-0,0127
20	Akdere - İmrahor - Türközü	-0,0193
16	Gülseren - Gülveren	-0,0204
TOTAL		**0,1093**

(Sorted Data From Table 6.1)

TABLE 6.3-) POPULATION INCREASES ACCORDING TO TOTAL GROWTH

Zones		% Increase In Zone Pops
4	Keçiören	35,38%
2	Etlik	26,19%
1	Karşıyaka	18,96%
3	Sanatoryum	14,30%
18	Kayaş	10,88%
8	Ziraat Fk. - Aydınlıkevler	8,06%
27	Balgat - Çukurambar	7,92%
7	Siteler - Ulubey	7,14%
23	Ayrancı	6,69%
25	Dikmen - Öveçler	6,25%
6	Hasköy	5,50%
28	Bahçelievler - Emek	5,25%
19	Mamak	3,48%
30	Söğütözü	3,45%
15	Cebeci	3,32%
24	Çankaya - Yıldız	2,21%
21	İncesu - Seyranbağları	1,27%
34	A.O.Ç. Fabrikalar	1,19%
17	Karaağaç	0,80%
5	Aktepe	0,57%
26	Devlet	-1,07%
29	Maltepe - Anıttepe	-1,26%
12	Altındağ	-1,56%
11	A.O.Ç. - Gazi Mah.	-1,63%
33	Kültür Aksı - Gençlik Parkı	-1,97%
9	Akköprü - Varlık Mah.	-2,28%
10	Yenimahalle - Demetevler	-2,40%
13	Aktaş - Asrimezarlık	-4,95%
14	Samanpazarı - Eski Ankara	-5,93%
31	Yenişehir	-7,23%
22	Küçük Esat - Kavaklıdere	-7,56%
32	Ulus	-7,73%
20	Akdere - İmrahor - Türközü	-11,24%
16	Gülseren - Gülveren	-12,00%

(Sorted Data From Table 6.1)

TABLE 6.4-) POPULATION INCREASES ACCORDING TO WITHIN-ZONE GROWTH

Zones		% Increase In Zone Pops
1	Karşıyaka	78,27%
2	Etlik	161,54%
3	Sanatoryum	126,23%
4	Keçiören	186,81%
5	Aktepe	5,25%
6	Hasköy	39,14%
7	Siteler - Ulubey	44,84%
8	Ziraat Fk. - Aydınlıkevler	40,94%
9	Akköprü - Varlık Mah.	-19,61%
10	Yenimahalle - Demetevler	-12,18%
11	A.O.Ç. - Gazi Mah.	-23,42%
12	Altındağ	-7,73%
13	Aktaş - Asrimezarlık	-18,32%
14	Samanpazarı - Eski Ankara	-35,48%
15	Cebeci	13,46%
16	Gülseren - Gülveren	-35,88%
17	Karaağaç	100,56%
18	Kayaş	60,42%
19	Mamak	16,59%
20	Akdere - İmrahor - Türközü	-35,34%
21	İncesu - Seyranbağları	6,79%
22	Küçük Esat - Kavaklıdere	-38,14%
23	Ayrancı	40,45%
24	Çankaya - Yıldız	14,95%
25	Dikmen - Öveçler	33,82%
26	Devlet	-42,56%
27	Balgat - Çukurambar	87,46%
28	Bahçelievler - Emek	21,23%
29	Maltepe - Anıttepe	-7,57%
30	Söğütözü	273,87%
31	Yenişehir	-50,16%
32	Ulus	-47,35%
33	Kültür Aksı - Gençlik Parkı	-29,44%
34	A.O.Ç. Fabrikalar	79,85%

(Sorted Data From Table 6.1)

Table 7.1-) COMPARISONS OF ENTROPIES: 1970 - 1990

	Zones	S(70)	S(90)	I(70)	I(90)	I(p90:q70)	S(90) - S(70)	I(90) - I(70)
1	Karşıyaka	0,4451132	0,6429321	-0,0084628	0,014698218	0,027443795	0,197818908	0,023160969
2	Etlik	0,3057145	0,6233236	-0,0134596	0,022430710	0,052170462	0,317609027	0,035890303
3	Sanatoryum	0,2124124	0,3802076	-0,0082659	0,009955825	0,025753539	0,167795195	0,018221715
4	Keçiören	0,3441482	0,7592600	-0,0027279	0,067998024	0,074607624	0,415111778	0,070725894
5	Aktepe	0,1996788	0,1796182	-0,0049148	0,012707211	-0,002073348	-0,020060631	0,017622041
6	Hasköy	0,2573846	0,2976442	-0,0042908	-0,000142608	0,004901029	0,040259540	0,004148171
7	Siteler - Ulubey	0,2874440	0,3444978	-0,0005182	0,006585851	0,007219899	0,057053772	0,007104032
8	Ziraat Fk. - Aydınlıkevler	0,3499295	0,4091373	0,0049994	0,013468635	0,007515978	0,059207820	0,008469250
9	Akköprü - Varlık Mah.	0,2157457	0,1521329	-0,0064558	-0,009988664	-0,005604040	-0,063612814	-0,003532845
10	Yenimahalle - Demetevler	0,3500434	0,2677131	0,0048370	-0,004420811	-0,008009474	-0,082330371	-0,009257817
11	A.O.Ç. - Gazi Mah.	0,1450196	0,0974355	-0,0192358	-0,016056233	-0,003611143	-0,047584153	0,003179526
12	Altındağ	0,2962068	0,2380871	0,0667642	0,044864451	-0,007181118	-0,058119770	-0,021899760
13	Aktaş - Asrimezarlık	0,4254878	0,3062816	0,0610548	0,029433399	-0,012694474	-0,119206144	-0,031621368
14	Samanpazarı - Eski Ankara	0,2519191	0,1474654	0,0493147	0,016725634	-0,010153901	-0,104453718	-0,032589024
15	Cebeci	0,3964673	0,3818538	0,0480234	0,044190110	-0,001840274	-0,014613522	-0,003833294
16	Gülseren - Gülveren	0,5634938	0,3256255	0,0391513	0,000812372	-0,020394910	-0,237868303	-0,038338917
17	Karaağaç	0,0184458	0,0299431	-0,0040491	-0,005549785	0,001310785	0,011497283	-0,001500726
18	Kayaş	0,3485393	0,4587365	-0,0240431	-0,018961050	0,013623488	0,110197108	0,005082042
19	Mamak	0,3626706	0,3577926	0,0159003	0,015085388	-0,000575756	-0,004878053	-0,000814862
20	Akdere - İmrahor - Türközü	0,5864521	0,3396633	-0,0130284	-0,026417993	-0,019300919	-0,246788791	-0,013389551
21	İncesu - Seyranbağları	0,2824497	0,2580102	0,0545135	0,045992135	-0,003188982	-0,024439526	-0,008521352
22	Küçük Esat - Kavaklıdere	0,3082118	0,1734333	0,0492040	0,013363789	-0,012351800	-0,134778529	-0,035840174
23	Ayrancı	0,2786735	0,3244846	0,0195485	0,029361654	0,006166955	0,045811165	0,009813140
24	Çankaya - Yıldız	0,2807862	0,2734414	-0,0146360	-0,014985517	-0,000772653	-0,007344828	-0,000349529
25	Dikmen - Öveçler	0,3488566	0,3896750	-0,0156561	-0,012990684	0,004708569	0,040818396	0,002665398
26	Devlet	0,0526470	0,0271629	-0,0074220	-0,005218474	-0,001617123	-0,025484110	0,002203510
27	Balgat - Çukurambar	0,1792169	0,2717066	-0,0159434	-0,013134772	0,012114273	0,092489675	0,002808581
28	Bahçelievler - Emek	0,3994527	0,4079932	0,0459343	0,048153596	0,001109616	0,008540434	0,002219305
29	Maltepe - Anıttepe	0,2771262	0,2223066	0,0228964	0,011974201	-0,005905055	-0,054819562	-0,010922167
30	Söğütözü	0,0285688	0,0818383	-0,0058876	-0,010199970	0,008396091	0,053269491	-0,004312337
31	Yenişehir	0,2310688	0,1069454	0,0289287	0,002531183	-0,009649744	-0,124123439	-0,026397472
32	Ulus	0,2577581	0,1254571	0,0363863	0,005384106	-0,010801241	-0,132300966	-0,031002208
33	Kültür Aksı - Gençlik Parkı	0,1203872	0,0755538	0,0001403	-0,003704000	-0,003787613	-0,044833386	-0,003844260
34	A.O.Ç. Fabrikalar	0,0342993	0,0503726	-0,0074114	-0,009519278	0,001741504	0,016073309	-0,002107908
	Totals	**9,4418196**	**9,5277319**	**0,3711883**	**0,304426653**	**0,10927004**		

From Tables 4.1, 5.1, 6.1

TABLE 7.2-) SPATIAL ENTROPY DIFFERENCES BY ZONES [S(90)-S(70)]

Zones		S(70)	Zones		S(90)	Zones		S(90)-S(70)
20	Akdere - İmrahor - Türközü	0,5865	4	Keçiören	0,7593	4	Keçiören	0,4151
16	Gülseren - Gülveren	0,5635	1	Karşıyaka	0,6429	2	Etlik	0,3176
1	Karşıyaka	0,4451	2	Etlik	0,6233	1	Karşıyaka	0,1978
13	Aktaş - Asrimezarlık	0,4255	18	Kayaş	0,4587	3	Sanatoryum	0,1678
28	Bahçelievler - Emek	0,3995	8	Ziraat Fk. - Aydınlıkevler	0,4091	18	Kayaş	0,1102
15	Cebeci	0,3965	28	Bahçelievler - Emek	0,4080	27	Balgat - Çukurambar	0,0925
19	Mamak	0,3627	25	Dikmen - Öveçler	0,3897	8	Ziraat Fk. - Aydınlıkevler	0,0592
10	Yenimahalle - Demetevler	0,3500	15	Cebeci	0,3819	7	Siteler - Ulubey	0,0571
8	Ziraat Fk. - Aydınlıkevler	0,3499	3	Sanatoryum	0,3802	30	Söğütözü	0,0533
25	Dikmen - Öveçler	0,3489	19	Mamak	0,3578	23	Ayrancı	0,0458
18	Kayaş	0,3485	7	Siteler - Ulubey	0,3445	25	Dikmen - Öveçler	0,0408
4	Keçiören	0,3441	20	Akdere - İmrahor - Türközü	0,3397	6	Hasköy	0,0403
22	Küçük Esat - Kavaklıdere	0,3082	16	Gülseren - Gülveren	0,3256	34	A.O.Ç. Fabrikalar	0,0161
2	Etlik	0,3057	23	Ayrancı	0,3245	17	Karaağaç	0,0115
12	Altındağ	0,2962	13	Aktaş - Asrimezarlık	0,3063	28	Bahçelievler - Emek	0,0085
7	Siteler - Ulubey	0,2874	6	Hasköy	0,2976	19	Mamak	-0,0049
21	İncesu - Seyranbağları	0,2824	24	Çankaya - Yıldız	0,2734	24	Çankaya - Yıldız	-0,0073
24	Çankaya - Yıldız	0,2808	27	Balgat - Çukurambar	0,2717	15	Cebeci	-0,0146
23	Ayrancı	0,2787	10	Yenimahalle - Demetevler	0,2677	5	Aktepe	-0,0201
29	Maltepe - Anıttepe	0,2771	21	İncesu - Seyranbağları	0,2580	21	İncesu - Seyranbağları	-0,0244
32	Ulus	0,2578	12	Altındağ	0,2381	26	Devlet	-0,0255
6	Hasköy	0,2574	29	Maltepe - Anıttepe	0,2223	33	Kültür Aksı - Gençlik Parkı	-0,0448
14	Samanpazarı - Eski Ankara	0,2519	5	Aktepe	0,1796	11	A.O.Ç. - Gazi Mah.	-0,0476
31	Yenişehir	0,2311	22	Küçük Esat - Kavaklıdere	0,1734	29	Maltepe - Anıttepe	-0,0548
9	Akköprü - Varlık Mah.	0,2157	9	Akköprü - Varlık Mah.	0,1521	12	Altındağ	-0,0581
3	Sanatoryum	0,2124	14	Samanpazarı - Eski Ankara	0,1475	9	Akköprü - Varlık Mah.	-0,0636
5	Aktepe	0,1997	32	Ulus	0,1255	10	Yenimahalle - Demetevler	-0,0823
27	Balgat - Çukurambar	0,1792	31	Yenişehir	0,1069	14	Samanpazarı - Eski Ankara	-0,1045
11	A.O.Ç. - Gazi Mah.	0,1450	11	A.O.Ç. - Gazi Mah.	0,0974	13	Aktaş - Asrimezarlık	-0,1192
33	Kültür Aksı - Gençlik Parkı	0,1204	30	Söğütözü	0,0818	31	Yenişehir	-0,1241
26	Devlet	0,0526	33	Kültür Aksı - Gençlik Parkı	0,0756	32	Ulus	-0,1323
34	A.O.Ç. Fabrikalar	0,0343	34	A.O.Ç. Fabrikalar	0,0504	22	Küçük Esat - Kavaklıdere	-0,1348
30	Söğütözü	0,0286	17	Karaağaç	0,0299	16	Gülseren - Gülveren	-0,2379
17	Karaağaç	0,0184	26	Devlet	0,0272	20	Akdere - İmrahor - Türközü	-0,2468
TOTALS		9,4418			9,5277			0,0859

(Sorted Data From Table 7.1)

TABLE 7.3-) INFORMATION GAIN DIFFERENCES BY ZONES [I(90)-I(70)]

Zones		I(70)	Zones		I(90)	Zones		I(90)-I(70)
12	Altındağ	0,0668	4	Keçiören	0,0680	4	Keçiören	0,0707
13	Aktaş - Asrimezarlık	0,0611	28	Bahçelievler - Emek	0,0482	2	Etlik	0,0359
21	İncesu - Seyranbağları	0,0545	21	İncesu - Seyranbağları	0,0460	1	Karşıyaka	0,0232
14	Samanpazarı - Eski Ankara	0,0493	12	Altındağ	0,0449	3	Sanatoryum	0,0182
22	Küçük Esat - Kavaklıdere	0,0492	15	Cebeci	0,0442	5	Aktepe	0,0176
15	Cebeci	0,0480	13	Aktaş - Asrimezarlık	0,0294	23	Ayrancı	0,0098
28	Bahçelievler - Emek	0,0459	23	Ayrancı	0,0294	8	Ziraat Fk. - Aydınlıkevler	0,0085
16	Gülseren - Gülveren	0,0392	2	Etlik	0,0224	7	Siteler - Ulubey	0,0071
32	Ulus	0,0364	14	Samanpazarı - Eski Ankara	0,0167	18	Kayaş	0,0051
31	Yenişehir	0,0289	19	Mamak	0,0151	6	Hasköy	0,0041
29	Maltepe - Anıttepe	0,0229	1	Karşıyaka	0,0147	11	A.O.Ç. - Gazi Mah.	0,0032
23	Ayrancı	0,0195	8	Ziraat Fk. - Aydınlıkevler	0,0135	27	Balgat - Çukurambar	0,0028
19	Mamak	0,0159	22	Küçük Esat - Kavaklıdere	0,0134	25	Dikmen - Öveçler	0,0027
8	Ziraat Fk. - Aydınlıkevler	0,0050	5	Aktepe	0,0127	28	Bahçelievler - Emek	0,0022
10	Yenimahalle - Demetevler	0,0048	29	Maltepe - Anıttepe	0,0120	26	Devlet	0,0022
33	Kültür Aksı - Gençlik Parkı	0,0001	3	Sanatoryum	0,0100	24	Çankaya - Yıldız	-0,0003
7	Siteler - Ulubey	-0,0005	7	Siteler - Ulubey	0,0066	19	Mamak	-0,0008
4	Keçiören	-0,0027	32	Ulus	0,0054	17	Karaağaç	-0,0015
17	Karaağaç	-0,0040	31	Yenişehir	0,0025	34	A.O.Ç. Fabrikalar	-0,0021
6	Hasköy	-0,0043	16	Gülseren - Gülveren	0,0008	9	Akköprü - Varlık Mah.	-0,0035
5	Aktepe	-0,0049	6	Hasköy	-0,0001	15	Cebeci	-0,0038
30	Söğütözü	-0,0059	33	Kültür Aksı - Gençlik Parkı	-0,0037	33	Kültür Aksı - Gençlik Parkı	-0,0038
9	Akköprü - Varlık Mah.	-0,0065	10	Yenimahalle - Demetevler	-0,0044	30	Söğütözü	-0,0043
34	A.O.Ç. Fabrikalar	-0,0074	26	Devlet	-0,0052	21	İncesu - Seyranbağları	-0,0085
26	Devlet	-0,0074	17	Karaağaç	-0,0055	10	Yenimahalle - Demetevler	-0,0093
3	Sanatoryum	-0,0083	34	A.O.Ç. Fabrikalar	-0,0095	29	Maltepe - Anıttepe	-0,0109
1	Karşıyaka	-0,0085	9	Akköprü - Varlık Mah.	-0,0100	20	Akdere - İmrahor - Türközü	-0,0134
20	Akdere - İmrahor - Türközü	-0,0130	30	Söğütözü	-0,0102	12	Altındağ	-0,0219
2	Etlik	-0,0135	25	Dikmen - Öveçler	-0,0130	31	Yenişehir	-0,0264
24	Çankaya - Yıldız	-0,0146	27	Balgat - Çukurambar	-0,0131	32	Ulus	-0,0310
25	Dikmen - Öveçler	-0,0157	24	Çankaya - Yıldız	-0,0150	13	Aktaş - Asrimezarlık	-0,0316
27	Balgat - Çukurambar	-0,0159	11	A.O.Ç. - Gazi Mah.	-0,0161	14	Samanpazarı - Eski Ankara	-0,0326
11	A.O.Ç. - Gazi Mah.	-0,0192	18	Kayaş	-0,0190	22	Küçük Esat - Kavaklıdere	-0,0358
18	Kayaş	-0,0240	20	Akdere - İmrahor - Türközü	-0,0264	16	Gülseren - Gülveren	-0,0383
TOTALS		0,3712			0,3044			-0,0668

(Sorted Data From Table 7.1)

APPENDIX - B

SOME CRITICAL NOTES ON URBAN DENSITY MODELS
(1998)

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INTRODUCTION

In my previous paper (1977), it was pointed out that one of the most significant theoretical contributions to the study of urban spatial structure has been made as an outcome of analysis of the pattern of population distribution. However, the term "Spatial Structure" is not readily defined and the estimating urban density functions, based on the "distance-decay" hypothesis have attracted many researchers. As it was presented more formally in the paper, the literature on the population–density functions up to the late 1975s can be grouped into 3-types of approaches:

1-) The first approach takes the problem from the standpoint of statistical distribution functions and aims to find the best–fit curve for the empirical findings. C. Clark's (1951) hypothesis that "residential population densities decline exponentially from the city centre" has been the origin of the researches in this group. The first revision of Clark's function was proposed by J. C. Tanner (1961) and by G. Sherratt (1960) who suggested that urban population densities decline exponentially with the square of the distance from the centre. Thus, the negative exponential density function of the first-degree has been generalized and statistical methods have been developed for the higher-degree distribution functions. (L. March, 1971, 1972).

2-) The second approach based on the entropy-maximizing method, provides a theoretical foundation for the empirically determined distance-decay hypothesis pioneered by A. G. Wilson (1967, 1970).

3-) The third approach, which can be called the behavioral, aims to relate the intra-urban land use and population distribution patterns to the consumer and producer behavior theories, utility functions and other complex models of the neoclassical economic theory. H. Richardson (1976a) evaluates that the branch of urban economic theory labeled the "New Urban Economics" attempts to integrate welfare economics and urban economics within a general equilibrium framework. Richardson (1976b) points out that most would mark either E. Mills' model (1967) or M. Beckmann's paper (1969) as the true beginning of a new age Beckmann's (1969) paper was evaluated and restated later by A. Montesano (1972) in the same journal. The important contributions by E. Casetti (1971) and G. Papageorgiou (1971) should also be regarded within this line of development. Casetti proposed and alternative formulation of the Alonso (1964) model for the derivation of continuous functions relating land prices and population densities to distance from and central point. However, both Alonso and Casetti (1971) assumed one central location to which accessibility was sought. Papageorgiou (1971) extended the monocentric urban theory to the multicentric formulation. L. King (1979) clarifies his positivism in quantitative-theoretic research and makes similar evaluations of these contributions.

DEVELOPMENTS DURING THE 1980/90 PERIOD AND AFTERWARDS

Studies under the three approaches outlined above have continued with developments and extensions during the last decade. Studies associated with the empirical / statistical approach make estimates of urban density functions for cities of both developed and developing countries (Mills & Tan, 1980), (B. Edmonston, et.al., 1985), (N. J. Glickman & Mr. White, 1979), (K. Zielinski, 1979); or extend the function which has been usually applied within the context of the urban area to the scale of a metropolitan region (J. Parr, 1985)

According to A. Anas & D. S. Dendrinos (1976), contributions to Alonso (1964) model made by Muth (1969) and Beckmann (1969), constituted the major "First-Generation Models" or "Standard Models", a term later coined by Solow. The "First-Generation Models" constituted the group of work completed in the 1960s. With the exception of the 1967 Mills Model, all of the "Second-Generation Models" appeared in the 1970s.

The Entropy-Maximization paradigm has attracted many researchers from different fields to use the concept of entropy from a Shannonian information theory viewpoint. Although Gould (1972) observed that A. G. Wilson's (1970) work "raises the gravity model phoenix-like from the ashes", the philosophic basis of entropy maximizing models are not easy or nonspecialists to understand. F.J. Cesario (1975) also complains that a lucid presentation on the potential applications in urban and regional planning is not available and the precise meaning of entropy is all but lost to the average reader. M. J. Webber (1977a) notes that the method is not widely understood partly because it is difficult to state in a non-mathematical manner and partly because it is a style of analysis that is radically different from that employed by geographers whose models of behavior are rooted in concepts from economics. Webber (1976, 1979) gives examples of misinterpretations and points out that such misunderstandings are primarily associated with the view that these models are merely analogies drawn from statistical mechanics and that they only describe systems at equilibrium. K. Haynes (et.al. 1980) stated that ambiguity and complexity in utilization of the entropy concept has been heightened by its multi-faceted heritage rooted in different sciences and mathematical forms. For these authors, geography has been a major borrower of isomorphic concepts from other fields and it appears to be more receptive and perhaps less critical in these borrowing activities. Yet, B. Berry (1978) draws a "Kuhnian Perspective" in the introduction of "The Nature of Changes in Geographical Ideas" and regards changes in geographical theory as paradigm shifts. Thus, T. Tocalis (1978) explores how the gravity concept of human interaction had great resiliency and ability to survive despite of drastic changes from Newtonian analogy to its reformulation by A. G. Wilson. To improve the theoretical basis of spatial interaction models, Wilson (1970, 1975) adopts a micro-behavioral level of resolution based on the concept of utility, as borrowed from various theories of consumer's behavior. In these investigations, Wilson shows that the entropy maximizer and the analyst of the utility-maximization will eventually arrive at the same answer. It can be noted here that the concept of utility has recently generated great controversy and debate in economics. P.Mirowski (1989) in his "More Heat Than Light: Economics As Social Physics; Physics As Nature's Economics '' ,according to A. Cohen (1992), claims that economics has largely and incompetently imitated physics and the concept of utility can be associated with the concept of potential energy of the Hamiltonian mechanics.

During the 1970s and 1980s, there were important contributions to apply the entropy concept in urban spatial context. M. Batty (1974a) in his "Spatial Entropy" develops several entropy statistics and interprets them. In one of his next papers, M. Batty (1974b) demonstrates the relations between urban population density and spatial entropy functions. E. S. Sheppard (1976) reviews first the Wilson's entropy maximizing method and then provides a Bayesian approach that relates theoretical ideas to empirical data in the spatial analysis. A. Anastassiadis (1986) derives the rank-size rule by using entropy-maximizing methods that finds relationships between the real and the estimated populations of the cities and the rank of the cities.

It should be added that Shannon-Wiener type of information represent the "Transmissional Approach" of the American Tradition that regards information as just another probability function. The British Tradition, on the other hand, has a "Meta-Semiotic Approach", developed by Mackay and Gabor, that looks for the relations between such a probability function and basic units of structural information. (D. Nauta, 1972).

Boltzmann, in 1877, conceived the remarkable idea of giving a statistical interpretation to the Second Law of Thermodynamics, i.e. "The entropy of the universe (or an isolated system) cannot decrease" (M. Dutta, 1968). C. Shannon (1948) showed that the Boltzmann's entropy function had great significance to the theory of communication. E. T. Jaynes (1957) demonstrated that the function had deeper meaning than had been supposed. (M. Tribus, 1969). According to A. Rapoport (1972), the connection between information and entropy is merely a by-product of mathematical formalism and the controversy is vacuous. C. H. Waddington (1977) also analyses the nature of the link between the Second Law and entropy. C. Joslyn (1991) states that the thermodynamic entropy concept is "content-full", but the statistical entropy is a property of probability distribution, not a real system; and therefore statistical entropy is a "content-free" concept. For C. Padet (1992), the significance of this idea is still increasing today and the analysis of connections between the interpretation of some physical (or other) phenomena and information theory; hence, every endeavor to unify the two theories is seen as a source of progress.

Hence, my second report shall concentrate and cover the following issues:

i-) Comparisons of the characteristics of the First and Second Generation Models of the urban structure developed during the 1960-1970 decade;

ii-) Review of the 1970-80 and 1980-1990 periods for the possible "Third-Generation Models" and other contributions;

iii-) Review of the developments in the Information Theory, Cybernetics and System Dynamics in relation to the urban structure concept;

iv-) Investigations on the "Spatial Entropy" methods in relation to the urban population density models.

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APPENDIX - C

SOME RECENT DEVELOPMENTS IN URBAN DENSITY FUNCTIONS (1977)

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REFERENCES AND FOOTNOTES

INTRODUCTION

The present extensive studies on the urban density functions can be grouped into three types of frameworks. The first group takes the problem of urban spatial structure from the point of statistical distribution functions. The second group of studies, entropy-maximizing methods aim to derive probability distributions subject to a set of constraints on the form of distribution. This method enables the model builder to obtain the least prejudiced model and thus extend the theoretical basis of the original hypothesis.

The third one, which is referred as the behavioral approach, uses theories of the urban land market, and is rooted in neoclassical microeconomics.

Any attempt to integrate these different approaches can be accepted as a long term strategy, and to achieve this a basis on which to begin such integration, it is necessary to review the basic issues they pose. From city planning standpoint, all three views have important implications for the development of intraurban growth and location policies.

For some authors, one of the most significant theoretical contributions to urban geography have been made as an outcome of analysis of the pattern of population distribution. With the exception of Muth's study in 1961, all researches in this subject started from the work of Colin Clark (1951), who first suggested that the pattern of population density with respect to distance from the city center has a negative exponential function. In the following sections this hypothesis is presented more formally and other family of distribution functions are introduced. The methods of the neoclassical economic theory and its basic assumptions in relation to the population distribution in an urban field are also reviewed. Figure-l

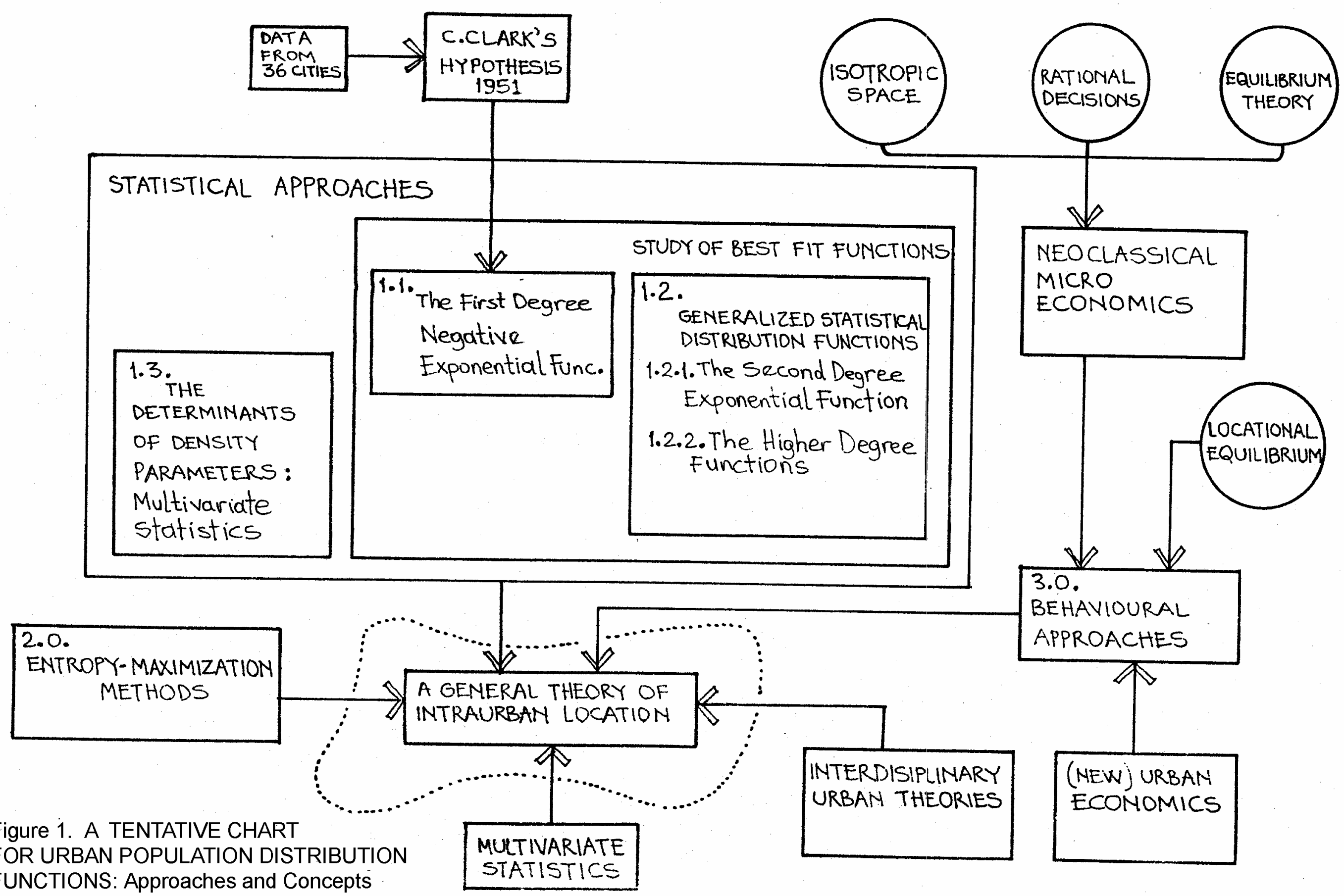


Figure 1. A TENTATIVE CHART FOR URBAN POPULATION DISTRIBUTION FUNCTIONS: Approaches and Concepts

is given to identify the related methods and research lines leading to a general theory of intraurban location and growth and numbered to correspond to those used in the sections of the paper.

This paper accepts that it has no purpose to build a "general theory", which is a formidable task, and all the discussions relate to a partial theory or model. However, its partial characteristics can be turned to advantage, because the arguments can be integrated in to a much more comprehensive framework for analyzing urban structure and growth as one of several partial theories.

Specifically, this paper aims to identify the underlying rationale concerning the "declining by distance" hypothesis from

a) Statistical,

b) Entropy-maximising, and

a) Behavioral

points of view. On the other hand, due to its exploratory nature, it hopes to raise questions about the possibility of integration of these three different views for a more comprehensive urban theory.

1.1.) HYPOTHESIS DEPENDING ON C. CLARK' S (1951) FINDINGS

The classical urban land use theories derived from urban-ecological models put forward by Burgess (1925), H. Hoyt (1939) and Harris-Ullman (1945) were highly morphological and limited in scope to explain the urban structure and growth processes. Since the contribution of Colin Clark in 1951, (1) extensive research were made to test the hypothesis imbedded in his statements and develop more comprehensive ways of explanations. C. Clark begins with two generalizations the va1idity of which he recognized as universal:

1.) In every large city, excluding the central business zone, which has few resident inhabitants, we have districts of dense population in the interior, with density falling of progressively as we proceed to the outer suburbs.

2.) In most (but not all) cities, as time goes on, density falls in the most populous inner suburbs, and to rise in the outer suburbs, and the whole city tends to “spread itself out".

He then produced evidence in support of his argument that regardless of time or place the spatial distribution of population densities within cities appears to conform to an empirically derived expression of exponential decline:

$$D_x = D_0 e^{-bx} \qquad (1)$$

where D_x is population density at distance x from the city center, D_0 is central dencity, as extrapolated, and b is the density gradient. When the natural logarithm of density is used, the equation becomes

$$\ln D_x = \ln D_0 - bx \qquad (2)$$

Clark (1951) provided 36 examples in which the equation (2) appeared to be a good fit to the sample data at his disposal. B. Berry (et.al) (1963) (2) notes that "almost a 100 cases are now (1963) available, with examples drawn from most parts of the world for the past 150 years, and no evidence has yet been advanced to counter Clark's assertion of the universal applicability of Equation-l". This consensus of opinion has been extended into system of axioms and equations by P.H. Rees (1970) (3) as follows.

<u>Axiom-l: The Decline of Population Densities With Distance From the City Center.</u>

Equations l and 2 are derived from this axiom.

Axiom-2: The Density Gradient Declines With Time

This axiom says that the density gradient falls in the course of time:

$$b_t = b_0 e^{-ct} \quad (3)$$

In natural logarithms, equation becomes:

$$\ln b_t = \ln b_0 - ct \quad (4)$$

where:

b_t = distance-density gradient at time *t*

b_0 = distance-density gradient at time *t*(0)

e = base of natural logarithms, or, 2.71818

c = exponent.

From the two axioms stated above, B. Newling (1966) (4) goes on to deduce a number of necessary consequences about the density of urban populations.

Theorem-l: The Intra-urban Growth is Allometric.

That is:

$$(1+R_x) = (1+R_0)\, e^{gx} \quad (5)$$

where:

R_x = the percentage rate of growth at distance (x)

R_0 = the percentage rate of growth at the center of the city.

g = the intra-urban growth gradient

The linear transformation of this function is:

$$\ln(1+R_x) = \ln(1+R_0) + gx \tag{6}$$

<u>Theorem-2: The Growth Rate of Density is Directly Related to the Level of Density</u>

That is:

$$(1+R_D) = AD^{-k} \tag{7}$$

where:

R_D = the percentage rate of growth during a given period when the density at the beginning of the period is D;

A = constant

k = the ratio of the intra-urban growth gradient (g) to the population density gradient (b).

B. Newling(1966) indicates that Equation-7 above, therefore, arrives deductively at a formal statement of the relationship between population density and the rate of growth, and, other things being equal, it shows that the two variables are inversely related.

<u>Theorem-3: There is A Critical Density Above Which Growth is Negative and Below Which Growth is Positive.</u>

By analyzing the results for Kingston, Jamaica, he argues for a link between such a critical density and social conditions. (5)

B. Newling (1966) shows that integration of Eq.-1 gives the total population within a given distance (x) from the center of the city as the solid of revolution generated by the density curve about the vertical axis. (Fig.-2): Hence,

$$Population(x) = \int_0^x D_0 e^{-br} (2\pi r)\, dr \tag{8}$$

which is evaluated as

$$Population(x) = 2\pi D_0 b^{r^2} \left[1 - e^{-bx}(1+bx)\right] \tag{9}$$

If we solve for (x) in Eq.-2 and substitute the solution, the Eq.-9 can be rewritten in terms of densities D_0 and D_x and gradient (b).

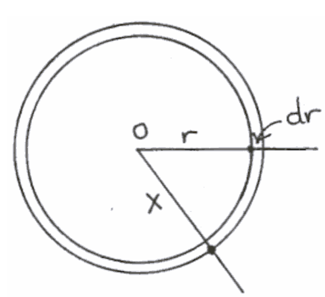


FIGURE.2

The calculation of total urban population requires an assumption for the peripheral (marginal) density. Usually, the urban density at the perimeter is taken as 8 persons per hectare (6) . However, E. Mills (1972, page 39) lets x goes to infinity to get the total number of people in the entire metropolitan area, which theoretically means peripheral density goes to zero. Hence, Eq.-9 becomes:

$$Population\left(\underset{x\to\infty}{x}\right)=\frac{2\pi D_0}{b^2} \tag{10}$$

E. Mills asserts that the assumption of zero density at the perimeter has a small bias. The two parameters, D_0, indicating concentration or crowding at the center; and b indicating compactness vary from city to city. In any temporal cross-section, central density appears to be determined by the growth history of the city up to that time; and the density gradient appears to be a function of city size. B.Berry (et.al.)(1963) makes cross-sectional and temporal comparisons among selected Western and Non-Western (particularly Indian) cities. Figure-3 shows how Western and Non-Western cities differ in the ways in which D_0 and b change through time. More recent research by John E. Brush (1970) (8) support to Berry's generalization that Non-Western cities follow a pattern of concentrated growth and increasing residential congestion in contrast to the Western patterns of growth, which are accompanied by residential dispersion or suburbanization.

1.2.) GENERALIZED STATISTICAL DISTRIBUTION FUNCTIONS

1.2.1.) The Second-Degree Exponential Curve

B. Newling (1969)(9) notes that the first revision of Clark's model was proposed J.C. Tanner in 1961 and by G. Sherratt in 1960 who suggested that urban population densities decline exponentially as the square of distance such that

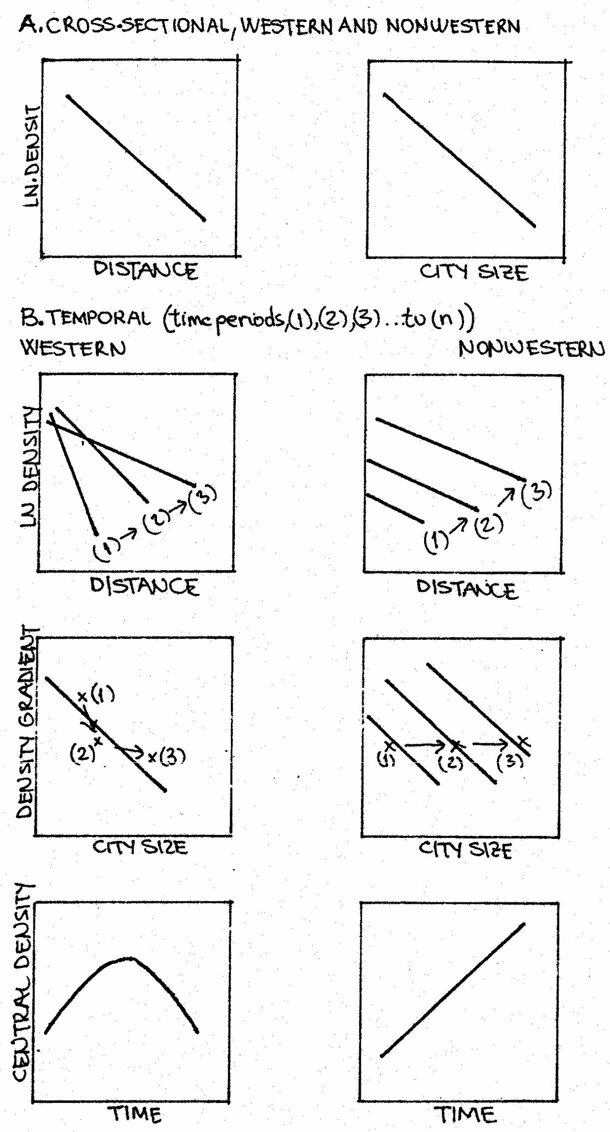


Figure 3. CROSS SECTIONAL AND TEMPORAL COMPARISONS

$$D_x = D_0 e^{-cx^2} \tag{11}$$

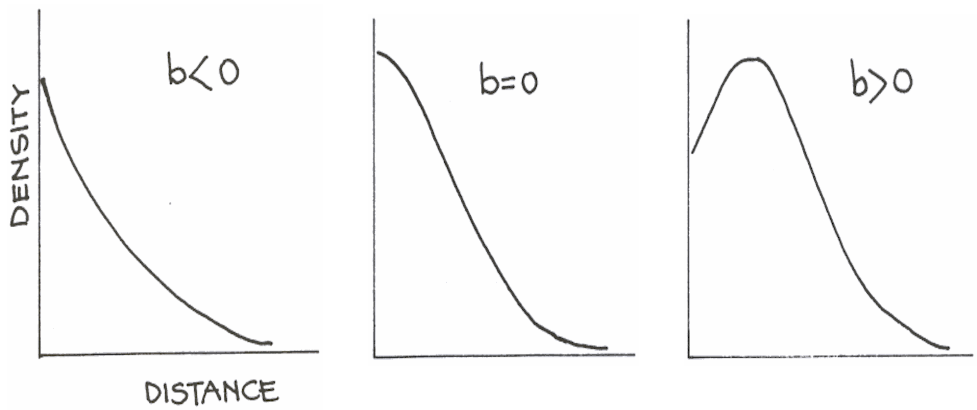
where c is a measure of the rate of change of the logarithm of density with distance squared. The density profile described by the Tanner-Sherratt model is thus a half-bell curve, and its logarithmic transformation produces a half parabola, concave downward. Tanner says of Clark's model that "examination and further analysis of Clark's data show that the plotted points tend to lie on curves, not on straight lines, on the log-linear diagrams, and that density in fact falls off very nearly exponentially with the square of the distance from the center". (L. March's citation, 1972) (l0).

Sherratt used the second-degree density model mainly of dwellings in Sydney. (1960)(11)

It has been suggested by Newling (1969) that a quadratic regression of the logarithm of density on distance is a better generalization, because it accounts for the density center at the center of the city. Eq.-l would thus become:

$$D_x = D_0 e^{bx-cx^2} \tag{12}$$

Figures-4, 5, and 6 show graphically where b can have either a positive or a negative sign in Eq.-12.



FIGURES - 4, 5, 6

The Tanner-Sherratt model is a special case of the more general quadratic exponential model, in which the linear term is equal to zero (Fig.-5). Newling also associates the absence of a density crater with an early stage of development and its presence with a later stage. Thus, by considering the sequential development of the city he shows how the rise and decline of density at the center of the city is most simply described if it is assumed that the central density is a quadratic exponential function of time, such that

$$D_{0,t} = D_{0,0}e^{mt-nt^2} \tag{13}$$

where,

$D_{0,t}$ = the central density at time t

m = A measure of the initial instantaneous rate of growth of the central density with time

n = a measure of the rate of change of the rate of growth with the passage of time.

After studying the shop locations aggregated by 250-meter rings, D. Sibley (1970) (12) finds that the second-degree function clearly constitutes a good model for Leicester (England).

In a more recent study,(1975), the second-degree negative exponential model was found to have better descriptive capabilities of density patterns of Tel-Aviv metropolitan area than the first-degree Clark function (13).

1.2.2.) The Higher Degree Functions

E. Casetti (1969)(4) develops procedures for determining which are among alternative families of functions, is more suited to given data. For 6 cities of different time periods, he carried out (a) on the complete sets of data, (b) on the data that refer to the central portion of the city, and (c) on the data referring to the outskirts and rural fringe. For each set of data the following regression was estimated:

$$D_x = (1+x)^n \exp\left(h + ax + bx^2 + cx^3\right) \tag{14}$$

Thus, by using Eq.-14 he determined which one of the fifteen families of functions subsumed in that equation is best suited to given data.

E. Casettits results supported weakly his first hypothesis that density crater functions are better suited to data for large cities than functions involving only a density decline, but supported strongly the second hypothesis that in large cities different functions are suited to different distance bands. He concluded that perhaps the negative exponential of first-degree, that lies between exponential functions of higher degree, better suited to central areas, and functions of lower degree, suited to peripheral areas, deserves its popularity because it is a compromise solution.

L. March (1971)(15) attempts to obtain a generalized distribution function and analyses first ten frequency distribution functions used in urban and regional studies in the form of

$$m\left(x,a,b,c\right) \qquad (15)$$

He notes that his ten fuctions are instances of

$$\mathsf{Y} = x^{a}\exp\left(-x^{b}\right) \qquad (16)$$

for particular values or ranges of a and b, where Y is related to the populatiun located at a distance x and where a and b are parameters.

L. March suggests, then, in its most general form, the generalized distribution £unction to be discussed has three parameters, a, b, c, and can be written

$$m\left(x,a,b,c\right) = \begin{cases} \dfrac{bc^{a/b}}{\Gamma\left(a/b\right)}x^{a-1}\exp\left(-cx^{b}\right), & x>0 \\ 0, & x\leq 0 \end{cases} \qquad (17)$$

where a, b, c > 0.

In Eq.-17, above ,

$m\left(x\right)\geq 0$ and if we let

$u = x^{b}$, then,

$\int_{-\infty}^{\infty} m\left(x\right)dx = 1$

since the substitution transforms *m*(x) to a simple gamma distribution in u. (16,17)

In addition to Newling's (1969) analysis of sequential developments of the city, the treatment of time and patterns of urban growth is seen in J. Amson's (1974)

(18) and J.Kain's studies. Amson describes first “equilibrium" modes of growth and shows how distance, density and price or rental of bundle of housing commodities functionally related. His "catastrophic" (or, decaying) modes of change are characterised by three distributions, i.e., density, rental bundle and a measure of civic wealth.

J. Kain and D. Harrison (1975) (19) propose an alternative model which emphasizes the durability of residential and non-residential capital and the "disequilibrium" nature of urban growth. Their alternative model depicts urban growth as a layering process and urban spatial structure at any point of time as the result of a cumulative process spanning decades. The density of a particular urban area at a point in time is then the sum of the density of development in each time period:

$$D_t = \frac{\sum_{i=0}^{t} D_i \, u_i}{\sum_{i=0}^{t} u_i} \qquad (18)$$

$(i),(t)$ = time period

D_t = average net residential density of the metropolitan area, at time period t,

D_i = net residential density of the dwelling units added to the area,

u_i = number of dwelling units

In contrast to the theoretical models developed by Muth and Mills, their formulation above implies no particular functional form for the relationship between distance and gross density. But to make their estimated density functions comparable to those reported by earlier authors, Kain and Harrison use the negative exponential function, shown by Eq.-l.

Although Clark and Newling are drawing some conclusions by interpreting their density functions and their gradients, their conclusions do not seem persuasive. Mills (1970)(20) points out that Clark's study is deficient in several ways: Because he does not indicate whether his densities are net or gross, how he handles topographical irregularities, and how he identifies CBD. His statistical procedure presents no multiple correlation coefficients, or tests for the linearity of his logarithmic regression equations. Mills also question Clark's conclusions about the relationship between transportation cost and the density pattern. Density profiles of Newling (1969) can be helpful, as he puts, “in comparisons between cities at any given time", or "can be associated with the movement of people from the central city to the suburbs”, but since the analysis do not have the explanatory variables, similar questions may also be raised.

In the previous sections we saw how Casetti and March provided us with a set of distribution functions which appear to fit empirical observations, and showed the communality of these functions. Both authors are not concerned with arguments for or against a casual principle, they point that the variety of these formulations poses a problem of choice that is not solved by relating families of density-distance functions to particular theoretical frameworks developed by Wingo {1961), Alonso (1964), Muth (1961, 1968), or the like. In fact, such a view cannot be tenable if we take what is said literary. Interestingly, independent research modes have resulted in the same or similar conclusions that the general pattern of population distribution follows the negative exponential function. Therefore, we shall not assume an independent "problem of choice”, but how the functions can be interrelated and integrated if possible with the findings of the independent disiplines. In the next sections this point shall be elaborated.

1.3.) THE DETERMINANTS OF DENSITY PARAMETERS: MULTIVARIATE STATISTICS

Determining the relationship between density and various characteristics of the city is highly complex and can be explored by the techniques of multivariate statistics (See Fig-l). For example, holding size constant, central density *D*(0) is related to the density gradient, which in turn may be influenced by a variety of additional factors. B. Berry (et.al.,1963) reports that Muth {1961) has provided the parameters *D*(0) and (b) for United States cities in 1950 and found that densities near the city center was a function of age and as a composite surropate for these other factors, of density gradient. A regression equation yielded the expression

$$D_0 = 0.5302 + 0.6362Age - 3.495b^{-1} \tag{19}$$

Both age, i.e., years since the city reached a population of 50,000, and density gradient were significant at the 0.01 level and 6l% of the variance of D_0 was accounted for. (21)

To provide an explanation for the determinants of urban density, H. Winsborough (1962)(22) develops an "accounting system" which gross population densities are regarded as the product of a series of components, themselves made up of a more basic set of variables. Thus Winsborough expresses the density variable with multiplicative components:

$$\frac{\text{Population}}{\text{Area}} = \frac{\text{Population}}{\text{Dwelling Units}} \times \frac{\text{Dwelling Units}}{\text{Structures}} \times \frac{\text{Structures}}{\text{Area}} \tag{20}$$

O.D. Duncan (1966)(23) explains how Winsborough develops a path analysis method. R. Treadway (1969)(24) also decomposes density into components in a similar manner. The effect of various housing and population characteristics on population density by distance from the center is examined by further expanding the negative exponential model. Population density (P/A) of a subarea has the following components:

$$\frac{U}{A} = \text{The ratio of total housing units to area}$$

$$\frac{O}{U} = \text{The ratio of occupied to total units}$$

$$\frac{H}{O} = \text{The ratio of household population to occupied housing units} \tag{23}$$

$$\frac{P}{H} = \text{The ratio of total to household population}$$

The interaction of these housing and population characteristics is given by the equation:

$$\frac{P}{A} = \frac{U}{A} \times \frac{O}{U} \times \frac{H}{O} \times \frac{P}{H} \tag{24}$$

Each component of population density influences the steepness of the population-density gradient to some extent; some make the decline of population density with distance from center more abrupt while others retard its decline. Since the gradients of the components of population density sum to the gradient of population density, the proportion of the gradient of population density that the gradient of each component comprises can be computed by dividing the component gradient by the population-density gradient. By this method R. Treadway determines the comparative importance and change by components between 1950 and 1960 for five U.S. cities.

B. Edmonston (1975)(25) groups the determinants of urban population density into 3 categories. Variables in Category-I are exogenous and their values are determined outside the system of relationships being investigated. Variables in Category-II are endogenous and their values are assumed to be determined by the exogenous variables. Finally, the density gradient in Category-III can be affected either directly by Category-I and Category-II variables or indirectly by Category-I variables through their effects on Category-II variables. (Fig.-7)

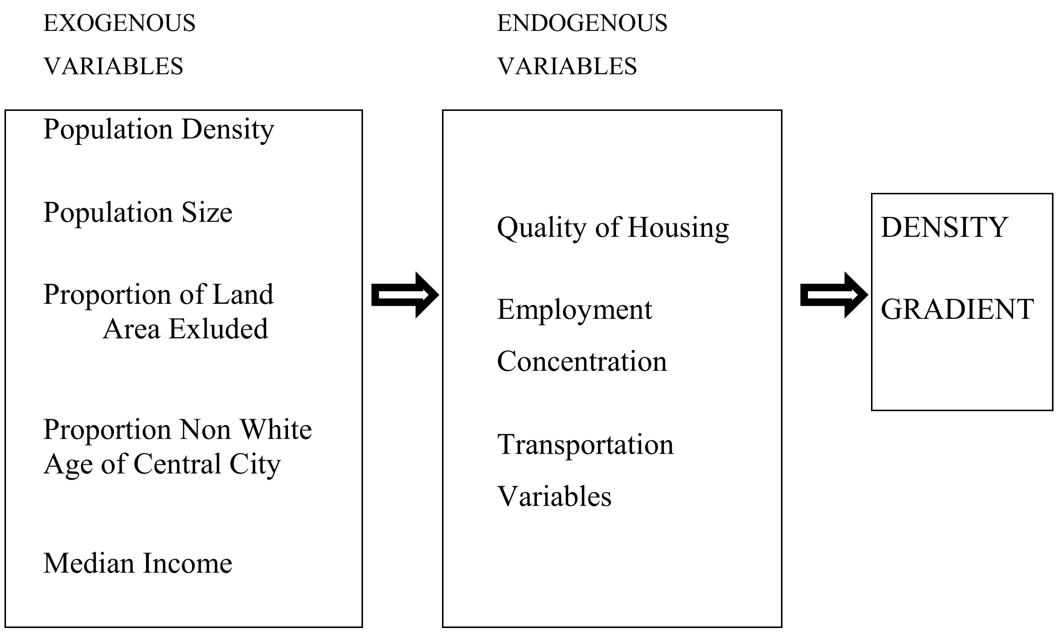


Figure-7. Three Categories of Variables That Affect the Density Gradient (Edmonston,1975)

Depending on the hypothesized relationships, Edmonston (1975) develops a recursive and 4 alternative non-recursive simultaneous models for U.S. cities between 1950-1970, and concludes that population concentration appear to stem largely from demographic factors, i.e., from population density, population size, and the age of the central city.

Edmonston uses the negative exponential function as originally presented by Clark (1951) as given in Eq.-l, and assumes that this is a good approximation.

In Winsborough’s and Treadway’s studies mentioned above, we see that the negative gradient of population density results, but it is necessary to explain why the proportion of land devoted to residences varies with distance from the city center and why the densities of people on residential land fall exponentially.

Mills (1972, p.38) indicates that the major problems are to decide which variables should be taken to be exogenous determinants of the density function parameters, the direction of the effect, and the form of the relationship. Since many explanatory variables in land use and land value theories are really endogenous to the urban economy, the problem whether a variable is endogenous or predetermined depends on the details of a simultaneous equation systems -and as Mills further notes, few urban economists are accustomed to thinking in terms of simultaneous equation systems.

2.0.) ENTROPY - MAXIMISING METHODS

Entropy-maximising methods are consistently related to a variety of maximizing methods used in urban modelling during the last decade and pioneered in this field by A.G. Wilson since 1967 (1967, 1970a, 1970b, 1974) (26, 27, 28, 29). Wilson (27) gives 3 alternative interpretations of "entropy”. The first alternative relates entropy to probability and uncertainty - which suits the framework of this paper. The second one shows how Shannon's (1948) information function is maximized subject to constraints. By maximizing entropy, or uncertainty, one is able to derive the most uncertain probable distribution which implies that information is minimized. Thirdly, Wilson relates the Bayesian methods of statistical inference to entropy. The next section gives some space for the first one.

2.1.) ENTROPY AS PROBABILITY AND UNCERTAINITY

A "state” of the system can be described at the microlevel with a high degree of detail and observing the behavior of each individual particle, or, at the macrolevel with a desired degree of accuracy to explain certain macroproperties of the system. Statistical mechanics is a branch of physics developed to study the state of a system, without having to refer to the behavior of each element. It can be proved and imagined that many microstates can give rise to the macrostate. For example, a total-transport-cost contains less information than the trip-distribution macrostate description, and so many trip distributions can give rise to the same locational distribution. If all microstates are equally probable, we can find the most probable state by calculating the number of microstates associated with each macrostate. The method establishes that the most probable macrostate of any system is the one which satisfies the known constraints and which maximizes its entropy. Maximum entropy is achieved by the macrostate which can be arrived at in the maximum number of microstates.

The mixed, or disordered state of, say, white and black powders in a jar, is more probable. This statistical probability is a measure of the number of different ways a given situation can occur. The greater its statistical probability, randomness and entropy. Entropy is the logarithm of a probability.(30)

To help the reader of Wilson's difficult work, "Entropy in Urban and Regional Modelling", (1970), P. Gould (1972)(31) elaborates some of the concepts presented.

M.H. Mogridge (1972)(32) points out that where we have large, complex and interacting systems, such as in an urban region, we must use theories based on the laws of large numbers and on probabilities. He notes how the maximum entropy distributions, the gamma, and the negative exponential distributions can be related under (dynamic) equilibrium conditions.

To indicate the relationship between entropy-maximizing methods and the negative exponential distribution further, we shall note Wilson's arguments (1970) in his explanation for the deterrence function, $F(c_{ij})$ used in the gravity models.

If O_i and D_j are given, the interaction T(i,j) can be estimated by

$$T(i,j) = A_i B_j O_i D_j F(c_{i,j}) \tag{25}$$

Where;

$$\sum_{j} T_{ij} = O_i \tag{26}$$

$$\sum_{i} T_{ij} = D_j \tag{27}$$

$$\sum T_{ij} c_{ij} = C \tag{28}$$

should be satisfied. The A_i's B_j's in Equation-25 are calculated to ensure that these equations are satisfied. The function $F(c_{i,j})$ represents the effect of travel costs and C the total expenditure. Hence the number of microstates $W(T_{i,j})$ giving rise to the most probable $T(i, j)$ is obtained by maximizing

$$\log W = \log \frac{T!}{\prod_{i,j} T(i,j)!} \tag{29}$$

subject to above equations. The maximum is obtained when

$$T(i,j) = A_i B_j O_i D_j \exp(-\beta c_{i,j}) \tag{30}$$

This equation has a negative exponential function as deterrence function. Wilson (1970) further notes that, this does not mean, according to the entropy-maximizing method, only the negative exponential function can be used as the deterrence function, but it does help us to interpret the deterrence function and to relate it to hypotheses about behavior.

The negative exponential function (NEF) arises because of the nature of the constraint Equation-28. In this equation it has been assumed so far that travellers perceive costs in the way in which they are (objectively) measured. This is a hypothesis about behavior. Thus if this is correct, then the entropy-maximizing method suggests that the NEF is the correct form of deterrence function. the NEF is likely to fit best where trip costs are generally small, while in study areas where trip costs are larger, a power function may give a better fit.

2.2.) TESTING C.CLARK'S HYPOTHESIS BY THE ENTROPY-MAXIMIZING METHOD

In this section, we shall review how Clark's hypothesis has been given a theoretical explanation by L. March (1972) (10) and discuss if the conclusions can be linked to the behavioral lines of thought. (Fig.-l) Summary of the procedure is as follows.

An urban region is divided into a number of discrete zones. Z(i). The population P persons is distributed in such a way that the probability for a person residing in Z(i) is proportional to the developed land area of Z{i). If this area is L(i) and the total area of the urban region is L, then we assume that the "a priori" probability is $k_i = L(i)/L$.

If P_1 persons reside in Z_1, P_2 persons in Z_2,............, we may speak of this particular distribution as a P(i) distribution defining the probability N($P_1, P_2, \ldots\ldots\ldots, P_n$), or, W($P_i$), as the proportion of all possible arrangements where this distribution occurs :

$$W(P_i) = KP!\prod_i \frac{k_i^{P_i}}{P_i!} \tag{31}$$

where K is a norma1ization constant. This fomulation is derived as finding the number of selection of P_1 persons but of P and multiplying the result by the a priori probability.

$$k_1^{P_1} k_2^{P_2} \ldots\ldots\ldots\ldots\ldots k_n^{P_n}$$

where

$$\sum_i k_i = 1 \tag{32}$$

The distribution which maximizes W(P_i), subject to the constraints

$$\sum_i P_i = P \tag{33}$$

$$\sum_i P_i c_i = R = \text{ Total Cost} \tag{34}$$

where c_i is the cost of locating in Z_i and we assume that the tota cost, R, to all residential location is constant for the region.

The most probable distribution will occur in that state where the possible number of arrangements is greatest.

We note that it is more convenient to maximize logW instead of W. From Eq.-31 we have

$$\log W = \log P! + \sum_i \left(P_i \log k_i - \log P_i!\right) + \text{ Constant} \tag{35}$$

To estimate a factorial term of a large number P, we may use Stirling's approximation:

$$\log P! \approx P \log P - P \tag{36}$$

Thus, by substituting, and considering constraint Eq.-32. and 33

$$\log W = -\sum_i P_i \log\left(\frac{P_i}{pk_i}\right) + \text{ constant} \tag{37}$$

By differentiating the constraints and the Eq.-37 , and applying the method of Lagrange multipliers,

$$\sum_i \partial P_i = 0$$

$$\sum_i c_i \, \partial P_i = 0 \tag{38}$$

$$\sum_i \left(\log\left(\frac{P_i}{pk_i} \right) + 1 \right) \partial P_i = 0$$

By applying the method of Lagrange multipliers λ for the first equation, and β for the second and adding all we obtain after simplifying

$$\log\left(\frac{P_i}{pk_i} \right) + \lambda + \beta c_i = 0 \tag{39}$$

$$\log P_i - \log P \, k_i + \lambda + \beta c_i = 0 \tag{40}$$

$$\log P_i = \log P \, k_i - \lambda - \beta c_i \tag{41}$$

This can be expressed as;

$$P_i = Pk_i \, \exp(-\lambda - \beta c_i) \tag{42}$$

$$\sum_i P_i = P = P \exp(-\lambda) \sum_i k_i \, \exp(-\beta c_i) \tag{43}$$

Usin the partition function (as explained by Wilson (1970b, p.l37))

$$e^{-\lambda} = \exp(-\lambda) = \left(\sum_i k_i \exp(-\beta c_i) \right)^{-1} = Q^{-1} \tag{44}$$

$$P_i = \frac{P\, k_i \exp(-\beta c_i)}{Q} \tag{45}$$

This is the fundmental equation of the location of population in an urban region subject only to the above constraints. In this equation there are no limitations on the form of the urban region considered.

To obtain Clark's equation, we must make further assumptions. Let;

a-) A monocentric urban region is divided into annuli of equal unit width about a single centre. (Fig.- 7)

b-) Z_1 is the first ring, Z_2 the next .(i =1, 2,........,N)

c-) The total land area to be perfectly homogeneous and equally available to development. (i.e., assumption of the Euclidean Space).

d-) The cost of location is directly proportional to the distance of the Z_i -th zone from the centre so that

$$c_i = (b/\beta)\, i \tag{46}$$

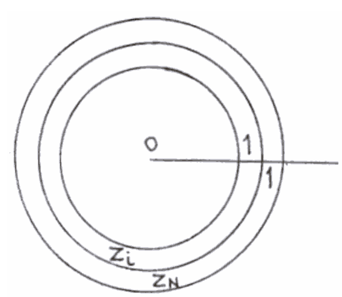


Figure – 7

Land area $L_i = 2\pi i \times 1$

Let $k_i = \frac{L_i}{L} = \frac{2\pi i}{L}$ as before .

We may rewrite Eq.-45 in the form

$$P_i = P\frac{2\pi i}{L} \cdot \frac{\exp(-bi)}{Q} \qquad (47)$$

$$P_i = \frac{P\ i\ \exp(-bi)}{Q'} \qquad (47)$$

where $Q' = \sum_i i\ \exp(-bi)$

in a similar fashion to partition function.

By taking the integral we may arrive at a good approximation for Q' .

$$Q' \approx \int_0^\infty x\ \exp(-bx)\ dx = \frac{1}{b^2}$$

Hence,

$$P_i \approx Pb^2 i \exp(-bi) \qquad (49)$$

The density D_i at i is dependent on the area $2\pi i$ of the annulus at (i), thus

$$D_i = \frac{Pb^2}{2\pi}\exp(-bi) \qquad (50)$$

which for i = x (continuous) reduces to Clark's Eq.-l. (For $D_0 = \frac{Pb^2}{2\pi}$, same equation obtained.)

The mean radial distance to the centre can be defined as:

$$\bar{x} = \frac{\sum_i P_i\ i}{\sum_i P_i} \qquad (51)$$

$$\bar{x} = b^2 \sum_i i^2 \exp(-bi) \tag{52}$$

Once again, we may approximate to the summation by using an integral approximation:

$$\int_0^\infty x^2 \exp(-bx)\,dx = \frac{2}{b^3} \tag{53}$$

We have from Eq.-52

$$\bar{x} = b^2 \frac{2}{b^3}, \text{ or, } b = \frac{2}{\bar{\bar{x}}} \tag{54}$$

whence Eq.-50 can be rewritten

$$D_i = \frac{2P}{\pi \bar{x}^2} \exp\left(\frac{-2i}{\bar{\bar{x}}}\right) \tag{55}$$

Thus we see that in an abstract (isotropic) space, where the conditions described above, Clark's parameter D_0 is directly proportional to the "density" of population with respect to a circular area of radius equal to the mean radial distance to the center, while parameter b is inversely proportional to the mean radial distance to the centre.(Eq.-54).

If we take natural logarithm in Eq.-55 we obtain a linear equation as we studied previously:

$$\ln D_i = \ln \frac{2P}{\pi} - 2\ln \bar{x} - \frac{2i}{\bar{\bar{x}}} \tag{56}$$

The assumptions to derive Clark's equation included the implication that "<u>Cost of Location</u>" is linearly related to the distance of residence away from the city centre.

If a "square-rate" city where cost of location is proportional to the square of the distance, Tanner's function can be derived. Above, the cost of location comprises two elements, the cost of land at the location and the cost of being at a distance from the centre. In a linear-rate city in which

rents decline from the centre and transportation costs increase, three possible curves (lines) for the cost of location depending on the relative gradients of the two components. For a situation in which rent and transport costs are non-linear, a second degree curve may give a better fit.

By using the entropy-maximizing method. L. March (1972) derives also Tanner's (196l) and M. Echenique's (1968) (32) hypothesis that floorspace ratio decays exponentially with distance. However, March concludes that the four hypothesis he tested in the most general situation -i.e., in heterogenous, polycentric urban regions -"beg the all important question of what we mean by cost of location".

Solutions to similar questions are expected to be provided by the behavioral approach; but this approach is not also free from important difficulties. Next section shall indicate some of them.

3.0.) BEHAVIORAL MODELS: Population Densities in Relation to Land Rent and Urban Land Use Theories

In the preceeding sections, two approaches were introduced and their contributions in terms of residential densities we argued. As the theoretical basis is concerned, we saw in what areas the previous approaches needed supports from land rent and urban land use theories. In these theories the boundary is large and loose; however, they aim to relate the urban land use and population distribution patterns to the consumer and producer behavior theories, utility functions, scale economies or other complex concepts.

To provide a theoretical rationale for the declining densities (33) is a difficult and interdisiplinary task. A general agreement existing among the urban economists can be stated as follows:

1-) Sites within cities offer two goods, i.e., land and location. Each urban activity derives utility from a site in accordance with the location of the site.

2-) Utility may be translated into ability to pay for that site.

3-) The must desirable locational property uf urban sites is centrality. The center has the maximum accessibility in the urban area.

4-) For any use, ability to pay directly related to centrality. The less central the location, the greater are the transport inputs incurred and the lower the net returns.

5-) Bid-rent functions thus decline with distance. However, the intercept (utility obtained from maximum centrality) and the slope (rent-distance trade-off) of this function differ for different activities, and in competitive locational equilibrium, with each site occupied by the use that pays most for the land.

6-) Land prices diminish outward; and as they do, land inputs will be substituted for other intensity of land use will diminish. If substitution between capital and land would not be possible, land use pattern and density would be the same every location, additionally, the rent-distance function would be linear.

7-) As a result of these factors, declining residential densities should be expected.

The basic steps outlined, is also given in a similar manner by B.Berry and F.Horton (1970, pp.299-302).

Most parts of a city are occupied by residential land uses of different kinds. Alonso (1964)(34) has shown that bid-rent functions are steeper for the poorer of any pair of households with identical tastes. Hence, in equilibrium, one expects the poor to live near the center on expensive land, consuming little of it, and the rich at the periphery, consuming more of it, (35). Since land consumed by each household increases with distance from the city center, population densities must drop, with due allowance for variation in size of household.

Muth (1961, 1969) (36) develops a model in which price per unit of housing, rent per unit of land, and output of housing per unit of land all decline; and per capita consumption of housing increases with distance from the city center. Net housing density must therefore also decline. Moreover, if the price-distance function is assumed to be negative exponential and the production function for housing logarithmically linear with constant returns to scale, then net population density must decline negative exponentially with distance from the city center.

J. Niedercorn (1971)(37) by assuming that the transport function implies a declining marigina1 costs with distance, shows that land rents follow a NEF. He also concludes that both net and gross employment densities and both net and gross population densities have an approximately NEF distribution.

E. Mills (1967)(1972)(38) presents a much more sophisticated version of this model for the derivation of land rent from transport cost, given certain simplifying assumptions. From a general equilibrium model encompassing production and transportation (with residential use) he deduces that:

$$R_x = R_0 e^{-bx} \qquad (57)$$

Where,

R_x = land rent at distance x

R_0 = a constant of integration interpreted as land rent at the city center.

Mills (1967) also deduces that all land use densities are proportional to land rent. In this model Mills almost excludes the demand side to focus attention on input substitution and technology; assumes the Cobb-Douglas production function.

4.0.) CONCLUSIONS: SOME MORE OBSERVATIONS

The three approaches that we studied in relation to the urban density functions may develop in their own ways: statistics may show better functions fitting to a given set of data; entropy-maximizing (i.e., information-minimizing) methods may indicate how to deal large and complex systems under constraints, and by developments in intraurban location theories, more realistic causal models may be build.

Urban economics is a developing science. H.Richardson (1976)(39) evaluates the branch of urban economic theory labelled the "New Urban Economics" that attempts to integrate welfare economics and urban economics within a general equilibrium framework. For Richardson, the partial equilibrium approach of the triumvirate of Alonso (1964) , Wingo {1961) and Muth(1961, 1969) are heralds of rather than participants in the general equilibrium model of the Nev Urban Economics. Richardson points out that most would mark either E. Mills' pap13r (1967) in the American Economic Review or M. Beckmann's model (1969)(40) published in the Journal of Economic Theory (which evaluated by A. Montesano (1972)(4l)) mark the true beginning of a new age.

Although there are important and recent developments in the field as summarized by Richardson (1976) well, we can classify the present studies also relating to urban densities into three groups.

i-) The first group of authors attempt to determine housing demand. Wingo's (1961)(42), and Alonso's (1964) models are well-known examples. Recently Beckmann (1969) associates himself with t.his group.

ii-) The second group approaches the problem from producer's point of view. Muth (1961, 1969) and Mills (1967, 1972) are the pioneering representatives.

iii-) The third group is concerned more with integrating these approaches and aiming to build dynamic models by relaxing the previous assumptions of microeconomics and urban structure models. Next step seems to develop a satisfactory explanation of the growth of subcenters as discussed by G. Papageorgiou (197l)(43) and Casetti (197l)(44). A. Evans (1975)(45) shows how buildings of different ages and heights can coexist in a growing city in dynamic equilibrium. Monocentricity and isotropic space assumptions are the key ones to be relaxed. Utility functions, treatment of externalities and mathematical techniques are also being reviewed. H.Richardson's recent article on "Discontinuous Densities" (1975)(46) attempts to introduce new analogies.

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27-) Wilson, A.G. – "Advances and Problems in Distribution Modelling", Transportation Research, Vol.4, 1970, (pp.1-18)

28-) Wilson, A.G. – Entropy in Urban and Regional Modelling, Pion Limited, London, 1970

29-) Wilson, A.G. – Urban and Regional Models in Geography and Planning, John Wiley and Sons, London, 1974

30-) Probability, *W*, is a multiplicative function, while entropy, *S*, is an additive function. The total probability is obtained by multiplying the individual probabilities;

$$W = W_1 \times W_2 \times .... \times W_n$$

while the total entropy is the sum of the individual entropies;

$$S = S_1 + S_2 + ... + S_n$$

The logarithm of a mulriplicative function is th sum of the individual logarithms;

$$\log W = \log W_1 + \log W_2 + ... + \log W_n$$

Thus, it is proposed that entropy, *S*, should be proportional to the logarithm of probability, log*W*. This simple but most important relationship was proposed in the late 1800's by Ludwig Boltzmann, and it is one of the fundamental equations of statistical thermodynamics. (S = k logW is inscribed on his gravestone)

31-) Gould, P. – "Pedagogic Review", Annals of the Association of American Geographers, Vo.62, No.4, 1972, (pp.689-700)

32-) Echenique, M. – Urban Systems: Towards An Explorative Model, LUBFS Working Paper 7, 1968, (reprinted as CES UWP-2, 1971), London

33-) H. Richardson (1975) seems to discuss increasing curves: "On the Possibility of Positive Rent Gradients", Journal of Urban Economics, (not available at METU library)

34-) Alonso, W. – Location and Land Use, Harvard University Press, 1964

35-) This is not true for the case of Turkish cities: The periphery is generally surrounded by "gecekondu" areas.

36-) Muth, R. – Cities and Housing, The Univ. of Chicago Press, 1969

37-) Niedercorn, J. – "a Negative Exponential Model of Urban Land Use Densities and Its Implications for Metropolitan Development", Journal of Regional Science, Vol.11, No.3, 1971, (pp.317-326)

38-) Mills, E. – "An Aggregative Model of Resource Allocation in Metropolitan Area", American Economic Review, Vol.57, 1967, (pp.197-210) (235-241)

39-) Richardson, H. – "The New Urban Economics: An Evaluation", Socio-Economic Planning Sciences, Vol.10, 1976, (pp.137-147)

40-) Beckmann, M. – "On the Distribution of Urban Rent and Residential Densities", Journal of Economic Theory, Vol.1, 1969, (pp.60-67)

41-) Montesano, A. – "A Restatement of Beckmann's Model on the Distribution of Urban Rent.", Journal of Economic Theory, Vol.4, 1972, (pp.329-354)

42-) Wingo, L. – Transportation and Urban Land, Resources for the Future, Wash. D.C., 1961

43-) Papageorgiou, W. – "The Population Density and Rent Distribution Models within A Multicentre Framework", Environment and Planning, Vol.3, 1971, (pp.267-282)

44-) Casetti, E. – "Equilibrium Land and Population Densities in An Urban Setting", Economic Geography, Vol.47, 1971, (pp.16-20)

45-) Evans, A. – "Rent and Housing in the Theory of Urban Growth", Journal of Regional Science, Vol.15, No.2, 1975, (pp.113-125)

46-) Richardson, H. – "Discontinuous Densities, Urban Spatial Structure and Growth: A New Approach", Land Economics, Nov.1975, Vol.51, (pp.305-315)

CURRICULUM VITAE

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EDUCATIONAL BACKGROUND

Middle East Technical University (METU), Ankara, **B.Architecture**, 1962; **M.Architecture** (M.Arch), 1963

Pratt Institute, New York, USA, **M.Science in Tropical Architecture**, 1965

Middle East Technical University (METU), Ankara, **M.City Planning**, 1966

LANGUAGES

English & French (Fair)

FIELDS OF INTEREST

Urban / Environmental Spatial Planning (1/250 000 to 1/1000 metric scales), Urban Renewal Planning, Urban Design, Sustainable Urban / Environmental Development, Systems Theory & Urban Modelling, Cybernetics, Urban Dynamics, Information Theory.

TEACHING : COURSES GIVEN

City Planning Studies: Where planning theories & methods are applied to real-life issues; at undergraduate and graduate studios, between years 1966-1977 & 1980-1999 Academic Years

Urban Planning Techniques & Research (CP 341-342), 1966-1979
Computer Applications In Urban Planning (CP 380), 1979/80 & 1980-82 (As a new course)

Urban System Dynamics (CP 384), 1980-84, (Development of a new course)
Models In Urban Planning (CP 451-452), 1982-2004
Second Year Section B-Studios (CP 201/B-202/B), 1984-89 (Development of an experimental studio).

Gazi University, Department of City & Regional Planning; Research Methods & Techniques (ŞP.331-332), & Planning Studios, (Assignment by the Higher Education Law No: 2547, Article: 40/b : Cooperation with other Universities), 1990-92.

Urban Design (Graduate) Studios (UD501-502), 2000-2004

EMPLOYMENT & WORK EXPERIENCE

March-September 1962: Bank of Provinces (İller Bankası), General Directorate, Ankara (as architect & planner)

1963-1964 : Ministry of Redevelopment & Resettlement, Regional Planning Directorate, Ankara (as architect & planner)

July-September 1965: United Nations, The Secretariat, Social & Economic Planning Section, New York, USA

September 1966-2004: METU, Faculty of Architecture, Ankara

UNIVERSITY COMMITTEE ASSIGNMENTS

Bursary & Aid Committee, for the students, METU Presidential Office, 1969/70.

Housing Allocation Committee, for the teaching staff members, METU Presidential Office, 1976/77 & 1980/81.

Journal of the Faculty of Architecture, JFA, Member of the Editorial Board, METU, 1973-1999.

Environment Area Committee, METU Presidential Office, 1996-1998

CONSULTING & PUBLIC SERVICES

75 (approximately) **"Expertise Reports"** submitted to the Supreme Court (Danıştay) or the Regional Administrative Supreme Courts, on the cases related to the architectural and urban plans. (Individual and/or team reports), between years 1973-2003

COMPETITIONS

International Competition for the Touristic Development of Side/Antalya (Turkey), 1969, with other two partners (No Award)

National Competition for the Planning of Zonguldak Metropolitan Area, 1972, (No Award)

NATIONAL RESEARCH PROJECT

Urban Renewal Project for the Antalya City Historical Core Area, sponsored by the Ministry of Tourism & Culture; With team members G.önül Tankut,Ülker Çopur, Murat Balamir ,Department,METU, 1977-1980 (Plan depended on the results of questionnaire made with 2742 persons in 869 house holds, survey of 664 out of total 861 buildings, 73 commercial shops and 1017 plots.)

INTERNATIONAL RESEARCH PROJECTS

METU Campus Design in the Turkish Republic of North Cyprus (TRNC): Urban Design Studio (UD 501-502)**,** With studio advisors Assoc.Prof.Dr.Baykan Günay, Assist. Prof.Dr. Adnan Barlas, Inst. Erhan Acar ; Res. Assist. Serdar Özbay, 2000-2001 Academic Year . (Campus is under construction)

Pogradec City Planning in Albania : Urban Design Studio (UD501-502), With advisors above, 2002-2003 Academic Year. (Trip to Pogradec,Ohrid Lake Region,Tirana, Durres & Thessaloniki (Selanik) /Greece included.)

INSURE: Flexible Framework for Indicators for Sustainability in Regions Using System Dynamics Modelling : Specific Targeted Research & Innovation Project , approved by the European Commission, Commencement Date: 01 April 2004, Duration : 30 months; 6th Framework Program (FP6) Priority Area: 1.1.6.3: Global Change & Ecosystems; by the Consortium of 8 partner organizations: TAU (Coordinator/ Madrid/Spain), Manchester Univ. (UK), Karlsruhe Univ. (Germany), Maastricht Univ. (Netherlands), FEEM (Milan/Italy), IREAS (Prague/Czech Rep.) & Joint Res. Center (Spain).

(Acting as the Team-Leader of the METU Group of three members, Assoc. Dr. Sibel Kalaycıoğlu & Assist. Dr. Helga R. Tılıç from Sociology Dept.; where "Antalya Region" shall be taken as one of the case studies in INSURE Project.)

MASTER THESES SUPERVISED
(At the Department, METU)

KAYILI, Ali Üçok – A Study in Environmental Pollution: A Case Study in İzmit Bay Region, CRP Dept., 1975

KOCHAK, İsmet – Petrochemical Complex in Kerkuk & Its Urban and Regional Interrelations, CRP Dept., 1979

KASSIR, İbrahim Abdüllatif – Fire Protection Methods & The İstanbul Naval Museum As A Case Study, Architecture Dept / Building Science, 1985

HEGAZY, Kamal – Solid Waste Site Selection Issues For the Metropolitan City of Ankara, City Planning Graduate Program,., Dec.2000

KOLAÇ, Enver – A New Campus for Middle East Technical University in the Turkish Republic of Northern Cyprus: Educational Program & Design Issues, Urban Design Graduate Program, Dec. 2002.

GRANTS RECEIVED

- FULBRIGHT Scholarship: During the "Student Exchange Program" between the METU & Pratt Institute (New York/USA), 1964/65 Academic Year.

- MUSTAFA PARLAR (Prof.Dr.) Education & Research Foundation (Ankara), Financial Support to participate the International System Dynamics Meeting, 22-24 Oct., 1986, Sevilla (Spain)

- TÜBİTAK, The Financial Support Program for the International Scientific Publications,Two Grants in 1992 (Publication No:19, ''Carmen…'') & in 1995 (Publication No:25 ,'' Transforming The Blue Plan…'')

- TÜBİTAK, The Financial Support Program for the European Commission FP6 Projects; for INSURE: Flexible Framework For Indicators For Sustainability in Regions Using System Dynamics Modelling, (in the FP6-Priority Area: 1.1.6.3 :Global Change & Ecosystems).Grant was given to the

METU Team as partner in the Consortium of 8 member organizations (March 2004).

PLATES RECEIVED

Plates/plaquettes were received for the following occasions:

- 25th Year of Service at METU, by the Presidential Office, (27 May 1991)
- The First Graduates of METU, by the Presidential Office, (10 gram Gold Coin Certificate), 5 July 1991
- Year 1962 Graduates of METU, by the Presidential Office, (METU Science Tree Emblem), 3 July 1992
- 30th Year in the Profession, by the Chamber of Architects, Ankara, (Chamber of Architects' Metal Emblem), 9 April 1993
- 40th Year of Graduation, by the Presidential Office,(Medal &Certificate), 02 June 2002 . (Photo at the top)
- 40th Year in the Profession, by the Chamber of Architects, at the State Guest House, Ankara, (Metal Ruler), 11 April 2003)
- Teaching Members of the First 10 Years of the Faculty of Architecture, METU, by the Dean's Office, (Metal Plate in box), 26 June 2004 . (Photo at the end)

PROFESSIONAL AND SOCIAL AFFILIATIONS

1) Chamber of Turkish Architects, Ankara, (since 1962).

2) Chamber of Turkish City Planners, Ankara (1968-1978).

3) The Institution of Fire Engineers, Leicester, England (1986). (Associate Membership, one year).

4) Sub-Commission of Fire-Fighting For The National Committee On The Minimization of Natural Disasters, Ministry of Internal Affairs, Ankara (1988-1992).

5) International Association For Cybernetics, Namur/Belgium, (1990-1999) (Scientific Member No: 134) (Association dissolved in 1999).

6) The Fulbright Alumni Association of Turkey, Ankara. (1992-) (Member No: 93).

7) Turkish National Committee For The Management of Coastal Areas of Turkey. METU, Ankara, (1996-)

8) Who's Who in Turkey, Biographical Encyclopedia, 1997/98, 5th Edition, Profesyonel Ltd., Istanbul, 1998; revised in 9th Edition, ibid, 2003 (A Short CV appeared in).

9) System Dynamics Society,Albany, NY, USA,(2004-)

PAPERS AND REPORTS
(T: Originally in Turkish; otherwise in English)

(1) A Report on the Housing Expenditures in Turkish Cities, Prepared for the Special Commission of Housing for the Third-Five Year Development Plan, State Planning Organization, Ankara, 1971 (Mimeographed, 8 pages) (T)

(2) An Overview of Nine Years' Period : Studio Works of City & Regional Planning Department,1963-71, June 1973, (Unpublished, 18 pages& photos) (T)

(3) A Report for the New Campus Location of METU Marine Science Department in Antalya, Submitted to the Presidential Office,(et.al.), Jan.1974 (T)

(4) The Structure and Location of Retail Centers in Ankara ,Paper read at the International Conference on Urban Development Models ;University of Cambridge, England, 22-26 July 1974

(5) Regional Development and Settlements- A Special Commission Report for the Fourth-Five Year Development Plan, State Planning Organization, Ankara, Nov., 1976 (pp.30-37) (et.al.) (T)

(6) Some Recent Developments in Urban Density Functions, April 1977, (Mimeographed,40 pages, prepared as a part of Ph.D. qualification exam).

(7) Utility Theory and its Implications in Urban Planning, 1978 (Unpublished, 45 pages)

(8) Four-Step Models in the Transportation Planning Process, Lecture Note Series No:11, Department of City & Regional Planning, METU, Ankara, Jan.1979 (Mimeographed,92 pages).

(9) Developments in the Theory, Techniques and Research Methods of Urban Planning, 1966-1979: Thirteen Years of Experience, (Partly in English) (This is an cross-sectional analysis of my own lectures during the above period.), Department,1979 (Mimeographed, 45 pages)

(10) ''Some Remarks on our Urban Planning Curriculum'' Occasional Papers, Faculty of Architecture, METU, August 1979 (T)

(11) Urban System Dynamics: Outline of a New Elective, Feb.1981, Department, (Mimeographed,13 pages).

(12) Eigenvalues and Eigenvectors: A Simplification for City Planners and Geographers, Department, 1981,(Unpublished , 20 pages).

(13) "A New Course Introduced: Urban System Dynamics", System Dynamics Newsletter, Vol. 19, Dec., 1981, Massachusetts Institute of Technology, Cambridge, Mass., USA, 1982 (pp.26-27).

(14) Some Remarks on the Final Conclusions of the European Urban Renaissance Review Conference, held in West-Berlin, 8-11 March, 1982, Department, April, 1982, (Mimeographed, 3 pages).

(15) "Curriculum 1980 of the METU City and Regional Planning Department: Cybernetic and Catastrophic Resolutions", paper Presented at the First National Design Congress held in Istanbul Technical University, 24-26 May, 1982 (Mimeographed ,28 pages) (T).

(16) "Urban Dynamics at METU", (Course Description), System Dynamics Newsletter, vol.21, (Jan, 1983-Dec. 1984), Massachusetts Institute of Technology, 1985, (p.26).

(17) "Carmen, Catastrophe Theory and System Dynamics: A Revitalization", Paper presented at the International Conference on System Dynamics, The System Dynamics Society, 22-24 Oct. 1986, University of Sevilla (Spain),(Unpublished) (Revised & enlarged in Proceedings, AIC, Namur/Belgique, 1990) (See the list of Publications No : 19)

(18) "What Happened at the Second-Year B-Studios, METU? An Educational Experiment in the City and Regional Planning Department", Paper presented at the Eleventh International Conference, IAPS, held at METU, 8-12 July 1990, Ankara. (Unpublished)

PUBLICATIONS

(T: Originally in Turkish; otherwise in English)

(1) Elements of Residential Density, May 1970, METU, (205 pages, 28 graphs, 19 Tables and 5 photos; Academic Promotion Study, mimeographed.)

(2) ‘’Urban Indicators and Methodological Issues In Data Collection’’, Bulletin, Faculty of Architecture, METU, Vol.1, No:4, 1974 (pp.322-349).

(3) ‘’Cybernetic Approach In Urban Planning and Management’’, Journal of The Faculty of Architecture, Vol, 1, No:1, 1975 (pp.129-143) (Turkish, summary in English)

(4) ‘’Spatial Dimensions of Crowding: An Analysis by the Density Gradients for Ankara, 1965-70”**,** in M. Gürkaynak and W. Ayhan Le Compte (edits) – Human Consequences of Crowding, Plenum Press, New York, Nato Conference Series-3, Vol.10, 1979 (pp.83-96).

(5) ‘’Some Issues on the Implementation and Control of the Antalya Historical Core Renewal Project”, Türk-İnşa, No: 6, May 1982, Ankara (pp.6-12) (T).

(6) “Renewal Planning : There in No ‘Pax Romana’ Any Longer!”, in Proceedings of Antalya seminar-1982, Meeting on Urban Renaissance Campaign, by the National Committee of Council of Europe, Ministry of Reconstruction, 1983, Ankara (pp.156-166) (T)

(7) “Renewal Project for the Historical Core-Antalya City: A Report” in Proceedings of Antalya Seminar-1982, ibid., (pp.156-166). (T) (With the other 3 authors as the team members of the project).

(8) “A Discussion on the Third Cybernetics” G. Ulusoy (et.al) (edits.) – Proceedings of the 6th National Congress on Operational Research, held in Ankara, 23-27 June, 1980, Operational Research Association, Gebze/Kocaeli, April 1983 (pp.363-373) (T).

(9) “A Meeting on Urban Planning by the Council of Europe. The Second Renaissance at Reichstag”. in Avrupa Konseyi Dergisi, 1983, No: 2 and 3, Ankara, (pp.5-14) (T).

(10) Fire Dynamics: An Attempt at Modelling”, in Ö. Esmer and S. Baran (edits) – Proceedings of the First National Congress on Fire-Fighting, held in

Ankara, 9-11 Dec., 1981, Middle East Technical University, Ankara, 1985. (pp.223-264) (T).

(11) "Waiting for the Jury of the Fifth Year Planning Studio", Media, No:2, April-Sept., 1984, Faculty of Architecture, METU, Ankara, (pp.5-7). (T).

(12) Esmer, Özcan; Baran, Ü. Savaş (edits.) – Proceedings of the First National Meeting on Fire Fighting, Dec. 9-11, 1981, METU-TUBİTAK (Scientific and Technical Research Council of Turkey); Faculty of Architecture, METU, Ankara, 1985. (Table of Contents and Abstracts of papers given in English) (444 pages, figures, tables, graphs and illustrations; Appendixes). (T&E).(The Meeting was accepted within the 100th Anniversary Year of Atatürk). (Acted as the Vice-President of the Meeting's Organization)

(13) "Abstracts" of 30 papers in English edited in the Proceedings-1981, (10 pages); compiling examples from the Supreme Court Decisions in relation to the fire cases; cover design and mise-en-page of the Proceedings, 1985 (ibid).

(14) "A Catastrophe Discourse On Urban Planning at the Turning Point", in Program and Paper Abstracts, Eight National Congress on Operational Research, Operational Research Association,held at METU 1983, (p.53-54). (T).

(15) "Topological Cybernetics: A Sketch for the Fourth Cybernetics", Proceedings of the Eleventh International Congress on Cybernetics, 25-29 August, 1986, Association Internationale De Cybernetique (AIC), 1987, Namur (Belgique), (pp.665-679).

(16) "A Discussion on the Fourth Cybernetics", in Paper Abstracts, of the Eleventh National Congress on Operations Research, by Operations Research Association, held at Marmara University, 1987, Istanbul, (p.97), (T).

(17) "Curriculum 1980 of the METU City and Regional Planning Department: Cybernetic and Catastrophic Resolutions", in Türk-İnşa, no:76-83 (in six succeeding issues), (Monthly periodical), 1988, Ankara. (T).

(18) "A First Discussion on the Fourth Cybernetics", Proceedings of the 12th International Congress on Cybernetics, Association Internationale De Cybernetique (AIC), 21-25 August 1989, Namur (Belgique), 1990, (pp.1274-1285).

(19) " 'Carmen', Catastrophe Theory and System Dynamics: A Second Look Through the Fourth Cybernetics", Proceedings of the 12th International

Congress on Cybernetics, (AIC), ibid, Namur (Belgique), 1990, (pp.1371-1384).

(20) "Is Theseus Still Fighting? Or, Running A Studio in the Absence of Planning Theory Courses…", in Zekiye Yenen (edit) Proceedings of the International Symposium on Architecture of Tourism in the Mediterranean, 13-16 Oct., 1991, Yıldız Technical University, Faculty of Architecture, Istanbul, 1991 (pp. 117-125).

(21) "City Planning in the Era of new Transformations and Paradigms", International Association for the Study of people and Physical Surroundings (IAPS), 12th Biennial International Conference, Volume of Abstracts, 11-14 July 1992, Aristotle University of Thessaloniki (Greece).

(22) "Transforming the Mediterranean Blue Plan into A System Dynamics Model", in E. Özhan (edit.) – MEDCOAST 93 Proceedings of the First International Conference on the Mediterranean Coastal Environment, (2-5 Nov., 1993, vol.1, (pp.273-290).

(23) "Kadınana Primary School Should be Protected Urgently", Afyon Zafer, (Daily Local Newspaper), 31 May 1995, no:1817. (T)

(24) "Once Upon A Time in 'Time' 1960", ODTÜ'LÜLER Bülteni, No:42, May 1995, METU Alumni Association, Ankara, (pp. 8-9), also in ODTÜ'LÜ Dergisi, No:10, 1995, METU, Ankara (pp.26-27) (T).

(25) "The Birth & Meaning of the METU Emblem: 1957", ODTÜ'LÜ Dergisi, No:11, 1996, METU, Ankara (pp.10-11) (T).

(26) "What Happened at the Battle of Termessos? Or, From 'System Dynamics' to the Mediterranean Dynamics Model", Coastal Area Management: Final Report, International Seminar, 25-29 Mart 1998, Antalya/Turkey, ISOCARP (Int. Society of City & Regional Planners), Netherlands, 1998 (pp.148-157).

(27) "Changing Foundations of Urban Modelling", Program & Abstracts, Eighteenth International Workshop on Maximum Entropy & Bayesian Methods (MaxEnt 98), 27-31 July 1998, Max-Planck Institut für Plasmaphysik, Garching/München/Germany, 1998, (p.28).

(28) "Can Istanbul Win the Competition For the 2008 Olympic Games?", in D. Camarda & L. Grassini (edits) – Proceedings of the Meeting on Coastal Zone Management in the Mediterranean Region, April 26-May 01, 2001, İzmir/Turkey, European Commission INCO-DC (DgXII), Politecnico di Bari, Italy, 2001 (pp.67-70).

(29) "Developing System Dynamics Viewpoint for the 'Terra Dinamica' ", Abstract Book, 5th EURO/INFORMS Joint International Meeting, July 6-10, 2003, Boğaziçi Univ., Istanbul, (p.66).

(30) "The Fourth Generation of Earth Probability Models At the Turn of the Third Millennium: A First Paper", in Ali Koçak (edit)-The Technical Congress of Küçükçekmece & Its Periphery : Earthquake & Planning, (Book of Proceedings), Vol.2, Technical Congress,8-10 Oct. 2003, Yıldız Technical University, Istanbul, 2003, (711-722); Also in Book of Abstracts,Earthquake Symposium, 12-14 March 2003, Kocaeli University, Izmit/Turkey, 2003, (32-33)