CP-VIOLATING EFFECTS IN B DECAYS BEYOND THE STANDARD MODEL

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ABSTRACT

CP-VIOLATING EFFECTS IN B DECAYS BEYOND THE STANDARD MODEL

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In this thesis, using a general model independent form of the effective Hamiltonian, the CP-violating asymmetries in the $b \to d\ell^+\ell^-$ transition, when one of the leptons is polarized, is investigated. The sensitivity of the CP-violating asymmetries on the new Wilson coefficients are analyzed.

Next, in the frame work of the same formalism, the polarized lepton pair forward–backward asymmetries in $B \to K^*\ell^+\ell^-$ decay are studied. We present the general expression for the nine double–polarization forward–backward asymmetries. It is obtained that, the zero point position of the forward–backward asymmetries of the doubly–polarized lepton pair does not depend on long distance effects but depends on short distance dynamics. Furthermore, it is shown that the zero position of A_{FB} is very sensitive to the sign of the new Wilson coefficients. When sign of the Wilson coefficients is positive (negative) the zero position of the forward–backward asymmetries shifts to the left(right) compared to the SM.

Moreover, the dependencies of the nine double–polarization forward–backward asymmetries on new Wilson coefficients, and the correlation of the averaged nine double–polarization forward–backward asymmetries with branching ratio, have been studied. It is observed that, the study of the nine double–polarization forward–backward asymmetries can serve as a good test in establishing new physics beyond the Standard Model. Finally, we observed that there are exist such regions of new Wilson coefficients for which the nine double–polarization forward–backward asymmetries considerably depart from the SM result, while the branching ratio coincides with that of the SM prediction. In other words, new physics effects can be established by analyzing polarized forward-backward asymmetry in this region of the new Wilson coefficients.

Keywords: B Physics, Rare decays, CP-Violation, Beyond The Standard Model.

ÖZ

STANDART MODEL ÖTESINDE B BOZUNUMLARINDA CP-BOZULUMU

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$b \to d\ell^+\ell^-$ bozunumunda, leptonların polarize olması durumunda, modelden bağımsız etkin Hamiltoniyenin en genel hali kulanılarak, CP-bozulumu araştırılmış ve CP-bozulum bakışımsızlığının yeni Wilson katsayılarına duyarlılığı incelenmiştir.

Aynı formulasyon çerçevesinde, $B \to K^*\ell^+\ell^-$ bozunumunda çift leptonların polarize olduğu durumunda ön-arka yönündeki bakışımsızlığa bakılmş ve 9 ön-arka çift polarize bakışımsızlığının ifadeleri bulunmuştur. Ayrıca ön-arka yönlerdeki çift polarize olmuş leptonların sıfır nokta konumunun ancak yakın mesafe etkilerinin dinamiğine bağlı olduğu saptanmıştır. Buna ek olarak, ön-arka yönlerdeki çift polarize olmuş leptonların sıfır nokta konumunun, Wilson katsayılarının artı(eksi) değerlerinde, Standart Modelin öngördüğü konumun sol (sağ) tarafına kaydığı gözlenmiştir.

Daha sonra 9 çift polarize ön-arka yönlerdeki bakışımsızlığın yeni Wilson katsayılarına olan bağlılığının yanısıra ortalamasının dallanma oranı ile olan bağıntısı incelenmiştir.

9 çift polarize ön-arka yönlerdeki bakışımsızlığın araştırılmasının Standart Model (SM) ötesindeki yeni fizik etkilerinin incelenmesi açısından yarar sağlayacağı gözlenmiştir.

Son olarak, Wilson katsayıları için dallanma oranının SM öngörüsü ile aynı olduğu ve çift polarize ön-arka yönlerdeki bakışımsızlığın SM öngörülerinden farklı olduğu bir bölge bulunmuştur. Böylece bu bölgede yeni fizik etkenlerinin ortaya çıkabilmesi için çift polarize bakışımsızlığının incelenmesi yeterlidir.

Anahtar Kelimeler: B fiziği, Nadir bozunumlar, CP-Bozulumu, Standart Model Ötesi.

aux mémoires de mon père, *Rıza Gulu*

et

ma mère, *Merziye*

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TABLE OF CONTENTS

ABSTRACT iv

ÖZ vi

DEDICATION viii

ACKNOWLEDGMENTS viii

TABLE OF CONTENTS x

LIST OF TABLES xii

LIST OF FIGURES xiii

1 INTRODUCTION 1

2 FLAVOR PHYSICS AND CP VIOLATION 5

2.1 Introduction 5

2.2 CP violation in the Standard Model 7

2.2.1 Phase Structure of the CKM Matrix 10

2.2.2 Unitarity Triangles of the CKM Matrix 17

2.2.3 Towards an Allowed Region in the $\bar{\rho}$–$\bar{\eta}$ Plane . . 20

2.3 DECAYS OF $\boldsymbol{B}$ MESONS 22

2.3.1 Leptonic Decays 22

2.3.2 Semileptonic Decays 25

3 GENERAL ANALYSIS OF CP VIOLATION IN POLARIZED $b \to d\ell^+\ell^-$ DECAY 27

3.1 Introduction 27

3.2 Formalism . . . 29

3.3 Numerical analysis . . . 41

4 POLARIZED LEPTON PAIR FORWARD–BACKWARD ASYMMETRIES IN $B \to K^* \ell^+ \ell^-$ DECAY BEYOND THE STANDARD MODEL . . . 57

4.1 Introduction . . . 57

4.2 Matrix element for the $B \to K^* \ell^+ \ell^-$ decay . . . 60

4.3 Polarized forward–backward asymmetries of leptons . . . 66

4.4 Numerical analysis . . . 75

5 CONCLUSION . . . 91

REFERENCES . . . 96

LIST OF TABLES

4.1 B meson decay form factors in a three-parameter fit, where the radiative corrections to the leading twist contribution and SU(3) breaking effects are taken into account. 76

LIST OF FIGURES

2.1 Hierarchy of the quark transitions mediated through charged-current processes. 12

2.2 The two non-squashed unitarity triangles of the CKM matrix, as explained in the text: (a) and (b) correspond to the orthogonality relations (2.43) and (2.46), respectively. 19

2.3 Contours in the $\bar{\rho}$–$\bar{\eta}$ plane, allowing us to determine the apex of the UT. 21

2.4 Feynman diagram contributing to the leptonic decay $B^- \to \ell\bar{\nu}$. . . . 23

2.5 Feynman diagram contributing to semileptonic $\bar{B}^0_d \to D^+(\pi^+)\ell\bar{\nu}$ decays. 25

3.1 The dependence of A_{CP} on $\hat{s}$ for the $b \to d\mu^+\mu^-$ transition, at fixed values of C_{LL}. 43

3.2 The same as in Fig. (3.1), but at fixed values of C_{LR}. 44

3.3 The same as in Fig. (3.1), but for the $b \to d\tau^+\tau^-$ transition, at fixed values of C_{LR}. 44

3.4 The same as in Fig. (3.3), but at fixed values of C_{LRRL}. 45

3.5 The same as in Fig. (3.3), but at fixed values of C_{LRLR}. 46

3.6 The same as in Fig. (3.3), but at fixed values of C_T. 46

3.7 The same as in Fig. (3.3), but at fixed values of C_{TE}. 47

3.8 The dependence of δA^L_{CP} on $\hat{s}$ for the $b \to d\mu^+\mu^-$ transition, at fixed values of C_{LL}, when one of the final leptons is longitudinally polarized. 47

3.9 The same as in Fig. (3.8), but at fixed values of C_{LRRL}. 48

3.10 The same as in Fig. (3.8), but for the $b \to d\tau^+\tau^-$ transition, at fixed values of C_T. 49

3.11 The same as in Fig. (3.10), but at fixed values of C_{TE}. 52

3.12 The same as in Fig. (3.10), but at fixed values of C_{LRRL}. 52

3.13 The same as in Fig. (3.10), but at fixed values of C_{LRLR}. 53

3.14 The same as in Fig. (3.10), but at fixed values of C_{RLRL}. 53

3.15 The same as in Fig. (3.10), but at fixed values of C_{RLLR}. 54

3.16 The same as in Fig. (3.10), but when one of the final leptons is transversally polarized, at fixed values of C_{LR}. 54

3.17 The same as in Fig. (3.16), but at fixed values of C_{RR}. 55

3.18 Parametric plot of the correlation between the partially integrated A_{CP}^{L} and δA_{CP}^{L} as a function of the new Wilson coefficients C_X, for the $b \to d\mu^+\mu^-$ transition, when one of the final leptons is longitudinally polarized. 55

3.19 The same as in Fig. (3.18), but for the $b \to d\tau^+\tau^-$ transition. . . 56

3.20 The same as in Fig. (3.19), but when one of the final leptons is transversally polarized. 56

4.1 The dependence of the double–lepton polarization asymmetry $\mathcal{A}_{FB}^{LL}$ on q^2 at four fixed values of C_{LL}, for the $B \to K^*\mu^+\mu^-$ decay. . . 77

4.2 The same as in Fig. (4.1), but at four fixed values of C_{LR}. 78

4.3 The dependence of the double–lepton polarization asymmetry $\mathcal{A}_{FB}^{LT}$ on q^2 at four fixed values of C_T, for the $B \to K^*\mu^+\mu^-$ decay. . . . 78

4.4 The same as in Fig. (4.3), but for $\mathcal{A}_{FB}^{TL}$. 79

4.5 The same as in Fig. (4.3), but at four fixed values of C_{TE}. 79

4.6 The same as in Fig. (4.4), but at four fixed values of C_{TE}. 80

4.7 The dependence of the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ on the new Wilson coefficients C_X, for the $B \to K^*\mu^+\mu^-$ decay. 82

4.8 The same as in Fig. (4.7), but for the $B \to K^*\tau^+\tau^-$ decay. 82

4.9 The same as in Fig. (4.8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$. 83

4.10 The same as in Fig. (4.8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$. 84

4.11 The same as in Fig. (4.8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle$. 85

4.12 The same as in Fig. (4.8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle$. 86

4.13 Parametric plot of the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ and the branching ratio for the $B \to K^* \mu^+ \mu^-$ decay. 86

4.14 Parametric plot of the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ and the branching ratio for the $B \to K^* \tau^+ \tau^-$ decay. 87

4.15 The same as in Fig. (4.14), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ and the branching ratio. 87

4.16 The same as in Fig. (4.15), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NT} \right\rangle$ and the branching ratio. 88

4.17 The same as in Fig. (4.16), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle$ and the branching ratio. 89

4.18 The same as in Fig. (4.17), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$ and the branching ratio. 90

CHAPTER 1

INTRODUCTION

Experimental discovery of the rare $B \to X_s\gamma$ and $B \to K^*\gamma$ decays opened a new window in investigation of the Flavour Changing Neutral Current (FCNC) processes [1]. On the experimental side, this is due to the fact that the study of the FCNC decays will provide a precise determination of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, which are free parameters of the Standard Model (SM), leptonic decay constants of heavy mesons etc. On the theoretical part, investigation of the FCNC decays allows us to check the predictions of the SM at the one-loop level [2]. For these reasons investigations on the rare radiative and semileptonic decays of the B meson receive special attention. Such decays are also very useful in looking for "New Physics" beyond the SM.

In this thesis, theoretical analysis of the semileptonic inclusive $b \to d\ell^+\ell^-$ transition[3] and exclusive $B \to K^*\ell^+\ell^-$ decay[4] is presented in a model independent way, using the most general form of the effective Hamiltonian including all possible forms of interactions. In $b \to d\ell^+\ell^-$ transition we investigate CP violation effects in branching ratio with and without lepton polarization, using the above–mentioned framework. We investigate forward-backward asymmetry in the exclusive $B \to K^* \ell^+ \ell^-$ decay by taking into account polarizations of both leptons.

As we noted, we mainly focuce on the polarization effects of the leptons, since measurement of the lepton polarizations is one of the most efficient way to establish new physics beyond the SM [5] - [17].

A contemporary tool in investigating weak decays is the Operator Product Expansion (OPE). Using OPE, an effective Hamiltonian [18]-[23] can be represented in the form , $H_{eff} \sim \sum C_i O_i$. The meaning of this expression is that any low energy weak process can be written in terms of the perturbative short distance Wilson coefficients, C_i, and the matrix elements $< O_i >$ of the operators which represent the long distance effects. Estimation of these matrix elements is hard since they belong to the non-perturbative sector of the QCD.

It is well known that theoretical analysis of inclusive decay channels, are rather easy but their experimental discovery is quite hard. For the exclusive decays, it is contrary to the case, i.e., their experimental detection is easy but theoretical studies have their own drawbacks. That is, basically, due to the fact that the matrix elements of the effective Hamiltonian between final and initial meson states, which are parameterized in terms of form factors, are needed.

The first step in the analysis of the Semileptonic B decays is the derivation of the effective Hamiltonian. The effective Hamiltonian, as mentioned above and discussed in 2.2, is first obtained by the Feynman diagram technique at large mass scale. Then using the re-normalization group equation we can calculate the effective Hamiltonian at low energy scale (in our case, $\mu = m_b$). These two steps are carried out in the framework of the perturbative approach. In further investigation of the exclusive $B \to (KK^*)\ell^+\ell^-$ decays, we need the matrix elements $\langle M \left| H^{eff} \right| B \rangle$. These matrix elements cannot be calculated in the framework of the perturbative approach and its calculation demands non-perturbative approach such as Chiral Theory [24], Three-point QCD Sum Rules [25], and Light Cone QCD Sum Rules [26],[27]. In this thesis, we have used the Light Cone QCD Sume Rules Method for predictions of the form factors.

The thesis is organized as follows; in Chapter 2, we present a brief overview of the SM of the electroweak interactions by introducing the theoretical framework in analyzing the tree level decays and the FCNC processes. In this chapter, we also discuss briefly Cabibbo-Kobayashi-Maskawa(CKM) matrix, as well as, appearance of CP violation in the standard model. In chapter 3 we present comprehensive analysis of CP–violating effects in branching ratio for the $b \to d\ell^+\ell^-$ decay using the most general form of the effective Hamitonian. We present analytical expression of the CP-violating effects in both cases and give numerical results. Chapter 4 is devoted to the analysis of the forward-backward asymmetry in $B \to K^*\ell^+\ell^-$ decay in a model independent way.

- FCNC: Flavor Changing Neutral Current.
- OPE: Operator Product Expansion.
- CKM: Cabibbo-Kobayashi-Maskawa matrix.
- QCD: Quantum Chromodynamics.
- CP: Charge, Parity operators.
- SM: Standard Model.

CHAPTER 2

FLAVOR PHYSICS AND CP VIOLATION

2.1 Introduction

Symmetries are the critical ingredients of any local field theory. Symmetry properties of a theory leads to the conserved quantities (Noether theorem). There are two type of symmetries: continuous(Lie group symmetries) and discrete symmetries(Parity, Time reversal, etc). We call those symmetries "Good" if physics is invariant under those transformations. Mathematically it means: We have an operator $\psi(\overrightarrow{x}, t)$ which under rotation it transforms in the following manner:

$$\psi(\overrightarrow{x}, t) \rightarrow R_{\varphi}\psi(\overrightarrow{x'}, t') \qquad x \rightarrow x'$$
$$t \rightarrow t'. \tag{2.1}$$

Under Parity transformation, the space and time coordinates transform as

$$P: \quad \overrightarrow{x} \rightarrow -\overrightarrow{x} \quad , t \rightarrow t, \tag{2.2}$$

respectively,

and under Time reversal, the transformation rules are

$$T: \quad \overrightarrow{x} \rightarrow \overrightarrow{x} \quad , t \rightarrow -t. \tag{2.3}$$

Parity operator acts on the Dirac spinor as:

$$
\begin{aligned}
P\psi(\overrightarrow{x},t) &= \gamma_o\psi(\overrightarrow{-x},t), \\
P\phi_{s,p}(\overrightarrow{x},t) &\Rightarrow \pm\phi_{s,p}(\phi_p(\overrightarrow{-x},t)), \\
P(1-\gamma_5)\psi(\overrightarrow{x},t) &\Rightarrow \gamma_0(1-\gamma_5)\psi(\overrightarrow{-x},t) = (1+\gamma_5)\gamma_0\psi(\overrightarrow{-x},t).
\end{aligned} \tag{2.4}
$$

Here the subscript s(p) in ϕ means scalar(pseudoscalar). And γ's are Dirac matrices. Thus under parity transformation left–handed particles are transformed to the right–handed ones.

Let us briefly discuss how charge conjugation operator C act on the Dirac spinor. Obviously, this operator transforms a particle to an antiparticle or vice versa. Under C operation, ψ transforms as

$$
\psi \to C\overline{\psi}^T, \tag{2.5}
$$

where $C = i\gamma_2\gamma_0$.

Using this expression let us consider how currents (for example we consider vector and axial vector currents which exist in the SM) transform under C operation. Using Eq.(2.5) we obtain

$$
\begin{aligned}
\overline{\psi}_1\gamma_\mu\psi_2 &\xrightarrow{\mathcal{C}} -\overline{\psi}_2\gamma_\mu\psi_1, \\
\overline{\psi}_1\gamma_\mu\gamma_5\psi_2 &\xrightarrow{\mathcal{C}} \overline{\psi}_2\gamma_\mu\gamma_5\psi_1.
\end{aligned} \tag{2.6}
$$

Let us consider the combined CP transformation

$$
\overline{\psi}_1\gamma_\mu\psi_2 \xrightarrow{\mathcal{CP}} \overline{\psi}_2\gamma_\mu\psi_1, \tag{2.7}
$$

$$\overline{\psi}_1\gamma_\mu\gamma_5\psi_2 \quad \xrightarrow{\mathcal{CP}} \quad \overline{\psi}_2\gamma_\mu\gamma_5\psi_1. \qquad (2.8)$$

All these results are summarized by the CPT theorem: any local, Lorentz invariant field theory is invariant under CPT transformation. From here we can conclude that if CP is violated, then T invariance must also be violated.

In this connection the following question might appear: How does CP violation appear in the Standard model. In the next section we try to answer this question.

2.2 CP violation in the Standard Model

It is well known that Lagrangian of the Standard model contains the following parts: kinetic term of Yang Mills(YM) fields $\mathcal{L}_{YM} = -\frac{1}{4}\overrightarrow{F}_{\mu\nu}\overrightarrow{F}^{\mu\nu}$, where $\overrightarrow{F}_{\mu\nu}$ is Yang-Mills field strength tensor,

$$\overrightarrow{F}_{\mu\nu} = \partial_\mu\overrightarrow{A}_\nu - \partial_\nu\overrightarrow{A}_\mu - ig[\overrightarrow{A}_\mu, \overrightarrow{A}_\nu], \qquad (2.9)$$

and vector arrow means that $\overrightarrow{F}^{\mu\nu}$ is a vector in isospin space. The next term in the Lagrangian of the Standard Model indicates the local interaction between fermions and gauge(YM) fields i.e.

$$\mathcal{L}_G = \sum \overline{Q}_i\, i\not{D}Q_i + \sum \overline{L}_i\, i\not{D}L_i. \qquad (2.10)$$

Here D_μ is the covariant derivative defined as

$$D_\mu = \partial_\mu - ig\overrightarrow{W}_\mu\overrightarrow{T} - ig\frac{Y}{2}B_\mu. \qquad (2.11)$$

Note that in this expression Q_i and L_i involve the left and right handed fermions, respectively.

The other term in the Lagrangian is the so-called Higgs part, which is responsible to give mass to the fermions:

$$\mathcal{L}_H = D_\mu H (D^\mu H)^\dagger - V(H) \tag{2.12}$$

And the last term

$$\mathcal{L}_Y = \sum \overline{Q}_{iL} H \Gamma_{ij} u_i + h.c. \tag{2.13}$$

is the so-called Yukawa term which describes interactions of fermions with Higgs fields and after spontaneously breaking of the symmetry the fermions obtain mass.

Usually, two basis have been used in the SM : interaction and mass basis. $\mathcal{L}_G$ is diagonal in the mass basis

$$\mathcal{L}_Y \to \mathcal{L}_M = \sum \overline{Q}_{iL} < H > \Gamma_{ij} u_i + h.c. = \sum \overline{u}_{Li} M_{ij} u_{Rj} + h.c. \tag{2.14}$$

Obviously, one basis can be obtained from another one by the unitary transformation, i.e.

$$d_L^0 \longrightarrow V_d^L d_L, \tag{2.15}$$

$$u_L^0 \longrightarrow V_u^L u_L. \tag{2.16}$$

where $(V_u^{L\dagger} V_u^L)_{ij} = \delta_{ij}$.

Under such transformation the mass matrices become diagonal, i.e.

$$V_d^{L\dagger} M^\alpha V_d^R = (m_d, m_s, m_b),$$

$$V_u^{L\dagger} M^\alpha V_u^R = (m_u, m_c, m_t). \tag{2.17}$$

However, in the mass basis $\mathcal{L}_G$ is not diagonal. Indeed,

$$\begin{aligned} \mathcal{L}_G &\approx \frac{g}{\sqrt{2}} \overline{u}^0_{Li} \gamma_\mu d^0_{Li} W^\mu + h.c. + \overline{u}^0_{Li} \gamma_\mu u^0_{Li} (Z, \gamma, g) \\ &\to \frac{g}{\sqrt{2}} \overline{u}_{Li} (V_u^{L\dagger} V_d^L)_{ij} d_j W^\mu + \overline{u}_{Li} (V_u^{L\dagger} V_u^L)_{ij} u_j (Z, \gamma, g). \end{aligned} \tag{2.18}$$

From this expression we get an essential result, namely, the neutral current Lagrangian is flavor diagonal in the mass basis, while charged current is not.

In other words, flavor changing neutral currents are absent in the SM at tree level.

Denoting $V = V_{CKM} = V_u^{L\dagger} V_d^L$, for the charged current Lagrangian we get

$$\mathcal{L}_G \sim V_{CKM} \bar{u}_L \gamma_\mu d_L W^\mu + h.c. \tag{2.19}$$

The matrix V_{CKM} is called as Cabibbo-Kobayashi-Maskawa matrix.

Let us analyze how this interaction Lagrangian transforms under CP transformation

$$\begin{aligned} \mathcal{L}_G &\sim W_\mu^- \bar{d}_{Lj} \gamma^\mu u_{Li} V_{ji}^\dagger + W_\mu^\dagger \bar{u}_{Li} \gamma^\mu d_{Li} V_{ij}^\dagger \\ &\stackrel{CP}{\to} W_\mu^+ \bar{u}_{Li} \gamma^\mu d_{Lj} V_{ji}^\dagger + W_\mu^- \bar{d}_{Lj} \gamma^\mu u_{Li} V_{ij}. \end{aligned} \tag{2.20}$$

If $\mathcal{L}_G$ is CP invariant we have

$$V_{ij} = V_{ij}^\dagger = V_{ij}^*. \tag{2.21}$$

Obviously, if V_{ij} is real then there is not any CP violation effect. The complex coupling, i.e. $V_{ij} \neq V_{ij}^*$ is a necessary condition to have CP violation.

2.2.1 Phase Structure of the CKM Matrix

We have the freedom to redefine the up- and down-type quark fields in the following manner[28]:

$$U \to \exp(i\xi_U)U, \quad D \to \exp(i\xi_D)D. \tag{2.22}$$

If we perform such transformations in (2.19), the invariance of the charged-current interaction Lagrangian implies the following phase transformations of the CKM matrix elements:

$$V_{UD} \to \exp(i\xi_U)V_{UD}\exp(-i\xi_D). \tag{2.23}$$

Using these transformations we can eliminate un-physical phases. It can be shown that the parametrization of the general $N \times N$ quark-mixing matrix, where N denotes the number of fermion generations, involves the following physical parameters:

$$\underbrace{\frac{1}{2}N(N-1)}_{\text{Euler angles}} + \underbrace{\frac{1}{2}(N-1)(N-2)}_{\text{complex phases}} = (N-1)^2. \tag{2.24}$$

If we apply this expression to the case of $N = 2$ generations, we observe that there is no phase, i.e. CP is conserved. And only one rotation angle – the Cabibbo angle θ_{C} [29] – is required for the parametrization of the 2×2 quark-mixing matrix, which can be written in the following form:

$$\hat{V}_{\mathrm{C}} = \begin{pmatrix} \cos\theta_{\mathrm{C}} & \sin\theta_{\mathrm{C}} \\ -\sin\theta_{\mathrm{C}} & \cos\theta_{\mathrm{C}} \end{pmatrix}, \tag{2.25}$$

where $\sin\theta_{\rm C} = 0.22$ can be determined from $K \to \pi\ell\bar{\nu}$ decays. We conclude that for CP violation in SM we need at least three generations, as pointed out by Kobayashi and Maskawa in 1973 [30]. On the other hand, in the case of $N = 3$ generations, the parametrization of the corresponding 3×3 quark-mixing matrix involves three Euler-type angles and a single *complex* phase.

In the "standard parametrization" advocated by the Particle Data Group (PDG) [37], the three-generation CKM matrix takes the following form:

$$\hat{V}_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \quad (2.26)$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. Performing appropriate redefinitions of the quark-field phases, the real angles θ_{12}, θ_{23} and θ_{13} can all be made to lie in the first quadrant. The advantage of this parametrization is that the generation labels $i, j = 1, 2, 3$ are introduced in such a manner that the mixing between two chosen generations vanishes if the corresponding mixing angle θ_{ij} is set to zero. In particular, for $\theta_{23} = \theta_{13} = 0$, the third generation decouples, and the 2×2 submatrix describing the mixing between the first and second generations takes the same form as (2.25).

Experimentally, It was established that there is hierarchy of the strengths of the charged-current processes at quark-level: transitions within the same generation are governed by CKM matrix elements of order $\mathcal{O}(1)$, those between the first

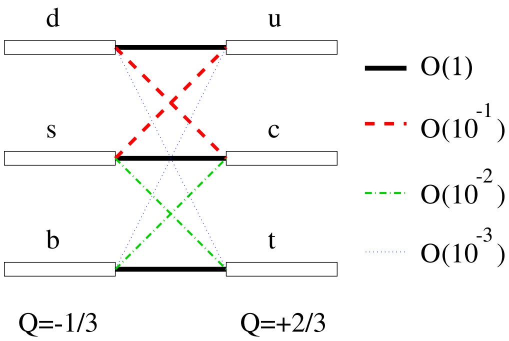

Figure 2.1: Hierarchy of the quark transitions mediated through charged-current processes.

and the second generation are suppressed by CKM factors of $\mathcal{O}(10^{-1})$, those between the second and the third generation are suppressed by $\mathcal{O}(10^{-2})$, and the transitions between the first and the third generation are even suppressed by CKM factors of $\mathcal{O}(10^{-3})$ Fig. 2.1. In the standard parametrization (2.26), this hierarchy is reflected by

$$s_{12} = 0.22 \gg s_{23} = \mathcal{O}(10^{-2}) \gg s_{13} = \mathcal{O}(10^{-3}). \tag{2.27}$$

For phenomenological applications, it would be useful to have a parametrization of the CKM matrix where it contains the hierarchy explicitly (see Eq.(2.29)). In order to derive such a parametrization, it is convenient to introduce a set of new parameters, λ, A, ρ and η, by imposing the following relations [32]:

$$s_{12} \equiv \lambda = 0.22, \quad s_{23} \equiv A\lambda^2, \quad s_{13}e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta). \tag{2.28}$$

If we now go back to the standard parametrization (2.26), we obtain an *exact* parametrization of the CKM matrix as a function of λ (and A, ρ, η), allowing us to expand each CKM element in powers of the small parameter λ. If we neglect terms of $\mathcal{O}(\lambda^4)$, we arrive at the famous "Wolfenstein parametrization" [31]:

$$\hat{V}_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{2.29}$$

which makes the hierarchical structure of the CKM matrix very transparent and is an important tool for phenomenological considerations.

For several applications, next-to-leading order(NLO) corrections in λ play an important role. Using the exact parametrization following from (2.26) and (2.28), they can be calculated straightforwardly by expanding each CKM element to the desired accuracy in λ [32]:

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + \mathcal{O}(\lambda^6), \quad V_{us} = \lambda + \mathcal{O}(\lambda^7), \quad V_{ub} = A\lambda^3(\rho - i\,\eta),$$
$$V_{cd} = -\lambda + \frac{1}{2}A^2\lambda^5\,[1-2(\rho+i\eta)] + \mathcal{O}(\lambda^7),$$
$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1+4A^2) + \mathcal{O}(\lambda^6), \tag{2.30}$$
$$V_{cb} = A\lambda^2 + \mathcal{O}(\lambda^8), \quad V_{td} = A\lambda^3\left[1-(\rho+i\eta)\left(1-\frac{1}{2}\lambda^2\right)\right] + \mathcal{O}(\lambda^7),$$
$$V_{ts} = -A\lambda^2 + \frac{1}{2}A(1-2\rho)\lambda^4 - i\eta A\lambda^4 + \mathcal{O}(\lambda^6), \quad V_{tb} = 1 - \frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6).$$

It should be noted that the matrix element

$$V_{ub} \equiv A\lambda^3(\rho - i\eta), \tag{2.31}$$

receives, *by definition*, no power corrections in λ within this prescription. If we follow [32] and introduce the generalized Wolfenstein parameters

$$\bar{\rho} \equiv \rho \left(1 - \frac{1}{2}\lambda^2\right), \quad \bar{\eta} \equiv \eta \left(1 - \frac{1}{2}\lambda^2\right), \tag{2.32}$$

we may simply write, up to corrections of $\mathcal{O}(\lambda^7)$,

$$V_{td} = A\lambda^3(1 - \bar{\rho} - i\,\bar{\eta}). \tag{2.33}$$

Moreover, to an excellent accuracy, we have

$$V_{us} = \lambda \quad \text{and} \quad V_{cb} = A\lambda^2, \tag{2.34}$$

as these quantities receive only corrections at the λ^7 and λ^8 levels, respectively. In comparison with other generalizations of the Wolfenstein parametrization found in the literature, the advantage of (2.30) is the absence of relevant corrections to V_{us} and V_{cb}, and that V_{ub} and V_{td} take forms similar to those in (2.29). Let us finally note that physical observables, for instance CP-violating asymmetries, *cannot* depend on the chosen parametrization of the CKM matrix, i.e. have to be invariant under the phase transformations specified in (2.23).

As we have just seen, in order to be able to accommodate CP violation within the framework of the SM through a complex phase in the CKM matrix, at least three generations are required. However, this feature is not sufficient for observable CP-violating effects. To this end, further conditions have to be satisfied, which can be summarized as follows [33]:

$$(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \times J_{\mathrm{CP}} \neq 0, \tag{2.35}$$

where

$$J_{\rm CP} = |{\rm Im}(V_{i\alpha}V_{j\beta}V^*_{i\beta}V^*_{j\alpha})| \quad (i \neq j,\, \alpha \neq \beta)\,. \tag{2.36}$$

The mass factors in (2.35) are related to the fact that the CP-violating phase of the CKM matrix could be eliminated through an appropriate unitary transformation of the quark fields if any two quarks with the same charge had the same mass. Consequently, the origin of CP violation is closely related to the "flavour problem" in elementary particle physics, and cannot be understood in a deeper way, unless we have fundamental insights into the hierarchy of quark masses and the number of fermion generations.

The second element of (2.35), the "Jarlskog invariants" $J_{\rm CP}$ [33], can be interpreted as a measure of the strength of CP violation in the SM. It does not depend on the chosen quark-field parametrization, i.e. it is invariant under (2.23), and the unitarity of the CKM matrix implies that all combinations $|{\rm Im}(V_{i\alpha}V_{j\beta}V^*_{i\beta}V^*_{j\alpha})|$ are equal to one another. Using the standard parametrization of the CKM matrix introduced in (2.26), we obtain

$$J_{\rm CP} = s_{12}s_{13}s_{23}c_{12}c_{23}c_{13}^2 \sin\delta_{13}. \tag{2.37}$$

Since the current experimental information on the CKM parameters implies a value of $J_{\rm CP}$ at the 10^{-5} level, CP violation is a small effect in the SM. However, new complex couplings are typically present in scenarios for new physics(NP) [36, 38], thereby yielding additional sources of CP violation. As far as the Jarlskog parameter introduced in (2.36) is concerned, we obtain the simple expression

$$J_{\rm CP} = \lambda^6 A^2 \eta, \tag{2.38}$$

which should be compared with (2.37). Note that all nine elements of CKM matrix, predicted from the theory, can be detrimined by experimental result. In order to determine the magnitudes $|V_{ij}|$ of the elements of the CKM matrix, we may use the following tree-level processes:

- Nuclear beta decays, neutron decays $\Rightarrow |V_{ud}|$.
- $K \to \pi \ell \bar{\nu}$ decays $\Rightarrow |V_{us}|$.
- ν production of charm off the valence d quarks $\Rightarrow |V_{cd}|$.
- Charm-tagged W decays (as well as ν production and semileptonic D decays) $\Rightarrow |V_{cs}|$.
- Exclusive and inclusive $b \to c \ell \bar{\nu}$ decays $\Rightarrow |V_{cb}|$.
- Exclusive and inclusive $b \to u \ell \bar{\nu}$ decays $\Rightarrow |V_{ub}|$.
- $\bar{t} \to \bar{b} \ell \bar{\nu}$ processes $\Rightarrow$ (crude direct determination of) $|V_{tb}|$.

If we use the corresponding experimental information, together with the CKM unitarity condition, and assume that there are only three generations, we arrive at the following 90% C.L. limits for the $|V_{ij}|$ [37]:

$$|\hat{V}_{\rm CKM}| = \left(\begin{array}{ccc} 0.9741\text{–}0.9756 & 0.219\text{–}0.226 & 0.0025\text{–}0.0048 \\ 0.219\text{–}0.226 & 0.9732\text{–}0.9748 & 0.038\text{–}0.044 \\ 0.004\text{–}0.014 & 0.037\text{–}0.044 & 0.9990\text{–}0.9993 \end{array}\right). \tag{2.39}$$

2.2.2 Unitarity Triangles of the CKM Matrix

The unitarity of the CKM matrix, which is described by

$$\hat{V}_{\rm CKM}^{\dagger} \cdot \hat{V}_{\rm CKM} = \hat{1} = \hat{V}_{\rm CKM} \cdot \hat{V}_{\rm CKM}^{\dagger}, \tag{2.40}$$

leads to a set of 12 equations, consisting of 6 normalization and 6 orthogonality relations. The latter can be represented as 6 triangles in the complex plane [34], all having the same area, $2S_{\Delta} = J_{\rm CP}$ [39]. Let us now have a closer look at these relations: those describing the orthogonality of different columns of the CKM matrix are given by

$$\underbrace{V_{ud}V_{us}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{cd}V_{cs}^*}_{\mathcal{O}(\lambda)} + \underbrace{V_{td}V_{ts}^*}_{\mathcal{O}(\lambda^5)} = 0, \tag{2.41}$$

$$\underbrace{V_{us}V_{ub}^*}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}V_{cb}^*}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{ts}V_{tb}^*}_{\mathcal{O}(\lambda^2)} = 0, \tag{2.42}$$

$$\underbrace{V_{ud}V_{ub}^*}_{(\rho+i\eta)A\lambda^3} + \underbrace{V_{cd}V_{cb}^*}_{-A\lambda^3} + \underbrace{V_{td}V_{tb}^*}_{(1-\rho-i\eta)A\lambda^3} = 0, \tag{2.43}$$

whereas those associated with the orthogonality of different rows take the following form:

$$\underbrace{V_{ud}^*V_{cd}}_{\mathcal{O}(\lambda)} + \underbrace{V_{us}^*V_{cs}}_{\mathcal{O}(\lambda)} + \underbrace{V_{ub}^*V_{cb}}_{\mathcal{O}(\lambda^5)} = 0, \tag{2.44}$$

$$\underbrace{V_{cd}^* V_{td}}_{\mathcal{O}(\lambda^4)} + \underbrace{V_{cs}^* V_{ts}}_{\mathcal{O}(\lambda^2)} + \underbrace{V_{cb}^* V_{tb}}_{\mathcal{O}(\lambda^2)} = 0, \tag{2.45}$$

$$\underbrace{V_{ud}^* V_{td}}_{(1-\rho-i\eta)A\lambda^3} + \underbrace{V_{us}^* V_{ts}}_{-A\lambda^3} + \underbrace{V_{ub}^* V_{tb}}_{(\rho+i\eta)A\lambda^3} = 0. \tag{2.46}$$

Here we have also indicated the structures that arise if we apply the Wolfenstein parametrization by keeping just the leading, non-vanishing terms. We observe that only in (2.43) and (2.46), which describe the orthogonality of the first and third columns and of the first and third rows, respectively, all three sides are of comparable magnitude, $\mathcal{O}(\lambda^3)$, while in the remaining relations, one side is suppressed with respect to the others by factors of $\mathcal{O}(\lambda^2)$ or $\mathcal{O}(\lambda^4)$. Consequently, we have to deal with only *two* non-squashed unitarity triangles in the complex plane. However, as we have already indicated in (2.43) and (2.46), the corresponding orthogonality relations agree with each other at the λ^3 level, yielding

$$[(\rho + i\eta) + (-1) + (1 - \rho - i\eta)]\, A\lambda^3 = 0. \tag{2.47}$$

Consequently, they describe the same triangle, which is usually referred to as *the* unitarity triangle of the CKM matrix [35]. Let us first have a closer look at the former relation. Including terms of $\mathcal{O}(\lambda^5)$, we obtain the following generalization of (2.47):

$$[(\bar\rho + i\bar\eta) + (-1) + (1 - \bar\rho - i\bar\eta)]\, A\lambda^3 + \mathcal{O}(\lambda^7) = 0, \tag{2.48}$$

where $\bar\rho$ and $\bar\eta$ are as defined in (2.32). If we divide this relation by the overall normalization factor $A\lambda^3$, and introduce

$$R_b \equiv \sqrt{\overline{\rho}^2 + \overline{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{ub}}{V_{cb}}\right|, \tag{2.49}$$

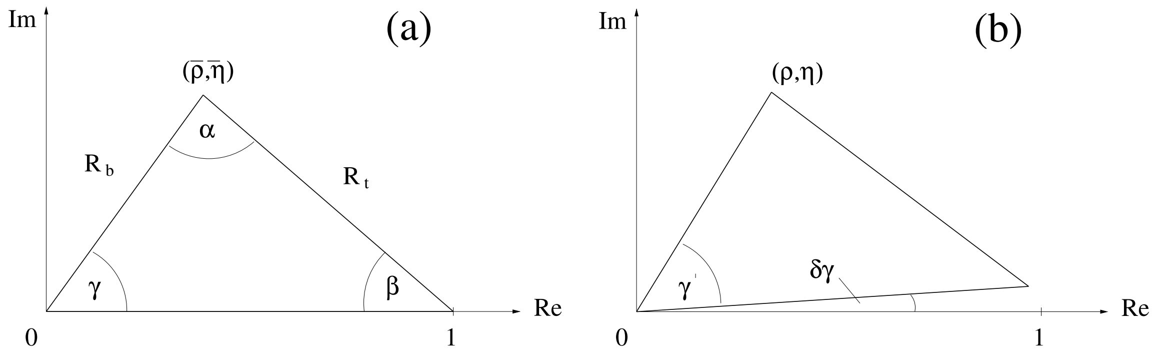

Figure 2.2: The two non-squashed unitarity triangles of the CKM matrix, as explained in the text: (a) and (b) correspond to the orthogonality relations (2.43) and (2.46), respectively.

$$R_t \equiv \sqrt{(1-\overline{\rho})^2 + \overline{\eta}^2} = \frac{1}{\lambda}\left|\frac{V_{td}}{V_{cb}}\right|, \tag{2.50}$$

we arrive at the unitarity triangle illustrated in Fig. 2.2 (a). It is a straightforward generalization of the leading-order case described by (2.47): instead of (ρ, η), the apex is now simply given by $(\bar{\rho}, \bar{\eta})$ [32]. The two sides R_b and R_t, as well as the three angles α, β and γ, will show up at several places throughout this thesis. Moreover, the relations

$$V_{ub} = |V_{ub}|\, e^{-i\gamma} = A\lambda^3 \left(\frac{R_b}{1-\lambda^2/2}\right) e^{-i\gamma}, \quad V_{td} = |V_{ub}|\, e^{-i\beta} = A\lambda^3 R_t e^{-i\beta}, \tag{2.51}$$

are also useful for phenomenological applications, since they make the dependences of γ and β explicit; they correspond to the phase convention chosen both in the standard parametrization (2.26) and in the generalized Wolfenstein parametrization (2.30).

2.2.3 Towards an Allowed Region in the $\bar{\rho}$–$\bar{\eta}$ Plane

It is possible to constrain – and even determine – the apex of the UT in the $\bar{\rho}$–$\bar{\eta}$ plane with the help of experimental data. However, it is useful to sketch the corresponding procedure – the "CKM fits" – already now, consisting of the following elements:

- The parameter R_b introduced in (2.49), which involves the ratio $|V_{ub}/V_{cb}|$. It can be determined experimentally through $b \to u\ell\bar{\nu}$ and $b \to c\ell\bar{\nu}$ decay processes. Following these lines, we may fix a circle in the $\bar{\rho}$–$\bar{\eta}$ plane that is centred at the origin $(0,0)$ and has the radius R_b.

- The parameter R_t introduced in (2.50), which involves the ratio $|V_{td}/V_{cb}|$. It can be determined with the help of the mass differences $\Delta M_{d,s}$ of the mass eigenstates of the neutral B_d- and B_s-meson systems. Experimental information on these quantities then allows us to fix another circle in the $\bar{\rho}$–$\bar{\eta}$ plane, which is centred at $(1,0)$ and has the radius R_t.

- Finally, we may convert the measurement of the observable ε, which describes the CP violation in the neutral kaon system that was discovered in 1964, into a hyperbola in the $\bar{\rho}$–$\bar{\eta}$ plane.

In Fig. 2.3, we have illustrated these contours; their intersection allows us to determine the apex of the UT within the SM. The curves that are implied by ΔM_d and ε depend on the CKM parameter A and the top-quark mass m_t, as well as on

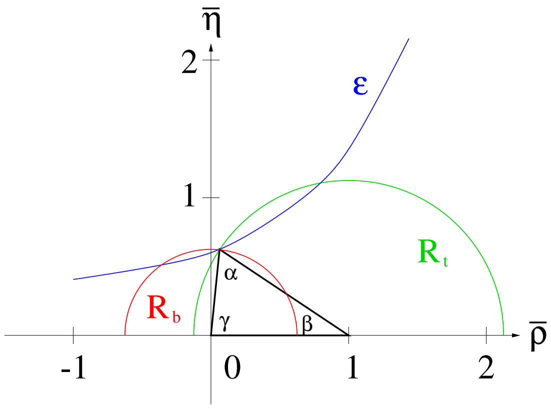

Figure 2.3: Contours in the $\bar{\rho}$–$\bar{\eta}$ plane, allowing us to determine the apex of the UT.

certain perturbatively calculable QCD corrections and non-perturbative parameters. Consequently, strong correlations between the theoretical and experimental uncertainties arise in the CKM fits. From the analysis of the experimental results, like $B^0 - \overline{B}^0$, $K^0 - \overline{K}^0$ mixing, $b \to c\ell\nu$, $b \to u\ell\nu$, etc, decays, the typical ranges for the UT angles are obtained:

$$70^\circ \lesssim \alpha \lesssim 130^\circ, \quad 20^\circ \lesssim \beta \lesssim 30^\circ, \quad 50^\circ \lesssim \gamma \lesssim 70^\circ. \tag{2.52}$$

On the other hand, CP violation in the B-meson system provides various strategies to determine these angles *directly*, thereby offering different ways to fix the apex of the UT in the $\bar{\rho}$–$\bar{\eta}$ plane. Following these lines, a powerful test of the CKM mechanism can be performed. This very interesting feature is also reflected by the tremendous efforts to explore CP violation in B decays experimentally in this decade. Before having a closer look at B mesons, their decays, the theoretical tools to deal with them and the general requirements for having non-vanishing CP asymmetries, let us first consider the classifications of the B decays.

2.3 DECAYS OF *B* MESONS

The B-meson system consists of charged and neutral B mesons, which are characterized by the

$$B^+ \sim u\,\bar{b}, \qquad B^- \sim \bar{u}\,b$$

$$B_c^+ \sim c\,\bar{b}, \qquad B_c^+ \sim \bar{c}\,b$$

and

$$B_d^0 \sim d\,\bar{b}, \qquad \bar{B}_d^0 \sim \bar{d}\,b$$

$$B_s^0 \sim s\,\bar{b}, \qquad \bar{B}_s^0 \sim \bar{s}\,b$$

valence-quark contents, respectively. The characteristic feature of the neutral B_q ($q \in \{d, s\}$) mesons is the phenomenon of B_q^0–$\bar{B}_q^0$ mixing (the counterpart of K^0–$\bar{K}^0$ mixing). As far as the weak decays of B mesons are concerned, we distinguish between leptonic, semileptonic and non-leptonic transitions.

2.3.1 Leptonic Decays

The simplest B-meson decay class is the leptonic decays of the kind $B^- \to \ell\bar{\nu}$, as illustrated in Fig. 2.4. If we evaluate the corresponding Feynman diagram, we arrive at the following transition amplitude:

$$T_{fi} = -\frac{g_2^2}{8} V_{ub} \underbrace{[\bar{u}_\ell \gamma^\alpha (1-\gamma_5) v_\nu]}_{\text{Dirac spinors}} \left[\frac{g_{\alpha\beta}}{k^2 - M_W^2}\right] \underbrace{\langle 0|\bar{u}\gamma^\beta(1-\gamma_5)b|B^-\rangle}_{\text{hadronic ME}}, \qquad (2.1)$$

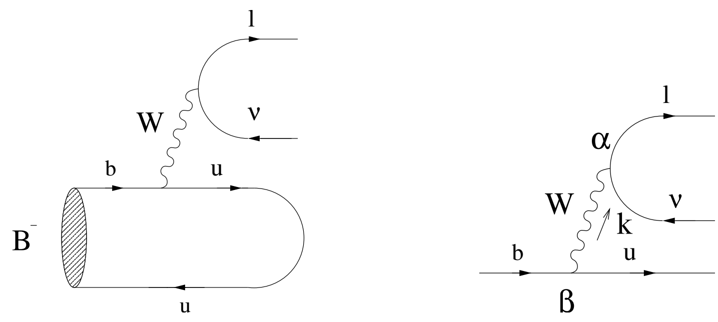

Figure 2.4: Feynman diagram contributing to the leptonic decay $B^- \to \ell\bar{\nu}$.

where g_2 is the $SU(2)_\mathrm{L}$ gauge coupling, V_{ub} the corresponding element of the CKM matrix, α and β are Lorentz indices, and M_W denotes the mass of the W gauge boson. It should be noted that in W-boson propagator we omitted the term proportional $k_\alpha k_\beta/m_W^2$ since using equation of motion one can show that this term is proportional to $m_\ell(m_b+m_q)/m_W^2 \; << 1$. So this term can be safely neglected. Since the four-momentum k that is carried by the W satisfies $k^2 = M_B^2 \ll M_W^2$, we may write

$$\frac{g_{\alpha\beta}}{k^2 - M_W^2} \quad \longrightarrow \quad -\frac{g_{\alpha\beta}}{M_W^2} \equiv -\left(\frac{8G_\mathrm{F}}{\sqrt{2}g_2^2}\right) g_{\alpha\beta}, \tag{2.2}$$

where G_F is Fermi's constant. Consequently, we may "integrate out" the W boson in (2.1), which yields

$$T_{fi} = \frac{G_\mathrm{F}}{\sqrt{2}} V_{ub} \left[\bar{u}_\ell \gamma^\alpha (1-\gamma_5) v_\nu\right] \langle 0|\bar{u}\gamma_\alpha(1-\gamma_5)b|B^-\rangle. \tag{2.3}$$

In this simple expression, *all* the hadronic physics is encoded in the *hadronic matrix element*

$$\langle 0|\bar{u}\gamma_\alpha(1-\gamma_5)b|B^-\rangle,$$

Since the B^- meson is a pseudoscalar particle, we have

$$\langle 0|\overline{u}\gamma_\alpha b|B^-\rangle = 0, \tag{2.4}$$

and may write

$$\langle 0|\bar{u}\gamma_\alpha\gamma_5 b|B^-(k)\rangle = if_B k_\alpha, \tag{2.5}$$

where f_B is the leptonic B-meson *decay constant* of the B-meson, which is an important input for phenomenological studies. In order to determine this quantity, which is a very challenging task, non-perturbative techniques, such as lattice [40] or QCD sum-rule analyses [42], are required. For example, QCD sum rules predicted that $f_B = 140\, MeV$[41]. If we use (2.3) with (2.4) and (2.5), and perform the corresponding phase-space integrations, we obtain the following expression for the decay rate:

$$\Gamma(B^- \to \ell\bar{\nu}) = \frac{G_{\rm F}^2}{8\pi}|V_{ub}|^2 M_B m_\ell^2 \left(1 - \frac{m_\ell^2}{M_B^2}\right)^2 f_B^2, \tag{2.6}$$

where M_B and m_ℓ denote the masses of the B^- and ℓ, respectively. Because of the tiny value of $|V_{ub}| \propto \lambda^3$ and a helicity-suppression mechanism, we obtain unfortunately very small branching ratios of $\mathcal{O}(10^{-10})$ and $\mathcal{O}(10^{-7})$ for $\ell = e$ and $\ell = \mu$, respectively [43]. The helicity suppression is not effective for $\ell = \tau$, but – because of the required τ reconstruction – these modes are also very challenging from an experimental point of view. A measurement of leptonic B-meson decays would nevertheless be very interesting, as it would allow an experimental determination of f_B, thereby providing tests of non-perturbative calculations

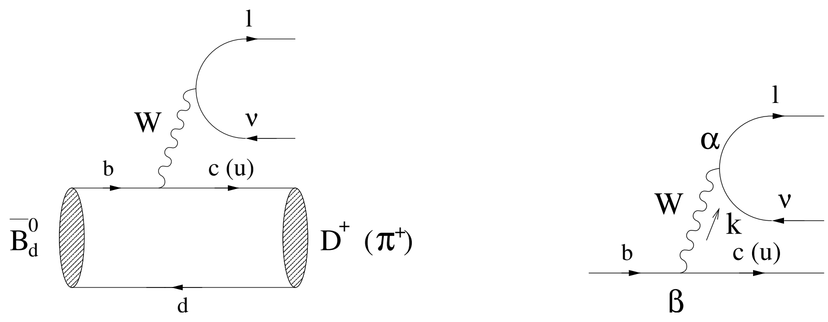

Figure 2.5: Feynman diagram contributing to semileptonic $\bar{B}^0_d \to D^+(\pi^+)\ell\bar{\nu}$ decays.

of this important parameter.[1] The CKM element $|V_{ub}|$ can be extracted from semileptonic B decays, which is our next topic.

2.3.2 Semileptonic Decays

Semileptonic B-meson decays of the kind shown in Fig. 2.5 have a structure that is more complicated than the one of the leptonic transitions. If we evaluate the corresponding Feynman diagram for the $b \to c$ case, we obtain

$$T_{fi} = -\frac{g_2^2}{8} V_{cb} \underbrace{\left[\bar{u}_\ell \gamma^\alpha (1-\gamma_5) v_\nu\right]}_{\text{Dirac spinors}} \left[\frac{g_{\alpha\beta}}{k^2 - M_W^2}\right] \underbrace{\langle D^+ | \bar{c} \gamma^\beta (1-\gamma_5) b | \bar{B}^0_d \rangle}_{\text{hadronic ME}} . \tag{2.7}$$

Because of $k^2 \sim M_B^2 \ll M_W^2$, we may again – as in (2.1) – integrate out the W boson with the help of (2.2), which yields

$$T_{fi} = \frac{G_{\text{F}}}{\sqrt{2}} V_{cb} \left[\bar{u}_\ell \gamma^\alpha (1-\gamma_5) v_\nu\right] \langle D^+ | \bar{c} \gamma_\alpha (1-\gamma_5) b | \bar{B}^0_d \rangle, \tag{2.8}$$

[1] Leptonic decays of $D_{(s)}$ mesons allow the extraction of the corresponding decay constants $f_{D_{(s)}}$, which are defined in analogy to (2.5). These measurements are an important element of the CLEO-c research programme [44].

where *all* the hadronic physics is encoded in the hadronic matrix element

$$\langle D^+|\bar{c}\gamma_\alpha(1-\gamma_5)b|\bar{B}^0_d\rangle,$$

Since the $\bar{B}^0_d$ and D^+ are pseudoscalar mesons, we have

$$\langle D^+|\bar{c}\gamma_\alpha\gamma_5 b|\bar{B}^0_d\rangle = 0, \tag{2.9}$$

and remaining matrix element can be parameterized in terms of two formfactors as follows:

$$\langle D^+(k)|\bar{c}\gamma_\alpha b|\bar{B}^0_d(p)\rangle = F_1(q^2)\left[(p+k)_\alpha - \left(\frac{M_B^2 - M_D^2}{q^2}\right) q_\alpha\right] + F_0(q^2)\left(\frac{M_B^2 - M_D^2}{q^2}\right) q_\alpha, \tag{2.10}$$

where $q \equiv p - k$, and the $F_{1,0}(q^2)$ denote the *form factors* of the $\bar{B} \to D$ transitions. In order to guarantee the finiteness of the form factors at $q^2 = 0$, the condition $F_1(q^2 = 0) = F_0(q^2 = 0)$ must be satisfied. In order to calculate these parameters, which depend on the momentum transfer q, again non-perturbative techniques (lattice, QCD sum rules, etc.) are required.

- FCNC: Flavor Changing Neutral Current.
- NP: New Physics.
- UT: Unitary Triangle.
- PDG: Particle Data Group.
- ME: Matrix Element.

CHAPTER 3

GENERAL ANALYSIS OF CP VIOLATION IN POLARIZED $b \to d\ell^+\ell^-$ DECAY

3.1 Introduction

Rare B meson decays, induced by the flavor changing neutral current (FCNC) $b \to s(d)$ transitions, provide one of the most promising research area in particle physics. Interest to rare B meson decays has its roots in their role being potentially the precision testing ground for the standard model (SM) at loop level and looking for new physics beyond the SM [45]. Experimentally, these decays will provide a more precise determination of the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, such as, V_{tq} $(q = d, s, b)$, V_{ub} and CP violation. In FCNC decays, any deviation over the SM results is an unambiguous indication for new physics. The first observation of the radiative $B \to X_s\gamma$ decay by CLEO [46], and later by ALEPH [47], have yielded $|V_{tb}V_{ts}^*| \sim 0.035$, which is in confirmation with the CKM estimates.

Rare semileptonic decays $b \to s(d)\ell^+\ell^-$ can provide alternative sources for searching new physics beyond the SM, and these decays are relatively clean compared to pure hadronic decays. The matrix elements of the $b \to s\ell^+\ell^-$ transition contain terms describing the virtual effects induced by the $t\bar{t}$, $c\bar{c}$ and $u\bar{u}$ loops, which are proportional to $|V_{tb}V_{ts}^*|$, $|V_{cb}V_{cs}^*|$ and $|V_{ub}V_{us}^*|$, respectively.

Using the unitarity condition of the CKM matrix and neglecting $|V_{ub}V_{us}^*|$ in comparison to $|V_{tb}V_{ts}^*|$ and $|V_{cb}V_{cs}^*|$, it is obvious that, the matrix element for the $b \to s\ell^+\ell^-$ decay involves only one independent CKM matrix element, namely, $|V_{tb}V_{ts}^*|$, so that CP–violation in this channel is strongly suppressed in the SM.

As has already been noted, $b \to q\ell^+\ell^-$ decay is a promising candidate for establishing new physics beyond the SM. New physics effects manifest themselves in rare B decays in two different ways; either through new contributions to the Wilson coefficients existing in the SM, or through the new structures in the effective Hamiltonian, which are absent in the SM. Note that, the semileptonic $b \to q\ell^+\ell^-$ decay has extensively been studied in numerous works [48]–[6], in the framework of the SM and its various extensions. Recently, the first measurement of the $b \to s\ell^+\ell^-$ decay has been reported by BELLE [7] and is in agreement with the SM expectation. Therefore, this result puts further constraint on any extension of the SM.

In this chapter we examine CP-violating effects for the case when one of the leptons is polarized, in model independent framework, by taking into account a more general form of the effective Hamiltonian. It should be noted here that similar calculation has been carried out in the SM in [8].

One efficient way in establishing new physics beyond the SM is the measurement of the lepton polarization [9]–[70]. This issue has been studied for the $b \to s\tau^+\tau^-$ transition and $B \to K^*\ell^+\ell^-$, $B \to K\ell^+\ell^-$ decays in a model independent way in [64] and [69, 70], respectively.

This chapter is organized as follows. In section 2, using the most general form of the four–Fermi interaction, we derive model independent expressions for the CP-violating asymmetry, for polarized and unpolarized leptons. In section 3 we present our numerical analysis.

3.2 Formalism

In this section we present the necessary expressions for CP-violating asymmetry when lepton is polarized and unpolarized, using the most general form of four–Fermi interactions. Following [62, 64], we write the matrix element of the $b \to d\ell^+\ell^-$ transition in terms of the twelve model independent four–Fermi interactions

$$
\begin{aligned}
\mathcal{M} = & \frac{G\alpha}{\sqrt{2}\pi} V_{tb}V_{td}^* \Bigg\{ C_{SL}\bar{d}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}Lb\bar{\ell}\gamma^\mu\ell + C_{BR}\bar{d}i\sigma_{\mu\nu}\frac{q^\nu}{q^2}Rb\bar{\ell}\gamma^\mu\ell + C_{LL}^{tot}\bar{d}_L\gamma_\mu b_L\bar{\ell}_L\gamma^\mu\ell_L \\
& + C_{LR}^{tot}\bar{d}_L\gamma_\mu b_L\bar{\ell}_R\gamma^\mu\ell_R + C_{RL}\bar{d}_R\gamma_\mu b_R\bar{\ell}_L\gamma^\mu\ell_L + C_{RR}\bar{d}_R\gamma_\mu b_R\bar{\ell}_R\gamma^\mu\ell_R \\
& + C_{LRLR}\bar{d}_L b_R\bar{\ell}_L\ell_R + C_{RLLR}\bar{d}_R b_L\bar{\ell}_L\ell_R + C_{LRRL}\bar{d}_L b_R\bar{\ell}_R\ell_L + C_{RLRL}\bar{d}_R b_L\bar{\ell}_R\ell_L
\end{aligned}
$$

$$+ C_T \bar{d}\sigma_{\mu\nu} b \bar{\ell}\sigma^{\mu\nu}\ell + iC_{TE}\epsilon_{\mu\nu\alpha\beta}\bar{d}\sigma^{\mu\nu} b \bar{\ell}\sigma^{\alpha\beta}\ell \Big\} , \tag{3.1}$$

where L and R stand for the chiral operators $L = (1-\gamma_5)/2$ and $R = (1+\gamma_5)/2$, respectively, and C_X are the coefficients of the four–Fermi interactions. The first two terms, C_{SL} and C_{BR} are the nonlocal Fermi interactions which correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, respectively. The next four terms are the vector type interactions with coefficients C_{LL}^{tot}, C_{LR}^{tot}, C_{RL} and C_{RR}. Two of these vector interactions containing the coefficients C_{LL}^{tot} and C_{LR}^{tot} do already exist in the SM in the forms $C_9^{eff} - C_{10}$ and $C_9^{eff} + C_{10}$, respectively. Therefore C_{LL}^{tot} and C_{LR}^{tot} can be represented as

$$\begin{aligned} C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL} , \\ C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR} , \end{aligned} \tag{3.2}$$

where C_{LL} and C_{LR} describe the contributions of new physics. The following four terms with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} describe the scalar type interactions, and the last two terms with the coefficients C_T and C_{TE} are the tensor type interactions. It should be noted here that, in further analysis we will assume that all new Wilson coefficients are real, as they are in the SM, while only C_9^{eff} contains imaginary part and it is parameterized in the following form

$$C_9^{eff} = \xi_1 + \lambda_u \xi_2 , \tag{3.3}$$

where

$$\lambda_u = \frac{V_{ub} V_{ud}^*}{V_{tb} V_{td}^*} ,$$

and

$$\begin{aligned}
\xi_1 &= 4.128 + 0.138\omega(\hat{s}) + g(\hat{m}_c, \hat{s}) C_0(\hat{m}_b) - \frac{1}{2} g(\hat{m}_d, \hat{s})(C_3 + C_4) \\
&- \frac{1}{2} g(\hat{m}_b, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6) , \\
\xi_2 &= [g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s})](3C_1 + C_2) , \qquad (3.4)
\end{aligned}$$

where $\hat{m}_q = m_q/m_b$, $\hat{s} = q^2$, $C_0(\mu) = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$, and

$$\begin{aligned}
\omega(\hat{s}) &= -\frac{2}{9}\pi^2 - \frac{4}{3} Li_2(\hat{s}) - \frac{2}{3}\ln(\hat{s})\ln(1-\hat{s}) - \frac{5+4\hat{s}}{3(1+2\hat{s})}\ln(1-\hat{s}) \\
&- \frac{2\hat{s}(1+\hat{s})(1-2\hat{s})}{3(1-\hat{s})^2(1+2\hat{s})}\ln(\hat{s}) + \frac{5+9\hat{s}-6\hat{s}^2}{3(1-\hat{s})(1+2\hat{s})} , \qquad (3.5)
\end{aligned}$$

represents the $O(\alpha_s)$ correction coming from one gluon exchange in the matrix element of the operator $\mathcal{O}_9$ [71], while the function $g(\hat{m}_q, \hat{s})$ represents one–loop corrections to the four–quark operators O_1–O_6 [72], whose form is

$$\begin{aligned}
g(\hat{m}_q, \hat{s}) &= -\frac{8}{9}\ln(\hat{m}_q) + \frac{8}{27} + \frac{4}{9} y_q - \frac{2}{9}(2 + y_q) \qquad (3.6) \\
&- \sqrt{|1-y_q|}\left\{\theta(1-y_q)\left[\ln\left(\frac{1+\sqrt{1-y_q}}{1-\sqrt{1-y_q}}\right) - i\pi\right] + \theta(y_q - 1)\arctan\left(\frac{1}{\sqrt{y_q-1}}\right)\right\} ,
\end{aligned}$$

where $y_q = 4\hat{m}_q^2/\hat{s}$.

In addition to the short distance contributions, $B \to X_d \ell^+ \ell^-$ decay also receives long distance contributions, which have their origin in the real $\bar{u}u$, $\bar{d}d$ and $\bar{c}c$ intermediate states, i.e., ρ, ω and J/ψ family. There are four different approaches in taking long distance contributions into consideration:

a) HQET based approach [73],

b) AMM approach [74],

c) LSW approach [75],

d) KS approach [70].

In the present work we choose the AMM approach, in which these resonance contributions are parameterized using the Breit–Wigner form for the resonant states. The effective coefficient C_9^{eff} including the ρ, ω and J/ψ resonances are defined as

$$C_9^{eff} = C_9(\mu) + Y_{res}(\hat{s}) \ , \tag{3.7}$$

where

$$\begin{aligned} Y_{res} = & -\frac{3\pi}{\alpha^2}\Bigg\{ \left(C^{(0)}(\mu) + \lambda_u \left[3C_1(\mu) + C_2(\mu)\right]\right) \sum_{V_i=\psi} K_i \frac{\Gamma(V_i \to \ell^+\ell^-) M_{V_i}}{M_{V_i}^2 - q^2 - i M_{V_i}\Gamma_{V_i}} \\ & - \lambda_u g(\hat{m}_u, \hat{s}) \left[3C_1(\mu) + C_2(\mu)\right] \sum_{V_i=\rho,\omega} \frac{\Gamma(V_i \to \ell^+\ell^-) M_{V_i}}{M_{V_i}^2 - q^2 - i M_{V_i}\Gamma_{V_i}} \Bigg\} \ . \end{aligned} \tag{3.8}$$

The phenomenological factor K_i has the universal value for the inclusive $B \to X_s \ell^+\ell^-$ decay $K_i \simeq 2.3$ [76], which we use in our calculations.

As we have already noted, CP asymmetry can appear both for the cases when lepton is polarized and unpolarized, and hence, along this line, we will present the expressions for the differential decay rate for both cases when the lepton is polarized and unpolarized. Starting with Eq. (3.1), after lengthy calculations we get the following expression for the unpolarized decay width:

$$\frac{d\Gamma}{d\hat{s}} = \frac{G^2\alpha^2 m_b^5}{32768\pi^5} \left|V_{tb}V_{td}^*\right|^2 \lambda^{1/2}(1,\hat{s},0) v \Delta(\hat{s}) \ , \tag{3.9}$$

where $\hat{s} = q^2/m_b^2$, $v = \sqrt{1 - 4\hat{m}_\ell^2/\hat{s}}$ is the velocity of the final lepton, $\hat{m}_\ell = m_\ell/m_b$, and $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ is the usual triangle function. The explicit expression of the function $\Delta(\hat{s})$ can be written as

$$
\begin{aligned}
\Delta(\hat{s}) =\; & 16(1-\hat{s})\,\mathrm{Re}\Bigg\{ \frac{8}{3\hat{s}^2}(2\hat{m}_\ell^2 + \hat{s})(2+\hat{s})\Big[|C_{BR}|^2 + 8\hat{s}\left(|C_T|^2 + 4|C_{TE}|^2\right)\Big] \\
& - \frac{8}{\hat{s}}(2\hat{m}_\ell^2 + \hat{s})\left(C_{LL}^{tot} + C_{LR}^{tot}\right) C_{BR}^* - \frac{32}{\hat{s}}\hat{m}_\ell(2+\hat{s})\left(C_T - 2C_{TE}\right) C_{BR}^* \\
& - \frac{4}{3\hat{s}}[2\hat{s}(\hat{m}_\ell^2 - \hat{s}) - (2\hat{m}_\ell^2 + \hat{s})]\left(|C_{LL}^{tot}|^2 + |C_{LR}^{tot}|^2 + |C_{RL}|^2 + |C_{RR}|^2\right) \\
& - 2(2\hat{m}_\ell^2 - \hat{s})\left(|C_{LRLR}|^2 + |C_{RLLR}|^2 + |C_{LRRL}|^2 + |C_{RLRL}|^2\right) \\
& + 8\hat{m}_\ell^2\Big[2\left(C_{LL}^{tot}(C_{LR}^{tot})^* + C_{RL}C_{RR}^*\right) - \left(C_{LRLR}C_{LRRL}^* + C_{RLLR}C_{RLRL}^*\right)\Big] \\
& - 4\hat{m}_\ell\Big[\left(C_{LL}^{tot} - C_{LR}^{tot}\right)\left(C_{LRLR}^* - C_{LRRL}^*\right) + \left(C_{RL} - C_{RR}\right)\left(C_{RLLR}^* - C_{RLRL}^*\right) \\
& - 12\left(C_{LL}^{tot} + C_{LR}^{tot}\right)\left(C_T^* - 2C_{TE}^*\right) - 12\left(C_{RL} + C_{RR}\right)\left(C_T^* + 2C_{TE}^*\right)\Big]\Bigg\}\,. \quad (3.10)
\end{aligned}
$$

Our result agrees with the one given in [62], except the term multiplying the coefficient $N_9(s)$ in [62]. The differential decay width for the CP conjugated process can be obtained from Eq. (3.8) by making the replacement

$$\Delta \to \bar{\Delta},\ \text{i.e.,}\ C_9^{eff} = \xi_1 + \lambda_u \xi_2 \to \bar{C}_9^{eff} = \xi_1 + \lambda_u^* \xi_2\ .$$

The lepton polarization has been firstly analyzed in the SM in [70] and [10], where it has been shown that additional information can be obtained about the quadratic functions of the Wilson coefficients C_7^{eff}, C_9^{eff} and C_{10}. In order to calculate the final lepton polarization, we define the orthogonal unit vectors $\vec{e}_L$,

$\vec{e}_T$ and $\vec{e}_N$ in such a way that, in the rest frame of leptons they can be expressed as

$$
\begin{aligned}
s_L^{-\mu} &= \left(0, \vec{e}_L^{\,-}\right) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right) ~, \\
s_N^{-\mu} &= \left(0, \vec{e}_N^{\,-}\right) = \left(0, \frac{\vec{p}_s \times \vec{p}_-}{|\vec{p}_s \times \vec{p}_-|}\right) ~, \\
s_T^{-\mu} &= \left(0, \vec{e}_T^{\,-}\right) = \left(0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}\right) ~, \\
s_L^{+\mu} &= \left(0, \vec{e}_L^{\,+}\right) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right) ~, \\
s_N^{+\mu} &= \left(0, \vec{e}_N^{\,+}\right) = \left(0, \frac{\vec{p}_s \times \vec{p}_+}{|\vec{p}_s \times \vec{p}_+|}\right) ~, \\
s_T^{+\mu} &= \left(0, \vec{e}_T^{\,+}\right) = \left(0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}\right) ~,
\end{aligned}
$$

where $\vec{p}_-$, $\vec{p}_+$ and $\vec{p}_s$ are the three–momenta of the leptons ℓ^-, ℓ^+, and the strange quark in the center of mass frame (CM) of $\ell^- \, \ell^+$, respectively, and the subscripts L, N and T stand for the longitudinal, normal and transversal polarization of the lepton. Boosting the unit vectors s_L^- and s_L^+ corresponding to longitudinal polarization by Lorentz transformation, from the rest frame of the corresponding leptons to the $\ell^- \, \ell^+$ CM frame, we get

$$
\begin{aligned}
\left(s_L^{-\mu}\right)_{CM} &= \left(\frac{|\vec{p}_-|}{m_\ell}, \frac{E\vec{p}_-}{m_\ell \, |\vec{p}_-|}\right) ~, \\
\left(s_L^{+\mu}\right)_{CM} &= \left(\frac{|\vec{p}_-|}{m_\ell}, -\frac{E\vec{p}_-}{m_\ell \, |\vec{p}_-|}\right) ~,
\end{aligned} \qquad (3.11)
$$

while $s_N^{\mp\mu}$ and $s_T^{\mp\mu}$ are not changed by the boost.

The differential decay rate of the $b \to d\ell^+\ell^-$ decay, for any spin direction $\vec{n}^{\mp}$ of $\ell^{\mp}$, where $\vec{n}^{\mp}$ is the unit vector in the rest frame of $\ell^{\mp}$, can be written in the following form

$$\frac{d\Gamma(s,\vec{n}^{\mp})}{d\hat{s}} = \frac{1}{2}\left(\frac{d\Gamma}{d\hat{s}}\right)_0 \left[1 + \left(P_L^{\mp}\vec{e}_L^{\,\mp} + P_N^{\mp}\vec{e}_N^{\,\mp} + P_T^{\mp}\vec{e}_T^{\,\mp}\right)\cdot\vec{n}^{\mp}\right] , \tag{3.12}$$

where $(d\Gamma/d\hat{s})_0$ corresponds to the unpolarized differential decay width (see Eq. (3.9)) for the $b \to d\ell^+\ell^-$ decay. The differential decay width for the $\bar{b} \to \bar{d}\ell^+\ell^-$ decay, can simply be obtained by making the replacement

$$\frac{d\Gamma(s,\vec{n}^{\mp})}{d\hat{s}} \to \frac{d\bar{\Gamma}(s,\vec{n}^{\mp})}{d\hat{s}} ,$$

where $d\bar{\Gamma}/d\hat{s}$ is obtained from $d\Gamma/d\hat{s}$ by replacing $C_9^{eff} = \xi_1 + \lambda_u\xi_2$ to $\bar{C}_9^{eff} = \xi_1 + \lambda_u^*\xi_2$. The polarizations P_L, P_N and P_T are defined as

$$P_i^{\mp}(\hat{s}) = \frac{\dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{\mp}=\vec{e}_i^{\,\mp}) - \dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{\mp}=-\vec{e}_i^{\,\mp})}{\dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{\mp}=\vec{e}_i^{\,\mp}) + \dfrac{d\Gamma}{d\hat{s}}(\vec{n}^{\mp}=-\vec{e}_i^{\,\mp})} = \frac{\Delta_i^{\mp}}{\Delta} , \tag{3.13}$$

with $i = L,\ N,\ T$.

The explicit expressions for the longitudinal polarization asymmetries P_L^- and P_L^+ are

$$\begin{aligned}
P_L^- &= \frac{32(1-\hat{s})v}{\Delta}\,\mathrm{Re}\Bigg\{4\left(C_{LL}^{tot} - C_{LR}^{tot}\right)C_{BR}^* \\
&- \frac{2}{3}(1+2\hat{s})\left(|C_{LL}^{tot}|^2 - |C_{LR}^{tot}|^2 + |C_{RL}|^2 - |C_{RR}|^2 - 128 C_T C_{TE}^*\right) \\
&+ \frac{16}{3\hat{s}}\hat{m}_\ell(2+\hat{s})\left(C_T - 2C_{TE}\right)C_{BR}^* + 2\hat{m}_\ell\Big[\left(C_{LL}^{tot} - C_{LR}^{tot}\right)\left(C_{LRLR}^* + C_{LRRL}^*\right) \\
&+ \left(C_{RL} - C_{RR}\right)\left(C_{RLLR}^* + C_{RLRL}^*\right) - 4\left(3C_{LL}^{tot} - C_{LR}^{tot}\right)\left(C_T^* - 2C_{TE}^*\right) \\
&- 4\left(C_{RL} - 3C_{RR}\right)\left(C_T^* + 2C_{TE}^*\right)\Big] - \hat{s}\Big(|C_{LRLR}|^2 - |C_{LRRL}|^2 \\
&+ |C_{RLLR}|^2 - |C_{RLRL}|^2 + 128 C_T^* C_{TE}\Big)\Bigg\},
\end{aligned}$$

and

$$
\begin{aligned}
P_L^+ &= \frac{32(1-\hat{s})v}{\Delta}\,\mathrm{Re}\Bigg\{ -4\left(\bar{C}_{LL}^{tot}-\bar{C}_{LR}^{tot}\right)C_{BR}^* \\
&+ \frac{2}{3}(1+2\hat{s})\left(|\bar{C}_{LL}^{tot}|^2-|\bar{C}_{LR}^{tot}|^2+|C_{RL}|^2-|C_{RR}|^2+128C_T C_{TE}^*\right) \\
&+ \frac{16}{3\hat{s}}\hat{m}_\ell(2+\hat{s})\left(C_T-2C_{TE}\right)C_{BR}^* + 2\hat{m}_\ell\Big[\left(\bar{C}_{LL}^{tot}-\bar{C}_{LR}^{tot}\right)\left(C_{LRLR}^*+C_{LRRL}^*\right) \\
&+ \left(C_{RL}-C_{RR}\right)\left(C_{RLLR}^*+C_{RLRL}^*\right) + 4\left(\bar{C}_{LL}^{tot}-3\bar{C}_{LR}^{tot}\right)\left(C_T^*-2C_{TE}^*\right) \\
&+ 4\left(3C_{RL}-C_{RR}\right)\left(C_T^*+2C_{TE}^*\right)\Big] - \hat{s}\Big(|C_{LRLR}|^2-|C_{LRRL}|^2 \\
&+ |C_{RLLR}|^2-|C_{RLRL}|^2+128C_T^*C_{TE}\Big)\Bigg\}.
\end{aligned}
$$

The normal asymmetries, P_N^- and P_N^+, are;

$$
\begin{aligned}
P_N^- &= -\frac{4\pi(1-\hat{s})v\sqrt{\hat{s}}}{\Delta}\,\mathrm{Im}\Bigg\{ -\frac{8}{\hat{s}}\hat{m}_\ell\left(C_{LL}^{tot}-C_{LR}^{tot}\right)C_{BR}^* \\
&+ 8\hat{m}_\ell\Big[C_{LL}^{tot}(C_{LR}^{tot})^* - C_{RL}C_{RR}^* - 2\left(C_{LRLR}+C_{LRRL}\right)\left(C_T^*-2C_{TE}^*\right) \\
&- 2\left(C_{RLLR}+C_{RLRL}\right)\left(C_T^*+2C_{TE}^*\right)\Big] + 4\Big[\left(C_{LRLR}+C_{LRRL}\right)C_{BR}^* \\
&+ C_{LL}^{tot}C_{LRRL}^* + C_{LR}^{tot}C_{LRLR}^* + C_{RL}C_{RLRL}^* + C_{RR}C_{RLLR}^* - 4C_{RL}\left(C_T^*+2C_{TE}^*\right) \\
&- 4C_{LR}^{tot}\left(C_T^*-2C_{TE}^*\right)\Big] - \frac{16}{\hat{s}}\left(C_T-2C_{TE}\right)C_{BR}^*\Bigg\},
\end{aligned}
$$

$$
\begin{aligned}
P_N^+ &= -\frac{4\pi(1-\hat{s})v\sqrt{\hat{s}}}{\Delta}\,\mathrm{Im}\Bigg\{\frac{8}{\hat{s}}\hat{m}_\ell\left(\bar{C}_{LL}^{tot}-\bar{C}_{LR}^{tot}\right)C_{BR}^* \\
&+ 8\hat{m}_\ell\Big[-\bar{C}_{LL}^{tot}(\bar{C}_{LR}^{tot})^* + C_{RL}C_{RR}^* + 2\left(C_{LRLR}+C_{LRRL}\right)\left(C_T^*-2C_{TE}^*\right)
\end{aligned}
$$

$$+ 2\left(C_{RLLR}+C_{RLRL}\right)\left(C_T^* + 2C_{TE}^*\right)\Big] - 4\Big[\left(C_{LRLR}+C_{LRRL}\right)C_{BR}^*$$

$$+ \bar{C}_{LL}^{tot}C_{LRLR}^* + \bar{C}_{LR}^{tot}C_{LRRL}^* + C_{RL}C_{RLLR}^* + C_{RR}C_{RLRL}^* + 4\bar{C}_{LL}^{tot}\left(C_T^* - 2C_{TE}^*\right)$$

$$+ 4C_{RR}\left(C_T^* + 2C_{TE}^*\right)\Big] + \frac{16}{\hat{s}}C_{BR}\left(C_T^* - 2C_{TE}^*\right)\Bigg\} \, .$$

The transverse asymmetries, P_T^- and P_T^+, are;

$$\begin{aligned}
P_T^- \; = & -\frac{8\pi(1-\hat{s})}{\Delta\sqrt{\hat{s}}}\,\mathrm{Re}\Bigg\{ -\frac{8}{\hat{s}}\hat{m}_\ell|C_{BR}|^2 + 4\hat{m}_\ell\left(3C_{LL}^{tot} + C_{LR}^{tot}\right)C_{BR}^* \\
& - 2(1+\hat{s})\hat{m}_\ell\left(|C_{LL}^{tot}|^2 - |C_{RR}|^2\right) + 4\hat{m}_\ell\hat{s}\Big[- C_{LL}^{tot}(C_{LR}^{tot})^* + C_{RL}C_{RR}^* \\
& + 2\left(C_{LRLR} - C_{LRRL}\right)\left(C_T^* - 2C_{TE}^*\right) + 2\left(C_{RLLR} - C_{RLRL}\right)\left(C_T^* + 2C_{TE}^*\right)\Big] \\
& + 2(1-\hat{s})\hat{m}_\ell\left(|C_{LR}^{tot}|^2 - |C_{RL}|^2\right) - 2\hat{s}\left(C_{LRLR} - C_{LRRL}\right)C_{BR}^* \\
& + \frac{8}{\hat{s}}(4\hat{m}_\ell^2 + \hat{s})C_{BR}\left(C_T^* - 2C_{TE}^*\right) + 4\hat{m}_\ell^2\Big[C_{LL}^{tot}C_{LRLR}^* - C_{LR}^{tot}C_{LRRL}^* \\
& + C_{RL}C_{RLLR}^* - C_{RR}C_{RLRL}^* - 12C_{LL}^{tot}\left(C_T^* - 2C_{TE}^*\right) + 12C_{RR}\left(C_T^* + 2C_{TE}^*\right)\Big] \\
& + 2(2\hat{m}_\ell^2 - \hat{s})\Big[C_{LL}^{tot}C_{LRRL}^* - C_{LR}^{tot}C_{LRLR}^* + C_{RL}C_{RLRL}^* - C_{RR}C_{RLLR}^* \\
& + 4C_{LR}^{tot}\left(C_T^* - 2C_{TE}^*\right) - 4C_{RL}\left(C_T^* + 2C_{TE}^*\right)\Big] + 256\hat{m}_\ell C_T C_{TE}^*\Bigg\} \, ,
\end{aligned}$$

and

$$\begin{aligned}
P_T^+ \; = & -\frac{8\pi(1-\hat{s})}{\Delta\sqrt{\hat{s}}}\,\mathrm{Re}\Bigg\{ -\frac{8}{\hat{s}}\hat{m}_\ell|C_{BR}|^2 + 4\hat{m}_\ell\left(\bar{C}_{LL}^{tot} + 3\bar{C}_{LR}^{tot}\right)C_{BR}^* \\
& + 2(1-\hat{s})\hat{m}_\ell\left(|\bar{C}_{LL}^{tot}|^2 - |C_{RR}|^2\right) - 4\hat{m}_\ell\hat{s}\Big[\bar{C}_{LL}^{tot}(\bar{C}_{LR}^{tot})^* - C_{RL}C_{RR}^* \\
& + 2\left(C_{LRLR} - C_{LRRL}\right)\left(C_T^* - 2C_{TE}^*\right) + 2\left(C_{RLLR} - C_{RLRL}\right)\left(C_T^* + 2C_{TE}^*\right)\Big] \\
& - 2(1+\hat{s})\hat{m}_\ell\left(|\bar{C}_{LR}^{tot}|^2 - |C_{RL}|^2\right) + 2\hat{s}\left(C_{LRLR} - C_{LRRL}\right)C_{BR}^*
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{8}{\hat{s}}(4\hat{m}_\ell^2 + \hat{s}) C_{BR}\left(C_T^* - 2C_{TE}^*\right) + 4\hat{m}_\ell^2 \Big[\bar{C}_{LL}^{tot} C_{LRRL}^* - \bar{C}_{LR}^{tot} C_{LRLR}^* \\
&+ C_{RL} C_{RLRL}^* - C_{RR} C_{RLLR}^* - 12\bar{C}_{LR}^{tot}\left(C_T^* - 2C_{TE}^*\right) + 12 C_{RL}\left(C_T^* + 2C_{TE}^*\right)\Big] \\
&+ 2(2\hat{m}_\ell^2 - \hat{s})\Big[\bar{C}_{LL}^{tot} C_{LRLR}^* - \bar{C}_{LR}^{tot} C_{LRRL}^* + C_{RL} C_{RLLR}^* - C_{RR} C_{RLRL}^* \\
&+ 4\bar{C}_{LL}^{tot}\left(C_T^* - 2C_{TE}^*\right) - 4C_{RR}\left(C_T^* + 2C_{TE}^*\right)\Big] + 256\hat{m}_\ell C_T C_{TE}^* \Bigg\} \, .
\end{aligned}
$$

It should be noted here that, these polarizations were calculated in [62], using the most general form of the effective Hamiltonian. Our results for P_L and P_N agree with the ones given in [62], while the transversal polarizations P_T^- and P_T^+ both differ from the ones given in the same work. In the SM case, our results for P_L, P_N and P_T coincide with the results of [8]. It is quite obvious from the expressions of P_i that, they involve various quadratic combinations of the Wilson coefficients and hence they are quite sensitive to the new physics. The polarizations P_N and P_T are proportional to m_ℓ and therefore can be significant for τ lepton only.

Having obtained all necessary expressions, we can proceed now to study the CP-violating asymmetries. In the unpolarized lepton case, the CP-violating differential decay width asymmetry is defined as

$$
A_{CP}(\hat{s}) = \frac{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 - \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0}{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 + \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0} = \frac{\Delta - \bar{\Delta}}{\Delta + \bar{\Delta}} \, , \qquad (3.14)
$$

where

$$
\frac{d\Gamma}{d\hat{s}} = \frac{d\Gamma(b \to d\ell^+\ell^-)}{d\hat{s}}, \text{ and, } \frac{d\bar{\Gamma}}{d\hat{s}} = \frac{d\bar{\Gamma}(b \to d\ell^+\ell^-)}{d\hat{s}} \, ,
$$

and $(d\bar{\Gamma}/d\hat{s})_0$ can be obtained from $(d\Gamma/d\hat{s})_0$ by making the replacement

$$C_9^{eff} = \xi_1 + \lambda_u \xi_2 \to \bar{C}_9^{eff} = \xi_1 + \lambda_u^* \xi_2 \; . \tag{3.15}$$

Using Eqs. (3.12) and (3.14), we get for the CP-violating asymmetry

$$\begin{aligned} A_{CP}(\hat{s}) &= -4\mathrm{Im}[\lambda_u] \frac{\Sigma(\hat{s})}{\Delta(\hat{s}) + \bar{\Delta}(\hat{s})} \; , \\ &\approx -2\mathrm{Im}[\lambda_u] \frac{\Sigma(\hat{s})}{\Delta(\hat{s})} \; , \end{aligned} \tag{3.16}$$

and $\Sigma(\hat{s})$, whose explicit form can easily be obtained using Eqs. (3.10) and (3.14).

$$\begin{aligned} \Sigma(\hat{s}) &= \frac{64}{3\hat{s}}(1-\hat{s}) \Big\{ 2(1+2\hat{s})(2\hat{m}_\ell^2 + \hat{s})\,\mathrm{Im}[\xi_1^* \xi_2] - 36 \hat{m}_\ell \hat{s}\,\mathrm{Im}[(C_T - 2C_{TE})\xi_2^*]) \\ &- (1+2\hat{s})(2\hat{m}_\ell^2 + \hat{s})\,\mathrm{Im}[(C_{LL} + C_{LR})\xi_2^*] + 12 C_7 (2\hat{m}_\ell^2 + \hat{s})\,\mathrm{Im}[\xi_2] \Big\}. \end{aligned} \tag{3.17}$$

In the presence of the lepton polarization, CP asymmetry is modified and the source of this modification can be attributed to the presence of new interference terms which contain C_9^{eff} (in our case C_{LL}^{tot} and C_{LR}^{tot}). We now proceed to calculate this new contribution.

In the polarized lepton case, CP asymmetry can be defined as

$$A_{CP}(\vec{s}) = \frac{\dfrac{d\Gamma}{d\hat{s}}(\hat{s}, \vec{n}^-) - \dfrac{d\bar{\Gamma}}{d\hat{s}}(\hat{s}, \vec{n}^+)}{\left(\dfrac{d\Gamma}{d\hat{s}}\right)_0 + \left(\dfrac{d\bar{\Gamma}}{d\hat{s}}\right)_0} \; , \tag{3.18}$$

where

$$\frac{d\Gamma}{d\hat{s}} = \frac{d\Gamma(b \to d\ell^+\ell^-(\vec{n}^-))}{d\hat{s}}, \text{ and, } \frac{d\bar{\Gamma}}{d\hat{s}} = \frac{d\Gamma(b \to d\ell^+(\vec{n}^+)\ell^-)}{d\hat{s}} \; .$$

The differential decay width with lepton polarization for the $b \to d\ell^+\ell^-$ channel is given by Eq. (3.12). Analogously, for the corresponding CP conjugated process we have the expression

$$\frac{d\bar{\Gamma}}{d\hat{s}}(\vec{n}^{\mp}) = \frac{1}{2}\left(\frac{d\bar{\Gamma}}{d\hat{s}}\right)_0 \left[1 + P_i^{+}(\vec{e}_i^{\,\mp} \cdot \vec{n}^{\mp})\right] \ . \tag{3.19}$$

With the choice $\vec{n}^- = \vec{n}^+$, P_i^+ can be constructed from the differential decay width analogous to Eq. (3.13). At this stage we have all necessary ingredients for calculation of the CP-violating asymmetry for the lepton ℓ^- with polarization $\vec{n} = \vec{e}_i$. Inserting Eqs. (3.12) and (3.19) into Eq. (3.18), and setting $\vec{n}^- = \vec{n}^+$, the CP-violating asymmetry when lepton is polarized is defined as

$$A_{CP} = \frac{\frac{1}{2}\left(\frac{d\Gamma}{d\hat{s}}\right)_0 \left[1 + P_i^{-}(\vec{e}_i^{\,-} \cdot \vec{n})\right] - \frac{1}{2}\left(\frac{d\bar{\Gamma}}{d\hat{s}}\right)_0 \left[1 + P_i^{+}(\vec{e}_i^{\,+} \cdot \vec{n})\right]}{\left(\frac{d\Gamma}{d\hat{s}}\right)_0 + \left(\frac{d\bar{\Gamma}}{d\hat{s}}\right)_0} \ .$$

Taking into account the fact that $\vec{e}_{L,N}^{\,+} = -\vec{e}_{L,N}^{\,-}$, and $\vec{e}_T^{\,+} = \vec{e}_T^{\,-}$, we obtain

$$A_{CP} = \frac{1}{2}\left\{ \frac{\left(\frac{d\Gamma}{d\hat{s}}\right)_0 - \left(\frac{d\bar{\Gamma}}{d\hat{s}}\right)_0}{\left(\frac{d\Gamma}{d\hat{s}}\right)_0 + \left(\frac{d\bar{\Gamma}}{d\hat{s}}\right)_0} \pm \frac{\left(\frac{d\Gamma}{d\hat{s}}\right)_0 P_i^- \mp \left(\frac{d\bar{\Gamma}}{d\hat{s}}\right)_0 P_i^+}{\left(\frac{d\Gamma}{d\hat{s}}\right)_0 + \left(\frac{d\bar{\Gamma}}{d\hat{s}}\right)_0} \right\} \ . \tag{3.20}$$

Using Eq. (3.9), we get from Eq. (3.20),

$$\begin{aligned} A_{CP}(\vec{n} = \mp\vec{e}_i^{\,-}) &= \frac{1}{2}\left\{ \frac{\Delta - \bar{\Delta}}{\Delta + \bar{\Delta}} \pm \frac{\Delta_i \mp \bar{\Delta}_i}{\Delta + \bar{\Delta}} \right\} \ , \\ &= \frac{1}{2}\left\{ A_{CP}(\hat{s}) \pm \delta A_{CP}^i(\hat{s}) \right\} \ , \end{aligned} \tag{3.21}$$

where the upper sign in the definition of δA_{CP} corresponds to L and N polarizations, while the lower sign corresponds to T polarization.

The $\delta A^i_{CP}(\hat{s})$ terms in Eq. (3.21) describe the modification to the unpolarized decay width, which can be written as

$$\delta A^i_{CP}(\hat{s}) = \frac{-4\mathrm{Im}[\lambda_u]\delta\Sigma^i(\hat{s})}{\Delta(\hat{s}) + \bar{\Delta}(\hat{s})} \; , \qquad \approx -2\mathrm{Im}[\lambda_u]\frac{\delta\Sigma^i(\hat{s})}{\Delta(\hat{s})} \; . \tag{3.22}$$

We present the explicit forms of the expressions for $\delta\Sigma^i(\hat{s})$, $(i = L, N, T)$, since their calculations are straightforward.

$$\delta\Sigma^L = \frac{64}{3}(1-\hat{s})(1+2\hat{s})v\,\mathrm{Im}[(C_{LL} - C_{LR} - 2C_{10})\xi_2^*] \tag{3.23}$$

$$\delta\Sigma^N(\hat{s}) = -8\pi(1-\hat{s})\sqrt{\hat{s}}v\Big\{ -\mathrm{Re}[(C_{LRRL} + C_{LRLR})\xi_2^*] + 2\hat{m}_\ell\,\mathrm{Re}[(C_{LL} - C_{LR} - 2C_{10})\xi_2^*]\Big\} \tag{3.24}$$

$$\delta\Sigma^T(\hat{s}) = -\frac{32\pi(1-\hat{s})}{\sqrt{\hat{s}}}\Big\{ -(4\hat{m}_\ell^2 + \hat{s})\,\mathrm{Im}[(C_T - 2C_{TE})\xi_2^*] - \hat{m}_\ell\hat{s}\,\mathrm{Im}[(C_{LL} + C_{LR})\xi_2^*] + 2\hat{m}_\ell\,\mathrm{Im}[2C_7\xi_2 + \hat{s}\xi_1^*\xi_2]\Big\} \tag{3.25}$$

3.3 Numerical analysis

In this section we will study the dependence of the CP asymmetries $A_{CP}(\hat{s})$ and $\delta A^i_{CP}(\hat{s})$ on $\hat{s}$ at fixed values of the new Wilson coefficients. Once again, we should remind the reader that in the present work all new Wilson coefficients are taken to be real. The experimental result on $b \to s\gamma$ decay put strong restriction on C_{BR}, i.e., practically it has the same value as it has in the SM. Therefore, in further numerical analysis we will set $C_{BR} = -2C_7^{eff}$. Throughout numerical analysis, we will vary all new Wilson coefficients in the range $-4 \leq C_X \leq 4$. The experimental bounds on the branching ratios of the $B \to K(K^*)\mu^+\mu^-$ [77, 78] and $B \to \mu^+\mu^-$ [79] suggest that this is the right order of magnitude for vector and scalar type interactions. Recently, BaBar and BELLE collaborations [10, 77] have presented new results on the branching ratios of $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays. When these results are used, stronger restrictions are imposed on some of the new Wilson coefficients. For example, $-2 \leq C_{LL} \leq 0$, $0 \leq C_{RL} \leq 2.3$, $-1.5 \leq C_T \leq 1.5$ and $-3.3 \leq C_{TE} \leq 2.6$, and rest of the coefficients vary in the region $-4 \leq C_X \leq 4$. Furthermore, in our analysis we will use the Wolfenstein parametrization [80] for the CKM matrix. The currently allowed range for the Wolfenstein parameters are: $0.19 \leq \rho \leq 0.268$ and $0.32 \leq \eta \leq 0.40$ [81, 28], where in the present analysis they are set to $\rho = 0.25$ and $\eta = 0.34$ [8].

We start our numerical analysis by first discussing the dependence of A_{CP} on $\hat{s}$ at fixed values of C_i, i.e., $C_i = -4; 0; 4$ which can be summarized as follows

- For the $b \to d\mu^+\mu^-$ case, far from resonance regions, A_{CP} depends strongly on C_{LL}. We observe that, when C_{LL} is positive (negative), the value of A_{CP} decreases (increases), while the situation for the C_{LR} case is the opposite

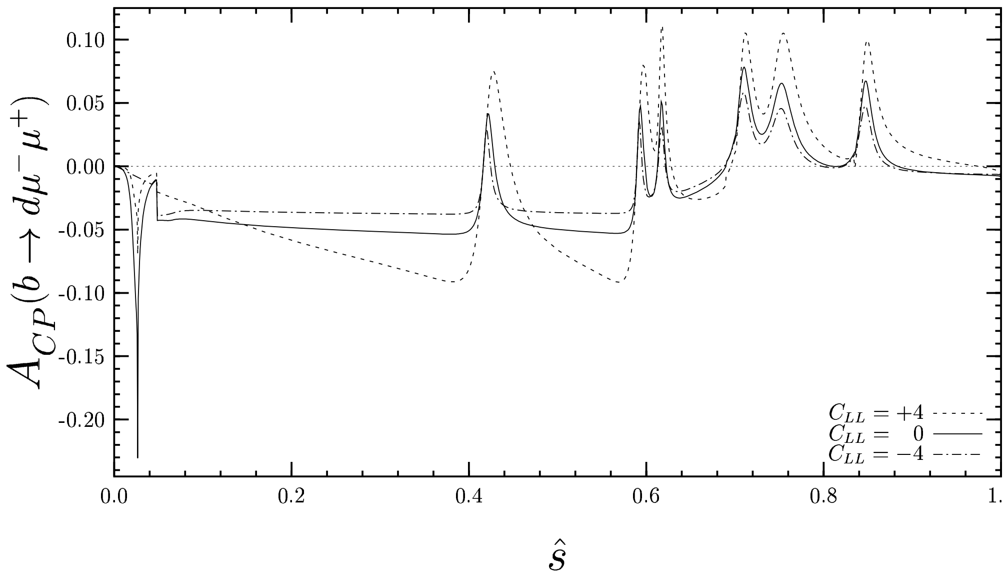

Figure 3.1: The dependence of A_{CP} on $\hat{s}$ for the $b \to d\mu^+\mu^-$ transition, at fixed values of C_{LL}.

way around (see Figs. (3.1) and (3.2)). If the tensor interaction is taken into account, A_{CP} practically seems to be zero for all values of C_T and C_{TE}. Furthermore, A_{CP} shows quite a weak dependence on all remaining Wilson coefficients and the departure from the SM result is very small.

- For the $b \to d\tau^+\tau^-$ case, in the region between second and third ψ resonances, A_{CP} is sensitive to C_{LR}, C_{LRLR}, C_{LRRL}, C_T, and C_{TE}, as can be seen from the Figs (3.3), (3.4), (3.5), (3.6) and (3.7), respectively. When C_{LR} and C_{LRLR} are positive (negative), they contribute destructively (constructively) to the SM result. The situation is contrary to this behavior for the C_{LRRL} scalar coupling. In the tensor interaction case, in the second and third resonance region, the magnitude of A_{CP} is smaller compared to that

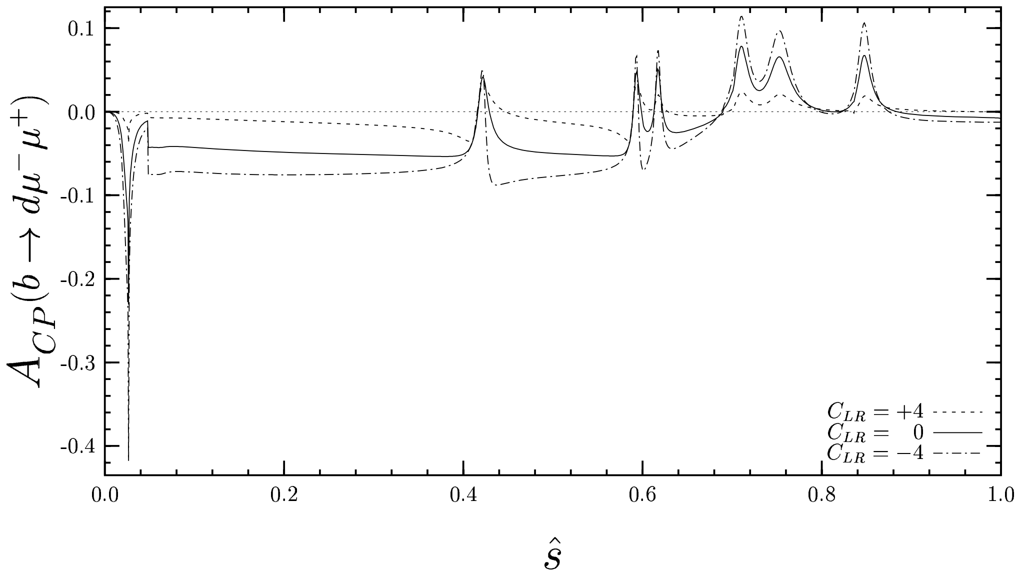

Figure 3.2: The same as in Fig. (3.1), but at fixed values of C_{LR}.

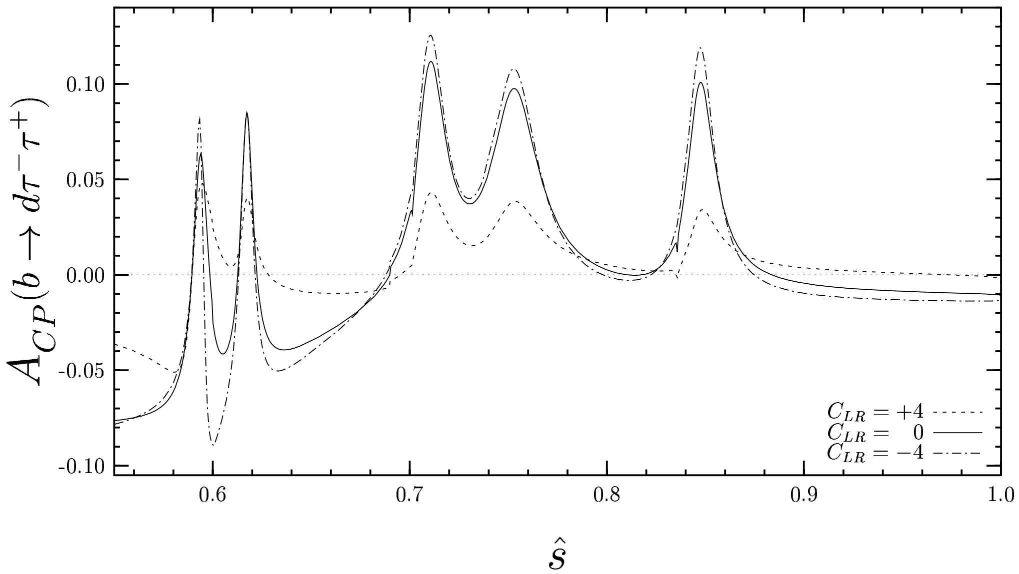

Figure 3.3: The same as in Fig. (3.1), but for the $b \to d\tau^+\tau^-$ transition, at fixed values of C_{LR}.

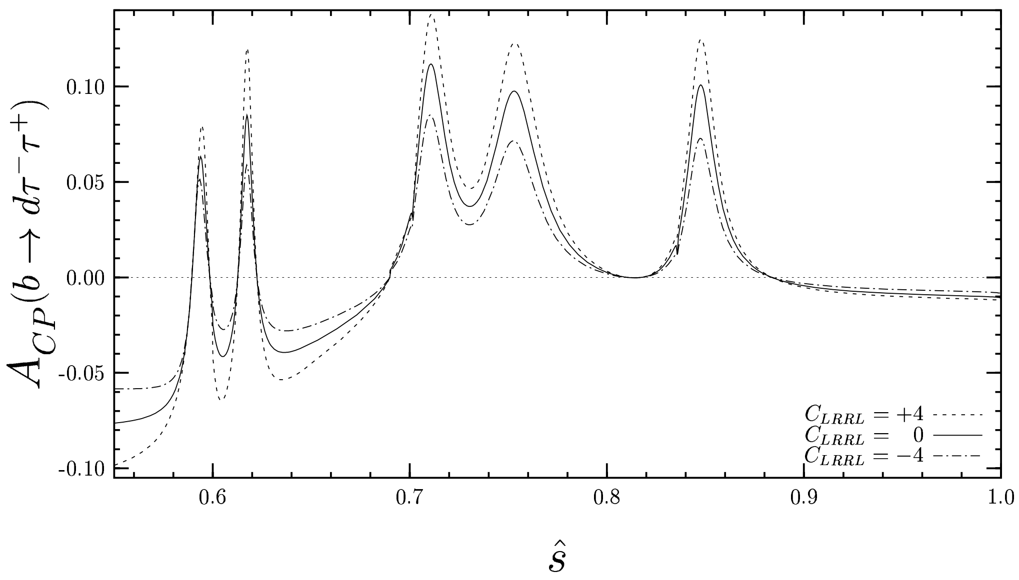

Figure 3.4: The same as in Fig. (3.3), but at fixed values of C_{LRRL}.

of the SM result. But, it is quite important to observe that A_{CP} asymmetry changes its sign, compared to its behavior in the SM, when C_T (C_{TE}) is negative (positive). Therefore, determination of the sign and magnitude of A_{CP} can give promising information about new physics.

The results concerning δA_{CP} for the $b \to d\mu^+\mu^-$ decay can be summarized as follows:

- In the region $1\ GeV^2/m_b^2 \leq \hat{s} \leq 8\ GeV^2/m_b^2$, which is free of resonance contribution, CP asymmetry due to the longitudinal polarization of μ lepton is dependent strongly on C_{LL}, and is practically independent of all remaining vector interaction coefficients. When C_{LL} is negative (positive), δA_{CP} is larger (smaller) compared to that of the SM result (see Fig. (3.8)).

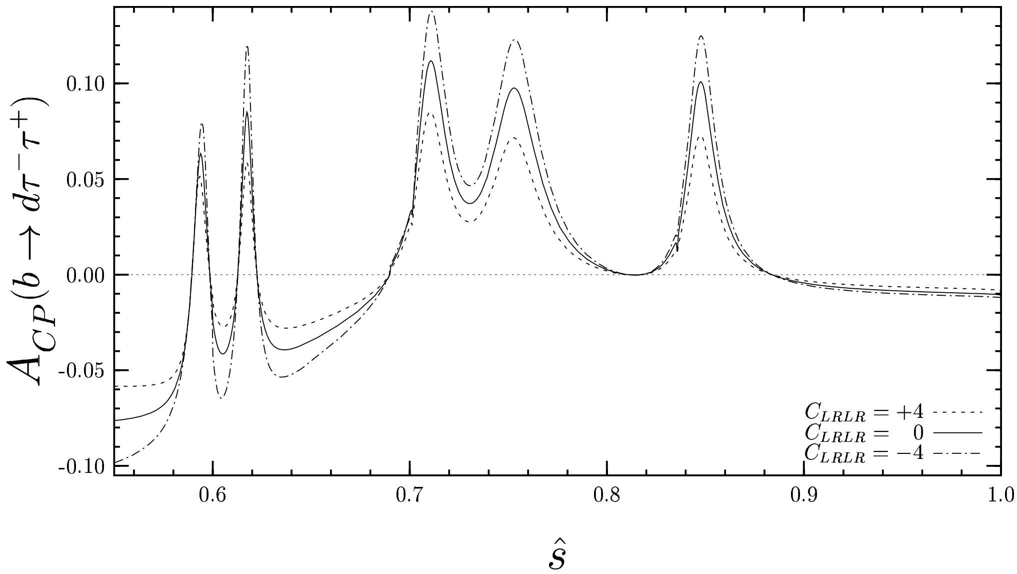

Figure 3.5: The same as in Fig. (3.3), but at fixed values of C_{LRLR}.

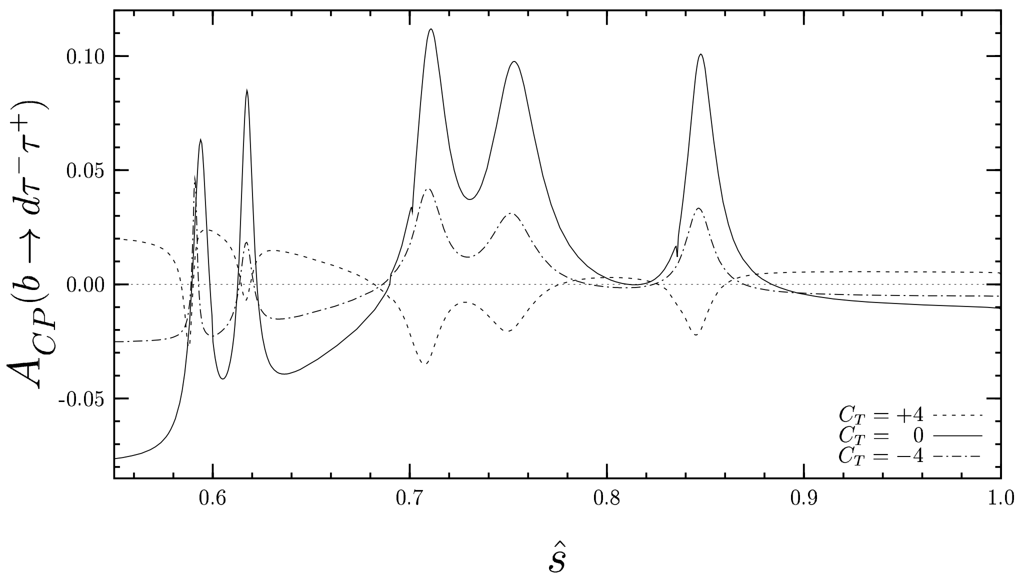

Figure 3.6: The same as in Fig. (3.3), but at fixed values of C_T.

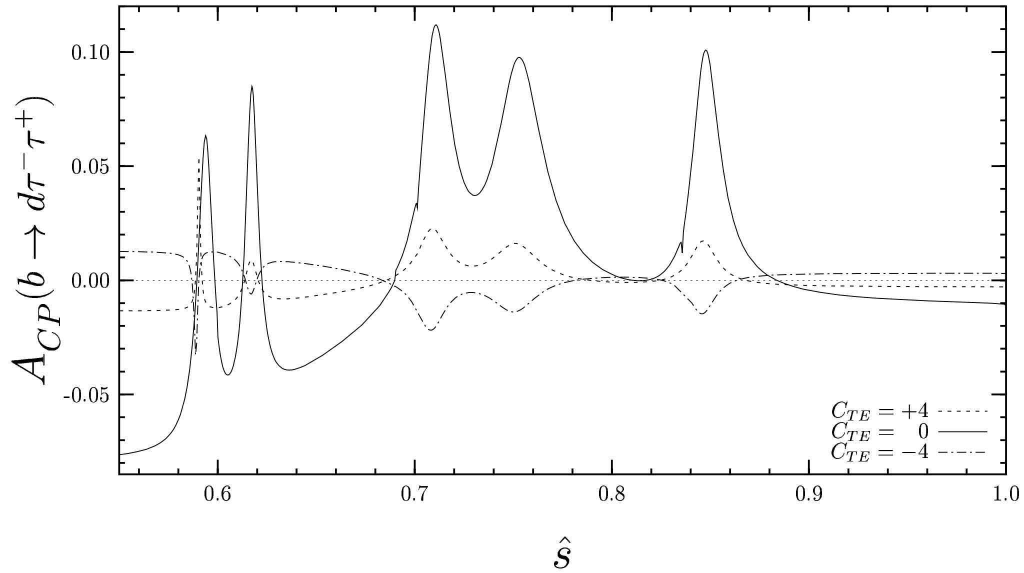

Figure 3.7: The same as in Fig. (3.3), but at fixed values of C_{TE}.

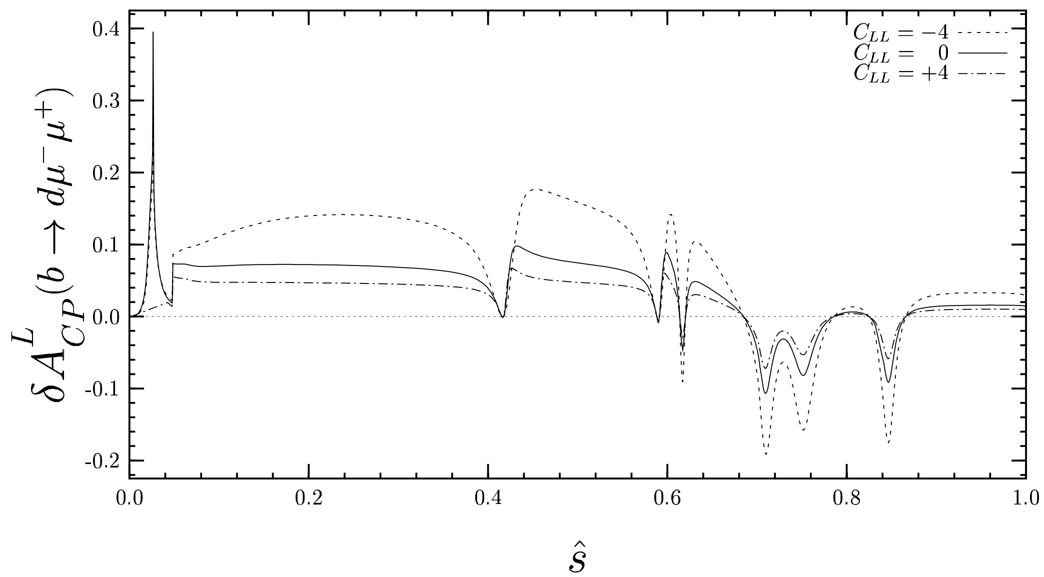

Figure 3.8: The dependence of δA_{CP}^{L} on $\hat{s}$ for the $b \to d\mu^+\mu^-$ transition, at fixed values of C_{LL}, when one of the final leptons is longitudinally polarized.

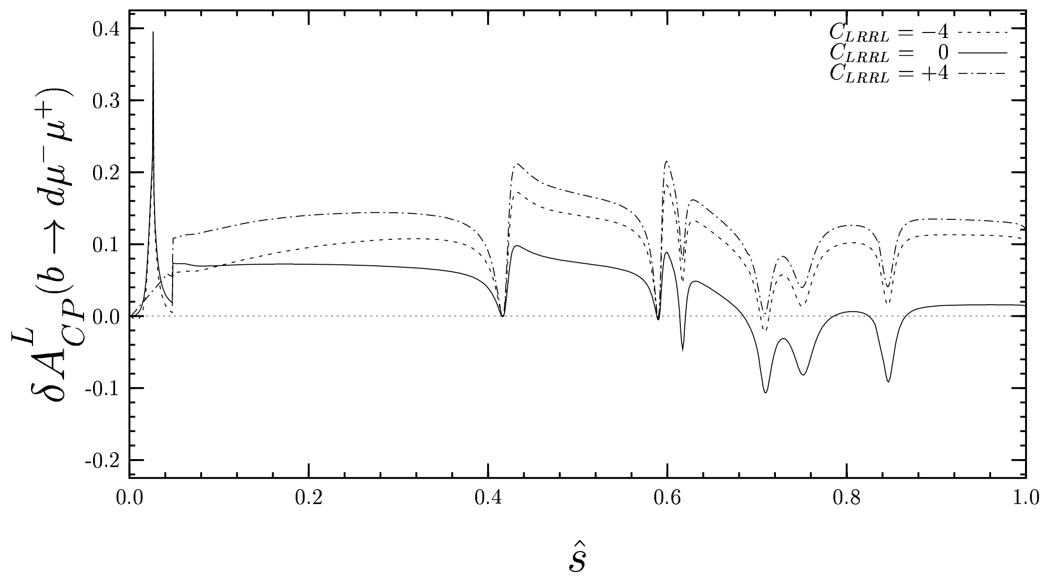

Figure 3.9: The same as in Fig. (3.8), but at fixed values of C_{LRRL}.

- δA^L_{CP} depends strongly on all scalar type interactions. As an example we present the dependence of δA_{CP} on C_{LRRL} in Fig. (3.9). The terms proportional to tensor interaction terms contribute destructively to the SM result.

For the $b \to d\tau^+\tau^-$ case, we obtain the following results:

- δA^L_{CP} depends strongly on the tensor type interactions and when C_T is negative (positive) it constructive (destructive) contribution to the SM result. For the other tensor interaction coefficient C_{TE}, the situation is contrary to this behavior (see Figs. (3.10) and (3.11)).

- Similar to the μ lepton case, δA^L_{CP} is quite sensitive to the existence of all scalar type interaction coefficients. When the signs of the coefficients C_{LRRL} and C_{LRLR} are negative (positive) the sign of δA^L_{CP} is positive (negative),

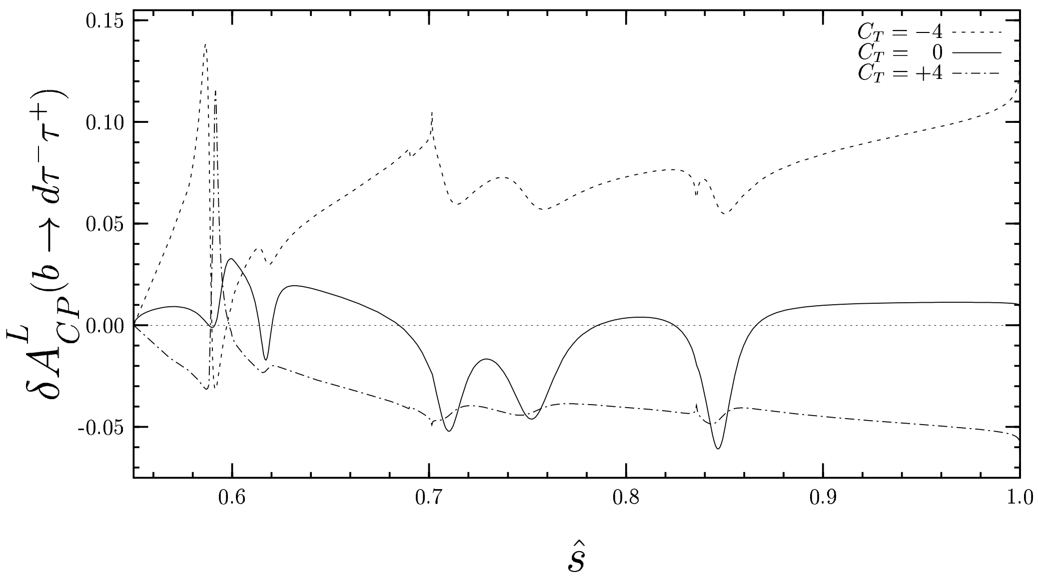

Figure 3.10: The same as in Fig. (3.8), but for the $b \to d\tau^+\tau^-$ transition, at fixed values of C_T.

in the region $\hat{s} \geq 0.6$ (see Figs. (3.12) and (3.13)). Note that in the SM case the sign of δA^L_{CP} can be positive or negative. Therefore in the region $\hat{s} \geq 0.6$, determination of the sign of δA^L_{CP} can give unambiguous information about the existence of new physics beyond the SM. For the remaining two scalar interaction coefficients C_{RLRL} (C_{RLLR}), the sign of δA^L_{CP} is negative (positive) (see Figs. (3.14) and (3.15)). Again, as in the previous case, determination of the sign and magnitude of δA^L_{CP} can give quite valuable hints for establishing new physics beyond the SM.

Since transversal and normal polarizations are proportional to the lepton mass, for the light lepton case, obviously, departure from the SM results is not substantial for all Wilson coefficients. On the other hand, for the $b \to d\ell^-\ell^+$ transition,

δA^i_{CP} ($i = T$ or N) is strongly dependent on C_{LR} (see Fig. (3.16)), C_{RR} (see Fig. (3.17)) and scalar type interactions. Note that, the dependence of δA^T_{CP} on C_T and C_{TE} is quite weak.

Finally, we would like to discuss the following question. As has already been mentioned, A_{CP}, as well as δA_{CP}, are very sensitive to the existence of new physics beyond the SM. The intriguing question is that, can we find a region of C_X, in which A_{CP} agrees with the SM result while δA_{CP} does not. A possible existence of such a region will allow us to establish new physics by only measuring δA_{CP}. In order to verify whether such a region of C_X does exist or not, we present the correlations between partially integrated A_{CP} and δA_{CP} in Figs. (3.18)–(3.20). The integration region for the $b \to d\mu^+\mu^-$ transition is chosen to be $1\ GeV^2/m_b^2 \le \hat{s} \le 8\ GeV^2/m_b^2$, and for the $b \to d\tau^+\tau^-$ transition it is chosen as $18\ GeV^2/m_b^2 \le \hat{s} \le 1$. These choices of the regions are dictated by the requirement that A_{CP} and δA_{CP} be free of resonance contributions.

In Figs. (3.18)–(3.19) we present the correlations δA^i_{CP} and A^i_{CP} asymmetries, when one of the leptons is longitudinally polarized, for the μ and τ lepton cases, respectively. In Fig. (3.20) we present the flows in the (A^T_{CP} and δA^T_{CP}) plane, when the final lepton is transversally polarized. From these figures we observe that, indeed, there exists a region of new Wilson coefficients in which A_{CP} agrees with the SM prediction, while δA_{CP} does not (in Figs (3.18)–(3.20), intersection point of all curves correspond to the SM case).

The numerical values of δA_{CP}^N and A_{CP}^N are very small and for this reason we do not present this correlation. As a final remark we would like to comment that, similar calculation can be carried out for the $B \to \rho \ell^+ \ell^-$ decay in search of new physics, since its detection in the experiments is easier compared to that of the inclusive $b \to d \ell^+ \ell^-$ decay.

In conclusion, we study the CP-violating asymmetries, when one of the final leptons is polarized, using the most general form of effective Hamiltonian. It is seen that, δA_{CP} and A_{CP} are very sensitive to various new Wilson coefficients. Moreover, we discuss the possibility whether there exist regions of new Wilson coefficients or not, for which A_{CP} coincides with the SM prediction, while δA_{CP} does not. In other words, if there exists such regions of C_X, this means new physics effects can only be established in δA_{CP} measurements. Our results confirm that, such regions of C_X do indeed exist.

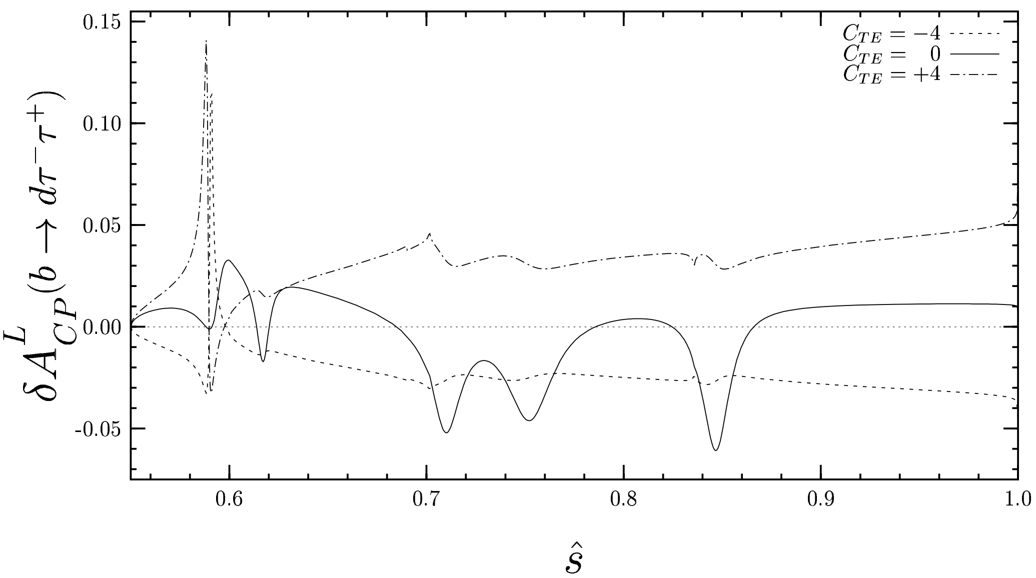

Figure 3.11: The same as in Fig. (3.10), but at fixed values of C_{TE}.

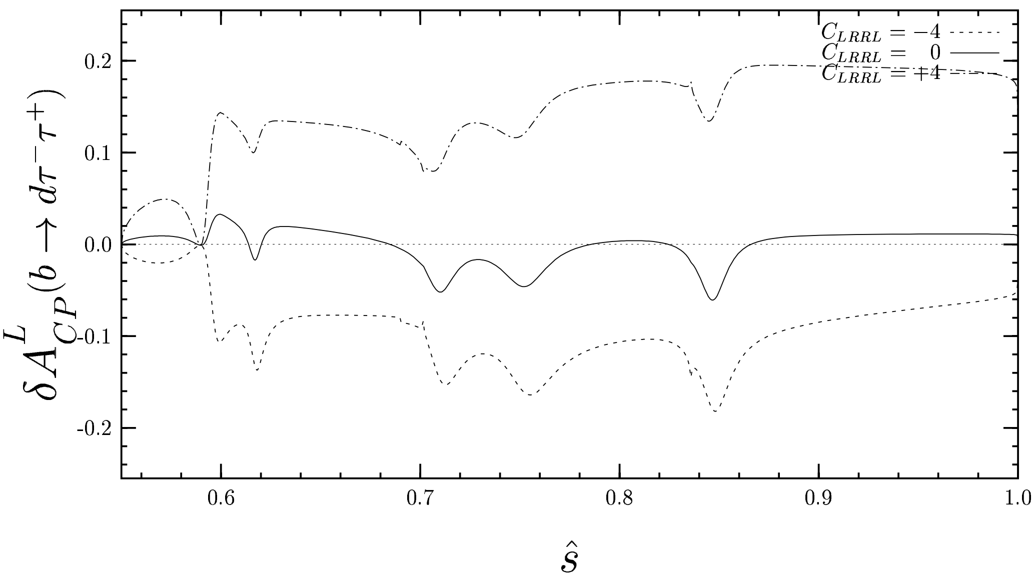

Figure 3.12: The same as in Fig. (3.10), but at fixed values of C_{LRRL}.

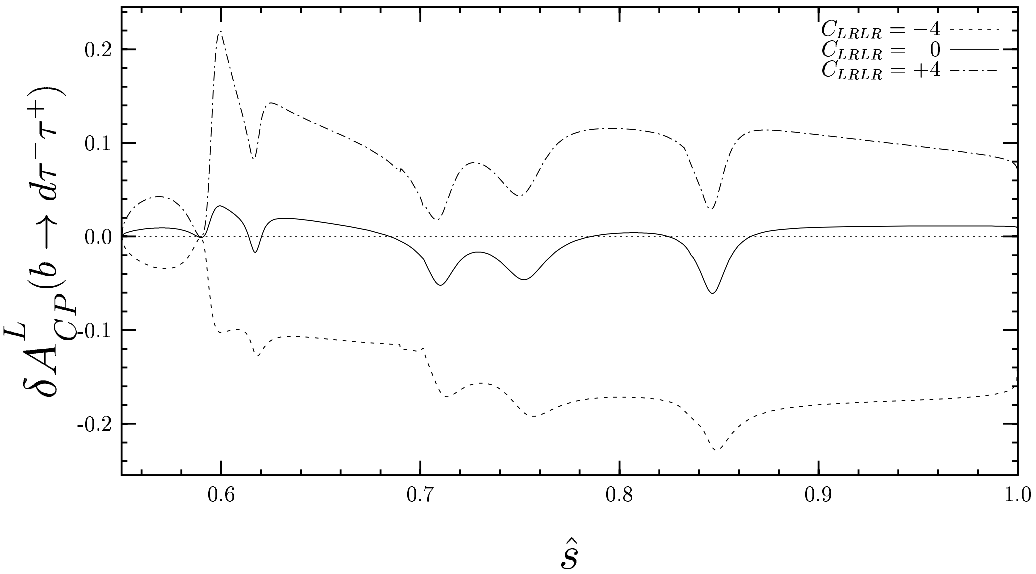

Figure 3.13: The same as in Fig. (3.10), but at fixed values of C_{LRLR}.

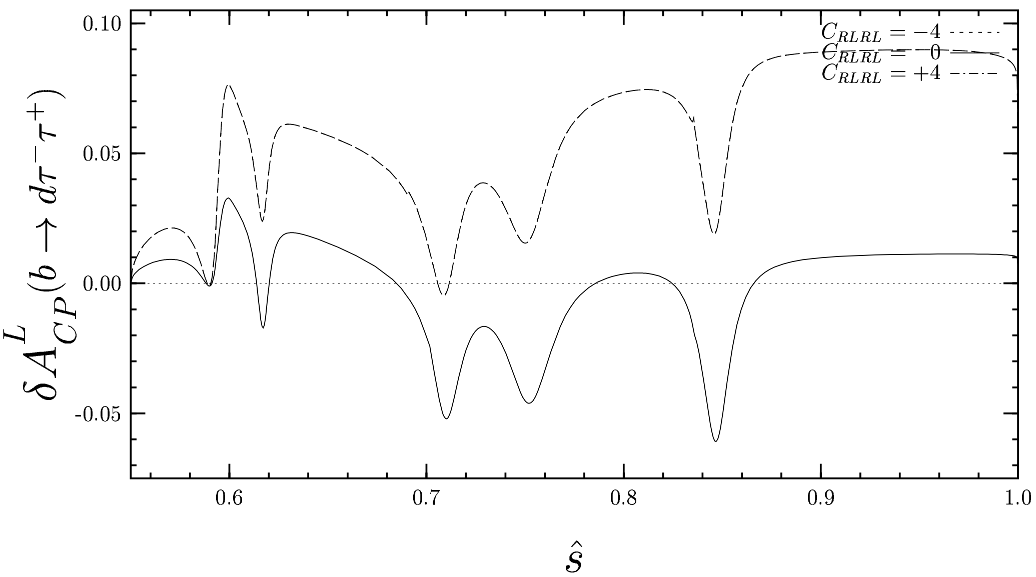

Figure 3.14: The same as in Fig. (3.10), but at fixed values of C_{RLRL}.

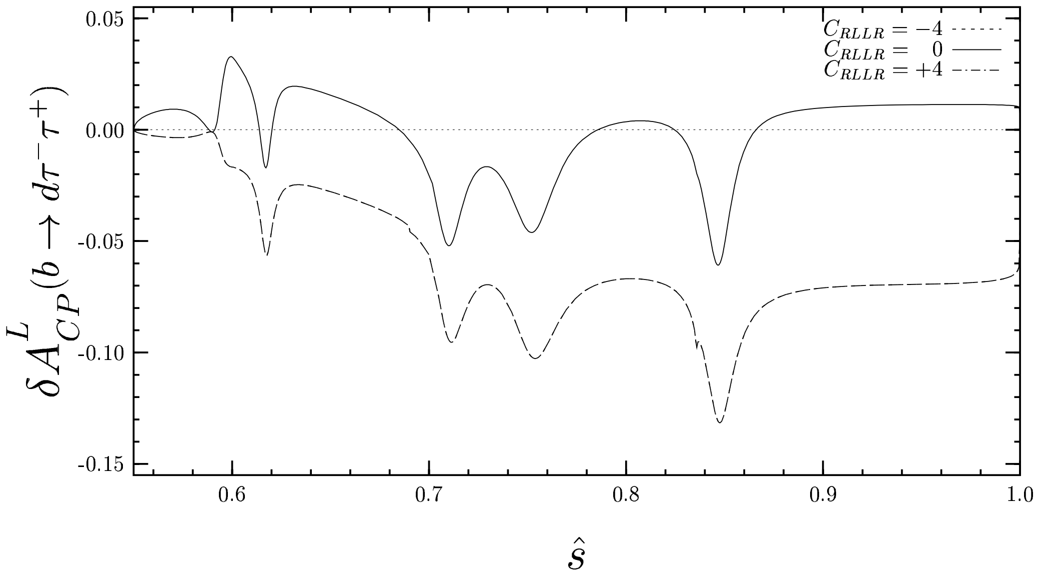

Figure 3.15: The same as in Fig. (3.10), but at fixed values of C_{RLLR}.

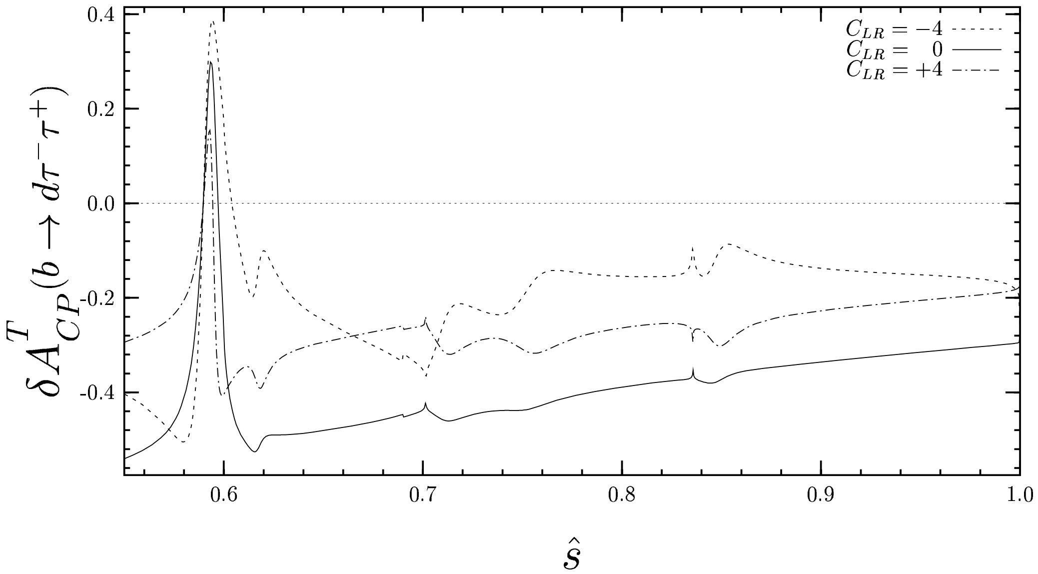

Figure 3.16: The same as in Fig. (3.10), but when one of the final leptons is transversally polarized, at fixed values of C_{LR}.

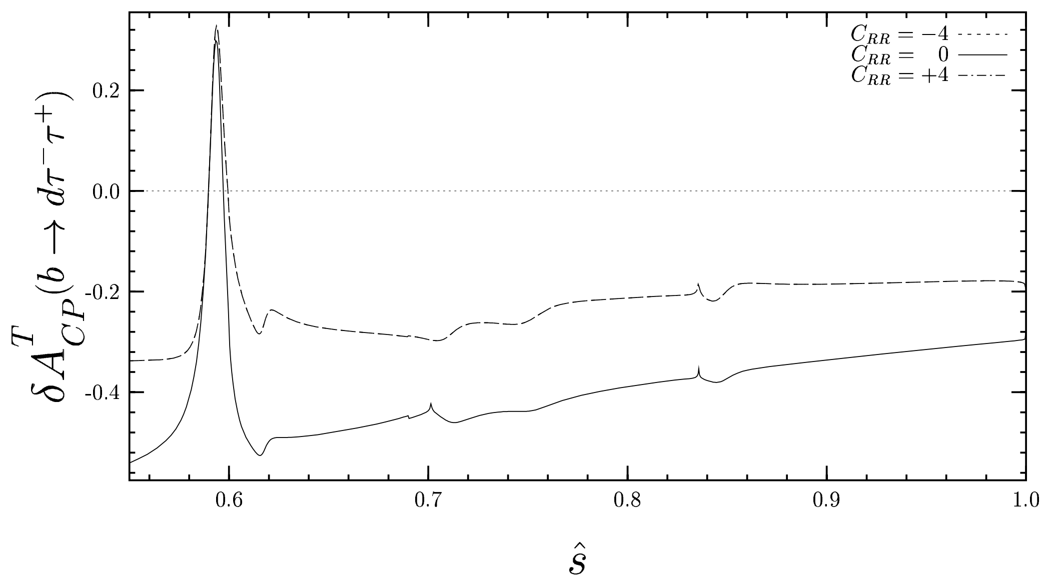

Figure 3.17: The same as in Fig. (3.16), but at fixed values of C_{RR}.

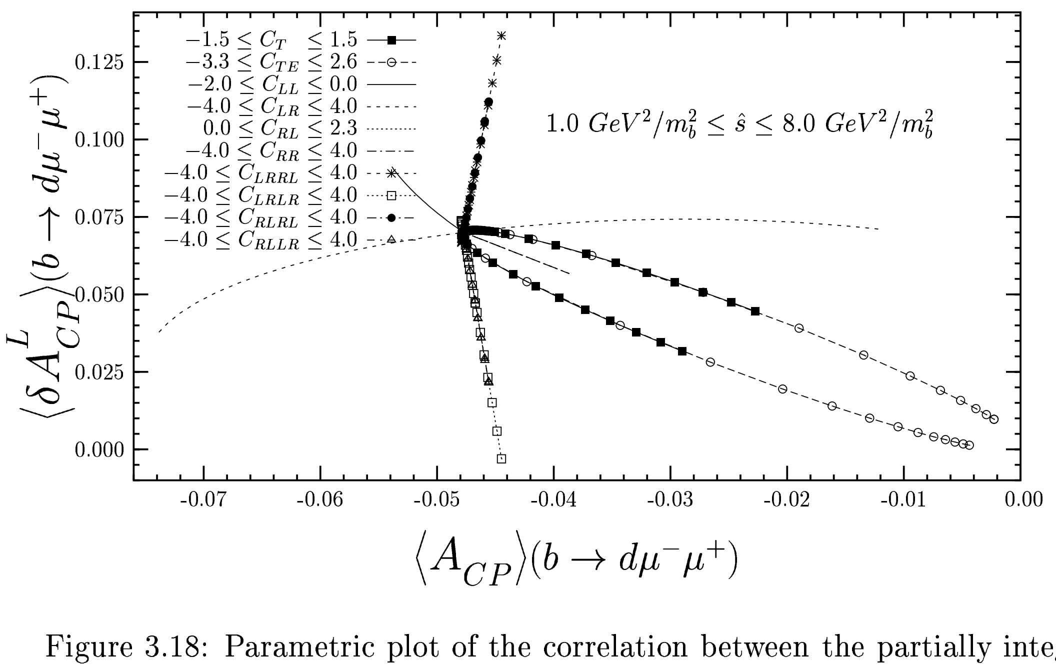

Figure 3.18: Parametric plot of the correlation between the partially integrated A_{CP}^{L} and δA_{CP}^{L} as a function of the new Wilson coefficients C_X, for the $b \to d\mu^+\mu^-$ transition, when one of the final leptons is longitudinally polarized.

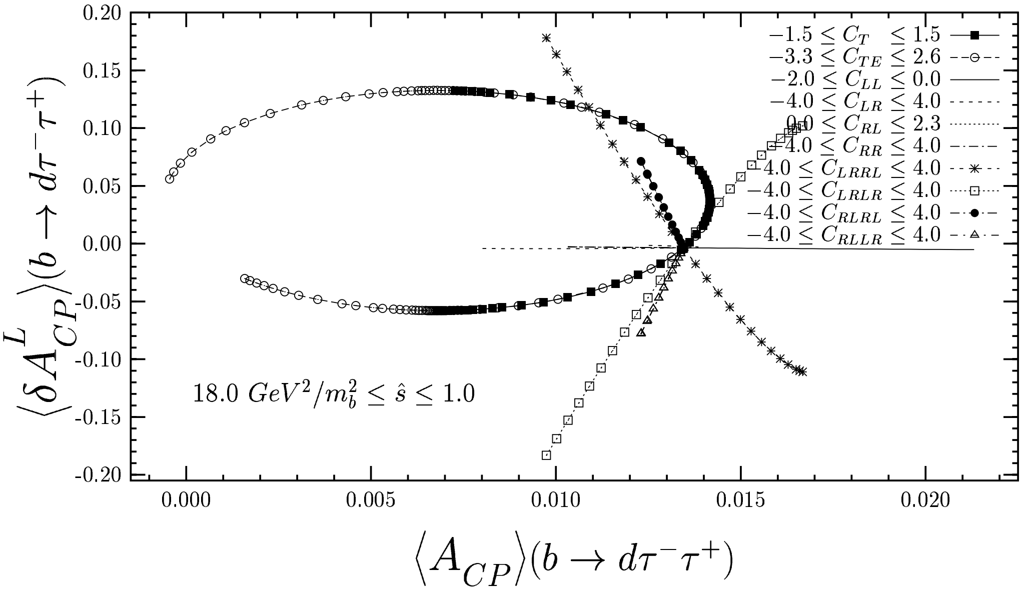

Figure 3.19: The same as in Fig. (3.18), but for the $b \to d\tau^+\tau^-$ transition.

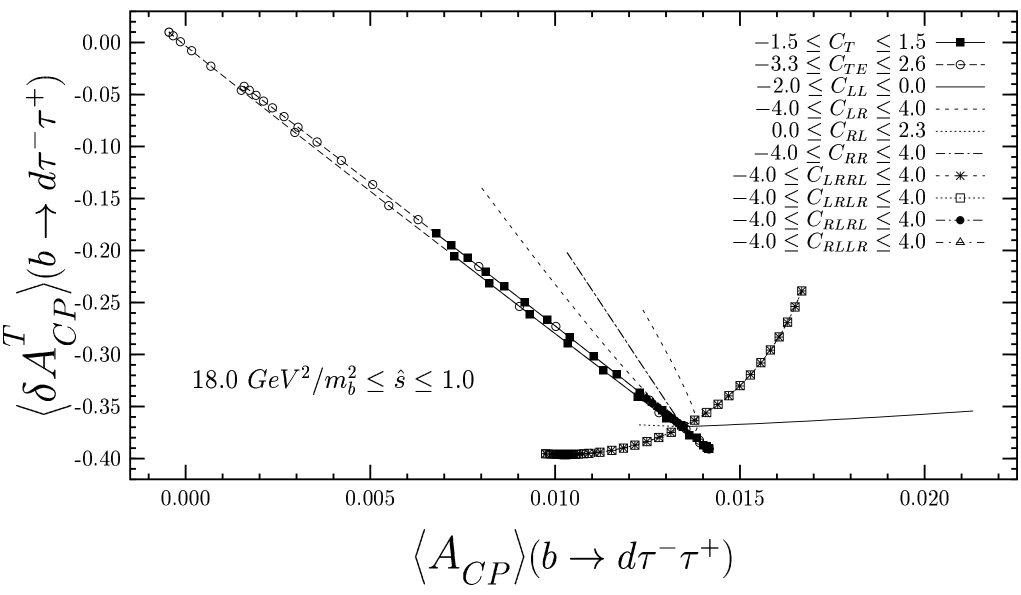

Figure 3.20: The same as in Fig. (3.19), but when one of the final leptons is transversally polarized.

CHAPTER 4

POLARIZED LEPTON PAIR FORWARD–BACKWARD ASYMMETRIES IN $B \to K^* \ell^+ \ell^-$ DECAY BEYOND THE STANDARD MODEL

4.1 Introduction

As we mentioned(see e.i, chapter 3), rare B meson decays, induced by flavor changing neutral current (FCNC) $b \to s(d)\ell^+\ell^-$ transitions provide a promising ground for testing the structure of weak interactions. These decays which are forbidden in the standard model (SM) at tree level, occur at loop level and are very sensitive to the gauge structure of the SM. Moreover, these decays are also quite sensitive to the existence of new physics beyond the SM, since loops with new particles can give considerable contribution to rare decays. As we see in chapter 3 the new physics effects in rare decays can appear in two ways; one via modification of the existing Wilson coefficients in the SM, or through the introducing of some operators with new coefficients. Theoretical investigation of the $B \to X_s \ell^+ \ell^-$ decays are relatively more clean compared to their exclusive counterparts, since they are not spoiled by nonperturbative long distance effects, while the corresponding exclusive channels are easier to measure experimentally. Some of the most important exclusive FCNC decays are $B \to K^*\gamma$ and $B \to (\pi, \rho, K, K^*)\ell^+\ell^-$ decays. The latter provides potentially a very rich set of experimental observable, such as, lepton pair forward–backward (FB) asymmetry, lepton polarizations, etc. Various kinematical distributions of such processes as $B \to K(K^*)\ell^+\ell^-$ [82, 83, 84], $B \to \pi(\rho)\ell^+\ell^-$ [85], $B_{s,d} \to \ell^+\ell^-$ [86] and $B_{s,d} \to \gamma\ell^+\ell^-$ [87] have already been studied. Experimentally measurable quantities such as forward–backward asymmetry, single polarization asymmetry, etc., have been studied for the $B \to K^*\ell^+\ell^-$ decay in [82, 88, 89, 90]. Study of these quantities can give useful information in fitting the parameters of the SM and put constraints on new physics [91, 92, 93]. It has been pointed out in [94] that the study of simultaneous polarizations of both leptons in the final state provide, in principle, measurement of many more observable which would be useful in further improvement of the parameters of the SM probing new physics beyond the SM. It should be noted here that both lepton polarizations in the $B \to K^*\tau^+\tau^-$ and $B \to K\ell^+\ell^-$ decays are studied in [95] and [96], respectively. As has already been noted, one efficient way of establishing new physics effects is studying forward–backward asymmetry in semileptonic $B \to K^*\ell^+\ell^-$ decay, since, $\mathcal{A}_{FB}$ vanishes at specific values of the dilepton invariant mass, and more essential than that, this zero position of $\mathcal{A}_{FB}$ is known to be practically free of hadronic uncertainties [93].

The aim of the present chapter is studying the polarized forward–backward asymmetry in the exclusive $B \to K^* \ell^+ \ell^-$ decay using a general form of the effective Hamiltonian, including all possible forms of interactions. Here we would like to remind the reader that the influence of new Wilson coefficients on various kinematical variables, such as branching ratios, lepton pair forward–backward asymmetries and single lepton polarization asymmetries for the inclusive $B \to X_{s(d)} \ell^+ \ell^-$ decays (see first references in [92, 94, 97]) and exclusive $B \to K \ell^+ \ell^-$, $K^* \ell^+ \ell^-$, $\gamma \ell^+ \ell^-$, $\pi \ell^+ \ell^-$, $\rho \ell^+ \ell^-$ [82, 83, 87, 90, 98, 99] and pure leptonic $B \to \ell^+ \ell^-$ decays [86, 100] have been studied comprehensively.

Recently, exiting results have been announced by the BaBar and Belle Collaborations for experimental study of the $B \to K^* \ell^+ \ell^-$ decay. As far as the results for the branching ratio of the $B \to K^* \ell^+ \ell^-$ decay measured by these Collaborations are gives as

$$\mathcal{B}(B \to K^* \ell^+ \ell^-) = \begin{cases} \left(11.5^{+2.6}_{-2.4} \pm 0.8 \pm 0.2\right) \times 10^{-7} & [101] \,, \\ \\ \left(0.88^{+0.33}_{-0.29}\right) \times 10^{-6} & [102] \,. \end{cases}$$

This chapter is organized as follows. In section 2, using a general form of the effective Hamiltonian, we obtain the matrix element in terms of the form factors of the $B \to K^*$ transition. In section 3 we derive the analytical results for the polarized forward–backward asymmetry. Last section is devoted to the numerical analysis, discussion and conclusions.

4.2 Matrix element for the $B \to K^* \ell^+ \ell^-$ decay

In this section we present the matrix element for the $B \to K^* \ell^+ \ell^-$ decay using a general form of the effective Hamiltonian. The $B \to K^* \ell^+ \ell^-$ process is governed by $b \to s\ell^+\ell^-$ transition at quark level. The effective Hamiltonian for the $b \to s\ell^+\ell^-$ can be written in terms of the twelve model independent four–Fermi interactions in the following form:

$$
\begin{aligned}
\mathcal{H}_{eff} = & \frac{G_F \alpha}{\sqrt{2}\pi} V_{ts} V_{tb}^* \Bigg\{ C_{SL}\, \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2}\, L\, b\, \bar{\ell}\gamma^\mu \ell + C_{BR}\, \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2}\, R\, b\, \bar{\ell}\gamma^\mu \ell \\
& + C_{LL}^{tot}\, \bar{s}_L \gamma_\mu b_L\, \bar{\ell}_L \gamma^\mu \ell_L + C_{LR}^{tot}\, \bar{s}_L \gamma_\mu b_L\, \bar{\ell}_R \gamma^\mu \ell_R + C_{RL}\, \bar{s}_R \gamma_\mu b_R\, \bar{\ell}_L \gamma^\mu \ell_L \\
& + C_{RR}\, \bar{s}_R \gamma_\mu b_R\, \bar{\ell}_R \gamma^\mu \ell_R + C_{LRLR}\, \bar{s}_L b_R\, \bar{\ell}_L \ell_R + C_{RLLR}\, \bar{s}_R b_L\, \bar{\ell}_L \ell_R \\
& + C_{LRRL}\, \bar{s}_L b_R\, \bar{\ell}_R \ell_L + C_{RLRL}\, \bar{s}_R b_L\, \bar{\ell}_R \ell_L + C_T\, \bar{s} \sigma_{\mu\nu} b\, \bar{\ell} \sigma^{\mu\nu} \ell \\
& + i C_{TE}\, \epsilon^{\mu\nu\alpha\beta}\, \bar{s} \sigma_{\mu\nu} b\, \bar{\ell} \sigma_{\alpha\beta} \ell \Bigg\} ,
\end{aligned}
\tag{4.1}
$$

as we see chapter 3, L and R in are defined as

$$
L = \frac{1-\gamma_5}{2} , \qquad R = \frac{1+\gamma_5}{2} ,
$$

and C_X are the coefficients of the four–Fermi interactions. Here, few words about the above Hamiltonian are in order. In principle, $\mathcal{O}_2$, being a member of the standard model operators, as well as operators of the type $\bar{s}_R b_L \bar{q}_L q_R$, where q represents a quark field, give contributions to the $b \to s\ell^+\ell^-$ transition at one–loop level. The Hamiltonian given in Eq. (4.1) should be understood as an effective version of the most general one, where the above–mentioned contributions are absorbed into effective Wilson coefficients which depend on q^2 in general. The first two coefficients in Eq. (4.1), C_{SL} and C_{BR}, are the nonlocal Fermi interactions, which correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, respectively. The following four terms with coefficients C_{LL}, C_{LR}, C_{RL} and C_{RR} are the vector type interactions. Two of these interactions containing C_{LL}^{tot} and C_{LR}^{tot} do already exist in the SM in the form $(C_9^{eff} - C_{10})$ and $(C_9^{eff} + C_{10})$. Hence, the explicit form of C_{LL}^{tot} and C_{LR}^{tot} are represented Eq. (3.2) allows us to conclude that C_{LL}^{tot} and C_{LR}^{tot} describe the sum of the contributions from SM and the new physics. The terms with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} describe the scalar type interactions. The remaining last two terms lead by the coefficients C_T and C_{TE}, obviously, describe the tensor type interactions.

The exclusive $B \to K^* \ell^+ \ell^-$ decay is described in terms of the matrix elements of the quark operators in Eq. (4.1) over meson states, which can be parameterized in terms of the form factors. Obviously, the following matrix elements

$$
\begin{aligned}
&\langle K^* \left| \bar{s}\gamma_\mu (1 \pm \gamma_5) b \right| B \rangle \ , \\
&\langle K^* \left| \bar{s} i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b \right| B \rangle \ , \\
&\langle K^* \left| \bar{s} (1 \pm \gamma_5) b \right| B \rangle \ , \\
&\langle K^* \left| \bar{s} \sigma_{\mu\nu} b \right| B \rangle \ ,
\end{aligned}
$$

are needed for the calculation of the $B \to K^* \ell^+ \ell^-$ decay. These matrix elements are defined as follows:

$$
\begin{aligned}
&\langle K^*(p_{K^*},\varepsilon)\,|\bar{s}\gamma_\mu(1\pm\gamma_5)b|\,B(p_B)\rangle = \\
&\quad -\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^\lambda_{K^*}q^\sigma\frac{2V(q^2)}{m_B+m_{K^*}} \pm i\varepsilon^*_\mu(m_B+m_{K^*})A_1(q^2) \\
&\quad \mp i(p_B+p_{K^*})_\mu(\varepsilon^*q)\frac{A_2(q^2)}{m_B+m_{K^*}} \mp iq_\mu\frac{2m_{K^*}}{q^2}(\varepsilon^*q)\left[A_3(q^2)-A_0(q^2)\right]\ ,
\end{aligned} \tag{4.2}
$$

$$
\begin{aligned}
&\langle K^*(p_{K^*},\varepsilon)\,|\bar{s}i\sigma_{\mu\nu}q^\nu(1\pm\gamma_5)b|\,B(p_B)\rangle = \\
&\quad 4\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^\lambda_{K^*}q^\sigma T_1(q^2) \pm 2i\left[\varepsilon^*_\mu(m_B^2-m_{K^*}^2)-(p_B+p_{K^*})_\mu(\varepsilon^*q)\right]T_2(q^2) \\
&\quad \pm 2i(\varepsilon^*q)\left[q_\mu-(p_B+p_{K^*})_\mu\frac{q^2}{m_B^2-m_{K^*}^2}\right]T_3(q^2)\ ,
\end{aligned} \tag{4.3}
$$

$$
\begin{aligned}
&\langle K^*(p_{K^*},\varepsilon)\,|\bar{s}\sigma_{\mu\nu}b|\,B(p_B)\rangle = \\
&\quad i\epsilon_{\mu\nu\lambda\sigma}\Bigg\{-2T_1(q^2)\varepsilon^{*\lambda}(p_B+p_{K^*})^\sigma+\frac{2}{q^2}(m_B^2-m_{K^*}^2)\big[T_1(q^2)-T_2(q^2)\big]\varepsilon^{*\lambda}q^\sigma \\
&\quad -\frac{4}{q^2}\left[T_1(q^2)-T_2(q^2)-\frac{q^2}{m_B^2-m_{K^*}^2}T_3(q^2)\right](\varepsilon^*q)p^\lambda_{K^*}q^\sigma\Bigg\}\ .
\end{aligned} \tag{4.4}
$$

where $q=p_B-p_{K^*}$ is the momentum transfer and ε is the polarization vector of K^* meson. In order to ensure finiteness of (4.2) at $q^2=0$, we assume that $A_3(q^2=0)=A_0(q^2=0)$ and $T_1(q^2=0)=T_2(q^2=0)$. The matrix element $\langle K^*\,|\bar{s}(1\pm\gamma_5)b|\,B\rangle$ can be calculated from Eq. (4.2) by contracting both sides of Eq. (4.2) with q^μ and using equation of motion. Neglecting the mass of the strange quark we get

$$
\langle K^*(p_{K^*},\varepsilon)\,|\bar{s}(1\pm\gamma_5)b|\,B(p_B)\rangle = \frac{1}{m_b}\big[\mp 2im_{K^*}(\varepsilon^*q)A_0(q^2)\big]\ . \tag{4.5}
$$

In deriving Eq. (4.5) we have used the relationship

$$2m_{K^*}A_3(q^2) = (m_B + m_{K^*})A_1(q^2) - (m_B - m_{K^*})A_2(q^2) \ ,$$

which follows from the equations of motion.

Using the definition of the form factors, as given above, the amplitude of the $B \to K^* \ell^+ \ell^-$ decay can be written as

$$\begin{aligned}
\mathcal{M}(B \to K^* \ell^+ \ell^-) &= \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{ts}^* \\
\times \Bigg\{ &\bar{\ell}\gamma^\mu(1-\gamma_5)\ell \Big[-2A_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_{K^*}^\lambda q^\sigma - iB_1 \varepsilon_\mu^* + iB_2 (\varepsilon^* q)(p_B + p_{K^*})_\mu + iB_3 (\varepsilon^* q) q_\mu \Big] \\
+ &\bar{\ell}\gamma^\mu(1+\gamma_5)\ell \Big[-2C_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_{K^*}^\lambda q^\sigma - iD_1 \varepsilon_\mu^* + iD_2 (\varepsilon^* q)(p_B + p_{K^*})_\mu + iD_3 (\varepsilon^* q) q_\mu \Big] \\
+ &\bar{\ell}(1-\gamma_5)\ell \Big[iB_4 (\varepsilon^* q) \Big] + \bar{\ell}(1+\gamma_5)\ell \Big[iB_5 (\varepsilon^* q) \Big] \\
+ &4\bar{\ell}\sigma^{\mu\nu}\ell \Big(iC_T \epsilon_{\mu\nu\lambda\sigma} \Big) \Big[-2T_1 \varepsilon^{*\lambda} (p_B + p_{K^*})^\sigma + B_6 \varepsilon^{*\lambda} q^\sigma - B_7 (\varepsilon^* q) {p_{K^*}}^\lambda q^\sigma \Big] \\
+ &16 C_{TE} \bar{\ell}\sigma_{\mu\nu}\ell \Big[-2T_1 \varepsilon^{*\mu} (p_B + p_{K^*})^\nu + B_6 \varepsilon^{*\mu} q^\nu - B_7 (\varepsilon^* q) {p_{K^*}}^\mu q^\nu \Bigg\} \ ,
\end{aligned} \tag{4.6}$$

where

$$\begin{aligned}
A_1 &= (C_{LL}^{tot} + C_{RL}) \frac{V}{m_B + m_{K^*}} - 2(C_{BR} + C_{SL}) \frac{T_1}{q^2} \ , \\
B_1 &= (C_{LL}^{tot} - C_{RL})(m_B + m_{K^*}) A_1 - 2(C_{BR} - C_{SL})(m_B^2 - m_{K^*}^2) \frac{T_2}{q^2} \ , \\
B_2 &= \frac{C_{LL}^{tot} - C_{RL}}{m_B + m_{K^*}} A_2 - 2(C_{BR} - C_{SL}) \frac{1}{q^2} \left[T_2 + \frac{q^2}{m_B^2 - m_{K^*}^2} T_3 \right] \ , \\
B_3 &= 2(C_{LL}^{tot} - C_{RL}) m_{K^*} \frac{A_3 - A_0}{q^2} + 2(C_{BR} - C_{SL}) \frac{T_3}{q^2} \ , \\
C_1 &= A_1 (C_{LL}^{tot} \to C_{LR}^{tot} \ , \ \ C_{RL} \to C_{RR}) \ , \\
D_1 &= B_1 (C_{LL}^{tot} \to C_{LR}^{tot} \ , \ \ C_{RL} \to C_{RR}) \ ,
\end{aligned}$$

$$
\begin{aligned}
D_2 &= B_2(C_{LL}^{tot} \to C_{LR}^{tot}\ , \quad C_{RL} \to C_{RR})\ , \\
D_3 &= B_3(C_{LL}^{tot} \to C_{LR}^{tot}\ , \quad C_{RL} \to C_{RR})\ , \\
B_4 &= -2(C_{LRRL} - C_{RLRL})\frac{m_{K^*}}{m_b}A_0\ , \\
B_5 &= -2(C_{LRLR} - C_{RLLR})\frac{m_{K^*}}{m_b}A_0\ , \\
B_6 &= 2(m_B^2 - m_{K^*}^2)\frac{T_1 - T_2}{q^2}\ , \\
B_7 &= \frac{4}{q^2}\left(T_1 - T_2 - \frac{q^2}{m_B^2 - m_{K^*}^2}T_3\right)\ .
\end{aligned}
\tag{4.7}
$$

From this expression of the decay amplitude, for the differential decay width we get the following result:

$$
\frac{d\Gamma}{d\hat{s}}(B \to K^* \ell^+ \ell^-) = \frac{G^2\alpha^2 m_B}{2^{14}\pi^5}\left|V_{tb}V_{ts}^*\right|^2 \lambda^{1/2}(1, \hat{r}_{K^*}, \hat{s}) v \Delta(\hat{s})\ , \tag{4.8}
$$

with

$$
\begin{aligned}
\Delta = \frac{2}{3\hat{r}_{K^*}\hat{s}} m_B^2 \,\mathrm{Re}\Big[&- 6 m_B \hat{m}_\ell \hat{s}\lambda(B_1 - D_1)(B_4^* - B_5^*) \\
&- 12 m_B^2 \hat{m}_\ell^2 \hat{s}\lambda\Big\{B_4 B_5^* + (B_3 - D_2 - D_3)B_1^* - (B_2 + B_3 - D_3)D_1^*\Big\} \\
&+ 6 m_B^3 \hat{m}_\ell \hat{s}(1 - \hat{r}_{K^*})\lambda(B_2 - D_2)(B_4^* - B_5^*) \\
&+ 12 m_B^4 \hat{m}_\ell^2 \hat{s}(1 - \hat{r}_{K^*})\lambda(B_2 - D_2)(B_3^* - D_3^*) \\
&+ 6 m_B^3 \hat{m}_\ell \lambda \hat{s}^2 (B_4 - B_5)(B_3^* - D_3^*) \\
&+ 48 \hat{m}_\ell^2 \hat{r}_{K^*}\hat{s}\Big\{3 B_1 D_1^* + 2 m_B^4 \lambda A_1 C_1^*\Big\} \\
&+ 48 m_B^5 \hat{m}_\ell \hat{s}\lambda^2 (B_2 + D_2) B_7^* C_{TE}^* \\
&- 16 m_B^4 \hat{r}_{K^*}\hat{s}(\hat{m}_\ell^2 - \hat{s})\lambda\Big\{\left|A_1\right|^2 + \left|C_1\right|^2\Big\} \\
&- m_B^2 \hat{s}(2\hat{m}_\ell^2 - \hat{s})\lambda\Big\{\left|B_4\right|^2 + \left|B_5\right|^2\Big\}
\end{aligned}
$$

$$
\begin{aligned}
&- 48m_B^3\hat{m}_\ell\hat{s}(1-\hat{r}_{K^*}-\hat{s})\lambda\Big\{(B_1+D_1)B_7^*C_{TE}^* + 2(B_2+D_2)B_6^*C_{TE}^*\Big\} \\
&- 6m_B^4\hat{m}_\ell^2\hat{s}\lambda\Big\{2(2+2\hat{r}_{K^*}-\hat{s})B_2D_2^* - \hat{s}\left|(B_3-D_3)\right|^2\Big\} \\
&+ 96m_B\hat{m}_\ell\hat{s}(\lambda+12\hat{r}_{K^*}\hat{s})(B_1+D_1)B_6^*C_{TE}^* \\
&+ 8m_B^2\hat{s}^2\Big\{v^2\left|C_T\right|^2 + 4(3-2v^2)\left|C_{TE}\right|^2\Big\}\Big\{4(\lambda+12\hat{r}_{K^*}\hat{s})\left|B_6\right|^2 \\
&- 4m_B^2\lambda(1-\hat{r}_{K^*}-\hat{s})B_6B_7^* + m_B^4\lambda^2\left|B_7\right|^2\Big\} \\
&- 4m_B^2\lambda\Big\{\hat{m}_\ell^2(2-2\hat{r}_{K^*}+\hat{s}) + \hat{s}(1-\hat{r}_{K^*}-\hat{s})\Big\}(B_1B_2^*+D_1D_2^*) \\
&+ \hat{s}\Big\{6\hat{r}_{K^*}\hat{s}(3+v^2)+\lambda(3-v^2)\Big\}\Big\{\left|B_1\right|^2+\left|D_1\right|^2\Big\} \\
&- 2m_B^4\lambda\Big\{\hat{m}_\ell^2[\lambda-3(1-\hat{r}_{K^*})^2]-\lambda\hat{s}\Big\}\Big\{\left|B_2\right|^2+\left|D_2\right|^2\Big\} \\
&+ 128m_B^2\Big\{4\hat{m}_\ell^2[20\hat{r}_{K^*}\lambda-12\hat{r}_{K^*}(1-\hat{r}_{K^*})^2-\lambda\hat{s}] \\
&+ \hat{s}[4\hat{r}_{K^*}\lambda+12\hat{r}_{K^*}(1-\hat{r}_{K^*})^2+\lambda\hat{s}]\Big\}\left|C_T\right|^2\left|T_1\right|^2 \\
&+ 512m_B^2\Big\{\hat{s}[4\hat{r}_{K^*}\lambda+12\hat{r}_{K^*}(1-\hat{r}_{K^*})^2+\lambda\hat{s}] \\
&+ 8\hat{m}_\ell^2[12\hat{r}_{K^*}(1-\hat{r}_{K^*})^2+\lambda(\hat{s}-8\hat{r}_{K^*})]\Big\}\left|C_{TE}\right|^2\left|T_1\right|^2 \\
&- 64m_B^2\hat{s}^2\Big\{v^2\left|C_T\right|^2+4(3-2v^2)\left|C_{TE}\right|^2\Big\}\Big\{2[\lambda+12\hat{r}_{K^*}(1-\hat{r}_{K^*})]B_6T_1^* \\
&- m_B^2\lambda(1+3\hat{r}_{K^*}-\hat{s})B_7T_1^*\Big\} \\
&+ 768m_B^3\hat{m}_\ell\hat{r}_{K^*}\hat{s}\lambda(A_1+C_1)C_T^*T_1^* \\
&- 192m_B\hat{m}_\ell\hat{s}[\lambda+12\hat{r}_{K^*}(1-\hat{r}_{K^*})](B_1+D_1)C_{TE}^*T_1^* \\
&+ 192m_B^3\hat{m}_\ell\hat{s}\lambda(1+3\hat{r}_{K^*}-\hat{s})\lambda(B_2+D_2)C_{TE}^*T_1^*\Big] \,, \qquad (4.9)
\end{aligned}
$$

where $\hat{s}=q^2/m_B^2$, $\hat{r}_{K^*}=m_{K^*}^2/m_B^2$ and $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$, $\hat{m}_\ell=m_\ell/m_B$, $v=\sqrt{1-4\hat{m}_\ell^2/\hat{s}}$ is the final lepton velocity.

The definition of the polarized FB asymmetries will be presented in the next section.

4.3 Polarized forward–backward asymmetries of leptons

In this section we calculate the polarized FB asymmetries. For this purpose, we define the following orthogonal unit vectors $s_i^{\pm\mu}$ in the rest frame of $\ell^\pm$, where $i = L, N$ or T correspond to longitudinal, normal, transversal polarization directions, respectively (see also [82, 89, 91, 95]),

$$\begin{aligned}
s_L^{-\mu} &= \left(0, \vec{e}_L^{\,-}\right) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right) ~, \\
s_N^{-\mu} &= \left(0, \vec{e}_N^{\,-}\right) = \left(0, \frac{\vec{p}_K \times \vec{p}_-}{|\vec{p}_K \times \vec{p}_-|}\right) ~, \\
s_T^{-\mu} &= \left(0, \vec{e}_T^{\,-}\right) = \left(0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}\right) ~, \\
s_L^{+\mu} &= \left(0, \vec{e}_L^{\,+}\right) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right) ~, \\
s_N^{+\mu} &= \left(0, \vec{e}_N^{\,+}\right) = \left(0, \frac{\vec{p}_K \times \vec{p}_+}{|\vec{p}_K \times \vec{p}_+|}\right) ~, \\
s_T^{+\mu} &= \left(0, \vec{e}_T^{\,+}\right) = \left(0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}\right) ~,
\end{aligned} \tag{4.10}$$

where $\vec{p}_\mp$ and $\vec{p}_K$ are the three–momenta of the leptons $\ell^\mp$ and K^* meson in the center of mass frame (CM) of $\ell^- \, \ell^+$ system, respectively. Transformation of unit vectors from the rest frame of the leptons to CM frame of leptons can be accomplished by the Lorentz boost. Boosting of the longitudinal unit vectors $s_L^{\pm\mu}$

yields

$$\left(s_L^{\mp\mu}\right)_{CM} = \left(\frac{|\vec{p}_\mp|}{m_\ell}, \frac{E_\ell \vec{p}_\mp}{m_\ell |\vec{p}_\mp|}\right) , \tag{4.11}$$

where $\vec{p}_+ = -\vec{p}_-$, E_ℓ and m_ℓ are the energy and mass of leptons in the CM frame, respectively. The remaining two unit vectors $s_N^{\pm\mu}$, $s_T^{\pm\mu}$ are unchanged under Lorentz boost.

The definition of the unpolarized and normalized differential forward–backward asymmetry is (see for example [103])

$$\mathcal{A}_{FB} = \frac{\displaystyle\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz}dz - \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz}dz}{\displaystyle\int_0^1 \frac{d^2\Gamma}{d\hat{s}dz}dz + \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz}dz} , \tag{4.12}$$

where $z = \cos\theta$ is the angle between B meson and ℓ^- in the center mass frame of leptons. When the spins of both leptons are taken into account, the $\mathcal{A}_{FB}$ will be a function of the spins of the final leptons and it is defined as

$$\begin{aligned}
\mathcal{A}_{FB}^{ij}(\hat{s}) = & \left(\frac{d\Gamma(\hat{s})}{d\hat{s}}\right)^{-1} \left\{\int_0^1 dz - \int_{-1}^0 dz\right\} \\
& \left\{\left[\frac{d^2\Gamma(\hat{s}, \vec{s}^- = \vec{i}, \vec{s}^+ = \vec{j})}{d\hat{s}dz} - \frac{d^2\Gamma(\hat{s}, \vec{s}^- = \vec{i}, \vec{s}^+ = -\vec{j})}{d\hat{s}dz}\right]\right. \\
& \left. - \left[\frac{d^2\Gamma(\hat{s}, \vec{s}^- = -\vec{i}, \vec{s}^+ = \vec{j})}{d\hat{s}dz} - \frac{d^2\Gamma(\hat{s}, \vec{s}^- = -\vec{i}, \vec{s}^+ = -\vec{j})}{d\hat{s}dz}\right]\right\} , \\
= & \mathcal{A}_{FB}(\vec{s}^- = \vec{i}, \vec{s}^+ = \vec{j}) - \mathcal{A}_{FB}(\vec{s}^- = \vec{i}, \vec{s}^+ = -\vec{j}) - \mathcal{A}_{FB}(\vec{s}^- = -\vec{i}, \vec{s}^+ = \vec{j}) \\
& + \mathcal{A}_{FB}(\vec{s}^- = -\vec{i}, \vec{s}^+ = -\vec{j}) .
\end{aligned} \tag{4.13}$$

Using these definitions for the double polarized FB asymmetries, we get the following results:

$$\mathcal{A}_{FB}^{LL} = \frac{2}{\hat{r}_{K^*}\Delta} m_B^3 \sqrt{\lambda} v \,\mathrm{Re}\Big[- m_B^3 \hat{m}_\ell \lambda \Big\{ 4(B_1 - D_1) B_7^* C_T^* - (B_4 + B_5)(B_2^* + D_2^*) \Big\}$$

$$+ \; 4 m_B^4 \hat{m}_\ell \lambda \Big\{ (1 - \hat{r}_{K^*})(B_2 - D_2) B_7^* C_T^* + \hat{s}(B_3 - D_3) B_7^* C_T^* \Big\}$$

$$- \; \hat{m}_\ell (1 - \hat{r}_{K^*} - \hat{s}) \Big\{ B_1^* (B_4 + B_5 - 8 B_6 C_T) + D_1^* (B_4 + B_5 + 8 B_6 C_T) \Big\}$$

$$+ \; 8 m_B \hat{r}_{K^*} \hat{s} (A_1 B_1^* - C_1 D_1^*) + 128 m_B^2 \hat{m}_\ell \hat{r}_{K^*} \hat{s} (A_1 - C_1) B_6^* C_{TE}^*$$

$$+ \; 2 m_B^3 \hat{s} \lambda \Big\{ (B_4 - B_5) B_7^* C_T^* + 2(B_4 + B_5) B_7^* C_{TE}^* \Big\}$$

$$- \; 8 m_B^2 \hat{m}_\ell (1 - \hat{r}_{K^*})(1 - \hat{r}_{K^*} - \hat{s})(B_2 - D_2) B_6^* C_T^*$$

$$- \; 4 m_B (1 - \hat{r}_{K^*} - \hat{s}) \hat{s} \Big\{ (B_4 - B_5) B_6^* C_T^* + 2(B_4 + B_5) B_6^* C_{TE}^*$$

$$+ \; 2 m_B \hat{m}_\ell (B_3 - D_3) B_6^* C_T^* \Big\} - 256 m_B^5 \hat{m}_\ell \hat{r}_{K^*} (1 - \hat{r}_{K^*})(A_1 - C_1) T_1^* C_{TE}^*$$

$$- \; 16 \hat{m}_\ell (1 - 5\hat{r}_{K^*} - \hat{s})(B_1 - D_1) T_1^* C_T^*$$

$$+ \; 16 m_B^2 \hat{m}_\ell (1 - \hat{r}_{K^*})(1 + 3\hat{r}_{K^*} - \hat{s})(B_2 - D_2) T_1^* C_T^*$$

$$+ \; 8 m_B (1 + 3\hat{r}_{K^*} - \hat{s}) \hat{s} \Big\{ 2(B_4 + B_5) T_1^* C_{TE}^* + (B_4 - B_5) T_1^* C_T^*$$

$$+ \; 2 m_B \hat{m}_\ell (B_3 - D_3) T_1^* C_T^* \Big\} \Big] \;, \tag{4.14}$$

$$\mathcal{A}_{FB}^{LN} = \frac{8}{3\hat{r}_{K^*}\hat{s}\Delta} m_B^2 \sqrt{\hat{s}} \lambda v \,\mathrm{Im}\Big[- \hat{m}_\ell (B_1 D_1^* + m_B^4 \lambda B_2 D_2^*) + 4 m_B^4 \hat{m}_\ell \hat{r}_{K^*} \sqrt{\hat{s}} A_1 C_1^*$$

$$- \; 2 m_B \hat{s} \Big\{ B_6 (C_T - 2C_{TE}) B_1^* + B_6 (C_T + 2C_{TE}) D_1^* \Big\}$$

$$- \; m_B^5 \hat{s} \lambda \Big\{ B_7 (C_T - 2C_{TE}) B_2^* + B_7 (C_T + 2C_{TE}) D_2^* \Big\}$$

$$- \; 16 m_B^2 \hat{m}_\ell \hat{s} \Big(4 \left| B_6 \right|^2 + m_B^4 \lambda \left| B_7 \right|^2 \Big) C_T C_{TE}^*$$

$$+ \; m_B^2 \hat{m}_\ell (1 - \hat{r}_{K^*} - \hat{s})(B_1 D_2^* + B_2 D_1^*)$$

$$
\begin{aligned}
&+\ m_B^3\hat{s}(1-\hat{r}_{K^*}-\hat{s})\Big\{(B_1^*B_7+2B_2^*B_6)(C_T-2C_{TE}) \\
&+\ (D_1^*B_7+2D_2^*B_6)(C_T+2C_{TE})\Big\} \\
&-\ 64m_B^2\hat{m}_\ell\hat{s}\Big\{-m_B^2(1-\hat{r}_{K^*}-\hat{s})\mathrm{Re}[B_6B_7^*]+4\left|T_1\right|^2-4\mathrm{Re}[B_6T_1^*] \\
&+\ 2m_B^2(1+3\hat{r}_{K^*}-\hat{s})\mathrm{Re}[B_7T_1^*]\Big\}C_TC_{TE}^* \\
&+\ 16m_B^3\hat{r}_{K^*}\hat{s}\Big\{(A_1-C_1)C_T^*T_1^*-2(A_1+C_1)C_{TE}^*T_1^*\Big\} \\
&+\ 4m_B\hat{s}\Big\{B_1^*(C_T-2C_{TE})T_1+D_1^*(C_T+2C_{TE})T_1\Big\} \\
&-\ 4m_B^3\hat{s}(1+3\hat{r}_{K^*}-\hat{s})\Big\{B_2^*(C_T-2C_{TE})T_1+D_2^*(C_T+2C_{TE})T_1\Big\}\Big]\ , \qquad (4.15)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{NL} &= \frac{8}{3\hat{r}_{K^*}\hat{s}\Delta}m_B^2\sqrt{\hat{s}}\lambda v\,\mathrm{Im}\Big[-\hat{m}_\ell(B_1D_1^*]+m_B^4\lambda B_2D_2^*)+4m_B^2\hat{m}_\ell\hat{r}_{K^*}\hat{s}A_1C_1^* \\
&+\ 2m_B\hat{s}\Big\{B_6(C_T+2C_{TE})B_1^*+B_6(C_T-2C_{TE})D_1^*\Big\} \\
&+\ m_B^5\hat{s}\lambda\Big\{B_7(C_T+2C_{TE})B_2^*+B_7(C_T-2C_{TE})D_2^*\Big\} \\
&+\ 16m_B^2\hat{m}_\ell\hat{s}\Big(4\left|B_6\right|^2+m_B^4\lambda\left|B_7\right|^2\Big)C_TC_{TE}^* \\
&+\ m_B^2\hat{m}_\ell(1-\hat{r}_{K^*}-\hat{s})(B_1D_2^*+B_2D_1^*) \\
&-\ m_B^3\hat{s}(1-\hat{r}_{K^*}-\hat{s})\Big\{(B_1^*B_7+2B_2^*B_6)(C_T+2C_{TE}) \\
&+\ (D_1^*B_7+2D_2^*B_6)(C_T-2C_{TE})\Big\} \\
&+\ 64m_B^2\hat{m}_\ell\hat{s}\Big\{-m_B^2(1-\hat{r}_{K^*}-\hat{s})\mathrm{Re}[B_6B_7^*]+4\left|T_1\right|^2-4\mathrm{Re}[B_6T_1^*] \\
&+\ 2m_B^2(1+3\hat{r}_{K^*}-\hat{s})\mathrm{Re}[B_7T_1^*]\Big\}C_TC_{TE}^* \\
&+\ 16m_B^3\hat{r}_{K^*}\hat{s}\Big\{(A_1-C_1)C_T^*T_1^*+2(A_1+C_1)C_{TE}^*T_1^*\Big\} \\
&-\ 4m_B\hat{s}\Big\{B_1^*(C_T+2C_{TE})T_1+D_1^*(C_T-2C_{TE})T_1\Big\}
\end{aligned}
$$

$$+\ 4m_B^3\hat{s}(1+3\hat{r}_{K^*}-\hat{s})\Big\{B_2^*(C_T+2C_{TE})T_1+D_2^*(C_T-2C_{TE})T_1\Big\}\Big]\ , \qquad (4.16)$$

$$\mathcal{A}_{FB}^{LT} = \frac{4}{3\hat{r}_{K^*}\hat{s}\Delta}m_B^2\sqrt{\hat{s}}\lambda\,\mathrm{Re}\Big[-\hat{m}_\ell\Big\{\left|B_1+D_1\right|^2+m_B^4\lambda\left|B_2+D_2\right|^2\Big\}$$

$$+\ 4m_B^4\hat{m}_\ell\hat{r}_{K^*}\hat{s}\Big\{\left|A_1+C_1\right|^2\Big\}$$

$$-\ 64m_B^2\hat{m}_\ell\hat{s}\left|C_{TE}\right|^2\Big\{4\left|B_6\right|^2+m_B^4\lambda\left|B_7\right|^2-4m_B^2(1-\hat{r}_{K^*}-\hat{s})B_6B_7^*\Big\}$$

$$+\ 2m_B^2\hat{m}_\ell(1-\hat{r}_{K^*}-\hat{s})(B_1+D_1)(B_2^*+D_2^*)$$

$$+\ 2m_B^3(1-\hat{r}_{K^*}-\hat{s})\Big\{4\hat{m}_\ell^2(2B_2^*B_6+B_1^*B_7)(C_T+2C_{TE})$$

$$-\ \hat{s}(2B_2^*B_6+B_1^*B_7)(C_T-2C_{TE})\Big\}$$

$$-\ 4m_B\Big\{4\hat{m}_\ell^2\Big[B_1^*B_6(C_T+2C_{TE})-B_6D_1^*(C_T-2C_{TE})\Big]$$

$$-\ \hat{s}\Big[B_1^*B_6(C_T-2C_{TE})-B_6D_1^*(C_T+2C_{TE})\Big]\Big\}$$

$$-\ 2m_B^5\lambda\Big\{4\hat{m}_\ell^2\Big[B_2^*B_7(C_T+2C_{TE})-B_7D_2^*(C_T-2C_{TE})\Big]$$

$$-\ \hat{s}\Big[B_2^*B_7(C_T-2C_{TE})-B_7D_2^*(C_T+2C_{TE})\Big]\Big\}$$

$$-\ 2m_B^3(1-\hat{r}_{K^*}-\hat{s})\Big\{4\hat{m}_\ell^2(2B_6D_2^*+B_7D_1^*)(C_T-2C_{TE})$$

$$-\ \hat{s}(2B_6D_2^*+B_7D_1^*)(C_T+2C_{TE})\Big\}$$

$$+\ 256m_B^2\hat{m}_\ell\Big\{2\hat{s}\left|C_{TE}\right|^2\Big[2B_6T_1^*-m_B^2(1+3\hat{r}_{K^*}-\hat{s})B_7T_1^*\Big]$$

$$+\ 4\left|T_1\right|^2\Big[\hat{r}_{K^*}\left|C_T\right|^2+(4\hat{r}_{K^*}-\hat{s})\left|C_{TE}\right|^2\Big]\Big\}$$

$$+\ 32m_B^3\hat{r}_{K^*}\Big\{4\hat{m}_\ell^2\Big[(A_1+C_1)C_T^*T_1^*+2(A_1-C_1)C_{TE}^*T_1^*\Big]$$

$$+\ \hat{s}\Big[A_1^*(C_T-2C_{TE})T_1+C_1^*(C_T+2C_{TE})T_1\Big]\Big\}$$

$$+\ 8m_B\Big\{4\hat{m}_\ell^2(C_T+2C_{TE})-\hat{s}(C_T-2C_{TE})\Big\}\Big\{B_1^*-m_B^2(1+3\hat{r}_{K^*}-\hat{s})B_2^*\Big\}T_1$$

$$- \; 8m_B\Big\{4\hat{m}_\ell^2(C_T - 2C_{TE}) - \hat{s}(C_T + 2C_{TE})\Big\}\Big\{D_1^* - m_B^2(1 + 3\hat{r}_{K^*} - \hat{s})D_2^*\Big\}T_1\Big] \; , \tag{4.17}$$

$$\begin{aligned}
\mathcal{A}_{FB}^{TL} = \; & \frac{4}{3\hat{r}_{K^*}\hat{s}\Delta} m_B^2 \sqrt{\hat{s}}\lambda \, \mathrm{Re}\Big[\hat{m}_\ell\Big\{ \left|B_1 + D_1\right|^2 + m_B^4\lambda \left|B_2 + D_2\right|^2 \Big\} \\
& - \; 4m_B^4 \hat{m}_\ell \hat{r}_{K^*}\Big\{ \left|A_1 + C_1\right|^2 \Big\} \\
& + \; 64 m_B^2 \hat{m}_\ell \hat{s} \left|C_{TE}\right|^2 \Big\{4\left|B_6\right|^2 + m_B^4\lambda\left|B_7\right|^2 - 4m_B^2(1 - \hat{r}_{K^*} - \hat{s})B_6 B_7^*\Big\} \\
& - \; 2m_B^2 \hat{m}_\ell (1 - \hat{r}_{K^*} - \hat{s})(B_1 + D_1)(B_2^* + D_2^*) \\
& + \; 2m_B^3 (1 - \hat{r}_{K^*} - \hat{s})\Big\{4\hat{m}_\ell^2(2B_2^* B_6 + B_1^* B_7)(C_T - 2C_{TE}) \\
& - \; \hat{s}(2B_2^* B_6 + B_1^* B_7)(C_T + 2C_{TE})\Big\} \\
& - \; 4m_B\Big\{4\hat{m}_\ell^2\Big[B_1^* B_6(C_T - 2C_{TE}) - B_6 D_1^*(C_T + 2C_{TE})\Big] \\
& - \; \hat{s}\Big[B_1^* B_6(C_T + 2C_{TE}) - B_6 D_1^*(C_T - 2C_{TE})\Big]\Big\} \\
& - \; 2m_B^5\lambda\Big\{4\hat{m}_\ell^2\Big[B_2^* B_7(C_T - 2C_{TE}) - B_7 D_2^*(C_T + 2C_{TE})\Big] \\
& - \; \hat{s}\Big[B_2^* B_7(C_T + 2C_{TE}) - B_7 D_2^*(C_T - 2C_{TE})\Big]\Big\} \\
& - \; 2m_B^3(1 - \hat{r}_{K^*} - \hat{s})\Big\{4\hat{m}_\ell^2(2B_6 D_2^* + B_7 D_1^*)(C_T + 2C_{TE}) \\
& - \; \hat{s}(2B_6 D_2^* + B_7 D_1^*)(C_T - 2C_{TE})\Big\} \\
& - \; 256 m_B^2 \hat{m}_\ell\Big\{2\hat{s}\left|C_{TE}\right|^2\Big[2B_6 T_1^* - m_B^2(1 + 3\hat{r}_{K^*} - \hat{s})B_7 T_1^*\Big] \\
& + \; 4\left|T_1\right|^2\Big[\hat{r}_{K^*}\left|C_T\right|^2 + (4\hat{r}_{K^*} - \hat{s})\left|C_{TE}\right|^2\Big]\Big\} \\
& - \; 32 m_B^3 \hat{r}_{K^*}\Big\{4\hat{m}_\ell^2\Big[(A_1 + C_1)C_T^* T_1^* - 2(A_1 - C_1)C_{TE}^* T_1^*\Big] \\
& + \; \hat{s}\Big[A_1^*(C_T + 2C_{TE})T_1 + C_1^*(C_T - 2C_{TE})T_1\Big]\Big\} \\
& - \; 8m_B\Big\{4\hat{m}_\ell^2(C_T + 2C_{TE}) - \hat{s}(C_T - 2C_{TE})\Big\}\Big\{D_1^* - m_B^2(1 + 3\hat{r}_{K^*} - \hat{s})D_2^*\Big\}T_1
\end{aligned}$$

$$+\ 8m_B\Big\{4\hat{m}_\ell^2(C_T-2C_{TE})-\hat{s}(C_T+2C_{TE})\Big\}\Big\{B_1^*-m_B^2(1+3\hat{r}_{K^*}-\hat{s})B_2^*\Big\}\ ,T_1\Big] \tag{4.18}$$

$$\begin{aligned}
\mathcal{A}_{FB}^{NT} = & \frac{2}{\hat{r}_{K^*}\hat{s}\Delta}m_B^2\sqrt{\lambda}\,\mathrm{Im}\Big[m_B^3\hat{m}_\ell\hat{s}\lambda\Big\{(B_4-B_5)(B_2^*+D_2^*)+8B_7C_{TE}(B_1^*-D_1^*) \\
& +\ 8m_B^2\hat{s}B_7^*C_{TE}^*(B_3-D_3)\Big\} \\
& -\ 2m_B^4\hat{m}_\ell^2\hat{s}\lambda(B_2+D_2)(B_3^*-D_3^*) \\
& +\ 4m_B^4\hat{m}_\ell(1-\hat{r}_{K^*})\lambda\Big\{2m_B\hat{s}B_7^*C_{TE}^*(B_2-D_2)+\hat{m}_\ell B_2D_2^*\Big\} \\
& +\ 2m_B^2\hat{m}_\ell^2\hat{s}(1+3\hat{r}_{K^*}-\hat{s})(B_1B_2^*-D_1D_2^*) \\
& +\ \hat{m}_\ell(1-\hat{r}_{K^*}-\hat{s})\Big\{m_B\hat{s}\Big[-B_1^*(B_4-B_5+16B_6C_{TE}) \\
& -\ D_1^*(B_4-B_5-16B_6C_{TE})+2m_B\hat{m}_\ell(B_1+D_1)(B_3^*-D_3^*)\Big] \\
& +\ 4\Big[\hat{m}_\ell B_1D_1^*+4m_B^3\hat{s}^2B_6C_{TE}(B_3^*-D_3^*)\Big]\Big\} \\
& -\ 16m_B^3\hat{m}_\ell\hat{s}(1-\hat{r}_{K^*})(1-\hat{r}_{K^*}-\hat{s})(B_2-D_2)B_6^*C_{TE}^* \\
& +\ 2m_B^2\hat{m}_\ell^2[\lambda+(1-\hat{r}_{K^*})(1-\hat{r}_{K^*}-\hat{s})](B_1^*D_2+B_2^*D_1) \\
& +\ 32m_B^3\hat{m}_\ell\hat{s}(1-\hat{r}_{K^*})(1+3\hat{r}_{K^*}-\hat{s})(B_2-D_2)C_{TE}^*T_1^* \\
& -\ 8m_B\hat{s}(1+3\hat{r}_{K^*}-\hat{s})\Big\{4\hat{m}_\ell(B_1-D_1)C_{TE}^*T_1^*-2m_B\hat{s}(B_4-B_5)C_{TE}^*T_1^* \\
& -\ 4m_B^2\hat{m}_\ell\hat{s}(B_3-D_3)C_{TE}^*T_1^*+m_B\hat{s}v^2(B_4+B_5)C_T^*T_1^*\Big\} \\
& -\ 4m_B^2\hat{s}^2(1-\hat{r}_{K^*}-\hat{s})\Big\{2(B_4-B_5)B_6^*C_{TE}^*-v^2(B_4+B_5)B_6^*C_T^*\Big\} \\
& +\ 2m_B^4\hat{s}^2\lambda\Big\{2(B_4-B_5)B_7^*C_{TE}^*-v^2(B_4+B_5)B_7^*C_T^*\Big\}\Big]\ ,
\end{aligned} \tag{4.19}$$

$$\mathcal{A}_{FB}^{TN} = \frac{2}{\hat{r}_{K^*}\hat{s}\Delta}m_B^2\sqrt{\lambda}\,\mathrm{Im}\Big[m_B^3\hat{m}_\ell\hat{s}\lambda\Big\{(B_4-B_5)(B_2^*+D_2^*)+8B_7C_{TE}(B_1^*-D_1^*)$$

$$
\begin{aligned}
&+ 8m_B^2\hat{s}B_7^*C_{TE}^*(B_3-D_3)\Big\} \\
&- 2m_B^4\hat{m}_\ell^2\hat{s}\lambda(B_2+D_2)(B_3^*-D_3^*) \\
&+ 4m_B^4\hat{m}_\ell(1-\hat{r}_{K^*})\lambda\Big\{2m_B\hat{s}B_7^*C_{TE}^*(B_2-D_2)+\hat{m}_\ell B_2D_2^*\Big\} \\
&+ 2m_B^2\hat{m}_\ell^2\hat{s}(1+3\hat{r}_{K^*}-\hat{s})\mathrm{Im}(B_1B_2^*-D_1D_2^*) \\
&+ \hat{m}_\ell(1-\hat{r}_{K^*}-\hat{s})\Big\{m_B\hat{s}\Big[B_1^*(B_4-B_5+16B_6C_{TE}) \\
&+ D_1^*(B_4-B_5-16B_6C_{TE})-2m_B\hat{m}_\ell(B_1+D_1)(B_3^*-D_3^*)\Big] \\
&+ 4\Big[\hat{m}_\ell B_1D_1^*+4m_B^3\hat{s}^2B_6C_{TE}(B_3^*-D_3^*)\Big]\Big\} \\
&- 16m_B^3\hat{m}_\ell\hat{s}(1-\hat{r}_{K^*})(1-\hat{r}_{K^*}-\hat{s})(B_2-D_2)B_6^*C_{TE}^* \\
&+ 2m_B^2\hat{m}_\ell^2[\lambda+(1-\hat{r}_{K^*})(1-\hat{r}_{K^*}-\hat{s})](B_1^*D_2+B_2^*D_1) \\
&+ 32m_B^3\hat{m}_\ell\hat{s}(1-\hat{r}_{K^*})(1+3\hat{r}_{K^*}-\hat{s})(B_2-D_2)C_{TE}^*T_1^* \\
&- 8m_B\hat{s}(1+3\hat{r}_{K^*}-\hat{s})\Big\{4\hat{m}_\ell(B_1-D_1)C_{TE}^*T_1^*-2m_B\hat{s}(B_4-B_5)C_{TE}^*T_1^* \\
&- 4m_B^2\hat{m}_\ell\hat{s}(B_3-D_3)C_{TE}^*T_1^*+m_B\hat{s}v^2(B_4+B_5)C_T^*T_1^*\Big\} \\
&- 4m_B^2\hat{s}^2(1-\hat{r}_{K^*}-\hat{s})\Big\{2(B_4-B_5)B_6^*C_{TE}^*-v^2(B_4+B_5)B_6^*C_T^*\Big\} \\
&+ 2m_B^4\hat{s}^2\lambda\Big\{2(B_4-B_5)B_7^*C_{TE}^*-v^2(B_4+B_5)B_7^*C_T^*\Big\}\Big]\ , \qquad (4.20)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{NN} &= \frac{2}{\hat{r}_{K^*}\Delta}m_B^3\sqrt{\lambda}v\,\mathrm{Re}\Big[-m_B^2\hat{m}_\ell\lambda\Big\{4(B_1-D_1)B_7^*C_T^*+(B_2+D_2)(B_4^*+B_5^*)\Big\} \\
&+ 4m_B^4\hat{m}_\ell\lambda\Big\{(1-\hat{r}_{K^*})(B_2-D_2)B_7^*C_T^*+\hat{s}(B_3-D_3)B_7^*C_T^*\Big\} \\
&+ 2m_B^3\hat{s}\lambda\Big\{(B_4-B_5)B_7^*C_T^*-2(B_4+B_5)B_7^*C_{TE}^*\Big\} \\
&+ \hat{m}_\ell(1-\hat{r}_{K^*}-\hat{s})\Big\{B_1^*(B_4+B_5+8B_6C_T)
\end{aligned}
$$

$$
\begin{aligned}
& + D_1^*(B_4 + B_5 - 8B_6C_T)\Big\} \\
& - 8m_B^2\hat{m}_\ell(1-\hat{r}_{K^*})(1-\hat{r}_{K^*}-\hat{s})(B_2-D_2)B_6^*C_T^* \\
& - 4m_B\hat{s}(1-\hat{r}_{K^*}-\hat{s})\Big\{(B_4-B_5)B_6^*C_T^* - 2(B_4+B_5)B_6^*C_{TE}^* \\
& + 2m_B\hat{m}_\ell(B_3-D_3)B_6^*C_T^*\Big\} \\
& + 16m_B^2\hat{m}_\ell(1-\hat{r}_{K^*})(1+3\hat{r}_{K^*}-\hat{s})(B_2-D_2)C_T^*T_1^* \\
& + 8m_B\hat{s}(1+3\hat{r}_{K^*}-\hat{s})\Big\{(B_4-B_5)C_T^*T_1^* - 2(B_4+B_5)C_{TE}^*T_1^*\Big\} \\
& - 16\hat{m}_\ell(1+3\hat{r}_{K^*}-\hat{s})(B_1-D_1)C_T^*T_1^* \\
& + 16m_B^2\hat{m}_\ell\hat{s}(1+3\hat{r}_{K^*}-\hat{s})(B_3-D_3)C_T^*T_1^*\Big] \ , \qquad (4.21)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{A}_{FB}^{TT} = & \ \frac{2}{\hat{r}_{K^*}\Delta} m_B^3 \sqrt{\lambda} v \,\mathrm{Re}\Big[m_B^2\hat{m}_\ell\lambda\Big\{4(B_1-D_1)B_7^*C_T^* + (B_2+D_2)(B_4^*+B_5^*)\Big\} \\
& - 4m_B^4\hat{m}_\ell(1-\hat{r}_{K^*})\lambda(B_2-D_2)B_7^*C_T^* \\
& - 2m_B^3\hat{s}\lambda\Big\{(B_4-B_5)B_7^*C_T^* - 2(B_4+B_5)B_7^*C_{TE}^* \\
& + 2m_B\hat{m}_\ell(B_3-D_3)B_7^*C_T^*\Big\} \\
& - 2(1-\hat{r}_{K^*}-\hat{s})\Big\{\hat{m}_\ell\Big[B_1^*(B_4+B_5+8B_6C_T) \\
& + D_1^*(B_4+B_5-8B_6C_T)\Big] - 4m_B\hat{s}\Big[(B_4-B_5)B_6^*C_T^* - 2(B_4+B_5)B_6^*C_{TE}^* \\
& + 2m_B\hat{m}_\ell(B_3-D_3)B_6^*C_T^*\Big]\Big\} \\
& + 8m_B^2\hat{m}_\ell(1-\hat{r}_{K^*})(1-\hat{r}_{K^*}-\hat{s})(B_2-D_2)B_6^*C_T^* \\
& - 16m_B^2\hat{m}_\ell(1-\hat{r}_{K^*})(1+3\hat{r}_{K^*}-\hat{s})(B_2-D_2)C_T^*T_1^* \\
& - 8m_B\hat{s}(1+3\hat{r}_{K^*}-\hat{s})\Big\{(B_4-B_5)C_T^*T_1^* - 2(B_4+B_5)C_{TE}^*T_1^*\Big\}
\end{aligned}
$$

$$+ 16\hat{m}_\ell(1+3\hat{r}_{K^*}-\hat{s})\Big\{(B_1-D_1)C_T^* T_1^* - m_B^2\hat{s}(B_3-D_3)C_T^* T_1^*\Big\}\Big] ~. \qquad (4.22)$$

In these expressions for $\mathcal{A}_{FB}^{ij}$, the first index in the superscript describes the polarization of lepton and the second index describes that of anti–lepton.

It should be noted here that, the double–polarized FB asymmetry for the $B \to K\tau^+\tau^-$ and $b \to s\tau^+\tau^-$ decays are calculated in the supersymmetric model in [104].

4.4 Numerical analysis

In this section we analyze the effects of the Wilson coefficients on the polarized FB asymmetry. The input parameters we use in our numerical calculations are: $|V_{tb}V_{ts}^*| = 0.0385$, $m_{K^*} = 0.892~GeV$, $m_\tau = 1.77~GeV$, $m_\mu = 0.106~GeV$, $m_b = 4.8~GeV$, $m_B = 5.26~GeV$ and $\Gamma_B = 4.22 \times 10^{-13}~GeV$. For the values of the Wilson coefficients we use $C_7^{SM} = -0.313$, $C_9^{SM} = 4.344$ and $C_{10}^{SM} = -4.669$. It should be noted that the above–presented value for C_9^{SM} corresponds only to short distance contributions. In addition to the short distance contributions, it receives long distance contributions which result from the conversion of $\bar{c}c$ to the lepton pair. In this work we neglect long distance contributions. The reason for such a choice is dictated by the fact that, in the SM the zero position of $\mathcal{A}_{FB}$ for the $B \to K^*\ell^+\ell^-$ decay is practically independent of the form factors and is determined in terms of short distance Wilson coefficients C_9^{SM} and C_7^{SM} (see [88, 93]) and $s_0 = 3.9~GeV^2$. For the form factors we have used the light cone

QCD sum rules results [105, 106]. As a result of the analysis carried out in this scheme, the q^2 dependence of the form factors can be represented in terms of three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2} ,$$

where the values of parameters $F(0)$, a_F and b_F for the $B \to K^*$ decay are listed in Table 4.1.

Table 4.1: B meson decay form factors in a three-parameter fit, where the radiative corrections to the leading twist contribution and SU(3) breaking effects are taken into account.

	$F(0)$	a_F	b_F
$A_1^{B\to K^*}$	0.34 ± 0.05	0.60	-0.023
$A_2^{B\to K^*}$	0.28 ± 0.04	1.18	0.281
$V^{B\to K^*}$	0.46 ± 0.07	1.55	0.575
$T_1^{B\to K^*}$	0.19 ± 0.03	1.59	0.615
$T_2^{B\to K^*}$	0.19 ± 0.03	0.49	-0.241
$T_3^{B\to K^*}$	0.13 ± 0.02	1.20	0.098

The new Wilson coefficients vary in the range $-|C_{10}| \le |C_i| \le |C_{10}|$. The experimental value of the branching ratio of the $B \to K^* \ell^+ \ell^-$ decay [101, 102] and the bound on the branching ratio of the $B \to \mu^+ \mu^-$ [8] suggest that this is the right order of magnitude for the vector and scalar interaction coefficients. It should be noted here that the experimental results lead to stronger restrictions on some of the Wilson coefficients, namely $-1.5 \le C_T \le 1.5$, $-3.3 \le C_{TE} \le 2.6$,

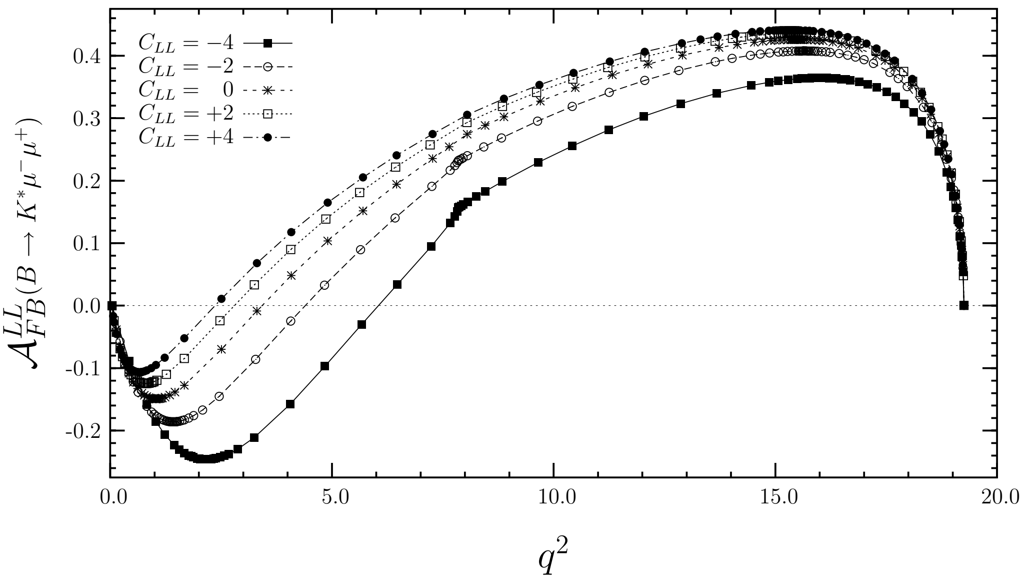

Figure 4.1: The dependence of the double–lepton polarization asymmetry $\mathcal{A}_{FB}^{LL}$ on q^2 at four fixed values of C_{LL}, for the $B \to K^* \mu^+ \mu^-$ decay.

$-2 \leq C_{LL},\, C_{RL} \leq 2.3$, while the remaining coefficients vary in the range $-4 \leq C_X \leq 4$.

In Fig. 4.1(4.2) we present the dependence of the $\mathcal{A}_{FB}^{LL}$ on q^2 for the $B \to K^* \mu^+ \mu^-$ at four fixed values of $C_{LL}(C_{LR})$: $-4, -2, 2, 4$. From these figures we see that nonzero values of the new Wilson coefficients shift the zero position of $\mathcal{A}_{FB}^{LL}$ corresponding to the SM result. When C_{LL} gets negative (positive) values, the zero position of $\mathcal{A}_{FB}^{LL}$ shifts to the left (right) in comparison to that of the zero position in the SM.

Our analysis shows that the zero position of $\mathcal{A}_{FB}^{LL}$ for the $B \to K^* \mu^+ \mu^-$ decay is practically independent of the existence of other Wilson coefficients. For this reason we do not present the dependence of $\mathcal{A}_{FB}^{LL}$ on q^2 at fixed values of the remaining Wilson coefficients.

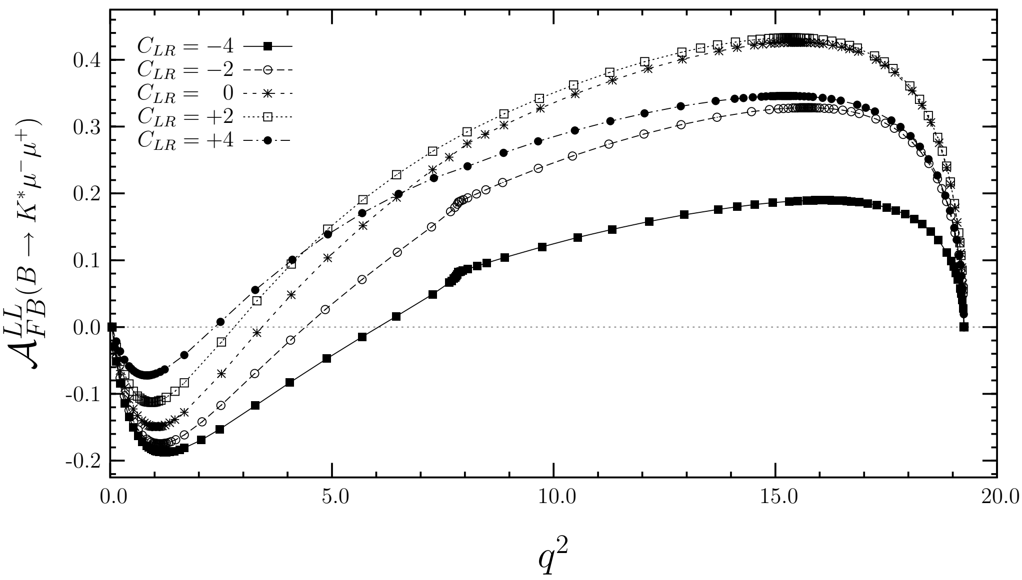

Figure 4.2: The same as in Fig. (4.1), but at four fixed values of C_{LR}.

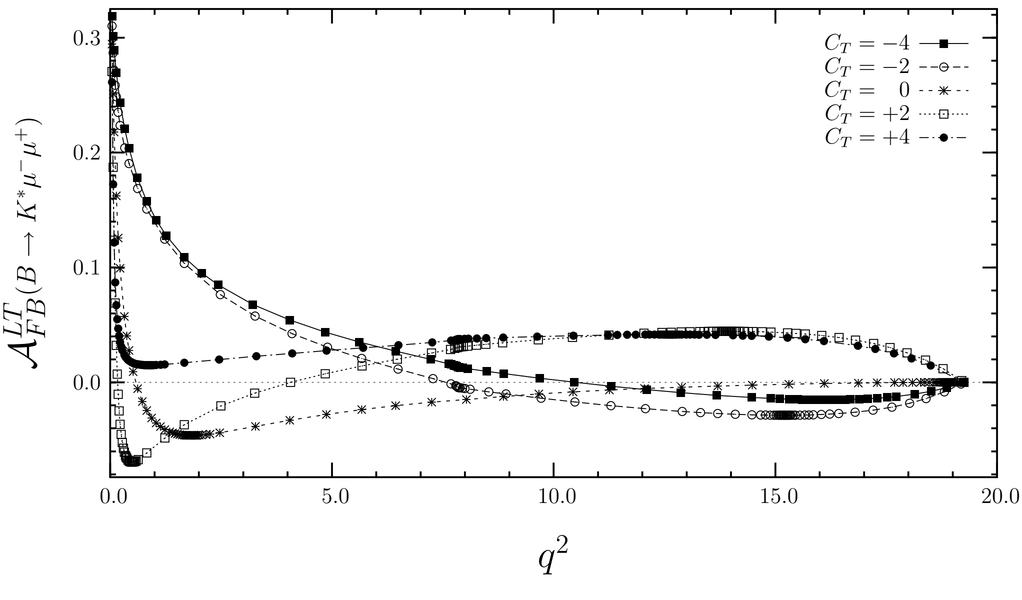

Figure 4.3: The dependence of the double–lepton polarization asymmetry $\mathcal{A}_{FB}^{LT}$ on q^2 at four fixed values of C_T, for the $B \to K^* \mu^+ \mu^-$ decay.

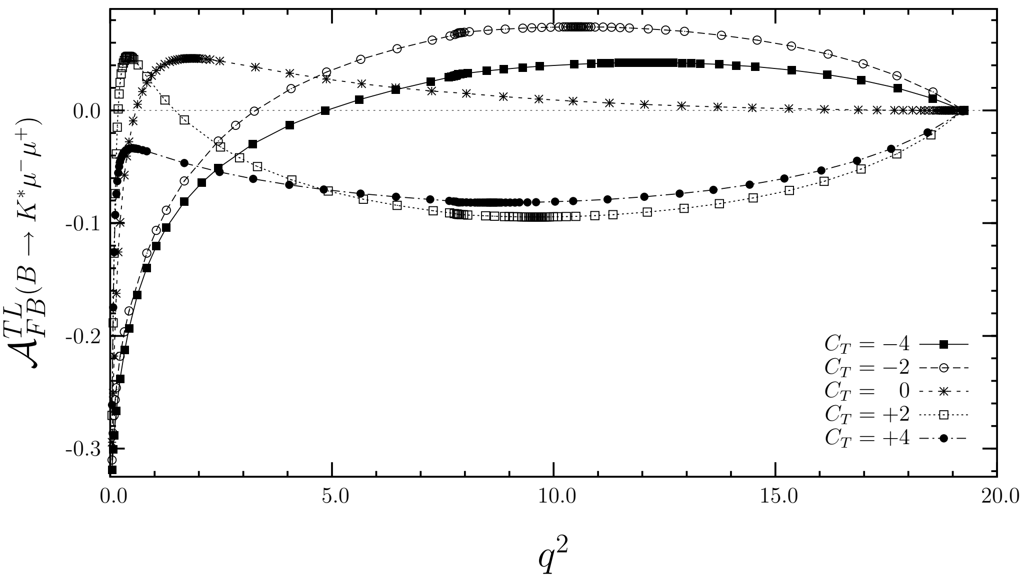

Figure 4.4: The same as in Fig. (4.3), but for $\mathcal{A}_{FB}^{TL}$.

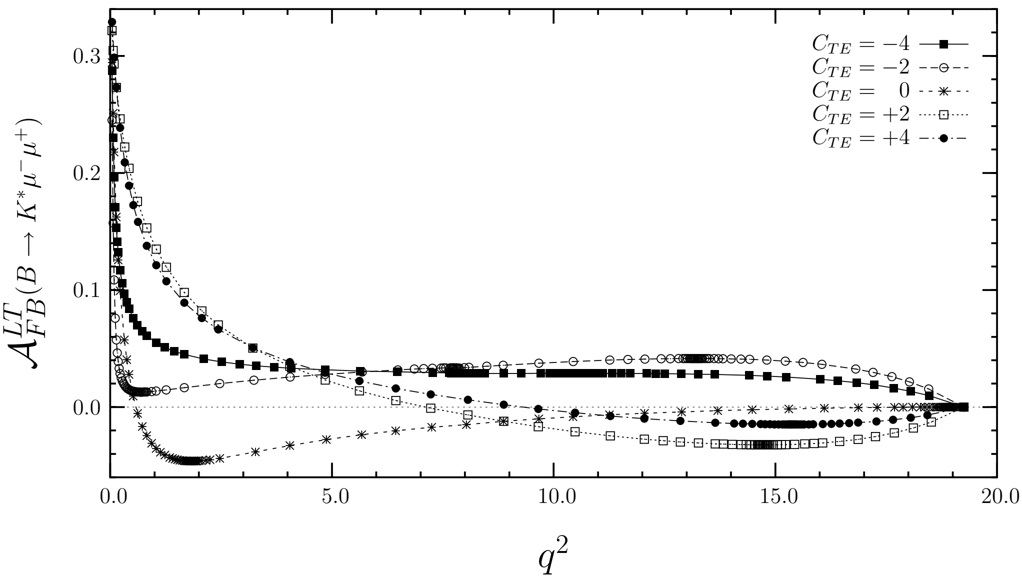

Figure 4.5: The same as in Fig. (4.3), but at four fixed values of C_{TE}.

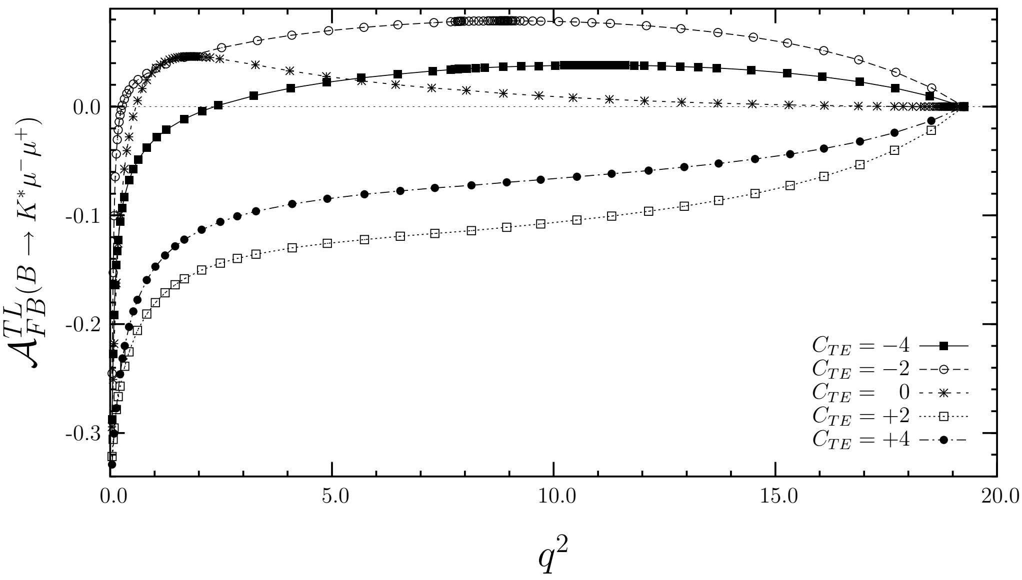

Figure 4.6: The same as in Fig. (4.4), but at four fixed values of C_{TE}.

Figs. 4.3(4.5) and 3.4(3.6) depict the dependence of $\mathcal{A}_{FB}^{LT}$ and $\mathcal{A}_{FB}^{TL}$ on q^2 at four fixed values of $C_T(C_{TE})$. We observe from these figures that the zero positions of $\mathcal{A}_{FB}^{LT}$ and $\mathcal{A}_{FB}^{TL}$ are very sensitive to the existence of tensor interactions. More essential than is that in the SM case $\mathcal{A}_{FB}^{LT}$ and $\mathcal{A}_{FB}^{TL}$ do not have zero values. Therefore, if zero values for the polarized $\mathcal{A}_{FB}^{LT}$ and $\mathcal{A}_{FB}^{TL}$ asymmetries are measured in the experiments in future, these results are unambiguous indication of the existence of new physics beyond the SM, more specifically, the existence of tensor interactions.

In the case of $B \to K^* \tau^+ \tau^-$ decay, the zero position for the double polarization asymmetries $\mathcal{A}_{FB}^{ij}$ is absent for most of the new Wilson coefficients, and hence, it could be concluded to be insensitive to the new physics beyond the SM, or the value of $\mathcal{A}_{FB}^{ij}$ is quite small, whose measurement in the experiments could practically be impossible. For this reason we do not present the dependencies of $\mathcal{A}_{FB}^{ij}$ on q^2 at fixed values of C_X for the $B \to K^* \tau^+ \tau^-$ decay.

As is obvious from the explicit expressions of the forward–backward asymmetries, they depend both on q^2 and the new Wilson coefficients C_X. As a result of this, it might be difficult to study the dependence of the polarized forward–backward asymmetries $\mathcal{A}_{FB}^{ij}$ on these parameters. However, we can eliminate the dependence of the polarized $\mathcal{A}_{FB}^{ij}$ on q^2 by performing integration over q^2 in the kinematically allowed region, so that the polarized forward–backward asymmetry is said to be averaged. The averaged polarized forward–backward asymmetry is defined as

$$\left\langle \mathcal{A}_{FB}^{ij} \right\rangle = \frac{\displaystyle\int_{4m_\ell^2}^{(m_B - m_{K^*})^2} \mathcal{A}_{FB}^{ij} \frac{d\mathcal{B}}{dq^2} dq^2}{\displaystyle\int_{4m_\ell^2}^{(m_B - m_{K^*})^2} \frac{d\mathcal{B}}{dq^2} dq^2} \; .$$

In Fig. (4.7), we present the dependence of $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ on C_X for the $B \to K^* \mu^+ \mu^-$ decay. The common intersection point of all curves corresponds to the SM case. We observe from this figure that, $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ has symmetric behavior on its dependence on C_T and C_{TE} with respect to zero position, and remains smaller compared to the SM result. The only case for which $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle > \left\langle \mathcal{A}_{FB}^{LL} \right\rangle_{SM}$ occurs for the positive values of the vector interaction coefficients. Therefore, if we measure in the experiments $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle > \left\langle \mathcal{A}_{FB}^{LL} \right\rangle_{SM}$, it is a direct indication of new physics beyond the SM, and this departure is to be attributed solely to the

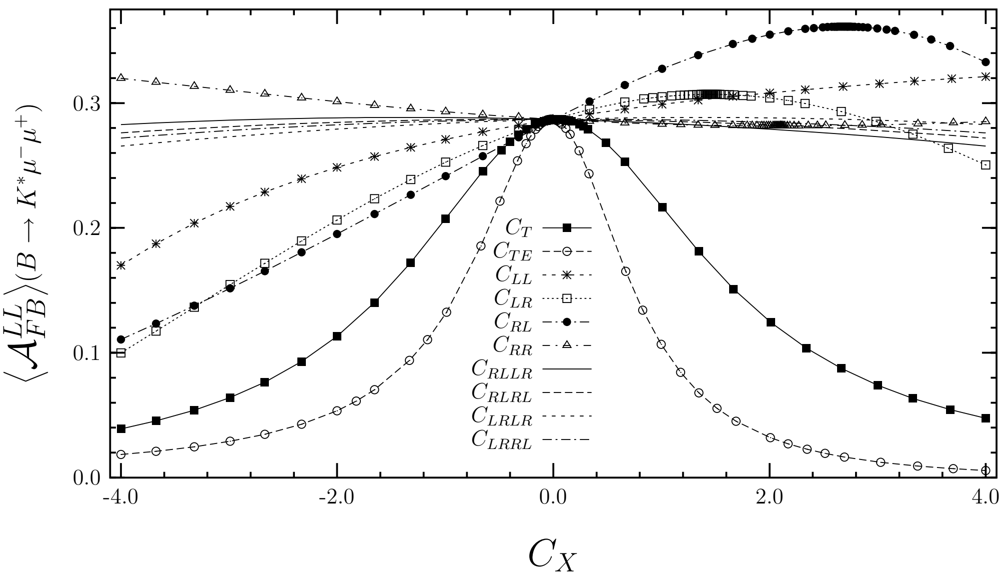

Figure 4.7: The dependence of the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ on the new Wilson coefficients C_X, for the $B \to K^* \mu^+ \mu^-$ decay.

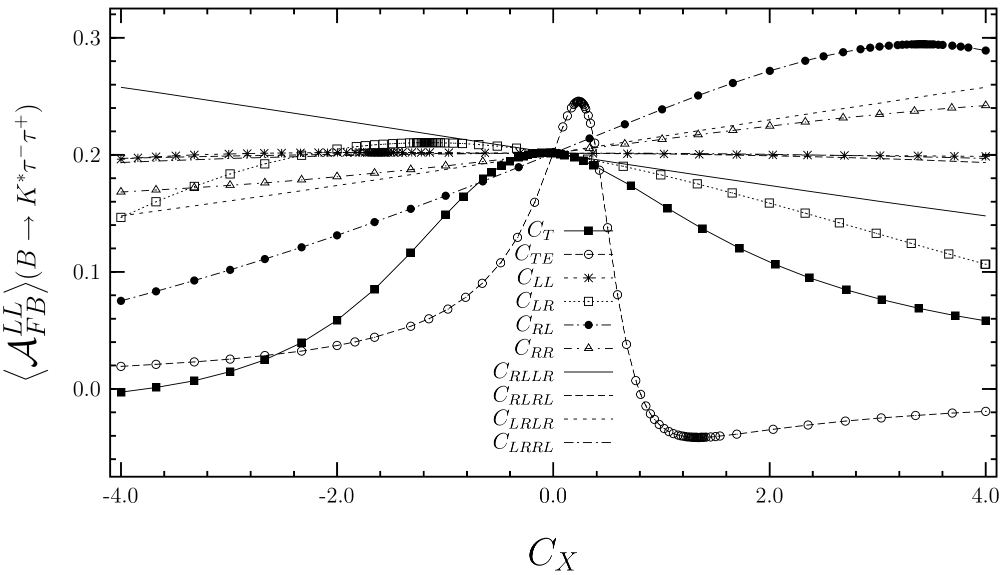

Figure 4.8: The same as in Fig. (4.7), but for the $B \to K^* \tau^+ \tau^-$ decay.

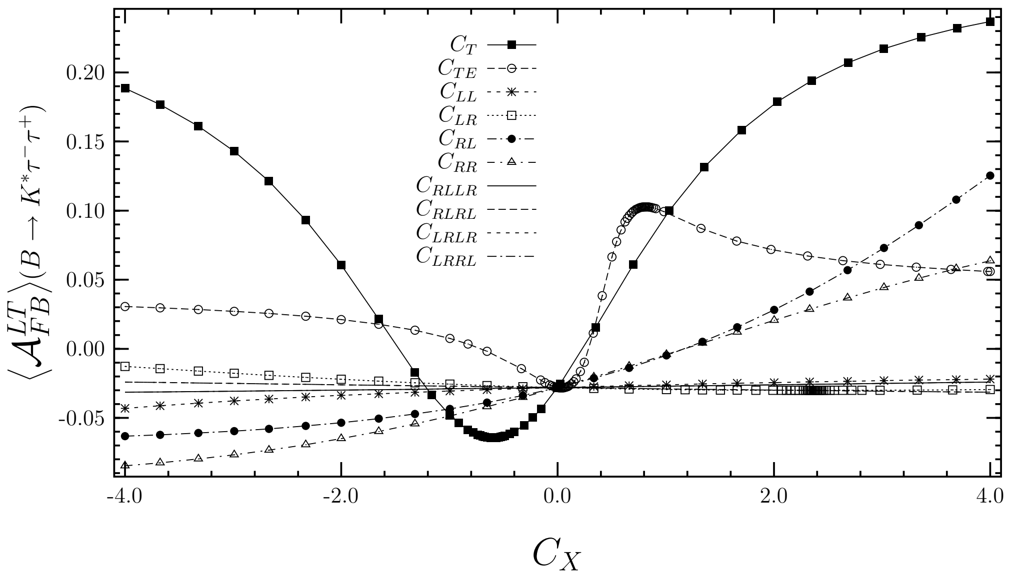

Figure 4.9: The same as in Fig. (4.8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$.

existence of vector type interactions.

The situation is even more informative for the $B \to K^* \tau^+ \tau^-$ case. In Figs. (4.9) and (4.10), we present the dependencies of $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ and $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ on the new Wilson coefficients C_X. From Fig. (4.9) we observe that, with respect to the zero value of the Wilson coefficients, $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ increases if C_{RL}, C_{LRLR} and C_{RR} increase, while it decreases when C_{RLLR} increases.

From Fig. (4.9) we see that the dependence of $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ on the tensor interaction is stronger. When C_T, C_{TE} and C_{LR} are negative (positive) and vary from -4 to zero (from zero to 4) $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ decrease (increase). Additionally, we observe that with increasing values of C_{RL} and C_{RR}, $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ increases. This figure further depicts that $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$, for practical purposes, is not sensitive to the existence of scalar interactions. On the other hand, $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle$ and $\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$ are very sensitive to

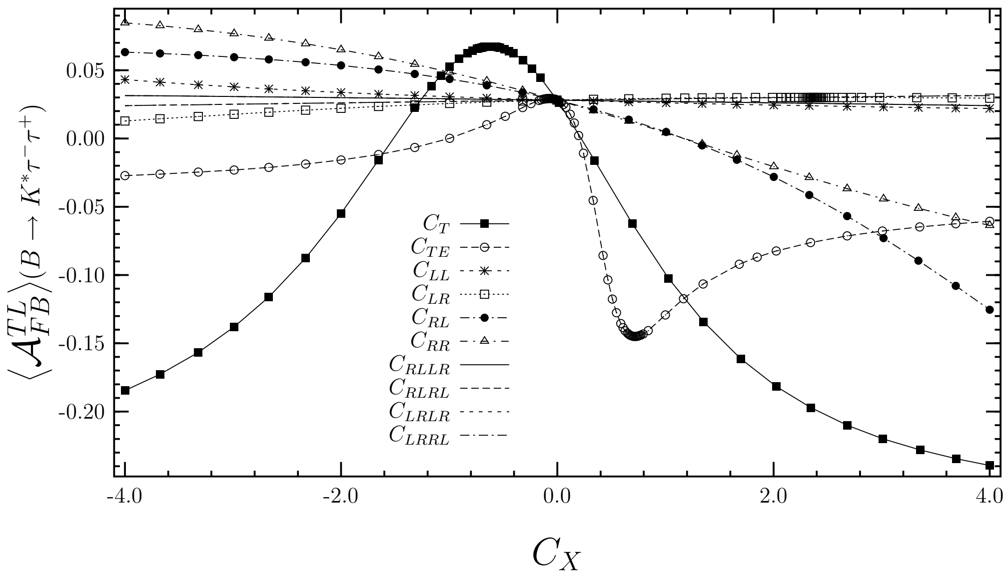

Figure 4.10: The same as in Fig. (4.8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$.

the presence of tensor and scalar interactions (see Figs. (4.10) and (4.11)).

It is clear from these results that several of the polarized forward–backward asymmetries show sizable departure from the SM results and they are sensitive to the existence of different type of interactions. therefore, study of these observable can be very useful in looking for new physics beyond the SM.

Obviously, if new physics beyond the SM exists, there hoped to be effects on the branching ratio besides the polarized $\mathcal{A}_{FB}$. Keeping in mind that the measurement of the branching ratio is easier, one could find it more convenient to study it for establishing new physics. But the intriguing question is, whether there could appear situations in which the value of the branching ratio coincides with that of the SM result, while polarized $\mathcal{A}_{FB}$ does not. In order to answer

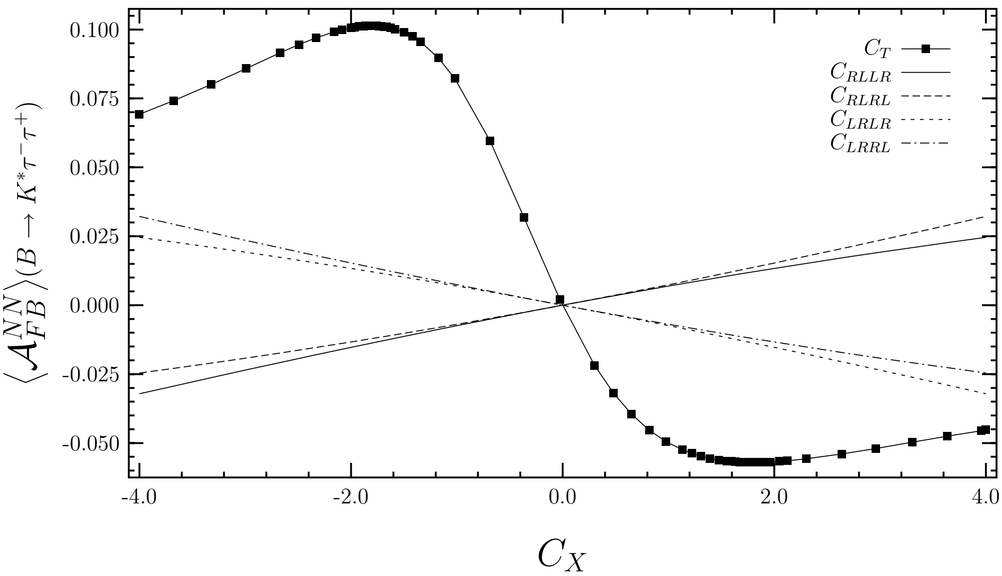

Figure 4.11: The same as in Fig. (4.8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle$.

this question we study the correlation between the averaged, polarized $\langle \mathcal{A}_{FB} \rangle$ and branching ratio. In further analysis we vary the branching ratio of $B \to K^* \mu^+ \mu^-$ $(K^* \tau^+ \tau^-)$ between the values $(1-3) \times 10^{-6}$ $[(1-3) \times 10^{-7}]$, which is very close to the SM calculations. Note that, we do not take into account the experimental results on branching ratio since they contain large errors, and it would be better to wait for more improved experimental results.

Our conclusion for the $B \to K^* \mu^+ \mu^-$ decay, in regard to the above–mentioned correlated relation, is as follows (remember that, the intersection of all curves corresponds to the SM value):

- for $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$, such a region is absent for all C_X,
- for $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$, such a region does exist for C_T and C_{TE} (see Fig. (4.13)).

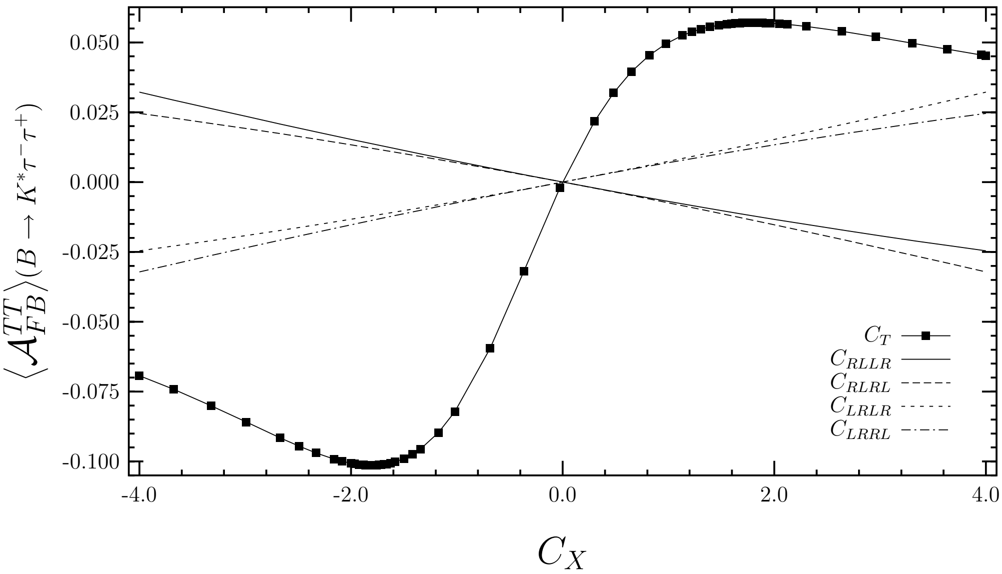

Figure 4.12: The same as in Fig. (4.8), but for the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle$.

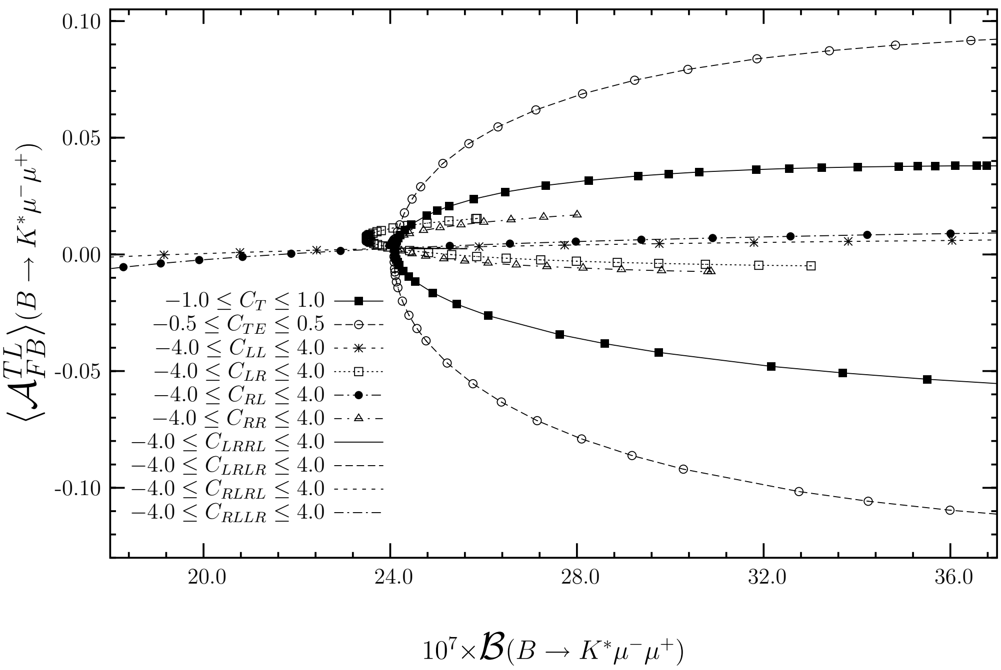

Figure 4.13: Parametric plot of the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle$ and the branching ratio for the $B \to K^* \mu^+ \mu^-$ decay.

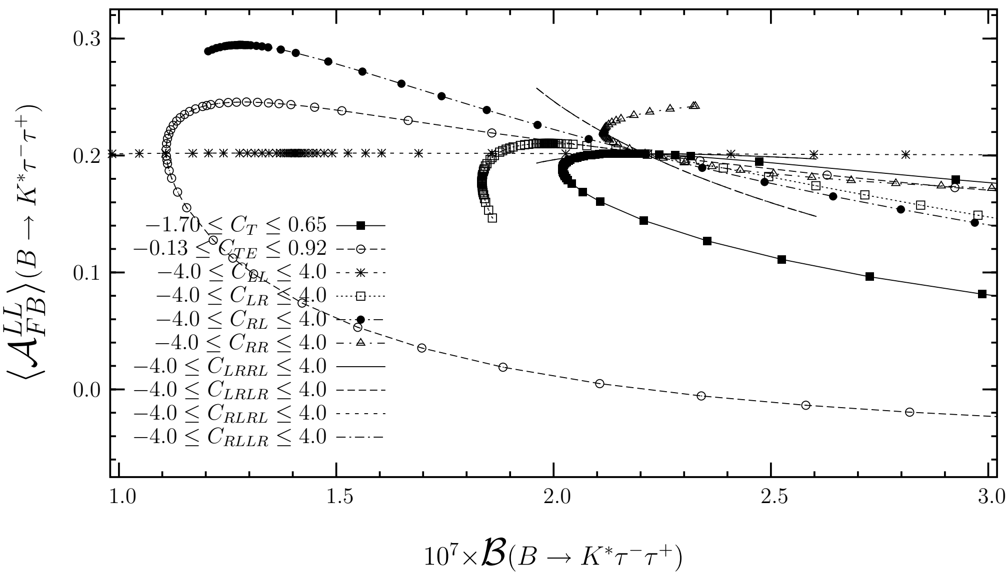

Figure 4.14: Parametric plot of the correlation between the averaged forward–backward double–lepton polarization asymmetry $\langle \mathcal{A}_{FB}^{LL} \rangle$ and the branching ratio for the $B \to K^* \tau^+ \tau^-$ decay.

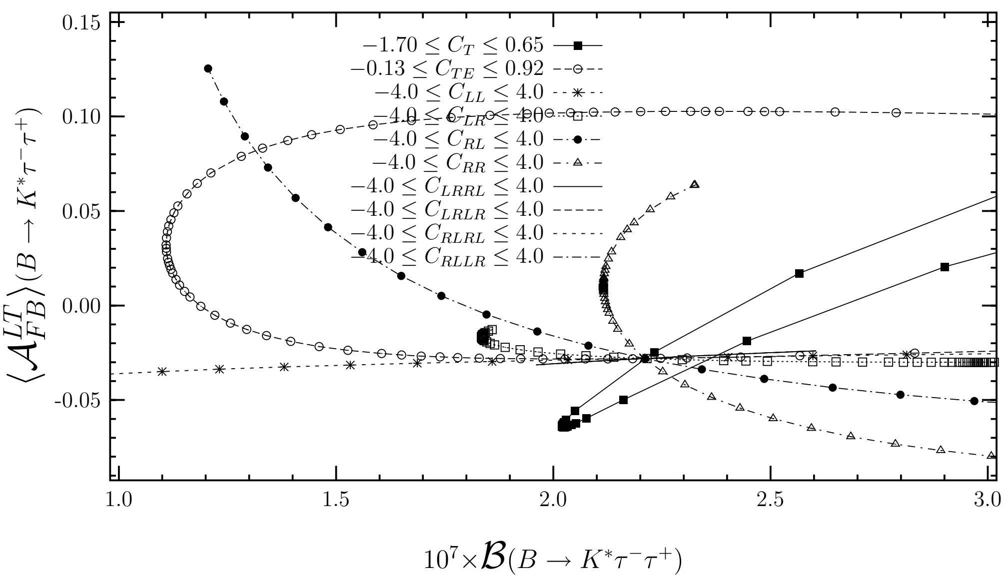

Figure 4.15: The same as in Fig. (4.14), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\langle \mathcal{A}_{FB}^{LT} \rangle$ and the branching ratio.

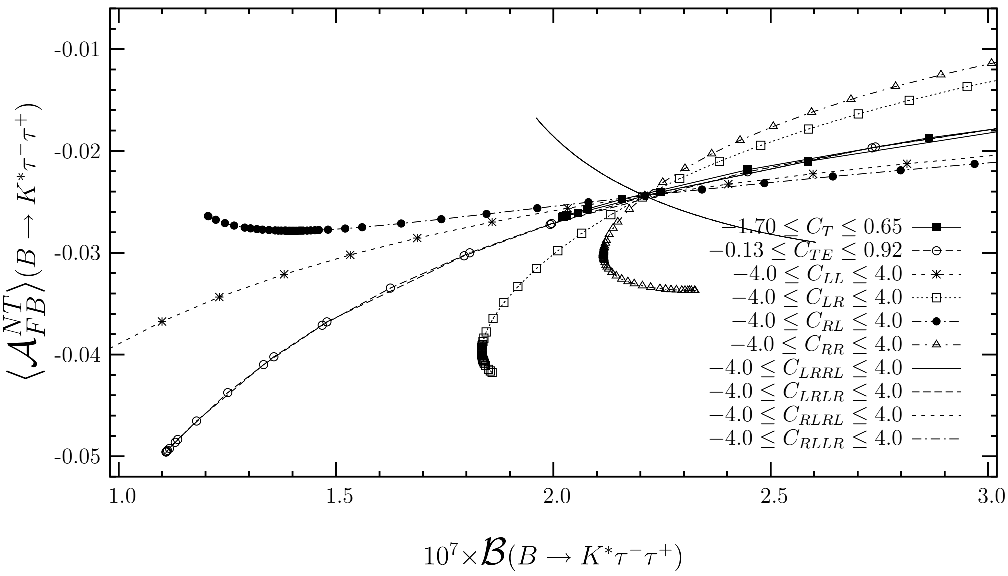

Figure 4.16: The same as in Fig. (4.15), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NT} \right\rangle$ and the branching ratio.

The situation is much more attractive for the $B \to K^* \tau^+ \tau^-$ decay. In Figs. (4.14)–(4.18), we depict the dependence of the averaged, forward–backward polarized asymmetries $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$; $\left\langle \mathcal{A}_{FB}^{LT} \right\rangle \approx -\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$; $\left\langle \mathcal{A}_{FB}^{NT} \right\rangle \approx \left\langle \mathcal{A}_{FB}^{TN} \right\rangle$; $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle$ and $\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$, on branching ratio. It follows from these figures that, indeed, there exist certain regions of the new Wilson coefficients for which, mere study of the polarized $\mathcal{A}_{FB}$ can give promising information about new physics beyond the SM.

In summary, in this chapter we present the analysis for the forward–backward asymmetries when both leptons are polarized, using a general, model independent form of the effective Hamiltonian. We obtained that the study of the zero position of $\left\langle \mathcal{A}_{FB}^{LL} \right\rangle$ can give unambiguous conformation of the new physics beyond the SM,

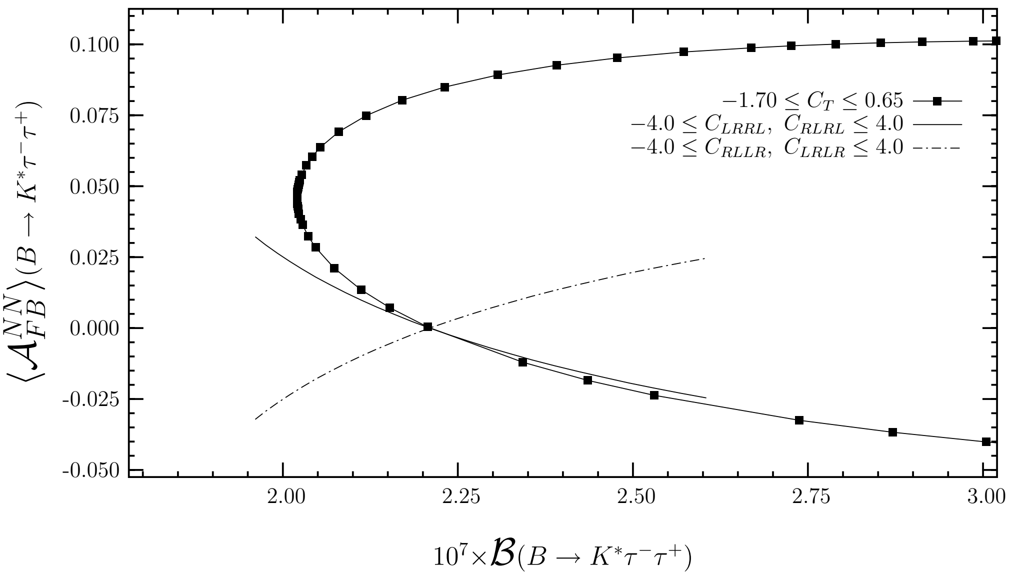

Figure 4.17: The same as in Fig. (4.16), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{NN} \right\rangle$ and the branching ratio.

since when new physics effects are taken into account, the results are shifted with respect to their zero positions in the SM. Moreover, we find that the polarized $\mathcal{A}_{FB}$ is quite sensitive to the existence of the tensor and vector interactions. Finally we obtain that there exist certain regions of the new Wilson coefficients for which, only study of the polarized forward–backward asymmetry gives invaluable information in establishing new physics beyond the SM.

- FB: forward–backward.
- L: Longitudinal.
- N: Normal.
- T: Transversal.

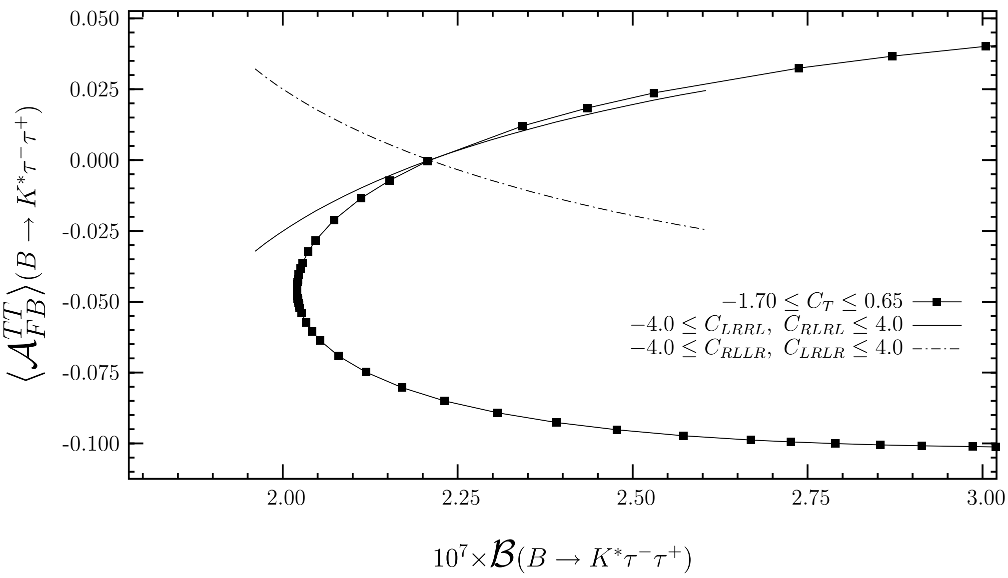

Figure 4.18: The same as in Fig. (4.17), but for the the correlation between the averaged forward–backward double–lepton polarization asymmetry $\left\langle \mathcal{A}_{FB}^{TT} \right\rangle$ and the branching ratio.

CHAPTER 5

CONCLUSION

In this thesis we study the CP-violating asymmetries in $b \to d\ell^+\ell^-$ decay, without and with one of the final leptons is polarized, using the most general form of effective Hamiltonian. We have obtained that, CP-asymmetries for unpolarized case (A_{CP}) and when one of the final lepton is polarized (δA_{CP}) are very sensitive to various new Wilson coefficients:

- For the $b \to d\mu^+\mu^-$ case, far from resonance regions, A_{CP} exhibit strong dependence only on vector type interactions with Wilson coefficients C_{LL} and C_{LR}. A_{CP} is zero for tensor type interactions and it shows quite weak dependence on all remaining Wilson coefficients and the value of A_{CP} very close to SM prediction.

- For the $b \to d\tau^+\tau^-$ case, in the region between second and third ψ resonances, A_{CP} is sensitive to C_{LR}, C_{LRLR}, C_{LRRL}, C_T, and C_{TE}.

  When C_{LR} and C_{LRLR} are positive (negative), they contribute destructively (constructively) to the SM result. The situation is contrary to this behavior for the C_{LRRL} scalar coupling. In the tensor interaction case, in the second and third resonance region, the magnitude of A_{CP} is smaller compared to that of the SM result. But, it is quite important to observe that A_{CP} asymmetry changes its sign, compared to its behavior in the SM, when C_T (C_{TE}) is negative (positive). Therefore, determination of the sign and magnitude of A_{CP} can give promising information about new physics.

The results concerning δA_{CP} for the $b \to d\mu^+\mu^-$ decay can be summarized as follows:

- In the region $1\ GeV^2/m_b^2 \leq \hat{s} \leq 8\ GeV^2/m_b^2$, which is free of resonance contribution, CP asymmetry due to the longitudinal polarization of μ lepton is dependent strongly on C_{LL}, and is practically independent of all remaining vector interaction coefficients. When C_{LL} is negative (positive), δA_{CP} is larger (smaller) compared to that of the SM result.

- δA_{CP}^L depends strongly on all scalar type interactions. The terms proportional to tensor interaction terms contribute destructively to the SM result.

For the $b \to d\tau^+\tau^-$ case, our main results are:

- δA_{CP}^L depends strongly on the tensor type interactions .

- Similar to the μ lepton case, δA_{CP}^L is quite sensitive to the existence of all scalar type interaction coefficients. Note that in the SM case the sign of δA_{CP}^L can be positive or negative. Therefore in the region $\hat{s} \geq 0.6$, determination of the sign of δA_{CP}^L can give unambiguous information about the existence of new physics beyond the SM. For the remaining two scalar interaction coefficients C_{RLRL} (C_{RLLR}), the sign of δA_{CP}^{L} is negative (positive). Again, as in the previous case, determination of the sign and magnitude of δA_{CP}^{L} can give quite valuable hints for establishing new physics beyond the SM.

Since transversal and normal polarizations are proportional to the lepton mass, for the light lepton case, obviously, departure from the SM results is not substantial for all Wilson coefficients. On the other hand, for the $b \to d\ell^-\ell^+$ transition, δA_{CP}^{i} ($i = T$ or N) is strongly dependent on C_{LR} and scalar type interactions. Note that, the dependence of δA_{CP}^{T} on C_T and C_{TE} is quite weak.

Finally, we have found the existence of regions where A_{CP} coincides with the SM prediction, while δA_{CP} does not. This indicates that the new physics effects can only be established in δA_{CP} measurements for those regions.

Using the same formalism we present the analysis for the forward–backward asymmetries for $B \to K^*\ell^+\ell^-$ when both leptons are polarized. We have found that several polarized forward–backward asymmetries show sizable departure from the SM results and they are sensitive to the existence of different type of interactions. Therefore, study of these observable can be very useful in looking for new physics beyond the SM.

The results for $B \to K^* \mu^+ \mu^-$ decay can be summarized as following:

- Nonzero values of C_{LL}, (C_{LR}) shift the zero position of $\mathcal{A}_{FB}^{LL}(q^2)$ corresponding to the SM result. Our analysis shows that the zero position of $\mathcal{A}_{FB}^{LL}(q^2)$ for the $B \to K^* \mu^+ \mu^-$ decay is practically independent of the existence of other Wilson coefficients.

- The zero positions of $\mathcal{A}_{FB}^{LT}(q^2)$ and $\mathcal{A}_{FB}^{TL}(q^2)$ are very sensitive to the existence of tensor interactions. More essential than is that in the SM case $\mathcal{A}_{FB}^{LT}(q^2)$ and $\mathcal{A}_{FB}^{TL}(q^2)$ do not have zero values. Therefore, if zero values for the polarized $\mathcal{A}_{FB}^{LT}(q^2)$ and $\mathcal{A}_{FB}^{TL}(q^2)$ asymmetries are measured in the experiments in future, these results are unambiguous indication of the existence of new physics beyond the SM, more specifically, the existence of tensor interactions.

In the case of $B \to K^* \tau^+ \tau^-$ decay, the zero position for the double polarization asymmetries $\mathcal{A}_{FB}^{ij}(q^2)$ is absent for most of the new Wilson coefficients, and hence, it could be concluded to be insensitive to the new physics beyond the SM, or the value of $\mathcal{A}_{FB}^{ij}(q^2)$ is quite small, whose measurement in the experiments could practically be impossible.

We also analyzed the problem where new physics can be established only by measurement of the polarization asymmetries, when branching ratio coincide with

SM result. For the $B \to K^* \mu^+ \mu^-$ decay, we obtained that such region is exist only for $\left\langle \mathcal{A}_{FB}^{TL} \right\rangle$ when tensor interactions are exist.

As for the $B \to K^* \tau^+ \tau^-$ decay, we obtained that there exist certain regions of the new Wilson coefficients for which, branching ratio coincide with SM predictions while polarized asymmetries exhibit strong departure from SM results. Therefor experimental study of the $\mathcal{A}_{FB}$ can give promising information about new physics beyond the SM.

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