THE GENERAL LOT SIZING AND SCHEDULING PROBLEM WITH SEQUENCE DEPENDENT CHANGEOVERS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

AYŞE KOÇLAR

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
INDUSTRIAL ENGINEERING

JUNE 2005

Approval of the Graduate School of Natural and Applied Sciences

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ABSTRACT

THE GENERAL LOT SIZING AND SCHEDULING PROBLEM WITH SEQUENCE DEPENDENT CHANGEOVERS

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June 2005, 162 pages

In this study, we consider the General Lot Sizing and Scheduling Problem in single level capacitated environments with sequence dependent item changeovers. Process industries may be regarded as suitable application areas of the problem. The focus on capacity utilization and intensively time consuming changeovers necessitate the integration of lot sizing and sequencing decisions in the production plan.

We present a mathematical model which captures the essence of cases in the most generic and realistic setting of the problem. We discuss the impact and validity of some of the assumptions commonly encountered in the related literature. We also represent the problem using an alternative formulation and attempt to enhance the formulations with the use of some additional inequalities. Finally, we develop a heuristic by restricting the number of possible changeovers. Computational results are discussed.

Keywords: Integrated Lot Sizing and Scheduling, Sequence Dependent Changeovers, Valid Inequalities, Mathematical Programming

ÖZ

SIRAYA BAĞLI HAZIRLIKLAR İÇEREN GENEL PARTİ BÜYÜKLÜĞÜ BELİRLEME VE SIRALAMA PROBLEMİ

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Haziran 2005, 162 sayfa

Bu çalışmada, ürünler arasında sıraya bağlı geçişler içeren, tek düzeyli ve kapasite kısıtlı ortamlarda Genel Parti Büyüklüğü Belirleme ve Sıralama Problemi ele alınmıştır. Proses endüstrisi tipi ortamlar, problemin uygulama alanları olarak düşünülebilir. Kapasiteyi verimli kullanma ihtiyacı ve uzun süren hazırlıklar, üretim planında parti büyüklüğü belirleme ile sıralama kararlarının beraberce verilmesini gerektirmektedir.

Problemin doğasının en genel ve gerçekçi yönlerini yansıtan durumlar için bir matematiksel model sunulmuş ve ilgili literatürde sıkça rastlanan varsayımların etki ve geçerliliği sorgulanmıştır. Bunun yanı sıra, problem alternatif bir formülasyon ile temsil edilmiş ve her iki formülasyon bir takım ilave eşitsizlikler yardımıyla iyileştirilmeye çalışılmıştır. Son olarak, ürünler arasında yapılabilecek geçişlerin sayısını kısıtlayan bir sezgisel yöntem geliştirilmiştir. Yapılan testlerin sonuçları sunulmuştur.

Anahtar Kelimeler: Bütünleşik Parti Büyüklüğü Belirleme ve Sıralama, Sıraya Bağlı Hazırlıklar, Geçerli Eşitsizlikler, Matematiksel Programlama

To my family

and

to the beloved memory of Seyhun Özbilen in particular

ACKNOWLEDGMENTS

I would like to thank my supervisor Asst. Prof. Dr. Haldun Süral sincerely for his guidance, suggestions and dedication throughout this thesis. He is one of the few people I know that have not lost a bit of their glittering enthusiasm about their job. Working with him was an interesting challenge, made up of hard work and fun at the same time. Although I must confess that I felt like playing the devil's advocate against him at times, I shall always recall the pleasure of setting out on a track with new ideas and loads of excitement.

I strongly felt the ceaseless faith and support of my family along the way, and I am especially obliged for their patient endurance against my sullen mood at home during the difficult stages of this study. I deem myself fortunate to have their presence and love. Special thanks go to my brother Akın Koçlar for making me discover my thesis theme song which accompanied me at times of trouble.

I am greatly indebted to my former roommates Melih Özlen and Bora Kat as well as to Fatma Kılınç for their help and guidance about the various problems I encountered during the study, in addition to their invaluable friendship. I would also like to thank Özlem Pınar for the companionship and similar dreams we shared, and to Münire Toksöz for her everlasting support even from way overseas.

I owe thanks to Prof. Dr. Murat Köksalan for his suggestions on part of this study and to Prof. Dr. Meral Azizoğlu for her morale support that reestablished my confidence many times throughout this thesis. Although their names do not appear individually here, I am grateful to all my professors, friends and colleagues at the department for their consideration and cheerful presence which created a genial atmosphere that I will always feel privileged to be part of.

Last but not the least, completion of this thesis would not be possible without Engin Volkan who went through almost every stage of this study together with me. When I look back on this journey, I see traces of your support in every possible way; with your encouragement, affection, consolation and most notably, with all those words of wisdom and patience that you attributed to Yoda to cheer me up. I find it needless to thank you here and had I tried, I would not have known how to, either.

"Soon will I rest. Yes, forever sleep. Earned it, I have."
Yoda

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CHAPTER 1

INTRODUCTION

This study is concerned with the General Lot Sizing and Scheduling Problem (GLSP) involving sequence dependent changeovers between items. The problem is related with determining the lot sizes and production sequences of multiple items with dynamic demand on a single facility subject to capacity constraints. The effect of changeovers is significant and is observed both in terms of costs and times. The prominent aspect about this problem is that changeovers are sequence dependent and the capacity is restrictive. Therefore, lot sizing and sequencing decisions need to be made simultaneously, unlike traditional approaches to production planning where these decisions are made sequentially.

Our aim in this thesis is to study the GLSP in its more generic setting with as few restricting assumptions as possible and develop solution methods that yield satisfactory results in reasonable times.

Motivation for the Study

In traditional production planning approaches, there is a hierarchy of decisions. Lot sizing decisions are made without considering production sequences, which are only handled in a secondary step based on the previously given lot sizing decisions. However, these kinds of approaches may not be suitable under certain conditions where the sequencing decisions cannot simply be disregarded until short term operational plans. The capacity may be restrictive with a few highly utilized lines, the setups or changeovers may be long and costly, as well as sequence-dependent. Process industries are very suitable environments for this setting and they are also the main inspiration for this study.

Process industries are characterized by special features that distinguish them from discrete part manufacturing systems, requiring different production planning and control approaches. The major concern in these environments is capacity availability and capacity utilization due to high capital intensity. Moreover, setups or changeovers are predominant in these environments, and are usually dependent upon the sequence in which items are produced in the facility. Despite the existence of a vast literature on the production planning approaches and principles specifically designed for discrete manufacturing systems, only limited amount of research has been carried out for process industry environments. Earlier studies on the process industries generally concentrated on the distinguishing characteristics of these environments from discrete manufacturing and involved a few real life applications with case-specific formulations and solution methods that were very limited in scope and in terms of applicability to other systems. Therefore, it is hard to talk about a consensus on the modeling assumptions, approaches and solution methods in this literature.

However, in the last decades, the number of studies dealing with integrated mathematical programming approaches for process industries started to increase. What is commonly agreed in these studies is the need to address capacity planning, lot sizing and sequencing issues in an integrated manner and study the interactions between them. In this line of thinking, the motivation for our study is to use an approach for simultaneous decision making for medium term planning in capacity-restrictive environments with predominant and sequence dependent changeovers.

Our perspective is mathematical programming oriented. Since we observed a gap in the field of related applications which were clear on their assumptions and capability, we primarily aim at developing a mathematical modeling formulation which is not only confined to a narrow range of systems, but is more generic in nature. We attempt to remove restrictive assumptions as much as possible and state our modeling framework openly and explicitly. This is the distinguishing aspect about our line-of-thinking, which makes our formulation a more practical alternative for representing real life situations compared with some earlier approaches.

With the need to develop a general model having the ability to represent different environments, we decided to use the most flexible formulation in large bucket, integrated lot sizing and scheduling models, which is the GLSP. In addition to the mathematical model, we have also relied on optimal seeking mathematical programming based approaches for the solution of the problem, due to their wider applicability compared with special-purpose approaches.

Problem Definition

The GLSP is an integrated lot sizing and scheduling model for dynamic and deterministic environments. The capacitated production facility is represented as a single-machine and the objective is to meet item demands without backlogging at minimal total cost.

Multiple items can be produced within a time period in the GLSP. The production lots of items are continuous, i.e., they can extend over to a new period without interruption and without the need for additional setups.

The prevalent feature of the GLSP is that the time periods within the planning horizon are considered to be composed of small positions to which items are assigned in order to be able to determine the production sequences. In this way, the number of positions within a time period determines the maximum number of items that may be produced in a period.

The property of continuous lots and the preservation of the setup state are especially important, because when combined with the two level time structure in the GLSP (featuring time periods and positions), they help represent solutions without making any approximations or aggregations due to bucketing, as is the case for some other lot sizing models.

The problem environment for our study contains predominant and sequence dependent changeover costs and times when the setup state of the facility changes. Our model is not only limited to the case where the assumption of triangle inequality holds for changeovers, which is commonly made in the related literature. In addition, there are minimum batch sizes and the possibility of using overtime.

As stated by Meyr (2002), the GLSP is known to be the broadest formulation among the single line models. This implies that it is the most general formulation with the flexibility to represent many other environments under slight modifications. However, it is also a highly complex model. Although our study is mainly formulation-oriented, it is for this reason that we also need to address the issues related with enhancing our formulation and developing reasonable approximations for its solution within the scope of this study.

Outline of the Chapters

The following chapters give a detailed account of our study on the GLSP.

In Chapter 2, we present a brief review of the literature related with our study. We focus on some of the important studies in the single-level, multi-item lot sizing literature and their basic features. Then, we introduce the main characteristics of process industries and discuss some of the fundamental issues regarding production planning pertaining to these environments. Finally, we examine some integrated mathematical applications that are comparable to our GLSP formulation.

In Chapter 3, we explain the structure of the GLSP in more detail and present its mathematical formulation. This formulation is based on previous GLSP models in the literature, however, it is partly modified to represent a wider variety of problem situations more realistically. In this chapter, we explicitly state the inherent assumptions in our formulation and also discuss how certain special cases are handled by our model. An equivalent alternative formulation of the GLSP based on the Transportation Problem is also presented at the end of the chapter.

Chapter 4 is devoted to our attempts at enhancing the GLSP formulations through the use of additional inequalities. The aim is to strengthen the LP relaxations and restrict the solution spaces to provide faster solutions. We propose a set of additional inequalities which are candidates for strong enhancements, and then present the results of some preliminary computational experiments conducted to select the best combination of these inequalities under different settings.

In Chapter 5 we present an analysis of the solution of the GLSP by using our enhanced formulation. The computational experiments involve testing the impact of problem size parameters and the level of minimum batch sizes. We also develop a heuristic approach, which we refer to as the k-Nearest heuristic, setting a limit on item changeovers. The performance of the heuristic is tested in comparison with the optimal solutions for several classes of problem instances.

Chapter 6 concludes the study with a summary of our major contributions and a few ideas on possible directions for future research.

CHAPTER 2

THE REVIEW OF RELATED LITERATURE

Multi-item capacitated lot sizing problems with deterministic dynamic demand have been the subject of innumerable studies in the mathematical programming literature. The major portion of the models developed within this context does not deal with sequencing decisions, which are only treated in a second step based on lot sizing decisions. Since our motivation is for developing an integrated model that is applicable in process industry type environments with highly utilized, expensive capacity lines and sequence dependent changeovers, we shall focus on integrated applications while reviewing the related lot sizing literature. The scope of production planning and sequencing is considered to be medium to short term decisions based on the master production schedule.

This chapter is organized as follows. In Section 2.1, we will present a brief introduction about single-machine, multi-item lot sizing problems in general, with reference to related review papers. Our emphasis will be on the distinction between small and large bucket models in this area. In Section 2.2, we will concentrate on specific aspects within this literature such as setup times, the use of overtime, modeling sequence dependency etc. Some commonly used strong formulations will also be mentioned within this section. In the remaining two sections, we will discuss the major characteristics of process industry type environments that are relevant to our problem and present some integrated studies that are comparable to the GLSP, while concentrating on their assumptions, basic modeling aspects and solution methods.

2.1 Single-Level, Multi-Item Lot Sizing Problems in General

The problem of determining the production lot sizes and their sequence in the facilities in the presence of scarce resources and the inherent tradeoffs has been a topic of interest for researchers in the fields of Operations Management and Operations Research for many decades. Over time, starting with the classical EOQ model, the lot sizing problem has extended to take many different forms with different properties and assumptions in a variety of environments, which have all strengthened its linkage to real world applications through increased richness and flexibility.

The crudest form of the dynamic single-level, multi-item lot sizing model determines the production quantity of different items on a single machine to meet customer demand on time (without backlogging) over multiple periods of time. The associated costs are those of inventory carriage and setup. Production costs are generally disregarded in many lot sizing problems, as they are fixed quantities in the objective function without any effect in optimization.

We refer to Drexl and Kimms (1997) and Gupta and Magnusson (2005) for reviews of single-level lot sizing and scheduling models and their basic assumptions.

The formulations for the lot sizing and scheduling problem can be mainly classified into two groups according to how time is treated. These groups are "small bucket (small time window)" and "large bucket (large time window)" models, respectively. In the former, the planning horizon is broken down into small intervals in which at most one item can be produced (i.e., a single mode of production). It needs to be stated that in small bucket models, there is an inherent sequencing decision. The most frequently studied small bucket model is the DLSP (Discrete Lot Sizing Problem). The DLSP does not allow the preservation of the setup state over idle periods and makes the restrictive assumption of "all-or-nothing" production, i.e., production of an item has to exhaust the full period capacity, if any (Fleischmann, 1990 and Salomon *et al.*, 1991). These two restrictions are removed in the CSLP (Continuous Setup Problem, sometimes referred to as the Product Cycling Problem), but again a single item may be produced within a time period (Karmarkar and Schrage, 1985).

The other class of problems, i.e., the large bucket models, feature larger time intervals where multiple items can be produced, for which the CLSP (Capacitated Lot Sizing Problem) is the most commonly studied. The classical CLSP only deals with lot sizing and it does not consider sequencing decisions within a period. As a result, setups cannot be carried over to the following periods and setups for items are incurred in every period where they are produced. The most frequently applied solution methods for the CLSP rely on Lagrangean relaxation approaches, different types of heuristics or reformulations and the addition of strong valid inequalities. Some reference studies for these approaches are by Barany *et al.* (1984), Thizy and Van Wassenhove (1985), Eppen and Martin (1987), Maes and Van Wassenhove (1988), Cattrysse *et al.* (1990) and Chen and Thizy (1990).

The GLSP is essentially a large bucket problem, since multiple items can be produced within a time period. However, its special structure involving positions (or microperiods) within time periods may be associated with a small bucket framework. While making the distinction between small and large bucket models, it should be kept in mind that as new features are incorporated into different types of formulations, differences between them in terms of modeling aspects start to decrease. We can provide many examples among the variants of the classical problems to justify this phenomenon. For instance, as it has been stated above, the restrictive "all-or-nothing" assumption in the classical DLSP is relaxed in the CSLP where the setup state is also preserved over idle periods. Another example is the DLSPSD (DLSP with sequence dependent setup costs) developed by Fleischmann (1994). The corresponding model involves a two level structure for time periods where the micro periods represent the classical small bucket intervals while the macro periods are used to account for the demand and inventory information, an approach that is very similar to that of the GLSP. However, it should also be noted that variant models with extended features usually suffer from increased complexity and when applied to these variants, the solution procedures developed for the original versions lose their computational advantages. For instance, "fast solution algorithms were developed for the classical DLSP, while computational experiments with large multi-item CSLP problems are rather disappointing" (Salomon *et al.*, 1991).

2.2 Special Features of Single-level, Multi-Item Lot Sizing Models

Before proceeding further, it may be worthwhile to shed some light on the terminology related with setups that we will frequently use in the remainder of this study.

A setup is an indication of the machine adjustments, preparations, cleaning activities etc. that are required before production of items. Setups may be incorporated into the models via cost penalties in the objective function, usually representing the opportunity cost of lost production, and/or setup times (production down time) consuming part of the capacity.

There are other terms related with setups used in the literature such as startups, changeovers, switch-offs etc. A startup is said to take place if a machine starts producing or switches between the production of two different items (Constantino, 1996). This implies that the facility is setup to produce an item for which it was not setup in the previous period. Alternatively, there are switch-off (or tear-down) variables, which indicate that the facility is setup to produce an item, while it is not setup for the same item in the following period (Belvaux and Wolsey, 2001). Similar to a startup, a changeover (or switchover) may be defined as a change of setup status, or alternatively as the switch-off of an item, followed immediately by the startup of another (Belvaux and Wolsey, 2001). The traditional distinction between a startup and a changeover is that the latter is usually sequence dependent (Wolsey, 1997), although there may be some alternative uses (Magnanti and Vachani, 1990 and Magnanti and Sastry, 2002). In many industries, changeover times are considered to comprise of both the actual time required to carry out the changeover (eg. for cleaning or tool adjustments) and the adjustment time until the new product lot begins to be produced at full line speed and efficiency.

In the following subsections, a brief review of some special components or aspects of single-level, multi-item lot sizing models that are relevant to our study are presented.

2.2.1 The Use of Overtime

The use of additional capacity in the form of overtime generally serves the purpose of a buffer against demand variability. An example for this kind of use for overtime is the study by Diaby *et al.* (1992), dealing with a generalized version of the CLSP which involves limited overtime as well as regular time. Anderson and Cheah (1993) include overtime options in their version of the CLSP with setup times and minimum batch sizes. They discuss that incorporating the possibility of overtime improves the feasibility problem and this benefit is especially crucial in the presence of setup times and minimum batch sizes in capacitated environments.

On the other hand, in some studies, overtime has been used to find initial feasible solutions or to check feasibility at intermediate steps of the solution procedure. In order words, unlimited use of overtime is allowed at the beginning and in each pass, the solutions are forced to eliminate it. These types of approaches have been devised by Trigeiro *et al.* (1989) and Gupta and Magnusson (2005).

It needs to be said that the use of overtime is not common in environments where capacity is strictly defined (eg. process industries), since the facilities are highly capital intensive and utilized to the limit and the addition of extra capacity to compensate for production losses is usually not possible (Smith-Daniels and Smith-Daniels, 1986).

2.2.2 Models with Setup Times

The incorporation of setup times increases the complexity of lot sizing problems to a great extent. It has been shown that the feasibility problem for the CLSP with non-zero setup times is NP-complete (Maes *et al.*, 1991). Because of this, many studies in the literature simply ignored setup times with the purpose of simplifying their models and reflected the effect of setups only through cost penalties in the objective function. In this regard, setup times are either assumed to be performed during off-shift periods or that they are insignificant compared with the corresponding setup costs. The latter perspective is the product of the underlying idea that setup costs may be used as surrogates for setup times (Kuik *et al.*, 1994). However, it is evident that "the problem with setup times is not just an extension of the problem without setup times", as it has been pointed out by Diaby *et al.* (1992). Especially in environments where production lots are continuous and setup times take up an important portion of the capacity, setup times need to be taken into account explicitly in the models for developing feasible production plans. Moreover, in the case where setup costs only represent the opportunity cost of not using the resources for value-added activity, and no other direct costs, the correct capacity requirements will not be evaluated in the absence of setup times and the true opportunity costs (i.e., setup costs) cannot be determined (Süral, 1996).

Manne (1958) studied the capacitated lot sizing problem with positive setup costs and times as well as overtime by using a set partitioning approach.

Sequence independent setup costs and times were modeled by Trigeiro *et al.* (1989) and the corresponding CLSP model was solved by using Lagrangean relaxation of the capacity constraints. Another study on the CLSP with sequence independent setup times is by Diaby *et al.* (1992), which was also solved by Lagrangean relaxation combined with Branch-and-Bound.

Note that CLSP models in their classical form assume that setups are not carried over between periods, i.e., the setup state cannot be preserved. Gopalakrishnan *et al.* (1995) address the need to model setup carryovers within a large bucket CLSP context for finding feasible solutions when setup times are significant. They assume sequence independent and constant setup times and their enhanced model keeps information regarding the first and last items in each period so that the setup state can be preserved between periods and over idle periods. This framework was later extended by Gupta and Magnusson (2005) to include sequence dependent setup costs, which require the model to keep information about the sequence of items within a period as well. Details about this study can be found in Section 2.4.

Salomon *et al*. (1997) study the small bucket DLSP with sequence dependent setup costs and times. Their model assumes that triangle inequality holds for setup times, production has an all-or-nothing nature and most importantly, that the setup times take up an integral number of time periods. The discrete nature of the problem facilitates the construction of a TSP network with time windows. Among the results obtained as a result of the experimentation is the fact that the inclusion of setup times facilitates the solution of the problem in terms of CPU time as the number of nodes in the network decrease and the time windows are tightened. However, it should be noted that prohibition of setup times to be fractions of period capacity is one of the most serious restrictions posed by small bucket models.

Other studies involving setup times (eg. Meyr, 2000, Smith-Daniels and Smith-Daniels, 1986) will be discussed in detail in the following sections.

2.2.3 Models Involving Sequence Dependency

The classical CLSP does not involve sequencing decisions, therefore it cannot model sequence dependent setups. A great body of work within the CLSP literature deals with setups that are only dependent on products and not the sequences. It is for this reason that the first attempts at incorporating sequence dependency into the models appears to start within the context of small bucket models. Although sequence dependency is encountered in many real life applications, there are not many studies in this area. As it has been pointed out by Fleischmann (1994) and Salomon *et al.* (1997), the primary reason for this is the extra difficulty caused by the interdependency of items as a result of sequence dependency, which prohibits the development of effective solution procedures.

One of the earliest studies with sequence dependent setup costs standing out in this area is the DLSPSD (DLSP with sequence dependent setup costs) by Fleischmann (1994). This model features an additional macro period structure but it cannot preserve the setup state over idle periods. Moreover, the assumption of triangle inequality need not be satisfied for setup costs. The model was reformulated as a TSP with time windows (TSPTW) and solved using Lagrangean relaxation. The TSPTW approach has also been applied to the DLSP with sequence dependent setup costs and times by Salomon *et al*. (1997) and the problem was solved to optimality for small to medium sized instances using DP.

Although the case with sequence dependent setups has been treated by other small time-bucket models (DLSP), no extensive research has been made for this setting using large time-bucket models. Some exceptions are noted below. The details regarding these studies can be found in Section 2.4.

Prabhakar (1974) produced one of the early studies for integrated lot sizing and sequencing with sequence dependent setup costs and times in the chemical industry, using a static MIP model for several parallel reactors.

Smith-Daniels and Smith-Daniels (1986) addressed sequence dependency in a different framework for items and families. Later, Haase (1996) modeled sequence dependent setup costs in his large bucket model CLSD for simultaneous lot sizing and scheduling. He pointed out that especially in the presence of an item-family structure in the production environment, setups between different families may have a substantial impact on the solution which has to be considered in the total cost.

In their large bucket model, Haase and Kimms (2000) considered sequence dependent setup costs and sequence dependent setup times, where the former was a function of the latter. Hill *et al.* (2000) considered sequence dependent changeover times in process industry type environments and their impact on MPS generation within a continuous time representation. Gupta and Magnusson (2005) used sequence dependent setup costs coupled with fixed setup times within an enhanced CLSP context.

Finally, for the sequence dependent GLSP, there are relevant studies by Fleischmann and Meyr (1997) and Meyr (2000, 2002).

2.2.4 Strong Formulations for Lot Sizing Models

The traditional formulations for the multi-item, capacitated lot sizing problem make use of classical production and inventory variables. However, it has been shown that the problems can be modeled using stronger equivalent formulations based on network representations.

Strength of a formulation is associated with the objective function value of its LP relaxation. In this regard, Denizel and Süral (2005) consider strong formulations for the CLSP with sequence independent setup times and no setup costs. They emphasize that coming up with the right formulation is central to obtaining good quality solutions for computationally challenging problems such as the CLSP with setup times. Among the strong formulations they address are the Transportation Problem (TP) and the Shortest Path

Problem (SP), where the latter is based on the reformulation by Eppen and Martin (1987). Their computational results show that the two strong formulations with the same LP relaxation values both improve the original formulations significantly and that the SP formulation performs slightly better than the TP.

As a result of experimentation, De Matta and Guignard (1994b) have shown that TP formulations featuring disaggregated production variables instead of the standard production and inventory variables yield stronger lower bounds for linear programming or Lagrangean relaxations.

The studies by Karmarkar and Schrage (1985), Wolsey (1989), Diaby *et al.* (1992), De Matta and Guignard (1994a, 1994b, 1995), and Stadtler (1996 and 2003) are only a few more references in the lot sizing literature that make use of the Transportation formulation.

One important drawback of these formulations is that the number of production variables and setup constraints grow quadratically with the number of time periods (Stadtler, 2003).

2.3 The Process Industry

Traditional production planning approaches focus on discrete parts manufacturing industries. However, process industry environments feature special aspects that require different perspectives and methods to solving production planning problems. Since we regard process industries as suitable application areas for the GLSP, we deemed it necessary to present a brief account of this type of environments within our review of the literature. This section is devoted to the examination of process industries, with an emphasis on the differences between them and discrete parts manufacturing systems in particular.

We refer to Crama *et al.* (2001), Fransoo and Rutten (1994) and Ashayeri *et al.* (1996) for the reviews of process industries and their prevalent characteristics.

The term "process industry" refers to environments featuring repetitive operations that involve physical and chemical reactions on non-discrete (bulk) materials. The APICS dictionary gives the following definition for such environments (Fransoo and Rutten, 1994):

> Process industries are businesses that add value to materials by mixing, separating, forming, or chemical reactions. Processes may be either continuous or batch and generally require rigid process control and high capital investment.

Chemical, pharmaceutical, steel, petroleum, food, beverage, paper, glass and semiconductor industries are some of the generally cited examples of process industries. The processes within these industries are usually high-volume and capital intensive. They are carried out in expensive facilities with a few specialized lines requiring high utilization levels. In addition to convergent material flows, divergent flows can also be observed in process industries, which implies that the processes may yield by-products and co-products. Consequently, in place of the classical “Bills of Materials” (BOM) used in MRP systems in discrete manufacturing, planning in process industries generally involves the use of “recipes” (formulas or ingredient lists). Flexible recipes can be used in processes with variable raw material quality, requiring approaches similar to those used for blending problems.

Process industries are usually classified between two extreme ends: flow (continuous) process industries and batch (mix) process industries. The former is usually characterized by heavy industries with high volume production and high changeover times. In these environments, there is usually no product differentiation in the process until containerization, all items follow the same routing and WIP constitutes the material on the entire production line, which may be considered as a single machine. Batch process industries, on the other hand, are closer in nature to the discrete manufacturing systems, in that their processes are more flexible and diversified.

However, it needs to be said that the traditional features that are usually associated with process industries, such as inflexible processes, commodity type products with limited variety, MTS production etc. are subject to a shift towards more customized production strategies for dynamic product mixes with enriched diversity. Therefore a strict one-dimensional classification of these environments may no longer be sufficient (Crama *et al.*, 2001). A more detailed classification of process industries was presented by Dennis and Meredith (2000) based on different characteristics of the P&IM (Production and Inventory Management) systems used.

Regarding production planning and control in process industries, the following excerpt from Günther and Van Beek (2003) perfectly highlights the issues frequently encountered in these environments that distinguish them from discrete manufacturing:

> Process industries show a considerably increased complexity compared to discrete parts manufacturing. For instance, the complexity of scheduling chemical processes is determined by such factors as batch size constraints, shared intermediates, flexible proportions of input and output goods, production of by-products, limited predictability of processing times and yields, mixing and blending processes, carrying

> out processes without interruption, use of multi-purpose equipment, sequence and usage dependent cleaning operations, finite intermediate storage and use of product specific storage devices, cyclical material flows, no-wait production of certain types of products, usage of secondary resources, such as energy, steam, or cooling water, and complex packaging and filling operations. Very often, time and cost intensive cleaning procedures as well as the necessity of detailed quality control after each batch are the major motivations for operating the production system in campaign mode, i.e. the equipment units required by a particular type of product or process are set up according to the corresponding recipe. When set-up and cleaning efforts are significant, e.g. in the production of specialty chemicals, only a few campaigns of a specific product, possibly, merely a single campaign are carried out each year. (…) As a consequence, most of the scheduling approaches developed for discrete parts manufacturing are hardly applicable. Hence, despite the analogy of the fundamental planning and scheduling problems, there are a number of major issues which have to be reflected by the mathematical models and solution algorithms employed.

The main point that needs to be emphasized in this discussion is that "planning and scheduling issues arising in the process industry are tightly intertwined." (Crama *et al*., 2001). That is why, unlike the disaggregated approaches in discrete manufacturing, medium term planning at the MPS level in process industries should also consider process details such as production sequences, changeovers and product routings etc., as well as deal with issues in forecasting, demand management and logistics in a proactive and integrated manner (Novitsky, 1984).

The point that is of concern for this study is the need to integrate sequencing decisions with production plans in the medium and short term. Since high utilization is required in process industries, capacity planning is critical and it should be considered at the MPS level before raw material constraints, an approach sometimes referred to as Capacity-Oriented Production Scheduling (CPS). The use of additional capacity in the form of overtime in order to compensate for the production loss is usually not possible, as in traditional approaches. Moreover, changeovers consume an important part of the capacity. In fact their dominance is so important that they should even be considered at the demand management level, since changeovers impact cycle times and thus the selection of the most profitable products (Fransoo, 1992). For all these reasons, commonly sequence dependent and predominant setups and changeovers have to be treated as a crucial part of the production plans (De Matta and Guignard, 1994a, Hill *et al.*, 2000).

Finally, we give a few examples for the solution methods used for process industry environments in the literature. While there are many papers describing the characteristics of process industries and their differences in comparison with discrete parts manufacturing, there are a limited number of studies that deal with generic production planning problems in process industries.

Although there exist some other approaches for tackling production planning and control problems in process industries, such spreadsheets, matrix data structures, network based frameworks, flow process scheduling techniques for recipe and raw material management, simulation and statistical methods etc., our focus in this study is on the mathematical programming applications in process industries. It has to be noted that most of the mathematical applications in this field are process-specific with special assumptions and restrictions.

When process industry applications in the literature are examined, one can find a big portion dealing with blending type of problems to determine the best recipes, in chemical and food processing industries. These models usually include special process restrictions such as nutritional requirements, raw material availability and variable quality, perishability etc. (*e.g.* Jensson, 1988, Munford, 1989, Rutten, 1993, Ashayeri *et al.*, 1994 and Al-Shammari and Dawood, 1997). Another class of applications addresses bulk storage problems in multi-stage environments with special storage facilities (eg. pipes, conveyors, tanks, silos etc.), usually featuring special conditioning restrictions, waiting times or minimum batch sizes etc. (*e.g.* Artiba and Riane, 1998 and Synder and Ibrahim, 1996). Finally, there are hierarchical models dealing with item and family relationships, disaggregation schemes, MPS generation over rolling horizons and integrated logistics decisions etc. These issues are usually studied by the use of mathematical models and are solved by optimization or disaggregation approaches. (*e.g.* Oliff and Burch, 1985, Rutten, 1993, Allen and Schuster, 1994 and Venkataraman and Nathan, 1994). All these studies may be regarded as small case study applications which are motivated by real life problems and they are usually dedicated to the characteristics of specific systems only, which is outside our scope.

In the next section, we will present and discuss in more detail some more examples for production planning applications in the process industry, with an emphasis on more generic mathematical applications for integrated decision making.

2.4 Mathematical Applications for Integrated Lot Sizing and Sequencing

Since our main motivation is to develop an integrated production planning model for capacitated environments with sequence dependent changeovers, we particularly concentrated on mathematical modeling applications in the process industries and the multi-item, single-machine lot sizing and sequencing literature in general in order to find models that are comparable to the GLSP. Below we present more detailed information on a few such studies in chronological order. The common point between these studies is that they are all integrated mathematical modeling approaches although they make different fundamental assumptions, approximations and use different solution methods. The justification for using mathematical programming approaches is that they are generic and versatile tools that can be applied to a great variety of problems, unlike only problem-specific methods with limited applicability (Crama *et al*., 2001)

One of the early studies in the area of integrating lot sizing and sequencing is by Prabhakar (1974), which considers simultaneous optimization of lot sizing and sequencing decisions with special constraints for a static environment in the chemical industry.

Smith-Daniels and Smith-Daniels (1986) describe a practical application emphasizing item-family structures in a two stage process industry framework. They work on the Packaging Line Scheduling Problem (a type of Joint Replenishment Problem), which is common in many process industry type applications, where batches of product are first processed in the main production line and then are packaged into various sizes of containers. Their model incorporates two types of setups: major setups between families and minor setups between the items in the same family and this model has a flexible nature in that it can consider a product as a major setup component and a package size as a minor setup component, and vice versa. Moreover, it is assumed that major setups (changeovers) are carried out in off-shift periods, i.e., they do not consume capacity in the form of setup time and that they are sequence independent, as opposed to their minor counterparts, which incur sequence dependent setup time consumption. The formulation treats the sequencing of items within families in each period as independent TSP's with explicit subtour elimination constraints. Other important assumptions are that backlogging is allowed and only one family is producible per period, which is restrictive for the feasibility of the problem. The MIP solution results are not promising except for very small problems and only in the absence of sequence dependencies, which highlights the level of complexity of this integrated problem.

Selen and Heuts (1990) address chemical process industry environments with sequence dependent switchover times, fixed batch sizes and extra plant restrictions such as storage capacities. They use a period-by-period heuristic approach (referred to as HS) for operational level planning. Their lot sizing and sequencing heuristic is based on the idea of modifying an initially found feasible solution by considering possible and potentially profitable item shifts between periods. Sequencing decisions are made based on the set of items assigned to periods and the best sequences are determined by using TSP approaches. Heuts *et al.* (1992) test the performance of HS heuristic against a similar improvement heuristic in the same process industry setting by using simulation experiments under rolling horizons.

De Matta and Guignard (1994a) highlight the importance of studying the effects of production loss during changeovers for dynamic production scheduling in process industries. They develop a small bucket model with sequence dependent changeover costs and times for a tile manufacturing company with multiple, non-identical processors and solve it using Lagrangean based procedures. The same authors later devise a similar approach to model the packaging operations in a pharmaceutical company, this time with no changeover times but with sequence dependent changeover costs in a rolling horizon environment (De Matta and Guignard, 1995).

Haase (1996) devises the formulation CLSD (Capacitated Lot Sizing Problem with Sequence Dependent Setup Costs), which is an integrated lot sizing and scheduling model with sequence dependent setup costs and continuous lot sizes. This model is similar in nature to the GLSP; however, it is a more restricted formulation with additional assumptions compared with it. Among these assumptions is the possibility of producing an item at most once within a period and that of setup costs satisfying triangle inequality. This model explicitly represents sequencing decisions in the form of a TSP with assignment and subtour elimination constraints. A backward oriented heuristic with priority based ordering of items and local search are used as solution methods with fast and satisfactory results.

Haase and Kimms (2000) present an application of production planning for a bottleneck process in high technology machine manufacturing where setup costs and times are both sequence dependent, and thus lot sizing and sequencing must be considered simultaneously. Their large bucket model, the LSPSD (Lot Sizing and Scheduling Problem with Sequence Dependent Setup Costs and Times), is different from most of the models in the related literature in that all efficient (undominated) production sequences are generated by solving TSP's prior to solving the main model and are used as input parameters. Moreover, triangle inequality is assumed to hold and zero-switch property applies for new production lots, i.e., production for an item in a period is only possible provided that the corresponding incoming inventory is zero, which is a rather restrictive assumption for capacitated environments with setup times. The study also concentrates on rescheduling opportunities through a rolling horizon and provides fast solutions using specially designed Branch-and-Bound procedures.

Hill *et al.* (2000) address the need to take capacity limitations and sequencing decisions into consideration during the preparation of the MPS in process industries. They propose a two-level scheme in which the MPS is exploded at the bottleneck stage and adjusted for capacity and sequence dependent changeover times via the use of a sequencing heuristic. Their approach adopts a continuous time representation and resembles the Job Scheduling Problem with setups. Their experiments indicate two important results; the first is that the two-level MPS with sequencing considerations results in lower changeover times and shortages, and the second is that the performance of this approach is not sensitive to changes in the minimum batch sizes. Later, the same authors use similar heuristic approaches to study the problem of revising the MPS in sequence dependent process industries and they present the results of a full factorial design which tests the effect of several factors such as changeover time variation, time between orders, MPS method, replanning frequency etc. (Hill *et al.*, 2003)

Gupta and Magnusson (2005) extend the idea of Gopalakrishnan *et al.* (1995) for the CLSP with setup carryover (i.e., preservation of the setup state) to include sequence dependent setup costs and fixed setup times. What this implies is that sequencing information needs to be kept within the large bucket CLSP context, which is uncommon in the literature. Their MIP formulation allows the possibility of setup carryovers and empty setups (i.e., setups which are not followed by immediate production in the corresponding period) through the use of different classes of binary variables. They assume that triangle inequality holds for setup costs and thus each item can be produced at most once in a period. Sequencing decisions can be associated with the TSP in this model and subtour elimination restrictions are also included. The authors propose a row aggregation scheme for lower bounding and a greedy heuristic with rather poor results.

Finally, we review the studies on our problem, the GLSP. It can be said that most large bucket models with integrated decisions and sequence dependency mentioned up to now assume that triangle inequality holds and thus, each item can be produced at most once in a period. This assumption is relaxed in GLSP models, which allow an item to be produced as many times as possible within a period as long as it is within the limit of the number of positions in that period.

Regarding GLSP, there are a few related studies in the literature, dealing with several variants of the problem. For instance, in their review paper, Drexl and Kimms

(1997) discuss a simpler version of the GLSP without sequence dependency or minimum batch sizes. Later, Fleischmann and Meyr (1997) present two different formulations for the GLSP with sequence dependent setup costs (i.e., changeover costs) and minimum batch sizes, namely the GLSP-CS and GLSP-LS. The distinction between the two formulations is that the former assumes the setup state is conserved after idle periods and the latter assumes it is lost. In this study, the associations between the GLSP and other lot sizing models such as the DLSP, CLSP, CSLP etc. are clearly explained and the justification for the term "General" is made by showing that the time structure in the GLSP may be used to represent all other lot sizing models under minor restrictions. This study also shows that the GLSP with non-zero minimum batch sizes is NP-complete. In a technical note, Koçlar and Süral (2005) have proposed an extension of the GLSP-CS formulation, related with minimum batch size splitting between periods (See Chapter 3 for details).

The GLSP-CS formulation, which is more general, is later extended to include sequence dependent changeover times (Meyr, 2000) in a formulation referred to as the GLSPST. Finally, non-identical parallel facilities were modeled within the GLSP framework with sequence dependent changeover costs and times (GLSPPL) (Meyr, 2002). Apart from these straightforward variations, the model formulations in all the three studies above are essentially the same. For solution methods, local search type heuristics (threshold accepting or simulated annealing) were employed to fix setup patterns. Fleischmann and Meyr (1997) also solve the remaining lot sizing problem heuristically using backward-processing techniques; but later, Meyr (2000, 2002) combines local search with dual reoptimization for the remaining lot sizing problem to increase the quality of the solutions and to avoid starting the solution each time from scratch.

Finally, Clark and Clark (2000) study the GLSP with sequence dependent setup times but no setup costs in environments with parallel facilities. Although they do not refer to their formulation as GLSP, their model contains the structure of positions within periods and a modeling framework very similar to the GLSP. However, their model allows backlogging, assumes that triangle inequality holds and includes no minimum batch sizes, differently from the GLSPPL (Meyr, 2002), which is another GLSP study for parallel facilities. An important contribution of this paper is through its examination of the rolling horizon solution of the GLSP. In their experiments, the authors develop approximate mathematical models by relaxing part of the constraints pertaining to time periods later in the horizon. However, their results show that the computational requirements of even the approximate models soon become prohibitive as problem size increases in terms of the number of items and periods as well as the number of positions within a period, which is a very important experimental setting. Thus, they address the need to use efficient heuristic or metaheuristic approaches.

CHAPTER 3

MATHEMATICAL FORMULATION OF THE GLSP

In this chapter, we present an introduction of the GLSP and its basic assumptions, followed by the verbal and mathematical models, and a brief account of its elements and properties. We shed light on some special cases handled by the model and finally present an equivalent alternative formulation based on the Transportation Problem.

The GLSP features integrated production planning for capacitated single facility environments with multiple items. It seeks to determine the sequence and quantity of production lots in order to meet deterministic and dynamic demand through the planning horizon without backlogging and at minimal total cost (which comprises changeover, production, inventory holding and overtime costs). The GLSP is essentially a large bucket model and it assumes continuous production lots. Our version of the GLSP also contains sequence dependent changeover costs and times and minimum batch sizes.

The structure of the GLSP consists of two time levels: time periods (macro-periods) and positions (micro-periods). As expressed compactly by Fleischmann and Meyr (1997), the former relates to the external system dynamics such as demand, inventory holding costs etc. whereas the latter represents internal dynamics of the system, such as the size and positioning of production lots, the setup status of the facility, etc. Each time period contains a fixed number of positions which are non-overlapping and in sequential order. One can visualize the two-level structure as in Figure 3.1 below.

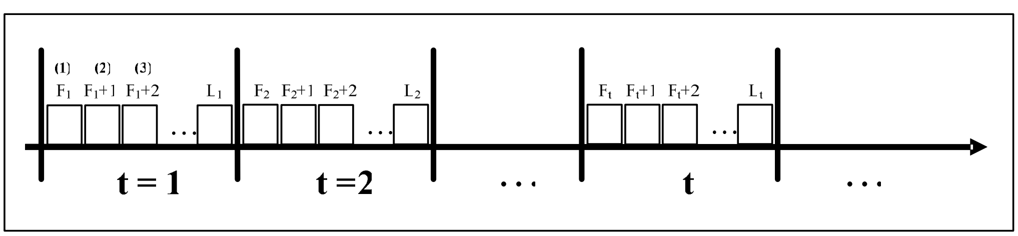


Figure 3.1 The Framework for Time Periods and Positions in the GLSP

In this figure, the small boxes represent positions and the hedges enclosing these positions represent time periods, where F_t (L_t) denotes the index number of the first (last) position in period t. The positions are ordered in a continuous sequence until the end of the planning horizon, which implies that the position following the last position of a given period is the first position of the next period, i.e., $L_t + 1 = F_{t+1}$.

Each position is reserved for the production of at most one item and thus a complete series of item-position assignments corresponds to a sequence of items to be produced. As a matter of fact, the positions may be regarded as micro-periods with variable lengths within fixed-length time periods. The length of a position depends on the size of the production lot assigned to it. Within this context, a production lot may be defined as "a sequence of consecutive positions over which the same item is produced" (Meyr, 2000). The basic assumption of the GLSP is that of continuous production lots, which implies that a lot may continue over several positions and several macro-time periods as well without interruption, i.e., without the need for further setups. This suggests that the need for setups (or changeovers) only arises when the facility switches to a different item. The setup state is also preserved over the idle positions that may remain in a period, where an idle position refers to a position with an assigned production quantity of zero.

As positions correspond to the production of a single item, the number of positions within a period (L_t-F_t+1) is an indication of the maximum number of production changeovers possible in that period. This value has to be determined in advance for each period and input to the model as a parameter.

Our GLSP formulation is mainly adopted from several similar studies in the literature (See Section 2.4). Among them are the GLSP-CS formulation by Fleischmann and Meyr (1997), which assumes that setup state is conserved over idle periods but does not involve setup times, and the GLSPST (GLSP with sequence dependent setup times) by Meyr (2000). However, compared with these earlier formulations, several aspects stand out as distinctive features of our formulation, which are briefly listed below:

- The existence of sequence dependent changeover times in addition to sequence dependent changeover costs (Although sequence dependent setup times were modeled by Meyr (2000) in GLSPST, they were only tested in sequence independent form during the computational experiments.)

- Enhanced ability to represent special cases related with minimum batch sizes more realistically in our formulation, by allowing minimum batch sizes to be split between periods (This point will be clarified later in this chapter.)
- Handling of item zero (the initial setup state) explicitly in the formulation and examination of special related cases (In previous GLSP models, there is no explicit information about how the initial setup state is incorporated in the formulations.)
- The incorporation of production costs in the objective function
- The possibility of resorting to overtime

For the last point, it may be argued that short time capacity expansions such as overtime are not used frequently in capital intensive environments such as process industries (Section 2.2.1). However, our primary motivation for incorporating the overtime option with considerably high costs was to come up with the most generic formulation for the problem in order to be able to cover possible variants of the problem and also to facilitate the process of finding feasible solutions.

One of the major strengths of the GLSP formulation is that it is the most general formulation for the integrated lot sizing and sequencing problems, with the flexibility to model many additional issues such as minimum batch sizes, sequence dependency, setup state preservation etc. This generic model may condense down to other specialized variants under certain assumptions (e.g. If only one position is allowed within a period, the model may be modified to represent the DLSP). Moreover, this exact formulation has the advantage of being solvable by general commercial packages, unlike case-specific algorithms that are limited in capability and applicability, which is a point emphasized by Maes and Van Wassenhove (1988) in favour of mathematical programming approaches.

3.1 Verbal Description of the GLSP

Our version of the GLSP is an integrated single-level, single-machine, multi-item capacitated lot sizing and sequencing model with sequence dependent changeover costs and times. Below is a verbal description of the model in terms of its basic assumptions, objective, parameters and decision variables.

Basic Assumptions:

- Item demands are dynamic and deterministic.

- Changeover, inventory holding and production costs are time independent.
- Backlogging demand is not allowed.
- Production lots are continuous, i.e., they can range over period borders without requiring a new setup in the consecutive period.
- Minimum batch sizes may be split between consecutive positions in two different periods if capacity is restricted.
- The setup state is preserved over idle periods and setups can be carried from one period to the next.
- Changeovers cannot be split between periods, as is the case in almost all large-bucket lot sizing formulations in the literature. This means that a changeover has to be started and completed within the same time period.
- Production for an item may be scheduled in a period even though the incoming inventory is not zero, thus the so called "zero-switch property" does not hold in our model, unlike some capacitated problems which make this restricting assumption. For instance, Haase and Kimms (2000) assume this property for new production lots in a period.
- Triangle inequality does not necessarily hold for changeover costs and times.

Objective:

To minimize the total cost which comprises costs associated with inventory holding, changeovers, production and overtime.

Basic Decisions:

- The quantity of each item to be produced in each period
- The amount of inventory to be kept for each item at the end of each period
- The production sequence of the items to be produced in each period, thus the amount of capacity to be allocated to changeovers in each period
- The overtime requirement in a period, if any

Parameters:

- Number of items and number of periods that are taken into consideration in the production plan
- Demand requirement of each item through the planning horizon
- Capacity available for production and setups in each period

- Unit capacity requirement for the production of each item (time-independent)
- Changeover time associated with item transitions (sequence dependent but time independent)
- Cost terms related with inventory holding, production, changeovers and overtime (time independent), where changeover costs are sequence dependent.
- Minimum batch size of each item (time-independent)
- Initial inventory levels of all items
- Maximum number of items producible in a period (i.e., number of positions in each period)
- The initial setup state of the facility (i.e., item zero)

3.2 Mathematical Model for the GLSP

In this section, we shall present the indices, parameters and decision variables along with the mathematical representation of the GLSP formulation. As previously stated, our formulation is mainly based on the GLSPST (Meyr, 2000) apart from the modifications which are listed at the beginning of this chapter.

Following the nomenclature used by Stadtler (1996), the formulation below will be referred to as the "Inventory & Lot Sizing (I&L)" formulation throughout the rest of the text.

Indices:

i,j: Items, $1,...,N$

(Note that the item zero ($i=0$) denotes the initial setup state of the facility)

t: Time periods (macro-periods), $1,...,K$

n: Positions (micro-periods), $1,...,L_K$

τ_n: The period to which position n belongs (i.e., $t|\ F_t \leq n \leq L_t$)

Parameters:

C_t: Capacity in period t

CO_t: Cost of overtime in period t

CP_j: Unit cost of production for item j

d_{jt}: Demand for item j in period t

F_t (L_t): Index for the first (last) position in period t

h_j: Unit inventory holding cost for item j

I_{j0}: Initial inventory level of item j at the beginning of the planning horizon

m_j: Minimum batch size of item j

M_{jt}: A very big number

(It stands for the maximum quantity of item j producible in period t)

P_j: Unit processing time of item j

SC_{ij}: Changeover cost for the transition from item i to item j, $SC_{ii}= 0$

ST_{ij}: Changeover time for the transition from item i to item j, $ST_{ii}= 0$

γ_t : Proportion of maximum allowable overtime to capacity in period t

Decision Variables:

X_{jn}: Quantity of item j produced in position n

I_{jt}: Inventory of item j at the end of period t

O_t: Amount of overtime used in period t

W_{jn}: $\begin{cases} 1, \text{ If item } j \text{ is assigned to position } n \\ 0, \text{ Otherwise} \end{cases}$ *(Setup Variable)*

δ_{ijn} : $\begin{cases} 1, \text{ If there is a changeover from item } i \text{ in position } n\text{-}1 \text{ to item } j \text{ in the next position } (n) \\ 0, \text{ Otherwise} \end{cases}$ *(Changeover Variable)*

Note: Variable δ_{ijn} is defined over the following special domain:

$$\text{For } n = 1,\ i = 0 \text{ and } j = 0, \dots, N$$

$$\text{For } n = 2, \dots, L_K,\ i = 0, \dots, N \text{ and } j = \begin{cases} 0, \dots, N & \text{if } i = 0 \\ 1, \dots, N & \text{if } i > 0 \end{cases}$$

This implies that for the first position, only changeovers from item zero are defined. Moreover, in the remaining positions, changeovers from actual items (*i>0*) into item zero (*j=0*) are not defined and thus, they do not appear in any part of the formulation. Related explanations are provided following the mathematical formulation of I&L.

[I&L]

Minimize $$\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{n=1}^{L_K} SC_{ij}\delta_{ijn} + \sum_{j=1}^{N}\sum_{t=1}^{K} h_j I_{jt} + \sum_{j=1}^{N}\sum_{n=1}^{L_K} CP_j X_{jn} + \sum_{t=1}^{K} CO_t O_t \quad (3.1)$$

Subject to $$I_{jt} = I_{j(t-1)} + \sum_{n=F_t}^{L_t} X_{jn} - d_{jt} \qquad \forall t, j \qquad (3.2)$$

$$X_{jn} \le M_{j\tau_n} W_{jn} \qquad \forall j, n \qquad (3.3)$$

$$\sum_{j=1}^{N}\sum_{n=F_t}^{L_t} P_j X_{jn} + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{n=F_t}^{L_t} ST_{ij}\delta_{ijn} \le C_t + O_t \qquad \forall t \qquad (3.4)$$

$$X_{jn} \ge m_j (W_{jn} - W_{j(n-1)}) \qquad \forall j, t, n \ne L_t \qquad (3.5)$$

$$X_{jn} + X_{j(n+1)} \ge m_j (W_{jn} - W_{j(n-1)}) \qquad \forall j, t, n = L_t \qquad (3.5')$$

$$\sum_{j=0}^{N} W_{jn} = 1 \qquad \forall n \qquad (3.6)$$

$$\delta_{ijn} \ge W_{i(n-1)} + W_{jn} - 1 \qquad \forall i, j = 0, \ldots, N,\ n \qquad (3.7)$$

$$\sum_{i=0}^{N}\sum_{j=0}^{N} \delta_{ijn} = 1 \qquad \forall n \qquad (3.8)$$

$$\delta_{00n} = \sum_{j=0}^{N} \delta_{0j(n+1)} \qquad \forall n = 1, \ldots, (L_K - 1) \qquad (3.9)$$

$$O_t \le \gamma_t C_t \qquad \forall t \qquad (3.10)$$

$$W_{jn} \in (0,1) \qquad \forall j = 0, \ldots, N$$

All other variables are non-negative. (3.11)

where $$M_{jt} = \frac{(1+\gamma_t)C_t}{P_j} \qquad \forall t, j \qquad (3.12)$$

In addition to N items, the I&L formulation also involves a dummy item (item zero) with no demand, only bearing the purpose of indicating the initial setup state of the facility at the beginning of the planning horizon. Note that this setup state actually may be for one of the N items available in the product mix, or a special starting state featuring completely different changeover costs and times. The distinguishing aspect about item zero is that it can only be part of a one-way changeover, i.e., a changeover from item zero to a different item, but not vice versa. This is why changeover variables δ_{i0n} $(i \neq 0)$ are not defined as part of the formulation. The facility remains in the initial setup state until production is started for one of the items, which requires a changeover from item zero once and for all. Changeover costs and times from item zero to all other items are appropriately defined and the constraints in the formulation are built in such a way so as to take this aspect into account. A more detailed discussion about how item zero is incorporated in the model can be found in Section 3.4.

In I&L, the objective function (3.1) minimizes the changeover, inventory holding, production and overtime costs. Note that our formulation includes minimum batch sizes, and thus the total production quantity of an item throughout the planning horizon is not a previously known constant value (such as the total demand over the planning horizon). Hence, although production costs are constant over time, they may not be disregarded as in most lot sizing models. (Note that in previous GLSP formulations (Fleischmann and Meyr, 1997, Meyr, 2000 and 2002) production costs are disregarded despite the existence of minimum batch sizes, which may not be a sound assumption especially if the minimum batch sizes are considerably large.)

Constraint (3.2) ensures that demand in a period is satisfied from inventory or production with no possibility of backlogging. It should be noted that an item can be produced in more than one position within a period. Besides, the term $I_{j(t-1)}$ is replaced by the initial inventory parameter for the first period.

Constraint (3.3) establishes the link between production and setup variables, i.e., an item can only be produced in a position if it has been set up for it. Here, the upper bound on the production quantity of an item in a position (big M term) is taken to be the capacity of the corresponding time period, inclusive of overtime (3.12).

Constraint (3.4) expresses the capacity limitations in period t and ensures that the capacity consumed in a period for production and changeovers does not exceed the available regular capacity and overtime. This constraint also ensures that changeovers are completed within a single period without being split over period borders.

Constraint set (3.5) imposes a minimum batch size restriction upon the startup of a production lot, i.e., the lot size should be greater than the minimum batch quantity in the first position (or the first two positions in (3.5')) where the production lot of an item commences. A more detailed discussion related with minimum batch sizes is presented in Section 3.4.

Constraint (3.6) guarantees that a unique item is assigned to each and every position. Note that item zero can also be assigned to positions possibly at the beginning of the planning horizon, before changing over to one of the actual items.

Constraint (3.7) enforces the sequence dependent changeover variable to take the value of 1 for the transitions between the setup states of items in consecutive positions. (Note that the domain of the constraint has been arranged so that one-way changeovers from item zero are properly handled.) Changeover costs and times associated with the transition from an item to itself are defined to be 0. In addition, constraints (3.6) and (3.7) together assure the preservation of the setup state over idle positions, by forcing a changeover from an item to itself.

Constraint (3.8) is an auxiliary restriction which prevents the changeover variable from taking on values greater than 1. Note that although changeovers are represented by real variables, positive changeover costs together with constraint (3.7) enforce them to take binary values in any optimal solution. However, exceptions may occur in the case of transitions from an item to itself, incurring no changeover costs or times. This implies that in the absence of (3.8), a self-transition changeover variable might take on any value greater than 1 without changing the optimal solution value and without violating any of the constraints. Thus, constraint (3.8) only helps establish binary values for self-transition changeover variables in this formulation.

Constraint (3.9) prevents the model from manipulating the asymmetric nature of item zero transformations. The reasons for including this constraint will be clarified in Section 3.4 with examples.

Constraint (3.10) limits the maximum overtime used in a period to be a specific portion of the corresponding period's capacity.

Finally, constraint (3.11) represents the non-negativity restrictions.

A side remark needs to be made here regarding the production bound in constraint (3.3). In many traditional lot sizing models, the bounds on production quantities are established by taking both demand and capacity information into consideration, which would be translated to the following expression in the presence of minimum batch sizes in our case:

$$M_{jt} = \begin{cases} \min\left\{ \max\left\{ \sum_{s=t}^{K} d_{js}, m_j \right\}, \frac{(1+\gamma_t)C_t}{P_j} \right\} & \text{if } \sum_{s=t}^{K} d_{js} \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.13}$$

The bound above considers capacity, minimum batch sizes and the unsatisfied demand remaining in future periods, if any. However, the most general formulation should allow for the possibility of production for the sake of saving changeover costs or times, without necessarily satisfying any demand, especially, even if $d_{it} = 0$. In this case, it is impossible to limit the production of any item by any other bound but the period capacity. (Nevertheless, these bounds could be somewhat tightened in our alternative formulation, which will be discussed at the end of this chapter.)

The total number of variables in this formulation is $O(N^2L_K + 3NL_K + NK)$, out of which $(N+1)L_K$ are binary (L_K denotes the total number of positions in the problem). It should be mentioned that determining the number of positions within periods is an important challenge inherent in the GLSP formulation. While too few positions would severely restrict possible changeovers and increase costs (or even make a feasible solution impossible), too many unnecessary positions which are doomed to remain idle would immensely increase problem size. Thus, several different values can be tested to come up with the most satisfactory setting in a comprehensive study.

3.3 The Properties of the GLSP Solution

In this section, we list some of the properties of an expected GLSP solution with brief remarks in order to gain more insight about the working logic of the model.

- **Remaining Inventory:** At the end of the planning horizon (at period K), a positive amount of inventory can remain on hand for some items due to the existence of minimum batch sizes.

- **Minimum Batch Size Splitting:** Minimum batch sizes may be split between consecutive positions in two different periods, i.e., if a production lot extends over a period boundary, the minimum batch size restriction applies to the total quantity produced in the last position of the first period plus the first position of the next period (Constraint (3.5')). See Section 3.4.1 for details.

- **Empty setups:** "Empty setups" (setup state changes without immediate production, i.e., setups to be used for future periods, as modeled by Gupta and Magnusson, 2005) cannot be handled in our model in the presence of non-zero minimum batch sizes unless the minimum batch size restriction is split between two consecutive positions at the period border (the case described above).

- **Triangle Inequality – Minimum Batch Sizes:** Since triangle inequality does not necessarily hold for changeovers in the most general case under consideration, it may be economically more attractive (in terms of changeover costs and/or times) to change over from one item to another via a third product which does not require production during the period, what we refer to as a "dummy changeover". In the absence of minimum batch sizes, such auxiliary items may appear in the plan without being actually produced, the case referred to as "a setup state change without a product change" (Fleischmann and Meyr, 1997). This case may or may not be realistic, depending on the nature of the process modeled. For example, referring to the example stated by Fleischmann and Meyr (1997), some chemical industries feature "rinsing products" that cleanse the facility during changeovers, acting as "dummy items" with no production. Note that by enforcing production of at least a minimum quantity of all items for which the facility is setup, minimum batch sizes reduce the occurrence of this phenomenon to a certain extent, which is the primary reason for incorporating them as part of the formulation. However, the effect of minimum batch sizes depends on their quantity, i.e., if they are too small so that the savings associated with "dummy changeovers" exceed the costs of unnecessary production and extra inventories, "dummy changeovers" are unavoidable. We incorporate this possibility as a property of the GLSP solution and construct our mathematical formulation accordingly (e.g. Recall how production upper bounds are established in I&L). On the other hand, if triangle inequality is assumed to hold for all items, the occurrence of "dummy changeovers" is completely eliminated and minimum batch sizes may be set to zero unless they are determined by the actual production process.

- **Multiple production lots of an item in a period:** The GLSP does not enforce each item to be produced at most once in a period. Provided that triangle inequality holds for changeovers, splitting the production lot of an item and incurring an extra changeover would not be beneficial. However, in the more general case where triangle inequality does not hold, some dummy changeovers (as discussed above) may become more appealing, leading to additional item transitions and the production of multiple lots of the same item within the same period. This property renders the problem of establishing the number of positions in a period more challenging.

3.4 Special Cases Handled by the GLSP Formulation

In this section, we provide brief remarks about some of the more involved aspects of the formulation and illustrate several special cases with the help of examples.

3.4.1 Splitting Minimum Batch Sizes for Two Consecutive Periods

If the minimum batch size constraints in I&L (3.5) are reexamined, it can be seen that there is a slight modification for the last positions of periods in (3.5’). The following discussion explains the need for making this distinction, based on a technical note raised by Koçlar and Süral (2005). The note draws attention to a possible shortcoming of the original GLSP formulation by Fleischmann and Meyr (1997) regarding minimum batch sizes and suggests some modifications in order to model continuous lots more realistically.

The GLSP-CS formulation by Fleischmann and Meyr (1997) ensures that the production quantity of any item in the first position of a production lot is greater than the minimum batch quantity. However, this limitation may be unrealistic in the following situation on which no explicit assumptions have been posted in the article: Considering the case where an item’s production extends over to a new period, if the first period is restricted in terms of capacity such that production of the minimum batch quantity is not possible, this model inevitably leaves the available capacity in the first period idle and forces production to start in the second period, as the following small-scale example demonstrates.

Example: Consider a problem with 2 items, 2 periods and 2 positions per period where the capacity per period is 2 units and changeover times are negligible. Let the other data be as shown in Table 3.1. The optimal solution of the problem is presented in Figure 3.2.

Table 3.1 Problem data for the Minimum Batch Size Splitting Example

	m_j	P_j	h_j	d_{j1}	d_{j2}
Item 1 (j = 1)	1	1	1	1	0
Item 2 (j = 2)	2	1	1	0	3

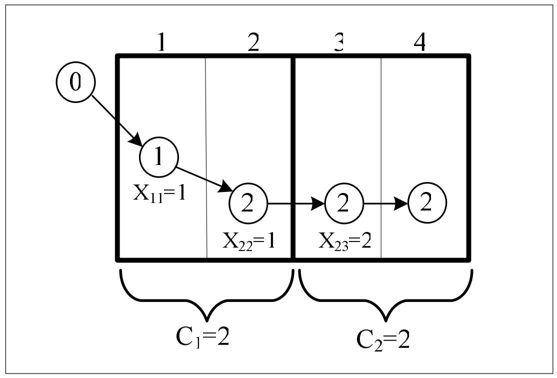


Figure 3.2 Optimal Solution of the Minimum Batch Size Splitting Example

As can be followed from Figure 3.2, the optimal solution of the example problem is X_{11}=1, X_{22}=1, X_{23} =2, I_{21}=1 with a total cost of 1. Here, the production of item 2 starts at the second position of period 1 using the remaining capacity of 1 unit. Nevertheless, with the original form of the minimum batch size constraints, the Fleischmann-Meyr model is unable to find a feasible solution in this example. The reason for this is the fact that the remaining capacity in period 1 (1 unit) cannot be used since it is smaller than the minimum batch size of item 2 (2 units) and the capacity in period 2 is insufficient for the production of the total demanded quantity of item 2 (3 units).

This example explains the reason why modifying the minimum batch size constraint (3.5) may be essential, especially if the capacity is critical and minimum batch sizes are considerably large. We therefore propose constraint (3.5') for the last positions of periods, enabling the extension of the production of the minimum batch quantity to the consecutive period.

However, it should be noted that this modification is only valid for two consecutive periods. If the total capacity of the two periods happens to be insufficient to cover the minimum batch quantity, then the modified formulation with constraint (3.5') may not be able to yield a feasible solution either. In order to accommodate this possibility as well, we need to extend the assumption of minimum batch size splitting to *k* consecutive periods (see Figure 3.3 below) by making some modifications to the formulation. We suggest the addition of constraints (3.14) in the model.

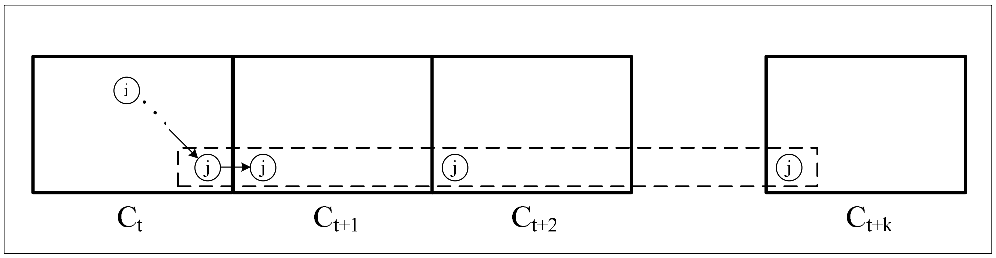


Figure 3.3 Minimum Batch Size Splitting over *k* Consecutive Periods

For each (*j*,*t*) pair, find the largest *k* s.t. $m_j \geq \left(\sum_{s=t}^{t+k-1} C_s - \min_{i|i \neq j} \{ST_{ij}\} \right)$

if $\exists k \geq 2$, then

$$X_{j(L_t)} + \sum_{s=t+1}^{t+k} X_{j(F_s)} \geq m_j (W_{j(L_t)} - W_{j(L_t - 1)}) \qquad \forall j,t,k \qquad (3.14)$$

Constraints (3.14) need only be written for special *(j,t)* and *k* combinations. As the condition in the constraint states, we check whether there exists a *k* value greater than or equal to 2 for a certain *(j,t)* pair, suggesting the largest interval [*t*, *t+k-1*] where total available capacity is insufficient to produce the minimum batch quantity, also considering the smallest possible changeover time from another item to *j* at the start of the lot. If such a *k* value can be found, the constraint is written in place of (3.5') so that the minimum batch size restriction is extended until period *t+k*, starting with position L_t, as displayed in Figure 3.3 above. The dashed line in the figure indicates the production lot of the item which needs to satisfy the minimum batch size restriction over *k+1* consecutive periods. If no *k* greater than or equal to 2 can be found, then the constraint is not written for that *(j,t)* pair and constraint (3.5') is included in the model instead. Note that *k*=1 would imply minimum batch size splitting over two consecutive periods (constraint (3.5')), whereas *k*=0 corresponds to the unextended version of the minimum batch size constraints (implying that the period capacity exceeds the minimum batch size on its own).

This constraint also needs to be adapted to the formulation by ensuring that the limited capacity inside the interval is totally consumed by the continuous lot of item j, leaving out no possibility to produce other items.

We have not included this rather cumbersome modification in our model, since the case where minimum batch sizes exceed total capacity for more than two consecutive macro periods in fact looks very unlikely. Nevertheless, should capacity critical environments to that extent be considered with considerably large minimum batch sizes and/or changeover times, our formulation will be able to handle the case with the addition of constraint set (3.14) along with some modifications.

3.4.2 Enabling Late Changeovers from Item Zero

In this part, we shall briefly explain the reasons for which we allow item zero changeovers in positions later than position 1.

Recall that the dummy item zero indicates the initial setup state of the facility at the beginning of the planning horizon. The incorporation of this item in the model poses some difficulties in the presence of sequence dependent changeovers. As it has been discussed in Section 3.2, changeovers involving this special item are not defined symmetrically, i.e., although we can change over from item zero to another item, changeovers from other items into item zero (δ_{i0n}) are not allowed and all the constraints in the model are appropriately built to consider this feature. As a matter of fact, what the model attempts to accomplish is to enforce a single transition from item zero (the initial state) into one of the items in the product mix at the beginning of the planning horizon before starting production, and never return to this state again. We need to point out that this single changeover does not necessarily take place in the very first position or even the first period. For this phenomenon, there are two reasons, both of which depend mainly on minimum batch size restrictions:

i) There may not be any demand early in the planning horizon. In this case, if the item zero changeover is performed right at the start, the positive minimum batch sizes oblige immediate production of an item in the first position. Note that empty setups, i.e., setups without immediately following production, cannot be accommodated for the first position unless it is the only position in period 1 (only then can production be delayed until the consecutive position in the next period via minimum batch size splitting). Thus, this situation would result in unnecessary early production and excessive inventory carrying costs.

ii) Case (i) may not even be feasible, since the facility may not be able to start production right away due to minimum batch size restrictions and restricted capacity. This case is highlighted below using an example.

Example: Consider a problem with two periods and two positions per period, where only one item (Item 1) has positive demand in period 1. Let the period capacities be 2 units each and the demand quantities of item 1 be one unit for each period. The changeover time between item zero and item 1 is 1 unit long and the minimum batch size of the item is 2 units. The optimal solution would look like the case in Figure 3.4, where the facility switches to item 1 in the last position of period 1 (i.e., position 2), where a production lot starts up and extends over to the consecutive period in order to take advantage of minimum batch size splitting (the case discussed in Section 3.4.1). However, by restricting the initial changeover to be at the very first position, we enforce immediate production of item 1 in position 1, which is not possible, since the sum of the initial changeover time (1 unit) and the minimum batch size (2 units) exceeds the capacity limit (2 units). Even though a feasible solution exists, the model is not able to obtain it in this case.

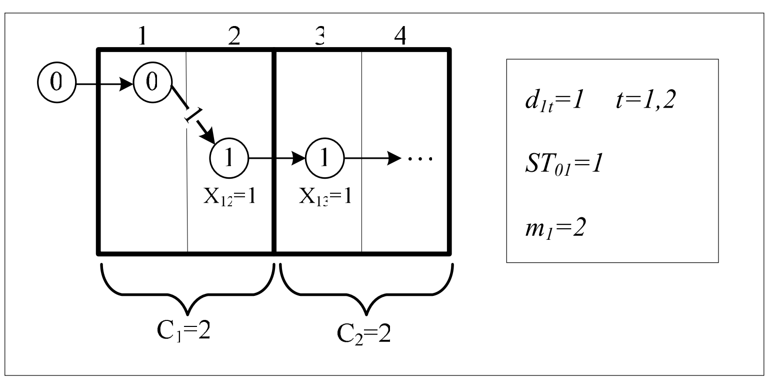


Figure 3.4 The Example Problem for Late Item Zero Changeovers

Formally, we can say that case (ii) is encountered when the following conditions are satisfied:

- $L_1 \neq 1$ (The first period contains more than one position)
- $\exists$ unique j s.t. $d_{j1}>0$ (There is a single item with positive demand in $t=1$)
- $m_j > C_1 - ST_{0j}$ (The minimum batch size of an item exceeds available capacity in period 1)
- $d_{11} - I_{10} \leq C_1 - ST_{0j}$ (The net demand can be produced with the available capacity in period 1, otherwise the case is infeasible)

Hence, the item zero changeover cannot be restricted to take place at the first position only, and in order to incorporate this aspect in the model, we need to define the changeover variables corresponding to item zero over all positions.

3.4.3 Prevention of the Manipulation of Item Zero Changeovers

In this part, we shall try to justify the need for adding constraint (3.9) in our model in order to be able prevent the manipulation of item zero changeovers.

Even though variable definitions and modifications in model constraints regarding item zero changeovers have been made properly, the model can still manipulate the solution in the absence of constraint (3.9). Let us illustrate the case with a small example problem.

Example: Consider the first three positions in a problem with 2 items (plus item zero). Suppose that the highlighted circles denote positions with positive production values and the arrows indicate changeovers between items. If constraint (3.9) is excluded, one can come up with a solution similar to that portrayed in Figure 3.5 on the next page.

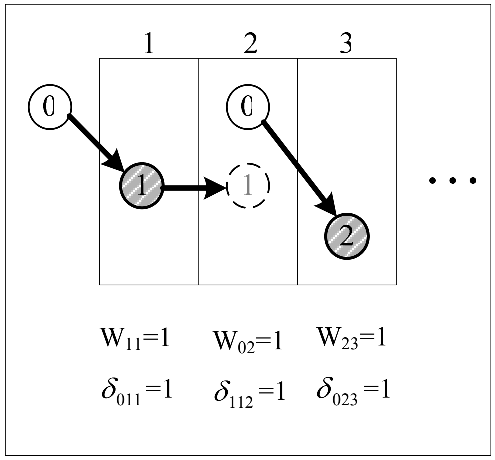


Figure 3.5 Example Denoting the Manipulation of Item Zero Changeovers

In the solution above, the facility initially switches to item 1 in the first position and performs the production of items 1 and 2 in the first and third positions respectively. However, note that in doing so, there is no direct transition between items 1 and 2; instead, item zero reappears in the plan in position 2, causing the logical link between item changeovers to be broken. In this way, the changeover cost between items 1 and 2 is not incurred. In fact, it is easy to verify that this situation does not violate any constraint and causes an incorrect interpretation of changeover penalties.

A closer look reveals that the problem occurs in position 2, in which there is no production. In this position, it looks as though the facility is set up for item zero (W_{02}=1) without a changeover into it (δ_{102}), when in fact such a changeover and the corresponding changeover constraint (3.7) are undefined. Instead, the changeover variable δ_{112} takes a positive value although the facility is not set up for item 1 in position 2. If we examine the corresponding changeover constraint (3.15) below, we can see the variable δ_{112} can take the value of 1 without causing any violations, as the constraint is in "greater than or equal to" form.

$$\delta_{112} \geq \underset{(1)}{W_{11}} + \underset{(0)}{W_{12}} - 1 \quad \Rightarrow \quad \delta_{112} \geq 0 \tag{3.15}$$

In order to resolve this problem, let us introduce constraint (9) into the model, which is reproduced below:

$$\delta_{00n} = \sum_{j=0}^{N} \delta_{0j(n+1)} \qquad \forall n = 1,\ldots,(L_K - 1) \qquad (3.9)$$

This constraint ensures that if there is a changeover in position ($n+1$) from item zero to another item or to itself, then the facility must have been in state zero in the previous position (position n) via an earlier transition from itself (i.e., $\delta_{00n} = 1$). In this way, the constraint helps establish an unbroken link of item zero changeovers and prevents the reappearance of item zero once the facility is setup for another item. It is easy to verify that the case displayed in Figure 3.5 violates constraint (3.9) for n=2 and that the problem is eliminated in this way.

3.5 Formulation of the GLSP as a Transportation Problem

In this section we present the alternative mathematical formulation of the GLSP, which is based on the Transportation Problem (TP). Similar formulations are also referred to as the Simple Plant Location Model (SPL) in some texts. This idea has been applied to many lot sizing problems, for which a few examples were provided in Section 2.2.4. However, to the best of our knowledge, this is the first study so far to use it within the GLSP context. The TP reformulation is known to be a strong formulation for lot sizing problems, as it has been briefly discussed in previous sections. For this reason we have decided to implement it for our problem in order to evaluate whether a change in formulation would bring about considerable improvement in the solution.

The basic idea in this alternative formulation is to disaggregate the production variables by relating each production quantity to the period at which it will be required. Considering our case in light of the TP reformulations in the literature, a first line of thinking would suggest that the following relationship holds:

$$X_{jn} = \sum_{t=\tau_n}^{K} Q_{jnt} \qquad \forall j,n \qquad (3.16)$$

where X_{jn} is the production variable in the I&L formulation and Q_{jnt} denotes the quantity of item *j* produced in position *n* to satisfy the demand of period *t*. The total production quantity of position *n* would thereby be distributed to the future periods of consumption. However, one should keep in mind that our formulation of the GLSP incorporates minimum batch sizes as well as the possibility of "dummy production", i.e., unnecessary production carried out for the sake of economizing on changeovers. This follows that we cannot associate every unit produced with the demand of some period, since part of the production quantities may remain unused at the end of the planning horizon. To take this case into consideration, we need to define an extra variable R_{jn} which denotes the quantity of the production lot in position *n* left unused at the end of the horizon. With this addition, relation (3.16) is replaced by:

$$X_{jn} = \sum_{t=\tau_n}^{K} Q_{jnt} + R_{jn} \qquad \forall j,n \qquad (3.17)$$

With the new disaggregated production variables, there is no need to include inventory variables in the model. Nevertheless, our model incorporates the possibility of positive initial item inventories, which are represented as the production of position zero (before the beginning of the planning horizon). The initially available inventories are also disaggregated according to their period of usage.

The extra decision variables and parameters used in the TP formulation are defined below, followed by the complete mathematical model.

Decision Variables:

Q_{jnt}: Quantity of item *j* produced in position *n* to satisfy the demand of period *t*, where $t \geq \tau_n$.

(Note that for initial inventory, the definition of the variable is changed as follows:

Q_{j0t}: Part of the demand of item *j* in *t* satisfied from the initial inventory on hand.)

R_{jn}: Unused portion of the quantity of item *j* produced in position *n*, in other words, the production carried to period *K+1* (beyond the planning horizon)

(Note that if *n*=0, it represents the unused portion of the initial inventory of item *j*.)

Parameters:

CQ_{jt}: Unit cost of producing item *j t* periods in advance of demand.

$$CQ_{jt} = CP_j + t * h_j$$

For the variables associated with initial inventory, the corresponding cost term becomes $(t-1) * h_j$

[TP]

Minimize $$\sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{n=1}^{L_K} SC_{ij}\delta_{ijn} + \sum_{j=1}^{N}\sum_{n=1}^{L_K}\sum_{t=\tau_n}^{K} CQ_{j(t-\tau_n)}Q_{jnt} + \sum_{j=1}^{N}\sum_{t=1}^{K}(t-1)h_j Q_{j0t}$$

$$+\sum_{j=1}^{N} Kh_j R_{j0} + \sum_{j=1}^{N}\sum_{n=1}^{L_K}(CP_j + (K-\tau_n+1)h_j)R_{jn} + \sum_{t=1}^{K} CO_t O_t \qquad (3.18)$$

Subject to $$\sum_{n=0}^{L_t} Q_{jnt} = d_{jt} \qquad \forall t, j \qquad (3.19)$$

$$\sum_{t=1}^{K} Q_{j0t} + R_{j0} = I_{j0} \qquad \forall j \qquad (3.20)$$

$$Q_{jnt} \le M_{j\tau_n t} W_{jn} \qquad \forall j, n, t \qquad (3.21)$$

$$\sum_{j=1}^{N}\sum_{n=F_t}^{L_t} P_j\left(\sum_{s=t}^{K} Q_{jns} + R_{jn}\right) + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{n=F_t}^{L_t} ST_{ij}\delta_{ijn} \le C_t + O_t \qquad \forall t \qquad (3.22)$$

$$\left(\sum_{s=\tau_n}^{K} Q_{jns} + R_{jn}\right) \ge m_j(W_{jn} - W_{j(n-1)}) \qquad \forall j, t, n \ne L_t \qquad (3.23)$$

$$\left(\sum_{s=\tau_n}^{K} Q_{jns} + R_{jn}\right) + \left(\sum_{s=\tau_{n+1}}^{K} Q_{j(n+1)s} + R_{j(n+1)}\right) \ge m_j(W_{jn} - W_{j(n-1)})$$

$$\forall j, t, n = L_t \qquad (3.23')$$

Constraints (3.6)-(3.11)

where $$M_{jst} = \min\left\{d_{jt}, \frac{(1+\gamma_s)C_s}{P_j}\right\} \qquad \forall j, s, t \qquad (3.24)$$

We provide brief remarks below regarding different aspects of the TP formulation in contrast with the original I&L formulation.

The objective function (3.18) again minimizes the total costs of changeovers, production, inventory holding and overtime, although in a somewhat different form. The costs of inventory holding and production are charged together (as CQ_{jt}) over the corresponding disaggregated production variables. Apart from these, note that the objective function also contains extra terms which are associated with:

- initial inventory which is used to satisfy the demand of some period, incurring inventory carrying costs until then,

  i.e., $\sum_{j=1}^{N}\sum_{t=1}^{K}(t-1)h_j Q_{j0t}$

- initial inventory which remains unused, incurring inventory carriage costs until period *K*,

  i.e., $\sum_{j=1}^{N} Kh_j R_{j0}$

- unused production realized during a position within the planning horizon, incurring a production cost in position *n* as well as inventory carriage costs until the end of the planning horizon,

  i.e., $\sum_{j=1}^{N}\sum_{n=1}^{L_K}(CP_j + (K-\tau_n+1)h_j)R_{jn}$

Constraint (3.19) guarantees that demand is satisfied for all items and periods.

Constraint (3.20) ensures that the available initial inventories are either assigned to satisfy the demand of some period, or they are left unused at the end of the planning horizon. Note that in the absence of this extra constraint, unused initial inventories may not take correct values, resulting in incorrect carrying charges. Recall that the I&L formulation incorporated inventory balance relations which automatically eliminated this possibility.

Another point to be highlighted is the upper bound used in constraint (3.21). Recall that the I&L formulation featured a bound on the total quantity produced in a position based on capacity. Here, the bound is established over the disaggregated production quantity, instead of the *total* production quantity, which allows us to take demand as well as capacity into account while determining a value for the big M term (3.24). Therefore, the resulting bound is much tighter compared with its I&L counterpart.

The rest of the formulation becomes essentially identical to the I&L model if all production variables are substituted by the disaggregated production variables and unused production variables according to relation (3.17).

Following the observed results in the literature, our expectation at this point is to achieve stronger LP relaxation values with the TP formulation due to tightened production bounds. However, a drawback of this formulation is the fact that the number of disaggregated production variables grows quadratically with the number of time periods, whereas this rate is only linear for I&L. The nature of the tradeoff between tightened relaxations versus increased complexity is evaluated through numerical experiments, where we expect to ascertain whether it pays off to use the alternative formulation for our problem.

CHAPTER 4

ENHANCEMENT OF THE GLSP FORMULATION AND COMPUTATIONAL RESULTS

It has been shown that the feasibility problem of the CLSP with positive setup times is NP-complete (Maes *et al.*, 1991). Since sequencing decisions are also taken into account in the GLSP as well as other additional features, the GLSP is a more general large bucket model than the CLSP. The feasibility problem for the GLSP with non-zero minimum batch sizes and without setup times is proved to be NP-complete (Fleischmann and Meyr, 1997). Thus, we can infer that for our version of the GLSP with positive changeover times and minimum batch sizes (also behaving as changeover times in a way), which is more general than the formulation used by Fleischmann and Meyr (1997), the feasibility problem is also NP-complete without the option of resorting to the use of overtime or in the case of limited overtime.

In order to attempt to solve the problem optimally, we have decided to make some enhancements to the two alternative formulations presented in Chapter 3. This chapter discusses the additional inequalities incorporated in our models for the purpose of restricting the solution space and facilitating the solution process. Some of these are valid inequalities adopted from the literature and some help in eliminating redundant solutions. All these inequalities in nature are redundant for the mixed integer solution of the problem. It has to be emphasized at this point that the approach adopted by the authors is one of single-session (*a-priori*) addition of additional inequalities. The purpose is to strengthen the LP relaxation of the problem, since a strong initial relaxation generally has a big impact on the performance of the solution algorithm, as has been pointed out by Aardal (1998). Instead of having to design special purpose Branch-and-Cut routines, a-priori addition of valid inequalities also enables us to easily make use of commercially available packages for the solution of the problem.

In what follows, the proposed inequalities to be used are introduced along with brief explanations. We then present the results of a preliminary set of experiments conducted so as to assess the impact of incorporating these inequalities in the formulations, which will eventually lead us to select the best combination as an experimental setting for the rest of the study.

4.1 The Proposed Inequalities

In this section, we introduce the set of extra constraints that we consider including in our model, followed by some explanations. The impact of incorporating some of these inequalities on the LP relaxation will be highlighted using small example problems.

We have classified the inequalities to be added to the model into two groups according to their purposes: i) valid inequalities related with setups and changeovers (abbreviated as VI) for strengthening the LP relaxation, and ii) elimination constraints to remove redundant integer solutions (abbreviated as EC). Note that although classified differently, some inequalities within the EC group are also proven to be valid inequalities that are able to strengthen the LP relaxations, as will be discussed in the upcoming parts of this section.

4.1.1 Valid Inequalities Related with Setups and Changeovers (VI)

Analysis of the results of a few initial tests with the formulations introduced in Chapter 3 revealed that the LP relaxations are poor in quality, mainly due to the assignment of non-integral values to the changeover and setup variables. We present a small size problem in order to demonstrate an example LP relaxation solution and the effect of incorporating valid inequalities.

Example: Consider a problem with 3 items and a single period with 3 positions. The period capacity is 6 units and each item has a demand quantity and minimum batch size of 1 unit. Let the changeover times between the items be as shown as in Table 4.1 on the next page.

Table 4.1 Changeover Times for the Example Problem

items	1	2	3
0	1	2	2
1	0	1	2
2	2	0	1
3	1	2	0

For simplicity, it is assumed that the changeover costs are equal to the changeover times. Quick examination of the changeover times is sufficient to realize that for each item, there is a single "ideal transition" incurring the lowest changeover time of 1 unit. Since all the items need to be produced once, the production sequence with the lowest total changeover time can easily be determined as 0 → 1 → 2 → 3. Thus, the optimal sequence looks like the following:

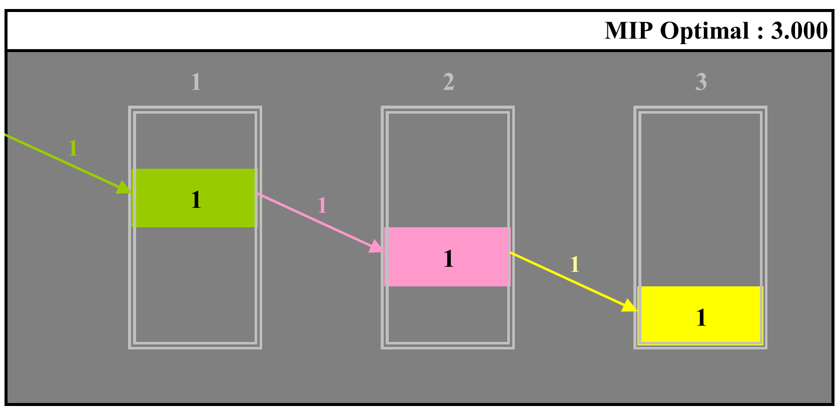


Figure 4.1 Optimal Solution for the Example Problem

In Figure 4.1, the gray boxes stand for the three positions. There are four rows in each box, each of which is reserved for a different item, with item zero on the top row and the other three items below in order. Items are differentiated with the use of alternative colours, where green, pink and yellow indicate items 1, 2 and 3 respectively. Item zero is denoted with white, even though it does not appear in the sequence in Figure 4.1. If an item is assigned to a certain position in the schedule, the row for the corresponding item in that position is highlighted in the item's colour. For instance, since item 1 is assigned to the first position, the second row of that position has been marked in green. Entries inside the rows of the items denote the value of the setup variables (W_{in}) while the arrows linking the positions stand for the changeover variables (δ_{ijn}) between the rows of the corresponding items. Therefore, from the figure, we can understand that after an initial changeover from item zero into item 1 in the first position, items 2 and 3 are produced in the remaining two positions. This sequence consumes the entire period capacity of 6 units and has an associated objective function value of 3.

The LP relaxation solution of the same problem is provided in Figure 4.2 below. Instead of the desired structure where a single item is assigned to each position and a single changeover takes place between each pair of consecutive positions, we see that multiple items are assigned to positions with partial (non-integral) values which add up to 1. Moreover, we realize that there is no direct link between setup and changeover variables, namely an item may be assigned partially to a position but there may be no incoming or outgoing changeovers related with it. See items 2 and 3 without any changeovers in Figure 4.2.

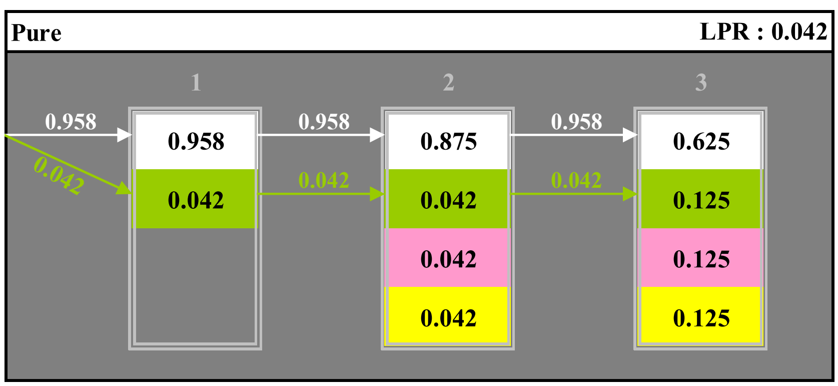


Figure 4.2 LPR of the Example Problem with the Pure Formulation

The poor LP relaxation quality of the pure formulations has led us to look for ways to enhance the structure between the setup and changeover variables by adding some extra inequalities, which will be discussed in detail with the use of the example problem.

It has to be emphasized that our approach in introducing these valid inequalities is not essentially one related with the details of polyhedral theory, but rather one for strengthening the LP relaxation as much as possible. Our focus is on the relationship between changeovers and setups, unlike the major portion of the related literature dealing with variants of (l,S) type valid inequalities introduced by Barany *et al.* (1984), which are out of our scope in this work. Therefore, we concentrated on a few studies dealing with start-ups and sequence dependent setups, such as those by Wolsey (1989, 1997, 2002) and Belvaux and Wolsey (2000, 2001). It needs to be said that the field for valid inequalities that deal with big bucket models with sequence dependent setups is mostly left open, as has been pointed out by Wolsey (2002).

4.1.1.1 VI-1 (Unit Flow Equalities)

a. $$\sum_{i=0}^{N} \delta_{ijn} = W_{jn} \qquad \forall j = 1,...,N, n = 1,...,L_K \qquad (4.1)$$

($\delta_{00n} = W_{0n}$ for $j = 0$ and $n = 1,...,L_K$)

b. $$\sum_{j=1}^{N} \delta_{ijn} = W_{i(n-1)} \qquad \forall i = 1,...,N, n = 2,...,L_K \qquad (4.2)$$

($\sum_{j=0}^{N} \delta_{0jn} = W_{0(n-1)}$ for $j = 0$ and $n = 1,...,L_K$)

The two equalities forming VI-1 provide the link between the setup and changeover variables, and in a way, they work as flow balance constraints. VI-1a is related with the incoming flow while its counterpart VI-1b handles the outgoing flow. The use of these valid inequalities can be traced back to the work of Karmarkar and Schrage (1985), where the changeovers were viewed within a network structure as the flow of a single unit between item setups. Mathematical proof of the validity of these equations can be given as follows:

Proof: For VI-1a, consider the changeover constraint (3.7),

i.e., $\delta_{ijn} \geq W_{i(n-1)} + W_{jn} - 1$.

If we sum these constraints over all i, we obtain:

$$\sum_i \delta_{ijn} \geq \sum_i W_{i(n-1)} + NW_{jn} - N$$
$$\sum_i \delta_{ijn} \geq 1 + N(W_{jn} - 1)$$

i) If $W_{jn} = 1$ then $\sum_i \delta_{ijn} \geq 1 \Rightarrow \sum_i \delta_{ijn} = 1 = W_{jn}$

ii) Otherwise if $W_{jn} = 0$ then $\exists k \neq j$ s.t. $W_{kn} = 1$

$$\sum_i \delta_{ikn} = 1 \quad \text{(From i)}$$
$$\Rightarrow \sum_i \delta_{ijn} = 0 = W_{jn}$$

□

The proof of VI-1b can be done in exactly the same manner.

Note that for the first two sets of valid inequalities, the constraints take a special form for item zero, which is due to the asymmetrical nature of its changeovers, as it was discussed extensively in Chapter 3.

If we add VI-1 to the pure formulation for the small example problem, we obtain a stronger LP relaxation solution with an objective value of 0.278. The resultant solution sequence with VI-1 is presented in Figure 4.3 on the next page along with the solutions corresponding to other valid inequalities that will be discussed next. Note that all valid inequalities are separately added to the pure formulation in this figure.

It can be seen that although the multi-item assignment to positions remains unchanged, the addition of VI-1 to the pure formulation has resulted in a structure where changeovers and setups are linked properly with balanced in-flows and out-flows. There are now changeovers into and out of items 2 and 3 which were otherwise unconnected to the other items in the pure formulation solution.

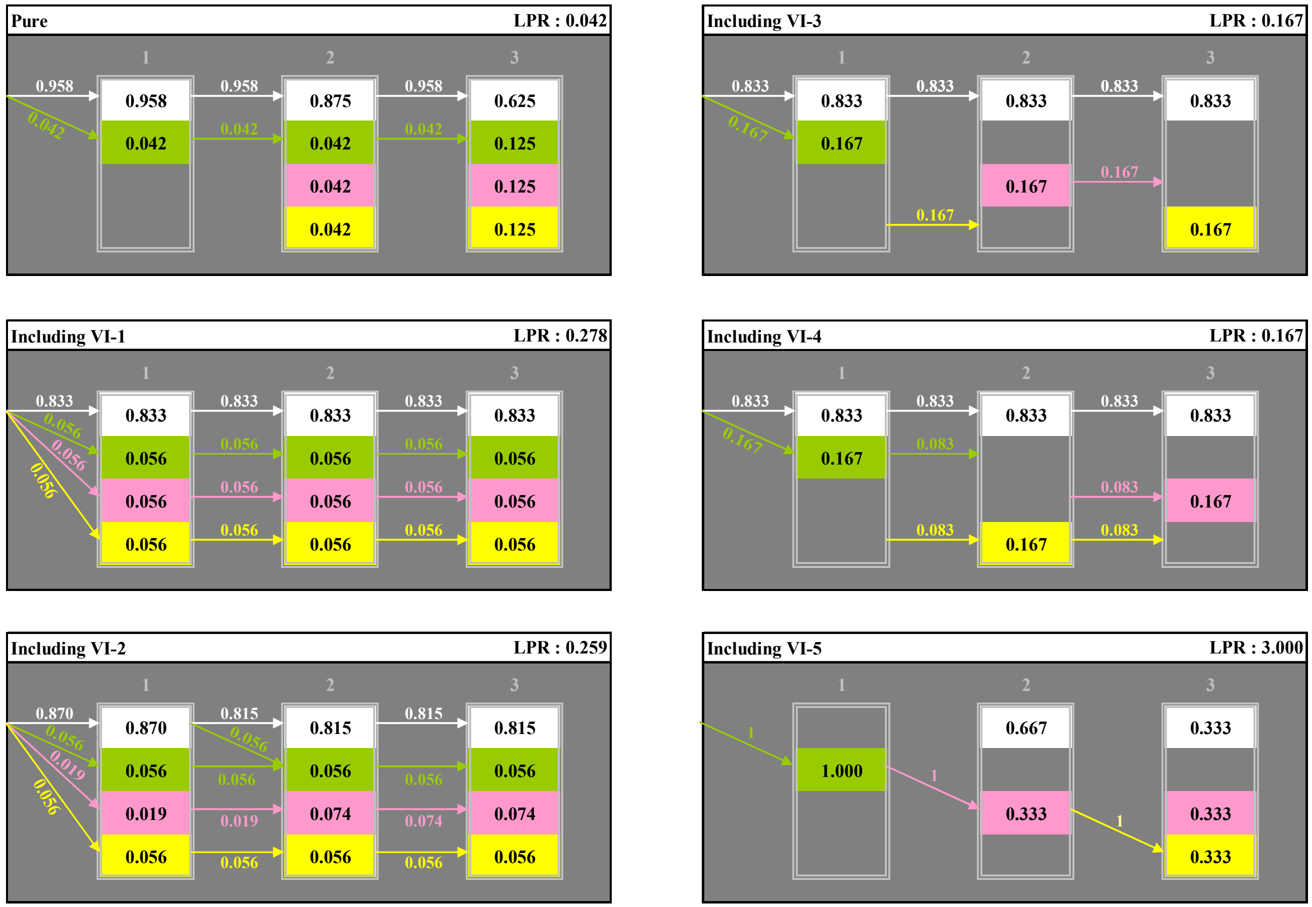


Figure 4.3 The Individual Effects of Valid Inequalities on the Solution of the Small Example Problem

4.1.1.2 VI-2 (Separate Unit-Flow Inequalities)

a. $$\delta_{ijn} \le W_{jn} \qquad \forall i = 0,...,N,\ j = 1,...,N,\ n \qquad (4.3)$$

($\delta_{00n} \le W_{0n}$ for *j = 0* and $n = 1,..., L_K$)

b. $$\delta_{ijn} \le W_{i(n-1)} \qquad \forall i = 0,...,N,\ j = 1,...,N,\ n \qquad (4.4)$$

($\delta_{00n} \le W_{0(n-1)}$ for *j = 0* and $n = 1,..., L_K$)

The two sets of inequalities in VI-2 work in a way similar to that of VI-1 constraints, but they are written separately for each *(i,j)* pair whereas VI-1 constraints are equalities expressed over the sum of items. Again the constraints take a special form for item zero. The validity of VI-2 follows from VI-1, as it is basically a separated form of the sum of changeover variables, each of which can take only non-negative values.

The results obtained as a result of incorporating VI-2 into the pure formulation for the small example problem are similar to those of VI-1, but somewhat poorer. If we examine Figure 4.3, we can see that although there is an enhanced flow structure in this case compared with the pure formulation, the flow balance equations do not necessarily hold for all item-position combinations. This is due to the fact that the link between setup and changeover variables is established on an individual basis in the form of inequalities in VI-2. The resulting LP relaxation value is slightly lower than that corresponding to the addition of VI-1.

4.1.1.3 VI-3 (Setup-Startup Inequality)

$$W_{i(n-1)} + \sum_{j=0|j\neq i}^{N} \delta_{jin} + \sum_{j=1|j\neq i}^{N} (W_{jn} - \sum_{k=0|k\neq j}^{N} \delta_{kjn}) \le 1 \qquad \forall i = 0,...,N,\ n \qquad (4.5)$$

VI-3 appears in the startup literature (for the results of Constantino, see Belvaux and Wolsey, 2001). The inequality is written for a specific item *i* and a position *n*, and is formed of four terms that represent the following:

$$\underbrace{W_{i(n-1)}}_{\text{setup of item } i \text{ in } (n\text{-}1)} + \underbrace{\sum_{\substack{j=0\\ j\neq i}}^{N} \delta_{jin}}_{\text{startup of item } i \text{ in } n} + \underbrace{\sum_{\substack{j=1\\ j\neq i}}^{N} W_{jn}}_{\text{setup of item } j \text{ in } n} - \underbrace{\sum_{\substack{j=1\\ j\neq i}}^{N} \sum_{\substack{k=0\\ k\neq j}}^{N} \delta_{kjn}}_{\text{startup of item } j \text{ in } n} \leq 1 \qquad \forall i, n$$

For each i and n pair, four mutually exclusive cases are possible, all of which can be handled by this valid inequality as shown below:

i) The facility is set up for item i in positions n-1 and n.
This logically implies that there can be no startups for item i or for another item in position n.

$$W_{i(n-1)} = 1,\ \sum_{j=0|j\neq i}^{N} \delta_{jin} = 0,\ \sum_{j=1|j\neq i}^{N} W_{jn} = 0,\ \sum_{j=1|j\neq i}^{N} \sum_{k=0|k\neq j}^{N} \delta_{kjn} = 0 \qquad \textit{(constraint satisfied)}$$

ii) The facility is set up for item i in position n-1, but not in n.

$$W_{i(n-1)} = 1 \text{ and } \sum_{j=1|j\neq i}^{N} W_{jn} = 1$$

$$\Rightarrow \sum_{j=1|j\neq i}^{N} \sum_{k=0|k\neq j}^{N} \delta_{kjn} = 1 \Rightarrow \sum_{j=0|j\neq i}^{N} \delta_{jin} = 0 \qquad \textit{(constraint satisfied)}$$

Examining the mathematical form of the constraint, we can see that both setup terms corresponding to items i and j ($j \neq i$) taking the value of 1 would force the last term with the negative sign corresponding to the startup of item j to be 1 as well, since the left hand side cannot exceed 1. Logically, this implies that the facility switches over from item i to some other item j in position n, which describes a startup.

iii) There is a startup of item i in n (i.e., item i is set up in position n, but not in n-1).

$$W_{i(n-1)} = 0 \text{ and } \sum_{j=0|j\neq i}^{N} \delta_{jin} = 1$$

$$\Rightarrow \sum_{j=1|j\neq i}^{N} W_{jn} = 0 \text{ (since } W_{in} = 1\text{) and } \sum_{j=1|j\neq i}^{N} \sum_{k=0|k\neq j}^{N} \delta_{kjn} = 0 \qquad \textit{(constraint satisfied)}$$

iv) The facility is set up for other items in *n-1* and *n*.

$$W_{i(n-1)} = 0, \quad \sum_{j=0|j\neq i}^{N} \delta_{jin} = 0 \text{ and } \sum_{j=1|j\neq i}^{N} W_{jn} = 1$$

$$\Rightarrow \sum_{j=1|j\neq i}^{N} \sum_{k=0|k\neq j}^{N} \delta_{kjn} = 0 \text{ or } 1 \qquad \textit{(constraint satisfied)}$$

This implies that item *i* does not appear in positions *n-1* or *n*, thus there can be no startup for item *i* in position *n* (the first two terms in the constraint are zero). The facility must be set up for some other item *j* in position *n* (the third term is 1). Hence, the left hand side of the constraint is already 1 and the last term with the negative sign can either take the value of 0 or 1 (i.e., there can be a startup for a different item in position *n*, or not).

Mathematical proof of the validity of VI-3 can be done by using VI-1 and VI-4, which is explained in the next subsection.

It can be seen from Figure 4.3 on page 50 that the pure formulation solution violates VI-3 for i = 0 and n = 2 and 3. When VI-3 is incorporated into the model, the LP relaxation value is improved to 0.167.

4.1.1.4 VI-4 (Lifted Version of the Changeover Constraint for *i=j*)

$$W_{in} + W_{i(n-1)} + \sum_{j=0|j\neq i}^{N} \delta_{jjn} \leq 1 + \delta_{iin} \qquad \forall i = 0,\ldots,N, n \qquad (4.6)$$

VI-4 is a lifted version of the changeover constraint (3.7) for *i=j*. This constraint appears in the context of small bucket lot sizing models (for the results of Constantino, see Belvaux and Wolsey, 2001).

Let us discuss the validity of the constraint on the basis of the possible values that the term $W_{in} + W_{i(n-1)}$ can take.

For the case where the facility is set up for item *i* in positions *n-1* and *n* (i.e., $W_{i(n-1)} = W_{in} = 1$), it enforces the corresponding changeover variable (δ_{iin}) to take the value of 1, as in the unlifted form. Besides, it automatically sets other self-changeover variables (δ_{jjn}) to zero, since the left hand side cannot take a value greater than 2.

If the sum $W_{in} + W_{i(n-1)}$ is 1, i.e., if there is a setup for item i in only one of the two positions *n-1* and *n*, variable δ_{iin} is forced to take the value of 0 (via VI-1). This again sets other self-changeover variables (δ_{jjn}) to zero, since the left hand side cannot exceed the value of the right hand side, which is 1.

For the final case where $W_{in} + W_{i(n-1)}$ equals 0, variable δ_{iin} is again forced to take on the value of 0, which implies that the sum of the other self-changeover variables is less than 1, i.e., $\sum_{j=0|j\neq i}^{N} \delta_{jjn} \leq 1$, which holds in any case and therefore is not an extra restriction.

At this point, we can show mathematically that VI-3 follows from VI-1 and VI-4 by substitution, as demonstrated below.

Proof: If we separate self changeovers from the sum in (4.1), we can express VI-1 in the following alternative form:

$$\delta_{jjn} + \sum_{i=0|i\neq j}^{N} \delta_{ijn} = W_{jn} \tag{4.7}$$

If we use equation (4.7) to substitute self-changeover terms in VI-4, we obtain:

$$W_{in} + W_{i(n-1)} + \sum_{j=0|j\neq i}^{N} (W_{jn} - \sum_{k=0|k\neq j}^{N} \delta_{kjn}) \leq 1 + (W_{in} - \sum_{j=0|j\neq i}^{N} \delta_{jin})$$

$$W_{i(n-1)} + \sum_{j=0|j\neq i}^{N} \delta_{jin} + \sum_{j=0|j\neq i}^{N} (W_{jn} - \sum_{k=0|k\neq j}^{N} \delta_{kjn}) \leq 1$$

We can exclude item zero from the domain of the second summation, since no startup is possible for this item. In this way, we obtain VI-3.

$$W_{i(n-1)} + \sum_{j=0|j\neq i}^{N} \delta_{jin} + \sum_{j=1|j\neq i}^{N} (W_{jn} - \sum_{k=0|k\neq j}^{N} \delta_{kjn}) \leq 1$$ □

Referring back to Figure 4.3 on page 50, we can see that the addition of VI-4 to the pure formulation improves the LP relaxation value of the example problem. The reader can check that VI-4 is violated in the pure formulation for *i*=2, 3 when *n*=2 and *i*=1, 2, 3 when *n*=3.

4.1.1.5 VI-5 (Initial Changeover Restriction)

$$\sum_{n=1}^{L_{t_j}} \sum_{i=0|i \neq j}^{N} \delta_{ijn} \geq 1 \qquad \forall j = 1,\ldots,N \qquad (4.8)$$

where t_j represents the smallest *t* that satisfies $\sum_{s=1}^{t} d_{js} > I_{0j}$.

VI-5 is a logical inequality developed by the authors with the purpose of enforcing an initial changeover into each item that requires production. This requirement is evaluated through the parameter t_j which, by definition, is the earliest period at which the cumulative demand exceeds the initially available inventory. This implies that for each *j*, there must be at least one production up to period t_j since backlogging is not allowed. Our aim in using this inequality is to link this production requirement to changeover variables, by suggesting that if production has to be performed for item *j* somewhere within the interval from the beginning of the planning horizon until the last position in period t_j, then there must also be at least one changeover from a different item into item *j* within the same interval in order to maintain feasibility.

In developing VI-5, our main motivation was to break the generally observed LP relaxation pattern where item zero has a series of predominant changeovers throughout the planning horizon and only a small portion of the unit changeover flow can visit the actual items. In this regard, reexamine Figure 4.3 for the pure solution as well as those with other inequalities but VI-5. It is easy to notice that the values for the self-changeovers of item zero are usually close to 1, while other changeovers take very small values around zero in relation to the corresponding *W* variables which also take small decimal values. We have realized that if we could enforce an initial transition from item zero to the actual items, this undesirable structure could be broken, since once item zero state has been quit, there is no possibility of changing back to it later in the planning horizon. Note that VI-5 is in the form of a greater than-or-equal to constraint; therefore it strives to bring the values of the changeover variables close to integrality.

The final case in Figure 4.3 on page 50 indicates the solution of the example problem including VI-5. Since there is a single period and each item has zero initial inventory and positive demand, it follows that $t_j=1$ for each *j*, i.e., each item needs production in the first period. This enforces a changeover into each item from another within the planning horizon, which results in a solution where all changeover variables take integral values. The inequality was successful in breaking the dominance of item zero which was prevalent in all other solutions. The LP relaxation value turns out to be 3, which is the optimal solution value; however, some of the setup variables are still partially assigned to positions. Therefore, in the scope of this small example problem, VI-5 has resulted in the strongest LP relaxation solution by far.

4.1.2 Elimination Constraints (EC)

In problems with high number of positions, there may be multiple ways to represent the same solution, as there may be some idle positions within periods and the production quantities may be arbitrarily assigned to the several consecutive positions forming the production lot of an item. Although these differences do not cause any changes in the objective function value or the resulting optimal production plan, they are sources of inherent redundancy in the model, which is undesirable from the point of view of modeling efficiency.

In order to avoid these redundant solutions and restrict the solution space without eliminating any feasible solution, we have incorporated two extra constraints EC-1 and EC-2 in our model, which will be introduced in the following subsections. These two constraints were originally proposed by Fleischmann and Meyr (1997). We have treated these constraints as a separate class, since their essential purpose is not one of strengthening the LP relaxation solution, as the previously discussed valid inequalities, but one of eliminating alternative solutions. However, it needs to be noted that EC inequalities may not always be totally ineffective in strengthening the LP relaxation values. In fact, we were able to demonstrate a few cases where EC-1 constraints happened to chop off LP relaxation solutions, acting as valid inequalities as well. No such inference can be made about EC-2 constraints as yet.

4.1.2.1 EC-1 (Idle Position Arrangement)

$$\sum_{j=1}^{N} \sum_{i=0, i \neq j}^{N} \delta_{ij(n-1)} \geq \sum_{j=1}^{N} \sum_{i=0, i \neq j}^{N} \delta_{ijn} \qquad \forall t, n = (F_t + 2), \ldots, (L_t - 1) \tag{4.9}$$

EC-1 ensures that idle positions within a period, if any, are placed at the end of that period. This is accomplished by keeping track of changeovers into different items (i.e., startups) over all consecutive position pairs within a period. Figure 4.4 below may provide a clearer understanding as to how certain situations are eliminated by the help of EC-1.

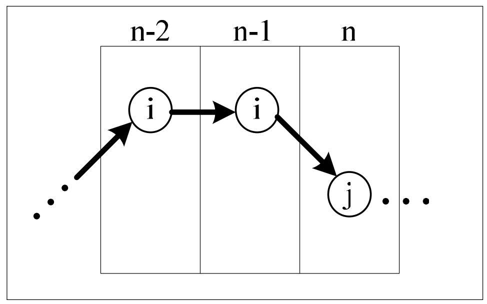


Figure 4.4 Example Case that Violates EC-1

Assume that in the case depicted above, all the three positions (*n-2*, *n-1* and *n*) belong to the same time period, which is automatically ensured by EC-1, since it is only written for positions greater than F_t+2 for any t. In the above figure, item i has been assigned to two consecutive positions (one of which can remain idle), followed by a changeover into a different item (item j) in position n. Then, it can be verified that the sums of the values of the changeover variables corresponding to a change of item status for positions *n-1* and *n* are 0 and 1, respectively, i.e.,

$$\sum_{j=1}^{N} \sum_{i=0, i \neq j}^{N} \delta_{ij(n-1)} = 0 \text{ and } \sum_{j=1}^{N} \sum_{i=0, i \neq j}^{N} \delta_{ijn} = 1$$

which implies that EC-1 is violated in this case.

The correct representation of the same situation without any loss of information would be as illustrated in Figure 4.5 on the next page.

In this way, the constraint ensures that all item production lots take up a single position at the beginning of a period and positions with self-changeovers (or idle positions), if any, are placed at the end of a period for only one item.

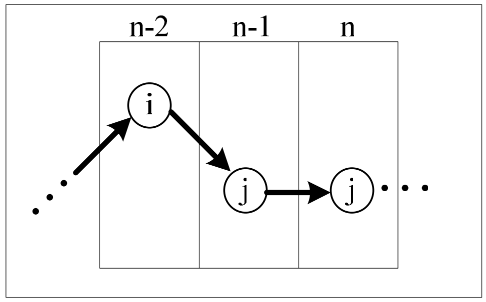


Figure 4.5 Correct Representation that Satisfies EC-1

Compared with its original version (Fleischmann and Meyr, 1997), the domain of EC-1 has been modified slightly so that the final positions of periods are not covered. In this way, we try to restrict the solution space for the GLSP in exactly the same way as Fleischmann and Meyr, while treating the last position within a period as a special case due to the possibility of partial production resulting from minimum batch size splitting discussed earlier (Section 3.4.1).

As it has been pointed out, EC-1 is actually a valid inequality, since there are cases where it can eliminate some LP relaxation solutions and strengthen the LP objective function value, although its effect is not as strong as those of VI presented earlier. For details, see Appendix A where the validity of EC-1 was demonstrated on an example problem in the presence of all other VI.

4.1.2.2 EC-2 (Production Quantity Distribution)

$$X_{jn} \leq M_{jt}(2 - \sum_{i=0}^{N} \delta_{ij(n-1)} - \delta_{jjn}) \qquad \forall t, j = 1,..., N, n = (F_t + 1),..., L_t \qquad (4.10)$$

In the case where the production lot of an item contains multiple positions, EC-2 assigns the entire production quantity to the first position of the lot in order to avoid arbitrary distribution of the total lot size. This is accomplished by checking the following two conditions:

- whether item *j* is assigned to position *n-1* (the penultimate term in the constraint),
- whether item *j* is assigned to position *n* as a result of a self-changeover from the previous position (the last term in the constraint).

If both conditions are satisfied, the production quantity of position *n* is set to zero; otherwise the constraint poses no restrictions upon the production variable. As an example, for the case illustrated in Figure 4.5, the entire production quantity of item *j*'s production lot would be assigned to position *n-1*, and position *n* as well as any other upcoming position for the same lot, are to be left idle. Observation of the domain reveals that this constraint is valid for positions inside the same period, and that the last position is not excluded as in EC-1, since the working logic of this inequality is not affected by minimum batch size splitting.

Note that for the TP formulation, the first type of valid inequalities VI and EC-1 can be used exactly as described, while EC-2 needs to be modified slightly by replacing the original production variable with the distributed and unused production variables characterizing the TP formulation, as follows:

$$\left(R_{jn} + \sum_{s=\tau_n}^{K} Q_{jns} \right) \leq M_{jt}(2 - \sum_{i=0}^{N} \delta_{ij(n-1)} - \delta_{jjn}) \qquad \forall t, j, n = (F_t + 1)..L_t \qquad (4.11)$$

4.2 Preliminary Experiments

In Section 4.1, we have presented a set of valid inequalities and elimination constraints that are candidates for enhancing the GLSP formulation. In this section, we will describe our preliminary experiments conducted in order to determine the experimental settings for the remainder of the study by testing the effect of alternative formulations as well as those of the additional inequalities.

In Section 4.2.1, we present the experimental settings in terms of the combinations of different formulations and additional inequalities tested as well as the test instances used. Section 4.2.2 contains the results of the preliminary experiments along with discussions.

4.2.1 Experimental Settings

Test Instances

For the preliminary test data, we have used the smallest twenty instances within the "Practical Industry Problems" used by Meyr (2000) with varying numbers of items and time periods. The following points can be made regarding the nature of the test instances:

- Information about item demands, period capacities, changeover (setup) costs, inventory holding costs, unit production requirements, initial inventories, minimum batch sizes are used exactly as given. In line with this, production costs are assumed to be zero and the use of overtime is not allowed. Note that the original parameters listed above all take real values.
- The instances contain sequence dependent changeover costs but sequence independent changeover times. The latter take up around 1-3% of the total available capacity in a period, which appears to be too low. For this reason, we have performed the experiments under different settings by making several modifications on the original data regarding the nature of changeover times and costs, which may be briefly depicted as follows:
    - **Case A**: The original data (Meyr, 2000) where the sequence independent setup times have been transformed into sequence dependent changeover times and also augmented in size. Augmenting changeover times tighten the test instances.
    - **Case B**: Data used in Case A without any changeover costs.
    - **Case C**: The original form of changeover costs and times as used by Meyr (2000).

  The details regarding how these cases differ from each other and how the modified data are generated will be clarified in the next section.
- In determining the number of positions within a period, we have used the following relation:

$$L_t - F_t = min\left\{N, k_t + 2\right\}$$

  where k_t is the largest integer satisfying $\sum_{j=1}^{k_t} m_j \leq C_t$, when items are ordered in non-decreasing order of their minimum batch sizes. Thus, k_t is an indication of the maximum number of items that may be assigned to a period under the extreme case where the production quantity of each item is as large as its minimum batch size. We need to add 2 to k_t in order to be able to consider the possibility of partial production (i.e., smaller than the minimum batch size) at the beginning and end of each period, due to minimum batch size splitting. In sum, we set the position limit by allowing each item to be produced once in a period provided that minimum batch sizes do not impose an extra restriction.

The preliminary test bed consists of 15 instances with K=4 periods and 5 instances with K=8 periods and different numbers of items. Information about instance characteristics and classification is provided in Table 4.2 below.

Table 4.2 Preliminary Instance Characteristics *

K = 4			K = 8		
N	# of instances	Instances	N	# of instances	Instances
2	1	3			
3	1	5		-	
4	1	6			
5	6	1,2,4,7,8,9	5	2	10,11
6	1	15		-	
7	2	16,17			
8	1	18	8	1	12
9	2	19,20	9	2	13,14

* The naming scheme used in the preliminary experiments is different from that of Meyr (2000), since the instances have been renumbered for convenience.

Among these twenty instances, we classify the smallest three (i.e., instances 3, 5 and 6) as "Easy" instances. In doing so, we have observed that for these three instances, the size of N and K, the special structure of changeover times and period capacities facilitate the solution of the problem by reducing solution times drastically to less than a few seconds. For this reason, their effect on overall averages may be misleading. By making such a distinction, we will be able to isolate the statistics for the easy instances from the overall averages and thereby we hope to arrive at more reliable conclusions.

Test Options

The purpose of the preliminary experiments is twofold: The first is to test the contribution of a change in formulation, and the second is to get insight about the effectiveness of the additional inequalities and then determine their best combination to be used as an experimental setting. In this light, we have developed a set of test options which include the two alternative formulations, and the candidate additional inequalities on their own as well as within several combinations. For easy referencing, we reproduce all additional inequalities as they have been used in the preliminary experiments in Figure 4.6.

VI-1 **a.**	$\sum_{i=0}^{N} \delta_{ijn} = W_{jn} \qquad \forall j = 1,...,N, n = 1,...,L_K$ ($\delta_{00n} = W_{0n}$ for $j = 0$ and $n = 1,...,L_K$)
b.	$\sum_{j=1}^{N} \delta_{ijn} = W_{i(n-1)} \qquad \forall i = 1,...,N, n = 2,...,L_K$ ($\sum_{j=0}^{N} \delta_{0jn} = W_{0(n-1)}$ for $j = 0$ and $n = 1,...,L_K$)
VI-2 **a.**	$\delta_{ijn} \le W_{jn} \qquad \forall i = 0,...,N, j = 1,...,N, n$ ($\delta_{00n} \le W_{0n}$ for $j = 0$ and $n = 1,...,L_K$)
b.	$\delta_{ijn} \le W_{i(n-1)} \qquad \forall i = 0,...,N, j = 1,...,N, n$ ($\delta_{00n} \le W_{0(n-1)}$ for $j = 0$ and $n = 1,...,L_K$)
VI-3	$W_{i(n-1)} + \sum_{j=0 \mid j \ne i}^{N} \delta_{jin} + \sum_{j=1 \mid j \ne i}^{N} (W_{jn} - \sum_{k=0 \mid k \ne j}^{N} \delta_{kjn}) \le 1 \qquad \forall i = 1,...,N, n$
VI-4	$W_{in} + W_{i(n-1)} + \sum_{j=0 \mid j \ne i}^{N} \delta_{jjn} \le 1 + \delta_{iin} \qquad \forall i = 0,...,N, n$
VI-5	$\sum_{n=1}^{L_{t_j}} \sum_{i=0 \mid i \ne j}^{N} \delta_{ijn} \ge 1 \qquad \forall j = 1,...,N$
EC-1	$\sum_{j=1}^{N} \sum_{i=0, i \ne j}^{N} \delta_{ij(n-1)} \ge \sum_{j=1}^{N} \sum_{i=0, i \ne j}^{N} \delta_{ijn} \qquad \forall t, n = (F_t + 2),...,(L_t - 1)$
EC-2	$X_{jn} \le M_{jt}(2 - \sum_{i=0}^{N} \delta_{ij(n-1)} - \delta_{jjn}) \qquad \forall t, j = 1,...,N, n = (F_t + 1),...,L_t$

Figure 4.6 The List of Additional Inequalities Used in the Preliminary Experiments

Table 4.3 below summarizes the three fields of test options used in the preliminary experiments.

Table 4.3 Legend for Test Options

Option1: Formulation type	
1	Original (I&L)
2	Transportation Problem (TP)
Option2: VI - Valid inequalities	
0	None
1	VI-1
2	VI-2
3	VI-3
4	VI-4
5	VI-5
6	VI-1 + VI-2
7	VI-1 + VI-2 + VI-3 + VI-4 + VI-5
8	VI-1 + VI-5
Option3: EC - Elimination Constraints	
0	None
5	EC-1 + EC-2

According to the notation we have developed, each solution setting is characterized by the sequential combination of these three test options. For instance, the setting 285 implies the representation of an instance using the TP formulation with an inclusion of VI-1 & VI-5 and EC-1 & EC-2.

The two EC-type inequalities were considered as a pair and were always added to the models together. Moreover, since we expected only a minor contribution from their incorporation, we have not tested them in every possible Option2 combination but only together with VI-1 & VI-5 (i.e., setting 8 for Option2) and on their own (setting 0 for Option2). Thus, overall, we have 22 options to test for each of the 20 instances in this preliminary study.

All models were coded with Turbo Pascal 7.0 (Borland) and solved by CPLEX 8.1.0 with default solution options on Pentium IV 1.6 GHz. PC's with 256 MB RAM running Windows NT Workstation 4.0. The computation times are given in CPU seconds on this machine setting. The Prime Modulus Multiplicative Linear Congruential Generator described by Law and Kelton (2000) was used as the random number generator.

For each of the tests, selected instances were solved with the selected options with a time limit of 2 CPU hours (7200 CPU seconds). In case the optimal solution cannot be obtained within this time limit, the time limit was extended for the sake of obtaining the optimal function value, if possible.

4.2.2 Preliminary Run Results

In this section, we provide a summary of the results obtained by solving the twenty instances under the three different settings (i.e., Cases A, B and C). For each case, the results of the corresponding mixed integer (MIP) as well as the linear programming relaxation (LPR) solutions are provided.

Below is the set of statistics that we mainly make use of for the preliminary analysis:

- **CPU**
  This statistic denotes the MIP solution time (in CPU seconds) for the corresponding option. Note that the termination limit for the solution time is 7200 CPU seconds.

- **CPU R** (CPU Ratio, i.e., CPU/CPU_r)
  This ratio indicates the proportion of the MIP solution time of the corresponding option to the reference MIP solution time, that is, the I&L formulation without any additional inequalities (option 100).

- **LPR t**
  This statistic denotes the LPR solution time (in CPU seconds) for the corresponding option.

- **LPR R** (LPR Ratio, i.e., LPR/LPR_r)
  This statistic denotes the ratio between the LP relaxation value of the corresponding option and the reference LP relaxation value (of option 100).

- **IGap%** (Integrality Gap, i.e., %(opt-LPR)/opt)
  It measures the percentage gap between the optimal solution of the instance and the LP relaxation value for the option being tested.

- **#opt**

  This statistic counts the number of times the optimal solution is found with the tested option within the time limit.

- **DU%** (Upper Bound Deviation, i.e., %(UB-opt)/opt)

  It measures the percentage gap between the best feasible solution (upper bound) obtained at the end of the time-limited run with the corresponding option and the optimal solution of the instance.

- **ULP%** (Upper Bound-LPR Deviation, i.e., %(UB-LPR)/LPR)

  It measures the percentage gap between the best feasible solution obtained for the tested option at the end of the time-limited run and the LP relaxation value under the same option.

- **Nodes**

  It denotes the total number of nodes generated by the tested option within the time-limited run.

- **Node R** (Node Ratio, i.e., $\#Nodes/\#Nodes_r$)

  This ratio represents the proportion of the total number of nodes generated by an option to the total number of nodes for the reference option (option 100).

4.2.2.1 Results of Case A (Augmented & Sequence Dependent Changeover Times)

As has been mentioned in Section 4.2.1, the data used Meyr (2000) includes sequence independent changeover times which are also relatively small in size compared with period capacities. For this reason, we have decided to base our preliminary analysis on a setting closer to what is featured by our mathematical formulation, namely where changeover times are higher and sequence dependent.

Regarding the modifications made on the original data of Meyr (2000), first of all, we have incorporated sequence dependency into changeover times. Sequence dependent changeover time values were generated in such a way so as to maintain the mean value for each item equal to its original sequence independent changeover time. Each sequence dependent changeover time entry was produced assuming an inverse relationship with the corresponding changeover cost with the purpose of creating an inherent tradeoff between the two. Although we expected the problem instances to become harder to solve in this case, the results of a few rough cut experiments have shown that the effects of sequence dependency are not significant, because the impact of changeover times is very small in these instances compared with changeover costs. Hence, secondly, we considered augmenting the size of the changeover times so that they have a more pronounced effect on the solution. In order to do so, we have distributed the available cumulative capacity margin (Cumulative Capacity-Cumulative Demands) to changeover times. Details related with the modification of the data to be used in this case are provided in Appendix B for reference.

Out of the total 20 instances tested under Case A, 4 turn out to be infeasible, due to increased changeover times and decreased capacity margins after modifications. Among the remaining 16 instances, the optimal solutions are known for all but one instance. Solution values of individual instances within the time limit are provided in Appendix C.1 along with extended solutions for the instances that cannot be solved within 2 hours.

Instances with Known Optimal Solutions

Table 4.4 on the next page contains detailed results regarding the LPR performance of the 22 test options for the 15 instances with known optimal solutions. Note that for some of these instances, the optimal solutions have been obtained outside the time limit. From these results, it can be seen that some of the options are capable of reducing the integrality gap of the pure formulations, i.e., those without any additional inequalities, considerably at the expense of increased LPR times. Examination of individual results as well averages reveals that the performance of options 150, 170, 180, 185, 250, 270, 280 and 285 appear to be promising in terms of their LPR performance, therefore these may be regarded as our favourable options at this point.

Another observation is that using the alternative TP formulation helps to reduce the integrality gaps only slightly. When the gaps for I&L and TP options are compared across individual instances, it can be seen that the difference does not go beyond 8%. (See options 140-240 for instance 5 and options 160-260 for instances 5 and 6). Thus, we can tentatively infer that although a change in formulation slightly improves the LPR performance, it is not as effective as the addition of a set of favorable additional inequalities.

Table 4.4 Detailed Results for Instances with Known Optimals (Case A)

		100		105		110		120		130		140		150		160		170		180		185	
instance		IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t
1	5x4	50.1	0.2	50.1	0.1	46.8	0.1	47.2	0.6	50.1	0.2	48.6	0.3	2.2	0.1	46.8	0.3	2.1	0.7	2.1	0.3	2.1	0.5
2	5x4	76.0	0.1	76.0	0.1	73.0	0.1	73.4	0.7	76.0	0.1	74.2	0.3	19.8	0.1	73.0	0.2	19.7	0.5	19.7	0.2	19.7	0.3
3	2x4	42.3	0.0	42.3	0.0	35.9	0.0	36.2	0.0	42.3	0.0	37.3	0.0	11.6	0.0	35.9	0.0	11.3	0.0	11.3	0.0	11.3	0.0
4	5x4	58.5	0.1	58.5	0.1	55.3	0.1	55.6	0.6	58.5	0.2	56.7	0.4	3.9	0.1	55.3	0.2	3.7	1.0	3.7	0.3	3.7	0.6
5	3x4	95.2	0.0	95.2	0.0	91.0	0.0	91.1	0.1	95.2	0.0	93.3	0.0	43.9	0.0	91.0	0.0	42.2	0.0	42.2	0.0	42.2	0.0
6	4x4	100.0	0.0	100.0	0.0	93.6	0.0	96.6	0.1	100.0	0.0	97.4	0.0	26.0	0.0	93.6	0.0	2.9	0.1	2.9	0.0	2.9	0.1
8	5x4	68.1	0.1	68.1	0.1	64.6	0.1	65.6	1.1	68.1	0.2	66.9	0.4	14.9	0.1	64.6	0.2	14.6	0.7	14.6	0.3	14.6	0.5
9	5x4	68.9	0.1	68.9	0.1	64.9	0.1	65.4	0.7	68.9	0.1	66.9	0.4	4.8	0.1	64.9	0.2	4.6	0.7	4.6	0.3	4.6	0.5
10	5x8	59.5	0.3	59.5	0.3	57.4	0.5	57.6	3.9	59.5	0.5	58.6	1.5	31.9	0.3	57.4	1.2	31.8	1.8	31.8	0.9	31.8	1.4
11	5x8	50.5	0.3	50.5	0.5	47.6	0.5	47.9	2.7	50.5	0.8	49.4	1.4	13.0	0.5	47.6	0.8	12.6	2.2	12.6	1.1	12.6	2.2
15	6x4	70.8	0.2	70.8	0.3	67.2	0.2	69.1	2.1	70.8	0.5	70.0	1.0	4.0	0.3	67.2	0.6	3.6	2.7	3.6	1.0	3.6	1.6
16	7x4	56.8	0.3	56.8	0.3	54.5	0.3	56.1	2.9	56.8	0.8	56.4	1.2	16.0	0.4	54.5	0.7	3.7	4.6	3.7	1.4	3.7	3.1
18	8x4	47.5	0.6	47.5	0.9	46.3	1.1	46.9	9.8	47.5	1.4	47.3	3.3	17.8	0.6	46.3	1.7	13.8	10.5	13.8	3.8	13.8	8.5
19	9x4	48.4	1.2	48.4	1.4	47.4	1.4	48.0	15.3	48.4	2.9	48.2	5.6	9.5	1.1	47.4	3.1	8.1	17.0	8.1	7.9	8.1	16.1
20	9x4	68.6	0.7	68.6	1.1	67.5	1.4	68.0	8.6	68.6	2.4	68.3	5.9	29.2	0.9	67.5	2.6	18.1	15.2	18.1	8.3	18.1	15.6
AVERAGE		64.1	0.3	64.1	0.4	60.9	0.4	61.6	3.3	64.1	0.7	62.6	1.4	16.6	0.3	60.9	0.8	12.8	3.8	12.8	1.7	12.8	3.4

		200		205		210		220		230		240		250		260		270		280		285	
instance		IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t
1	5x4	50.1	0.2	50.1	0.2	45.6	0.2	46.1	1.2	50.1	0.3	47.9	0.5	2.2	0.3	45.6	0.4	2.0	0.9	2.0	0.4	2.0	0.7
2	5x4	75.9	0.2	75.9	0.2	69.2	0.3	69.9	1.6	75.9	0.4	71.4	0.6	19.8	0.2	69.2	0.5	19.5	0.9	19.5	0.4	19.5	0.7
3	2x4	36.3	0.0	36.3	0.0	33.3	0.0	33.3	0.0	36.3	0.0	35.9	0.0	11.4	0.0	33.3	0.0	11.3	0.0	11.3	0.0	11.3	0.0
4	5x4	58.4	0.1	58.4	0.1	53.4	0.1	54.0	1.0	58.4	0.2	55.8	0.5	3.9	0.2	53.4	0.4	3.6	0.9	3.6	0.4	3.6	0.5
5	3x4	94.8	0.0	94.8	0.0	82.5	0.0	83.4	0.1	94.8	0.0	85.0	0.0	43.9	0.0	82.5	0.0	41.7	0.1	41.7	0.0	41.7	0.0
6	4x4	99.7	0.0	99.7	0.0	85.3	0.0	90.5	0.1	99.7	0.1	92.2	0.0	25.9	0.0	85.3	0.1	2.5	0.1	2.5	0.1	2.5	0.1
8	5x4	68.1	0.2	68.1	0.2	63.0	0.2	64.4	1.8	68.1	0.5	65.6	0.8	14.9	0.2	63.0	0.4	14.6	1.1	14.6	0.5	14.6	0.7
9	5x4	68.7	0.2	68.7	0.2	59.8	0.2	60.7	1.3	68.7	0.4	64.7	0.5	4.8	0.2	59.8	0.5	4.5	1.1	4.5	0.6	4.5	0.7
10	5x8	59.4	1.3	59.4	1.5	55.5	1.5	56.1	11.1	59.4	2.4	57.2	3.5	31.9	1.3	55.5	3.0	31.7	4.1	31.7	2.5	31.7	3.2
11	5x8	50.4	0.7	50.4	0.7	46.5	0.6	46.9	4.4	50.4	1.0	48.7	1.6	13.0	0.5	46.5	0.8	12.6	3.9	12.6	1.7	12.6	2.2
15	6x4	70.6	0.7	70.6	0.8	64.7	0.6	67.3	3.8	70.6	1.1	68.9	1.9	3.8	0.6	64.7	1.0	3.3	4.3	3.3	1.5	3.3	2.7
16	7x4	56.7	0.7	56.7	0.7	53.1	0.8	55.2	4.7	56.7	1.7	55.9	1.6	16.0	0.9	53.1	1.5	3.7	3.9	3.7	2.9	3.7	3.5
18	8x4	47.5	1.4	47.5	2.5	44.4	3.3	45.6	21.1	47.5	3.7	46.4	5.6	17.8	1.8	44.4	4.8	13.3	13.8	13.3	8.4	13.3	13.4
19	9x4	48.4	3.3	48.4	3.9	45.7	5.9	46.9	30.7	48.4	7.4	47.5	9.0	9.4	4.3	45.7	8.4	7.5	24.4	7.5	13.4	7.5	23.4
20	9x4	68.6	3.2	68.6	3.0	65.3	4.2	66.5	17.2	68.6	5.7	67.2	10.7	29.2	4.3	65.3	6.7	17.6	25.0	17.6	14.5	17.6	23.3
AVERAGE		63.6	0.8	63.6	0.9	57.8	1.2	59.1	6.7	63.6	1.7	60.7	2.4	16.5	1.0	57.8	1.9	12.6	5.6	12.6	3.1	12.6	5.0

Table 4.5 below displays the average statistics for the 22 test options over 12 instances (excluding the 3 easy instances) with known optimal solutions. The three easy instances have been eliminated, since their effect on overall averages may be misleading, as it has been pointed out previously in Section 4.2.1. The averages over all 15 instances may be found in Appendix C.3.

Table 4.5 Case A- Average Results for the 12 Instances with Known Optimals (Excluding Easy Instances) *

	CPU	CPU R	LPR R	IGap%	#opt *	DU%	ULP%	Nodes	Node R
100	limit	1.00	1.00	60.3	5 (0)	3.2	183.3	514166	1.00
105	limit	1.00	1.00	60.3	5 (0)	5.6	189.1	470912	0.92
110	5843.4	0.81	1.07	57.7	8 (5)	2.2	153.7	151330	0.34
120	6860.3	0.95	1.05	58.4	6 (1)	2.8	166.5	61857	0.12
130	limit	1.00	1.00	60.3	5 (0)	2.6	181.6	289766	0.58
140	limit	1.00	1.03	59.3	5 (0)	3.4	167.5	266027	0.51
150	4753.1	0.66	2.30	13.9	6 (6)	6.2	25.2	214875	0.63
160	5834.9	0.81	1.07	57.7	8 (5)	2.2	153.7	153326	0.35
170	3218.0	0.45	2.36	11.4	9 (7)	0.9	15.3	13704	0.06
180	2839.4	0.39	2.36	11.4	9 (8)	0.1	14.2	26073	0.13
185	2548.4	0.35	2.36	11.4	9 (8)	0.7	15.0	12534	0.06
200	limit	1.00	1.00	60.2	7 (0)	3.4	182.6	486033	0.91
205	limit	1.00	1.00	60.2	5 (0)	5.6	189.0	419029	0.79
210	5753.2	0.80	1.14	55.5	8 (4)	0.6	134.7	118656	0.27
220	6811.9	0.95	1.11	56.6	7 (1)	1.6	144.1	61449	0.11
230	limit	1.00	1.00	60.2	5 (0)	3.8	183.7	267953	0.52
240	limit	1.00	1.06	58.1	6 (0)	4.4	159.1	244575	0.46
250	5602.2	0.78	2.30	13.9	6 (4)	5.1	23.6	243706	0.58
260	5761.0	0.80	1.14	55.5	8 (4)	0.6	134.7	119104	0.27
270	2800.5	0.39	2.37	11.2	9 (8)	0.2	14.0	9818	0.05
280	2542.9	0.35	2.37	11.2	9 (8)	0.1	13.9	15591	0.08
285	2490.3	0.35	2.37	11.2	8 (8)	0.1	13.9	9473	0.05

* "limit" denotes 7200 CPU seconds, the termination limit for the solution time.
#opt denotes the number of times the optimal solution was found, where the entry in parentheses denotes the number of verified optimal solutions.

Regarding LP relaxations, cross comparison of the average LPR ratios reveals that the performance of the options with the TP formulation are close to those of the corresponding I&L options. Our tentative inference that a change in formulation only slightly improves the integrality gap, whereas the incorporation of strong additional inequalities has a considerable effect, is in fact verified by the average results excluding the easy instances. Following Table 4.5, we observe that the gap corresponding to the pure I&L formulation is 60.3% which only reduces to 60.2% with the pure TP formulation but drastically to around 11.2% for the options enhanced with favourable inequalities. Generally speaking, the favourable options (namely options x70, x80 and x85) yield the strongest LPR. The LPR statistics for these options turn out to be identical and we expect to observe a distinction between them only by observing the MIP performances.

Our assessment of the MIP performance relies mainly on two criteria: the average solution time (CPU) and the deviation between the final UB & the optimal solution (DU%). We do not use the information about the best lower bound (LB) obtained at the end of the MIP solution, since this measure essentially reflects the performance of the commercial solver and its inherent solution mechanisms, e.g. automatically generated cuts, on which we have no direct control.

Regarding average CPU, we can see that the incorporation of any additional inequality except VI-3, VI-4 and EC's on their own causes a decrease compared with the reference solution time (which is 7200, the time limit). The fastest solutions can be obtained by options 285 (2490 sec), 280 (2543 sec) and 185 (2548 sec), as can be checked from Table 4.5. Another observation is that for VI added on their own, resorting to the TP formulation generally causes reductions in solution time.

#opt column displays two types of information, the first is the number of times the best UB at the end of the time limit is equal to the optimal solution (regardless of whether there is a solution gap or not) and the second inside the parentheses denotes the verified optimal solutions, i.e. solutions that terminate within the time limit with zero solution gap. #opt statistic reveals that the highest number of optimal solutions are found by the use of the favourable options, which is 8 and 9 times out of the 12 instances.

The percent gap between the UB and the optimal solution (DU%) is less than 6% for almost all options, indicating that the solver is capable of obtaining good quality feasible solutions within the time limit. The incorporation of the favourable additional inequalities causes a decrease in the size of this gap. However, the effect of a change in formulation does not seem to be significant.

Since UB's are very close to optimal solutions, examination of the gap between UB and LPR values (ULP%) leads to conclusions which are similar to those obtained as a result of observing the integrality gap. We also note that for small size instances (those smaller than 6x4), there is hardly any differentiation between the UB values obtained for different options within the time limit, as can be seen from the detailed individual statistics for each instance in Appendix C.2. The effect of using different formulations and additional inequality combinations starts to reflect itself in the UB's as the size of the instances increases.

Observation of the average number of nodes and the Node R (ratio of the number of nodes) reveals the TP formulation generates much fewer nodes compared with the I&L formulation. Fewest nodes are generated by option 285 on the average. If we refer back to Table 4.4, we can see that the average LPR time of this option is among the highest among all options. This implies that during the MIP solution, option 285 generates fewer nodes, spends considerable amount of time in dealing with each and results in the shortest average MIP solution times.

Instances for which the Optimal Solutions Cannot be Obtained within the Time Limit

There are five instances in Case A that cannot be solved within the time limit under any option. Four of these (instances 10, 18, 19 and 20) could be solved optimally by extending the time limit but for one (instance 13), the optimal solution is not known.

The LPR performance of the four instances that are solved in extended times are already included in Table 4.4. For the unsolved instance 13, LPR R and LPR t statistics are provided in Table 4.6 below.

Table 4.6 LPR Statistics of Instances for Which the Optimal Solution is Not Known (Case A) *

	LPR R	LPR t
100	1.00	1.52
105	1.00	3.70
110	1.02	4.83
120	1.01	54.49
130	1.00	6.15
140	1.01	23.23
150	1.74	1.36
160	1.02	7.45
170	1.90	30.73
180	1.90	20.48
185	1.90	25.24
200	1.00	4.85
205	1.00	8.69
210	1.07	11.72
220	1.04	87.20
230	1.00	11.51
240	1.02	27.83
250	1.74	5.67
260	1.07	17.52
270	1.92	71.64
280	1.92	38.54
285	1.92	59.29

* For instance 13 only

Again we can infer that LPR is strengthened with the favourable options. LPR solution times have increased considerably compared with those of optimally solvable instances, which is an expected result. Although LPR strength does not differ much between the favourable options, x70 options give the longest LPR times compared with x80 and x85.

Table 4.7 on the next page displays the detailed statistics for all five instances that require extension of the time limit (including the unsolvable instance 13). Information regarding the solutions obtained within extended times for each instance can be found in Appendix C.1.

Since the time limit is exceeded for all options and all instances, we have not included any statistics about MIP solution times in Table 4.7. (The average extended solution time per instance until optimality is close to 24 CPU hours.) The deviation between the upper bound and the optimal solutions for instance 13 cannot be computed as the optimal solution is not known. Moreover, for one of the instances (instance 19), no feasible solution could be obtained within the time limit for several options. As a result, the corresponding statistics involving UB's cannot be computed and these cases are marked as "*nofeas*" in the table.

DU% statistic is lowest for the favourable options (smaller than 1% for TP formulations), which indicates that the quality of the upper bounds obtained at the end of the time limit is close to optimality. This in fact is a phenomenon that is frequently observed during the solution of such hard models with considerable size, where the algorithm is able to find a good quality upper bound within a reasonable amount of time, but then starts to increase the lower bound with minuscule improvements until the upper bound is finally proven to be optimal.

When the ULP% statistic is analyzed for the five instances in Table 4.7, it can be seen that it is considerably smaller for the favourable options (26% for option 285) than for the reference case (191% for option 100, the pure I&L formulation). This improvement is partly due to increased upper bound quality, and partly to stronger LPR values.

Since we do not know the optimal solution for instance 13, we cannot compute the exact deviation of the upper bound. However, for the instances with known optimal solutions (excluding easy instances), the average gap between the optimal and the LPR is close 14% of the LPR value under the favourable options (x70, x80 and x85). For instance 13, the smallest value for the ULP% statistic is 37.8% under option 285. This suggests that the UB is approximately 24% (of the LPR) away from the optimal solution.

Table 4.7 Detailed Results for Instances Exceeding the Time Limit (Case A) *

		100			105			110			120			130			140			150			160			170			180			185		
instance		DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t
10	5x8	8.6	167.9	0.3	15.0	183.7	0.3	1.8	138.8	0.5	8.4	155.8	3.9	9.2	169.3	0.5	10.0	165.7	1.5	9.0	60.0	0.3	1.8	138.8	1.2	8.6	59.1	1.8	0.7	47.6	0.9	7.4	57.4	1.4
18	8x4	10.2	109.9	0.6	22.5	133.5	0.9	8.6	102.3	1.1	3.4	94.6	9.8	8.1	106.0	1.4	5.0	99.2	3.3	6.4	29.4	0.6	8.6	102.3	1.7	0.2	16.2	10.5	0.0	16.0	3.8	0.0	16.0	8.5
19	9x4	nofeas	nofeas	1.2	nofeas	nofeas	1.4	6.0	101.7	1.4	nofeas	nofeas	15.3	nofeas	nofeas	2.9	18.4	128.6	5.6	39.1	53.6	1.1	6.0	101.7	3.1	0.0	8.8	17.0	0.7	9.5	7.9	0.4	9.2	16.1
20	9x4	15.9	268.6	0.7	23.0	291.3	1.1	9.4	236.5	1.4	15.7	261.2	8.6	9.7	248.8	2.4	7.2	238.0	5.9	12.9	59.5	0.9	9.4	236.5	2.6	2.1	24.7	15.2	0.2	22.4	8.3	0.4	22.6	15.6
AVERAGE		11.6	182.1	0.7	20.2	202.8	0.9	6.5	144.8	1.1	9.2	170.5	9.4	9.0	174.7	1.8	10.2	157.9	4.1	16.8	50.6	0.7	6.5	144.8	2.2	2.7	27.2	11.1	0.4	23.9	5.2	2.0	26.3	10.4
13	9x8	-	218.1	1.5	-	284.8	3.7	-	190.6	4.8	-	229.8	54.5	-	227.7	6.2	-	216.7	23.2	-	86.3	1.4	-	190.6	7.5	-	47.4	30.7	-	45.8	20.5	-	64.6	25.2
OVERALL		11.6	191.1	0.9	20.2	223.3	1.5	6.5	154.0	1.8	9.2	185.3	18.4	9.0	188.0	2.7	10.2	169.6	7.9	16.8	57.8	0.9	6.5	154.0	3.2	2.7	31.3	15.0	0.4	28.2	8.3	2.0	34.0	13.3

		200			205			210			220			230			240			250			260			270			280			285		
instance		DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t
10	5x8	8.4	167.3	1.3	18.8	192.9	1.5	0.8	126.7	1.5	8.2	146.2	11.1	8.7	168.0	2.4	2.5	139.6	3.5	1.9	49.6	1.3	0.8	126.7	3.0	1.1	47.9	4.1	0.8	47.5	2.5	0.0	46.4	3.2
18	8x4	15.4	119.8	1.4	12.7	114.7	2.5	3.5	86.2	3.3	7.5	97.7	21.1	10.1	109.7	3.7	11.3	107.7	5.6	6.9	29.9	1.8	3.5	86.2	4.8	0.2	15.6	13.8	0.2	15.6	8.4	0.6	16.1	13.4
19	9x4	nofeas	nofeas	3.3	nofeas	nofeas	3.9	2.1	88.1	5.9	0.6	89.6	30.7	nofeas	nofeas	7.4	33.0	153.5	9.0	39.8	54.4	4.3	2.1	88.1	8.4	0.0	8.1	24.4	0.0	8.1	13.4	0.3	8.4	23.4
20	9x4	12.1	256.6	3.2	24.5	296.0	3.0	0.5	189.3	4.2	2.9	206.8	17.2	13.5	260.9	5.7	5.9	222.7	10.7	11.3	57.3	4.3	0.5	189.3	6.7	1.2	22.9	25.0	0.2	21.7	14.5	0.6	22.2	23.3
AVERAGE		12.0	181.2	2.3	18.7	201.2	2.7	1.7	122.6	3.7	4.8	135.1	20.0	10.8	179.5	4.8	13.2	155.9	7.2	15.0	47.8	2.9	1.7	122.6	5.7	0.6	23.6	16.8	0.3	23.2	9.7	0.4	23.3	15.8
13	9x8	-	225.8	4.9	-	293.1	8.7	-	164.1	11.7	-	171.3	87.2	-	220.1	11.5	-	213.8	27.8	-	82.9	5.7	-	164.1	17.5	-	40.8	71.6	-	41.9	38.5	-	37.8	59.3
OVERALL		12.0	192.4	2.8	18.7	224.2	3.9	1.7	130.9	5.3	4.8	142.3	33.4	10.8	189.7	6.1	13.2	167.5	11.3	15.0	54.8	3.5	1.7	130.9	8.1	0.6	27.1	27.8	0.3	27.0	15.5	0.4	26.2	24.5

* "nofeas" indicates that no feasible solution could be obtained within the time limit, therefore the corresponding statistic cannot be computed. In this table, the optimal solution could be obtained in extended time for the 4 instances at the top (i.e. , 10, 18, 19, 20), but it is not known for instance 13.

Conclusions Regarding the TP formulation and VI

Below are a few concluding remarks about the preliminary results for Case A.

Among the VI tested on their own, VI-5 seems to have the strongest impact, followed by VI-1. VI-2 and VI-4 also bring about improvements on the pure formulations, but their effect is not as strong as those of VI-1 and VI-5. VI-3 only seems to have a weak impact.

In all runs and with both formulations, VI-1 is better than VI-2 in terms of the strength of the LPR, integrality gap, UB deviation and MIP solution time. Moreover, we observe that the performance of VI-1 and VI-2 together (i.e., option x6x) is exactly the same as using VI-1 on its own (Option x1x), indicating that VI-2 does not bring an extra contribution on top of VI-1 when both are present. Therefore, we can conclude that VI-1 outperforms VI-2.

The effect of EC's on the LPR performance is negligible. In terms of UB deviation and the integrality gap, the performances of the options enhanced with EC's (Options 285 and 185) are close to their respective counterparts (Options 280 and 180) for optimally solvable instances. However, the former generate fewer nodes and they can be solved in shorter times on the average. Although the UB performance of the options enhanced with the EC's (Option x85) is observed to be slightly worse in comparison with Option x80 for the instances exceeding the time limit, the results at this point indicate that the incorporation of EC's seems to be effective. However, further tests may be necessary to justify their usage in the case of harder problem instances.

The collective performance of all VI together (Option x70) is the same in terms of LPR statistics compared with that of VI-1 and VI-5 together (Option x80). The difference between the two is that the former requires longer solution times but generates fewer nodes in return. However, options x70 perform worse than options x85 in terms of number of nodes, solution times and upper bound deviation.

Hence, we can say that out of all the options tested, 180, 185, 280 and 285 stand out as viable alternatives. 280 and 285 yield stronger LPR and thus smaller optimality gaps. Among the two, 285 generates fewer nodes and provides shorter MIP solution times. Moreover, observation of the number of nodes reveals that generally the TP formulation generates fewer nodes and spends more time on each node compared with the I&L formulation.

One final remark is that the incorporation of useful additional inequalities also contributes to the detection of infeasibility in this case. When we examine the individual LPR and MIP statistics for the infeasible instances in Appendix C.2, we see that the options without the effective inequalities generally reach the time termination limit without even a sign of infeasibility whereas the favored options detect infeasibility only in a few seconds.

4.2.2.2 Results of Case B (Augmented & Sequence Dependent Changeover Times, No Changeover Costs)

In this extension case, we have used the same instances in Case A but we set all changeover costs to zero, while everything else remains the same. In doing so, we intend to test the performance of the favorable options under the situation where the effect of changeover times is not dominated by those of changeover costs in the objective function. The corresponding results will be briefly discussed below.

Since the same instances are used in both cases, the same 4 instances in Case A turn out to be infeasible in Case B as well. Among the remaining 16, the optimal solutions are known for 14 instances.

If the optimal solution values of the instances are examined individually from Appendix D.1, one can realize that they have decreased compared to Case A results (Appendix C.1). This is evident, since changeover costs constituted a major term in the objective function for the first experimental case.

Table 4.8 on the next page contains detailed results regarding the LPR performance of the 22 test options for the 14 instances with known optimal solutions and Table 4.9 following it displays the average statistics for the same instances excluding the easy ones.

Interesting phenomena can be observed regarding LPR statistics. Here, it can be seen that the LPR values across different options are no more different. Moreover, these LPR vales are close to those obtained in Case A for the options without the favourable additional inequalities, such as option 100, while the stronger options have much lower LPR values in Case B compared with Case A. This is an expected result, as in the previous case, the effect of additional inequalities in terms of strengthened changeover and setup variables showed itself in the objective function in the form of changeover costs, which is not possible any more.

Table 4.8 Detailed Results for Instances with Known Optimals (Case B)

		100		105		110		120		130		140		150		160		170		180		185	
instance		IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t
1	5x4	2.2	0.1	2.2	0.1	2.2	0.1	2.2	0.7	2.2	0.2	2.2	0.2	2.2	0.1	2.2	0.2	2.2	0.7	2.2	0.3	2.2	0.4
2	5x4	17.0	0.1	17.0	0.1	17.0	0.1	17.0	0.6	17.0	0.1	17.0	0.1	17.0	0.1	17.0	0.2	16.9	0.4	16.9	0.2	16.9	0.3
3	2x4	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0
4	5x4	6.6	0.1	6.6	0.1	6.6	0.1	6.6	0.8	6.6	0.2	6.6	0.2	6.6	0.1	6.6	0.2	6.5	0.7	6.5	0.3	6.5	0.5
5	3x4	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0
6	4x4	100.0	0.0	100.0	0.0	100.0	0.0	100.0	0.1	100.0	0.0	100.0	0.0	100.0	0.0	100.0	0.0	98.3	0.1	98.3	0.0	98.3	0.0
8	5x4	7.4	0.1	7.4	0.1	7.4	0.1	7.4	0.8	7.4	0.2	7.4	0.2	7.4	0.1	7.4	0.2	6.9	0.6	6.9	0.3	6.9	0.3
9	5x4	9.2	0.1	9.2	0.1	9.2	0.1	9.2	0.7	9.2	0.1	9.2	0.2	9.2	0.1	9.2	0.3	9.1	0.6	9.1	0.3	9.1	0.4
10	5x8	6.6	0.3	6.6	0.3	6.6	0.4	6.6	2.6	6.6	0.5	6.6	0.5	6.6	0.3	6.6	0.7	6.6	1.2	6.6	0.7	6.6	1.0
11	5x8	5.4	0.2	5.4	0.3	5.4	0.4	5.4	2.7	5.4	0.7	5.4	0.9	5.4	0.3	5.4	0.5	5.4	1.3	5.4	0.9	5.4	1.3
15	6x4	10.3	0.1	10.3	0.2	10.3	0.2	10.3	1.5	10.3	0.3	10.3	0.6	10.3	0.2	10.3	0.6	10.1	2.4	10.1	0.9	10.1	1.1
16	7x4	5.1	0.2	5.1	0.2	5.1	0.2	5.1	3.3	5.1	0.3	5.1	0.6	5.1	0.3	5.1	0.5	4.9	3.0	4.9	1.0	4.9	1.9
18	8x4	7.1	0.3	7.1	0.4	7.0	1.0	7.1	10.0	7.1	0.6	7.1	1.7	7.1	0.3	7.0	2.4	6.9	7.5	6.9	4.0	6.9	6.4
20	9x4	15.7	0.4	15.7	0.5	15.7	1.3	15.7	7.5	15.7	1.5	15.7	1.5	15.7	0.6	15.7	1.7	15.6	8.5	15.6	7.5	15.6	14.4
AVERAGE		20.6	0.1	20.6	0.2	20.6	0.3	20.6	2.2	20.6	0.3	20.6	0.5	20.6	0.2	20.6	0.5	20.4	1.9	20.4	1.2	20.4	2.0

		200		205		210		220		230		240		250		260		270		280		285	
instance		IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t
1	5x4	2.2	0.2	2.2	0.2	2.2	0.2	2.2	0.9	2.2	0.4	2.2	0.3	2.2	0.2	2.2	0.3	2.2	0.5	2.2	0.2	2.2	0.5
2	5x4	17.0	0.2	17.0	0.2	17.0	0.3	17.0	1.6	17.0	0.4	17.0	0.3	17.0	0.3	17.0	0.6	16.9	0.9	16.9	0.5	16.9	0.5
3	2x4	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0	8.8	0.0
4	5x4	6.6	0.1	6.6	0.2	6.6	0.1	6.6	1.1	6.6	0.3	6.6	0.3	6.6	0.2	6.6	0.4	6.5	0.8	6.5	0.3	6.5	0.5
5	3x4	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.0	87.1	0.1	87.1	0.0	87.1	0.0
6	4x4	92.0	0.0	92.0	0.0	92.0	0.0	92.0	0.1	92.0	0.0	92.0	0.0	92.0	0.0	92.0	0.1	92.0	0.1	92.0	0.0	92.0	0.1
8	5x4	7.4	0.2	7.4	0.2	7.3	0.2	7.3	1.3	7.4	0.4	7.4	0.4	7.3	0.2	7.3	0.5	6.9	0.7	6.9	0.4	6.9	0.6
9	5x4	9.2	0.2	9.2	0.2	9.2	0.4	9.2	1.5	9.2	0.3	9.2	0.4	9.2	0.2	9.2	0.4	9.1	1.0	9.1	0.5	9.1	0.5
10	5x8	6.6	1.1	6.6	1.4	6.6	1.1	6.6	8.7	6.6	2.1	6.6	2.2	6.6	0.9	6.6	1.8	6.6	3.5	6.6	1.5	6.6	2.2
11	5x8	5.4	0.6	5.4	0.5	5.4	0.7	5.4	3.0	5.4	1.1	5.4	1.2	5.4	0.5	5.4	0.9	5.4	1.5	5.4	0.9	5.4	1.4
15	6x4	10.1	0.6	10.1	0.8	10.0	0.8	10.0	3.4	10.1	1.0	10.1	0.9	10.1	0.6	10.0	1.3	9.6	2.5	9.6	1.4	9.6	2.8
16	7x4	5.1	0.4	5.1	0.4	5.0	0.7	5.0	6.6	5.1	1.3	5.1	1.0	5.1	0.5	5.0	1.3	4.9	2.0	4.9	1.5	4.9	1.7
18	8x4	7.1	2.1	7.1	2.1	6.9	3.2	7.0	18.1	7.1	3.8	7.1	3.6	7.1	1.8	6.9	6.5	6.8	13.2	6.8	5.9	6.8	7.9
20	9x4	15.7	3.2	15.7	2.8	15.6	5.4	15.6	17.8	15.7	7.7	15.7	4.8	15.7	3.2	15.6	5.1	15.5	16.4	15.5	11.7	15.5	19.1
AVERAGE		20.0	0.6	20.0	0.6	20.0	0.9	20.0	4.6	20.0	1.3	20.0	1.1	20.0	0.6	20.0	1.4	19.9	3.1	19.9	1.8	19.9	2.7

Table 4.9 Case B- Average Results for the 11 Instances with Known Optimals (Excluding Easy Instances) *

	CPU	CPU R	LPR R	IGap%	#opt *	DU%	ULP%	Nodes	Node R
100	3532.7	1.00	1.00	8.4	8 (7)	0.3	9.8	177357	1.00
105	3258.0	0.92	1.00	8.4	8 (7)	0.9	10.4	142850	0.88
110	2272.0	0.49	1.00	8.4	8 (8)	0.3	9.8	46715	0.25
120	4186.8	1.43	1.00	8.4	8 (6)	0.7	10.2	40737	0.24
130	4634.9	1.98	1.00	8.4	8 (5)	0.4	9.9	177372	1.28
140	4279.9	1.62	1.00	8.4	8 (6)	0.5	10.1	159274	1.07
150	4707.7	2.30	1.00	8.4	8 (5)	0.4	9.9	254159	1.89
160	2281.9	0.49	1.00	8.4	8 (8)	0.3	9.8	47281	0.25
170	2803.9	0.72	1.00	8.3	8 (8)	0.2	9.5	24591	0.13
180	2281.4	0.45	1.00	8.3	8 (8)	0.2	9.5	30753	0.15
185	2143.0	0.41	1.00	8.3	8 (8)	0.3	9.6	19940	0.09
200	3684.5	1.07	1.00	8.4	7 (6)	0.2	9.6	159206	0.92
205	3034.1	0.72	1.00	8.4	8 (8)	0.6	10.1	99515	0.54
210	2159.3	0.38	1.00	8.3	9 (8)	0.0	9.4	28745	0.15
220	3297.9	0.91	1.00	8.4	8 (8)	0.2	9.6	19993	0.11
230	4135.4	1.46	1.00	8.4	8 (6)	0.1	9.5	124188	0.78
240	4247.8	1.45	1.00	8.4	8 (7)	0.3	9.7	137902	0.84
250	3670.9	1.55	1.00	8.4	8 (7)	0.3	9.7	151500	1.14
260	2159.2	0.38	1.00	8.3	9 (8)	0.0	9.4	28648	0.14
270	2552.4	0.57	1.00	8.2	8 (8)	0.1	9.3	16675	0.08
280	2365.2	0.46	1.00	8.2	9 (8)	0.0	9.2	26494	0.11
285	2122.0	0.37	1.00	8.2	9 (8)	0.1	9.3	13345	0.06

* #opt denotes the number of times the optimal solution was found, where the entry in parentheses denotes the number of verified optimal solutions.

Parallel to these observations, we note that the average integrality gap has also reduced drastically from around 60% for option 100 in Case A (Table 4.5) down to approximately 8% for the same option in Case B (Table 4.9). The primary reason for this could be the drop in the optimal solution value. In Case A, we were able to observe striking differences between pure options and the options with effective additional inequalities in terms of average integrality gap (60.3% vs. 11.2%), whereas in Case B, the difference virtually melts away (8.4% vs. 8.2%). The gap values between the UB and the LPR (ULP%) are almost identical for different options as well, due to the same reason.

Regarding MIP solution performance, we notice a decrease in average CPU time for this case compared with Case A. It may be conjectured that the problems have become easier to solve in the absence of changeover costs, which is actually contrary to our expectations. The reason for this may be the fact that changeover times are small compared with capacity; therefore their effect is reduced even more without costs. Moreover, in Case

A, we established a negative relation between changeover costs and times, which is also removed in Case B, and this might have facilitated the solution process even more.

DU% statistics have decreased below 1% for all options, which indicates that the upper bounds are almost optimal. Option 285 again results in the lowest solution time and smallest number of nodes and the previously selected favourable options perform close to it. However, in addition to these favorable options, options 110 and 210 also perform considerably well in this case, both in terms of MIP solution time and number of nodes generated. The TP formulation generates much fewer nodes compared with the I&L options in this case as well.

Among the five instances that cannot be solved within the time limit under any option, we were able to solve three (instances 10, 18 and 19) by extending the time limit. The LPR performance of these instances, as well as two for which the optimal is not known (instances 13 and 19), are provided in Table 4.10 on the next page. Following it, LPR R and LPR t statistics are provided in Table 4.11 for the two unsolvable instances.

The results in the two tables also support the fact that there is no significant difference between different options in terms of LPR performance or the quality of upper bounds obtained. However, LPR times are higher for the options which were previously selected as favourable, in line with the observation of lower solution times and fewer nodes for these options.

Since we do not know the optimal solution for two instances (13 and 19), we cannot compute the exact deviation of the upper bound from the optimal solution. Hence, we can also make tentative inferences as we did in Section 4.2.2.1 for Case A. Since the average gap between the optimal and the LPR is around 9% of the LPR for the instances with known optimal solutions (excluding easy instances) for the favourable TP options and the average ULP% statistic for the two unsolvable instances is around 12% for the favourable options, we can consider UB to be approximately 3% (of the LPR) away from the optimal solution, which is quite small.

Table 4.10 Detailed Results for Instances Exceeding the Time Limit (Case B) *

		100			105			110			120			130			140			150			160			170			180			185		
instance		DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t
10	5x8	0.3	7.4	0.3	0.6	7.7	0.3	0.0	7.1	0.4	0.5	7.6	2.6	0.5	7.7	0.5	0.9	8.1	0.5	0.2	7.3	0.3	0.0	7.1	0.7	0.7	7.8	1.2	0.1	7.2	0.7	0.0	7.1	1.0
18	8x4	1.2	8.9	0.3	3.2	11.1	0.4	2.5	10.3	1.0	3.2	11.0	10.0	0.7	8.4	0.6	1.8	9.5	1.7	2.4	10.2	0.3	2.5	10.3	2.4	1.0	8.4	7.5	1.4	8.9	4.0	1.2	8.7	6.4
20	9x4	2.3	21.3	0.4	5.7	25.3	0.5	1.0	19.9	1.3	3.7	23.0	7.5	2.7	21.9	1.5	3.2	22.4	1.5	1.6	20.5	0.6	1.0	19.9	1.7	1.0	19.6	8.5	1.2	19.9	7.5	2.1	20.9	14.4
AVERAGE		1.3	12.5	0.3	3.2	14.7	0.4	1.2	12.4	0.9	2.4	13.9	6.7	1.3	12.6	0.8	2.0	13.3	1.2	1.4	12.7	0.4	1.2	12.4	1.6	0.9	11.9	5.7	0.9	12.0	4.1	1.1	12.2	7.3
13	9x8	-	16.0	1.0	-	18.0	1.4	-	13.0	2.6	-	19.9	18.6	-	18.2	2.6	-	16.7	4.0	-	16.8	1.5	-	13.0	3.6	-	14.3	15.0	-	13.3	8.1	-	16.9	13.7
19	9x4	-	14.5	0.4	-	nofeas	0.6	-	14.3	2.2	-	nofeas	9.3	-	nofeas	1.6	-	25.2	2.5	-	15.0	0.7	-	14.3	3.0	-	15.0	12.1	-	11.9	7.8	-	15.3	11.8
AVERAGE		-	15.2	0.7	-	18.0	1.0	-	13.6	2.4	-	19.9	13.9	-	18.2	2.1	-	20.9	3.2	-	15.9	1.1	-	13.6	3.3	-	14.7	13.6	-	12.6	8.0	-	16.1	12.8
OVERALL		1.3	13.6	0.5	3.2	15.5	0.6	1.2	12.9	1.5	2.4	15.4	9.6	1.3	14.0	1.3	2.0	16.4	2.0	1.4	14.0	0.7	1.2	12.9	2.3	0.9	13.0	8.9	0.9	12.2	5.6	1.1	13.8	9.5

		200			205			210			220			230			240			250			260			270			280			285		
instance		DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t
10	5x8	0.3	7.4	1.1	0.9	8.0	1.4	0.1	7.2	1.1	0.0	7.1	8.7	0.0	7.1	2.1	0.6	7.8	2.2	0.3	7.5	0.9	0.1	7.2	1.8	0.0	7.1	3.5	0.0	7.1	1.5	0.1	7.2	2.2
18	8x4	0.4	8.0	2.1	2.7	10.5	2.1	0.0	7.5	3.2	0.4	7.9	18.1	0.4	8.0	3.8	1.8	9.5	3.6	1.5	9.2	1.8	0.0	7.5	6.5	1.0	8.5	13.2	0.4	7.7	5.9	0.0	7.3	7.9
20	9x4	1.2	20.1	3.2	3.1	22.3	2.8	0.3	18.8	5.4	1.8	20.6	17.8	0.9	19.7	7.7	0.8	19.6	4.8	1.3	20.2	3.2	0.3	18.8	5.1	0.3	18.8	16.4	0.0	18.4	11.7	1.1	19.6	19.1
AVERAGE		0.6	11.8	2.2	2.2	13.6	2.1	0.1	11.2	3.2	0.7	11.9	14.9	0.4	11.6	4.5	1.1	12.3	3.5	1.0	12.3	2.0	0.1	11.2	4.5	0.5	11.4	11.0	0.1	11.1	6.4	0.4	11.4	9.7
13	9x8	-	14.7	3.6	-	21.8	3.9	-	12.8	6.7	-	17.8	34.0	-	15.6	7.0	-	16.3	7.3	-	16.0	4.2	-	12.8	9.9	-	13.0	21.4	-	13.4	15.7	-	14.2	24.8
19	9x4	-	nofeas	3.3	-	nofeas	3.6	-	15.1	6.8	-	16.0	30.5	-	nofeas	7.7	-	nofeas	6.1	-	nofeas	4.6	-	15.1	8.6	-	11.4	66.3	-	13.0	9.8	-	15.7	16.2
AVERAGE		-	14.7	3.4	-	21.8	3.7	-	14.0	6.7	-	16.9	32.2	-	15.6	7.3	-	16.3	6.7	-	16.0	4.4	-	14.0	9.2	-	12.2	43.8	-	13.2	12.8	-	14.9	20.5
OVERALL		0.6	12.6	2.7	2.2	15.6	2.8	0.1	12.3	4.6	0.7	13.9	21.8	0.4	12.6	5.6	1.1	13.3	4.8	1.0	13.2	2.9	0.1	12.3	6.4	0.5	11.7	24.2	0.1	11.9	8.9	0.4	12.8	14.0

* "nofeas" indicates that no feasible solution could be obtained within the time limit, therefore the corresponding statistic cannot be computed. In this table, the optimal solution could be obtained in extended time for the 3 instances at the top (i.e. , 10, 18, 20), but it is not known for instances 13 and 19.

Table 4.11 LPR Statistics of Instances for Which the Optimal Solution is Not Known (Case B) *

	LPR R	LPR t
100	1.00	0.7
105	1.00	1.0
110	1.00	2.4
120	1.00	13.9
130	1.00	2.1
140	1.00	3.2
150	1.00	1.1
160	1.00	3.3
170	1.00	13.6
180	1.00	8.0
185	1.00	12.8
200	1.00	3.4
205	1.00	3.7
210	1.00	6.7
220	1.00	32.2
230	1.00	7.3
240	1.00	6.7
250	1.00	4.4
260	1.00	9.2
270	1.00	43.8
280	1.00	12.8
285	1.00	20.5

* Averages for instances 13 and 19 only

As a result of analyzing the findings for this second case, our expectations that the problems become harder to solve without explicit changeover costs in the objective function have not been confirmed, probably because changeover times only constitute a low level of capacity consumption and the inverse relationship between changeover costs and times are also removed in this case. As a result, changeovers have become ineffective without costs. The differences in the performance of different options, which were clearly pronounced in the previous case, seem to disappear. This suggests that a change of formulation and the incorporation of a favorable set of additional inequalities only pays off in environments featuring changeovers which have a major effect in terms of both cost and capacity consumption. These initial test results have provided a better understanding of the underlying difficulties in solving this problem. However, further experimentation is needed to justify the tentative conclusions regarding the difficulty of solving the problems under different situations depending on whether changeover costs and times are present or how they are correlated with each other etc.

4.2.2.3 Results of Case C (The Original Instances with Sequence Independent Change-over Times)

In our final set of preliminary experiments, we have used changeover times and costs exactly as provided by Meyr (2000) and no modifications were made on the data (Recall Appendix B for the changes made in Case A). Thus, in this setting, changeover times are sequence independent and relatively small in size compared with period capacities and item production requirements.

Out of the total 20 instances tested, no infeasibilities are encountered and the optimal solutions are known for 16 instances.

Table 4.12 on the next page contains detailed results regarding the LPR performance of the 22 test options for the 16 instances with known optimal solutions. These results indicate noticeable differences in performance in favour of the options previously selected as favourable. For instance, the integrality gap of the pure I&L formulation is 66.5% on the average, which falls down to 11.5% for options 270, 280 and 285, coupled with increased LPR times.

Table 4.13 on page 82 contains the average statistics for optimally solvable instances excluding the 3 easy instances. (The averages over all instances may be found in Appendix E.3). These results verify that the options with the TP formulation are stronger than the corresponding I&L options in terms of LPR values. Regarding the performance of the favourable options, we observe results that are very similar to those in Case A, namely that they yield the strongest LP relaxations, lowest UB deviations and smallest solution times. Here, the solution time of option 185 turns out to be lower than that of option 285, but the fewest number of nodes are again generated by option 285. It can also be said that the average solution times are generally lower than those of Case A, presumably because the problems are more relaxed and changeover times are sequence-independent. The highest number of optimal solutions was obtained as a result of using the favourable options.

Table 4.12 Detailed Results for Instances with Known Optimals (Case C)

		100		105		110		120		130		140		150		160		170		180		185	
instance		IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t
1	5x4	49.5	0.1	49.5	0.1	46.2	0.1	46.6	0.7	49.5	0.1	48.0	0.3	1.0	0.1	46.2	0.3	1.0	0.8	1.0	0.4	1.0	0.5
2	5x4	74.9	0.1	74.9	0.1	71.7	0.1	72.2	0.7	74.9	0.1	73.1	0.4	16.2	0.1	71.7	0.3	16.1	0.6	16.1	0.3	16.1	0.2
3	2x4	37.1	0.0	37.1	0.0	30.1	0.0	30.8	0.0	37.1	0.0	31.7	0.0	3.6	0.0	30.1	0.0	3.3	0.0	3.3	0.0	3.3	0.0
4	5x4	57.2	0.1	57.2	0.1	53.9	0.1	54.3	0.6	57.2	0.2	55.4	0.4	1.0	0.1	53.9	0.2	0.9	0.8	0.9	0.3	0.9	0.5
5	3x4	95.0	0.0	95.0	0.0	90.8	0.0	90.9	0.0	95.0	0.0	93.1	0.0	42.5	0.0	90.8	0.0	41.0	0.0	41.0	0.0	41.0	0.0
6	4x4	100.0	0.0	100.0	0.0	93.6	0.0	96.6	0.1	100.0	0.0	97.4	0.1	26.0	0.0	93.6	0.0	2.9	0.1	2.9	0.1	2.9	0.1
7	5x4	100.0	0.1	100.0	0.1	96.6	0.1	97.7	0.4	100.0	0.1	99.0	0.2	42.9	0.1	96.6	0.1	42.7	0.2	42.7	0.1	42.7	0.1
8	5x4	67.6	0.1	67.6	0.1	64.0	0.1	65.1	0.7	67.6	0.2	66.4	0.4	13.5	0.1	64.0	0.2	13.3	1.3	13.3	0.4	13.3	0.5
9	5x4	67.9	0.1	67.9	0.1	63.8	0.1	64.3	0.6	67.9	0.2	65.9	0.4	1.8	0.1	63.8	0.2	1.7	0.7	1.7	0.2	1.7	0.4
10	5x8	57.5	0.3	57.5	0.4	55.3	0.5	55.5	3.7	57.5	0.6	56.6	1.5	28.5	0.2	55.3	1.1	28.4	1.6	28.4	0.8	28.4	1.5
11	5x8	48.9	0.3	48.9	0.4	45.8	0.4	46.2	3.2	48.9	0.7	47.7	1.7	10.0	0.4	45.8	0.8	9.8	3.1	9.8	1.6	9.8	2.0
15	6x4	70.2	0.3	70.2	0.3	66.6	0.3	68.5	2.5	70.2	0.4	69.4	1.2	2.2	0.3	66.6	0.5	2.1	2.7	2.1	1.1	2.0	1.4
16	7x4	56.6	0.3	56.6	0.4	54.3	0.4	56.0	2.5	56.6	0.6	56.3	1.7	15.7	0.5	54.3	0.5	3.3	3.3	3.3	1.5	3.3	2.1
17	7x4	88.6	0.4	88.6	0.5	85.5	0.4	86.1	5.2	88.6	1.0	87.6	2.4	2.2	0.7	85.5	0.9	2.1	6.0	2.1	2.6	2.1	4.0
18	8x4	46.6	0.4	46.6	0.5	45.3	0.9	45.9	10.6	46.6	1.5	46.3	3.9	16.3	0.6	45.3	1.7	12.5	9.6	12.5	3.8	12.5	7.4
19	9x4	46.7	0.8	46.7	1.2	45.7	1.8	46.3	15.0	46.7	3.7	46.5	7.6	12.7	1.6	45.7	3.1	5.1	14.5	5.1	7.4	5.1	13.6
AVERAGE		66.5	0.2	66.5	0.3	63.1	0.3	63.9	2.9	66.5	0.6	65.0	1.4	14.8	0.3	63.1	0.6	11.6	2.8	11.6	1.3	11.6	2.1

		200		205		210		220		230		240		250		260		270		280		285	
instance		IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t	IGap%	LPR t
1	5x4	49.5	0.2	49.5	0.2	44.9	0.2	45.5	1.1	49.5	0.2	47.3	0.5	1.0	0.2	44.9	0.3	1.0	1.0	1.0	0.5	1.0	0.6
2	5x4	74.8	0.2	74.8	0.2	67.8	0.2	68.6	1.4	74.8	0.4	70.1	0.5	16.2	0.2	67.8	0.4	15.9	0.9	15.9	0.5	15.9	0.7
3	2x4	30.5	0.0	30.5	0.0	27.4	0.0	27.4	0.0	30.5	0.0	30.1	0.0	3.5	0.0	27.4	0.0	3.3	0.0	3.3	0.0	3.3	0.0
4	5x4	57.2	0.1	57.2	0.1	52.0	0.2	52.6	0.7	57.2	0.2	54.4	0.3	1.0	0.1	52.0	0.3	0.9	0.7	0.9	0.4	0.9	0.4
5	3x4	94.7	0.0	94.7	0.0	82.0	0.0	82.9	0.1	94.7	0.0	84.6	0.0	42.5	0.0	82.0	0.0	40.7	0.1	40.7	0.0	40.7	0.0
6	4x4	99.7	0.0	99.7	0.0	85.3	0.0	90.5	0.1	99.7	0.1	92.2	0.1	25.9	0.0	85.3	0.1	2.5	0.1	2.5	0.1	2.5	0.1
7	5x4	99.7	0.2	99.7	0.2	90.6	0.2	93.7	1.0	99.7	0.4	95.3	0.3	42.9	0.2	90.6	0.3	42.5	0.5	42.5	0.3	42.5	0.4
8	5x4	67.6	0.3	67.6	0.3	62.4	0.3	63.9	1.7	67.6	0.5	65.1	0.6	13.5	0.2	62.4	0.4	13.2	1.3	13.2	0.5	13.2	0.9
9	5x4	67.7	0.1	67.7	0.2	58.5	0.2	59.5	1.2	67.7	0.4	63.6	0.7	1.8	0.1	58.5	0.4	1.7	0.8	1.7	0.5	1.7	0.5
10	5x8	57.4	1.1	57.4	1.4	53.4	1.2	53.9	8.2	57.4	1.8	55.1	3.7	28.5	1.2	53.4	2.4	28.3	3.3	28.3	1.8	28.3	2.5
11	5x8	48.8	0.5	48.8	0.6	44.7	0.5	45.1	3.9	48.8	1.0	46.9	2.4	10.0	0.7	44.7	0.9	9.7	3.1	9.7	1.6	9.7	2.2
15	6x4	70.0	0.6	70.0	0.9	64.0	0.5	66.6	3.7	70.0	1.1	68.2	1.8	2.1	0.8	64.0	1.1	1.9	3.2	1.9	1.8	1.9	2.9
16	7x4	56.5	0.5	56.5	0.8	52.9	0.7	55.1	5.6	56.5	1.4	55.7	1.9	15.6	0.8	52.9	1.4	3.3	4.7	3.3	2.2	3.3	3.2
17	7x4	88.4	1.2	88.4	1.9	80.8	0.9	82.9	8.1	88.4	2.5	85.9	3.1	2.1	1.3	80.8	1.9	2.0	7.9	2.0	4.3	2.0	6.0
18	8x4	46.6	1.2	46.6	2.4	43.4	2.5	44.7	20.7	46.6	4.4	45.4	5.0	16.3	2.4	43.4	5.1	12.2	12.3	12.2	6.3	12.2	9.5
19	9x4	46.7	3.5	46.7	3.7	43.9	4.3	45.2	28.8	46.7	9.9	45.8	10.3	12.6	3.7	43.9	7.4	4.6	83.5	4.6	11.5	4.6	18.0
AVERAGE		66.0	0.6	66.0	0.8	59.6	0.7	61.1	5.4	66.0	1.5	62.9	1.9	14.7	0.7	59.6	1.4	11.5	7.7	11.5	2.0	11.5	3.0

Table 4.13 Case C- Average Results for the 13 Instances with Known Optimals (Excluding Easy Instances) *

	CPU	CPU R	LPR R	IGap%	#opt *	DU%	ULP%	Nodes	Node R
100	6776.7	1.00	1.00	64.0	5 (1)	2.4	139117.7	518341	1.00
105	6564.1	0.93	1.00	64.0	5 (2)	3.4	139118.8	437677	0.84
110	5606.0	0.80	47.88	61.1	9 (6)	0.3	382.0	138480	0.30
120	6575.5	1.03	33.18	61.9	7 (2)	1.7	495.2	71513	0.15
130	6924.6	1.09	1.00	64.0	6 (1)	2.5	139116.6	312347	0.67
140	6873.7	1.06	14.79	62.9	6 (1)	2.2	948.7	282547	0.59
150	4977.5	0.70	795.54	12.6	7 (5)	1.8	19.4	254848	0.63
160	5615.9	0.80	47.88	61.1	9 (6)	0.3	382.0	138163	0.30
170	3102.0	0.43	799.35	10.7	9 (9)	0.2	14.9	16428	0.06
180	2106.2	0.29	799.35	10.7	10 (10)	0.1	14.7	21455	0.10
185	1851.2	0.26	799.35	10.7	10 (10)	0.1	14.8	11023	0.05
200	6879.4	1.06	4.59	63.9	7 (1)	2.7	3102.2	523911	0.99
205	6759.6	0.99	4.59	63.9	7 (1)	2.9	3102.1	391840	0.75
210	5316.5	0.75	131.13	58.4	10 (6)	0.1	210.8	113767	0.26
220	6354.0	0.93	88.95	59.8	7 (2)	1.1	262.6	60543	0.11
230	7066.8	1.17	4.60	63.9	7 (1)	2.2	3095.9	281786	0.59
240	7066.6	1.17	66.93	61.5	7 (1)	1.8	322.5	283667	0.58
250	4213.2	0.60	795.54	12.6	8 (7)	2.0	19.5	168012	0.45
260	5316.3	0.75	131.13	58.4	10 (6)	0.1	210.8	113431	0.26
270	3117.0	0.43	801.35	10.6	10 (8)	0.0	14.5	13339	0.05
280	2233.6	0.31	801.35	10.6	11 (10)	0.0	14.5	16576	0.08
285	1892.7	0.26	801.35	10.6	10 (10)	0.1	14.6	9210	0.04

* #opt denotes the number of times the optimal solution was found, where the entry in parentheses denotes the number of verified optimal solutions.

Regarding the instances that cannot be solved within the time limit under any option, the detailed results are provided in Table 4.14. There are 7 such options in Case C, 4 of which are unsolved (instances 12, 13, 14 and 20), while the remaining 3 can be solved optimally by extending the time limit (instances 10, 18, 19). The average extended solution time for these instances turns out to be close to 8.6 CPU hours.

From Table 4.14, we can see that the DU% statistic for the 3 optimally solvable instances decreases as options are enhanced with favourable inequalities. Overall averages indicate that the gap between the UB and the LPR (ULP%) also improves considerably for the favourable options and the LPR times are increased. However, it needs to be said that the lowest ULP% statistics are obtained for options 180, 270 and 280 in this case, contrary to the results in Case A where option 285 was the best setting in this regard.

Table 4.14 Detailed Results for Instances Exceeding the Time Limit (Case C) *

		100			105			110			120			130			140			150			160			170			180			185		
instance		DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t
10	5x8	5.5	148.0	0.3	16.4	173.5	0.4	2.1	128.1	0.5	9.8	147.0	3.7	11.0	160.9	0.6	7.2	146.7	1.5	10.5	54.6	0.2	2.1	128.1	1.1	2.0	42.5	1.6	0.9	41.0	0.8	1.2	41.4	1.5
18	8x4	6.5	99.3	0.4	13.3	112.2	0.5	1.4	85.5	0.9	8.6	100.9	10.6	9.1	104.3	1.5	8.0	101.3	3.9	2.6	22.6	0.6	1.4	85.5	1.7	0.0	14.3	9.6	0.0	14.2	3.8	0.0	14.2	7.4
19	9x4	13.7	113.3	0.8	7.1	100.9	1.2	0.7	85.4	1.8	2.9	91.4	15.0	5.0	96.9	3.7	8.8	103.4	7.6	9.0	24.8	1.6	0.7	85.4	3.1	0.4	5.9	14.5	0.1	5.5	7.4	0.2	5.6	13.6
AVERAGE		8.5	120.2	0.5	12.3	128.9	0.7	1.4	99.7	1.1	7.1	113.1	9.7	8.4	120.7	1.9	8.0	117.1	4.3	7.4	34.0	0.8	1.4	99.7	2.0	0.8	20.9	8.6	0.3	20.2	4.0	0.5	20.4	7.5
12	8x8	-	614.7	2.2	-	765.3	2.0	-	471.7	2.9	-	644.0	53.5	-	546.7	5.0	-	528.1	15.1	-	88.3	1.8	-	471.7	4.0	-	62.1	16.2	-	58.9	8.1	-	70.8	11.6
13	9x8	-	210.5	1.7	-	265.8	3.2	-	178.3	5.3	-	192.4	45.3	-	223.5	4.4	-	208.2	22.3	-	91.6	1.2	-	178.3	8.0	-	40.2	37.7	-	41.6	21.1	-	52.2	25.3
14	9x8	-	700.6	1.9	-	841.6	4.1	-	449.2	4.9	-	534.3	117.3	-	658.4	6.3	-	603.9	21.3	-	85.3	2.1	-	449.2	9.5	-	79.6	41.5	-	53.1	20.1	-	103.3	43.6
20	9x4	-	216.3	0.9	-	284.3	0.9	-	201.6	1.1	-	227.3	8.8	-	232.5	2.3	-	227.9	8.4	-	47.4	1.0	-	201.6	2.9	-	19.7	19.5	-	19.0	8.0	-	19.0	14.9
AVERAGE		-	435.5	1.7	-	539.3	2.6	-	325.2	3.5	-	399.5	56.2	-	415.3	4.5	-	392.1	16.8	-	78.1	1.5	-	325.2	6.1	-	50.4	28.7	-	43.1	14.3	-	61.3	23.8
OVERALL		8.5	300.4	1.2	12.3	363.4	1.7	1.4	228.6	2.5	7.1	276.8	36.3	8.4	289.0	3.4	8.0	274.2	11.4	7.4	59.2	1.2	1.4	228.6	4.3	0.8	37.7	20.1	0.3	33.3	9.9	0.5	43.8	16.8

		200			205			210			220			230			240			250			260			270			280			285		
instance		DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t	DU%	ULP%	LP t
10	5x8	10.9	160.6	1.1	8.3	154.6	1.4	1.5	117.6	1.2	4.8	127.5	8.2	13.3	166.3	1.8	8.1	140.9	3.7	6.5	49.0	1.2	1.5	117.6	2.4	0.0	39.6	3.3	0.3	39.9	1.8	1.3	41.3	2.5
18	8x4	10.0	105.8	1.2	12.8	111.2	2.4	0.0	76.7	2.5	3.0	86.1	20.7	8.4	103.0	4.4	8.0	98.0	5.0	5.3	25.8	2.4	0.0	76.7	5.1	0.2	14.1	12.3	0.0	13.8	6.3	0.0	13.8	9.5
19	9x4	12.3	110.5	3.5	13.6	112.9	3.7	0.2	78.7	4.3	5.5	92.5	28.8	5.6	97.9	9.9	6.2	95.9	10.3	12.5	28.7	3.7	0.2	78.7	7.4	0.0	4.9	83.5	0.0	4.9	11.5	0.0	4.9	18.0
AVERAGE		11.0	125.6	1.9	11.6	126.2	2.5	0.6	91.0	2.7	4.4	102.0	19.2	9.1	122.4	5.3	7.4	111.6	6.3	8.1	34.5	2.4	0.6	91.0	5.0	0.1	19.5	33.0	0.1	19.5	6.5	0.4	20.0	10.0
12	8x8	-	553.4	3.6	-	591.2	4.8	-	429.5	6.5	-	495.5	80.4	-	627.1	8.3	-	507.9	12.3	-	107.3	3.8	-	429.5	9.1	-	67.3	36.7	-	67.5	15.8	-	65.2	23.3
13	9x8	-	211.4	5.2	-	266.7	7.3	-	159.6	10.1	-	180.1	71.9	-	183.9	12.6	-	175.9	26.1	-	80.0	4.9	-	159.6	17.2	-	38.1	54.4	-	36.2	35.3	-	34.7	52.5
14	9x8	-	566.0	13.5	-	888.3	24.6	-	411.2	23.9	-	600.4	268.1	-	708.8	38.9	-	543.5	65.2	-	118.3	16.4	-	411.2	40.0	-	57.1	444.8	-	62.5	65.8	-	109.0	73.7
20	9x4	-	238.6	2.9	-	259.0	3.0	-	181.2	3.4	-	217.4	20.7	-	243.5	8.5	-	206.6	10.8	-	47.6	3.7	-	181.2	6.4	-	17.4	27.4	-	17.4	13.0	-	18.0	25.6
AVERAGE		-	392.3	6.3	-	501.3	9.9	-	295.4	11.0	-	373.3	110.3	-	440.8	17.0	-	358.5	28.6	-	88.3	7.2	-	295.4	18.2	-	45.0	140.8	-	45.9	32.5	-	56.7	43.8
OVERALL		11.0	278.0	4.4	11.6	340.6	6.7	0.6	207.8	7.4	4.4	257.1	71.2	9.1	304.4	12.0	7.4	252.7	19.1	8.1	65.2	5.1	0.6	207.8	12.5	0.1	34.1	94.6	0.1	34.6	21.4	0.4	41.0	29.3

* In this table, the optimal solution could be obtained in extended time for the 3 instances at the top (i.e. 10, 18, 19), but it is not known for the 4 instances below (12, 13, 14, 20).

Finally, we present the LPR statistics of the four unsolved instances in Case C in Table 4.15 below.

Table 4.15 LPR Statistics of Instances for Which the Optimal Solution is Not Known (Case C) *

	LPR R	LPR t
100	1.00	1.7
105	1.00	2.6
110	1.06	3.5
120	1.03	56.2
130	1.00	4.5
140	1.01	16.8
150	2.84	1.5
160	1.06	6.1
170	2.97	28.7
180	2.97	14.3
185	2.97	23.8
200	1.00	6.3
205	1.00	9.9
210	1.16	11.0
220	1.10	110.3
230	1.00	17.0
240	1.06	28.6
250	2.84	7.2
260	1.16	18.2
270	2.98	140.8
280	2.98	32.5
285	2.98	43.8

* Averages for instances 12, 13, 14 and 20

Since we do not know the optimal solution for instances 12, 13, 14 and 20, we cannot compute the exact deviation of the upper bound. However, the same inference that was made for the previous cases can also be made about the quality of the upper bounds for this case by using the average gap between the optimal and the LPR for the instances with known optimal solutions (excluding easy instances). This gap turns out to be 14.5% of the LPR for the favourable options. The average ULP% statistic for the four unsolvable instances is 43.1% (corresponding to option 180 in Table 4.14), which suggests that the UB is approximately 29% (of the LPR) away from the optimal solution. This result is also close to our inference about the unsolved instances in Case A.

4.2.2.4 General Conclusions

Generally speaking, our initial conclusions about the favourable options for Case A, which was the most ideal setting among the three experimental cases, were not disproved in later experiments and all results were more or less in line with each other.

It can be said that although their performance was not very striking in Case B where changeover costs were removed, the options with favourable additional inequalities give satisfactory results in terms of LPR values, CPU times, UB quality and number of nodes in all cases. Although a change in formulation does not have a strong impact, the combination of VI-1 and VI-5 clearly enhances the formulation quality. The incorporation of other VI on top of these does not bring an important improvement to the solution. On the other hand, the elimination constraints (EC) were generally observed to be effective both in terms of solution times and the number of nodes generated. Thus, in light of all the experiments carried out so far, we have decided that the best experimental setting to be used for the optimal solution of the GLSP would be 285, while options 180, 185 and 280 might be considered close alternatives.

CHAPTER 5

SOLVING THE GLSP

After having developed a viable mathematical model and determined its applicability and inherent assumptions, we have attempted at enhancing it through the addition of some additional inequalities. However, it needs to be said that as the size of the instances increases, the computational effort required to solve the problem optimally becomes prohibitive even after making strong enhancements. For this reason, it is important to develop reasonable approximations in order to attack the solution of such a challenging problem.

Our main aim in this chapter is to gain more insight about the intricacies of the GLSP and consider how best we can make use of the tools and information we have developed thus far. In doing so, we used a test bed taken from the literature that is different from the preliminary data set and tried to evaluate how the complexity of the problem is affected by an increase in the number of items and number of time periods. We also examined the effect of minimum batch sizes, which were otherwise unrestrictive in the preliminary experiments. In light of all these observations, we intend to come up with a satisfactory heuristic approach that is able to provide good feasible solutions within reasonable solution times.

5.1 Experimentation on the GLSP

In this section we describe the computational experiments performed to test the effectiveness of our GLSP formulation. The section is organized as follows: First, we describe the test problems taken from a reference study in the literature and the modifications we have made on them. Then, we state which experimental settings were used for these tests. Finally, we present the computational results with an emphasis on the effects of alternative minimum batch size settings and those of problem size parameters.

5.1.1 Choice of Data Set

For the preliminary set of experiments, our instances were based on the "Practical Industry Problems" used by Meyr (2000). However, this data set has a number of disadvantages, among which the most important is that setup times are sequence independent and relatively small in size compared to capacities, while the so-called sequence dependent setup costs only take several levels of values and are not truly sequence dependent. Moreover, further examination of the data reveals that although overall capacity utilization appears to be high, the demand-capacity distribution structure takes a special form such that all item demands are concentrated late in the planning horizon. This special structure facilitates the construction of feasible schedules, as the first few periods with no item demands can be reserved for early production for later periods.

For all these reasons, we have considered it appropriate to base our experiments on an alternative data set from the literature. We have decided to use the data generation scheme proposed by Haase and Kimms (2000), which features sequence dependent setup times taking up approximately 10% of period capacities. Setup costs are expressed as multiples of setup times, therefore they are also sequence dependent. However, there are a number of extra assumptions and restrictions in this study, such as the assumption of triangle inequality for setups and that of the zero-switch property (i.e., production of an item is only possible if the incoming inventory is zero). For this reason, we needed to make some modifications to the data in order to fit it within our framework. The points below describe the nature of the data used and some of the modifications we have made on them.

- Unit production requirements (P_i) for all items are equal to 1. Item demands per period, inventory holding costs and setup times are uniformly distributed with parameters [40,60], [2,10] and [2,10], respectively. Setup times are sequence dependent and $ST_{jj} = 0$.
- Setup costs are dependent on the setup times through the relation $SC_{ij}=50 \times ST_{ij}$.
- The facility is initially set up for item 1 (i.e., item zero in our formulation is equal to item 1).
- Period capacities are calculated by considering the maximum total demand in any period and a utilization coefficient, as follows:

$$C_t = C = \frac{\max_t \left\{ \sum_i d_{it} \right\}}{Utilization}$$

The utilization coefficient (*Utilization*) is taken as 0.8, which is the highest setting used by Haase and Kimms (2000). This coefficient accounts for the expected utilization of the capacity for production quantities only, since setup times are not considered. Following from the original paper, overtime is not allowed.

- Triangle inequality does not necessarily hold for setup costs and times.
- Zero-switch property assumption is removed.
- In the original data, there are no minimum batch sizes. Therefore, we have incorporated this parameter (m_i) into the model and determined its value according to the following relation:

$$m_i = MBS \times \sum_t d_{it}$$

where the minimum batch size of an item is expressed as a portion of its total demand quantity throughout the planning horizon. This portion is determined based on the coefficient MBS, which is set at different levels for the experiments. For instance, setting 0.1 indicates that item minimum batch sizes are set to be 10% of their total demands. In this way, minimum batch sizes depend on the number of time periods, *K*. The longer the planning horizon, the greater the item minimum batch sizes.

- The number of positions within a period are determined exactly as in preliminary experiments, i.e., by using the following relation:

$$L_t - F_t = min\{N, k_t + 2\}$$

The reader is referred to Section 4.2.1 for details.

In proposing this generation scheme, the authors Haase and Kimms (2000) aimed at achieving tight instances. Compared with our preliminary data set, this generation scheme provides much tighter instances, since we have a full demand matrix (with no zero entries) and higher setup times.

Using the scheme described above, we have generated 7 different classes of instances, where the number of items range from 6 to 9 and number of periods range from 2 to 5. Each class contains 10 instance replications. Instance properties including our naming scheme are summarized in Table 5.1 below.

Table 5.1 Characteristics of HK Instances

N x K	Instance #
6 x 2	46,…,55
6 x 3	56,…,65
6 x 4	66,…,75
6 x 5	76,…,85
7 x 3	1,…,5 and 31,…,35
8 x 3	16,…,20 and 36,…,40
9x 3	26,…,30 and 41,…,45

As it has been stated above, we randomly generated the changeover times and did not enforce triangle inequality. However, in order to gain more insight about the nature of the instances, we have examined triangle inequality violations in our generated data. For this purpose, we checked the changeover times for each triple combination possible between different items and counted the violations of the inequality. For each instance, %Violations is expressed as a ratio of the number of violations to the total number of triple combinations possible. The calculations are presented in Appendix F. The average %Violation per instance turns out to be 7.8%, which is rather low. Therefore, this data set may be regarded as close to the case where triangle inequality assumption is satisfied.

5.1.2 Experimental Settings

For the experiments, all models were coded with Turbo Pascal 7.0 (Borland) and solved by CPLEX 8.1.0 with default solution options on Pentium IV 1.8 GHz. PC's with 256 MB RAM running Windows NT Workstation 4.0, which is a slightly superior configuration compared with the one used for our preliminary experiments. The computation times are given in CPU seconds on this machine setting. The Prime Modulus Multiplicative Linear Congruential Generator described by Law and Kelton (2000) was used as the random number generator.

All 70 instances were solved with option 285 (i.e., TP formulation enhanced with VI-1, VI-5 and EC constraints) and with a time limit of 2 CPU hours (7200 CPU seconds). In case the optimal solution cannot be obtained within this time limit, the time limit was extended for the sake of obtaining the optimal function value, if possible.

The minimum batch sizes are determined for each item by using the MBS parameter, as explained in Section 5.1.1. For the extended set of experiments, we tested alternative MBS settings. The basic MBS setting was 0.1 (i.e., the minimum batch size of each item is equal to 10% of its total demand throughout the horizon), but we also used lower and higher settings of 0.025 and 0.25, respectively. Some preliminary analysis showed that increasing MBS to 0.4 resulted in infeasibilities and required huge amounts of computational effort, so we decided that increasing MBS further would not be reasonable. Therefore, we have 3 MBS settings to test for each of the 70 instances, namely 0.025, 0.1 and 0.25.

5.1.3 Results of the Experiments

Details of the experiments with the 3 different MBS settings of 0.1, 0.025 and 0.25 are presented in Appendix G, H and I, respectively. These appendices also contain information about the solution values obtained at the end of extended solution times corresponding to the instances that could not be solved within the time limit.

Average results for the experiments with the 2 hr limit are provided in Table 5.2 below.

Table 5.2 Average Results for Experiments on HK Instances with Alternative MBS Settings *

	MBS=0.1				MBS=0.025				MBS=0.25			
	CPU	IGap%	#opt	DU%	CPU	IGap%	#opt	DU%	CPU	IGap%	#opt	DU%
6 x 2	4.8	41.1	10 (10)	0.0	4.4	41.1	10 (10)	0.0	4.8	41.1	10 (10)	0.0
6 x 3	56.0	61.8	10 (10)	0.0	53.1	61.8	10 (10)	0.0	48.6	61.8	10 (10)	0.0
6 x 4	1301.9	71.9	10 (10)	0.0	1342.7	71.9	10 (10)	0.0	3061.0	73.3	10 (8)	0.0
6 x 5	limit	76.5	2 (0)	0.9	6979.6	76.5	1 (1)	1.0	limit	82.4	4 (0)	3.4
7 x 3	404.5	62.6	10 (10)	0.0	465.9	62.6	10 (10)	0.0	398.6	62.6	10 (10)	0.0
8 x 3	4156.8	63.9	9 (9)	0.0	3517.6	63.9	10 (9)	0.0	3902.0	63.9	8 (8)	0.1
9 x 3	7037.4	64.1	2 (1)	0.9	7085.5	64.1	5 (1)	1.0	6750.2	64.1	3 (1)	0.7
OVERALL	2880.2	63.1	53 (50)	0.3	2778.4	63.1	56 (51)	0.3	3052.2	64.2	55 (47)	0.6

* #opt denotes the number of times the optimal solution was found, where the entry in parentheses denotes the number of verified optimal solutions.

The column for #opt statistic contains two sets of entries. The first is a measure of the number of times the best UB at the end of the time limit is equal to the optimal solution, while the second only counts the best UB's that were verified to be optimal, i.e., only those instances that stopped with zero solution gaps were considered.

Since our main experimental setting for MBS is 0.1, we will examine the results for this case more closely. In this setting, the optimal solutions are known for all instances; therefore all gaps can be computed. As it can be seen from Appendix G.1, 20 instances could be solved by extending the time limit. The average solution time for these instances is 6.5 CPU hours.

Table 5.2 shows that the integrality gap increases both as the number of items and the number of periods increase. However, the number of time periods seems to be more effective in this respect. For instance, we observe that the IGap% value of 61.8% for class 6x3 increases to 71.9% for 6x4, while it only increases to 62.6% for class 7x3.

Solution times also increase as instances get larger. In fact, we can say with 99% confidence that the number of periods and the number of items both increase CPU times significantly (Appendix J). Class 6x5 results in the highest solution times, integrality gaps, upper bound deviations and smallest number of optimal solutions.

DU% statistics are all close to zero, which indicates that 2 hr. solution limit is sufficient to obtain good quality upper bounds for these instances. Out of the total 70 instances, the optimal solutions were found for 53 within 2 hours, and the overall average deviation of the UB is 0.3%.

When we compare the optimal solutions for MBS settings 0.1 and 0.025, we can see that they are exactly identical for all instance classes. This indicates that decreasing the minimum batch size further was not effective in changing the solutions. This observation also applies for MBS=0.25 for instances with small number of periods (i.e., K=2,3). However as number of time periods increases, minimum batch size restriction starts to take effect and MBS=0.25 setting results in higher solution values compared with the other two settings. The reader may compare the optimal solution values of instance class 6x4 for MBS settings 0.1 and 0.25 in Appendix G.1 and I.1, respectively for confirmation of this finding. Therefore, it can be claimed that the minimum batch size scheme we have used only changed the solutions for instances with time periods greater than 4.

Regarding the average solution statistics of MBS settings 0.025 and 0.25, we again refer to Table 5.2. Parallel to the observations we have made above, the statistics of MBS setting 0.025 are very close to those of MBS=0.1. The integrality gaps and DU% statistics are almost identical, while the solution times turn out to be lower for some instance classes. Moreover, higher number of optimal solutions was found in this setting. This implies that although decreasing minimum batch sizes did not cause changes in the optimal solutions, the solution effort required to solve these problems decreased for some classes.

Since the optimal solutions are only affected by number of periods greater than 4, we can examine the statistics for MBS setting 0.25 in two parts. For the first, the statistics for instance classes with small number of periods indicate results that are similar to those for our MBS=0.025 analysis above. In order words, integrality gap values and UB deviations are almost identical, while for some classes, average solution times turn out to be smaller than the MBS=0.1 setting. On the other hand, for instance classes 6x4 and 6x5, the integrality gaps and solution times are much higher compared to MBS=0.1. In fact, none of the ten 6x5 instances could be solved optimally with MBS=0.25 even with extended solution times and the average deviation between the best upper bound and the LPR is approximately 490% on the average, which is rather high. This suggests that as minimum batch sizes start to take influence, the instances become harder to solve.

Note that when our results are compared to those of Haase and Kimms (2000), our solution times turn out to be much higher. However, such a comparison would not be reasonable, since their approach requires solving all TSP's previously in order to enumerate all efficient sequences and these solution times are not included in their results.

5.2 k-Nearest Heuristic

Although computational results have shown that the average solution quality is satisfactory within a 2 hour time limit for our enhanced GLSP formulation, it is evident that attacking the optimal solution of this problem is not reasonable for larger problem instances. Therefore, one must develop some practical approximations in order to obtain good solutions in reasonable times.

Unlike other heuristic approaches previously applied to the GLSP or to similar models, which either used metaheuristics or backward oriented procedures for obtaining feasible solutions, we decided to employ a mathematical programming based approximation, which will be referred to as the "k-Nearest" heuristic.

This approach is based on our enhanced GLSP formulation (option 285) with a restriction on the number of item changeovers possible. This idea has been initiated from some rough analysis of the solutions of the experiments, which showed that in the optimal solutions, there are usually only a few preferred changeovers between items, while a great deal of changeover combinations are hardly ever used. Since the main drawback of the GLSP is the size in terms of number of variables, we thought it would be wise to simply reduce the number of changeover variables by allowing only k transitions from an item into another but itself. This implies that for each item, we need to define a restricted set of allowable items which contains k elements. The elements of these sets are determined based on changeover times in our experiments, thus, for each item, we only allow changeovers to the k different items that are closest to it in terms of changeover times. As an example, let us consider a problem with four items and let the changeover times from item 1 to the other items be given as 2, 4 and 3 units, respectively. For this problem, if k is set as 2, then the restricted set of allowable items corresponding to item 1 would include items 2 and 4, since they are the closest items according to the criterion of changeover time. In this case, the changeover from item 1 to item 3 will not be allowed, since it has the longest changeover time of 4 units.

However, we make an exception to this rule for the following two cases:

i) the item under consideration is item zero, or

ii) we are at a period boundary (i.e., $n=L_t$)

This means that all changeovers are allowed for the two exceptional cases above. The first exception is clearly justifiable. The second is necessary in order to facilitate finding feasible solutions. Put in another way, the procedure that we propose only restricts the number of changeovers within a period, but allows all transitions between periods.

Another issue is how to set the value of input parameter k. As k approaches $N\text{-}1$ (i.e., the original number of different changeovers possible in the unrestricted problem- the maximum value that k can take), the solution is expected to approach the optimal at the expense of greater solution effort, and vice versa. In order to verify this conjecture, we have decided to use 3 alternative settings for parameter k. We aim at restricting the number of changeovers to be approximately 80, 60 and 40% of the original, respectively. The procedure we use to determine the value of k depends on the number of items in the problem instance under consideration. For each possible k, we compute the proportion of k over $N\text{-}1$ and express it as a percentage. Then, among all percentages computed in this way, we select the 3 settings that are closest to the approximate percentages of 80, 60 and 40. Table 5.3 on the next page presents the calculations and the selected k values corresponding to each N that is used in the computational experiments. To clarify, for the problems with 8 items, the 3 settings of k are determined to be 6, 4 and 3, respectively according to the 3 percentage levels. For example *k*=6 corresponds to using approximately 80% of different item changeovers and this setting will be used for problems with 8 items when solved with k-Nearest heuristic 80%, as referred to in the experiments that follow.

Table 5.3 Procedure Used to Determine the Value of k *

N	k	%	
6	4	80.00	✓
	3	60.00	✓
	2	40.00	✓
7	5	83.33	✓
	4	66.67	✓
	3	50.00	
	2	33.33	✓
8	6	85.71	✓
	5	71.43	
	4	57.14	✓
	3	42.86	✓
	2	28.57	
9	7	87.50	✓
	6	75.00	
	5	62.50	✓
	4	50.00	
	3	37.50	✓
	2	25.00	

* % represents k/(N-1) x 100.

We anticipate that this heuristic will yield better results as the number of items increases. This is because the reduction in the number of changeovers for only a few items would be a severe restriction whereas with greater number of items, this reduction is expected to pay off with more pronounced effects. Moreover, the heuristic is expected to become more effective under situations where changeovers satisfy the triangle inequality assumption because there will be no more need to perform dummy changeovers between items.

For the experiments with the k-Nearest heuristic, we have used the 70 experimental HK instances with the basic MBS=0.1 setting and a time limit of 2 CPU hours. Recall that all instances could be solved optimally for this case, which means that the performance of the heuristic can be evaluated in comparison to the original (i.e., *k*=*N-1*) optimal solutions.

Table 5.4 displays the average statistics for the 2 hour performance of the 3 k-Nearest heuristic settings as well as those for the original versions of the instances. The detailed statistics may be found in Appendix K.

Table 5.4 Results of k-Nearest Heuristic Experiments on HK Instances with MBS=0.1 *

	Original			k-Nearest 80%			k-Nearest 60%			k-Nearest 40%		
	CPU	DU%	#opt	CPU	DU%	#opt	CPU	DU%	#opt	CPU	DU%	#opt
6 x 2	4.8	0.0	10	4.2	0.0	10	3.7	1.5	7	2.6	6.0	4
6 x 3	56.0	0.0	10	40.5	0.1	9	33.3	0.6	6	16.4	2.8	5
6 x 4	1301.9	0.0	10	773.1	0.0	10	453.5	0.2	9	136.7	5.4	3
6 x 5	limit	0.9	2	6998.8	0.5	5	6439.8	1.3	5	703.8	3.6	3
7 x 3	404.5	0.0	10	458.3	0.0	10	283.8	0.0	10	70.1	3.2	3
8 x 3	4156.8	0.0	9	3092.7	0.0	9	2353.3	0.2	9	973.7	1.8	7
9 x 3	7037.4	0.9	2	7102.9	0.9	4	6732.2	0.8	4	2629.0	0.2	8
OVERALL	2880.2	0.3	53	2638.6	0.2	57	2328.5	0.6	50	647.5	3.3	33

* Original denotes the unrestricted optimal MIP solution within a 2 hr limit.
#opt statistics are calculated by comparing the best UB at the end of the time limit with the optimal solution of the unrestricted problem.

The results verify our expectations. As the number of changeovers is restricted, solution times generally decrease at the expense of worse upper bounds and reduced number of optimal solutions. Exceptions to this observation are instance classes 7x3 and 9x3 for which k-Nearest 80% resulted in slightly higher average solution times compared with the original solutions.

k-Nearest 80% yields results that are very close to the original solutions, in that the solutions times are high and upper bounds are very close to the optimal solutions. In fact, upper bound deviations are even slightly lower and the number of optimal solutions found is greater for this case compared with the original solutions for the hardest instance classes of 6x5 and 9x3. This may be explained by the speed of the heuristic, i.e., it spends less time on each node and works for a slightly restricted solution space, and hence it is able to scan more solutions within the time limit.

For the other two heuristic settings of k-Nearest 60% and 40%, upper bound quality starts to deteriorate and the number of optimal solutions decreases for small instance classes. As a matter of fact, the use of this heuristic is not reasonable for very small instances, especially those with only a few items, since the original solutions are already satisfactory and by approximating, we give up on optimality. However, the picture changes as instances become harder. Then, the reduction of solution times becomes considerable and upper bound deviations start to take tolerable values. As an example, instances within class 6x5 could not be solved at all within the time limit and the optimal solutions were obtained only for 2 instances in the original case. However, k-Nearest 40% solves this class of problems within 703 seconds on the average and provides 3 optimal solutions, with an average upper deviation of 3.6%, which is acceptable. The performance of this heuristic setting becomes even better as number of items increase. Compare the results for instance classes 7x3, 8x3 and 9x3 to verify that excellent quality upper bounds could be found within reasonable solution times. Especially for class 9x3, k-Nearest 40% performs best among all other settings in terms of both CPU and upper bound quality, which is almost zero and with a striking count of 8 optimal solutions out of 10, whereas the original solution was only able to identify 2.

Therefore, as a result of this analysis, we have shown that there is sufficient evidence for resorting to restricted number of changeovers for the GLSP with sequence dependent changeovers. In this light, the k-Nearest heuristic opens up a promising path to obtain effective solutions within reasonable times for problems with high number of items.

CHAPTER 6

CONCLUSIONS AND DIRECTIONS FOR FURTHER STUDY

In this study, we have presented a mathematical formulation for the GLSP, which is an integrated multi-item, single-level capacitated lot sizing and sequencing problem with sequence dependent changeover costs and times. During the formulation stage, we focused on the elimination of restrictive modeling assumptions as much as possible in order to be able to represent continuous production lots more realistically under strict capacity limitations. We represented the problem using a stronger alternative formulation and also considered some enhancements through the incorporation of some additional inequalities. Preliminary experiments were conducted to select the best combination of these inequalities to be used for the solution of the GLSP.

In our computational experiments, we tested the LP relaxation and MIP performance of our enhanced formulation using a set of instances taken from the literature. We checked the effect of alternative minimum batch size settings which depend on specific portions of total item demands over the planning horizon. Our results have shown that increasing minimum batch sizes increases the solution times and gaps for large instances, while for the major portion of instances, minimum batch sizes did not have a considerable effect on the solution.

Recognizing the need to develop viable approximations for the GLSP in order to tackle larger problems, we considered posing some restrictions on item changeovers, in what we referred to as the k-Nearest heuristic. We tested this approach under different settings and the results show that the highest restriction, i.e., reducing the number of different changeovers possible for each item to be 40% of the original, yielded very promising results with low computation times and upper bounds that are very close to optimal solutions. This shows that especially for problems with many items, using this kind of an approximation is a potentially sound option against striving in vain for optimality.

The fundamental contribution of this thesis is, in our opinion, in the area of developing a complete mathematical formulation with explicit assumptions and the capability to represent special cases more effectively, compared with earlier GLSP formulations. We provide the first numerical results for the tests including both sequence dependent changeover times and changeover costs for the single level GLSP. Moreover, we develop an alternative TP formulation for the problem and combine it with some effective additional inequalities, some of which are original to our work. These approaches have been applied to other lot sizing models, but to our knowledge, this is the first time they appear within the GLSP context. The TP formulation combined with unit flow equalities, our original valid inequality and several redundancy elimination constraints was shown to improve the MIP performance of the pure GLSP considerably. Finally, the k-Nearest approach proposed may be regarded as a new line of thinking in the area of lot sizing with sequence dependent changeovers, with potentially strong impacts.

Our experiments were confined to considerably small instances with known optimal solutions. For the continuation of the study, the potency of the k-Nearest heuristic should be tested for larger instances. The quality of its solutions may be evaluated in comparison with the LP relaxation bounds in this case, and inferences about its effectiveness can be made using similar gap information available from smaller instances.

In our experiments with the k-Nearest heuristic, we used linearly dependent changeover costs and times, which enabled the selection of k items based on a single criterion. Thus, another possible extension is apparent; the k-Nearest heuristic should be designed for use in the context of independent or negatively associated changeover costs and times, and a procedure should be developed for selecting the best changeovers with respect to both criteria using an appropriate weighting scheme.

An important setting that needs to be evaluated during an extensive test would be the number of positions within time periods. In all our problems, we used a constant setting for this parameter depending on the number of items and minimum batch sizes. Nevertheless, other values should be tested to come up with the most satisfactory setting. Note that a very low setting would severely restrict the number of production lots and would increase the occurrence of infeasibilities, whereas too many positions would needlessly increase the size and complexity of the formulation.

During our computational experiments, we have observed the strong impact of capacity-demand distributions in the test data. The HK data set used for the experiments included tight instances with full demand matrices, which may have clouded the effects of the model features tested, such as the minimum batch sizes. Thus, in a thorough analysis, alternative demand and capacity patterns should be used as well as different schemes for the determination of minimum batch sizes.

Regarding the mathematical formulation, an extension of our model would be the case where changeover times are also allowed to be split between periods in addition to the production lots and minimum batch sizes. This modification would be suitable especially if changeover times are excessively long with respect to capacity and it would render the production truly continuous over time. However, such a modification is not straightforward and is expected to require extensive changes to the original framework.

For further research on the solution of the GLSP, one can improve our enhancement scheme using strong valid inequalities. Improving the flow-based setup and changeover inequalities as well as the incorporation of (l,S) type inequalities rooting back to Barany *et al.* (1984) may be considered (especially for the I&L formulation) as possible alternatives in this area.

Apart from our k-Nearest heuristic approach, other alternative approximations may be considered for this problem. For instance, one may envisage an a-priori tour between the items based on the smallest changeover times (or costs), i.e., the optimal TSP tour. If certain items are not part of the sequence in a certain period, then a plausible approximation for the best tour between the remaining items would be to keep the same order enforced by the a-priori tour with unused items skipped. In this way, sequencing decisions will be substituted with this predefined tour and they can be easily integrated within the lot sizing model. However, such an approximation is expected to produce satisfactory results only under special changeover patterns and especially if triangle inequality is satisfied.

Although Lagrangean relaxation approaches are commonly used solution methods for lot sizing problems, they have not been frequently applied to integrated models involving sequencing decisions. As a matter of fact, examination of the model structure suggests that the relaxation of a single constraint will not be sufficient to condense it down to easier subproblems, because there are many interdependencies between the items and the time periods. For example, even if the capacity restriction is relaxed, items will still be related through changeover constraints. This requires the relaxation of a set of fundamental constraints together, but as such, the remaining problems will have lost their essential nature. Alternatively, decomposition approaches may be considered for this problem rather than Lagrangean relaxation.

Another heuristic approach could be to consider the mathematical decomposition of the lot sizing and sequencing decisions in two sequential steps, while establishing the links between them iteratively. A rough initial discussion on such an approach (which is referred to as the TSH for the **T**wo-**S**tep **H**euristic) is presented in Appendix L for a simpler version of our problem. It needs to be said that the results obtained after a rudimentary study are not very promising and they call for substantial improvement effort.

REFERENCES

Aardal, K., 1998. Reformulation of Capacitated Facility Location Problems: How Redundant Information Can Help. *Annals of Operations Research* **82**, 289-308.

Allen, S.J. and Schuster, E.W., 1994. Practical Production Scheduling with Capacity Constraints and Dynamic Demand: Family Planning and Disaggregation. *Production and Inventory Management Journal* **35** (4), 15-21.

Al-Shammari, M. and Dawood, I., 1997. Linear Programming Applied to a Production Blending Problem: A Spreadsheet Modeling Approach. *Production and Inventory Management Journal* **38** (1), 1-7.

Anderson, E.J. and Cheah, B.S., 1993. Capacitated Lot-Sizing with Minimum Batch Sizes and Setup Times. *International Journal of Production Economics* **30-31**, 137-152.

Artiba, A. and Riane, F., 1998. An Application of a Planning and Scheduling Multi-Model Approach in the Chemical Industry. *Computers in Industry* **36**, 209-229.

Ashayeri, J., Teelen, A. and Selen, W., 1996. Computer-Integrated Manufacturing in the Chemical Industry. *Production and Inventory Management Journal* **37** (1), 52-57.

Ashayeri, J., Van Eijs, A.G.M and Nederstigt, P., 1994. Blending Modelling in a Process Manufacturing: A Case Study. *European Journal of Operational Research* **72**, 460-468.

Barany, I., Van Roy, T.J. and Wolsey, L.A, 1984. Strong Formulations for Multi-Item Capacitated Lot Sizing. *Management Science* **30** (10), 1255-1261.

Belvaux, G. and Wolsey, L.A, 2000. bc-prod: A Specialized Branch-and-Cut System for Lot-Sizing Problems. *Management Science* **46** (5), 724-738.

Belvaux, G. and Wolsey, L.A, 2001. Modelling Practical Lot-Sizing Problems as Mixed-Integer Programs. *Management Science* **47** (7), 993-1007.

Cattrysse, D., Maes, J. and Van Wassenhove, L.N., 1990. Set Partitioning and Column Generation Heuristics for Capacitated Dynamic Lotsizing. *European Journal of Operational Research* **46**, 38-47.

Chen, W.H. and Thizy, J.M, 1990. Analysis of Relaxations for the Multi-Item Capacitated Lot-Sizing Problem. *Annals of Operations Research* **26**, 29-72.

Clark, A.R. and Clark, S.J., 2000. Rolling-horizon Lot-sizing When Set-up Times are Sequence-Dependent. *International Journal of Production Research* **38** (10), 2287-2307.

Constantino, M., 1996. A Cutting Plane Approach to Capacitated Lot-Sizing with Start-up Costs. *Mathematical Programming* **75**, 353-376.

Crama, Y., Pochet, Y. and Wera, Y., 2001. A Discussion of Production Planning Approaches in the Process Industry. CORE Discussion Paper, 2001/42, University of Liége.

De Matta, R. and Guignard, M., 1994a. Studying the Effects of Production Loss Due to Setup in Dynamic Production Scheduling. *European Journal of Operational Research* **72**, 62-73.

De Matta, R. and Guignard, M., 1994b. Dynamic Production Scheduling For a Process Industry. *Operations Research* **42** (3), 492-503.

De Matta, R. and Guignard, M., 1995. The Performance of Rolling Production Schedules in a Process Industry. *IIE Transactions* **27**, 564-573.

Denizel, M. and Süral, H., 2005. On Alternative Mixed Integer Programming Formulations and LP-based Heuristics for Lot-sizing with Setup Times. *Journal of the Operational Research Society,* Forthcoming.

Dennis, D.R. and Meredith, J.R., 2000. An Analysis of Process Industry Production and Inventory Management Systems. *Journal of Operations Management* **18**, 683-699.

Diaby, M., Bahl, H.C., Karwan, M.H. and Zionts, S., 1992. Capacitated Lot-Sizing and Scheduling by Lagrangean Relaxation. *European Journal of Operational Research* **59**, 444-458.

Drexl, A. and Kimms, A., 1997. Lot Sizing and Scheduling - Survey and Extensions. *European Journal of Operational Research* **99**, 221-235.

Eppen, G.D. and Martin, R.K., 1987. Solving Multi-Item Capacitated Lot-Sizing Problems Using Variable Redefinition. *Operations Research* **35** (6), 832-848.

Fleischmann, B., 1990. The Discrete Lot-sizing and Scheduling Problem. *European Journal of Operational Research* **44**, 337-348.

Fleischmann, B., 1994. The Discrete Lot-sizing and Scheduling Problem with Sequence-dependent Setup Costs. *European Journal of Operational Research* **75**, 395-404.

Fleischmann, B. and Meyr, H., 1997. The General Lotsizing and Scheduling Problem. *OR Spektrum* **19** (1), 11-21.

Fransoo, J.C., 1992. Demand Management and Production Control in Process Industries. *International Journal of Operations & Production Management* **12** (7), 187-196.

Fransoo, J.C. and Rutten, W.G.M.M., 1994. A Typology of Production Control Situations in Process Industries. *International Journal of Operations & Production Management* **14** (12), 47-57.

Gopalakrishnan, M., Miller, D.M. and Schmidt, C.P., 1995. A Framework for Modelling Setup Carryover in the Capacitated Lot Sizing Problem. *International Journal of Production Research* **33** (7), 1973-1988.

Gupta, D. and Magnusson, T., 2005. The Capacitated Lot-Sizing and Scheduling Problem with Sequence-Dependent Setup Costs and Setup Times. *Computers & Operations Research* **32**, 727-747.

Günther, H.-O. and Van Beek, P., 2003. Advanced Planning and Scheduling in Process Industry *in Advanced Planning and Scheduling Solutions in Process Industry*. Springer-Verlag, Berlin, 1-9.

Haase, K., 1996. Capacitated Lot-Sizing with Sequence Dependent Setup Costs. *OR Spektrum* **18**, 51-59.

Haase, K. and Kimms, A., 2000. Lot Sizing and Scheduling with Sequence-dependent Setup Costs and Times and Efficient Rescheduling Opportunities. *International Journal of Production Economics* **66**, 159-169.

Heuts, R.M.J, Seidel, H.P. and Selen, W.J., 1992. A Comparison of Two Lot Sizing-Sequencing Heuristics for the Process Industry. *European Journal of Operational Research* **59**, 413-424.

Hill, J.A, Berry, W.L., Leong, G.K. and Schilling, D.A., 2000. Master Production Scheduling in Capacitated Sequence-Dependent Process Industries. *International Journal of Production Research* **38** (18), 4743-4761.

Hill, J.A, Berry, W.L. and Schilling, D.A., 2003. Revising the Master Production Schedule in Sequence Dependent Processes. *International Journal of Production Research* **41** (9), 2021-2035.

Jensson, P., 1988. Daily Production Planning in Fish Processing Firms. *European Journal of Operational Research* **36**, 410-415.

Karmarkar, U. and Schrage, L., 1985. The Deterministic Dynamic Product Cycling Problem. *Operations Research* **33** (2), 326-345.

Koçlar, A. and Süral, H., 2004. Development of a Production Planning Model for Process Industry Environments (Proses Endüstrisi Tipi Ortamlar için bir Üretim Planlama Modelinin Geliştirilmesi). *Proceedings of the 24th National Conference on Operational Research and Industrial Engineering,* Gaziantep University Press, 40-42.

Koçlar, A. and Süral, H., 2005. A Note on "The General Lot Sizing and Scheduling Problem". *OR Spektrum* **27** (1), 145-146.

Kuik, R. and Salomon, M., 1994. Batching Decisions: Structure and Models. *European Journal of Operational Research* **75**, 243-263.

Law, A.M. and Kelton, W.D., 2000. *Simulation Modeling and Analysis (3rd edition)*. McGraw-Hill, Singapore, 428-429.

Maes, J. and Van Wassenhove, L.N., 1988. Multi-item Single-Level Capacitated Dynamic Lot-Sizing Heuristics: A General Review. *The Journal of the Operational Research Society* **39** (11), 991-1004.

Maes, J., McClain, J.O. and Van Wassenhove, L.N., 1991. Multilevel Capacitated Lotsizing Complexity and LP-based Heuristics. *European Journal of Operational Research* **53**, 131-148.

Magnanti, T.L. and Sastry, T., 2002. Facets and Reformulations for Solving Production Planning with Changeover Costs. *Operations Research* **50** (4), 708-719.

Magnanti, T.L. and Vachani, R., 1990. A Strong Cutting Plane Algorithm for Production Scheduling with Changeover Costs. *Operations Research* **38** (3), 456-473

Manne, A.S., 1958. Programming of Economic Lot Sizes. *Management Science* **4** (2), 115-135.

Meyr, H., 2000. Simultaneous Lotsizing and Scheduling by Combining Local Search with Dual Reoptimization. *European Journal of Operational Research* **120**, 311-326.

Meyr, H., 2002. Simultaneous Lotsizing and Scheduling on Parallel Machines. *European Journal of Operational Research* **139**, 277-292.

Munford, A.G., 1989. A Microcomputer System for Formulating Animal Diets Which May Involve Liquid Raw Materials. *European Journal of Operational Research* **41**, 270-276.

Novitsky, M.P., 1984. Master Production Scheduling in Process Industries. *Production and Inventory Management Journal* **25** (1), 48-53.

Oliff, M.D. and Burch, E.E., 1985. Multiproduct Production Scheduling at Owens-Corning Fiberglass. *Interfaces* **15**, 25-34.

Prabhakar, T., 1974. A Production Scheduling Problem with Sequencing Considerations. *Management Science* **21** (1), 34-42.

Rutten, W.G.M.M., 1993. Hierarchical Mathematical Programming for Operational Planning in a Process Industry. *European Journal of Operational Research* **64**, 363-369.

Salomon, M., Kroon, L.G., Kuik, R. and Van Wassenhove, L.N., 1991. Some Extensions of the Discrete Lotsizing and Scheduling Problem. *Management Science* **37** (7), 801-812.

Salomon, M., Solomon, M.M, Van Wassenhove, L.N., Dumas, Y. and Dauzère-Pérès, S., 1997. Solving the Discrete Lotsizing and Scheduling Problem with Sequence Dependent Set-up Costs and Times Using the Travelling Salesman Problem with Time Windows. *European Journal of Operational Research* **100**, 494-513.

Selen, W.J. and Heuts, R.M.J., 1990. Operational Production Planning in a Chemical Manufacturing Environment. *European Journal of Operational Research* **45**, 38-46.

Smith-Daniels, V.L. and Smith-Daniels, D.E., 1986. A Mixed Integer Programming Model for Lot Sizing and Sequencing Packaging Lines in the Process Industries, *IIE Transactions* **18**, 278-285.

Stadtler, H., 1996. Mixed Integer Programming Model Formulations for Dynamic Multi-Item Multi-Level Capacitated Lotsizing. *European Journal of Operational Research* **94**, 561-581.

Stadtler, H., 2003. Multi-Level Lot Sizing with Setup Times and Multiple Constrained Resources: Internally Rolling Schedules with Lot-Sizing Windows. *Operations Research* **51** (3), 487-502.

Süral, H., 1996. Multi-Item Lot-Sizing With Setup Times. *PhD Thesis,* Middle East Technical University, Ankara.

Synder, R.L. and Ibrahim, H.A., 1996. Capacity Planning and Operational Assessment of Bulk Storage Systems. *Production and Inventory Management Journal* **37** (4), 14-18.

Thizy, J.M. and Van Wassenhove, L.N., 1985. Lagrangean Relaxation for the Multi-Item Capacitated Lot-Sizing Problem: A Heuristic Implementation, *IIE Transactions* **17** (4), 308-313.

Trigeiro, W.W., Thomas, L.J. and McClain, J.O., 1989. Capacitated Lot Sizing with Setup Times, *Management Science* **35** (3), 353-366.

Venkataraman, R. and Nathan, J., 1994. Master Production Scheduling for a Process Industry Environment. A Case Study. *International Journal of Operations & Production Management* **14** (10), 44-53.

Wolsey, L.A., 1989. Uncapacitated Lot-Sizing Problems with Start-up Costs, *Operations Research* **37** (5), 741-747.

Wolsey, L.A., 1997. MIP Modelling of Changeovers in Production Planning and Scheduling Problems, *European Journal of Operational Research* **99**, 154-165.

Wolsey, L.A., 2002. Solving Multi-Item Lot-Sizing Problems with an MIP Solver Using Classification and Reformulation, *Management Science* **48** (12), 1587-1602.

APPENDIX A

AN EXAMPLE INSTANCE FOR WHICH EC-1 IS SHOWN TO BE VALID

Below we provide an example instance for which the LPR solution changes upon the addition of EC-1 into the model. The example instance is instance 15 from our preliminary data set for Case A, which is based on the study by Meyr (2000) with augmented and sequence dependent changeover times, as explained in Section 4.2.2.1. The instance involves 6 items and 4 periods. Related input parameters are provided in the three tables that follow.

Table A.1 Input Data for Preliminary Instance 15 – Case A *

Demands		Period				h	P	m
		1	2	3	4			
Item	1	21.926	30.724	30.724	97.981	33.332	1	2
	2	0	0	36.525	172.560	46.935	1	2
	3	0	0	0	0.843	58.467	1	2
	4	0	0	5.975	69.067	46.688	1	2
	5	0.248	0.078	0.078	0.609	85.174	1	2
	6	0	0	0	0.662	51.392	1	2
Capacity		140.741	133.333	133.333	133.333			

* h, P and m stand for inventory holding cost, unit production requirement and minimum batch size, respectively. Case A is the first case tested for Preliminary Experiments in Chapter 4.

Table A.2 Changeover Cost Data for Preliminary Instance 15 – Case A

SC	1	2	3	4	5	6
0	6986.2	5279.3	5495.1	5246.8	2606.5	3625.7
1	0	6516.9	6732.7	6484.4	4709.9	4709.9
2	8070.4	0	6579.3	6331.0	4709.9	4709.9
3	8070.4	6363.5	0	6331.0	4709.9	4709.9
4	8070.4	6363.5	6579.3	0	4709.9	4709.9
5	8070.4	6516.9	6732.7	6484.4	0	4709.9
6	8070.4	6516.9	6732.7	6484.4	4709.9	0

Table A.3 Changeover Time Data for Preliminary Instance 15 – Case A

ST	1	2	3	4	5	6
0	4.358	6.409	3.897	4.358	4.358	3.897
1	0	4.358	2.650	2.964	4.358	3.897
2	4.358	0	5.767	6.450	4.358	3.897
3	4.358	9.485	0	6.450	4.358	3.897
4	4.358	9.485	5.767	0	4.358	3.897
5	4.358	4.358	2.650	2.964	0	3.897
6	4.358	4.358	2.650	2.964	4.358	0

In order to show the effect of incorporating EC-1, we have solved the LP relaxation of instance 15 described above with and without EC-1 in the presence of all VI (i.e., together with VI-1, VI-2, VI-3, VI-4 and VI-5). This is due to the fact that EC constraints are not usually effective on their own, but only make a difference together with other stronger valid inequalities. The resulting setup-changeover sequences for the first period (which involves 6 positions) are provided on the next page for the LP relaxation solutions. Note that we have only included the values of the self-changeover variables in order to keep the figures simple.

The initial observation is that the setup-changeover sequence is modified in the presence of EC-1 and as a result, the LPR value has improved by 3 units, which proves that EC-1 is not redundant for the LP solution.

Let us examine the effect of EC-1 more closely. Recall EC-1:

$$\sum_{j=1}^{N} \sum_{i=0, i\neq j}^{N} \delta_{ij(n-1)} \geq \sum_{j=1}^{N} \sum_{i=0, i\neq j}^{N} \delta_{ijn} \qquad \forall t, n = (F_t + 2)..(L_t - 1)$$

It is easy to verify that the first solution on the top violates EC-1 by checking the values of the changeover variables for some positions within period 1. For instance, let us select the two consecutive positions 2 and 3. Since the values of all changeover variables entering a position sum up to 1, we can evaluate the sum of the values of the changeover variables between different items (i.e., the terms in EC-1) by subtracting the values of self-changeover variables from 1. This implies that,

$$\sum_{j=1}^{N} \sum_{i=0, i\neq j}^{N} \delta_{ij2} = 1 - \sum_{i=0}^{N} \delta_{ii2} = 1 - 0.076 = 0.924$$

$$\sum_{j=1}^{N} \sum_{i=0, i\neq j}^{N} \delta_{ij3} = 1 - \sum_{i=0}^{N} \delta_{ii3} = 1 - 0.023 = 0.977$$

EC-1 is violated since $0.924 < 0.977$. The reader can check that there is also a violation for the consecutive position pair (3,4).

When EC-1 is included (the bottom case in Figure A.1), the values of the setup and changeover variables are changed so that the above violations are eliminated. For instance,

$$\sum_{j=1}^{N} \sum_{i=0, i\neq j}^{N} \delta_{ij2} = 1 - \sum_{i=0}^{N} \delta_{ii2} = 1 - 0.003 = 0.997$$

$$\sum_{j=1}^{N} \sum_{i=0, i\neq j}^{N} \delta_{ij3} = 1 - \sum_{i=0}^{N} \delta_{ii3} = 1 - (0.001 + 0.002) = 0.997$$

Thus, EC-1 is satisfied. The same result is obtained for consecutive positions 3 and 4.

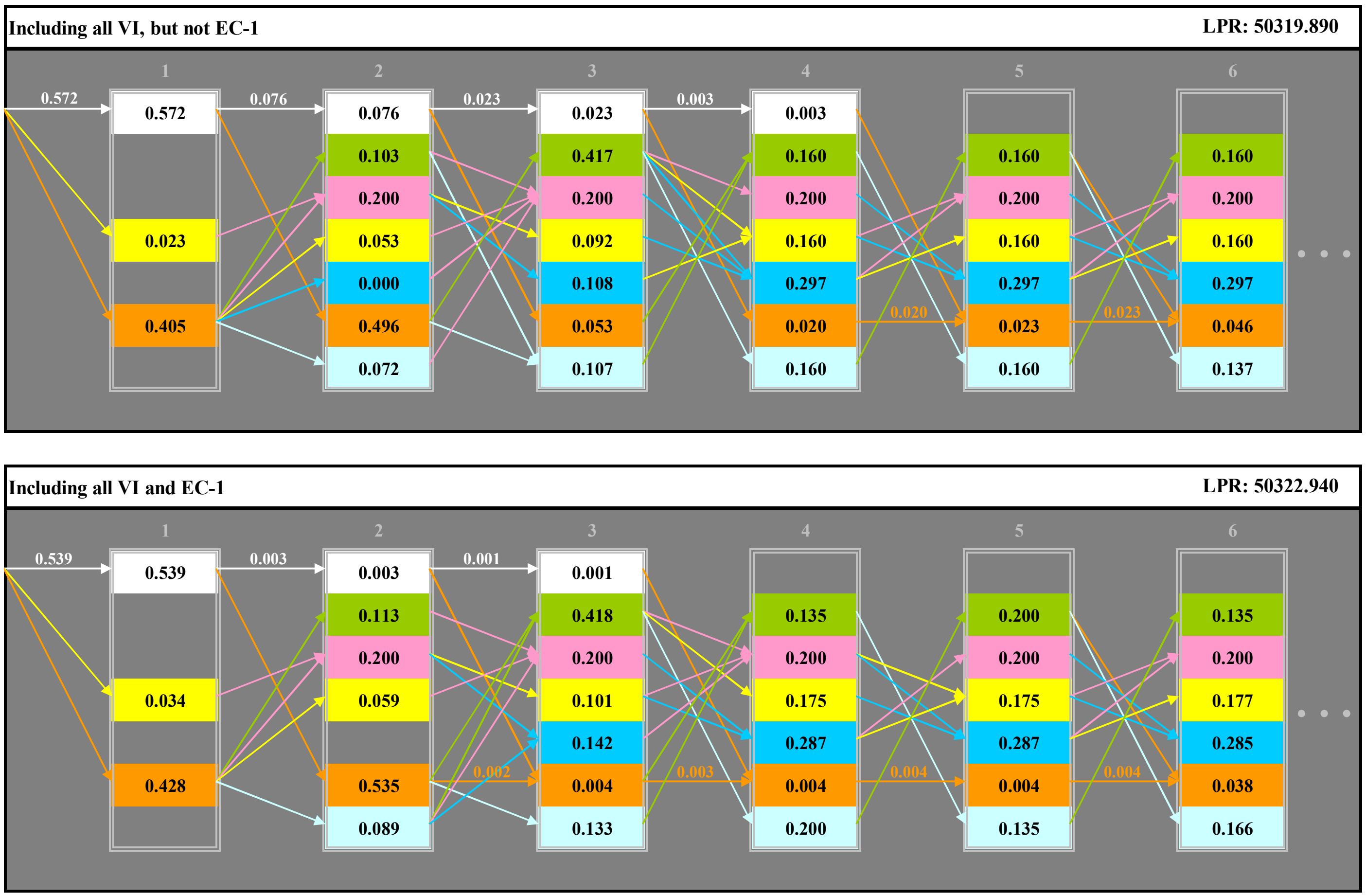


Figure A.1 The Effect of EC-1 on the LPR of Instance 15 (Case A)

APPENDIX B

MODIFICATION OF THE CHANGEOVER TIME DATA FOR PRELIMINARY TESTS – CASE A

Below is a brief account of the ad-hoc method by which we have transformed the original data used by Meyr (2000) into a form used in our Preliminary Case A. The modifications follow two main steps, namely creating sequence dependent changeover times and augmenting the size of the changeover times.

a) Creating Sequence Dependency

Our aim is to generate the sequence dependent changeover time parameter ST_{ij} from its sequence independent counterpart ST_j while assuming an inverse relationship with the corresponding changeover cost SC_{ij}. The applied procedure is briefly described below.

$$E(SC_j) = \frac{\sum_{i=1|i\neq j}^{N} SC_{ij}}{N-1}$$

$$Range(SC_j) = \underset{\substack{i=1..N \\ i\neq j}}{Max}\{SC_{ij}\} - \underset{\substack{i=1..N \\ i\neq j}}{Min}\{SC_{ij}\}$$

$$SC_{ij}^{'} = \frac{SC_{ij} - E(SC_j)}{Range(SC_j)/2} \quad \text{(the dispersion coefficient)}$$

$$E(ST_j) = ST_j = ST_{0j}$$

If $ST_j < 1$ then

$$ST_{ij} = ST_j - SC_{ij}^{'} * 0.4 * ST_j$$

If $ST_j \geq 1$ then

$$ST_{ij} = ST_j - SC_{ij}^{'} * 0.6 * ST_j$$

Firstly, the average changeover cost for switching into a certain item is computed and the range between the minimum and maximum changeover cost into that item is determined. These are used for computing a coefficient for each (*i,j*) pair, which depends on the dispersion of the corresponding sequence dependent changeover cost entry from the mean in terms of the relevant range. Here, the range is divided by two in order to consider a single-sided dispersion. Note that the resultant coefficient may also be negative, depending on the whether the changeover cost in question is smaller or larger than the average value.

The expected value of the changeover time into an item is equal to the available sequence independent setup time value. The changeover time from item zero into each item is taken to be the original sequence independent setup time parameter. The remaining sequence dependent changeover time entries are computed by subtracting (since we would like to create an inverse relationship) a given proportion of the mean setup time value times the dispersion coefficient from the mean setup time. The proportion to be used is dependent on the size of the mean changeover time.

b) Augmenting the Size of Changeover Times

Our aim is to increase the quantity of the changeover times by allocating the available capacity margin to items while keeping the sequence dependent structure generated using the above procedure. Here, we assume that only N^* items out of N are producible in a period (where $N^* = \lfloor 0.75 * N \rfloor$). Below is a description of the steps followed.

$$\text{Cum.Margin} = \sum_{t=1}^{K} C_t - \sum_{t=1}^{K}\sum_{i=1}^{N} d_{it}$$

$$\overline{ST} = \frac{\sum_{j=1}^{N} ST_j}{N} \quad \text{(Average changeover time)}$$

$$\sum ST = \overline{ST} \times N^* \times K \quad \text{(Total changeover time required through the planning horizon)}$$

$$\text{Rem.Gap} = \text{Cum.Margin} - \sum ST \quad \text{(Remaining capacity available for changeover time augmentation)}$$

$$\text{Rem.Gap/item-period} = \frac{\text{Rem.Gap}}{\text{N}^* \times \text{K}} \quad \text{(Augmentation for each changeover time entry)}$$

$$ST_{ij}^{\circ} = ST_{ij} + \frac{ST_{ij}}{\overline{ST}} \times \text{Rem.Gap/item-period} \quad \text{(Augmented changeover time)}$$

APPENDIX C

PRELIMINARY EXPERIMENTS UNDER CASE A

Table C.1 Solutions of Preliminary Instances Under Case A *

inst#	Original Instance #	N	K	Best sol. at the end of time limit	Extended CPU	Final Gap% at the end of extended time	Best sol.
1	11	5	4	36898.8	-	-	36898.8
2	21	5	4	31495.4	-	-	31495.4
3	41	2	4	17846.0	-	-	17846.0
4	51	5	4	32413.2	-	-	32413.2
5	63	3	4	6191.0	-	-	6191.0
6	83	4	4	8332.4	-	-	8332.4
7	94	5	4	INFEASIBLE	-	-	INFEASIBLE
8	104	5	4	55963.3	-	-	55963.3
9	111	5	4	27583.2	-	-	27583.2
10	11	5	8	64062.2	35941.7	-	64047.3
11	41	5	8	47012.5	-	-	47012.5
12	94	8	8	INFEASIBLE	-	-	INFEASIBLE
13	53	9	8	39020.3	290613.8	11.51	39020.3
14	64	9	8	INFEASIBLE	-	-	INFEASIBLE
15	74	6	4	52021.2	-	-	52021.2
16	13	7	4	31367.4	-	-	31367.4
17	124	7	4	INFEASIBLE	-	-	INFEASIBLE
18	102	8	4	32538.6	22635.8	-	32538.6
19	103	9	4	42830.9	48763.7	-	42830.9
20	123	9	4	27831.3	236201.7	-	27784.5

* Shaded cells indicate solutions with non-zero solution gaps. Best sol. refers to the best known solution for the instance. Best sol. at the end of the time limit is the best known solution among all options. However, extended solutions were only performed with option 285.

Table C.2 Detailed Results of Experiments on Preliminary Instances Under Case A *

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
36898.80	1	5 x 4	100	7200.0	28198.32	36898.80	23.58	1077971	562852	18405.93	0.2	1.0	1.0	50.1	0.0	100.5	1.0
			105	7200.0	31617.37	36898.80	14.31	944748	322423	18405.93	0.1	1.0	1.0	50.1	0.0	100.5	0.9
			110	2260.9	36898.49	36898.80	0.00	105995	5	19612.56	0.1	0.3	1.1	46.8	0.0	88.1	0.1
			120	7200.0	33680.72	36898.80	8.72	110667	45057	19478.14	0.6	1.0	1.1	47.2	0.0	89.4	0.1
			130	7200.0	27800.81	36898.80	24.66	577813	340848	18405.93	0.2	1.0	1.0	50.1	0.0	100.5	0.5
			140	7200.0	29512.25	36898.80	20.02	387729	213911	18957.28	0.3	1.0	1.0	48.6	0.0	94.6	0.4
			150	1780.9	36898.43	36898.80	0.00	251311	28	36081.14	0.1	0.2	2.0	2.2	0.0	2.3	0.2
			160	2260.3	36898.49	36898.80	0.00	105995	5	19612.56	0.3	0.3	1.1	46.8	0.0	88.1	0.1
			170	47.7	36898.80	36898.80	0.00	750	0	36136.37	0.7	0.0	2.0	2.1	0.0	2.1	0.0
			180	49.8	36898.80	36898.80	0.00	1620	0	36136.37	0.3	0.0	2.0	2.1	0.0	2.1	0.0
			185	35.5	36898.80	36898.80	0.00	947	0	36136.37	0.5	0.0	2.0	2.1	0.0	2.1	0.0
			200	7200.0	29141.68	36898.80	21.02	1150165	543993	18416.09	0.2	1.0	1.0	50.1	0.0	100.4	1.1
			205	7200.0	31858.73	36898.80	13.66	991658	302982	18416.09	0.2	1.0	1.0	50.1	0.0	100.4	0.9
			210	1890.5	36898.48	36898.80	0.00	133428	6	20079.48	0.2	0.3	1.1	45.6	0.0	83.8	0.1
			220	7200.0	34833.44	36898.80	5.60	135616	37620	19879.42	1.2	1.0	1.1	46.1	0.0	85.6	0.1
			230	7200.0	27790.78	36898.80	24.68	673507	342668	18416.09	0.3	1.0	1.0	50.1	0.0	100.4	0.6
			240	7200.0	31522.28	36898.80	14.57	483975	242844	19236.85	0.5	1.0	1.0	47.9	0.0	91.8	0.4
			250	1382.7	36898.44	36898.80	0.00	150968	61	36081.14	0.3	0.2	2.0	2.2	0.0	2.3	0.1
			260	1851.6	36898.48	36898.80	0.00	133428	6	20079.48	0.4	0.3	1.1	45.6	0.0	83.8	0.1
			270	61.5	36898.80	36898.80	0.00	654	0	36157.07	0.9	0.0	2.0	2.0	0.0	2.1	0.0
			280	49.0	36898.80	36898.80	0.00	1846	0	36157.07	0.4	0.0	2.0	2.0	0.0	2.1	0.0
			285	38.7	36898.66	36898.80	0.00	846	1	36157.07	0.7	0.0	2.0	2.0	0.0	2.1	0.0
31495.36	2	5 x 4	100	7200.0	16986.92	31495.36	46.07	638027	356893	7564.94	0.1	1.0	1.0	76.0	0.0	316.3	1.0
			105	7200.0	19704.48	31542.10	37.53	549983	281912	7564.94	0.1	1.0	1.0	76.0	0.1	317.0	0.9
			110	7032.3	31495.05	31495.36	0.00	496739	12	8514.43	0.1	1.0	1.1	73.0	0.0	269.9	0.8
			120	7200.0	26177.17	31495.36	16.89	133184	71209	8370.63	0.7	1.0	1.1	73.4	0.0	276.3	0.2
			130	7200.0	17802.71	31542.10	43.56	399422	229895	7564.94	0.1	1.0	1.0	76.0	0.1	317.0	0.6
			140	7200.0	19960.93	31495.36	36.62	393753	257126	8122.52	0.3	1.0	1.1	74.2	0.0	287.8	0.6
			150	1925.1	31495.05	31495.36	0.00	191875	7	25247.88	0.1	0.3	3.3	19.8	0.0	24.7	0.3
			160	6942.3	31495.05	31495.36	0.00	496739	12	8514.43	0.2	1.0	1.1	73.0	0.0	269.9	0.8
			170	357.8	31495.07	31495.36	0.00	7465	9	25292.72	0.5	0.0	3.3	19.7	0.0	24.5	0.0
			180	148.8	31495.20	31495.36	0.00	5375	11	25292.72	0.2	0.0	3.3	19.7	0.0	24.5	0.0
			185	133.1	31495.36	31495.36	0.00	3190	0	25292.72	0.3	0.0	3.3	19.7	0.0	24.5	0.0
			200	7200.0	17609.55	31495.36	44.09	650833	398778	7580.60	0.2	1.0	1.0	75.9	0.0	315.5	1.0
			205	7200.0	22763.96	31495.36	27.72	589395	251496	7580.60	0.2	1.0	1.0	75.9	0.0	315.5	0.9
			210	7200.0	30439.67	31495.36	3.35	203092	18464	9690.67	0.3	1.0	1.3	69.2	0.0	225.0	0.3
			220	7200.0	26446.32	31495.36	16.03	130962	69466	9471.63	1.6	1.0	1.3	69.9	0.0	232.5	0.2
			230	7200.0	17156.37	31495.36	45.53	361165	228275	7580.60	0.4	1.0	1.0	75.9	0.0	315.5	0.6
			240	7200.0	20689.64	31495.36	34.31	289547	192274	9011.38	0.6	1.0	1.2	71.4	0.0	249.5	0.5
			250	1540.6	31495.05	31495.36	0.00	99056	4	25247.88	0.2	0.2	3.3	19.8	0.0	24.7	0.2
			260	7200.0	30506.20	31495.36	3.14	205297	17150	9690.67	0.5	1.0	1.3	69.2	0.0	225.0	0.3
			270	239.3	31495.36	31495.36	0.00	2765	0	25349.12	0.9	0.0	3.4	19.5	0.0	24.2	0.0
			280	64.2	31495.17	31495.36	0.00	2055	1	25349.12	0.4	0.0	3.4	19.5	0.0	24.2	0.0
			285	52.1	31495.36	31495.36	0.00	767	0	25349.12	0.7	0.0	3.4	19.5	0.0	24.2	0.0
17846.03	3	2 x 4	100	0.1	17846.03	17846.03	0.00	29	0	10295.83	0.0	1.0	1.0	42.3	0.0	73.3	1.0
			105	0.1	17846.03	17846.03	0.00	23	0	10295.83	0.0	1.2	1.0	42.3	0.0	73.3	0.8
			110	0.1	17846.03	17846.03	0.00	22	0	11441.34	0.0	0.9	1.1	35.9	0.0	56.0	0.8
			120	0.2	17846.03	17846.03	0.00	20	0	11387.94	0.0	2.0	1.1	36.2	0.0	56.7	0.7
			130	0.1	17846.03	17846.03	0.00	20	0	10295.83	0.0	1.2	1.0	42.3	0.0	73.3	0.7
			140	0.2	17846.03	17846.03	0.00	26	0	11182.69	0.0	1.9	1.1	37.3	0.0	59.6	0.9
			150	0.1	17846.03	17846.03	0.00	16	0	15782.09	0.0	1.1	1.5	11.6	0.0	13.1	0.6
			160	0.1	17846.03	17846.03	0.00	22	0	11441.34	0.0	0.9	1.1	35.9	0.0	56.0	0.8
			170	0.1	17846.03	17846.03	0.00	14	0	15837.93	0.0	1.0	1.5	11.3	0.0	12.7	0.5
			180	0.1	17846.03	17846.03	0.00	13	0	15837.93	0.0	0.8	1.5	11.3	0.0	12.7	0.4
			185	0.1	17846.03	17846.03	0.00	13	0	15837.93	0.0	0.9	1.5	11.3	0.0	12.7	0.4
			200	0.1	17846.03	17846.03	0.00	14	0	11372.49	0.0	1.3	1.1	36.3	0.0	56.9	0.5
			205	0.1	17846.03	17846.03	0.00	28	0	11372.49	0.0	1.4	1.1	36.3	0.0	56.9	1.0
			210	0.1	17846.03	17846.03	0.00	15	0	11895.19	0.0	1.4	1.2	33.3	0.0	50.0	0.5
			220	0.2	17846.03	17846.03	0.00	29	0	11895.19	0.0	2.6	1.2	33.3	0.0	50.0	1.0
			230	0.1	17846.03	17846.03	0.00	30	0	11372.49	0.0	1.3	1.1	36.3	0.0	56.9	1.0
			240	0.2	17846.03	17846.03	0.00	28	0	11439.03	0.0	1.8	1.1	35.9	0.0	56.0	1.0
			250	0.1	17846.03	17846.03	0.00	36	0	15816.37	0.0	1.4	1.5	11.4	0.0	12.8	1.2
			260	0.1	17846.03	17846.03	0.00	15	0	11895.19	0.0	1.4	1.2	33.3	0.0	50.0	0.5
			270	0.2	17846.03	17846.03	0.00	21	0	15837.93	0.0	1.8	1.5	11.3	0.0	12.7	0.7
			280	0.1	17846.03	17846.03	0.00	29	0	15837.93	0.0	1.4	1.5	11.3	0.0	12.7	1.0
			285	0.1	17846.03	17846.03	0.00	28	0	15837.93	0.0	1.6	1.5	11.3	0.0	12.7	1.0

Table C.2 (continued)

Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
				MIP						LPR							
32413.16	4	5 x 4	100	7200.0	23024.28	32413.16	28.97	801698	460387	13465.64	0.1	1.0	1.0	58.5	0.0	140.7	1.0
			105	7200.0	24856.47	32413.16	23.31	635227	277963	13465.64	0.1	1.0	1.0	58.5	0.0	140.7	0.8
			110	4248.9	32412.97	32413.16	0.00	178244	6	14493.87	0.1	0.6	1.1	55.3	0.0	123.6	0.2
			120	7200.0	29014.53	32413.16	10.49	89651	38629	14389.59	0.6	1.0	1.1	55.6	0.0	125.3	0.1
			130	7200.0	22284.96	32413.16	31.25	362558	225318	13465.64	0.2	1.0	1.0	58.5	0.0	140.7	0.5
			140	7200.0	25614.58	32413.16	20.97	467524	236367	14018.96	0.4	1.0	1.0	56.7	0.0	131.2	0.6
			150	2270.4	32412.84	32413.16	0.00	228033	31	31144.21	0.1	0.3	2.3	3.9	0.0	4.1	0.3
			160	4235.9	32412.97	32413.16	0.00	178244	6	14493.87	0.2	0.6	1.1	55.3	0.0	123.6	0.2
			170	242.0	32413.16	32413.16	0.00	2771	0	31201.37	1.0	0.0	2.3	3.7	0.0	3.9	0.0
			180	95.8	32413.16	32413.16	0.00	2584	0	31201.37	0.3	0.0	2.3	3.7	0.0	3.9	0.0
			185	64.1	32413.16	32413.16	0.00	927	0	31201.37	0.6	0.0	2.3	3.7	0.0	3.9	0.0
			200	7200.0	23596.89	32413.16	27.20	932021	478693	13475.89	0.1	1.0	1.0	58.4	0.0	140.5	1.2
			205	7200.0	25397.80	32413.16	21.64	727332	267331	13475.89	0.1	1.0	1.0	58.4	0.0	140.5	0.9
			210	3525.7	32412.87	32413.16	0.00	177645	7	15096.69	0.1	0.5	1.1	53.4	0.0	114.7	0.2
			220	7200.0	30396.24	32413.16	6.22	146722	43084	14897.85	1.0	1.0	1.1	54.0	0.0	117.6	0.2
			230	7200.0	22492.76	32486.34	30.76	449892	234637	13475.89	0.2	1.0	1.0	58.4	0.2	141.1	0.6
			240	7200.0	27694.81	32413.16	14.56	422641	181911	14333.31	0.5	1.0	1.1	55.8	0.0	126.1	0.5
			250	4391.6	32412.85	32413.16	0.00	534399	81	31144.21	0.2	0.6	2.3	3.9	0.0	4.1	0.7
			260	3557.8	32412.87	32413.16	0.00	177645	7	15096.69	0.4	0.5	1.1	53.4	0.0	114.7	0.2
			270	111.2	32413.16	32413.16	0.00	1019	0	31234.56	0.9	0.0	2.3	3.6	0.0	3.8	0.0
			280	60.8	32413.16	32413.16	0.00	1198	0	31234.56	0.4	0.0	2.3	3.6	0.0	3.8	0.0
			285	36.5	32413.16	32413.16	0.00	410	0	31234.56	0.5	0.0	2.3	3.6	0.0	3.8	0.0
6190.97	5	3 x 4	100	1.1	6190.97	6190.97	0.00	458	0	299.86	0.0	1.0	1.0	95.2	0.0	1964.6	1.0
			105	1.5	6190.97	6190.97	0.00	657	0	299.86	0.0	1.4	1.0	95.2	0.0	1964.6	1.4
			110	0.8	6190.97	6190.97	0.00	116	0	554.69	0.0	0.7	1.8	91.0	0.0	1016.1	0.3
			120	2.0	6190.97	6190.97	0.00	176	0	551.60	0.1	1.9	1.8	91.1	0.0	1022.4	0.4
			130	1.0	6190.97	6190.97	0.00	203	0	299.86	0.0	0.9	1.0	95.2	0.0	1964.6	0.4
			140	2.7	6190.97	6190.97	0.00	525	0	416.22	0.0	2.5	1.4	93.3	0.0	1387.4	1.1
			150	1.1	6190.97	6190.97	0.00	306	0	3473.27	0.0	1.0	11.6	43.9	0.0	78.2	0.7
			160	0.7	6190.97	6190.97	0.00	116	0	554.69	0.0	0.7	1.8	91.0	0.0	1016.1	0.3
			170	1.3	6190.97	6190.97	0.00	83	0	3576.52	0.0	1.2	11.9	42.2	0.0	73.1	0.2
			180	0.9	6190.97	6190.97	0.00	71	0	3576.52	0.0	0.8	11.9	42.2	0.0	73.1	0.2
			185	0.7	6190.97	6190.97	0.00	40	0	3576.52	0.0	0.6	11.9	42.2	0.0	73.1	0.1
			200	0.5	6190.97	6190.97	0.00	155	0	318.90	0.0	0.5	1.1	94.8	0.0	1841.3	0.3
			205	0.9	6190.97	6190.97	0.00	356	0	318.90	0.0	0.9	1.1	94.8	0.0	1841.3	0.8
			210	0.3	6190.97	6190.97	0.00	19	0	1083.95	0.0	0.3	3.6	82.5	0.0	471.1	0.0
			220	1.3	6190.97	6190.97	0.00	89	0	1030.65	0.1	1.2	3.4	83.4	0.0	500.7	0.2
			230	0.8	6190.97	6190.97	0.00	137	0	318.90	0.0	0.7	1.1	94.8	0.0	1841.3	0.3
			240	1.8	6190.97	6190.97	0.00	361	0	931.63	0.0	1.7	3.1	85.0	0.0	564.5	0.8
			250	0.5	6190.97	6190.97	0.00	92	0	3473.27	0.0	0.4	11.6	43.9	0.0	78.2	0.2
			260	0.3	6190.97	6190.97	0.00	19	0	1083.95	0.0	0.3	3.6	82.5	0.0	471.1	0.0
			270	0.6	6190.97	6190.97	0.00	15	0	3607.84	0.1	0.6	12.0	41.7	0.0	71.6	0.0
			280	0.4	6190.97	6190.97	0.00	6	0	3607.84	0.0	0.4	12.0	41.7	0.0	71.6	0.0
			285	0.6	6190.97	6190.97	0.00	26	0	3607.84	0.0	0.6	12.0	41.7	0.0	71.6	0.1
8332.36	6	4 x 4	100	0.3	8332.36	8332.36	0.00	77	0	0.39	0.0	1.0	1.0	100.0	0.0	2137012.3	1.0
			105	0.3	8332.36	8332.36	0.00	72	0	0.39	0.0	1.2	1.0	100.0	0.0	2137012.3	0.9
			110	0.3	8332.36	8332.36	0.00	21	0	534.97	0.0	1.1	1372.1	93.6	0.0	1457.6	0.3
			120	0.5	8332.36	8332.36	0.00	25	0	283.59	0.1	1.9	727.4	96.6	0.0	2838.2	0.3
			130	0.3	8332.36	8332.36	0.00	82	0	0.39	0.0	1.3	1.0	100.0	0.0	2137012.3	1.1
			140	0.4	8332.36	8332.36	0.00	46	0	215.01	0.0	1.7	551.5	97.4	0.0	3775.3	0.6
			150	0.3	8332.36	8332.36	0.00	46	0	6166.57	0.0	1.1	15816.2	26.0	0.0	35.1	0.6
			160	0.3	8332.36	8332.36	0.00	21	0	534.97	0.0	1.2	1372.1	93.6	0.0	1457.6	0.3
			170	0.4	8332.36	8332.36	0.00	16	0	8093.54	0.1	1.6	20758.6	2.9	0.0	3.0	0.2
			180	0.3	8332.36	8332.36	0.00	13	0	8093.54	0.0	1.4	20758.6	2.9	0.0	3.0	0.2
			185	0.3	8332.36	8332.36	0.00	14	0	8093.54	0.1	1.3	20758.6	2.9	0.0	3.0	0.2
			200	0.8	8332.36	8332.36	0.00	143	0	21.46	0.0	3.2	55.1	99.7	0.0	38719.6	1.9
			205	0.7	8332.36	8332.36	0.00	97	0	21.46	0.0	2.9	55.1	99.7	0.0	38719.6	1.3
			210	0.7	8332.36	8332.36	0.00	82	0	1221.51	0.0	2.7	3133.0	85.3	0.0	582.1	1.1
			220	2.5	8332.36	8332.36	0.00	172	0	788.74	0.1	10.2	2023.0	90.5	0.0	956.4	2.2
			230	0.8	8332.36	8332.36	0.00	52	0	21.46	0.1	3.4	55.1	99.7	0.0	38719.6	0.7
			240	1.0	8332.36	8332.36	0.00	95	0	652.52	0.0	4.0	1673.6	92.2	0.0	1177.0	1.2
			250	0.8	8332.36	8332.36	0.00	45	0	6178.39	0.0	3.3	15846.5	25.9	0.0	34.9	0.6
			260	0.7	8332.36	8332.36	0.00	82	0	1221.51	0.1	2.8	3133.0	85.3	0.0	582.1	1.1
			270	1.5	8332.36	8332.36	0.00	22	0	8122.24	0.1	5.9	20832.2	2.5	0.0	2.6	0.3
			280	0.9	8332.36	8332.36	0.00	30	0	8122.24	0.1	3.6	20832.2	2.5	0.0	2.6	0.4
			285	1.0	8332.36	8332.36	0.00	26	0	8122.24	0.1	4.0	20832.2	2.5	0.0	2.6	0.3

Table C.2 (continued)

Opt. sol.	inst#		Option	MIP						LPR							
				CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
INFEASIBLE	7	5 x 4	100	1.8	infeasible	infeasible	-	35	0	2.58	0.1						
			105	1.3	infeasible	infeasible	-	15	0	2.58	0.1						
			110	2.7	infeasible	infeasible	-	0	0	1571.73	0.1						
			120	5.1	infeasible	infeasible	-	6	0	1079.70	0.3						
			130	4.2	infeasible	infeasible	-	8	0	2.58	0.1						
			140	2.5	infeasible	infeasible	-	16	0	463.91	0.2						
			150	0.0	infeasible	infeasible	-	0	0	infeasible	0.0						
			160	2.8	infeasible	infeasible	-	0	0	1571.73	0.1						
			170	0.1	infeasible	infeasible	-	0	0	infeasible	0.1						
			180	0.1	infeasible	infeasible	-	0	0	infeasible	0.1						
			185	0.1	infeasible	infeasible	-	0	0	infeasible	0.1						
			200	1.8	infeasible	infeasible	-	25	0	123.00	0.2						
			205	1.6	infeasible	infeasible	-	11	0	123.00	0.2						
			210	0.1	infeasible	infeasible	-	0	0	4380.25	0.2						
			220	0.6	infeasible	infeasible	-	0	0	3504.02	1.3						
			230	4.9	infeasible	infeasible	-	12	0	123.22	0.4						
			240	3.1	infeasible	infeasible	-	22	0	2269.63	0.4						
			250	0.0	infeasible	infeasible	-	0	0	infeasible	0.1						
			260	0.1	infeasible	infeasible	-	0	0	4380.25	0.4						
			270	0.1	infeasible	infeasible	-	0	0	infeasible	1.0						
			280	0.1	infeasible	infeasible	-	0	0	infeasible	0.3						
			285	0.1	infeasible	infeasible	-	0	0	infeasible	0.3						
55963.26	8	5 x 4	100	7200.0	35055.51	55963.26	37.36	832413	468353	17844.65	0.1	1.0	1.0	68.1	0.0	213.6	1.0
			105	7200.0	41477.21	55963.26	25.88	748664	336209	17844.65	0.1	1.0	1.0	68.1	0.0	213.6	0.9
			110	4909.2	55962.73	55963.26	0.00	308089	21	19821.83	0.1	0.7	1.1	64.6	0.0	182.3	0.4
			120	7200.0	48438.91	55963.26	13.45	139083	63978	19270.58	1.1	1.0	1.1	65.6	0.0	190.4	0.2
			130	7200.0	37157.45	55963.26	33.60	552632	308029	17844.65	0.2	1.0	1.0	68.1	0.0	213.6	0.7
			140	7200.0	41619.08	55963.26	25.63	408133	223665	18518.66	0.4	1.0	1.0	66.9	0.0	202.2	0.5
			150	3626.8	55962.73	55963.26	0.00	362351	38	47610.82	0.1	0.5	2.7	14.9	0.0	17.5	0.4
			160	4917.3	55962.73	55963.26	0.00	308089	21	19821.83	0.2	0.7	1.1	64.6	0.0	182.3	0.4
			170	193.3	55963.26	55963.26	0.00	4883	0	47775.57	0.7	0.0	2.7	14.6	0.0	17.1	0.0
			180	97.8	55963.26	55963.26	0.00	3692	0	47775.57	0.3	0.0	2.7	14.6	0.0	17.1	0.0
			185	54.5	55963.26	55963.26	0.00	1289	0	47775.57	0.5	0.0	2.7	14.6	0.0	17.1	0.0
			200	7200.0	37945.18	55963.26	32.20	589944	324231	17853.95	0.2	1.0	1.0	68.1	0.0	213.5	0.7
			205	7200.0	35799.37	56214.50	36.32	435820	211268	17853.95	0.2	1.0	1.0	68.1	0.4	214.9	0.5
			210	5387.0	55962.86	55963.26	0.00	263786	14	20696.60	0.2	0.7	1.2	63.0	0.0	170.4	0.3
			220	7200.0	47402.74	55963.26	15.30	98672	51817	19918.32	1.8	1.0	1.1	64.4	0.0	181.0	0.1
			230	7200.0	35465.19	55963.26	36.63	315839	206897	17853.95	0.5	1.0	1.0	68.1	0.0	213.5	0.4
			240	7200.0	37059.40	56013.74	33.84	395952	257009	19223.61	0.8	1.0	1.1	65.6	0.1	191.4	0.5
			250	7200.0	55744.46	55963.26	0.39	573900	18520	47622.19	0.2	1.0	2.7	14.9	0.0	17.5	0.7
			260	5488.2	55962.86	55963.26	0.00	263786	14	20696.60	0.4	0.8	1.2	63.0	0.0	170.4	0.3
			270	197.1	55963.26	55963.26	0.00	2942	0	47815.13	1.1	0.0	2.7	14.6	0.0	17.0	0.0
			280	118.9	55963.26	55963.26	0.00	3682	0	47815.13	0.5	0.0	2.7	14.6	0.0	17.0	0.0
			285	87.4	55963.26	55963.26	0.00	1224	0	47815.13	0.7	0.0	2.7	14.6	0.0	17.0	0.0
27583.18	9	5 x 4	100	7200.0	22032.60	27583.18	20.12	958893	359085	8589.45	0.1	1.0	1.0	68.9	0.0	221.1	1.0
			105	7200.0	26524.81	27583.18	3.84	1085070	111152	8589.45	0.1	1.0	1.0	68.9	0.0	221.1	1.1
			110	1268.1	27582.99	27583.18	0.00	49647	2	9669.63	0.1	0.2	1.1	64.9	0.0	185.3	0.1
			120	3122.4	27583.04	27583.18	0.00	59831	1	9553.11	0.7	0.4	1.1	65.4	0.0	188.7	0.1
			130	7200.0	18956.84	27583.18	31.27	488608	275290	8589.45	0.1	1.0	1.0	68.9	0.0	221.1	0.5
			140	7200.0	25016.20	27583.18	9.31	592854	157096	9131.42	0.4	1.0	1.1	66.9	0.0	202.1	0.6
			150	310.3	27583.02	27583.18	0.00	29944	4	26257.42	0.1	0.0	3.1	4.8	0.0	5.0	0.0
			160	1261.9	27582.99	27583.18	0.00	49647	2	9669.63	0.2	0.2	1.1	64.9	0.0	185.3	0.1
			170	62.8	27583.18	27583.18	0.00	1541	0	26315.37	0.7	0.0	3.1	4.6	0.0	4.8	0.0
			180	36.3	27583.18	27583.18	0.00	1330	0	26315.37	0.3	0.0	3.1	4.6	0.0	4.8	0.0
			185	58.9	27583.18	27583.18	0.00	1326	0	26315.37	0.5	0.0	3.1	4.6	0.0	4.8	0.0
			200	7200.0	20559.86	27583.18	25.46	918132	421176	8625.15	0.2	1.0	1.0	68.7	0.0	219.8	1.0
			205	7200.0	24778.50	27583.18	10.17	945529	171416	8625.15	0.2	1.0	1.0	68.7	0.0	219.8	1.0
			210	634.4	27582.93	27583.18	0.00	26658	2	11101.97	0.2	0.1	1.3	59.8	0.0	148.5	0.0
			220	2541.8	27582.99	27583.18	0.00	40089	1	10834.33	1.3	0.4	1.3	60.7	0.0	154.6	0.0
			230	7200.0	21991.26	27583.18	20.27	483129	236535	8625.15	0.4	1.0	1.0	68.7	0.0	219.8	0.5
			240	7200.0	26169.81	27583.18	5.12	462186	64598	9735.48	0.5	1.0	1.1	64.7	0.0	183.3	0.5
			250	2309.8	27582.91	27583.18	0.00	277101	15	26257.42	0.2	0.3	3.1	4.8	0.0	5.0	0.3
			260	633.3	27582.93	27583.18	0.00	26658	2	11101.97	0.5	0.1	1.3	59.8	0.0	148.5	0.0
			270	34.3	27583.18	27583.18	0.00	598	0	26342.82	1.1	0.0	3.1	4.5	0.0	4.7	0.0
			280	19.3	27583.18	27583.18	0.00	516	0	26342.82	0.6	0.0	3.1	4.5	0.0	4.7	0.0
			285	23.1	27583.18	27583.18	0.00	248	0	26342.82	0.7	0.0	3.1	4.5	0.0	4.7	0.0

Table C.2 (continued)

Opt. sol.	inst#		Option	MIP						LPR							
				CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
64047.26	10	5 x 8	100	7200.0	29807.49	69572.09	57.16	445405	338569	25965.63	0.3	1.0	1.0	59.5	8.6	167.9	1.0
			105	7200.0	29737.27	73656.96	59.63	362378	258948	25965.63	0.3	1.0	1.0	59.5	15.0	183.7	0.8
			110	7200.0	41639.37	65230.26	36.17	93794	70586	27314.97	0.5	1.0	1.1	57.4	1.8	138.8	0.2
			120	7200.0	38501.73	69421.21	44.54	44653	34791	27141.27	3.9	1.0	1.0	57.6	8.4	155.8	0.1
			130	7200.0	29528.04	69928.62	57.77	228572	172928	25965.63	0.5	1.0	1.0	59.5	9.2	169.3	0.5
			140	7200.0	30303.81	70467.96	57.00	162740	141495	26516.98	1.5	1.0	1.0	58.6	10.0	165.7	0.4
			150	7200.0	44923.13	69810.07	35.65	329577	230568	43640.84	0.3	1.0	1.7	31.9	9.0	60.0	0.7
			160	7200.0	41635.67	65230.26	36.17	93412	70262	27314.97	1.2	1.0	1.1	57.4	1.8	138.8	0.2
			170	7200.0	53513.05	69554.12	23.06	29200	20626	43708.60	1.8	1.0	1.7	31.8	8.6	59.1	0.1
			180	7200.0	52754.65	64494.90	18.20	97503	68945	43708.60	0.9	1.0	1.7	31.8	0.7	47.6	0.2
			185	7200.0	55407.69	68780.20	19.44	67318	44568	43708.60	1.4	1.0	1.7	31.8	7.4	57.4	0.2
			200	7200.0	30274.06	69421.90	56.39	271663	214088	25975.79	1.3	1.0	1.0	59.4	8.4	167.3	0.6
			205	7200.0	29905.25	76074.57	60.69	286300	215264	25975.79	1.5	1.0	1.0	59.4	18.8	192.9	0.6
			210	7200.0	42451.97	64541.77	34.23	68343	55341	28473.61	1.5	1.0	1.1	55.5	0.8	126.7	0.2
			220	7200.0	38492.59	69289.11	44.45	36495	31309	28139.58	11.1	1.0	1.1	56.1	8.2	146.2	0.1
			230	7200.0	29589.10	69621.19	57.50	162600	142206	25975.79	2.4	1.0	1.0	59.4	8.7	168.0	0.4
			240	7200.0	30779.88	65619.67	53.09	146462	121803	27388.25	3.5	1.0	1.1	57.2	2.5	139.6	0.3
			250	7200.0	44887.18	65276.65	31.24	207386	138368	43640.84	1.3	1.0	1.7	31.9	1.9	49.6	0.5
			260	7200.0	42442.83	64541.77	34.24	68106	55139	28473.61	3.0	1.0	1.1	55.5	0.8	126.7	0.2
			270	7200.0	52276.66	64729.96	19.24	36869	27174	43761.41	4.1	1.0	1.7	31.7	1.1	47.9	0.1
			280	7200.0	56158.55	64541.77	12.99	79286	55838	43761.41	2.5	1.0	1.7	31.7	0.8	47.5	0.2
			285	7200.0	59219.56	64062.21	7.56	58081	30225	43761.41	3.2	1.0	1.7	31.7	0.0	46.4	0.1
47012.46	11	5 x 8	100	7200.0	26994.05	47044.83	42.62	309544	240153	23249.74	0.3	1.0	1.0	50.5	0.1	102.3	1.0
			105	7200.0	30171.89	47437.90	36.40	294428	176889	23249.74	0.5	1.0	1.0	50.5	0.9	104.0	1.0
			110	7200.0	38402.23	47012.46	18.31	88594	67343	24621.97	0.5	1.0	1.1	47.6	0.0	90.9	0.3
			120	7200.0	35669.95	48679.06	26.72	36296	26373	24472.84	2.7	1.0	1.1	47.9	3.5	98.9	0.1
			130	7200.0	28656.55	47735.23	39.97	171519	124400	23249.74	0.8	1.0	1.0	50.5	1.5	105.3	0.6
			140	7200.0	28569.53	47088.33	39.33	144334	112035	23794.76	1.4	1.0	1.0	49.4	0.2	97.9	0.5
			150	7200.0	41347.24	50127.29	17.52	385965	287792	40918.62	0.5	1.0	1.8	13.0	6.6	22.5	1.2
			160	7200.0	38394.78	47012.46	18.33	88145	67009	24621.97	0.8	1.0	1.1	47.6	0.0	90.9	0.3
			170	7200.0	46702.63	47012.46	0.66	49046	8186	41085.73	2.2	1.0	1.8	12.6	0.0	14.4	0.2
			180	3969.6	47012.15	47012.46	0.00	73209	9	41085.73	1.1	0.6	1.8	12.6	0.0	14.4	0.2
			185	1047.1	47012.28	47012.46	0.00	9936	2	41085.73	2.2	0.1	1.8	12.6	0.0	14.4	0.0
			200	7200.0	26934.14	47556.95	43.36	339688	268436	23299.90	0.7	1.0	1.0	50.4	1.2	104.1	1.1
			205	7200.0	27483.17	49487.76	44.46	252721	187272	23299.90	0.7	1.0	1.0	50.4	5.3	112.4	0.8
			210	7200.0	39660.73	47012.46	15.64	87998	64604	25161.28	0.6	1.0	1.1	46.5	0.0	86.8	0.3
			220	7200.0	35891.59	47012.46	23.66	35456	26783	24967.62	4.4	1.0	1.1	46.9	0.0	88.3	0.1
			230	7200.0	27192.82	51087.63	46.77	206949	154507	23299.90	1.0	1.0	1.0	50.4	8.7	119.3	0.7
			240	7200.0	28920.30	47055.95	38.54	173992	145764	24125.04	1.6	1.0	1.0	48.7	0.1	95.1	0.6
			250	7200.0	41636.81	47276.73	11.93	282882	185883	40918.62	0.5	1.0	1.8	13.0	0.6	15.5	0.9
			260	7200.0	39670.79	47012.46	15.62	88570	65016	25161.28	0.8	1.0	1.1	46.5	0.0	86.8	0.3
			270	3693.4	47012.00	47012.46	0.00	30498	8	41085.63	3.9	0.5	1.8	12.6	0.0	14.4	0.1
			280	1066.8	47012.33	47012.46	0.00	18022	2	41085.63	1.7	0.1	1.8	12.6	0.0	14.4	0.1
			285	654.1	47012.03	47012.46	0.00	7370	1	41085.63	2.2	0.1	1.8	12.6	0.0	14.4	0.0
INFEASIBLE	12	8 x 8	100	7200.0	21907.13	no feas sol	inf	113193	98622	17825.10	1.9						
			105	7200.0	21679.42	no feas sol	inf	104933	83191	17825.10	2.7						
			110	7200.0	44206.84	no feas sol	inf	21212	13415	19554.31	2.1						
			120	422.0	infeasible	infeasible	-	140	0	18786.36	47.1						
			130	7200.0	25759.53	no feas sol	inf	77997	58103	17825.10	5.2						
			140	7200.0	21104.53	no feas sol	inf	31720	26561	18150.89	16.7						
			150	0.4	infeasible	infeasible	-	0	0	infeasible	1.0						
			160	7200.0	44204.48	no feas sol	inf	21173	13394	19554.31	4.7						
			170	4.0	infeasible	infeasible	-	0	0	infeasible	3.0						
			180	2.9	infeasible	infeasible	-	0	0	infeasible	1.7						
			185	4.2	infeasible	infeasible	-	0	0	infeasible	2.6						
			200	7200.0	20939.22	no feas sol	inf	74252	62536	17831.68	3.5						
			205	7200.0	22779.59	no feas sol	inf	56541	39856	17831.68	6.9						
			210	7200.0	47987.05	no feas sol	inf	8919	2862	21602.07	6.9						
			220	7200.0	46946.16	no feas sol	inf	3298	2039	20168.46	60.6						
			230	7200.0	22126.39	no feas sol	inf	60944	50580	17831.74	8.7						
			240	7200.0	21128.72	no feas sol	inf	41965	37167	19311.14	14.1						
			250	0.6	infeasible	infeasible	-	0	0	infeasible	1.6						
			260	7200.0	47991.75	no feas sol	inf	8946	2871	21602.07	12.0						
			270	3.9	infeasible	infeasible	-	0	0	infeasible	6.4						
			280	4.5	infeasible	infeasible	-	0	0	infeasible	4.9						
			285	5.6	infeasible	infeasible	-	0	0	infeasible	7.3						

Table C.2 (continued)

Opt. sol.	inst#		Option	MIP						LPR							
				CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
39020.25	13	9 x 8	100	7200.0	15451.03	46922.94	67.07	66863	58082	14752.07	1.5	1.0	1.0			218.1	1.0
			105	7200.0	15461.34	56771.76	72.77	69974	59894	14752.07	3.7	1.0	1.0			284.8	1.0
			110	7200.0	19881.38	43764.25	54.57	17541	15588	15060.25	4.8	1.0	1.0			190.6	0.3
			120	7200.0	16812.82	49233.01	65.85	3047	2871	14926.94	54.5	1.0	1.0			229.8	0.0
			130	7200.0	15448.88	48346.92	68.05	38188	34349	14752.07	6.2	1.0	1.0			227.7	0.6
			140	7200.0	15459.35	46958.92	67.08	18010	17154	14829.49	23.2	1.0	1.0			216.7	0.3
			150	7200.0	25613.70	47696.50	46.30	66840	60153	25605.77	1.4	1.0	1.7			86.3	1.0
			160	7200.0	19881.38	43764.25	54.57	17540	15587	15060.25	7.5	1.0	1.0			190.6	0.3
			170	7200.0	31084.75	41280.15	24.70	5170	4179	28006.78	30.7	1.0	1.9			47.4	0.1
			180	7200.0	31509.98	40837.37	22.84	13767	11226	28006.78	20.5	1.0	1.9			45.8	0.2
			185	7200.0	31526.46	46103.77	31.62	8350	5532	28006.78	25.2	1.0	1.9			64.6	0.1
			200	7200.0	15451.26	48075.62	67.86	60398	55690	14754.09	4.9	1.0	1.0			225.8	0.9
			205	7200.0	15457.76	57995.05	73.35	53734	43912	14754.09	8.7	1.0	1.0			293.1	0.8
			210	7200.0	21837.42	41767.49	47.72	11407	10022	15817.01	11.7	1.0	1.1			164.1	0.2
			220	7200.0	16779.41	41753.77	59.81	3530	3008	15391.05	87.2	1.0	1.0			171.3	0.1
			230	7200.0	15450.50	47222.39	67.28	31804	29686	14754.09	11.5	1.0	1.0			220.1	0.5
			240	7200.0	15465.30	47366.20	67.35	29380	28009	15096.73	27.8	1.0	1.0			213.8	0.4
			250	7200.0	25670.14	46830.42	45.18	59341	55494	25606.83	5.7	1.0	1.7			82.9	0.9
			260	7200.0	21837.42	41767.49	47.72	11397	10012	15817.01	17.5	1.0	1.1			164.1	0.2
			270	7200.0	31673.80	39872.46	20.56	4268	3553	28317.42	71.6	1.0	1.9			40.8	0.1
			280	7200.0	32132.71	40195.37	20.06	8492	6211	28317.42	38.5	1.0	1.9			41.9	0.1
			285	7200.0	32297.73	39020.25	17.23	5074	4478	28317.42	59.3	1.0	1.9			37.8	0.1
INFEASIBLE	14	9 x 8	100	7200.0	21265.57	no feas sol	inf	34610	22087	17266.72	2.0						
			105	7200.0	21418.34	no feas sol	inf	23573	13034	17266.72	3.6						
			110	7200.0	49739.48	no feas sol	inf	12457	7664	18665.07	4.1						
			120	496.2	infeasible	infeasible	-	36	0	18093.93	94.5						
			130	7200.0	21266.53	no feas sol	inf	7593	4352	17266.72	4.7						
			140	7200.0	20483.51	no feas sol	inf	7239	5638	17610.29	25.2						
			150	0.6	infeasible	infeasible	-	0	0	infeasible	1.0						
			160	7200.0	49744.80	no feas sol	inf	12515	7700	18665.07	9.9						
			170	20.2	infeasible	infeasible	-	0	0	infeasible	7.6						
			180	6.2	infeasible	infeasible	-	0	0	infeasible	3.5						
			185	5.9	infeasible	infeasible	-	0	0	infeasible	3.7						
			200	7200.0	24824.82	no feas sol	inf	7089	3225	17339.78	16.5						
			205	1546.0	infeasible	infeasible	-	1245	0	17339.78	24.7						
			210	7200.0	37857.74	no feas sol	inf	6515	5480	21282.12	25.2						
			220	7200.0	35513.33	no feas sol	inf	724	433	19961.90	232.7						
			230	835.5	infeasible	infeasible	-	274	0	17339.78	35.5						
			240	7200.0	19136.24	no feas sol	inf	6104	5333	18562.15	56.7						
			250	1.0	infeasible	infeasible	-	0	0	infeasible	5.4						
			260	7200.0	37857.74	no feas sol	inf	6502	5467	21282.12	44.3						
			270	14.3	infeasible	infeasible	-	0	0	infeasible	11.4						
			280	8.3	infeasible	infeasible	-	0	0	infeasible	5.7						
			285	4.4	infeasible	infeasible	-	0	0	infeasible	19.9						
52021.20	15	6 x 4	100	7200.0	25663.87	52022.08	50.67	462652	295305	15170.34	0.2	1.0	1.0	70.8	0.0	242.9	1.0
			105	7200.0	33929.37	52021.20	34.78	429282	221624	15170.34	0.3	1.0	1.0	70.8	0.0	242.9	0.9
			110	7200.0	49751.67	52021.20	4.36	219527	35180	17048.88	0.2	1.0	1.1	67.2	0.0	205.1	0.5
			120	7200.0	40682.21	52021.20	21.80	56179	33530	16088.81	2.1	1.0	1.1	69.1	0.0	223.3	0.1
			130	7200.0	29327.59	52021.20	43.62	274920	200976	15170.34	0.5	1.0	1.0	70.8	0.0	242.9	0.6
			140	7200.0	29882.61	52022.08	42.56	259285	188879	15600.31	1.0	1.0	1.0	70.0	0.0	233.5	0.6
			150	3923.0	52020.69	52021.20	0.00	230859	26	49951.96	0.3	0.5	3.3	4.0	0.0	4.1	0.5
			160	7200.0	49811.03	52021.20	4.25	221740	34515	17048.88	0.6	1.0	1.1	67.2	0.0	205.1	0.5
			170	414.2	52021.20	52021.20	0.00	4670	1	50130.44	2.7	0.1	3.3	3.6	0.0	3.8	0.0
			180	244.2	52021.20	52021.20	0.00	6371	0	50130.44	1.0	0.0	3.3	3.6	0.0	3.8	0.0
			185	72.0	52021.20	52021.20	0.00	873	0	50130.44	1.6	0.0	3.3	3.6	0.0	3.8	0.0
			200	7200.0	28235.92	52021.20	45.72	355179	266641	15281.95	0.7	1.0	1.0	70.6	0.0	240.4	0.8
			205	7200.0	28888.53	52021.20	44.47	289503	181147	15281.95	0.8	1.0	1.0	70.6	0.0	240.4	0.6
			210	7200.0	51140.45	52021.20	1.69	188920	12104	18340.58	0.6	1.0	1.2	64.7	0.0	183.6	0.4
			220	7200.0	40199.32	52021.20	22.73	44479	28970	17035.07	3.8	1.0	1.1	67.3	0.0	205.4	0.1
			230	7200.0	28136.12	52021.20	45.91	215499	151045	15281.95	1.1	1.0	1.0	70.6	0.0	240.4	0.5
			240	7200.0	30544.64	52021.20	41.28	199636	151478	16197.41	1.9	1.0	1.1	68.9	0.0	221.2	0.4
			250	7200.0	51294.38	52021.20	1.40	303942	122411	50024.71	0.6	1.0	3.3	3.8	0.0	4.0	0.7
			260	7200.0	51142.81	52021.20	1.69	188988	12060	18340.58	1.0	1.0	1.2	64.7	0.0	183.6	0.4
			270	208.0	52021.20	52021.20	0.00	1565	0	50319.89	4.3	0.0	3.3	3.3	0.0	3.4	0.0
			280	157.5	52021.20	52021.20	0.00	2005	0	50319.89	1.5	0.0	3.3	3.3	0.0	3.4	0.0
			285	57.5	52021.20	52021.20	0.00	336	0	50322.94	2.7	0.0	3.3	3.3	0.0	3.4	0.0

Table C.2 (continued)

Opt. sol.	inst#		Option	MIP CPU	MIP LB	MIP UB	MIP Gap (%)	MIP Total Nodes	MIP Rem. Nodes	LPR LPR	LPR LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
31367.40	16	7 x 4	100	7200.0	15994.86	31476.44	49.18	317821	231550	13566.32	0.3	1.0	1.0	56.8	0.3	132.0	1.0
			105	7200.0	17931.41	31434.93	42.96	306065	203332	13566.32	0.3	1.0	1.0	56.8	0.2	131.7	1.0
			110	7200.0	27333.92	31367.40	12.86	156211	95318	14269.93	0.3	1.0	1.1	54.5	0.0	119.8	0.5
			120	7200.0	20273.36	31371.01	35.38	43600	32172	13762.60	2.9	1.0	1.0	56.1	0.0	127.9	0.1
			130	7200.0	16876.97	31471.84	46.37	231081	182379	13566.32	0.8	1.0	1.0	56.8	0.3	132.0	0.7
			140	7200.0	19111.63	31371.01	39.08	235326	182894	13665.74	1.2	1.0	1.0	56.4	0.0	129.6	0.7
			150	7200.0	27394.33	31371.01	12.68	262235	182271	26349.10	0.4	1.0	1.9	16.0	0.0	19.1	0.8
			160	7200.0	27107.06	31367.40	13.58	178637	107059	14269.93	0.7	1.0	1.1	54.5	0.0	119.8	0.6
			170	1296.8	31367.30	31367.40	0.00	17166	4	30216.80	4.6	0.2	2.2	3.7	0.0	3.8	0.1
			180	630.4	31367.31	31367.40	0.00	11413	1	30216.80	1.4	0.1	2.2	3.7	0.0	3.8	0.0
			185	315.2	31367.40	31367.40	0.00	3814	0	30216.80	3.1	0.0	2.2	3.7	0.0	3.8	0.0
			200	7200.0	17438.02	31367.40	44.41	382171	296531	13579.86	0.7	1.0	1.0	56.7	0.0	131.0	1.2
			205	7200.0	17804.04	31434.93	43.36	274410	195970	13579.86	0.7	1.0	1.0	56.7	0.2	131.5	0.9
			210	7200.0	26843.14	31367.40	14.42	188763	137412	14720.15	0.8	1.0	1.1	53.1	0.0	113.1	0.6
			220	7200.0	22871.15	31369.74	27.09	40120	29410	14044.32	4.7	1.0	1.0	55.2	0.0	123.4	0.1
			230	7200.0	16800.72	31514.07	46.69	179822	136976	13579.86	1.7	1.0	1.0	56.7	0.5	132.1	0.6
			240	7200.0	17393.79	31367.40	44.55	240416	194147	13833.71	1.6	1.0	1.0	55.9	0.0	126.7	0.8
			250	7200.0	27633.40	31434.93	12.09	281470	172126	26360.00	0.9	1.0	1.9	16.0	0.2	19.3	0.9
			260	7200.0	27188.97	31367.40	13.32	191145	131753	14720.15	1.5	1.0	1.1	53.1	0.0	113.1	0.6
			270	259.5	31367.29	31367.40	0.00	1844	1	30217.58	3.9	0.0	2.2	3.7	0.0	3.8	0.0
			280	177.0	31367.16	31367.40	0.00	2815	1	30217.58	2.9	0.0	2.2	3.7	0.0	3.8	0.0
			285	133.0	31367.40	31367.40	0.00	1081	0	30217.58	3.5	0.0	2.2	3.7	0.0	3.8	0.0
INFEASIBLE	17	7 x 4	100	7200.0	19636.84	no feas sol	inf	187715	70918	5423.95	0.3						
			105	2165.8	infeasible	infeasible	-	46356	0	5423.95	0.4						
			110	7200.0	32660.77	no feas sol	inf	62595	44771	6891.20	0.4						
			120	7200.0	44465.81	no feas sol	inf	33663	24603	6602.23	5.8						
			130	55.8	infeasible	infeasible	-	348	0	5423.95	0.6						
			140	7200.0	18136.09	no feas sol	inf	118392	73509	5909.23	2.4						
			150	0.3	infeasible	infeasible	-	0	0	infeasible	0.2						
			160	7200.0	31400.43	no feas sol	inf	66062	49438	6891.20	1.0						
			170	2.4	infeasible	infeasible	-	0	0	infeasible	2.7						
			180	1.4	infeasible	infeasible	-	0	0	infeasible	0.8						
			185	1.5	infeasible	infeasible	-	0	0	infeasible	2.1						
			200	7200.0	15091.44	no feas sol	inf	226098	137012	5515.53	1.2						
			205	7200.0	47480.91	no feas sol	inf	190141	33694	5515.53	1.3						
			210	7200.0	33849.79	no feas sol	inf	61160	44781	9125.57	1.1						
			220	81.2	infeasible	infeasible	-	20	0	8110.50	13.9						
			230	339.3	infeasible	infeasible	-	3321	0	5515.53	3.2						
			240	438.5	infeasible	infeasible	-	1869	0	6689.24	4.3						
			250	0.3	infeasible	infeasible	-	0	0	infeasible	0.7						
			260	7200.0	33850.09	no feas sol	inf	61200	44813	9125.57	2.3						
			270	2.8	infeasible	infeasible	-	0	0	infeasible	2.6						
			280	1.2	infeasible	infeasible	-	0	0	infeasible	1.1						
			285	2.7	infeasible	infeasible	-	0	0	infeasible	2.7						
32538.64	18	8 x 4	100	7200.0	17886.13	35843.45	50.10	147118	123349	17074.15	0.6	1.0	1.0	47.5	10.2	109.9	1.0
			105	7200.0	18102.82	39859.90	54.58	126435	99161	17074.15	0.9	1.0	1.0	47.5	22.5	133.5	0.9
			110	7200.0	23860.01	35347.53	32.50	53485	45218	17473.48	1.1	1.0	1.0	46.3	8.6	102.3	0.4
			120	7200.0	21226.43	33629.87	36.88	15060	12894	17285.19	9.8	1.0	1.0	46.9	3.4	94.6	0.1
			130	7200.0	18019.11	35173.67	48.77	81040	71662	17074.15	1.4	1.0	1.0	47.5	8.1	106.0	0.6
			140	7200.0	18067.87	34179.09	47.14	70243	61212	17161.18	3.3	1.0	1.0	47.3	5.0	99.2	0.5
			150	7200.0	26947.46	34628.29	22.18	124385	93973	26756.48	0.6	1.0	1.6	17.8	6.4	29.4	0.8
			160	7200.0	23860.44	35347.53	32.50	53586	45306	17473.48	1.7	1.0	1.0	46.3	8.6	102.3	0.4
			170	7200.0	30259.66	32602.28	7.19	23394	17079	28050.05	10.5	1.0	1.6	13.8	0.2	16.2	0.2
			180	7200.0	30039.74	32538.64	7.68	56986	42392	28050.05	3.8	1.0	1.6	13.8	0.0	16.0	0.4
			185	7200.0	31637.73	32538.64	2.77	29538	12016	28050.05	8.5	1.0	1.6	13.8	0.0	16.0	0.2
			200	7200.0	17850.10	37551.89	52.47	95405	86700	17081.28	1.4	1.0	1.0	47.5	15.4	119.8	0.6
			205	7200.0	17893.09	36676.43	51.21	93297	81412	17081.28	2.5	1.0	1.0	47.5	12.7	114.7	0.6
			210	7200.0	24543.39	33691.01	27.15	32962	27787	18097.25	3.3	1.0	1.1	44.4	3.5	86.2	0.2
			220	7200.0	21455.55	34982.64	38.67	13275	11935	17697.09	21.1	1.0	1.0	45.6	7.5	97.7	0.1
			230	7200.0	17963.16	35812.56	49.84	65982	57082	17081.28	3.7	1.0	1.0	47.5	10.1	109.7	0.4
			240	7200.0	18180.42	36226.08	49.81	53960	47897	17444.78	5.6	1.0	1.0	46.4	11.3	107.7	0.4
			250	7200.0	26841.03	34768.51	22.80	80788	59195	26757.23	1.8	1.0	1.6	17.8	6.9	29.9	0.5
			260	7200.0	24552.12	33691.01	27.13	33271	28055	18097.25	4.8	1.0	1.1	44.4	3.5	86.2	0.2
			270	7200.0	30061.38	32602.28	7.79	16235	12405	28204.82	13.8	1.0	1.7	13.3	0.2	15.6	0.1
			280	7200.0	30984.91	32602.28	4.96	29883	19265	28204.82	8.4	1.0	1.7	13.3	0.2	15.6	0.2
			285	7200.0	31606.87	32743.89	3.47	20459	10390	28204.82	13.4	1.0	1.7	13.3	0.6	16.1	0.1

Table C.2 (continued)

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
42830.94	19	9 x 4	100	7200.0	22873.08	no feas sol	inf	94300	87087	22099.89	1.2	1.0	1.0	48.4	-	-	1.0
			105	7200.0	22888.13	no feas sol	inf	75941	67384	22099.89	1.4	1.0	1.0	48.4	-	-	0.8
			110	7200.0	32606.63	45406.41	28.19	33842	28630	22508.71	1.4	1.0	1.0	47.4	6.0	101.7	0.4
			120	7200.0	25021.45	no feas sol	inf	4041	3758	22284.74	15.3	1.0	1.0	48.0	-	-	0.0
			130	7200.0	22932.20	no feas sol	inf	53639	50112	22099.89	2.9	1.0	1.0	48.4	-	-	0.6
			140	7200.0	22878.60	50706.32	54.88	28788	25831	22177.31	5.6	1.0	1.0	48.2	18.4	128.6	0.3
			150	7200.0	38915.48	59578.54	34.68	78970	71796	38777.12	1.1	1.0	1.8	9.5	39.1	53.6	0.8
			160	7200.0	32606.63	45406.41	28.19	33824	28616	22508.71	3.1	1.0	1.0	47.4	6.0	101.7	0.4
			170	7200.0	40956.87	42830.94	4.38	7682	4436	39375.38	17.0	1.0	1.8	8.1	0.0	8.8	0.1
			180	7200.0	41062.51	43113.10	4.76	18400	11765	39375.38	7.9	1.0	1.8	8.1	0.7	9.5	0.2
			185	7200.0	41520.31	43008.64	3.46	12210	7621	39375.38	16.1	1.0	1.8	8.1	0.4	9.2	0.1
			200	7200.0	22898.04	no feas sol	inf	65265	59320	22111.72	3.3	1.0	1.0	48.4	-	-	0.7
			205	7200.0	22909.97	no feas sol	inf	64374	56480	22111.72	3.9	1.0	1.0	48.4	-	-	0.7
			210	7200.0	30933.53	43730.02	29.26	26343	20283	23249.44	5.9	1.0	1.1	45.7	2.1	88.1	0.3
			220	7200.0	25892.61	43099.97	39.92	7101	5852	22726.25	30.7	1.0	1.0	46.9	0.6	89.6	0.1
			230	7200.0	23040.66	no feas sol	inf	48827	45234	22111.72	7.4	1.0	1.0	48.4	-	-	0.5
			240	7200.0	22877.09	56977.74	59.85	30810	28269	22474.16	9.0	1.0	1.0	47.5	33.0	153.5	0.3
			250	7200.0	38954.87	59893.48	34.96	57700	51645	38784.15	4.3	1.0	1.8	9.4	39.8	54.4	0.6
			260	7200.0	30937.79	43730.02	29.25	26440	20368	23249.44	8.4	1.0	1.1	45.7	2.1	88.1	0.3
			270	7200.0	41561.24	42830.94	2.96	8540	4594	39635.43	24.4	1.0	1.8	7.5	0.0	8.1	0.1
			280	7200.0	41431.03	42830.94	3.27	25707	15860	39635.43	13.4	1.0	1.8	7.5	0.0	8.1	0.3
			285	7200.0	41610.64	42975.74	3.18	10440	5425	39635.44	23.4	1.0	1.8	7.5	0.3	8.4	0.1
27784.54	20	9 x 4	100	7200.0	9470.60	32196.35	70.58	84150	78949	8735.75	0.7	1.0	1.0	68.6	15.9	268.6	1.0
			105	7200.0	9502.09	34184.96	72.20	92719	82081	8735.75	1.1	1.0	1.0	68.6	23.0	291.3	1.1
			110	7200.0	14181.84	30405.09	53.36	31788	28473	9036.15	1.4	1.0	1.0	67.5	9.4	236.5	0.4
			120	7200.0	11428.61	32148.64	64.45	10036	9021	8900.38	8.6	1.0	1.0	68.0	15.7	261.2	0.1
			130	7200.0	9480.41	30466.44	68.88	55391	47263	8735.75	2.4	1.0	1.0	68.6	9.7	248.8	0.7
			140	7200.0	9529.16	29791.01	68.01	41609	38322	8813.16	5.9	1.0	1.0	68.3	7.2	238.0	0.5
			150	7200.0	19668.94	31359.57	37.28	102996	90232	19659.63	0.9	1.0	2.3	29.2	12.9	59.5	1.2
			160	7200.0	14183.02	30405.09	53.35	31848	28529	9036.15	2.6	1.0	1.0	67.5	9.4	236.5	0.4
			170	7200.0	23689.81	28369.76	16.50	15878	13863	22744.87	15.2	1.0	2.6	18.1	2.1	24.7	0.2
			180	7200.0	24256.38	27831.28	12.84	34392	27688	22744.87	8.3	1.0	2.6	18.1	0.2	22.4	0.4
			185	7200.0	24457.16	27886.55	12.30	19039	14420	22744.87	15.6	1.0	2.6	18.1	0.4	22.6	0.2
			200	7200.0	9561.43	31154.27	69.31	81931	75261	8736.56	3.2	1.0	1.0	68.6	12.1	256.6	1.0
			205	7200.0	9606.27	34596.68	72.23	78009	67353	8736.56	3.0	1.0	1.0	68.6	24.5	296.0	0.9
			210	7200.0	16031.10	27929.07	42.60	25930	22087	9654.58	4.2	1.0	1.1	65.3	0.5	189.3	0.3
			220	7200.0	11631.44	28592.29	59.32	8403	7452	9320.09	17.2	1.0	1.1	66.5	2.9	206.8	0.1
			230	7200.0	9568.77	31532.69	69.65	52222	49316	8736.64	5.7	1.0	1.0	68.6	13.5	260.9	0.6
			240	7200.0	9553.31	29424.21	67.53	35318	31804	9116.81	10.7	1.0	1.0	67.2	5.9	222.7	0.4
			250	7200.0	19743.27	30927.36	36.16	74877	68615	19659.78	4.3	1.0	2.3	29.2	11.3	57.3	0.9
			260	7200.0	16031.10	27929.07	42.60	25916	22077	9654.58	6.7	1.0	1.1	65.3	0.5	189.3	0.3
			270	7200.0	24506.89	28120.81	12.85	14285	12193	22883.79	25.0	1.0	2.6	17.6	1.2	22.9	0.2
			280	7200.0	25213.10	27843.02	9.45	20082	15680	22883.79	14.5	1.0	2.6	17.6	0.2	21.7	0.2
			285	7200.0	25125.83	27961.87	10.14	12408	8516	22884.26	23.3	1.0	2.6	17.6	0.6	22.2	0.1

* LB, UB and Gap (%) refer to the final values of the lower bound, upper bound and the corresponding gap, respectively at the end of the time limit (7200 sec.). If the optimal solution of an instance is not known, the best known solution is provided under Opt.Sol. column in underlined form.

Table C.3 Case A- Average Results for 15 Instances with Known Optimals *

	CPU	CPU R	LPR R	IGap%	#opt *	DU%	ULP%	Nodes	Node R
100	5760.3	1.00	1.00	64.1	8 (3)	2.5	152933.3	411370	1.00
105	5760.2	1.05	1.00	64.1	8 (3)	4.4	152937.9	376779	0.94
110	4674.8	0.83	92.53	60.9	11 (8)	1.7	291.6	121074	0.36
120	5488.4	1.15	49.53	61.6	9 (4)	2.2	410.6	49500	0.19
130	5760.2	1.03	1.00	64.1	8 (3)	2.1	152932.0	231833	0.61
140	5760.3	1.21	37.75	62.6	8 (3)	2.7	482.2	212861	0.58
150	3802.6	0.74	1057.13	16.6	9 (9)	4.9	28.6	171925	0.62
160	4668.0	0.83	92.53	60.9	11 (8)	1.7	291.6	122671	0.36
170	2574.5	0.61	1386.70	12.8	12 (10)	0.7	18.1	10971	0.11
180	2271.6	0.51	1386.70	12.8	12 (11)	0.1	17.2	20865	0.15
185	2038.8	0.47	1386.70	12.8	12 (11)	0.5	17.9	10032	0.10
200	5760.2	1.14	4.62	63.6	10 (3)	2.6	3044.8	388847	0.91
205	5760.2	1.15	4.62	63.6	8 (3)	4.4	3049.8	335255	0.83
210	4602.7	0.94	210.09	57.8	11 (7)	0.5	181.3	94932	0.33
220	5449.8	1.69	136.06	59.1	10 (4)	1.3	215.7	49179	0.32
230	5760.2	1.16	4.62	63.6	8 (3)	3.0	3045.6	214377	0.55
240	5760.3	1.30	112.71	60.7	9 (3)	3.5	247.1	195692	0.57
250	4481.8	0.97	1059.15	16.5	9 (7)	4.0	27.3	194976	0.60
260	4608.9	0.94	210.09	57.8	11 (7)	0.5	181.3	95291	0.33
270	2240.5	0.86	1391.61	12.6	12 (11)	0.2	17.0	7858	0.11
280	2034.4	0.64	1391.61	12.6	12 (11)	0.1	16.9	12477	0.16
285	1992.3	0.69	1391.61	12.6	11 (11)	0.1	16.9	7583	0.13

* #opt denotes the number of times the optimal solution was found, where the entry in parentheses denotes the number of verified optimal solutions.

APPENDIX D

PRELIMINARY EXPERIMENTS UNDER CASE B

Table D.1 Solutions of Preliminary Instances Under Case B *

inst#	Original Instance #	N	K	Best sol. at the end of time limit	Extended CPU	Final Gap% at the end of extended time	Best sol.
1	11	5	4	18813.5	-	-	18813.5
2	21	5	4	9107.4	-	-	9107.4
3	41	2	4	10714.2	-	-	10714.2
4	51	5	4	14402.8	-	-	14402.8
5	63	3	4	309.8	-	-	309.8
6	83	4	4	154.1	-	-	154.1
7	94	5	4	INFEASIBLE	-	-	INFEASIBLE
8	104	5	4	19255.6	-	-	19255.6
9	111	5	4	9436.5	-	-	9436.5
10	11	5	8	27826.2	19626.8	-	27794.5
11	41	5	8	24569.2	-	-	24569.2
12	94	8	8	INFEASIBLE	-	-	INFEASIBLE
13	53	9	8	16653.4	328500.1	5.6	16603.1
14	64	9	8	INFEASIBLE	-	-	INFEASIBLE
15	74	6	4	16909.8	-	-	16909.8
16	13	7	4	14299.7	-	-	14299.7
17	124	7	4	INFEASIBLE	-	-	INFEASIBLE
18	102	8	4	18372.0	21419.0	-	18372.0
19	103	9	4	24648.0	995786.1	1.8	24340.5
20	123	9	4	10363.3	138490.7	-	10363.3

* Shaded cells indicate solutions with non-zero solution gaps. Best sol. refers to the best known solution for the instance. Best sol. at the end of the time limit is the best known solution among all options. However, extended solutions were only performed with option 285.

Table D.2 Detailed Results of Experiments on Preliminary Instances Under Case B *

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
18813.50	1	5 x 4	100	143.8	18813.31	18813.50	0.00	28374	6	18394.64	0.1	1.0	1.0	2.2	0.0	2.3	1.0
			105	96.4	18813.39	18813.50	0.00	10912	2	18394.64	0.1	0.7	1.0	2.2	0.0	2.3	0.4
			110	14.8	18813.50	18813.50	0.00	572	0	18395.21	0.1	0.1	1.0	2.2	0.0	2.3	0.0
			120	117.7	18813.50	18813.50	0.00	1745	0	18395.21	0.7	0.8	1.0	2.2	0.0	2.3	0.1
			130	351.5	18813.34	18813.50	0.00	39227	11	18394.64	0.2	2.4	1.0	2.2	0.0	2.3	1.4
			140	104.0	18813.46	18813.50	0.00	9497	3	18394.64	0.2	0.7	1.0	2.2	0.0	2.3	0.3
			150	103.4	18813.32	18813.50	0.00	14892	5	18394.64	0.1	0.7	1.0	2.2	0.0	2.3	0.5
			160	14.8	18813.50	18813.50	0.00	572	0	18395.21	0.2	0.1	1.0	2.2	0.0	2.3	0.0
			170	54.6	18813.50	18813.50	0.00	749	0	18398.75	0.7	0.4	1.0	2.2	0.0	2.3	0.0
			180	14.9	18813.50	18813.50	0.00	595	0	18398.75	0.3	0.1	1.0	2.2	0.0	2.3	0.0
			185	18.0	18813.50	18813.50	0.00	432	0	18398.75	0.4	0.1	1.0	2.2	0.0	2.3	0.0
			200	119.6	18813.31	18813.50	0.00	18350	10	18394.64	0.2	0.8	1.0	2.2	0.0	2.3	0.6
			205	98.0	18813.39	18813.50	0.00	13423	3	18394.64	0.2	0.7	1.0	2.2	0.0	2.3	0.5
			210	13.3	18813.50	18813.50	0.00	418	0	18396.35	0.2	0.1	1.0	2.2	0.0	2.3	0.0
			220	88.4	18813.50	18813.50	0.00	948	0	18396.35	0.9	0.6	1.0	2.2	0.0	2.3	0.0
			230	237.8	18813.32	18813.50	0.00	22732	4	18394.64	0.4	1.7	1.0	2.2	0.0	2.3	0.8
			240	229.9	18813.32	18813.50	0.00	23724	9	18394.64	0.3	1.6	1.0	2.2	0.0	2.3	0.8
			250	151.2	18813.36	18813.50	0.00	16963	4	18394.64	0.2	1.1	1.0	2.2	0.0	2.3	0.6
			260	13.1	18813.50	18813.50	0.00	418	0	18396.35	0.3	0.1	1.0	2.2	0.0	2.3	0.0
			270	40.0	18813.37	18813.50	0.00	490	1	18398.76	0.5	0.3	1.0	2.2	0.0	2.3	0.0
			280	14.1	18813.50	18813.50	0.00	641	0	18398.76	0.2	0.1	1.0	2.2	0.0	2.3	0.0
			285	11.0	18813.50	18813.50	0.00	170	0	18398.76	0.5	0.1	1.0	2.2	0.0	2.3	0.0
9107.41	2	5 x 4	100	1693.5	9107.33	9107.41	0.00	233529	22	7559.71	0.1	1.0	1.0	17.0	0.0	20.5	1.0
			105	1669.5	9107.35	9107.41	0.00	224851	16	7559.71	0.1	1.0	1.0	17.0	0.0	20.5	1.0
			110	781.0	9107.34	9107.41	0.00	57701	4	7559.71	0.1	0.5	1.0	17.0	0.0	20.5	0.2
			120	4043.8	9107.32	9107.41	0.00	133944	15	7559.71	0.6	2.4	1.0	17.0	0.0	20.5	0.6
			130	4772.0	9107.33	9107.41	0.00	463005	32	7559.71	0.1	2.8	1.0	17.0	0.0	20.5	2.0
			140	5285.9	9107.32	9107.41	0.00	560509	47	7559.71	0.1	3.1	1.0	17.0	0.0	20.5	2.4
			150	7200.0	8815.81	9107.41	3.20	785974	258763	7561.38	0.1	4.3	1.0	17.0	0.0	20.4	3.4
			160	773.0	9107.34	9107.41	0.00	57701	4	7559.71	0.2	0.5	1.0	17.0	0.0	20.5	0.2
			170	1898.9	9107.39	9107.41	0.00	44294	2	7567.83	0.4	1.1	1.0	16.9	0.0	20.3	0.2
			180	776.5	9107.35	9107.41	0.00	36161	5	7567.83	0.2	0.5	1.0	16.9	0.0	20.3	0.2
			185	430.4	9107.41	9107.41	0.00	10978	0	7567.83	0.3	0.3	1.0	16.9	0.0	20.3	0.0
			200	1390.2	9107.32	9107.41	0.00	184053	17	7559.71	0.2	0.8	1.0	17.0	0.0	20.5	0.8
			205	523.4	9107.35	9107.41	0.00	48695	4	7559.71	0.2	0.3	1.0	17.0	0.0	20.5	0.2
			210	275.7	9107.41	9107.41	0.00	13513	1	7559.71	0.3	0.2	1.0	17.0	0.0	20.5	0.1
			220	1454.1	9107.41	9107.41	0.00	23971	0	7559.71	1.6	0.9	1.0	17.0	0.0	20.5	0.1
			230	3958.2	9107.32	9107.41	0.00	280168	28	7559.71	0.4	2.3	1.0	17.0	0.0	20.5	1.2
			240	4048.7	9107.32	9107.41	0.00	350307	30	7559.71	0.3	2.4	1.0	17.0	0.0	20.5	1.5
			250	3616.8	9107.32	9107.41	0.00	345216	25	7561.38	0.3	2.1	1.0	17.0	0.0	20.4	1.5
			260	272.2	9107.41	9107.41	0.00	13513	1	7559.71	0.6	0.2	1.0	17.0	0.0	20.5	0.1
			270	740.0	9107.37	9107.41	0.00	12796	1	7568.28	0.9	0.4	1.0	16.9	0.0	20.3	0.1
			280	498.8	9107.41	9107.41	0.00	14361	0	7568.28	0.5	0.3	1.0	16.9	0.0	20.3	0.1
			285	268.1	9107.41	9107.41	0.00	4552	0	7568.28	0.5	0.2	1.0	16.9	0.0	20.3	0.0
10714.24	3	2 x 4	100	0.4	10714.24	10714.24	0.00	12	0	9776.09	0.0	1.0	1.0	8.8	0.0	9.6	1.0
			105	0.4	10714.24	10714.24	0.00	12	0	9776.09	0.0	1.1	1.0	8.8	0.0	9.6	1.0
			110	0.1	10714.24	10714.24	0.00	5	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	0.4
			120	0.2	10714.24	10714.24	0.00	8	0	9776.09	0.0	0.5	1.0	8.8	0.0	9.6	0.7
			130	0.1	10714.24	10714.24	0.00	12	0	9776.09	0.0	0.3	1.0	8.8	0.0	9.6	1.0
			140	0.2	10714.24	10714.24	0.00	7	0	9776.09	0.0	0.4	1.0	8.8	0.0	9.6	0.6
			150	0.1	10714.24	10714.24	0.00	3	0	9776.09	0.0	0.1	1.0	8.8	0.0	9.6	0.3
			160	0.1	10714.24	10714.24	0.00	5	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	0.4
			170	0.1	10714.24	10714.24	0.00	6	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	0.5
			180	0.1	10714.24	10714.24	0.00	6	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	0.5
			185	0.1	10714.24	10714.24	0.00	6	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	0.5
			200	0.1	10714.24	10714.24	0.00	12	0	9776.09	0.0	0.3	1.0	8.8	0.0	9.6	1.0
			205	0.1	10714.24	10714.24	0.00	17	0	9776.09	0.0	0.3	1.0	8.8	0.0	9.6	1.4
			210	0.1	10714.24	10714.24	0.00	15	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	1.3
			220	0.2	10714.24	10714.24	0.00	12	0	9776.09	0.0	0.4	1.0	8.8	0.0	9.6	1.0
			230	0.1	10714.24	10714.24	0.00	9	0	9776.09	0.0	0.3	1.0	8.8	0.0	9.6	0.8
			240	0.2	10714.24	10714.24	0.00	31	0	9776.09	0.0	0.5	1.0	8.8	0.0	9.6	2.6
			250	0.1	10714.24	10714.24	0.00	6	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	0.5
			260	0.1	10714.24	10714.24	0.00	15	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	1.3
			270	0.1	10714.24	10714.24	0.00	11	0	9776.09	0.0	0.3	1.0	8.8	0.0	9.6	0.9
			280	0.1	10714.24	10714.24	0.00	7	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	0.6
			285	0.1	10714.24	10714.24	0.00	7	0	9776.09	0.0	0.2	1.0	8.8	0.0	9.6	0.6

Table D.2 (continued)

Opt. sol.	inst#		Option	MIP CPU	MIP LB	MIP UB	MIP Gap (%)	MIP Total Nodes	MIP Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
14402.77	4	5 x 4	100	282.7	14402.71	14402.77	0.00	40370	4	13456.04	0.1	1.0	1.0	6.6	0.0	7.0	1.0
			105	132.4	14402.70	14402.77	0.00	17088	2	13456.04	0.1	0.5	1.0	6.6	0.0	7.0	0.4
			110	53.0	14402.77	14402.77	0.00	3188	0	13456.95	0.1	0.2	1.0	6.6	0.0	7.0	0.1
			120	245.1	14402.77	14402.77	0.00	4583	0	13456.95	0.8	0.9	1.0	6.6	0.0	7.0	0.1
			130	337.2	14402.77	14402.77	0.00	22783	0	13456.04	0.2	1.2	1.0	6.6	0.0	7.0	0.6
			140	299.0	14402.62	14402.77	0.00	26073	7	13456.04	0.2	1.1	1.0	6.6	0.0	7.0	0.6
			150	1669.6	14402.65	14402.77	0.00	189293	26	13457.71	0.1	5.9	1.0	6.6	0.0	7.0	4.7
			160	52.6	14402.77	14402.77	0.00	3188	0	13456.95	0.2	0.2	1.0	6.6	0.0	7.0	0.1
			170	208.9	14402.76	14402.77	0.00	3334	1	13468.33	0.7	0.7	1.0	6.5	0.0	6.9	0.1
			180	68.1	14402.63	14402.77	0.00	1926	1	13468.33	0.3	0.2	1.0	6.5	0.0	6.9	0.0
			185	48.4	14402.75	14402.77	0.00	1361	1	13468.33	0.5	0.2	1.0	6.5	0.0	6.9	0.0
			200	257.4	14402.67	14402.77	0.00	37666	9	13456.04	0.1	0.9	1.0	6.6	0.0	7.0	0.9
			205	150.6	14402.67	14402.77	0.00	14423	2	13456.04	0.2	0.5	1.0	6.6	0.0	7.0	0.4
			210	50.3	14402.77	14402.77	0.00	2073	0	13459.22	0.1	0.2	1.0	6.6	0.0	7.0	0.1
			220	276.5	14402.77	14402.77	0.00	4686	0	13459.22	1.1	1.0	1.0	6.6	0.0	7.0	0.1
			230	117.8	14402.76	14402.77	0.00	8443	3	13456.04	0.3	0.4	1.0	6.6	0.0	7.0	0.2
			240	339.4	14402.71	14402.77	0.00	23715	2	13456.04	0.3	1.2	1.0	6.6	0.0	7.0	0.6
			250	651.5	14402.66	14402.77	0.00	55744	5	13457.71	0.2	2.3	1.0	6.6	0.0	7.0	1.4
			260	50.7	14402.77	14402.77	0.00	2073	0	13459.22	0.4	0.2	1.0	6.6	0.0	7.0	0.1
			270	97.9	14402.77	14402.77	0.00	1527	0	13468.41	0.8	0.3	1.0	6.5	0.0	6.9	0.0
			280	62.7	14402.77	14402.77	0.00	1798	0	13468.41	0.3	0.2	1.0	6.5	0.0	6.9	0.0
			285	40.8	14402.77	14402.77	0.00	947	0	13468.41	0.5	0.1	1.0	6.5	0.0	6.9	0.0
309.77	5	3 x 4	100	0.3	309.77	309.77	0.00	15	0	39.97	0.0	1.0	1.0	87.1	0.0	674.9	1.0
			105	0.3	309.77	309.77	0.00	30	0	39.97	0.0	1.1	1.0	87.1	0.0	674.9	2.0
			110	0.2	309.77	309.77	0.00	17	0	39.97	0.0	0.8	1.0	87.1	0.0	674.9	1.1
			120	1.0	309.77	309.77	0.00	16	0	39.97	0.0	3.8	1.0	87.1	0.0	674.9	1.1
			130	0.4	309.77	309.77	0.00	21	0	39.97	0.0	1.5	1.0	87.1	0.0	674.9	1.4
			140	0.7	309.77	309.77	0.00	71	0	39.97	0.0	2.6	1.0	87.1	0.0	674.9	4.7
			150	0.3	309.77	309.77	0.00	20	0	39.97	0.0	1.1	1.0	87.1	0.0	674.9	1.3
			160	0.2	309.77	309.77	0.00	17	0	39.97	0.0	0.8	1.0	87.1	0.0	674.9	1.1
			170	0.3	309.77	309.77	0.00	8	0	39.97	0.0	1.3	1.0	87.1	0.0	674.9	0.5
			180	0.3	309.77	309.77	0.00	6	0	39.97	0.0	1.2	1.0	87.1	0.0	674.9	0.4
			185	0.3	309.77	309.77	0.00	0	0	39.97	0.0	1.0	1.0	87.1	0.0	674.9	0.0
			200	0.1	309.77	309.77	0.00	0	0	39.97	0.0	0.5	1.0	87.1	0.0	674.9	0.0
			205	0.2	309.77	309.77	0.00	7	0	39.97	0.0	0.6	1.0	87.1	0.0	674.9	0.5
			210	0.2	309.77	309.77	0.00	0	0	39.97	0.0	0.7	1.0	87.1	0.0	674.9	0.0
			220	0.5	309.77	309.77	0.00	12	0	39.97	0.0	2.1	1.0	87.1	0.0	674.9	0.8
			230	0.3	309.77	309.77	0.00	16	0	39.97	0.0	1.1	1.0	87.1	0.0	674.9	1.1
			240	0.5	309.77	309.77	0.00	37	0	39.97	0.0	1.8	1.0	87.1	0.0	674.9	2.5
			250	0.3	309.77	309.77	0.00	10	0	39.97	0.0	1.1	1.0	87.1	0.0	674.9	0.7
			260	0.2	309.77	309.77	0.00	0	0	39.97	0.0	0.7	1.0	87.1	0.0	674.9	0.0
			270	0.5	309.77	309.77	0.00	9	0	39.97	0.1	2.0	1.0	87.1	0.0	674.9	0.6
			280	0.3	309.77	309.77	0.00	10	0	39.97	0.0	1.1	1.0	87.1	0.0	674.9	0.7
			285	0.3	309.77	309.77	0.00	11	0	39.97	0.0	1.0	1.0	87.1	0.0	674.9	0.7
154.06	6	4 x 4	100	0.2	154.06	154.06	0.00	12	0	0.00	0.0	1.0	undefined	100.0	0.0	undefined	1.0
			105	0.2	154.06	154.06	0.00	15	0	0.00	0.0	1.1	undefined	100.0	0.0	undefined	1.3
			110	0.2	154.06	154.06	0.00	16	0	0.00	0.0	0.8	undefined	100.0	0.0	undefined	1.3
			120	0.6	154.06	154.06	0.00	32	0	0.00	0.1	3.0	undefined	100.0	0.0	undefined	2.7
			130	0.2	154.06	154.06	0.00	19	0	0.00	0.0	1.1	undefined	100.0	0.0	undefined	1.6
			140	0.2	154.06	154.06	0.00	13	0	0.00	0.0	1.2	undefined	100.0	0.0	undefined	1.1
			150	0.3	154.06	154.06	0.00	32	0	0.00	0.0	1.5	undefined	100.0	0.0	undefined	2.7
			160	0.2	154.06	154.06	0.00	16	0	0.00	0.0	1.1	undefined	100.0	0.0	undefined	1.3
			170	0.3	154.06	154.06	0.00	17	0	2.58	0.1	1.6	undefined	98.3	0.0	5880.6	1.4
			180	0.3	154.06	154.06	0.00	23	0	2.58	0.0	1.3	undefined	98.3	0.0	5880.6	1.9
			185	0.5	154.06	154.06	0.00	26	0	2.58	0.0	2.3	undefined	98.3	0.0	5880.6	2.2
			200	0.3	154.06	154.06	0.00	17	0	12.30	0.0	1.4	undefined	92.0	0.0	1152.2	1.4
			205	0.3	154.06	154.06	0.00	14	0	12.30	0.0	1.4	undefined	92.0	0.0	1152.2	1.2
			210	0.5	154.06	154.06	0.00	35	0	12.30	0.0	2.4	undefined	92.0	0.0	1152.2	2.9
			220	1.2	154.06	154.06	0.00	22	0	12.30	0.1	5.9	undefined	92.0	0.0	1152.2	1.8
			230	0.4	154.06	154.06	0.00	15	0	12.30	0.0	1.8	undefined	92.0	0.0	1152.2	1.3
			240	0.4	154.06	154.06	0.00	25	0	12.30	0.0	2.1	undefined	92.0	0.0	1152.2	2.1
			250	0.4	154.06	154.06	0.00	11	0	12.30	0.0	1.9	undefined	92.0	0.0	1152.2	0.9
			260	0.5	154.06	154.06	0.00	35	0	12.30	0.1	2.3	undefined	92.0	0.0	1152.2	2.9
			270	0.8	154.06	154.06	0.00	19	0	12.30	0.1	3.9	undefined	92.0	0.0	1152.2	1.6
			280	0.7	154.06	154.06	0.00	26	0	12.30	0.0	3.3	undefined	92.0	0.0	1152.2	2.2
			285	0.6	154.06	154.06	0.00	18	0	12.30	0.1	2.8	undefined	92.0	0.0	1152.2	1.5

Table D.2 (continued)

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
INFEASIBLE	7	5 x 4	100	2.4	infeasible	infeasible	-	76	0	0.00	0.0						
			105	5.3	infeasible	infeasible	-	236	0	0.00	0.0						
			110	2.2	infeasible	infeasible	-	0	0	0.00	0.1						
			120	55.3	infeasible	infeasible	-	740	0	0.00	0.1						
			130	4.3	infeasible	infeasible	-	76	0	0.00	0.1						
			140	14.1	infeasible	infeasible	-	640	0	0.00	0.1						
			150	0.0	infeasible	infeasible	-	0	0	infeasible	0.0						
			160	2.2	infeasible	infeasible	-	0	0	0.00	0.1						
			170	0.1	infeasible	infeasible	-	0	0	infeasible	0.2						
			180	0.1	infeasible	infeasible	-	0	0	infeasible	0.1						
			185	0.1	infeasible	infeasible	-	0	0	infeasible	0.1						
			200	6.7	infeasible	infeasible	-	315	0	0.00	0.1						
			205	2.7	infeasible	infeasible	-	26	0	0.00	0.3						
			210	1.1	infeasible	infeasible	-	0	0	0.00	0.2						
			220	8.3	infeasible	infeasible	-	8	0	0.00	0.7						
			230	3.9	infeasible	infeasible	-	29	0	0.00	0.3						
			240	3.5	infeasible	infeasible	-	46	0	0.00	0.4						
			250	0.0	infeasible	infeasible	-	0	0	infeasible	0.1						
			260	1.1	infeasible	infeasible	-	0	0	0.00	0.3						
			270	0.1	infeasible	infeasible	-	0	0	infeasible	0.6						
			280	0.1	infeasible	infeasible	-	0	0	infeasible	0.3						
			285	0.1	infeasible	infeasible	-	0	0	infeasible	0.4						
19255.63	8	5 x 4	100	466.7	19255.46	19255.63	0.00	53431	6	17825.08	0.1	1.0	1.0	7.4	0.0	8.0	1.0
			105	635.8	19255.49	19255.63	0.00	91073	15	17825.08	0.1	1.4	1.0	7.4	0.0	8.0	1.7
			110	257.2	19255.45	19255.63	0.00	17341	7	17825.46	0.1	0.6	1.0	7.4	0.0	8.0	0.3
			120	820.4	19255.50	19255.63	0.00	21523	1	17825.46	0.8	1.8	1.0	7.4	0.0	8.0	0.4
			130	1098.1	19255.47	19255.63	0.00	86544	7	17825.08	0.2	2.4	1.0	7.4	0.0	8.0	1.6
			140	1184.2	19255.44	19255.63	0.00	105860	17	17825.08	0.2	2.5	1.0	7.4	0.0	8.0	2.0
			150	776.6	19255.45	19255.63	0.00	88591	31	17830.42	0.1	1.7	1.0	7.4	0.0	8.0	1.7
			160	261.0	19255.45	19255.63	0.00	17341	7	17825.46	0.2	0.6	1.0	7.4	0.0	8.0	0.3
			170	250.6	19255.48	19255.63	0.00	6130	1	17931.04	0.6	0.5	1.0	6.9	0.0	7.4	0.1
			180	110.5	19255.55	19255.63	0.00	4684	2	17931.04	0.3	0.2	1.0	6.9	0.0	7.4	0.1
			185	97.1	19255.51	19255.63	0.00	3599	1	17931.04	0.3	0.2	1.0	6.9	0.0	7.4	0.1
			200	423.9	19255.46	19255.63	0.00	46861	9	17836.45	0.2	0.9	1.0	7.4	0.0	8.0	0.9
			205	362.9	19255.44	19255.63	0.00	37352	5	17836.45	0.2	0.8	1.0	7.4	0.0	8.0	0.7
			210	78.8	19255.60	19255.63	0.00	4528	1	17848.64	0.2	0.2	1.0	7.3	0.0	7.9	0.1
			220	431.4	19255.63	19255.63	0.00	7658	0	17842.04	1.3	0.9	1.0	7.3	0.0	7.9	0.1
			230	701.8	19255.45	19255.63	0.00	47739	6	17836.45	0.4	1.5	1.0	7.4	0.0	8.0	0.9
			240	587.3	19255.47	19255.63	0.00	55461	11	17836.45	0.4	1.3	1.0	7.4	0.0	8.0	1.0
			250	542.7	19255.45	19255.63	0.00	39158	6	17841.79	0.2	1.2	1.0	7.3	0.0	7.9	0.7
			260	78.5	19255.60	19255.63	0.00	4528	1	17848.64	0.5	0.2	1.0	7.3	0.0	7.9	0.1
			270	165.6	19255.58	19255.63	0.00	3076	1	17935.19	0.7	0.4	1.0	6.9	0.0	7.4	0.1
			280	114.1	19255.46	19255.63	0.00	2874	1	17935.19	0.4	0.2	1.0	6.9	0.0	7.4	0.1
			285	64.5	19255.63	19255.63	0.00	1413	0	17935.19	0.6	0.1	1.0	6.9	0.0	7.4	0.0
9436.48	9	5 x 4	100	254.8	9436.39	9436.48	0.00	27293	3	8570.92	0.1	1.0	1.0	9.2	0.0	10.1	1.0
			105	408.4	9436.39	9436.48	0.00	38665	55	8570.92	0.1	1.6	1.0	9.2	0.0	10.1	1.4
			110	151.7	9436.48	9436.48	0.00	6833	0	8570.93	0.1	0.6	1.0	9.2	0.0	10.1	0.3
			120	700.0	9436.48	9436.48	0.00	12473	0	8570.93	0.7	2.7	1.0	9.2	0.0	10.1	0.5
			130	1224.8	9436.39	9436.48	0.00	93851	16	8570.92	0.1	4.8	1.0	9.2	0.0	10.1	3.4
			140	916.2	9436.39	9436.48	0.00	63051	18	8570.92	0.2	3.6	1.0	9.2	0.0	10.1	2.3
			150	1403.2	9436.40	9436.48	0.00	107755	11	8570.92	0.1	5.5	1.0	9.2	0.0	10.1	3.9
			160	151.3	9436.48	9436.48	0.00	6833	0	8570.93	0.3	0.6	1.0	9.2	0.0	10.1	0.3
			170	218.7	9436.48	9436.48	0.00	4374	0	8577.93	0.6	0.9	1.0	9.1	0.0	10.0	0.2
			180	95.7	9436.39	9436.48	0.00	4684	1	8577.93	0.3	0.4	1.0	9.1	0.0	10.0	0.2
			185	115.8	9436.48	9436.48	0.00	3156	0	8577.93	0.4	0.5	1.0	9.1	0.0	10.0	0.1
			200	515.4	9436.47	9436.48	0.00	47527	1	8570.92	0.2	2.0	1.0	9.2	0.0	10.1	1.7
			205	164.2	9436.48	9436.48	0.00	12991	0	8570.92	0.2	0.6	1.0	9.2	0.0	10.1	0.5
			210	53.7	9436.48	9436.48	0.00	2018	0	8570.95	0.4	0.2	1.0	9.2	0.0	10.1	0.1
			220	304.1	9436.48	9436.48	0.00	3825	1	8570.95	1.5	1.2	1.0	9.2	0.0	10.1	0.1
			230	826.7	9436.43	9436.48	0.00	47669	2	8570.92	0.3	3.2	1.0	9.2	0.0	10.1	1.7
			240	528.2	9436.39	9436.48	0.00	27215	1	8570.92	0.4	2.1	1.0	9.2	0.0	10.1	1.0
			250	1259.1	9436.39	9436.48	0.00	113922	13	8570.92	0.2	4.9	1.0	9.2	0.0	10.1	4.2
			260	53.7	9436.48	9436.48	0.00	2018	0	8570.95	0.4	0.2	1.0	9.2	0.0	10.1	0.1
			270	161.4	9436.48	9436.48	0.00	2364	0	8579.17	1.0	0.6	1.0	9.1	0.0	10.0	0.1
			280	90.1	9436.48	9436.48	0.00	2239	0	8579.17	0.5	0.4	1.0	9.1	0.0	10.0	0.1
			285	72.2	9436.48	9436.48	0.00	1282	0	8579.17	0.5	0.3	1.0	9.1	0.0	10.0	0.0

Table D.2 (continued)

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
27794.55	10	5 x 8	100	7200.0	26470.56	27870.98	5.02	468853	306312	25954.34	0.3	1.0	1.0	6.6	0.3	7.4	1.0
			105	7200.0	26453.69	27964.19	5.40	364414	235190	25954.34	0.3	1.0	1.0	6.6	0.6	7.7	0.8
			110	7200.0	27184.48	27794.55	2.19	236440	146054	25954.91	0.4	1.0	1.0	6.6	0.0	7.1	0.5
			120	7200.0	26846.61	27924.94	3.86	58247	43153	25954.91	2.6	1.0	1.0	6.6	0.5	7.6	0.1
			130	7200.0	26410.66	27942.24	5.48	252925	161451	25954.34	0.5	1.0	1.0	6.6	0.5	7.7	0.5
			140	7200.0	26312.20	28049.82	6.19	237284	171597	25954.34	0.5	1.0	1.0	6.6	0.9	8.1	0.5
			150	7200.0	26507.19	27844.23	4.80	342314	239841	25954.34	0.3	1.0	1.0	6.6	0.2	7.3	0.7
			160	7200.0	27185.67	27794.55	2.19	238357	147277	25954.91	0.7	1.0	1.0	6.6	0.0	7.1	0.5
			170	7200.0	27055.05	27978.08	3.30	80986	57476	25958.46	1.2	1.0	1.0	6.6	0.7	7.8	0.2
			180	7200.0	27166.87	27820.41	2.35	155220	101303	25958.46	0.7	1.0	1.0	6.6	0.1	7.2	0.3
			185	7200.0	27449.25	27794.55	1.24	137413	63281	25958.46	1.0	1.0	1.0	6.6	0.0	7.1	0.3
			200	7200.0	26441.13	27866.47	5.11	297923	202304	25954.34	1.1	1.0	1.0	6.6	0.3	7.4	0.6
			205	7200.0	26501.07	28031.77	5.46	287971	209228	25954.34	1.4	1.0	1.0	6.6	0.9	8.0	0.6
			210	7200.0	27262.34	27828.65	2.03	141504	91665	25956.06	1.1	1.0	1.0	6.6	0.1	7.2	0.3
			220	7200.0	26943.44	27794.55	3.06	52130	35240	25956.06	8.7	1.0	1.0	6.6	0.0	7.1	0.1
			230	7200.0	26453.05	27794.55	4.83	189211	133636	25954.34	2.1	1.0	1.0	6.6	0.0	7.1	0.4
			240	7200.0	26404.52	27972.04	5.60	195555	157938	25954.34	2.2	1.0	1.0	6.6	0.6	7.8	0.4
			250	7200.0	26355.61	27890.71	5.50	317093	239717	25954.34	0.9	1.0	1.0	6.6	0.3	7.5	0.7
			260	7200.0	27261.40	27828.65	2.04	140754	91201	25956.06	1.8	1.0	1.0	6.6	0.1	7.2	0.3
			270	7200.0	27165.82	27794.55	2.26	54574	32635	25958.46	3.5	1.0	1.0	6.6	0.0	7.1	0.1
			280	7200.0	27308.69	27794.55	1.75	154806	91876	25958.46	1.5	1.0	1.0	6.6	0.0	7.1	0.3
			285	7200.0	27453.09	27826.18	1.34	86490	39524	25958.46	2.2	1.0	1.0	6.6	0.1	7.2	0.2
24569.25	11	5 x 8	100	7200.0	24166.23	24569.25	1.64	359511	156799	23232.12	0.2	1.0	1.0	5.4	0.0	5.8	1.0
			105	7200.0	24223.48	24569.25	1.41	377719	79542	23232.12	0.3	1.0	1.0	5.4	0.0	5.8	1.1
			110	623.5	24569.00	24569.25	0.00	20618	12	23232.58	0.4	0.1	1.0	5.4	0.0	5.8	0.1
			120	7200.0	24469.40	24569.25	0.41	79507	11733	23232.58	2.7	1.0	1.0	5.4	0.0	5.8	0.2
			130	7200.0	24306.45	24569.25	1.07	252499	68120	23232.12	0.7	1.0	1.0	5.4	0.0	5.8	0.7
			140	7200.0	24217.03	24569.25	1.43	200998	69783	23232.12	0.9	1.0	1.0	5.4	0.0	5.8	0.6
			150	7200.0	24313.87	24569.25	1.04	388670	117942	23232.12	0.3	1.0	1.0	5.4	0.0	5.8	1.1
			160	624.3	24569.00	24569.25	0.00	20618	12	23232.58	0.5	0.1	1.0	5.4	0.0	5.8	0.1
			170	3632.0	24569.03	24569.25	0.00	46391	13	23240.24	1.3	0.5	1.0	5.4	0.0	5.7	0.1
			180	994.8	24569.01	24569.25	0.00	29216	18	23240.24	0.9	0.1	1.0	5.4	0.0	5.7	0.1
			185	530.2	24569.05	24569.25	0.00	9245	5	23240.24	1.3	0.1	1.0	5.4	0.0	5.7	0.0
			200	7200.0	24034.70	24676.46	2.60	450411	239950	23232.12	0.6	1.0	1.0	5.4	0.4	6.2	1.3
			205	5257.6	24569.00	24569.25	0.00	262178	54	23232.12	0.5	0.7	1.0	5.4	0.0	5.8	0.7
			210	540.4	24569.01	24569.25	0.00	25029	7	23234.71	0.7	0.1	1.0	5.4	0.0	5.7	0.1
			220	6168.6	24569.00	24569.25	0.00	54928	8	23234.71	3.0	0.9	1.0	5.4	0.0	5.7	0.2
			230	7200.0	24491.47	24569.25	0.32	265293	25628	23232.12	1.1	1.0	1.0	5.4	0.0	5.8	0.7
			240	7108.7	24569.01	24569.25	0.00	259602	66	23232.12	1.2	1.0	1.0	5.4	0.0	5.8	0.7
			250	7200.0	24320.59	24569.25	1.01	388339	101641	23232.12	0.5	1.0	1.0	5.4	0.0	5.8	1.1
			260	538.3	24569.01	24569.25	0.00	25029	7	23234.71	0.9	0.1	1.0	5.4	0.0	5.7	0.1
			270	2204.1	24569.04	24569.25	0.00	40173	12	23240.25	1.5	0.3	1.0	5.4	0.0	5.7	0.1
			280	544.5	24569.04	24569.25	0.00	17884	5	23240.25	0.9	0.1	1.0	5.4	0.0	5.7	0.0
			285	263.1	24569.23	24569.25	0.00	5904	1	23240.25	1.4	0.0	1.0	5.4	0.0	5.7	0.0
INFEASIBLE	12	8 x 8	100	7200.0	18103.33	no feas sol	inf	61662	45234	17825.08	1.1						
			105	7200.0	18005.59	no feas sol	inf	84750	69870	17825.08	1.2						
			110	7200.0	18351.21	no feas sol	inf	32609	27786	17825.32	1.7						
			120	7200.0	18130.37	no feas sol	inf	5413	4533	17825.32	19.8						
			130	7200.0	17988.34	no feas sol	inf	24660	18158	17825.08	3.6						
			140	7200.0	17977.23	no feas sol	inf	46285	40921	17825.08	4.5						
			150	0.5	infeasible	infeasible	-	0	0	infeasible	0.4						
			160	7200.0	18350.99	no feas sol	inf	32476	27675	17825.32	2.7						
			170	4.0	infeasible	infeasible	-	0	0	infeasible	2.6						
			180	3.4	infeasible	infeasible	-	0	0	infeasible	1.7						
			185	4.7	infeasible	infeasible	-	0	0	infeasible	2.8						
			200	7200.0	18089.64	no feas sol	inf	45055	29234	17830.59	3.0						
			205	7200.0	18314.00	no feas sol	inf	39543	21723	17830.59	4.5						
			210	7200.0	18395.55	no feas sol	inf	28370	23735	17832.78	5.0						
			220	7200.0	18063.72	no feas sol	inf	2733	2302	17832.39	31.5						
			230	7200.0	18052.01	no feas sol	inf	29765	22093	17830.59	6.7						
			240	7200.0	18313.60	no feas sol	inf	15803	9317	17830.59	9.4						
			250	0.6	infeasible	infeasible	-	0	0	infeasible	2.2						
			260	7200.0	18395.58	no feas sol	inf	28468	23805	17832.78	7.9						
			270	7.7	infeasible	infeasible	-	0	0	infeasible	6.0						
			280	3.6	infeasible	infeasible	-	0	0	infeasible	3.5						
			285	6.2	infeasible	infeasible	-	0	0	infeasible	7.1						

Table D.2 (continued)

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
16603.13	13	9 x 8	100	7200.0	14800.15	17112.32	13.51	56097	46189	14752.07	1.0	1.0	1.0			16.0	1.0
			105	7200.0	14823.22	17402.06	14.82	47011	32186	14752.07	1.4	1.0	1.0			18.0	0.8
			110	7200.0	15030.98	16666.41	9.81	19755	16163	14753.39	2.6	1.0	1.0			13.0	0.4
			120	7200.0	14818.56	17689.71	16.23	1901	1736	14753.28	18.6	1.0	1.0			19.9	0.0
			130	7200.0	14768.50	17435.48	15.30	26617	23939	14752.07	2.6	1.0	1.0			18.2	0.5
			140	7200.0	14800.34	17208.68	13.99	19019	15953	14752.07	4.0	1.0	1.0			16.7	0.3
			150	7200.0	14810.70	17236.79	14.08	48970	42317	14752.07	1.5	1.0	1.0			16.8	0.9
			160	7200.0	15030.74	16666.41	9.81	19741	16151	14753.39	3.6	1.0	1.0			13.0	0.4
			170	7200.0	14987.11	16896.15	11.30	8480	6912	14783.11	15.0	1.0	1.0			14.3	0.2
			180	7200.0	15041.48	16748.84	10.19	17600	15563	14783.11	8.1	1.0	1.0			13.3	0.3
			185	7200.0	15043.02	17277.62	12.93	7455	5645	14783.11	13.7	1.0	1.0			16.9	0.1
			200	7200.0	14822.21	16925.43	12.43	32759	25150	14753.13	3.6	1.0	1.0			14.7	0.6
			205	7200.0	14788.63	17970.65	17.71	49854	33341	14753.13	3.9	1.0	1.0			21.8	0.9
			210	7200.0	15162.27	16653.45	8.95	17151	14585	14765.40	6.7	1.0	1.0			12.8	0.3
			220	7200.0	14815.74	17384.94	14.78	1520	1395	14763.99	34.0	1.0	1.0			17.8	0.0
			230	7200.0	14782.53	17053.45	13.32	27267	23717	14753.22	7.0	1.0	1.0			15.6	0.5
			240	7200.0	14784.06	17156.35	13.83	20319	18001	14753.13	7.3	1.0	1.0			16.3	0.4
			250	7200.0	14777.81	17117.00	13.67	39480	34446	14753.13	4.2	1.0	1.0			16.0	0.7
			260	7200.0	15162.27	16653.45	8.95	17140	14574	14765.40	9.9	1.0	1.0			12.8	0.3
			270	7200.0	15075.92	16715.87	9.81	4018	3276	14793.15	21.4	1.0	1.0			13.0	0.1
			280	7200.0	15091.92	16780.36	10.06	5846	4922	14793.15	15.7	1.0	1.0			13.4	0.1
			285	7200.0	15160.66	16888.84	10.23	3280	2645	14793.15	24.8	1.0	1.0			14.2	0.1
INFEASIBLE	14	9 x 8	100	7200.0	19141.91	no feas sol	inf	13869	2104	17266.46	1.2						
			105	7200.0	18536.62	no feas sol	inf	18590	7192	17266.46	1.8						
			110	191.7	infeasible	infeasible	-	12	0	17266.46	3.6						
			120	1053.2	infeasible	infeasible	-	24	0	17266.46	33.2						
			130	1325.4	infeasible	infeasible	-	1269	0	17266.46	3.8						
			140	6532.0	infeasible	infeasible	-	1430	0	17266.46	6.0						
			150	0.4	infeasible	infeasible	-	0	0	infeasible	1.1						
			160	192.2	infeasible	infeasible	-	12	0	17266.46	5.4						
			170	16.2	infeasible	infeasible	-	0	0	infeasible	6.0						
			180	7.6	infeasible	infeasible	-	0	0	infeasible	3.4						
			185	4.8	infeasible	infeasible	-	0	0	infeasible	3.0						
			200	115.7	infeasible	infeasible	-	14	0	17309.48	14.3						
			205	7200.0	18593.64	no feas sol	inf	2175	631	17309.48	21.6						
			210	7200.0	17739.23	no feas sol	inf	5397	4140	17313.74	30.2						
			220	7200.0	17728.17	no feas sol	inf	564	475	17313.44	152.6						
			230	7200.0	18136.11	no feas sol	inf	1389	424	17309.51	34.0						
			240	7200.0	18709.95	no feas sol	inf	1383	294	17309.48	29.4						
			250	1.1	infeasible	infeasible	-	0	0	infeasible	4.6						
			260	7200.0	17739.23	no feas sol	inf	5372	4117	17313.74	36.3						
			270	16.6	infeasible	infeasible	-	0	0	infeasible	9.8						
			280	8.5	infeasible	infeasible	-	0	0	infeasible	5.7						
			285	13.0	infeasible	infeasible	-	0	0	infeasible	23.2						
16909.84	15	6 x 4	100	4386.7	16909.67	16909.84	0.00	337521	40	15165.90	0.1	1.0	1.0	10.3	0.0	11.5	1.0
			105	2936.4	16909.68	16909.84	0.00	157632	25	15165.90	0.2	0.7	1.0	10.3	0.0	11.5	0.5
			110	1090.5	16909.73	16909.84	0.00	35029	3	15166.04	0.2	0.2	1.0	10.3	0.0	11.5	0.1
			120	7200.0	16685.42	16909.84	1.33	64305	18178	15166.04	1.5	1.6	1.0	10.3	0.0	11.5	0.2
			130	7200.0	16471.50	16909.84	2.59	302100	130493	15165.90	0.3	1.6	1.0	10.3	0.0	11.5	0.9
			140	7200.0	16785.02	16909.84	0.74	234661	40242	15165.90	0.6	1.6	1.0	10.3	0.0	11.5	0.7
			150	7200.0	16464.51	16909.84	2.63	368195	174984	15167.93	0.2	1.6	1.0	10.3	0.0	11.5	1.1
			160	1094.5	16909.73	16909.84	0.00	35029	3	15166.04	0.6	0.2	1.0	10.3	0.0	11.5	0.1
			170	1743.0	16909.72	16909.84	0.00	23025	5	15204.78	2.4	0.4	1.0	10.1	0.0	11.2	0.1
			180	1076.2	16909.74	16909.84	0.00	27975	2	15204.78	0.9	0.2	1.0	10.1	0.0	11.2	0.1
			185	452.6	16909.83	16909.84	0.00	5722	2	15204.78	1.1	0.1	1.0	10.1	0.0	11.2	0.0
			200	7200.0	16380.00	16909.84	3.13	413331	211330	15206.04	0.6	1.6	1.0	10.1	0.0	11.2	1.2
			205	4600.3	16909.70	16909.84	0.00	223869	26	15206.04	0.8	1.0	1.0	10.1	0.0	11.2	0.7
			210	691.4	16909.78	16909.84	0.00	21181	4	15220.56	0.8	0.2	1.0	10.0	0.0	11.1	0.1
			220	4405.8	16909.74	16909.84	0.00	38237	6	15220.56	3.4	1.0	1.0	10.0	0.0	11.1	0.1
			230	7200.0	16876.34	16909.84	0.20	244803	8967	15206.04	1.0	1.6	1.0	10.1	0.0	11.2	0.7
			240	7200.0	16829.41	16909.84	0.48	209989	19867	15206.04	0.9	1.6	1.0	10.1	0.0	11.2	0.6
			250	3361.4	16909.68	16909.84	0.00	142927	16	15208.55	0.6	0.8	1.0	10.1	0.0	11.2	0.4
			260	690.5	16909.78	16909.84	0.00	21181	4	15220.56	1.3	0.2	1.0	10.0	0.0	11.1	0.1
			270	1572.8	16909.73	16909.84	0.00	19442	2	15281.65	2.5	0.4	1.0	9.6	0.0	10.7	0.1
			280	2433.2	16909.75	16909.84	0.00	36652	7	15281.65	1.4	0.6	1.0	9.6	0.0	10.7	0.1
			285	710.3	16909.83	16909.84	0.00	6265	1	15281.65	2.8	0.2	1.0	9.6	0.0	10.7	0.0

Table D.2 (continued)

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
14299.67	16	7 x 4	100	2829.9	14299.53	14299.67	0.00	177372	40	13566.20	0.2	1.0	1.0	5.1	0.0	5.4	1.0
			105	1158.3	14299.53	14299.67	0.00	65200	19	13566.20	0.2	0.4	1.0	5.1	0.0	5.4	0.4
			110	419.8	14299.54	14299.67	0.00	14603	4	13566.34	0.2	0.1	1.0	5.1	0.0	5.4	0.1
			120	4127.6	14299.53	14299.67	0.00	45417	16	13566.34	3.3	1.5	1.0	5.1	0.0	5.4	0.3
			130	7200.0	14258.09	14299.67	0.29	307403	42695	13566.20	0.3	2.5	1.0	5.1	0.0	5.4	1.7
			140	3289.5	14299.53	14299.67	0.00	161712	48	13566.20	0.6	1.2	1.0	5.1	0.0	5.4	0.9
			150	4631.5	14299.53	14299.67	0.00	274674	81	13566.20	0.3	1.6	1.0	5.1	0.0	5.4	1.5
			160	529.4	14299.54	14299.67	0.00	18086	9	13566.34	0.5	0.2	1.0	5.1	0.0	5.4	0.1
			170	1235.0	14299.67	14299.67	0.00	20197	0	13603.13	3.0	0.4	1.0	4.9	0.0	5.1	0.1
			180	358.4	14299.59	14299.67	0.00	10758	7	13603.13	1.0	0.1	1.0	4.9	0.0	5.1	0.1
			185	280.2	14299.57	14299.67	0.00	4394	2	13603.13	1.9	0.1	1.0	4.9	0.0	5.1	0.0
			200	1822.2	14299.53	14299.67	0.00	99048	35	13577.10	0.4	0.6	1.0	5.1	0.0	5.3	0.6
			205	617.7	14299.56	14299.67	0.00	33928	9	13577.10	0.4	0.2	1.0	5.1	0.0	5.3	0.2
			210	447.8	14299.56	14299.67	0.00	13211	9	13580.25	0.7	0.2	1.0	5.0	0.0	5.3	0.1
			220	1547.2	14299.53	14299.67	0.00	15159	4	13580.24	6.6	0.5	1.0	5.0	0.0	5.3	0.1
			230	3646.4	14299.53	14299.67	0.00	140685	42	13577.10	1.3	1.3	1.0	5.1	0.0	5.3	0.8
			240	5083.0	14299.53	14299.67	0.00	256808	89	13577.10	1.0	1.8	1.0	5.1	0.0	5.3	1.4
			250	1996.3	14299.53	14299.67	0.00	94768	17	13577.10	0.5	0.7	1.0	5.1	0.0	5.3	0.5
			260	453.3	14299.56	14299.67	0.00	13113	9	13580.25	1.3	0.2	1.0	5.0	0.0	5.3	0.1
			270	1494.2	14299.66	14299.67	0.00	25117	1	13603.78	2.0	0.5	1.0	4.9	0.0	5.1	0.1
			280	659.3	14299.55	14299.67	0.00	14108	4	13603.78	1.5	0.2	1.0	4.9	0.0	5.1	0.1
			285	310.8	14299.67	14299.67	0.00	4575	0	13603.78	1.7	0.1	1.0	4.9	0.0	5.1	0.0
INFEASIBLE	17	7 x 4	100	12.9	infeasible	infeasible	-	15	0	5414.09	0.2						
			105	8.8	infeasible	infeasible	-	23	0	5414.09	0.3						
			110	22.1	infeasible	infeasible	-	58	0	5414.57	0.4						
			120	43.9	infeasible	infeasible	-	20	0	5414.57	5.9						
			130	14.8	infeasible	infeasible	-	21	0	5414.09	0.4						
			140	31.7	infeasible	infeasible	-	18	0	5414.09	0.5						
			150	0.2	infeasible	infeasible	-	0	0	infeasible	0.1						
			160	17.2	infeasible	infeasible	-	18	0	5414.57	1.1						
			170	4.2	infeasible	infeasible	-	0	0	infeasible	1.8						
			180	2.0	infeasible	infeasible	-	0	0	infeasible	0.9						
			185	3.0	infeasible	infeasible	-	0	0	infeasible	1.0						
			200	7200.0	6486.92	no feas sol	inf	211101	33411	5455.31	1.2						
			205	13.1	infeasible	infeasible	-	26	0	5455.31	1.5						
			210	7200.0	6688.93	no feas sol	inf	31813	9730	5461.95	2.2						
			220	7200.0	6268.15	no feas sol	inf	26628	18900	5461.95	10.8						
			230	7200.0	6068.41	no feas sol	inf	186136	97343	5455.37	2.1						
			240	41.8	infeasible	infeasible	-	76	0	5455.31	2.3						
			250	0.5	infeasible	infeasible	-	0	0	infeasible	0.4						
			260	7200.0	6687.39	no feas sol	inf	31516	9655	5461.95	2.7						
			270	6.9	infeasible	infeasible	-	0	0	infeasible	2.1						
			280	2.6	infeasible	infeasible	-	0	0	infeasible	0.9						
			285	3.3	infeasible	infeasible	-	0	0	infeasible	2.3						
18371.96	18	8 x 4	100	7200.0	17175.32	18592.98	7.62	153465	115775	17074.08	0.3	1.0	1.0	7.1	1.2	8.9	1.0
			105	7200.0	17178.29	18965.30	9.42	139011	101995	17074.08	0.4	1.0	1.0	7.1	3.2	11.1	0.9
			110	7200.0	17507.50	18831.86	7.03	88746	72706	17078.32	1.0	1.0	1.0	7.0	2.5	10.3	0.6
			120	7200.0	17200.91	18954.02	9.25	22209	18017	17075.33	10.0	1.0	1.0	7.1	3.2	11.0	0.1
			130	7200.0	17152.77	18500.97	7.29	88445	53713	17074.08	0.6	1.0	1.0	7.1	0.7	8.4	0.6
			140	7200.0	17144.55	18700.75	8.32	101595	77217	17074.08	1.7	1.0	1.0	7.1	1.8	9.5	0.7
			150	7200.0	17128.34	18807.30	8.93	146442	115519	17074.08	0.3	1.0	1.0	7.1	2.4	10.2	1.0
			160	7200.0	17508.00	18831.86	7.03	89558	73377	17078.32	2.4	1.0	1.0	7.0	2.5	10.3	0.6
			170	7200.0	17677.38	18546.97	4.69	33201	24907	17106.10	7.5	1.0	1.0	6.9	1.0	8.4	0.2
			180	7200.0	17665.32	18629.77	5.18	49871	37617	17106.10	4.0	1.0	1.0	6.9	1.4	8.9	0.3
			185	7200.0	17750.30	18589.65	4.52	35531	25085	17106.10	6.4	1.0	1.0	6.9	1.2	8.7	0.2
			200	7200.0	17135.95	18446.28	7.10	90155	64009	17074.83	2.1	1.0	1.0	7.1	0.4	8.0	0.6
			205	7200.0	17176.73	18866.03	8.95	93561	71262	17074.83	2.1	1.0	1.0	7.1	2.7	10.5	0.6
			210	7200.0	17937.18	18371.96	2.37	65795	40765	17095.29	3.2	1.0	1.0	6.9	0.0	7.5	0.4
			220	7200.0	17282.49	18439.40	6.27	13100	9591	17082.41	18.1	1.0	1.0	7.0	0.4	7.9	0.1
			230	7200.0	17176.34	18447.10	6.89	74458	51061	17074.84	3.8	1.0	1.0	7.1	0.4	8.0	0.5
			240	7200.0	17126.95	18700.38	8.41	66428	56185	17074.83	3.6	1.0	1.0	7.1	1.8	9.5	0.4
			250	7200.0	17111.16	18644.64	8.22	90270	73714	17074.83	1.8	1.0	1.0	7.1	1.5	9.2	0.6
			260	7200.0	17937.13	18371.96	2.37	65688	40705	17095.29	6.5	1.0	1.0	6.9	0.0	7.5	0.4
			270	7200.0	17798.03	18564.82	4.13	17995	13166	17114.63	13.2	1.0	1.0	6.8	1.0	8.5	0.1
			280	7200.0	17843.20	18439.76	3.24	38728	29838	17114.63	5.9	1.0	1.0	6.8	0.4	7.7	0.3
			285	7200.0	18113.49	18371.96	1.41	29260	10426	17114.63	7.9	1.0	1.0	6.8	0.0	7.3	0.2

Table D.2 (continued)

Opt. sol.	inst#		Option	MIP						LPR							
				CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
<u>24340.48</u>	19	9 x 4	100	7200.0	22192.18	25294.22	12.26	96978	79050	22099.88	0.4	1.0	1.0			14.5	1.0
			105	7200.0	22242.88	no feas sol	inf	84591	74397	22099.88	0.6	1.0	1.0			-	0.9
			110	7200.0	22551.88	25262.63	10.73	59574	50648	22102.11	2.2	1.0	1.0			14.3	0.6
			120	7200.0	22271.79	no feas sol	inf	8862	7856	22102.11	9.3	1.0	1.0			-	0.1
			130	7200.0	22145.64	no feas sol	inf	78102	72792	22099.88	1.6	1.0	1.0			-	0.8
			140	7200.0	22157.05	27679.50	19.95	72278	64820	22099.88	2.5	1.0	1.0			25.2	0.7
			150	7200.0	22188.64	25405.01	12.66	83509	72874	22099.88	0.7	1.0	1.0			15.0	0.9
			160	7200.0	22551.35	25262.63	10.73	58965	50115	22102.11	3.0	1.0	1.0			14.3	0.6
			170	7200.0	22707.36	25435.88	10.73	9584	8184	22114.78	12.1	1.0	1.0			15.0	0.1
			180	7200.0	22912.31	24753.54	7.44	12636	10793	22114.78	7.8	1.0	1.0			11.9	0.1
			185	7200.0	22875.09	25505.20	10.31	10097	8476	22114.78	11.8	1.0	1.0			15.3	0.1
			200	7200.0	22184.19	no feas sol	inf	75014	69130	22105.45	3.3	1.0	1.0			-	0.8
			205	7200.0	22234.78	no feas sol	inf	59623	49917	22105.45	3.6	1.0	1.0			-	0.6
			210	7200.0	22966.43	25462.87	9.80	32959	28757	22114.04	6.8	1.0	1.0			15.1	0.3
			220	7200.0	22431.33	25658.88	12.58	6285	4976	22113.18	30.5	1.0	1.0			16.0	0.1
			230	7200.0	22199.68	no feas sol	inf	56379	50986	22105.45	7.7	1.0	1.0			-	0.6
			240	7200.0	22230.22	no feas sol	inf	35422	32419	22105.45	6.1	1.0	1.0			-	0.4
			250	7200.0	22208.33	no feas sol	inf	67010	60642	22105.45	4.6	1.0	1.0			-	0.7
			260	7200.0	22967.26	25462.87	9.80	33210	28976	22114.04	8.6	1.0	1.0			15.1	0.3
			270	7200.0	22934.49	24647.99	6.95	4800	3372	22128.26	66.3	1.0	1.0			11.4	0.0
			280	7200.0	23001.10	24994.87	7.98	10011	8120	22128.26	9.8	1.0	1.0			13.0	0.1
			285	7200.0	22961.69	25608.45	10.34	4199	3508	22128.26	16.2	1.0	1.0			15.7	0.0
10363.31	20	9 x 4	100	7200.0	8786.25	10599.24	17.10	71206	53514	8735.73	0.4	1.0	1.0	15.7	2.3	21.3	1.0
			105	7200.0	8862.74	10949.58	19.06	84786	55250	8735.73	0.5	1.0	1.0	15.7	5.7	25.3	1.2
			110	7200.0	9281.12	10471.64	11.37	32792	26713	8737.19	1.3	1.0	1.0	15.7	1.0	19.9	0.5
			120	7200.0	8847.20	10746.65	17.67	4156	3623	8737.18	7.5	1.0	1.0	15.7	3.7	23.0	0.1
			130	7200.0	8776.34	10645.58	17.56	42310	33308	8735.73	1.5	1.0	1.0	15.7	2.7	21.9	0.6
			140	7200.0	8851.63	10691.70	17.21	50772	34768	8735.73	1.5	1.0	1.0	15.7	3.2	22.4	0.7
			150	7200.0	8775.17	10529.82	16.66	88953	67138	8735.73	0.6	1.0	1.0	15.7	1.6	20.5	1.2
			160	7200.0	9281.12	10471.64	11.37	32803	26724	8737.19	1.7	1.0	1.0	15.7	1.0	19.9	0.5
			170	7200.0	9206.03	10468.41	12.06	7821	5926	8749.81	8.5	1.0	1.0	15.6	1.0	19.6	0.1
			180	7200.0	9245.06	10491.57	11.88	17189	13328	8749.81	7.5	1.0	1.0	15.6	1.2	19.9	0.2
			185	7200.0	9302.01	10580.66	12.08	7509	5164	8749.81	14.4	1.0	1.0	15.6	2.1	20.9	0.1
			200	7200.0	8806.55	10490.61	16.05	65942	50428	8735.88	3.2	1.0	1.0	15.7	1.2	20.1	0.9
			205	7200.0	8848.14	10681.07	17.16	66275	50328	8735.88	2.8	1.0	1.0	15.7	3.1	22.3	0.9
			210	7200.0	9496.53	10394.21	8.64	26928	21512	8750.03	5.4	1.0	1.0	15.6	0.3	18.8	0.4
			220	7200.0	8986.67	10547.16	14.80	5279	4633	8747.90	17.8	1.0	1.0	15.6	1.8	20.6	0.1
			230	7200.0	8891.93	10455.60	14.96	44872	37205	8735.88	7.7	1.0	1.0	15.7	0.9	19.7	0.6
			240	7200.0	8835.62	10447.11	15.43	48122	38468	8735.88	4.8	1.0	1.0	15.7	0.8	19.6	0.7
			250	7200.0	8813.76	10499.07	16.05	62105	48019	8735.88	3.2	1.0	1.0	15.7	1.3	20.2	0.9
			260	7200.0	9495.91	10394.21	8.64	26809	21413	8750.03	5.1	1.0	1.0	15.6	0.3	18.8	0.4
			270	7200.0	9313.13	10397.44	10.43	5870	4060	8755.47	16.4	1.0	1.0	15.5	0.3	18.8	0.1
			280	7200.0	9433.40	10363.31	8.97	7348	5114	8755.47	11.7	1.0	1.0	15.5	0.0	18.4	0.1
			285	7200.0	9602.30	10472.92	8.31	5939	4242	8755.47	19.1	1.0	1.0	15.5	1.1	19.6	0.1

* LB, UB and Gap (%) refer to the final values of the lower bound, upper bound and the corresponding gap, respectively at the end of the time limit (7200 sec.). If the optimal solution of an instance is not known, the best known solution is provided under Opt.Sol. column in underlined form.

Table D.3 Case B- Average Results for 14 Instances with Known Optimals *

	CPU	CPU R	LPR R	IGap%	#opt *	DU%	ULP%	Nodes	Node R
100	2775.7	1.00	1.00	20.6	11 (10)	0.3	61.0	139355	1.00
105	2559.9	0.96	1.00	20.6	11 (10)	0.7	61.5	112243	0.99
110	1785.2	0.51	1.00	20.6	11 (11)	0.3	60.9	36707	0.40
120	3289.8	1.64	1.00	20.6	11 (9)	0.5	61.3	32012	0.50
130	3641.8	1.77	1.00	20.6	11 (8)	0.3	61.0	139367	1.29
140	3362.9	1.57	1.00	20.6	11 (9)	0.4	61.2	125150	1.29
150	3699.0	2.00	1.00	20.6	11 (8)	0.3	61.0	199701	1.79
160	1793.0	0.53	1.00	20.6	11 (11)	0.3	60.9	37152	0.40
170	2203.1	0.79	1.00	20.4	11 (11)	0.2	476.4	19324	0.27
180	1792.6	0.54	1.00	20.4	11 (11)	0.2	476.4	24165	0.32
185	1683.9	0.57	1.00	20.4	11 (11)	0.2	476.5	15669	0.26
200	2895.0	0.99	1.00	20.0	10 (9)	0.2	138.8	125093	0.90
205	2384.0	0.74	1.00	20.0	11 (11)	0.5	139.1	78193	0.64
210	1696.6	0.53	1.00	20.0	12 (11)	0.0	138.6	22589	0.41
220	2591.3	1.31	1.00	20.0	11 (11)	0.2	138.7	15712	0.34
230	3249.3	1.37	1.00	20.0	11 (9)	0.1	138.7	97580	0.84
240	3337.7	1.45	1.00	20.0	11 (10)	0.2	138.8	108359	1.17
250	2884.3	1.44	1.00	20.0	11 (10)	0.2	138.8	119038	1.04
260	1696.6	0.52	1.00	20.0	12 (11)	0.0	138.6	22512	0.41
270	2005.6	0.88	1.00	19.9	11 (11)	0.1	138.5	13105	0.28
280	1858.5	0.69	1.00	19.9	12 (11)	0.0	138.4	20820	0.33
285	1667.3	0.58	1.00	19.9	12 (11)	0.1	138.5	10488	0.25

* #opt denotes the number of times the optimal solution was found, where the entry in parentheses denotes the number of verified optimal solutions.

APPENDIX E

PRELIMINARY EXPERIMENTS UNDER CASE C

Table E.1 Solutions of Preliminary Instances Under Case C *

inst#	Original Instance #	N	K	Best sol. at the end of time limit	Extended CPU	Final Gap% at the end of extended time	Best sol.
1	11	5	4	36461.0	-	-	36461.0
2	21	5	4	30123.8	-	-	30123.8
3	41	2	4	16374.9	-	-	16374.9
4	51	5	4	31457.1	-	-	31457.1
5	63	3	4	6037.3	-	-	6037.3
6	83	4	4	8332.4	-	-	8332.4
7	94	5	4	46587.1	-	-	46587.1
8	104	5	4	55051.6	-	-	55051.6
9	111	5	4	26736.6	-	-	26736.6
10	11	5	8	61071.5	31628.5	-	61045.5
11	41	5	8	45466.2	-	-	45466.2
12	94	8	8	99582.8	329280.2	2.2	91285.6
13	53	9	8	38088.0	411080.9	8.3	37927.5
14	64	9	8	102512.8	381131.4	21.1	100537.1
15	74	6	4	50971.5	-	-	50971.5
16	13	7	4	31246.3	-	-	31246.3
17	124	7	4	47540.1	-	-	47540.1
18	102	8	4	31970.3	41198.8	-	31970.3
19	103	9	4	41452.6	20105.5	-	41452.6
20	123	9	4	26700.2	439204.0	3.82	26639.5

* Shaded cells indicate solutions with non-zero solution gaps. Best sol. refers to the best known solution for the instance. Best sol. at the end of the time limit is the best known solution among all options. However, extended solutions were only done with option 285.

Table E.2 Detailed Results of Experiments on Preliminary Instances Under Case C *

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
36460.97	1	5 x 4	100	7200.0	27621.97	36460.97	24.24	921698	473838	18405.93	0.1	1.0	1.0	49.5	0.0	98.1	1.0
			105	7200.0	30114.64	36460.97	17.41	1072765	453941	18405.93	0.1	1.0	1.0	49.5	0.0	98.1	1.2
			110	5159.3	36460.70	36460.97	0.00	178066	7	19612.56	0.1	0.7	1.1	46.2	0.0	85.9	0.2
			120	7200.0	33966.07	36460.97	6.84	120164	40799	19478.14	0.7	1.0	1.1	46.6	0.0	87.2	0.1
			130	7200.0	27593.60	36460.97	24.32	507607	305091	18405.93	0.1	1.0	1.0	49.5	0.0	98.1	0.6
			140	7200.0	29186.17	36460.97	19.95	397503	229573	18957.28	0.3	1.0	1.0	48.0	0.0	92.3	0.4
			150	1543.9	36460.61	36460.97	0.00	179353	34	36081.14	0.1	0.2	2.0	1.0	0.0	1.1	0.2
			160	5189.9	36460.70	36460.97	0.00	178066	7	19612.56	0.3	0.7	1.1	46.2	0.0	85.9	0.2
			170	135.9	36460.74	36460.97	0.00	2757	3	36098.70	0.8	0.0	2.0	1.0	0.0	1.0	0.0
			180	65.5	36460.72	36460.97	0.00	2187	2	36098.70	0.4	0.0	2.0	1.0	0.0	1.0	0.0
			185	39.8	36460.97	36460.97	0.00	820	0	36098.70	0.5	0.0	2.0	1.0	0.0	1.0	0.0
			200	7200.0	29729.07	36460.97	18.46	1207876	531026	18416.09	0.2	1.0	1.0	49.5	0.0	98.0	1.3
			205	7200.0	32046.42	36460.97	12.11	862605	231437	18416.09	0.2	1.0	1.0	49.5	0.0	98.0	0.9
			210	2055.8	36460.68	36460.97	0.00	94352	7	20079.48	0.2	0.3	1.1	44.9	0.0	81.6	0.1
			220	7200.0	33781.85	36460.97	7.35	123096	43893	19876.14	1.1	1.0	1.1	45.5	0.0	83.4	0.1
			230	7200.0	29382.14	36460.97	19.41	582755	272015	18416.09	0.2	1.0	1.0	49.5	0.0	98.0	0.6
			240	7200.0	30718.49	36460.97	15.75	476997	248746	19233.07	0.5	1.0	1.0	47.3	0.0	89.6	0.5
			250	2893.9	36460.60	36460.97	0.00	301632	92	36081.14	0.2	0.4	2.0	1.0	0.0	1.1	0.3
			260	2075.6	36460.68	36460.97	0.00	94352	7	20079.48	0.3	0.3	1.1	44.9	0.0	81.6	0.1
			270	106.9	36460.97	36460.97	0.00	1623	0	36106.35	1.0	0.0	2.0	1.0	0.0	1.0	0.0
			280	67.4	36460.61	36460.97	0.00	1982	3	36106.35	0.5	0.0	2.0	1.0	0.0	1.0	0.0
			285	56.4	36460.97	36460.97	0.00	1331	0	36106.35	0.6	0.0	2.0	1.0	0.0	1.0	0.0
30123.81	2	5 x 4	100	7200.0	18334.98	30512.56	39.91	765511	448625	7564.94	0.1	1.0	1.0	74.9	1.3	303.3	1.0
			105	7200.0	20043.76	30123.81	33.46	597131	266201	7564.94	0.1	1.0	1.0	74.9	0.0	298.2	0.8
			110	4825.2	30123.52	30123.81	0.00	289820	12	8514.43	0.1	0.7	1.1	71.7	0.0	253.8	0.4
			120	7200.0	25419.14	30123.81	15.62	142558	75063	8365.09	0.7	1.0	1.1	72.2	0.0	260.1	0.2
			130	7200.0	16465.52	30216.84	45.51	339343	215828	7564.94	0.1	1.0	1.0	74.9	0.3	299.4	0.4
			140	7200.0	21615.21	30123.81	28.25	400890	229315	8115.30	0.4	1.0	1.1	73.1	0.0	271.2	0.5
			150	1040.4	30123.53	30123.81	0.00	77215	3	25247.88	0.1	0.1	3.3	16.2	0.0	19.3	0.1
			160	4863.7	30123.52	30123.81	0.00	289820	12	8514.43	0.3	0.7	1.1	71.7	0.0	253.8	0.4
			170	163.2	30123.81	30123.81	0.00	3410	0	25278.64	0.6	0.0	3.3	16.1	0.0	19.2	0.0
			180	73.0	30123.81	30123.81	0.00	2879	0	25278.64	0.3	0.0	3.3	16.1	0.0	19.2	0.0
			185	47.0	30123.81	30123.81	0.00	790	0	25278.64	0.2	0.0	3.3	16.1	0.0	19.2	0.0
			200	7200.0	20300.11	30123.81	32.61	709379	375339	7580.60	0.2	1.0	1.0	74.8	0.0	297.4	0.9
			205	7200.0	24218.39	30123.81	19.60	616076	260085	7580.60	0.2	1.0	1.0	74.8	0.0	297.4	0.8
			210	6810.0	30123.57	30123.81	0.00	236906	11	9690.67	0.2	0.9	1.3	67.8	0.0	210.9	0.3
			220	7200.0	25775.50	30123.81	14.43	130998	66149	9465.09	1.4	1.0	1.3	68.6	0.0	218.3	0.2
			230	7200.0	18019.38	30123.81	40.18	372886	220181	7580.60	0.4	1.0	1.0	74.8	0.0	297.4	0.5
			240	7200.0	20790.26	30123.81	30.98	429604	258106	9003.76	0.5	1.0	1.2	70.1	0.0	234.6	0.6
			250	776.1	30123.68	30123.81	0.00	52245	8	25247.88	0.2	0.1	3.3	16.2	0.0	19.3	0.1
			260	6829.6	30123.57	30123.81	0.00	236906	11	9690.67	0.4	0.9	1.3	67.8	0.0	210.9	0.3
			270	240.9	30123.81	30123.81	0.00	4146	0	25322.53	0.9	0.0	3.3	15.9	0.0	19.0	0.0
			280	43.7	30123.81	30123.81	0.00	1116	0	25322.53	0.5	0.0	3.3	15.9	0.0	19.0	0.0
			285	32.1	30123.81	30123.81	0.00	488	0	25322.53	0.7	0.0	3.3	15.9	0.0	19.0	0.0
16374.87	3	2 x 4	100	0.1	16374.87	16374.87	0.00	16	0	10295.83	0.0	1.0	1.0	37.1	0.0	59.0	1.0
			105	0.1	16374.87	16374.87	0.00	14	0	10295.83	0.0	1.0	1.0	37.1	0.0	59.0	0.9
			110	0.1	16374.87	16374.87	0.00	8	0	11441.34	0.0	0.6	1.1	30.1	0.0	43.1	0.5
			120	0.2	16374.87	16374.87	0.00	12	0	11336.41	0.0	2.1	1.1	30.8	0.0	44.4	0.8
			130	0.1	16374.87	16374.87	0.00	15	0	10295.83	0.0	1.1	1.0	37.1	0.0	59.0	0.9
			140	0.2	16374.87	16374.87	0.00	16	0	11182.69	0.0	1.6	1.1	31.7	0.0	46.4	1.0
			150	0.1	16374.87	16374.87	0.00	11	0	15782.09	0.0	1.3	1.5	3.6	0.0	3.8	0.7
			160	0.1	16374.87	16374.87	0.00	8	0	11441.34	0.0	0.6	1.1	30.1	0.0	43.1	0.5
			170	0.1	16374.87	16374.87	0.00	4	0	15837.93	0.0	0.8	1.5	3.3	0.0	3.4	0.3
			180	0.1	16374.87	16374.87	0.00	4	0	15837.93	0.0	0.7	1.5	3.3	0.0	3.4	0.3
			185	0.1	16374.87	16374.87	0.00	0	0	15837.93	0.0	0.6	1.5	3.3	0.0	3.4	0.0
			200	0.1	16374.87	16374.87	0.00	12	0	11372.49	0.0	1.0	1.1	30.5	0.0	44.0	0.8
			205	0.1	16374.87	16374.87	0.00	16	0	11372.49	0.0	1.2	1.1	30.5	0.0	44.0	1.0
			210	0.1	16374.87	16374.87	0.00	3	0	11895.19	0.0	1.1	1.2	27.4	0.0	37.7	0.2
			220	0.2	16374.87	16374.87	0.00	8	0	11895.19	0.0	1.9	1.2	27.4	0.0	37.7	0.5
			230	0.1	16374.87	16374.87	0.00	8	0	11372.49	0.0	0.9	1.1	30.5	0.0	44.0	0.5
			240	0.2	16374.87	16374.87	0.00	16	0	11439.03	0.0	1.6	1.1	30.1	0.0	43.1	1.0
			250	0.2	16374.87	16374.87	0.00	14	0	15801.25	0.0	1.5	1.5	3.5	0.0	3.6	0.9
			260	0.1	16374.87	16374.87	0.00	3	0	11895.19	0.0	1.1	1.2	27.4	0.0	37.7	0.2
			270	0.1	16374.87	16374.87	0.00	0	0	15837.93	0.0	1.1	1.5	3.3	0.0	3.4	0.0
			280	0.1	16374.87	16374.87	0.00	4	0	15837.93	0.0	0.9	1.5	3.3	0.0	3.4	0.3
			285	0.1	16374.87	16374.87	0.00	0	0	15837.93	0.0	0.9	1.5	3.3	0.0	3.4	0.0

Table E.2 (continued)

				MIP						LPR							
Opt. sol.	inst#		Option	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
31457.08	4	5 x 4	100	7200.0	23113.64	31457.08	26.52	765058	372423	13465.64	0.1	1.0	1.0	57.2	0.0	133.6	1.0
			105	7200.0	24481.64	31546.70	22.40	670900	256592	13465.64	0.1	1.0	1.0	57.2	0.3	134.3	0.9
			110	5477.8	31456.88	31457.08	0.00	152291	7	14493.87	0.1	0.8	1.1	53.9	0.0	117.0	0.2
			120	7200.0	28262.15	31457.08	10.16	96808	43286	14389.59	0.6	1.0	1.1	54.3	0.0	118.6	0.1
			130	7200.0	21191.31	31457.08	32.63	396433	243023	13465.64	0.2	1.0	1.0	57.2	0.0	133.6	0.5
			140	7200.0	24840.23	31457.08	21.03	362914	202433	14018.96	0.4	1.0	1.0	55.4	0.0	124.4	0.5
			150	3988.9	31456.77	31457.08	0.00	450249	125	31144.21	0.1	0.6	2.3	1.0	0.0	1.0	0.6
			160	5474.3	31456.88	31457.08	0.00	152291	7	14493.87	0.2	0.8	1.1	53.9	0.0	117.0	0.2
			170	112.0	31456.82	31457.08	0.00	2717	3	31165.14	0.8	0.0	2.3	0.9	0.0	0.9	0.0
			180	77.8	31456.80	31457.08	0.00	2603	8	31165.14	0.3	0.0	2.3	0.9	0.0	0.9	0.0
			185	46.6	31457.08	31457.08	0.00	875	0	31165.14	0.5	0.0	2.3	0.9	0.0	0.9	0.0
			200	7200.0	22606.71	31457.08	28.13	958492	521039	13475.89	0.1	1.0	1.0	57.2	0.0	133.4	1.3
			205	7200.0	25071.36	31457.08	20.30	757411	295907	13475.89	0.1	1.0	1.0	57.2	0.0	133.4	1.0
			210	2544.9	31456.81	31457.08	0.00	131370	9	15096.69	0.2	0.4	1.1	52.0	0.0	108.4	0.2
			220	7200.0	29143.25	31457.08	7.36	134324	44964	14897.85	0.7	1.0	1.1	52.6	0.0	111.2	0.2
			230	7200.0	22093.39	31457.08	29.77	396424	238101	13475.89	0.2	1.0	1.0	57.2	0.0	133.4	0.5
			240	7200.0	25253.54	31457.08	19.72	434824	227695	14330.20	0.3	1.0	1.1	54.4	0.0	119.5	0.6
			250	1987.6	31456.77	31457.08	0.00	249442	52	31144.21	0.1	0.3	2.3	1.0	0.0	1.0	0.3
			260	2537.8	31456.81	31457.08	0.00	131370	9	15096.69	0.3	0.4	1.1	52.0	0.0	108.4	0.2
			270	62.2	31457.08	31457.08	0.00	898	1	31173.91	0.7	0.0	2.3	0.9	0.0	0.9	0.0
			280	74.3	31457.07	31457.08	0.00	1673	1	31173.91	0.4	0.0	2.3	0.9	0.0	0.9	0.0
			285	20.7	31457.06	31457.08	0.00	330	1	31173.91	0.4	0.0	2.3	0.9	0.0	0.9	0.0
6037.33	5	3 x 4	100	1.6	6037.33	6037.33	0.00	706	0	299.86	0.0	1.0	1.0	95.0	0.0	1913.4	1.0
			105	2.2	6037.33	6037.33	0.00	972	0	299.86	0.0	1.4	1.0	95.0	0.0	1913.4	1.4
			110	0.9	6037.33	6037.33	0.00	151	0	554.69	0.0	0.6	1.8	90.8	0.0	988.4	0.2
			120	1.9	6037.33	6037.33	0.00	257	0	551.60	0.0	1.2	1.8	90.9	0.0	994.5	0.4
			130	2.6	6037.33	6037.33	0.00	607	0	299.86	0.0	1.6	1.0	95.0	0.0	1913.4	0.9
			140	3.0	6037.33	6037.33	0.00	700	0	416.22	0.0	1.9	1.4	93.1	0.0	1350.5	1.0
			150	0.7	6037.33	6037.33	0.00	204	0	3473.27	0.0	0.4	11.6	42.5	0.0	73.8	0.3
			160	0.9	6037.33	6037.33	0.00	151	0	554.69	0.0	0.6	1.8	90.8	0.0	988.4	0.2
			170	0.8	6037.33	6037.33	0.00	47	0	3560.16	0.0	0.5	11.9	41.0	0.0	69.6	0.1
			180	0.8	6037.33	6037.33	0.00	73	0	3560.16	0.0	0.5	11.9	41.0	0.0	69.6	0.1
			185	0.6	6037.33	6037.33	0.00	53	0	3560.16	0.0	0.4	11.9	41.0	0.0	69.6	0.1
			200	1.6	6037.33	6037.33	0.00	735	0	318.90	0.0	1.0	1.1	94.7	0.0	1793.1	1.0
			205	1.5	6037.33	6037.33	0.00	641	0	318.90	0.0	1.0	1.1	94.7	0.0	1793.1	0.9
			210	0.6	6037.33	6037.33	0.00	26	0	1083.95	0.0	0.4	3.6	82.0	0.0	457.0	0.0
			220	1.6	6037.33	6037.33	0.00	134	0	1030.65	0.1	1.0	3.4	82.9	0.0	485.8	0.2
			230	2.5	6037.33	6037.33	0.00	839	0	318.90	0.0	1.6	1.1	94.7	0.0	1793.1	1.2
			240	2.2	6037.33	6037.33	0.00	504	0	931.63	0.0	1.4	3.1	84.6	0.0	548.0	0.7
			250	0.6	6037.33	6037.33	0.00	143	0	3473.27	0.0	0.4	11.6	42.5	0.0	73.8	0.2
			260	0.5	6037.33	6037.33	0.00	26	0	1083.95	0.0	0.3	3.6	82.0	0.0	457.0	0.0
			270	0.9	6037.33	6037.33	0.00	56	0	3579.64	0.1	0.5	11.9	40.7	0.0	68.7	0.1
			280	0.6	6037.33	6037.33	0.00	37	0	3579.64	0.0	0.4	11.9	40.7	0.0	68.7	0.1
			285	0.6	6037.33	6037.33	0.00	59	0	3579.64	0.0	0.4	11.9	40.7	0.0	68.7	0.1
8332.36	6	4 x 4	100	0.3	8332.36	8332.36	0.00	96	0	0.39	0.0	1.0	1.0	100.0	0.0	2137012.3	1.0
			105	0.3	8332.36	8332.36	0.00	108	0	0.39	0.0	1.1	1.0	100.0	0.0	2137012.3	1.1
			110	0.3	8332.36	8332.36	0.00	25	0	534.97	0.0	1.0	1372.1	93.6	0.0	1457.6	0.3
			120	0.5	8332.36	8332.36	0.00	26	0	283.59	0.1	1.8	727.4	96.6	0.0	2838.2	0.3
			130	0.4	8332.36	8332.36	0.00	109	0	0.39	0.0	1.3	1.0	100.0	0.0	2137012.3	1.1
			140	0.4	8332.36	8332.36	0.00	45	0	215.01	0.1	1.4	551.5	97.4	0.0	3775.3	0.5
			150	0.4	8332.36	8332.36	0.00	51	0	6164.52	0.0	1.2	15811.0	26.0	0.0	35.2	0.5
			160	0.3	8332.36	8332.36	0.00	25	0	534.97	0.0	1.0	1372.1	93.6	0.0	1457.6	0.3
			170	0.4	8332.36	8332.36	0.00	14	0	8093.54	0.1	1.3	20758.6	2.9	0.0	3.0	0.1
			180	0.4	8332.36	8332.36	0.00	16	0	8093.54	0.1	1.2	20758.6	2.9	0.0	3.0	0.2
			185	0.4	8332.36	8332.36	0.00	15	0	8093.54	0.1	1.2	20758.6	2.9	0.0	3.0	0.2
			200	0.5	8332.36	8332.36	0.00	59	0	21.46	0.0	1.7	55.1	99.7	0.0	38719.6	0.6
			205	0.6	8332.36	8332.36	0.00	85	0	21.46	0.0	2.0	55.1	99.7	0.0	38719.6	0.9
			210	0.6	8332.36	8332.36	0.00	44	0	1221.36	0.0	2.0	3132.6	85.3	0.0	582.2	0.5
			220	2.5	8332.36	8332.36	0.00	165	0	788.60	0.1	8.7	2022.6	90.5	0.0	956.6	1.7
			230	0.6	8332.36	8332.36	0.00	67	0	21.46	0.1	2.1	55.1	99.7	0.0	38719.6	0.7
			240	0.4	8332.36	8332.36	0.00	36	0	652.52	0.1	1.4	1673.6	92.2	0.0	1177.0	0.4
			250	0.6	8332.36	8332.36	0.00	40	0	6177.38	0.0	2.2	15844.0	25.9	0.0	34.9	0.4
			260	0.6	8332.36	8332.36	0.00	44	0	1221.36	0.1	2.0	3132.6	85.3	0.0	582.2	0.5
			270	1.0	8332.36	8332.36	0.00	20	0	8122.24	0.1	3.5	20832.2	2.5	0.0	2.6	0.2
			280	0.7	8332.36	8332.36	0.00	15	0	8122.24	0.1	2.2	20832.2	2.5	0.0	2.6	0.2
			285	0.8	8332.36	8332.36	0.00	23	0	8122.24	0.1	2.8	20832.2	2.5	0.0	2.6	0.2

Table E.2 (continued)

Opt. sol.	inst#		Option	MIP						LPR							
				CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
46587.12	7	5 x 4	100	1696.0	46586.84	46587.12	0.00	300541	5	2.58	0.1	1.0	1.0	100.0	0.0	1805948.9	1.0
			105	506.6	46587.09	46587.12	0.00	101784	2	2.58	0.1	0.3	1.0	100.0	0.0	1805948.9	0.3
			110	594.1	46586.67	46587.12	0.00	53491	2	1571.73	0.1	0.4	609.3	96.6	0.0	2864.1	0.2
			120	3394.5	46586.83	46587.12	0.00	182385	3	1079.70	0.4	2.0	418.6	97.7	0.0	4214.8	0.6
			130	3618.9	46586.72	46587.12	0.00	519860	17	2.58	0.1	2.1	1.0	100.0	0.0	1805948.9	1.7
			140	2957.3	46586.79	46587.12	0.00	473215	7	463.91	0.2	1.7	179.8	99.0	0.0	9942.3	1.6
			150	248.8	46586.71	46587.12	0.00	54853	8	26590.20	0.1	0.1	10308.3	42.9	0.0	75.2	0.2
			160	623.5	46586.67	46587.12	0.00	53491	2	1571.73	0.1	0.4	609.3	96.6	0.0	2864.1	0.2
			170	45.0	46587.12	46587.12	0.00	1773	0	26716.70	0.2	0.0	10357.3	42.7	0.0	74.4	0.0
			180	30.6	46587.12	46587.12	0.00	1967	0	26716.70	0.1	0.0	10357.3	42.7	0.0	74.4	0.0
			185	21.4	46587.12	46587.12	0.00	677	0	26716.70	0.1	0.0	10357.3	42.7	0.0	74.4	0.0
			200	3029.8	46586.66	46587.12	0.00	420725	8	123.00	0.2	1.8	47.7	99.7	0.0	37775.4	1.4
			205	1474.1	46586.70	46587.12	0.00	161385	4	123.00	0.2	0.9	47.7	99.7	0.0	37775.4	0.5
			210	253.8	46586.72	46587.12	0.00	15895	1	4360.53	0.2	0.1	1690.5	90.6	0.0	968.4	0.1
			220	1231.5	46587.12	46587.12	0.00	34469	0	2947.60	1.0	0.7	1142.7	93.7	0.0	1480.5	0.1
			230	5467.2	46586.68	46587.12	0.00	488827	11	123.22	0.4	3.2	47.8	99.7	0.0	37709.6	1.6
			240	5465.0	46586.68	46587.12	0.00	554400	25	2211.10	0.3	3.2	857.2	95.3	0.0	2007.0	1.8
			250	434.2	46586.76	46587.12	0.00	41941	4	26590.20	0.2	0.3	10308.3	42.9	0.0	75.2	0.1
			260	254.4	46586.72	46587.12	0.00	15895	1	4360.53	0.3	0.2	1690.5	90.6	0.0	968.4	0.1
			270	44.1	46587.12	46587.12	0.00	796	0	26783.43	0.5	0.0	10383.2	42.5	0.0	73.9	0.0
			280	53.7	46587.12	46587.12	0.00	955	0	26783.43	0.3	0.0	10383.2	42.5	0.0	73.9	0.0
			285	33.8	46587.12	46587.12	0.00	435	0	26783.43	0.4	0.0	10383.2	42.5	0.0	73.9	0.0
55051.64	8	5 x 4	100	7200.0	37319.42	55051.64	32.21	939200	575979	17844.65	0.1	1.0	1.0	67.6	0.0	208.5	1.0
			105	7200.0	40342.75	55051.64	26.72	787207	324391	17844.65	0.1	1.0	1.0	67.6	0.0	208.5	0.8
			110	5946.0	55051.11	55051.64	0.00	358293	21	19821.83	0.1	0.8	1.1	64.0	0.0	177.7	0.4
			120	7200.0	46139.83	55051.64	16.19	114013	58273	19217.80	0.7	1.0	1.1	65.1	0.0	186.5	0.1
			130	7200.0	35393.33	55051.64	35.71	482701	291989	17844.65	0.2	1.0	1.0	67.6	0.0	208.5	0.5
			140	7200.0	39332.74	55051.64	28.55	374416	231057	18518.66	0.4	1.0	1.0	66.4	0.0	197.3	0.4
			150	7200.0	51255.50	55051.64	6.90	783781	250927	47610.18	0.1	1.0	2.7	13.5	0.0	15.6	0.8
			160	5977.9	55051.11	55051.64	0.00	358293	21	19821.83	0.2	0.8	1.1	64.0	0.0	177.7	0.4
			170	207.7	55051.12	55051.64	0.00	5972	3	47730.42	1.3	0.0	2.7	13.3	0.0	15.3	0.0
			180	238.3	55051.26	55051.64	0.00	10232	6	47730.42	0.4	0.0	2.7	13.3	0.0	15.3	0.0
			185	138.8	55051.64	55051.64	0.00	4378	0	47730.42	0.5	0.0	2.7	13.3	0.0	15.3	0.0
			200	7200.0	36242.93	55051.64	34.17	553202	328428	17853.95	0.3	1.0	1.0	67.6	0.0	208.3	0.6
			205	7200.0	35382.83	55051.64	35.73	478038	257986	17853.95	0.3	1.0	1.0	67.6	0.0	208.3	0.5
			210	6455.3	55051.16	55051.64	0.00	285833	7	20696.60	0.3	0.9	1.2	62.4	0.0	166.0	0.3
			220	7200.0	46168.11	55051.64	16.14	114046	59799	19884.74	1.7	1.0	1.1	63.9	0.0	176.9	0.1
			230	7200.0	34451.88	55051.64	37.42	328836	210900	17853.95	0.5	1.0	1.0	67.6	0.0	208.3	0.4
			240	7200.0	38526.11	55051.64	30.02	402738	253045	19223.61	0.6	1.0	1.1	65.1	0.0	186.4	0.4
			250	2223.7	55051.10	55051.64	0.00	174463	56	47621.55	0.2	0.3	2.7	13.5	0.0	15.6	0.2
			260	6419.3	55051.16	55051.64	0.00	285833	7	20696.60	0.4	0.9	1.2	62.4	0.0	166.0	0.3
			270	370.4	55051.23	55051.64	0.00	7013	2	47764.91	1.3	0.1	2.7	13.2	0.0	15.3	0.0
			280	109.1	55051.27	55051.64	0.00	2892	2	47764.91	0.5	0.0	2.7	13.2	0.0	15.3	0.0
			285	126.6	55051.64	55051.64	0.00	2207	0	47764.95	0.9	0.0	2.7	13.2	0.0	15.3	0.0
26736.57	9	5 x 4	100	7200.0	21859.59	26736.57	18.24	1004466	405502	8589.45	0.1	1.0	1.0	67.9	0.0	211.3	1.0
			105	5626.0	26736.33	26736.57	0.00	733267	22	8589.45	0.1	0.8	1.0	67.9	0.0	211.3	0.7
			110	475.3	26736.34	26736.57	0.00	23641	2	9669.63	0.1	0.1	1.1	63.8	0.0	176.5	0.0
			120	2885.9	26736.50	26736.57	0.00	66291	5	9553.11	0.6	0.4	1.1	64.3	0.0	179.9	0.1
			130	7200.0	20742.31	26736.57	22.42	572435	273433	8589.45	0.2	1.0	1.0	67.9	0.0	211.3	0.6
			140	7200.0	24850.22	26736.57	7.06	583489	105246	9129.59	0.4	1.0	1.1	65.9	0.0	192.9	0.6
			150	284.4	26736.35	26736.57	0.00	35379	3	26257.42	0.1	0.0	3.1	1.8	0.0	1.8	0.0
			160	477.0	26736.34	26736.57	0.00	23641	2	9669.63	0.2	0.1	1.1	63.8	0.0	176.5	0.0
			170	43.4	26736.57	26736.57	0.00	836	0	26273.75	0.7	0.0	3.1	1.7	0.0	1.8	0.0
			180	20.3	26736.57	26736.57	0.00	557	0	26273.75	0.2	0.0	3.1	1.7	0.0	1.8	0.0
			185	18.7	26736.57	26736.57	0.00	256	0	26273.75	0.4	0.0	3.1	1.7	0.0	1.8	0.0
			200	7200.0	23159.05	26736.57	13.38	1120584	394258	8625.15	0.1	1.0	1.0	67.7	0.0	210.0	1.1
			205	7200.0	20898.39	26736.57	21.84	744358	273722	8625.15	0.2	1.0	1.0	67.7	0.0	210.0	0.7
			210	593.3	26736.57	26736.57	0.00	22788	1	11101.97	0.2	0.1	1.3	58.5	0.0	140.8	0.0
			220	2168.9	26736.49	26736.57	0.00	39825	1	10834.33	1.2	0.3	1.3	59.5	0.0	146.8	0.0
			230	7200.0	22153.95	26736.57	17.14	464409	162656	8625.15	0.4	1.0	1.0	67.7	0.0	210.0	0.5
			240	7200.0	23644.56	26736.57	11.56	508565	115883	9734.82	0.7	1.0	1.1	63.6	0.0	174.6	0.5
			250	161.2	26736.38	26736.57	0.00	16486	12	26257.42	0.1	0.0	3.1	1.8	0.0	1.8	0.0
			260	594.2	26736.57	26736.57	0.00	22788	1	11101.97	0.4	0.1	1.3	58.5	0.0	140.8	0.0
			270	23.1	26736.57	26736.57	0.00	259	0	26281.00	0.8	0.0	3.1	1.7	0.0	1.7	0.0
			280	16.4	26736.57	26736.57	0.00	422	0	26281.00	0.5	0.0	3.1	1.7	0.0	1.7	0.0
			285	13.4	26736.57	26736.57	0.00	149	0	26281.00	0.5	0.0	3.1	1.7	0.0	1.7	0.0

Table E.2 (continued)

	inst#		Option	MIP						LPR							
				CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
61045.51	10	5 x 8	100	7200.0	29161.02	64387.09	54.71	324709	269334	25965.63	0.3	1.0	1.0	57.5	5.5	148.0	1.0
			105	7200.0	29705.65	71027.37	58.18	340480	276951	25965.63	0.4	1.0	1.0	57.5	16.4	173.5	1.0
			110	7200.0	40515.10	62302.93	34.97	101970	75192	27314.97	0.5	1.0	1.1	55.3	2.1	128.1	0.3
			120	7200.0	38182.59	67027.11	43.03	39230	32606	27141.27	3.7	1.0	1.0	55.5	9.8	147.0	0.1
			130	7200.0	29744.41	67745.31	56.09	195089	161142	25965.63	0.6	1.0	1.0	57.5	11.0	160.9	0.6
			140	7200.0	30481.34	65429.08	53.41	117261	95728	26516.98	1.5	1.0	1.0	56.6	7.2	146.7	0.4
			150	7200.0	44581.45	67450.06	33.90	296517	223585	43640.84	0.2	1.0	1.7	28.5	10.5	54.6	0.9
			160	7200.0	40513.92	62302.93	34.97	101854	75092	27314.97	1.1	1.0	1.1	55.3	2.1	128.1	0.3
			170	7200.0	54574.96	62275.47	12.37	38616	23218	43694.67	1.6	1.0	1.7	28.4	2.0	42.5	0.1
			180	7200.0	54307.47	61589.36	11.82	102506	63785	43694.67	0.8	1.0	1.7	28.4	0.9	41.0	0.3
			185	7200.0	56841.82	61801.67	8.03	63677	29013	43694.67	1.5	1.0	1.7	28.4	1.2	41.4	0.2
			200	7200.0	29289.66	67695.00	56.73	282565	239294	25975.79	1.1	1.0	1.0	57.4	10.9	160.6	0.9
			205	7200.0	29783.60	66141.75	54.97	229871	168494	25975.79	1.4	1.0	1.0	57.4	8.3	154.6	0.7
			210	7200.0	41282.45	61956.42	33.37	71116	51887	28473.61	1.2	1.0	1.1	53.4	1.5	117.6	0.2
			220	7200.0	39124.06	63982.78	38.85	34265	29007	28130.10	8.2	1.0	1.1	53.9	4.8	127.5	0.1
			230	7200.0	29849.97	69180.61	56.85	175016	149058	25975.79	1.8	1.0	1.0	57.4	13.3	166.3	0.5
			240	7200.0	31650.74	65963.45	52.02	128893	116139	27382.60	3.7	1.0	1.1	55.1	8.1	140.9	0.4
			250	7200.0	48721.54	65034.02	25.08	242370	177169	43640.84	1.2	1.0	1.7	28.5	6.5	49.0	0.7
			260	7200.0	41282.68	61956.42	33.37	71152	51919	28473.61	2.4	1.0	1.1	53.4	1.5	117.6	0.2
			270	7200.0	54049.40	61071.52	11.50	39852	24957	43760.13	3.3	1.0	1.7	28.3	0.0	39.6	0.1
			280	7200.0	59493.15	61207.65	2.80	67094	15782	43760.13	1.8	1.0	1.7	28.3	0.3	39.9	0.2
			285	7200.0	56366.63	61846.73	8.86	59413	32029	43760.13	2.5	1.0	1.7	28.3	1.3	41.3	0.2
45466.25	11	5 x 8	100	7200.0	27241.96	46355.68	41.23	323590	236433	23249.74	0.3	1.0	1.0	48.9	2.0	99.4	1.0
			105	7200.0	27627.33	47842.75	42.25	307401	232842	23249.74	0.4	1.0	1.0	48.9	5.2	105.8	0.9
			110	7200.0	38310.81	45466.25	15.74	85468	57319	24621.97	0.4	1.0	1.1	45.8	0.0	84.7	0.3
			120	7200.0	35773.77	45483.89	21.35	26504	19379	24472.84	3.2	1.0	1.1	46.2	0.0	85.9	0.1
			130	7200.0	27610.36	48756.15	43.37	210980	175808	23249.74	0.7	1.0	1.0	48.9	7.2	109.7	0.7
			140	7200.0	28452.94	47665.87	40.31	152770	130254	23794.76	1.7	1.0	1.0	47.7	4.8	100.3	0.5
			150	7200.0	41702.67	46153.71	9.64	286400	133784	40918.62	0.4	1.0	1.8	10.0	1.5	12.8	0.9
			160	7200.0	38319.06	45466.25	15.72	85878	57574	24621.97	0.8	1.0	1.1	45.8	0.0	84.7	0.3
			170	7200.0	44031.65	45466.25	3.16	34593	9473	41028.93	3.1	1.0	1.8	9.8	0.0	10.8	0.1
			180	2307.9	45466.00	45466.25	0.00	31172	4	41028.93	1.6	0.3	1.8	9.8	0.0	10.8	0.1
			185	1193.0	45465.95	45466.25	0.00	12023	2	41028.93	2.0	0.2	1.8	9.8	0.0	10.8	0.0
			200	7200.0	27648.05	46293.66	40.28	420341	315761	23299.90	0.5	1.0	1.0	48.8	1.8	98.7	1.3
			205	7200.0	27829.41	46421.44	40.05	311923	220293	23299.90	0.6	1.0	1.0	48.8	2.1	99.2	1.0
			210	7200.0	38832.73	45466.25	14.59	90475	63100	25161.28	0.5	1.0	1.1	44.7	0.0	80.7	0.3
			220	7200.0	36044.47	45560.84	20.89	32335	24171	24967.44	3.9	1.0	1.1	45.1	0.2	82.5	0.1
			230	7200.0	27337.79	45619.65	40.07	194108	147875	23299.90	1.0	1.0	1.0	48.8	0.3	95.8	0.6
			240	7200.0	29107.05	45560.84	36.11	155540	123619	24123.70	2.4	1.0	1.0	46.9	0.2	88.9	0.5
			250	7200.0	41331.37	46066.51	10.28	303895	151499	40918.62	0.7	1.0	1.8	10.0	1.3	12.6	0.9
			260	7200.0	38810.63	45466.25	14.64	88920	62068	25161.28	0.9	1.0	1.1	44.7	0.0	80.7	0.3
			270	7200.0	43794.71	45466.25	3.68	30541	9918	41035.73	3.1	1.0	1.8	9.7	0.0	10.8	0.1
			280	796.9	45466.15	45466.25	0.00	9478	1	41035.73	1.6	0.1	1.8	9.7	0.0	10.8	0.0
			285	1537.7	45465.94	45466.25	0.00	13263	3	41036.70	2.2	0.2	1.8	9.7	0.0	10.8	0.0
99582.78	12	8 x 8	100	7200.0	20575.62	127388.53	83.85	77047	71482	17825.10	2.2	1.0	1.0			614.7	1.0
			105	7200.0	20589.13	154241.61	86.65	68170	60728	17825.10	2.0	1.0	1.0			765.3	0.9
			110	7200.0	39558.75	111792.21	64.61	25839	22468	19554.31	2.9	1.0	1.1			471.7	0.3
			120	7200.0	28952.31	139764.40	79.28	5267	4622	18786.36	53.5	1.0	1.1			644.0	0.1
			130	7200.0	20779.78	115270.83	81.97	58141	51469	17825.10	5.0	1.0	1.0			546.7	0.8
			140	7200.0	20533.44	114009.85	81.99	42398	38273	18150.89	15.1	1.0	1.0			528.1	0.6
			150	7200.0	62850.18	117841.28	46.67	75180	59606	62585.38	1.8	1.0	3.5			88.3	1.0
			160	7200.0	39546.65	111792.21	64.62	25758	22390	19554.31	4.0	1.0	1.1			471.7	0.3
			170	7200.0	70723.29	101570.01	30.37	10603	7455	62677.50	16.2	1.0	3.5			62.1	0.1
			180	7200.0	71084.91	99582.78	28.62	20679	15287	62677.50	8.1	1.0	3.5			58.9	0.3
			185	7200.0	71576.28	107083.31	33.16	12797	10699	62677.50	11.6	1.0	3.5			70.8	0.2
			200	7200.0	20749.94	116515.35	82.19	84386	72243	17831.68	3.6	1.0	1.0			553.4	1.1
			205	7200.0	20672.81	123249.93	83.23	71120	48760	17831.68	4.8	1.0	1.0			591.2	0.9
			210	7200.0	43951.64	114389.13	61.58	21262	18529	21602.07	6.5	1.0	1.2			429.5	0.3
			220	7200.0	31297.15	120110.76	73.94	4900	4386	20168.27	80.4	1.0	1.1			495.5	0.1
			230	7200.0	20578.93	129653.44	84.13	48126	45365	17831.74	8.3	1.0	1.0			627.1	0.6
			240	7200.0	20596.43	117390.29	82.45	30328	28032	19311.14	12.3	1.0	1.1			507.9	0.4
			250	7200.0	62749.66	129753.25	51.64	64852	58743	62590.89	3.8	1.0	3.5			107.3	0.8
			260	7200.0	43951.64	114389.13	61.58	21307	18574	21602.07	9.1	1.0	1.2			429.5	0.3
			270	7200.0	71908.01	104876.05	31.44	5483	4743	62701.17	36.7	1.0	3.5			67.3	0.1
			280	7200.0	74881.64	105042.93	28.71	11867	10248	62701.17	15.8	1.0	3.5			67.5	0.2
			285	7200.0	72542.10	103553.95	29.95	7445	6331	62701.26	23.3	1.0	3.5			65.2	0.1

Table E.2 (continued)

Opt. sol.	inst#		Option	MIP CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
37927.55	13	9 x 8	100	7200.0	15449.23	45807.02	66.27	55335	48542	14752.07	1.7	1.0	1.0			210.5	1.0
			105	7200.0	15451.44	53958.48	71.36	67456	54139	14752.07	3.2	1.0	1.0			265.8	1.2
			110	7200.0	19818.75	41908.40	52.71	16442	15172	15060.25	5.3	1.0	1.0			178.3	0.3
			120	7200.0	16417.62	43647.34	62.39	4456	4092	14926.47	45.3	1.0	1.0			192.4	0.1
			130	7200.0	15448.96	47722.82	67.63	32777	31235	14752.07	4.4	1.0	1.0			223.5	0.6
			140	7200.0	15460.24	45708.16	66.18	16353	14574	14829.49	22.3	1.0	1.0			208.2	0.3
			150	7200.0	25670.52	49057.38	47.67	75308	72021	25605.77	1.2	1.0	1.7			91.6	1.4
			160	7200.0	19818.75	41908.40	52.71	16454	15184	15060.25	8.0	1.0	1.0			178.3	0.3
			170	7200.0	31063.43	39256.71	20.87	5702	3571	28000.37	37.7	1.0	1.9			40.2	0.1
			180	7200.0	31164.23	39634.63	21.37	11041	8222	28000.37	21.1	1.0	1.9			41.6	0.2
			185	7200.0	31431.98	42629.31	26.27	8235	5384	28000.37	25.3	1.0	1.9			52.2	0.1
			200	7200.0	15453.85	45941.75	66.36	56281	50923	14754.09	5.2	1.0	1.0			211.4	1.0
			205	7200.0	15454.98	54109.78	71.44	61539	55846	14754.09	7.3	1.0	1.0			266.7	1.1
			210	7200.0	22196.34	41051.30	45.93	12597	11637	15814.80	10.1	1.0	1.1			159.6	0.2
			220	7200.0	17423.43	43093.04	59.57	3189	3005	15387.25	71.9	1.0	1.0			180.1	0.1
			230	7200.0	15455.41	41881.17	63.10	37104	35344	14754.09	12.6	1.0	1.0			183.9	0.7
			240	7200.0	15487.17	41646.76	62.81	22557	20317	15096.39	26.1	1.0	1.0			175.9	0.4
			250	7200.0	25637.20	46095.90	44.38	70400	66056	25606.83	4.9	1.0	1.7			80.0	1.3
			260	7200.0	22196.34	41051.30	45.93	12616	11656	15814.80	17.2	1.0	1.1			159.6	0.2
			270	7200.0	31060.20	39060.65	20.48	3284	2282	28275.19	54.4	1.0	1.9			38.1	0.1
			280	7200.0	32020.44	38504.46	16.84	11583	7507	28275.19	35.3	1.0	1.9			36.2	0.2
			285	7200.0	32429.87	38088.04	14.86	5876	3346	28275.19	52.5	1.0	1.9			34.7	0.1
102512.84	14	9 x 8	100	7200.0	20365.55	138229.89	85.27	58408	51614	17266.72	1.9	1.0	1.0			700.6	1.0
			105	7200.0	24180.87	162591.94	85.13	35500	25051	17266.72	4.1	1.0	1.0			841.6	0.6
			110	7200.0	40559.35	102512.84	60.43	18352	15096	18665.07	4.9	1.0	1.1			449.2	0.3
			120	7200.0	26567.69	114767.64	76.85	1811	1317	18093.85	117.3	1.0	1.0			534.3	0.0
			130	7200.0	20642.58	130953.44	84.24	32764	30687	17266.72	6.3	1.0	1.0			658.4	0.6
			140	7200.0	20429.84	123964.02	83.52	20934	18875	17610.29	21.3	1.0	1.0			603.9	0.4
			150	7200.0	67083.09	123676.99	45.76	62877	58967	66737.23	2.1	1.0	3.9			85.3	1.1
			160	7200.0	40559.35	102512.84	60.43	18333	15077	18665.07	9.5	1.0	1.1			449.2	0.3
			170	7200.0	69533.85	120274.65	42.19	2678	2096	66980.12	41.5	1.0	3.9			79.6	0.0
			180	7200.0	71796.79	102579.55	30.01	6661	4955	66980.12	20.1	1.0	3.9			53.1	0.1
			185	7200.0	74829.96	136390.79	45.14	4489	3820	67078.50	43.6	1.0	3.9			103.3	0.1
			200	7200.0	20475.02	115481.51	82.27	32353	30377	17339.78	13.5	1.0	1.0			566.0	0.6
			205	7200.0	21158.35	171362.29	87.65	22067	18557	17339.78	24.6	1.0	1.0			888.3	0.4
			210	7200.0	39365.40	108798.41	63.82	8448	7783	21282.12	23.9	1.0	1.2			411.2	0.1
			220	7200.0	23717.03	139805.59	83.04	914	824	19961.26	268.1	1.0	1.2			600.4	0.0
			230	7200.0	20791.70	140245.85	85.17	17866	16704	17339.78	38.9	1.0	1.0			708.8	0.3
			240	7200.0	19050.07	119445.18	84.05	13330	12399	18562.15	65.2	1.0	1.1			543.5	0.2
			250	7200.0	67052.35	145794.90	54.01	33153	31431	66796.18	16.4	1.0	3.9			118.3	0.6
			260	7200.0	39365.40	108798.41	63.82	8461	7796	21282.12	40.0	1.0	1.2			411.2	0.1
			270	7200.0	71020.53	105471.42	32.66	1559	1393	67120.03	444.8	1.0	3.9			57.1	0.0
			280	7200.0	72668.44	109087.36	33.39	3448	2947	67120.03	65.8	1.0	3.9			62.5	0.1
			285	7200.0	71768.21	140611.78	48.96	2338	2137	67276.60	73.7	1.0	3.9			109.0	0.0
50971.50	15	6 x 4	100	7200.0	31599.24	50980.04	38.02	544809	409708	15170.34	0.3	1.0	1.0	70.2	0.0	236.1	1.0
			105	7200.0	28505.19	51007.64	44.12	343487	176512	15170.34	0.3	1.0	1.0	70.2	0.1	236.2	0.6
			110	7200.0	47650.31	50971.50	6.52	188154	53574	17048.85	0.3	1.0	1.1	66.6	0.0	199.0	0.3
			120	7200.0	40143.21	50971.50	21.24	42431	25578	16071.04	2.5	1.0	1.1	68.5	0.0	217.2	0.1
			130	7200.0	26758.33	50971.50	47.50	275966	202232	15170.34	0.4	1.0	1.0	70.2	0.0	236.0	0.5
			140	7200.0	31019.52	50980.04	39.15	299021	196369	15600.31	1.2	1.0	1.0	69.4	0.0	226.8	0.5
			150	7200.0	50752.44	50971.50	0.43	401860	77352	49843.06	0.3	1.0	3.3	2.2	0.0	2.3	0.7
			160	7200.0	47670.18	50971.50	6.48	188931	53412	17048.85	0.5	1.0	1.1	66.6	0.0	199.0	0.3
			170	255.4	50971.50	50971.50	0.00	3450	0	49924.67	2.7	0.0	3.3	2.1	0.0	2.1	0.0
			180	222.7	50971.50	50971.50	0.00	4077	0	49924.67	1.1	0.0	3.3	2.1	0.0	2.1	0.0
			185	107.2	50971.50	50971.50	0.00	1222	0	49945.07	1.4	0.0	3.3	2.0	0.0	2.1	0.0
			200	7200.0	28743.91	50971.50	43.61	317570	217312	15281.95	0.6	1.0	1.0	70.0	0.0	233.5	0.6
			205	7200.0	25854.13	50971.50	49.28	298281	186437	15281.95	0.9	1.0	1.0	70.0	0.0	233.5	0.5
			210	7200.0	46354.15	50971.50	9.06	157951	63682	18340.58	0.5	1.0	1.2	64.0	0.0	177.9	0.3
			220	7200.0	39774.47	50971.50	21.97	51699	34817	17035.07	3.7	1.0	1.1	66.6	0.0	199.2	0.1
			230	7200.0	24579.48	50971.50	51.78	212623	157290	15281.95	1.1	1.0	1.0	70.0	0.0	233.5	0.4
			240	7200.0	28511.92	50971.50	44.06	199003	155151	16197.41	1.8	1.0	1.1	68.2	0.0	214.7	0.4
			250	3094.5	50970.99	50971.50	0.00	164396	89	49885.94	0.8	0.4	3.3	2.1	0.0	2.2	0.3
			260	7200.0	46277.96	50971.50	9.21	154594	63225	18340.58	1.1	1.0	1.2	64.0	0.0	177.9	0.3
			270	377.2	50971.49	50971.50	0.00	3999	1	49990.91	3.2	0.1	3.3	1.9	0.0	2.0	0.0
			280	222.1	50971.32	50971.50	0.00	3370	7	49990.91	1.8	0.0	3.3	1.9	0.0	2.0	0.0
			285	112.4	50971.50	50971.50	0.00	789	0	50023.71	2.9	0.0	3.3	1.9	0.0	1.9	0.0

Table E.2 (continued)

Opt. sol.	inst#		Option	MIP CPU	MIP LB	MIP UB	MIP Gap (%)	MIP Total Nodes	MIP Rem. Nodes	LPR LPR	LPR LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
31246.28	16	7 x 4	100	7200.0	16408.32	31304.28	47.58	330855	255065	13566.32	0.3	1.0	1.0	56.6	0.2	130.7	1.0
			105	7200.0	17713.62	31373.84	43.54	277011	173504	13566.32	0.4	1.0	1.0	56.6	0.4	131.3	0.8
			110	7200.0	27767.63	31246.28	11.13	205689	119854	14269.93	0.4	1.0	1.1	54.3	0.0	119.0	0.6
			120	7200.0	20899.71	31319.50	33.27	47623	37006	13762.32	2.5	1.0	1.0	56.0	0.2	127.6	0.1
			130	7200.0	16890.15	31347.91	46.12	226114	179105	13566.32	0.6	1.0	1.0	56.6	0.3	131.1	0.7
			140	7200.0	17246.10	31304.28	44.91	233681	194393	13665.60	1.7	1.0	1.0	56.3	0.2	129.1	0.7
			150	7200.0	27284.82	31304.28	12.84	269973	191579	26349.10	0.5	1.0	1.9	15.7	0.2	18.8	0.8
			160	7200.0	27773.43	31246.28	11.11	206685	120373	14269.93	0.5	1.0	1.1	54.3	0.0	119.0	0.6
			170	4809.8	31245.97	31246.28	0.00	53966	24	30214.34	3.3	0.7	2.2	3.3	0.0	3.4	0.2
			180	561.7	31245.99	31246.28	0.00	12970	7	30214.34	1.5	0.1	2.2	3.3	0.0	3.4	0.0
			185	339.4	31246.24	31246.28	0.00	4476	2	30214.34	2.1	0.0	2.2	3.3	0.0	3.4	0.0
			200	7200.0	16003.19	31304.28	48.88	345366	265504	13579.86	0.5	1.0	1.0	56.5	0.2	130.5	1.0
			205	7200.0	15681.41	31370.69	50.01	244595	168091	13579.86	0.8	1.0	1.0	56.5	0.4	131.0	0.7
			210	7200.0	27822.86	31246.28	10.96	229282	148426	14720.15	0.7	1.0	1.1	52.9	0.0	112.3	0.7
			220	7200.0	23191.38	31367.19	26.06	46439	36532	14044.32	5.6	1.0	1.0	55.1	0.4	123.3	0.1
			230	7200.0	16786.06	31350.31	46.46	192681	153747	13579.86	1.4	1.0	1.0	56.5	0.3	130.9	0.6
			240	7200.0	16500.91	31347.91	47.36	150979	119514	13833.71	1.9	1.0	1.0	55.7	0.3	126.6	0.5
			250	7200.0	27457.63	31246.28	12.13	279393	189964	26360.00	0.8	1.0	1.9	15.6	0.0	18.5	0.8
			260	7200.0	27823.87	31246.28	10.95	229510	148568	14720.15	1.4	1.0	1.1	52.9	0.0	112.3	0.7
			270	3294.9	31246.01	31246.28	0.00	26547	6	30215.71	4.7	0.5	2.2	3.3	0.0	3.4	0.1
			280	1397.8	31246.02	31246.28	0.00	25092	5	30215.71	2.2	0.2	2.2	3.3	0.0	3.4	0.1
			285	470.8	31246.08	31246.28	0.00	5511	2	30215.71	3.2	0.1	2.2	3.3	0.0	3.4	0.0
47540.07	17	7 x 4	100	7200.0	10447.72	48768.06	78.58	299592	235835	5423.95	0.4	1.0	1.0	88.6	2.6	799.1	1.0
			105	7200.0	18871.76	48028.93	60.71	251910	164821	5423.95	0.5	1.0	1.0	88.6	1.0	785.5	0.8
			110	7200.0	32036.38	47540.41	32.61	83671	61721	6891.20	0.4	1.0	1.3	85.5	0.0	589.9	0.3
			120	7200.0	25889.77	47544.06	45.55	26995	21329	6602.23	5.2	1.0	1.2	86.1	0.0	620.1	0.1
			130	7200.0	11906.81	47540.07	74.95	191194	160731	5423.95	1.0	1.0	1.0	88.6	0.0	776.5	0.6
			140	7200.0	14942.33	47540.07	68.57	171062	146284	5908.84	2.4	1.0	1.1	87.6	0.0	704.6	0.6
			150	7200.0	47020.73	47540.07	1.09	257657	126141	46482.49	0.7	1.0	8.6	2.2	0.0	2.3	0.9
			160	7200.0	32729.54	47540.07	31.15	77649	56239	6891.20	0.9	1.0	1.3	85.5	0.0	589.9	0.3
			170	5752.0	47539.69	47540.07	0.00	32905	7	46545.95	6.0	0.8	8.6	2.1	0.0	2.1	0.1
			180	2182.2	47539.63	47540.07	0.00	26042	121	46545.95	2.6	0.3	8.6	2.1	0.0	2.1	0.1
			185	512.6	47540.07	47540.07	0.00	3606	0	46546.10	4.0	0.1	8.6	2.1	0.0	2.1	0.0
			200	7200.0	16963.17	47791.39	64.51	315882	246431	5515.53	1.2	1.0	1.0	88.4	0.5	766.5	1.1
			205	7200.0	12858.98	47544.06	72.95	229577	144802	5515.53	1.9	1.0	1.0	88.4	0.0	762.0	0.8
			210	7200.0	34757.36	47540.07	26.89	68301	47576	9125.57	0.9	1.0	1.7	80.8	0.0	421.0	0.2
			220	7200.0	26408.12	47540.07	44.45	22656	18450	8110.50	8.1	1.0	1.5	82.9	0.0	486.2	0.1
			230	7200.0	11768.79	47544.97	75.25	145668	120137	5515.53	2.5	1.0	1.0	88.4	0.0	762.0	0.5
			240	7200.0	13076.66	47899.25	72.70	148596	130904	6689.24	3.1	1.0	1.2	85.9	0.8	616.1	0.5
			250	7200.0	46966.38	47540.07	1.21	225261	120509	46523.74	1.3	1.0	8.6	2.1	0.0	2.2	0.8
			260	7200.0	34749.55	47540.07	26.90	68083	47435	9125.57	1.9	1.0	1.7	80.8	0.0	421.0	0.2
			270	7200.0	47479.42	47540.07	0.13	32482	3150	46593.96	7.9	1.0	8.6	2.0	0.0	2.0	0.1
			280	4654.5	47539.65	47540.07	0.00	43067	17	46593.96	4.3	0.6	8.6	2.0	0.0	2.0	0.1
			285	600.4	47540.07	47540.07	0.00	2233	0	46597.26	6.0	0.1	8.6	2.0	0.0	2.0	0.0
31970.33	18	8 x 4	100	7200.0	17886.39	34033.90	47.45	134810	115156	17074.15	0.4	1.0	1.0	46.6	6.5	99.3	1.0
			105	7200.0	17871.21	36227.69	50.67	114454	100467	17074.15	0.5	1.0	1.0	46.6	13.3	112.2	0.8
			110	7200.0	23556.56	32413.46	27.32	47659	29323	17472.90	0.9	1.0	1.0	45.3	1.4	85.5	0.4
			120	7200.0	20629.30	34729.76	40.60	16530	14466	17283.31	10.6	1.0	1.0	45.9	8.6	100.9	0.1
			130	7200.0	18067.91	34875.80	48.19	86323	77273	17074.15	1.5	1.0	1.0	46.6	9.1	104.3	0.6
			140	7200.0	18014.38	34540.78	47.85	67123	61490	17161.18	3.9	1.0	1.0	46.3	8.0	101.3	0.5
			150	7200.0	26911.90	32797.17	17.94	132479	87709	26756.48	0.6	1.0	1.6	16.3	2.6	22.6	1.0
			160	7200.0	23554.28	32413.46	27.33	47521	29209	17472.90	1.7	1.0	1.0	45.3	1.4	85.5	0.4
			170	7200.0	30430.25	31976.07	4.83	21424	12309	27984.67	9.6	1.0	1.6	12.5	0.0	14.3	0.2
			180	7200.0	30199.04	31970.33	5.54	55756	36276	27984.67	3.8	1.0	1.6	12.5	0.0	14.2	0.4
			185	7200.0	30897.47	31970.33	3.36	37581	17248	27984.67	7.4	1.0	1.6	12.5	0.0	14.2	0.3
			200	7200.0	17850.35	35161.57	49.23	91954	81777	17081.28	1.2	1.0	1.0	46.6	10.0	105.8	0.7
			205	7200.0	17933.50	36075.67	50.29	96837	81345	17081.28	2.4	1.0	1.0	46.6	12.8	111.2	0.7
			210	7200.0	24788.78	31970.33	22.46	45512	39251	18093.69	2.5	1.0	1.1	43.4	0.0	76.7	0.3
			220	7200.0	21906.93	32915.14	33.44	15154	11944	17690.82	20.7	1.0	1.0	44.7	3.0	86.1	0.1
			230	7200.0	17923.76	34667.82	48.30	62957	56812	17081.28	4.4	1.0	1.0	46.6	8.4	103.0	0.5
			240	7200.0	18161.43	34535.81	47.41	57547	51095	17442.66	5.0	1.0	1.0	45.4	8.0	98.0	0.4
			250	7200.0	26851.01	33650.86	20.21	76822	57742	26757.23	2.4	1.0	1.6	16.3	5.3	25.8	0.6
			260	7200.0	24797.25	31970.33	22.44	45974	39649	18093.69	5.1	1.0	1.1	43.4	0.0	76.7	0.3
			270	7200.0	30704.27	32041.39	4.17	16350	10818	28085.84	12.3	1.0	1.6	12.2	0.2	14.1	0.1
			280	7200.0	29675.63	31970.33	7.18	34152	25022	28085.84	6.3	1.0	1.6	12.2	0.0	13.8	0.3
			285	7200.0	31517.07	31970.33	1.42	22692	8742	28085.84	9.5	1.0	1.6	12.2	0.0	13.8	0.2

Table E.2 (continued)

Opt. sol.	inst#		Option	MIP: CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR: LPR	LPR t	CPU R	LPR R	IGap%	DU%	ULP%	Node R
41452.58	**19**	9 x 4	100	7200.0	22822.17	47128.88	51.57	83593	73677	22099.89	0.8	1.0	1.0	46.7	13.7	113.3	1.0
			105	7200.0	22949.59	44395.09	48.31	92001	65787	22099.89	1.2	1.0	1.0	46.7	7.1	100.9	1.1
			110	7200.0	30535.66	41742.13	26.85	32030	25754	22508.71	1.8	1.0	1.0	45.7	0.7	85.4	0.4
			120	7200.0	24385.15	42638.61	42.81	8142	6300	22276.56	15.0	1.0	1.0	46.3	2.9	91.4	0.1
			130	7200.0	22890.03	43522.87	47.41	56462	49229	22099.89	3.7	1.0	1.0	46.7	5.0	96.9	0.7
			140	7200.0	23056.13	45115.50	48.90	39771	36484	22177.31	7.6	1.0	1.0	46.5	8.8	103.4	0.5
			150	7200.0	36302.54	45172.59	19.64	87312	77846	36204.68	1.6	1.0	1.6	12.7	9.0	24.8	1.0
			160	7200.0	30534.94	41742.13	26.85	32002	25732	22508.71	3.1	1.0	1.0	45.7	0.7	85.4	0.4
			170	7200.0	40261.36	41635.13	3.30	11148	8407	39328.03	14.5	1.0	1.8	5.1	0.4	5.9	0.1
			180	7200.0	40683.95	41490.60	1.94	25966	16559	39328.03	7.4	1.0	1.8	5.1	0.1	5.5	0.3
			185	7200.0	40646.77	41540.20	2.15	12919	7111	39328.03	13.6	1.0	1.8	5.1	0.2	5.6	0.2
			200	7200.0	22862.37	46537.62	50.87	66904	56689	22111.72	3.5	1.0	1.0	46.7	12.3	110.5	0.8
			205	7200.0	23032.65	47072.12	51.07	62968	46116	22111.72	3.7	1.0	1.0	46.7	13.6	112.9	0.8
			210	7200.0	31038.62	41533.04	25.27	29196	24545	23248.13	4.3	1.0	1.1	43.9	0.2	78.7	0.3
			220	7200.0	26432.48	43729.49	39.55	7750	6363	22717.84	28.8	1.0	1.0	45.2	5.5	92.5	0.1
			230	7200.0	23110.94	43762.01	47.19	46027	38214	22111.72	9.9	1.0	1.0	46.7	5.6	97.9	0.6
			240	7200.0	23032.64	44030.49	47.69	39983	37247	22473.02	10.3	1.0	1.0	45.8	6.2	95.9	0.5
			250	7200.0	36349.08	46624.70	22.04	55811	47544	36219.53	3.7	1.0	1.6	12.6	12.5	28.7	0.7
			260	7200.0	31039.18	41533.04	25.27	29232	24571	23248.13	7.4	1.0	1.1	43.9	0.2	78.7	0.3
			270	7200.0	40809.71	41452.58	1.55	8900	5091	39530.24	83.5	1.0	1.8	4.6	0.0	4.9	0.1
			280	7200.0	40944.85	41452.58	1.22	24197	15163	39530.24	11.5	1.0	1.8	4.6	0.0	4.9	0.3
			285	7200.0	41027.92	41452.98	1.03	10893	5000	39530.24	18.0	1.0	1.8	4.6	0.0	4.9	0.1
<u>26639.52</u>	**20**	9 x 4	100	7200.0	9446.24	27632.72	65.82	83291	72183	8735.75	0.9	1.0	1.0			216.3	1.0
			105	7200.0	9503.38	33573.27	71.69	96854	82308	8735.75	0.9	1.0	1.0			284.3	1.2
			110	7200.0	14029.26	27254.68	48.53	36997	31217	9036.14	1.1	1.0	1.0			201.6	0.4
			120	7200.0	11281.53	29133.80	61.28	8079	7038	8900.36	8.8	1.0	1.0			227.3	0.1
			130	7200.0	9509.74	29049.34	67.26	68227	62223	8735.75	2.3	1.0	1.0			232.5	0.8
			140	7200.0	9525.20	28900.14	67.04	44028	40753	8812.72	8.4	1.0	1.0			227.9	0.5
			150	7200.0	19675.54	28976.28	32.10	114452	101728	19659.63	1.0	1.0	2.3			47.4	1.4
			160	7200.0	14029.26	27254.68	48.53	37035	31255	9036.14	2.9	1.0	1.0			201.6	0.4
			170	7200.0	23368.44	27159.79	13.96	15510	12719	22695.30	19.5	1.0	2.6			19.7	0.2
			180	7200.0	23398.37	27005.80	13.36	35062	29085	22695.30	8.0	1.0	2.6			19.0	0.4
			185	7200.0	23510.45	26997.36	12.92	18607	14769	22695.30	14.9	1.0	2.6			19.0	0.2
			200	7200.0	9503.32	29581.41	67.87	78196	68777	8736.56	2.9	1.0	1.0			238.6	0.9
			205	7200.0	9664.94	31365.20	69.19	84890	74150	8736.56	3.0	1.0	1.0			259.0	1.0
			210	7200.0	15022.40	27144.72	44.66	30277	27204	9654.58	3.4	1.0	1.1			181.2	0.4
			220	7200.0	11648.86	29578.92	60.62	9295	8619	9320.09	20.7	1.0	1.1			217.4	0.1
			230	7200.0	9600.76	30010.44	68.01	51300	47922	8736.64	8.5	1.0	1.0			243.5	0.6
			240	7200.0	9624.69	27947.83	65.56	36601	33615	9115.47	10.8	1.0	1.0			206.6	0.4
			250	7200.0	19758.05	29022.89	31.92	81281	69412	19659.78	3.7	1.0	2.3			47.6	1.0
			260	7200.0	15020.23	27144.72	44.67	30183	27119	9654.58	6.4	1.0	1.1			181.2	0.4
			270	7200.0	24518.80	26700.25	8.17	13490	10570	22734.22	27.4	1.0	2.6			17.4	0.2
			280	7200.0	24599.96	26700.25	7.87	23473	18215	22734.22	13.0	1.0	2.6			17.4	0.3
			285	7200.0	24481.78	26834.81	8.77	13234	9796	22734.63	25.6	1.0	2.6			18.0	0.2

* LB, UB and Gap (%) refer to the final values of the lower bound, upper bound and the corresponding gap, respectively at the end of the time limit (7200 sec.). If the optimal solution of an instance is not known, the best known solution is provided under Opt.Sol. column in underlined form.

Table E.3 Case C- Average Results for 16 Instances with Known Optimals *

	CPU	CPU R	LPR R	IGap%	#opt *	DU%	ULP%	Nodes	Node R
100	5506.2	1.00	1.00	66.5	8 (4)	2.0	246719.6	421203	1.00
105	5333.5	0.97	1.00	66.5	8 (5)	2.7	246720.6	355681	0.90
110	4555.0	0.79	124.84	63.1	12 (9)	0.3	466.0	112527	0.31
120	5342.8	1.15	72.61	63.9	10 (5)	1.3	644.6	58123	0.21
130	5626.4	1.14	1.00	66.5	9 (4)	2.1	246718.7	253827	0.73
140	5585.1	1.16	46.64	65.0	9 (4)	1.8	1094.0	229617	0.63
150	4044.3	0.75	1635.38	14.8	10 (8)	1.5	22.8	207081	0.61
160	4563.0	0.79	124.84	63.1	12 (9)	0.3	466.0	112269	0.30
170	2520.4	0.51	1947.73	11.6	12 (12)	0.2	16.9	13352	0.08
180	1711.3	0.39	1947.73	11.6	13 (13)	0.1	16.7	17438	0.11
185	1504.1	0.34	1947.73	11.6	13 (13)	0.1	16.8	8961	0.06
200	5589.6	1.09	7.31	66.0	10 (4)	2.2	5055.3	425728	0.96
205	5492.3	1.06	7.31	66.0	10 (4)	2.3	5055.2	318417	0.78
210	4319.7	0.82	302.63	59.6	13 (9)	0.1	238.6	92441	0.25
220	5162.9	1.48	198.98	61.1	10 (5)	0.9	305.9	49210	0.24
230	5742.0	1.23	7.31	66.0	10 (4)	1.8	5050.2	229008	0.63
240	5741.8	1.23	159.25	62.9	10 (4)	1.5	372.6	230514	0.60
250	3423.3	0.74	1637.45	14.7	11 (10)	1.6	22.8	136522	0.46
260	4319.6	0.82	302.63	59.6	13 (9)	0.1	238.6	92168	0.25
270	2532.7	0.67	1953.95	11.5	13 (11)	0.0	16.4	10843	0.06
280	1814.9	0.47	1953.95	11.5	14 (13)	0.0	16.4	13472	0.09
285	1537.9	0.47	1953.95	11.5	13 (13)	0.1	16.5	7489	0.05

* #opt denotes the number of times the optimal solution was found, where the entry in parentheses denotes the number of verified optimal solutions.

APPENDIX F

VIOLATIONS OF TRIANGLE INEQUALITY IN THE HK INSTANCES

Table F.1 Details of Triangle Inequality Violations in HK Instances

	inst#	# Violations	# Triple Combinations	% Violation
6 x 2	46	15	150	10.00
	47	16	150	10.67
	48	17	150	11.33
	49	14	150	9.33
	50	9	150	6.00
	51	6	150	4.00
	52	14	150	9.33
	53	9	150	6.00
	54	14	150	9.33
	55	4	150	2.67
6 x 3	56	16	150	10.67
	57	6	150	4.00
	58	11	150	7.33
	59	16	150	10.67
	60	6	150	4.00
	61	5	150	3.33
	62	16	150	10.67
	63	12	150	8.00
	64	10	150	6.67
	65	14	150	9.33
6 x 4	66	16	150	10.67
	67	12	150	8.00
	68	15	150	10.00
	69	11	150	7.33
	70	13	150	8.67
	71	7	150	4.67
	72	8	150	5.33
	73	6	150	4.00
	74	5	150	3.33
	75	9	150	6.00
6 x 5	76	17	150	11.33
	77	23	150	15.33
	78	15	150	10.00
	79	8	150	5.33
	80	15	150	10.00
	81	7	150	4.67
	82	9	150	6.00
	83	13	150	8.67
	84	12	150	8.00
	85	16	150	10.67

	inst#	# Violations	# Triple Combinations	% Violation
7 x 3	1	25	252	9.92
	2	12	252	4.76
	3	24	252	9.52
	4	28	252	11.11
	5	25	252	9.92
	31	12	252	4.76
	32	21	252	8.33
	33	18	252	7.14
	34	12	252	4.76
	35	41	252	16.27
8 x 3	16	32	392	8.16
	17	11	392	2.81
	18	25	392	6.38
	19	34	392	8.67
	20	13	392	3.32
	36	33	392	8.42
	37	23	392	5.87
	38	21	392	5.36
	39	39	392	9.95
	40	41	392	10.46
9 x 3	26	48	576	8.33
	27	59	576	10.24
	28	56	576	9.72
	29	36	576	6.25
	30	32	576	5.56
	41	52	576	9.03
	42	29	576	5.03
	43	33	576	5.73
	44	67	576	11.63
	45	46	576	7.99

Average %Violation 7.81

APPENDIX G

EXPERIMENTS ON HK INSTANCES WITH MBS = 0.1

Table G.1 Solutions of HK Instances with MBS = 0.1 *

Size	inst#	Best sol. at the end of time limit	Extended CPU	Final Gap% at the end of extended time	Best sol.
6 x 2	46	1977.72	-	-	1977.72
6 x 2	47	1810.80	-	-	1810.80
6 x 2	48	1662.24	-	-	1662.24
6 x 2	49	1869.05	-	-	1869.05
6 x 2	50	1815.52	-	-	1815.52
6 x 2	51	2364.26	-	-	2364.26
6 x 2	52	1600.74	-	-	1600.74
6 x 2	53	1639.21	-	-	1639.21
6 x 2	54	1611.00	-	-	1611.00
6 x 2	55	2031.71	-	-	2031.71
6 x 3	56	2444.79	-	-	2444.79
6 x 3	57	2822.08	-	-	2822.08
6 x 3	58	2325.74	-	-	2325.74
6 x 3	59	2421.72	-	-	2421.72
6 x 3	60	2726.10	-	-	2726.10
6 x 3	61	2948.37	-	-	2948.37
6 x 3	62	2388.69	-	-	2388.69
6 x 3	63	2765.01	-	-	2765.01
6 x 3	64	2539.12	-	-	2539.12
6 x 3	65	2454.52	-	-	2454.52
6 x 4	66	3045.04	-	-	3045.04
6 x 4	67	3639.85	-	-	3639.85
6 x 4	68	3075.65	-	-	3075.65
6 x 4	69	3476.68	-	-	3476.68
6 x 4	70	3401.50	-	-	3401.50
6 x 4	71	3964.52	-	-	3964.52
6 x 4	72	3561.53	-	-	3561.53
6 x 4	73	4431.01	-	-	4431.01
6 x 4	74	3926.11	-	-	3926.11
6 x 4	75	3155.77	-	-	3155.77
6 x 5	76	4058.46	24099.6	0.00	3992.83
6 x 5	77	3317.44	9709.2	0.00	3317.44
6 x 5	78	4779.35	109064.3	0.00	4667.45
6 x 5	79	5074.30	23287.6	0.00	4998.03
6 x 5	80	3793.34	9305.7	0.00	3793.34
6 x 5	81	5156.68	17797.3	0.00	5150.04
6 x 5	82	4568.39	21175.1	0.00	4489.33
6 x 5	83	4702.02	10634.6	0.00	4665.45
6 x 5	84	4426.91	14610.1	0.00	4423.78
6 x 5	85	3895.34	15106.6	0.00	3870.14
7 x 3	1	2633.32	-	-	2633.32
7 x 3	2	3210.22	-	-	3210.22
7 x 3	3	3012.42	-	-	3012.42
7 x 3	4	2705.11	-	-	2705.11
7 x 3	5	3040.29	-	-	3040.29
7 x 3	31	3221.19	-	-	3221.19
7 x 3	32	2809.13	-	-	2809.13
7 x 3	33	3288.23	-	-	3288.23
7 x 3	34	2929.66	-	-	2929.66
7 x 3	35	2242.64	-	-	2242.64
8 x 3	16	3652.51	11413.9	0.00	3640.63
8 x 3	17	4158.24	-	-	4158.24
8 x 3	18	3592.83	-	-	3592.83
8 x 3	19	2923.44	-	-	2923.44
8 x 3	20	4210.04	-	-	4210.04
8 x 3	36	3218.06	-	-	3218.06
8 x 3	37	3202.54	-	-	3202.54
8 x 3	38	3755.65	-	-	3755.65
8 x 3	39	3229.05	-	-	3229.05
8 x 3	40	3273.34	-	-	3273.34
9 x 3	26	3758.91	28427.0	0.00	3733.70
9 x 3	27	3354.74	8119.0	0.00	3354.74
9 x 3	28	3655.15	19071.7	0.00	3549.33
9 x 3	29	3660.51	23605.5	0.00	3645.96
9 x 3	30	4017.30	35784.1	0.00	3947.71
9 x 3	41	3442.34	13079.8	0.00	3430.02
9 x 3	42	3926.71	24564.7	0.00	3910.75
9 x 3	43	3714.66	15478.4	0.00	3690.26
9 x 3	44	3049.08	-	-	3049.08
9 x 3	45	3674.80	35345.8	0.00	3598.29

* Shaded cells indicate solutions with non-zero solution gaps. Best sol. refers to the best known solution for the instance.

Table G.2 Detailed Results of Experiments on HK Instances with MBS = 0.1 *

	inst#	Opt. sol.	MIP CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	IGap%	DU%	ULP%
6 x 2	46	1977.72	7.0	1977.72	1977.72	0.00	86	0	1157.41	0.3	41.5	0.0	70.9
	47	1810.80	4.6	1810.80	1810.80	0.00	42	0	1059.91	0.2	41.5	0.0	70.8
	48	1662.24	4.8	1662.24	1662.24	0.00	83	0	955.01	0.3	42.5	0.0	74.1
	49	1869.05	5.2	1869.05	1869.05	0.00	78	0	1106.54	0.2	40.8	0.0	68.9
	50	1815.52	4.8	1815.52	1815.52	0.00	75	0	1044.41	0.2	42.5	0.0	73.8
	51	2364.26	5.4	2364.26	2364.26	0.00	62	0	1351.89	0.2	42.8	0.0	74.9
	52	1600.74	4.3	1600.74	1600.74	0.00	52	0	905.41	0.2	43.4	0.0	76.8
	53	1639.21	4.5	1639.21	1639.21	0.00	102	0	957.15	0.2	41.6	0.0	71.3
	54	1611.00	3.7	1611.00	1611.00	0.00	33	0	991.58	0.3	38.4	0.0	62.5
	55	2031.71	4.0	2031.71	2031.71	0.00	52	0	1295.26	0.3	36.2	0.0	56.9
6 x 3	56	2444.79	45.6	2444.79	2444.79	0.00	988	0	950.85	0.6	61.1	0.0	157.1
	57	2822.08	133.8	2822.08	2822.08	0.00	2926	0	1077.49	0.7	61.8	0.0	161.9
	58	2325.74	23.5	2325.74	2325.74	0.00	382	0	907.17	0.7	61.0	0.0	156.4
	59	2421.72	37.4	2421.72	2421.72	0.00	646	0	862.15	0.6	64.4	0.0	180.9
	60	2726.10	116.1	2726.10	2726.10	0.00	3037	0	1030.69	0.5	62.2	0.0	164.5
	61	2948.37	32.5	2948.37	2948.37	0.00	683	0	1202.51	0.5	59.2	0.0	145.2
	62	2388.69	29.3	2388.69	2388.69	0.00	522	0	955.18	0.6	60.0	0.0	150.1
	63	2765.01	39.7	2765.01	2765.01	0.00	693	0	1012.49	0.6	63.4	0.0	173.1
	64	2539.12	43.3	2539.12	2539.12	0.00	675	0	943.08	0.6	62.9	0.0	169.2
	65	2454.52	58.6	2454.52	2454.52	0.00	1363	0	920.75	0.7	62.5	0.0	166.6
6 x 4	66	3045.04	191.8	3045.04	3045.04	0.00	3291	0	856.53	1.2	71.9	0.0	255.5
	67	3639.85	2384.3	3639.84	3639.85	0.00	46754	1	1069.86	1.2	70.6	0.0	240.2
	68	3075.65	587.3	3075.65	3075.65	0.00	11111	0	798.75	1.1	74.0	0.0	285.1
	69	3476.68	2197.6	3476.66	3476.68	0.00	41798	1	981.82	1.0	71.8	0.0	254.1
	70	3401.50	541.1	3401.47	3401.50	0.00	8266	1	967.22	1.1	71.6	0.0	251.7
	71	3964.52	1359.3	3964.52	3964.52	0.00	25930	0	1074.78	1.1	72.9	0.0	268.9
	72	3561.53	370.0	3561.53	3561.53	0.00	6535	0	993.78	1.2	72.1	0.0	258.4
	73	4431.01	939.0	4430.97	4431.01	0.00	17171	1	1192.30	1.0	73.1	0.0	271.6
	74	3926.11	4239.6	3926.11	3926.11	0.00	111932	1	1057.80	1.0	73.1	0.0	271.2
	75	3155.77	209.5	3155.77	3155.77	0.00	3621	0	1019.36	1.0	67.7	0.0	209.6
6 x 5	76	3992.83	limit	3492.34	4058.46	13.95	89274	54043	978.77	2.2	75.5	1.6	314.7
	77	3317.44	limit	3116.64	3317.44	6.05	106257	31606	751.38	1.8	77.4	0.0	341.5
	78	4667.45	limit	3691.79	4779.35	22.76	80911	58228	1020.20	1.8	78.1	2.4	368.5
	79	4998.03	limit	4407.30	5074.30	13.14	134837	79536	1155.34	2.2	76.9	1.5	339.2
	80	3793.34	limit	3552.08	3793.34	6.36	117537	33481	914.55	2.0	75.9	0.0	314.8
	81	5150.04	limit	4557.60	5156.68	11.62	93696	51666	1173.14	2.1	77.2	0.1	339.6
	82	4489.33	limit	4075.60	4568.39	10.79	70170	40175	1120.48	1.8	75.0	1.8	307.7
	83	4665.45	limit	4327.81	4702.02	7.96	96529	36171	1151.75	1.6	75.3	0.8	308.2
	84	4423.78	limit	4000.40	4426.91	9.63	90359	43833	1051.37	2.2	76.2	0.1	321.1
	85	3870.14	limit	3541.22	3895.34	9.09	112541	57465	856.46	1.7	77.9	0.7	354.8

Note: in the printed table, "MIP" spans the columns CPU through Rem. Nodes, and "LPR" spans the columns LPR and LPR t.

Table G.2 (continued)

			MIP						LPR				
	inst#	Opt. sol.	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	IGap%	DU%	ULP%
7 x 3	1	2633.32	339.4	2633.32	2633.32	0.00	3961	0	967.57	1.5	63.3	0.0	172.2
	2	3210.22	681.0	3210.22	3210.22	0.00	8069	0	1172.35	1.4	63.5	0.0	173.8
	3	3012.42	373.7	3012.42	3012.42	0.00	6948	0	1077.56	1.5	64.2	0.0	179.6
	4	2705.11	491.9	2705.11	2705.11	0.00	6185	0	995.35	1.8	63.2	0.0	171.8
	5	3040.29	368.2	3040.29	3040.29	0.00	4484	1	1146.76	1.9	62.3	0.0	165.1
	31	3221.19	304.3	3221.19	3221.19	0.00	3726	0	1267.59	1.4	60.6	0.0	154.1
	32	2809.13	423.0	2809.13	2809.13	0.00	4394	0	1089.74	1.4	61.2	0.0	157.8
	33	3288.23	657.5	3288.21	3288.23	0.00	8088	2	1149.20	1.9	65.1	0.0	186.1
	34	2929.66	164.5	2929.66	2929.66	0.00	1772	0	1150.34	1.4	60.7	0.0	154.7
	35	2242.64	241.2	2242.64	2242.64	0.00	2576	0	851.75	1.5	62.0	0.0	163.3
8 x 3	16	3640.63	limit	3363.39	3652.51	7.92	65553	25543	1360.57	4.0	62.6	0.3	168.5
	17	4158.24	6380.0	4158.20	4158.24	0.00	54576	7	1428.61	3.9	65.6	0.0	191.1
	18	3592.83	1889.2	3592.80	3592.83	0.00	16284	1	1244.85	4.6	65.4	0.0	188.6
	19	2923.44	1856.4	2923.44	2923.44	0.00	12837	0	1087.04	6.1	62.8	0.0	168.9
	20	4210.04	6712.1	4210.01	4210.04	0.00	71397	5	1499.18	3.1	64.4	0.0	180.8
	36	3218.06	3763.6	3218.03	3218.06	0.00	35545	3	1141.58	5.1	64.5	0.0	181.9
	37	3202.54	3143.0	3202.54	3202.54	0.00	28266	1	1148.32	4.4	64.1	0.0	178.9
	38	3755.65	2572.1	3755.63	3755.65	0.00	21430	4	1427.86	4.6	62.0	0.0	163.0
	39	3229.05	3343.9	3229.02	3229.05	0.00	28434	3	1167.43	4.9	63.8	0.0	176.6
	40	3273.34	4707.6	3273.33	3273.34	0.00	41686	1	1196.46	4.7	63.4	0.0	173.6
9 x 3	26	3733.70	limit	3211.15	3758.91	14.57	38575	22991	1369.51	9.6	63.3	0.7	174.5
	27	3354.74	limit	3229.11	3354.74	3.74	31718	6447	1261.67	8.2	62.4	0.0	165.9
	28	3549.33	limit	3085.17	3655.15	15.59	25856	16174	1215.41	9.6	65.8	3.0	200.7
	29	3645.96	limit	3170.09	3660.51	13.40	33416	20118	1311.70	10.4	64.0	0.4	179.1
	30	3947.71	limit	3283.02	4017.30	18.28	41615	25149	1432.31	8.2	63.7	1.8	180.5
	41	3430.02	limit	3100.39	3442.34	9.93	31149	14022	1248.60	8.6	63.6	0.4	175.7
	42	3910.75	limit	3407.68	3926.71	13.22	33340	19939	1395.75	9.9	64.3	0.4	181.3
	43	3690.26	limit	3288.65	3714.66	11.47	33131	16949	1329.30	8.8	64.0	0.7	179.4
	44	3049.08	5573.6	3049.06	3049.08	0.00	29251	1	1087.81	8.3	64.3	0.0	180.3
	45	3598.29	limit	3031.66	3674.80	17.50	31462	21268	1252.42	11.1	65.2	2.1	193.4

* LB, UB and Gap (%) refer to the final values of the lower bound, upper bound and the corresponding gap, respectively at the end of the time limit.

APPENDIX H

EXPERIMENTS ON HK INSTANCES WITH MBS = 0.025

Table H.1 Solutions of HK Instances with MBS = 0.025 *

	inst#	Best sol. at the end of time limit	Extended CPU	Final Gap% at the end of extended time	Best sol.
6 x 2	46	1977.72	-	-	1977.72
	47	1810.80	-	-	1810.80
	48	1662.24	-	-	1662.24
	49	1869.05	-	-	1869.05
	50	1815.52	-	-	1815.52
	51	2364.26	-	-	2364.26
	52	1600.74	-	-	1600.74
	53	1639.21	-	-	1639.21
	54	1611.00	-	-	1611.00
	55	2031.71	-	-	2031.71
6 x 3	56	2444.79	-	-	2444.79
	57	2822.08	-	-	2822.08
	58	2325.74	-	-	2325.74
	59	2421.72	-	-	2421.72
	60	2726.10	-	-	2726.10
	61	2948.37	-	-	2948.37
	62	2388.69	-	-	2388.69
	63	2765.01	-	-	2765.01
	64	2539.12	-	-	2539.12
	65	2454.52	-	-	2454.52
6 x 4	66	3045.04	-	-	3045.04
	67	3639.85	-	-	3639.85
	68	3075.65	-	-	3075.65
	69	3476.68	-	-	3476.68
	70	3401.50	-	-	3401.50
	71	3964.52	-	-	3964.52
	72	3561.53	-	-	3561.53
	73	4431.01	-	-	4431.01
	74	3926.11	-	-	3926.11
	75	3155.77	-	-	3155.77
6 x 5	76	4017.88	15607.5	0.00	3992.83
	77	3354.01	15821.3	0.00	3317.44
	78	4792.45	90263.0	0.00	4667.45
	79	5061.07	16560.2	0.00	4998.03
	80	3793.34	-	-	3793.34
	81	5156.68	17349.7	0.00	5150.04
	82	4500.74	18393.2	0.00	4489.33
	83	4702.02	10883.7	0.00	4665.45
	84	4502.35	19174.5	0.00	4423.78
	85	3910.79	8934.4	0.00	3870.14

	inst#	Best sol. at the end of time limit	Extended CPU	Final Gap% at the end of extended time	Best sol.
7 x 3	1	2633.32	-	-	2633.32
	2	3210.22	-	-	3210.22
	3	3012.42	-	-	3012.42
	4	2705.11	-	-	2705.11
	5	3040.29	-	-	3040.29
	31	3221.19	-	-	3221.19
	32	2809.13	-	-	2809.13
	33	3288.23	-	-	3288.23
	34	2929.66	-	-	2929.66
	35	2242.64	-	-	2242.64
8 x 3	16	3640.63	-	-	3640.63
	17	4158.24	-	-	4158.24
	18	3592.83	-	-	3592.83
	19	2923.44	-	-	2923.44
	20	4210.04	16242.1	0.00	4210.04
	36	3218.06	-	-	3218.06
	37	3202.54	-	-	3202.54
	38	3755.65	-	-	3755.65
	39	3229.05	-	-	3229.05
	40	3273.34	-	-	3273.34
9 x 3	26	3733.70	35462.9	0.00	3733.70
	27	3405.97	11938.8	0.00	3354.74
	28	3549.33	8059.5	0.00	3549.33
	29	3721.70	22448.7	0.00	3645.96
	30	4082.90	47147.2	0.00	3947.71
	41	3446.47	11413.6	0.00	3430.02
	42	4007.49	63276.6	0.00	3910.75
	43	3690.26	13095.2	0.00	3690.26
	44	3049.08	-	-	3049.08
	45	3598.29	13079.4	0.00	3598.29

* Shaded cells indicate solutions with non-zero solution gaps. Best sol. refers to the best known solution for the instance.

Table H.2 Detailed Results of Experiments on HK Instances with MBS = 0.025 *

			MIP							LPR				
	inst#	Opt. sol.	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	IGap%	DU%	ULP%
6 x 2	46	1977.72	4.7	1977.72	1977.72	0.00	91	0	1157.41	0.2	41.5	0.0	70.9
	47	1810.80	4.5	1810.80	1810.80	0.00	37	0	1059.91	0.2	41.5	0.0	70.8
	48	1662.24	5.3	1662.24	1662.24	0.00	105	0	955.01	0.2	42.5	0.0	74.1
	49	1869.05	4.7	1869.05	1869.05	0.00	62	0	1106.54	0.2	40.8	0.0	68.9
	50	1815.52	4.0	1815.52	1815.52	0.00	50	0	1044.41	0.2	42.5	0.0	73.8
	51	2364.26	4.4	2364.26	2364.26	0.00	66	0	1351.89	0.3	42.8	0.0	74.9
	52	1600.74	4.5	1600.74	1600.74	0.00	51	0	905.41	0.4	43.4	0.0	76.8
	53	1639.21	5.1	1639.21	1639.21	0.00	91	0	957.15	0.3	41.6	0.0	71.3
	54	1611.00	3.6	1611.00	1611.00	0.00	42	0	991.58	0.3	38.4	0.0	62.5
	55	2031.71	3.4	2031.71	2031.71	0.00	64	0	1295.26	0.3	36.2	0.0	56.9
6 x 3	56	2444.79	35.9	2444.77	2444.79	0.00	596	1	950.85	0.7	61.1	0.0	157.1
	57	2822.08	114.3	2822.08	2822.08	0.00	2297	0	1077.49	0.8	61.8	0.0	161.9
	58	2325.74	21.5	2325.74	2325.74	0.00	307	0	907.17	0.6	61.0	0.0	156.4
	59	2421.72	39.0	2421.72	2421.72	0.00	617	0	862.15	0.6	64.4	0.0	180.9
	60	2726.10	80.9	2726.10	2726.10	0.00	1868	0	1030.69	0.6	62.2	0.0	164.5
	61	2948.37	55.0	2948.37	2948.37	0.00	921	0	1202.51	0.6	59.2	0.0	145.2
	62	2388.69	32.8	2388.69	2388.69	0.00	564	0	955.18	0.6	60.0	0.0	150.1
	63	2765.01	44.5	2765.01	2765.01	0.00	974	0	1012.49	0.8	63.4	0.0	173.1
	64	2539.12	34.8	2539.12	2539.12	0.00	464	0	943.08	0.7	62.9	0.0	169.2
	65	2454.52	72.0	2454.52	2454.52	0.00	1352	0	920.75	0.7	62.5	0.0	166.6
6 x 4	66	3045.04	399.5	3045.04	3045.04	0.00	7824	0	856.53	1.2	71.9	0.0	255.5
	67	3639.85	3646.7	3639.83	3639.85	0.00	75741	5	1069.86	1.0	70.6	0.0	240.2
	68	3075.65	1027.9	3075.64	3075.65	0.00	16535	1	798.75	1.0	74.0	0.0	285.1
	69	3476.68	2217.7	3476.66	3476.68	0.00	43576	2	981.82	1.0	71.8	0.0	254.1
	70	3401.50	794.8	3401.48	3401.50	0.00	12342	2	967.22	1.0	71.6	0.0	251.7
	71	3964.52	1452.8	3964.48	3964.52	0.00	38710	3	1074.78	1.0	72.9	0.0	268.9
	72	3561.53	790.0	3561.53	3561.53	0.00	12147	0	993.78	0.9	72.1	0.0	258.4
	73	4431.01	996.1	4431.01	4431.01	0.00	17162	0	1192.30	1.0	73.1	0.0	271.6
	74	3926.11	1929.0	3926.08	3926.11	0.00	34600	2	1057.80	0.9	73.1	0.0	271.2
	75	3155.77	172.6	3155.77	3155.77	0.00	2494	0	1019.36	0.9	67.7	0.0	209.6
6 x 5	76	3992.83	limit	3642.97	4017.88	9.33	73395	36020	978.77	2.2	75.5	0.6	310.5
	77	3317.44	limit	2991.66	3354.01	10.80	80221	41443	751.38	2.0	77.4	1.1	346.4
	78	4667.45	limit	3825.52	4792.45	20.18	75770	52946	1020.20	2.3	78.1	2.7	369.8
	79	4998.03	limit	4450.91	5061.07	12.06	83224	44344	1155.34	1.9	76.9	1.3	338.1
	80	3793.34	4996.5	3793.31	3793.34	0.00	67761	6	914.55	2.2	75.9	0.0	314.8
	81	5150.04	limit	4666.70	5156.68	9.50	75823	36045	1173.14	2.0	77.2	0.1	339.6
	82	4489.33	limit	4035.87	4500.74	10.33	79585	38757	1120.48	1.6	75.0	0.3	301.7
	83	4665.45	limit	4350.66	4702.02	7.47	96930	41229	1151.75	1.8	75.3	0.8	308.2
	84	4423.78	limit	3988.64	4502.35	11.41	118005	66277	1051.37	2.1	76.2	1.8	328.2
	85	3870.14	limit	3683.34	3910.79	5.82	93295	26280	856.46	1.5	77.9	1.1	356.6

Table H.2 (continued)

	inst#	Opt. sol.	MIP						LPR		IGap%	DU%	ULP%
			CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t			
7 x 3	1	2633.32	274.1	2633.32	2633.32	0.00	2837	0	967.57	1.5	63.3	0.0	172.2
	2	3210.22	480.5	3210.22	3210.22	0.00	6891	0	1172.35	1.3	63.5	0.0	173.8
	3	3012.42	670.6	3012.42	3012.42	0.00	12083	0	1077.56	1.4	64.2	0.0	179.6
	4	2705.11	1139.0	2705.11	2705.11	0.00	15138	0	995.35	1.5	63.2	0.0	171.8
	5	3040.29	383.7	3040.29	3040.29	0.00	4801	0	1146.76	1.6	62.3	0.0	165.1
	31	3221.19	316.1	3221.19	3221.19	0.00	3669	0	1267.59	1.8	60.6	0.0	154.1
	32	2809.13	247.0	2809.13	2809.13	0.00	2179	0	1089.74	1.7	61.2	0.0	157.8
	33	3288.23	466.4	3288.23	3288.23	0.00	7392	0	1149.20	2.0	65.1	0.0	186.1
	34	2929.66	265.2	2929.63	2929.66	0.00	3175	1	1150.34	1.8	60.7	0.0	154.7
	35	2242.64	415.9	2242.62	2242.64	0.00	5170	1	851.75	1.8	62.0	0.0	163.3
8 x 3	16	3640.63	4487.0	3640.61	3640.63	0.00	29741	1	1360.57	4.5	62.6	0.0	167.6
	17	4158.24	4179.0	4158.24	4158.24	0.00	36517	0	1428.61	4.9	65.6	0.0	191.1
	18	3592.83	2916.1	3592.83	3592.83	0.00	19067	0	1244.85	4.4	65.4	0.0	188.6
	19	2923.44	3450.7	2923.44	2923.44	0.00	24416	0	1087.04	3.6	62.8	0.0	168.9
	20	4210.04	limit	3777.66	4210.04	10.27	49095	23498	1499.18	4.7	64.4	0.0	180.8
	36	3218.06	3016.6	3218.04	3218.06	0.00	29013	2	1141.58	5.1	64.5	0.0	181.9
	37	3202.54	2745.1	3202.52	3202.54	0.00	18751	1	1148.32	3.7	64.1	0.0	178.9
	38	3755.65	1188.4	3755.65	3755.65	0.00	9227	0	1427.86	5.2	62.0	0.0	163.0
	39	3229.05	2812.1	3229.02	3229.05	0.00	18285	1	1167.43	4.2	63.8	0.0	176.6
	40	3273.34	3180.5	3273.33	3273.34	0.00	24771	1	1196.46	5.0	63.4	0.0	173.6
9 x 3	26	3733.70	limit	3140.68	3733.70	15.88	22695	15494	1369.51	9.7	63.3	0.0	172.6
	27	3354.74	limit	3091.46	3405.97	9.23	23708	11042	1261.67	8.6	62.4	1.5	170.0
	28	3549.33	limit	3386.75	3549.33	4.58	27064	5610	1215.41	8.7	65.8	0.0	192.0
	29	3645.96	limit	3200.12	3721.70	14.01	29987	17709	1311.70	8.8	64.0	2.1	183.7
	30	3947.71	limit	3285.09	4082.90	19.54	29978	20998	1432.31	10.0	63.7	3.4	185.1
	41	3430.02	limit	3160.68	3446.47	8.29	34879	13797	1248.60	9.1	63.6	0.5	176.0
	42	3910.75	limit	3219.77	4007.49	19.66	51146	36372	1395.75	7.2	64.3	2.5	187.1
	43	3690.26	limit	3321.33	3690.26	10.00	32556	13589	1329.30	10.1	64.0	0.0	177.6
	44	3049.08	6055.5	3049.07	3049.08	0.00	24020	1	1087.81	8.1	64.3	0.0	180.3
	45	3598.29	limit	3330.07	3598.29	7.45	37418	16071	1252.42	9.8	65.2	0.0	187.3

* LB, UB and Gap (%) refer to the final values of the lower bound, upper bound and the corresponding gap, respectively at the end of the time limit.

APPENDIX I

EXPERIMENTS ON HK INSTANCES WITH MBS = 0.25

Table I.1 Solutions of HK Instances with MBS = 0.25 *

	inst#	Best sol. at the end of time limit	Extended CPU	Final Gap% at the end of extended time	Best sol.
6 x 2	46	1977.72	-	-	1977.72
	47	1810.80	-	-	1810.80
	48	1662.24	-	-	1662.24
	49	1869.05	-	-	1869.05
	50	1815.52	-	-	1815.52
	51	2364.26	-	-	2364.26
	52	1600.74	-	-	1600.74
	53	1639.21	-	-	1639.21
	54	1611.00	-	-	1611.00
	55	2031.71	-	-	2031.71
6 x 3	56	2444.79	-	-	2444.79
	57	2822.08	-	-	2822.08
	58	2325.74	-	-	2325.74
	59	2421.72	-	-	2421.72
	60	2726.10	-	-	2726.10
	61	2948.37	-	-	2948.37
	62	2388.69	-	-	2388.69
	63	2765.01	-	-	2765.01
	64	2539.12	-	-	2539.12
	65	2454.52	-	-	2454.52
6 x 4	66	3254.18	-	-	3254.18
	67	3777.81	9108.0	0.00	3777.81
	68	3373.48	-	-	3373.48
	69	3794.04	8187.8	0.00	3794.04
	70	3543.03	-	-	3543.03
	71	4292.75	-	-	4292.75
	72	3865.97	-	-	3865.97
	73	4727.42	-	-	4727.42
	74	4218.58	-	-	4218.58
	75	3391.61	-	-	3391.61
6 x 5	76	5776.91	174389.6	25.48	5637.14
	77	5155.98	95620.4	23.94	4733.06
	78	6604.82	7200.0	43.10	6604.82
	79	7409.43	228218.9	20.57	6979.24
	80	6674.06	57354.4	36.20	6436.21
	81	7656.04	183566.2	26.71	6984.18
	82	6094.26	47682.0	31.14	6094.26
	83	7086.17	172692.1	24.98	6856.32
	84	6551.95	7200.0	44.19	6551.95
	85	5928.75	7200.0	42.91	5928.75

	inst#	Best sol. at the end of time limit	Extended CPU	Final Gap% at the end of extended time	Best sol.
7 x 3	1	2633.32	-	-	2633.32
	2	3210.22	-	-	3210.22
	3	3012.42	-	-	3012.42
	4	2705.11	-	-	2705.11
	5	3040.29	-	-	3040.29
	31	3221.19	-	-	3221.19
	32	2809.13	-	-	2809.13
	33	3288.23	-	-	3288.23
	34	2929.66	-	-	2929.66
	35	2242.64	-	-	2242.64
8 x 3	16	3640.63	-	-	3640.63
	17	4178.72	13416.0	0.00	4158.24
	18	3592.83	-	-	3592.83
	19	2923.44	-	-	2923.44
	20	4224.34	23155.6	0.00	4210.04
	36	3218.06	-	-	3218.06
	37	3202.54	-	-	3202.54
	38	3755.65	-	-	3755.65
	39	3229.05	-	-	3229.05
	40	3273.34	-	-	3273.34
9 x 3	26	3764.59	22710.2	0.00	3733.70
	27	3354.74	13087.2	0.00	3354.74
	28	3564.66	11813.2	0.00	3549.33
	29	3660.51	15838.9	0.00	3645.96
	30	4008.59	34499.6	0.00	3947.71
	41	3430.02	9822.2	0.00	3430.02
	42	3973.20	23257.0	0.00	3910.75
	43	3720.47	26583.1	0.00	3690.26
	44	3049.08	-	-	3049.08
	45	3650.37	18901.8	0.00	3598.29

* Shaded cells indicate solutions with non-zero solution gaps. Best sol. refers to the best known solution for the instance.

Table I.2 Detailed Results of Experiments on HK Instances with MBS = 0.25 *

			MIP						LPR				
	inst#	Opt. sol.	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	IGap%	DU%	ULP%
6 x 2	46	1977.72	5.8	1977.72	1977.72	0.00	86	0	1157.41	0.2	41.5	0.0	70.9
	47	1810.80	5.2	1810.80	1810.80	0.00	48	0	1059.91	0.3	41.5	0.0	70.8
	48	1662.24	5.1	1662.24	1662.24	0.00	103	0	955.01	0.4	42.5	0.0	74.1
	49	1869.05	6.0	1869.05	1869.05	0.00	92	0	1106.54	0.2	40.8	0.0	68.9
	50	1815.52	3.7	1815.52	1815.52	0.00	38	0	1044.41	0.3	42.5	0.0	73.8
	51	2364.26	4.7	2364.26	2364.26	0.00	57	0	1351.89	0.3	42.8	0.0	74.9
	52	1600.74	4.2	1600.74	1600.74	0.00	53	0	905.41	0.3	43.4	0.0	76.8
	53	1639.21	5.1	1639.21	1639.21	0.00	85	0	957.15	0.2	41.6	0.0	71.3
	54	1611.00	3.9	1611.00	1611.00	0.00	35	0	991.58	0.3	38.4	0.0	62.5
	55	2031.71	4.6	2031.71	2031.71	0.00	57	0	1295.26	0.3	36.2	0.0	56.9
6 x 3	56	2444.79	44.8	2444.79	2444.79	0.00	807	0	950.85	0.5	61.1	0.0	157.1
	57	2822.08	111.2	2822.08	2822.08	0.00	2942	0	1077.49	0.5	61.8	0.0	161.9
	58	2325.74	26.3	2325.74	2325.74	0.00	469	0	907.17	0.7	61.0	0.0	156.4
	59	2421.72	33.4	2421.72	2421.72	0.00	725	0	862.15	0.8	64.4	0.0	180.9
	60	2726.10	69.0	2726.10	2726.10	0.00	1498	0	1030.69	0.5	62.2	0.0	164.5
	61	2948.37	43.2	2948.37	2948.37	0.00	639	0	1202.51	0.6	59.2	0.0	145.2
	62	2388.69	27.6	2388.69	2388.69	0.00	497	0	955.18	0.6	60.0	0.0	150.1
	63	2765.01	49.0	2765.01	2765.01	0.00	964	0	1012.49	0.5	63.4	0.0	173.1
	64	2539.12	33.8	2539.12	2539.12	0.00	601	0	943.08	0.7	62.9	0.0	169.2
	65	2454.52	48.2	2454.52	2454.52	0.00	973	0	920.75	0.6	62.5	0.0	166.6
6 x 4	66	3254.18	1130.2	3254.17	3254.18	0.00	22671	1	881.33	1.5	72.9	0.0	269.2
	67	3777.81	limit	3580.60	3777.81	5.22	140747	36731	1085.40	1.4	71.3	0.0	248.1
	68	3373.48	2211.8	3373.48	3373.48	0.00	41036	0	812.15	1.3	75.9	0.0	315.4
	69	3794.04	limit	3659.56	3794.04	3.54	135895	20816	1016.51	1.3	73.2	0.0	273.2
	70	3543.03	741.8	3543.02	3543.03	0.00	10302	1	977.00	1.3	72.4	0.0	262.6
	71	4292.75	3981.4	4292.75	4292.75	0.00	89083	1	1075.57	1.5	74.9	0.0	299.1
	72	3865.97	1192.1	3865.95	3865.97	0.00	18964	3	997.59	1.6	74.2	0.0	287.5
	73	4727.42	2211.2	4727.38	4727.42	0.00	31800	3	1195.86	1.3	74.7	0.0	295.3
	74	4218.58	4065.7	4218.55	4218.58	0.00	68010	2	1128.86	1.3	73.2	0.0	273.7
	75	3391.61	675.6	3391.61	3391.61	0.00	11098	0	1023.07	1.7	69.8	0.0	231.5
6 x 5	76	5637.14	limit	3388.99	5776.91	41.34	61110	36440	1024.41	3.7	81.8	2.5	463.9
	77	4733.06	limit	3044.84	5155.98	40.95	67574	49180	803.56	3.4	83.0	8.9	541.6
	78	6604.82	limit	3758.09	6604.82	43.10	74700	53345	1144.13	3.0	82.7	0.0	477.3
	79	6979.24	limit	4555.49	7409.43	38.52	67823	43033	1233.71	3.4	82.3	6.2	500.6
	80	6436.21	limit	3570.99	6674.06	46.49	93667	71567	992.44	3.5	84.6	3.7	572.5
	81	6984.18	limit	4294.48	7656.04	43.91	80170	62881	1304.17	3.5	81.3	9.6	487.0
	82	6094.26	limit	3753.61	6094.26	38.41	82306	55730	1206.44	3.2	80.2	0.0	405.1
	83	6856.32	limit	4182.14	7086.17	40.98	75060	47498	1225.01	3.3	82.1	3.4	478.5
	84	6551.95	limit	3656.66	6551.95	44.19	83294	65495	1199.89	3.7	81.7	0.0	446.0
	85	5928.75	limit	3384.53	5928.75	42.91	75616	59246	942.81	3.6	84.1	0.0	528.8

Table I.2 (continued)

			MIP						LPR				
	inst#	Opt. sol.	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	LPR	LPR t	IGap%	DU%	ULP%
7 x 3	1	2633.32	389.1	2633.30	2633.32	0.00	3702	1	967.57	1.8	63.3	0.0	172.2
	2	3210.22	406.9	3210.22	3210.22	0.00	4554	0	1172.35	1.7	63.5	0.0	173.8
	3	3012.42	544.5	3012.42	3012.42	0.00	7530	0	1077.56	1.8	64.2	0.0	179.6
	4	2705.11	423.5	2705.11	2705.11	0.00	6138	0	995.35	1.7	63.2	0.0	171.8
	5	3040.29	414.9	3040.29	3040.29	0.00	4944	0	1146.76	1.7	62.3	0.0	165.1
	31	3221.19	310.7	3221.19	3221.19	0.00	3194	0	1267.59	2.1	60.6	0.0	154.1
	32	2809.13	478.1	2809.13	2809.13	0.00	4804	0	1089.74	2.0	61.2	0.0	157.8
	33	3288.23	555.2	3288.23	3288.23	0.00	7878	0	1149.20	1.8	65.1	0.0	186.1
	34	2929.66	235.7	2929.66	2929.66	0.00	2709	0	1150.34	1.7	60.7	0.0	154.7
	35	2242.64	227.5	2242.64	2242.64	0.00	2766	0	851.75	2.1	62.0	0.0	163.3
8 x 3	16	3640.63	3951.0	3640.61	3640.63	0.00	32760	2	1360.57	3.7	62.6	0.0	167.6
	17	4158.24	limit	3790.14	4178.72	9.30	53268	25157	1428.61	4.4	65.6	0.5	192.5
	18	3592.83	4190.9	3592.81	3592.83	0.00	29604	4	1244.85	5.7	65.4	0.0	188.6
	19	2923.44	2334.9	2923.42	2923.44	0.00	15404	1	1087.04	3.9	62.8	0.0	168.9
	20	4210.04	limit	3674.27	4224.34	13.02	55608	32041	1499.18	3.9	64.4	0.3	181.8
	36	3218.06	3608.4	3218.03	3218.06	0.00	32992	2	1141.58	3.4	64.5	0.0	181.9
	37	3202.54	2663.9	3202.53	3202.54	0.00	19519	2	1148.32	4.6	64.1	0.0	178.9
	38	3755.65	1460.3	3755.65	3755.65	0.00	14847	0	1427.86	4.0	62.0	0.0	163.0
	39	3229.05	2635.6	3229.03	3229.05	0.00	27926	2	1167.43	4.4	63.8	0.0	176.6
	40	3273.34	3774.9	3273.31	3273.34	0.00	31576	2	1196.46	4.0	63.4	0.0	173.6
9 x 3	26	3733.70	limit	3270.65	3764.59	13.12	28406	16633	1369.51	10.0	63.3	0.8	174.9
	27	3354.74	limit	3012.05	3354.74	10.22	27981	12274	1261.67	8.5	62.4	0.0	165.9
	28	3549.33	limit	3209.55	3564.66	9.96	34904	15433	1215.41	8.8	65.8	0.4	193.3
	29	3645.96	limit	3251.42	3660.51	11.18	30832	14684	1311.70	9.7	64.0	0.4	179.1
	30	3947.71	limit	3407.06	4008.59	15.01	31179	18440	1432.31	9.2	63.7	1.5	179.9
	41	3430.02	limit	3227.26	3430.02	5.91	30612	10825	1248.60	7.3	63.6	0.0	174.7
	42	3910.75	limit	3377.56	3973.20	14.99	36322	21069	1395.75	9.4	64.3	1.6	184.7
	43	3690.26	limit	3171.79	3720.47	14.75	37122	23810	1329.30	9.3	64.0	0.8	179.9
	44	3049.08	2702.3	3049.08	3049.08	0.00	14135	0	1087.81	8.7	64.3	0.0	180.3
	45	3598.29	limit	3165.43	3650.37	13.28	45096	26400	1252.42	10.7	65.2	1.4	191.5

* If the optimal solution is not known for the instance, the best known solution is given instead in underlined form.
LB, UB and Gap (%) refer to the final values of the lower bound, upper bound and the corresponding gap, respectively at the end of the time limit.

APPENDIX J

STATISTICAL SIGNIFICANCE TESTS FOR THE EFFECT OF *N* AND *K* ON CPU

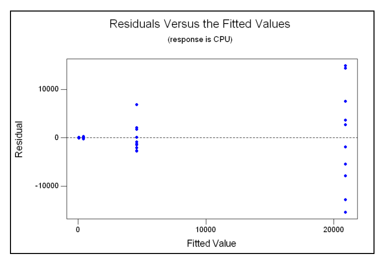
Below we present the results of one-way ANOVA tests for testing the effect of the number of items (N) and the number of time periods (K) on the MIP solution time (CPU) of the experiment instances (HK) under MBS=0.1 setting. The residual plots are also provided alongside ANOVA outcomes. Since the data classes do not show the same amount of variability, we have also performed the analysis on transformed data. The analysis of transformed data shows that both K and N are significant for CPU at 0.01 confidence level. (Note: $F_{0.01}(3,36) = 4.39$)

Effect of the Number of Items (*N*) on Solution Time (CPU)

One-way ANOVA: CPU versus N

```
Analysis of Variance for CPU
Source     DF        SS        MS        F        P
N           3 2.899E+09 966254644    32.28    0.000
Error      36 1.077E+09  29930119
Total      39 3.976E+09
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  -----+---------+---------+---------+-
6          10        56        38   (---*---)
7          10       404       166   (----*---)
8          10      4578      2934        (----*---)
9          10     20905     10540                           (---*----)
                                   -----+---------+---------+---------+-
Pooled StDev =     5471                 0      8000     16000     24000
```

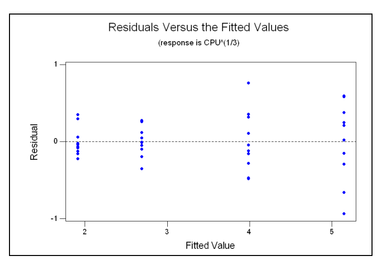
Residuals vs Fits for CPU



One-way ANOVA: CPU^(1/3) versus N

```
Analysis of Variance for CPU^(1/3
Source     DF        SS        MS        F        P
N           3    61.033    20.344   166.33    0.000
Error      36     4.403     0.122
Total      39    65.436
                                  Individual 95% CIs For Mean
                                  Based on Pooled StDev
Level       N      Mean     StDev  ----+---------+---------+---------+--
6          10    1.9115    0.1872   (-*-)
7          10    2.6896    0.1920           (-*-)
8          10    3.9850    0.3932                        (-*-)
9          10    5.1450    0.5125                                   (-*--)
                                  ----+---------+---------+---------+--
Pooled StDev =   0.3497               2.0       3.0       4.0       5.0
```

Residuals vs Fits for CPU^(1/3)

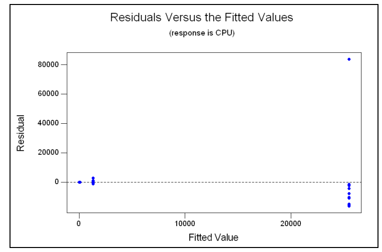


Effect of the Number of Time Periods (*K*) on Solution Time (CPU)

One-way ANOVA: CPU versus K

```
Analysis of Variance for CPU
Source     DF        SS          MS        F        P
K           3  4.708E+09   1.569E+09    7.02    0.001
Error      36  8.042E+09   223394835
Total      39  1.275E+10
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  -------+---------+---------+--------
2          10         5         1   (-----*-----)
3          10        56        38   (-----*-----)
4          10      1302      1296   (------*-----)
5          10     25479     29865                    (-----*-----)
                                   -------+---------+---------+--------
Pooled StDev =    14946                   0     15000     30000
```

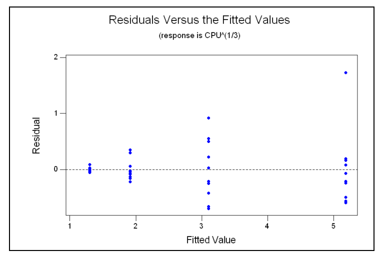
Residuals vs Fits for CPU



One-way ANOVA: CPU^(1/3) versus K

```
Analysis of Variance for CPU^(1/3
Source     DF        SS        MS        F        P
K           3    87.865    29.288   148.75    0.000
Error      36     7.088     0.197
Total      39    94.954
                                   Individual 95% CIs For Mean
                                   Based on Pooled StDev
Level       N      Mean     StDev  ----+---------+---------+---------+--
2          10    1.2973    0.0389   (-*-)
3          10    1.9115    0.1872       (-*-)
4          10    3.1008    0.5453               (-*-)
5          10    5.1811    0.6736                             (-*)
                                   ----+---------+---------+---------+--
Pooled StDev =   0.4437               1.5       3.0       4.5       6.0
```

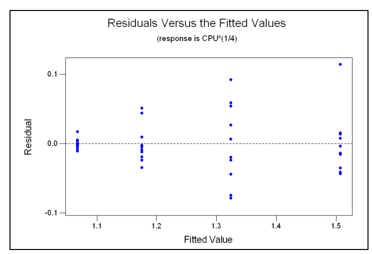
Residuals vs Fits for CPU^(1/3)



One-way ANOVA: CPU^(1/4) versus K

```
Analysis of Variance for CPU^(1/4
Source     DF        SS        MS        F        P
K           3   1.09105   0.36368   230.16    0.000
Error      36   0.05689   0.00158
Total      39   1.14793
                                  Individual 95% CIs For Mean
                                  Based on Pooled StDev
Level       N      Mean     StDev  -+---------+---------+---------+-----
2          10    1.0671    0.0079  (-*-)
3          10    1.1749    0.0280          (*-)
4          10    1.3235    0.0581                    (*-)
5          10    1.5068    0.0458                               (*-)
                                  -+---------+---------+---------+-----
Pooled StDev =   0.0398         1.05      1.20      1.35      1.50
```

Residuals vs Fits for CPU^(1/4)



APPENDIX K

k-NEAREST EXPERIMENTS ON HK INSTANCES WITH MBS = 0.1

Table K.1 Solutions of HK Instances for k-Nearest 80% *

	inst#	Best sol. at the end of time limit	Extended CPU	Best sol.
6 x 2	46	1977.72	-	1977.72
	47	1810.80	-	1810.80
	48	1662.24	-	1662.24
	49	1869.05	-	1869.05
	50	1815.52	-	1815.52
	51	2364.26	-	2364.26
	52	1600.74	-	1600.74
	53	1639.21	-	1639.21
	54	1611.00	-	1611.00
	55	2031.71	-	2031.71
6 x 3	56	2444.79	-	2444.79
	57	2822.08	-	2822.08
	58	2325.74	-	2325.74
	59	2421.72	-	2421.72
	60	2760.85	-	2760.85
	61	2948.37	-	2948.37
	62	2388.69	-	2388.69
	63	2765.01	-	2765.01
	64	2539.12	-	2539.12
	65	2454.52	-	2454.52
6 x 4	66	3045.04	-	3045.04
	67	3639.85	-	3639.85
	68	3075.65	-	3075.65
	69	3476.68	-	3476.68
	70	3401.50	-	3401.50
	71	3964.52	-	3964.52
	72	3561.53	-	3561.53
	73	4431.01	-	4431.01
	74	3926.11	-	3926.11
	75	3155.77	-	3155.77
6 x 5	76	4017.88	9421.3	3992.83
	77	3317.44	9093.9	3317.44
	78	4762.10	48853.8	4667.45
	79	5014.53	10075.0	4998.03
	80	3793.34	-	3793.34
	81	5222.06	38387.8	5150.04
	82	4489.33	12260.4	4489.33
	83	4702.02	12214.8	4665.45
	84	4423.78	8237.6	4423.78
	85	3870.14	-	3870.14

	inst#	Best sol. at the end of time limit	Extended CPU	Best sol.
7 x 3	1	2633.32	-	2633.32
	2	3210.22	-	3210.22
	3	3012.42	-	3012.42
	4	2705.11	-	2705.11
	5	3040.29	-	3040.29
	31	3221.19	-	3221.19
	32	2809.13	-	2809.13
	33	3288.23	-	3288.23
	34	2929.66	-	2929.66
	35	2242.64	-	2242.64
8 x 3	16	3640.63	-	3640.63
	17	4158.24	-	4158.24
	18	3592.83	-	3592.83
	19	2923.44	-	2923.44
	20	4224.16	8580.5	4210.04
	36	3218.06	-	3218.06
	37	3202.54	-	3202.54
	38	3755.65	-	3755.65
	39	3229.05	-	3229.05
	40	3273.34	-	3273.34
9 x 3	26	3749.13	50175.7	3733.70
	27	3354.74	8085.3	3354.74
	28	3549.33	-	3549.33
	29	3663.56	14845.8	3645.96
	30	3999.82	22462.5	3947.71
	41	3504.01	15505.1	3430.02
	42	3910.75	12169.7	3910.75
	43	3842.22	16861.0	3690.26
	44	3049.08	-	3049.08
	45	3604.26	20310.1	3598.29

* Shaded cells indicate solutions with non-zero solution gaps. Best sol. refers to the best known solution for the instance.

Table K.2 Solutions of HK Instances for k-Nearest 60% *

	inst#	Best sol. at the end of time limit	Extended CPU	Best sol.
6 x 2	46	1977.72	-	1977.72
	47	1810.80	-	1810.80
	48	1705.29	-	1705.29
	49	1869.05	-	1869.05
	50	1815.52	-	1815.52
	51	2364.26	-	2364.26
	52	1600.74	-	1600.74
	53	1701.60	-	1701.60
	54	1611.00	-	1611.00
	55	2202.58	-	2202.58
6 x 3	56	2511.47	-	2511.47
	57	2833.96	-	2833.96
	58	2325.74	-	2325.74
	59	2421.72	-	2421.72
	60	2760.85	-	2760.85
	61	2948.37	-	2948.37
	62	2388.69	-	2388.69
	63	2765.01	-	2765.01
	64	2539.12	-	2539.12
	65	2489.87	-	2489.87
6 x 4	66	3045.04	-	3045.04
	67	3639.85	-	3639.85
	68	3075.65	-	3075.65
	69	3476.68	-	3476.68
	70	3401.50	-	3401.50
	71	4045.76	-	4045.76
	72	3561.53	-	3561.53
	73	4431.01	-	4431.01
	74	3926.11	-	3926.11
	75	3155.77	-	3155.77
6 x 5	76	3992.83	-	3992.83
	77	3390.58	8797.5	3390.58
	78	4810.48	10826.7	4782.12
	79	4998.03	7937.8	4998.03
	80	3793.34	9196.2	3793.34
	81	5156.68	10683.8	5150.04
	82	4489.33	-	4489.33
	83	4665.45	7665.8	4665.45
	84	4588.87	13522.5	4423.78
	85	4013.97	9851.3	3945.26

	inst#	Best sol. at the end of time limit	Extended CPU	Best sol.
7 x 3	1	2633.32	-	2633.32
	2	3210.22	-	3210.22
	3	3012.42	-	3012.42
	4	2705.11	-	2705.11
	5	3040.29	-	3040.29
	31	3221.19	-	3221.19
	32	2809.13	-	2809.13
	33	3288.23	-	3288.23
	34	2929.66	-	2929.66
	35	2242.64	-	2242.64
8 x 3	16	3640.63	-	3640.63
	17	4158.24	-	4158.24
	18	3592.83	-	3592.83
	19	2923.44	-	2923.44
	20	4275.28	-	4275.28
	36	3218.06	-	3218.06
	37	3202.54	-	3202.54
	38	3755.65	-	3755.65
	39	3229.05	-	3229.05
	40	3273.34	-	3273.34
9 x 3	26	3747.57	14780.3	3733.70
	27	3354.74	8677.5	3354.74
	28	3549.33	8757.3	3549.33
	29	3722.20	14860.2	3645.96
	30	4037.01	43559.7	4010.12
	41	3446.47	13570.9	3430.02
	42	3925.68	19090.3	3910.75
	43	3690.26	-	3690.26
	44	3049.08	-	3049.08
	45	3688.94	10855.3	3598.29

* Shaded cells indicate solutions with non-zero solution gaps. Best sol. refers to the best known solution for the instance.

Table K.3 Solutions of HK Instances for k-Nearest 40% *

	inst#	Best sol. at the end of time limit	Extended CPU	Best sol.
6 x 2	46	2316.35	-	2316.35
	47	1810.80	-	1810.80
	48	1885.09	-	1885.09
	49	2036.99	-	2036.99
	50	1926.94	-	1926.94
	51	2364.26	-	2364.26
	52	1600.74	-	1600.74
	53	1701.60	-	1701.60
	54	1611.00	-	1611.00
	55	2247.74	-	2247.74
6 x 3	56	2539.65	-	2539.65
	57	2915.93	-	2915.93
	58	2325.74	-	2325.74
	59	2724.46	-	2724.46
	60	2899.27	-	2899.27
	61	2948.37	-	2948.37
	62	2388.69	-	2388.69
	63	2765.01	-	2765.01
	64	2539.12	-	2539.12
	65	2497.49	-	2497.49
6 x 4	66	3391.78	-	3391.78
	67	3668.09	-	3668.09
	68	3297.11	-	3297.11
	69	3824.79	-	3824.79
	70	3401.50	-	3401.50
	71	4045.76	-	4045.76
	72	3561.53	-	3561.53
	73	4611.30	-	4611.30
	74	3926.11	-	3926.11
	75	3749.06	-	3749.06
6 x 5	76	4282.70	-	4282.70
	77	3434.96	-	3434.96
	78	4967.64	-	4967.64
	79	5119.16	-	5119.16
	80	3793.34	-	3793.34
	81	5273.43	-	5273.43
	82	4489.33	-	4489.33
	83	4885.54	-	4885.54
	84	4423.78	-	4423.78
	85	4211.81	-	4211.81
7 x 3	1	2741.28	-	2741.28
	2	3349.07	-	3349.07
	3	3034.00	-	3034.00
	4	2944.37	-	2944.37
	5	3226.52	-	3226.52
	31	3333.78	-	3333.78
	32	2809.13	-	2809.13
	33	3435.41	-	3435.41
	34	2929.66	-	2929.66
	35	2242.64	-	2242.64
8 x 3	16	3640.63	-	3640.63
	17	4158.24	-	4158.24
	18	3838.10	-	3838.10
	19	3011.86	-	3011.86
	20	4543.96	-	4543.96
	36	3218.06	-	3218.06
	37	3202.54	-	3202.54
	38	3755.65	-	3755.65
	39	3229.05	-	3229.05
	40	3273.34	-	3273.34
9 x 3	26	3733.70	-	3733.70
	27	3354.74	-	3354.74
	28	3549.33	-	3549.33
	29	3645.96	-	3645.96
	30	4010.12	-	4010.12
	41	3430.02	-	3430.02
	42	3910.75	-	3910.75
	43	3712.33	-	3712.33
	44	3049.08	-	3049.08
	45	3598.29	-	3598.29

* Best sol. refers to the best known solution for the instance. Since all instances were solved with the time limit, the entries in the first and third columns are all equal for this setting.

Table K.4 Detailed Results for k-Nearest Experiments on HK Instances *

		k-Nearest 80%							k-Nearest 60%							k-Nearest 40%						
	inst#	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%
6 x 2	46	4.0	1977.72	1977.72	0.0	78	0	0.0	3.6	1977.72	1977.72	0.00	67	0	0.0	1.0	2316.35	2316.35	0.00	24	0	17.1
	47	3.6	1810.80	1810.80	0.0	38	0	0.0	3.5	1810.80	1810.80	0.00	40	0	0.0	3.1	1810.80	1810.80	0.00	48	0	0.0
	48	4.8	1662.24	1662.24	0.0	83	0	0.0	4.2	1705.29	1705.29	0.00	105	0	2.6	3.3	1885.09	1885.09	0.00	46	0	13.4
	49	5.2	1869.05	1869.05	0.0	95	0	0.0	4.0	1869.05	1869.05	0.00	73	0	0.0	3.0	2036.99	2036.99	0.00	57	0	9.0
	50	4.1	1815.52	1815.52	0.0	60	0	0.0	3.5	1815.52	1815.52	0.00	55	0	0.0	2.8	1926.94	1926.94	0.00	17	0	6.1
	51	4.1	2364.26	2364.26	0.0	39	0	0.0	4.2	2364.26	2364.26	0.00	69	0	0.0	2.0	2364.26	2364.26	0.00	27	0	0.0
	52	4.7	1600.74	1600.74	0.0	91	0	0.0	4.1	1600.74	1600.74	0.00	57	0	0.0	2.5	1600.74	1600.74	0.00	35	0	0.0
	53	4.6	1639.21	1639.21	0.0	79	0	0.0	3.3	1701.60	1701.60	0.00	75	0	3.8	2.3	1701.60	1701.60	0.00	38	0	3.8
	54	4.1	1611.00	1611.00	0.0	63	0	0.0	3.1	1611.00	1611.00	0.00	40	0	0.0	3.5	1611.00	1611.00	0.00	32	0	0.0
	55	3.1	2031.71	2031.71	0.0	33	0	0.0	3.7	2202.58	2202.58	0.00	61	0	8.4	2.3	2247.74	2247.74	0.00	30	0	10.6
6 x 3	56	38.6	2444.79	2444.79	0.0	709	0	0.0	37.3	2511.47	2511.47	0.00	707	0	2.7	12.8	2539.65	2539.65	0.00	206	0	3.9
	57	80.8	2822.08	2822.08	0.0	1793	0	0.0	36.7	2833.96	2833.96	0.00	985	0	0.4	22.5	2915.93	2915.93	0.00	543	0	3.3
	58	23.7	2325.74	2325.74	0.0	345	0	0.0	18.9	2325.74	2325.74	0.00	347	0	0.0	11.2	2325.74	2325.74	0.00	130	0	0.0
	59	41.6	2421.72	2421.72	0.0	1045	0	0.0	35.1	2421.72	2421.72	0.00	754	0	0.0	14.3	2724.46	2724.46	0.00	343	0	12.5
	60	64.9	2760.85	2760.85	0.0	1368	0	1.3	55.8	2760.85	2760.85	0.00	1339	0	1.3	24.5	2899.27	2899.27	0.00	721	0	6.4
	61	33.6	2948.37	2948.37	0.0	632	0	0.0	36.2	2948.37	2948.37	0.00	751	0	0.0	12.6	2948.37	2948.37	0.00	274	0	0.0
	62	17.9	2388.69	2388.69	0.0	321	0	0.0	21.0	2388.69	2388.69	0.00	396	0	0.0	12.8	2388.69	2388.69	0.00	167	0	0.0
	63	34.3	2765.01	2765.01	0.0	601	0	0.0	30.6	2765.01	2765.01	0.00	772	0	0.0	26.9	2765.01	2765.01	0.00	643	0	0.0
	64	22.8	2539.12	2539.12	0.0	403	0	0.0	28.0	2539.12	2539.12	0.00	620	0	0.0	10.2	2539.12	2539.12	0.00	222	0	0.0
	65	46.4	2454.52	2454.52	0.0	1112	0	0.0	33.4	2489.87	2489.87	0.00	811	0	1.4	15.9	2497.49	2497.49	0.00	378	0	1.8
6 x 4	66	212.7	3045.04	3045.04	0.0	3187	0	0.0	220.4	3045.04	3045.04	0.00	5097	0	0.0	62.4	3391.78	3391.78	0.00	1989	0	11.4
	67	1698.4	3639.85	3639.85	0.0	43782	2	0.0	866.2	3639.83	3639.85	0.00	18951	1	0.0	128.6	3668.09	3668.09	0.00	2766	0	0.8
	68	492.4	3075.65	3075.65	0.0	8285	0	0.0	408.7	3075.62	3075.65	0.00	9259	1	0.0	178.2	3297.11	3297.11	0.00	3488	0	7.2
	69	2082.6	3476.65	3476.68	0.0	40967	4	0.0	1082.1	3476.68	3476.68	0.00	17745	1	0.0	84.4	3824.79	3824.79	0.00	1718	0	10.0
	70	451.7	3401.50	3401.50	0.0	8080	0	0.0	163.2	3401.50	3401.50	0.00	3815	0	0.0	79.7	3401.50	3401.50	0.00	1939	0	0.0
	71	703.5	3964.52	3964.52	0.0	17711	0	0.0	97.7	4045.76	4045.76	0.00	2918	0	2.0	106.8	4045.76	4045.76	0.00	2699	0	2.0
	72	276.7	3561.53	3561.53	0.0	4705	0	0.0	370.2	3561.50	3561.53	0.00	6194	1	0.0	80.0	3561.53	3561.53	0.00	2077	0	0.0
	73	840.7	4430.98	4431.01	0.0	16673	1	0.0	657.4	4431.01	4431.01	0.00	11685	0	0.0	133.7	4611.30	4611.30	0.00	2615	0	4.1
	74	864.3	3926.11	3926.11	0.0	17934	0	0.0	561.4	3926.11	3926.11	0.00	13306	0	0.0	301.1	3926.11	3926.11	0.00	6092	0	0.0
	75	108.3	3155.75	3155.77	0.0	1868	1	0.0	107.6	3155.77	3155.77	0.00	1659	0	0.0	212.3	3749.06	3749.06	0.00	5163	0	18.8

Table K.4 (continued)

		k-Nearest 80%							k-Nearest 60%							k-Nearest 40%						
	inst#	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%
6 x 5	76	limit	3814.44	4017.88	5.1	82522	23227	0.6	5573.0	3992.81	3992.83	0.00	77526	2	0.0	247.1	4282.70	4282.70	0.00	5913	0	7.3
	77	limit	3163.82	3317.44	4.6	96012	24512	0.0	limit	3207.03	3390.58	5.41	111143	28948	2.2	885.0	3434.96	3434.96	0.00	23291	1	3.5
	78	limit	3860.24	4762.10	18.9	118150	84517	2.0	limit	4540.20	4810.48	5.62	131999	49138	3.1	277.7	4967.64	4967.64	0.00	6945	1	6.4
	79	limit	4706.06	5014.53	6.2	73861	25249	0.3	limit	4809.38	4998.03	3.77	95715	17672	0.0	1799.6	5119.16	5119.16	0.00	21896	0	2.4
	80	5245.3	3793.34	3793.34	0.0	62987	2	0.0	limit	3545.63	3793.34	6.53	96502	28699	0.0	997.9	3793.34	3793.34	0.00	14973	0	0.0
	81	limit	4350.20	5222.06	16.7	84321	54139	1.4	limit	4807.91	5156.68	6.76	148297	52335	0.1	764.5	5273.43	5273.43	0.00	14428	0	2.4
	82	limit	4118.39	4489.33	8.3	94142	40353	0.0	1224.6	4489.32	4489.33	0.00	26347	2	0.0	518.4	4489.33	4489.33	0.00	9116	0	0.0
	83	limit	4272.75	4702.02	9.1	93939	38000	0.8	limit	4539.37	4665.45	2.70	130506	17654	0.0	493.0	4885.54	4885.54	0.00	9645	0	4.7
	84	limit	4249.40	4423.78	3.9	95334	21030	0.0	limit	4045.65	4588.87	11.84	104626	55971	3.7	954.7	4423.78	4423.78	0.00	15983	0	0.0
	85	7142.8	3870.11	3870.14	0.0	78587	5	0.0	limit	3691.44	4013.97	8.04	88312	35621	3.7	100.4	4211.81	4211.81	0.00	2077	0	8.8
7 x 3	1	288.3	2633.32	2633.32	0.0	3883	0	0.0	273.7	2633.32	2633.32	0.00	2740	0	0.0	52.5	2741.28	2741.28	0.00	717	0	4.1
	2	583.7	3210.19	3210.22	0.0	8002	1	0.0	380.0	3210.22	3210.22	0.00	6719	0	0.0	111.4	3349.07	3349.07	0.00	1549	0	4.3
	3	347.3	3012.42	3012.42	0.0	4845	0	0.0	289.8	3012.42	3012.42	0.00	4051	0	0.0	46.1	3034.00	3034.00	0.00	687	0	0.7
	4	367.6	2705.09	2705.11	0.0	5171	1	0.0	406.0	2705.11	2705.11	0.00	3578	0	0.0	45.5	2944.37	2944.37	0.00	846	0	8.8
	5	689.6	3040.29	3040.29	0.0	8057	0	0.0	321.0	3040.29	3040.29	0.00	3745	0	0.0	97.0	3226.52	3226.52	0.00	1497	0	6.1
	31	330.7	3221.19	3221.19	0.0	4110	0	0.0	200.2	3221.19	3221.19	0.00	2856	0	0.0	88.4	3333.78	3333.78	0.00	939	0	3.5
	32	422.9	2809.13	2809.13	0.0	5589	0	0.0	338.4	2809.13	2809.13	0.00	3615	0	0.0	34.4	2809.13	2809.13	0.00	411	0	0.0
	33	1170.1	3288.21	3288.23	0.0	16402	1	0.0	300.0	3288.23	3288.23	0.00	4341	0	0.0	118.7	3435.41	3435.41	0.00	1339	0	4.5
	34	147.3	2929.65	2929.66	0.0	1570	1	0.0	152.6	2929.66	2929.66	0.00	1885	0	0.0	55.4	2929.66	2929.66	0.00	953	0	0.0
	35	235.0	2242.64	2242.64	0.0	2891	0	0.0	176.6	2242.64	2242.64	0.00	2534	0	0.0	51.7	2242.64	2242.64	0.00	690	0	0.0
8 x 3	16	2449.5	3640.63	3640.63	0.0	29199	0	0.0	2161.9	3640.61	3640.63	0.00	24216	1	0.0	1214.7	3640.63	3640.63	0.00	13708	0	0.0
	17	6559.4	4158.22	4158.24	0.0	65933	2	0.0	6613.8	4158.23	4158.24	0.00	64537	1	0.0	1706.8	4158.24	4158.24	0.00	14517	0	0.0
	18	2658.8	3592.83	3592.83	0.0	26369	0	0.0	1077.0	3592.83	3592.83	0.00	10519	1	0.0	287.6	3838.10	3838.10	0.00	3525	0	6.8
	19	1752.4	2923.44	2923.44	0.0	12562	1	0.0	1170.1	2923.44	2923.44	0.00	10055	0	0.0	1035.8	3011.85	3011.86	0.00	7610	1	3.0
	20	limit	4027.91	4224.16	4.7	54183	14373	0.3	2817.5	4275.26	4275.28	0.00	20988	1	1.5	362.1	4543.96	4543.96	0.00	4476	0	7.9
	36	2441.6	3218.06	3218.06	0.0	27232	0	0.0	1997.3	3218.03	3218.06	0.00	19549	1	0.0	862.3	3218.06	3218.06	0.00	10941	0	0.0
	37	2548.3	3202.52	3202.54	0.0	21648	1	0.0	1332.2	3202.54	3202.54	0.00	11102	0	0.0	744.7	3202.54	3202.54	0.00	8056	0	0.0
	38	1462.4	3755.65	3755.65	0.0	14040	0	0.0	823.9	3755.65	3755.65	0.00	5881	0	0.0	687.8	3755.65	3755.65	0.00	7504	0	0.0
	39	1749.8	3229.05	3229.05	0.0	16448	0	0.0	2435.8	3229.05	3229.05	0.00	21487	0	0.0	1425.6	3229.05	3229.05	0.00	11650	0	0.0
	40	2104.4	3273.32	3273.34	0.0	19667	1	0.0	3103.5	3273.34	3273.34	0.00	24751	0	0.0	1410.0	3273.34	3273.34	0.00	12451	0	0.0

Table K.4 (continued)

		k-Nearest 80%							k-Nearest 60%							k-Nearest 40%						
	inst#	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%	CPU	LB	UB	Gap (%)	Total Nodes	Rem. Nodes	DU%
9 x 3	26	limit	3123.74	3749.13	16.7	42679	26579	0.4	limit	3255.12	3747.57	13.14	36522	17826	0.4	3114.9	3733.70	3733.70	0.00	15416	0	0.0
	27	limit	3231.60	3354.74	3.7	32789	6827	0.0	limit	3156.32	3354.74	5.91	35836	9308	0.0	3470.3	3354.73	3354.74	0.00	15045	1	0.0
	28	6228.5	3549.31	3549.33	0.0	37494	1	0.0	limit	3337.45	3549.33	5.97	32651	8443	0.0	1868.1	3549.33	3549.33	0.00	7230	0	0.0
	29	limit	3223.22	3663.56	12.0	27679	13477	0.5	limit	3233.16	3722.20	13.14	42229	21218	2.1	3503.9	3645.95	3645.96	0.00	22682	1	0.0
	30	limit	3414.86	3999.82	14.6	41325	22619	1.3	limit	3396.83	4037.01	15.86	49200	31612	2.3	1754.8	4010.12	4010.12	0.00	9321	0	1.6
	41	limit	3023.15	3504.01	13.7	37174	20631	2.2	limit	3067.13	3446.47	11.01	48849	23127	0.5	2126.2	3430.01	3430.02	0.00	14046	1	0.0
	42	limit	3589.89	3910.75	8.2	30445	13478	0.0	limit	3454.60	3925.68	12.00	35051	18797	0.4	1526.7	3910.74	3910.75	0.00	12685	1	0.0
	43	limit	3238.11	3842.22	15.7	31970	18982	4.1	4849.9	3690.25	3690.26	0.00	35458	2	0.0	2977.6	3712.33	3712.33	0.00	20127	0	0.6
	44	limit	3049.07	3049.08	0.0	14998	1	0.0	4871.8	3049.07	3049.08	0.00	25208	1	0.0	1486.4	3049.08	3049.08	0.00	9345	0	0.0
	45	limit	3155.59	3604.26	12.5	40692	21417	0.2	limit	3301.35	3688.94	10.51	36882	16835	2.5	4461.3	3598.26	3598.29	0.00	27549	2	0.0

* LB, UB and Gap (%) refer to the final values of the lower bound, upper bound and the corresponding solution gap, respectively at the end of the time limit.
DU% is computed according to the formula (UB-opt)/opt x 100, where opt refers to the optimal solution value of the instance without any item changeover restrictions.

APPENDIX L

THE TWO-STEP HEURISTIC APPROACH (TSH)

As it is, the integrated lot sizing and scheduling problem with sequence dependent changeovers is difficult, because several decisions which are linked together need to be made simultaneously. The solution of this model can be approximated by a two-step heuristic (TSH) which decomposes the problem into lot sizing (LSM) and sequencing (SM) steps and solves them in an iterative manner (Koçlar and Süral, 2004).

Description of the Approach

In this section, we base our discussion of the TSH on a simple version of the lot sizing and sequencing problem, which assumes that an item can be produced at most once in a period and minimum batch sizes cannot be split between periods. The production, setup and changeover variables are defined over time periods (not positions), as follows:

X_{jt}: Quantity of item j produced in period t

Y_{jt}: $\begin{cases} 1, \text{ If item } j \text{ is set up in period } t \\ 0, \text{ Otherwise} \end{cases}$ *(Setup Variable)*

Z_{ijt} : $\begin{cases} 1, \text{ If there is a changeover from item } i \text{ to item } j \text{ in period } t \\ 0, \text{ Otherwise} \end{cases}$ *(Changeover Variable)*

The remaining decision variables and problem parameters are used exactly as in the original GLSP formulation.

On the next page, we present a non-linear formulation of the problem, which has a nested structure. The link between the lot sizing and sequencing steps is established through setup costs and setup times, where:

$f(Y_t)$= cost of the optimal TSP tour among the items assigned to period t

$g(t) = g(Y_t)$ = setup time required by the optimal TSP tour in period t

$$\text{Minimize } \sum_{i,t} h_i I_{it} + \sum_t CO_t O_t + \sum_{i,t} CP_i X_{it} + \sum_t f(Y_t)$$

Subject to

$$I_{i(t-1)} + X_{it} - I_{it} = d_{it} \qquad \forall i,t$$

$$X_{it} \le M_{it} Y_{it} \qquad \forall i,t$$

$$\sum_i P_i X_{it} \le C_i + O_t - g(Y_t) \qquad \forall t$$

$$X_{it} \ge m_i Y_{it} \qquad \forall i,t$$

$$O_t \le \gamma_t C_t \qquad \forall t$$

$$Y_{it} \in (0,1) \qquad \forall i,t$$

All other variables are non-negative.

For each time period *t*, the function $f(Y_t)$ can be defined mathematically by:

$$f(Y_t) = \text{Minimize} \quad \sum_{ij} SC_{ij} Z_{ijt}$$

$$\sum_{j=0|j\ne i}^{N} Z_{jit} = Y_{it} \qquad \forall i = 1,\ldots,N$$

$$\sum_{j=0|j\ne i}^{N} Z_{ijt} = Y_{it} \qquad \forall i = 1,\ldots,N$$

$$\sum_{i=0}^{N} Z_{0it} = 1$$

$$\sum_{i=0}^{N} Z_{i0t} = 1$$

$$\sum_{ij\in SxS}^{N} Z_{ijt} \le |S| - 1 \qquad S \subseteq N(Y_t), 2 \le |S| \le N$$

$$Z_{ijt} \in (0,1) \qquad \forall i,j$$

where $g(Y_t) = g(t) = \sum_{i,j} ST_{ij} Z_{ijt} + ST_{(last\ \text{item}_{t-1}, \text{first item}_t)}$

The TSH approach is approximation to this non-linear model. It considers the two levels in the nested framework separately as two linked models for lot sizing (LSM) and sequencing (SM), respectively.

For the solution, an iterative procedure is employed. The TSH procedure initially starts off with zero setup costs and times for all items and solves the lot sizing problem to meet demands subject to capacity limitations and minimum batch sizes. The LSM solves the lot sizing problem without considering setups. The corresponding *Y* variables are used as inputs in the second model (SM), which is solved for each time period to determine the optimal TSP tour among the items selected for production by the first model. Using lifted MTZ constraints for subtour elimination, the SM problem for each period is solved sequentially in order to be able to take into consideration the setups between periods. In doing so, node zero is assumed to represent the initial setup state in each period and the setup costs associated with variable Z_{0j} are calculated accordingly,

i.e., $ST_{0j} = ST_{(\text{first item in the period, } j)}$

In this way, the sequencing decision will be made given the initial setup state each period as opposed to the option of solving each period independently. Then, the setup cost and time information obtained by the solution of the SM for each period is fed back to the LSM. The actual cost of the solution at each iteration is equal to the objective function of the LSM and all SM's added together.

In the subsequent steps, the setup time required by the lot sizing solution is deducted from the available capacities in the LSM and the model is resolved. In these consecutive runs, the LSM faces reduced capacity and it may be obliged to shift the production of some items to other periods with some inventory carrying in order to offset the effect of setups, which is the basic tradeoff in a lot sizing model. Note that with the use of overtime option, we avoid the possibility of hitting a capacity infeasible solution for the LSM in any step.

The algorithm is designed to stop with a feasible solution whenever the capacity required by a solution (including production and setup requirements) does not exceed the remaining available capacity determined by the LSM. If the algorithm does not stop after a certain number of iterations, a feasible solution can easily be obtained by charging extra overtime penalty for each period with excess capacity requirements, as long as they do not violate overtime limits. To be more precise;

$$Exceed_t\text{:}\ \underbrace{\sum_i P_i X_{it} - O_t - C_t}_{\text{at the end of the LSM solution}} + g(t)$$

where $Exceed_t$ indicates the amount by which the total requirement of an iteration solution (with production and setup times) exceeds the actual period capacity plus the overtime determined at the LSM step. If this quantity is negative, it means that the LSM and SM decisions are compatible, i.e., the sequencing scheme does not require additional overtime, and the algorithm stops with a feasible solution. Otherwise, if $Exceed_t$ is positive, then extra overtime needs to be assigned with additional cost, provided that it is within limits. As long as $Exceed_t$ remains positive, the algorithm continues by changing the capacity levels in the LSM and recomputing sequences. However, at any intermediate stage with positive excess quantities, one may find a converted feasible solution with the following objective function value:

$$Obj.Func = Obj.Func_{iteration} + \sum_t \left(CO_t Exceed_t \right)$$

Rudimentary Experiments

A few rudimentary experiments were conducted to gain more insight about the working logic of the TSH and to evaluate its performance.

For these tests, we have mainly used the data from the study by Trigeiro *et al.* (1989) with some modifications and additions. Some of these are:

- Two data sets with size 6x7 and 6x15 were used. 20 instances were generated from each.
- The given sequence independent setup cost and time parameters were transformed into sequence dependent data as follows: For each item, a deviation quantity was generated within the range (0%,25%) of the mean using unbiased uniform random variates. Then, two realizations of data were obtained by increasing and decreasing the mean by the generated deviation quantity, respectively. The data generated in this way were randomly ordered among items to avoid bias.
- The overtime cost for all periods was taken to be the maximum setup cost among all items and the minimum batch size of each item was taken as its minimum non-zero demand through the planning horizon.

If we examine the typical behaviour of the heuristic solution, we observe that there are three solution cases:

- The procedure starts off with a very high solution value with excessive use of overtime, which is reduced iteratively. A feasible solution is obtained at the last iteration. (The most common case)
- TSH enters a loop (the solution value displays zigzagging behaviour), and the best converted feasible solution is one of the solutions inside the loop.
- TSH enters a loop and the best converted feasible solution is one of the solutions obtained prior to entering the loop.

Once in a loop, TSH continuously fluctuates between a set of previously obtained solutions. Therefore, in our experiments we have a set a limit of 10 iterations in order to stop continuous loops. In the experiments, we compared the best TSH solutions with the best upper bound obtained by the simplified GLSP within a 0.5 hour time limit (UB). The original data set as well as a restricted version (with a 10% reduction in period capacities) was used for the tests. The results are shown in Table L.1 below.

Table L.1 TSH Results under Original and Reduced Capacities *

		Original			10% Red. Cap.		
Inst		Gap	Time (sec)	Iter#	Gap	Time (sec)	Iter#
6x7	1	13.0	0.1	2	16.0	0.1	2
	2	-10.4	0.3	5	120.0	0.2	3
	3	64.2	0.1	2	89.6	0.1	2
	4	-1.4	0.1	2	741.3	1.1	15
	5	49.7	0.1	2	116.7	0.1	2
	6	7.0	0.1	2	1021.3	1.1	15
	7	21.3	0.1	2	14.9	0.3	4
	8	40.3	0.8	10	72.1	0.1	2
	9	10.6	0.1	2	168.5	1.2	15
	10	1.5	0.1	2	1.7	0.1	2
	11	7.4	0.1	2	142.2	1.2	15
	12	-10.3	0.0	1	30.3	0.2	2
	13	-5.3	0.2	2	222.2	1.6	15
	14	14.3	0.1	1	333.5	7.7	15
	15	-1.1	0.0	1	180.0	1.3	15
	16	-72.1	0.1	1	31.4	0.2	2
	17	-4.6	0.1	2	60.0	0.2	2
	18	23.2	0.1	2	106.9	0.1	2
	19	4.5	0.2	2	156.4	0.1	2
	20	-5.1	0.1	1	-79.7	0.2	2
Aver.		21.4 -13.8	0.1	2.3	190.8 -79.7	0.8	6.7

		Original			10% Red. Cap.		
Inst		Gap	Time (sec)	Iter#	Gap	Time (sec)	Iter#
6x15	21	10.5	0.3	2	338.8	1.7	15
	22	528.3	0.5	4	419.4	0.5	4
	23	-18.9	0.2	2	199.7	1.4	15
	24	18.4	1.4	10	352.9	2.2	15
	25	-23.6	0.2	2	129.5	0.2	2
	26	9.1	0.3	2	721.0	0.4	3
	27	2.8	0.3	2	399.3	0.5	3
	28	233.9	1.3	10	368.1	1.5	15
	29	1.9	0.2	2	249.4	1.9	15
	30	308.8	1.2	10	195.1	1.8	15
	31	9.9	0.1	2	176.1	0.3	2
	32	-21.1	0.2	2	310.6	0.4	2
	33	-2.2	0.1	1	123.7	1.9	15
	34	-7.1	0.1	1	180.7	4.0	15
	35	-2.8	0.2	1	116.8	5.5	15
	36	8.7	0.1	1	149.1	0.3	2
	37	4.3	0.2	2	317.1	1.9	15
	38	23.5	17.9	10	437.6	0.6	3
	39	-0.1	0.2	2	103.7	0.5	3
	40	-8.1	0.1	1	23.5	0.2	2
Aver.		96.7 -10.5	1.2	3.5	265.6 -	1.4	8.8

* Shaded cells indicate that TSH entered a loop. The iteration limit is 10 for these tests.
Gap= (TSH-UB)/UB%. The averages have been computed separately for instances with positive and negative gaps.

The results indicate that the heuristic is able to find feasible solutions in very small solution times, however, the quality of the solutions is arguable. For small instances with normal capacity, the TSH yielded lower solution values compared with the UB of GLSP in 8 of the 20 instances and the average gap for these instances is -13.8%. For the remaining instances, TSH solutions were worse than GLSP by 21.4% on the average. For larger instances and when the capacity is restricted, the performance of the heuristic sharply deteriorates. There are only a few instances where TSH gave better results than the GLSP. Moreover, the number of instances with looping behaviour increases under the case with tight capacity restriction. It can be said that the instances with loops usually yield worse solutions compared with the GLSP (i.e., they have positive gaps). Therefore, we understand that looping is a severe drawback of this algorithm and one must find ways of detecting and eliminating this behaviour in order to improve heuristic performance.

In its crudest form, TSH is a myopic heuristic that usually yields fast but poor quality solutions. Substantial effort is needed for the adjusting the iteration solutions using improvement steps to make it comparable with the GLSP. Among these improvement possibilities that are open for further examination is the adjustment of minimum batch sizes to allow splitting between periods if capacity is restrictive, the elimination of the cases where minimum batch sizes are needlessly forced to be produced twice over period boundaries etc. Alternatively, one can find ways of incorporating capacity limitations within the SM stage. In this way, the TSP's will be solved under some consideration of the available capacity and the link between capacity consumption decisions in both models can be established more closely.