

DEVELOPING A COMPUTER PROGRAM FOR EVALUATING  
UNCERTAINTY OF SOME TYPICAL DIMENSIONAL MEASURING AND  
GAUGING DEVICES

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**HASAN EMRAH ÇELEBİOĞLU**

## **ABSTRACT**

### **DEVELOPING A COMPUTER PROGRAM FOR EVALUATING UNCERTAINTY OF SOME TYPICAL DIMENSIONAL MEASURING AND GAUGING DEVICES**

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In dimensional measurements, it is required to specify uncertainty in the measurement as the range of possible deviation for the measurement result. In this thesis, a computer program is developed for evaluating uncertainty in measurement of commonly used dimensional measuring devices like vernier callipers, micrometers, comparators, and gauge blocks.

In evaluation of the uncertainty in measurement, some uncertainty sources like temperature difference between the measured part and the instrument, uncertainty in reference gauge block's dimension, mechanical effects, etc. are considered. The program developed, employs the EAL, NIST and GUM uncertainty evaluation equations as standard equations. However, the program can also be used for other measuring instruments and the users can define their own uncertainty equation. In the evaluations, for the standard uncertainty of the variables considered, symmetric distributions are used.

The program gives the uncertainty budget and to compare the contribution of each variable on the overall uncertainty of the measurement, the uncertainty effect ratio is also given. In this thesis the evaluation process for uncertainty in

measurement, the difference between the measurement error and uncertainty in measurement and the structure of the program are discussed. Also, a set of experiments has been made to illustrate the application of the program for evaluating the measurement uncertainty of vernier callipers with 1/50 and 1/20 resolutions, digital vernier calliper and 25 mm micrometer.

**Keywords:** Uncertainty, Uncertainty in Measurement, Dimensional Measurement

## ÖZ

### BAZI TİPİK BOYUTSAL ÖLÇÜM CİHAZLARININ ÖLÇÜM BELİRSİZLİĞİNİ HESAPLAMAK İÇİN BİLGİSAYAR PROGRAMI GELİŞTİRİLMESİ

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Boyutsal ölçümlerde, ölçümdeki belirsizliği ölçüm sonucunun olası aralığını görmek amacıyla belirtmek gerekmektedir. Bu tezde mikrometreler, sürmeli kumpaslar, blok masterlar, komparatörler gibi boyutsal ölçüm cihazlarının ölçüm belirsizliğini hesaplamak için bir bilgisayar programı geliştirilmiştir.

Ölçüm belirsizliğini hesaplanmasında, ölçülen parça ile ölçüm aleti arasındaki sıcaklık farkı, referans olarak kullanılan masterların boyutlarındaki belirsizlik, mekanik etkiler vb belirsizlik kaynakları göz önüne alınmıştır. Geliştirilen programda standart olarak EAL, NIST ve GUM'ın belirsizlik formülleri kullanılmıştır. Bununla birlikte geliştirilmiş olan program diğer ölçü aletleri için de kullanılabilir ve kullanıcılar kendi belirsizlik formüllerini tanımlayabilirler. Hesaplamalarda değişkenlerin standart belirsizlikleri için simetrik dağılımlar alınmıştır.

Program, belirsizlik bütçelerini ve aynı zamanda her etmenin toplam belirsizlik üzerindeki etkisini karşılaştırmak için belirsizlik etki oranını da vermektedir. Bu tez çalışmasında, belirsizlik hesaplama işlemi, ölçüm hatası ile

belirsizliđi arasındaki fark ve programın yapısı tartıřılmıştır. Ayrıca, çözünürlükleri 1/20, 1/50, 1/100 vernier ve dijital sürmeli kumpaslar ile 25 mm'lik mikrometrenin ölçüm belirsizliđinin hesaplanmasında programın kullanımını göstermek ve kullanıcılara örnek olmak amacıyla bir dizi deney yapılmıştır.

**Anahtar Kelimeler:** Belirsizlik, Ölçüm Belirsizliđi, Boyutsal Ölçme

To My Parents and My Sister

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## LIST OF SYMBOLS

|                    |                                       |
|--------------------|---------------------------------------|
| $\text{cov}$       | : covariance                          |
| $c_i$              | : sensitivity coefficient             |
| $k$                | : coverage factor                     |
| $n$                | : number of observations              |
| $S$                | : experimental standard deviation     |
| $u$                | : standard uncertainty                |
| $u_c$              | : combined standard uncertainty       |
| $U$                | : expanded uncertainty                |
| $V$                | : variance                            |
| $x_i$              | : variable                            |
| $X_i$              | : independent observations            |
| $X_M$              | : arithmetic mean of the observations |
| $y$                | : measurand                           |
| $\mu$              | : expectation                         |
| $\nu_{\text{eff}}$ | : effective degrees of freedom        |
| $\nu_i$            | : degrees of freedom of each variable |

## **CHAPTER 1**

### **INTRODUCTION**

In this chapter, dimensional measuring and gauging instruments will be introduced, literature survey on the importance of uncertainty in measurement will be discussed, and the aim of the thesis will be given.

#### **1.1 Literature Survey**

##### **1.1.1 Some Typical Dimensional Measuring and Gauging Devices**

For measurement of linear dimensions, a large variety of display type measurement devices are available which differ in their principle of function. However, the achievable measurement uncertainty and the difficulty of handling are different. In this thesis study, commonly used measuring and gauging devices, like vernier callipers, micrometers, and gauge blocks, are considered.

Vernier Callipers: The most common inspection device for workshop applications is the calliper. The beam of the calliper carries the measure embodiment, e.g. a yardstick, and the slider a mechanism that scans the beam shows the actual measurement value. The measured length is transmitted mechanically and a scale with millimeter divisions that can be read absolutely is used. Use of a vernier scale provides an additional means of displaying 1/10, 1/20, or 1/50 mm graduations. The function, e.g. of the 1/10 mm vernier scale is based on providing a length of 39 mm with ten graduation marks at equal intervals. The point at which a graduation mark on the main scale is aligned with a graduation mark on the vernier scale indicates the number of 1/10 mm on the measured length. Sometimes a division with 20 graduation marks or a circular scale is used instead of the vernier scale with ten graduation marks. The various designs of callipers are used for external, internal, and depth measurements (Figure 1.1).

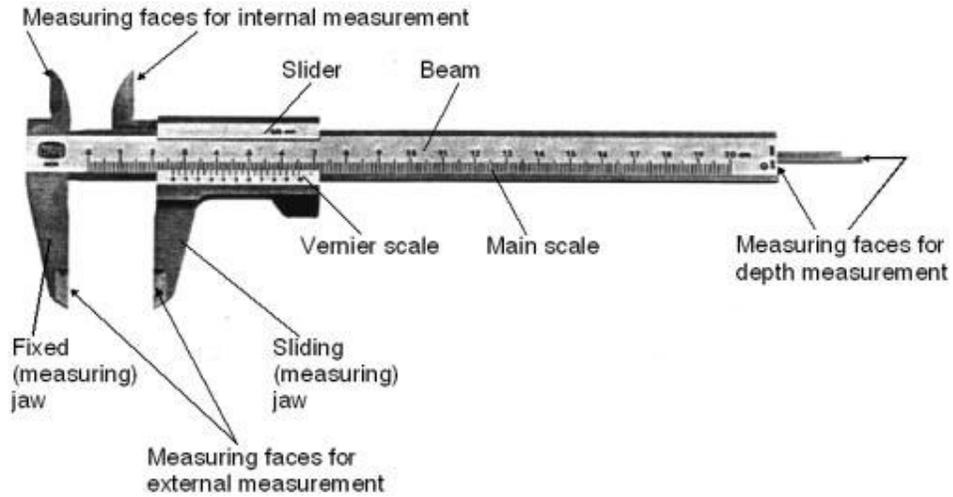


Figure 1.1 Vernier calliper for External, Internal, and Depth Measurement [1]

Except for a depth gauge, the scale of a calliper and the measuring object cannot be fully aligned as shown in Figure 1.3. This violation of Abbe's comparator principle causes a sine deviation between the scale and the slider due to an angular deviation as shown in Figure 1.3. In order to avoid unnecessarily high values of measurement deviations caused by the sine deviation, it is advisable to measure the object as close as possible to the beam of the calliper.

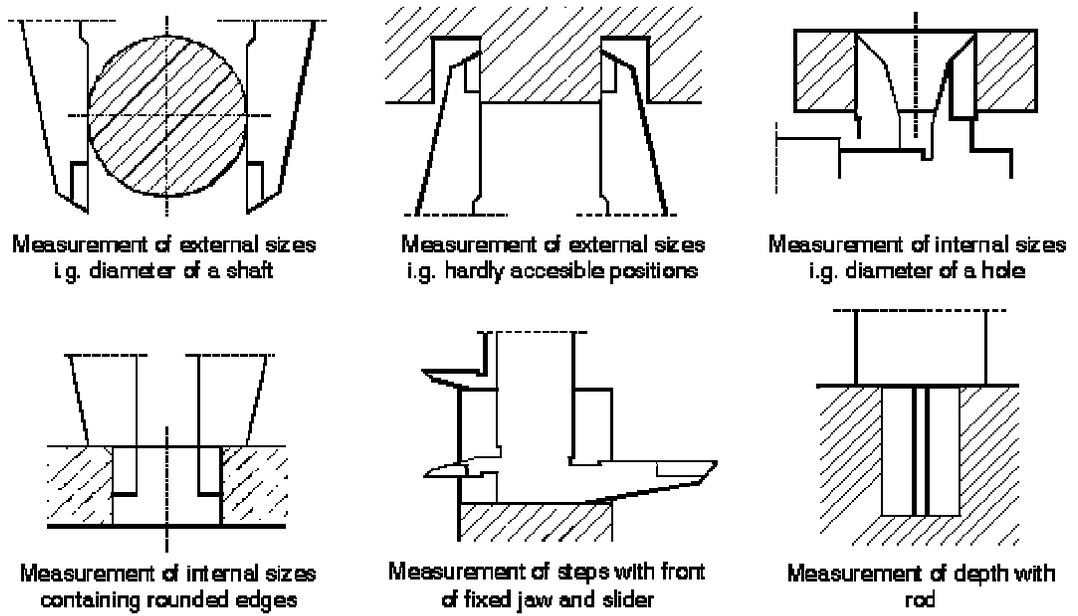


Figure 1.2 Examples for Measurements with a Calliper [1]

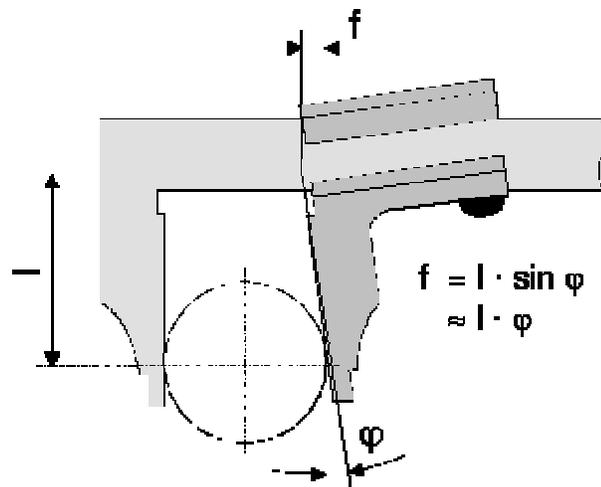


Figure 1.3 Violation of Comparator Principle Using a Calliper [1]

Micrometers: External micrometers are the measuring instruments that run by lead screw mechanical amplification principle. They have two measuring faces supported by a rigid frame. In common use, the micrometers have 0,01 mm of minimum scale division. The measuring ranges of micrometers are beginning with 0-25 mm and extending to 500 mm, with increments of 25 mm. Some types of micrometer gauges (ISO 3611) can be used for the same tasks as callipers. Micrometer calipers, as shown in Figure 1.4, are used for external and internal measurements (the measuring range is usually about 25 mm as standard and up to 500 mm for special gauges) and depth micrometers for depth measurements. Drill-hole diameters can be measured using three-point internal micrometer gauges [2].

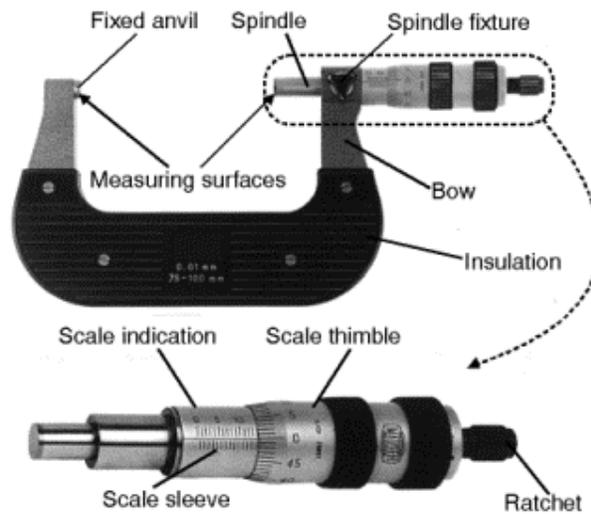


Figure 1.4 Micrometer Calliper with Measuring Head [1]

Micrometer gauges ensure that the measuring object and the scale are aligned. Since the comparator principle is not violated, no first-order measuring error can occur; only a second-order error remains (also called a cosine deviation)

and this is much less significant. According to the measuring range the maximum permissible error MPE is specified between 4  $\mu\text{m}$  and 13  $\mu\text{m}$  (ISO 3611). The fixed anvil bends up to a maximum of 15  $\mu\text{m}$  under a measuring force of 10 N.

**Gauge Blocks:** Gauges are testing tools that embody the size and/or the form of the feature that is to be checked. Gauges can also be designed to check the position of two workpiece features with reference to each other. During the testing process, i.e. the attempt to mat the workpiece and gauge, a Boolean decision is made if the limits of size are met or not. A quantitative statement about the value of the deviations is not possible with gauging [3].

### **1.1.2 Uncertainty in Measurement**

According to the international “vocabulary of basic and general terms in metrology”, uncertainty of measurement is a parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand. This parameter could be a standard deviation or another part of an interval indicating a certain confidence range. It is important that one does not only consider the single measurement but also the overall result of a test. In this case, uncertainty of measurement embraces all components of a test. Some of them may be obtained by interpreting the statistical spread of results of a series of measurements. Other components have to be worked out from complementary methods (sampling plans, experience). Testing results should be the best approximation to the true value. Statistical Random and systematic factors effect contribute to the uncertainty of measurement of the testing results. If possible, the latter should be eliminated by using for instance correction factors.

Uncertainty of measurement has to be taken into account when testing procedures and/or testing results are compared with each other or against specifications. An understanding of the concept of uncertainty of measurement is important in order to be able to choose testing methods that are fit for purpose. The

overall uncertainty of measurement should be consistent with the given requirements. The economic aspects related to the methods have always to be taken into consideration. According to ISO/IEC 17025, testing laboratories must report uncertainty estimates where specified by the method, where required by the client and/ or where the interpretation of the result could be compromised by a lack of knowledge of the uncertainty. This should at least be the case where testing results have to be compared to other testing results or other numerical values, such as specifications. In any case, laboratories should know the uncertainty associated with a measurement whether it is reported or not.

Dimensional measurement laboratories are expected to estimate the uncertainty of measurement for all calibrations carried out on measuring equipment (i.e., equipment which is used to take measurements such as dimension, force, extension, temperature and mass). The practicability of this will largely depend on whether the calibration involves measurements by equipment for which uncertainty values are available or readily determinable. To determine the uncertainty associated with a calibration, the procedure should first be broken down into its component measurements. The significant sources of all uncertainties should then be identified and quantified. In most cases, uncertainties may then be combined by an appropriate method to produce an overall uncertainty value.

Every time a measurement is taken, random effects from various sources contribute uncertainty to the value of the reading taken. These include variability resulting from imprecise definition of the calibration (e.g., poor accessibility for taking a length measurement), uncertainty in discrimination (e.g., interpolation on a scale) and random fluctuations (e.g., fluctuation in an influencing parameter such as temperature).

## **1.2 Importance of Uncertainty in Mechanical Testing, Measurement and Calibration Laboratories**

A measurement result is complete only when accompanied by a quantitative statement of its uncertainty. About a hundred and fifty years ago, Lord Kelvin said “I often say that when you can measure what you are speaking about and express it in numbers you know something about it but when you cannot measure it when you cannot express it in numbers your knowledge is of meager and unsatisfactory kind” [4]. If he lives today Lord Kelvin recognizes that measure itself is nothing without knowing the uncertainty of the measuring device.

There are thousands of lengths measuring tools in the small and medium size industrials companies, and with these measuring tools, there are millions of measurements made during manufacturing processes. Without knowing the uncertainty, many fault parts are wrongly accepted and many regular parts wrongly rejected. To avoid this every measurement device must be calibrated and their uncertainty must be either calculated or estimated with the guidance of ISO 17025 “General requirements for the competence of testing and calibration laboratories”. The calculation of uncertainty for a measurement is an effort to set reasonable bounds for the measurement result according to standardized rules. Since every measurement produces only an estimate of the answer, the primary requisite of an uncertainty statement is to inform the reader of how sure the answer is in a certain range.

### **1.2.1 History of Uncertainty in Measurement**

The requirement to test and measure the form of workpieces has constantly increased in importance since industrial manufacturing of technological products started. At the time when the first steam engines (James Watt, 1769) were made, it was only necessary to ascertain the deviations in dimensions and form of manually and individually produced parts with the aim of ensuring that the part fulfilled its

function. With the technological development the importance of the measurement science is understood in the 1780 when the steam engine was developed 1-2 mm uncertainty is sufficient but nowadays with the help of computers, 1nm uncertainty is sufficient (Figure 1.5). With the introduction of machine tools for series production of single parts (1785) that could be used in the assembly of any individual product of a series, interchangeability also became an objective. The required precision was in the tenth of a millimeter range. Ever more complex production machines, ever-smaller tolerances, the necessity to determine the measurement uncertainty, and growth in the variety of the influencing parameters that have to be considered influence the requirements for process-integrated measurement in the 21st century. This demands worker-oriented and process-chain-oriented metrology.

The demands on production metrology with respect to precision, variety of quantities to be measured, and permissible measuring times are oriented toward production structures and methods because measurement is not an end in itself but a means to achieve defined economic objectives. These objectives are constantly being further developed in an effort to boost cost-effectiveness and improve performance. With reference to the changes in production technology, the demands put on production metrology have also changed.

The demand for measurement results which are easy to evaluate, for automatic functioning, integration into the production process and networking has led to the development of automatic measuring machines with electrical and digital result output. The measurement methods used for testing the macro form in single-part, small-batch, and large-series, testing can be subdivided as follows

- Conventional testing (manual measuring instruments),
- Testing with form measuring machines
- Testing with coordinate measuring machines

- Testing with gear measuring machines
- Point-by-point control measurement with multi-gauging measuring instruments and automatic measuring machines

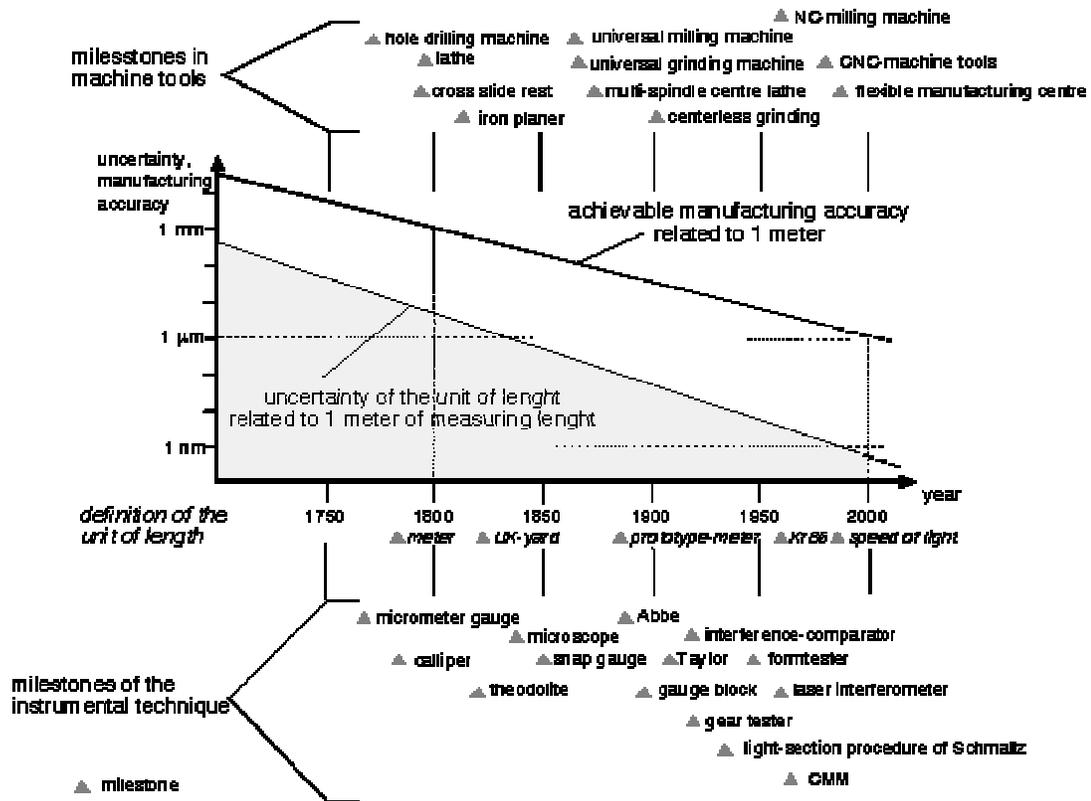


Figure 1.5 Development of Manufacturing Accuracy and Uncertainty of the Unit “Meter” [1]

Finished parts always show deviations from the designed ideal feature caused by the imperfectness resulting from the production methods. Form deviations can be divided into deviations of the macrostructure (deviations of size, form and position) and deviations of the microstructure (surface roughness). The

task of testing is to measure the value of the part's actual deviations by the use of adequate measurement tools and testing tools and to prove the fulfillment (or failure) of the specifications, considering the measurement uncertainty.

Subjective testing and gauging, only leads to a qualitative statement about fulfillment or not fulfillment of a specification. A numeric statement concerning the degree of the deviation of the actual value from the ideal value is only achievable by measurement. The decision for the implementation of a certain testing method is made according to the kind of property that is to be measured as well as the maximum acceptable uncertainty, which should not be exceeded during measurement. Testing of a narrow tolerance deviation of form, demands more involved measurement methods than testing a simple tolerance of size (Figure 1.6) [9].

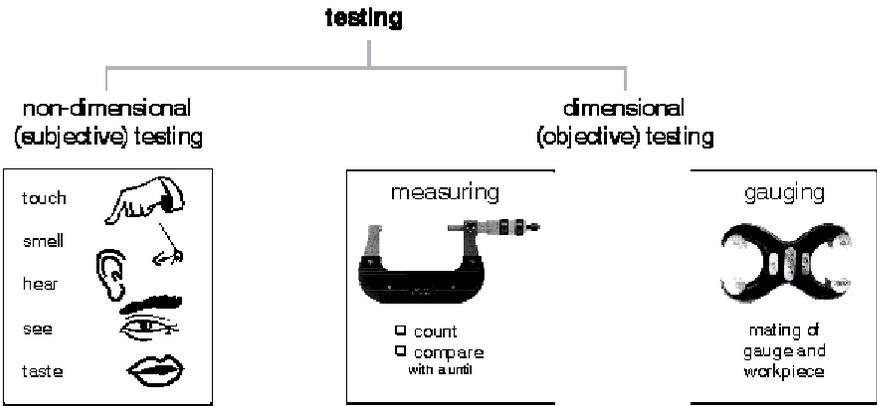


Figure 1.6 Differences in Testing of Workpieces [1]

### 1.2.2 Necessity of This Thesis

Engineering activities like research, design, development, manufacturing, calibration or testing, rely on measurements and all measurements involve error, uncertainty and risk of wrongly accepted and wrongly rejected parts. To control errors and uncertainty it is necessary to identify error sources, estimate and combine contributing component uncertainties, estimate biases, compute tolerance specifications and more without training in statistics or probability.

This thesis computer program makes compliance with ISO/IEC 17025[5], ISO 3650 [3], ISO3611 [2], ISO 6906 [6], ISO 3599 [7], ISO 14253-1 [8], QS9000. This is because this computer program implements the methods and techniques of the ISO/TAG4/WG3 “Guide to the Expression of Uncertainty in Measurement” (the "GUM") in a comprehensive user-friendly desktop tool. Using the GUM requires a strong statistical background, but using this computer program does not. This computer program produces results that normally require a high-priced consultant. A scientist, design engineer, production engineer, test engineer, metrologist, or anyone else who is concerned with measurement accuracy, need to know only information that falls within their technical specialty. This computer program furnishes the statistical expertise, while user furnishes the technical knowledge by using the help menu. It is assured that the analysis will cover all sources of measurement uncertainty and results will be based on techniques and methods that represent the leading edge in uncertainty analysis. This computer program makes this possible through easy to follow on-screen procedures and user-interactive worksheets that cover a wide spectrum of uncertainty sources. The worksheets carry the user through analysis in a structured way that ensures covering all the sources of uncertainty.

User will probably find that many of the uncertainty analysis problems can run into are similar except for a few details. This means that, with this computer program, a new analysis may take only seconds. Simply call up a saved file from a previous analysis, enter the changes, and save the new analysis under a different

file name. In this way, user can build a library of solutions for the problems encounter day-to-day.

### **1.3 Aim of the Thesis**

This thesis is about a developing a computer program for uncertainty analysis for dimensional measurements. The extensive application of management models aimed at the assurance and improvement of company process quality levels has produced simulating debates on the role of test and measurement in the development of a documented quality system. A prominent position has been taken by the widely accepted models described in the ISO 9000:2000 series of standards which have been applied worldwide with steadily growing rates. Even in many other guidance norms and in specific standards the importance of test and measurement activities has always been highlighted. Moreover the recognition by the international agencies of the impelling need to regulate such a complicated matter has led to the publication of an European Pre standard known as the “Guide to the expression of Uncertainty in Measurement ” and of an important norm regarding accreditation of laboratories which now supersedes ISO/IEC Guide 25 1990. The solution proposed in the GUM [1] was pragmatically founded on widely accepted practices that were referable to the law of propagation of errors.

This thesis establishes general rules for evaluating and expressing uncertainty in measurement that can be followed at various levels of accuracy and in many fields from the shop floor to fundamental research. Therefore, the principles of this thesis intended to be applicable to a broad range of dimensional measurements, including those required for;

- Maintaining quality control and quality assurance in production
- Complying with and enforcing laws and regulations
- Conducting basic research, and applied research and development, in science and engineering

- Calibrating standards, instruments and performing tests throughout a national measurement system in order to achieve tractability to national standards
- Developing, maintaining, and comparing international and national physical reference standards, including reference materials

In this thesis study, a software program will be developed for small and medium size industrial companies for helping them to evaluate and/or to estimate the uncertainty of some typical mechanical measuring and gauging devices like micrometer, vernier calliper, comparator, gauge block. This computer program, not only a computational program but also a training program, it gives information not only about the terms to calculate uncertainty but also method used to calculate and /or estimate the uncertainty and make comments about the results.

This thesis primarily concerned with the expression of uncertainty in the measurement of dimension of a part the measurand that can be characterized by an essentially unique value. In the thesis, the computer program is also, applicable to evaluating and expressing the uncertainty associated with the conceptual design and theoretical analysis of experiments, methods of measurement, and complex components and systems. In addition, this thesis provides general rules for evaluating and expressing uncertainty in measurement rather than detailed, technology specific instructions. Further, it does not discuss how the uncertainty of a particular measurement result, once evaluated, may be used for different purposes, for example, to draw conclusions about the compatibility of that result with other similar results, to establish tolerance limits in a manufacturing process, or to decide if a certain course of action may be safely undertaken. It may therefore particular standards based from standards that deal with the problems peculiar to specific fields of measurement or with the various uses of qualitative expressions of uncertainty.

## CHAPTER 2

### UNCERTAINTY IN MEASUREMENT

In this chapter, the definition of uncertainty in measurement, the difference between error and uncertainty, the process of uncertainty in measurement evaluation, the basic sources of uncertainty in measurement will be presented..

#### 2.1 Errors and Uncertainty

It is important to distinguish between the error and the uncertainty. Error is defined as the difference between an individual result and the true value of the measurand. In principle, the value of a known error can be applied as a correction to the result. Error is an idealized concept and cannot be known exactly.

Uncertainty, on the other hand, takes the form of a range, and, if estimated for a measurement procedure and defined sample, may apply to all determinations so described. Uncertainty in measurement, in the International Vocabulary of Basic and General Terms in Metrology, is defined as “a parameter associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand”. In general, the value of the uncertainty cannot be used to correct a measurement result. To illustrate the further difference, the result of an analysis after correction may by chance be very close to the value of the measurand, and hence have a negligible error. However, the uncertainty may still be very large, simply because the analyst is very unsure of how close that result is to the value. The uncertainty of the result of a measurement should never be interpreted as representing the error itself, or the error remaining after correction. An error regarded as having two components, namely, a random component, and a systematic component.

Figure 2.1 depicts some of the ideas about error and uncertainty. The exact error of a result of a measurement is, in general, unknown and unknowable. All one can do is estimate the values of input quantities, including corrections for recognized systematic effects, together with their standard uncertainties. Standard uncertainties can be found either from unknown probability distributions that are sampled by means of repeated observations or from subjective or priori distributions based on the pool of available information. Then calculate the measurement result from the estimated values of the input quantities and the combined standard uncertainty of that result from the standard uncertainties of those estimated values. Only if there is a sound basis for believing that all of this done properly, one can assume that the measurement result is a reliable estimate of the value of the measurand and that its combined standard uncertainty is a reliable measure of its possible error.

The uncertainty of the result of a measurement reflects the lack of exact knowledge of the measurand. The result of a measurement after correction for recognized systematic effects are still an estimate of the value of the measurand because of the uncertainty arising from the random effects and from imperfect correction of the result for systematic effects. The result of a measurement can unknowably be very close to the value of the measurand even though it may have a large uncertainty. Thus, the uncertainty of the result should not be confused with the remaining unknown error.

*Concepts based on observable quantities*

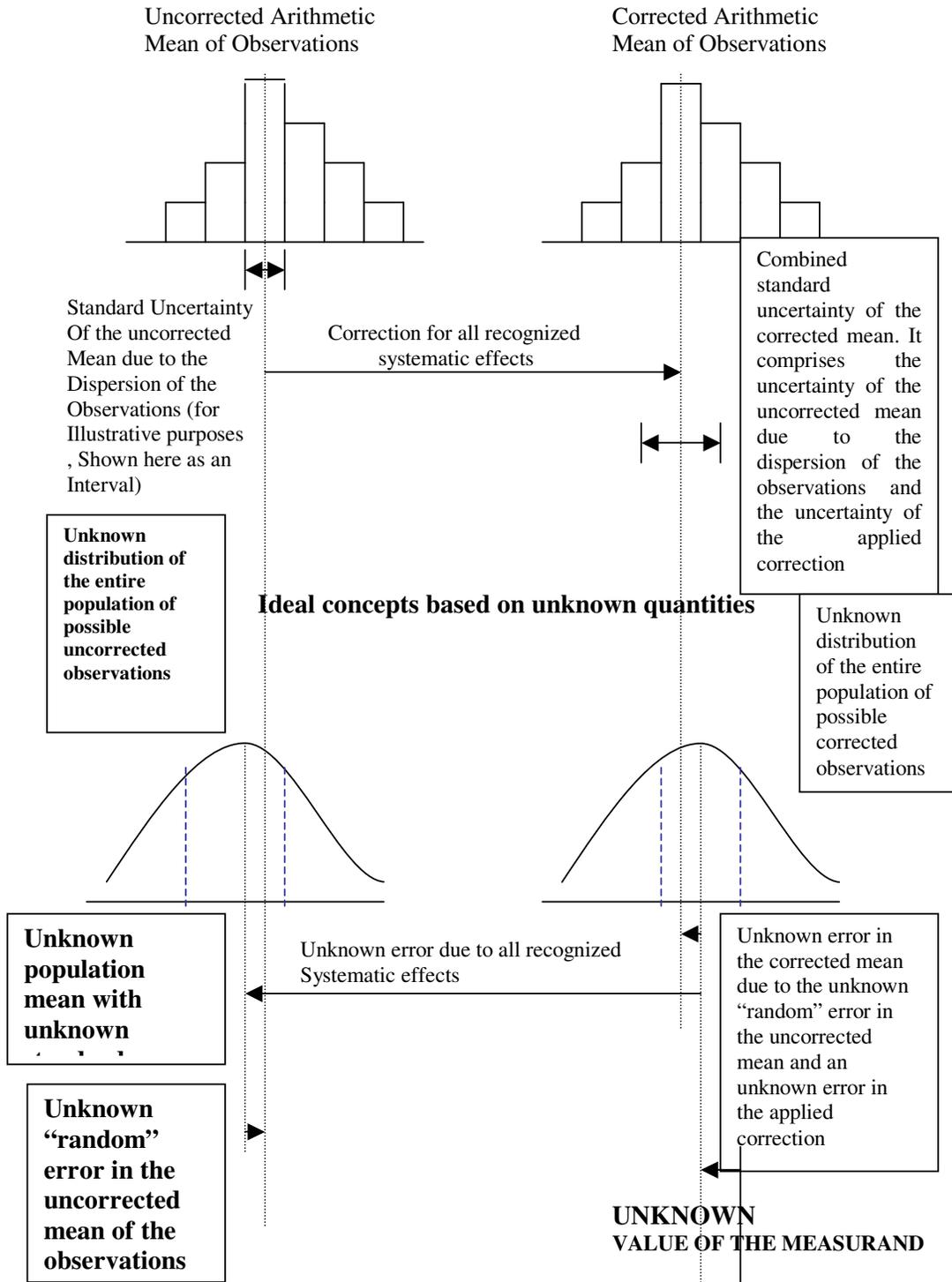


Figure 2.1 Graphical Illustrations of Value, Error and Uncertainty [10]

### **2.1.1 Random Error**

Random error in measurement typically arises from unpredictable variations of influence quantities. These random effects give rise to variations in repeated observations of the measurand. The random error of the measurement cannot be compensated for, but increasing the number of observations can usually reduce it.

### **2.1.2 Systematic Error**

It is defined as a component of error that, in the course of a number of analyses of the same measurand, remains constant or varies in a predictable way. It is independent of the number of measurements made and cannot therefore be reduced by increasing the number of analyses under constant measurement conditions.

Constant systematic errors, such as failing to make an allowance for a reagent blank in an assay, or inaccuracies in a multi-point instrument calibration, are constant for a given level of the measurement value. Systematic errors may vary with the level of the measurement value. Effects that change systematically in magnitude during a series of analyses, caused, for example by inadequate control of experimental conditions, give rise to systematic errors that are not constant. The result of the measurement should be corrected for all recognized significant effects. Measuring instruments and systems are calibrated using measurement standards and reference materials to correct the systematic effects. The uncertainties associated with these standards and materials and the uncertainty in the correction must still be taken into account.

The further type of error is spurious error, or blunder. Errors of this type invalidate a measurement and typically arise through human failure or instrument malfunction. While recording data, transposing digits in a number is the most common example of this type error.

Measurements for which errors such as these have been detected should be rejected and no attempt should be made to incorporate the errors into any statistical analysis. However, errors such as digit transposition can be corrected, particularly if they occur in the leading digits. Spurious errors are not always obvious and, where a sufficient number of replicate measurements are available, it is usually appropriate to apply an outlier test to check for the presence of suspect members in the data set. Any positive result obtained from such a test should be considered with care and, where possible, referred back to the originator for confirmation. It is generally not wise to reject a value on purely statistical grounds [11].

## 2.2 The Evaluation Process of Uncertainty in Measurement

Evaluation of uncertainty in measurement is simple in principle. The following paragraphs summarize the tasks that need to be performed in order to obtain an evaluation of uncertainty associated with a measurement result. Evaluation process provides additional guidance applicable in different circumstances, particularly relating to the use of data from method validation studies and the use of formal uncertainty propagation principles. The steps involved are given as [12]:

- **Specify Measurand:** Write down a clear statement of what is being measured, including the relationship between the measurand and the input quantities upon which it depends. Where possible, include corrections for known systematic effects.
- **Identify Uncertainty Sources:** List the possible sources of uncertainty. This will include sources that contribute to the uncertainty on the parameters in the relationship specified in Step 1.
- **Quantify Uncertainty Components:** Measure or estimate the size of the uncertainty component associated with each potential source of uncertainty identified. It is often possible to estimate or determine a single contribution to uncertainty associated with a number of separate

sources. It is also important to consider whether available data accounts sufficiently for all sources of uncertainty, and plan additional experiments and studies carefully to ensure that all sources of uncertainty are adequately accounted for.

- **Calculate Combined Uncertainty:** The information obtained in step 3 consist a number of quantified contributions to overall uncertainty, whether associated with individual sources or with the combined effects of several sources. The contributions have to be expressed as standard deviations and combined according to the appropriate rules, to give a combined standard uncertainty. The appropriate coverage factor should be applied to give an expanded uncertainty.

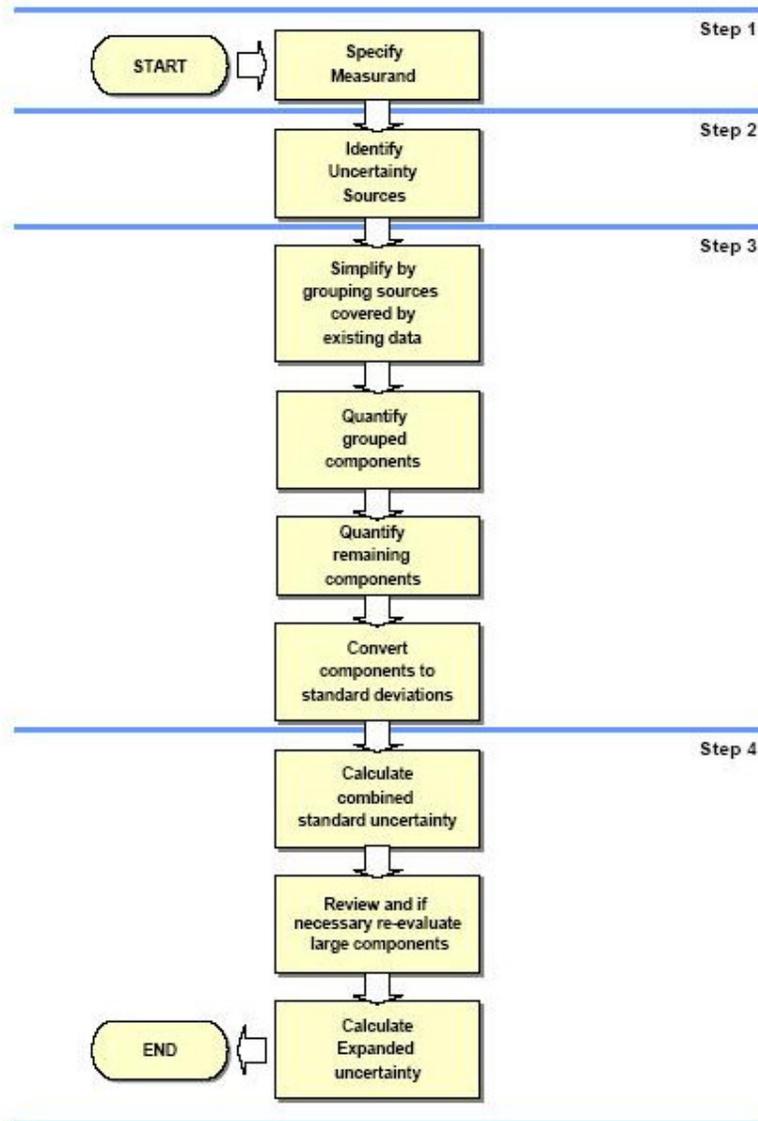


Figure 2.2 Uncertainty Evaluation Process [11]

### 2.3 Specifying the Measurand

In most cases, a measurand  $y$  is not measured directly, determined from  $N$  other variables  $x_1, x_2, \dots, x_N$  through a functional relationship  $f$ :

$$y=f(x_1, x_2, \dots, x_N) \quad (2.1)$$

The variables  $x_1, x_2, \dots, x_N$  upon which the output variable  $y$  depends may themselves be viewed as measurands and may themselves depend on other quantities, including corrections and correction factors for systematic effects, thereby leading to a complicated functional relationship  $f$  that may never be written down explicitly. Further,  $f$  may be determined experimentally or exists only as an algorithm that must be evaluated numerically.

Thus, if data indicate that  $f$  does not model the measurement to the degree imposed by the required accuracy of the measurement result, additional input variables must be included in  $f$  to eliminate the inadequacy. This may require introducing a variable reflect incomplete knowledge of a phenomenon that effects the measurand.

The set of variables  $x_1, x_2, \dots, x_N$  may be categorized as:

- Variables whose values and uncertainties are directly determined in the current measurement. These values and uncertainties may be obtained from, for example, a single observation, repeated observations, or judgment based on experience, and may involve the determination of corrections to instrument readings and corrections for influence quantities, such as ambient temperature, barometric pressure, and humidity.
- Variables whose values and uncertainties are brought into the measurement from external sources, such as quantities associated with calibrated measurement standards, certified reference materials, and reference data obtained from handbooks.

#### **2.4 Identifying Sources of Uncertainty in Measurement**

A comprehensive list of relevant sources of uncertainty should be assembled. At this stage, it is not necessary to be concerned about the quantification of individual components; the aim is to be completely clear about what should be considered.

In forming the required list of uncertainty sources, it is usually convenient to start with the basic expression used to calculate the measurand from intermediate values. All the parameters in this expression may have an uncertainty associated with their value and are therefore potential uncertainty source. In addition there may be other parameters that do not appear explicitly in the expression used to calculate the value of the measurand, but which nevertheless affect the measurement results, e.g., extraction time or temperature. These are also potential sources of uncertainty.

Once the list of uncertainty sources is assembled, in principle their effects on the result can be represented by a formal measurement model, in which each effect is associated with a parameter or variable in an equation. The equation then forms a complete model of the measurement process in terms of all the individual factors affecting the result. This function may be very complicated and it may not be possible to write it down explicitly. Where possible, however, this should be done, as the form of expression will generally determine the method of combining individual uncertainty contributions.

It may additionally be useful to consider a measurement procedure as a series of discrete operations, each of which may be assessed separately to obtain associated with them. This is a particularly useful approach where similar measurement procedures share common unit operation then form contributions to the overall uncertainty. In practice, it is more usual in analytical uncertainty evaluation to consider associated with elements of overall method performance, such as observable precision and bias measured with respect to appropriate reference materials. These contributions generally form the dominant contributions to the uncertainty evaluation. In addition, they are best modeled as separate effects on the result [6].

- Sampling

Where sampling forms part of the specified procedure, effects such as random variations between different samples and any potential for bias in the sampling procedure form components of uncertainty affecting the result.

- Storage Conditions

Where test items stored for any period before analysis, the storage conditions may affect the results. The duration of storage as well as conditions during storage should therefore be considered as uncertainty sources.

- Instrument Effects

Instrument effects may include for example the limits of accuracy on the calibration of an analytical balance; a temperature controller that may maintain a mean temperature which differs (within specification) from its indicated set-point an auto-analyzer that could be subject to carry-over effects.

- Measurement Conditions

The instrument that may be used at ambient conditions would be at different conditions than at which it is calibrated. Gross temperature effects should be corrected for uncertainty. Similarly, humidity may be important where materials are sensitive to possible changes in humidity. In addition, the thermometer used to measure the temperature of the gauge has some uncertainty. If the measurement is not made at exactly 20 °C, a thermal expansion correction must be made using the thermal expansion coefficient of the gauge. However, the uncertainty in this coefficient is also a source of uncertainty. In comparison, in dimensional calibrations there can be a temperature difference between the master gauge and the test gauge. [13].

- Mechanical Deformation

All mechanical measurements involve contact of surfaces and all surfaces in contact are deformed. In some cases, the deformation is unwanted, in gauge block comparisons for example, and applying a correction to get the undeformed length is very suitable [13].

- Instrument Geometry

Each instrument has a characteristic motion or geometry that, if not perfect, will lead to errors. The specific uncertainty depends on the instrument, but the sources fall into a few broad categories: reference surface geometry, alignment, and motion errors. Reference surface geometry includes the flatness and parallelism of the anvils of micrometers used in ball and cylinder measurements, the roundness of the contacts in gauge block and ring comparators, and the sphericity of the probe balls on coordinate measuring machines. It also includes the flatness of reference flats used in many interferometric measurements. An instrument such as a micrometer or coordinate measuring machine has a moving probe, and motion in any single direction has six degrees of freedom and thus six different error motions. The scale error is the error in the motion direction. The straightness errors are the motions perpendicular to the motion direction. The angular error motions are rotations about the axis of motion (roll) and directions perpendicular to the axis of motion (pitch and yaw). If the scale is not exactly along the measurement axis, the angle errors produce measurement errors called Abbe errors [13].

- Sample Effects

The recovery for an analyte from a complex matrix, or an instrument response, may be affected by other elements of the matrix. Analyte specification may further compound this effect. When a spike is used to estimate recovery, the recovery of the analyte from the sample, may differ from the recovery of the spike introducing an uncertainty that needs to be evaluated.

- Computational Effects

Selection of the calibration model, e.g. using a straight-line calibration on a curved response, leads to poorer fit and gives higher uncertainty. Truncation and round off in calculations can lead to inaccuracies in the result. Since these are rarely predicable, an uncertainty allowance may be necessary.

- Operator Effects

Operator effect would appear as reading a meter or scale consistently high or low. Possibility of making a slightly different interpolation of the method effects the measurements also the uncertainty.

- Random Effects

Random effects contribute to the uncertainty in all determinations. This entry should be included in the list as a matter of case.

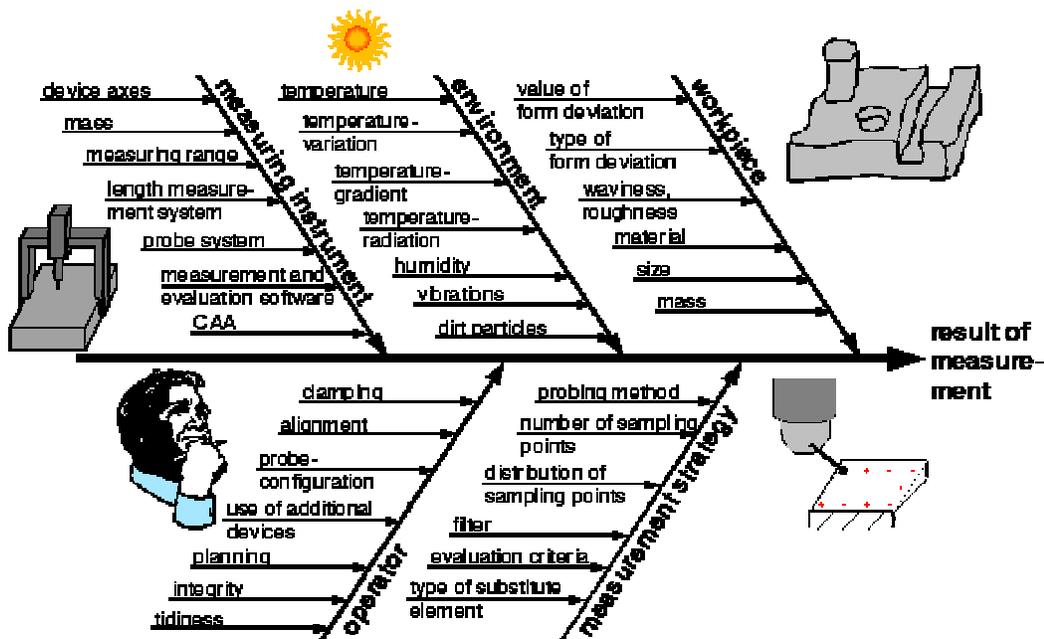


Figure 2.3 Basic Sources of Uncertainty in Measurement [1]

## CHAPTER 3

### CALCULATION PRINCIPLES FOR UNCERTAINTY IN MEASUREMENT

In this chapter uncertainty in measurement components, step-by-step procedure for uncertainty in measurement evaluation will be discussed.

#### 3.1 Components of Uncertainty in Measurement

In evaluating the overall uncertainty, it may be necessary to take each source of the uncertainty and treat it separately to obtain the contribution from that source. Each of the separate contributions to uncertainty referred to as an uncertainty component. When expressed as a standard uncertainty, an uncertainty component known as a standard uncertainty.

The uncertainty of measurement associated with the input estimates is evaluated according to either Type A or a Type B method of evaluation. The Type A evaluation of standard uncertainty is the method of evaluating the uncertainty by the statistical analysis of a series of observations. In this case, the standard uncertainty is the experimental standard deviation of the mean that follows from an averaging procedure or an appropriate regression analysis [9].

The Type B evaluation of standard uncertainty is the method of evaluating the uncertainty by means other than statistical analysis of a series of observations. In this case, the evaluation of the standard uncertainty is based on some other scientific knowledge.

For a measurement result, the total uncertainty is termed as combined standard uncertainty. For most purposes, an expanded uncertainty should be used. The expanded uncertainty provides an interval within which the value of the measurand is believed to lie with a higher level of confidence. Expanded

uncertainty is obtained by multiplying combined standard uncertainty, by a coverage factor  $k$ . The choice of the factor  $k$  is based on the level of confidence and degrees of freedom desired [11].

### 3.1.1 Type A Evaluation of Standard Uncertainty

Type A evaluation of standard uncertainty is the method of evaluating the uncertainty by the statistical analysis series of observations. In this case the standard uncertainty is the experimental standard deviation of the mean that follows from an averaging procedure or an appropriate regression analysis. The Type A evaluation of standard uncertainty can be applied when several independent observations have been made for one of the input quantities under the same conditions of measurement. If there is sufficient, resolution in the measurement process there will be an observable scatter or spread in the values obtained. For calculating the type A standard uncertainty, only 5 to 25 independent single observations needed [12]. The arithmetic mean of these observations is calculated as;

$$X_M = 1/n (\sum X_i) \quad (3.1)$$

Where  $X_i$  is the independent observation and  $X_M$  is the arithmetic mean of these observations

Variance is simply a measure of dispersion that is the sum of the squared deviations of observations from their average divided by one less than the number of observations. The variance given by;

$$V = 1/(n-1)(\sum (X_i^2 - X_M^2)) \quad (3.2)$$

The square root of the variance is defined as “experimental standard deviation” which can be defined as ‘a parameter characterizes the variability of the observed values  $X_i$  or more specifically their dispersion above mean  $X_m$ . And is

given by

$$S=\sqrt{V} \quad (3.3)$$

After calculating “the experimental standard deviation”, “the experimental standard deviation of the mean” is also called as the standard uncertainty is found by;

$$u=S/\sqrt{n} \quad (3.4)$$

### 3.1.2 Type B Evaluation of Standard Uncertainty

The Type B evaluation of standard uncertainty is associated with by means other than the statistical analysis of series of observations. Type B calculation is based on experience and general knowledge and is a skill that can be learned with practice. The variables needed for calculating the standard uncertainty of the variable can be found from handbooks, or calibration certificates etc. The standard uncertainty  $u(x_i)$  is evaluated by scientific judgment based on all available information on the possible variability of the variable. Values belonging to this category may be derived from [14]

- Previous measurement data
- Experience with or general knowledge of the behavior and properties of relevant materials and instruments
- Manufacturer’s specifications
- Data provided in calibration and other certificates
- Uncertainties assigned to reference data taken from handbooks

For calculating the Type B standard uncertainty of the variable, expanded uncertainty and coverage factor or the bounds and the distribution type of the variable must be known. If the expanded uncertainty,  $U(x_i)$ , and the coverage factor of the variable inserted, then the standard uncertainty of the variable found from the equation given below:

$$u(x_i) := \frac{U(x_i)}{k} \quad (3.5)$$

If the bounds of the variable and the distribution type are given, then the standard uncertainty can be obtained from the table given in Appendix F.

### 3.1.3 Combined Standard Uncertainty

The combined standard uncertainty of a measurement result is taken to represent the estimated standard deviation of the result. It is obtained by combining the individual standard uncertainties  $u_c$ , whether arising from a Type A evaluation or a Type B evaluation, using the usual method for combining standard deviations. It is assumed that a correction (or correction factor) is applied to compensate for each recognized systematic effect that significantly influences the measurement result and that every effort has been made to identify such effects. The relevant uncertainty to associate with each recognized systematic effect is then the standard uncertainty of the applied correction. The correction may be either positive, negative, or zero, and its standard uncertainty in some cases be obtained from a Type A evaluation while in other cases by a Type B evaluation.

The uncertainty of a correction applied to a measurement result, to compensate for a systematic effect is not the systematic error in the measurement result due to the effect. Rather, it is a measure of the uncertainty of the result due to incomplete knowledge of the required value. Although it is strongly recommended that corrections be applied for all recognized significant systematic effects, in some cases it may not be practical because of limited resources. Nevertheless, the expression of uncertainty in such cases should conform to these guidelines fully. The combined standard uncertainty  $u_c$ , is a widely employed measure of uncertainty.

After calculating the standard uncertainties of Type A and Type B, the

sensitivity coefficients of each variable will be calculated. Sensitivity coefficients can describe by how the output  $y$  varies with changes in the values of the input variables  $x_1, x_2, \dots, x_N$ . Considering the sources of uncertainty, once the uncertainty evaluation equation,  $f(x_1, x_2, \dots, x_N)$ , is formed, then sensitivity coefficient can defined as  $c_i = \partial f / \partial x_i$ . To calculate the sensitivity coefficients, the nominal values of the variables are required.

After calculating the sensitivity coefficient, user can calculate the combined standard uncertainty that can be described as the standard uncertainty of the equation of the measuring tool and it is equal to;

$$u_c^2(y) = \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \quad (3.6)$$

The equation above is only valid if the input quantities are independent or uncorrelated (the random variables, not the physical quantities that are assumed to be invariants). If some of the input quantities are significantly correlated, the correlations must be taken into account. The correlation generally occurs in electrical and chemical measurements. When the input quantities are correlated, the appropriate expression for the combined standard uncertainty associated with result of the measurement is

$$u_c(y) = \sqrt{\sum_{i=1}^N \left[ \frac{\partial f}{\partial x_i} \right]^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)} \quad (3.7)$$

where  $x_i$  and  $x_j$  are the estimates of  $X_i$  and  $X_j$  and  $u(x_i, x_j) = u(x_j, x_i)$  is the estimated covariance associated with  $x_i$  and  $x_j$  (see Appendix D).

### 3.1.4 Degrees of Freedom for Uncertainty in Measurement

The combined uncertainty will, in general, be determined from Type A and Type B evaluations of uncertainty. In Type A evaluations of uncertainty, it is possible that the best estimate of each quantity is obtained from a different number

of repeated measurements. The degrees of freedom of a standard uncertainty,  $u(x_i)$ , obtained from a Type A evaluation is determined by appropriate statistical methods. In the common case where  $u(x_i)$ , the degrees of freedom of is  $n - 1$ .

The degrees of freedom to associate with a standard uncertainty  $u(x_i)$  obtained from a Type B evaluation is more problematic. It is common practice to carry out such evaluations in a manner ensures that an underestimation is avoided. For example, when the lower and upper limits,  $a$  and  $a_+$ , are usually chosen in such a way, the probability of the quantity in question lying outside these limits is in fact extremely small. Under the assumption that this practice is followed, the degrees of freedom of  $u(x_i)$  may be taken to be infinitely large. The reliability of the standard uncertainty assigned to the output estimate is determined by its effective degrees of freedom. However, the reliability criterion is always met, if none of the uncertainty contributions is obtained from Type A evaluation based on less than ten repeated observations.

After finding degrees of freedom of each variable, one can calculate the effective degrees of freedom,  $\nu_{\text{eff}}$ , using the Welch- Satterthwaite formula, which can be expressed as [15]:

$$\nu_{\text{eff}} = \frac{u_c^4(y)}{\sum_{i=1}^N \frac{c_i^4 u^4(x_i)}{\nu_i}}, \quad (3.8)$$

After finding the effective degrees of freedom, coverage factor,  $k$ , can be found from t-distribution table given in Appendix B.

### 3.1.5 Confidence Interval

A confidence interval gives an estimated range of values that is likely to include an unknown population parameter, the estimated range being calculated from a given set of sample data. The width of the confidence interval gives some idea about how uncertain the unknown parameter is. If independent samples are

taken repeatedly from the same population, and a confidence interval calculated for each sample, then a certain percentage (confidence level) of the intervals will include the unknown population parameter. Confidence intervals are usually calculated so that this percentage is 95%, but one can produce 90%, 99%, 99.9% (or whatever) confidence intervals for the unknown parameter [16].

### 3.1.6 Coverage Factor

Coverage Factor is a numerical factor used as a multiplier of the standard uncertainty of measurement in order to obtain an expanded uncertainty of the measurement. In general, the value of the coverage factor,  $k$ , is chosen on the basis of the desired level of confidence to be associated with the interval defined by  $U = k u_c$ . Typically,  $k$  is in the range 2 to 3. When the normal distribution applies and  $u_c$  is a reliable estimate of the standard deviation of  $y$ ,  $U = 2 u_c$  (i.e.,  $k = 2$ ) defines an interval having a level of confidence of approximately 95 %.  $U = 3 u_c$  (i.e.,  $k = 3$ ) defines an interval having a level of confidence greater than 99 %. For a given degrees of freedom and the level of confidence, the coverage factor values can be obtained from the table given in Appendix B [16].

### 3.1.7 Expanded Uncertainty

Expanded uncertainty of measurement,  $U$ , is obtained by multiplying the combined standard uncertainty,  $u_c (y)$ , of the output estimate,  $y$ , by a coverage factor,  $k$ ,

$$U = k u_c(y) \tag{3.9}$$

In cases where a normal distribution can be attributed to the measurand and the standard uncertainty associated with the output estimate has sufficient reliability, the standard coverage factor  $k=2$  shall be used. The assigned expanded uncertainty corresponds to a coverage probability of approximately 95%. These

conditions are fulfilled in the majority of cases encountered in calibration work. The assumption of a normal distribution cannot always be easily confirmed experimentally. However, in the cases where several (i.e.  $N \geq 3$ ) uncertainty components derived from well-behaved probability distributions of independent quantities, normal distributions or rectangular probability distributions contribute to the standard uncertainty associated with the output estimate by comparable amounts the conditions of the central limit theorem are met. It can be assumed to a high degree of approximation that the distribution of the output quantity is normal [16].

If one of these conditions (normality or sufficient reliability) is not fulfilled, the standard coverage factor  $k=2$  can yield an expanded uncertainty corresponding to a coverage probability less than 95%. In these cases in order to ensure that a value of the expanded uncertainty is quoted corresponding to the same coverage probability as in the normal case other procedures have to be followed. The use of approximately the same coverage probability is essential whenever two results of measurement of the same quantity have to be compared e.g. when evaluating the results of an inter-laboratory comparison or assessing compliance with a specification

Even if a normal distribution can be assumed, it may still occur that the standard uncertainty associated with the output estimate is of insufficient reliability. If in this case, it is not expedient to increase the number  $n$  of repeated measurements or to use a Type-B evaluation instead of Type-A evaluation of poor reliability. For the remaining cases, i.e. all cases where the assumption of a normal distribution cannot be justified, information on the actual probability distribution of the output estimate must be used to obtain a value of the coverage factor  $k$  that corresponds to a coverage probability of approximately 95% [16].

### 3.2 Preparing Uncertainty Budget in Measurement

In the uncertainty analysis for a measurement, for the sake of clarity, it is recommended to present the data relevant to this analysis in the form of a table that is called as the uncertainty budget of the measurement. In this table, a physical symbol  $X_i$  or a short identifier should reference all sources of uncertainty. For each of them, at least the estimate  $x_i$ , the associated standard uncertainty of measurement  $u(x_i)$ , the sensitivity coefficient  $c_i$ , and the different uncertainty contributions  $u_i(y)$  should be specified. The dimension of each of the quantities should also be stated with the numerical values given in the table.

A formal example of such an uncertainty budget is given in Table 3.1 and is applicable to the case of uncorrected input quantities. The standard uncertainty associated with the measurement result,  $u(y)$ , given in bottom right corner of the table is the root sum square of all the uncertainty contributions in the outer right column  $X_i$  [16].

Table 3.1 Sample Uncertainty Budget

| Quantity | Estimate | Standard Uncertainty | Sensitivity Coefficient | Contribution to the Standard Uncertainty |
|----------|----------|----------------------|-------------------------|--|
| $X_i$    | $x_i$    | $u(x_i)$             | $c_i$                   | $u_i(y)$                                 |
| $X_1$    | $x_1$    | $u(x_1)$             | $c_1$                   | $u_1(y)$                                 |
| $X_2$    | $x_2$    | $u(x_2)$             | $c_2$                   | $u_2(y)$                                 |
| $X_N$    | $x_N$    | $u(x_N)$             | $c_N$                   | $u_N(y)$                                 |
| Y        | y        |                      |                         | $u(y)$                                   |

## **CHAPTER 4**

### **COMPUTER AIDED MANAGEMENT OF EVALUATING UNCERTAINTY IN DIMENSIONAL MEASUREMENTS**

In this chapter, the developed program for the evaluation of uncertainty of will be described and the structure of the program will be discussed. After the discussions, sample uncertainty evaluations for the commonly used dimensional measuring and gauging devices will be given.

#### **4.1 Preparation of Program**

Today's computer technology in PC basis makes it possible to manage all the requirements of evaluating measurement uncertainty with reasonably low investment costs [17]. The computer aided management and calculation of measurement uncertainty becomes a beneficial tool if it contains database structures useful uncertainty reporting system and appropriate work planning [18].

This software package is designed for using in calibration laboratories, small and medium size industrial companies, or factories that are activated are limited with controlling the measuring equipment and the product quality. EA (European co-operation for Accreditation), GUM (Guide to Measurement Uncertainty) uncertainty equations are accepted as general methods for evaluating uncertainty for gauge blocks, micrometers, and vernier calipers. Before preparing the package, other standards like ISO 3650, ISO/DIN 6906, ISO 3599, and ISO 3611 have been investigated. The user defined menu designed for evaluating uncertainty of measuring equipment whose uncertainty equation can be developed by the user.

## 4.2 Structure of the Program

This package is suitable for Personal Computers running at preferably Windows 98, Windows 2000, Windows Me and Windows XP. The program is consisted of five main files that are linked each other and totally about 16 MB memory. This program was designed by, Microsoft Visual Basic 6.0 Enterprise Edition.

The hardware requirements are as are as follows

- At least 50 MB of free hard disk space
- Pentium III 600 MHz CPU
- At least 128 MB Ram
- VGA color monitor

The main part of the program is evaluating the uncertainty of vernier callipers, gauge blocks, micrometers, comparators, and user defined. Each of this measurement equipment has different uncertainty equation and graphical demonstrations.

The help of the prepared calendar subroutine automatically calculates the present day and calibration overdue date. The calendar gives the overdue date in the form of day/month/year and program do recalling for measurement equipment whose evaluation of uncertainty expired.

The results of measurement data are recorded in data files sequentially. These data files hold the information of preceding measurement uncertainty evaluation results, overdue date, evaluation date, history, code for measurement instrument, and necessary data for the variables in the uncertainty equation. The computer program automatically runs the searching subroutine whenever the user logs in to the program, in order to check and to find any overdue instrument exists in that day.

### 4.3 Description of the Program Facilities

This computer program was developed mainly for evaluating the uncertainties of dimensional measuring devices like micrometers, vernier callipers, gauge blocks, and comparators. For vernier callipers, gauge blocks, micrometer and comparator menu the measurement equation is fixed and user cannot change this equation. On the other hand, if the user selects the user-defined menu, users can create their own measurement equation.

In the flowchart of the computer program developed is given in Appendix A. The user firstly enters the confidence level, and the measurement uncertainty equation of the variable. Then, the nominal values of each variable are inserted and according to its standard uncertainty evaluation type, Type A or Type B, the necessary fields of each variable will be selected. If the variables standard uncertainty evaluated by Type A, total 5-25 independent observations have been inserted and the computer program evaluates the mean value, variance, standard deviation, and standard uncertainty of the variable. If the variables standard uncertainty is evaluated by Type B, the user selects the evaluation type. If bounds and distribution type is chosen the bounds of the distribution is inserted and the distribution type is selected by the user. If the expanded uncertainty and the coverage factor are chosen, the user inserts the expanded uncertainty and the coverage factor. After entering these, the degrees of freedom of the each variable are inserted. Then the user clicks the evaluate button and the program evaluates the sensitivity coefficient of each variable. The combined standard uncertainty of the measurement and effective degrees of freedom of the measurement is evaluated. With the effective degrees of freedom and the confidence level, the program finds the coverage factor of the measurement from the t-distribution chart given in Appendix B. Then, the computer program evaluates the expanded uncertainty of the measurement.

For evaluating the uncertainty of measurement with using user defined menu user must click “User Defined” button from the “New Evaluation” screen. After

selecting “User Defined” instrument uncertainty evaluation screen appears to the screen. Then user must follow the instructions:

1. User enters the instrument id and the name of the instrument.
2. User selects the coverage probability from the screen. Here it is selected 95.45 %.
3. Then user is going enter the uncertainty equation. Here, first user enters the coefficient of the first variable and then enters the symbol for the variable and the type of function for that variable. The built-in basic function types can be selected from “x”, “x<sup>2</sup>”, “x<sup>3</sup>”, “lnx”, “e<sup>x</sup>”, “1/x”, “x<sup>1/2</sup>”, and “x<sup>1/3</sup> “. Then the operation between the terms can be select from “+”, “-“, “\*”, “/” and “=”. For example, in writing the uncertainty equation for the first variable user selects 1 for the coefficient, then writes m<sub>s</sub> for the symbol, selects x from the function type and selects + in order to write the second variable. After entering the entire terms, user selects “=” term to finish the writing the uncertainty equation for the instrument. After writing the uncertainty equation, the user enters the nominal values of the variables.
4. User selects unit from the combo box of the variables.
5. User selects the evaluation type for the variables as Type A or Type B. In Type B, Expanded Uncertainty and Coverage Factor or Bounds and the Distribution Type are entered.
6. For Type A variables, user enters the degrees of freedom of the variables, which comes to the text box automatically n-1 where n is the number of observation. After entering these fields, the screen looks like as given in Figure 4.10 where the calibration equation of nominal weight of 10 kg given.
7. User clicks the “Evaluate” button and the results come to the screen.
8. After clicking “Save” button, results and the uncertainty equation are saved to the database and the results screen appears. Here user can see the effects of uncertainty contribution of the variable with a graph by clicking “Plot Graph” button. As it can be seen from the graph that the uncertainty of reference standard effects more than any other variable in the uncertainty equation.

9. User can take print out by clicking “Print” button.
10. User can take excel report by clicking “Go To Excel” button. The report screen is shown in Figure 4.13

#### **4.4 Sample Evaluation of Measurement Uncertainty for Calibration of a Vernier Calliper**

In this section, evaluation of the uncertainty of a vernier calliper with the measurement range of 150 mm and the reading interval of 0,05 mm will be made by without using the computer program. Then these results will be compared with the results taken from the computer program. A vernier calliper made of steel is calibrated against gauge blocks used as working standards. Several gauge blocks with nominal lengths in the range 0,5-100 mm are used in calibration. They are selected in such a way that the measurement points are spaced at nearly equal distances (for example at 0 mm, 50 mm, 100 mm, 150 mm). Before calibration, several checks on the condition of the calliper are made. These include dependence of the measurement on the size of the measured item of the vernier beam (Abbe Error), quality of the measuring faces of the jaws (flatness, parallelism, squareness), and function of the locking mechanism. Instrumental error of the external measurement can be finding by using a gauge block. It is inserted between the faces for the external measurement and the measurement readings at the root and the tip of the measuring faces are noted. The instrumental error is obtained by reducing the dimension of the gauge block from the reading of the calliper. This process is repeated using block gauges of different length of to cover the entire measuring length of the scale.

The measurement equation is given by [16];

$$E_x = l_{ix} - l_s + L_s * \alpha * \Delta t + \delta l_{ix} + \delta l_m$$

(4.1)

where;

$l_{ix}$  : Indication of the calliper.

$l_s$  : Length of the actual gauge block. From the calibration certificates of the gauge block, the central length of the gauge block coincides within  $\pm 0.8 \mu\text{m}$  with the rectangular limits.

$L_s$ : Nominal length of the actual gauge block

$\alpha$  : Average thermal expansion coefficient of the calliper and the gauge block. The average thermal expansion coefficient is  $11,5 * 10^{-6} \text{ }^\circ\text{C}^{-1}$

$\Delta t$  : Difference in temperature between the calliper and the gauge block. After adequate stabilization time, the temperatures of the calliper and the gauge block are equal within  $\pm 2 \text{ }^\circ\text{C}$  with rectangular limits.

$\delta l_{ix}$  : Correction due to the finite resolution of the calliper . Scale interval is 0,05 mm. Thus from various due to the finite resolution are estimated to have rectangular limits of  $\pm 25 \mu\text{m}$  so;

$\delta l_m$  : Correction due to mechanical effects, such as applied measurement force, abbe error, flatness and parallelism errors of the measurement faces. The total variation for the mechanical effects estimated to have rectangular limits of  $\pm 50 \mu\text{m}$ .

After finding the standard uncertainties of the variables, the results can be shown in the form of uncertainty budget as given in Table 4.1

Table 4.1 Uncertainty Budget for Vernier Calliper [16]

| Quantity        | Estimate  | Standard Uncertainty | Probability Distribution | Sensitivity Coefficient | Degrees of Freedom | Uncertainty Contribution |
|-----------------|-----------|----------------------|--------------------------|-------------------------|--------------------|--------------------------|
| $l_{ix}$        | 150,10 mm | 0                    | -----                    | -----                   | -----              | -----                    |
| $l_s$           | 150,00 mm | 0,46 $\mu\text{m}$   | Rectangular              | -1                      | $\infty$           | -0,46 $\mu\text{m}$      |
| $\Delta t$      | 0         | 1,15 C               | Rectangular              | 1,7 $\mu\text{mC}^{-1}$ | $\infty$           | 2,0 $\mu\text{m}$        |
| $\delta l_{ix}$ | 0         | 14,44 $\mu\text{m}$  | Rectangular              | 1                       | $\infty$           | 14,44 $\mu\text{m}$      |
| $\delta l_m$    | 0         | 28,86 $\mu\text{m}$  | Rectangular              | 1                       | $\infty$           | 28,86 $\mu\text{m}$      |
| $E_x$           | 0,10 mm   |                      |                          |                         |                    | 32,34 $\mu\text{m}$      |

From t-distribution table given in Appendix B, for the confidence level of %95.45, the coverage factor is 2. So the expanded uncertainty becomes;

$$U = k * u(E_x) = 2 * 32,34 = 64,68 \mu\text{m}$$

The result is reported as;

“At 150 mm, the error of indication of the calliper is  $(0,10 \pm 0,064) \text{ mm}$ ”

For sample calculation of uncertainty in measurement for calibration of a vernier calliper, the uncertainty evaluation input screen and the results screen are given in Figures 4.1 and 4.3, respectively. Also in Figure 4.2 Bounds and The Distribution Type Selection Screen can be seen. The uncertainty effect of the each variable considered on the overall measurement uncertainty of the comparator is expressed in the form of ratio as given in Figure 4.4. The Excel report screen which gives the uncertainty budget together with the uncertainty effects ratio of the variables considered for the comparator is given in 4.5.

Company Name EMRAH A.Ş.

Instrument Id: Ver-Emr-150-1 Date: 17.03.2005

| Nominal Values |           |                         |                 |   |                    |
|----------------|-----------|-------------------------|-----------------|---|--------------------|
| $l_s$          | 150,1     | mm                      | $\Delta t$      | 0 | $^{\circ}\text{C}$ |
| $l_n$          | 150       | mm                      | $\delta l_{ix}$ | 0 | mm                 |
| L              | 150       | mm                      | $\delta l_m$    | 0 | mm                 |
| $\alpha$       | 0,0000115 | $^{\circ}\text{C}^{-1}$ |                 |   |                    |

Uncertainty Function:  $l_n - l_s + L \alpha \Delta t + \delta l_{ix} + \delta l_m$

Coverage Probability: 95,45

TYPE A

n = 5 Ok

| Type B Evaluation Fields |                    |                             |                      |
|--------------------------|--------------------|-----------------------------|----------------------|
|                          | Degrees of Freedom | Evaluation Type             | Standard Uncertainty |
| $l_n$                    | $\infty$           | Bounds and Distribution Typ | 0,00046              |
| $\Delta t$               | $\infty$           | Bounds and Distribution Typ | 1,1547               |
| $\delta l_{ix}$          | $\infty$           | Bounds and Distribution Typ | 0,01444              |
| $\delta l_m$             | $\infty$           | Bounds and Distribution Typ | 0,02886              |

F3 - Calculate F4 - Clear ESC - Quit

Figure 4.1 Uncertainty Evaluation Screen for Vernier Calliper

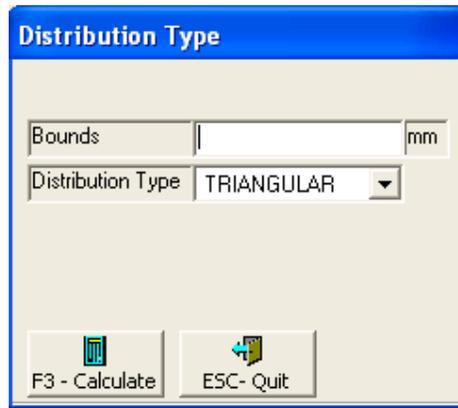


Figure 4.2 Bounds and Distribution Type Screen

|                           |                                |                        |                                    |
|---------------------------|--------------------------------|------------------------|------------------------------------|
| <b>No</b> 2               | <b>Company Info</b>            |                        | <b>Type</b> Vernier Calliper       |
| <b>Name</b> EMRAH         | <b>Company Name</b> EMRAH A.Ş. | <b>Name</b> EMRAH      | <b>Instrument No</b> Ver-Emr-150-1 |
| <b>Surname</b> CELEBIOĞLU | <b>Surname</b> CELEBIOĞLU      | <b>Date</b> 17.03.2005 |                                    |

### Uncertainty Budget

| Variable     | Nominal Value | Evaluation Type                     | Sensitivity Coefficient | Standard Uncertainty | Uncertainty Contribution | Degrees of Freedom |
|--------------|---------------|-------------------------------------|-------------------------|----------------------|--------------------------|--------------------|
| $l_s$        | 150,1         | TYPE A                              | 1                       | 0                    | 0                        | 4                  |
| $l_n$        | 150           | TYPE B Bounds and Distribution Type | -1                      | 0,00046              | -0,00046                 | $\infty$           |
| $\Delta t$   | 0             | TYPE B Bounds and Distribution Type | 0,0015                  | 1,1547               | 0,00173                  | $\infty$           |
| $\delta l_x$ | 0             | TYPE B Bounds and Distribution Type | 1                       | 0,01444              | 0,01444                  | $\infty$           |
| $\delta l_m$ | 0             | TYPE B Bounds and Distribution Type | 1                       | 0,02886              | 0,02886                  | $\infty$           |
| <b>E(x)</b>  | <b>0,1</b>    |                                     |                         |                      |                          |                    |

| Combined Uncertainty | Degrees of Freedom | Coverage Factor | Coverage Probability | Expanded Uncertainty |
|----------------------|--------------------|-----------------|----------------------|----------------------|
| 0,03232              | $\infty$           | 2               | 95,45                | 0,06464              |

|           |          |                 |                 |            |           |            |
|-----------|----------|-----------------|-----------------|------------|-----------|------------|
| F2 - Back | F3 - New | F4 - Goto Excel | F5 - Plot Graph | F6 - Print | F1 - Save | ESC - Quit |
|-----------|----------|-----------------|-----------------|------------|-----------|------------|

Figure 4.3 Uncertainty Evaluation Result Screen for Vernier Calliper

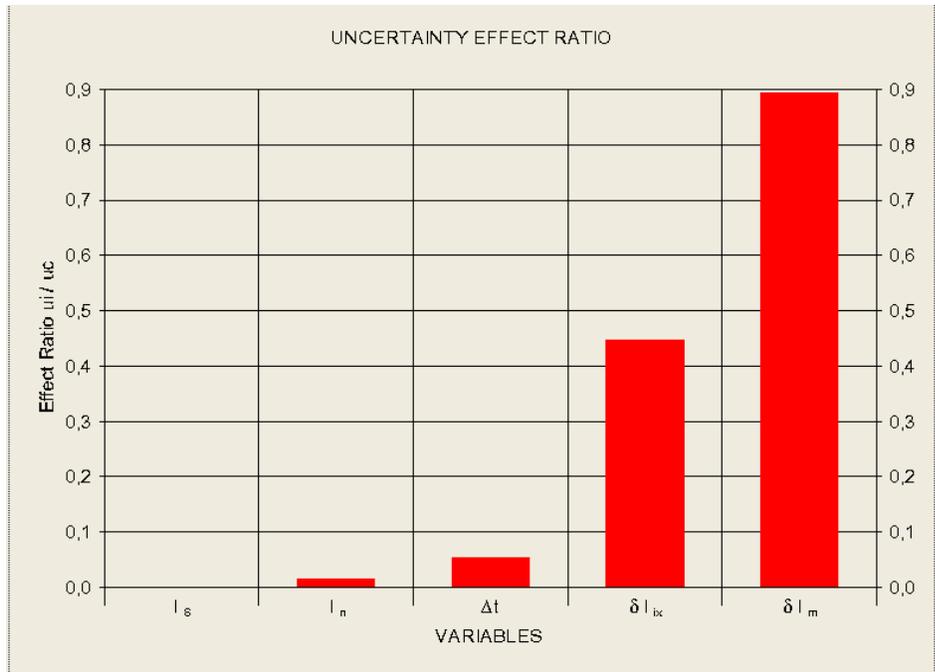


Figure 4.4 Graph of Measurement Uncertainty Effect Ratio for Vernier Calliper

## UNCERTAINTY BUDGET

**Type** : Vernier Calliper

**Ins. No** : Ver-Emr-150-1

**Date** : 17.03.2005

| Evaluation Made By |              | Company Info     |              |
|--------------------|--------------|------------------|--------------|
| <b>No</b>          | : 2          | <b>Com. Name</b> | : EMRAH A.Ş. |
| <b>Name</b>        | : EMRAH      | <b>Name</b>      | : EMRAH      |
| <b>Surname</b>     | : ÇELEBİOĞLU | <b>Surname</b>   | : ÇELEBİOĞLU |

| Variable        | Nominal Value | Evaluation Type                     | Sensitivity Coefficient | Standard Uncertainty | Uncertainty Contribution | Degrees of Freedom |
|-----------------|---------------|-------------------------------------|-------------------------|----------------------|--------------------------|--------------------|
| $l_{ix}$        | 150,1         | TYPE A                              | 1                       | 0                    | 0                        | 4                  |
| $l_a$           | 150           | TYPE B Bounds and Distribution Type | -1                      | 0,00046              | -0,00046                 | $\infty$           |
| $\Delta t$      | 0             | TYPE B Bounds and Distribution Type | 0,0015                  | 1,1547               | 0,00173205               | $\infty$           |
| $\delta l_{ix}$ | 0             | TYPE B Bounds and Distribution Type | 1                       | 0,01444              | 0,01444                  | $\infty$           |
| $\delta l_m$    | 0             | TYPE B Bounds and Distribution Type | 1                       | 0,02886              | 0,02886                  | $\infty$           |
| $E(x)$          | 0,1           |                                     |                         |                      |                          |                    |

| Com. Stan. Uncer. | Eff. Degrees of Freedom | Coverage Factor | Coverage Probability | Expanded Uncertainty |
|-------------------|-------------------------|-----------------|----------------------|----------------------|
| 0,03232           | $\infty$                | 2               | 95,45                | 0,06464              |

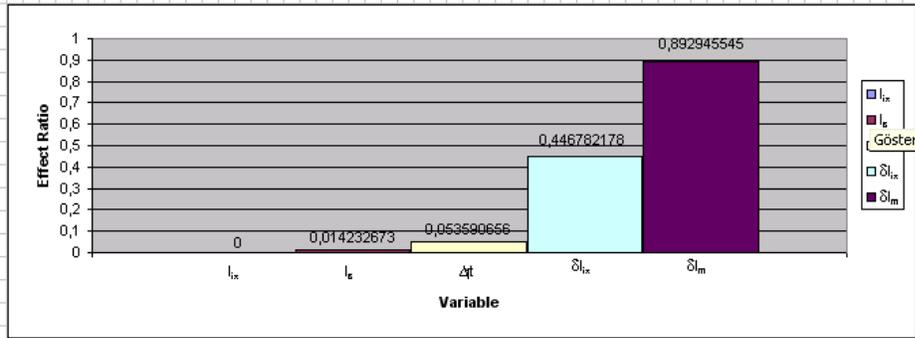


Figure 4.5 Excel Report Screen for Vernier Calliper

#### 4.5 Sample Evaluation of Uncertainty in Measurement for Calibration of a Gauge Block of Nominal Length 50 mm

In this section, evaluation of the uncertainty in measurement of a gauge block of nominal length 50 mm is described. The calibration of the gauge block of 50 mm nominal length is carried out by comparison, using a comparator and a calibrated gauge block of the same nominal length and the same material as reference standard. The difference in central length is determined in vertical position of the two gauge blocks using two length indicators contacting upper and lower measuring faces. In this method, the gauge block, to be calibrated, and the reference gauge block are wrung on a reference surface plate and compared using the comparator.

For gauge block, the measurement equation that equation is given as [16]

$$l_x = l_s + \delta l_d + \delta l + \delta l_c - L * (\alpha * \delta t + \delta \alpha * \Delta t) - \delta l_v \quad (4.2)$$

where;

$l_s$  : Length of the reference gauge block at reference temperature  $t_0 = 20$  °C. According to its calibration certificate as 50,00002 mm  $\pm$  30 nm (coverage factor  $k = 2$ )

$\delta l_d$  : Change in length of the reference gauge block due to drift since its last calibration. The temporal drift of the length of the reference gauge block is estimated 0 with the limits  $\pm$  30 nm of rectangular limits

$\delta l$  : Observed difference in length between the unknown and the reference gauge block.

$\delta l_c$  : Correction in non-linearity and offset of the comparator. The maximum length difference with the limits  $\pm$  32 nm with rectangular limits

$L$  : Nominal lengths of the gauge blocks considered.

$\alpha$  : Average of the thermal expansion coefficients of the unknown and the reference gauge blocks.

$\delta t$  : Temperature difference between the reference and the unknown gauge blocks. The remaining difference in temperature between the standard and the unknown gauge block is estimated within 0,05 °C with rectangular limits

$\delta\alpha$  : Difference in the thermal expansion coefficients between the unknown and the reference gauge blocks

$\Delta t$  : Deviation of the average temperature of the unknown and the reference gauge blocks

$\delta l_v$  : Correction for non-central contacting of the measuring faces of the unknown gauge blocks. The correction due to central misalignment of the contacting point is estimated to be within  $\pm 6,7$  nm

Table 4.2 Uncertainty Budget for Gauge Block [19]

| Quantity                | Estimate     | Standard Uncertainty | Probability Distribution | Sensitivity Coefficient | Degrees of Freedom | Uncertainty Contribution |
|-------------------------|--------------|----------------------|--------------------------|-------------------------|--------------------|--------------------------|
| $l_s$                   | 50,00002 mm  | 15 nm                | Normal                   | 1                       | $\infty$           | 15 nm                    |
| $\delta l_D$            | 0 mm         | 17,3 nm              | Rectangular              | 1                       | $\infty$           | 17,3 nm                  |
| $\delta l$              | 0,000092 mm  | 3,74 nm              | Normal                   | 1                       | 4                  | 3,74 nm                  |
| $\delta l_C$            | 0 mm         | 18,47 nm             | Rectangular              | 1                       | $\infty$           | 18,47 nm                 |
| $\delta t$              | 0°C          | 0,0288°C             | Rectangular              | -575 nm/°C              | $\infty$           | -16,6 nm                 |
| $\delta\alpha*\Delta t$ | 0            | $0,23*10^{-6}$       | Rectangular              | -50 mm                  | $\infty$           | -11,8 nm                 |
| $\delta l_v$            | 0 mm         | 3,87 nm              | Rectangular              | -1                      | $\infty$           | -3,87 nm                 |
| $l_x$                   | 49,999928 mm |                      |                          |                         |                    | 36,6 nm                  |

From t-distribution table, for confidence level of %95,45 the coverage factor is

2. So the expanded uncertainty becomes;

$$U = k * u(E_x) = 2 * 36,6 = 73,2 \text{ nm}$$

The result is reported as;

“The measured value of the nominal 50 mm gauge block is 49,999928 mm  $\pm$  73.2 nm”

For sample calculation of uncertainty in measurement for calibration of a gauge block, the uncertainty evaluation input screen and the results screen are given in Figures 4.6 and 4.7, respectively. The uncertainty effect of the each variable considered on the overall measurement uncertainty of the comparator is expressed in the form of ratio as given in Figure 4.8. The Excel report screen which gives the uncertainty budget together with the uncertainty effects ratio of the variables considered for the comparator is given in 4.9.

**Company Name:** EMRAH A.Ş.  
**Name:** EMRAH  
**Surname:** ÇELEBİOĞLU

**Nominal Values:**

|              |           |    |                 |          |                  |
|--------------|-----------|----|-----------------|----------|------------------|
| $l_s$        | 50,00002  | mm | $\alpha$        | 0,000015 | °C <sup>-1</sup> |
| $\delta l_d$ | 0         | mm | $\delta t$      | 0        | °C               |
| $\delta l$   | -0,000094 | mm | $\delta \alpha$ | 0        | °C <sup>-1</sup> |
| $\delta l_s$ | 0         | mm | $\Delta t$      | 0        | °C               |
| $L$          | 50        | mm | $\delta l_v$    | 0        | mm               |

**Uncertainty Function:**  $l_s + \delta l_d + \delta l + \delta l_s - L [(\alpha \delta t) + (\delta \alpha \Delta t)] - \delta l$

**Coverage Probability:** 95,45

**Type A:** n = 5

**Type B Evaluation Fields:**

|                 | Degrees of Freedom | Evaluation Type             | Standard Uncertainty |
|-----------------|--------------------|-----------------------------|----------------------|
| $l_s$           | $\infty$           | Expanded Uncertainty and C  | 0,000015             |
| $\delta l_d$    | $\infty$           | Bounds and Distribution Typ | 0,000017             |
| $\delta l_s$    | $\infty$           | Bounds and Distribution Typ | 0,000018             |
| $\delta t$      | $\infty$           | Bounds and Distribution Typ | 0,02887              |
| $\delta \alpha$ | $\infty$           | Bounds and Distribution Typ | 0,0000008            |
| $\Delta t$      | $\infty$           | Bounds and Distribution Typ | 0,28867              |
| $\delta l_v$    | $\infty$           | Bounds and Distribution Typ | 0,000003             |

Figure 4.6 Uncertainty Evaluation Screen for Gauge Block

| No 2                           |               | Company Info            |                         | Type Gauge Block     |                          |                    |
|--------------------------------|---------------|-------------------------|-------------------------|----------------------|--------------------------|--------------------|
| Name EMRAH                     |               | Company Name EMRAH A.Ş. |                         | Instrument 1         |                          |                    |
| Surname ÇELEBİOĞLU             |               | Name EMRAH              |                         | Date 03.04.2005      |                          |                    |
|                                |               | Surname ÇELEBİOĞLU      |                         |                      |                          |                    |
| Uncertainty Budget             |               |                         |                         |                      |                          |                    |
| Variable                       | Nominal Value | Evaluation Type         | Sensitivity Coefficient | Standard Uncertainty | Uncertainty Contribution | Degrees of Freedom |
| $\delta l$                     | -0,000094     | TYPE A                  | 1                       | 0,0000037            | 0,0000037                | 4                  |
| $l_s$                          | 50,00002      | TYPE B Expanded         | 1                       | 0,000015             | 0,000015                 | $\infty$           |
| $\delta l_d$                   | 0             | TYPE B Bounds and       | 1                       | 0,000017             | 0,000017                 | $\infty$           |
| $\delta l_o$                   | 0             | TYPE B Bounds and       | 1                       | 0,000018             | 0,000018                 | $\infty$           |
| $\delta l$                     | 0             | TYPE B Bounds and       | -0,00075                | 0,02887              | -0,0000217               | $\infty$           |
| $\delta \alpha \cdot \Delta t$ | 0             | SPECIAL                 | -50                     | 0,0000002            | -0,000001                | $\infty$           |
| $\delta l_v$                   | 0             | TYPE B Bounds and       | -1                      | 0,000003             | -0,000003                | $\infty$           |
| E(x) 49,999926                 |               |                         |                         |                      |                          |                    |
| Combined Uncertainty           |               | Degrees of Freedom      | Coverage Factor         | Coverage Probability | Expanded Uncertainty     |                    |
| 0,0000382                      |               | $\infty$                | 2                       | 95,45                | 0,0000764                |                    |

Figure 4.7 Uncertainty Evaluation Results Screen for Gauge Block

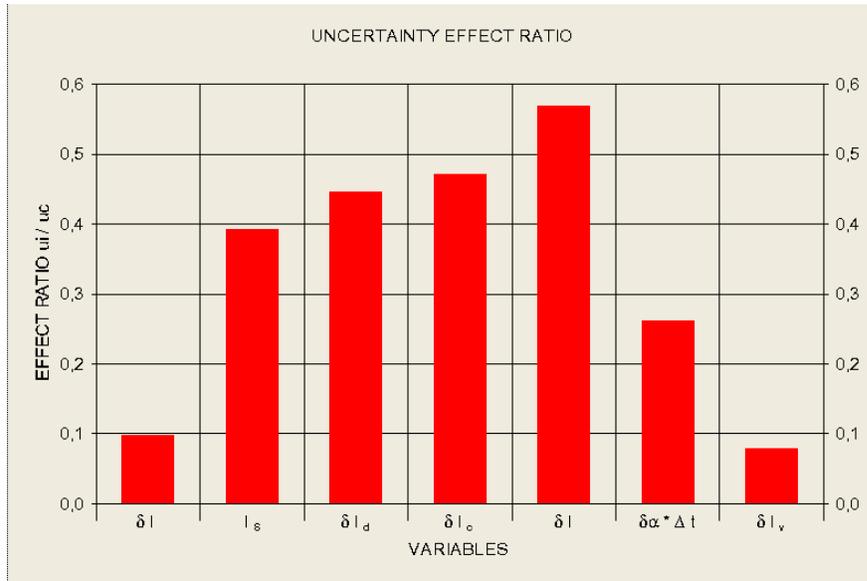


Figure 4.8 Graph of Measurement Uncertainty Effect Ratio for Gauge Block

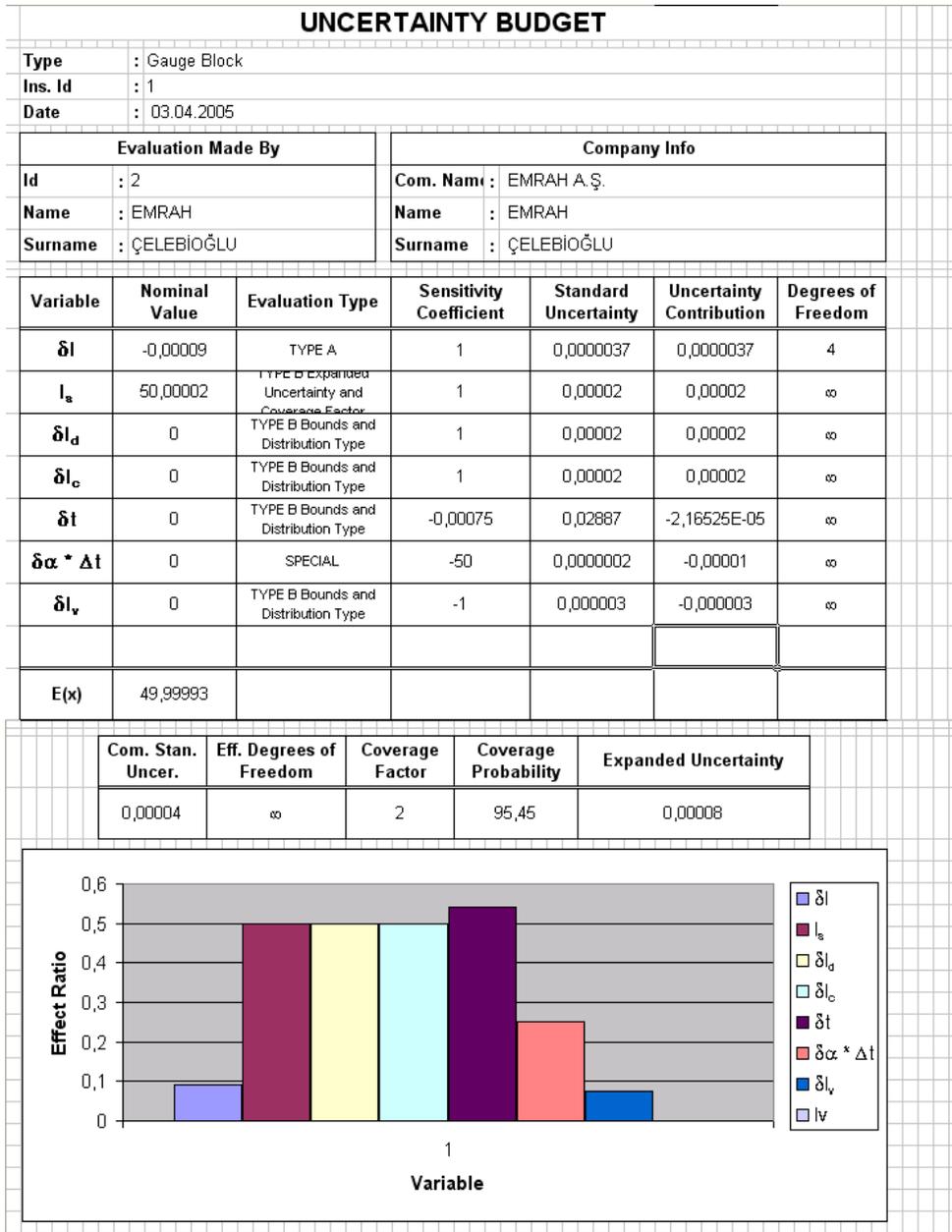


Figure 4.9 Excel Report Screen for Gauge Block

#### 4.6 Sample Evaluation of Uncertainty in Measurement with Using User Defined Option

Here evaluation of the measurement uncertainty for weight of nominal value 10 kg is described. This uncertainty evaluation is carried out by comparison to a reference standard of the same nominal value using a mass comparator whose performance characteristics have previously been determined.

The measurement equation is given by [16]:

$$m_x = m_s + \delta m_d + \delta m + \delta m_c + \delta B \quad (4.3)$$

where ;

$m_s$  : Conventional mass of the standard. From the calibration certificate for the reference standard gives a value of 10000,005 g with an associated expanded uncertainty of 45 mg with coverage factor  $k = 2$

$\delta m_d$  : Drift of value of the standard since its last calibration. The drift of the value estimated from the previous evaluation to be zero with  $\pm 15$  mg with rectangular limits

$\delta m$  : Observed difference in mass between the unknown mass and standard.

$\delta m_c$  : Correction for eccentricity and magnetic effects. Variations due to eccentricity and magnetic effects are estimated to have rectangular limits of  $\pm 10$  mg

$\delta B$  : Correction for air buoyancy. The limits of deviation are estimated to be  $\pm 1 \times 10^{-6}$  of the nominal value with rectangular limits

Table 4.3 Uncertainty Budget for 10 Kg Weight [16]

| Quantity     | Estimate    | Standard Uncertainty | Probability Distribution | Sensitivity Coefficient | Degrees of Freedom | Uncertainty Contribution |
|--------------|-------------|----------------------|--------------------------|-------------------------|--------------------|--------------------------|
| $m_s$        | 10000,006g  | 22,5 mg              | Normal                   | 1                       | $\infty$           | 22,5 mg                  |
| $\delta m_D$ | 0           | 8,6 mg               | Rectangular              | 1                       | $\infty$           | 8,6 mg                   |
| $\delta m_c$ | 0           | 5,77 mg              | Rectangular              | 1                       | $\infty$           | 5,77 mg                  |
| $\delta m$   | 0,03g       | 7,07 mg              | Rectangular              | 1                       | 4                  | 7,07 mg                  |
| $\delta B$   | 0           | 5,77 mg              | Rectangular              | 1                       | $\infty$           | 5,77 mg                  |
| $m_x$        | 10000,036 g |                      |                          |                         |                    | 26,41mg                  |

From t-distribution table, for the confidence level of 95,45%,  $k = 2$ . So the expanded uncertainty for this instrument;

$$U(m_x) = 26,41 * 2 = 52,82 \text{ mg}$$

Thus, the measured value of the nominal 10 kg weight is 10000,036 g  $\pm$  52,82 mg.

For sample calibration for weight of nominal value 10 kg, the uncertainty evaluation input screen and the results screen are given in Figures 4.10 and 4.11, respectively. The uncertainty effect of the each variable considered on the overall measurement uncertainty of the comparator is expressed in the form of ratio as given in Figure 4.12. The Excel report screen which gives the uncertainty budget together with the uncertainty effects ratio of the variables considered for the comparator is given in 4.13.

| Company Data            |              |          |           | Date  | 15.03.2005 | Combined Uncertainty           |                      |                      |      |
|-------------------------|--------------|----------|-----------|---|------------|--------------------------------|----------------------|----------------------|------|
| Company Name EMRAH A.Ş. |              |          |           | Instrument Id   | Emr-10kg-1 | Eff. Degrees of Freedom        |                      |                      |      |
| Name                    | EMRAH        |          |           | Instrument Name                                       | 10kg       | Expanded Uncertainty           |                      |                      |      |
| Surname                 | ÇELEBIOĞLU   |          |           | Probability   | 95,45      | Coverage Factor                |                      |                      |      |
| Uncertainty Equation    |              |          |           | $m_s + \delta m_a + \delta m + \delta m_c + \delta B$ |            |                                | Nominal Value        |                      |      |
| Coefficient             | Symbol       | Function | Operation | Nominal Value   | Birim      | Evaluation Type                | Standard Uncertainty | Degrees of Freedom   | Undo |
| 1                       | $m_s$        |          | +         | 10000,005   | N/A        | TYPE B Expanded Uncertainty    | 0,0225               | 20000000000000000000 |      |
| 1                       | $\delta m_a$ |          | +         | 0   | N/A        | TYPE B Bounds and Distribut... | 0,00866              | 20000000000000000000 |      |
| 1                       | $\delta m$   |          | +         | 0,03  | N/A        | TYPE A                         | 0,00707              | 4                    |      |
| 1                       | $\delta m_c$ |          | +         | 0   | N/A        | TYPE B Bounds and Distribut... | 0,00577              | 20000000000000000000 |      |
| 1                       | $\delta B$   |          | +         | 0   | N/A        | TYPE B Bounds and Distribut... | 0,00577              | 20000000000000000000 |      |

Figure 4.10 Evaluation of Uncertainty with Using User Defined Menu

| Calculation Made By       |                              | Company Info  |                          | Type                 | User Defined    |                  |
|---------------------------|------------------------------|---|--------------------------|----------------------|-----------------|------------------|
| No                        | 2                            | Company Name  | EMRAH A.Ş.               | Ins. No              | Emr-10kg-1      |                  |
| Name                      | EMRAH                        | Name  | EMRAH                    | Ins. Name            | 10kg            |                  |
| Surname                   | ÇELEBIOĞLU                   | Surname   | ÇELEBIOĞLU               | Date                 | 15.03.2005      |                  |
| <b>Uncertainty Budget</b> |                              |   |                          |                      |                 |                  |
| Symbol                    | Nominal Va...                | Evaluation Type                                       | Sensitivity Co...        | Standard Un...       | Uncertainty ... | Degrees of Fr... |
| $m_s$                     | 10000,005                    | TYPE B Expanded Uncert...                             | 1                        | 0,0225               | 0,0225          | 2E+21            |
| $\delta m_a$              | 0                            | TYPE B Bounds and Distri...                           | 1                        | 0,00866              | 0,00866         | 2E+21            |
| $\delta m$                | 0,03                         | TYPE A  | 1                        | 0,00707              | 0,00707         | 4                |
| $\delta m_c$              | 0                            | TYPE B Bounds and Distri...                           | 1                        | 0,00577              | 0,00577         | 2E+21            |
| $\delta B$                | 0                            | TYPE B Bounds and Distri...                           | 1                        | 0,00577              | 0,00577         | 2E+21            |
| TOPLAM :                  | 10000,035                    |   |                          |                      |                 |                  |
| Uncertainty Equation      |                              | $m_s + \delta m_a + \delta m + \delta m_c + \delta B$ |                          |                      |                 |                  |
| Com. Standard Uncertainty | Effective Degrees of Freedom | Coverage Factor                                       | Probability Distribution | Expanded Uncertainty |                 |                  |
| 0,02642                   | 780,034                      | 2   | 95,45                    | 0,05284              |                 |                  |
| F3 - New                  | F4 - Goto Excel              | F5 - Plot Graph                                       | F6 - Print               | ESC - Quit           |                 |                  |

Figure 4.11 Result Screen of User Defined Menu

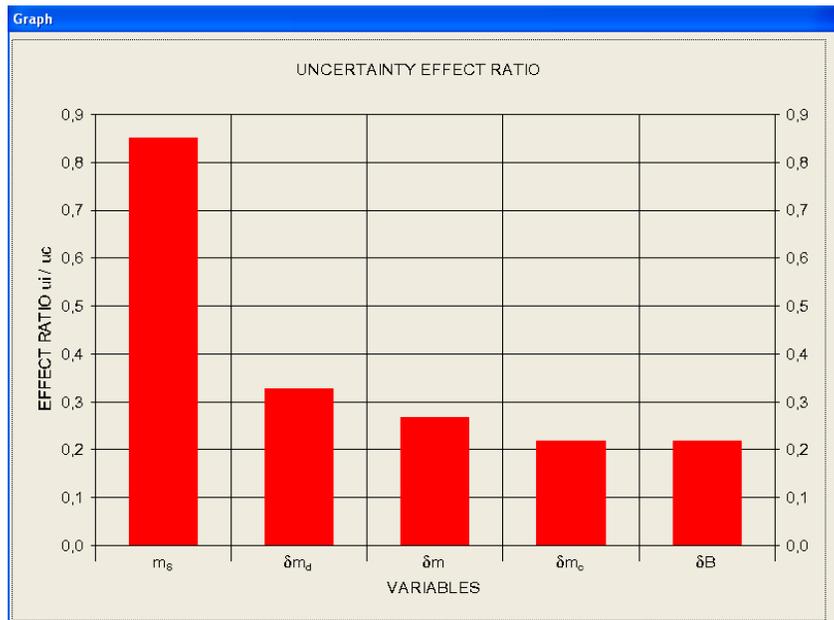


Figure 4.12 Graph of Measurement Uncertainty Effect Ratio for User Defined Menu



#### **4.7 Sample Evaluation of Uncertainty in Measurement for Calibration of a Micrometer of Nominal Length 100 mm**

Here evaluation of measurement uncertainty for a micrometer made of steel is calibrated against gauge blocks used as working standards. Several gauge blocks with nominal lengths in the range 0,5-100 mm are used in calibration. They are selected in such a way that the measurement points are spaced at nearly equal distances (e.g. at 0 mm, 50 mm, 100 mm, 150 mm). Before calibration, several checks of the condition of the micrometer are made. These include the quality of the measuring faces of the jaws (flatness, parallelism, squareness), and function of the locking mechanism and instrumental error will be found. Instrumental error is defined as a value that is obtained by subtracting the true value from the value that the micrometer indicates. First, the flatness of measuring faces is found. An optical flat or optical parallel is set in close contact with the measuring face and the minimum number of red interference fringes produced by light is noted. After this, parallelism of measuring faces is found. For finding it optical parallel or a combination of optical parallel and gauge block is set in contact with the measuring face of anvil until interference fringes get one color or closed curve appears. Then the number of red fringes produced by white light, appearing on the measuring face of the spindle under measurement force of micrometer is noted. Measurements are made at successive integral number of spindle turnings and at four places corresponding to the number of spindle turning where the revolution fraction equals to a multiple of  $\frac{1}{4}$  revolutions. The largest value of red fringes obtained is noted for obtaining parallelism the instrument under calibration. After these, instrumental error will be found. After zero setting of the instrument at the minimum measuring length, gauge blocks of different lengths are inserted in succession between the measuring faces of the micrometer under calibration. In each of above settings, the difference between the readings of the micrometer and the length of the gauge block under the normal measuring force is noted, which represents instrumental error of the micrometer under calibration. Micrometer measurement equation is equal to, taken from [20].

$$E_x = l_m - l_s + L * \alpha * \Delta t \quad (4.4)$$

where;

$l_s$  : Length of the reference gauge block at reference temperature  $t_0 = 20 \text{ }^\circ\text{C}$  according to calibration certificate. According to its calibration certificate as  $100,00 \text{ mm} \pm 0,8 \text{ }\mu\text{m}$ .

$l_m$  : Indication of the micrometer. The measurement is repeated several times without detecting any scatter in the observations. Thus, uncertainty due to limited repeatability does not give a contribution. The result of the measurement for the 100 mm gauge block is 100,05 mm.

$L$  : Nominal lengths of the gauge blocks considered.

$\alpha$  : Average of the thermal expansion coefficients of the unknown and the reference gauge blocks.  $11,5 \cdot 10^{-6} \pm 2 \cdot 10^{-6} \text{ }^\circ\text{C}^{-1}$  with triangular limits.

$\Delta t$  : Deviation of the average temperature of the unknown and the reference gauge blocks. The deviation of the mean temperature of measurement from the reference temperature is estimated to be within  $\pm 0,05 \text{ }^\circ\text{C}$  with rectangular limits.

Table 4.4 Uncertainty Budget for Micrometer [20]

| Quantity   | Estimate                        | Standard Uncertainty    | Probability Distribution | Sensitivity Coefficient | Degrees of Freedom | Uncertainty Contribution |
|------------|---------------------------------|-------------------------|--------------------------|-------------------------|--------------------|--------------------------|
| $l_s$      | 100,00mm                        | 0,4 $\mu\text{m}$       | Normal                   | -1                      | $\infty$           | - 0,4 $\mu\text{m}$      |
| $l_m$      | 100,05 mm                       | -----                   | -----                    | -----                   | -----              | -----                    |
| $\alpha$   | 0,0000115 $^\circ\text{C}^{-1}$ | $0,816 \cdot 10^{-6}$   | Triangular               | 50                      | $\infty$           | 0,040 $\mu\text{m}$      |
| $\Delta t$ | 0,5 $^\circ\text{C}$            | 0,0288 $^\circ\text{C}$ | Rectangular              | 0,00115                 | $\infty$           | 0,03 $\mu\text{m}$       |
| $E_x$      | 0,05 mm                         |                         |                          |                         |                    | 0,40 $\mu\text{m}$       |

From t distribution table, for the confidence level of %95,45, the coverage factor is 2. So the expanded uncertainty becomes

$$U = k * u(E_x) = 2 * 0,403 = 0,80 \mu\text{m}$$

Thus, the measured value of the nominal 100 mm micrometer is  $0,05 \text{ mm} \pm 0,80 \mu\text{m}$ .

For sample calibration of a micrometer of nominal Length 100, the uncertainty evaluation input screen and the results screen are given in Figures 4.14 and 4.15, respectively. The uncertainty effect of the each variable considered on the overall measurement uncertainty of the comparator is expressed in the form of ratio as given in Figure 4.16. The Excel report screen which gives the uncertainty budget together with the uncertainty effects ratio of the variables considered for the comparator is given in 4.17.

| Company Name |  | EMRAH A.Ş. |  |
|--------------|--|------------|--|
| Name         |  | EMRAH      |  |
| Surname      |  | CELEBIOĞLU |  |

Instrument Id: emr-mik-100-1 | Date: 03.04.2005

Nominal Values

|          |           |                  |       |     |    |
|----------|-----------|------------------|-------|-----|----|
| $l_m$    | 100,05    | mm               | $t_m$ | 0,5 | °C |
| $l_r$    | 100       | mm               |       |     |    |
| L        | 100       | mm               |       |     |    |
| $\alpha$ | 0,0000115 | °C <sup>-1</sup> |       |     |    |

Uncertainty Function:  $l_m - l_r + L \cdot \alpha \cdot \Delta t$

Coverage Probability: 95,45

TYPE A

n = 5

|                |          |          |
|----------------|----------|----------|
| Value1: 100,05 | Value6:  | Value11: |
| Value2: 100,05 | Value7:  | Value12: |
| Value3: 100,05 | Value8:  | Value13: |
| Value4: 100,05 | Value9:  | Value14: |
| Value5: 100,05 | Value10: | Value15: |
| Value16:       | Value21: |          |
| Value17:       | Value22: |          |
| Value18:       | Value23: |          |
| Value19:       | Value24: |          |
| Value20:       | Value25: |          |

Typ B Hesaplamaları

|            | Degrees of Freedom | Evaluation Type             | Standard Uncertainty |
|------------|--------------------|-----------------------------|----------------------|
| $l_m$      | $\infty$           | Expanded Uncertainty and C  | 0,0004               |
| $\Delta t$ | $\infty$           | Bounds and Distribution Typ | 0,0288               |
| $\alpha_m$ | $\infty$           | Bounds and Distribution Typ | 0,000001             |

F3 - Calculate | F4 - Clear | ESC - Quit

Figure 4.14 Uncertainty Evaluation Screen for Micrometer

| No         |  | Company Info |  | Type                        |  |
|------------|--|--------------|--|-----------------------------|--|
| 2          |  | EMRAH A.Ş.   |  | Micrometer                  |  |
| EMRAH      |  | EMRAH        |  | Instrument No emr-mik-100-1 |  |
| ÇELEBİOĞLU |  | ÇELEBİOĞLU   |  | Date 03.04.2005             |  |

| Uncertainty Budget |               |  |                         |                      |                          |                    |
|--------------------|---------------|--|-------------------------|----------------------|--------------------------|--------------------|
| Variable           | Nominal Value | Evaluation Type                          | Sensitivity Coefficient | Standard Uncertainty | Uncertainty Contribution | Degrees of Freedom |
| $l_r$              | 100,05        | TYPE A                                   | 1                       | 0                    | 0                        | 4                  |
| $l_m$              | 100           | TYPE B Expanded Uncertainty and Coverage | 1                       | 0,0004               | 0,0004                   | $\infty$           |
| $t_m$              | 0,5           | TYPE B Bounds and Distribution Type      | 0,001                   | 0,0288               | 0,00003                  | $\infty$           |
| $\alpha_m$         | 0,0000115     | TYPE B Bounds and Distribution Type      | 50                      | 0,000001             | 0,00005                  | $\infty$           |
| $E(x)$             | 0,05          |  |                         |                      |                          |                    |

| Combined Uncertainty | Degrees of Freedom | Coverage Factor | Coverage Probability | Expanded Uncertainty |
|----------------------|--------------------|-----------------|----------------------|----------------------|
| 0,0004               | $\infty$           | 2               | 95,45                | 0,0008               |

|           |          |                 |                 |            |           |            |
|-----------|----------|-----------------|-----------------|------------|-----------|------------|
| F2 - Back | F3 - New | F4 - Goto Excel | F5 - Plot Graph | F6 - Print | F1 - Save | ESC - Quit |
|-----------|----------|-----------------|-----------------|------------|-----------|------------|

Figure 4.15 Uncertainty Evaluation Results Screen for Micrometer

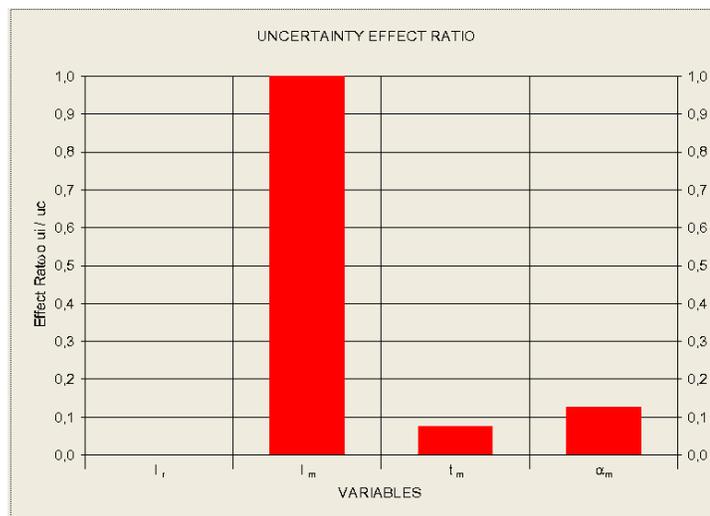


Figure 4.16 Graph of Measurement Uncertainty Effect Ratio for Micrometer

## UNCERTAINTY BUDGET

**Type** : Micrometer  
**Ins. No** : emr-mik-100-1  
**Date** : 03.04.2005

| Evaluation Made By |              | Company Info    |              |
|--------------------|--------------|-----------------|--------------|
| <b>No</b>          | : 2          | <b>Com. Nam</b> | : EMRAH A.Ş. |
| <b>Name</b>        | : EMRAH      | <b>Name</b>     | : EMRAH      |
| <b>Surname</b>     | : ÇELEBİOĞLU | <b>Surname</b>  | : ÇELEBİOĞLU |

| Variable   | Nominal Value | Evaluation Type                                 | Sensitivity Coefficient | Standard Uncertainty | Uncertainty Contribution | Degrees of Freedom |
|------------|---------------|---|-------------------------|----------------------|--------------------------|--------------------|
| $l_m$      | 100,05        | TYPE A  | 1                       | 0                    | 0                        | 4                  |
| $l_a$      | 100           | TYPE B Expanded Uncertainty and Coverage Factor | 1                       | 0,0004               | 0,0004                   | $\infty$           |
| $\Delta t$ | 0,5           | TYPE B Bounds and Distribution Type             | 0,001                   | 0,0288               | 0,0000288                | $\infty$           |
| $\alpha$   | 0,00001       | TYPE B Bounds and Distribution Type             | 50                      | 0,000001             | 0,00005                  | $\infty$           |
|            |               |   |                         |                      |                          |                    |
| $E(x)$     | 0,05          |   |                         |                      |                          |                    |

| Com. Stan. Uncer. | Eff. Degrees of Freedom | Coverage Factor | Coverage Probability | Expanded Uncertainty |
|-------------------|-------------------------|-----------------|----------------------|----------------------|
| 0,0004            | $\infty$                | 2               | 95,45                | 0,0008               |

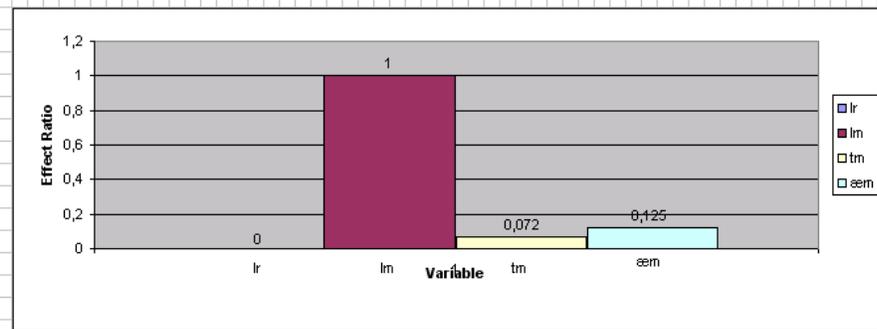


Figure 4.17 Excel Report Screen for Micrometer

## 4.8 Sample Calculation of Uncertainty in Measurement for Calibration of a Comparator

In this section, evaluation of the uncertainty of a comparator is described. The calibration of the comparator is carried out by comparison, using a calibrated comparator and a calibrated gauge block of the same nominal length and the same material as reference standard. The difference in central length is determined in vertical position of the two gauge blocks using two length indicators contacting upper and lower measuring faces. The uncertainty equation according to [21];

$$E_x = -l_s - l_{ix} + L_s * \alpha * \Delta t + \delta l_{ix} + \delta l_m \quad (4.5)$$

where;

$l_{ix}$  : Indication of the comparator.

$l_s$  : Length of the actual gauge block. From the calibration certificates of the gauge block the central length of the gauge block coincides with in  $\pm 0,8 \mu\text{m}$  with the rectangular limits.

$L_s$  : Nominal length of the actual gauge block

$\alpha$  : Average thermal expansion coefficient of the comparator and the gauge block. The average thermal expansion coefficient is  $11,5 * 10^{-6} \text{ }^\circ\text{C}^{-1}$

$\Delta t$  : Difference in temperature between the comparator and the gauge block. After adequate stabilization time, the temperatures of the comparator and the gauge block are equal within  $\pm 0,5 \text{ }^\circ\text{C}$  with rectangular limits.

$\delta l_{ix}$  : Correction due to the finite resolution of the comparator . Resolution error is estimated to have rectangular limits of  $\pm 0,02 \mu\text{m}$

$\delta l_m$  : Correction due to mechanical effects, such as applied measurement force, Abbe error, flatness and parallelism errors of the measurement faces. The total variation for the mechanical effects estimated to have rectangular limits of a  $\pm 0,05 \mu\text{m}$ .

Table 4.5 Uncertainty Budget for Comparator [21]

| Quantity        | Estimate      | Standard Uncertainty | Probability Distribution | Sensitivity Coefficient | Degrees of Freedom | Uncertainty Contribution |
|-----------------|---------------|----------------------|--------------------------|-------------------------|--------------------|--------------------------|
| $l_{ix}$        | -0,000016     | 0,000004 mm          | Normal                   | -1                      | 4                  | -0,000004                |
| $l_s$           | 150,00002     | 0,00046 mm           | Rectangular              | 1                       | $\infty$           | 0,00046                  |
| $\Delta t$      | 0             | 0,288                | Rectangular              | 0,0018                  | $\infty$           | 0,00051                  |
| $\delta l_{ix}$ | 0             | 0,00001 mm           | Rectangular              | 1                       | $\infty$           | 0,00001                  |
| $\delta l_m$    | 0             | 0,000028 mm          | Rectangular              | 1                       | $\infty$           | 0,000028                 |
| $E_x$           | 150,000004 mm |                      |                          |                         |                    | 0,00069                  |

From t-distribution table, for the confidence level of 95%, the coverage factor is  $k = 1,96$ . So the expanded uncertainty for this instrument;

$$U(E_x) = 0,00136 \text{ mm}$$

For sample calculation of uncertainty in measurement for calibration of a comparator, the uncertainty evaluation input screen and the results screen are given in Figures 4.18 and 4.19, respectively. The uncertainty effect of the each variable considered on the overall measurement uncertainty of the comparator is expressed in the form of ratio as given in Figure 4.20. The Excel report screen which gives the uncertainty budget together with the uncertainty effects ratio of the variables considered for the comparator is given in 4.21.

Company Name: EMRAH A.Ş.  
 Name: EMRAH  
 Surname: ÇELEBİOĞLU

Instrument Id: 1      Date: 03.04.2005

Nominal Values

|          |           |    |                 |                  |    |
|----------|-----------|----|-----------------|------------------|----|
| $l_{ix}$ | -0,000016 | mm | $\Delta t$      | 0                | °C |
| $l_s$    | 150,00002 | mm | $\delta l_{ix}$ | 0                | mm |
| L        | 150       | mm | $\delta l_m$    | 0                | mm |
| $\alpha$ | 0,0000115 |    |                 | °C <sup>-1</sup> |    |

Uncertainty Function:  $l_s - l_{ix} - L * \alpha * \Delta t + \delta l_{ix} + \delta l_m$   
 Coverage Probability: 95,45

TYPE A

n = 5      Ok

|          |          |          |  |          |  |
|----------|----------|----------|--|----------|--|
| Value1:  | -0,00002 | Value6:  |  | Value11: |  |
| Value2:  | -0,00001 | Value7:  |  | Value12: |  |
| Value3:  | -0,00003 | Value8:  |  | Value13: |  |
| Value4:  | -0,00001 | Value9:  |  | Value14: |  |
| Value5:  | -0,00001 | Value10: |  | Value15: |  |
| Value16: |          | Value21: |  |          |  |
| Value17: |          | Value22: |  |          |  |
| Value18: |          | Value23: |  |          |  |
| Value19: |          | Value24: |  |          |  |
| Value20: |          | Value25: |  |          |  |

Type Evaluation Side

|                 | Degrees of Freedom | Evaluation Type              | Standard Uncertainty |
|-----------------|--------------------|------------------------------|----------------------|
| $l_s$           | $\infty$           | Expanded Uncertainty and C   | 0,000461             |
| $\Delta t$      | $\infty$           | Bounds and Distribution Type | 0,288                |
| $\delta l_{ix}$ | $\infty$           | Bounds and Distribution Type | 0,000029             |
| $\delta l_m$    | $\infty$           | Bounds and Distribution Type | 0,000011             |

Figure 4.18 Uncertainty Evaluation Screen for Comparator

Company Info

|         |            |
|---------|------------|
| No      | 2          |
| Name    | EMRAH      |
| Surname | ÇELEBİOĞLU |

|              |            |
|--------------|------------|
| Company Name | EMRAH A.Ş. |
| Name         | EMRAH      |
| Surname      | ÇELEBİOĞLU |

|               |               |
|---------------|---------------|
| Type          | Comparator    |
| Instrument No | Emr-kom-150-1 |
| Date          | 03.03.2005    |

### Uncertainty Budget

| Variable        | Nominal Value     | Evaluation Type                          | Sensitivity Coefficient | Standard Uncertainty | Uncertainty Contribution | Degrees of Freedom |
|-----------------|-------------------|--|-------------------------|----------------------|--------------------------|--------------------|
| $l_{ix}$        | -0,000016         | TYPE A                                   | -1                      | 0,000004             | -0,000004                | 4                  |
| $l_s$           | 150,00002         | TYPE B Expanded Uncertainty and Coverage | 1                       | 0,000461             | 0,000461                 | $\infty$           |
| $\Delta t$      | 0                 | TYPE B Bounds and Distribution Type      | 0,0018                  | 0,288                | 0,000518                 | $\infty$           |
| $\delta l_{ix}$ | 0                 | TYPE B Bounds and Distribution Type      | 1                       | 0,000029             | 0,000029                 | $\infty$           |
| $\delta l_m$    | 0                 | TYPE B Bounds and Distribution Type      | 1                       | 0,000011             | 0,000011                 | $\infty$           |
| <b>E(x)</b>     | <b>150,000036</b> |  |                         |                      |                          |                    |

|                      |                    |                 |                      |                      |
|----------------------|--------------------|-----------------|----------------------|----------------------|
| Combined Uncertainty | Degrees of Freedom | Coverage Factor | Coverage Probability | Expanded Uncertainty |
| 0,000694             | $\infty$           | 1,96            | 95                   | 0,00136              |

F2 - Back    F3 - New    F4 - Goto Excel    F5 - Plot Graph    F6 - Print    F1 - Save    ESC - Quit

Figure 4.19 Uncertainty Evaluation Results Screen for Comparator

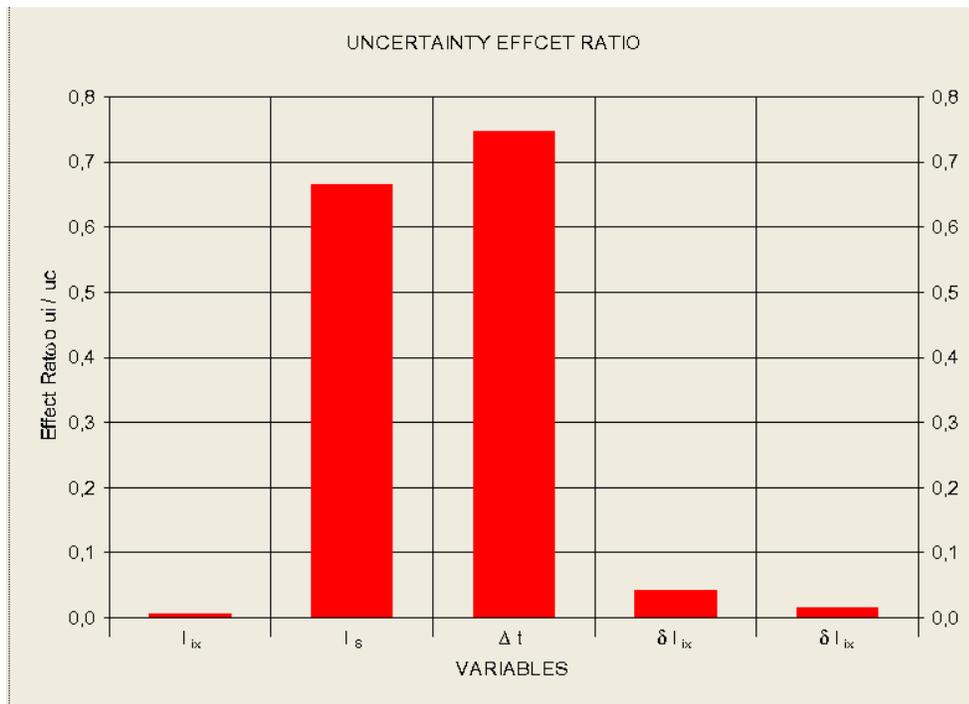


Figure 4.20 Graph of Measurement Uncertainty Effect Ratio for Comparator

## UNCERTAINTY BUDGET

Type : Comparator  
 Ins. Id : Emr-kom-150-1  
 Date : 08.03.2005

| Evaluation Made By |              | Company Info |              |
|--------------------|--------------|--------------|--------------|
| NO                 | : 2          | Com. Nam     | : EMRAH A.Ş. |
| Name               | : EMRAH      | Name         | : EMRAH      |
| Surname            | : ÇELEBİOĞLU | Surname      | : ÇELEBİOĞLU |

| Variable        | Nominal Value | Evaluation Type                                 | Sensitivity Coefficient | Standard Uncertainty | Uncertainty Contribution | Degrees of Freedom |
|-----------------|---------------|---|-------------------------|----------------------|--------------------------|--------------------|
| $I_{ix}$        | -0,00002      | TYPE A  | -1                      | 0,00004              | -0,00004                 | 4                  |
| $I_a$           | 150,00002     | TYPE B Expanded Uncertainty and Coverage Factor | 1                       | 0,00046              | 0,00046                  | $\infty$           |
| $\Delta t$      | 0             | TYPE B Bounds and Distribution Type             | 0,0018                  | 0,288                | 0,0005184                | $\infty$           |
| $\delta I_{ix}$ | 0             | TYPE B Bounds and Distribution Type             | 1                       | 0,00003              | 0,00003                  | $\infty$           |
| $\delta I_m$    | 0             | TYPE B Bounds and Distribution Type             | 1                       | 0,00001              | 0,00001                  | $\infty$           |
| E(x)            | 150,00004     |   |                         |                      |                          |                    |

| Com. Stan. Uncer. | Eff. Degrees of Freedom | Coverage Factor | Coverage Probability | Expanded Uncertainty |
|-------------------|-------------------------|-----------------|----------------------|----------------------|
| 0,00069           | $\infty$                | 1,96            | 95                   | 0,00136              |

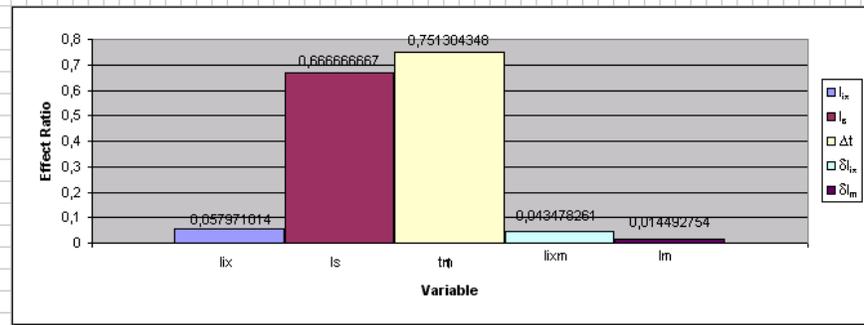


Figure 4.21 Excel Report Screen for Comparator

## CHAPTER 5

### AN EXPERIMENT ABOUT EVALUATING UNCERTAINTY IN DIMENSIONAL MEASUREMENT

In this chapter, an experiment for evaluating the uncertainty in measurement of a vernier calliper, and micrometer will be given. After evaluating the uncertainty in measurement, the results will be discussed.

#### 5.1 Instruments Used in Experiments

In the experiments, an external micrometer with 25 mm range is used. Typical micrometer has an anvil face that is the reference point, and a spindle face that is the measuring point. An accurate screw (40 threads per inch for British Standards and 0.5 mm pitch length for metric standards), revolving in affixed nut varies the distance between the spindle and the anvil. The scale on barrel makes it possible to read 0.01 mm discrimination properly.

Also in the experiments, M type 300-mm vernier calliper with 1/20 resolution, CM type 300-mm vernier calliper with 1/50 resolution and a digital vernier calliper are used. Various gauge blocks are used for reference standard.

#### 5.2 Analysis of Experimental Data

The measurements are carried out by a group of students in ME 433 course in the metrology laboratory of Mechanical Engineering Department, METU. Before the students take the measurements on the part, they have gained some experience by making many measurements with vernier callipers and micrometers on different parts. Each student measured four basic dimensions of the same part given in Figure 5.1. Micrometer is fixed to an apparatus table in order to reduce the measurement reading

errors [22]. During each measurement, the temperature of the test part and the instrument are measured. The data taken from the measurements are given in Appendix F.

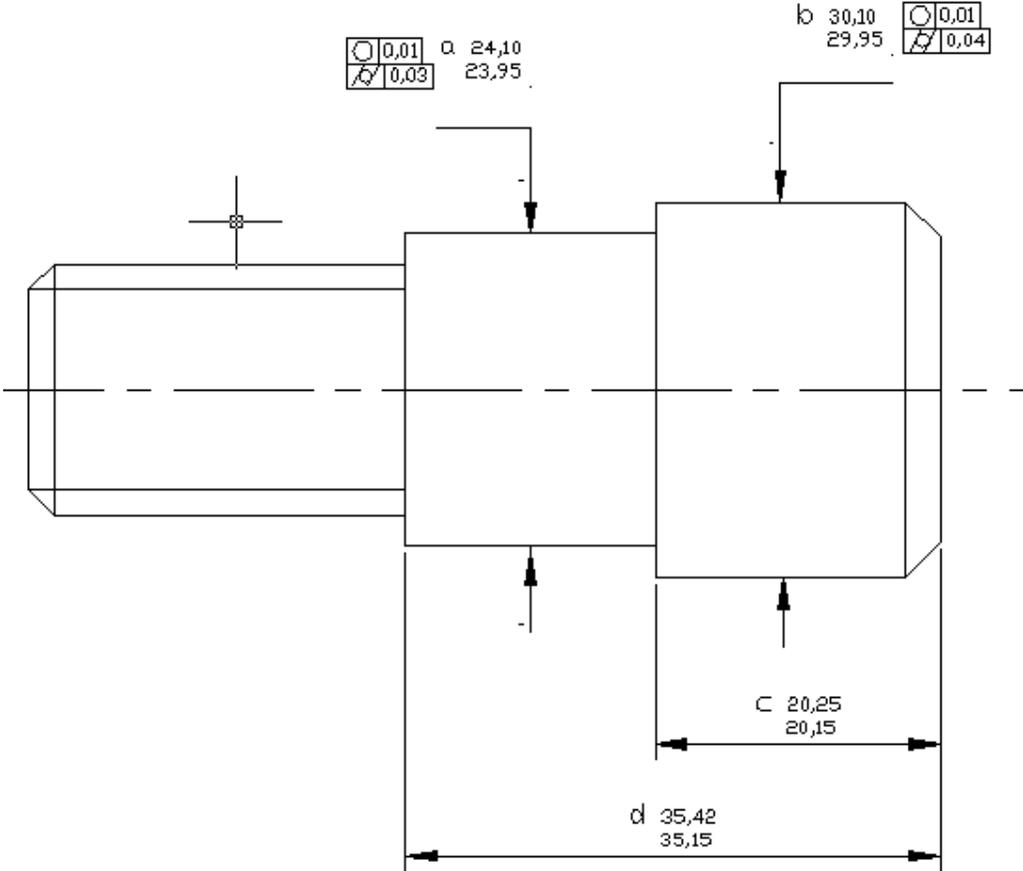


Figure 5.1 Dimensions Measured on the Test Part

Before evaluating the measurement uncertainty, the corrections must be made. Firstly, the cylindricity error was measured by using a pasameter and found to be 0,033 mm. This error in measurements was found by measuring the base diameter and tip diameter of the part a, b, c, d dimension. Then by using triangle rule shown

in Figure 5.2, the error of each measurement data is found and then the corrections are made.

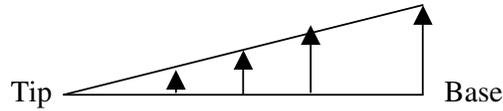


Figure 5.2 Application of Triangle Rule to the Measurements

The error correction procedure is as follows:

- Firstly, the number of different values of the repeated measurements on each dimension was determined. For example, for the vernier calliper with 1/20 resolution and part dimension “a”, the only measurement results are 24.00, 24.05, 24.10, and 24.15 mm. Here only four different measurements exist.
- The total cylindricity error is divided by the number of different values of the repeated measurements on each dimension measured, in order to find the error in each measurement step. For example, for the vernier calliper with 1/20 resolution and part dimension a, it is equal to  $0,033 / 4 = 0,00825$  mm
- The effect of cylindricity error of different values of the repeated measurements on each dimension was found. Here, it is assumed that the smallest measurement result is the tip diameter and the largest measurement result is the base diameter of the part. For example, for the vernier calliper with 1/20 resolution and the part dimension a, by using the triangle rule, the corrected values can be calculated as;

$$24 - 0,00825 = 23,992$$

$$24,05 - 2*(0,00825)=24,034$$

$$24,10 - 3*(0,00825)=24,075$$

$$24,15 - 4*(0,00825)=24,117$$

The uncertainty evaluations are made by using the corrected values of the experimental data that are given in Appendix F.

### 5.3 Evaluation of Uncertainty in Measurement for the Experimental Data

After the corrections made, the uncertainty of each measurement can be evaluated. For the measuring instruments, the uncertainty equation is taken to be the same;

$$U(x) = I_m + \delta I_i + L_s * \alpha * \Delta t \quad (5.1)$$

where;

$I_m$  : Observed values taken from independent observations. The main sources of measurement uncertainty taken from independent observations are the applied force, Abbe error, flatness and parallelism errors of the measurement faces, the measurement readings at the root and tip of measuring faces, operator misreading, measured part uncertainty.

$\delta I_i$  : Uncertainty of the measuring instruments. For the vernier calliper with 1/20 resolution and with 1/50 resolution, the expanded uncertainties are obtained from the calibration certificates as 10  $\mu\text{m}$  and for the digital micrometer used the expanded uncertainty is equal to 1.5  $\mu\text{m}$  with the coverage factor of 2.

$L$  : Nominal length of the measured part and has no uncertainty.

$\alpha$  : Average thermal expansion coefficient of the part  $11.5 * 10^{-6} \text{ } ^\circ\text{C}^{-1}$  and measuring instrument  $11.5 * 10^{-6} \text{ } ^\circ\text{C}^{-1}$  and has no uncertainty.

$\Delta t$  : Difference in temperature between the part and measuring instrument. The difference in temperature between the part and measuring instrument is assumed to be zero with rectangular limits  $\pm 1 \text{ } ^\circ\text{C}$ .

The uncertainty values are evaluated by using the computer program developed and the expanded uncertainties of each reading with different instruments are given in Table 5.1

Table 5.1 Uncertainty Values of the Measured Dimensions on the Test Part

| INSTRUMENT               | DIMENSION |           |           |           |
|--------------------------|-----------|-----------|-----------|-----------|
|                          | a         | b         | c         | d         |
| Vernier Calliper 1/20    | 0,0199 mm | 0,0124 mm | 0,0179 mm | 0,0296 mm |
| Vernier Calliper 1/50    | 0,0203 mm | 0,0122 mm | 0,0188 mm | 0,0179 mm |
| Digital Vernier Calliper | 0,0159 mm | 0,0138 mm | 0,0150 mm | 0,0205 mm |
| Micrometer               | 0,0065 mm | -----     | 0,0129 mm | -----     |

The uncertainty of a measurement is very important in decision-making process for rejection or acceptance of the part. The effect of uncertainties on the measurements can be shown clearly in graphical form. Figures 5.3 to 5.16 show the number of repeating for the dimensions measured and the instruments used. The readings on the measured dimensions are divided into equal intervals. The numbers of readings in those dimension intervals are plotted. On the same graphs, the upper and lower tolerance limits (UTL and LTL) together with the evaluated uncertainty values above and below the tolerance limits are shown on each dimension. In the graphs red, green, and the blue lines denote;

-  Mean Value
-  Lower and Upper Tolerance Limits of the Measured Dimension
-  Uncertainty Above and Below the Tolerance Limits

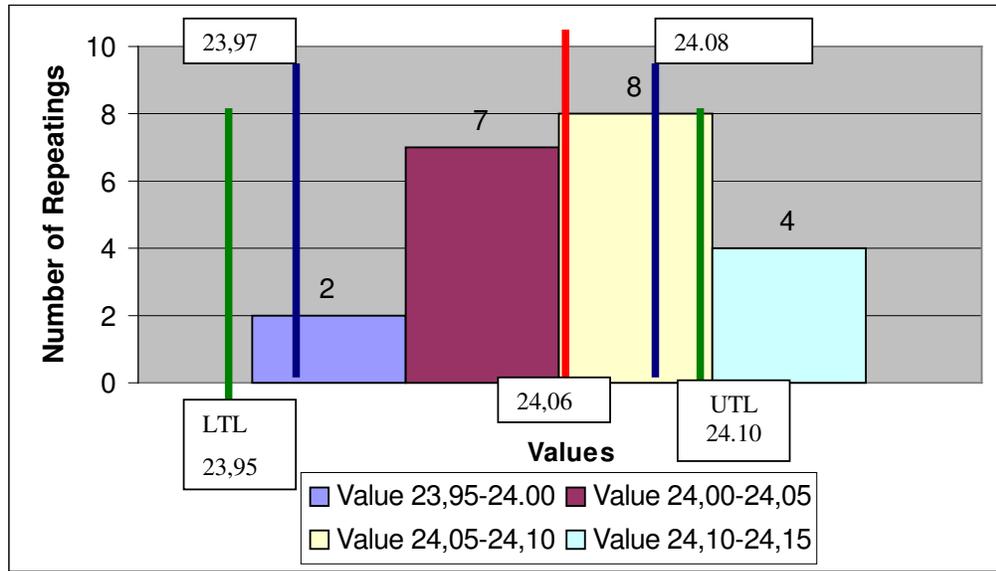


Figure 5.3 Uncertainty in the Measurement of Dimension “a” Using Vernier Calliper with 1/20 Resolution

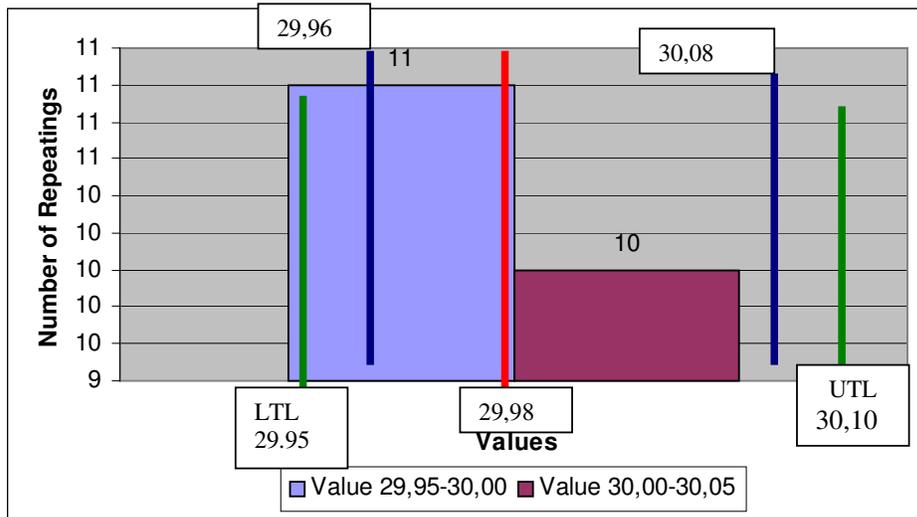


Figure 5.4 Uncertainty in the Measurement of Dimension “b” Using Vernier Calliper with 1/20 Resolution

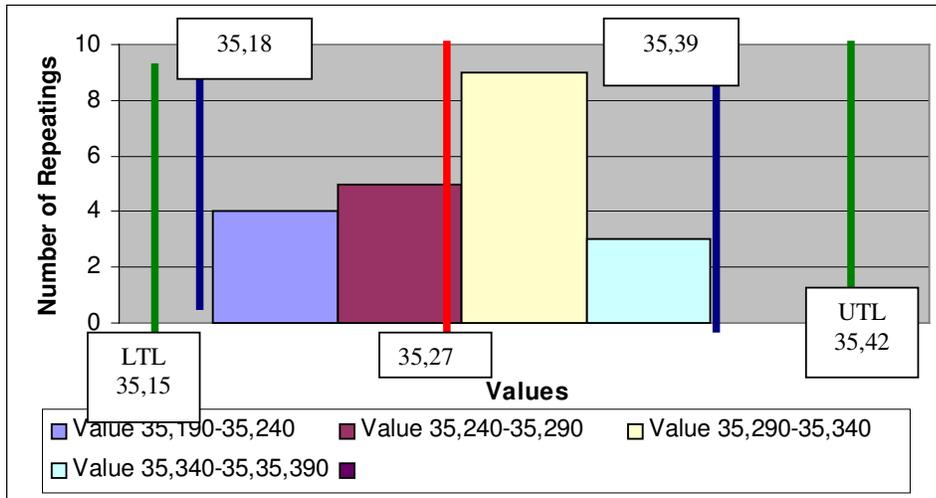


Figure 5.5 Uncertainty in the Measurement of Dimension “d” Using Vernier Calliper with 1/20 Resolution

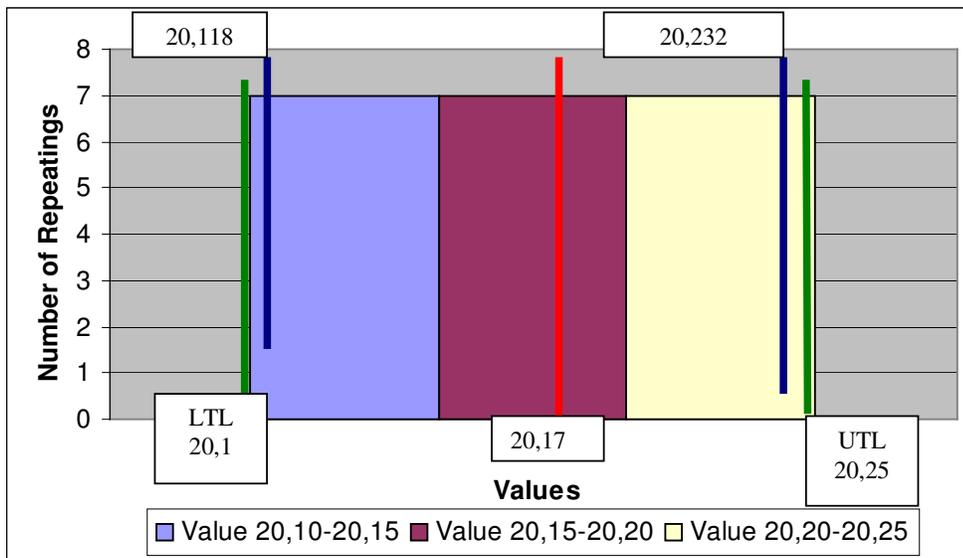


Figure 5.6 Uncertainty in the Measurement of Dimension “c” Using Vernier Calliper with 1/20 Resolution

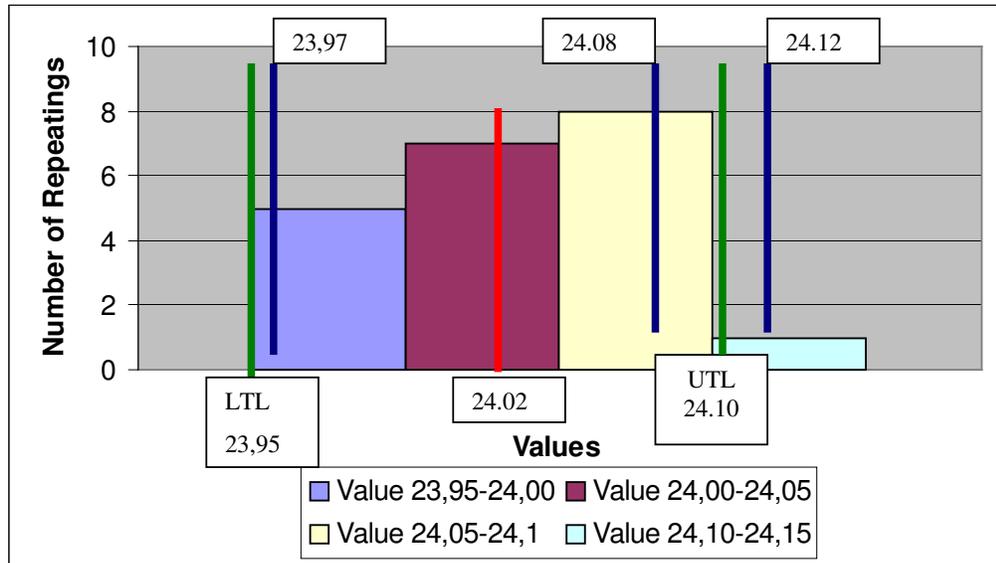


Figure 5.7 Uncertainty in the Measurement of Dimension “a” Using Vernier Calliper with 1/50 Resolution

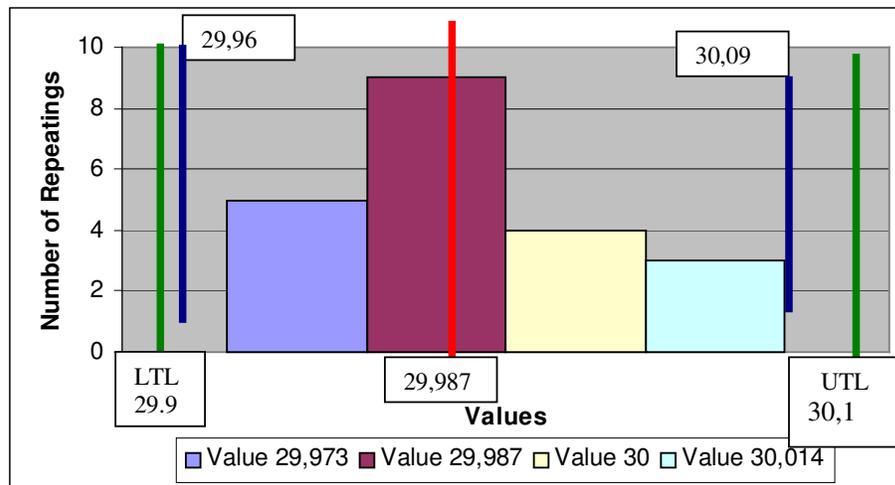


Figure 5.8 Uncertainty in the Measurement of Dimension “b” Using Vernier Calliper with 1/50 Resolution

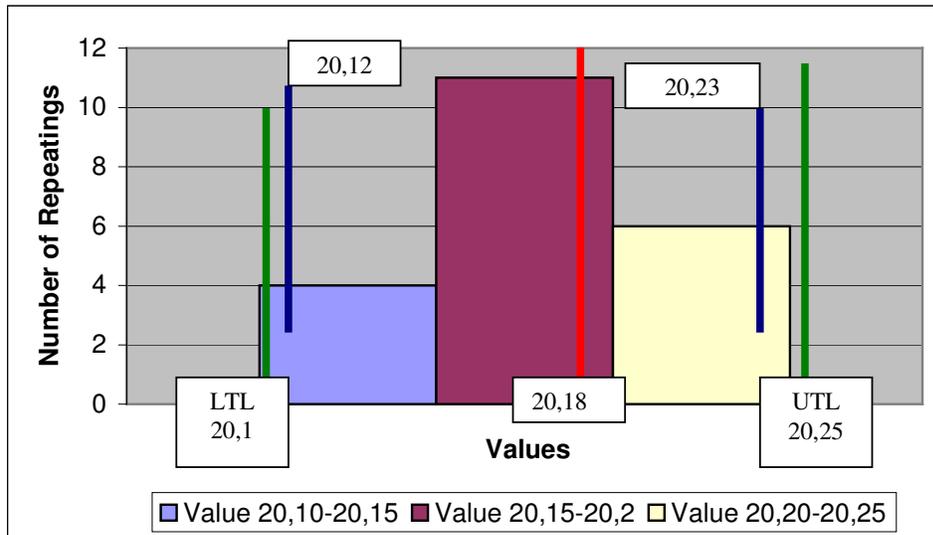


Figure 5.9 Uncertainty in the Measurement of Dimension “c” Using Vernier Calliper with 1/50 Resolution

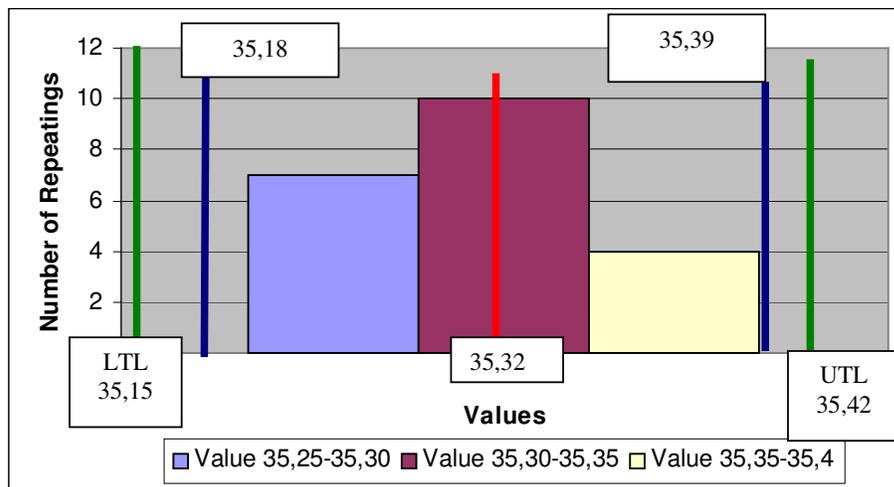


Figure 5.10 Uncertainty in the Measurement of Dimension “d” Using Vernier Calliper with 1/50 Resolution

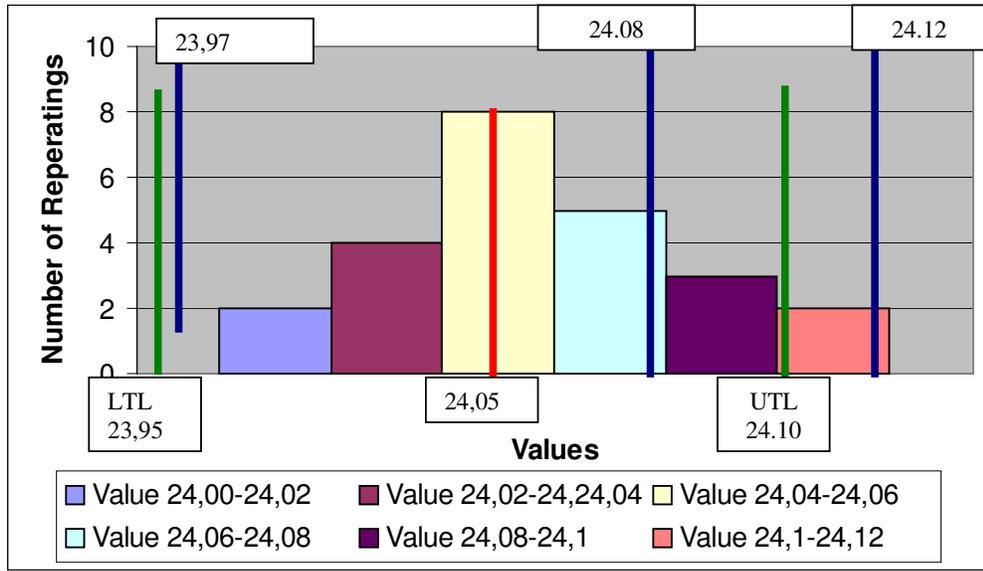


Figure 5.11 Uncertainty in the Measurement of Dimension “a” Using Digital Vernier Calliper

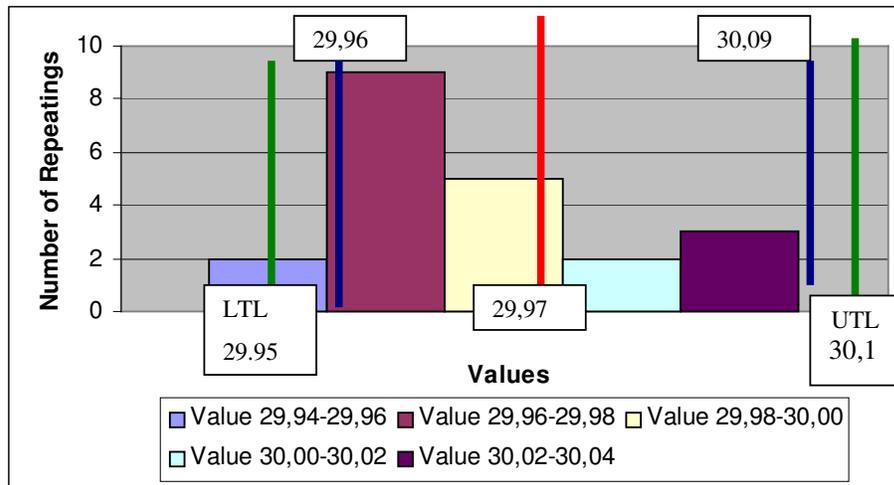


Figure 5.12 Uncertainty in the Measurement of Dimension “b” Using Digital Vernier Calliper

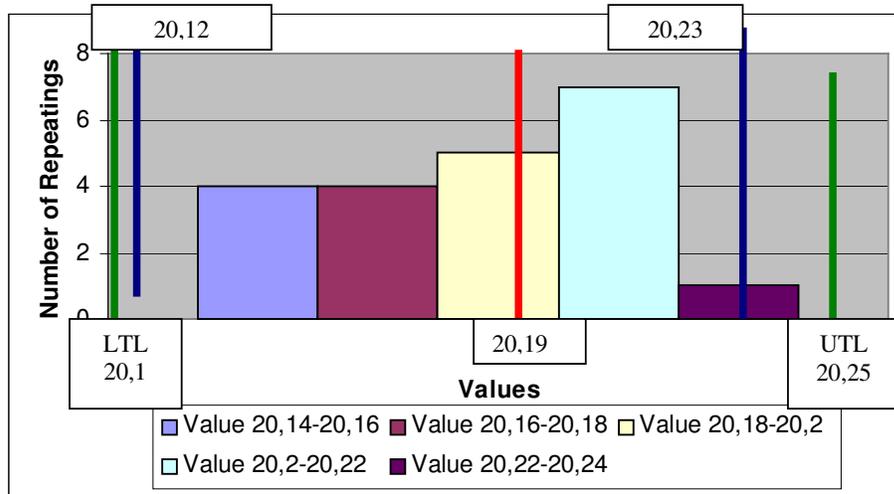


Figure 5.13 Uncertainty in the Measurement of Dimension “c” Using Digital Vernier Calliper

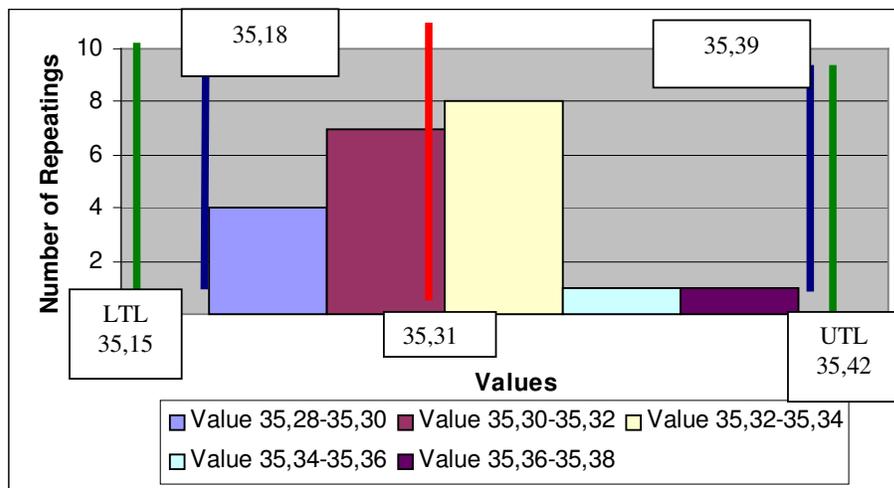


Figure 5.14 Uncertainty in the Measurement of Dimension “d” Using Digital Vernier Calliper

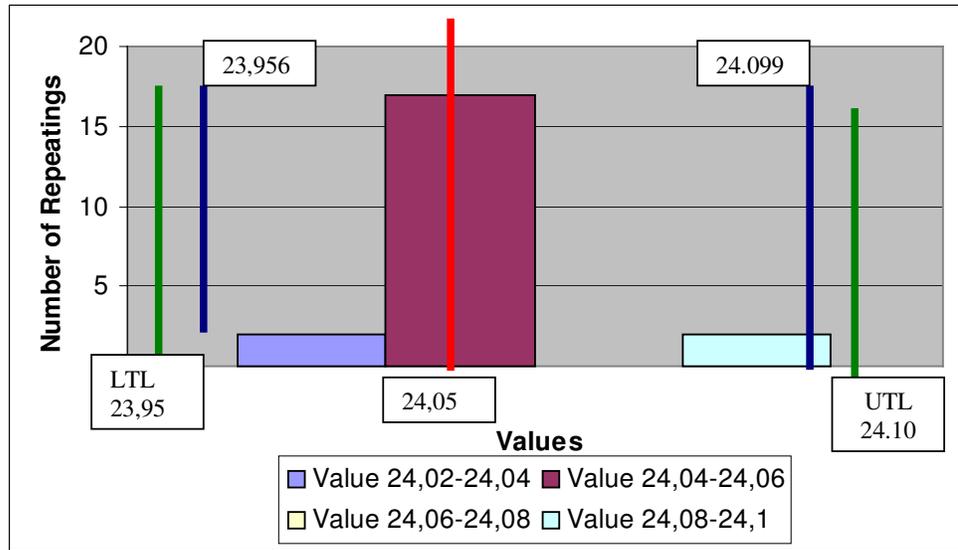


Figure 5.15 Uncertainty in the Measurement of Dimension “a” Using Micrometer

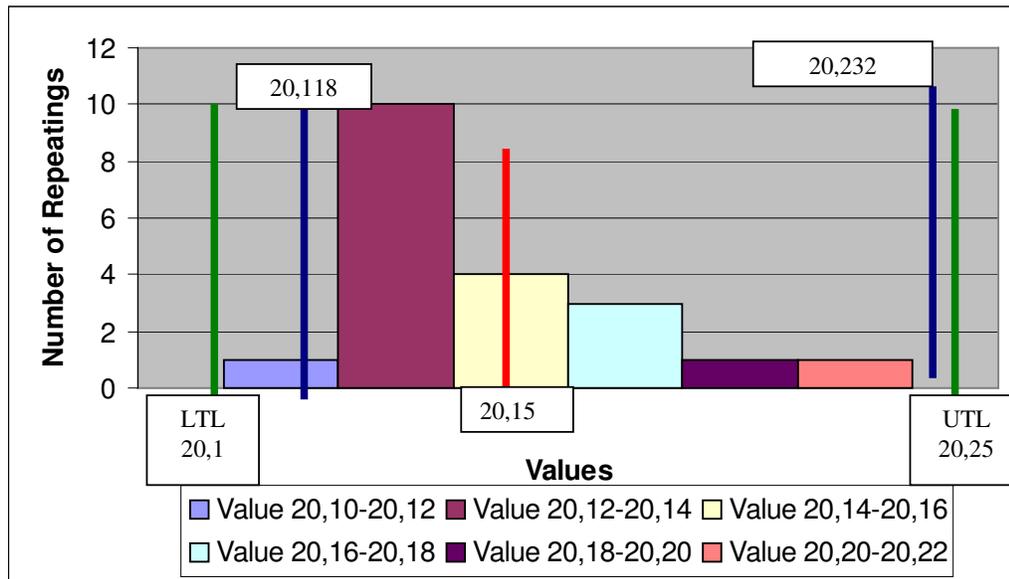


Figure 5.16 Uncertainty in the Measurement of Dimension “c” Using Micrometer

#### 5.4 Discussion of the Results

In all measurements of the dimensions a, b, c, and d of the test part given above, except the dimension given in Figures 5.1, 5.3, 5.7 5,11, 5.12, there is not a decision problem for the operator, since the calculated uncertainty limits are within the upper and lower tolerance limits of the dimension given on the engineering drawing. All the readings are in the conformance zone defined by the uncertainty in measurements and UTL and LTL of the measured dimension. However, there are decision-making problems in the other readings [23]. As it can clearly be seen from Figure 5.3, there is a problem due to the calculated measurement uncertainty of 0.02 mm, because the upper tolerance limit is 24.10 mm, the lower tolerance limit is 23.95 mm, and the 24.117 mm reading is out of the tolerance range. The person who made the measurement must decide to accept or to reject the part with the dimension 24.117 mm reading according to ISO 14253-1 “Decision rules for proving conformance or non-conformance with specifications”. ISO 14253-1 contains decision rules that require the tolerances to be reduced by the measuring uncertainty when measurements are made to prove conformance to a specification and expanded by the measuring uncertainty when attempting to prove non-conformance to a specification as it can be seen from Figure 5.17. The rules, basically state that a supplier has to subtract his measuring uncertainty from the tolerance limits to find the interval in which he can prove conformance with specification. The rules also state that a customer has to add the uncertainty to the tolerance in order to find the interval that has to be exceeded in order to prove non-conformance.

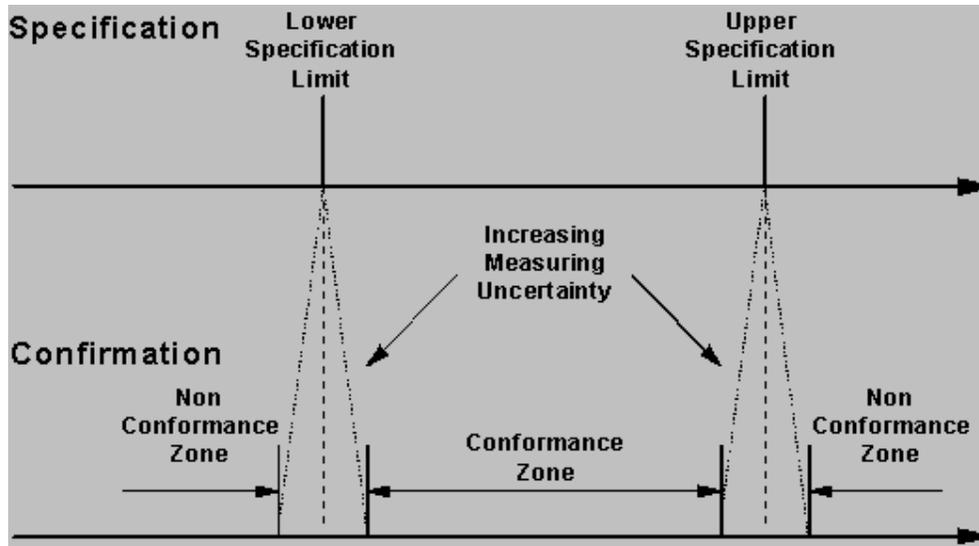


Figure 5.17 Conformance - Non Conformance Zones According to ISO 14352-1

Similarly, in Figure 5.7 the upper tolerance limit 24.10 mm and the lower tolerance limit 23.95 mm and the uncertainty is 0.02 mm. There is not a problem in the lower tolerance limit, but in the upper tolerance limit 24.091 mm, 24.107 mm reading is out of the tolerance range. The person who made the measurement must decide to accept or to reject the part with the dimension of 24,091 mm and 24,107 mm readings according to ISO 14253-1. In addition, in Figure 5.12 the upper tolerance limit is 30.1 mm, the lower tolerance limit is 29.95 mm, and the measurement uncertainty is 0,012 mm. There is not a problem in the upper tolerance limit, but in the lower tolerance limit, 29.956 mm reading is out of the tolerance range. The person who made the measurement must decide to accept or to reject the part with the dimension of 29,956 mm according to ISO 14253-1.

## CHAPTER 6

### RESULTS AND CONCLUSIONS

#### 6.1 Discussion of the Results

In this thesis a computer program is developed for evaluating uncertainty in measurement of vernier callipers, micrometers, comparators, and gauge blocks. When evaluating the uncertainty in measurement of these instruments EAL, NIST and GUM uncertainty evaluation equations are used as standard equations. In these equations temperature difference between the measured part and the instrument, uncertainty in reference gauge block's dimension, mechanical effects are considered as sources of measurement uncertainty. In addition, calibration limits, experience of operators in measurement, repeatability of the measuring instruments, and the geometry, condition of the measured workpiece, and other environmental conditions can be considered as other common factors. If users do not prefer to use the built-in standard equations or to evaluate uncertainty in measurement of an instrument other than the given instruments, then they may define their own special uncertainty equation, then the program evaluates the uncertainty in measurement.

The program gives the uncertainty budget details in the form of a table. In the budget table, the variables that contribute to the overall uncertainty of the instrument are listed together with the entered nominal value and standard uncertainty, the evaluation type, distribution type and/or bounds, the degrees of freedom. The effect ratio graph is presented to compare the effect of each variable considered in the overall uncertainty.

In preparing the calibration certificates of instruments, it is necessary to report the uncertainty of the instrument therefore the uncertainty in measurement results must be taken printouts. In the program developed, users can take the printout of the results either from Excel form or the results can be taken directly

from a printer. If the user prefers to see the uncertainty effect ratio in their printouts, they have to take the prints from Excel form. If they do not prefer to see the effect ratio in their printout, then they can use the print button in the program.

In this thesis, a set of experiments has been made to illustrate the application of the program for evaluating the measurement uncertainty of vernier callipers with 1/50 and 1/20 resolutions, digital vernier calliper and 25 mm micrometer. The sources of uncertainty in measurement such as temperature difference between the measured part and the instrument measured, mechanical effects on the instrument, and human effects are investigated in the experiments. The temperature difference between the measured part and the instrument measured is found by using a electronic thermometer, the mechanical effects are found from the instrument's calibration certificate and the human effects are found by making independent measurements. . It was observed from the standard uncertainties obtained from the experiments that the main source of uncertainty is human effects. With less experienced personnel, the uncertainty becomes higher.

The numbers of measurements in a certain dimension range are also given in the graphical form. The difference in the lower tolerance limit and the upper tolerance limit is divided into equal intervals. On those graphs, the upper and lower limits specified on the measured part are shown together with the calculated uncertainty limits. In this experiment, the uncertainty in each measurement has been calculated by the program developed, then by using the results of the experiment, decision making rules in the ISO 14253-1, "Geometrical Product Specifications (GPS)", and the lower and the upper tolerance limits the user decides to accept or to reject the measured part.

## **6.2 Conclusions**

In this thesis, a software program was developed for small and medium size industrial companies to evaluate and/or to estimate the uncertainty of some typical mechanical measuring and gauging devices like micrometer, vernier calliper, comparator, gauge block. Also this computer program not only a computational program but also a training program, it gives information not only about the terms to calculate uncertainty but also method used to calculate and /or estimate the uncertainty and make comments about the results. Also with the help of the program user can see the weight of each parameter by the help of the uncertainty effect ratio, then can investigate its reasons.

In this thesis, the program is also applicable to evaluating and expressing the uncertainty associated with the conceptual design and theoretical analysis of experiments, methods of measurement, and complex components and systems. In addition, this thesis shows general rules for evaluating and expressing uncertainty in measurement rather than detailed specific instructions.

## **6.3 Future Work of the Thesis**

- In this thesis, the computer program has a database that was designed with Microsoft Access. This database stores up to 10000 files. A database that will be designed with Oracle will be more useful, because in Oracle there is no file storage limitation.
- In this thesis, the computer program was written in Visual Basic 6.0 Enterprise Edition. In this programming language, taking the partial derivatives of the variables is difficult and complex. By using a mathematics function solver programs like MathCAD, Maple, vs. which can runs under Visual Basic may be more useful to solve the derivatives

- In this thesis, symmetric distribution types were considered, because when evaluating the uncertainty in dimensional measurement only these distributions are used in literature. Unsymmetrical distributions may be used and converted to the computer program

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APPENDIX A

FLOWCHART OF THE MEASUREMENT UNCERTAINTY COMPUTER PROGRAM

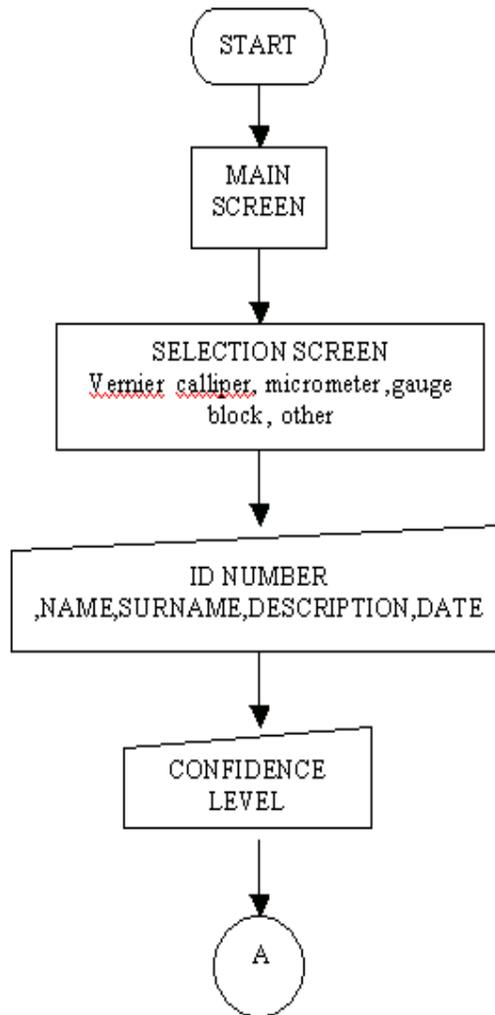


Figure A.1 Flowchart of the Computer Program

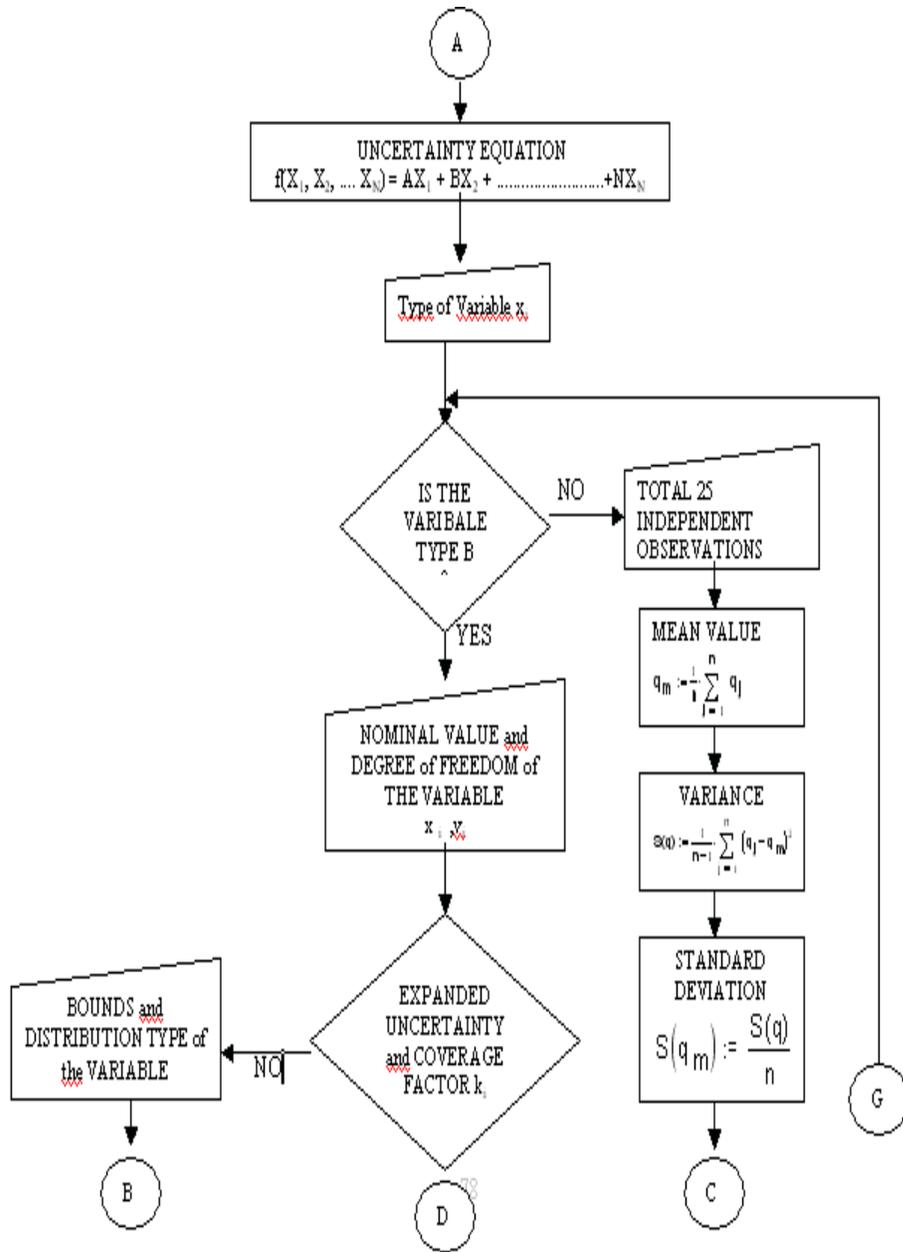


Figure A.1 Flowchart of the Computer Program (Continued)

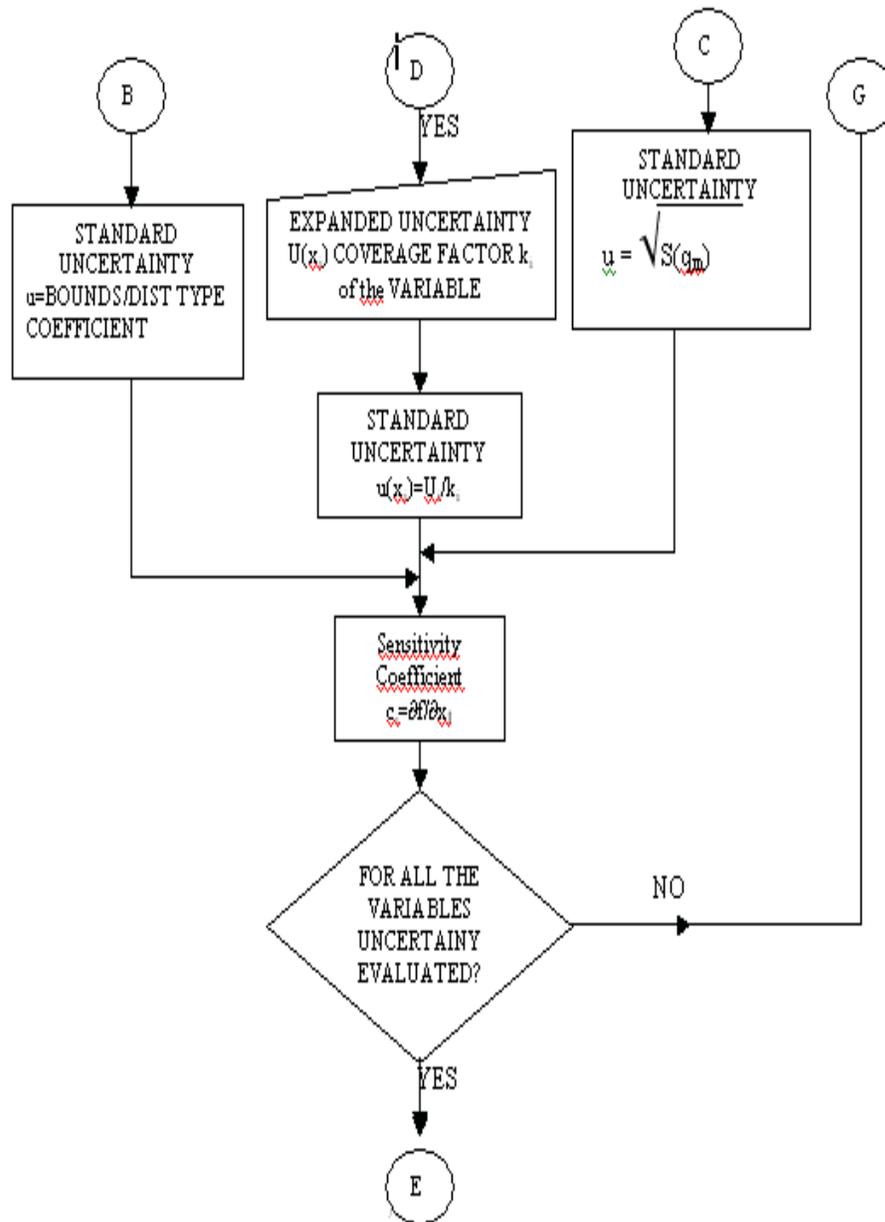
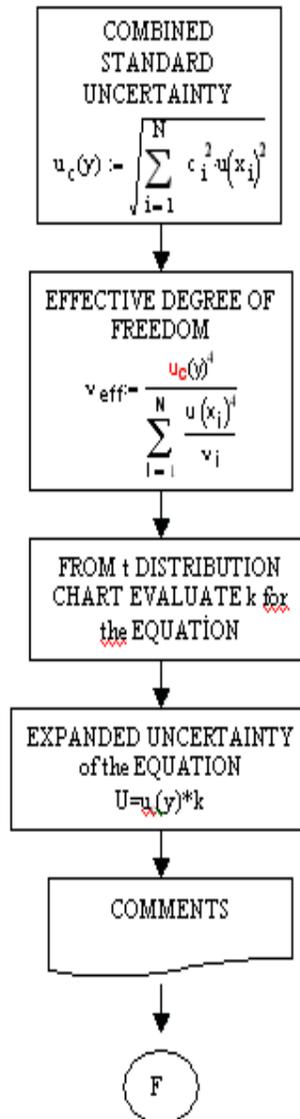


Figure A.1 Flowchart of the Computer Program (Continued)



80  
 Figure A.1 Flowchart of the Computer Program (Continued)

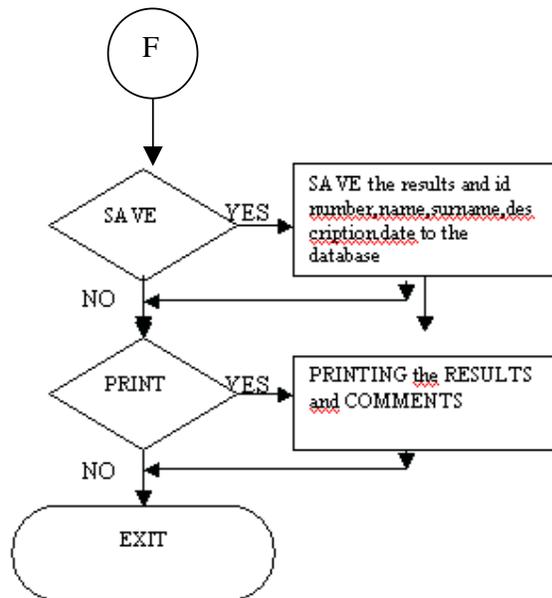


Figure A.1 Flowchart of the Computer Program (Continued)

## APPENDIX B

### COVERAGE FACTORS

Table B.1 Coverage Factors [24]

| Degrees of freedom<br>$\nu$ | Fraction $p$ in percent |       |       |       |       |        |
|-----------------------------|-------------------------|-------|-------|-------|-------|--------|
|                             | 68.27                   | 90    | 95    | 95.45 | 99    | 99.73  |
| 1                           | 1.84                    | 6.31  | 12.71 | 13.97 | 63.66 | 235.80 |
| 2                           | 1.32                    | 2.92  | 4.30  | 4.53  | 9.92  | 19.21  |
| 3                           | 1.20                    | 2.35  | 3.18  | 3.31  | 5.84  | 9.22   |
| 4                           | 1.14                    | 2.13  | 2.78  | 2.87  | 4.60  | 6.62   |
| 5                           | 1.11                    | 2.02  | 2.57  | 2.65  | 4.03  | 5.51   |
| 6                           | 1.09                    | 1.94  | 2.45  | 2.52  | 3.71  | 4.90   |
| 7                           | 1.08                    | 1.89  | 2.36  | 2.43  | 3.50  | 4.53   |
| 8                           | 1.07                    | 1.86  | 2.31  | 2.37  | 3.36  | 4.28   |
| 9                           | 1.06                    | 1.83  | 2.26  | 2.32  | 3.25  | 4.09   |
| 10                          | 1.05                    | 1.81  | 2.23  | 2.28  | 3.17  | 3.96   |
| 11                          | 1.05                    | 1.80  | 2.20  | 2.25  | 3.11  | 3.85   |
| 12                          | 1.04                    | 1.78  | 2.18  | 2.23  | 3.05  | 3.76   |
| 13                          | 1.04                    | 1.77  | 2.16  | 2.21  | 3.01  | 3.69   |
| 14                          | 1.04                    | 1.76  | 2.14  | 2.20  | 2.98  | 3.64   |
| 15                          | 1.03                    | 1.75  | 2.13  | 2.18  | 2.95  | 3.59   |
| 16                          | 1.03                    | 1.75  | 2.12  | 2.17  | 2.92  | 3.54   |
| 17                          | 1.03                    | 1.74  | 2.11  | 2.16  | 2.90  | 3.51   |
| 18                          | 1.03                    | 1.73  | 2.10  | 2.15  | 2.88  | 3.48   |
| 19                          | 1.03                    | 1.73  | 2.09  | 2.14  | 2.86  | 3.45   |
| 20                          | 1.03                    | 1.72  | 2.09  | 2.13  | 2.85  | 3.42   |
| 25                          | 1.02                    | 1.71  | 2.06  | 2.11  | 2.79  | 3.33   |
| 30                          | 1.02                    | 1.70  | 2.04  | 2.09  | 2.75  | 3.27   |
| 35                          | 1.01                    | 1.70  | 2.03  | 2.07  | 2.72  | 3.23   |
| 40                          | 1.01                    | 1.68  | 2.02  | 2.06  | 2.70  | 3.20   |
| 45                          | 1.01                    | 1.68  | 2.01  | 2.06  | 2.69  | 3.18   |
| 50                          | 1.01                    | 1.68  | 2.01  | 2.05  | 2.68  | 3.16   |
| 100                         | 1.005                   | 1.660 | 1.984 | 2.025 | 2.626 | 3.077  |
| $\infty$                    | 1.000                   | 1.645 | 1.960 | 2.000 | 2.576 | 3.000  |

## APPENDIX C

### GENERAL METROLOGICAL TERMS

#### Measurable Quantity

Attribute of a phenomenon, body, or substance that may be distinguished qualitatively and determined quantitatively.

#### Value of a Quantity

Magnitude of a particular quantity generally expressed as a unit of measurement multiplied number. The value of a quantity may be positive, negative or zero. The value of a quantity may be expressed in more than a way. The values of quantities of dimension one are generally express as pure numbers. A quantity that cannot be expressed as a unit of measurement multiplied by a number may be expressed by reference to a conventional reference scale or to a measurement procedure or to both [25].

#### True Value of a Quantity

Value consistent with the definition of a given particular quantity. This value would be obtained by a perfect measurement. True values are by nature indeterminate.

#### Conventional True Value of a Quantity

Value attributed to a particular quantity and accepted sometimes by conventional having an uncertainty appropriate for given purpose. Frequently a number of results of measurements of a quantity is used to establish a conventional true value. For example the CODATA recommended value for the Avogadro constant:  $6.022\ 136\ 7 \times 10^{23}\ \text{mol}^{-1}$ .

## Measurement

Set of operations having the object of determining a value of a quantity.

## Method of Measurement

Logical sequence of operations described generically used in the performance of measurements. Methods of measurement may be qualified in various ways such as substitution method, differential method, and null method.

## Measurement Procedure

Set of operations described specifically used in the performance of particular measurements according to given method. A measurement procedure is usually recorded in a document that is sometimes itself called a measurement procedure and is usually in sufficient detail to enable an operator to carry out a measurement without additional information.

## Measurand

It is the particular quantity subject to measurement. The specification of a measurand may require statements about quantities such as time temperature and pressure.

## Influence Quantity

Quantity that is not the measurand but that affects the result of the measurand For example temperature of a micrometer used to measure length or frequency in the measurement of the amplitude of an alternating electric potential difference or bilirubin concentration in the measurement of hemoglobin concentration in a sample of human blood plasma [26].

## Result of a Measurement:

Value attributed to a measurement. When a result is given, it should be made clear whether it refers to the indication, the uncorrected result, the corrected

result and whether several values are averaged. A complete statement of the result of a measurement includes information about the uncertainty of the measurement.

#### Accuracy of Measurement

It is the closeness of the agreement between the result of a measurement and a true value of the measurand.

#### Repeatability (of results of measurements)

Closeness of the agreement between the results of successive measurements of the same measurand carried out under the same conditions of measurement. Repeatability conditions include the same measurement procedure, the same observer, and the same measuring instrument under the same conditions, the same location, and repetition over a short period. Repeatability may be expressed quantitatively in terms of the dispersion characteristics of the results.

#### Reproducibility (of results of measurements)

Closeness of the agreement between the results of measurements of the same measurand carried out under changed conditions of measurement. A valid statement of reproducibility requires specification of the conditions changed. The changed conditions may include principle of measurement, method of measurement, observer, measuring instrument, reference standard, location, conditions of use, time.

#### Experimental Standard Deviation

For a series of  $n$  measurements of the same measurand, the quantity  $s(q_k)$  characterizing the dispersion of the results and given by the formula

$$s_{q^{(k)}} := \sqrt{\frac{\sum_{k=1}^n (q_k - q_m)^2}{n-1}} \quad (C.1)$$

$q_k$  being the result of the  $k$ th measurement and  $q_m$  being the arithmetic mean of the  $n$  results considered. Considering the series of  $n$  values as sample of a distribution,  $q_m$  is an unbiased estimate of the mean  $\mu_q$ , and  $s^2(q_k)$  is an unbiased estimate of the variance  $V^2$  of that distribution [27].

**Error (of Measurement):**

It is the result of a measurement minus a true value of the measurand. Since a true value cannot be determined in practice, a conventional true value is used.

**Relative Error**

Error of measurement divided by a true value of the measurand.

**Random Error**

Result of a measurement minus the mean that would result from an infinite number of measurements of the same measurand carried out under repeatability condition. Random error is equal to error minus systematic error. Because only a finite number of measurements can be made it is possible to determine only an estimate of random error.

**Systematic Error**

Mean that would result from an infinite number of measurements of the same measurand carried out under repeatability conditions minus a true value of the measurand. Systematic error is equal to error minus random error. Like true value, systematic error and its causes cannot be completely known.

**Correction**

Value added algebraically to the uncorrected result of measurement to compensate for systematic error. The correction is equal to the negative of the estimate systematic error. Since the systematic error cannot be known perfectly compensation cannot be complete.

Correction Factor:

It is a numerical factor by which the uncorrected result of a measurement is multiplied to compensate for systematic error.

## APPENDIX D

### BASIC STATISTICAL TERMS AND CONCEPTS

#### Probability

A real number in the scale 0 to 1 attached to a random event. It can be related to a long-run relative frequency of occurrence or to a degree of belief that an event occurs. For a high degree of belief, the probability is near one.

#### Random Variable

A variable that may take any of the values of specified set of values and with which associated a probability distribution. A random variable that may take only isolated values is to be discrete. A random variable, which may take any value within a finite or infinite interval, is said to be continuous.

#### Probability Distribution:

A function giving the probability that a random variable takes any given value or belongs to a given set of values. The probability overall set of values of the random variable equals one.

#### Distribution Function:

A function giving for every value  $x$  the probability that the random variable  $X$  be less than or equal to  $x$ .

$$F(x) := \Pr(X \leq x) \tag{D.1}$$

#### Probability Density Function

The derivative (when it exists) of the distribution function

$$f(x) := \frac{d}{dx} F(x) \tag{D.2}$$

$f(x)dx$  is the probability element.

### Correlation

It is the relationship between two or several random variables within a distribution of two or more random variables. Most statistical measures of correlation measure only the degree of linear relationship.

### Expectation:

For a discrete random variable  $X$  taking the value  $x_i$  with the probabilities  $p_i$  the expectation if it exists is

$$\mu(i) := \sum_i p_i \cdot x_i \quad (D.3)$$

the sum being extended over all the values  $x_i$  which can be taken by  $X$

2. For a continuous random variable  $X$  having the probability density function  $f(x)$  the expectation if it exists is

$$\mu := \int x \cdot f(x) dx \quad (D.4)$$

the integral being extended over the intervals of variation of  $X$ .

### Centered Random Variable:

It is a random variable the expectation of which equals to zero. If the random variable  $X$  has an expectation equal to  $\mu$  the corresponding centered random variable is  $X-\mu$ .

### Variance

It is the expectation of the square of the centered random variable.

### Covariance

The covariance of two random variables is a measure of their mutual dependence.

The covariance of random variables  $y$  and  $z$  defined by

$$\text{cov}(y,z) = \text{cov}(z,y) = E[(y-E(y))(z-E(z))]$$

which leads to

$$\text{cov}(y,z) := \int \int (y - \mu_y) \cdot (z - \mu_z) dy dz \quad (\text{D.5})$$

or

equals to

$$\text{cov}(y,z) := \int \int y \cdot z \cdot p(y,z) dy dz - \mu_y \cdot \mu_z \quad (\text{D.6})$$

where  $p(y,z)$  is the joint probability density function of two variables  $y$  and  $z$ . The covariance  $\text{cov}(y,z)$  may be estimated by  $s(y_i, z_i)$  obtained from  $n$  independent pairs of simultaneous observations  $y_i$  and  $z_i$  of  $y$  and  $z$ .

### Population

It is the totality of items under consideration. In case of a random variable the probability distribution is considered to define the population of that variable.

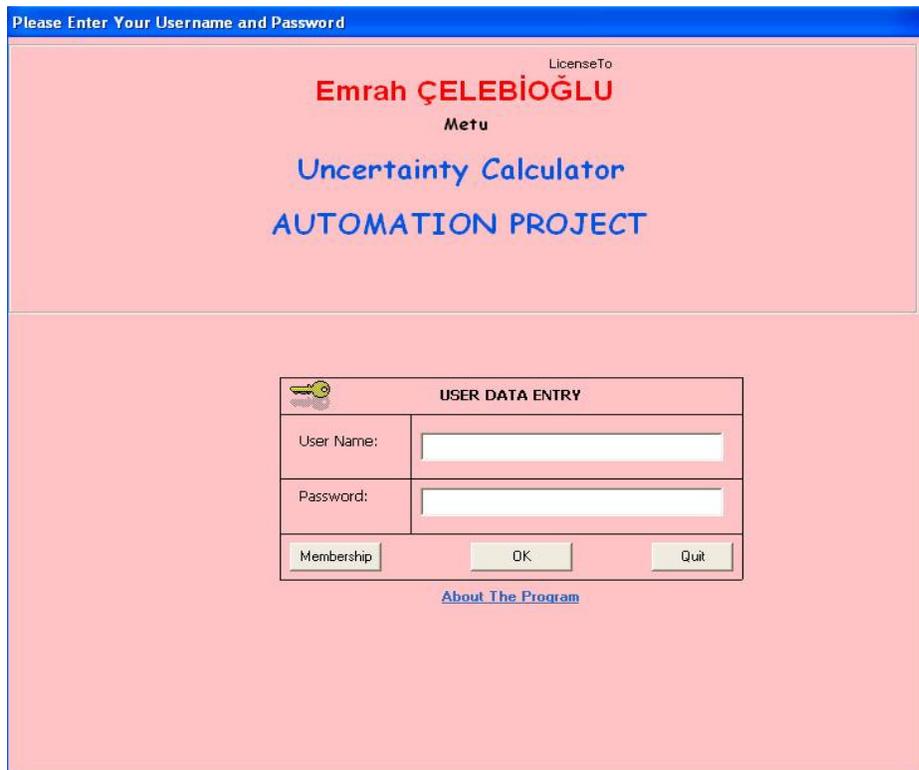
### Frequency

It is the number of occurrences of a given type of event or the number of observations falling into a specified class.

## APPENDIX E

### USER MANUAL OF THE PROGRAM

The program is executed by double clicking the “UNCERTAINTY CALCULATOR.EXE”. After clicking the program asks the user code and password (see Figure E.1)



The screenshot shows a window titled "Please Enter Your Username and Password". The window has a pink background. At the top, it says "LicenseTo" in small text, followed by "Emrah ÇELEBİOĞLU" in large red letters, "Metu" in smaller black letters, "Uncertainty Calculator" in blue letters, and "AUTOMATION PROJECT" in blue letters. Below this, there is a "USER DATA ENTRY" form with a key icon. The form has two input fields: "User Name:" and "Password:". Below the input fields are three buttons: "Membership", "OK", and "Quit". At the bottom of the form, there is a blue link that says "About The Program".

| USER DATA ENTRY |                                     |
|-----------------|-------------------------------------|
| User Name:      | <input type="text"/>                |
| Password:       | <input type="password"/>            |
| Membership      | <input type="button" value="OK"/>   |
|                 | <input type="button" value="Quit"/> |

[About The Program](#)

Figure E.1 Opening Page of the Computer Program

If the user does not registered before, user can register by clicking “OK” button. In the second screen computer asks the user if he/she wants to register to the program or not (see Figure E.2).



Figure E.2 Asking for User Registration

If user clicks “ YES I WANT ” button registration menu comes to the screen. User enters the necessary fields and by clicking “OK” user registers the program (See Figure E.3)



The image shows a 'Sign In' dialog box with a blue title bar. Inside, there is a form with the following fields: Name, Surname, E - Mail, Ev Telefonu, İş Telefonu, Cep Telefonu, Adress (with a vertical scrollbar), Country, and Nation. At the bottom, there are two buttons: 'OK' and 'Quit'.

|              |  |
|--------------|--|
| Name         |  |
| Surname      |  |
| E - Mail     |  |
| Ev Telefonu  |  |
| İş Telefonu  |  |
| Cep Telefonu |  |
| Adress       |  |
| Country      |  |
| Nation       |  |

OK      Quit

Figure E.3 User Registration Menu

After Clicking “OK” button the program gives automatically to the user a usercode and a password (See Figure E.4)

The image shows a 'Sign In' window with a blue title bar. Inside, there is a form with the following fields:

|                      |         |
|----------------------|---------|
| Müracaat Numaranuz : | 26      |
| User Name            | EMRAH26 |
| Password             | EMRAH26 |

Figure E.4 Giving Usercode and Password Menu

By clicking “OK” opening screen of the computer program comes again (See Figure E.1) and in this screen again by clicking “OK” user enters to the main screen of the computer program In this screen the computer program has a main menu. In this menu, there are “Evaluate”, “Help” and “ Quit ” buttons. By clicking “Quit” button user can exit the computer program, by clicking “Help” button user can get general information about the program and the measurement uncertainty, or by clicking “Evaluation” button user can reach the sub buttons of the program. In this sub buttons, there are “New Evaluation”, “Add Company”, “Search”, “Evaluation Date Expires”, and “Change Password”. By clicking “New Evaluation” button user, open file for a new firm or new department (see Figure E.5). In this window user, enter company name, company address, company’s authorized person name, surname, and telephone number.

| Company Data Entry                  |                                     |
|-------------------------------------|-------------------------------------|
| Company Name                        | <input type="text"/>                |
| Company Adress                      | <input type="text"/>                |
|                                     |                                     |
| Name                                | <input type="text"/>                |
| Surname                             | <input type="text"/>                |
| Telephone                           | <input type="text"/>                |
| Telephone                           | <input type="text"/>                |
|                                     |                                     |
| <input type="button" value="Save"/> | <input type="button" value="Quit"/> |

Figure E.5 Firm Data Entry Window

If user clicks, “Change Password” button user enters the password changer menu he/she can change his/her password (see FigureE.6)

| Password Change    |       |
|--------------------|-------|
| 2 EMRAH ÇELEBİOĞLU |       |
| User Name          | emrah |
| Old Password       | 2     |
| New Password       |       |
| Confirm            |       |
| Ok                 | Exit  |

Figure E.6 Password Changer Screen

By clicking “New Evaluation” button a new screen that is called “Customer Search” comes to the window. Here user selects the company or department whose measurement instrument uncertainty is going to evaluate (see Figure 7)

|              |  |    |  |
|--------------|--|----|--|
| Company Name |  | EM |  |
| Name         |  |    |  |
| Surname      |  |    |  |

| No | Id | Company Name | Name Surname     |
|----|----|--------------|------------------|
| 1  | 25 | EMRAH A.Ş.   | EMRAH ÇELEBİOĞLU |

|   |  |  |
|---|--|--|
|  F3 - Search |  F4 - Clear |  ESC-Quit |
|---|--|--|

Figure E.7 Company Searching Screen

In this screen user can find the firm by entering the first three letters of the company's name and then by clicking "Search" button. After finishing this procedure the name of the company and the authorized person name comes to the screen. After this by clicking on the company name the opening file screen appears to the screen (see Figure 5.8). In this screen, there is a combo box. In this, combo box user selects the measuring instrument that he/she wants to evaluate the uncertainty. In this combo box, user can select "Vernier Calliper", "Gauge Block", "Comparator", "Micrometer", and "User Defined".

|               |                               |
|---------------|-------------------------------|
| Firm Name     | EMRAH A.Ş.                    |
| Firm's Adress | DHFJHDSHFJKHSDJ DHSHFJHDSJHFJ |

|                  |                |
|------------------|----------------|
| Name             | EMRAH          |
| Surname          | ÇELEBİOĞLU     |
| Telephone Number | (11) 111 11 11 |
| Telephone Number | (11) 111 11 11 |

SELECT THE INSTRUMENT TYPE

Figure E.8 Instrument Selection Menu

If user selects “Vernier Calliper” vernier caliper uncertainty evaluation screen comes (see Figure E.9). In vernier caliper uncertainty evaluation screen the uncertainty equation is taken from EA (European Co-operation for Accreditation). Here computer program evaluates the standard uncertainties of each variable whether the evaluation of uncertainty method is Type A or Type B also combined standard uncertainty and expanded uncertainty of the vernier caliper is evaluated automatically by the computer program.

Company Name EMRAH A.Ş.

Name EMRAH

Surname ÇELEBIOĞLU

Instrument Id \_\_\_\_\_ Date 09.2004

Nominal Values

|          |                  |                 |    |
|----------|------------------|-----------------|----|
| $l_0$    | mm               | $\Delta t$      | °C |
| $l_n$    | mm               | $\delta l_{ix}$ | mm |
| $L$      | mm               | $\delta l_m$    | mm |
| $\alpha$ | °C <sup>-1</sup> |                 |    |

Function  $l_m = l_0 + L \alpha \Delta t + \delta l_{ix} + \delta l_m$

Coverage Probability \_\_\_\_\_

TYPE A

n = \_\_\_\_\_ Ok

|          |          |          |
|----------|----------|----------|
| Value1:  | Value6:  | Value11: |
| Value2:  | Value7:  | Value12: |
| Value3:  | Value8:  | Value13: |
| Value4:  | Value9:  | Value14: |
| Value5:  | Value10: | Value15: |
| Value16: | Value21: |          |
| Value17: | Value22: |          |
| Value18: | Value23: |          |
| Value19: | Value24: |          |
| Value20: | Value25: |          |

Type B Evaluation Fields

|                 | Degrees of Freedom | Evaluation Type | Standard Uncertainty |
|-----------------|--------------------|-----------------|----------------------|
| $l_n$           |                    |                 |                      |
| $\Delta t$      |                    |                 |                      |
| $\delta l_{ix}$ |                    |                 |                      |
| $\delta l_m$    |                    |                 |                      |

F3 - Evaluate    F4 - Clear    ESC - Quit

Figure E.9 Vernier Caliper Uncertainty Evaluation Screen

If user selects “Micrometer” micrometer uncertainty evaluation screen comes (see Figure E.10). In micrometer uncertainty evaluation screen the uncertainty equation is taken from EA (European Co-operation for Accreditation). Here computer program evaluates the standard uncertainties of each variable whether the evaluation of uncertainty method is Type A or Type B also combined standard uncertainty and expanded uncertainty of the micrometer is evaluated automatically by the computer program

Company Name EMRAH A.Ş.

Name EMRAH  
Surname ÇELEBİOĞLU

Instrument Id \_\_\_\_\_ Date 10.09.2004

Nominal Values

|          |                  |       |    |
|----------|------------------|-------|----|
| $l_m$    | mm               | $t_m$ | °C |
| $l_r$    | mm               |       |    |
| L        | mm               |       |    |
| $\alpha$ | °C <sup>-1</sup> |       |    |

Function  $l_m - l_r + L \alpha \Delta t$

Coverage Probability \_\_\_\_\_

TYPE A

n = \_\_\_\_\_ Ok

|                |                |                |
|----------------|----------------|----------------|
| Value1: 1      | Value6: _____  | Value11: _____ |
| Value2: 1      | Value7: _____  | Value12: _____ |
| Value3: 2      | Value8: _____  | Value13: _____ |
| Value4: 3      | Value9: _____  | Value14: _____ |
| Value5: 1      | Value10: _____ | Value15: _____ |
| Value16: _____ | Value21: _____ |                |
| Value17: _____ | Value22: _____ |                |
| Value18: _____ | Value23: _____ |                |
| Value19: _____ | Value24: _____ |                |
| Value20: _____ | Value25: _____ |                |

TYPE B Uncertainty Evaluation Fields

|            | Degrees of Freedom | Evaluation Type | Standard Uncertainty |
|------------|--------------------|-----------------|----------------------|
| $l_m$      |                    |                 |                      |
| $\Delta t$ |                    |                 |                      |
| $\alpha_m$ |                    |                 |                      |

Lütfen Serbestlik Derecesine Sayısal Değer Giriniz

F3 - Evaluate F4 - Clear ESC - Quit

Figure E.10 Micrometer Uncertainty Evaluation Screen

If user selects “Comparator” comparator uncertainty evaluation screen comes (see Figure E.11). In comparator uncertainty evaluation screen the uncertainty equation is taken from EA (European Co-operation for Accreditation). Here computer program evaluates the standard uncertainties of each variable whether the evaluation of uncertainty method is Type A or Type B also combined standard uncertainty and expanded uncertainty of the comparator is evaluated automatically by the computer program

Company Name EMRAH A.Ş.

Instrument Id: [ ] Date: 08.09.2004

Name EMRAH

Surname CELEBIOGLU

Function:  $L \pm \alpha \cdot \Delta t + \delta l_{ix} + \delta l_m$

Coverage Probability: [ ]

TYPE A

n = [ ] Ok

|              |              |              |
|--------------|--------------|--------------|
| Value1: [ ]  | Value6: [ ]  | Value11: [ ] |
| Value2: [ ]  | Value7: [ ]  | Value12: [ ] |
| Value3: [ ]  | Value8: [ ]  | Value13: [ ] |
| Value4: [ ]  | Value9: [ ]  | Value14: [ ] |
| Value5: [ ]  | Value10: [ ] | Value15: [ ] |
| Value16: [ ] | Value21: [ ] |              |
| Value17: [ ] | Value22: [ ] |              |
| Value18: [ ] | Value23: [ ] |              |
| Value19: [ ] | Value24: [ ] |              |
| Value20: [ ] | Value25: [ ] |              |

Nominal Values

|          |                  |                 |    |
|----------|------------------|-----------------|----|
| $l_{ix}$ | mm               | $\Delta t$      | °C |
| $l_{ix}$ | mm               | $\delta l_{ix}$ | mm |
| $l_m$    | mm               | $\delta l_m$    | mm |
| $\alpha$ | °C <sup>-1</sup> |                 |    |

Type B Uncertainty Evaluation Fields

|                 | Degrees of Freedom | Evaluation Type | Standard Uncertainty |
|-----------------|--------------------|-----------------|----------------------|
| $l_{ix}$        | [ ]                | [ ]             | [ ]                  |
| $\Delta t$      | [ ]                | [ ]             | [ ]                  |
| $\delta l_{ix}$ | [ ]                | [ ]             | [ ]                  |
| $\delta l_m$    | [ ]                | [ ]             | [ ]                  |

F3 - Evaluate F4 - Clear ESC -Quit

Figure E.11 Comparator Uncertainty Evaluation Screen

If user selects “Gauge Block” gauge block uncertainty evaluation screen comes (see Figure E.12). Gauge block uncertainty evaluation screen the uncertainty equation is taken from EA (European Co-operation for Accreditation). Here computer program evaluates the standard uncertainties of each variable whether the evaluation of uncertainty method is Type A or Type B also combined standard uncertainty and expanded uncertainty of the Gauge Block is evaluated automatically by the computer program.

If user selects “User Defined” User Defined uncertainty evaluation screen comes (see Figure 5.13). In this screen user writes his/her own uncertainty equation, by the help of this equation the computer program evaluates the uncertainty of the measuring instrument. Here computer program evaluates the sensitivity coefficients and standard uncertainties of each variable also effective degrees of freedom, combined standard uncertainty and expanded uncertainty is evaluated automatically by the computer program



In this computer program, user can find the measuring instrument, which he/she evaluates its measurement uncertainty before. If the user clicks the “Search” button, this search screen appears on the screen (see Figure E.14).

|              |  |    |  |
|--------------|--|----|--|
| Company Name |  | EM |  |
| Name         |  |    |  |
| Surname      |  |    |  |

| No | Id | Company Name | Name Surname     |
|----|----|--------------|------------------|
| 1  | 25 | EMRAH A.Ş.   | EMRAH ÇELEBİOĞLU |

F3 - Search    F4 - Clear    ESC-Quit

Figure E.14 Company Searching Screen

Firstly, user selects the company or department name by entering the first three letters of the firm or the department name. After this user selects the company or department by clicking on it, then the instrument searching form comes to the screen (see Figure E.15). In this form user selects the instrument type

whether it is vernier caliper, micrometer, comparator, gauge block, or user defined.

| No | Id | Company Name | Name-Surname |
|----|----|--------------|--------------|
|----|----|--------------|--------------|

Figure E.15 Instrument Selection Screen

After this application user can search the measurement instrument with respect to instrument code or uncertainty evaluation date of the instrument (see Figure E.16). User find the measuring instrument by entering the code of the instrument or entering the date after clicking “Search” the instrument’s uncertainty specifications comes to the screen. If the user clicks on the instrument code the values of evaluation comes to the screen (see Figure E.17)

|            |  |               |  |      |  |
|------------|--|---------------|--|------|--|
| Company    |  | Instrument Id |  | Date |  |
| EMRAH A.Ş. |  | 1             |  |      |  |

| No | Id | Date       | Instrument Id | Overall Nominal... | Combined Uncer... | Degrees Of Free... | Coverage Factor | Coverage P... | Expanded Uncert... |
|----|----|------------|---------------|--------------------|-------------------|--------------------|-----------------|---------------|--------------------|
| 1  | 2  | 21.08.2004 | 1             | 300                | 0,571547399       | 2,92418814         | 5,84            | 99            | 3,33783681         |

F3 - Search    F4 - Clear    ESC - Quit

Figure E.16 Instrument Searching Screen

|              |            |               |  |                  |  |
|--------------|------------|---------------|--|------------------|--|
| EMRAH 2      |            | EMRAH EMRAH   |  | EMRAH ÇELEBİOĞLU |  |
| Company Info |            | Company Name  |  | Surname          |  |
| EMRAH A.Ş.   |            | EMRAH         |  | ÇELEBİOĞLU       |  |
| Type         | Micrometer | Instrument No |  | Date             |  |
|              |            |               |  | 21.08.2004       |  |

**EMRAH A.Ş.**

| Variable      | Nominal Value | Evaluation Type                  | Sensitivity Coefficient | Standard Uncertainty | Uncertainty Contribution | Degrees of Freedom |
|---------------|---------------|----------------------------------|-------------------------|----------------------|--------------------------|--------------------|
| $l_r$         | 100           | TYPE A                           | 1                       | 2,92418814           | 2,92419                  | 4                  |
| $l_m$         | 100           | TYPE B DAĞILIM ŞEKLİ VE SINIRLAR | 1                       | 0,40824829           | 0,40825                  | 2                  |
| $l_m$         | 100           | TYPE B DAĞILIM ŞEKLİ VE SINIRLAR | 1                       | 0,40824829           | 0,40825                  | 3                  |
| $\alpha_m$    | 1             | TYPE B DAĞILIM ŞEKLİ VE SINIRLAR | 100                     | 0,40824829           | 40,82483                 | 4                  |
| <b>Toplam</b> | <b>300</b>    |                                  |                         |                      |                          |                    |

|                      |                    |                 |                      |                      |
|----------------------|--------------------|-----------------|----------------------|----------------------|
| Combined Uncertainty | Degrees of Freedom | Coverage Factor | Coverage Probability | Expanded Uncertainty |
| 0,571547399          | 2,92418814         | 5,84            | 99                   | 3,33783681           |

Figure E.17 Instrument Results Screen

In this computer program the user can look for measuring instruments whose measurement uncertainty validity is expired. When the user clicks “Evaluation Expiry Search”, the computer program shows user the instruments whose measurement uncertainty validity is expired (see Figure E.18).

| <b>CAUTION !!!</b>  |    |            |            |                |                |  |
|---|----|------------|------------|----------------|----------------|--|
| Sayın EMRAH ÇELEBİOĞLU  |    |            |            |                |                |  |
| There are some instruments that needs calculation of uncertainty again  |    |            |            |                |                |  |
| Please click the related record to achive the necessary documents that enables you to communicate with the firms which owns the related datas |    |            |            |                |                |  |
| NO  | Id | Firm       | Date       | Type           | Instrument Id  |  |
| 1   | 25 | EMRAH A.Ş. | 03.09.2004 | SURMELI KUMPAS | emr_sur.150.01 |  |
| 2   | 25 | EMRAH A.Ş. | 03.09.2004 | DİŞER          | 1              |  |
| 3   | 25 | EMRAH A.Ş. | 08.09.2004 | DİŞER          | 1              |  |
| 4   | 25 | EMRAH A.Ş. | 16.09.2004 | DİŞER          | emr.A10kg.1    |  |
| 5   | 25 | EMRAH A.Ş. | 17.09.2004 | KOMPARATÖR     | 1              |  |
| 6   | 25 | EMRAH A.Ş. | 17.09.2004 | KOMPARATÖR     | 1              |  |
| 7   | 25 | EMRAH A.Ş. | 17.09.2004 | KOMPARATÖR     | 1              |  |

Figure E.18 Uncertainty Validity Search Screen

## APPENDIX F

### MEASUREMENTS TAKEN FROM EXPERIMENTS

The basic dimensions of the part “a”, “b”, “c”, and “d” are shown in Figure 5.1.

Table F.1 Measurements Obtained from Vernier Calliper with 1/20 Resolution

| Obs No | a(mm) | b(mm) | c(mm) | d(mm) | Temp.(C°) |
|--------|-------|-------|-------|-------|-----------|
| 1      | 24,00 | 30,00 | 20,15 | 35,20 | 24        |
| 2      | 24,00 | 30,00 | 20,15 | 35,20 | 24        |
| 3      | 24,05 | 30,00 | 20,15 | 35,20 | 24        |
| 4      | 24,05 | 30,00 | 20,15 | 35,20 | 24        |
| 5      | 24,05 | 30,00 | 20,15 | 35,25 | 24        |
| 6      | 24,05 | 30,00 | 20,15 | 35,30 | 24        |
| 7      | 24,05 | 30,00 | 20,15 | 35,30 | 24        |
| 8      | 24,05 | 30,00 | 20,20 | 35,30 | 24        |
| 9      | 24,05 | 30,00 | 20,20 | 35,35 | 24        |
| 10     | 24,10 | 30,00 | 20,20 | 35,35 | 24        |
| 11     | 24,10 | 30,00 | 20,20 | 35,35 | 24        |
| 12     | 24,10 | 30,05 | 20,20 | 35,35 | 24        |
| 13     | 24,10 | 30,05 | 20,20 | 35,35 | 25        |
| 14     | 24,10 | 30,05 | 20,20 | 35,35 | 25        |
| 15     | 24,10 | 30,05 | 20,25 | 35,35 | 25        |
| 16     | 24,10 | 30,05 | 20,25 | 35,35 | 25        |
| 17     | 24,10 | 30,05 | 20,25 | 35,35 | 25        |
| 18     | 24,15 | 30,05 | 20,25 | 35,40 | 25        |
| 19     | 24,15 | 30,05 | 20,25 | 35,40 | 25        |
| 20     | 24,15 | 30,05 | 20,25 | 35,40 | 25        |
| 21     | 24,15 | 30,05 | 20,25 | 35,40 | 25        |

Table F.2 Measurements Obtained from Vernier Calliper with 1/50 Resolution

**INSTRUMENT2 Vernier Calliper 1/50**

| <b>Obs No</b> | <b>a(mm)</b> | <b>b(mm)</b> | <b>c(mm)</b> | <b>d(mm)</b> | <b>Temp.(C°)</b> |
|---------------|--------------|--------------|--------------|--------------|------------------|
| 1             | 24,00        | 29,98        | 20,12        | 35,26        | 24               |
| 2             | 24,00        | 29,98        | 20,14        | 35,28        | 24               |
| 3             | 24,00        | 29,98        | 20,16        | 35,28        | 24               |
| 4             | 24,00        | 29,98        | 20,16        | 35,30        | 24               |
| 5             | 24,00        | 29,98        | 20,18        | 35,30        | 24               |
| 6             | 24,02        | 30,00        | 20,18        | 35,30        | 24               |
| 7             | 24,02        | 30,00        | 20,18        | 35,30        | 24               |
| 8             | 24,04        | 30,00        | 20,18        | 35,32        | 24               |
| 9             | 24,04        | 30,00        | 20,20        | 35,32        | 24               |
| 10            | 24,06        | 30,00        | 20,20        | 35,32        | 24               |
| 11            | 24,06        | 30,00        | 20,20        | 35,32        | 24               |
| 12            | 24,06        | 30,00        | 20,20        | 35,34        | 24               |
| 13            | 24,08        | 30,00        | 20,20        | 35,34        | 25               |
| 14            | 24,08        | 30,00        | 20,22        | 35,34        | 25               |
| 15            | 24,10        | 30,02        | 20,22        | 35,36        | 25               |
| 16            | 24,12        | 30,02        | 20,24        | 35,36        | 25               |
| 17            | 24,12        | 30,02        | 20,24        | 35,36        | 25               |
| 18            | 24,12        | 30,02        | 20,24        | 35,38        | 25               |
| 19            | 24,12        | 30,04        | 20,26        | 35,40        | 25               |
| 20            | 24,12        | 30,06        | 20,28        | 35,40        | 25               |
| 21            | 24,14        | 30,06        | 20,28        | 35,40        | 25               |

Table F.3 Measurements Obtained from Digital Vernier Calliper

**INSTRUMENT3 Digital Vernier Calliper**

| <b>Obs No</b> | <b>a(mm)</b> | <b>b(mm)</b> | <b>c(mm)</b> | <b>d(mm)</b> | <b>Temp.(C<sup>o</sup>)</b> |
|---------------|--------------|--------------|--------------|--------------|-----------------------------|
| 1             | 24,02        | 29,96        | 20,15        | 35,30        | 24                          |
| 2             | 24,02        | 29,96        | 20,16        | 35,30        | 24                          |
| 3             | 24,03        | 29,98        | 20,16        | 35,30        | 24                          |
| 4             | 24,05        | 29,98        | 20,16        | 35,30        | 24                          |
| 5             | 24,05        | 29,98        | 20,17        | 35,31        | 24                          |
| 6             | 24,05        | 29,98        | 20,18        | 35,31        | 24                          |
| 7             | 24,06        | 29,98        | 20,18        | 35,31        | 24                          |
| 8             | 24,06        | 29,98        | 20,19        | 35,32        | 24                          |
| 9             | 24,06        | 29,99        | 20,20        | 35,32        | 24                          |
| 10            | 24,06        | 29,99        | 20,20        | 35,32        | 24                          |
| 11            | 24,06        | 29,99        | 20,21        | 35,33        | 24                          |
| 12            | 24,08        | 30,00        | 20,22        | 35,34        | 24                          |
| 13            | 24,08        | 30,01        | 20,22        | 35,34        | 25                          |
| 14            | 24,09        | 30,02        | 20,23        | 35,34        | 25                          |
| 15            | 24,09        | 30,02        | 20,23        | 35,35        | 25                          |
| 16            | 24,10        | 30,02        | 20,23        | 35,35        | 25                          |
| 17            | 24,11        | 30,03        | 20,24        | 35,36        | 25                          |
| 18            | 24,12        | 30,03        | 20,24        | 35,36        | 25                          |
| 19            | 24,13        | 30,06        | 20,25        | 35,36        | 25                          |
| 20            | 24,14        | 30,06        | 20,25        | 35,37        | 25                          |
| 21            | 24,14        | 30,06        | 20,26        | 35,41        | 25                          |

Table F.4 Measurements Obtained with Micrometer

**INSTRUMENT4**  
**Micrometer**

| <b>Obs No</b> | <b>a(mm)</b> | <b>c(mm)</b> | <b>Temp.(C°)</b> |
|---------------|--------------|--------------|------------------|
| 1             | 24,050       | 20,120       | 24               |
| 2             | 24,050       | 20,125       | 24               |
| 3             | 24,055       | 20,130       | 24               |
| 4             | 24,057       | 20,130       | 24               |
| 5             | 24,058       | 20,135       | 24               |
| 6             | 24,060       | 20,140       | 24               |
| 7             | 24,060       | 20,140       | 24               |
| 8             | 24,060       | 20,140       | 24               |
| 9             | 24,060       | 20,145       | 24               |
| 10            | 24,060       | 20,150       | 24               |
| 11            | 24,060       | 20,150       | 24               |
| 12            | 24,060       | 20,155       | 24               |
| 13            | 24,060       | 20,160       | 25               |
| 14            | 24,060       | 20,165       | 25               |
| 15            | 24,061       | 20,170       | 25               |
| 16            | 24,065       | 20,170       | 25               |
| 17            | 24,070       | 20,180       | 25               |
| 18            | 24,070       | 20,200       | 25               |
| 19            | 24,070       | 20,210       | 25               |
| 20            | 24,120       | 20,220       | 25               |
| 21            | 24,120       | 20,260       | 25               |

Table F.5 Corrected Values for Vernier Calliper with 1/20 Resolution  
Measurements

| <b>Obs No</b> | <b>a(mm)</b> | <b>b(mm)</b> | <b>c(mm)</b> | <b>d(mm)</b> |
|---------------|--------------|--------------|--------------|--------------|
| 1             | 23,992       | 29,984       | 20,139       | 35,193       |
| 2             | 23,992       | 29,984       | 20,139       | 35,193       |
| 3             | 24,034       | 29,984       | 20,139       | 35,193       |
| 4             | 24,034       | 29,984       | 20,139       | 35,193       |
| 5             | 24,034       | 29,984       | 20,139       | 35,237       |
| 6             | 24,034       | 29,984       | 20,139       | 35,280       |
| 7             | 24,034       | 29,984       | 20,139       | 35,280       |
| 8             | 24,034       | 29,984       | 20,178       | 35,280       |
| 9             | 24,034       | 29,984       | 20,178       | 35,324       |
| 10            | 24,075       | 29,984       | 20,178       | 35,324       |
| 11            | 24,075       | 29,984       | 20,178       | 35,324       |
| 12            | 24,075       | 30,017       | 20,178       | 35,324       |
| 13            | 24,075       | 30,017       | 20,178       | 35,324       |
| 14            | 24,075       | 30,017       | 20,178       | 35,324       |
| 15            | 24,075       | 30,017       | 20,217       | 35,324       |
| 16            | 24,075       | 30,017       | 20,217       | 35,324       |
| 17            | 24,075       | 30,017       | 20,217       | 35,324       |
| 18            | 24,117       | 30,017       | 20,217       | 35,367       |
| 19            | 24,117       | 30,017       | 20,217       | 35,367       |
| 20            | 24,117       | 30,017       | 20,217       | 35,367       |
| 21            | 24,117       | 30,017       | 20,217       | 35,367       |

Table F.6 Corrected Values for Vernier Calliper with 1/50 Resolution  
Measurements

| <b>Obs No</b> | <b>a(mm)</b> | <b>b(mm)</b> | <b>c(mm)</b> | <b>d(mm)</b> |
|---------------|--------------|--------------|--------------|--------------|
| 1             | 23,996       | 29,973       | 20,116       | 35,256       |
| 2             | 23,996       | 29,973       | 20,133       | 35,272       |
| 3             | 23,996       | 29,973       | 20,149       | 35,272       |
| 4             | 23,996       | 29,973       | 20,149       | 35,288       |
| 5             | 23,996       | 29,973       | 20,165       | 35,288       |
| 6             | 24,012       | 29,987       | 20,165       | 35,288       |
| 7             | 24,012       | 29,987       | 20,165       | 35,288       |
| 8             | 24,028       | 29,987       | 20,165       | 35,304       |
| 9             | 24,028       | 29,987       | 20,182       | 35,304       |
| 10            | 24,044       | 29,987       | 20,182       | 35,304       |
| 11            | 24,044       | 29,987       | 20,182       | 35,304       |
| 12            | 24,044       | 29,987       | 20,182       | 35,319       |
| 13            | 24,059       | 29,987       | 20,182       | 35,319       |
| 14            | 24,059       | 29,987       | 20,198       | 35,319       |
| 15            | 24,075       | 30,000       | 20,198       | 35,335       |
| 16            | 24,091       | 30,000       | 20,214       | 35,335       |
| 17            | 24,091       | 30,000       | 20,214       | 35,335       |
| 18            | 24,091       | 30,000       | 20,214       | 35,351       |
| 19            | 24,091       | 30,014       | 20,231       | 35,367       |
| 20            | 24,091       | 30,027       | 20,247       | 35,367       |
| 21            | 24,107       | 30,027       | 20,247       | 35,367       |

Table F.7 Corrected Values for Digital Vernier Calliper Measurements

| <b>obs no</b> | <b>a(mm)</b> | <b>b(mm)</b> | <b>c(mm)</b> | <b>d(mm)</b> |
|---------------|--------------|--------------|--------------|--------------|
| 1             | 24,017       | 29,956       | 20,147       | 35,296       |
| 2             | 24,017       | 29,956       | 20,155       | 35,296       |
| 3             | 24,024       | 29,972       | 20,155       | 35,296       |
| 4             | 24,041       | 29,972       | 20,155       | 35,296       |
| 5             | 24,041       | 29,972       | 20,162       | 35,303       |
| 6             | 24,041       | 29,972       | 20,169       | 35,303       |
| 7             | 24,048       | 29,972       | 20,169       | 35,303       |
| 8             | 24,048       | 29,972       | 20,176       | 35,309       |
| 9             | 24,048       | 29,978       | 20,184       | 35,309       |
| 10            | 24,048       | 29,978       | 20,184       | 35,309       |
| 11            | 24,048       | 29,978       | 20,191       | 35,315       |
| 12            | 24,065       | 29,984       | 20,198       | 35,222       |
| 13            | 24,065       | 29,989       | 20,198       | 35,222       |
| 14            | 24,072       | 29,995       | 20,205       | 35,222       |
| 15            | 24,072       | 29,995       | 20,205       | 35,328       |
| 16            | 24,079       | 29,995       | 20,205       | 35,328       |
| 17            | 24,086       | 30,001       | 20,213       | 35,334       |
| 18            | 24,093       | 30,001       | 20,213       | 35,334       |
| 19            | 24,100       | 30,027       | 20,22        | 35,334       |
| 20            | 24,107       | 30,027       | 20,22        | 35,341       |
| 21            | 24,107       | 30,027       | 20,227       | 35,377       |

Table F.8 Corrected Values for Micrometer Measurements

| <b>Obs No</b> | <b>a(mm)</b> | <b>c(mm)</b> |
|---------------|--------------|--------------|
| 1             | 24,046       | 20,118       |
| 2             | 24,046       | 20,121       |
| 3             | 24,048       | 20,124       |
| 4             | 24,046       | 20,124       |
| 5             | 24,043       | 20,127       |
| 6             | 24,042       | 20,130       |
| 7             | 24,042       | 20,130       |
| 8             | 24,042       | 20,130       |
| 9             | 24,042       | 20,133       |
| 10            | 24,042       | 20,136       |
| 11            | 24,042       | 20,139       |
| 12            | 24,042       | 20,141       |
| 13            | 24,042       | 20,144       |
| 14            | 24,042       | 20,147       |
| 15            | 24,039       | 20,155       |
| 16            | 24,039       | 20,155       |
| 17            | 24,041       | 20,173       |
| 18            | 24,041       | 20,181       |
| 19            | 24,041       | 20,181       |
| 20            | 24,087       | 20,189       |
| 21            | 24,087       | 20,227       |

## APPENDIX G

### DISTRIBUTION TYPES USED IN MEASUREMENT UNCERTAINTY EVALUATION

#### Rectangular Distribution

This type of distribution is used when a certificate or other specification gives limits without specifying a level of confidence e.g.  $100\text{mm} \pm 0.05\mu\text{m}$ . or an estimate is made in a form of a maximum range with no knowledge of the shape of the distribution. One can assume that it is equally probable for  $X_i$

Procedure for Standard Uncertainty Estimation for Rectangular Distribution: Estimate lower and upper limits  $a$  and  $a_+$  for the value of the input quantity in question such that the probability that the value lies in the interval  $a$  and  $a_+$  is, for all practical purposes, 100 %. Provided that there is no contradictory information, treat the quantity as if it is equally probable for its value to lie anywhere within the interval  $a$  to  $a_+$ ; that is, model it by a uniform (i.e., rectangular) probability distribution. The best estimate of the value of the quantity is then  $(a_+ + a)/2$  with  $u_j = a/\sqrt{3}$ , where  $a = (a_+ - a)/2$  is the half-width of the interval [28].

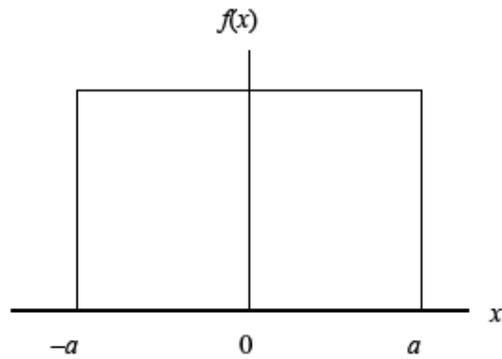


Figure G.1 Graph of Rectangular Distribution

### Triangular Distribution

The rectangular distribution is a reasonable default model in the absence of any other information. But if it is known that values of the quantity in question near the center of the limits are more likely than values close to the limits, a normal distribution or, for simplicity, a triangular distribution, may be a better model.

Procedure for Standard Uncertainty Estimation for Triangular Distribution: Estimate lower and upper limits  $a$  and  $a_+$  for the value of the input quantity in question such that the probability that the value lies in the interval  $a$  to  $a_+$  is, for all practical purposes, 100 %. Provided that there is no contradictory information, model the quantity by a triangular probability distribution. The best estimate of the value of the quantity is then  $(a_+ + a)/2$  with  $u_j = a\sqrt{6}$ , where  $a = (a_+ - a)/2$  is the half-width of the interval [28].

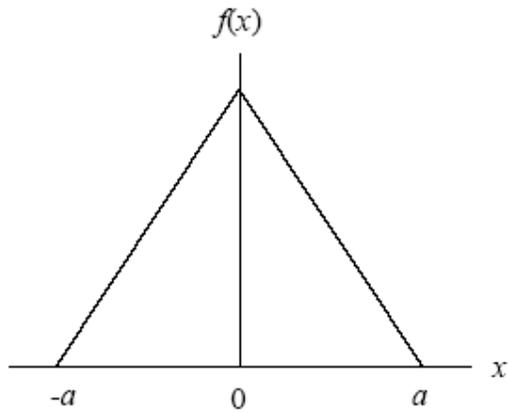


Figure G.2 Graph of Triangular Distribution

### Normal Distribution

Normal distributions are a family of distributions that have the same general shape. They are symmetric with scores more concentrated in the middle than in the tails. Normal distributions are sometimes described as bell shaped. Examples of normal distributions are shown to the right. Notice that they differ in how spread out they are. The area under each curve is the same. The height of a normal distribution can be specified mathematically in terms of two parameters the mean and the standard deviation. This distribution is used when an estimate is made from repeated observations of a randomly varying process [29].

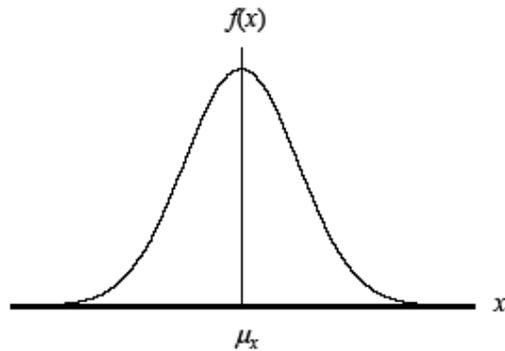


Figure G.3 Graph of Normal Distribution

### U-Shaped Distribution

The U-Shaped Distribution generally applied to the temperature effects. The uncertainty of the U-shaped Distribution equals to;

$$u = \frac{a}{\sqrt{2}}. \quad (G.1)$$

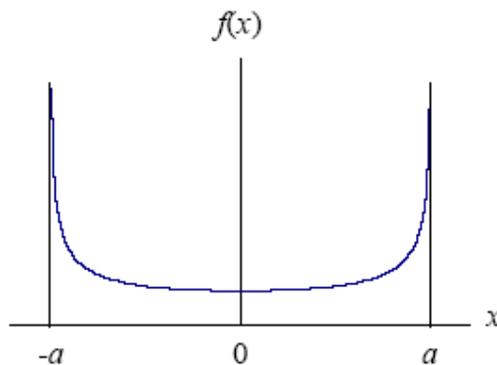


Figure G.4 Graph of a U-Shaped Distribution

### Student T Distribution

If the underlying distribution is normal, and a Type A estimate and degrees of freedom are available, confidence limits for measurement errors or parameter deviations may be obtained using the Student's t distribution. This distribution is available in statistics textbooks and popular spreadsheet applications. Its pdf is

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\pi\nu}\Gamma\left(\frac{\nu}{2}\right)}(1+x^2/\nu)^{-(\nu+1)/2}, \quad \text{G(2)}$$

where  $\nu$  is the degrees of freedom and  $\Gamma(\cdot)$  is the gamma function. The degrees of freedom quantifies the amount of knowledge used in estimating uncertainty. This knowledge is incomplete if the limits  $\pm a$  are approximate and the containment probability  $p$  is estimated from recollected experience. Since the knowledge is incomplete, the degrees of freedom associated with a Type B estimate is not infinite. If the degrees of freedom variable is finite but unknown, the uncertainty estimate cannot be rigorously used to develop confidence limits, perform statistical tests or make decisions. This limitation has often precluded the use of Type B estimates as statistical quantities and has led to such discomforting artifices as fixed coverage factors [29].

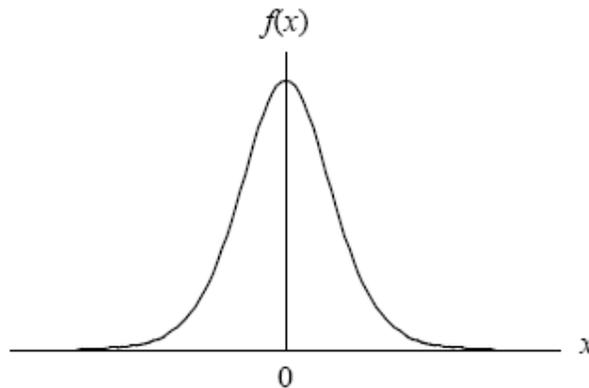


Figure G.5 Graph of Student's T Distribution