KINEMATIC SYNTHESIS OF SPATIAL MECHANISMS USING ALGEBRA OF EXPONENTIAL ROTATION MATRICES

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ÖZ

ÜSTEL DÖNME MATRİSLERİ ARACILIĞI İLE UZAYSAL MEKANİZMALARIN SENTEZİ

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Bu tezin büyük bir kısmı uzaysal mekanizmaların yörünge ve hareket üretimi sentezine adanmıştır. İlk kez üstel dönme matrisleri aracılığıyla bir kinematik sentez metodu ortaya konulmuştur. Ayrıca küresel, silindrik ve Hook gibi uzaysal eklemler , döner ve kayar eklemlerin bileşimi biçiminde modellenerek ve Denavit-Hartenberg kuralını kullanarak, tüm alt kinematik çift içeren uzaysal mekanizmaların yörünge ve hareket üretiminin sentezi için genel döngü kapanım denklemleri sunulmuştur. Mevcut sentez metodlarıyla kıyasladığımızda, bu tezde sunulmuş olan metodun en aşikar avantajı, ortaya konulmuş olan döngü kapanım denklemlerinin, tüm alt kinematik çift içeren uzaysal mekanizmalarda kullanışlı olmasıdır. Ayrıca bu metodda tasarımcı üstel dönme matrisleri cebirinin avantajlarından faydalanabilir.

Bu tezde sunulmuş olan sentez yönteminin kullanışlı olduğunu göstermek için RSHR, RCCR ve RSSR-SC mekanizmalarının döngü kapanım denklemleri elde edilmiş, bu denklemleri kullanarak 6 sayısal örnek çözülmüştür. Döngü kapanım denklemleri esas alınarak, bu mekanizmaların kinematik sentezinde tanımlanabilecek noktaların ve konumların sayısı hakkında faydalı bilgiler tablolar biçiminde sunulmuştur.

Sayısal örneklerde mekanizmalar, tezde elde edilmiş olan döngü kapanım denklemleri esas alınarak, çözülmüştür. Bazı örneklerde yarı analitik çözümler elde edilse de, örneklerin çoğunda döngü kapanım denklemleri **Mathcad'le** yazılmış olan programlarla çözülmüştür. Her sayısal örneğin sonunda girdi-çıktı açılarının diyagramı çizilmiş ve dallanmanın engellenmiş olduğu gösterilmiştir. Yazılmış olan bilgisayar programları hakkında detaylı bilgi verilmiş, denklemleri çözerken ortaya çıkabilecek sorunlar tartışılmış, çözümler üretilmiştir.

Yukarıda belirlenmiş olan mevzulara ilaveten, RCCR mekanizmasının üzerinde bir hareket kabiliyeti analizi yapılmıştır ve uzuv uzunluklarına bağlı olan eşitsizlikler elde edilmiştir. RCCR mekanizmasının salınım açısının diyagramı da çizilmiştir.

Anahtar kelimeler : Kinematik Sentezi, Uzaysal Mekanizma, Üstel Deveran Matrisleri , Döngü Kapanma Denklemleri.

ABSTRACT

KINEMATIC SYNTHESIS OF SPATIAL MECHANISMS USING ALGEBRA OF EXPONENTIAL ROTATION MATRICES

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The major part of this thesis has been devoted to path and motion generation synthesis of spatial mechanisms. For the first time kinematic synthesis methods have been presented based on the algebra of exponential rotation matrices. Besides modeling spatial pairs such as spheric , cylindric and Hook's joints by combinations of revolute and prismatic joints and applying Denavit-Hartenberg's convention , general loop closure equations have been presented for path and motion generation synthesis of any spatial mechanism with lower kinematic pairs. In comparison to the exsisting synthesis methods the main advantage of the methods presented in this thesis is that , general loop closure equations have been presented for any kind of spatial linkage consisting of lower kinematic pairs. Besides these methods enable the designer to benefit the advantages of the algebra of exponential rotation matrices.

In order to verify the applicability of the synthesis methods presented in the thesis , the general loop closure equations of RSHR , RCCR and RSSR-SC mechanisms have been determined and then using these equations six numerical examples have been solved. Some tables have been presented based on the determined loop closure equations which reveal useful information about the number of precision points or positions that can be considered for the kinematic synthesis of the above mentioned mechanisms and the number of free parameters.

In numerical examples , the mechanisms have been synthesized based on the general loop closure equations and the synthesis algorithms presented in the thesis. Although in some cases semi-analytical solutions have been obtained, in most of the cases, the loop closure equations were solved by computer programs written by **Mathcad.** The input angle-output angle diagrams drawn at the end of each numerical example illustrate the motion continuity of the mecahnisms and that branching has been avoided. Detailed information has been given about the computer programs and the difficulties which may arise while synthesizing spatial mechanisms.

In addition to the above mentioned points, a mobility analysis has been done for the RCCR mechanism and some inequalities have been obtained in terms of the link lengths. The swing angle diagram of the RCCR linkage has been drawn too.

Key words : Kinematic Synthesis , Spatial Mechanism , Algebra of Exponential Rotation Matrices, Loop Closure Equations.

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CHAPTER ONE

INTRODUCTION

1.1) GENERAL

Most of the mechanical linkages used in various machines and instruments are planar mechanisms. However there are many cases where spatial motion is needed. A mechanism whose motion is not limited to a fixed plane is considered to be spatial. Like planar mechanisms , spatial mechanisms are useful for generating various paths , motions , functions , or for transfering force and torque. Some spatial mechanisms have been illustrated in figures (1.1) to (1.3) .

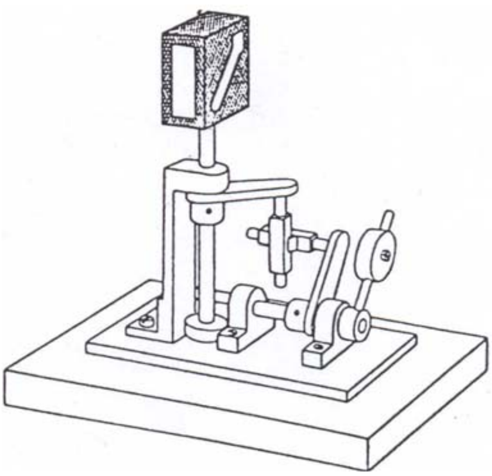


Figure (1.1) : Railway signal mechanism. A function generator (ref.29) .

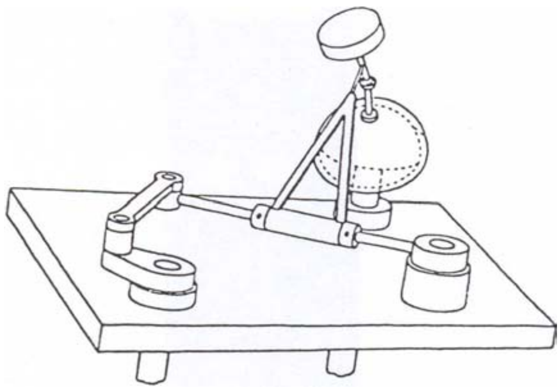


Figure (1.2) : Lens polishing machine. A motion generator (ref.29) .

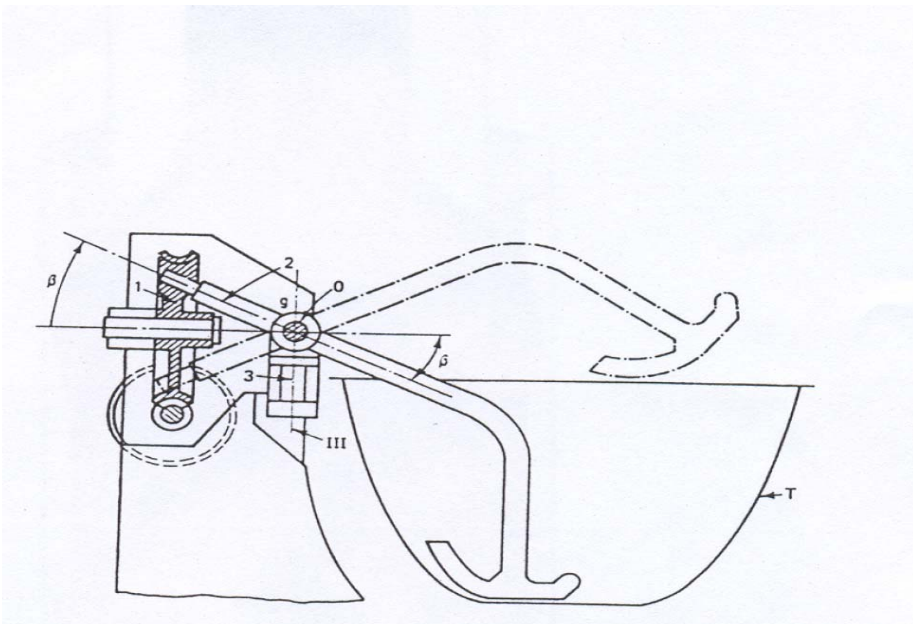


Figure (1.3) : Dough kneeding mechanism. A path generator (ref.29)

While there are only two types of lower kinematic joints in planar mechanisms , spatial mechanisms consist of various joint types. However the motion caused by any spatial joint can be modeled by a combination of revolute and prismatic joints. The following figure illustrates some spatial joints and their equivalent P-R (prismatic-revolute) combinations.

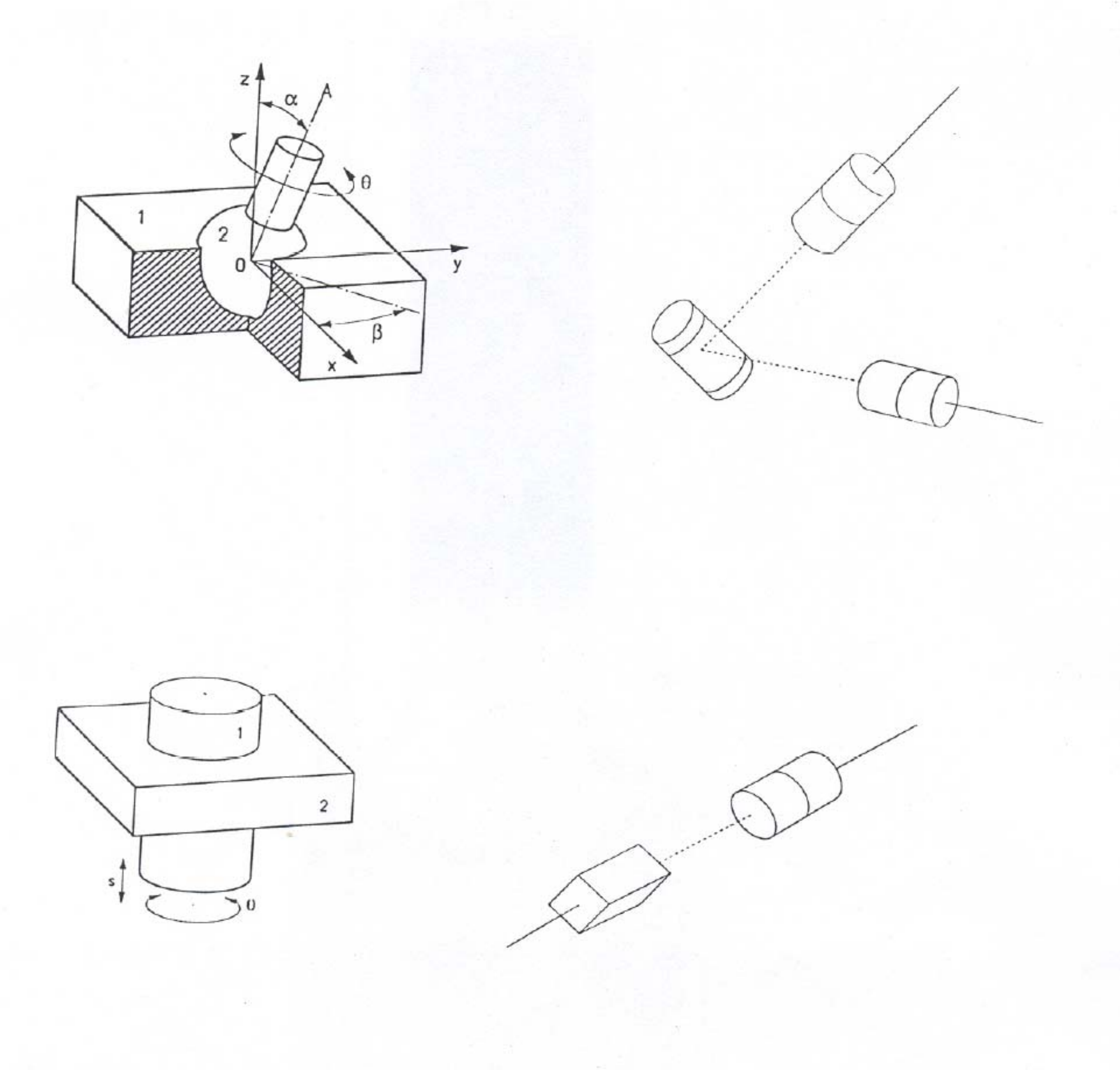
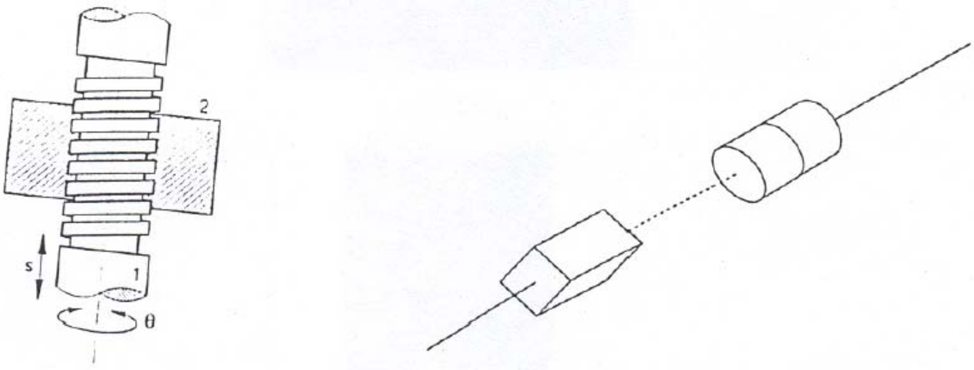


Figure (1.4) : Cylindric and spheric joints and their equivalent P-R combinations.



$$s = n\theta$$

Figure (1.5) : A screw joint and its equivalent P-R combination.

In order to present general loop closure equations for any spatial linkage with lower kinematic pairs, in this thesis all spatial joints have been replaced by their equivalent P-R combinations. Thus applying Denavit-Hartenberg's convention , the author succeeded to present general loop closure equations in chapter two. These equations can be applied to any spatial linkage with lower kinematic pairs.

1.2) KINEMATIC SYNTHESIS

In the kinematic synthesis of a mechanism the designer aims to synthesize a mechanism whose dimensions satisfy the desired motion of one of its links. There are three types of kinematic synthesis problems, *Motion Generation , Path Generation and Function Generation.* In motion generation synthesis the aim is to design a mechanism with a floating link passing through prescribed positions. In path generation synthesis it is desired to synthesize a mechanism so that some point on one of its floating links passes through prescribed points. In function generation synthesis , rotation or sliding motion of input and output links are correlated.

The major part of this thesis has been devoted to path and motion generation synthesis of spatial linkages and new methods of synthesis have been presented based on the algebra of exponential rotation matrices. This algebra has been described in detail by M.K.Özgören (ref.1,2). The properties of the algebra of exponential rotation matrices have been presented in the second chapter of this thesis too.

1.3) LITERATURE SURVEY

So far many analytical methods have been presented for kinematic synthesis of spatial mechanisms. Novodvorski (ref.7) formulated the function generation synthesis problem of the RSSR mechanism whose axes of ground pivots were skewed and nonintersecting. Rao et al (ref.9)used the principle of linear super position to synthesize several function generation mechanisms for the maximum number of precision points. Wilson (ref.10) derived the relationships to calculate centerpoint and spheric point curves for guiding a rigid body by means of an R-S link. Roth (ref.13) investigated the loci of special lines and points associated with spatial motion . Roth and Chen (ref.14,15) and Roth (ref.11,12) proposed a general theory for computing the number and locus of points in a rigid body in finite or infinitesimal motion. Sandor (ref.16) and Sandor and Bisshopp (ref.17) introduced methods of dual number , quaternions and stretch rotation tensors to find loop closure equations of spatial mechanisms . Suh (ref.18,19) employed 4 by 4 matrices for the synthesis of spatial mechanisms where design equations are expressed as constraint equations in order to obtain constrained motion. Kohli and Soni (ref.20,21) employed matrix methods to synthesize spherical four link and six link mechanisms for multiply separated positions of a rigid body in spherical motion. Alizade et al (ref.23,24) described the basis for a new method of type synthesis with the use of single loop structural groups having zero degrees of freedom. Jimenez et al (ref.26) used a set of fully Cartesian coordinates to describe a mechanism by a set of geometric constraints and introduced the design requirements by a set of functional constraints and finally Shih and Yan (ref.25) presented a synthesis method for the rigid body guidance between two prescribed positions based on descriptive geometry.

1.4) MOTIVATION

Up to now many mathematical methods such as the algebra of complex numbers , the algebra of dual numbers , quaternions , screw algebra and the algebra of exponential rotation matrices have been used to develop the theory of kinematics. The algebra of exponential rotation matrices which is an efficient and elegant tool for working with matrice equations has been used in the analysis of robot manipulators (ref.1,2,4,6) and spatial mechanisms (ref.5) . However so far no body has used the algebra of exponential rotation matrices for the purpose of synthesis in an official text. Since the author of this thesis had worked with this mathematical tool when analyzing serial robot manipulators and he was fully aware of its fantastic capabilities , he decided to use it for synthesizing spatial mechanisms for the first time.

Most of the synthesis methods presented for synthesizing spatial mechanisms describe general synthesis techniques but not general formulas. In this thesis , using the algebra of exponential rotation matrices and Denavit-Hartenberg's convention the author succeeded to present general formulas for the path and motion generation synthesis of spatial mechanisms which can be applied to both single and multiloop spatial mechanisms . The synthesis methods presented in this thesis , enable the mechanism designer to benefit the advantages of the algebra of exponential rotation matrices. Besides in these methods the designer directly deals with link lengths and link angles which make more sense while in some other synthesis methods (ref. 18,19,26) the designer works with X,Y and Z coordinates.

Three spatial mechanisms have been chosen as examples in the thesis . The first mechanism is an RSHR linkage which is a simple single loop spatial mechanism and was chosen just for its simplicity and in order to explain how the synthesis methods are applied to a single loop spatial mechanism. The second mechanism is an RCCR linkage which is an overconstrained mechanism . This mechanism was selected because of its constraints and their effects on the synthesis procedure of the mechanism. The last example is an RSSR-SC linkage which is a two-loop spatial mechanism. It was chosen to verify that the synthesis methods and formulas presented in the thesis are appliable to multiloop linkages too.

CHAPTER TWO
MATHEMATICAL TOOLS AND CONVENTIONS

2.1) GENERAL

Analysis and synthesis of spatial mechanisms always require solving nonlinear equations which are usually lengthy and complicated.The usual mathematical methods like matrix and vector algebra can be used for analyzing and synthesizing mechanisms but since these usual methods are time consuming, mechanism designers have tried to develope more efficient mathematical methods in kinematics . Among the various developed methods the following methods are noteworthy,

1) METHOD OF COMPLEX NUMBERS

This method was developed by Coolidge[1940] , Zwikker[1950] , Morley[1954] and Beris[1958]. Although some interesting results were found in the method the fact is that it is quite limited to planar motion (ref.28).

2) ALGEBRA OF DUAL NUMBERS

This algebra was introduced by Clifford[1850] and it was systematically applied to kinematics by Kotelnikov[1895] . It can be applied to both planar and spatial kinematics (ref.28).

3) ALGEBRA OF QUATERNIONS

Algebra of quaternions is an elegant tool to describe spherical displacements and has been used by Blaschke[1960] and H.R.Müller[1962].

4) SCREW ALGEBRA

Screw algebra has been employed in kinematics for more than two centuries. Mozzi studied this algebra in the eighteenth century. It was rediscovered in 1960's by Hunt and Phillips then Walderon and Hunt employed this theory to search for overconstrained mechanisms. The last decades witnessed the publication of several studies from Duffy concerning the kinematic and dynamic analysis of spatial linkages via screw theory (ref.27,34) .

5) ALGEBRA OF EXPONENTIAL ROTATION MATRICES

This algebra which was developed by Özgören (ref.1,2) enables us to efficiently simplify the matrix and vector equations involved in the synthesis and analysis procedures of spatial mechanisms. The algebra of exponential rotation matrices has been successfully used in the kinematic analysis of robot manipulators by M.K. Özgören (ref.1,4,6). He has also written a paper on the analysis of spatial mechanisms (ref.5) by means of the algebra of exponential rotation matrices. In this thesis , using the algebra of exponential rotation matrices , the author succeeded to develope a synthesis method for the path and motion generation synthesis of spatial mechanisms.

2.2) PROPERTIES OF THE ALGEBRA OF EXPONENTIAL ROTATION MATRICES

Let $\bar{n} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$, $\tilde{n} = \begin{bmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{bmatrix}$, $\hat{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then

$$e^{\tilde{n}\theta} = \hat{I}\cos\theta + \tilde{n}\sin\theta + \bar{n}\bar{n}^t(1-\cos\theta)$$

Where $e^{\tilde{n}\theta}$ is the rotation matrix about an axis of unit vector $\bar{n}$ through angle θ .

Assuming $\bar{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\bar{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\bar{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ the following equations are obtained (ref.6),

$$e^{\tilde{u}_1\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} , \quad e^{\tilde{u}_2\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} , \quad e^{\tilde{u}_3\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

1) $\tilde{n}^2 = \bar{n}\bar{n}^t - \hat{I}$ and $\tilde{n}^3 = -\tilde{n}$

2) $\tilde{n}\tilde{m} = \bar{m}\bar{n}^t - (\bar{m}^t\bar{n})\hat{I}$

3) If $\bar{u} = \tilde{n}\bar{m}$ then $\tilde{u} = \tilde{n}\tilde{m} - \tilde{m}\tilde{n} = \bar{m}\bar{n}^t - \bar{n}\bar{m}^t$

4) $\bar{r}^t\tilde{n}\bar{r} = 0$ but $\bar{r}^t\tilde{n}^2\bar{r} = (\bar{n}^t\bar{r})^2 - \bar{r}^t\bar{r}$

5) $(e^{\hat{n}\theta})^{-1} = (e^{\tilde{n}\theta})^t = e^{-\tilde{n}\theta}$

6) $e^{\tilde{n}\theta}e^{\tilde{n}\phi} = e^{\tilde{n}\phi}e^{\tilde{n}\theta} = e^{\tilde{n}(\theta+\phi)}$

7) $e^{\tilde{n}\theta}\overline{n} = \overline{n}$ and $\overline{n}^t e^{\tilde{n}\theta} = \overline{n}^t$

8) $e^{\tilde{n}\theta}\tilde{n} = \tilde{n}e^{\tilde{n}\theta}$

9) If $\overline{m} = e^{\tilde{n}\theta}\overline{u}$ then $\tilde{m} = e^{\tilde{n}\theta}\tilde{u}e^{-\tilde{n}\theta}$ and $e^{\tilde{m}\phi} = e^{\tilde{n}\theta}e^{\tilde{u}\phi}e^{-\tilde{n}\theta}$

10) $e^{\tilde{m}\beta}e^{\tilde{n}\theta} = e^{\tilde{p}\theta}e^{\tilde{m}\beta}$ where $\overline{p} = e^{\tilde{m}\beta}\overline{n}$

11) $e^{\tilde{n}\theta}e^{\tilde{m}\beta} = e^{\tilde{m}\beta}e^{\tilde{q}\theta}$ where $\overline{q} = e^{-\tilde{m}\beta}\overline{n}$

12) $e^{\tilde{u}_i\theta}\overline{u}_j = \overline{u}_j \cos\theta + \tilde{u}_i\overline{u}_j \sin\theta$

13) $\overline{u}_j^{\,t} e^{\tilde{u}_i\theta} = \overline{u}_j^{\,t} \cos\theta + (\tilde{u}_j\overline{u}_i)^t \sin\theta$

2.3) DENAVIT-HARTENBERG'S CONVENTION

Denavit-Hartenberg's convention has been applied to all fixed and moving frames in both synthesis and analysis processes in this thesis .The convention can be shortly explained as follows (ref.33) ,

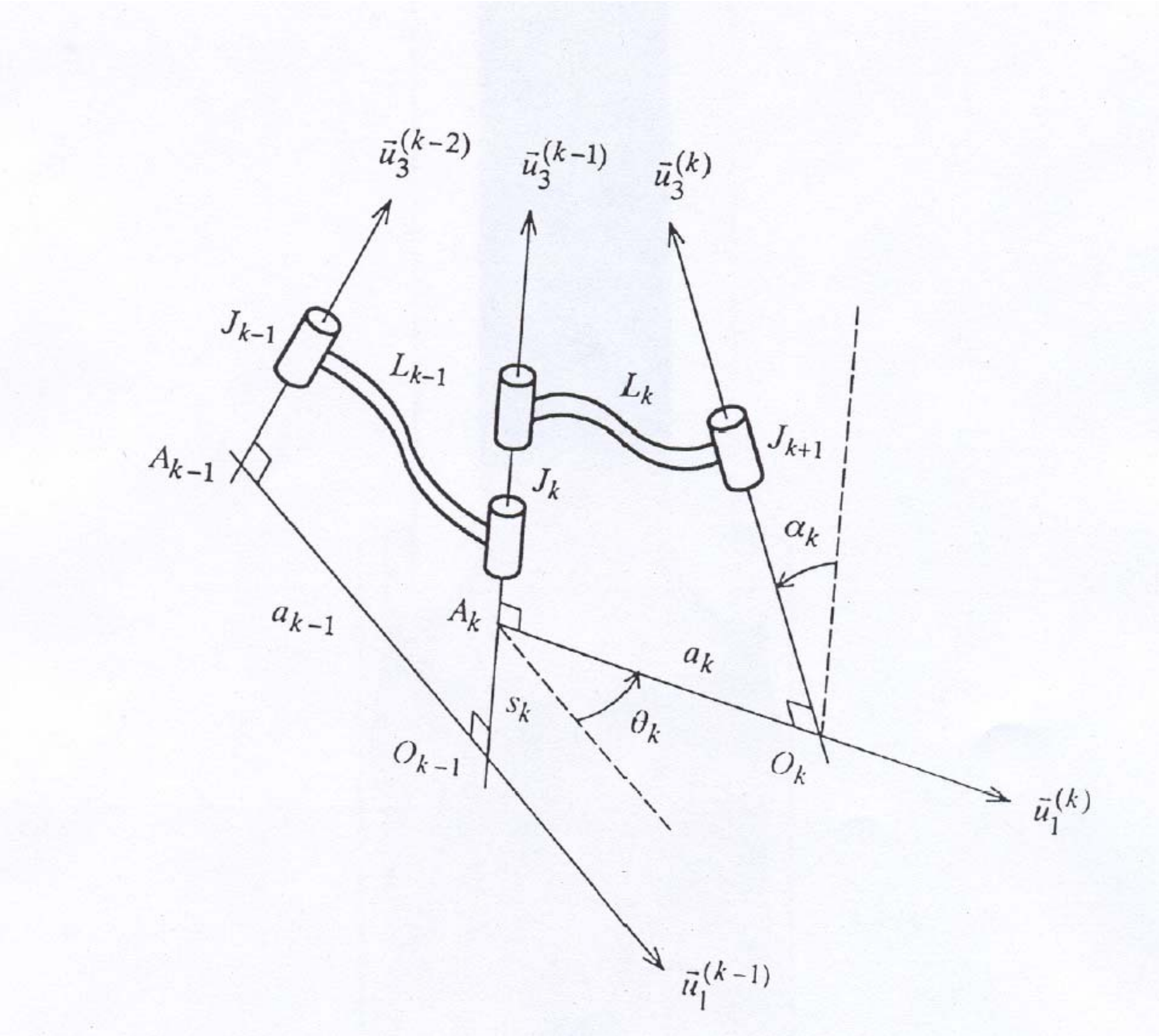


Figure (2.1) : Installation of reference frames according to Denavit-Hartenberg's convention. (ref.6)

As illustrated in Figure(2.1) the third axis lies along the joint axis and the first axis is the common normal of the two neighbor joint axes and finally the orientation of the second axis is determined according to the right hand rule.

The link parameters are ,

$a_k = A_k O_k$: the effective length of link (k) .

α_k : the angle between $\vec{u}_3^{(k)}$ and $\vec{u}_3^{(k-1)}$ about $\vec{u}_1^{(k)}$.

s_k : the translational distance of link (k) with respect to link (k-1) along $\vec{u}_3^{(k-1)}$.

θ_k : the rotational angle of link (k) with respect to link (k-1) about $\vec{u}_3^{(k-1)}$.

2.4) LOOP CLOSURE EQUATIONS

Consider a single loop spatial mechanism with n links in which the fixed link is named as both 0′th and n′th link. In other words link zero and link n both address the fixed link. Installing coordinate systems according to Denavit-Hartenberg's convention the following loop closure equations , are written (ref.5),

a) ORIENTATION LOOP CLOSURE EQUATION :

$$\hat{C}^{(0,1)}\hat{C}^{(1,2)}\hat{C}^{(2,3)}...\hat{C}^{(n-1,n)} = \hat{C}^{(0,n)} = \hat{I} \tag{2.1}$$

where $\hat{C}^{(k-1,k)}$ is the orientation matrix of coordinate system (k) with respect to coordinate system (k-1) and $\hat{I}$ is the 3 by 3 identity matrix.

Considering Denavit-Hartenberg's convention the folllowing equation can be written,

$$\hat{C}^{(k-1,k)} = e^{\tilde{u}_3\theta_k} e^{\tilde{u}_1\alpha_k}$$

Thus equation (2.1) can be presented in the form below,

$$e^{\tilde{u}_3\theta_1} e^{\tilde{u}_1\alpha_1} e^{\tilde{u}_3\theta_2} e^{\tilde{u}_1\alpha_2} ...e^{\tilde{u}_3\theta_n} e^{\tilde{u}_1\alpha_n} = \hat{I} \tag{2.2}$$

This equation has been written based on the fact that the orientation of the fixed link is always constant.

b) POSITION LOOP CLOSURE EQUATION

$$\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \ldots + \vec{r}_n = \vec{0} \tag{2.3}$$

where $\vec{r}_k$ is a vector drawn from the origin of the (k-1)'th frame to the origin of the k'th frame.

The following equation is obtained according to Denavit-Hartenberg's convention,

$$\overline{r}_k^{(0)} = d_k \hat{C}^{(0,k-1)} \overline{u}_3 + a_k C^{(0,k)} \overline{u}_1$$

where $\overline{r}_k^{(0)}$ represents the column matrix form of $\vec{r}_k$ defined in zero'th frame.

Thus equation (2.3) can be written in the following form ,

$$d_1 \overline{u}_3 + a_1 \hat{C}^{(0,1)} \overline{u}_1 + d_2 \hat{C}^{(0,1)} \overline{u}_3 + a_2 \hat{C}^{(0,2)} \overline{u}_1 + \ldots + d_n \hat{C}^{(0,n-1)} \overline{u}_3 + a_n \hat{C}^{(0,n)} \overline{u}_1 = \overline{0} \tag{2.4}$$

The loop closure equations above are two fundamental matrix equations by which the general displacement equation of any spatial linkage is obtained. These equations also play a key role in the synthesis method which will be explained in the next chapter. The following example shows how the general displacement equation of a spatial linkage is determined by means of loop closure equations explained above.

Example (2.1) : Determine the general displacement equation of the RSHR mechanism illustrated in Figure (2.2) .

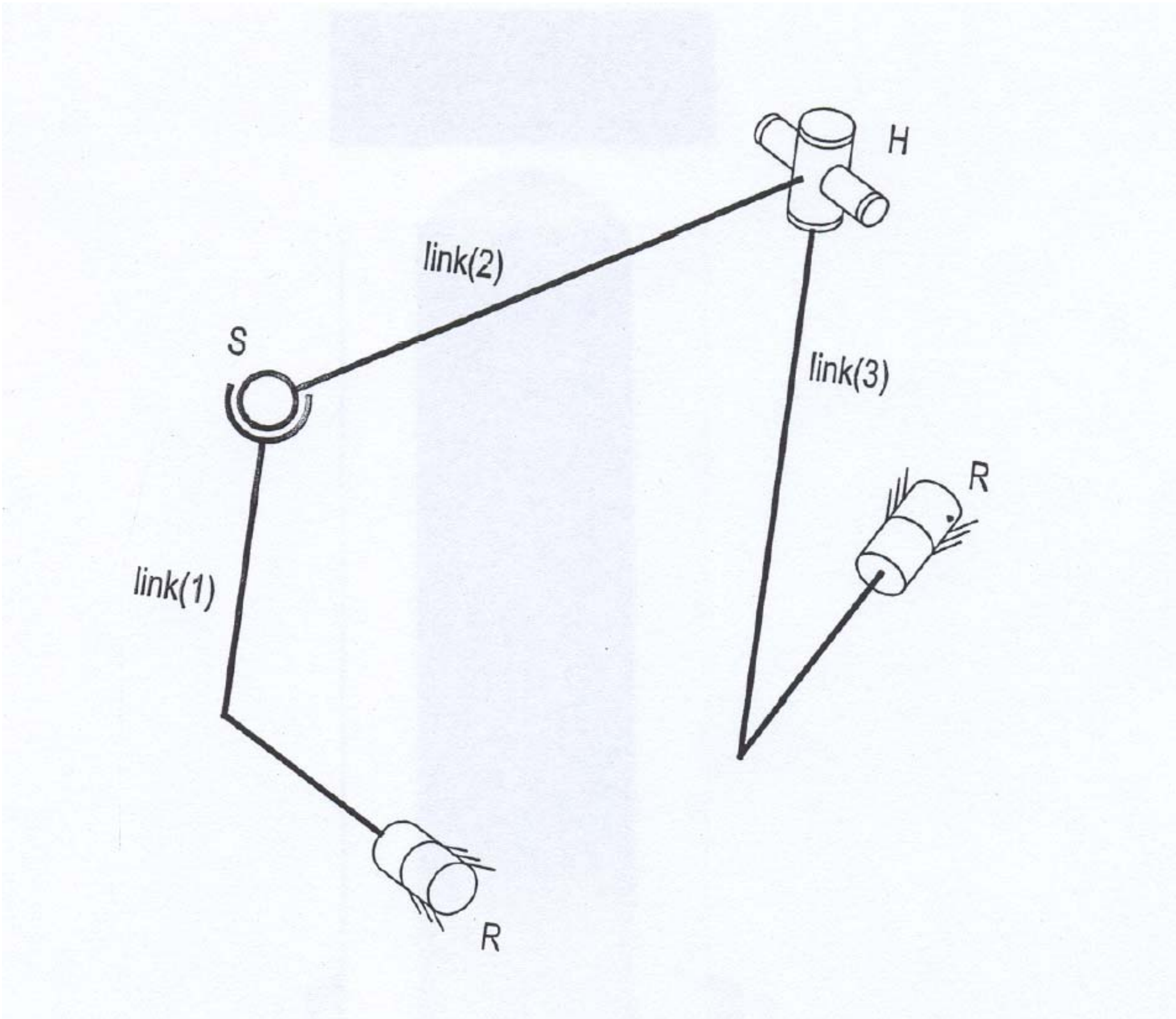
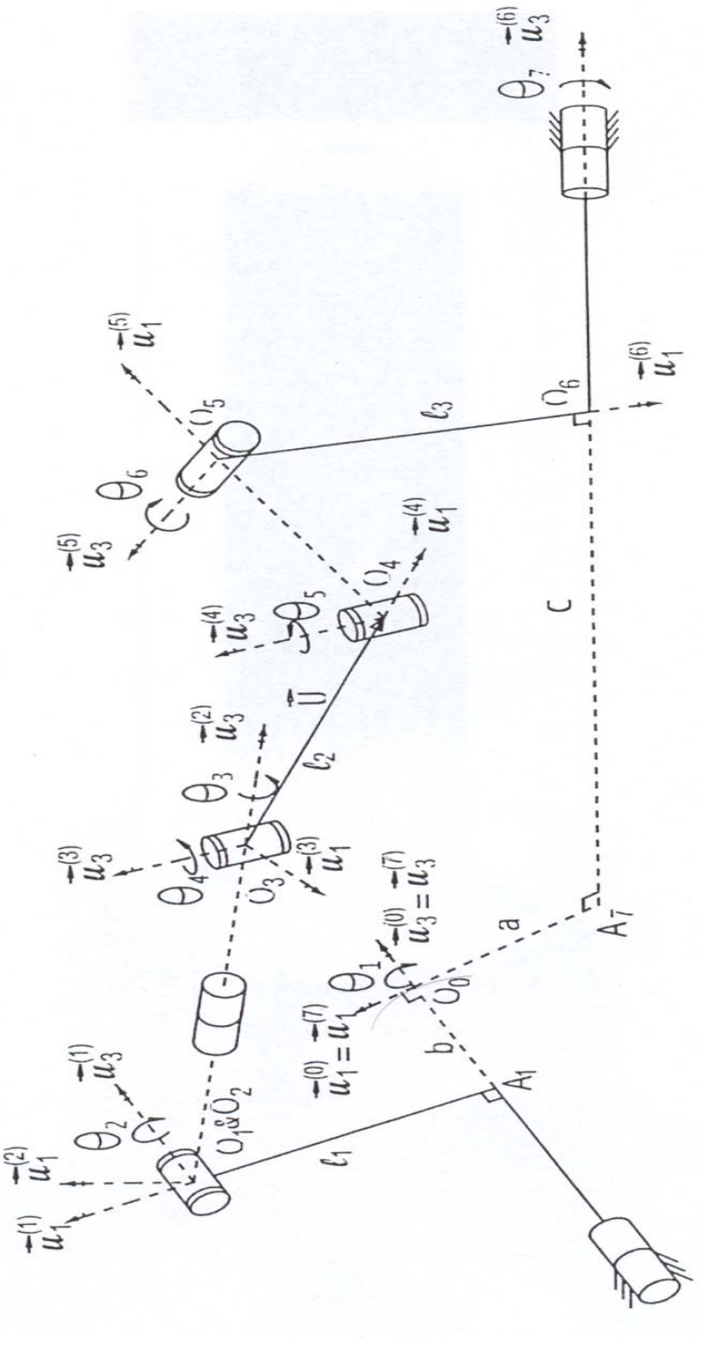


Figure (2.2) : An RSHR linkage

The RSHR linkage can be redrawn as illustrated in Figure(2.3) . Note that links 2,3,5 are virtual and their lengths are equal to zero.



Figure(2.3) : P_R combination of the RSHR linkage. The reference frames have been installed according to Denavit-Hartenberg's convention.

Table (2.1) has been constructed according to Figure(2.3) . Considering this table the following equations are obtained,

$$\hat{C}^{(0,1)} = e^{\tilde{u}_3\theta_1} \quad \hat{C}^{(0,2)} = e^{\tilde{u}_3(\theta_1+\theta_2)}e^{-\tilde{u}_1\pi/2} \quad \hat{C}^{(0,3)} = e^{\tilde{u}_3(\theta_1+\theta_2)}e^{-\tilde{u}_1\pi/2}e^{\tilde{u}_3\theta_3}e^{\tilde{u}_1\pi/2} = e^{\tilde{u}_3(\theta_1+\theta_2)}e^{\tilde{u}_2\theta_3}$$

$$\hat{C}^{(0,4)} = e^{\tilde{u}_3(\theta_1+\theta_2)}e^{\tilde{u}_2\theta_3}e^{\tilde{u}_3\theta_4} \quad \hat{C}^{(0,5)} = e^{\tilde{u}_3(\theta_1+\theta_2)}e^{\tilde{u}_2\theta_3}e^{\tilde{u}_3(\theta_4+\theta_5)}e^{-\tilde{u}_1\pi/2}$$

$$\hat{C}^{(0,6)} = e^{\tilde{u}_3(\theta_1+\theta_2)}e^{\tilde{u}_2\theta_3}e^{\tilde{u}_3(\theta_4+\theta_5)}e^{\tilde{u}_2\theta_6} \quad \hat{C}^{(0,7)} = \hat{I}$$

Table (2.1) :Joint variables and Denavit-Hartenberg parameters of the RSHR linkage.

Link	θ	α	a	d
1	variable	0	l_1	-b
2	variable	$-\pi/2$	0	0
3	variable	$\pi/2$	0	0
4	variable	0	l_2	0
5	variable	$-\pi/2$	0	0
6	variable	$\pi/2$	l_3	0
7	variable	$\pi/2$	a	-c

The equation below can be written according to equation (2.4)

$$-b\overline{u}_3 + l_1\hat{C}^{(0,1)}\overline{u}_1 + l_2\hat{C}^{(0,4)}\overline{u}_1 + l_3\hat{C}^{(0,6)}\overline{u}_1 - c\hat{C}^{(0,6)}\overline{u}_3 + a\hat{C}^{(0,7)}\overline{u}_1 = \overline{0} \quad (2.5)$$

Considering equation (2.1) the following equations are obtained,

$$\hat{C}^{(0,4)} = (\hat{C}^{(4,7)})^{-1} = (e^{\tilde{u}_3\theta_5}e^{\tilde{u}_2\theta_6}e^{\tilde{u}_3\theta_7}e^{\tilde{u}_1\pi/2})^{-1} = e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7}e^{-\tilde{u}_2\theta_6}e^{-\tilde{u}_3\theta_5}$$

$$\hat{C}^{(0,5)} = (\hat{C}^{(5,7)})^{-1} = (e^{\tilde{u}_2\theta_6} e^{\tilde{u}_3\theta_7} e^{\tilde{u}_1\pi/2})^{-1} = e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7} e^{-\tilde{u}_2\theta_6}$$

$$\hat{C}^{(0,6)} = (\hat{C}^{(6,7)})^{-1} = (e^{\tilde{u}_3\theta_7} e^{\tilde{u}_1\pi/2})^{-1} = e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7}$$

$$\hat{C}^{(0,7)} = \hat{I}$$

Substituting $\hat{C}^{(0,4)}, \hat{C}^{(0,5)}, \hat{C}^{(0,6)}$ and $\hat{C}^{(0,7)}$ from equations above into equation (2.5) equation below is got,

$$-b\bar{u}_3 + l_1 e^{\tilde{u}_3\theta_1}\bar{u}_1 + l_2 e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7} e^{-\tilde{u}_2\theta_6} e^{-\tilde{u}_3\theta_5}\bar{u}_1 + l_3 e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7}\bar{u}_1 - c e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7}\bar{u}_3 + a\bar{u}_1 = \bar{0} \tag{2.6}$$

or

$$-b\bar{u}_3 + l_1 \cos\theta_1 \bar{u}_1 + l_1 \sin\theta_1 \bar{u}_2 + l_2(\cos\theta_5 \cos\theta_6 \cos\theta_7 - \sin\theta_5 \sin\theta_7)\ \bar{u}_1$$
$$+ l_2 \cos\theta_5 \sin\theta_6 \bar{u}_2 + l_2(\cos\theta_5 \cos\theta_6 \sin\theta_7 + \sin\theta_5 \cos\theta_7)\bar{u}_3 + l_3 \cos\theta_7 \bar{u}_1$$
$$+ l_3 \sin\theta_7 \bar{u}_3 - c\bar{u}_2 + a\bar{u}_1 = \bar{0}$$

which results in the following equations,

$$l_1 \cos\theta_1 + l_2(\cos\theta_5 \cos\theta_6 \cos\theta_7 - \sin\theta_5 \sin\theta_7) + l_3 \cos\theta_7 + a = 0 \tag{2.7}$$

$$l_1 \sin\theta_1 + l_2 \cos\theta_5 \sin\theta_6 - c = 0 \tag{2.8}$$

$$-b + l_2(\cos\theta_5 \cos\theta_6 \sin\theta_7 + \sin\theta_5 \cos\theta_7) + l_3 \sin\theta_7 = 0 \tag{2.9}$$

From equations (2.7) and (2.9) the following equation is derived,

$$l_2{}^2 \cos^2\theta_5 \cos^2\theta_6 + \sin^2\theta_5 = l_1{}^2 \cos^2\theta_1 + l_3{}^2 - 2bl_3 \sin\theta_7 + a^2 + b^2 + 2l_1 l_3 \cos\theta_1 \cos\theta_7$$
$$+ 2al_1 \cos\theta_7 + 2al_3 \cos\theta_7 \tag{2.10}$$

and from equation (2.8) equation below can be obtained,

$$l_2^2 \cos^2 \theta_5 \sin^2 \theta_6 = l_1^2 \sin^2 \theta_1 + c^2 - 2 l_1 c \sin \theta_1 \tag{2.11}$$

Now adding equations (2.10) and (2.11) the following equation is gained,

$$-2l_3(l_1 \cos\theta_1 + a)\cos\theta_7 + 2bl_3 \sin\theta_7 = l_1^2 - l_2^2 + l_3^2 + a^2 + b^2 + c^2 + 2al_1 \cos\theta_1 - 2l_1 c \sin\theta_1$$

Let $\theta_1 = \psi$ and $\theta_7 = \phi$ .Hence equation above can be rewritten in the following form,

$$-2l_3(l_1 \cos\psi + a)\cos\phi + 2bl_3 \sin\phi = l_1^2 - l_2^2 + l_3^2 + a^2 + b^2 + c^2 + 2al_1 \cos\psi$$
$$-2l_1 c \sin\psi \tag{2.12}$$

Equation (2.12) is called the general displacement equation of the RSHR linkage and plays a key role in mobility analysis and function generation synthesis of the linkage.

Example (2.2) :Consider an RSHR mechanism with the following dimensions ,

$l_1 = 5 \quad l_2 = 11 \quad l_3 = 20 \quad a = 4 \quad b = 15 \quad c = 3$

Determine the general displacement equation of the linkage and draw its output angle diagram versus input angle.

Equation below can be written according to equation (2.12)

$$-40(5\cos\psi + 4)\cos\phi + 600\sin\phi = 40\cos\psi - 30\sin\psi + 554$$

Let $p(\psi) = -2l_3(l_1 \cos\psi + a)$, $q = 2bl_3$,

$r(\psi) = l_1^{\,2} - l_2^{\,2} + l_3^{\,2} + a^2 + b^2 + c^2 + 2al_1 \cos\psi - 2l_1 c \sin\psi$

Thus equation (2.12) is written in the following form ,

$$p(\psi)\cos\phi + q\sin\phi = r(\psi) \tag{2.13}$$

Now let $\cos\phi = \dfrac{1-t^2}{1+t^2}$ and $\sin\phi = \dfrac{2t}{1+t^2}$. Hence equation (2.13) is written in the form below ,

$$(r(\psi) + p(\psi))t^2 - 2qt + r(\psi) - p(\psi) = 0$$

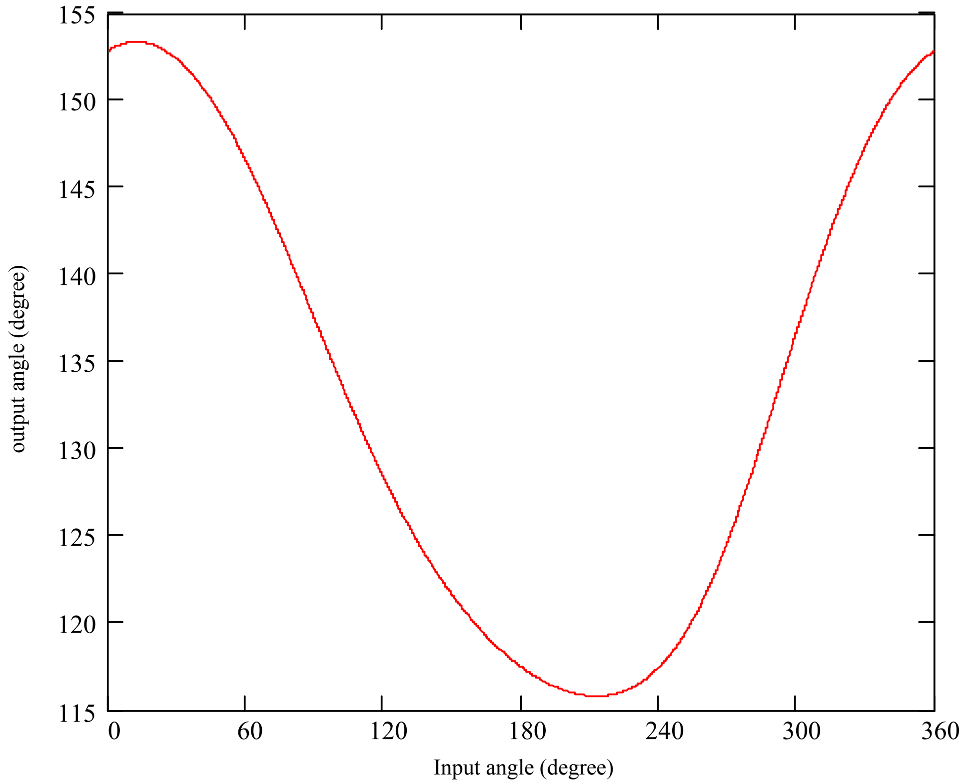
which results in ,

$$t = \frac{q \pm \sqrt{p^2(\psi) + q^2 - r^2(\psi)}}{r(\psi) + p(\psi)}$$

Thus the equation below is derived ,

$$\phi = angle(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2})$$

For the RSHR linkage whose dimensions have been given above the output-input angle curve has been illustrated in Figure(2.4).



Figure(2.4) : As illustrated by the diagram, the RSHR linkage acts as a crank_rocker

CHAPTER THREE

PATH AND MOTION GENERATION SYNTHESIS OF SPATIAL MECHANISMS

3.1) GENERAL

The path and motion generation synthesis methods which will be explained in this chapter are the first synthesis methods developed by means of the algebra of exponential rotation matrices. The main advantage of these methods is that general dyad equations have been presented for single loop spatial linkages with n links (equations (3.1) and (3.2)) and very similar equations can be written for multiloop spatial linkages. Besides in these methods the mechanism designer directly deals with link lengths and joint angles which make more sense while in some synthesis methods ,designers work with X ,Y and Z coordinates (ref.18,19,26). Finally using these synthesis methods designers can benefit the advantages of the algebra of exponential rotation matrices.

3.2) PATH GENERATION SYNTHESIS

Consider a single loop spatial mechanism with n links. Assume that a coordinate system is attached to each link according to Denavit-Hartenberg's convention . Now consider a point P on the k'th link .This point -which is called the path tracer point- is supposed to pass through points $P_0, P_1, ..., P_{j-1}$ which are called precision points.

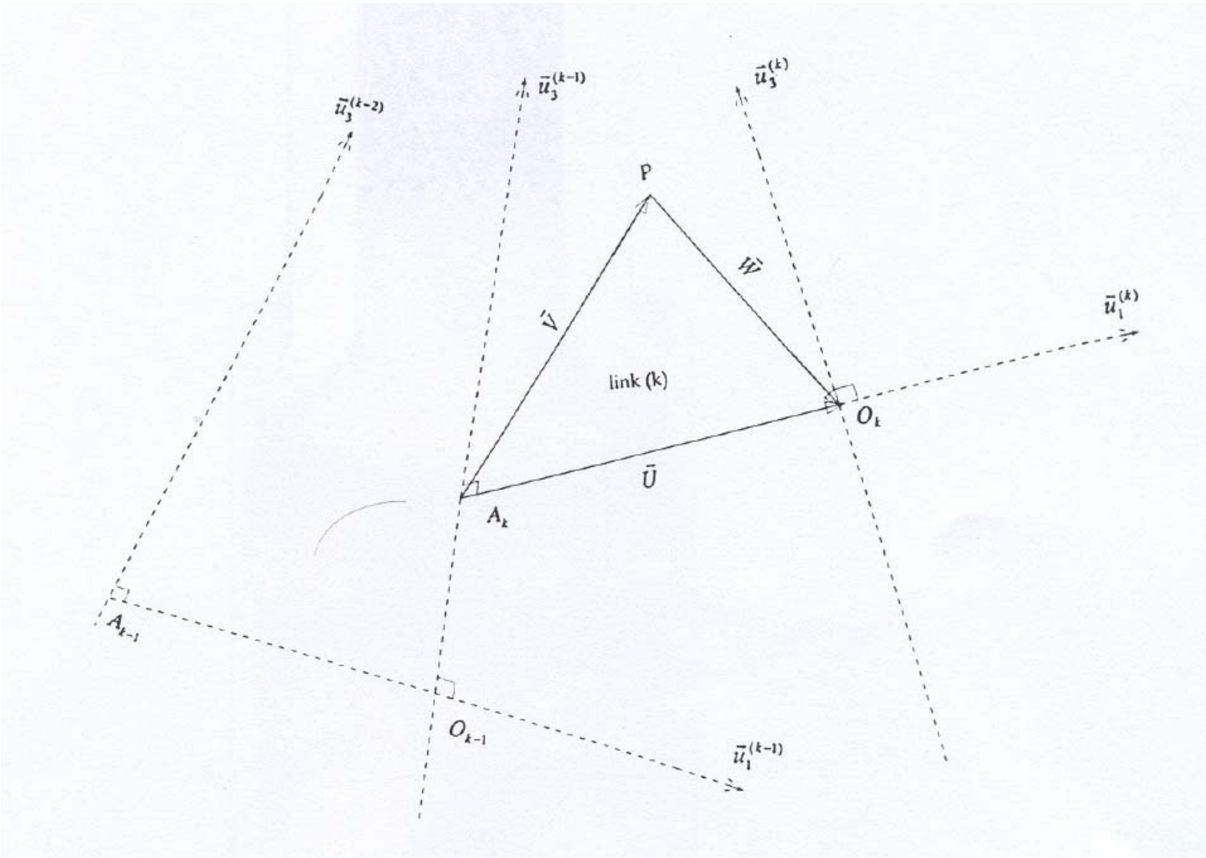


Figure (3.1) : The vectors which construct the k'th link .

As illustrated in Figure (3.1) vectors $\vec{U}, \vec{V}, \vec{W}$ have been defined as constant vectors in the k'th frame because the k'th frame is attached to the k'th link .

Let $\vec{R}_0, \vec{R}_1, ..., \vec{R}_{j-1}$ be vectors which have been drawn from the origin of a global frame to the precision points $P_0, P_1, ..., P_{j-1}$ respectively . Where j is the number of precision points. Assuming that the fixed frame attached to the linkage is called as both zero'th frame and n'th frame, the following loop closure equations can be written for the left and right dyads of the linkage ,

$$\bar{r} + d_1 \hat{C}^{(g,0)} \bar{u}_3 + a_1 \hat{C}_i^{(g,1)} \bar{u}_1 + d_2 \hat{C}_i^{(g,1)} \bar{u}_3 + a_2 \hat{C}_i^{(g,2)} \bar{u}_1 + ... + d_{k-1} \hat{C}_i^{(g,k-2)} \bar{u}_3 + a_{k-1} \hat{C}_i^{(g,k-1)} \bar{u}_1 + d_k C_i^{(g,k-1)} \bar{u}_3 + \hat{C}_i^{(g,k)} \bar{V}^{(k)} = \bar{R}_i \quad (3.1)$$

and

$$\hat{C}_i^{(g,k)} \overline{W}^{(k)} + d_{k+1} \hat{C}_i^{(g,k)} \bar{u}_3 + a_{k+1} \hat{C}_i^{(g,k+1)} \bar{u}_1 + ... + d_n \hat{C}_i^{(g,n-1)} \bar{u}_3 + a_n \hat{C}_i^{(g,n)} \bar{u}_1 - \bar{r} = -\bar{R}_i \quad (3.2)$$

and

$$\bar{V}^{(k)} + \overline{W}^{(k)} = \bar{U}^{(k)} \quad (3.3)$$

where ,

$i = 0,1,..., j-1$ and j is the number of precision points .

$\hat{C}_i^{(g,k)} = \hat{C}^{(g,0)} \hat{C}_i^{(0,k)}$

$\hat{C}^{(g,0)}$ is the orientation matrix of the zero'th frame with respect to the global frame and since both frames are fixed ,obviously $\hat{C}^{(g,0)}$ will be a constant matrix which can be defined by an arbitrary sequence of rotations . For example by $\hat{C}^{(g,0)} = e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}$ or $\hat{C}^{(g,0)} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}$.

$\hat{C}_i^{(0,k)}$ is the orientation matrix of the k'th coordinate system with respect to the zero'th frame when the path tracer point is coincident to the i'th precision point and is defined as follows,

$$\hat{C}_i^{(0,k)} = e^{\tilde{u}_3 \theta_1^i} e^{\tilde{u}_1 \alpha_1} e^{\tilde{u}_3 \theta_2^i} e^{\tilde{u}_1 \alpha_2} \ldots e^{\tilde{u}_3 \theta_k^i} e^{\tilde{u}_1 \alpha_k} \tag{3.4}$$

$\theta_1^i, \theta_2^i, \ldots, \theta_k^i$ are the joint variables when the path tracer point is coincident to the i'th precision point and α_k is the angle between $\overline{u}_3^{(k)}$ and $\overline{u}_3^{(k-1)}$ about $\overline{u}_1^{(k)}$.

$\overline{V}^{(k)}$, $\overline{W}^{(k)}$ and $\overline{U}^{(k)}$ have been illustrated in Figure (3.1) .

Note that when the locations and orientations of the fixed joints are prescribed ,without loss of generality the global frame can be chosen to be coincident to the zero'th frame . In this case the following equalities will be available,

$\hat{C}^{(g,0)} = \hat{I}$ and $\overline{r} = \overline{0}$

where $\hat{I}$ is the 3 by 3 identity matrix and $\overline{0}$ is the null matrix .

3.3) PATH GENERATION SYNTHESIS OF AN RSHR LINKAGE

a) THE POSITIONS OF THE GROUND PIVOTS ARE PRESCRIBED.

Considering Figure (3.3) the associated Denavit-Hartenberg parameters have been determined as listed in table (3.1) .

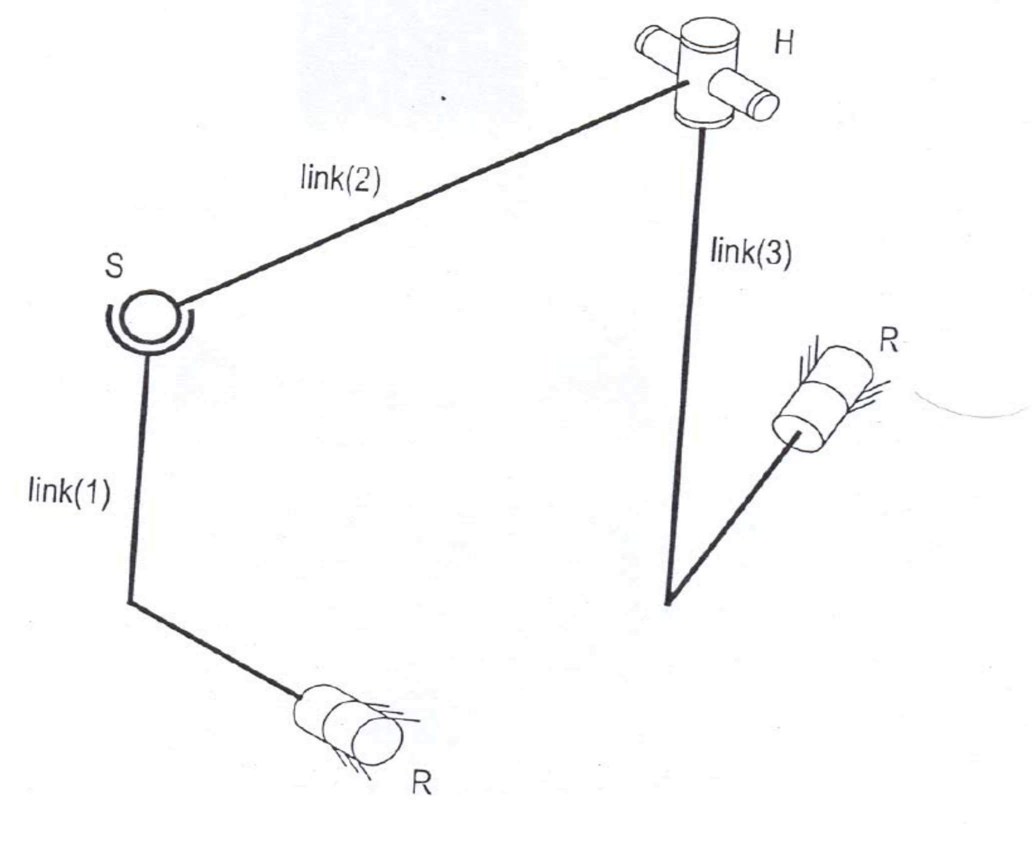


Figure (3.2) : An RSHR linkage.

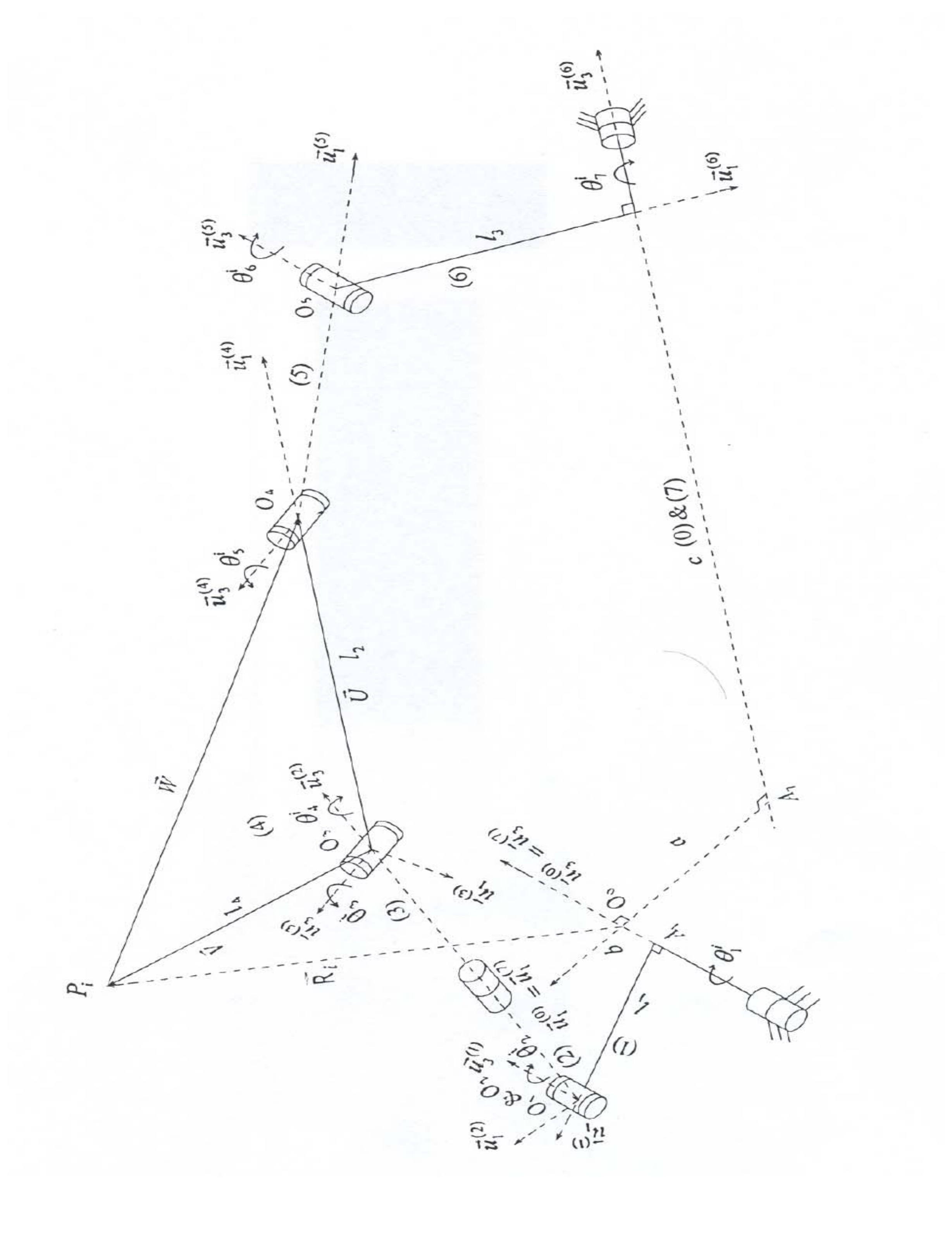


Figure (3.3) : The right and left dyads of the RSHR linkage .Note that links 2,3 and 5 are virtual links and their lengths are equal to zero.

Table(3.1) : Joint variables and Denavit-Hartenberg parameters of the RSHR linkage.

link	θ	α	a	d
1	θ_1^i	0	l_1	$-b$
2	θ_2^i	$\frac{-\pi}{2}$	0	0
3	θ_3^i	$\frac{\pi}{2}$	0	0
4	θ_4^i	0	l_2	0
5	θ_5^i	$\frac{-\pi}{2}$	0	0
6	θ_6^i	$\frac{\pi}{2}$	l_3	0
7	θ_7^i	$\frac{\pi}{2}$	a	$-c$

According to equation (3.1) the equation below can be written for the left dyad ,

$$-b\overline{u}_3 + l_1\hat{C}_i^{(0,1)}\overline{u}_1 + \hat{C}_i^{(0,4)}\overline{V}^{(4)} = \overline{R}_i \tag{3.5}$$

and considering equation (3.2) the following equation is written, for the right dyad,

$$\overline{R}_i + \hat{C}_i^{(0,4)}\overline{W}^{(4)} + l_3\hat{C}_i^{(0,6)}\overline{u}_1 - c\hat{C}_i^{(0,6)}\overline{u}_3 + a\hat{C}_i^{(0,7)}\overline{u}_1 = \overline{0} \tag{3.6}$$

Regarding equation (2.1), equations below are obtained ,

$$\hat{C}_i^{(0,4)} = (\hat{C}_i^{(4,7)})^{-1}$$

$$\hat{C}_i^{(0,6)} = (\hat{C}_i^{(6,7)})^{-1}$$

and

$$\hat{C}_i^{(0,7)} = \hat{I}$$

Thus considering Denavit-Hartenberg parameters in table (3.1) the following equations are derived ,

$$\hat{C}_i^{(0,1)} = e^{\tilde{u}_3 \theta_i}$$

$$\hat{C}_i^{(0,4)} = (\hat{C}_i^{(4,7)})^{-1} = (e^{\tilde{u}_3 \theta_5^i} e^{\tilde{u}_2 \theta_6^i} e^{\tilde{u}_3 \theta_7^i} e^{\tilde{u}_1 \pi/2})^{-1} = e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i}$$

$$\hat{C}_i^{(0,6)} = (\hat{C}_i^{(6,7)})^{-1} = (e^{\tilde{u}_3 \theta_7^i} e^{\tilde{u}_1 \pi/2})^{-1} = e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i}$$

Hence equations (3.5) and (3.6) can be written in the form below ,

$$-b\overline{u}_3 + l_1 e^{\tilde{u}_3 \theta_1^i} \overline{u}_1 + e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} \overline{V}^{(4)} = \overline{R}_i \tag{3.7}$$

$$e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} \overline{W}^{(4)} + l_3 e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} \overline{u}_1 - c e^{-\tilde{u}_1 \pi/2} \overline{u}_3 + a\overline{u}_1 = -\overline{R}_i \tag{3.8}$$

The following equation can be written according to equation (3.3) ,

$$\overline{V}^{(4)} + \overline{W}^{(4)} = \overline{U}^{(4)}$$

From Figure (3.3) it is seen that $\overline{U}^{(4)} = l_2 \overline{u}_1$. Thus equation above can be written in the following form ,

$$\overline{V}^{(4)} + \overline{W}^{(4)} = l_2 \overline{u}_1 \tag{3.9}$$

Table (3.2) has been constructed according to equations (3.7) to (3.9) .

Table (3.2) : According to the table below when the locations and orientations of the fixed joints of the RSHR linkage are prescribed , the path tracer point can pass through at most three precision points.

number of precision points	number of scalar equations	number of unknowns	number of free parameters
1	9	13 $(l_1, l_2, l_3, \overline{V}^{(4)}, \overline{W}^{(4)}, \theta_1^0, \theta_5^0, \theta_6^0, \theta_7^0)$	4
2	15	17 (above + $\theta_1^1, \theta_5^1, \theta_6^1, \theta_7^1$)	2
3	21	21 (above + $\theta_1^2, \theta_5^2, \theta_6^2, \theta_7^2$)	0

Assume that it is desired to synthesize an RSHR linkage whose path tracer point is supposed to pass through three precision points . For i=0,1,2 the following equations are obtained ,

$$-b\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1^0}\overline{u}_1 + e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^0} e^{-\tilde{u}_2\theta_6^0} e^{-\tilde{u}_3\theta_5^0} \overline{V}^{(4)} = \overline{R}_0 \qquad (3.10)$$

$$-b\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1^1}\overline{u}_1 + e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^1} e^{-\tilde{u}_2\theta_6^1} e^{-\tilde{u}_3\theta_5^1} \overline{V}^{(4)} = \overline{R}_1 \qquad (3.11)$$

$$-b\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1^2}\overline{u}_1 + e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^2} e^{-\tilde{u}_2\theta_6^2} e^{-\tilde{u}_3\theta_5^2} \overline{V}^{(4)} = \overline{R}_2 \qquad (3.12)$$

$$e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^0} e^{-\tilde{u}_2\theta_6^0} e^{-\tilde{u}_3\theta_5^0} \overline{W}^{(4)} + l_3 e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^0}\overline{u}_1 - c e^{-\tilde{u}_1\pi/2}\overline{u}_3 + a\overline{u}_1 = -\overline{R}_0 \qquad (3.13)$$

$$e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^1}e^{-\tilde{u}_2\theta_6^1}e^{-\tilde{u}_3\theta_5^1}\overline{W}^{(4)}+l_3e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^1}\overline{u}_1-ce^{-\tilde{u}_1\pi/2}\overline{u}_3+a\overline{u}_1=-\overline{R}_1 \tag{3.14}$$

$$e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^2}e^{-\tilde{u}_2\theta_6^2}e^{-\tilde{u}_3\theta_5^2}\overline{W}^{(4)}+l_3e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^2}\overline{u}_1-ce^{-\tilde{u}_1\pi/2}\overline{u}_3+a\overline{u}_1=-\overline{R}_2 \tag{3.15}$$

The following equations are obtained from equation (3.10) and (3.13) ,

$$\overline{V}^{(4)}=e^{\tilde{u}_3\theta_5^0}e^{\tilde{u}_2\theta_6^0}e^{\tilde{u}_3\theta_7^0}e^{\tilde{u}_1\pi/2}(\overline{R}_0-l_1e^{\tilde{u}_3\theta_1^0}\overline{u}_1+b\overline{u}_3) \tag{3.16}$$

$$\overline{W}^{(4)}=e^{\tilde{u}_3\theta_5^0}e^{\tilde{u}_2\theta_6^0}e^{\tilde{u}_3\theta_7^0}e^{\tilde{u}_1\pi/2}(-\overline{R}_0-l_3e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^0}\overline{u}_1-a\overline{u}_1+ce^{-\tilde{u}_1\pi/2}\overline{u}_3) \tag{3.17}$$

Substituting $\overline{V}^{(4)}$ and $\overline{W}^{(4)}$ from equations above into equations (3.11) , (3.12) , (3.14) and (3.15) equations below are obtained,

$$-b\overline{u}_3+l_1e^{\tilde{u}_3\theta_1^1}\overline{u}_1+e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^1}e^{-\tilde{u}_2\theta_6^1}e^{\tilde{u}_3(\theta_5^0-\theta_5^1)}e^{\tilde{u}_2\theta_6^0}e^{\tilde{u}_3\theta_7^0}e^{\tilde{u}_1\pi/2}(\overline{R}_0-l_1e^{\tilde{u}_3\theta_1^0}\overline{u}_1+b\overline{u}_3)=\overline{R}_1 \tag{3.18}$$

$$-b\overline{u}_3+l_1e^{\tilde{u}_3\theta_1^2}\overline{u}_1+e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^2}e^{-\tilde{u}_2\theta_6^2}e^{\tilde{u}_3(\theta_5^0-\theta_5^2)}e^{\tilde{u}_2\theta_6^0}e^{\tilde{u}_3\theta_7^0}e^{\tilde{u}_1\pi/2}(\overline{R}_0-l_1e^{\tilde{u}_3\theta_1^0}\overline{u}_1+b\overline{u}_3)=\overline{R}_2 \tag{3.19}$$

$$\begin{aligned}&e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^1}e^{-\tilde{u}_2\theta_6^1}e^{\tilde{u}_3(\theta_5^0-\theta_5^1)}e^{\tilde{u}_2\theta_6^0}e^{\tilde{u}_3\theta_7^0}e^{\tilde{u}_1\pi/2}(-a\overline{u}_1+ce^{-\tilde{u}\pi/2}\overline{u}_3-\overline{R}_0-l_3e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^0})\\&+l_3e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^1}\overline{u}_1=-a\overline{u}_1+ce^{-\tilde{u}_1\pi/2}\overline{u}_3-\overline{R}_1\end{aligned} \tag{3.20}$$

$$\begin{aligned}&e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^2}e^{-\tilde{u}_2\theta_6^2}e^{\tilde{u}_3(\theta_5^0-\theta_5^2)}e^{\tilde{u}_2\theta_6^0}e^{\tilde{u}_3\theta_7^0}e^{\tilde{u}_1\pi/2}(-a\overline{u}_1+ce^{-\tilde{u}\pi/2}\overline{u}_3-\overline{R}_0-l_3e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^0})\\&+l_3e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^2}\overline{u}_1=-a\overline{u}_1+ce^{-\tilde{u}_1\pi/2}\overline{u}_3-\overline{R}_2\end{aligned} \tag{3.21}$$

Adding equations (3.16) and (3.17) equation below is derived ,

$$\begin{aligned}&\overline{V}^{(4)}+\overline{W}^{(4)}=e^{\tilde{u}_3\theta_5^0}e^{\tilde{u}_2\theta_6^0}e^{\tilde{u}_3\theta_7^0}e^{\tilde{u}_1\pi/2}(-l_1e^{\tilde{u}_3\theta_1^0}\overline{u}_1+b\overline{u}_3-l_3e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^0}\overline{u}_1-a\overline{u}_1+ce^{-\tilde{u}_1\pi/2}\overline{u}_3)\\&=l_2\overline{u}_1\end{aligned} \tag{3.22}$$

Thus ,

$$\overline{u}_2{}^t e^{\tilde{u}_3\theta_5^0} e^{\tilde{u}_2\theta_6^0} e^{\tilde{u}_3\theta_7^0} e^{\tilde{u}_1\pi/2} (-l_1 e^{\tilde{u}_3\theta_1^0}\overline{u}_1 + b\overline{u}_3 - l_3 e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^0}\overline{u}_1 - a\overline{u}_1 + ce^{-\tilde{u}_1\pi/2}\overline{u}_3) = 0 \qquad (3.23)$$

$$\overline{u}_3{}^t e^{\tilde{u}_3\theta_5^0} e^{\tilde{u}_2\theta_6^0} e^{\tilde{u}_3\theta_7^0} e^{\tilde{u}_1\pi/2} (-l_1 e^{\tilde{u}_3\theta_1^0}\overline{u}_1 + b\overline{u}_3 - l_3 e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^0}\overline{u}_1 - a\overline{u}_1 + ce^{-\tilde{u}_1\pi/2}\overline{u}_3) = 0 \qquad (3.24)$$

Using a proper numerical method, equations (3.18) to (3.24) can be solved for $\theta_1^0, \theta_1^1, \theta_1^2, \theta_5^0, \theta_5^1, \theta_5^2, \theta_6^0, \theta_6^1, \theta_6^2, \theta_7^0, \theta_7^1, \theta_7^2, l_1$ and l_3 , then $\overline{V}^{(4)}$ and $\overline{W}^{(4)}$ can be determined from equations (3.16) and (3.17) respectively and l_2 is found from equation (3.9) .

b) THE POSITIONS OF THE GROUND PIVOTS ARE NOT PRESCRIBED.

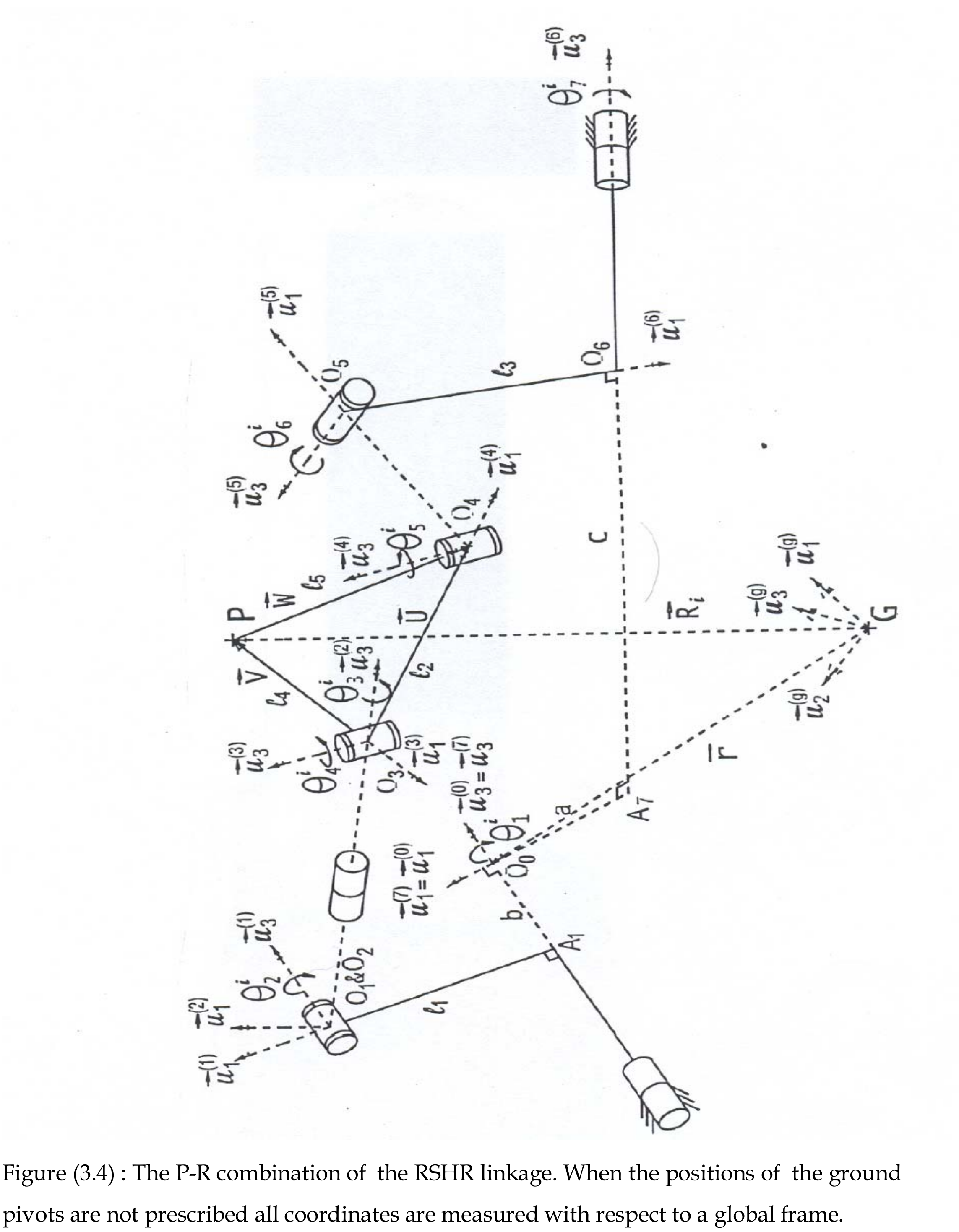


Figure (3.4) : The P-R combination of the RSHR linkage. When the positions of the ground pivots are not prescribed all coordinates are measured with respect to a global frame.

As illustrated in Figure (3.4) ,in this case all locations and orientations are defined in a global coordinate system .According to equations (3.1) and (3.2) the following equations are obtained ,

$$\overline{r} - b\hat{C}^{(g,0)}\overline{u}_3 + l_1\hat{C}_i^{(g,1)}\overline{u}_1 + \hat{C}_i^{(g,4)}\overline{V}^{(4)} = \overline{R}_i \tag{3.25}$$

$$\overline{R}_i + \hat{C}_i^{(g,4)}\overline{W}^{(4)} + l_3\hat{C}_i^{(g,6)}\overline{u}_1 - c\hat{C}_i^{(g,6)}\overline{u}_3 + a\hat{C}_i^{(g,7)}\overline{u}_1 = \overline{r} \tag{3.26}$$

where ,

$$\hat{C}^{(g,0)} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}$$

$$\hat{C}_i^{(g,1)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,1)} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 (z+\theta_1^i)}$$

$$\hat{C}_i^{(g,4)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,4)} = \hat{C}^{(g,0)}(\hat{C}_i^{(4,7)})^{-1} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^i}e^{-\tilde{u}_2\theta_6^i}e^{-\tilde{u}_3\theta_5^i}$$

$$\hat{C}_i^{(g,6)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,6)} = \hat{C}^{(g,0)}(\hat{C}_i^{(6,7)})^{-1} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^i}$$

$$\hat{C}^{(g,7)} = \hat{C}^{(g,0)}\hat{C}^{(0,7)} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}\hat{I} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}$$

Thus equations (3.25) and (3.26) can be written in the form below ,

$$\overline{r} - be^{\tilde{u}_3 x}e^{\tilde{u}_2 y}\overline{u}_3 + l_1 e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 (z+\theta_1^i)}\overline{u}_1 + e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^i}e^{-\tilde{u}_2\theta_6^i}e^{-\tilde{u}_3\theta_5^i}\overline{V}^{(4)} = \overline{R}_i \tag{3.27}$$

$$e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^i}e^{-\tilde{u}_2\theta_6^i}e^{-\tilde{u}_3\theta_5^i}\overline{W}^{(4)} + l_3 e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}e^{-\tilde{u}_1\pi/2}e^{-\tilde{u}_3\theta_7^i}\overline{u}_1 - ce^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}e^{-\tilde{u}_1\pi/2}\overline{u}_3$$
$$+ ae^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}\overline{u}_1 - \overline{r} = -\overline{R}_i \tag{3.28}$$

In equations (3.27) and (3.28) the summations of the vectors which do not include subscript (i) can be considered as constant vectors .That is let

$$\overline{r} - be^{\tilde{u}_3 x} e^{\tilde{u}_2 y} \overline{u}_3 = \overline{r}_1 \tag{3.29}$$

and

$$ae^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} \overline{u}_1 - ce^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} \overline{u}_3 - \overline{r} = \overline{r}_2 \tag{3.30}$$

Hence equations (3.27) and (3.28) are written in the following forms ,

$$\overline{r}_1 + l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i)} \overline{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} \overline{V}^{(4)} = \overline{R}_i \tag{3.31}$$

$$\overline{r}_2 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} \overline{W}^{(4)} + l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} \overline{u}_1 = -\overline{R}_i \tag{3.32}$$

and recalling equation (3.3) the following equation is obtained,

$$\overline{V}^{(4)} + \overline{W}^{(4)} = l_2 \overline{u}_1 \tag{3.33}$$

Thus Table (3.3) can be constructed according to equations (3.31) to (3.33) .

Table (3.3) : The table below shows that when the positions of the ground pivots of the RSHR linkage are not prescribed its path tracer point can pass through at most seven precision points.

number of precision points	number of scalar equations	number of unknowns	number of free parameters
1	9	22 ($\overline{r}_1, \overline{r}_2, \overline{V}^{(4)}, \overline{W}^{(4)}, l_1, l_2, l_3, x, y, z, \theta_1^0, \theta_5^0, \theta_6^0, \theta_7^0$)	13
2	15	26 (above + $\theta_1^1, \theta_5^1, \theta_6^1, \theta_7^1$)	11
3	21	30 (above + $\theta_1^2, \theta_5^2, \theta_6^2, \theta_7^2$)	9
4	27	34 (above + $\theta_1^3, \theta_5^3, \theta_6^3, \theta_7^3$)	7
5	33	38(above + $\theta_1^4, \theta_5^4, \theta_6^4, \theta_7^4$)	5
6	39	42 (above + $\theta_1^5, \theta_5^5, \theta_6^5, \theta_7^5$)	3
7	45	46 (above + $\theta_1^6, \theta_5^6, \theta_6^6, \theta_7^6$)	1

Let's consider the case of five precision points. The following equations are obtained from equations (3.31) and (3.32) for i=0 ,

$$\overline{r}_1 = \overline{R}_0 - l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_0)} \overline{u}_1 - e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^0} e^{-\tilde{u}_2 \theta_6^0} e^{-\tilde{u}_3 \theta_5^0} \overline{V}^{(4)} \tag{3.34}$$

$$\overline{r}_2 = -\overline{R}_0 - l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^0} \overline{u}_1 - e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^0} e^{-\tilde{u}_2 \theta_6^0} e^{-\tilde{u}_3 \theta_5^0} \overline{W}^{(4)} \tag{3.35}$$

Substituting $\overline{r}_1$ and $\overline{r}_2$ from equations (3.34) and (3.35) into equations (3.31) and (3.32) for i=1,2,3,4 the following equalities are obtained ,

$$l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} (e^{\tilde{u}_3 \theta_1^i} - e^{\tilde{u}_3 \theta_1^0}) \overline{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} (e^{-\tilde{u}_3 \theta_7^i} - e^{\tilde{u}_3 \theta_7^0}) \overline{V}^{(4)} = \overline{R}_i - \overline{R}_0 \quad (3.36)$$

$$l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} (e^{-\tilde{u}_3 \theta_7^i} - e^{-\tilde{u}_3 \theta_7^0}) \overline{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} (e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} - e^{-\tilde{u}_3 \theta_7^0} e^{-\tilde{u}_2 \theta_6^0} e^{-\tilde{u}_3 \theta_5^0}) \overline{W}^{(4)}$$
$$= \overline{R}_0 - \overline{R}_i \quad (3.37)$$

Solving equations (3.36) , (3.37) and (3.33) with five free parameters $l_1, l_2, l_3, x, y, z, \theta_1^i, \theta_5^i, \theta_6^i, \theta_7^i, \overline{V}^{(4)}, \overline{W}^{(4)}$ can be determined and consequently $\overline{r}_1$ and $\overline{r}_2$ can be found from equations (3.34) and (3.35) then a, b, c and $\overline{r}$ are determined from equations (3.29) and (3.30) as follows ,

Adding equations (3.29) and (3.30) the following equation is obtained ,

$$a e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} \overline{u}_1 - b e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} \overline{u}_3 - c e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} \overline{u}_3 = \overline{r}_1 + \overline{r}_2$$

or

$$a \overline{u}_1 - b \overline{u}_3 - c e^{-\tilde{u}_1 \pi/2} \overline{u}_3 = e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (\overline{r}_1 + \overline{r}_2) \quad (3.38)$$

and premultiplying equation (3.38) by $\overline{u}_1^{\,t}, \overline{u}_2^{\,t}$ and $\overline{u}_3^{\,t}$ equations below are gained ,

$$a = \overline{u}_1^{\,t} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (\overline{r}_1 + \overline{r}_2)$$

$$b = -\overline{u}_3^{\,t} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (\overline{r}_1 + \overline{r}_2)$$

$$c = -\overline{u}_2^{\,t} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (\overline{r}_1 + \overline{r}_2)$$

c) THE ORDERS OF PRECISION POINTS ARE PRESCRIBED

In kinematic synthesis there are some cases where the order of the precision points or prescribed positions are important. In Examples (3.1) and (3.2) presented in section (3.7) no order has been prescribed for the precision points however the same loop closure equations and solution procedure can be applied to the cases where the order of precision points is important. The only difference is that the prescribed order should be imposed to the computer program as inequalities in terms of the input angles. For example assume that in Example (3.2) the path tracer point is desired to pass through points P_0 to P_3 successively. In this case adding the following inequalities to the program the required condition will be satisfied ,

$$0 \prec \theta_1^0 \prec \theta_1^1 \prec \theta_1^2 \prec \theta_1^3 \prec 2\pi \qquad \text{or} \qquad 0 \prec \theta_1^3 \prec \theta_1^2 \prec \theta_1^1 \prec \theta_1^0 \prec 2\pi$$

Example (3.3) presented in section (3.7) verifies the above discussion.

3.4) MOTION GENERATION SYNTHESIS

Consider a single loop spatial linkage with n links. Assume that a coordinate system is attached to each link according to Denavit-Hartenberg's convention. Now consider a point P on the k'th link. A coordinate system (p) is attached to the k'th link so that its origin coincides with the path tracer point (P) . The coordinate system is supposed to lie in prescribed positions when the path tracer point passes through the precision points.

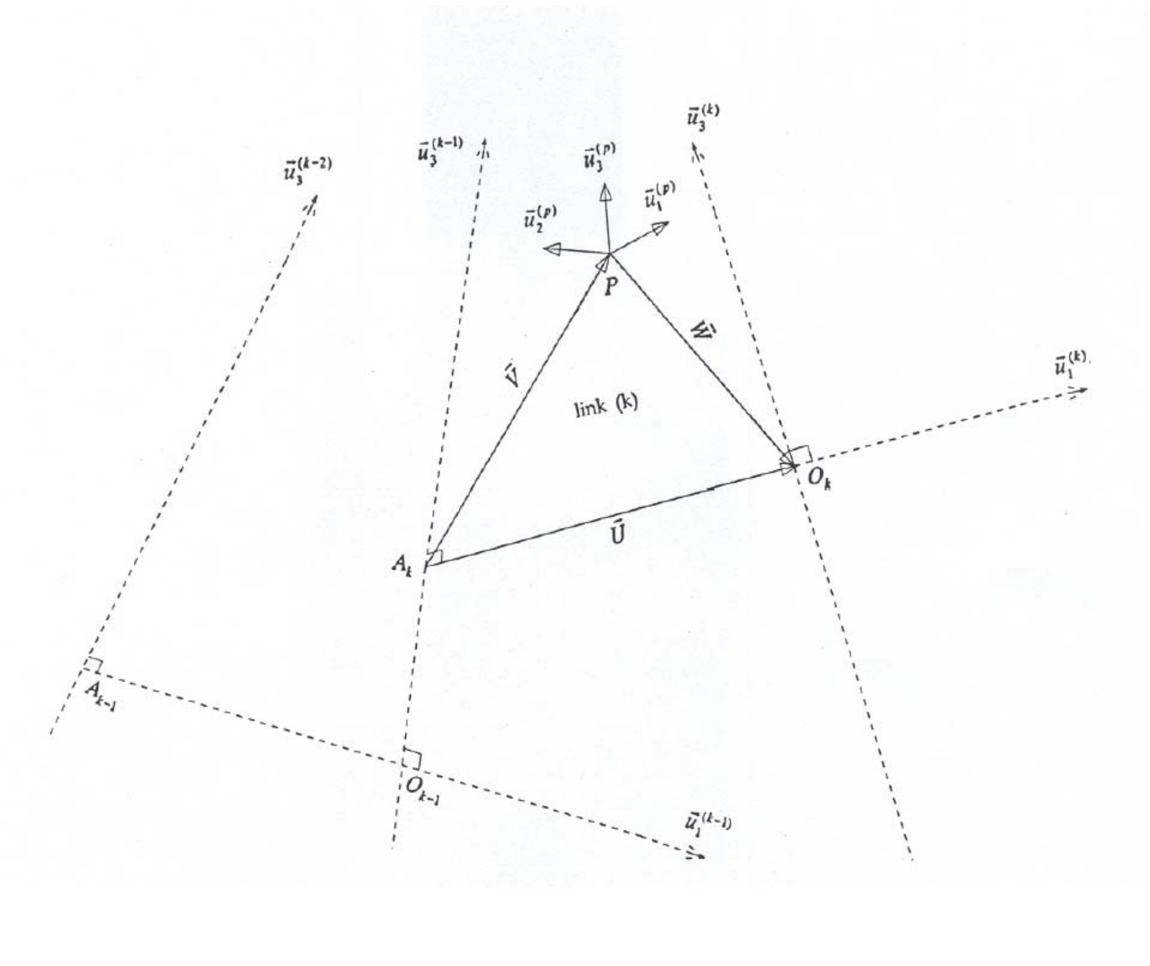


Figure (3.5) : k'th link of an n link spatial mechanism.

Let $\hat{C}_i^{(g,p)}$ be the orientation matrix of frame (p) with respect to a global frame when frame (p) lies in the i'th prescribed position . Hence in addition to the loop closure equations (3.1) , (3.2) and (3.3) the following equation can be written ,

$$\hat{C}_i^{(g,k)}\hat{C}^{(k,p)} = \hat{C}_i^{(g,p)} \tag{3.39}$$

Where ,

$\hat{C}_i^{(g,k)}$ is the orientation matrix of the k'th frame with respect to the global frame when frame (p) is in the i'th prescribed position.

$\hat{C}^{(k,p)}$ is the orientation matrix of frame (p) with respect to the k'th frame . Since both frames are attached to the k'th link obviously $\hat{C}^{(k,p)}$ must be a constant matrix and can be defined as follows ,

$$\hat{C}^{(k,p)} = e^{\tilde{u}_3\alpha}e^{\tilde{u}_2\beta}e^{\tilde{u}_3\gamma} \tag{3.40}$$

and finally $\hat{C}_i^{(g,p)}$ is the orientation matrix of frame (p) with respect to the global frame when frame (p) lies in the i'th position and is always prescribed.

3.5) MOTION GENERATION SYNTHESIS OF AN RSHR LINKAG

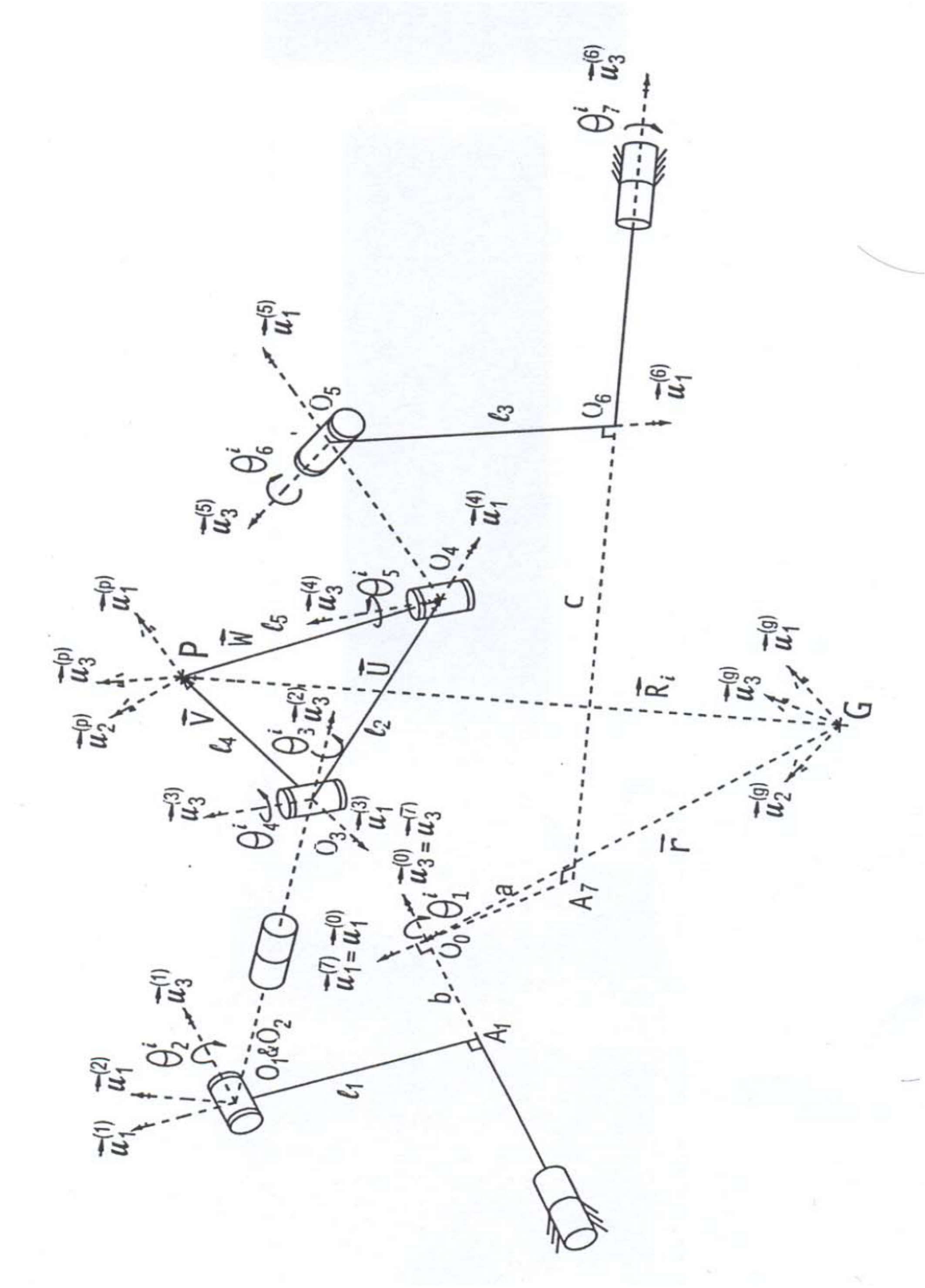


Figure (3.6) : The right and left dyads of the P-R combination of the RSHR linkage.

Considering Figure (3.6) and recalling equations (3.1) , (3.2) and (3.3) equations below are derived ,

$$\bar{r} - b\hat{C}^{(g,0)}\bar{u}_3 + l_1\hat{C}_i^{(g,1)}\bar{u}_1 + \hat{C}_i^{(g,4)}\bar{V}^{(4)} = \bar{R}_i \tag{3.41}$$

$$\bar{R}_i + \hat{C}_i^{(g,4)}\bar{W}^{(4)} + l_3\hat{C}_i^{(g,6)}\bar{u}_1 - c\hat{C}_i^{(g,6)}\bar{u}_3 + a\hat{C}_i^{(g,7)}\bar{u}_1 = \bar{r} \tag{3.42}$$

$$\bar{V}^{(4)} + \bar{W}^{(4)} = \bar{U}^{(4)} \tag{3.43}$$

and from equation (3.39) the following equality is gained ,

$$\hat{C}_i^{(g,4)}\hat{C}^{(4,p)} = \hat{C}_i^{(g,p)} \tag{3.44}$$

Where ,

$$\hat{C}^{(g,0)} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}$$

$$\hat{C}_i^{(g,1)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,1)} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i)}$$

$$\hat{C}_i^{(g,4)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,4)} = \hat{C}^{(g,0)}(\hat{C}_i^{(4,7)})^{-1} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i}$$

$$\hat{C}_i^{(g,6)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,6)} = \hat{C}^{(g,0)}(\hat{C}_i^{(6,7)})^{-1} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i}$$

$$\hat{C}^{(g,7)} = \hat{C}^{(g,0)}\hat{C}^{(0,7)} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}\hat{I} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}$$

$$\hat{C}^{(4,p)} = e^{\tilde{u}_3 \alpha} e^{\tilde{u}_2 \beta} e^{\tilde{u}_3 \gamma}$$

Note that $\hat{C}_i^{(g,p)}$ is prescribed.

Thus equations (3.41) and (3.42) can be written in the form below ,

$$\bar{r} - be^{\tilde{u}_3 x} e^{\tilde{u}_2 y} \bar{u}_3 + l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i)} \bar{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} \overline{V}^{(4)} = \overline{R}_i \quad (3.45)$$

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} \overline{W}^{(4)} + l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} \bar{u}_1 - ce^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} \bar{u}_3$$

$$+ ae^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} \bar{u}_1 - \bar{r} = -\overline{R}_i \quad (3.46)$$

In equations (3.45) and (3.46) the summation of the vectors which do not include subscript (i) can be considered as constant vectors .That is let

$$\bar{r} - be^{\tilde{u}_3 x} e^{\tilde{u}_2 y} \bar{u}_3 = \bar{r}_1 \quad (3.47)$$

and

$$ae^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} \bar{u}_1 - ce^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} \bar{u}_3 - \bar{r} = \bar{r}_2 \quad (3.48)$$

Hence equations (3.45) and (3.46) are written in the following form ,

$$\bar{r}_1 + l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i)} \bar{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} \overline{V}^{(4)} = \overline{R}_i \quad (3.49)$$

$$\bar{r}_2 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} \overline{W}^{(4)} + l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} \bar{u}_1 = -\overline{R}_i \quad (3.50)$$

and considering Figure (3.6) the following equation is obtained ,

$$\overline{V}^{(4)} + \overline{W}^{(4)} = l_2 \bar{u}_1 \quad (3.51)$$

and from equation (3.44) the following equation is got ,

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^i} e^{-\tilde{u}_2 \theta_6^i} e^{-\tilde{u}_3 \theta_5^i} e^{\tilde{u}_3 \alpha} e^{\tilde{u}_2 \beta} e^{\tilde{u}_3 \gamma} = \hat{C}_i^{(g,p)} \qquad (3.52)$$

Regarding equations (3.49) to (3.52) the following table is constructed,

Table (3.4) : According to the table below when the locations and orientations of the fixed joints are not prescribed , the coupler link of the RSHR linkage can lie at most in three prescribed positions.

number of prescribed orientations	number of scalar equations	number of unknowns	number of free parameters
1	12	25 $(l_1, l_2, l_3, \theta_1^0, \theta_5^0, \theta_6^0, \theta_7^0, \alpha, \beta, \gamma, \bar{r}_1, \bar{r}_2, x, y, z, \overline{V}^{(4)}, \overline{W}^{(4)})$	13
2	21	29 (above + $\theta_1^1, \theta_5^1, \theta_6^1, \theta_7^1$)	8
3	30	33 (above + $\theta_1^2, \theta_5^2, \theta_6^2, \theta_7^2$)	3

Let's consider the case in which the coupler link of RSHR linkage is supposed to take two prescribed orientations . Hence assuming i=0,1 equations (3.49) and (3.50) are written in the form below ,

$$\bar{r}_1 + l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^0)} \bar{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^0} e^{-\tilde{u}_2 \theta_6^0} e^{-\tilde{u}_3 \theta_5^0} \overline{V}^{(4)} = \overline{R}_0 \qquad (3.53)$$

$$\bar{r}_1 + l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^1)} \bar{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^1} e^{-\tilde{u}_2 \theta_6^1} e^{-\tilde{u}_3 \theta_5^1} \overline{V}^{(4)} = \overline{R}_1 \qquad (3.54)$$

$$\bar{r}_2 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^0} e^{-\tilde{u}_2 \theta_6^0} e^{-\tilde{u}_3 \theta_5^0} \overline{W}^{(4)} + l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^0} \bar{u}_1 = -\overline{R}_0 \qquad (3.55)$$

$$\overline{r}_2 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^1} e^{-\tilde{u}_2 \theta_6^1} e^{-\tilde{u}_3 \theta_5^1} \overline{W}^{(4)} + l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^1} \overline{u}_1 = -\overline{R}_1 \quad (3.56)$$

and regarding equation (3.52) equations below are obtained ,

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^0} e^{-\tilde{u}_2 \theta_6^0} e^{-\tilde{u}_3 \theta_5^0} e^{\tilde{u}_3 \alpha} e^{\tilde{u}_2 \beta} e^{\tilde{u}_3 \gamma} = \hat{C}_0^{(g,p)} \quad (3.57)$$

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \pi/2} e^{-\tilde{u}_3 \theta_7^1} e^{-\tilde{u}_2 \theta_6^1} e^{-\tilde{u}_3 \theta_5^1} e^{\tilde{u}_3 \alpha} e^{\tilde{u}_2 \beta} e^{\tilde{u}_3 \gamma} = \hat{C}_1^{(g,p)} \quad (3.58)$$

Let's choose $\alpha, \beta, \gamma, \theta_5^0, \theta_6^0, \theta_7^0$ arbitrarily . Thus from equation (3.57) the following equation is obtained ,

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} = \hat{C}_0^{(g,p)} e^{-\tilde{u}_3 \gamma} e^{-\tilde{u}_2 \beta} e^{-\tilde{u}_3 \gamma} e^{\tilde{u}_3 \theta_5^0} e^{\tilde{u}_2 \theta_6^0} e^{\tilde{u}_3 \theta_7^0} e^{\tilde{u}_1 \pi/2} \quad (3.59)$$

Let $\hat{C}_0^{(g,p)} e^{-\tilde{u}_3 \gamma} e^{-\tilde{u}_2 \beta} e^{-\tilde{u}_3 \gamma} e^{\tilde{u}_3 \theta_5^0} e^{\tilde{u}_2 \theta_6^0} e^{\tilde{u}_3 \theta_7^0} e^{\tilde{u}_1 \pi/2} = \hat{M}$ then postmultiplying equation (3.59) by $\overline{u}_3$ equation below is gained ,

$$\cos x \sin y \overline{u}_1 + \sin x \sin y \overline{u}_2 + \cos y \overline{u}_3 = M_{13} \overline{u}_1 + M_{23} \overline{u}_2 + M_{33} \overline{u}_3 \quad (3.60)$$

Which results in ,

$$\cos y = M_{33} \quad , \quad \cos x \sin y = M_{13} \quad , \quad \sin x \sin y = M_{23}$$

and consequently x and y can be determined with one ambiguity related to y as follows ,

$$\sin y = \pm \sqrt{1 - M_{33}^{\ 2}}$$

$$y = angle(M_{33}, \sqrt{1-M_{33}^{\ 2}}) \quad \text{or} \quad y = angle(M_{33}, -\sqrt{1-M_{33}^{\ 2}})$$

$$x = angle(\frac{M_{13}}{\sin y}, \frac{M_{23}}{\sin y})$$

Now premultiplying equation (3.59) by $\overline{u}_3^{\ t}$ equation below is gained ,

$$-\cos y \cos z \overline{u}_1 + \sin y \sin z \overline{u}_2 + \cos y \overline{u}_3 = M_{31}\overline{u}_1 + M_{32}\overline{u}_2 + M_{33}\overline{u}_3$$

From which it can be derived that ,

$$z = angle(\frac{M_{32}}{-\sin y}, \frac{M_{31}}{\sin y})$$

Note that if $y = 0$ a singularity takes place. In this case other elements of $\hat{M}$ should be used.

From equation (3.58) the following equation is obtained ,

$$e^{\tilde{u}_3\theta_5^1} e^{\tilde{u}_2\theta_6^1} e^{\tilde{u}_3\theta_7^1} = e^{\tilde{u}_1\alpha} e^{\tilde{u}_2\beta} e^{\tilde{u}_3\gamma} (\hat{C}_1^{(g,p)})^{-1} e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1\pi/2} \tag{3.61}$$

Let $e^{\tilde{u}_1\alpha} e^{\tilde{u}_2\beta} e^{\tilde{u}_3\gamma} (\hat{C}_1^{(g,p)})^{-1} e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1\pi/2} = \hat{N}$ then using a technique similar to the one used above , $\theta_7^1, \theta_6^1, \theta_5^1$ can be found as follows ,

$$\theta_6^1 = angle(N_{33}, \sqrt{1-N_{33}^{\ 2}}) \quad \text{or} \quad \theta_6^1 = angle(N_{33}, -\sqrt{1-N_{33}^{\ 2}})$$

$$\theta_5^1 = angle(\frac{N_{13}}{\sin\theta_6^1}, \frac{N_{23}}{\sin\theta_6^1}) \quad \text{and} \quad \theta_7^1 = angle(\frac{N_{32}}{-\sin\theta_6^1}, \frac{N_{31}}{\sin\theta_6^1})$$

From equations (3.49) and (3.50) for i=0 the following equations can be obtained ,

$$\overline{V}^{(4)} = e^{\tilde{u}_3\theta_5^1} e^{\tilde{u}_2\theta_6^1} e^{\tilde{u}_3\theta_7^1} e^{\tilde{u}_1\pi/2} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (\overline{R}_0 - \overline{r}_1 - l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_0)} \overline{u}_1) \qquad (3.62)$$

$$\overline{W}^{(4)} = e^{\tilde{u}_3\theta_5^1} e^{\tilde{u}_2\theta_6^1} e^{\tilde{u}_3\theta_7^1} e^{\tilde{u}_1\pi/2} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (-\overline{R}_0 - \overline{r}_2 - l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1\pi/2} \overline{u}_1) \qquad (3.63)$$

Substituting $\overline{V}^{(4)}$ and $\overline{W}^{(4)}$ from equations above into equations (3.51) , (3.54) and (3.56) the following equalities are gained ,

$$\overline{r}_1 + l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^1)} \overline{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^1} e^{-\tilde{u}_2\theta_6^1} e^{\tilde{u}_3 (\theta_5^0 - \theta_5^1)} e^{\tilde{u}_2\theta_6^0} e^{\tilde{u}_3\theta_7^0}$$
$$e^{\tilde{u}_1\pi/2} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (\overline{R}_0 - \overline{r}_1 - l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^0)} \overline{u}_1) = \overline{R}_1 \qquad (3.64)$$

$$\overline{r}_2 + l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^0} \overline{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^1} e^{-\tilde{u}_2\theta_6^1} e^{\tilde{u}_3 (\theta_5^0 - \theta_5^1)} e^{\tilde{u}_2\theta_6^0}$$
$$e^{\tilde{u}_3\theta_7^0} e^{\tilde{u}_1\pi/2} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (-\overline{R}_0 - \overline{r}_2 - l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^0} \overline{u}_1) = -\overline{R}_2 \qquad (3.65)$$

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^1} e^{-\tilde{u}_2\theta_6^1} e^{\tilde{u}_3 (\theta_5^0 - \theta_5^1)} e^{\tilde{u}_2\theta_6^0} e^{\tilde{u}_3\theta_7^0} e^{\tilde{u}_1\pi/2} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (-\overline{r}_1 - \overline{r}_2$$
$$- l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^0)} - l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1\pi/2} e^{-\tilde{u}_3\theta_7^0} \overline{u}_1) = l_2 \overline{u}_1 \qquad (3.66)$$

Using a proper numerical method equations (3.64) to (3.66) can be solved for $l_1, l_2, l_3, \theta_1^0, \theta_1^1, \overline{r}_1, \overline{r}_2$ with two free parameters then $\overline{V}^{(4)}$ and $\overline{W}^{(4)}$ can be determined from equations (3.62) and (3.63) . Now adding up equations (3.47) and (3.48) equation below is obtained ,

$$a e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} \overline{u}_1 - b e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} \overline{u}_3 - c e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1\pi/2} \overline{u}_3 = \overline{r}_1 + \overline{r}_2$$

or

$$a\overline{u}_1 - b\overline{u}_3 - ce^{-\tilde{u}_1\pi/2}\overline{u}_3 = e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_3 x}(\overline{r}_1 + \overline{r}_2)$$

Premultiplying equation (3.67) by $\overline{u}_1^{\,t}, \overline{u}_2^{\,t}$ and $\overline{u}_3^{\,t}$ the following equations are obtained ,

$$a = \overline{u}_1^{\,t} e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_3 x}(\overline{r}_1 + \overline{r}_2)$$

$$b = -\overline{u}_3^{\,t} e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_3 x}(\overline{r}_1 + \overline{r}_2)$$

$$c = -\overline{u}_2^{\,t} e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_3 x}(\overline{r}_1 + \overline{r}_2)$$

and for $\overline{r}$ it can be written ,

$$\overline{r} = \overline{r}_1 + be^{\tilde{u}_3 x}e^{\tilde{u}_2 y}\overline{u}_3$$

3.6) NUMERICAL EXAMPLES

All numerical examples of this thesis were solved by a Pentium 4 with a processor frequency of 2.8 Ghz and a RAM of 128MB. Detailed information has been presented in chapter 6 about the computer programs , trial and error steps and the difficulties which may arise while solving equations.

Example (3.1) : Synthesize an RSHR linkage whose path tracer point on its coupler link passes through points $P_0 = \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$, $P_1 = \begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix}$, $P_2 = \begin{bmatrix} -10 \\ 10 \\ 0 \end{bmatrix}$ **defined in the zero'th frame. The following data is available , a= 30 , b=2 , c=5**

Applying the solution algorithm explained in part (a) of section (3.3) a computer program can be written for solving this problem. Here using Mathcad the following results were determined (Running Time : 21 seconds).

Inputs

$$\overline{R}_0 = \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} \quad , \quad \overline{R}_1 = \begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix} \quad , \quad \overline{R}_2 = \begin{bmatrix} -10 \\ 10 \\ 0 \end{bmatrix} \quad , \quad a = 30 \quad , \quad b = 2 \quad , \quad c = 5$$

Initial Values

$$l_1 = 20 \quad , \quad l_3 = 50 \quad , \quad \theta_1^0 = 4 \quad , \quad \theta_1^1 = 7.7 \quad , \quad \theta_1^2 = 8.7 \quad , \quad \theta_5^0 = 0 \quad , \quad \theta_5^1 = 0.5$$

$$\theta_5^2 = 1 \quad , \quad \theta_6^0 = 0.5 \quad , \quad \theta_6^1 = 1 \quad , \quad \theta_6^2 = 1.5 \quad , \quad \theta_7^0 = 5 \quad , \quad \theta_7^1 = 11 \quad , \quad \theta_7^2 = 17$$

Outputs

$l_1 = 19.36108$, $l_2 = 49.50558$, $l_3 = 48.91563$, $l_4 = 29.2786$, $l_5 = 42.38448$

$\theta_1^0 = 4.4365$, $\theta_1^1 = 6.96546$, $\theta_1^2 = 6.53532$

$\theta_5^0 = -0.56212$, $\theta_5^1 = 0.25014$, $\theta_5^2 = 3.13484$

$\theta_6^0 = 3.60277$, $\theta_6^1 = 4.08354$, $\theta_6^2 = 5.24591$

$\theta_7^0 = 4.15402$, $\theta_7^1 = 4.19076$, $\theta_7^2 = 4.16323$

$$\overline{V}^{(4)} = \begin{bmatrix} 15.26699 \\ 24.42313 \\ -5.25987 \end{bmatrix} , \quad \overline{W}^{(4)} = \begin{bmatrix} 34.23871 \\ -24.42312 \\ 5.25989 \end{bmatrix}$$

Now it should be checked to see if the path tracer point passes through the precision points continuously or not . Recalling equation (2.12) the diagram of the output angle versus input angle of the above mentioned RSHR linkage can be drawn .According to Figure (3.7) the RSHR linkage acts as a crank-rocker and since the input and output angles determined through the synthesis procedure all lie on a continuous curve it can be deduced that the path tracer point will pass through the precision points continuously.

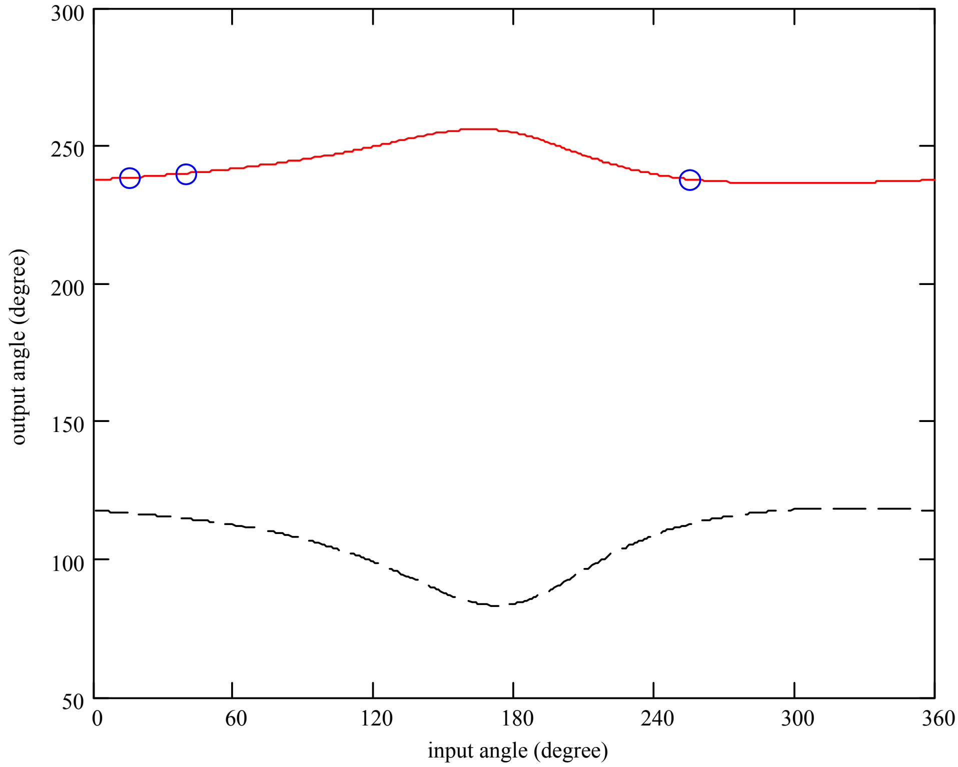


Figure (3.7) : The circles illustrate the input and output angles at which the path tracer point passes through the precision points. The dashed curve displays the other loop closure curve of the mechanism.

Example (3.2) : Synthesize an RSHR linkage whose path tracer point on its coupler link passes through the following points ,

$$P_0 = \begin{bmatrix} 60 \\ -30 \\ 20 \end{bmatrix} \quad , \quad P_1 = \begin{bmatrix} 50 \\ -20 \\ 30 \end{bmatrix} \quad , \quad P_2 = \begin{bmatrix} 50 \\ -40 \\ 25 \end{bmatrix} \quad , \quad P_3 = \begin{bmatrix} 55 \\ -25 \\ 35 \end{bmatrix} \quad , \quad P_4 = \begin{bmatrix} 55 \\ -35 \\ 35 \end{bmatrix}$$

Applying the solution procedure explained in part (b) of section (3.3) and using Mathcad the following results were determined (Running Time : 124 seconds).

Inputs

$$R_0 = \begin{bmatrix} 60 \\ -30 \\ 20 \end{bmatrix} \quad , \quad R_1 = \begin{bmatrix} 50 \\ -20 \\ 30 \end{bmatrix} \quad , \quad R_2 = \begin{bmatrix} 50 \\ -40 \\ 25 \end{bmatrix} \quad , \quad R_3 = \begin{bmatrix} 55 \\ -25 \\ 35 \end{bmatrix} \quad , \quad R_4 = \begin{bmatrix} 55 \\ -35 \\ 35 \end{bmatrix}$$

Initial Values

$l_1 = 100$, $l_2 = 100$, $l_3 = 100$, $\theta_1^0 = 5\pi/6$, $\theta_1^1 = 3\pi/2$, $\theta_1^2 = 5\pi/3$

$\theta_1^3 = 7\pi/6$, $\theta_1^4 = 7\pi/4$, $\theta_5^0 = 5\pi/6$, $\theta_5^1 = 2\pi/3$, $\theta_5^2 = 5\pi/4$, $\theta_5^3 = 3\pi/2$

$\theta_5^4 = 7\pi/6$, $\theta_6^0 = \pi/4$, $\theta_6^1 = 2\pi/3$, $\theta_6^2 = \pi$, $\theta_6^3 = \pi/2$, $\theta_6^4 = -\pi/4$

$\theta_7^0 = -\pi/6$, $\theta_7^1 = \pi/3$, $\theta_7^2 = \pi$, $\theta_7^3 = 0$, $\theta_7^4 = 3\pi/2$

Imposed Conditions

$10 \prec l_1 \prec 120$, $20 \prec l_2 \prec 120$, $l_2 \succ l_3$, $l_3 \succ l_1$

$-0.5 \prec \theta_7^1 - \theta_7^0 \prec 0.5$, $-0.5 \prec \theta_7^2 - \theta_7^0 \prec 0.5$, $-0.5 \prec \theta_7^3 - \theta_7^0 \prec 0.5$

$-0.5 \prec \theta_7^4 - \theta_7^0 \prec 0.5$

Outputs

$l_1 = 12.47711$, $l_2 = 120$, $l_3 = 56.38387$, $l_4 = 28.92972$, $l_5 = 91.43267$

$\theta_1^0 = 6.185$, $\theta_1^1 = 4.86693$, $\theta_1^2 = 1.89699$, $\theta_1^3 = -1.57486$, $\theta_1^4 = 3.1647$

$\theta_5^0 = 3.99495$, $\theta_5^1 = 0.65539$, $\theta_5^2 = 1.02014$, $\theta_5^3 = -2.49228$, $\theta_5^4 = -2.33563$

$\theta_6^0 = 0.47608$, $\theta_6^1 = -3.46868$, $\theta_6^2 = 2.72237$, $\theta_6^3 = 0.30943$, $\theta_6^4 = 0.35106$

$\theta_7^0 = 2.2672$, $\theta_7^1 = 1.95996$, $\theta_7^2 = 2.76719$, $\theta_7^3 = 1.95451$, $\theta_7^4 = 2.47549$

$x = -2.776$, $y = 1.86581$, $z = -0.14$, $a = 20.9615$, $b = -45.81694$, $c = -81.58719$

$$\bar{r} = \begin{bmatrix} 83.68423 \\ -41.35241 \\ 35.88041 \end{bmatrix} , \quad \bar{V}^{(4)} = \begin{bmatrix} 28.65415 \\ -2.41027 \\ -3.17163 \end{bmatrix} , \quad \bar{W}^{(4)} = \begin{bmatrix} 91.34585 \\ 2.41027 \\ 3.17193 \end{bmatrix}$$

Now the continuity of the motion of the path tracer point when it passes through the precision points should be checked. As illustrated in Figure (3.8) the input and output angles at which the path tracer point passes through the precision points all lie on a continuous curve . This proves that the path tracer point will pass through the precision points continuously.

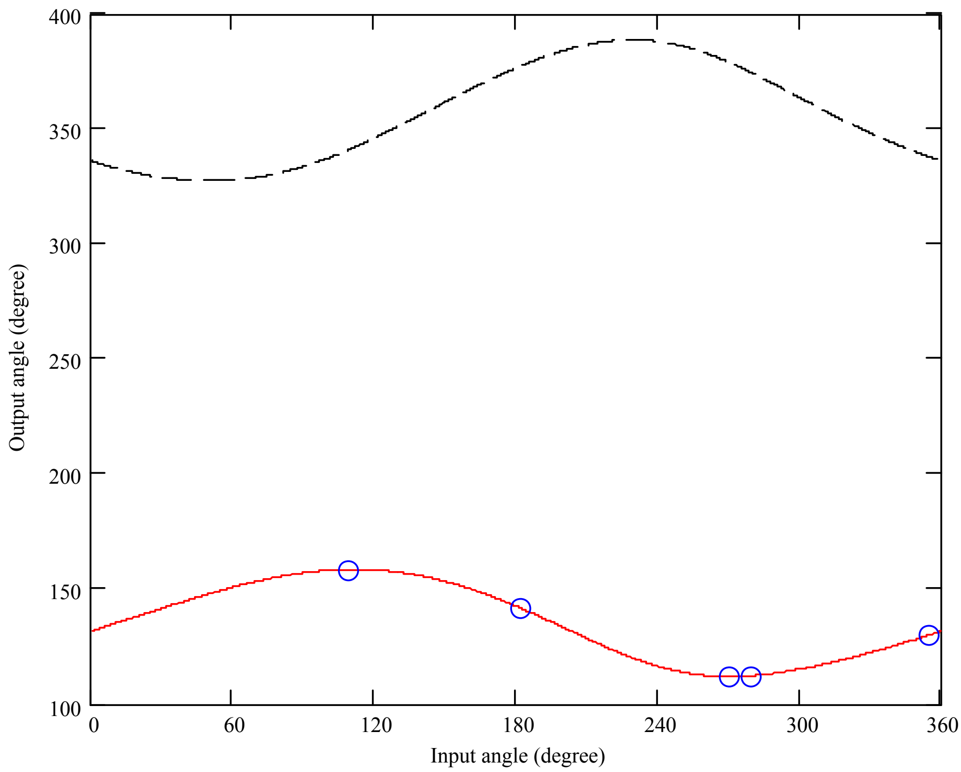


Figure (3.8) : The circles illustrates the input and output angles at which the path tracer point passes through the precision points . The dashed curve displays the other loop closure curve of the mechanism.

Example (3.3) : Synthesize an RSHR linkage whose path tracer point on its coupler link passes through the following points successively,

$$P_0 = \begin{bmatrix} 60 \\ -30 \\ 20 \end{bmatrix} \quad , \quad P_1 = \begin{bmatrix} 50 \\ -20 \\ 30 \end{bmatrix} \quad , \quad P_2 = \begin{bmatrix} 50 \\ -40 \\ 25 \end{bmatrix} \quad , \quad P_3 = \begin{bmatrix} 55 \\ -25 \\ 35 \end{bmatrix}$$

Applying the solution procedure explained in part (b) of section (3.3) and using Mathcad the following results were determined (Running Time : 55 seconds) .

Inputs

$$R_0 = \begin{bmatrix} 60 \\ -30 \\ 20 \end{bmatrix} \quad , \quad R_1 = \begin{bmatrix} 50 \\ -20 \\ 30 \end{bmatrix} \quad , \quad R_2 = \begin{bmatrix} 50 \\ -40 \\ 25 \end{bmatrix} \quad , \quad R_3 = \begin{bmatrix} 55 \\ -25 \\ 35 \end{bmatrix}$$

Initial Values

$l_1 = 100$, $l_2 = 100$, $l_3 = 100$, $\theta_1^0 = 1.2$, $\theta_1^1 = 3\pi/2$, $\theta_1^2 = 5\pi/3$

$\theta_1^3 = 11\pi/10$, $\theta_5^0 = 5\pi/6$, $\theta_5^1 = 2\pi/3$, $\theta_5^2 = 5\pi/4$, $\theta_5^3 = 3\pi/2$

$\theta_6^0 = \pi/4$, $\theta_6^1 = 2\pi/3$, $\theta_6^2 = \pi$, $\theta_6^3 = \pi/2$, $\theta_7^0 = -\pi/6$, $\theta_7^1 = \pi/3$

$\theta_7^2 = \pi$, $\theta_7^3 = 0$, $x = -\pi/4$, $y = \pi$, $z = -\pi/6$

Imposed Conditions

$20 \prec l_1 \prec 90$, $20 \prec l_2 \prec 120$, $20 \prec l_3 \prec 110$, $l_2 \succ l_3$, $l_3 \succ l_1$

$-0.55 \prec \theta_7^1 - \theta_7^0 \prec 0.55$, $-0.55 \prec \theta_7^2 - \theta_7^0 \prec 0.55$, $-0.55 \prec \theta_7^3 - \theta_7^0 \prec 0.55$

$0 \prec \theta_1^0 \prec 1$, $1.5 \prec \theta_1^1 \prec 2.5$, $3 \prec \theta_1^2 \prec 4$, $4.5 \prec \theta_1^3 \prec 5.5$

Outputs

$l_1 = 22.00465$, $l_2 = 120$, $l_3 = 80.99909$, $l_4 = 93.19316$, $l_5 = 31.00092$

$\theta_1^0 = 1$, $\theta_1^1 = 1.8063$, $\theta_1^2 = 4$, $\theta_1^3 = 5.5$

$\theta_5^0 = 3.4735$, $\theta_5^1 = 0.36469$, $\theta_5^2 = 0.02166$, $\theta_5^3 = 0.03051$

$\theta_6^0 = -0.27479$, $\theta_6^1 = 3.46975$, $\theta_6^2 = 3.46351$, $\theta_6^3 = 3.3332$

$\theta_7^0 = 0.70151$, $\theta_7^1 = 0.942$, $\theta_7^2 = 1.25151$, $\theta_7^3 = 0.91661$

$x = 0.65$, $y = 1.189$, $z = -1.083$, $a = -11.36594$, $b = -43.069$, $c = -19.11834$

$$\bar{r} = \begin{bmatrix} 104.62349 \\ -18.31878 \\ 15.86019 \end{bmatrix} , \quad \overline{V}^{(4)} = \begin{bmatrix} 92.18295 \\ 13.68441 \\ -0.07453 \end{bmatrix} , \quad \overline{W}^{(4)} = \begin{bmatrix} 27.81705 \\ -13.68441 \\ 0.07453 \end{bmatrix}$$

Considering the input angles of the RSHR mechanism when the path tracer point passes through the precision points, it is seen that the path tracer point will pass from point P_0 to point P_3 successively. Besides Figure (3.9) verifies the motion continuity of the mechanisms when its path tracer point passes through the precision points.

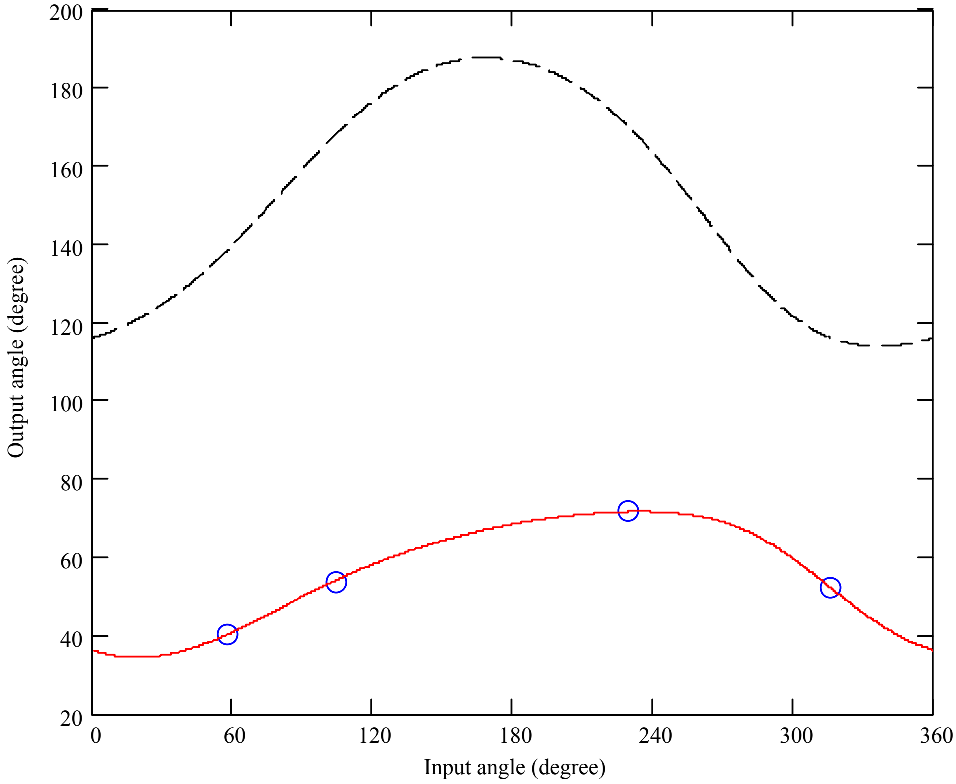


Figure (3.9) : The circles illustrate the input and output angles at which the path tracer point passes through the precision points . The dashed curve displays the other loop closure curve of the mechanism.

Example (3.4) : Synthesize an RSHR linkage whose coupler link moves a cube from position (1) to position (2) as shown in Figure (3.10) .

Position (1) : $\{\overline{R}_0 = \begin{bmatrix} 50 \\ 80 \\ 30 \end{bmatrix}, \hat{C}_0 = e^{\tilde{u}_3 \pi/6} e^{\tilde{u}_2 3\pi/4} e^{\tilde{u}_1 \pi/3}\}$

Position (2) : $\{\overline{R}_1 = \begin{bmatrix} 30 \\ 0 \\ 40 \end{bmatrix}, \hat{C}_1 = e^{\tilde{u}_3 \pi/2} e^{\tilde{u}_2 -\pi/4} e^{\tilde{u}_1 2\pi/3}\}$

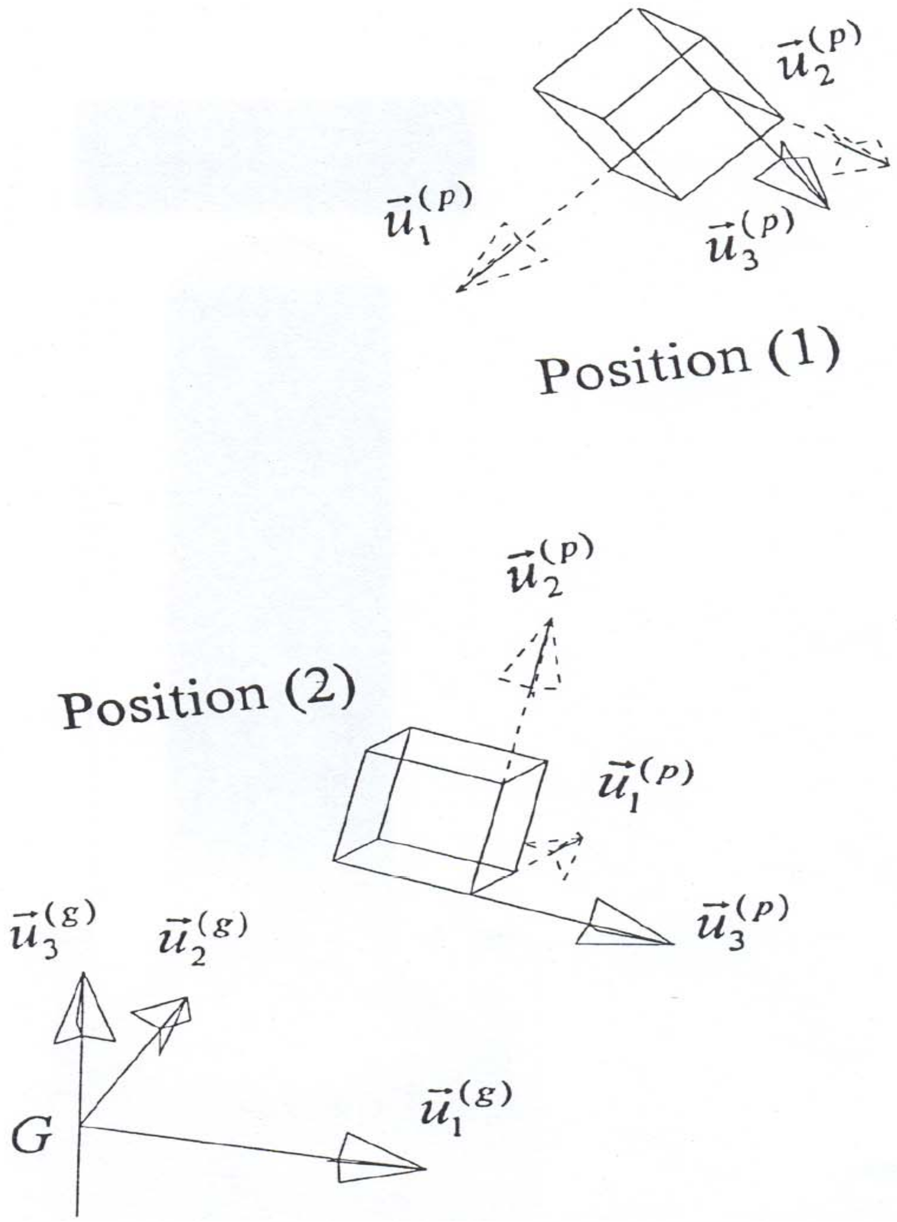


Figure (3.10) : The two prescribed positions of an object which is supposed to be carried by the RSHR linkage.

Applying the solution procedure explained in section (3.5) and using Mathcad program the following results were determined . Note that since this problem has been solved semianalytically some of the free parameters were selected beforehand (Running Time : 14 seconds).

Inputs

Prescribed values ,

$$\overline{R}_0 = \begin{bmatrix} 50 \\ 80 \\ 30 \end{bmatrix} \quad , \quad \overline{R}_1 = \begin{bmatrix} 30 \\ 0 \\ 40 \end{bmatrix} \quad , \quad \hat{C}_0 = e^{\tilde{u}_3 \pi/6} e^{\tilde{u}_2 3\pi/4} e^{\tilde{u}_1 \pi/3} \quad , \quad \hat{C}_1 = e^{\tilde{u}_3 \pi/2} e^{\tilde{u}_2 -\pi/4} e^{\tilde{u}_1 2\pi/3}$$

Arbitrarily chosen values ,

$$\alpha = \pi/7 \quad , \quad \beta = 5\pi/7 \quad , \quad \gamma = \pi/5 \quad , \quad \theta_5^0 = 2\pi/5 \quad , \quad \theta_6^0 = \pi/10 \quad , \quad \theta_7^0 = -\pi/6$$

Initial Values

$$\overline{r}_1 = \begin{bmatrix} 100 \\ 50 \\ 30 \end{bmatrix} \quad , \quad \overline{r}_2 = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix} \quad , \quad l_1 = 2 \quad , \quad l_2 = 30 \quad , \quad l_3 = 2 \quad , \quad \theta_1^0 = \pi/2 \quad , \quad \theta_1^1 = \pi/2$$

Imposed Condition

$$-\pi/12 \prec \theta_1^0 - \theta_1^1 \prec \pi/12$$

Outputs

$$l_1 = 63.05401 \;\; , \;\; l_2 = 44.33913 \;\; , \;\; l_3 = 42.91135 \;\; , \;\; l_4 = 109.30748 \;\; , \;\; l_5 = 68.70836$$

$$x = 3.38236 \quad , \quad y = 5.36427 \quad , \quad z = 2.68679 \quad , \quad \theta_1^0 = 0.87232 \quad , \quad \theta_1^1 = 1.13412$$

$$\theta_6^1 = 3.02012 \quad , \quad \theta_5^1 = 2.60402 \quad , \quad a = -110.07879 \quad , \quad b = 8.54829$$

$c = 52.52234$

$$\bar{r} = \begin{bmatrix} -72.45168 \\ 15.00284 \\ 105.29682 \end{bmatrix} \quad , \quad \overline{V}^{(4)} = \begin{bmatrix} 103.66968 \\ 7.84179 \\ 33.75247 \end{bmatrix} \quad , \quad \overline{W}^{(4)} = \begin{bmatrix} -59.33065 \\ -7.84185 \\ -33.75231 \end{bmatrix}$$

Figure (3.11) illustrates the continuity of the coupler link's motion when it passes from the first prescribed position to the second one.

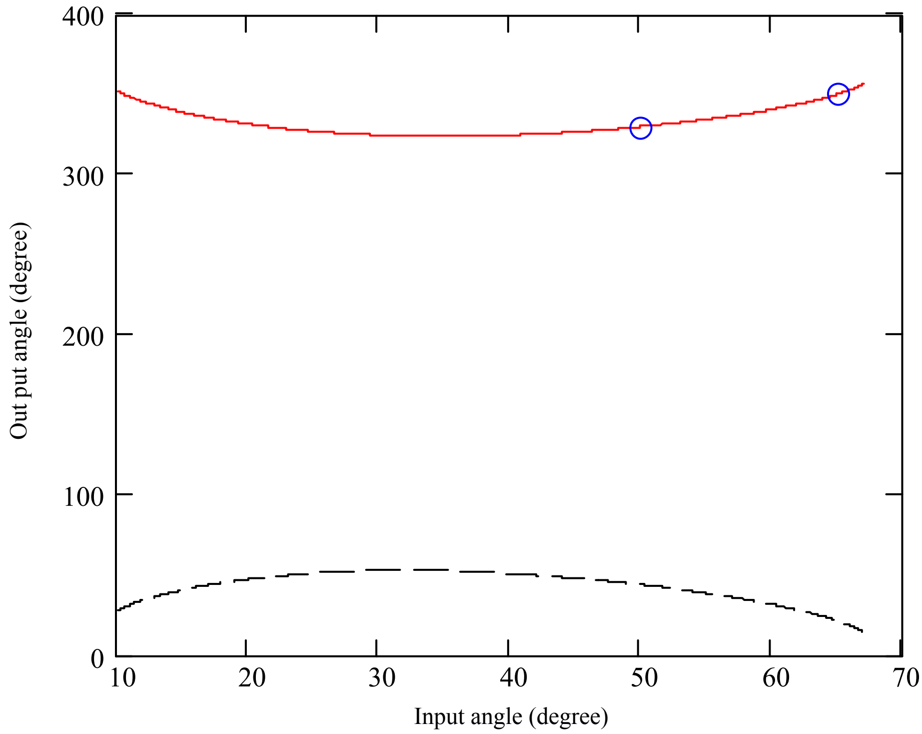


Figure (3.11) : The circles illustrate the input and output angles at which frame (p) lies in the prescribed positions. The dashed curve displays the other loop closure curve of the RSHR mechanism.

CHAPTER FOUR
OVER CONSTRAINED SPATIAL MECHANISMS

4.1) GENERAL

The degree of freedom of a spatial mechanism can be calculated by the following formula ,

$$D.O.F = 6(n-1) - 5R - 5P - 4C - 3S \qquad (4.1)$$

Where ,

n is the number of links .

R is the number of revolute joints .

P is the number of prismatic joints .

C is the number of cylindric joints .

S is the number of spheric joints .

When the degree of freedom of a mechanism is less than one , it is expected to be immobile .However there are some spatial mechanisms whose degrees of freedom - according to formula (4.1) -are less than unity and still they can move under some specific conditions . Such mechanisms are called over constrained mechanisms and the conditions under which they move are called the constraints of the mechanism.

Over constrained mechanisms are attractive to mechanism designers for their higher capacity-in comparison to the similar mechanisms-to carry loads and that they are cheaper .A.J. Shih(ref.30) and J.E.Baker(ref.31,32) recently have worked on some over constrained mechanisms. In this chapter using the algebra of exponential rotation matrices ,the mobility and kinematic synthesis of an over constrained spatial mechanism has been studied.

4.2) RCCR LINKAGE

According to equation (4.1) the degree of freedom of the RCCR linkage illustrated in Figure (4.1) is equal to zero but it has been proved that this mechanism is able to move under a constraint .

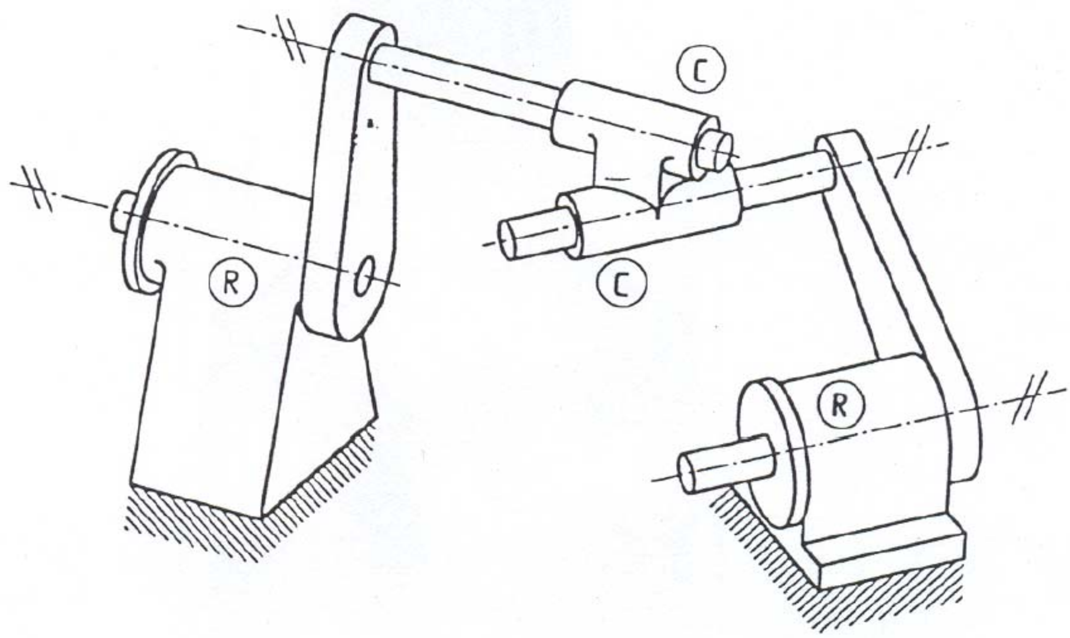


Figure (4.1) : An RCCR linkage

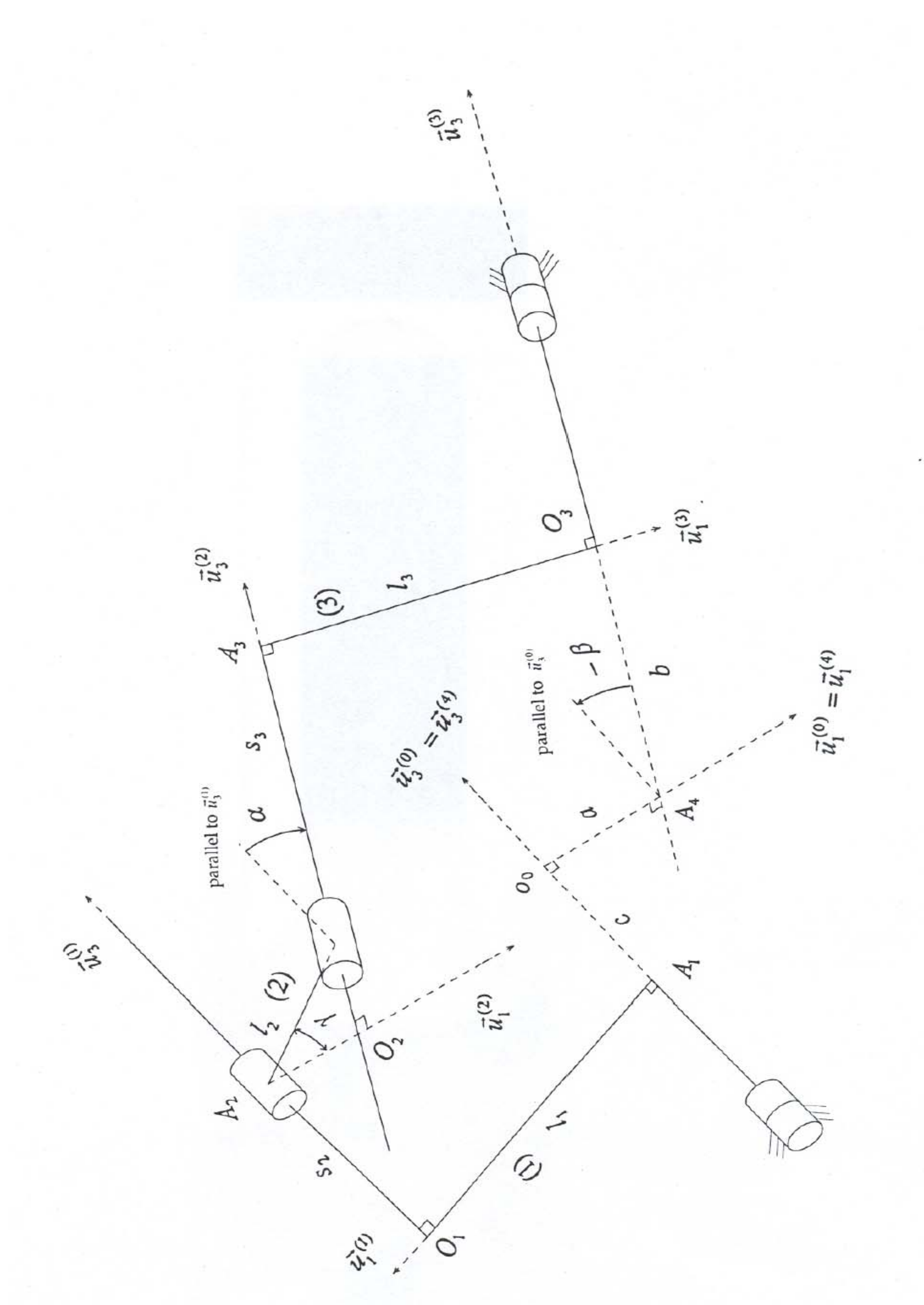


Figure (4.2) : Schematic figure of the RCCR linkage.

Considering Figure (4.2) and Denavit-Hartenberg's convention table (4.1) can be constructed. Hence the rotation matrices between the link frames are determined as follows ,

Table (4.1) : Denavit-Hartenberg parameters of the RCCR linkage.

Link	θ	α	a	d
1	variable	0	l_1	$-c$
2	variable	α	$l_2 \cos\gamma$	s_2
3	variable	0	l_3	s_3
4	variable	$-\beta$	$-a$	$-b$

$$\hat{C}^{(0,1)} = e^{\tilde{u}_3\theta_1} \quad ; \quad \hat{C}^{(1,2)} = e^{\tilde{u}_3\theta_2} e^{\tilde{u}_1\alpha} \quad ; \quad \hat{C}^{(2,3)} = e^{\tilde{u}_3\theta_3} \quad ; \quad \hat{C}^{(3,4)} = e^{\tilde{u}_3\theta_4} e^{-\tilde{u}_1\beta}$$

The following equation can be written according to equation (2.1) ,

$$\hat{C}^{(0,4)} = \hat{I}$$

Thus equation below is derived ,

$$\hat{C}^{(0,2)}\hat{C}^{(2,4)} = \hat{I} \Rightarrow \hat{C}^{(0,2)} = (\hat{C}^{(2,4)})^{-1} \Rightarrow e^{\tilde{u}_3(\theta_1+\theta_2)} e^{\tilde{u}_1\alpha} = e^{\tilde{u}_1\beta} e^{-\tilde{u}_3(\theta_3+\theta_4)} \tag{4.2}$$

From equation (4.2) the following equation is resulted ,

$$\begin{bmatrix} \cos(\theta_1+\theta_2) & -\cos\alpha\sin(\theta_1+\theta_2) & \sin\alpha\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos\alpha\cos(\theta_1+\theta_2) & -\sin\alpha\cos(\theta_1+\theta_2) \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos(\theta_3+\theta_4) & \sin(\theta_3+\theta_4) & 0 \\ -\cos\beta\sin(\theta_3+\theta_4) & \cos\beta\cos(\theta_3+\theta_4) & -\sin\beta \\ -\sin\beta\sin(\theta_3+\theta_4) & \sin\beta\cos(\theta_3+\theta_4) & \cos\beta \end{bmatrix} \tag{4.3}$$

which results in ,

$$\cos\alpha = \cos\beta \Rightarrow \alpha = \beta \text{ or } \alpha = -\beta$$

If $\alpha = \beta$ then according to equation (4.3) constraint below is obtained ,

$$\cos(\theta_1 + \theta_2) = 1 \Rightarrow \theta_1 + \theta_2 = 0$$

and

$$\cos(\theta_3 + \theta_4) = 1 \Rightarrow \theta_3 + \theta_4 = 0$$

If $\alpha = -\beta$ then ,

$$\cos(\theta_1 + \theta_2) = -1 \Rightarrow \theta_1 + \theta_2 = \pi$$

and

$$\cos(\theta_3 + \theta_4) = -1 \Rightarrow \theta_3 + \theta_4 = \pi$$

From Figure (4.2) it can be seen that ,

$$\alpha = \beta$$

Therefore it can be deduced that the RCCR linkage will be movable if and only if the axes of the fixed joints are parallel to the axes of the moving joints.

Recalling equation (2.4) the following equation is written ,

$$d_1\overline{u}_3 + a_1\hat{C}^{(0,1)}\overline{u}_1 + d_2\hat{C}^{(0,1)}\overline{u}_1 + a_2\hat{C}^{(0,2)}\overline{u}_1 + d_3\hat{C}^{(0,2)}\overline{u}_3 + a_3\hat{C}^{(0,3)}\overline{u}_1 + d_4\hat{C}^{(0,3)}\overline{u}_3 + a_4\hat{C}^{(0,4)}\overline{u}_1 = \overline{0} \tag{4.4}$$

Substituting a_i and d_i from table (4.1) equation (4.4) can be written in the form below ,

$$l_1\hat{C}^{(0,1)}\overline{u}_1 - c\overline{u}_3 + l_2\cos\gamma\hat{C}^{(0,2)}\overline{u}_1 + s_2\hat{C}^{(0,1)}\overline{u}_3 + l_3\hat{C}^{(0,3)}\overline{u}_1 + s_3\hat{C}^{(0,2)}\overline{u}_3 - a\hat{C}^{(0,4)}\overline{u}_1 - b\hat{C}^{(0,3)}\overline{u}_3 = \overline{0} \tag{4.5}$$

Let $\alpha = \beta = \delta$ then equation below is deduced ,

$$\hat{C}^{(0,4)} = \hat{I} \Rightarrow \hat{C}^{(0,3)}\hat{C}^{(3,4)} = \hat{I} \Rightarrow \hat{C}^{(0,3)} = (\hat{C}^{(3,4)})^{-1} = (e^{\tilde{u}_3\theta_4}e^{-\tilde{u}_1\delta})^{-1} = e^{\tilde{u}_1\delta}e^{-\tilde{u}_3\theta_4}$$

Replacing $\hat{C}^{(0,4)}$ and $\hat{C}^{(0,3)}$ by $\hat{I}$ and $e^{\tilde{u}_1\delta}e^{-\tilde{u}_3\theta_4}$ in equation (4.5) respectively the following equation is obtained,

$$\begin{aligned} &l_1e^{\tilde{u}_3\theta_1}\overline{u}_1 - c\overline{u}_3 + l_2\cos\gamma e^{\tilde{u}_3(\theta_1+\theta_2)}e^{\tilde{u}_1\delta}\overline{u}_1 + s_2e^{\tilde{u}_3\theta_1}\overline{u}_3 + s_3e^{\tilde{u}_3(\theta_1+\theta_2)}e^{\tilde{u}_1\delta}\overline{u}_3 + l_3e^{\tilde{u}_1\delta}e^{-\tilde{u}_3\theta_4}\overline{u}_1 \\ &- a\overline{u}_1 - be^{\tilde{u}_1\delta}e^{-\tilde{u}_3\theta_4}\overline{u}_3 = \overline{0} \end{aligned} \tag{4.6}$$

Regarding the constraint of the mechanism ($\theta_1 + \theta_2 = 0$) equation (4.6) results in the scalar equations below ,

$$l_1\cos\theta_1 + l_2\cos\gamma + l_3\cos\theta_4 - a = 0 \tag{4.7}$$

$$l_1\sin\theta_1 - l_3\cos\delta\sin\theta_4 - s_3\sin\delta + b\sin\delta = 0$$

$$-c + s_2 - l_3\sin\delta\sin\theta_4 + s_3\cos\delta - b\cos\delta = 0$$

Let $\theta_1 = \psi$ and $\theta_4 = \phi$. Hence according to equation (4.7) the general displacement equation of the RCCR linkage will be ,

$$l_1 \cos\psi + l_2 \cos\gamma + l_3 \cos\phi - a = 0 \tag{4.8}$$

Note that γ is a constant angle and belongs to the structure of the mechanism.

4.3) MOBILITY ANALYSIS OF THE RCCR LINKAGE

1) CRANK-ROCKER (ψ varies from 0 to 2π)

According to equation (4.8) the inequality below is derived,

$$-l_1 \le l_2 \cos\gamma + l_3 \cos\phi - a \le l_1 \Rightarrow \frac{a - l_1 - l_2 \cos\gamma}{l_3} \le \cos\phi \le \frac{a + l_1 - l_2 \cos\gamma}{l_3} \qquad (4.9)$$

In this case the inequality $-1 \le \cos\phi \le 1$ must be never violated. Hence the following inequality has to be satisfied,

$$-1 \le \frac{a - l_1 - l_2 \cos\gamma}{l_3} \le \frac{a + l_1 - l_2 \cos\gamma}{l_3} \le 1 \qquad (4.10)$$

which results in the following inequalities ,

$$l_1 - l_3 \le a - l_2 \cos\gamma \le l_3 - l_1 \quad \text{and} \quad l_3 > l_1 \qquad (4.11)$$

Therefore under condition above the RCCR linkage will act as a crank-rocker.

Example (4.1) : Draw the output angle-input angle diagrams of the RCCR linkages with the following dimensions ,

$l_1 = 20 \quad ; \quad l_3 = 40 \quad ; \quad f = a - l_2 \cos\gamma = -20, -10, \ldots, 20$

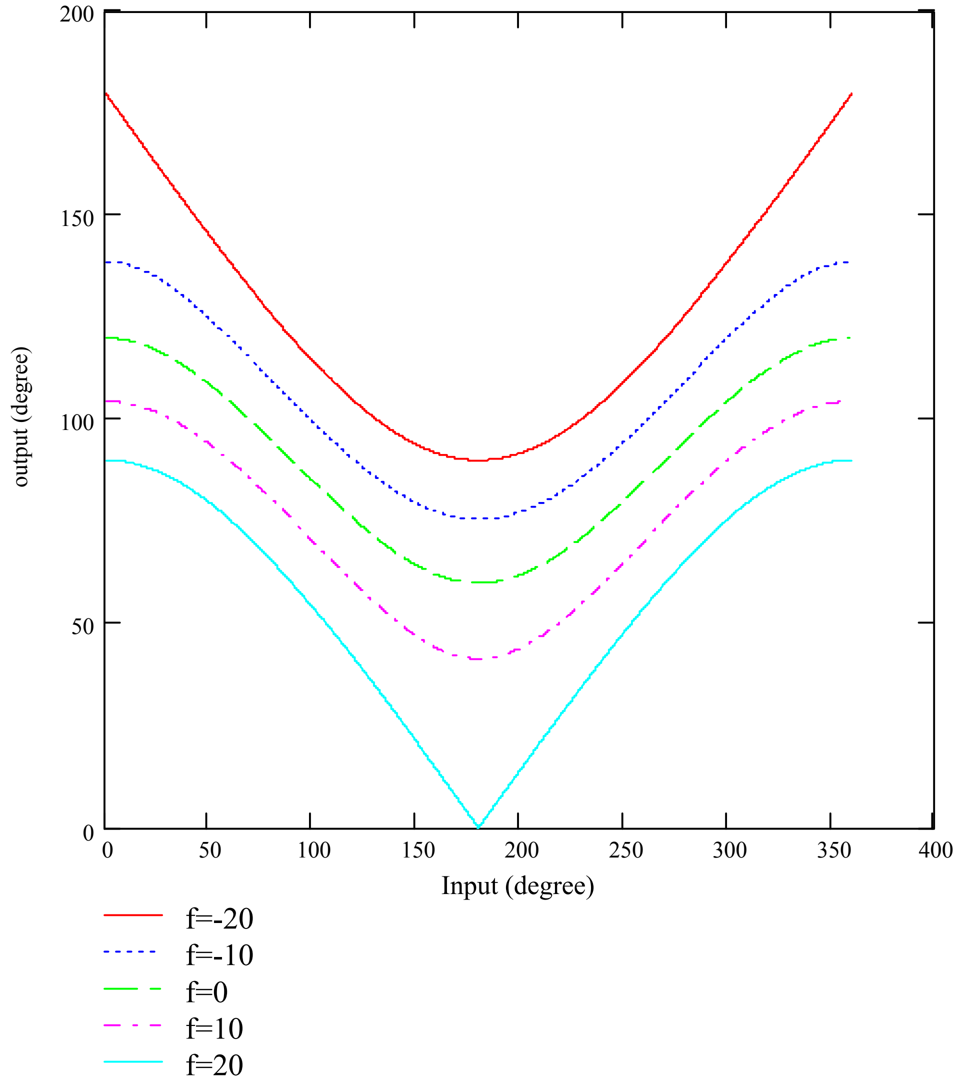


Figure (4.3) : The angular displacement of the output link versus the angular displacement of the input link for $l_1 = 20$, $l_3 = 40$ and various f's .

As shown in the diagram above , all of the RCCR linkages whose dimensions satisfy inequality (4.11) act as crank-rockers.

2) ROCKER-CRANK (ϕ varies from 0 to 2π)

From equation (4.8) equation below is derived ,

$$-l_3 \le l_1 \cos\psi + l_2 \cos\gamma - a \le l_3 \Rightarrow \frac{a - l_2 \cos\gamma - l_3}{l_1} \le \cos\psi \le \frac{a - l_2 \cos\gamma + l_3}{l_1} \qquad (4.12)$$

In the case of rocker-crank inequality $-1 \le \cos\psi \le 1$ must be never violated. Thus inequality ,

$$-1 \le \frac{a - l_3 - l_2 \cos\gamma}{l_1} \le \frac{a + l_3 - l_2 \cos\gamma}{l_1} \le 1 \qquad (4.13)$$

Has to be satisfied which results in ,

$$l_3 - l_1 \le a - l_2 \cos\gamma \le l_1 - l_3 \quad \text{and} \quad l_1 > l_3 \qquad (4.14)$$

Hence under condition above the mechanism will act as a rocker-crank.

Example (4.2) : Draw the output angle-input angle diagrams of the RCCR linkages with the following dimensions ,

$l_1 = 40$; $l_3 = 20$; $f = a - l_2 \cos\gamma = -20, -10, \ldots, 20$

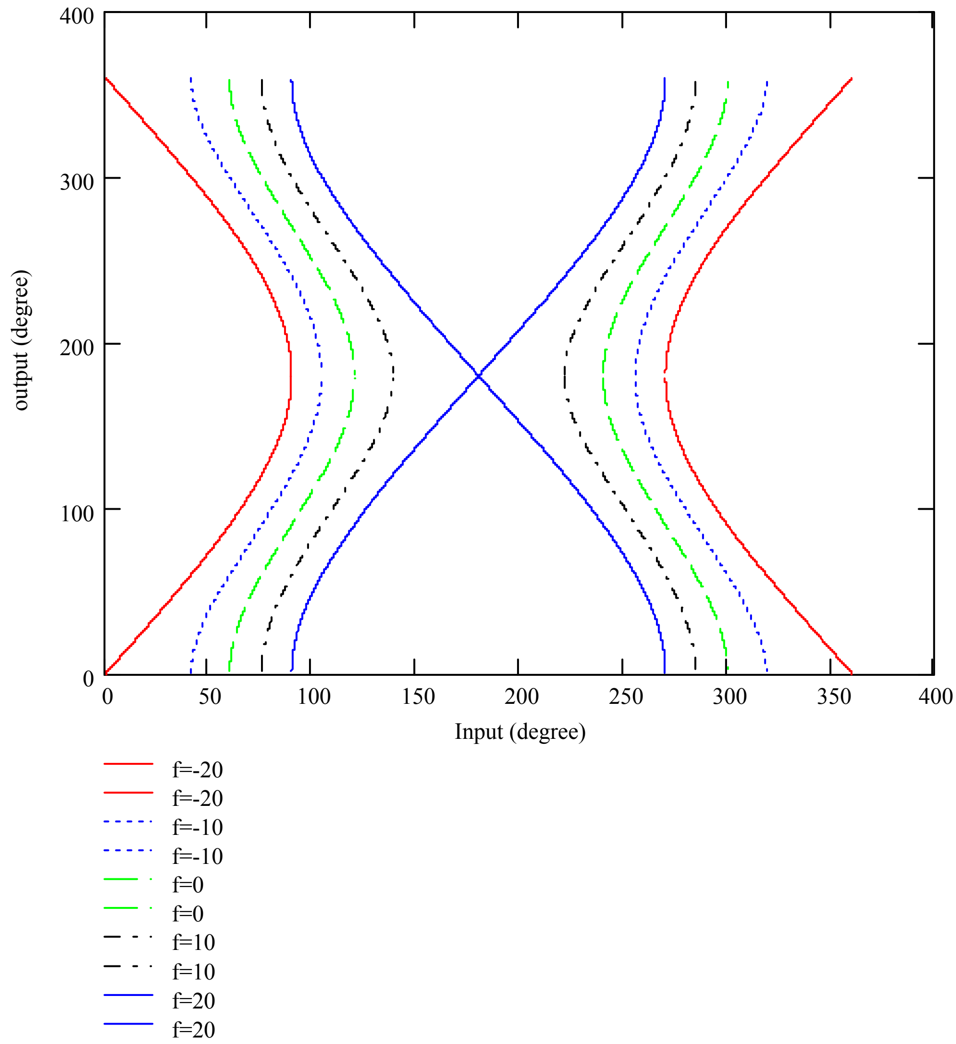


Figure (4.4) : As illustrated in the diagram all the RCCR linkages whose dimensions satisfy inequality (4.14) act as rocker-cranks.

3) DOUBLE-CRANK (both ψ and ϕ vary from 0 to 2π)

So far the conditions under which the RCCR linkage acts as crank-rocker and rocker-crank were determined. The common part of these two conditions will give us the condition under which the linkage acts as a double crank. Considering inequalities (4.11) and (4.14) it can be seen that both inequalities are satisfied if and only if ,

$$l_1 = l_3 \quad \text{and} \quad a = l_2 \cos\gamma \tag{4.15}$$

Thus under condition above the mechanism will be a double crank.

Example (4.3) : Draw the output angle-input angle diagram of the RCCR linkages with the following dimensions ,

$$l_1 = l_3 \quad \text{and} \quad a = l_2 \cos\gamma$$

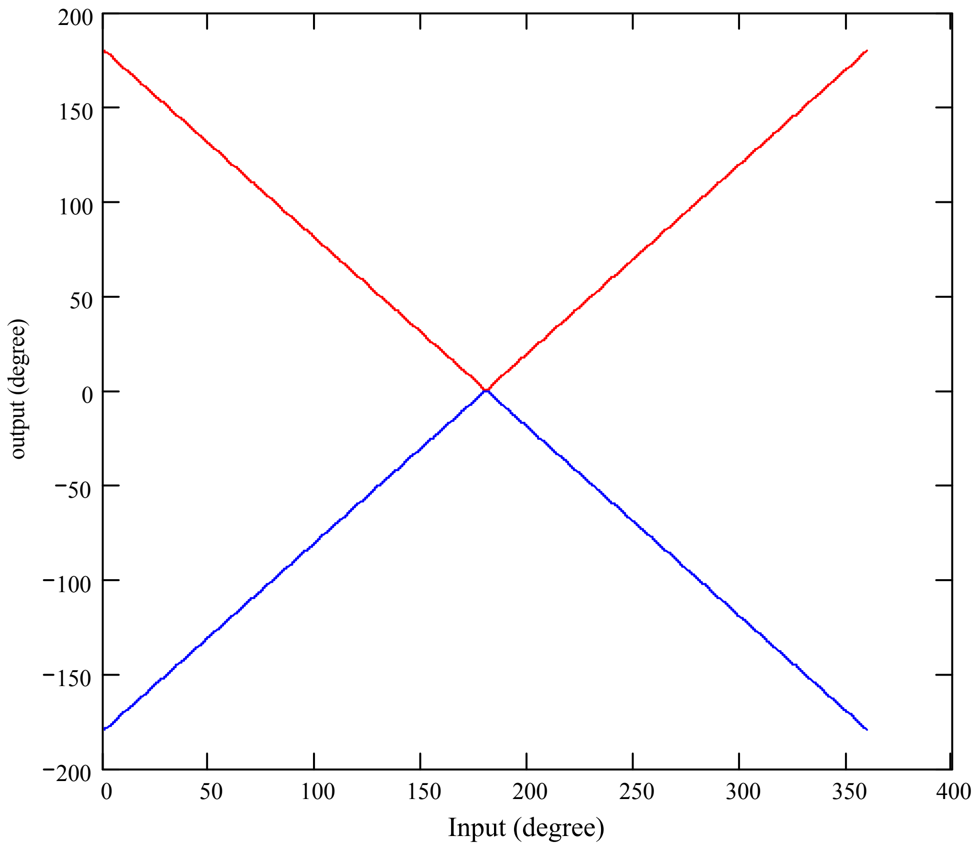


Figure (4.5) : When condition (4.15) is satisfied the RCCR linkage acts as a double crank.

4) DOUBLE-ROCKER (both ψ and ϕ vary in a range less than 2π)

Recalling the conditions under which the RCCR linkage acts as crank-rocker and rocker-crank it can be deduce that the opposite conditions are required for the linkage to act as a double rocker. That is ,

$$a - l_2 \cos\gamma \geq l_3 - l_1 \quad \text{or} \quad a - l_2 \cos\gamma \leq l_1 - l_3 \quad \text{when} \quad l_3 > l_1 \tag{4.16}$$

$$a - l_2 \cos\gamma \geq l_1 - l_3 \quad \text{or} \quad a - l_2 \cos\gamma \leq l_3 - l_1 \quad \text{when} \quad l_1 > l_3 \tag{4.17}$$

However conditions (4.16) and (4.17) can be considered only as necessary conditions for the RCCR linkage to act as a double rocker. The reason is that so far the conditions under which the linkage becomes locked have not been regarded. From inequality (4.9) it can be stated that the mechanism won't be locked if and only if ,

$$\frac{a + l_1 - l_2 \cos\gamma}{l_3} \geq -1 \quad \text{and} \quad \frac{a - l_1 - l_2 \cos\gamma}{l_3} \leq 1$$

which result in

$$-(l_1 + l_3) \leq a - l_2 \cos\gamma \leq l_1 + l_3 \tag{4.18}$$

Hence inequality (4.18) together with inequality (4.16) or (4.17) gives us the conditions under which the linkage acts as a double rocker. That is ,

$$-(l_1 + l_3) \leq a - l_2 \cos\gamma \leq l_1 - l_3 \quad \text{or} \quad l_3 - l_1 \leq a - l_2 \cos\gamma \leq l_1 + l_3 \quad \text{when} \quad l_3 > l_1 \tag{4.19}$$

or

$$-(l_1+l_3)\le a-l_2\cos\gamma\le l_3-l_1 \quad \text{or} \quad l_1-l_3\le a-l_2\cos\gamma\le l_1+l_3 \quad \text{when} \quad l_1>l_3 \qquad (4.20)$$

Example (4.4) : Draw the output angle-input angle diagram of the RCCR linkages with the following dimensions ,

a) $l_1=20$, $l_3=40$, $f=a-l_2\cos\gamma=-30,-40,-50$

b) $l_1=40$, $l_3=20$, $f=a-l_2\cos\gamma=30,40,50$

a) $l_1=20$, $l_3=40$, $f=a-l_2\cos\gamma=-30,-40,-50$

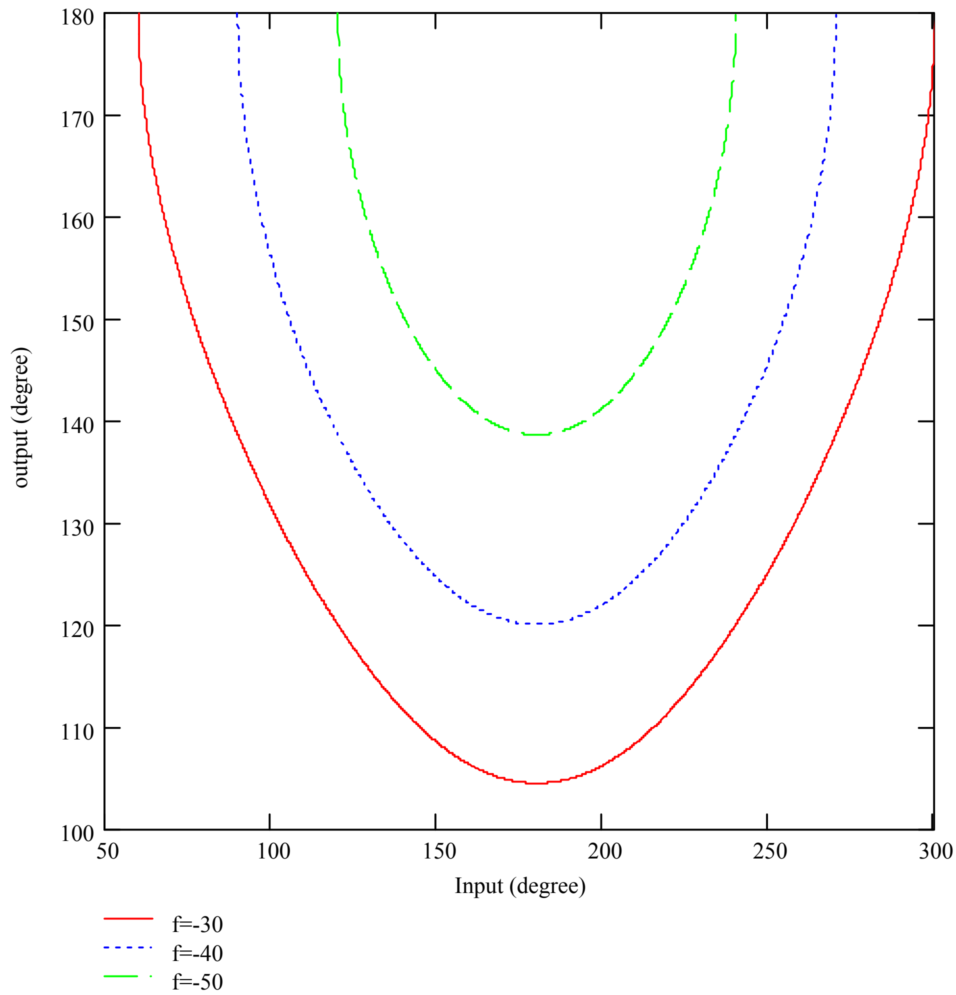


Figure (4.6) : All RCCR linkages whose dimensions satisfy inequality (4.19) are double rockers .

b) $l_1 = 40 \quad , \quad l_3 = 20 \quad , \quad f = a - l_2 \cos\gamma = 30,40,50$

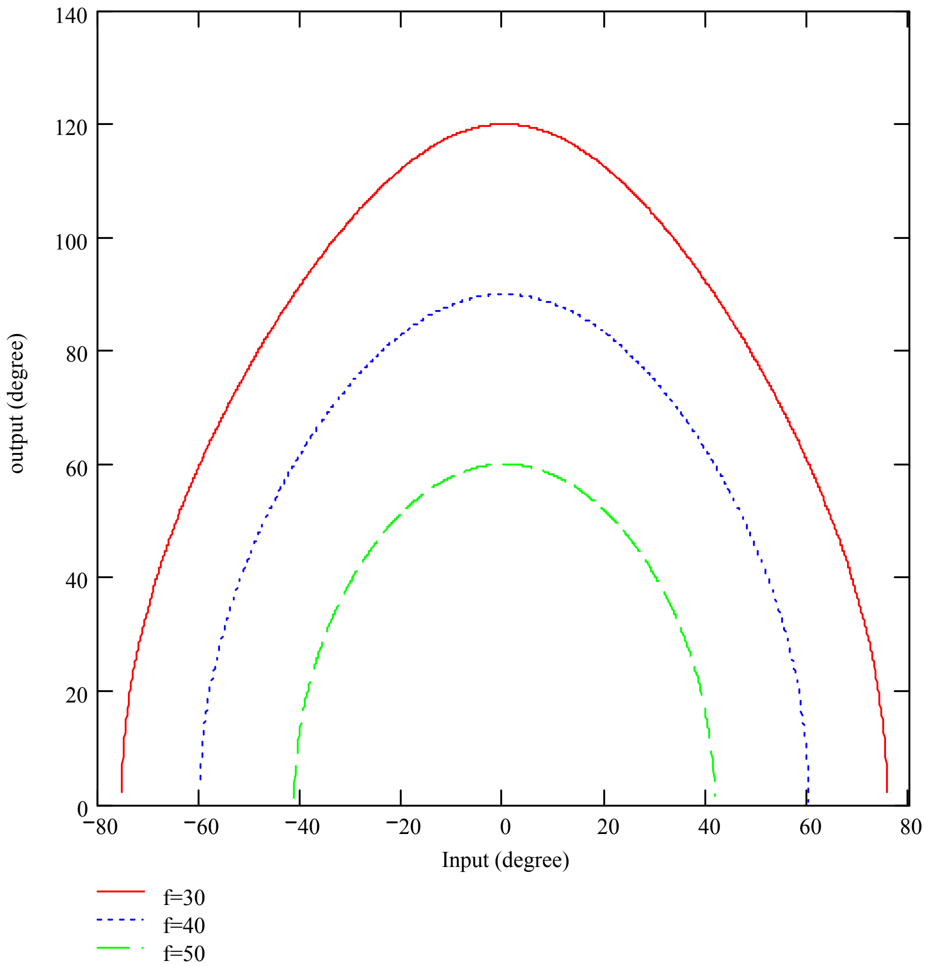


Figure (4.7) : All RCCR linkages whose dimensions satisfy inequality (4.20) act as double rockers.

4.4) SWING ANGLE OF THE RCCR LINKAGE

The swing angle of a crank-rocker mechanism is the variation range of the output angle for a complete rotation of the input link . Considering that in return positions the angular speed of the output link is equal to zero , the swing angle of the mechanism is determined as follows ,

Differentiating equation (4.8) in terms of ψ the following equation is derived,

$$-l_3 \sin\phi (\frac{d\phi}{d\psi}) = l_1 \sin\psi \tag{4.21}$$

The following equation is available in return positions,

$$(\frac{d\phi}{d\psi}) = 0 \Rightarrow \sin\psi = 0 \Rightarrow \psi_1 = 0 \text{ or } \psi_2 = \pi$$

If $\psi_1 = 0$ then the following equation is resulted from equation (4.8) ,

$$\cos\phi_1 = \frac{-l_1}{l_3} + \frac{a - l_2 \cos\gamma}{l_3} \Rightarrow \phi_1 = \cos^{-1}(\frac{-l_1}{l_3} + \frac{a - l_2 \cos\gamma}{l_3}) \tag{4.22}$$

If $\psi_2 = \pi$ then ϕ_2 is determined as follows ,

$$\phi_2 = \cos^{-1}(\frac{l_1}{l_3} + \frac{a - l_2 \cos\gamma}{l_3}) \tag{4.23}$$

Hence the swing angle is determined as follows ,

$$\Delta\phi = |\phi_2 - \phi_1|$$

Let $\frac{l_1}{l_3} = p$ and $\frac{a - l_2 \cos\gamma}{l_3} = q$ then ,

$$\Delta\phi = \left|\cos^{-1}(p+q) - \cos^{-1}(-p+q)\right| \tag{4.24}$$

Now the swing angle diagram of the RCCR linkage can be drawn by means of equation (4.24).

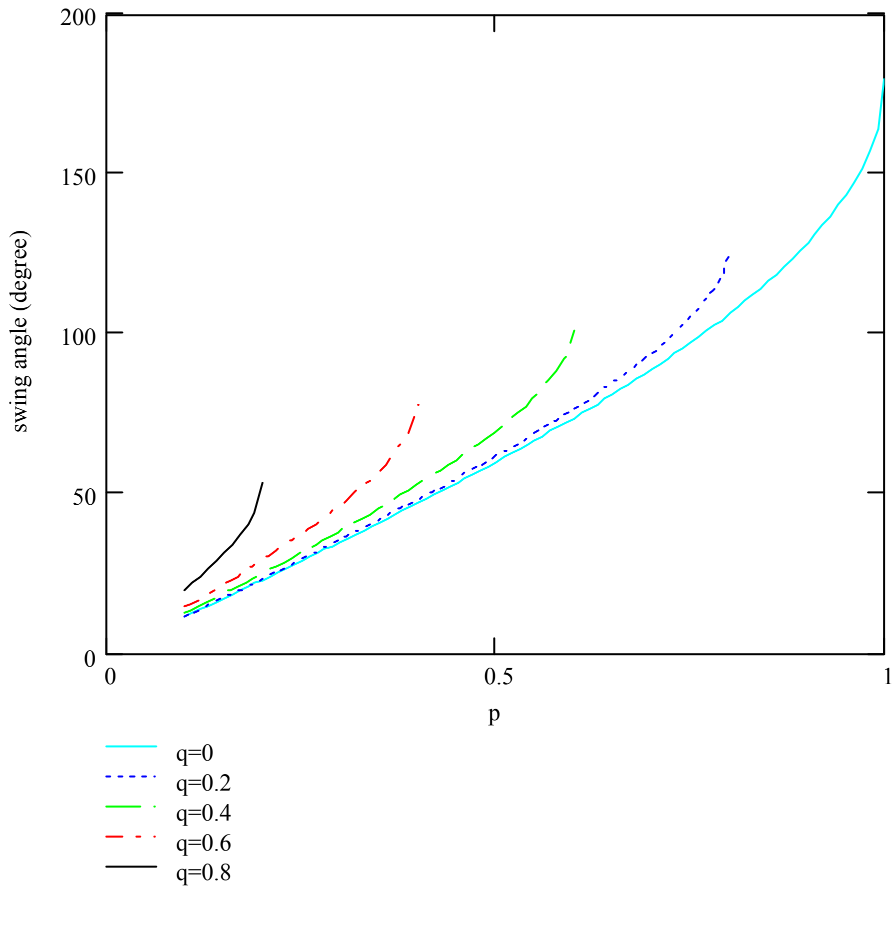


Figure (4.8) : Swing angle diagram of the RCCR linkage for various p's and q's. Here $p = \frac{l_1}{l_3}$ and $q = \frac{a - l_2 \cos\gamma}{l_3}$.

4.5) PATH GENERATION SYNTHESIS OF THE RCCR LINKAGE

Let's consider the case in which the locations and orientations of the fixed joints are not prescribed. In this case a global coordinate system is used to specify the locations of the precision points as shown in the figure below ,

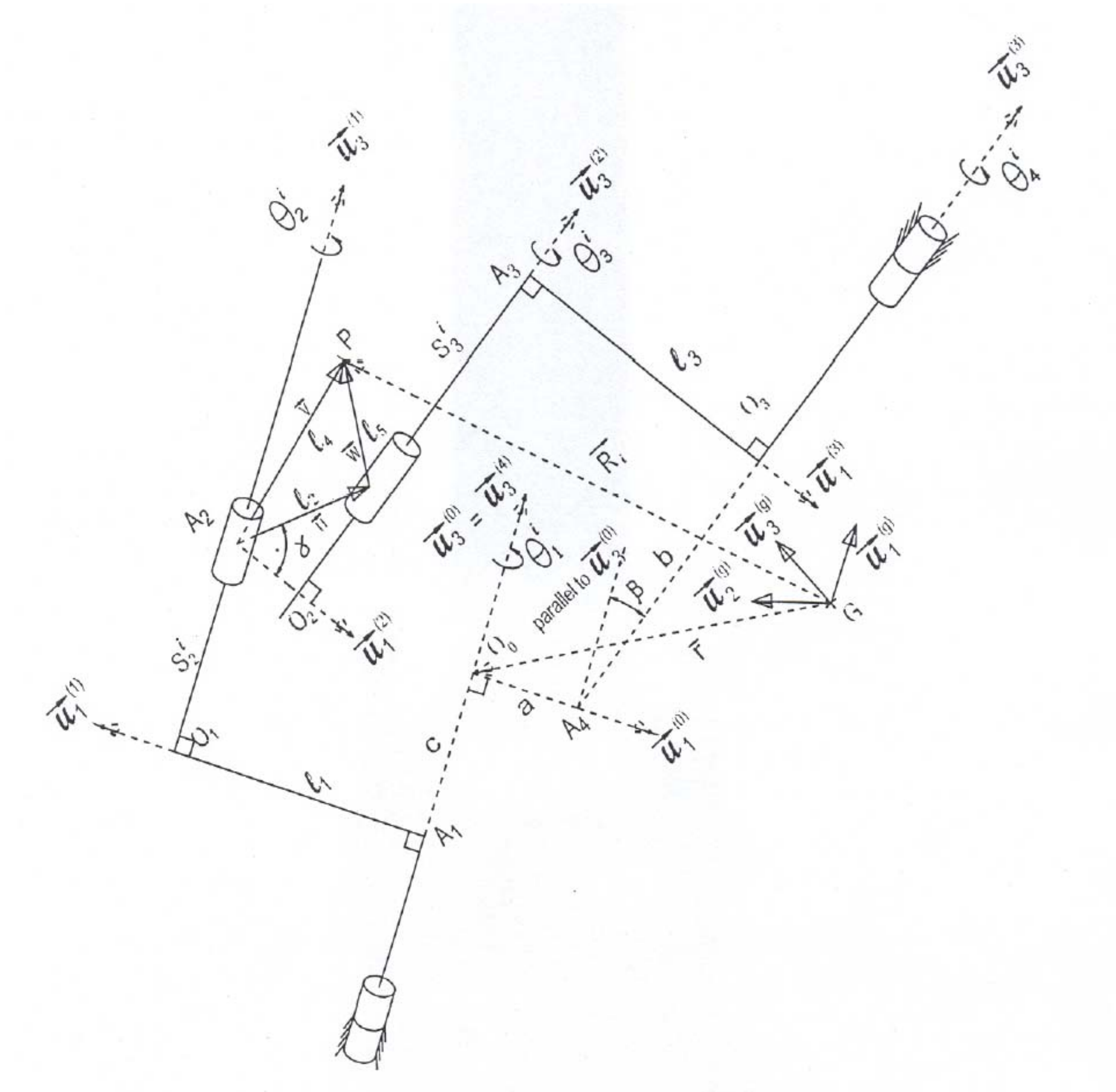


Figure (4.9) : When the locations and orientations of the fixed joints are not prescribed all positions and orientations are defined based on a global coordinate system.

Note that for the synthesis procedure the link variables shown in table (4.1) are changed as shown in table (4.2) ,

Table (4.2) : Joint variables and Denavit-Hartenberg parameters of the RCCR linkage.

Link	θ	α	a	d
1	θ_1^i	0	l_1	$-c$
2	θ_2^i	δ	$l_2 \cos\gamma$	s_2^i
3	θ_3^i	0	l_3	s_3^i
4	θ_4^i	$-\delta$	$-a$	$-b$

After dividing the mechanism into left and right dyads the loop closure equations are written for the left and right dyads as follows ,

$$\bar{r} - c\hat{C}^{(g,0)}\bar{u}_3 + l_1\hat{C}_i^{(g,1)}\bar{u}_1 + s_2^i\hat{C}_i^{(g,1)}\bar{u}_3 + \hat{C}_i^{(g,2)}\bar{V}^{(2)} = \bar{R}_i \tag{4.25}$$

$$\bar{R}_i + \hat{C}_i^{(g,2)}\bar{W}^{(2)} + s_3^i\hat{C}_i^{(g,2)}\bar{u}_3 + l_3\hat{C}^{(g,3)}\bar{u}_1 - b\hat{C}_i^{(g,3)}\bar{u}_3 - a\hat{C}^{(g,0)}\bar{u}_1 = \bar{r} \tag{4.26}$$

$$\bar{V}^{(2)} + \bar{W}^{(2)} = \bar{U}^{(2)} \tag{4.27}$$

Where ,

$$\hat{C}^{(g,0)} = e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}$$

$\bar{r}$ has been illustrated in Figure (4.9) and $\bar{r} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}$

$$\hat{C}_i^{(g,1)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,1)} = e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_3 \theta_1^i}$$

$$\hat{C}_i^{(g,2)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,2)} = e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_3(\theta_1^i+\theta_2^i)} e^{\tilde{u}_1 \delta}$$

$$\hat{C}_i^{(g,3)} = \hat{C}^{(g,0)}\hat{C}_i^{(0,3)} = e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} (\hat{C}_i^{(3,4)})^{-1} = e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} (e^{\tilde{u}_3\theta_4^i} e^{-\tilde{u}_1\delta})^{-1} = e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} e^{-\tilde{u}_3\theta_4^i}$$

As it was deduced from equation (4.3) , $\theta_1^i + \theta_2^i = 0$ and $\theta_3^i + \theta_4^i = 0$ are constraints of the RCCR linkage . Hence equations (4.25) and (4.26) can be written in the forms below ,

$$\overline{r} - c e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} \overline{u}_3 + l_1 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i)} \overline{u}_1 + s_2^i e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} \overline{u}_3 + e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \overline{V}^{(2)} = \overline{R}_i \tag{4.28}$$

$$\begin{aligned} &\overline{R}_i + e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \overline{W}^{(2)} + s_3^i e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \overline{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_4^i} \overline{u}_1 \\ &- b e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \overline{u}_3 - a e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} \overline{u}_1 = \overline{r} \end{aligned} \tag{4.29}$$

The following equation is written according to Figure (4.9) ,

$$\overline{V}^{(2)} + \overline{W}^{(2)} = \overline{U}^{(2)} = \begin{bmatrix} l_2 \cos\gamma \\ l_2 \sin\gamma \\ 0 \end{bmatrix} \tag{4.30}$$

Let

$$\overline{r} - c e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} \overline{u}_3 + e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \overline{V}^{(2)} = \overline{r}_1 \tag{4.31}$$

and

$$- a e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} \overline{u}_1 - b e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \overline{u}_3 + e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \overline{W}^{(2)} - \overline{r} = \overline{r}_2 \tag{4.32}$$

Hence equations (4.28) and (4.29) can be written in the following form ,

$$\overline{r}_1 + l_1 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i)} \overline{u}_1 + s_2^i e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} \overline{u}_3 = \overline{R}_i \tag{4.33}$$

$$\bar{r}_2 + s_3^i e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_4^i} \bar{u}_1 = -\bar{R}_i \quad (4.34)$$

According to equations (4.33) and(4.34) the following table is constructed,

Table (4.3) : According to table (4.3) when the locations and orientations of the fixed joints are not prescribed the path tracer point of the RCCR linkage will pass through at most six precision points.

number of precision points	number of scalar equations	number of unknowns	number of free parameters
1	6	$16(\bar{r}_1, \bar{r}_2, l_1, l_3, x, y, z, \delta, s_2^0, s_3^0, \theta_1^0, \theta_4^0)$	10
2	12	20(above + $s_2^1, s_3^1, \theta_1^1, \theta_4^1$)	8
3	18	24(above + $s_2^2, s_3^2, \theta_1^2, \theta_4^2$)	6
4	24	28(above + $s_2^3, s_3^3, \theta_1^3, \theta_4^3$)	4
5	30	32(above + $s_2^4, s_3^4, \theta_1^4, \theta_4^4$)	2
6	36	36(above + $s_2^5, s_3^5, \theta_1^5, \theta_4^5$)	0

Assume that it is desired to synthesize an RCCR linkage whose path tracer point is capable of passing through five precision points . That is i=0,1,2,3,4 .

Considering equation (4.33) the following equations are derived ,

$$\bar{r}_1 + l_1 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^0)} \bar{u}_1 + s_2^0 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} \bar{u}_3 = \bar{R}_0 \quad (4.35)$$

$$\bar{r}_1 + l_1 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^1)} \bar{u}_1 + s_2^1 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} \bar{u}_3 = \bar{R}_1 \quad (4.36)$$

$$\overline{r}_1 + l_1 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 (z+\theta_1^2)} \overline{u}_1 + s_2^2 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} \overline{u}_3 = \overline{R}_2 \tag{4.37}$$

$$\overline{r}_1 + l_1 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 (z+\theta_1^3)} \overline{u}_1 + s_2^3 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} \overline{u}_3 = \overline{R}_3 \tag{4.38}$$

$$\overline{r}_1 + l_1 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 (z+\theta_1^4)} \overline{u}_1 + s_2^4 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} \overline{u}_3 = \overline{R}_4 \tag{4.39}$$

Substituting $\overline{r}_1$ from equation (4.35) into equations (4.36) to (4.39) equations below are obtained ,

$$l_1 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 z} (e^{\widetilde{u}_3 \theta_1^1} - e^{\widetilde{u}_3 \theta_1^0}) \overline{u}_1 + (s_2^1 - s_2^0) e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} \overline{u}_3 = \overline{R}_1 - \overline{R}_0$$

$$l_1 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 z} (e^{\widetilde{u}_3 \theta_1^2} - e^{\widetilde{u}_3 \theta_1^0}) \overline{u}_1 + (s_2^2 - s_2^0) e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} \overline{u}_3 = \overline{R}_2 - \overline{R}_0$$

$$l_1 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 z} (e^{\widetilde{u}_3 \theta_1^3} - e^{\widetilde{u}_3 \theta_1^0}) \overline{u}_1 + (s_2^3 - s_2^0) e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} \overline{u}_3 = \overline{R}_3 - \overline{R}_0$$

$$l_1 e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 z} (e^{\widetilde{u}_3 \theta_1^4} - e^{\widetilde{u}_3 \theta_1^0}) \overline{u}_1 + (s_2^4 - s_2^0) e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} \overline{u}_3 = \overline{R}_4 - \overline{R}_0$$

or

$$l_1 (e^{\widetilde{u}_3 \theta_1^1} - e^{\widetilde{u}_3 \theta_1^0}) \overline{u}_1 + (s_2^1 - s_2^0) \overline{u}_3 = e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_1 - \overline{R}_0) \tag{4.40}$$

$$l_1 (e^{\widetilde{u}_3 \theta_1^2} - e^{\widetilde{u}_3 \theta_1^0}) \overline{u}_1 + (s_2^2 - s_2^0) \overline{u}_3 = e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_2 - \overline{R}_0) \tag{4.41}$$

$$l_1 (e^{\widetilde{u}_3 \theta_1^3} - e^{\widetilde{u}_3 \theta_1^0}) \overline{u}_1 + (s_2^3 - s_2^0) \overline{u}_3 = e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_3 - \overline{R}_0) \tag{4.42}$$

$$l_1 (e^{\widetilde{u}_3 \theta_1^4} - e^{\widetilde{u}_3 \theta_1^0}) \overline{u}_1 + (s_2^4 - s_2^0) \overline{u}_3 = e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_4 - \overline{R}_0) \tag{4.43}$$

The following equations are obtained from equation (4.34),

$$\bar{r}_2 + s_3^0 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_4^0} \bar{u}_1 = -\bar{R}_0 \tag{4.44}$$

$$\bar{r}_2 + s_3^1 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_4^1} \bar{u}_1 = -\bar{R}_1 \tag{4.45}$$

$$\bar{r}_2 + s_3^2 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_4^2} \bar{u}_1 = -\bar{R}_2 \tag{4.46}$$

$$\bar{r}_2 + s_3^3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_4^3} \bar{u}_1 = -\bar{R}_3 \tag{4.47}$$

$$\bar{r}_2 + s_3^4 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_4^4} \bar{u}_1 = -\bar{R}_4 \tag{4.48}$$

Substituting $\bar{r}_2$ from equation (4.44) into equations (4.45) to (4.48) equations below are derived ,

$$(s_3^1 - s_3^0) e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} (e^{-\tilde{u}_3 \theta_4^1} - e^{-\tilde{u}_3 \theta_4^0}) \bar{u}_1 = \bar{R}_0 - \bar{R}_1$$

$$(s_3^2 - s_3^0) e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} (e^{-\tilde{u}_3 \theta_4^2} - e^{-\tilde{u}_3 \theta_4^0}) \bar{u}_1 = \bar{R}_0 - \bar{R}_2$$

$$(s_3^3 - s_3^0) e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} (e^{-\tilde{u}_3 \theta_4^3} - e^{-\tilde{u}_3 \theta_4^0}) \bar{u}_1 = \bar{R}_0 - \bar{R}_3$$

$$(s_3^4 - s_3^0) e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} \bar{u}_3 + l_3 e^{\tilde{u}_1 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{\tilde{u}_1 \delta} (e^{-\tilde{u}_3 \theta_4^4} - e^{-\tilde{u}_3 \theta_4^0}) \bar{u}_1 = \bar{R}_0 - \bar{R}_4$$

or

$$(s_3^1 - s_3^0) \bar{u}_3 + l_3 (e^{-\tilde{u}_3 \theta_4^1} - e^{-\tilde{u}_3 \theta_4^0}) \bar{u}_1 = e^{-\tilde{u}_1 \delta} e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_1 x} (\bar{R}_0 - \bar{R}_1) \tag{4.49}$$

$$(s_3^2 - s_3^0)\overline{u}_3 + l_3(e^{-\tilde{u}_3\theta_4^2} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_2) \quad (4.50)$$

$$(s_3^3 - s_3^0)\overline{u}_3 + l_3(e^{-\tilde{u}_3\theta_4^3} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_3) \quad (4.51)$$

$$(s_3^4 - s_3^0)\overline{u}_3 + l_3(e^{-\tilde{u}_3\theta_4^4} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_4) \quad (4.52)$$

Multiplying equations (4.40) to (4.43) and (4.49) to (4.52) by $\overline{u}_1^{\,t}$ and $\overline{u}_2^{\,t}$ the following equations are obtained ,

$$l_1\overline{u}_1^{\,t}(e^{\tilde{u}_3\theta_1^1} - e^{\tilde{u}_3\theta_1^0})\overline{u}_1 = \overline{u}_1^{\,t}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_1 - \overline{R}_0) \quad (4.53)$$

$$l_1\overline{u}_1^{\,t}(e^{\tilde{u}_3\theta_1^2} - e^{\tilde{u}_3\theta_1^0})\overline{u}_1 = \overline{u}_1^{\,t}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_2 - \overline{R}_0) \quad (4.54)$$

$$l_1\overline{u}_1^{\,t}(e^{\tilde{u}_3\theta_1^3} - e^{\tilde{u}_3\theta_1^0})\overline{u}_1 = \overline{u}_1^{\,t}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_3 - \overline{R}_0) \quad (4.55)$$

$$l_1\overline{u}_1^{\,t}(e^{\tilde{u}_3\theta_1^4} - e^{\tilde{u}_3\theta_1^0})\overline{u}_1 = \overline{u}_1^{\,t}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_4 - \overline{R}_0) \quad (4.56)$$

$$l_1\overline{u}_2^{\,t}(e^{\tilde{u}_3\theta_1^1} - e^{\tilde{u}_3\theta_1^0})\overline{u}_1 = \overline{u}_2^{\,t}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_1 - \overline{R}_0) \quad (4.57)$$

$$l_1\overline{u}_2^{\,t}(e^{\tilde{u}_3\theta_1^2} - e^{\tilde{u}_3\theta_1^0})\overline{u}_1 = \overline{u}_2^{\,t}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_2 - \overline{R}_0) \quad (4.58)$$

$$l_1\overline{u}_2^{\,t}(e^{\tilde{u}_3\theta_1^3} - e^{\tilde{u}_3\theta_1^0})\overline{u}_1 = \overline{u}_2^{\,t}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_3 - \overline{R}_0) \quad (4.59)$$

$$l_1\overline{u}_2^{\,t}(e^{\tilde{u}_3\theta_1^4} - e^{\tilde{u}_3\theta_1^0})\overline{u}_1 = \overline{u}_2^{\,t}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_4 - \overline{R}_0) \quad (4.60)$$

$$l_3\overline{u}_1^{\,t}(e^{-\tilde{u}_3\theta_4^1} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = \overline{u}_1^{\,t}e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_1) \quad (4.61)$$

$$l_3\overline{u}_1{}^t(e^{-\tilde{u}_3\theta_4^2} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = \overline{u}_1{}^t e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_2) \tag{4.62}$$

$$l_3\overline{u}_1{}^t(e^{-\tilde{u}_3\theta_4^3} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = \overline{u}_1{}^t e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_3) \tag{4.63}$$

$$l_3\overline{u}_1{}^t(e^{-\tilde{u}_3\theta_4^4} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = \overline{u}_1{}^t e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_4) \tag{4.64}$$

$$l_3\overline{u}_2{}^t(e^{-\tilde{u}_3\theta_4^1} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = \overline{u}_2{}^t e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_1) \tag{4.67}$$

$$l_3\overline{u}_2{}^t(e^{-\tilde{u}_3\theta_4^2} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = \overline{u}_2{}^t e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_2) \tag{4.68}$$

$$l_3\overline{u}_2{}^t(e^{-\tilde{u}_3\theta_4^3} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = \overline{u}_2{}^t e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_3) \tag{4.69}$$

$$l_3\overline{u}_2{}^t(e^{-\tilde{u}_3\theta_4^4} - e^{-\tilde{u}_3\theta_4^0})\overline{u}_1 = \overline{u}_2{}^t e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_0 - \overline{R}_4) \tag{4.70}$$

Using a proper numerical method equations (4.53) to (4.70) can be solved for $(x, y, z, \delta, l_1, l_3, \theta_1^0, \theta_1^1, \theta_1^2, \theta_1^3, \theta_1^4, \theta_4^0, \theta_4^1, \theta_4^2, \theta_4^3, \theta_4^4)$. Then choosing s_2^0 and s_3^0 arbitrarily $s_2^1, s_2^2, s_2^3, s_2^4, s_3^1, s_3^2, s_3^3, s_3^4$ can be determined as follows ,

Multiplying equations (4.40) to (4.43) and (4.49) to (4.52) by $\overline{u}_3{}^t$ equations below are derived ,

$$s_2^1 = s_2^0 + \overline{u}_3{}^t e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_1 - \overline{R}_0) \tag{4.71}$$

$$s_2^2 = s_2^0 + \overline{u}_3{}^t e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_2 - \overline{R}_0) \tag{4.72}$$

$$s_2^3 = s_2^0 + \overline{u}_3{}^t e^{-\tilde{u}_3 z}e^{-\tilde{u}_2 y}e^{-\tilde{u}_1 x}(\overline{R}_3 - \overline{R}_0) \tag{4.73}$$

$$s_2^4 = s_2^0 + \overline{u}_3{}^t e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_4 - \overline{R}_0) \tag{4.74}$$

$$s_3^1 = s_3^0 + \overline{u}_3{}^t e^{-\widetilde{u}_1 \delta} e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_0 - \overline{R}_1) \tag{4.75}$$

$$s_3^2 = s_3^0 + \overline{u}_3{}^t e^{-\widetilde{u}_1 \delta} e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_0 - \overline{R}_2) \tag{4.76}$$

$$s_3^3 = s_3^0 + \overline{u}_3{}^t e^{-\widetilde{u}_1 \delta} e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_0 - \overline{R}_3) \tag{4.77}$$

$$s_3^4 = s_3^0 + \overline{u}_3{}^t e^{-\widetilde{u}_1 \delta} e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{R}_0 - \overline{R}_4) \tag{4.78}$$

Now $\overline{r}_1$ and $\overline{r}_2$ can be found from equations (4.35) and (4.44) respectively then adding equations (4.31) and (4.32) the following equation is resulted ,

$$-ae^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 z} \overline{u}_1 - be^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 z} e^{\widetilde{u}_1 \delta} \overline{u}_3 - ce^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} \overline{u}_3 + e^{\widetilde{u}_1 x} e^{\widetilde{u}_2 y} e^{\widetilde{u}_3 z} e^{\widetilde{u}_1 \delta} (\overline{V}^{(2)} + \overline{W}^{(2)}) = \overline{r}_1 + \overline{r}_2 \tag{4.79}$$

but from equation (4.30) it can be seen that ,

$$\overline{V}^{(2)} + \overline{W}^{(2)} = \overline{U}^{(2)} = \begin{bmatrix} l_2 \cos\gamma \\ l_2 \sin\gamma \\ 0 \end{bmatrix}$$

Hence equation (4.79) can be written in the following form ,

$$-a\overline{u}_1 - be^{\widetilde{u}_1 \delta} \overline{u}_3 - c\overline{u}_3 = e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x} (\overline{r}_1 + \overline{r}_2) - \overline{U}^{(2)} \tag{4.80}$$

Thus choosing l_2 and γ arbitrarily , a, b and c are determined from equation (4.80) as follows ,

$$a = -\overline{u}_1^{\,t}[e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x}(\overline{r}_1 + \overline{r}_2) - e^{\widetilde{u}_1 \delta}\overline{U}^{(2)}] \tag{4.81}$$

$$b = \frac{\overline{u}_2^{\,t}}{\sin\delta}[e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x}(\overline{r}_1 + \overline{r}_2) - e^{\widetilde{u}_1 \delta}\overline{U}^{(2)}] \tag{4.82}$$

$$c = -\overline{u}_3^{\,t}[e^{-\widetilde{u}_3 z} e^{-\widetilde{u}_2 y} e^{-\widetilde{u}_1 x}(\overline{r}_1 + \overline{r}_2) - e^{\widetilde{u}_1 \delta}\overline{U}^{(2)}] - b\cos\delta \tag{4.83}$$

Now choosing $\overline{V}^{(2)}$ arbitrarily $\overline{W}^{(2)}$ can be determined from equation (4.30) .

Example (4.5) : Synthesize an RCCR linkage whose path tracer point passes through the following precision points ,

$$P_0 = \begin{bmatrix} 50 \\ -20 \\ 15 \end{bmatrix} \;,\; P_1 = \begin{bmatrix} 35 \\ -4 \\ 0 \end{bmatrix} \;,\; P_2 = \begin{bmatrix} 40 \\ -20 \\ 20 \end{bmatrix} \;,\; P_3 = \begin{bmatrix} 20 \\ 0 \\ 30 \end{bmatrix} \;,\; P_4 = \begin{bmatrix} 20 \\ 10 \\ 20 \end{bmatrix}$$

Applying the solution procedure explained in this chapter and using Mathcad the following results were determined (Running Time : 84 seconds). Detailed information has been presented in chapter 6 about the computer programs and trial and error steps and the difficulties which may arise while solving equations.

Inputs

$$R_0 = \begin{bmatrix} 50 \\ -20 \\ 15 \end{bmatrix} \quad , \quad R_1 = \begin{bmatrix} 35 \\ -4 \\ 0 \end{bmatrix} \quad , \quad R_2 = \begin{bmatrix} 40 \\ -20 \\ 20 \end{bmatrix} \quad , \quad R_3 = \begin{bmatrix} 20 \\ 0 \\ 30 \end{bmatrix} \quad , \quad R_4 = \begin{bmatrix} 20 \\ 10 \\ 20 \end{bmatrix}$$

Arbitrarily chosen values ,

$$s_2^0 = 40 \qquad s_3^0 = 15 \qquad \overline{V}^{(2)} = \begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix} \qquad l_2 = 30 \qquad \gamma = \tfrac{\pi}{3}$$

Initial Values

$$l_1 = 50 \quad , \quad l_3 = 20 \quad , \quad \theta_1^0 = 7\pi/6 \quad , \quad \theta_1^1 = 5\pi/3 \quad , \quad \theta_1^2 = 7\pi/4 \quad , \quad \theta_1^3 = 2\pi$$

$$\theta_1^4 = 7\pi/3 \quad , \quad \theta_7^0 = \pi/6 \quad , \quad \theta_7^1 = \pi \quad , \quad \theta_7^2 = \pi/2 \quad , \quad \theta_7^3 = 2\pi/3 \quad , \quad \theta_7^4 = 5\pi/6$$

$$x = \pi/4 \quad , \quad y = \pi \quad , \quad z = \pi/6 \quad , \quad \delta = \pi/3$$

Imposed Conditions

$$-2 \prec \theta_7^0 - \theta_7^1 \prec 2 \quad , \quad -2 \prec \theta_7^0 - \theta_7^2 \prec 2 \quad , \quad -2 \prec \theta_7^0 - \theta_7^3 \prec 2 \quad , \quad -2 \prec \theta_7^0 - \theta_7^4 \prec 2$$

$$\delta \prec 0$$

Outputs

$l_1 = 13.18833 \quad l_2 = 30 \quad l_3 = 14.42179 \quad l_4 = 36.05551 \quad l_5 = 36.05551$

$\theta_1^0 = 2.2313 \quad \theta_1^1 = 5.78843 \quad \theta_1^2 = 2.93902 \quad \theta_1^3 = 4.01633 \quad \theta_1^4 = 5.14723$

$\theta_4^0 = 1.18097 \quad \theta_4^1 = 2.97302 \quad \theta_4^2 = 0.77451 \quad \theta_4^3 = 1.15344 \quad \theta_4^4 = 2.17274$

$s_2^0 = 40 \quad s_2^1 = 33.68528 \quad s_2^2 = 33.56116 \quad s_2^3 = 6.78575 \quad s_2^4 = 6.26432$

$s_3^0 = 15 \quad s_3^1 = 29.09742 \quad s_3^2 = 24.54516 \quad s_3^3 = 54.04923 \quad s_3^4 = 55.4552$

$a = 12.38945 \quad b = 55.56276 \quad c = -1.19062 \quad \overline{V}^{(2)} = \begin{bmatrix} 30 \\ 0 \\ 20 \end{bmatrix} \quad \overline{W}^{(2)} = \begin{bmatrix} -15 \\ 25.98076 \\ -20 \end{bmatrix}$

$\delta = -0.54975$

Now it should be checked that if the path tracer point passes through the precision points continuously or not. For this purpose the output angle – input angle diagram of the RCCR linkage should be drawn as shown below,

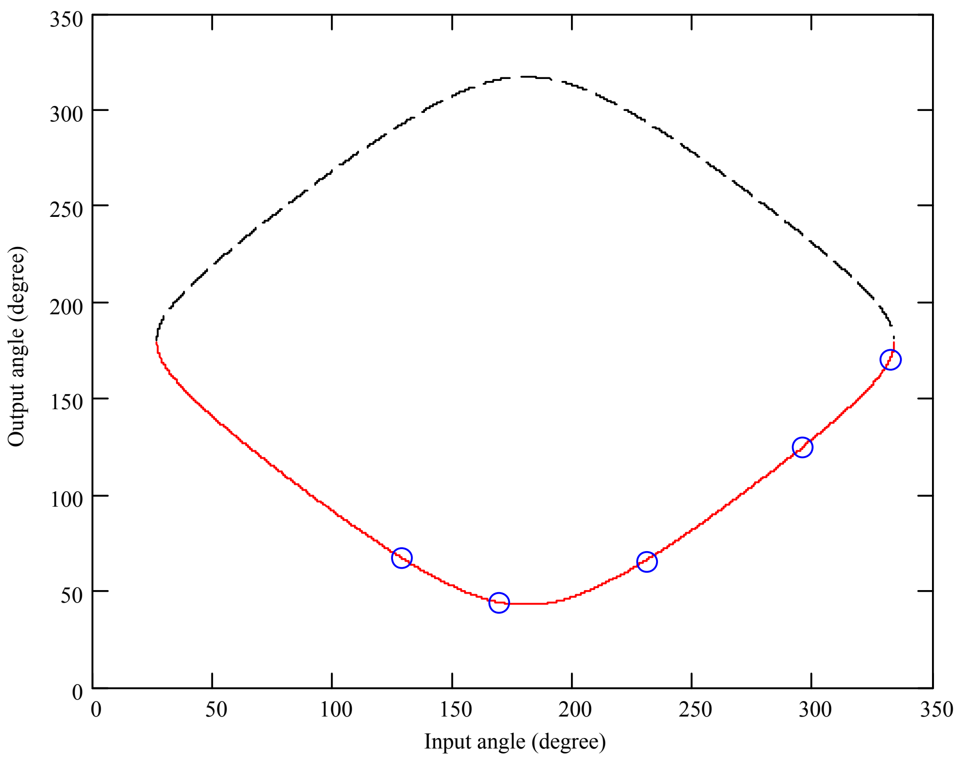


Figure (4.10) : The circles indicate the input and output angles at which the path tracer point passes through the precision points. The dashed curve displays the other loop closure curve.

As illustrated in the Figure (4.10) since the input and output angles at which the path tracer point passes through the precision points all lie on a continuous curve ,it can be deduced that the path tracer point will pass through the precision points continuously.

CHAPTER FIVE

MULTILOOP SPATIAL LINKAGES

5.1) GENERAL

In many cases multiloop spatial linkages are preferred to single loop spatial linkages , because in comparison to single loop spatial linkages they have a higher capacity to carry loads and besides that since the number of joints in multiloop spatial linkages is greater than that of the single loop spatial linkages , in the synthesis cases where a high number of points or positions have been prescribed , the multiloop spatial linkages are more preferable.

In this chapter the general displacement equations of a two loop spatial linkage have been obtained and then a procedure has been presented for the motion generation synthesis of this linkage.

Consider the RSSR-SC linkage illustrated in Figure (5.1) . Replacing the spheric joints by equivalent combinations of revolute joints the RSSR-SC linkage is redrawn as shown in Figure (5.2) . Note that in this case there are two fixed coordinate systems attached to the fixed link and that the fixed link is called L_0, L_8 and L'_{10} . In other words L_0, L_8 and L'_{10} all address the fixed link .

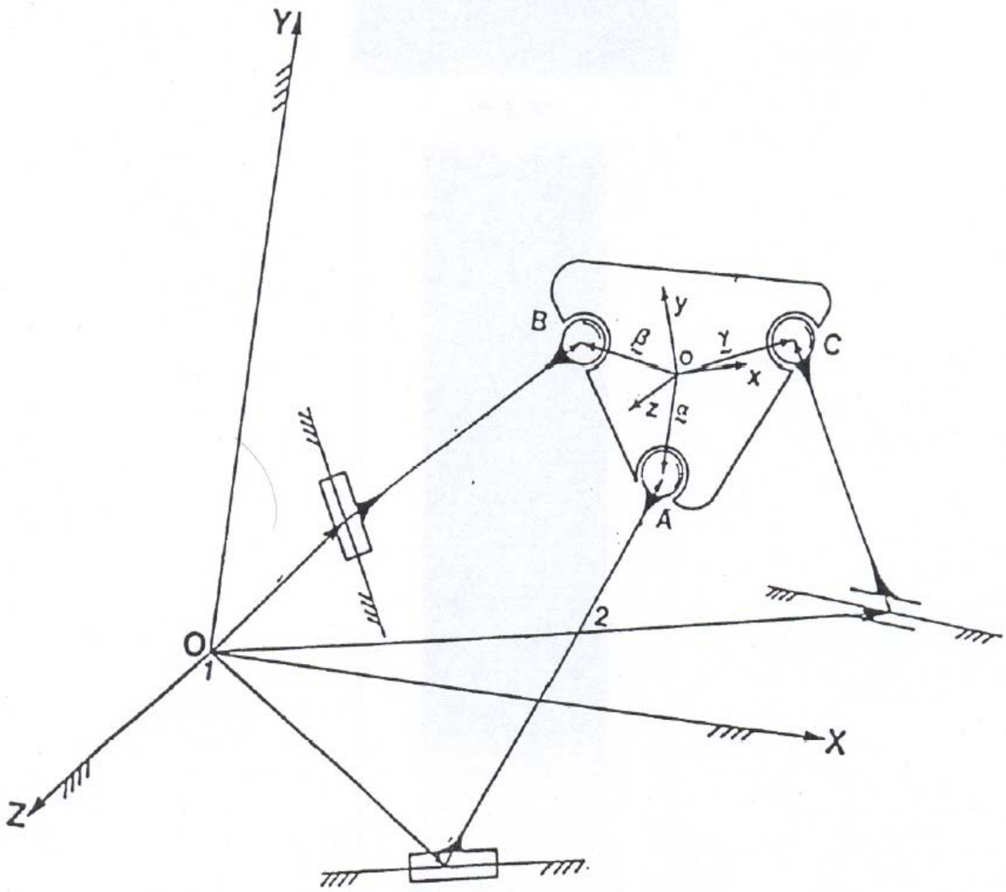


Figure (5.1) : An RSSR_SC linkage (ref.29)

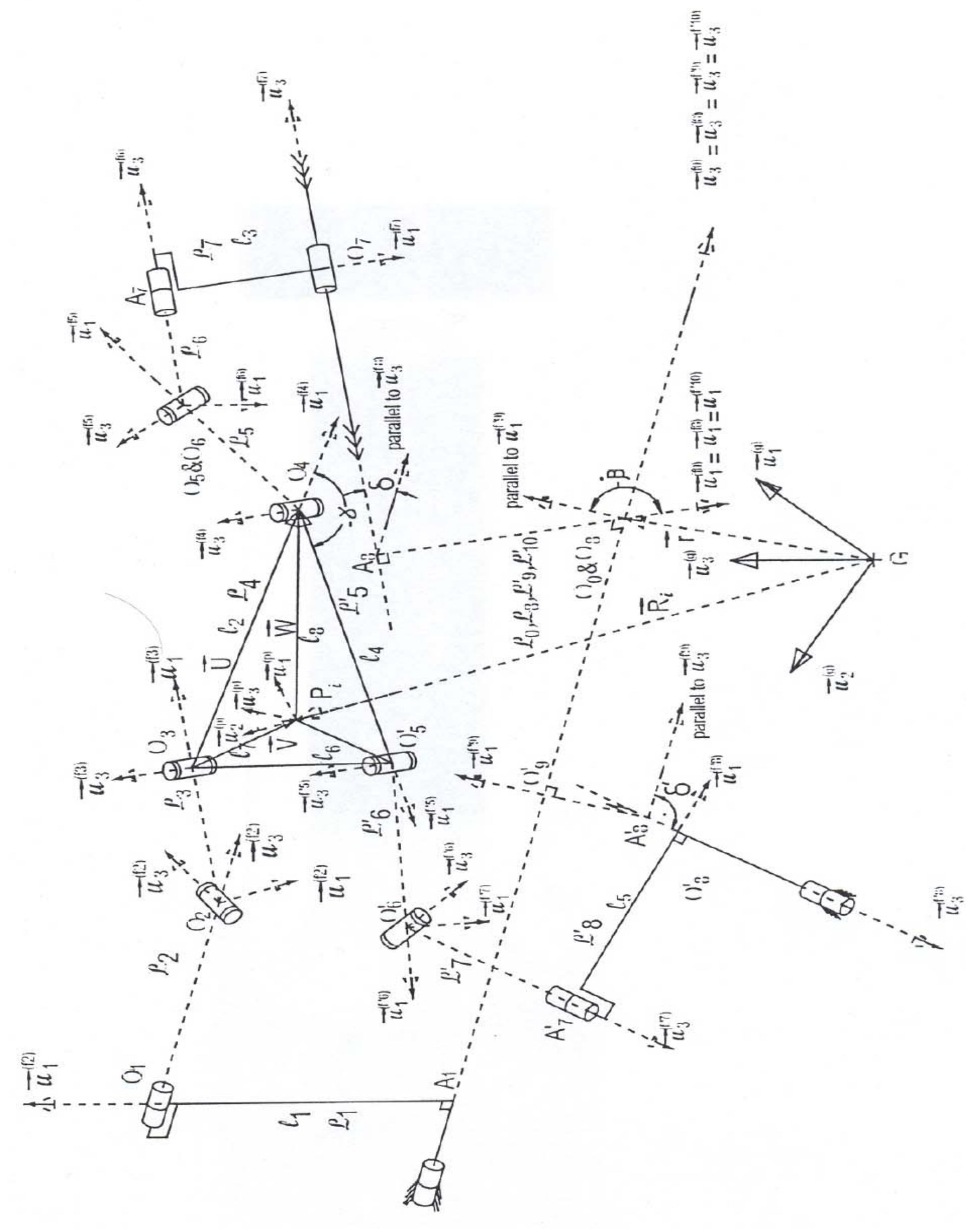


Figure (5.2) : In this figure the spheric joints of the RSSR-SC have been replaced by equivalent combinations of revolute joints. Note that links L_2, L_3, L_5, L_6, L_6' and L_7' are virtual and their lengths are equal to zero.

5.2) LOOP CLOSURE EQUATIONS

Regarding Figure (5.2) it is seen that two orientation loop closure equations and two position loop closure equations can be written for the RSSR_SC linkage as follows ,

a) ORIENTATION LOOP CLOSURE EQUATIONS

$$\hat{C}^{(f_0,f_1)}\hat{C}^{(f_1,f_2)}...\hat{C}^{(f_7,f_8)} = \hat{C}^{(f_0,f_8)} = \hat{I} \qquad (5.1)$$

$$\hat{C}^{(f_0,f_1)}\hat{C}^{(f_1,f_2)}...\hat{C}^{(f_4,f_5')}\hat{C}^{(f_5',f_6')}...\hat{C}^{(f_9',f_{10}')} = \hat{C}^{(f_0,f_{10}')} = \hat{I} \qquad (5.2)$$

where $\hat{C}^{(f_{k-1},f_k)}$ is the orientation matrix of frame (k) with respect to frame (k-1) .

The equation below is written according to Denavit-Hartenberg's convention,

$$\hat{C}^{(f_0,f_k)} = e^{\tilde{u}_3\theta_1}e^{\tilde{u}_1\alpha_1}e^{\tilde{u}_3\theta_2}e^{\tilde{u}_1\alpha_2}...e^{\tilde{u}_3\theta_k}e^{\tilde{u}_1\alpha_k} \qquad (5.3)$$

where θ_i is joint variable and α_i is link parameter.

b) POSITION LOOP CLOSURE EQUATIONS

$$\vec{r}_{o_0o_1} + \vec{r}_{o_1o_2} + \ldots + \vec{r}_{o_{n-1}o_n} = \vec{0} \qquad (5.4)$$

$$\vec{r}_{o_0o_1} + \vec{r}_{o_1o_2} + \ldots + \vec{r}_{o_4o'_5} + \vec{r}_{o'_5o'_6} + \ldots + \vec{r}_{o'_9o'_{10}} = \vec{0} \qquad (5.5)$$

where

$\vec{r}_{o_{k-1}o_k}$ is the vector drawn from the origin of frame (k-1) to the origin of frame (k) .

Considering Denavit-Hartenberg's convention equations (5.4) and (5.5) can be written in the following form ,

$$d_1\overline{u}_3 + a_1\hat{C}^{(f_0,f_1)}\overline{u}_1 + d_2\hat{C}^{(f_0,f_1)}\overline{u}_3 + a_2\hat{C}^{(f_1,f_2)}\overline{u}_1 + \ldots + d_n\hat{C}^{(f_0,f_{n-1})}\overline{u}_3 + a_n\hat{C}^{(f_0,f_n)}\overline{u}_1 = \overline{0} \qquad (5.6)$$

$$d_1\overline{u}_3 + a_1\hat{C}^{(f_0,f_1)}\overline{u}_1 + d_2\hat{C}^{(f_0,f_1)}\overline{u}_3 + a_2\hat{C}^{(f_1,f_2)}\overline{u}_1 + \ldots + d_5\hat{C}^{(f_0,f_4)}\overline{u}_3 + a'_5\hat{C}^{(f_0,f'_5)}\overline{u}_1 + \ldots + d'_{10}\hat{C}^{(f_0,f'_9)}\overline{u}_3$$
$$+ a'_{10}\hat{C}^{(f_0,f'_{10})}\overline{u}_1 = \overline{0} \qquad (5.7)$$

Regarding Figure (5.2) the link parameters and joint variables of the RSSR-SC linkage have been listed in tables (5.1) and (5.2) .

Table (5.1): Link parameters and joint variables of the first loop.

Link	θ	α	a	d
L_1	θ_1	0	l_1	-b
L_2	θ_2	$-\pi/2$	0	0
L_3	θ_3	$\pi/2$	0	0
L_4	θ_4	0	l_2	0
L_5	θ_5	$-\pi/2$	0	0
L_6	θ_6	$\pi/2$	0	0
L_7	θ_7	0	l_3	0
L_8	θ_8	δ	a	$-s_8$

Table (5.2) : Link parameters and joint variables of the second loop.

Link	θ	α	a	d
L_1	θ_1	0	l_1	-b
L_2	θ_2	$-\pi/2$	0	0
L_3	θ_3	$\pi/2$	0	0
L_4	θ_4	0	l_2	0
L'_5	$-\gamma$	0	l_4	0
L'_6	θ'_6	$-\pi/2$	0	0
L'_7	θ'_7	$\pi/2$	0	0
L'_8	θ'_8	0	l_5	0
L'_9	θ'_9	δ'	a'	$-c'$
L'_{10}	$-\beta$	0	0	b'

Considering equations (5.6) and (5.7) and according to the link parameters and joint variables listed in tables (5.1) and (5.2) it can be written that ,

$$-b\bar{u}_3 + l_1\hat{C}^{(f_0,f_1)}\bar{u}_1 + l_2\hat{C}^{(f_0,f_4)}\bar{u}_1 + l_3\hat{C}^{(f_0,f_7)}\bar{u}_1 - s_8\hat{C}^{(f_0,f_7)}\bar{u}_3 + a\hat{C}^{(f_0,f_8)}\bar{u}_1 = \bar{0} \tag{5.8}$$

$$-b\bar{u}_3 + l_1\hat{C}^{(f_0,f_1)}\bar{u}_1 + l_2\hat{C}^{(f_0,f_4)}\bar{u}_1 + l_4\hat{C}^{(f_0,f_5')}\bar{u}_1 + l_5\hat{C}^{(f_0,f_8')}\bar{u}_1 - c'\hat{C}^{(f_0,f_8')}\bar{u}_3 + a'\hat{C}^{(f_0,f_9')}\bar{u}_1$$
$$+ b'\hat{C}^{(f_0,f_{10}')}\bar{u}_3 = \bar{0} \tag{5.9}$$

Considering table (5.1) and according to equation (5.3) the following equations are obtained,

$$\hat{C}^{(f_0,f_1)} = e^{\tilde{u}_3\theta_1}$$

$$\hat{C}^{(f_0,f_4)} = e^{\tilde{u}_3(\theta_1+\theta_2)}e^{\tilde{u}_2\theta_3}e^{\tilde{u}_3\theta_4}$$

According to equation (5.1) equations below are written ,

$$\hat{C}^{(f_0,f_7)}\hat{C}^{(f_7,f_8)} = \hat{C}^{(f_0,f_8)} = \hat{I} \Rightarrow \hat{C}^{(f_0,f_7)} = [\hat{C}^{(f_7,f_8)}]^{-1} = [e^{\tilde{u}_3\theta_8}e^{\tilde{u}_1\delta}]^{-1} = e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3\theta_8}$$

$$\hat{C}^{(f_0,f_8)} = \hat{I}$$

Now regarding table (5.2) and according to equations (5.2) and (5.3) the following equations are derived ,

$$\hat{C}^{(f_0,f_5')} = \hat{C}^{(f_0,f_4)}\hat{C}^{(f_4,f_5')} = e^{\tilde{u}_3(\theta_1+\theta_2)}e^{\tilde{u}_2\theta_3}e^{\tilde{u}_3(\theta_4-\gamma)}$$

$$\hat{C}^{(f_0,f_8')}\hat{C}^{(f_8',f_{10}')} = \hat{C}^{(f_0,f_{10}')} = \hat{I} \Rightarrow \hat{C}^{(f_0,f_8')} = [\hat{C}^{(f_8',f_{10}')}]^{-1} = [e^{\tilde{u}_3\theta_9'}e^{\tilde{u}_1\delta'}e^{-\tilde{u}_3\beta}]^{-1} = e^{\tilde{u}_3\beta}e^{-\tilde{u}_1\delta'}e^{-\tilde{u}_3\theta_9'}$$

$$\hat{C}^{(f_0,f_9')}\hat{C}^{(f_9',f_{10}')} = \hat{C}^{(f_0,f_{10}')} = \hat{I} \Rightarrow \hat{C}^{(f_0,f_9')} = [\hat{C}^{(f_9',f_{10}')}]^{-1} = [e^{-\tilde{u}_3\beta}]^{-1} = e^{\tilde{u}_3\beta}$$

$$\hat{C}^{(f_0,f_{10}')} = \hat{I}$$

Thus equations (5.8) and (5.9) can be written in the form below ,

$$-b\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1}\overline{u}_1 + l_2 e^{\tilde{u}_3(\theta_1+\theta_2)} e^{\tilde{u}_2\theta_3} e^{\tilde{u}_3\theta_4}\overline{u}_1 + l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8}\overline{u}_1 - s_8 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8}\overline{u}_3 + a\overline{u}_1 = \overline{0} \qquad (5.10)$$

$$-b\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1}\overline{u}_1 + l_2 e^{\tilde{u}_3(\theta_1+\theta_2)} e^{\tilde{u}_2\theta_3} e^{\tilde{u}_3\theta_4}\overline{u}_1 + l_4 e^{\tilde{u}_3(\theta_1+\theta_2)} e^{\tilde{u}_2\theta_3} e^{\tilde{u}_3(\theta_4-\gamma)}\overline{u}_1 + l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'}\overline{u}_1$$
$$-c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'}\overline{u}_3 + a' e^{\tilde{u}_3\beta}\overline{u}_1 + b'\overline{u}_3 = \overline{0} \qquad (5.11)$$

5.3) GENERAL DISPLACEMENT EQUATIONS

From equations (5.10) and (5.11) the following equations are obtained ,

$$-l_2 e^{\tilde{u}_3(\theta_1+\theta_2)} e^{\tilde{u}_2\theta_3} e^{\tilde{u}_3\theta_4}\overline{u}_1 = a\overline{u}_1 - b\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1}\overline{u}_1 + l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8}\overline{u}_1 - s_8 e^{-\tilde{u}_1\delta}\overline{u}_3 \qquad (5.12)$$

$$e^{\tilde{u}_3(\theta_1+\theta_2)} e^{\tilde{u}_2\theta_3} e^{\tilde{u}_3\theta_4}[-l_2\overline{u}_1 - l_4 e^{-u_3\gamma}\overline{u}_1] = a' e^{\tilde{u}_3\beta}\overline{u}_1 + (b'-b)\overline{u}_3 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'}\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1}\overline{u}_1$$
$$+ l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'}\overline{u}_1 \qquad (5.13)$$

and subtracting equation (5.10) from equation (5.11) equations below are derived,

$$l_4 e^{\tilde{u}_3(\theta_1+\theta_2)} e^{\tilde{u}_2\theta_3} e^{\tilde{u}_3(\theta_4-\gamma)}\overline{u}_1 + l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'}\overline{u}_1 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'}\overline{u}_3 + a' e^{\tilde{u}_3\beta}\overline{u}_1 + b'\overline{u}_3 - l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8}\overline{u}_1$$
$$+ s_8 e^{-\tilde{u}_1\delta}\overline{u}_3 - a\overline{u}_1 = \overline{0}$$

or

$$-l_4 e^{\tilde{u}_3(\theta_1+\theta_2)} e^{\tilde{u}_2\theta_3} e^{\tilde{u}_3(\theta_4-\gamma)}\overline{u}_1 = l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'}\overline{u}_1 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'}\overline{u}_3 + a' e^{\tilde{u}_3\beta}\overline{u}_1 + b'\overline{u}_3$$
$$- l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8}\overline{u}_1 + s_8 e^{-\tilde{u}_1\delta}\overline{u}_3 - a\overline{u}_1 \qquad (5.14)$$

Taking the transposes of equations (5.12) to (5.14) the following equations are obtained ,

$$-l_2\overline{u}_1{}^t e^{-\tilde{u}_3\theta_4} e^{-\tilde{u}_2\theta_3} e^{-\tilde{u}_3(\theta_1+\theta_2)} = [a\overline{u}_1 - b\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1}\overline{u}_1 + l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8}\overline{u}_1 - s_8 e^{-\tilde{u}_1\delta}\overline{u}_3]^t \qquad (5.15)$$

$$-[l_2\overline{u}_1 + l_4 e^{-\tilde{u}_3\gamma}\overline{u}_1]^t e^{-\tilde{u}_3(\theta_1+\theta_2)} e^{-\tilde{u}_2\theta_3} e^{-\tilde{u}_3\theta_4} = [a' e^{\tilde{u}_3\beta}\overline{u}_1 + (b'-b)\overline{u}_3 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'}\overline{u}_3 + l_1 e^{\tilde{u}_3\theta_1}\overline{u}_1$$

$$+ l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'}\overline{u}_1]^t \qquad (5.16)$$

$$-\bar{u}_1^{\,t} l_4 e^{-\tilde{u}_3(\theta_1+\theta_2)} e^{-\tilde{u}_2\theta_3} e^{-\tilde{u}_3(\theta_4-\gamma)} = [l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'} \bar{u}_1 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} \bar{u}_3 + a' e^{\tilde{u}_3\beta} \bar{u}_1$$
$$+ b'\bar{u}_3 - l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8} \bar{u}_1 + s_8 e^{-\tilde{u}_1\delta} \bar{u}_3 - a\bar{u}_1]^t \tag{5.17}$$

Multiplying equations (5.12) to (5.14) by equations (5.15) to (5.17) respectively the following equations are derived ,

$$l_2^{\,2} = [a\bar{u}_1 - b\bar{u}_3 + l_1 e^{\tilde{u}_3\theta_1} \bar{u}_1 + l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8} \bar{u}_1 - s_8 e^{-\tilde{u}_1\delta} \bar{u}_3]^t [a\bar{u}_1 - b\bar{u}_3 + l_1 e^{\tilde{u}_3\theta_1} \bar{u}_1 + l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8} \bar{u}_1$$
$$- s_8 e^{-\tilde{u}_1\delta} \bar{u}_3] \tag{5.18}$$

$$(l_2 + l_4\cos\gamma)^2 + l_4^{\,2}\sin^2\gamma = [a' e^{\tilde{u}_3\beta} \bar{u}_1 + (b'-b)\bar{u}_3 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} \bar{u}_3 + l_1 e^{\tilde{u}_3\theta_1} \bar{u}_1$$
$$+ l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'} \bar{u}_1]^t \, [a' e^{\tilde{u}_3\beta} \bar{u}_1 + (b'-b)\bar{u}_3 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} \bar{u}_3 + l_1 e^{\tilde{u}_3\theta_1} \bar{u}_1$$
$$+ l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'} \bar{u}_1] \tag{5.19}$$

$$l_4^{\,2} = [l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'} \bar{u}_1 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} \bar{u}_3 + a' e^{\tilde{u}_3\beta} \bar{u}_1 + b'\bar{u}_3 - l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8} \bar{u}_1 + s_8 e^{-\tilde{u}_1\delta} \bar{u}_3 - a\bar{u}_1]^t$$
$$[l_5 e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} e^{-\tilde{u}_3\theta_9'} \bar{u}_1 - c' e^{\tilde{u}_3\beta} e^{-\tilde{u}_1\delta'} \bar{u}_3 + a' e^{\tilde{u}_3\beta} \bar{u}_1 + b'\bar{u}_3 - l_3 e^{-\tilde{u}_1\delta} e^{-\tilde{u}_3\theta_8} \bar{u}_1 + s_8 e^{-\tilde{u}_1\delta} \bar{u}_3 - a\bar{u}_1] \tag{5.20}$$

Equations (5.18) to (5.20) are the general displacement equations of the RSSR_SC linkage. Note that the only variables in equations above are , θ_1, θ_8, s_8 and θ_9' and the rest of the terms are parameters.

Example (5.1) : Considering the sliding motion of the cylindric joint of the RSSR-SC linkage shown in Figure (5.1) as the input of the linkage ,draw the input-output diagrams of the RSSR-SC linkage whose dimensions have been given as follows ,

$l_1 = 10$, $l_2 = 41$, $l_3 = 10$, $l_4 = 8$, $l_5 = 5$, $a = 19$, $a' = 18$, $b = 16$

$b' = 8$, $c' = 11$, $\beta = 0$, $\gamma = 4\pi/5$, $\delta = \pi/3$, $\delta' = \pi/6$

Let $\theta_1 = \psi(s)$, $\theta_8 = \Omega(s)$, $\theta_9' = \Phi(s)$. By solving equations (5.18) to (5.20) simultaneously the following diagrams were drawn ,

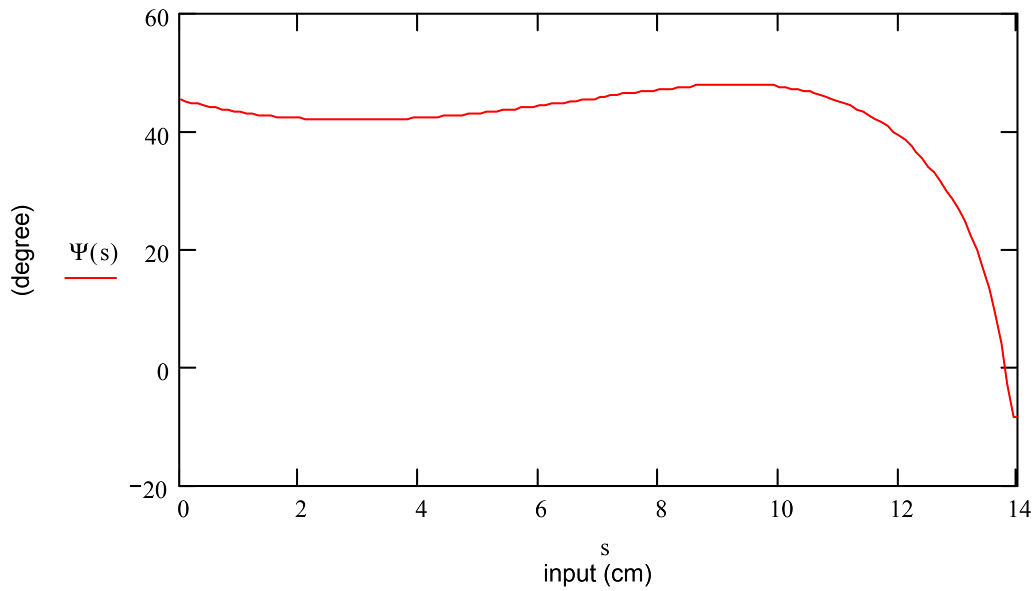


Figure (5.3) : The input angle–output angle diagram of the RSSR-SC linkage . Note that S has been considered as the input of the system.

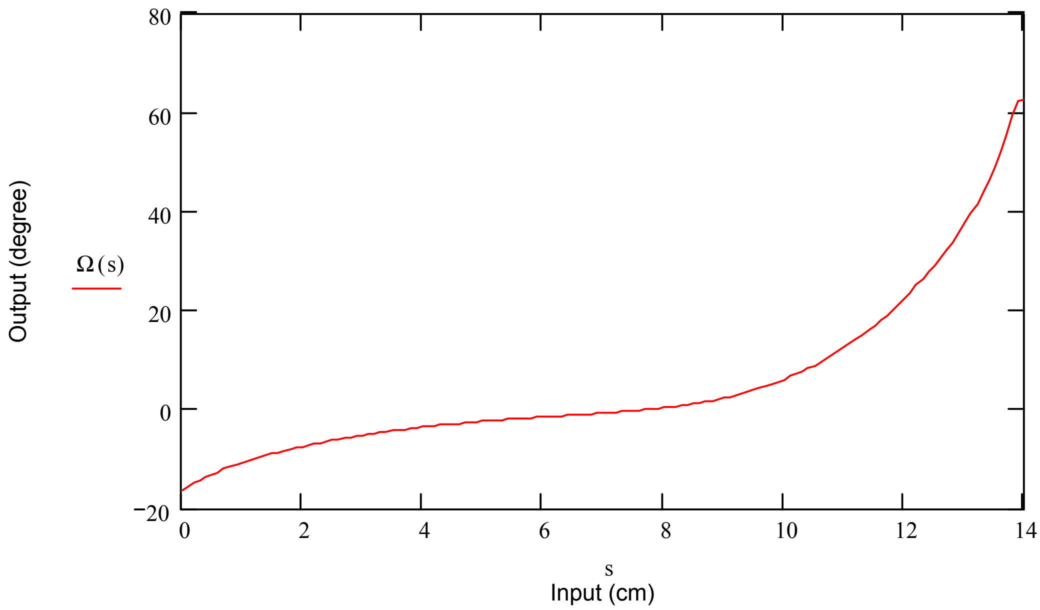


Figure (5.4) : The input angle–output angle diagram of the RSSR-SC linkage . Note that S has been considered as the input of the system.

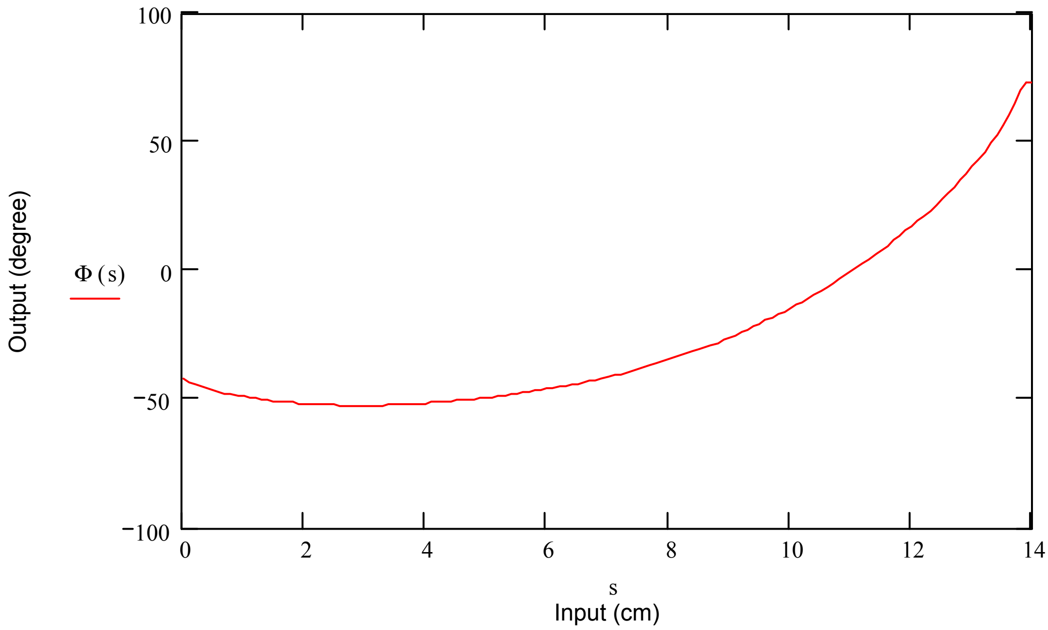


Figure (5.5) : The input angle–output angle diagrams of the RSSR-SC linkage . Note that S has been considered as the input of the system.

5.4) MOTION GENERATION SYNTHESIS OF THE RSSR-SC MECHANISM

Considering Figure (5.2) and tables (5.1) and (5.2) the following loop closure equations can be written,

$$\bar{r} - b\hat{C}^{(g,f_0)}\bar{u}_3 + l_1\hat{C}_i^{(g,f_1)}\bar{u}_1 + \hat{C}_i^{(g,f_4)}\bar{V}^{(f_4)} = \bar{R}_i \tag{5.21}$$

$$\hat{C}^{(g,f_4)}\overline{W}^{(f_4)} + l_3\hat{C}_i^{(g,f_7)}\bar{u}_1 - s_8\hat{C}_i^{(g,f_7)}\bar{u}_3 + a\hat{C}_i^{(g,f_8)}\bar{u}_1 - \bar{r} = -\bar{R}_i \tag{5.22}$$

$$\hat{C}_i^{(g,f_4)}\overline{W}^{(f_4)} + l_4\hat{C}_i^{(g,f_5')}\bar{u}_1 + l_5\hat{C}_i^{(g,f_8')}\bar{u}_1 - c'\hat{C}_i^{(g,f_8')}\bar{u}_3 + a'\hat{C}_i^{(g,f_9')}\bar{u}_1 + b'\hat{C}_i^{(g,f_9')}\bar{u}_3 - \bar{r} = -\bar{R}_i \tag{5.23}$$

where

$$\hat{C}^{(g,f_0)} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}$$

$$\hat{C}_i^{(g,f_1)} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i)}$$

$$\hat{C}_i^{(g,f_4)} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i+\theta_2^i)} e^{\tilde{u}_2 \theta_3^i} e^{\tilde{u}_3 \theta_4^i}$$

$$\hat{C}_i^{(g,f_7)} = \hat{C}^{(g,f_0)}\hat{C}_i^{(f_0,f_7)} = \hat{C}^{(g,f_0)}[\hat{C}_i^{(f_7,f_8)}]^{-1} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}[e^{\tilde{u}_3 \theta_8^i} e^{\tilde{u}_1 \delta}]^{-1} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_8^i}$$

$$\hat{C}_i^{(g,f_8)} = \hat{C}^{(g,f_0)}\hat{C}_i^{(f_0,f_8)} = \hat{C}^{(g,f_0)}\hat{I} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}$$

$$\hat{C}_i^{(g,f_5')} = \hat{C}^{(g,f_0)}\hat{C}_i^{(f_0,f_5')} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i+\theta_2^i)} e^{\tilde{u}_2 \theta_3^i} e^{\tilde{u}_3 (\theta_4^i-\gamma)}$$

$$\hat{C}_i^{(g,f_8')} = \hat{C}^{(g,f_0)}\hat{C}_i^{(f_0,f_8')} = \hat{C}^{(g,f_0)}[\hat{C}_i^{(f_8',f_{10}')}]^{-1} = e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z}[e^{\tilde{u}_3 \theta_9'} e^{\tilde{u}_1 \delta'} e^{-\tilde{u}_3 \beta}]^{-1}$$
$$= e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\beta)} e^{-\tilde{u}_1 \delta'} e^{-\tilde{u}_3 \theta_9'}$$

$$\hat{C}_i^{(g,f_9')} = \hat{C}^{(g,f_0)}\hat{C}_i^{(f_0,f_9')} = \hat{C}^{(g,f_0)}[\hat{C}_i^{(f_9',f_{10}')}]^{-1} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}[e^{-\tilde{u}_3\beta}]^{-1} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\beta)}$$

$$\hat{C}_i^{(g,f_{10}')} = \hat{C}^{(g,f_0)}\hat{C}_i^{(f_0,f_{10}')} = \hat{C}^{(g,f_0)}\hat{I} = e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}$$

and finally the following orientation loop closure equation is derived,

$$\hat{C}_i^{(g,p)} = \hat{C}_i^{(g,f_4)}\hat{C}^{(f_4,p)} \tag{5.24}$$

where $\hat{C}_i^{(g,p)}$ is the orientation matrix of frame (p) with respect to the global frame when frame (p) lies in the i'th position and it is a prescribed matrix. $\hat{C}^{(f_4,p)}$ is the orientation matrix of frame (p) with respect to the fourth frame and is a constant matrix. This orientation matrix can be defind by an arbitrary sequence of triple solutions . For example , $\hat{C}^{(f_4,p)} = e^{\tilde{u}_3\phi}e^{\tilde{u}_2\psi}e^{\tilde{u}_3\omega}$. Note that ϕ,ψ and ω are constant angles.

Hence equations (5.21) to (5.24) can be written in the form below ,

$$\overline{r} - be^{\tilde{u}_3 x}e^{\tilde{u}_2 y}\overline{u}_3 + l_1 e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\theta_1^i)}\overline{u}_1 + e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\theta_1^i+\theta_2^i)}e^{\tilde{u}_2\theta_3^i}e^{\tilde{u}_2\theta_4^i}\overline{V}^{(f_4)} = \overline{R}_i \tag{5.25}$$

$$e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\theta_1^i+\theta_2^i)}e^{\tilde{u}_2\theta_3^i}e^{\tilde{u}_3\theta_4^i}\overline{W}^{(f_4)} + l_3 e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}e^{-\tilde{u}_1\delta}e^{-\tilde{u}_3\theta_i^8}\overline{u}_1 - s_8^i e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}e^{-\tilde{u}_1\delta}\overline{u}_3$$
$$+ ae^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3 z}\overline{u}_1 - \overline{r} = -\overline{R}_i \tag{5.26}$$

$$e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\theta_1^i+\theta_2^i)}e^{\tilde{u}_2\theta_3^i}e^{\tilde{u}_3\theta_4^i}\overline{W}^{(f_4)} + l_4 e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\theta_1^i+\theta_2^i)}e^{\tilde{u}_2\theta_3^i}e^{\tilde{u}_3(\theta_4^i-\gamma)}\overline{u}_1 +$$
$$l_5 e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\beta)}e^{-\tilde{u}_1\delta'}e^{-\tilde{u}_3\theta_9'^i}\overline{u}_1 - c'e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\beta)}e^{-\tilde{u}_1\delta'}\overline{u}_3 + a'e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}e^{\tilde{u}_3(z+\beta)}\overline{u}_1 + b'e^{\tilde{u}_3 x}e^{\tilde{u}_2 y}\overline{u}_3$$
$$= -\overline{R}_i \tag{5.27}$$

Let ,

$$\overline{r} - b e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} \overline{u}_3 = \overline{r}_1$$

$$a e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} \overline{u}_1 - \overline{r} = \overline{r}_2$$

$$-c' e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\beta)} e^{-\tilde{u}_1 \delta'} \overline{u}_3 + a' e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\beta)} \overline{u}_1 + b' e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} \overline{u}_3 = \overline{r}_3$$

Note that now an additional equation is obtained ,

$$\overline{u}_2{}^t e^{-\tilde{u}_3 z} e^{-\tilde{u}_2 y} e^{-\tilde{u}_3 x} (\overline{r}_1 + \overline{r}_2) = 0 \tag{5.28}$$

Thus equations (5.25) to (5.27) can be written in the following form ,

$$\overline{r}_1 + l_1 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i)} \overline{u}_1 + e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i+\theta_2^i)} e^{\tilde{u}_2 \theta_3^i} e^{\tilde{u}_3 \theta_4^i} \overline{V}^{(f_4)} = \overline{R}_i \tag{5.29}$$

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i+\theta_2^i)} e^{\tilde{u}_2 \theta_3^i} e^{\tilde{u}_3 \theta_4^i} \overline{W}^{(f_4)} + l_3 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \delta} e^{-\tilde{u}_3 \theta_i^8} \overline{u}_1 - s_8^i e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 z} e^{-\tilde{u}_1 \delta} \overline{u}_3$$
$$+ \overline{r}_2 = -\overline{R}_i \tag{5.30}$$

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i+\theta_2^i)} e^{\tilde{u}_2 \theta_3^i} e^{\tilde{u}_3 \theta_4^i} \overline{W}^{(f_4)} + l_4 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i+\theta_2^i)} e^{\tilde{u}_2 \theta_3^i} e^{\tilde{u}_3 (\theta_4^i - \gamma)} \overline{u}_1 +$$
$$l_5 e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\beta)} e^{-\tilde{u}_1 \delta'} e^{-\tilde{u}_3 \theta_9'^i} \overline{u}_1 + \overline{r}_3 = -\overline{R}_i \tag{5.31}$$

$$e^{\tilde{u}_3 x} e^{\tilde{u}_2 y} e^{\tilde{u}_3 (z+\theta_1^i+\theta_2^i)} e^{\tilde{u}_2 \theta_3^i} e^{\tilde{u}_3 (\theta_4^i+\phi)} e^{\tilde{u}_2 \psi} e^{\tilde{u}_3 \omega} = \hat{C}_i^{(g,p)} \tag{5.32}$$

$$\overline{V}^{(f_4)} + \overline{W}^{(f_4)} = l_2 \overline{u}_1 \tag{5.33}$$

Now regarding equations (5.28) to (5.33) the following table is constructed , Table (5.3) : According to the table below, the floating link of the RSSR-SC linkage can pass through at most five prescribed positions.

number of prescribed positions	number of scalar equations	number of unknowns	number of free parameters
1	16	37 $(x, y, z, l_1, l_2, l_3, l_4, l_5$ $\theta_1^0, \theta_2^0, \theta_3^0, \theta_4^0, \theta_8^0, \theta_9'^0$ $s_8^0, \beta, \gamma, \delta, \delta', \phi, \psi, \omega$ $\bar{r}_1, \bar{r}_2, \bar{r}_3, \overline{V}^{(f_4)}, \overline{W}^{(f_4)})$	21
2	28	44 (above + θ_1^1, θ_2^1 $\theta_3^1, \theta_4^1, \theta_8^1, \theta_9'^1, s_8^1$)	16
3	40	51 (above + θ_1^2, θ_2^2 $\theta_3^2, \theta_4^2, \theta_8^2, \theta_9'^2, s_8^2$)	11
4	52	58 (above + θ_1^3, θ_2^3 $\theta_3^3, \theta_4^3, \theta_8^3, \theta_9'^3, s_8^3$)	6
5	64	65 (above + θ_1^4, θ_2^4 $\theta_3^4, \theta_4^4, \theta_8^4, \theta_9'^4, s_8^4$)	1

Example (5.2) : Synthesize an RSSR-SC linkage whose floating link guides a rigid body through the following positions ,

$$P_0 = \{R_0 = \begin{bmatrix} 30 \\ -10 \\ 50 \end{bmatrix}, \hat{C}_0 = e^{\tilde{u}_3\pi/6} e^{\tilde{u}_2\pi/3} e^{\tilde{u}_3\pi/6}\} \qquad P_1 = \{R_1 = \begin{bmatrix} 40 \\ -20 \\ 40 \end{bmatrix}, \hat{C}_1 = e^{\tilde{u}_3\pi/2} e^{\tilde{u}_2\pi/4} e^{\tilde{u}_3\pi/2}\}$$

$$P_2 = \{R_2 = \begin{bmatrix} 20 \\ 0 \\ 60 \end{bmatrix}, \hat{C}_2 = e^{\tilde{u}_3\pi/5} e^{\tilde{u}_2\pi/3} e^{\tilde{u}_3\pi/5}\}$$

Solving equations (5.28) to (5.33) by Mathcad the following results were determined (Running Time : 22 minutes). Detailed information has been presented in chapter 6 about the computer programs and trial and error steps and the difficulties which may arise while solving equations.

Inputs

$$R_0 = \begin{bmatrix} 30 \\ -10 \\ 50 \end{bmatrix} \quad , \quad R_1 = \begin{bmatrix} 40 \\ -20 \\ 40 \end{bmatrix} \quad , \quad R_2 = \begin{bmatrix} 20 \\ 0 \\ 60 \end{bmatrix} \quad , \quad \hat{C}_0 = e^{\tilde{u}_3\pi/6} e^{\tilde{u}_2\pi/3} e^{\tilde{u}_3\pi/6}$$

$$\hat{C}_1 = e^{\tilde{u}_3\pi/2} e^{\tilde{u}_2\pi/4} e^{\tilde{u}_3\pi/2} \quad , \quad \hat{C}_2 = e^{\tilde{u}_3\pi/5} e^{\tilde{u}_2\pi/3} e^{\tilde{u}_3\pi/5}$$

Initial Values

$l_1 = 72.76656$, $l_2 = 12.77803$, $l_3 = 12.5$, $l_4 = 10$, $l_5 = 23.02063$

$\theta_1^0 = 1.62006$, $\theta_1^1 = 1.38569$, $\theta_1^2 = 1.86238$, $\theta_2^0 = 0.28054$

$\theta_2^1 = -0.05636$, $\theta_2^2 = 0.63721$, $\theta_3^0 = -0.01675$, $\theta_3^1 = -0.02962$

$\theta_3^2 = 0.97325$, $\theta_4^0 = -0.61632$, $\theta_4^1 = 0.1463$, $\theta_4^2 = 0.7902$

$\theta_8^0 = 1.07288$, $\theta_8^1 = -0.22137$, $\theta_8^2 = -1.31715$, $\theta_9'^0 = -1.21868$

$s_8^0 = 2.52774$, $s_8^1 = 19.5692$, $s_8^2 = -14.48906$, $x = -0.11656$

$y = 0.73979$, $z = -0.54102$, $\phi = 0.39492$, $\psi = 0.25304$

$\omega = -0.33561$, $\gamma = 1.58053$, $\delta = 0.29146$, $\delta' = -0.03164$

$\beta = 0.19118$, $\overline{V}^{(f_4)} = \begin{bmatrix} -0.01137 \\ -6.51512 \\ -8.37842 \end{bmatrix}$, $\overline{W}^{(f_4)} = \begin{bmatrix} -1.13456 \\ 6.51512 \\ 8.37842 \end{bmatrix}$

Imposed Conditions

$10 \prec l_1 \prec 150$, $10 \prec l_2 \prec 150$, $10 \prec l_3 \prec 150$, $10 \prec l_4 \prec 150$, $10 \prec l_5 \prec 150$

$-20 \prec s_8^1 - s_8^0 \prec 20$, $-20 \prec s_8^2 - s_8^0 \prec 20$, $-0.4 \prec \theta_9'^1 - \theta_9'^0 \prec 0.4$

$-0.4 \prec \theta_9'^2 - \theta_9'^0 \prec 0.4$, $-0.4 \prec \theta_1^1 - \theta_1^0 \prec 0.4$, $-0.4 \prec \theta_1^2 - \theta_1^0 \prec 0.4$

$-0.4 \prec \theta_8^1 - \theta_8^0 \prec 0.4$, $-0.4 \prec \theta_8^2 - \theta_8^0 \prec 0.4$, $\pi / 2 \prec \gamma \prec 5\pi / 6$

Outputs

$l_1 = 43.7904$, $l_2 = 10.02003$, $l_3 = 55.50913$, $l_4 = 11.58281$, $l_5 = 75.16235$

$\theta_1^0 = 1.86537$, $\theta_1^1 = 1.46546$, $\theta_1^2 = 2.1773$, $s_8^0 = 4.627$, $s_8^1 = -0.11931$

$s_8^2 = -10.21372$, $\theta_8^0 = 1.19713$, $\theta_8^1 = 1.50914$, $\theta_8^2 = 0.79963$, $\theta_9'^0 = 1.06688$

$\theta_9'^1 = 1.29441$, $\theta_9'^2 = 0.86831$, $x = 0.63504$, $y = 1.30835$, $z = -1.02597$

$\phi = 1.36086$, $\psi = 0.75934$, $\omega = -1.55676$, $\delta = 0.27067$, $\delta' = 0.12176$

$\beta = -0.40544$, $\gamma = 2.61799$, $a = -6.66059$, $b = 13.39069$, $a' = -11.37168$

$b' = -223.35849$, $c' = -233.77873$, $\theta_2^0 = 0.90257$, $\theta_2^1 = -1.17766$, $\theta_2^2 = 0.25739$

$\theta_3^0 = -0.76975$, $\theta_3^1 = 0.80603$, $\theta_3^2 = 0.9435$, $\theta_4^0 = -0.95437$, $\theta_4^1 = 1.35643$

$\theta_4^2 = 0.5064$

$$\overline{V}^{(4)} = \begin{bmatrix} -0.16055 \\ -4.14199 \\ 2.70903 \end{bmatrix} , \quad \overline{W}^{(4)} = \begin{bmatrix} 10.18057 \\ 4.14199 \\ -2.70903 \end{bmatrix} , \quad \bar{r} = \begin{bmatrix} 51.04917 \\ -30.09171 \\ 84.71729 \end{bmatrix}$$

Figures (5.6) to (5.8) show that the rigid body is guided through the prescribed positions continuously .

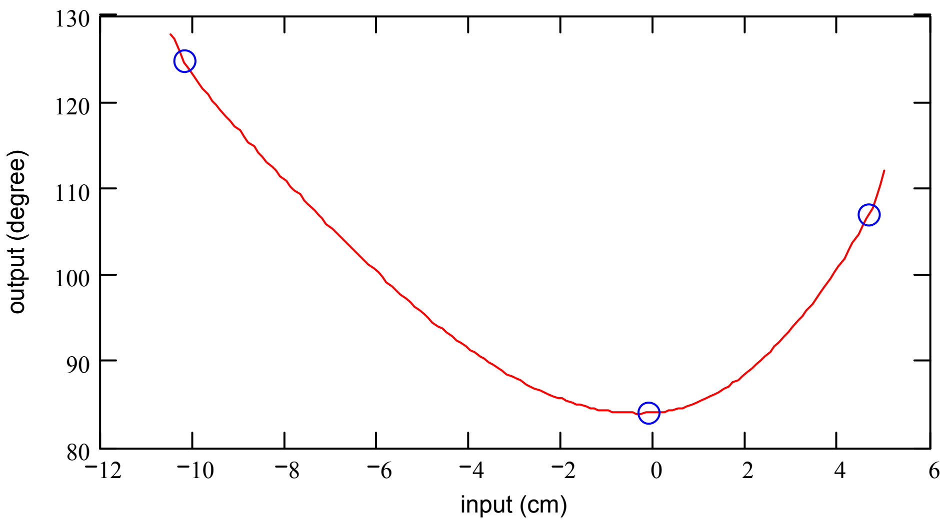


Figure (5.6) : The variations of $\psi(s)$ ($\theta_1(s)$) versus s . The circles illustrate the s and $\psi(s)$ at which the rigid body attached to the floating link lies in the prescribed positions.

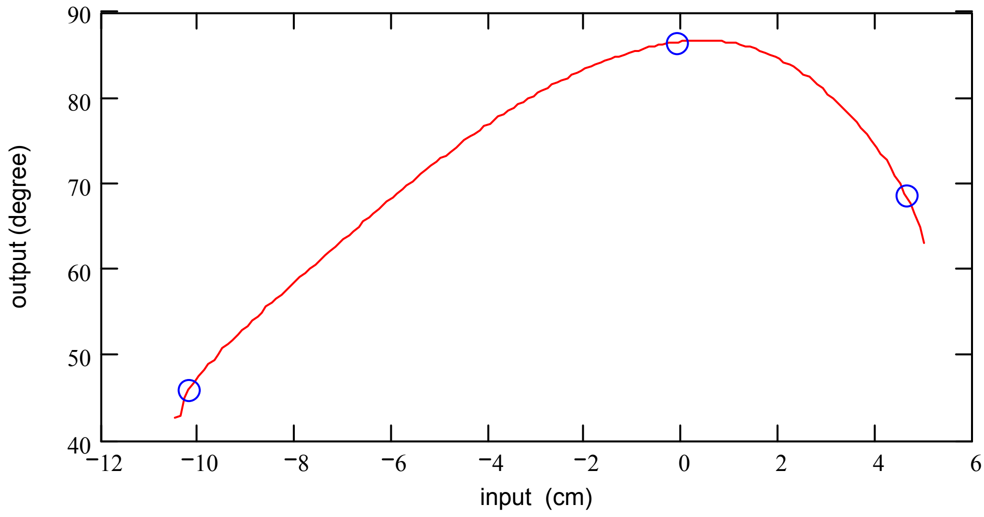


Figure (5.7) : The variations of $\Omega(s)$ $(\theta_8(s))$ versus s . The circles illustrate the s and $\Omega(s)$ at which the rigid body attached to the floating link lies in the prescribed positions

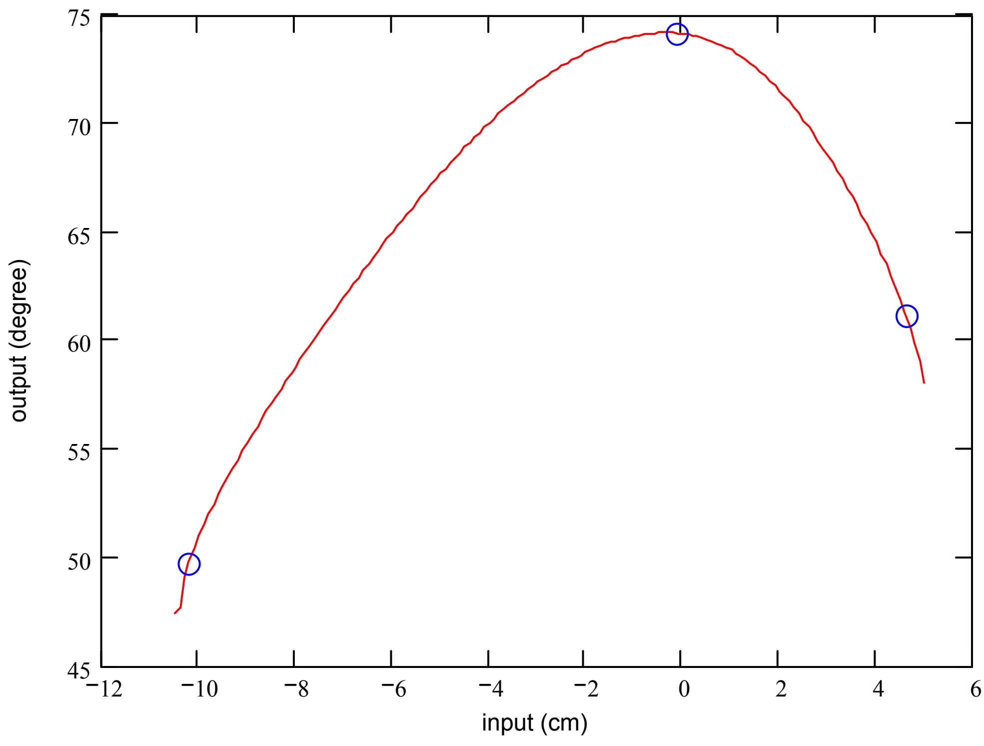


Figure (5.8) : The variations of $\Phi(s)$ $(\theta_9'(s))$ versus s . The circles illustrate the s and $\Phi(s)$ at which the rigid body attached to the floating link lies in the prescribed positions

CHAPTER SIX

SOME REMARKS ON THE COMPUTER PROGRAMS

6.1) GENERAL

In this thesis all nonlinear equations resulted from the synthesis procedures were solved by Mathcad's **Find** command . The auto-select feature of Mathcad automatically determines the kind of problem which is aimed to be solved and tries various appropriate solution algorithms to find a valid solution .The Conjugate Gradient, Quasi-Newton and Levenberg-Marguaret are three methods which are used by Mathcad for solving nonlinear equations. The accuracy of the results can be adjusted by changing the convergence tolerance of the program. The convergence tolerance of the programs used in this thesis was chosen to be 0.001. It was observed that with this convergence tolerance, the closed loop equations were satisfied with errors less than 0.01. Considering that in all programs the dimensions are in centimeters , one will vrify that a 0.01 cm error is an acceptable error. Note that a smaller convergence tolerance will considerably increase the running time of the programs.

The major difficulties which arised while solving numerical synthesis problems , can be classified as follows,

a) No solution case.
b) Case of unreasonable link lengths.
c) Branching case.

6.2) NO SOLUTION CASE

It is well known that in numerical methods ,the matter of finding a solution for a system of nonlinear equations highly depends on the selected initial values. The more the number of equations is the more difficult it would be to find a solution. Besides note that in the cases where no free parameter exists the existence of a solution can not be guarantied . However, leaving some free parameters –if there are any- to the computer and allowing the computer to select values for the free parameters, will increase the probabality of finding a solution.

As seen from table (3.2) no free parameter exists in the solution procedure of Example (3.1). That is why it was very difficult and time consuming to find a solution for this problem and the only thing the author could do was changing the initial values. However in other examples it was not that difficult to find a solution because some free parameters were present, and allowing the computer to select the values of these free parameters made it easy for the computer to find a solution.

Now one may ask " Is it reasonable to let the computer choose values for all free parameters while some of them may be needed to meet design conditions or for other reasons?" The answer is that, the program can be forced to meet our conditions by imposing constraints to the solution block of the computer program. For example by adding inequalities to the program it can be forced to find the link lengths in a suitable range or determine the input angles can be determined in a prescribed order.

6.3) CASE OF UNREASONABLE LINK LENGTHS

In many cases it was observed that the link lengths found by the program were not reasonable . For example, sometimes the lengths of links were determined as negative values or in some cases the length of a link was equal or very close to zero or the values of link lengths were too big. In order to resolve this problem some inequalities were imposed to the programs . For example the following inequalities were added to the solution block of the program which was written for Example (3.2),

$$10 \prec l_1 \prec 120 \quad , \quad 20 \prec l_2 \prec 120 \quad , \quad l_2 \succ l_3 \quad , \quad l_3 \succ l_1$$

Without the above inequalities , with the same initial values and with the same inputs the link lengths in Example (3.2) had been determined as follows,

$$l_1 = 2470.18 \quad , \quad l_2 = 504.12453 \quad , \quad l_3 = -173.6554$$

Note that adding inequalities to the program may result in no solution case. In such cases either the initial values or the inequalities should be modified.

6.4) BRANCHING CASE

As presented at the end of all numerical synthesis examples, the input angle-output angle curves of the synthesized mechanisms illustrate if the obtained dimensions result in branchings or not. According to the experience of the author, in most of the cases the determined link lengths and angles result in branchings. The following diagrams illustrate the branchings which took place in some of the numerical examples solved in this thesis.

A BRANCHING CASE OF EXAMPLE (3.1) :

Initial Values

$l_1 = 70$, $l_3 = 50$, $\theta_1^0 = 2$, $\theta_1^1 = 3$, $\theta_1^2 = 4.5$, $\theta_5^0 = 0$, $\theta_5^1 = 0.5$

$\theta_5^2 = 2$, $\theta_6^0 = 1$, $\theta_6^1 = 3.5$, $\theta_6^2 = 4$, $\theta_7^0 = 1$, $\theta_7^1 = 3$, $\theta_7^2 = 4.5$

No condition was imposed

Outputs

$l_1 = 108.45268$, $l_2 = 117.92025$, $l_3 = 49.49181$, $l_4 = 118.81177$

$l_5 = 42.20453$, $\theta_1^0 = -2.39481$, $\theta_1^1 = -1.72046$, $\theta_1^2 = 6.29182$

$\theta_5^0 = 0$, $\theta_5^1 = 9.04164$, $\theta_5^2 = 1.46564$, $\theta_6^0 = 2.35073$, $\theta_6^1 = 5.02261$

$\theta_6^2 = 3.28258$, $\theta_7^0 = -2.13781$, $\theta_7^1 = 9.02332$, $\theta_7^2 = 2.15583$

$$\overline{V}^{(4)} = \begin{bmatrix} 111.26244 \\ -41.67377 \\ 0.45227 \end{bmatrix} , \quad \overline{W}^{(4)} = \begin{bmatrix} 6.6578 \\ 41.67363 \\ -0.45217 \end{bmatrix}$$

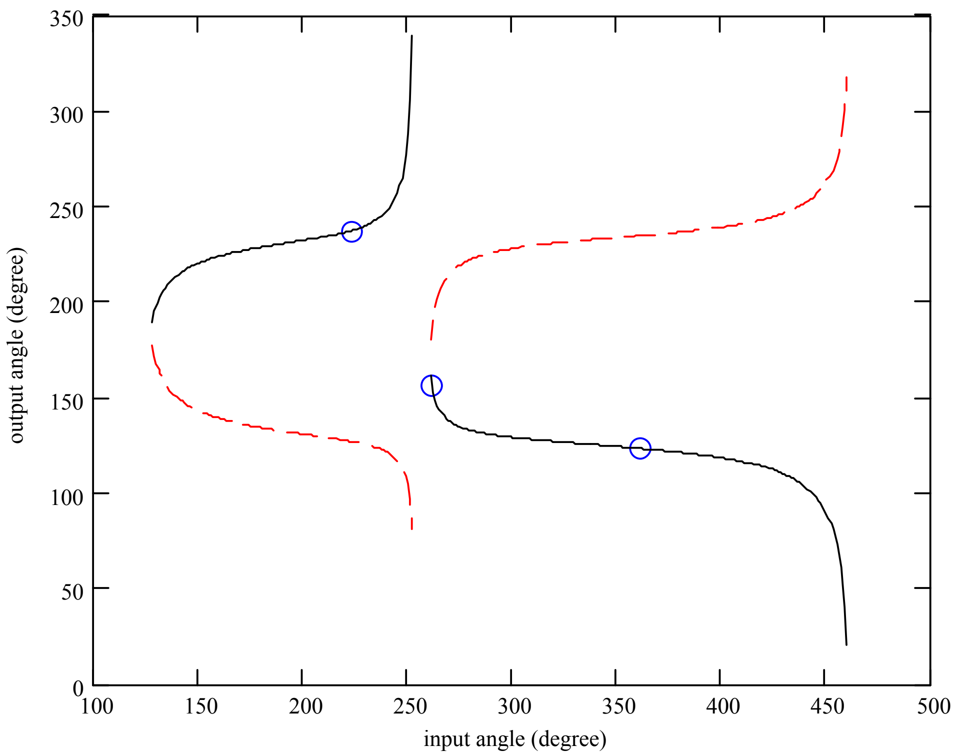


Figure (6.1) : The input angle- output angle diagram of the RSHR mechanism synthesized according to the inputs given in Example (3.1) and initial values presented on the last page. The circles illustrate the input and output angles at which the path tracer point passes through the precision points.

A BRANCHING CASE OF EXAMPLE (3.2) :

Initial Values

The same as presented in Example (3.2)

Imposed Conditions

$10 \prec l_1 \prec 120$, $20 \prec l_2 \prec 120$, $l_2 \succ l_3$, $l_3 \succ l_1$

$-0.7 \prec \theta_7^1 - \theta_7^0 \prec 0.7$, $-0.7 \prec \theta_7^2 - \theta_7^0 \prec 0.7$, $-0.7 \prec \theta_7^3 - \theta_7^0 \prec 0.7$

$-0.7 \prec \theta_7^4 - \theta_7^0 \prec 0.7$

Outputs

$l_1 = 21.79941$, $l_2 = 55.33151$, $l_3 = 55.33139$, $\theta_1^0 = 2.03393$

$\theta_1^1 = 7.86249$, $\theta_1^2 = 4.75337$, $\theta_1^3 = 2.46689$, $\theta_1^4 = 6.66991$

$\theta_5^0 = -0.33098$, $\theta_5^1 = -6.08571$, $\theta_5^2 = 4.8283$, $\theta_5^3 = 6.86444$

$\theta_5^4 = -0.81489$, $\theta_6^0 = 5.14063$, $\theta_6^1 = 5.1159$, $\theta_6^2 = 23.4037$

$\theta_6^3 = 5.15857$, $\theta_6^4 = 5.00675$, $\theta_7^0 = 1.55803$, $\theta_7^1 = 1.3082$

$\theta_7^2 = 2.25401$, $\theta_7^3 = 0.85804$, $\theta_7^4 = 2.25803$, $x = 4.189$

$y = 7.354$, $z = 6.759$, $\overline{V}^{(4)} = \begin{bmatrix} 102.74421 \\ 7.52747 \\ 78.1695 \end{bmatrix}$, $\overline{W}^{(4)} = \begin{bmatrix} -47.4127 \\ -7.52747 \\ -78.1695 \end{bmatrix}$

$\bar{r} = \begin{bmatrix} 98.69214 \\ 5.7064 \\ 42.98439 \end{bmatrix}$

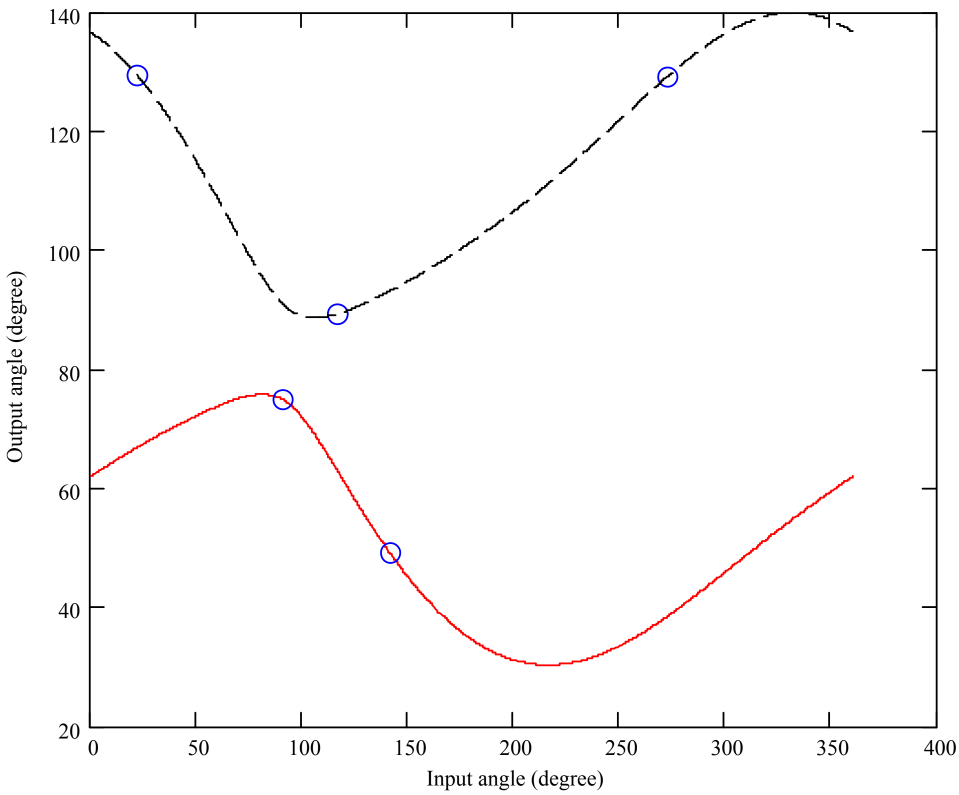


Figure (6.2) : The input angle- output angle diagram of the RSHR mechanism synthesized according to the inputs given in Example (3.2) and initial values presented on the last page. The circles illustrate the input and output angles at which the path tracer point passes through the precision points.

A BRANCHING CASE OF EXAMPLE (3.3) :

Initial Values

$$\theta_1^0 = 5\pi / 6$$

The rest of the initial values are the same as presented in Example (3.3)

Imposed Conditions

$10 \prec l_1 \prec 120$, $20 \prec l_2 \prec 120$, $l_2 \succ l_3$, $l_3 \succ l_1$

$-0.5 \prec \theta_7^1 - \theta_7^0 \prec 0.5$, $-0.5 \prec \theta_7^2 - \theta_7^0 \prec 0.5$, $-0.5 \prec \theta_7^3 - \theta_7^0 \prec 0.5$

$0 \prec \theta_1^0 \prec \theta_1^1 \prec \theta_1^2 \prec \theta_1^3 \prec 2\pi$

Outputs

$\theta_1^0 = 0$, $\theta_1^1 = 1.34653$, $\theta_1^2 = 2.35198$, $\theta_1^3 = 3.3745$, $\theta_5^0 = 1.9064$

$\theta_5^1 = 2.52747$, $\theta_5^2 = 5.33945$, $\theta_5^3 = 4.84228$, $\theta_6^0 = 2.98133$, $\theta_6^1 = 1.86137$

$\theta_6^2 = 4.79906$, $\theta_6^3 = 1.949$, $\theta_7^0 = 1.55958$, $\theta_7^1 = 1.35241$, $\theta_7^2 = 2.05462$

$\theta_7^3 = 2.05958$, $x = -2.776$, $y = 8.149$, $z = -0.14$, $\overline{V}^{(4)} = \begin{bmatrix} 59.43716 \\ -6.46413 \\ -10.14035 \end{bmatrix}$

$\overline{W}^{(4)} = \begin{bmatrix} 21.0334 \\ 6.46413 \\ 10.14035 \end{bmatrix}$, $\bar{r} = \begin{bmatrix} 2.86529 \\ -46.58887 \\ 9.67999 \end{bmatrix}$

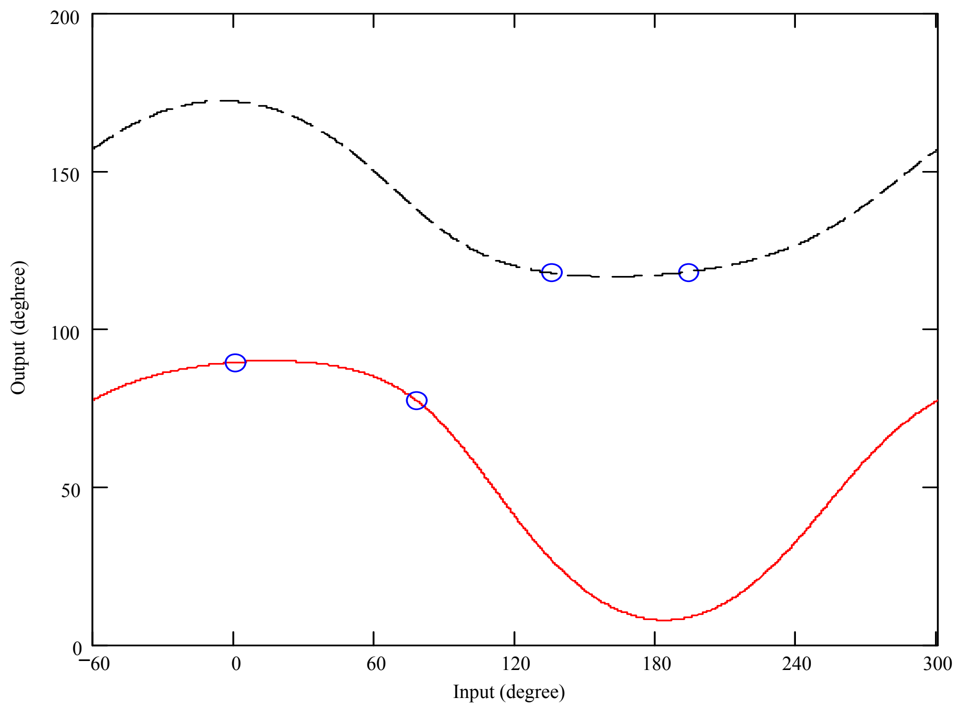
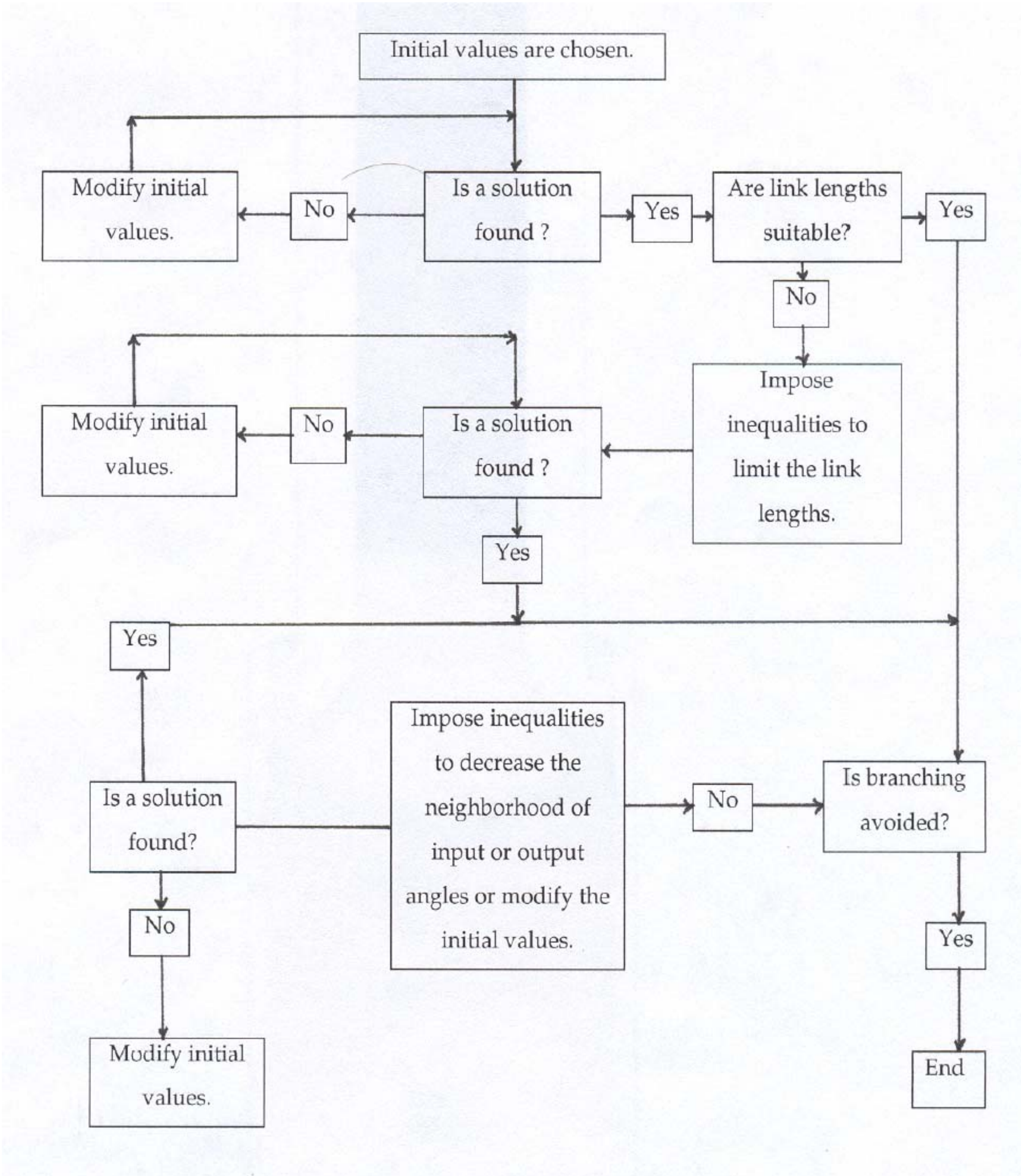


Figure (6.3) : The input angle- output angle diagram of the RSHR mechanism synthesized according to the inputs given in Example (3.3) and initial values presented on the last page. The circles illustrate the input and output angles at which the path tracer point passes through the precision points.

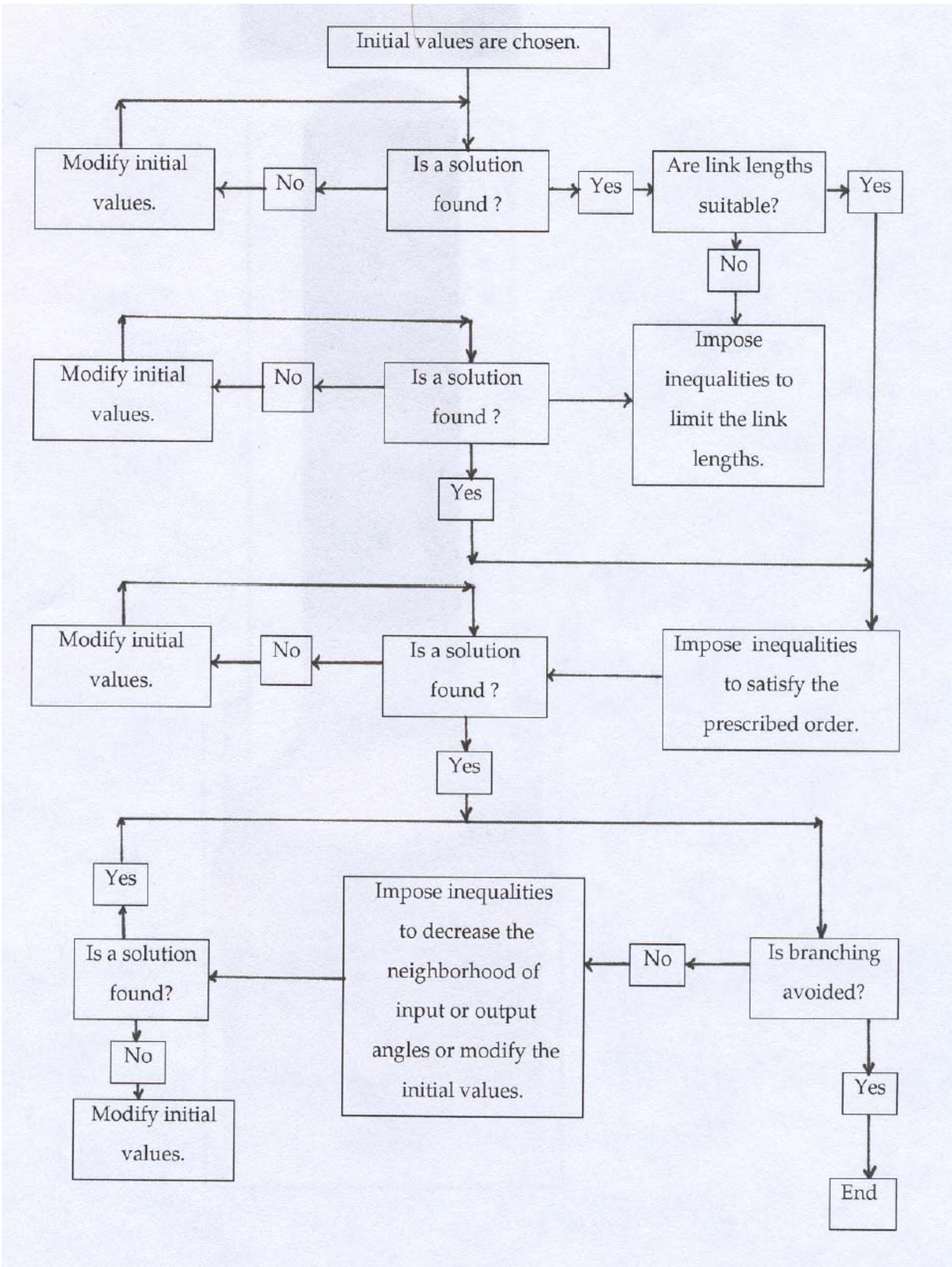
The problem of branching can be resolved by forcing the program to choose input or output angles- at which the path tracer point passes through the precision points -close to one another. This is done by adding inequalities in terms of input or output angles to the program. In the cases where free parameters are present the program can be forced to choose free parameters according to the inequalities which result in close input or output angles. For example from figure (6.1) and (6.2) it is observed that choosing output angles closer to each other may make all circles lie on a single branch. Thus decreasing the neighborhood of the output angles as presented in Examples (3.1) and (3.2) branching was avoided. However decreasing the neighborhood of output or input angles may lead to no solution case. In such cases the initial values should be modified.

6.5) TECHNIQUES USED IN COMPUTER PROGRAMS

The general algorithm used in computer programs of the numerical examples is as follows,



The algorithm used for Example (3.3) which requires a prescribed order for the precision points is as follows,



The algorithm used for Example (5.2) is different. Since the number of equations in Example (5.2) was more than other examples it was more difficult to find suitable initial values for this example. In order to determine proper initial values for the program, at the first step the equations were solved by **Minerr** command. The difference between **Minerr** and **Find** is that in the case of **Find** command, if the precision of the determined values is below the prescribed accuracy limit, the program will state that "No solution was found" . While in the case of **Minerr** command a solution will be found any way, however the error of the results may be great. In other words **Minerr** finds the results with the minimum error but this minimum error may still be unacceptable. After solving the equations by **Minerr** command the results were used as initial values and the equations were resolved by **Find** according to the new initial values. Then the same algorithm which was presented on page 123 was applied .

At the end of each program, the determined values were substituted into the equations to verify the accuracy of solutions. It was observed that all equations were satisfied with an error ratio less than 0.01.

CHAPTER SEVEN
CONCLUSION

Considering the general loop closure equations which were obtained based on Denavit-Hartenberg's convention and the algebra of exponential rotation matrices , it can be deduced that these loop closure equations and the path and motion generation synthesis methods presented in the thesis are sufficiently general so that they can be applied to the synthesis of any spatial linkage with lower kinematic pairs. Besides, regarding the presented examples which include path and motion generation synthesis of three different types of spatial mechanisms based on the synthesis methods presented in the thesis , one will verify the applicability of these methods.

Since the general loop closure equations obtained in chapter 3 ,have been written based on a systematic convention (Denavit-Hartenberg's convention) , the presented synthesis methods can be easily used in computer programs. While solving synthesis examples presented in the thesis the properties of the algebra of exponential rotation matrices have been used to simplify the loop closure equations as much as possible. These properties facilitated the simplification procedures and made it easy to work with matrix equations. In other words the algebra of exponential rotation matrices which was used to develope the kinematic synthesis methods presented in the thesis , enables the designer to efficiently simplify the loop closure equations.

Although the kinematic synthesis methods described in the present study are considerably general methods and cover all spatial mechanisms with lower kinematic pairs, still they can not be applied to the spatial mechanisms with higher kinematic pairs such as mechanisms including gears , cam and followers and etc. More work is required to modify the loop closure equations and the synthesis methods so that they can be applied to all types of spatial mechanisms including those which consist of higher kinematic pairs.

Although the algebra of exponential rotation matrices which is an efficient and elegant mathematical tool for simplifying matrix equations resulted from the kinematic analysis and synthesis, has been used in the analysis of robot manipulators (ref.1,2,3,4,6) very little work exists on the application of this algebra to analysis or synthesis of spatial mechanisms. Applying this valuable algebra to more analysis and synthesis problems of spatial mechanisms , other mechanism designers can be encouraged to use this productive algebra.

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