STAR MODELS: AN APPLICATION TO TURKISH INFLATION AND EXCHANGE RATES

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DİLEM YILDIRIM

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Prof. Dr. Sencer Ayata
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Erol Çakmak
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Associate Prof. Dr. Nadir Öcal
Supervisor

Examining Committee Members

Associate Prof. Dr. Yılmaz Akdi (AU, STAT)

Associate Prof. Dr. Nadir Öcal (METU, ECON)

Assist. Prof. Dr. Elif Akbostancı (METU, ECON)

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : Dilem Yıldırım

Signature :

ABSTRACT

STAR MODELS: AN APPLICATION TO TURKISH INFLATION AND EXCHANGE RATES

YILDIRIM, Dilem

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The recent empirical literature has shown that the dynamic generating mechanism of macroeconomic variables can be asymmetric. Inspiring from these empirical results, this thesis uses a class of nonlinear models called smooth transition autoregressive models to investigate possible asymmetric dynamics in inflation and nominal exchange rate series of Turkey. Estimation results imply that variables under consideration contain strong nonlinearities and these can be modeled by STAR models.

Key Words: Smooth transition autoregressive (STAR) model, Inflation, Nominal exchange rate, Asymmetry, Nonlinearity.

ÖZ

YUMUŞAK GEÇİŞLİ OTOREGRESSİF MODELLER: TÜRKİYE ENFLASYONU VE DÖVİZ KURU ÜZERİNE BİR UYGULAMA

YILDIRIM, Dilem

Yüksek Lisans Tezi, İktisat Anabilim Dalı

Tez Yöneticisi: Doç. Dr. Nadir ÖCAL

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Son yıllarda gelişmekte olan ampirik yazın, makroekonomik değişkenlerin dinamiğini oluşturan mekanizmanın asimetrik olabileceğini göstermektedir. Bu ampirik bulgulardan esinlenerek, bu tez yumuşak geçişli otoregresif modelleri kullanarak Türkiye'nin enflasyon ve nominal döviz kurları verilerindeki olası asimetrik dinamiği araştırmaktadır. Tahmin sonuçları, analiz edilen değişkenlerin güçlü doğrusal olmayan yapılar içerdiğini ve bu yapıların yumuşak geçişli otoregresif (STAR) modellerle modellenebileceğini göstermektedir.

Anahtar Kelimeler: Yumuşak geçişli otoregresif (STAR) modeller, Enflasyon, Nominal döviz kuru, Asimetri, Doğrusalsızlık.

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CHAPTER I

INTRODUCTION

Modeling of business cycles is a key feature of macroeconomic time series analysis. Recent empirical literature has shown that the business cycle is asymmetric in the sense that the economy behaves differently during expansions and recessions. It has been established that downturns in the business cycle are sharper than recoveries in key macroeconomic variables. For example, output and employment fall more sharply than they rise. Following this feature, the unemployment rate is likely to rise sharply in recessions and than slowly decline to its long-run value in expansion periods, implying that dynamic adjustment of the unemployment rate depends on the phases of business cycle.

Cyclical asymmetry is actually a nonlinear phenomenon, therefore it can not be represented by linear models with symmetric error distributions because such models can only generate realizations with symmetric fluctuations and they are incapable of generating asymmetric cycles, which seems to be the characteristics of business cycle data.

Recently several nonlinear models have been suggested in the literature to capture observed asymmetries. A common feature of these models is that they assume the presence of different regimes, within which the time series under scrutiny can have different means, variances and (auto-) correlation structures. Threshold autoregressive (TAR) models, smooth transition autoregressive (STAR) models and Markov-switching regime models have been the most popular nonlinear models in applied literature. Threshold autoregressive model is a set of different linear AR models, changing according to the value of threshold variable(s) relative to fixed threshold(s) and indicating different regimes. Although the process is linear in each regime, the possibility of regime switching means that the entire process is nonlinear. In the smooth transition autoregressive (STAR) model, the fixed thresholds of a standard threshold autoregressive (TAR) model are replaced with a smooth transition function, which needs to be continuous and non-decreasing. The Markov-switching regime model imposes entirely stochastic breaks for the model while other two have exogenous breaks. In the Markov model, the inferences are drawn on the base of probabilistic estimates of the most likely state prevailing at each point of time during the observation period.

This study aims to provide a review of these three nonlinear models with emphasis being on the application of the smooth transition autoregressive models. The study concentrates on STAR models since they are flexible enough to allow several different types of dynamics that could be observed in macroeconomic time series. For example, a STAR model with a logistic transition function can describe an economy with its dynamic properties in expansion being different than that of contraction, whereas with an exponential transition function it represents an economy where the contractions and expansions have rather similar dynamic structures, but the middle ground can have different dynamics. Moreover, while the Hamilton type Markov switching models and threshold models imply that the economy can only be in two different regimes, namely recession and expansion. STAR models allow a continuum of regimes between these two extremes.

These models have been employed to analyze not only economic variables used to examine business cycle fluctuations but also to investigate the possible nonlinearities in the financial variables such as exchange rate and interest rates. We also wanted to examine macroeconomic variables but because of sample size problems we consider monthly inflation and nominal exchange rate data of Turkey with sample period January 1987-June 2001. The first reason of selecting these variables is the sufficient length of the sample. Secondly, the studies concerning PPP hypothesis suggests that as a consequence of nonzero transaction, transportation costs and other impediments to trade, deviations from the PPP should contain significant nonlinearities. The empirical studies in this field contain Liew et al. (2003), Sarno (2000), Taylor and Peel (2000) and Baum et al. (2001). They find strong evidence of nonlinear behavior of real exchange rate and characterized the data by smooth transition autoregressive (STAR) model, a type of nonlinear time series model that allows the real exchange rate to adjust smoothly every moment in between two regimes, which may be either appreciating and depreciating or undervaluation and overvaluation regimes. Sarno, Taylor and Chowdhury (2004) test empirically the validity of the law of one price. Using threshold autoregressive (TAR) models, they find significant nonlinear mean reversion in deviations from the law of one price. The other empirical studies follow the same procedure are Micheal et al. (1994), Obstfeld and Taylor (1997) and Taylor

(2000). Sarno (1997) finds evidence of nonlinearity in real exchange rate of Turkey but does not provide any information about the possible source of nonlinearity that can be observed either one of the component series or both. In our study, to examine the source of this possible nonlinear structure in PPP hypothesis we decided to analyze the component series, namely inflation and nominal exchange rate, separately. The nonlinear structure can be a result of only one variable or both. This is important because if one does not consider the source of nonlinearity one can make wrong inferences about the dynamics of variables. Therefore, the first step of modeling cycle for transformed variables such as real exchange rates, TL / $ should be to uncover the source of nonlinearity. We estimate STAR models for inflation, TL / $ and TL / £ series and examine their dynamic properties and forecast performances. Adequacy of these models is checked by a group of misspecification tests. Our findings indicate that STAR models adequately describe the dynamics of the variables. This implies that possible asymmetric dynamics of these variables should not be neglected when they are used in macroeconomic modeling.

This study is quite important on the following grounds. First, it indicates that if the data show significant nonlinearity, than the use of linear models will not be appropriate and the constructed nonlinear model will probably give better forecasts. Second, the identification of nonlinearity will be quite useful in determining macroeconomic policy changes. That is the effectiveness of macroeconomic policies may change from one regime to another. Other implication of this study is that if the data follow nonlinear process, than the dynamic structure may change according to the regimes and this may cause a nonlinear stationary process to be taken as a linear nonstationary process.

The plan of this study is as follows. Chapter two discusses the main properties of three nonlinear models. The modeling cycle of smooth transition autoregressive model, the data and evaluation of empirical findings are provided in Chapter three. Finally, Chapter four gives an evaluation of the study.

CHAPTER II

NONLINEAR TIME SERIES MODELS

2.1 Introduction

The assumption of linearity has long been dominating macro econometric model building. If linear approximations to economic relationships had not been successful in empirical work they would no doubt have been abandoned long ago. However, in the last three decades, the adequacy of linear model in capturing the dynamics of business cycle data has been debated very often in the literature. It has been observed that the business cycle is asymmetric in the sense that the economy behaves differently during expansions and recessions. A wide variety of linear models have been employed to model business cycle features, but linear time series models with symmetric error terms cannot capture asymmetric dynamics. To describe such dynamics one needs nonlinear models.

In the last two decades, many nonlinear models have been suggested, though only a few commonly applied in economics. These models are the threshold autoregressive (TAR) model due to Tong (1983), the Markov switching model used by Hamilton (1989) and Engle and Hamilton (1990) and the smooth-transition autoregressive (STAR) models promoted by Teräsvirta (1994) and his co-authors as a smooth generalization of TAR models. The main objective of this chapter is to provide a brief review of these models.

This chapter is organized as follows. In Section 2.2, the threshold autoregressive model is introduced and modelling procedure is explained. Markov-switching models and two different approaches for this model are provided in Section 2.3. Section 2.4 considers the main topic of this study, smooth transition autoregressive models. Finally, the last section is devoted to the comparison of nonlinear time series models.

2.2 Threshold Autoregressive Models

The threshold autoregressive model was first proposed by Tong (1978) and discussed in detail by Tong and Lim (1980), Tong (1983) and Tsay (1989). The procedure proposed by Tong and Lim (1980) is quite complex. It involves several computing-intensive stages and there were no diagnostic statistics available to assess the need for a threshold model for a given data set. Because of these reasons their estimation procedure has not received much attention in applied studies. Tsay (1989) suggested a simple method to estimate TAR models. The simplicity of the procedure comes from the usage of familiar linear regression techniques. Potter (1995) and Tiao and Tsay (1994) used this modeling procedure to describe US real GNP and found good results in favor of TAR specification.

2.2.1 Representation of Basic TAR Model

A TAR model is a piecewise linear autoregressive model in the space of the threshold variable. A simple TAR model of a time series Y_t with the threshold variable r_{t-d} is defined as follows, where r_t is a stationary variable from inside or outside of the model and d is referred as threshold lag. Partition the space of r_{t-d}, the real line, by $-\infty = \lambda_0 < \lambda_1 < \ldots\ldots\ldots < \lambda_g < \lambda_{g+1} = \infty$

Then a TAR model of order p for y_t is defined as

$$Y_t = \Phi_0^{(h)} + \Phi_1^{(h)} Y_{t-1} + \ldots\ldots\ldots + \Phi_1^{(h)} Y_{t-p} + \varepsilon_{h,t} \quad \text{for } \lambda_{h-1} \leq r_{t-d} < \lambda_h \tag{2.1}$$

$\varepsilon_{h,t} \sim iid(0,\sigma^2)$ h = 1,...g, g: number of regimes in the model

where λ_h's are referred to as the thresholds and g is the positive integer.

In a two-regime case, the model becomes,

$$Y_t = \begin{cases} \alpha_{10} + \alpha_{11} Y_{t-1} + \ldots\ldots\ldots\ldots \alpha_{1p} Y_{t-p} + \varepsilon_{1,t} & \text{if } r_{t-d} > \tau \\ \alpha_{20} + \alpha_{211} Y_{t-1} + \ldots\ldots\ldots\ldots \alpha_{2p} Y_{t-p} + \varepsilon_{2,t} & \text{if } r_{t-d} \leq \tau \end{cases} \tag{2.2}$$

where r_{t-d} is the transition variable, d is the delay parameter and τ is the threshold parameter

On one side of the threshold, the Y_t sequence is governed by one autoregressive process and on the other side of the threshold, by another AR process. Although Y_t is linear in each regime, the possibility of regime switching implies that the entire sequence is nonlinear and therefore TAR model is said to be a piecewise linear autoregressive model. This property of these models allows modeling asymmetric behavior observed in empirical data.

2.2.2 Modeling TAR Models

Modeling procedure of threshold autoregressive models contains five steps.

In the first step an appropriate AR model is selected, for the all data points in use to construct a basis for nonlinear modeling. AR order p can be selected either by considering the partial autocorrelation function (PACF) or some other information criteria such as Akaike information criteria (AIC) or Schwarz Bayesian criteria (SBC). However, PACF is more preferable over other information criteria because if the process in indeed is nonlinear than the information criteria could be misleading[1].

Second step of TAR modeling is threshold nonlinearity test and finding the threshold variable. To detect nonlinearity and the threshold variable, arranged autoregression procedure is applied. An arranged autoregression is an autoregression with cases rearranged, based on the values of a particular regressor. For the TAR model, arranged autoregression becomes useful by arranging the values of the variables according to ascendingly ordered values of the threshold variable. This arrangement effectively transforms a threshold model into a change point problem. That is the data points are grouped in such a way that all of the observations in a group follow the same linear AR model.

The procedure to be applied in the third step can be summarized as follows. An arranged autoregression based on the threshold variable r_{t-d} is constructed. This rearrangement procedure is done for all possible values of the delay parameter, d. To start threshold nonlinearity test, predictive residuals are obtained from the each arranged autoregression. These predictive residuals play a crucial role in this step,

[1]See, Tsay (1989)

because if the appropriate model is linear than standardized predictive residuals and regressors will be orthogonal. Based on this, an auxiliary regression is constructed by regressing predictive residuals on the regressors. After estimating the auxiliary regression, LM test is applied. Then, for all possible variables and delay parameters auxiliary regressions are constructed and LM test is applied. The variable having smallest p-value (maximum power) is selected as the optimum threshold variable for the TAR model**.**

After detecting threshold nonlinearity and specifying delay parameter, the next step is to determine location of threshold value and the number of regimes. For this purpose, scatter plots of the standardized predictive residuals or ordinary predictive residuals versus determined threshold variable or the scatter plots of t-ratios of recursive estimates of an linear AR coefficient versus the threshold variable can be used. The use of a scatter plot of recursive t-ratios of an AR coefficient versus the threshold variable has two functions. Firstly, they show the significance of that particular AR coefficient. Secondly, if the process is linear and the coefficient is significant, t-ratios gradually and smoothly converge to a fixed value as the recursion continues. However, if the process is nonlinear the predictive residuals will be biased at the threshold value and the convergence of the t-ratio will be destroyed. This biasedness and convergence destruction indicates the location of threshold value and major changes in the slope of the t-ratio suggest regime partitions.

Chan (1993) introduced a similar technique for finding the consistent estimate of the threshold. His method is also heavily based on the concept of arranged autoregression. In that technique, the sum of squared residuals from any TAR model are thought as being a function of the threshold value used in the estimation. According to this technique, the smallest value for sum of squared of residuals is a sign of being close to the true threshold value. Hence, sum of squares of residuals should be minimized at the value of the threshold. The true threshold value is captured in the trough of the graph of the sum of squared residuals and if the graph has several minima, than this means that the model has several thresholds and several regimes. One difficulty with this approach is the fact that one has to estimate a TAR model for each possible threshold value and variable.

Having found the appropriate AR order, threshold variable, possible threshold value and the number of regimes, the modeling cycle of the TAR model comes to the estimation stage. Since TAR model is a locally linear model, ordinary least squares techniques are useful in studying the process. TAR model can be estimated with two available least squares estimation techniques. According to the number of regimes, the series is divided into the groups and OLS technique is applied for each group. A single regression is constructed for the whole series with indicator functions given by the single index multiplying lags of the time series. Second method is useful when there is a need to impose a restriction of equality of certain estimated coefficients across regimes or to use exogenous variables, which might have same or different coefficients across regimes.

Having estimated the TAR model, validity of the model must be checked. First check is the location of the threshold value. The only limitation is that a threshold should not be too close to the 0th or 100th percentile. For these extreme points there will be not enough observations to provide an efficient estimate. To decide the adequacy of the model a battery of misspecification tests must be carried out and these are tests for no residual autocorrelation, heteroscedasticity, remaining nonlinearity, skewness, kurtosis and normality. Final evaluation criterion is post-sample forecasting and size of the combined error variance. To be able to choose the TAR model over linear AR model, TAR model should be statistically adequate, should have better forecast performance and smaller combined error variance. Otherwise there is no need to use this complex modeling cycle. Post-sample forecast performance comparison can be based on the comparison of the mean squared error of forecasts. In combined error variance case there is a quite important point. A reduction in the combined error variance obtained from the TAR model does not always indicate a better performance of the TAR model. There can be two reasons for this reduction, real improvement or just over fitting in the sample. In order to decide, linear and nonlinear models should be estimated recursively and the correlation between recursive forecasts and actual data should be obtained. If the correlation is at maximum in the TAR model, than the reduction is actually because of real improvement.

2.3 Markov Switching Models

The interest in Markov switching models in describing business cycle data has risen following the study by Hamilton (1989) and Filardo (1994). Hamilton type Markov models allow for two states to exist with a series of shifts between the states occurring in a probabilistic fashion. Thus, the shifts occur endogenously rather than being imposed from outside. In the Markov regime switching process, the probability of being in a particular state is only depend on the state of the process in the previous period. It is assumed that the probabilities of switching from one regime to the other are fixed over time. The Hamilton two state Markov switching regime AR(1) model is as follows

$$Y_t = \mu_{st} + \alpha_{st} + \varepsilon_t \qquad (2.3)$$

where ε_t is $N(0, \sigma^2_{st})$

It is assumed that that regime path (s_{t-1}, s_{t-2},…) follows a first order Markov process with time homogeneous transition probabilities,

$P(s_t = j \mid s_{t-1} = i) = pij$,

for i and j =1,2.

Thus pij is equal to the probability that the Markov chain model moves from state *i* at time t-1 to state j at time t. In application to business cycle data s_t =1 can be thought as recession regime and s_t = 2 can be thought as expansion regime. This Hamilton process is completely described by the following constant transition probabilities:

$P(s_t = 1 \mid s_{t-1} = 1) = p_{11}$

$P(s_t = 2 \mid s_{t-1} = 1) = p_{21} = 1 - p_{11}$

$P(s_t = 2 \mid s_{t-1} = 2) = p_{22}$

$P(s_t = 1 \mid s_{t-1} = 2) = p_{12} = 1-p_{22}$

So p_{11} is the probability of continuing in recession regime, p_{21} is the probability of switching from recession to expansion, p_{22} is the probability of continuing in expansion regime and finally p_{12} is the probability of switching from expansion to recession.

The regime path process s_t depends on regime path s_t only through previous value of regime path s_{t-1}. The unconditional probabilities, found by Bayesian method, of the stationary distributions that the process is in each of the regimes are given by

$$P(s_t = 1) = \frac{1 - p_{22}}{2 - p_{11} - p_{22}}, \text{ and}$$

$$P(s_t = 2) = \frac{1 - p_{11}}{2 - p_{11} - p_{22}},$$

To obtain estimates of the parameters and the transition probabilities governing the Markov chain of the unobserved state, one needs an iterative estimation technique. Estimates of parameters for the two most likely regimes are calculated using maximum likelihood estimation techniques. The likelihood is calculated for each possible state, and the probability that a particular state is prevailing is obtained by dividing the likelihood of particular state by the total likelihood for both states.

One important shortcoming of Hamilton process is fixed probabilities. This is important because it is assumed that, for example, the probability of moving from recession to expansion fixed irrespective of whether the economy is at the beginning or end of the recession. As a solution to this drawback Filardo (1994) extended the Markov chain component by allowing the transition probabilities to fluctuate over time with movements in an indicator variable, x_t. He uses a single indicator specification of the logistic form:

P_{11} = Prob(S_t = 1|S_{t-1}= 1, x_{t-K1}) = $\dfrac{1}{1+\exp\{-(\beta_{10}+\beta_{11}x_{t-K1})\}}$

P_{00} = Prob(S_t = 1|S_{t-1}= 1, x_{t-K0}) = $\dfrac{1}{1+\exp\{-(\beta_{00}+\beta_{01}x_{t-K0})\}}$

where β_{10} and β_{00} give rise to constant transition probabilities for regime 1 and regime 0 respectively when x = 0, while β_{11} and β_{01} are the regime 1 and regime 0 coefficients on the (respective) lagged value of the leading indicator. Thus, the probability of remaining in a regime is conditional on the lagged value of the leading indicator, x_{t-j}, as well as the lagged regime S_{t-1}. Although Filardo's approach provides a flexible modeling we do not consider this method due to its complexity.

2.4 Smooth Transition Autoregressive Models

Smooth transition autoregressive (STAR) models are, due to their flexibility, have been frequently used for modeling economic data. The term 'smooth transition' in its present meaning first appeared by Bacon and Watts (1971). They presented their smooth transition model as a generalization to models of two intersecting straight line with an abrupt change from one linear regression to another at some unknown change point. They also apply the model to two sets of physical data. One year later, Goldfeld and Quant (1972) generalized the so-called two-regime switching regression model by using the same idea. Goldfeld and Quandt (1972) and Chan and Tong (1986) both proposed that the smooth transition between regimes be modeled by using the cumulative distribution function of a standard normal variable as the transition function. Bacon and Watts (1971) used the hyperbolic tangent function. Luukkonen, Saikkonen and Teräsvirta (1988) proposed the logistic function, which is a popular choice in this field. Following these developments, Teräsvirta (1994) devised a data-based technique for specification, estimation and evaluation of STAR models. This modeling technique has been applied to several economic time series in Teräsvirta and Anderson (1992). The STAR models are discussed in detail in Granger and Teräsvirta (1993), Teräsvirta (1998) and Potter (1999).

2.4.1 Representation of the Basic STAR Model

In the smooth transition autoregressive model, the fixed thresholds of a standard threshold autoregressive (TAR) model are replaced with a smooth function, which needs to be continuous and non-decreasing. This implies that STAR models are more flexible than TAR and Hamilton type Markov Switching regime models and therefore preferred in this study.

A two-regime STAR model of order p is given by

$$Y_t = \pi_{10} + \sum_{i=1}^{p} \pi_{1i} Y_{t-i} + (\theta_{20} + \sum_{i=1}^{p} \theta_{2i} Y_{t-i}) * F(Y_{t-d}; \gamma, c) + \varepsilon_t \qquad (2.4)$$

or

$$Y_t = \pi' \omega_t + \theta' \omega_t * F(Y_{t-d}; \gamma, c) + \varepsilon_t \qquad (2.5)$$

with $\pi = (\pi_{10}, \ldots\ldots, \pi_{1p})'$, $\theta = (\theta_{20}, \ldots\ldots, \theta_{2p})'$ and $\omega_t = (1, y_{t-1}, \ldots\ldots, y_{t-p})'$

The ε_t's are assumed to be a martingale difference sequence with respect to the history of the time series up to time t-1, which is denoted as $\Omega_{t-1} = \{ y_{t-1}, \ldots\ldots y_{t-p} \}$ that is E($\varepsilon_t | \Omega_{t-1}$) = 0. Also it is assumed that the conditional variance of ε_t is constant, E($\varepsilon_t^{\,2} | \Omega_{t-1}$) = σ^2. An extension of STAR model, which allows for a change in variance, that is autoregressive conditional heteroscedasticity (ARCH) is considered in Lundbergh and Teräsvirta (1998). In this study any ARCH effect is tested by Engle (1982) ARCH test.[2]

$F(Y_{t-d};\gamma,c)$ is the transition function that is bounded between 0 and 1. The extreme value 0, corresponds to regime 1 with coefficients π_{1i}, i = 0,1,….,p and the other extreme value 1, indicates regime 2 with coefficients π_{1i} + θ_{2i}, i = 0,1,….,p. Intermediate values of the transition function define situations in which the process is a mixture of two linear AR(p) processes of regimes 1 and 2. Hence STAR model allows for a 'continuum' of regimes, each associated with a different value of $F(Y_{t-d};\gamma,c)$ between 0 and 1.

Different choices for the transition function $F(Y_{t-d};\gamma,c)$ give rise to different types of regime-switching behavior. There are two popular choices for the transition function, logistic function and exponential function. The first-order logistic function is,

$$F_L(y_{t-d}) = (1+\exp(-\gamma_L(y_{t-d}-c_L)))^{-1},\ \gamma_L>0 \qquad (2.6)$$

where y_{t-d} is transition variable, γ_L is transition or smoothness parameter and c_L is threshold parameter.

STAR model with this function is called logistic STAR (LSTAR) model. The parameter γ determines the smoothness of change in the value of the logistic function and, thus, the smoothness of the transition from one regime to the other. As γ becomes very large, the logistic function $F_L(y_{t-d})$ goes to the indicator function

[2]See, Lundbergh and Teräsvirta (1998)

$I(y_{t-d} > c_L)$ defined as I(A) = 1 if A is true and I(A) = 0 otherwise, and, consequently the change of $F_L(y_{t-d})$ from 0 to 1 becomes instantaneous at $y_{t-d} = c_L$. Hence the LSTAR model goes to the self-exciting TAR (SETAR) model, discussed in Section 2.2. When $\gamma \to 0$, the logistic function approaches a constant (equal to 0.5) and when $\gamma = 0$, the LSTAR model reduces to a linear AR model with parameters $\pi_{1i} + (½)\theta_{2i}$, i = 0,1,….,p. In the LSTAR model, the two regimes are associated with small and large values of the transition variable y_{t-d} relative to c_L and this permits business cycle expansions ($F_L(y_{t-d}) =$ 1) and contractions ($F_L(y_{t-d}) =$ 0) to have different dynamics. This type of regime switching can be convenient for modeling, for example, business cycle asymmetry where the regimes of the LSTAR are related to expansions and recessions.[3]

In certain applications it is more convenient to specify the transition function such that regimes are associated with small and large absolute values of y_{t-d} relative to the threshold value. This can be achieved by using the second popular choice for the transition function, the exponential function,

$$F_E(y_{t-d}) = 1 - \exp(-\gamma_E (y_{t-d} - c_E)^2) \quad , \ \gamma_E > 0 \tag{2.7}$$

and the resultant model is called exponential STAR (ESTAR) model. The exponential function has the property that $F_E(y_{t-d})$ goes to 1 both as y_{t-d} goes to ∞ and $-\infty$, whereas $F_E(y_{t-d})$ = 0 for $y_{t-d} = c_E$. In the ESTAR model, only the distance from the location parameter is important, so that the regimes are effectively

[3]See, Teräsvirta and Anderson (1993) and Skalin and Teräsvirta (2001)

defined by values close to c_E and far from c_E. For either $\gamma \to 0$ or $\gamma \to \infty$, the exponential function approaches to a constant (equal to 0 and 1, respectively). Hence, the model collapses to a linear model in both cases. The resultant exponential STAR (ESTAR) model has been applied to real (effective) exchange rates by Michael (1997), Sarantis (1999) and Taylor (2001), motivated by the argument that the behavior of the real exchange rate depends nonlinearly on the size of the deviation from purchasing power parity.

2.4.2 Modeling Cycle of STAR Model

Following the procedure suggested in Teräsvirta (1994), there are five steps in modeling procedure.

2.4.2.1 Specification of a Linear Autoregressive Model

In the first step an appropriate linear AR model is constructed to obtain a basis for the nonlinear model. To specify a linear AR(p) model, one can employ an order selection criteria such as AIC (Akaike 1974) or SBIC (Rissanen 1978; Schwarz 1978). However, SBIC, which is dimension-consistent, sometimes leads to too parsimonious models in the sense that the estimated residuals of the selected model are not free from serial correlation. In that sense, AIC may be more preferable. The procedure using AIC for order selection is as follows. To select an appropriate AR model, an AR model with 24 lags is considered and the maximum order of lag is restricted by AIC. However, it is important that the use of any model selection procedure should be accompanied by a proper test for residual autocorrelation, like the Portmanteau test of Ljung and Box (1978). This is important, because omitted autocorrelation can detoriate the remaining steps of modeling cycle.

2.4.2.2 Linearity Test Against STAR Model

Testing linearity against STAR model constitutes a crucial step towards building STAR models. The null hypothesis of linearity can be expressed by the equivalence of the autoregressive parameters in the two regimes of the STAR model in (2.4).

The testing problem is complicated by the presence of unidentified nuisance parameters under the null hypothesis. Formally, the STAR model contains parameters, which are not restricted by the null hypothesis, but nothing can be learned from the data when the null hypothesis holds true. The null hypothesis of equivalence of two regime parameters does not give a restriction on the parameters of transition function, γ and c. An alternative way to illustrate the presence of unidentified parameters in this case is to note that the null hypothesis for the linearity test can be formulated in different ways. In addition to the equality of the AR parameters in the two regimes, the alternative null hypothesis $H_0': \gamma = 0$ also indicates a linear model for each choice of the transition function. If H_0' is used, the location parameter c and the AR parameters of two regimes will be unidentified parameters.

The problem of unidentified nuisance parameters under the null hypothesis was first considered by Davies (1977; 1987). His solution is to derive the test statistic by keeping the unidentified parameters fixed. The main consequence of the presence of such nuisance parameters is that the conventional statistical theory is not available for obtaining the asymptotic null distribution of the classical likelihood ratio, Lagrange Multiplier and Wald statistic. Instead, these test statistics tend to have nonstandard distributions for which analytic expressions are most often not available, but which have to be obtained by the help of simulation. Luukkonen, Saikkonen and Teräsvirta (1988) encountered the same problem in an alternative way. Their proposed solution is to replace the transition function $F_L(y_{t-d};\gamma,c)$ by a suitable Taylor approximation. In the reparametrized equation, the identification problem is solved and linearity can be tested by means of Lagrange Multiplier (LM) statistic with a standard χ^2 distribution under the null hypothesis. This technique has two main advantages. First, the model under the alternative hypothesis need not be estimated and standard asymptotic theory is available for obtaining asymptotic critical values for the test statistic.

2.4.2.2.1 Test Against LSTAR

Consider the LSTAR model,

$$Y_t = \pi_{10} + \sum_1^p \pi_{1i} Y_{t-i} + (\theta_{20} + \sum_1^p \theta_{2i} Y_{t-i}) * F_L(Y_{t-d};\gamma_L, c_L) + \varepsilon_t \tag{2.8}$$

Luukkonen suggests approximating the logistic function

$F_L(y_{t-d}) = (1+\exp(-\gamma_L(y_{t-d} - c_L)))^{-1}$

with a first-order Taylor series approximation around $\gamma_L = 0$ and obtained the following auxiliary regression

$$Y_t = \beta_0 + \sum_1^p \beta_{1i} Y_{t-i} + \sum_1^p \beta_{2i} Y_{t-i} Y_{t-d} + e_t \tag{2.9}$$

The parameters β_{1i}, β_{2i}; i = 1,……,p in the auxiliary regression are functions of the parameters in the LSTAR model such that the restriction $\gamma_L = 0$ implies $\beta_0 \neq 0, \beta_{1i} \neq 0$ and $\beta_{2i} = 0$ for i = 1,……,p. Hence, testing the null hypothesis $H_0': \gamma_L = 0$ turns to testing the null hypothesis $H_0'': \beta_{2i} = 0$ for i = 1,……,p.

However, this auxiliary regression does not have good power in situations where only the intercept differs across regimes. A test which does have power against this situation is constructed by approximating the transition function $F_L(y_{t-d};\gamma,c)$ by a third-order Taylor approximation. The resultant auxiliary regression is

$$Y_t = \beta_0 + \sum_{i=1}^{p}\beta_{1i}Y_{t-i} + \sum_{i=1}^{p}\beta_{2i}Y_{t-i}Y_{t-d} + \sum_{i=1}^{p}\beta_{3i}Y_{t-i}Y^2{}_{t-d} + \sum_{i=1}^{p}\beta_{3i}Y_{t-i}Y^3{}_{t-d} + e_t \qquad (2.10)$$

Again, β_{2i}, β_{3i} and β_{4i} for i = 1,…..,p are functions of the parameters in the LSTAR model such that the null hypothesis $H_0': \gamma_L = 0$ now corresponds to $H_0'': \beta_{2i} = \beta_{3i} = \beta_{4i} = 0$, i = 1,….,p. This type of approximation is more general and more applicable.[4]

2.4.2.2.2 Test Against ESTAR

Consider the ESTAR model,

$$Y_t = \pi_{10} + \sum_{i=1}^{p}\pi_{1i}Y_{t-i} + (\theta_{20} + \sum_{i=1}^{p}\theta_{2i}Y_{t-i}) * F_E(Y_{t-d};\gamma_E, c_E) + \varepsilon_t \qquad (2.11)$$

Saikkonen and Luukkonen (1988) suggest testing linearity against an ESTAR model by using the auxiliary regression

$$Y_t = \beta_0 + \sum_{i=1}^{p}\beta_{1i}Y_{t-i} + \sum_{i=1}^{p}\beta_{2i}Y_{t-i}Y_{t-d} + \sum_{i=1}^{p}\beta_{3i}Y_{t-i}Y_{t-d}{}^2 + e_t \qquad (2.12)$$

Equation (2.12) is obtained by using first-order Taylor approximation for the exponential transition function. The restriction $\gamma_E = 0$ corresponds with $\beta_{2i} = \beta_{3i} = 0$, i = 1,……,p.

[4]See, Teräsvirta (1994, 1998) and Granger and Teräsvirta (2001) for more detail.

Escribano and Jorda (1999) claim that a first-order Taylor approximation of the exponential function is not sufficient to capture its characteristic features, the two inflection points of this function in particular. They suggest that a second-order Taylor approximation is needed. Using second-order Taylor approximation causes extra variables in the auxiliary regression.

Since this effect is neutralized by the increase in the dimension of the null hypothesis, neither of the approximations dominates the other in terms of power and first-order Taylor approximation becomes more applicable.

2.4.2.2.3 Application of Linearity Test

The linearity test is carried out by using LM test. The procedure for obtaining LM statistic is as follows. First, the model is estimated under the null hypothesis of linearity by regressing y_t on $(1, y_{t-1}, \ldots, y_{t-p})$ and the residuals $\hat{e}_t$ are obtained. Then, the auxiliary regressions are constructed by using $\hat{e}_t$ as dependent variable. For the LSTAR model equation (2.13) and for the ESTAR model equation (2.14) are used.

$$\hat{e}_t = \beta_0 + \sum_{i=1}^{p} \beta_{1i} Y_{t-i} + \sum_{i=1}^{p} \beta_{2i} Y_{t-i} Y_{t-d} + \sum_{i=1}^{p} \beta_{3i} Y_{t-i} Y^2{}_{t-d} + \sum_{i=1}^{p} \beta_{4i} Y_{t-i} Y^3{}_{t-d} + v_t \qquad (2.13)$$

$$\hat{e}_t = \beta'_0 + \sum_{i=1}^{p} \beta'_{1i} Y_{t-i} + \sum_{i=1}^{p} \beta'_{2i} Y_{t-i} Y_{t-d} + \sum_{i=1}^{p} \beta'_{3i} Y_{t-i} Y^2{}_{t-d} + v_t \qquad (2.14)$$

The null hypotheses and corresponding test statistics are

$H_0 = \beta_{2i} = \beta_{3i} = \beta_{4i} = 0$, i = 1,…..,p; LM_3 for LSTAR model

$H'_0 = \beta'_{2i} = \beta'_{3i} = 0$, i = 1,…..,p; LM_2 for ESTAR model

The test statistics have asymptotic χ^2 distribution. However, in small samples it is recommended to use F versions of the LM test statistics, because these have better size properties than χ^2 variant, which may be heavily oversized.

2.4.2.2.4 Determining the Transition Variable

If linearity is rejected, the next objective is to select the appropriate transition variable. Even though, the LM_3 statistic was developed as a test against the LSTAR alternative, it has power against ESTAR alternatives as well. An intuitive way to understand this is to note that all variables in the first-order approximation to the ESTAR model in (2.13) are contained in (2.14). This means that the appropriate transition variable can be determined, without specifying the form of the transition function, by computing LM_3 statistic for various choices of delay parameter, d = 1,….,p, and selecting the one with smallest p-value. The logic behind this procedure is that if the correct transition variable is used, than the test should have maximum power

2.4.2.3 Choosing Between LSTAR and ESTAR Model

The selection between LSTAR and ESTAR is based on testing a sequence of hypothesis in equation (2.13), using the selected transition variable Y_{t-d} from step 2. The sequence of hypothesis tested is

$H_{01} : \beta_{4i} = 0$
$H_{02} : \beta_{3i} = 0 | \beta_{4i} = 0$

$H_{03}: \beta_{2i} = 0 | \beta_{3i} = \beta_{4i} = 0$ where i =1,……,p

These hypotheses are tested by ordinary F tests. The decision rule is as follows: If H_{02} is rejected and H_{03} not rejected, then appropriate model is ESTAR. If H_{02} is not rejected and H_{03} is rejected, then LSTAR model is preferable. However, it is better to compare relative strengths of the rejections. Then the decision criteria is as follows: If H_{02} has the minimum p-value ESTAR model is chosen and if H_{03} has the minimum p-value LSTAR model is more preferable. However, if the p-values corresponding to F_3 and F_2 or F_4 and F_3 are close to each other in relative terms, both LSTAR and ESTAR models should be considered.

Discrimination between these models can also be done by using the structure of the data. If the observations are symmetrically distributed around the transition value, than ESTAR model is selected correctly almost without exception. If most of the observations lie on one side of the threshold value, LSTAR and ESTAR models give similar implications and they can be used as close substitutes. So ESTAR models provide a good approximation to LSTAR models if most of the observations lie on the right hand side of threshold.

Recent increases in computational power have made these decision rules less important in practice. It is now easy to estimate both LSTAR and ESTAR models and selection between them can be implemented by misspecification tests. Therefore we prefer to estimate both type models and leave model selection to post-estimation stage.

2.4.2.4 Estimation

Once the transition variable y_{t-d} and the transition function $F(y_{t-d};\gamma,c)$ have been selected, the next stage in the modeling cycle is estimation of the parameters in the STAR model. The estimation procedure is carried out using nonlinear least squares (NLS). The parameters,

$\theta = (\pi_{10}, \pi_{1i}, \theta_{20}, \theta_{2i}, \gamma, c)$ i = 1,…….,p can be estimated as

$$\hat{\theta} = \arg\min_{\theta} Q_T(\theta) = \arg\min_{\theta} \sum_{t=1}^{T} (y_t - F(y_{t-1},......., y_{t-p};\theta))^2$$

where $F(y_{t-1},......., y_{t-p};\theta) = \pi_{10} + \sum_{1}^{p} \pi_{1i} Y_{t-i} + (\theta_{20} + \sum_{1}^{p} \theta_{2i} Y_{t-i}) * F(Y_{t-d};\gamma,c) + \varepsilon_t$

However, this high dimensionality of NLS estimation can cause computational problems. To simplify this estimation problem, Leybourne, Newbold and Vougas (1998) suggested concentrating on the sum of squares function. When parameters γ and c in the transition function are known and fixed, the STAR model is linear in the autoregressive parameters of two regimes. In this case the estimation of autoregressive parameters become conditional upon γ and c and this reduces the dimensionality of the NLS estimation problem considerably.

Teräsvirta proposed a different approach. As a first step of estimation procedure, Teräsvirta suggested to standardize the exponent of $F(Y_{t-d};\gamma,c)$ by division the sample variance and standard deviation, respectively for the estimation of ESTAR and LSTAR models. The logic of this scaling procedure is to make the smoothness of transition variable approximately scale-free, and this provides easiness in determining a set of initial values for this parameter. The initial values for remaining parameters should also be determined. To obtain the initial values, an extensive two-dimensional grid search for pairs of γ and c is constructed. The set of grid values for γ is ranged between 1 and 150, and if the minimizing value for γ is found close to 150, the range is extended. A reasonable set of grid values for the location parameter c may be defined as sample percentiles of the transition variable y_{t-d}. This guarantees that the values of the transition function contain enough sample variation for each choice of γ and c. The combination yielding the lowest residual sum of squares is used as starting value for all parameters. However, at least 30 different sets of initial values, selected from the grid search, are used as an attempt to find the global minimum.

The only problem in estimation procedure can be seen in estimation of transition variable γ. When the value of γ is very large, than it may be difficult to obtain an accurate estimate of this parameter. This is due to the fact that for such large values of γ, the STAR is similar to a threshold model, as the transition function turns to be a step function. To obtain an accurate estimate of γ, one then needs many observations in the immediate neighborhood of c, because even large changes in γ only have a small effect on the shape of transition function. The estimate of γ may therefore be rather imprecise and often appear to be insignificant when judged by its t-statistic.[5] This should, however, not be thought as evidence for weak nonlinearity, as the t-statistic does not have its customary asymptotic t-distribution under the

[5] See Bates and Watts (1987, 1988)

hypothesis that $\gamma = 0$, due to the identification problem discussed in Section 2.4.2.2. In this case, the causes of a large standard error estimate are purely numerical. Besides, large changes in γ have only minor effect on the transition function; high accuracy in estimating γ is not necessary.

2.4.2.5 Evaluation

After estimating the parameters in a STAR model, the next stage is model evaluation. First of all, the location of threshold value is important. If threshold value is far outside of the observed range, then the estimated model is not satisfactory. Secondly, high standard deviations of the parameters, except that of the transition parameter and threshold, indicate redundant parameter problem in the model. If the model does not have these problems, its adequacy should be checked by a set of misspecification tests. These tests are test for skewness, kurtosis, normality, heteroscedasticity, autocorrelation and parameter instability. Moreover evidence of unmodeled nonlinearity should be examined. Eitrheim and Teräsvirta (1996) design several tests for testing the evidence of autocorrelation, parameter consistency and additive nonlinearity where the residuals are that of a STAR model and these are discussed below.

2.4.2.5.1 Testing the Hypothesis of No Error Autocorrelation

The general residual autocorrelation test, the customary portmanteau test of Ljung and Box (1978) is inapplicable because its asymptotic null distribution is unknown if the test is based on estimated residuals of a STAR model. Eitrheim and Teräsvirta (1996) proposed a test procedure for STAR type models. The procedure is as follows

A STAR model can be defined as

$y_t = G(\omega_t, \Psi) + u_t$, where

$G(\omega_t, \Psi) = G(\omega_t; \pi, \theta, \gamma, c) = \pi' \omega_t + \theta' \omega_t * F(y_{t-d}; \gamma, c)$

with $\pi = (\pi_{10}, \ldots\ldots, \pi_{1p})'$, $\theta = (\theta_{20}, \ldots\ldots, \theta_{2p})'$ and $\omega_t = (1, y_{t-1}, \ldots\ldots, y_{t-p})'$

The procedure of no residual autocorrelation test is as follows:

i) The STAR model is estimated by NLS under the assumption of uncorrelated errors and the residuals are obtained

ii) Following the logic of LM test, the derivatives $\dfrac{\partial \widehat{G}}{\partial \Psi}$ are computed. Because of the structure of nonlinearity, the regressors, essential for the test, are obtained by using derivatives[6] . If the model is a LSTAR model, the derivatives are

$$\frac{\partial \widehat{G}}{\partial \gamma} = \widehat{g}_{\gamma}(t) = (1 + \exp(-\widehat{\gamma}(y_{t-d} - \widehat{c})))^{-2} \exp(-\widehat{\gamma}(y_{t-d} - \widehat{c}))(y_{t-d} - \widehat{c})\widehat{\theta}' \omega_t$$

$$\frac{\partial \widehat{G}}{\partial c} = \widehat{g}_{c}(t) = \widehat{\gamma}(1 + \exp(-\widehat{\gamma}(y_{t-d} - \widehat{c})))^{-2} \exp(-\widehat{\gamma}(y_{t-d} - \widehat{c}))\widehat{\theta}' \omega_t$$

$$\frac{\partial \widehat{G}}{\partial \pi} = \widehat{g}_{\pi}(t) = \omega_t$$

$$\frac{\partial \widehat{G}}{\partial \theta} = \omega_t F(y_{t-d}; \widehat{\gamma}, \widehat{c})$$

In the case of ESTAR model, the derivatives become

$$\frac{\partial \widehat{G}}{\partial \gamma} = \widehat{g}_{\gamma}(t) = \exp(-\widehat{\gamma}(y_{t-d} - \widehat{c})^2)(y_{t-d} - \widehat{c})^2 \widehat{\theta}' \omega_t$$

$$\frac{\partial \widehat{G}}{\partial c} = \widehat{g}_{c}(t) = 2\widehat{\gamma} \exp(-\widehat{\gamma}(y_{t-d} - \widehat{c})^2)(y_{t-d} - \widehat{c})^2 \widehat{\theta}' \omega_t$$

[6]See Eitrheim and Teräsvirta (1996) for more detail

$$\frac{\partial \hat{G}}{\partial \pi} = \hat{g}_{\pi}(t) = \omega_t$$

$$\frac{\partial \hat{G}}{\partial \theta} = \hat{g}_{\theta}(t) = \omega_t F(y_{t-d}; \hat{\gamma}, \hat{c})$$

iii) After that, residuals obtained from step i are regressed on q lagged residuals, $\hat{g}_{\gamma}(t)$, $\hat{g}_{c}(t)$, $\hat{g}_{\pi}(t)$ and $\hat{g}_{\theta}(t)$ and ordinary LM test is applied. This is actually a generalization of the LM test for serial correlation in an AR(p) model of Godfrey (1979), which is obtained by setting $G(\omega_t, \Psi) = \theta' \omega_t$.

2.4.2.5.2 Testing the Hypothesis of No Remaining Nonlinearity

To discuss this test it is appropriate to introduce multiple regime (two-transition) STAR models. Single-transition STAR model (2.4) cannot accommodate more than two regimes, regardless of the specific functional form of the $F(y_{t-d}; \gamma, c)$. Even though two regimes might be sufficient in many applications, it can be desirable to allow multiple regimes.

An obvious extension of the STAR model in equation (2.4) is to include two additive transition functions. The resulting model permits more than two underlying regimes and its equation can be given as

$$Y_t = \pi' \omega_t + \theta' \omega_t * F_1(Y_{t-d}; \gamma_1, c_1) + \phi' \omega_t * F_2(Y_{t-e}; \gamma_2, c_2) + u_t, \qquad (2.15)$$

$u_t \sim \text{iid}(0, \sigma^2)$

This extension leads to additive STAR model. The model has been parameterized in such a way that F_1 and F_2 are logistic or exponential functions as appropriate.

Moreover, F_1 and F_2 are specified such that the location values satisfy $c_1 < c_2$, in order to separately identify the two transition functions. It may appear that the extension to two transition functions allows the possibility of four regimes, as first the function F_1 changes from 0 to 1, followed by a similar change of F_2. While it may technically be possible to distinguish four regimes in this way when d and e take different values, when they are same then y_{t-d} determines both transitions and at least one of the four extreme cases will be ruled out.[7] As an example, assume that over a range of y_{t-d} the two extremes $F_1 = 0,1$ are possible while $F_2 = 1$ through this range. Then the distinct values for c_1 and c_2 (which allow $F_1 = 0,1$ while $F_2 = 1$) must logically rule out $F_1 = F_2 = 0$ because $F_2 = 1$ in the range of y_{t-d} where $F_1 = 0$

Another way of obtaining a four-regime model is 'encapsulating' two different two-regime LSTAR models as follows:

$$\begin{aligned} Y_t &= (\pi'\omega_t + \theta'\omega_t * F_1(Y_{t-d};\gamma_1,c_1)) + (\phi'\omega_t + \delta'\omega_t * F_1(Y_{t-d};\gamma_1,c_1)) \\ &* F_2(Y_{t-e};\gamma_2,c_2) + \varepsilon_t \end{aligned} \tag{2.16}$$

with $\phi = (\phi_{10},..........\phi_{1p})'$ and $\delta = (\delta_{10},..........\delta_{1p})'$

The effective relationship between y_t and its lagged values is given by a linear combination of four linear AR models, each associated with a particular combination of $F_1(Y_{t-d};\gamma_1,c_1)$ and $F_2(Y_{t-e};\gamma_2,c_2)$ being equal to 0 or 1. This is so-called multiple regime STAR (MRSTAR) model and it is discussed in detail in van Dijk and Franses (1999). The MRSTAR model in (2.16) reduces to a (SE)TAR model with four-regimes determined by two sources when the smoothness parameters γ_1 and γ_2

[7]See, Ocal and Osborn (2000)

become very large, such that the logistic functions F_1 and F_2 approach indicator functions I($y_{t-d} > c_1$) and I($y_{t-e} > c_2$), respectively. The resultant nested TAR (NeTAR) model is discussed in Astatkie et al. (1997). It is now appropriate to discuss additional nonlinearity test in detail.

Eitrheim and Teräsvirta (1996) develop an LM statistic to test the two-regime LSTAR model against the alternative of an additive STAR model defined in (2.15). However, it is well known from the linearity test part of the modeling cycle that the LM test has also power against the case of exponential transition function. Consider the additive STAR model:

$$Y_t = \pi' \omega_t + \theta' \omega_t * F_1(Y_{t-d}; \gamma_1, c_1) + \phi' \omega_t * F_2(Y_{t-e}; \gamma_2, c_2) + u_t$$

The null hypothesis of a two-regime model can be expressed either by $H_0 : \gamma_2 = 0$ or $H_0 : \theta = \phi$. Evidently, this testing problem suffers from a similar identification problem as encountered in testing linearity against a two-regime STAR model, see Section 2.4.2.2. To circumvent this identification problem, the transition function $F_2(Y_{t-e}; \gamma_2, c_2)$ is replaced by a Taylor series approximation around $\gamma_2 = 0$. Hence, it is assumed that F_2 is logistic and replaced by its third-order Taylor approximation about $\gamma_2 = 0$. After some reparametrizations, equation (2.15) takes the form:

$$\begin{aligned} y_t &= \beta_0' \omega_t + (\theta' \omega_t) * F_1(y_{t-d}; \gamma_1, c_1) + \beta'_1 \varpi_t y_{t-e} + \beta'_2 \varpi_t y_{t-e}{}^2 \\ &+ \beta'_3 \varpi_t y_{t-e}{}^3 + r_t \end{aligned} \tag{2.17}$$

where $\varpi_t = (y_{t-1}, \ldots.., y_{t-p})$ and the parameters β_i , i = 0,1,2,3, are functions of the parameters $\pi, \theta, \phi, \gamma_2$ and c_2. The null hypothesis of no additional nonlinearity $H_0 : \gamma_2 = 0$ translates into $H_0' : \beta_1 = \beta_2 = \beta_3 = 0$. The test statistic can be computed as nR^2 from the auxiliary regression of the residuals, which are obtained from the estimated model under the null hypothesis of $\gamma_2 = 0$, on the partial derivatives of regression function with respect to parameters in the two-regime model, $\pi, \theta, \phi, \gamma_1$ and c_1, evaluated under the null hypothesis and auxiliary regressors $\varpi_t y^i_{t-e}$, i = 1,2,3. The partial derivatives with respect to the parameters $\pi, \theta, \phi, \gamma_1$ and c_1 used in the auxiliary regression are same with the derivatives used for no error autocorrelation test for LSTAR case, see Section 2.4.2.5.1.1. The derived test statistic has an asymptotic χ^2 distribution with 3p degrees of freedom and the degrees of freedom in F test are 3p and T-n-3p, respectively.

Note that in deriving the test it is assumed that the delay parameter e in the second transition function is known. Because the value of e might be unknown, the test should be carried out for various values of e and to be able to use the two-regime STAR model, the null hypothesis should not be rejected for all possible values of e. Carrying out the LM test for all possible values of e is helpful in checking that the delay parameter d in the two-regime model is correctly specified. In other words, it is useful to check whether y_{t-d} is an appropriate transition variable for the single-transition STAR model. If the result of the test requires constructing an additive STAR model, the functional form of F_2 is selected as discussed in Section 2.4.2.3.

In the case of MRSTAR model, van Dijk and Franses (1999) derive an LM test for testing the null of the two-regime LSTAR model against the MRSTAR alternative given in (2.16). The null hypothesis can be expressed either $H_0 : \gamma_2 = 0$ or $H_0' : \pi = \phi, \theta = \delta$. In the case the transition function $F_2(Y_{t-e}; \gamma_2, c_2)$ is replaced with a third-order Taylor approximation, the result of the corresponding approximation can be written as

$$y_t = \theta_1{}'\omega_t + (\theta_2{}'\omega_t) * F_1(y_{t-d};\gamma_1, c_1) + \beta_1{}'\varpi_t y_{t-e} + \beta_2{}'\varpi_t y^2{}_{t-e} + \beta_3{}'\varpi_t y^3{}_{t-e}$$
$$+ (\beta_4{}'\varpi_t y_{t-e} + \beta_5{}'\varpi_t y^2{}_{t-e} + \beta_6{}'\varpi_t y^3{}_{t-e}) * F_1(y_{t-d};\gamma_1, c_1) + e_t \quad (2.19)$$

The null hypothesis can be reformulated as $H_0{}': \beta_i = 0$, i =1,…6.

The resultant test statistic is asymptotically χ^2 distributed with 6p.[8]

2.4.2.5.3 Test of Parameter Consistency

Because STAR models are estimated under the assumption of constant parameters, parameter stability test is important in checking the adequacy of the model.

To test consistency of parameters, it is assumed that the transition function has constant parameters, whereas both π, θ are subject to changes over time. Consider the following nonlinear model

$$Y_t = \pi(t)'\omega_t + \theta(t)'\omega_t * F\ (Y_{t-d};\gamma\ ,c\ ) + u_t$$

The time varying parameter vectors are

$\pi(t) = \tilde{\pi} + \lambda_1 H_j(t;\gamma_1, c_1)$, where λ_1 is a (k*1) vector

$\theta(t) = \tilde{\theta} + \lambda_2 H_j(t;\gamma_1, c_1)$, where λ_2 is a (ℓ*1) vector

[8]See , Eitrheim and Teräsvirta (1996) for more detail

The null hypothesis of parameter consistency is H_0: $H_j(t;\gamma_1,c_1) \equiv 0$ (or $H_j(t;\gamma_1,c_1)$ $\equiv$ constant). Lin and Teräsvirta (1994) use three functional forms for H_j. They are

$$H_1(t;\gamma_1,c_1) = (1+\exp(-\gamma_1(t-c_1)))^{-1} - 0.5 \tag{2.20}$$

$$H_2(t;\gamma_1,c_1) = (1-\exp(-\gamma_1(t-c_1)^2) \tag{2.21}$$

$$H_3(t;\gamma_1,c_1) = (1+\exp(-\gamma_1(t^3+c_{12}t^2+c_{11}t+c_{10})))^{-1} - 0.5 \tag{2.22}$$

where γ_1 >0 and c_1 =(c_{10}, c_{11}, c_{12})'. So, the null hypothesis of parameter consistency may be indicated as H_0: γ_1 = 0. Transition function (2.20) postulates a smooth monotonic parameter change. Function (2.21) represents a nonmonotonic change which is symmetric about t = c and (2.22) is the most flexible function allowing both monotonically and nonmonotonically changing parameters.

General test statistic for parameter consistency is derived for testing H_0 with H_3 as the alternative, the tests when either H_1 or H_2 is assumed under the alternative will be special cases of this test. As the first step of testing procedure, the STAR model is estimated under the assumption of parameter consistency and residuals are obtained. After that, an auxiliary regression is constructed by regressing residuals on $\hat{z}_t = (1, \hat{w}_t, \overline{w}_t F(y_{t-d};\hat{\gamma},\hat{c}), \hat{g}_\gamma(t), \hat{g}_c(t))'$ and

$\hat{v}_t = (t\tilde{w}'_t, t^2\tilde{w}_t', t^3\tilde{w}_t', t\overline{w}_t' F(y_{t-d};\hat{\gamma},\hat{c}), t^2\overline{w}_t' F(y_{t-d};\hat{\gamma},\hat{c}), t^3\overline{w}_t' F(y_{t-d};\hat{\gamma},\hat{c}))'$ where $\tilde{w}_t$ is the vector of including lags, after insignificant ones are excluded.

If the alternative is specified to be (a), than $t^3\tilde{w}_t'$ and $t^3\overline{w}_t' F(y_{t-d};\hat{\gamma},\hat{c})$ regressors are excluded from the auxiliary regression and if monotonic change in parameters, defined in (c), is allowed, $t^2\tilde{w}_t'$ and $t^2\overline{w}_t' F(y_{t-d};\hat{\gamma},\hat{c})$ are also eliminated from the regression

2.4.2.5.4 Autoregressive Conditional Heteroscedasticity (ARCH) Test

The LM tests assume constant (conditional) variance. Neglected heteroscedasticity has similar effects on tests for nonlinearity as residual autocorrelation, in that it may lead to spurious rejection of the null hypothesis.

For testing the ARCH effect, the Lagrange Multiplier test of Engle (1982) is used, and the procedure is as follows:

i) The STAR model is estimated by NLS under the assumption of no heteroscedasticity (homoscedasticity), the residuals and square of the residuals are obtained.

ii) An auxiliary regression is constructed by regressing the square of the residuals on the q-lagged values of the dependent variable. Ordinary LM test is applied on this auxiliary regression and $LM_{ARCH}(q)$ statistic is obtained.

Wooldridge (1990; 1991) developed specification tests that can be used in the presence of heteroscedasticity, without need to satisfy the form of the heteroscedasticity explicitly. These procedures may be readily applied to robust the tests against STAR nonlinearity, see also Granger and Teräsvirta (1993). However, Lundbergh and Teräsvirta (1998) present simulation evidence suggesting that in some cases this robustification removes most of the power of the linearity test, as a result existing nonlinearity may not be detected. If the aim of the analysis is to find and model nonlinearity in the conditional mean, robustification therefore cannot be recommended.

After carrying all these tests the soundness of model is assessed. First, estimated values have to be examined. Large standard deviations of the estimated coefficients (except γ_L or γ_E) indicate that the model includes redundant parameters. Furthermore if the estimated threshold value does not lie within the observed range of Y_t, then the model is not satisfactory. The next step is the interpretation of the individual parameters. Although the interpretation of them is very difficult for STAR models, the roots of the associated characteristic polynomials are quite informative about their dynamic properties. The roots can be calculated for various values of F_L and F_E, but roots of polynomials corresponding to the extreme regimes (F_L and F_E = 0,1) are particularly interesting ones for describing the local dynamics of different regimes.

2.4.3 Smooth Transition Regressive (STR) Models

STAR models, analyzed in this thesis, are especially a special case of smooth transition regression (STR) models. They are straightforward generalization of univariate smooth transition autoregressive models to a multivariate framework. STAR models capture nonlinearity in a univariate context by modeling the transition between states or regimes as a function of a lagged value of the variable of interest. However, these univarite models cannot predict regime changes; they can only respond to the signal given by a past value that the regime has changed.

A STR model gives the opportunity of examing the effects of one variable on another within a nonlinear, asymmetric dynamics framework. In this context, the single transition STR model can be defined as:

$$y_t = \phi_{10} + \sum_{i=1}^{p} \phi_{1i} y_{t-i} + \sum_{j=1}^{q} \delta_{1j} x_{t-j} + F(r_{t-d})(\phi_{20} + \sum_{i=1}^{p} \phi_{2i} y_{t-i} + \sum_{j=1}^{q} \delta_{2j} x_{t-j}) + \varepsilon_t \quad (2.23)$$

where $\varepsilon_t \sim$ i.i.d $(0, \sigma^2)$ and r_{t-d} is the transition variable (leading indicator) and d is the delay parameter. Transition variable r_{t-d} can either be a lagged term of the dependent variable or a lagged term of the exogenous variable. The transition function F(.) satisfies all properties mentioned in Section 2.4.1.

In the context of modeling nonlinearities between two variables, y_t and x_t, there are two possibilities, both of which are plausible. That is, a change in y_t may contribute to regime changes in x_t or the effects of the former differ over the regimes in the latter, and vice versa. Combining this with the fact that no priori assumptions regarding the form of the transition function and the possible transition variable are made, suggests eight models. These models are:

$$y_t = \phi_{10} + \sum_{i=1}^{p} \phi_{1i} y_{t-i} + \sum_{j=1}^{q} \delta_{1j} x_{t-j} + F(x_{t-d})(\phi_{20} + \sum_{i=1}^{p} \phi_{2i} y_{t-i} + \sum_{j=1}^{q} \delta_{2j} x_{t-j}) + \varepsilon_t \quad (2.24)$$

$$y_t = \phi_{10} + \sum_{i=1}^{p} \phi_{1i} y_{t-i} + \sum_{j=1}^{q} \delta_{1j} x_{t-j} + F(y_{t-d})(\phi_{20} + \sum_{i=1}^{p} \phi_{2i} y_{t-i} + \sum_{j=1}^{q} \delta_{2j} x_{t-j}) + \varepsilon_t \quad (2.25)$$

$$x_t = \phi_{10} + \sum_{i=1}^{p} \phi_{1i} y_{t-i} + \sum_{j=1}^{q} \delta_{1j} x_{t-j} + F(y_{t-d})(\phi_{20} + \sum_{i=1}^{p} \phi_{2i} y_{t-i} + \sum_{j=1}^{q} \delta_{2j} x_{t-j}) + \varepsilon_t \quad (2.26)$$

$$x_t = \phi_{10} + \sum_{i=1}^{p} \phi_{1i} y_{t-i} + \sum_{j=1}^{q} \delta_{1j} x_{t-j} + F(x_{t-d})(\phi_{20} + \sum_{i=1}^{p} \phi_{2i} y_{t-i} + \sum_{j=1}^{q} \delta_{2j} x_{t-j}) + \varepsilon_t \quad (2.27)$$

where F(.) represents either the exponential or logistic function.

By using y_{t-d} and x_{t-d} as the transition variables in models (2.24)-(2.27), it is considered that regimes for both variables are defined either in terms of past values of the dependent variable or past values of the leading indicator. Models (2.24) and (2.26) implies that transition between different regimes in the variable of interest is a function of a lagged value of the leading indicator and therefore define the situation that one variable impose a regime change in the other variable. Models (2.21) and (2.23), on the other hand, describe the transition between different regimes as a function of a lagged value of the variable of interest with the other variable having different effects in these different regimes. As a result, they imply that the effects of one variable change over the regimes in the other variable.[9]

The method for specification and estimation of STR models is based on Teräsvirta (1994; 1998) and almost same with STAR model, the only difference is that at each step the second variable, x_t, is also taken into the account. Although a brief review of these models is presented, they are not considered in this thesis.

2.4 Comparision of Three Nonlinear Time Series Models

The first distinction among the discussed nonlinear models is seen in their empirical applications. While STAR models do not assume a sharp switch from one regime to the other, TAR and the Hamilton's Markov switching regime models indicate a sharp change. Smooth transition seems to be more appropriate for macroeconomic time series because it is unlikely that economic agents change their behavior simultaneously. Hence, as Teräsvirta (1994) notes, for aggregated processes the change in regime may be smooth rather than discrete. Although TAR and the

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[9]See, Sensier, Osborn and Ocal (2002), and Ocal (2002, 2003) for empirical examples

Hamilton's Markov switching regime models differ from STAR models by indicating a sharp switch, Filardo's Markov switching regime model shows a similar structure to smooth transition. Filardo's approach allows for smooth change under a Markov switching framework and therefore may be more appropriate for modeling macroeconomic variables than Hamilton's modeling approach. However, the theory and application procedure of this approach is quite complex and not considered here. It seems that smooth transition autoregressive models are very flexible and easy to implement and therefore employed in this study.

The second feature that discriminate STAR model from TAR and the Hamilton's Markov regime switching models is the state of economy determined by the models. In the Hamilton' Markov switching and TAR models the economy must be within a single regime in each time period. However, the smooth transition model allows the possibility that the economy may be in an intermediate state between, say, recession and expansion. Filardo's Markov switching model indicates again similar implications with STAR model by allowing the probability of remaining in a regime to be conditional on the lagged value of the leading indicator and lagged regime.

Another nice feature of the STAR models over other nonlinear models is that they nest linear regression model, and we can thus use linear Lagrange Multiplier (LM) tests for testing the null of linearity before fitting any nonlinear model. We can also use LM tests for choosing between the alternative STAR specifications.

Although STAR models are flexible and easy to implement it is difficult to reach a conclusion regarding the performances of nonlinear models in practice. One approach could be the use of all nonlinear models for the same data set and find out which one best fit the data. However, this procedure is very time consuming and we will only employ STAR models to examine the evidence of nonlinearity in our data. Next chapter discusses empirical findings.

2.5 Conclusion

Recent empirical econometrics literature has shown that economic variables may contain asymmetric cycles in their generating mechanisms. To capture this asymmetric behavior some empirical models have been developed. In this chapter the most prominent ones, Markov- switching regime model, threshold autoregressive (TAR) model and smooth transition autoregressive (STAR) model are briefly explained. In the Markov-switching models, the regimes associated with business cycle expansions and contractions and the switch between regimes described by a probabilistic fashion. Threshold autoregressive models specify the switch as a function of past values and STAR models are the smooth transition generalization of TAR models. We put more emphasis on STAR models in this chapter. This is because of the fact that STAR models are more flexible than TAR and Markov-switching models. In contrast to TAR and Hamilton type Markov-switching models, STAR model allow the possibility that the economy may be in intermediate states. These features make smooth transition models more appropriate for macroeconomic time series. Therefore, this study focuses on STAR models and the next chapter contains applications of them to certain financial variables.

CHAPTER III

EMPRICAL INVESTIGATIONS

3.1 Introduction

This chapter provides the empirical modeling of our data set using STAR models. We have considered three macroeconomic time series of Turkey which includes consumer price index (CPI) and nominal exchange rates; TL/$ and TL/£. All series are seasonally unadjusted monthly series and taken from the Central Bank of Turkey. As mentioned before there are two reasons to select these macroeconomic variables. The sufficient length of the sample size is the first reason and second is to analyze whether the variables used to examine PPP hypothesis contain nonlinearity. This is important because nonlinearity found in real exchange rate could be due to each of the component series or both. Sarno (1997) finds evidence of nonlinearity in real exchange rate of Turkey but does not provide any information about the possible nonlinearity in component series. Our objective is to close this gap.

The plan of this chapter is as follows. In Section 3.2, consumer price index of Turkey is analyzed and estimated STAR model is examined for its adequacy in all perspectives. The same analysis is performed for US Dollar, in Section 3.3 and for British Pound in Section 3.4.

3.2 STAR Modeling for Consumer Price Index

There are lots of studies concerning Turkish inflation rate. One of these studies is presented by Erlat. He tested whether inflation is stationary but exhibits long-memory and found a significant long-memory component. Other studies on Turkish inflation concerns the possible impacts of inflation and construct multivariate models. Onis and Ozmucur (1990) explore inflationary dynamics in Turkey with a VAR and a simultaneous equation model and find supply-side factors seem to have significant effects on inflation. Metin (1995) finds that fiscal expansion dominates Turkish inflation from 1950 to 1988. Diboglu and Kibritcioglu (2002) show terms of trade, monetary and balance of payment shocks figure prominently in the inflationary process. Unlike the recent literature, however, we believe that inflation indicates nonlinear behavior and constructed a STAR model for the data.

The series we consider in this section is the logarithmic form of the consumer price index (CPI) of Turkey, at the monthly frequency covering the period January 1987 - June 2001. The series, which is shown in Figure 3.1, show nonstationary behavior and requires a differencing procedure. After taking first difference of the logarithmic form, as shown in Figure 3.2, stationary is approximately obtained. Figure 3.2 indicates that the inflation data contains a pronounced seasonal pattern. Typically, inflation rate is above its average value during the winter, late summer and fall (January-March, July-December) and below average during spring (April-June). We assume that structure of seasonality is deterministic and the systematic component of seasonality can be adequately captured by monthly dummy variables, which are denoted as $S_{s,t}$, s = 1,…11. Where $S_{s,t}$ = 1 if observation t corresponds to month s and $S_{s,t}$ = 0 otherwise. In the light of the assumption of deterministic seasonality, seasonally adjusted data is obtained as the residuals from the regression of differenced data on a constant and $S_{s,t}$, s = 1,….11. Final form of data, which is used in modeling procedure, is shown in Figure 3.3.

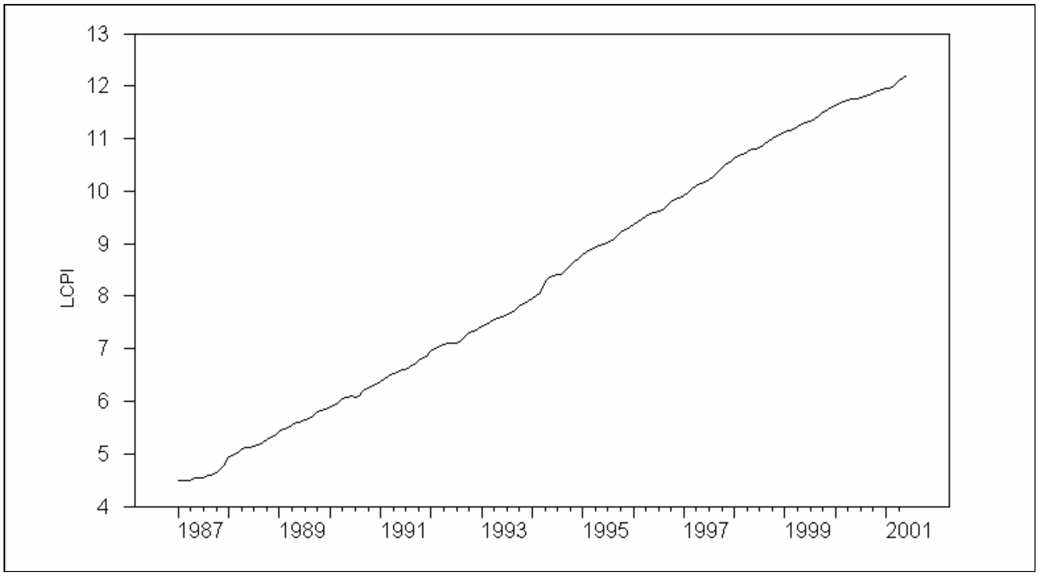

Figure 3.1: Logarithmic Form of CPI

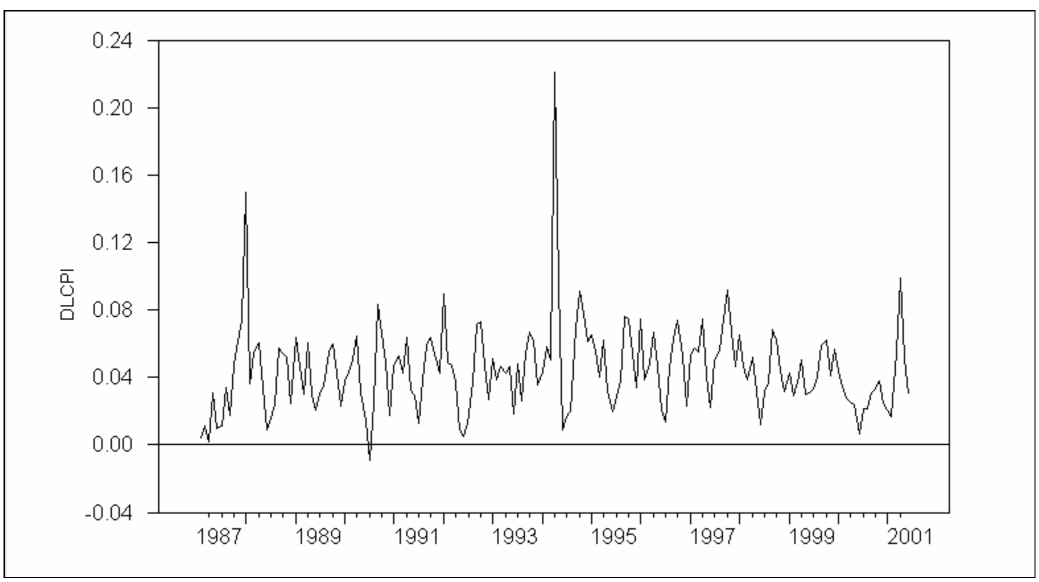

Figure 3.2: Differenced Form of CPI

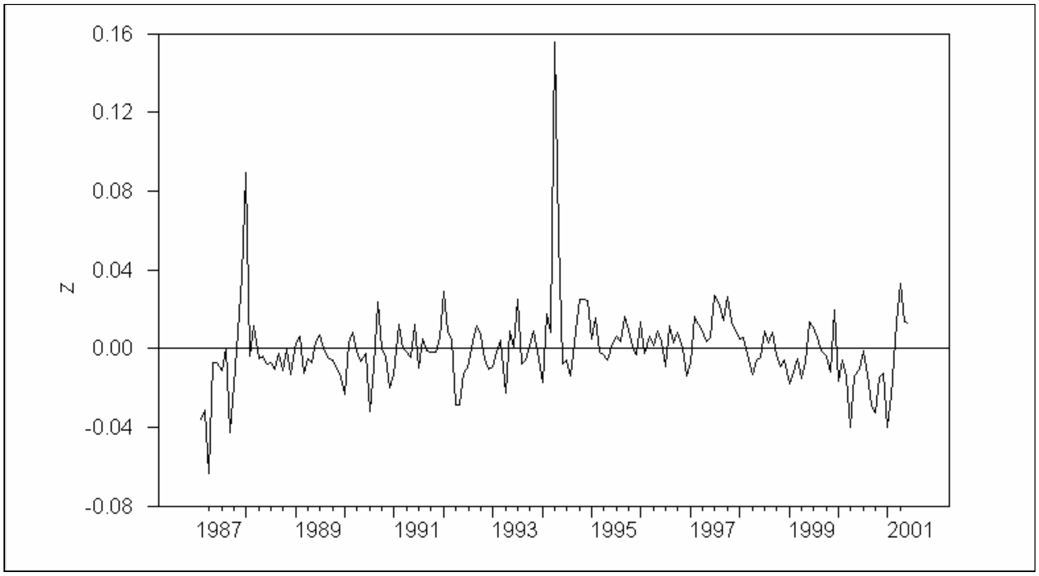

Figure 3.3: Seasonally Adjusted Form of CPI

Table 3.1 presents various statistics of seasonally adjusted inflation rate. The results show that the distribution is skewed, exhibits excess kurtosis and does not satisfy normality.

Table 3.1: Statistics of Seasonally Adjusted CPI

Sample Mean	-0.000	Significance Levels
Standard Error	0.021	
t-statistic	-0.000	1.000
Skewness	2.772	0.000
Kurtosis	20.076	0.000
Jarque-Bera	3127.048	0.000

The seasonally unadjusted data indicates that, Turkey is a high inflation country but the inflation in Turkey is not hyperinflation. In other words, it does not reach large three-digit levels but remains around a figure which is, consistently, greater than fifty percent but never goes beyond a hundred percent except for a couple of months in

1994. The extreme values are, located especially in 1988:1, 1994:4 and 2001:4, the result of economic crises experienced in Turkish economy. Common explanations of these episodes in inflation rate are devaluations, oil-price shocks, balance-of-payment crises, public sector deficits, the Persian Gulf crisis in 1990-1991, financial crises at home and abroad and recent earthquakes. If the determination of outliers is carried out statistically, that is, if we take the values above and below three standard deviation from its mean as extraordinary values, than 1987:4, 1988:1 and 1994:4 values are outliers. Because the extraordinary structure of 2001:4 value is not supported by standard deviation rule, we continue modeling cycle both with and without 2001:4 dummy variable.

Following the modeling cycle as outlined in Section 2.4.2, we start by specifying a linear AR model. AR model is parameterized by allowing for a maximum of 15 lags, p =1,……,15. According to the minimum AIC and SBC criteria and misspecification tests results, the best linear model contains only the first and fifth lagged term and the estimated equation is given in Table 3.2.

Table 3.2: Estimated Linear Models for CPI

$$\underset{(0.350)}{Z_t = -0.0009} + \underset{(0.000)}{0.3307Z_{t-1}} + \underset{(0.006)}{0.1299Z_{t-5}} + \underset{(0.000)}{0.0783D881} + \underset{(0.000)}{0.1523D944}$$

$$+ \underset{(0.013)}{0.0316D20014} + \hat{\varepsilon}_t \tag{3.1}$$

$\hat{\sigma}_\varepsilon$ =0.012 SK = 0.631 EK = 0.096 JB = 0.209 Q(12) = 0.831

AIC = -1457.96 SBC = -8.680

where p-values are given below the parameter estimates, $\hat{\varepsilon}_t$ denotes the regression residual at time t, $\hat{\sigma}_\varepsilon$ is the residual standard deviation, SK is skewness, EK is excess kurtosis, JB is the Jarque-Bera test of normality of the residuals, Q(12) is the Ljung-Box Q-Statistics for no residual autocorrelation. The results of these tests show that the estimated model passes basic diagnostic tests. There seems to be a weak evidence of excess kurtosis but very negligible.

The next stage of modeling cycle is to test linearity against STAR type nonlinearity using LM statistics, see Section 2.4.2.2. The results of linearity tests with and without dummy variables are reported in Table 3.3.

Table 3.3: Linearity Tests Results for CPI

F_{TS}-test (p-values)				
Variable	d	Without Dummy	With Dummy (88:1, 94:4)	With Dummy (88:1, 94:4,2001:4)
Z	1	0.214	0.054	0.056
	2	0.159	0.055	0.056
	3	0.590	0.048	0.046
	4	0.930	0.812	0.792
	5	0.760	0.104	0.108

Test results clearly show how outliers may affect linearity test results. As long as the results with dummies considered there is evidence of nonlinearity at first, second and third lag of the variable of interest. Whereas when dummy variables are excluded, than there remain no evidence of nonlinearity and STAR type nonlinearity is rejected for all possible values of delay parameter, d. Although according to results with dummy variables third lag seems to be optimal delay parameter, first and second lag can also be taken as delay parameters since they also provide significant p-value lower than 10%. We do not make model selection tests and estimate both ESTAR and LSTAR models to be able to see results of different specifications.

Twelve nonlinear models are estimated according to the procedure explained in Section 2.4.2.4. All nonlinear models are initially specified with maximum lag orders five, and then insignificant lags are deleted one by one (starting with the least statistically significant one according to the t-ratio) provided that such deletions reduce Akaike Information Criteria (AIC). Misspecification tests are applied to the estimated models as described in Section 3.4.2.5.1 and finally following adequate nonlinear models are obtained.

Table 3.4: Estimated Nonlinear Models for CPI

LSTAR Model:

$$\underset{}{Z_t} = \underset{(0.281)}{-0.0058} + \underset{(0.000)}{0.459} Z_{t-1} - \underset{(0.107)}{0.292} Z_{t-2} + \underset{(0.005)}{0.133} Z_{t-5} + \underset{(0.000)}{0.0765} D881 + \underset{(0.000)}{0.148} D944$$

$$+ \underset{(0.028)}{0.0274} D20014 + (\underset{(0.058)}{0.058} - \underset{(0.010)}{0.771} Z_{t-1} - \underset{(0.070)}{0.287} Z_{t-3})$$

$$* (1/(1+\exp((\underset{(0.071)}{-1.60}/0.0188) * (Z_{t-2} - \underset{(0.000)}{0.031})))) \qquad (3.2)$$

AIC = -1457.32   SK = 0.993   KU = 0.048 $LM_{ARCH}(12)$ = 0.879 $F_{AC}(12)$ = 0.333 F_{PC} = 0.326
$F_{NL,1}$ = 0.662 $F_{NL,2}$ =0.352 $F_{NL,3}$ = 0.084 $F_{NL,4}$ = 0.690 $F_{NL,5}$ =0.390 $\hat{\sigma}_{NL}$ = 0.0117

ESTAR Model:

$$Z_t = -0.002 + 0.471Z_{t-1} - 0.063Z_{t-3} + 0.075D881 + 0.150D944$$
(0.109) (0.000) (0.233) (0.000) (0.000)

$$+ 0.026D20014 + (0.0127 - 0.724Z_{t-1})$$
(0.041) (0.107) (0.005) (0.070)

$$* (1 - \exp(-0.204 / 0.00035) * (Z_{t-2} + 0.007)^2) \qquad (3.3)$$
(0.170) (0.077)

AIC = -1473.28 SK = 0.305 KU = 0.004 $LM_{ARCH}(12)$ = 0.836 $F_{AC}(12)$ = 0.166 F_{PC} = 0.936
$F_{NL,1}$ = 0.409 $F_{NL,2}$ =0.190 $F_{NL,3}$ = 0.170 $F_{NL,4}$ = 0.594 $F_{NL,5}$ =0.229 $\hat{\sigma}_{NL}$ =0.0120

LSTAR Model:

$$Z_t = -0.006 + 0.475Z_{t-1} - 0.345Z_{t-2} + 0.129Z_{t-5} + 0.076D881 + 0.148D944$$
(0.312) (0.000) (0.092) (0.007) (0.000) (0.000)

$$+ (0.065 - 0.796Z_{t-1} - 0.2307Z_{t-3})$$
(0.057) (0.009) (0.056)

$$* (1/(1 + \exp((-1.503 / 0.0188) * (Z_{t-2} - 0.032)))) \qquad (3.4)$$
(0.074) (0.000)

AIC = -1455.21 SK = 0.973 KU = 0.077 $LM_{ARCH}(12)$ = 0.552 $F_{AC}(12)$ = 0.328 F_{PC} = 0.281
$F_{NL,1}$ = 0.542 $F_{NL,2}$ =0.084 $F_{NL,3}$ = 0.099 $F_{NL,4}$ = 0.633 $F_{NL,5}$ =0.305 $\hat{\sigma}_{NL}$ = 0.0119

ESTAR Model:

$$Z_t = -0.002 + 0.487Z_{t-1} - 0.140Z_{t-3} + 0.142Z_{t-5} + 0.072D881 + 0.146D944$$
(0.046) (0.000) (0.012) (0.003) (0.000) (0.000)

$$+ (0.023 - 0.872Z_{t-1})$$
(0.033) (0.002)

$$* (1 - \exp((-0.152 / 0.00035) * (Z_{t-2} + 0.008)))) \qquad (3.5)$$
(0.103) (0.028)

AIC = -1462.24 SK = 0.886 KU = 0.068 $LM_{ARCH}(12)$ = 0.505 $F_{AC}(12)$ = 0.228 F_{PC} = 0.215
$F_{NL,1}$ = 0.227 $F_{NL,2}$ =0.106 $F_{NL,3}$ = 0.226 $F_{NL,4}$ = 0.649 $F_{NL,5}$ =0.3173 $\hat{\sigma}_{NL}$ =0.0118

According to the minimum AIC criteria and misspecification test results the final model which best fits to the data is an ESTAR with d = 2. The estimated equation and the result of diagnostic tests are given in Equation 3.3.

We start to analyze the estimated nonlinear model with interpretable coefficients, which are transition parameter γ_E and location parameter c_E. Estimated threshold value $\hat{c}_E$ is equal to -0.007 and this means that, estimated model is in the middle regime (moderate inflation) when the transition variable, Z_{t-2}, is about -0.007 and in the outer regime (low or high inflation) when Z_{t-2} value goes to far away from this value. In other words, the closeness of the two month lagged value to the value -0.007 determines the state of economy. The other important point to notice is that there are many observations lying on both sides of location parameter $\hat{c}_E$ = -0.007, which is shown in Figure 3.4, and this creates a symmetric and U-shaped transition function, supporting the ESTAR model without any doubt. Moreover small values of F_E are more common than large ones implying the possibility that the usual state of economy is the moderate inflation period. Indeed, with a few exceptions, Turkey has never experienced a very high or low inflation periods compared to the periods represented by the middle regime.

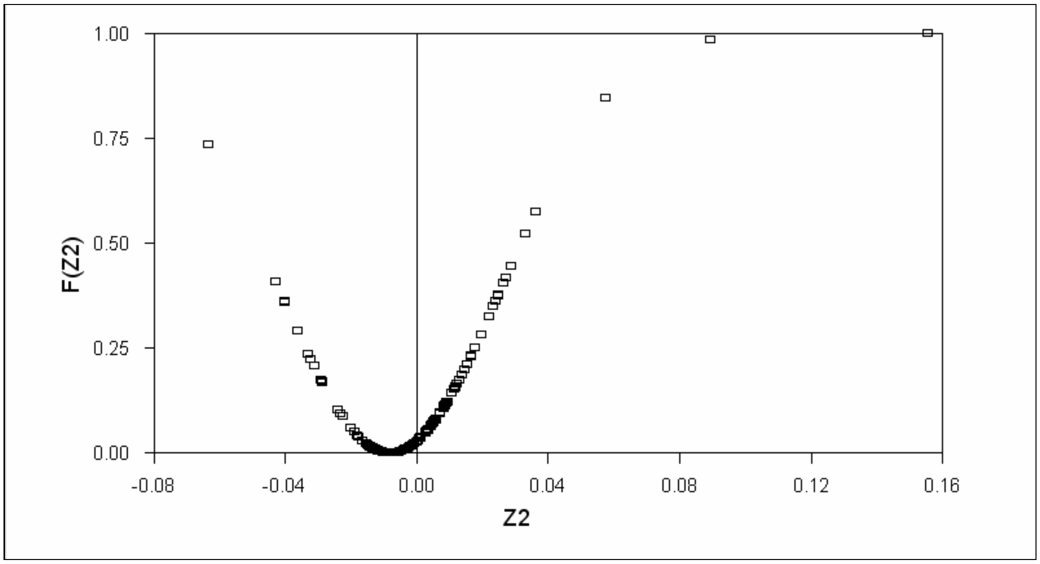

Figure 3.4: Transition Function for CPI

To highlight the dynamic behavior of the ESTAR model, the models corresponding to $F_E = 0$ and 1 need to be analyzed. In our model, middle regime refers to moderate inflation case and outer regime means high and low-inflation period. The corresponding linear models for $F_E = 0$ and $F_E = 1$ are

$F_E = 0$;

$$Z_t = -0.002 + 0.471 Z_{t-1} - 0.063 Z_{t-3} + 0.075 D881 + 0.150 D944 + 0.26 D20013$$

$F_E = 1$;

$$Z_t = 0.0107 - 0.253 Z_{t-1} - 0.063 Z_{t-3} + 0.075 D881 + 0.150 D944 + 0.26 D20013$$

As seen, although same variables contained in both linear models corresponding to different phases, the coefficients show great differences. For example, the coefficient of Z_{t-1} shows a change not only in size but also in sign, implying asymmetric behavior of inflation rate. This means that the dynamics of inflation differs according to the period in which the economy is in.

Table 3.5: Regime Analysis Statistics for CPI

Variable	Regime	F_E	F_L	Root	Modules
CPI	Middle	0	—	-0.288	0.288
				$0.379 \mp 0.273i$	0.467
	Outer	1	—	0.503	0.503
				$0.125 \mp 0.331i$	0.354

The roots of the characteristic equations corresponding to each regime or linear model are presented in Table 3.5. The mid regime contains a pair of complex roots with modulus 0.467 and one real root, which is smaller than one in absolute value. So, the mid-regime is locally stationary. The outer regime indicates the same dynamic with a real root and a pair of complex roots having modulus 0.354. This implies that to move inflation from one regime to another a big external shock or a sequence of minor ones with same signs are necessary.

To compare estimated linear and nonlinear models, we first check the reduction in the estimated residual variances. As shown in the Table 3.6, the nonlinear ESTAR model provides lower residual variance, $\hat{\sigma}_{NL}{}^{2} / \hat{\sigma}_{L}{}^{2} = 0.998$. So the gain of the model over its linear counterpart in terms of residual variance is 0.2 %. This gain is not very high but this is not surprising considering linearity test results. That is there is a mild or no evidence of nonlinearity in inflation rate. This may be because of the fact that CPI is an aggregate variable and this aggregation procedure may smooth the nonlinear structure. That is the component series of CPI may contain nonlinearities and aggregation may have smoothed out these nonlinearities.

Table 3.6: Comparison Statistics for Linear and Nonlinear Models for CPI

	Forecast Performance		
$\hat{\sigma}_{NL}^{2} / \hat{\sigma}_{L}^{2}$	Nonlinear RMSE	Nonlinear/Linear RMSE	S_1(p-value)
0.966	0.0169	1.096	0.875

We also compare forecast performances of the specified linear and nonlinear models. For this purpose, we use 18 months as the forecast period. The models are specified using the whole sample period, but the forecasts are made by recursively re-estimating the models as each observation is added during this period. The forecasts are evaluated according to the two criteria, the RMSE and the test of equal forecast accuracy due to Diebold and Mariano (1995). The test of forecast accuracy considers a sample path of a loss-differential series, $d_t=\{g(e_{i,t})-g(e_{j,t})\}$, for rival forecasts i and j, t = 1,…..,T. We use the mean-square error as the Standard of forecast quality; that is, $d_t = \hat{e}^2{}_{1,t} - \hat{e}^2{}_{2,t}$.

Table 3.6 provides the RMSE of the 18 months forecasts for linear and nonlinear models and the results for the null hypothesis of equal forecast accuracy between linear and nonlinear ESTAR models. As expected from the linearity test results, nonlinear models do not provide any forecast gain over the linear model. The forecast equality test also suggests that there is no evidence of nonlinearity in terms of the predictive performance of the nonlinear model. The reason for this outcome could be the fact that forecast performance of nonlinear models depends on the period in which forecast was made.[1] The other important point shown in the Figure

[1] See, Granger and Terasvirta (1993) and Ocal and Osborn (2000)

3.5 is that ESTAR model seems to be capturing most of the turning points while linear model continue as a straight line during the forecast period. This means that nonlinear model may signal turning points, while linear model does not. As seen from the Figure 3.5, nonlinearity is needed mainly to capture the periods indicating high price changes and forecast periods include only a few such data points. To summarize we may conclude that nonlinearity is needed only to capture several high inflation period and therefore does not provide big substantial improvement over the linear model. Nevertheless, the estimated ESTAR model may be suffering from over fitting and a longer sample may help to remedy this.

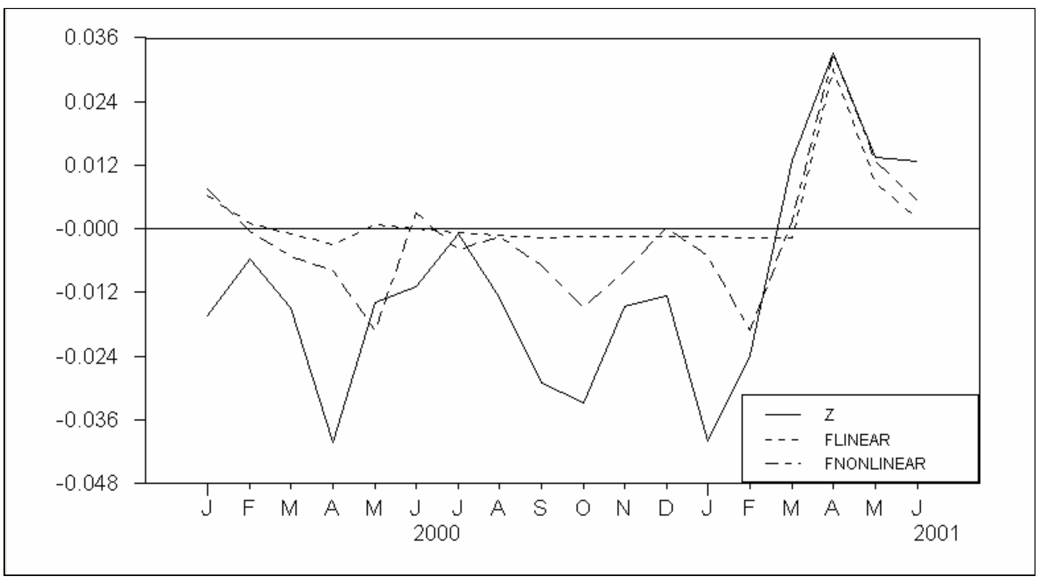

Figure 3.5: Forecast Results for CPI

3.3 STAR Modeling for US Dollar

In this section we analyze the logarithmic form of the TL/$ series with sample period January 1987 to June 2001. As seen from the Figure 3.6, like consumer price index, logarithmic form of the exchange rate indicates nonstationarity. To solve this nonstationarity problem, differencing procedure is applied and stationarity is obtained, as indicated in Figure 3.7. After this procedure, seasonality effect on the data appears evidently. It is assumed seasonality is deterministic and deseasonalization procedure is applied by regressing differenced data on a constant and $S_{s,t}$, s = 1,…..11. However, the regression results indicate that only S_1 and S_{11} monthly dummies are significant, therefore the regression is run once more with only these dummy variables and seasonally adjusted data is obtained. Final form of the data that will be used in modeling is shown in Figure 3.8.

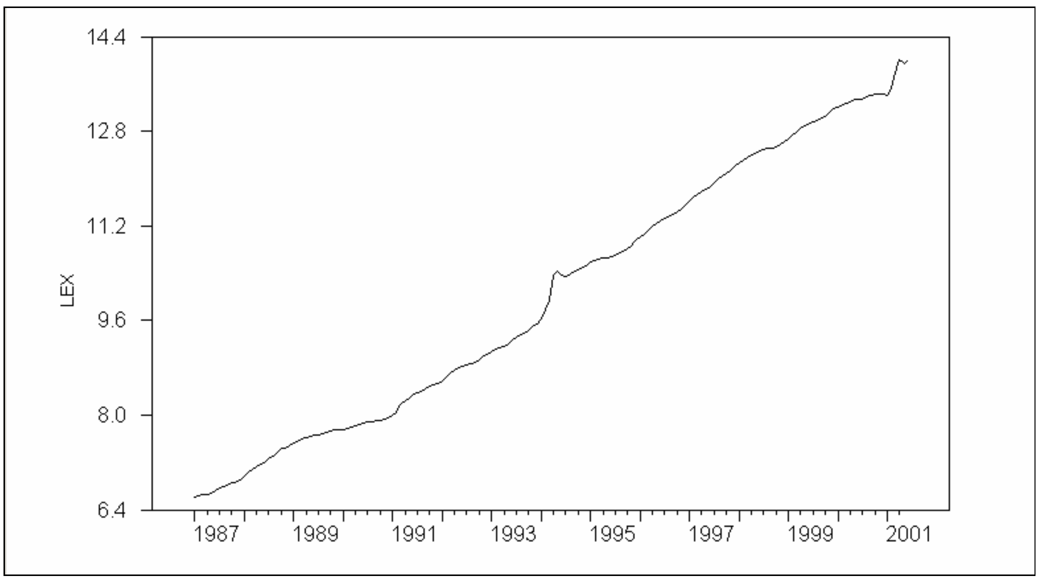

Figure 3.6: Logarithmic Form of TL/$

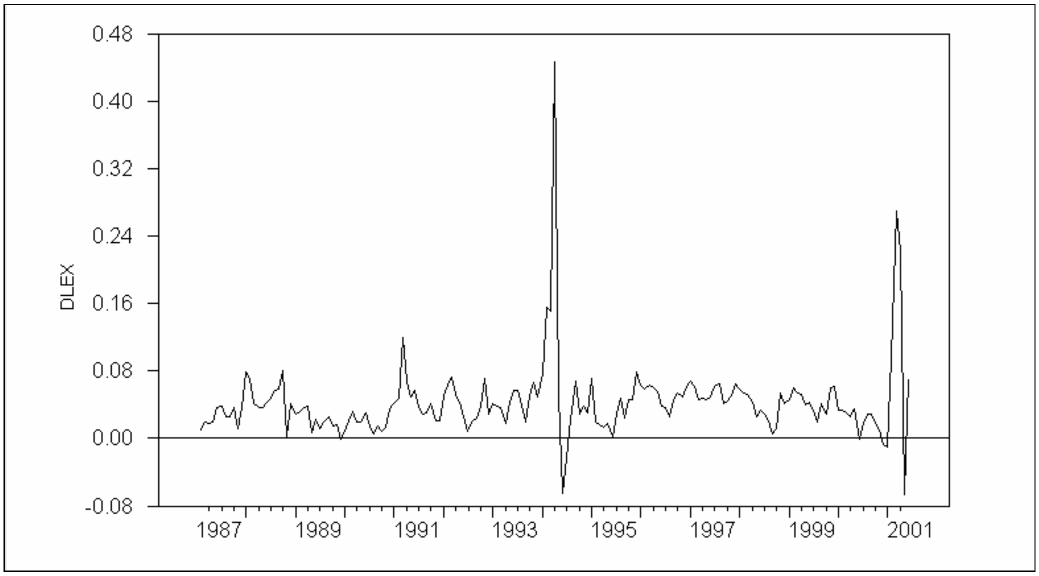

Figure 3.7: Differenced Form of TL/$

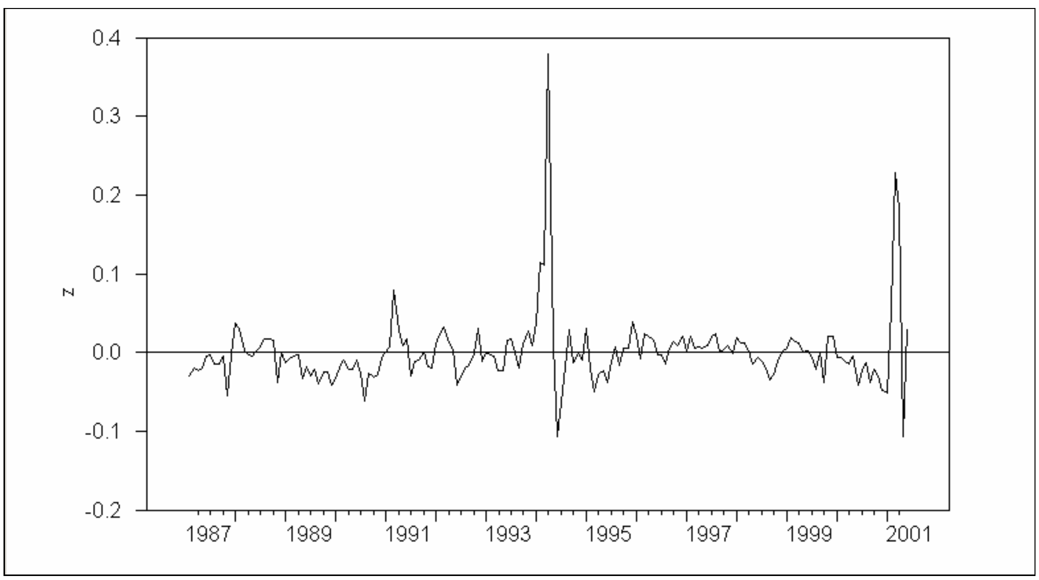

Figure 3.8: Seasonally Adjusted Form of TL/$

Combining Figure 3.8 with the statistics in Table 3.7, it is obvious that the data in use have significant skewness, excess kurtosis and normality problems

Table 3.7: Statistics of Seasonally Adjusted TL/$

Sample Mean	-0.000	Significance Levels
Standard Error	0.046	
t-statistic	-0.000	1.000
Skewness	4.345	0.000
Kurtosis	30.830	0.000
Jarque-Bera	7396.475	0.000

Like the case of consumer price index, exchange rate series suffers from abnormal values. The values being outside of the three standard deviation band, are taken as outliers. The data points are April 1994, March 2001 and April 2001 and these correspond to the economic crises in the economy.

The modeling procedure applied is as follows. Firstly, a linear AR model is specified to construct a basis for STAR type nonlinear model. We start with a maximum of p_{max} = 15 lags of variable of interest and restrict the general model according to the AIC and SBC values. Misspecification tests results of the residuals are also considered and three suitable linear AR models are obtained, and these are given in Table 3.8.

Table 3.8: Estimated Linear Models for TL/$

$$\underset{}{Z_t} = \underset{(0.940)}{0.0002} + \underset{(0.000)}{0.382} Z_{t-1} \tag{3.6}$$

$\hat{\sigma}_{\varepsilon}$=0.042 SK = 0.000 EK = 0.000 JB = 0.000 Q(12) = 0.990

AIC = -1080.19 SBC = -6.28

$$Z_t = -0.0039 + 0.2030 Z_{t-1} - 0.1620 Z_{t-2} + 0.379 D944 + 0.2130 D20013$$
$$\quad (0.056) \quad (0.000) \quad (0.001) \quad (0.000) \quad (0.000)$$

$$+ 0.149 D20014 + \hat{\varepsilon}_t \qquad (3.7)$$
$$(0.000)$$

$\hat{\sigma}_\varepsilon$ =0.026   SK = 0.001   EK = 0.000   JB = 0.000   Q(12) = 0.001

AIC = -1226.92   SBC = -7.170

$$Z_t = -0.003 + 0.211 Z_{t-1} - 0.186 Z_{t-2} + 0.030 Z_{t-3} + 0.004 Z_{t-4} + 0.0169 Z_{t-5} \qquad (3.8)$$
$$\quad (0.099) \quad (0.000) \quad (0.003) \quad (0.629) \quad (0.948) \quad (0.794)$$

$$- 0.061 Z_{t-6} + 0.025 Z_{t-7} + 0.026 Z_{t-8} + 0.381 D944 + 0.212 D20013$$
$$\quad (0.351) \quad (0.699) \quad (0.026) \quad (0.000) \quad (0.000)$$

$$+ 0.150 D20014 + \hat{\varepsilon}_t$$
$$(0.000)$$

$\hat{\sigma}_\varepsilon$ =0.026   SK = 0.001   EK = 0.000   JB = 0.000   Q(12) = 0.001

AIC = -1161.77   SBC = -7.040

The first model is constructed without considering outlier effect and the other two are specified with dummy variables. As seen all three models' residuals have skewness, excess kurtosis and normality problem. In addition to these problems, second and third model have a significant autocorrelation problem. Two reasons for the poor results could be the treatment of outliers and deseasonalization procedure. The treatment of outliers is an important practical problem in nonlinear economic modelling[2], but since they are not central of interest to this study and they are on their own a very comprehensive topic, we removed them directly by using corresponding dummy variables.

[2]See van Dijk et al., 1999

Second, deseasonalization is applied in the light of the deterministic seasonality assumption without being supported with any test result. However, the autocorrelation problem, shared by the models with dummy variables, can be the result of the necessity of a nonlinear structure. Linear autoregressive models constructed with dummy variables up to order 24 continue to exhibit autocorrelation which is significant at 5 % level and this points out that residual autocorrelation problem is not a simple consequence of order misspecification but may be a consequence of neglected nonlinearity.

If we ignore misspecification problems for the present, according to the minimum AIC criteria, the best linear model seems to be second model. However to have a flexible modeling, we decide to take third model as our base linear model. Nonlinearity test is carried out for all possible delay parameters d = 1,....,8. The results of linearity test with and without dummy variables are reported in Table 3.9. It should be noted that although we selected a maximum lag length of 15 for linear modeling, an AR (8) is preferred for nonlinear specification simply due to possible degrees of freedom problem in nonlinearity tests. Moreover, as noted before, high orders do not bring solution to the autocorrelation problem observed in linear models.

Table 3.9: Linearity Tests Results for TL / $

F_{TS}-test (p-values)			
Variable	D	Without Dummy	With Dummy
Z	1	0.000	0.000
	2	0.000	0.000
	3	0.000	0.000
	4	0.000	0.000
	5	0.000	0.000
	6	0.004	0.001
	7	0.001	0.001
	8	0.000	0.000

Table 3.9 shows that there is strong evidence of nonlinearity irrespective of outliers. This evidence of nonlinearity is found at all possible delay parameters d = 1,....,8. Therefore, we search for a suitable nonlinear STAR model for all possible lagged terms.

Following the procedure discussed in Section 2.4.2.4, thirty-two STAR models are estimated. More specifically, eight models are constructed with logistic transition function and eight with exponential transition function and these models are analyzed both with and without dummy variables. The general case of these models is constructed with maximum lag order of eight, than insignificant lags are reduced one by one (starting with the most statistically insignificant one according to the t-ratio) provided that such reductions decrease AIC value. The adequacy of reduced models is checked by misspecification tests, mentioned in Section 2.4.2.5.1. However, most of the reduced models are eliminated because of the residual autocorrelation and additional nonlinearity problem. The reasons of these poor diagnostics could be handling of outliers and deseasonalization procedure as discussed in the previous section. Nevertheless, another important reason could be the fact that a multiple regime model may be necessary to capture the dynamic structure. If we do not consider these problems, the most suitable nonlinear STAR model according to AIC is an ESTAR model with d = 2 and its equation is provided in Table 3.10. The only problem with the selected model is additional nonlinearity for delay of seven month, skewness and excess kurtosis in the residuals.

Table 3.10: Selected STAR Model for TL / $

ESTAR Model:

$$\underset{}{Z_t} = \underset{(0.709)}{-0.001} + \underset{(0.000)}{0.683Z_{t-1}} - \underset{(0.034)}{0.154Z_{t-6}} + \underset{(0.028)}{0.189Z_{t-8}} + \underset{(0.000)}{0.393D944} + \underset{(0.000)}{0.217D20013}$$

$$+ \underset{(0.034)}{0.058D20014} + (\underset{(0.238)}{-0.022} - \underset{(0.000)}{1.693Z_{t-1}} - \underset{(0.012)}{0.487Z_{t-4}} + \underset{(0.005)}{1.739Z_{t-6}} - \underset{(0.002)}{2.965Z_{t-8}})$$

$$*(1-\exp(-\underset{(0.000)}{0.121}/0.0019)*(Z_{t-2} - \underset{(0.002)}{0.018})^2) \qquad (3.9)$$

AIC = -1240.41 SK = 0.002 KU = 0.000 $LM_{ARCH}(12)$ = 0.138 $F_{AC}(12)$ = 0.742 F_{PC} = 0.931
$F_{NL,1}$ = 0.187 $F_{NL,2}$ =0.216 $F_{NL,3}$ = 0.210 $F_{NL,4}$ = 0.195 $F_{NL,5}$ =0.846 $F_{NL,6}$ =0.878
$F_{NL,7}$ =0.035 $F_{NL,8}$ =0.107 $\hat{\sigma}_{NL}$ =0.0205

According to the estimation results, the switch between two regimes, transition from middle to outer regime, occurs at the value of 0.018. The middle regime applies for values of the variable about 0.018 and the outer regime is for values far away from this value. Combining Figure 3.8 with Figure 3.9, we see that most of the observations are below the threshold value and the ones greater than the value belong to the crises periods. This nonsymmetrical structure of the transition function may be the sign of the need of a second transition function, namely a two-transition STAR model.[3]

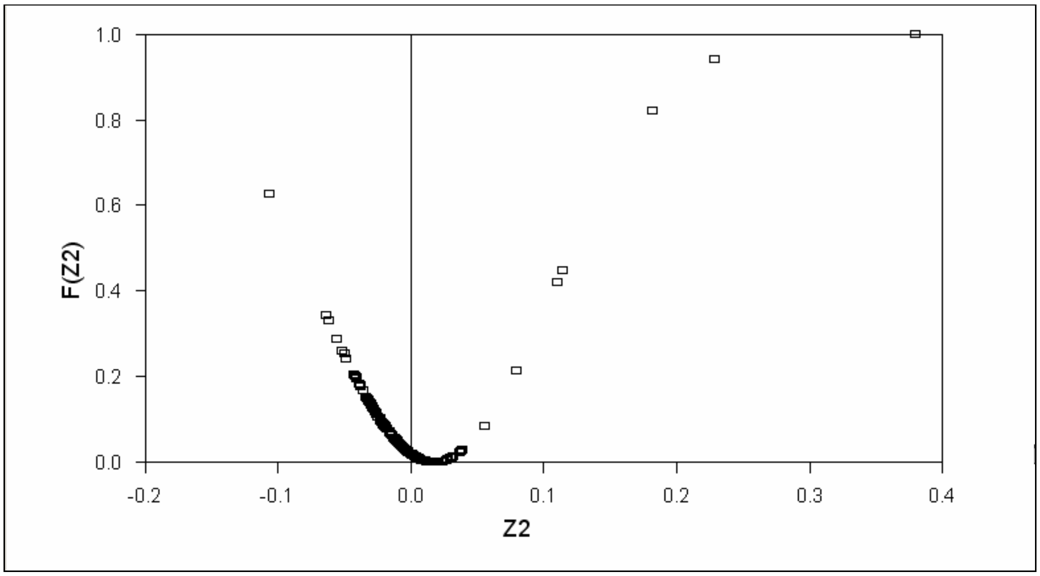

Figure 3.9: Transition Function of TL/$

To be able to see the different dynamics corresponding to two distinct regimes, two extreme cases of the transition function is considered. These cases correspond to two different linear AR models, one for middle regime ($F_E = 0$) and second for outer regime ($F_E = 1$). The models are

[3]See, Ocal and Osborn (2000) for a discussion

$F_E = 0$;

$$Z_t = -0.001 + 0.683Z_{t-1} - 0.154Z_{t-6} + 0.189Z_{t-8} + 0.393D944 + 0.217D20013 + 0.058D20014$$

$F_E = 1$;

$$Z_t = -0.023 - 1.010Z_{t-1} - 0.487Z_{t-4} + 1.585Z_{t-6} - 2.776Z_{t-8} + 0.393D944 + 0.217D20013 + 0.058D20014$$

Comparing these two linear models, it can be obviously seen that the estimated coefficients show great differences. Moving from the mid-regime to the outer regime, the coefficients of variables, Z_{t-2}, Z_{t-6}, Z_{t-8}, change both in size and in sign. So, exchange rate series exhibit different dynamics in different phases.

Table 3.11: Regime Analysis Statistics for TL / $

Variable	Regime	F_E	F_L	Root	Modules
CPI	Middle	0	—	0.87	0.87
				$0.72 \mp 0.48i$	0.86
				$0.08 \mp 0.83i$	0.83
				$-0.54 \mp 0.53i$	0.76
				-0.69	0.69
	Outer	1	—	$-1.18 \mp 0.12i$	1.19
				$-0.66 \mp 1.06i$	1.25
				$0.41 \mp 1.04i$	1.12
				$0.928 \mp 0.37i$	0.99

Table 3.11 provides the roots of the corresponding equations. The mid-regime with $Y_{t-2} = c_E = 0.018$ and $F_E = 0$, contains three pairs of complex roots and two real roots. According to the modules and absolute value of real roots, the mid-regime is locally

stationary. Unlike the mid-regime, outer regime with four pairs of complex roots shows nonstationary behavior with high modules. This suggests that when the process is in the outer regime it moves on to the middle regime very quickly but not vice versa. This is an expected behavior of exchange rate data. Such that if we consider the location parameter as the equilibrium point, one way or another the process has to turn this value. That is a highly appreciated or depreciated currency will have to move its equilibrium value. At that point nonlinearity indicates its importance once more. Ignoring nonlinear behavior of US Dollar series may result in wrong conclusions about stationary properties of the series. These conclusions may be due to the fact that the variable exhibits distinct dynamics in different regimes. That is we may have nonstationary and stationary regimes but overall a stationary model.

To make a comparison between linear and nonlinear models, we first inspect the reduction in the estimated residual variances. In Table 3.12, it can be seen that nonlinear model gives lower residual variance with the ratio of $\hat{\sigma}_{NL}^{2} / \hat{\sigma}_{L}^{2} = 0.628$ and the gain obtained from using nonlinear specification is 37.2 %, which is quite large. It is worthy of note that although our nonlinear model has poor diagnostics, it provides a better characterization of the data. This might suggest that considering multiple regime models may not only improve the diagnostics but also description of the data. However, we do not consider multiple regime models considering the large number of parameters to be estimated and sample size. Moreover these poor diagnostics may be a result of highly erratic structure of data.

Table 3.12: Comparison Statistics for Linear and Nonlinear Models for TL/$

	Forecast Performance		
$\hat{\sigma}_{NL}^{2} / \hat{\sigma}_{L}^{2}$	Nonlinear RMSE	Nonliear/Linear RMSE	S_1(p-value)
0.628	0.01697	1.0955	0.848

The forecast performances of the estimated linear and nonlinear models are also considered. As in the consumer price index, 18 months period is used for this purpose and the forecasts are evaluated according to the RMSE and the test of equal forecast accuracy results.

According to the Table 3.12, the forecast equality test result indicates that there is no gain in using nonlinear structure for prediction purpose, linear model is adequate. However, the estimated nonlinear ESTAR model gives quite lower residual variances and better forecasts in crises periods, see Figure 3.10. This may be because of the fact that nonlinear model exhibits a better forecast performance for the periods showing unexpected behavior and forecast period does not contain such periods frequently. Also change points seem to be better described by the estimated ESTAR model. This is an important point because the identification of the appropriate nonlinear dynamics of exchange rate is crucial as exchange rate may serve as one of potential intermediate policy tools in the economy. Moreover, additive nonlinearity test results show that there is a need for two threshold model with second delay parameter being d = 7. So, two-threshold model may be needed. However, two-threshold models contain more parameters to be estimated and therefore need larger samples. As a result, we assume that poor diagnostics and evidence of additional nonlinearity could be due to the highly erratic structure of the data as can be observed in exchange rate data.

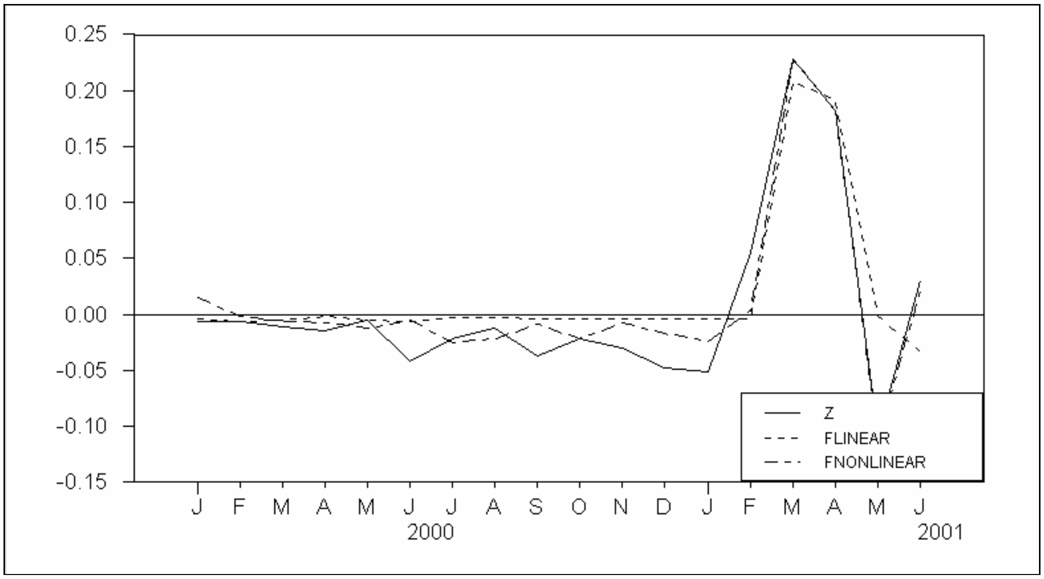

Figure 3.10: Forecast Results for TL/$

3.3 STAR Modeling for British Pound

British Pound series with sample period January 1987 to June 2001 is examined in this part of the study. Similar to other two applied series, logarithmic form of the TL/£ is nonstationary and needs a differencing procedure to be able to reach a stationary series, as indicated in Figure 3.11 and 3.12. After that, to remove seasonality effect, an auxiliary regression is constructed with constant and significant seasonal dummies, as mentioned in Section 3.2 and 3.3. Final form of the data that will be used in modeling procedure is shown in Figure 3.13.

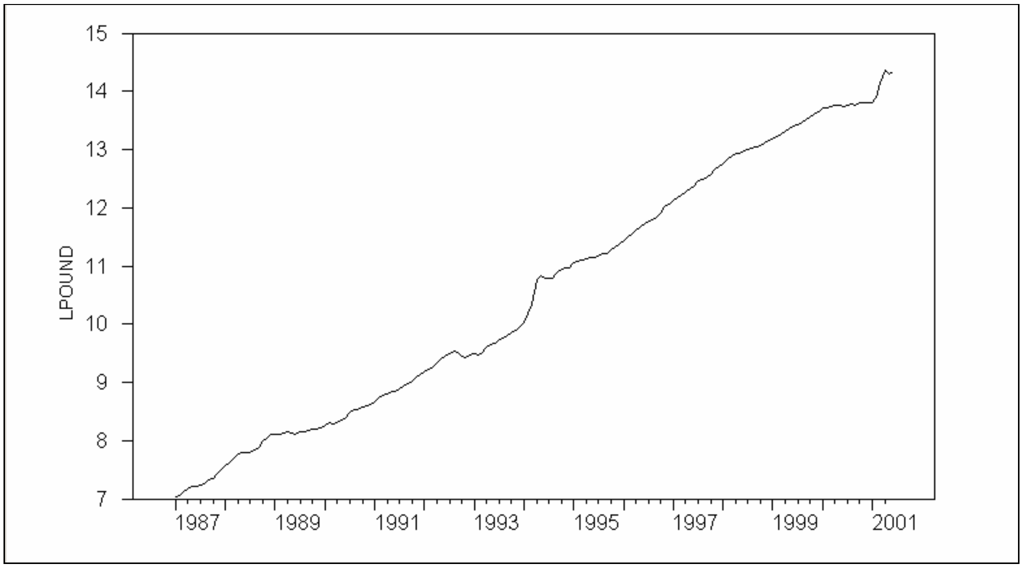

Figure 3.11: Logarithmic Form of TL/£

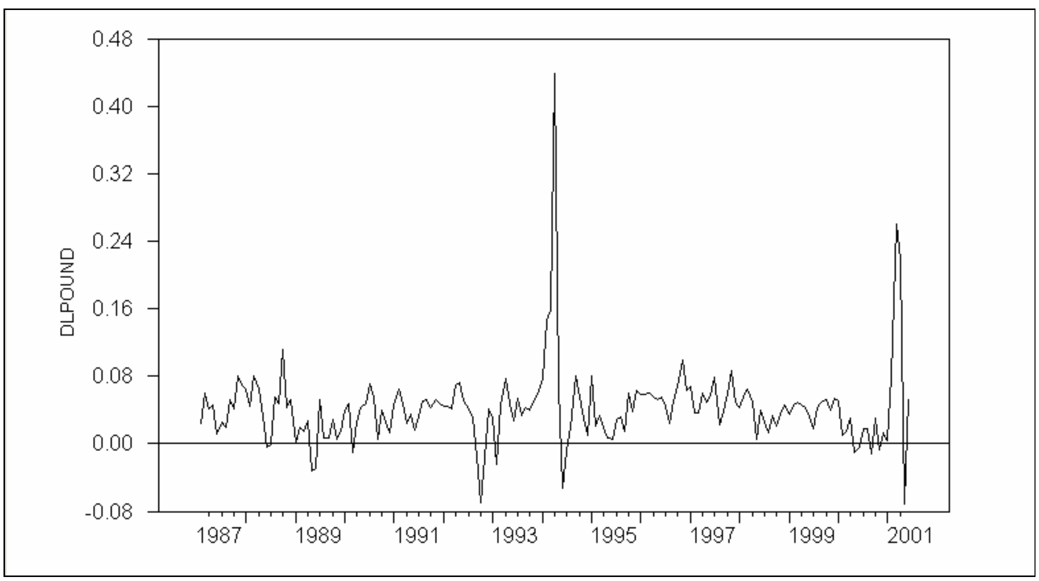

Figure 3.12: Differenced Form of TL/£

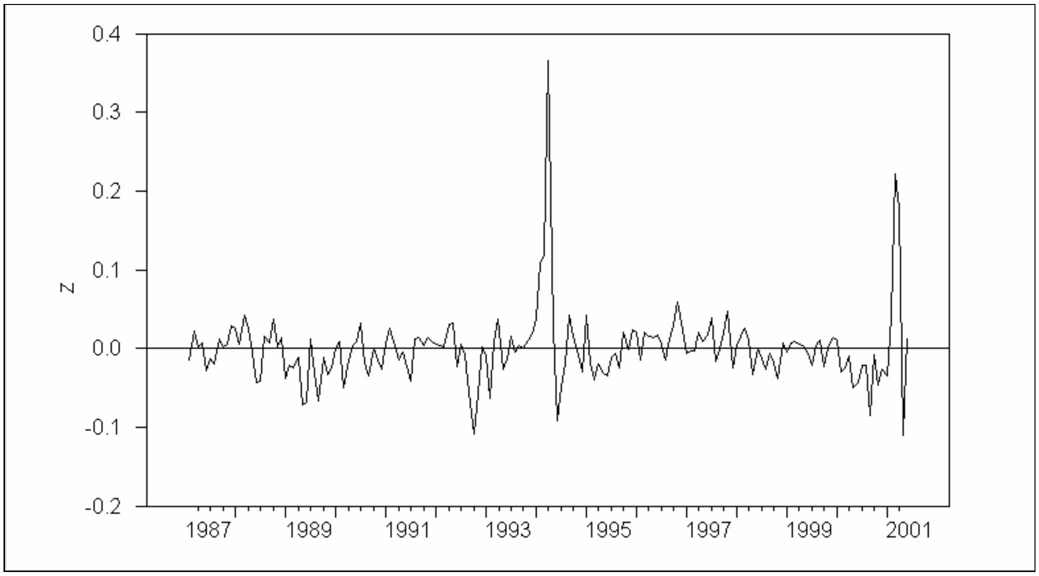

Figure 3.13: Seasonally Adjusted Form of TL/£

Table 3.13 contains some statistics of the seasonally adjusted British Pound series and we see that there is significant skewness, excess kurtosis and normality problems. The other important point to be mentioned in the structure of the data is outlier problem. According to the band constructed with three standard deviation from the mean, the values April 1994, March 2001 and April 2001 are abnormal and dummy variables are used for these data points.

Table 3.13 : Statistics of Seasonally Adjusted TL/£

Sample Mean	0.000	Significance Levels
Standard Error	0.047	
t-statistic	0.000	1.000
Skewness	3.450	0.000
Kurtosis	23.519	0.000
Jarque-Bera	4330.678	0.000

Modeling cycle procedure is started with specifying a suitable linear AR model. Again we start with a maximum of p_{max} = 15 lagged of variable of interest and impose restrictions to the general model with considering AIC, SBC values and misspecification tests results of the residuals. Three linear AR model is obtained as shown in Table 3.14. First model contains no outlier effect and constructed without dummy variables, while other two are specified with suitable dummies. Misspecification tests results indicate that excess kurtosis and normality problems are common features of all three models. Skewness problem is present in the first model and residual autocorrelation problem is significant in the second and third model. As mentioned in Section 3.3, the reasons of poor diagnostics may be the treatment of outliers and applied deseasonalization procedure. However, linear AR models constructed with dummy variables up to order 24 continue to have significant residual autocorrelation problem, this may be due to the nonlinear structure. If we ignore these problems, the best linear AR model to use for STAR type modeling is model 3.12 in Table 3.14.

Table 3.14: Estimated Linear Models for British Pound

$$Z_t = \underset{(0.972)}{0.00012} + \underset{(0.000)}{0.372} Z_{t-1} \tag{3.10}$$

$\hat{\sigma}_\varepsilon$ =0.044 SK = 0.000 EK = 0.000 JB = 0.000 Q(12) = 0.586

AIC = -1069.24 SBC = -6.22

$$Z_t = -0.004 + 0.219 Z_{t-1} - 0.162 Z_{t-2} + 0.360 D944 + 0.2109 D20013$$
$$\quad (0.080) \quad (0.000) \qquad (0.003) \qquad (0.000) \qquad (0.000)$$

$$+ 0.1404 D20014 + \hat{\varepsilon}_t \tag{3.11}$$
$$(0.000)$$

$\hat{\sigma}_\varepsilon$ =0.0297 SK = 0.511 EK = 0.000 JB = 0.000 Q(12) = 0.000

AIC = -1191.57 SBC = -6.927

$$Z_t = -0.004 + 0.237 Z_{t-1} - 0.212 Z_{t-2} + 0.923 Z_{t-3} + 0.023 Z_{t-4} - 0.0153 Z_{t-5}$$
$$\quad (0.113) \quad (0.000) \qquad (0.001) \qquad (0.172) \qquad (0.739) \qquad (0.826)$$

$$- 0.0499 Z_{t-6} - 0.005 Z_{t-7} + 0.028 Z_{t-8} + 0.359 D944 + 0.216 D20013 \tag{3.12}$$
$$\quad (0.481) \qquad (0.936) \qquad (0.658) \qquad (0.000) \qquad (0.000)$$

$$+ 0.142 D20014 + \hat{\varepsilon}_t$$
$$(0.000)$$

$\hat{\sigma}_\varepsilon$ =0.0298 SK = 0.741 EK = 0.000 JB = 0.000 Q(12) = 0.000

AIC = -1123.96 SBC = -6.81

Nonlinearity test is carried out for d = 1,…,8. As noted before this order is selected due to degrees of freedom considerations. Moreover, the evidence of autocorrelation in linear model is not a result of short lag length since an AR(24) model does not solve the problem. The use of dummy variables does not cause a significant change in the test results. There is a significant nonlinearity for all possible values of delay parameter and therefore all alternatives are considered.

Table 3.15: Linearity Tests Results for TL/£

F_{TS}-test (p-values)			
Variable	d	Without Dummy	With Dummy
Z	1	0.000	0.000
	2	0.000	0.000
	3	0.000	0.000
	4	0.000	0.001
	5	0.014	0.019
	6	0.003	0.006
	7	0.037	0.048
	8	0.031	0.027

Thirty-two STAR models are estimated. Following the model reduction procedure as in Section 2.2.4, the most parsimonious model is obtained. The most suitable nonlinear model is selected according to the misspecification tests results and AIC and SBC values. However, like US Dollar case, most of the estimated models suffer from residual autocorrelation, which may be the result of the deseasonalization procedure, handling of outliers and the need of additive nonlinearity and/or highly erratic structure of the data. Among the estimated models, the one which best fits the data is an ESTAR model with d = 2 and its estimated equation is presented in Table 3.16.

Table 3.16: Selected STAR Model for TL/£

ESTAR Model:

$$Z_t = -0.007 + 0.5725Z_{t-1} + 0.236Z_{t-4} - 0.321Z_{t-5} + 0.366D944 + 0.208D20013$$
$$\quad (0.765)\ (0.000) \quad (0.007) \quad (0.002) \quad (0.000) \quad (0.000)$$

$$+ 0.082D20014 + (0.00004 - 0.7656Z_{t-1} - 0.417Z_{t-2} + 0.792Z_{t-3} - 1.166Z_{t-4}$$
$$(0.012) \quad (0.998) \quad (0.000) \quad (0.000) \quad (0.000) \quad (0.027)$$

$$+ 2.174Z_{t-5}) * (1 - \exp(-0.202/0.00207) * (Z_{t-2} - 0.001)^2) \qquad (3.13)$$
$$(0.029) \quad (0.053) \quad (0.818)$$

AIC = -1195.74 SK = 0.761 KU = 0.114 $LM_{ARCH}(12)$ = 0.055 $F_{AC}(12)$ = 0.069 F_{PC} = 0.638
$F_{NL,1}$ = 0.023 $F_{NL,2}$ =0.465 $F_{NL,3}$ = 0.694 $F_{NL,4}$ = 0.241 $F_{NL,5}$ =0.164 $F_{NL,6}$ =0.426
$F_{NL,7}$ =0.419 $F_{NL,8}$ =0.804 $\hat{\sigma}_{NL}$ =0.02492

The estimation results indicate that, second lag of variable of interest is the transition variable of the model. Model diagnostics show evidence of autocorrelation, heteroscedasticity and there is evidence of additive nonlinearity at d = 1. The transition from middle regime to outer regime occurs about the value of 0.001. So, when the growth rate is about zero we are in middle regime and values far away from zero represents the dynamics of outer regime. The estimated value of transition variable is $\hat{\gamma}_E$ = 0.202 / 0.00207 and the data points are equally spreaded around the estimated threshold value, as seen in Figure 3.14. This implies that exponential form provides an adequate description of the data.

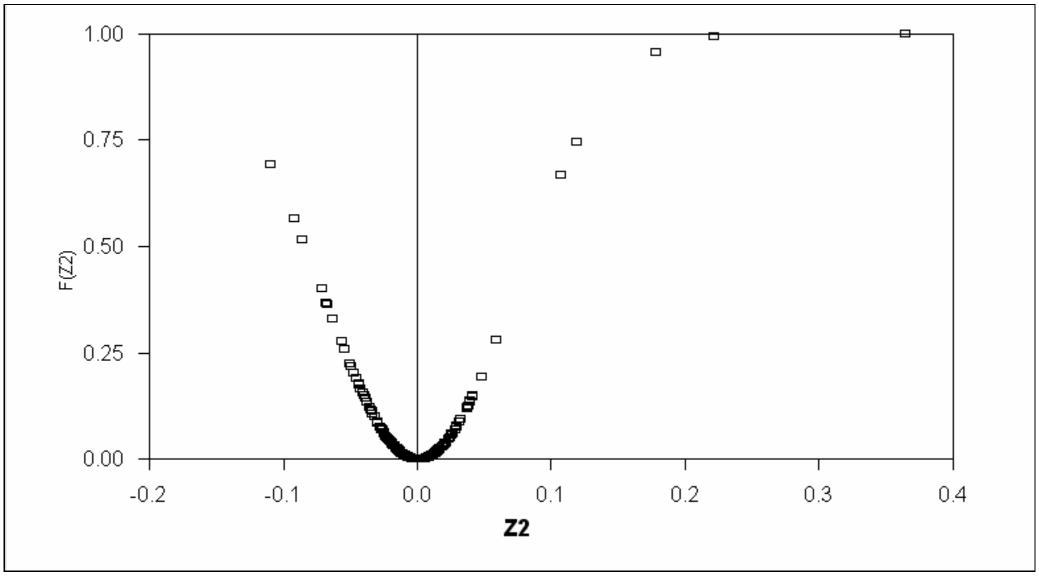

Figure 3.14: Transition Function of TL/£

In the third stage, we analyze the dynamic structure of the selected ESTAR model. ESTAR model indicates two different linear AR models according to the two extreme values of the exponential transition function, first for middle regime ($F_E = 0$) and second for outer regime ($F_E = 1$).

These models are as follows,

$F_E = 0$;

$$Z_t = -0.0007 + 0.572Z_{t-1} + 0.236Z_{t-4} - 0.3213Z_{t-5} + 0.366D944 + 0.208D20013 + 0.082D20014$$

$F_E = 1$;

$$Z_t = -0.000069 - 0.193Z_{t-1} - 0.417Z_{t-2} + 0.792Z_{t-3} - 0.930Z_{t-4} + 1.853Z_{t-5} + 0.393D944 + 0.217D20013 + 0.058D20014$$

As seen, the linear models corresponding to different phases differ from each other in the number of regressors, size and sign of the coefficients. While middle regime is constructed with Z_{t-1}, Z_{t-4} and Z_{t-5}, outer regime includes additional variables, Z_{t-2} and Z_{t-3}. Moreover, the coefficients of variables show great differences. Switching from the middle to outer regime, the coefficients of Z_{t-1} and Z_{t-4} turns to be negative and the coefficient of Z_{t-5} becomes positive and this indicates that different phases have different dynamic structures.

Table 3.17: Regime Analysis Statistics for TL / £

Variable	Regime	F_E	F_L	Root	Modules
CPI	Middle	0	—	-0.78	0.78
				$-0.07 \mp 0.78i$	0.78
				$0.75 \mp 0.32i$	0.82
	Outer	1	—	$0,31 \mp 0.98i$	1.027
				1.02	1.016
				$-0.91 \mp 0.94i$	1.31

According to the Table 3.17, the middle regime with one real and two pairs of complex roots having modules 0.78 and 0.82 is locally stationary. On the other hand, the outer regime with modules 1.027 and 1.31 has locally explosive structure. This means that a highly appreciated or depreciated currency cannot last forever, the process one way or another move to the middle regime represented by the values about the threshold value $c_E = 0.001$.

To be able to compare the estimated linear and nonlinear models, we use estimated residual variance values and forecast performances. We first inspect the reduction in the estimated residual variances from the estimated ESTAR model. Table 3.18 indicates that, nonlinear model gives lower residual variance with the ratio $\hat{\sigma}_{NL}^{2} / \hat{\sigma}_{L}^{2} = 0.699$ and the gain obtained over the nonlinear specification is 30.1 %. Secondly, we compare forecast performances of the estimated linear and nonlinear models by using RMSE values and the test of equal forecast accuracy result.

According to the Table 3.18, the RMSE value of the ESTAR model is higher than the estimated linear model. Also test of equal forecast accuracy result says that there is no significant difference between the forecast performances of these two estimated models.

Table 3.18: Comparison Statistics for Linear and Nonlinear Models for TL/£

	Forecast Performance		
$\hat{\sigma}_{NL}^{\ 2} / \hat{\sigma}_{L}^{\ 2}$	Nonlinear RMSE	Nonlinear/Linear RMSE	S_1(p-value)
0.699	0.01697	1.0955	0.835

However, the estimated nonlinear ESTAR model gives lower residual variances and lower RMSE value in especially crises periods and seems to imitate the wrinkles in the data. This supports the fact that the forecast performance of nonlinear models depends on the forecast period. The nonlinear dynamics captured by our model may not be exhibited during the forecast period and therefore these poor results might be arising. This can be supported with the fact that while linear models can be adequate for normal periods, for crises periods nonlinear models are more preferable, which can be seen also from the Figure 3.15. Moreover, the additional nonlinearity test results show that multiple regime models may be needed but short sample size does not allow to specify such a model with large number of parameters.

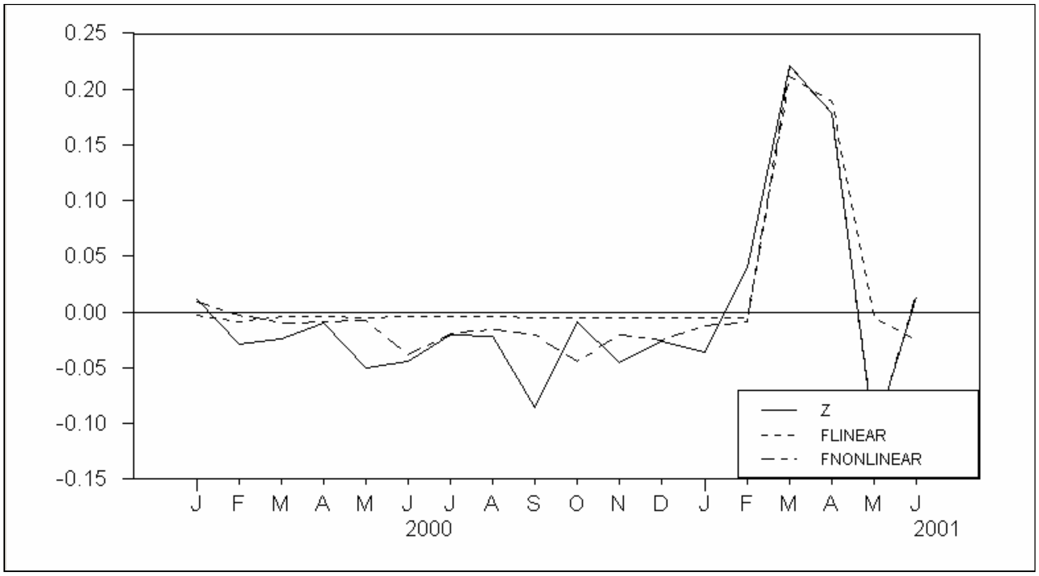

Figure 3.15: Forecast Results for TL/£

3.5 Conclusion

Our examination of nonlinearity in the three monthly seasonally adjusted Turkey macroeconomic series showed that while consumer price index series does not seem to be exhibiting nonlinear behavior, exchange rate series contain nonlinearity in their generating mechanisms and they can adequately be characterized by STAR models. The results show that exchange rates series can be represented by ESTAR models and does have similar dynamics in the outer regimes, with middle ground having distinct dynamics.

In general, the estimated models provide substantial improvements over the linear counterparts. Therefore, we conclude that STAR models are more adequate than linear models for describing the characteristics of variables considered here.

CHAPTER IV

CONCLUSION

Over the last fifteen years, the interest in nonlinear time series models has been steadily incresing. The reason behind this rising interest is the inadequacy of linear models in capturing the observed asymmetric in macroeconomic data in practice. Therefore, in applications to economic time series, models which allow for state-dependent or regime switching behaviour have been most popular in the literature.

This study provides a survey of most prominent nonlinear models in the literature. These are Markov-switching regime model, threshold autoregressive (TAR) model and smooth Transition autoregressive (STAR) model. In the study the emphasis is on STAR models because they are more flexible than TAR and Hamilton type Markov-switching models and therefore more appropriate for modelling macroeconomic variables. We attempt to model three monthly macroeconomic variables of Turkey which includes consumer price index (CPI), nominal exchange rates; US Dollar and British Pound. There are two main reasons behind this selection. Firstly, nonlinear modelling requires a large sample and these varaiables are appropriate for modelling with large number of data points. Secondly, inflation and nominal exchange rate are the forming variables of PPP hypothesis. Recent literature indicates that deviation from the PPP should contain significant nonlinearities. To pinpoint the source of nonlinearity we decided to examine the component variables seperately. The logarithmic form of the series with sample period January 1987 to June 2001 is considered. Under the deterministic seasonality assumption, all series are deseasonalized.

The conclusion for consumer price index gives the evidence that there is a mild or no significant nonlinearity. However, the estimation results indicate that the selected nonlinear model is as good as the linear one. The most appropriate model which best fits to the data is an ESTAR model. The estimation results show that the dynamics of inflation differs according to the period in which economy is in. Altough same variables contained in both linear models corresponding to different phases, the coefficients show great differences both in size and sign. High and low inflation periods are represented in one regime and moderate inflation period in other regime. Both regimes are locally stationary, which implies that to move inflation from one regime to another a huge external shock is needed. According to the estimation results, ESTAR model seems to be more preferable than the linear one because it gives low residual variances and better forecast performance especially in abnormal periods and imitating the fluctuations in the data.

The empirical results for TL / $ and TL / £ series show that there are strong evidence of nonlinearities and the most suitable models are ESTAR models. According to the estimation results, the series exhibit different dynamics in different phases. That is a highly depreciated or appreciated currency shows a nonstationary behaviour. However, if the process is near to its equilibrium point, it does not tend to move to outer regime unless there is a big external shock or a sequence of minor ones with same signs. Ignoring nonlinearity can result in a conclusion that exchange rate series are nonstationary. However, the dynamic structure of ESTAR models indicate that the series can be stationary or nonstationary depending on the regime in which the currency is in. The need for nonlinear models is also supported with lower residual variances obtained from the nonlinear specifications. Moreover, the estimated ESTAR models give better forecasts than their linear counterparts in crises periods and change points.

To sum up, we conclude that the variables under consideration have nonlinear structures in their generating mechanisms. This is important on the following grounds. First, if one use linear models than it is very likely to have poor forecasts compared nonlinear ones. Second, since these variables contain different regimes, one should argue the effect of macroeconomic policies in the different regimes. Because unlike linear models, the impulse response function is allowed to be time varying. In other words, a current shock will have a different impact on future observations depending on the sign and/or magnitude of this shock, as well as, on the past observations. This implies that there is no room for fixed fiscal and monetary policies. Third, it is now heavily discussed in the literature that a nonlinear process may be identified as a nonstationary process. This in turn may lead to misleading inferences about the dynamics of data. Fourth, if economic variables are nonlinear, economists should consider nonlinear economic models.

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