THE EFFECT OF INSTRUCTION WITH CONCRETE MODELS ON EIGHTH GRADE STUDENTS' GEOMETRY ACHIEVEMENT AND ATTITUDES TOWARD GEOMETRY

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ABSTRACT

THE EFFECT OF INSTRUCTION WITH CONCRETE MODELS ON EIGHTH GRADE STUDENTS' GEOMETRY ACHIEVEMENT AND ATTITUDES TOWARD GEOMETRY

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The purpose of the study was to investigate the effects of concrete models on eighth grade students' geometry achievement and attitudes toward geometry.

The study was conducted on 106 eighth grade students in one of the private school in Ankara. The subjects of the study received instruction with concrete models, and by the traditional method. Cooperative learning method and discovery learning method were also used to provide better classroom environment and to create exciting classroom atmosphere for the use of concrete models.

The following measuring instruments were used to collect data: The Geometry Attitude Scale (GAS), Geometry Achievement Test (GAT) and open ended questions. The present study was a matching-only pre-test- post-test control group design.

The data of the present study were analyzed by Analysis of Co-Variance and by two-way Analysis of Variance. The results of the study indicated that: (1) There was a statistically significant mean difference between students received instruction with concrete models and those received instruction with traditional method in terms of the GAch; (2) there was no statistically significant mean difference between girls and boys in terms of GAch; (3) there was no statistically significant interaction between treatment and gender on GAch; (4) there was no statistically significant mean difference between students received instruction with concrete models and those received instruction with traditional method in terms of ATG; (5) there was no statistically significant mean difference between girls and boys in terms of ATG; and (6) there was no statistically significant interaction between treatment and gender on ATG.

Key Words: Geometry Attitude Scale, Geometry Achievement Test, Instruction with Concrete Models, Traditional Method, Gender.

ÖZ

SOMUT MODELLERLE ÖĞRETİMİN SEKİZİNCİ SINIF ÖĞRENCİLERİNİN GEOMETRİ BAŞARISINA VE GEOMETRİYE YÖNELİK TUTUMUNA ETKİSİ

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Bu çalışmanın amacı somut modelle öğretimin sekizinci sınıf öğrencilerinin geometri başarısına ve geometriye yönelik tutumuna etkisini araştırmaktır.

Araştırma Ankara da ki bir özel okulun 106 sekizinci sınıf öğrencisi ile yürütülmüştür. Çalışmanın denekleri somut modeller (SM) ve Geleneksel Yöntem (GY) ile öğretim almışlardır. Ayrıca bu çalışmada somut modellerin sınıfta kullanılmasına uygun ortam sağladığı ve zevkli bir çalışma atmosferi yarattığı için İşbirliğine Dayalı Öğrenme Yöntemi (İDÖY) ve Keşfetme Yöntemi (KY) kullanılmıştır.

Bu araştırmada veri toplamak için şu ölçme araçları kullanılmıştır. Geometri Tutum Ölçeği (GTÖ), Geometri Başarı Testi (GBT) ve açık uçlu sorular. Bu çalışmada eşleştirilmiş ön-test son-test kontrol grup deseni kullanılmıştır.

Bu araştırmanın verileri kovaryans analizi ve iki yönlü varyans analizi ile analiz edilmiştir. Çalışmanın sonuçları şunları göstermiştir: (1) geometri başarısı açısından somut modelerle öğretim alan öğrenciler ile geleneksel yöntem ile öğretim alan öğrencilerin ortalamaları arsında anlamlı fark vardır; (2) geometri başarısı açısından kız ve erkek öğrencilerin ortalamaları arasında anlamlı fark yoktur; (3) geometri başarısı açısından öğretim yöntemi ile cinsiyet arasında anlamlı etkileşim yoktur; (4) geometriye yönelik tutum açısından somut modelerle öğretim alan öğrenciler ile geleneksel yöntem ile öğretim alan öğrencilerin ortalamaları arsında anlamlı fark yoktur; (5) geometriye yönelik tutum açısından kız ve erkek öğrencilerin ortalamaları arasında anlamlı bir fark yoktur; (6) geometriye yönelik tutum açısından öğretim yöntemi ile cinsiyet arasında anlamlı etkileşim yoktur.

Anahtar Kelimeler: Geometri Tutum Ölçeği, Geometri Başarı Testi, Somut Modeller, Geleneksel Yöntem, Cinsiyet

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ABBREVIATIONS

GAS: Geometry Attitude Scale

GAT: Geometry Achievement Test

GAch: Subjects Score Obtained From GAT

ATG: Subjects Score Obtained From GAS

ICM: Instruction with Concrete Models

TM: Traditional Method

EG: Experimental Group

CG: Control Group

SD: Standard Deviation

α: Level of Significance

ANOVA: Analysis of Variance

ANCOVA: Analysis of Covariance

CHAPTER I

INTRODUCTION

Teaching mathematics is much like building a house (Gluck, 1991). If the foundation is weak, many difficulties will appear later. Students' understanding of basic mathematical concepts helps them move to the next logically connected concepts. Traditional method used in most of mathematics classes does not allow students enough time to fully reach that understanding. According to Hartshorn and Boren (1990), one way to strengthen students' understanding of mathematics is the use of manipulatives. Recent studies show the importance of the use of concrete models at all grade levels (Suydam and Higins, 1977; Sowell, 1989; Gluck, 1991; Thomson and Lambdin, 1994; Dienes and Golding, 1971; Reys, 1971). The National Council of Teachers of Mathematics (NCTM) has also encouraged the use of concrete models at all levels. The NCTM's Curriculum and Evaluation Standards (1989) for grades 5 through 8 emphasize the use of concrete models in representing mathematical concepts and processes: "Learning should be grounded the use of concrete materials designed to reflect underlying mathematical ideas"(p.87). It was underlined that engaging students in examining, measuring, comparing, and contrasting a wide variety of shapes develops essential learning skills (NCTM, 1989).

Piaget (1973), Bruner (1966), the Van Hieles (1958), and Dienes (1967) developed the strongest arguments in favor of concrete models. Piaget (1973) studied the stages of cognitive development of children from birth to maturity. According to Piaget, understanding comes from actions performed by an individual in response to his or her environment. These actions change over time from very physical actions to partially internalized actions that can be performed with symbols. According to Piaget's theory, this is a continuous process of accommodation to and assimilation of the individual environment. The cognitive development starts with the use of physical actions to form schemas, followed by the use of symbols. Piaget emphasized that learning involves both physical actions and symbols that represent previously performed actions. Therefore, learning environments should include both concrete and symbolic models of the ideas to be learned. However, these models should be consistent with the development of schemas at the various development levels. A child's learning at the beginning of his or her cognitive development should be made meaningful with concrete models, while at more advanced levels concrete models may be replaced by symbolic models. Children up to the age of 12 can use symbols only after they have experienced the ideas to be learned through the manipulation of concrete models. Hence, at these ages concrete experiences should facilitate the learning of most of mathematical ideas.

Bruner's studies support Piaget's findings. Bruner (1966) described three ways of knowing: enactive, iconic, and symbolic. He said that a growing human being acts toward his or her environment through direct actions, imagery, and language. A child starts to play with objects. By touching, smelling, and tasting, he/she experiences the characteristics of the objects. Later, the child develops mental images and remembers the objects. Even later he/she connects names with the objects. According to Bruner, after children learn to distinguish objects by color, size, and shape they begin mastering the concept of numbers. Later in school, when children learn new mathematics concepts, they need to go in the same sequence from concrete objects to pictorial and then to abstract symbols.

The Van Hieles' research (1958) explained how students go through series of levels as they learn mathematics. The Van Hieles' levels range from concrete structure (level 0) to visual geometric structure (levels 1-2) and then to abstract structure (levels 3-4). According to Van Hieles, the learner cannot achieve a higher level of thinking without having passed through the previous level. Instructional experiences at each level are essential for effective progress.

One possible reason for students' difficulties with mathematical concepts may be a mismatch between teaching strategies and students' cognitive level (Wiebe, 1986). Lawson (1978) and Chiapetta (1977) stated that junior high and high school students are informal thinkers, so the symbolic approach to teaching mathematics concepts at these levels would not, in general, be successful. However, cognitive theorists believe that if topics are presented through physical situations and manipulations, concrete thinkers can master most parts of these topics (Wiebe, 1986). Bledsoe (1974) compared the use of manipulative materials with more abstract methods of teaching mathematical concepts to students in junior high and high school. The studies showed positive effect of the manipulative approach.

Dienes (1967) stated four principles of concept learning in mathematics, which support the use of concrete materials. He stated that the manipulation of concrete materials takes students from concept to concept and helps them build the conceptual structure of mathematics in their own minds.

According to Anderson (1983b), a student takes what the teacher says and makes a declarative encoding of what he/she has heard. If the declarative encoding is transformed into executable knowledge, then such coding becomes procedural knowledge. The Procedural Analogy Theory discussed in the present study is concerned with the movement from declarative to procedural knowledge (Hall, 1998). The use of concrete materials allows the proceduralization to begin. Additional activities with concrete materials are essential for the transfer of declarative knowledge to procedural knowledge. Because concrete materials help students construct procedural knowledge, prior learning through concrete materials makes new learning easier and more meaningful. Teachers provide declarative knowledge and let the students use concrete materials to transfer this knowledge into procedural knowledge. However, guidance is needed so that the students reach the desired outcome. The Procedural Analogy theory instructs how to use concrete materials to achieve a particular goal.

The model of instruction that goes from concrete to abstract is widely accepted by scholars and teachers. The gap between concrete and abstract functioning is considered as a continuum (Heddens, 1986). She stated that bridging this gap results from the process of internalization. Students build on many concrete experiences and develop mathematical concepts at the abstract symbolic level. The role of the teacher is to provide activities involving many concrete experiences to help students make this transition. The teacher is also responsible for selecting appropriate models, organizing classroom environment, and planning for successful use of models.

Elswick (1995) stated that the use of concrete experiences helps instill in students a sense of confidence in their ability to think and communicate mathematically, especially when group work is involved. Working with concrete models in groups encourages all group members to actively participate. Students can be asked to discuss ideas in the group, justify, solve, and apply these ideas. Students working in groups often share information with each other, which increases the likelihood of constructing a new knowledge. Concrete models provide an excellent opportunity to use cooperative groups in the classroom.

Discovery learning method creates an exciting classroom atmosphere, encourages and increases participation, enthusiasm and inquiry, and improves the students' ability to learn new content. The recent researches show many advantages of discovery learning (Gagne & Brown, 1961; Guthrie, 1967; Kersh, 1962; Wittrock, 1963; Anthony, 1973; Kuhfitting, 1974). Discovery learning develops students' cognitive and critical thinking skills, which allow students to learn quicker and deeper once they mastered learning skills.

The Second International Mathematics Study (SIMS, 1982-1983) shows that gender differences do not appear in mathematical learning except in the least taught areas, such as geometry and measurement. In these areas, prior out-of-class experience is significant. In many societies, girls often do not play games that enhance their visual spatial knowledge. According to Gaulin (1985), girls are therefore disadvantaged when these topics are taught in class. However, using concrete materials in the mathematics class would provide opportunities for all children to develop their skills in measurement and geometry. The NCTM (1989, p. 70) attracts attention of educators to reasoning in spatial contexts and reasoning deductively. The activities involving concrete models would give students an opportunity to engage in learning, both mentally and physically, by practicing spatial reasoning through visualization.

Many scholars who studied achievement differences between boys and girls found little variation in mathematics achievement during the elementary school year (Fennema, 1974; Hyde, Fennema, and Lamon, 1990). However, significant gender differences appear as students advance to the middle school. Boys outperformed girls in some mathematical skills and girls outperformed boys in the others (Campbell and Beaudry, 1998; Brandon, Newton, and Hammond, 1987; Fennema and Carpenter, 1981).

A number of studies focused on gender differences from the point of view of geometry achievement. They presented conflicting findings regarding the superiority of boys or girls in geometry. For example, Hanna (1986), and Fennema (1981) reported that boys had higher scores than girls in geometry and measurement. On the other hand, Senk and Usiskin (1983) noticed no significant difference between geometry scores of boys and girls. Therefore, another goal of the present study is to investigate the effect of gender differences on students' attitudes toward geometry and achievement in geometry.

Consequently, the aim of the study is to investigate the effect of instruction with concrete models and gender on eighth grade studets' geometry achievement and attitudes toward geometry.

CHAPTER 2

REVIEW OF LITERATURE

In this chapter, the theoretical background for the instructional methods used in the present study is explained and the literature related to the present study is reviewed and discussed.

2.1 The Procedural Analogy Theory and Concrete Representation

Many teachers and educators appreciate the value of concrete materials in teaching and learning mathematics. However, for some teachers the purpose of using the concrete materials is not clear, and tends not to realize that students need structure for effective use of concrete materials (Lesh, et al., 1987a). According o Hall (1998), teaching using concrete materials generally begins with activities, which aim to explore the properties of the materials. While manipulating the materials students are acquainted with the mathematical concepts. Then they start to use the materials systematically and move to a symbolic representation, which reflects the structure of the materials and their actions. The target procedure is completing an algorithm or problem solving sequence.

There are different types of concrete materials and a variety of ways to use them. Sometimes teachers do not know which teaching approaches are appropriate and which are inappropriate for a particular teaching situation. Szendrei (1996) has noted that if teachers do not know the proper use of the materials, such materials might do more harm than good. In this case, a procedural analogy theory can provide a set of guiding principles for the use of concrete materials in a particular situation. Hall (1998) stated that the aim of a procedural analogy theory is to guide instruction and look for both theoretical and practical guidelines, which will bring changes to students' cognitive structures.

There are two kinds of knowledge: Declarative and procedural knowledge. According to Anderson (1983b) students make a declarative encoding of verbal instructions given by the teacher and they cannot put this knowledge into practice until they transform this declarative encoding to procedural encoding. Declarative encoding takes place on hearing the teacher's description of a new concept or relationship in school learning. This declarative knowledge can only be transferred to procedural knowledge through teacher instruction, demonstration and example. The aim of the teacher talk is to help students to form declarative knowledge and then transfer to procedural knowledge. The use of concrete material helps students to perceive declarative and procedural knowledge easily by providing a bridge between symbol systems about them. Concrete materials can help students move from the declarative knowledge to procedural knowledge if they are given by guided instruction in how to use the concrete materials. In this way, students can transform declarative knowledge into procedural knowledge, which they can use to develop, apply and remember algorithms (symbolic systems).

Millward (1980) and Ohlsson (1991) describe declarative knowledge and procedural knowledge. Declarative knowledge is descriptive and includes facts, events and generalizations. Procedural knowledge is prescriptive and includes strategies, tactics, and plans. Since declarative knowledge is made up of generalizations, students may be able to repeat the knowledge to the teacher, but cannot put the knowledge into practice or use it in operations (Mostow, 1983; Neves and Anderson, 1981). According to Millward (1980) and Ohlsson (1991) students need procedures in order to transform this declarative knowledge correctly into useful operational knowledge. The initial declarative knowledge is very important. It can be inefficient if it is coded incorrectly (Ohlsson and Hall, 1990). To avoid misinterpretation and misconceptions, teachers should ask students to repeat what was said, to write it down, and to read it out.

What students need in everyday situations is procedural knowledge. Procedural knowledge is the knowledge needed to put what the students know declaratively into practice by following a set of rules (Hall, 1998). Anderson (1986) has noted the importance of procedural skills in completing tasks. He also pointed out that procedural knowledge allows students to produce solutions to problems and algorithms in an effective way. When students combine steps in procedures, they increase their performance speed (Newell and Rosenbloom, 1981). In this way, students automate procedures (Schneider and Fisk, 1983) and simplify problem solving without having to memorize declarative knowledge and without having to refer to long-term memory (Anderson, 1986).

The teaching sequence in using concrete materials involves many instances of declarative and procedural knowledge. As students progress from one step to another new declarative knowledge has to be formed and then presented through action on concrete materials. Procedural Analogy Theory is concerned with the movement from the declarative to the procedural knowledge and concrete materials allow the proceduralization process begins. Teachers should explain the procedure using the concrete materials instead of using abstract ideas. This process helps teachers to see what the students are thinking when they are manipulating their thoughts. In other words materials provide a visible analogy of the students' working memory. It also allows teachers some access to the student's cognitive processes and gives a chance for the teacher to intervene and increase learning efficiency. As a result, the purpose of the theory is to apply analogies so that students are able to move from concrete representation of a process to a symbolic representation of that process (Hall, 1998). In this process, concrete materials will be very helpful because it will be easier for the teacher to describe actions on physical objects than to describe operations on symbols and for students to proceduralize such a description correctly.

2.2 Piaget's Cognitive Development Theory

Human growth and development falls into four categories: cognitive, social, psychological, and physical. Growth and development brings about changes in structure and function of human characteristics. Most psychologists agree that learning in school is mainly cognitive (Ornstein, 1988) and that cognitive development depends on interaction between the child and learning environment (Sprinthall & Sprinthall, 1977)

Cognitive development theories fold that growth and development occurs in progressive stages. Piaget (1972) presents the most comprehensive overview of these theories and provides us with a broad outline of the cognitive system that children use at different periods in their lives.

Piaget's study was based on careful and detailed observation of children in natural settings and used repeated naturalistic observations. By carefully examining the functioning of intelligence in children, Piaget found out that at certain ages children have difficulties in understanding "simple ideas." For example, children do not understand that when they move beans from a short fat glass into a tall thin glass, the number of beans stays constant. Piaget (1973) examined the thinking patterns of children from birth through adolescence and found consistent systems within certain broad age ranges. He described four periods, or stages, of cognitive development. They are:

1. Sensorimotor Stage (birth to age 2),
2. Preoperational Stage (ages 2 to 7),
3. Concrete Operations Stage (ages 7 to 11), and
4. Formal Operations Stage (age 11 onwards).

There are two important things to remember about these stages. First of all, each major stage is a system of thinking which is qualitatively different from the preceding stage. Second, the child must go through each stage in a regular sequence. It is not possible to jump over or miss a stage or by-pass a stage. Children need to have enough experience at each stage and enough time to internalize this experience before they can move on.

Educators need to understand the most important parts of each stage before they decide what to teach and how to teach. To maximize the effect of the teacher's help to a child, it is also important to realize how the cognitive systems develop or, in other words, when a child is ready to learn.

1. Sensorimotor stage: This stage can also be viewed as presymbolic and preverbal. Children gain experience through their senses, and the major intellectual activity at this stage is interaction between the senses and the environment. Activities are practical. Children see and feel what is happening, but they have no way of categorizing their experience. Children develop the concept of the permanence of objects and start to establish simple relations between similar objects. A rich sensory environment prepares children to move to the next stage.
2. Preoperational Stage (Intuitive): At this stage, objects and events begin to assume symbolic meaning. Language development begins and increases quickly. Children's natural speech is dominated by monologues. The predominant learning mode at this stage is intuitive; children are not overly concerned with precision but enjoy imitating sounds and trying a lot of different words. Children show an increased ability to learn more complex concepts from experience if they are provided with familiar examples that have properties common to the ones that were explored at the previous stage. Children's capacity to store images increases. Thought processes are based on perceptual cues, and children are not aware of contradictory statements.
3. Concrete Operations Stage: The child starts to organize data into logical relationships and gains ability in manipulating data in problem solving situations.

This learning situation happens, however, only if concrete objects are present. At this stage, the child can make judgments in terms of reversibility and reciprocal relations.

4. Formal Operations Stage: The child develops full formal patterns of thinking and is able to develop logical, rational, and abstract strategies. The child can understand symbolic meanings and similes. The more active the symbolic process is, the more it improves cognitive growth. The learner can formulate hypotheses and deduce possible results from them, construct theories, and reach conclusions without having had a direct experience in the subject. Learning depends on his or her intellectual potential and environmental experiences.

Piaget's cognitive development theory aims at explaining the mechanisms and processes by which the infant and then the child develops into an individual who can reason and think using hypotheses. Piaget described three basic processes, which affect cognitive development: assimilation, accommodation, and equilibration (Sprinthall and Sprinthall, 1977).

According to Piaget, assimilation is the incorporation of new experiences into existing experiences. Assimilation involves integration of new data with the existing internal structures that can make use of the new information. Piaget (1973) stated that young children often do not integrate new data because they do not have an appropriate assimilatory structure.

Accommodation is the adjustment of internal structures to the particular characteristics of specific situations. Accommodation and assimilation function together in encounters with the environment at all levels of knowing. Children reorganize prior ways of thinking if they find that events in the environment contradict them. This modification brings about a higher level of thinking.

According to Piaget, at particular stages children assimilate certain experiences. If children assimilate these experiences from their environment, they can later internalize them, which consequently results in a complete development. This overall dual process of assimilation and accommodation can be represented by the concept of equilibration. Equilibration is the process of attaining balance between the ideas which were understood and those yet to be understood. In cognitive development, equilibration allows the individual to grow, develop, and change while maintaining stability (Piaget, 1932). Equilibration is an important factor in the cognitive growth because it maintains stability during a process of continuous interactions and continuous change.

In order to facilitate cognitive growth, curriculum should supply specific educational experiences based on the children's developmental level. Curriculum materials should also be within the children's level of understanding. Children at the preoperational stage cannot assimilate abstract experiences, which are beyond the level of their mental development.

Applying Piaget's theory to instruction requires the teacher to be sensitive to several important points:

1. Classroom instruction must allow time for children to make their own mistakes and to correct these errors by themselves. In so doing, children are using the processes of assimilation and accommodation while constructing new knowledge.
2. Student experimentation is an important part of instruction at all ages. Only through experimentation a learner can acquire skills necessary for formal operational thought. Experimentation also often generates new ideas, which later in life can lead to original discoveries.
3. When applying Piaget's theory to instruction, one must remember that knowledge is a construction of the learner (Piaget, 1970). Knowledge involves operative processes that lead to a transformation of reality, either in actions or in thought. It is different from copying a reality. The situations that lead to the highest progress allow students compare various ways of thinking. When students confront the conflict between different ways of thinking, they are forced to explain their

hypotheses and test them. This process is slow as students revise their previous ideas and adopt the new ones. Equilibration occurs only after many alternative ideas and explanations are tried and failed.

Piaget (1973) commented that education should be characterized by the use of methods that support spontaneous research by the child and adolescent. In traditional mathematics and science teaching, ideas are presented as a set of truths that can be understood only through an abstract language. However, Piaget noted that mathematics involves actions and operations; therefore, understanding mathematics should begin with action. He suggested that this process should start in nursery school with concrete exercises related to lengths, surfaces, numbers, etc., and then progressing to physical mechanical experiments in secondary school (Piaget, 1973). Piaget (1973) believed that education should introduce students to experimental procedures and free activity. He also drew attention to the need for collaboration and interchange among the students. Piaget noted that classroom learning should include both independent and collaborative student activities. He concluded that spontaneous activity, based on small groups of students working together because of their mutual interest in a particular activity, should be a major feature of classroom learning.

2.3. Bruner's Theory of Instruction

Jerome Bruner's developed a theory of instruction rather than a learning theory. Bruner (1966) believed that the teacher has to teach the subject so that the students understand the general nature, or "structure," of the subject matter rather than details and facts. Learning that is based on structure is more permanent and lasting. According to Bruner, if the learner has a structural pattern, the information can be transferred to new situations and used later. Bruner placed heavy emphasis on science and mathematics as major disciplines for teaching structure (Ornstein, and Hunkins, 1988).

Bruner's instructional theory includes four main principles (1966): (1) Motivation, (2) Structure, (3) Sequence, and (4) Reinforcement.

1. Motivation: According to Bruner, most children have an inborn desire to learn. While external reinforcement is important to motivate children, the will to learn is primarily sustained through intrinsic motivation. Teachers must facilitate and regulate their students' exploration of alternatives. Learning and problem solving require exploration of alternatives, which constitutes the very core of instruction and creates a long-term interest in learning. According to Bruner, exploration of alternatives consists of three phases: *activation, maintenance,* and *direction*. The teacher must first provide students with problems that are difficult just enough to make child's intrinsic curiosity *activate* exploration. Once activated, exploration must be *maintained* under the guiding hand of the teacher. Meaningful exploration must have *direction*. Learners should know what the goal is and how close they are to achieving it.

In sum, Bruner's first principle indicates that children have an inborn will to learn, and teachers must manage and enhance this will so that students would see that guided exploration is more satisfying than spontaneous learning on their own (Sprinthall, and Sprinthall, 1977).

2. Structure: Bruner's second principle states that any body of knowledge can be appropriately structured so that almost any learner can understand it (Sprinthall, and Sprinthall, 1977). Bruner (1966) recognizes three types of the structure of knowledge: *mode of presentation, economy of presentation,* and *power of presentation*.

Information can be presented in three *modes*—through actions, icons, and symbols. The youngest children can understand things the best in actions. When children are in the enactive stage of thinking, the best messages are the wordless ones. Older children learn to think at the iconic level. Iconic representation involves the use of pictures or diagrams to transfer information. At the most advanced, the symbolic, stage, children can translate their experience into language. Symbolic representation helps children make logical derivations and think more compactly. The choice of the *mode of presenting* of knowledge depends on the learners' age and background as well as on the subject matter. According to Bruner (1966), mathematics can and should be represented by all three modes.

Economy of presentation is concerned with the amount of information that children need in order to continue learning. Concise summaries are needed to provide economy in teaching. Because children have fewer facts and bits of information to keep in their minds, greater economy is required.

A *powerful presentation* is a simple presentation that can be easily understood. In mathematics, a powerful presentation is very important because it helps children see new relationships and connect separate facts.

3. Sequence: Because intellectual development is sequential and moves from enactive to iconic and then to symbolic representation gradually, teachers should teach any new subject in the same order. First, the teacher should introduce new subject with wordless messages. Then, the students should be encouraged to explore by using diagrams and different pictorial representations. Finally, the teacher should communicate messages symbolically through words, numbers and other symbols.

4. Reinforcement: Bruner (1966) points out that learning requires reinforcement. In order to solve a problem, one needs a feedback as to how he/she is doing. The timing and clarity of reinforcement are very important. Reinforcement should be given at the appropriate time to yield successful results. Reinforcement given too early may discourage exploration. If reinforcement is delayed, the learner might have already incorporated false information. It is also significant that the reinforcement is given in a form, which the learner can understand. If a learner is at the enactive level, symbolic or iconic reinforcement will not be understood. At the same time, teachers should avoid giving unnecessary reinforcement because it

might make learners so dependent on a teacher that they will learn only when rewarded.

Finally, Bruner emphasizes that guided instruction from a teacher is only a temporary state that aims at making the learner or problem solver self-sufficient. Therefore, raising a "life-long learner" is the goal of all formal schooling.
These four principals are set for providing learning based on understanding and meaning (Sprinthall & Sprinthall, 1977). Bruner (1960) stated that learning how things are related means learning the structure of knowledge and therefore the final goal of teaching is to promote the "general understanding of the structure of a subject matter. So, teachers should try to provide conditions in which students can perceive the structure of subject matters easily.

2.4 The Van Hiele Model of Thinking in Geometry

Many secondary school students have difficulties understanding geometry. Research studies have been carried out to better understand those difficulties. The studies of Jean Piaget and Pierre M. van Hiele play especially important role in the improvement of teaching geometry (Fuys et al, 1988).

Pierre M. van Hiele (1958) studied the role of intuition in the learning of geometry. His theory, the so-called van Hiele Model of Thinking in Geometry Among Adolescents, focused on the levels of thinking that students go through in understanding geometry, as well as how teachers can assist students in moving from one level to another. This theory was used to revise the geometry curriculum in the former Soviet Union.

The van Hieles (Pierre M. van Hiele and Dina van Hiele-Geldof) studied the difficulties that secondary school students encounter when learning geometry. They have emphasized the fact that secondary school students need a higher level of thinking in geometry although they have not had enough experiences in thinking at lower levels. The van Hieles' research summarizes the levels of thinking in geometry and the role of instruction that helps students move from one level to the next (Fuys et al, 1988).

The van Hieles (1958) stated that students need to pass through five levels of thinking and they cannot progress to the next level unless they have succeeded at the previous lower level. These levels are described as level 0, level 1, level 2, level 3, and level 4. The properties of the van Hiele thinking levels can be summarized as follows:

At level 0, the students are expected to identify, name, compare, and operate on geometric figures on the basis on the appearance of the figures.
At level 1, the students examine figures in terms of their parts and relationships among those parts and classify the figures according to their properties by experimenting.
At level 2, the students logically interrelate the properties and rules they discovered by giving or following informal arguments.
At level 3, the students prove theorems deductively by setting up relationships among networks of theorems.
At level 4, a higher level of thinking is required and students are expected to establish theorems.

The van Hieles briefly categorize these levels as follows: level 0 - concrete, levels 1-2 - visual geometric structures, and levels 3-4 - abstract structures (van Hiele and van Hile- Geldof, 1958; van Hiele, 1959/1984; Wirszup, 1976).

Other scholars used different terminology to categorize these levels, for example: level 0- visualization; level 1-analysis, level 2- informal deduction, level 3- formal deduction, and level 4-rigor (Mistretta, 2000).

The van Hieles claimed that progress from one level to the next depends mainly on instruction rather than on age or biological maturation, and instructional experiences have a crucial impact on this progress. The van Hiele (1984) stated, "it is possible that certain methods of teaching do not permit the attainment of the higher levels, so that methods of thought used at these levels remain inaccessible to the students." They developed instructional modules based on the model designed as a research tool in a one-to-one instructional/testing setting.

According to the van Hiele (1984), movement from one level to another includes five phases: (1) information, (2) guided orientation, (3) explication, (4) free orientation, and (5) integration. The modules they designed follow these phases. Each activity begins with informal work that gives students an idea of the topic at information phase. Then, during the guided orientation phase, activities involving a series of manipulations and questions help students discover the properties by themselves and reach the target goals. When students complete the task they move to the explication phase, where they are asked to express their findings in words. At the next phase, the free orientation phase, the students are presented with problems and tasks that can be approached in many ways. Students are expected to explore the problems using the concepts they have just learned. The final phase is the integration phase where students summarize what they learned about the topics during the lesson. Activities at this phase involve the use of materials that allow students who are less verbal to express their ideas.

According to the van Hieles (1958), learning is a discontinuous process. The jumps in the learning curve occur when the learner moves from one level to the next. Students may need a long time to pass from one level to another, but they must go through all levels. Sometimes, the learning process may seem to stop, but it continues later when students mature and jump to a new level. Until students have reached the next level of thinking, teachers may feel that their instruction is not understood.

The van Hieles (1958) also studied general nature of these levels of thinking and their relationship to teaching. They emphasized that each level has its own language, set of symbols, and system of relations that connects these symbols. This explains why two people sometimes cannot understand each other or follow the thought process of the other. This situation is sufficient to explain why at times teachers fail to help students in geometry learning. The students and teachers have their own languages, and often teachers use a language of a higher level, which students do not understand. The van Hieles noted that providing students with information which is above their actual thought level would not help the students to move to the next higher level. On the contrary, it will take them to a lower level.

In many middle and high schools, students do not have enough experience in reasoning about geometric ideas (Carrol, 1998; Fuys et al, 1988). Some students develop misconceptions, while others can only visualize geometric figures. These students cannot move to a higher level of geometric thinking because geometry is generally taught at symbolic level only. Teaching techniques presented by the van Hieles allow students to learn geometry by means of hands-on activities. In so doing, students can combine their concrete experiences with problem-solving strategies and reach the higher order thinking skills at an abstract level (Fuys et al, 1988).

The van Hieles method was successfully tested in practice. For example, the research of Mistretta (2000) is a field trial of a supplemental geometry unit aimed at increasing the van Hieles thinking levels in a group of 23 eighth grade students by training them to use thinking skills of a higher order. The results showed the increase in the van Hieles thinking levels of the students. The opinion survey also revealed that most of the students changed their attitudes toward geometry. The students commented that geometry was more enjoyable, interesting and easier to learn with hands-on activities. In the light of these findings, the present study uses the van Hieles model as a basis for each class activity. Each lesson opens with a concrete activity that helps the students to grasp the concepts of geometric shapes before they start to analyze and explore a pattern that resulted in a formula.

2.5 The Concrete Models in Teaching Mathematics

In this section, concrete models will be defined and literature review on the effect of the use of concrete models in mathematics classroom will be presented.

2.5.1 What are Concrete Models?

In mathematics classes, different types of tools have been used to improve student achievement and develop students' positive attitude toward mathematics. These tools have been classified and defined in different ways. Some researchers defined them as materials (e.g. Sowell), others named them as models (e.g. Schultz, 1986; Fennema, 1972).

According to Sowell (1974), there are three kinds of materials: concrete, pictorial, and abstract. Concrete materials can be moved around or manipulated by students. Materials that are basically visual and include pictures, diagrams and charts are defined as pictorial. Numerals and words are called abstract materials.

Schultz (1986) put concrete, pictorial, and symbolic models in the category of representational models. Blocks, sticks, chips, Cuisenaire rods and Diene blocks are examples of concrete models. Pictures of the very same items represented on worksheets, textbook pages, papers or cards are examples of pictorial models. Numerals on worksheets, textbook pages, papers, cards, chalkboards, or bulletin boards are examples of symbolic models.

Similarly, Fennema (1972) stated that three types of models could represent mathematical ideas: concrete, symbolic, and pictorial. A concrete model represents a mathematical idea through the three-dimensional objects. A symbolic model represents a mathematical idea of commonly accepted numerals and signs that show mathematical operation or relationships. The third type, the pictorial models, attributes both concrete and symbolic models.

In the present study, the experimental group was instructed through the use of different kinds of tools, or "concrete models." They consist of tools constructed for educational purposes (geoboards, cubes, solid figures, etc.), and real life objects (sugar, water, rice and colored paper). The term "concrete models" is consistent with the terminology of Fennema (1972) and Schultz (1986). Other studies provide different names, such as manipulatives (Hartshorn and Boren, 1990; Bohan, 1971; Shawaker and Bohan, 1994; Lewis, 1985), or concrete materials (Sowell, 1974; Howden, 1989).

2.5.2 Effects of the use of Concrete Models in Mathematics Classroom

Many educators and researchers emphasized the importance of using concrete models in mathematics classes (Thompson and Lambdin, 1994; Berman and Friederwitzer, 1983; Gluck, 1991; McBride and Lamb, 1986; Driscoll, 1984). The concrete models have been receiving increased attention of scholars since 1960s, after the publication of theoretical justifications of the use of manipulatives by Dienes (1960) and Bruner (1961). However, there are mixed opinions regarding the effectiveness of concrete models on students' achievement and attitudes toward subjects. The results of the early studies on concrete representation in teaching mathematics from 1950s and 1960s were inconclusive (Fennema, 1972). Almost half of the studies (7 out of 15) showed no significant differences between manipulative and non-manipulative treatments, four favored the manipulative groups; three showed mixed results and one favored the non-manipulative group. A majority of more recent research supports the importance of using concrete models in developing mathematical concepts (Dienes and Golding, 1971; Reys, 1971; Suydam and Dessart, 1976). Suydam and Higgins (1976) in their study of the activity-based mathematics learning in grades K-8 determined that mathematics achievement increased when manipulatives were used. A research review conducted by Suydam (1984a) suggests that manipulatives enhance mathematics achievement across a variety of topics, grade levels, and achievement and ability levels.

Educators have different attitudes toward the use of concrete models in mathematics classrooms for children of different ages. For example, Fennema (1972) claims that while beginning learners usually benefit from the use of concrete materials, older learners not always do. On the contrary, Suydam and Higgins (1977) reported benefits for all learners. While middle and upper primary students observed by Labinowics (1985) experienced considerable difficulty making sense of base-ten-blocks, Fuson and Briars (1990) who investigated the use of base-ten-blocks in teaching addition and subtraction algorithms reported that their students had amazing success. In a different study, base-ten-blocks had little effect on upper primary students' understanding or use of their already memorized whole-number addition and subtraction algorithms (Thompson, 1992). On the contrary, Wearne and Niebert (1988) reported consistent success in students' understanding of decimal fractions and decimal numeration when concrete materials were used.

Thompson and Lambdin (1994) suggested that mixed and contradictory results might result from the studies that do not investigate instructional methods and student engagement. It is obvious that mere use of concrete materials is not enough to guarantee success. Only through the examination of total instructional environment one can understand the effective use of concrete materials, especially of teachers' images of what they try to teach and of students' images of the activities in which they are asked to engage.

The aspects of teacher preparation for the use of concrete models, including teaching approaches and lessons content were investigated in detail by Fuson and Briars (1990). They emphasized the link between the action on the base-ten-blocks and written symbols, and the use of much verbalization about the blocks, in everyday English and in base-ten terms. This research provides strong support for the educational value of concrete materials, and for the need to use them in particular ways to meet specific objectives.

The detailed description of how the concrete models are used in the classroom is important for making them work successfully. Sowell (1989), Fennema (1972), and Scott and Neufeld (1976) criticized the studies that do not provide a clear feedback on the use of concrete materials. The scholars outlined the points that need clarification, such as the details of the instruction, the specificity of what was actually compared between the control group and the experimental group, the treatment, which the groups did, and its difference with the control group, and the meaning of concreteness (Hall, 1998).

The use of concrete models yields the best result when applied as a long-term project. Sowell (1989) conducted a meta-analysis of 60 studies, which ranged from kindergarten children to university students and used a wide range of manipulatives and mathematical topics. Sowell came to a conclusion that the long-term use of manipulatives was more effective than the short-term use. He stated that the short-term treatment with manipulatives caused no difference in the post-test scores of the manipulative and non-manipulative groups. Besides this, when manipulatives are used over an extended period of time, teacher's training critically influences their effectiveness. Sowell noted that groups taught by teachers trained in the long-term use of manipulatives have higher scores (Sowell, 1989).

In Turkey, we could have reached only one research study conducted on the use of manipulatives in mathematics classes at school. Yıldız (2004) has recently studied perceptions, beliefs, and expectations of the preservice teachers regarding the use of manipulatives in mathematics classes as well as the influence of the field experience on the use of manipulatives. She reported that after taking the method course the preservice teachers developed positive attitudes toward manipulatives in mathematics classes.

Although educational research indicates that manipulatives can be very effective, not many teachers are using them. Gilbert and Bush (1988) studied the recognition, availability, and use of 11 manipulatives among primary teachers. The results showed that the less experienced teachers tend to use manipulatives more often than the more experienced ones, probably because experienced teachers lack the training that more recent graduates have. Directed in-service training in application of manipulatives, however, increases their use among all teachers. To assure the best results, the teacher training should not only teach the content and introduce various manipulatives, but also develop good classroom organization skills (Szendrie, 1996).

Boulton-Lewis (1992) held that concrete materials are especially helpful in the situations where the structure of the material and the structure of the concept correspond to each other. She extended her argument to suggest that the effectiveness of concrete materials is a function of the concrete processing load required in their use. When the materials and analogies are unfamiliar or inappropriate, and the students lack declarative and procedural knowledge, the processing load for the students increases and the effectiveness decreases. In other words, a teacher who attempts to assist students in their learning may unwillingly hinder the learning by providing perceptually compelling but misleading cues. Therefore, the use of concrete materials should be combined with careful instruction. The teacher has to make sure that concrete aids as tools for teaching and learning do not create a barrier between learners and their construction of mathematical knowledge (Hall, 1998).

Fennema (1972) pointed out that children could learn better if their learning environment includes experiences with models that are suited to the children's level of cognitive development. As children progress through the elementary school, the concrete models should be replaced with the symbolic models in order to facilitate the comprehension of mathematical ideas. At the concrete-operational stage of cognitive development (up to the age of twelve), learners can learn with symbols only if the symbols represent the actions that the learners had experienced before. Fennema reported that because children enter the elementary school with a few concrete experiences, they need a concrete representation to facilitate their learning of mathematical ideas. At the upper school levels, where the experiential background of the learners is much richer, the symbolic representation is more adequate.

In the middle school, teachers tend to apply concrete approaches to teaching mathematics less often than in the lower grades. However, according to Piaget's (1973) classification of ages and learning stages, the average student is still at the concrete to semiconcrete learning stage in the fifth grade and just begins to understand abstract concepts in the seventh grade. Thus, the mismatch between the teaching methods and the learning stages, which frequently occurs in the middle school, creates a discouraging factor in the study of mathematics (Boling, 1991).

In the middle school, the content of mathematics becomes more abstract and remote from the everyday experience of the students. Solving word problems demands from the fifth- and sixth-graders a higher level of thinking than is generally required by other subjects. This circumstance discourages students from studying mathematics. In addition, because many of the middle school students are still at the concrete learning stage, they lose their interest in mathematics or have troubles learning it (Boling, 1991).

What is taught is not as important as how it is presented. Teachers need to use concrete models to introduce and reinforce concepts to be learned. Since most middle school students have not yet moved from the concrete and semi-concrete stages to the abstract stage of learning, they do not consider manipulatives "childish" if the materials are used with appropriate activities. Indeed, the students seem to be more interested in using manipulatives rather than in following the traditionally dominant paper-and-pencil activities. When teaching a new mathematics topic, teachers need to consider adding a concrete activity and a pictorial representation to the symbolic mathematical expression of the topic. This approach allows students who are not at the symbolic level yet move along with the lesson and eventually understand the topic. It also promotes better retention of the topic by all students (Fennema, 1972).

The use of concrete models in the mathematics classroom increases the responsibilities of the teachers. Teachers should have good classroom organization skills. They should carefully select the manipulatives, which are the best for the learning objective, and organize concrete models for easy use and distribution to the class. However, more is needed to start the mathematics learning process. The teacher should also give children precise instructions on what to do with the manipulatives. Szendrie (1996) stated that without proper instruction children would play with the materials rather than use them for learning.

Teachers should also be aware of how children interpret the manipulative materials. Assuming that students understand materials in the same way as teacher does may jeopardize the communication between the teacher and the students in the situations when students' understanding is different. There is a danger of misusing concrete models by teachers. This usually happens when the teacher has a prescribed activity in mind and rejects students' findings that do not correspond to the convention. In such situations, students are made to believe that "to understand" means to memorize a prescribed activity (Thompson and Thompson, 1994; Szendrie, 1996). Therefore, the misuse often happens when students are forced to memorize.

Teachers should learn the proper use of concrete models and reserve enough classroom time to teach students how to use the models. Concrete models should be selected carefully to be meaningful and acceptable to the children. Fielder (1989) outlined some selection criteria. The materials should

- serve the purpose, for which they were intended,

- be multipurpose if possible,
- allow for proper storage and easy access by teachers and students,
- prompt the proper mental image of the mathematical concept,
-be attractive and motivating,
-be safe to use,
-offer a variety of embodiments for a concept,
-be durable,
-be age-appropriate in size,
-model real problem-solving situations.

Children should be free to choose from alternative types of models so that they can find one, which is helpful to them at their developmental level. (Fennma, 1972)
The advantages of the use of concrete models can be summarized as follows. Concrete models;

1. facilitate the development of initial concepts, procedures and other aspects of mathematics,
2. encourage the use of correct language and symbolism,
3. provide means to learn thinking strategies,
4. help students develop some skills in spatial relations (3-dimensional geometry), Euclidean geometry, probability, and measurement which are not equally developed through the out-of-class experience, and
5. help students make connections between mathematical topics (Rathmell, 1978, Szendrie).

Elswick (1995) argued that in a long-term run manipulative help students develop a sense of confidence in their ability to think and communicate mathematically. The introduction of manipulatives into the curriculum helps teachers teach mathematics in a meaningful way and increases teachers' confidence and skills in teaching mathematics (Bohan and Shawaker, 1994; Hollis, 1985). Concrete models should be use with a care (Bobis, 1992; Lesh, Post, Behr, 1987a). The advantages and disadvantages of the concrete models should be examined carefully. The following points should be taken into consideration before start to use the concrete models:

1. Teachers should plan the use models according to the needs of the society and the educational philosophies of the school (Szendrei, 1996).
2. Teacher training should be given for the use of models. The role of teacher training at any level should be not only to teach the content and introduce the different types of concrete models but to develop good classroom organization skills as well (Szendrei, 1996)
3. Students should have enough experiences with the concrete models before they start to learn a mathematical concept. The experiences with concrete models are not enough. Students need to connect this experience with abstract mathematical forms of the concept. (Leitze and Kitt, 2000).
4. Teachers should tell the importance of the use of concrete models in learning of abstract mathematical concepts (Cain- Caston, 1996).

Meaningful learning of mathematical ideas is likely to happen when concrete and symbolic models are used properly. When the needs of middle school students are understood and methods of teaching mathematics are adjusted to fit these needs, both students and teachers enjoy mathematics classes more and the students' achievement often improves. Students are also able to better apply what they have learned to new situations and easily master the highly abstract content of advanced mathematics.

2.5.3 Bridging the Gap between Concrete and Abstract Thinking

Many students have difficulty understanding mathematics because they cannot make a connection between the physical world and the world of thoughts, in other words, between concrete and abstract (Heddens, 1996)

The stage between concrete and abstract attracts increased attention of the scholars and is interpreted in a number of different ways. Underhill (1977) defines the learning stage between the concrete and the abstract levels as the semi-concrete stage. Heddens (1984) adds one more level, the semi-abstract level, to this scheme. The semi-concrete level is a representation of a real situation, that is, the pictures of the real objects are used instead of the actual items. The semi-abstract level includes a symbolic representation of concrete items. The symbols or pictures represent the objects but do not always look like them. Tallies are used to represent the objects. According to Heddens (1986), the gap between concrete and abstract functioning should be considered as a continuum.

Similarly, Sowell (1989) divides the intermediate stage into concrete-abstract and pictorial-abstract. At the concrete-abstract level, the students begin to notice the relationships. At the pictorial-abstract level, pictures or diagrams are used in conjunction with the written symbols. At the end of this continuum, the learning experience is completely abstract. Eventually, students are expected to formulate the relationships and use them to solve related problems.

According to Piaget (1977), learners cannot understand an abstract representation of new knowledge until they internalize this knowledge. He defined the two processes of interaction between the reality and the mind as accommodation and assimilation. Some children can assimilate new knowledge very rapidly, while others need considerably more time to accommodate, or reorganize, their mental structures to incorporate new knowledge (Sprinthall and Sprinthall, 1977; Heddens, 1986; Hartshorn and Boren, 1990; Sowell, 1974; Driscoll, 1984). Likewise, Sowell (1989) underlined that children should have enough concrete experiences before they are asked to work with abstract matters. Children usually learn to operate at the abstract level over a period of time, after acquiring different experiences at other levels. At the concrete-abstract level, students should be encouraged to record what they are doing as a group and as individuals (Sowell, 1989).

Scholars, such as Howden (1986), Heddens (1986), Sowell (1989), and Berman and Friederwitzer (1983) provided various suggestions about bridging the gap between concrete and abstract. All of them emphasized the crucial role that teachers play in this process. Heddens (1986) claimed that children begin to develop their own thought techniques and use them in their own thinking through a systematic questioning from the teacher. Teachers should ask questions to the students and evaluate the quality and level of questions posed by the students. The questions of the teachers should guide children's thinking through the studied mathematical concepts. Questions can show new directions of thought, encourage children to continue their current line of thought, and provide clues that will stimulate thinking when the progress has been temporarily blocked. Heddens reported that the use of concrete materials enhances children's thought-processing skills, for example, logical thinking, and facilitates the transition from concrete to abstract (Heddens, 1986; Stanic and McKillip, 1989). With concrete experiences, the students can internalize mathematical concepts and develop them at the abstract, or symbolic, level. Otherwise, students will see mathematics as rules to be memorized rather than as a unique and helpful way to look at the world. If the students are asked to explain their procedures, they should be expected to use their own words and show the understanding based on their work. If students memorize the procedures without understanding, they will quickly confused or forget. Stanic and McKillip (1989), stated that real understanding will be expressed in the students' own words and will be last longer. According to Heddens (1986), verbalization is also important in developing thought-processing skills of students. Students should be given opportunities to verbalize their thought process to clarify their own thinking.

Howden (1986) and Suydam and Higgins (1977) also suggest the use of manipulatives, as a mean of bridging the gap between the concrete and abstract levels, although doing so requires careful approach. If the bridge has not been structured through a careful choice of manipulatives, children may not be able to solve the problem at the abstract level even though they can solve the same problem at the concrete level.

Sowell (1989) stated that comprehension of new mathematical concepts should begin with concrete learning experience. Students need to manipulate the objects by themselves. Teacher's demonstration alone does not provide the concrete learning experience. In another research, Sowell (1989) reemphasized the idea of using manipulatives and underlined the importance of their long-term application. He noted that children should have enough concrete experiences before they are asked to work abstractly. He also suggested that the teacher's training for effectiveness.

Berman and Friederwitzer (1983) noted that mathematical concepts are best taught by activities, which involve the transition from concrete to abstract. At the first stage of concept development, children should take part in experiences using concrete materials. Later these remembered concrete experiences would provide the basis for understanding and performing abstract paper-pencil activities. The scholars pointed out that when children engage in experiences with manipulative materials, they should use semi-concrete or pictorial representations of the same materials. In this way, children can transfer their knowledge derived from manipulating concrete objects to the pictures of these objects. The next step in moving children from concrete to abstract involves activities that use semi-abstract diagrams. Concrete experiences—actual or recalled—should constitute the first step in the development and symbolization of the new abstract concepts. If students receive the initial idea of the concepts by manipulating concrete objects, they can later develop their own internal mental images of the concept. When mathematical symbols are introduced, students can accept them as a code to represent ideas that they had already understood (Skemp, 1971).

Bohan (1971) held that since most children cannot proceed directly from the concrete model to abstract symbolism of mathematics, using pictures of the objects (semi-concrete models) when children are engaged in actual manipulations could guide children along the road to abstract symbolism. Diagrams or illustrations (semi-abstract models) of concrete materials continue this process. Once a concept has been introduced with concrete materials, pictures and diagrams, the learner is ready to comprehensively use numerals and symbols (abstract models).

The intellectual development is the transformation of the overt actions into mental operations. Physical objects that exemplify given concepts, patterns, or operations would help students carry out actions. Pictorial or semi-symbolic representations and, then, imaged objects and operations should be given after physical objects. Subsequently, the abstract concepts emerge in a form that can be not only used meaningfully, but can also be reinterpreted in terms of the previous levels of representation instead of existing solely at the symbolic level (Harrison and Harrison, 1986)

In the middle school mathematics classes, students are often expected to think at the abstract (symbolic) level without experiencing the concrete level first. The content of the middle school mathematics becomes more abstract and remote from the students' everyday experience. While instructors often use concrete materials in teaching lower-grade mathematics, they tend to disregard them in the middle school. Boling (1991) suggested that teachers should consider incorporating concrete activities and pictorial representations, in addition to symbolic mathematical expressions, into the middle school instruction. This recommendation can satisfy a twofold purpose: to help the students who are not advanced enough for the symbolic level move along with the lesson, and to assure that all students gain a deeper understanding of the topic.

2.6 Cooperative Learning

In this section components and necessary situations for cooperative learning, advantages and disadvantages of cooperative learning are explained and the literature related to cooperative learning is reviewed and discussed.

2.6.1 Components and Necessary Situations for Cooperative Learning

Cooperative learning has been defined in different ways. The most general definition provides that cooperative learning involves small groups of learners who work together as a team to solve a problem, complete a task, or accomplish a common goal (Artz and Newman, 1990a). In traditional education systems, the students are assigned a passive role. They listen to the teacher, absorb what the teacher says, and reproduce what the teacher have said at a later time. Teacher is the presenter of the information and he/she is in the center of the classroom. There is a clear boundary between the teacher and students and the interaction between them is highly limited.

In cooperative learning, the roles of teachers and students have changed (Anglin, 1995). Students are at the center and the teacher becomes a coordinator or facilitator of learning resources. The students are encouraged to be successful active learners. Group discussions techniques are used to encourage students to develop their own thinking and support each other's ideas. Classroom interactions become situations, which involve a different power relationship between the teacher and the students. Cooperative learning receives bigger emphasis. The members of the group interact with each other as they share a common goal and set of standards, which provides direction and limits to their activity.

Cooperative learning also helps students develop self-esteem and enhances their ability to learn. Low achieving students can imitate the study skills and work habits of more proficient students. By explaining the material to the others, higher achieving students often develop a deeper understanding of the task or master a sharper skill. Since explanation is one of the best means for establishing connections, and students in cooperative settings often give explanations to each other, the likelihood of constructing rich networks of knowledge under these conditions increases.

Parker (1985) stated that team activities, such as mathematics relays and small group activities, particularly in cooperative learning groups, can be used in teaching every topic in the mathematics classes. Group-oriented activities can be used to attract interest and attention of students and to involve students into activities.

According to Johnson and Johnson and Holubec (1990), cooperative groups must have the following five essential components in order to allow all students master-learning goals:

1. Positive interdependence—students must learn the assigned material by themselves and ensure that all members of their group complete the assignment.
2. Face-to-face promotive interaction—students interact with each other to promote each other's success.
3. Individual accountability—each student is responsible for his or her own learning and for his own part in the group assignment.
4. Appropriate use of interpersonal and small group skills—in order to achieve mutual goals, students in cooperative groups must trust each other, communicate well, assist each other, and learn to manage conflicts.
5. Group processing—it is important for all group members to discuss how the group is functioning together and how well they are achieving their goals.

2.6.2 Advantages and Disadvantages of Cooperative Learning

Many researchers have mentioned advantages and disadvantages of cooperative learning. One of the advantages is that cooperative learning enhances opportunities for mathematical learning because students learn from each other's ideas (Good, Reys, Grouws, and Mulryan, 1989/90). In a similar way, cooperative learning supports critical thinking and higher level processing skills as students challenge each other while reaching a group decision (Rottier and Ogan, 1991). Students improve their communication and social skills and often gain self-esteem as they work toward a common goal (Good et al, 1989-90; Artz and Newman, 1990b; Slavin, 1990; Griffith, 1990). In addition, cooperative learning allows students to move from concrete to abstract thinking and often makes it easier to learn difficult tasks (Rottier and Ogan, 1991; Block, 1971). Cooperative learning also improves long-term retention (Guyton, 1991; Whicker, Bol, and Nunnery, 1997).

On the other hand, researchers have also identified disadvantages of cooperative learning. Classrooms become noisy and teachers have less control as the students begin working independently. Cooperative learning requires extra time and may lead to discontinuity in the curriculum(Good et al, 1989/1990). Time is needed to teach students procedures and skills of working effectively in cooperative groups (Tyrrell, 1990). A teacher needs classroom experience using cooperative learning (Docterman and Synder, 1991). However, when these disadvantages are seen as surmountable, the advantages easily outweigh the disadvantages.

2.6.3 Cooperative Learning in Mathematics Classes

Working within a group at school helps students develop efficient team skills. It improves their communication abilities needed in cooperative learning settings.

Forsyth, Lolliffe and Stevensens (1999) underlined some of the objectives of cooperative learning method, such as actively involving learners in the learning process, increasing their motivation, encouraging learners to learn from each other, giving learners the opportunities to express their opinions and ideas, improving oral communication among the learners, allowing learners to work independently of the larger group, and encouraging learners to take responsibility for their own learning.

Cooperative learning method has recently attracted more scholarly attention than other teaching methods because of its sociological, psychological, educational, and pedagogical benefits. Many research findings underlined the positive cognitive and affective results of cooperative group instruction. It was noted that cooperative group instruction could enhance students' mathematics achievement, develop friendships between students, and enhance self-esteem (Blaney, Stephan, Rosenfield, Aronson, and Sikes, 1977; Johnson and Johnson, 1981; Oickle, 1980; Slavin and Karweit, 1981; Slavin, 1989).

Cooperative learning helps develop a perception of self and its relationships with the others. Curzon (1997) stated that "students are aware of their dependency on one another in achieving a common goal; the success; their positive contacts with one another in group discussion build understanding and tolerance; isolation of students is diminished when all members of the group fell that their contributions are of significance, and self esteem increased." According to Mulryan (1992), cooperative small group instruction in mathematics improves students' self esteem and fosters non-cognitive behavior, such as developing peer relations, in addition to increasing students' achievement. Guyton (1991) and Platte (1991) found out that cooperative learning has positive effects on attitudinal variables as well as on achievement.

Cooperative learning provides incentive for learning. Johnson and his colleagues (1981) reviewed 122 studies and concluded that cooperation was considerably more effective than interpersonal competition and individualistic efforts. Cooperative learning method improves thinking abilities. The researchers stated that cooperative learning improves achievement in many subject areas and at all age levels, especially for those activities that require concept attainment. Forsyth, Jolliffe and Stevens (1999) concluded that group instruction allows students to achieve cognitive skills of higher order such as synthesis and analysis and to develop attitudinal skills. Likewise, Stewards and McCormack (1997) emphasized that in a positive motivational environment that involves interaction with the others, students learn better. According to Kutnick and Rogers (1994), effective small-group work provides a good climate for the learner. Slavin (1983) reviewed 46 studies on cooperative learning in different areas and reported that cooperative learning increased students' achievement.

Turkish scholar came to similar results. Erdem (1993) stated that cooperative learning method increased achievement scores of the university students. Similarly, Bulut (1994) found that students taught by cooperative learning method have higher test scores in probability than students taught by traditional learning method.

Cooperative learning also increases students' motivation to learn. Good et al. (1989-90) reported that students who worked in cooperative learning setting were more active, motivated and enthusiastic about mathematics. Good's study showed that students respond to work in cooperative groups in different ways. Some students become active whereas others prefer to be passive and show minimal involvement in the group activities. While most of low achievers manifested a passive behavior, some high achievers tended to work alone. Webb's research (1980) demonstrates similar students' differential responses in cooperative group. According to Webb, in mixed ability groups middle achievers were less active than the high and low achievers.

Some differences in students' responses have been found to be gender-related (Peterson and Fennema, 1985). Girls benefit more from involvement in cooperative learning in regard to achievement gains in higher order mathematics tasks. Mulryan (1992) examined responses of the fifth and sixth graders and identified group processes that influence student involvement in the group activities in cooperative learning. She reported that students were more actively involved in the cooperative small-group context than in the whole-class mathematics context. However, low achievers were relatively passive comparing to their higher achieving peers. There were no significant gender differences in students' attendance and participation.

While yielding obvious benefits to the students, group work increases the responsibilities of the teachers. For example, teachers should plan tasks and activities and set up a proper classroom environment for group work (Kutnick and Rogers, 1994; Bennet and Dunne, 1994). A teacher should decide the group size, assign the groups, arrange and prepare the classroom, evaluate the group performance, and guide students to fulfil the essential components as defined by Johnson, Johnson, and Holubec (1990) in order to increase the effectiveness of the group work.

Educators should pay special attention to the size of the group. There are different suggestions for the group size. While some educators, like Biott (1984), believe that there should be no rules for the group size, others, like Kagan (1989), are very clear about the group size. The number of children in a group determines the number of communication lines. Groups of four are ideal because they provide communication lines that increase learning potential. Groups of five leave an odd person out and allow less time for individual participation. Groups of three and four members with mixed ability are insistently recommended because when high achievers involve themselves into activities requiring higher order skills, they can also draw their lower achieving peers into these activities (Webb, 1980).

The educator is also responsible for orchestrating cooperation among students. For example, Slavin (1994) notes that mere telling students to cooperate are not enough. A program based on cooperation among children must be engineered to overcome the problems that emerge during group work and adapt cooperative activities to the need of students and the limitations of the classroom. Teachers should be aware of the individual differences between the students in order to promote active involvement of all students into the classroom activities. At the same time, the teacher must assure that the group has the resources, such as intellectual skills, relevant information and properly prepared task instructions, necessary to complete the assignment successfully. Students' prerequisite skills are important factors influencing the success of both individual student and a group. The task instructions should be clear to avoid misunderstanding.

Finally, groups should be provided with corrective feedback that helps avoid misunderstandings and misconceptions. Providing feedback-correctives would encourage all group members to help each other achieve the common goal (Mevarech, 1991).

Cooperative learning method is one of the most effective learning methods. Current scholarship holds that this method is very effective in increasing students' achievement in mathematics. In addition, cooperative learning method gives students an opportunity to develop their personal, social and psychological skills. However, one should remember that benefits of cooperative learning could be properly achieved only in a long-term application. The short-term treatment only may bring about an improper outcome.

2.7 Discovery Learning

Discovery learning is one of the approaches that show promise for improving learning of mathematics. In the last two decades, discovery learning in mathematics has been receiving increased attention.

Discovery learning is defined in different terms. Bruner (1960) presents discovery learning as a "matter of rearranging a transforming evidence so reassembled to additional new insights." Ausubel (1963) notes that in discovery learning "the principal content of what is to be learned is not give." According to Bawell (1967), discovery learning is a learning in which students join pieces of knowledge together to get new knowledge from the new whole. Kersh (1962) describes discovery learning as "learner's goal-directed behaviour when he is forced to complete a learning task without help from teacher." According to Jones and Arbor (1970), discovery learning is a teaching process which helps a learner to understand a mathematical fact or relationship not perceived previously without having been told about this fact or relationship by another person.

Recent research demonstrates the advantages of discovery learning as compared to other learning methods. For example, many studies compare discovery learning and expository learning, leaning, completely or partially, toward the discovery learning. (Gagne and Brown, 1961; Guthrie, 1967; Kersh, 1962; Wittrock, 1963; Anthony, 1973). Kuhfitting, 1974) stated that regardless of how much help the teacher provides, discovery learning can be defined as "guided discovery learning". According to Weimer (1975), guided discovery is one of the discovery learning types, which falls in between of expository and pure discovery. The guidance can be given in a form of rules, praise, answers, hints, instructions, encouragement, or concrete models that are provided to students. Gagne and Brown (1961) and Wittrock (1963) pointed out that even a little use of discovery learning could result in a high success in mathematical learning.

Bruner (1960) pointed out several advantages of discovery learning. Discovery learning provides for better transfer and retention, creates exciting classroom atmosphere, encourages and increases participation, provokes enthusiasm and inquiry, and helps students learn new content. Bruner underlined that discovery may not always occur and students need background knowledge, such as declarative, procedural and conditional knowledge. When students possess prerequisite knowledge, well-structured material can allow them to discover facts easily. Discovery learning also develops cognitive and critical thinking skills. Students can learn quickly and deeply because they learn the ability to learn. Discovery learning creates exciting atmosphere and enables students to develop confidence in their ability to overcome problems by themselves. As a result, self-confidence encourages students to go further in their learning.

Discovery learning also has certain disadvantages, which have been mentioned in the literature. One of the disadvantages, for example, is that discovery learning is time-consuming (Skinner, 1968) and difficult to use in large classes.

Discovery learning puts extra responsibilities on the teachers, such as selecting, preparing, choosing and developing materials to facilitate students' discovery. Teachers should keep students working productively in any setting and understand how students think in different situations (Binter and Deway, 1968). In order to use discovery-learning method effectively, teachers must be very knowledgeable about the topic and flexible in planning to be able to respond to the needs of students.

2.8 Gender Differences in Geometry

Educational scholarship had often focused on gender issues in mathematics education. Scholars examined a number of contributing factors in different areas of mathematics. Some of these factors include *algebra* (Swafford, 1981 and Kirsher, 1989), *reasoning skills* (Linn and Pulos, 1983), *counting skills* (Callahan and Clements, 1984), *spatial area* (Fennema and Tartre, 1985, Ferrini-Mundy, 1987, and Battista, 1990), *computers* (Noss, 1987), *comprehension* (Curcio, 1987), *affective factors* (Wolleat, Pedro, Becker and Fennema, 1980), *mathematics anxiety (*Hackett and Betz, 1989, Hart, 1989, and Elliot, 1990, Hembree, 1990), and *the nature and quality of teacher-students interactions* (Becker, 1981).

Many scholars investigated links between gender and mathematics learning. Some studies reported a significant difference in mathematical ability between students of different gender (Benbow and Stanley, 1983), whereas others stated that the difference was insignificant (Fennema and Carpenter, 1981). Researchers have found that a few gender differences manifest themselves already in elementary school mathematics (Fennema, 1974; Hyde, Fennema and Lamon, 1990). In general, girls fall behind boys in mathematics learning in the middle school, and, further on, in the high school (Armstrong, 1981; Crosswhite, Dossey, Swafford, McKnight and Cooney, 1985; Ethinghton and Wolfle, 1984; Fennema, 1974, 1980, 1984; Fox, 1980; Leder, 1985; Peterson and Fennema, 1985; Fennema and Sherman, 1977). In the later elementary school years, girls a better at calculation and boys are better at problem solving (Marshall, 1984). More gender differences begin to emerge in the junior-high school (Hall and Hoff (1988)). For instance, moderate-sized differences in problem solving favoring boys start to appear in high school (Hyde, 1981). These findings suggest that girls and boys have different mathematics skills and knowledge.

Gender differences are also related to specific skills or tasks. Girls and boys make different errors when they solve problems (Marshall and Smith, 1987). A number of empirical studies have shown that boys tend to outperform girls in measurement, proportion, geometry, spatial geometry, analytic geometry, trigonometry, and application of mathematics (Battista, 1990; Fennema, 1980; Fennema and Carpentre, 1981; Hanna, 1986; Linn and Pulos, 1983; Marshall, 1983; Martin and Hoover, 1987; Pattison and Grieve, 1984; Sabers, Cushing and Saber, 1987; Wood, 1976; Ma, 1995). On the other hand, girls perform better than boys in computation, set operation, and symbolic relations (Brandon and Newton, 1987; Johnson, 1987; Pattison and Grieve, 1984; Wood, 1976). Overall, Ethington and Wollfle (1984) found out that women scored somewhat lower than men on a combined mathematics test even after controlling for the effects of parental education, spatial and perceptual abilities, and high school grades, attitudes towards mathematics and exposure towards mathematics courses.

Leder (1990) proposed a model, which emphasized variables that are important to educators in learning of mathematics. These factors are associated with environment and the learner. The environmental factors include situational factors (society, home, school, classroom variables), personal variables (parents, peers, teachers), and curriculum variables (contents of mathematics, types of items and methods of assessment and instruction). The learner-related factors consist of cognitive variables (spatial ability, verbal ability and mathematical ability) and psychosocial variables (achievement, motivation, confidence, conformity, self-esteem, and interdependence).

Having applied this model to the middle school mathematics classes, Hanna (1989) reported that at the age of thirteen gender differences are likely due to the out-of-class experience and psychosocial development rather than to biological differences. She concluded that differences among the thirteen-years-olds are mostly insignificant. Hanna pointed out that since it is unlikely that biological differences between the sexes vary from country to country, the SIMS data tends to contradict those theories that attempt to explain boys' superiority in mathematics on the basis of biological differences. When the superiority of boys is observed, like in geometry and measurement, one should remember that these mathematics topics are taught the least in classes.

Becker (1981) focused on the treatment of male and female students in high school geometry classes. He stated that teachers treat students differently on the basis of gender and that student's respond differently in class complying with the expectations of the teacher and the wider society. In order to develop high-level cognitive skills in mathematics, a student must think creatively and autonomously. Our society nurtures these qualities in males more than in females (Fennema and Peterson 1985). However, Fennema and Peterson (1985) suggested that girls differ from boys in a way that girls do mathematics in a rote fashion while boys are more autonomous.

Hanna (1986) researched the gender-related differences in mathematics achievement of the middle school students. One of her articles (1986) studied eighth-grade students in Ontario, Canada. The mean percent of correct responses in geometry and measurement was slightly higher for boys than for girls, although the difference was not large. Hanna stated that boys had some previous informal training through out-of-class activities that are not normally pursued by girls (for example, following instructions for building models, reading charts and graphs, etc). These differences in informal training could explain the differences in geometry achievement, especially in measurement. Likewise, McLean (1983), who studied all geometry topics taught at throughout the school years, supports the idea that out-of-class activities contributed to the differences in achievement between the sexes.

In many societies, girls and boys are encouraged to play with different types of toys, which are chosen according to their gender. Girls usually play with dolls and kitchen tools that do not help them develop their three-dimensional geometry skills. Instead, these toys develop an orientation in a micro world with many objects, and improve topological skills. Boys, on the other hand, are encouraged to play with construction games (for example, LEGO), billiards, darts, and the like, to learn woodcutting, to build cars, airplanes, and railway models. All of this develops boys' geometric problem solving skills. By disassembling mechanical items, participating in strategy memory games, competing in math contests, and playing geometrical or trigonometrically sports, such assailing or billiards, boys more than girls develop and use reasoning powers that are useful in mathematics (Scot and Neufeld, 1976).

Maccoby and Jacklin (1974) focused on learner-related variables in their research that explores gender differences in mathematics learning. Two cognitive areas, intelligence and spatial abilities, especially attracted their attention. Females were found to be stronger in verbal abilities and males performed better in non-verbal activities that involved spatial visualization ability. Similarly, according to Linn and Petersen (1985), gender differences in spatial ability relate to fairly specific tasks such as those that require rapid rotation of visually presented figures or distracting information to be discounted to allow recognition of the vertical and horizontal. Many mathematical tasks do not require these skills. Spatial skills are useful in solving only certain types of problems, primarily those that involve perception or assimilation of patterns and use of diagrams or graphs.

The longitudinal study of Fennema and Tartre (1985) explained the link between spatial differences and gender differences in mathematics achievement. The scholars found out that those females who have low spatial skills but high verbal skills consistently receive the lowest scores. However, the males who have low spatial and high verbal skills obtained the highest score each year. Thus, low spatial ability seemed to disadvantage females but not males with respect to mathematics achievements. Fennema and Tartre pointed out that the relationship between the spatial skill and mathematics achievement is not simple. When relevant factors were controlled, gender-related differences in favor of males did not appear often, and when they did they were not large. Students who differed in spatial visualization skill did not differ in their ability to find correct problem solutions. Meanwhile, students with a higher level of spatial visualization skill tended to use this skill in problem solving more often than their peers with the lower level of spatial visualization skill. Girls tended to use more pictures, but it did not help them to always reach correct solutions (Fennema and Tartre, 1985).

According to Armstrong (1981), females enter high school with the same or greater mathematical skills than males. Sometimes during the high school years boys catch up with girls and even do better than the girls in certain areas. Armstrong claims that these differences are related neither to differences in participation nor to spatial visualization.

A number of experimental studies show the effects of training and treatment on girls' achievement. After an eight–week spatial training course, girls increased their average in calculus course (Ferrini-Mundy, 1987). Noss (1987) conducted a research on geometrical concepts, particularly length and angle that children learn through logo programming. The result of the study shows a consistent trend toward a differential beneficial effect in favor of the girls.

Battista (1990) examined the extent to which spatial visualization and logical reasoning skills and gender- and teacher-linked differences affect performance in geometry. He stated briefly that for both males and females' spatial visualization and logical reasoning are important determinants of geometry achievement, success in problem solving, and strategies used. However, spatial visualization and logical reasoning appear to contribute differentially to the performance of males and females. While no evidence is found of gender differences in logical reasoning or use of geometry problem solving strategies, male high-school students on average scored significantly higher than their female peers in spatial visualization, geometry achievement, and geometric problem-solving tasks. Battista argues that certain instructional practices may either exacerbate or minimize gender differences in geometry learning.

Specific attention must be given to improving mathematics achievement for females. Individual school systems need to implement programs that are designed to eliminate achievement differences between males and females, and teachers should seek out information about programs and procedures or develop their own programs. The success of such programs assures that equitable education for males and females can be achieved. Activities in the mathematics classroom should be specifically prepared to give an equal opportunity to all children to develop their skills in all areas (Szendrie, 1996).

In sum, research indicates that girls and boys have different experiences outside of school and this affects their academic success in mathematics classes. The differences in mathematics performance do not show up significantly until high school, when they are especially apparent in spatial-visualization and problem solving. Because both of these topics are not given much emphasis at lower grade levels, the differences do not show up until later in the student's mathematics education. Teachers need to be aware of these factors and try to help girls develop their skills properly by providing them with extra opportunities to manipulate materials and use reasoning and problem solving in their classroom activities at an early age.

2.9 Attitudes toward Geometry

Girls and boys in middle school tend to have different attitudes towards mathematics. Studies have shown that more boys than girls have positive attitudes at the middle school and high school levels. (Fennema, 1974). Aiken (1972) established a positive correlation between mathematics achievement and attitude toward mathematics. Furthermore, Perl (1982) emphasized that for both males and females ability and achievement in mathematics result in positive attitudes to mathematics. The perceived usefulness of mathematics for educational and career goals, and the positive influence of key reference groups, such as parents, teaches, counselors, and peers, were found to be particularly important in forming positive attitudes to mathematics.

Most of the studies focus on the relationship between the students' mathematics achievement and students' attitude toward mathematics. In Turkey, Aksu (1985) and Tuncer (1993) focused on the effects of gender on students' attitude toward mathematics. Although their subjects were from different grade levels (secondary school and university), they have reported that there is no significant mean difference between attitude scores of girls and boys.

At the same time, we found only one study in Turkey dedicated to possible effects of students' attitudes toward geometry on their geometry achievement. Bulut (2002) develops an attitude scale to measure students' attitudes toward geometry. The present study applies Bulut's geometry attitude scale.

It is widely believed that a teacher's attitude towards mathematics affects students' attitude. A study conducted by Clark, Quisenberry, and Mouw (1982) established that prospective teachers for the lower grade levels (pre-K to grade 9) have less favorable attitudes towards mathematics than prospective high school mathematics teachers. Since students tend to form lasting attitudes towards mathematics during their middle school years (Anttonen, 1969; Callahan, 1971), it is essential that their teachers have a positive attitude towards mathematics. When concrete materials are used in mathematics lessons, both teachers and students report that they enjoy mathematics learning more (Fielder, 1989). This is only one of many ways that teachers can create a positive attitude in the math classroom. Because mathematics achievement is closely connected to the attitude, improving achievement necessarily improves the attitude (Aiken, 1972).

CHAPTER 3

METHOF OF THE STUDY

This chapter explains the main problem and the hypotheses of the present study, research design, and subjects of the study, definitions of terms used in the study, statement of the variables, measurement instruments, procedures followed, and tools used for analyzing the data.

3.1 Research Design of the Study

The present study uses a matching-only pre-test – post-test control group design, which is one of the methods of the quasi-experimental design (Fraenkel and Wallen, 1996). The Geometry Achievement Test (GAT) and the Geometry Attitude Scale (GAS) and were also administered during the present study.

Table 3.1 Research Design of the Present Study

Group	Pre-test	Treatment	Post-test
EG	T1,T2	ICM	T1,T2
CG	T1,T2	TM	T1,T2

In Table 3.1, the abbreviations have the following meanings: EG represent experimental group, which received instruction with the “Concrete Models”; CG represent the control group, which received instruction with the "Traditional Method" (TM).

The measuring instruments in Table 3.1 are the following: T1—Geometry Achievement Test (GAT); T2—Geometry Attitude Scale (GAS). The GAT and GAS were administered as pre-tests and post-tests.

3.2 Main and Sub-problems and Associated Hypotheses

This section presents the main problem and related sub-problems of the thesis, and examines relevant hypotheses.

The main problem of the present study is the following:

- MP: What is the *effect of instruction with concrete models and gender* on students' attitudes toward geometry and geometry achievement?

The main problem has been divided into two sub-problems:

- *SP1: What is the effect of instruction with concrete models and gender on* students' geometry achievement?
- SP2: What is the *effect of instruction with concrete models and gender* on students' attitudes toward geometry?

Before studying the first sub-problem SP1, the following three hypotheses (H1.1-H1.3) were stated:

- H1.1: There is no significant difference among the mean scores of students received instruction with concrete models and those received instruction with traditional method in terms of geometry achievement (GAch).
- H1.2: There is no significant difference between the mean scores of girls and boys in terms of GAch.
- H1.3: There is no significant interaction between treatment and gender on GAch.

To study the second sub-problem SP2, the following three hypotheses (H2.1-H2.3) were tested:

- H2.1: There is no significant difference between the mean scores of the students received instruction with concrete models and those received instruction with traditional method in terms of attitudes toward geometry (ATG).

- H2.2: There is no significant difference between the mean scores of girls and boys in terms of ATG.
- H2.3: There is no significant interaction between treatment and gender on ATG.

As shown above, the hypotheses are defined in the null form. They will be tested at the level of significance $\alpha=0.05$ after the treatment of subjects in the experimental and control groups.

3.3 Subjects of the Study

The present study used a convenient form of sampling. The study involved 106 eighth grade students enrolled in one of the private schools in Ankara-Turkey in 2003-2004 academic year. The students in the study sample were 51 girls and 55 boys. There were 72 students in the experimental group and 34 students in the control group. All the students learn the same mathematical content with the same textbook in the same period of time. The students were assigned to classes randomly by the school authorities when they started the sixth grade and the classes were heterogeneous. The distribution of the subjects is given in Table 3.2.

Table 3.2 Distributions of Subjects of the Present Study

	Experimental Group	**Control Group**	**Total**
Teaching Method	ICM	TM	
Girls	36	15	51
Boys	36	19	55
Total	72	34	106

3.4 Definition of Terms

In this section, some of terms that were used in this study are defined to prevent any misunderstandings.

1. Geometry Achievement refers to subjects' achievement scores on the "Geometry Achievement Test".
2. Attitude toward Geometry: refers to subjects' attitude scores on the "Geometry Attitude Scale".
3. Concrete Model refers the tools, which are constructed for educational purposes (geoboards, cubes, solid figures etc.), and real life objects (sugar, water, rice and colored paper)
4. Treatment refers to the method of instruction; either instruction given by Traditional Method (TM) or instruction with concrete models (CM).
5. Control Group (CG) refers to the group who received instruction with the Traditional Method.
6. Experimental Group (EG) refers to the group received instruction with Concrete Models.

3.5 Procedure

In this section procedure of the study is explained.

3.5.1. Steps of the Study

1. The study began with the review of literature about various aspects and current state of questions researched in the current study.
2. Prior to beginning the study, all necessary permissions were obtained from the General Directorate of the private high school.
3. The geometry attitude scale (GAS), developed by extending the scale developed by Bulut, Ekici, İşeri and Helvacı (2002). The researcher developed the geometry achievement test (GAT).
4. Both GAT and GAS were piloted with 90 ninth and tenth grade students at the private high school in April 2003, which allowed testing the reliability and

validity of GAT and GAS. According to the results of this pilot study, the GAS and the GAT were revised.

5. Activities were prepared using appropriate concrete models as recommended by reports of research found in the literature (see Appendix D).

6. Mathematics teachers administered the GAS and GAT to the students before and after the treatment during a mathematics lesson.

7. Two teachers taught the control groups, but only the researcher taught the experimental groups.

8. The study ran for a period of two years beginning in May 2003 at the private middle school in Ankara with 93 eighth-grade students. During the first year of the study, some problems were encountered due to a late start in the academic year. Many students were absent during the study, and some of the activities were hard to implement. Based on this experience, the researcher made revisions and continued the study in the spring of 2004.

9. In the second year (2004), the study began earlier in the academic year, in April, to avoid the problem of absent students. Some of the activities were also revised for use in the second year.

10. In the second year, the study was carried out at the same school with 106 eighth-grade students.

11. The data obtained from the GAS and GAT after the second year of the study was analyzed and used in reaching conclusions about the problem.

3.5.2 Problems Encountered and Revisions

During the administration of the activities, there were some problems. Accordingly, the revisions were made and activities were improved.

The opening activity took too much time and students did not have enough prior knowledge to feel successful at the task. In order to reduce time required for this activity and to allow the teacher more time to support students, the activity was re-designed as follows:

The shapes were grouped on the large poster board. Their definitions and properties were written on separate small cards, which were given to each group. Groups studied the cards and passed them to other groups. After reviewing each card, groups were asked to decide where each card belongs on the poster board.

Another problem was that some activities did not help the students enough so that they could move from concrete to abstract (symbolic) levels of thinking. These activities were reorganized to begin with concrete representations and to end with symbolic representations. In addition, verbalization was emphasized in each of the activities.

3.5.3 Choosing Groups and Group Structure

Researchers suggested that groups should be formed to enable student's work together more effectively (Kutnick, 1994). Some, like Biott (1984), believe that there should be no fixed rule for group size. Others, for example, Bennett and Dunne (1994), are very clear about the group size. They point out that the number of children in a group will determine the number of lines of communication and suggest that "teams of four are ideal". They believe that increasing communication lines increases the learning potential. In traditional classrooms, there often is only one line of communication – from the teacher to the student and back.

For the present study, groups of four were formed carefully to ensure effective group work. Each group had at least one high, one average, and one low achiever. The groups were heterogeneous in nature, both academically and gender-wise. The groups were told that they were responsible for learning of all group members, so that the students had to work together and help each other. They were also informed that the quality of their group work would be evaluated.

3.6 The Development of the Activities

Activities incorporating concrete models were used during the study in the experimental group. The activities were prepared so that the discovery learning and the cooperative learning methods could be used. The use of concrete models started each activity. As underlined by Sowell (1989), children should have enough concrete experiences before they are asked to work abstractly. Once a concept has been introduced using the concrete models, pictures, and diagrams, the learner are ready to use numerals and symbols (abstract models) with understanding. Therefore, to help students move from concrete to abstract, activities for the experimental group were designed using concrete models (Berman and Friederwitzer, 1983). Every step of the activities was designed to make the transfer from concrete to abstract (symbolic) level in the learning continuum easy. In so doing, we relied on the method of Howden (1986) and Suydam and Higgins (1977), who suggested using concrete models in activities to bridge the gap between the concrete and abstract levels.

In the present study, all of the activities were prepared using the model suggested by the van Hieles (1958), which consists of Information, Guided Orientation, Explicitation, Free Orientation, and Integration. Each activity was started with informal work to give students an idea of the topic (Information). Then, activities involving a series of manipulations and questions were used to help students discover the properties by themselves and reach the target goals (Guided Orientation). When the students completed the task they were asked to express their findings in words using terms such as height or base (Explicitation). Next, different types of problems were presented to the students for exploration and students were expected to use the concept they have just learned (Free Orientation). Finally, students summarize all that they have learned about the topic (Integration). Each activity includes two steps. The first step involves discovering a formula and the second step requires the students to use the formulas in problem solving.

Discovery learning increases the students' ability to learn new content, develops cognitive and critical thinking skills, and allows students to learn in greater depth because students discover that they are in charge of their own learning (Bruner, 1961). In the present study, we used guided discovery. When discovery learning was used, teacher's help was available. The teacher also guided the discovery during each activity.

Each activity involved the discovery of formulas and problem solving. Students were asked to write their findings first in verbal and then symbolic language. Verbalization is important in developing thought process skills of students. As Heddens (1986) suggested, providing students with opportunities to verbalize their thought process helps to clarify their own thinking.

Each activity was completed by problem solving. For example, students were asked to solve the problem by using the formulas they discovered. Considering these theories developed the following activities.

3.6.1 Opening Activity

Before the beginning of the unit, an hour of class time was spent to revise the relevant seventh grade topic on properties of plane figures. Student groups were asked to organize a set of colored papers into a concept map. On each paper there was a plane figure, a definition, or a property. By organizing the papers students demonstrated their prior knowledge of the relationships between shapes, properties, and the definitions.

3.6.2 Activities for Plane Figures

The activities for finding the areas of plane figures generally include cutting out the shapes and pasting them together in order to see similar component parts of each plane figure. Plane figures drawn on colored papers were used as concrete models.

To find out the formulas students applied their prior experience studying the area of rectangles and squares in the seventh grade. At the end of these activities, the students discovered formulas for the area of the parallelogram, triangle, trapezoid, deltoid, rhombus, regular polygon, and circular regions.

3.6.3 Activities for Solids

The activities for the surface area and volume of the solid figures were conducted using colored papers, colored unit cubes, and cubes, models of prisms, pyramids, cones, and spheres. The concepts of surface area and volume were discussed at the beginning of the activities. Students were encouraged to hold, observe, and compare the solids before working on the activities. The teacher asked many questions to guide the students in exploring the properties of the shapes before each activity. Transparent relational solids were used for finding the volume of the solids. Since these shapes have a removable base, and can be filled with water or rice, students could easily explore volume relationships among the shapes by filling one solid and pouring its contents into another solid. Each group manipulated the solids, calculated their surface area and volume by using their properties, recorded their findings, and compared their findings with other groups. Using the geometric solids, students made concrete connections between the shapes and their formulas for the volume and surface area. They also developed an understanding of the relationships between the various shapes.

Formulas became easier to remember when students discovered that only the method for calculating the area of the base changes from formula to formula, and that all other variables are calculated the same way regardless of the shape. As students went through formulas, they visualized them and understood why the formulas worked and how they evolved.

3.7 Development of the Measuring Instruments

In the present study, a geometry achievement test and an attitude toward geometry scale were administered. The attitude scale and the achievement test were presented in Turkish in order to overcome the language barrier.

1. Geometry Attitude Scale (GAS)
2. Geometry Achievement Test (GAT)
3. Open-ended questions

3.7.1 Geometry Attitude Scale

The Geometry Attitude Scale (see Appendix C) was used to assess the students' attitudes toward geometry before and after the treatment and to test the equivalence of the experimental and control groups before the treatment. The Geometry Attitude Scale was improved by using the scale developed by Bulut, Ekici, İşeri and Helvacı (2002). They measured attitude in three dimensions: "enjoyment", "usefulness," and "anxiety". They suggested adding more items to the second and the third dimensions to increase reliability. Hence a pilot study was conducted to test the construct validity of the scale and find out its sub-dimensions.

As Bulut, Ekici, İşeri and Helvacı (2002) suggested, in order to increase the internal validity of the two dimensions, "usefulness" and "anxiety", the new items were added and the new scale with 29 items was developed. The scale consisted of 17 positive and 12 negative items and the attitude was recorded on a five-point Likert type scale: Strongly Agree, Agree, and Undecided, Disagree, and Strongly Disagree. The positively worded items were scored starting from Strongly Agree as 5, to Strongly Disagree as 1, and the negatively worded items were reversed to a positive direction for scoring purposes. Approximately 90 ninth and tenth grade students participated in the pilot study of the measuring instrument because they had already studied these subjects. The pilot study was conducted to determine the validity and reliability of the attitude scale.

The present study uses the factor analysis to test the construct validity of the GAS and to examine its sub-dimensions. According to the initial principal factor solution with iterations, the first three eigenvalues were 10.60, 2.91, and 1.98. Factor loading of the GAS in the first (general) factor ranged between 0.44 and 0.83. The first factor accounted for 36.5% of the total variation in the GAS scores, the second factor accounted for 10.0% of the total variation in the GAS scores, and the third factor accounted for 6.8% of the total variation in the GAS scores. The varimax rotation was performed to analyze the factor structure of the scale more precisely. The eigenvalues were obtained as 5.64, 5.12, and 4.73. The first factor explained 19.5% of the variation of total scores of the GAS, the second factor accounted for 17.6 %, and the third factor explained 16.3% of the total variation. The factor loadings with the values at 0.43 or above are presented in Table 3.3.

Table 3.3 First Varimax Rotated Factor Loadings for the GAS

Item No	**Enjoyment**	**Usefulness**	**Anxiety**
7	.80		
29	.74		
8	.72		
1	.71		
22	.66		
5	.63		
13	.62		
17	.50		
26	.49		
23	.40		
24		.85	
19		.81	
9		.75	
14		.71	
16		.67	
11		.65	
21		.59	
4		.55	
28		.50	

Table 3.3. (Continued)

Item No	Enjoyment	Usefulness	Anxiety
3			.77
10			.73
12			.71
18			.59
25			.59
15			.56
20			.55
27			.51
6			.50
2			.49

The items of the attitude scale were examined. As seen in table 3.3, item 22, which shows usefulness, was loaded in the first factor so we decided to eliminate it. Item 20, 18, 28 and 2 were loaded in two factors almost equally so they were eliminated. The second varimax factor analysis was performed to the new attitude scale with 24 items. The eigenvalues were obtained as 4.80, 4.56, and 3.77. The first factor explained 20.0% of the variation of total scores of the GAS, the second factor explained 19.0 %, and the third factor accounted for 15.7% of the total variation. The factor loading with the values at 0.43 or above are presented in Table 3.4.

Table 3.4. Second Varimax Rotated Factor Loadings for the GAS

Item No	Enjoyment	Usefulness	Anxiety
7	.84		
29	.80		
8	.76		
1	.71		
5	.68		
13	.60		
26	.52		
17	.51		
23	.43		
24		.86	
19		.78	
9		.75	
14		.72	
16		.68	
11		.66	
21		.61	
4		.58	

Table 3.4 (Continued)

Item No	Enjoyment	Usefulness	Anxiety
3			.80
12			.72
10			.65
25			.64
15			.63
6			.59
27			.55

As seen from table 3.4, the first factor was called "enjoyment", the second factor was named "usefulness," and the third factor was entitled "anxiety". In the first factor, the items were 1, 5, 7, 8, 13, 17, 23, 26, and 29. In the second factor, the items were 4, 9, 11, 14, 16, 19, 21, and 24. In the third factor, the items were 3, 6, 10, 12, 15, 25, and 27. The alpha (Cronbach) reliability coefficients of sub-dimensions were 0.89, 0.89, and 0.83 respectively. There were 14 negative items and 10 positive items. The reliability coefficient of the GAS with 24 items was found as 0.89 with the SPSS package program. The total score of GAS was between 24 and120.

3.7.2 Geometry Achievement Test

In the present study, we developed and applied the geometry achievement test (GAT) (see Appendix B). It was used to determine the students' geometry achievement and to assess the students' degree of attainment of the course objectives. It was also used to test the equivalence of the experimental and control groups in terms of geometry achievement before and after the treatment.

The section below explains the design procedure and the process used in developing the measuring instrument. Course content was determined according to the curriculum program published by the Ministry of Education (see Appendix A).

1. Objectives were written at the comprehension, application and analysis levels as defined by Bloom' s Taxonomy (see Appendix A).

2. A table of specification was prepared (see Appendix A).

3. Writing problems at the different cognitive levels formed an item bank. It consisted of 154 problems, in English and in Turkish. The researcher classified them according to basic geometry concepts and levels in the cognitive domain of Bloom's Taxonomy.

4. 24 problems were selected from the item bank according to the table of specification. The achievement test was prepared in Turkish to overcome the language barrier. If the answer of the problem was correct, it was scored as 1. If not, it was scored as 0.

5. A mathematics education researcher and the middle school mathematics teachers reviewed the 24-item GAT. Based on their comments, the test was reorganized.

6. A pilot study was conducted to determine the validity and reliability of the test. Approximately 90 ninth and tenth grade students in the private high school were chosen for the pilot study.

7. An item analysis of the data from the pilot study was conducted using the ITEMAN program. The ITEMAN program indicated item discrimination power as a biserial coefficient and item difficulty power as the percentage of correct responses to each item. The criterion was that the item discrimination power should be greater or equal to 0.2. The criterion for the item difficulty power was that the coefficient should be between 0.2 and 0.8. The item discrimination powers and item difficulty powers of each item were analyzed according to these criteria. Item 18 was eliminated because the scale had a parallel item for the same dimension. Although both item discrimination powers and item difficulty powers of item 11 were less than 0.2 and the item difficulty powers of item 19 was less than 0.2, to satisfy the content validity of the test we decided to keep these two items instead of eliminating them from the GAT. These two items assess basic concepts of volume of solid figures. The pilot study of the GAT was also used to test whether any of the items of the GAT had grammatical mistakes or was ambiguous or unclear according to students' answer to questions or their questions asked during the administration of the GAT. None of the items were ambiguous; therefore, all items

except item 18 were included in the test. Then, we checked the validity of the GAT by reviewing the course content, course objectives, and table of specification. The above-mentioned expert and mathematics teacher reviewed the validity of the GAT too. The ITEMAN program showed alpha reliability coefficient of the GAT with 24 items as 0.88. After piloting, the total score of the GAT was out of 23.

3.7.3 Open-ended questions:

A questionnaire was developed to learn the students' feelings and thoughts about the teaching method used in the Surface Area and Volume Unit in the eighth grade mathematics classes involved in this study. After the treatment, the experimental group was asked the following questions about cooperative learning method, discovery learning, and activities and models used in the treatment. The students were given the questions in Turkish and were asked to respond in writing:

1. What do you think about cooperative learning method?
2. What do you think about the use of concrete models in your mathematics classes?
3. What do you think about being active in mathematics classes?
4. What do you think about the activities?
5. You were responsible for your own learning while working in-group. What do you think about it?
6. At the end of each activity you were expected to write your findings first in verbal and then in symbolic language. The formulas were not given to you directly. You tried to find the formulas. How do you think this helped you?

3.8 Treatments

Different treatments were administered to the control and the experimental groups, but both the experimental groups and the control groups received instruction from their own mathematics teacher. The two groups were taught the same content to reach exactly the same objectives, which are presented in Appendices A. These objectives covered basic concepts of geometry including area of plane figures and the surface area and volume of solid figures. There were four control groups and four experimental groups, which received the treatments described below.

3.8.1 Treatment of the Control Group (CG):

The instruction given to the CG was called the Traditional Method (TM) because the teacher taught concepts and skills directly to the whole class. The subjects were taught in a teacher-centered way. The only interaction between students and the teacher occurred when students asked questions. This class received 800 minutes of instruction during four weeks. The subjects in this group were both girls and boys. Students did not use concrete models in the control group. The teacher only brought solid figures to the class and showed them to the students. Students worked individually during the class. The control groups were given the GAS and the GAT before and after the unit on surface area and volume. The teacher explained to the students the purpose of the attitude scale and achievement test.

3.8.2 Treatment of the Experimental Group (EG):

The EG was instructed using concrete models (ICM). The researcher for the ICM treatment developed a packet of models. The instruction of the ICM groups lasted 800 minutes during four weeks. Of the subjects in this group, 36 were girls and 36 were boys. One day before the treatment the students were explained the purpose of the treatment, procedure to be followed, expected collaborative behavior as well as the definition of group success. The following instructions were given to the groups:

1. Work together as a group
2. Complete the activities as a group
3. Share the work
4. Make sure that each group member participate in the group work

5. Ask for help from your group-mates whenever you need
6. Don’t forget learning the content is your responsibility
7. Group work will be evaluated.

The teacher tried to be flexible time-wise to facilitate the discovery learning.
In the experimental group, students worked in small groups throughout the study. Their regular mathematics teacher formed the groups. The small groups worked together, helped each other and shared the work in order to complete a task during each class period. The students were encouraged to work together, complete the worksheet, share concrete models, and share the work when writing the results.

The teacher guided and monitored the groups by asking questions like "Is everyone participating?” and "Are you helping each other?” The teacher chose a spokesperson to present the group results during the study. The teacher also encouraged every member of the group to participate in the group work.

Each topic of the unit was taught using activities, which asked the students to explore the formulas, relationships, and properties of the plane and solid figures. By the end of the first set of activities, students were expected to discover the relationships and formulas for the area and the volume of the figures and express their findings orally, in writing, and through symbols. Using simple discovery activities instead of memorizing the formulas, the students learned how to develop the formulas and memorization became unnecessary. The second part of the activity involved related problems. While completing the problem sheet, the students were encouraged to work together and help each other, and to check their results with each other. Each member of a group answered at least one of the questions on each problem sheet. Also, each group was asked to submit a single answer sheet for the entire group that was accepted by all group members for the questions on the problem sheet. Students were given feedback about their group work. When each student in the class completed his or her work, the teacher discussed the results in the class. The mathematics teacher checked the answers and, if needed, explained some questions.

3.9. Variables

Four variables were considered in the present study. Two are dependent variables, and two are independent variables. The dependent variables are the following:

1. Geometry Achievement, and
2. Attitude toward Geometry.

The independent variables of the present study are considered in two groups:

1. Teaching Method, this includes
    (i) Traditional Method (TM) and
    (ii) Instruction with Concrete Models (ICM)
2. Gender.

3.10 Data Analysis

We analyzed the data of the present study using the following statistical techniques. Reliability analysis was used to test the reliability of the administered tests and scales. Factor Analysis was used to test the validity of the scales of attitude and to determine the dimensions of these scales. T-test was used to test the pre-treatment mean differences between girls and boys, and between treatment groups in terms of geometry achievement and attitude toward geometry. After the treatment, Analysis of Co-Variance (ANCOVA) was used to test the effect of instruction with concrete models and gender on students' geometry achievement with previous geometry achievement as a covariate. Analysis of Variance (ANOVA) was used to test the effect of ICM and gender on students' attitudes toward geometry.

The data of the present study was analyzed with the SPSS package program and the ITEMAN program.

3.11 Assumptions and Limitations

As in other studies there are several assumptions and limitations in the present study.

3.11.1 Assumptions

The main assumptions of the present study are the following:

1. There was no interaction between the experimental and control groups to affect the results of the present study.
2. No outside event occurred during the experimental study to affect the results.
3. The instructors were not biased during the treatment.
4. The instructors were considered as equal.
5. The administration of the tests, scales, and questionnaire were completed under standard conditions.
6. All subjects of the control and experimental groups answered the measurement instruments accurately and sincerely.

3.11.2 Limitations

The limitations of the present study are as listed below:

1. This study was limited to the eighth-grade students in a private middle school in Ankara during the spring semesters of 2003 and 2004 academic years.
2. Self-report techniques, which require the subject to respond truthfully and willingly, were applied.

CHAPTER 4

RESULTS AND CONCLUSIONS

In the previous chapters, the theoretical background of the study, the review of previous studies and the method of the present study were stated. In this chapter, the results of the analyses of pre-treatment and post-treatment measures with respect to treatment and gender. Conclusions are also presented. Its hypotheses were stated as a null form and tested at the level of significance 0.05.

4.1 The Results of Pre-treatment Measures with respect to Treatment and Gender

Before the treatment Geometry Achievement Test (GAT) and Geometry Attitudes Scale (GAS) was administered to the subjects. The results of t-test of the pre-treatment measures scores with respect to treatment were given in Table 4.1.

Table 4.1. The Results of t-tests of Pre-treatment Measures Scores with respect to Treatment

Variables	TM		ICM		t-value
	Mean	SD	Mean	SD	
GAch	5.47	2.29	3.79	2.79	3.06*
ATG	84.38	12.65	78.99	15.33	1.78

P<0.05

As seen in Table 4.1 there was statistically significant mean difference between students who received instruction with concrete models (ICM) and those received instruction with traditional method (TM) in terms of GAch ($p<0.05$). In addition, there was no statistically significant mean difference between students received instruction with concrete models and those received instruction with TM in terms of ATG ($p>0.05$).

The results of t-test of the pre-treatment measures scores with respect to gender were given in Table 4.2.

Table 4.2. The Results of t-tests of Pre-treatment Measures Scores with respect to Gender

Variables	Girls		Boys		t-value
	Mean	SD	Mean	SD	
GAch	4.08	2.47	4.56	2.97	-0.91
ATG	81.71	13.02	79.80	16.14	0.67

As seen in Table 4.2 there was no statistically significant mean difference between girls and boys in terms of GAch and ATG ($p>0.05$). As a result of analyses of pre-treatment measures, covariate variable was determined as prior geometry achievement scores to test the hypotheses related to post-geometry achievement.

4.2. The Results of Post-treatment Measures with respect to Treatment and Gender

In this section, the problems of the present study will be examined by means of their associated hypotheses.

4.2.1 The Results of Post- Geometry Achievement with respect to Treatment and Gender

As a result of analyses of pre-treatment measures, prior geometry achievement was taken as a covariate variable. Using ANCOVA at the level of significance 0.05

tested hypotheses related to post-geometry achievement. The results were given in Table 4.3.

Table 4.3. The Results of ANCOVA of Post-Geometry Achievement Test with respect to Treatment and Gender

Source	Type III Sum of Squares	df	Mean Square	F
Corrected Model	372.675	4	93.169	5.04*
Intercept	1145.711	1	1145.711	62.03*
Prior GAch	289.893	1	289.893	15.70*
Treatment	175.868	1	175.868	9.52*
Gender	0.595	1	0.595	0.03
Treatment * Gender	7.126	1	7.126	0.39
Error	1865.561	101	18.471	
Total	13019.000	106		
Corrected Total	2238.236	105		

*$p<0.05$

As seen in the Table 4.3 prior GAch was a statistically significant covariate variable ($p<0.05$). After testing the hypothesis H1.1, it was found that there was a statistically significant mean difference between students received instruction with concrete models and those received instruction with the TM in terms of the GAch in the favor of ICM ($Mean_{ICM}= 10.68$, $SD_{ICM}= 4.99$; $Mean_{TM} =8.82$, $SD_{TM} = 3.44$, $p<0.05$).

As can be seen from Table 4.3, after testing the hypothesis H1.2, it was found that there was no statistically significant mean difference between girls and boys in terms of GAch ($p>0.05$). The mean score of girls was slightly lower than the mean score of boys ($Mean_{girl}= 9.98$, $SD_{girl}= 4.55$; $Mean_{boy}= 10.18$, $SD_{boy}= 4.72$).

As seen in Table 4.3, after testing the hypothesis H1.3, it was found that there was no statistically significant interaction between treatment and gender on GAch ($p>0.05$). The mean scores and standard deviations were given with respect to treatment by gender in Table 4.4. The mean scores of girls and boys in each treatment group were almost equal ($Mean_{ICM\&girl}$= 10.47, $SD_{ICM\&girl}$= 5.19; $Mean_{ICM\&boy}$= 10.89, $SD_{ICM\&boy}$= 4.85; $Mean_{TM\&girl}$= 8.80, $SD_{TM\&girl}$= 2.11; $Mean_{TM\&boy}$= 8.84, $SD_{TM\&boy}$= 4.27).

Table 4.4. The mean scores and standard deviations of Post-Geometry Achievement with respect to treatment by gender.

Treatment	Gender	Mean	SD
CM	Girl	10.47	5.19
	Boy	10.89	4.85
TM	Girl	8.80	2.11
	Boy	8.84	4.27

4.2.2 The Results of Post-Attitudes toward Geometry with respect to Treatment and Gender

As a result of analyses of pre-attitudes toward Geometry, it was not taken as covariate variable. Using ANOVA at the level of significance 0.05 tested hypotheses related to post-attitudes toward geometry. The results of the study were given in Table 4.5.

Table 4.5. The Results of ANOVA of Post-Attitudes toward Geometry with respect to Treatment and Gender

Source	Type III Sum of Squares	df	Mean Square	F
Corrected Model	491.090	3	163.697	0.651
Intercept	593321.127	1	593321.127	2358.815*
Treatment	313.696	1	313.696	1.247
Gender	186.862	1	186.862	0.743
Treatment * Gender	112.657	1	112.657	0.448
Error	25656.419	102	251.534	
Total	726482.000	106		
Corrected Total	26147.509	105		

* $p < 0.05$.

As seen in the Table 4.5 after testing the hypothesis H2.1, it was found that there was no statistically significant mean difference between students received instruction with concrete models and those received instruction with the traditional methot in terms of the ATG ($Mean_{ICM}$= 82.36, SD_{ICM}= 15.44; $Mean_{TM}$ =78.97, SD_{TM} = 16.47, $p>0.05$).

As can be seen from Table 4.5, after testing the hypothesis H2.2, it was found that there was no statistically significant mean difference between girls and boys in terms of ATG ($p>0.05$). The mean score of girls was slightly lower than the mean score of boys ($Mean_{girl}$= 80.31, SD_{girl}= 14.90; $Mean_{boy}$= 82.18, SD_{boy}= 16.64).

As seen in Table 4.5, after testing the hypothesis H2.3, it was found that there was no statistically significant interaction between treatment and gender on ATG ($p>0.05$). The mean scores and standard deviations were given with respect to treatment by gender in Table 4.6. In the ICM the mean scores of girls and boys in

ICM were almost equal ($Mean_{ICM\&girl}$= 82.06, $SD_{ICM\&girl}$= 15.54; $Mean_{ICM\&boy}$= 82.69, $SD_{ICM\&boy}$= 15.55). However, in the TM the mean score of girls was lower than the mean score and the mean score of boys ($Mean_{TM\&girl}$= 76.13, $SD_{TM\&girl}$= 12.78; $Mean_{TM\&boy}$= 81.21, $SD_{TM\&boy}$= 18.93).

Table 4.6. The mean scores and standard deviations with respect to treatment by gender.

Treatment	Gender	Mean	SD
ICM	Girl	82.06	15.54
	Boy	82.69	15.55
TM	Girl	76.13	12.78
	Boy	81.21	18.93

4.3 Conclusions

In the light of the above findings obtained by testing of each hypothesis, the following conclusions can be deduced:

1. There was a statistically significant mean difference between students received instruction with concrete models (ICM) and those received instruction with the traditional method (TM) in terms of GAch. Students taught by ICM had higher mean scores than the students taught by TM.
2. There was no statistically significant mean difference between girls and boys in terms of GAch. The mean score of girls was slightly lower than mean score of boys.
3. There was no statistically significant interaction between treatment and gender on GAch.
4. There was no statistically significant mean difference between students received instruction with concrete models (ICM) and those received instruction with the traditional method (TM) in terms of ATG.

5. There was no statistically significant mean difference between girls and boys in terms of ATG. The mean score of girls was slightly lower than mean score of boys.
6. There was no statistically significant interaction between treatment and gender on ATG.

CHAPTER 5

DISCUSSION AND RECOMMENDATIONS

This chapter includes discussion and interpretation of the results as well as recommendations. The first section restates and discusses results. The second section discusses internal and external validity of the study. Finally, the third section draws recommendations.

5.1 Discussion

The main problem of this study was to investigate the effectiveness of the instruction with concrete materials on student's geometry achievement and attitudes towards geometry. In addition, the effect of gender on student's geometry achievement and attitudes towards geometry was researched. In order to investigate these problems, a quasi-experimental design was used. Data was gathered from eighth-grade heterogeneously grouped math classes using concrete materials with the experimental group, and using traditional teaching methods (no concrete materials) in the control group. Achievement was measured using a test (GAT) developed by the researcher, and attitude was measured using a Geometry Attitude Scale (GAS).

5.1.1 Geometry Achievement

In the present study, a pre-test given to both the experimental and control groups showed that the control group had a statistically higher mean score on the Geometry Achievement Test (GAT). At the end of the study, however, the experimental group had a statistically high mean score on the GAT. The difference between the number of subjects in experiemental group and control group was not small. This might be the reason why the treatment resulted in favor of the experimental group. The results of this study regarding the effectiveness of concrete models in geometry learning are consistent with findings of previous research studies. For example, Piaget (1973) and Bruner (1966) recommended that students use concrete materials to enhance learning both when their thinking is at the concrete level and when they move through the concrete-abstract continuum. Piaget (1973) underlined that “the true case of failure in formal education is essentially the fact that one begins with language (accompanied by drawings) instead of beginning with real and material action (103-104).” Dutch educators Pierre and Dina Van Hiele (1958) also emphasized that students need help from concrete materials as they move through the levels of thinking required for understanding geometry. Likewise, Copeland (1984) stated “children at the concrete operational level should have concrete objects as a basis for abstracting mathematical ideas.”

Concrete models are commonly used in primary school. Boling (1991) argued that concrete models should continue to be used in middle school, as many students are still at the concrete level as late as the seventh grade. He suggested there often may be a mismatch of teaching methods and learning stages in middle school mathematics classes, and that this may be a discouraging factor in the study of mathematics for students at that level. Other scholars, for example, Sowell (1989), also claimed that mathematics achievement is increased through the long-term use of concrete-instructional materials. The results of the present study support Boling’s hypothesis by showing that students whose learning was enhanced by concrete materials achieved statistically higher mean scores.

The improved results on the achievement test in the experimental group may be explained by the instructional method used with this group. The teacher formed cooperative groups and students worked together to accomplish the required tasks. Cooperative learning enhances critical thinking and higher level processing skills as students challenge each other to reach a group decision (Rottier and Ogan, 1991).

Rottier and Ogan supported the idea of Block (1971) when they suggested that cooperative learning helps students move from concrete to abstract thinking and often makes learning difficult tasks easier.

At the same time, students in the control group learned individually without the use of cooperative groups. According to Johnson and his colleagues (1981), Slavin (1983b), and Kagan (1989), cooperative learning assures higher achievements than competitive and individualistic learning structures. The lower achievement scores of the control group may be explained by this factor.

Students in the experimental group, while working with concrete materials, were expected to use a discovery approach to accomplish the goals of the activity. Bruner (1961) stated that discovery learning encourages and increases participation, enthusiasm, and inquiry, and improves the students' ability to learn new content. Students learn quickly and deeply as they use cognitive and critical thinking skills. They master learning skills and gain confidence in their own abilities. These factors explain higher achievement test scores in the experimental group.

5.1.2 Attitudes toward Geometry

A pre-test given to both experimental and control groups showed that there was no change in attitudes toward geometry in the course of study in either group. This result is inconsistent with Sowell (1989), who found that students' attitudes toward mathematics are improved when knowledgeable teachers provide instruction using concrete materials. In the present study, the use of concrete materials showed higher achievement even when the use was short-term. However, no immediate change in attitude was recorded. Perhaps a long-term use of manipulatives provided by knowledgeable teachers could show a favorable change in the attitude.

5.1.3 Gender Differences

The present study established that there were no statistically significant differences between girls and boys with respect to geometry achievement. This result is consistent with findings of Senk and Usiskin (1983). At the same time, research studies conducted by Hanna (1986) and Fennema (1981) yield different results that show significant statistical differences in favor of boys regarding in terms of geometry achievement. As we discussed before, there are many factors and reasons to explain the difference in the results. Social factors such as family, school, peer interaction, or economic status of the students can account for some of them. Most of the subjects of the present study are from educated families with high socio-economic status and are already motivated to perform better and learn more. Even more, the subjects of this study were eighth-graders who were prepared for High School Entrance Examination. Their preparation for the exam might have helped them to recall their previous geometry knowledge. This exam could cause competition among most of the students in the school. Besides this, we should also consider the impact of the school on students' success. The school involved in the study has a very good reputation in Turkey. Its learning environment was an important factor in students' achievement and motivation.

5.1.4 Open-ended questionnaire

At the end of this study, students in the experimental group answered open-ended questions concerning their feelings and thoughts about the teaching method used in the study. Questions address cooperative learning method, use of concrete models, and discovery learning.

Most of students said that cooperative learning method was pleasant and relaxing. Students stated that they were more willing to ask questions of their classmates than they would be in a large class discussion with a teacher. Many of the students did not hesitate to ask questions of their classmate who had a better understanding of the material. They also mentioned that they learned through the discussion that took place in the group. They also said that thanks to the activities it was easier to concentrate on learning. However, others complained that it was difficult to concentrate because of the noise and frequent unrelated conversations between friends. They remarked that it was important that everyone in the group participated in the group tasks and discussion. They felt that they gradually learned how to work effectively in a group. S Some students recognized that it was easier to work in a group of friends, but with more experience they would learn that there are effective ways to work with nearly everyone. Majority of the students identified the need to be individually accountable when working in groups. It was noted that in the traditional classroom environment it is easier for students not to learn and not to affect the learning of others. Overall, students indicated that they had positive attitudes toward working in the groups.

Most of the students' responses about the use of concrete models were also positive. They mentioned that they learn better when they can manipulate and see an object rather than a two-dimensional drawing on the chalkboard. While they were manipulating the materials they were learning, and they liked active learning. They felt that they remembered the information better because they used concrete models in the learning process. They said that they were familiar with the figures and could visualize them in their mind. Connecting formulas with concrete objects made it easier to memorize the formulas. They stated that they more often forget formulas learned through direct instruction than formulas mastered through guided discovery and concrete models.

Most of the students found learning activities used in the study enjoyable, helpful, and creative. They indicated that they enjoyed doing the activities. However, a number of students mentioned that the activities were repetitive as they progressed through the different shapes.

According to students' responses, discovery learning was difficult because students were not accustomed to this type of learning. Students were afraid of making mistakes, and often did not risk proposing a hypothesis and testing its validity. Some students preferred to learn the "right answer" from the teacher, rather than discover it on their own and risk a chance to make a mistake. They recognized that they sometimes needed a little help as they progressed through the activities. On the positive side, most of the students said that they learned better when they discovered information on their own. Discovery learning allowed students to learn in their own way instead of the teacher's way. Some students said they would have preferred more information at the beginning of the activity. But most of them mentioned that discovery learning was better than rote memorization and they felt they would remember the material better.

Therefore, the results of the open-ended questionnaire indicate that students recognized the benefits of cooperative learning method, concrete models, and guided discovery learning. The students' strong support for these methods may also help to explain their higher achievement scores on the post-test. Often the "study group" recognizes its special status and increases its efforts in order to be successful. (Hawthorne Effect, 2000)

5.2 Recommendations

As a result of the study, statistical analysis of the result of the study and researcher's experiences during the study following recommendations were stated.

1. Students between the age of 11 and 14 are described as ranging from concrete to abstract in their thinking (Piaget, 1973). They are highly sociable with their peers, and very restless due to rapid physical growth. Middle school mathematics students could use concrete models to overcome a number of difficulties encounter. Concrete thinkers could use concrete models as well as symbols to increase their understanding of mathematics. Sociable students could

work with peers while using concrete models to solve mathematical problems. Restless students could stand up and move around while using concrete models. Therefore, concrete models seem to address the challenges of many different types of learners.

2. Activities involving the use of concrete models should be developed and varied.

3. Teachers should have enough experiences and knowledge about the use of concrete models, cooperative learning and discovery learning methods in mathematics classes. So teacher training is needed for the effective use of these methods.

4. Teachers should clarify the some of the intents during the group work. Instructions should be given for the use of concrete models. Also guided instruction is needed during the cooperative group work and discovery learning activities.

5. The class time should be planned carefully to provide immediate feedback for the group work. Students should be provided with corrective feedback to avoid misunderstanding and lead misconceptions during class time.

6. One of the obstacles of the group work activities in our study was that students had no prior experience about the group work. It took long time for them to get used to new learning method. The students had little experience in working in-groups and in using manipulative. Although, they were informed how to use manipulative and guided instruction were given for group working during the study in experimental group, it was observed that some of the students could not manage to work cooperatively in groups. Some of the students had stated that they lost their motivation in-group working but most of them had positive thoughts toward group work because they learn from each other and are comfortable about asking questions to their peers. So to develop positive attitudes toward the use of concrete models, and to increase the effectiveness of cooperative learning and discovery learning methods, students should be informed about the methods before the treatment.

7. The present study focused on teaching middle school mathematics. So the findings reported here cannot be generalized to other grade levels, other subject matters of mathematics and other subject areas. Further investigation is needed to examine the effects of the methods on different subject matters and different grade levels.

8. No comparison was made between other middle schools and between different types of schools (public and private schools). Similar studies should be conducted with other school to determine the effects of school types on students' attitudes toward geometry and achievement in geometry.

9. There was only one mathematics teacher, teaching the experimental groups. Considering the possible influence of teachers on students' attitude and achievement, further studies should be conducted with other teachers or educators.

10. Since the sample is limited to three classes, any generalization drawn from this investigation should be considered with caution. Further investigation should be conducted with a large sample of students to increase the validity of the results.

11. Investigation of the effects of different reward systems on students' attitudes in cooperative small-group context can be useful to increase students' motivation.

12. Further research should be conducted to examine the long-term use of concrete models and cooperative learning method.

13. Further research should be conducted to examine the effects of cooperative learning and discovery learning method The positive result obtained in students' achievement in this study might be due to the positive effects of group work and discovery learning

14. Continue to use concrete models in the teaching of this unit and see if the same conclusions will be drawn. As the teacher gains experiences in teaching with cooperative groups and guided discovery, see whether the students improve their performance or they do the same as.

15. Further research might be conducted to determine whether students use skills they developed during cooperative group work and discovery learning method in other subject areas and later on higher-grade levels.

16. Further research studies should be conducted to determine how discovery learning method and the use of concrete models help for the retention of the knowledge.

5.3 Internal and External Validity

In this section, the internal and external validity of the present study will be discussed and how the treats to internal and external validity are controlled will be explained.

5.3.1 Internal Validity of the Study

Internal validity of a study means that observed differences on the dependent variable are directly related to the independent variable, but not due to some other unintended variable (Frankel and Wallen, 1996).

In the present study, the possible treats to internal validity were subject characteristics, location, data collector characteristics, data collector bias, confidentiality, and Hawthorne effect.

In the present study, the students were at the same grade level and the number of girls and boys were almost equal. So those characteristics did not affect research results unintentionally. The subjects were selected from a restricted range of ability, i.e. high-ability group. Therefore, variance of ability would not be a treat for the internal validity. Almost all of the subjects were from educated families with high socioeconomic status.

One of the subjects' characteristics could have been be a factor treating the results. As mentioned before they were preparing for a nationwide exam called "High School Entrance Exam (HSEE)". Most of the students were attending a course outside the school. They might have learned the topic before they were taught in school. As a result of analyses of pre-treatment measures, prior geometry achievement was taken as covariate variable. So, the possible effect of subject's prior knowledge on the results was eliminated.

The testing locations, i.e. classrooms were the same in terms of physical conditions. Classrooms were at the same building having the same position.

Data collector characteristics and data collector bias would not be treats of the present study. The researcher who's the mathematics teacher of the classes instructed all of the experimental groups. The other mathematics teacher of the same school taught the experimental groups. The researcher and other teachers of the same mathematics department followed the same mathematics program written by the Ministry of Education. Since only the researcher taught the experimental group, the control groups did not affected from the possible bias of the researcher. Computer checked the tests.

The subject names were taken just for matching the pre-test and post-test results and kept secret. The subjects did not write their names when they were answering the open-ended questions. The students were informed about the secrecy of the results. So confidentiality was satisfied.

All the students either in experimental group or control group were already motivated to learn due to the HSEE. Hawthorne effect could have been a treat.

5.3.2 External Validity

External validity is the extent to which the results of a study can be generalized (Frankel and Wallen, 1996).

The subjects of the study were selected from one of the private schools in Ankara. Convenience sampling was used. Therefore, Generalization of findings of the study was limited. The generalization can be done on subjects having the same characteristics mentioned in this study. The treatments and tests were given in regular classroom conditions. The results of the present study can be generalized to classrooms settings similar to this study.

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APPENDIX A

TABLE OF SPECIFICATION

Table A1: Table of Specification.

Content	Objectives (Percentages of items)		
	Comprehension	Application	Analysis
A. Perimeters and Areas of Plane Figures			
a. Rectangle, Square, Parallelogram	4		
b. Triangles	4		
c. Trapezoids	4		
d. Rhombuses	4		
e. Deltoids	4		
f. Regular Polygons and Circles		4	
g. Combination of parts of section A		14	
B. The Surface Area and Volume of a Right Prisms.	4	10	
C. The Surface Area and Volume of Cylinders		4	
D. The Surface Area and Volume of Right Square pyramids.	4		
E. The Surface Area and Volume of Right Circular Cones.		4	4
F. The Surface Area and Volume of Spheres	4		
G. Combination of parts B, C, D, and E.		23	4

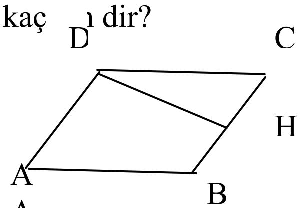
APPENDIX B

GEOMETRY ACHIEVEMENT TEST

1. Bir ikizkenar üçgenin taban kenarının uzunluğu 24 cm ve taban açılarının her biri 45° olduğuna göre alanı kaç cm^2 dir?

A) 72 **B)** 144 **C)** 288 **D)**576

2. ABCD paralekenarının alanı 48 cm^2 dir. $[DH]\perp[BC]$ ve $|AD| = 6$ cm olduğuna göre $|DH|$ kaç ı dir?



A) 6 **B)** 8 **C)** 9 **D)** 12

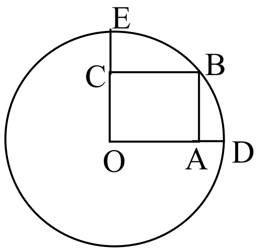
3. ABCD eşkenar dörtgeninde $|AB| = |DB| = 4$cm olduğuna göre **eşkenar dörtgenin alanı** kaç cm^2 dir?

A) $4\sqrt{3}$ **B)** $8\sqrt{3}$ **C)** $16\sqrt{3}$ **D)** $20\sqrt{3}$

4. ABCD deltoidinde $|AD|$ =4cm ve $|AB| = |AC| = 4\sqrt{3}$ cm olduğuna göre **deltoidin alanı** kaç cm^2 dir?

A) $4\sqrt{3}$ **B)** $6\sqrt{3}$ **C)** $8\sqrt{3}$ **D)** $16\sqrt{3}$

5. O merkezli çemberin içine çizilen OABC karesinin bir kenarı 2 cm dir. **Dairenin alanı** kaç cm^2 dir? (π=3 alınacak)

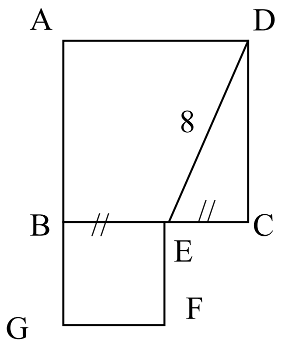


A) 12 **B)** 14 **C)** 24 **D)**48

6. Bir dik yamuğun üst ve alt tabanları sırayla 8 cm ve 10 cm dir. Yamuğun yüksekliği 2 cm olduğuna göre **alanı** kaç cm^2 dir?

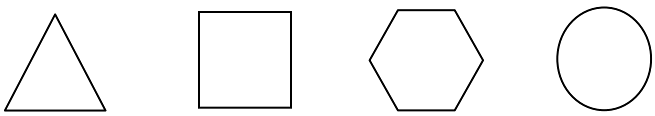
A)18 **B)** 17 **C)** 16 **D)** 14

7. Şekilde ABCD ve BEFG birer karedir. $|BE| = |EC|$ ve $|ED| = 8$ cm ise karelerin **alanları toplamı** kaç cm^2 dir?



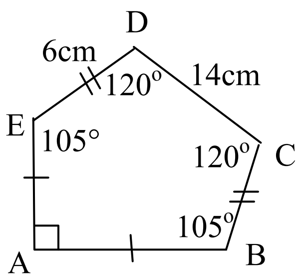
A) 16 **B)** 32 **C)** 64 **D)** 96

8. Çevreleri eşit olan aşağıdaki geometrik şekillerden hangisinin alanı **<u>en büyüktür</u>**?(π=3 alınacak)



A) Eşkenar Üçgen **B)** Kare **C)** Düzgün Altıgen **D)** Daire

9. Şekildeki ABCD **çokgeninin alanı** kaç cm^2 dir?



A) $51\sqrt{3}+100$ **B)**)$17\sqrt{3}+200$ **C)**)$51\sqrt{3}+200$ **D)**)$17\sqrt{3}+50$

10. Bir dik prizmanın tabanı kenar uzunlukları 3,4,5 cm olan dik üçgendir. Prizmanın yanal alanı 84 cm^2 olduğuna göre **hacmi** kaç cm^3 tür?

A) 42 **B)** 48 **C)** 54 **D)** 64

11. Bir kenarı 12 cm olan kare dik bir silindir oluşturulmak üzere bükülüyor. Oluşan **silindirin alanı** kaç cm^2 dir?($\pi = 3$)

A) 144 **B)** 156 **C)** 168 **D)** 288

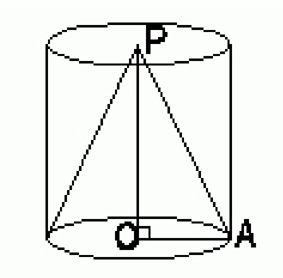
12. Tabanının alanı 16 cm^2 ve bütün ayrıtlarının toplamı 64 cm olan **kare dik prizmanın hacmi** kaç cm^3 tür?

A) 124 **B)** 128 **C)** 148 **D)** 150

13. Tabanını bir kenarı 10 cm ve yüksekliği $5\sqrt{3}$ cm olan **eşkenar üçgen dik prizmanın bütün yüz alanı** kaç cm^2 dir?

A) $200\sqrt{3}$ **B)** $100\sqrt{3}$ **C)** $150\sqrt{3}$ **D)** $50\sqrt{3}$

14. Taban yarıçapı 3 cm, yüksekliği 4 cm olan silindirin içine silindirin yüzeylerine teğet bir dik koni yerleştiriliyor. **Koninin hacmi** kaç cm^3 tür?

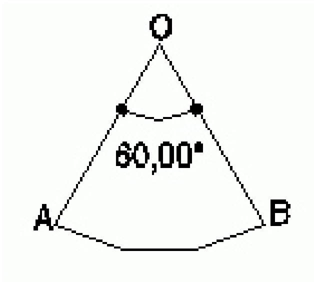


A) 16π **B)** 12π **C)** 36π **D)** 48π

15. Tabanının bir kenarının uzunluğu 10 cm ve yüksekliği $\sqrt{11}$ cm olan **kare dik piramidin bütün alanı** kaç cm^2 dir?

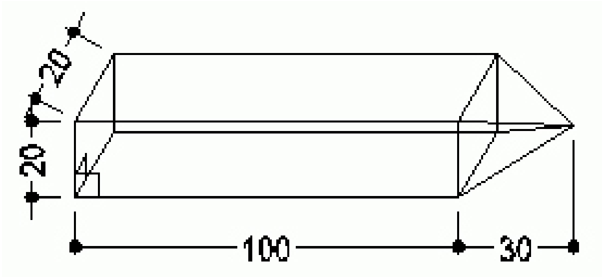
A) 75 **B)**110 **C)** 220 **D)**320

16. Aşağıdaki şekilde $|OA| = |OB| = 30\,cm$ ve m $\angle$AOB = 60° olduğuna göre, daire diliminin bükülmesiyle elde edilecek **cismin(koninin) taban yarıçapı** kaç cm dir? (π= 3 alınacak)



A) 5 **B)** 30 **C)** 6 **D)** 15

17. Şekildeki katı cismin hacmi kaç cm^3 tür?



A)44000 **B)**52000 **C)**80000 **D)**88000

18. Bir ayrıtı a cm olan bir küp ile yarıçapı r, yüksekliği h olan silindirin **hacimlerinin eşit** olabilmesi için aşağıdaki hangi şartın sağlanması gerekir? (π=3 alınacak)

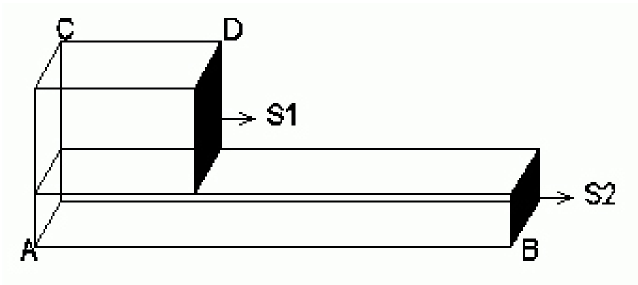
A) r= a ve $h = \frac{a}{3}$

B) h= a ve $r = \frac{a}{3}$

C) h= a ve r=3a

D) r= a ve h= 3a

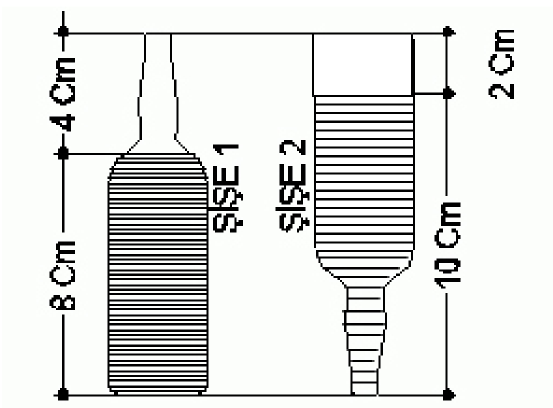
19. Şekilde üst üste konmuş iki dikdörtgenler prizmasının taralı yüzeylerinin arasında $S_1 = 4S_2$ bağıntısı vardır. $|AB| = 3$ birim ve $|CD| = 1$ birim ise **alttaki prizmanın hacmi üsttekinin kaç katıdır**?



A) $\frac{3}{4}$ **B)** $\frac{7}{3}$ **C)** $\frac{4}{3}$ **D)** $\frac{1}{4}$

20. Taban yarıçapı 2 cm olan 12 cm boyundaki bir şişe içinde yüksekliği 8 cm kadar olan su vardır. Şişe 2. duruma geçtiğinde suyun yüksekliği 10 oluyor. Buna göre, **şişenin hacmi** kaç cm^3 tür?

A) 24π **B)** 36π **C)** 40π **D)** 48π



21. Düzlemdeki bir AB doğru parçasının **A noktası sabit tutularak B noktası etrafında 360° döndürülüyor**. Oluşan şekil için ne söylenebilir?

A) B merkezli $[AB]$ yarıçaplı bir daire oluşur.

B) Tabanı B merkezli, yüksekliği $[AB]$ olan bir koni oluşur.

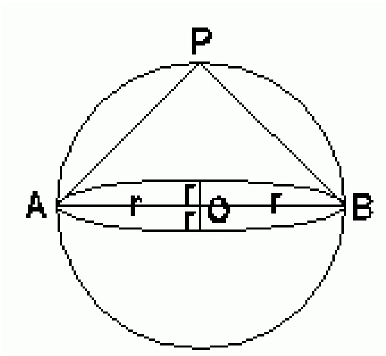
C) A merkezli $[AB]$ yarıçaplı bir daire oluşur.

D) Tabanı A merkezli, yüksekliği $[AB]$ olan bir koni oluşur.

22. Hacmi $4000cm^3$ olan **kürenin alanı** kaç cm^3 dir? (π=3 alınacak)

A)300 **B)** 900 **C)** 1000 **D)** 1200

23. Kürenin merkezi ile tabanının merkezi aynı olan küre ile dik koni şekildeki gibidir. **Koninin hacmi kürenin hacmine oranı kaçtır**?



A) $\frac{1}{3}$ **B)** $\frac{1}{4}$ **C)** $\frac{2}{5}$ **D)** $\frac{3}{4}$

APPENDIX C

GEOMETRY ATTITUDE SCALE

Genel Açıklama: Aşağıda geometriye ilişkin tutum cümleleri ile her cümlenin karşısında "Tamamen Katılıyorum", "Katılıyorum", "Kararsızım", "Katılmıyorum" ve "Hiç Katılmıyorum" olmak üzere beş seçenek verilmiştir. Her bir cümleyi dikkatli okuyarak boş bırakmadan bu cümlelere ne ölçüde katılıp katılmadığınızı seçeneklerdn birini işaretleyerek belirtiniz.

	Tamamen Katılıyorum	Katılıyorum	Kararsızım	Katılmıyorum	Hiç Katılmıyorum
1. Geometri konularını tartışmaktan hoşlanırım.	O	O	O	O	O
2. Geometri gerçek yaşamda kullanılmayan bir konudur.	O	O	O	O	O
3. Zor geometri problemleri ile uğraştığımı düşündüğüm zaman, kendimi çaresiz hissederim.	O	O	O	O	O
4. Geometri konularını severek çalışırım.	O	O	O	O	O
5. Geometri bilmek hayatımı kazanmama yardım edecektir.	O	O	O	O	O
6. Geometri ilgimi çeker.	O	O	O	O	O
7. Geometri benim için zevklidir.	O	O	O	O	O

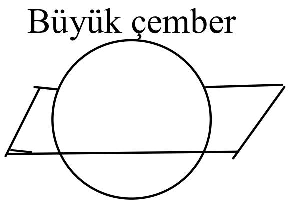
	Tamamen Katılıyorum	Katılıyorum	Kararsızım	Katılmıyorum	Hiç Katılmıyorum
8. Geometri sınavları süresince genellikle rahatımdır.	O	O	O	O	O
9. Geometri öğrenmek zaman kaybıdır.	O	O	O	O	O
10. Geometri konusundan korkarım.	O	O	O	O	O
11. Geometri konuları zihin gelişimine yardımcı olmaz.	O	O	O	O	O
12. Geometri ile ilgili ileri düzeyde bilgi edinmek isterim.	O	O	O	O	O
13. Geometri konularını öğrenmekte zorlanırım.	O	O	O	O	O
14. Yararlı olduğunu bildiğim için geometri çalışıyorum.	O	O	O	O	O
15. Geometri çalışırken aklım karışır.	O	O	O	O	O
16. Çalışma zamanımın çoğunu geometriye ayırmak isterim.	O	O	O	O	O
17. Geometri problemlerini çözebilmek konusunda genellikle hiç endişelenmem.	O	O	O	O	O
18. Geometri genellikle beni sinirlendirir.	O	O	O	O	O
19. Geometri konusuna çalışmak içimden gelmez.	O	O	O	O	O
20. Geometri sınavı beni korkutur.	O	O	O	O	O
21. Geometri değerli ve gerekli bir alandır.	O	O	O	O	O
22. Geometri dersinde zaman benim için çabuk geçer.	O	O	O	O	O
23. Gelecekteki çalışmalarım için geometriye ihtiyacım olacaktır.	O	O	O	O	O
24. Geometri konuları benim için eğlencelidir.	O	O	O	O	O

APPENDIX D

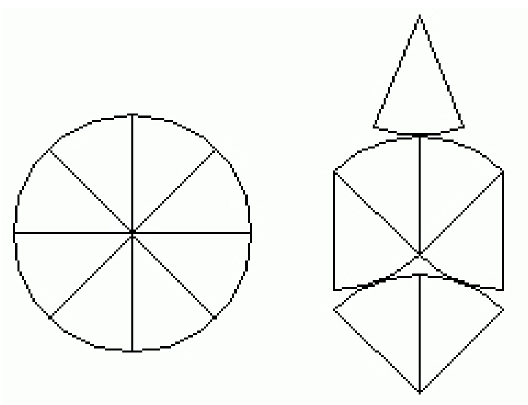
SAMPLES for ACTIVITY SHEETS

Grup No: .... Grup İsmi: .... Tarih:...............

KÜRENİN YÜZEY ALANI



- Elinizdeki topu büyük çember boyunca ikiye bölün. Meydana gelen büyük çemberi renkli kağıda çizin ve kesin.
- Çemberi sekiz eşit parçaya ayırın ve parçaları kesin. Parçaları şekildeki gibi bantla birleştirin.



- Bant veya toplu iğne kullanarak sekiz parçayı kürenizin çevresine sarın.
- Kaç tane daha çember kullanırsanız kürenizi tamamen kaplarsınız.

- Kürenizin yüzey alanını bulmak için ne yaparsınız. Sözel olarak ifade ediniz.

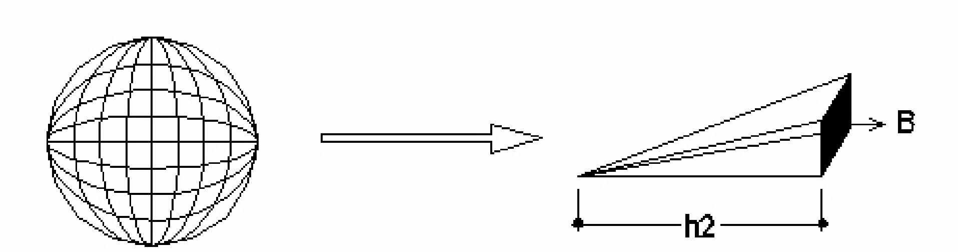
- Kürenizin yüzey alanını ile dairenin alanı arasında ilişki varmıdır? Sözel olarak ifade ediniz.

- Kürenizin yüzey alanını bulmak için kullanabileceğiniz ifadeyi matematiksel semboller kullanarak ifade ediniz(Formül ile).

Grup No: .... Grup İsmi: .... Tarih:...............

KÜRENİN HACMİ

- Kürenin hacmini bulmak için piramidin hacmini kullanabiliriz.



- Kürenizi tepe noktaları kürenin merkezi olan sonsuz sayıda piramitlere ayırdığınızı düşünün. Kürenin hacmi ile bu piramitlerin toplam hacmi eşit olacaktır.

Piramidin yüksekliği nedir?

Piramidin taban alanı B olsun, hacmini nasıl hesaplarız?

- Kürenin hacmini piramitlerin hacmini toplayarak bulmaya çalışalım.

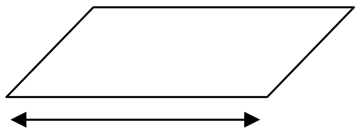
<u>Küre ile ilgili aşağıdaki soruları çözünüz.</u>

1. Yarıçapı 6 cm olan kürenin yüzey alanını bulunuz(π=3,14).
2. Hacmi 2304π cm^3 olan kürenin yarıçapını bulunuz(π=π).
3. Hacmi 18π cm^3 olan yarım kürenin yüzey alanını bulunuz(π=π).

Grup no: Grup ismi: Tarih:

PARALELKENARIN ALANI

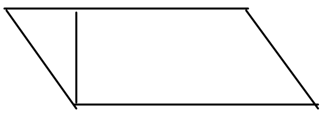
- Paralelkenarınızın alt tabanını ölçün. Taban uzunluğu=



- Paralelkenarın üst geniş açısından alt tabanına olan yüksekliğini çizin ve ölçün.

  Yüksekliği=

- Paralelkenarınızı yüksekliği boyunca kesin. Şimdi elinizde iki parça var.
- Kesilen küçük parçayı nereye taşırsanız bildiğiniz bir şekil elde edebilirsiniz?



- Elde ettiğiniz şeklin alanını nasıl hesaplarsınız? Açıklayınız.

- Paralelkenarınız ile oluşturduğunuz şeklin alanları arasında bir ilişki var mı?Açıklayınız.

  ____

- Paralelkenarın alanının nasıl bulabiliriz?(yazarak ifade etmeye çalışın)

  ____

- Paralelkenarın alanını matematiksel olarak semboller kullanarak nasıl ifade edersiniz?(Formül yazarak)

Ödev: Paralelkenarın alanını dikdörtgenden bulmaya çalışınız.

Paralelkenar ile ilgili aşağıdaki soruları çözünüz.

1. ABCD paralelkenarında $|AB| = 16$ cm, $|BC| = 14$ cm ve A noktasından $[DC]$ kenarına çizilen yükseklik 10 cm ise paralelkenarın alanını bulunuz.

2. ABCD paralelkenarında $|AD| = 6$ cm, $|CD| = 7$ cm ve $m\angle D = 30°$ ise paralelkenarın alanını ve çevre uzunluğunu bulunuz.

3. Alanı 72 cm^2 olan bir paralelkenarın yükseklikleri 8 cm ve 6 cm ise paralelkenarın çevre uzunluğunu bulunuz.