## ABSTRACT

# NUMERICAL ANALYSIS OF SETTLEMENT, AND STRESS CONCENTRATION RATIO IN CLAYEY SOILS REINFORCED BY FLOATING SINGLE AGGREGATE PIERS

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This study discusses the results of numerical modeling aspect of aggregate pier foundations (aggregate piers) in soft, compressible soils. *FLAC* 2D (Fast Lagrangian Analysis of Continua), a finite difference code is utilized in the analyses. Use of axisymmetry enabled to visualize a three dimensional model throughout this research.

The primary objective of this research is to make comparisons for stress concentration ratio '*n*' , and settlement reduction ratio b for given variables consisting of length, diameter, elastic modulus of the aggregate piers, and foundation pressures.

Analyses have been carried out with 1, 1.5, 2, and 3 m long piers with diameters of 60 cm, and 80 cm, placed under a circular footing in 1.30 m diameter. Two values for elastic modulus of the piers have been used to reflect the effect of pier stiffness on settlement behavior. Analysis and design methodology have been carried out in three stages. The first stage consists of modeling the matrix soil with an elastic constitutive model and exerting foundation pressures to first check the accuracy of the mesh by comparing the effective vertical stress and settlement values by analytical methods. Once satisfactory results are achieved, modeling of a rigid foundation is carried out. Consequently, aggregate piers are modeled and loaded. For foundation pressures, a range of values consisting of 25, 50, 75, and 100 kPa have been chosen to see the behavior of piers under variable foundation pressures.

There are solid outcomes of this study. It concludes by stating that the settlement behavior of piers having L/d ratios greater than 3.75, are alike. Thus, there is almost no additional settlement improvement achieved with piers longer than 3 m with 60 cm pier diameter.

Key Words : aggregate piers, stress concentration ratio, settlement reduction ratio, ground improvement

# ÖZ

## YÜZER, TEK GEO-KOLONLARLA GÜÇLENDİRİLMİŞ KİLLİ ZEMİNLERİN OTURMA, VE GERİLME KONSANTRASYON ORANININ NUMERİK ANALİZİ

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Bu araştırma, geo-kolonların, yumuşak zeminlerdeki numerik sonuçlarını tartışmaktadır. Analiz için, FLAC 2D (Fast Lagrangian Analysis of Continua) sonlu farklar prensibi ile hazırlanmış bilgisayar programı kullanılmıştır. *FLAC*'taki axisymmetry komutu, problemin yarısı modellenip, bir eksen etrafında döndürülmek suretiyle üç boyutlu bir model elde edilmesine izin vermiştir.

Bu araştırmanın temel amacı, gerilme konsantrasyon oranı, ve oturma azaltma oranları ile taş kolon boyu, çapı, elastik modülü, ve uygulanan temel yükleri arasında bir bağıntı bulmaktır.

Analizlerde, 1.30 m çapında dairesel temel altına yerleştirilmiş, çapları 60 cm, ve 80 cm olan, boyları sırası ile 1 m, 1.5 m, 2 m, 3 m olan geo-kolonlar kullanılmıştır. Kolon rijiditesinin oturma davranışına etkisini gözlemlemek açısından iki adet kolon elastik modulü seçilmiştir. Modelleme üç aşamadan oluşmaktadır. Birincisi, *FLAC*'ta oluşturulan ağın (mesh) doğruluğunu kontrol etmek için, elastik parametrelerle modellenmiş yumuşak zemine uygulanan temel yüklerinin meydana getirdiği oturma değerlerinin analitik yollarla bulunanlarla karşılaştırılması. Tatmin edici sonuçlar elde edildiği takdirde, rijit bir temel, ve geo-kolonların modellenmesi suretiyle yüklemeye devam edilmesi ikinci ve üçüncü aşamaları oluşturmaktadır. Temel yükleri olarak, kolonların değişik yükler altındaki davranışlarını görmek açısından, 25 kPa, 50 kPa, 75 kPa, ve 100 kPa alınmıştır.

Bu araştırmanın somut sonuçları vardır. L/d oranı 3.75'ten fazla olan kolonların oturmaya ekstra etkisinin hemen hemen hiç olmadığı, ve bu suretle 60 cm çapta 3 m'den fazla boyu olan kolonlar ile ekstra oturma iyileştirmesi yapılamayacağı sonuç olarak özetlenebilir.

Anahtar Kelimeler : geo-kolonlar, gerilme konsantrasyon oranı, oturma azaltma oranı, zemin iyileştirme.

Dedicated to Ayhan Kemaloglu Sr., the most devoted and principle oriented person I have ever known – may his dedication be an inspiration to us all...

Sarp Kemaloglu ‘04

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Eventually, I would like to extend my sincere gratitude to my alma mater – University of Wisconsin at Madison where I have acquired my Bachelor of Science degree in Civil Engineering, for giving me all the background I need to complete a Master of Science degree here in the Middle East Technical University.


> *“ What’s important in life is not how many breaths we get to take, but those moments that take our breath away “*
>
> Jacques Cousteau

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## NOMENCLATURE

| | | |
|---|---|---|
| $A_g$ | : | area of the pier elements |
| $A_r$ | : | area ratio, ratio of the area of the aggregate pier element to the area of the foundation |
| $A_s$ | : | area of the matrix soil below footing |
| b | : | settlement reduction ratio |
| c | : | cohesion |
| d | : | diameter of pier elements |
| E | : | elastic modulus |
| $E_b$ | : | bulk modulus |
| G | : | shear modulus |
| g | : | unit weight |
| $q_g$ | : | stress at top of pier elements |
| $q_s$ | : | vertical stress in matrix soil below footing |
| $k_g$ | : | aggregate pier modulus of subgrade reaction |
| $k_s$ | : | matrix soil modulus of subgrade reaction |

**L** : length of aggregate pier element

**n** : stress concentration ratio

**n** : poisson's ratio

**f** : internal friction angle

**Q** : total downward force exerted by the footing

**$Q_g$** : total downward force resisted by the piers

**$Q_s$** : total downward force resisted by the matrix soil

**RAP** : rammed aggregate piers (in the text it implies aggregate piers)

**$R_a$** : area ratio of $A_g$ to A

**$R_s$** : stiffness ratio

**S** : settlement

# CHAPTER 1

# INTRODUCTION

## 1.1 General

Sites that contain soft compressible soils extending to a substantial amount of depth typically require the installation of deep foundations to transfer foundation loads to firm, relatively stiffer strata as it is the case for end bearing piles, or improving the zone beneath the tip and rely on skin friction developed around the perimeter of the pile as it is the case for friction piles. In most of the cases where the overlying structure is of small nature such as a warehouse, or a single or double story residential or commercial building, the foundation system makes up a great proportional amount economically, to the overall cost of the structure. Thus, economically substantial savings can be made by comparing the cons and pros of that particular foundation system and selecting the one that is the most suitable for a given project (Fox, 2002).

Aggregate piers also known as geopiers – a trademark of Geopier® Company have been an advantageous and a successful alternative solution for a deep foundation system in highly organic soils over the others since 1989. This system has been used to reinforce a variety of soil types including peat, debris soil, uncompacted soil fills, loose silt, organic clays, loose sand, medium dense, and dense sands.

The technique comprises drilling a 45- 90 cm diameter cavity within a compressible layer to a depth of 2.5 -8 m and backfilling with clean crushed stone and densifying in layers typically 30 cm, with a tamper to form an aggregate pile – or an aggregate pier as referred to in this study- to improve compressibility and strength characteristics of the surrounding soil. Construction technique will be elaborated in the coming chapter.

In most of the cases, due to the presence of more stable and stiff layer in deeper elevations, aggregate piers are often designed to be of floating type – not end bearing type. Nevertheless, as will be illustrated on the following chapter, as a result of stress concentration on aggregate piers, vertical stress distribution due to foundation load below the pier is also said to be improved as relatively much less load is carried by the compressible zone preventing excessive, differential settlements. As a matter of fact, the main idea behind the application of aggregate pier elements is to introduce a media with higher elastic modulus (densified crushed stone) than the surrounding soil, and prestress the surrounding soil (will be referred to as 'matrix soil' from hereon) by impact energy, hence improving the composite properties of the compressible soil stratum. As a result, the long term total and differential settlements are limited to structural design tolerances.

## 1.2 Background

The aggregate pier method was invented by Dr. Nathaniel Fox in the mid 1980's. It did not take long for it to be approved and granted a patent, in fact by the mid 1990's, both US and foreign patents were granted. Since then, many aggregate piers have been sought to be a solution to various geotechnical engineering challenges. To express this in figures, in 2001 almost 55,000 aggregate piers have been installed in variety of projects. By the end of 2001, nearly 200,000 aggregate piers were used overall.

For over a decade and a half, this innovative method has been applied in numerous projects in the Unites States and recently gaining popularity in Asia and Europe. This method has been applied within a wide variety of soil conditions including peat to support compressive loads applied by the overlying foundation or structure. Supported structures up to date, have been noted to range from two to four story residential and commercial buildings, to warehouses. In over 250 places of applications, aggregate piers have been used as reinforcement in support of foundations of office buildings, parking lots, storage tanks, schools, warehouses, manufacturing buildings, roadway & railway embankments, floor slabs, commercial and industrial structures, earth slopes, mechanically stabilized earth walls etc.

Table 1.1 – Case histories of settlement behavior of aggregate piers in the US (Fox, Lawton '94)

| Project Description | Typical Foundation Description | Load kN | Bearing Pressure kPa | Settlement, mm (in) | | |
|---|---|---|---|---|---|---|
| | | | | Unreinforced Matrix Soil | Reinforced with Piers | Actual Settlement |
| **5 story office building, Columbia, SC** | 3.66 m square footing | 3,560 | 266 | 33 to 102 | 18 | < 1.5 |
| **12 m (40') tall milk silo Atlanta, GA** | 4.57 m square footing | 3,010 | 144 | 48 to 104 | 13 | < 1.8 |
| **46 × 91 m greenhouse, Atlanta, GA** | 0.91 m diameter footing | 160 | 244 | 58 to 79 | 5 | < 6 |
| **Industrial warehouse, Winterset, IA** | 1.52 m square footing | 445 | 193 | 150 to 230 | 23 | < 19 |

Table 1.1 cont.

| | | | | | | |
|---|---|---|---|---|---|---|
| **Office addition, Orenburg, SC** | 1.07 × 2.13 m footing | 801 | 352 | 41 to 112 | 13 | < 13 |
| **hospital addition, Hickory, NC** | 12.2 m square mat | 47,150 | 317 | 61 to 109 | 10 | 3.3 |
| **hospital addition, Hickory, NC** | 2.74 m square footing | 1,824 | 242 | 30 to 104 | 13 | < 6 |
| **16 story tower, Atlanta, GA** | 15.2 × 30.5 m mat | 66,720 | 144 | 20 to 89 | 10 | 6 |
| **12 story tower, Atlanta, GA** | 3.66 m square footing | 4,448 | 332 | 61 to 66 | 10 | < 6 |
| **7 story parking deck, Marietta, GA** | 4.27 m square footing | 5,782 | 318 | 124 to 188 | 38 | 20 to 33 |

# CHAPTER 2

# AN OVERVIEW OF AGGREGATE PIERS

## 2.1 Floating Foundations

Unlike end-bearing foundations, floating foundations are designed to terminate in soft, compressible layers. By definition, a deep foundation consists of a stiff composite layer that extends to a certain depth to reduce the applied foundation pressure and settlements as a result of consolidation of the compressible layer. Aggregate piers elements are designed to act as these stiff layers that increase the composite stiffness of the surrounding soil to depths that are most influenced by the stresses induced by the overlying footing. The aim is to limit long-term total and differential settlements to meet the design tolerances.

## 2.2 Aggregate Piers

Aggregate piers are crushed stone aggregate columns referred to as geopiers, constructed to support compressive loads on weak soils to reduce time dependent settlements and to act as composite bearing material. The effectiveness of aggregate piers can be attributed to the lateral prestressing that is induced on the matrix soil that during impact ramming of the piers by a beveled tamper (Hall Blake and Assoc., 2002). Due to the high stiffness and shear strength of the aggregate piers, they have a wide region of application from slope failure prevention to support of foundation loads.

### 2.2.1 *Properties of aggregate piers* (Lien & Fox, 2001)

i. Stone columns are installed using vibratory methods, aggregate piers are installed using impact ramming,

ii. Aggregate piers are designed mainly to stiffen subgrade soil. Secondary considerations include increased radial drainage within subgrade soil, thus increased consolidation rate and reduced construction commencement time ;

iii. Aggregate piers are relatively short, they extend about 3 to 4 times their width. As mentioned earlier, they are mostly of floating type and do not extend to stiffer zones and typical element lengths are in between 2 to 8 m ;

iv. Aggregate pier construction involves matrix soil displacement by means of forming a cavity by augers rather than horizontal or vertical soil displacement ;

v. Aggregate piers are compacted in thin lifts of 30 cm, prestraining and prestressing the adjacent matrix soil to form a stiff composite material for vertical and horizontal load supports ;

vi. Typical in-situ densities of aggregate piers are measured to vary around 2.2 to 2.4 tons / $m^3$ ;

vii. Typical void ratio values are as low as 0.07 to 0.23 ;

viii. Average value for internal friction angle Ø ranges between 45 to 55 degrees ;

ix. Allowable bearing pressures up to 300 kPa can be supported ;

x. Aggregate piers are ductile hence experience deformations without reduction in their strength ;

xi. Pier integrity is retained during an earthquake.

### 2.2.2 *Construction Procedure of Aggregate Piers*

The construction process of aggregate pier elements are fairly simple and do not require complex machinery and equipment. It consists of a 5-step construction procedure clearly illustrated below:

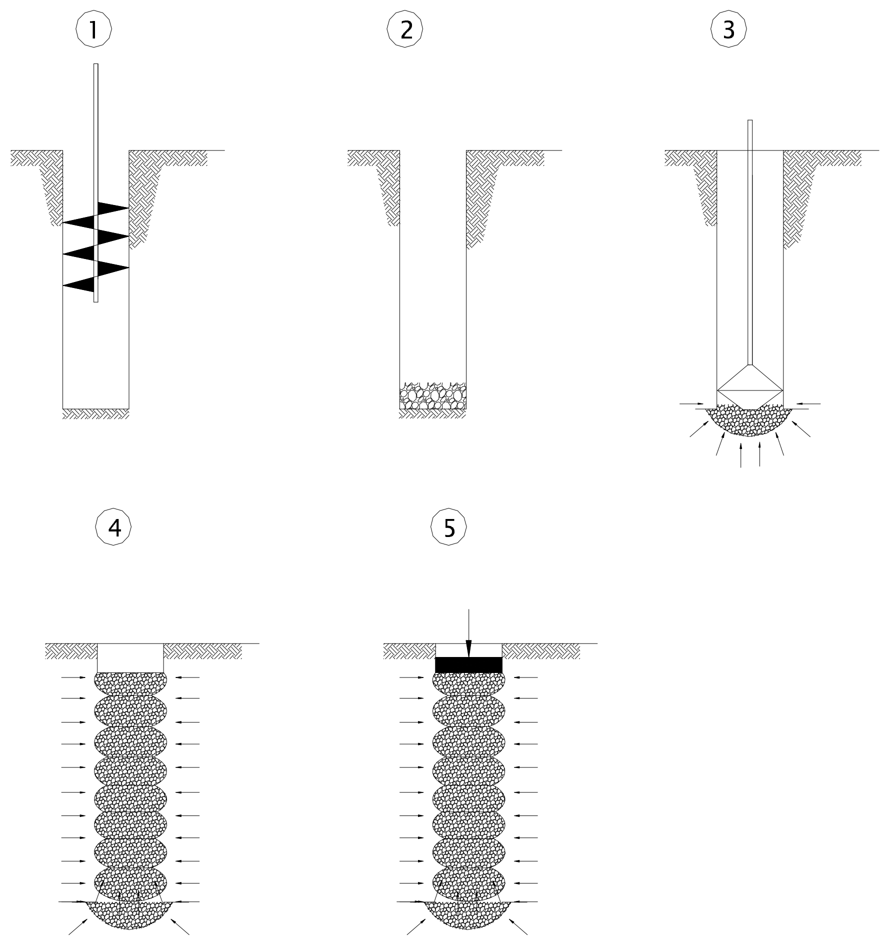
i. **Phase 1 :** This phase involves drilling of a cavity of 600 to 900 mm in diameter by augers. A common figure for a diameter is 750 mm. Typically, the depth of the cavity ranges between 2.5 to 8 m depending on the design. Shall there be cave-ins occurring, temporary casing is installed to prevent collapse of sidewalls. This casing is placed down to the design depth of the pier and advanced upward as backfilling lifts require which is typically 300 mm per lift.

ii. **Phase 2 :** To stabilize cavity bottom, graded clean crushed stone of 3 ¾ to 7 ½ cm sizes are backfilled into the cavity.

iii. **Phase 3 :** To form a bottom bulb using a beveled tamper. It should be noted that the energy applied is impact ramming energy and not vibration energy. Amplitudes of about 10 mm are applied with a frequency value ranging from 300 to 600 cycles per minute. Typically, bottom bulb is formed by application of impact for about 1 minute.

iv. **Phase 4 :** To form a stiff layer of aggregate piers, lifts of 300 mm aggregate layers are rammed with a beveled tamper. This provides lateral stress build up on matrix soil leading prestraining and prestressing of the matrix soil. Thus, providing additional stiffness.

v. **Phase 5 :** The last step involves preloading of the pile to take care of the secondary time dependent settlements of the matrix soil. The preloading phase especially accelerates the rate of the consolidation because the aggregate pier acts as a vertical drain to dissipate the excess pore water pressures.




1. Drill cavity using augers, install casing if cave-ins occur
2. Place crushed stone at bottom of cavity
3. Ram bottom stone with beveled tamper to produce bulb
4. Densify crushed stone in layers of 30 cm with tamper
5. Preload top of RAP element

Figure 2.1 – 5 Step construction procedure of aggregate pier elements

### *2.2.3 Construction Outcomes*

The ramming process described above enables an aggregate pier element to achieve a high angle of friction (Ø) leading to an increase in shear strength and stiffness. In fact, recent studies on aggregate pier elements reveal stiffness-wise they are about 10 to 55 times stiffer than the pre-reinforced matrix soil (B.H. Lien and N.S. Fox, 2001). Increased lateral earth pressure on matrix soil results in increased stiffness of the whole composite system. Introducing an open graded granular material to a fine-grained soil provides piers to act as vertical drains that speed up the consolidation process.

Aggregate piers are an effective solution to support earthquake loads due to their high shear strength characteristics, and their ductile behavior, which substantially reduces the potential for liquefaction. Also, by installing steel harnesses for uplift, aggregate pier elements act as effective anchorages to resist uplift forces.

### *2.2.4 Testing of Aggregate Piers*

Testing of aggregate piers is done with full scale in-situ models. Stiffness is defined by the term 'pier modulus', and this term is essential in computing the design strength and settlement estimate of the piers. Pier stiffness coefficient is denoted as $k_g$ in the preceding chapter where theory is elaborated. The set-up of the experiment apparatus is shown in figure 2.2. In this test, full cross section area of a pier element is applied a compressive load resembling that of a load exerted by the overlying footing. The load is applied at the top of the element. Load is then increased gradually and top deflection of the element is measured. As seen in the figure, installment of a telltale at the bottom of the element permits the recording of the movement at the bottom of the element. *The Aggregate pier Company* states that in most cases applied pressure is 150 % of the design stress at the top of the element. However, composite settlement is based on the stress at the design load.

Stiffness is defined by the applied stress at top of pier divided by the movement at top, and is calculated at design load.

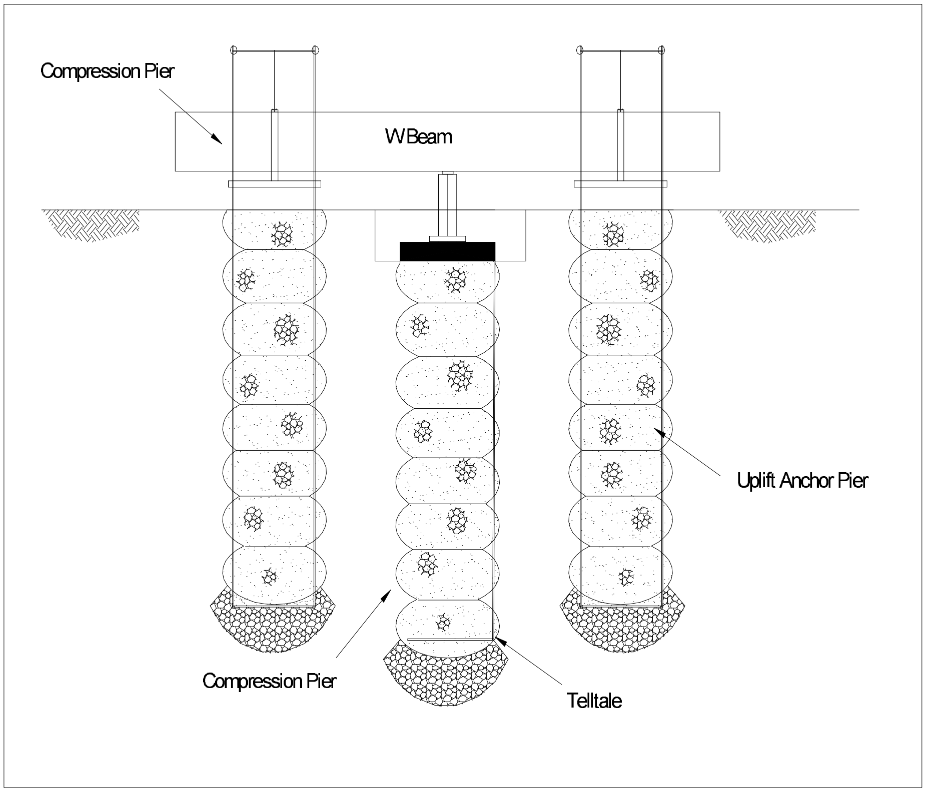



Figure 2.2 – Set-up for modulus test

Up to date, about 400 modulus tests have been performed on aggregate piers. The extent of these tests covers a range of soils for the past 10 years or so. According to the outcomes of the experiments of Fox and Lien, it is seen that reinforced soil stiffness is substantially higher (about 10 to 50 times) than that of pre-reinforced matrix soil. As Fox and Lien's synopsis states, vertical stresses concentrated on piers is 10 to 50 times greater than that on the matrix soils. Figure 2.3 illustrates the graph of typical data for a modulus load test.

Table 2.1 – Stiffness modulus for clay and sand (Wismann, Fitzpatrick, White, Lien, 2002)

| Soil Classification | Unconfined Compressive Strength (kPa) | Stiffness Modulus, kg (MN/m³) | Soil Classification | SPT N-Value | Stiffness Modulus, $k_g$ (MN/m³) |
|---|---|---|---|---|---|
| Clay | 10 - 110 | 34 - 48 | Sand | 1 - 6 | 45 - 71 |
| | 111 - 220 | 48 - 68 | | 7 - 12 | 71 - 77 |
| | 221 - 380 | 68 - 75 | | 13 - 25 | 77 - 88 |

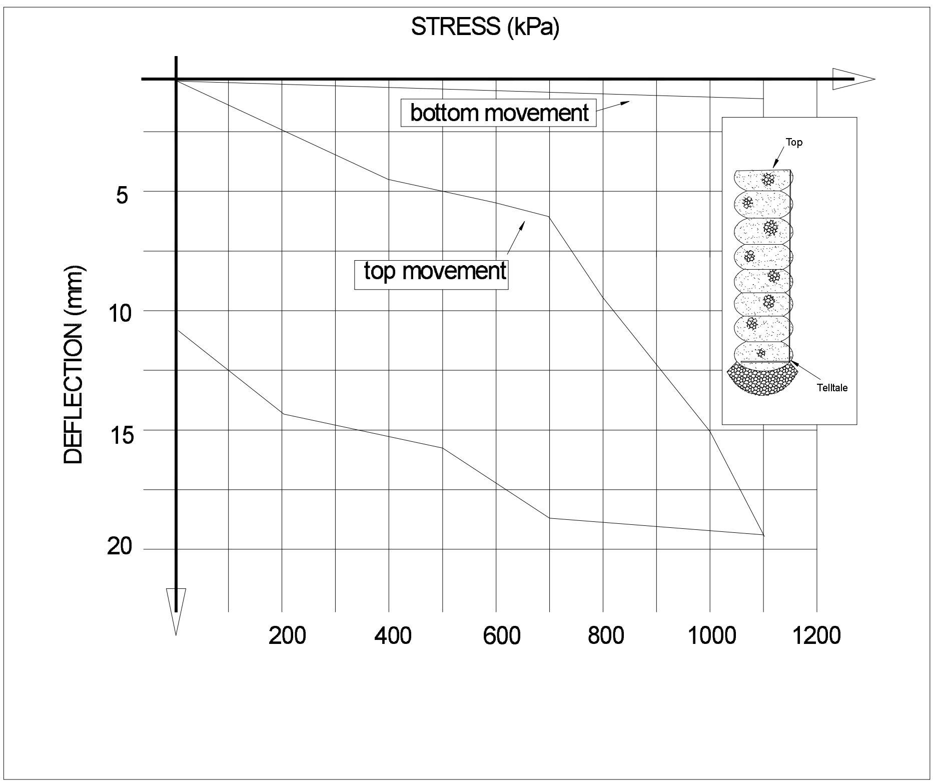



Figure 2.3 – Typical graph for stress vs deflection in a pier modulus test

As far as estimating shear strength is concerned, full scale direct shear tests are performed at site. Samples may also be tested in lab. Triaxial testing has also shown reliable results as compared to field results. Figure 2.4 is a gathering of data from Wissmann, Patrick, White, and Lien's (2002) full scale experiments. The slopes of the two straight lines give the friction angles for well and open graded aggregates respectively. Open graded meaning, fine content of the aggregates is zero and the latter meaning 5 to 10 % fines are included. Just for comparison, friction angle achieved in triaxial tests for well-graded aggregates is 51º.

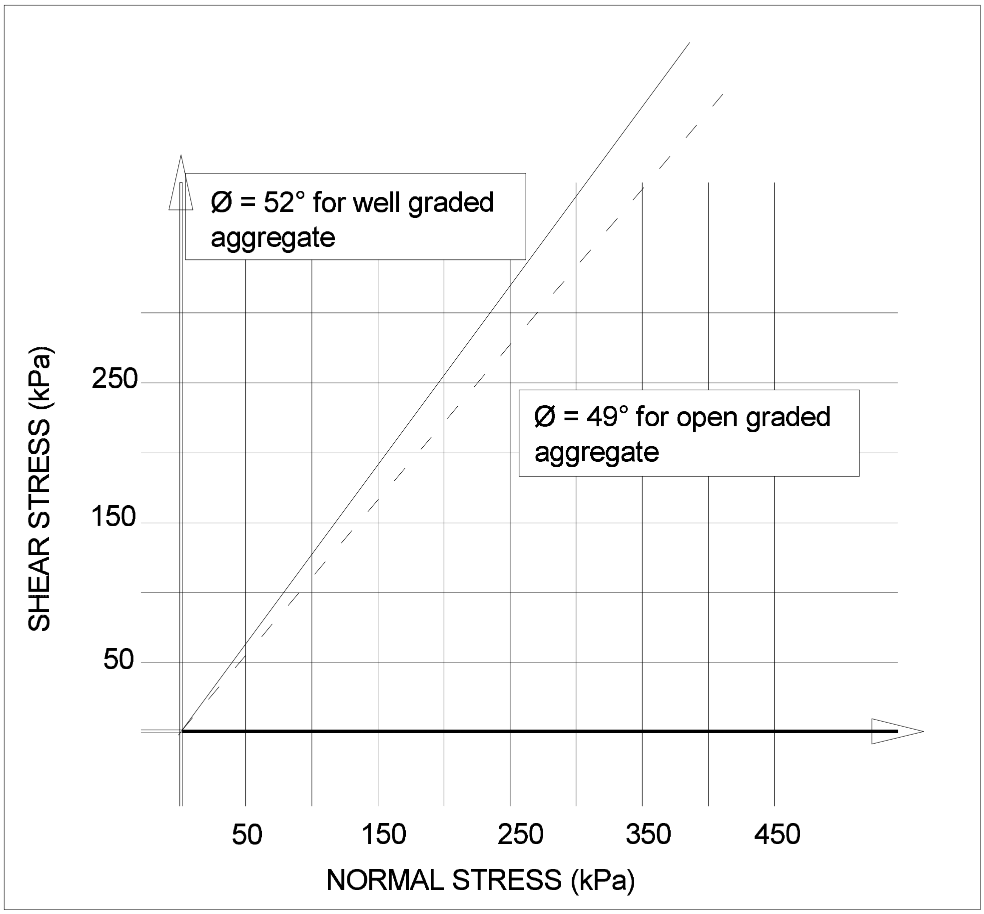



Figure 2.4 – Full scale direct shear test results (Wissmann et al, 2002)

One more testing that is performed on aggregate piers is the uplift capacity test. Though this is not a popular test, it is performed to determine the uplift force capacity of a pier element. The set up is not very different from the modulus test only tensile force is applied at the top of the pier. Because of the geometry of the pier, and the irregular skin, very high capacity for uplift is developed around the piers. A typical figure according to Fox and Lien is for 3 to 4 m long pier, uplift capacity is from 20 to 40 tons per pier.

### *2.2.5 Resistance to Lateral Loads*

Lateral loads may be induced on an aggregate pier element due to winds, earthquakes, and lateral earth pressures. As discussed in the foregoing text, due to high shear resistance of the piles, and as a result of stress concentration on aggregate piers which, will be discussed in the coming chapter, aggregate piers provide adequate resistance to lateral loads. Fox and Cowell (1998) defines the sliding resistance for an aggregate pier reinforced footing is the sum of the sliding resistance between the footing and the top of the aggregate pier element plus the sliding resistance between the footing and the matrix soil. Resistance between the footing and the pier element is the governing design criteria for the sliding resistance of the pier element.

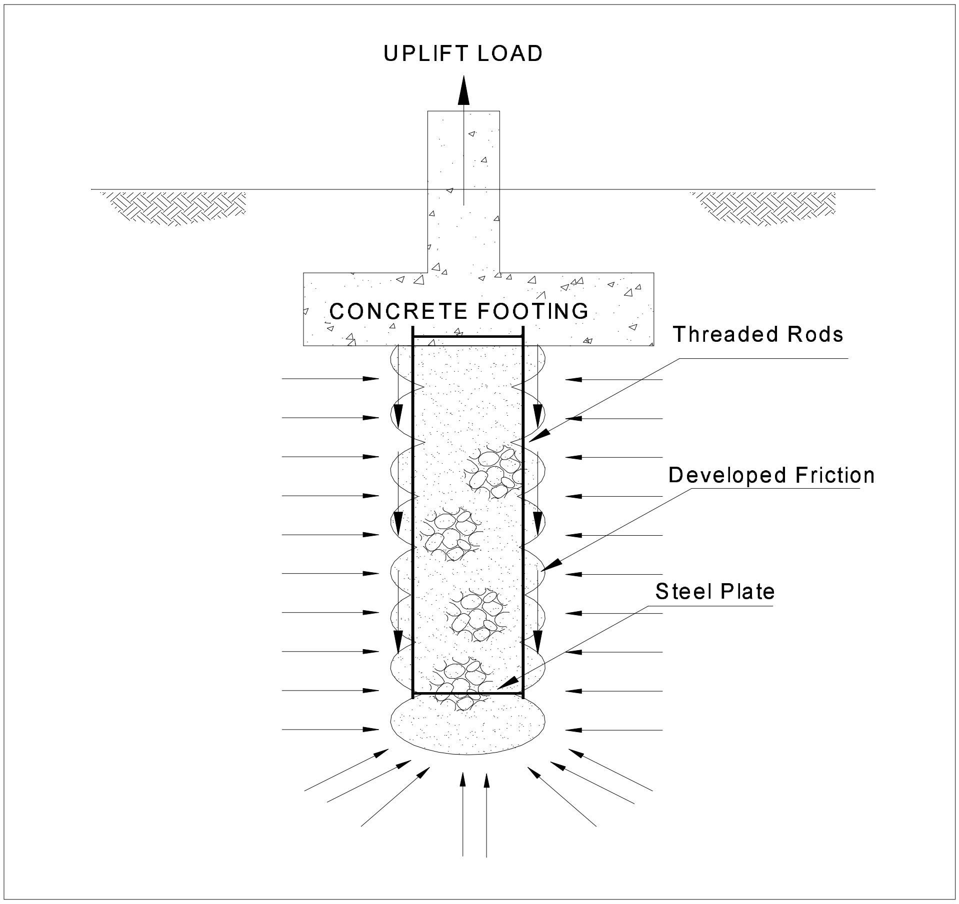



Figure 2.5 – Set-up for uplift capacity test

## 2.3 Advantages of Aggregate Piers

As mentioned earlier, when deep foundation systems are of concern, economics is the main driving force in choosing the right system once all design criteria are fulfilled. Based on this principle, aggregate pier system has been developed to be an alternate solution to deep foundation systems such as over-excavation, pile foundations, and vibratory stone columns.

### 2.3.1 *Comparison of Aggregate Piers versus Over-excavation*

Over-excavation is a traditional method involves replacing the soft compressible soil layer with a stronger backfill material. Often times it turns out to be the most expensive methods among the four methods. As compared to over-excavation, the biggest advantage aggregate piers hold is their lower cost and time savings. Although both of the methods are utilized to stiffen the existing soil, aggregate piers are more efficient to stiffen and improve soft shallow and deep soils with a lower cost of construction, more reliable, and quicker application. Both of the systems may be used to support loads from shallow strip and spread footings. Aggregate piers however, are able to support heavier loads than the over-excavated and replaced soil. To express this in figures, 300 to 500 kPa bearing pressures can be resisted by aggregate piers whereas this value is as low as 150 to 300 kPa for the over-excavated and backfilled soil. Construction procedure obviously is completely different in the two methods. Heavy grading equipment such as scrapers and bull-dozers are used to haul away a massive volume of soil in the over-excavation method. Backfilling of the material and re-compaction is also accomplished using heavy machinery. In aggregate pier applications, a cavity is drilled by means of augers to remove the compressible soil, and clean crushed stone is backfilled into the shaft and rammed by a 25 ton rammer hammer.

One rule of thumb that needs to be noted is that, in over-excavation method, construction schedule is highly dependent on the weather conditions. In rainy seasons, there can be a great deal of delays regarding excavation. Consistency of the backfill material is also another factor that determines the quality between these two methods. Inadequacy of the consistency of the backfill material in the over-excavation method prevents the backfill to present a homogenous behavior whereas the imported crushed stone in aggregate pier applications are always consistent due to the small volume.

As far as associated risks are concerned with these two methods, aggregate pier applications have a relatively low risk potential. Associated risk would include possible caving in but this only delays the project for a little bit once the casing installation is completed. However, in over-excavation method, due to presence of poor soil in specified design depth, deeper excavation may be needed that leads to a great deal of time delays. Moreover, where working below groundwater horizon, dewatering and excavation support may be required. Bottom of excavated area also needs stabilization for bottom heaves and collapses.

At certain places, where environmental regulations are strict, these two methods differ from each other regarding environmental impacts as well. Again, aggregate pier applications impose low disturbance to the environment. Low dust emission with bobcats and excavators, quieter construction with tapping of the hammer, small volume of the soil that is disturbed puts this technique on top of the latter regarding environmental point of view.

As discussed in the above paragraphs aggregate pier applications over over-excavation replacement method have more favorable sides, such as reduced costs and construction time.

### *2.3.2 Comparison of Aggregate Piers versus Pile Foundations*

Pile foundations are a conventional, and a widely accepted method that have served geotechnical engineers' purposes for many years. Although, they are a very reliable resolution for a deep foundation system, for cost savings, and due to the complexity of the construction procedure, they are usually selected as the last foundation option among the other improvement methods.

The biggest benefit of using an aggregate pier improvement method over pile foundations is time savings. Once acquired the required permits, performance rates in the field is about two times in lengthwise. Although piles are either designed as end-bearing or friction piles, occasionally, due to presence of the firm strata at deeper elevations, it becomes costly to design the piles as end-bearing piles. Common materials that are used as aggregate piers are 1 ½ to 3 inch open graded stone on bottom bulb and places below the groundwater horizon. Below groundwater table, ¾ to 1 ½ inch Class 2 AB aggregates are used.

The governing criterion in a structural design is the settlement criterion in most of the cases. In soft soils in particular, when long term consolidation is of concern, limiting settlement criterion is 25 mm. Hence, both of these systems behave well to reduce settlements and keep the settlements below benchmark limits

The element 'group' effect is also considered as an important issue regarding design of deep foundations. For pile foundations, capacity per pile is reduced for pile drag on matrix soil, on the contrary for aggregate piers, this capacity per element is increased due to horizontal stress reflection in matrix soil. Often times, piles are required to translate horizontal forces due to seismic activities by means of tie beams, and necessity for these beams is eliminated when aggregate piers are in concern.

Again, as this was the case above, where environmental issues are considered, especially, where there are adjacent structures, driving of piles in some cases may lead to damage due to vibration. Aggregate Pier Foundation Company cites the noise level climbs up to 110 decibels at 50 feet during driving a pile. They mention that this level is held steady at levels 75-85 decibels for aggregate pier applications. The biggest benefit of the aggregate piers with respect to the environmental issues is that ground vibrations are lower and the aggregate pier application is safer where working area is tight and surrounded by adjacent structures. Figure 2.6 sketches deep foundation systems discussed in the foregoing paragraphs.

### *2.3.3 Comparison of Aggregate Piers versus Stone Columns*

Although, the idea of aggregate piers does seem similar with the concept of *stone columns,* which is also another cost effective and widely used ground improvement method, one should comprehend that the application principles and the geotechnical design approaches are completely different. The intended use of stone columns is to support of flexible structures such as tanks, and embankments. On the other hand, aggregate piers are designed to support both rigid and flexible structures. Basically, all types of buildings, embankments are known to have been successfully supported by aggregate piers.

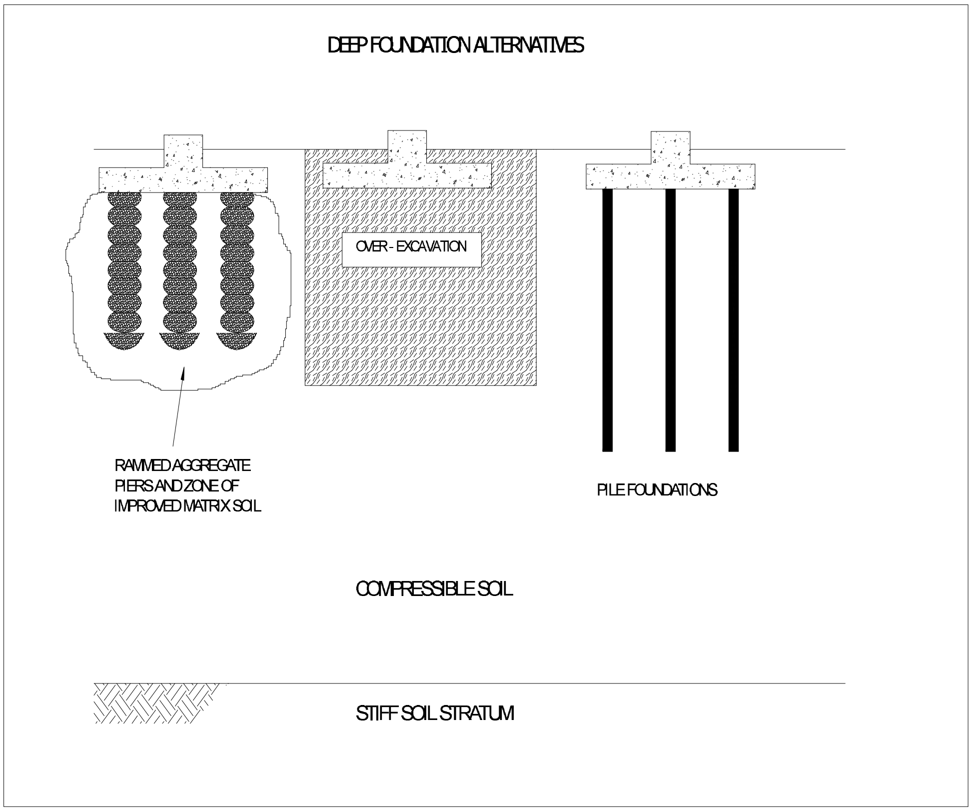



Figure 2.6 – Deep foundation alternatives in soft soils

Perhaps, the biggest difference between these two methods is their applications. Stone columns are installed using vibratory methods whereas aggregate piers are rammed into the drilled shaft using impact energy, thus the surrounding matrix soil is also improved due to lateral prestressing (bulging out of layers). On the contrary, stone columns form stiff, homogeneous stone piles with no improved zone. Typical element length for a stone column is between 5 to 15 m and it is between 2 to 8 m for aggregate piers. Table 2.2 summarizes the other notable differences between aggregate piers and stone columns.

Table 2.2 – Comparison of aggregate piers versus stone columns (courtesy of Geopier Co., 2003)

| | **Stone Columns** | **Aggregate piers** |
|---|---|---|
| **Typical length** | 5 - 15 m | 2 - 8 m |
| **Typical center to center spacing** | 4d | 2d |
| **Thickness of lifts** | 1.5 - 3 m | 20 - 30 cm |
| **Allowable foundation pressure** | 25 - 150 kPa | 250 - 300 kPa |
| **Typical length to diameter ratio** | 5 - 30 | 2 – 4 |
| **Construction equipment** | 6 m probe mounted crane | backhoe with 4 m long tamper & accs. |

### *2.3.4 Limitations of Aggregate Piers*

Disadvantages associated with aggregate piers can be categorized into two consisting of economic limitations and performance limitations.

The requirement of a drilled cavity, and the fact that almost all the soils requiring improvement with aggregate piers, being very soft and compressible, cavity collapse is an inevitable issue. To prevent this, temporary casing is placed, and advanced once the backfilling stage onsets. This slows down the application rate and increases the cost per element.

Additionally, where treatment zone depths are required to be greater than say 8 m, aggregate piers shall not be considered as a solution because they give best performance when used in compressible strata as a floating pile to depths up to 8 m.

## 2.4 Characteristics of Compressible Soils

Soils referred to as peat vary from a fibrous vegetative material that when undergoes a drying process is suitable for mulch or fuel, to finely divided semi carbonized organic material intermixed and interlayered with mineral soils, sands and silts in particular.

The analogy of the behavior of an aggregate pier in peat and a cylindrical shaped aggregates sample in a triaxial test provides a fair estimate of determining the lab strength of the piers. As a result of construction principles, high friction angle is the basis for design leading to higher shear strength value. According to the findings of Fox and Cowell, from their conducted full scale field experiments in 1998, measured internal friction angle of the aggregate piers came out to be greater than 50°.

Pier deflection is identified as the bulging of the pier out into the matrix soil layer. Since the aggregate pier is horizontally sustained from moving, lateral displacements are minimized once the construction process is carried out.

# CHAPTER 3

# THEORY

## 3.1 Load Transfer Mechanisms

Load transfer from the overlying footing to the piers is an essential issue when conducting settlement calculations. Aggregate piers are designed to have a certain area ratio with the footing area. As a general case, one can categorize load transfer mechanisms in three cases :

i. only one pier supports the overlying footing with the same diameter or width ;
ii. only one pier supports the overlying footing ;
iii. the overlying footing is supported by more than one pier with smaller diameters or widths than those of the footing.

The first case mentioned above is not very common in application. Rather, cases 2 and 3 are more widely used because of the reasons listed below.

### *3.1.1 Case 1*

Whatever load is induced by the footing is directly translated to the pier. Deflection response is dependent on the density, and granulometry of the aggregates. Modeling of this case is standardized in ASTM D-1994 by static load tests. Developed lateral pressures along the interface between the pier and the matrix soil is variable due to various reasons such as, stress relaxation characteristics of the soil after drilling, and initial state of stress before drilling. Generated lateral pressure against the matrix soil can be set to a limit equal to the passive resistance of the matrix soil.

### *3.1.2 Case 2 & 3*

These two cases may be analyzed together since the aggregate pier element area is expressed as a percentage of the total area of the footing. A very common value for this proportion is between 20 – 40%. Before the formulas are elaborated below, assuming a typical relative stiffness ratios among the range of 10 to 20, the portion of the load carried by the piers as a group (if more than one is installed) will vary between 71 – 93%. Bowles (1998) states that the ratio of the bearing stress applied to the piers to the bearing stress applied to the adjacent matrix soil is roughly equal to the relative stiffness ratio.

### *3.1.3 Load Transfer Mechanism to the Matrix Soil*

Behavior of a floating aggregate pier and a friction pile is analogous in the sense that they both rely on the developed skin friction around the perimeter of the elements. However, due to the undulating surface of the aggregate piers, bearing resistance is also created out into the matrix soil shear interface. This leads to a substantial amount of shear resistance of the piers. Another issue to keep in mind is that once the settlement onsets, the matrix soil bulges outward the pier element leading to an increased confinement pressure on the pier element as in a lab triaxial test. This also stiffens the pier element resembling stiffening that occurs in a strain-hardening material.

## 3.2 Settlement

There are a lot of interaction mechanisms involved in a settlement of a footing supported by an aggregate pier. These interactions include the ones in between footing and piers, footing and the matrix soil, and eventually matrix soil and piers. Experimental procedures in determining accurately these interactions and estimating the settlement more precisely is still under research. Meantime, the present formulas provide a rough estimate for settlement of a footing supported by aggregate piers. Figure 3.1 on the following page, illustrates the schematic of the developed pressures on a typical aggregate pier element.

Having supported with aggregate piers, settlement of the footing is reduced due to two factors: (1) composite stiffness of the soil plus the piers is substantially higher than that of the unreinforced soil itself, (2) vertical stresses transmitted below the pier reinforced zone are lower than that of unreinforced soil. Having said this, settlement analysis is disected into two sections: an analysis for the upper zone and an analysis for the lower zone. Total foundation settlement is computed adding these two partitions. Figure 3.2, on the next page is a sketch of division of the zones.

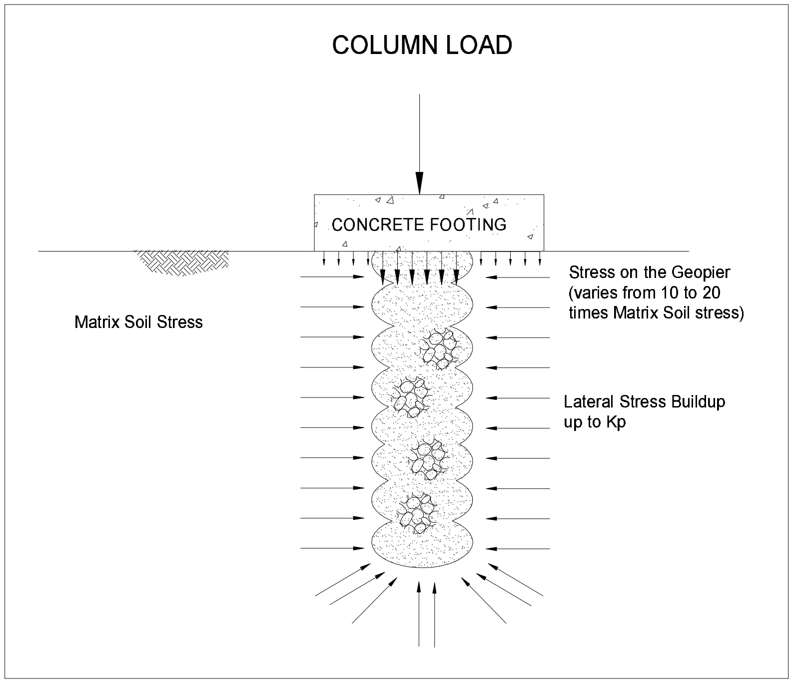



Figure 3.1 – Load transfer mechanism on a pier element

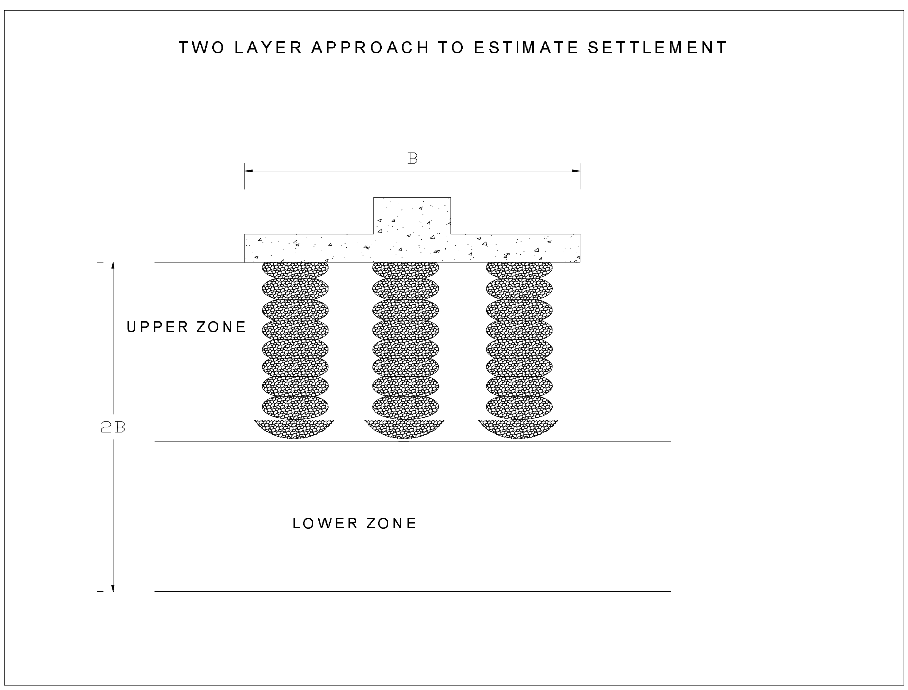



Figure 3.2 – Aggregate pier design approach to estimate settlement

*3.2.1 Upper Zone Analysis*

As seen from figure 3.2, the upper zone also referred to as the 'aggregate pier influence zone' is identified by the region whose depth is equal to the length of the aggregate pier elements. This zone is analogous to a spring acting as a stiff material whereas the matrix soil is considered to be a soft spring. Having mentioned this, the following formulas are relevant in estimation of the settlement occurring in the upper zone.

The assumptions made here are that the footing is perfectly rigid compared to the other foundation materials. Hence, the following expression can be assumed :

$\mathbf{Q = qA}$ may also be written as $\mathbf{Q = Q_g + Q_s = q_gA_g + q_sA_s}$ ....... **(1)**

where Q is the total downward force exerted by the footing
$Q_g$ is the total downward force resisted by the piers
$Q_s$ is the total downward force resisted by the matrix soil
$A_g$ is the area of the pier elements
$A_s$ is the area of the matrix soil below footing
$q_g$ is the stress at top of pier elements
$q_s$ is the vertical stress in matrix soil below footing

As may be perceived from the above equation, stresses induced by the footing to the composite foundation material (pier elements + matrix soil), depend on their relative stiffnesses ($R_s$) and areas.

As stated above, due to the footing being rigid compared to the other materials, the settlement of the pier elements will be equal to the settlement of the matrix soil. Thus, the following expression will hold true :

$\mathbf{s = q_g / k_g = q_s / k_s}$ .................... ............................ **(2)**

where s is the settlement of the footing

kg is the aggregate pier modulus of subgrade reaction

ks is the matrix soil modulus of subgrade reaction

Rearranging terms, the above equation becomes :

$$\mathbf{q_s = q_g(k_s / k_g) = q_g / (k_g / k_s) = q_g / R_s} \quad \ldots\ldots\ldots\ldots \quad \mathbf{(3)}$$

Equations (1) and (3) are combined to give the following expression :

$$q = \{q_gA_g / A + q_gA_s / (AR_s)\} = [q_gR_a + q_g(1\text{-}R_a) / R_s\}$$

$$\mathbf{q = \{q_g [ R_a + 1/R_s - R_a/R_s\} = \{q_g[R_aR_s + 1\text{-}R_a] / R_s\}} \quad \ldots\ldots\ldots \quad \mathbf{(4)}$$

$$\mathbf{q_g = \{qR_s / [R_aR_s + 1 - R_a]\}} \quad \ldots\ldots\ldots\ldots \quad \mathbf{(5)}$$

where $R_a$ is the area ratio of $A_g$ to A

$R_s$ is the stiffness ratio

so the ultimate upper zone settlement expression becomes :

$$\mathbf{s = (qR_s / [R_aR_s + 1 - R_a]) / k_g} \quad \ldots\ldots\ldots\ldots \quad \mathbf{(6)}$$

**Note: the subscript "s" is replaced by "m" denoting the matrix soil in some other texts.**

*3.2.2 Lower Zone Analysis*

Settlements contributed from the lower zone portion of the system are derived using the conventional geotechnical stress distribution expressions. This conventional stress distribution approach to estimate total settlement of the footing is conservative because presence of the pier elements results in more efficient stress dissipation with respect to depth below the footing bottom. Also, it does not account for the fact that the upper zone of the soil (the reinforced zone) is stiffened so that the compressibility of the soil is also the same as the matrix soil.

### *3.2.3 Concerns about Factor of Safety*

There are various issues regarding the forecasting of the settlement of the system reinforced with aggregate pier elements. The first one is that when calculating the upper zone settlements, for modulus values of the piers, the value corresponding to the 2.5 cm top deflection is taken. However, actual modulus values corresponding to a 1.25 cm top deflection comes up higher, so that by using a lesser value, settlement predictions are overestimated. As mentioned, the actual concentration of stress on the aggregate pier elements is neglected by assuming a conventional geotechnical engineering approach such as the Westergaard vertical distribution, where in actually stresses dissipate considerably with increasing depth. Likewise stated earlier in this text, due to prestraining and prestressing of the matrix soil beneath and around aggregate pier elements, additional effects regarding these on the settlement behavior are also neglected in the design process. Eventually, lower zone modulus of the matrix soil is usually underestimated due to lack of data on the preconsolidation pressures.

## 3.3 Case Histories

To compare the predicted and the actual settlements, and to obtain a better idea about the typical values come up in the aggregate pier design stage, several case histories are presented below. Up at the beginning of this text on table 1.1, 10 case histories have been summarized. The ones below are more detailed.

### *3.3.1 5-Story Office Building, Columbia, SC (Lawton & Fox, 1993)*

Dimensions of this building are 91 m × 49 m. The site location is categorized as Piedmont geological province, and the bearing soil is virgin soil. SPT blow counts within the upper 9 to 12 m of the soil stratum varied between 2 to 12, averaging 8. This stratum consisted of very loose to firm silty fine to medium sand. Underlying this stratum lied stiff clayey fine to coarse sand with SPT blow counts varying from 12 to 37, averaging 20. No groundwater had been encountered. Column loads varied in between 222 and 3,560 kN, with wall loads 58 and 102 kN/m. After all the decision process for choosing the most efficient system to support the spread footings, the owner of the project chose aggregate piers as a result of a value engineering process. Total cost savings of 250,000 USD have been estimated.

Static load tests implied a subgrade modulus of 149 MN/m³, which was twice the initial estimate of 76 MN/m³. The final design bearing pressure for the aggregate piers was 287 kPa which came out to be 4 times (72 kPa) the allowable bearing pressure for unreinforced soil. The estimated total settlement with this system was 18 mm which is less than the tolerable limit of 25 mm.

Dimensions of the installed aggregate piers were 0.76 m and 0.91 in diameter, and their heights were 1.5 m and 1.8 m respectively. All lifts were compacted to a blow count between 18 and 46, and the optimum blow count that corresponded to maximum dry density was measured to be 15 blows.

Six months after the completion of the building, settlement surveys were held on twelve instrumented columns. The maximum settlement has been recorded 1.6 mm and most giving away zero settlement values.

### *3.3.2 Industrial Manufacturing Building, Winterset, IO (Lawton & Fox, 1993)*

This is a very special project in the history of the aggregate pier applications. The soil conditions were the poorest in this project, among the other aggregate pier application projects. This building consisted of a large, one-story steel frame manufacturing building with column loads varying in between 180 and 800 kN designed to lie on top of underlying soft aeolian silts (loess), and overlain by stiffer glacial till. Groundwater table was present at elevations 0.08 to 0.9 m below ground level. To support the foundation system, over-excavation of the existing material was first commenced, but wall collapses ceased the process. Eventually, short aggregate piers were selected to support foundation loads.

Several tests including borehole shear tests were conducted on the loess fraction at the site. The results imply a drained friction angle of 37º, and cohesion of 10 kPa. Combining this data with the unit weight of the loess (14.3 kN/m³), the passive pressure came out to be 90 kPa, and these values agree with the results of the Ko stepped blade tests.

Aggregate piers of 0.76 m and 0.61 m have been installed. The final design bearing pressure of 192 kPa with an estimated settlement of 23 mm came out to be twice the bearing pressure without the aggregate piers. The heights of the piers were equal to twice their diameters. Again, as the previous case study, settlement reading taken after six months of completion, recorded 19 mm readings.

### *3.3.3. Puget Sound Condominium, Anacortes, WA (Lawton & Fox, 1993)*

In this project, aggregate pier elements were installed to support the foundation of a structure exerting 160 tons of column loads and 15 tons/m of wall loads on to the foundation. Soil formation consisted of sand and silt up to 3 m depth with SPT N values corresponding to 3 to 13, underlain by very soft to firm clay down to 22 m with SPT N values of 2 to 7. Groundwater table was close to the ground level.

Aggregate pier elements with lengths of 3.5 m to 4.5 m have been installed with measured modulus values of 82 MN/m³, and subsequently the used values were 35 NM/m³ for the design of the upper zone settlements. Upper zone contributions to the settlement were calculated to be 10 to 12 mm, whereas lower zone was calculated to be less than 13 mm. With modulus test, all this estimations have been successfully confirmed, yielding a very successful application of the aggregate piers.

### *3.3.4 Marriott Courtyard Hotel, Portland, OR (Lawton & Fox, 1993)*

This is a five-story concrete and wood frame hotel building whose column load values lie in between 100 and 175 tons. The ground formation consists of 14 m of very soft, compressible silty clay with SPT N values 1 to 2. Groundwater was present 3 m below the ground surface. Aggregate pier elements have been installed to support footings with design bearing pressures of 215 kN/m². The depth of aggregate pier elements was 3.7 m. Once the modulus load test was performed, it showed that 285 kN/m² would be feasible to confine the upper zone settlements to 12 mm or less. Based on this, upper zone calculations for settlement came out to be 10 – 12 mm, and lower zone settlement predictions ranged from 10 to 13 mm.

There are many more examples of projects regarding aggregate pier performance. The examples above are present just to give an overview about the typical values, site conditions, and places of applications.

# CHAPTER 4

# OVERVIEW ON FLAC (FAST LAGRANGIAN ANALYSIS OF CONTINUA)

## 4.1 Introduction

FLAC (Fast Lagrangian Analysis of Continua) used for this paper is a 2D finite difference code for modeling soil, rock, structural members like struts, beams, anchors, footings, etc. that is widely utilized by mining and geotechnical engineers. This explicit finite difference formulation of the code enables staged modeling. For instance, sequential excavations can be modeled to eliminate the effect of "wished in-place" excavation to simulate the real in-situ conditions, staged backfilling can also be performed that comes in handy for modeling embankments and dam cores.

This formulation can accommodate large strains and deformations associated with a number of constitutive models that will be discussed briefly in the below text. Modeling non-linear behavior is another advantage that provides more accurate modeling when a certain material in concern undergoes plastic, non-linear response.

## 4.2 Features

When utilizing FLAC in non-menu driven mode as used for this paper, because of the non-user friendliness of the code, one has to understand the principles and the theory that lead to conclusions. A useful tool that can be used efficiently in FLAC is the FISH feature that is the built-in language for the user to add defined variables and features.

Although this version of FLAC is a 2D code, use of *'axisymmetry'* configuration can be used to revolve a 2D drawing about an axis to create a solid model. Hence, only half of the model is specified and drawn.

FLAC can also be used to model interfaces where there are 2 or more materials in contact. Several analysis options are available in FLAC that are listed below :

i. Dynamic Analysis
ii. Thermal Analysis
iii. Creep Analysis
iv. Two-Phase Flow
v. User Defined Constitutive Models by C++

The above options provide the user to analyze a problem for different behaviors. For instance, in dynamic analysis option, the code enables modeling of the system for full dynamic response for a specified duration of time. This option is utilized when response spectrum outputs are needed for identifying liquefaction potential of the system. The next option declared above is the thermal option where displacements and generated stresses due to heat conduction of a material are of concern. In the creep option, is used where the material undergoes a time dependent deformation. When the model contains to immiscible fluids in a porous medium, for coupled simulations, two-phase flow option can be used. Finally, C++ programming option provides the user to edit custom constitutive models.

### 4.3 Constitutive Models

As mentioned in the foregoing paragraphs, there are a number of built-in constitutive models available in FLAC. Plus, the code allows for the utilization of the custom models edited and saved in C++. Each model is developed to represent specific behavior associated by different materials.

### *4.3.1 Null Model*

This model parameter corresponds to a void medium, where the user is modeling an excavation, a borehole. It can be used to model staged excavations where new material will be added at a later stage.

### *4.3.2 Elastic Model*

This model is utilized where the material used exhibits a linear stress strain behavior until strength limit. When dealing with manufactured materials such as steel, concrete, etc. and the continuum is homogenous and isotropic. This model can also be used for factor of safety calculations loaded below strength limit.

### *4.3.3 Transversely Isotropic Elastic Model*

This model is similar to elastic model described above, only it is used where thinly laminated material exhibiting a well-defined anisotropy, such as slate is being modeled for loading below strength limit.

### *4.3.4 Drucker – Prager Plasticity Model*

Where plastic deformations are likely to occur and where the failure criterion in which the shear yield stress is a function of isotropic stress, this model can be utilized. However, it has a limited application and is considered in soft clays with low internal friction angles. This model is served as a common model for comparison to implicit finite-element programs.

#### *4.3.5 Mohr – Coulomb Plasticity Model*

This model is widely used for general rock and soil mechanics where excavation and slope stability are to be modeled. Application of this model includes loose and cemented granular materials, soils, rock, and concrete that yield when subjected to shear loading, but the yield stress depends on the major and minor principal stresses only.

#### *4.3.6 Strain Hardening / Softening Mohr – Coulomb*

Where granular materials are used in a system that exhibit non-linear hardening or softening, this model gives more accurate results. When studies after failure is of concern such as progressive collapse, yielding, etc., this model gives better results.

#### *4.3.7 Ubiquitous – Joint Model*

Representative material for this model is a thinly laminated material that exhibits anisotropy in strength. This model is based on a Mohr – Coulomb material that exhibits well-defined strength anisotropy due to embedded planes of weakness. Application field for this model consists of excavation in closely bedded strata.

#### *4.3.8 Bilinear Strain Hardening / Softening Ubiquitous – Joint Model*

This model is a combination of strain hardening / softening model with the ubiquitous model where a thinly laminated material exhibiting non-linear material hardening or softening is of concern for post failure studies.

#### *4.3.9 Double – Yield Model*

In places where permanent volume decrease is caused by pressure exerted on lightly cemented granular material for hydraulically placed backfill, this model yields better results. This model is an extension of the strain-softening model to simulate irreversible compaction as well as shear yielding.

### *4.3.10 Modified Cam – Clay*

This model is used where shear strength, and deformability is a function of volume change. A typical place of application for this model is geotechnical construction on clay.

## 4.4 Selecting the Best Constitutive Model

It is recommended that attempting the problem with the simplest model is the best way to approach the problem once the properties of the materials are figured out. In fact, one is advised to model the problem using an elastic model for simplicity as it requires only two parameters- bulk and shear moduli. This provides a simple perspective of the behavior of the problem and also saves time for it runs the fastest. It is often helpful to start off with a simple model, by observing the behavior, grid densities may be chosen accordingly. At later stages, according to the nature of the problem, and the materials in concern, more complex constitutive models may be selected.

# CHAPTER 5

# *FLAC* MODELING & ANALYSIS OF AGGREGATE PIERS

The main purpose of this study is to reflect the settlement behavior of aggregate piers through precise modeling. This settlement behavior of aggregate piers will later be expressed in terms of stress concentration ratio ‘n’, and settlement reduction ratio ‘b’.

## 5.1 Problem & Model Definition

There are several parameters that have been used as variables throughout the analysis. These variables are the elastic modulus of the aggregate piers, the length and diameter of the aggregate piers, and finally the foundation load. 64 runs have been carried out in *FLAC* 2D for the analysis of aggregate piers.

It is crucial to choose the material properties and set-up a correct geometry by choosing an appropriate grid pattern for the sake of accuracy, and time saving. The foregoing paragraphs elaborate on the specific phases before the modeling phase.

## 5.2 Model Parameters

For the compressible layer modeled in *FLAC*, the parameters are presented in table 5.4. Note that the analysis is carried on for a 'long-term' analysis, hence drained parameters are used in the model. According to the values illustrated in the above tables, the compressible stratum is classified as soft clay, thus excessive and differential settlements are most likely to occur upon loading due to foundation, embankment, road, etc. Therefore, the soil requires treatment to improve settlement and strength characteristics.

Table 5.1 – Values used in the FLAC model for the compressible soil

| | |
|---|---|
| **Soil Type** | Soft Clay |
| **Condition** | Drained |
| **Elastic Modulus, E (MPa)** | 2.45 |
| **Shear Modulus, G (MPa)** | 0.875 |
| **Bulk Modulus, $E_b$ (MPa)** | 4.1 |
| **$q_c$ (MPa)** | 0.7 |
| **Unit Weight, $g_{sat}$ (kN / m³)** | 18 |
| **c' (kPa)** | 2 |
| **Ø' (degrees)** | 26 |
| **Groundwater Level** | 0.5 m below surface |
| **Poisson's Ratio, $n$** | 0.4 |
| **Depth of Compressible Layer (m)** | 8 |

The following table summarizes the parameters used for modeling an aggregate pier. From the field results available were the elastic modulus itself. To figure out the bulk and shear modulus required for modeling, equations (5.1) and (5.2) have been used.

If the values for modulus for these two components of the system were to be compared, it could be seen that the modulus values for the aggregate pier element are much higher than that of the compressible soil. The ratio is in the range of 20 – 50 which agrees with the values discussed in the literature survey. Hence, all the parameters used in the model are consistent with the actual values derived from conducted field experiments.

Table 5.2 – Aggregate pier parameters used in the FLAC model

| | |
|---|---|
| **Pier Material** | Clean Crushed Stone of 15 - 30 mm |
| **Diameter (cm)** | Ø60, Ø80 |
| **Elastic Modulus, E (MPa)** | 25, 100 |
| **Shear Modulus, G (MPa)** | 9, 36 |
| **c (kPa)** | 0 |
| **Ø (degrees)** | 45 |
| **Poisson' s Ratio, n** | 0.3 |
| **Bulk Modulus, $E_b$ (MPa)** | 42, 166 |
| **Unit Weight, g (kN / m³)** | 16 |
| **Foundation Load (kPa)** | 25, 50, 75, 100 |
| **Height of Pier (m)** | 1, 1.5, 2, 3 |

Table 5.3 below denotes the variables used in the *FLAC* model. These parameters have been chosen as variables to cover the range of values that they may acquire and to have a better understanding of the behavior of the system in general.

$$E_b = \frac{E_s}{3(1-2n)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{(5.1)}$$

$$G = \frac{E_s}{2(1+n)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{(5.2)}$$

where

Es is the elastic modulus expressed in terms of MPa

Eb is the bulk modulus expressed in terms of MPa

G is the shear modulus expressed in terms of MPa

n is the dimensionless Poisson's ratio

Table 5.3 – Variables used in the FLAC model

| Variable Parameter | Medium | Values |
|---|---|---|
| | | |
| **Elastic Modulus, E (MPa)** | Aggregate Pier Element | 25, 100 |
| **Length (m)** | Aggregate Pier Element | 1, 1,5, 2, 3 |
| **Diameter** | Aggregate Pier Element | Ø60 cm, Ø80 cm |
| **Foundation Load (kPa)** | Aggregate Pier Element | 25, 50, 75, 100 |

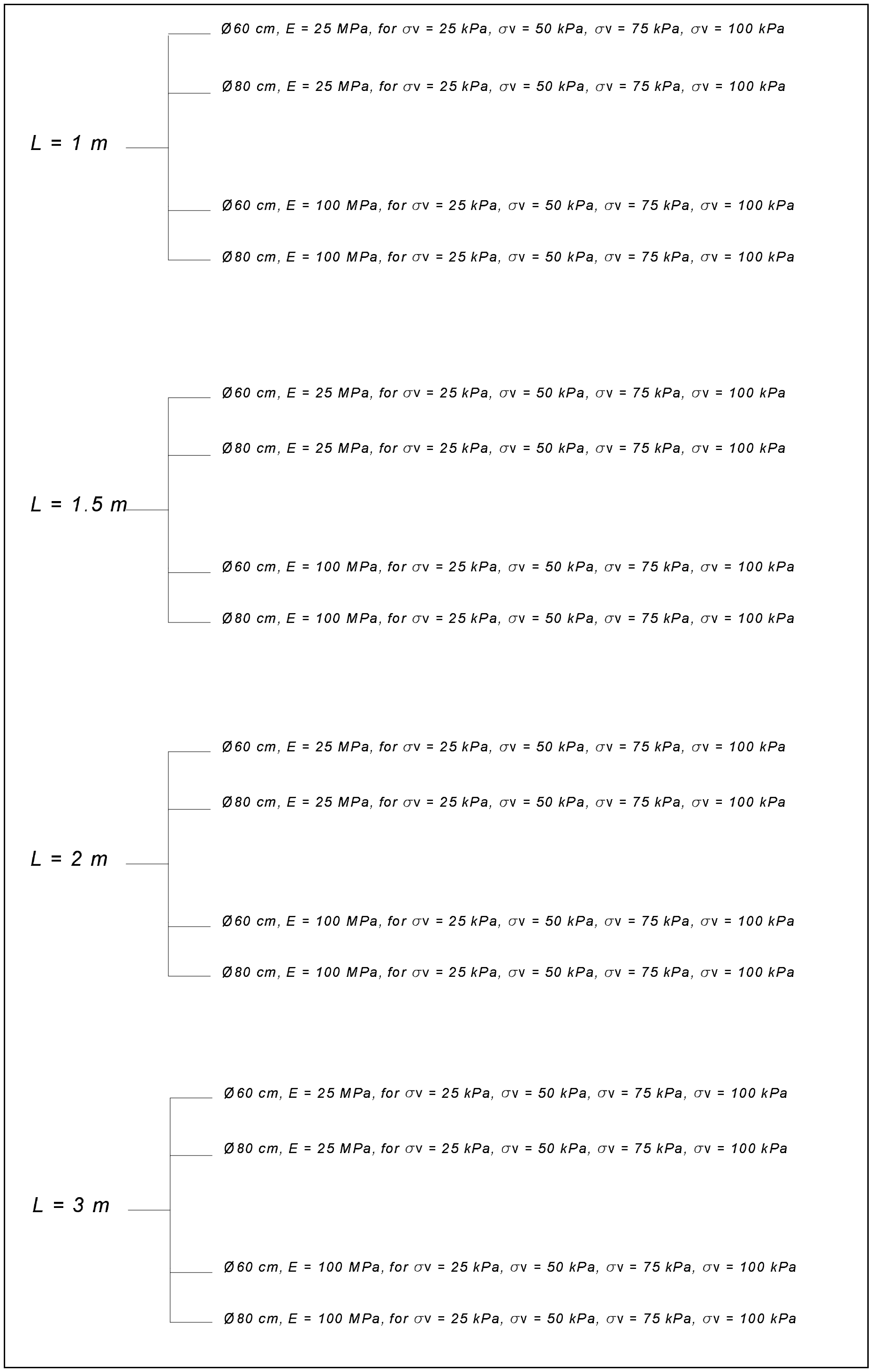



Figure 5.1 – Model scenarios

Figure 5.1 is a schematic illustration of the particular cases that have been modeled and run in *FLAC*. For each of the cases, displacement, vertical stress distribution, horizontal distribution, and settlement plots have been generated. Next section discusses the grid, constitutive model, and geometry selection for the system.

## 5.3 Grid Selection & Problem Geometry

It is crucial for a finite difference code to select an appropriate mesh size to represent accurately the system that is to be analyzed. Fine meshes are for greater accuracy, but the duration of the runtime is relatively higher. Coarse meshes are easy to run but they tend to lack the required accuracy. What it boils down to in the end is that it really depends on what is to be modeled and the degree of the required accuracy. As a rule of thumb, grid sizes shall be chosen with an aspect ratio as close to 1 as possible. Up to aspect ratios of 4 to 1 do yield fair and accurate results, however exceedence of that threshold will lead in inaccurate results.

For this research, models have been described with mesh aspect ratio of 2.5 to 1, so it is within the region described above. Square grids, nevertheless have been proven to give the best results. But sometimes, the user is obliged to use rectangular elements in accordance with the problem geometry. For instance for this research, the spacing between vertical gridlines is chosen as 5 cm because every dimension to be modeled including the diameter of foundation, diameter of aggregate piers are divisible by 5 so vertical gridlines will denote whole dimensions. On the other hand, spacing between horizontal gridlines is chosen as 12.5 cm. The same principle applies here. Every dimension to be modeled including the depth of the foundation, depth of piers, level of the groundwater table, and the depth of the soil stratum are divisible by 12.5 so horizontal gridlines will denote whole dimensions as well. Therefore, mesh aspect ratio is determined keeping in mind the principles discussed above. It really depends on the geometry of the model.

All analysis has been carried out using a grid size of 160 by 64. As seen from the profile drawings, the depth of the compressible layer is 8 m. Underlying stratum is taken as rigid with fixities both constrained in x and y directions

Making use of the axisymmetric modeling option in *FLAC* provides a realistic 3D approach to the system. Figure 5.2 is a representation of the 3D system revolved to a full 360º about i=1 line. A slicing plane provides a more detailed view of the cross section of the formed 3D solid. Concrete footing and the underlying aggregate pier element is shown at the right. Only half of the system geometry is defined in the *FLAC* code, thus revolving it 360º in space makes it a full 3D solid system with the required geometry.

## 5.4 Selection of Constitutive Models

As mentioned in Chapter 4, there are number of constitutive models built in ready in *FLAC* to represent the behavior of certain materials. Obviously, it plays a great deal of role for the accuracy of the results. There are two types of constitutive models used in this research: Mohr – Coulomb Model, and Elastic Model.

Concrete foundation is modeled as an elastic material having the density of concrete which is 2500 N / $m^3$ and having appropriate elasticity parameters such as shear modulus of 8.4 GPa, and bulk modulus of 14.9 GPa. Since this foundation material is rigid relative to the underlying soil having a bulk modulus of 2.45 MPa, (approx. 6000 fold), the 25 cm concrete foundation settles the same every at node which will be illustrated when the particular scenarios are discussed in the preceding paragraphs.

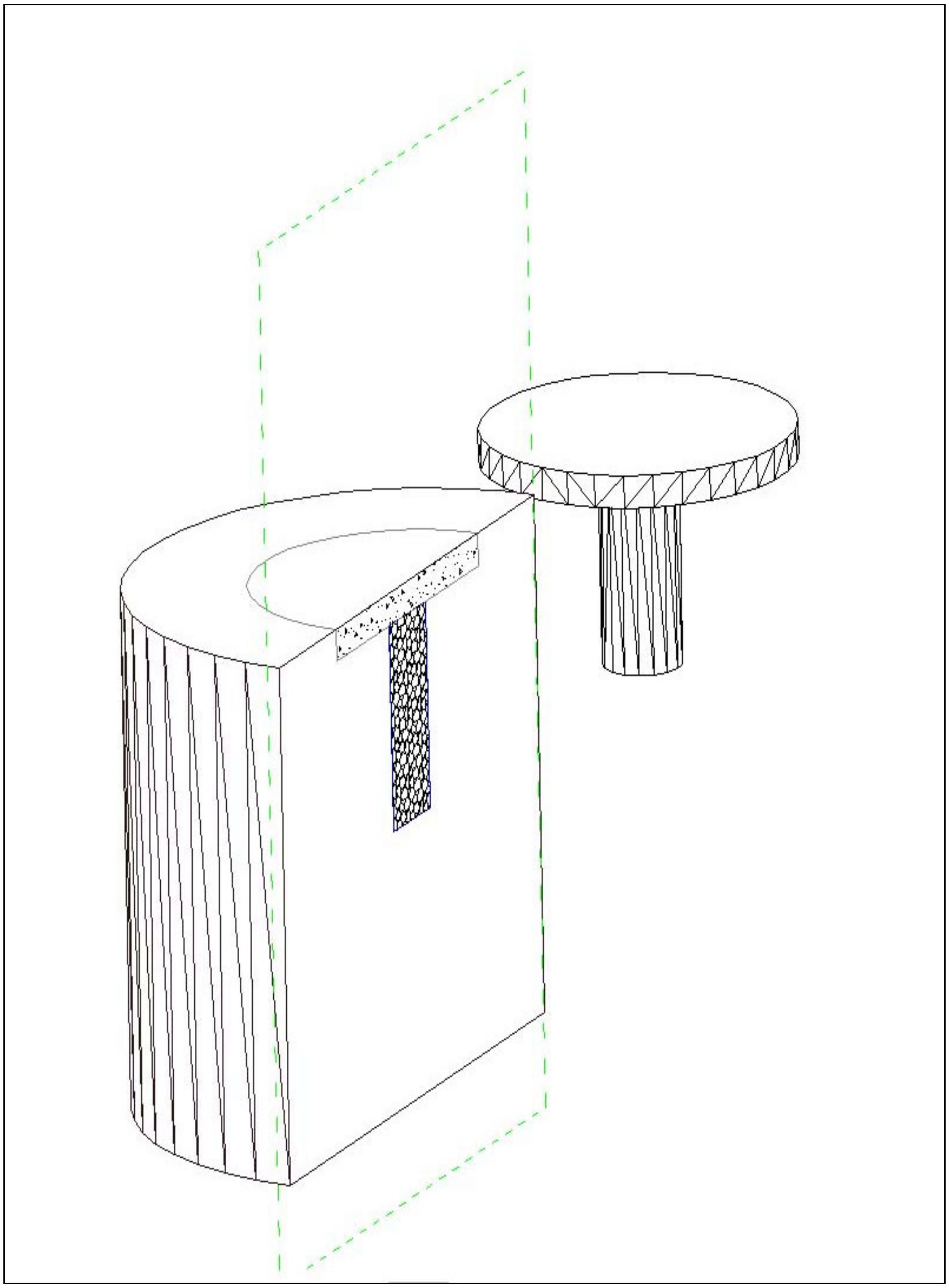


Figure 5.2 – 3D modeling approach represented by axisymmetry in FLAC (not to scale)

Constitutive model for the compressible clay layer and the aggregate pier element is the Mohr – Coulomb Model. All parameters have been entered in FLAC according to the appropriate constitutive model that, that material is attributed to.

Table 5.4 – Summary of constitutive models

| **Medium** | **Attributed Constitutive Model** |
|---|---|
| Compressible Soil | Mohr - Coulomb |
| Aggregate Pier | Mohr - Coulomb |
| Foundation Footing | Elastic |

## 5.5 Boundary Conditions

A crucial phase of a modeling process is to decide on the boundary conditions for the system. Defining the right boundary conditions lead to more accurate results. For the case presented in figure 5.3, the bottom and the right portion of the compressible soil stratum are constrained from displacing in both x and y directions. However, the line of axisymmetry is fixed with rollers allowing movement in y direction but preventing any horizontal movement. By constraining the bottom boundary, it is assumed that there is a firm, rigid stratum underlying the soft. For stability, center line of the axis of symmetry is constrained from x displacement rather than vertical displacement is because, all the vertical stress and settlement values are examined through this axis on the output figures.

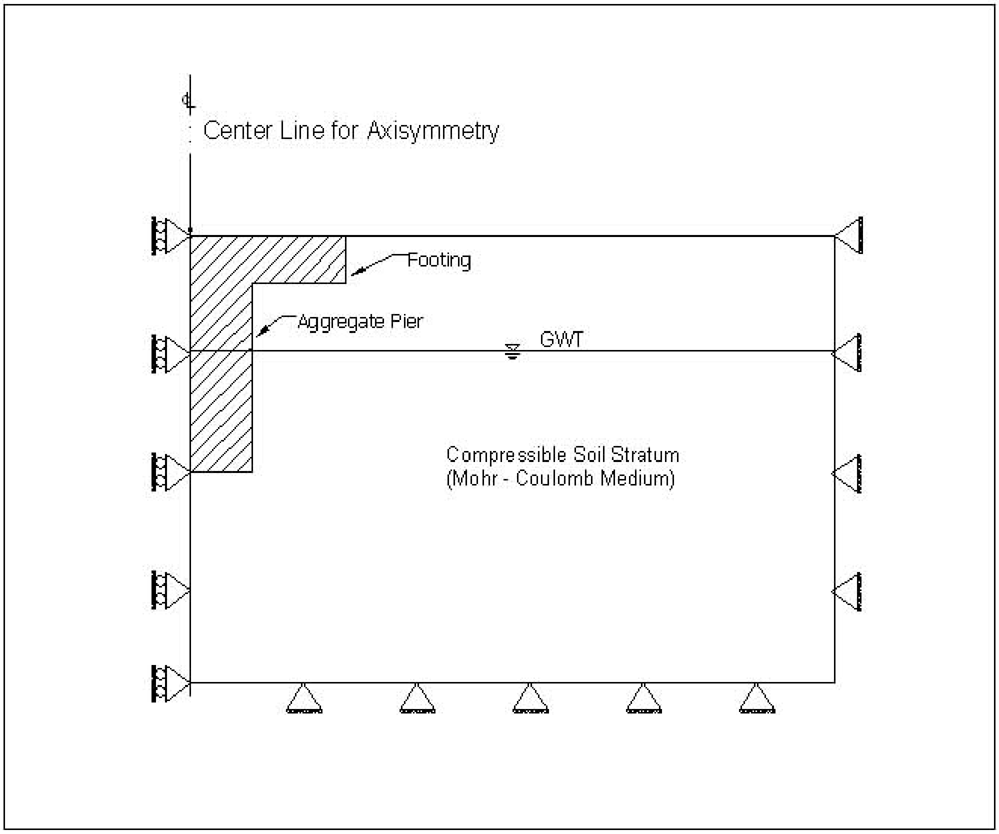



Figure 5.3 – Model boundary conditions

## 5.6 Selection of Output Values

As mentioned earlier, the aim here is to find whether or not the length of an aggregate pier element has an influence on settlement reduction, and if so, what is the optimum length of a pier element.

The following individual case studies correspond to the ones illustrated in figure 5.1. In addition to the presented cases, there are a few more extra cases to represent the behavior of longer piles up to a length of 5 m.

## 5.7 Accuracy of the Model

It is very important in FLAC to do the modeling step by step. It is always practical and helpful to start with a simple model and then build upon that more complex elements so that one can keep track of the changes he is making, and not get lost where he cannot find his mistake onwards. Having said all this, to start off with, a simple all elastic model of the soil and the foundation is set up in FLAC. There are no aggregate piers present in this trial. This is to check whether or not the mesh set-up, and the geometry of the model give the required accuracy. By setting the properties such as density, elastic modulus, shear modulus, both the foundation and the surrounding soil is let to be of the same material. The groundwater table level is set 50 cm below surface as it would be set in the final model.

The idea behind doing this is to compare settlement values obtained in FLAC with the analytical solutions from Lambe Whitman. Foundation pressures of 25 kPa, 50 kPa, 75 kPa, and 100 kPa have been applied and the corresponding maximum settlement values are shown below on table 5.8. The analytical formula in Lamb Whitman for a simple expression for the settlement of a circular footing at the center line is :

$$r = Dq_s\, R/E\ 2(1-n^2) \quad \text{.........................................} \qquad \mathbf{(5.3)}$$

Table 5.5 – Comparison of elastic FLAC settlements vs analytical values

| **LOAD (kPa)** | **FLAC settlement (mm)** | **Lamb & Whitman (mm)** | **% Difference** |
|---|---|---|---|
| 25 | 10,8 | 11,0 | 1,9 |
| 50 | 21,6 | 21,4 | 0,9 |
| 75 | 32,5 | 32,1 | 1,2 |
| 100 | 43,3 | 43,0 | 0,7 |

By interpreting the above percent differences, it can be seen that FLAC has provided accurate estimates for the maximum settlement values. To take it one step further, additional plots for effective vertical stress distribution, and settlement values on the central axis corresponding to the four load scenarios have been presented below for the circular rigid footing resting on untreated matrix soil (with no aggregate piers installed).

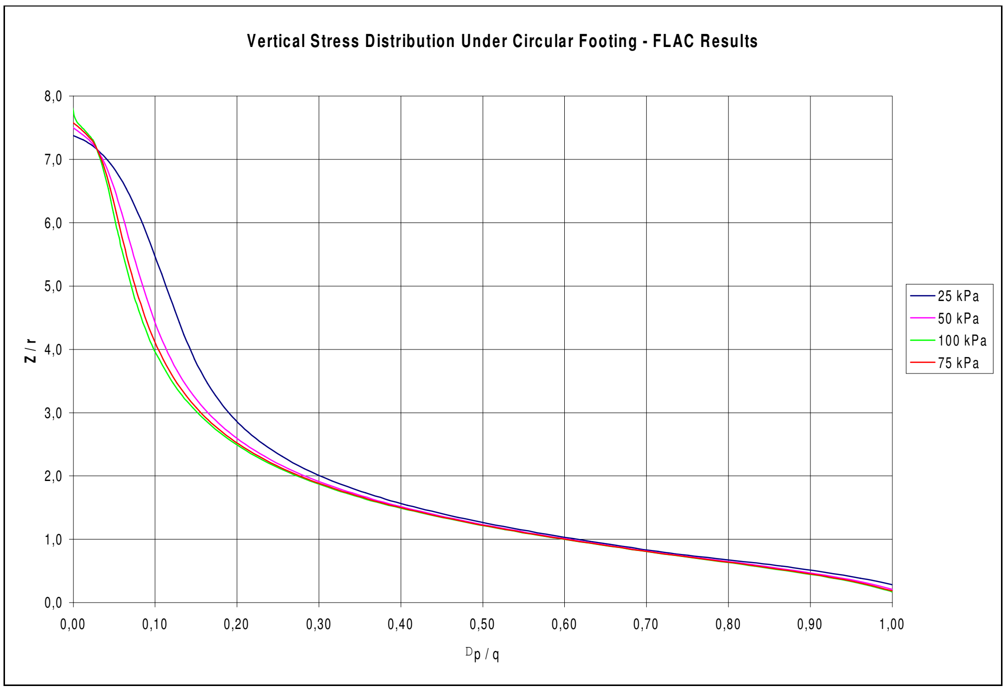



Figure 5.4 – Vertical effective stress distribution at the center line for an elastic medium

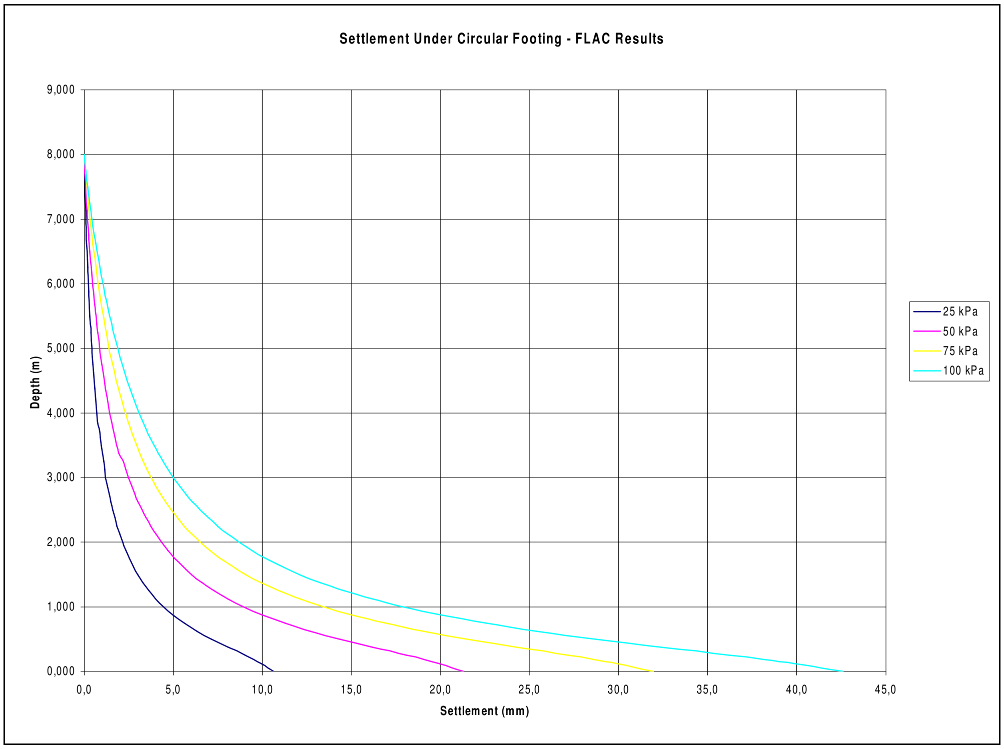



Figure 5.5 – Settlement at the center line for an elastic medium

## 5.8 Settlement Values for Untreated Soil

Once satisfactory results are achieved and that the geometry does not give an error when FLAC is run, the next step will be to model all media with the most appropriate constitutive model. This phase consists of identifying the foundation as an elastic concrete medium, the compressible soil as a Mohr-Coulomb medium, with the relevant properties as will be shown in the foregoing paragraphs. Here are the settlement values for the untreated soil :

Under 25 kPa pressure - *8.1 mm*

Under 50 kPa pressure - *20.3 mm*

Under 75 kPa pressure - *40.0 mm*

Under 100 kPa pressure - *68.9 mm*

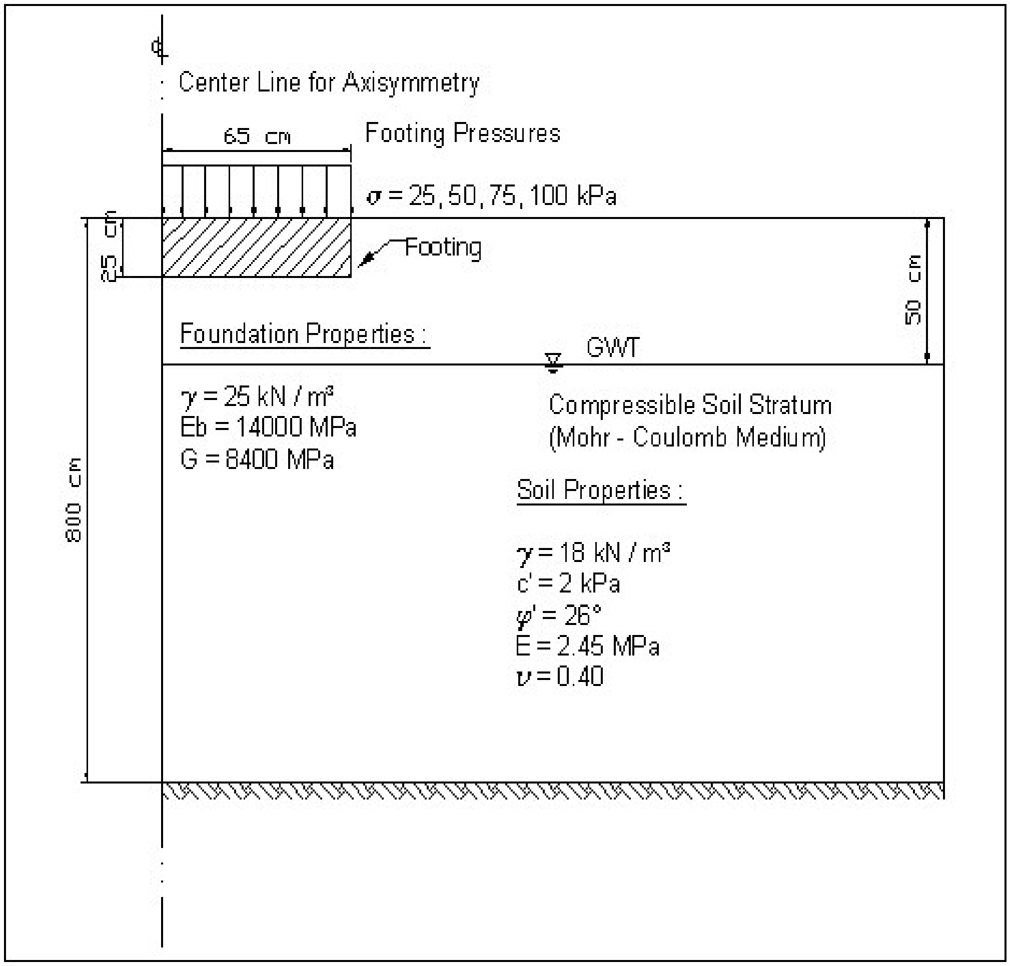



Figure 5.6 – Problem geometry for untreated soil

## 5.9 Improvement with Aggregate Pier Elements

Now that untreated soil settlement values are available, the next step will be to model aggregate pier elements complying with the scenarios presented in figure 5.1. The procedure for this is to model only half of the system and use axisymmetry for a 3 dimensional representation. There are four parameters that vary in this approach: The length of the pier element that is 1 m, 1.5 m, 2 m, and 3 m, the modulus of the pier elements, that is 25 MPa, and 100 MPa, the diameter of the pier elements, that is 60 cm, and 80 cm, and finally, the applied foundation pressure, that is 25 kPa, 50 kPa, 75 kPa, and 100 kPa.

The readings for the settlement values are taken at the central axis which is the axis of symmetry. The stress concentration ratio *'n'* which is described as the average effective vertical stress generated within the aggregate pier element divided by the average effective vertical stress generated right beneath the circular foundation has been calculated by averaging the readings from FLAC output tables, the effective vertical stress generated at the axis of symmetry, and the one generated at the right edge (to avoid edge disturbances, the reading has been taken 5 cm inwards off the edge). For calculations for the settlement reduction ratio, settlement values at the center line have been recorded.

### 5.9.1 Model Scenarios

#### 5.9.1.1 Case 1

This particular case consists of modeling 1 m long aggregate pier elements right below the circular footing. Surrounding soil as described, is modeled to be of a Mohr-Coulomb medium, and so is the aggregate pier element. The circular footing is elastic. Groundwater table is set 50 cm below surface level. Circular footing has a diameter of 1.3 m, and aggregate pier element has diameters of Ø60 cm and Ø80 cm. Applied foundation pressure varies in between 25 kPa, and 100 kPa. Figure 5.7 is an illustration of the problem dimensions and properties for case 1.

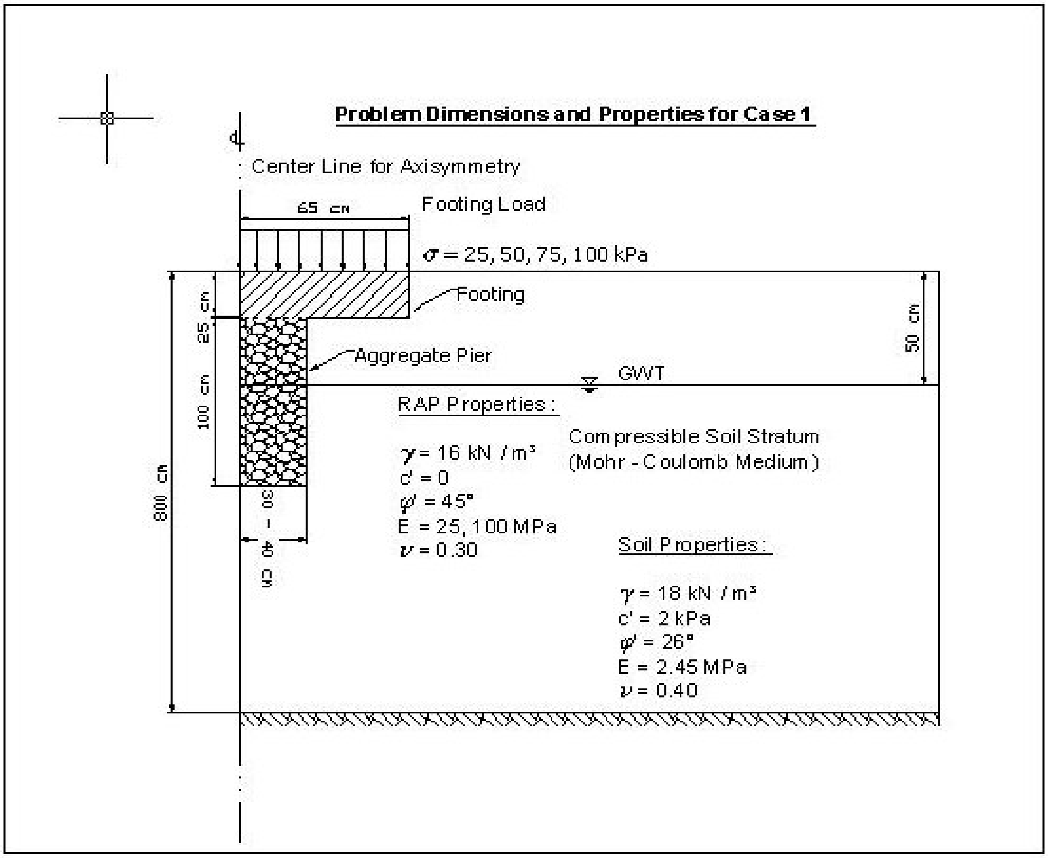



Figure 5.7 – Problem dimensions and properties for case *1*

*5.9.1.1.1 Case 1.1*

The variables for this case is presented in the below table. This case pertains to a 1 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is 1.67. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.6, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 6.5 mm. Although, the current settlement is not substantial (under 25 mm), still an aggregate pier element provides a 20% settlement reduction.

*5.9.1.1.2 Case 1.2*

This particular case pertains to a 1 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 1.67. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.6, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 15.5 mm. Although, the current settlement is not substantial (under 25 mm), still an aggregate pier element provides a 24% settlement reduction.

*5.9.1.1.3 Case 1.3*

This particular case pertains to a 1 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 1.67. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.6, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 35.0 mm, yielding a 13% settlement reduction.

Table 5.6 – Description of cases 1-1, 1-2, and 1-3

| **Case No** | ***1-1*** | | **Case No** | ***1-2*** | | **Case No** | ***1-3*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 1 | | **RAP Length (m)** | 1 | | **RAP Length (m)** | 1 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 | | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 6,5 | | 0,00 | 15,5 | | 0,00 | 35,0 |
| 0,50 | 6,3 | | 0,50 | 14,9 | | 0,50 | 34,2 |
| 1,00 | 5,8 | | 1,00 | 13,8 | | 1,00 | 32,6 |
| 1,50 | 4,4 | | 1,50 | 9,8 | | 1,50 | 27,0 |
| 2,00 | 3,0 | | 2,00 | 5,9 | | 2,00 | 12,7 |
| 2,50 | 2,2 | | 2,50 | 4,3 | | 2,50 | 5,4 |
| 3,00 | 1,6 | | 3,00 | 3,2 | | 3,00 | 4,4 |
| 3,50 | 1,3 | | 3,50 | 2,5 | | 3,50 | 3,5 |
| 4,00 | 1,0 | | 4,00 | 2,0 | | 4,00 | 2,7 |
| 4,50 | 0,8 | | 4,50 | 1,5 | | 4,50 | 2,2 |
| 5,00 | 0,6 | | 5,00 | 1,2 | | 5,00 | 1,7 |
| 5,50 | 0,5 | | 5,50 | 0,9 | | 5,50 | 1,3 |
| 6,00 | 0,3 | | 6,00 | 0,7 | | 6,00 | 0,9 |
| 6,50 | 0,2 | | 6,50 | 0,5 | | 6,50 | 0,6 |
| 7,00 | 0,1 | | 7,00 | 0,3 | | 7,00 | 0,4 |
| 7,50 | 0,1 | | 7,50 | 0,1 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

#### *5.9.1.1.4 Case 1.4*

This particular case pertains to a 1 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 1.67. Maximum settlement that was achieved in the presence of no aggregate pier elements under

100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.7, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 56.1 mm, yielding a 19% settlement reduction.

*5.9.1.1.5 Case 1.5*

This particular case pertains to a 1 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 1.67. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.7, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 6.2 mm, yielding a 23% settlement reduction, although , the current settlement is not substantial (under 25 mm).

*5.9.1.1.6 Case 1.6*

This particular case pertains to a 1 m long, Ø60 cm aggreg ate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 1.67. Maxi mum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.7, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 14.4 mm, yielding a 29% settlement reduction, although , the current settlement is not substantial (under 25 mm).

Table 5.7 – Description of cases 1-4, 1-5, and 1-6

| Case No | *1-4* | | Case No | *1-5* | | Case No | *1-6* |
|---|---|---|---|---|---|---|---|
| RAP Length (m) | 1 | | RAP Length (m) | 1 | | RAP Length (m) | 1 |
| RAP Diameter (cm) | Ø60 | | RAP Diameter (cm) | Ø60 | | RAP Diameter (cm) | Ø60 |
| $E_{RAP}$ (MPa) | 25 | | $E_{RAP}$ (MPa) | 100 | | $E_{RAP}$ (MPa) | 100 |
| $s_{FOUND}$ (kPa) | 100 | | $s_{FOUND}$ (kPa) | 25 | | $s_{FOUND}$ (kPa) | 50 |
| | | | | | | | |
| Depth (m) | S (mm) | | Depth (m) | S (mm) | | Depth (m) | S (mm) |
| | | | | | | | |
| 0,00 | 56,1 | | 0,00 | 6,2 | | 0,00 | 14,4 |
| 0,50 | 54,5 | | 0,50 | 6,1 | | 0,50 | 14,1 |
| 1,00 | 51,0 | | 1,00 | 5,7 | | 1,00 | 12,1 |
| 1,50 | 39,9 | | 1,50 | 4,4 | | 1,50 | 9,0 |
| 2,00 | 20,8 | | 2,00 | 3,0 | | 2,00 | 5,9 |
| 2,50 | 6,5 | | 2,50 | 2,2 | | 2,50 | 4,2 |
| 3,00 | 5,0 | | 3,00 | 1,6 | | 3,00 | 3,2 |
| 3,50 | 4,1 | | 3,50 | 1,3 | | 3,50 | 2,5 |
| 4,00 | 3,3 | | 4,00 | 1,0 | | 4,00 | 1,9 |
| 4,50 | 2,6 | | 4,50 | 0,8 | | 4,50 | 1,5 |
| 5,00 | 2,1 | | 5,00 | 0,6 | | 5,00 | 1,2 |
| 5,50 | 1,6 | | 5,50 | 0,5 | | 5,50 | 0,9 |
| 6,00 | 1,2 | | 6,00 | 0,3 | | 6,00 | 0,7 |
| 6,50 | 0,8 | | 6,50 | 0,2 | | 6,50 | 0,5 |
| 7,00 | 0,5 | | 7,00 | 0,1 | | 7,00 | 0,3 |
| 7,50 | 0,2 | | 7,50 | 0,0 | | 7,50 | 0,1 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

#### *5.9.1.1.7 Case 1.7*

This particular case pertains to a 1 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 1.67. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.8, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 30.8 mm, yielding a 23% settlement reduction.

*5.9.1.1.8 Case 1.8*

This particular case pertains to a 1 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is also 1.67. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.8, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 47.9 mm, yielding a 30% settlement reduction.

Table 5.8 – Description of cases 1-7, and 1-8

| **Case No** | ***1-7*** | | **Case No** | ***1-8*** |
|---|---|---|---|---|
| **RAP Length (m)** | 1 | | **RAP Length (m)** | 1 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 30,8 | | 0,00 | 47,9 |
| 0,50 | 30,4 | | 0,50 | 47,5 |
| 1,00 | 26,2 | | 1,00 | 43,6 |
| 1,50 | 20,7 | | 1,50 | 35,9 |
| 2,00 | 9,8 | | 2,00 | 18,3 |
| 2,50 | 5,8 | | 2,50 | 7,1 |
| 3,00 | 4,6 | | 3,00 | 5,6 |
| 3,50 | 3,6 | | 3,50 | 4,4 |

| Table 5.8 cont. | | | | |
|---|---|---|---|---|
| 4,00 | 2,8 | | 4,00 | 3,5 |
| 4,50 | 2,2 | | 4,50 | 3,0 |
| 5,00 | 1,7 | | 5,00 | 2,2 |
| 5,50 | 1,3 | | 5,50 | 1,7 |
| 6,00 | 1,0 | | 6,00 | 1,2 |
| 6,50 | 0,7 | | 6,50 | 0,8 |
| 7,00 | 0,4 | | 7,00 | 0,5 |
| 7,50 | 0,2 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 |

*5.9.1.1.9 Case 1.9*

This particular case pertains to a 1 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is 1.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.9, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 5.9 mm, yielding a 27% settlement reduction, although , the current settlement is not substantial (under 25 mm).

*5.9.1.1.10 Case 1.10*

This particular case pertains to a 1 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the above table. Length to diameter ratio ‘ $L_d$ ’ for this case is also 1.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.9, the maximum

settlement with the presence of a 1 m aggregate pier element is reduced to 15.2 mm, yielding a 25% settlement reduction.

### *5.9.1.1.11 Case 1.11*

This particular case pertains to a 1 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$ ' for this case is also 1.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40 mm as mentioned earlier. From table 5.9, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 32.2 mm, yielding a 20% settlement reduction.

Table 5.9 – Description of cases 1-9, 1-10, and 1-11

| **Case No** | ***1-9*** | | **Case No** | ***1-10*** | | **Case No** | ***1-11*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 1 | | **RAP Length (m)** | 1 | | **RAP Length (m)** | 1 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 | | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 5,9 | | 0,00 | 15,2 | | 0,00 | 32,2 |
| 0,50 | 5,7 | | 0,50 | 14,7 | | 0,50 | 31,5 |
| 1,00 | 5,4 | | 1,00 | 13,9 | | 1,00 | 30,3 |
| 1,50 | 4,3 | | 1,50 | 11,1 | | 1,50 | 25,2 |
| 2,00 | 3,0 | | 2,00 | 6,6 | | 2,00 | 15,7 |
| 2,50 | 2,1 | | 2,50 | 4,4 | | 2,50 | 5,8 |
| 3,00 | 1,6 | | 3,00 | 3,3 | | 3,00 | 4,5 |
| 3,50 | 1,2 | | 3,50 | 2,5 | | 3,50 | 3,5 |

| Table 5.9 cont. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4,00 | 1,0 | | 4,00 | 2,0 | | 4,00 | 2,8 |
| 4,50 | 0,8 | | 4,50 | 1,5 | | 4,50 | 2,2 |
| 5,00 | 0,6 | | 5,00 | 1,2 | | 5,00 | 1,7 |
| 5,50 | 0,4 | | 5,50 | 0,9 | | 5,50 | 1,3 |
| 6,00 | 0,3 | | 6,00 | 0,7 | | 6,00 | 0,9 |
| 6,50 | 0,2 | | 6,50 | 0,5 | | 6,50 | 0,7 |
| 7,00 | 0,1 | | 7,00 | 0,3 | | 7,00 | 0,4 |
| 7,50 | 0,1 | | 7,50 | 0,1 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

##### *5.9.1.1.12 Case 1.12*

This particular case pertains to a 1 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$ ' for this case is also 1.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.10, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 52.3 mm, yielding a 24% settlement reduction.

##### *5.9.1.1.13 Case 1.13*

This particular case pertains to a 1 m long, Ø80 cm aggregate pier eleme nt with an elastic modulus of 100 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$ ' for this case is also 1.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.10, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 5.5 mm, yielding a 32% settlement reduction, although , the current settlement is not substantial (under 25 mm).

#### *5.9.1.1.14 Case 1.14*

This particular case pertains to a 1 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$ ' for this case is also 1.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.10, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 13 mm, yielding a 36% settlement reduction, although , the current settlement is not substantial (under 25 mm).

Table 5.10 – Description of cases 1-12, 1-13, and 1-14

| **Case No** | ***1-12*** | | **Case No** | ***1-13*** | | **Case No** | ***1-14*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 1 | | **RAP Length (m)** | 1 | | **RAP Length (m)** | 1 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 100 | | **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 52,3 | | 0,00 | 5,5 | | 0,00 | 13,0 |
| 0,50 | 51,4 | | 0,50 | 5,4 | | 0,50 | 12,9 |
| 1,00 | 49,5 | | 1,00 | 5,3 | | 1,00 | 12,0 |
| 1,50 | 42,0 | | 1,50 | 4,4 | | 1,50 | 9,2 |
| 2,00 | 26,9 | | 2,00 | 3,0 | | 2,00 | 6,1 |
| 2,50 | 10,4 | | 2,50 | 2,2 | | 2,50 | 4,3 |
| 3,00 | 4,9 | | 3,00 | 1,6 | | 3,00 | 3,2 |

| Table 5.10 cont. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3,50 | 4,2 | | 3,50 | 1,3 | | 3,50 | 2,5 |
| 4,00 | 3,4 | | 4,00 | 1,0 | | 4,00 | 1,9 |
| 4,50 | 2,7 | | 4,50 | 0,8 | | 4,50 | 1,5 |
| 5,00 | 2,1 | | 5,00 | 0,6 | | 5,00 | 1,2 |
| 5,50 | 1,6 | | 5,50 | 0,4 | | 5,50 | 0,9 |
| 6,00 | 1,2 | | 6,00 | 0,3 | | 6,00 | 0,7 |
| 6,50 | 0,8 | | 6,50 | 0,2 | | 6,50 | 0,5 |
| 7,00 | 0,5 | | 7,00 | 0,1 | | 7,00 | 0,3 |
| 7,50 | 0,2 | | 7,50 | 0,1 | | 7,50 | 0,1 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

#### *5.9.1.1.15 Case 1-15*

This particular case pertains to a 1 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 1.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.11, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 27.6 mm, yielding a 31% settlement reduction.

#### *5.9.1.1.16 Case 1-16*

This particular case pertains to a 1 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 1.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.11, the maximum settlement with the presence of a 1 m aggregate pier element is reduced to 43.8 mm, yielding a 36% settlement reduction.

Table 5.11 – Descriptions of cases 1-15, and 1-16

| **Case No** | ***1-15*** | | **Case No** | ***1-16*** |
|---|---|---|---|---|
| **RAP Length (m)** | 1 | | **RAP Length (m)** | 1 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 27,6 | | 0,00 | 43,8 |
| 0,50 | 27,4 | | 0,50 | 43,5 |
| 1,00 | 25,7 | | 1,00 | 39,5 |
| 1,50 | 20,3 | | 1,50 | 32,2 |
| 2,00 | 12,4 | | 2,00 | 20,9 |
| 2,50 | 6,0 | | 2,50 | 7,9 |
| 3,00 | 4,7 | | 3,00 | 5,6 |
| 3,50 | 3,6 | | 3,50 | 4,5 |
| 4,00 | 2,8 | | 4,00 | 3,6 |
| 4,50 | 2,2 | | 4,50 | 2,8 |
| 5,00 | 1,7 | | 5,00 | 2,2 |
| 5,50 | 1,3 | | 5,50 | 1,7 |
| 6,00 | 1,0 | | 6,00 | 1,2 |
| 6,50 | 0,7 | | 6,50 | 0,9 |
| 7,00 | 0,4 | | 7,00 | 0,5 |
| 7,50 | 0,2 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 |

### 5.9.1.2 Case 2

This particular case consists of modeling 2 m long aggregate pier elements right below the circular footing. Surrounding soil as described, is modeled to be of a

Mohr-Coulomb medium, and so is the aggregate pier element. The circular footing is elastic. Groundwater table is set 50 cm below surface level. Circular footing has a diameter of 1.3 m, and aggregate pier element has diameters of Ø60 cm and Ø80 cm. Applied foundation pressure varies in between 25 kPa, and 100 kPa. Below, is an illustration of the problem dimensions and properties for case 2.

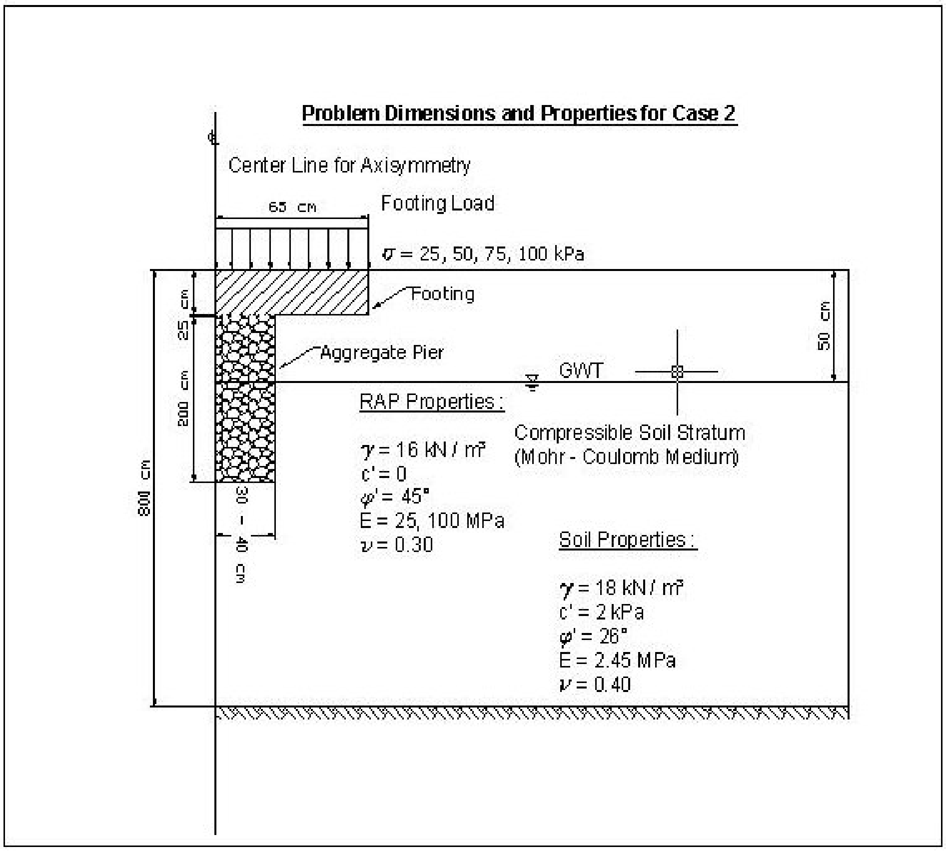



Figure 5.8 – Problem dimensions and properties for case 2

*5.9.1.2.1 Case 2.1*

This particular case pertains to a 2 m long, Ø6 0 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 25 kPa.

Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is 3.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.12, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 5.6 mm, yielding a 31% settlement reduction, although, the current settlement is not substantial (under 25 mm).

##### *5.9.1.2.2 Case 2.2*

This particular case pertains to a 2 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 3.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.12, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 13.7 mm, yielding a 33% settlement reduction, although, the current settlement is not substantial (under 25 mm).

##### *5.9.1.2.3 Case 2.3*

This particular case pertains to a 2 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 3.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.12, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 23.9 mm, yielding a 40% settlement reduction.

Table 5.12 – Description of cases 2-1, 2-2, and 2-3

| **Case No** | ***2-1*** | | **Case No** | ***2-2*** | | **Case No** | ***2-3*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 2 | | **RAP Length (m)** | 2 | | **RAP Length (m)** | 2 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 | | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 5,6 | | 0,00 | 13,7 | | 0,00 | 23,9 |
| 0,50 | 5,3 | | 0,50 | 13,0 | | 0,50 | 22,9 |
| 1,00 | 4,6 | | 1,00 | 11,1 | | 1,00 | 19,5 |
| 1,50 | 4,0 | | 1,50 | 7,5 | | 1,50 | 11,9 |
| 2,00 | 3,6 | | 2,00 | 6,8 | | 2,00 | 10,5 |
| 2,50 | 2,8 | | 2,50 | 5,3 | | 2,50 | 7,9 |
| 3,00 | 1,9 | | 3,00 | 3,7 | | 3,00 | 5,4 |
| 3,50 | 1,4 | | 3,50 | 2,7 | | 3,50 | 4,0 |
| 4,00 | 1,1 | | 4,00 | 2,1 | | 4,00 | 3,1 |
| 4,50 | 0,8 | | 4,50 | 1,6 | | 4,50 | 2,4 |
| 5,00 | 0,6 | | 5,00 | 1,2 | | 5,00 | 1,8 |
| 5,50 | 0,5 | | 5,50 | 0,9 | | 5,50 | 1,4 |
| 6,00 | 0,3 | | 6,00 | 0,7 | | 6,00 | 1,0 |
| 6,50 | 0,2 | | 6,50 | 0,5 | | 6,50 | 0,7 |
| 7,00 | 0,1 | | 7,00 | 0,3 | | 7,00 | 0,4 |
| 7,50 | 0,1 | | 7,50 | 0,1 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

#### *5.9.1.2.4 Case 2.4*

This particular case pertains to a 2 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 3.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.13, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 43.3 mm, yielding a 37% settlement reduction.

*5.9.1.2.5 Case 2.5*

This particular case pertains to a 2 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 3.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.13, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 12.3 mm, yielding a 37% settlement reduction, although, the current settlement is not substantial (under 25 mm).

*5.9.1.2.6 Case 2.6*

This particular case pertains to a 2 m long, Ø60 cm aggregate pi er element with an elastic modulus of 100 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 3.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.13, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 12.3 mm, yielding a 39% settlement reduction.

Table 5.13 – Description of cases 2-4, 2-5, and 2-6

| **Case No** | ***2-4*** | **Case No** | ***2-5*** | **Case No** | ***2-6*** |
|---|---|---|---|---|---|
| **RAP Length (m)** | 2 | **RAP Length (m)** | 2 | **RAP Length (m)** | 2 |
| **RAP Diameter (cm)** | Ø60 | **RAP Diameter (cm)** | Ø60 | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 25 | **$E_{RAP}$ (MPa)** | 100 | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 100 | **$s_{FOUND}$ (kPa)** | 25 | **$s_{FOUND}$ (kPa)** | 50 |
| | | | | | |
| **Depth (m)** | **S (mm)** | **Depth (m)** | **S (mm)** | **Depth (m)** | **S (mm)** |
| | | | | | |
| 0,00 | 43,3 | 0,00 | 5,1 | 0,00 | 12,3 |
| 0,50 | 42,1 | 0,50 | 5,0 | 0,50 | 12,1 |
| 1,00 | 35,7 | 1,00 | 4,4 | 1,00 | 8,8 |
| 1,50 | 23,4 | 1,50 | 4,0 | 1,50 | 7,1 |
| 2,00 | 21,1 | 2,00 | 3,9 | 2,00 | 6,9 |
| 2,50 | 16,1 | 2,50 | 3,1 | 2,50 | 5,6 |
| 3,00 | 6,8 | 3,00 | 2,1 | 3,00 | 3,8 |
| 3,50 | 4,9 | 3,50 | 1,5 | 3,50 | 2,8 |
| 4,00 | 3,8 | 4,00 | 1,1 | 4,00 | 2,1 |
| 4,50 | 3,0 | 4,50 | 0,8 | 4,50 | 1,6 |
| 5,00 | 2,3 | 5,00 | 0,6 | 5,00 | 1,2 |
| 5,50 | 1,8 | 5,50 | 0,5 | 5,50 | 0,9 |
| 6,00 | 1,3 | 6,00 | 0,4 | 6,00 | 0,7 |
| 6,50 | 0,9 | 6,50 | 0,2 | 6,50 | 0,5 |
| 7,00 | 0,5 | 7,00 | 0,1 | 7,00 | 0,3 |
| 7,50 | 0,2 | 7,50 | 0,1 | 7,50 | 0,1 |
| 8,00 | 0,0 | 8,00 | 0,0 | 8,00 | 0,0 |

##### *5.9.1.2.7 Case 2.7*

This particular case pertains to a 2 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 3.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.14, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 23.0 mm, yielding a 43% settlement reduction.

*5.9.1.2.8 Case 2.8*

This particular case pertains to a 2 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 3.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.14, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 43.0 mm, yielding a 38% settlement reduction.

Table 5.14 – Description of cases 2-7, and 2-8

| **Case No** | ***2-7*** | | **Case No** | ***2-8*** |
|---|---|---|---|---|
| **RAP Length (m)** | 2 | | **RAP Length (m)** | 2 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 23,0 | | 0,00 | 43,0 |
| 0,50 | 22,6 | | 0,50 | 42,4 |
| 1,00 | 17,2 | | 1,00 | 33,8 |
| 1,50 | 12,0 | | 1,50 | 24,6 |
| 2,00 | 11,0 | | 2,00 | 21,9 |
| 2,50 | 8,2 | | 2,50 | 16,9 |
| 3,00 | 5,5 | | 3,00 | 6,9 |
| 3,50 | 4,0 | | 3,50 | 3,8 |

| Table 5.14 cont. | | | | |
|---|---|---|---|---|
| 4,00 | 3,1 | | 4,00 | 3,0 |
| 4,50 | 2,4 | | 4,50 | 2,3 |
| 5,00 | 1,8 | | 5,00 | 1,8 |
| 5,50 | 1,4 | | 5,50 | 1,3 |
| 6,00 | 1,0 | | 6,00 | 0,9 |
| 6,50 | 0,7 | | 6,50 | 0,5 |
| 7,00 | 0,4 | | 7,00 | 0,2 |
| 7,50 | 0,2 | | 7,50 | 0,0 |
| 8,00 | 0,0 | | 8,00 | |

*5.9.1.2.9 Case 2.9*

This particular case pertains to a 2 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.15, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 4.9 mm, yielding a 40% settlement reduction.

*5.9.1.2.10 Case 2.10*

This particular case pertains to a 2 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.15, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 11.0 mm, yielding a 46% settlement reduction.

### 5.9.1.2.11 Case 2.11

This particular case pertains to a 2 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.15, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 19.4 mm, yielding a 52% settlement reduction.

Table 5.15 – Description of cases 2-9, 2-10, and 2-11

| Case No | *2-9* | | Case No | *2-10* | | Case No | *2-11* |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 2 | | **RAP Length (m)** | 2 | | **RAP Length (m)** | 2 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 | | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 4,9 | | 0,00 | 11,0 | | 0,00 | 19,4 |
| 0,50 | 4,6 | | 0,50 | 10,4 | | 0,50 | 18,6 |
| 1,00 | 4,1 | | 1,00 | 9,0 | | 1,00 | 16,3 |
| 1,50 | 3,6 | | 1,50 | 7,6 | | 1,50 | 11,4 |
| 2,00 | 3,4 | | 2,00 | 7,0 | | 2,00 | 10,5 |
| 2,50 | 2,8 | | 2,50 | 5,6 | | 2,50 | 8,2 |
| 3,00 | 1,9 | | 3,00 | 3,9 | | 3,00 | 5,7 |
| 3,50 | 1,4 | | 3,50 | 2,8 | | 3,50 | 4,2 |
| 4,00 | 1,1 | | 4,00 | 2,1 | | 4,00 | 3,2 |
| 4,50 | 0,8 | | 4,50 | 1,6 | | 4,50 | 2,4 |
| 5,00 | 0,6 | | 5,00 | 1,3 | | 5,00 | 1,9 |

| Table 5.15 cont. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 5,50 | 0,5 | | 5,50 | 0,9 | | 5,50 | 1,4 |
| 6,00 | 0,3 | | 6,00 | 0,7 | | 6,00 | 1,0 |
| 6,50 | 0,2 | | 6,50 | 0,5 | | 6,50 | 0,7 |
| 7,00 | 0,1 | | 7,00 | 0,3 | | 7,00 | 0,4 |
| 7,50 | 0,1 | | 7,50 | 0,1 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

*5.9.1.2.12 Case 2.12*

This particular case pertains to a 2 m long, Ø80 cm aggregate pier element w ith an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the below table. Length to diameter ratio ‘ $L_d$ ’ for this case is also 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.16, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 36.4 mm, yielding a 47% settlement reduction.

*5.9.1.2.13 Case 2.13*

This particular case pertains to a 2 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the below table. Length to diameter ratio ‘ $L_d$ ’ for this case is also 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.16, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 4.2 mm, yielding a 48% settlement reduction.

*5.9.1.2.14 Case 2.14*

This particular case pertains to a 2 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the below table. Length to diameter ratio ‘ $L_d$ ’ for this case is also 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.16, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 10.3 mm, yielding a 49% settlement reduction.

Table 5.16 – Description of cases 2-12, 2-13, and 2-14

| **Case No** | ***2-12*** | | **Case No** | ***2-13*** | | **Case No** | ***2-14*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 2 | | **RAP Length (m)** | 2 | | **RAP Length (m)** | 2 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 100 | | **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 36,4 | | 0,00 | 4,2 | | 0,00 | 10,3 |
| 0,50 | 35,3 | | 0,50 | 4,1 | | 0,50 | 10,2 |
| 1,00 | 31,6 | | 1,00 | 3,9 | | 1,00 | 9,1 |
| 1,50 | 23,2 | | 1,50 | 3,8 | | 1,50 | 7,1 |
| 2,00 | 21,8 | | 2,00 | 3,7 | | 2,00 | 6,9 |
| 2,50 | 17,3 | | 2,50 | 3,1 | | 2,50 | 5,7 |
| 3,00 | 9,5 | | 3,00 | 2,1 | | 3,00 | 3,9 |
| 3,50 | 5,2 | | 3,50 | 1,5 | | 3,50 | 2,8 |
| 4,00 | 4,1 | | 4,00 | 1,1 | | 4,00 | 2,1 |
| 4,50 | 3,2 | | 4,50 | 0,8 | | 4,50 | 1,6 |
| 5,00 | 2,4 | | 5,00 | 0,6 | | 5,00 | 1,3 |
| 5,50 | 1,8 | | 5,50 | 0,5 | | 5,50 | 0,9 |
| 6,00 | 1,3 | | 6,00 | 0,4 | | 6,00 | 0,7 |
| 6,50 | 0,9 | | 6,50 | 0,2 | | 6,50 | 0,5 |

| Table 5.16 cont. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7,00 | 0,6 | | 7,00 | 0,1 | | 7,00 | 0,3 |
| 7,50 | 0,2 | | 7,50 | 0,1 | | 7,50 | 0,1 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

*5.9.1.2.15 Case 2.15*

This particular case pertains to a 2 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the below table. Length to diameter ratio ' $L_d$ ' for this case is also 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.17, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 17.8 mm, yielding a 56% settlement reduction.

*5.9.1.2.16 Case 2.16*

This particular case pertains to a 2 m long, Ø80 cm aggregate pier e lement with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the above table. Length to diameter ratio ' $L_d$ ' for this case is also 2.5. Maximum settle ment that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.21, the maximum settlement with the presence of a 2 m aggregate pier element is reduced to 35.1 mm, yielding a 49% settlement reduction.

Table 5.17 – Description of cases 2-15, and 2-16

| **Case No** | ***2-15*** | | **Case No** | ***2-16*** |
|---|---|---|---|---|
| **RAP Length (m)** | 2 | | **RAP Length (m)** | 2 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 17,8 | | 0,00 | 35,1 |
| 0,50 | 17,5 | | 0,50 | 34,8 |
| 1,00 | 14,8 | | 1,00 | 30,5 |
| 1,50 | 11,8 | | 1,50 | 24,2 |
| 2,00 | 11,3 | | 2,00 | 22,2 |
| 2,50 | 8,3 | | 2,50 | 17,3 |
| 3,00 | 5,8 | | 3,00 | 9,6 |
| 3,50 | 4,2 | | 3,50 | 5,2 |
| 4,00 | 3,2 | | 4,00 | 4,1 |
| 4,50 | 2,4 | | 4,50 | 3,2 |
| 5,00 | 1,9 | | 5,00 | 2,4 |
| 5,50 | 1,4 | | 5,50 | 1,8 |
| 6,00 | 1,0 | | 6,00 | 1,3 |
| 6,50 | 0,7 | | 6,50 | 0,9 |
| 7,00 | 0,4 | | 7,00 | 0,6 |
| 7,50 | 0,2 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 |

#### 5.9.1.3 Case 3

This particular case consists of modeling 3 m long aggregate pier elements right below the circular footing. Surrounding soil as described, is modeled to be of a Mohr-Coulomb medium, and so is the aggregate pier element. The circular footing is elastic. Groundwater table is set 50 cm below surface level. Circular footing has a diameter of 1.3 m, and aggregate pier element has diameters of Ø60 cm and Ø80 cm. Applied foundation pressure varies in between 25 kPa, and 100 kPa. Below, is an illustration of the problem dimensions and properties for case 3.

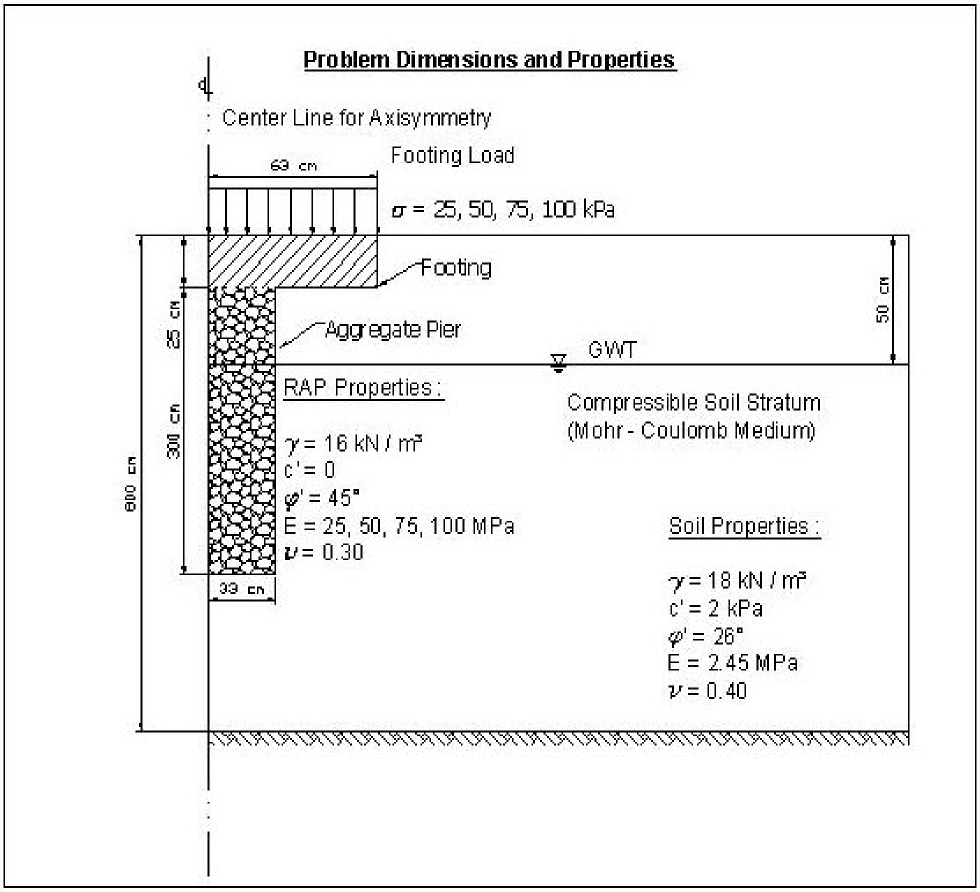



Figure 5.9 - Problem dimensions and properties for case 3

*5.9.1.3.1 Case 3.1*

This particular case pertains to a 3 m long, Ø60 cm aggregate pier elemen t with an elastic modulus of 25 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.18, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 5.3 mm, yielding a 35% settlement reduction.

*5.9.1.3.2 Case 3.2*

This particular case pertains to a 3 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is also 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.18, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 13.5 mm, yielding a 33% settlement reduction.

*5.9.1.3.3 Case 3.3*

This particular case pertains to a 3 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the above table. Length to diameter ratio ' $L_d$ ' for this case is also 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.22, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 24.4 mm, yielding a 39% settlement reduction.

Table 5.18 – Description of cases 3-1, 3-2, and 3-3

| **Case No** | ***3-1*** | **Case No** | ***3-2*** | **Case No** | ***3-3*** |
|---|---|---|---|---|---|
| **RAP Length (m)** | 3 | **RAP Length (m)** | 3 | **RAP Length (m)** | 3 |
| **RAP Diameter (cm)** | Ø60 | **RAP Diameter (cm)** | Ø60 | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 25 | **$E_{RAP}$ (MPa)** | 25 | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 25 | **$s_{FOUND}$ (kPa)** | 50 | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | |
| **Depth (m)** | **S (mm)** | **Depth (m)** | **S (mm)** | **Depth (m)** | **S (mm)** |
| | | | | | |
| 0,00 | 5,3 | 0,00 | 13,5 | 0,00 | 24,4 |
| 0,50 | 4,9 | 0,50 | 12,8 | 0,50 | 23,6 |
| 1,00 | 4,1 | 1,00 | 11,2 | 1,00 | 19,9 |
| 1,50 | 3,5 | 1,50 | 6,6 | 1,50 | 11,5 |
| 2,00 | 2,9 | 2,00 | 5,2 | 2,00 | 7,5 |
| 2,50 | 2,6 | 2,50 | 4,6 | 2,50 | 6,6 |
| 3,00 | 2,3 | 3,00 | 4,3 | 3,00 | 6,1 |
| 3,50 | 1,9 | 3,50 | 3,4 | 3,50 | 4,9 |
| 4,00 | 1,3 | 4,00 | 2,4 | 4,00 | 3,5 |
| 4,50 | 0,9 | 4,50 | 1,7 | 4,50 | 2,6 |
| 5,00 | 0,7 | 5,00 | 1,3 | 5,00 | 1,9 |
| 5,50 | 0,5 | 5,50 | 1,0 | 5,50 | 1,4 |
| 6,00 | 0,4 | 6,00 | 0,7 | 6,00 | 1,0 |
| 6,50 | 0,2 | 6,50 | 0,5 | 6,50 | 0,7 |
| 7,00 | 0,1 | 7,00 | 0,3 | 7,00 | 0,4 |
| 7,50 | 0,0 | 7,50 | 0,1 | 7,50 | 0,2 |
| 8,00 | 0,0 | 8,00 | 0,0 | 8,00 | 0,0 |

#### *5.9.1.3.4 Case 3.4*

This particular case pertains to a 3 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.19, the maximum

settlement with the presence of a 3 m aggregate pier element is reduced to 37.5 mm, yielding a 46% settlement reduction.

*5.9.1.3.5 Case 3.5*

This particular case pertains to a 3 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is also 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.19, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 4.8 mm, yielding a 41% settlement reduction.

*5.9.1.3.6 Case 3.6*

This particular case pertains to a 3 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is also 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.19, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 11.7 mm, yielding a 42% settlement reduction.

Table 5.19 – Description of cases 3-4, 3-5, and 3-6

| **Case No** | ***3-4*** | | **Case No** | ***3-5*** | | **Case No** | ***3-6*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 3 | | **RAP Length (m)** | 3 | | **RAP Length (m)** | 3 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 100 | | **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 37,5 | | 0,00 | 4,8 | | 0,00 | 11,7 |
| 0,50 | 36,3 | | 0,50 | 4,7 | | 0,50 | 11,5 |
| 1,00 | 31,0 | | 1,00 | 3,8 | | 1,00 | 8,2 |
| 1,50 | 18,0 | | 1,50 | 3,0 | | 1,50 | 5,9 |
| 2,00 | 11,3 | | 2,00 | 2,9 | | 2,00 | 5,1 |
| 2,50 | 8,9 | | 2,50 | 2,7 | | 2,50 | 4,9 |
| 3,00 | 8,1 | | 3,00 | 2,6 | | 3,00 | 4,8 |
| 3,50 | 6,5 | | 3,50 | 2,2 | | 3,50 | 3,9 |
| 4,00 | 4,6 | | 4,00 | 1,4 | | 4,00 | 2,6 |
| 4,50 | 3,4 | | 4,50 | 1,0 | | 4,50 | 1,9 |
| 5,00 | 2,5 | | 5,00 | 0,7 | | 5,00 | 1,4 |
| 5,50 | 1,9 | | 5,50 | 0,5 | | 5,50 | 1,0 |
| 6,00 | 1,4 | | 6,00 | 0,4 | | 6,00 | 0,7 |
| 6,50 | 0,9 | | 6,50 | 0,3 | | 6,50 | 0,5 |
| 7,00 | 0,6 | | 7,00 | 0,2 | | 7,00 | 0,3 |
| 7,50 | 0,3 | | 7,50 | 0,1 | | 7,50 | 0,1 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

##### *5.9.1.3.7 Case 3.7*

This particular case pertains to a 3 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is also 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.24, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 22.1 mm, yielding a 45% settlement reduction.

*5.9.1.3.8 Case 3.8*

This particular case pertains to a 3 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio ‘ $L_d$ ’ for this case is also 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.24, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 37.1 mm, yielding a 46% settlement reduction.

Table 5.20 – Description of cases 3-7, and 3-8

| **Case No** | ***3-7*** | | **Case No** | ***3-8*** |
|---|---|---|---|---|
| **RAP Length (m)** | 3 | | **RAP Length (m)** | 3 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 22,1 | | 0,00 | 37,1 |
| 0,50 | 21,7 | | 0,50 | 36,4 |
| 1,00 | 15,7 | | 1,00 | 27,9 |
| 1,50 | 9,8 | | 1,50 | 17,6 |
| 2,00 | 7,0 | | 2,00 | 10,6 |
| 2,50 | 6,7 | | 2,50 | 8,3 |
| 3,00 | 6,5 | | 3,00 | 8,1 |

| | | | | |
|---|---|---|---|---|
| Table 5.20 cont. | | | | |
| 3,50 | 5,4 | | 3,50 | 6,6 |
| 4,00 | 3,6 | | 4,00 | 4,6 |
| 4,50 | 2,6 | | 4,50 | 3,4 |
| 5,00 | 2,0 | | 5,00 | 2,5 |
| 5,50 | 1,5 | | 5,50 | 1,9 |
| 6,00 | 1,1 | | 6,00 | 1,4 |
| 6,50 | 0,7 | | 6,50 | 0,9 |
| 7,00 | 0,4 | | 7,00 | 0,6 |
| 7,50 | 0,2 | | 7,50 | 0,3 |
| 8,00 | 0,0 | | 8,00 | 0,0 |

*5.9.1.3.9 Case 3.9*

This particular case pertains to a 3 m long, Ø8 0 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ' $L_d$ ' for this case is 3.7 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.21, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 4.4 mm, yielding a 46% settlement reduction.

*5.9.1.3.10 Case 3.10*

This particular case pertains to a 3 m long, Ø80 cm aggregate pier element with a n elastic modulus of 25 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio ' $L_d$ ' for this case is 3.75. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.21, the maximum

settlement with the presence of a 3 m aggregate pier element is reduced to 10.4 mm, yielding a 49% settlement reduction.

*5.9.1.3.11 Case 3.11*

This particular case pertains to a 3 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio ‘ $L_d$ ’ for this case is 3.75. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.21, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 18.4 mm, yielding a 54% settlement reduction.

Table 5.21 – Description of cases 3-9, 3-10, and 3-11

| **Case No** | ***3-9*** | | **Case No** | ***3-10*** | | **Case No** | ***3-11*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 3 | | **RAP Length (m)** | 3 | | **RAP Length (m)** | 3 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 | | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 4,4 | | 0,00 | 10,4 | | 0,00 | 18,4 |
| 0,50 | 4,2 | | 0,50 | 9,9 | | 0,50 | 17,6 |
| 1,00 | 3,6 | | 1,00 | 8,4 | | 1,00 | 15,1 |
| 1,50 | 3,1 | | 1,50 | 6,6 | | 1,50 | 10,2 |
| 2,00 | 2,7 | | 2,00 | 5,2 | | 2,00 | 7,5 |
| 2,50 | 2,4 | | 2,50 | 4,8 | | 2,50 | 6,9 |
| 3,00 | 2,2 | | 3,00 | 4,5 | | 3,00 | 6,4 |
| 3,50 | 1,8 | | 3,50 | 3,7 | | 3,50 | 5,3 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Table 5.21 cont. | | | | | | | |
| 4,00 | 1,3 | | 4,00 | 2,6 | | 4,00 | 3,7 |
| 4,50 | 0,9 | | 4,50 | 1,9 | | 4,50 | 2,7 |
| 5,00 | 0,7 | | 5,00 | 1,4 | | 5,00 | 2,0 |
| 5,50 | 0,5 | | 5,50 | 1,0 | | 5,50 | 1,5 |
| 6,00 | 0,4 | | 6,00 | 0,7 | | 6,00 | 1,1 |
| 6,50 | 0,2 | | 6,50 | 0,5 | | 6,50 | 0,7 |
| 7,00 | 0,1 | | 7,00 | 0,3 | | 7,00 | 0,4 |
| 7,50 | 0,1 | | 7,50 | 0,1 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

##### *5.9.1.3.12 Case 3.12*

This particular case pertains to a 3 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio ‘ $L_d$ ’ for this case is 3.75. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.22, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 28.9 mm, yielding a 58% settlement reduction.

##### *5.9.1.3.13 Case 3.13*

This particular case pertains to a 3 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio ‘ $L_d$ ’ for this case is 3.75. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.22, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 3.4 mm, yielding a 58% settlement reduction.

*5.9.1.3.14 Case 3.14*

This particular case pertains to a 3 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio '$L_d$' for this case is 3.75. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.22, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 8.9 mm, yielding a 56% settlement reduction.

Table 5.22 – Description of cases 3-12, 3-13, and 3-14

| **Case No** | ***3-12*** | **Case No** | ***3-13*** | **Case No** | ***3-14*** |
|---|---|---|---|---|---|
| **RAP Length (m)** | 3 | **RAP Length (m)** | 3 | **RAP Length (m)** | 3 |
| **RAP Diameter (cm)** | Ø80 | **RAP Diameter (cm)** | Ø80 | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 25 | **$E_{RAP}$ (MPa)** | 100 | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 100 | **$s_{FOUND}$ (kPa)** | 25 | **$s_{FOUND}$ (kPa)** | 50 |
| | | | | | |
| **Depth (m)** | **S (mm)** | **Depth (m)** | **S (mm)** | **Depth (m)** | **S (mm)** |
| | | | | | |
| 0,00 | 28,9 | 0,00 | 3,4 | 0,00 | 8,9 |
| 0,50 | 27,7 | 0,50 | 3,3 | 0,50 | 8,7 |
| 1,00 | 24,1 | 1,00 | 3,1 | 1,00 | 7,4 |
| 1,50 | 16,4 | 1,50 | 3,0 | 1,50 | 5,6 |
| 2,00 | 10,5 | 2,00 | 2,8 | 2,00 | 5,3 |
| 2,50 | 9,2 | 2,50 | 2,7 | 2,50 | 5,1 |
| 3,00 | 8,6 | 3,00 | 2,7 | 3,00 | 5,0 |
| 3,50 | 7,0 | 3,50 | 2,2 | 3,50 | 4,2 |
| 4,00 | 5,0 | 4,00 | 1,5 | 4,00 | 2,9 |
| 4,50 | 3,6 | 4,50 | 1,1 | 4,50 | 2,0 |
| 5,00 | 2,7 | 5,00 | 0,8 | 5,00 | 1,5 |
| 5,50 | 2,0 | 5,50 | 0,6 | 5,50 | 1,1 |

| Table 5.22 cont. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6,00 | 1,4 | | 6,00 | 0,4 | | 6,00 | 0,8 |
| 6,50 | 1,0 | | 6,50 | 0,3 | | 6,50 | 0,5 |
| 7,00 | 0,6 | | 7,00 | 0,2 | | 7,00 | 0,3 |
| 7,50 | 0,3 | | 7,50 | 0,1 | | 7,50 | 0,1 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

*5.9.1.3.15 Case 3.15*

This particular case pertains to a 3 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is 3.75. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.23, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 16.4 mm, yielding a 59% settlement reduction.

*5.9.1.3.16 Case 3.16*

This particular case pertains to a 3 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is 3.75. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.23, the maximum settlement with the presence of a 3 m aggregate pier element is reduced to 27.8 mm, yielding a 60% settlement reduction.

Table 5.23 – Description of cases 3-15, and 3-16

| Case No | ***3-15*** | | Case No | ***3-16*** |
|---|---|---|---|---|
| **RAP Length (m)** | 3 | | **RAP Length (m)** | 3 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 16,4 | | 0,00 | 27,8 |
| 0,50 | 16,2 | | 0,50 | 27,5 |
| 1,00 | 12,9 | | 1,00 | 23,2 |
| 1,50 | 8,8 | | 1,50 | 15,0 |
| 2,00 | 7,4 | | 2,00 | 9,6 |
| 2,50 | 7,1 | | 2,50 | 8,9 |
| 3,00 | 7,0 | | 3,00 | 8,7 |
| 3,50 | 5,9 | | 3,50 | 7,3 |
| 4,00 | 4,0 | | 4,00 | 5,1 |
| 4,50 | 2,9 | | 4,50 | 3,7 |
| 5,00 | 2,1 | | 5,00 | 2,7 |
| 5,50 | 1,6 | | 5,50 | 2,0 |
| 6,00 | 1,1 | | 6,00 | 1,5 |
| 6,50 | 0,8 | | 6,50 | 1,0 |
| 7,00 | 0,5 | | 7,00 | 0,6 |
| 7,50 | 0,2 | | 7,50 | 0,3 |
| 8,00 | 0,0 | | 8,00 | 0,0 |

### 5.9.1.4 Case 4

This particular case consists of modeling 1.5 m long aggregate pier elements right below the circular footing. Surrounding soil as described, is modeled to be of a Mohr-Coulomb medium, and so is the aggregate pier element. The circular footing is elastic. Groundwater table is set 50 cm below surface level. Circular footing has

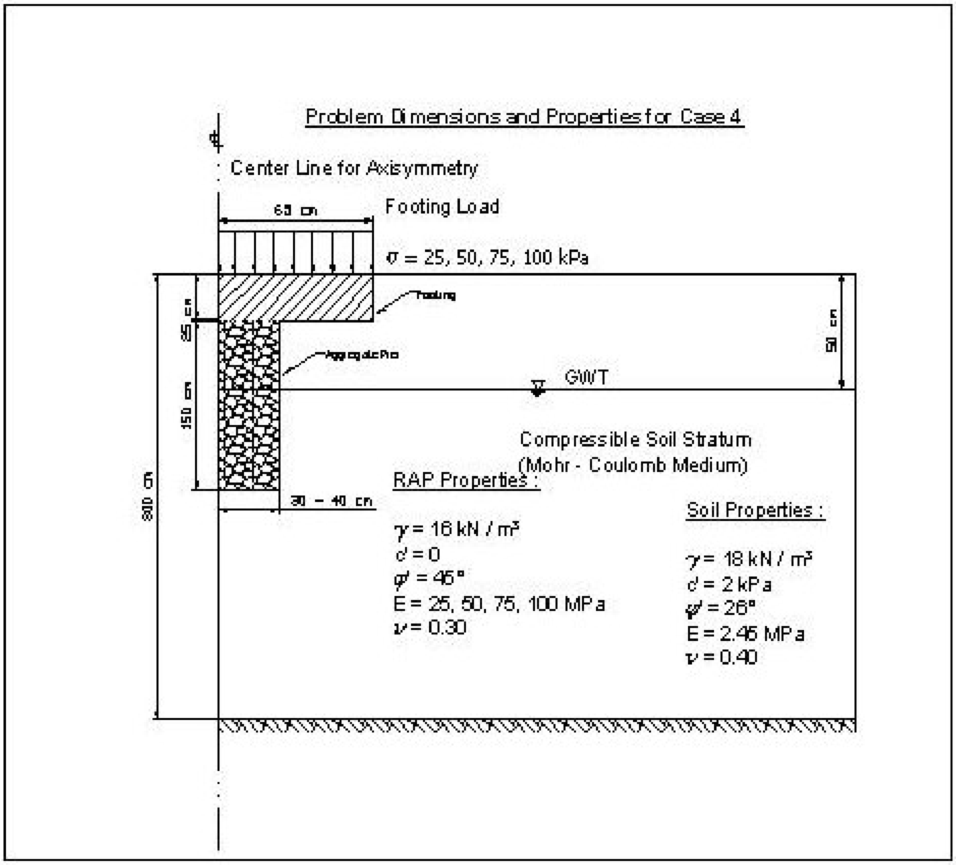



Figure 5.10 – Problem dimensions and properties for case 4

a diameter of 1.3 m, and aggregate pier element has diameters of Ø60 cm and Ø80 cm. Applied foundation pressure varies in between 25 kPa, and 100 kPa. Figure 5.10, is an illustration of the problem dimensions and properties for case 4.

*5.9.1.4.1 Case 4.1*

This particular case pertains to a 1.5 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is 2. 5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.2, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 6.0 mm, yielding a 26% settlement reduction.

*5.9.1.4.2 Case 4-2*

This particular case pertains to a 1.5 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio '$L_d$' for this case is 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.28, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 14.3 mm, yielding a 30% settlement reduction.

*5.9.1.4.3 Case 4-3*

This particular case pertains to a 1.5 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio '$L_d$' for this case is 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.28, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 26.3 mm, yielding a 34% settlement reduction.

Table 5.24 – Description of cases 4-1, 4-2, and 4-3

| **Case No** | ***4-1*** | | **Case No** | ***4-2*** | | **Case No** | ***4-3*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 | | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 6,0 | | 0,00 | 14,3 | | 0,00 | 26,7 |
| 0,50 | 5,6 | | 0,50 | 13,8 | | 0,50 | 25,8 |
| 1,00 | 5,0 | | 1,00 | 11,5 | | 1,00 | 22,0 |
| 1,50 | 4,5 | | 1,50 | 9,9 | | 1,50 | 18,7 |
| 2,00 | 3,5 | | 2,00 | 7,1 | | 2,00 | 14,1 |
| 2,50 | 2,4 | | 2,50 | 4,7 | | 2,50 | 6,7 |
| 3,00 | 1,7 | | 3,00 | 3,4 | | 3,00 | 5,0 |
| 3,50 | 1,3 | | 3,50 | 2,6 | | 3,50 | 3,8 |
| 4,00 | 1,0 | | 4,00 | 2,0 | | 4,00 | 3,0 |
| 4,50 | 0,8 | | 4,50 | 1,6 | | 4,50 | 2,3 |
| 5,00 | 0,6 | | 5,00 | 1,2 | | 5,00 | 1,8 |
| 5,50 | 0,5 | | 5,50 | 0,9 | | 5,50 | 1,4 |
| 6,00 | 0,3 | | 6,00 | 0,7 | | 6,00 | 1,0 |
| 6,50 | 0,2 | | 6,50 | 0,5 | | 6,50 | 0,7 |
| 7,00 | 0,1 | | 7,00 | 0,3 | | 7,00 | 0,4 |
| 7,50 | 0,1 | | 7,50 | 0,1 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0 | | 8,00 | 0,0 |

#### *5.9.1.4.4 Case 4-4*

This particular case pertains to a 1.5 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio ‘ $L_d$ ’ for this case is 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.25, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 49.2 mm, yielding a 29% settlement reduction.

*5.9.1.4.5 Case 4-5*

This particular case pertains to a 1.5 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio '*R*' for this case is 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.25, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 5,5 mm, yielding a 32% settlement reduction.

*5.9.1.4.6 Case 4-6*

This particular case pertains to a 1.5 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio '*R*' for this case is 2.5. Maximu m settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.25, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 13,2 mm, yielding a 35% settlement reduction.

Table 5.25 – Description of cases 4-4, 4-5, and 4-6

| **Case No** | ***4-4*** | | **Case No** | ***4-5*** | | **Case No** | ***4-6*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 100 | | **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 49,2 | | 0,00 | 5,5 | | 0,00 | 13,2 |
| 0,50 | 47,9 | | 0,50 | 5,4 | | 0,50 | 12,9 |
| 1,00 | 40,3 | | 1,00 | 4,9 | | 1,00 | 10,2 |
| 1,50 | 34,3 | | 1,50 | 4,7 | | 1,50 | 8,9 |
| 2,00 | 26,6 | | 2,00 | 3,7 | | 2,00 | 6,7 |
| 2,50 | 11,6 | | 2,50 | 2,5 | | 2,50 | 4,6 |
| 3,00 | 5,2 | | 3,00 | 1,8 | | 3,00 | 3,4 |
| 3,50 | 4,4 | | 3,50 | 1,3 | | 3,50 | 2,6 |
| 4,00 | 3,5 | | 4,00 | 1,0 | | 4,00 | 2,0 |
| 4,50 | 2,8 | | 4,50 | 0,8 | | 4,50 | 1,5 |
| 5,00 | 2,2 | | 5,00 | 0,6 | | 5,00 | 1,2 |
| 5,50 | 1,7 | | 5,50 | 0,5 | | 5,50 | 0,9 |
| 6,00 | 1,2 | | 6,00 | 0,3 | | 6,00 | 0,7 |
| 6,50 | 0,8 | | 6,50 | 0,2 | | 6,50 | 0,5 |
| 7,00 | 0,5 | | 7,00 | 0,1 | | 7,00 | 0,3 |
| 7,50 | 0,2 | | 7,50 | 0,1 | | 7,50 | 0,1 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

*5.9.1.4.7 Case 4-7*

This particular case pertains to a 1.5 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.26, the maximum settlement with

the presence of a 1.5 m aggregate pier element is reduced to 26.6 mm, yielding a 34% settlement reduction.

*5.9.1.4.8 Case 4-8*

This particular case pertains to a 1.5 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is 2.5. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.26, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 45.7 mm, yielding a 34% settlement reduction.

Table 5.26 – Description of cases 4-7, and 4-8

| **Case No** | ***4-7*** | | **Case No** | ***4-8*** |
|---|---|---|---|---|
| **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 26,6 | | 0,00 | 45,7 |
| 0,50 | 26,2 | | 0,50 | 45,2 |
| 1,00 | 20,5 | | 1,00 | 36,8 |
| 1,50 | 18,4 | | 1,50 | 32,5 |
| 2,00 | 14,0 | | 2,00 | 25,5 |
| 2,50 | 6,6 | | 2,50 | 11,1 |

| Table 5.26 cont. | | | | |
|---|---|---|---|---|
| 3,00 | 5,0 | | 3,00 | 5,5 |
| 3,50 | 3,8 | | 3,50 | 4,5 |
| 4,00 | 2,9 | | 4,00 | 3,6 |
| 4,50 | 2,3 | | 4,50 | 2,8 |
| 5,00 | 1,8 | | 5,00 | 2,2 |
| 5,50 | 1,3 | | 5,50 | 1,7 |
| 6,00 | 1,0 | | 6,00 | 1,2 |
| 6,50 | 0,7 | | 6,50 | 0,9 |
| 7,00 | 0,4 | | 7,00 | 0,5 |
| 7,50 | 0,2 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 |

#### *5.9.1.4.9 Case 4-9*

This particular case pertains to a 1.5 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is 1.9. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.27, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 5.3 mm, yielding a 35% settlement reduction.

#### *5.9.1.4.10 Case 4-10*

This particular case pertains to a 1.5 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio '$L_d$' for this case is 1.9. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.27, the maximum settlement with

the presence of a 1.5 m aggregate pier element is reduced to 12.1 mm, yielding a 40% settlement reduction.

*5.9.1.4.11 Case 4-11*

This particular case pertains to a 1.5 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio ‘ $L_d$ ’ for this case is 1.9. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.27, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 22.3 mm, yielding a 44% settlement reduction.

Table 5.27 – Description of cases 4-9, 4-10, and 4-11

| **Case No** | ***4-9*** | | **Case No** | ***4-10*** | | **Case No** | ***4-11*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 | | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 5,3 | | 0,00 | 12,1 | | 0,00 | 22,3 |
| 0,50 | 5,0 | | 0,50 | 11,6 | | 0,50 | 21,5 |
| 1,00 | 4,5 | | 1,00 | 10,4 | | 1,00 | 19,3 |
| 1,50 | 4,2 | | 1,50 | 9,5 | | 1,50 | 16,9 |
| 2,00 | 3,4 | | 2,00 | 6,9 | | 2,00 | 12,8 |
| 2,50 | 2,4 | | 2,50 | 4,7 | | 2,50 | 7,6 |
| 3,00 | 1,7 | | 3,00 | 3,5 | | 3,00 | 5,2 |

| Table 5.27 cont. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3,50 | 1,3 | | 3,50 | 2,6 | | 3,50 | 3,9 |
| 4,00 | 1,0 | | 4,00 | 2,0 | | 4,00 | 3,0 |
| 4,50 | 0,8 | | 4,50 | 1,6 | | 4,50 | 2,4 |
| 5,00 | 0,6 | | 5,00 | 1,2 | | 5,00 | 1,8 |
| 5,50 | 0,5 | | 5,50 | 0,9 | | 5,50 | 1,4 |
| 6,00 | 0,3 | | 6,00 | 0,7 | | 6,00 | 1,0 |
| 6,50 | 0,2 | | 6,50 | 0,5 | | 6,50 | 0,7 |
| 7,00 | 0,1 | | 7,00 | 0,3 | | 7,00 | 0,4 |
| 7,50 | 0,1 | | 7,50 | 0,1 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

*5.9.1.4.12 Case 4-12*

This particular case pertains to a 1.5 m long, Ø8 0 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio '$L_d$' for this case is 1.9. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.28, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 42.1 mm, yielding a 39% settlement reduction.

*5.9.1.4.13 Case 4-13*

This particular case pertains to a 1.5 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 25 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio '$L_d$' for this case is 1.9. Maximum settlement that was achieved in the presence of no aggregate pier elements under 25 kPa pressure was 8.1 mm as mentioned earlier. From table 5.28, the maximum settlement with

the presence of a 1.5 m aggregate pier element is reduced to 4.8 mm, yielding a 41% settlement reduction.

#### 5.9.1.4.14 Case 4-14

This particular case pertains to a 1.5 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 50 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio ‘ $L_d$ ’ for this case is 1.9. Maximum settlement that was achieved in the presence of no aggregate pier elements under 50 kPa pressure was 20.3 mm as mentioned earlier. From table 5.28, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 10.7 mm, yielding a 47% settlement reduction.

Table 5.28 – Description of cases 4-12, 4-13, and 4-14

| **Case No** | ***4-12*** | | **Case No** | ***4-13*** | | **Case No** | ***4-14*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 100 | | **$s_{FOUND}$ (kPa)** | 25 | | **$s_{FOUND}$ (kPa)** | 50 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 42,1 | | 0,00 | 4,8 | | 0,00 | 10,7 |
| 0,50 | 41,0 | | 0,50 | 4,7 | | 0,50 | 10,5 |
| 1,00 | 37,5 | | 1,00 | 4,5 | | 1,00 | 9,6 |
| 1,50 | 32,2 | | 1,50 | 4,4 | | 1,50 | 8,7 |
| 2,00 | 26,0 | | 2,00 | 3,7 | | 2,00 | 6,9 |
| 2,50 | 15,7 | | 2,50 | 2,5 | | 2,50 | 4,8 |
| 3,00 | 5,8 | | 3,00 | 1,8 | | 3,00 | 3,5 |

| Table 5.28 cont. | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3,50 | 4,7 | | 3,50 | 1,3 | | 3,50 | 2,6 |
| 4,00 | 3,7 | | 4,00 | 1,0 | | 4,00 | 2,0 |
| 4,50 | 2,9 | | 4,50 | 0,8 | | 4,50 | 1,6 |
| 5,00 | 2,3 | | 5,00 | 0,6 | | 5,00 | 1,2 |
| 5,50 | 1,7 | | 5,50 | 0,5 | | 5,50 | 1,0 |
| 6,00 | 1,3 | | 6,00 | 0,3 | | 6,00 | 0,7 |
| 6,50 | 0,9 | | 6,50 | 0,2 | | 6,50 | 0,5 |
| 7,00 | 0,5 | | 7,00 | 0,1 | | 7,00 | 0,3 |
| 7,50 | 0,2 | | 7,50 | 0,1 | | 7,50 | 0,1 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

*5.9.1.4.15 Case 4-15*

This particular case pertains to a 1.5 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented on the table below. Length to diameter ratio '$L_d$' for this case is 1.9. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.33, the maximum settlement with the presence of a 1.5 m aggregate pier element is reduced to 22.6 mm, yielding a 44% settlement reduction.

*5.9.1.4.16 Case 4-16*

This particular case pertains to a 1.5 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented on the table below. Length to diameter ratio '$L_d$' for this case is 1.9. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.33, the maximum settlement with

the presence of a 1.5 m aggregate pier element is reduced to 22.6 mm, yielding a 44% settlement reduction.

Table 5.29 – Description of cases 4-15, and 4-16

| **Case No** | ***4-15*** | | **Case No** | ***4-16*** |
|---|---|---|---|---|
| **RAP Length (m)** | 1,5 | | **RAP Length (m)** | 1,5 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 22,6 | | 0,00 | 41,2 |
| 0,50 | 22,3 | | 0,50 | 40,8 |
| 1,00 | 19,6 | | 1,00 | 36,8 |
| 1,50 | 17,9 | | 1,50 | 34,3 |
| 2,00 | 13,7 | | 2,00 | 27,5 |
| 2,50 | 7,9 | | 2,50 | 16,8 |
| 3,00 | 5,2 | | 3,00 | 6,1 |
| 3,50 | 3,9 | | 3,50 | 4,7 |
| 4,00 | 3,0 | | 4,00 | 3,8 |
| 4,50 | 2,3 | | 4,50 | 3,0 |
| 5,00 | 1,8 | | 5,00 | 2,3 |
| 5,50 | 1,4 | | 5,50 | 1,8 |
| 6,00 | 1,0 | | 6,00 | 1,3 |
| 6,50 | 0,7 | | 6,50 | 0,9 |
| 7,00 | 0,4 | | 7,00 | 0,5 |
| 7,50 | 0,2 | | 7,50 | 0,2 |
| 8,00 | 0,0 | | 8,00 | 0,0 |

### 5.9.2 Additional Case (L = 5 m)

In addition to the scenarios explained above, there are a few more scenarios that have been modeled for aggregate pier length of 5 m. Case studies below are only for 75 kPa, and 100 kPa foundation pressures. General layout for this particular case is presented in figure 5.11.

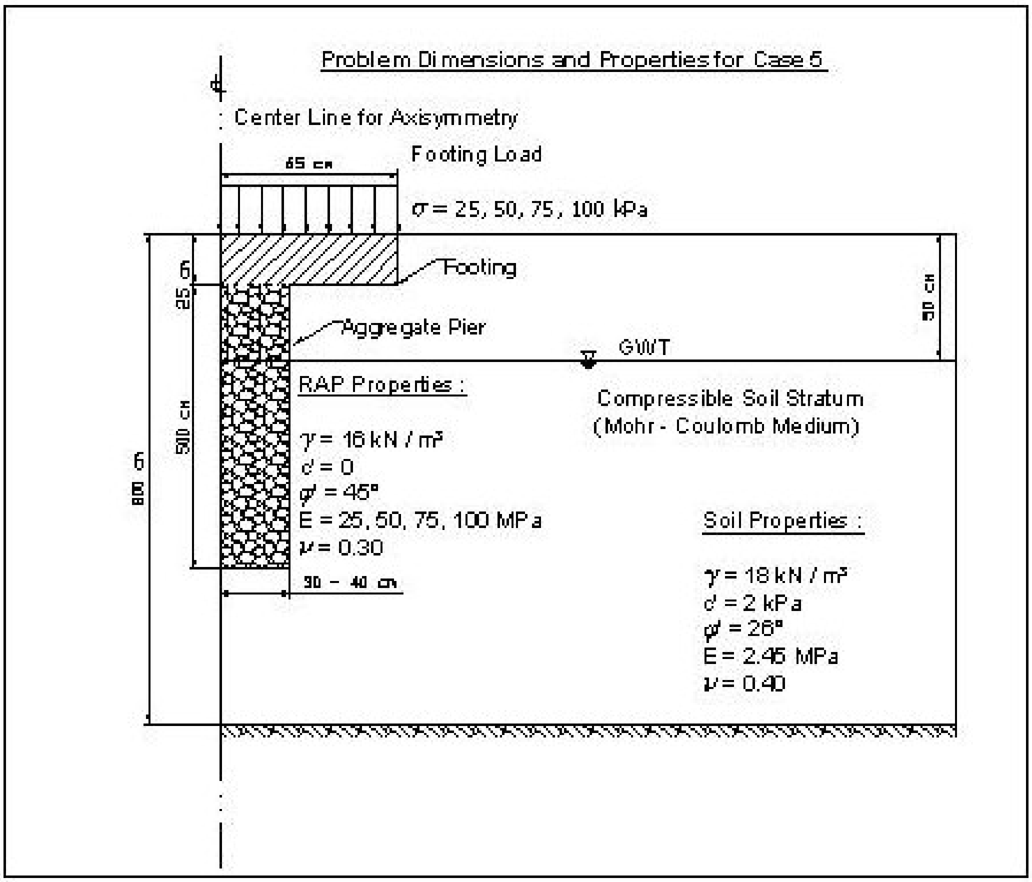



Figure 5.11 - Problem dimensions and properties for case 5

#### *5.9.2.1 Case 5.1*

This particular case pertains to a 5 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa.

Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio *'R'* for this case is 8.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.30, the maximum settlement with the presence of a 5 m aggregate pier element is reduced to 24.7 mm, yielding a 38% settlement reduction.

#### *5.9.2.2 Case 5.2*

This particular case pertains to a 5 m long, Ø60 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio *'R'* for this case is 8.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.30, the maximum settlement with the presence of a 5 m aggregate pier element is reduced to 37.7 mm, yielding a 45% settlement reduction.

#### *5.9.2.3 Case 5.3*

This particular case pertains to a 5 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio *'R'* for this case is 8.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.30, the maximum settlement with the presence of a 5 m aggregate pier element is reduced to 22.2 mm, yielding a 45% settlement reduction.

Table 5.30 – Description of cases 5-1, 5-2, and 5-3

| **Case No** | ***5-1*** | | **Case No** | ***5-2*** | | **Case No** | ***5-3*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 5 | | **RAP Length (m)** | 5 | | **RAP Length (m)** | 5 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø60 |
| **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 | | **$s_{FOUND}$ (kPa)** | 75 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 24,7 | | 0,00 | 37,7 | | 0,00 | 22,2 |
| 0,50 | 23,8 | | 0,50 | 36,5 | | 0,50 | 21,7 |
| 1,00 | 20,5 | | 1,00 | 30,6 | | 1,00 | 15,6 |
| 1,50 | 11,6 | | 1,50 | 17,9 | | 1,50 | 9,1 |
| 2,00 | 6,5 | | 2,00 | 10,7 | | 2,00 | 5,4 |
| 2,50 | 5,3 | | 2,50 | 7,4 | | 2,50 | 4,5 |
| 3,00 | 4,6 | | 3,00 | 5,8 | | 3,00 | 4,1 |
| 3,50 | 3,9 | | 3,50 | 5,1 | | 3,50 | 3,9 |
| 4,00 | 3,4 | | 4,00 | 4,4 | | 4,00 | 3,7 |
| 4,50 | 3,0 | | 4,50 | 3,9 | | 4,50 | 3,5 |
| 5,00 | 2,7 | | 5,00 | 3,5 | | 5,00 | 3,4 |
| 5,50 | 2,2 | | 5,50 | 2,8 | | 5,50 | 2,8 |
| 6,00 | 1,4 | | 6,00 | 1,8 | | 6,00 | 1,7 |
| 6,50 | 0,9 | | 6,50 | 1,1 | | 6,50 | 1,0 |
| 7,00 | 0,5 | | 7,00 | 0,6 | | 7,00 | 0,6 |
| 7,50 | 0,2 | | 7,50 | 0,3 | | 7,50 | 0,3 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

#### *5.9.2.4 Case 5.4*

This particular case pertains to a 5 m long, Ø60 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio *'R'* for this case is 8.33. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.31, the maximum settlement with the presence of a 5 m aggregate pier element is reduced to 38.0 mm, yielding a 45% settlement reduction

*5.9.2.5 Case 5.5*

This particular case pertains to a 5 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio *'R'* for this case is 6.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.31, the maximum settlement with the presence of a 5 m aggregate pier element is reduced to 17.6 mm, yielding a 56% settlement reduction.

*5.9.2.6 Case 5.6*

This particular case pertains to a 5 m long, Ø80 cm aggregate pier element with an elastic modulus of 25 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio *'R'* for this case is 6.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.31, the maximum settlement with the presence of a 5 m aggregate pier element is reduced to 28.7 mm, yielding a 58% settlement reduction.

Table 5.31 – Description of cases 5-4, 5-5, and 5-6

| Case No | ***5-4*** | | Case No | ***5-5*** | | Case No | ***5-6*** |
|---|---|---|---|---|---|---|---|
| **RAP Length (m)** | 5 | | **RAP Length (m)** | 5 | | **RAP Length (m)** | 5 |
| **RAP Diameter (cm)** | Ø60 | | **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 25 | | **$E_{RAP}$ (MPa)** | 25 |
| **$s_{FOUND}$ (kPa)** | 100 | | **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | | | | |
| 0,00 | 38,0 | | 0,00 | 17,6 | | 0,00 | 28,7 |
| 0,50 | 37,3 | | 0,50 | 16,8 | | 0,50 | 27,6 |
| 1,00 | 28,6 | | 1,00 | 14,3 | | 1,00 | 24,0 |
| 1,50 | 17,5 | | 1,50 | 8,5 | | 1,50 | 16,1 |
| 2,00 | 9,7 | | 2,00 | 6,3 | | 2,00 | 9,7 |
| 2,50 | 5,7 | | 2,50 | 5,5 | | 2,50 | 7,2 |
| 3,00 | 4,8 | | 3,00 | 4,8 | | 3,00 | 6,0 |
| 3,50 | 4,5 | | 3,50 | 4,1 | | 3,50 | 5,3 |
| 4,00 | 4,3 | | 4,00 | 3,6 | | 4,00 | 4,7 |
| 4,50 | 4,1 | | 4,50 | 3,2 | | 4,50 | 4,2 |
| 5,00 | 4,0 | | 5,00 | 3,0 | | 5,00 | 3,9 |
| 5,50 | 3,3 | | 5,50 | 2,4 | | 5,50 | 3,1 |
| 6,00 | 2,0 | | 6,00 | 1,6 | | 6,00 | 2,0 |
| 6,50 | 1,3 | | 6,50 | 1,0 | | 6,50 | 1,3 |
| 7,00 | 0,7 | | 7,00 | 0,6 | | 7,00 | 0,7 |
| 7,50 | 0,3 | | 7,50 | 0,2 | | 7,50 | 0,3 |
| 8,00 | 0,0 | | 8,00 | 0,0 | | 8,00 | 0,0 |

#### *5.9.2.7 Case 5.7*

This particular case pertains to a 5 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 75 kPa. Settlement value corresponding to a certain depth is also presented in the table below. Length to diameter ratio *'R'* for this case is 6.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 75 kPa pressure was 40.0 mm as mentioned earlier. From table 5.32, the maximum settlement with the presence of a 5 m aggregate pier element is reduced to 16.5 mm, yielding a 59% settlement reduction.

### *5.9.2.8 Case 5.8*

This particular case pertains to a 5 m long, Ø80 cm aggregate pier element with an elastic modulus of 100 MPa resisting an applied footing pressure of 100 kPa. Settlement value corresponding to a certain depth is also presented in the table above. Length to diameter ratio *'R'* for this case is 6.25. Maximum settlement that was achieved in the presence of no aggregate pier elements under 100 kPa pressure was 68.9 mm as mentioned earlier. From table 5.32, the maximum settlement with the presence of a 5 m aggregate pier element is reduced to 28.8 mm, yielding a 58% settlement reduction.

Table 5.32 – Description of cases 5-7, and 5-8

| **Case No** | ***5-7*** | | **Case No** | ***5-8*** |
|---|---|---|---|---|
| **RAP Length (m)** | 5 | | **RAP Length (m)** | 5 |
| **RAP Diameter (cm)** | Ø80 | | **RAP Diameter (cm)** | Ø80 |
| **$E_{RAP}$ (MPa)** | 100 | | **$E_{RAP}$ (MPa)** | 100 |
| **$s_{FOUND}$ (kPa)** | 75 | | **$s_{FOUND}$ (kPa)** | 100 |
| | | | | |
| **Depth (m)** | **S (mm)** | | **Depth (m)** | **S (mm)** |
| | | | | |
| 0,00 | 16,5 | | 0,00 | 28,8 |
| 0,50 | 16,3 | | 0,50 | 28,4 |
| 1,00 | 13,0 | | 1,00 | 22,5 |
| 1,50 | 7,9 | | 1,50 | 14,2 |
| 2,00 | 5,0 | | 2,00 | 7,4 |
| 2,50 | 4,6 | | 2,50 | 5,6 |
| 3,00 | 4,3 | | 3,00 | 5,1 |

| Table 5.32 cont. | | | | |
|---|---|---|---|---|
| 3,50 | 4,1 | | 3,50 | 4,8 |
| 4,00 | 3,9 | | 4,00 | 4,6 |
| 4,50 | 3,8 | | 4,50 | 4,5 |
| 5,00 | 3,7 | | 5,00 | 4,4 |
| 5,50 | 3,1 | | 5,50 | 3,7 |
| 6,00 | 1,9 | | 6,00 | 2,3 |
| 6,50 | 1,2 | | 6,50 | 1,4 |
| 7,00 | 0,7 | | 7,00 | 0,8 |
| 7,50 | 0,3 | | 7,50 | 0,3 |
| 8,00 | 0,0 | | 8,00 | 0,0 |

## 5.10 Summary of Scenarios

From the above table, for case 1, the average improvement in terms of settlement comes out to be 26%. There is a dramatic difference between having a 2 m long aggregate pier installed and a 1 m long one. When an intermediate length is considered as in case 4, average settlement improvement comes out to be 36% as shown in table 5.33. As illustrated in table 5.32, the average percent settlement improvement achieved by 2 m long aggregate piers is 43%. This improvement percentage further increases to 48 % with 3 m long piers. Evidently, from table 5.34, 5 m long aggregate pier elements provided a 51% average improvement. However, if an average settlement improvement percentage is to be computed with respect to foundation pressures of 75 kPa, and 100 kPa for the 3 m aggregate pier elements, the percent improvement value comes out to be 51%, which is the same as the one for 5 m aggregate piers.

Rather than mentioning a percentage in settlement reduction, in the next chapter, concept of ‘ b‘ will be explained, which denotes a ratio of the treated settlement value to the untreated value for maximum settlements.
The average values for settlement reduction is summarized in table 5.36.

All of the above scenarios are summarized in the following tables:

Table 5.33 – Summary of case 1

| Case No | RAP Length (m) | RAP Diameter (cm) | $E_{RAP}$ (MPa) | $s_{FOUND}$ (kPa) | Untreated Settlement | Treated Settlement | % Reduction |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| 1-1 | 1 | 60 | 25 | 25 | 8,1 | 6,5 | 19,8 |
| 1-2 | | | | 50 | 20,3 | 15,5 | 23,6 |
| 1-3 | | | | 75 | 40,0 | 35 | 12,5 |
| 1-4 | | | | 100 | 68,9 | 56,1 | 18,6 |
| 1-5 | | | 100 | 25 | 8,1 | 6,2 | 23,5 |
| 1-6 | | | | 50 | 20,3 | 14,4 | 29,1 |
| 1-7 | | | | 75 | 40,0 | 30,8 | 23,0 |
| 1-8 | | | | 100 | 68,9 | 47,9 | 30,5 |
| 1-9 | | 80 | 25 | 25 | 8,1 | 5,9 | 27,2 |
| 1-10 | | | | 50 | 20,3 | 15,2 | 25,1 |
| 1-11 | | | | 75 | 40,0 | 32,2 | 19,5 |
| 1-12 | | | | 100 | 68,9 | 52,3 | 24,1 |
| 1-13 | | | 100 | 25 | 8,1 | 5,5 | 32,1 |
| 1-14 | | | | 50 | 20,3 | 13 | 36,0 |
| 1-15 | | | | 75 | 40,0 | 27,6 | 31,0 |
| 1-16 | | | | 100 | 68,9 | 43,8 | 36,4 |

Table 5.34 – Summary of case 2

| Case No | RAP Length (m) | RAP Diameter (cm) | $E_{RAP}$ (MPa) | $s_{FOUND}$ (kPa) | Untreated Settlement | Treated Settlement | % Reduction |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| **2-1** | 2 | 60 | 25 | 25 | 8,1 | 5,6 | 30,9 |
| **2-2** | | | | 50 | 20,3 | 13,7 | 32,5 |
| **2-3** | | | | 75 | 40,0 | 23,9 | 40,3 |
| **2-4** | | | | 100 | 68,9 | 43,3 | 37,2 |
| **2-5** | | | 100 | 25 | 8,1 | 5,1 | 37,0 |
| **2-6** | | | | 50 | 20,3 | 12,3 | 39,4 |
| **2-7** | | | | 75 | 40,0 | 23 | 42,5 |
| **2-8** | | | | 100 | 68,9 | 43 | 37,6 |
| **2-9** | | 80 | 25 | 25 | 8,1 | 4,9 | 39,5 |
| **2-10** | | | | 50 | 20,3 | 11 | 45,8 |
| **2-11** | | | | 75 | 40,0 | 19,4 | 51,5 |
| **2-12** | | | | 100 | 68,9 | 36,4 | 47,2 |
| **2-13** | | | 100 | 25 | 8,1 | 4,2 | 48,1 |
| **2-14** | | | | 50 | 20,3 | 10,3 | 49,3 |
| **2-15** | | | | 75 | 40,0 | 17,8 | 55,5 |
| **2-16** | | | | 100 | 68,9 | 35,1 | 49,1 |

Table 5.35 – Summary of case 3

| Case No | RAP Length (m) | RAP Diameter (cm) | $E_{RAP}$ (MPa) | $s_{FOUND}$ (kPa) | Untreated Settlement | Treated Settlement | % Reduction |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| 3-1 | 3 | 60 | 25 | 25 | 8,1 | 5,3 | 34,6 |
| 3-2 | | | | 50 | 20,3 | 13,5 | 33,5 |
| 3-3 | | | | 75 | 40,0 | 24,4 | 39,0 |
| 3-4 | | | | 100 | 68,9 | 37,5 | 45,6 |
| 3-5 | | | 100 | 25 | 8,1 | 4,8 | 40,7 |
| 3-6 | | | | 50 | 20,3 | 11,7 | 42,4 |
| 3-7 | | | | 75 | 40,0 | 22,1 | 44,8 |
| 3-8 | | | | 100 | 68,9 | 37,1 | 46,2 |
| 3-9 | | 80 | 25 | 25 | 8,1 | 4,4 | 45,7 |
| 3-10 | | | | 50 | 20,3 | 10,4 | 48,8 |
| 3-11 | | | | 75 | 40,0 | 18,4 | 54,0 |
| 3-12 | | | | 100 | 68,9 | 28,9 | 58,1 |
| 3-13 | | | 100 | 25 | 8,1 | 3,4 | 58,0 |
| 3-14 | | | | 50 | 20,3 | 8,9 | 56,2 |
| 3-15 | | | | 75 | 40,0 | 16,4 | 59,0 |
| 3-16 | | | | 100 | 68,9 | 27,8 | 59,7 |

Table 5.36 – Summary of case 4

| Case No | RAP Length (m) | RAP Diameter (cm) | $E_{RAP}$ (MPa) | $s_{FOUND}$ (kPa) | Untreated Settlement | Treated Settlement | % Reduction |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| 4-1 | 1,5 | 60 | 25 | 25 | 8,1 | 6,0 | 26 |
| 4-2 | | | | 50 | 20,3 | 14,3 | 30 |
| 4-3 | | | | 75 | 40 | 26,7 | 33 |
| 4-4 | | | | 100 | 68,9 | 49,2 | 29 |
| 4-5 | | | 100 | 25 | 8,1 | 5,5 | 32 |
| 4-6 | | | | 50 | 20,3 | 13,2 | 35 |
| 4-7 | | | | 75 | 40 | 26,6 | 34 |
| 4-8 | | | | 100 | 68,9 | 45,7 | 34 |
| 4-9 | | 80 | 25 | 25 | 8,1 | 5,3 | 35 |
| 4-10 | | | | 50 | 20,3 | 12,1 | 40 |
| 4-11 | | | | 75 | 40 | 22,3 | 44 |
| 4-12 | | | | 100 | 68,9 | 42,1 | 39 |
| 413 | | | 100 | 25 | 8,1 | 4,8 | 41 |
| 4-14 | | | | 50 | 20,3 | 10,7 | 47 |
| 4-15 | | | | 75 | 40 | 22,6 | 44 |
| 4-16 | | | | 100 | 68,9 | 41,2 | 40 |

Table 5.37 – Summary of case 5

| Case No | RAP Length (m) | RAP Diameter (cm) | $E_{RAP}$ (MPa) | $s_{FOUND}$ (kPa) | Untreated Settlement | Treated Settlement | % Reduction |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| 5-1 | 5 | 60 | 25 | 75 | 40,0 | 24,1 | 39,8 |
| 5-2 | | | | 100 | 68,9 | 37,7 | 45,3 |
| 5-3 | | | 100 | 75 | 40,0 | 22,2 | 44,5 |
| 5-4 | | | | 100 | 68,9 | 38,0 | 44,8 |
| 5-5 | | 80 | 25 | 75 | 40,0 | 17,6 | 56,0 |
| 5-6 | | | | 100 | 68,9 | 28,7 | 58,3 |
| 5-7 | | | 100 | 75 | 40,0 | 16,5 | 58,8 |
| 5-8 | | | | 100 | 68,9 | 28,8 | 58,2 |

To give a better idea for the settlement behavior of aggregate piers, following graphs have been generated. Settlement values are taken on the axis of symmetry, thus centerline of the piers.

Table 5.38 – Summary of average settlement reductions for all cases

| Case No | Pier Length (m) | Average Settlement Reduction (%) |
|---|---|---|
| Case 1 | 1 | 26 |
| Case 2 | 2 | 43 |
| Case 3 | 3 | 48 |
| Case 4 | 1,5 | 36 |
| Case 5 ** | 5 | 51 |

** Case 5 is only modeled using 75, and 100 kPa foundation pressures.

Table 5.39 – Summary of average settlement reductions for $s_{FOUND}$ = 75, 100 kPa

| Case No | Pier Length (m) | Average Settlement Reduction (%) |
|---|---|---|
| Case 1 | 1 | 24 |
| Case 2 | 2 | 45 |
| Case 3 | 3 | 51 |
| Case 4 | 1,5 | 36 |
| Case 5 | 5 | 51 |

Note in the above table the proximity of the values for cases 2, 3, and 5. As the height of the aggregate pier reaches to a certain threshold, average settlement reduction value stabilizes around 50%.

If the graphs illustrated below are examined, it can be seen that as foundation pressures increase, the difference between the maximum settlements for L = 2m, 3m, and 5 m are not that substantial.  For lower foundation pressures such as 25 kPa, and 50 kPa, settlement vs depth graphs tend to yield greater gaps between particular aggregate pier lengths.

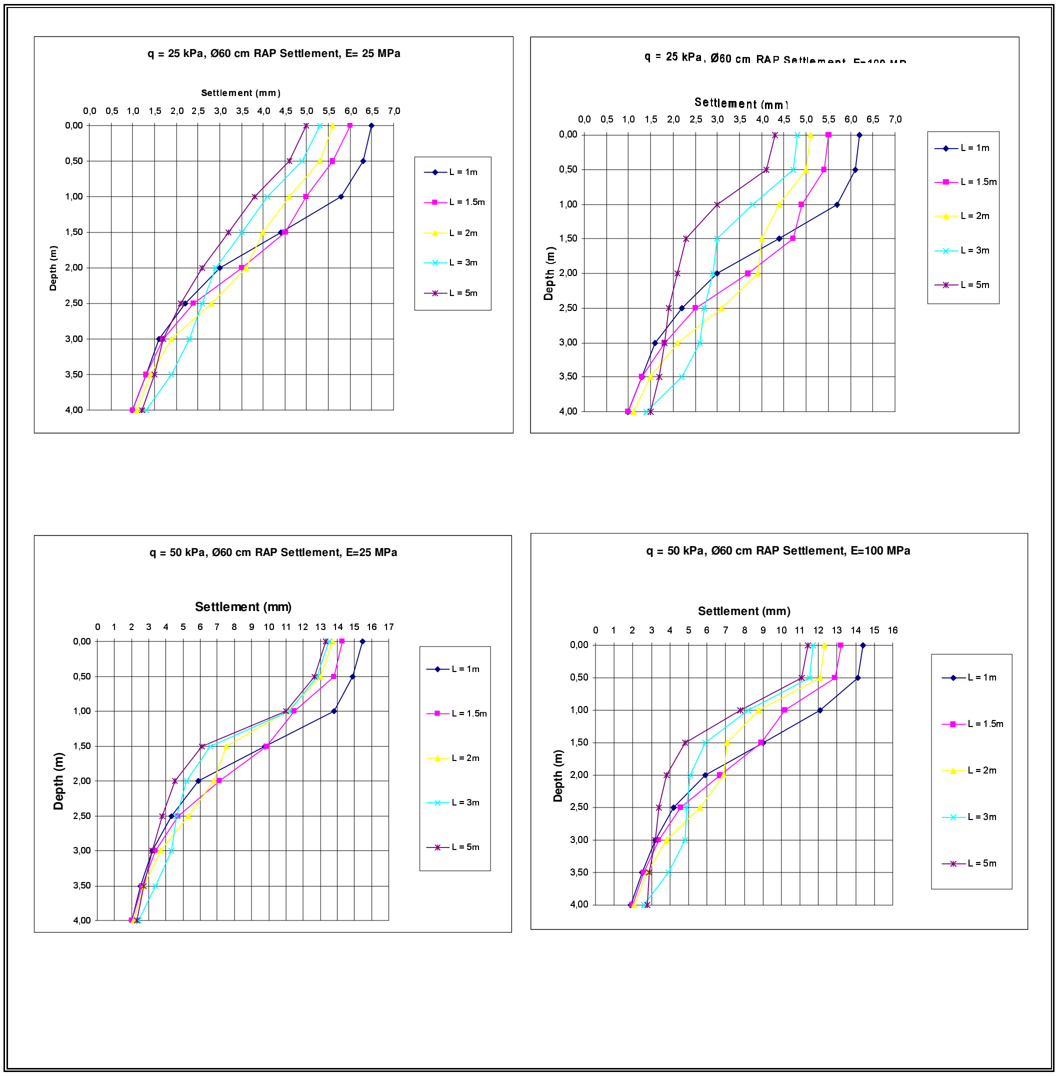



Figure 5.12 – Settlement vs depth plots for 60 cm dia. aggregate piers under 25, and 50 kPa

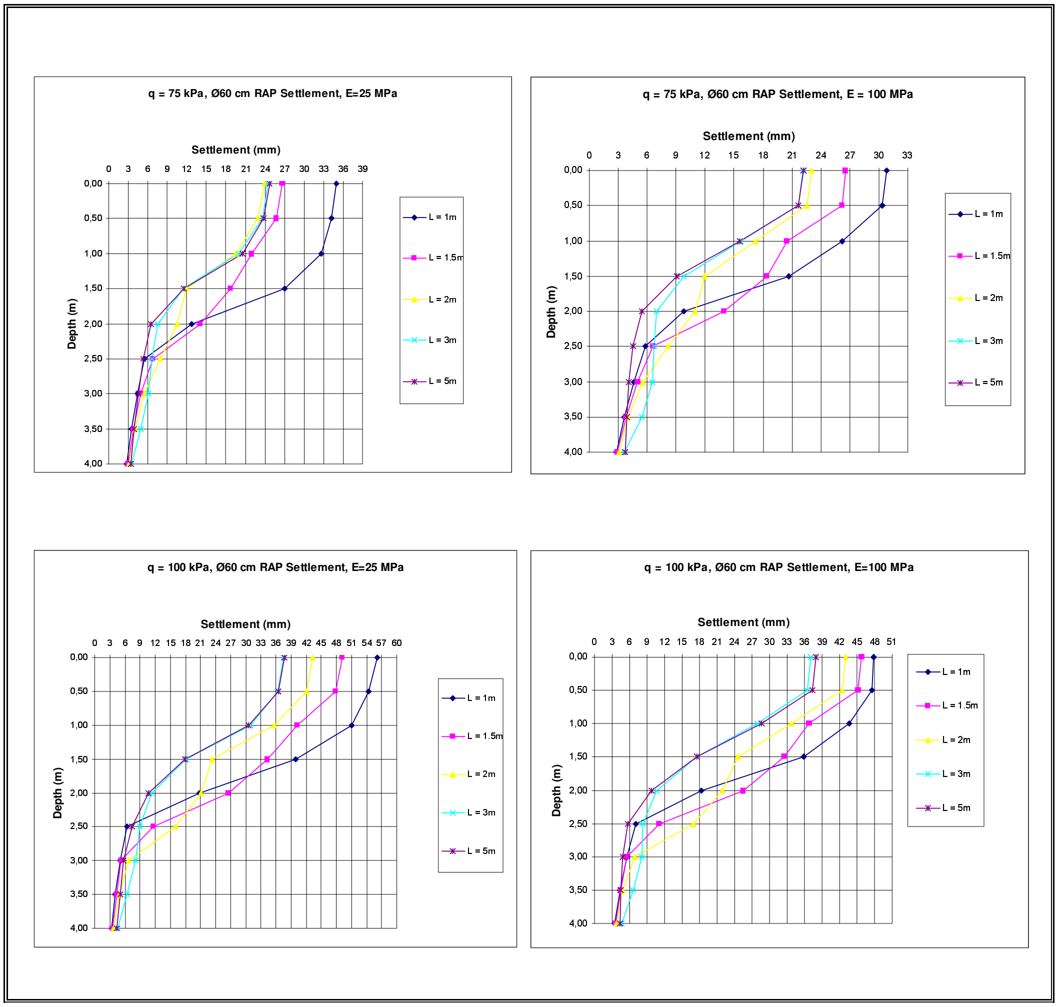



Figure 5.13– Settlement vs depth plots for 60 cm dia. aggregate piers under 75, and 100 kPa

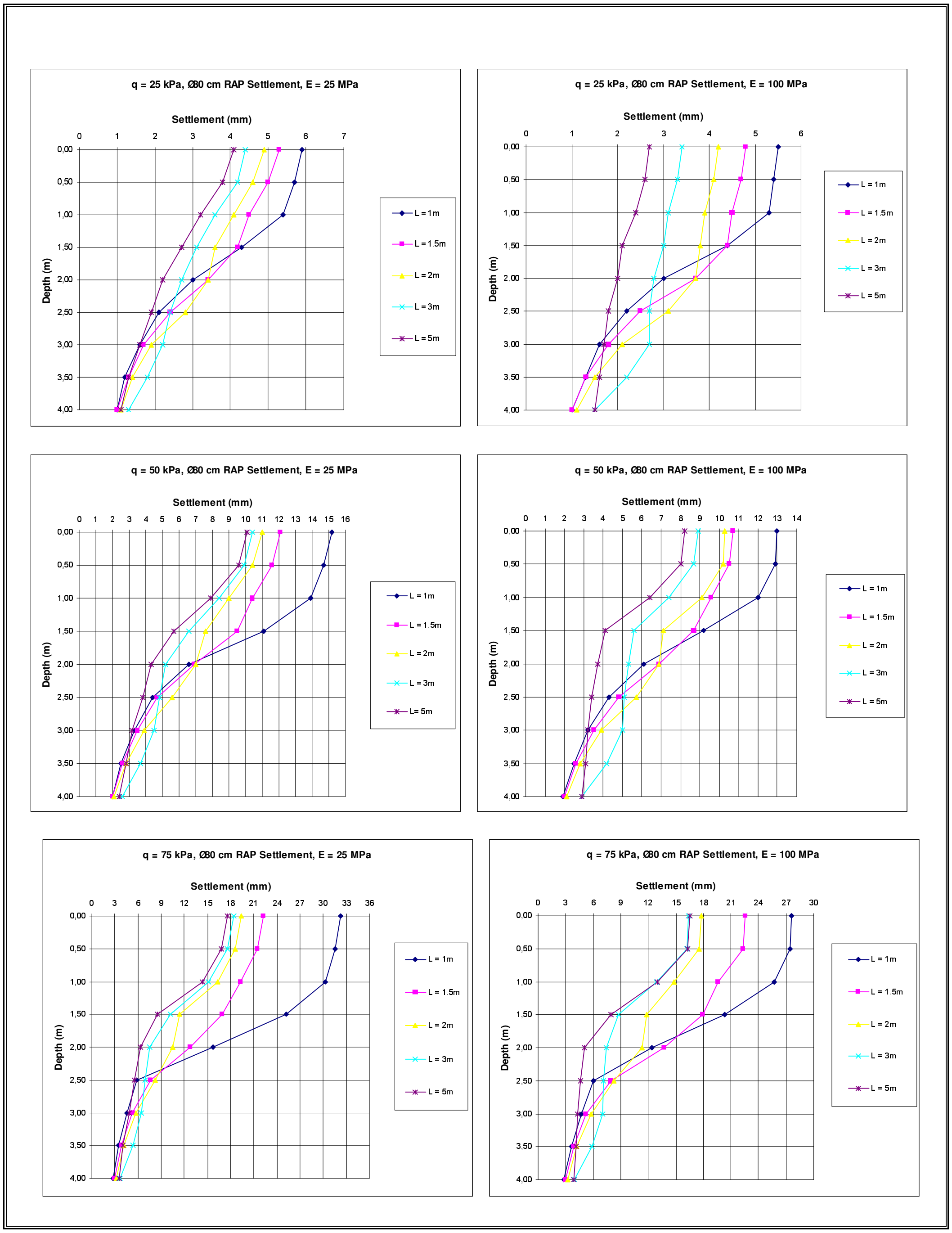



Figure 5.14 - Settlement vs depth plots for 80 cm dia. aggregate piers under 25, 50, and 75 kPa

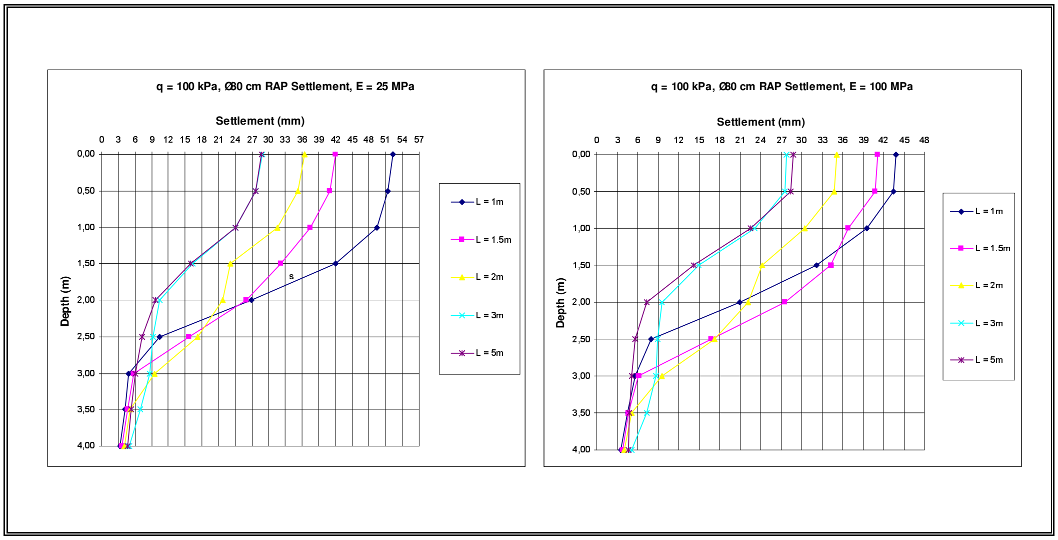



Figure 5.15 – Settlement vs depth plots for 80 cm dia. aggregate piers under 100 kPa

## CHAPTER 6

## DISCUSSION OF RESULTS

### 6.1 Settlement Reduction Ratio, '**b**'

As mentioned in the previous chapter, settlement improvement is often denoted by the letter ' b ', implying a settlement reduction ratio, which is the ratio of the treated settlement to the untreated settlement value.

To see a relationship between b and the applied pressure better, and derive a trend, the following plots have been prepared with varying diameter, length, elastic modulus, and the applied foundation pressures.

When aggregate piers are examined for their settlement reduction ratio versus the applied foundation pressure, like the one whose plot is illustrated in figure 6.1, one thing that comes up so obvious is that, the behavior of short and long piers, and relatively stiff piers tend to differ.

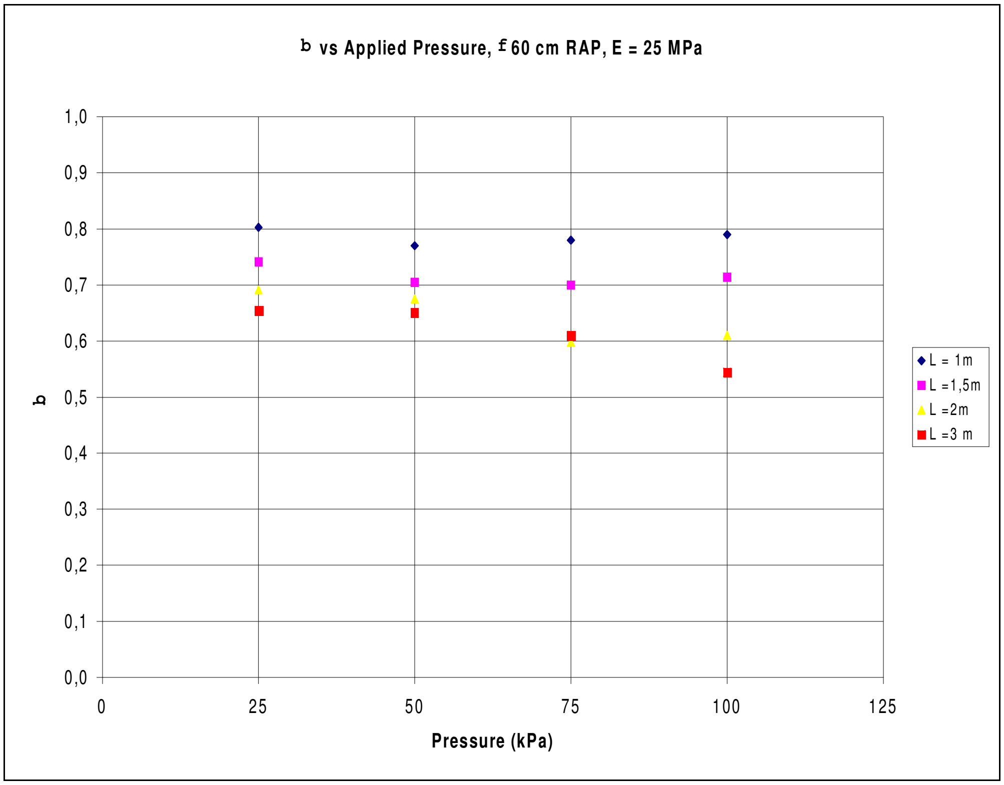



Figure 6.1 – Relationship with b and the applied pressure for f=60 cm, E = 25 MPa

Like in figure 6.1, for L = 1 m, L = 1.5m, b can be taken as constant for increasing foundation pressure. The average b value for the case presented above for L=1 m is 0.79. For L =1.5 m, the average b value is 0.71 When greater lengths are in concern, for instance, 2m, and 3 m, the trend varies, as foundation pressure increases, b decreases. The average values for 2m, and 3 m piers are, 0.64, and 0.61, respectively. The same trend also appears for the same parameters with E = 100 MPa shown . Only, the b values are different. The average value for 1m, is 0.75, it decreases to 0.66 for L = 1.5 m. For L = 2 m, and 3 m, the values are 0.60, and 0.56, respectively. So as a rule of thumb, one may say that as the aggregate piers get stiffer, b decreases, which makes sense. In simple terms, as aggregate piers get stronger, their settlement reducing ability becomes greater.

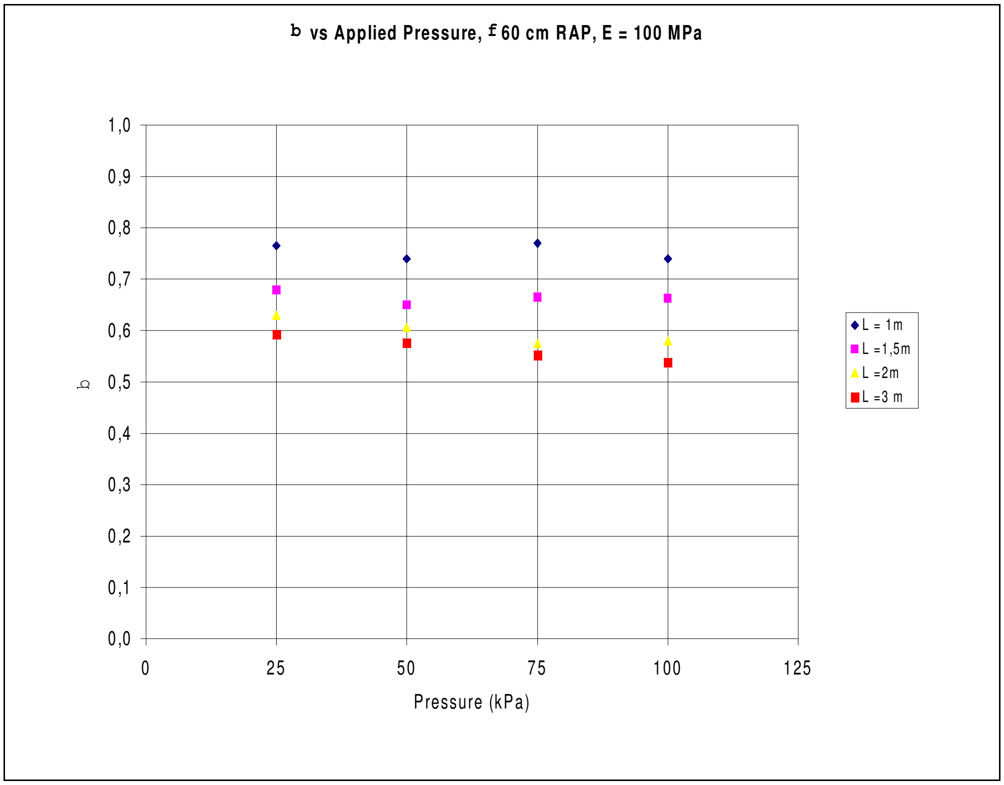



Figure 6.2 – Relationship with b and the applied pressure for f=60 cm, E = 100 MPa

When the soil is relatively weaker (E = 25 MPa), the behavior of b versus corresponding foundation pressure is clearly illustrated in figure 6.3. For aggregate piers up to a length of 2 m ,(shorter piers), we have a constant relationship for b versus increasing foundation pressure. As in previous plots, b tends to decrease with pressure for 2m, and 3 m piers. Nevertheless, as the aggregate piers get stiffer (E = 100 MPa), this trend also slightly changes to having a constant relationship of b versus pressure for both short and long piers as it is the case in figure 6.4.

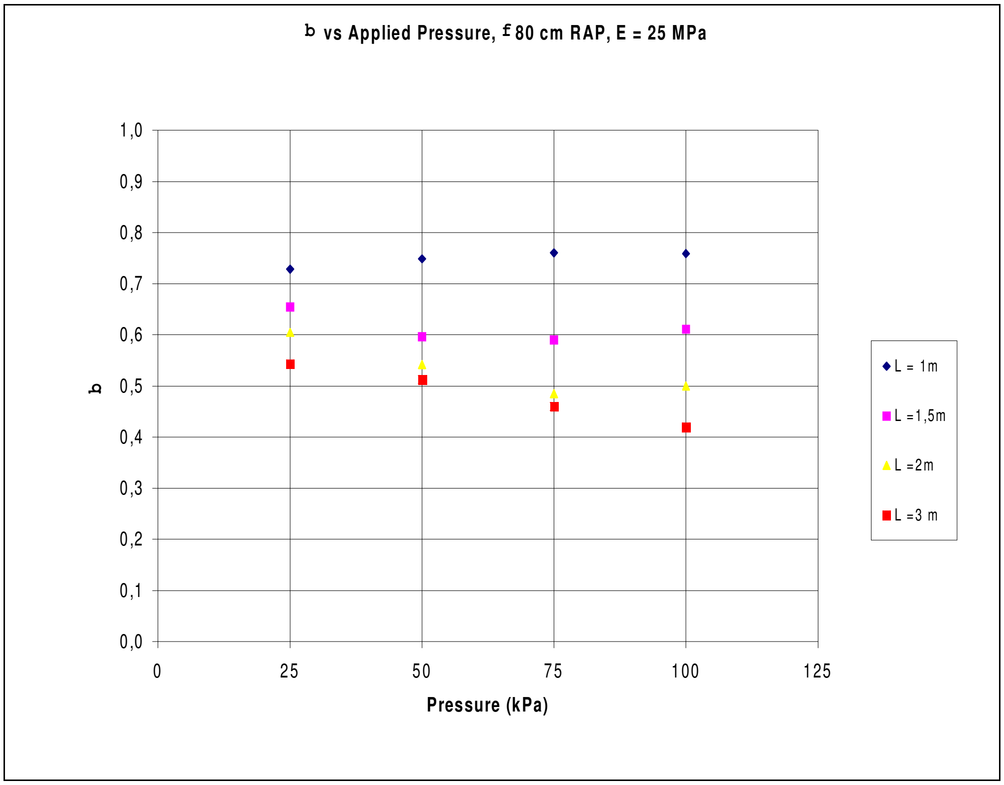



Figure 6.3 – Relationship with b and the applied pressure for f=80 cm, E = 25 MPa

To reflect this trend better, a specific case study is presented below to derive a trend for the effect of elastic modulus, diameter of piers, and length of piers on b. Figure 6.5 clearly illustrates that as elastic modulus of the piers increases, b decreases, however, for increasing pressure, bstays constant. Figure 6.6 represents the effect of change of diameter of the piers. Obviously, there is also a constant trend here. For increasing pressure, b stays constant for F60 cm and F80 cm aggregate piers. The b values however, tend to be lower for greater diameter.

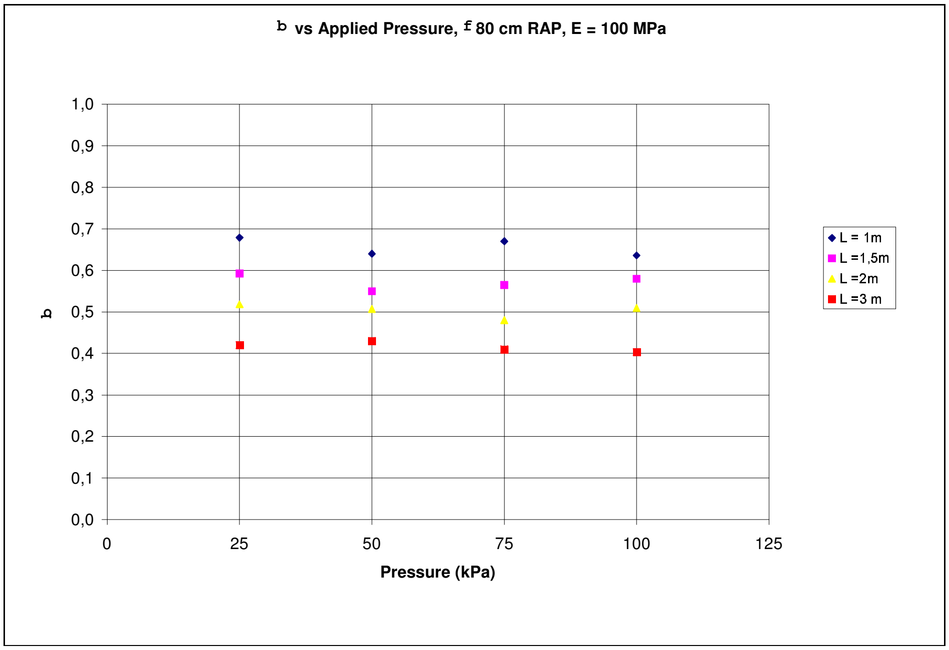



Figure 6.4 – Relationship with b and the applied pressure for f=80 cm, E = 100 MPa

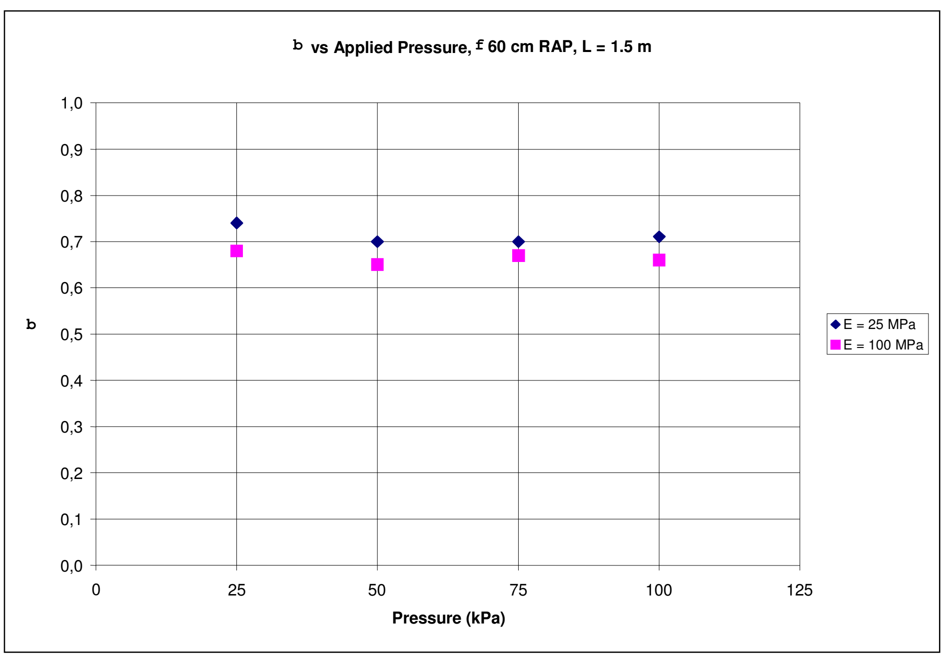



Figure 6.5 – Relationship with b and elastic modulus of aggregate piers

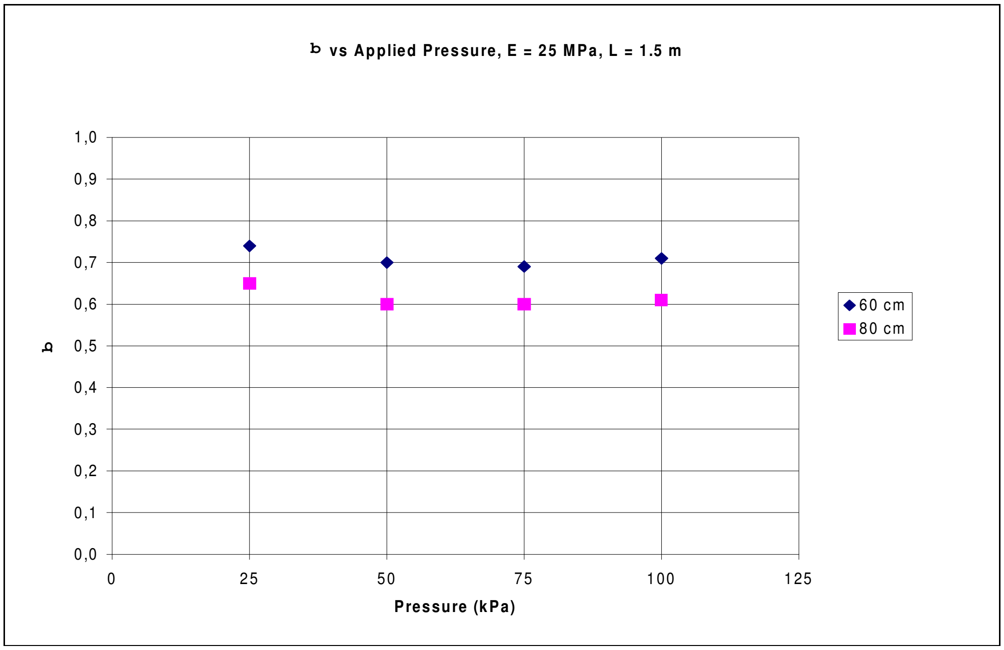



Figure 6.6 – Relationship with b and diameter of aggregate piers for E = 25 MPa

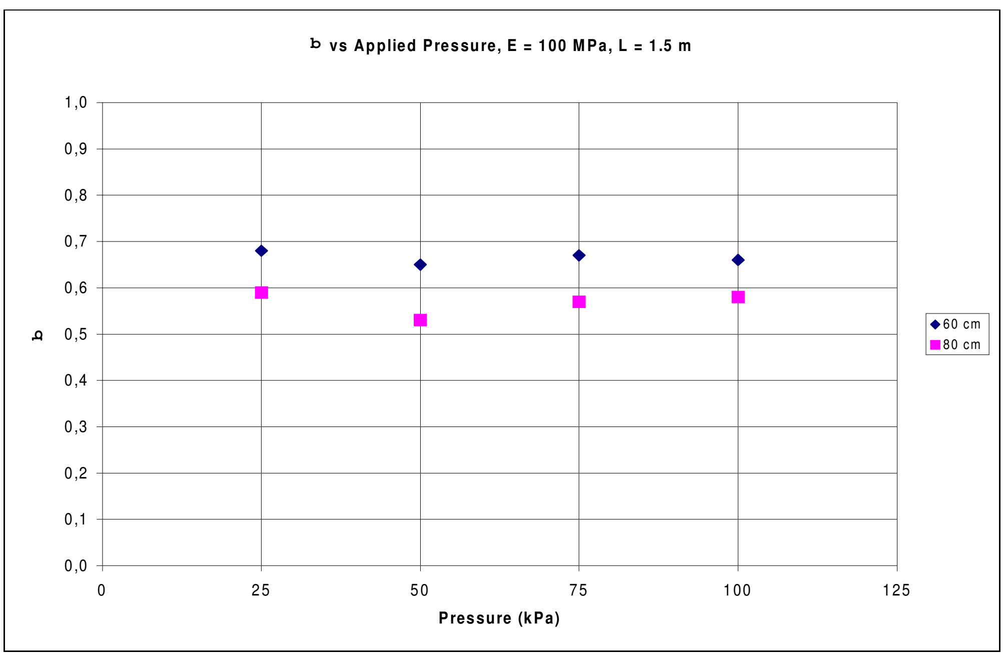



Figure 6.7 – Relationship with b and diameter of aggregate piers for E = 100 MPa

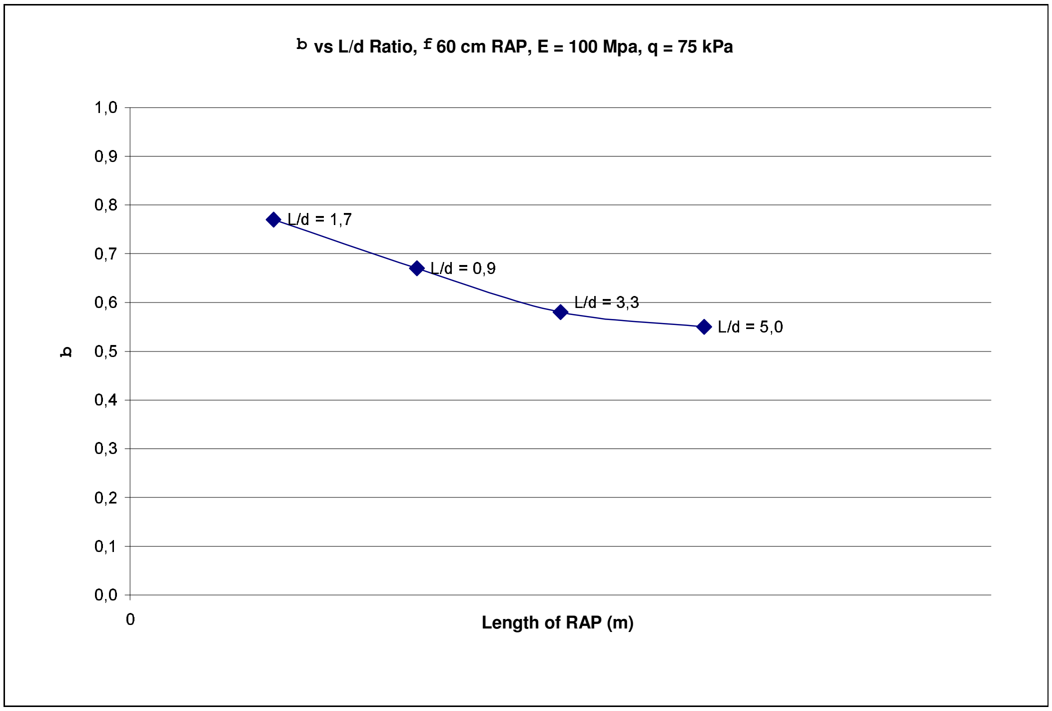



Figure 6.8 – Relationship with b and length of aggregate piers

When effect of length is considered as in figure 6.8, as length to diameter ratio of aggregate piers increase, the curve is flattened. There is a decreasing trend here for b value with increasing length. Nevertheless, at length = 3 m, the curve flattens out yielding a constant relationship from thereon.

### 6.2 Stress Concentration Ratio, 'n'

As mentioned before, 'n' is described as the stress concentration ratio, which is the ratio of the average effective vertical stress at the centerline, and the edge of the aggregate pier element to the average effective vertical stress developed at the bottom of the foundation. To avoid edge effect disturbance, these readings have been taken 12.5 cm below the footing. To reflect the trend for n versus the variables used, such as elastic modulus, length, and diameter, the following plots have been generated.

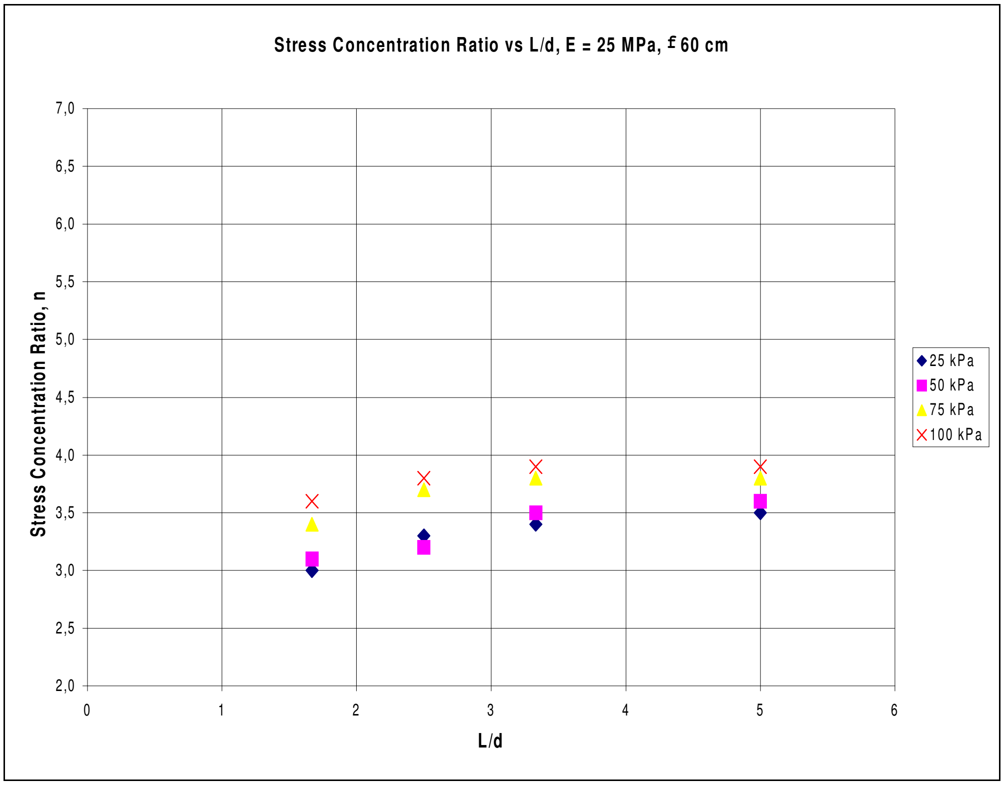



Figure 6.9 – Relationship with n and L / d for 60 cm piers, E = 25 MPa

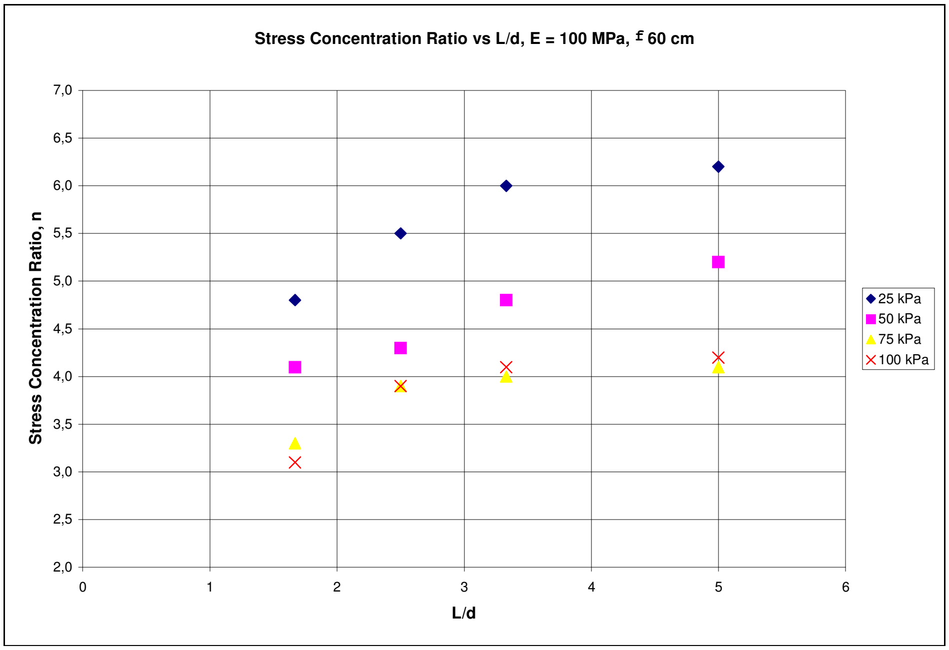



Figure 6.10 – Relationship with n and L / d for 60 cm piers, E = 100 MPa

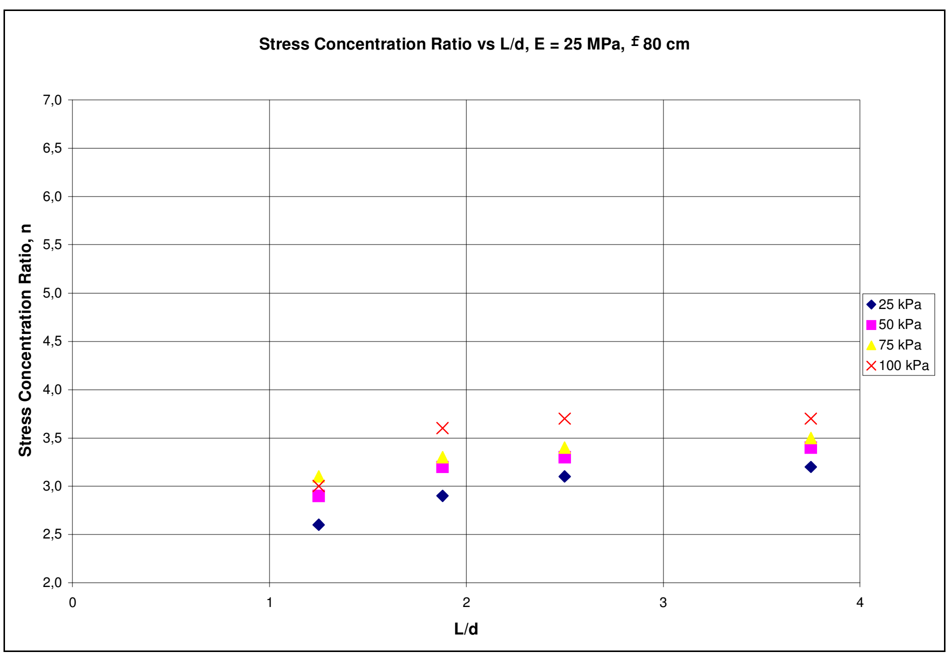



Figure 6.11 – Relationship with n and L / d for 80 cm piers, E = 25 MPa

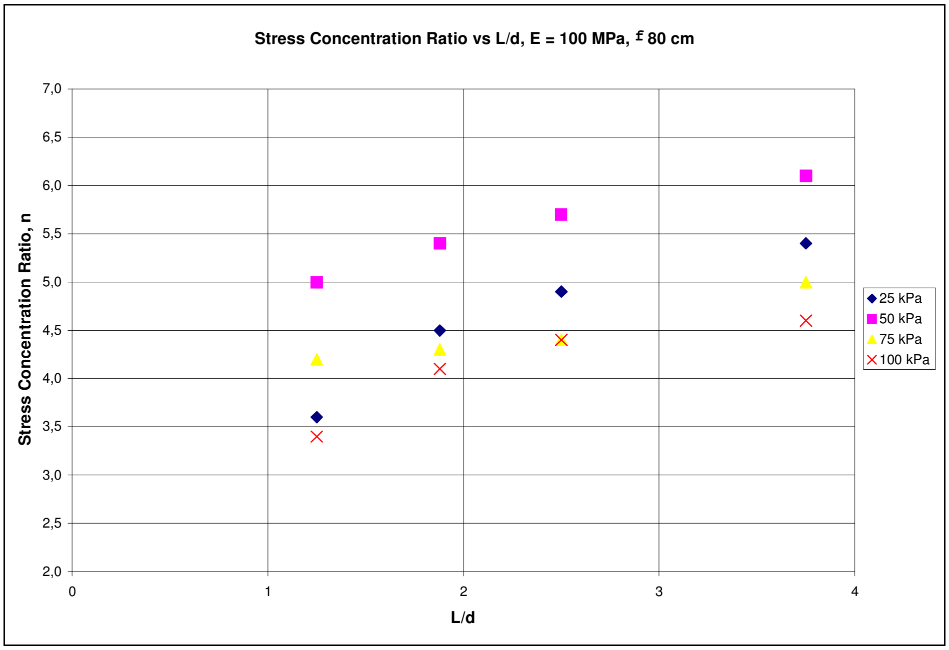



Figure 6.12 – Relationship with n and L / d for 80 cm piers, E = 100 MPa

From the plots illustrated above, it can be seen that stress concentration ratio increases as L / d increases for a set diameter, for given stress levels. As elastic modulus varies, for a stiffer aggregate pier having an elastic modulus of 100 MPa, the plots reveal that the pier shares more stress for smaller pressures such as 25 kPa. However, as the pier gets more stiff (E = 100 MPa), the pier shares more stress for higher loads such as 100 kPa. Nevertheless, as diameter of the pier increases, the shared pier stress (n ratio) increases. To see this for a specific case, two plots, one for applied foundation pressure of 25 kPa representing light load, and one for 75 kPa representing a relatively higher load both of the cases for £60 cm piers have been illustrated below. In agreement with the current discussion, as lighter loads are imposed on the piers, stronger piers tend to share more stress (higher n value), on the contrary, as magnitude of the loads gets greater, both stiff and less stiff piers get approximately the same amount of share of stress. In simple words, from the graphs below, as load increases from 25 kPa to 75 kPa, n ratio for stronger piers decreases. For less stiff piers, as load increases, n ratio increases. Range of n values has been presented in table 6.1.

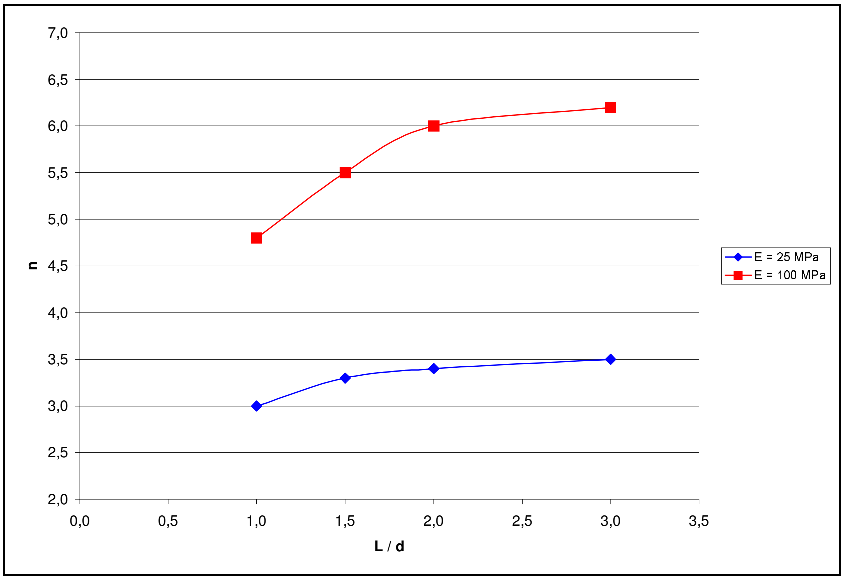



Figure 6.13 – Relationship with n and L / d for 60 cm piers, p = 25 kPa

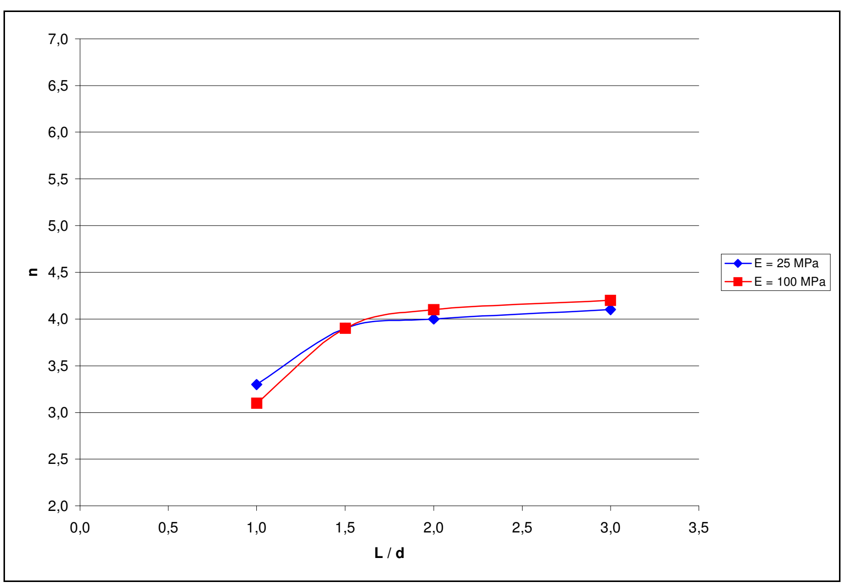



Figure 6.14 – Relationship with n and L / d for 60 cm piers, p = 75 kPa

Table 6.1 – Range of n values

| Dia (cm) | E (MPa) | L/d | n |
|---|---|---|---|
| [redacted] | | | |
| 60 | 25 | 1,7 | 3,0 - 3,6 |
| | | 2,5 | 3,2 - 3,7 |
| | | 3,3 | 3,4 - 3,8 |
| | | 5,0 | 3,5 - 3,8 |
| | 100 | 1,7 | 3,2 - 4,8 |
| | | 2,5 | 3,8 - 5,5 |
| | | 3,3 | 4,0 - 6,0 |
| | | 5,0 | 4,1 - 6,2 |
| 80 | 25 | 1,7 | 2,6 - 3,1 |
| | | 2,5 | 2,8 - 3,6 |
| | | 3,3 | 3,1 - 3,7 |
| | | 5,0 | 3,2 - 3,7 |
| | 100 | 1,7 | 3,4 - 5,0 |
| | | 2,5 | 4,2 - 5,4 |
| | | 3,3 | 4,4 - 5,7 |
| | | 5,0 | 4,6 - 6,1 |

## 6.3 Settlement

As one of the objectives of this research, the graph on the next page reflects the settlement improvement of the compressible soil with aggregate piers. The blue curve represents the untreated soil settlement under foundation loads. Note that, the curves pertaining to 3 m, and 5 m long piles are almost identical, meaning that after 3 m, there will not be any improvement on the settlement behavior of the soil no matter how deep you make your aggregate piers. To do further improvement, more densely spacing of piers may be selected, or bigger diameter piers may be installed.

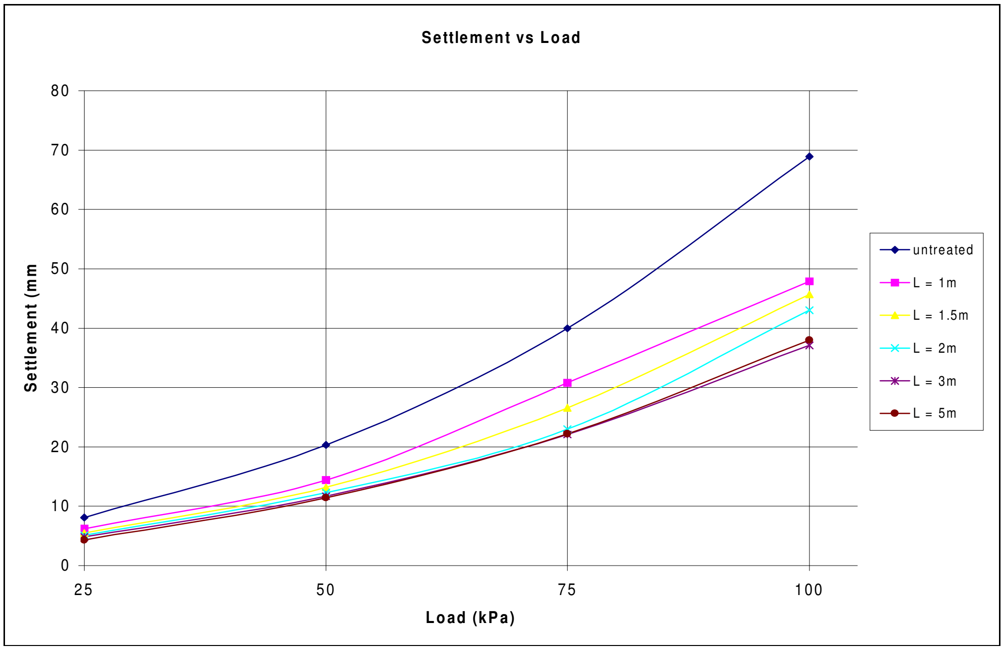



Figure 6.15 – Settlement vs load curves for all cases

Table 6.2 – Range of b values

| Dia (cm) | E (MPa) | Length (m) | b |
|---|---|---|---|
| [redacted] | | | |
| 60 | 25 | 1,0 | 0,79 |
| | | 1,5 | 0,71 |
| | | 2,0 | 0,61 - 0,69 |
| | | 3,0 | 0,54 - 0,65 |
| | 100 | 1,0 | 0,75 |
| | | 1,5 | 0,66 |
| | | 2,0 | 0,58 - 0,63 |
| | | 3,0 | 0,54 - 0,59 |
| 80 | 25 | 1,0 | 0,75 |
| | | 1,5 | 0,61 |
| | | 2,0 | 0,50 - 0,60 |
| | | 3,0 | 0,42 - 0,54 |
| | 100 | 1,0 | 0,66 |
| | | 1,5 | 0,57 |
| | | 2,0 | 0,5 |
| | | 3,0 | 0,42 |

# CHAPTER 7

# CONCLUSION

## 7.1 General

The aim of this research is to find relationships for settlement behavior of the piers with depth, settlement reduction ratio, and stress concentration ratio for varying length, diameter, and elastic modulus for aggregate piers, and foundation pressures. For a generalized approach, ratio of elastic modulus of the aggregate pier to compressible soil ($E_a$ / $E_s$) can be stated as follows:

$E_a$ is a variable with values 25 MPa, and 100 MPa
$E_s$ is fixed having a value of 2.45 MPa

So, $E_a$ / $E_s$ for small $E_a$ yields a ratio of 10.2, subsequently for greater $E_a$, the ratio of the elastic moduli becomes 40.8. As a rule of thumb, it would be better to express these terms that have been referred to in the previous graphs in terms of ratios. As a result, on the graphs that have been illustrated in previous chapters, whenever smaller elastic modulus (25 MPa) is in concern, modulus ratio of 10.2 shall be considered. When there is greater elastic modulus (100 MPa), modulus ratio of 40.8 shall be considered.

Several cases have been investigated throughout this research providing us with solid outcomes that will be summarized as follows :

## 7.2 Stress Concentration Ratio

The outcomes of the analysis as discussed in the previous chapter, show that as length to diameter ratio increases for given stress levels, stress concentration ratio increases. When the magnitude of the foundation pressure is relatively low, aggregate piers having a greater elastic modulus tend to have a greater n value. In the case of a relatively higher foundation pressure imposed on the aggregate pier element, the sharing of the stress between the soil and the pier element is more like identical regardless of the pier stiffness, thus n value decreases.

## 7.3 Settlement

Accepted throughout the world as a viable solution for settlement improvement of shallow foundations, the outcomes of the load versus settlement analysis of aggregate piers conclude that, up to 3 m of length of aggregate pier elements, there is a great reduction in settlement values. Greater depths results in almost no settlement improvement. As a result, by using 60 and 80 cm diameter piers having lengths between 2 to 3 m, 50 % improvement in terms of settlement may be accomplished. Compared to other ground improvement methods, aggregate pier system has a lot of advantages in terms of ease of application, cost of application, performance, and duration.

## 7.4 Settlement Reduction Ratio

As elaborated in the previous chapter, this research has concluded solid outcomes to derive a trend for settlement reduction ratio with regard to length, diameter, and elastic modulus of aggregate piers, as well as the applied foundation pressures.

Here, the effect of short and long piers is in concern. In particular, when 60 cm aggregate pier elements, both for low and high elastic modulus, are loaded, as the magnitude of the load increases for short piers having lengths 1 m, and 1.5 m, settlement reduction value `b` stays constant. The shortest pier tends to have the highest `b` value of 0.79. Followed by the 1.5 m pier with `b` value of 0.71, `b` value decreases with increasing pressure as the greater length pier elements reach to 2 m, and 3 m. Range of `b` values is illustrated below. When diameter of the piers increases to 80 cm, for the low modulus value, the above trend applies, but when the elastic modulus value increases, `b` values show a constant trend for increasing loads.

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