TEACHING LOGARITHM BY GUIDED DISCOVERY LEARNING AND REAL LIFE APPLICATIONS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

YÜCEL ÇETİN

IN PARTIAL FULFILLMENT OF REQUIREMENTS FOR THE DEGREE
OF
MASTER OF SCIENCE
IN
THE DEPARTMENT OF SECONDARY SCIENCE AND MATHEMATICS EDUCATION

APRIL 2004

Approval of the Graduate School of Natural and Applied Sciences

____

Prof. Dr. Canan Özgen
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science

____

Prof. Dr. Ömer Geban
Head of Department

This is to certify that we have read this thesis and that in are opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

____

Assist. Prof Dr. Erdinç Çakıroğlu
Supervisor

Examining Committe Members

Prof. Dr. Ömer Geban ____________________

Assoc. Prof. Safure Bulut ____________________

Assist. Prof. Dr. Ceren Tekkaya ____________________

Assist. Prof. Dr. Kürşat Çağıltay ____________________

Assist. Prof. Dr. Erdinç Çakıroğlu ____________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name: Yücel Çetin

Signature :

ABSTRACT

TEACHING LOGARITHM BY GUIDED DISCOVERY LEARNING AND REAL LIFE APPLICATIONS

ÇETİN, Yücel

M.S., Department of Secondary Science and Mathematics Education

Supervisor: Assist. Prof. Dr. Erdinç Çakıroğlu

April, 2004, 123 pages

The purpose of the study was to investigate the effects of discovery and application based instruction (DABI) on students' mathematics achievement and also to explore opinions of students toward DABI. The research was conducted by 118 ninth grade students from Etimesgut Anatolian High School, in Ankara, during the spring semester of 2001-2002 academic year.

During the study, experimental groups received DABI and control groups received Traditionally Based Instruction (TBI). The treatment was completed in three weeks. Mathematics Achievement Test (MAT) and Logarithm Achievement Test (LAT) were administered as pre and posttest respectively. In addition, a questionnaire, Students' Views and Attitudes About DABI (SVA) and interviews were administered to determine students' views and attitudes toward DABI.

Analysis of Covariance (ANCOVA), independent sample t-test and descriptive statistics were used for testing the hypothesis of the study.

No significant difference was found between LAT mean scores of students taught with DABI and traditionally based instruction when MAT test scores were controlled. In addition, neither students' field of study nor gender was a significant factor for LAT scores.

Students' gender was not a significant factor for SVA scores. However, there was significant effect of math grades and field selections of students on SVA scores.

Key Words: Discovery and Application Based Instruction, Mathematics Achievement, Logarithm Achievement, Views and Attitudes of Students

ÖZ

KEŞFEDEREK VE UYGULAYARAK LOGARİTMA ÖĞRETİMİ

ÇETİN, Yücel

Yüksek Lisans, Fen ve Matematik Alanları Eğitimi

Tez Danışmanı: Y. Doç. Dr. Erdinç Çakıroğlu

Nisan 2004, 123 Sayfa

Bu çalışmanın amacı, keşfederek ve uygulayarak logaritma öğretiminin lise öğrencilerinin matematik başarısı üzerine etkisini ve öğrencilerin bu öğretim yöntemi hakkında tutum ve görüşlerini araştırmaktır.

Araştırma, Etimesgut Anadolu Lisesinden 118 dokuzuncu sınıf öğrencisiyle 2001-2002 öğretim yılı, ilkbahar döneminde yürütülmüştür. Çalışmada, deney gruplarına Keşfederek ve Uygulayarak Logaritma Öğretimi Etkinlikleri (KULE), deney gruplarına ise Geleneksel Matematik Öğretimi (GMÖ) yöntemleri uygulanmıştır. Çalışma 3 hafta sürmüş, Matematik Başarı Testi (MBT) ile Logaritma Başarı Testi (LBT) ön test ve son test olarak kullanılmıştır.

Araştırmanın hipotezlerini test edebilmek için Kovaryans Analizi, t-test ve betimsel istatistik yöntemleri kullanılmıştır. Çalışmanın sonuçları şöyledir:

KULE ve GMÖ gruplarının logaritma başarı puan ortalamaları arasında anlamlı bir farklılık bulunamamıştır. Bununla birlikte, cinsiyet ve alan seçiminin LBT puanları üzerinde anlamlı bir etkisi olmadığı görülmüştür. Cinsiyetin, KULE hakkında Öğrenci Görüş ve Tutumlarını ölçen (ÖGT) anketi puanları üzerinde anlamlı bir etkisi olmadığı saptanmıştır. Fakat, öğrencilerin matematik notlarının ve alan seçimlerinin ÖGT puanları üzerinde anlamlı bir etkisi olduğu belirlenmiştir.

Anahtar Kelimeler: Keşfederek ve Uygulayarak Öğretim, Matematik Başarısı, Logaritma Başarısı, Öğrenci Görüş ve Tutumları

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

$\overline{X}$: Mean of the Sample

BAL: Bayburt Anatolian High School

CG : Control Group

DABI: Discovery and Application Based Instruction

DF : Degree of Freedom

EAL: Etimesgut Anatolian High School

EG : Experimental Group

H_0 i : Hypothesis indexed with i, where i is a positive integer.

LAT: Logarithm Achievement Test

MAT: Mathematics Achievement Test-1

MS : Mean Square

n : Sample Size

NCTM: National Council of Teachers of Mathematics

p : Significance Level

SD: Standard Deviation

SPSS: Statistical Packages for Social Sciences

SS : Sum of Squares

SVA: Students Views and Attitudes About DABI

TBI: Traditionally Based Instruction

CHAPTER I

INTRODUCTION

1.1 Rationale

In a traditional and common math lesson, teacher writes the peculiarities on the blackboard, and then, goes on solving the problems related to it. The students prepare for the exam by memorizing these concepts and formulas, and by solving the related problems. But, meanwhile, some of the students can not comprehend the concept, some others are not interested in the subject as they think that it is no useful to them, and the others are like spectators while few students come to the blackboard and solve the problems. Most of the students do not participate lesson actively and can not comprehend the concept. They are forced to study the lessons for the sake of exams. Teacher only expects them to write, memorize and solve questions. In the end, math lesson becomes a boring, meaningless, abstract, hard and problematic.

Some of the basic problems that mathematics education face with are rote learning, abstraction of subjects, lack of teaching utility of algebraic concepts and lack of adapting technology.

Even though for more than three decades there has been need for mathematics reform for the way mathematics is being taught and learned, Martin (1996) argues that in most of the traditional mathematics classrooms, teaching mathematics has still been made using a method that he calls "the ABC method," that is austere, boring and colorless. Erol (1989) asserted that many students perceive mathematics as a strange field including incomprehensible rules that must be learned and mass of numbers, origin of which are not known. Furthermore, students are generally afraid of mathematics and they dislike it. Whereas, the aim of a lesson should not only be learning the subject, but also be the enjoyment and appreciation of the knowledge. In short, the students should learn how to learn and be happy with that.

Olivier (1999) asserted that although instruction clearly affects what children learn, it does not determine it, because the child is an active participant in the construction of his own knowledge. He further states that new ideas interpreted and understood in the light of child's current knowledge, built up out of his previous experiences. In the same line, Freudenthal (cited in Erbaş,1999) stated that we should not teach students something that they could discover by themselves. Similarly, Thoumasis (1993) promoted that teaching should be concerned with helping students make connections between ideas and discover their logical interrelations. Teaching process should include exploring answers to the questions such as, "Where did it come from?"

To get rid of rote learning and involve students in lessons, students could be guided to discover rather than being told. Confrey (1991) states that mathematics educators should stress in importance of ;

1. involving the student actively in the learning process,
2. emphasizing the process of "coming to know" over rapid production of correct answers,

3. extracting and making increasingly visible the structure of a concept.

Another problem of students in mathematics lesson is abstraction of concepts. Erbaş (1999) stated that the mathematics curriculum, from elementary school to college, increases in abstraction, symbolism and obscurity. As students progress from year to year in mathematics, the letters they use, the concepts they learn, become increasingly abstract and they become ambiguous to them. This high level of abstraction and the extensive use of symbolism keep many students away from mathematics and it is a difficult and challenging task for many of the students who attempt to deal with it. Therefore, there is need for instructional approaches to help children better understand abstract concepts in mathematics.

Another gap in mathematics education is neglecting making connections among mathematics concepts. In mathematics instruction, usually no sufficient emphasis is given to the applications of mathematics in other disciplines and in daily life. Therefore, students can not be motivated toward mathematics and they frequently ask questions like "where do we get this formula?", "why do we learn this subject?" If these questions are not answered then students may become strangers to mathematics.

Erol (1989) contended that the students often want to know how useful what they have learned. In other words, they want to be sure that the information they have learned are used in daily life. As the type of medicine changes depending on the illness of patient and as the clothes change depending on seasons, likewise; in this age, the technology has improved so much that the affairs which had taken too much time to be done in the past are now easily accomplished in a relatively very short time through improvements in transportation and telecommunication. Like that, in mathematics, all mathematical operations can be made by using computers and calculators. So, manipulating operations in mathematics has lost its importance any longer. Due to these technologies available in 21st century, mathematics educators should save more time to apply, explore, discover and interpret results. However, in math lessons in Turkey, we still only dwell on the ability of operation and calculations, but not on concept, research, practice, and commenting. Whereas, new opinions and thoughts show that we must save much more time for the comprehension of concept, practice and commenting than the time for operations and calculations. For example; Solow (1994) asserted that we should now have more time to teach students how to explore, think, reason, investigate, and conjecture mathematically, because technology will free us from the burden of teaching mindless, boring, and time consuming paper and pencil skills that are not needed by society. Sutherland and Healy (cited in Işıksal, 2002) stated that students become very motivated and engaged in a problem while using technology.

One focus in mathematics education is the use of graphical calculator as a tool in the mathematics instruction. Alexander (1993) emphasized that the results of most studies suggested that the use of graphing calculator in teaching and learning is beneficial in terms of students' level of understanding and achievement in algebra and precalculus. In addition, Hembree and Desart (1986) contended that students using calculators possess a better attitude toward mathematics and an especially better self concept in mathematics than non-calculator students. This statement applies all grades and ability levels.

In 1989, National Council of Teaching Mathematics (NCTM) in USA–which is one of the largest professional organization of mathematics educators in the world–published the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). These standards included important critiques of the traditional mathematics instruction and provided suggestions for the school mathematics. The standards tried to answer; i) why traditional mathematics lesson should be changed, ii) how this change should occur, iii) how to assess the progress of the students. According to the standards, the rationale for a change in school mathematics are (NCTM, 1989):

- There is no longer a need for the usual operational arithmetic as we moved to an age of information and technology. Therefore, math educators should make more use of technology in classrooms.
- Math educators should incorporate the knowledge of how students learn into their teaching strategies.
- Math teachers should create a learning environment by selecting task that allow students to construct new meaning by building on and extending prior knowledge.
- Math educators should encourage students for full participation and emphasize connections between mathematics and daily life.
- Math educators should promote students' confidence, flexibility, curiosity and inventiveness in doing mathematics.

The 1989 NCTM standards encouraged the mathematics educators to become familiar with what and how students learn and use the new approaches as a result

(NCTM, 1989). Erbaş (1999) observed that in order to meet the needs of information society, mathematics curriculum and mathematics educators need to be improved in various dimensions.

Haladayna (1997) argues that student learning should occur in a meaningful context where students perform for their own good and see the intrinsic merit in what they do. The alternative is performing tasks that seem mindless, meaningless or irrelevant. Where can we find good material for this meaningful, contextualized relevant learning? This question was a starting point of this study. Finding good learning tasks and relevant materials for teaching mathematics is a challenge for many mathematics educators. One of the aims of this study was to produce such learning tasks and materials. Therefore, we started by preparing some activities to enhance mathematics lesson. Many researchers argue that logarithm is a problematic and hard concept for high school students. Many students even hate the concept of logarithm. They think that it is meaningless and including unintelligible rules. Thoumasis (1993) asserted that for students who approaches logarithm concept with a routine definition for the first time, it is impossible to understand its relation to the real world. The term "logarithm" seems completely arbitrary, unconnected as it were to the mathematical process. Therefore, we selected the topic logarithm to study and to teach this topic by discovery and applications. Finally, a number of activities prepared for the concept of logarithm based on the principles of (i) helping students to make connection among concepts and to the real life, (ii) utilizing information technology, (iii) helping students to figure out concepts by themselves.

1.2 Activities

In this study a series of activities were developed for the logarithm unit. In the first activity, the aim was helping students to make connections and discover logical interrelations between their prior knowledge and properties of logarithm by means of three discovery sheets. In addition, active participation of students were planned and the tasks were planned to be enjoyable to students.

In the second activity, students were motivated by solving application examples of logarithm. Moreover, daily life problems were solved to prevent students from mindless, meaningless and irrelevant learning. In this activity, we also planned to apply data analysis activity in which we used graphical calculator TI-83 to enhance the lesson .In this activity, we wanted logarithm properties to be applied by students on two examples from daily life.

In the third activity, we planned to help students to discover properties of logarithm and overcome possible misconceptions in logarithm by using graphing calculators. In this activity, students saw properties about logarithm and possible conflicts in their conception by making graphical analysis.

1.3 Purpose of the Study

The main goal of the study was to investigate the effect of discovery and application based logarithm instruction (DABI) on students' logarithm achievement and determine opinions of students about DABI.

Cankoy (1998) contended that teachers need to understand mathematical difficulties before students can be guided successfully through the necessary accommodations. Medical doctors, before applying a treatment, try to diagnose illnesses of patient. In the same sense, before applying a new treatment in logarithm, in the first stage, prior to developing instructional activities, we aimed to determine possible misconceptions of high school students in applying properties of logarithm. In the second stage, meaningful and effective instructional activities were designed to enhance mathematics lesson. In the third stage, after applying activities, it would be possible to observe the effects of them on students in terms of achievement and attitudes. Consequently ,the main purpose of this study was to investigate effects of these activities on students' opinions toward mathematics lesson, effect of them on students' logarithm achievement.

Problem statement, main and subproblems, and hypotheses are given in the third chapter.

1.4 Significance of the Study

Discovery learning takes place most notably in problem solving situations where the learner draws on his own experience and prior knowledge to discover the truths that are to be learned. Bruner (1971) identified three stages of cognitive growth and discovery learning allows students to move through these three stages as they encounter new information. First, the students manipulate and act on materials; then they form images as they note specific features and make observations ; and finally, they abstract general ideas and principles from these experiences and observations. When students are motivated and participate in discovery project, discovery learning leads to superior learning. On the other hand, there are some criticisms about discovery learning which claim that discovery learning is impractical (Tomei,2003). In theory, discovery learning seems ideal, but in practice there are problems. To be successful, discovery projects often require special materials and extensive preparations. To respond these requirements, several activity sheets were prepared and administered in the study. As mentioned above, preparing discovery learning materials is difficult process. In this sense, this study may be beneficial in contributing to prepare discovery materials and it may be a model for other mathematics topics. Moreover, students perceive the math lessons as a strange field including incomprehensible rules the origins of which is not known. To get rid of rote learning and involve students in the lessons, students can be guided to discover rather than be told. In this respect, developing such instructional units may be useful for students.

The current and classic form of the classroom is not designed to be interactive. Students are on the most part keep quiet, not allowed to collaborate, sit in assigned seats arranged in rows facing one way towards teacher. In most schools, during lessons, teacher’s voice is heard. It is asking a question and then answering it , lecturing, yelling or just rambling. In the same way, In mathematics lesson, teaching is often interpreted as an activity mainly carried out by the teachers. He or she introduces the subject gives one or two examples, may ask a question and invites the students who have been passive listeners to become active by starting to complete exercises from the book. They were intended to show the relevance of mathematics in the concrete real world . The most obvious are immediate uses in every day living.

As stated earlier, in mathematics instruction usually no sufficient emphasis is given to the applications of mathematics in other disciplines and in daily life. According to many researchers using applications in mathematics instruction may help us to better motivate our students to learning mathematics. Moreover, if the real word problems are used as the starting point for conceptual development, then, students will learn mathematics better (De Lange,1996). As known, people do not appreciate something which is not beneficial for themselves. Therefore, application based instruction may help students to see the benefits of what they have learned and in turn motivate students toward mathematics.

Furthermore, technological development makes more real applications reachable for classroom experiences. In recent years, using technology in mathematics education had become important. Hanafin (cited in Işıksal,2002) stated that many educators advocated that using technology to create more learner centered, open ended learning environments. Also, NCTM (2002) asserted that technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning. However, Allison (1995) stated bringing technology into our classrooms is not enough, we must find ways to effectively use technology as a tool in the learning environments. In this sense, portable and practical tools, calculators suggested by educators to use technology effectively in mathematics education, and it is frequently suggested that use of a calculator frees to focus on strategic issues when tackling problems. Dunham & Dick (1994) cited evidence that upper secondary students with experience of graphing technology have more flexible approaches to problem solving and show more engagement and persistence in it.

Also, as stated earlier, many educators contended that there is a need for a radical change in traditional mathematics instruction, but there are few research studies to produce and apply various instructional strategies. In addition, in Turkey, there are a few attempts to use graphical calculators in mathematics instruction. Therefore, this study may be useful and beneficial to show effect of a new instructional strategy and utility of graphing calculators in mathematics lesson.

On the other hand, from a constructivist perspective misconceptions are crucially important to learning and teaching, because misconceptions form part of pupil's conceptual structures that will interact with new concepts, and influence new learning, mostly in a negative way, because misconceptions generate errors. İşeri (1997) promoted that teaching experiments should be conducted to find suitable ways of recovering students from their misconceptions. In addition, Lochead & Mestre (1988) described an effective inductive technique inducing conflict by drawing out the contradictions in students' misconceptions.

There are several studies which aimed to diagnose high school students' misconceptions in logarithm but very few have some attempts in overcoming those misconceptions. This study proposed a suitable way of recovering students from their misconceptions. In this respect, it may be beneficial in contributing to the related field.

1.5 Definition of Terms

Control Group (CG): It refers to the group which continued to study logarithm with the traditional approach.

Experimental Group (EG): It refers to the group which continued to study logarithm using DABI.

Discovery and Application Based Instruction (DABI): It refers to the instruction, in which students learn logarithm with activities designed by the researcher. Students carry out their work with activity sheets and graphing calculator TI-83.

Traditionally Based Instruction (TBI): It refers to the instruction in the classroom without any equipment. Students carry out their work with paper and pencil.

Misconception: Difference between errors and misconceptions should be distinguished. Errors are wrong answers due to planning; they are systematic in that they are applied regularly in the same circumstances. Errors are the symptoms of the underlying conceptual structures that are the cause of the underlying conceptual errors that is called misconceptions. Whenever the conception held by someone contradicts its counterpart, we will refer to it, is a misconception. A misconception is an underlying belief which governs mistake or error (Cankoy, 1998).

3 CHAPTER 2

REVIEW OF LITERATURE

The primary goal of this study is to determine the effect of a discovery and application based logarithm instruction (DABI) on students' logarithm achievement and the perceptions of students toward logarithm activities.

This chapter is devoted to the presentation of theoretical background of this study. The concepts that will be covered in this chapter are; discovery learning, mathematics teaching based on applications, graphic calculators, teaching logarithm and significance of the study.

2.1 Discovery Learning

Recent calls to education reform in mathematics in developed countries, discuss both changes in the content and pedagogy of mathematics teaching. Traditional high school curricula, in many countries gives less emphasis to mechanical symbol manipulation abilities, in part because this kind of mathematics can be done by computer, and because of an increasing concern for more flexible problem-solving skills. New curriculum proposals also reject traditional teacher-centered pedagogy and favor student-centered approaches. In particular, there is a growing consensus that students should learn through inquiry and through the construction of their own mathematics.

The term inquiry learning is used interchangeably with discovery learning by some educators. One distinction often being made between the two is as follows: In discovery learning, the students are provided with data. By questioning of the teacher, they are expected to ascertain the particular principle hidden in the lesson objective. In inquiry Learning, the goal is to make students to develop their own strategies to manipulate and process information.

Discovery learning encompasses the scientific model which matches cognitive development. Bruner defined discovery as "all forms of obtaining knowledge for oneself by the use of one's own mind" (Bruner,1961, p. 22). In essence, this is a matter of "rearranging or transforming evidence in such a way that one is enabled to go beyond the evidence so assembled to additional new insights" (Bruner,1961). Bruner believed that the process of discovery contributes significantly to intellectual development and that the heuristics of discovery can only be learned through the exercise of problem solving. That being so, he proposed discovery learning as a pedagogic strategy with such important human implications that it must be tested in schools.

A true act of discovery, Bruner contended, is not a random event. It involves an expectation of finding regularities and relationships in the environment. With this expectation, learners devise strategies for searching and finding out what the regularities and relationships are.. According to Bruner, if students' information gathering lacks connectivity and organization and, their ability to solve problems would be deficient. By contrast, using a connectionist approach where information is collected in a systematic and organized way would help solving the problem.

In discovery learning, the teacher must carefully plan the questions which should be asked in order to help students to attain the principle or abstraction being taught, order the examples in the lesson, and be certain that the reference materials and equipment are readily available. However these preparations do not always guarantee success. In order to benefit from a discovery situation, students must have basic knowledge about the problem and must know how to apply problem-solving strategies. Without this knowledge and skill, they will flounder and grow frustrated. Instead of learning from the materials, they may simply play with them. Then, valuable classroom time will be wasted.

Discovery learning encourages students to actively use their intuition, imagination, and creativity because the approach starts with the specific and moves to the general. The teacher presents examples and the students work with the examples until they discover the interrelationships. Bruner (1961) believes that classroom learning should take place through inductive reasoning, that is, by using specific examples to formulate a general principle. For instance, if students are presented with enough examples of triangles and non-triangles, they will eventually discover what the basic properties of triangles must be.

An inductive approach requires intuitive thinking on the part of students. Bruner suggests that teachers can nurture this intuitive thinking by encouraging students to make guesses based on incomplete evidence and then to confirm or disprove the guesses systematically. The students could check their guesses through systematic research.

Tomei (2003) stated that Bruner's ideas for discovery learning can be implemented in the classroom as follows:

- Present both examples and nonexamples of the concepts you are teaching.
- Help students see connections among concepts.
- Use diagrams, outlines, and summaries to point out conclusions.
- Pose a question and let students try to find the answer.
- Encourage students to make intuitive guesses by administering following suggestions:

a. Instead of giving a word's definition, say, "Let's guess what it might mean by looking at the words around it."

b. Don't comment after the first few guesses. Wait for several ideas before giving the answer.

c. Use guiding questions to focus students when their discovery has led them too far astray.

In particular, he emphasized that discovery is not haphazard; it proceeds systematically toward a model which is there all the time. "The constant provision of a model, the constant response to the individual's response after response, back and forth between two people, constitute Invention' learning guided by an accessible model" (Bruner, 1973b, p. 70).

The provision of models is important for discovery in another aspect. By asking certain kinds of questions or by prompting certain hypotheses during problem solving, the teacher also models the conduct of inquiry. It is necessary, according to Bruner, to teach children how to cut their losses, to pose good testable guesses, to persist in seeking appropriate evidence, and to be concise. Guided practice in inquiry and sufficient prior knowledge, then, constitute minimum conditions for discovery learning to be successful. Bruner (1973) also adds reflection and contrast. The need for reflection occurs when children can accomplish some task but are not able to represent to themselves what they did. In other words, they may successfully solve a problem but have little clue as to why they were successful. Reflecting back on the problem and recasting what occurred in a mode of thought understood by learners may help them to figure it out, to make the knowledge their own. Contrasts which lead to cognitive conflicts can set the stage for discovery. That is, "readiness to explore contrasts provides a choice among the alternatives that might be relevant" in a discovery learning situation (Bruner, 1973).

Bruner's recommendation for contrasts that cause cognitive conflict parallels that made by Piaget and the information processing theorists who have focused on restructuring as the major developmental process. Although they have all offered different explanations for why the strategy works, the important point is that it does and can be reliably used in instruction.

Schulman (1965) stated some goals of discovery learning as following:

- To give students experience in discovering patterns in abstract situations.
- Students to know that mathematics really and truly is discoverable.
- Each student, as a part of the task of knowing himself, to get a realistic assessment of his own personal ability in discovering mathematics.
- Students to have a feeling that mathematics is fun or exiting. or
- Students to posses considerable facility in relating the various parts of

mathematics one to another – for example, using algebra as a tool in geometry, or recognizing the structure of the algebra.

- Students to posses an easy skill in relating mathematics to the applications of mathematics in physics and elsewhere.

Finally, Bruner (1973) spoke to the instructional issues of reinforcement and motivation. Although feedback which can be used for correction is obviously important, Bruner contended that it must be provided in a mode that is both meaningful and within the information processing capacity of the learner. Extrinsic reinforcement, on the other hand can develop in which children look for cues to the right answer or right way of doing things. Exposing children to discovery learning can therefore promote a sense of self-reward in which students become motivated to learn because of the intrinsic pleasure of discovery.

2.2 Using and Applying Mathematics in Education

In the mid eighties, mathematics educators propagating the teaching of mathematics by applications represented a small and unique group. The purpose of applied mathematics is to elucidate scientific concepts and describe scientific phenomena through the use of mathematics, and to stimulate development of new mathematics through such studies. At present, this ”movement” towards more applications has gained quite some momentum (Keitel,1993). The aim of this chapter is to establish a reasonably accurate picture of the applied mathematics and actual state of applying mathematics in schools.

In this part, necessity of applied mathematics, importance of the applied mathematics in education, applied mathematics models, obstacles for implementation of applied mathematics curriculum and a sample study of applied mathematics on secondary school students will be mentioned.

In the first half of this century pure mathematics stood higher than applied mathematics. Dieudonne (1970) stated that mental universe stood higher than the physical universe. These ideas lead to more pure mathematics which in turn leads to critical remarks from people outside the mathematical scene. The idea arose that it was not necessary to undertake problems of the real world. Abstract mathematics would prove useful. On the other hand, the divergence from reality in the study of mathematics provoked much discussions about the nature of mathematics.

Many physicists and mathematicians warned against the danger of mathematics becoming more and more isolated (Klein, 1895; Poincare, 1905). However, in their isolated manner, pure mathematicians did not feel the necessity to the society and avoid answering the questions about the meaning of their work. Bishop (1994) expressed the circumstance as following: Most mathematicians feel that mathematics has meaning but it bores them to try to find out what it is. However, in recent decades, there seems to be a change in attitudes. Not only there is a trend towards unification in mathematics but there is also the feeling that true applied mathematics may be an art as well (Hilton,1976).

There are some factors that have contributed more positive attitudes toward applied mathematics. One of the factors is the attitude of society towards mathematics. De Lange (1996) stated that society tolerates mathematics because of practical importance of applying mathematics. Another factor is that using more applied mathematics contributed the development of pure mathematics. According to Freudenthal (1973) occurrence of many inventions between 1200 and 1500 led to growing of pure mathematics. Searching the secrets of nature caused the sudden growth of pure mathematics with applying mathematics more.. As known, necessity leads to progress. Furthermore, the rise of information technology has made more real applications accessible to mathematics, which opened new applications, like cryptography. Therefore new applications are important factors which caused positive attitude toward applied mathematics. Besides, in recent decades, it has become a useful tool in more disciplines.

Consequently, social need, relevance and technological requirements drive the development and transmission of mathematical knowledge, thus indicating that applied mathematics is the pre-eminent for mathematical in society.

During the last 80 years, there has continuously been discussion about the desirability of including applications in mathematics education (De Lange,1996). And certainly during recent decades there is an obvious trend in literature towards more applications. To explain well the desirability of including applications we should address the question that why should applications be a part of or integrated in a mathematics curriculum? One can answer this question in many different ways. In the first place, as Freudenthal noted that;

"Mathematics is distinguished from other teaching subjects by the fact that it is comparatively small body of knowledge, of such a generality that is applied to a richer variety of situations than any other teaching subject." Besides this, according the authors like Engel and Polak (cited in Driscoll,1993) usefulness is one of the main reasons for society to support mathematics. In this sense, Niss (1991) also argues the ultimate reason for giving substantial mathematics education to the general public is that mathematics is being used extensively and ever increasingly in society in such a way that people's professions and lives are strongly influenced by it.

In addition, applying mathematics might help to prepare competent citizens for 21th century society. As Keitel (1993) observes that artificial problems are replaced by real problems in real situations, where the process of problem posing refers to students' interest and environmental care.

In the Netherlands, the goals for the majority of children very much resemble the set of goals stated by the British Committee of Inquiry into the Teaching of Mathematics in Schools in (Cockcfoft,1982). They are as follows:

1) To become an intelligent and competent citizen for democratic life.

2) To prepare for the workplace and future education.

3) To understand mathematics as a discipline.

Moreover, at ICME (1980) four applications were discussed in several groups. During the conference, most people become convinced that including many applications and stressing the usefulness of mathematics is particularly desirable to motivate students towards mathematics. This point also emphasized by Howson (1973) and Van der Blij (1968).

We must not forget that each child or adult has already an implicit definition of her or his own real world, which may not be known to the outside world, including teachers and curriculum designers. In this regard, Thompson (1992) notes from the constructivist perspective that if students do not become engaged imaginistically in the ways that relate mathematical reasoning to principled experience, then they will come to see their worlds in anyway mathematical.

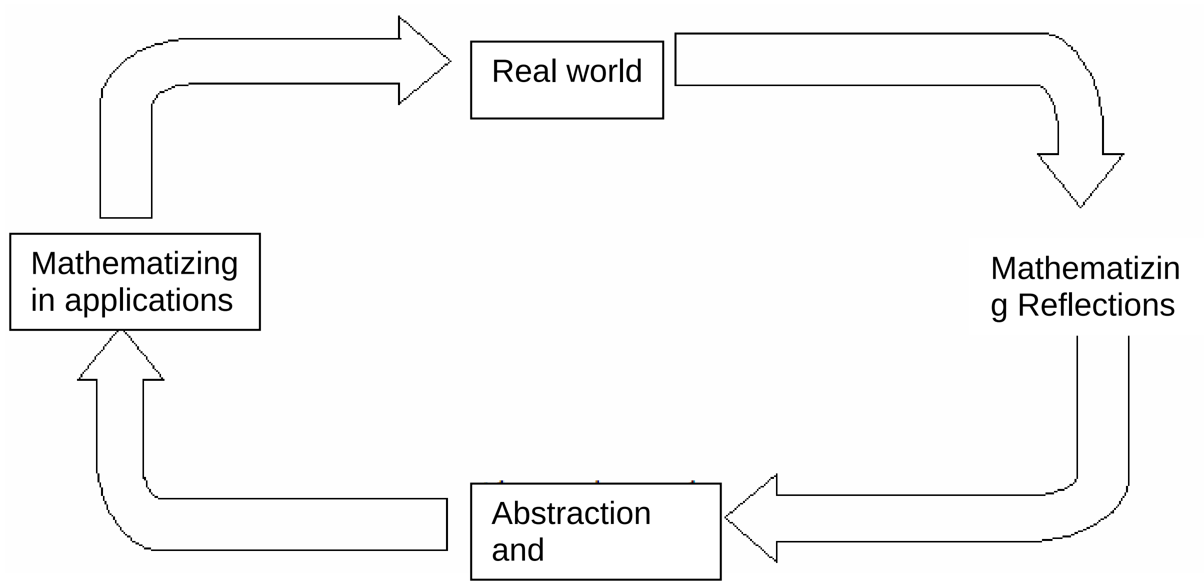
In the same way Cobb (1994) described the starting points of instructional sequences should be experimentally real to students so that they can immediately engage in personally meaningful mathematical activity (Gravemeijer (1990); Streefland (1991).

Ball (1993) inquires "how valuable are the students' interests to connect them to mathematical ideas?" As an answer for this question, De Lange(1996), reported that: Many countries have found that motivating students is not a very serious problem when applications are used as one of the possible motivations.

In addition to motivation, another benefit of including applications in mathematics education is to help better understanding of mathematical concepts. The process of mathematization will force the students to explore the situation find and identify relevant mathematics, schematize and to discover regularities and develop resulting in mathematical concept.

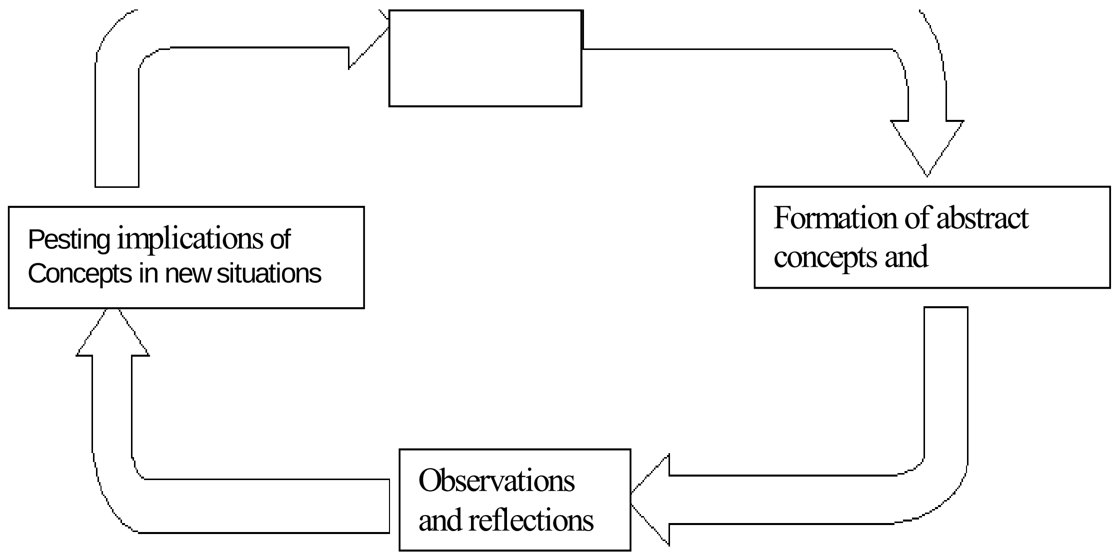
Vergnaud (1982) points out concepts develop gradually through applying. In the same way De Lange (1996) expressed that mathematics will be more successful as a school discipline when applied mathematics is used in education. In summary, benefits of applying mathematics are practical importance, motivation and helping students understand the concept.

An important argument about application of mathematics is that; instead of starting the learning of mathematics by introducing abstract concepts -as in the curricular proposal related to the introduction the new mathematics–new contexts are emphasized as starting points (Keitel,1993). Hooke and Shaffer (cited in De Lange, 1996) stated that in mathematics courses the talk is usually about problems. Not much is said about where the problems come from or what is done with answers. The process of developing mathematical concepts and ideas starting from the real world can be called 'Conceptual Mathematization' (De Lange,1987). A schematic model for the learning process is given in Figure 2.I and 2.2.



F i g u r e 2.1 Conceptual Mathematization Model of De Lange (1989)

This shows a remarkable similarity to the Experimental learning model of Lewin (1951) (Figure 2.2):



F i g u r e 2.2 Experimental Learning Model of Lewin (1973)

Two aspects of this learning model are particularly noteworthy: Firstly its emphasis on concrete experience to validate and test abstract concepts. Secondly, the feedback principle in the process. Lewin used the concept of the feedback to describe social learning and problem solving process that generates valid information to assess deviations from desired goals. In a more recent study Kolb (1984) adapts this Lewinian model and compares it with Dewey's Model of experimental learning and Piaget's model of learning and cognitive development. In his opinion all the models suggest the idea that learning is by its very nature a tension – and conflict filled process. New knowledge, skills, or attitudes are achieved through confrontation among four modes experimental learning. Learners need four different kinds of abilities- 'concrete experience', 'reflective observation', 'abstract conceptualization' and 'active experimentation'. This means they must be able to involve themselves in new experiences and to reflect on and observe their experiences from many perspectives. Kolb's definition of learning fits with this perspective of mathematization for concept development that learning is the process whereby knowledge is created through the transformation of experience.

According to van Hiele (1973), the process of learning proceeds through three levels:

1. A pupil reaches the first level of thinking as soon as he can manipulate the known characteristics of a pattern that is familiar to him.

2. As soon as he learns to manipulate the interrelatedness of the characteristics he will reach the second level.

3. He will reach the third level of thinking when he starts manipulating the intrinsic characteristics of relations.

Trekkers (cited in De Lange,1989) points out that van Hiele did not sufficiently answer question of “how should we concretize the phenomenological exploration?” Freduenthal’s didactical phenomenology (1983) helps to answer the questions:

What a didactical phenomenology can do is prepare the following approach: starting from those phenomena that beg to be organized and from that starting point teaching the learner to manipulate these means of organizing.

The socio-constructivist approach is motivated by desire to understand student’s mathematical learning as it occurs in the social situation or the classroom. This approach relies on real world application and modeling.

Prager (1972) expressed that applied mathematics is a bridge that connected pure mathematics with science and technology.

Applications and modeling should be part of the mathematics curriculum in order to generate, develop and quality a critical potential in students towards the use and misuse of mathematics in extra-mathematical contexts.

2.2.1 Using, Applying, Modelling, Mathematization

De Lange (1996) stated that different people tend to use the same language for different concepts. One doesn't need a detailed analysis of the literature to find out that there are many local interpretations of applying, using and modeling in mathematics education.

Polak (1976) suggests two categories about applied mathematics:

1. Applied mathematics means beginning with a situation in some other field or in real life, making a mathematical interpretation, or model, doing some mathematics on the model and applying the result to the original solution. The 'other field' is by no means restricted to physical science and includes areas of application in biology, social studies, management, business studies, and so on.

2. Applied Mathematics means people to what they actually do in their livelihood.

In a real world problem, we are studying with a collection of objects, relations between them, and structures belonging to the area that we are studying. Next, we translate these into mathematical objects, relations and structures, which represent the original ones. This process is often called mathematization. Often the term mathematization is used as a synonym for 'modelling'. But a more precise, meaning is to define it as the translation part of the modeling process (De Lange,1996).

Freduenthal (1973) argue that mathematizing is an essential part of the curriculum, and certainly at the lowest Van Hiele level, where it applies to nonmathematical matter, to guarantee the applicability of mathematics.

We can identify the mathematization aimed at transferring the problem to a mathematically stated problem. Via schematizing and visualizing we try to discover regularities and relations, for which it is necessary to identify the specific mathematics in a general context.

Mathematization always goes together with reflection De Lange,1996). This reflection must take place in all phases of mathematization. The students must reflect on their personal processes of mathematization, discuss their activities with other students, must evaluate the products of their mathematization, and interpret the result.

An example study about applying mathematics in education carried out by Joe Boaler (1997) on Amber Hill and Phoenix Park students. While students in Amber Hill were instructed mathematics with traditionally based instruction, Phoenix Park students were instructed with applications based instruction. In order to determine differences existed in the extent, nature or form of students' understanding a variety of assessments were used. These included applied assessments, long term learning tests, short term contextualized questions and the GCSE examinations. These assessments were administered on ninth year students until their eleventh. These assessments revealed that Phoenix Park students had developed a mathematical understanding and they were more able to make use of what they learned than the Amber Hill students. It means that the students at the two schools had developed different kind of mathematics knowledge. The Phoenix Park students did not have a greater knowledge of mathematical facts, rules and procedures, but they were more able to make use of the knowledge they did have in different situations. However, Amber Hill students had developed a broad knowledge of mathematical facts, rules and procedures that they demonstrated in their textbook questions, but they found it difficult remembering these methods to base decisions on when or how to use and adapt them.

2.2.2 Problems and Obstacles :

When implementing a problem-or application-oriented curriculum one can expect a lot of problems (De Lange,1996). So, we need to engage into the areas where we still see problems and obstacles that are in our way.

According to De Lange (1996), at classroom level one finds the same problems over and over, again. A well known problem is that mathematics teachers from school to university are afraid of not having enough time to deal with problem solving and applications in addition to implementation of traditional mathematics curriculum. Phrased this way mathematics and applications are seen as different subjects. Usiskin (1993) pointed this view as causing an obstacle.

Another obstacle mentioned by Blum and Niss (cited in De Lange, 1996) is that the mathematics lesson become more demanding for both teachers and students. For many students this obstacle quickly becomes an advantage as they get very motivated by the challenging problems. But this shows that the quality of the problems is a key issue here, and the way the mathematics is embedded as well.

De Lange (1996) reported that apart from these obstacles the following 'problems' were encountered when implementing problem oriented curricula:

· the 'loss' of teaching

· the 'loss' of basic skills and routines

· the 'loss' of structure

· the 'loss' of clarity of goals

· the complexity of 'authentic' assessment

· the use of technology

Each of these points will be discussed.

a. Loss of Teaching : Different strategies often involve more than one level of mathematical thinking, forcing the teacher into a discussion about the values of the strategies Teachers may often find it difficult, if not impossible, to design their own tests, suitable with the new strategy.

b. Loss of Basic Skills and Routines: For many teachers, the role of basic skills hasn't been part of their daily practice, basic skills are a matter of fact, and form the kernel of mathematics education. (De Lange,1996)

According to De Lange (1996) the attentive teacher notices that in the occasional situation where the students need a basic skill, they often lack it. Of course at one moment in their career they had mastered this skill, but with little need to use it, this outcome was predictable. It seems necessary to analyze the implemented curricula, the ‘real’ world problems that we think are relevant, and the skills that are necessary to solve them. In the United States the Curriculum and Evaluation Standards (NCTM,1989) make clear that decreased attention should be given to rote practice, rote memorization of rules, written practice, long division, memorizing rules and algorithms, and manipulating formulas. This is not cure for the problems and it will take a lot of practice, experiments, developmental research and vision how to integrate technology into the teaching and learning process.

c. Loss of Structure : Quite often, if the curriculum starts in the real world, we’ll notice a ‘loss’ of structure, compared with traditional curricula.

d. Loss of Clarity of Goals : In the programs where applications are emphasized the intended goals are not always immediately clear for both the teacher and the student. Real problems, in the more complex sense, obscure mathematical goals also. We may even not know the goals precisely because the problem is so real and therefore so open ended that the goals can only be reconstructed afterwards (De Lange,1995).

e. Complexity of Assessment : Popper (1968) and Philips (1987) have argued that a theory can only be tested in terms of its own tenets. Constructivist or realist mathematics education teaching and learning can only be evaluated by assessment procedures derived from the same principle.The goals of assessment, or better, the goals and principles of assessment have to be changed too, which adds to the problem of matching assessment to the teaching and learning process.

f) The Use of Technology : Technology will help us advance towards better mathematics education. And the advance of technology itself has made more real applications reachable for classroom experiences. However, according to De Lange (1996) the teachers see two main obstacles in using technology. First, they may not seen any substantial gain in conceptual understanding. The use of the computer just as a computing or graphing tool does not fully justify the problems encountered.

The fact that the use of technology creates additional obstacles makes technology itself an obstacle in getting more applications and modeling into the curriculum.

2.3 Graphics Calculator

Hand-held calculators appeared in the 1960s, soon became popular, and were brought into the mainstream of economic and technical life (Zand and Crowe,1997). This situation, however, did not last long: the rise of the personal computer, together with the availability of software for a variety of educational purposes, overshadowed the calculator. But, in a world of rapid technological change, ever more sophisticated calculators were designed, increasingly resembling portable computers, with tool systems which provided for arithmetic, algebra, geometry and statistics. These are now available at affordable prices in many parts of the world and have changed many traditional mathematical activities carried out by students in schools and colleges across the globe. More recently, a new generation of these machines has come on to the market, usually referred to as "graphic calculators", which can display mathematical expressions, data and a variety of graphs; moreover most are programmable.

Graphic calculator is best described as an extremely portable, hand-held, mini computer. ‘Graphic calculators are a quite powerful new technology for mathematics teaching (Demana & Waits, 1990). Moreover, by means of graphic calculators technology becomes part of the normal classroom activity and not just a special activity.

2.3.1 Learning Outcomes

Some of the commonly agreed learning outcomes of graphic calculators as following:

- Graphic calculators help the students represent their mathematics in different forms i.e. numerical, algebraic and graphical using .

- Students developing an intelligent partnership between themselves and the calculator. It is used to extend their understanding, rather than to do their work for them

- The calculators improving classroom efficiencies in some areas of mathematics. Many tedious calculations are being removed allowing more time for interpretation and evaluation.

- Students continuing to move away from the traditional “school mathematics” that is based on pencil and paper technology towards doing real mathematics.

- Students gaining considerable proficiency in using a graphics calculator. They are making excellent use of this tool in projects, tests, problem solving assignments and examinations.
- Graphic calculators impelling students to be active learners.
- Graphic calculators helping students to develop confidence about their ability to think about and do mathematics.
- Students being encouraged to read, write, and discuss mathematical ideas.
- Enhance students' understanding of the fundamental concepts underlying the calculus.
- Helping students to use calculus in other disciplines. Inspire students to continue their study of mathematics.
- Provide an environment where students enjoy learning and doing mathematics.

2.3.2 Discussions About Using Calculators in Mathematics Education

Using calculators and graphic calculators in mathematics education has been still controversial. Therefore, importance of using calculators should be mentioned.

In form of computers, technology enters more and more areas of life. Kutzler (1999) stated that the two areas are mathematics (intellectual) and moving (physical) can be compared to explain the importance and significance of technology therein. The most elementary method of moving is walking. Walking is a physical achievement obtained with mere muscle power. The corresponding activity in mathematics is mental calculation. Mental calculation requires nothing but “brain power”.

Riding a bicycle is a method of moving, where we employ a mechanical device for making more effective use of our muscle power. Compared to walking we can move greater distances or faster. The corresponding activity mathematics is paper and pencil calculation. We use paper and pencil as external memory” which allows us to use our brain power more efficiently.

Another method of moving is driving a car. The car is a device that produces movement. The driver needs (almost) no muscle power for driving, but needs new skills: He must be able to start the engine, to accelerate, to steer, to brake, to stick to the traffic regulations, etc. The corresponding activity in mathematics is calculator/computer calculation. The calculator or computer produces the result, while its user needs to know how to operate it.

Kutzler (1999) asks What method of moving is sensible in which situation? If we ask a colleague to get a newspaper from a 250 meter distant newsstand, he probably will walk. In case the newsstand is 1,000 meters away, a bicycle may be the most reasonable means of transportation. In case the distance to the shop is 10,000 meters, one will use a car. Kutzler argues that in mathematics, the sensible use of technology is accordingly: The multiplication of two one-digit numbers is best done mentally. Two two-digit numbers can well be multiplied using paper and pencil, while for the product of two five-digit numbers one will use a calculator. Using a calculator to obtain the product of 7 and 9 is a clear case of improper use of technology. We should not banish calculators and computers just because some students might use them improperly.

2.3.3 Research Related to Graphic Calculators

Schoenfeld (1989) mentioned that views and attitudes of the students regarding mathematics and mathematics classes are increasingly seen as crucial factors affecting their performance. How much curriculum innovations can address these factors have been great interest to educators in recent years.

In a recent experience of a new national curriculum, Ponte (1992) reported that 7^{th} grade students' views and attitudes were found to improve significantly, in close relation with the introduced methodological changes. However, in the same experience, the views and attitudes of tenth grade students showed no positive change, but rather an increase in anxiety and distrust regarding the new national curriculum.

Ruthven (1990) studied the effects on the mathematics performance of extended use of graphing calculators of students. He reported that this technology had a strong influence both on mathematical attainment and on students' approaches to specific tasks, especially what he called symbolization items.

About effects of graphic calculators on students' achievement, Dunham and Dick (1994) cited three studies yielding positive effects, three showing neutral effects, and one produce negative effect. On the other hand, the results of most studies suggested that the use of graphic calculator in teaching and learning is beneficial in terms of students' level of understanding and achievement in algebra and precalculus (Alexander,1993;Chandler,1993). However, Emese and Hall(1993) reported that there is no significant differences between the treatment group using graphing calculators and the control group. In addition, in only rare instances, Giamati (1991) and Upshaw (1994) mentioned that graphing calculators had a negative impact on achievement and they do not facilitate learning. On the other hand, in the United States, NCTM reported that using calculators develop a better attitude and higher levels of achievement on number skills and facts.

Hembree and Dessart (1992) identified general trends from seventy nine calculator studies and draw some conclusions that high school students using calculators possess a better attitude toward mathematics and especially better self concept in mathematics than non-calculator students. This statements applies all grades and ability levels. On the other hand, on student achievement no effect of calculator use on conceptual knowledge was found; on computation skills and problem solving skills.

In a questionnaire study monitored parental attitudes to a calculator project at the middle school level, Bitter and Hatfield (1992) received the agreement of around 80% of respondents that the calculators stimulate a child to learn mathematics and the calculator makes mathematics fun. There also have been claims that graphic calculator can increase the accessibility of realistic problems to students. Also use of a calculator frees students to focus on strategical issues when tackling problems.

Dunham and Dick (1994) cited evidence that secondary and tertiary students with experience of graphing technology have more flexible approaches to problem solving. They reported that in the former group, pupils exhibited more exploratory behaviors in problem solving and spent more time attacking problems and less time computing. (Evidence was gathered through clinical interviews.)

Furthermore, a systematic study of teacher change in a project conducted by Graves (1993) suggests that most teachers claimed that to have substantial changes in their teaching of mathematics; adapting more open ended approach, using calculators to facilitate problem solving, often set in a realistic context.

Milou (1999) investigated secondary mathematics teachers' use of graphing calculator in their classroom with 146 high school and middle school teachers in a large north eastern US city. A survey methodology was used in this study and findings revealed that a majority of algebra teachers found the graphic calculator as a motivational tool for students.

Another research carried by Zand and Crowe (1997), on distance learners, who took the first part of the new open university mathematics foundation course, open mathematics (OM). In the research, a questionnaire and interview were used to investigate implications of using graphic calculator on learning concepts and developing their mathematical skills. OM extensively uses the graphic facilities of the calculator to introduce concepts and their applications, in modeling certain "real world" phenomena. Analysis of the participants' experience the calculator helped them in three distinct but related ways, namely visualization, exploration and computation. Moreover, according the findings of the study, calculator increased students' interest in studying mathematics course and graphic calculator reduced the common feeling of anxiety among distance learners of mathematics.

Ponte and Canavarro (1999) carried a research about the views and attitudes of students of a low achieving 11th grade class who were in an innovative experience with graphic calculators for all academic year. This study took place in a suburban school in Lisbon and 15 students attended the study. The results were obtained from questionnaire and from interviews, revealed that students tend to point some improvements in the mathematics class and the calculator was seen useful but not radically changed the nature of the work. The calculator did not improve dramatically the global achievement of the class. It was not regarded by the students as a major influence in their way of learning mathematics.

2.4 Teaching Logarithm

Hammack and Lyons (1995) reported that many students had difficulties mastering the logarithm concept, more so than with other functions. In a similar way, Soptick (1984) mentioned that many students said that "I hate logarithms" or "I just can not understand logarithms". Therefore, some useful methods and activities developed and administered by researchers to teach logarithms. We will mention some of these studies.

First of them, "Teaching Logarithm Via Their History" is written by Thoumasis (1993). In recent years, the importance of the history of mathematics in relation to the teaching of mathematics has been widely recognized and promoted (Jones, 1975; Grantan,1978). Many excellent works can also be used as sources to introduce historical material in the teaching of high school mathematics (Walter,1975). This article presents a specific example of the use of historical materials in developing an important topic, the concept of logarithm.

In this study, the students were presented with the following progression.

Arithmetic Progression (A.P.) 1 2 3 4 5 6 7 8

Geometric Progression (G.P.)   2   4   8 16   32 64 128 256

Each term of the arithmetic progression is the number which expresses the order of the respective term in the geometric one. After this observation, the students were asked to multiply and divide any two terms of the geometric progression.

For instance;

3 + 4 = 7   5 - 2 = 3

β   β   β   β

8.16=128   32 / 4 = 8

After this observation, the students were asked to multiply and divide any two terms of the geometric progression. He pointed out the sum of two numbers in the first exponent row is the exponent of the power of 2 that represents the product of the corresponding numbers in the second row. In addition, subtraction of numbers in row one corresponds to division of numbers in row two. After solving examples like above, students discovered multiplication and division rules of logarithm by observing relationship between two rows.

In addition, the students were presented with the two logarithmic systems based on the two progressions with ratio 2 and 3 respectively.

$\log_2 1=0$ $\log_2 2=1$ $\log_2 4=2$ $\log_2 8=3$ ...

After some other examples of this kind, they discovered $\log_b b^n = n$

The advantages of historical approach to teaching logarithms were mentioned in the article as following;

1. Connects directly the new topic with the previous one of progression.

2. Helps them to realize the initial utility of logarithms in numerical computation.

3. Shows students the practical need of genesis of logarithm.

Thoumasis (1993) defended that teaching should be concerned with helping students make connections between ideas and discover their logical interrelations. Also, teaching a concept should be exploring answers to questions such as: “Where did it come from ? Who, why and how did someone did come up with that?”

According to Thoumasis (1993) to achieve the main goals of mathematics education, learning materials incorporating the history of mathematics designed as guided activities seem to be a very suitable means.

A similar study was carried out by Hammack and Lyons (1995). In this study, researchers used the conceptual way to help students to understand the function $y=\log_a x$ was to view it on the inverse of $y=a^x$. Their approach was based on a-□, simple change of notation, they replaced $\log_a$ by a-box and begun with examples rather than definitions. Next, they questions such as;

2□8=? “What number goes in the box so that 2 raised to that power is 8?” They used this approach in classes and found great improvement in the students’ comprehension of logarithms.

Similarly, Soptick (1984) used the arrow method to teach logarithm concept. An example;

Change the following equations to exponential form.

$\log_3 243$

Solution:

$\log_3 243 = 5 \Rightarrow 3^5 = 243$

Power functions and exponential functions often describe the relationship between variables in natural phenomena. By using this relationship, Rahn and Berndes (1994) prepared some activities that have helped students make generalizations about physical phenomena and reported them in the article called "Using Logarithms to Explore Power and Exponential Functions ". Furthermore, they studied on some methods that have helped students determine an approximate function represented by data. They aimed with the study to develop students' graphing sense, their understanding of logarithms and their knowledge of two important functions that are used to represent many physical phenomena. As known , logarithm can be used to if a nonlinear function is a exponential function ($y=kb^x$) or power function ($y=ax^n$), then logarithm can be used to determine the constants a, b, k, n in this study, data was given to students to graph (x, y) and (logx,logy) and they saw the logarithmic graph was linear. Then, they predicted actual function generating data which made by the students as following:

$y = ax^n$

$logy = log\ ax^n$

$logy = loga + nlogx$

Making substitions of Y=logy and X= logx yields Y= nX +loga

This result is an equation of a straight line with a slope of n and y- intercept of log a. Next, students calculate the slope, y- intercept and predict the function. There are two activities used in the article. First one is finding relationship between the period of oscillating spring and mass suspended on the spring. Another activity is about modeling exponential decay.

Second activity in DABI was prepared by taking into account this study.

In recent years, a current focus in mathematics education has been graphical approach by using computer and graphic calculator in the exploration of mathematics. So, many researches and studies has been made. One of them was made by Mayes(1994). In his article, titled Discovering Relationships: Logarithmic and Exponential Functions, he reported that the graphs of logarithmic and exponential functions can be used to determined important properties of these functions. Therefore, some activities were prepared in which graphical analysis approach was used to discover relationships between logarithmic and exponential functions. In these activities, the software tool "Derive" was used to investigate graphs of these functions.

In application of activities some questions were directed students to help them discover the properties of logarithm. In the second activity, the equation ($b^x = \log_b x$) which cannot be solved using basic algebraic techniques, was solved by using graphical analysis. Thus, activity was focused on analyzing graphical data and problem solving.

2.5 Significance of The Study In Literature

Although, discovery learning seems theoretically ideal to get rid of rote learning and involve students in the lessons, in practice there are problems. To be successful, discovery projects often require special materials and extensive preparations. In this sense, this study may be beneficial in contributing to prepare discovery materials and it may be a model for other mathematics topics.

According to many researchers using applications in mathematics instruction may help us to better motivate our students to learning mathematics. As known, people do not appreciate something which is not beneficial for themselves. Therefore, application based instruction may help students to see the benefits of what they have learned and in turn motivate students toward mathematics.

Also, as stated earlier, many educators contended that there is a need for a radical change in traditional mathematics instruction, but there are few research studies to produce and apply various instructional strategies. In addition, in Turkey, there are few attempts to use graphical calculators in mathematics instruction. Therefore, this study may be useful and beneficial to show effect of a new instructional strategy and utility of graphing calculators in mathematics lesson.

There are several studies which aimed to diagnose high school students' misconceptions in logarithm but very few have some attempts in overcoming those misconceptions. This study proposed a suitable way of recovering students from their misconceptions. In this respect, it may be beneficial in contributing to the related field.

CHAPTER 3

METHODOLOGY

This chapter will describe research method followed in this study, including the utilized measuring instruments and the other procedures.

3.1 Main and Subproblems

The main problem (primary goal) of this study is to determine the effect of a discovery and application based logarithm instruction (DABI) on students' logarithm achievement and the perceptions of students toward logarithm activities.

3.1.1 Subproblems:

1. Is there a significant difference between logarithm achievement test (LAT) mean scores of students taught with DABI and traditionally based instruction (TBI) when MAT scores were controlled?
2. Is there a significant mean difference between students in science field and Turkish-mathematics field regarding the mean scores of logarithm achievement test?
3. Is there a significant mean difference between LAT mean scores of boys and girls?

4. Is there a significant effect of interaction between gender and field of study regarding mean scores of LAT when MAT scores were controlled.
5. Is there a significant mean difference between the SVA mean scores of students with higher math grades and lower math grades?
6. Is there a significant mean difference between the SVA mean scores of students selecting science field and language-math field?
7. Is there a significant mean difference between SVA mean scores of boys and girls?

3.2 Hypotheses

In the study, hypotheses are stated in null form at a significance level of 0.05

H_0 1: There is no significant difference between LAT mean scores of students taught with DABI and traditionally based instruction when MAT scores were controlled.

H_0 2: There is no significant mean difference between students in science field and Turkish-mathematics field regarding the mean scores of logarithm achievement test when MAT scores were controlled.

H_0 3: There is no significant mean difference between LAT mean scores of boys and girls when MAT scores were controlled.

H_0 4: There is no significant effect of interaction between gender and field of study regarding mean scores of LAT when MAT scores were controlled.

$H_0$5: There is no significant mean difference between the SVA mean scores of students with higher math grades and lower math grades.

$H_0$6**:** There is no significant mean difference between the SVA mean scores of students selecting science field and language-math field.

$H_0$7: There is no significant mean difference between SVA mean scores of boys and girls.

3.3 Variables

In this study variables are categorized as independent and dependent variables.

Independent Variable: The independent variables in this study were gender, achievement level, field of study, and treatment. Treatment included two dimensions:

i. Discovery and Application Based Instruction (DABI)

ii. Traditionally Based Instruction (TBI)

Dependent Variables: The dependent variables of students were students' logarithm achievement and SVA mean scores.

Mathematics achievement pretest marks were considered as a covariate in statistical analysis.

3.4 Research Design

To determine the effect of the discovery and application based logarithm instruction on students' achievement, nonequivalent control group

(quasi -experimental group) design was used. In this study, groups were not formed by random assignment. Preformed groups existing in the school were used as control group and experimental group. The groups in this school are not formed based on any affective or cognitive characteristic or ability. In this study, groups assigned randomly to one of the treatments. Experimental groups received DABI and control groups received TBI.

3.5 Subjects of the Study

The subject of this study consisted of 118 students from four classes in Etimesgut Anatolian High School. Classes were selected among the researcher's classes whose pretest scores were close. On the other hand, in terms of validity of the research, one class as an experimental group was selected among another mathematics teacher's classes. The distribution of the subjects was given in Table3.1.

Table 3.1**.** Experimental Groups, Control Groups and number of students

Groups	Number of Students	Gender		Field of Study	
	Total	Male	Female	Science	Turkish-Math
EG	62	36	26	45	16
CG	56	34	22	38	14
Total	118	70	48	83	30

Note: Experimental groups were denoted EG, control groups were denoted CG

3.6 Instruments

In this study, following measuring instruments were used.

- Mathematics Achievement Test (MAT- Pretest)
- Logarithm Achievement Test (LAT- Posttest)
- Questionnaire (SVA)

- Interview Form

3.6.1 Mathematics Achievement Test (MAT)

The purpose of Mathematics Achievement Test (MAT) was to determine mathematics achievement level of students and use its scores as a covariate in the analysis of Logarithm Achievement Test (LAT). We prepared and administered MAT as a pretest to control effect of mathematics achievement of students. The researcher and his colleague prepared MAT . There are eight open ended essay type items in the test. MAT is presented in Appendix A. The topics included in the pretest are: sets and functions because the students were instructed these topics before administration of MAT. The validity of a measuring instrument was defined as appropriateness of the interpretations made from test scores. Moreover, the test was reviewed by two experienced mathematics teachers. They expressed that test was suitable for ninth grade students. In addition, topics included in MAT were suitable to measure mathematical background knowledge of students to learn logarithm topic.. Furthermore, to minimize any possible inconsistencies in scoring, the researcher and a mathematics teacher scored the test separately and they used to scoring rubric which is prepared by the researcher. The scoring rubric is presented in Appendix B.

3.6.2 Logarithm Achievement Test (LAT)

The aim of logarithm achievement test was to determine logarithm achievement level of students. LAT was prepared and administered as a post-test in the study to investigate effect of treatments, which were applied in the research, on logarithm achievement of students'. LAT is presented in Appendix C. Pilot study of LAT was conducted in Bayburt Anatolian High School (BAL) in spring 2000 by 35 10^{th} grade students. The administration of post test, LAT, was held in one class hour, which was forty minutes. There were 10 questions in the test. After pilot study, 5 of the items were eliminated due to their difficulties. Instead of these eliminated items three new items were added to the test. LAT, was out of 100. The scoring rubric was prepared by the researcher. In terms of validity, the test was reviewed by two experienced mathematics teachers. They expressed that test was suitable for the purpose it was designed. The questions in LAT were based on operation skill in properties of logarithm. It was concluded by the mathematics teachers that, questions in LAT were suitable to measure effect of the treatment on students' logarithm achievement. For inter rater reliability, the researcher and a mathematics teacher scored the test separately by using the scoring rubric.

3.6.3 Questionnaire (SVA)

In order to determine students' views about DABI, a questionnaire developed by Çetin, Ersoy & Çakıroğlu (2002). The questionnaire which is called Students' Views and Attitudes about DABI (SVA), is presented in Appendix D. The questionnaire, was consisted of two parts. First part was in Likert Type Scale, using a five point scale ranging from "strongly agree" to "strongly disagree and there were thirteen questions in this section. In this part, we aimed to determine opinions and views of students toward logarithm activities. The items in the first part of the SVA include statements like;

“logarithm activities helped me to improve my skills in operations.”

The second part of SVA contained four open ended questions that aimed to find out students’ detailed views about the activities.

Pilot study of the questionnaire was conducted in Bayburt Anatolian High School (BAL) in spring 2000 by 35 students and next in Etimesgut Anatolian High School (EAL) in fall 2001 by 26, 10th grade students. After first part of the pilot study conducted in BAL, items in the questionnaire were revised and second item in part C was changed due to ambiguity of expressions. In addition, some corrections and alterations were made on items in the first part. Afterwards, at the end of the second part of the pilot study conducted in EAL, Cronbach’s alpha reliability estimate for the first part of the questionnaire were found to be 0.88.

In calculating scores of participants in the first part, the response to each question was scored as; 1 points for “strongly disagree,” 2 points for “disagree,” 3 points for “no opinion,” 4 points for “agree,” and 5 points “strongly agree.” A score for a participant was obtained by calculating the sum of scores from each item for all questions of part A. The questionnaire was administered at the end of the treatments.

3.6.4 Interview Form

Interview form consisted of 8 (semi-structured) open-ended questions. However, four open-ended questions were added for the interviews with the participants of the experimental group. In order to understand students’ attitudes toward logarithm activities and to determine differences of perception among students about treatments and logarithm topic, we did interviews with students from control groups and experimental groups.

Four students from each control and experimental group were chosen and interviewed to obtain in depth information about their perception toward logarithm concept and logarithm activities administered in experimental groups. When choosing students to do interview, the researcher chose students from different achievement levels. The selection were based on the scores of students in the MAT which was administered as a pretest.

There were two parts in interview schedule. Questions of first part were asked all of students. Questions of second part were asked only experimental groups' students. A typical interview schedule was as follows.

PART I.

1. What did you gain by learning logarithm?

2. What do you understand from the term logarithm of a number?

3. What does logarithm connotates you?

4. When you see a logarithm problem, how do you feel ?

PART II:

5. What are the reasons in making error in LAT?

6. Which do you prefer between learning by listening and applying? Why?

7. What do you think about demonstrating application examples of logarithm?

8. Which was more beneficial teaching how to make operations or applying subject? Why?

Each interviewee was asked 4 or 8 questions in a period of 15-25 minutes.

3.7 Procedure

This study was conducted in 3 weeks during 2001-2002 spring semester. Four ninth grade classes were selected at EAL in Ankara. Three of classes were selected among researcher's classes and one of them among his colleague's. However, all of the classes were instructed by the researcher. A total of 118 students participated in this study. It was not possible to form an extra class with random sampling in the school. Four classes were randomly assigned to two instructions**,** DABI and TBI. Two experimental groups were instructed by DABI and two control groups were instructed by TBI. The treatments were administered during regular class hours. One class hour was 45 minutes.

Before implementation of the treatments, students were unfamiliar with logarithm topics. Teacher prepared a lesson plan on logarithm topics and applied same plan in both of the experimental groups. Other than daily life problems solved in experimental groups, all students in all groups (both experimental and control) solved same exercises, to reach the same objectives. The difference was only treatment. Experimental groups took DABI and control groups took TBI. At the end of the treatment students took LAT, questionnaire and researcher interviewed with selected students from the groups.

3.7.1 Treatment of Experimental Group (DABI)

3.7.1.1 First Activity:

In experimental group, as a first group of activities; discovery sheets were given to students to discovered the properties of logarithm. Each of the discovery sheets consisted 3 sheets and there were 6 questions in each sheet. The Discovery Sheets are presented in Appendix E.

The first discovery sheet was a preparation activity, which was aimed to help students comprehend the concept of logarithm. Second discovery sheet was aimed to help students discover multiplication property of logarithm. Finally, the third discovery sheet was aimed help students to discover the division property of logarithm.

In application of discovery sheets, students filled in the blanks in each question which is designed to help students to be discovered properties. During the lesson, teacher guided students about how to proceed with the steps of the discovery sheets because they have never been taught with activity sheets and they were stranger to them.. Then, students completed the exercises in the discovery sheets by themselves, which were planned to help them in comprehending and discovering the logarithm concept and multiplication and division properties of logarithm. After applying the discovery sheets, teacher explained other properties of logarithm and solved related examples. The discovery sheets required individual work of students. Interaction among students was minimized during the activity. These discovery sheets were designed to allow students' inquiry while guiding, prompting and helping students to comprehend properties of logarithm. At first, teacher guided students about discovery sheets. These discovery sheets were developed by the researcher. They were prepared by taking into account three study from literature. First of these studies were Hammack and Lyons (1995). In this article, researchers used the conceptual way to understand the function $y=\log_a x$ and to view it as the inverse of $y=a^x$ .Their approach was based simple change of notation. They replaced $\log_a$ by a-box and begun with examples rather than definitions. At first, they wrote following equation;

2□8=?

Next, they asked class some questions to teach logarithm concept. For example; “What number goes in the box so that 2 raised to that power is 8?” Since $2^3=8$, blank is filled with a “3”.

They used this approach in classes and found great improvement in the students’ comprehension of logarithms. Similarly, Soptick (1984) used the arrow method to teach logarithm concept.

An example;

Change the following equations to exponential form.

$\log_3 243$

Solution:

$\log_3 243 = 5 \Rightarrow 3^5 = 243$

In the same way, Thoumasis (1993) presented an example of the use of historical materials in developing the concept of logarithm. In this study, the students were presented with the following progression.

Arithmetic Progression (A.P.)	1	2	3	4	5	6	7	8
Geometric Progression (G.P.)	2	4	8	16	32	64	128	256

Each term of the arithmetic progression is the number which expresses the order of the respective term in the geometric one. After this observation, the students were asked to multiply and divide any two terms of the geometric progression.

For instance;

3 + 4 = 7 5 - 2 = 3

β β β β

8.16=128 32 / 4 = 8

After solving some exercises of this kind the students discovered that addition of two terms in arithmetic progression corresponds to multiplication of two terms in geometric progression and subtraction in arithmetic progression corresponds two division in geometric one. Observing the relationship between arithmetic and geometric progression constitutes the theoretical background of logarithms. Next, the terms of arithmetic progression are called "logarithms" of the corresponding terms of geometric one. Then, he said that addition in arithmetic progression corresponds to multiplication in the geometric one, that is, if we write $l(b^n)=na$
we obtain $l(x)+ l(y)=l(x.y)$ where x,y are any positive integral powers of b. Similarly, we get the other standard rules $l(x)- l(y)=l(x/y)$ *and* $l(x^n)=n.\ l(x)$

Finally this session ended with writing down the previous important properties about logarithms but in accordance with new symbolization. That is, x, y are terms of the geometric progression with ratio, a>0 then, the following rules are discovered by students:

1. log(a·b)= log(a) + log(b)
2. log(a/b)= log(a) – log(b)
3. log(a^n)=n. log(a)

Like that in these studies, in our approach we used relationship between exponential and logarithmic functions to be discovered some properties of logarithm. However, in opposition to TBI, we begun examples, which were used to be discovered properties of logarithm, rather than general definitions.

3.7.1.2 Second Activity:

In experimental groups, teacher solved problems including real life applications by using the properties logarithm, which were new to the students. Students found a chance to transfer and apply their basic skills to more complex situations related to real life. Afterwards, the teacher taught students formulization method in which logarithm properties were used. Then, students were grouped into groups of two or three, for the activity that require the use of graphic calculator, TI-83. In EG-2, had 24 students, each group had 2 students, in EG-1, had 36 students, groups consisted 3 pupils. Later, one student was selected from each group and they were taught for the functions of TI-83 buttons and all the needed information they needed to use while applying activity. Also, teacher guided students in using calculator during the activity. This activity sheet is presented in Appendix E. In this activity, teacher gave data showing weight and blood pressure. At first, students saved data in graphic calculator and plotted graph of data in TI-83. Afterwards, they plotted the logarithm of data and they saw that the graph of the data was parabolic and graph of logarithm of data was linear. Then, students tried to find a formula showing relation between weight and blood pressure by using data, equations, logarithm properties and TI-83. After finding formula, they used it to find out their own and their friends' blood pressure. Also, students found their own blood pressure by zooming on graph of weight- blood pressure. At the end of the activity, teacher gave students homework to apply formula to find their parents' blood pressure.

Third activity was prepared by taking into account an article called "Using Logarithms to Explore Power and Exponential Functions. " In this article, Rahn and Berndes (1994) studied with activities have helped students make visual generalizations about power functions and exponential functions. Also they studied on some methods that have helped students determine an approximate function represented by data. They aimed to develop students' graphing sense, their understanding of logarithms and their knowledge of two important functions that are used to represent many physical phenomena. As known , if a nonlinear function is an exponential function ($y=kb^{x}$) or power function ($y=ax^{n}$) then logarithm can be used to determine the constants a, b, k, n. In this study, data was given to students to graph (x, y) and (logx, logy) and they saw the logarithmic graph was linear. Then, they predicted actual function generating data which made by the students:

$y = ax^{n}$

$\log y = \log ax^{n}$

$\log y = \log a + n\log x$

Making substitions of Y=logy and X= logx yields Y= nX +loga

This result is an equation of a straight line with a slope of n and y- intercept of loga. Next, students calculate the slope, y- intercept and predict the function. There are two activities. First one is finding relationship between the period of oscillating spring and and mass suspended on the spring. Another activity is about modeling exponential decay.

3.7.1.3 Third Activity:

At last activity, "data show" activity was used by teacher to be overcome possible misconceptions in properties of logarithm. In this activity, students saw conflicts and similarities among graphs of expressions given below by drawing their graphs with TI-83. For example; They saw on graphs that $\log(a \cdot b) = \log(a) + \log(b)$, and $\log(a \cdot b) \neq \log(a) \cdot \log(b)$ Examples of graphing activities applied can be seen below.

1. $\log(a \cdot b)$, $\log(a) + \log(b)$, $\log(a) \cdot \log(b)$
2. $\log(a/b)$, $\log(a) - \log(b)$, $\log(a) \div \log(b)$
3. $\log(x^2)$, $(\log x)^2$

Afterwards, same activity was applied by the teacher. Teacher reflected images of graphs of expressions on a large screen by using the teacher calculator. Students saw image of graphs showing conflicts in misconceptions in a larger extent.

Third activity was based on Rahn and Berndes (1994) and R.L. Mayes (1994). Purpose of Mayes in the article was to present an application of the software tool "Derive" in the exploration and visualization of a relationship between logarithmic and exponential functions. He suggests to pose some questions were directed to students to explore and discover a variety of logarithmic relationships by investigating the graphical representations of the exponential and logarithmic functions.

3.7.2 Treatment of Control Group (TBI)

The instruction of the control groups was traditionally based instruction. Students in these groups were taught by teacher centered instruction in which teacher only used lecture method and students tried to solve questions with related topics without using any technological tools.

3.8 Data Analysis

For data analysis, descriptive statistics, independent sample t-test, analysis of Covariance (ANCOVA) and qualitative data analysis were used. ANCOVA was used to control effects of students' mathematics achievement.

CHAPTER 4

RESULTS

The purpose of this chapter is to present the results of this study. At first, results of the achievement tests will be given. Next, results of SVA will be presented. Finally, results of open ended questions in SVA and interview results would be presented.

4.1 Descriptive Statistics

Basic descriptive statistics about the dependent variable is given in Table 4.1. The results are presented based on the groups in the experimental study.

Table 4.1 Descriptive statistics of control groups and experimental groups based on MAT scores.

Variables	N	Mean	SD	Skewness	Kurtosis
EG	62	57,26	27,25	-0,173	-1,022
CG	56	55,13	28,97	-0,080	-1,092

Note: Experimental groups were denoted as EG, control groups were named CG

Table 4.2 Descriptive statistics of control groups and experimental groups based on LAT scores.

Variables	N	Mean	SD	Skewness	Kurtosis
EG	62	59,61	26,66	-0,454	-0,863
CG	56	53,16	36,53	-0,004	-1,179

Note: Experimental groups were denoted as EG, control groups were named CG

The descriptive statistics on LAT shows that means of students scores in LAT ranged from 53.16 to 59,61.

4.2 Results of testing hypotheses

$H_0$1: There is no significant difference between LAT mean scores of students taught with DABI and traditionally based instruction when MAT scores were controlled.

H_0 2: There is no significant mean difference between students in science field and Turkish-mathematics field regarding the mean scores of logarithm achievement test when MAT scores were controlled.

H_0 3: There is no significant mean difference between LAT mean scores of boys and girls when MAT scores were controlled.

H_0 4: There is no significant effect of interaction between gender and field of study regarding mean scores of LAT when MAT scores were controlled.

4.2.1 Results of testing of first hypothesis

Analysis of Covariance (ANCOVA) was conducted to explore the impact of treatment, field of study and gender on logarithm achievement level as measured by Logarithm Achievement Test (LAT). In ANCOVA, LAT scores were used as the dependent variable, treatment, field of study and gender were used as independent variables, and MAT scores of the students were used as the covariate in this analysis. Because there was a little difference between MAT mean scores of experimental and control group students (Table 4.1). Also, analysis in a correlation Pearson Correlation Coefficient between MAT and LAT mean scores were found .727 which was high. Therefore, to reduce students' mathematical background effect on logarithm achievement, pretest scores were used as a covariate.

Subject were divided into two groups according to their treatment Group1:DABI, Group2:TBI. There were two experimental and two control groups. ANCOVA was run to test the effectiveness of treatment on LAT mean scores (Table 4.4). The results revealed that, although mean scores of DABI group were higher than mean scores of TBI group, there was no significant mean difference between DABI and TBI groups with respect to logarithm achievement, when MAT scores were controlled. There was no significant effect of treatment on students' logarithm achievement ($F(1,106)=0.022$, $p= .881$). Analysis of the data was summarized in the Table 4.3.

4.2.2 Results of testing of second hypothesis

As can be seen in the table 4.3, it was found that there is no significant mean difference between students in science field and Turkish-math field regarding mean scores of logarithm achievement test. (F(2,106)=1.166, p=.315), after the analysis of covariance (ANCOVA) where MAT scores were used as covariate.

4.2.3 Results of testing of third hypothesis

As can be seen in the table 4.3, it was found that there was no significant mean difference between the LAT mean scores of female students and male students (F(1,106)= 0.279 p= .599) with respect to logarithm achievement when MAT scores were controlled.

4.2.4 Results of testing of fourth hypothesis

As can be seen in the table 4.3, it was found that there is no significant effect of interaction between gender and field of study (F(2,106)=1.133 p=.326) regarding mean scores of logarithm achievement test when MAT scores were controlled.

Table 4.3. Analysis of Covariance regarding LAT scores.

Source	df	*F*	η^2	*P*
MAT (Pretest)	1	70.513	.399	.000
Treatment (T)	1	0.022	.000	.881
Gender (G)	1	0.279	.003	.599
Field (F)	2	1.166	.022	.315
Gender*Field	2	1.133	.021	.326
T within group error	106	(422.744)		

Note. Value enclosed in parantheses represents mean square error.

4.3 Analysis of Questionnaire

4.3.1 Results of SVA

Means of SVA scores were above 4 points which indicate generally positive views of students about DABI (Table 4.4). Means of SVA scores for all questions was found to be "4.16". Descriptive statistics results can be seen below in table 4.5.

The items 3, 9, 6, and 2 got the highest means among others. This result revealed that students think that activities helped them to develop better skills in operations, were enjoyable, were understandable, helped them retain what they learned and helped them learn properties of logarithm. On the other hand, the items that have lowest mean scores were 8., 11., and 12.

Table 4.4 Means and standard deviations about each item in student views about activities scale.

The instruction helped me to do		
Items	**Mean**	**SD**
1. Understand logarithm concept	4.42	0.91
2. Learn properties of logarithm	4.33	1.07
3. Develop my skills in operations	4.43	1.06
4. Be interested	4.20	0.97
5. Grasp properties of logarithm easily.	4.07	1.00
6. Retain what I learned	4.35	1.02
7. Increase my thinking and interpretation power	4.12	0.92
8. Recover from memorization	3.72	1.28
9. Enjoy from learning	4.42	0.83
10. Assemble relationship with daily life	4.18	1.08
11. Relate think the concept more concretely	3.92	1.06
12. Help learn myself	3.95	1.06
13. Think in different forms	4.00	1.09

Table 4.5 Descriptive Statistics of SVA Mean Scores

Variables	N	Mean	SD	Min.	Max.
EG	60	4.16	0.69	1.85	5.00

4.3.2 Results of testing of fifth hypothesis

$H_0$5: There is no significant mean difference between the SVA scores of students with higher math grades and lower math grades.

An Independent samples t-test was conducted to compare SVA scores of students with high mathematics grades and low mathematics grades. There was significant difference between SVA scores of students with high grade and low grade (t=-2.78, p=.007). The magnitude of the differences in the means was big (eta squared=.12) according to Cohen (1988) criteria. Analysis results can be seen below in table 4.6.

Table 4.6. Independent samples t-test to compare SVA scores of students with high mathematics grades and low mathematics grades.

Math Grade	N	SVA Mean Scores	t	p	η^2
High	45	4,31	-2,78	.007	0.12
Low	16	3,76			

Next, we examined effect of field selection on students' attitudes toward logarithm activities.

4.3.3 Results of testing of sixth hypothesis

$H_0$6: There is no significant mean difference between the SVA scores of students selecting science field and Turkish-math field.

An Independent samples t-test was conducted to compare SVA scores of students with high mathematics grades and low mathematics grades. there was significant effect of field selections of students on SVA scores**.** There was significant difference between SVA scores of students selected Science field and Turkish-Math field (t=2.46, p=.017). The magnitude of the differences in the means was big (eta squared=.96). Analysis results can be seen below in table 4.7.

Table 4.7 Independent samples t-test to compare SVA scores of students from science and Turkish-math fields.

Field	N	SVA Mean Scores	t	p	η^2
Science-Math	45	4,29	2,46	.017	.096
Turkish-Math	16	3,81			

4.3.4 Results of testing of seventh hypothesis

$H_0$7: There is no significant mean difference between SVA scores of boys and girls.

An Independent samples t-test was conducted to compare SVA scores of female and male students. Although SVA mean scores of girls were higher than mean scores of boys', there was no significant mean difference between SVA scores of boys and girls (t=-0,77, p=.444). However, the magnitude of the differences in the means was very small (eta squared=.01). Analysis results can be seen below in Table 4.8.

Table 4.8 Independent samples t-test to compare SVA scores of male and female students.

Field	N	SVA Mean Scores	t	p	η^2
Female	25	4,24	-0,77	.444	0.01
Male	35	4,10			

4.4 Interviews and answers of open-ended questions

In order to investigate the opinions and views of the students in the experimental group about the implementation of DABI in mathematics lessons, students were asked to respond to open-ended questions in SVA and interviewed after the treatment. This section will present the results of these qualitative data.

Most of the students expressed that they liked DABI (As given in Table 4.9). There were also few ones who found it difficult to understand The students who reported they liked DABI were not only upper level (had grade 4 or 5 for maths in the previous semester) but there were also students from average (had 2 or 3) level of achievement. Views of students about DABI were gathered basically under four headlines:

1. Enjoyment
2. Increasing Motivation Toward Lesson
3. Retention of Subject
4. Increasing Self Confidence

There were also different opinions which were not accumulated under headlines above. They would be presented under headline that “Other Views and Opinions of Students”.

One major category in student responses were about enjoyment. Students frequently expressed how they enjoyed the activities. Almost all of the students indicated that they enjoyed mathematics lessons with DABI. They found lessons very enjoyable and interesting. One student expressed that “now I like logarithm. Of course sometimes I can’t solve some logarithm problems but this doesn’t prevent me from liking it. This might be because we know where logarithm is used and that we actually implement it.” Another student replied that “I think a course about applications like DABI should be given teachers and preservice-teachers. They should learn how to make lessons funnier.”

Another major category in student responses were about motivation. They frequently expressed that the activities increased their motivation toward mathematics lesson. There was one student who expressed that “when you learn it yourself, you say, I can do it, I am successful in logarithm . You can pay attention, you enjoy and your interest increases.” One student replied that learning benefits of logarithm in daily life took our attention.

- Third major category in student responses were about retention. They frequently expressed that the activities was effective in terms of retention of the subject. One student said that “I will never forget the formulas for log(a.b) and log(a/b).” Similarly, another student replied that “instead of memorizing the formulas that the teacher gave us, we understand it better by discovering it on our own.

Another interesting expression was that “instead of being out of the event or hearing activity from the second person, being in the activity not only provide it being unforgettable, but also , I believe that, it make mathematics funnier.”

Another interesting major conclusion was increasing self confidence of the students. One student replied that “My self-esteem has increased. Having the responsibility of something, overcoming it and believing that I can manage to do has taught me that I shouldn’t be afraid of the problems in life.” Another similar answer was that “When I find something on my own, I get courage and of course this brings on more successes. Otherwise I feel like I have to do something and this worries me.”

To sum up, frequency of expressions, made by the students.in the interviews and open-ended questions are presented in Table 4.9. In addition, detailed expressions of students about DABI is presented in Appendix F.

Table 4.9 Frequency of Experimental Group Interviewers’ Answers About DABI.

Views about DABI	Frequency for Interviewers	Frequency for Open-ended Questions
Retention	10	26
Enjoyment	19	52
Increasing motivation	9	23
Increasing self confidence	5	9

Note. Each value above represents frequency of comments made by the students.

In interviews, questions about logarithm concept were directed to students who belong to experimental and control groups to compare attitudes of students toward logarithm concept. At first, students were asked “What did you gain by learning logarithm?”. One student from control group replied that “it didn’t provide us to gain anything. I learnt the subject at the end. But I didn’t consider it important since I didn’t think I would use it in the future.” Another similar answer of a student from control group replied that “It can't be said that it has provided me to gain anything.” Most of the interviewers from control group gave similar answers (As given in Table 4.10). On the other hand, experimental group interviewers gave substantially different answers. For example, one interviewee of the experimental group replied that “It told me that everything was in a ratio, For example; it told me that there was a ratio between our weight and blood pressure.” They also reported that logarithm was enjoyable.

Second question was asked students that **“**What do you understand from the term logarithm of a number?” Almost all of the control group students replied that **“**it doesn’t mean anything.**”** However, most of the experimental group students reported that “It is the opposite of exponential function.”

The third question was “What does logarithm connotates you?”. Most of the control group students replied again that **“**it doesn’t mean anything.**”** In contrast, experimental group students mentioned about applications of logarithm in daily life. One student experimental group student reported that “The calculation of the brightness of the stars in astronomy, usage of logarithm in photography, the ratio of our weight and blood pressure occur in my mind.”

Finally, as a fourth question, “when you see a logarithm problem, how do you feel?” were directed students. Some of the control group students replied that “At first, it becomes too difficult. After a while, I can do because I realize that it is easy.” On the other hand, one experimental group student reported that logarithm explained us function of numbers in science. Similarly, another student from EG replied that “I became aware that everything I have learnt is applicable to a real life situation. Therefore, the questions become attractive and enjoyable.” Expressions of students and their frequencies are given below. In addition, detailed expressions of students about logarithm concept is presented in Appendix F.

Table 4.10 Frequency of students' answers for the comparative questions about logarithm concept.

Questions	Answers	Frequency EG	Frequency CG
What did you gain by learning logarithm?	Nothing	–	3
	Beneficial for Lise-2	–	2
	Usage of Logarithm in Daily Life	4	–
	Enjoyable	2	–
What do you understand from the term logarithm of a number?	Nothing	–	6
	It is inverse of exponential function.	6	1
What does logarithm connotates you?	Nothing	–	5
	It is only a mathematics subject.	–	1
	Benefits of Logarithm in Daily Life	4	–
	Enjoyable	1	–
When you see a logarithm problem, how do you feel ?	At first, it is difficult but I can solve later.	1	3
	Difficult	–	1
	Easy or Enjoyable	4	2

Note. Each value above represents repetition numbers of answers by the students.

The interview and SVA results of the experimental groups students were supported the effectiveness of DABI and revealed they had positive views and opinions about the implementation of this instruction technique in mathematics. lessons.

CHAPTER 5

CONCLUSIONS AND DISCUSSION

This chapter states the conclusions of the research, discussion of the results, recommendations for further studies, and implications. A discussion of limitations, internal and external validity is also included.

The main purpose of this study was to determine the effect of a discovery and application based logarithm instruction on students' logarithm achievement and also to investigate the views of students about this instruction. A quasi-experimental design was used, which last three weeks. There were four groups. Two of them were experimental groups and the other two groups were control groups. Experimental groups were taught the concept of logarithm and its properties through DABI. The control groups were taught by TBI. Both of the experimental and control groups were instructed in their classrooms. Experimental groups' instruction were focused on discovering properties by students, application of logarithm properties and visualization of the concept.

Experimental groups' students got opportunities to discover and apply the properties of logarithm during the instruction. In experimental instruction there were process of discovery, utilization of technology and applications of the properties of logarithm. During implementation of DABI and TBI, with a few exceptions, same examples were used for all groups. The difference only was logarithm activities applied in DABI.

Mathematics achievement (MAT) and logarithm achievement (LAT) tests were administered to all groups as pre and post-tests. However, questionnaire (SVA) was administered to only experimental group students. In addition, the data was collected by interviews, conducted with four students from each group. These interviews were made to describe their thoughts about logarithm concept and their opinions about implementation of DABI in mathematics lessons.

On the other hand, students were taught by DABI, expressed great enjoyment and showed great enthusiasm during the activities. They stated that;

- "Now, logarithm is in a special place in my life because I strived to learn it and I had an active role."

- "DABI was good method to make us love logarithm."

- "Students will love mathematics. We enjoyed while learning. We understood that mathematics is not as difficult as we believe".

Also, students were willing to solve logarithm problems using calculator. They had enthusiasm to discover mathematical issues and they were more interested in the tasks than before. Especially, having responsibility and completing tasks motivated them very much. Therefore, students became very happy when they discovered new ideas. When we examined students' expressions in SVA and interviews, we observed positive feelings of students about DABI. This result was consistent with the findings of Zand and Crowe (1997) who reported that using calculator in mathematics applications increased students' interest in studying mathematics. Results also consistent with Canavarro and Ponte (1992) who stated that use of graphic calculators found to be useful and interesting by the students.

After the statistical analysis, it was found that there was no significant mean difference between boys and girls in terms of logarithm achievement.

5.1 Conclusions

In order to achieve the aims of this study, three instruments were used and interviews were made after the treatment. Results of them were given detailed in chapter four. The conclusions of this study can be summarized as following:

1. The statistical analysis revealed that logarithm achievement of students instructed with DABI and TBI were not significantly different. However, students, instructed with DABI, get higher scores than TBI students. It means that DABI did not result in difference in logarithm achievement. Previous similar studies reported that application based instruction did not improved significantly students' knowledge of mathematical rules and procedures (Boaler, 1997).
2. Logarithm achievement scores of female and male students were not significantly different after the treatment. This means that discovery and application based instruction did not result in different benefits for male and female students.
3. No significant difference was found for science and Turkish-math students with respect to logarithm achievement. It means that DABI did not cause different advantages in logarithm achievement for science and Turkish-math students.
4. According to the findings of this study, there is no significant effect of interaction between gender and field of study regarding mean scores of LAT when MAT scores were controlled.
5. According to the findings of this study, SVA scores of the students with high mathematics grade were significantly higher than scores of the students with low mathematics grade. This result reveals that DABI resulted in difference in the views of students with high grade and low grade toward logarithm activities.

6. Results of the study showed that although, female students had higher SVA scores, SVA scores of female and male students were not significantly different. This result means that DABI did not result in difference in the views of male and female students toward logarithm activities.
7. According to SVA and interview results, majority of students found DABI beneficial and enjoyable. On the other hand, SVA scores of science section students were found higher than Turkish-math students'. It means that science section students' views about DABI were more positive than Turkish-math section students. It may be due to the factor that most of the Turkish-math students had problems and difficulties in mathematics lesson because of deficiencies of mathematical background knowledge and they did not like mathematics. Therefore, they did not showed great enthusiasm toward DABI.
8. According to open-ended questions of SVA and interview results revealed that DABI provided permanence of logarithm subject, enjoyment, self-confidence and facilitation of the subject. It means that DABI helped students to develop their affective aspect.
9. According to open-ended questions of SVA and interview results, students using graphic calculators in logarithm activities possessed a better attitude toward mathematics and self-concept in mathematics.

5.2 Conclusions Based on Interviews and SVA

In order to investigate opinions and views of the students in the experimental groups about implementation of DABI, we administered a questionnaire (SVA) and interviewed with students after the treatment. First part of SVA was in Likert type scale, using a five point scale ranging from "strongly agree" to “strongly disagree”. The results of this part showed that students in experimental groups supported the effectiveness of DABI and revealed that they had positive opinions about implementation of this instructional unit. As it was shown in table 4.4, students stated that they enjoyed the activities which helped better understanding of logarithm concept, learning properties of logarithm and developing operation skill by using logarithm properties.

Moreover, students believed that activities provide long-term learning, helped making better interpretation of concepts, increase interpretation and helped relate coordinate the subject to daily life. Furthermore, expressions of students in open-ended questions and interviews supported that DABI resulted in increased motivation of students toward mathematics lessons and developed students in terms of affective aspect (e.g., in terms of self-concept, responsibility). Also, applications in DABI were found to be more practical than the operations in TBI.

To compare differences in attitudes of the students toward logarithm subject, four questions in the interview were asked students in both groups. When we examine the answers of these questions, we observed that students in control groups used dull and lifeless expressions about logarithm concept. However, students in experimental groups used rich, vivid, animated and meaningful expressions about logarithm concept. These colorful and rich expressions revealed that logarithm activities and DABI developed affective aspect of students and they were happy with DABI. Although educators clarified that developing students’ feelings was difficult, results of this research revealed that DABI successful in influencing attitudes. As a matter of fact, main purpose of preparing logarithm activities was to instruct mathematics meaningfully and save mathematics lesson from dull, poor and meaningless experiences.

Consequently, there will be need for instruments to measure sentimental developments of students for similar studies in the future. Indeed, researchers should deepen measurements in sentimental developments in terms of self efficacy, enjoyment, retention of the subject and attitudes of students toward subject.

5.3 Discussion

According to the statistical analysis, there was no significant mean difference between students taught by DABI and TBI with respect to logarithm achievement, when mathematics achievement scores were controlled. Although experimental group students yielded higher scores than the control group, no significant mean difference was found between students who took DABI and TBI. This result consistent with the study carried out by Boaler (1997) on Amber Hill and Phoenix Park school students. In Amber Hill, students were instructed with TBI and Phoenix Park school students were taught mathematics by application based instruction (ABI). Results of the study revealed that students taught with ABI did not have a greater knowledge of mathematical facts, rules and procedures but they were more able to make use of the knowledge they did have in different situations. Moreover, students, instructed with TBI, found it difficult remembering rules and procedures to base decisions on when or how to use and adapt them. These results means that application based instruction had developed different kind of mathematics knowledge.

In both of the groups, no significant difference between mean scores may be due to factor that application of the treatment was very limited. Also, students were put a burden of exams as treatments was being instructed. Therefore, treatment, could not produce significant result with respect to logarithm achievement. It may also be the case that, development of students' skills and abilities was not initially measured by traditional school exams. Indeed, there will be need for an instrument to measure sentimental developments of students.

According to the findings of this study, SVA scores of the students with high mathematics grade were significantly higher than scores of the students with low mathematics grade. It may be due to the factor that real life applications of logarithm were more difficult for students than solving traditional logarithm examples. Also, students with low mathematics grade had difficulties and deficiencies in mathematical operations. In this respect, they had difficulties in doing mathematical operations in the activities. Therefore, they did not expressed great enthusiasm toward DABI.

According to results based on interviews and SVA, DABI helped students to improve in terms of affective aspect. It may be due to factor that DABI encouraged students for full participation and emphasized connections between mathematics and daily life. In addition, it promoted students' confidence, curiosity and inventiveness in doing mathematics.

According to open-ended questions of SVA and interview results, students using graphic calculators in logarithm activities possessed a better attitude toward mathematics and self-concept in mathematics. It may be due to factor that graphics calculators provided an environment where students enjoy learning and doing mathematics.

5.4 Limitations

5.4.1 Internal Validity

Fraenkel and Wallen (1996) stated that observed differences on the dependent variable are directly related to the independent variable, and not due to some unintended variable were identified as internal validity. In this part, possible threats to the internal validity will be discussed.

Students were at the same age, all of which were ninth grade students. Students' were from families with similar socio-economic-status. Thus, subject characteristics could not be a threat. In analyzing data, students' pretest scores were used as a covariate. Therefore, their educational background should not be a problem. All of the subjects were present during the collection of data. They attained and completed pre and post achievement tests and questionnaire. Hence, mortality was not a threat.

For this study, location and history could not be a threat, because all measuring instruments administered in the classrooms almost at the same time. Also, physical conditions were not a problem, because all the classes were in the same floor with equal conditions. Implementation could not be a threat because the researcher applied both of treatments in all groups. However, biased behavior of the researcher during instruction might be a threat. To reduce and control this threat, an observer, researcher's colleague in the school, observed a lesson in a control group and he found instruction suitable and not biased. Besides, in applying treatments, mathematics teacher followed the same plan and solved same exercises. In addition, while scoring pretest and posttest, researcher reviewed scoring rubric together with another mathematics teacher from EAL who found scoring rubric suitable. Therefore, data collector characteristics and bias could not be a threat for this study.

According to open-ended questions of SVA and interview results, graphics calculators provided an environment where students enjoy learning and doing mathematics. However, this result may be due to novelty factor. This was the limitation of the study.

5.4.2 External and Population Validity

Both the nature of the sample and the environmental conditions –the setting- within which study takes place must be considered in thinking about generalizability. The extent to which the results of a study can be generalized determines the external validity of the study.

In this research, convenience sampling was used. So, generalization of the results was limited. Generalization can be done to subjects who have similar characteristics to that of the subjects in this study.

5.4.3 Ecological Validity

Ecological validity refers to degree which results of a study can be extended to other settings or conditions (Fraenkel & Wallen,1996).

In this study experimental and control groups were instructed in classrooms. The results of this study can be generalized to classroom settings similar to the present study.

5.5 Implications

In this part, implications of the present study could be stated as follows:

- According to open-ended questions of SVA and interview results, students demonstrated a high degree of motivation and interest towards the use of application based instruction. Therefore, the use of relevant applications of mathematics in mathematics instruction, and forming active learning environments enhance mathematics lessons.

- The meaning of what has been learned in math for students' lives is usually a question mark. Application based instruction helped students think more about this question and consequently enabled them to find out reasonable and satisfactory answers. In addition, students developed a better intrinsic motivation towards learning mathematics.
- The content and teaching styles of mathematics curriculum must be changed. There must be increased focused on importance of discovering, applying and using technology in mathematics lessons.
- Alike science lessons, in mathematics lessons teacher should give opportunities students to apply subject. Even, mathematics application and laboratory lessons should be planned for science students in Anatolian High Schools.
- Graphic calculator facilitated problem solving in a realistic context and helped to develop self-concept of students in mathematics. Therefore, calculators should be used in mathematics applications.

5.6 Recommendations

On the basis of this study, it can be recommended that:

- Further studies can be conducted to see the effects of DABI on developing affective aspects of students by using different and suitable instruments.

- Similar research studies can be conducted for different mathematics subjects and grade levels and high school .
- New activity sheets and mathematics applications for other mathematics subjects should be designed and administered.
- To rescue mathematics lessons monotone, dull and meaningless condition, educators should design new instruction models in a meaningful context for each mathematics subject.
- Application time of this study was very limited. An expanded study, should be conducted in order to investigate further effects on achievement and attitude.

CHAPTER 5

CONCLUSIONS AND DISCUSSION

This chapter states the conclusions of the research, discussion of the results, recommendations for further studies, and implications. A discussion of limitations, internal and external validity is also included.

The main purpose of this study was to determine the effect of a discovery and application based logarithm instruction on students' logarithm achievement and also to investigate the views of students about this instruction. A quasi-experimental design was used, which last three weeks. There were four groups. Two of them were experimental groups and the other two groups were control groups. Experimental groups were taught the concept of logarithm and its properties through DABI. The control groups were taught by TBI. Both of the experimental and control groups were instructed in their classrooms. Experimental groups' instruction were focused on discovering properties by students, application of logarithm properties and visualization of the concept.

Experimental groups' students got opportunities to discover and apply the properties of logarithm during the instruction. In experimental instruction there were process of discovery, utilization of technology and applications of the properties of logarithm. During implementation of DABI and TBI, with a few exceptions, same examples were used for all groups. The difference only was logarithm activities applied in DABI.

Mathematics achievement (MAT) and logarithm achievement (LAT) tests were administered to all groups as pre and post-tests. However, questionnaire (SVA) was administered to only experimental group students. In addition, the data was collected by interviews, conducted with four students from each group. These interviews were made to describe their thoughts about logarithm concept and their opinions about implementation of DABI in mathematics lessons.

On the other hand, students were taught by DABI, expressed great enjoyment and showed great enthusiasm during the activities. They stated that;

- "Now, logarithm is in a special place in my life because I strived to learn it and I had an active role."
- "DABI was good method to make us love logarithm."
- "Students will love mathematics. We enjoyed while learning. We understood that mathematics is not as difficult as we believe".

Also, students were willing to solve logarithm problems using calculator. They had enthusiasm to discover mathematical issues and they were more interested in the tasks than before. Especially, having responsibility and completing tasks motivated them very much. Therefore, students became very happy when they discovered new ideas. When we examined students' expressions in SVA and interviews, we observed positive feelings of students about DABI. This result was consistent with the findings of Zand and Crowe (1997) who reported that using calculator in mathematics applications increased students' interest in studying mathematics. Results also consistent with Canavarro and Ponte (1992) who stated that use of graphic calculators found to be useful and interesting by the students.

After the statistical analysis, it was found that there was no significant mean difference between boys and girls in terms of logarithm achievement.

5.1 Conclusions

In order to achieve the aims of this study, three instruments were used and interviews were made after the treatment. Results of them were given detailed in chapter four. The conclusions of this study can be summarized as following:

1. The statistical analysis revealed that logarithm achievement of students instructed with DABI and TBI were not significantly different. However, students, instructed with DABI, get higher scores than TBI students. It means that DABI did not result in difference in logarithm achievement. Previous similar studies reported that application based instruction did not improved significantly students' knowledge of mathematical rules and procedures (Boaler, 1997).
2. Logarithm achievement scores of female and male students were not significantly different after the treatment. This means that discovery and application based instruction did not result in different benefits for male and female students.
3. No significant difference was found for science and Turkish-math students with respect to logarithm achievement. It means that DABI did not cause

different advantages in logarithm achievement for science and Turkish-math students.

4. According to the findings of this study, there is no significant effect of interaction between gender and field of study regarding mean scores of LAT when MAT scores were controlled.

5. According to the findings of this study, SVA scores of the students with high mathematics grade were significantly higher than scores of the students with low mathematics grade. This result reveals that DABI resulted in difference in the views of students with high grade and low grade toward logarithm activities.
6. Results of the study showed that although, female students had higher SVA scores, SVA scores of female and male students were not significantly different. This result means that DABI did not result in difference in the views of male and female students toward logarithm activities.
7. According to SVA and interview results, majority of students found DABI beneficial and enjoyable. On the other hand, SVA scores of science section students were found higher than Turkish-math students'. It means that science section students' views about DABI were more positive than Turkish-math section students. It may be due to the factor that most of the Turkish-math students had problems and difficulties in mathematics lesson because of deficiencies of mathematical background knowledge and they did not like mathematics. Therefore, they did not showed great enthusiasm toward DABI.

8. According to open-ended questions of SVA and interview results revealed that DABI provided permanence of logarithm subject, enjoyment, self-confidence and facilitation of the subject. It means that DABI helped students to develop their affective aspect.
9. According to open-ended questions of SVA and interview results, students using graphic calculators in logarithm activities possessed a better attitude toward mathematics and self-concept in mathematics.

5.2 Conclusions Based on Interviews and SVA

In order to investigate opinions and views of the students in the experimental groups about implementation of DABI, we administered a questionnaire (SVA) and interviewed with students after the treatment. First part of SVA was in Likert type scale, using a five point scale ranging from "strongly agree" to "strongly disagree". The results of this part showed that students in experimental groups supported the effectiveness of DABI and revealed that they had positive opinions about implementation of this instructional unit. As it was shown in table 4.4, students stated that they enjoyed the activities which helped better understanding of logarithm concept, learning properties of logarithm and developing operation skill by using logarithm properties.

Moreover, students believed that activities provide long-term learning, helped making better interpretation of concepts, increase interpretation and helped relate coordinate the subject to daily life. Furthermore, expressions of students in open-ended questions and interviews supported that DABI resulted in increased motivation of students toward mathematics lessons and developed students in terms of affective aspect (e.g., in terms of self-concept, responsibility). Also, applications in DABI were found to be more practical than the operations in TBI.

To compare differences in attitudes of the students toward logarithm subject, four questions in the interview were asked students in both groups. When we examine the answers of these questions, we observed that students in control groups used dull and lifeless expressions about logarithm concept. However, students in experimental groups used rich, vivid, animated and meaningful expressions about logarithm concept. These colorful and rich expressions revealed that logarithm activities and DABI developed affective aspect of students and they were happy with DABI. Although educators clarified that developing students' feelings was difficult, results of this research revealed that DABI successful in influencing attitudes. As a matter of fact, main purpose of preparing logarithm activities was to instruct mathematics meaningfully and save mathematics lesson from dull, poor and meaningless experiences.

Consequently, there will be need for instruments to measure sentimental developments of students for similar studies in the future. Indeed, researchers should deepen measurements in sentimental developments in terms of self efficacy, enjoyment, retention of the subject and attitudes of students toward subject.

5.3 Discussion

According to the statistical analysis, there was no significant mean difference between students taught by DABI and TBI with respect to logarithm achievement, when mathematics achievement scores were controlled. Although experimental group students yielded higher scores than the control group, no significant mean difference was found between students who took DABI and TBI. This result consistent with the study carried out by Boaler (1997) on Amber Hill and Phoenix Park school students. In Amber Hill, students were instructed with TBI and Phoenix Park school students were taught mathematics by application based instruction (ABI). Results of the study revealed that students taught with ABI did not have a greater knowledge of mathematical facts, rules and procedures but they were more able to make use of the knowledge they did have in different situations. Moreover, students, instructed with TBI, found it difficult remembering rules and procedures to base decisions on when or how to use and adapt them. These results means that application based instruction had developed different kind of mathematics knowledge.

In both of the groups, no significant difference between mean scores may be due to factor that application of the treatment was very limited. Also, students were put a burden of exams as treatments was being instructed. Therefore, treatment, could not produce significant result with respect to logarithm achievement. It may also be the case that, development of students' skills and abilities was not initially measured by traditional school exams. Indeed, there will be need for an instrument to measure sentimental developments of students.

According to the findings of this study, SVA scores of the students with high mathematics grade were significantly higher than scores of the students with low mathematics grade. It may be due to the factor that real life applications of logarithm were more difficult for students than solving traditional logarithm examples. Also, students with low mathematics grade had difficulties and deficiencies in mathematical operations. In this respect, they had difficulties in doing mathematical operations in the activities. Therefore, they did not expressed great enthusiasm toward DABI.

According to results based on interviews and SVA, DABI helped students to improve in terms of affective aspect. It may be due to factor that DABI encouraged students for full participation and emphasized connections between mathematics and daily life. In addition, it promoted students' confidence, curiosity and inventiveness in doing mathematics.

According to open-ended questions of SVA and interview results, students using graphic calculators in logarithm activities possessed a better attitude toward mathematics and self-concept in mathematics. It may be due to factor that graphics calculators provided an environment where students enjoy learning and doing mathematics.

5.4 Limitations

5.4.1 Internal Validity

Fraenkel and Wallen (1996) stated that observed differences on the dependent variable are directly related to the independent variable, and not due to some unintended variable were identified as internal validity. In this part, possible threats to the internal validity will be discussed.

Students were at the same age, all of which were ninth grade students. Students' were from families with similar socio-economic-status. Thus, subject characteristics could not be a threat. In analyzing data, students' pretest scores were used as a covariate. Therefore, their educational background should not be a problem. All of the subjects were present during the collection of data. They attained and completed pre and post achievement tests and questionnaire. Hence, mortality was not a threat.

For this study, location and history could not be a threat, because all measuring instruments administered in the classrooms almost at the same time. Also, physical conditions were not a problem, because all the classes were in the same floor with equal conditions. Implementation could not be a threat because the researcher applied both of treatments in all groups. However, biased behavior of the researcher during instruction might be a threat. To reduce and control this threat, an observer, researcher's colleague in the school, observed a lesson in a control group and he found instruction suitable and not biased. Besides, in applying treatments, mathematics teacher followed the same plan and solved same exercises. In addition, while scoring pretest and posttest, researcher reviewed scoring rubric together with another mathematics teacher from EAL who found scoring rubric suitable. Therefore, data collector characteristics and bias could not be a threat for this study.

According to open-ended questions of SVA and interview results, graphics calculators provided an environment where students enjoy learning and doing mathematics. However, this result may be due to novelty factor. This was the limitation of the study.

5.4.2 External and Population Validity

Both the nature of the sample and the environmental conditions –the setting- within which study takes place must be considered in thinking about generalizability. The extent to which the results of a study can be generalized determines the external validity of the study.

In this research, convenience sampling was used. So, generalization of the results was limited. Generalization can be done to subjects who have similar characteristics to that of the subjects in this study.

5.4.3 Ecological Validity

Ecological validity refers to degree which results of a study can be extended to other settings or conditions (Fraenkel & Wallen,1996).

In this study experimental and control groups were instructed in classrooms. The results of this study can be generalized to classroom settings similar to the present study.

5.5 Implications

In this part, implications of the present study could be stated as follows:

- According to open-ended questions of SVA and interview results, students demonstrated a high degree of motivation and interest towards the use of application based instruction. Therefore, the use of relevant applications of mathematics in mathematics instruction, and forming active learning environments enhance mathematics lessons.

- The meaning of what has been learned in math for students' lives is usually a question mark. Application based instruction helped students think more about this question and consequently enabled them to find out reasonable and satisfactory answers. In addition, students developed a better intrinsic motivation towards learning mathematics.
- The content and teaching styles of mathematics curriculum must be changed. There must be increased focused on importance of discovering, applying and using technology in mathematics lessons.
- Alike science lessons, in mathematics lessons teacher should give opportunities students to apply subject. Even, mathematics application and laboratory lessons should be planned for science students in Anatolian High Schools.
- Graphic calculator facilitated problem solving in a realistic context and helped to develop self-concept of students in mathematics. Therefore, calculators should be used in mathematics applications.

5.6 Recommendations

On the basis of this study, it can be recommended that:

- Further studies can be conducted to see the effects of DABI on developing affective aspects of students by using different and suitable instruments.
- Similar research studies can be conducted for different mathematics subjects and grade levels and high school .

- New activity sheets and mathematics applications for other mathematics subjects should be designed and administered.
- To rescue mathematics lessons monotone, dull and meaningless condition, educators should design new instruction models in a meaningful context for each mathematics subject.
- Application time of this study was very limited. An expanded study, should be conducted in order to investigate further effects on achievement and attitude.

REFERENCES

Allison (1995) The Status of Computer Technology in Classroom Using The Integrated Mathematic Instructional Model. International Journal of Instructional Media,22(1),3343

Barnett, A. R.; Ziegler, M.R.; Byleen, K.E. (1999). Precalculus: Functions and Graphs. New York: Mc-Graw Hill

Barnett, A. R.; Ziegler, M.R.; Byleen, K.E. (2000). Precalculus: A Graphing Approach. New York: Mc-Graw Hill

Bendigo Senior Secondary College (1999) Changing Teaching and Learning in Mathematics. The Graphics Calculator Project

Berry, J.; Wainwright, P. (1991) Foundation of Mathematics For Engineers. Hampshire: Mc Millan

Bishop, A.;Clements, K.; Keitel, C.; Kilpatrick,J.(1996) International Handbook. Dordrehct: Kluwer Academic Publishers

Boaler, J.(1997) Experiencing School Mathematics. Philadelphia: Open University Press.

Canavarro, A. P.; Ponte, J. P. (1992) Graphic Calculators in the Classroom : Students' View Points http://www.educ.fc.ul.pt/docentes/jponte/ind_uk.htm

Cankoy (1998) Determining and Overcoming Preservice Elementary Teachers' Misconceptions in Interpreting and Applying Decimals. A Thesis Submitted to the Graduate School of Social Sciences of The Middle East Technical University , for the Degree of Doctor of Philosophy in The Department of Educational Sciences.

Confrey ,J. (1999) Learning to Listen: A Student's Understanding of Powers of Ten E.von Glasersfeld (ed.) Radical Constructivism in Mathematics Education, Kluwer Academic Publishers, Printed in Netherlands

Crowe, W. D.; Zand, H. (1997) Novices Entering Mathematics 2: The graphic Calculator and Distance Learners. Milton Keynes: Elsevier Science Ltd.

Driscoll, M. P. (1994) Psychology of Learning for Instruction: Learning and Instructional Technology. Needham Heights: Allyn Bacon.

Erbaş,A. K. (1999) An Investigation into Students' Performances, Difficulties and Misconceptions in Elementary Algebra, A Thesis Submitted to the Faculty of Education, The Middle East Technical University of Ankara, for the Degree of Master, Turkey

Erol, E. (1989) Matematik Dersinde Bulgulama, İstanbul: FKM Yayınları, ,

Fraenkel, J.R.,& Wallen N.E. (1996) How to Design and Evaluate Research in Education , New York, Mc Graw- Hill, Inc.

Haladayna, T.M.(1997) Writing Test Items to Evaluate Higher Order Thinking. Needham Heights, Aviacom Company)

Hammack, R. & Lyons, D., (1995). A Simple Way to Teach Logarithm, Mathematics Teacher 88(5), 174-175

Hastings, N..(1999) Workshop Calculus with Graphing Calculators. Binghampton : Verlag, Inc.

Hembree,R.& Dessart, (1986)Effects of Hand held Calculators in Precollege Mathematics Education A Meta Analysis. Journal for Research in Mathematics Education,17,83-99

Hilton, P. J.; Young, G. S. (1980) New Directions in Applied Mathematics. New York: Springer – Verlag.

İşeri, A. (1997) Diagnosis of Students' Misconceptions in Interpreting and Applying Decimals, A Thesis Submitted to the Faculty of Education, The Middle East Technical University of Ankara, for the Degree of Master, Turkey

Işıksal,M.(2002) The effect of Spreadsheet and Dynamic Geometry Software On The Mathematics Achievement and Mathematics Self Efficacy of 7th Grade Students, A Thesis Submitted to the Faculty of Education, The Middle East Technical University of Ankara, for the Degree of Master, Turkey

Keislar, E.R.;Shulman, L.S.(1966) Learning By Discovery : A Critical Appraisal. Chicago: Rand McNally Company.

Kutzler (1999)The Algebraic Calculator as a Pedagogical Tool for Teaching Mathematics. Ohio: Teachers Teaching with Technology College Course Program

Lin, C.C.; Segel, L.A.; Handelman, G.H. (1988) Mathematics Applied To Deterministic Problems in Natural Sciences, V.1. New York Kensselaer Polytechnic Institute.

Lochead, P. & Mestre , J. (1988) From Words to Algebra :Mending Misconceptions In A. Coxford & A.Shulte (Eds) The Ideas of Algebra, K-12

Milou,E. (1999) The Graphing Calculator: A Survey of Classroom Usage, School Science and Mathematics,99,(3),133-140

National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA.

Olivier, A. (1989)Handling Pupils Misconceptions, Thirteenth National Convention on Mathematics, Physical Science and Biology Education

Solow, A. (1994) Preparing For a New Calculus. Reston: The Mathematical Association of America.

Soptick, J. (1984). The Arrow Method of Teaching Logarithms, Mathematics Teacher (12), 690-692

Thoumasis, C. (1993). Teaching Logarithm Via Their History, School Science and Mathematics 8(12), 428-434

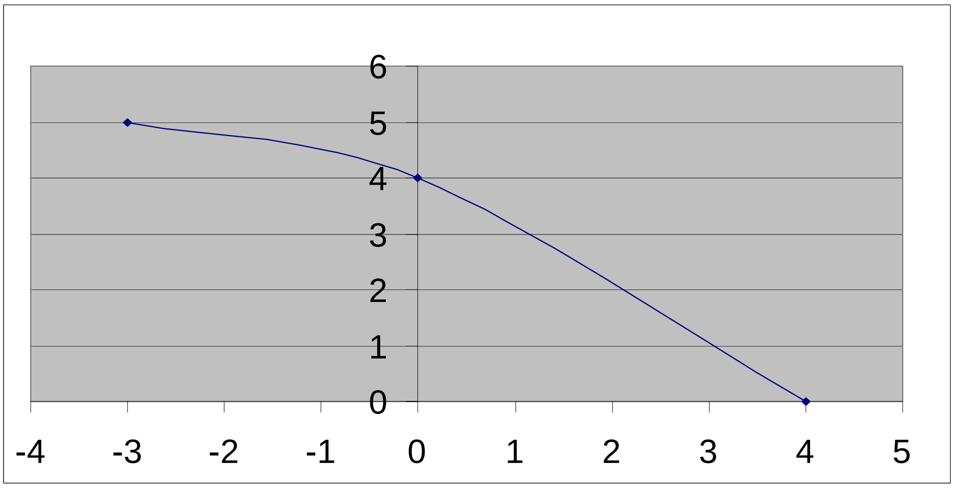
Tomei (2003) http:// www.daq.edu/~tomei/ed711psy/c_bruner.htm

APPENDIX A

MATHEMATICS ACHIEVEMENT TEST

1. $f(x) = 2^{4X+2} + 16^{X}$ olduğuna göre, $f^{-1}(320) = ?$

2. Aşağıdaki grafik f(x) fonksiyonuna aittir. $fof(4) + f^{-1}(5)$ toplamının değeri kaçtır kaçtır?



3. Reel sayılar kümesinde $*$ işlemi $a*b = 2a + b - 2(b*a)$ olarak tanımlanıyor.Buna göre, $3*2$ işleminin sonucu kaçtır?

4. Bir A kümesinin 3 ten az elemanlı alt kümelerinin sayısı 16 olduğuna göre, A kümesinin eleman sayısı kaçtır?

5. $f(x) = \frac{1}{2x+3}$ olduğuna göre, $f^{-1}(x)$ in $f(x)$ türünden değeri nedir?

6. R de $a \oplus b = 2a - b$, $a * b = 3ab - 1$ işlemleri veriliyor $(2 \oplus 3) * k = 14$ ise k kaçtır?

7. R den R ye tanımlanan f ve g fonksiyonları için $f(x) = x - 1$, $fog(x) = 3x - 7$ ise $g^{-1}(4)$ ün değeri nedir?

8. $f(x+3) = 4x+7$ ise $f(x)$ nedir?

APPENDIX B

SCORING RUBRIC

Points	Description
12	Completed all operations and computations correctly.
9	Despite of one or more errors in details, completed operations and computations.
6	Despite of some missing values,completed at least half the studies.
3	Incompleted most of the operations and computations.
0	Failed to attempt the studies.

APPENDIX C

LOGARITHM ACHIEVEMENT TEST

1-) $\log\left(\dfrac{x+999}{10}\right)=2$ ise x nedir?

2-) $\dfrac{\log 50+\log 16-\log 2}{\log 20}=?$

3-) $\log_2(\log_{10} 2)=1$ ise x nedir?

4-) $\dfrac{1}{\log_3 12}+\dfrac{1}{\log_8 12}+\dfrac{1}{\log_6 12}=?$

5-) $\log 2=a$, $\log 3=b$, $\log 1656=c$ ise $\log 23$'ün a,b ve c türünden değeri neye eşit olur?

6-) $\log 8-2\log x=2\log\dfrac{1}{x}+\log x$ denkleminin çözüm kümesini bulunuz.

7-) $\log 40=\dfrac{1}{2}(\log 20+\log b)$ olduğuna göre b neye eşittir?

8-) $6\log(x^2)=3(\log x)^2$ olduğuna göre $\log x=?$

APPENDIX D

ÖĞRENCİ GÖRÜŞ VE TUTUM ANKETİ (SVA)

Genel Açıklama: Derslerde yaptığınız 'LOGARİTMA' etkinlikleriyle ilgili düşüncelerinizi ve önerilerinizi bilmek ve Matematik derslerinde size yardımcı olmak istiyoruz. Bu nedenle, sizden bazı bilgiler edinmek istedik.
Aşağıdaki önermeleri dikkatlice okuyun ve kendi düşüncenizi yansıtacak biçimde cevaplayınız. Cevap verirken aşağıda kullanılan kısaltmalara, TA, KA,U, KD, TD bakınız ve birini seçiniz. Bu önermelerin doğru ya da yanlış diye bir yanıtı yoktur. Düşüncelerinizi ayraç içine tik işareti (√) koyarak belirtiniz .

Kısaltmalar: **TA:**Tümüyle katılıyorum. **KA**:Kısmen katılıyorum.
U:Çekimserim
KD:Kısmen katılmıyorum **TD:**Tümüyle katılmıyorum.

A. KİŞİSEL BİLGİLER

I. Dönem Matematik Karne Notu:	....	Adı-Soyadı:	....	
Seçmek İstediğiniz Meslek:	....	Cinsiyet :	[ ] Kız [ ] Erkek	
Alan:	Ön Test:	Sınıf/Şube:	............/............	

B1. LOGARİTMA ÖĞRETİMİ ETKİNLİKLERİ

	Hazırlanan ve Sınıfta Uygulanan Logaritma Öğrenme/Öğretme Etkinlikleri:	TA	KA	U	KD	TD
1	• Logaritma kavramını kazandırıyor.	( )	( )	( )	( )	( )
2	• Logaritmanın özelliklerini öğrenmeyi sağlar.	( )	( )	( )	( )	( )
3	• Logaritmanın özellikleriyle işlem yapma becerisini geliştirir.	( )	( )	( )	( )	( )
4	• İlgi çekecek niteliktedir.	( )	( )	( )	( )	( )
5	• Logaritmanın özelliklerini kolayca kavramayı sağlar.	( )	( )	( )	( )	( )
6	• Konuyu anlayarak ve daha kalıcı öğrenmemizi sağlıyor.	( )	( )	( )	( )	( )
7	• Düşünme ve yorumlama gücümüzü artırıcıdır.	( )	( )	( )	( )	( )
8	• Kavramları ve kuralları ezberlemeden kurtarıyor.	( )	( )	( )	( )	( )
9	• Dersler daha zevkli hale geliyor.	( )	( )	( )	( )	( )
10	• Konunun günlük hayatla birleştirilmesi öğrenmeyi kolaylaştırıyor.	( )	( )	( )	( )	( )
11	• Konuyu somutlaştırıyor.	( )	( )	( )	( )	( )
12	• Konuyu kendi kendimize öğrenmede bize yardımcı oluyor.	( )	( )	( )	( )	( )
13	• Ufkumuzu açıyor, düşünce boyutumuzu genişletiyor.	( )	( )	( )	( )	( )

C1. ETKİNLİKLERDEKİ KONU İŞLENİŞLERİ: DÜŞÜNCE VE ÖNERİLER

1. **Önceki konuların işlenişi ile bu konunun işleniş tarzını karşılaştırarak görüş ve önerilerinizi açık ve öz bir biçimde yazınız.**

2. **Etkinlikler sırasında yapmış olduğunuz bir dizi çalışmayı ve kendi vazifenizi ne derece benimsediniz? Niçin?**

3. **Logaritma konusunu öğrenirken sizin daha aktif olarak derse katılmanızın olumlu ya da olumsuz sonuçları ne oldu? Niçin?**

4. **Öğretmenin kuralları doğrudan (direkt) ve hazır vermesiyle, formülleri kendiniz keşfederek bulmanız arasında bir karşılaştırma yaparak konu ile ilgili düşüncelerinizi yazınız.**

APPENDIX E

ACTIVITY SHEETS IN EXPERIMENTAL GROUP

DISCOVERY SHEETS

HAZIRLIK ETKİNLİĞİ: Fonksiyon ve Ters Fonksiyon Değerlerini Bulma

Aşağıdaki alıştırmalarda , soruları aşağıdaki tablolar yardımıyla çözüp, cevabını yandaki boşluklara yazınız.

f: x→2^x olsun. $f(x) = 2^x$ $f^-1(y)$: 2^x→x $f^{-1}(y) = x$

x	1	2	3	4	5	6	7
$f(x)$	2	4	8	16	32	64	128

Y	2	4	8	16	32	64	128
$f^{-1}(y)$	1	2	3	4	5	6	7

g: x→3^x olsun. $g(x) = 3^x$ g^{-1}: 3^x→x $g^{-1}(y)=x$

X	1	2	3	4	5	6
g(x)	3	9	27	81	243	729

Y	3	9	27	81	243	729
$g^{-1}(y)$	1	2	3	4	5	6

A1. $f(x) = 2^x$ ise

1. $f(2)$ = ? ----- f^{-1} (4) =? -----

2. $f(4)$ = ? ----- $f^{-1}(16)$ =? -----

3. $f^{-1}(y) = x = \log_2 y$ olduğuna göre, $f^{-1}(32) = \log_2 32$ =? -----

A2.
1. g (2) = ? ----- g^{-1}(9) = ? -----

2. g (3) = ? ----- $g^{-1}(27)$ =? -----

3. $g^{-1}(y)=x=\log_3 y$ olduğuna göre, $g^{-1}(243) = \log_3 243$ = ? -----

ETKİNLİK -I Logaritmanın Çarpım Özelliği

Aşağıdaki alıştırmalarda, soruları aşağıdaki tablolar yardımıyla çözüp, cevabını yandaki boşluklara yazınız.

$f: x \to 2^x$ olsun. $f(x) = 2^x$ $\qquad$ $f^{-}1(y) : 2^x \to x$ $\quad f^{-1}(y) = x$

x	1	2	3	4	5	6	7
$f(x)$	2	4	8	16	32	64	128

Y	2	4	8	16	32	64	128
$f^{-1}(y)$	1	2	3	4	5	6	7

g: $x \to 3^x$ olsun. $g(x) = 3^x$ $\qquad$ $g^{-1}: 3^x \to x$ $\quad g^{-1}(y)=x$

X	1	2	3	4	5	6
g(x)	3	9	27	81	243	729

Y	3	9	27	81	243	729
$g^{-1}(y)$	1	2	3	4	5	6

g: $x \to 3^x$ olsun. $g(x) = 3^x$ $\qquad$ $g^{-1}: 3^x \to x$ $\quad g^{-1}(y)=x$

1. $f^{-1}(8)= f^{-1}(2.4) =?$ -------- $f^{-1}(2) + f^{-1}(4) = ?$ --------

2. $f^{-1}(128)= f^{-1}(8.16) =?$ -------- $f^{-1}(8) + f^{-1}(16) = ?$ --------

3. $f^{-1}(32)= f^{-1}(4.8)=\log_2(4.8) = ?$ --------- $f^{-1}(4) + f^{-1}(8) = \log_2 4 + \log_2 8 = ?$ ---------

4. $g^{-1}(81)= g^{-1}(3.27) =?$ ------- $g^{-1}(3) + g^{-1}(27) = ?$ -------

5.. $g^{-1}(729)= g^{-1}(9.81) =?$ ------- $g^{-1}(9) + g^{-1}(81) = ?$ -------

6. $g^{-1}(243)= g^{-1}(9.27) = \log_3 (9.27) = ?$ ----- $g^{-1}(9) + g^{-1}(27) = \log_3 9 + \log_3 27 = ?$ ---------

I. $f^{-1}(y_1 . y_2)$ hakkında nasıl bir tahmin yapabilirsiniz?$\mathbf{f^{-1}(y_1 . y_2) = ?}$ ------

II. $f^{-1}(y) = \log_2 y$ $\quad g^{-1}(y) = \log_3 y$ olduğuna göre log**a** $(y_1.y_2)$hakkında nasıl bir tahmin yapabilirsiniz? **loga** $\mathbf{(y_1.y_2) = ?}$ ------------

ETKİNLİK-II Logaritmanın Bölüm Özelliği

Aşağıdaki alıştırmalarda , soruları aşağıdaki tablolar yardımıyla çözüp, cevabını yandaki boşluklara yazınız.

f: $x \to 2^x$ olsun. $f(x) = 2^x$ $f^{-}1(y) : 2^x \to x$ $f^{-1}(y) = x$

x	1	2	3	4	5	6	7
f(x)	2	4	8	16	32	64	128

Y	2	4	8	16	32	64	128
f^{-1}(y)	1	2	3	4	5	6	7

g:

$x \to 3^x$ olsun. $g(x) = 3^x$ $g^{-1}: 3^x \to x$ $g^{-1}(y)=x$

X	1	2	3	4	5	6
g(x)	3	9	27	81	243	729

Y	3	9	27	81	243	729
g^{-1}(y)	1	2	3	4	5	6

1. $f^{-1}(16/2) = f^{-1}(8) = ?$ -------- $f^{-1}(16) - f^{-1}(2) = ?$ --------

2. $f^{-1}(128/32) = f^{-1}(4) = ?$ -------- $f^{-1}(128) - f^{-1}(32) = ?$ --------

3. $f^{-1}(8) = f^{-1}(32/4) = \log_2 (32/4) = ?$ ---------- $f^{-1}(32) - f^{-1}(4) = \log_2 32 - \log_2 4 = ?$ ----------

4. $g^{-1}(27/9) = g^{-1}(3) = ?$ -------- $g^{-1}(27) - g^{-1}(9) = ?$ --------

5. $g^{-1}(81/3) = g^{-1}(27) = ?$ -------- $g^{-1}(81) - g^{-1}(3) = ?$ --------

6. $g^{-1}(81) = g^{-1}(729/9) = \log_3 (729/9) = ?$ ------- $g^{-1}(729) - g^{-1}(9) = \log_3 729 - \log_3 9 = ?$ ----------

I. $f^{-1}(y_1 / y_2)$ hakkında nasıl bir tahmin yapabilirsiniz? $\mathbf{f^{-1}(y_1 / y_2) = ?}$ --------

II. log**a**(y_1/y_2) hakkında nasıl bir tahmin yapabiliriz ?

loga(y_1/y_2) =?---------

ÖĞRETMEN REHBERİ-1

Ünitenin adı : Logaritma

Konunun adı : Logaritmanın çarpım özelliği

Sınıf : 9-10.

Süre : 15 dk.

Eğitim Malzemeleri :Kağıt ve kalem ,Sınıf tahtası, Etkinlik kağıdı

Gerekli Ön Bilgiler:

a-) Öğrenciler fonksiyon kavramını ve üstel fonksiyonu bilmeli .

b-) Öğrenciler ters fonksiyon kavramını bilmeli.

Hedefler :

1-) Logaritma kavramını üstel fonksiyonun ters fonksiyonu olarak kavramak.

2-) Logaritmanın çarpım özelliğini keşfetmek

İşleniş:

Öğrenciler en fazla üç kişilik gruplarda çalışmalı. Onlara işlemleri basamak basamak yapmalarını söyleyin . Başlangıçta üstel ve ters fonksiyon kavramlarını hatırlatın .

ÖĞRETMEN REHBERİ-2

Ünitenin adı : Logaritma

Konunun adı : Logaritmanın bölüm özelliği

Sınıf : 9-10.

Süre : 15 dk.

Eğitim Malzemeleri :Kağıt ve kalem ,Sınıf tahtası, Etkinlik kağıdı

Gerekli Ön Bilgiler:

a-) Öğrenciler fonksiyon kavramını ve üstel fonksiyonu bilmeli .

b-)Öğrenciler ters fonksiyon kavramını bilmeli.

Hedefler :

1-) Logaritma kavramını üstel fonksiyonun ters fonksiyonu olarak kavramak.

2-) Logaritmanın bölüm özelliğini keşfetmek.

İşleniş:

Öğrenciler en fazla üç kişilik gruplarda çalışmalı. Onlara işlemleri basamak basamak yapmalarını söyleyin . Başlangıçta üstel ve ters fonksiyon kavramlarını hatırlatın .

CEVAP KAĞIDI – 1 Logaritmanın Çarpım Özelliği

1. $f^{-1}(2.4) = f^{-1}(8) = 3$ $\quad f^{-1}(2) + f^{-1}(4) = 1+2=3$

2. $f^{-1}(8.16) = f^{-1}(128)= 7$ $\quad f^{-1}(8) + f^{-1}(16) = 3+4 = 7$

3. $f^{-1}(4.8) = f^{-1}(32) = 5$ $\quad f^{-1}(4) + f^{-1}(8) = 2 +3 = 5$

4. $g^{-1}(3.27) = g^{-1}(81)= 4$ $\quad g^{-1}(3) + g^{-1}(27) = 1 + 3 = 4$

5..$g^{-1}(9.81) = g^{-1}(729)= 6$ $\quad g^{-1}(9) + g^{-1}(81) = 2 + 4 = 6$

6. $g^{-1}(9.27) = g^{-1}(243) = 5$ $\quad g^{-1}(9) + g^{-1}(27) = 2 + 3 = 5$

$f^{-1}(y) =x= \log_2 y$, $g^{-1}(y)=x=\log_3 y$ olduğuna göre,

bu örneklerden $\log_2(y_1 .y_2) = \log_2 y_1 + \log_2 y_2$ ve $\log_3(y_1 .y_2) = \log_3 y_1 + \log_3 y_2$ eşitlikleri ortaya çıkıyor.

Demek ki $\log_a(y_1 .y_2) = \log_a y_1 + \log_a y_2$ eşitliği daima sağlanıyor. Yani iki sayının çarpımlarının logaritması bu sayıların ayrı ayrı logaritmalarının toplamına eşittir.

CEVAP KAĞIDI – 2 Logaritmanın Bölüm Özelliği

1. $f^{-1}(16/2) = f^{-1}(8) = ?$ 3 $f^{-1}(16) - f^{-1}(2) = 4 - 1 = 3$
2. $f^{-1}(128/32) = f^{-1}(4) =$ 2 $f^{-1}(128) - f^{-1}(4) = 7 – 2 = 5$
3. $f^{-1}(32/4) = f^{-1}(8) =$ 3 $f^{-1}(32) - f^{-1}(4) = 5 – 2 = 3$
4. $g^{-1}(27/9) = g^{-1}(3) =$ 1 $g^{-1}(27) - g^{-1}(9) = 3 – 2 = 1$
5. $g^{-1}(81/3) = g^{-1}(27) =$ 3 $g^{-1}(81) - g^{-1}(3) = 4 – 1 = 3$
6. $g^{-1}(729/9) = g^{-1}(81) =$ 4 $g^{-1}(729) - g^{-1}(9) = 6 – 2 = 4$

$f^{-1}(y) = x = \log_2 y$, $g^{-1}(y) = x = \log_3 y$ olduğuna göre ,

bu örneklerden $\log_2(y_1 / y_2) = \log_2 y_1 - \log_2 y_2$ ve $\log_3(y_1 . y_2) = \log_3 y_1 - \log_3 y_2$ eşitlikleri ortaya çıkıyor.

Demek ki $\log_a(y_1 . y_2) = \log_a y_1 - \log_a y_2$ eşitliği daima sağlanıyor. Yani iki sayının bölümlerinin logaritması bu sayıların ayrı ayrı logaritmalarının farkına eşittir.

LOGARİTMANIN KULLANIM ALANLARI

- **13,75.276,2. 0,04075 =?**

ÇÖZÜM:

13,75.276,2.0,04075 =A olsun.

log(3,75.276,2.0,04075) =logA

log(13,75.276,2. 0,04075) =log13,75+log276,2+log0,04075 =2,1896

Antilog2,1896=154,7=A

13,75.276,2. 0,04075 =154,7

- $\sqrt{827,2}$ **= ?**

ÇÖZÜM:

$\sqrt{827,2}$ = A olsun.

log $\sqrt{827,2}$ = logA

$= \frac{1}{2} \cdot \log 827,2 = \frac{1}{2} \cdot 2,9176$

=1,4588

Antilog1,4588 = 28,76 =A

$\sqrt{827,2}$ = 28,76

- 17 Ağustos depremi sonucu 6.10^{15} joule enerji açığa çıktığı bilindiğine göre , bu deprem rihter ölçeğine göre kaç şiddetinde olmuştur?

ÇÖZÜM:

$$M = \frac{2}{3} \log \frac{E}{E_0}$$

M= rihter ölçeğine göre deprem şiddeti

E= deprem sonucunda ortaya

çıkan enerji

$E_0 = 10^{4.40}$ joule

$$M = \frac{2}{3} \log \frac{6.10^{15}}{10^{4,40}} \cong 7,6 \text{ rihter}$$

- Van gölü yüzeyindeki ışığın yoğunluğu 12 lümen ,

k= -0,0125 ise 30 metre derinlikte ışığın yoğunluğu ne olur?

ÇÖZÜM:

$$\log \frac{I}{I_0} = k.\ X$$

X = derinlik

I_0 : ışığın baştaki yoğunluğu

I : ışığın maddenin içinden geçtikten sonraki yoğunluğu

$$\log \frac{12}{I} = -0,0125.\ 30$$

$$\log \frac{12}{I} = -0,375$$

$$\frac{12}{I} = 10^{-0,375}$$

$$I= \frac{12}{1,43} = 8,35 \text{ lümen}$$

- 50000 nüfuslu bir ilçenin yıllık ortalama %2,5 nüfus artış oranıyla nüfusu ne kadar bir sürede 100.000 ‘e ulaşır?

ÇÖZÜM:

$P(t)= P_0.(1+r)^t$

P_0: şimdiki nüfus

P(t):gelecekteki nüfus

r: nüfus artış yüzdesi

t: yıl

$$100.000= 50.000.(1+0,025)^t$$

$$2 = 1,025^t$$

$$\log 2 = \log 1,025^t$$

$$\log 2 = t.\log 1,025$$

$$t = \frac{\log 2}{\log 1,025} = 28,071$$

28 sene ve birkaç gün

VERİ ANALİZİ ETKİNLİĞİ

Aşağıdaki eksersizlerde veri analizini logaritma ve grafikler yardımıyla yapmaya çalışacağız.

Grafik Hesap Makinesini ya da EXCEL'i kullanarak.

1. Aşağıdaki verileri girin.

Kütle(kg.)	20	50	80	110	125	140
Kan Basıncı(mm.Hg.)	106	133	150	162	167	172

2. Bu verilerin 'e' tabanına göre logaritmalarını alın.

3. x-y dağılım grafiğini çizin. Nasıl bir grafik çıkıyor ?

4. Aynı şekilde (lnx, lny) grafiğini çizin. Nasıl bir grafik çıkıyor ?

5. Yukarıdaki bilgileri kullanarak kan basıncını ağırlığa bağlı bir fonksiyon olarak ifade et. Üstel bir fonksiyon çıkıyor mu ?

6. Bulduğun bu fonksiyonu $\ln y = m.\ln x + c$ şekline dönüştür.

7. Yukarıdaki verilerden bir tanesini formülü kullanarak test et. Formül doğru sonuç veriyor mu ?

8. Formülü kullanarak kendi kan basıncını bul.

APPENDIX F

VIEWS and OPINIONS of STUDENTS ABOUT DABI

1. Retention of the Subject

- I will never forget the formulas for log(a.b) and log(a/b).
- Instead of memorizing the formulas which the teacher gave us, we understand it better by discovering it on our own because I have heard that students remember 10% of what they read and 90% of what they see and use.
- We acquired long term learning when the subject is suited with the current topic. On the other hand, it sometimes confuses our minds.
- Instead of being out of the event or hearing event from the second person, being in the event not only help it being unforgettable, but also, I believe that, it make lesson funnier.
- What we learn in math is only solving the problems that are given to us. But it would be more clear and unforgettable if we learn when or how we get the formulas-in other words if we learn all of the things in DABI that we used in learning logarithm.
- Using graphing calculators, discovering rules make the learning long-term

- In traditional instruction, you use your mind only for listening task but when you learn it by applying you try and this is the long-term learning.
- Certainly applying better. (Why?) If I put into practice something which I have just learned, it is kept in my mind for a long time and I think it is really important for students. In my opinion,. If I do not know where to apply the subject I have learned then why should remember it after examinations.
- I see it and do it myself –it is different. Unless we don't practice and apply ourselves, it won't be more different than opening an empty box with a hole and fill it.
- Sometimes it is difficult to concentrate on the subject. But with logarithm activities concentration is not a problem because we actively participate to the lesson. So I think it will be more retentive.

2. Enjoyment

- Discovering formulas gives a different satisfaction..
- I feel as if I was dealing with an exciting career, trying to finish an important project.
- It was funny and interesting because we used calculator. Also it was clear for us because we saw the graphs of the operations.
- Now I have an enthusiasm to discover something.

- Now I like logarithm. Of course sometimes I can not solve logarithm problems but this does not prevent me from liking it. This might be because we learned where logarithm is used and that we actually apply it.
- I like this activities because I had a chance to implement the rules.
- I understood that logarithm destroyed the words “write, read, study” that we use in other lesson.
- Logarithm activities succeeded to make us love logarithm.
- I was more interested because lessons were funnier. Also I thought that it was my responsibility and I adopted it.
- If only every subjects are taught like that.
- I think teachers and preservice teachers should be given a course about applications like that. They should learn how to make lessons funnier.
- We saw logarithm’s pleasure when we saw that tools. Drawing graphics was where enjoyable.
- Learning it by applying is more enjoyable than only listening. Because it is more interesting. I think technology makes it interesting.
- At first making it with only formulas, I was confused. But after using calculator, I solved the problems. I understood that it wasn‘t confusing. It was very enjoyable and retentive. Before we solved application examples, I didn’t know why we were learning it. But when I learnt where it is used, it became more enjoyable.

- At first l didn't like it and l thought it was very confusing. But after using calculator, l realized that it was very enjoyable.
- Students will love mathematics. They enjoy while learning. They understood that mathematics is not as difficult as that they believe.

3. About Traditional Instruction

- We learned where the subject is used. However, in traditional method, we just make operation and it is a little bit boring.

- Learning without learning where the subject is used not only leaves questions in our minds but also makes, us don't understand the subject so we think that the subject is not useful.

- In traditional method you only learn to make operations.

- When the teacher solves the problems on the board, you must memorize the types of problems. But when you learn it by applying you remember the graphics and formulas.

- We ask ourselves how we can use this subject, when it can be useful. If we imagine, we can have some answers. If we apply it, we can reply ourselves. If we try to take in something with no reason, we fail.

- Most of schools give lessons with reciting, like this: " take this number, add it, divide with that etc. "What if I subtract it, why do I have to make it this way?

- I'd like to know what topic is used in which area; I think nothing is useless in mathematics.

-What happens when it is not told?

- I don't enjoy the lesson and I give it up. (This is the reason why students sometimes fall asleep during lesson times)

- Applying the subject made me keep the knowledge in my mind, and this is a good thing. It is hard but good. I can't take any pleasure without understanding the lesson; only using numbers, and it takes away my desires.

- I want to know where it is used; I don't want to solve logarithm problems, which I don't understand as in the exam.

- Normally, I don't really like mathematics. Solving problems, learning examples don't make me feel so good. But if I really understand the subject and apply it, I can feel better. Maybe I will like mathematics in the future with the help of this instructional method (DABI).

- Learning by only listening makes us memorize it. Sometimes we don't listen and we can't pay attention. But when we learn it by applying the probability of not paying attention is removed, because it is enjoyable.
- When we learn logarithm with traditional method, we should memorize everything about it. But when we learn its advantages, it's different. We study other mathematic subject for only grades but now we study them in order to learn.

4. Self - Efficacy

- When I find something on my own, I get courage and of course this brings on more successes. Otherwise I feel like I have to do something and this worries me.
- My self-esteem has increased. Having the responsibility of something, overcoming it and believing that I can manage to do has taught me that I shouldn't be afraid of the problems in life.
- I had more responsibility.
- It is also the activity that develops us socially.
- We have belief in ourselves.
- I feel that l can succeed. I learn that nothing is frightened; l can make everything if l wants.
- Discovering the formula by ownself makes it enjoyable. And this increases our self-confidence.
- : How would it be if you apply the logarithm problems?

A: We would gain self-confidence, we would be able to solve problems by ownself, and we will do something without teachers` help.

- (About data analyzing) I got self confidence

(What made you get it?) To be successful.

5. Increasing Motivation Toward Lesson

- Our enthusiasm to study mathematics increases when we learn that the subject we are taught is necessary and will help us we will have in the future.

- Now, logarithm have a special place in my life because I strived to learn it, I had an active role.

- With the help of logarithm activities we add the subject to the daily life.

- We not only studied on a subject but also learnt it.

- If we don't know where we will use it, it can be boring. But if we know where it is used , our motivation increases.

- When you learn it yourself, you say, " I can do it, I am successful in logarithm ". You can pay attention, you enjoy and your interest increase.

- Learning it by applying, take our attention and make learning easier.

- Of course learning it by applying and learning where we use it in future is better because we can't say why we learn it if we won't use it in the future and learning its applications takes our attention.

- When you learn applicationsyou feel that you don't do meaningless things

(What benefits does it give when you imagine it in your mind?)

At least I realize what I am doing, so I try to be more successful.

- Logarithm activities made me keep my eyes on and feel myself a part of the lessons. We all made something really useful –noticing what is going on, I mean the control was in our hands so we studied harder to create good things and have the others` congratulate. That was a remembrance harmony.

6. Facilitation

- You discover, you find, you learn, briefly you are in the subject in all prospects. So, students don’t have any difficulty.
- I have learnt without memorizing.
- We look into the subject and get the basics. It is better than scribbling nonsense on paper.
- I won’t push myself too hard next year with questions like “what is logarithm?” and “where did it come from?”
- Any teacher can give the formulas but not every teacher can teach them.
- When teachers give us information, we don’t think about it much. Even if we have trouble in finding it on our own, when we find it by ourselves we understand it better.

- I think, putting it into practicel make the learning easier. Because while we were only listening, we could learn only specific subjects. We perceived logarithm easily when we learn it by applying.

- It is easier for you to learn something if you know what you are going to do with it. We are not solving problems in vain. We know how it works.

- Students will love math. They enjoy while learning. They understood that mathematics Isn't as difficult as they thought.

- I understand better when origins of the properties and rules are explained with their reasons. In this way, I don't have to recite some formulas without understanding.

7. About Errors in Logarithm

- (When origins of the properties and rules are explained with their reasons) I realize the formula better and the probability of making mistakes decreases.

- In the last example l remembered the two graphics.

- That graphics and tools about logarithm decrease the probability of making mistake.

- I think, making it in practical make the understanding easier. Because while we were only listening, we could learn only specific subjects. We perceived logarithm easily when we learn it in practical.

8. About Uninterested Students in Lessons

- With this style of studying, even students who were lazy without self-trust understood that they could do it if they wanted, and it gave them the habit of trusting themselves. When the teacher used traditional method, he or she was focusing (reluctantly) on some students. But when we used those machines, everyone understood the lesson. I think this is more effective than traditional method.

Comparative answers to the questions about "Logarithm Concept" during features

Q-1) What did you gain by learning logarithm?

CG-1

B- It can't be said that it has helped me to gain anything

R- It provided great benefit to the high school-2 students

CG-2

Bu- In my opinion, it didn't help us to gain anything. I learnt the subject in the end. But I didn't consider it important since I didn't think I would use it in the future.

F- Above all, mathematics broadens our horizon, K- It provided me easiness for the next year,

The answers of experiment group students

EG-1

Rü- I have learnt that everything is in an order,

Ç- It told me that everything was in a ratio, For example, It told me that there was a ratio between our weight and blood pressure.

EG-2

H- It helpd us to learn better by applying. Even if it became difficult to do with those calculators, it was better than learning uselessly.

E- understood that the problems in mathematics could be solved easier. I learnt that logarithm included many subjects.

S- It was enjoyable to learn the logarithm subject.

Q-2) What do you understand from the term logarithm of a number?

CG-2

Bu- It doesn't mean anything. When I say Log 5, I get confused.

Ö- It doesn't mean anything.

K- It tells the base of that number

CG-1

B- Nothing occurs in my mind.

A- Nothing.

R- It doesn't remind me of anything.

Experimental group

EG-1

Ç- The logarithm of a number means the opposite of that number's basic function.

T- Actually It is a part of a formula.

Ru- It is the opposite of a function.

EG-2

S- When I see that process, I remember the base of that number. In any way, we had learnt it as the opposite of basic function.

H- It reminds me of the complexity and width of the mathematics.

E- Especially basic numbers and easiness occur in my mind.

Q-3) What does logarithm connotates you?

The answers of control groups

CG-2

Ö- It doesn’t mean anything.

K- Personally, it doesn’t mean anything.

B- It is only a subject in mathematics.

CG-1

B- It doesn’t mean anything because I don’t know anything about logarithm.

E- Complex numbers.

R- It doesn’t mean anything.

The answers of experiment groups

EG-1

T- It explains the function of numbers in our daily life. It reminds me three dimensional things.

M- It reminds me of the graphics in the classroom.

Ru- The calculation of the brightness of the stars in astronomy, usage of logarithm in photography, the ratio of our weight and blood pressureoccur in my mind.

EG-2

S- The base of a number occurs in my mind.

H- The formulas which you made to calculate the periods between planets and the calculation of the blood pressure were very benefical.

Q-4) When you see a logarithm problem, how do you feel ?

The answers of control groups

CG-2

Ö- It is something no different from other questions.

K- It sounds as interesting and as solving a secret.

Bu- I say “ I won't be able to solve this “ to myself. I lose my self-confidence.

F- I think it is easy.

CG-1

B- I get scared when I see it for the first time because it sounds too difficult. But I feel that it gets easier when I begin to solve.

A- At first, it is too difficult. After a while, I can do it because I realize that it is easy.

R- I get scared at first but I like it when I realize that it is easy.

Ay- It doesn’t cause any problem. It is nice.

The answers of experiment groups

EG-1

M- Although logarithm was difficult, I could do it better than other things.

Rü- It was complex when done with only formulas at first. I said “ What will we do by using this? “ Why do we use logarithm “ to myself. But later, I realized that it was enjoyable when I used calculator.

EG-2

S- It was boring at first. I thought that it was very enjoyable when I learnt all the subjects. I learnt that everything was used in somewhere so I could solve easily.

H- I try to do if I can. I don’t want to solve the problems uselessly as it is during the exams. I want to know their functions.