MODEL INDEPENDENT ANALYSIS OF RARE, EXCLUSIVE B–MESON DECAYS

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MEHMET KERIM ÇAKMAK

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Approval of the Graduate School of Natural and Applied Sciences.

Prof. Dr. Canan Özgen
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Sinan Bilikmen
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

Prof. Dr. Mustafa Savcı
Supervisor

Examining Committee Members

Prof. Dr. Mustafa Savcı

Prof. Dr. Erhan Onur İltan

Prof. Dr. Takhmassib Aliev

Prof. Dr. Hüseyin Koru

Prof. Dr. Satılmış Atağ

ABSTRACT

MODEL INDEPENDENT ANALYSIS OF RARE, EXCLUSIVE B–MESON DECAYS

Çakmak, Mehmet Kerim

PhD, Department of Physics

Supervisor: Prof. Dr. Mustafa Savcı

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Using the general, model independent form of the effective Hamiltonian, the general expressions of the longitudinal, normal and transversal polarization asymmetries for ℓ^- and ℓ^+ for the exclusive $B \to K(K^*)\ell^-\ell^+$ decays has been calculated. Existence of regions of Wilson coefficients for which the branching ratio coincides with the Standard Model result, while the lepton polarizations differ from the standard model prediction is expected. Hence, studying lepton polarizations in these regions of new Wilson coefficients may be helpful in establishing new physics beyond the standard model.

Keywords: B–Meson Decays, Branching Ratio, Lepton Polarization Asymmetry

ÖZ

NADIR B–MEZON BOZUNUMLARININ MODEL'DEN BAĞIMSIZ ANALIZI

Çakmak, Mehmet Kerim

Doktora, Fizik Bölümü

Tez Yöneticisi: Prof. Dr. Mustafa Savcı

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Efektif Hamiltonun en genel ve model bağımsız formu kullanılarak, $B \to K(K^*)\ell^-\ell^+$ bozunumları için, ℓ^- ve ℓ^- leptonlarının dik, paralel ve çapraz polarizasyon asimetrileri hesaplanmıştır. Wilson katsayıları için, dallanma oranlarının standart modelde hesaplanan oranlarla uyum gösterdiği fakat lepton polarizasyonları açısından farklılık olduğu bölglerin olması beklenmektedir. Böylece, Wilson katsayılarının bu bölgelerinde, lepton polarizasyonları üzerinde yapılacak çalışmaların standart modelin ötesinde, yeni teorilerin kurulmasında yardımcı olacağı düşünülmektedir.

Anahtar Kelimeler: B–Meson Bozunumları, Dallanma Oranı, Lepton Polarizasyon Asimetrisi

to wife with love...

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CHAPTER 1

INTRODUCTION

Experimental discovery of the rare $B \to X_s\gamma$ and $B \to K^*\gamma$ decays opened a new window in investigation of Flavor Changing Neutral Currents (FCNC) processes [1]. On the experimental side, this is due to the fact that the study of the FCNC decays will provide a precise determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, which are free parameters of the Standard Model (SM). On the theoretical part, investigation of the FCNC decays allows us to check the predictions of the SM the quantum i.e., at one-loop level [2]. For these reasons investigations on the rare radiative and semileptonic decays of B mesons received special attention. Such decays are also very useful in looking for new physics beyond SM. Especially the inclusive decay channel $b \to s(d)\ell^+\ell^-$ is known to be very sensitive to various extensions of the SM.

In this thesis, a theoretical analysis of the semileptonic exclusive $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays are carried out in a model independent way. These decays at quark level are described by the FCNC $b \to s$ transition, which has been argued in detail in section 2.1. Note that recently, BaBar and Belle Collaboration announced observation of the $B \to K\ell^+\ell^-$ decay [3]. The semileptonic decays of B mesons are much clearer compared to the non-leptonic decay modes. Since in these modes, there does not exist any problems connected with the presence of the third strong interacting particle. For this reason, in the present thesis our main interest is on semileptonic decays.

In our analysis, we mainly focused on the polarizations of the leptons, since measurement of the lepton polarizations is one of the most efficient ways to establish new physics beyond the SM [4] - [13]. We use the most general form of the effective Hamiltonian, where in addition to the penguin and vector type interactions are included in SM, we also included the scalar and tensor type interactions in the effective Hamiltonian. Using the Operator Product Expansion (OPE), method, an effective Hamiltonian can be represented as (see section 2.2) [14]-[18], $H_{eff} \sim \sum C_i O_i$. This representation have the following advance, namely one can factorize low energy weak processes in terms of perturbative short distance Wilson coefficients, C_i [19], from the long distance operator matrix elements $< O_i >$.

It is well known that theoretical analysis of inclusive decay channels, are rather easy but their experimental discovery is quite problematic. For the exclusive decays, the case is contrary, i.e., their experimental detection is easy but theoretical studies have their own drawbacks. The main problem in analyzing exclusive decays is the appearance of the form factors, i.e. the matrix elements of the effective Hamiltonian between final and initial meson states are needed. Obviously, these matrix elements as we already noted, belong to the perturbative (long distance) part of the theory.

The analysis of the semileptonic B decays includes as a first step, the derivation of the effective Hamiltonian. The effective Hamiltonian, as mentioned above and discussed in 2.2, is first obtained by Feynman diagram technique at large mass scale. Then using the renormalization group equation we can calculate the effective Hamiltonian at low energy scale (in our case, $\mu = m_b$). These two steps are calculated in the framework of the perturbative approach. In further investigation of the $B \to (KK^*)\ell^+\ell^-$ transitions, we need the matrix elements $\langle M \left| H^{eff} \right| B \rangle$. These matrix elements cannot be calculated in the framework of the perturbative approach and its calculation demands non-perturbative approach (see chapter3). These matrix elements have been studied in the framework of different approaches, such as Chiral Theory [20], Three-point QCD Sum Rules [21], and Light Cone QCD Sum Rules [22],[23]. In this thesis, we have used the Light Cone QCD Sum Rules Method as described in chapter 3.

The aim of our work is to present a rigorous study of the lepton polarizations in the exclusive $B \to (KK^*)\ell^+\ell^-$ decays for a general form of the effective Hamiltonian, including tensor and scalar type interactions as well as the vector type interaction and without forcing concrete values for Wilson coefficients corresponding to any specific model. Investigations on the lepton polarizations itself, might lead to strong indications to new physics. In our analysis, we try to answer the following question: Do certain regions of Wilson coefficients exist for which the value of the Branching ratio of the corresponding decay coincides with that of the SM prediction but its lepton polarizations do not? We have found out that such regions of Wilson coefficients indeed do exist, i.e. the study of the lepton polarizations itself can give promising information for establishing new physics beyond SM.

The thesis is organized as follows; in Chapter 2, we present a brief overview of the SM of the electroweak interactions by introducing the theoretical framework in analyzing the tree level decays and the FCNC processes. In this chapter, we also discuss briefly a more formal and more complete approach based on the Operator Product Expansion (OPE) and the renormalization group. We present the classification of all operators relevant for further analysis as well as Feynman diagrams from which they originate. We then introduce the effective Hamiltonian, and the coefficients appearing in the expression as well. Chapter 3 is devoted to the QCD Sum Rules method, which is an effective approach in calculating the form factors of the transitions from heavy to light quark systems. In Chapter 4 we present the our calculations for the lepton polarization asymmetries for the decays $B \to K \ell^+ \ell^-$ and $B \to K^* \ell^+ \ell^-$ decays. Finally, Chapter 5 contains our conclusion.

CHAPTER 2

STANDART MODEL AND FLAVOR CHANGING NEUTRAL CURRENTS

2.1 The Flavor Sector and Flavor Changing Neutral Currents

At all events, the weak and electromagnetic interactions of both quarks and leptons are described in a partially unified way by the electroweak theory ($SU(2)_L \otimes U(1)_Y$) which is based on the spontaneous breaking of the $SU(2)_L \otimes U(1)_Y$ to $U(1)_{em}$. This theory is a non-Abelian gauge theory which is based on the local gauge invariance. The new Abelian $U(1)$ group is associated with a weak analogue of hypercharge, just as $SU(2)_L$ was associated with weak isospin. The quark transitions can be represented in the left handed doublets,

$$q_L = \begin{pmatrix} u \\ d' \end{pmatrix}_L , \begin{pmatrix} c \\ s' \end{pmatrix}_L \text{ and } \begin{pmatrix} t \\ b' \end{pmatrix}_L .$$

For the weak isospin we use symbols T, T_3 and we can make specific lepton assignments as

$$T = \frac{1}{2}, \qquad T_3 = \begin{cases} +1/2 \\ -1/2 \end{cases} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L. \tag{2.1}$$

For quarks we assign in a corresponding sequence of generations,

$$T = \frac{1}{2}, \qquad T_3 = \begin{cases} +1/2 \\ -1/2 \end{cases} \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L. \tag{2.2}$$

The subscript L denotes that only the left handed parts of the wave functions enter into these weak transitions. For this reason the weak isospin group is usually referred to as $SU(2)_L$. The primes will be discussed below.

The relation of weak hypercharge with weak isospin is [24]:

$$Q = (T_3 + Y/2), \tag{2.3}$$

where Q is the electric charge (in units of e), T_3 is the third component of weak isospin and Y is the weak hypercharge. Clearly, the lepton doublets (ν_e, e^-) , etc. have $y = -1$, while the quark doublets (u, d'), etc. have $Y = 1/3$. The Electroweak interactions of quarks and leptons are mediated by massive gauge bosons $W^\pm$ and Z^0 and by the photon γ. The dynamics of the theory is described by the fundamental Lagrangian [25]:

$$\mathcal{L} = \mathcal{L}(QCD) + \mathcal{L}(SU(2)_L \otimes U(1)_Y). \tag{2.4}$$

Let us here state a few more things about the fermion-gauge-boson electroweak interactions resulting from (2.4). These interactions are summarized by the Lagrangian

$$\mathcal{L}_{int} = \mathcal{L}_{cc} + \mathcal{L}_{nc}, \tag{2.5}$$

where

$$\mathcal{L}_{cc} = \frac{g}{2\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}), \tag{2.6}$$

describes the charged current interactions and

$$\mathcal{L}_{nc} = -eJ_\mu^{em} A^\mu + \frac{g}{2\cos\theta_W} J_\mu^0 Z^{0\mu}, \tag{2.7}$$

the neutral current interactions. Here θ_W is the Weinberg angle and g is the coupling constant for weak interactions. The currents are given as follows:

$$\begin{aligned} J_\mu^+ &= (ud')_{V-A} + (cs')_{V-A} + (tb')_{V-A} + (\nu_e e)_{V-A} + (\nu_\mu \mu)_{V-A} \\ &\quad +(\nu_\tau \tau)_{V-A}, \qquad (2.8) \\ J_\mu^{em} &= \sum_f Q_f \bar{f}\gamma_\mu f, \qquad (2.9) \\ J_\mu^0 &= \sum_f \bar{f}\gamma_\mu (v_f - a_f\gamma_5) f, \qquad (2.10) \end{aligned}$$

with

$$v_f = T_3^f - 2Q_f \sin^2\theta_W, \qquad a_f = T_3^f, \tag{2.11}$$

where Q_f and T_3^f denote the charge and the third component of weak isospin, respectively. Here V and A represent the vector and axial vector currents.

Additionally, $\mathcal{L}(QCD)$ in (2.4) can be written as follows

$$\mathcal{L}(QCD) = J_{\mu\alpha}^{QCD} G^{\mu\alpha}, \tag{2.12}$$

with

$$J_{\mu\alpha} = g_s(\bar{q}\gamma_\mu \frac{\lambda_\alpha}{2} q), \tag{2.13}$$

where λ_α denotes the 8 Gell-Mann λ matrices and $G^{\mu\alpha}$ is the gluon field, g_s is the coupling constant for strong interactions, μ is Lorentz and α is the color index.

We represent the elementary interaction vertices in Fig. 2.1 which follows from the interaction Lagrangian, (see 2.5–2.13)

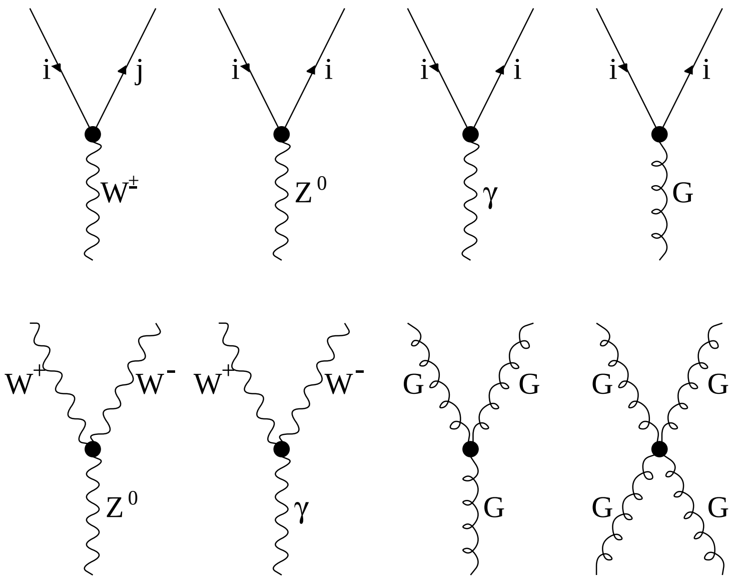

Figure 2.1: Elementary Vertices

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The striking property of the interactions listed above is the flavor conservation in the vertices involving neutral gauge bosons, Z^0, γ and G. This fact implies the absence of FCNC transitions at tree level [27]. However the charged current processes mediated by $W^{\pm}$ are obviously flavor violating with the strength of violation given by the gauge coupling g and effectively at low energies by the Fermi constant

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2},$$

and a unitary 3×3 CKM matrix [28, 29]. This matrix connects the weak eigenstates (d', s', b') and the corresponding mass eigenstates (d, s, b) through the transformation

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \times \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{2.14}$$

so that by using Eqs. (2.5–2.11) we get

$$d \xrightarrow{W^+} t = i\frac{g}{2\sqrt{2}} V_{td}\gamma_\mu(1-\gamma_5), \qquad t \xrightarrow{W^-} d = i\frac{g}{2\sqrt{2}} V^*_{td}\gamma_\mu(1-\gamma_5). \tag{2.15}$$

In the leptonic sector the analogous mixing matrix is a unit matrix due to the masslessness of neutrinos in the SM.

The unitary condition of the CKM matrix reads as

$$\sum_j V_{ij} V^*_{jk} = 0. \tag{2.16}$$

This condition also assures the absence of FCNC transitions at tree level. Moreover, the fact that V_{ij}'s can *a priori* be complex numbers allows the introduction of CP–violation in SM. FCNC transitions only occur in the one loop level in SM. The FCNC processes can be summarized by a set of basic triple and quartic effective vertices. In literature, they appear under the names of penguin (Fig. 2.2) and box (Fig. 2.3) diagrams.

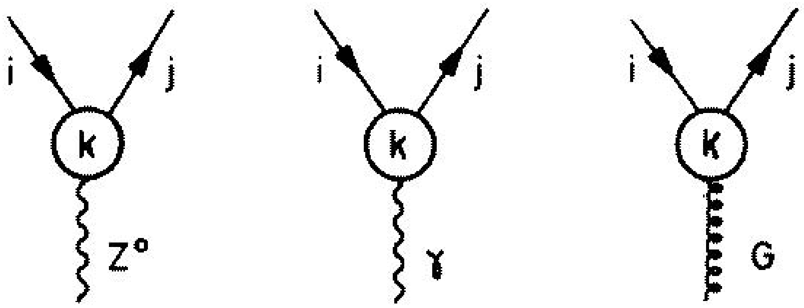

Figure 2.2: Penguin vertices

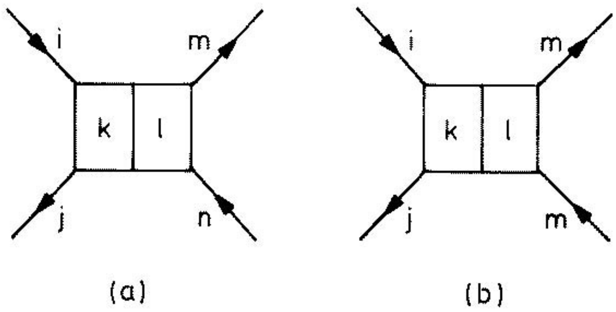

Figure 2.3: Box vertices

where i and j denote the quarks with different flavor but same charge and k is the internal quark whose charge is different from i and j, and i, j, m and n in Fig. 2.3, stand for external quarks or leptons and k and l denote the internal quarks and leptons.

Those effective vertices can be calculated by using the elementary vertices and propagators. Important examples to penguin and box vertices are given in Fig. 2.4 and Fig. 2.5, respectively.

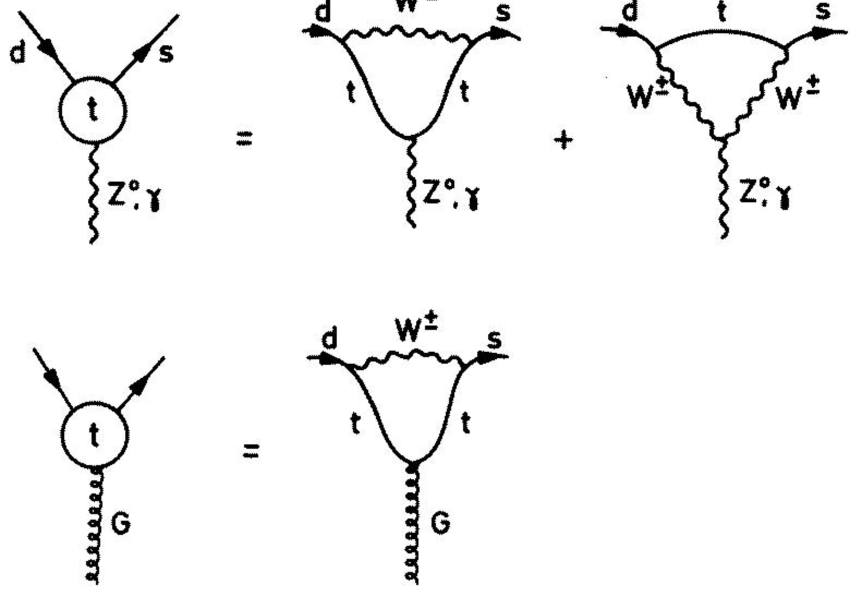

Figure 2.4: Penguin vertices resolved in terms of basic vertices

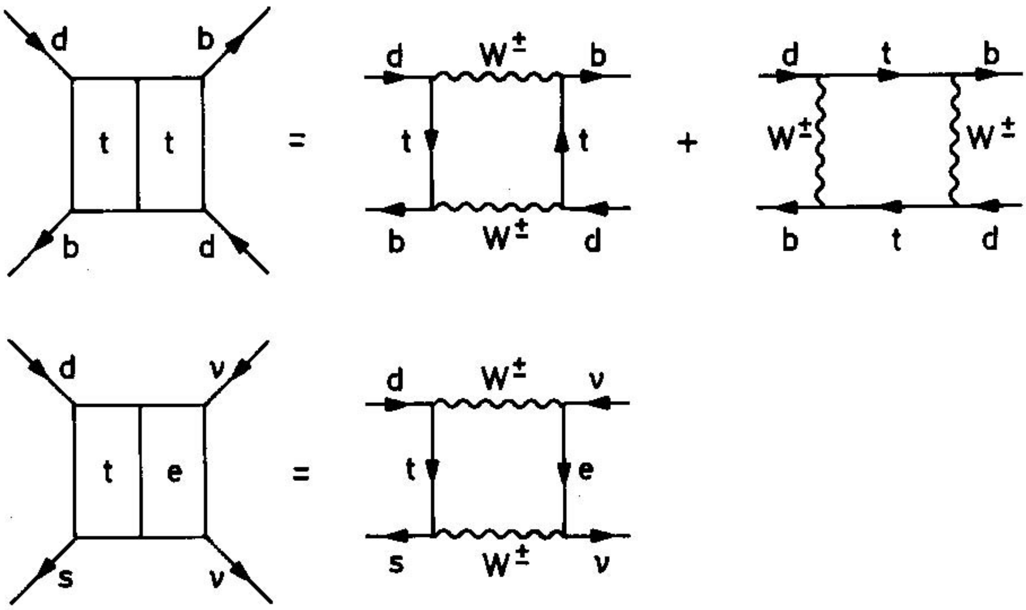

Figure 2.5: Box vertices resolved in terms of elementary vertices

2.2 The Effective Hamiltonian

For the investigation of the $b \to s\ell^+\ell^-$ transition, we will use a very powerful method, namely the effective Hamiltonian approach. Generally an effective Hamiltonian for a FCNC transition in the absence of QCD corrections is [25],

$$H_{eff}^{FCNC} = \sum_k C_k O_k, \tag{2.17}$$

where O_k denote a set of the local operators and the C_k are the coefficients of these operators called as Wilson coefficients. Firstly, these coefficients are calculated at high scale, namely $\mu = m_W$, using Feynman diagrams which describes $b \to s\ell^+\ell^-$ transition. Then, using the renormalization group equation, we can carry out the calculations in the low energy scale ($\mu = m_b$).

For the sake of the completeness of the theory, we classify the operators (O_k) and represent the typical diagrams in Fig. 2.6, which are originated by these operators as [25]:

Current-Current Operators (Fig. 2.6a):

$$Q_1 = (\bar{c}_\alpha b_\beta)_{V-A}(\bar{s}_\beta c_\alpha)_{V-A}, \qquad Q_2 = (\bar{c}b)_{V-A}(\bar{s}c)_{V-A},$$

QCD-Penguin Operators (Fig. 2.6b)

$$Q_3 = (\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V-A}, \qquad Q_4 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A},$$

$$Q_5 = (\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V+A}, \qquad Q_6 = (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A},$$

Electroweak-Penguin Operators (Fig. 2.6c)

$$Q_7 = \frac{3}{2}(\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} e_q(\bar{q}q)_{V+A}, \qquad Q_8 = \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q(\bar{q}_\beta q_\alpha)_{V+A},$$

$$Q_9 = \frac{3}{2}(\bar{s}b)_{V-A} \sum_{q=u,d,s,c,b} e_q(\bar{q}q)_{V-A}, \qquad Q_{10} = \frac{3}{2}(\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q(\bar{q}_\beta q_\alpha)_{V-A},$$

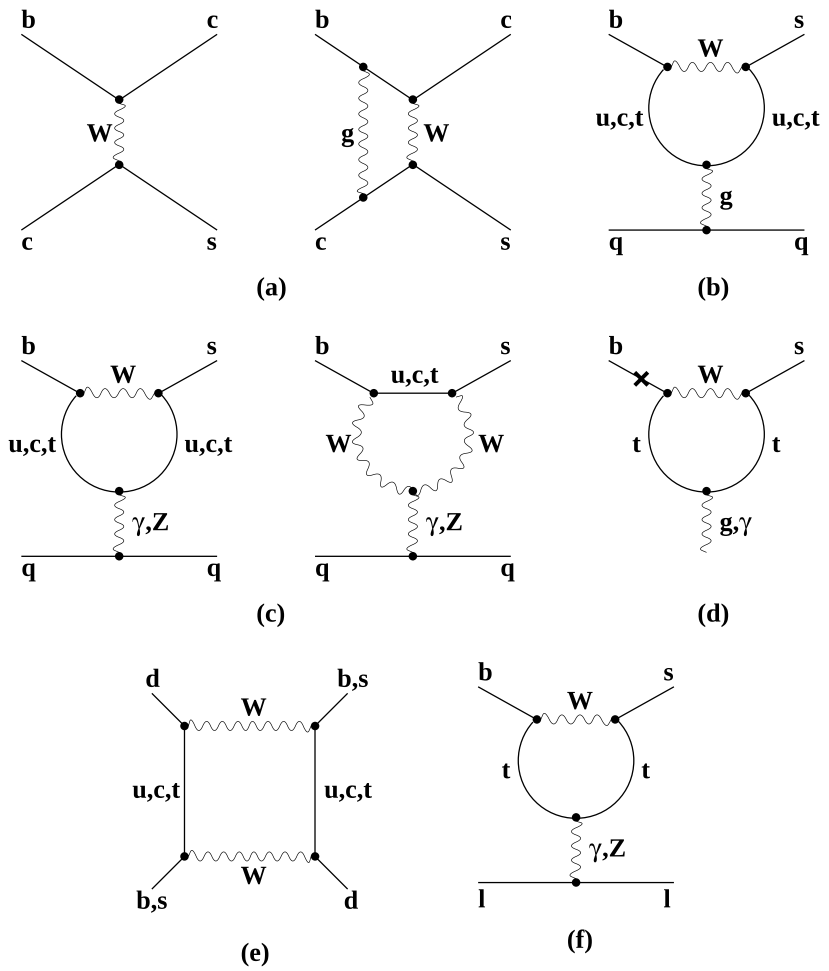

Figure 2.6: Typical Penguin and Box Diagrams.

Magnetic-Penguin Operators (Fig 2.6d)

$$Q_{7\gamma} = \frac{e}{8\pi^2} m_b \bar{s} \sigma^{\mu\nu}(1+\gamma_5) b F_{\mu\nu},$$

$$Q_{8G} = \frac{9}{8\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu}(1+\gamma_5) T^a_{\alpha\beta} b_\beta G^a_{\mu\nu},$$

$\Delta S = 2$ and $\Delta B = 2$ Operators (Fig. 2.6e)

$$Q(\Delta S = 2) = (\bar{s}b)_{V-A}(\bar{s}b)_{V-A}, \qquad Q(\Delta B = 2) = (\bar{s}b)_{V-A}(\bar{s}b)_{V-A},$$

Semi-Leptonic Operators (Fig. 2.6f)

$$Q_{9V} = (\bar{s}b)_{V-A}(\bar{l}l)_V, \qquad Q_{10A} = (\bar{s}b)_{V-A}(\bar{l}l)_A,$$

$$Q_{\nu\bar{\nu}} = (\bar{s}b)_{V-A}(\bar{\nu}\nu)_{V-A}, \qquad Q_{l\bar{l}} = (\bar{s}b)_{V-A}(\bar{l}l)_{V-A}, \qquad (2.18)$$

with α and β denoting the color indices and V and A representing the vector and axial vector currents such that $(\bar{q}q)_V$, $(\bar{q}q)_A$ and $(\bar{q}q)_{V\pm A}$ are $\bar{q}\gamma_\mu q$, $\bar{q}\gamma_\mu\gamma_5 q$ and $\bar{q}\gamma_\mu(1\pm\gamma_5)q$, respectively, also in $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are field strengths of photon and gluon fields.

The Wilson coefficients at high energy scale are [30, 31, 32]

$$\begin{aligned}
C_1(m_W) &= \frac{11\alpha_s(m_W)}{8\pi}, \qquad C_2(m_W) = 1 - \frac{11\alpha_s(m_W)}{24\pi}, \\
C_3(m_W) &= \frac{-C_4(m_W)}{3} = C_5(m_W) = \frac{-C_6(m_W)}{3} = \left(\frac{2}{3} - E(x)\right)\frac{\alpha_s(m_W)}{12\pi}, \\
C_7(m_W) &= x\frac{7-5x-8x^2}{24(x-1)^3} + \frac{x^2(3x-2)}{4(x-1)^4}\ln x, \\
C_8(m_W) &= \frac{-x(x^2-5x-2)}{8(x-1)^3} - \frac{3x^2}{4(x-1)^4}\ln x, \\
C_9(m_W) &= -\frac{B(x)}{\sin^2\theta_W} + \frac{1-4\sin^2\theta_W}{\sin^2\theta_W}C(x) \\
&+ \frac{-19x^3+25x^2}{36(x-1)^3} + \frac{-3x^4+30x^3-54x^2+32x-8}{18(x-1)^2}\ln x + \frac{4}{9}, \\
C_{10}(m_W) &= \frac{1}{\sin^2\theta_W}(B(x)-C(x)),
\end{aligned} \tag{2.19}$$

where

$$\begin{aligned}
B(x) &= -\frac{x}{4(x-1)} + \frac{x}{4(x-1)^2}\ln x, \\
C(x) &= -\frac{x}{4}\left(\frac{x-6}{3(x-1)} + \frac{3x+2}{2(x-1)^2}\ln x\right), \\
E(x) &= \frac{-9x^2+16x-4}{6(x-1)^4}\ln x + \frac{x^3+11x^2-18x}{12(x-1)^3}, \\
x &= \frac{m_t^2}{m_W^2},
\end{aligned} \tag{2.20}$$

and $\sin^2\theta_W = 0.23$ is the Weinberg angle. With $\mu = m_W$, the large logarithms are in the matrix elements of the operators $O_1 - O_8$, which are transferred from the matrix elements of the operators to their coefficients C_j by scaling the subtraction point μ down from m_W to m_b using the renormalization group equation [33],

$$\mu\frac{d}{d\mu}C_j(\mu)-\sum_{i=1}^{8}\gamma_{ij}C_i(\mu)=0, \tag{2.21}$$

where γ_{ij} is the anomalous dimension matrix [33]. The Wilson coefficients for the operators O_1-O_7 are given in the logarithmic approximation by [32, 34, 35, 36]

$$C_j(\mu) = \sum_{i=1}^{8}k_{ij}\eta^{a_i} \qquad (j=1,...,6), \tag{2.22}$$

$$C_7^{eff}(\mu) = \eta^{\frac{16}{23}}C_7(m_W)+\frac{8}{3}\left(\eta^{\frac{14}{23}}-\eta^{\frac{16}{23}}\right)C_8(m_W)+\sum_{i=1}^{8}h_i\eta^{a_i}, \tag{2.23}$$

with

$$\eta=\frac{\alpha_s(m_W)}{\alpha_s(\mu)}, \tag{2.24}$$

and the numbers a_i, k_{ij} and h_i are given by

$$\begin{aligned}
a_i &= (14/23,16/23,6/23,-12/23,0.4086,-0.423,-0.8994,0.1456),\\
k_{1i} &= (0,0,1/2,-1/2,0,0,0,0),\\
k_{2i} &= (0,0,1/2,1/2,0,0,0,0),\\
k_{3i} &= (0,0,-1/14,1/6,0.051,-0.1403,-0.0113,0.0054),\\
k_{4i} &= (0,0,-1/14,-1/6,0.0984,0.1214,0.0156,0.0026),\\
k_{5i} &= (0,0,0,0,-0.0397,0.0117,-0.0025,0.0304),\\
k_{6i} &= (0,0,0,0,0.0335,0.0239,-0.0462,-0.0112),\\
h_i &= (2.2996,-1.088,-3/7,-1/14,-0.6494,-0.038,-0.0186,-0.0057),
\end{aligned} \tag{2.25}$$

where

$$\alpha_s(\mu)=\frac{1}{4\pi\beta_0\ln(\mu^2/\Lambda^2)},$$

with β_0 is the lowest order coefficient of Gell-Mann–Low β–function and Λ is QCD scale parameter [34]. The coefficient $C_8(m_b)$ does not appear in the effective Hamiltonian of $b \to s\ell^+\ell^-$ decay and $C_{10}(m_W) = C_{10}(m_b)$ since O_{10} does not renormalize under QCD [34]. Finally the explicit form of the coefficient of O_9 at $\mu = m_b$ scale is [37]

$$
\begin{aligned}
C_9^{eff} &= C_9(m_W) + g(\hat{m}_c, \hat{s})[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) \\
&+ C_4(\mu) + 3C_5(\mu) + C_6(\mu)] \\
&+ \lambda_u[g(\hat{m}_c, \hat{s}) - g(\hat{m}_u, \hat{s})][3C_1(\mu) + C_2(\mu)] - \frac{1}{2}g(\hat{m}_s, \hat{s})[C_3(\mu) + 3C_4(\mu)] \\
&- \frac{1}{2}g(\hat{m}_b, \hat{s})[4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)] \\
&+ \frac{2}{9}[3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)], \qquad (2.26)
\end{aligned}
$$

where

$$\lambda_u = \frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}, \qquad (2.27)$$

and the one loop-functions,

$$
\begin{aligned}
g(\hat{m}_i, \hat{s}) &= -\frac{8}{9}\ln(m_i/m_b) + \frac{8}{27} + \frac{4}{9}y_i - \frac{2}{9}(2 + y_i)\sqrt{|1 - y_i|} \qquad (2.28) \\
&\times \left\{\Theta(1 - y_i)\left[\ln\left(\frac{1 + \sqrt{1 - y_i}}{1 - \sqrt{1 - y_i}}\right) - i\pi\right] + \Theta(y_i - 1)2\arctan\frac{1}{\sqrt{y_i - 1}}\right\}
\end{aligned}
$$

with $y_i = 4\hat{m}_i^2/\hat{s}$, $\hat{m}_i^2 = m_i^2/m_b^2$ and $\hat{s} \equiv \hat{p}^2 = p^2/m_B^2$ and Θ's being the step functions.

With these remarks, the effective Hamiltonian for the standard model for $b \to s\ell^+\ell^-$ transition can be written as,

$$
\begin{aligned}
H_{eff} &= \frac{G_F\alpha}{2\pi\sqrt{2}}V_{tb}V_{ts}^*\left\{C_9^{eff}\bar{s}\gamma_\mu(1 - \gamma_5)b\bar{\ell}\gamma^\mu\ell + C_{10}\bar{s}\gamma_\mu(1 - \gamma_5)b\bar{\ell}\gamma_5\gamma^\mu\ell\right. \\
&\left. -2C_7\frac{m_b}{p^2}\bar{s}i\sigma_{\mu\nu}p^\nu(1 + \gamma_5)b\bar{\ell}\gamma^\mu\ell\right\}, \qquad (2.29)
\end{aligned}
$$

where p is the 4-momentum transfer, V_{tb} and V_{ts} are the CKM matrix factors and C_9^{eff}, C_{10} and C_7 are the Wilson coefficients.

In the Standard Model, the left handed parts of quarks and leptons are doublets whereas the right handed contributors are singlets. So, only the left handed parts of the wave functions enter into the weak transitions. That is why we did not included the right handed contributors of the relevant wave functions into the above effective Hamiltonian representation. In order to consider a model independent representation, the SM Hamiltonian given in Eq. (2.29) can be rewritten as,

$$\begin{aligned} H_{eff} &= \frac{G_F \alpha}{2\pi\sqrt{2}} V_{tb} V_{ts}^* \Big\{ (C_9^{eff} - C_{10})(\bar{s_L} \gamma_\mu b_L \bar{\ell_L} \gamma^\mu \ell_L) \\ &+ (C_9^{eff} + C_{10})(\bar{s_L} \gamma_\mu b_L \bar{\ell_R} \gamma^\mu \ell_R) - 2C_7 \frac{m_b}{p^2} \bar{s} i \sigma_{\mu\nu} p^\nu \, R \, b \bar{\ell} \gamma^\mu \ell \Big\} . \end{aligned} \tag{2.30}$$

To construct a most general representation of the effective Hamiltonian above, we include the right handed contributions of leptons and quarks as well as the left handed ones to get,

$$\begin{aligned} H_{eff} &= \frac{G_F \alpha}{\sqrt{2}\pi} \Big\{ C_{SL} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} L b \bar{l} \gamma^\mu l + C_{BR} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} R b \bar{l} \gamma^\mu l + C_{LL}^{tot} \bar{s_L} \gamma_\mu b_L \bar{l_L} \gamma^\mu l_L \\ &+ C_{LR}^{tot} \bar{s_L} \gamma_\mu b_L \bar{l_R} \gamma^\mu l_R + C_{RL} \bar{s_R} \gamma_\mu b_R \bar{l_L} \gamma^\mu l_L + C_{RR} \bar{s_R} \gamma_\mu b_R \bar{l_R} \gamma^\mu l_R + \\ &+ C_{LRLR} \bar{s_L} b_R \bar{l_L} l_R + C_{RLLR} \bar{s_R} b_L \bar{l_L} l_R + C_{LRRL} \bar{s_L} b_R \bar{l_R} l_L + C_{RLRL} \bar{s_R} b_L \bar{l_R} l_L \\ &+ C_T \bar{s} \sigma_{\mu\nu} b \bar{l} \sigma^{\mu\nu} l + i C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{s} \sigma_{\mu\nu} b \bar{l} \sigma_{\alpha\beta} l \Big\} \end{aligned} \tag{2.31}$$

where the chiral projection operators L and R in (2.31) are defined as

$$L = \frac{1 - \gamma_5}{2} , \qquad R = \frac{1 + \gamma_5}{2} ,$$

and C_X are the coefficients of the four–Fermi interactions. The first two of these coefficients, C_{SL} and C_{BR}, are the nonlocal Fermi interactions which correspond to $-2m_s C_7^{eff}$ and $-2m_b C_7^{eff}$ in the SM, respectively. The following four terms in this expression are the vector type interactions with coefficients C_{LL}, C_{LR}, C_{RL} and C_{RR}. Two of these vector interactions containing C_{LL}^{tot} and C_{LR}^{tot} do already exist in the SM in combinations of the form $(C_9^{eff} - C_{10})$ and $(C_9^{eff} + C_{10})$. Therefore by writing

$$
\begin{aligned}
C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL} \ , \\
C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR} \ ,
\end{aligned}
$$

one concludes that C_{LL}^{tot} and C_{LR}^{tot} describe the sum of the contributions from SM and the new physics. The terms with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} describe the scalar type interactions. The remaining two terms leaded by the coefficients C_T and C_{TE}, obviously, describe the tensor type interactions.

So, our next task is to calculate the matrix elements for the $B \to K \ell^+ \ell^-$ and $B \to K^* \ell^+ \ell^-$ decays. In other words, we need the matrix elements

$$
\begin{aligned}
&\langle K | \bar{s} \gamma_\mu b | B \rangle \\
&\langle K | \bar{s} i \sigma_{\mu\nu} q^\nu b | B \rangle \\
&\langle K | \bar{s} b | B \rangle \\
&\langle K | \bar{s} \sigma_{\mu\nu} b | B \rangle
\end{aligned}
\tag{2.32}
$$

and

$$
\begin{aligned}
&\langle K^* | \bar{s} \gamma_\mu (1 \pm \gamma_5) b | B \rangle \\
&\langle K^* | \bar{s} i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b | B \rangle \\
&\langle K^* | \bar{s} (1 \pm \gamma_5) b | B \rangle \\
&\langle K^* | \bar{s} \sigma_{\mu\nu} b | B \rangle \ ,
\end{aligned}
\tag{2.33}
$$

respectively.

For calculation of these matrix elements, we need some non–perturbative approach. Among non–perturbative approaches QCD sum rules method is the most powerful one since it is based on the first principle of the fundamental QCD Lagrangian. In the next chapter, we will give a brief introduction to the QCD sum rules method and state the calculation of the form factors with respect to these matrix elements.

CHAPTER 3

QCD SUM RULES

In order to calculate the lepton polarizations for the decays $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$, first we have to calculate the matrix elements for these decays, which is nothing but the effective Hamiltonian sandwiched between final and initial states. The calculations of these exclusive decays, require an additional knowledge of the decay form factors for the relevant decays. In the first section, we will introduce the parametrizations of these matrix elements. In the second section, we will explain the Light Cone QCD Sum Rules method which we will use to calculate the corresponding form factors. Finally, the third section will include calculations for these form factors.

3.1 The Effective Hamiltonian and Matrix Elements

In our calculations, we use the most general form of the effective Hamiltonian as follows:

$$
\begin{aligned}
H_{eff} &= \frac{G_F \alpha}{\sqrt{2}\pi} \Bigg\{ C_{SL} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} L b \bar{l} \gamma^\mu l + C_{BR} \bar{s} i \sigma_{\mu\nu} \frac{q^\nu}{q^2} R b \bar{l} \gamma^\mu l + C_{LL}^{tot} \bar{s_L} \gamma_\mu b_L \bar{l_L} \gamma^\mu l_L \\
&+ C_{LR}^{tot} \bar{s_L} \gamma_\mu b_L \bar{l_R} \gamma^\mu l_R + C_{RL} \bar{s_R} \gamma_\mu b_R \bar{l_L} \gamma^\mu l_L + C_{RR} \bar{s_R} \gamma_\mu b_R \bar{l_R} \gamma^\mu l_R + \\
&+ C_{LRLR} \bar{s_L} b_R \bar{l_L} l_R + C_{RLLR} \bar{s_R} b_L \bar{l_L} l_R + C_{LRRL} \bar{s_L} b_R \bar{l_R} l_L + C_{RLRL} \bar{s_R} b_L \bar{l_R} l_L \\
&+ C_T \bar{s} \sigma_{\mu\nu} b \bar{l} \sigma^{\mu\nu} l + i C_{TE} \epsilon^{\mu\nu\alpha\beta} \bar{s} \sigma_{\mu\nu} b \bar{l} \sigma_{\alpha\beta} l \Bigg\}. \qquad (3.1)
\end{aligned}
$$

The matrix elements for the decay $B \to K l^+ l^-$ and $B \to K^* l^+ l^-$ can be written as $M = \langle K | H_{eff} | B \rangle$ and $\langle K^* | H_{eff} | B \rangle$, respectively. Thus, to calculate the matrix element for $B \to K l^+ l^-$ decay, the following matrix elements are needed.

$$
\begin{aligned}
&\langle K | \bar{s} \gamma_\mu b | B \rangle, \\
&\langle K | \bar{s} i \sigma_{\mu\nu} q^\nu b | B \rangle, \\
&\langle K | \bar{s} b | B \rangle, \qquad (3.2) \\
&\langle K | \bar{s} \sigma_{\mu\nu} b | B \rangle.
\end{aligned}
$$

Note that the matrix elements of operations involving γ_5 are zero due to the parity arguments. Then the first and the fourth matrix elements for the relevant decays can be parametrized as follows,

$$
\begin{aligned}
\langle K | \bar{s} \gamma_\mu b | B \rangle &= f_+ \left[(p_B + p_K)_\mu - \frac{m_B^2 - m_K^2}{q^2} q^\mu \right] + f_0 \frac{m_B^2 - m_K^2}{q^2} q_\mu, \qquad (3.3) \\
\langle K | \bar{s} \sigma_{\mu\nu} b | B \rangle &= -i \frac{f_T}{m_B + m_K} \left[(p_B + p_K)_\mu q_\nu - q_\mu (p_B + p_K)_\nu \right]. \qquad (3.4)
\end{aligned}
$$

The matrix elements $\langle K | \bar{s} i \sigma_{\mu\nu} q^\nu b | B \rangle$ and $\langle K | \bar{s} b | B \rangle$ can be calculated by contracting both sides of Eqs. (3.3) and (3.4) with q^μ. Using the equation of motion, we get

$$\langle K|\bar{s}i\sigma_{\mu\nu}q^{\nu}B\rangle = \frac{f_T}{m_B+m_K}\Big[(p_B+p_K)_\mu q^2 - q_\mu(m_B^2-m_K^2)\Big], \tag{3.5}$$

$$\langle K|\bar{s}b|B\rangle = f_0\frac{m_B^2-m_K^2}{m_b-m_s}, \tag{3.6}$$

where $q = p_B - p_K$ is the momentum transfer.

Similarly, we need the matrix elements for the decay $B \to K^* l^+ l^-$ as

$$\begin{aligned}
&\langle K^*|\bar{s}\gamma_\mu(1\pm\gamma_5)b|B\rangle,\\
&\langle K^*|\bar{s}i\sigma_{\mu\nu}q^\nu(1\pm\gamma_5)b|B\rangle,\\
&\langle K^*|\bar{s}(1\pm\gamma_5)b|B\rangle,\\
&\langle K^*|\bar{s}\sigma_{\mu\nu}b|B\rangle,
\end{aligned} \tag{3.7}$$

which are parametrized in terms of form factors as follows,

$$\begin{aligned}
\langle K^*(p_{K^*},\varepsilon)|\bar{s}\gamma_\mu(1\pm\gamma_5)b|B(p_B)\rangle = -\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p_{K^*}^\lambda q^\sigma\frac{2V(q^2)}{m_B+m_{K^*}} \pm \\
i\varepsilon_\mu^*(m_B+m_{K^*})A_1(q^2) \mp i(p_B+p_{k^*})_\mu(\varepsilon^*q)\frac{A_2(q^2)}{m_B+m_{K^*}} \mp \\
iq_\mu\frac{2m_{K^*}}{q^2}(\varepsilon^*q)\Big[A_3(q^2)-A_0(q^2)\Big],
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
\langle K^*(p_{K^*},\varepsilon)|\bar{s}i\sigma_{\mu\nu}q^\nu(1\pm\gamma_5)b|B(p_B)\rangle = 4\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\mu}p_{K^*}^\lambda q^\sigma T_1(q^2) \pm \\
2i\Big[\varepsilon_\mu^*(m_B^2-m_{K^*}^2)-(p_B+p_{K^*})_\mu(\varepsilon^*q)\Big]T_2(q^2) \mp \\
2i(\varepsilon^*q)\left[q_\mu-\Big(p_B+p_{K^*}\Big)_\mu\frac{q^2}{m_B^2-m_{K^*}^2}\right]T_3(q^2),
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\langle K^*(p_{K^*},\varepsilon)|\bar{s}\sigma_{\mu\nu}b|B(p_B)\rangle = i\epsilon_{\mu\nu\lambda\sigma}\Big[-2T_1(q^2)\varepsilon^{*\lambda}(p_B+p_{K^*})^\sigma + \\
\frac{2}{q^2}(m_B^2-m_{K^*}^2)\varepsilon^{*\lambda}q^\sigma - \frac{4}{q^2}\Big(T_1(q^2)-T_2(q^2)-\frac{q^2}{m_B^2-m_{K^*}^2}T_3(q^2)\Big)(\varepsilon^*q)p_{K^*}^\lambda q^\sigma\Big],
\end{aligned} \tag{3.10}$$

where $q = p_B - p_{K^*}$ is the momentum transfer and ε is the polarization vector of K^* meson. The matrix element $\langle K^*|\bar{s}(1 \pm \gamma_5)b|B\rangle$ can be calculated from Eq. (3.8) by contracting both sides of Eq. (3.8) with q^μ and using equation of motion. Neglecting the mass of the strange quark, we get

$$\langle K^*(p_{K^*}, \varepsilon)|\bar{s}(1 \pm \gamma_5)b|B(p_B)\rangle = \frac{1}{m_b}\Big[\mp 2im_{K^*}(\varepsilon^* q)A_0(q^2)\Big]. \tag{3.11}$$

In deriving this equation, we have used the relation

$$2m_{K^*}A_3(q^2) = (m_B + m_{K^*})A_1(q^2) - (m_B - m_{K^*})A_2(q^2). \tag{3.12}$$

For calculating both of these decays, the form factors f_0, f_+, f_T, A_1, A_2, A_0, T_1, T_2, and T_3 need to be calculated. We will use the Light Cone QCD Sum Rules method to calculate these form factors.

3.2 Light Cone QCD Sum Rules

The SVZ sum rules [53], proposed more than twenty years ago, is one of the most powerful analytical non-perturbative approaches. Compared to lattice calculations, the main power of SVZ sum rules (and its extensions) is in the analyticity of the methods. In this approach, deep connection is established between the low energy process and the non-trivial QCD vacuum through several condensates, the quark condensate $\langle \bar{q}q\rangle$, the quark gluon mixed condensate $\langle \bar{q}\sigma Gq\rangle$, the gluon condensate $\langle g^2G^2\rangle$ and other higher dimensional condensates. These condensates are either calculated in other approaches or else obtained from the sum rules itself.

The starting point of QCD sum rules approaches is that, hadrons are represented by their interpolating quark currents taken at large virtuality. Then the correlation function of these currents is introduced and treated within the framework of the operator product expansion (OPE), where the short and long distance quark–gluon interactions are separated. In SVZ Sum Rules, OPE is performed with respect to dimension of operators.

In this section, the general machinery of the SVZ sum rules and of its extensions, the light cone QCD sum rules will be described. For illustrative purposes the mass sum rule for the B meson will be considered.

3.2.1 The Correlator Function

In order to study the properties of quarks in the vacuum, what is done is to inject quarks into the QCD vacuum at the space-time point $x = 0$ and study its evolution. This process is described by the correlation functions:

$$\Pi(q^2) = i \int d^4x \; e^{iqx} \langle 0|Tj(x)\bar{j}(0)|0\rangle, \tag{3.13}$$

where T is the time ordering operator, $j(x)$ is the current that injects quarks into the vacuum at space time point x, and q is the total momentum of the quarks. Eq. (3.13) is an example of a two-point correlation function which leads to mass sum rules.

For large negative $q^2 = -Q^2 << -\Lambda^2_{QCD}$, the main contribution to the function (3.13) comes from short spatial distances and short times [54] and hence can be calculated in terms of quarks and gluons. To see that this is indeed so, in the case of massless quarks, first note that the vacuum expectation value of the correlator can only depend on the space-time interval x^2. Introducing the Fourier transform of the vacuum expectation value through

$$\langle 0|Tj(x)\bar{j}(0)|0\rangle = \int d\ell \; e^{i\ell x^2} f(\ell), \tag{3.14}$$

and inserting Eq. (3.14) into Eq. (3.13) one obtains

$$\Pi(q^2) = i \int d\ell \int d^4x \; e^{i\ell(x+\frac{q}{2\ell})^2} \; e^{i\frac{Q^2}{4\ell}} f(\ell). \tag{3.15}$$

The contribution of the integrand in Eq. (3.15) is suppressed if at least one of the exponential functions oscillates rapidly. Hence, for large Q^2, the main contribution comes from the region $\ell \sim Q^2$ and $x^2 \sim \frac{1}{\ell}$ which implies that

$$x^2 \sim \frac{1}{Q^2}, \tag{3.16}$$

i.e. the quarks propagate near the light cone. This does not, by itself, imply that the main contribution comes from short spatial distances and short times. For $q^2 < 0$ it is always possible to choose a reference frame in which $q_0 = 0$ so that $q^2 = Q^2$. In this frame, the exponential in Eq. (3.13) is simply e^{iqx}. Again, to avoid a fast oscillating integrand, it is required that

$$|x| \sim \frac{1}{\sqrt{Q^2}}, \tag{3.17}$$

which, when combined with the previous result, gives

$$|x| \sim x_0 \sim \frac{1}{\sqrt{Q^2}}. \tag{3.18}$$

For massive quarks, the analysis is even more simpler since the quark masses $m_{c,b} >> \Lambda_{QCD}$ introduces an intrinsic high energy scale and the and the distance that a quark can propagate is determined by its inverse mass.

As q^2 approaches positive values, the quarks tend to move to larger spatial separations and eventually for sufficiently large positive values of q^2, the quarks start to form hadrons. In this regime, the correlator function Eq. (3.13) can be calculated by inserting a complete set of hadronic states between the two currents

$$\begin{aligned} 2Im\Pi(q^2) &= \sum_n \langle 0|\bar{j}(0)|n\rangle\langle n|j(0)\rangle d\ell_n (2\pi)^4 \delta^{(4)}(q-p_n) \\ &= 2\pi f_H^2 \delta(q^2-m_H^2) + 2\pi\rho^h(q^2)\theta(q^2-s_0^2), \end{aligned} \quad (3.19)$$

where the sum rules goes over all possible hadronic states that can be created by its currents, $d\ell$ denotes integration over all phase–space volume of the hadron. In Eq. (3.19), in the second line, H is the hadron with the lowest mass that can be created by current $j(x)$, $\langle H|j(0)|0\rangle = f_H$, and $\rho^h(q^2)$ denotes the contributions of the higher states and continuum, where s_0 is their threshold. In general little is known about $\rho^h(q^2)$, and one approximates $\rho^h(q^2)$ using the quark-hadron duality.

3.2.2 Dispersion Relation

The correlation function (3.13) is an analytic function of its argument q^2. Hence using the Cauchy formula for analytical functions it is possible to link the values of $\Pi(q^2)$ for positive values of q^2, which can be expressed in terms of hadron properties as in Eq. (3.19), to its values at negative values of q^2.

For this purpose, consider the contour shown in Fig 3.1. Using the Cauchy formula for analytical functions, one can write,

$$\begin{aligned} \Pi(q^2) &= \frac{1}{2\pi i}\oint_C dz\, \frac{\Pi(z)}{z-q^2} \\ &= \frac{1}{2\pi i}\oint_{|z|=R} dz\, \frac{\Pi(z)}{z-q^2} + \frac{1}{2\pi i}\int_{t_{min}}^{R} dz \frac{Pi(z+i\varepsilon)-\Pi(z-i\varepsilon)}{z-q^2}, \end{aligned} \quad (3.20)$$

where t_{min} is the threshold for creation of real states.

Eventually, the radius R of the circular part of the contour will be sent to infinity. Now, let us consider the first term, i.e. the integral over the infinite

Figure 3.1: The contour integral

circle. If $\Pi(z)$ vanishes sufficiently last at $|z| \to \infty$, then the integral vanishes. On the other case, if it does not vanish sufficiently fast, or if it does not vanish at all, one can expand the denominator in terms of q^2/z, and eventually, at some order n, $\Pi(q^2)/z^n$ would vanish sufficiently fast and hence the remaining terms in the expansion will not contribute. Thus, one sees that in the limit $R \to \infty$, the first term reduces to a polynomial in q^2, the so called *subtraction terms*.

Using a theorem from complex analysis, the so-called Schwartz reflection principle and noting that $\Pi(q^2)$ is real for $q^2 < t_{min}$, one can conclude that $\Pi(z+i\varepsilon) - \Pi(z-i\varepsilon) = 2i \; Im\Pi(q^2)$ at $q^2 > t_{min}$. Hence we obtain the dispersion relation:

$$\Pi(q^2) = \frac{1}{\pi}\int_{t_{min}}^{\infty} ds \; \frac{Im\Pi(s)}{s - q^2 - i\varepsilon} + subtraction \; terms, \tag{3.21}$$

where $Im\Pi(s)/\pi = \rho(s)$ is called the spectral density. The dispersion relation can be used to link the values of $\Pi(q^2)$ for positive values of q^2 to the values of

$\Pi(q^2)$ at negative values of q^2.

The spectral density for positive q^2 can be expressed in terms of the hadronic parameters using Eq. (3.19). It can also be expressed in terms of quarks and gluons by evaluating the correlation function in the large Euclidean momenta limit and than extracting the corresponding spectral density, $\rho^{qg}(s)$, from:

$$\Pi(q^2) = \int_0^\infty ds \, \frac{\rho^{qg}(s)}{s - q^2}, \tag{3.22}$$

for large Euclidean $Q^2 = -q^2 >> \Lambda_{QCD}$. Using Eqs. (3.19) and (3.22), one obtains:

$$\int_0^\infty ds \, \frac{\rho^{qg}(s)}{s - q^2} = \frac{f_H^2}{m_H^2 - q^2} + \int_{s_0^h}^\infty ds \, \frac{\rho^h(s)}{s - q^2} + subtraction\ terms, \tag{3.23}$$

To get rid of the subtraction terms which are polynomials in q^2, and suppress the contribution of the higher states and the continuum, one applies Borel transformation on $Q^2 = -q^2$ to both sides of Eq. (3.23). The Borel transformation is defined as,

$$B_{M^2}(q^2) f(q^2) = \lim_{Q^2, n \to \infty} \frac{(-q^2)^n}{(n-1)!} \left(\frac{d}{dq^2} \right)^{n-1} f(q^2). \tag{3.24}$$

Any polynomial gives zero after Borel transformation. Two important examples are,

$$B_{M^2}(q^2) \left(\frac{1}{(m^2 - q^2)^k} \right) = \frac{1}{(k-1)!} \frac{e^{-m^2/M^2}}{(M^2)^{k-1}}, \tag{3.25}$$

$$B_{M^2}(q^2) e^{-\alpha q^2} = \delta\Big(\frac{1}{M^2} - \alpha \Big). \tag{3.26}$$

After Borel transformation, Eq. (3.23) takes the form

$$\int_0^\infty ds \, \rho^{qg}(s) e^{-s/M^2} = f_H^2 \, e^{-m_H^2/M^2} + \int_{s_0^h}^\infty ds \, \rho^h(s) e^{-s/M^2}. \tag{3.27}$$

Indeed, the subtraction terms are eliminated and the contribution of higher states and the continuum are exponentially suppressed.

In order to get an approximation of the contribution of the higher states and the continuum to the r.h.s. of Eq. (3.23), note that in the limit $Q^2 \to \infty$, the correlation function is given completely in terms of the perturbative part. Hence, one assumes that for sufficiently large Q^2

$$\int_{s_0^h}^{\infty} ds \; \frac{\rho^h(s)}{s - q^2} \simeq \int_{s_0}^{\infty} ds \; \frac{\rho^{qg}(s)}{s - q^2}, \tag{3.28}$$

which is called the local quark hadron duality approximation. In Eq. (3.28), s_0, which is called the continuum threshold, is a parameter to be fitted to the available data. Applying Borel transformation of Eq. (3.28) and substituting the result into Eq. (3.27), one obtains the following sum rules:

$$f_H^2 \; e^{-\frac{m_H^2}{M^2}} = \int_0^{s_0} ds \; \rho^{qg}(s) e^{-\frac{s}{M^2}}, \tag{3.29}$$

In Eq. (3.29), there are two unknowns: the Borel mass parameter, M^2, and the continuum threshold, s_0. The continuum threshold is not completely arbitrary, being related to the energy of the excited states. It is in general taken to be around $(m_H + 0.7GeV)^2$, but the result should be stable with respect to small variations of this quantity. M^2 is in general completely arbitrary. But due to the approximations used, it is restricted to a window; out of this window, either the contributions of the continuum or the contributions of the neglected higher dimensional operators become large. It cannot be too small since in this case, the contributions of higher dimensional operators, which are inversely proportional to powers of M^2, becomes important and hence one cannot neglect them. A lower limit on M^2 is obtained by demanding that the contribution of the highest dimensional operator in the expansion is not more than a small fraction of the total result. On the other hand, the parameter cannot be too large either, since in this case, the quark-hadron duality cannot be trusted and the exponential suppression of the contributions of higher states is reduced. The upper limit on M^2 is obtained by demanding that the contributions of excited states to the sum rules remains a small part of the total dispersion integral. In this window, it should be checked that any physical quantity calculated using the sum rules is almost independent of the value of M^2.

After introducing the QCD sum rules, let us now show the calculations of the form factors for $B \to K$ and $B \to K^*$ decays. In this thesis, instead of the SVZ version of Sum Rules, we will employ Light Cone QCD Sum Rules. The main reasons why we use Light Cone version of Sum Rules are as follows,

1. In the SVZ Sum Rules, OPE breaking upsets counting in the large momentum/mass. As an example, the sum rule for pion electromagnetic form factor [50],[51] schematically can be written as

$$F_\pi(Q^2) \sim \frac{1}{Q^2} + \frac{\langle g^2 G^2 \rangle}{M^4} + Q^2 \frac{\langle \bar{q} q \rangle^2}{M^8}, \tag{3.30}$$

where M is the Borel parameter which is of the order 1 GeV^2. The first term describes contribution of perturbation theory and we see that at large Q^2, $F_\pi(Q^2)$ increases. Such behavior clearly is unphysical and indicates that at large Q^2 OPE breaks down.

2. The Three–Point Sum Rules for heavy-light decays, which we are interested in our thesis, have similar problem at large recoil, i.e. around $q^2 = 0$ point. For example, Sum Rules for form factor A_1 in $B \to \rho \ell \nu$ decay [52] at maximum recoil

$q^2 = 0$ has the following structure,

$$A_1(q^2 = 0) \sim \frac{\alpha}{m_B^{3/2}} + \beta m_B^{1/2} \langle \bar{q}q \rangle + \gamma m_B^{3/2} m_0^2 \langle \bar{q}q \rangle + ... \quad (3.31)$$

and at $m_b \to \infty$, $A_1(q^2 = 0) \to \infty$. Obviously this is also unphysical.

3. Another problem with the Three–Point Sum Rules is the contamination of the sum rule by non-diagonal transitions of the ground state to the excited states. In other words, these transitions are exponentially not suppressed. Therefore, contribution of continuum and Sum Rule prediction are not reliable.

For solving these problems, Light Cone QCD Sum Rules are introduced. For these reasons we preferred to work with the Light Cone version of the Sum Rules. Note that OPE is done in terms of twist of the operators, rather than their dimensions as in SVZ version of the Sum Rules.

3.3 Calculation of Form Factors in $B \to K l^+ l^-$ Decay

For calculating the form factors appearing in the parametrizations (Eqs. 3.3-3.6) of the matrix element in the $B \to K l^+ l^-$ decay, we consider the following correlation functions

$$\Pi_\mu^{(1)} = i \int d^4x e^{iqx} \langle K(p_K) | T\{\bar{s}(x)\gamma_\mu b(x)\bar{b}(0) i\gamma_5 q(0)\} |0\rangle, \quad (3.32)$$

$$\Pi_\mu^{(2)} = i \int d^4x e^{iqx} \langle K(p_K) | T\{\bar{s}(x) i\sigma_{\mu\nu} q^\nu b(x) barb(0) i\gamma_5 q(0)\} |0\rangle, \quad (3.33)$$

where p_K is the K-meson momentum and q is the transfer momentum. $\Pi^{(1)}$ is the relevant correlation function for calculating the form factors f_+ and f_-, and

$\Pi^{(2)}$ is the relevant one for calculating f_T.

It is helpful to note here that the form factor f_0, which appears in the parametrizations of the matrix elements of the $B \to K l^+ l^-$ decay is nothing but the combination of the form factors f_+ and f_- which can be written in the form,

$$\langle K(p_K)|\bar{q}\gamma_\mu b|B(p_B)\rangle = f_+(q^2)(p_B + p_K)_\mu + f_-(q^2)q_\mu,$$

where $f_- = (f_0 - f_+)(m_B^2 - m_K^2)/q^2$, from Eq. (3.31).

A formal expression can be represented as,

$$\Pi_\mu(P_K, q) = \int_0^\infty \frac{d\alpha}{16\pi^2\alpha^2} \int d^4x e^{iqx - m_b^2\alpha + \frac{x^2}{4\alpha}} \times$$
$$\left(m_b \langle K(p_K)|\bar{s}(x)\gamma_\mu\gamma_5 u(0)|0\rangle + \frac{ix^\rho}{2\alpha}\langle K(p_K)|\bar{s}(x)\gamma_\mu\gamma_\rho\gamma_5 u(0)|0\rangle \right) \quad (3.34)$$

where we have made use of the following representation of the free propagator S_b^0

$$\begin{aligned} \langle 0|T\{b(x)\bar{b}(0)\}|0\rangle &= iS_b^0(x) \\ &= -\int_0^\infty \frac{d\alpha}{16\pi^2\alpha^2}\Big(m_b + i\frac{\hat{x}}{2\alpha}\Big)e^{-m_b^2 x^2}. \end{aligned} \quad (3.35)$$

Considering the first term in Eq. (3.34), the matrix element for the non-local operator is given by [38], [39]

$$\langle K(p_K)|\bar{s}(x)\gamma_\mu\gamma_5 q(0)|0\rangle = -iP_{K\mu} f_K \int_0^1 du e^{iup_K x}\Big(\psi_K(u) + \frac{5\delta_K^2}{36}x^2\psi_{4K}(u)\Big), \quad (3.36)$$

where $\psi_K(u)$ is the K-meson light cone wave function of the leading twist–2 and $\psi_{4K}(u)$ represents one of the next-to-leading twist–4 wave functions. All wave functions, normalized to unity and δ_K^2 is a dimensionfull parameter which is $\simeq$ $0.2\ GeV^2$.

We can split the matrix element appearing in the second term of the Eq. (3.34) into two using the identity $\gamma_\mu \gamma_\rho = -i\sigma_{\mu\rho} + g_{\mu\rho}$. These matrix elements are determined by the wave functions of twist–3:

$$\langle K(p_K)|\bar{s}(x) i\gamma_5 q(0)|0\rangle = \frac{f_K m_K^2}{m_q + m_s} \int_0^1 du e^{iup_K x} \psi_{pK}(u), \qquad (3.37)$$

$$\langle K(p_K)|\bar{s}(x)\sigma_{\mu\nu}\gamma_5 q(0)|0\rangle = -i(p_{K\mu}x_\nu - p_{K\nu}x_\mu)\frac{f_K m_K^2}{6(m_q + m_s)} \int_0^1 du e^{iup_K x} \psi_{\sigma K}(u). \qquad (3.38)$$

Substituting (3.36), (3.37) and (3.38) into (3.34) and integrating over x and the auxiliary parameter α, we obtain,

$$\begin{aligned} \Pi_{QCD}(q^2, (q+p_K)^2) &= -f_K m_b \int_0^1 du \Bigg(\frac{\psi_K(u)}{(q+p_K u)^2 - m_b^2} - \frac{10\delta_K^2 m_b^2 \psi_{4K}(u)}{9((q+p_k u)^2 - m_b^2)^3} \Bigg) \\ &- \frac{f_K m_K^2}{m_q + m_s} \int_0^1 du \Bigg[\frac{\psi_{\rho K}(u)}{(q+p_K u)^2 - m_b^2} \\ &+ \frac{\psi_{\sigma K}(u)}{6((q+p_K u)^2 - m_b^2)} \Big(2 - \frac{q^2 + m_b^2}{(q+p_K u)^2 - m_b^2} \Big) \Bigg]. \end{aligned} \qquad (3.39)$$

In addition to quark–antiquark wave functions described above, there are also contributions from multi–particle wave functions. The most important corrections arise from quark–gluon operators in the OPE. The leading contributor arises from twist–3 operator.

$$\begin{aligned} \langle K(p_K)|\bar{s}(x) g_s G_{\mu\nu}(z)\sigma_{\rho\lambda}\gamma_5 q(0)|0\rangle = i f_{3K} \Big[p_{K\mu}(p_{K\rho} g_{\lambda\nu} - p_{K\lambda} g_{\rho\nu}) - \\ p_{K\nu}(p_{K\rho} g_{\lambda\mu} - p_{K\lambda} g_{\rho\mu}) \Big] \int D\alpha_i \psi_{3K}(\alpha_i) e^{ip_K(x\alpha_1 + z\alpha_3)}, \end{aligned} \qquad (3.40)$$

where $G_{\mu\nu}(z) = (\lambda^c/2) G^c_{\mu\nu}(z)$, λ^c and $G^c_{\mu\nu}$ being usual color matrices and the gluon field tensor, and $D\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(\alpha_1 + \alpha_2 + \alpha_3 - 1)$. $\psi_{3K}(\alpha_i) =$

$\psi_{3K}(\alpha_1, \alpha_2, \alpha_3)$ is the three particle wave function and f_{3K} is the corresponding coupling constant which is introduced and discussed in [38], [40].

In the calculation of the gluonic correction, instead of the free propagator, we use the b-quark propagator including the interactions with gluons in the first order,

$$\langle 0|T\{b(x)\bar{b}(0)\}|0\rangle = i\hat{S}_b^0(x) - ig\int dz \hat{S}_b^0(x-z)\gamma^\mu \frac{\lambda^c}{2} A_\mu^c(z)\hat{S}_b^0(z), \qquad (3.41)$$

where $\hat{S}_b^0$ is the free b-quark propagator defined in [41]. In the fixed point gauge, $x^\mu A_\mu^c = 0$, the gluon field A_μ^c can be represented directly in terms of the field strength $G_{\mu\nu}^c$:

$$A_\mu^c(z) = z_\nu \int_0^1 udu G_{\nu\mu}^c(uz). \qquad (3.42)$$

Now, substituting (3.41) into (3.32) and using (3.40), (3.42) and integrating over x and z, yields the following expression for the quark-gluon contribution as,

$$\Pi_{QCD}^G(q^2, (q+p_K)^2) = 4f_{3K}\int_0^1 udu \int D\alpha_i \frac{(q.p_K)\psi_{3K}(\alpha_i)}{\Big((q+(\alpha_1+u\alpha_3)p_K)^2 - m_b^2\Big)^2}. \qquad (3.43)$$

Using Borel transformation (see Appendix for details of Borel transformations) for Eq. (3.39) and the correction (Eq. (3.43)), we get for the $f_+(p^2)$ form factor,

$$\begin{aligned}
f^+(q^2) \;&=\; \frac{m_b f_K}{2f_B m_B^2} e^{m_B^2/M^2}\Big\{\int_\delta^1 exp\Big(-\frac{m_b^2 - q^2(1-u) + p_K^2 u(1-u)}{uM^2}\Big)\frac{du}{u} \\
&\Big[m_b\Big(\psi_K(u) - \frac{8m_b^2[g_1(u)+G_2(u)]}{2u^2M^4} + \frac{2g_2(u)}{M^2}\Big) \\
&+\; \mu_K\Big(u\psi_\rho(u) + \frac{\psi_\sigma(u)}{6}\Big(2 + \frac{m_B^2 + q^2 - p_K^2 u^2}{uM^2}\Big)\Big)\Big] \\
&+\; f_{3K}\int_0^1 du \int D\alpha_i \Theta(\omega-\delta) exp\Big(-\frac{m_b^2 - q^2(1-\omega) + p_K^2\omega(1-\omega)}{\omega M^2}\Big) \\
&\Big[\frac{(2u-1)\psi_{3K}}{f_K}\frac{3p_K^2}{\omega M^2} + \frac{2u\psi_{3K}}{f_K}\Big(-\frac{1}{\omega^2} + \frac{m_b^2 - q^2 - p_K^2\omega^2}{\omega^3 M^2}\Big)
\end{aligned}$$

$$+ \quad \frac{m_b}{f_{3K}}\Big(\frac{2\psi_\perp - \psi_\parallel + 2\tilde{\psi_\perp} - \tilde{\psi_\parallel}}{\omega^2 M^2} + \frac{2p_K^2\alpha_3(\tilde{\Psi}_\perp + \tilde{\Psi}_\parallel + \Psi_\perp + \Psi_\parallel)}{\omega^2 M^4}\Big)\Big]\Big\}. \tag{3.44}$$

The sum rules for f_0, which is just the combination of f_+ and f_- can be calculated in the same manner to get,

$$\begin{aligned} f_+(q^2) \quad + \quad & f_-(q^2) = \\ & \frac{m_b f_K}{f_B m_B^2} e^{m_b^2/M^2}\Big\{\int_\delta^1 exp\Big(-\frac{m_b^2 - q^2(1-u) + p_K^2 u(1-u)}{uM^2}\Big)\frac{du}{u} \\ \times \quad & \Big[2m_b\frac{g_2(u)}{uM^2} + \mu_K\Big(\psi_\rho(u) + \frac{\psi_\sigma(u)}{6u} - \frac{\psi_\sigma(u)}{6u^2M^2}\big(m_b^2 - q^2 + p_K^2u^2\big)\Big)\Big] \\ + \quad & \int_0^1 du \int D\alpha_i \Theta(\omega - \delta) exp\Big(-\frac{m_b^2 - q^2(1-\omega) + p_K^2\omega(1-\omega)}{\omega M^2}\Big) \\ \times \quad & \frac{p_K^2}{\omega^2 M^2}\Big[\frac{f_{3K}(2u-3)\psi_{3K}}{f_K} + 2m_b\alpha_3\frac{\Psi_{perp} + \Psi_\parallel + \tilde{\Psi}_{perp} + \tilde{\Psi}_\parallel}{\omega M^2}\Big]\Big\}. \end{aligned} \tag{3.45}$$

Also using (3.33) with the similar discussions above, one can calculate the sum rules for the form factor f_T as,

$$\begin{aligned} f_T(q^2) \quad = \quad & \frac{m_b(m_B + m_K)f_K}{f_B m_B^2} e^{m_b^2/M^2} \\ & \Big\{\int_\delta^1 du\ exp\Big(-\frac{m_b^2 - q^2(1-u) + p_K^2u(1-u)}{uM^2}\Big) \\ \times \quad & \Big[-\mu_K\frac{m_b\psi_\sigma}{6u^2M^2} - \frac{1}{2}\frac{\psi_K(u)}{u} + 2\big(\frac{m_b^2}{uM^2} + 1\big)\frac{[g_1(u) + G_2(u)]}{u^2M^2}\Big] \\ \times \quad & \int_0^1 du \int D\alpha_i\Theta(\omega - \delta)\ exp\Big(-\frac{m_b^2 - q^2(1-\omega) + p_K^2\omega(1-\omega)}{\omega M^2}\Big) \\ \times \quad & \Big[\frac{u(\psi_\parallel - 2\tilde{\psi_\perp})}{\omega^2M^2} + \frac{\psi_\parallel + \tilde{\psi_\parallel} - 2\psi_\perp - 2\tilde{\psi_\perp}}{\omega^2M^2}\Big]\Big\}, \end{aligned} \tag{3.46}$$

where $\delta = (m_b^2 - q^2)/(s_0 - q^2)$, $p_K^2 = m_K^2$ and $\mu_K = m_K^2/(m_s - m_q)$. The functions $\Psi_\perp(\tilde{\Psi}_\perp)$, $\Psi_\parallel(\tilde{\Psi}_\parallel)$ in Eqs. (3.44)-(3.46) are defined in the following way,

$$\Psi_\perp(\tilde{\Psi}_\perp) \quad = \quad -\int_0^u \psi_\perp(v)(\tilde{\psi}_\perp(v))dv,$$

	f_+	f_0	f_T
$F(0)$	0.319	0.319	0.355
c_1	1.465	0.633	1478
c_2	0.372	-0.095	0.373
c_3	0.782	0.591	0.7

Table 3.1: Parameters for the form factors

$$\Psi_{\|}(\tilde{\Psi}_{\|}) = -\int_0^u \psi_{\|}(v)(\tilde{\psi}_{\|}(v))dv.$$

For kaon wave functions, the results of [42], [43], [44], [45] are used as follows,

$$\begin{aligned}
\psi_K &\simeq 6u(1-u)\Big\{1+0.52[5(2u-1)^2-1]+0.34[21(2u-1)^4- \\
&\quad 14(2u-1)^2+1], \\
\psi_\rho &\simeq 1, \\
\psi_\sigma &\simeq 6u(1-u), \\
g_1 &\simeq \frac{5}{2}\delta^2 u^2(1-u)^2, \\
g_2 &\simeq \frac{10}{3}\delta^2 u(1-u)(2u-1), \\
\psi_{3K}(\alpha_i) &\simeq 360\ \alpha_1\ \alpha_2\ \alpha_3^2, \\
\psi_\perp(\alpha_i) &\simeq 10\ \delta^2(\alpha_1-\alpha_2)\alpha_3^2, \\
\psi_{\|}(\alpha_i) &\simeq 120\ \delta^2\varepsilon\ (\alpha_1-\alpha_2)\alpha_1\ \alpha_2\ \alpha_3, \\
\tilde{\psi}_\perp(\alpha_i) &= 10\ \delta^2\ \alpha_3^2(1-\alpha_3), \\
\tilde{\psi}_{\|}(\alpha_i) &= -40\ \delta^2\ \alpha_1\ \alpha_2\ \alpha_3,
\end{aligned}$$

where, $\delta^2(\mu_b) \simeq 0.17GeV^2$ at $\mu_b \simeq \sqrt{m_B^2-m_b^2} \simeq 2.4GeV$, $\varepsilon \simeq 0.36$. For the values of the form factors, we have used the results of [46]. The q^2 dependence of the form factors can be represented in terms of three parameters as,

$$F(s) = F(0)e^{c_1 s + c_2 s^2 + c_3 s^3},$$

where the values of parameters $F(0),\ c_1,\ c_2,\ c_3$ for the $B \to K$ decay are listed in Table 3.1.

3.4 Calculation of Form Factors in $B \to K^* l^+ l^-$ Decay

For calculating the form factors appearing in the parametrizations of the matrix element in the $B \to K^* l^+ l^-$ decay, (Eqs. 3.8-3.11), we consider the following correlation functions,

$$\Pi_\mu^{(1)}(p,q) = i \int d^4x\ e^{iqx} \langle K^*(p)|\bar{s}(x)\gamma_\mu(1-\gamma_5)b(x)\bar{b}(0)i\gamma_5 q(0)|0\rangle, \qquad (3.47)$$

$$\Pi_\mu^{(2)}(p,q) = i \int d^4x\ e^{iqx} \langle K^*(p)|\bar{s}(x)i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b(x)\bar{b}(0)i\gamma_5 q(0)|0\rangle. \qquad (3.48)$$

The first correlator is relevant to calculate the form factors $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $A_0(q^2)$ and the second is relevant to calculate $T_1(q^2)$, $T_2(q^2)$, $T_3(q^2)$.

To derive the sum rule, let us start by considering the hadronic representation of Eq. (3.48) with the following matrix element of the time-ordered product of two currents between the vacuum state and the K^*–meson at momentum p,

$$i \int d^4x\ e^{iqx}\ \langle K^*(p,\varepsilon)|T\{\bar{\psi}(x)\sigma_{\mu\nu}q^\nu b(x)\bar{b}(0)i\gamma_5\psi(0)\}|0\rangle = i\epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\mu}q^\rho p^\sigma\ T((p+q)^2), \qquad (3.49)$$

and

$$\begin{aligned}\Pi_\mu(q,p) &= i \int d^4x\ e^{iqx}\ \langle K^*(p,\varepsilon)|T\{\bar{\psi}(x)\sigma_{\mu\nu}\gamma_5 q^\nu b(x)\bar{b}(0)i\gamma_5\psi(0)\}|0\rangle \\ &= \varepsilon^*_{\alpha\beta}\frac{p_B^\alpha p_B^\beta}{m_B}F(q^2,(p+q)^2)p_\mu + \varepsilon^*_{\alpha\beta}\frac{p_B^\alpha p_B^\beta}{m_B}H(q^2,(p+q)^2)q_\mu \\ &\quad +\varepsilon^*_{\mu\alpha}\frac{p_B^\alpha}{m_B}G(q^2,(p+q)^2). \qquad (3.50)\end{aligned}$$

Making use of the free propagator $S_b^0(x)$,

$$\begin{aligned} \langle 0|T\{b(x)\bar{b}(0)\}|0\rangle &= iS_b^0(x) = i\int \frac{d^4p}{(2\pi)^4}\, e^{-ipx}\frac{m_b + \not{p}}{p^2 - m_b^2} \\ &= -\int_0^\infty \frac{d\alpha}{16\pi^2\alpha^2}\Big(m_b + \frac{i\not{x}}{2\alpha}\Big)e^{-m_b^2\alpha + x^2/4\alpha}, \end{aligned} \tag{3.51}$$

a formal expression is obtained from Eq. (3.49) as in [47],

$$\int d^4x e^{iqx}\int \frac{d^4k}{(2\pi)^4}\frac{q^\nu}{m_b^2 - k^2}\langle K^*(p,\varepsilon)|T\{\bar{\psi}(x)\sigma_{\mu\nu}(m_b + \not{k})i\gamma_5\psi(0)|0\rangle, \tag{3.52}$$

and from (3.50) as in [48]

$$\int \frac{d\alpha}{16\pi^2\alpha^2}\int dx\; e^{iqx - m_b^2\alpha + x^2/4\alpha}\langle K^*(p,\varepsilon)|T\{\bar{\psi}(x)\sigma_{\mu\nu}\gamma_5 q^\nu\Big(m_b + \frac{i\not{x}}{2\alpha}\Big)i\gamma_5\psi(0)\}|0\rangle. \tag{3.53}$$

In general, Eqs. (3.52) and (3.53) are expressed through matrix elements of non-local operators sandwiched between the vacuum and meson state. The first term in Eq. (3.52) is given by [47]

$$\langle 0|\bar{\psi}(0)\sigma_{\mu\nu}\psi(x)|K^*(p,\varepsilon)\rangle = i(\varepsilon_\mu p_\nu - \varepsilon_\nu p_\mu)f_{K^*}^\perp \times \int_0^1 du\; e^{-iupx}\phi_\perp(u,\mu^2). \tag{3.54}$$

Likewise,

$$\begin{aligned} \langle 0|\bar{\psi}(0)\gamma_\mu\psi(x)|K^*(p,\varepsilon)\rangle &= f_{K^*}m_{K^*}\int_0^1 du\; e^{-iupx} \times \Bigg[p_\mu\frac{(\varepsilon.x)}{(p.x)}\phi_\parallel(u,\mu^2) \\ &+ \Bigg(\varepsilon_\mu - p_\mu\frac{(\varepsilon.x)}{p.x}\Bigg)g_\perp^{(v)}(u,\mu^2)\Bigg], \end{aligned} \tag{3.55}$$

$$\langle 0|\bar{\psi}(0)\gamma_\mu\gamma_5\psi(x)|K^*(p,\varepsilon)\rangle = -\frac{1}{4}\;\epsilon_{\mu\nu\rho\sigma}\varepsilon^\nu p^\rho x^\sigma f_{K^*}m_{K^*}\int_0^1 du\; e^{-iupx}g_\perp^{(a)}(u,\mu^2). \tag{3.56}$$

Also, the first term in Eq. (3.53) is given by [48]

$$\langle K^*(p,\varepsilon)|T\{\bar{\psi}(x)\sigma_{\mu\nu}\gamma_5 q^\nu\gamma_5\psi(0)\}|0\rangle = -i\; p_\mu f_{K^*}\int_0^1 du e^{iuqx}\phi_\parallel(u,\mu^2), \tag{3.57}$$

where, the functions $\phi_\perp(u,\mu^2)$ and $\phi_{||}(u,\mu^2)$ give the leading–twist distributions in the fraction of total momentum carried by the quark in transversely and longitudinally polarized mesons, respectively. The functions $g_\perp^{(v)}$ and $g_\perp^{(a)}$ are twist–3 wave functions and are given as,

$$g_\perp^{(v)}(u) = \frac{3}{4}[1+(2u-1)^2], \tag{3.58}$$

$$g_\perp^{(a)}(u) = 6u(1-u). \tag{3.59}$$

For the explicit form of $\phi_{||}$ we shall use the results of [47]

$$\begin{aligned} \phi_{||}(u,\mu^2) &= 6u(1-u)\Big\{1+a_1(\mu)(2u-1)+a_2(\mu)\left[(2u-1)^2-\frac{1}{5}\right] \\ &+ a_3(\mu)\left[\frac{7}{3}(2u-1)^3-(2u-1)\right]+....\Big\}, \end{aligned} \tag{3.60}$$

with

$$a_n(\mu) = a_n(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\gamma_n/b},$$

where $b = 11/3\ N_c - 2/3\ n_f$ and

$$\gamma_n = C_F\left(1+4\sum_{j=2}^{n+1}\frac{1}{j}\right).$$

Here, $C_F = (N_c^2-1)/(2N_c)$, N_c and n_f being the number colors and active flavors, respectively. Also for $\phi_{||}$ we use [48]

$$\phi_{||} = 6u(1-u). \tag{3.61}$$

Considering Eq. (3.49) using Eqs. (3.54) - (3.56) in Eq. (3.52), we obtain the invariant amplitude T as,

$$\begin{aligned} T((p+q)^2) &= \int_0^1 \frac{du}{\Delta}\Big[m_b\ f_{K^*}\ \phi_\perp(u) + u\ m_{K^*}\ f_{K^*}\ g_\perp^{(v)}(u)\ + \\ & 1/4\ m_{K^*}\ f_{K^*}\ g_\perp^{(a)}(u)\Big] + \frac{1}{4}\int_0^1 du\ \frac{m_b^2-u^2m_{K^*}^2}{\Delta^2}\ m_{K^*}^2\ f_{K^*}\ g_\perp^{(a)}(u), \end{aligned} \tag{3.62}$$

where $\Delta = m_b^2 - (q+pu)^2$.

Since we have splited (3.48) into two terms, we similarly obtain the contribution from second term also, to get the whole expression as [49],

$$\begin{aligned}
\Pi_\mu^{(2)} &= \epsilon_{\mu\nu\rho\sigma}\ \varepsilon^{*\nu} p^\rho q^\sigma \Big\{ m_b\ f_{K^*} \int_0^1 \frac{du}{\Delta}\phi_\perp - m_{K^*}\ f_{K^*} \Big[\int_0^1 \frac{du}{\Delta}\big(\Phi_\parallel - G_\parallel^{(v)}\big) \\
&- \int_0^1 \frac{du}{\Delta} u g_\perp^{(v)} - \int_0^1 \frac{du}{2\Delta} g_\perp(\Delta + q^2 + 2pqu)\Big]\Big\} + i\big[\varepsilon_\mu^*(p.q) - (q.\varepsilon^*)p_\mu\big] \\
&\times \Big\{ m_b\ f_{K^*} \int_0^1 \frac{du}{\Delta}\phi_\perp + m_{K^*}\ f_{K^*} \int_0^1 \frac{du}{\Delta}\Big[-\big(\Phi_\parallel - G_\parallel^{(v)}\big) + u g_\perp^{(v)} \\
&+ \frac{g_\perp^{(a)}}{2} + \frac{g_\perp^{(a)} u(p.q)}{2\Delta}\Big]\Big\} + i m_{K^*}\ f_{K^*}\big[\varepsilon_\mu^* q^2 - (q.\varepsilon^*)q_\mu\big] \int_0^1 \frac{du}{\Delta}\Big[g_\perp^{(v)} - \frac{p^2 u}{2\Delta} g_\perp^{(a)}\Big] \\
&+ 2 i m_{K^*}\ f_{K^*}(q.\varepsilon^*)\big[p_\mu q^2 - (p.q)q_\mu\big] \int_0^1 \frac{du}{\Delta^2}\big(\Phi_\parallel - G_\parallel^{(a)}\big). \qquad (3.63)
\end{aligned}$$

We can also calculate the theoretical part of the sum rules for the correlation Eq. (3.47) as [49]

$$\begin{aligned}
\Pi_\mu^{(1)} &= -i m_b\ f_{K^*}\ m_{K^*} \int_0^1 \frac{du}{\Delta}\Big[\varepsilon_\mu^* g_\perp^{(v)} + 2(q.\varepsilon^*)p_\mu \frac{1}{\Delta}\big(\Phi_\parallel - G_\parallel^{(v)}\big)\Big] \\
&- \epsilon_{\mu\nu\rho\sigma}\ \varepsilon^{*\nu} p^\rho q^\sigma \Big[m_b \frac{1}{2}\ f_{K^*}\ m_{K^*}\ \int_0^1 \frac{du}{\Delta^2}\ g_\perp^{(a)} + f_{K^*}\ \int_0^1 du \frac{\phi_\perp}{\Delta}\Big] \\
&- i\ f_{K^*} \int_0^1 du \frac{\phi_\perp}{\Delta}\big[\varepsilon_\mu^*(p.q + p^2 u) - p_\mu(q.\varepsilon^*)\big]. \qquad (3.64)
\end{aligned}$$

Having calculated the theoretical part of the correlation functions, let us focus on their hadronic representations. The hadronic representation is obtained by inserting a complete set of states including the B-meson ground state as for Eq. (3.49) for instance

$$\begin{aligned}
i\epsilon_{\mu\nu\rho\sigma}\ \varepsilon^{*\nu} q^\rho p^\sigma T((p+q)^2) &= \frac{\langle K^*(p,\varepsilon)|\bar{\psi}(x)\sigma_{\mu\nu}q^\nu b(x)|B\rangle\langle B|\bar{b}(0)i\gamma_5\psi(0)|0\rangle}{m_B^2 - (p+q)^2} \\
&+ \sum_h \frac{\langle K^*(p,\varepsilon)|\bar{\psi}(x)\sigma_{\mu\nu}q^\nu b(x)|h\rangle\langle h|\bar{b}(0)i\gamma_5\psi(0)|0\rangle}{m_B^2 - (p+q)^2}. \qquad (3.65)
\end{aligned}$$

If we separate the contribution of the B-meson mass as the pole contribution to the invariant function $T((p+q)^2)$ to get,

$$T((p+q)^2) = \frac{f_B m_B^2}{m_b} + \frac{2F_1^{K^*}(0)}{m_B^2 - (p+q^2)} + higher - mass\ resonance\ terms... \tag{3.66}$$

where

$$\langle 0|\bar{\psi}\gamma_\mu\gamma_5 b|B(p_B)\rangle = i\ p_{B\mu}\ f_B, \tag{3.67}$$

is used. Similar arguments on Eq. (3.50) can be carried on to get the invariant amplitudes for G and H. Considering so, the hadronic representation of (3.48) can be written as [49]

$$\Pi_\mu^{(2)} = \frac{f_B m_B^2}{m_b[m_B^2 - (q+p)^2]}\langle K^*(p,\varepsilon)|\bar{s}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)q(0)|B(p+q)\rangle, \tag{3.68}$$

and similarly (3.47) as

$$\Pi_\mu^{(1)} = \frac{f_B m_B^2}{m_b[m_B^2 - (q+p)^2]}\langle K^*(p,\varepsilon)|\bar{s}\gamma_\mu(1-\gamma_5)q(0)|B(p+q)\rangle. \tag{3.69}$$

Using definitions of form factors in Eqs. (3.8) - (3.11) and equating these to (3.63) and (3.68), we get the sum rules for the form factors [49] after applying the Borel transformations,

$$\begin{aligned} V(q^2) &= \frac{m_B + m_{K^*}}{2}\frac{m_b}{f_B m_B^2}\ e^{m_B^2/M^2}\int_\delta^1 du\ exp\left(-\frac{m_b^2 + p^2 u\bar{u} - q^2\bar{u}}{uM^2}\right) \\ &\times \left\{m_b f_{K^*} m_{K^*}\frac{g_\perp^{(a)}}{2u^2M^2} + \frac{f_{K^*}\phi_\perp}{u}\right\}, \end{aligned} \tag{3.70}$$

$$\begin{aligned} A_1(q^2) &= \frac{1}{m_B + m_{K^*}}\frac{m_b}{f_B m_B^2}\ e^{m_B^2/M^2}\int_\delta^1 du\ exp\left(-\frac{m_b^2 + p^2 u\bar{u} - q^2\bar{u}}{uM^2}\right) \\ &\times \left\{m_b f_{K^*} m_{K^*}\frac{g_\perp^{(v)}}{u} + \frac{f_{K^*}\phi_\perp(m_b^2 - q^2 + p^2u^2)}{2u^2}\right\}, \end{aligned} \tag{3.71}$$

$$A_2(q^2) = -(m_B+m_{K^*})\frac{m_b}{f_B m_B^2}\ e^{m_B^2/M^2}\int_\delta^1 du\ exp\Big(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\Big) \times \Big\{\frac{m_b f_{K^*}m_{K^*}}{u^2M^2}\Big(\Phi_\parallel - G_\perp^{(v)}\Big)-\frac{1}{2}f_{K^*}\frac{\phi_\perp}{u}\Big\}, \quad (3.72)$$

$$A_3(q^2)-A_0(q^2) = \frac{q^2}{2m_{K^*}}\frac{m_b}{f_B m_B^2}\ e^{m_B^2/M^2}\int_\delta^1 du\ exp\Big(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\Big) \times \Big\{\frac{m_b f_{K^*}m_{K^*}}{u^2M^2}\Big(\Phi_\parallel - G_\perp^{(v)}\Big)-\frac{1}{2}f_{K^*}\frac{\phi_\perp}{u}\Big\}, \quad (3.73)$$

$$T_1(q^2) = \frac{1}{4}\frac{m_b}{f_B m_B^2}\ e^{m_B^2/M^2}\int_\delta^1 \frac{du}{u}\ exp\Big(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\Big)\times\Big\{m_b f_{K^*}\phi_\perp - f_{K^*}m_{K^*}\Big[\Phi_\perp - G_\perp^{(v)} - u g_\perp^{(v)} - \frac{g_\perp^{(a)}}{4} - \frac{g_\perp^{(a)}(m_b^2+q^2-p^2u^2)}{4uM^2}\Big]\Big\}, \quad (3.74)$$

$$T_2(q^2) = \frac{1}{2(m_B^2-m_{K^*}^2)}\frac{m_b}{f_B m_B^2}\ e^{m_B^2/M^2}\int_\delta^1 \frac{du}{u}\ exp\Big(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\Big) \times \Big\{f_{K^*}m_{K^*}\Big[g_\perp^{(v)}-\frac{p^2}{2M^2}g_\perp^{(a)}\Big]q^2+\frac{m_b f_{K^*}\phi_\perp}{2u}(m_b^2-q^2-p^2u^2) + f_{K^*}m_{K^*}\Big[\frac{(m_b^2-q^2-p^2u^2)}{2u}\times\Big(-\Big[\Phi_\perp - G_\perp^{(v)}\Big] + u g_\perp^{(v)} + \frac{(m_b^2-q^2-p^2u^2)g_\perp^{(a)}}{4uM^2}\Big)\Big]\Big\}, \quad (3.75)$$

$$T_3(q^2) = \frac{1}{4}\frac{m_b}{f_B m_B^2}\ e^{m_B^2/M^2}\int_\delta^1 \frac{du}{u}\ exp\Big(-\frac{m_b^2+p^2u\bar{u}-q^2\bar{u}}{uM^2}\Big)\times\Big\{m_{K^*}f_{K^*}\Big[\frac{g_\perp^{(a)}}{4} + \frac{(m_b^2-q^2p^2u^2)}{4uM^2}g_\perp^{(a)}\Big] - 2m_{K^*}f_{K^*}\Big[\frac{g_\perp^{(v)}}{2}(2-u)-\frac{p^2 g_\perp^{(a)}}{2M^2}\Big] - 2m_{K^*}f_{K^*}\Big[\frac{\Phi_\parallel - G_\perp^{(v)}}{uM^2}\Big(\frac{m_b^2-q^2-p^2u^2}{u}+q^2-M^2+\frac{uM^2}{2}\Big)\Big] + m_b f_{K^*}\phi_\perp\Big\}. \quad (3.76)$$

Using the equations of motion, we can relate T_3 and $A_3 - A_0$ by,

$$A_3(q^2)-A_0(q^2) = -\frac{A_2(q^2)\ q^2}{2m_{K^*}(m_B+m_{K^*})}, \quad (3.77)$$

	$F(0)$	a_F	b_F
$A_1^{B\to K^*}$	0.34 ± 0.05	0.6	-0.023
$A_2^{B\to K^*}$	0.28 ± 0.04	1.18	0.281
$V^{B\to K^*}$	0.46 ± 0.07	1.55	0.575
$T_1^{B\to K^*}$	0.19 ± 0.03	1.59	0.615
$T_2^{B\to K^*}$	0.19 ± 0.03	0.49	-0.241
$T_3^{B\to K^*}$	0.13 ± 0.02	1.20	0.098

Table 3.2: B-meson form factors in a three parameter fit, where the radiative corrections to the leading twist contribution and SU(3) breaking effects are taken into account.

$$T_3(q^2) = m_{K^*}(m_b - m_s)\frac{A_3(q^2) - A_0(q^2)}{q^2}, \tag{3.78}$$

where M^2 is the Borel mass parameter. The q^2 dependence of the form factors is given in [52], [45], [46] in terms of three parameters as,

$$F(q^2) = \frac{F(0)}{1 - a_F \frac{q^2}{m_B^2} + b_F \left(\frac{q^2}{m_B^2}\right)^2}, \tag{3.79}$$

where the values of parameters $F(0)$, a_F and b_F for the $B \to K^*$ decay are listed in Table 3.2.

CHAPTER 4

MODEL INDEPENDENT ANALYSIS OF SEMILEPTONIC B MESON DECAYS

In this chapter, we study the lepton polarizations in the exclusive $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays using the most general form of the effective Hamiltonian including all possible forms of interactions. As we noted in the Introduction section, theoretical study of exclusive decays is rather more difficult than the relevant inclusive decays, but the experimental detection of exclusive decays is easier than the inclusive ones.

The main drawback of the theoretical calculation of exclusive decays is the calculation of the matrix elements of the effective Hamiltonian sandwiched between final and initial meson states of the corresponding decay. In this chapter we use the results for relevant form factors which were calculated in the Light Cone QCD Sum Rules in the previous chapter.

One main goal of the B physics program in the current B factories and in the forthcoming LHC–B mechanics, is to find inconsistencies within the SM, in particular to find indications for new physics in the flavor and CP violating sectors [55]. In general, new physics effects manifest themselves in rare B meson decays either through new contributions to the Wilson coefficients that exist in the SM or by introducing new structures in the effective Hamiltonian which are absent in the SM.

During the theoretical analysis of these two exclusive decays given above, we will consider the most general model independent four-Fermi interactions, governed by the effective Hamiltonian given as,

$$\begin{aligned}
\mathcal{H}_{eff} &= \frac{G_F\alpha}{\sqrt{2}\pi} V_{ts}V_{tb}^* \Bigg\{ C_{SL}\,\bar{s} i\sigma_{\mu\nu}\frac{q^\nu}{q^2}\,L\,b\,\bar{\ell}\gamma^\mu\ell + C_{BR}\,\bar{s} i\sigma_{\mu\nu}\frac{q^\nu}{q^2}\,R\,b\,\bar{\ell}\gamma^\mu\ell \\
&+ C_{LL}^{tot}\,\bar{s}_L\gamma_\mu b_L\,\bar{\ell}_L\gamma^\mu\ell_L + C_{LR}^{tot}\,\bar{s}_L\gamma_\mu b_L\,\bar{\ell}_R\gamma^\mu\ell_R + C_{RL}\,\bar{s}_R\gamma_\mu b_R\,\bar{\ell}_L\gamma^\mu\ell_L \\
&+ C_{RR}\,\bar{s}_R\gamma_\mu b_R\,\bar{\ell}_R\gamma^\mu\ell_R + C_{LRLR}\,\bar{s}_L b_R\,\bar{\ell}_L\ell_R + C_{RLLR}\,\bar{s}_R b_L\,\bar{\ell}_L\ell_R \qquad (4.1) \\
&+ C_{LRRL}\,\bar{s}_L b_R\,\bar{\ell}_R\ell_L + C_{RLRL}\,\bar{s}_R b_L\,\bar{\ell}_R\ell_L + C_T\,\bar{s}\sigma_{\mu\nu}b\,\bar{\ell}\sigma^{\mu\nu}\ell \\
&+ iC_{TE}\,\epsilon^{\mu\nu\alpha\beta}\bar{s}\sigma_{\mu\nu}b\,\bar{\ell}\sigma_{\alpha\beta}\ell \Bigg\} ,
\end{aligned}$$

where q^2 is the invariant dilepton mass, and $L(R) = [1 - (+)\gamma_5]/2$ are the projection operators, C_X are the coefficients of the four-Fermi interactions and $q = p_B - p_K$ is the momentum transfer. Note that among twelve Wilson coefficients several already exist in the SM. For instance, the coefficients C_{SL} and C_{BR}, which are the non-local Fermi interactions, correspond to $-2m_sC_7^{eff}$ and $-2m_bC_7^{eff}$, in the SM respectively. The following four terms in this expression are the vector type interactions with coefficients C_{LL}, C_{LR}, C_{RL} and C_{RR}. Two of these vector interactions containing C_{LL}^{tot} and C_{LR}^{tot} do already exist in the SM in combinations of the form $(C_9^{eff} - C_{10})$ and $(C_9^{eff} + C_{10})$. Therefore by writing

$$\begin{aligned} C_{LL}^{tot} &= C_9^{eff} - C_{10} + C_{LL} \ , \\ C_{LR}^{tot} &= C_9^{eff} + C_{10} + C_{LR} \ , \end{aligned}$$

one concludes that C_{LL}^{tot} and C_{LR}^{tot} describe the sum of the contributions from SM and the new physics. The terms with coefficients C_{LRLR}, C_{RLLR}, C_{LRRL} and C_{RLRL} describe the scalar type interactions. The remaining two terms leaded by the coefficients C_T and C_{TE}, obviously, describe the tensor type interactions.

The decay rate of any exclusive decay is determined by the following expression:

$$\Gamma^{B\to M} = \frac{(2\pi)^4}{2m_B} \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \frac{d^3\vec{p}_M}{(2\pi)^3 2E_M} |\mathcal{M}_M|^2 \delta^4(q - p_1 - p_2) \qquad (4.2)$$

where M is the final meson state, $q = p_B - p_M$, p_1 and p_2 are the 4-momenta of the final leptons and $\mathcal{M}$ is the matrix element of the decay, which is the effective Hamiltonian sandwiched between final and initial states. Since our main interest is to calculate the polarization asymmetries of the rare B meson decays, after calculating the decay rate in Eq. (4.2) our next task is to compute the final lepton polarizations. For this purpose, we define the following orthogonal unit vectors, $S_L^{-\ell}$ in the rest frame of ℓ^- and $S_L^{+\ell}$ in the rest frame of ℓ^+, for the polarization of the leptons along the longitudinal (L), transversal (T) and normal (N) directions:

$$\begin{aligned} S_L^{-\ell} &\equiv (0, \vec{e}_L^{\,-}) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right), \\ S_N^{-\ell} &\equiv (0, \vec{e}_N^{\,-}) = \left(0, \frac{\vec{p}_K \times \vec{p}_-}{|\vec{p}_K \times \vec{p}_-|}\right), \\ S_T^{-\ell} &\equiv (0, \vec{e}_T^{\,-}) = \left(0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}\right), \end{aligned}$$

$$
\begin{aligned}
S_L^{+\ell} &\equiv (0, \vec{e}_L^{\,+}) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right), \\
S_N^{+\ell} &\equiv (0, \vec{e}_N^{\,+}) = \left(0, \frac{\vec{p}_K \times \vec{p}_+}{|\vec{p}_K \times \vec{p}_+|}\right), \\
S_T^{+\ell} &\equiv (0, \vec{e}_T^{\,+}) = \left(0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}\right),
\end{aligned}
\tag{4.3}
$$

where $\vec{p}_\mp$ and $\vec{p}_K$ are the three momenta of $\ell^\mp$ and K meson in the center of mass (CM) frame of the $\ell^+\ell^-$ system, respectively. The longitudinal unit vectors S_L^- and S_L^+ are boosted to the CM frame of $\ell^+\ell^-$ by Lorentz transformation,

$$
\begin{aligned}
S_{L,CM}^{-\ell} &= \left(\frac{|\vec{p}_-|}{m_\ell}, \frac{E_\ell\, \vec{p}_-}{m_\ell |\vec{p}_-|}\right), \\
S_{L,CM}^{+\ell} &= \left(\frac{|\vec{p}_-|}{m_\ell}, -\frac{E_\ell\, \vec{p}_-}{m_\ell |\vec{p}_-|}\right).
\end{aligned}
\tag{4.4}
$$

The vectors $\vec{S}_N$ and $\vec{S}_T$ are not changed by boost.

The differential decay rate of the $B \to K(K^*)\ell^+\ell^-$ decay for any spin direction $\vec{n}^{(\mp)}$ of the $\ell^{(\mp)}$, where $\vec{n}^{(\mp)}$ is the unit vector in the $\ell^{(\mp)}$ rest frame, can be written as

$$
\frac{d\Gamma(\vec{n}^{(\mp)})}{dq^2} = \frac{1}{2}\left(\frac{d\Gamma}{dq^2}\right)_0 \left[1 + \left(P_L^{(\mp)} \vec{e}_L^{\,(\mp)} + P_N^{(\mp)} \vec{e}_N^{\,(\mp)} + P_T^{(\mp)} \vec{e}_T^{\,(\mp)}\right) \cdot \vec{n}^{(\mp)}\right], \tag{4.5}
$$

where $(d\Gamma/dq^2)_0$ corresponds to the unpolarized differential decay rate which is given in Eq. (4.2), and P_L, P_N, and P_T represent longitudinal, normal and transversal polarizations, respectively. The polarizations P_L, P_N, and P_T are defined as:

$$
P_i^{(\mp)}(q^2) = \frac{\frac{d\Gamma}{dq^2}(\vec{n}^{(\mp)} = \vec{e}_i^{\,(\mp)}) - \frac{d\Gamma}{dq^2}(\vec{n}^{(\mp)} = -\vec{e}_i^{\,(\mp)})}{\frac{d\Gamma}{dq^2}(\vec{n}^{(\mp)} = \vec{e}_i^{\,(\mp)}) + \frac{d\Gamma}{dq^2}(\vec{n}^{(\mp)} = -\vec{e}_i^{\,(\mp)})}, \tag{4.6}
$$

where $P^{(\mp)}$ represents the charged lepton $\ell^{(\mp)}$ polarization asymmetry for $i = L,\ N,\ T$, i.e., P_L and P_T are the longitudinal and transversal asymmetries in the decay plane, respectively, and P_N is the normal component to both of them.

With respect to the direction of the lepton polarization, P_L and P_T are P–odd, T–even, while P_N is P–even, T–odd and CP–odd.

4.1 Exclusive $B \to K\ell^+\ell^-$ Decay

Exclusive $B \to K\ell^+\ell^-$ decay is described by the matrix element of effective Hamiltonian over B and K meson states, which can be parametrized in terms of form factors. It follows from Eq. (4.1) that in order to calculate the amplitude of the $B \to K\ell^+\ell^-$ decay, the following matrix elements are needed

$$\langle K|\bar{s}\gamma_\mu b|B\rangle \ ,$$

$$\langle K|\bar{s}i\sigma_{\mu\nu}q^\nu b|B\rangle \ ,$$

$$\langle K|\bar{s}b|B\rangle \ ,$$

$$\langle K|\bar{s}\sigma_{\mu\nu}b|B\rangle \ .$$

These matrix elements are defined in Eqs. (3.3) - (3.6) as follows

$$\langle K|\bar{s}\gamma_\mu b|B\rangle = f_+\left[(p_B+p_K)_\mu - \frac{m_B^2-m_K^2}{q^2}q^\mu\right] + f_0\frac{mB^2-m_K^2}{q^2}q_\mu, \quad (4.7)$$

$$\langle K|\bar{s}\sigma_{\mu\nu}b|B\rangle = -i\frac{f_T}{m_B+m_K}\left[(p_B+p_K)_\mu q_\nu - q_\mu(p_B+p_K)_\nu\right], \quad (4.8)$$

$$\langle K|\bar{s}i\sigma_{\mu\nu}q^\nu B\rangle = f_0\frac{m_B^2-m_K^2}{m_b-m_s}, \quad (4.9)$$

$$\langle K|\bar{s}b|B\rangle = \frac{f_T}{m_B+m_K}\left[(p_B+p_K)q^2 - q_\mu(m_B^2-m_K^2)\right], \quad (4.10)$$

where $q = p_B - p_K$ is the momentum transfer. So, the matrix element of the $B \to K\ell^+\ell^-$ decay can be written as,

$$\mathcal{M}(\mathcal{B} \to \mathcal{K}\ell^+\ell^-) = \frac{G_F\alpha}{4\sqrt{2}\pi}V_{tb}V_{ts}^*\Big\{\bar{\ell}\gamma^\mu\ell\big[A(p_B+p_K)_\mu + Bq_\mu\big]$$

$$
\begin{aligned}
&+ \quad \bar{\ell}\gamma^\mu\gamma_5\ell\Big[C(p_B+p_K)_\mu + Dq_\mu\Big] + \bar{\ell}\ell Q + \bar{\ell}\gamma_5\ell N \\
&+ \quad 4\bar{\ell}\sigma^{\mu\nu}\ell(-iG)\Big[(p_B+p_K)_\mu q_\nu - (p_B+p_K)_\nu q_\mu\Big] \\
&+ \quad 4\bar{\ell}\sigma^{\alpha\beta}\ell\epsilon_{\mu\nu\alpha\beta}H\Big[(p_B+p_K)_\mu q_\nu - (p_B+p_K)_\nu q_\mu\Big]\Big\}, \qquad (4.11)
\end{aligned}
$$

where the auxiliary functions above are defined as,

$$
\begin{aligned}
A &= (C_{LL}^{tot} + C_{LR}^{tot} + C_{RL} + C_{RR})\, f_+ + 2(C_{BR} + C_{SL})\,\frac{f_T}{m_B + m_K}, \\
B &= (C_{LL}^{tot} + C_{LR}^{tot} + C_{RL} + C_{RR})\, f_- - 2(C_{BR} + C_{SL})\,\frac{f_T}{(m_B + m_K)q^2}\,(m_B^2 - m_K^2), \\
C &= (C_{LR}^{tot} + C_{RR} - C_{LL}^{tot} - C_{RL})\, f_+, \\
D &= (C_{LR}^{tot} + C_{RR} - C_{LL}^{tot} - C_{RL})\, f_-, \qquad (4.12) \\
Q &= f_0\,\frac{m_B^2 - m_K^2}{m_b - m_s}\,(C_{LRLR} + C_{RLLR} + C_{LRRL} + C_{RLRL}), \\
N &= f_0\,\frac{m_B^2 - m_K^2}{m_b - m_s}\,(C_{LRLR} + C_{RLLR} - C_{LRRL} - C_{RLRL}), \\
G &= \frac{C_T}{m_B + m_K}\, f_T, \\
H &= \frac{C_{TE}}{m_B + m_K}\, f_T,
\end{aligned}
$$

where

$$
f_- = (f_0 - f_+)\frac{m_B^2 - m_K^2}{q^2}.
$$

It follows from Eq. (4.11) that the difference from the SM is due to the last four terms only, namely, scalar and tensor type interactions. Using Eq. (4.11) we next calculate the final lepton polarizations for the $B \to K\ell^+\ell^-$ decay.

In order to calculate the final lepton polarizations, firstly we have to find the decay rate expression for the unpolarized leptons. The detailed calculations can be found in the Appendix. We present here only the results. Taking the modulus square of the Eq. (4.11), we find,

$$
\begin{aligned}
|M|^2 &= \left|\frac{G_F\alpha}{2\sqrt{2}\pi}\right|^2 \left|V_{tb}V_{ts}^*\right|^2 \Big\{ |A|^2[2(p\cdot p_1)(p\cdot p_2)-p^2(p_1\cdot p_2)-m_l^2p^2] \\
&+ 2Re(AB^*)[(p\cdot p_1)(q\cdot p_2)+(p\cdot p_2)(q\cdot p_1)-(p\cdot q)(p_1\cdot p_2)-m_l^2(p\cdot q)] \\
&+ |B|^2[2(q\cdot p_1)(q\cdot p_2)-q^2(p_1\cdot p_2)-m_l^2q^2] \\
&+ 2Re(AQ^*)m_l[(p\cdot p_2)-(p\cdot p_1)]+2Re(BQ^*)m_l[(q\cdot p_2)-(q\cdot p_1)] \\
&+ 16Re(AG^*)m_l[-(p\cdot q)(p_1\cdot p)-(p\cdot q)(p_2\cdot p)+p^2(p_1\cdot q)+p^2(p_2\cdot q)] \\
&+ 16Re(BG^*)m_l[-q^2(p_1\cdot p)-q^2(p_2\cdot p)+(p\cdot q)(p_1\cdot q)+(p\cdot q)(p_2\cdot q)] \\
&+ |C|^2[2(p\cdot p_1)(p\cdot p_2)-p^2(p_1\cdot p_2)+m_l^2p^2] \\
&+ 2Re(CD^*)[(p\cdot p_1)(q\cdot p_2)+(p\cdot p_2)(q\cdot p_1)-(p\cdot q)(p_1\cdot p_2)+m_l^2(p\cdot q)] \\
&+ |D|^2[2(q\cdot p_1)(q\cdot p_2)-q^2(p_1\cdot p_2)+m_l^2q^2] \\
&- 2Re(CN^*)m_l[(p_1\cdot p)+(p_2\cdot p)]-2Re(DN^*)m_l[(p_1\cdot q)+(p_2\cdot q)] \\
&+ |Q|^2(p_1\cdot p_2)+|N|^2(p_1\cdot p_2)-4Re(QG^*)[(p\cdot p_2)(q\cdot p_1)-(p\cdot p_1)(q\cdot p_2)] \\
&+ 128|G|^2[-2(p\cdot p_1)(q\cdot p_2)(p\cdot q)-2(q\cdot p_1)(p\cdot p_2)(p\cdot q)+2(p\cdot p_1)(p\cdot p_2)q^2 \\
&\quad +2(q\cdot p_1)(q\cdot p_2)p^2-(p_1\cdot p_2)p^2q^2+(p_1\cdot p_2)(p\cdot q)(p\cdot q) \\
&\quad +m_l^2p^2q^2-m_l^2(p\cdot q)(p\cdot q)] \\
&+ 256|H|^2[2(p\cdot p_1)(q\cdot p_2)(p\cdot q)+2(q\cdot p_1)(p\cdot p_2)(p\cdot q)-2(p\cdot p_1)(p\cdot p_2)q^2 \\
&\quad -2(q\cdot p_1)(q\cdot p_2)p^2+(p_1\cdot p_2)p^2q^2-(p_1\cdot p_2)(p\cdot q)(p\cdot q)] \Big\}, \qquad (4.13)
\end{aligned}
$$

where we called $p = (p_B + p_K)$ for simplicity, and $q = p_B - p_K$.

Using Eq. (4.13) in Eq. (4.2), and after performing integration over final lepton momenta, we find the unpolarized differential decay rate as, (we present these integration techniques in Appendix B)

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\alpha^2}{2^{14}\pi^5 m_B}\left|V_{tb}V_{ts}^*\right|^2\lambda^{1/2}(1,r,s)v\Delta, \tag{4.14}$$

where $\lambda(1,r,s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $s = q^2/m_B^2$, $r = m_K^2/m_B^2$ and $v = \sqrt{1 - 4m_l^2/q^2}$ is the lepton velocity. The explicit form for Δ for massless leptons is,

$$\begin{aligned} \Delta &= 4m_B^2 s\,|N|^2 + \frac{1024}{3}\lambda m_B^6 s\,|H|^2 \\ &+ 4m_B^2 s\,|Q|^2 + \frac{8}{3}\lambda m_B^4 s\,|A|^2 + \frac{512}{3}\lambda m_B^6 s\,|G|^2 \\ &+ \frac{4}{3}m_B^4 s\Big\{2\lambda\Big[2\lambda - 3(1-r)^2\Big]\Big\}\,|C|^2. \end{aligned} \tag{4.15}$$

The calculations for massive leptons are represented in the Appendix section.

4.1.1 Polarization Asymmetries in $B \to K\ell^+\ell^-$ Decay

We next calculate the differential decay rate when the leptons are polarized longitudinally or transversally. In order to do this calculations, we make use of the expression in (4.6) where the differential decay rate here, is given in (4.5).

From the matrix element as given in Eq. (4.11), for the longitudinally polarized decay, using Eq. (4.3) we find the module square as,

$$\begin{aligned} |M_\mp|^2 &= \left|\frac{G_F\alpha}{2\sqrt{2}\pi}\right|^2\left|V_{tb}V_{ts}^*\right|^2\Big\{2Re(AC^*)\Big[\pm 8m_l(p.s)((p_1.p) + (p_2.p)) \\ &\quad \pm 4m_l(p.s)((p_2.q) + (p_1.q)) \mp 4m_l p^2((p_1.s) + (p_2.s)) \mp 8m_l p^2(p_2.s) \\ &\quad \mp 8m_l(p.q)(p_2.s) \mp 2m_l(p_2.s)q^2 \pm 8m_l(q.s)((p.p_2) + (p.p_1)) \\ &\quad \pm 4m_l(q.s)((p_1.q) + (p_2.q))\Big] \\ &+ 2Re(AD^*)\Big[\pm 4m_l(p.s)((p_1.q) + (p_2.q)) \pm 4m_l(p.q)(p_2.s) \mp 2m_l q^2(p_2.s) \end{aligned}$$

$$
\begin{aligned}
& \pm 4m_l(q.s)((p_2.q)+(p_1.q))\Big] \\
+ \quad & 2Re(BC^*)\Big[\mp 4m_l(p.q)(p_2.s) \mp 2m_l q^2(p_2.s) \pm 4m_l(q.s)((p_1.p)+(p_2.p)) \\
& \pm 4m_l(q.s)((p_2.q)+(p_1.q))\Big] \\
+ \quad & 2Re(BD^*)\Big[\mp 2m_l q^2(p_2.s) \pm 4m_l(q.s)((p_2.q)+(p_1.q))\Big] \\
+ \quad & 2Re(AQ^*)\Big[\pm 2(p.p_2)(p_1.s) \pm (p_1.s)(p_2.q)\Big] \\
+ \quad & 2Re(AN^*)\Big[\pm 2m_l^2(p.s) \pm 2(p.s)(p_1.p_2) \mp 2(p_1.p)(p_2.s) \mp (p_1.q)(p_2.s) \\
& \pm m_l^2(q.s) \pm (q.s)(p_1.p_2)\Big] \\
+ \quad & 2Re(AG^*)\Big[\mp 32m_l^2(p.q)(p.s) \pm 32(p.q)(p.s)(p_1.p_2) \\
& \mp 32(p.s)((p_2.p)+(p_2.q))((p_1.q)+(p_1.p)) \mp 32(p.p2)(p.q)(p_1.s) \\
& \mp 32(p.s)(p_1.q)(p_2.q) \pm 32p^2(p_1.s)(p_2.q) \pm 16(p.q)(p_1.s)(p_2.q) \\
& \mp 16m_l^2q^2(p.s) \pm 16q^2(p.s)(p_1.p_2) \mp 16q^2(p_1.s)(p_2.p) \pm 32m_l^2p^2(q.s) \\
& \pm 64(p_1.p)(p_2.p)(q.s) \pm 16m_l^2(p.q)(q.s) \mp p^2(p_1.p_2)(q.s) \\
& \mp 16(p.q)(p_1.p_2)(q.s) \pm 32(p_1.p)(p_2.q)(q.s)\Big] \\
+ \quad & 2Re(AH^*)\Big[\pm 256(p.s)((p.p_1)+(p.p_2))((q.p_1)+(q.p_2)) \\
& \pm 64(p.p_1)(p.q)(p_2.s) \mp 64p^2(p_1.q)(p_2.s) \\
& \mp 32(p.q)(p_1.q)(p_2.s) \pm 32q^2(p_1.p)(p_2.s)\Big] \\
+ \quad & 2Re(BQ^*)\Big[\pm (p_1.s)(p_2.q)\Big] \\
+ \quad & 2Re(BN^*)\Big[\mp (p_1.q)(p_2.s) \pm (m_l^2+(p_1.p_2))(q.s)\Big] \\
+ \quad & 2Re(BG^*)\Big[\mp 32(p.s)(p_1.q)(p_2.q) \pm 16m_l(p.q)(p_1.s)(p_2.q) \\
& \mp 16m_l^2q^2(p.s) \pm 16(p.s)(p_1.p_2)q^2 \mp 16(p_2.p)(p_1.s)q^2 \pm 16m_l^2(p.q)(q.s) \\
& \mp 16(p.q)(p_1.p_2)(q.s) \pm 32(p_1.p)(p_2.q)(q.s)\Big]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad 2Re(BH^*)\Big[\mp 32(p.q)(p_1.q)(p_2.s) \pm 32(p_1.p)(p_2.s)q^2\Big] \\
&+ \quad 2Re(CQ^*)\Big[\pm 2(p_2.p)(p_1.s) \pm (p_1.s)(p_2.q)\Big] \\
&+ \quad 2Re(CN^*)\Big[\pm 2m_l^2(p.s) \pm 2(p.s)(p_1.p_2) \mp 2(p_1.p)(p_2.s) \mp (p_1.q)(p_2.s) \\
&\quad \pm m_l^2(q.s) \pm (p_1.p_2)(q.s)\Big] \\
&+ \quad 2Re(CG^*)\Big[\mp 32m_l^2(p.q)(p.s) \pm 32(p.q)(p.s)(p_1.p_2) \\
&\quad \mp 32(p.p_2)(p.q)(p_1.s) \mp 32(p_2.p)(p.s)(p_1.q) \mp 32(p.s)(p_1.q)(p_2.q) \\
&\quad \pm 32p^2(p_1.s)(p_2.q) \pm 16(p.q)(p_1.s)(p_2.q) \mp 16m_l^2(p.s)q^2 \\
&\quad \pm 16(p.s)(p_1.p_2)q^2 \mp 16(p_2.p)(p_1.s)q^2 \pm 32m_l^2p^2(q.s) \\
&\quad \pm 64(p_1.p)(p_2.p)(q.s) \pm 16m_l^2(p.q)(q.s) \mp 32p^2(p_1.p_2)(q.s) \\
&\quad \mp 16(p.q)(p_1.p_2)(q.s) \pm 32(p_1.p)(p_2.q)(q.s)\Big] \\
&+ \quad 2Re(CH^*)\Big[\pm 64(p_2.p)(p.s)(p_1.q) \pm 64(p_1.p)(p.q)(p_2.s) \\
&\quad \mp 64p^2(p_1.q)(p_2.s) \mp 32(p.q)(p_1.q)(p_2.s) \pm 64q^2(p_1.p)(p_2.s)\Big] \\
&+ \quad 2Re(DQ^*)\Big[\pm (p_1.s)(p_2.q) \mp (p_1.q)(p_2.s)\Big] \\
&+ \quad 2Re(DN^*)\Big[\mp (p_1.q)(p_2.s) \mp m_l^2(q.s) \pm (p_1.p_2)(q.s)\Big] \\
&+ \quad 2Re(DG^*)\Big[\mp 32(p.s)(p_1.q)(p_2.q) \pm 16(p.q)(p_1.s)(p_2.q) \\
&\quad \mp 16m_l^2q^2(p.s) \pm 16q^2(p.s)(p_1.p_2) \mp 16(p_2.p)(p_1.s)q^2 \\
&\quad \pm 16m_l^2(p.q)(q.s) \mp 16(p.q)(p_1.p_2)(q.s) \pm 32(p_1.p)(p_2.q)(q.s)\Big] \\
&+ \quad 2Re(DH^*)\Big[\mp 32(p.q)(p_1.q)(p_2.s) \pm 32q^2(p_1.p)(p_2.s)\Big] \\
, &+ \quad 2Re(QN^*)\Big[\mp 2m_lk(p_2.s)\Big] \\
&+ \quad 2Re(NG^*)\Big[\mp 32m_l(p.s)((p_1.q) + (p_2.q)) \\
&\quad \pm 32m_l(q.s)((p_1.q) + (p_2.p))\Big]
\end{aligned}
$$

$$+ \quad 2Re(GH^*)\Big[\pm 1024 m_l (p.q)(p.s)((p_1.q)+(p_2.q))$$
$$\mp 512 m_l (p.q)^2 (p_2.s) \pm 512 p^2 q^2 (p_2.s)\Big]. \tag{4.16}$$

For the longitudinally polarized state, the branching ratio for the $B \to K\ell^+\ell^-$ decay can be written as,

$$\frac{d\Gamma_\pm}{ds} = \frac{G_F^2 \alpha^2 |V_{tb}V_{ts}^*|^2}{2^{14}\pi^5 m_B} v\Delta_\pm, \tag{4.17}$$

where $\Delta_\pm$ for longitudinally polarized decay for massless leptons is,

$$\Delta_\pm^L = 2Re(AC^*)\Big[\mp \frac{8}{3}\lambda m_B^4\Big] + 2Re(GH^*)\Big[\frac{512}{3}\lambda m_B^6 s\Big]$$
$$+ \quad 2Re(QN^*)\Big[-4m_B^2 s\Big]. \tag{4.18}$$

Considering the transversal polarizations, Eq. (4.17) is still valid but we write $\Delta_\pm$ for massless leptons as,

$$\Delta_\pm^T = m_B^3 \pi \sqrt{\lambda s}\Big\{4Re(CQ^*) + 2Re(AN^*)\Big\}. \tag{4.19}$$

It is now easy to calculate the polarization asymmetries for massless leptons, given in Eq. (4.6), where the polarized differential decay rates are given in Eqs. (4.18, 4.19). So, calculations lead to the following results for the longitudinal, transversal and normal polarization of the $\ell^{(\mp)}$:

$$P_L^\mp = \frac{4m_B^2}{\Delta}\Big\{\pm \frac{4}{3}\lambda m_B^2 \,\mathrm{Re}(AC^*) + \frac{256}{3}\lambda m_B^4 s \,\mathrm{Re}(GH^*)$$
$$- \quad 2\,\mathrm{Re}(NQ^*)\Big\} \tag{4.20}$$
$$P_T^\mp = \frac{\pi m_B^3 \sqrt{s\lambda}}{\Delta}\Big\{\pm 2\,\mathrm{Re}(AN^*) + 2\,\mathrm{Re}(CQ^*)\Big\}, \tag{4.21}$$
$$P_N^\mp = \frac{m_B^3 v \sqrt{s\lambda}}{\Delta}\Big\{2\,\mathrm{Im}(CN^*) \mp 2\,\mathrm{Im}(AQ^*)\Big\}, \tag{4.22}$$

where, Δ is given in Eq. (4.15). The results we have presented here are for the massless leptons. In the Appendix, we present the results for massive leptons also.

From these expressions we can make the following conclusion. Contributions coming from the SM to P_L^- and P_L^+ are exactly the same but with the opposite sign. However contributions coming from new interactions to P_L^- and P_L^+ can have same or opposite sign. This can be useful in looking for new physics.

From Eq. (4.21) we observe that at zero lepton mass limit, contributions coming from scalar interactions survive. Similarly terms coming from scalar and tensor interactions survive in the massless lepton limit for $P_L^{(\mp)}$. Therefore, experimentally measured value of $P_{L,T}^{(\mp)}$ for the $B \to K\mu^+\mu^-$ can give a very promising hint in looking new physics beyond SM. About normal polarization we can comment as follows. One can see from Eq. (4.22) the difference between P_N^- and P_N^+ (for which SM predicts $P_N^- = -P_N^+$) can again be attributed to the existence of the scalar and tensor interactions. Incidentally, we should note that a similar situation takes place for the lepton polarizations in the $B \to K^*\ell^+\ell^-$ decay (see next section). It follows from this discussion that a measurement of the lepton polarization of each lepton and combined analysis of lepton and antilepton polarizations $P_L^- + P_L^+$, $P_T^- - P_T^+$ and $P_N^- + P_N^+$ can give very useful information to constraint or to discover new physics beyond SM, which are all zero in the SM in the limit of massless leptons. Therefore if in experiments nonzero value of the above mentioned combined lepton asymmetries were observed, this can be considered as an indication of the new physics beyond SM.

4.1.2 Numerical Analysis of the $B \to K\ell^+\ell^-$ Decay

First of all we introduce the values of the input parameters used in the present work: $|V_{tb}V_{ts}^*| = 0.0385$, $\alpha^{-1} = 129$, $G_F = 1.17 \times 10^{-5}\ GeV^{-2}$, $\Gamma_B = 4.22 \times 10^{-13}\ GeV$, $C_9^{eff} = 4.344$, $C_{10} = -4.669$. It is well known that the Wilson coefficient C_9^{eff} receives short as well as long distance contributions coming from the real $\bar{c}c$ intermediate states, i.e., with the J/ψ family: But in this work we consider only short distance contributions. Experimental data on $\mathcal{B}(B \to X_s\gamma)$ fixes only the modulo of C_7^{eff}. For this reason throughout our analysis we have considered both possibilities, i.e., $C_7^{eff} = \mp 0.313$, where the upper sign corresponds to the SM prediction.

For the values of the form factors, we have used the results of [57] (see also [58], [45]). The q^2 dependence of the form factors can be represented in terms of three parameters as

$$F(s) = F(0)\, exp(c_1 s + c_2 s^2 + c_3 s^3)\ , \tag{4.23}$$

where the values of parameters $F(0)$, c_1, c_2 and c_3 for the $B \to K$ decay are listed in Table 2.1.

From the expressions of the lepton polarizations we see that they all depend on q^2 and the new Wilson coefficients. It may be experimentally difficult to study the dependence of the the polarizations of each lepton on both quantities. Therefore we eliminate the dependence of the lepton polarizations on q^2, by performing integration over q^2 in the allowed kinematical region, so that the lepton polarizations are averaged. The averaged lepton polarizations are defined as

$$\langle P_i \rangle = \frac{\displaystyle\int_{4m_\ell^2}^{(m_b-m_K)^2} P_i \frac{d\mathcal{B}}{dq^2} dq^2}{\displaystyle\int_{4m_\ell^2}^{(m_b-m_K)^2} \frac{d\mathcal{B}}{dq^2} dq^2} \, . \tag{4.24}$$

We present our results in a series figures. Note that in all figures we presented the value of C_7^{eff} which is chosen to have its SM value, i.e., $C_7^{eff} = -0.313$.

Figs. (4.1) and (4.2) depict the dependence of the averaged longitudinal polarization $\left\langle P_L^- \right\rangle$ of ℓ^- and the combination $\left\langle P_L^- + P_L^+ \right\rangle$ on new Wilson coefficients, at $C_7^{eff} = -0.313$ for $B \to K\mu^+\mu^-$ decay. From these figures we see that $\left\langle P_L^- \right\rangle$ is sensitive to the existence of all new interactions except to vector and scalar interactions with coefficients C_{LL}, C_{RL} and C_{RLLR}, C_{LRLR}, respectively, while the combined average $\left\langle P_L^- + P_L^+ \right\rangle$ is sensitive to scalar type interactions only. It is interesting that contributions from C_{RLLR}, C_{LRLR} (C_{LRRL}, C_{RLRL}) to the combined asymmetry is always negative (positive). Therefore determination of the sign of $\left\langle P_L^- + P_L^+ \right\rangle$ can be useful in discriminating the type of the interaction. From Fig. (4.2) we see that $\left\langle P_L^- + P_L^+ \right\rangle = 0$ at $C_X = 0$, which confirms the SM result as expected. For the other choice of C_7^{eff}, i.e., $C_7^{eff} = 0.313$ the situation is similar to the previous case, but the magnitude of $\left\langle P_L^- + P_L^+ \right\rangle$ is smaller.

Figs. (4.3) and (4.4) are the same as Figs.(4.1) and (4.2) but for the $B \to K\tau^+\tau^-$ decay. In this case the difference of the dependence of the longitudinal polarization $\left\langle P_L^- \right\rangle$ on new Wilson coefficients from the muon case is as follows: In the muon case $\left\langle P_L^- \right\rangle$ is negative for all values of the new Wilson coefficients while for the tau case $\left\langle P_L^- \right\rangle$ can receive both values, for example for $C_T < 1$, $\left\langle P_L^- \right\rangle$ is positive, and for $C_T > 1$, $\left\langle P_L^- \right\rangle$ is negative.

It is obvious from Fig. (4.4) that if the values of the new Wilson coefficients

C_{LRRL}, C_{LRLR}, C_{RLLR}, C_{RLRL} and C_{TE} are negative (positive), $\left\langle P_L^- + P_L^+ \right\rangle$ is negative (positive). Absolutely similar situation takes place for $C_7^{eff} > 0$. For these reasons determination of the sign and of course magnitude of $\left\langle P_L^- + P_L^+ \right\rangle$ can give promising information about new physics.

In Figs. (4.5) and (4.6) the dependence of the average transversal polarization $\left\langle P_T^- \right\rangle$ and the combination $\left\langle P_T^- - P_T^+ \right\rangle$ on the new Wilson coefficients are presented for the $B \to K\mu^+\mu^-$ decay, respectively. We observe from Fig. (4.5) that the average transversal polarization is strongly dependent only on C_{LRRL} and C_{RLRL} and quite weakly to remaining Wilson coefficients. It is also interesting to note that for the negative (positive) values of these scalar coefficients $\left\langle P_T^- \right\rangle$ is negative (positive). For the $\left\langle P_T^- - P_T^+ \right\rangle$ case, there appears strong dependence on all four scalar interactions with coefficients C_{LRRL}, C_{RLLR}, C_{LRLR}, C_{RLRL}. The behavior of this combined average transversal polarization is identical for the coefficients C_{LRLR}, C_{RLLR} and C_{LRRL}, C_{RLRL} in pairs, so that four lines responsible for these interactions appear only to be two. Moreover $\left\langle P_T^- - P_T^+ \right\rangle$ is negative (positive) for the negative (positive) values of the new Wilson coefficients C_{LRRL} and C_{RLRL} and positive (negative) for the coefficients C_{LRLR} and C_{RLLR}. Remembering that in SM, in massless lepton case $\left\langle P_T^- \right\rangle \approx 0$ and $\left\langle P_T^- - P_T^+ \right\rangle \approx 0$. Therefore determination of the signs and magnitudes of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ can give quite a useful information about the existence of new physics. For the choice of $C_7^{eff} = 0.313$, apart from the minor differences in their magnitudes, the behaviors of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ are similar as in the previous case.

As is obvious from Figs. (4.7) and (4.8), $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ show stronger dependence only on C_T for the $B \to K\tau^+\tau^-$ decay. Again $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ change sign at $C_T \approx -1$. As has already been noted, determination of the sign and magnitude of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ are useful hints in looking for new physics.

Note that for simplicity all new Wilson coefficients in this work are assumed to be real. Under this condition $\left\langle P_N^- \right\rangle$ and $\left\langle P_N^- + P_N^+ \right\rangle$ have non-vanishing values coming from the imaginary part of SM, i.e., from C_9^{eff}. From Figs. (4.9) and (4.10) we see that $\left\langle P_N^- \right\rangle$ and $\left\langle P_N^- + P_N^+ \right\rangle$ are strongly dependent on all scalar type interactions for the $B \to K\mu^+\mu^-$ decay. Similar behavior takes place for the $B \to K\tau^+\tau^-$ decay as well. The change in sign and magnitude of both $\left\langle P_N^- \right\rangle$ and $\left\langle P_N^- + P_N^+ \right\rangle$ that are observed in these figures is an indication of the fact that an experimental verification of them can give unambiguous information about new physics.

In the present work we analyze the possibility of pinning down new physics beyond SM by studying lepton polarizations only. It follows from Eq. (4.14) that the branching ratio of the $B \to K\ell^+\ell^-$ decay depends also on the new Wilson coefficients and hence we expect that it can give information about new physics. In this connection there follows the question: Can one establish new physics by studying the lepton polarizations only? In other words, are there regions of the new Wilson coefficients C_X in which the value of the branching ratio coincides with that of the SM prediction, but the lepton polarizations would not? In order to answer this question, we present in Figs. (4.11)–(4.14) the dependence of the branching ratio on the average and combined average polarizations of the leptons. In these figures the value of the branching ratio ranges between the values $10^{-7} \leq \mathcal{B}(B \to K\tau^+\tau^-) \leq 5 \times 10^{-7}$. These figures depict that there indeed exist such regions of C_X in which the value of the branching ratio does agree with the SM result, while the lepton polarizations differ from the SM prediction. It follows from the pair of Figs. (4.3), (4.11); (4.7), (4.13) and (4.8), (4.14), that if C_T lies in the region $-2 \leq C_T \leq 0$, the above–mentioned condition, i.e., mismatch of the polarizations in the standard model and the new physics, is fulfilled. On the other hand one can immediately see from Fig. (4.12) that such a region for the combined average longitudinal lepton polarization does not exist and hence it is not suitable in search of new physics. Note that in all figures intersection point of all curves correspond to the SM case. This analysis allows us to conclude that there exists certain regions of new Wilson coefficients for which study of the lepton polarization itself can give promising information about new physics.

Finally, a few words about the detectibilty of the lepton polarization asymmetries at B factories or future hadron colliders, are in order. As an estimation, we choose the averaged values of the longitudinal polarization of muon and transversal and normal polarizations of the τ lepton, which are approximately close to the SM prediction, i.e., $\langle P_L \rangle \simeq -0.9$, $\langle P_T \rangle \simeq 0.6$ and $\langle P_N \rangle \simeq -0.01$. Experimentally, to measure an asymmetry $\langle P_i \rangle$ of a decay with the branching ratio B at the $n\sigma$ level, the required number of events is given by the formula $N = n^2/(\mathcal{B} \langle P_i \rangle^2$. It follows from this expression that to observe the lepton polarizations $\langle P_L \rangle$, $\langle P_T \rangle$ and $\langle P_N \rangle$ in $B \to K\tau^+\tau^-$ decay at 1σ level, the expected number of events are $N = (1; 3; 10^4) \times 10^7$, respectively. On the other hand, the number of $B\bar{B}$ pairs that is expected to be produced at B factories is about $N \sim 5 \times 10^8$. A comparison of these numbers allows us to conclude that while measurement of the normal polarization of the τ lepton is impossible, measurements of the longitudinal polarization of muon and transversal polarization of τ lepton could be accessible at B factories.

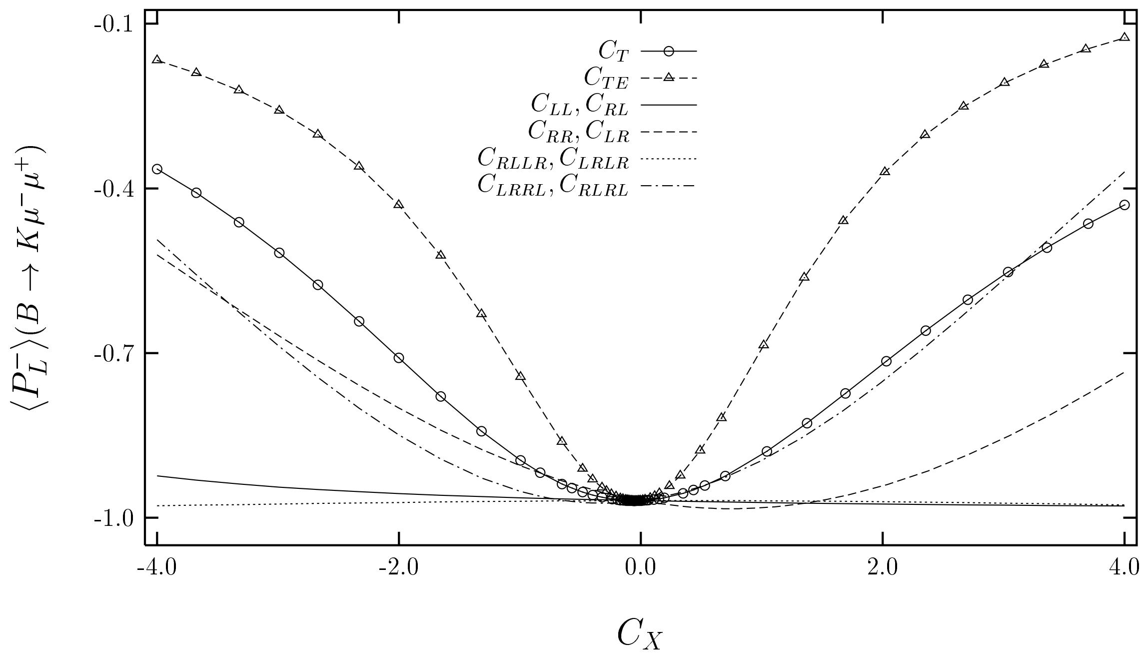

Figure 4.1: The dependence of the average longitudinal polarization asymmetry $\langle P_L^- \rangle$ of muon on the new Wilson coefficients.

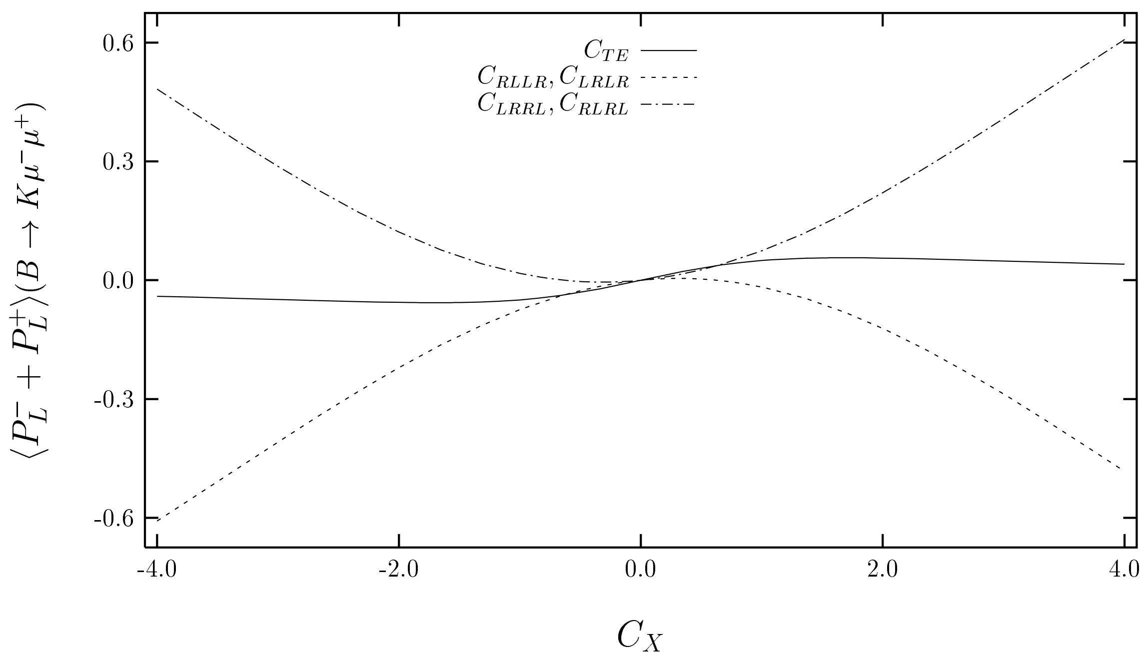

Figure 4.2: The dependence of the combined average longitudinal polarization asymmetry $\langle P_L^- + P_L^+ \rangle$ of $\ell^+ \, \ell^-$ on the new Wilson coefficients for the $B \to K\mu^+\mu^-$ decay.

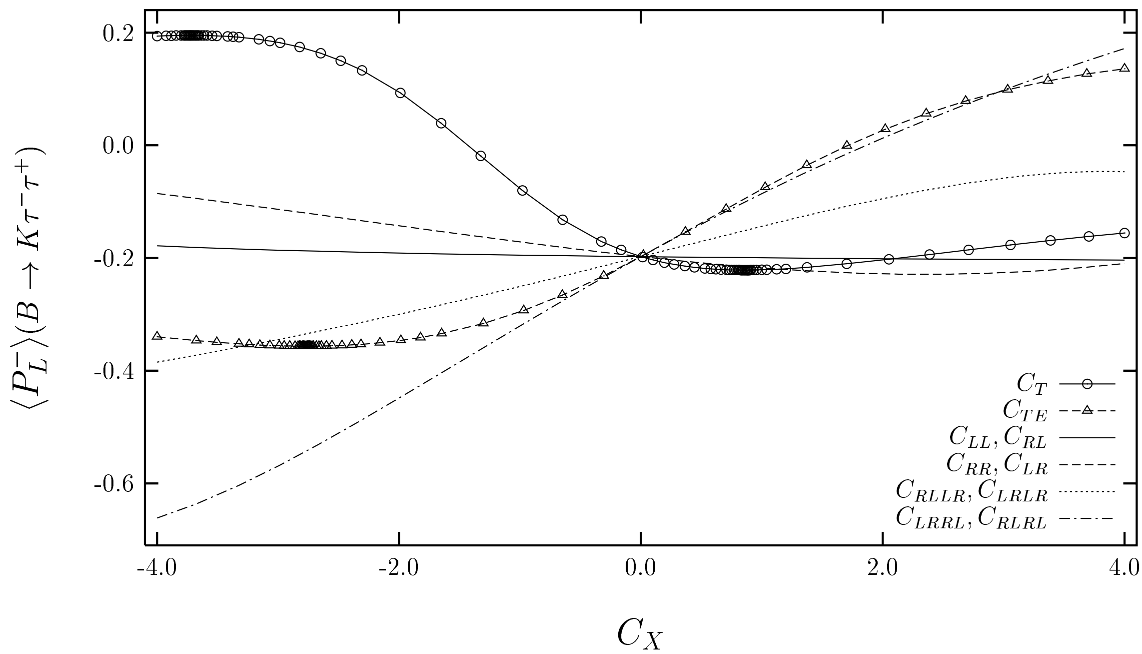

Figure 4.3: The dependence of the average longitudinal polarization asymmetry $\langle P_L^- \rangle$ of tau on the new Wilson coefficients.

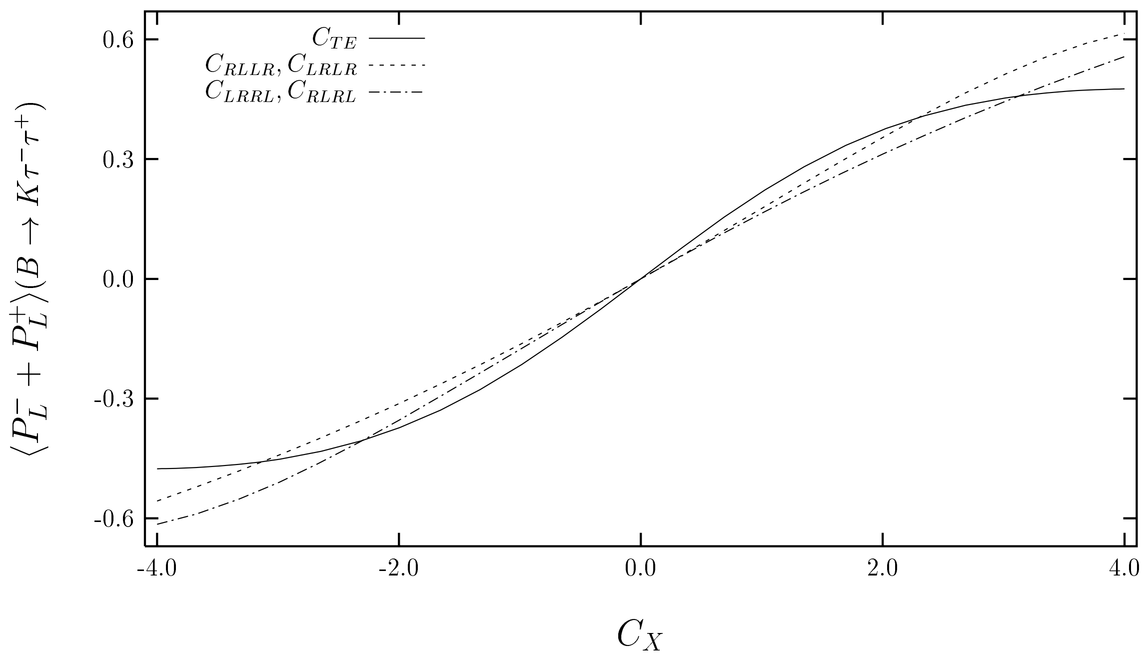

Figure 4.4: The dependence of the combined average longitudinal polarization asymmetry $\langle P_L^- + P_L^+ \rangle$ of $\ell^+ \, \ell^-$ on the new Wilson coefficients for the $B \to K\tau^+\tau^-$ decay.

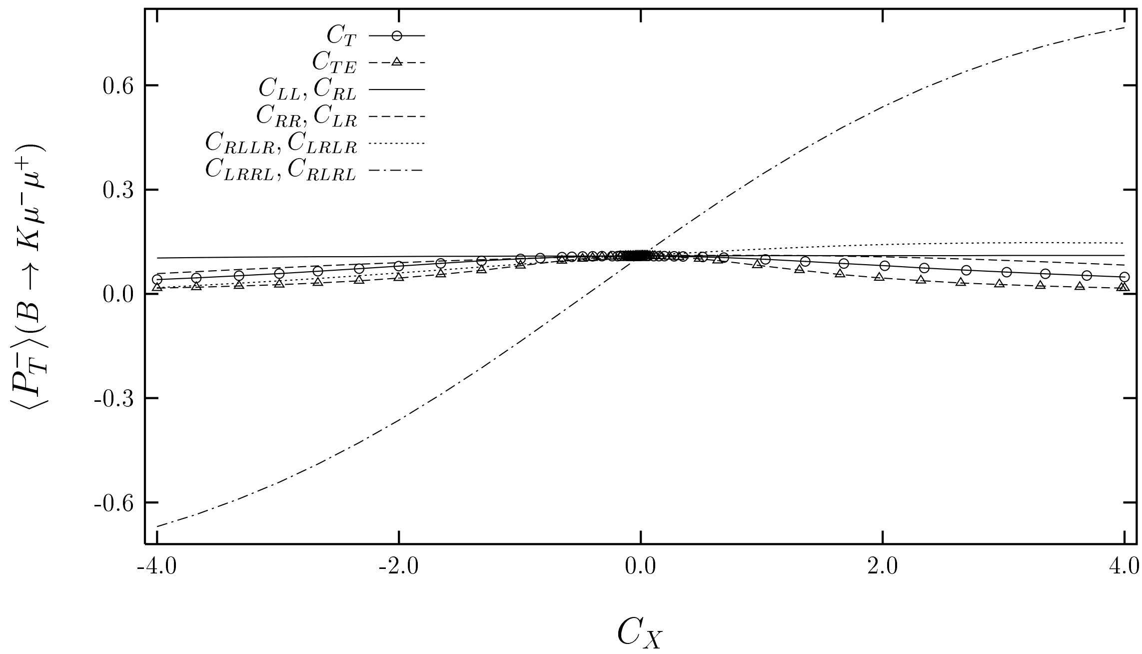

Figure 4.5: The dependence of the average transversal polarization asymmetry $\langle P_T^- \rangle$ of muon on the new Wilson coefficients.

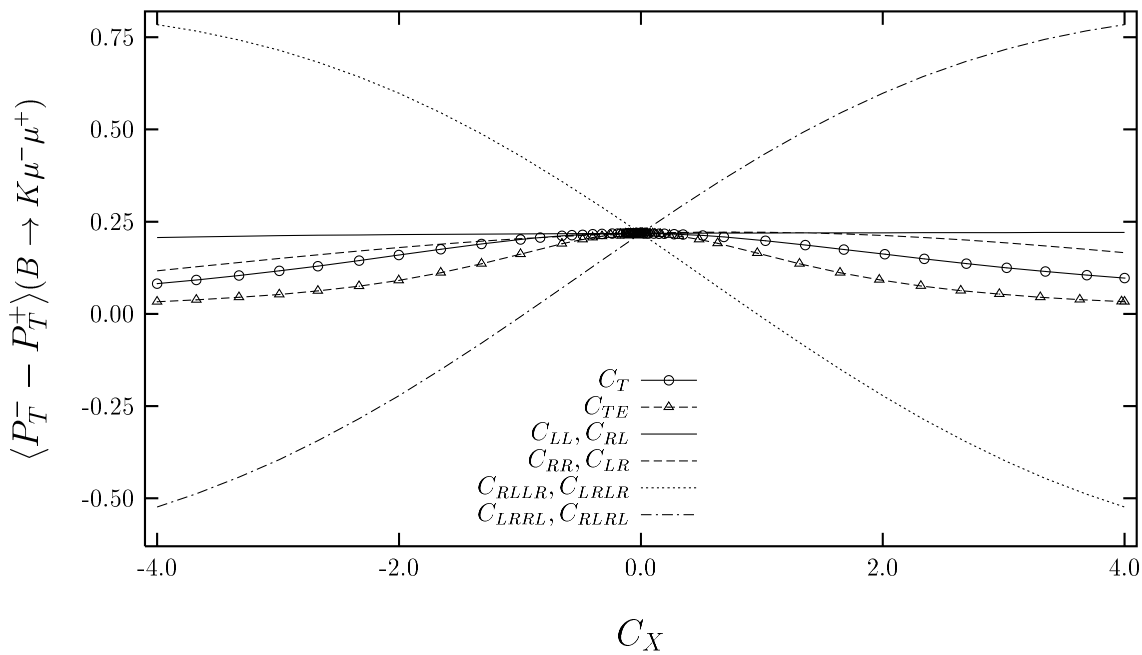

Figure 4.6: The dependence of the combined average transversal polarization asymmetry $\langle P_T^- + P_T^+ \rangle$ of $\ell^+ \ \ell^-$ on the new Wilson coefficients for the $B \to K \mu^+ \mu^-$ decay.

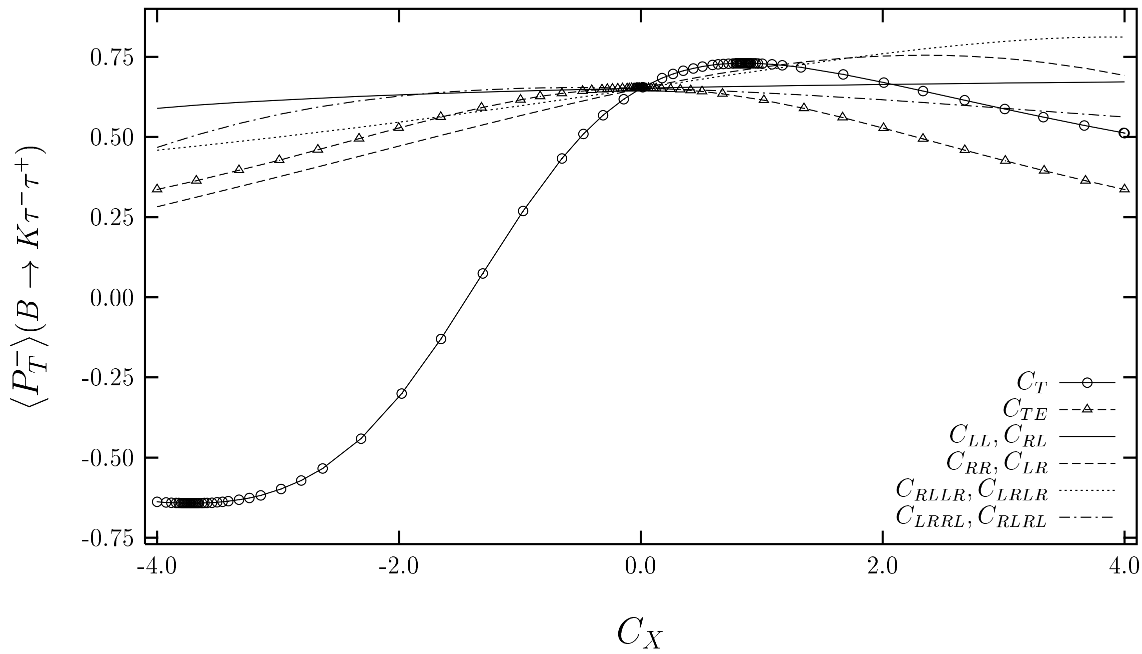

Figure 4.7: The dependence of the average transversal polarization asymmetry $\langle P_T^- \rangle$ of tau on the new Wilson coefficients.

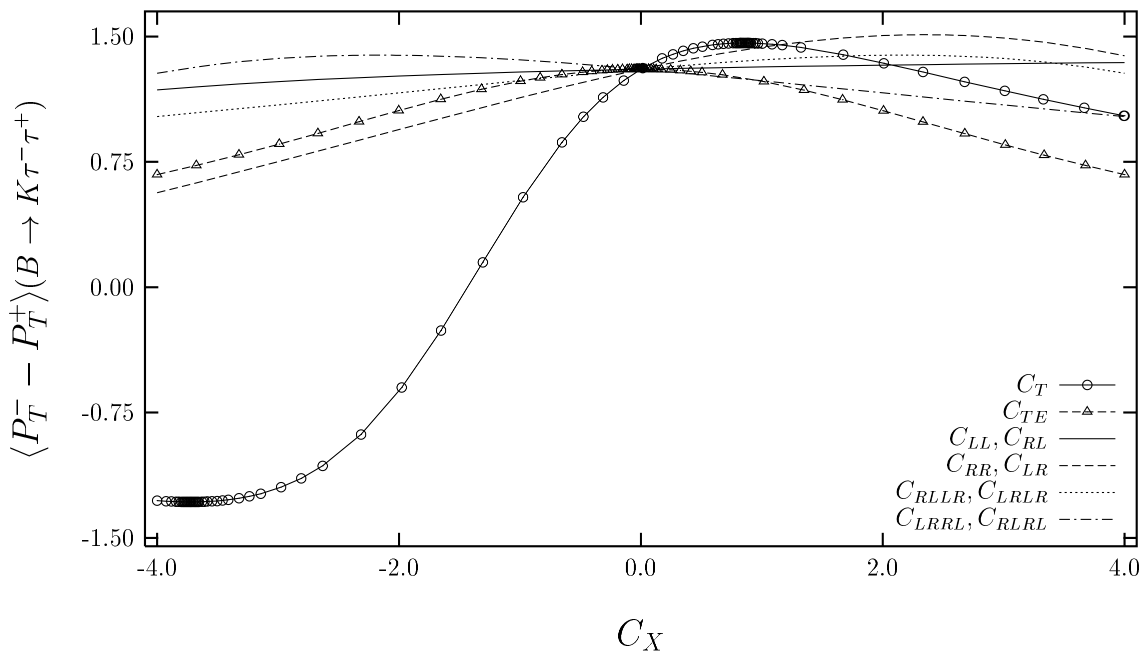

Figure 4.8: The dependence of the combined average transversal polarization asymmetry $\langle P_T^- + P_T^+ \rangle$ of $\ell^+ \ell^-$ on the new Wilson coefficients for the $B \to K\tau^+\tau^-$ decay.

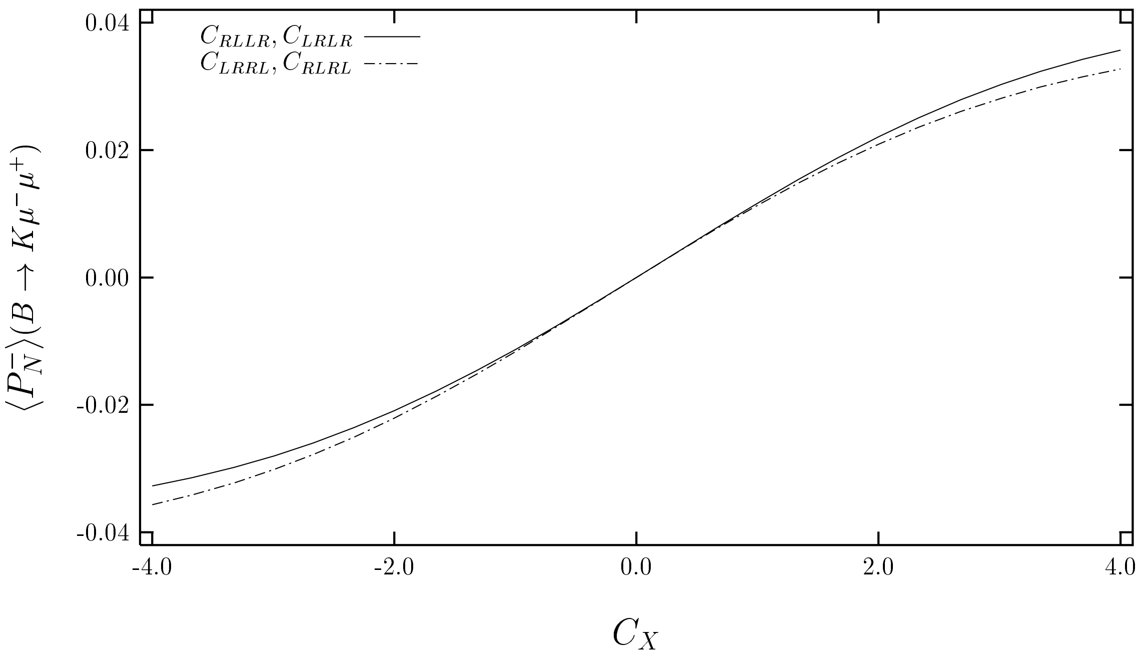

Figure 4.9: The dependence of the average normal polarization asymmetry $\langle P_N^- \rangle$ of muon on the new Wilson coefficients.

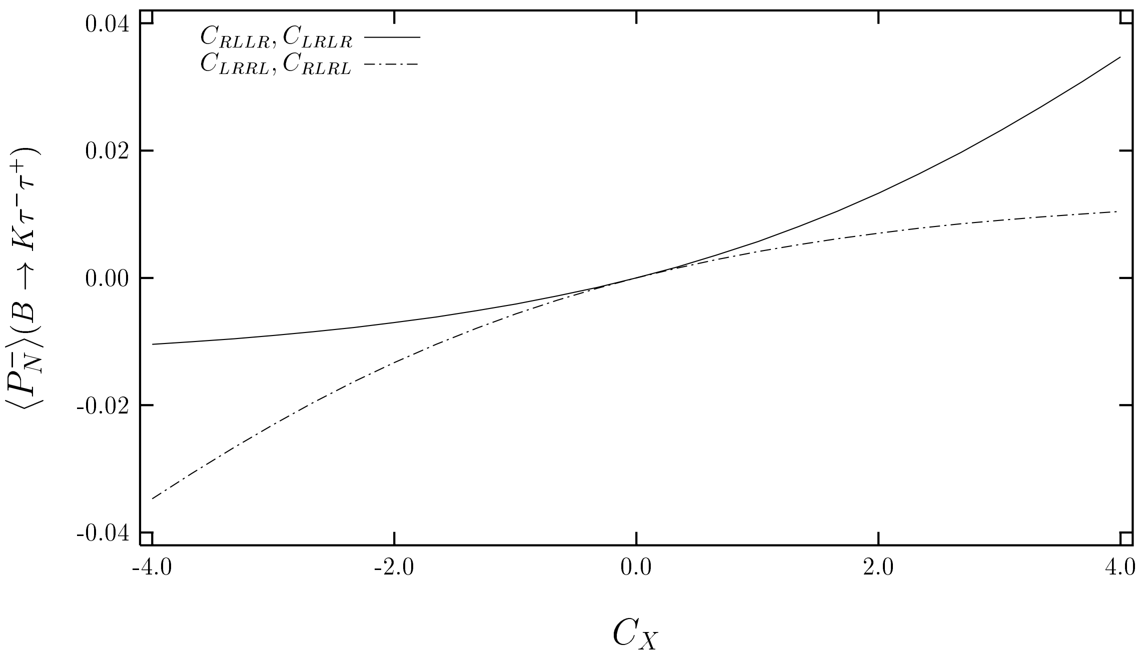

Figure 4.10: The dependence of the combined average normal polarization asymmetry $\langle P_N^- + P_N^+ \rangle$ of $\ell^+ \ \ell^-$ on the new Wilson coefficients for the $B \to K\mu^+\mu^-$ decay.

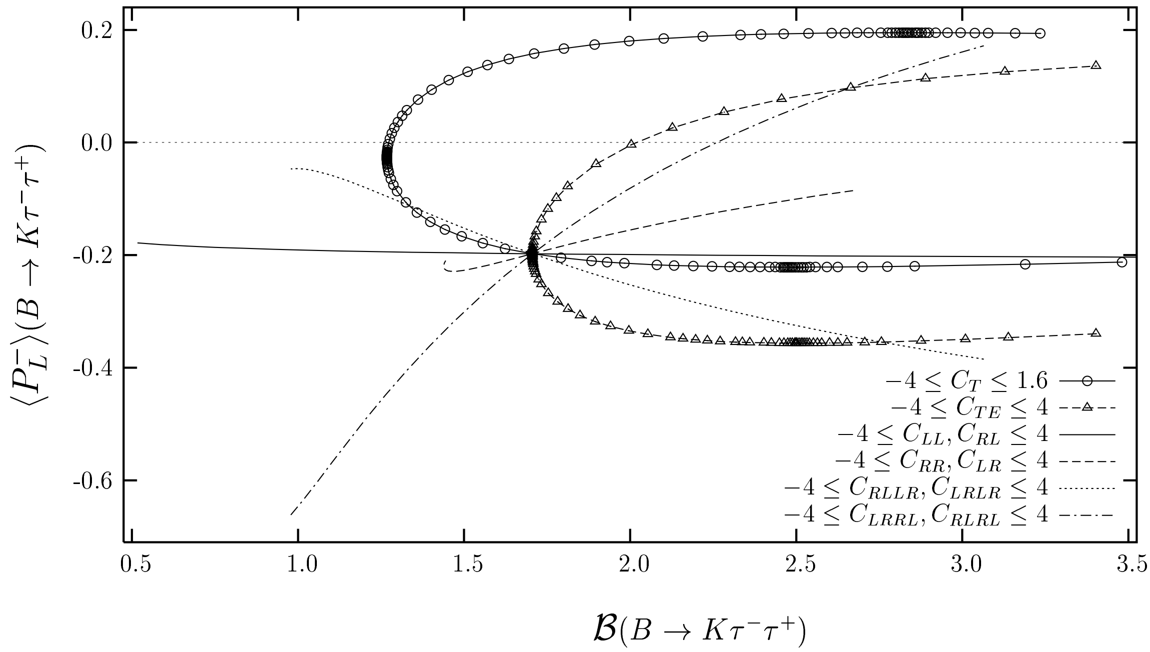

Figure 4.11: Parametric plot of the correlation between the integrated branching ratio $\mathcal{B}$ (in units of 10^{-7}) and the average longitudinal lepton polarization asymmetry $\langle P_L^- \rangle$ as function of the new Wilson coefficients as indicated in the figure for the $B \to K\tau^-\tau^+$ decay.

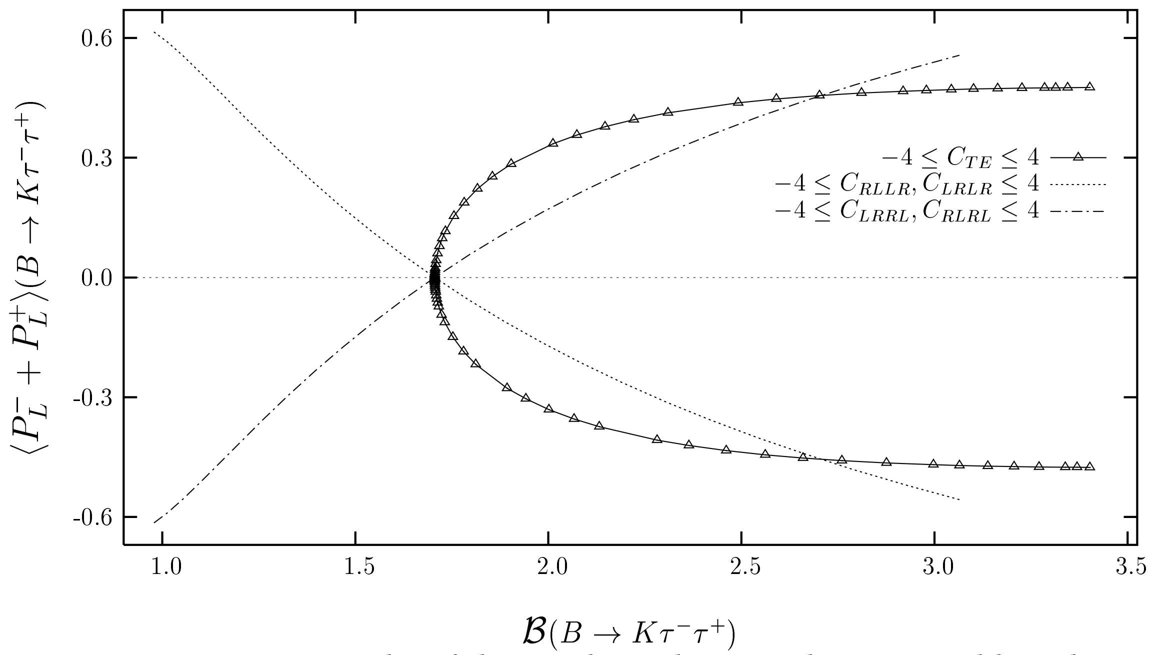

Figure 4.12: Parametric plot of the correlation between the integrated branching ratio $\mathcal{B}$ (in units of 10^{-7}) and the combined average longitudinal lepton polarization asymmetry $\langle P_L^- + P_L^+ \rangle$ as function of the new Wilson coefficients as indicated in the figure for the $B \to K\tau^-\tau^+$ decay.

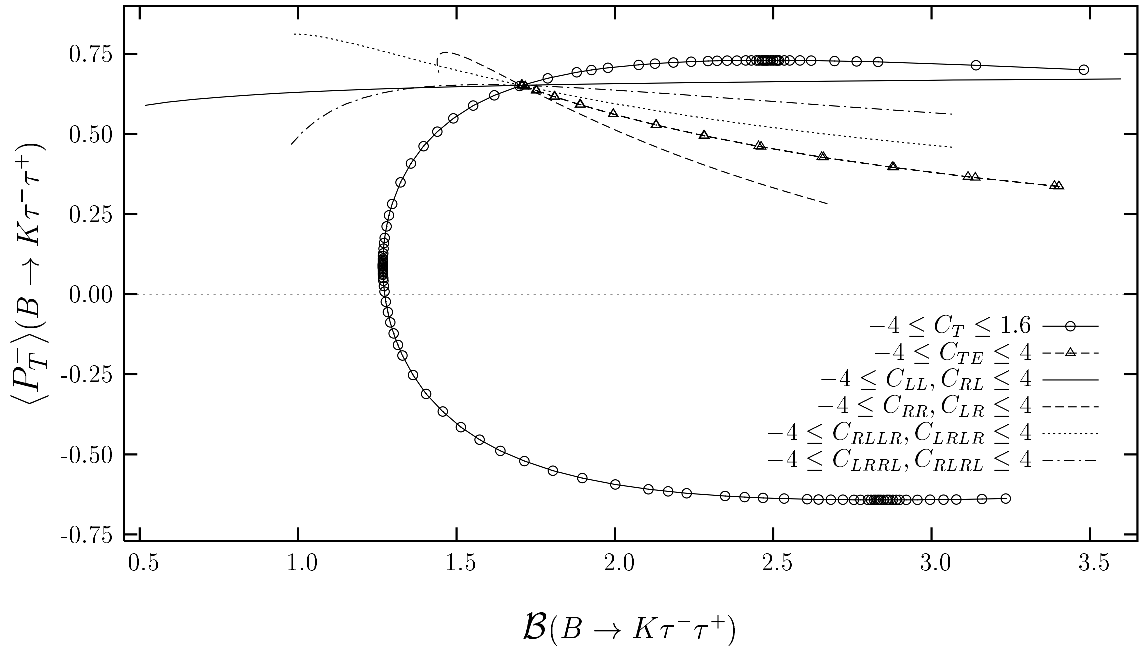

Figure 4.13: Parametric plot of the correlation between the integrated branching ratio $\mathcal{B}$ (in units of 10^{-7}) and the average transversal lepton polarization asymmetry $\langle P_T^- \rangle$ as function of the new Wilson coefficients as indicated in the figure for the $B \to K\tau^-\tau^+$ decay.

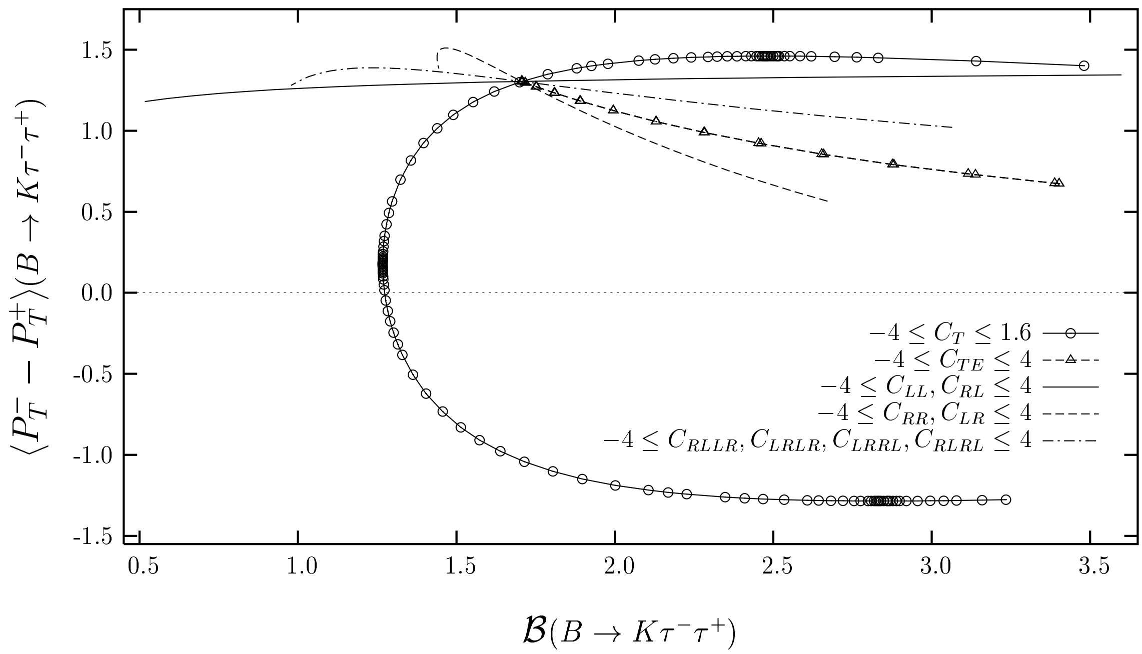

Figure 4.14: Parametric plot of the correlation between the integrated branching ratio $\mathcal{B}$ (in units of 10^{-7}) and the combined average transversal lepton polarization asymmetry $\langle P_T^- + P_T^+ \rangle$ as function of the new Wilson coefficients as indicated in the figure for the $B \to K\tau^-\tau^+$ decay.

4.2 Exclusive $B \to K^* \ell^+ \ell^-$ Decay

Exclusive $B \to K^* \ell^+ \ell^-$ decay is described by the matrix element of effective Hamiltonian over B and K^* meson states, which can be parametrized in terms of form factors. It follows from Eq. (4.1) that in order to calculate the amplitude of the $B \to K^* \ell^+ \ell^-$ decay, the following matrix elements are needed

$$\langle K^* | \bar{s}\gamma_\mu (1 \pm \gamma_5) b | B \rangle \ ,$$

$$\langle K^* | \bar{s} i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b | B \rangle \ ,$$

$$\langle K^* | \bar{s} (1 \pm \gamma_5) b | B \rangle \ ,$$

$$\langle K^* | \bar{s} \sigma_{\mu\nu} b | B \rangle \ ,$$

These matrix elements are defined in (3.8) - (3.11) as follows

$$\begin{aligned}
&\langle K^*(p_{K^*}, \epsilon) | \bar{s}\gamma_\mu (1 \pm \gamma_5) b | B(p_B) \rangle = \\
&\quad -\epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} p_{K^*}^\lambda q^\sigma \frac{2V(q^2)}{m_B + m_{K^*}} \pm i \epsilon_\mu^* (m_B + m_{K^*}) A_1(q^2) \\
&\quad \mp i (p_B + p_{K^*})_\mu (\epsilon^* q) \frac{A_2(q^2)}{m_B + m_{K^*}} \mp i q_\mu \frac{2 m_{K^*}}{q^2} (\epsilon^* q) \left[A_3(q^2) - A_0(q^2) \right] \ ,
\end{aligned} \tag{4.25}$$

$$\begin{aligned}
&\langle K^*(p_{K^*}, \epsilon) | \bar{s} i \sigma_{\mu\nu} q^\nu (1 \pm \gamma_5) b | B(p_B) \rangle = \\
&\quad 4 \epsilon_{\mu\nu\lambda\sigma} \epsilon^{*\nu} p_{K^*}^\lambda q^\sigma T_1(q^2) \pm 2i \left[\epsilon_\mu^* (m_B^2 - m_{K^*}^2) - (p_B + p_{K^*})_\mu (\epsilon^* q) \right] T_2(q^2) \\
&\quad \pm 2i (\epsilon^* q) \left[q_\mu - (p_B + p_{K^*})_\mu \frac{q^2}{m_B^2 - m_{K^*}^2} \right] T_3(q^2) \ ,
\end{aligned} \tag{4.26}$$

$$\begin{aligned}
&\langle K^*(p_{K^*}, \epsilon) | \bar{s} \sigma_{\mu\nu} b | B(p_B) \rangle = \\
&\quad i \epsilon_{\mu\nu\lambda\sigma} \Bigg[- 2 T_1(q^2) \epsilon^{*\lambda} (p_B + p_{K^*})^\sigma + \frac{2}{q^2} (m_B^2 - m_{K^*}^2) \epsilon^{*\lambda} q^\sigma \\
&\quad - \frac{4}{q^2} \left(T_1(q^2) - T_2(q^2) - \frac{q^2}{m_B^2 - m_{K^*}^2} T_3(q^2) \right) (\epsilon^* q) p_{K^*}^\lambda q^\sigma \Bigg] \ .
\end{aligned} \tag{4.27}$$

$$\langle K^*(p_{K^*},\epsilon)|\bar{s}(1\pm\gamma_5)b|B(p_B)\rangle = \frac{1}{m_b}\Big[\mp 2im_{K^*}(\epsilon^* q)A_0(q^2)\Big] \ . \qquad (4.28)$$

where $q = p_B - p_{K^*}$ is the four-momentum transfer and ε is the polarization vector of K^* meson. In deriving the Eq. (4.28) it is assumed that $A_3(q^2=0) = A_0(q^2=0)$ and $T_1(q^2=0) = T_2(q^2=0)$. Also, we have used the relation

$$2m_{K^*}A_3(q^2) = (m_B + m_{K^*})A_1(q^2) - (m_B - m_{K^*})A_2(q^2) \ .$$

The form factors in Eqs. (4.25)-(4.28) have already been calculated in the second chapter presented in the Eqs. (3.70)-(3.78) and tabulated in Table (3.2).

Taking into account the Eqs. (4.1) and (4.25)-(4.28), the matrix element of the $B \to K^*\ell^+\ell^-$ decay can be written as,

$$\begin{aligned}
\mathcal{M}(B\to K^*\ell^+\ell^-) &= \frac{G\alpha}{4\sqrt{2}\pi}V_{tb}V_{ts}^* \\
\times &\Bigg\{\bar{\ell}\gamma^\mu(1-\gamma_5)\ell\Big[-2A_1\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}p_{K^*}^\lambda q^\sigma - iB_1\epsilon_\mu^* + iB_2(\epsilon^* q)(p_B+p_{K^*})_\mu \\
&+iB_3(\epsilon^* q)q_\mu\Big] \\
+ &\quad \bar{\ell}\gamma^\mu(1+\gamma_5)\ell\Big[-2C_1\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}p_{K^*}^\lambda q^\sigma - iD_1\epsilon_\mu^* + iD_2(\epsilon^* q)(p_B+p_{K^*})_\mu \\
&+iD_3(\epsilon^* q)q_\mu\Big] \\
+ &\quad \bar{\ell}(1-\gamma_5)\ell\Big[iB_4(\epsilon^* q)\Big] + \bar{\ell}(1+\gamma_5)\ell\Big[iB_5(\epsilon^* q)\Big] \\
+ &\quad 4\bar{\ell}\sigma^{\mu\nu}\ell\Big(iC_T\epsilon_{\mu\nu\lambda\sigma}\Big)\Big[-2T_1\epsilon^{*\lambda}(p_B+p_{K^*})^\sigma + B_6\epsilon^{*\lambda}q^\sigma - B_7(\epsilon^* q){p_{K^*}}^\lambda q^\sigma\Big] \\
+ &\quad 16C_{TE}\bar{\ell}\sigma_{\mu\nu}\ell\Big[-2T_1\epsilon^{*\mu}(p_B+p_{K^*})^\nu + B_6\epsilon^{*\mu}q^\nu - B_7(\epsilon^* q){p_{K^*}}^\mu q^\nu\Bigg\} \ , \qquad (4.29)
\end{aligned}$$

where the auxiliary functions in (4.29) are given by

$$
\begin{aligned}
A_1 &= (C_{LL}^{tot} + C_{RL}) \frac{V}{m_B + m_{K^*}} - 2(C_{BR} + C_{SL}) \frac{T_1}{q^2} \;, \\
B_1 &= (C_{LL}^{tot} - C_{RL})(m_B + m_{K^*}) A_1 - 2(C_{BR} - C_{SL})(m_B^2 - m_{K^*}^2) \frac{T_2}{q^2} \;, \\
B_2 &= \frac{C_{LL}^{tot} - C_{RL}}{m_B + m_{K^*}} A_2 - 2(C_{BR} - C_{SL}) \frac{1}{q^2} \left[T_2 + \frac{q^2}{m_B^2 - m_{K^*}^2} T_3 \right] \;, \\
B_3 &= 2(C_{LL}^{tot} - C_{RL}) m_{K^*} \frac{A_3 - A_0}{q^2} + 2(C_{BR} - C_{SL}) \frac{T_3}{q^2} \;, \\
C_1 &= A_1(C_{LL}^{tot} \to C_{LR}^{tot} \;, \;\; C_{RL} \to C_{RR}) \;, \\
D_1 &= B_1(C_{LL}^{tot} \to C_{LR}^{tot} \;, \;\; C_{RL} \to C_{RR}) \;, \\
D_2 &= B_2(C_{LL}^{tot} \to C_{LR}^{tot} \;, \;\; C_{RL} \to C_{RR}) \;, \\
D_3 &= B_3(C_{LL}^{tot} \to C_{LR}^{tot} \;, \;\; C_{RL} \to C_{RR}) \;, \\
B_4 &= -2(C_{LRRL} - C_{RLRL}) \frac{m_{K^*}}{m_b} A_0 \;, \\
B_5 &= -2(C_{LRLR} - C_{RLLR}) \frac{m_{K^*}}{m_b} A_0 \;, \\
B_6 &= 2(m_B^2 - m_{K^*}^2) \frac{T_1 - T_2}{q^2} \;, \\
B_7 &= \frac{4}{q^2} \left(T_1 - T_2 - \frac{q^2}{m_B^2 - m_{K^*}^2} T_3 \right) \;. \qquad (4.30)
\end{aligned}
$$

Here, we carry out similar steps with the previous section. Since the results are too long to display here, we preferred to present modulus square of the matrix element (4.29) in Appendix E. Referring to the result (E.1) we perform integration over final lepton momenta and find the unpolarized differential decay rate using Eq. (4.2) as,

$$
\frac{d\Gamma}{dq^2} = \frac{G_F^2 \alpha^2}{2^{14} \pi^5 m_B} \left| V_{tb} V_{ts}^* \right|^2 \lambda^{1/2}(1, r, s) v \Delta \qquad (4.31)
$$

where $\lambda(1,r,s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $s = q^2/m_B^2$, $r = m_{K^*}^2/m_B^2$ and $v = \sqrt{1 - 4m_l^2/q^2}$ is the lepton velocity. The explicit form of Δ for massless leptons is,

$$
\begin{aligned}
\Delta_{m_\ell=0} &= \frac{32}{3} m_B^6 s\lambda\Big(|A_1|^2+|C_1|^2\Big)+\frac{2}{r} m_B^4 s\lambda\left(|B_4|^2+|B_5|^2\right) \\
&- \frac{8}{3r} m_B^4 \lambda\,(1-r-s)\Big[\mathrm{Re}(B_1B_2^*)+\mathrm{Re}(D_1D_2^*)\Big] \\
&+ \frac{4}{3r}\, m_B^2(\lambda+12rs)\Big(|B_1|^2+|D_1|^2\Big) \\
&+ \frac{4}{3r} m_B^6\lambda^2\left(|B_2|^2+|D_2|^2\right) \\
&+ \frac{256}{3r}|T_1|^2|C_T|^2 m_B^2\Big(-12r(2+2r-s)]+m_B^2\,[\lambda(16r+s) \\
&\quad +12rs(2+2r-s)]\Big) \\
&+ \frac{1024}{3r}|T_1|^2|C_{TE}|^2 m_B^2\Big(12r(2+2r-s)] \\
&\quad +m_B^2\,[\lambda(16r+s)+12rs(2+2r-s)]\Big) \\
&+ \frac{16}{3r} m_B^2\Big(4m_B^2 s|C_{TE}|^2+m_B^2 s v^2|C_T|^2\Big)\times\Big(4(\lambda+12rs)|B_6|^2 \\
&+ m_B^4\lambda^2|B_7|^2-4m_B^2(1-r-s)\lambda\,\mathrm{Re}(B_6B_7^*) \\
&- 16\,[\lambda+12r(1-r)]\,\mathrm{Re}(T_1B_6^*) \\
&+ 8m_B^2(1+3r-s)\lambda\,\mathrm{Re}(T_1B_7^*)\Big)\ . \qquad (4.32)
\end{aligned}
$$

Here we have excluded the lepton masses. We have rewritten this expression for massive leptons in Appendix E.

4.2.1 Polarization Asymmetries in $B \to K^*\ell^+\ell^-$ Decay

We now calculate the differential decay rate when the leptons are polarized longitudinally and/or transversally. In order to do these calculations, we make use of the expression in Eq. (4.6), where the differential decay rate can be calculated as in Eq. (4.5).

From the matrix element as given in Eq. (4.29), we now would like to calculate the polarization asymmetries of the decay $B \to K^* \ell^+ \ell^-$. When we take the lepton polarizations into consideration, the square of the matrix element of the decay should be modified. This modification is presented in Appendix E in details. We preferred to present the results here for massless leptons, for the sake of simplicity.

Considering the leptons to be polarized longitudinally, that is we use the orthogonal unit vectors

$$
\begin{aligned}
S_L^{-\ell} &= (0, \vec{e}_L^{\,-}) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right) , \\
S_L^{+\ell} &= (0, \vec{e}_L^{\,+}) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right) ,
\end{aligned}
$$

as given in Eq. (4.3). The decay rate expression calculated from the polarized decay matrix element expression given in Eq. (E.3) will be,

$$
\begin{aligned}
\left(\frac{d\Gamma(\vec{e}_L^{\,\mp})}{dq^2}\right)_{m_l=0} &= \frac{G_F^2\alpha^2}{2^{14}\pi^5 m_B}\left|V_{tb}V_{ts}^*\right|^2 \Bigg\{\Big[|A_1|^2 - |C_1|^2 + 2Re(A_1C_1^*)\Big]\Big[\pm\frac{32}{3}\lambda m_B^6 s\Big] \\
&+ \Big[\mp|B_1|^2 \mp |D_1|^2 + 2Re(B_1D_1^*)\Big]\Big[-\frac{4}{3r}\lambda m_B^2 - 16 m_B^2 s\Big] \\
&+ \Big[\pm 2Re(B_1B_2^*) + 2Re(B_1D_2^*) + 2Re(B_2D_1^*) \pm 2Re(D_1D_2^*)\Big] \\
&\quad \times\Big[-\frac{4}{3}\lambda m_B^4 + \frac{4}{3r}\lambda m_B^4 - \frac{4}{3r}\lambda m_B^4 s\Big] \\
&+ \Big[\pm|B_2|^2 \mp |D_2|^2 + 2Re(B_2D_2^*)\Big]\Big[-\frac{4}{3r}\lambda^2 m_B^6\Big] \\
&+ \Big[|B_4|^2 - |B_5|^2\Big]\frac{4\lambda s m_B^4}{r} \\
&+ \Big[|B_6|^2\frac{\lambda + 12rs}{3r} + |B_7|^2\frac{16\lambda^2 m_B^6}{3r}\Big]\Big[\pm|C_T|^2 m_B^2 s \\
&\quad \mp 4|C_{TE}|^2 m_B^2 s - 4Re(C_TC_{TE}^*) m_B^2 s\Big] \\
&+ |T_1|^2\frac{256 m_B^2}{3rs}\Big[\mp 4|C_{TE}|^2\Big(12m_B^2 r s^2(2+2r-s) \\
&\quad + m_B^2 s(16r+s)\Big) \\
&\quad \pm|C_T|^2\Big(12m_B^2 r s^2(2+2r-s) + m_B^2 s(16r+s))\Big) \\
&\quad + 2Re(C_TC_{TE}^*)\Big(m_B^2 s(12rs(-2-2r+s) - \lambda(16r+s))\Big)
\end{aligned}
$$

$$+ \quad 2Re(B_6 B_7^*) \frac{32\lambda m_B^4(-1+r+s)}{3r} \Big[\pm |C_T|^2 m_B^2 s$$

$$\mp 4|C_{TE}|^2 m_B^2 s + 4Re(C_T C_{TE}^*) m_B^2 s \Big]$$

$$+ \quad 2Re(B_6 T_1^*) \frac{-128 m_B^2(\lambda - 12(-1+r)r)}{3r} \Big[\pm |C_T|^2 m_B^2 s$$

$$\mp 4|C_{TE}|^2 m_B^2 s + 4Re(C_T C_{TE}^*) m_B^2 s \Big]$$

$$+ \quad 2Re(B_7 T_1^*) \frac{64\lambda m_B^4(1+3r-s)}{3r} \Big[\pm |C_T|^2 m_B^2 s$$

$$\mp 4|C_{TE}|^2 m_B^2 s + 4Re(C_T C_{TE}^*) m_B^2 s \Big] \Bigg\} , \qquad (4.33)$$

where the superscripts (+) and (−) in $\vec{e}_L^{\,\pm}$ is to represent the decay rate expression when the leptons ℓ^+ and ℓ^- are polarized longitudinally, respectively. The corresponding result for massive leptons can be found in Appendix E. In order to calculate the polarizations of the leptons, we use Eq. (4.5) in such a way that, for the longitudinally polarized lepton, Eq. (4.5) becomes,

$$\frac{1}{\Big(d\Gamma/dq^2\Big)_0} \left[2\frac{d\Gamma(\vec{e}_L^{\,\pm})}{dq^2} - \left(\frac{d\Gamma}{dq^2}\right)_0 \right] = P_L^{\,\pm}.$$

We use Eq. (4.33) and Eq. (4.32) here and for the longitudinal polarization of the ℓ^- and get (again for the massless leptons; the corresponding result for massive leptons can be found in Appendix E).

$$\begin{aligned}
\Big(P_L^-\Big)_{m_l=0} &= \frac{4}{\Delta_{m_l=0}} m_B^2 \Bigg\{ \frac{1}{3r}\lambda^2 m_B^4 \Big[|B_2|^2 - |D_2|^2 \Big] + \frac{8}{3}\lambda m_B^4 s \Big[|A_1|^2 - |C_1|^2 \Big] \\
&- \frac{1}{2r}\lambda m_B^2 s \Big[|B_4|^2 - |B_5|^2 \Big] \\
&- \frac{2}{3r}\lambda m_B^2 (1-r-s) \Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*) \Big] \\
&+ \frac{1}{3r}(\lambda + 12rs) \Big[|B_1|^2 - |D_1|^2 \Big] \\
&+ \frac{16}{3r}\lambda^2 m_B^6 s \, |B_7|^2 \, \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{64}{3r}(\lambda + 12rs) m_B^2 s \, |B_6|^2 \, \mathrm{Re}(C_T C_{TE}^*)
\end{aligned}$$

$$
\begin{aligned}
&- \frac{64}{3r}\lambda m_B^4 s(1-r-s)\,\mathrm{Re}(B_6B_7^*)\mathrm{Re}(C_TC_{TE}^*) \\
&+ \frac{128}{3r}\lambda m_B^4 s(1+3r-s)\,\mathrm{Re}(B_7T_1^*)\mathrm{Re}(C_TC_{TE}^*) \\
&- \frac{256}{3r} m_B^2 s[\lambda + 12r(1-r)]\,\mathrm{Re}(B_6T_1^*)\mathrm{Re}(C_TC_{TE}^*) \\
&+ \left.\frac{256}{3r} m_B^2 [\lambda(4r+s) + 12r(1-r)^2]\,\left|T_1\right|^2\,\mathrm{Re}(C_TC_{TE}^*)\right\} . \qquad (4.34)
\end{aligned}
$$

Similarly, we found for the longitudinal polarization for ℓ^+ as,

$$
\begin{aligned}
\left(P_L^+\right)_{m_l=0} &= \frac{4}{\Delta_{m_l=0}} m_B^2 \Bigg\{ -\frac{1}{3r}\lambda^2 m_B^4 \Big[\left|B_2\right|^2 - \left|D_2\right|^2 \Big] - \frac{8}{3}\lambda m_B^4 s \Big[\left|A_1\right|^2 - \left|C_1\right|^2 \Big] \\
&- \frac{1}{2r}\lambda m_B^2 s \Big[\left|B_4\right|^2 - \left|B_5\right|^2 \Big] \\
&+ \frac{2}{3r}\lambda m_B^2 (1-r-s)\Big[\mathrm{Re}(B_1B_2^*) - \mathrm{Re}(D_1D_2^*)\Big] \\
&- \frac{1}{3r}(\lambda + 12rs)\Big[\left|B_1\right|^2 - \left|D_1\right|^2 \Big] \\
&+ \frac{16}{3r}\lambda^2 m_B^6 s \left|B_7\right|^2 \mathrm{Re}(C_TC_{TE}^*) \\
&+ \frac{64}{3r}(\lambda + 12rs) m_B^2 s \left|B_6\right|^2 \mathrm{Re}(C_TC_{TE}^*) \\
&- \frac{64}{3r}\lambda m_B^4 s(1-r-s)\,\mathrm{Re}(B_6B_7^*)\mathrm{Re}(C_TC_{TE}^*) \\
&+ \frac{128}{3r}\lambda m_B^4 s(1+3r-s)\,\mathrm{Re}(B_7T_1^*)\mathrm{Re}(C_TC_{TE}^*) \\
&- \frac{256}{3r} m_B^2 s[\lambda + 12r(1-r)]\,\mathrm{Re}(B_6T_1^*)\mathrm{Re}(C_TC_{TE}^*) \\
&+ \frac{256}{3r} m_B^2 [\lambda(4r+s) + 12r(1-r)^2]\,\left|T_1\right|^2\,\mathrm{Re}(C_TC_{TE}^*)\Bigg\} , \qquad (4.35)
\end{aligned}
$$

where Δ is given in Eq. (4.32). Both of these last two results are calculated when the mass of the leptons are neglected. we have re-calculated these polarizations for massive leptons in Appendix E. From Eqs. (4.34) and (4.35) we observe that the terms containing "pure" SM contribution, i.e., the terms containing C_{BR}, C_{SL}, C_{LL}^{tot} and C_{LR}^{tot} are the same for both lepton and antilepton but with opposite sign. However for the terms containing new physics effects this does not hold. In other words, such terms may have same or different signs for lepton and antilepton. In due course this difference can be a useful tool for looking new physics effects.

Back to the matrix element of the polarized decay (please refer to Eq. (E.3)), we now consider transversal polarizations. That is to say, in Eq. (E.3) the orthogonal unit vectors (S) are now defined as,

$$\begin{aligned}
S_T^{-\ell} &\equiv (0, \vec{e}_T^{\,-}) = \left(0, \vec{e}_N^{\,-} \times \vec{e}_L^{\,-}\right), \\
S_T^{+\ell} &\equiv (0, \vec{e}_T^{\,+}) = \left(0, \vec{e}_N^{\,+} \times \vec{e}_L^{\,+}\right),
\end{aligned}$$

for the polarizations of the leptons ℓ^- and ℓ^+, respectively. We can now calculate the decay rate expression for transversally polarized massless leptons as,

$$\begin{aligned}
\left(\frac{d\Gamma(\vec{e}_T^{\,\mp})}{dq^2}\right)_{m_l=0} &= \frac{G_F^2\alpha^2}{2^{14}\pi^5 m_B}\left|V_{tb}V_{ts}^*\right|^2 \times \Bigg\{\left[|A_1|^2+|C_1|^2\right]\left(\frac{32\,\lambda\, m_B^6\, s}{3}\right) \\
&+ \left[|B_1|^2-|D_1|^2\right]\left(\frac{4\,\lambda\, {m_B}^2}{3\,r}+16\,{m_B}^2\,s\right) \\
&+ \left[Re(B_1B_2^*)-Re(D_1D_2^*)\right]\left(-\frac{4\,\lambda\, {m_B}^4\,(1-r-s)}{3\,r}\right) \\
&+ \left[|B_2|^2-|D_2|^2\right]\left(\frac{4\,\lambda^2\, {m_B}^6}{3\,r}\right) \\
&+ \left[2Re(B_1B_4^*)-2Re(D_1B_5^*)\right]\left(\mp\frac{\sqrt{\lambda}\, {m_B}^3\,\pi\,(1-r-s)\,\sqrt{s}}{2\,r}\right) \\
&+ \left[2Re(D_2B_5^*)-2Re(B_2B_4^*)\right]\left(\mp\frac{\lambda^{\frac{3}{2}}\, {m_B}^5\,\pi\,\sqrt{s}}{2\,r}\right) \\
&+ \left[|B_4|^2-|B_5|^2\right]\left(\frac{2\lambda\, {m_B}^4\, s}{r}\right) \\
&+ \left[2Re(B_1T_1^*C_{TE}^*)-2Re(D_1T_1^*C_{TE}^*)\right]\left(\pm 32\,\sqrt{\lambda}\, {m_B}^3\,\pi\,\sqrt{s}\right) \\
&+ \left[2Re(B_1T_1^*C_T^*)+2Re(D_1T_1^*C_T^*)\right]\left(\pm 16\,\sqrt{\lambda}\, {m_B}^3\,\pi\,\sqrt{s}\right) \\
&+ \left[2Re(A_1B_6^*C_{TE}^*)+2Re(C_1B_6C_{TE}^*)\right]\left(\pm 16\,\sqrt{\lambda}\, {m_B}^5\,\pi\, s^{\frac{3}{2}}\right) \\
&+ \left[2Re(A_1B_6^*C_T^*)-2Re(C_1B_6^*C_T^*)\right]\left(\pm 8\,\sqrt{\lambda}\, {m_B}^5\,\pi\, s^{\frac{3}{2}}\right)
\end{aligned}$$

$$
\begin{aligned}
&+ \Big[2Re(A_1 T_1^* C_{TE}^*) + 2Re(C_1 T_1^* C_{TE}^*)\Big] \Big(\mp 32\, \sqrt{\lambda}\, {m_B}^5\, \pi\, s^{\frac{3}{2}} \\
&\mp 32 \sqrt{\lambda}\, m_B^5\, \pi\, \sqrt{s}\sqrt{\lambda + 4rs}\Big) \\
&+ \Big[2Re(A_1 T_1^* C_T^*) - 2Re(C_1 T_1^* C_T^*)\Big] \Big(\mp 16\, \sqrt{\lambda}\, {m_B}^5\, \pi\, s^{\frac{3}{2}} \\
&\mp 16\, \sqrt{\lambda}\, m_B^5\, \pi\, \sqrt{s}\, \sqrt{\lambda + 4\, r\, s}\Big) \\
&+ \frac{256}{3rs} |T_1|^2 |C_T|^2 m_B^2 \Big(m_B^2 s \,[\lambda(16r + s) + 12rs(2 + 2r - s)]\Big) \\
&+ \frac{1024}{3rs} |T_1|^2 |C_{TE}|^2 m_B^2 \Big(m_B^2 s \,[\lambda(16r + s) + 12rs(2 + 2r - s)]\Big) \\
&+ \frac{16}{3r} m_B^2 \Big(4 m_B^2 s |C_{TE}|^2 + m_B^2 s |C_T|^2\Big) \times \Big(4(\lambda + 12rs)|B_6|^2\Big) \\
&+ m_B^4 \lambda^2 |B_7|^2 - 4 m_B^2 (1 - r - s)\lambda\, \mathrm{Re}(B_6 B_7^*) \\
&- 16\,[\lambda + 12r(1 - r)]\, \mathrm{Re}(T_1 B_6^*) \\
&+ 8 m_B^2 (1 + 3r - s)\lambda\, \mathrm{Re}(T_1 B_7^*)\Big)\Bigg\} \,, \qquad (4.36)
\end{aligned}
$$

where the superscripts $(-)$ and $(+)$ in $\vec{e}_T^{\,\mp}$ is to represent the decay rate expression when the leptons ℓ^+ and ℓ^- are polarized transversally, respectively. In order to calculate the transversal polarizations, we again use Eq. (4.5) to get the transversal polarizations as,

$$
\begin{aligned}
\Big(P_T^-\Big)_{m_l=0} &= \frac{\pi}{\Delta_{m_l=0}} m_B \sqrt{s\lambda} \Bigg\{ \frac{-1}{rs}(1 - r - s) m_B^2 s \Big[\mathrm{Re}(B_1 B_4^*) - \mathrm{Re}(D_1 B_5^*)\Big] \\
&- \frac{1}{rs} m_B^2 \lambda m_B^2 s \Big[\mathrm{Re}(D_2 B_5^*) - \mathrm{Re}(B_2 B_4^*)\Big] \\
&+ 16 m_B^4 s\, \mathrm{Re}[A_1^* (C_T - 2C_{TE}) B_6] \\
&- 16 m_B^4 s\, \mathrm{Re}[C_1^* (C_T + 2C_{TE}) B_6] \\
&- \frac{32}{s} m_B^4\, s\, (1 - r)\, \mathrm{Re}[A_1^* (C_T - 2C_{TE}) T_1] \\
&+ \frac{32}{s} m_B^4\, s\, (1 - r)\, \mathrm{Re}[C_1^* (C_T + 2C_{TE}) T_1] \\
&+ 64 m_B^2\, \mathrm{Re}[(B_1 - D_1)(T_1 C_{TE})^*]
\end{aligned}
$$

$$- \quad 32m_B^2\,\mathrm{Re}[(B_1+D_1)(T_1C_T)^*]\Bigg\}\ , \tag{4.37}$$

and,

$$\begin{aligned}
\Big(P_T^+\Big)_{m_l=0} &= \frac{\pi}{\Delta_{m_l=0}} m_B\sqrt{s\lambda}\Bigg\{ -\frac{1}{r}m_B^2 s(1-r-s)\Big[\mathrm{Re}(B_1B_5^*)-\mathrm{Re}(D_1B_4^*)\Big] \\
&+ \frac{1}{r}m_B^4\lambda\Big[\mathrm{Re}(B_2B_5^*)-\mathrm{Re}(D_2B_4^*)\Big] \\
&- 16m_B^4\, s\,\mathrm{Re}[A_1^*(C_T+2C_{TE})B_6] \\
&+ 16m_B^4\, s\,\mathrm{Re}[C_1^*(C_T-2C_{TE})B_6] \\
&+ 32m_B^4(1-r)\,\mathrm{Re}[A_1^*(C_T+2C_{TE})T_1] \\
&- 32m_B^4(1-r)\,\mathrm{Re}[C_1^*(C_T-2C_{TE})T_1] \\
&- 64m_B^2\,\mathrm{Re}[(B_1-D_1)(T_1C_{TE})^*] \\
&- 32m_B^2\,\mathrm{Re}[(B_1+D_1)(T_1C_T)^*]\Bigg\}\ ,
\end{aligned} \tag{4.38}$$

where Δ is given in Eq. (4.32). Here, again it is useful to denote that we neglected the lepton masses in these results, for the sake of simplicity. The relevant results for massive leptons can be found in Appendix E.

Finally for normal asymmetries for massless leptons, we get

$$\begin{aligned}
\Big(P_N^-\Big)_{m_l=0} &= \frac{1}{\Delta_{m_l=0}}\pi m_B^3\sqrt{s\lambda}\Bigg\{ -\frac{1}{r}m_B^2\lambda\,\mathrm{Im}[(B_2^*B_4)+(D_2^*B_5)] \\
&- 16m_B^2 s\Big(\mathrm{Im}[A_1^*(C_T-2C_{TE})B_6]+\mathrm{Im}[C_1^*(C_T+2C_{TE})B_6]\Big) \\
&+ 32m_B^2(1-r)\Big(\mathrm{Im}[A_1^*(C_T-2C_{TE})T_1]+\mathrm{Im}[C_1^*(C_T+2C_{TE})T_1]\Big) \\
&+ 32\Big(\mathrm{Im}[B_1^*(C_T-2C_{TE})T_1]-\mathrm{Im}[D_1^*(C_T+2C_{TE})T_1]\Big)\Bigg\}\ ,
\end{aligned} \tag{4.39}$$

$$\begin{aligned}
\Big(P_N^+\Big)_{m_l=0} &= \frac{1}{\Delta_{m_l=0}}\pi m_B^3\sqrt{s\lambda}\Bigg\{\frac{1}{r}m_B^2\lambda\,\mathrm{Im}[(B_2^*B_5)+(D_2^*B_4)] \\
&- \frac{1}{r}(1-r-s)\,\mathrm{Im}[(B_1^*B_5)+(D_1^*B_4)]
\end{aligned}$$

$$
\begin{aligned}
&- \; 16m_B^2 s\Big(\mathrm{Im}[A_1^*(C_T+2C_{TE})B_6] + \mathrm{Im}[C_1^*(C_T-2C_{TE})B_6]\Big) \\
&+ \; 32m_B^2(1-r)\Big(\mathrm{Im}[A_1^*(C_T+2C_{TE})T_1] + \mathrm{Im}[C_1^*(C_T-2C_{TE})T_1]\Big) \\
&- \; 32\Big(\mathrm{Im}[B_1^*(C_T+2C_{TE})T_1] - \mathrm{Im}[D_1^*(C_T-2C_{TE})T_1]\Big)\Bigg\} \, . \qquad (4.40)
\end{aligned}
$$

Again Δ is given in Eq. (4.32).

Concerning expressions $P_L^{(\pm)}$, $P_T^{(\pm)}$ and $P_N^{(\pm)}$, taking the results presented in the Appendix E into consideration also, few remarks are in order. The difference between P_L^- and P_L^+ results from the scalar and tensor type interactions. Similar situation takes place for the normal polarization $P_N^{(\pm)}$ of leptons and antileptons. In the $m_\ell \to 0$ limit, the difference between P_T^- and P_T^+ is due to again existence of new physics, i.e., scalar and tensor type interactions. For these reasons the experimental study of $P_L^{(\pm)}$ and $P_T^{(\pm)}$ can give essential information about new physics. Note that similar situation takes place for the inclusive channel $b \to s\ell^+\ell^-$ (see [60]).

Combined analysis of the lepton and antilepton polarizations can also give very useful hints in search of new physics, since in the SM $P_L^- + P_L^+ = 0$, $P_N^- + P_N^+ = 0$ and $P_T^- - P_T^+ \approx 0$.

Using Eqs. (4.34), (4.35) we get

$$
\begin{aligned}
\Big(P_L^- + P_L^+\Big)_{m_l=0} &= \frac{4}{\Delta_{m_l=0}} m_B^2 \Bigg\{ - \frac{1}{r} m_B^2 s\lambda \Big(\left|B_4\right|^2 - \left|B_5\right|^2 \Big) \\
&+ \frac{32}{3r} m_B^6 s\lambda^2 \left|B_7\right|^2 \mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{128}{3r} m_B^4 s\lambda(1-r-s)\,\mathrm{Re}(B_6 B_7^*)\,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{128}{3r} m_B^2 s(\lambda+12rs)\, \left|B_6\right|^2 \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{512}{3r} m_B^2 \left[\lambda(4r+s) + 12r(1-r)^2\right] \left|T_1\right|^2 \, \mathrm{Re}(C_T C_{TE}^*)
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{512}{3r} m_B^2 s\, [\lambda + 12r(1-r)]\, \mathrm{Re}(T_1 B_6^*)\, \mathrm{Re}(C_T C_{TE}^*) \\
&+ \left. \frac{256}{3r} m_B^4 s \lambda (1 + 3r - s)\, \mathrm{Re}(T_1 B_7^*)\, \mathrm{Re}(C_T C_{TE}^*) \right\} . \qquad (4.41)
\end{aligned}
$$

For the case of transversal polarization, it is the difference of the lepton and antilepton polarizations that is relevant and it can be calculated from Eqs. (4.37) and (4.38)

$$
\begin{aligned}
\left(P_T^- - P_T^+\right)_{m_l=0} &= \frac{\pi}{\Delta_{m_l=0}} m_B \sqrt{s\lambda} \left\{ \frac{1}{r} m_B^4 \lambda\, \mathrm{Re}[(B_2 + D_2)(B_4^* - B_5^*)] \right. \\
&- \frac{1}{r} m_B^2 (1 - r - s) \mathrm{Re}[(B_1 + D_1)(B_4^* - B_5^*)] \\
&+ 32 m_B^4 s\, \mathrm{Re}[(A_1 - C_1)(B_6 C_T)^*] \\
&- 64 m_B^4 (1 - r)\, \mathrm{Re}[(A_1 - C_1)(T_1 C_T)^*] \\
&+ \left. 128 m_B^2\, \mathrm{Re}[(B_1 - D_1)(T_1 C_{TE})^*] \right\} . \qquad (4.42)
\end{aligned}
$$

In the same manner it follows from Eqs. (4.39) and (4.40)

$$
\begin{aligned}
\left(P_N^- + P_N^+\right)_{m_l=0} &= \frac{1}{\Delta_{m_l=0}} \pi m_B^3 \sqrt{s\lambda} \left\{ -\frac{1}{r} (1 - r - s)\, \mathrm{Im}[(B_1 - D_1)(B_4^* - B_5^*)] \right. \\
&+ \frac{1}{r} m_B^2 \lambda\, \mathrm{Im}[(B_2 - D_2)(B_4^* - B_5^*)] \\
&+ 32 m_B^2 s\, \mathrm{Im}[(A_1 + C_1)(B_6 C_T)^*] \\
&- 64 m_B^2 (1 - r)\, \mathrm{Im}[(A_1 + C_1)(T_1 C_T)^*] \\
&+ \left. 128\, \mathrm{Im}[(B_1 + D_1)(T_1 C_{TE})^*] \right\} . \qquad (4.43)
\end{aligned}
$$

It is evident from Eq. (4.41) that the "pure" SM contribution to the $P_L^- + P_L^+$ completely disappears. Therefore a measurement of the nonzero value of $P_L^- + P_L^+$ in future experiments, is an indication of the discovery of new physics beyond SM.

4.2.2 Numerical Analysis for the $B \to K^* \ell^+ \ell^-$ Decay

The input parameters we used here is the same with the ones presented in the previous chapter as: $|V_{tb}V_{ts}^*| = 0.0385$, $\alpha^{-1} = 129$, $G_F = 1.17 \times 10^{-5}\ GeV^{-2}$, $\Gamma_B = 4.22 \times 10^{-13}\ GeV$, $C_9^{eff} = 4.344$, $C_{10} = -4.669$. It should be noted here that the above–value of the Wilson coefficient C_9^{eff} we have used in our numerical calculations corresponds only to short distance contribution. In addition to the short distance contribution C_9^{eff} also receives long distance contributions associated with the real $\bar{c}c$ intermediate states, i.e., with the J/ψ family. In this work we restricted ourselves only to short distance contributions. As far as C_7^{eff} is concerned, experimental results fixes only the modulo of it. For this reason throughout our analysis we have considered both possibilities, i.e., $C_7^{eff} = \mp 0.313$, where the upper sign corresponds to the SM prediction. The values of the input parameters which are summarized above, have been fixed by their central values.

For the values of the form factors, we have used the results of [23], where the radiative corrections to the leading twist contribution and $SU(3)$ breaking effects are also taken into account. The q^2 dependence of the form factors can be represented in terms of three parameters as

$$F(q^2) = \frac{F(0)}{1 - a_F \dfrac{q^2}{m_B^2} + b_F \left(\dfrac{q^2}{m_B^2} \right)^2} ,$$

where the values of parameters $F(0)$, a_F and b_F for the $B \to K^*$ decay are listed in Table 2.2.

Note that in the present analysis the final state Coulomb interactions of the leptons with the other charged particles are neglected since this effect is known to be much smaller than the averaged values of the SM (see [13]). Furthermore the final state interaction of the lepton polarization for the $K_L \to \pi^+\mu^-\bar{\nu}_\mu$ or $K^+ \to \pi^+\mu^-\mu^+$ decays is estimated to be of the order of $\alpha(m_\mu/m_K) \approx 10^{-3}$ [59]. For this reason the final state interaction effect is neglected as well.

We observe from the explicit form of the expressions of the lepton polarizations that they all depend on q^2 and the new Wilson coefficients. Therefore it may be experimentally difficult to study the dependence of the the polarizations of each lepton on all $\ell^+\ell^-$ center of mass energies and on new Wilson coefficients. So we eliminate the dependence of the lepton polarizations on one of the variables, namely q^2, by performing integration over q^2 in the allowed kinematical region, so that the lepton polarizations are averaged. The averaged lepton polarizations are defined as

$$\langle P_i \rangle = \frac{\displaystyle\int_{4m_\ell^2}^{(m_b - m_{K^*})^2} P_i \frac{d\mathcal{B}}{dq^2} dq^2}{\displaystyle\int_{4m_\ell^2}^{(m_b - m_{K^*})^2} \frac{d\mathcal{B}}{dq^2} dq^2} \ . \tag{4.44}$$

We present our analysis in a series Figures. Figs. (4.15) and (4.16) depict the dependence of the averaged longitudinal polarization $\left\langle P_L^- \right\rangle$ of ℓ^- and the combination $\left\langle P_L^- + P_L^+ \right\rangle$ on new Wilson coefficients, at $C_7^{eff} = -0.313$ for $B \to K^*\mu^+\mu^-$ decay. From these figures we observe that $\left\langle P_L^- \right\rangle$ is more sensitive to the existence of the tensor interaction, while the combined average $\left\langle P_L^- + P_L^+ \right\rangle$ is to both scalar and tensor type interactions. As has already been noted, this is an expected result since vector type interactions are canceled when the combined longitudinal polarization asymmetry of the lepton and antilepton is considered. From Fig. (4.16) we see that $\left\langle P_L^- + P_L^+ \right\rangle = 0$ at $C_X = 0$, which confirms the SM result as expected. For the other choice of C_7^{eff}, i.e., $C_7^{eff} = 0.313$ the situation is similar to the previous case, but the magnitude of $\left\langle P_L^- + P_L^+ \right\rangle$ is smaller.

Figs. (4.17) and (4.18) are the same as Figs.(4.15) and (4.16) but for the $B \to K^* \tau^+ \tau^-$ decay. Similar to the muon longitudinal polarization, $\left\langle P_L^- \right\rangle$ is strongly dependent on the tensor interaction coefficients C_T and C_{TE}. It is very interesting to observe that for $C_{TE} > 0.5$ $\left\langle P_L^- \right\rangle$ changes sign, but for all other cases $\left\langle P_L^- \right\rangle$ is negative.

From Fig. (4.18) we see that the dependence of $\left\langle P_L^- + P_L^+ \right\rangle$ on C_T is stronger. Furthermore if the values of the new Wilson coefficients C_{LRRL}, C_{LRLR} and C_T are negative (positive) so is $\left\langle P_L^- \right\rangle$ negative (positive). The situation is to the contrary for the coefficients C_{RLRL}, C_{RLLR}, i.e., $\left\langle P_L^- + P_L^+ \right\rangle$ is positive (negative) when the corresponding Wilson coefficients are negative (positive). Absolutely similar situation takes place for $C_7^{eff} > 0$. For these reasons determination of the sign and of course magnitude of $\left\langle P_L^- \right\rangle$ and $\left\langle P_L^- + P_L^+ \right\rangle$ can give promising information about new physics.

In Figs. (4.19) and (4.20) the dependence of the average transversal polarization $\left\langle P_T^- \right\rangle$ and the combination $\left\langle P_T^- - P_T^+ \right\rangle$ on the new Wilson coefficients, respectively, for the $B \to K^* \mu^+ \mu^-$ decay and at $C_7^{eff} = -0.313$ are presented. We observe from Fig. (4.19) that the average transversal polarization is strongly dependent on C_T, C_{TE}, C_{LRRL} and C_{RLRL} and quite weakly to remaining Wilson coefficients. It is also interesting to note that for the negative (positive) values of the coefficients C_{TE} and C_{LRRL}, $\left\langle P_T^- \right\rangle$ is negative (positive) while it follows the opposite path for the coefficients C_T and C_{RLRL}. For the $\left\langle P_T^- - P_T^+ \right\rangle$ case, there appears strong dependence on the tensor interactions C_T and C_{TE}, as well as all four scalar interactions with coefficients C_{LRRL}, C_{RLLR}, C_{LRLR}, C_{RLRL}. The behavior of this combined average is identical for the coefficients C_{LRLR}, C_{RLRL} and C_{LRRL}, C_{RLLR} in pairs, so that four lines responsible for these interactions appear only to be two. Moreover $\left\langle P_T^- - P_T^+ \right\rangle$ is negative (positive) for the negative (positive) values of the new Wilson coefficients C_{TE}, C_{LRRL} and C_{RLLR}. The situation is the other way around for the coefficients C_T, C_{LRLR} and C_{RLRL}. Remembering that in SM, in massless lepton case $\left\langle P_T^- \right\rangle \approx 0$ and $\left\langle P_T^- - P_T^+ \right\rangle \approx 0$, determination of the signs of the $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ can give quite a useful information about the existence of new physics. For the choice of $C_7^{eff} = 0.313$, apart from the minor differences in their magnitudes, the behaviors of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ are similar as in the previous case.

As is obvious from Figs. (4.21) and (4.22), $\left\langle P_T^- \right\rangle$ shows stronger dependence on C_T and $\left\langle P_T^- - P_T^+ \right\rangle$ on C_T and C_{TE}, respectively, at $C_7^{eff} = -0.313$ for the $B \to K^* \tau^+ \tau^-$ decay. Again change in signs of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ are observed depending on the change in the tensor interaction coefficients. As has already been noted, determination of the sign and magnitude of $\left\langle P_T^- \right\rangle$ and $\left\langle P_T^- - P_T^+ \right\rangle$ are useful tools in looking for new physics.

Note that for simplicity all new Wilson coefficients in this work are assumed to be real. Under this condition $\left\langle P_N^- \right\rangle$ and $\left\langle P_N^- + P_N^+ \right\rangle$ have non-vanishing values coming from the imaginary part of SM, i.e., from C_9^{eff}. From Fig. (4.23) we see that $\left\langle P_N^- \right\rangle$ is strongly dependent on all tensor and scalar type interactions. On the other hand Fig. (4.24) depicts that the behavior $\left\langle P_N^- + P_N^+ \right\rangle$ is determined by only the tensor interactions, for $B \to K^* \mu^+ \mu^-$ decay. Similar behavior takes place for the $B \to K^*\tau^+\tau^-$ decay as well, as can easily be seen in Figs. (4.25) and (4.26). The change in sign and magnitude of both $\left\langle P_N^- \right\rangle$ and $\left\langle P_N^- + P_N^+ \right\rangle$ that are observed in these figures is an indication of the fact that an experimental verification of them can give unambiguous information about new physics.

In Figs. (4.27), (4.28) and (4.29) we present parametric plot of the correlations between the integrated branching ratio and averaged lepton polarization asymmetries of τ^- and τ^+ as a function of the new Wilson coefficients. In Fig. (4.27) we present the flows in the $\left(\mathcal{B},\ \left\langle P_L^- + P_L^+ \right\rangle\right)$ plane by varying the coefficients of the tensor and scalar type interactions. Fig. (4.28) shows the flows in $\left(\mathcal{B},\ \left\langle P_T^- - P_T^+ \right\rangle\right)$ plane by varying the coefficients of vector, scalar and tensor type interactions. Finally, Fig. (4.29) depicts the flows in $\left(\mathcal{B}, \left\langle P_N^- + P_N^+ \right\rangle\right)$ plane by changing the coefficients of the tensor type interactions only.

It should be noted that the influence of the variation of various coefficients confirms our previous results, i.e., the influence of the tensor interactions is quite large. The ranges of variation of the new Wilson coefficients are determined by assuming that the value of the branching ratio is about the SM prediction. For example if branching ratio is restricted to have the values in the range $10^{-7} \leq \mathcal{B}(B \to K^*\tau^+\tau^-) \leq 5 \times 10^{-7}$, then it follows from Fig. (4.27) that the new Wilson coefficients of the tensor interactions lie in the region $-2.6 \leq C_T \leq 1.55$ or $-0.35 \leq C_{TE} \leq 1.15$, while all scalar interaction coefficients vary in the range between -4 and 4 (in the present work we vary all coefficients in the range -4 and 4).

Finally we would like to discuss briefly the detectibilty of the lepton polarization asymmetries. Experimentally, to be able to measure an asymmetry $\langle P_i \rangle$

of a decay with the branching ratio B at the $n\sigma$, the required number of events are $N = n^2/(\mathcal{B}\langle P_i\rangle^2$. As an example for detecting $\langle P_T\rangle \simeq 0.3$ the number of events expected is $N \simeq 6 \times 10^7 n^2$ events. Therefore at B factories detection of polarization asymmetries for τ could be accessible.

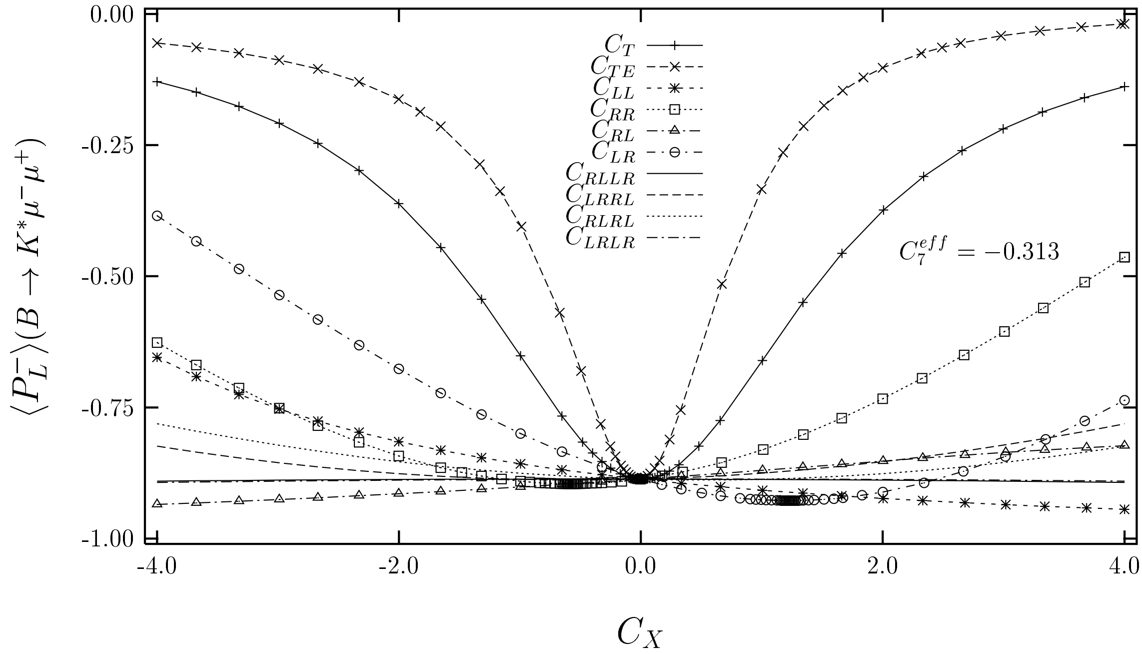

Figure 4.15: The dependence of the average longitudinal polarization asymmetry $\langle P_L^- \rangle$ of ℓ^- on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^+ \mu^-$ decay.

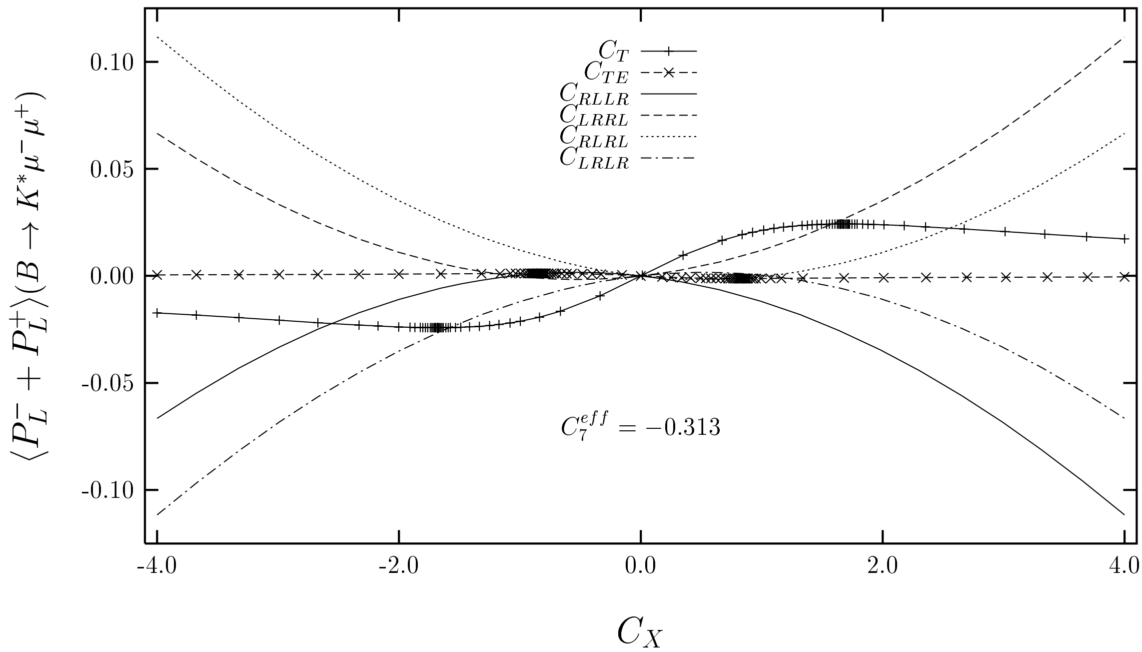

Figure 4.16: The dependence of the combined average longitudinal polarization asymmetry $\left\langle P_L^- + P_L^+ \right\rangle$ of ℓ^- and ℓ^+ on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^- \mu^+$ decay.

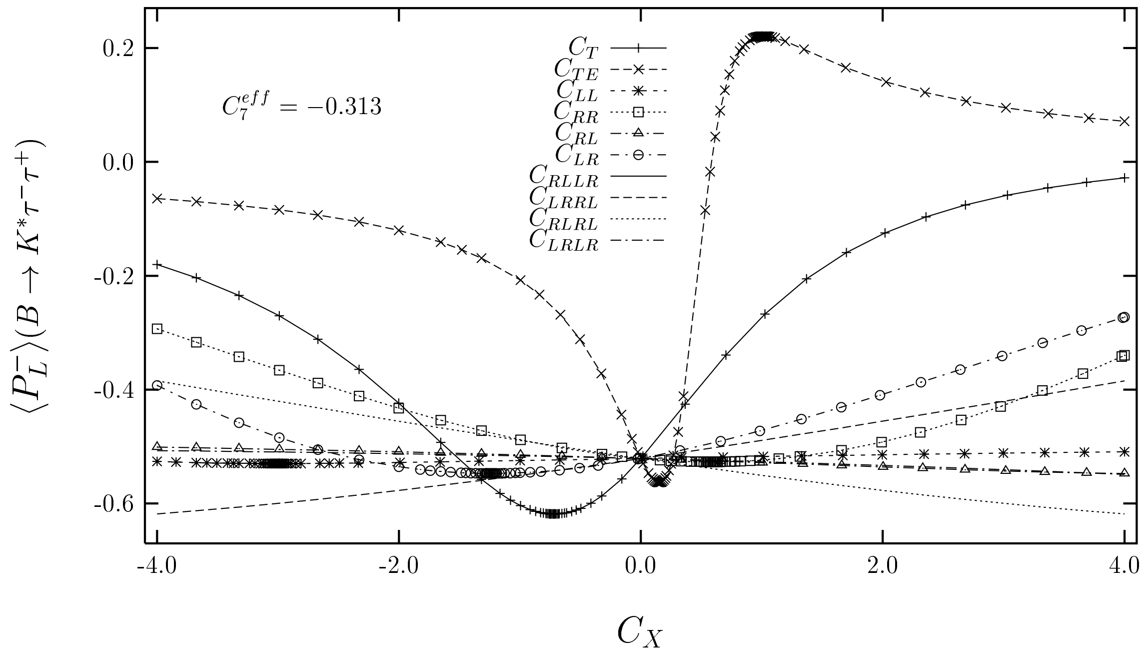

Figure 4.17: The dependence of the average longitudinal polarization asymmetry $\langle P_L^- \rangle$ of ℓ^- on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \tau^+ \tau^-$ decay.

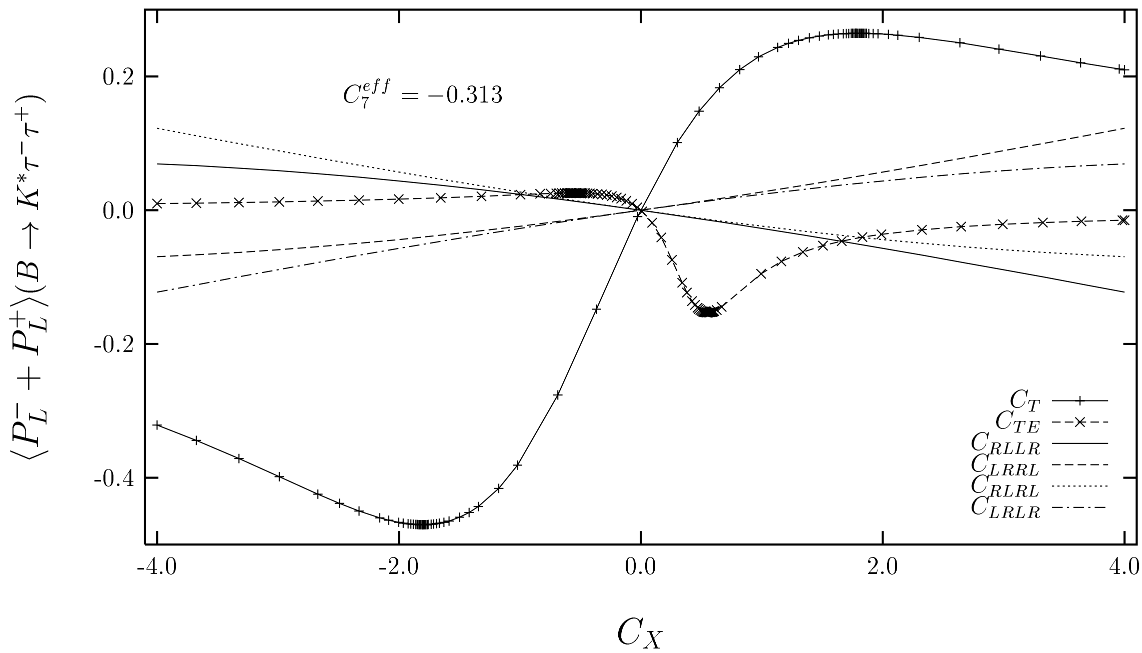

Figure 4.18: The dependence of the combined average longitudinal polarization asymmetry $\left\langle P_L^- + P_L^+ \right\rangle$ of ℓ^- and ℓ^+ on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \tau^- \tau^+$ decay.

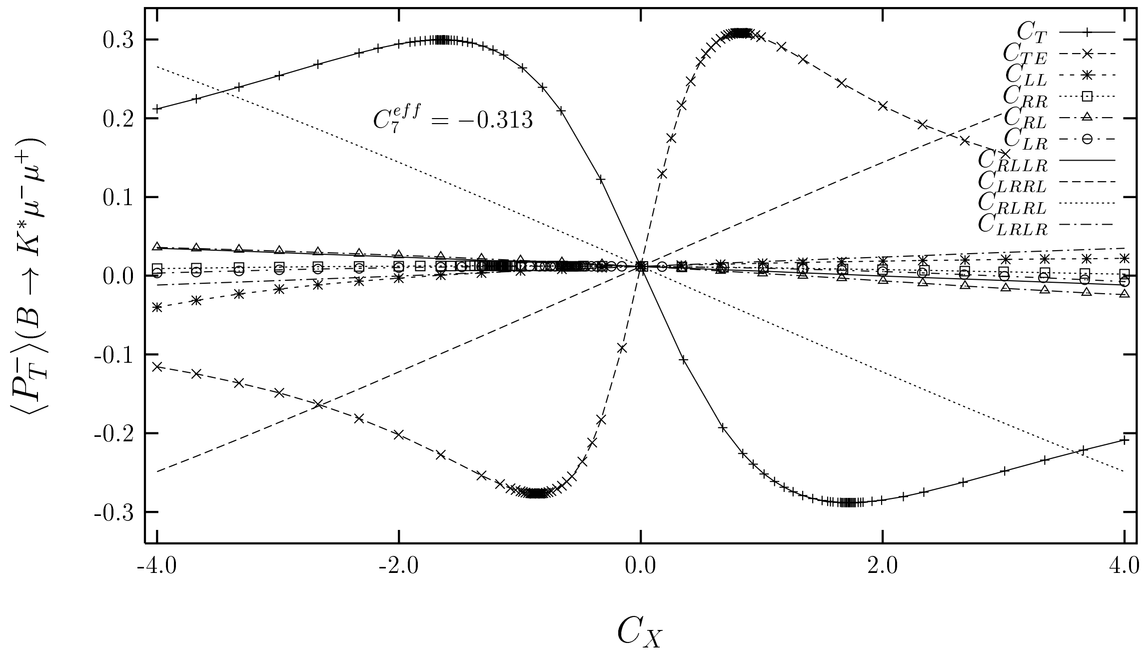

Figure 4.19: The dependence of the average transversal polarization asymmetry $\langle P_T^- \rangle$ of ℓ^- on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^+ \mu^-$ decay.

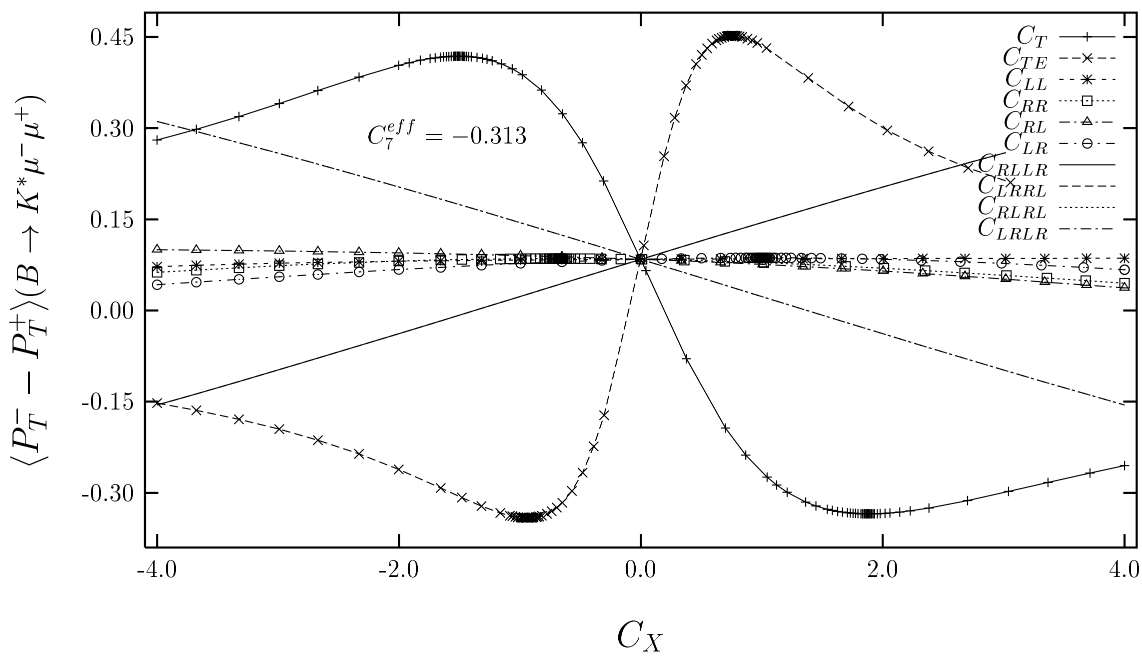

Figure 4.20: The dependence of the combined average transversal polarization asymmetry $\left\langle P_T^- + P_T^+ \right\rangle$ of ℓ^- and ℓ^+ on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^- \mu^+$ decay.

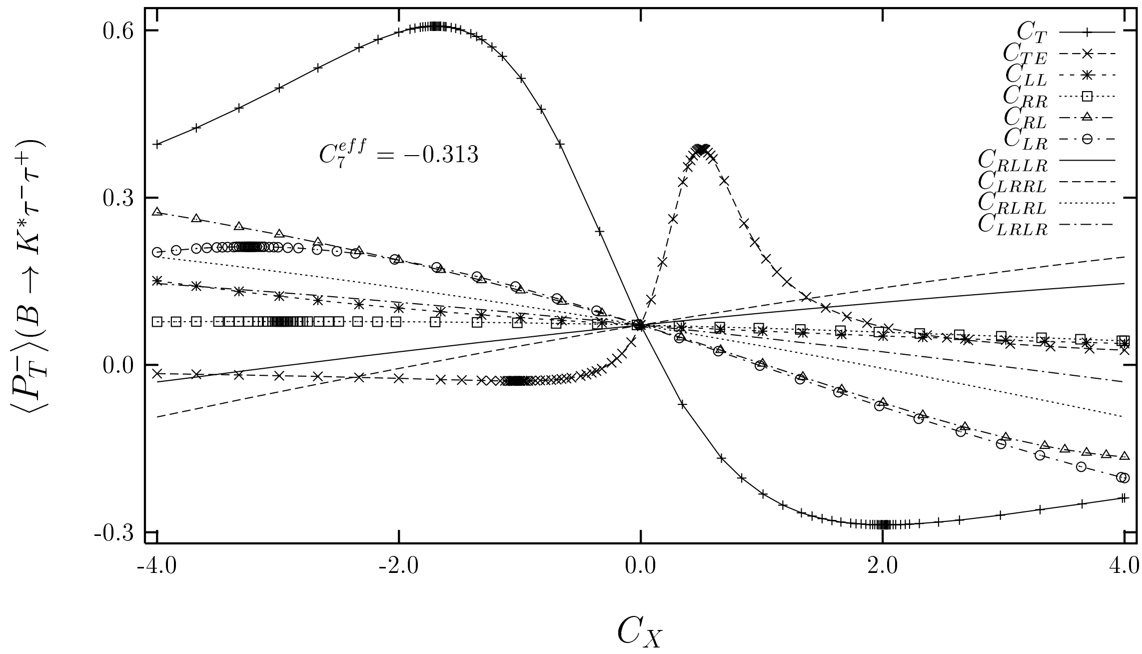

Figure 4.21: The dependence of the average transversal polarization asymmetry $\langle P_T^- \rangle$ of ℓ^- on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \tau^+ \tau^-$ decay.

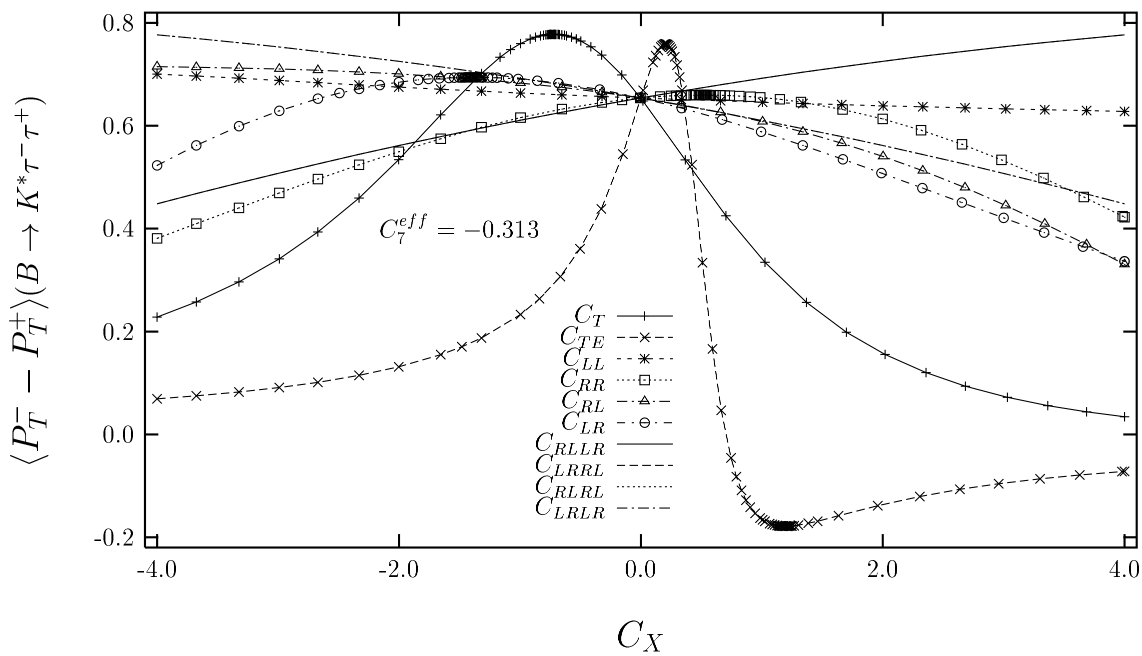

Figure 4.22: The dependence of the combined average transversal polarization asymmetry $\left\langle P_T^- + P_T^+ \right\rangle$ of ℓ^- and ℓ^+ on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \tau^- \tau^+$ decay.

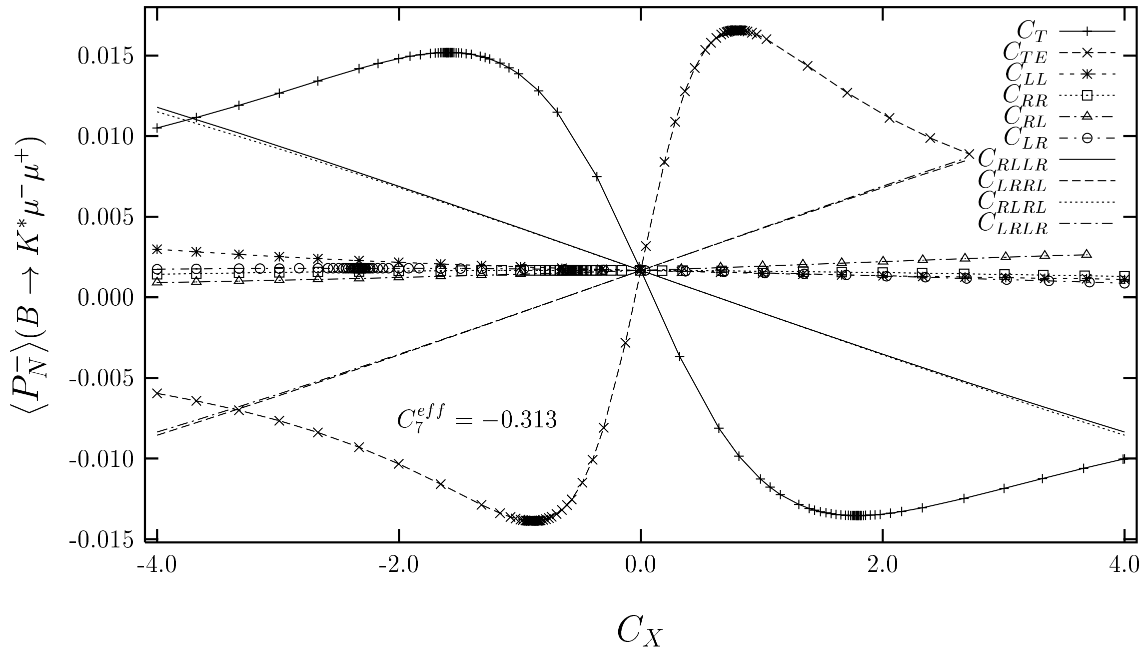

Figure 4.23: The dependence of the average normal polarization asymmetry $\langle P_N^- \rangle$ of ℓ^- on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^+ \mu^-$ decay.

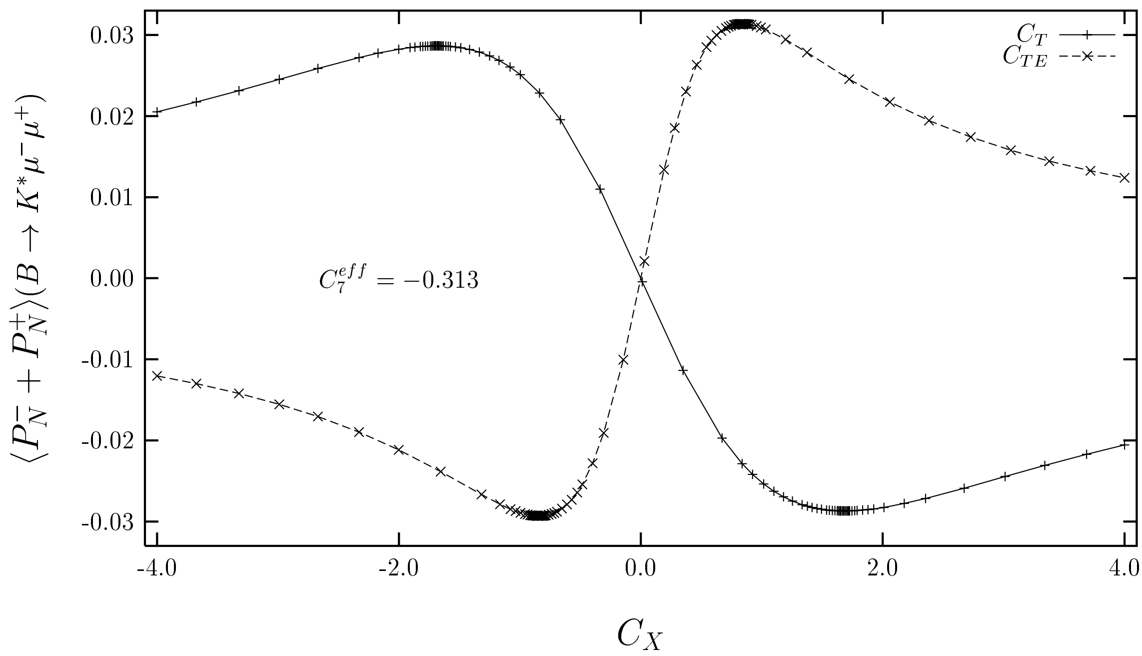

Figure 4.24: The dependence of the combined average normal polarization asymmetry $\left\langle P_N^- + P_N^+ \right\rangle$ of ℓ^- and ℓ^+ on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \mu^- \mu^+$ decay.

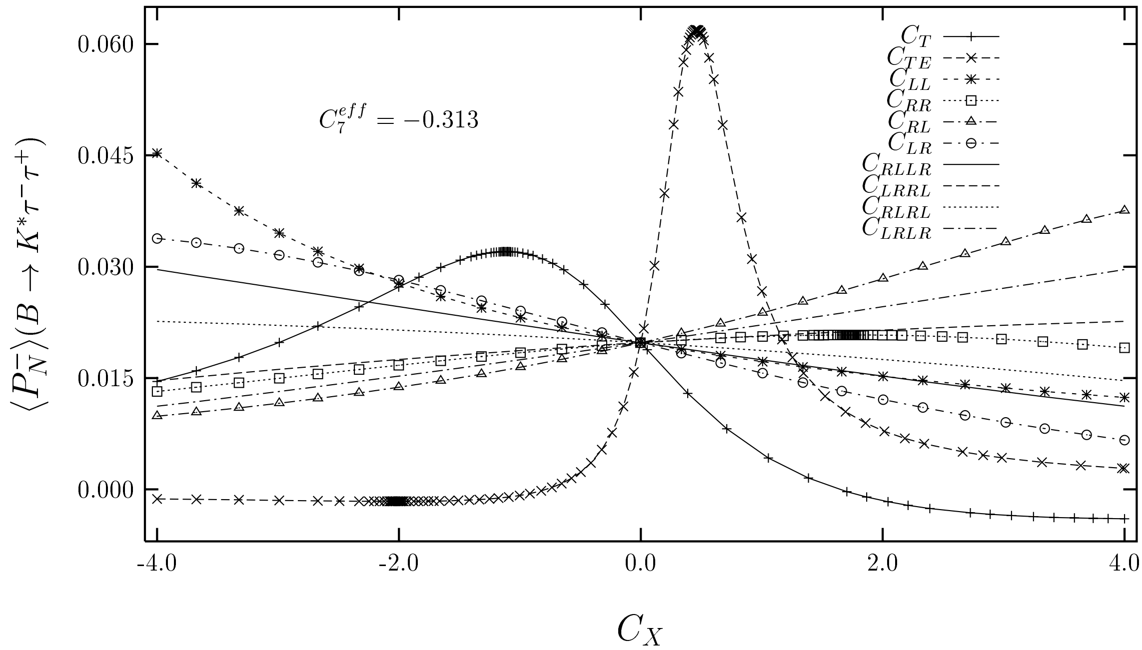

Figure 4.25: The dependence of the average normal polarization asymmetry $\langle P_N^- \rangle$ of ℓ^- on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \tau^+ \tau^-$ decay.

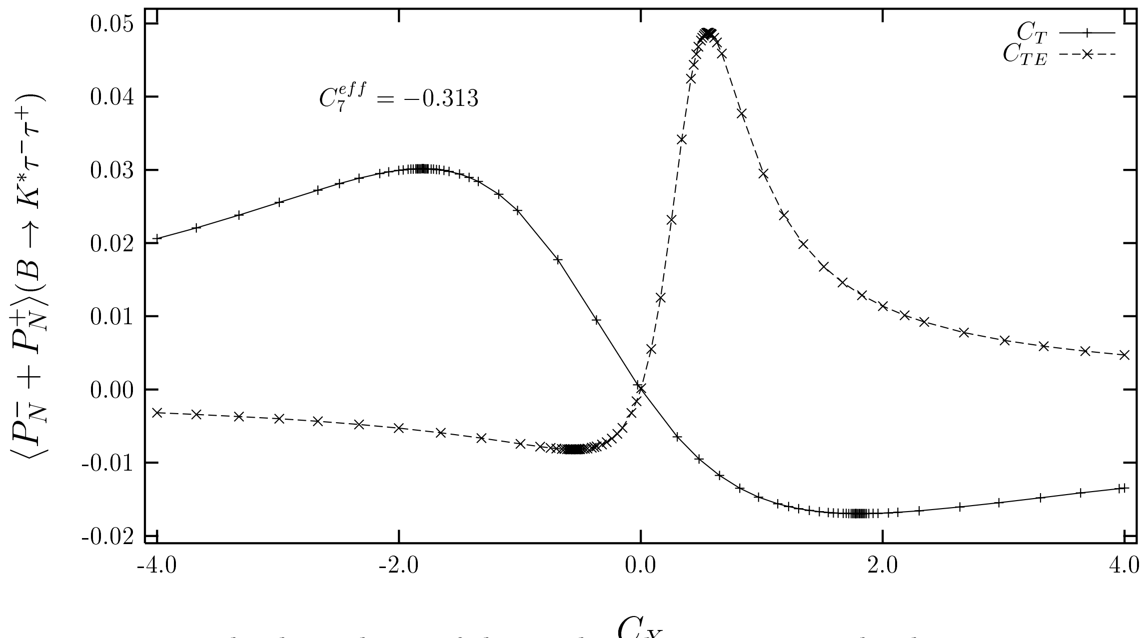

Figure 4.26: The dependence of the combined average normal polarization asymmetry $\left\langle P_N^- + P_N^+ \right\rangle$ of ℓ^- and ℓ^+ on the new Wilson coefficients at $C_7^{eff} = -0.313$ for the $B \to K^* \tau^- \tau^+$ decay.

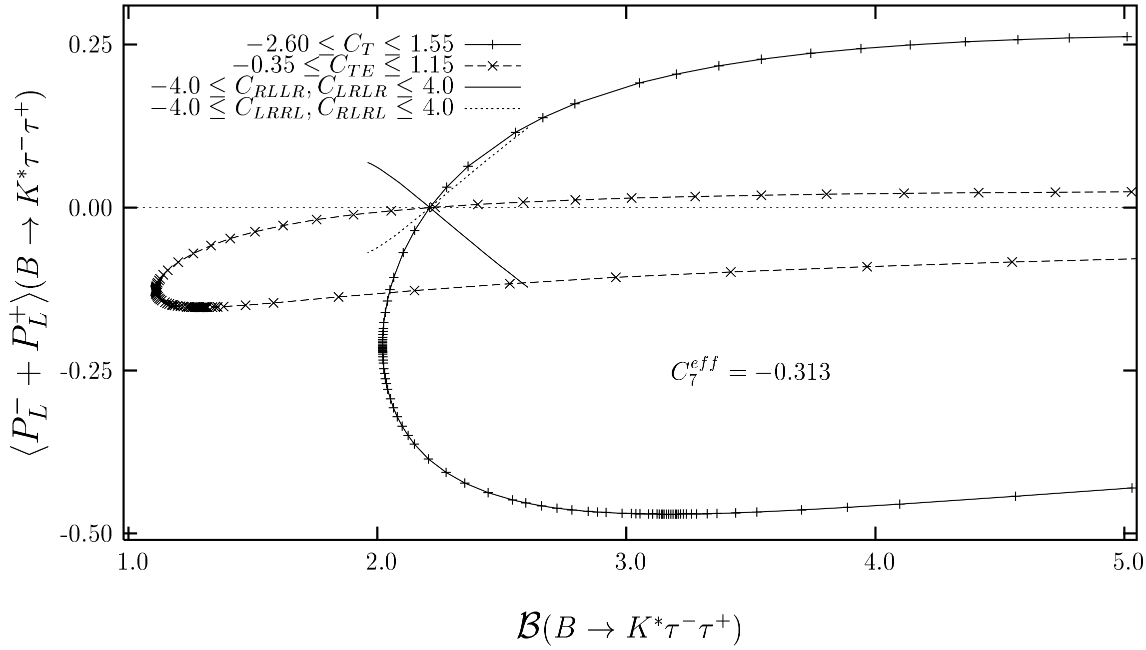

Figure 4.27: Parametric plot of the correlation between the integrated branching ratio $\mathcal{B}$ (in units of 10^{-7}) and the average longitudinal lepton polarization asymmetry $\langle P_L^+ + P_L^- \rangle$ at $C_7^{eff} = -0.313$ as function of the new Wilson coefficients as indicated in the figure, for the $B \to K^* \tau^+ \tau^-$ decay.

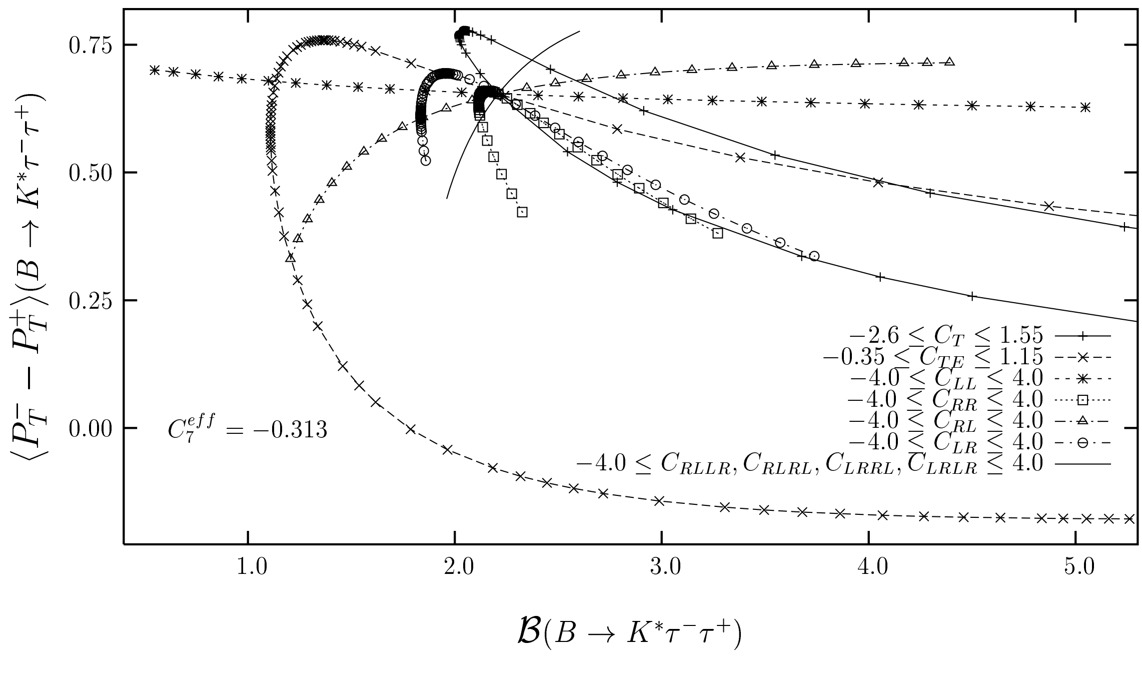

Figure 4.28: The same as in figure 4.27, but for the combined average transversal lepton polarization asymmetry $\langle P_T^+ + P_T^- \rangle$.

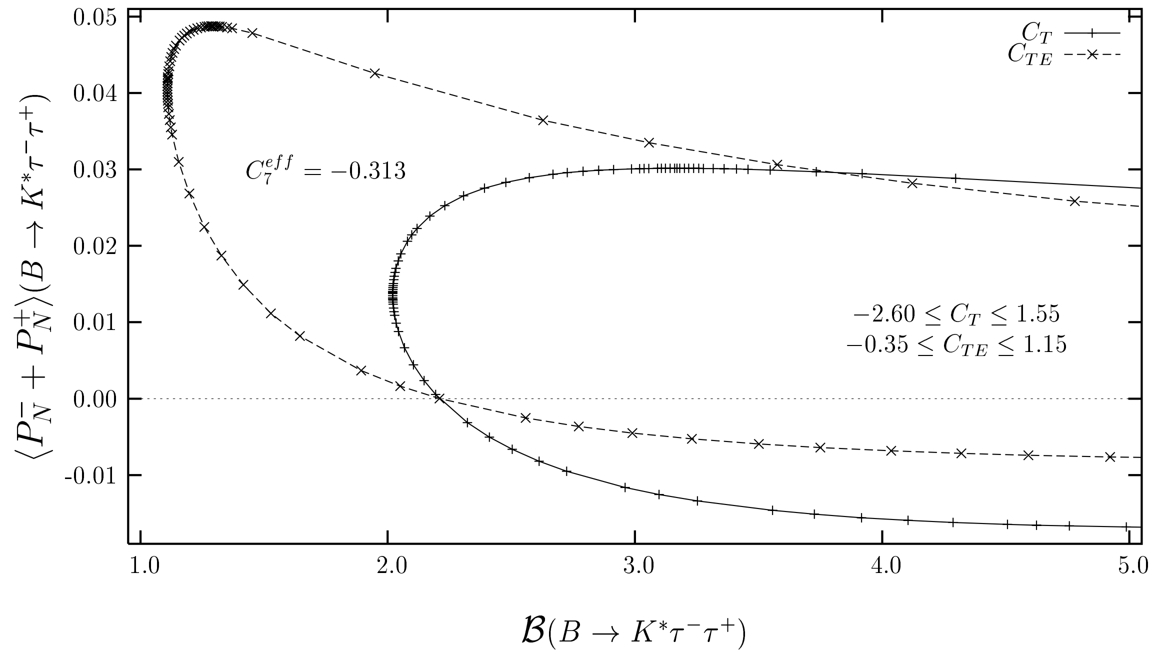

Figure 4.29: The same as in figure 4.27, but for the combined average normal lepton polarization asymmetry $\langle P_N^+ + P_N^- \rangle$.

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CHAPTER 5

CONCLUSION

In this thesis, we investigated the $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ decays in a model independent way. We have mainly focused our analysis to the lepton polarization asymmetries of the relevant decays. This is because we expected that studying the lepton polarizations only, would give promising information on new physics.

The analysis of each decay constituted of two parts. At first part, we have calculated the branching ratios of these decays using the most general form of the effective Hamiltonian. Without forcing the concrete values for the Wilson coefficients corresponding to any model, we found that the branching ratios are in agreement with the SM predictions for some certain values of the Wilson coefficients.

In the second part, we included the lepton polarizations in our calculations. We have found some regions of Wilson coefficients for which the branching ratio of the corresponding decay agrees with the SM prediction while the lepton polarizations do not. This would be a strong indication that in establishing new physics, the study of lepton polarizations would be an effective tool.

We studied the dependence of the longitudinal, transversal and normal polarization asymmetries of the ℓ^+ and ℓ^- and their combined asymmetries on the Wilson coefficients. We have found that, besides the above-mentioned results, the lepton polarizations are very sensitive to the existence of the vector and scalar type interactions which do not take place in the SM.

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APPENDIX A

INPUT VALUES

$m_b = 4.8\,GeV, \quad m_c = 1.4\,GeV, \quad m_t = 176\,GeV, \quad m_u = m_d = 10\,MeV,$

$m_B = 5.27\,GeV, \quad m_K = 0.49\,GeV, \quad m_{K^*} = 0.89\,GeV, \quad m_W = 80.2\,GeV,$

$\alpha = 1/129, \quad A = 0.81, \quad \lambda = 0.22, \quad \tau_B = 1.6 \times 10^{-12} s, \quad m_\tau = 1.78\,GeV,$

$|V_{tb} V_{ts}^*| = 0.0385\ , \quad G_F = 1.17 \times 10^{-5} GeV^{-2}\ , \quad \Gamma_B = 4.22 \times 10^{-13} GeV,$

$C_9^{eff} = 4.344\ , \quad C_{10} = -4.669.$

APPENDIX B

BOREL TRANSFORMATIONS

Consider a function $f(x)$ and introduce $\bar{f}(\lambda)$ which is defined via;

$$\bar{f}(\lambda) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{\lambda/x} f(x) x d(\frac{1}{x}) \ , \tag{B.1}$$

where the integration contour runs to the right of all the singularities of the function $f(x)$. The function $\bar{f}(x)$ is called Borel transform of $f(x)$. The inverse transformation is given by

$$f(x) = \int_0^\infty \bar{f}(\lambda) e^{-\lambda/x} \frac{d\lambda}{x}. \tag{B.2}$$

To clarify the meaning of the Borel transform, assume that $f(x)$ is given as a power series of "x":

$$f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_k x^k + ... \ , \tag{B.3}$$

Then the expansion of $\bar{f}(\lambda)$ is,

$$\bar{f}(\lambda) = a_0 + \frac{a_1 \lambda}{1!} + \frac{a_2 \lambda^2}{2!} + ... + \frac{a_k \lambda^k}{k!} + ... \tag{B.4}$$

so that the coefficients are factorially suppressed as compared to the case (B.3). This implies that the approximation of the whole series by the few first terms is more reliable for $\bar{f}(\lambda)$ than for $f(x)$.

APPENDIX C

CALCULATION OF THE INTEGRAL OVER FINAL LEPTON MOMENTA

To perform integration over final lepton momenta we will use the following invariant integration method, which is true for any arbitrary reference frame. Let us consider the following integral

$$I_{\alpha\beta} = \int \frac{d^3\vec{p_1}}{2E_1}\frac{d^3\vec{p_2}}{2E_2} p_{1\alpha}p_{2\beta}\delta^4(p - p_1 - p_2). \tag{C.1}$$

This integral can be represented as

$$I_{\alpha\beta} = g_{\alpha\beta}K_1 + p_\alpha p_\beta K_2, \tag{C.2}$$

where K_1 and K_2 are two unknown coefficients. In order to calculate K_1 and K_2, we multiply $I_{\alpha\beta}$ with the metric tensor and with $p_\alpha p_\beta$, after which we get the two equations

$$\begin{aligned} g_{\alpha\beta}I_{\alpha\beta} &= \int \frac{d^3\vec{p_1}}{2E_1}\frac{d^3\vec{p_2}}{2E_2}(p_1 \cdot p_2)\delta^4(p - p_1 - p_2), \\ &= 4K_1 + p^2K_2, \end{aligned} \tag{C.3}$$

and

$$p_\alpha p_\beta I_{\alpha\beta} = \int \frac{d^3\vec{p_1}}{2E_1}\frac{d^3\vec{p_2}}{2E_2}(p_1\cdot p)(p_2\cdot p)\delta^4(p-p_1-p_2),$$
$$= p^2K_1 + p^4K_2. \tag{C.4}$$

From momentum conservation we have

$$p_1\cdot p_2 = \frac{1}{2}\left[p^2 - 2m_\ell^2\right],$$

and

$$p_1\cdot p = p_2\cdot p = \frac{1}{2}p^2.$$

Using these relations we have

$$4K_1 + p^2K_2 = \frac{1}{2}\left[p^2 - 2m_\ell^2\right] I_0,$$
$$p^2K_1 + p^4K_2 = \frac{1}{4}p^4 I_0.$$

Making the replacement $\hat{s} \equiv \hat{p}^2 = p^2/m_B^2$ and $\hat{r}_\pi = m_\pi^2/m_B^2$ we can write,

$$\frac{m_B^2\hat{s}}{2}\left[1 - \frac{2m_\ell^2}{m_B^2\hat{s}}\right] I_0 = 4K_1 + m_B^2\hat{s}K_2,$$
$$\frac{m_B^4\hat{s}^2}{4} I_0 = m_B^2\hat{s}K_1 + m_B^4\hat{s}^2K_2.$$

Solving these two equations for K_1 and K_2 we get,

$$K_1 = \frac{m_B^2\hat{s}}{12} I_0\left[1 - \frac{4m_\ell^2}{m_B^2\hat{s}}\right],$$
$$K_2 = \frac{I_0}{6}\left[1 + \frac{2m_\ell^2}{m_B^2\hat{s}}\right].$$

Therefore, (C.2) can be written as

$$I_{\alpha\beta} = g_{\alpha\beta}\frac{m_B^2\hat{s}I_0}{12}\left[1 - \frac{4m_\ell^2}{m_B^2\hat{s}}\right] + \frac{I_0}{6}\left[1 + \frac{2m_\ell^2}{m_B^2\hat{s}}\right] p_\alpha p_\beta, \tag{C.5}$$

where

$$I_0 = \int \frac{d^3\vec{p_1}}{2E_1}\frac{d^3\vec{p_2}}{2E_2}\delta^4(p-p_1-p_2),$$
$$= \frac{\pi}{2}v, \tag{C.6}$$

with

$$v = \sqrt{1 - \frac{4m_\ell^2}{m_B^2\hat{s}}}\ ,$$

is the lepton velocity. We present the calculation of I_0 below.

C.1 Calculation of the Integral I_0

We present I_0 in Eq (C.6) as,

$$I_0 = \int \frac{d^3\vec{p_1}}{2E_1}\frac{d^3\vec{p_2}}{2E_2}\delta^4(p-p_1-p_2)\ . \tag{C.7}$$

In calculating this integral, we consider the center of mass frame of the final leptons, i.e.,

$$\vec{p_1} + \vec{p_2} = 0\ .$$

Performing integration over $d^3\vec{p_2}$, we get

$$I_0 = \int \frac{d^3\vec{p_1}}{4E_1E_2}\delta(E-E_1-E_2). \tag{C.8}$$

Since $\vec{p_1} = -\vec{p_2}$, so $E_2 = E_1$. Using this fact, we have

$$I_0 = \int \frac{|\vec{p_1}|\, dE_1\, 4\pi}{4E_1}\delta(E-2E_1)$$
$$= \frac{\pi}{2}\frac{\sqrt{(E^2/4) - m_\ell^2}}{E/2} = \frac{\pi}{2}\sqrt{1 - \frac{4m_\ell^2}{E^2}}\ . \tag{C.9}$$

This expression can be rewritten in any arbitrary frame by making the replacement,

$$E^2 \rightarrow p^2,$$

from which it follows that

$$I_0 = \frac{\pi}{2}\, v. \tag{C.10}$$

APPENDIX D

EXPRESSIONS FOR $B \to K\ell^+\ell^-$ DECAY FOR MASSIVE LEPTONS

In this chapter, we would like to include the details of the calculations presented in Section 3.1. We will also like to include the lepton masses here which we neglect in our results presented in Section 3.1.

Performing for massive lepton case, the differential branching ratio is

$$\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{G_F^2\alpha^2}{2^{14}\pi^5 m_B}\Big|V_{tb}V_{ts}^*\Big|^2\lambda^{1/2}(1,r,s)v\times\Bigg\{-128\lambda m_B^4 m_\ell\,\mathrm{Re}(AG^*) \\
&+ 32m_B^2m_\ell^2(1-r)\,\mathrm{Re}(CD^*)+16m_B^2m_\ell(1-r)\,\mathrm{Re}(CN^*) \\
&+ 16m_B^2m_\ell^2 s\,|D|^2+4m_B^2 s\,|N|^2+16m_B^2m_\ell s\,\mathrm{Re}(DN^*)+\frac{1024}{3}\lambda m_B^6 sv^2\,|H|^2 \\
&+ 4m_B^2sv^2\,|Q|^2+\frac{4}{3}\lambda m_B^4 s(3-v^2)\,|A|^2+\frac{256}{3}\lambda m_B^6 s(3-v^2)\,|G|^2 \\
&+ \frac{4}{3}m_B^4 s\Big\{2\lambda-(1-v^2)\Big[2\lambda-3(1-r)^2\Big]\Big\}\,|C|^2\Bigg\}, \qquad \text{(D.1)}
\end{aligned}$$

with lepton masses taken into consideration here. These integration techniques are presented in Appendix C.

D.1 Explicit Expressions for Lepton Polarization Asymmetries in $B \to K\ell^+\ell^-$ Decay

Using Eq. (4.3), we calculated the module square of the matrix element for the polarized decay to find Eq. (4.16). When longitudinally polarizations are considered, again integrating over final lepton momenta, we find the decay rate expression for the massive leptons as,

$$\begin{aligned}
\left(\frac{d\Gamma_\pm}{ds}\right)_L &= \frac{G_F^2\alpha^2|V_{tb}V_{ts}^*|^2}{2^{14}\pi^5 m_B} v\Big\{2Re(AC^*)\Big[\mp\frac{8}{3}\lambda m_B^4 v\Big] \\
&+ 2Re(CG^*)\Big[\pm\frac{128}{3}\lambda m_B^4 m_l v\Big] + 2Re(AH^*)\Big[-\frac{128}{3}\lambda m_B^4 m_l v\Big] \\
&+ 2Re(CQ^*)\Big[-8m_B^2 m_l v(1-r)\Big] + 2Re(GH^*)\Big[\frac{512}{3}\lambda m_B^6 s v\Big] \\
&+ 2Re(DQ^*)\Big[-8m_B^2 m_l v\Big] + 2Re(QN^*)\Big[-4m_B^2 s v\Big]\Big\}. \qquad \text{(D.2)}
\end{aligned}$$

Where, $v = \sqrt{1-4m_l^2/q^2}$ is the lepton velocity. Similarly, when the leptons are transversally polarized, we find the decay rate expression as,

$$\begin{aligned}
\left(\frac{d\Gamma_\pm}{ds}\right)_T &= \frac{G_F^2\alpha^2|V_{tb}V_{ts}^*|^2\sqrt{\lambda s} m_B^2}{2^{14}\pi^4} v\Big\{m_B^3\pi\sqrt{\lambda s}\Big\{2Re(AC^*)\Big[\mp\frac{2}{s}m_l(1-r)\Big] \\
&+ 2Re(CG^*)\Big[\pm\frac{32}{s}m_l^2(1-r)\Big] \\
&+ 2Re(CQ^*)\Big[2(1-m_l^2/(m_B^2 s))\Big] + 2Re(AD^*)\Big[\mp 2m_l\Big] \\
&+ 2Re(DG^*)\Big[\pm 32m_l^2\Big] + 2Re(AN^*) + 2Re(GN^*)\Big[\pm 16m_l\Big]. \qquad \text{(D.3)}
\end{aligned}$$

So, the longitudinal, transversal and normal polarization asymmetries, for massive leptons are calculated respectively, as follows,

$$\begin{aligned}
P_L^\mp &= \frac{4m_B^2 v}{\Delta}\Big\{\pm\frac{4}{3}\lambda m_B^2\,\mathrm{Re}(AC^*) \mp \frac{64}{3}\lambda m_B^2 m_\ell\,\mathrm{Re}(CG^*) - \frac{64}{3}\lambda m_B^2 m_\ell\,\mathrm{Re}(AH^*) \\
&- 4m_\ell(1-r)\,\mathrm{Re}(CQ^*) + \frac{256}{3}\lambda m_B^4 s\,\mathrm{Re}(GH^*) \\
&- 4m_\ell s\,\mathrm{Re}(DQ^*) - 2\,\mathrm{Re}(NQ^*)\Big\} \qquad \text{(D.4)}
\end{aligned}$$

$$
\begin{aligned}
P_T^{\mp} &= \frac{\pi m_B^3 \sqrt{s\lambda}}{\Delta}\Bigg\{ \pm \frac{4}{s} m_\ell (1-r)\,\mathrm{Re}(AC^*) \mp \frac{64}{s}(1-r) m_\ell^2\,\mathrm{Re}(CG^*) \\
&\pm 4m_\ell\,\mathrm{Re}(AD^*) \mp 64 m_\ell^2\,\mathrm{Re}(DG^*) \pm 2\,\mathrm{Re}(AN^*) \mp 32 m_\ell\,\mathrm{Re}(GN^*) \\
&+ 2v^2\,\mathrm{Re}(CQ^*)\Bigg\}\,, \qquad \text{(D.5)}
\end{aligned}
$$

$$
P_N^{\mp} = \frac{m_B^3 v \sqrt{s\lambda}}{\Delta}\Bigg\{4m_\ell\,\mathrm{Im}(CD^*) + 2\,\mathrm{Im}(CN^*) \mp 2\,\mathrm{Im}(AQ^*) \pm 32 m_\ell\,\mathrm{Im}(CG^*)\Bigg\}\,. \qquad \text{(D.6)}
$$

APPENDIX E

EXPRESSIONS FOR $B \to K^*\ell^+\ell^-$ DECAY FOR MASSIVE LEPTONS

In this chapter, we would like to present modulus square of the matrix element for the $B \to K^*\ell^+\ell^-$ decay, including the lepton masses which we have neglected in Section 3.2.

After lengthy but straightforward calculations for the modulus square of the matrix element given in Eq. (4.29), we obtain the following expression;

$$
\begin{aligned}
|\mathcal{M}|^2 \;=\; & \frac{G_F^2\alpha^2}{32\pi^2}\left|V_{tb}V_{ts}^*\right|^2\Big\{(|A_1|^2+|C_1|^2)\Big[-64(e.q)(e^*.q)(p.p_1)(p.p_2) \\
& +32(e.q)(e^*.p_2)(p.p_1)(p.q)+32(e.p_2)(e^*.q)(p.p_1)(p.q) \\
& +32(e.q)(e^*.p_1)(p.p_2)(p.q)+32(e.p_1)(e^*.q)(p.p_2)(p.q) \\
& -32(e.p_2)(e^*.p_1)(p.q)^2-32(e.p_1)(e^*.p_2)(p.q)^2 \\
& +32(e.q)(e^*.q)p^2(p_1.p_2)-32(e.q)(e^*.p_2)p^2(p_1.q) \\
& -32(e.p_2)(e^*.q)p^2(p_1.q)-64(e.e^*)(p.p_2)(p.q)(p_1.q) \\
& -32(e.q)(e^*.p_1)p^2(p_2.q)-32(e.p_1)(e^*.q)p^2(p_2.q)
\end{aligned}
$$

$$-64(e.e^*)(p.p_1)(p.q)(p_2.q) + 64(e.e^*)p^2(p_1.q)(p_2.q)$$

$$+32(e.p_2)(e^*.p_1)p^2q^2 + 32(e.p_1)(e^*.p_2)p^2q^2$$

$$+64(e.e^*)(p.p_1)(p.p_2)q^2 - 32(e.e^*)p^2(p_1.p_2)q^2\Big]$$

$$+ \quad (|B_1|^2 + |D_1|^2)\Big[8(e.p_2)(e^*.p_1)8(e.p_1)(e^*.p_2) - 6(e.e^*)(p_1.p_2)$$

$$+ \quad (|B_2|^2 + |D_2|^2)\Big[64(e.q)(e^*.q)(p.p_1)(p.p_2) - 32(e.q)(e^*.q)p^2(p_1.p_2)$$

$$+32(e.q)(e^*.q)(p.p_2)(p_1.q) + 32(e.q)(e^*.q)(p.p_1)(p_2.q)$$

$$+16(e.q)(e^*.q)(p_1.q)(p_2.q) - 8(e.q)(e^*.q)(p_1.p_2)q^2\Big]$$

$$+ \quad (|B_3|^2 + |D_3|^2)\Big[16(e.q)(e^*.q)(p_1.q)(p_2.q) - 8(e.q)(e^*.q)(p_1.p_2)q^2\Big]$$

$$+ \quad 2\Big(Re(B_1B_2^*) + Re(D_1D_2^*)\Big)\Big[-16(e.q)(e^*.2)(p.p_1) - 16(e.q)(e^*.p_1)(p.p_2)$$

$$+8(e.q)(e^*.q)(p_1.p_2) - 8(e.q)(e^*.p_2)(p_1.q) - 8(e.q)(e^*.p_1)(p_2.q)\Big]$$

$$+ \quad 2\Big(Re(B_1B_3^*) + Re(D_1D_3^*)\Big)\Big[8(e.q)(e^*.q)(p_1.p_2) - 8(e.p_2)(e^*.q)(p_1.q)$$

$$-8(e.p_1)(e^*.q)(p_2.q)\Big]$$

$$+ \quad 2\Big(Re(B_2B_3^*) + Re(D_2D_3^*)\Big)\Big[-16(e.q)(e^*.q)(p.q)(p_1.p_2)$$

$$+16(e.q)(e^*.q)(p.p_2)(p_1.q) + 16(e.q)(e.^*.q)(p.p_1)(p_2.q)$$

$$+16(e.q)(e^*.q)(p_1.q)(p_2.q) - 16(e.q)(e^*.q)(p_1.p_2)q^2\Big]$$

$$+ \quad 2Re(B_1D_1^*)\Big[-8m_l^2(e.e^*)\Big]$$

$$+ \quad 2\Big(Re(B_2D_1^*) + Re(B_1D_2^*) + Re(B_3D_1^*) + Re(B_1D_3^*)\Big)\Big[8m_l^2(e.q)(e^*.q)\Big]$$

$$+ \quad 2Re(B_2D_2^*)\Big[-32m_l^2(e.q)(e^*.q)p^2 - 32m_l^2(e.q)(e^*.q)(p.q)$$

$$-8m_l^2(e.q)(e^*.q)q^2\Big]$$

$$+ \quad 2\Big(Re(B_3D_2^*) + Re(B_2D_3^*)\Big)\Big[-16m_l^2(e.q)(e^*.q)(p.q)$$

$$-8m_l^2(e.q)(e^*.q)q^2\Big] + 2Re(B_3D_3^*)\Big[-8m_l^2(e.q)(e^*.q)q^2\Big]$$

$$
\begin{aligned}
+ \quad & 2Re(A_1C_1^*)\Big[-32m_l^2(e.q)(e^*.q)p^2 - 32m_l^2(e.e^*)(p.q)^2 + 32m_l^2(e.e^*)p^2q^2\Big] \\
+ \quad & 2\Big(Re(A_1B_1^*) - Re(C_1D_1^*)\Big)\Big[-16(e.q)(e^*.p_2)(p.p_1) \\
& +16(e.q)(e^*.p_1)(p.p_2) - 16(e.e^*)(p.p_2)(p_1.q) + 16(e.e^*)(p.p_1)(p_2.q)\Big] \\
+ \quad & 2\Big(Re(A_1B_2^*) - Re(C_1D_2^*)\Big)\Big[32(e.q)(e^*.p_2)(p.p_1)(p.q) \\
& -32(e.q)(e^*.p_1)(p.p_2)(p.q) - 32(e.q)(e^*.p_2)p^2(p_1.q) \\
& +16(e.q)(e^*.q)(p.p_2)(p_1.q) + 32(e.q)(e^*.p_1)p^2(p_2.q) \\
& -16(e.q)(e^*.q)(p.p_1)(p_2.q) + 16(e.q)(e^*.p_1)(p.q)(p_2.q) \\
& -16(e.q)(e^*.p_1)(p.p_2)q^2\Big] \\
+ \quad & 2\Big(Re(A_1B_3^*) - Re(C_1D_3^*)\Big)\Big[16(e.q)(e^*.q)(p.p_2)(p_1.q) \\
& -16(e.q)(e^*.p_2)(p.q)(p_1.q) - 16(e.q)(e^*.q)(p.p_1)(p_2.q) \\
& +16(e.q)(e^*.p_1)(p.q)(p_2.q) + 32(e.q)(e^*.p_2)(p.p_1)q^2\Big] \\
+ \quad & 2\Big(Re(B_1B_4^*) - (D_1B_4^*)\Big)\Big[-8m_l(e.p_2)(e^*.q)\Big] \\
+ \quad & 2\Big(Re(B_1B_5^*) - (D_1B_5^*)\Big)\Big[8m_l(e.p_1)(e^*.q)\Big] \\
+ \quad & 2\Big(Re(B_2B_4^*) - Re(D_2B_4^*)\Big)\Big[16m_l(e.q)(e^*.q)(p.p_2) \\
& +8m_l(e.q)(e^*.q)(p_1.q)\Big] \\
+ \quad & \Big(Re(B_2B_5^*) - Re(D_2B_5^*)\Big)\Big[-16m_l(e.q)(e^*.q)(p.p_2) \\
& -8m_l(e.q)(e^*.q)(p_2.q)\Big] \\
+ \quad & \Big(Re(B_3B_4^*) - Re(D_3B_4^*) + Re(B_3B_5^*) - Re(D_3B_5^*)\Big) \\
& \Big[8m_l(e.q)(e^*.q)(p_2.q)\Big] \\
+ \quad & 2\Big(Re(B_1B_6^*C_{TE}^*) + Re(D_1B_6^*C_{TE}^*)\Big)\Big[64m_l(e.p_1)(e^*.q) \\
& +64m_l(e.p_2)(e^*.q) - 64m_l(e.e^*)(p_1.q) - 64m_l(e.e^*)(p_2.q)\Big]
\end{aligned}
$$

$$+ \quad 2\Big(Re(B_1 B_6^* C_T^*) - Re(D_1 B_6^* C_T^*)\Big)\Big[-32m_l(e.p_1)(e^*.q)$$

$$+32m_l(e.p_2)(e^*.q) + 32m_l(e.e^*)(p_1.q) - 32m_l(e.e^*)(p_2.q)\Big]$$

$$+ \quad 2\Big(Re(B_2 B_6^* C_{TE}^*) + Re(D_2 B_6 C_{TE}^*)\Big)\Big[-128m_l(e.p_1)(e^*.q)(p.q)$$

$$-128m_l(e.p_2)(e^*.q)(p.q) + 64m_l(e.q)(e^*.q)(p_1.q)$$

$$+64m_l(e.q)(e^*.q)(p_2.q) - 64m_l(e.p_1)(e^*.q)q^2 - 64m_l(e.p_2)(e^*.q)q^2\Big]$$

$$+ \quad 2\Big(Re(B_1 B_6^* C_T^*) - Re(D_1 B_6^* C_T^*)\Big)\Big[64m_l(e.p_1)(e^*.q)(p.q)$$

$$-64m_l(e.p_2)(e^*.q)(p.q) - 32m_l(e.q)(e^*.q)(p_1.q) + 32m_l(e.q)(e^*.q)(p_2.q)$$

$$+32m_l(e.p_1)(e^*.q)q^2 - 32m_l(e.p_2)(e^*.q)q^2\Big]$$

$$+ \quad 2\Big(Re(B_3 B_6^* C_{TE}^*) + Re(D_3 B_6^* C_{TE}^*)\Big)\Big[64m_l(e.q)(e^*.q)(p_1.q)$$

$$+64m_l(e.q)(e^*.q)(p_2.q) - 64m_l(e.p_1)(e^*.q)q^2 - 64m_l(e.p_2)(e^*.q)q^2\Big]$$

$$+ \quad 2\Big(Re(B_3 B_6^* C_T^*) - Re(D_3 B_6^* C_T^*)\Big)\Big[-32m_l(e.q)(e^*.q)(p_1.q)$$

$$+32m_l(e.q)(e^*.q)(p_2.q) + 32m_l(e.p_1)(e^*.q)q^2 - 32m_l(e.p_2)(e^*.q)q^2\Big]$$

$$+ \quad 2\Big(Re(B_1 B_7^* C_{TE}^*) + Re(D_1 B_7^* C_{TE}^*)\Big)\Big[-64m_l(e.q)(e^*.q)((p_1.p) + (p_2.p))\Big]$$

$$+ \quad 2\Big(Re(B_1 B_7^* C_T^*) - Re(D_1 B_7^* C_T^*)\Big)\Big[32m_l(e.q)(e^*.q)((p.p_1) - (p.p_2))\Big]$$

$$+ \quad 2\Big(Re(B_2 B_7^* C_{TE}^*) + Re(D_2 B_7^* C_{TE}^*)\Big)\Big[128m_l(e.q)(e^*.q)((p.p_1)$$

$$+(p.p_2))(p.q) + 128m_l(e.q)(e^*.q)p^2((p_1.q) + (p_2.q))$$

$$-64m_l(e.q)(e^*.q)(p.q)((p_1.q) + (p_2.q))$$

$$+64m_l(e.q)(e^*.q)((p.p_1) + (p.p_2))q^2\Big]$$

$$+ \quad 2\Big(Re(B_2 B_7^* C_T^*) - Re(D_2 B_7^* C_T^*)\Big)\Big[64m_l(e.q)(e^*.q)((p.p_1) - (p.p_2))(p.q)$$

$$-64m_l(e.q)(e^*.q)p^2((p_1.q) - (p_2.q)) - 32m_l(e.q)(e^*.q)(p.q)((p_1.q)$$

$$-(p_2.q)) + 32m_l(e.q)(e^*.q)((p_1.p) - (p_2.p))q^2\Big]$$

$$
\begin{aligned}
+ \quad & 2\Big(Re(B_3B_7^*C_{TE}^*) + D_3B_7^*C_{TE}^*\Big)\Big[-64m_l(e.q)(e^*.q)(p.q)((p_1.q)+(p_2.q)) \\
& +64m_l(e.q)(e^*.q)((p.p_1)+(p.p_2))q^2\Big] \\
+ \quad & 2\Big(Re(B_3B_7^*C_T^*) - D_3B_7^*C_T^*\Big)\Big[-32m_l(e.q)(e^*.q)(p.q)((p_1.q)-(p_2.q)) \\
& +32m_l(e.q)(e^*.q)((p.p_1)-(p.p_2))q^2\Big] \\
+ \quad & 2\Big(Re(B_1T_1^*C_{TE}^*) + Re(D_1T_1^*C_{TE}^*)\Big)\Big[-128m_l((e.p_1)+(e.p_2))(e^*.q) \\
& +256m_l(e.e^*)((p.p_1)+(p.p_2)) + 128m_l(e.e^*)((p_1.q)+(p_2.q))\Big] \\
+ \quad & 2\Big(Re(B_1T_1^*C_T^*) - Re(D_1T_1^*C_T^*)\Big)\Big[64m_l((e.p_1)-(e.p_2))(e^*.q) \\
& +128m_l(e.e^*)((p.p_1)-(p.p_2)) + 64m_l(e.e^*)((p_1.q)-(p_2.q))\Big] \\
+ \quad & 2\Big(Re(B_2T_1^*C_{TE}^*) + Re(D_2T_1^*C_{TE}^*)\Big)\Big[512m_l((e.p_1)+(e.p_2))(e^*.q)p^2 \\
& -256m_l(e.q)(e^*.q)((p.p_1)+(p.p_2)) + 512m_l((e.p_1)+(e.p_2))(e^*.q)(p.q) \\
& -128m_l(e.q)(e^*.q)((p_1.q)+(p_2.q)) + 128m_l(e.q)(e^*.q)((p_1.q)+(p_2.q))\Big] \\
+ \quad & 2\Big(Re(B_2T_1^*C_T^*) - Re(D_2T_1^*C_T^*)\Big)\Big[256m_l((e.p_1)-(e.p_2))(e^*.q)p^2 \\
& +128m_l(e.q)(e^*.q)((p.p_1)-(p.p_2)) + 256m_l((e.p_1)-(e.p_2))(e^*.q)(p.q) \\
& -64m_l(e.q)(e^*.q)((p_1.q)-(p_2.q)) + 64m_l(e.q)(e^*.q)((p_1.q)-(p_2.q))\Big] \\
+ \quad & 2\Big(Re(B_3T_1^*C_{TE}^*) + Re(B_3T_1C_{TE}^*)\Big)\Big[-256m_l(e.q)(e^*.q)((p.p_1)+(p.p_2)) \\
& +256m_l((e.p_1)+(e.p_2))(e^*.q)(p.q) - 128m_l(e.q)(e^*.q)((p_1.q)+(p_2.q)) \\
& +128m_l((e.p_1)+(e.p_2))(e^*.q)q^2\Big] \\
+ \quad & 2\Big(Re(B_3T_1^*C_T^*) - Re(B_3T_1C_T^*)\Big)\Big[128m_l(e.q)(e^*.q)((p.p_1)-(p.p_2)) \\
& +128m_l((e.p_1)-(e.p_2))(e^*.q)(p.q) - 64m_l(e.q)(e^*.q)((p_1.q)-(p_2.q)) \\
& +64m_l((e.p_1)-(e.p_2))(e^*.q)q^2\Big] \\
+ \quad & 2\Big(Re(A_1B_6^*C_{TE}^*) - Re(C_1B_6^*C_{TE}^*)\Big)\Big[-128m_l(e.q)(e^*.q)((p.p_1)-(p.p_2))
\end{aligned}
$$

$$+128m_l(e.q)((e^*.p_1)-(e^*.p_2))(p.q)-128m_l(e.e^*)(p.q)((p_1.q)-(p_2.q))$$

$$+128m_l(e.e^*)((p.p_1)-(p.p_2))q^2\Big]$$

$$+\quad 2\Big(Re(A_1B_6^*C_T^*)+Re(C_1B_6^*C_T^*)\Big)\Big[-64m_l(e.q)(e^*.q)((p.p_1)+(p.p_2))$$

$$+64m_l(e.q)((e^*.p_1)+(e^*.p_2))(p.q)-64m_l(e.e^*)(p.q)((p_1.q)+(p_2.q))$$

$$+64m_l(e.e^*)((p.p_1)+(p.p_2))q^2\Big]$$

$$+\quad 2\Big(Re(A_1B_7^*C_{TE}^*)-Re(C_1B_7^*C_{TE}^*)\Big)\Big[128m_l(e.q)(e^*.q)((p.p_1)$$

$$-(p.p_2))(p.q)-128m_l(e.q)((e^*.p_1)-(e^*.p_2))(p.q)^2$$

$$-128m_l(e.q)(e^*.q)p^2((p_1.q)-(p_2.q))$$

$$+128m_l(e.q)((e^*.p_1)-(e^*.p_2))p^2q^2\Big]$$

$$+\quad 2\Big(Re(A_1B_7^*C_T^*)+Re(C_1B_7^*C_T^*)\Big)\Big[64m_l(e.q)(e^*.q)((p.p_1)+(p.p_2))(p.q)$$

$$-64m_l(e.q)((e^*.p_1)+(e^*.p_2))(p.q)^2-64m_l(e.q)(e^*.q)p^2((p_1.q)+(p_2.q))$$

$$+64m_l(e.q)((e^*.p_1)+(e^*.p_2))p^2q^2\Big]$$

$$+\quad 2\Big(Re(A_1T_1^*C_{TE}^*)-Re(C_1T_1^*C_{TE}^*)\Big)\Big[512m_l(e.q)((e^*.p_1)-(e^*.p_2))p^2$$

$$-256m_l(e.q)(e^*.q)((p.p_1)-(p.p_2))-256m_l(e.q)((e^*.p_1)-(e^*.p2))(p.q)$$

$$+512m_l(e.e^*)p^2((p_1.q)-(p_2.q))-512m_l(e.e^*)(p.q)((p_1.q)-(p_2.q))\Big]$$

$$+\quad 2\Big(Re(A_1T_1^*C_T^*)+Re(C_1T_1^*C_T^*)\Big)\Big[256m_l(e.q)((e^*.p_1)+(e^*.p_2))p^2$$

$$-128m_l(e.q)(e^*.q)((p.p_1)+(p.p_2))-128m_l(e.q)((e^*.p_1)+(e^*.p2))(p.q)$$

$$+256m_l(e.e^*)p^2((p_1.q)+(p_2.q))-256m_l(e.e^*)(p.q)((p_1.q)+(p_2.q))\Big]$$

$$+\quad |B_4|^2\Big[8(e.q)(e^*.q)(p_1.p_2)\Big]+|B_5|^2\Big[m_l^2(e.q)(e^*.q)\Big]$$

$$+\quad 2Re(B_4B_5^*)\Big[-8m_l^2(e.q)(e^*.q)\Big]$$

$$+\quad 2Re((B_4+B_5)B_6^*C_{TE}^*)\Big[-64(e.p_2)(e^*.q)(p_1.q)-64(e.p_1)(e^*.q)(p_2.q)\Big]$$

$$
\begin{aligned}
&+ \quad 2Re((B_4 - B_5)B_6^* C_T^*)\Big[32(e.p_2)(e^*.q)(p_1.q) - 32(e.p_1)(e^*.q)(p_2.q)\Big] \\
&+ \quad 2Re((B_4 + B_5)B_7^* C_{TE}^*)\Big[-64(e.q)(e^*.p_2)(p_1.q) - 64(e.q)(e^*.p_1)(p_2.q)\Big] \\
&+ \quad 2Re((B_4 - B_5)B_7^* C_T^*)\Big[32(e.q)(e^*.p_2)(p_1.q) - 32(e.q)(e^*.p_1)(p_2.q)\Big] \\
&+ \quad 2\Big(Re(B_4 T_1^* C_{TE}^*) + Re(B_5 T_1^* C_{TE}^*)\Big)\Big[256(e.p_2)(e^*.q)(p.p_1) \\
&\quad\quad -256(e.p_1)(e^*.q)(p.p2) + 128(e.p_2)(e^*.q)(p_1.q) - 128(e.p_1)(e^*.q)(p_2.q)\Big] \\
&+ \quad 2\Big(Re(B_4 T_1^* C_T^*) - Re(B_5 T_1^* C_T^*)\Big)\Big[-128(e.p_2)(e^*.q)(p.p_1) \\
&\quad\quad +128(e.p_1)(e^*.q)(p.p2) - 64(e.p_2)(e^*.q)(p_1.q) + 64(e.p_1)(e^*.q)(p_2.q)\Big] \ ,
\end{aligned}
\tag{E.1}
$$

where, $p = p_B + p_{K^*}$ and $q = p_B - p_{K^*}$. Performing integration over final lepton momenta, we find the unpolarized differential decay rate, for massive leptons as,

$$
\begin{aligned}
\left(\frac{d\Gamma}{dq^2}\right)_0 &= \frac{G_F^2\alpha^2}{2^{14}\pi^5 m_B}|V_{tb}V_{ts}^*|^2\lambda^{1/2}v \\
\times &\quad \Bigg\{\frac{32}{3}m_B^4\lambda\Big[(m_B^2 s - m_\ell^2)\Big(|A_1|^2 + |C_1|^2\Big) + 6m_\ell^2\,\mathrm{Re}(A_1C_1^*)\Big] \\
+ &\quad 96m_\ell^2\,\mathrm{Re}(B_1D_1^*) - \frac{4}{r}m_B^2 m_\ell\lambda\,\mathrm{Re}[(B_1 - D_1)(B_4^* - B_5^*)] \\
+ &\quad \frac{8}{r}m_B^2 m_\ell^2\lambda\,\Big(\mathrm{Re}[B_1(-B_3^* + D_2^* + D_3^*)] + \mathrm{Re}[D_1(B_2^* + B_3^* - D_3^*)] - \mathrm{Re}(B_4B_5^*)]\Big) \\
+ &\quad \frac{4}{r}m_B^4 m_\ell(1-r)\lambda\,\Big(\mathrm{Re}[(B_2 - D_2)(B_4^* - B_5^*)] + 2m_\ell\,\mathrm{Re}[(B_2 - D_2)(B_3^* - D_3^*)]\Big) \\
- &\quad \frac{8}{r}m_B^4 m_\ell^2\lambda(2 + 2r - s)\,\mathrm{Re}(B_2D_2^*) + \frac{4}{r}m_B^4 m_\ell s\lambda\,\mathrm{Re}[(B_3 - D_3)(B_4^* - B_5^*)] \\
+ &\quad \frac{4}{r}m_B^4 m_\ell^2 s\lambda\,|B_3 - D_3|^2 + \frac{2}{r}m_B^2(m_B^2 s - 2m_\ell^2)\lambda\,\Big(|B_4|^2 + |B_5|^2\Big) \\
- &\quad \frac{8}{3rs}m_B^2\lambda\,\Big[m_\ell^2(2 - 2r + s) + m_B^2 s(1 - r - s)\Big]\Big[\mathrm{Re}(B_1B_2^*) + \mathrm{Re}(D_1D_2^*)\Big] \\
+ &\quad \frac{4}{3rs}\,\Big[2m_\ell^2(\lambda - 6rs) + m_B^2 s(\lambda + 12rs)\Big]\Big(|B_1|^2 + |D_1|^2\Big) \\
+ &\quad \frac{4}{3rs}m_B^4\lambda\,\Big(m_B^2 s\lambda + m_\ell^2[2\lambda + 3s(2 + 2r - s)]\Big)\Big(|B_2|^2 + |D_2|^2\Big) \\
+ &\quad \frac{32}{r}m_B^6 m_\ell\lambda^2\,\mathrm{Re}[(B_2 + D_2)(B_7C_{TE})^*]
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{32}{r} m_B^4 m_\ell \lambda(1-r-s)\Big(\mathrm{Re}[(B_1+D_1)(B_7 C_{TE})^*] + 2\,\mathrm{Re}[(B_2+D_2)(B_6 C_{TE})^*]\Big) \\
&+ \frac{64}{r}(\lambda + 12rs) m_B^2 m_\ell\, \mathrm{Re}[(B_1+D_1)(B_6 C_{TE})^*] \\
&+ \frac{256}{3rs}|T_1|^2 |C_T|^2 m_B^2 \Big(4m_\ell^2\,[\lambda(8r-s) - 12rs(2+2r-s)] \\
&+ m_B^2 s\,[\lambda(16r+s) + 12rs(2+2r-s)]\Big) \\
&+ \frac{1024}{3rs}|T_1|^2 |C_{TE}|^2 m_B^2 \Big(8m_\ell^2\,[\lambda(4r+s) + 12rs(2+2r-s)] \\
&+ m_B^2 s\,[\lambda(16r+s) + 12rs(2+2r-s)]\Big) \\
&- \frac{128}{r} m_B^2 m_\ell\,[\lambda + 12r(1-r)]\,\mathrm{Re}[(B_1+D_1)(T_1 C_{TE})^*] \\
&+ \frac{128}{r} m_B^4 m_\ell \lambda(1+3r-s)\,\mathrm{Re}[(B_2+D_2)(T_1 C_{TE})^*] \\
&+ 512 m_B^4 m_\ell \lambda\, \mathrm{Re}[(A_1+C_1)(T_1 C_T)^*] \\
&+ \frac{16}{3r} m_B^2 \Big(4(m_B^2 s + 8m_\ell^2)|C_{TE}|^2 + m_B^2 s v^2 |C_T|^2\Big) \times \Big(4(\lambda+12rs)|B_6|^2 \\
&+ m_B^4 \lambda^2 |B_7|^2 - 4m_B^2(1-r-s)\lambda\,\mathrm{Re}(B_6 B_7^*) - 16\,[\lambda + 12r(1-r)]\,\mathrm{Re}(T_1 B_6^*) \\
&+ 8m_B^2(1+3r-s)\lambda\,\mathrm{Re}(T_1 B_7^*)\Big)\Bigg\}\,, \qquad \text{(E.2)}
\end{aligned}
$$

where $\lambda(1,r,s) = 1 + r^2 + s^2 - 2r - 2s - 2rs$, $s = q^2/m_B^2$, $r = m_{K^*}^2/m_B^2$ and $v = \sqrt{1 - 4m_l^2/q^2}$ is the lepton velocity.

E.1 Expressions for Lepton Polarization Asymmetries in $B \to K^* \ell^+ \ell^-$ Decay

In the calculations we presented in the text, we neglected the mass of the leptons. In this section, we would like to include the mass of leptons in our calculations.

$$
\begin{aligned}
|\mathcal{M}_\pm|^2 &= \left|\frac{G_F \alpha}{2\sqrt{2}\pi}\right|^2 \left|V_{tb}V_{ts}^*\right|^2 \Big\{ \\
& \times\Big| -2A_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_{K^*}^{\lambda} q^{\sigma} - iB_1 \varepsilon_\mu^* + iB_2(\varepsilon^*.q)(p_B + p_{K^*})_\mu + iB_3(\varepsilon^*.q) q_\mu \Big|^2 \\
& \times Tr[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})\gamma_\mu(1-\gamma_5)(\not{p}_2 - m_l)\gamma_\alpha(1-\gamma_5)] \\
&+ [-2A_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_{K^*}^{\lambda} q^{\sigma} - iB_1 \varepsilon_\mu^* + iB_2(\varepsilon^*.q)(p_B + p_{K^*})_\mu + iB_3(\varepsilon^*.q) q_\mu] \\
& \times[-2C_1^* \epsilon_{\alpha\beta\rho\delta} \varepsilon^{*\beta} p_{K^*}^{\rho} q^{\delta} + iD_1^* \varepsilon_\alpha^* - iD_2^*(\varepsilon.q)(p_B + p_{K^*})_\alpha - iD_3^*(\varepsilon.q) q_\alpha] \\
& \times Tr[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})\gamma_\mu(1-\gamma_5)(\not{p}_2 - m_l)\gamma_\alpha(1+\gamma_5)] \\
&+ [-2C_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_{K^*}^{\lambda} q^{\sigma} - iD_1 \varepsilon_\mu^* + iD_2(\varepsilon^*.q)(p_B + p_{K^*})_\mu + iD_3(\varepsilon^*.q) q_\mu] \\
& \times[-2A_1^* \epsilon_{\alpha\beta\rho\delta} \varepsilon^{*\beta} p_{K^*}^{\rho} q^{\delta} + iB_1^* \varepsilon_\alpha^* - iB_2^*(\varepsilon.q)(p_B + p_{K^*})_\alpha - iB_3^*(\varepsilon.q) q_\alpha] \\
& \times Tr[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})\gamma_\mu(1+\gamma_5)(\not{p}_2 - m_l)\gamma_\alpha(1-\gamma_5)] \\
&+ [-2A_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_{K^*}^{\lambda} q^{\sigma} - iB_1 \varepsilon_\mu^* + iB_2(\varepsilon^*.q)(p_B + p_{K^*})_\mu + iB_3(\varepsilon^*.q) q_\mu] \\
& \times[-iB_4^*(\varepsilon.q)] \times Tr[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})\gamma_\mu(1-\gamma_5)(\not{p}_2 - m_l)(1-\gamma_5)] \\
&+ [-2A_1^* \epsilon_{\alpha\beta\rho\delta} \varepsilon^{*\beta} p_{K^*}^{\rho} q^{\delta} + iB_1^* \varepsilon_\alpha^* - iB_2^*(\varepsilon.q)(p_B + p_{K^*})_\alpha - iB_3^*(\varepsilon.q) q_\alpha] \\
& \times[iB_4(\varepsilon^*.q)] \times Tr[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})(1-\gamma_5)(\not{p}_2 - m_l)\gamma_\alpha(1-\gamma_5)] \\
&+ [-2A_1 \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\nu} p_{K^*}^{\lambda} q^{\sigma} - iB_1 \varepsilon_\mu^* + iB_2(\varepsilon^*.q)(p_B + p_{K^*})_\mu + iB_3(\varepsilon^*.q) q_\mu] \\
& \times[-iB_5^*(\varepsilon.q)] \times Tr[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})\gamma_\mu(1-\gamma_5)(\not{p}_2 - m_l)(1+\gamma_5)] \\
&+ [-2A_1^* \epsilon_{\alpha\beta\rho\delta} \varepsilon^{*\beta} p_{K^*}^{\rho} q^{\delta} + iB_1^* \varepsilon_\alpha^* - iB_2^*(\varepsilon.q)(p_B + p_{K^*})_\alpha - iB_3^*(\varepsilon.q) q_\alpha] \\
& \times[iB_5(\varepsilon^*.q)] \times Tr[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})(1+\gamma_5)(\not{p}_2 - m_l)\gamma_\alpha(1-\gamma_5)]
\end{aligned}
$$

$$
\begin{aligned}
+ \quad & [-2A_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^{\lambda}_{K^*}q^{\sigma} - iB_1\varepsilon^*_{\mu} + iB_2(\varepsilon^*.q)(p_B+p_{K^*})_{\mu} + iB_3(\varepsilon^*.q)q_{\mu}] \\
& \times[-4iC^*_T\epsilon_{\alpha\beta\rho\delta}\Big[-2T^*_1\varepsilon^{\rho}(p_B+p_{K^*})^{\delta} + B^*_6\varepsilon^{\rho}q^{\delta} - B^*_7(\varepsilon.q)p^{\rho}_{K^*}q^{\delta}\Big]] \\
& \times Tr[(\not p_1 + m_l)(1-\gamma_5 \not S)\gamma_{\mu}(1-\gamma_5)(\not p_2 - m_l)\sigma_{\alpha\beta}] \\
+ \quad & [4iC_T\epsilon_{\mu\nu\lambda\sigma}\Big[-2T_1\varepsilon^{*\lambda}(p_B+p_{K^*})^{\sigma} + B_6\varepsilon^{*\lambda}q^{\sigma} - B_7(\varepsilon^*.q)p^{\lambda}_{K^*}q^{\sigma}\Big]] \\
& \times[-2A^*_1\epsilon_{\alpha\beta\rho\delta}\varepsilon^{*\beta}p^{\rho}_{K^*}q^{\delta} + iB^*_1\varepsilon^*_{\alpha} - iB^*_2(\varepsilon.q)(p_B+p_{K^*})_{\alpha} - iB^*_3(\varepsilon.q)q_{\alpha}] \\
& \times Tr[(\not p_1 + m_l)(1-\gamma_5 \not S)\sigma_{\mu\nu}(\not p_2 - m_l)\gamma_{\alpha}(1-\gamma_5)] \\
+ \quad & [-2A_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^{\lambda}_{K^*}q^{\sigma} - iB_1\varepsilon^*_{\mu} + iB_2(\varepsilon^*.q)(p_B+p_{K^*})_{\mu} + iB_3(\varepsilon^*.q)q_{\mu}] \\
& \times[16C^*_{TE}\Big[-2T^*_1\varepsilon^{*\alpha}(p_B+p_{K^*})^{\beta} + B^*_6\varepsilon^{\alpha}q^{\beta} - B^*_7(\varepsilon.q)p^{\alpha}_{K^*}q^{\beta}\Big]] \\
& \times Tr[(\not p_1 + m_l)(1-\gamma_5 \not S)\gamma_{\mu}(1-\gamma_5)(\not p_2 - m_l)\sigma_{\alpha\beta}] \\
+ \quad & [16C_{TE}\Big[-2T_1\varepsilon^{*\mu}(p_B+p_{K^*})^{\nu} + B_6\varepsilon^{*\mu}q^{\nu} - B_7(\varepsilon^*.q)p^{\mu}_{K^*}q^{\nu}\Big]] \\
& \times[-2A^*_1\epsilon_{\alpha\beta\rho\delta}\varepsilon^{*\beta}p^{\rho}_{K^*}q^{\delta} + iB^*_1\varepsilon^*_{\alpha} - iB^*_2(\varepsilon.q)(p_B+p_{K^*})_{\alpha} - iB^*_3(\varepsilon.q)q_{\alpha}] \\
& \times Tr[(\not p_1 + m_l)(1-\gamma_5 \not S)\sigma_{\mu\nu}(\not p_2 - m_l)\gamma_{\alpha}(1-\gamma_5)] \\
+ \quad & \Big|-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^{\lambda}_{K^*}q^{\sigma} - iD_1\varepsilon^*_{\mu} + iD_2(\varepsilon^*.q)(p_B+p_{K^*})_{\mu} + iD_3(\varepsilon^*.q)q_{\mu}\Big|^2 \\
& \times Tr[(\not p_1 + m_l)(1-\gamma_5 \not S)\gamma_{\mu}(1+\gamma_5)(\not p_2 - m_l)\gamma_{\alpha}(1+\gamma_5)] \\
+ \quad & \Big[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^{\lambda}_{K^*}q^{\sigma} - iD_1\varepsilon^*_{\mu} + iD_2(\varepsilon^*.q)(p_B+p_{K^*})_{\mu} + iD_3(\varepsilon^*.q)q_{\mu}\Big] \\
& \times\Big[-iB^*_4(\varepsilon.q)\Big] \times Tr\Big[(\not p_1 + m_l)(1-\gamma_5 \not S)\gamma_{\mu}(1+\gamma_5)(\not p_2 - m_l)(1-\gamma_5)\Big] \\
+ \quad & \Big[-2C^*_1\epsilon_{\alpha\beta\rho\delta}\varepsilon^{*\beta}p^{\rho}_{K^*}q^{\delta} + iD^*_1\varepsilon^*_{\alpha} - iD^*_2(\varepsilon.q)(p_B+p_{K^*})_{\alpha} - iD^*_3(\varepsilon.q)q_{\alpha}\Big] \\
& \times\Big[iB_4(\varepsilon^*.q)\Big] \times Tr\Big[(\not p_1 + m_l)(1-\gamma_5 \not S)(1-\gamma_5)(\not p_2 - m_l)\gamma_{\alpha}(1+\gamma_5)\Big] \\
+ \quad & \Big[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^{\lambda}_{K^*}q^{\sigma} - iD_1\varepsilon^*_{\mu} + iD_2(\varepsilon^*.q)(p_B+p_{K^*})_{\mu} + iD_3(\varepsilon^*.q)q_{\mu}\Big] \\
& \times\Big[-iB^*_5(\varepsilon.q)\Big] \times Tr\Big[(\not p_1 + m_l)(1-\gamma_5 \not S)\gamma_{\mu}(1+\gamma_5)(\not p_2 - m_l)(1+\gamma_5)\Big] \\
+ \quad & \Big[-2C^*_1\epsilon_{\alpha\beta\rho\delta}\varepsilon^{*\beta}p^{\rho}_{K^*}q^{\delta} + iD^*_1\varepsilon^*_{\alpha} - iD^*_2(\varepsilon.q)(p_B+p_{K^*})_{\alpha} - iD^*_3(\varepsilon.q)q_{\alpha}\Big]
\end{aligned}
$$

$$
\begin{aligned}
& \times\Big[iB_5(\varepsilon^*.q)\Big] \times Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1+\gamma_5)(\not p_2-m_l)\gamma_\alpha(1+\gamma_5)\Big] \\
+ & \Big[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^\lambda_{K^*}q^\sigma - iD_1\varepsilon^*_\mu + iD_2(\varepsilon^*.q)(p_B+p_{K^*})_\mu + iD_3(\varepsilon^*.q)q_\mu\Big] \\
& \times\Big[4iC_T\epsilon_{\mu\nu\lambda\sigma}\Big[-2T_1\varepsilon^{*\lambda}(p_B+p_{K^*})^\sigma + B_6\varepsilon^{*\lambda}q^\sigma - B_7(\varepsilon^*.q)p^\lambda_{K^*}q^\sigma\Big]\Big] \\
& \times Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\gamma_\mu(1+\gamma_5)(\not p_2-m_l)\sigma_{\alpha\beta}\Big] \\
+ & \Big[4iC_T\epsilon_{\mu\nu\lambda\sigma}\Big[-2T_1\varepsilon^{*\lambda}(p_B+p_{K^*})^\sigma + B_6\varepsilon^{*\lambda}q^\sigma - B_7(\varepsilon^*.q)p^\lambda_{K^*}q^\sigma\Big]\Big] \\
& \times\Big[-2C_1^*\epsilon_{\alpha\beta\rho\delta}\varepsilon^{*\beta}p^\rho_{K^*}q^\delta + iD_1^*\varepsilon^*_\alpha - iD_2^*(\varepsilon.q)(p_B+p_{K^*})_\alpha - iD_3^*(\varepsilon.q)q_\alpha\Big] \\
& \times Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\sigma_{\mu\nu}(\not p_2-m_l)\gamma_\alpha(1+\gamma_5)\Big] \\
+ & \Big[-2C_1\epsilon_{\mu\nu\lambda\sigma}\varepsilon^{*\nu}p^\lambda_{K^*}q^\sigma - iD_1\varepsilon^*_\mu + iD_2(\varepsilon^*.q)(p_B+p_{K^*})_\mu + iD_3(\varepsilon^*.q)q_\mu\Big] \\
& \times\Big[16C^*_{TE}\Big[-2T_1^*\varepsilon^{*\alpha}(p_B+p_{K^*})^\beta + B_6^*\varepsilon^\alpha q^\beta - B_7^*(\varepsilon.q)p^\alpha_{K^*}q^\beta\Big]\Big] \\
& \times Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\gamma_\mu(1+\gamma_5)(\not p_2-m_l)\sigma_{\alpha\beta}\Big] \\
+ & \Big[16C_{TE}\Big[-2T_1\varepsilon^{*\mu}(p_B+p_{K^*})^\nu + B_6\varepsilon^{*\mu}q^\nu - B_7(\varepsilon^*.q)p^\mu_{K^*}q^\nu\Big]\Big] \\
& \times\Big[-2C_1^*\epsilon_{\alpha\beta\rho\delta}\varepsilon^{*\beta}p^\rho_{K^*}q^\delta + iD_1^*\varepsilon^*_\alpha - iD_2^*(\varepsilon.q)(p_B+p_{K^*})_\alpha - iD_3^*(\varepsilon.q)q_\alpha\Big] \\
& \times Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\sigma_{\mu\nu}(\not p_2-m_l)\gamma_\alpha(1+\gamma_5)\Big] \\
+ & \Big|iB_4(\varepsilon^*.q)\Big|^2 \times Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1-\gamma_5)(\not p_2-m_l)(1-\gamma_5)\Big] \\
+ & \Big[iB_4(\varepsilon^*.q)\Big]\Big[-iB_5^*(\varepsilon.q)\Big] \\
& Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1-\gamma_5)(\not p_2-m_l)(1+\gamma_5)\Big] \\
+ & \Big[iB_5(\varepsilon^*.q)]\Big]\Big[-iB_4^*(\varepsilon.q)\Big] \\
& Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1+\gamma_5)(\not p_2-m_l)(1-\gamma_5)\Big] \\
+ & \Big[-4iC_T^*\epsilon_{\alpha\beta\rho\delta}\Big[-2T_1^*\varepsilon^\rho(p_B+p_{K^*})^\delta + B_6^*\varepsilon^\rho q^\delta - B_7^*(\varepsilon.q)p^\rho_{K^*}q^\delta\Big]\Big] \\
& \times\Big[iB_4(\varepsilon^*.q)\Big]Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1-\gamma_5)(\not p_2-m_l)\sigma_{\alpha\beta}\Big] \\
+ & \Big[4iC_T\epsilon_{\mu\nu\lambda\sigma}\Big[-2T_1\varepsilon^{*\lambda}(p_B+p_{K^*})^\sigma + B_6\varepsilon^{*\lambda}q^\sigma - B_7(\varepsilon^*.q)p^\lambda_{K^*}q^\sigma\Big]\Big]
\end{aligned}
$$

$$
\begin{aligned}
& \Big[-iB_4^*(\varepsilon.q)\Big]Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\sigma_{\mu\nu}(\not p_2-m_l)(1-\gamma_5)\Big] \\
+ & \Big[16C_{TE}^*\Big[-2T_1^*\varepsilon^{*\alpha}(p_B+p_{K^*})^\beta+B_6^*\varepsilon^\alpha q^\beta-B_7^*(\varepsilon.q)p_{K^*}^\alpha q^\beta\Big]\Big] \\
& \Big[iB_4(\varepsilon^*.q)\Big]Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1-\gamma_5)(\not p_2-m_l)\sigma_{\alpha\beta}\Big] \\
+ & \Big[16C_{TE}\Big[-2T_1\varepsilon^{*\mu}(p_B+p_{K^*})^\nu+B_6\varepsilon^{*\mu}q^\nu-B_7(\varepsilon^*.q)p_{K^*}^\mu q^\nu\Big]\Big] \\
& \Big[-iB_4^*(\varepsilon.q)\Big]Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\sigma_{\mu\nu}(\not p_2-m_l)(1-\gamma_5)\Big] \\
+ & \Big|iB_5(\varepsilon^*.q)\Big|^2Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1+\gamma_5)(\not p_2-m_l)(1+\gamma_5)\Big] \\
+ & \Big[-4iC_T^*\epsilon_{\alpha\beta\rho\delta}\Big[-2T_1^*\varepsilon^\rho(p_B+p_{K^*})^\delta+B_6^*\varepsilon^\rho q^\delta-B_7^*(\varepsilon.q)p_{K^*}^\rho q^\delta\Big]\Big] \\
& \times\Big[iB_5(\varepsilon^*.q)\Big]Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1+\gamma_5)(\not p_2-m_l)\sigma_{\alpha\beta}\Big] \\
+ & \Big[4iC_T\epsilon_{\mu\nu\lambda\sigma}\Big[-2T_1\varepsilon^{*\lambda}(p_B+p_{K^*})^\sigma+B_6\varepsilon^{*\lambda}q^\sigma-B_7(\varepsilon^*.q)p_{K^*}^\lambda q^\sigma\Big]\Big] \\
& \times\Big[-iB_5^*(\varepsilon.q)\Big]Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\sigma_{\mu\nu}(\not p_2-m_l)(1+\gamma_5)\Big] \\
+ & \Big[16C_{TE}^*\Big[-2T_1^*\varepsilon^{*\alpha}(p_B+p_{K^*})^\beta+B_6^*\varepsilon^\alpha q^\beta-B_7^*(\varepsilon.q)p_{K^*}^\alpha q^\beta\Big]\Big] \\
& \times\Big[iB_5(\varepsilon^*.q)\Big]Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)(1+\gamma_5)(\not p_2-m_l)\sigma_{\alpha\beta}\Big] \\
+ & \Big[16C_{TE}\Big[-2T_1\varepsilon^{*\mu}(p_B+p_{K^*})^\nu+B_6\varepsilon^{*\mu}q^\nu-B_7(\varepsilon^*.q)p_{K^*}^\mu q^\nu\Big]\Big] \\
& \times\Big[-iB_5^*(\varepsilon.q)\Big]Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\sigma_{\mu\nu}(\not p_2-m_l)(1+\gamma_5)\Big] \\
+ & \Big|4iC_T\epsilon_{\mu\nu\lambda\sigma}\Big[-2T_1\varepsilon^{*\lambda}(p_B+p_{K^*})^\sigma+B_6\varepsilon^{*\lambda}q^\sigma-B_7(\varepsilon^*.q)p_{K^*}^\lambda q^\sigma\Big]\Big|^2 \\
& \times Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\sigma_{\mu\nu}(\not p_2-m_l)\sigma_{\alpha\beta}\Big] \\
+ & \Big[4iC_T\epsilon_{\mu\nu\lambda\sigma}\Big[-2T_1\varepsilon^{*\lambda}(p_B+p_{K^*})^\sigma+B_6\varepsilon^{*\lambda}q^\sigma-B_7(\varepsilon^*.q)p_{K^*}^\lambda q^\sigma\Big]\Big] \\
& \times\Big[16C_{TE}^*\Big[-2T_1^*\varepsilon^{*\alpha}(p_B+p_{K^*})^\beta+B_6^*\varepsilon^\alpha q^\beta-B_7^*(\varepsilon.q)p_{K^*}^\alpha q^\beta\Big]\Big] \\
& \times Tr\Big[(\not p_1+m_l)(1-\gamma_5\ \not S)\sigma_{\mu\nu}(\not p_2-m_l)\sigma_{\alpha\beta}\Big] \\
+ & \Big[-4iC_T^*\epsilon_{\alpha\beta\rho\delta}\Big[-2T_1^*\varepsilon^\rho(p_B+p_{K^*})^\delta+B_6^*\varepsilon^\rho q^\delta-B_7^*(\varepsilon.q)p_{K^*}^\rho q^\delta\Big]\Big] \\
& \times\Big[16C_{TE}\Big[-2T_1\varepsilon^{*\mu}(p_B+p_{K^*})^\nu+B_6\varepsilon^{*\mu}q^\nu-B_7(\varepsilon^*.q)p_{K^*}^\mu q^\nu\Big]\Big]
\end{aligned}
$$

$$\times Tr\Big[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})\sigma_{\mu\nu}(\not{p}_2 - m_l)\sigma_{\alpha\beta}\Big]$$

$$+ \quad \Big|16C_{TE}\Big[-2T_1\varepsilon^{*\mu}(p_B+p_{K^*})^\nu + B_6\varepsilon^{*\mu}q^\nu - B_7(\varepsilon^*.q)p^\mu_{K^*}q^\nu\Big]\Big|^2$$

$$Tr\Big[(\not{p}_1 + m_l)(1-\gamma_5 \not{S})\sigma_{\mu\nu}(\not{p}_2 - m_l)\sigma_{\alpha\beta}\Big], \qquad \text{(E.3)}$$

*****

Using the trace theorems and performing integration over final lepton momenta we find the expression for the decay rate, when leptons are longitudinally polarized as,

$$\begin{aligned}
\frac{d\Gamma(\vec{e}^{\mp}_L)}{dq^2} &= \frac{G_F^2\alpha^2}{2^{14}\pi^5 m_B}\Big|V_{tb}V_{ts}^*\Big|^2 \times \Bigg\{\Big[|A_1|^2 - |C_1|^2 + 2Re(A_1C_1^*)\Big]\Big[\pm\frac{32}{3}\lambda m_B^6 sv\Big] \\
&+ \Big[\mp|B_1|^2 \mp |D_1|^2 + 2Re(B_1D_1^*)\Big]\Big[-\frac{4}{3r}\lambda m_B^2 v - 16m_B^2 sv\Big] \\
&+ \Big[\pm 2Re(B_1B_2^*) + 2Re(B_1D_2^*) + 2Re(B_2D_1^*) \pm 2Re(D_1D_2^*)\Big] \\
&\times\Big[-\frac{4}{3}\lambda m_B^4 v + \frac{4}{3r}\lambda m_B^4 v - \frac{4}{3r}\lambda m_B^4 sv\Big] \\
&+ \Big[\pm|B_2|^2 \mp |D_2|^2 + 2Re(B_2D_2^*)\Big]\Big[-\frac{4}{3r}\lambda^2 m_B^6 v\Big] \\
&+ \Big[2Re(A_1T_1^*C_{TE}^*) + 2Re(C_1T_1^*C_{TE}^*)\Big]\frac{64}{3m_l}\Big[m_B^2 s(-1+v) \\
&+4m_l^2(-3+s)v\Big] \\
&+ \Big[\pm 2Re(A_1T_1^*C_T^*) \mp 2Re(C_1T_1^*C_T^*)\Big]\frac{32}{3m_l}\Big[m_B^2 s(-1+v) \\
&+4m_l^2(6+(3+s)v)\Big] \\
&+ \Big[\pm 2Re(B_1T_1^*C_{TE}^*) \pm 2Re(D_1T_1^*C_{TE}^*)\Big]\frac{16}{3m_l r}\Big[48m_l^2(-1+r)r(3+2v) \\
&+\lambda(m_B^2 s(-1+v) + 4m_l^2(-3+(-3+s)v)) \\
&+ \Big[2Re(B_1T_1^*C_T^*) + 2Re(D_1T_1^*C_T^*)\Big]\frac{8}{3m_l r}\Big[-12m_l^2(-1+r)r(-3+4v) \\
&+\lambda(m_B^2 s(-1+v) + m_l^2(-3+4sv))\Big] \\
&+ \Big[\pm 2Re(B_2T_1^*C_{TE}^*) \pm 2Re(D_2T_1^*C_{TE}^*)\Big]\frac{-16(1+3r-s)\lambda}{3m_l r}
\end{aligned}$$

$$\times\Big[m_B^2 s(-1+v) + 4m_l^2(-3(-3+s)v)\Big]$$

$$+ \quad \Big[2Re(B_2 T_1^* C_T^*) + 2Re(D_2 T_1^* C_T^*)\Big]\frac{-8(1+3r-s)\lambda}{3m_l r}$$

$$\times\Big[m_B^2 s(-1+v) + m_l^2(-3+4sv)\Big]$$

$$+ \quad \Big[\pm 2Re(B_1 B_6^* C_{TE}^*) \pm 2Re(D_1 B_6^* C_{TE}^*)\Big]\frac{-8m_B^2}{3m_l r}\Big[-48m_l^2 rs(3+2v)$$

$$+\lambda(m_B^2 s(-1+v)) + 4m_l^2(-3+(-3+s))\Big]$$

$$+ \quad \Big[2Re(B_1 B_6^* C_T^*) + 2Re(D_1 B_6^* C_T^*)\Big]\frac{-4m_B^2}{3m_l r}\Big[12m_l^2 rs(-3+4v)$$

$$+\lambda(m_B^2 s(-1+v) + m_l^2(-3+4sv))\Big]$$

$$+ \quad \Big[\pm 2Re(B_2 B_6^* C_{TE}^*) \pm 2Re(D_2 B_6^* C_{TE}^*)\Big]\frac{-8\lambda(-1+r+s)}{3m_l r}$$

$$\Big[m_B^2 s(-1+v) + 4m_l^2(-3(-3+s)v)\Big]$$

$$+ \quad \Big[2Re(B_2 B_6^* C_T^*) + 2Re(D_2 B_6^* C_T^*)\Big]\frac{-4\lambda(-1+r+s)}{3m_l r}$$

$$\times\Big[m_B^2 s(-1+v) + m_l^2(-3+4sv)\Big]$$

$$+ \quad \Big[\pm 2Re(B_1 B_7^* C_{TE}^*) \pm 2Re(D_1 B_7^* C_{TE}^*)\Big]\frac{-4\lambda m_B^4(-1+r+s)}{3m_l r}$$

$$\Big[m_B^2 s(-1+v) + 4m_l^2(-3(-3+s)v)\Big]$$

$$+ \quad \Big[2Re(B_1 B_7^* C_T^*) + 2Re(D_1 B_7^* C_T^*)\Big]\frac{-2\lambda m_B^4(-1+r+s)}{3m_l r}$$

$$\times\Big[m_B^2 s(-1+v) + m_l^2(-3+4sv)\Big]$$

$$+ \quad \Big[\pm 2Re(B_2 B_7^* C_{TE}^*) \pm 2Re(D_2 B_7^* C_{TE}^*)\Big]\frac{-4\lambda^2 m_B^6}{3m_l r}\Big[m_B^2 s(-1+v)$$

$$+4m_l^2(-3+(-3+s)v)\Big]$$

$$+ \quad \Big[2Re(B_2 B_7^* C_T^*) + 2Re(D_2 B_7^* C_T^*)\Big]\frac{-2\lambda^2 m_B^6}{3m_l r}\Big[m_B^2 s(-1+v)$$

$$+m_l^2(-3+4sv)\Big]$$

$$+ \quad \Big[2Re((B_1 - D_1)B_4^*)\Big]\frac{2}{r}\lambda m_B^2 m_l v$$

$$+ \quad \Big[2Re(B_1 - D_1)B_5^*)\Big]\frac{2m_B^2\lambda m_l}{r}(-2+v)$$

$$
\begin{aligned}
+ & \Big[2Re((B_2-D_2)B_4^*)\Big]\frac{2\lambda m_B^4 m_l v}{r}(-1+r) \\
- & \Big[2Re((B_3-D_3)B_4^*)\Big]\frac{\lambda m_B^4 m_l s v}{r} \\
+ & \Big[2Re((B_2-D_2)B_5^*)\Big]\frac{2\lambda m_B^4 m_l}{r}(2-2r-v-vr) \\
+ & \Big[2Re((B_3-D_3)B_5^*)\Big]\frac{2\lambda m_B^4 m_l s}{r}(2-v) \\
+ & \Big[|B_4|^2-|B_5|^2\Big]\frac{2\lambda m_B^2}{r}\Big[-2m_l^2+m_B^2 s(1+v)\Big] \\
- & 2Re(B_4 B_5^*)\frac{-4\lambda m_B^2 m_l^2}{r} \\
+ & \Big[|B_6|^2\frac{\lambda+12rs}{3r}+|B_7|^2\frac{16\lambda^2 m_B^6}{3r}\Big]\Big[\pm|C_T|^2(-4m_l^2+m_B^2 s) \\
& \mp 4|C_{TE}|^2(8m_l^2+m_B^2 s)+2Re(C_T C_{TE}^*)(-2m_B^2 s v)\Big] \\
+ & |T_1|^2\frac{256 m_B^2}{3rs}\Big[\mp 4|C_{TE}|^2\Big(12r(2+2r-s)s(8m_l^2+m_B^2 s)) \\
& +\lambda(8m_l^2(4r+s)+m_B^2 s(16r+s)\Big) \\
& \pm|C_T|^2\Big(12r(2+2r-s)s(-4m_l^2+m_B^2 s) \\
& +\lambda(4m_l^2(8r-s)+m_B^2 s(16r+s))\Big) \\
& +2Re(C_T C_{TE}^*)\Big(m_B^2 s(12rs(-2-2r+s)-\lambda(16r+s))v\Big) \\
+ & 2Re(B_6 B_7^*)\frac{32\lambda m_B^4(-1+r+s)}{3r}\Big[\pm|C_T|^2(-4m_l^2+m_B^2 s) \\
& \mp 4|C_{TE}|^2(8m_l^2+m_B^2 s)-2Re(C_T C_{TE}^*)(-2m_B^2 s v)\Big] \\
+ & 2Re(B_6 T_1^*)\frac{-128 m_B^2(\lambda-12(-1+r)r)}{3r}\Big[\pm|C_T|^2(-4m_l^2+m_B^2 s) \\
& \mp 4|C_{TE}|^2(8m_l^2+m_B^2 s)-2Re(C_T C_{TE}^*)(-2m_B^2 s v)\Big] \\
+ & 2Re(B_7 T_1^*)\frac{64\lambda m_B^4(1+3r-s)}{3r}\Big[\pm|C_T|^2(-4m_l^2+m_B^2 s) \\
& \mp 4|C_{TE}|^2(8m_l^2+m_B^2 s)-2Re(C_T C_{TE}^*)(-2m_B^2 s v)\Big]\Bigg\}\,, \qquad \text{(E.4)}
\end{aligned}
$$

where the superscripts (+) and (−) in $\vec{e}_L^{\,\pm}$ is to represent the decay rate expression when the leptons ℓ^+ and ℓ^- are polarized longitudinally, respectively. We find the longitudinal polarization of the ℓ^- for massive leptons as,

$$
\begin{aligned}
P_L^- &= \frac{4}{\Delta} m_B^2 v \Bigg\{ \frac{1}{3r}\lambda^2 m_B^4 \Big[\left|B_2\right|^2 - \left|D_2\right|^2 \Big] + \frac{1}{r}\lambda m_\ell \, \mathrm{Re}[(B_1 - D_1)(B_4^* + B_5^*)] \\
&- \frac{1}{r}\lambda m_B^2 m_\ell (1-r)\, \mathrm{Re}[(B_2 - D_2)(B_4^* + B_5^*)] + \frac{8}{3}\lambda m_B^4 s \Big[\left|A_1\right|^2 - \left|C_1\right|^2 \Big] \\
&- \frac{1}{2r}\lambda m_B^2 s \Big[\left|B_4\right|^2 - \left|B_5\right|^2 \Big] - \frac{1}{r}\lambda m_B^2 m_\ell s \, \mathrm{Re}[(B_3 - D_3)(B_4^* + B_5^*)] \\
&- \frac{2}{3r}\lambda m_B^2 (1-r-s) \Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*) \Big] \\
&+ \frac{1}{3r}(\lambda + 12 r s) \Big[\left|B_1\right|^2 - \left|D_1\right|^2 \Big] \\
&+ \frac{256}{3}\lambda m_B^2 m_\ell \Big(\mathrm{Re}[A_1^*(C_T + C_{TE}) T_1] - \mathrm{Re}[C_1^*(C_T - C_{TE}) T_1] \Big) \\
&+ \frac{4}{3r}\lambda^2 m_B^4 m_\ell \Big(\mathrm{Re}[B_2^*(C_T + 4C_{TE}) B_7] + \mathrm{Re}[D_2^*(C_T - 4C_{TE}) B_7] \Big) \\
&- \frac{8}{3r}\lambda m_B^2 m_\ell (1-r-s) \Big(\mathrm{Re}[B_2^*(C_T + 4C_{TE}) B_6] + \mathrm{Re}[D_2^*(C_T - 4C_{TE}) B_6] \Big) \\
&- \frac{4}{3r}\lambda m_B^2 m_\ell (1-r-s) \Big(\mathrm{Re}[B_1^*(C_T + 4C_{TE}) B_7] + \mathrm{Re}[D_1^*(C_T - 4C_{TE}) B_7] \Big) \\
&+ \frac{8}{3r}(\lambda + 12 r s) m_\ell \Big(\mathrm{Re}[B_1^*(C_T + 4C_{TE}) B_6] + \mathrm{Re}[D_1^*(C_T - 4C_{TE}) B_6] \Big) \\
&- \frac{16}{3r} m_\ell [\lambda + 12 r (1-r)] \Big(\mathrm{Re}[B_1^*(C_T + 4C_{TE}) T_1] + \mathrm{Re}[D_1^*(C_T - 4C_{TE}) T_1] \Big) \\
&+ \frac{16}{3r}\lambda m_B^2 m_\ell (1+3r-s) \Big(\mathrm{Re}[B_2^*(C_T + 4C_{TE}) T_1] + \mathrm{Re}[D_2^*(C_T - 4C_{TE}) T_1] \Big) \\
&+ \frac{16}{3r}\lambda^2 m_B^6 s \left|B_7\right|^2 \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{64}{3r}(\lambda + 12 r s) m_B^2 s \left|B_6\right|^2 \mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{64}{3r}\lambda m_B^4 s (1-r-s)\, \mathrm{Re}(B_6 B_7^*) \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{128}{3r}\lambda m_B^4 s (1+3r-s)\, \mathrm{Re}(B_7 T_1^*) \mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{256}{3r} m_B^2 s [\lambda + 12 r (1-r)]\, \mathrm{Re}(B_6 T_1^*) \mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{256}{3r} m_B^2 [\lambda (4r + s) + 12 r (1-r)^2] \left|T_1\right|^2 \, \mathrm{Re}(C_T C_{TE}^*) \Bigg\} . \qquad \text{(E.5)}
\end{aligned}
$$

Similarly, the longitudinal polarization of the ℓ^+ for massive leptons becomes,

$$
P_L^+ = \frac{4}{\Delta} m_B^2 v \Bigg\{ -\frac{1}{3r}\lambda^2 m_B^4 \Big[\left|B_2\right|^2 - \left|D_2\right|^2 \Big] + \frac{1}{r}\lambda m_\ell \, \mathrm{Re}[(B_1 - D_1)(B_4^* + B_5^*)]
$$

$$
\begin{aligned}
&- \frac{1}{r}\lambda m_B^2 m_\ell (1-r)\,\mathrm{Re}[(B_2-D_2)(B_4^*+B_5^*)] - \frac{8}{3}\lambda m_B^4 s\Big[\,|A_1|^2-|C_1|^2\Big] \\
&- \frac{1}{2r}\lambda m_B^2 s\Big[\,|B_4|^2-|B_5|^2\Big] - \frac{1}{r}\lambda m_B^2 m_\ell s\,\mathrm{Re}[(B_3-D_3)(B_4^*+B_5^*)] \\
&+ \frac{2}{3r}\lambda m_B^2(1-r-s)\Big[\mathrm{Re}(B_1B_2^*)-\mathrm{Re}(D_1D_2^*)\Big] - \frac{1}{3r}(\lambda+12rs)\Big[\,|B_1|^2-|D_1|^2\Big] \\
&- \frac{256}{3}\lambda m_B^2 m_\ell\Big(\mathrm{Re}[A_1^*(C_T-C_{TE})T_1]-\mathrm{Re}[C_1^*(C_T+C_{TE})T_1]\Big) \\
&+ \frac{4}{3r}\lambda^2 m_B^4 m_\ell\Big(\mathrm{Re}[B_2^*(C_T-4C_{TE})B_7]+\mathrm{Re}[D_2^*(C_T+4C_{TE})B_7]\Big) \\
&- \frac{8}{3r}\lambda m_B^2 m_\ell(1-r-s)\Big(\mathrm{Re}[B_2^*(C_T-4C_{TE})B_6]+\mathrm{Re}[D_2^*(C_T+4C_{TE})B_6]\Big) \\
&- \frac{4}{3r}\lambda m_B^2 m_\ell(1-r-s)\Big(\mathrm{Re}[B_1^*(C_T-4C_{TE})B_7]+\mathrm{Re}[D_1^*(C_T+4C_{TE})B_7]\Big) \\
&+ \frac{8}{3r}(\lambda+12rs)m_\ell\Big(\mathrm{Re}[B_1^*(C_T-4C_{TE})B_6]+\mathrm{Re}[D_1^*(C_T+4C_{TE})B_6]\Big) \\
&- \frac{16}{3r}m_\ell[\lambda+12r(1-r)]\Big(\mathrm{Re}[B_1^*(C_T-4C_{TE})T_1]+\mathrm{Re}[D_1^*(C_T+4C_{TE})T_1]\Big) \\
&+ \frac{16}{3r}\lambda m_B^2 m_\ell(1+3r-s)\Big(\mathrm{Re}[B_2^*(C_T-4C_{TE})T_1]+\mathrm{Re}[D_2^*(C_T+4C_{TE})T_1]\Big) \\
&+ \frac{16}{3r}\lambda^2 m_B^6 s\,|B_7|^2\,\mathrm{Re}(C_TC_{TE}^*) \\
&+ \frac{64}{3r}(\lambda+12rs)m_B^2 s\,|B_6|^2\,\mathrm{Re}(C_TC_{TE}^*) \\
&- \frac{64}{3r}\lambda m_B^4 s(1-r-s)\,\mathrm{Re}(B_6B_7^*)\mathrm{Re}(C_TC_{TE}^*) \\
&+ \frac{128}{3r}\lambda m_B^4 s(1+3r-s)\,\mathrm{Re}(B_7T_1^*)\mathrm{Re}(C_TC_{TE}^*) \\
&- \frac{256}{3r}m_B^2 s[\lambda+12r(1-r)]\,\mathrm{Re}(B_6T_1^*)\mathrm{Re}(C_TC_{TE}^*) \\
&+ \frac{256}{3r}m_B^2[\lambda(4r+s)+12r(1-r)^2]\,|T_1|^2\,\mathrm{Re}(C_TC_{TE}^*)\Bigg\}\,, \qquad \text{(E.6)}
\end{aligned}
$$

where the Δ, appearing in Eqs. (E.5) and (E.6) is the differential decay rate when the leptons are unpolarized and is presented in Eq. (E.2).

We also calculate the decay rate expression for transversally polarized lepton case and find,

$$
\frac{d\Gamma(\vec{e}_T^{\mp})}{dq^2} = \frac{G_F^2\alpha^2}{2^{14}\pi^5 m_B}\Big|V_{tb}V_{ts}^*\Big|^2 \times \Bigg\{\Big[|A_1|^2+|C_1|^2\Big]\Bigg(\frac{-32\,\lambda\,m_B^4\,m_l^2}{3}+\frac{32\,\lambda\,m_B^6\,s}{3}\Bigg)
$$

$$
\begin{aligned}
+ & \Big[Re(A_1 B_1^*) + Re(A_1 D_1^*) + Re(C_1 B_1^*) + Re(C_1 D_1^*)\Big] \times \\
& \Big(-8\sqrt{\lambda}\, m_B^3\, m_l\, \pi\, \sqrt{s}\Big) \\
+ & \Big[|B_1|^2 - |D_1|^2\Big] \Bigg(-16\, m_l{}^2 + \frac{4\,\lambda\, m_B{}^2}{3\, r} + \frac{8\,\lambda\, m_l{}^2}{3\, r\, s} + 16\, m_B{}^2\, s \\
& \pm \frac{\sqrt{\lambda}\, m_B\, m_l\, \pi\ (1-r-s)}{r\,\sqrt{s}}\Bigg) - 96 Re(B_1 D_1^*) m_l^2 \\
+ & \Big[Re(B_1 B_2^*) - Re(D_1 D_2^*)\Big] \Bigg(\frac{-4\,\lambda\, m_B{}^2\, m_l{}^2}{r} - \frac{4\,\lambda\, m_B{}^4\ (1-r-s)}{3\, r} \\
& - \frac{8\,\lambda\, m_B{}^2\, m_l{}^2\ (1-r-s)}{3\, r\, s} \pm \frac{\lambda^{\frac{3}{2}}\, m_B{}^3\, m_l\, \pi}{r\,\sqrt{s}} \pm \\
& 2\,\sqrt{\lambda}\, m_B{}^3\, m_l\, \pi\, \sqrt{s} \pm \frac{\sqrt{\lambda}\, m_B{}^3\, m_l\, \pi\ (1-r-s)\ \sqrt{s}}{2\, r}\Bigg) \\
+ & \Big[2Re(B_1 B_3) - 2Re(D_1 D_3^*) - 2Re(B_1 D_3^*) + 2Re(B_3 D_1^*)\Big] \\
& \Bigg(\frac{-4\,\lambda\, m_B{}^2\, m_l{}^2}{r} \mp \frac{\sqrt{\lambda}\, m_B{}^3\, m_l\, \pi\ (1-r-s)\ \sqrt{s}}{2\, r}\Bigg) \\
+ & \Big[2Re(B_1 D_2^*) - 2Re(B_2 D_1^*)\Big] \Bigg(\frac{4\,\lambda\, m_B{}^2\, m_l{}^2}{r} + \\
& \mp 2\,\sqrt{\lambda}\, m_B{}^3\, m_l\, \pi\, \sqrt{s} \mp \frac{\sqrt{\lambda}\, m_B{}^3\, m_l\, \pi\ (1-r-s)\ \sqrt{s}}{2\, r}\Bigg) \\
+ & \Big[|B_2|^2 - |D_2|^2\Big] \Bigg(16\,\lambda\, m_B{}^4\, m_l{}^2 + \frac{4\,\lambda^2\, m_B{}^6}{3\, r} + \frac{8\,\lambda\, m_B{}^4\, m_l{}^2\ (1-r-s)}{r} \\
& + \frac{8\,\lambda^2\, m_B{}^4\, m_l{}^2}{3\, r\, s} + \frac{4\,\lambda\, m_B{}^4\, m_l{}^2\, s}{r} \mp \frac{\lambda^{\frac{3}{2}}\, m_B{}^5\, m_l\, \pi\ (1-r-s)}{r\,\sqrt{s}} \\
& \mp \frac{\lambda^{\frac{3}{2}}\, m_B{}^5\, m_l\, \pi\, \sqrt{s}}{r}\Bigg) \\
+ & \Big[2Re(B_2 B_3^*) - 2Re(B_2 D_3^*) - 2Re(D_2 D_3^*) + 2Re(B_3 D_2^*)\Big] \times \\
& \Bigg(\frac{4\,\lambda\, m_B{}^4\, m_l{}^2\ (1-r-s)}{r} + \frac{4\,\lambda\, m_B{}^4\, m_l{}^2\, s}{r} \mp \frac{\lambda^{\frac{3}{2}}\, m_B{}^5\, m_l\, \pi\, \sqrt{s}}{2\, r}\Bigg) \\
+ & \Big[2Re(B_2 D_1^*) - 2Re(B_1 D_2^*)\Big] \times \\
& \Bigg(-16\,\lambda\, m_B{}^4\, m_l{}^2 - \frac{8\,\lambda\, m_B{}^4\, m_l{}^2\ (1-r-s)}{r} - \frac{4\,\lambda\, m_B{}^4\, m_l{}^2\, s}{r}\Bigg) \\
+ & \Big[|B_3|^2 - |D_3|^2 - 2Re(B_3 D_3^*)\Big] \frac{4\,\lambda\, m_B{}^4\, m_l{}^2\, s}{r} \\
+ & \Big[2Re(B_1 B_4^*) - 2Re(D_1 B_5^*)\Big] \Bigg(\frac{-2\,\lambda\, m_B{}^2\, m_l}{r}
\end{aligned}
$$

$$\pm\frac{\sqrt{\lambda}\,m_B\,{m_l}^2\,\pi\ (1-r-s)}{r\,\sqrt{s}} \mp \frac{\sqrt{\lambda}\,{m_B}^3\,\pi\ (1-r-s)\ \sqrt{s}}{2\,r}\Bigg)$$

$$+ \quad \Big[2Re(B_1B_5^*) - 2Re(D_1B_4^*)\Big]\left(\frac{2\,\lambda\,{m_B}^2\,m_l}{r} \pm \frac{\sqrt{\lambda}\,m_B\,{m_l}^2\,\pi\ (1-r-s)}{r\,\sqrt{s}}\right)$$

$$+ \quad \Big[2Re(D_2B_5^*) - 2Re(B_2B_4^*)\Big]\Bigg(\frac{2\,\lambda\,{m_B}^4\,m_l\ (1-r-s)}{r} + \frac{2\,\lambda\,{m_B}^4\,m_l\,s}{r}$$

$$\pm\frac{\lambda^{\frac{3}{2}}\,{m_B}^3\,{m_l}^2\,\pi}{r\,\sqrt{s}} \mp \frac{\lambda^{\frac{3}{2}}\,{m_B}^5\,\pi\,\sqrt{s}}{2\,r}\Bigg)$$

$$+ \quad \Big[2Re(B_2B_5^*) - 2Re(D_2B_4^*)\Big]\Bigg(\frac{-2\,\lambda\,{m_B}^4\,m_l\ (1-r-s)}{r} - \frac{2\,\lambda\,{m_B}^4\,m_l\,s}{r}$$

$$\mp\frac{\lambda^{\frac{3}{2}}\,{m_B}^3\,{m_l}^2\,\pi}{r\,\sqrt{s}}\Bigg)$$

$$+ \quad \Big[2Re(B_3B_4^*) - 2Re(D_3B_4^*) + 2Re(B_3B_5^*) - 2Re(D_3B_5^*)\Big]\frac{2\,\lambda\,{m_B}^4\,m_l\,s}{r}$$

$$+ \quad \Big[|B_4|^2 - |B_5|^2\Big]\left(\frac{-4\,\lambda\,{m_B}^2\,{m_l}^2}{r} + \frac{2\lambda\,{m_B}^4\,s}{r}\right)$$

$$+ \quad 2Re(B_4B_5^*)\left(-\frac{4\lambda\,{m_B}^2\,{m_l}^2}{r}\right)$$

$$+ \quad \Big[2Re(B_4T_1^*C_{TE}^*) - 2Re(B_5T_1^*C_{TE}^*)\Big]\Big(\mp 64\,\sqrt{\lambda}\,{m_B}^3\,m_l\,\pi\,\sqrt{s}$$

$$\mp\frac{16\,\sqrt{\lambda}\,{m_B}^3\,m_l\,\pi\ (1-r-s)\ \sqrt{s}}{r}\Bigg)$$

$$+ \quad \Big[2Re(B_4B_6^*C_{TE}^*) - 2Re(B_5B_6^*C_{TE}^*)\Big]\frac{\pm 8\sqrt{\lambda}\,{m_B}^3\,m_l\,\pi\ (1-r-s)\ \sqrt{s}}{r}$$

$$+ \quad \Big[2Re(B_4B_7^*C_{TE}^*) - 2Re(B_5B_7^*C_{TE}^*)\Big]\frac{\mp 4\,\lambda^{\frac{3}{2}}\,{m_B}^5\,m_l\,\pi\sqrt{s}}{r}$$

$$+ \quad \Big[2Re(B_1B_6^*C_{TE}^*) - 2Re(D_1B_6^*C_{TE}^*)\Big]\Bigg(\frac{-32\,\lambda\,{m_B}^2\,m_l}{r} - 384\,{m_B}^2\,m_l\,s$$

$$\pm\frac{16\,\sqrt{\lambda}\,m_B\,{m_l}^2\,\pi\,\sqrt{\lambda+4\,r\,s}}{r\,\sqrt{s}}\Bigg)$$

$$+ \quad \Big[2Re(B_2B_6^*C_{TE}^*) - 2Re(D_2B_6^*C_{TE}^*)\Big]\Bigg(\frac{32\,\lambda\,{m_B}^4\,m_l\,\sqrt{\lambda+4\,r\,s}}{r}$$

$$\mp\frac{16\,\lambda^{\frac{3}{2}}\,{m_B}^3\,{m_l}^2\,\pi}{r\,\sqrt{s}} \mp 64\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s}$$

$$\mp\frac{16\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s}\,\sqrt{\lambda+4\,r\,s}}{r}\Bigg)$$

$$+ \quad \Big[2Re(B_3B_6^*C_{TE}^*) - 2Re(D_3B_6^*C_{TE}^*)\Big]\frac{\mp 16\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s}\,\sqrt{\lambda+4\,r\,s}}{r}$$

$$
\begin{aligned}
+ \quad & \Big[2Re(B_1 B_7^* C_{TE}^*) - 2Re(D_1 B_6^* C_{TE}^*)\Big] \left(\frac{16\,\lambda\,{m_B}^4\,m_l\,\sqrt{\lambda+4\,r\,s}}{r}\right. \\
& \left.\mp \frac{8\,\lambda^{\frac{3}{2}}\,{m_B}^3\,{m_l}^2\,\pi}{r\,\sqrt{s}}\right) \\
+ \quad & \Big[2Re(B_2 B_7^* C_{TE}^*) - 2Re(D_2 B_7^* C_{TE}^*)\Big] \left(\frac{-16\,\lambda^2\,{m_B}^6\,m_l}{r}\right. \\
& \left.\pm \frac{8\,\lambda^{\frac{3}{2}}\,{m_B}^5\,{m_l}^2\,\pi\,\sqrt{s}}{r} \pm \frac{8\,\lambda^{\frac{3}{2}}\,{m_B}^5\,{m_l}^2\,\pi\,\sqrt{\lambda+4\,r\,s}}{r\,\sqrt{s}}\right) \\
+ \quad & \Big[2Re(B_3 B_7^* C_{TE}^*) - 2Re(D_3 B_7^* C_{TE}^*)\Big] \frac{\pm 8\lambda^{\frac{3}{2}}\,{m_B}^5\,{m_l}^2\,\pi\,\sqrt{s}}{r} \\
+ \quad & \Big[2Re(B_1 T_1^* C_{TE}^*) - 2Re(D_1 T_1^* C_{TE}^*)\Big] \left(\frac{64\,\lambda\,{m_B}^2\,m_l}{r} + 768\,{m_B}^2\,m_l\,s\right. \\
& +768\,{m_B}^2\,m_l\,\sqrt{\lambda+4\,r\,s} \mp \frac{256\,\sqrt{\lambda}\,m_B\,{m_l}^2\,\pi}{\sqrt{s}} \pm 32\,\sqrt{\lambda}\,{m_B}^3\,\pi\,\sqrt{s} \\
& \left.\mp \frac{32\,\sqrt{\lambda}\,m_B\,{m_l}^2\,\pi\,\sqrt{\lambda+4\,r\,s}}{r\,\sqrt{s}}\right) \\
+ \quad & \Big[2Re(B_1 T_1^* C_T^*) + 2Re(D_1 T_1^* C_T^*)\Big] \times \\
& \left(\frac{\pm 64\,\sqrt{\lambda}\,m_B\,{m_l}^2\,\pi}{\sqrt{s}} \pm 16\,\sqrt{\lambda}\,{m_B}^3\,\pi\,\sqrt{s}\right) \\
+ \quad & \Big[2Re(B_2 T_1 C_{TE}^*) - 2Re(D_2 T_1 C_{TE}^*)\Big] \left(-256\,\lambda\,{m_B}^4\,m_l - \right. \\
& -\frac{64\,\lambda\,{m_B}^4\,m_l\,\sqrt{\lambda+4\,r\,s}}{r} \pm \frac{32\,\lambda^{\frac{3}{2}}\,{m_B}^3\,{m_l}^2\,\pi}{r\,\sqrt{s}} \pm 256\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s} \\
& \left.\mp \frac{128\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{\lambda+4\,r\,s}}{\sqrt{s}} \pm \frac{32\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s}\,\sqrt{\lambda+4\,r\,s}}{r}\right) \\
+ \quad & \Big[2Re(B_3 T_1^* C_{TE}^*) - 2Re(D_3 T_1^* C_{TE}^*)\Big] \left(\pm 128\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s}\right. \\
& \left.\pm \frac{32\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s}\,\sqrt{\lambda+4\,r\,s}}{r}\right) \\
+ \quad & \Big[2Re(A_1 B_6^* C_{TE}^*) + 2Re(C_1 B_6 C_{TE}^*)\Big] \\
& \left(\pm 64\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s} \pm 16\,\sqrt{\lambda}\,{m_B}^5\,\pi\,s^{\frac{3}{2}}\right) \\
+ \quad & \Big[2Re(A_1 B_6^* C_T^*) - 2Re(C_1 B_6^* C_T^*)\Big] \times \\
& \left(\mp 32\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s} \pm 8\,\sqrt{\lambda}\,{m_B}^5\,\pi\,s^{\frac{3}{2}}\right) \\
+ \quad & \Big[2Re(A_1 T_1^* C_{TE}^*) + 2Re(C_1 T_1^* C_{TE}^*)\Big] \left(\mp 128\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \mp 32\,\sqrt{\lambda}\,{m_B}^5\,\pi\,s^{\frac{3}{2}} \mp \frac{128\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{\lambda+4\,r\,s}}{\sqrt{s}} \\
& \mp 32\,\sqrt{\lambda}\,{m_B}^5\,\pi\,\sqrt{s}\,\sqrt{\lambda+4\,r\,s}\Big) \\
+ & \Big[2Re(A_1T_1^*C_T^*) - 2Re(C_1T_1^*C_T^*)\Big]\Big(-256\,\lambda\,{m_B}^4\,m_l \\
& \pm 64\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{s} \mp 16\,\sqrt{\lambda}\,{m_B}^5\,\pi\,s^{\frac{3}{2}} \\
& \pm \frac{64\,\sqrt{\lambda}\,{m_B}^3\,{m_l}^2\,\pi\,\sqrt{\lambda+4\,r\,s}}{\sqrt{s}} \mp 16\,\sqrt{\lambda}\,{m_B}^5\,\pi\,\sqrt{s}\,\sqrt{\lambda+4\,r\,s}\Big) \\
+ & \frac{256}{3rs}|T_1|^2|C_T|^2m_B^2\Big(4m_\ell^2\,[\lambda(8r-s) - 12rs(2+2r-s)] \\
& +m_B^2s\,[\lambda(16r+s) + 12rs(2+2r-s)]\Big) \\
+ & \frac{1024}{3rs}|T_1|^2|C_{TE}|^2m_B^2\Big(8m_\ell^2\,[\lambda(4r+s) + 12rs(2+2r-s)] \\
& +m_B^2s\,[\lambda(16r+s) + 12rs(2+2r-s)]\Big) \\
+ & \Big[|T_1|^2Re(C_TC_{TE}^*)\Big]\frac{4096}{s}m_B^3m_l\pi\sqrt{s\lambda}(1-r) \\
+ & \frac{16}{3r}m_B^2\Big(4(m_B^2s+8m_\ell^2)|C_{TE}|^2 + m_B^2sv^2|C_T|^2\Big) \times \Big(4(\lambda+12rs)|B_6|^2 \\
+ & m_B^4\lambda^2|B_7|^2 - 4m_B^2(1-r-s)\lambda\,\mathrm{Re}(B_6B_7^*) \\
- & 16\,[\lambda+12r(1-r)]\,\mathrm{Re}(T_1B_6^*) \\
+ & 8m_B^2(1+3r-s)\lambda\,\mathrm{Re}(T_1B_7^*)\Big) \\
- & \Big[Re(T_1C_TB_6^*C_{TE}^*)\Big]2048m_B^3m_l\pi\sqrt{s\lambda}\Big\},
\end{aligned}
\tag{E.7}
$$

where the superscripts $(-)$ and $(+)$ in $\vec{e}_T^{\,\mp}$ is to represent the decay rate expression when the leptons ℓ^+ and ℓ^- are polarized transversally, respectively. We used Eq. (4.5) to calculate the transversal polarization of massive leptons and found for ℓ^-,

$$
P_T^- = \frac{\pi}{\Delta}m_B\sqrt{s\lambda}\Big\{ - 8m_B^2m_\ell\,\mathrm{Re}[(A_1+C_1)(B_1^*+D_1^*)]
$$

$$
\begin{aligned}
&+ \frac{1}{r} m_B^2 m_\ell (1+3r-s) \Big[\mathrm{Re}(B_1 D_2^*) - \mathrm{Re}(B_2 D_1^*)\Big] \\
&+ \frac{1}{rs} m_\ell (1-r-s) \Big[\, |B_1|^2 - |D_1|^2 \,\Big] \\
&+ \frac{2}{rs} m_\ell^2 (1-r-s) \Big[\mathrm{Re}(B_1 B_5^*) - \mathrm{Re}(D_1 B_4^*)\Big] \\
&- \frac{1}{r} m_B^2 m_\ell (1-r-s)\, \mathrm{Re}[(B_1 + D_1)(B_3^* - D_3^*)] \\
&- \frac{2}{rs} m_B^2 m_\ell^2 \lambda \Big[\mathrm{Re}(B_2 B_5^*) - \mathrm{Re}(D_2 B_4^*)\Big] \\
&+ \frac{1}{rs} m_B^4 m_\ell (1-r) \lambda \Big[\, |B_2|^2 - |D_2|^2 \,\Big] + \frac{1}{r} m_B^4 m_\ell \lambda\, \mathrm{Re}[(B_2 + D_2)(B_3^* - D_3^*)] \\
&- \frac{1}{rs} m_B^2 m_\ell [\lambda + (1-r-s)(1-r)] \Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*)\Big] \\
&+ \frac{1}{rs} (1-r-s)(2m_\ell^2 - m_B^2 s) \Big[\mathrm{Re}(B_1 B_4^*) - \mathrm{Re}(D_1 B_5^*)\Big] \\
&+ \frac{1}{rs} m_B^2 \lambda (2m_\ell^2 - m_B^2 s) \Big[\mathrm{Re}(D_2 B_5^*) - \mathrm{Re}(B_2 B_4^*)\Big] \\
&- \frac{16}{rs} \lambda m_B^2 m_\ell^2\, \mathrm{Re}[(B_1 - D_1)(B_7 C_{TE})^*] \\
&+ \frac{16}{rs} \lambda m_B^4 m_\ell^2 (1-r)\, \mathrm{Re}[(B_2 - D_2)(B_7 C_{TE})^*] \\
&+ \frac{8}{r} \lambda m_B^4 m_\ell\, \mathrm{Re}[(B_4 - B_5)(B_7 C_{TE})^*] \\
&+ \frac{16}{r} \lambda m_B^4 m_\ell^2\, \mathrm{Re}[(B_3 - D_3)(B_7 C_{TE})^*] \\
&+ \frac{32}{rs} m_\ell^2 (1-r-s)\, \mathrm{Re}[(B_1 - D_1)(B_6 C_{TE})^*] \qquad \text{(E.8)} \\
&- \frac{32}{rs} m_B^2 m_\ell^2 (1-r)(1-r-s)\, \mathrm{Re}[(B_2 - D_2)(B_6 C_{TE})^*] \\
&- \frac{16}{r} m_B^2 m_\ell (1-r-s)\, \mathrm{Re}[(B_4 - B_5)(B_6 C_{TE})^*] \\
&- \frac{32}{r} m_B^2 m_\ell^2 (1-r-s)\, \mathrm{Re}[(B_3 - D_3)(B_6 C_{TE})^*] \\
&- 16 m_B^2 \Big(4m_\ell^2\, \mathrm{Re}[A_1^*(C_T + 2C_{TE})B_6] - m_B^2 s\, \mathrm{Re}[A_1^*(C_T - 2C_{TE})B_6]\Big) \\
&+ 16 m_B^2 \Big(4m_\ell^2\, \mathrm{Re}[C_1^*(C_T - 2C_{TE})B_6] - m_B^2 s\, \mathrm{Re}[C_1^*(C_T + 2C_{TE})B_6]\Big) \\
&+ \frac{32}{s} m_B^2 (1-r) \Big(4m_\ell^2\, \mathrm{Re}[A_1^*(C_T + 2C_{TE})T_1] - m_B^2 s\, \mathrm{Re}[A_1^*(C_T - 2C_{TE})T_1]\Big) \\
&- \frac{32}{s} m_B^2 (1-r) \Big(4m_\ell^2\, \mathrm{Re}[C_1^*(C_T - 2C_{TE})T_1] - m_B^2 s\, \mathrm{Re}[C_1^*(C_T + 2C_{TE})T_1]\Big)
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{64}{rs} m_B^2 m_\ell^2 (1-r)(1+3r-s)\,\mathrm{Re}[(B_2-D_2)(T_1 C_{TE})^*] \\
&+ \quad \frac{64}{r} m_B^2 m_\ell^2 (1+3r-s)\,\mathrm{Re}[(B_3-D_3)(T_1 C_{TE})^*] \\
&+ \quad \frac{32}{r} m_B^2 m_\ell (1+3r-s)\,\mathrm{Re}[(B_4-B_5)(T_1 C_{TE})^*] \\
&+ \quad \frac{64}{rs} [m_B^2 rs - m_\ell^2 (1+7r-s)]\,\mathrm{Re}[(B_1-D_1)(T_1 C_{TE})^*] \\
&- \quad \frac{32}{s} (4m_\ell^2 + m_B^2 s)\,\mathrm{Re}[(B_1+D_1)(T_1 C_T)^*] \\
&- \quad 2048 m_B^2 m_\ell\,\mathrm{Re}[(C_T T_1)(B_6 C_{TE})^*] \\
&+ \quad \left.\frac{4096}{s} m_B^2 m_\ell (1-r)\,|T_1|^2\ \mathrm{Re}(C_T C_{TE}^*)\right\}\,,
\end{aligned}
$$

and for ℓ^+,

$$
\begin{aligned}
P_T^+ \quad = \quad & \frac{\pi}{\Delta} m_B \sqrt{s}\lambda \Bigg\{ -8 m_B^2 m_\ell\,\mathrm{Re}[(A_1+C_1)(B_1^*+D_1^*)] \\
- \quad & \frac{1}{r} m_B^2 m_\ell (1+3r-s) \Big[\mathrm{Re}(B_1 D_2^*) - \mathrm{Re}(B_2 D_1^*)\Big] \\
- \quad & \frac{1}{rs} m_\ell (1-r-s) \Big[\,|B_1|^2 - |D_1|^2\,\Big] \\
+ \quad & \frac{1}{rs} (2m_\ell^2 - m_B^2 s)(1-r-s) \Big[\mathrm{Re}(B_1 B_5^*) - \mathrm{Re}(D_1 B_4^*)\Big] \\
+ \quad & \frac{1}{r} m_B^2 m_\ell (1-r-s)\,\mathrm{Re}[(B_1+D_1)(B_3^*-D_3^*)] \\
- \quad & \frac{1}{rs} m_B^2 \lambda (2m_\ell^2 - m_B^2 s) \Big[\mathrm{Re}(B_2 B_5^*) - \mathrm{Re}(D_2 B_4^*)\Big] \\
- \quad & \frac{1}{rs} m_B^4 m_\ell (1-r)\lambda \Big[\,|B_2|^2 - |D_2|^2\,\Big] - \frac{1}{r} m_B^4 m_\ell \lambda\,\mathrm{Re}[(B_2+D_2)(B_3^*-D_3^*)] \\
+ \quad & \frac{1}{rs} m_B^2 m_\ell [\lambda + (1-r-s)(1-r)] \Big[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*)\Big] \\
+ \quad & \frac{2}{rs} m_\ell^2 (1-r-s) \Big[\mathrm{Re}(B_1 B_4^*) - \mathrm{Re}(D_1 B_5^*)\Big] \\
+ \quad & \frac{2}{rs} m_B^2 m_\ell^2 \lambda \Big[\mathrm{Re}(D_2 B_5^*) - \mathrm{Re}(B_2 B_4^*)\Big] \\
+ \quad & \frac{16}{rs} \lambda m_B^2 m_\ell^2\,\mathrm{Re}[(B_1-D_1)(B_7 C_{TE})^*] \\
- \quad & \frac{16}{rs} \lambda m_B^4 m_\ell^2 (1-r)\,\mathrm{Re}[(B_2-D_2)(B_7 C_{TE})^*] \\
- \quad & \frac{8}{r} \lambda m_B^4 m_\ell\,\mathrm{Re}[(B_4-B_5)(B_7 C_{TE})^*]
\end{aligned}
$$

$$
\begin{aligned}
&- \frac{16}{r}\lambda m_B^4 m_\ell^2 \,\mathrm{Re}[(B_3 - D_3)(B_7 C_{TE})^*] \\
&- \frac{32}{rs} m_\ell^2 (1-r-s)\,\mathrm{Re}[(B_1 - D_1)(B_6 C_{TE})^*] \qquad \text{(E.9)} \\
&+ \frac{32}{rs} m_B^2 m_\ell^2 (1-r)(1-r-s)\,\mathrm{Re}[(B_2 - D_2)(B_6 C_{TE})^*] \\
&+ \frac{16}{r} m_B^2 m_\ell (1-r-s)\,\mathrm{Re}[(B_4 - B_5)(B_6 C_{TE})^*] \\
&+ \frac{32}{r} m_B^2 m_\ell^2 (1-r-s)\,\mathrm{Re}[(B_3 - D_3)(B_6 C_{TE})^*] \\
&+ 16 m_B^2 \Big(4 m_\ell^2 \,\mathrm{Re}[A_1^*(C_T - 2C_{TE})B_6] - m_B^2 s\,\mathrm{Re}[A_1^*(C_T + 2C_{TE})B_6]\Big) \\
&- 16 m_B^2 \Big(4 m_\ell^2 \,\mathrm{Re}[C_1^*(C_T + 2C_{TE})B_6] - m_B^2 s\,\mathrm{Re}[C_1^*(C_T - 2C_{TE})B_6]\Big) \\
&- \frac{32}{s} m_B^2 (1-r)\Big(4 m_\ell^2 \,\mathrm{Re}[A_1^*(C_T - 2C_{TE})T_1] - m_B^2 s\,\mathrm{Re}[A_1^*(C_T + 2C_{TE})T_1]\Big) \\
&+ \frac{32}{s} m_B^2 (1-r)\Big(4 m_\ell^2 \,\mathrm{Re}[C_1^*(C_T + 2C_{TE})T_1] - m_B^2 s\,\mathrm{Re}[C_1^*(C_T - 2C_{TE})T_1]\Big) \\
&- \frac{64}{rs} m_B^2 m_\ell^2 (1-r)(1+3r-s)\,\mathrm{Re}[(B_2 - D_2)(T_1 C_{TE})^*] \\
&- \frac{64}{r} m_B^2 m_\ell^2 (1+3r-s)\,\mathrm{Re}[(B_3 - D_3)(T_1 C_{TE})^*] \\
&- \frac{32}{r} m_B^2 m_\ell (1+3r-s)\,\mathrm{Re}[(B_4 - B_5)(T_1 C_{TE})^*] \\
&- \frac{64}{rs}[m_B^2 rs - m_\ell^2(1+7r-s)]\,\mathrm{Re}[(B_1 - D_1)(T_1 C_{TE})^*] \\
&- \frac{32}{s}(4 m_\ell^2 + m_B^2 s)\,\mathrm{Re}[(B_1 + D_1)(T_1 C_T)^*] \\
&- 2048 m_B^2 m_\ell \,\mathrm{Re}[(C_T T_1)(B_6 C_{TE})^*] \\
&+ \frac{4096}{s} m_B^2 m_\ell (1-r)\,|T_1|^2 \;\mathrm{Re}(C_T C_{TE}^*)\Bigg\}\;.
\end{aligned}
$$

Normal asymmetries for massive leptons were also calculated as,

$$
\begin{aligned}
P_N^- \;=\; & \frac{1}{\Delta}\pi v m_B^3 \sqrt{s\lambda}\Bigg\{ 8 m_\ell \,\mathrm{Im}[(B_1^* C_1) + (A_1^* D_1)] \\
&- \frac{1}{r} m_B^2 \lambda \,\mathrm{Im}[(B_2^* B_4) + (D_2^* B_5)] \\
&+ \frac{1}{r} m_B^2 m_\ell \lambda \,\mathrm{Im}[(B_2 - D_2)(B_3^* - D_3^*)] \\
&- \frac{1}{r} m_\ell \,(1+3r-s)\,\mathrm{Im}[(B_1 - D_1)(B_2^* - D_2^*)]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{1}{r}(1-r-s)\,\mathrm{Im}[(B_1^* B_4) + (D_1^* B_5)] \\
&- \quad \frac{1}{r} m_\ell\,(1-r-s)\,\mathrm{Im}[(B_1 - D_1)(B_3^* - D_3^*)] \\
&- \quad \frac{8}{r} m_B^2 m_\ell \lambda\,\mathrm{Im}[(B_4 + B_5)(B_7 C_{TE})^*] \qquad \text{(E.10)} \\
&+ \quad \frac{16}{r} m_\ell\,(1-r-s)\,\mathrm{Im}[(B_4 + B_5)(B_6 C_{TE})^*] \\
&- \quad \frac{32}{r} m_\ell\,(1+3r-s)\,\mathrm{Im}[(B_4 + B_5)(T_1 C_{TE})^*] \\
&- \quad 16 m_B^2 s\Big(\,\mathrm{Im}[A_1^*(C_T - 2C_{TE})B_6] + \mathrm{Im}[C_1^*(C_T + 2C_{TE})B_6]\Big) \\
&+ \quad 32 m_B^2 (1-r)\Big(\,\mathrm{Im}[A_1^*(C_T - 2C_{TE})T_1] + \mathrm{Im}[C_1^*(C_T + 2C_{TE})T_1]\Big) \\
&+ \quad 32\Big(\mathrm{Im}[B_1^*(C_T - 2C_{TE})T_1] - \mathrm{Im}[D_1^*(C_T + 2C_{TE})T_1]\Big) \\
&+ \quad 512 m_\ell\,\left(\left|C_T\right|^2 - 4\left|C_{TE}\right|^2\right)\,\mathrm{Im}(B_6^* T_1)\Bigg\}\;,
\end{aligned}
$$

$$
\begin{aligned}
P_N^+ \quad &= \quad \frac{1}{\Delta}\pi v m_B^3 \sqrt{s\lambda}\Bigg\{ - 8 m_\ell\,\mathrm{Im}[(B_1^* C_1) + (A_1^* D_1)] \\
&+ \quad \frac{1}{r} m_B^2 \lambda\,\mathrm{Im}[(B_2^* B_5) + (D_2^* B_4)] \\
&+ \quad \frac{1}{r} m_B^2 m_\ell \lambda\,\mathrm{Im}[(B_2 - D_2)(B_3^* - D_3^*)] \\
&- \quad \frac{1}{r} m_\ell\,(1+3r-s)\,\mathrm{Im}[(B_1 - D_1)(B_2^* - D_2^*)] \\
&- \quad \frac{1}{r}(1-r-s)\,\mathrm{Im}[(B_1^* B_5) + (D_1^* B_4)] \\
&- \quad \frac{1}{r} m_\ell\,(1-r-s)\,\mathrm{Im}[(B_1 - D_1)(B_3^* - D_3^*)] \\
&+ \quad \frac{8}{r} m_B^2 m_\ell \lambda\,\mathrm{Im}[(B_4 + B_5)(B_7 C_{TE})^*] \qquad \text{(E.11)} \\
&- \quad \frac{16}{r} m_\ell\,(1-r-s)\,\mathrm{Im}[(B_4 + B_5)(B_6 C_{TE})^*] \\
&+ \quad \frac{32}{r} m_\ell\,(1+3r-s)\,\mathrm{Im}[(B_4 + B_5)(T_1 C_{TE})^*] \\
&- \quad 16 m_B^2 s\Big(\,\mathrm{Im}[A_1^*(C_T + 2C_{TE})B_6] + \mathrm{Im}[C_1^*(C_T - 2C_{TE})B_6]\Big) \\
&+ \quad 32 m_B^2 (1-r)\Big(\,\mathrm{Im}[A_1^*(C_T + 2C_{TE})T_1] + \mathrm{Im}[C_1^*(C_T - 2C_{TE})T_1]\Big) \\
&- \quad 32\Big(\mathrm{Im}[B_1^*(C_T + 2C_{TE})T_1] - \mathrm{Im}[D_1^*(C_T - 2C_{TE})T_1]\Big)
\end{aligned}
$$

$$+ \quad 512 m_\ell \left(|C_T|^2 - 4\,|C_{TE}|^2 \right) \mathrm{Im}(B_6^* T_1) \Big\} \; .$$

It would also be very useful to present here the combined analysis of lepton and antilepton polarizations for massive leptons, such that,

$$\begin{aligned}
P_L^- + P_L^+ &= \frac{4}{\Delta} m_B^2 v \Big\{ \frac{2}{r} m_\ell \lambda \,\mathrm{Re}[(B_1 - D_1)(B_4^* + B_5^*)] \\
&- \frac{2}{r} m_B^2 m_\ell \lambda (1-r)\,\mathrm{Re}[(B_2 - D_2)(B_4^* + B_5^*)] \\
&- \frac{1}{r} m_B^2 s \lambda \Big(|B_4|^2 - |B_5|^2 \Big) - \frac{2}{r} m_B^2 m_\ell s \lambda \,\mathrm{Re}[(B_3 - D_3)(B_4^* + B_5^*)] \\
&+ \frac{8}{3r} m_B^4 m_\ell \lambda^2 \,\mathrm{Re}[(B_2 + D_2)(B_7 C_T)^*] \\
&+ \frac{32}{3r} m_B^6 s \lambda^2 \,|B_7|^2 \,\mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{8}{3r} m_B^2 m_\ell \lambda (1-r-s)\,\mathrm{Re}[(B_1 + D_1)(B_7 C_T)^*] \\
&- \frac{16}{3r} m_B^2 m_\ell \lambda (1-r-s)\,\mathrm{Re}[(B_2 + D_2)(B_6 C_T)^*] \\
&- \frac{128}{3r} m_B^4 s \lambda (1-r-s)\,\mathrm{Re}(B_6 B_7^*)\,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{16}{3r} m_\ell (\lambda + 12 r s)\,\mathrm{Re}[(B_1 + D_1)(B_6 C_T)^*] \qquad \text{(E.12)} \\
&+ \frac{128}{3r} m_B^2 s (\lambda + 12 r s)\; |B_6|^2 \,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{512}{3r} m_B^2 \left[\lambda(4r + s) + 12 r (1-r)^2\right]\; |T_1|^2 \;\mathrm{Re}(C_T C_{TE}^*) \\
&- \frac{512}{3r} m_B^2 s \left[\lambda + 12 r(1-r)\right] \mathrm{Re}(T_1 B_6^*)\,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{256}{3r} m_B^4 s \lambda (1 + 3r - s)\,\mathrm{Re}(T_1 B_7^*)\,\mathrm{Re}(C_T C_{TE}^*) \\
&+ \frac{512}{3} m_B^2 m_\ell \lambda \,\mathrm{Re}[(A_1 + C_1)(T_1 C_{TE})^*] \\
&- \frac{32}{3r} m_\ell \left[\lambda + 12 r(1-r)\right] \mathrm{Re}[(B_1 + D_1)(T_1 C_T)^*] \\
&+ \frac{32}{3r} m_B^2 m_\ell \lambda (1 + 3r - s)\,\mathrm{Re}[(B_2 + D_2)(T_1 C_T)^*] \Big\} \; .
\end{aligned}$$

$$P_T^- - P_T^+ = \frac{\pi}{\Delta} m_B \sqrt{s\lambda} \Big\{ \frac{2}{rs} m_B^4 m_\ell (1-r) \lambda \Big[|B_2|^2 - |D_2|^2 \Big]$$

$$
\begin{aligned}
&+ \frac{1}{r} m_B^4 \lambda \,\mathrm{Re}[(B_2+D_2)(B_4^*-B_5^*)] \\
&+ \frac{2}{r} m_B^4 m_\ell \lambda \,\mathrm{Re}[(B_2+D_2)(B_3^*-D_3^*)] \\
&+ \frac{2}{r} m_B^2 m_\ell (1+3r-s) \left[\mathrm{Re}(B_1 D_2^*) - \mathrm{Re}(B_2 D_1^*)\right] \\
&+ \frac{2}{rs} m_\ell (1-r-s) \left[\, |B_1|^2 - |D_1|^2 \,\right] \\
&- \frac{1}{r} m_B^2 (1-r-s) \mathrm{Re}[(B_1+D_1)(B_4^*-B_5^*)] \\
&- \frac{2}{r} m_B^2 m_\ell (1-r-s) \mathrm{Re}[(B_1+D_1)(B_3^*-D_3^*)] \\
&- \frac{2}{rs} m_B^2 m_\ell [\lambda + (1-r)(1-r-s)] \left[\mathrm{Re}(B_1 B_2^*) - \mathrm{Re}(D_1 D_2^*)\right] \\
&- \frac{32}{rs} m_B^2 m_\ell^2 \lambda \,\mathrm{Re}[(B_1-D_1)(B_7 C_{TE})^*] \\
&+ \frac{32}{rs} m_B^4 m_\ell^2 \lambda (1-r) \,\mathrm{Re}[(B_2-D_2)(B_7 C_{TE})^*] \qquad \text{(E.13)} \\
&+ \frac{16}{r} m_B^4 m_\ell \lambda \,\mathrm{Re}[(B_4-B_5)(B_7 C_{TE})^*] \\
&+ \frac{32}{r} m_B^4 m_\ell^2 \lambda \,\mathrm{Re}[(B_3-D_3)(B_7 C_{TE})^*] \\
&+ \frac{64}{rs} m_\ell^2 (1-r-s) \,\mathrm{Re}[(B_1-D_1)(B_6 C_{TE})^*] \\
&- \frac{64}{rs} m_B^2 m_\ell^2 (1-r)(1-r-s) \,\mathrm{Re}[(B_2-D_2)(B_6 C_{TE})^*] \\
&- \frac{32}{r} m_B^2 m_\ell (1-r-s) \,\mathrm{Re}[(B_4-B_5)(B_6 C_{TE})^*] \\
&- \frac{64}{r} m_B^2 m_\ell^2 (1-r-s) \,\mathrm{Re}[(B_3-D_3)(B_6 C_{TE})^*] \\
&+ 32 m_B^4 s v^2 \,\mathrm{Re}[(A_1-C_1)(B_6 C_T)^*] \\
&+ \frac{64}{r} m_B^2 m_\ell (1+3r-s) \,\mathrm{Re}[(B_4-B_5)(T_1 C_{TE})^*] \\
&- 64 m_B^4 (1-r) v^2 \,\mathrm{Re}[(A_1-C_1)(T_1 C_T)^*] \\
&+ \frac{128}{rs} [m_B^2 rs - m_\ell^2 (1+7r-s)] \,\mathrm{Re}[(B_1-D_1)(T_1 C_{TE})^*] \\
&+ \frac{128}{rs} m_B^2 m_\ell^2 (1-r)(1+3r-s) \mathrm{Re}[(B_2-D_2)(T_1 C_{TE})^*] \\
&+ \frac{128}{r} m_B^2 m_\ell^2 (1+3r-s) \mathrm{Re}[(B_3-D_3)(T_1 C_{TE})^*] \Bigg\} \,.
\end{aligned}
$$

$$
\begin{aligned}
P_N^- + P_N^+ &= \frac{1}{\Delta}\pi v m_B^3 \sqrt{s\lambda}\bigg\{ -\frac{2}{r} m_\ell (1+3r-s)\,\mathrm{Im}[(B_1-D_1)(B_2^*-D_2^*)] \\
&- \frac{2}{r} m_\ell (1-r-s)\,\mathrm{Im}[(B_1-D_1)(B_3^*-D_3^*)] \\
&- \frac{1}{r} (1-r-s)\,\mathrm{Im}[(B_1-D_1)(B_4^*-B_5^*)] \\
&+ \frac{2}{r} m_B^2 m_\ell \lambda\,\mathrm{Im}[(B_2-D_2)(B_3^*-D_3^*)] \qquad \text{(E.14)} \\
&+ \frac{1}{r} m_B^2 \lambda\,\mathrm{Im}[(B_2-D_2)(B_4^*-B_5^*)] \\
&+ 32 m_B^2 s\,\mathrm{Im}[(A_1+C_1)(B_6 C_T)^*] \\
&+ 1024 m_\ell \Big(|C_T|^2 - |4C_{TE}|^2 \Big) \mathrm{Im}(B_6^* T_1) \\
&- 64 m_B^2 (1-r)\,\mathrm{Im}[(A_1+C_1)(T_1 C_T)^*] \\
&+ 128\,\mathrm{Im}[(B_1+D_1)(T_1 C_{TE})^*] \bigg\} \,.
\end{aligned}
$$

VITA

Mehmet Kerim Çakmak was born in Ankara, Turkey, on January 23th of 1973. He graduated from Yükseliş Koleji in 1991. He received his B.S. degree in Physics from the Middle East Technical University (METU) in 1997 and his M.S. degree in 1999, on High Energy Physics again from the METU.