# ANALYSIS OF SALTWATER INTRUSION AND INVESTIGATIONS ON PREVENTION TECHNIQUES IN COASTAL AQUIFERS

**A THESIS SUBMITTED TO**
**THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES**
**OF**
**THE MIDDLE EAST TECHNICAL UNIVERSITY**

**BY**

**NÜVİT BERKAY BAŞDURAK**

**IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF**

**MASTER OF SCIENCE**

**IN**

**THE DEPARTMENT OF CIVIL ENGINEERING**

**JANUARY 2004**

# ABSTRACT

## ANALYSIS OF SALTWATER INTRUSION AND INVESTIGATIONS ON PREVENTION TECHNIQUES IN COASTAL AQUIFERS

BAŞDURAK, Nüvit Berkay
M.S., Department of Civil Engineering
Supervisor: Prof.Dr.Halil ÖNDER

January 2004, 73 Pages

In this study the saltwater intrusion in coastal aquifers is briefly described and the prevention techniques are discussed. By using two computer models SWI and SHARP, the movement of freshwater and saltwater is analyzed on hypothetical cases and on one real aquifer in Marmaris in the coast of Mediterranean Region. Artificial recharge and subsurface barrier techniques are applied to hypothetical cases as means of controling  the saltwater intrusion. The performance of strip recharge and subsurface barrier in reducing the salt water intrusion is analyzed by simulation of the groundwater flow with the codes mentioned above. The results obtained are compared with each other. The results of hypothetical cases are relatively in good agreement. For the real aquifer the results show discrepancy that cannot be ignored.

Keywords: Saltwater intrusion, Coastal aquifers, MODFLOW, SWI, SHARP

# ÖZ

## KIYI AKİFERLERİNDE TUZLU SU GİRİŞİMİ ANALİZİ VE ÖNLEME TEKNİKLERİ ÜZERİNE ARAŞTIRMALAR

BAŞDURAK, Nüvit Berkay
Yüksek Lisans, İnşaat Mühendisliği Bölümü
Tez Danışmanı: Prof.Dr.Halil ÖNDER

Ocak 2004, 73 Pages

Bu çalışmada kıyı akiferlerindeki tuzlu su girişimi tanımlanmış ve önleme teknikleri tartışılmıştır. SWI ve SHARP adında iki farklı bilgisayar programı kullanılarak, hipotetik ideal bir akiferde ve Marmaris'deki gerçek bir akiferde, tatlı su ve tuzlu su hareketi analiz edilmiştir. Tuzlu su girişimini önlemek amacıyla hipotetik akiferlere suni besleme ve yüzeyaltı perdeleme teknikleri uygulanmıştır. Bu tekniklerin tuzlu su girişimini azaltıcı performansı yukarıda adı geçen iki farklı bilgisayar programı simülasyonu ile analiz edilmiştir. Programlardan elde edilen sonuçlar birbiriyle karşılaştırılmıştır. Hipotetik akifer için elde edilen sonuçlar birbiriyle uyumludur. Gerçek akifer için elde edilen sonuçlar arasında yadsınamayacak bir fark olduğu gözlenmiştir.

Anahtar kelimeler: Tuzlu su girişimi, Kıyı akiferleri, MODFLOW, SWI, SHARP

# ACKNOWLEDGEMENTS

I express sincere appreciation to Prof. Dr. Halil ÖNDER who guided me at every stage of this thesis.

I would like to thank to my parents for their support and encouragement.

# TABLE OF CONTENTS

# LIST OF TABLES

TABLE

# LIST OF FIGURES

FIGURE

# LIST OF SYMBOLS

$b$ :saturated thickness of aquifer

$B_f$ :thickness of freshwater zone

$B_s$ :thicknesses of saltwater zone

$B'$ :thickness of confining layer

$c$ :resistance to vertical flow

$d$ :depth of aquifer

$h$ :hydraulic head

$h_c$ :head at the bottom of aquifer c

$h_d$ :head at the top of aquifer d

$\bar{h}_f$ :vertically averaged freshwater head

$\bar{h}_s$ :vertically averaged saltwater head

$j$ :grid number

$K$ :hydraulic conductivity tensor

$K_f$ :freshwater hydraulic conductivity

$K_s$ :saltwater hydraulic conductivity

$\bar{K}_f$ :vertically averaged freshwater hydraulic conductivity tensor

$\bar{K}_s$ :vertically averaged saltwater hydraulic conductivity tensor

$K'$ :hydraulic conductivity of confining layer

$k_v$ :vertical hydraulic conductivity of the resistance layer

$L$ :length of aquifer in x direction

$n$ :effective porosity

$p$ :zone name

$q$ :specific discharge vector

$q_l$ :vertical leakage

$q_f$ :freshwater specific discharge

$q_s$ :saltwater specific discharge

$q_{fz}$ :vertical component of freshwater flux

$q_{sz}$ :vertical component of saltwater flux

$Q_{lf}$ :total freshwater leakage across top and bottom of aquifer

$Q_{ls}$ :total saltwater leakage across top and bottom of aquifer

$S$ :storage coefficient

$S_r$ :specific storage

$S_f$ :freshwater specific storage

$S_s$ :saltwater specific storage

$t$ :time

$T$ :transmissivity

$U$ :horizontal flow vector

$W$ :volumetric flux per unit horizontal area

$W_f$ :freshwater source/sink terms

$W_s$ :saltwater source/sink terms

$v$ :slope

$w$ : volumetric flux per unit volume

$x_l$ :interface tip projection distance

$\omega$ : weighting factor

$\rho$ :three dimensional density distribution.

$\rho_c$ :density below confining layer

$\rho_d$ :density above confining layer

$\rho_f$ :density of freshwater

$\rho_s$ :density of saltwater

$\sigma$ :total transmissivity of all zones

$\zeta$ :elevation

$\zeta_b$ :elevation of bottom of aquifer

$\zeta_c$ :elevation of top of confining layer

$\zeta_d$ :elevation of bottom of confining layer

$\zeta_m$ :elevation of interface

$\zeta_t$ :elevation of top of aquifer

$\alpha$ :a parameter representing the type of aquifer (confined/unconfined)

$\beta$ :dimensionless density relation between freshwater and saltwater

$\nu$ :dimensionless density relation between freshwater and any fluid

$\delta$ :measure for the variation of density between zones

$\phi$ :dependent variable

$\Delta\zeta$ :elevation difference

$\Delta x$ :length of grid along x direction

# LIST OF ABBREVATIONS

BAS : Basic package

BCF : Block centered flow package

DE4 : Direct solution based on alternating diagonal ordering

DELC : The grid widths along a column

DELR : The grid widths along a row

DIS : Discretization package

DRN : Drain Package

HY : Hydraulic conductivity

IBOUND : Boundary array

LAYCBD : The layer type code

LPF : Layer Property Flow Package

MULT : Multiplier array file

NSTP : Stress period number

OC : Output control package

PCG : Preconditioned Conjugate-Gradient package
PHIF : Initial freshwater heads

RCH : Recharge Package

RIV : River Package

SIP : The strongly implicit procedure

SOR : Slice successive overrelaxation solver

STORF : Freshwater specific storage

STORS : Saltwater specific storage

TRAN : Transmissivity

TRPY : Horizontal anisotropy factor

VCONT : Vertical hydraulic conductivity

WELL : Well Package

ZONE : Zone array file

# CHAPTER 1

# INTRODUCTION AND LITERATURE REVIEW

## 1.1 Preliminary remarks

In near future, the freshwater demand will become an important issue because of the increase in its usage and global warming. The coastal aquifers are frequently used as freshwater resource to meet the demands for various uses such as domestic, industrial and agricultural purposes. As freshwater in coastal aquifers is in contact with saline water, which if drawn into the freshwater aquifer system, can diminish the water's potability as well as its usefulness for other purposes. Thus, a quantitive understanding of the patterns of movement and mixing between fresh and saline water is necessary to manage and protect these resources for future use.

In Turkey, surrounded by seas and having a coastline of approximately 8400 km, especially at the coasts of Mediterranean and Aegean regions, freshwater demand increases depending on the increase in the population in summer seasons and in the water requiring activities. Coastal aquifers should be managed properly to meet the water demand.

## 1.2 Problem definition

The zone separating freshwater and saltwater is a transition zone created by the mixing of waters due to the effects of diffusion and mechanical dispersion. Cooper (1959) and Kohout (1964) have shown that in the zone of mixing, the diluted saltwater is less dense than the original seawater, causing it to rise and move seaward along the interface. This induces a cyclic flow of saltwater from the sea through the sea floor, to the zone of mixing and back to the sea (Figure 1.1).

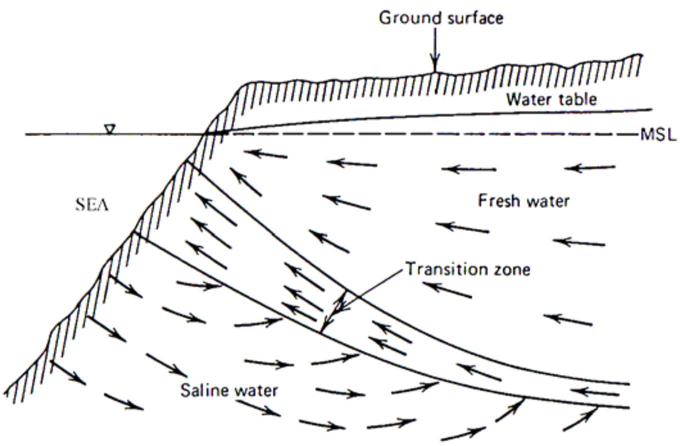



**Figure 1.1** Schematic vertical cross section showing freshwater, saltwater circulation (Todd,1980)

Under certain conditions the width of the transition zone is relatively small when compared with the thickness of the aquifer so that a sharp interface approximation can be used (Figure1.2). In sharp interface approach, freshwater and saltwater are assumed as immiscible fluids and the effects of diffusion and mechanical dispersion are neglected. This approach simplifies the problem in many cases of practical interest.

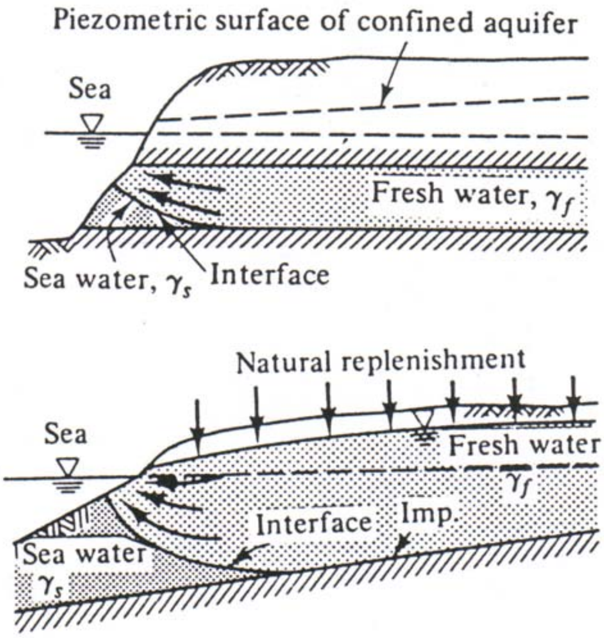
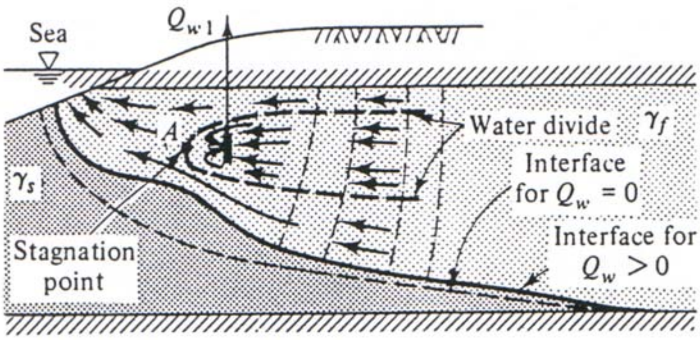
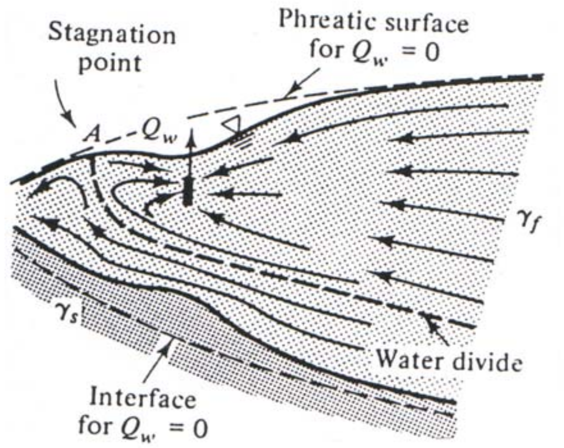



**Figure 1.2** Sharp interfaces in confined and unconfined coastal aquifers (Bear,1979)

Under natural undisturbed conditions in coastal aquifer, a state of equilibrium is maintained, with a stationary interface and a freshwater flow to the sea above it. At every point on this interface, the elevation and the slope are determined by the freshwater potential and gradient. The continuous change of slope results from the fact that as the sea is approached, the specific discharge of freshwater tangent to the interface increases. By pumping from a coastal aquifer in excess of replenishment, the water table in the vicinity of the coast is lowered to the extent that the piezometric head in the freshwater body becomes less than in the adjacent seawater wedge, and the interface starts to advance inland until a new equilibrium is reached. This phenomenon is called seawater intrusion. When pumping takes place in a well located above the interface, the latter upcones towards the pumping well (Figure 1.3). Unless the rate of pumping is controlled, seawater will enter the pumped well.




**a)**




**b)**

**Figure 1.3** Upconing of interface a) Confined b) Unconfined coastal aquifers (Bear,1979)

Methods for controlling intrusion vary widely depending on the source of the saline water, the extent of intrusion, local geology, water use, and economic factors. There are six generally recognized methods in coping with saltwater intrusion (Todd,1980): i) Modification of pumping pattern method includes changing of well locations or reduction in pumping rate. ii) In direct artificial surface recharge method, surface spreading and recharge wells are used for unconfined and confined aquifers, respectively. iii) Extraction barrier method requires continuous pumping trough with a line of wells adjacent to the sea. iv) Injection barrier method maintains a pressure ridge along the coast by a line of recharge wells. Injected freshwater flows both seaward and landward (Figure 1.4). v) Subsurface barrier method involves establishment of an impermeable subsurface barrier parallel to the coast and through the vertical extent of the aquifer. This can effectively prevent the inflow of seawater into the aquifer. (Figure 1.5). vi) Combination of extraction and injection barrier.

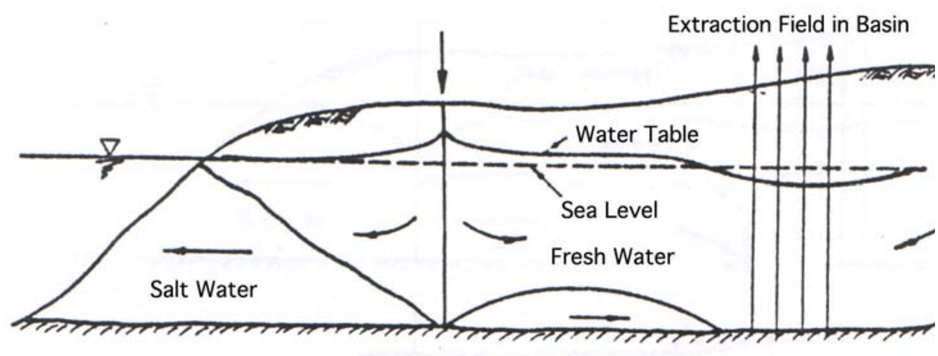



**Figure 1.4** Control of saltwater intrusion in unconfined aquifer by an injection barrier (Todd, 1974)

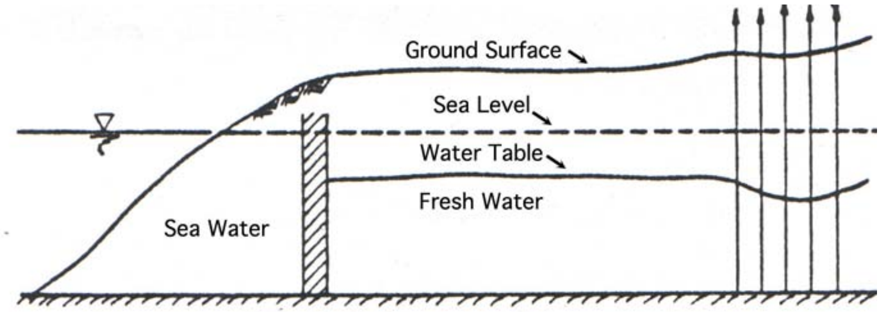



**Figure 1.5** Control of saltwater intrusion in unconfined aquifer by subsurface barrier (Todd, 1974)

To analyze seawater intrusion in coastal aquifers, two general approaches have been used: the disperse zone representing the transition between salt and fresh water and the sharp interface. Sharp interface models generally fall into two categories: one dynamic fluid approach which models freshwater flow only, and two fluid approach which models coupled freshwater and saltwater flow.

Badon-Ghyben (1889) and Herzberg (1901) related the freshwater head above sea level to the depth to the interface below sea level for a system in static equilibrium; that is steady horizontal freshwater flow and stationary saltwater. At the interface, the pressure due to the overlying column of freshwater must be equivalent to that due to the column of saltwater. According to that approximation for freshwater and saltwater having densities of 1000 kg/m$^3$, 1025 kg/m$^3$ respectively, at any distance from the sea, the depth of a stationary interface below sea level, is forty times the height of the freshwater table above it.

The sharp interface models that simulate flow in the freshwater region only, incorporate the Ghyben-Herzberg relation assuming that at each time step saltwater adjusts instantaneously to changes in the freshwater zone, and an equilibrium is achieved. For the long term studies if the interface can respond quickly to applied stress, the one fluid flow approach may be a reasonable assumption. However to reproduce the short term response of a coastal aquifer, it is necessary to include the influence of saltwater flow (Essaid, 1986).

In the two fluid approach, the freshwater and the saltwater flow equations are coupled by the interface boundary conditions defined as; same specific discharge on both sides and same pressures on both sides. On the basis of the principle of continuity of pressure, the interface elevation can be expressed as a function of the freshwater and saltwater head.

In this study, two programs are used to analyze the interface movement. One of them is the Sea Water Intrusion (SWI) Package, a computer model that simulates three dimensional regional seawater intrusion in coastal multiaquifer systems (Bakker and Schaars,2003). Although it considers both freshwater and saltwater flow it differs from the two fluid approach presenting a new formulation that depends on potential flow. The formulation is based on a vertical discretization of the groundwater into zones of either constant density (stratified flow) or constinuously varying density and is written in terms of vertically integrated fluxes. The flow in the aquifer is steady state, but the position of the interface changes over time in case the aquifer is approximated as incompressible.

The other computer model used, is SHARP (Essaid, 1990b). It is designed especially for layered coastal aquifer systems simulating three dimensional two fluid flow seperated by a sharp interface. Within each aquifer vertically integrated freshwater and saltwater flow equations coupled by the boundary conditions at the interface, are solved. Lekage between aquifers is calculated by applying Darcy’s Law.

### 1.3 Objectives

To control the seawater intrusion there are several ways like artificial recharge. The aim of this study is to analyze saltwater intusion in coastal aquifers by using two different computer models, on hypothetical schemes and on one real case in Marmaris. The other purpose of this thesis is to analyze the control techniques on saltwater inrusion.

### 1.4 Description of the thesis

This thesis is composed of six chapters. The first chapter covers the brief definition of the problem, the objective of the study and literature review involving the past studies about the subject.

The theoretical background is explained in the second chapter. The governing equations and approximations of saltwater intrusion for two different models, SWI and SHARP, are presented.

The third chapter contains simulations of a hypothetical coastal aquifer, whereas in the fourth chapter simulations are made on a real coastal aquifer. The simulations are made by using two different computer codes SWI and SHARP.

The fifth chapter covers the comparison of the results of two models and discussion of the techniques on prevention of saltwater intrusion in coastal aquifers. And the last chapter includes the conclusion and recommendations.

### 1.5 Literature review

Although much progress have been made toward the mathematical description of saltwater-freshwater relationships in groundwater systems since the late $19^{th}$ century, the advective and dispersive mechanisms involved are still incompletely understood.

In the late 1800's Badon Ghyben and Herzberg developed a formula relating the elevation of the water table in an unconfined aquifer to the elevation of the boundary or interface between the salt and fresh groundwater. Although the formula contains oversimplification assuming that, the head at the water table is the same as the head of the freshwater at the interface, which implies that there is no vertical head gradient, it formed the beginning of quantitive analysis of the interface position.

Muskat (1937) and Hubbert (1940) showed that the continuity of pressure in the flow field must be maintained across the assumed interface. Thus the interface can be treated as a boundary surface that couples two separate flow fields. They formulated the physics of the relations in terms of a sharp interface, which was recognized as the limiting streamline of the freshwater system. It also opened the path for better designed field studies, because the type of information required to define the flow situation had been defined.

Glover (1959) developed a formula to describe the saltwater freshwater sharp interface in a coastal aquifer that accounts for the movement and discharge of the freshwater.

Cooper (1959) developed a hypothesis to explain zone of dispersion and the associated perpetual circulation of seawater observed in the various field investigations. He also defined the amount of mixing attributed to tidal fluctuations in coastal environments.

Henry (1959) also developed some solutions for determining the sharp interface under various conditions and also made the first attempt to quantitatively determine the effects of dispersion and density dependent fluid flow on saltwater intrusion in coastal aquifers, by investigating a two dimensional hypothetical cross-section.

The analysis of the transient movement of interfaces was first investigated by Bear and Dagan (1964b).

Applications that use two dimensional cross-sectional analysis on various forms continued. Mualem and Bear (1974) investigated the role and effect of semipervious layers in an aquifer system.

Benetti and Giusti (1971) used electrical analog simulation to study the saltwater interface.

Numerical solutions using the sharp interface approach were done by Shamir and Dagan (1971). For density-dependent flow some computer codes were developed by McCracken (1977) and Voss (1985).

Two dimensional areal analysis has been undertaken to account for horizontal movement of the freshwater and saltwater; and both horizontal and vertical movement of the saltwater interface. Vanden Berg (1974) and Andrews (1981) analyzed the problem only in terms of horizontal movement of the boundary neglecting density effects. For transient cases Bonnet and Sauty (1975), Pinder and Page (1977), Mercer et al. (1980a), Garza (1982), and Wilson and Sa Da Costa (1982) used sharp interface approach and developed computer models.

Guswa and Le Blanc (1981), assuming static saltwater with a sharp interface, used a three dimensional freshwater flow model, and used Hubbert’s equation to determine the position of interface. Weiss (1982), Kuiper (1983) have used three dimensional simulations in which fluid density is specified in advance as function of position. Essaid (1990a) developed a quasi three dimensional code SHARP for a multilayered sharp interface model.

Three dimensional, density dependent solute transport codes which solve the flow and transport equations using a numerical method, have been developed by Huyakorn et al. (1987), Kipp (1987) but are limited in their application to regional coastal systems by computational constraints. SUTRA which can simulate density dependent groundwater flow with energy transport or chemically reactive solute transport, has become the widely accepted 3D variable-density groundwater flow model throughout the world (Voss and Souza, 1987). SWICHA is a 3D finite element code which can simulate variable-density fluid flow and solute transport processes in saturated porous media (Huyakorn et al., 1987; Lester, 1991).

There are programs that solve the flow and transport equation separately using a particle tracking approach and compute a new flow field only every so often. Some of them are MOCDENS3D (Oude Essink ,1998,2001), a link between MOC3D (Konikow et al. 1996) and MODFLOW (McDonald and Harbaugh, 1988) with density effects; SEAWAT (Gua and Langevin, 2002), a link between MT3DMS (Zhang and Wang, 1999) and MODFLOW. Recently Bakker and Schaars (2003) developed a package code SWI for the modelling of regional seawater intrusion with MODFLOW.

# CHAPTER 2

## THEORETICAL BACKGROUND

In this study, three package programs are used. A brief description of these packages and the mathematical models they adopted, are given below.

### 2.1 Governing equations

#### 2.1.1 Governing equation used in MODFLOW

The water movement through a homogenous and isotropic aquifer is described emprically by Darcy's Law (Bear, 1979) which indicates that discharge per unit area is proportional to the gradient of hydraulic head. In the case where principle axes of conductivity are parallel to the principle coordinate axes, Darcy's Law may be summarized for an unisotropic aquifer in the Cartesian coordinate system as;

$$q_x = -K_x \frac{\partial h}{\partial x} \quad , q_y = -K_y \frac{\partial h}{\partial y} \quad , q_z = -K_z \frac{\partial h}{\partial z} \tag{2.1}$$

where,

x, y, z are the coordinates in the Cartesian coordinate system

$q_x$ , $q_y$ , $q_z$ are the components of specific discharge vector, **q** in x, y, z coordinates respectively

$K_x$ ,$K_y$ ,$K_z$ are the components of hydraulic conductivity tensor K in x, y, z coordinates respectively and h is the hydraulic head

The continuity equation for groundwater flows can be written by using the principle of conservation of mass as,

$$-\frac{\partial q_x}{\partial x}-\frac{\partial q_y}{\partial y}-\frac{\partial q_z}{\partial z}+w=S_r\frac{\partial h}{\partial t} \tag{2.2}$$

Combining the mathematical form of the continuity equation (2.2) with the Darcy's Law (2.1), the following governing equation which indicates the three-dimensional movement of groundwater of constant density through porous material, is obtained.

$$\frac{\partial}{\partial x}(K_x\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(K_y\frac{\partial h}{\partial y})+\frac{\partial}{\partial z}(K_z\frac{\partial h}{\partial z})+w=S_r\frac{\partial h}{\partial t} \tag{2.3}$$

where,

w is a volumetric flux per unit volume representing sources and/or sinks of water, with $w<0.0$ for flow out of the groundwater system (extraction), and $\omega>0.0$ for flow in to the groundwater system (injection)

$S_r$ is the specific storage of the porous material, and t is the time

This equation describes groundwater flow under nonequilibrium conditions in a heterogeneous and anisotropic medium, provided that the principal axes of hydraulic conductivity are aligned with the coordinate directions.

If the flow is assumed to be horizontal, equation (2.3) may be integrated in z direction and takes the following form.

$$\frac{\partial}{\partial x}\left(T_X \frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right)+W = S\frac{\partial h}{\partial t} \tag{2.4}$$

where,

$T_x = K_x b$ ,$T_y = K_y b$ ; are the transmissivities in x and y direction

b is the saturated thickness of the aquifer

W = bw ; is sink-source term expressed as volumetric rate rate per unit horizontal area, and $S = bS_r$ is the storage coefficient

### 2.1.2 Governing equation used in SWI

A conceptualized vertical cross-section of an aquifer used in SWI is presented in Figure 2.1.a. There are two alternative options (Bakker,2003). For the first option called stratified flow option, the water has a constant density in each zone and the density is discontinuous from zone to zone (Figure 2.1.b). For the variable density flow option, the surface bounding the zones are iso-surfaces of the density; the density varies linearly in the vertical direction in each zone and is continuous from zone to zone (Figure 2.1.c).

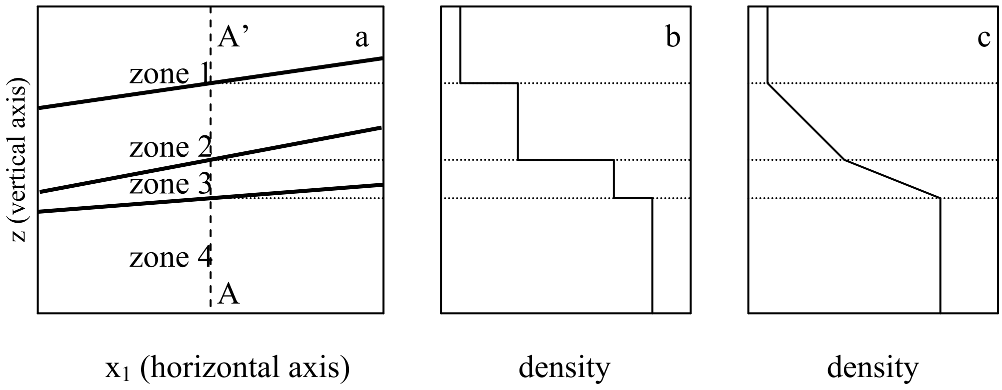



**Figure 2.1** Conceptual models a) Vertical cross section of an aquifer b) density distribution along A-A' for stratified flow c) density distribution along A-A' for variable density flow (Bakker,2003)

The assumptions are : 1) The Dupuit approximations is adopted and will be interpreted to mean that the resistance to flow in vertical direction is neglected (Strack, 1989). 2) The mass balance equation is replaced by the continuity of flow equation in the computation of the flow field; density effects are taken into account through Darcy's law. 3)Effects of dispersion and diffusion are not taken into account. 4) Unstable stratification is not allowed.

The groundwater is discetized vertically into N zones. Zones and surfaces are numbered from the top to down; zone n is bounded on top by surface n (Figure 2.2). The elevation of surface n is represented by the function $\zeta_n(x,y)$; the elevation of the bottom of the aquifer is called $\zeta_{N+1}$ and the elevation of the top of the saturated aquifer (i.e. the phreatic aquifer) is called $\zeta_1$.

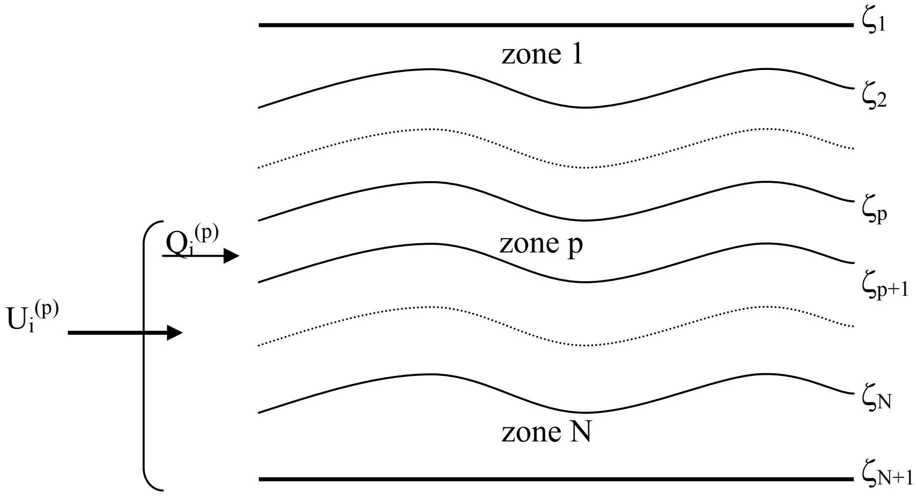



**Figure 2.2** Schematic vertical cross section of an aquifer

The three-dimensional density distribution is written in dimensionless form as;

$$\upsilon(x,y,z) = \frac{\rho(x,y,z) - \rho_f}{\rho_f} \tag{2.5}$$

where $\rho_f$ is the density of freshwater and $\rho$ (x,y,z) is the three dimensional density distribution.

Under the Dupuit approximation, the pressure distribution is hydrostatic, so the vertical gradient of the freshwater head is (Strack, 1995);

$$\frac{\partial h}{\partial z} = -\upsilon \tag{2.6}$$

Integration of (2.6) between surface z and $\zeta_n$ gives;

$$h = h_n - \int_{\zeta_n}^{z} \upsilon(x, y, z')dz' \tag{2.7}$$

where $h_n$ is the head at elvation $z = \zeta_n$

Substitution of (2.7) for h in Darcy's Law (2.1) gives the specific discharge as;

$$q_x = -K_f \frac{\partial h_n}{\partial x} + K_f \frac{\partial}{\partial x} \int_{\zeta_n}^{z} \upsilon(x, y, z')dz' \tag{2.8.a}$$

$$q_y = -K_f \frac{\partial h_n}{\partial y} + K_f \frac{\partial}{\partial y} \int_{\zeta_n}^{z} \upsilon(x, y, z')dz' \tag{2.8.b}$$

where $K_f$ is the freshwater hydraulic conductivity

Considering constant density in each zone, the specific discharge and discharge vectors in zone p can be written respectively as;

$$q_x^{(p)} = -K_f \frac{\partial h_p}{\partial x} - K_f \upsilon_p \frac{\partial \zeta_p}{\partial x} \quad , \quad q_y^{(p)} = -K_f \frac{\partial h_p}{\partial y} - K_f \upsilon_p \frac{\partial \zeta_p}{\partial y} \quad \text{(2.9a ; 2.9b)}$$

$$Q_x^{(p)} = \int_{\zeta_{p+1}}^{\zeta_p} q_x^{(p)} dz \quad , \quad Q_y^{(p)} = \int_{\zeta_{p+1}}^{\zeta_p} q_y^{(p)} dz \quad \text{(2.10a ; 2.10b)}$$

Using equation (2.7), the head at the top of zone p is expressed in terms of $h_1$ as;

$$h_p = h_1 + \sum_{n=1}^{p-1} \upsilon_n (\zeta_n - \zeta_{n+1}) \tag{2.11}$$

The comprehensive discharge vector $U^{(p)}$,which is defined as the vertically integrated flow below surface p, may be written in x and y direction as;

$$U_x^{(p)} = \sum_{n=p}^{N} Q_x^{(n)} \quad , \quad U_y^{(p)} = \sum_{n=p}^{N} Q_y^{(n)} \qquad \text{(2.12a ; 2.12b)}$$

Continuity of the total flow in the aquifer and the continuity of flow below surface p (p=2,…,N) may be redefined respectively as;

$$\frac{\partial U_x^{(1)}}{\partial x} + \frac{\partial U_y^{(1)}}{\partial y} = -S \frac{\partial h_1}{\partial t} + W \tag{2.13}$$

$$\frac{\partial U_x^{(p)}}{\partial x} + \frac{\partial U_y^{(p)}}{\partial y} = -n \frac{\partial \zeta_p}{\partial t} + W_p \tag{2.14}$$

where,

n is the effective porosity

$W_p$ is a source term below surface p

Combination of (2.13), (2.14) with (2.12) result in;

$$\frac{\partial}{\partial x}\left[\sigma_1 \frac{\partial h_1}{\partial x}\right] + \frac{\partial}{\partial y}\left[\sigma_1 \frac{\partial h_1}{\partial y}\right] = S\frac{\partial h_1}{\partial t} - W + R_1 \qquad (2.15)$$

$$\delta_p \frac{\partial}{\partial x}\left[\sigma_p \frac{\partial \zeta_p}{\partial x}\right] + \delta_p \frac{\partial}{\partial y}\left[\sigma_p \frac{\partial \zeta_p}{\partial y}\right] = n\frac{\partial \zeta_p}{\partial t} - W_p + R_p \qquad (2.16)$$

$$\sigma_p = \sum_{n=p}^{N} T_n = \sum_{n=p}^{N} K_f(\zeta_n - \zeta_{n+1}) \qquad (2.17)$$

where,

$T_n$ is the transmissivity of zone n

$\sigma_p$ is defined as the comprehensive transmissivity below surface p

$\delta_p$ is a measure for the variation of density between zones p and p-1

For stratified flow, $R_1$ and $R_p$ in equation 2.15 and 2.16 are defined as;

$$R_1 = -\sum_{n=1}^{N} \delta_n \frac{\partial}{\partial x}\left[\sigma_n \frac{\partial \zeta_n}{\partial x}\right] - \sum_{n=1}^{N} \delta_n \frac{\partial}{\partial y}\left[\sigma_n \frac{\partial \zeta_n}{\partial y}\right] \qquad (2.18)$$

$$\begin{aligned} R_p = &-\frac{\partial}{\partial x}\left[\sigma_p \frac{\partial h_1}{\partial x}\right] - \sum_{n=1}^{p-1} \delta_n \frac{\partial}{\partial x}\left[\sigma_p \frac{\partial \zeta_n}{\partial x}\right] - \sum_{n=p+1}^{N} \delta_n \frac{\partial}{\partial x}\left[\sigma_n \frac{\partial \zeta_n}{\partial x}\right] \\ &-\frac{\partial}{\partial y}\left[\sigma_p \frac{\partial h_1}{\partial y}\right] - \sum_{n=1}^{p-1} \delta_n \frac{\partial}{\partial y}\left[\sigma_p \frac{\partial \zeta_n}{\partial y}\right] - \sum_{n=p+1}^{N} \delta_n \frac{\partial}{\partial y}\left[\sigma_n \frac{\partial \zeta_n}{\partial y}\right] \end{aligned} \qquad (2.19)$$

where;

$$\delta_1 = \upsilon_1, \qquad \delta_n = \upsilon_n - \upsilon_{n-1} \qquad n = 2,3,...,N \tag{2.20}$$

For leakage, the approach used in SWI is that when water of a certain zone, for example p, flows through leaky layer into another aquifer, it is added as a source to zone p. For multiaquifer systems the leakage terms are included in source terms in the continuity equation.

The vertical upward flow $q_L$ from aquifer d to aquifer c is approximated as;

$$q_L = \frac{h_d - h_c}{C_d} - \frac{1}{2} k_v (\upsilon_d - \upsilon_c) \tag{2.21}$$

where,

$h_c$ is the head at the bottom of aquifer c

$h_d$ is the head at the top of aquifer d

$k_v$ is the vertical hydraulic conductivity of the leaky layer

$C_d$ is the resistance to vertical flow of leaky layer d ($C_d$=B'/K')

Flow in a system with N zones is governed by the set of N differential equations (2.15) and (2.16) with terms (2.18) and (2.19). For the solution of these equations, initial values of the freshwater head $h_1$ at the saturated top of aquifer, and the initial elevations $\zeta_n$ of surfaces 2 through N in each aquifer are specified. Boundary conditions are specified for the head, the head gradient, or a combination thereof, along all the boundaries of the model.

### 2.1.3 Governing equation used in SHARP

Within each aquifer of a layered coastal system the freshwater and saltwater domains are coupled by the common boundary at the interface (Figure 2.3).

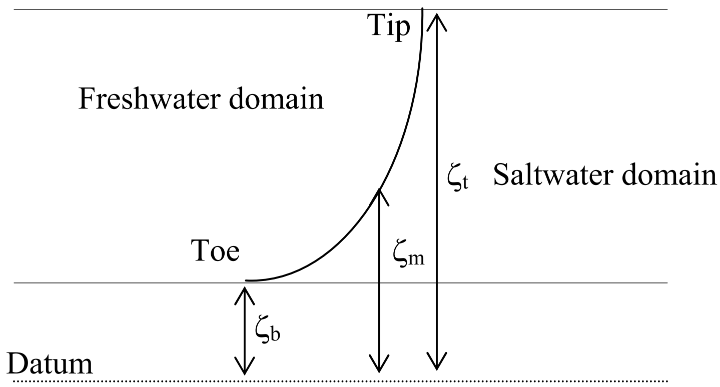



**Figure 2.3** Freshwater and saltwater flow domain in a single aquifer

The vertically integrated continuity equations for each flow domain are respectively (Bear,1979);

$$-\nabla'(B_f \overline{K}_f \nabla' \overline{h}_f) + (n\beta + S_f B_f)\frac{\partial \overline{h}_f}{\partial t} - n(1+\beta)\frac{\partial \overline{h}_s}{\partial t} - q_f' \Big|_{\zeta_t} \nabla' \zeta_t + q_{fz} \Big|_{\zeta_t} = 0 \quad (2.22)$$

$$-\nabla'(B_s \overline{K}_s \nabla' \overline{h}_s) + \left[S_s B_s + n(1+\beta)\right]\frac{\partial \overline{h}_s}{\partial t} - n\beta\frac{\partial \overline{h}_f}{\partial t} + q_s' \Big|_{\zeta_b} \nabla' \zeta_b - q_{sz} \Big|_{\zeta_b} = 0 \quad (2.23)$$

$$\beta = \frac{\rho_f}{(\rho_s - \rho_f)} \tag{2.24}$$

$$q' = q_x e_x + q_y e_y \tag{2.25}$$

$$\nabla'(\ ) = \frac{\partial(\ )}{\partial x} x + \frac{\partial(\ )}{\partial y} y \tag{2.26}$$

where,

$\overline{h}_f$ , $\overline{h}_s$ are the vertically averaged freshwater and saltwater heads

$\rho_s$ is the density of saltwater

$S_f$ , $S_s$ are the freshwater and saltwater specific storages

$B_f$ , $B_s$ are the thicknesses of freshwater and saltwater zones

$\overline{K}_f$ , $\overline{K}_s$ are the vertically averaged freshwater and saltwater hydraulic conductivity tensors

$q_f$ ,$q_s$ are the freshwater and saltwater specific discharges

$q_{fz}$ ,$q_{sz}$ are the vertical components of freshwater and saltwater fluxes

$\zeta_t$ , $\zeta_b$, $\zeta_m$ are the elevations of top, bottom of aquifer and interface respectively

Invoking continuity of pressure at the interface, the interface elevation can be calculated from the freshwater and saltwater heads.

$$\zeta_m = (1+\beta)\overline{h}_s - \overline{h}_f \tag{2.27}$$

In equation (2.22) and (2.23), respectively, the terms;

$$-q_f' \,\Big|_{\zeta_t} \nabla' \zeta_t + q_{fz} \,\Big|_{\zeta_t}$$

$$+q_s' \,\Big|_{\zeta_b} \nabla' \zeta_b - q_{sz} \,\Big|_{\zeta_b}$$

represent the boundary conditions at the top and the bottom of the aquifer. If the boundaries are impermeable, these terms will equal to zero. If the boundaries are leaky, these terms will be given by the leakage through the overlying and underlying confining layers.

Introducing boundary conditions and accounting for source/sink terms, the vertically integrated equations for freshwater and saltwater flow, respectively , become

$$\begin{aligned} &\underset{(1)}{S_f B_f \frac{\partial \bar{h}_f}{\partial t}} + \underset{(2)}{n\alpha \frac{\partial \bar{h}_f}{\partial t}} + \underset{(3)}{\left[ n\beta \frac{\partial \bar{h}_f}{\partial t} - n(1+\beta)\frac{\partial \bar{h}_s}{\partial t} \right]} \\ &= \underset{(4)}{\frac{\partial}{\partial x}(B_f K_{fx} \frac{\partial \bar{h}_f}{\partial x})} + \underset{(5)}{\frac{\partial}{\partial y}(B_f K_{fy} \frac{\partial \bar{h}_f}{\partial y})} + \underset{(6)}{W_f} + \underset{(7)}{Q_{Lf}} \end{aligned} \tag{2.28}$$

$$\begin{aligned} &\underset{(1)}{S_s B_s \frac{\partial \bar{h}_s}{\partial t}} + \underset{(3)}{\left[ n(1+\beta)\frac{\partial \bar{h}_s}{\partial t} - n\beta \frac{\partial \bar{h}_f}{\partial t} \right]} \\ &= \underset{(4)}{\frac{\partial}{\partial x}(B_s K_{sx} \frac{\partial \bar{h}_s}{\partial x})} + \underset{(5)}{\frac{\partial}{\partial y}(B_s K_{sy} \frac{\partial \bar{h}_s}{\partial y})} + \underset{(6)}{W_s} + \underset{(7)}{Q_{Ls}} \end{aligned} \tag{2.29}$$

where,

$K_{fx}$, $K_{sx}$ are the freshwater and saltwater hydraulic conductivities in x direction

$K_{fy}$, $K_{sy}$ are the freshwater and saltwater hydraulic conductivities in y direction

$W_f$, $W_s$ are the freshwater and saltwater source/sink terms

$Q_{lf}$, $Q_{ls}$ are the sums of freshwater and saltwater leakage terms across the top and bottom of the aquifer

$\alpha$ is a parameter equal to 1 for an unconfined aquifer and 0 for a confined aquifer.

In equations (2.28) and (2.29) the type 1, 2, 3, 4, 5, and 6 terms represent the change in elastic storage, change in freshwater storage due to drainage at the water table, change in storage due to movement of the interface, divergence of the fluxes in the x and y directions, sources and sinks (recharge and pumpage), leakage, respectively. Further discussion on the leakage term will be provided in the following paragraph.

Considering the importance of vertical density gradients when waters of different density occur on either side of the confining layer, Darcy's Law is formulated in terms of pressure (Essaid,1990a). Evaluating the pressures above and below the confining layer, the leakage term can be written as

$$q_L = -\frac{K'}{\rho_f B'}\left[\rho_c h_c - \rho_d h_d + \frac{(-\rho_c + \rho_d)(\zeta_c + \zeta_d)}{2}\right] \qquad (2.30)$$

where,

$K'$ is the hydraulic conductivity of the confining layer

$B'$ is the thickness of the confining layer

$\zeta_c$ , $\zeta_d$ are the elevations of the top and bottom of the confining layer

$\rho_c$ , $\rho_d$ are the densities above and below the confining layer

For aquifer systems with predominantly horizontal flow components, it is assumed that when water of one type leaks into a zone of another, the amount of leakage is small relative to the water in place.

## 2.2 Numerical method

The differential equations used in all three codes are very similar. In SWI, equations (2.15) and (2.16); in SHARP, equations (2.28) and (2.29) are another form of equation (2.4) used in MODFLOW including leakage terms and they may be solved using block centered fully implicit finite difference scheme (McDonald and Harbaugh, 1988).

In SWI, equation (2.15) is solved to compute the head at time $t + \Delta t$ , using the elevations of the surfaces at time t to compute $R_1$. Next, equation (2.16) are solved to compute the elevations of the surfaces at time $t + \Delta t$. the terms $R_p$ are computed using the head values at time $t + \Delta t$ and the elevations of the surfaces at time t.

In SHARP, equations (2.28) and (2.29) are first solved for the freshwater head and the saltwater head and then the interface elvation can be obtained from (2.27).

Since the moving boundary is considered at the interface, the position of the interface tip and the interface toe will not always coincide with the block boundaries. To overcome this, tip-toe tracking algorithms are provided.

### 2.2.1 Tip toe tracking algorithm used in SWI

When a surface intersects the bottom of an aquifer or a vertical no-flow boundary, the normal flux is set to zero. When a surface intersects the top of an aquifer, the normal flux is set to the normal component of the comprehensive flow in the aquifer. At the end of each time step, cells along boundary of each surface are evaluated to determine whether the boundary should be moved horizontally. The algorithm for dealing with a toe is presented below (Figure 2.4); the algorithm for a tip is analogous.

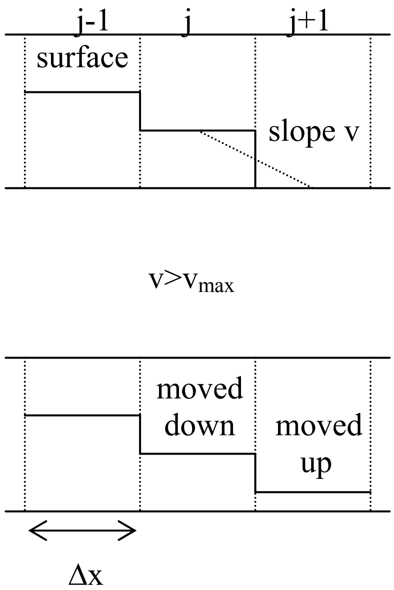



**Figure 2.4** Toe tracking algorithm

The procedure used in SWI is that if the slope v is larger than $v_{max}$, the surface is moved into the adjacent empty cell j+1, by lowering the surface elevation in cell j by a small amount $\Delta\zeta$ and at the same time specifying an elevation $\Delta\zeta$ at the empty cell j+1. Conversely, if the elevation of the surface in cell j is smaller than $\Delta\zeta_{min}$, the surface is moved into adjacent nonempty cell j-1, by lowering surface in cell j to the bottom of the aquifer and at the same time increasing the surface elevation in cell j-1 by the same amount. The maximum slope $v_{max}$ is a representative value of the slope of a surface in an aquifer. Then $\Delta\zeta$ and $\Delta\zeta_{min}$ can be expressed as

$$\Delta\zeta=0.1\, v_{max}\, \Delta x \qquad , \qquad \Delta\zeta_{min}=0.1\Delta\zeta \tag{2.31}$$

### 2.2.2 Tip toe tracking algorithm used in SHARP

SHARP incorporates a tip and toe tracking algorithm based on a weighted extrapolation of the interface slope at finite difference blocks containing the tip and toe. Following the determination of the positions of the tip and toe within the finite-difference grid for each aquifer, the tip and toe are located by linearly projecting the interface, based on the interface slope until they intersect respectively the top and the bottom of the aquifer. Tip projection is carried out at locations where there is a transition from a block containing some freshwater to a block containing no freshwater (Figure 2.5). Similarly, toe projection is carried out where there is a transition from a block containing some saltwater to a block containing no saltwater.

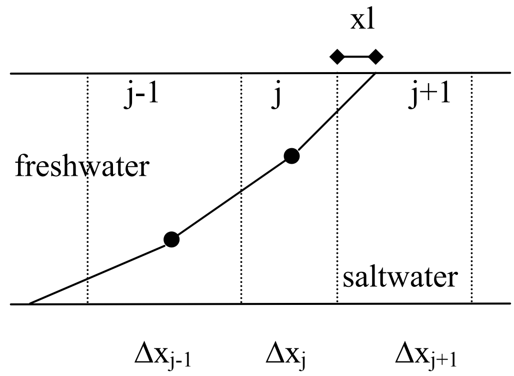



**Figure 2.5** Interface tip projection ( Essaid, 1990b)

In the vicinity of the tip and toe the interface slope in the x and y directions can be calculated on the basis of weighted freswater and saltwater head derivatives.

The weighting factor, $\omega$ is necessary to prevent abrupt changes in slope as the interface tip or toe crosses from one block to another. For the smooth movement of the interface w must be in range ($0 \leq \omega \leq 1$). That is when xl = 0, $\omega = 0$ and whan xl = $\Delta x_{j+1}$, $\omega=1$. Once the interface is located, the leakages across the overlying and underlying confining layers are calculated and the net leakage into a block is obtained by the sum of these leakages.

### 2.3 Model informations of MODFLOW

MODFLOW is a computer program that simulates three-dimensional ground-water flow through a porous medium by using a finite difference method (McDonald and Harbaugh,1988). There are several versions called MODFLOW-84-88-96.

Frequently there is a need to solve additional equations; for example, transport equations and equations for estimating parameter values that produces the closest match between model calculated heads and flows and measured values. To accommodate the solution of equations in addition to the groundwater flow equation a new version of MODFLOW, called MODFLOW-2000, is designed.

To faciliate the capability to simulate different kinds of groundwater flow problems the program is divided into pieces called packages. Each hydrologic capability, such as leakage to rivers, recharge, and evapotranspiration, that is included within the groundwater flow equation is a separate package. Each package is an independent, self-contained unit with its own data input and output.

Basic Package (BAS) provides overall control for the simulation. The BAS package takes care of many administrative tasks such as allocating memory for arrays, keeping track of simulation time, controlling output, printing head, drawdown and volumetric budget output.

There are two internal flow package options. Block Centered Flow Package (BCF) computes flows from cell to cell within the groundwater flow system. It includes the capability to convert dry cells to wet and alternate methods for calculating interblock transmissivity. Layer Property Flow Package (LPF) calculates conductance coefficients and is an alternative to the BCF Package.

Source Term Packages which are also called Stress Packages, are divided according to whether they are head dependent or head independent. Head dependent packages include River Package (RIV), Drain Package (DRN), General Head Boundary Package (GHB) and Evapotranspiration Package (EVT). Head independent packages include Well Package (WEL) and Recharge Package (RCH).

Four solvers are included in MODFLOW-2000. These are strongly implicit procedure (SIP), slice successive overrelaxation (SOR), preconditioned conjugate gradient (PCG) and direct solution based on alternating diagonal ordering (DE4).

Other than those there are discretization file (DIS), multiplier array file (MULT), zone array file (ZONE), output control option file (OC).

MODFLOW-2000 can produce either one or two listing output files depending on whether the processes are used or not. The listing files are named as GLOBAL and LIST.

There is a single NAME file that contains the names of most of the files used by the Ground Water Flow, Observation, Sensitivity, and Parameter Estimation Processes. The NAME file also includes file types that determine which program options are activated. MODFLOW-2000 has a preprocesser, MFI2K, a data input program.

# CHAPTER 3

## HYPOTHETICAL CASE STUDY

A simple idealized rectangular coastal aquifer is used for the three cases wich are going to be presented in the following sections. There are two zones (a freshwater zone and a seawater zone) and the flow is sharp interface flow for an unconfined aquifer. The problems are first solved by SWI and then by SHARP.

### 3.1 Application of SWI

The input files are produced by using MFI2K. The packages used in the case studies are DIS, BAS, BCF, RCH, WELL, OC and PCG. To simulate seawater intrusion with MODFLOW and the SWI package, one additional input file SWI, which is written by using an external editor, is required. This file contains information on the initial density distribution, the initial interface elevations and the parameters for the transient evolution of the density distribution in the aquifer. The name of this file having the extension '.swi' and the name of a binary output file having the extension '.zta' and containing the elevations of the surfaces at the desired times, are added as extra lines in the NAME file.

In SWI, for a coastal aquifer, the ocean is modeled by specifying a thin additional layer that represents the ocean floor, on top of top aquifer (Figure 3.1). The head in this additional fictitious layer is fixed to the average water level in the ocean.

For the DIS package of MFI2K, the main data inputs are as follows: The number of layers, rows and columns and stress periods are respectively; 2, 5, 15, 1. The first 3 columns represent the ocean . Although the hypothetical aquifer is assumed as unconfined, the aquifer is assumed as 2 layered by adding the fictitious layer (Figure 3.1). The time unit and the length unit of model are specified as undefined. The layer type code (LAYCBD) is given for the first additional layer as “1” which is the code for unconfined aquifers, and for the second layer as “0” which is the code for confined aquifers. The grid widths along a row (DELR) and along a column (DELC) are both taken as 100 m. The thickness of the first fictitious layer is taken as 0 m at an elevation of  0.5 m and the second layers thickness is taken as 20 m below seawater level having an elevation of 0m. The length of the stress period is 10000 days and the number of time steps in a stress period is 200, meaning that the time step is 50 days . The Ss/tr parameter in this package is set to Ss meaning that the stress period is steady-state.

For all cases solved by SWI, the comprehensive flow balance is solved assuming steady-state conditions. But as it is stated in the theory section, although the flow in the aquifer is at steady state, the position of the interface changes over time. Hence, SWI  produces different steady-state solutions for different time periods.

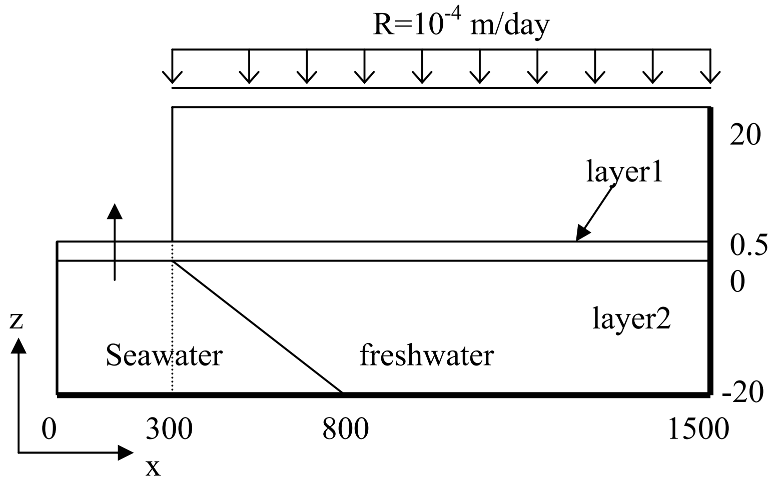



**Figure 3.1** The cross sectional view of SWI model showing the initial conditions

For the BAS package of MFI2K, the main data inputs are as follows: The boundary array (IBOUND), specified by the user and read by the Basic Package, contains a code for each cell which indicates whether (1) the head varies with time (active cell), (2) the head is constant (constant head cell), or (3) no flow takes place within the cell (inactive cell). Here the cells at the upper layer takes a code of “-1” which means constant head and the second layer takes a code of “1” which means variable head. And also the initial heads are defined in this package. For all layers the initial (starting) head is 0.

For the BCF package of MFI2K, the data input can be summarized in the following manner: The horizontal anisotropy factor (TRPY) is defined for each layer. Here for every layer it is taken as 1.0 assuming an isotropic condition. The hydraulic conductivity (HY) along rows is specified for an unconfined aquifer and the transmissivity (TRAN) along rows is specified. In order to obtain HY and TRAN along columns, these values have to be multiplied by TRPY. Here for the upper layer, the hydraulic conductivity is taken as $10^{-5}$m/day and the transmissivity for the confined layer is 200 $m^2$/day. The vertical resistance to outflow in the ocean is c=25 days. The VCONT value which is defined as the vertical hydraulic conductivity divided by the thickness from a layer to the layer below, is 0.02. This value is used only for the upper aquifer because VCONT cannot be specified for the bottom layer.

The recharge flux is $10^{-4}$ m/day and defined in the second layer meaning that the vertical distribution of uniform recharge is applied to second layer. The wells are located at different columns in second layer, along third row. The Q is the volumetric recharge rate. A positive value indicates injection and a negative value indicates pumping. Here the pumping rate is 20 $m^3$/day.

To solve the system of linear algebraic equations Preconditioned Conjugate-Gradient (PCG) package which is the default solution procedure of SWI, is chosen.

For the output control package OC, the numeric codes are used. The format in which heads and drowdowns are printed, is specified by choosing the default option. The unit number for the heads to be saved is 30. The file containing head values is also specified in the NAME file by adding an extra line including the name of the binary file having an extension of ".hds". Head and drowdown are not printed for the corresponding layer. Head is saved for the corresponding layer but drowdown is not. The cell-by-cell flow terms are written to the LIST file.

After the MFI2K is terminated and the packages are generated, the SWI file is added to the NAME file. For SWI package, the number of surfaces, which equals to the number of zones minus 1, is 1. Density distribution is assumed constant. Number of steps between output recordings is 100. It means that considering the total length of stress period as 10000 days, the interface elevations are printed out NSTP/NPRN (200/100) times. The elevations are printed out for 5000$^{th}$ and 10000$^{th}$ days. Maximum slope of tip cells and toe cells are assumed as 0.05. And minimum elevation of a plane before it is removed from a cell $\Delta\zeta$ and $\Delta\zeta_{min}$ are calculated with respect to equation (2.21) and are respectively; 0.25 and 0.025. The dimensionless density values for freshwater and saltwater are respectively; 0 and 0.025. For each layer initial elevations of the interface is written. The elevation of the first layer is constant and is 0.5m. For second layer, the initial interface elevations which are identical in all 5 rows, are given in Table 3.1. The effective porosity for the aquifer is 0.2.

**Table 3.1** Initial interface elevations of layer 2 in SWI model

| Column numbers | Distances along x direction (m) | Initial Interface Elevations (m) |
|---|---|---|
| 1 | 100 | 0 |
| 2 | 200 | 0 |
| 3 | 300 | 0 |
| 4 | 400 | -2.5 |
| 5 | 500 | -7.5 |
| 6 | 600 | -12.5 |
| 7 | 700 | -17.5 |
| 8 | 800 | -20 |
| 9 | 900 | -20 |
| 10 | 1000 | -20 |
| 11 | 1100 | -20 |
| 12 | 1200 | -20 |
| 13 | 1300 | -20 |
| 14 | 1400 | -20 |
| 15 | 1500 | -20 |

For all the cases the output is in binary format because SWI produces output files in binary format. To convert them into ascii format MATLAB is used.

### 3.1.1 Case 1: Rectangular aquifer with a pumping well

In Case 1 with the parameters defined above, the well locations are changed in order to analyze the interface elevation movement. The general input data are summarized in the Table 3.2 for simplification.

**Table 3.2** The general input data for case 1

| | |
|---|---|
| Aquifer length | 1500 m |
| Aquifer width | 500 m |
| Thickness 1st layer | 0.5 m |
| Thickness 2nd layer | 20 m |
| Recharge | 0.0001 m/day |
| Discharge from well | -20 $m^3$/day |
| Hydraulic conductivity of 1st layer | $10^{-5}$ m/day |
| Hydraulic conductivity of 2nd layer | 10 m/day |
| Vertical conductivity | 0.02 m/day |
| Effective porosity | 0.2 |

### 3.1.1.1 Case1-a

The well is placed at 600 m distance. In Figure 3.2, the black borders represent the impervious boundary and the first three columns represent the sea. The interface elevations at 10000th days along 3rd row is plotted in Figure 3.3.

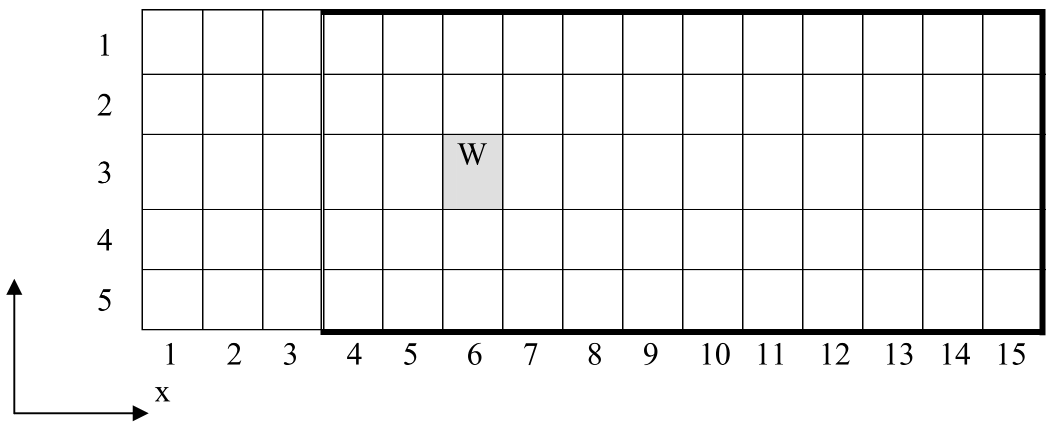



**Figure 3.2** Plan view of idealized rectangular aquifer for case 1-a

Note that there is an upconing at 600m where the pumping well is located.

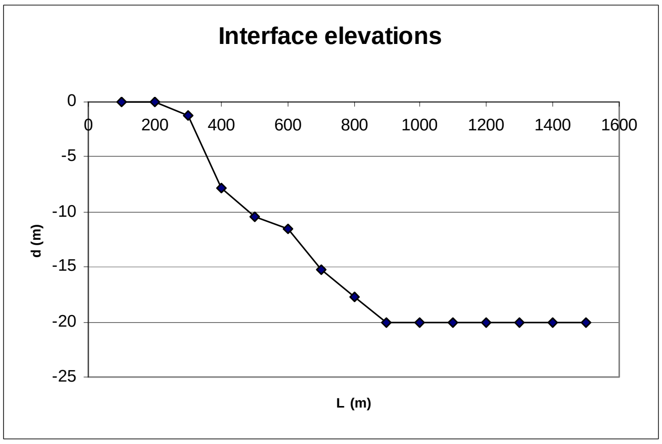



**Figure 3.3** The interface elevations along 3rd row for case 1-a

### 3.1.1.2 Case1-b

The well is shifted to the 8th column along 3rd row by changing the locations in WELL package. The interface elevations at 10000th day are plotted in Figure 3.4. Note that there is an upconing at 800m where the pumping well is located.

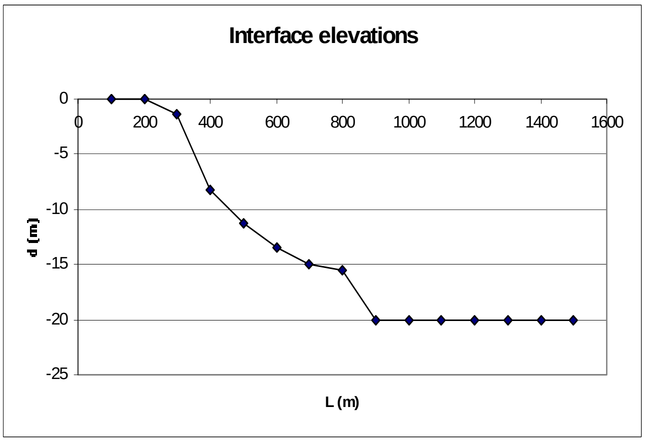



**Figure 3.4** The interface elevation along 3rd row for case 1-b

#### 3.1.1.3 Case1-c

For this case the location of the well is shifted to 10th column along 3rd row. The interface elevations at 10000th day are plotted in Figure 3.5.

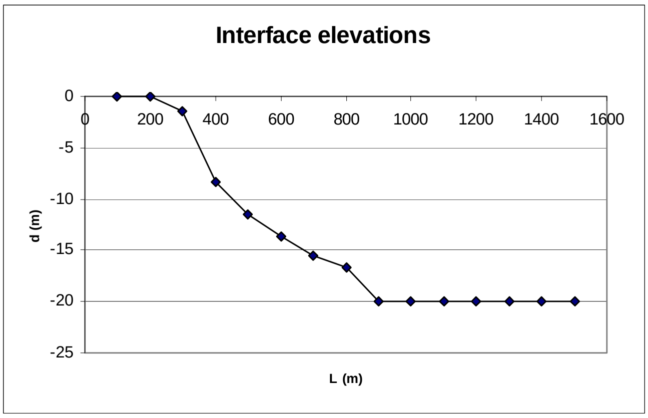



**Figure 3.5** The interface elevation along 3rd row for case 1-c

### 3.1.2 Case 2: Rectangular aquifer with a pumping well and strip recharge

For the second case in addition to natural uniform recharge, a strip recharge is applied along the 6th column to see the effects of artificial recharge on interface movement. The well is located at the 8th column of 3rd row (Figure 3.6). For two different strip recharge rates the solutions are obtained. The recharge package is modified for these cases.

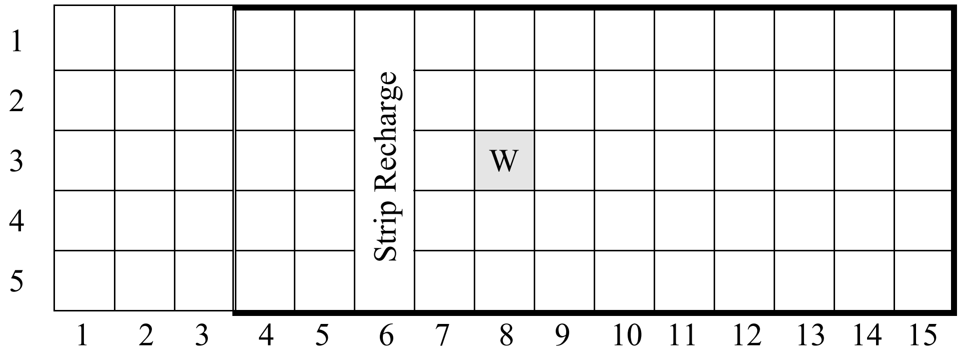



**Figure 3.6** Plan view of idealized rectangular aquifer for case 2

### 3.1.2.1 Case 2-a

The strip recharge rate, $R_{strip}$, is taken as 0.001m/day, 10 times the uniform recharge rate. The interface elevations along 3rd row at 10000th day are plotted in Figure 3.7.

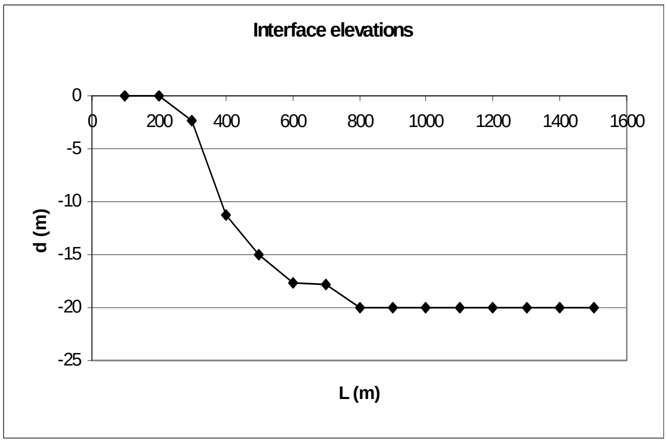



**Figure 3.7** The interface elevation along 3rd row for case 2-a

#### 3.1.2.2 Case 2-b

The strip recharge rate, $R_{strip}$, is increased to 0.0015m/day. The interface elevations along 3$^{rd}$ row at 10000$^{th}$ day are plotted in Figure 3.8.

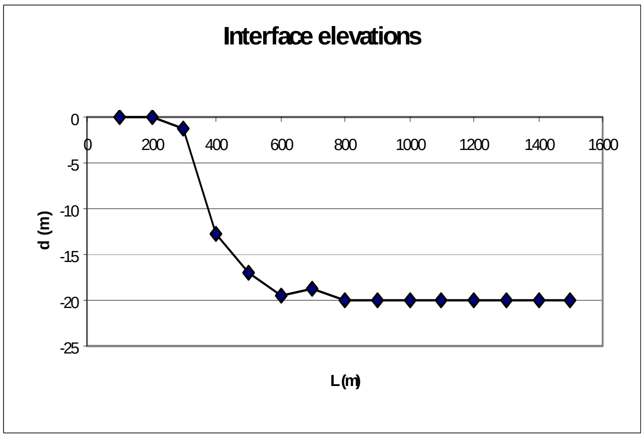



**Figure 3.8** The interface elevation along 3$^{rd}$ row for case 2-b

### 3.1.3 Case 3: Rectangular aquifer with a pumping well and subsurface barrier

For this case to anlyze the interface movement, an impermeable subsurface barrier is applied along the 6$^{th}$ column, having height of 20m from the bottom elevation of coastal aquifer. This method involves establishment of a subsurface barrier to reduce the permeability of the aquifer sufficiently to prevent saltwater intrusion. In order to represent the barrier, the hydraulic conductivity of the 6$^{th}$ column is reduced to $10^{-6}$m/day from 10m/day. The well is located at the 8$^{th}$ column of 3$^{rd}$ row (Figure 3.9).

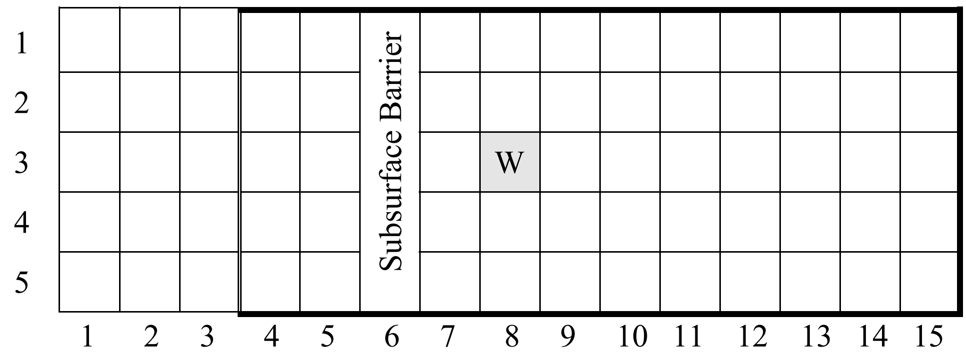



**Figure 3.9** Plan view of idealized rectangular aquifer for case 3

The interface elevations along 3rd row are plotted at 10000th day in Figure 3.10.

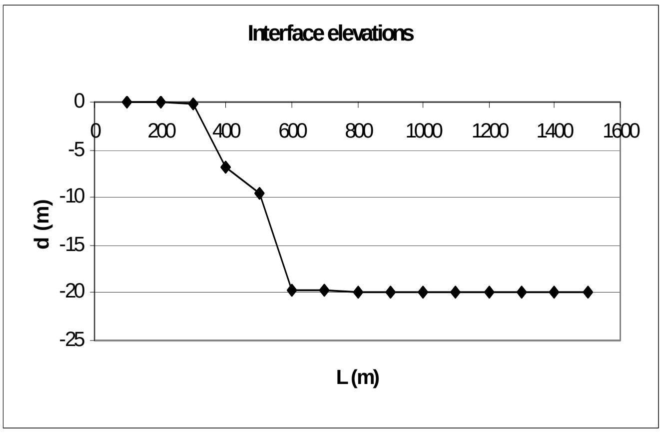



**Figure 3.10** The interface elevation along 3rd row for case 3

### 3.2 Application of SHARP

The hypothetical cases are solved with SHARP to compare the results. To run SHARP an input file named as INPUT is needed. This file is written according to the restrictions defined in the manual (Essaid, 1990b). All the values used in SWI, are same but having time units of seconds instead of days. The plain is discretized by 7 rows and 22 columns. The domain is divided into smaller grids near the well locations so as to get the exact shape of the interface. The outer columns and rows of domain indicate no flow boundary (inactive cells), which prevents inflow or outflow from the system and, has to be specified for SHARP. These no flow boundaries can be assumed as impermeable dikes (Figure 3.11). The 200m distance after the first no flow boundary column represents sea (Figure 3.12).

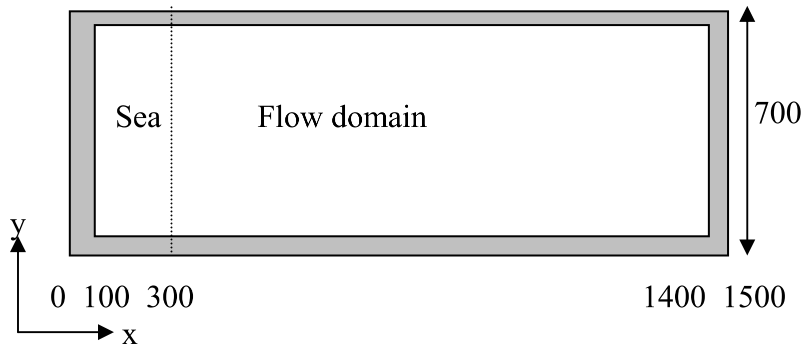



**Figure 3.11** Plan view of idealized rectangular aquifer of SHARP model

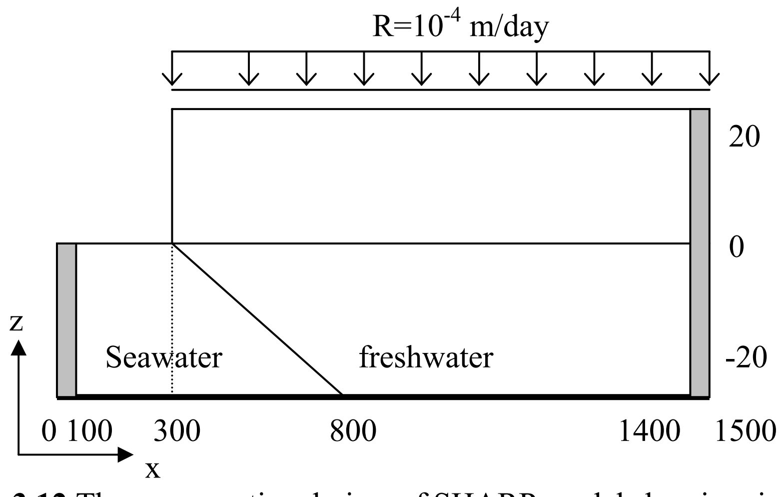



**Figure 3.12** The cross sectional view of SHARP model showing initial conditions

The grid widths in y direction are 100m but the grid widths in x direction differ. The interface slope is same with the one used in SWI. The initial interface elevations and the widths of columns are given in Table 3.3.

**Table 3.3** Initial interface elevations in SHARP model

| Column numbers | Distances along x direction (m) | Initial Interface Elevations (m) |
|---|---|---|
| 1 | 100 | 0 |
| 2 | 300 | 0 |
| 3 | 400 | -2.500 |
| 4 | 425 | -3.750 |
| 5 | 450 | -5.000 |
| 6 | 475 | -6.250 |
| 7 | 500 | -7.500 |
| 8 | 525 | -8.750 |
| 9 | 550 | -10.000 |
| 10 | 575 | -11.250 |
| 11 | 600 | -12.500 |
| 12 | 625 | -13.7500 |
| 13 | 650 | -15.000 |
| 14 | 675 | -16.250 |
| 15 | 700 | -17.500 |
| 16 | 725 | -18.125 |
| 17 | 750 | -18.750 |
| 18 | 775 | -19.375 |
| 19 | 800 | -20.000 |
| 20 | 1000 | -20.000 |
| 21 | 1400 | -20.000 |
| 22 | 1500 | -20.000 |

The hydraulic conductivity in x direction is defined as $1.157*10^{-4}$m/s for active nodes, and 0 for inactive nodes. There is no need to specify the inactive nodes as 0 for the hydraulic conductivity in y direction (Essaid,1990b).

A constant saltwater and freshwater head block is specified by assigning a negative specific storage to that location in the matrix both STORS and STORF. Fixing both freshwater and saltwater head at a node also fixes the interface position in that node. For the second column that represents sea, the heads are fixed.

Initial freshwater heads (PHIF) are obtained by dividing initial interface elevations by “–40” (Ghyben Herzberg interface). Since the aquifer is unconfined the aquifer thickness is greater than maximum saturated thickness and has a value of 40 m (Essaid,1990b).

The recharge is applied all over the domain except the second column representing the sea. In SHARP the well pumpage is defined as positive. Also the elevation of top and bottom of open interval of well is specified respectively as; 20 m, -10 m. In MFI2K only the identification of layer in where the well is located, is sufficient.

The strongly implicit procedure (SIP) is used in the solution of equations. The general input datas are summarized in the Table 3.4 for simplification.

**Table 3.4** The general input data for SHARP

| | |
|---|---|
| Active aquifer length | 1500 m |
| Active aquifer width | 500 m |
| Thickness of the aquifer | 40 m |
| The bottom elevation of aquifer | -20 m |
| Recharge | $1.157*10^{-9}$m/sec |
| Discharge from well (pumpage) | $2.31*10^{-4}$ $m^3$/sec |
| Hydraulic conductivity | $1.157*10^{-4}$m/sec |
| Effective porosity | 0.2 |

### 3.2.1 Case 1: Rectangular aquifer with a pumping well

Owing to the fact that different grid spacing is used, the well locations are at same distance but different columns. For Case 1-a, Case 1-b, Case 1-c the wells are at 11th ,19th and 20th columns respectively. Figures below represent the transient movement of interface at the end of 10000th day. No steady state solution is reached for the specified pumping period,10000 days. (Figure 3.13, Figure 3.14, Figure 3.15)

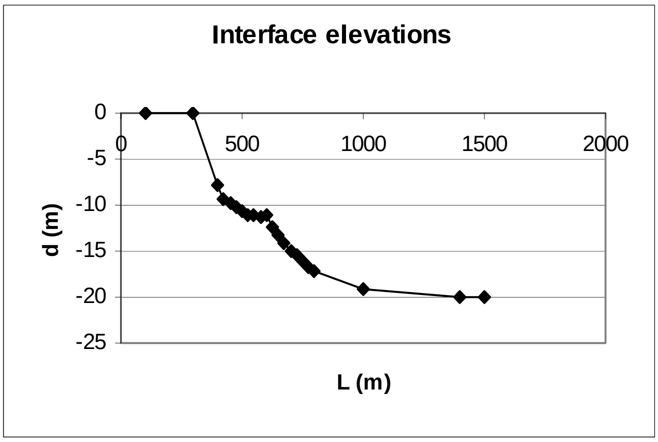



**Figure 3.13** The interface elevation along 3rd row for case 1-a

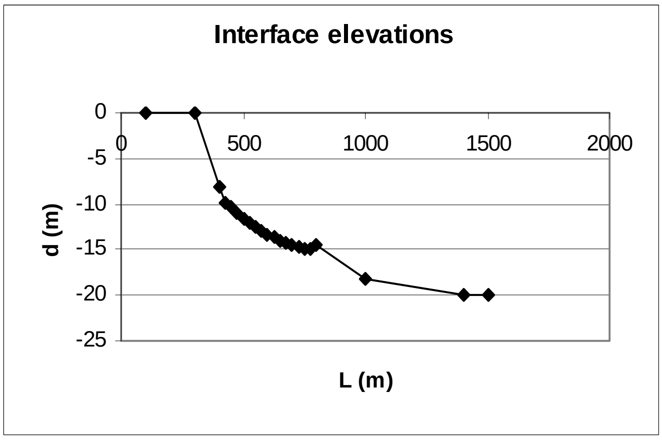



**Figure 3.14** The interface elevation along 3rd row for case 1-b

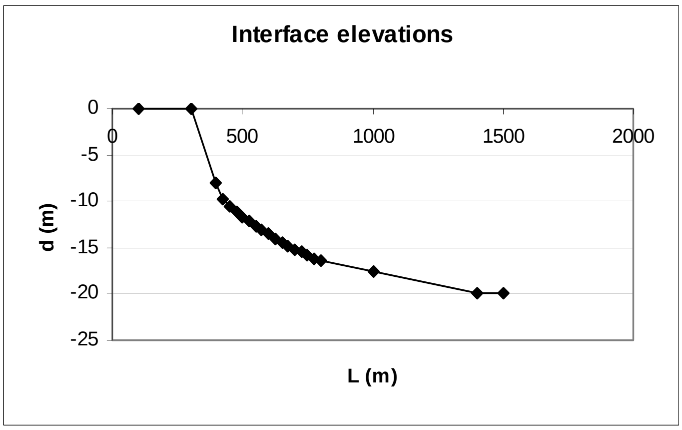



**Figure 3.15** The interface elevation along 3rd row for case 1-c

### 3.2.2 Case 2: Rectangular aquifer with a pumping well and strip recharge

For the second case different grid spacing in x direction is used. In order to obtain more accurate values, the grids spacing near the well and near the strip recharge area, is decreased (Table 3.5).

**Table 3.5** Initial interface elevations in SHARP model for Case 2 and Case 3

| Column numbers | Distances along x direction (m) | Initial Interface Elevations (m) |
|---|---|---|
| 1 | 100 | 0 |
| 2 | 300 | 0 |
| 3 | 400 | -2.500 |
| 4 | 500 | -7.500 |
| 5 | 537.5 | -9.375 |
| 6 | 550 | -10.000 |
| 7 | 575 | -11.250 |
| 8 | 600 | -12.500 |
| 9 | 625 | -13.750 |
| 10 | 637.5 | -14.375 |
| 11 | 650 | -15.000 |
| 12 | 700 | -17.500 |
| 13 | 725 | -18.125 |
| 14 | 750 | -18.750 |
| 15 | 775 | -19.375 |
| 16-22 | 800-1500 | -20.000 |

The well is at 16$^{th}$ column and the strip recharge of $11.57*10^{-9}$m/sec is applied to 6$^{th}$, 7$^{th}$, 8$^{th}$, 9$^{th}$ and 10$^{th}$ columns in order to have the identical area of 500m*100m specified in SWI model. Figure 3.16 shows the effects of strip recharge at the end of 10000$^{th}$ day.

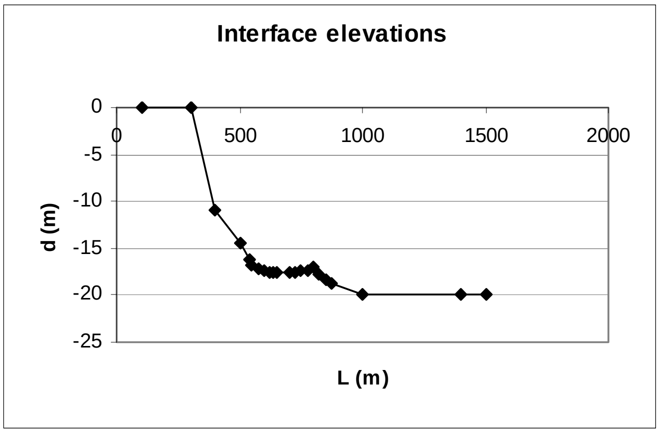



**Figure 3.16** The interface elevation along $3^{rd}$ row for case 2-a

### 3.2.3 Case 3: Rectangular aquifer with a pumping well and subsurface barrier

For the this case, again the grid spacing specified in Table 3.15 is used. The well is at $16^{th}$ column and the hydraulic conductivity of the $6^{th}$, $7^{th}$, $8^{th}$, $9^{th}$ and $10^{th}$ columns is $1.157*10^{-11}$m/sec representing the identical area of subsurface barrier specified in SWI model. Figure 3.17 shows the effects of subsurface barrier at the end of $10000^{th}$ day.

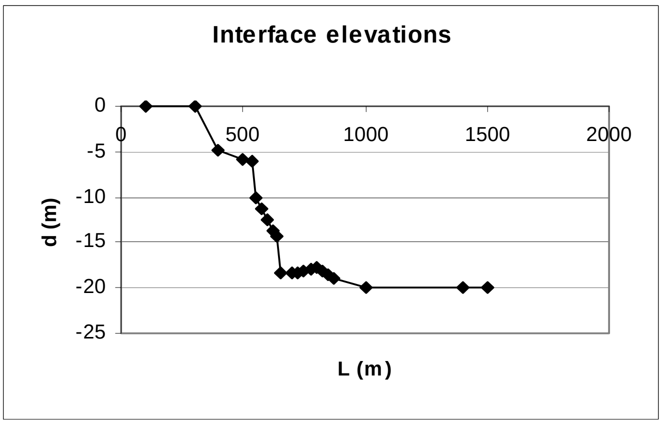



**Figure 3.17** The interface elevation along $3^{rd}$ row for case 3

# CHAPTER 4

# REAL CASE STUDY

## 4.1 Description of the study area

The study area is near Kocaalan Creek in Çamlı Köyü, Marmaris,Muğla and is located between 36° 57’ to 37° 00’ latitude north, and between 28° 15’ to 28° 18’ longitude east. The area is approximately 25 $km^2$. The average mean annual precipitation is 1193.4 mm and average annual temperature is 18.56°C.

The most important river in the region is Kocaalan Creek. The discharge values around Kocaalan Creek are measured on 4 different points for 18 months. By these measurements the annual average discharge of Kocaalan Creek is estimated as 2.27 $m^3/s$. The sources at around the study area, flow seasonally and diminish by summer. The hydraulic conductivity of the aquifer is 0.000292 m/s in both horizontal and vertical directions. The bottom elevation of the aquifer with respect to the mean sea level is taken as -61.5 m. The location of the site is given in Figure 4.1, 4.2.

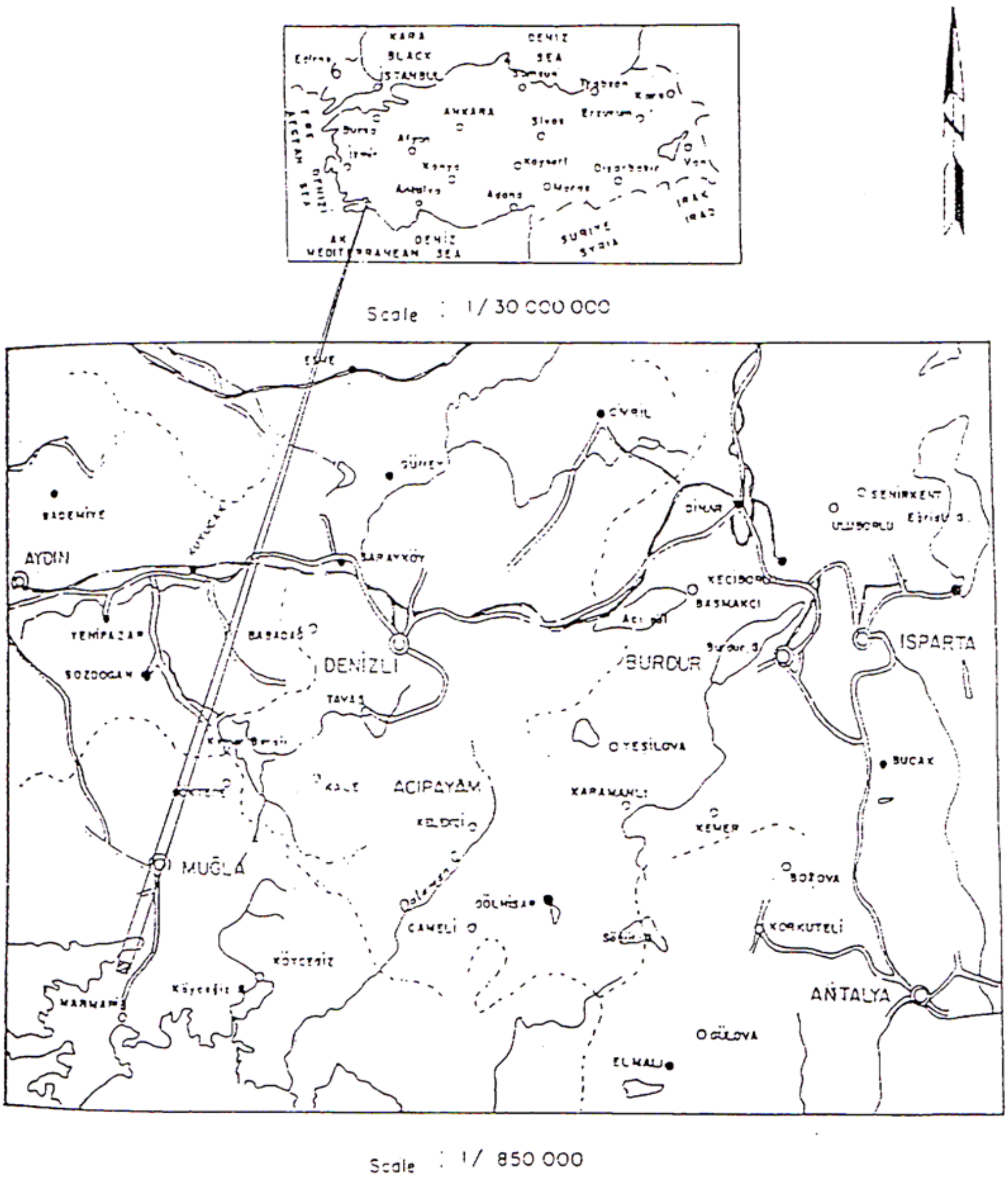



**Figure 4.1** The study area and its location in Turkey

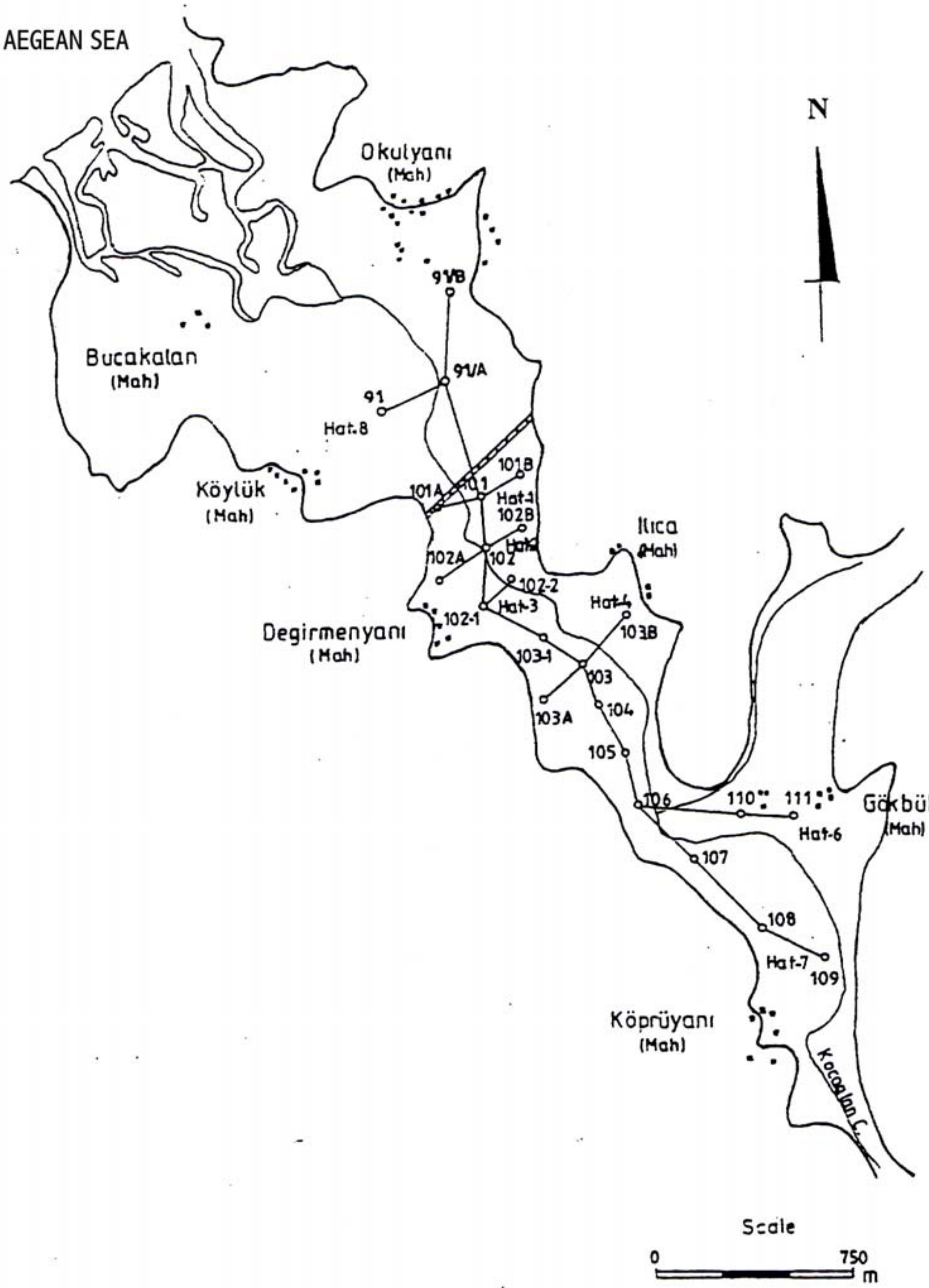



**Figure 4.2** Map of the aquifer and the construction site

## 4.2 Construction of model

The flow domain is discretized by 20 rows and 40 columns. The grid scales are 100 m along x direction and 125 m along y direction. There is a uniform recharge and there are two pumping wells along the $12^{th}$ row at columns 19 and 24.

The problem is solved by SWI and by SHARP. For SWI, steady satate solutions at the end of $3990^{th}$, $7980^{th}$ days are obtained. In SHARP, the steady state condition is reached at the end of $7980^{th}$ day. The transient case output for $3990^{th}$ day is also added in order to compare with the output of SWI.

### 4.2.1 Application of SWI

The discretized domain is modified to obtain the SWI model. Two columns are inserted to the leftside of the domain so as to represent the sea. By adding these columns, the flow domain having 20 rows and 42 columns is obtained (Figure 4.3).

Although the aquifer is unconfined and 1 layered, the fictitious layer having 1 m depth, is added at the top of top aquifer as a result of SWI restrictions.

Some of the inputs that should be used in SWI were lacking in the report. Therefore values consistent with the site area are selected. The inputs are summarized in Tables 4.1 and 4.2 for simplicity. The additional cell locations are shown with "-" sign in Table 4.1.

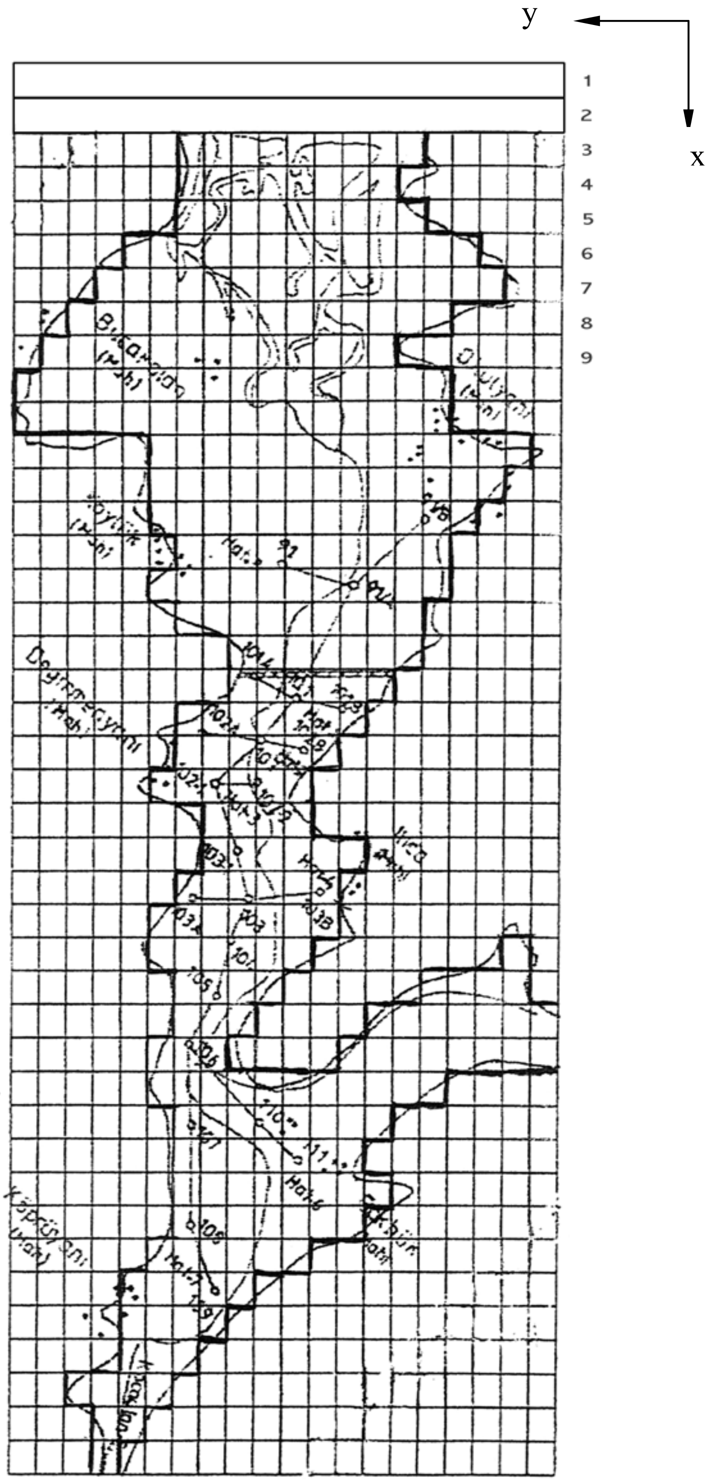



**Figure 4.3** Finite difference grid and approximation of aquifer boundaries for SWI model

**Table 4.1** Initial interface elevations in SWI model

| Column numbers | Distances along x direction (m) | Initial Interface Elevations (m) |
|---|---|---|
| 1 | -100 | 0 |
| 2 | -0 | 0 |
| 3 | 100 | -2.2 |
| 4 | 200 | -4.4 |
| 5 | 300 | -6.65 |
| 6 | 400 | -8.8 |
| 7 | 500 | -11 |
| 8 | 600 | -13.2 |
| 9 | 700 | -15.4 |
| 10 | 800 | -17.6 |
| 11 | 900 | -19.8 |
| 12 | 1000 | -22 |
| 13 | 1100 | -24.2 |
| 14 | 1200 | -26.4 |
| 15 | 1300 | -28.6 |
| 16 | 1400 | -30.8 |
| 17 | 1500 | -32.9 |
| 18 | 1600 | -35.1 |
| 19 | 1700 | -37.3 |
| 20 | 1800 | -39.5 |
| 21 | 1900 | -41.7 |
| 22 | 2000 | -43.9 |
| 23 | 2100 | -46.1 |
| 24 | 2200 | -48.3 |
| 25 | 2300 | -50.5 |
| 26 | 2400 | -52.7 |
| 27 | 2500 | -54.9 |
| 28 | 2600 | -57.1 |
| 29 | 2700 | -59.3 |
| 30-42 | 2800-4000 | -61.5 |

**Table 4.2** The general input data of SWI model

| | |
|---|---|
| Aquifer length | 4200 m |
| Aquifer width | 2500 m |
| Thickness of 1st layer | 1 m |
| Thickness of 2nd layer | 61.5 m |
| Recharge | $8.175*10^{-4}$ m/day |
| Discharge from well 1 | -900 $m^3$/day |
| Discharge from well 2 | -900 $m^3$/day |
| Hydraulic conductivity of 1st layer | $10^{-5}$ m/day |
| Hydraulic conductivity of 2nd layer | 25.23 m/day |
| Vertical conductivity | 0.02 m/day |
| Effective porosity | 0.3 |

The resulting interface elevations along x and y directions at 7980th day are given in Figures 4.4 and 4.5.

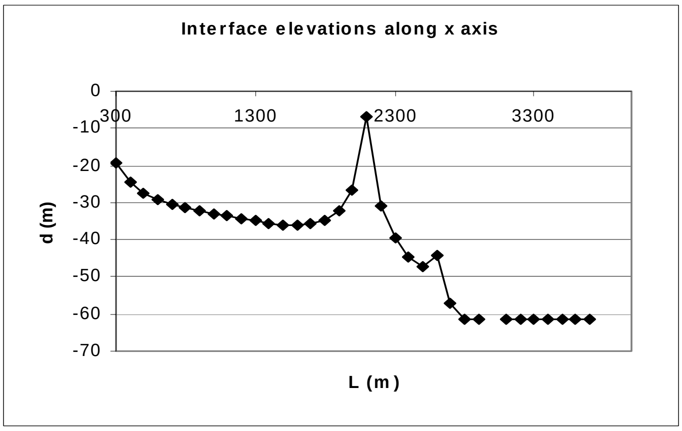



**Figure 4.4** Interface elevations at 7980th day along 12th row

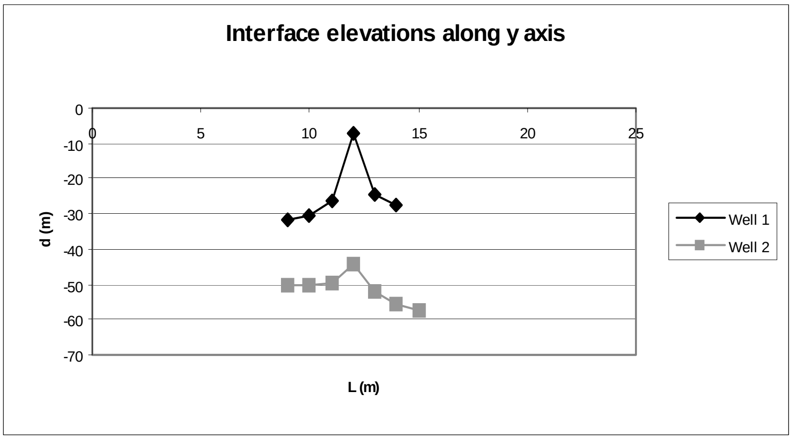



**Figure 4.5** Interface elevations at 7980$^{th}$ day along 21$^{th}$ and 26$^{th}$ columns

### 4.2.2 Application of SHARP

The discretized domain is modified to obtain the SHARP model. Two columns and two rows are added around the domain so as to represent the no flow boundary. These no flow boundaries can be assumed as impermeable dikes. And also one more column is added to the left side as a second column having length of 1000 m in x direction to represent sea. By adding these columns, the flow domain having 22 rows and 43 columns is obtained (Figure 4.6).

The elevation of top and bottom of open interval of well is specified respectively as; 10, -61.5. And the land surface is specified as 60m from the mean sea level. As the inputs should be in time units of seconds, the units are converted. The inputs are summarized in Table 4.3 for simplicity. Initial interface elevations are exactly the same with the ones used in SWI model.

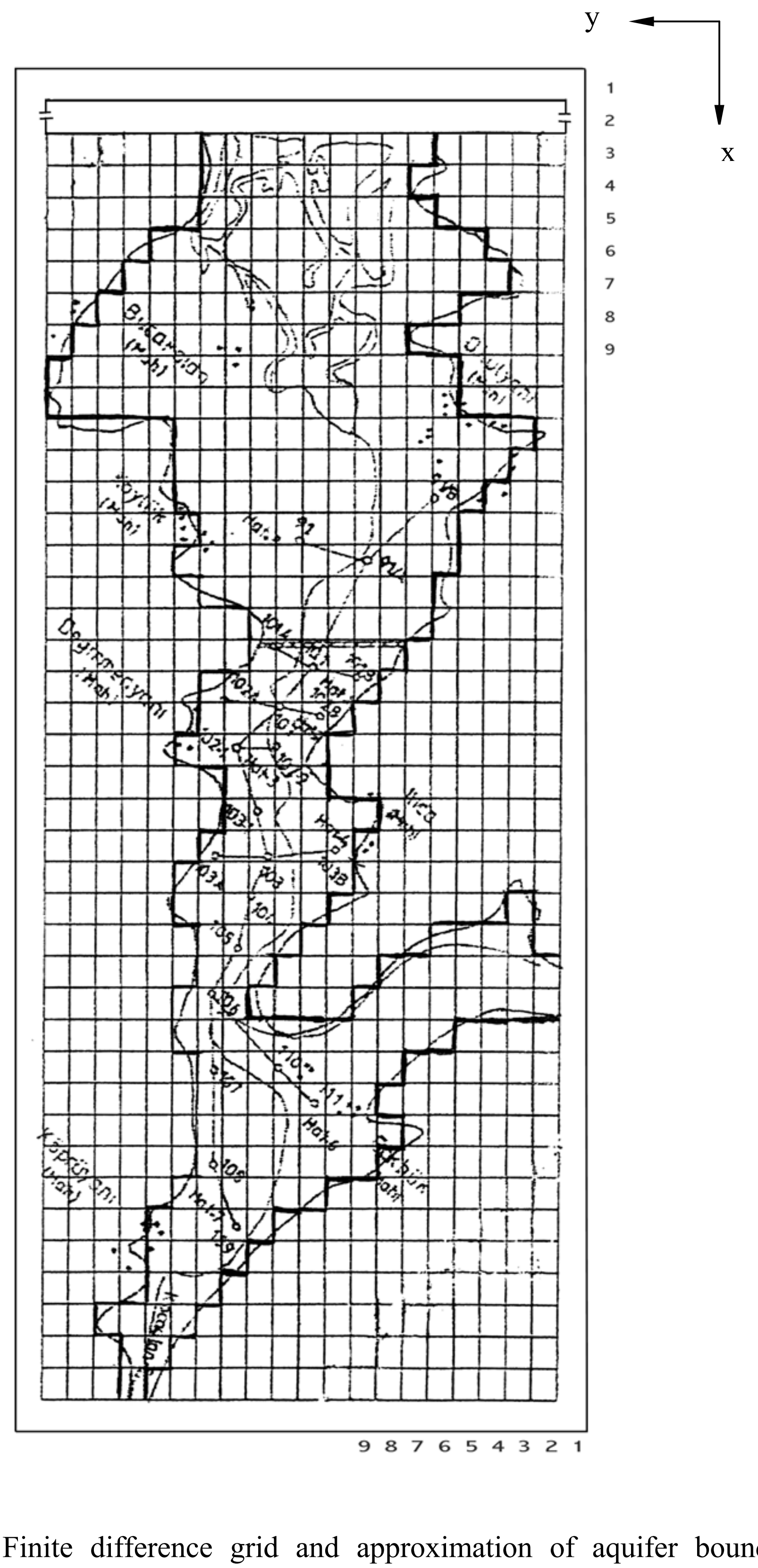


**Figure 4.6** Finite difference grid and approximation of aquifer boundaries for SHARP model

**Table 4.3** The general input data for SHARP

| | |
|---|---|
| Active aquifer length | 5000 m |
| Active aquifer width | 2000 m |
| Thickness of the aquifer | 120.5 m |
| The bottom elevation of aquifer | -61.5 m |
| Recharge | $9.462*10^{-9}$m/sec |
| Discharge from well 1 (pumpage) | $1.042*10^{-2}$ $m^3$/sec |
| Discharge from well 2 (pumpage) | $1.042*10^{-2}$ $m^3$/sec |
| Hydraulic conductivity | $2.92*10^{-4}$m/sec |
| Effective porosity | 0.3 |

The resulting interface elevations along x and y directions at 7980$^{th}$ day are given in Figures 4.7 ,4.8.

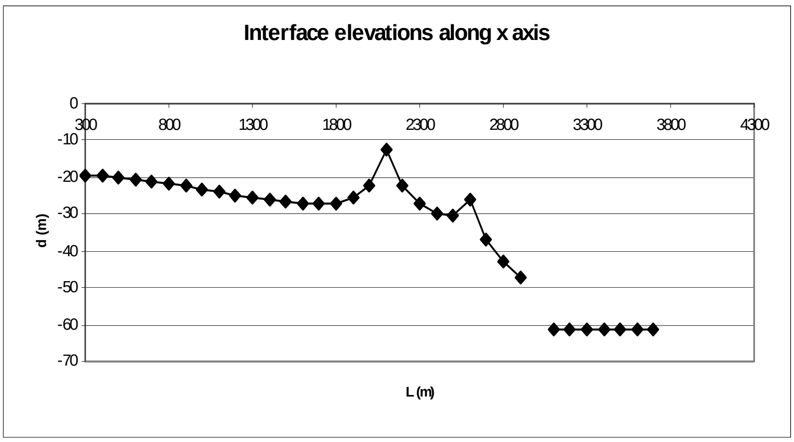



**Figure 4.7** Interface elevations at 7980$^{th}$ day along 13$^{th}$ row

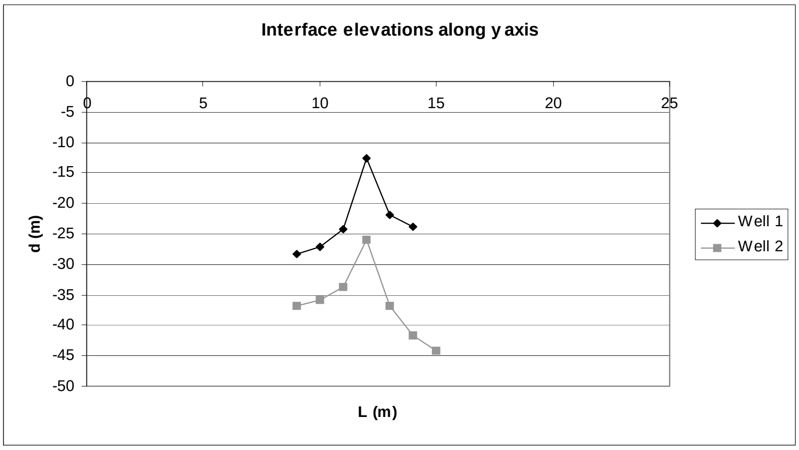



**Figure 4.8** Interface elevations at 7980$^{th}$ day along 21$^{th}$ and 26$^{th}$ columns

# CHAPTER 5

## DISCUSSION OF RESULTS

The interface elevations obtained for Case 1-a, containing uniform recharge and a well at the $6^{th}$ column 300 m inland from the coast line, are similar in SWI and SHARP applications, resulting with an upconing below pumping well. In SHARP along the $3^{rd}$ row, the interface rises to the elevation of -11.10 m, and in SWI the interface rises to the elevation of -11.49 m below pumping well (Figure 5.1). Considering the fact that SHARP model gives more accurate results when smaller sized grids are used in the pumping area, variable grid widths are used for SHARP model. From the results it can be seen that the variable grid spacing is an important point in SHARP model, as both simulations give similar results although same grid spacing of 100m is used in SWI. The similar results are also obtained for Case 1-b and Case 1-c.

For Case 2-a, by applying strip recharge in addition to the uniform recharge and well at the $8^{th}$ column, the upconing is lowered down. Both models give the same reaction to strip recharge application and the interface elevation below the pumping well is -17.05 m -20.00 m in SHARP and SWI, respectively (Figure 5.2). For Case 2-b, by increasing the strip recharge rate, the upconing is fully blocked in SWI.

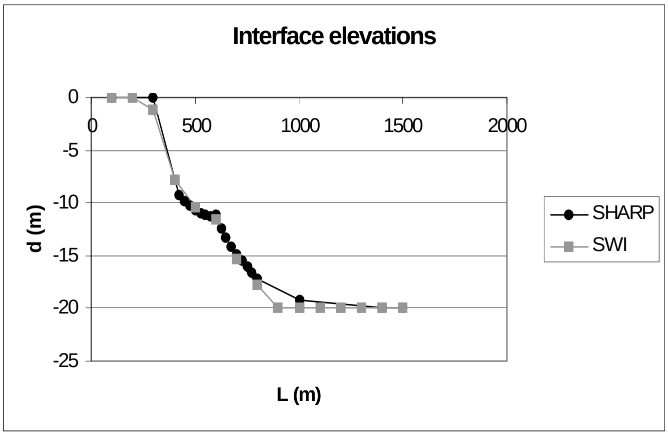



**Figure 5.1** Comparison of SWI and SHARP models for case 1-a at 10000$^{th}$ day

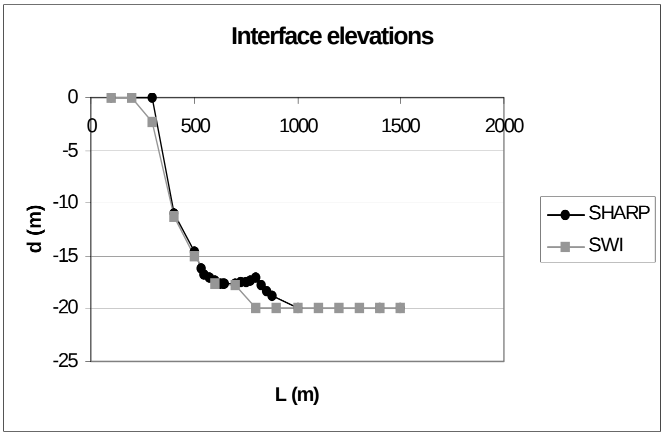



**Figure 5.2** Comparison of SWI and SHARP models for Case2-a at 10000$^{th}$ day

For Case 3, by applying a subsurface barrier, which is simulated by assigning very low hydraulic conductivity, the upconing is again lowered down in both models. The interface elevation below the pumping well is -17.73 m  -20.00 m in SHARP and SWI, respectively (Figure 5.3).

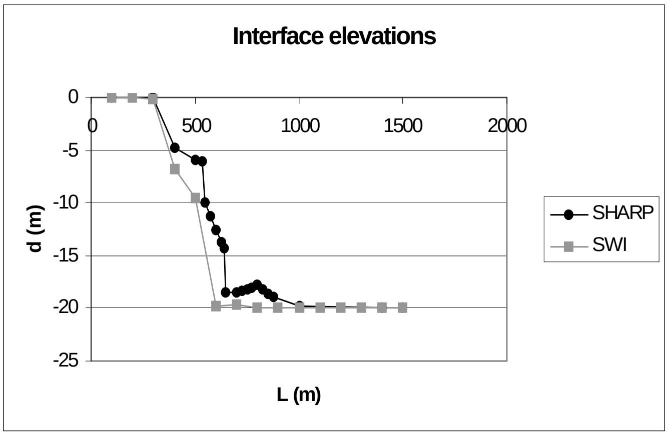



**Figure 5.3** Comparison of SWI and SHARP models for Case3 at 10000$^{th}$ day

In real case study, different than the hypothetical cases, the irregular domain is used. The interface elevations obtained for the real case are different in SWI and SHARP models. The results are obtained for transient condition at 3990$^{th}$ day (Figure 5.4), and for steady state case at 7980$^{th}$ day (Figure 5.5).

At the end of 3990 days, there is a 6m difference between the interface elevations of SWI and SHARP models, approximately. The upconing in the first well located at 21$^{th}$ column, reaches to elevation of –16.88m and  –14.75m in SWI and SHARP, respectively. For the second well located at 26th column, the upconing reaches to elevations of –44.40m and –35.10m in SWI and SHARP, respectively.

At the end of 7980 days, the difference of interface elevations between SWI and SHARP models, becomes larger. The upconing in the first well located at 21th column, reaches to elevation of –6.92m and –12.60m in SWI and SHARP, respectively. For the second well located at 26th column, the upconing reaches to elevation of –44.30m and –25.90m in SWI and SHARP, respectively.

The difference in interface elevations of SWI and SHARP models, under the effects of uniform recharge and two pumping wells, increases with time. This variation may be a result of different tip and toe tracking algorithms that the models adopted in solution procedure and the effects of these algorithms on irregular domains.

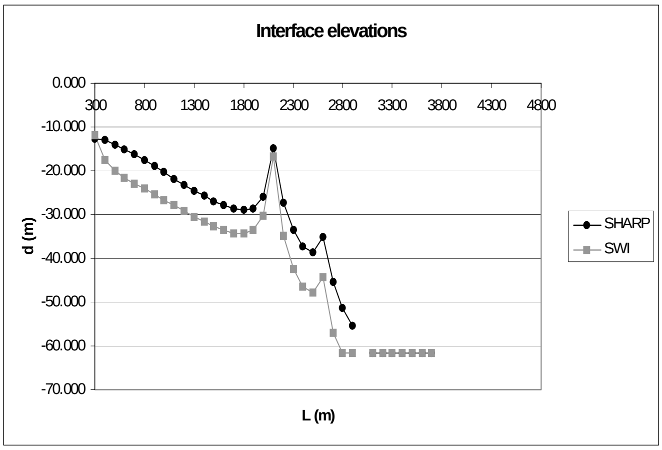



**Figure 5.4** Comparison of SWI and SHARP models for real case at 3990$^{th}$ day

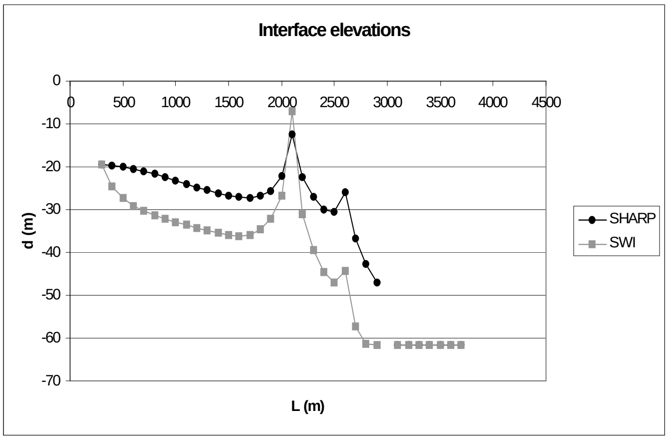



**Figure 5.5** Comparison of SWI and SHARP models for real case at 7980$^{th}$ day

# CHAPTER 6

## CONCLUSION AND RECOMMENDATIONS

In this study, the saltwater intrusion problem in coastal aquifers is discussed. Considering sharp interface approach, the movement of interface is analyzed for hypothetical and real cases by using two computer codes, SWI and SHARP. For hypothetical cases, an idealized rectangular unconfined coastal aquifer is considered. For the real case study, a site in Çamlı Köyü, Marmaris, Muğla is used. For the real case study some of the input data necessary for modeling were lacking. In order to overcome this shortage, appropriately estimated values are used.

Two techniques are discussed in prevention of saltwater intrusion. The strip recharge and subsurface barrier techniques are applied to hypothetical cases. And for all cases the comparison of the results of two computer codes are done. The resulting conclusions are summarized as follows;

The computer models SWI and SHARP give similar results for hypothetical cases.

By applying strip recharge and subsurface barrier, the interface elevation upconing, as a result of pumping, is lowered down. It may be concluded that these two techniques are efficient in controlling the saltwater intrusion.

For real case study, there are differences in interface elevations obtained by SWI and SHARP. This difference may be a result of the impact of different tip and toe tracking algorithms that are adopted by SWI and SHARP, over the irregular flow domain.

The computer models give more accurate results if smaller sized grids are used near the extraction areas and the areas where the prevention techniques will be applied.

Both models have some advantages and disadvantages. Since SWI is a package program using MODFLOW 2000 and its preprocessor MFI2K, it is easy to use. The older versions work under DOS, but MFI2K works under WINDOWS. The grid sizes are not much important as in SHARP. SWI will be upgraded to overcome the difficulties in adding an extra layer. SHARP uses DOS editor in preparation of input file but the output files are in ASCII format. In SWI, to obtain the output files another external program is used to convert them to ASCII. SHARP has some restrictions on some parameter sizes. Sometimes the model needs to be recompiled after rearranging the array sizes to satisfy your conditions. This is an additional effort on getting the results.

Based on the conclusion achieved during the thesis, followings are the suggestions for the further studies in saltwater intrusion in coastal aquifers.

- The computer models can be tested by using smaller sized grids near the extraction areas and near the areas where the prevention techniques will be applied.

- In the thesis, no case representing the multi layered coastal aquifer system, is used. The layered aquifer simulations can be done, by using these computer codes. Particularly the computer model SHARP is written for layered coastal aquifer systems.

- In prevention of saltwater intrusion, there are different techniques described in Chapter 2. They may also be applied and modeled by using SWI and SHARP.

# REFERENCES


Anderson, P.F., White, H.O., Mercer, J.W., Truschel, A.D., Huyakorn, P.S., 1986, Numerical modeling of groundwater flow and saltwater transport in Northern Pinellas Country, Florida, Proceedings of the focus conference on southeastern groundwater issues, pp. 419-449

Anderson, P.F., White, H.O. and Mercer, J.W., 1988, Numerical modeling of saltwater intrusion at Hollandale, Florida, Ground Water, 26(5), 619-630

Andrews, R.W., 1981, Saltwater intrusion in the Costa de Hermosillo, Mexico: A numerical analysis of water management proposals, Ground Water, 19(6), 635-647

Badon Ghyben, W., 1889, Notes on the probable results of the proposed well drilling near Amsterdam, K. Inst. Ing. Tijdschr., The Hague, p.21

Bakker, M., 1998, Transient Dupuit interface flow with partially penetrating features, Water Resour. Res., 34(11), 2911-2918

Bakker M, Schaars F., 2003, The Sea Water Intrusion (SWI) Package Manual Version 0.2, The University of Georgia

Bakker, M., 2003, A Dupuit formulation for modeling seawater intrusion in regional aquifer systems, Water Resour. Res., 39(5), 1131-1140

Bear, J. and Dagan, G., 1964b, Some exact solutions of interface problems by means of the hydrograph method, J.Geophys. Res., 69(8), 1563-1572

Bear, J., 1979, Hydraulics of groundwater, New York, McGraw-Hill, 569p

Bennett, G.D. and Giusti, E.V., 1971, Coastal groundwater flow near Ponce, Puerto Rico, U.S. Geol. Surv., 206-211

Bruington, A.E., 1969, Control of sea water intrusion in a groundwater aquifer, Ground Water J., 7(3), 180-187

Cooper, H.H., 1959, A hypothesis concerning the dynamic balance of freshwater and saltwater in a coastal aquifer, J. Geophys. Res., 64(4), 461-467

Essaid, H. I., 1986, A comparison of the coupled freshwater-saltwater flow and the Ghyben Herzberg sharp interface approaches to modeling of transient behavior in coastal aquifer systems, J. Hydrol., 86, 169-193

Essaid, H. I., 1990a, A multilayered sharp interface model of coupled freshwater and saltwater flow in coastal systems: model development and application, Water Resour. Res., 26(7), 1431-1454

Essaid, H. I., 1990b, The computer model SHARP, a quasi three-dimensionalfinite difference model to simulate fresh water and saltwater flow in layered coastal aquifer systems, U.S. Geological Survey Water Resources Investigations Report 90-4130, 130p.

Garza, S., 1982, Projected effects of proposed salinity control projects on shallow groundwater, U.S. Geol. Surv., Open File Rep. 82-908, 46pp

Glover, R.E., 1959, The pattern of freshwater flow in a coastal aquifer, J. Geophys. Res., 64(4), 457-459

Guo, W., and Langevin, C.D., 2002, Users guide to SEAWAT: A computer program for simulation of three dimensional variable density groundwater flow, U.S. Geological Survey Open-File, 01-434

Guswa, J.H. and LeBlanc, D.R., 1981, Digital models of groundwater flow in the Cape Cod aquifer system, Massachusetts, Water Resour. Invest., Open-File Rep., 80-67,127pp

Harbough, A. W., Banta, E. R., Hill, M. C.,and McDonald, M. G., 2000, User guide to modularization concepts and the ground-water flow process, the U.S. Geological Survey modular ground-water model, U.S. Geological Survey Open-File Report 00-92, 121 p.

Henry, H.R., 1959, Salt intrusion into freshwater aquifers, J. Geophys. Res., 64(11), 1911-1919

Hubbert, M.K., 1940, The theory of groundwater motion, J.Geol., 48(8), 785-944
Jacob, B., 1979, Hydraulics of groundwater, New York, McGraw-Hill, 569p

Huyakorn, P.S., Anderson, P.F., White, H.O., Mercer, J.W., Truschel, 1987, Saltwater intrusion in aquifers, Water Resour. Res., 23(2), 293-312

Kipp, K.L., 1987, HST3D, A computer code for simulation of head and solute transport in three dimensional groundwater flow systems, U.S. Geol. Surv. Water Resour. Invest., 780-26

Kohout, F.A., 1964, The flow of freshwater and saltwater in the Biscayne aquifer of the Miami area, Florida, U.S. Geol. Surv., Water-Supply Pap. 1613-C, p.12-32

Konikow, L.F., Goode, D.J., Hornberger, G.Z., 1996, A three dimensional method of characteristics solute transport model (MOC3D), U.S. Geol. Surv. Water Resour. Invest., 96-4267

Kuiper, L.K., 1983, A numerical procedure for the solution of the steady state variable density groundwater flow equation, Water Resour. Res., 19(1), 234-240

Lester, B., 1991, SWICHA, A Three-Dimensional Finite-Element Code for Analyzing Seawater Intrusion in Coastal Aquifers, Version 5.05. GeoTrans, Inc., Sterling, Virginia, U.S.A., IGWMC, International Ground Water Modeling Center, Delft, the Netherlands: 178 pp.

McCracken, G., Voss, C., Pinder, G. and Ungs, M., 1977, Block iterative finite element preprocessed scheme, a package for simulation of nonlinear transient problems with one or two governing equations in two or three dimensions, Princeton University Water Resour. Program, 77-WR-12, 450pp

McDonald, M.G., and Harbough A.W., 1988, A modular three-dimensional finite difference groundwater flow model, U.S. Geol. Surv. Techniques Water Resour. Invest., 6(A1)

Mercer, J.W., Larson, S.P. and Faust, C.R., 1980a, Finite difference model to simulate the areal fflow of saltwater and freshwater separated by an interface, U.S. Geol. Surv., Open-File Rep., 80-407, 88pp

Mualem, Y. and Bear, J., 1974, The shape of the interface in steady flow in a stratified aquifer, Water Resour. Res., 10(6), 1207-1215

Muskat, M., 1937, The flow of homogeneous fluids through porous media, McGraw-Hill, New York, 763pp

Oude Essink, G.H.P., 2001, Saltwater intrusion in a three-dimensional groundwater system in the Netherlands, Transp. Porous Media, 43, 137-158

Pinder, G. F. and Page, R. H., 1977, Finite element simulation of saltwater intrusion on the south fork of Long Island, First International Conference on Finite Elements in Water Resources, Pentech, London, 2.51-2.69pp

Reilly, T. E., Goodman A. S., 1985, Quantitive analysis of saltwater-freshwater relationships in groundwater systems-a historical perspective, J. Hydrol., 80, 125-160

Shamir, U. nad Dagan, G., 1971, Motion of the seawater interface in coastal aquifers, Water Resour. Res., 7(3), 644-657

Todd, D.K., 1974, Salt water intrusion and its control, J. AWWA, 180-187

Todd, D.K., 1980, Groundwater Hydrology, Wiley, New York, pp. 494-520

Vanden Berg, A., 1974, A digital simulation of horizontal saltwater encroachment induced by freshwater pumping, Inland Waters Directorate, Water Resour. Branch, Ottowa, Ont., 41, 27 pp

Voss, C. I.,1984, SUTRA, A finite element simulation for saturated-unsaturated, fluid-density-dependent ground-water flow with energy transport or chemically reactive single-species solute transport, U.S.G.S. Water-Resources Investigations Report 84-4369: 409

Voss, C. I., 1985, A finite element simulation model for saturated-unsaturated fluid density dependent groundwater flow with energy transport or chemically-reactive single species solute transport, U.S. Geol. Surv., Water Resour. Invest. Rep., 84-4369, 409pp

Voss, C. I., and Souza, W.R. ,1987, Variable density flow and solute transport simulation of regional aquifers containing a narrow freshwater-saltwater transition zone; Water Resour. Res., 23 (10): 1851-1866.13

Weiss, E., 1982, A model for the simulation of flow of variable density groundwater in three dimensions under steady state conditions, , U.S. Geol. Surv., Open-File Rep., 82-352, 59pp

Wilson, J.L. and Sa Da Costa, A., 1982, Finite element simulation of saltwater freshwater interface with indirect toe tracking, Water Resour. Res., 18(4), 1069-1080

Zhang, C., Wang P.P., 1999, A modular three dimensional multi species transport model for simulation of advection, dispersion and chemical reactions of contaminants in groundwater systems; documentation and users guide, U.S. AERDCC, Rep. SERDP-99-1