

SHORT -TERM STATISTICS OF WIND WAVES  
AROUND THE TURKISH COAST

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Approval of the Graduate School of Natural and Applied Sciences

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## **ABSTRACT**

### **SHORT-TERM STATISTICS OF WIND-WAVES AROUND THE TURKISH COAST**

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In this thesis, the wind-wave records obtained at three locations along the Turkish coasts (Alanya, Dalaman and Hopa) are analyzed. Probability distributions of individual wave characteristics (wave height, wave period and wave steepness) are obtained and compared with the model distributions. Goodness of fit of the observed distributions is checked by Chi-square test and Kolmogorov-Smirnov tests. Joint probability distribution of individual wave heights and periods is also studied and compared with the theoretical distributions. The relationships among various statistical wave height parameters and statistical wave period parameters are investigated and compared with the theoretical and reported values.

**Keywords:** Wind waves, probability distribution, wave height, wave period, wave record, Mediterranean, Black Sea.

## ÖZ

# TÜRKİYE KIYILARINDAKİ RÜZGAR DALGALARININ KISA DÖNEM İSTATİSTİĞİ

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Bu tez çalışmasında Türkiye kıyılarındaki üç merkezde (Alanya, Dalaman ve Hopa) alınan rüzgar dalgası kayıtlarının analizi yapılmıştır. Birey dalga özelliklerinin (dalga yüksekliği, dalga dönemi ve dalga dikliği) olasılık dağılımları hesaplanmış ve bu dağılımlar model dağılımlarla karşılaştırılmıştır. Gözlenen dağılımların model dağılımlarla uyum iyiliği  $\chi^2$  testi ve Kolmogorov-Smirnov testleri kullanılarak kontrol edilmiştir. Birey dalga yüksekliği ve dönemlerinin ortak olasılık dağılımı da incelenmiş ve model dağılımlarla karşılaştırılmıştır. Çeşitli istatistiksel dalga yüksekliği değerleri arasındaki ve çeşitli istatistiksel dalga dönemi değerleri arasındaki ilişkiler araştırılmış ve elde edilen sonuçlar, model dağılımlardan elde edilen teorik değerler ve daha önce bildirilmiş bulunan ölçüm değerleri ile karşılaştırılmıştır.

Anahtar sözcükler: Rüzgar dalgaları, olasılık dağılımı, dalga yüksekliği, dalga dönemi, Akdeniz, Karadeniz.

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## LIST OF SYMBOLS

$a$	dimensionless parameter for the modified form of Rayleigh distribution
$a_n$	amplitude of the $n^{\text{th}}$ sinusoidal component
$B$	bias error
$B_n$	normalized bias error
$D_n$	maximum difference between sample distribution and hypothetical distribution for Kolmogorov-Smirnov test
$D_n^\alpha$	critical value for Kolmogorov-Smirnov test
$e_i$	expected frequency for a certain data interval
$f$	wave frequency
$f_n$	frequency of the $n^{\text{th}}$ component
$f_i$	observed frequency for a certain data interval
$g$	gravitational acceleration (=9.81 m/sec <sup>2</sup> )
$H$	wave height
$H^*$	reference wave height
$H_i$	individual wave height
$H_{\text{avg}}$	mean wave height
$H_{\text{rms}}$	root mean square wave height
$H_{m0}$	significant wave height computed from wave energy spectrum
$H_{1/3}$	significant wave height computed from wave record (average of the highest 1/3 of the waves)

$H_{1/10}$	average of the highest 1/10 of the waves
$H_{\max}$	maximum wave height in a record
$H_{m+}$	wave height defined as the twice the maximum water surface elevations in '+' direction
IQR	interquartile, difference between the third and the first quartile value in a data set
$k$	wave number ( $=2\pi/L$ )
$L$	wave length
$L_0$	deep water wave length
$m$	degree of freedom
$m_0$	zerorth moment of the wave spectrum (the area under the spectral curve)
$m_n$	$n^{\text{th}}$ moment of the spectrum
$N_a$	total number of data in a data set
$N$	total number of individual waves in a wave record
$N_x$	number of digital surface elevations
$N_c$	number of crests in a wave record
$N_z$	number of zero up-crossing waves in a record
$n_r$	number of individual waves in a certain interval
$n$	dimensionless parameter for the modified form of Rayleigh distribution
$nt$	number of data intervals
$P(x)$	cumulative probability of $x$
$P(x)$	sample distribution function for Kolmogorov-Smirnov test
$P_0(x)$	hypothetical distribution function for Kolmogorov-Smirnov test
$p(x)$	probability density function of $x$
$r$	correlation coefficient between individual wave heights and periods
$rg$	range, difference between the maximum and minimum value in a data set

$Q_1$	first quartile value in a data set
$Q_3$	third quartile value in a data set
RMS	root mean square difference
$RMS_n$	normalized root mean square difference
$S(f)$	wave energy spectrum function
$S$	wave steepness ( $=H/L$ )
$S_0$	deep water wave steepness ( $=H/L_0$ )
$S_*$	steepness of the wave having significant height and mean period.
SI	scatter index
$SI_n$	normalized scatter index
$t$	time variable
$T$	wave period
$T_{avg}$	mean wave period
$T_{1/3}$	significant wave period (average of the periods of the highest 1/3 of the waves in a wave record)
$T_{1/10}$	average of the periods of the highest 1/10 of the waves in a record
$T_{max}$	period of the highest wave in a wave record
$T_{hmax}$	period of the highest wave in a wave record
$T_i$	individual wave period
$T_m$	mean period of maximum surface elevations in '+' direction
$T_z$	period of zero up-crossing waves
$T_c$	crest to crest wave period
$T_{01}$	definition of $T_z$ by 'zeroth' and 'first' moments of wave spectrum
$T_{02}$	definition of $T_z$ by 'zeroth' and 'second' moments of wave spectrum
$x$	dimensionless wave height

$z$	random variable
$x_{\max}$	maximum value in a data set
$x_{\min}$	minimum value in a data set
$\chi^2$	calculated chi-square value
$\chi_{\alpha,m}$	expected value of $\chi^2$ for significance level of $\alpha$ and for degree of freedom of $m$
$y$	reference value
$\alpha$	significance level for $\chi^2$ test
$\gamma$	significance level for Kolmogorov-Smirnov test
$\alpha_i$	parameter for the relationships between representative wave height parameters
$\beta_j$	parameter for the relationships between representative wave period parameters
$\Delta f$	frequency interval
$\varepsilon$	spectral width parameter ( $\approx 2\nu$ )
$\eta(t)$	wave profile
$\eta$	instantaneous surface elevation
$\eta^2$	variance of surface elevations ' $\eta$ '
$\eta_{rms}$	root mean square of ' $\eta$ '
$\eta_{m+}$	maximum surface elevation in '+' direction
$\lambda$	parameter of wave height distribution given by Forristall (=2.126)
$\mu$	dimensionless wave period given by Cavanier-Arhan-Ezraty
$\psi$	parameter of wave height distribution given by Forristall (=8.42)
$\nu$	spectral width parameter
$\sigma_a$	standard deviation for dimensionless parameter $a$

$\sigma_n$	standard deviation for dimensionless parameter n
$\xi$	dimensionless wave height ( $=H/H_{avg}$ )
$\xi^*$	dimensionless wave height ( $=H/H_{rms}$ )
$\tau$	dimensionless wave period ( $=T/T_{avg}$ )
$\tau^*$	dimensionless wave period given by Cavanier-Arhan-Ezraty
$\phi$	dimensionless wave steepness
$\phi_n$	phase angle

## **CHAPTER 1**

### **INTRODUCTION**

#### **1.1. GENERAL DESCRIPTION**

Wind-waves can generally be described as continuously changing irregular surface forms, observed on the sea surface, which are generated by the wind blowing over the sea.

From the coastal engineering point of view, wind-waves are the most important phenomenon to be considered among the environmental conditions affecting maritime structures and other marine and coastal activities. One of the basic steps of design procedure for maritime structures is to calculate the disturbing forces affecting the structure. Among these forces, the one that is due to wind-waves usually has the greatest influence. Beside the design and construction of maritime structures, obtaining reliable information on the characteristics of wind-waves is a crucial need for other coastal engineering applications such as coastal erosion, control and protection of special coastal and marine areas. Furthermore, various activities related to the sea or the coastal zone like fishery development and fishing operations, navigation, tourism and recreation, coastal zone planning and management and national defense planning and operations also require reliable information on the wave conditions of the regions where these activities take place.



Characteristic wave parameters like significant wave height, mean wave period and mean wave direction are the basic factors mostly used for describing the wave conditions. The quantitative description of the long term distribution of wave conditions by using these parameters is termed as the wave climate.

In order to define the wave climate of a region, data sets collected for a long period of time (several tens of years) are required. These data sets can be obtained from various sources such as visual observations, instrumental measurements and wave predictions. All available data are analyzed to make estimates for the probability of occurrence of different sea states. This analysis is generally called long-term wave statistics.

Instrumental measurement of sea waves is the most reliable and the most accurate source for wave data. Instrumental measurements can be classified into three main types:

- a. Measurements from below the water surface;
- b. Measurements at or across the water surface;
- c. Measurements from above the water surface.

Fluctuations of the water surface or any of the associated effects such as wave pressure and water particle motion are usually measured for relatively short periods (15-20 minutes) and stored as wave records. These records of raw data are analyzed to compute the statistical distributions of individual wave heights and periods, the characteristic wave parameters and the wave spectra. The statistical analysis of waves in a wave record is called short-term wave statistics.

Obtaining information on the wind-wave characteristics of the Turkish coast is essential for efficient use of the coastal area of the country. A major project, called NATO TU-WAVES Project (Özhan and Abdalla 1992, 1993a and 1993b, and Özhan et.al. 1995a and 1995b) was carried out under the leadership of the Coastal and Harbor Engineering Research Center of Middle East Technical University (METU-KLARE) to provide reliable information on the wave climate of Turkish coastline.

## **1.2. THE SCOPE AND EXTENT OF THIS STUDY**

In this thesis study, short-term statistical analysis of wind-waves measured at three locations on the Turkish coast (namely: Alanya, Dalaman, and Hopa) was carried out. The data used in this study were provided from the measurements taken in the context of the NATO TU-WAVES Project.

The aim of this study is to investigate the statistical properties of the individual wave characteristics in a wave record for all available data sets and consequently to obtain reliable information about the short-term wind-wave characteristics of coastal regions of Turkey. In order to achieve this aim, the following tasks have been carried out:

- Probability distributions of individual wave characteristics (individual wave height, wave period, wave steepness) were obtained and compared with theoretical distributions. Results were studied to define the properties of the measured wave data.

- Joint probability distribution of individual wave heights and periods was investigated and compared with the model distributions.

In the second chapter of the thesis, general description of the theory supporting this study is presented by giving information about wave measurements, short-term wave statistics, theoretical probability distributions and the wave spectrum. The third chapter describes the NATO TU-WAVES Project, in particular the wave measurements and management of the data. The results obtained during this study are presented and discussed in detail in the fourth chapter. The fifth chapter provides the conclusions derived from this study.

## **CHAPTER 2**

### **MEASUREMENT AND ANALYSIS OF WIND-WAVES**

#### **2.1. WAVE MEASUREMENTS**

Instrumental measurements are the most reliable and accurate source for wind-wave data. Instrumental measurements can be classified according to the elevation of gaging with respect to the water surface as:

- a. Measurements from below the surface;
- b. Measurements at or across the surface;
- c. Measurements from above the surface.

##### **2.1.1. MEASUREMENTS FROM BELOW THE WATER SURFACE**

Wave measuring devices are placed below the water surface either on the bottom directly (in the case of relatively shallow water) or attached to a pier. The advantage of this kind of wave measurements is that the sensor is not exposed to many of the dangers which an instrument placed at the surface can face. The measured data are generally stored by the electronic hardware of the instrument for a period of time. Alternatively, radio transmission can also be used to send the data, but there is a possibility of data loss through radio interference. Some of the basic techniques used in this type of measurements are:

Pressure measurements: In this technique, a pressure transducer is placed either on the sea bed if the depth is shallow enough, or at an appropriate elevation if not. This transducer measures the changes in the dynamic pressure that result as the waves pass overhead. Pressure fluctuations are converted into the corresponding variation of the water surface elevation. Although this is a reliable device, attenuation with depth is severely observed. This attenuation also causes the loss of high frequency waves (i.e. truncation of the wave spectrum) for automatically performed spectral analysis.

Inverted echo sounding: A successfully used method is that of placing an inverted narrow-band echo sounder at or above the sea bed. Sound waves are sent to the water surface and the time needed by the sound to come back is converted into distance. There is no problem of wave attenuation with depth in this method but sound waves can be scattered by air bubbles formed at the water surface due to wave breaking during severe storms. In areas where the waves are predominantly swell, this method is very successful.

### **2.1.2. MEASUREMENTS AT OR ACROSS THE WATER SURFACE**

For this group of measurement techniques, either a surface-piercing vertical sensor which responds to its depth of immersion, or a float which senses its vertical (and sometimes horizontal) movement, is utilized.

There are different devices used in surface measurements depending on the water depth. Various types of resistance wave gauges are utilized in shallow water. These gauges utilize an insulated resistance wire. The wire is held vertically through the water surface. The water would short circuit the part of the wire located below the water surface, and thus measuring the resistance change of the wire. The change in resistance is converted into water surface elevation and recorded. Alternatively, capacitance is measured instead of resistance in some types of gauges. The errors in the wave-gauge measurements are mainly attributed to the vibration of the gauge, water splashes and water surface tension. Loss of reliability occurs in such devices due to fouling after some time.

For the case of deep water, the only practical method is to use a floating wave-rider buoy that measures the accelerations (directional wave buoys usually measure three independent accelerations). The displacements can be calculated by performing double integration of these accelerations. Because this type of devices is deployed far away from the shore, they are vulnerable to dangers like ship collision, mooring failure and being lost during severe storms. The measured data are either stored on-board on magnetic media or transmitted through a wireless to a shore station equipped with a receiver and usually a computer. There is a risk of having problems with the reception of the wireless signal if the system is installed in a region with intensive wireless communication signals. On the other hand, the on-board storage requires the change of the storage media from time to time.

### **2.1.3. MEASUREMENTS FROM ABOVE THE WATER SURFACE**

Signals of various types like the sound waves, laser beams and the radar signals from various sources located above the water surface are used in this type of measuring techniques. The vertical motion of the water surface or its geometry is measured from the time interval during which the signal is sent to water surface and come back. There may be some problems in using acoustic signals differences resulting from the change of signal velocity due to air temperature differences and water spray. However, the devices using laser beams or radar signals usually produce good results. This group of devices is mounted on a platform, airplane or satellite.

## **2.2. WAVE RECORDS AND WAVE PARAMETERS**

There are some commonly used methods for defining the individual waves in an irregular wave record such as zero up-crossing or zero down-crossing methods. According to zero up-crossing method, an individual wave is identified as the disturbance between two successive crossings of the mean water level upward by the surface profile. The vertical difference between the highest and lowest points of the profile in the interval between two successive zero up-crossings is considered as the

individual wave height and the time difference between these crossings is taken as the individual wave period.

In order to describe the sea state for a certain period of time, some representative wave parameters are defined. The most commonly used parameters are as follows:

$H_{\max}, T_{\max}$  : are respectively the height and the period of the highest individual wave in a wave record. (Period of the highest wave is also denoted as  $T_{h\max}$ )

$H_{1/10}, T_{1/10}$  : are respectively the average height and the period of the highest one-tenth of the waves in a wave record.

$H_{1/3}, T_{1/3}$  : are respectively the average height and period of the highest one-third of the waves in a wave record. (They are also called as significant wave height and significant wave period respectively).

$H_{\text{avg}}, T_{\text{avg}}$  : are respectively the average height and period of all individual waves in a wave record.

The significant wave height  $H_{1/3}$  is said to be roughly equal to the representative wave height observed visually during a storm and it is the most frequently used characteristic wave parameter.

### **2.3. SHORT TERM WAVE STATISTICS**

Short term wave statistics implies the statistical properties of individual wave characteristics of a single wave record. In this section, the proposed probability distributions for wave height, wave period and wave steepness, as well as the joint probability distribution of wave heights and periods are reviewed.

### 2.3.1. PROBABILITY DISTRIBUTION OF WAVE HEIGHTS

It is generally accepted that marginal probability distribution of individual wave heights in a wave record follows the Rayleigh distribution (Goda,1979). In its general form, the Rayleigh distribution is given as:

$$P(x) = 1 - \exp(-a^2 x^2) \quad (2.1)$$

where,

- $P(x)$  : The cumulative probability that the value of the variable 'x' is not exceeded
- $x = H/H^*$  : Dimensionless wave height
- $H^*$  : A reference wave height
- $a = H^*/(8m_0)^{1/2}$  : A dimensionless coefficient
- $m_0$  : Zeroth moment of the wave energy spectrum this is equivalent to the mean of the squares of surface elevations.

Depending on the reference wave height  $H^*$ , the value of the dimensionless coefficient 'a' is given as:

$$a = \begin{cases} 1/\sqrt{8} & \text{for } H^* = \sqrt{m_0} = \eta_{rms} \\ \sqrt{\pi/2} & \text{for } H^* = H_{avg} \\ 1 & \text{for } H^* = H_{rms} \\ \sqrt{2} & \text{for } H^* = H_{1/3} \end{cases} \quad (2.2)$$

where:

$\eta_{rms}$  : square root of mean of the squared water surface fluctuations  $\eta_i$  (i.e. the root mean square surface elevation with reference to the mean water level) given by :

$$\eta_{rms} = \left( \frac{1}{N} \sum_1^N \eta_i^2 \right)^{1/2} \quad (2.3)$$

$H_{rms}$ : Square root of the mean of squared wave heights (i.e. the root mean square wave height)

The applicability of Rayleigh distribution to wind-waves was first proven theoretically by Longuet-Higgins (1952) for the case of narrow-band spectrum (i.e. small changes in the individual wave periods). However, random sea waves exhibit a wide-band spectrum due to the fairly wide range of wave periods. Therefore, the applicability of the Rayleigh distribution to real sea waves is questionable. Nevertheless, subsequent wave measurements showed that the probability distribution of individual wave heights, defined by zero crossing methods, is quite close to the Rayleigh distribution. Therefore, Rayleigh distribution has been accepted to represent the individual wave height distribution irrespective of the spectral width.

From Eqn. (2.1), the probability density function is obtained as:

$$p(x) = \frac{dP(x)}{dx} = 2 a^2 x \exp(-a^2 x^2) \quad (2.4)$$

Rayleigh distribution yields the following relationships between the significant wave height and other characteristic wave heights:

$$\begin{aligned} H_{avg} &= 0.625 H_{1/3} \\ H_{rms} &= H_{1/3} / \sqrt{2} \\ H_{1/10} &= 1.27 H_{1/3} \\ H_{1/100} &= 1.67 H_{1/3} \\ H_{max} &= (\ln N/2)^{1/2} H_{1/3} \end{aligned} \quad (2.5)$$

where, N is the number of individual waves in the record.



Forristal (1978) proposes a slight empirical modification to the Rayleigh distribution as:

$$P(\xi) = 1 - \exp(-\xi^{-\lambda} / \psi) \quad (2.6)$$

where,

$$\xi = H / \sqrt{m_0}, \quad \lambda = 2.126, \quad \psi = 8.42$$

Fig. 2.1 compares the Rayleigh distribution and the distribution proposed by Forristal.

### 2.3.2. DISTRIBUTION OF WAVE PERIODS

The most commonly used theoretical probability distributions to represent the actual distribution of individual wave periods in a wave record are the Bretschneider, Longuet-Higgins, Cavanier-Arhan-Ezraty distributions.

#### a. Bretschneider Distribution:

Bretschneider (1959) observed empirically that the squares of wave periods approximately fit to the Rayleigh distribution. The empirical distribution suggested by Bretschneider is as follows:

$$P(\tau) = 1 - \exp(-0.675 \tau^4) \quad (2.7)$$

where,

$P(\tau)$  : The cumulative probability that the dimensionless wave period does not exceed the given  $\tau$  value

$\tau = T/T_{avg}$  : Dimensionless wave period

Hence, the probability density function obtained from Eqn. (2.7) is given as:

$$p(\tau) = \frac{dP(\tau)}{d\tau} = 2.7 \tau^3 \exp(-0.675 \tau^4) \quad (2.8)$$

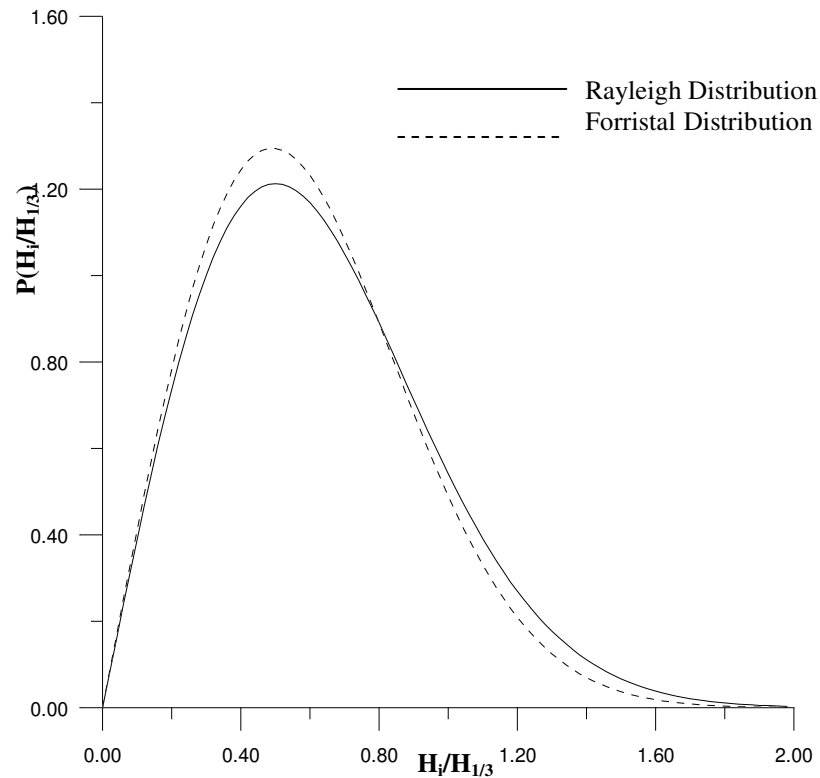


Fig. 2.1. Comparison of the Rayleigh distribution and the Forristal distribution

### b. Longuet-Higgins Distribution :

Longuet-Higgins (1952) has theoretically derived the joint distribution of heights and periods of waves that are characterized by a narrow band energy spectrum. The marginal distribution of wave periods derived from this joint distribution can be written as the following probability density function:

$$p(\tau) = \frac{\nu^2}{2[\nu^2 + (\tau - 1)^2]^{3/2}} \quad (2.9)$$

where,

$$\nu = \left[ \left( \frac{m_0 m_2}{m_1^2} \right) - 1 \right]^{1/2} \quad : \text{Spectral width parameter}$$

$$m_n \quad : n^{\text{th}} \text{ moment of wave energy spectrum}$$

The probability distribution given in Eqn. (2.9) is symmetrical around  $\tau = 1$ . The domain for  $\tau$  is  $(-\infty, +\infty)$ . There is a small error in this distribution, because there is a small probability for negative  $\tau$  values.

### c. Cavanier-Arhan-Ezraty Distribution:

The distribution given by Longuet-Higgins and Bretschneider yields no correlation between wave heights and wave periods. On the other hand, analysis of wind wave records have shown that there exists weak positive correlation. The group of Cavanier, Arhan and Ezraty (1976) derived a joint probability distribution for wave heights and wave periods that takes this correlation into account. The marginal distribution for wave periods can be derived from the proposed joint distribution as:

$$p(\tau^*) = \frac{\alpha^3 a^2 \mu^2 \tau^*}{[(\mu^2 \tau^{*2} - \alpha^2)^2 + a^2 \alpha^4]^{3/2}} \quad (2.10)$$

where,

$$\tau^* = T/T_{avg} \quad : \text{Dimensionless wave period}$$

$$T \quad : \text{Wave period}$$

$$T_{avg} \quad : \text{Mean wave period}$$

$$\alpha = \left\{ \frac{1 + (1 - \varepsilon^2)^{1/2}}{2} \right\}$$

$$a = \varepsilon / (1 - \varepsilon^2)^{1/2}$$

$$\varepsilon = \left\{ \frac{1 - (m_2^2 / m_0 m_4)}{2} \right\}^{1/2} : \text{Another spectral width parameter}$$

$$\mu = 1$$

Longuet-Higgins (1952) has shown that  $\varepsilon = 2\nu$  for narrow band spectrum.

The comparison among the period distributions mentioned above was given by Özhan (1981). This comparison is shown in Fig. 2.2 for Bretschneider, Longuet-Higgins ( $\nu = 0.25$  and  $\nu = 0.35$ ) and Cavanier-Arhan-Ezraty ( $\varepsilon = 0.50$  and  $\varepsilon = 0.70$ ) distributions.

As it can be seen from the figure, the distribution suggested by Longuet-Higgins reaches its peak value for  $T/T_{avg} = 1$  (mean wave period), while this value is a bit greater ( $T/T_{avg} = 1.05$ ) for Bretschneider distribution. There is a shift in the location of the peak, in the direction of smaller periods, for Cavanier-Arhan-Ezraty distribution as  $\varepsilon$  increases. The distribution suggested by Bretschneider gives almost the same peak density value with those given by Longuet-Higgins for  $\nu = 0.35$  and by Cavanier-Arhan-Ezraty group for  $\varepsilon = 0.7$ .

Although there is no theoretical relationship between characteristic period parameters, it has been empirically found that these parameters are interrelated. In literature, from the analysis of field wave data, the following results have been reported (Goda,1974) :

$$\begin{aligned}
 T_{max} &= (0.6 - 1.3) T_{1/3} \\
 T_{1/10} &= (0.9 - 1.1) T_{1/3} \\
 T_{avg} &= (0.7 - 1.1) T_{1/3}
 \end{aligned}
 \tag{2.11}$$

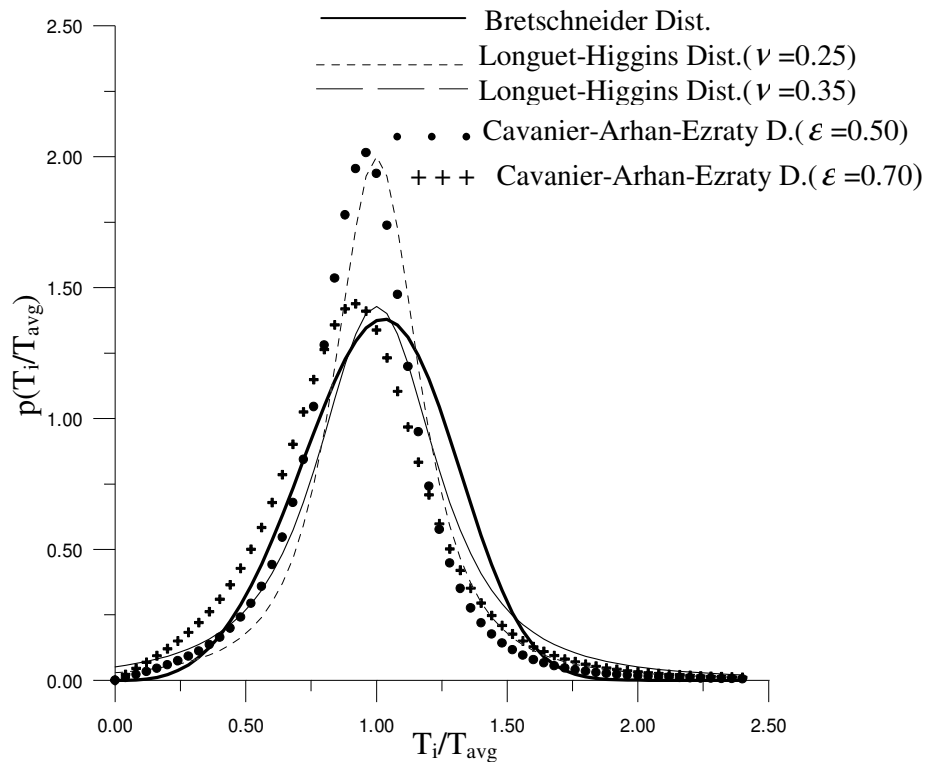


Fig. 2.2. Comparison of theoretical distributions of wave periods

### 2.3.3. JOINT DISTRIBUTION OF WAVE HEIGHTS AND WAVE PERIODS

An example to the distribution of observed individual wave heights and periods, illustrated by probability contours on the H-T plane is given in Fig. 2.3. A certain correlation between wave heights and periods can be observed in this figure. Among the three theoretical probability distributions, which are proposed for joint probability distribution of wave heights and periods, only the equation of Cavanier- Arhan-Ezraty group considers a positive correlation like the one observed in Fig. 2.3. The other two distributions consider no significant correlation between wave heights and periods.

There are three theoretical distributions proposed to represent the joint probability distribution of individual wave heights and periods. These distributions are given as follows:

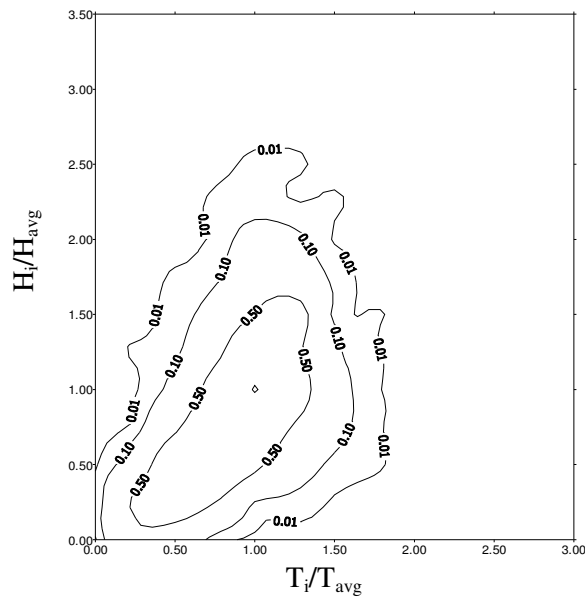


Fig.2.3. Probability contours for joint distribution of observed wave heights and periods

**a. Bretschneider Joint Probability Distribution :**

The distribution proposed by Bretschneider (1959) assumes that the wave heights and wave periods are independent variables (i.e. they are un-correlated). Therefore, this distribution can be written as

$$p(\xi, \tau) = p(\xi) p(\tau) = 1.35 \pi \xi \tau^3 \exp\left(-\frac{\pi}{4} \xi^2 - 0.675 \tau^4\right) \quad (2.12)$$

where,

$$\xi = H/H_{avg} \quad \text{and} \quad \tau = T/T_{avg}$$

**b. Longuet-Higgins Joint Probability Distribution :**

Assuming that the wave spectrum is narrow-banded, Longuet-Higgins (1975) proposed the following joint probability distribution of wave heights and periods:

$$p(\xi, \tau) = \frac{\pi}{4\nu} \xi^2 \exp\left(-\frac{\pi}{4} \xi^2 \left(1 + \frac{(\tau-1)^2}{\nu^2}\right)\right) \quad (2.13)$$

**c. Cavanier-Arhan-Ezraty Joint Probability Distribution :**

The theoretical distribution, proposed by Cavanier-Arhan-Ezraty group (1976) is given as:

$$p(\xi^*, \tau^*) = \frac{\pi \alpha^3 \xi^{*2} \tau^{*-5}}{2 \varepsilon (1 - \varepsilon^2) \mu^4} \exp\left[-\frac{\pi \xi^{*2} \tau^{*-4}}{4 \varepsilon^2 \mu^4} \left((\mu^2 \tau^{*2} - \alpha^2)^2 + \alpha^2 \alpha^4\right)\right] \quad (2.14)$$

The dimensionless wave height  $\xi^*$  in Eqn. (2.14) is theoretically calculated by using the maximum deviations of the sea surface in positive direction:

$$\xi^* = 2 \eta_{m+} / H_{avg} = H_{m+} / H_{avg} \quad (2.15)$$

$H_{m+}$ : is the wave height that is obtained by multiplying the maximum deviation of the sea surface in positive direction by two. The other symbols are similar to those in Eqn. (2.10).

The distribution given by Bretschneider does not depend on the spectral width parameter ' $\epsilon$ '. Fig. 2.4 shows the probability contours of this distribution. In Fig. 2.5 and Fig. 2.6, the distributions given by Longuet-Higgins and by Cavanier-Arhan-Ezraty group are presented for different values of spectral width parameters. Two distributions can be compared by accepting that  $\epsilon = 2\nu$ .

Bretschneider and Longuet-Higgins distributions are symmetrical about the mean wave period ( $T=T_{avg}$ ). Longuet-Higgins distribution shows higher probability for high waves when compared to the Bretschneider distribution. On the other hand, the Bretschneider distribution foresees small probabilities for negative wave heights which is unrealistic.

The distribution given by Cavanier-Arhan-Ezraty is not symmetrical. It depicts the correlation between wave heights and wave periods especially for waves with a broad-band spectrum. Among these three distributions, the distribution of Cavanier-Arhan-Ezraty is the only one depicting the observed positive correlation between wave heights and wave periods.

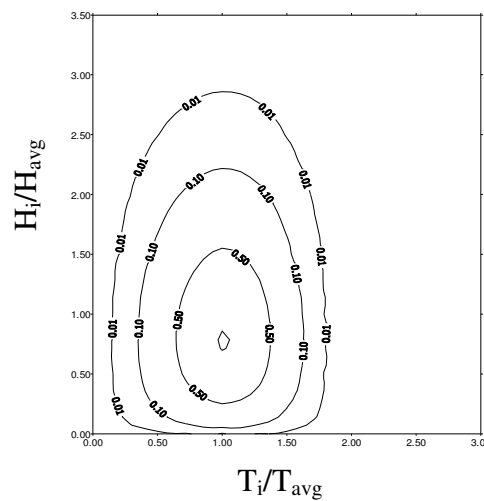


Fig.2.4. Probability contours for the Bretschneider joint distribution

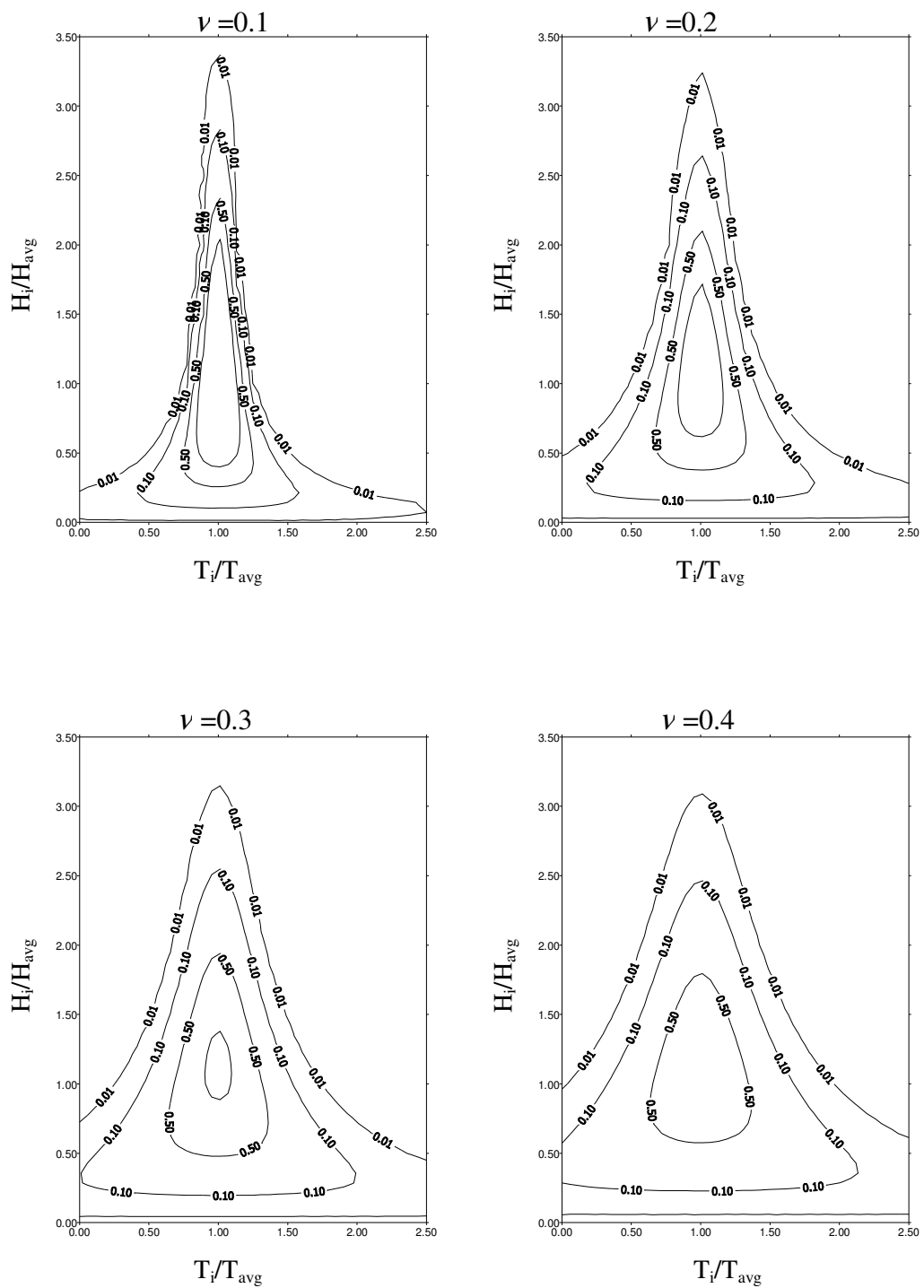


Fig.2.5. Probability contours for the Longuet-Higgins joint distribution



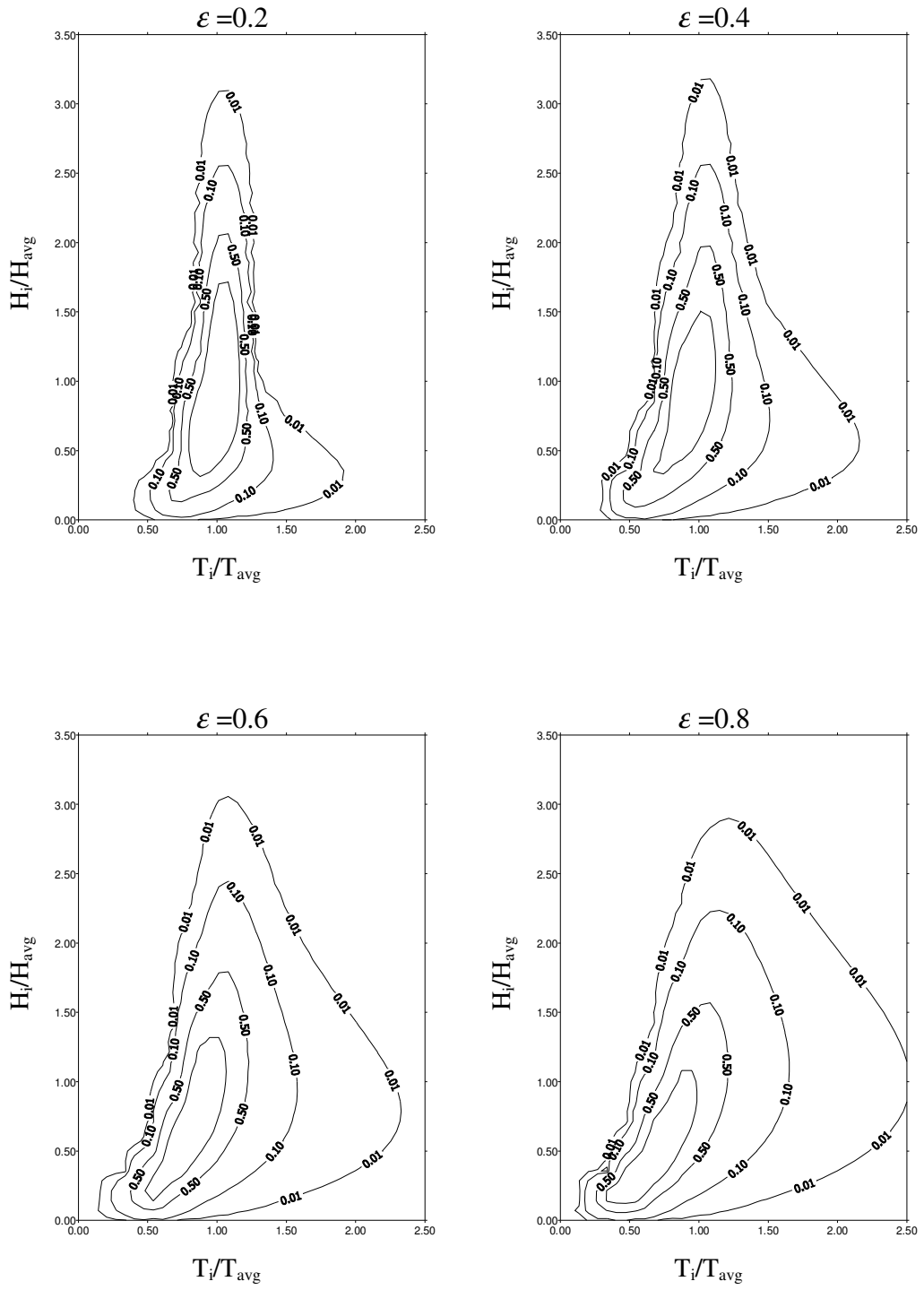


Fig.2.6. Probability contours for the Cavanaugh-Arhan-Ezraty joint distribution

### 2.3.4. DISTRIBUTION OF WAVE STEEPNESS

Wave steepness,  $S$ , is defined as the ratio of the wave height,  $H$ , to wave length,  $L$ , namely:

$$S = H / L \quad (2.16)$$

Therefore, the deep water wave steepness is:

$$S_0 = H/L_0 = \frac{2 \pi H}{g T^2} \quad (2.17)$$

The dimensionless wave steepness,  $\phi$ , may be defined as:

$$\phi = S/S_* = \frac{H / H_{1/3}}{(T / T_{avg})^2} \quad (2.18)$$

where,

$$S_* = \frac{2 \pi H_{1/3}}{g T_{avg}^2} \quad (2.19)$$

$S_*$  represents the steepness of the wave having significant wave height and mean wave period.

Cumulative probability distribution of dimensionless wave steepness can be calculated using the joint distributions of wave heights and wave periods as:

$$P(\phi) = P(\xi < \phi \tau) = \int_0^{\infty} d\tau \int_0^{\phi \tau} P(\xi, \tau) d\xi \quad (2.20)$$

where,

$P(\phi)$  : Probability that the dimensionless wave steepness is equal to or smaller than  $\phi$  value

For the distribution of Longuet-Higgins (Eqn. 2.13) and Cavanier-Arhan-Ezraty (Eqn. 2.14), the integral in Eqn. (2.20) can only be calculated numerically. Battjes

(1977) gave the probability distribution of wave steepness in closed form, which is obtained from the joint probability distribution of Bretschneider (Eqn. 2.12) as:

$$P(\phi) = \frac{4\phi^2}{1.35 + 4\phi^2} \quad (2.21)$$

The probability density function for this distribution is:

$$p(\phi) = \frac{dP(\phi)}{d\phi} = \frac{10.8\phi}{(1.35 + 4\phi^2)^2} \quad (2.22)$$

Overvik and Houmb (1977) proposed another wave steepness distribution by making use of the distribution of the curvatures of sea profile at wave crests as given by Rice (1944):

$$p(\phi) = 4(1 - \varepsilon^2)\phi \exp[-2(1 - \varepsilon^2)\phi^2] \quad (2.23)$$

The distributions given in Eqn. (2.22) and Eqn. (2.23) are compared in Fig. 2.7. Eqn. (2.23) is plotted for two values of spectral width parameter:  $\varepsilon = 0.2$  and  $\varepsilon = 0.70$ . For  $\varepsilon = 0.2$ , the distribution of Overvik and Houmb is seen to be comparable with that of Battjes.

## 2.4. WAVE SPECTRUM

Another way of representing the random fluctuations of the sea-surface elevation is by means of the wave spectrum. The concept of the wave spectrum is based on the assumption that the shape of the sea surface during a storm may be expressed as the superposition of an infinite number of sinusoidal waves with various heights, frequencies, phases and directions of propagation. The wave spectrum is the distribution of wave energy with respect to these assumed sinusoidal components.

The harmonic analysis or Fourier analysis is a tool to decompose the random sea surface into its sinusoidal components. This can be represented mathematically by:

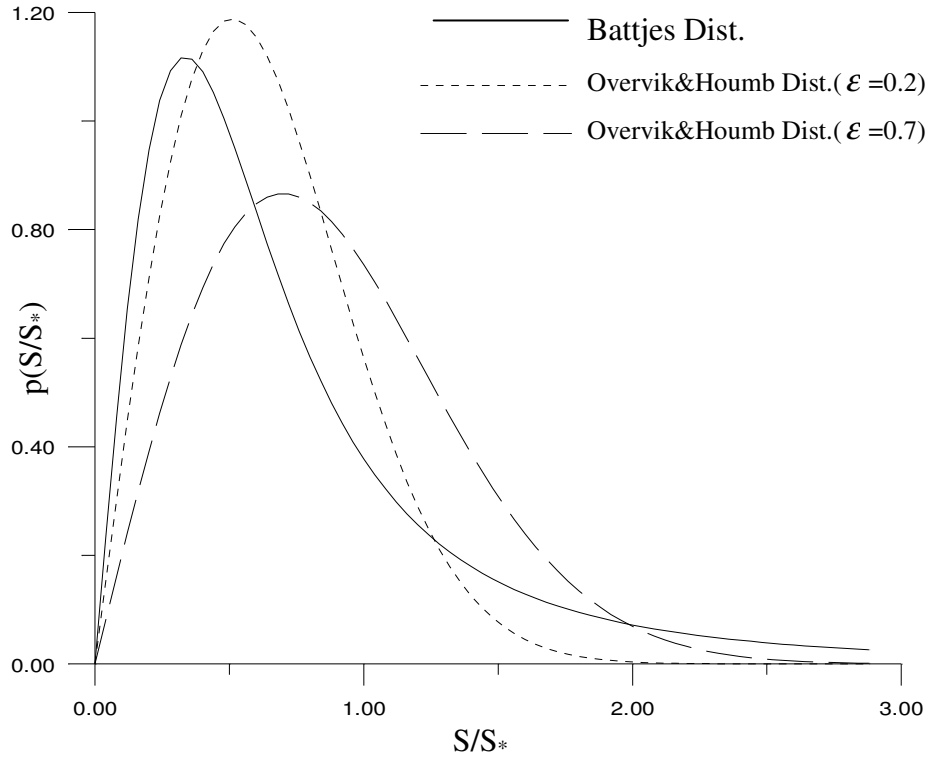


Fig.2.7. Comparison of theoretical distributions of wave steepness

$$\eta(t) = \sum_{n=1}^N a_n \sin(2\pi f_n t + \phi_n) \quad (2.24)$$

where,

- $\eta(t)$  : Surface elevation at time t, at a fixed point
- $a_n$  : The amplitude of the n<sup>th</sup> sinusoidal component
- $f_n$  : The frequency of the n<sup>th</sup> sinusoidal component
- $\phi_n$  : Phase angle of the n<sup>th</sup> sinusoidal component
- $N_x$  : number of all possible wave components

It is worthwhile to mention that this is a simplified description of the harmonic analysis of wind-waves since all components with  $a_n$ ,  $f_n$  and  $\phi_n$  coming from various directions are assumed as one wave component.

As the result of Fourier analysis, the squares of amplitudes 'a<sub>n</sub>' of the sinusoidal wave components can be obtained. When these values are multiplied by 1/2 and then plotted against frequencies, the obtained result is the wave variance spectrum:

$$S ( f_n ) \Delta f_n = 1/2 a_n^2 \quad (2.25)$$

where,

S(f<sub>n</sub>) : Energy density of the wave component with frequency f<sub>n</sub>

Δf<sub>n</sub> : Frequency interval

From Eqns. (2.24) and (2.25), the following relation can be obtained:

$$\overline{\eta^2} = \sum_{n=1}^N S ( f_n ) \Delta f_n \quad (2.26)$$

Eqn. (2.26) implies that the area under the spectral curve gives the variance of water surface fluctuations.

Wave spectrum is usually given as a continuous curve connecting the discrete points found from Fourier analysis. A typical measured wave spectrum is shown in Fig. 2.8.

Several characteristic parameters to describe the sea state can be defined in terms of the moments of the wave spectrum. In general, the n<sup>th</sup> moment of the spectrum is given by:

$$m_n = \int_0^{\infty} f^n S ( f ) df \quad (2.27)$$

In this formula, S(f) denotes the energy at the frequency f per unit interval of f so that S(f)df represents the energy contained in the frequency interval between f-df/2 and f+df/2 (energy density). The zeroth moment of the spectrum, m<sub>0</sub>, gives the total area under the spectral curve and represents the total energy of the waves in the wave record.

The commonly used spectral wave height parameter, H<sub>m0</sub>, is related to m<sub>0</sub> as:

$$H_{m_0} = 4 \sqrt{m_0} \quad (2.28)$$

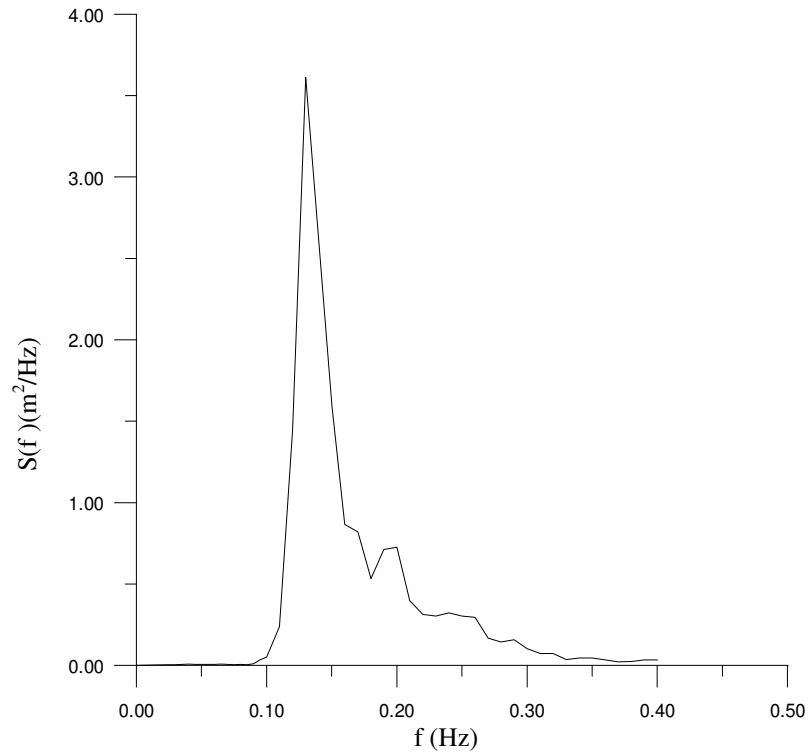


Fig.2.8. A typical measured wave spectrum

Theoretically,  $H_{m0}$  corresponds to  $H_{1/3}$  only for very narrow spectra. In fact, the significant wave height is better expressed by the relation:

$$H_{1/3} \approx 3.8 \sqrt{m_0} \quad (2.29)$$

for deep water (Goda, 1979). Eqns. (2.28) and (2.29) shows that there is a difference of about 5% between  $H_{m0}$  and  $H_{1/3}$  for deep water waves even for narrow spectra.

There exists two approximate spectral wave period parameters for estimating the average wave period. They are given in terms of the spectral moments of spectrum as:

$$T_{01} = m_0 / m_1 \quad (2.30)$$

$$T_{02} = \sqrt{m_0 / m_2} \quad (2.31)$$

The width of the wave spectrum, i.e. spectral width, is used as a measure of irregularity of the sea state. Its definition is given as (Goda, 1979) :

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \quad (2.32)$$

The spectral width parameter  $\varepsilon$  varies theoretically between  $\varepsilon = 0$  (very narrow spectrum, regular waves) and  $\varepsilon = 1$  (broad spectrum, white noise).

The spectral width parameter  $\varepsilon$  can be obtained by using the raw wave data and zero up-crossing method from (Longuet-Higgins, 1952) :

$$\varepsilon = \sqrt{1 - \left( \frac{\bar{T}_c}{\bar{T}} \right)^2} = \sqrt{1 - (N_z / N_c)^2} \quad (2.33)$$

where,

$\bar{T}_c$  : Average of the crest to crest periods including local crests between successive zero up-crossings

$\bar{T}$  : Mean period of the zero up-crossing waves

$N_z$  : Number of zero up-crossing waves

$N_c$  : Number of all crests in the record including local crests

A more robust definition of the spectral width parameter has been proposed by Longuet-Higgins (1952) as:

$$\nu = \sqrt{\frac{m_0 m_2}{m_1^2} - 1} \quad (2.34)$$

For small values of  $\nu$ , it is nearly equal to 1/2 of the spectral width parameter  $\varepsilon$  defined by Eqn.2.32 (Longuet-Higgins, 1952). Therefore, the values of  $\nu$  changes between  $\nu = 0$  and  $\nu = 0.5$ .

## 2.5. STATISTICAL ERRORS

In order to present the difference between two series of data, the following statistical parameters are defined and used throughout this work:

Root-mean-square difference (error) (RMS): represents a measure for the mean absolute difference between the two data sets. This difference is given by:

$$RMS = \left[ \frac{1}{N-1} \sum (y-x)^2 \right]^{1/2} \quad (2.35)$$

Scatter index (SI) represents a measure for the mean relative difference between the two data sets. Scatter index is given by:

$$SI = \left[ \frac{1}{N-1} \sum \left( \frac{y-z}{y} \right)^2 \right]^{1/2} \quad (2.36)$$

Bias (B) is the mean difference between the two data sets. Bias is given by:

$$B = \frac{1}{N} \sum (y-z) \quad (2.37)$$

where, in Eqs. (2.35) to (2.37)

$z$  : value to be compared

$y$  : reference value

$N_a$  : the number of data in a series

The summations are over the whole data set (i.e. from 1 to N).



On the other hand, the difference parameters are normalized by using a weighted average rather than simple arithmetic averaging. Therefore, the following can be defined:

Normalized root-mean-square difference ( $RMS_n$ ) :

$$RMS_n = \left[ \frac{\sum (y - z)^2 z}{\sum z} \right]^{1/2} \quad (2.38)$$

Normalized scatter index ( $SI_n$ ) :

$$SI_n = \left[ \frac{\sum \left( \frac{y - z}{z} \right)^2 z}{\sum z} \right] \quad (2.39)$$

Normalized bias ( $B_n$ ) :

$$B_n = \frac{\sum (y - z) z}{\sum z} \quad (2.40)$$

## 2.6. CORRELATION ANALYSIS

The term correlation analysis indicates the analysis of a relationship in a set of paired data  $(z_1, y_1)$ ,  $(z_2, y_2)$ ,  $(z_3, y_3)$ , ..., and  $(z_n, y_n)$ ; where  $z_i$  and  $y_i$  are values assumed by corresponding random variables  $z$  and  $y$  for  $i=1, 2, 3, \dots, n$ . The strength of the linear relationship between the two variables is calculated by the use of the formula:

$$r = \frac{n \sum_{i=1}^n z_i y_i - \left( \sum_{i=1}^n z_i \right) \left( \sum_{i=1}^n y_i \right)}{\sqrt{n \sum_{i=1}^n z_i^2 - \left( \sum_{i=1}^n z_i \right)^2} \sqrt{n \sum_{i=1}^n y_i^2 - \left( \sum_{i=1}^n y_i \right)^2}} \quad (2.41)$$

where;

- $z_i$  : discrete values of the random variable  $z$
- $y_i$  : discrete values of the random variable  $y$
- $n$  : number of pairs,  $(z_i, y_i)$

The estimate,  $r$ , is called the sample correlation coefficient and its value changes in the range  $[-1, 1]$ . When  $r=0$ , two random variables are totally uncorrelated, and in the case of bivariate normal distribution they are also independent. Two variables are positively correlated as  $r$  approaches 1 and negatively correlated as  $r$  approaches -1. In the case of full correlation, the points  $(z_i, y_i)$  actually fall on a straight line, and  $r$  equals to +1 or -1, depending on whether this line has a positive or negative slope.

## 2.7. TEST OF GOODNESS OF FIT

The term "goodness of fit" here indicates the tests through which it is determined whether a set of data may be looked upon as values assumed by a random variable having a certain distribution. In other words, goodness of fit tests examine the assumption (or the "null hypothesis" as it is statistically termed) that a given set of data comes from a population having a model distribution.

### 2.7.1. CHI-SQUARE TEST

One of the most widely used goodness of fit tests is the  $\chi^2$  test. According to the this test, a parameter,  $\chi^2$  is calculated for the observed data from the following equation:

$$\chi^2 = \sum_{i=1}^m \frac{(f_i - e_i)^2}{e_i} \quad (2.42)$$

where,

- $m$  : the number of terms (i.e. number of data intervals) or the number of independent outcomes
- $f_i$  : observed frequency for the  $i^{\text{th}}$  interval

$e_i$  : expected frequency for the  $i^{\text{th}}$  interval

Observed frequency,  $f_i$ , is defined as the number of individual waves recorded in the  $i^{\text{th}}$  data interval. The expected frequency for the  $i^{\text{th}}$  data interval,  $e_i$ , can be calculated as :

$$e_i = (P_2 - P_1) * N \quad (2.43)$$

where,

$P_1$  : Cumulative probability obtained from the model distribution for the start value of the  $i^{\text{th}}$  interval

$P_2$  : Cumulative probability obtained from the model distribution for the end value of the  $i^{\text{th}}$  interval

$N$  : Total number of individual waves in a wave record

For example, the expected frequency of normalized wave heights for Rayleigh distribution for the interval of  $H/H_{1/3} = [0.9-1.0)$  for a data of 100 individual waves can be calculated as:

$$[ ( 1 - \exp( - a^2 * ( 1.0 )^2 ) ) - ( 1 - \exp( - a^2 * ( 0.9 )^2 ) ) ] * 100 \quad (2.44)$$

The calculated  $\chi^2$  value is compared with the theoretical  $\chi^2$  value which is obtained from the  $\chi^2$ -distribution for a certain significance level (probability of rejecting a true hypothesis),  $\alpha$ , and for  $m-k-1$  degrees of freedom;  $k$  denoting the number of parameters of the distribution estimated from the data. These theoretical values are given in tabular form in almost any statistics book. The hypothesis that the observed data follows a given model distribution is rejected if:

$$\chi^2 \geq \chi_{\alpha, m-k-1}^2 \quad (2.45)$$

### 2.7.2. KOLMOGOROV-SMIRNOV TEST

In the application of the Chi-square test, some of the information is lost due to the grouping of the data into intervals. In order to avoid this loss, alternative tests are developed. The most important of these alternatives to chi-square test is the Kolmogorov-Smirnov test. The test that was developed by Kolmogorov, involves the comparison between the experimental cumulative probability distribution and an assumed theoretical distribution function. In this test, the maximum difference between sample cumulative probability distribution function,  $P(x)$ , from the hypothetical cumulative probability distribution function,  $P_0(x)$ , is the measure of discrepancy between the theoretical model and the observed data. The maximum difference,  $D_n$ , is denoted by

$$D_n = \max|P(x) - P_0(x)|$$

For a specified significance level,  $\gamma$ , the Kolmogorov-Smirnov test compares the observed maximum difference with the critical  $D_n^\gamma$  which can be obtained from many statistical textbooks for different significance levels. If the observed  $D_n$  is less than the critical value, the assumed distribution is accepted; otherwise, it is rejected at the specified significance level  $\gamma$ .

Use of the Kolmogorov-Smirnov test may be more efficient than the Chi-square test for a small size sample. The advantage of the Kolmogorov-Smirnov test over the Chi-square test is that it is not necessary to group the data into intervals. Hence, the problems associated with the Chi-square approximation for small  $e_i$  or small number of class intervals would not be faced in the Kolmogorov-Smirnov test.

## CHAPTER 3

### NATO TU-WAVES PROJECT

#### 3.1. INTRODUCTION

Turkey has a coastline length of approximately 8300 km. To make efficient and sustainable use of such a lengthy coastal area, reliable information about wind wave characteristics affecting the Turkish coasts is needed. Reliable wave data was missing for the Turkish coasts. In order to fill this gap, a major project, called NATO TU-WAVES Project (Özhan and Abdalla, 1992, 1993a and 1993b and Özhan et al., 1995a and 1995b), which included systematical wave measurements, wave modeling and wave climate computations, was carried out. Financial support for the project was provided by the NATO Science for Stability (SfS) Programme – Phase III.

The main objectives of the project were:

- To obtain detailed knowledge on and to establish a reliable data bank of wind waves affecting the Turkish coasts and the whole of the Black Sea,
- To verify and implement a third generation wind-wave model for the seas surrounding Turkey (Black Sea, Sea of Marmara, Aegean Sea and the Eastern Mediterranean), and

- To construct a wave atlas for the Turkish coasts and the whole of the Black Sea in order to provide statistical information on sea state parameters.

Coastal and Harbor Engineering Research Center of Middle East Technical University (METU-KLARE) was the leading organization of the NATO TU-WAVES Project. Three other national organizations; namely: Department of Navigation, Hydrography and Oceanography of the Turkish Navy (TN-DNHO), General Directorate of State Meteorological Services (SMS), and Railway, Harbor and Airport Construction General Directorate of Ministry of Transport (MT-RHAC GD); were the other contributors in the project.

There was also an international dimension of the project. The Black Sea wind-wave climate was investigated in collaboration with the following institutes from four Black Sea riparian countries: Institute of Oceanology, Bulgaria; Rumanian Marine Research Institute and National Institute of Meteorology and Hydrology, Rumania; P.P.Shirshov Institute of Oceanology, State Oceanographical Institute and Moscow Civil Engineering University, Russia; and Marine Hydrophysical Institute, Ukraine.

### **3.2. WAVE MEASUREMENTS**

Obtaining reliable wave measurements was needed to achieve the objectives of the NATO TU-WAVES Project. For this purpose, the proper locations of gaging stations were selected by considering several criteria. Since an expensive system consisting of a buoy (deployed in the sea) and a receiver unit (at the shore) was used for wave measurements; the safety of this system was essential. Some infrastructure facilities were required for setting up the computers and transmitting the data to the project center at METU-KLARE. Therefore, it was decided to set up the gaging systems at the meteorological stations of State Meteorological Services. The gaging stations were selected to cover most of the possible wave regimes affecting the Turkish coasts within the above mentioned constraint. As it can be seen in Fig. 3.1, the national wave gaging network consisted of five directional waverider buoys and one non-directional wave gage. Wave measurements along the Black Sea coast were carried out

by using three directional wave buoys (two of them are part of the national gaging network) and three non-directional wave gages. The Black Sea wave gaging stations are shown in Fig. 3.2.

The instrument that was used to collect the directional wave data was a spherical buoy of 90 cm diameter (called "Directional Waverider Buoy"). It provided the directional wave spectra. The instrument contains a heave-pitch-roll sensor, three-axis fluxgate compass, two fixed x and y accelerometers, and a microprocessor. The accelerations measured in the x and y directions of the moving buoy reference frame are used to calculate the accelerations along the fixed north and west axes. All three accelerations (vertical, north and west) are then digitally integrated to displacements and filtered to a high frequency cut-off (0.6 Hz). As the last step of process, Fast Fourier Transform (FFT) is performed every 30 minutes. The displacements of the buoy are measured for 20 minutes at every two hours and these displacements are compressed to motion vertical, motion north and motion west. The records of these motions are called as raw data. There are 1536 data samples in a raw data record of 20 minutes (i.e. a measurement is taken at each 0.78 seconds). Some spectral parameters like spectral energy density, main direction, directional spread, skewness, curtosis and the normalized second harmonic of the directional distribution for each frequency band, and some other sea-state parameters like  $H_{m0}$  (significant wave height calculated from wave spectrum) and  $T_z$  (mean wave period) are computed on-board and called as processed data. The buoy transmits both raw and processed data. The buoy measures individual wave heights up to 40 m (with a resolution of 1 cm) and periods between 1.6 s - 30 s. Directional resolution is  $1.5^\circ$  and the frequency resolution is 0.005 Hz for frequencies less than 0.1 Hz, and 0.01 Hz otherwise. The standard buoy also measures sea surface water temperature in the range  $-5^\circ\text{C}$  to  $+45^\circ\text{C}$ . Because the directional waverider buoy measures the horizontal motions instead of wave slopes, the measurements are not affected by the roll motions of the buoy. This also justifies the small size of the buoy. On the other hand, the current velocities over 2.5 m/s could cause some distortions on the buoy measurements.

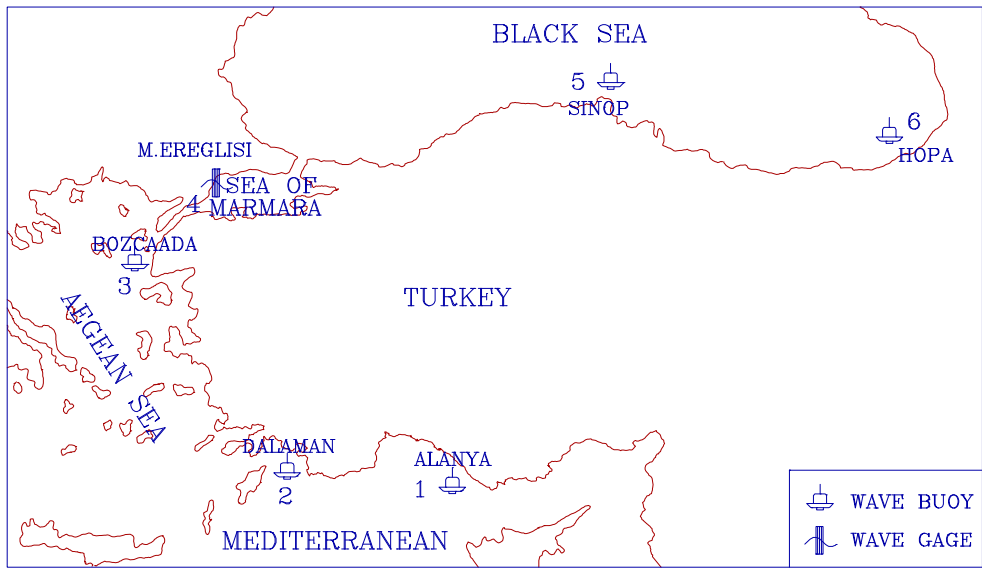


Fig. 3.1. The Turkish (national) wave gaging network (Özhan et. al. 1995a)

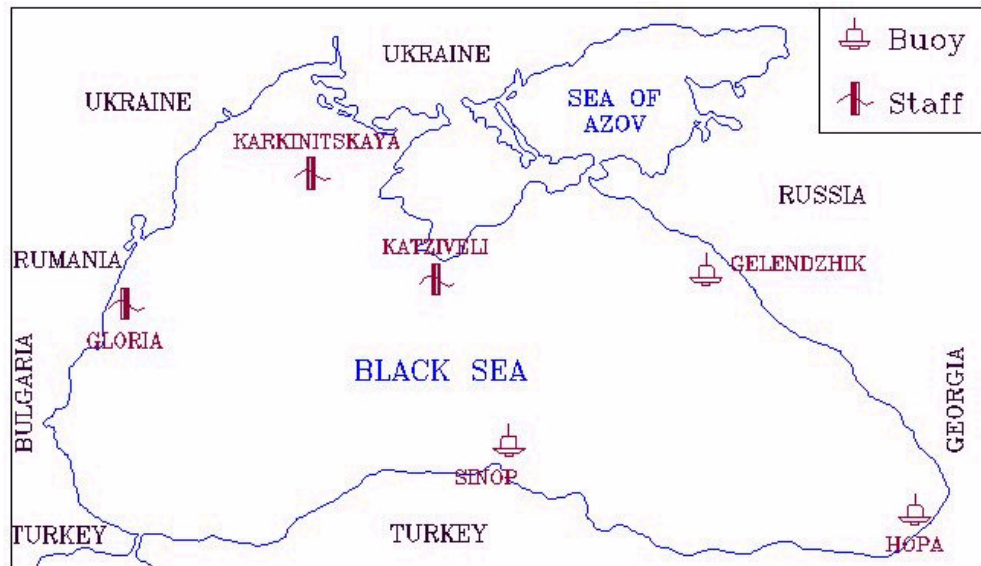


Fig. 3.2. The Black Sea (outreach) wave gaging network (Özhan et. al. 1995a)



For non-directional wave measurements, a resistance wire gage produced by the Marine Hydrophysical Institute (MHI), Ukraine, was utilized. The wire was wound around a flexible cable to maintain its safety against breakage. The gage was 10 meter long. The gage measured the instantaneous fluctuations in water surface elevation for 17 minutes. A built-in microprocessor was responsible for controlling the operation of the gage, collecting the surface elevation data, processing the raw data and transmitting the data to the shore-station or storing the data in a built-in hard memory. The microprocessor utilized the FFT for transforming the raw data into wave energy spectra. The gage could provide the wave spectra within the following two frequency ranges: 0.016 - 1 Hz or 0.031 - 2 Hz. The accuracy of the water elevation measurements was 2 cm. The maximum individual wave height that could be measured was 10 m (Özhan et al., 1995a and 1995b).

All directional wave buoys were deployed in locations having water depths of at least 100 m except the one at Bozcaada, which was deployed at a water depth of 60 m, and the one at Gelendzhik which was deployed at a water depth of 90 m. The water depth at the locations of non-directional gages varied between 17-35 m depending on the available structure at which the gage was mounted.

### **3.3. DATA MANAGEMENT**

The wave buoys transmitted (using wireless communication) the raw and the processed data to a nearby shore station equipped with a receiver, a PC (computer), a back-up unit, a modem and a telephone line. For all stations, the PC received the data from the receiver (or from the wave gage) and stored in its hard disk. The PC then transmitted the processed data to the project center at METU-KLARE every two hours (almost in real time) through the modem and telephone line. Both raw and processed data were backed up, on site, at regular intervals (about three months). The whole data set was stored on magnetic tapes, and brought to the project center a few times a year. The data was archived in a form of an evolving wave data bank at METU-KLARE.

### **3.4. DATA PROCESSING**

Raw data received from the buoy could be converted to individual waves, by means of zero up-crossing method, by operating a software which was provided by Datawell Inc. Representative wave height and period values (e.g.  $H_{\max}$ ,  $H_{1/3}$ ,  $H_{\text{avg}}$ ,  $T_{\text{hmax}}$ ,  $T_{\text{avg}}$ ...etc.) were also computed by the same software for each wave record of 20 minutes. The process gave files named as \*.WAV. By using this files as the inputs, statistical analysis was carried out.

## CHAPTER 4

### RESULTS AND DISCUSSIONS

#### 4.1. DATA SELECTION FOR ANALYSIS

The analyzed wave data were obtained from three gaging stations; Alanya, Dalaman and Hopa. Stations and the periods of data used in this thesis work are given in Table 4.1.

Table 4.1 Periods of measured wave data

Stations	Data Periods
ALANYA	May 1995-December 1995
DALAMAN	March 1996-July 1996
HOPA	January 1994
	January 1995-May 1995
	July 1996-December 1996
	January 1997
	May 1997-December 1997
	January 1998-December 1998
	January 1999-April 1999

All the measured waves used in this study were deep water waves and statistical analysis were made for the deep water case.

A certain threshold value has to be defined in order to select the storms from the wave data. This value should represent a reasonably high sea state. Besides, there should be sufficient number of waves above this threshold in order to make reliable analysis. As a result, the records over which the significant wave height is above 2.0 m were analyzed as storms in this thesis research.

The wave data were investigated to detect the records that satisfy the requirement mentioned above. At Alanya 25 wave records, Dalaman 9 and Hopa 311 wave records were identified respectively totaling 345 records, which have significant wave heights above 2.0 m.

Considering the small number of records especially for Dalaman and Alanya, the records for which the significant wave height is in the range of 0.50 m to 2.0 m were also analyzed at a later stage. Satisfying this condition, at Alanya 819 wave records, Dalaman 288 and Hopa 2942 wave records were identified respectively totaling 4049 records.

## 4.2. PROBABILITY DISTRIBUTION OF INDIVIDUAL WAVE HEIGHTS

The individual wave heights in a wave record were normalized by the significant wave height,  $H_{1/3}$ . By using an increment of  $\Delta(H_i/H_{1/3}) = 0.1$ , the number of waves in each  $(H_i/H_{1/3})$  interval was computed. Afterwards, the probability density of the individual wave height in each interval was calculated as:

$$p(H_i / H_{1/3}) = \frac{n_r}{N * \Delta H} \quad (4.1)$$

where,

$p(H_i/H_{1/3})$  : Probability density of normalized wave height in a certain interval. (Assuming that the probability density is constant over the interval.)

- $n_r$  : Number of individual waves in that interval.  
 $N$  : Total number of individual waves in a wave record.  
 $\Delta H$  : Increment for ( $H_i/H_{1/3}$ )

Rayleigh distribution was used for comparison with the probability distribution of the measured data.

While deciding the normalized wave height increment to be used for obtaining the observed probability distribution, several considerations were made. A single record contains 150 to 300 individual waves. On the average, there are roughly 200 individual waves in a wave record. The formula (Freedman and Diaconis 1981) that gives the number of intervals to be used in a statistical frequency distribution as:

$$nt = \frac{rg.N^{1/3}}{2.IQR} \quad (4.2)$$

where,

- $nt$  : Number of intervals  
 $rg$  : Range, (the difference between the maximum and minimum values in the data set,  $x_{\max} - x_{\min}$ )  
 $N$  : Total number of data in the data set  
 $IQR$  : Interquartile  $Q_3 - Q_1$   
 $Q_3$  : Third quarter value in the data set  
 $Q_1$  : First quarter value in the data set

For example, if there are 100 data in a data set, 100 is divided to quarters. Then, the first quarter value (i.e. 25<sup>th</sup> value) is called as  $Q_1$  and the third quarter value (i.e. 75<sup>th</sup> value) is called as  $Q_3$ .

The range of normalized wave heights was taken between 0.0 and 2.0 since very few waves were observed in the range of  $H_i/H_{1/3} > 2.0$ . Representative samples were selected from the wave records and inserted in Eqn.(4.2). The number of intervals so

computed was in the range of 20 to 25.

Another consideration was given to Kolmogorov-Smirnov test. Data intervals of 5, 10, 15, 20 were used in the Kolmogorov-Smirnov test. It was observed that when the number of data intervals decreased, this test gave worse results statistically.

According to these considerations, the number of data intervals was selected as 20. This means that the wave height increment was used as  $\Delta(H_i / H_{1/3}) = 0.1$ .

#### **4.2.1. STATISTICAL TESTS FOR GOODNESS OF FIT**

##### **A. KOLMOGOROV-SMIRNOV TEST**

Kolmogorov-Smirnov test was used for testing the goodness of fit of the probability distribution of the individual wave heights to the Rayleigh distribution. The analysis was made by using significance level of  $\gamma = 0.01$ . According to this significance level, 100% of individual wave height records for storms with  $H_{1/3} > 2.0$  m. fit to the Rayleigh distribution for Alanya, 88.9% for Dalaman and 98.4% for Hopa. For the other records ( $0.50 \text{ m.} < H_{1/3} < 2.0 \text{ m.}$ ), 98.2% of the records fit to the Rayleigh distribution for Alanya, 97.6% for Dalaman and 98.0% for Hopa. This shows that distribution of individual wave heights fits very well to the Rayleigh distribution.

The analysis was repeated for different significance levels as well. The results showed that even for greater significance levels, the probability distribution of individual wave heights fits well to the Rayleigh distribution. The percentages of individual wave records that fit to the theoretical distribution for different significance levels for each station are given in Table 4.2 and Table 4.3.

As it is seen from Table 4.2 and Table 4.3, for Dalaman, there is a difference between the percentages of individual wave records that fit to the theoretical distribution. There are nine records for the severe storms for Dalaman. Therefore,

just a few number of wave records might affect the result greatly.

Table 4.2. The percentages of individual wave records that fit to the theoretical distribution for different significance levels of Kolmogorov-Smirnov test ( $H_{1/3} > 2.0$  m.)

Station	Significance level				
	$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.10$	$\gamma = 0.20$
ALANYA	100.0	100.0	92.0	80.0	76.0
DALAMAN	88.9	88.9	66.7	66.7	66.7
HOPA	98.4	97.7	94.9	89.7	82.0

Table 4.3. The percentages of individual wave records that fit to the theoretical distribution for different significance levels of Kolmogorov-Smirnov test ( $0.50\text{m.} < H_{1/3} < 2.0$  m.)

Station	Significance level				
	$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.10$	$\gamma = 0.20$
ALANYA	98.2	96.7	93.3	86.3	78.8
DALAMAN	97.6	95.1	92.4	86.8	77.4
HOPA	98.0	96.7	92.8	87.0	78.9

## B. CHI-SQUARE TEST

Another statistical test that was used for testing the goodness of fit of the probability distribution of the individual wave heights to the Rayleigh distribution was Chi-square test. The analysis was made by using significance level of  $\alpha = 0.01$ .

According to this significance level, 100% of individual wave height records for storms with  $H_{1/3} > 2.0$  m. fits to the Rayleigh distribution for Alanya, 88.9% for Dalaman and 95.8% for Hopa. For the other records, ( $0.50 \text{ m.} < H_{1/3} < 2.0$  m.), 98.7% of individual wave height records fits to the Rayleigh distribution for Alanya, 97.2% for Dalaman and 97.6% for Hopa. This test also shows that the distribution of individual wave heights fit very well to the Rayleigh distribution.

The analysis was repeated for different significance levels as well. The results showed that even for greater significance levels, the probability distribution of individual wave heights fits well to the Rayleigh distribution. The percentages of individual wave records that fit to the theoretical distribution for different significance levels for each station are given in Table 4.4 and Table 4.5.

#### 4.2.2. ANALYSIS WITH A MODIFIED RAYLEIGH DISTRIBUTION

For individual wave height distribution, there is a strong agreement between the observed distribution and the Rayleigh distribution. It was decided to modify the theoretical distribution in order to find the probability distribution that provides the best fit to the observed distribution.

Table 4.4. The percentages of individual wave records that fit to the theoretical distribution for different significance levels of chi-square test ( $H_{1/3} > 2.0$  m.)

Station \ Significance level	$\alpha = 0.005$	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	$\alpha = 0.10$
ALANYA	100.0	100.0	96.0	96.0	88.0
DALAMAN	88.9	88.9	88.9	77.8	77.8
HOPA	98.1	95.8	93.2	91.6	85.5

Table 4.5. The percentages of individual wave records that fit to the theoretical distribution for different significance levels of chi-square test ( $0.50$  m.  $< H_{1/3} < 2.0$  m.)

Station \ Significance level	$\alpha = 0.005$	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	$\alpha = 0.10$
ALANYA	98.7	97.7	95.2	91.6	81.8
DALAMAN	97.2	95.1	93.1	91.0	79.9
HOPA	97.6	96.3	93.7	91.5	83.0

The cumulative Rayleigh distribution has the following form:



$$P(x) = 1 - \exp(-a^2 x^n) \quad (4.3)$$

where:

$$a = \sqrt{2}, \quad x = (H_i / H_{1/3}), \quad \text{and } n = 2$$

The probability density function is:

$$p(x) = na^2 x^{n-1} \exp(-a^2 x^n) \quad (4.4)$$

The modification mentioned above was done only on the value of the parameters  $a$  and  $n$  without changing the general form of the equation. Trials for the modification were compared with the wave records.

The root-mean-square difference, defined by Eqn.(2.35) and denoted as RMS, is probably the most suitable indicator to be used in the comparison study, since it gives the average of the absolute differences between two sets of data by considering all  $(H_i/H_{1/3})$  intervals as equally important.

The Root-Mean-Square differences between the measured and the theoretical data sets were calculated by taking the Rayleigh distribution as the reference. It was also investigated how RMS values change through changing the values of two parameters of the modified Rayleigh distribution.

The value of 'a' was changed between 0.4142 and 4.4142 with an increment of  $\Delta a=0.01$  and the value of 'n' was changed between 1.50 and 6.50 with an increment of  $\Delta n=0.01$  in order to find the optimum value of 'a' and 'n'.

For the relatively more severe storm records, the maximum value of the 'a' was found as 1.5142 and the minimum one as 1.3642. The maximum value of 'n' was found as 3.01 and the minimum as 1.60. Maximum, minimum and average values of the two parameters and RMS differences for each station are given in Table 4.6.

For the other records (0.50 m.  $<H_{1/3}<2.0$  m.), the maximum value of the 'a' was found as 1.5242 and the minimum as 1.3242. The maximum value of 'n' was found as

2.92 and the minimum one as 1.59. The maximum, minimum and average values of the two parameters and RMS differences for each station are given in Table 4.7.

As it is seen from Table 4.6 and Table 4.7 the average of the optimum values of the parameter 'a' is very slightly less than the theoretical value of 1.4142 for all stations and for mild and severe storms. Standard deviations of this parameter are small. For the parameter 'n', the average of the optimum values of the parameter 'n' is consistently larger than the theoretical value of 2. This indicates a slightly more peaked distribution. Standard deviations are also larger for this parameter.

Table 4.6. The optimum values of the Rayleigh distribution parameters and the RMS differences of the measured probability distributions with Rayleigh and modified Rayleigh distributions. ( $H_{1/3} > 2.0$  m.) (Severe storms)

Stations	a	n	RMS (Rayleigh)	RMS (Modified Ray.)
<b>Station: ALANYA</b>				
Maximum 'n'	1.51	2.49	0.166	0.101
Minimum 'n'	1.39	1.75	0.228	0.210
Average of 25 records	1.40	2.08	-	-
Standard deviations	$\sigma_a = 0.06$	$\sigma_n = 0.20$	-	-
<b>Station: DALAMAN</b>				
Maximum 'n'	1.45	3.01	0.297	0.173
Minimum 'n'	1.38	1.87	0.131	0.126
Average of 9 records	1.39	2.17	-	-
Standard deviations	$\sigma_a = 0.047$	$\sigma_n = 0.382$	-	-
<b>Station: HOPA</b>				
Maximum 'n'	1.47	2.79	0.254	0.180
Minimum 'n'	1.36	1.60	0.217	0.163
Average of 311 records	1.39	2.11	-	-
Standard deviations	$\sigma_a = 0.06$	$\sigma_n = 0.20$	-	-
<b>THEORETICAL</b>	1.4142	2.00		

The modified Rayleigh distributions of individual wave heights for all individual storm wave records in each analyzed station were plotted together in order to obtain the envelope of the modified Rayleigh distributions for a station. Besides, the averages of all modified distributions were also plotted. These plots, together with the theoretical Rayleigh distribution are given in Fig. 4.1. It is seen from the graphs that the Rayleigh distribution lays inside the envelope of the modified Rayleigh distributions for individual wave records. The cumulative probability for each modified distribution was checked (it should be 1.0 theoretically) by integrating the equation of the corresponding curve for the data range. It was found that the positive and negative deviations from 1.0 are  $6.0 \times 10^{-6}$  and  $-1.0 \times 10^{-5}$  respectively for the areas under the modified distributions.

Table 4.7. The optimum values of the Rayleigh distribution parameters and the RMS differences of the measured probability distributions with Rayleigh and modified Rayleigh distributions. (0.50m.  $<H_{1/3}<2.0$  m.) (Mild storms)

Date of analyzed wave records	a	n	RMS (Rayleigh)	RMS (Modified Ray .)
<b>Station: ALANYA</b>				
Maximum 'n'	1.43	2.70	0.222	0.112
Minimum 'n'	1.32	1.65	0.204	0.170
Average of 819 records	1.39	2.10	-	-
Standard deviations	$\sigma_a=0.04$	$\sigma_n=0.16$	-	-
<b>Station: DALAMAN</b>				
Maximum 'n'	1.52	2.58	0.211	0.145
Minimum 'n'	1.40	1.83	0.147	0.134
Average of 288 records	1.41	2.12	-	-
Standard deviations	$\sigma_a=0.04$	$\sigma_n=0.14$	-	-
<b>Station: HOPA</b>				
Maximum 'n'	1.42	2.92	0.301	0.196
Minimum 'n'	1.38	1.59	0.196	0.121
Average of 2942 records	1.40	2.11	-	-
Standard deviations	$\sigma_a=0.06$	$\sigma_n=0.18$	-	-
<b>THEORETICAL</b>	1.4142	2.00		

Since RMS difference indicates the average of the absolute difference between two sets of data, it has the same unit with the compared data. Therefore, the importance of the value of RMS difference is only clear relative to a characteristic value representing the compared data. Hence, by taking the theoretical distribution as the reference, it may be suitable to compare the obtained RMS differences with the average of the theoretical probability values. Result of this comparison can be given in percentage as:

$$\% \text{ RMS error} = \frac{\text{RMS error}}{\text{Average probability density}} * 100 \quad (4.5)$$

$$\text{Average probability density} = \frac{1}{\left(\frac{H_i}{H_{1/3}}\right)_{\max}} \left[ \int_0^{\left(\frac{H_i}{H_{1/3}}\right)_{\max}} p(H_i / H_{1/3}) d(H_i / H_{1/3}) \right] = 0.5$$

Since  $(H_i/H_{1/3})_{\max}=2.0$ , and the value of the integral is very close to 1. Thus;

$$\% \text{ RMS error} = \frac{\text{RMS}}{0.5} * 100 = 200\text{RMS} \quad (4.6)$$

For the severe storm records ( $H_{1/3}>2.0$  m.), the RMS errors for each of the analyzed wave records changed from 0.0732 to 0.297. Hence, the distribution of individual wave heights for the analyzed wave records gives a percent RMS error of 14.64 % to 59.40 % when compared to the average probability density. On the other hand, RMS errors for the modified Rayleigh distributions change from 0.056 to 0.255. Then, the modified Rayleigh distributions give a percent RMS error of 11.20 % to 51.00 %. The maximum and minimum values of the RMS errors this measured distributions from the Rayleigh and the modified Rayleigh distributions for each station are given in Table 4.8.

As seen from Table 4.8, the maximum difference between the Rayleigh distribution and modified Rayleigh distribution is 24.80%, and the minimum difference is 0.58%.

Station: ALANYA

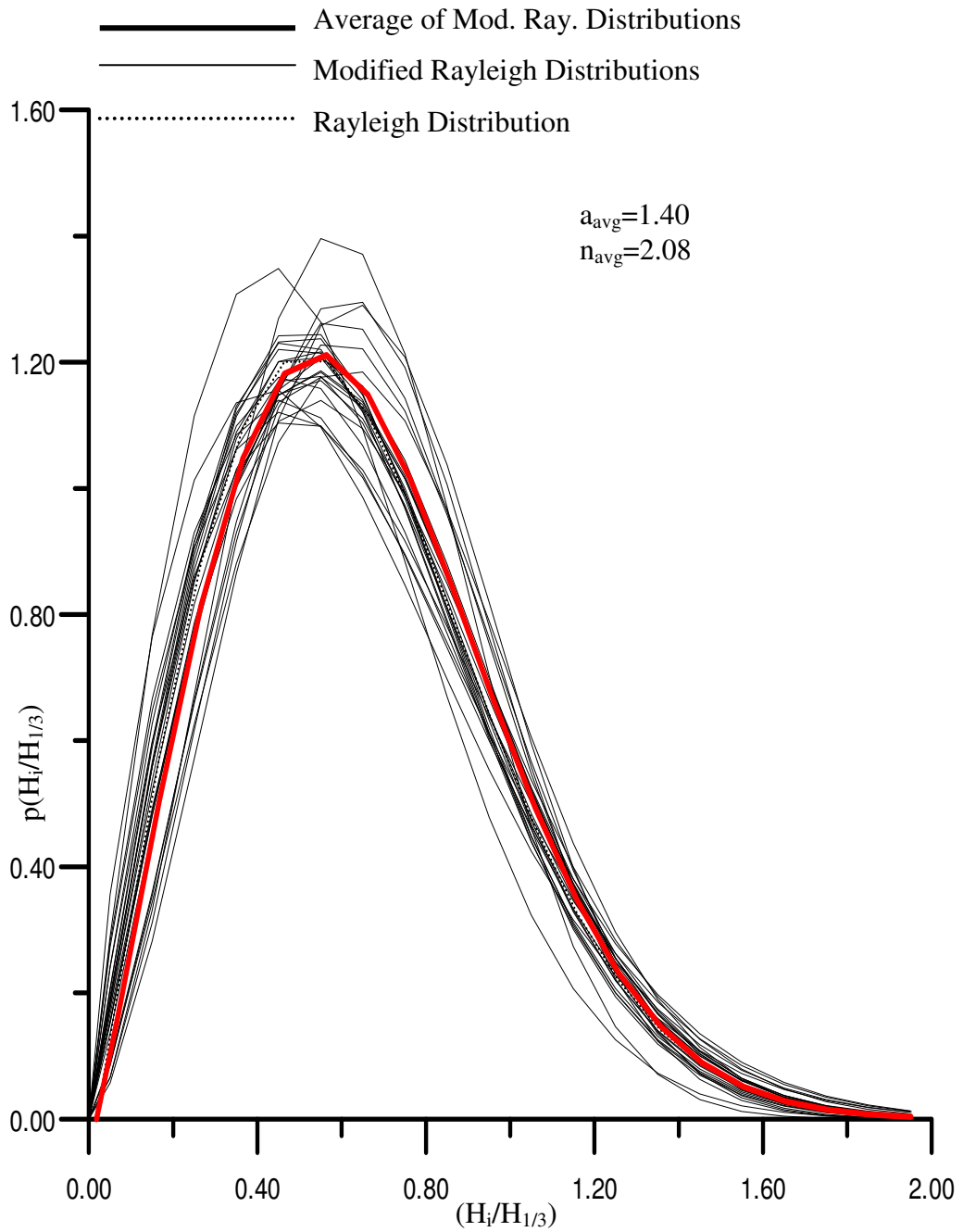


Fig.4.1. Comparison of Modified Rayleigh distributions with the Rayleigh distribution ( $H_{1/3} > 2.0$  m.)

Station:DALAMAN

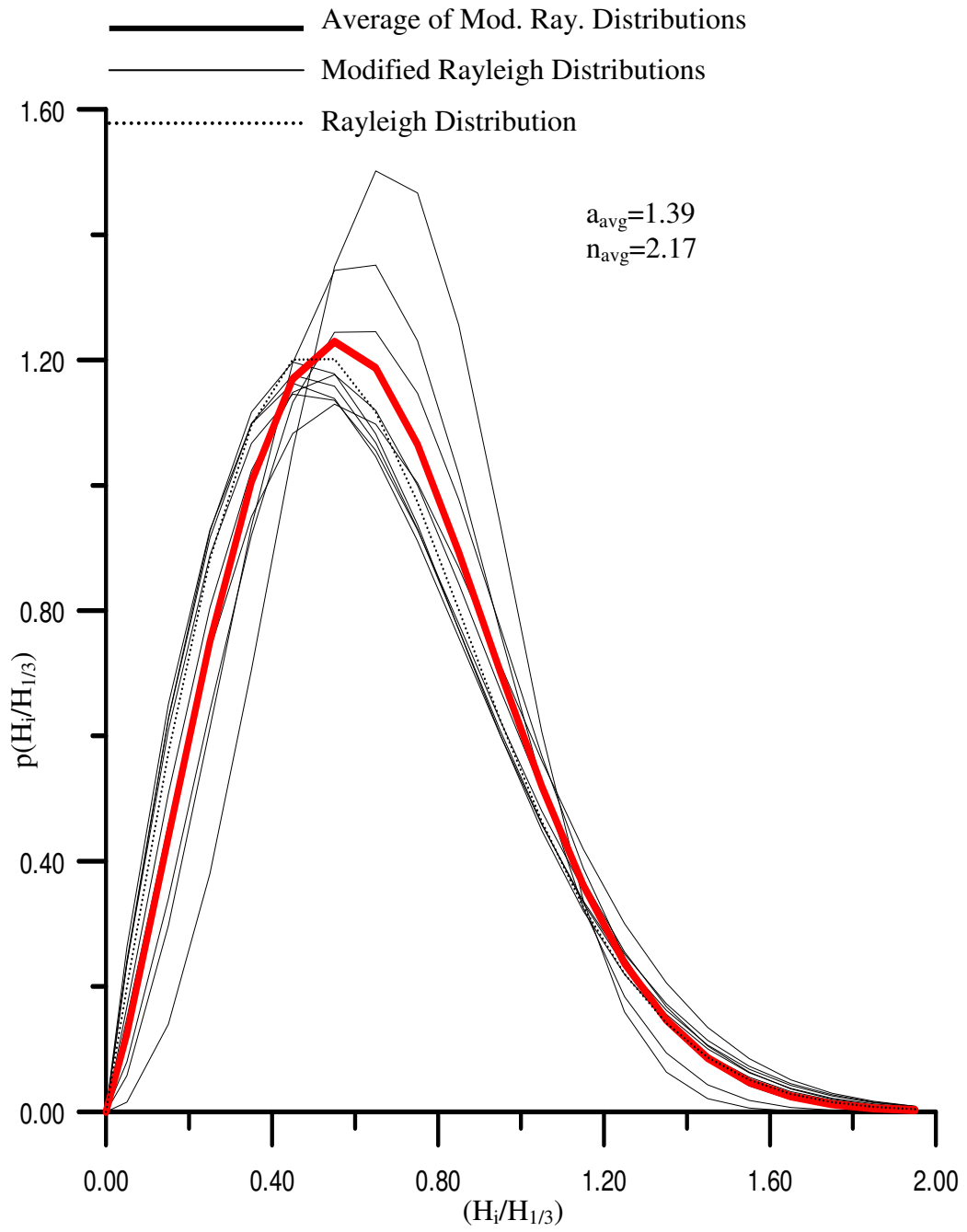


Fig.4.1. (Continued)

Station:HOPA

———— Average of Mod. Ray. Distributions

———— Modified Rayleigh Distributions

..... Rayleigh Distribution

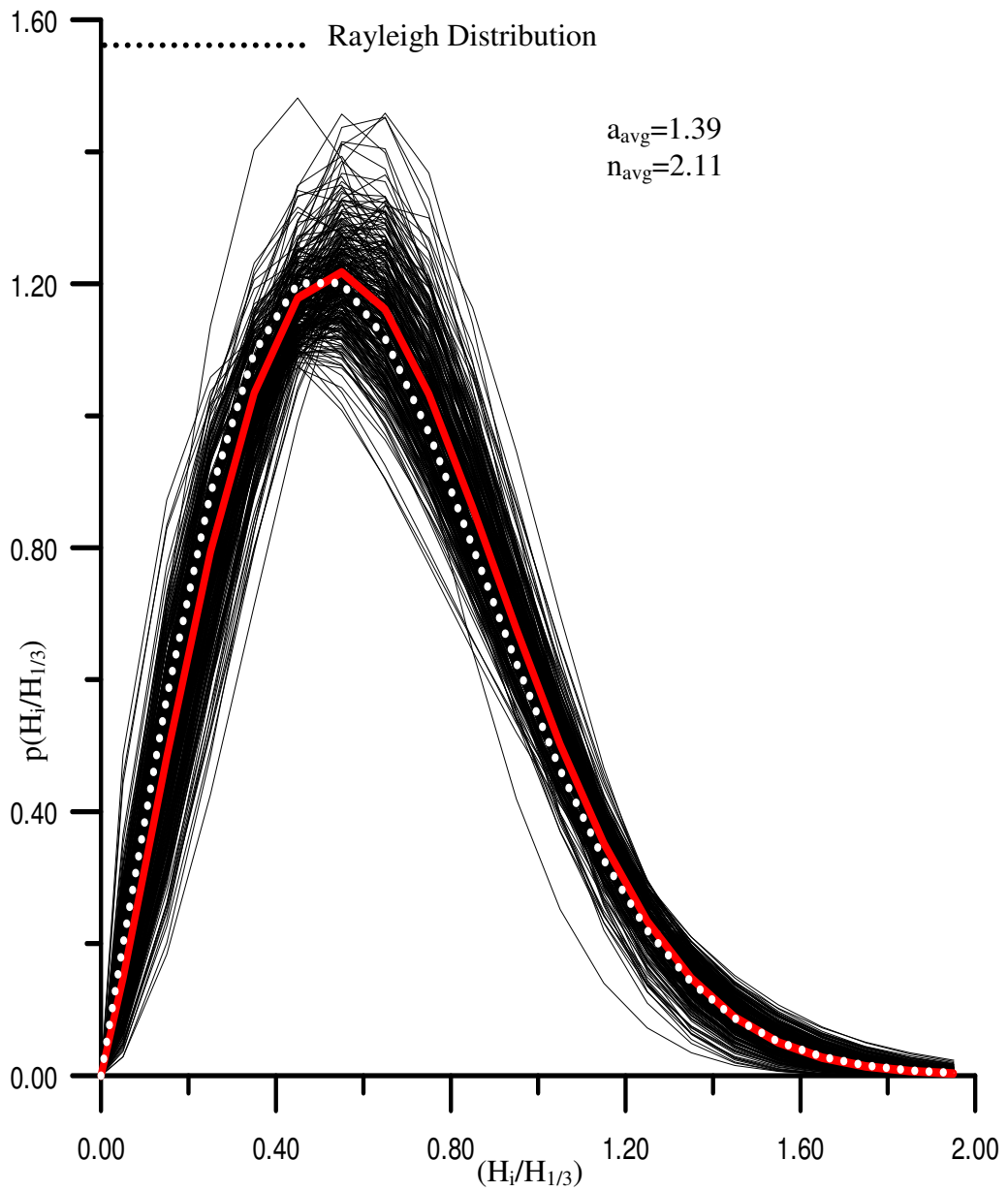


Fig.4.1. (Continued)

For the records of the milder storms, ( $0.50 \text{ m.} < H_{1/3} < 2.0 \text{ m.}$ ), the RMS errors for each of the analyzed wave records changed from 0.048 to 0.435. Hence, the distribution of individual wave heights for the analyzed wave records gives a percent RMS error of 9.60 % to 87.00 % when compared to the average probability density. On the other hand, RMS errors for the modified Rayleigh distributions change from 0.041 to 0.376. Then, the modified Rayleigh distributions give a percent RMS error of 8.20 % to 75.20 %. The maximum and minimum values of the RMS errors of this measured distributions from the Rayleigh and the modified Rayleigh distributions for each station are given in Table 4.9.

Table 4.8. Maximum and minimum statistical differences for measured distributions with Rayleigh and modified Rayleigh distributions ( $H_{1/3} > 2.0 \text{ m.}$ )

Station	RMS error (Rayleigh)	%RMS error (Rayleigh)	RMS error (Modified Ray.)	%RMS error (Modified Ray.)
ALANYA				
Minimum	0.104	20.80	0.0917	18.34
Maximum	0.228	45.60	0.210	42.00
DALAMAN				
Minimum	0.0957	19.14	0.0887	17.74
Maximum	0.297	59.40	0.173	34.60
HOPA				
Minimum	0.0732	14.64	0.0703	14.06
Maximum	0.270	54.00	0.252	50.40

As seen from Table 4.9, the maximum difference between the Rayleigh distribution and modified Rayleigh distribution is 11.80%, and the minimum difference is 0.20%.

In conclusion, the results of the goodness of fit tests indicate that the probability distribution of individual wave heights measured in all three locations fits rather well to the Rayleigh distribution and yields a shape that is very close to the Rayleigh



distribution. For a few wave records, the probability distribution of individual wave heights does not follow the Rayleigh distribution very well.

Table 4.9. Maximum and minimum statistical differences for measured distributions with Rayleigh and modified Rayleigh distributions (0.50m. <math>H\_{1/3}</math><math><2.0\text{ m.}</math>)

Station	RMS error (Rayleigh)	%RMS error (Rayleigh)	RMS error (Modified Ray.)	%RMS error (Modified Ray.)
ALANYA				
Minimum	0.064	12.80	0.063	12.60
Maximum	0.254	50.80	0.238	47.60
DALAMAN				
Minimum	0.054	10.80	0.053	10.60
Maximum	0.276	55.20	0.236	47.20
HOPA				
Minimum	0.048	9.60	0.041	8.20
Maximum	0.435	87.00	0.376	75.20

Since the distribution of individual wave heights is Rayleighian, the wave height parameters ( $H_{\max}$ ,  $H_{1/10}$ ,  $H_{1/3}$  and  $H_{\text{avg}}$ ) calculated from the measured wave data should satisfy the relationships given in Eqn.(2.5). By taking  $H_{1/3}$  as the reference, values of  $H_{\max}$ ,  $H_{1/10}$  and  $H_{\text{avg}}$  were compared with  $H_{1/3}$  for each record. Then, the equations of the wave height parameters were obtained as:

$$H^* = \alpha_i H_{1/3} \quad (4.4)$$

where,

$H^*$  : the wave height parameter which is compared with  $H_{1/3}$

- $i = 1$  for  $H^* = H_{\text{avg}}$
- $2$  for  $H^* = H_{1/10}$
- $3$  for  $H^* = H_{\max}$

The theoretical values of these parameters obtained from the Rayleigh distribution; the maximum, minimum and mean values obtained from the records are given in Table 4.10 and Table4.11 for each station.

Table 4.10. Constants for the equations of the relationships between representative wave height parameters ( $H_{1/3} > 2.0$  m.)

$$H^* = \alpha_i H_{1/3}$$

Parameter ( $H^*$ )	$H_{avg}$		$H_{1/10}$		$H_{max}$	
Station	$\alpha_1$		$\alpha_2$		$\alpha_3$	
	Min.	Max.	Min.	Max.	Min.	Max.
ALANYA (25 wave records)						
	0.626	0.673	1.17	1.37	1.333	2.059
Mean	0.638		1.25		1.709	
Standard deviation	0.023		0.043		0.185	
DALAMAN (9 wave records)						
	0.619	0.693	1.21	1.28	1.466	1.848
Mean	0.645		1.24		1.611	
Standard deviation	0.034		0.039		0.115	
HOPA (311 wave records)						
	0.591	0.692	1.17	1.51	1.307	2.174
Mean	0.641		1.25		1.636	
Standard deviation	0.024		0.046		0.157	
THEORETICAL	0.625		1.27		1.546-1.653	

Table 4.11. Constants for the equations of the relationships between representative wave height parameters (0.50m.  $<H_{1/3}<2.0$  m.)

$$H^* = \alpha_i H_{1/3}$$

Parameter ( $H^*$ )	$H_{avg}$		$H_{1/10}$		$H_{max}$	
	Min.	Max.	Min.	Max.	Min.	Max.
Station	$\alpha_1$		$\alpha_2$		$\alpha_3$	
ALANYA (819 wave records)						
	0.591	0.683	1.160	1.347	1.318	2.324
Mean	0.640		1.25		1.696	
Standard deviation	0.022		0.038		0.165	
DALAMAN (288 wave records)						
	0.609	0.675	1.180	1.376	1.431	3.151
Mean	0.640		1.26		1.744	
Standard deviation	0.020		0.031		0.186	
HOPA (2942 wave records)						
	0.539	0.734	1.028	1.474	1.047	3.250
Mean	0.640		1.25		1.686	
Standard deviation	0.024		0.040		0.179	
THEORETICAL	0.625		1.27		1.547-1.727	

The results obtained from the records are rather similar with the theoretical values. As seen from Table 4.10 and Table 4.11, average values of the coefficient  $\alpha_1$  are larger than the theoretical value. Average values of the coefficient  $\alpha_2$  are slightly less than the theoretical value. Average values of the coefficient  $\alpha_3$  are between the theoretical limits. However, for Alanya (for severe storms,  $H_{1/3}>2.0$  m.) and Dalaman (for milder storms,  $0.50$  m.  $<H_{1/3}<2.0$  m.) average values of  $\alpha_3$  are larger than the theoretical limit.

### 4.3. PROBABILITY DISTRIBUTION OF THE INDIVIDUAL WAVE PERIODS

Similar to what has been done for individual wave heights, the distribution of periods of individual waves in a wave record was studied. The individual wave periods in a wave record were normalized by the mean wave period,  $T_{avg}$ . By using an increment of  $\Delta(T_i/T_{avg})=0.1$ , the number of waves in each  $(T_i/T_{avg})$  interval was calculated. The probability density of the individual wave period in each interval was computed as:

$$p\left(T_i / T_{avg}\right) = \frac{n_r}{N * \Delta T} \quad (4.8)$$

where,

- $p(T_i/T_{avg})$  : Probability density of normalized wave period in a certain interval  
(Assuming that the probability density is constant over the interval.)
- $n_r$  : Number of individual waves in that  $(T_i/T_{avg})$  interval
- $N$  : Total number of individual waves in a wave record
- $\Delta T$  : Increment for  $(T_i/T_{avg})$

As it was discussed in Section (2.3.2), there are three theoretical probability distributions proposed for individual wave periods. These are namely: Bretschneider, Longuet-Higgins and Cavanier-Arhan-Ezraty distributions. Bretschneider distribution does not include any parameter and yields a single curve for  $T_i/T_{avg}$ . The models of Longuet-Higgins and Cavanier-Arhan-Ezraty however, provide different distributions depending on the spectral width parameters  $\nu$  and  $\varepsilon$ ; respectively. The value of  $\varepsilon$  for each wave record was calculated by using Eqn.(2.33). The spectral width parameter of Longuet-Higgins,  $\nu$ , was taken as  $\nu = 1/2 \varepsilon$  (Section (2.4)).

### 4.3.1. STATISTICAL TESTS FOR GOODNESS OF FIT

#### A. KOLMOGOROV-SMIRNOV TEST

Kolmogorov-Smirnov test was used for testing the goodness of fit of the probability distribution of the individual wave periods to the Bretschneider distribution. The analysis were made by using significance level of  $\gamma = 0.01$ . According to this significance level, for severe storm records ( $H_{1/3} > 2.0$  m.), 100% of individual wave period records fits to the distribution for Alanya, 100% for Dalaman and 93.2% for Hopa. For mild storm records ( $0.50 \text{ m.} < H_{1/3} < 2.0 \text{ m.}$ ), 79.6% of individual wave period records fits to the distribution for Alanya, 85.8% for Dalaman and 84.3% for Hopa. This shows that distribution of individual wave periods fits very well to the Bretschneider distribution.

The analysis was repeated for different significance levels as well. The results showed that even for greater significance levels, the probability distribution of individual wave periods fits well to the Bretschneider distribution. The percentages of individual wave records that fit to the theoretical distribution for different significance levels for each station are given in Table 4.12 and Table 4.13.

Table 4.12. The percentages of individual wave records that fit to the Bretschneider distribution for different significance levels of Kolmogorov-Smirnov test ( $H_{1/3} > 2.0$  m.)

Significance level \ Station	$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.10$	$\gamma = 0.20$
ALANYA	100.0	100.0	76.0	68.0	52.0
DALAMAN	100.0	100.0	88.9	66.7	55.6
HOPA	93.2	87.8	78.5	66.9	53.1

When the two tables were compared, severe storm records ( $H_{1/3} > 2.0$  m.) fit the Bretschneider distribution better than the mild storm records ( $0.50 \text{ m.} < H_{1/3} < 2.0 \text{ m.}$ ).

Table 4.13. The percentages of individual wave records that fit to the Bretschneider distribution for different significance levels of Kolmogorov-Smirnov test ( $0.50 \text{ m.} < H_{1/3} < 2.0 \text{ m.}$ )

Station \ Significance level	$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.10$	$\gamma = 0.20$
ALANYA	79.6	72.9	62.3	50.2	38.8
DALAMAN	85.8	80.2	73.3	61.5	45.8
HOPA	84.3	79.1	70.7	61.5	49.0

The analysis was repeated by using significance level of  $\gamma = 0.01$  for Longuet-Higgins distribution. According to this significance level, for severe storm records, 96.0% of individual wave period records fits to the distribution for Alanya, 100% for Dalaman and 89.4% for Hopa. For mild storm records, 49.8% of individual wave period records fits to the distribution for Alanya, 15.6% for Dalaman and 59.4% for Hopa. This shows that distribution of individual wave periods fits well to the Longuet-Higgins distribution.

The analysis was repeated for different significance levels as well. The results showed that even for greater significance levels the probability distribution of individual wave periods fits well to the Longuet-Higgins distribution. The percentages of individual wave records that fit to the theoretical distribution for different significance levels for each station are given in Table 4.15 and Table 4.16.

Table 4.15. The percentages of individual wave records that fit to the Longuet-Higgins distribution for different significance levels of Kolmogorov-Smirnov test ( $H_{1/3} > 2.0 \text{ m.}$ )

Station \ Significance level	$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.10$	$\gamma = 0.20$
ALANYA	96.0	80.0	60.0	40.0	20.0
DALAMAN	100.0	77.8	66.7	55.6	0.0
HOPA	89.4	81.4	66.6	46.0	18.0

Table 4.16. The percentages of individual wave records that fit to the Longuet-Higgins distribution for different significance levels of Kolmogorov-Smirnov test (0.50 m.  $<H_{1/3}<2.0$  m.)

Station \ Significance level	$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.10$	$\gamma = 0.20$
ALANYA	49.8	40.4	29.1	18.3	8.6
DALAMAN	15.6	9.4	4.9	2.4	1.0
HOPA	59.4	52.2	40.8	28.1	15.4

When the two tables were compared, the severe storm records ( $H_{1/3}>2.0$  m.) fit the Longuet-Higgins distribution better than the mild storm records (0.50 m.  $<H_{1/3}<2.0$  m.). For Dalaman, there are nine severe storm records. Therefore, the difference between the two tables is great for that station.

The analysis was also made by using significance level of  $\gamma = 0.01$  for Cavanier-Arhan-Ezraty distribution. According to this significance level, for severe storm records, none of the individual wave period records fits to the distribution for Alanya and Dalaman. For Hopa, only 1.6% of the wave records fits to the theoretical distribution. For mild storm records, none of records fits to the distribution for Dalaman. For Hopa, 3.5% of the wave records fits to the theoretical distribution and for Alanya 1.0%. This indicates that probability distribution of individual wave periods does not generally fit to the Cavanier-Arhan-Ezraty distribution.

## B. CHI-SQUARE TEST

The second statistical test that was used for testing the goodness of fit of the probability distribution of the individual wave periods to the Bretschneider distribution was the Chi-square test. The analysis was made by using significance level of  $\alpha = 0.01$ .

According to this significance level, for severe storm records, 24% of individual wave period records fits to the Bretschneider distribution for Alanya, 33.3% for Dalaman and 46.3% for Hopa. For mild storm records, 27.5% of

individual wave period records fits to the Bretschneider distribution for Alanya, 19.8% for Dalaman and 40.8% for Hopa. This test also indicates that the distribution of individual wave periods fits to the Bretschneider distribution moderately.

The analysis was repeated by using significance level of  $\alpha=0.01$  for Longuet-Higgins distribution as well. According to analysis, for severe storm records, 20.0% of individual wave period records fits to the distribution for Alanya, 33.3% for Dalaman and 27.3% for Hopa. For mild storm records, 6.7% of wave records fits to the distribution for Alanya, 1.4% for Dalaman and 4.8% for Hopa. This shows that distribution of individual wave periods fits to the Longuet-Higgins distribution moderately for severe storm records. On the other hand, it does not fit to the Longuet-Higgins distribution for mild storm records.

The analysis was repeated by using significance level of  $\alpha=0.01$  for Cavanier-Arhan-Ezraty distribution as well. According to analysis, for severe storm records, none of the wave records fits to the theoretical distribution for Alanya, 11.11% for Dalaman, and 4.18% for Hopa. For mild storm records, none of the individual wave period records fits to the theoretical distribution for Dalaman, 1.22% for Alanya, and 15.98% for Hopa. That indicates that probability distribution of individual wave periods does not fit generally to the Cavanier-Arhan-Ezraty distribution.

Chi-square test gave worse results than the Kolmogorov-Smirnov test. This is due to the algorithm of the Chi-square test. For example, let us consider that there are 200 individual waves in a wave record. After individual wave periods are normalized; there might be only one wave in the interval of  $(T_i/T_{avg})=[1.9,2.0)$ . The expected number of waves for this interval for Bretschneider distribution is 0.026. Chi-square value of this pair is 36.49. This shows that even a single wave can greatly change the result of Chi-square test.

In general, it can be concluded for the analyzed data, the probability distribution of individual wave periods in single wave records fits to the proposed theoretical distributions by Bretschneider and Longuet-Higgins with same levels of confidence. On the other hand, it does not generally fit to the Cavanier-Arhan-Ezraty distribution.



Besides studying the probability distribution of individual wave periods, the relationships between the wave period parameters ( $T_{\max}$ ,  $T_{1/10}$  and  $T_{\text{avg}}$ ) computed from the measured wave data were also analyzed by taking  $T_{1/3}$  as the reference for each wave record.

Then, the equations of the wave period parameters were obtained as:

$$T_* = \beta_j T_{1/3} \quad (4.9)$$

where,

$T_*$  : the wave height parameter which is compared with  $H_{1/3}$

$j = 1$  for  $T_* = T_{\text{avg}}$

$2$  for  $T_* = T_{1/10}$

$3$  for  $T_* = T_{\max}$

Reported values of these parameters given in Section (2.3.2), maximum and minimum values for wave records for each station are given in Table 4.17 and Table 4.18.

As seen from the Table 4.17, for severe storm records, the minimum, maximum and average values for each coefficient calculated from observed data from all three stations are between the limits reported earlier. Only for Hopa, the maximum value of  $\beta_3$  constant slightly exceed the earlier limits.

As shown from the Table 4.18, for mild storm records, the average values for each coefficient calculated from observed data from all three stations are also between the limits reported earlier.

Table 4.17. Constants for the equations of the relationships between representative wave period parameters ( $H_{1/3} > 2.0$  m.)

$$T_* = \beta_j T_{1/3}$$

Parameter ( $T_*$ )	$T_{avg}$		$T_{1/10}$		$T_{max}$	
	Min.	Max.	Min.	Max.	Min.	Max.
Station	$\beta_1$		$\beta_2$		$\beta_3$	
	Min.	Max.	Min.	Max.	Min.	Max.
ALANYA						
	0.787	0.915	0.949	1.053	0.803	1.282
Mean	0.847		1.016		0.991	
Standard deviation	0.032		0.026		0.127	
DALAMAN						
	0.824	0.909	0.915	1.045	0.806	1.227
Mean	0.849		0.986		0.999	
Standard deviation	0.029		0.042		0.132	
HOPA						
	0.712	0.939	0.922	1.082	0.694	1.348
Mean	0.847		1.007		0.997	
Standard deviation	0.037		0.030		0.107	
Reported Range	0.7-1.1		0.9-1.1		0.6-1.3	

#### 4.4. PROBABILITY DISTRIBUTION OF THE INDIVIDUAL WAVE STEEPNESSES

As it was explained in Section (2.3.4), wave steepness is defined as the ratio of wave height to wave length. Dimensionless steepnesses of the individual waves were calculated by using Eqn.(2.18).

The same procedure that was carried out for the individual wave heights and periods was repeated for the wave steepness. An increment of  $\Delta (S/S_*)=0.1$  was used for calculation of the probability densities. The distributions of individual wave steepnesses

Table 4.18. Constants for the equations of the relationships between representative wave period parameters (0.50 m. <math>H\_{1/3}</math><math><2.0</math> m.)

$$T_* = \beta_j T_{1/3}$$

Parameter (T*)	T <sub>avg</sub>		T <sub>1/10</sub>		T <sub>max</sub>	
Station	$\beta_1$		$\beta_2$		$\beta_3$	
	Min.	Max.	Min.	Max.	Min.	Max.
ALANYA						
	0.729	0.946	0.906	1.143	0.579	1.462
Mean	0.848		1.010		0.997	
Standard deviation	0.036		0.034		0.156	
DALAMAN						
	0.730	0.953	0.925	1.186	0.571	1.608
Mean	0.866		1.020		1.009	
Standard deviation	0.036		0.037		0.189	
HOPA						
	0.644	1.141	0.859	1.341	0.587	1.957
Mean	0.858		1.011		1.001	
Standard deviation	0.045		0.037		0.149	
Reported Range	0.7-1.1		0.9-1.1		0.6-1.3	

for single wave records were computed and compared with the theoretical distributions of Battjes and Overvik-Houmb.

#### 4.4.1. STATISTICAL TESTS FOR GOODNESS OF FIT

##### A. KOLMOGOROV-SMIRNOV TEST

Kolmogorov-Smirnov test was used for testing the goodness of fit of the probability distribution of the individual wave steepnesses to the Battjes distribution. The analysis was made by using significance level of  $\gamma = 0.01$ . For severe storm

records, the analysis resulted as; 32% of individual wave period records fits to the theoretical distribution for Alanya, 11.1% for Dalaman and 36.3% for Hopa. For mild storm records, 38.1% of individual wave period records fits to the theoretical distribution for Alanya, 47.9% for Dalaman and 42.5% for Hopa. This indicates that probability distribution of individual wave steepnesses fit to the Battjes distribution moderately.

The analysis was repeated by using significance level of  $\gamma = 0.01$  for Overvik-Houmb distribution. According to the analysis, for severe storm records, 12% of individual wave period records fits to the distribution for Alanya, 0% for Dalaman and 12.5% for Hopa. For mild storm records, 38.2% of individual wave period records fits to the distribution for Alanya, 70.8% for Dalaman and 37.5% for Hopa. That results show that probability distribution of individual wave steepnesses does not fit generally to the Overvik-Houmb distribution for the severe storm records. However, it fits moderately to the Overvik-Houmb distribution for mild storm records ( $0.50 \text{ m.} < H_{1/3} < 2.0 \text{ m.}$ ). There are less wave records for the severe storms than the mild storm records. For that reason, severe storm records might give worse results than the mild storm records.

## **B. CHI-SQUARE TEST**

Another statistical test that was used for testing the goodness of fit of the probability distribution of the individual wave steepnesses to the Battjes distribution was the Chi-square test. The analysis was made by using significance level of  $\alpha = 0.01$ .

According to this significance level, for severe storm records, 16 % of individual wave period records fits to the Battjes distribution for Alanya, 0 % for Dalaman and 14.8% for Hopa. For mild storm records ( $0.50 \text{ m.} < H_{1/3} < 2.0 \text{ m.}$ ), 7.9 % of individual wave period records fits to the Battjes distribution for Alanya, 9.4 % for Dalaman and 11.6% for Hopa. From the analysis, probability distribution of individual wave steepnesses fits to the theoretical distribution slightly.

The analysis was repeated by using significance level of  $\alpha = 0.01$  for Overvik-Houmb distribution. For severe storm records, the analysis resulted as; 8% of individual wave period records fits to the theoretical distribution for Alanya, 0% for Dalaman and 10.6 % for Hopa. For mild storm records, 2.0% of individual wave period records fits to the theoretical distribution for Alanya, 0% for Dalaman and 6.0 % for Hopa. That indicates that probability distribution of individual wave steepnesses does not generally fit to the Overvik-Houmb distribution.

In conclusion, the probability distribution of individual wave steepnesses in single wave records fits moderately the proposed theoretical distribution by Battjes. On the other hand, it does not fit generally to the Overvik-Houmb distribution.

#### 4.5. ANALYSIS ON THE JOINT DISTRIBUTION OF INDIVIDUAL WAVE HEIGHTS AND PERIODS

As it was mentioned in Section (2.3.3), it is generally accepted that a weak correlation exists between individual wave heights and wave periods. For this reason, the correlation between the heights and the periods of the individual waves and the joint distribution of these two basic wave parameters were studied.

The correlation coefficient 'r' as defined by Eqn.(2.40) was calculated for the wave records. The results are given in Table 4.19 and Table 4.20.

Table 4.19. Correlation coefficients between wave heights and periods for a single wave record ( $H_{1/3} > 2.0$  m.)

Station	Minimum 'r'	Maximum 'r'	Mean 'r'
ALANYA	0.421	0.726	0.584
DALAMAN	0.483	0.598	0.549
HOPA	0.247	0.749	0.577

Table 4.20. Correlation coefficients between wave heights and periods for a single wave record (0.50 m. <math>H\_{1/3}</math><math>< 2.0\text{ m.}</math>)

Station	Minimum 'r'	Maximum 'r'	Mean 'r'
ALANYA	0.309	0.728	0.530
DALAMAN	0.256	0.688	0.462
HOPA	0.135	0.805	0.516

As it is seen from the tables there is a weak correlation between the normalized individual wave heights and periods.

The correlation coefficients for different wave height ranges were calculated to see if there was a difference among small and large waves. The normalized wave height range ( $H_i/H_{avg}$ ) was divided into intervals with  $\Delta(H_i/H_{avg})=0.50$ , and the correlation coefficient for each interval was calculated. Results are presented in Table 4.21 and Table 4.22. The change of the average value of the correlation coefficient,  $r$ , for different ( $H_i/H_{avg}$ ) intervals and for each station can be seen in Figure 4.2 for the severe storm records.

It is seen from Fig. 4.2 that the correlation coefficient between the normalized wave heights and periods decreases as the normalized wave height increases. Up to ( $H_i/H_{avg}$ )  $\approx 1.75$ , the correlation coefficient was computed as positive. After ( $H_i/H_{avg}$ )  $\approx 1.75$ , it was computed as negative. This indicates that the period of an individual wave is weakly positive correlated with its height up to a certain height of ( $H_i/H_{avg}$ )  $\approx 1.50$ . For higher waves, these two basic parameters are nearly uncorrelated.

Due to the presence of correlation between wave heights and periods, the joint probability distribution proposed by Cavanier-Arhan-Ezraty was chosen as the model distribution for comparison. A sample cross plot of normalized wave heights and periods and model distribution are shown in Figure 4.3. Equal probability levels for joint distribution of individual wave heights and periods (probability contours) were plotted as ( $H_i/H_{avg}$ ) versus ( $T_i/T_{avg}$ ) for that wave record shown in Figure 4.4.

Table 4.21. Correlation coefficients between normalized wave heights and wave periods, for different  $(H_i/H_{avg})$  intervals. ( $H_{1/3} > 2.0$  m.)

	$r((H_i/H_{avg}), (T_i/T_{avg}))$			
Interval for $(H_i/H_{avg})$	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0
Station: ALANYA				
Minimum	0.117	-0.017	0.090	-0.399
Average	0.490	0.333	0.067	0.028
Maximum	0.730	0.579	0.389	0.334
Station: DALAMAN				
Minimum	0.308	0.122	-0.179	-0.354
Average	0.537	0.347	0.077	-0.030
Maximum	0.671	0.475	0.240	0.176
Station: HOPA				
Minimum	0.027	-0.088	-0.294	-0.629
Average	0.517	0.342	0.098	0.009
Maximum	0.924	0.643	0.460	0.569

Table 4.22. Correlation coefficients between normalized wave heights and wave periods, for different  $(H_i/H_{avg})$  intervals. ( $0.50$  m.  $< H_{1/3} < 2.0$  m.)

	$r((H_i/H_{avg}), (T_i/T_{avg}))$			
Interval for $(H_i/H_{avg})$	0.0-0.5	0.5-1.0	1.0-1.5	1.5-2.0
Station: ALANYA				
Minimum	-0.167	-0.078	-0.381	-0.585
Average	0.373	0.301	0.093	0.004
Maximum	0.739	0.591	0.478	0.603
Station: DALAMAN				
Minimum	-0.050	0.033	-0.246	-0.509
Average	0.308	0.249	0.085	0.042
Maximum	0.706	0.501	0.454	0.479
Station: HOPA				
Minimum	-0.544	-0.730	-0.791	-0.984
Average	0.401	0.286	0.078	0.007
Maximum	0.997	0.902	0.924	0.882

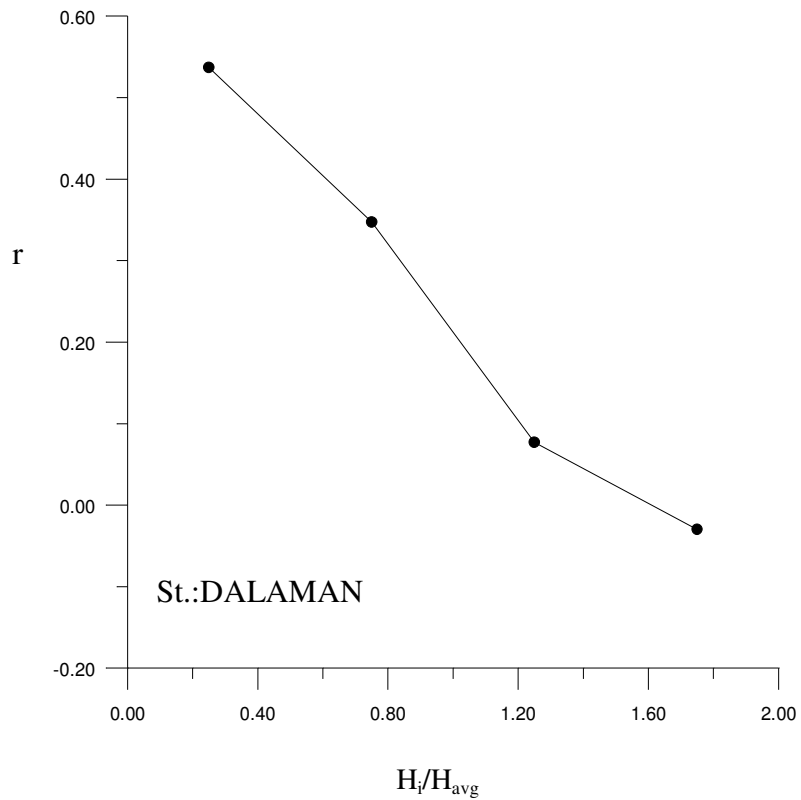
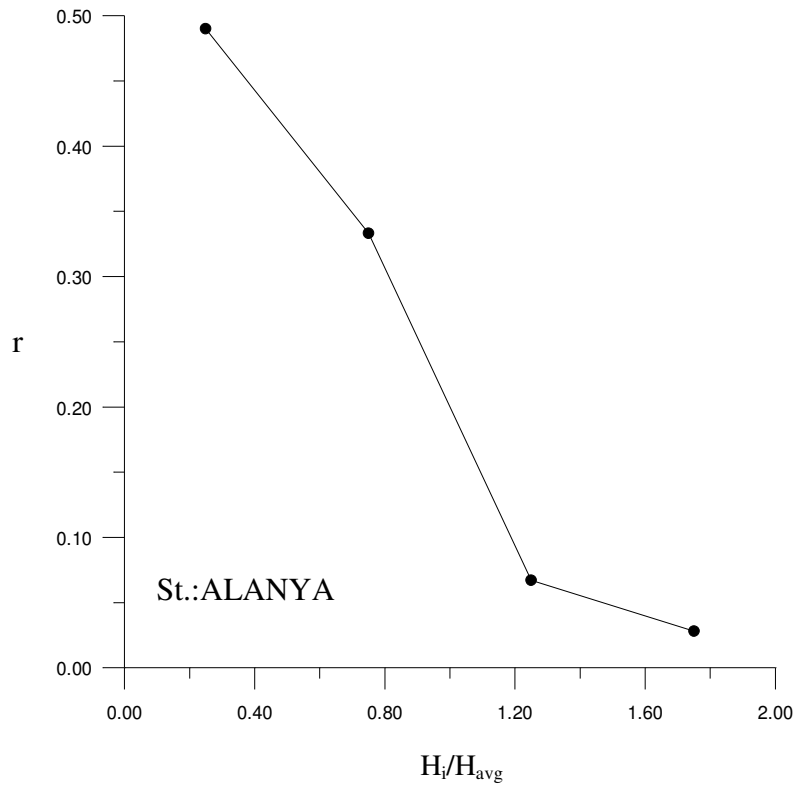


Fig.4.2. The change of the average value of the correlation coefficient,  $r$ , with respect to  $(H_i/H_{avg})$  intervals for  $H_{1/\beta} > 2.0$  m.



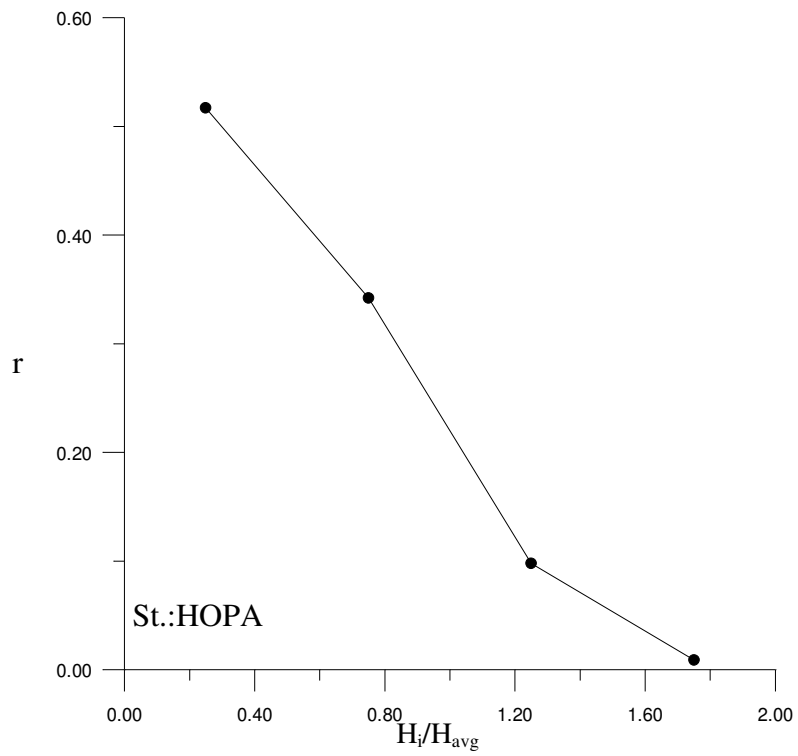


Fig.4.2. (Continued)

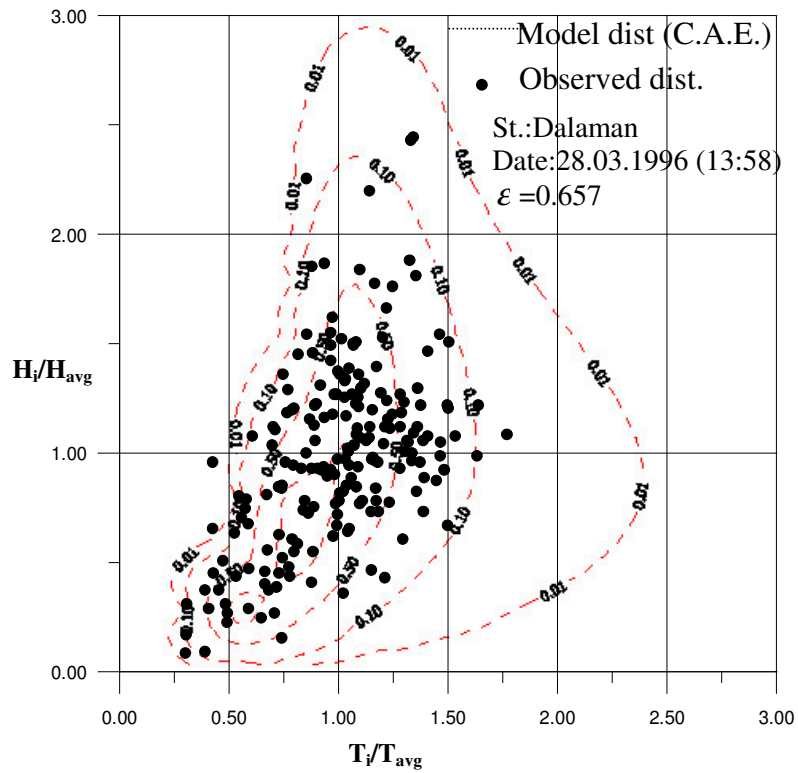


Fig.4.3. Cross plot of normalized wave heights and periods

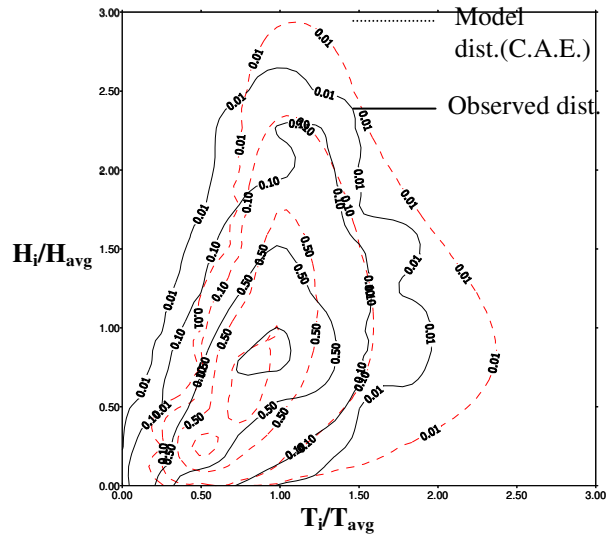


Fig.4.4. Comparison of theoretical and observed joint distribution of individual wave heights and periods (Station: Dalaman, Date: 28.03.1996 (13:58))

As it is shown in Figure 4.4, the observed joint distribution of wave heights and periods in a single wave record does not fit to the model distribution. The observed distribution has a skewed shape with respect to the mean wave period, this indicates the correlation between the wave heights and periods.

In the literature, a goodness of fit test which is applicable to joint probability distribution of two variables has not been found. For that reason, a proper goodness of fit test could not be carried out for the joint distribution of individual wave heights and periods. As engineering approaches, two methods were suggested in this study.

The joint distribution plane of individual wave heights and periods were divided into cells by using the increment of  $\Delta(H_i/H_{avg})=0.50$  and  $\Delta(T_i/T_{avg})=0.50$ , resulting in 42 cells. The number of observed waves was compared with the expected number of waves in each cell. This comparison was made by Chi-square test by using a significance level of  $\alpha=0.01$ . For the severe storm records, the percentage of waves satisfying this significance level is; 16.00 % for Alanya, 11.11 % for Dalaman and 12.12 % for Hopa. However, this poor agreement is caused by the occurrence of a few waves in the record. When the waves between the normalized wave periods of 0.0 to

0.50 and normalized wave heights between 0.50 to 1.0 and 1.0 to 1.50 were discarded the test gave better results. The number of cells that were excluded and the percent of wave records that fit the theoretical distribution are given in Table 4.23. For the mild storm records, the percentage of wave records satisfying that significance level is; 2.93 % for Alanya, 1.04 % for Dalaman and 7.34 % for Hopa. After exclusion of the cells, these records also fitted to the distribution better as seen in Table 4.24. When the severe storm records and the mild storm records are compared, it is seen that the severe storm records fit the Cavanier-Arhan-Ezraty joint distribution better.

Table 4.23. The number of cells that are excluded and percent of wave records that fit the theoretical distribution with the confidence limits  $\alpha = 0.01$ . ( $H_{1/3} > 2.0$  m.)

Number of cells that excluded	0	1 ( $H_i/H_{avg}=0.5-1.0$ )	2 ( $H_i/H_{avg}=1.0-1.5$ )
Station	%	%	%
Alanya (25 wave records)	16.00	68.00	72.00
Dalaman (9 wave records)	11.11	77.78	100.00
Hopa (311 wave records)	12.12	76.21	81.67

Table 4.24. The number of cells that are excluded and percent of wave records that fit the theoretical distribution with the confidence limits  $\alpha = 0.01$ . ( $0.50$  m.  $< H_{1/3} < 2.0$  m.)

Number of cells that excluded	0	1 ( $H_i/H_{avg}=0.5-1.0$ )	2 ( $H_i/H_{avg}=1.0-1.5$ )
Station	%	%	%
Alanya (819 wave records)	2.93	46.15	53.60
Dalaman (288 wave records)	1.04	27.78	34.72
Hopa (2542 wave records)	7.34	54.35	60.84

Second comparison method for the goodness of fit of the joint probability distribution of individual wave heights and periods was proposed. After dividing H-T joint distribution plane into the cells; the theoretical probability density and the

observed probability density value was compared for each cell by means of Root-Mean-Square (RMS) error as defined in section 2.3.5. RMS error of probability densities for each station is given in Table 4.25 and Table 4.26. As it is seen from Table 4.25 and Table 4.26 the observed distribution does not fit to the theoretical distribution.

Table 4.25. Maximum and minimum RMS error of probability densities ( $H_{1/3} > 2.0$  m.)

Station	Probability density	
	RMS-min	RMS-max
ALANYA	0.101	0.199
DALAMAN	0.115	0.163
HOPA	0.092	0.205

Table 4.26. Maximum and minimum RMS error of probability densities  
( $0.50 \text{ m.} < H_{1/3} < 2.0 \text{ m.}$ )

Station	Probability density	
	RMS-min	RMS-max
ALANYA	0.076	0.187
DALAMAN	0.076	0.183
HOPA	0.079	0.300

In conclusion, as seen from the analysis; the joint distribution of individual wave heights and periods fits to the Cavanier-Arhan-Ezraty distribution when the two cells between the normalized wave periods 0.00 to 0.50 and normalized wave heights between 0.50 to 1.00 and 1.00 to 1.50 were excluded from H-T joint distribution plane.

## CHAPTER 5

### CONCLUSIONS

Short-term statistical analysis of the available wind-wave data, which were measured at three locations (Alanya, Dalaman and Hopa) along the Turkish coast, was carried out for the periods given in Section (4.1). The following conclusions can be made as the result of this study:

1. The observed probability distribution of individual wave heights, analyzed in this thesis, fits to the Rayleigh distribution.

2. The theoretical relationships to relate each pair of the statistical wave height parameters ( $H_{\max}$ ,  $H_{1/10}$ ,  $H_{1/3}$  and  $H_{\text{avg}}$ ) are verified by the data from the Turkish stations.

3. The observed probability distribution of individual wave periods, analyzed in this thesis, fits to the Bretschneider and Longuet-Higgins theoretical distributions. On the other hand, it does not fit to the Cavanier-Arhan-Ezraty distribution.

4. The empirical relationships proposed earlier (Goda,1974) to relate each pair of the statistical wave period parameters ( $T_{\max}$ ,  $T_{1/3}$ ,  $T_{1/10}$  and  $T_{\text{avg}}$ ) are verified by the data from the studied Turkish stations.

5. The observed probability distribution of individual wave steepnesses, analyzed in this thesis, fits moderately to the theoretical distribution proposed by Battjes. On the other hand, it does not fit to the Overvik-Houmb distribution.

6. There is a weak correlation between the heights and the periods of the individual waves in the records analyzed in this thesis. This correlation becomes even weaker as normalized wave height increases. There is almost no apparent correlation for the waves having  $(H_i/H_{avg}) > 2.0$  m.

7. There is no correlation between the heights and the periods of the individual waves for both of the Bretschneider and Longuet-Higgins distributions. On the other hand, the observed distribution have a skewed shape in the direction of higher periods, similar to Cavanier-Arhan-Ezraty distribution, which implies the weak correlation between wave heights and wave periods.

8. The joint distribution of measured wave heights and wave periods does not fit the Cavanier-Arhan-Ezraty distribution. However, when some parts ( $T_i/T_{avg}$  0.00-0.50,  $H_i/H_{avg}$  =0.50-1.00 and 1.00 -1.50) are excluded from the H-T joint distribution plane, the agreement with the theoretical distribution becomes better.

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