THE STANDARD MODEL ANALYSIS OF THE CP VIOLATION IN THE INCLUSIVE SEMILEPTONIC B-MESON DECAYS

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ABSTRACT

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Being a flavor changing neutral current process, $B \to X_d \, \ell^+ \ell^-$decays provide reliable testing grounds for the Standard Model at the loop level. They are also important in the CKM phenomenology and investigating the CP violation due to the existence of the sizable interference terms in the decay amplitude. In this work, the rare $B \to X_d \, \ell^+ \ell^-$decays ($\ell = e, \mu, \tau$) are investigated in the context of the Standard Model. The differential branching ratio, forward-backward asymmetry, CP-violating asymmetry and CP-violating asymmetry in the forward-backward asymmetry in these processes are examined. The dependencies of these physical parameters on the Standard Model parameters are analyzed by paying a special attention to the long distance effects. Although the branching ratios predicted for the $B \to X_d \, \ell^+ \ell^-$decays are relatively small because of CKM suppression, it has been found that there is a significant A_{CP} and $A_{CP}(A_{FB})$ for these processes.

Keywords: Standard Model, Flavor Changing Neutral Current, B-meson Decays, CP asymmetry.

ÖZ

İNKLUSİF B-MEZON BOZUNUMLARINDAKİ CP BOZULMASININ STANDART MODEL ANALİZİ

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$B \to X_d \ell^+\ell^-$bozunumları çeşni değiştiren nötr akım prosesleri oldukları için Standart Modelin halka seviyesinde test edilmesinde güvenilir bir zemin sağlamaktadırlar. Bu bozunumlar ayrıca, bozunum genliğindeki oldukça büyük karışım terimleri yüzünden CKM fenomenolojisinde ve CP asimetrisinin araştırılmasında da önemlidirler. Bu çalışmada, Standart Model çerçevesinde nadir $B \to X_d \ell^+\ell^-(\ell = e, \mu, \tau)$ bozunumları incelendi. Bu prosesin difransiyel dallanma oranı, ileri-geri asimetrisi, CP bozulma asimetrisi ve ileri-geri asimetrisindeki CP bozulma asimetrisi çalışıldı. Bu fiziksel parametrelerin Standard Model model parametrelerine bağlılıkları uzun mesafe etkileri özellikle dikkate alınarak incelendi. $B \to X_d \ell^+\ell^-$bozunumu için, CKM bastırması yüzünden dallanma oranının göreceli olarak küçük olmasına karşın önemli ölçüde A_{CP} ve $A_{CP}(A_{FB})$ olduğu gösterildi.

Anahtar Sözcükler: Standart Model, Çeşni Değiştiren Nötr Akımlar, B-mezon Bozunumları, CP Asimetrisi.

TO MY FAMILY

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CHAPTER 1

INTRODUCTION

The Standard Model (SM) of elementary physics provides a very successful framework in understanding the weak, electromagnetic and strong interactions. The basic ingredients of the SM are the quarks, which are the known building blocks of the strongly interacting particles, and the fundamental fermions that are not influenced by the strong interactions. The quarks and leptons are classified into three families. Each family is made up of a charged lepton, its associated neutrino and two quarks, one quark with charge -1/3 and one with charge +2/3. Of the three forces that mentioned above the electromagnetism was the first to be described by an accurate theory, namely, quantum electrodynamics (QED). In QED, interaction of two charged particles, such as two electrons, is related to the exchange of a third particle: photon. Photon is massless and the electromagnetic force is long-range in nature. The quarks are distinguished from leptons by possessing another kind of a charge known as color. The corresponding quantum theory of strong interactions which is modelled directly on QED, is called Quantum Chromodynamics (QCD) and acts not between the electric charges but between the color charges. QCD permits calculations of a wide range of properties of hadrons, which are the finite structures formed by the strong force, and has been validated by the discovery of its force-carrier, the gluon.

QED initially encountered divergent quantities but then they were controlled by a procedure known as renormalisation, leading to successful estimates of quantities such as the anomalous magnetic moment of the electron and the Lamb shift in hydrogen. By contrast, weak interactions as formulated up to the mid-1960s involved interactions of two currents exchanged by heavy bosons and this interaction is very singular and cannot be renormalized. The use of the Higgs mechanism to break the electroweak symmetry converted this phenomenological theory into one suitable for higher-order calculations. This electroweak theory has been very successful, leading to the prediction and observation of the W and Z bosons.

Since all the known elementary particles are subject to the weak interaction its phenomenology is very rich and has always provided valuable information about the nature of elementary particle interactions. Among the weak interactions between the elementary particles, the B-meson systems represent an ideal framework for the study of the structure of the SM, especially for its poorly studied aspects, particularly Cabbibo-Kobayashi-Maskawa matrix elements, the leptonic decay constants, etc. Since the b quark mass is much larger than the typical scale of the strong interaction, long-distance strong interactions are less important and are under better control than in for example kaon physics. Thus, for example the CP violation in the B system will give an important independent test of the SM description of the CP violation.

The so-called rare B decays are of particular interest. It is believed that almost all b-quarks decay weakly into charm quarks, via the W-emission process, $b \to c\, W^-$. Then, the rare decays are those which do not include the release of a c quark onto the final state. These may include both the so-called Cabibbo-suppressed decays, such as those mediated by the transition $b \to u\, W^-$, and flavor changing neutral current (FCNC) decays, that is, the decays via the currents that change the flavor but not the charge of the quark. In the SM at tree level, unitarity implies that FCNC processes are absent. However, they may appear at one loop level through the so-called box and/or penguin diagrams in the SM. Therefore, these decays have been always good candidates for testing the SM at loop level.

The aim of this work is to perform a quantitative analysis on the SM CP violation and the related observables, such as the forward-backward asymmetry and CP violation aysmmetry in the forward-backward asymmetry in the $B \to X_d \ell^+ \ell^-$ decays. As being an inclusive mode, this decay provides theoretically clean observables because no specific model is needed to describe the hadronic final states. In our work, we study the abovemensioned observables to consider all three lepton modes by mainly focusing on LD effects and also their dependence on the SM parameters ρ and η.

The plan of the thesis is as follows: Chapter 2 contains an overview of the main features of the SM in which we have performed our calculations. Chapter 3 attempts to summarize the SM picture of the CP violation in general. Chapter 4 is devoted to the analysis of the SM CP violation in the inclusive $B \to X_d \, \ell^+\ell^-$decay. There we have calculated CP violation aysmmetry , forward-backward asymmetry and CP violation aysmmetry in the forward-backward asymmetry in the $B \to X_d\ell^+\ell^-$ decays. We present the conclusion of the thesis in Chapter V.

CHAPTER 2

THE STANDARD MODEL

The Standard Model (SM) [1]-[4] provides a framework to describe the "point-like" particles of nature and the interactions between them. It has been known for quite some time now that the particles such as protons, neutrons, pions, kaons etc. are not fundamental. The fact that the neutron possesses a magnetic moment is evidence enough that the neutron must be a composite particle.

All existing particles can be divided into two groups: fermions (which obey the Pauli Exclusion Principle) and bosons (which don't). The SM seeks to incorporate the fundamental interactions between these particles in terms of the exchange of intermediate particles.

Four different forces act between the particles:

The Gravitational Force is too weak to play a major role in any particle physics scenario except in the earliest stages of universe. It comes about due to the exchange of gravitons. It is neglected in the particle physics problems.

The Weak Force is evident in decays of particles and nuclei (beta decay, etc.) as well as in the interactions of neutrinos. It comes about due to the exchange of spin-1 $W^{\pm}$ and Z^0 bosons in the SM.

The Electromagnetic Force is responsible for just about all of the everyday physics we see around us. It results from the exchange of massless photons.

The Residual Strong Nuclear Force was first observed in the late 40s and is due to the exchange of pions. From the uncertainty principle the lower the mass of the exchanged particle the greater the range of the force. As the pion is the lightest nuclear active particle, this pion exchange force has the largest range

($\sim 10^{-15}\,m$) and hence was the earliest to observe. It is however not fundamental as it can be explained at a lower level in terms of exchanged quarks.

The Fundamental Strong Nuclear Force is the basic force between quarks resulting from exchange of gluons.

The SM incorporates the unified theory of electroweak interactions and the theory of strong interactions, QCD. It therefore handles the strong, electromagnetic and weak forces in one description, although the weak phenomena and the QCD does not derive naturally from the same theory. The crucial step for finding a Lagrangian that describes the electroweak interactions is to identify the local symmetry group. The electroweak theory invokes a "weak isospin",T, rather like ordinary spin but relating to the weak species type. The mathematics is identical to that of ordinary spin and is summarized by SU(2). Also invoked is a "weak hypercharge" Y, which is a scalar field just like ordinary charge. The group describing this is U(1). The discovery of the weak neutral current established that the local symmetry group of the electroweak theory is $SU(2) \times U(1)$. There is an exchange particle conveying differences in weak hypercharge from place to place called B^0 vector boson. The particle conveying differences in weak isospin are the W^+ , W^- and W^0 vector bosons. As a result of the Higgs mechanism, whose details will be summarized in the next section, these "primitive" filed bosons absorb the Goldstone bosons and mix giving the set of observable filed bosons:

$$\begin{aligned} & W^+ \ , \ W^- \ , \\ & Z^0 = W^0 \cos\theta_W - B^0 \sin\theta_W \ , \\ & A^0 = W^0 \sin\theta_W + B^0 \cos\theta_W \ . \end{aligned}$$

Here, θ_W is called the Weinberg-Salam mixing angle and has been measured in a wide variety of experiments.

As for the theory of strong interactions, it is best described by the exchange of gluons, changing the color of the quarks in the process. The group theory used is SU(3) and there are three basic colors which are called red, green, and blue. For SU(n) there are $(n^2 - 1)$ independent messengers required - so there are 8

Table 2.1: The known quarks and leptons. Masses in GeV except where indicated otherwise. Here and elsewhere we take $\hbar$ and c= 1 .

Quarks	Mass	Charge	Leptons	Mass	Charge
u	$0.001 - 0.005$	2/3	e	0.000511	-1
d	$0.003 - 0.009$	$-1/3$	μ	0.106	-1
c	$1.15 - 1.35$	2/3	τ	1.77	-1
s	$0.075 - 0.175$	$-1/3$	ν_e	0	0
b	$4.0 - 4.4$	$-1/3$	ν_μ	0	0
t	174.3 ± 5.1	2/3	ν_τ	0	0

gluons in the theory and these carry "color charge" making this a non-abelian group theory. This makes it different in many ways to QED which is Abelian as the photons carry no charge.

The group theory summary of the SM is therefore

$$SU(2) \otimes U(1) \otimes SU(3)\,.$$

We summarize the properties of the fundamental quarks and leptons in Tables (2.1) and (2.2). We note from these tables that:

(a) the relationship between T, Y and Q satisfies $Q = T_3 + Y/2$, the so-called Gell-Mann-Nishijima relation [5],

(b) the hypercharge is the same for each isospin doublet,

(c) the fundamental particles seem to occur in left handed (LH) doublets but right handed (RH) singlets.

(d) the prime indicates Cabbibo mixing in that the defined member of the doublets does not have a well defined downness or strangeness, or beauty.

In fact

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \qquad (2.1)$$

Table 2.2: The isospin, hypercharge and charge values of the fundamental particles.

Particle			T_3	Y	Q/e
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$	$\pm 1/2$	-1	$0;-1$
e_R	μ_R	τ_R	0	-2	-1
$\begin{pmatrix} u \\ d' \end{pmatrix}$	$\begin{pmatrix} c \\ s' \end{pmatrix}$	$\begin{pmatrix} t \\ b' \end{pmatrix}$	$\pm 1/2$	1/3	$2/3 - 1/3$
u_R	c_R	t_R	0	4/3	2/3
d_R	s_R	b_R	0	$-2/3$	$-1/3$
$(\nu_e)_R$	$(\nu_\mu)_R$	$(\nu_\tau)_R$	0	0	0

where the matrix is called as the "Cabibbo - Kobayashi - Maskawa (CKM) Matrix" .

(e) Note also that all the parameters are zero for right handed neutrinos. However right handed neutrinos are needed for neutrinos to have a non zero mass; so any observation of masses would take us beyond the SM. There is now strong evidence that neutrino flavor oscillations take place spontaneously requiring a small non zero mass for at least some of the types.

2.1 Spontaneous Symmetry Breaking and the Higgs Field

As it is well known symmetry has always played an important role in development of physics. It has played a central role in classifying the known particles and in predicting new ones.

An important result for the field theory and particle physics is provided by the Noether's Theorem : if an action is invariant under some group of transformations (symmetry), then there exist one or more conserved quantities which are associated to these transformations. The SM is a gauge theory constructed by using the invariance of its Lagrangian under local transformations. For its QED sector, we require the invariance under local gauge transformations of the U (1) group, for QCD sector, under SU(3) and finally for the weak interactions, under SU (2).

In quantum field theory, the dynamics of a system is encoded in a function of the fields called Lagrangian, which is related to the energy of the system [6]. The Lagrangian is the most convenient means for studying the symmetries of the theory because it is usually a simple task to check if Lagrangian remains unchanged under particular symmetry operations. Exact symmetries give rise, in general, to exact conservations laws. In this case both Lagrangian and the vacuum (the ground state of the system) are invariant. However there are some conservations laws which are not exact; for example, isospin, stangeness, etc. These situations can be described by adding to the invariant Lagrangian a "small term" that violates the symmetry. The second kind of symmetry can be obtained from an exactly symmetric Lagrangian, provided that the physical vacuum is not invariant under symmetry group. Such a symmetry is called "the spontaneously broken symmetry".

As a simple example of this phenomenon, let us consider a scalar self - interacting real field with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \,, \tag{2.2}$$

with

$$V(\Phi) = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4 \ \text{ and } \ \lambda > 0 \,. \tag{2.3}$$

The Lagrangian is invariant under the discrete transformation

$$\Phi \to -\Phi \,. \tag{2.4}$$

Let's check if the vacuum is also invariant under this transformation in Eq.(2.4):

Let Φ_0 be constant corresponding to the minimum of $V(\Phi)$, which satisfies

$$\Phi_0(\mu^2 + \lambda\Phi_0^2) = 0 \,. \tag{2.5}$$

- For $\mu^2 > 0$, we have just one vacuum at $\Phi_0 = 0$ and it is also invariant under Eq.(2.4).

- For $\mu^2 < 0$, we have two vacuum states corresponding to $(\Phi_0)^{\pm} = \pm\sqrt{-\mu^2/\lambda}$.

Since the Lagrangian is invariant under Eq. (2.4), the choice between Φ_0^+ or Φ_0^- is irrelevant. Nevertheless once one choice is made the symmetry is spontaneously broken since Lagrangian is invariant but vacuum is not. One may define a new field $\Phi^{'}$ by shifting the old field by

$$\upsilon = \sqrt{-\mu^2/\lambda}\,, \tag{2.6}$$

$$\Phi^{'} \equiv \Phi - \upsilon\,, \tag{2.7}$$

where the vacuum state of new field is $\Phi^{'} = 0$. Then the Lagrangian can be written as

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^{'} \partial^\mu \Phi^{'} - \frac{1}{2}\left(\sqrt{-2\mu^2}\right)^2 \Phi^{'2} - \lambda\upsilon\Phi^{'3} - \frac{1}{4}\lambda\Phi^{'4}\,. \tag{2.8}$$

This Lagrangian describes a scalar field $\Phi^{'}$ with real and positive mass, $M_{\Phi'} = \sqrt{-2\mu^2}$, but it losts the original symmetry due to the $\Phi^{'3}$ term.

We can now investigate a new phenomenon which happens when a continuous symmetry is spontaneously broken. The Lagrangian for such a field is given by

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi^*, \Phi)\,, \tag{2.9}$$

and

$$V(\Phi^*, \Phi) = \mu^2(\Phi^*\Phi) + \lambda(\Phi^*\Phi)^2\,. \tag{2.10}$$

Eq. (2.9) is invariant under global phase transformation (U(1) symmetry)

$$\Phi \to \exp(-i\theta)\Phi\,. \tag{2.11}$$

If we define the complex field in terms of two real fields by

$$\Phi = \frac{\Phi_1 + i\Phi_2}{\sqrt{2}}\,, \tag{2.12}$$

then the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi_1 \partial^\mu \Phi_1 + \partial_\mu \Phi_2 \partial^\mu \Phi_2) - V(\Phi_1, \Phi_2)\,, \tag{2.13}$$

which is invariant under SO(2) rotations

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \to \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} . \tag{2.14}$$

- For $\mu^2 > 0$, the vacuum is at $< \Phi_1 >_0 = < \Phi_2 >_0 = 0$ which means that we have two scalar fields Φ_1 and Φ_2 with mass $m^2 = \mu^2 > 0$.

- For $\mu^2 < 0$, we have a continuum of distinct vacua at

$$< |\Phi|^2 > = \frac{(< |\Phi_1|^2 > + < |\Phi_2|^2 >)}{2} = \frac{-\mu^2}{2\lambda} \equiv \frac{\upsilon^2}{2} , \tag{2.15}$$

which is invariant under SO(2) unless a choice of vacuum is made. If we choose

$$\begin{aligned} \Phi_1 &= \upsilon , \\ \Phi_2 &= 0 , \end{aligned}$$

and define the new fields as ,

$$\begin{aligned} \Phi_1^{'} &= \Phi_1 - \upsilon , \\ \Phi_2^{'} &= \Phi_2 , \end{aligned}$$

then the Lagrangian becomes

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi_1^{'} \partial^\mu \Phi_1^{'} - \frac{1}{2}(-2\mu^2)\Phi_1^{'\,2} + \frac{1}{2}\partial_\mu \Phi_2^{'} \partial^\mu \Phi_2^{'} + \text{interaction terms} . \tag{2.16}$$

Now we have a scalar field $\Phi_1^{'}$ with real and positive mass and a scalar boson $\Phi_2^{'}$.

In summary, when an exact continuous global symmetry is spontaneously broken, that is, it is not a symmetry of the physical vacuum, the theory contains one massless scalar particle for each broken generator of the original symmetry group. This is called Goldstone Theorem [7] and massless scalar particles are called Goldstone bosons.

The SM is a gauge theory so it must be invariant under the local gauge transformations of the fermion fields. Therefore, the spontaneous symmetry breaking mechanism through the Goldstone model outlined above must be generalized to be invariant under local gauge transformations, and in this way the so-called Higgs mechanism [8] operates. As before we start with the complex scalar filed having the Lagrangian given by Eq.(2.9) and require it to be invariant under a U(1) group of local transformations

$$\Phi \rightarrow \exp(-i\theta(x))\Phi\,, \tag{2.17}$$

and after the symmetry is broken down, the gauge bosons become massive.

Let us introduce a gauge field A_μ and replace the ordinary four derivative by the covariant derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu\,. \tag{2.18}$$

Then we obtain

$$\mathcal{L} = [(\partial_\mu + iA_\mu)\Phi^*(\partial^\mu - ieA^\mu)\Phi] - \mu^2\Phi^*\Phi - \lambda(\Phi^*\Phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \tag{2.19}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu\,, A_\mu \tag{2.20}$$

is the free gauge field. Under local transformations

$$\Phi(x) \rightarrow \Phi^{'}(x) = \exp(-i\theta(x))\Phi(x)\,, \tag{2.21}$$

$$\Phi^*(x) \rightarrow \Phi^{'*}(x) = \exp(i\theta(x))\Phi^*(x)\,, \tag{2.22}$$

$$A_\mu \rightarrow A^{'}_\mu = A_\mu - \frac{1}{e}\partial_\mu\theta(x)\ . \tag{2.23}$$

- If $\mu^2 > 0$, Eq. (2.19) is just the Lagrangian for charged scalar electrodynamics.

- If $\mu^2 < 0$, vacuum state is not unique, leading to spontaneous symmetry breaking. We shift the fields to write Lagrangian in terms of those with vanishing expectations values $<\Phi>_0 = \frac{\upsilon}{\sqrt{2}}$

$$\Phi(x) = \frac{1}{\sqrt{2}}[\upsilon + \xi(x) + i\chi(x)] \tag{2.24}$$

where ξ and χ are the new real fields with

$$\upsilon = \sqrt{\frac{-\mu^2}{\lambda}} \,. \tag{2.25}$$

Then Eq.(2.19) becomes

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{e^2\upsilon^2}{2}A_\mu A^\mu + \frac{1}{2}(\partial_\mu\xi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}(2\lambda\upsilon^2)\xi^2 - e\upsilon A_\mu\partial_\mu\chi \\ &+ \text{ int. terms} \,. \end{aligned} \tag{2.26}$$

The term involving $A_\mu A^\mu$ is a great surprise since in a quantum picture it looks as if the gauge vector A_μ has acquired a mass. If we look at the structure of $\mathcal{L}$ in Eq. (2.26) it now seems to describe the interaction of a massive vector field A_μ and two scalars, the massive ξ field and the massless χ field. In Eq. (2.19) there is one massless vector field and one complex scalar field. Since the theory does not change with any choice of the transformation function $\theta(x)$ in Eq.(2.17) let us choose at each space-time point to equal the phase of $\Phi(x)$. Then a gauge transformation of the form $\Phi'(x) = \exp(-i\theta(x))\Phi(x)$ can be found which transforms $\Phi(x)$ into a real field of the form

$$\Phi'(x) = \frac{1}{\sqrt{2}}[\upsilon + \eta(x)] \,, \tag{2.27}$$

where η is the Higgs boson. The Lagrangian in Eq.(2.19) now becomes, instead of Eq.(2.26) ,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}{}'F^{\mu\nu\prime} + \frac{1}{2}e^2\upsilon^2 A'_\mu A'^\mu + \frac{1}{2}(\partial_\mu\eta)^2 - \frac{1}{2}(2\lambda\upsilon^2)\eta^2 - \frac{1}{4}\lambda\eta^4 + \frac{1}{2}e^2(A'_\mu)^2(2\upsilon\eta + \eta^2) \,, \tag{2.28}$$

where

$$F_{\mu\nu}{}' = \partial_\mu A'_\nu - \partial_\nu A'_\mu \,. \tag{2.29}$$

This form describes the interaction of the massive vector boson A'_μ with the massive, real, scalar field η, whose mass squared is given by

$$m_\eta^2 = 2\lambda\upsilon^2 = -2\mu^2 \,. \tag{2.30}$$

What has happened is that in the spontaneously broken symmetry, the gauge boson has acquired mass, by eating the Goldstone Boson.

2.2 The Glashow-Weinberg-Salam Model

The gauge theory of the electroweak interactions based on the symmetry group $SU(2)_L \otimes U(1)$ is known as the Glashow-Weinberg-Salam theory [1]. In this model, as a result of the Higgs mechanism, the gauge bosons $W^{\pm}$, Z^0 become massive while photon remains massless. Since parity is violated in the weak interaction only the LH states participate in the weak interactions. Therefore, the leptons and quarks which feel the weak interaction are placed in LH weak isospin doublets, which may defined as

$$Ł = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L = \begin{pmatrix} L\nu_\ell \\ L\ell \end{pmatrix} = \begin{pmatrix} \nu_{\ell L} \\ \ell_L \end{pmatrix}, \tag{2.31}$$

with $\ell = e, \mu, \tau$, and

$$Q_{L1} = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad Q_{L2} = \begin{pmatrix} c \\ s \end{pmatrix}_L, \quad Q_{L3} = \begin{pmatrix} t \\ b \end{pmatrix}_L.$$

On the other hand, the states which feel only electromagnetic interactions must be in weak isospin singlets. For leptons, these are e_R , μ_R , τ_R, which form RH $SU(2)$ singlets

$$\Re = R\ell = \ell_R\,, \tag{2.32}$$

and for quarks they are given by $u_{R1} = u_R$, $u_{R2} = c_R$, $u_{R3} = t_R$, $d_{R1} = d_R$, $d_{R2} = s_R$, $d_{R3} = b_R$. Here, L and R are the LH and RH projection operators, which are defined as

$$\begin{aligned} L &= \frac{1}{2}(1-\gamma_5)\,, \\ R &= \frac{1}{2}(1+\gamma_5)\,. \end{aligned} \tag{2.33}$$

The group generated by T_3 and Y is $SU(2) \otimes U(1)$ with $[T_3^i, Y] = 0$. We have to also introduce the gauge field for this gauge group. These are, a triplet $W_\mu^i\,(i = 1, 2, 3)$ associated with $SU(2)$ and a singlet B_μ associated with the U(1) subgroup.

The SM Lagrangian can be written as a sum of four pieces:

$$\mathcal{L}_{SM} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}} + \mathcal{L}_{\text{quarks}} + \mathcal{L}_{\text{scalar}} \,. \tag{2.34}$$

Here,

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^{i}_{\mu\nu}F^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{2.35}$$

with

$$\begin{aligned} F^{i}_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + ig\epsilon^{ijk}W^j_\mu W^k_\nu \,, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \,. \end{aligned} \tag{2.36}$$

The lepton part of Lagrangian is given by

$$\mathcal{L} = \overline{\text{Ł}} i \not{D} \text{Ł} + \overline{\Re} i \not{D} \Re \tag{2.37}$$

The convenient covariant derivative for the LH and the RH components are

$$\begin{aligned} \text{Ł} &: D = \partial_\mu + i\frac{g}{2}\tau_i W_\mu{}^i + i\frac{g'}{2}YB_\mu \,, \\ \Re &: D = \partial_\mu + i\frac{g'}{2}YB_\mu \,, \end{aligned} \tag{2.38}$$

where g and g' are the coupling constants of $SU(2)_L$ and U(1), respectively and τ_i are the Pauli matrices. By using these definitions in Eq.(2.38) the Lagrangian in Eq. (2.37) becomes

$$\mathcal{L}_{\text{lepton}} \rightarrow \mathcal{L}_{\text{lepton}} + \overline{\text{Ł}} i\gamma^\mu (i\frac{g}{2}\tau^i W_\mu{}^i + i\frac{g'}{2}YB_\mu)\text{Ł} + \overline{\Re} i\gamma^\mu (i\frac{g'}{2}YB_\mu)\Re \,. \tag{2.39}$$

Let us first pick up the "left" piece of Eq.(2.39):

$$\mathcal{L}^L_{\text{lepton}} = -g\overline{\text{Ł}}\gamma^\mu(\frac{\tau^1}{2}W_\mu{}^1 + \frac{\tau^2}{2}W_\mu{}^2)\text{Ł} - g\text{Ł}\gamma^\mu\frac{\tau_3}{2}\text{Ł}W_\mu{}^3 - \frac{g'}{2}Y\overline{\text{Ł}}\gamma^\mu\text{Ł}B_\mu \tag{2.40}$$

whose first term is charged and can be written as

$$\mathcal{L}^{\text{Ł}\pm}_{\text{leptons}} = \frac{-g}{2}\overline{\text{Ł}}\gamma^\mu \begin{pmatrix} 0 & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & 0 \end{pmatrix} \text{Ł} \,, \tag{2.41}$$

by using the explicit form of the Pauli matrices

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ,\tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \,,\ \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

If we define

$$W_\mu^\pm = \frac{1}{2}(W_\mu^1 \mp W_\mu{}^2)\,, \tag{2.42}$$

and put the definition of Ł into the Lagrangian in Eq.(2.2), it takes the form

$$\mathcal{L}^{\text{Ł}\pm}_{\text{leptons}} = \frac{-g}{\sqrt{2}}\left[\overline{\nu}\gamma^\mu(1-\gamma_5)\ell W_\mu{}^+ + \overline{\ell}\gamma^\mu(1-\gamma_5)\nu W_\mu W_\mu{}^-\right]. \tag{2.43}$$

Let us investigate the neutral part of $\mathcal{L}_{\text{leptons}}$:

$$\mathcal{L}^{(\text{Ł}+\Re)(0)}_{\text{leptons}} = -g\overline{\text{Ł}}(\gamma_\mu\frac{\tau_3}{2})\text{Ł}W_\mu{}^3 - \frac{g^{'}}{2}(\overline{\text{Ł}}\gamma_\mu Y\text{Ł} + \overline{\Re}\gamma_\mu Y\Re)B_\mu\,, \tag{2.44}$$

$$\begin{aligned}\mathcal{L}^{(\text{Ł}+\Re)(0)}_{\text{leptons}} &= \frac{-g}{2}(\overline{\nu_L}\gamma^\mu\nu_L - \overline{\ell_L}\gamma^\mu\ell_L)W_\mu{}^3 + \frac{g\prime}{2}(\overline{\nu_L}\gamma^\mu\nu_L + \overline{\ell_L}\gamma^\mu\ell_L)\\ &+ (2\overline{\ell_R}\gamma^\mu\ell_R)B_\mu\,.\end{aligned} \tag{2.45}$$

To obtain right combination of fields that couples the electromagnetic current, let us define the neutral fields

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} B_\mu \\ W_\mu{}^3 \end{pmatrix}. \tag{2.46}$$

The relation between the SU(2) and U(1) coupling constants is given by

$$\sin\theta_W = \frac{g^{'}}{\sqrt{g^2+g^{'\,2}}}\ ;\ \cos\theta_W = \frac{g}{\sqrt{g^2+g^{'\,2}}}\,. \tag{2.47}$$

If we put Eq. (2.46) into Eq. (2.45) we get the neutral part of the Lagrangian:

$$\begin{aligned}\mathcal{L}^{(\text{Ł}+\Re)(0)}_{\text{leptons}} &= \Big(-\frac{g}{2}\sin\theta_W(\overline{\nu_L}\gamma^\mu\nu_L - \overline{\ell_L}\gamma^\mu\ell_L) - \frac{g^{'}}{2}\cos\theta_W(\overline{\nu_L}\gamma^\mu\nu_L\overline{\ell_L}\gamma^\mu\ell_L\\ &+ 2\overline{\ell_L}\gamma\mu\ell_R)\Big)A_\mu + \Big(\frac{-g}{2}\cos\theta_W(\overline{\nu_L}\gamma^\mu\nu_L - \overline{\ell_L}\gamma^\mu\ell_L)\\ &- \frac{g^{'}}{2}\sin\theta_W(\overline{\ell_L}\gamma^\mu\ell_L + 2\overline{\ell_L}\gamma\mu\ell_R)\Big)Z_\mu\,.\end{aligned} \tag{2.48}$$

If we write the Eq.(2.48) by using vector (V) and axial (A) couplings we get

$$\mathcal{L}^{(\text{Ł}+\Re)(0)}_{\text{leptons}} = -g\sin\theta_W(\overline{\ell}\gamma_\mu\ell)A_\mu - \frac{g}{2\cos\theta_W}\sum_{\psi_i=\nu,\ell}\overline{\psi_i}\gamma^\mu(g_V{}^i - g_A{}^i\gamma_5)\psi_i Z_\mu\,, \tag{2.49}$$

where

$$
\begin{aligned}
g_V^i &= T_3^i - 2Q_i \sin^2\theta_W \,, \\
g_A^i &= T_3^i \,.
\end{aligned}
\tag{2.50}
$$

We can identify the coupling of the electromagnetic current to the photon field A_μ via the electromagnetic charge

$$
e = g\sin\theta_W = g^{'}\cos\theta_W \,. \tag{2.51}
$$

The quark part of the $\mathcal{L}_{SM}$ in Eq. (2.34) can be derived by following very similar steps as the corresponding lepton part. Therefore we do not give the details of this calculation here. In the next chapter, when we consider the formulation of the SM CP violation, we will derive the Yukawa interaction term between the quarks and the scalar Higgs fields only.

Now, the next step is to add scalar fields in order to break the symmetry spontaneously and use the Higgs mechanism to give mass to $W_\pm$ and Z^0. The scalar Lagrangian is

$$
\mathcal{L}_{\text{scalar}} = \partial_\mu\Phi^\dagger\partial_\mu\Phi - \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \,. \tag{2.52}
$$

We should introduce the covariant derivative for maintaining the gauge invariance under $SU(2)_L \otimes U(1)$:

$$
\partial_\mu \to \partial_\mu + ig\frac{\tau^i}{2}W_\mu^i \quad + \quad \frac{ig^{'}}{2}Y\,B_\mu
$$

Then we rewrite Eq.(2.52) as

$$
\begin{aligned}
\mathcal{L}_{\text{scalar}} &= (\partial_\mu\Phi^\dagger + \frac{i\,g}{2}\tau^i W^i\Phi^\dagger + \frac{ig^i}{2}YB_\mu\Phi^\dagger)(\partial^\mu\Phi + \frac{ig^i}{2}YB_\mu\Phi - \frac{i\,g}{2}\tau^i W^i\Phi) \\
&- \mu^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2 \,.
\end{aligned}
\tag{2.53}
$$

When $\mu^2 < 0$, one component, which we choose to be neutral component of Φ, develops a vacuum-expectation value

$$
\Phi_0 = \Phi_{\text{vac}} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \,, \tag{2.54}
$$

which breaks both the $SU(2)$ and the hypercharge $U(1)$ symmetry. The surviving symmetry operator is the combination of Q (Eq.(4.28)) and $\upsilon = \sqrt{\frac{-\mu^2}{\lambda}}$ as defined before. Next, we redefine the scalar fields, associating a new field with each broken generator. So, we define

$$U(\xi) = \exp(\frac{-i\vec{\xi}\cdot\vec{\tau}}{2\upsilon}) \,, \tag{2.55}$$

and write

$$\begin{aligned} \Phi &\rightarrow \Phi^{'} = U(\xi)\Phi = \begin{pmatrix} 0 \\ \frac{\upsilon+\eta}{\sqrt{2}} \end{pmatrix} , \\ Ł &\rightarrow Ł^{'} = U(\xi)Ł \,, \\ W_\mu &\rightarrow W^{'}_\mu \,. \end{aligned} \tag{2.56}$$

When we rewrite Eq.(2.53) through these definitions, it becomes

$$\begin{aligned} \mathcal{L}_{\text{scalar}} &= \left| \left((\partial_\mu + \frac{g^{'}}{2} Y B_\mu + \frac{i\,g}{2}\tau^i W^i_\mu)(\frac{\upsilon+\eta}{\sqrt{2}}) \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 \\ &- \mu^2 \frac{(\upsilon+\eta)^2}{2} - \lambda \frac{(\upsilon+\eta)^4}{4} \,. \end{aligned} \tag{2.57}$$

Using Eq.(2.42) and Eq.(2.46) for $W^\pm$ and Z_μ, respectively, in Eq.(2.46) the first part of equation Eq.(2.57) becomes

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}\partial_\mu \eta \partial^\mu \eta + \frac{g^2}{4}(\upsilon+\eta)^2 (W^+_\mu W^{-\,\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu) \,. \tag{2.58}$$

The quadratic terms in the vector fields are in the form

$$\frac{g^2 \upsilon^2}{4} W^+{}_\mu W^{-\mu} + \frac{g^2\upsilon^2}{8\cos^2\theta_W} Z_\mu Z^\mu \,. \tag{2.59}$$

If we compare this with the usual mass terms for a charged and neutral vector bosons.

$$M^2{}_W W^+{}_\mu W^{-\,\mu} + \frac{1}{2} M^2{}_Z Z_\mu \,, Z^\mu \tag{2.60}$$

we can identify that

$$M_W = \frac{g\upsilon}{2} \quad \text{and} \quad M_Z = \frac{g\upsilon}{2\cos\theta_W} = \frac{M_W}{\cos\theta_W} \,. \tag{2.61}$$

Thus the SM predicts the $W^{\pm}$ and Z^0 masses in terms of three experimentally well known quantities: the fine structure constant $\alpha = e^2/4\pi = 1/137$, the Fermi coupling constant $G_F = 1.166 \times 10^{-5}\, GeV^{-2}$, and the weak mixing angle θ_W, which is determined from neutrino scattering experiments and given by $\sin^2\theta_W = 0.231 \pm 0.014$. Since the Fermi Constant G is related to g/M_W, one finds

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} = \frac{1}{2v^2} \;\Rightarrow\; v = 2^{-1/4} G_F^{-1/2} = 246\, GeV\,. \tag{2.62}$$

Then,

$$\begin{aligned} M_W^2 &= \frac{e^2 v2}{4\sin^2\theta_W} \simeq \left(\frac{\pi\alpha}{\sin^2\theta_W}\right) v^2 \simeq (\frac{37.2\,\mathrm{GeV}}{\sin\theta_W})^2\,, \\ M_W &\sim 80\,\mathrm{GeV}\,, \end{aligned} \tag{2.63}$$

$$M_Z^2 \simeq \left(\frac{37.2}{\sin\theta_W\cos\theta_W}\,\mathrm{GeV}\right)^2 \Rightarrow M_Z \simeq 90\,\mathrm{GeV}\,, \tag{2.64}$$

which are in a good agreement with the experimentally measured masses [9].

The mass of the Higgs boson is determined by the coupling in the self energy part of the potential

$$m_H^2 = \lambda\, v^2\,, \tag{2.65}$$

and it can not be predicted in the SM since the coupling λ is an unknown parameter. However, there are some arguments to constrain the Higgs mass. For example, if the SM is required to remain as a perturbative theory up to the scale of the so-called GUT (Grand Unified Theory), which is $\mathcal{O}(10^{16})$ GeV, an upper bound of the Higgs mass is given by ~ 200 GeV. For $\Lambda \sim 1$ TeV and the constraint $m_H \leq \Lambda$ predicts an upper bound of ~ 700 GeV. With a top quark of mass 175 GeV, and $\Lambda \sim 1$ TeV , a lower bound on the Higgs mass, which follows from the requirement of vacuum stability, is given by ~ 55 GeV. For $\Lambda \sim M_{GUT}$ the lower bound increases to 130 GeV. The direct Higgs boson search in the $e^+\, e^- \rightarrow H^0\, Z^0$ process at CERNs LEP experiment indicates that $m_H > 114$ GeV. A new machine at CERN, namely Large Hadron Collider (LHC) is expected to operate in the year 2005 and its main goal is to search for the Higgs particles .

CHAPTER 3

CP SYMMETRY VIOLATION

3.1 Introduction

The CP violation is the violation of the combined conservation laws associated with parity P and charge conjugation C by weak nuclear force and it is one of the most interesting topics in high-energy physics.

The parity operation is the spatial inversion of the coordinates ; $(x, y, z) \longrightarrow (-x, -y, -z)$ and it is a discrete transformation. The vector and the axial-vector fields transform as:

$$V^\mu(\vec{r}, t) \to V^\mu(-\vec{r}, t) \ \ , \ \ A^\mu(\vec{r}, t) \to -A^\mu(-\vec{r}, t)\,, \tag{3.1}$$

and the vector and the axial-vector currents transform similarly. LH components of fermions, $\psi_L = \frac{1}{2}(1 - \gamma_5)\psi$ transform into RH ones, $\psi_R = \frac{1}{2}(1 + \gamma_5)\psi$, and vice-versa. Since weak interactions only involve the LH components, parity is not a good symmetry of the weak force.

The charge conjugation operation reverses the sign of the charge and magnetic moment of a particle, leaving coordinates untouched. Thus, it converts each particle into its antiparticle. Charge conjugation implies that every charged particle has an oppositely charged antiparticle. The antiparticle of an electrically neutral particle may be identical to the particle, as in the case of the neutral π-meson, or it may be distinct, as the antineutron. Unlike P, most of the particles in nature are not eingestates of C since $C^2 = I$,

$$C|p>=|\overline{p}>=\pm|p>\,, \tag{3.2}$$

only those particles that are their own antiparticles can be eigenstates of C. Strong and electromagnetic interactions are found experimentally to be invariant under the C conjugation operation. On the other hand, it is not a symmetry of weak interactions, because when it is applied to a neutrino (LH) it gives a LH antineutrino which does not exist. It can be seen from the decay of π^+ or π^-:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \,. \tag{3.3}$$

In this decay the emitted antimuon always comes out LH. Under C operation this reaction would be

$$\pi^- \rightarrow \mu^- + \overline{\nu}_\mu \,, \tag{3.4}$$

with a LH muon, whereas in fact the muon always comes out RH.

Although weak interactions are neither invariant under P, nor invariant under C, it was originally believed that the product CP was preserved. However the observation of the decay $K_L \rightarrow \pi\pi$ by Christenson, Cronin, Fitch and Turlay in 1964 [10] changed this view. K mesons (kaons) are unstable and produced through the strong nuclear force, but they decay via the weak interaction. Neither K nor $\overline{K}$ leads a life of its own. Instead, each transforms repeatedly into others. There are two neutral strange mesons, K^0 and $\overline{K^0}$. The existence of these states results in a second property called strangeness oscillations: a pure strangeness eingenstate, say K^0, produced at a given time becomes a later time a mixture of K^0 and $\overline{K^0}$. These states are mixed by the weak interaction, which does not conserve strangeness, to produce two states quite similar in their masses (which differ only by $\Delta m = 3.49 \times 10^{-6}$ eV $= 5.30 \times 10^9$ s^{-1}), but very dissimilar in their distinctive decay modes and their life times. The two mesons K^0 and $\overline{K^0}$ are quite distinct in the presence of strong interactions which conserve strangeness. Both K^0 and $\overline{K^0}$ can decay into pions via strangeness-violating weak transitions. Thus the transmutation of K^0 into $\overline{K^0}$, or inversely of $\overline{K^0}$ into K^0 can proceed through common intermediate states of pions as in $K^0 \rightarrow (2\pi, 3\pi) \rightarrow \overline{K^0}$. The quark content of the K^0 is $\overline{s}d$, and $\overline{K^0}$ $s\overline{d}$. If the P and C operates K^0 and $\overline{K^0}$:

$$P|K^0 >= -|K^0 > \quad \text{and} \quad P|\overline{K^0} >= -|\overline{K^0} > , \tag{3.5}$$

C changes particle to its antiparticle ,

$$C|K^0>=|\overline{K^0}> \quad \text{and} \quad C|\overline{K^0}>=|K^0>, \tag{3.6}$$

then,

$$CP|K^0>=-|\overline{K^0}> \text{ and } CP|\overline{K^0}>=-|K^0> \ . \tag{3.7}$$

With the definition of Eq. (3.7), we can create CP eingenstates of neutral K mesons, labelled with 1 and 2 as linear combination of K^0 and $\overline{K^0}$ as

$$|K_1^0>=\frac{1}{\sqrt{2}}(|K^0>-|\overline{K^0}>) \ , \ |K_2^0>=\frac{1}{\sqrt{2}}(|K^0>+|\overline{K^0}>) \tag{3.8}$$

where CP eingenvalues of $|K_1>$ and $|K_2>$ are +1 and -1, respectively; that is,

$$CP|K_1^0>=|K_1^0> \ , \ CP|K_2^0>=-|K_2^0> \ . \tag{3.9}$$

K_1^0 should decay into states with CP=1 and the K_2^0 should decay into states with CP $=-1$. The neutral kaons decay into two or three pions ,

$$K_1^0 \rightarrow \pi^+\pi^-, \, \pi^0\pi^0 \quad \text{and} \quad K_2^0 \rightarrow \pi^+\pi^-\pi^0, \, \pi^0\pi^0\pi^0 \tag{3.10}$$

are allowed by CP conservation, while decays

$$K_1^0 \rightarrow \pi^+\pi^-\pi^0, \pi^0\pi^0\pi^0 \quad \text{and} \quad K_2^0 \rightarrow \pi^+\pi^-, \, \pi^0\pi^0 \tag{3.11}$$

are forbidden. K mesons decayed into two different hadronic channels at two different time scales. The first type goes through two -pion channels, with life time $\tau_S = 8.92 \times 10^{-11}\, s$, which is called K_S. The second type, which can decay into three pions with characteristic time $\tau_L = 5.17 \times 10^{-8}\, s$ is called K_L. Assuming CP conservation, one may identify K_S with the CP even state K_1^0, and K_L with the CP odd state K_2^0. It is also measured that K_S decays into pions and K_L decays into three pions. Since CP eingenvalues of two pion state is $+1$ and that of three pions is -1 the common consideration is that the K_S corresponds to the K_1, and the K_L corresponds to the K_2, respectively. The branching fraction was order of $\sim 10^{-3}$ from the Cronin and Fitch's experiment in 1964. Despite of this tiny fraction, it was an evidence of CP violation and an evidence for that K_S and K_L are not real CP eingenstates and should be written as :

$$
\begin{aligned}
|K_S> &= \frac{1}{\sqrt{1+|\epsilon^2|}}(|K_1> + \epsilon|K_2>), \\
|K_L> &= \frac{1}{\sqrt{1+|\epsilon^2|}}(|K_2> + \epsilon|K_1>)\,,
\end{aligned}
\tag{3.12}
$$

where the coefficient ϵ's experimental value is $\epsilon = 2.3 \times 10^{-3}$.

In comparison with kaons, the B meson system has several features which makes it well-suited to study SM CP violation. Since in the B meson decays, the top quark in loop diagrams is neither GIM nor CKM suppressed, large CP violating effects are expected in many different B meson decay modes. Further, in the SM, there is only one CP violating parameter, so that all CP violating effects in this theory are related. There are predicted relationships between these effects, and between the other CP conserving SM parameters. Thus the patterns of the B decays, as well as their relationships to the observed CP violation in K-decays, provide ways to test for the physics beyond the SM. In addition, in some B-meson decay channels, influence from strong interactions are relatively small so, the SM can make more precise predictions. These are what makes it so interesting to test the pattern of CP violation in B decays.

Till very recently, CP violation has only been measured in the kaon system, and there exists still few experimental data in this phenomenon. However, the first observation of CP violation in the B-meson system have been reported by the e^+e^- B factories [16]. In the near future, more experimental tests will be possible at the B-factories.

3.2 CP Violation in the Standard Model

In the SM quarks acquire mass through a gauge invariant way called Yukawa coupling of the quarks with the Higgs field, Φ:

$$
\mathcal{L}_Y = -Y^d_{ij}\,\overline{Q}^I_{Li}\,\Phi\,d^I_{Rj} - Y^u_{ij}\,\overline{Q}^I_{Li}\,\widetilde{\Phi}\,u^I_{Rj} + \text{h.c.}\,, \qquad \widetilde{\Phi} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\Phi^*\,, \tag{3.13}
$$

where i, j label the three generations, and the superscripts I denote that the quark fields in the weak interaction basis:

$$\begin{aligned} Q^I_{L1} &= \begin{pmatrix} u^I \\ d^I \end{pmatrix}_L , \; Q^I_{L2} = \begin{pmatrix} c^I \\ s^I \end{pmatrix}_L , \; Q^I_{L3} = \begin{pmatrix} t^I \\ b^I \end{pmatrix}_L , \\ u^I_{R1} &= u^I_R , \; u^I_{R2} = c^I_R , \; u^I_{R3} = t^I_R , \; d^I_{R1} = d^I_R , \; d^I_{R2} = s^I_R , \; d^I_{R3} = b^I_R . \end{aligned} \tag{3.14}$$

Consider now the terms

$$Y_{ij}\, \overline{\psi}_{Li}\, \Phi\, \psi_{Rj} + Y^*_{ij}\, \overline{\psi}_{Rj}\, \Phi^\dagger\, \psi_{Li} . \tag{3.15}$$

Under CP transformation, it becomes

$$Y_{ij}\, \overline{\psi}_{Rj}\, \Phi^\dagger\, \psi_{Li} + Y^*_{ij}\, \overline{\psi}_{Li}\, \Phi\, \psi_{Rj} . \tag{3.16}$$

Comparing Eqs. (3.15) and (3.16), we see that they are identical if a basis for the quark fields can be chosen such that $Y_{ij} = Y^*_{ij}$, i.e., that Y_{ij} are real.

After spontaneous symmetry breaking, by inserting the vacuum expectation values of Φ and $\widetilde{\Phi}$ in Eq. (3.13), we obtain mass terms for the quarks,

$$\mathcal{L}_{\rm mass} = M^u_{ij}\, \overline{u}^I_{Li}\, u^I_{Rj} + M^d_{ij}\, \overline{d}^I_{Li} d^I_{Rj} + \text{h.c.} , \tag{3.17}$$

where $M^u = (v/\sqrt{2})\, Y^u$ and $M^d = (v/\sqrt{2})\, Y^d$ stand for the mass matrices for up- and down-type quarks, respectively. To obtain the physical mass eigenstates, we must diagonalize the matrices M^d and M^u. As any complex matrix, they can be diagonalized by two unitary matrices, $U_{L,R}$ and $D_{L,R}$, respectivelly:

$$\begin{aligned} M^u_{\rm diag} &\equiv U_L\, M^u\, U^\dagger_R , \\ M^d_{\rm diag} &\equiv D_L\, M^d\, D^\dagger_R . \end{aligned} \tag{3.18}$$

Let us rewrite the up-quarks mass term from Eq. (3.17):

$$\overline{u}^I_{Li} M^u_{ij}\, u^I_{Rj} + \text{h.c.} \equiv \overline{u}^I_L\, U^+_L U_L M^u U^+_R U_R\, u^I_R + \text{h.c.} = \overline{u}_L\, M^u_{\rm diag}\, u_R + \text{h.c.} = \overline{u}\, M^u_{\rm diag}\, u ,$$

where we identified the mass eigenstates u_L and d_R according to the following formulas:

$$u_L = U_L u^I_L , \qquad u_R = U_R u^I_R . \tag{3.19}$$

Applying the same procedure to matrix M^d, we observe that it becomes diagonal as well in the new rotated basis:

$$d_L = D_L d_L^I \ , \qquad d_R = D_R d_R^I \, . \tag{3.20}$$

In summary, we start from the quark fields in the weak interaction basis and find that they should be rotated by four unitary matrices U_L, U_R, D_L and D_R in order to obtain mass eigenstates with diagonal masses. Since kinetic energies and interactions with the vector fields W_μ^3, B_μ and gluons are diagonal in the quark fields, these terms remain diagonal in the new basis, too. The only term in the SM Lagrangian where the matrices U and D show up is charged current interaction with the emission of W-boson:

$$L_{int}^{CC} \;=\; -\frac{g}{\sqrt{2}} W_\mu^+ (\overline{u}_L^I,\ \overline{c}_L^I,\ \overline{t}_L^I)\, \gamma^\mu + \begin{pmatrix} d_L^I \\ s_L^I \\ b_L^I \end{pmatrix} + \text{h.c.}\,,$$

which becomes in the new basis

$$L_{int}^{CC} \;=\; -\frac{g}{\sqrt{2}} W_\mu^+ (\overline{u}_L,\ \overline{c}_L,\ \overline{t}_L)\, \gamma^\mu\, U_L^+ D_L \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}\,,$$

where the unitary matrix $V_{\text{CKM}} \equiv U_L^+ D_L$ is called the Cabibbo-Kobayashi-Maskawa quark mixing matrix [3].

3.3 Parametrization of the CKM Matrix

The quark mixing matrix, V_{CKM}, is unitary. Unitary matrices of dimension N form a Lie group $SO(N)$, whose elements may be specified by $(N-1)^2$ real parameters. With 2 quark generations, V_{CKM} is defined by a single real parameter, the Cabbibo angle θ. However, with 3 quark generations, 4 parameters are required. The real rotations may be taken to be 3 Euler angles, and the remaining extra parameter is an irreducible complex phase. This phase is the only source of CP violation in flavor changing transitions in the SM.

In the "standard parametrization" [11] recommended by the Particle Data Group [12], the three-generation CKM matrix takes the form

$$
\begin{aligned}
V_{\rm CKM} &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} ,
\end{aligned}
$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. It has been observed experimentally that the CKM matrix has a hierarchical structure reflected by

$$
s_{12} = 0.22 \gg s_{23} = \mathcal{O}(10^{-2}) \gg s_{13} = \mathcal{O}(10^{-3}). \tag{3.21}
$$

Thus, if in the standard parametrization above, we introduce new parameters λ, A, ρ and η by imposing the relations

$$
s_{12} \equiv \lambda = 0.22 \ , \ s_{23} \equiv A\lambda^2 \ , \ s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta), \tag{3.22}
$$

we arrive at

$$
V_{\rm CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \ldots . \tag{3.23}
$$

This is the "Wolfenstein parametrization" of the CKM matrix [13], and is valid to order λ^4.

Concerning the test of the CKM picture of CP violation, the central target is the unitarity of the CKM matrix, described by

$$
V_{\rm CKM}^\dagger \cdot V_{\rm CKM} = V_{\rm CKM} \cdot V_{\rm CKM}^\dagger = \hat{1} \, , \tag{3.24}
$$

which imposes the following conditions on the matrix elements:

$$
\sum_{j=1}^{3} |V_{ij}|^2 = 1 \ , \ \sum_{i=1}^{3} |V_{ij}|^2 = 1 \ , \ \sum_{k=1}^{3} V_{ik}^* V_{kj} = 0 \ . \tag{3.25}
$$

It is very convenient to discuss the predictions of the unitarity by using the unitarity triangle, which is just a geometrical representation of the relation in Eq.(3.25) which equals 0 in the complex plane:

$$
\begin{aligned}
V_{cd}V_{ud}^* + V_{cs}V_{us}^* + V_{cb}V_{ub}^* &= 0\,, \\
V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* &= 0\,, \\
V_{us}V_{ud}^* + V_{cs}V_{cd}^* + V_{ts}V_{td}^* &= 0\,, \\
V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* &= 0\,, \\
V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* &= 0\,, \\
V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* &= 0\,.
\end{aligned}
$$

However, in only last two of these relations, all three sides are of comparable magnitude $\mathcal{O}(10^{-3})$, while in the remaining ones, one side is suppressed relative to the others by $\mathcal{O}(10^{-4})$. At the leading order in λ, these relations agree with each other, and yield

$$
(\rho + i\eta)A\lambda^3 - A\lambda^3 - (1 - \rho - i\eta)A\lambda^3 = 0\,. \tag{3.26}
$$

Consequently, they describe the same triangle in the $\rho - \eta$ plane, which is usually referred to as the unitarity triangle of the CKM matrix [14]. The present situation about the knowledge of the element of the CKM matrix can be summarized by [15]

$$
\begin{aligned}
|V_{us}| = \lambda = 0.2196 \pm 0.0026 \quad &, \quad |V_{cb}| = (41.2 \pm 2.0) \times 10^{-3} \ , \\
\frac{|V_{ub}|}{\lambda |V_{cb}|} = 0.40 \pm 0.08 \quad &, \quad |V_{ub}| = (35.7 \pm 3.1) \times 10^{-4} \ ,
\end{aligned}
$$

implying

$$
A = 0.85 \pm 0.04\,.
$$

CHAPTER 4

STANDARD MODEL CP VIOLATION IN $B \to X_d \ell^+\ell^-$DECAYS

4.1 Introduction

The B system represents an ideal framework for determining the parameters of the SM accurately, testing its subtle properties and searching for signatures of new physics beyond it. Since b quark mass is larger than the typical scale of the strong interaction, long-distance strong interactions are generally less important and are under better control in B systems. Thus, for example the CP violation in the B system will yield an important independent test of the SM description of CP violation. B meson decays also allow for a rich CKM phenomenology and a strict test of the unitarity constraints.

The so-called rare decays are of particular interest. While the dominant charged current decays change the b quark into an either charge +2/3 c or u quark, the rare processes are those which do not include the release of a c quark onto the final state. In the SM, they represent flavour changing neutral currents (FCNC) that change the flavor but not the charge of the quark and are induced by one-loop diagrams. Therefore, these decays have been always good candidates for testing the SM at loop level. For example, Fig. (4.1) contains four types of Feynman loop (penguin) diagrams that describes the transitions of a b quark into a charged -1/3 d quark, which is effectively a neutral current transition.

Among the rare B-meson decays, the inclusive B $\to X_{s,d}\ell^+\ell^-$ modes are prominent because of their relative cleanness compared to the pure hadronic decays. In the SM, $B \to X_{s(d)}\ell^+\ell^-$ decays are dominated by the parton level processes

$b \to s(d)\ell^+\ell^-$, which occur through an intermediate u, c or t quarks. They can be described in term of an effective Hamiltonian which contains the information about the short and long distance effects.

The FCNC decays are also relevant to the CKM phenomenology; and $b \to d\ell^+\ell^-$ modes are especially important in this respect. In case of the $b \to s\ell^+\ell^-$ decays, the matrix element receives a combination of various contributions from the intermediate t, c or u quarks with factors $V_{tb}V_{ts}^* \sim \lambda^2$, $V_{cb}V_{cs}^* \sim \lambda^2$ and $V_{ub}V_{us}^* \sim \lambda^4$, respectively, where $\lambda = \sin\theta_C \cong 0.22$. Since the last factor is extremely small compared to the other two we can neglect it and this reduces the unitarity relation for the CKM factors to the form $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* \approx 0$. Hence, the matrix element for the $b \to s\ell^+\ell^-$ decays involve only one independent CKM factor so that CP violation would not show up. On the other hand, as pointed out before [17, 18], for $b \to d\ell^+\ell^-$ decay, all the CKM factors $V_{tb}V_{td}^*$, $V_{cb}V_{cd}^*$ and $V_{ub}V_{ud}^*$ are at the same order λ^3 in the SM and the matrix element for these processes would have sizable interference terms, so as to induce a CP violating asymmetry between the decay rates of the reactions $b \to d\ell^+\ell^-$ and $\bar{b} \to \bar{d}\ell^+\ell^-$. Therefore, $b \to d\ell^+\ell^-$ decays seem to be suitable for establishing CP violation in B mesons.

We note that the inclusive $B \to X_s\ell^+\ell^-$ decays have been widely studied in the framework of the SM and its various extensions [19]-[37]. As for $B \to X_d\ell^+\ell^-$ modes, they were first considered within the SM in [17] and [18]. In ref. [17], together with the branching ratio, the CP violating asymmetry for the $B \to X_d\,\ell^+\ell^-$decays has been studied including the long-distance (LD) effects, but only for $\ell = e$ mode. In [18], a SM analysis for the forward-backward asymmetry is given again only for $\ell = e$ mode and neglecting the LD contributions. The general two Higgs doublet model contributions and minimal supersymmetric extension of the SM (MSSM) to the CP asymmetries were discussed in refs. [38] and [39], respectively. Ref. [39] contains a comparative study of the CP asymmetries in the inclusive $B \to X_d\,\ell^+\ell^-$and exclusive $B \to \gamma\,\ell^+\ell^-$ decays for $\ell = \tau$ only, by mainly focusing on the effects of the scalar interactions in the framework of the MSSM. Recently, CP violation in the polarized $b \to d\ell^+\ell^-$ decay has been also investigated in the SM [40] and also in a general model independent way [41].

The aim of this work is to perform a quantitative analysis on the SM CP violation and the related observables, such as the forward-backward asymmetry and CP violation aysmmetry in the forward-backward asymmetry in the $B \to X_d \ell^+ \ell^-$ decays, some of which have already addressed in [17], [18] and [39], as pointed out above. However, in this work we extend the investigation of the above-mentioned observables to consider all three lepton modes by mainly focusing on LD effects and also their dependence on the SM parameters ρ and η.

¿From the experimental side, the branching ratio (BR) of the $B \to X_s \ell^+ \ell^-$ decay has been reported by the BELLE Collaboration [42], $BR(B \to X_s \ell^+ \ell^-) = (6.1 \pm 1.4)^{+1.4}_{-1.1}$, which is very close to the value predicted by the SM [43], and may be used to put further constraint on the models beyond the SM.

The rest of this chapter is organized as follows:Following this brief introduction, in section 4.2, the effective Hamiltonian is represented. Then, the basic formulas of the double and differential decay rates, CP violation asymmetry, A_{CP}, forward-backward asymmetry, A_{FB}, and CP violating asymmetry in forward-backward asymmetry $A_{CP}(A_{FB})$ for $B \to X_d \, \ell^+ \ell^-$decay will be introduced . Section 4.3 is devoted to the numerical analysis and discussion.

4.2 The theoretical framework of $B \to X_d \ell^+ \ell^-$ decays

Inclusive decay rates of the heavy hadrons can be calculated in the heavy quark effective theory (HQET) [44] and the important result from this procedure is that the leading terms in $1/m_q$ expansion turn out to be the decay of a free quark, which can be calculated in the perturbative QCD; while the corrections to the partonic decay rate start with $1/m_q^2$ only. On the other hand, the powerful framework for both the inclusive and the exclusive modes into which the perturbative QCD corrections to the physical decay amplitude are incorporated in a systematic way is the effective Hamiltonian method. In this approach, heavy degrees of freedom, namely t quark and $W^{\pm}$ bosons in the present case, are integrated out. The procedure is to take into account the QCD corrections through matching the full theory with the effective low energy one at the high scale $\mu = m_W$ and evaluating the Wilson coefficients from m_W down to the lower scale $\mu \sim \mathcal{O}(m_b)$. The effective Hamiltonian obtained in this way for the process $b \to d\,\ell^+\ell^-$, is given by [23, 31, 46]:

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}}\,V_{tb}V_{td}^{*}\Bigg\{\sum_{i=1}^{10} \quad C_i(\mu)\,O_i(\mu) - \lambda_u\{C_1(\mu)[O_1^u(\mu) - O_1(\mu)] \\ +C_2(\mu)[O_2^u(\mu) - O_2(\mu)]\}\Bigg\} \tag{4.1}$$

where

$$\lambda_u = \frac{V_{ub}V_{ud}^{*}}{V_{tb}V_{td}^{*}}, \tag{4.2}$$

obtained using the unitarity of the CKM matrix i.e. $V_{tb}V_{td}^{*} + V_{ub}V_{ud}^{*} = -V_{cb}V_{cd}^{*}$. The operator basis in the SM for the process under consideration is given by [45, 46]

$$\begin{aligned}
O_1 &= (\bar{d}_{L\alpha}\gamma_\mu c_{L\beta})(\bar{c}_{L\beta}\gamma^\mu b_{L\alpha}), \\
O_2 &= (\bar{d}_{L\alpha}\gamma_\mu c_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu b_{L\beta}), \\
O_3 &= (\bar{d}_{L\alpha}\gamma_\mu b_{L\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\beta}), \\
O_4 &= (\bar{d}_{L\alpha}\gamma_\mu b_{L\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}), \\
O_5 &= (\bar{d}_{L\alpha}\gamma_\mu b_{L\alpha})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\beta}), \\
O_6 &= (\bar{d}_{L\alpha}\gamma_\mu b_{L\beta})\sum_{q=u,d,s,c,b}(\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}), \\
O_7 &= \frac{e}{16\pi^2}\bar{d}_\alpha\sigma_{\mu\nu}(m_b R + m_s L)b_\alpha\mathcal{F}^{\mu\nu}, \\
O_8 &= \frac{g}{16\pi^2}\bar{d}_\alpha T^a_{\alpha\beta}\sigma_{\mu\nu}(m_b R + m_s L)b_\beta\mathcal{G}^{a\mu\nu}\ , \\
O_9 &= \frac{e}{16\pi^2}(\bar{d}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{l}\gamma^\mu l)\ , \\
O_{10} &= \frac{e}{16\pi^2}(\bar{d}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{l}\gamma^\mu\gamma_5 l)\ ,
\end{aligned} \tag{4.3}$$

where α and β are $SU(3)$ color indices and $\mathcal{F}^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$ are the field strength tensors of the electromagnetic and strong interactions, respectively.

O_1^u and O_2^u are the new operators for $b \to d$ transitions which are absent in the $b \to s$ decays and given by

$$\begin{aligned} O_1^u &= (\bar{d}_\alpha \gamma_{mu} P_L u_\beta)(\bar{u}_\beta \gamma^{mu} P_L d_\alpha)\,, \\ O_2^u &= (\bar{d}_\alpha \gamma_{mu} P_L u_\alpha)(\bar{u}_\beta \gamma^{mu} P_L d_\beta). \end{aligned}$$

The initial values of the Wilson coefficients for the relevant process in the SM are [22, 23]

$$\begin{aligned} C^{SM}_{1,3,...6,11,12}(m_W) &= 0\ , \\ C^{SM}_2(m_W) &= 1\ , \\ C^{SM}_7(m_W) &= \frac{3x^3-2x^2}{4(x-1)^4}\ln x + \frac{-8x^3-5x^2+7x}{24(x-1)^3}\ , \\ C^{SM}_8(m_W) &= -\frac{3x^2}{4(x-1)^4}\ln x + \frac{-x^3+5x^2+2x}{8(x-1)^3}\ , \\ C^{SM}_9(m_W) &= -\frac{1}{sin^2\theta_W}B(x) + \frac{1-4\sin^2\theta_W}{\sin^2\theta_W}C(x) - D(x) + \frac{4}{9}\,, \\ C^{SM}_{10}(m_W) &= \frac{1}{\sin^2\theta_W}(B(x)-C(x))\,, \end{aligned} \tag{4.4}$$

with

$$x = \frac{m_t^2}{m_W^2} \quad . \tag{4.5}$$

The explicit forms of the functions $A(x), B(x), C(x), D(x)$ are given as

$$\begin{aligned} A(x) &= \frac{x(8x^2+5x-7)}{12(x-1)^3} + \frac{x^2(2-3x)}{2(x-1)^4}\ln x\,, \\ B(x) &= \frac{x}{4(1-x)} + \frac{x}{4(x-1)^2}\ln x\,, \\ C(x) &= \frac{x(x-6)}{x(x-1)} + \frac{x(3x+2)}{8(x-1)^2}\ln x\,, \\ D(x) &= \frac{-19x^3+25x^2}{36(x-1)^3} + \frac{x^2(5x^2-2x-6)}{18(x-1)^4}\ln x - \frac{4}{9}\ln x\,. \end{aligned} \tag{4.6}$$

In Eq.(4.1), $C_i(\mu)$ are the Wilson coefficients calculated at a renormalization point μ and their evolution from the higher scale $\mu = m_W$ down to the low-energy scale $\mu = m_b$ is described by the renormalization group equation. Wilson coefficients that play the essential role in this process are $C_7^{SM}(\mu)$, $C_9^{SM}(\mu)$, and $C_{10}^{SM}(\mu)$. They are given by [31, 45, 46]:

$$
\begin{aligned}
C_7^{SM}(\mu) &= \eta^{16/23} C_7^{SM}(m_W) + (8/3)(\eta^{14/23} - \eta^{16/23}) C_8^{SM}(m_W) \\
&+ C_2^{SM}(m_W) \sum_{i=1}^{8} h_i \eta^{a_i} ,
\end{aligned} \tag{4.7}
$$

and $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, h_i and a_i are the numbers which appear during the evaluation [31].

The Wilson coefficient $C_9(\mu)$ contains as well as a perturbative part, a part coming from long distance (LD) effects due to conversion of the real $\bar{c}c$ resonances into lepton pair $\ell^+\ell^-$, i.e. with the reaction chain $B \to X_d + V(c\bar{c}) \to X_d \ell^+\ell^-$. This additional contributions appear as exclusive modes for which the momentum scale of the intermediate quarks is a strong interaction scale and not the short distance scale m_W. This forces us to view the intermediate states as hadrons rather than quarks. To calculate this LD contributions, an effective Lagrangian $\mathcal{L}_{res}$ corresponding to these kind of $c\bar{c}$ resonances is added to the original effective Lagrangian for the process $B \to X_d\, \ell^+\ell^-$. The resulting structure of $\mathcal{L}_{res}$ is the same as that of the operator O_9 in (4.1). It is then convenient to include the resonance contribution by simply making the replacement

$$
C_9^{eff}(\mu) \to C_9^{eff}(\mu) + Y_{reson}(s) \ , \tag{4.8}
$$

where

$$
\begin{aligned}
C_9^{eff}(\mu) &= C_9 + h(u,s)[3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu) \\
&+ \lambda_u(3C_1 + C_2)] - \frac{1}{2} h(1,s)\,(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\
&- \frac{1}{2} h(0,s)\,[C_3(\mu) + 3C_4(\mu) + \lambda_u(6C_1(\mu) + 2C_2(\mu))] \\
&+ \frac{2}{9}\,(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \ ,
\end{aligned} \tag{4.9}
$$

and

$$
\begin{aligned}
Y_{reson}(s) &= -\frac{3}{\alpha^2} \kappa \sum_{V_i = \psi_i} \frac{\pi \Gamma(V_i \to \ell^+\ell^-) m_{V_i}}{m_B^2 s - m_{V_i}^2 + i m_{V_i} \Gamma_{V_i}} \\
&\times \ [(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\
&+ \ \lambda_u(3C_1(\mu) + C_2(\mu))] \, .
\end{aligned} \tag{4.10}
$$

Table 4.1: Charmonium ($\bar{c}c$) masses and widths [12].

Meson	Mass (GeV)	BR(V→ $\ell^+\ell^-$)	Γ (MeV)
$J/\Psi(1s)$	3.097	6.0×10^{-2}	0.088
$\Psi(2s)$	3.686	8.3×10^{-3}	0.277
$\Psi(3770)$	3.770	1.1×10^{-5}	23.6
$\Psi(4040)$	4.040	1.4×10^{-5}	52
$\Psi(4160)$	4.159	1.0×10^{-5}	78
$\Psi(4415)$	4.415	1.1×10^{-5}	43

In Eq.(4.9), $s = q^2/m_B^2$ where q is the momentum transfer, $u = \frac{m_c}{m_b}$ and the functions $h(u,s)$ arise from one loop contributions of the four-quark operators $O_1 - O_6$ and are given by

$$
\begin{aligned}
h(u,s) &= -\frac{8}{9}\ln\frac{m_b}{\mu} - \frac{8}{9}\ln u + \frac{8}{27} + \frac{4}{9}y \qquad (4.11)\\
&\quad -\frac{2}{9}(2+y)|1-y|^{1/2}\begin{cases} \left(\ln\left|\frac{\sqrt{1-y}+1}{\sqrt{1-y}-1}\right| - i\pi\right), & \text{for } y \equiv \frac{4u^2}{s} < 1 \\ 2\arctan\frac{1}{\sqrt{y-1}}, & \text{for } y \equiv \frac{4u^2}{s} > 1, \end{cases}\\
h(0,s) &= \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s + \frac{4}{9}i\pi \ . \qquad (4.12)
\end{aligned}
$$

The phenomenological parameter κ in Eq. (4.10) is taken as 2.3 (see e.g. [47]). There are six known resonances in the $\bar{c}c$ system that can contribute to the decay modes $B \to X_d\,\ell^+\ell^-$. Their properties are summarized in Table (4.1).

The next step is to calculate the matrix element of the $B \to X_d\ell^+\ell^-$ decay. The relevant one-loop diagrams contributing to this decay in the SM are given in Fig.(4.1). Neglecting the mass of the d quark, the effective short distance Hamiltonian in Eq.(4.1) leads to the following QCD corrected matrix element:

$$
\begin{aligned}
\mathcal{M} &= \frac{G_F\alpha}{2\sqrt{2}\pi}V_{tb}V_{td}^*\Big\{C_9^{eff}(m_b)\ \bar{d}\gamma_\mu(1-\gamma_5)b\,\bar{\ell}\gamma^\mu\ell + C_{10}(m_b)\ \bar{d}\gamma_\mu(1-\gamma_5)b\,\bar{\ell}\gamma^\mu\gamma_5\ell\\
&- \ 2C_7^{eff}(m_b)\ \frac{m_b}{q^2}\bar{d}i\sigma_{\mu\nu}q^\nu(1+\gamma_5)b\,\bar{\ell}\gamma^\mu\ell\Big\}. \qquad (4.13)
\end{aligned}
$$

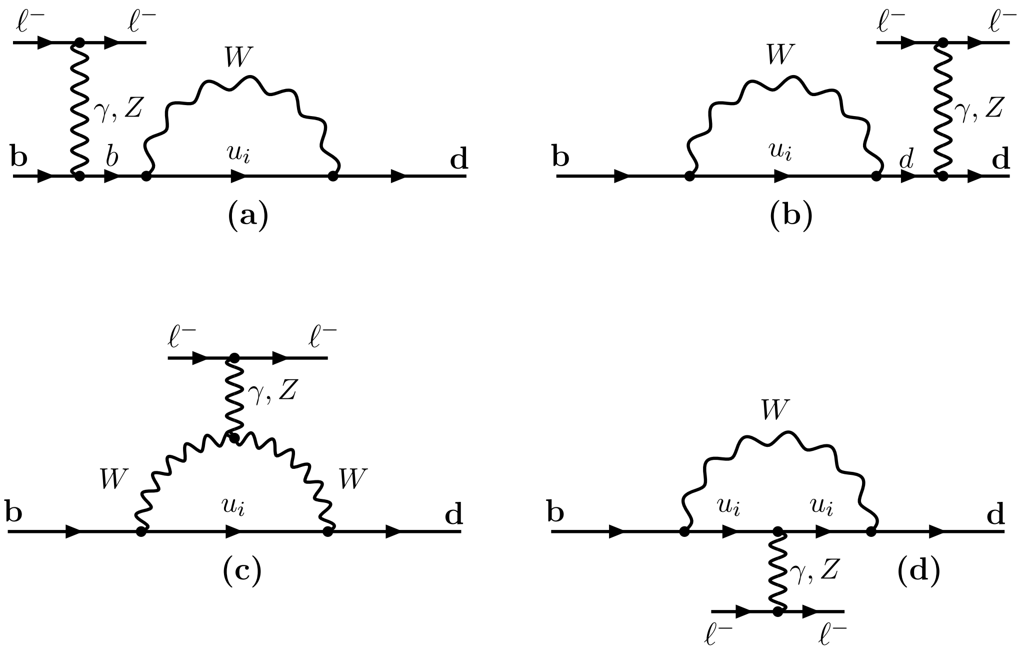

Figure 4.1: The one-loop Feynman diagrams contributing the decay $b \to d\ell^+\ell^-$ in the SM. Here, $u_i = u, c, t$ quarks.

In order to calculate the analytical expressions of the physical observables, such as the branching ratio, forward-backward asymmetry, etc., we need to find the decay rate of the process $B \to X_d\ell^+\ell^-$, whose general formula is given by

$$d\Gamma = \frac{(2\pi)^4}{2E_b}\delta^4(\mathcal{P}_b - \mathcal{P}_d - \mathcal{P}_1 - \mathcal{P}_2)\frac{d^3\vec{P}_1}{(2\pi)^3E_1}\frac{d^3\vec{P}_2}{(2\pi)^3E_2}\frac{d^3\vec{P}_d}{(2\pi)^3E_d}|\mathcal{M}|^2 \,. \tag{4.14}$$

where $\mathcal{P}_b, \mathcal{P}_d, \mathcal{P}_1, \mathcal{P}_2$ and E_b, E_d, E_1, E_2 are the momentum four-vectors, and energies of the b and d quarks, and ℓ^+, ℓ^- leptons, respectively. In the CM frame, since $\vec{P}_1 + \vec{P}_2 = 0$ we have $\vec{P}_1 = -\vec{P}_2$. Since $m_1 = m_2 = m_\ell$ we also have $E_1 = E_2$. Thus,

$$\begin{aligned}\int d^3\vec{P}_2\delta^4(\mathcal{P}_b - \mathcal{P}_d - \mathcal{P}_1 - \mathcal{P}_2) &= \delta(E_b - E_d - E_1 - E_2) \\ &\quad \int d^3\vec{P}_2\delta^3(\vec{P}_b - \vec{P}_d - \vec{P}_1 - \vec{P}_2) \\ &= \delta(E_b - E_d - E_1 - E_2)\Big|_{\vec{P}_b=(\vec{P}_d+\vec{P}_1+\vec{P}_2)} \end{aligned} \tag{4.15}$$

Therefore, Eq.(4.14) takes the form given by

$$d\Gamma = \frac{1}{2^9\pi^5}\frac{1}{E_b}\delta(E_b - E_d - 2E_1)\frac{d^3\vec{P}_1}{E_1^2}\frac{d^3\vec{P}_d}{E_d}|\mathcal{M}|^2 \,. . \tag{4.16}$$

By using

$$P_1 = \sqrt{E_1^2 - m_\ell^2} \implies dP_1 \; = \; \frac{E_1\, dE_1}{P_1}\,, \tag{4.17}$$

we write

$$\frac{d^3\vec{P}_1}{E_1^2} \; = \; \frac{P_1^2\, dP_1\; d\Omega}{E_1^2} \quad = 2\pi\frac{\sqrt{E_1^2 - m_\ell^2}}{E_1} dE_1\, dz\,. \tag{4.18}$$

Here, we have replaced the integration over the solid angle $d\Omega$ with $2\pi dz$, where $z = \cos\theta$ and, θ is the angle between $\vec{P}_1$ and $\vec{P}_b$, which are conveniently chosen along the z-axis. Then Eq.(4.16) becomes

$$d\Gamma = \frac{1}{2^8\,\pi^4}\frac{1}{E_b}\frac{\sqrt{E_1^2 - m_\ell^2}}{E_1}\frac{d^3\vec{P}_d}{E_d}dz|\mathcal{M}|^2\,. \tag{4.19}$$

Now, we again write $d^3\vec{P}_d$ in Eq.(4.19) in spherical coordinates

$$\frac{d^3\vec{P}_d}{E_d} = \frac{P_d^2\, dP_d\, d\Omega}{E_d} = 4\pi\, E_d\, dE_d\,. \tag{4.20}$$

It is possible to write the above relation in terms of the momentum transfer, q. Since, in the CM of the decaying lepton pair we have

$$q^2 \; = \; (E_b - E_d)^2 - (\vec{P}_b - \vec{P}_d)^2 = (E_b - E_d)^2\,, \tag{4.21}$$

we can obtain a dimensionless quantity related to the momentum transfer, s:

$$s \equiv \frac{q^2}{m_b^2} = \frac{(E_b - E_d)^2}{m_b^2}\,. \tag{4.22}$$

In addition, we can write E_d , dE_d and E_b in terms of s in the following way

$$E_d = \frac{m_b(1-s)}{2\sqrt{2}} \quad , \quad dE_d = \frac{m_b(1+s)}{4s\sqrt{s}}ds \quad , \quad \text{and} \quad E_b = \frac{m_b(1+s)}{2\sqrt{s}}\,. \tag{4.23}$$

We now put the pieces together in Eq.(4.19) and rewrite it in the form given by

$$\frac{d\Gamma}{dsdz} = \frac{1}{2^9\pi^3}m_b(1-s)\upsilon|\mathcal{M}|^2 ds\, dz\,, \tag{4.24}$$

where

$$v \equiv \sqrt{1 - \frac{4t}{s}} \quad , \quad t = \frac{m_\ell^2}{m_b^2}. \tag{4.25}$$

The next step is to calculate the square of the matrix elements $|\mathcal{M}|^2 = \mathcal{M}\mathcal{M}^*$. When the incident b-quark beam is unpolarized and the final state polarizations are not measured, we sum over final state polarizations and average over the initial spins:

$$|\overline{\mathcal{M}}|^2 = \frac{1}{2} \sum_{spins} |\mathcal{M}|^2. \tag{4.26}$$

More explicitly it can be written as

$$\begin{aligned}
|\mathcal{M}|^2 &= \left|\frac{G_F V_{tb} V_{td}^*}{\sqrt{2}}\right|^2 \Big\{ |C_9^{eff}|^2 Q_{\mu\nu}^{(1)} L^{(1)\mu\nu} + |C_{10}|^2 Q_{\mu\nu}^{(1)} L^{(2)\mu\nu} \\
&+ 4\frac{m_b^2}{q^4} |C_7^{eff}|^2 Q_{\mu\nu}^{(2)} L^{(1)\mu\nu} + \mathrm{Re}(C_9^{eff} C_{10}^*) Q_{\mu\nu}^(1) L^{(3)\mu\nu} \\
&+ i\frac{m_b}{q^2} \mathrm{Re}(C_9^{eff} C_7^{eff\,*}) Q_{\mu\nu}^{(3)} L^{(1)\mu\nu} \\
&+ i\frac{m_b}{q^2} \mathrm{Re}(C_{10} C_7^{eff\,*}) Q_{\mu\nu}^{(3)} L^{(3)\mu\nu} \Big\},
\end{aligned} \tag{4.27}$$

where

$$\begin{aligned}
Q_{\mu\nu}^{(1)} &= Tr[(\not{p}_d + m_d)\gamma_\mu L(\not{p}_b + m_b)\gamma_\nu L]\,, \\
Q_{\mu\nu}^{(2)} &= Tr[(\not{p}_d + m_d)\sigma_{\mu\alpha} q^\alpha R(\not{p}_b + m_b)\sigma_{\nu\beta} q^\beta L]\,, \\
Q_{\mu\nu}^{(3)} &= Tr[(\not{p}_d + m_d)\gamma_\mu L(\not{p}_b + m_b)\sigma_{\nu\beta} q^\beta L]\ ,
\end{aligned} \tag{4.28}$$

give the hadronic tensors and

$$\begin{aligned}
L^{(1)\mu\nu} &= Tr[(\not{p}_1 - m_\ell)\gamma_\mu(\not{p}_2 + m_\ell)\gamma_\nu]\,, \\
L^{(2)\mu\nu} &= Tr[(\not{p}_1 - m_\ell)\gamma_\mu\gamma_5 L(\not{p}_2 + m_\ell)\gamma_\nu \gamma 5]\,, \\
L^{(3)\mu\nu} &= Tr[(\not{p}_1 + m_\ell)\gamma_\mu(\not{p}_2 + m_\ell)\gamma_\nu\gamma_5]\,,
\end{aligned} \tag{4.29}$$

give the leptonic tensors. In deriving Eqs.(4.28) and (4.29) the following operators summed over spin states have been used:

$$\sum_{spin} q(p_q)\overline{q}(p_q) = \not{p}_q + m_q \quad \text{for } q = b, d$$

$$\sum_{spin} \ell(\mathcal{P}_1)\bar{\ell}(\mathcal{P}_1) = \not{p}_1 - m_\ell \,,$$
$$\sum_{spin} \ell(\mathcal{P}_2)\bar{\ell}(\mathcal{P}_2) = \not{p}_2 + m_\ell \,. \quad (4.30)$$

Calculating the traces in Eqs.(4.28) and (4.29) and substituting the decay rate in Eq.(4.24), it becomes

$$\begin{aligned}\frac{d^2\Gamma}{ds\,dz} &= \Gamma(B \to X_c \ell\nu)\frac{\alpha^2}{4\pi^2 f(u)k(u)}(1-s)^2\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2}\, v\,\Big\{6\,v\,z\,\mathrm{Re}(C_7^{eff}C_{10}^*) \\ &+ 6\Big(1+\frac{2t}{s}\Big)\,\mathrm{Re}(C_7^{eff}C_9^{eff\,*}) + 3\,v\,sz\mathrm{Re}(C_{10}C_9^{eff\,*}) \\ &+ \frac{3}{4}\Big[(1+s)-(1-s)\,v^2z^2+4t\Big]|C_9^{eff}|^2 \\ &+ 3\Big[\Big(1+\frac{1}{s}\Big)-\Big(1-\frac{1}{s}\Big)\,v^2z^2+\frac{4t}{s}\Big]|C_7^{eff}|^2 \\ &+ \frac{3}{4}\Big[(1+s)-(1-s)\,v^2z^2-4t\Big]|C_{10}|^2\Big\}\,. \end{aligned} \quad (4.31)$$

In Eq. (4.31),

$$\Gamma(B \to X_c\ell\nu) = \frac{G_F^2 m_b^5}{192\pi^3}|V_{cb}|^2 f(u)k(u)\,, \quad (4.32)$$

where

$$f(u) = 1 - 8u + 8u^4 - u^8 - 24u^4 ln(u)\,, \quad (4.33)$$
$$k(u) = 1 - \frac{2\alpha_s(m_b)}{3\pi}\left[\left(\pi^2 - \frac{31}{4}\right)(1-u^2) + \frac{3}{2}\right], \quad (4.34)$$

are the phase space factor and the QCD corrections to the semi-leptonic decay rate, respectively, which is used to normalize the decay rate of $B \to X_d \ell^+\ell^-$ to remove the uncertainties in the value of m_b.

After integrating the double differential decay rate in Eq.(4.31) over the angle variable z, we find

$$\frac{d\Gamma}{ds} = \Gamma(B \to X_c\ell\nu)\frac{\alpha^2}{4\pi^2 f(u)k(u)}(1-s)^2\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2}\sqrt{1-\frac{4t}{s}}\,\Delta(s)\,, \quad (4.35)$$

where

$$\begin{aligned}\Delta(s) &= \frac{(s+2s^2+2t-8st)}{s}|C_{10}|^2 + \frac{4}{s^2}(2+s)(s+2t)|C_7^{eff}|^2 \\ &+ (1+2s)(1+\frac{2t}{s})|C_9^{eff}|^2 + \frac{12}{s}(s+2t)\mathrm{Re}(C_7^{eff}C_9^{eff\,*})\,. \end{aligned} \quad (4.36)$$

We start with calculating the CP asymmetry A_{CP} between the $B \to X_d \ell^+ \ell^-$ and the conjugated one $\bar{B} \to \bar{X}_d \ell^+ \ell^-$, which is defined as

$$A_{CP}(s) = \frac{\frac{d\Gamma}{ds} - \frac{d\bar{\Gamma}}{ds}}{\frac{d\Gamma}{ds} + \frac{d\bar{\Gamma}}{ds}} \tag{4.37}$$

where

$$\frac{d\Gamma}{ds} = \frac{d\Gamma(B \to X_d \ell^+ \ell^-)}{ds} \quad , \quad \frac{d\bar{\Gamma}}{ds} = \frac{d\Gamma(\bar{B} \to \bar{X}_d \ell^+ \ell^-)}{ds} . \tag{4.38}$$

Let us rewrite $A_{CP}(s)$ as follows

$$A_{CP}(s) = \frac{\Delta(s) - \overline{\Delta}(s)}{\Delta(s) + \overline{\Delta}(s)} , \tag{4.39}$$

where

$$\begin{aligned} \Delta(s) &= A|C_7|^2 + B|C_9|^2 + C|C_{10}|^2 + D\text{Re}(C_7 C_9^{eff\,*}) \\ \overline{\Delta}(s) &= A|\overline{C}_7|^2 + B|\overline{C}_9|^2 + C|\overline{C}_{10}|^2 + D\text{Re}(\overline{C}_7 C_9^{eff\,*}) \end{aligned} \tag{4.40}$$

Here, A, B, C and D are the symbols of the coefficients of the $C_7(\overline{C}_7)$, $C_9(\overline{C}_9)$, $C_{10}(\overline{C}_{10})$, $\text{Re}(C_7\, C_9^{eff\,*})$ and $\text{Re}(\overline{C_7}\, \overline{C_9}^{eff\,*})$ in Eq. (4.36). Since in the SM only C_9^{eff} contains imaginary part, we can represent $C_9^{eff}, C_9^{eff\,*}, \overline{C}_9^{eff}, \overline{C}_9^{eff\,*}$ symbolically as

$$\begin{aligned} C_9^{eff} &= \xi_1 + \lambda_u \xi_2 \quad , \quad C_9^{eff\,*} = \xi_1^* + \lambda_u^* \xi_2^* , \\ \overline{C}_9^{eff} &= \xi_1 + \lambda_u^* \xi_2 \quad , \quad \overline{C}_9^{eff\,*} = \xi_1^* + \lambda_u \xi_2^* . \end{aligned} \tag{4.41}$$

Let us calculate the numerator of the Eq.(4.39):

$$\Delta(s) - \overline{\Delta}(s) = B(|C_9|^2 - |\overline{C}_9|^2) + D\text{Re}(C_7\, C_9^{eff\,*} - C_7\, \overline{C}_9^{eff\,*}) , \tag{4.42}$$

using definitions in Eq. (4.41) we get

$$|C_9|^2 - \overline{|C_9|}^2 = (\lambda_u^* - \lambda_u)(\xi_1 \xi_2^* - \xi_2 \xi_1^*) , \tag{4.43}$$

and

$$\begin{aligned} \text{Re}(C_7 C_9^{eff\,*}) &= C_7 \text{Re}[\xi_1^* + \lambda_u^* \xi_2^*] , \\ \text{Re}(C_7 \overline{C}_9^{eff\,*}) &= -C_7 \text{Re}[\xi_1^* + \lambda_u \xi_2^*] . \end{aligned} \tag{4.44}$$

Writing the complex quantities ξ_1 , ξ_2 and λ_u explicitly as

$$\begin{aligned}
\xi_1 &= \mathrm{Re}\xi_1 + i\mathrm{Im}\xi_1 \quad , \quad \xi_1^* = \mathrm{Re}\xi_1 - i\mathrm{Im}\xi_1 \,, \\
\xi_2 &= \mathrm{Re}\xi_2 + i\mathrm{Im}\xi_2 \quad , \quad \xi_2^* = \mathrm{Re}\xi_2 - i\mathrm{Im}\xi_2 \,, \\
\lambda_u &= \mathrm{Re}\lambda_u + i\mathrm{Im}\lambda_u \,,
\end{aligned} \tag{4.45}$$

the second term in Eq.(4.42) becoms

$$\mathrm{Re}(C_7 C_9^{eff\,*}) - \mathrm{Re}(C_7 \overline{C}_9^{eff\,*}) = -2C_7 \mathrm{Im}\lambda_u \mathrm{Im}\xi_2 \,. \tag{4.46}$$

Substituting Eq.(4.43) and Eq.(4.46) in Eq.(4.42) and simplifying we obtain that

$$\Delta(s) - \overline{\Delta}(s) \;=\; -2\mathrm{Im}\lambda_u \{2B\mathrm{Im}(\xi^*\xi_2) + C_7 D \mathrm{Im}\xi_2\} \,. \tag{4.47}$$

When we put the explicit forms of B and D into Eq.(4.47) it becomes

$$\Delta(s) - \overline{\Delta}(s) \;=\; -4\mathrm{Im}(\lambda_u)\Sigma \,, \tag{4.48}$$

where

$$\Sigma \;=\; \Big(1 + \frac{2t}{s}\Big) \left[(1+2s)\,\mathrm{Im}(\xi_1^*\xi_2) + 6\, C_7^{eff}\,\mathrm{Im}(\xi_2)\right]\,\mathrm{Im}(\lambda_u) \,. \tag{4.49}$$

It is possible to calculate the denominator of Eq.(4.39) by following the same steps summarized through Eqs. (4.42-4.48). The result is given by

$$\Delta(s) + \overline{\Delta}(s) = 2\Delta(s) + 4\mathrm{Im}(\lambda_u)\Sigma \,. \tag{4.50}$$

Then, $A_{CP}(s)$ for $B \to X_d\,\ell^+\ell^-$takes the form given by

$$A_{CP}(s) = \frac{-2\mathrm{Im}\lambda_u \Sigma}{\Delta(s) + 2\mathrm{Im}\lambda_u \Sigma} \quad . \tag{4.51}$$

We next consider the forward-backward asymmetry, A_{FB}, in $B \to X_d\,\ell^+\ell^-$, which is another physical quantity that may be useful to test the theoretical models. Using the definition of differential $A_{FB}(s)$

$$A_{FB}(s) \;=\; \frac{\int_0^1 dz \frac{d^2\Gamma}{dsdz} - \int_{-1}^0 dz \frac{d^2\Gamma}{dsdz}}{\int_0^1 dz \frac{d^2\Gamma}{dsdz} + \int_{-1}^0 dz \frac{d^2\Gamma}{dsdz}} \,, \tag{4.52}$$

we find

$$A_{FB}(s) = \frac{-3\ v}{\Delta(s)} \, \mathrm{Re}[C_{10}(2C_7^{eff} + s\, C_9^{eff\,*})], \qquad (4.53)$$

which agrees with the result given by ref. [18], but not by [39].

We have also a CP violating asymmetry in A_{FB}, $A_{CP}(A_{FB})$, in $B \to X_d\, \ell^+\ell^-$ decay. Since in the limit of CP conservation, one expects $A_{FB} = -\bar{A}_{FB}$ [18, 48], where A_{FB} and $\bar{A}_{FB}$ are the forward-backward asymmetries in the particle and antiparticle channels, respectively, $A_{CP}(A_{FB})$ is defined as

$$A_{CP}(A_{FB}) \;=\; A_{FB} + \bar{A}_{FB} \quad . \qquad (4.54)$$

Here, $\bar{A}_{FB}$ can be obtained by the replacement,

$$C_9^{eff}(\lambda_u) \to \bar{C}_9^{eff}(\lambda_u \to \lambda_u^*). \qquad (4.55)$$

Using Eqs.(4.53) we can find

$$\begin{aligned} A_{CP}(A_{FB}) \;&=\; \frac{6\ v\ \mathrm{Im}(\lambda_u)}{\Delta(\Delta + 4\mathrm{Im}(\lambda_u)\ \Sigma)}\ C_{10} \\ &\cdot \Big[2\Sigma\ (2C_7^{eff} + s(\mathrm{Re}(\xi_1) + \mathrm{Re}(\xi_2)\ \mathrm{Re}(\lambda_u) - \mathrm{Im}(\xi_2)\ \mathrm{Im}(\lambda_u))) - s\ \Delta\ \mathrm{Im}(\xi_2)\Big]\,, \end{aligned} \qquad (4.56)$$

which is slightly different from the one given [39].

4.3 Numerical analysis and discussion

In this section, we present results of our calculations related to $B \to X_d\, \ell^+\ell^-$ decays, for two different sets of the Wolfenstein parameters. For this we first give the Wolfenstein parametrization [13] of the CKM factor in Eq.(4.2)

$$\lambda_u = \frac{\rho(1-\rho) - \eta^2 - i\eta}{(1-\rho)^2 + \eta^2} + O(\lambda^2)\,, \qquad (4.57)$$

and also

$$\frac{|V_{tb}V_{td}^*|^2}{|V_{cb}|^2} \;=\; \lambda^2[(1-\rho)^2 + \eta^2] + \mathcal{O}(\lambda^4)\,. \qquad (4.58)$$

The updated fitted values for the parameters ρ and η are given in ref.[49] as

$$\begin{aligned}\bar{\rho} &= 0.22 \pm 0.07\ (0.25 \pm 0.07)\,, \\ \bar{\eta} &= 0.34 \pm 0.04\ (0.34 \pm 0.04)\,, \end{aligned} \qquad (4.59)$$

with (without) including the chiral logarithm uncertainties. In our numerical analysis, we have used $(\rho,\, \eta) = (0.15;\, 0.30)$ and $(0.32;\, 0.38)$, which are the lower and higher allowed values of the parameters given in Eq. (4.59) above, and present the dependence of the A_{CP}, A_{FB} and $A_{CP}(A_{FB})$ on the dimensionles photon energy s for the $B \to X_d\, \ell^+\ell^-(\ell = e,\, \mu,\, \tau)$ decays in Figs. (4.2)-(4.7).

In order to investigate the dependence of the observables we consider on the SM parameters, we eliminate the other parameter, namely s, by performing s-integration over the allowed kinematical region. In this way we obtain the average values of the corresponding observables, that is, for example,

$$< A_{CP} > \;=\; \frac{\int A_{CP} \frac{d\Gamma}{ds} ds}{\int \frac{d\Gamma}{ds} ds}. \qquad (4.60)$$

We have evaluated the average values of CP asymmetry $< A_{CP} >$, forward-backward asymmetry $< A_{FB} >$ and CP asymmetry in the forward-backward asymmetry $< A_{CP}(A_{FB}) >$ in $B \to X_d\, \ell^+\ell^-$decay for the above sets of parameters $(\rho,\, \eta)$, and our results are displayed in Table 4.2 and 4.3 without and with including the long distance effects, respectively.

The input parameters and the initial values of the Wilson coefficients we used in our numerical analysis are as follows:

$$\begin{aligned}
& m_b = 4.8\, GeV\ ,\ m_c = 1.4\, GeV\ , m_t = 175\, GeV\ , \\
& m_e = 0.511\, MeV\ \ ,\ m_\tau = 1.78\, GeV,\ m_\mu = 0.105\, GeV\ , \\
& BR(B \to X_c e \bar{\nu}_e) = 10.4\%\ ,\ \alpha = 1/129\ ,\ m_W = 80.4\, GeV\ ,\ m_Z = 91.1\, GeV\ , \\
& C_1 = -0.245,\ C_2 = 1.107,\ C_3 = 0.011,\ C_4 = -0.026,\ C_5 = 0.007, \\
& C_6 = -0.0314,\ C_7^{eff} = -0.315,\ C_9 = 4.220,\ C_{10} = -4.619.
\end{aligned} \qquad (4.61)$$

In our numerical analysis, we take into account five possible resonances for the LD effects coming from the reaction $b \to d\, \psi_i \to d\, \ell^+\ell^-$, where $i = 1, ..., 5$

Table 4.2: The average values of A_{CP}, A_{FB} and $A_{CP}(A_{FB})$ in $B \to X_d\, \ell^+\ell^-$ for the three distinct lepton modes without including the long distance effects. The first and the second data lines correspond to $(\rho,\, \eta) = (0.15;\, 0.30)$ and $(0.32;\, 0.38)$, respectively.

$< A_{CP} >$			$< A_{FB} >$			$< A_{CP}(A_{FB}) >$		
$\ell = e$	$\ell = \mu$	$\ell = \tau$	$\ell = e$	$\ell = \mu$	$\ell = \tau$	$\ell = e$	$\ell = \mu$	$\ell = \tau$
0.030	0.036	0.134	−0.124	−0.151	−0.182	−0.009	−0.009	0.001
0.051	0.061	0.169	−0.129	−0.156	−0.180	−0.015	−0.015	0.002

Table 4.3: The same as Table (4.2), but including the long distance effects.

$< A_{CP} >$			$< A_{FB} >$			$< A_{CP}(A_{FB}) >$		
$\ell = e$	$\ell = \mu$	$\ell = \tau$	$\ell = e$	$\ell = \mu$	$\ell = \tau$	$\ell = e$	$\ell = \mu$	$\ell = \tau$
0.032	0.036	0.144	−0.119	−0.139	−0.157	−0.017	−0.017	−0.004
0.051	0.059	0.230	−0.125	−0.140	−0.150	−0.031	−0.030	−0.009

and divide the integration region into two parts for $\ell = \tau$: $(2m_\ell/m_B)^2 \leq s \leq ((m_{\psi_1} - 0.02)/m_B)^2$ and $((m_{\psi_1} + 0.02)/m_B)^2 \leq s \leq 1$, where $m_{\psi_1} = 3.097$ GeV is the mass of the first resonance. As for $\ell = e$ and μ modes, the integration region is divided into three parts : $(2m_\ell/m_B)^2 \leq s \leq ((m_{\psi_1} - 0.02)/m_B)^2$, $((m_{\psi_1}+0.02)/m_B)^2 \leq s \leq ((m_{\psi_2}-0.02)/m_B)^2$ and $((m_{\psi_2}+0.02)/m_B)^2 \leq s \leq 1$, where $m_{\psi_2} = 3.686$ GeV is the mass of the second resonance.

For reference, we present our SM predictions with long distance effects

$$BR(B \to X_d\, \ell^+\, \ell^-) \;=\; (3.01, 2.61, 0.11) \times 10^{-7}\,, \qquad (4.62)$$

for $\ell = e, \mu, \tau$, respectively, with $(\rho; \eta) = (0.30; 0.34)$, which is in agreement with the results of [17].

In Fig.(4.2) and Fig.(4.3), we present the dependence of A_{CP} on the dimensionless photon energy s, for $B \to X_d\, \ell^+\ell^-$ decay for the Wolfenstein parameters $(\rho;\, \eta) = (0.15;\, 0.30)$ and $(\rho;\, \eta) = (0.32;\, 0.38)$, respectively. The three distinct lepton modes $\ell = e,\ \mu,\ \tau$ are represented by the dashed, dotted and solid curves, respectively. We observe that the A_{CP} for $\ell = e,\ \mu$ cases almost coincide, reaching up to 25 % for the larger values of s. The A_{CP} for $\ell = \tau$ mode exceeds the

Table 4.4: The SM predictions for the average CP-violating asymmetry in the forward-backward asymmetry $< A_{CP}(A_{FB}) > \times 10^{-2}$ for different regions of the dimensionless photon energy s with $(\rho; \eta) = (0.15; 0.30)$.

ℓ	SD contribution	$(2m_l/m_B)^2 \le s \le ((m_{\psi_1} - 0.02)/m_B)^2$	$((m_{\psi_1} + 0.02)/m_B)^2 \le s \le ((m_{\psi_2} - 0.02)/m_B)^2$	$((m_{\psi_2} + 0.02)/m_B)^2 \le s \le 1$	SD+LD contribution
e	-0.92	-0.29	-0.25	-1.20	-1.78
μ	-0.91	-0.29	-0.25	-1.20	-1.78
τ	-0.11	-0.42	3.10×10^{-3}		-0.42

Table 4.5: Same as Table (4.4), but with $(\rho; \eta) = (0.32; 0.38)$.

ℓ	SD contribution	$(2m_l/m_B)^2 \le s \le ((m_{\psi_1} - 0.02)/m_B)^2$	$((m_{\psi_1} + 0.02)/m_B)^2 \le s \le ((m_{\psi_2} - 0.02)/m_B)^2$	$((m_{\psi_2} + 0.02)/m_B)^2 \le s \le 1$	SD+LD contribution
e	-1.59	-0.51	-0.43	-2.15	-3.10
μ	-1.57	-0.51	-0.43	-2.15	-3.09
τ	0.20	-0.94	3.30×10^{-3}		-0.94

values of the other modes and reaches 40 %. We also observe from Tables 1 and 2 that including the LD effects in calculating $< A_{CP} >$ does not change the results for $\ell = e, \mu$ modes, while $\ell = \tau$ mode, it is quite sizable, $8 - 36\%$, depending on the sets of the parameters used for $(\rho; \eta)$.

The s dependence of A_{FB} for the $B \to X_d \ell^+ \ell^- (\ell = e, \mu, \tau)$ decays are plotted in Figs.(4.4) and (4.5) for $(\rho; \eta) = (0.15; 0.30)$ and $(\rho; \eta) = (0.32; 0.38)$, respectively. We see that A_{FB} is negative for almost all values of s, except in the resonance and very small-s regions. $< A_{FB} >$ takes the values between $-(12 - 15)\%$ depending on the sets of the parameters used for $(\rho; \eta)$. The LD effects on $< A_{FB} >$ are about 10%, but in reverse manner, decreasing its magnitude in comparison to the values without LD contributions.

We present the dependence of the $A_{CP}(A_{FB})$ of $B \to X_d \ell^+ \ell^-$decay on s in Fig.(4.6) and Fig.(4.7), again for two different sets of the Wolfenstein parameters. As for A_{CP}, $A_{CP}(A_{FB})$ for $\ell = e$, and $\ell = \mu$ modes almost coincide. We see that

$A_{CP}(A_{FB})$ is all negative except in a very small region for the intermediate values of s for $\ell = e, \mu$ cases. LD effects seem to be quite significant for $< A_{CP}(A_{FB}) >$, enhancing its value twice (four times) for $\ell = e, \mu$ ($\ell = \tau$) modes. To see this LD contributions more closely, we present the $< A_{CP}(A_{FB}) >$ for different regions of s in Table (4.4) and (4.5), for $(\rho; \eta) = (0.15; 0.30)$ and $(\rho; \eta) = (0.32; 0.38)$, respectively. We see that for the light lepton modes, $\ell = e, \mu$, $A_{CP}(A_{FB})$ is more sizable in the high-dilepton mass region of s, $((m_{\psi_2} + 0.02)/m_B)^2 \leq s \leq 1$. However, for $\ell = \tau$, the contribution from the high-dilepton mass region of s is negligible and the contribution to $< A_{CP}(A_{FB}) >$ comes effectively from the low-dilepton mass region, $(2m_l/m_B)^2 \leq s \leq ((m_{\psi_1} - 0.02)/m_B)^2$ and amounts to -1%.

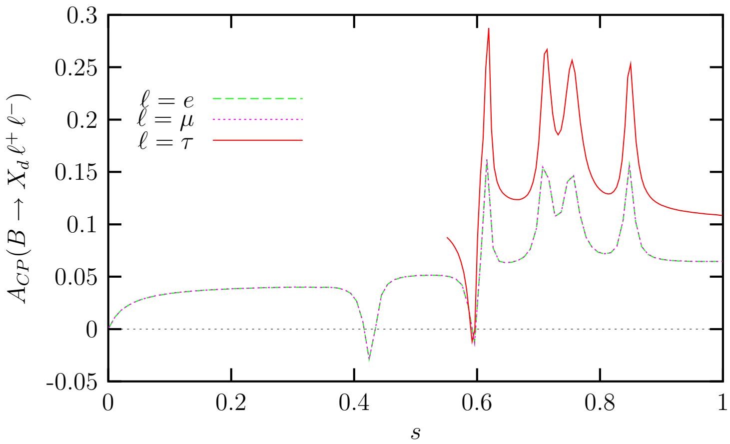

Figure 4.2: A_{CP} for $B \to X_d \ell^+ \ell^-$decay for the Wolfenstein parameters $(\rho, \eta) = (0.15; 0.30)$. The three distinct lepton modes $\ell = e, \mu, \tau$ are represented by the dashed, dotted and solid curves, respectively.

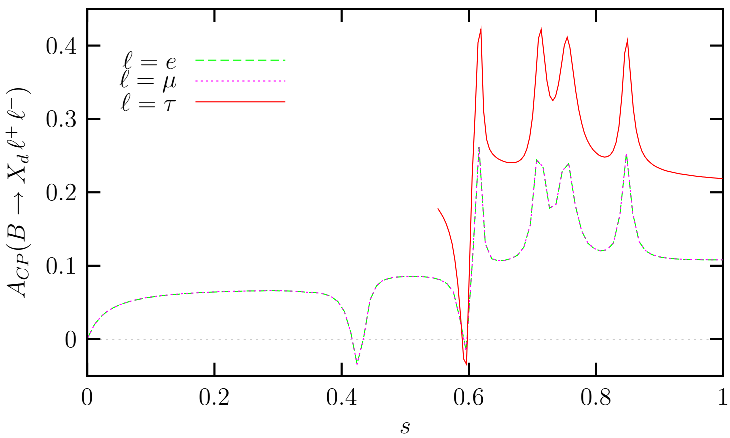

Figure 4.3: The same as Fig.(4.2) but for the Wolfenstein parameters $(\rho, \eta) = (0.32; 0.38)$

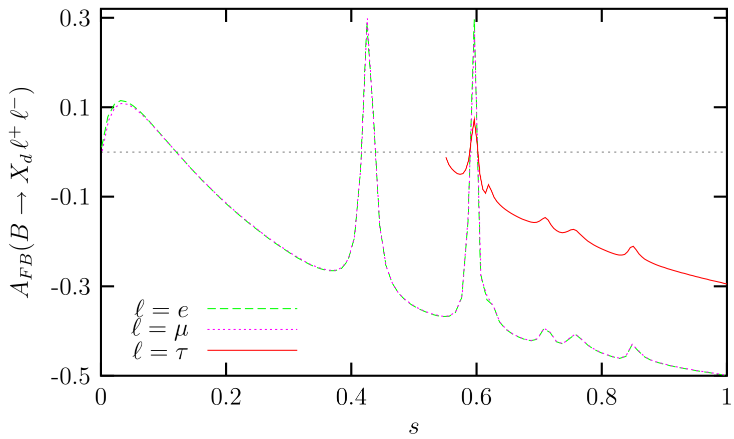

Figure 4.4: A_{FB} for $B \to X_d\,\ell^+\ell^-$decay for the Wolfenstein parameters $(\rho,\,\eta) = (0.15;\,0.30)$. The three distinct lepton modes $\ell =\; e,\; \mu,\; \tau$ are represented by the dashed, dotted and solid curves, respectively.

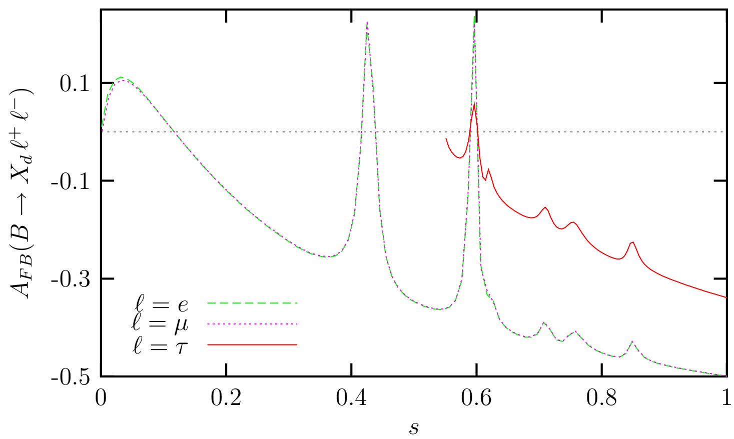

Figure 4.5: The same as Fig.(4.4) but for the Wolfenstein parameters $(\rho,\,\eta) = (0.32;\,0.38)$

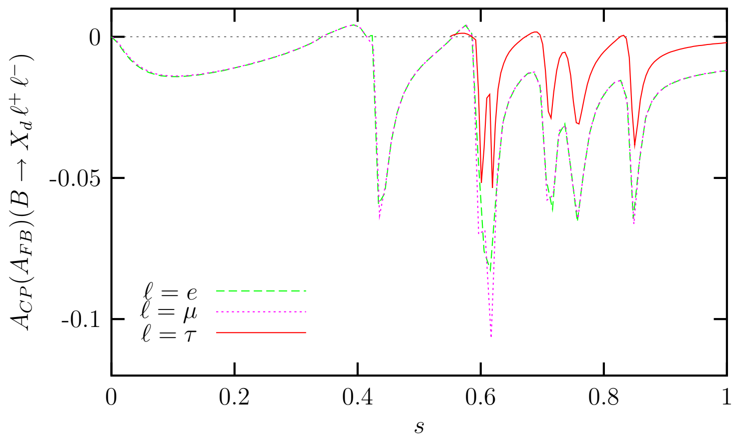

Figure 4.6: $A_{CP}(A_{FB})$ for $B \to X_d \ell^+ \ell^-$decay for the Wolfenstein parameters $(\rho, \eta) = (0.15; 0.30)$. The three distinct lepton modes $\ell = e, \mu, \tau$ are represented by the dashed, dotted and solid curves, respectively.

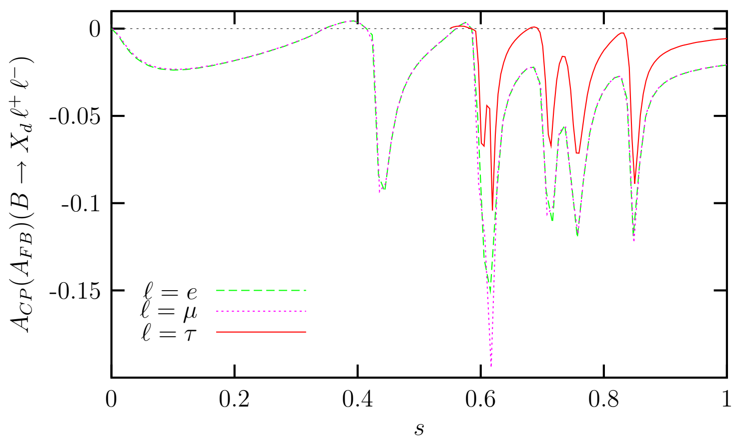

Figure 4.7: The same as Fig.(4.6) but for the Wolfenstein parameters $(\rho, \eta) = (0.32; 0.38)$

CHAPTER 5

CONCLUSION

The weak interactions exhibit the most different and complicated pattern among the all fundamental forces of nature. For example, it is very puzzling that whereas the discrete symmetries C, P, CP, and T are respected by strong and electromagnetic interactions, the weak force violates them all. Although the SM of the strong and electroweak forces, which is the present day understanding of particle physics in general, can describe very successfully a huge amount of experimental data, there are many and big questions left unanswered. The most important ones like the problem of electroweak symmetry breaking and the origin of fermion masses and quark mixing are closely related to the part of the SM describing weak interactions. For these reasons, big efforts are being spent to develop our theoretical understanding of weak interaction phenomena, its basic mechanism and parameters. The very rich phenomenology of weak meson decays provide an excellent laboratory for such types of studies.

The B system represents especially an ideal framework for determining the parameters of the SM accurately, testing its subtle properties and searching for signatures of new physics beyond it. Since b quark mass is larger than the typical scale of the strong interaction, long-distance strong interactions are generally less important and are under better control in B systems. Thus, for example the CP violation in the B system will yield an important independent test of the SM description of CP violation. B meson decays also allow for a rich CKM phenomenology and a strict test of the unitarity constraints.

The so-called rare decays, which represent flavour changing neutral currents, are of particular interest since they are induced by loop diagrams. Therefore, these decays have been always good candidates for testing the SM at loop level.

Among the rare B-meson decays, the inclusive B $\rightarrow X_{s,d}\ell^+\ell^-$ modes are prominent because of their relative cleanness compared to the pure hadronic decays. In the SM, $B \rightarrow X_{s(d)}\ell^+\ell^-$ decays are dominated by the parton level processes $b \rightarrow s(d)\ell^+\ell^-$, which occur through an intermediate u, c or t quarks. They can be described in term of an effective Hamiltonian which contains the information about the short and long distance effects.

The FCNC decays are relevant to the CKM phenomenology as well and $b \rightarrow d\ell^+\ell^-$ modes are especially important in this respect due to the existence of sizable interference terms in the decay amplitude that can induce considerable CP violation between the decay rates of the reactions $b \rightarrow d\ell^+\ell^-$ and $\bar{b} \rightarrow \bar{d}\ell^+\ell^-$.

In this work we have performed a quantitative analysis on the SM CP violation and the related observables, such as the forward-backward asymmetry and CP violation aysmmetry in the forward-backward asymmetry in the $B \rightarrow X_d\ell^+\ell^-$ decays for all three lepton modes by mainly focusing on LD effects and also their dependence on the SM parameters ρ and η. The important conclusions that can be extracted from this work can be summarized as follows:

- The A_{CP} for $\ell = e,\ \mu$ cases almost coincide, reaching up to 25 % for the larger values of s. The A_{CP} for $\ell = \tau$ mode exceeds the values of the other modes and reaches 40 %. Further, including the LD effects in calculating $< A_{CP} >$ does not change the results for $\ell = e,\ \mu$ modes, while $\ell = \tau$ mode, it is quite sizable, $8 - 36\%$, depending on the sets of the parameters used for $(\rho; \eta)$.

- A_{FB} is negative for almost all values of s, except in the resonance and very small-s regions. $< A_{FB} >$ takes the values between $-(12 - 15)\%$ depending on the sets of the parameters used for $(\rho; \eta)$. The LD effects on $< A_{FB} >$ are about 10%, but in reverse manner, decreasing its magnitude in comparison to the values without LD contributions.

- As for A_{CP}, $A_{CP}(A_{FB})$ for $\ell = e$, and $\ell = \mu$ modes almost coincide. $A_{CP}(A_{FB})$ is all negative except in a very small region for the intermediate values of s for $\ell = e, \mu$ cases. LD effects seem to be quite significant for $< A_{CP}(A_{FB}) >$, enhancing its value twice (four times) for $\ell = e, \mu$ ($\ell = \tau$) modes. For the light lepton modes, $\ell = e, \mu$, $A_{CP}(A_{FB})$ is more sizable in the high-dilepton mass region of s, $((m_{\psi_2} + 0.02)/m_B)^2 \leq s \leq 1$. However, for $\ell = \tau$, the contribution from the high-dilepton mass region of s is negligible and the contribution to $< A_{CP}(A_{FB}) >$ comes effectively from the low-dilepton mass region, $(2m_l/m_B)^2 \leq s \leq ((m_{\psi_1} - 0.02)/m_B)^2$ and amounts to -1%.

As a conclusion we can say that there is a significant A_{CP} and $A_{CP}(A_{FB})$ for the $B \to X_d\, \ell^+\ell^-$decay, although the branching ratios predicted for these channels are relatively small because of CKM suppression. So, $B \to X_d\, \ell^+\ell^-$decays seem promising for investigating CP violation.

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