RADIAL AND NONRADIAL OSCILLATIONS IN STARS WITH GRAVITATIONAL POTENTIAL PERTURBATION

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ABSTRACT

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In this study, the linear adiabatic radial and nonradial oscillations of evolutionary stellar model with $1.80M_{\odot}$,representing the star V1162 Ori, has been investigated. The already working oscillation program with Cowling approximation, which ignores the Eulerian perturbation in gravitational potential, has been modified to carry out the radial and nonradial linear adiabatic perturbations of stellar models using the Eulerian perturbation in gravitational potential. The new set of equations were solved without Cowling approximation.

In the last part, the calculated oscillation frequencies with and without Cowling approximation have been compared with the observed frequency spectrum of the variable δ-Scuti star V1162 Ori.

Keywords: Stellar Evolution, Oscillations, Variable Stars, V1162 Ori

ÖZ

YILDIZLARDA KÜTLE ÇEKİM POTANSİYEL TEDİRGİNLİĞİNDE

ÇAPSAL VE ÇAPSAL OLMAYAN SALINIMLAR

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Bu çalışmada, V1162 Ori'nin özelliklerini taşıdığını varsaydığımız 1.80 güneş kütlesinde evrimleştirilmiş bir modelin çapsal ve çapsal olmayan salınımları araştırılmıştır. Kütle çekim potansiyeli üzerindeki Eulerian tedirginliğini hesaba katarak çapsal ve çapsal olmayan lineer adiabatik yıldız modellerinin tedirginliklerini incelemek için kütle çekim potansiyeli üzerindeki Eulerian tedirginliğinin ihmal edildiği mevcut salınım programı üzerinde değişiklik yapıldı. Yeni salınım eşitlikleri Cowling yaklaşımı yapılmadan çözüldü.

Son bölümde değişken bir δ Scuti yıldızı olan V1162 Ori'nin Cowling yaklaşımı varken ve yokken hesaplanmış salınım frekansları gözlemlenmiş frekanslarla karşılaştırıldı.

Anahtar Kelimeler: Evrim Modeli, Salınımlar, Degişken Yıldızlar, V1162 Ori

TO MY FATHER

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TABLE OF CONTENTS

ABSTRACT iii

ÖZ v

DEDICATION vi

ACKNOWLEDGMENTS vii

TABLE OF CONTENTS viii

LIST OF TABLES x

LIST OF FIGURES xi

1 INTRODUCTION 1

2 GENERAL EQUATIONS OF HYDRODYNAMICS 7

2.1 The material derivative 7

2.2 The continuity equation 8

2.3 Euler's equation 9

2.4 The mechanical and thermal energy equations 12

2.5 Heat gains and losses 15

3 RADIAL OSCILLATIONS 16

3.1 The Models 17

3.2 Linear Theory 18

3.3 Eulerian and Lagrangian variations 18

3.4 Linearized equations of Radial pulsation 19

3.5 Adiabatic Radial Pulsations 24

4 NONRADIAL OSCILLATIONS 31
- 4.1 Basic Properties of Non-radial Oscillations 32
- 4.2 Dimensionless Formulation of Equations 33
- 4.3 Propagation Diagram 37
- 4.4 Rotation 45
- 4.5 The Eigenvalues and Eigenfunctions 50

5 CONCLUSION 52

REFERENCES 59

LIST OF TABLES

3.1 The characteristic of nonrotating and uniformly rotating models with mass 1.80M$_\odot$ compatible approximately with the characteristic of V1162 Ori. 17

4.1 Some pulsational properties of a rotating models with l=1, $\phi' \neq 0$ 51

5.1 The characteristic of nonrotatig and uniformly rotating models with masses 1.80M$_\odot$ compatible approximately with the characteristic of V1162 Ori. 53

5.2 Radial Oscillation frequencies for the nonrotating 1.80$M_\odot$ model (l=0). 53

5.3 Radial Oscillation frequencies for the rotating 1.80M$_\odot$ model (l=0). 54

5.4 Nonradial Oscillation frequencies for the nonrotating 1.80M$_\odot$ model (l=1). 54

5.5 Nonradial Oscillation frequencies for the rotating 1.80M$_\odot$ model (l=1). 54

5.6 Nonradial Oscillation frequencies for the nonrotating 1.80M$_\odot$ model (l=2). 55

5.7 Nonradial Oscillation frequencies for the rotating 1.80M$_\odot$ model (l=2). 55

5.8 Oscillation frequencies for 1.80M$_\odot$, V_{rot}=57.8 kms^{-1}, $\phi' = 0$ 58

5.9 Oscillation frequencies for 1.80M$_\odot$, V_{rot}=57.8 kms^{-1}, $\phi' \neq$0 58

LIST OF FIGURES

3.1 Eigenfunctions of the relative radial displacement ξ_r/r as a function of fractional stellar radius for a nonrotating 1.80M⊙ model.(l=0 with Cowling approximation. 27

3.2 Eigenfunctions of the relative radial displacement ξ_r/r as a function of stellar radius for a nonrotating 1.80M⊙ model.(l=0 without Cowling approximation. 27

3.3 The behavior of the potential energy perturbation in units of erg for a nonrotating 1.80M⊙ model; l=0. 28

3.4 Eigenfunctions of the relative radial displacement ξ_r/r for a rotating 1.80M⊙ model. (l=0 with Cowling approximation) 29

3.5 Eigenfunctions of the relative radial displacement ξ_r/r for a rotating 1.80M⊙ model. (l=0 without Cowling approximation) 29

3.6 The behavior of the potential energy perturbation for rotating model V_{rot}=57.8 km/s, l=0 with the units of erg. 30

4.1 Propagation diagram for the model star for l=1, R being of the radius of the star. 37

4.2 Eigenfunctions of the relative radial displacement ξ_r/r for the nonrotating 1.80M⊙ model (l=1 with Cowling approximation). 39

4.3 Eigenfunctions of the relative radial displacement ξ_r/r for the nonrotating 1.80M⊙ model (l=1 without Cowling approximation). . . 39

4.4 Eigenfunctions of the relative radial displacement ξ_r/r only showing f and g modes in figure 4.2 for the nonrotating 1.80M⊙ model (l=1 with Cowling approximation). 40

4.5 Eigenfunctions of the relative radial displacement ξ_r/r only showing f and g modes in figure 4.3 for the nonrotating 1.80M⊙ model (l=1 without Cowling approximation). 40

4.6 The behavior of the potential energy perturbation in units of erg for the nonrotating 1.80M⊙ model (l=1). 41

4.7 Eigenfunctions of the relative radial displacement ξ_r/r for the rotating 1.80M⊙ model (l=1 with Cowling approximation). 42

4.8 Eigenfunctions of the relative radial displacement ξ_r/r for the same model above for l=1 without Cowling approximation. 42

4.9 Eigenfunctions of the relative radial displacement ξ_r/r only showing f and g modes in figure 4.7 for the rotating 1.80M$\odot$ model (l=1 with Cowling approximation). 43
4.10 Eigenfunctions of the relative radial displacement ξ_r/r only showing f and g modes in figure 4.8 for the rotating 1.80M$\odot$ model (l=1 without Cowling approximation). 43
4.11 The behavior of the potential energy perturbation in units of erg for the rotating 1.80M$\odot$ model (l=1). 44

CHAPTER 1

INTRODUCTION

Every one of the stars including our sun is a self-gravitating gaseous sphere that radiates an enormous amount of energy to the outer space. Energy radiated from the surface of a star is generated in the deep interior by thermonuclear reactions. A star, born out of an interstellar cloud, spends most of its life in the hydrogen-burning main-sequence stage. As a star consumes its nuclear fuel, it evolves by changing its internal structure. Some stars blow out stellar winds from their surfaces with speeds ranging up to a few thousand kilometers per second, while some others are pulsating variables. A pulsating variable is a star that changes its brightness periodically by changing its volume just as a human body breathes rhythmically. Physical properties of pulsating stars change with time. They are sometimes called intrinsic variable stars.

By intrinsically varying stars, we mean those that vary in their light output in a periodic or semi-periodic fashion where the cause of variability is due to internal processes within the star. They are uniquely valuable in stellar astronomy because temporal variations in the behavior of their surface layers allow us to probe deep into their interiors and thus enrich our understanding of stellar structure. The variability of stars was first explained in terms of stellar pulsations by Ritter (1880). The early history of studies on stellar pulsation was concisely described by Rosseland (1949) in his famous textbook, *The Pulsation Theory of Variable Stars* .

A non-variable star in hydrostatic and thermal equilibrium when perturbed in some way so as to induce mass motions will gradually return to its original state. The reason for this is that a parcel of mass that has been compressed, for example, is hotter than its surrounding and thus (normally) radiates to its surrounding. Thus the PdV work that went into the compressing parcel is effectively dissipated and the parcel loses some of the "bounce" that might cause it to reexpand as in a necessary prerequisite for a sound wave. If this behavior mostly rules in the star, then the energy that originally went into the perturbation of the star will gradually leak out through the star, eventually being radiated from the surface.

In a variable star, the situation is reversed. There are some regions where compression results in heat effectively leaking into the compressed parcel and thus giving the parcel more bounce than usual. If this region dominates, then a perturbation will be driven and grow in time and not be damped out. The regions in a variable star that are responsible for this are those in which atomic ionization and recombination of abundant elements are actively taking place. Such "ionization zones" are important for more reasons than just stellar variability. It is the effect of ionization and opacity that eventually makes variable stars pulsate, i.e., expand and contract in radius and wax and wane in light output.

The study of pulsating stars constitutes a relatively small, but highly important, area of modern stellar astrophysics. It is interesting to see that the theory of nonradial pulsation developed by Lord Kelvin (1863) preceded the theory of radial pulsation developed by Ritter (1879). The pulsation hypothesis was presented in a definite form by Shapley (1914). Then some important works by Eddington (1918, 1919, 1926) laid the basis for the theory of adiabatic radial oscillations of gaseous spheres. Eddington (1919, 1926) demonstrated that in a pulsating star some mechanism is continuously acting so as to convert thermal energy into mechanical energy of pulsations. For stellar conditions, Eddington indicates two possible mechanisms for exciting oscillations of a star. These are nuclear driving mechanism and κ (absorbtion) mechanism. First one is due to the nuclear reactions near the stellar center and the second one is a result of the ionization of an abundant element, such as hydrogen or helium, at a critical depth below the stellar surface. Such pulsation are believed to be result of critical double ionization of helium (Zhevakin 1953). Accurate calculations were then carried out by some researchers (Baker and Kippenhahn, 1962, Cox 1963) and the effectiveness of He^{+} ionization as a pulsation driving mechanism was demonstrated.

Pekeris (1938) obtained the exact analytic solution for adiabatic nonradial oscillations in the homogeneous compressible model. Cowling (1941) extended the study to the polytrope model. Summary of the history of the pulsation theory has been given by Rosseland (1949), Eddington (1926), Ledoux and Walrevan (1958). The development of the study of nonradial pulsations may have started with the work of Ledoux (1951). He suggested that nonradial oscillations could explain the double periodicity observed in β Canis Majoris. Osaki (1971) examined Ledoux's theory by calculating line profiles for a star doing nonradial oscillations and compared the result with observations at that time.

The theory of stellar pulsations was originally developed in order to explain the pulsation of classical variable stars such as Cepheids and RR Lyrae stars. These variables are thought to be radial pulsators. The origin and maintenance of β Cephei pulsation have been discussed by Osaki (1974). He suggested a possible mechanism for their oscillations based on nonradial oscillation. There is an argument that the main pulsation of β Cephei stars is radial (Smith,1980). Even so, some β Cephei stars show multi-periodicity, and this indicates that nonradial oscillations must be involved as well.

The discovery of the solar five-minute oscillations by Leighton, Noyes, Simon (1962) opened a new era in stellar pulsation theory. Today, scientists probe the solar deep interior by using oscillations. Improvement of observations resulted with the detection of nonradial oscillations. In recent years, pulsations and relevant phenomena have also been discovered in many stars that were regarded as non-pulsating stars before, including δ Scuti, white dwarfs, Ap stars, early type O and B stars and classical novae.

In the HR diagram, the δ Scuti stars are located in the lower part of the classical instability strip where stellar pulsations are excited by the well-known κ-mechanism acting in zones of partial ionization of hydrogen and helium (Baker & Kippenhahn 1962, Cox 1963, Zhevakin 1963). Chevalier (1971) was the first who explicitly studied pulsations of δ Scuti star models and concluded that they are excited by κ-mechanism acting in the second helium ionization zone. Fitch (1976), (Breger 1979, Breger et al. 1987) emphasized the requirement of very long observation times in order to understand the extremely complex behavior of light variation and low amplitude of many δ Scuti stars.

Kurtz (1988) published an observational review on δ Scuti pulsators, while Shibahashi (1991) and Dziembowski et. al. (1992) reviewed the theoretical side of the δ Scuti pulsation. Breger et. al. (1991) presented multisite campaigns on some δ Scuti stars, and Dziembowski and Kroliowska (1985) investigated the mode selection mechanism of δ Scuti stars. Furthermore, Dziembowski et. al. (1992) studied the mechanism which limits amplitudes in δ Scuti stars.

However, pulsation properties of δ Scuti stars remained unsolved. Most of the δ Scuti stars are believed to pulsate with a mixture of radial and nonradial modes. The reason why some of the δ Scuti stars are pulsating at considerably low amplitudes with respect to the some others is still unknown. This may be due to a limited database of observations. Besides theoretical modelling for stellar evolution and pulsations may also need improving.

The aim of this work is to study the effects of the gravitational potential perturbation on the linear adiabatic radial and nonradial oscillation frequency calculations. For this purpose, the rotating and nonrotating models of $1.80M_\odot$ are studied.

In chapter II, we introduce some theoretical information on hydrodynamics. The method relevant to our case of radial and nonradial pulsation analyses are given in Chapter III and IV, respectively. Our findings on the behavior of the oscillations for Eulerian perturbation on both radial and nonradial pulsation cases are explained and compared with observations in later Chapter.

CHAPTER 2

GENERAL EQUATIONS OF HYDRODYNAMICS

A hydrodynamical system is characterized by specifying the physical quantities as functions of position $\mathbf{r}$ and time t. These properties include the local density $\rho(\mathbf{r}, t)$, the local pressure P($\mathbf{r}$,t) , and other thermodynamic quantity that may be needed as well as the local instantaneous velocity $\mathbf{V}$($\mathbf{r}$,t). Here $\mathbf{r}$ denotes the position vector to a given point in space, and the description therefore corresponds to what is seen by a stationary observer. This is known as the *Eulerian* description. In addition, we shall also use the so-called *Lagrangian* description,following the motion of a parcel of fluid. These descriptions correspond to the time derivative $\frac{\partial}{\partial t}$ seen by a stationary observer, and the derivative $\frac{d}{dt}$ observed when following the motion.

2.1 The material derivative

$\frac{\partial}{\partial t}$ will denote the rate of change of some quantity with respect to time *at a fixed position in space.* $\frac{d}{dt}$ (The material derivative or Stokes) will denote the rate of change of some quantity with respect to time but traveling along with fluid.

Let f be any quantity (e.g. temperature). Then

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v}.\nabla f \tag{2.1}$$

where $\mathbf{v}(\mathbf{r}$,t) is the velocity of the fluid at position $\mathbf{r}$ and time.

2.2 The continuity equation

We shall now derive the fundamental equations of fluid dynamics. We will begin with the equation which expresses the conservation of matter (Landau and Lifshitz, 1959). Consider a volume V_0, which is fixed in space. The total mass of fluid in V_0 is $\int \rho dV$,where $\rho(\mathbf{r}, t)$ is the density of the fluid and integration is taken over the volume V_0. The mass of fluid flowing in unit time through an element d$\mathbf{S}$ of the surface bounding this volume $\rho\mathbf{v}.d\mathbf{S}$, the magnitude of the vector d$\mathbf{S}$ is equal to the area of the surface element and its direction is along the normal. By convention, we take d$\mathbf{S}$ along the outward normal. Then $\rho\mathbf{v}$.d$\mathbf{S}$ is positive if the fluid is flowing out of the volume and negative if the flow is into the volume. The total mass of the fluid flowing out of the volume V_0 in unit time is therefore

$$\oint \rho\mathbf{v}.d\mathbf{S}$$

where the integration is taken over the whole of the closed surface surrounding the volume in question.

Next, the decrease per unit time in the mass of fluid in the volume V_0 can be written

$$-\frac{\partial}{\partial t}\int \rho d V$$

Equating the two expressions, we have

$$\frac{\partial}{\partial t}\int \rho d V = -\oint \rho \mathbf{v}.d\mathbf{S} \tag{2.2}$$

The surface integral can be transformed by Green's formula to a volume integral

$$\oint \rho \mathbf{v}.\mathbf{dS} = \int \mathbf{div}(\rho \mathbf{v})\mathbf{d} V \tag{2.3}$$

Thus,

$$\int [\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v})] d V = 0$$

Since this equation must hold for any volume, the integrand must vanish,i.e.

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0 \tag{2.4}$$

This is the equation of continuity.

2.3 Euler's equation

One can similarly derive the momentum equation (Landau and Lifshitz, 1959), or equation of motion, for the fluid by considering the rate of change of the total momentum of the fluid inside a volume V. It turns out to be easiest to consider a volume moving with the fluid, so that no fluid is flowing across its surface into or out of V. The momentum of the fluid in V is $\int_V \rho \mathbf{u} dV$, and the rate of change of this momentum is equal to the net force acting on the fluid in volume V. The forces acting on the fluid are of two kinds:

a) Surface forces,

b) Body forces.

The surface forces are exerted on the surface S of V by the surrounding fluid. In an *inviscid fluid* , we are generally considering, the surface force that acts in the direction of the normal to the surface and its net effect equals to the integral

$$-\oint PdS,$$

p being the pressure. Transforming above equation to a volume integral, we have

$$-\oint Pd\mathbf{S} = -\int \nabla P dV.$$

Hence we see that the fluid surrounding any volume element dV exerts on that element a force -dV**grad**p. In other words, we can say that a force **-grad**p acts on unit volume of the fluid. Body forces, such as gravity, which act on the particles inside V: their net effect is a force

$$\int_V \rho \mathbf{f} dV$$

where $\mathbf{f}$ is the body force per unit mass (force per unit mass has dimensions of acceleration). E.g, $\mathbf{f}$ could be the gravitational acceleration $\mathbf{g}$.

We can now write down the equation of motion of a volume element in the fluid by equating the -∇P to the product of the mass per unit volume (ρ) and the acceleration d**v**/dt:

$$\rho d\mathbf{v}/dt = -\nabla P \tag{2.5}$$

The derivative d**v**/dt which appears here denotes not the rate of change of the fluid velocity at a fixed point in space, but the rate of change the velocity of a given fluid particle as it moves about in space. This derivative has to be expressed in terms of quantities referring to points fixed in space. To do so, we notice that the change d**u** in the velocity of the given fluid particle during the time dt is composed of two parts, namely the change during dt in the velocity at a point fixed in space, and the difference between the velocities (at the same instant) at two points d**r** apart, where d**r** is the distance moved by the given fluid particle during the time dt. The first part is $(\partial\mathbf{v}/\partial t)$ is taken for constant x,y,z i.e. at the given point in space. The second part is

$$dx\frac{\partial\mathbf{v}}{\partial x} + dy\frac{\partial\mathbf{v}}{\partial y} + dz\frac{\partial\mathbf{v}}{\partial z} = (d\mathbf{r}.\nabla)\mathbf{v}.$$

Thus

$$d\mathbf{v} = (\partial\mathbf{v}/\partial t) + (d\mathbf{r}.\nabla)\mathbf{v},$$

or, dividing both sides by dt

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} \tag{2.6}$$

Substituting this in (2.5), we find

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\frac{\mathbf{1}}{\rho}\nabla P \tag{2.7}$$

This is the required equation of motion of the fluid; it was first obtained by L.Euler in 1755. It is called *Euler's equation* and one of the fundamental equations of fluid dynamics.

If the fluid is in a gravitational field, an additional force $\rho\mathbf{g}$, where $\mathbf{g}$ is the acceleration due to gravity, acts on any unit volume. This force must be added to the right hand side of equation (2.5) so that equation (2.7) takes the form

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\frac{\nabla P}{\rho} + \mathbf{g} \tag{2.8}$$

This is the momentum equation of motion for an *inviscid* fluid. In deriving the equation of motion we have taken no account of processes of energy dissipation, which may occur in moving fluid in consequence of internal friction (viscosity) in the fluid and heat exchange between different parts of it. The absence of heat exchange between different parts of the fluid means that the motion is adiabatic throughout the fluid. Thus the motion of an ideal fluid must necessarily be supposed adiabatic.

2.4 The mechanical and thermal energy equations

If one takes Newton's third law, $F=ma=m(dv/dt)$ and multiplies by velocity v , one obtains that rate of work done by the forces, Fv, is equal to the rate of change of kinetic energy, $(\frac{1}{2}mv^2)/dt$.

Similarly. taking the dot product of the equation of motion for a fluid, (2.8) with the fluid velocity $\mathbf{v}$ yields

$$\frac{d}{dt}(\frac{1}{2}v^2) = -\frac{1}{\rho}\mathbf{v}.(\nabla P) + \mathbf{v}.\mathbf{f} \tag{2.9}$$

where $v^2 = \mathbf{v}.\mathbf{v}$ and $\mathbf{f}$ is the body force per unit mass. Equation (2.9) says that the rate of change of the kinetic energy of a unit mass of fluid is equal to the rate of work which is done on the fluid by pressure and body forces. This is sometimes called the mechanical energy equation.

An equation for the total energy - kinetic and internal thermal energy - can be derived in the same manner as was the momentum equation. Let the internal energy per unit mass of fluid be U. Then the rate of change of kinetic plus internal energy of a material volume (i.e. one moving with the fluid) must be equal to the rate of work done on the fluid by surface and body forces, plus the rate of which heat is added to the fluid. Heat can be added in two ways:

a) By being generated at a rate ϵ per unit mass within the fluid volume (e.g. by nuclear reactions),

b) By the flux of heat $\mathbf{F}$ into the volume from the surroundings (e.g. by radiation).

Thus

$$\frac{d}{dt}\int_V (\frac{1}{2}\mathbf{v}^2 + U)\rho dV = -\int_S \mathbf{v}.Pd\mathbf{S} + \int_V \mathbf{v}.\mathbf{f}\rho dV + \int_V \epsilon\rho dV - \int_S \mathbf{F}.\mathbf{dS} \quad (2.10)$$

All the surface integrals in this equation are rewritten as a volume integrals, using divergence theorem. The resulting equation holds for an arbitrary volume V and we obtain

$$\rho\frac{d}{dt}(\frac{1}{2}\mathbf{v}^2 + U) = -\nabla.(P\mathbf{v}) + \rho\mathbf{v}.\mathbf{f} + \rho\epsilon - \nabla.\mathbf{F}. \quad (2.11)$$

One can derive an equation for the thermal energy alone by dividing (2.11) by the density and then subtracting the kinetic energy equation (2.9):

$$\frac{dU}{dt} = \frac{p}{\rho^2}\frac{d\rho}{dt} + \epsilon - \frac{1}{\rho}\nabla.\mathbf{F}. \quad (2.12)$$

Note that the divergence of $\mathbf{v}$ has been replaced by -$\rho^{-1}d\rho/dt$ using the continuity equation. Noting that the volume per unit mass is just the reciprocal of the density, i.e. $V = \rho^{-1}$. The principle of conservation of internal energy is just the first law of thermodynamics.

$$dU = TdS - pdV \quad (2.13)$$

where S is thermodynamic state variable, the *specific entropy* (i.e. the entropy per unit mass), T is the temperature.

The appropriate equation can easily be obtained by combining (2.12) and (2.13) which express the conservation of thermal energy.

$$\rho T(\frac{\partial}{\partial t} + \mathbf{u}.\nabla)\mathbf{S} = \rho\epsilon - \nabla.\mathbf{F} \qquad (2.14)$$

2.5 Heat gains and losses

We must be able to calculate dQ/dt, the net rate of gain of heat per unit mass. In general, we may say that the net heat gain by an element of matter is just the difference between the heat gained from the heat sources and the heat lost from the sinks. Let ϵ denote the total rate of gain of heat per unit mass from the sources (i.e., thermonuclear sources). Also, let **F** denote the total vector flux (energy flow across unit area normal to the direction of flow per unit time) of heat due to all transport mechanism that might be operate (radiation, conduction, convection, neutrino losses, mass losses, etc). then we may write thermonuclear energy generation rate is

$$\frac{dQ}{dt} = \epsilon - \frac{1}{\rho}\nabla.\mathbf{F}$$

The most important contributions to the total heat flux **F** in astrophysics are usually the radiative and convective photon flux.

CHAPTER 3

RADIAL OSCILLATIONS

The simplest motion is purely radial pulsation, in which the stars maintain a spherical shape at all times, but changes its volume, as if it were breathing.

In this chapter we consider the case of small oscillations of a star about its equilibrium configuration. The term equilibrium must be understood as the spherically symmetric star, in thermal and hydrostatic equilibrium. In other words the equilibrium configuration, which may also be called as the static situation, implies that there is no change in the physical parameters of the star with respect to time unless they are perturbed. Namely, we treat the star as a nonrotating gaseous sphere with no magnetic field and neglect all the terms involving the viscosity. The equilibrium model is also adiabatic, which means that there is no heat exchange of the stellar material with the surroundings.

We shall summarize some of the relevant equations first in general spherical coordinates, and shall then specialize to the case of spherically symmetric radial motion.

3.1 The Models

The evolutionary stellar models given in Table 3.1 are investigated for radial and nonradial oscillation. Star models with 1.8 $M_\odot$ were constructed using a modified version of Ezer's stellar evolution code (Ezer and Camenon, 1967). OPAL opacity tables (Iglesias et al., 1992) and the the equation of state developed by Mihalas et al. (1990) (MHD EOS) were applied to the stellar models. The numerical integration procedure and the physical inputs are described elsewhere (Yıldız& Kızıoğlu, 1997). The evolutionary study was carried out using the Henyey method which was applied from the center all the way to the surface. The evolutionary sequences of models were started at the threshold of the stability point at which the models become stable against gravitational collapse and were evolved up to the point where the luminosity and the effective temperature and so on given in Table 3.1. For the chemical composition we assumed fractional hydrogen abundance of X=0.699 a heavy element abundance of Z=0.19 and the mixing length parameter $\alpha = 1.74$.

Table 3.1: The characteristic of nonrotating and uniformly rotating models with mass 1.80$M_\odot$ compatible approximately with the characteristic of V1162 Ori.

Model No.	M ($M_\odot$)	L ($L_\odot$)	R ($R_\odot$)	T_e (K)	$\rho_c/<\rho>$ $\times 10^2$	Rot.Vel. (km/s)
1	1.80	13.35	2.167	7500	3.41	0.0
2	1.80	13.38	2.195	7450	3.49	57.8

3.2 Linear Theory

If a particular solution, which we shall call the "unperturbed" solution, is known, we are often interested in finding another solution, which we shall call "perturbed" solution that differ only slightly from the unperturbed solution.

If two solutions differ from each other only slightly, then each dependent variable for the perturbed solution can be expressed as the sum of the corresponding dependent variable for the unperturbed solution and a small correction term. That is, a small variation or perturbation substituting the dependent variables so expressed into the equations. The great advantage of the resulting set of equations is that they are linear, and well-known mathematical procedures can then be applied to obtain solutions for the variations.

Two types of the variations, *Eulerian* and *Lagrangian* are in general employed in the linear theory.

3.3 Eulerian and Lagrangian variations

We consider the "unperturbed" equilibrium state of a star and superimpose on it "small" perturbation. There are two different ways to express perturbation. The *Eulerian* perturbation is defined as a perturbation of a physical quantity at a given position, denoted by prime, while the *Lagrangian* perturbation (variation) is defined by for a given fluid element denoted by symbol δ. Any physical quantity f is expressed by either

$$f'(\mathbf{r},t) = f(\mathbf{r},t) - f_0(\mathbf{r},t), \tag{3.1}$$

or

$$\delta f(\mathbf{r},t) = f(\mathbf{r},t) - f_0(\mathbf{r}_0,t) \tag{3.2}$$

Where zero subscripts denote that the reference frame is unperturbed configuration. The connection between the Eulerian and Lagrangian variations f' and δf, respectively, is obtained from equation (3.2) subtracting and adding $f_0(\mathbf{r},t)$ and using Taylor series about $\mathbf{r_0}$ and keep terms up to the first order in δr we obtaine,

$$\delta f = f' + \delta\mathbf{r}.\nabla f_0 \tag{3.3}$$

3.4 Linearized equations of Radial pulsation

To obtain the linear set of equations governing the radial adiabatic oscillations classical linear theory is applied (Ledoux and Walraven, 1958). Beginning from the basic equations related with the structure of a star, physical parameters are perturbed and neglecting the higher order terms linear set of radial pulsation equations are obtained.

The basic equations that describe oscillations of a star are conservation of mass, momentum, and energy equations, which are given by

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{u}) = 0 \tag{3.4}$$

$$\rho(\frac{\partial}{\partial t} + \mathbf{u}.\nabla)\mathbf{u} = -\nabla \mathbf{P} - \rho \nabla \phi \tag{3.5}$$

$$\rho T(\frac{\partial}{\partial t} + \mathbf{u}.\nabla)\mathbf{S} = \rho \epsilon - \nabla.\mathbf{F} \tag{3.6}$$

Where ρ denotes the density, P the pressure, T the temperature, $\mathbf{u}$ the fluid velocity, S the specific entropy, ϕ the gravitational potential, ϵ the energy generation rate, ∇ the gradient operator. Supplementary equations are needed to complete the description of a system. The first one of them is the Poisson equation that relates the gravitational potential to the distribution of matter; it is written as

$$\nabla^2 \phi = 4\pi G \rho \tag{3.7}$$

where G is the gravitational constant and ∇^2 is the Laplacian operator. The radiative flux is given by the radiative diffusion equation

$$F_R = -K\nabla T, K = \frac{4acT^3}{3\kappa\rho}, \tag{3.8}$$

where the radiative conductivity K is written in terms of opacity κ and a is the radiation density constant, and c the velocity of the light.

There are convection zone in most stars, either in the interior or the outer envelope. The convective motion in stars is considered to be a states of fully developed turbulence. When convection is present, the treatment of stellar oscillation becomes very difficult because the separation of the velocity into convective motion and oscillation and their mutual interaction are very complicated. Here we consider a non-rotating, non-magnetic star without convection.The values of all physical variables (i.e., velocity, density) in the unperturbed star as a function of $\mathbf{r}$. As the fluid moves, each fluid element in the star is displaced from its equilibrium position at $\mathbf{r}$ by an arbitrary, but infinitesimal, vector distance $\xi(\mathbf{r}, \mathbf{t})$.

When no motion exits in the unperturbed state (i.e., $\mathbf{v} = \mathbf{0}$), the Eulerian and Lagrangian perturbations of $\mathbf{v}$, denoted respectively by $\mathbf{v}'$ and $\delta\mathbf{v}$ are equal and are given by

$$\mathbf{v}' = \delta\mathbf{v} = \frac{\partial\vec{\xi}}{\partial t} = \frac{d\vec{\xi}}{dt} \tag{3.9}$$

where d/dt is the stokes derivative. When a small perturbation is applied to the equilibrium model, variation in physical quantities can be found in terms of Eulerian variation. As the fluid is displaced, all other physical variables are perturbed accordingly. Thus, for example, the pressure $P(\mathbf{r})$, which was originally associated with the fluid parcel at $\mathbf{r}$, becomes $P(\mathbf{r}) + P'(\mathbf{r}, t)$ when the parcel is moved to $\mathbf{r} + \xi(\mathbf{r}, t)$.

After perturbing and linearizing the above equations derived in the Eulerian form as follows:

$$\frac{\partial\rho'}{\partial t} + \nabla.(\rho_0\mathbf{v}) = 0, \tag{3.10}$$

$$\rho_0\frac{\partial\mathbf{v}}{\partial t} + \nabla p' + \rho_0\nabla\phi' + \rho'\nabla\phi_0 = 0, \tag{3.11}$$

$$\rho_0 T_0\frac{\partial}{\partial t}(S' + \xi.\nabla S_0) = (\rho\epsilon_N)' - \nabla.\mathbf{F}', \tag{3.12}$$

$$\nabla^2 \phi' = 4\pi G \rho'. \tag{3.13}$$

Equations (3.10)-(3.13) are linear, homogeneous, partial differential equations with respect to time t and space coordinates **r** for perturbed variables with prime (such as ρ', ϕ',...) and velocity vector **v**. The quantities at equilibrium such as ρ_0, ϕ_0 are functions of radial coordinate r only.

The dynamical properties of stellar oscillations can be studied assuming that the specific entropy is conserved during the oscillations; i.e., $\delta S = 0$. We omit the subscript 0 for equilibrium quantities. The equation of continuity (3.10) is written as

$$\rho' + \nabla.(\rho\xi) = 0 \tag{3.14}$$

or

$$\delta\rho/\rho + \nabla.\xi = 0 \tag{3.15}$$

while the equation of motion (3.11) is written as

$$\frac{\partial \mathbf{v}'}{\partial t} = -\frac{1}{\rho}\nabla P' + \frac{\rho'}{\rho^2}\nabla P - \nabla\phi' \tag{3.16}$$

We express the density perturbation ρ' in terms of P', ξ by the use of the thermodynamic relation (3.34),

$$\frac{\rho'}{\rho} = \frac{1}{\Gamma_1}\frac{P'}{P} - A\xi \tag{3.17}$$

Where the quantity A is the Schwarzschild discriminant used by (Ledoux and Walraven, 1958) to denote the degree of convective instability (i.e.,$A > 0$) or stability (i.e.,$A < 0$)

$$A = -N^2/g = \frac{dln\rho}{dr} - \frac{1}{\Gamma_1}\frac{dlnp}{dr} \tag{3.18}$$

Since the equilibrium star is spherical, we shall use spherical polar coordinates of the form, which is given in detail in the next chapter

$$f'(t, r, \theta, \psi) = f'(r) Y_m^l(\theta, \psi) e^{i\sigma t} \tag{3.19}$$

After doing some algebraic manipulation these basic equations are finally reduced to

$$\frac{1}{r^2}\frac{d}{dr}(r^2\xi_r) - \frac{g}{c^2}\xi_r + (1 - \frac{L_l^2}{\sigma^2})\frac{p'}{\rho c^2} = \frac{l(l+1)}{\sigma^2 r^2}\phi', \tag{3.20}$$

$$\frac{1}{\rho}\frac{dp'}{dr} + \frac{g}{\rho c^2}p' + (N^2 - \sigma^2)\xi_r = -\frac{d\phi'}{dr}, \tag{3.21}$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d\phi'}{dr}) - \frac{l(l+1)}{r^2}\phi' = 4\pi G\rho(\frac{p'}{\rho c^2} + \frac{N^2}{g}\xi_r). \tag{3.22}$$

where $c = (\Gamma_1 p/\rho)^{1/2}$ is the sound velocity and l is the spherical harmonic degree. L_l^2 and N^2 are the Lamb frequency related with sound wave and the Brunt-Väisälä frequency related with gravity wave, respectively, and given by

$$L_l^2 = \frac{l(l+1)c^2}{r^2} \tag{3.23}$$

and

$$N^2 = -gA = g(\frac{1}{\Gamma_1}\frac{dlnp}{dr} - \frac{dln\rho}{dr}). \tag{3.24}$$

The equations (3.20)-(3.22) with proper boundary conditions give an eigenvalue problem with an eigenvalue σ^2. The solutions of the equations have to satisfy the boundary conditions. We will consider the boundary conditions at the center (at r=0, δr has to remain finite) and at the surface (at r=R , $\delta p = 0$) where the equations are singular.

3.5 Adiabatic Radial Pulsations

Radial pulsation is the particular case with l=0. For radial pulsations equation (3.20) is reduced to (Cox, 1980)

$$\frac{p'}{\rho c^2} = -\frac{1}{r^2}\frac{d}{dr}(r^2\xi_r) + \frac{g}{c^2}\xi_r. \tag{3.25}$$

Substituting equation (3.25) into equation (3.22) with l=0,we obtain

$$\frac{d}{dr}(r^2\frac{d\phi'}{dr}) + 4\pi G[\rho\frac{d}{dr}(r^2\xi_r) + r^2\xi_r\frac{d\rho}{dr}] = 0, \tag{3.26}$$

where the relation $N^2/g = -dln\rho/dr - g/c^2$ was used. Integrating equation (3.26) under the condition that $d\phi'/dr$ is non-singular at r=0, we obtain

$$\frac{d\phi'}{dr} + 4\pi G\rho\xi_r = 0 \tag{3.27}$$

Substituting equations (3.25) and (3.27) into equation (3.21) and using the relation for equilibrium structure such as

$$\frac{dp}{dr} = -\rho g,$$
$$\frac{dM_r}{dr} = 4\pi r^2\rho.$$

We finally obtain a standard differential equation of radial pulsations (Ledoux and Walraven, 1958):

$$\frac{d}{dr}(\Gamma_1 p r^4 \frac{d}{dr}(\frac{\xi_r}{r})) + (\sigma^2 \rho r^4 + r^3 \frac{d}{dr}((3\Gamma_1 - 4)p))(\frac{\xi_r}{r}) = 0 \tag{3.28}$$

This equation with the boundary conditions forms a Strum-Liouville type eigenvalue problem with the eigenvalue σ^2. We can rewrite above equation as coupled two first-order differential equations (Hansen and Kawaler, 1995) as follows

$$\frac{d\zeta}{dr} = -\frac{1}{r}(3\zeta + \frac{1}{\Gamma_1}\frac{\delta P}{P}), \tag{3.29}$$

$$\frac{\delta P/P}{dr} = -\frac{dlnP}{dr}(4\zeta + \frac{\sigma^2 r^3}{GM_r}\zeta + \frac{\delta P}{P}) \tag{3.30}$$

where $\zeta = \xi_r/r$. We now need boundary conditions. The first of these is simple because we require that δr be zero at the center (r=0). Physical regularity of the solutions also requires that both ζ and $d\zeta/dr$ be finite at the center. This gives the first boundary condition

$$3\zeta + \frac{1}{\Gamma_1}\frac{\delta P}{P} = 0 \text{ at r=0.} \tag{3.31}$$

The second boundary condition is applied at the surface. We assume density and pressure vanish at the stellar surface. The relative pressure perturbation, $\delta P/P$, remains finite.

We must have

$$4\zeta + \frac{\sigma^2 R^3}{GM}\zeta + \frac{\delta P}{P} \text{ at r=R.} \tag{3.32}$$

The assumptions of adiabatic oscillations are usually used as an approximation. The adiabatic relations are expressed as

$$\frac{\delta P}{P} = \Gamma_1 \frac{\delta\rho}{\rho} \tag{3.33}$$

$$\frac{\delta T}{T} = (\Gamma_3 - 1)\frac{\delta\rho}{\rho} \tag{3.34}$$

In above equations

$$\Gamma_1 = (\frac{dlnP}{dln\rho})_{ad}, \tag{3.35}$$

$$\Gamma_3 = (\frac{dlnT}{dln\rho})_{ad} + 1. \tag{3.36}$$

The further work done on solving the equations (3.29) and (3.30) with two boundary conditions (3.31) and (3.32) is mainly the computational work evaluating these equations and calculating numerical solutions shown in the following figures presented in this chapter.

The solution of the system of equations were computed using Runge-Kutta method. At the beginning, a minimum value for the dimensionless frequency ω^2 was given and the solutions were evaluated. The parameter ω^2, or σ^2, was the eigenvalue of the system. If these solutions did not satisfy the boundary condition at the surface, then another ω^2 was assigned by adding a small increment and so on. This continued until the desired accuracy was obtained at the boundary condition, at which time the value of ω^2 was that of a certain mode. The numerical accuracy in obtaining frequencies was about 10^{-3}. To get an accurate description of the oscillation in the inner propagation zone, we used typically 4000 mass zones. Computation were carried out for low spherical harmonic degres $l \leq 2$.

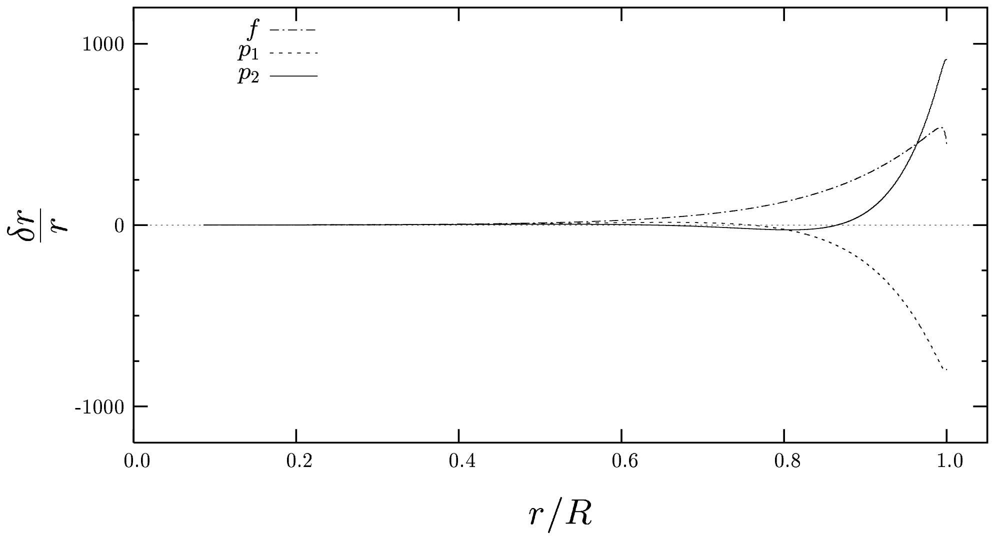

Figure 3.1: Eigenfunctions of the relative radial displacement ξ_r/r as a function of fractional stellar radius for a nonrotating 1.80M$\odot$ model.(l=0 with Cowling approximation.

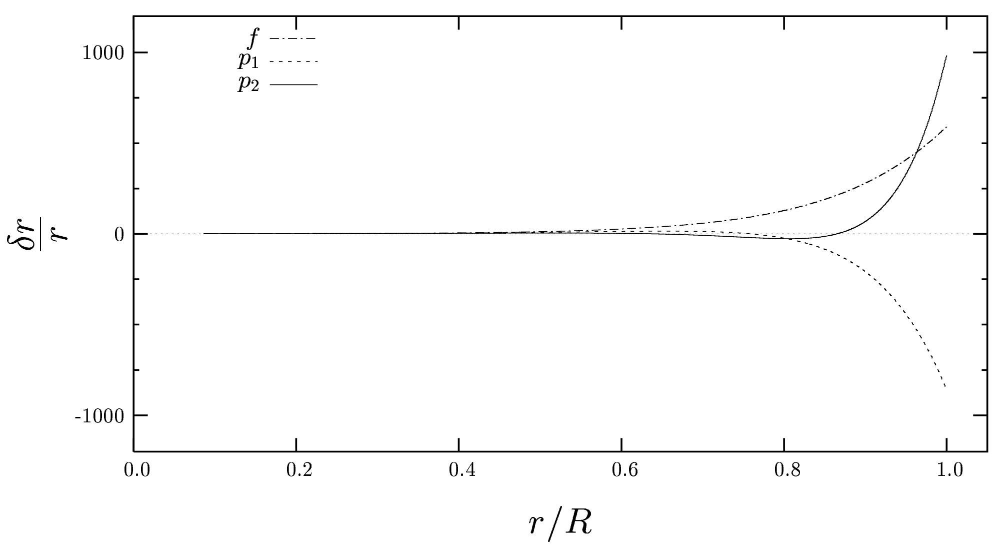

Figure 3.2: Eigenfunctions of the relative radial displacement ξ_r/r as a function of stellar radius for a nonrotating 1.80M$\odot$ model.(l=0 without Cowling approximation.

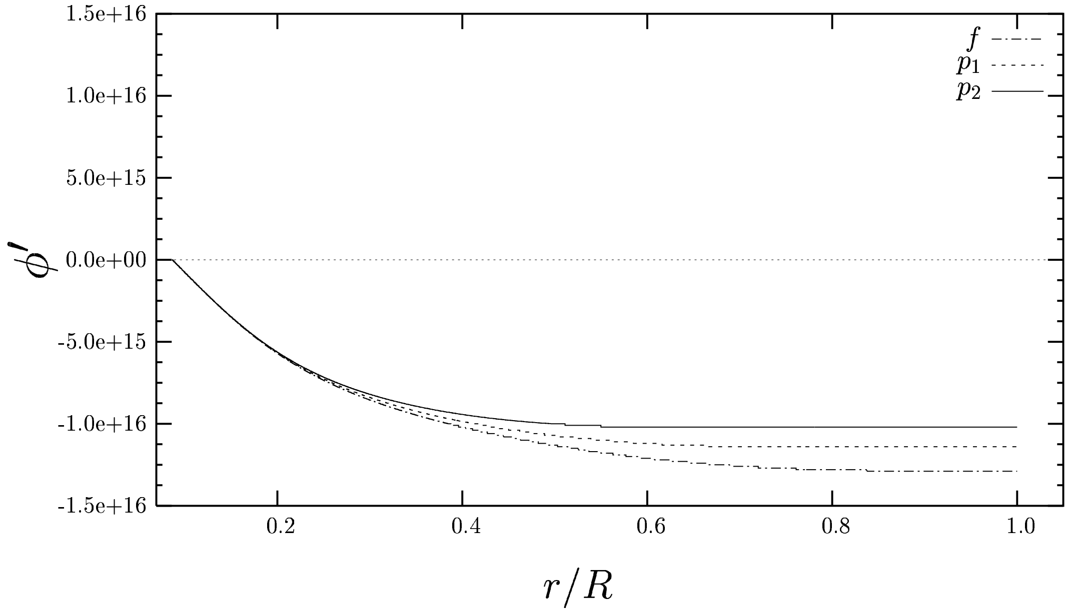

Figure 3.3: The behavior of the potential energy perturbation in units of erg for a nonrotating 1.80M⊙ model; l=0.

In Figure 3.1 , f-mode is equal to 149.4283 μHz, f_{p_1}=193.0086 μHz, f_{p_2}=238.8877 μHz. The analysis of this model was done when the Cowling approximation is adopted. In figure 3.2, f_f=149.1872 μHz, f_{p_1}=192.8302 μHz and f_{p_2}=238.6701 μHz. There are convection zone in most stars, either in the interior or the outer envelope. The convective motion in stars is considered to be a states of fully developed turbulence. When convection is present, the treatment of stellar oscillation becomes very difficult because the separation of the velocity into convective motion and oscillation and their mutual interaction are very complicated. Starting point of the nonrotating 1.80M⊙ model in the figure 3.1 and 3.2 is 0.086 where we restrict ourselves to the case without convection.

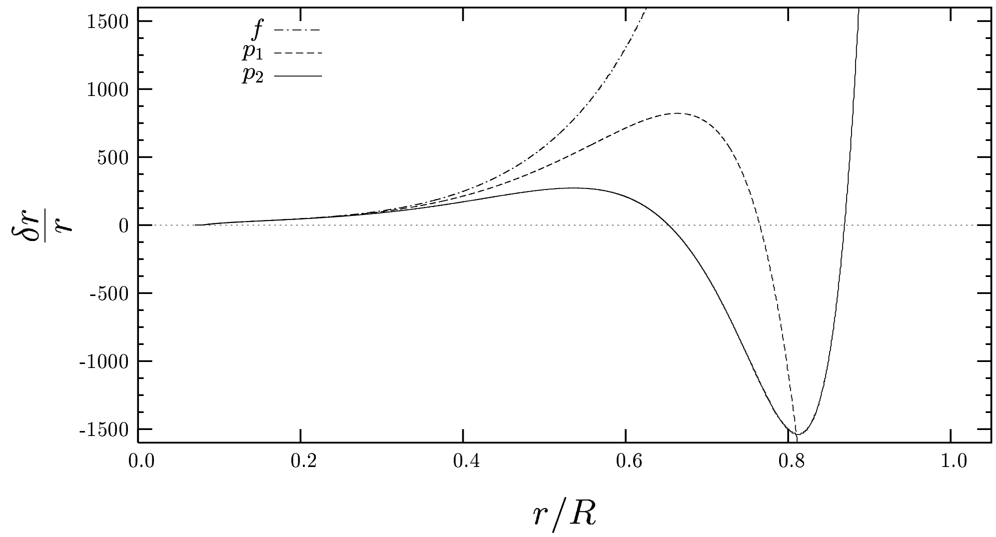

Figure 3.4: Eigenfunctions of the relative radial displacement ξ_r/r for a rotating 1.80M$\odot$ model. (l=0 with Cowling approximation)

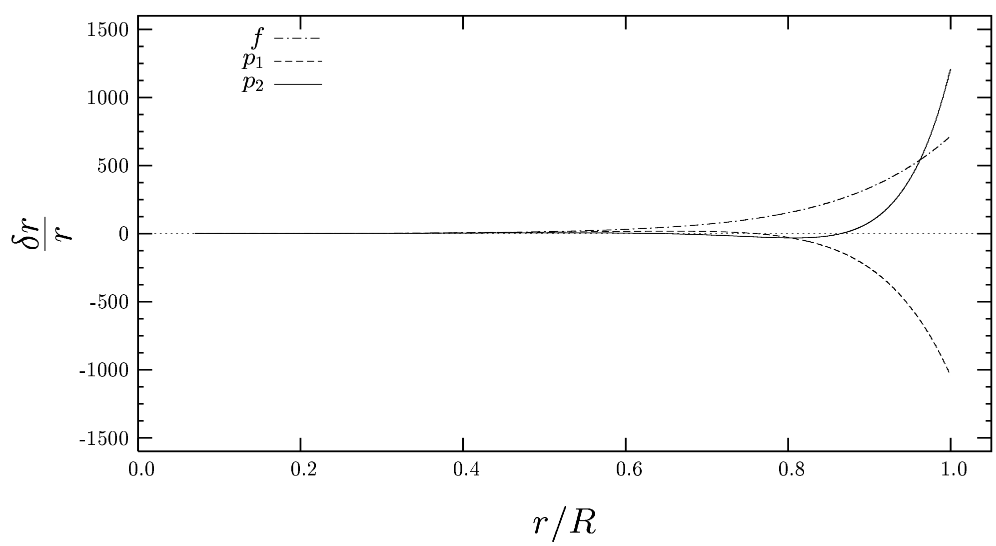

Figure 3.5: Eigenfunctions of the relative radial displacement ξ_r/r for a rotating 1.80M$\odot$ model. (l=0 without Cowling approximation)

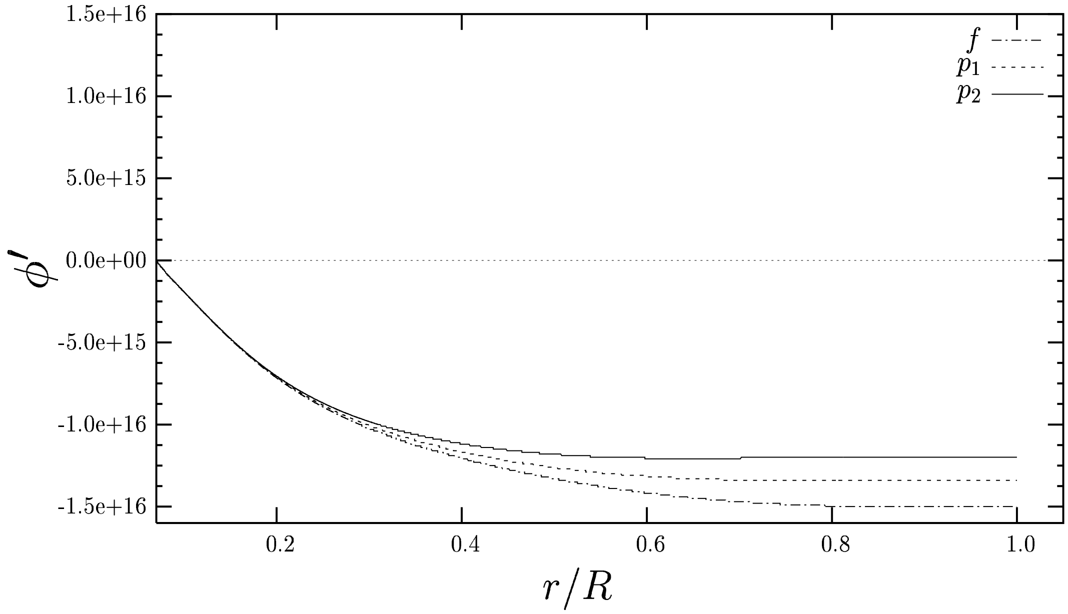

Figure 3.6: The behavior of the potential energy perturbation for rotating model V_{rot}=57.8 km/s, l=0 with the units of erg.

In the analysis of the other model that the behavior of the radial displacement is shown in figure 3.4 having the V_{rot}=57.8 km/s it is found that f-mode equal to 146.3737 μHz, f_{p_1}=189.4125 μHz, f_{p_2}=235.6222 μHz.

For the same model but not considering the Cowling approximation in the figure 3.5 it is found that f_f=147.1394 μHz, f_{p_1}=189.7685 μHz and f_{p_2}=234.7954 μHz. Starting point of the figures 3.4 and 3.5 is 0.070.

CHAPTER 4

NONRADIAL OSCILLATIONS

Non-radial oscillations mean that stars oscillate in modes which do not preserve their radial symmetries. As some types of stars pulsate, their surfaces do not move uniformly in and out in a simple " breathing" motion. Instead the star executes a more complicated motion in which some regions of its surface expand while others areas contract.The displacement of a typical mass element from its unperturbed position may be now in any direction at all. Consequently, now this displacement must be described by a vector in three dimensions.

Mainly, it is considered that stars in their unperturbed (nonoscillating) states are spherically symmetric, static and in hydrostatic equilibrium without large scale magnetic fields and no rotation. In a dynamically stable star, a small overall contraction tends to increase the pressure gradient in such a way that the resulting force exceeds the increased gravitational force. So, there is a tendency to come back to the equilibrium configuration. If a perturbation is applied to a dynamically stable configuration, oscillation will take place. The amplitude of these oscillations can eventually increase with time , in which case the configuration is said to be vibrationally unstable or they can decrease with time, in which case the configuration is vibrationally stable.

The radial pulsations of stars was attributed to standing sound waves in the stellar interior. For the case of nonradial oscillations the sound waves can propagate horizontally as well as radially to produce waves that travel around the star. Because pressure provides the restoring force for the compression and expansion of a sound wave, these nonradial oscillations are called p-modes. Gravity is the source of the restoring force for another class of nonradial oscillations called g-modes that are produced by internal gravity waves.

4.1 Basic Properties of Non-radial Oscillations

A pulsating star has various modes of oscillation and tones. The normal modes in a spherically symmetric star are characterized by the eigenfunctions that are proportional to the spherical harmonics: $Y_m^l(\theta, \psi)$ (l=0,1,2,...; m= -l, -l+1,..., l-1, l). Thus for a given value of l there are only 2l+1 permitted values of m. l=0 is the radial mode, l=1 and l=2 harmonics are called dipole and quadrupole oscillations, respectively. The eigenfrequencies depend on l but are degenerate by (2l+1)-folds in m. Normal modes are distinguished by number of nodes, n, in the radial component of displacement from the center to the surface. n=0 is the fundamental mode, n=1 is the first overtone and n=2 is the second overtone. In short normal modes are classified by the radial quantum number n and the angular quantum number l. When rotation is involved, the azimuthal quantum number m is added.

In radial oscillations only pressure can act as a restoring force therefore their spectrum contains pressure mode (p-mode) or the acoustic wave mode.In nonradial oscillations gravity is also a restoring force, so that they have the spectrum of both pressure and gravity (g-mode) mode.

Assuming that the unperturbed state is in time-independent equilibrium. We take the perturbation of physical variables to be proportional to $Y_l^m(\theta, \psi)e^{i\sigma t}$. In such a theory the temporal and spatial dependence of a small perturbation can be represented by using spherical polar coordinates of the form

$$f'(t, r, \theta, \psi) = f'(r)Y_m^l(\theta, \psi)e^{i\sigma t} \tag{4.1}$$

The corresponding expression for the displacement vector ξ is given by

$$\xi = [\xi_r(r), \xi_h(r)\frac{\partial}{\partial\theta}, \xi_h(r)\frac{\partial}{sin\theta\partial\psi}]Y_l^m(\theta, \psi)e^{i\sigma t}, \tag{4.2}$$

where ξ_h is given below

$$\xi_h = \frac{1}{\sigma^2 r}(\frac{p'}{\rho} + \phi') \tag{4.3}$$

4.2 Dimensionless Formulation of Equations

The general method used in stellar stability is the small perturbation method (Ledoux 1958, 1949, Unno et al. 1989, Cox 1980) in which a small perturbation is applied to the model and the problem consists of following the evolution of the perturbed model. This is solved by linearizing all physical equations (mass, momentum, thermal energy conservations and Poisson equation).

To linearize these equations one takes each variable, adds a perturbation to it and introduces it in the equation, keeping only the linear terms with respect to the perturbations and taking into account that these equations are satisfied by the unperturbed variables.

These algebra are done in the previous chapter. The basic equation are equations (3.20)-(3.22). The oscillation is assumed to be adiabatic and the Cowling approximation is not adopted throughout in this study. In numerical analysis, it is preferred the dimensionless variables. We thus rewrite the basic equations (3.20)-(3.22) in four first-order differential equations with four dimensionless variables. The following four variables are used (Dziembowski, 1971):

$$y_1 = \frac{\xi_r}{r}, \tag{4.4}$$

$$y_2 = \frac{1}{gr}\left(\frac{p'}{\rho} + \phi'\right), \tag{4.5}$$

$$y_3 = \frac{1}{gr}\phi', \tag{4.6}$$

$$y_4 = \frac{1}{g}\frac{d\phi'}{dr}. \tag{4.7}$$

Further, the dimensionless radius x is defined by

$$x = r/R \tag{4.8}$$

The substitution of above equations into equations (3.20)-(3.22) gives

$$x\frac{dy_1}{dx} = (V_g - 3)y_1 + \left(\frac{l(l+1)}{c_1\omega^2} - V_g\right)y_2 + V_g y_3, \tag{4.9}$$

$$x\frac{dy_2}{dx} = (c_1\omega^2 - A^*)y_1 + (A^* - U + 1)y_2 - A^* y_3, \tag{4.10}$$

$$x\frac{dy_3}{dx} = (1 - U)y_3 + y_4, \tag{4.11}$$

and

$$x\frac{dy_4}{dx} = UA^*y_1 + UV_g y_2 + (l(l+1) - UV_g)y_3 - Uy_4, \tag{4.12}$$

where

$$V_g = \frac{V}{\Gamma_1} = -\frac{1}{\Gamma_1}\frac{dlnP}{dlnr} = \frac{gr}{c^2}, U \equiv \frac{dlnM_r}{dlnr} = \frac{4\pi\rho}{M_r^3}, \tag{4.13}$$

$$c_1 \equiv (r/R)^3/(M_r/M), \tag{4.14}$$

$$\omega^2 = \sigma^2 R^3/(GM), \tag{4.15}$$

and V is the homology invariant defined by $-dlnP/dlnr = GM_r\rho/rP$,

$$A^* \equiv -rA = rg^{-1}N^2. \tag{4.16}$$

A is the Schwarzchild discriminant.

Near the center, dimensionless quantities of stellar equilibrium structure approach their central values as follows:

$$\left.\begin{array}{llll} U \rightarrow & 3 & + & O(x^2) \\ V \rightarrow & 0 & + & O(x^2) \\ A^* \rightarrow & 0 & + & O(x^2) \end{array}\right\} \text{as x} \rightarrow 0 \tag{4.17}$$

Equations (4.9)-(4.12) may then be regarded as differential equations with constant coefficients near the center. We finally obtain two homogeneous relations near the center by using Cauchy-Euler equation which is given below

$$det(a_{ij} - \lambda\delta_{ij}) = 0. \tag{4.18}$$

where a_{ij} is the constant coefficient matrix

$$a_{ij} = \begin{pmatrix} -3 & \frac{l(l+1)}{c_1\omega^2} & 0 & 0 \\ c_1\omega^2 & -2 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & l(l+1) & -3 \end{pmatrix} \tag{4.19}$$

According to the new variables we now have the following boundary conditions near the center

$$\frac{c_1\omega^2}{l} y_1 - y_2 = 0, \tag{4.20}$$

$$l y_3 - y_4 = 0. \tag{4.21}$$

These equations (4.20)-(4.21) are the two inner boundary conditions.

The proper outer boundary conditions can similarly be obtained. The surface properties that

$$\left.\begin{array}{llll} U \rightarrow & 0 & , V \rightarrow & V_g(x=1) \\ A^* \rightarrow & A^*(x=1) & , c_1 \rightarrow & 1 \end{array}\right\} \text{as x} \rightarrow 1 \tag{4.22}$$

Equations (4.9)-(4.12) are reduced to differential equations with constant coefficients near the surface. In this case the constant coefficient matrix:

$$b_{ij} = \begin{pmatrix} V_g - 3 & l(l+1)/\omega^2 - V_g & V_g & 0 \\ \omega^2 - A^* & 1 + A^* & -A^* & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & l(l+1) & 0 \end{pmatrix} \tag{4.23}$$

After doing some algebra, we obtain two homogeneous relations among $y_1, y_2, y_3,$ and y_4 near the surface as follows:

$$(l+1)y_3 + y_4 = 0, \tag{4.24}$$

$$y_1 - y_2 + y_3 = 0. \tag{4.25}$$

These equations (4.24)-(4.25) are the two outer boundary conditions

4.3 Propagation Diagram

In order to understand the condition of wave trapping in the stellar models, a diagram that L_l^2 and N^2 being two critical frequencies are plotted below as functions of the fraction of stellar radius. The ordinate ω^2 denotes the square of dimensionless frequency $[\omega^2 = \sigma^2/(GM/R^3)]$, and L_l^2 and N^2 are measured.

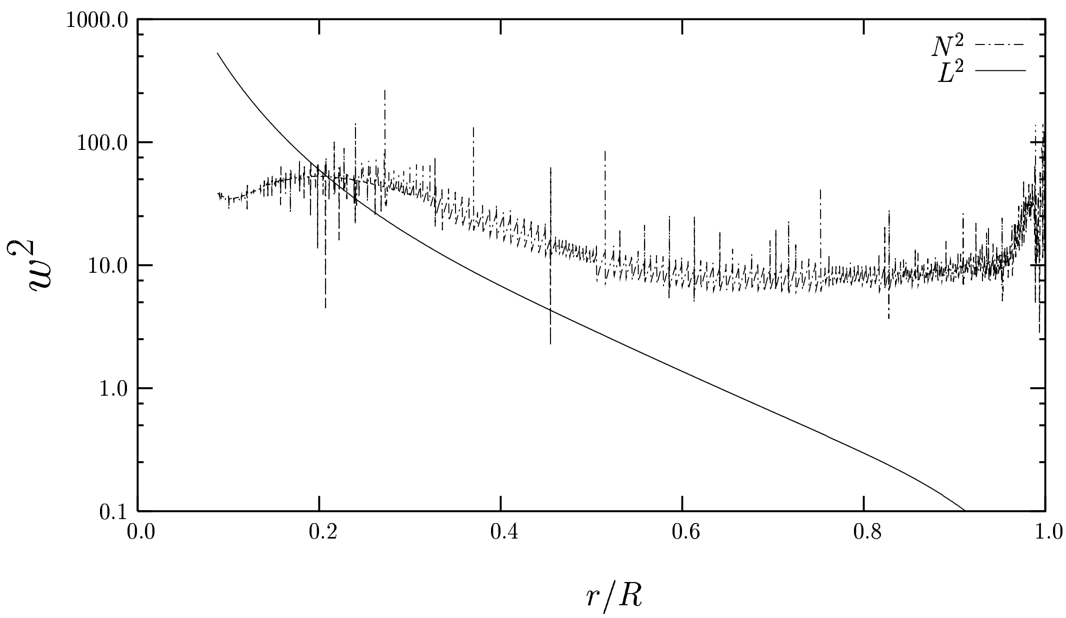

Figure 4.1: Propagation diagram for the model star for l=1, R being of the radius of the star.

The behavior of L_l^2 is not much different from star to star, being infinite at r=0 and decreasing monotonically as r increases. But the behavior of N^2 changes with evolution.For a given value of σ^2, we can easily see in which region of the star a wave can propagate and in which region it is evanescent. if σ^2 is very large, i.e., if $\sigma^2 > N^2, L_l^2$ in radiative region and $\sigma^2 \geq 0$ we may deduce that oscillations are acoustic, p mode oscillations. In the case of where $\sigma^2 < N^2, L_l^2$ for sufficiently small σ^2 it is possible to say that the gravity g modes dominates. The case of σ^2 lying in between the two frequencies $N^2 < \sigma^2 < L_l^2$ or $L_l^2 < \sigma^2 < N^2$ corresponds to the mixed case.

In figures 4.2-4.5, the eigenfunctions of the fundamental, p and g modes versus the fractional radius have been plotted for the nonrotating 1.80M$\odot$ model. Also, I have done the same work for the rotating 1.80M$\odot$ model in figures 4.7-4.10. Introducing the gravitational potential perturbation into the equations governing the nonradial oscillations, I have displayed the behavior of gravitational potential perturbation versus the fractional radius in figure 4.6 for the nonrotating 1.80M$\odot$ model and in figure 4.11 for rotating 1.80M$\odot$ model.

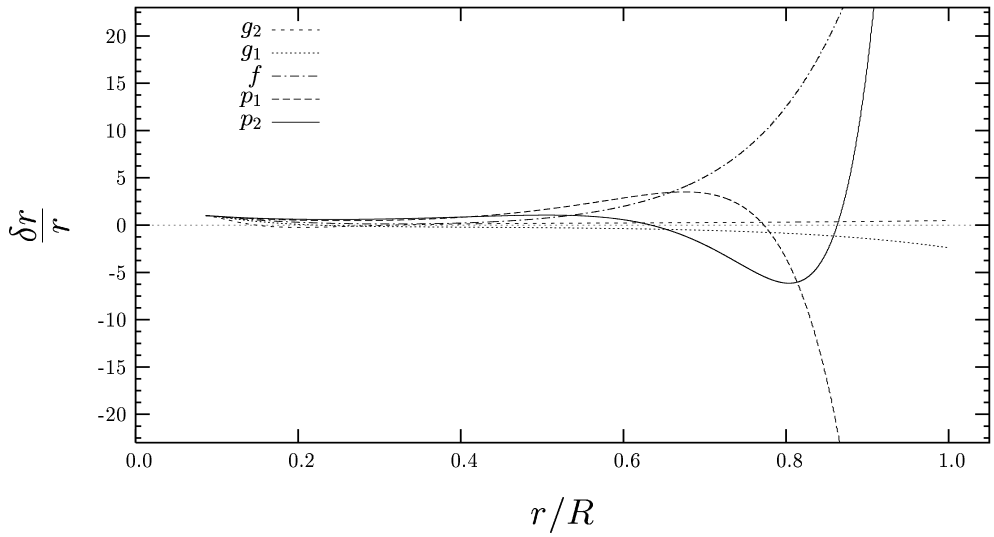

Figure 4.2: Eigenfunctions of the relative radial displacement ξ_r/r for the nonrotating 1.80M⊙ model (l=1 with Cowling approximation).

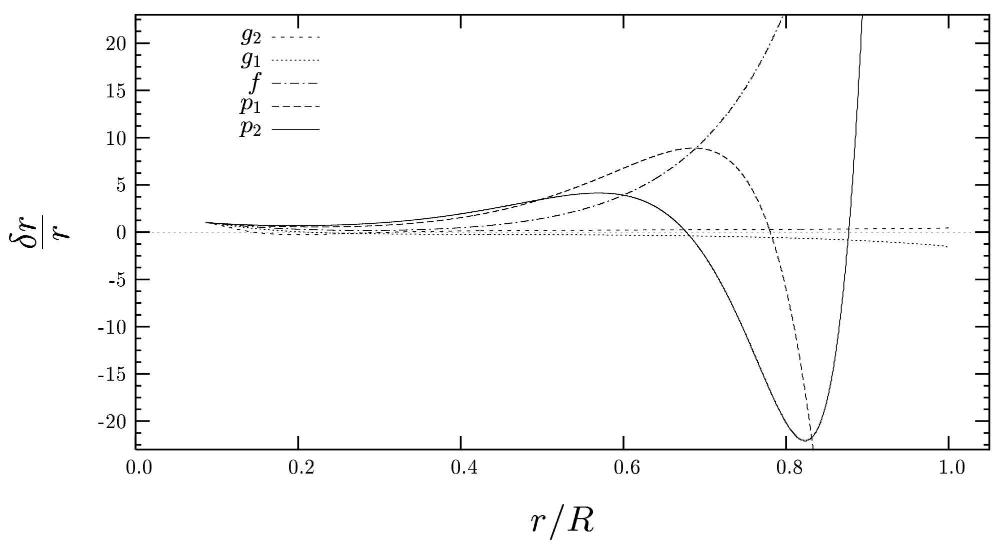

Figure 4.3: Eigenfunctions of the relative radial displacement ξ_r/r for the nonrotating 1.80M⊙ model (l=1 without Cowling approximation).

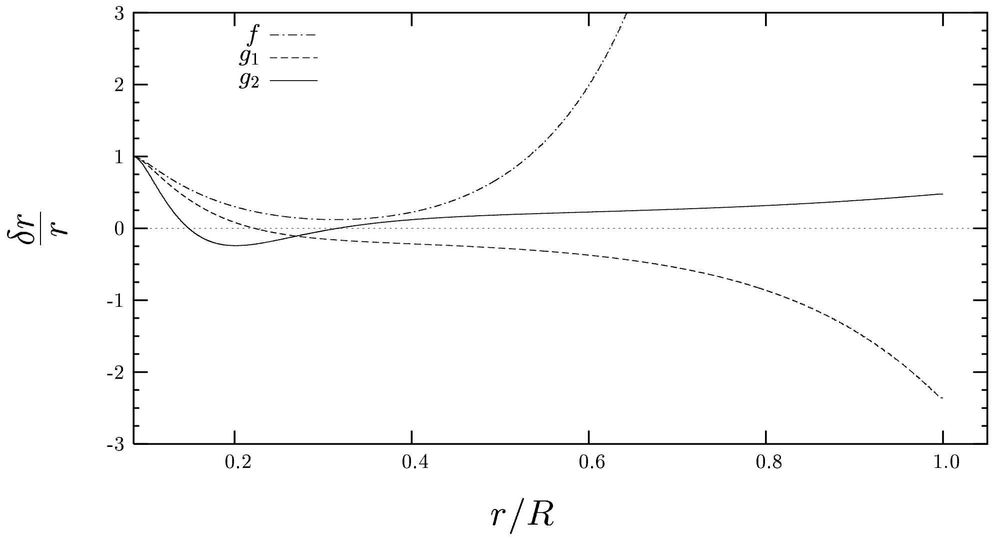

Figure 4.4: Eigenfunctions of the relative radial displacement ξ_r/r only showing f and g modes in figure 4.2 for the nonrotating 1.80M⊙ model (l=1 with Cowling approximation).

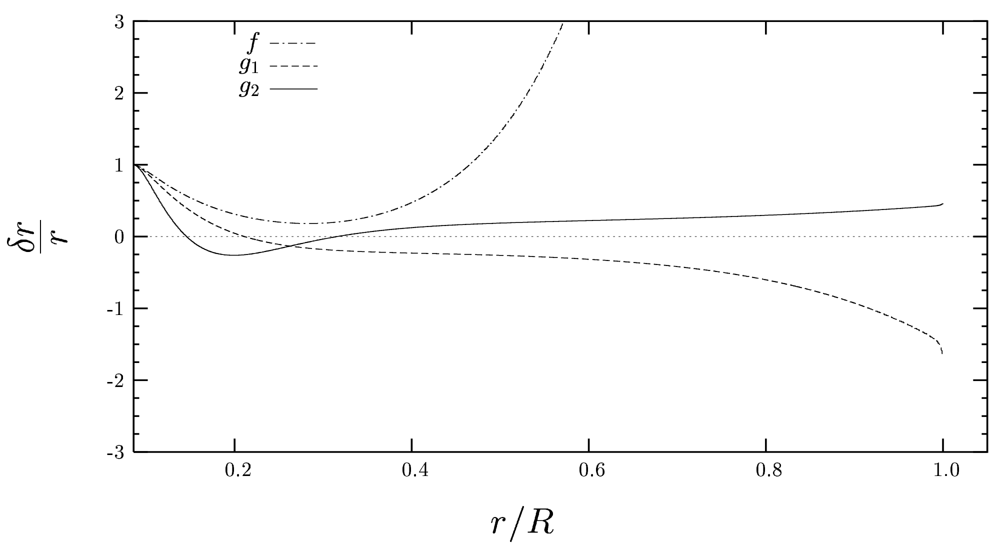

Figure 4.5: Eigenfunctions of the relative radial displacement ξ_r/r only showing f and g modes in figure 4.3 for the nonrotating 1.80M⊙ model (l=1 without Cowling approximation).

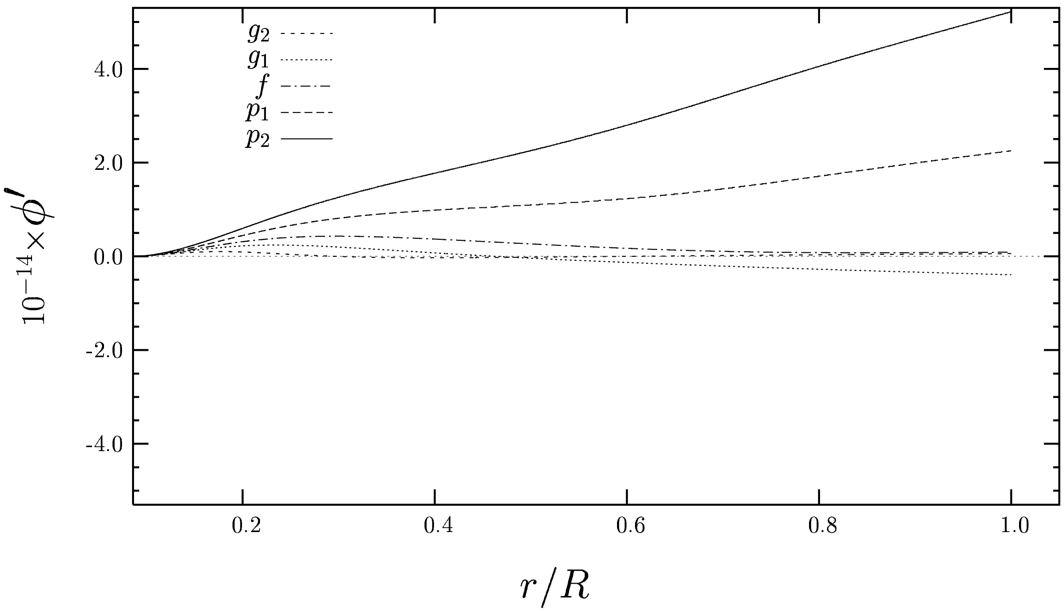

Figure 4.6: The behavior of the potential energy perturbation in units of erg for the nonrotating 1.80M$\odot$ model (l=1).

In order to disentengle the behavior of the radial displacement for the fundamental and gravity modes in figure 4.2 and 4.3, figures 4.4 and 4.5 are plotted again, respectively. Starting point of the figure 4.2 and 4.3 is 0.086.

In figure 4.2, f_{g_2}=87.7765 μHz, f_{g_1}=127.6145 μHz,the fundamental mode (f-mode) is equal to 154.294 μHz, the acoustic modes are f_{p_1}=196.9406 μHz and f_{p_2}=237.7214 μHz

In figure 4.3, f_{g_2}=86.0775 μHz, f_{g_1}=123.5632 μHz, f_f=153.5993 μHz, f_{p_1}=199.2699 μHz and f_{p_2}=248.3962 μHz.

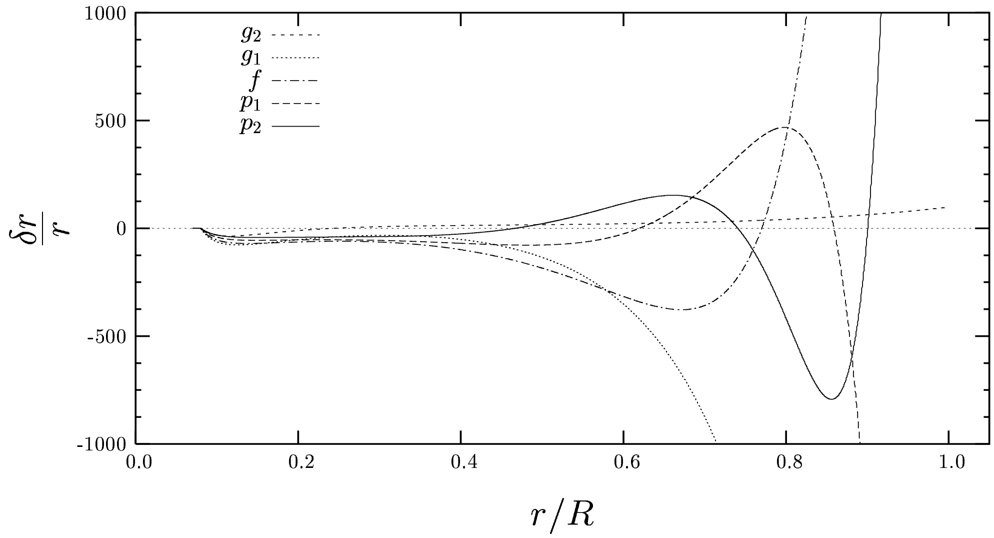

Figure 4.7: Eigenfunctions of the relative radial displacement ξ_r/r for the rotating 1.80M$\odot$ model (l=1 with Cowling approximation).

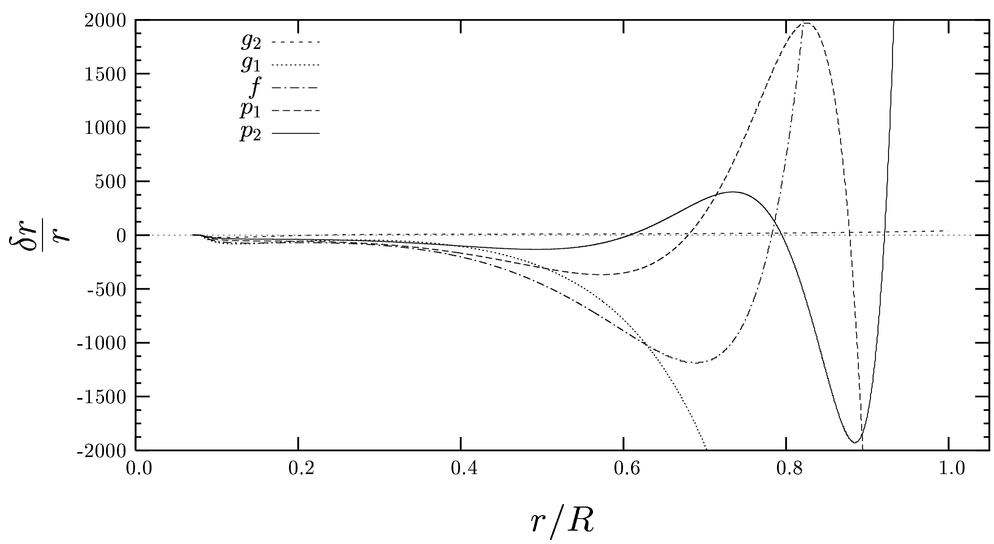

Figure 4.8: Eigenfunctions of the relative radial displacement ξ_r/r for the same model above for l=1 without Cowling approximation.

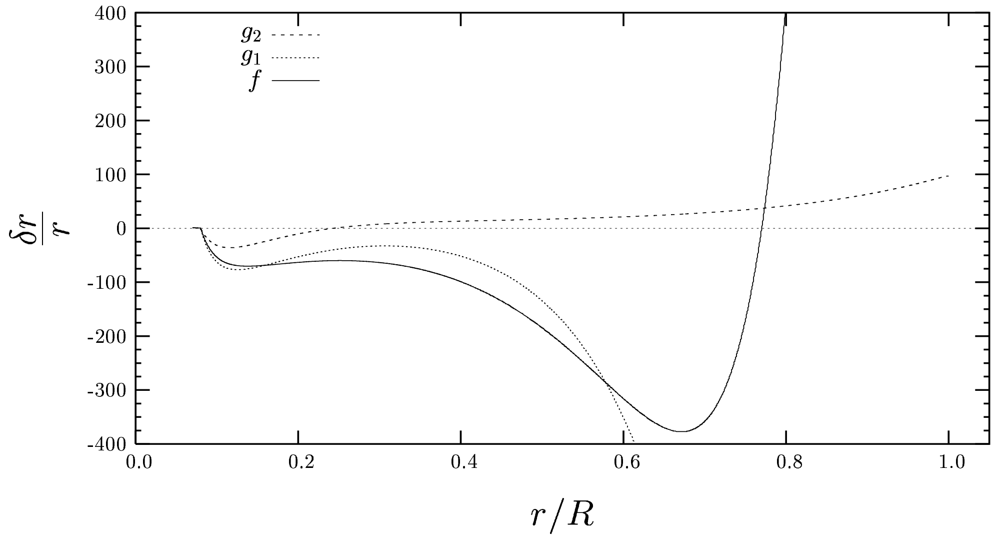

Figure 4.9: Eigenfunctions of the relative radial displacement ξ_r/r only showing f and g modes in figure 4.7 for the rotating 1.80M⊙ model (l=1 with Cowling approximation).

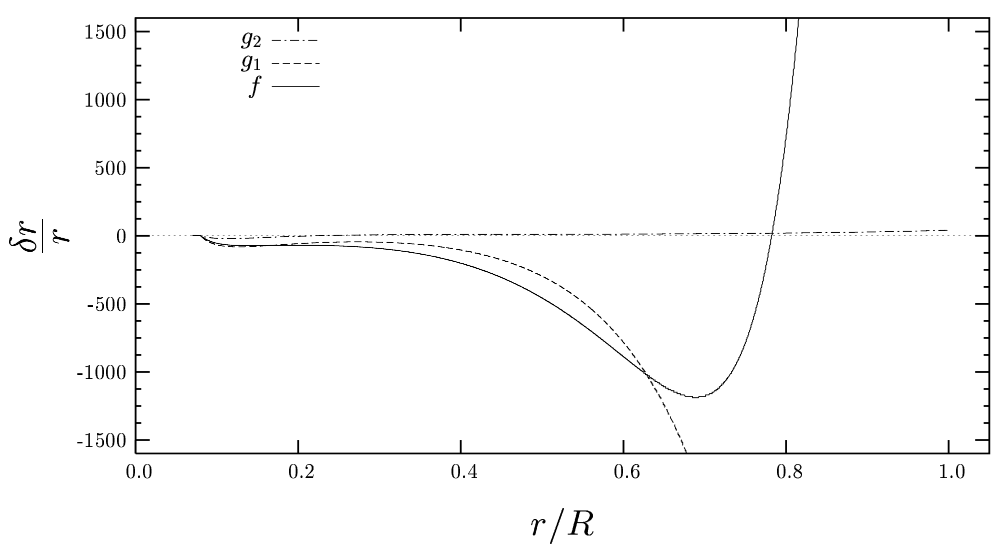

Figure 4.10: Eigenfunctions of the relative radial displacement ξ_r/r only showing f and g modes in figure 4.8 for the rotating 1.80M⊙ model (l=1 without Cowling approximation).

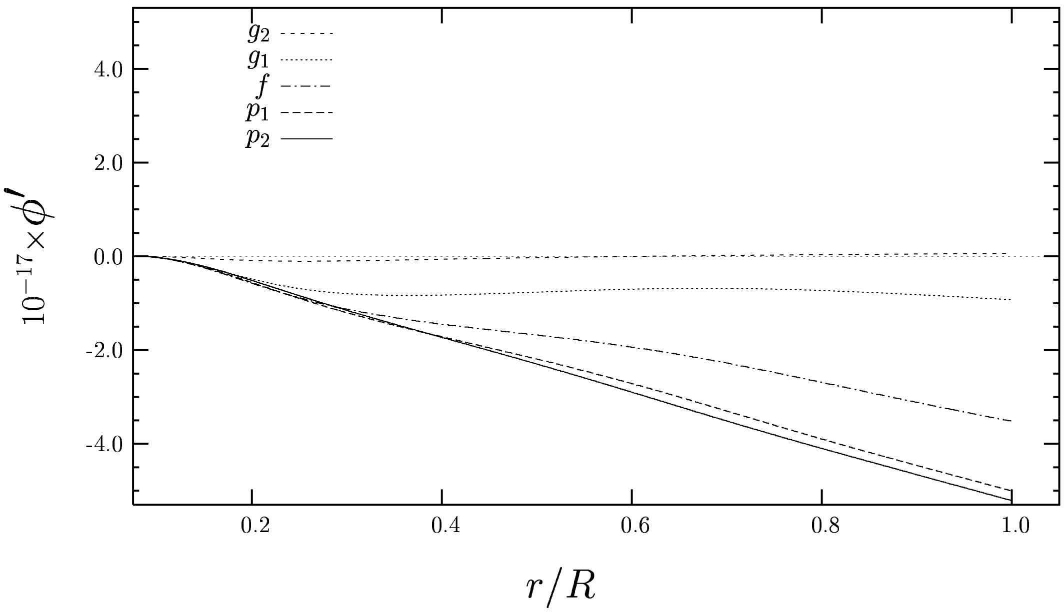

Figure 4.11: The behavior of the potential energy perturbation in units of erg for the rotating 1.80M⊙ model (l=1).

Figure 4.9 and 4.10 are replotted for the figures 4.7 and 4.8,respectively, to display the behavior of the fundamental (f-mode) and gravity modes (g-modes). The frequencies of all the modes shown in the figure 4.7 are found as follows: f_{g_2}=115.6273 μHz, f_{g_1}=149.9141 μHz, f_f=191.0577 μHz, f_{p_1}=228.6916 μHz, f_{p_2}=264.0007 μHz. In figure 4.8 which is plotted without Cowling approximation, the frequencies are f_{g_2}=109.1142 μHz, f_{g_1}=149.9020 μHz, f_f=194.5573 μHz, f_{p_1}=242.6869 μHz, f_{p_2}=291.7452 μHz, respectively.

4.4 Rotation

For a nonrotating star, the eigenfunction and the eigenfrequency σ of a normal mode are independent of the order m associating Legendre polynomial $P^m(Cos\theta)$ of degree l and order m. In the case of rotation, departures from spherical symmetry lift this degeneracy, causing a frequency splitting according to m. There is (2l+1)- fold degeneracy in m, which simply comes from the symmetry of the equilibrium structure around the rotation axis. For nonradial oscillations it is known (Cowling &Newing 1949, also Ledoux 1949) that groups of oscillations exist with very close periods, provided that the star is in slow rotation.

The equation of motion in the presence of a velocity field is given in the rotating frame of reference

$$\left(\frac{d\mathbf{v}}{dt}\right)_r = \left(\frac{d\mathbf{v}}{dt}\right)_0 + 2(\mathbf{\Omega} \times \mathbf{v}) + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}). \tag{4.26}$$

The second and third terms in the right-hand side represent the Coriolis and centrifugal force which can be neglected in the case of slow rotation, respectively. Then the equation becomes (3.16) becomes

$$\frac{\partial \mathbf{v}'}{\partial t} + 2(\mathbf{\Omega} \times \mathbf{v}') = -\frac{\mathbf{1}}{\rho}\nabla \mathbf{P}' + \frac{\rho'}{\rho^{\mathbf{2}}}\nabla \mathbf{P} - \nabla \phi' \tag{4.27}$$

If we suppose that the dependence with respect to the time t is the form of $e^{i\sigma t}$ given below,

$$\xi(\mathbf{r}, t) = \xi(\mathbf{r})e^{i\sigma t} \tag{4.28}$$

then the Eulerian equation of motion, with respect to rotating axis with angular velocity of the star $\mathbf{\Omega}$ can be written

$$-\sigma'^2\xi + \mathbf{2i}\sigma'(\mathbf{\Omega}\times\xi) = -\frac{\mathbf{1}}{\rho}\nabla\mathbf{P}' + \frac{\rho'}{\rho^{\mathbf{2}}}\nabla\mathbf{P} - \nabla\phi' \tag{4.29}$$

We must take into account the equation of continuity (3.14), and the adiabatic relation,

$$P' = -\frac{\Gamma_1 P}{\rho}\nabla.\xi - \xi.\nabla P = \frac{\Gamma_1 P}{\rho}\rho' - \xi\left(\nabla P - \frac{\Gamma_1 P}{\rho}\nabla\rho\right) \tag{4.30}$$

Now the eigenvalues and the eigenfunctions in the new set of equations (4.29, 4.30, 3.13, 3.17) are considered for the rotating frame of reference. These equations can be solved by the perturbation method if the effect of rotation is considered to be small. The eigenvalue σ' and the eigenfunction f are expanded in powers of the angular velocity Ω (Simon,1969)

If it is denoted, the solutions for $\Omega = 0$, it can be easily verified that the corresponding displacements ξ_0 are orthogonal.

$$\int_0^M \xi_{k,0}.\xi_{l,0}^* dm = 0, (k \neq l) \tag{4.31}$$

If we retain the terms in Ω, σ_0 is increased by a small amount σ_1 and ξ_0 by a small vector ξ_1 which can be represented as

$$\sigma' = \sigma_0 + \sigma_1 + \sigma_2 \tag{4.32}$$

$$f = f_0 + f_1 + f_2 \tag{4.33}$$

where quantities with subscripts 1 and 2 denote quantities of order Ω and Ω^2, respectively. The quantities with zero subscript represent for the case of no rotation. Furthermore, to solve the equation (4.29), one expands $\xi_{\mathbf{1}}$ and the other perturbed quantities into eigenfunctions of the nonrotating configuration (Zahn 1966, Borthamieu et al. 1978, Lee&Saio 1986) which can be represented by a series of them.

$$\xi_1 = \sum_i a_i \xi_{i,0} \tag{4.34}$$

$$f_1' = \sum_i a_i f_{i,0} \tag{4.35}$$

Introducing (4.32)-(4.33) and above expressions into equation (4.29) and keeping first-order terms only, we obtain

$$2\sigma_0\sigma_1\xi_0 = 2i\sigma_0(\mathbf{\Omega} \times \xi_0) - \sum_i a_i(\sigma_0^2 - \sigma_{i,0}^2)\xi_{i,0}. \tag{4.36}$$

If we multiply this equation ξ^* and integrate the sum over the whole mass,and use the orthogonality relation, we find

$$\sigma_1 = \frac{i\int_0^M (\mathbf{\Omega} \times \xi_0).\xi_0^* dm}{\int_0^M (\xi_0.\xi_0^*) dm} \tag{4.37}$$

Since

$$\xi = \left(\xi_r, \xi_h \frac{\partial}{\partial\theta}, \xi_h \frac{1}{Sin\theta}\frac{\partial}{\partial\psi}\right) Y_l^m(\theta,\psi) e^{-i\sigma t} \tag{4.38}$$

and

$$\mathbf{\Omega} = (\Omega Cos\theta, -\Omega Sin\theta, 0) \tag{4.39}$$

the scalar product $-i(\mathbf{\Omega} \times \xi).\xi^*$ is given by

$$-i(\mathbf{\Omega} \times \xi).\xi^* = -m\Omega(2\xi_{r,0}\xi_{h,0}|Y_l^m|^2 + \xi_{h,0}^2 \frac{Cos\theta}{Sin\theta}\frac{\partial}{\partial\theta}|Y_l^m|^2) \tag{4.40}$$

According to the properties of spherical harmonics $Y_l^m(\theta, \psi$ we have

$$-i \int_O^M (\Omega \times \xi_0).\xi_0^* dm = -m\Omega \int_0^R (2\xi_{r,0}\xi_{h,0} + \xi_{h,0}^2)\rho r^2 dr \tag{4.41}$$

and finally,

$$\sigma_1 = -m\sigma C \tag{4.42}$$

where C is a constant depending on the model and on the mode.

$$C = \frac{\int (2\xi_{r,0}\xi_{h,0} + \xi_{h,0}^2)\rho r^2 dr}{\int (\xi_{r,0}^2 + l(l+1)\xi_{h,0}^2)\rho r^2 dr} \tag{4.43}$$

According to (4.32) we have

$$\sigma' = \sigma_0 - m\Omega C \tag{4.44}$$

σ' is the frequency of oscillation in the rotating system. If it is assumed that the ψ dependence of the perturbed quantities is represented by exp(imψ), according to a fixed observer the frequency of oscillation σ will be

$$\sigma = \sigma' + m\Omega \tag{4.45}$$

Finally, we have

$$\sigma = \sigma_0 + m\Omega(1 - C) \tag{4.46}$$

To obtain numerical solution of the equations (3.13, 3.17, 4.29,4.30) we need to rewrite these equations in four first-order differential equations with dimensionless variables.

$$x\frac{dy_1}{dx} = (V_g - 3)y_1 + (\frac{l(l+1)}{c_1\omega^2} - V_g)y_2 + V_g y_3 +$$

$$+\frac{2m\Omega}{\sigma_0}(y_{0,1}+(\frac{1}{c_1\omega_0^2}-\frac{\sigma_1}{m\Omega}\frac{l(l+1)}{c_1\omega_0^2})y_{0,2}), \tag{4.47}$$

$$x\frac{dy_2}{dx}=(c_1\omega^2-A^*)y_1+(A^*-U+1)y_2-A^*y_3+$$

$$\frac{2m\Omega}{\sigma_0}(\frac{\sigma_1}{m\Omega}c_1\omega_0^2y_{0,1}-y_{0,2}) \tag{4.48}$$

$$x\frac{dy_3}{dx}=(1-U)y_3+y_4 \tag{4.49}$$

$$\frac{dy_4}{dx}=UA^*y_1+UV_gy_2+(l(l+1)-UV_g)y_3-Uy_4 \tag{4.50}$$

where $y_{0,i}$(i=1,...4) are the variables in the nonrotating star. The equations (4.47)-(4.50) are subject to the following boundary conditions:

$$c_1\omega_0^2y_1-ly_2+\frac{2m\Omega}{\sigma_0}c_1\omega_0^2(\frac{\sigma_1}{m\Omega}-\frac{1}{l})y_{0,1}=0 \tag{4.51}$$

and

$$ly_3-y_4=0 \tag{4.52}$$

at the center; and

$$(1-\frac{4+c_1\omega_0^2}{V})y_1+(\frac{l(l+1)}{c_1\omega_0^2V}-1)y_2+(1-\frac{l+1}{V})y_3$$

$$+\frac{2m\Omega}{\sigma_0V}((1-\frac{\sigma_1}{m\Omega}c_1\omega_0^2)y_{0,1}+(1-\frac{\sigma_1}{m\Omega}l(l+1)+c_1\omega_0^2)\frac{y_{0,2}}{c_1\omega_0^2})=0 \tag{4.53}$$

and

$$(l+1)y_3+y_4=0 \tag{4.54}$$

at the surface.

4.5 The Eigenvalues and Eigenfunctions

For a pulsating star, the period-mean density relation is expressed as

$$Q = \tau \sqrt{\frac{\rho}{\rho_\odot}} \tag{4.55}$$

where τ is the pulsation period in days and ρ and $\rho_\odot$ denote the mean stellar density and solar density, Q is called the pulsation constant. However, it is not actually a constant, it weakly depends on the structure of the star. In table 4.1 Q-values for different modes have been given for the rotating 1.80M⊙ model. They reasonably agree with observations.

Table 4.1: Some pulsational properties of a rotating models with l=1, $\phi' \neq 0$

Mode	m	f_c (d^{-1})	ω^2	Q-Value (days)
	-1	9.178	6.6530	0.04
g_2	0	9.427	7.0190	0.04
	1	9.677	7.3947	0.04
	-1	12.440	12.2205	0.03
g_1	0	12.952	13.2472	0.03
	1	13.464	14.3153	0,03
	-1	16.288	20.9523	0.03
f	0	16.810	22.3155	0.02
	1	17.331	23.7216	0.02
	-1	20.452	33.0324	0.02
p_1	0	20.968	34.7214	0.02
	1	21.484	36.4525	0.02
	-1	24.710	48.2200	0.02
p_2	0	25.207	50.1801	0.02
	1	25.704	52.1792	0.02

CHAPTER 5

CONCLUSION

The models given in Table 5.1 were investigated for radial and nonradial oscillations. In this table, the characteristics of nonrotating and uniformly rotating models with mass 1.80 $M_\odot$ are given. As seen from the Table 5.2, for non-rotating model, the gravitational potential perturbation has small effect on the radial oscillation frequencies. The mean difference between the calculated frequencies (f_c) with ($\phi' = 0$) and the frequencies with ($\phi' \neq 0$) is about 0.014 d^{-1}. The calculated radial oscillation frequencies for 1.80$M_\odot$ are given in Table 5.3 for a 1.80 $M_\odot$ star which rotates with 57.8 kms^{-1}. The difference between the calculated frequencies with ($\phi' = 0$) and with ($\phi' \neq 0$) is about 0.079.

For nonradial calculation we considered only the low degree ($l \leq 2$). Table 5.4 lists the selected modes of nonradial oscillation frequencies for spherical harmonic degree l=1 for model 1. For comparison, we selected only the frequencies between g_4 and p_4. As seen from the table, the difference between calcutated frequency (f_c) with Cowling approximation ($\phi' = 0$) and (f_c) without Cowling approximation ($\phi' \neq 0$) has smallest value at g_4 mode. The difference increases as the number

Table 5.1: The characteristic of nonrotatig and uniformly rotating models with masses 1.80M$_\odot$ compatible approximately with the characteristic of V1162 Ori.

Model No.	M ($M_\odot$)	L ($L_\odot$)	R ($R_\odot$)	T_e (K)	$\rho_c/<\rho>$ $\times 10^2$	Rot.Vel. (km/s)
1	1.80	13.35	2.167	7500	3.41	0.0
2	1.80	13.38	2.195	7450	3.49	57.8

Table 5.2: Radial Oscillation frequencies for the nonrotating 1.80$M_\odot$ model (l=0).

Model	V_{rot} kms^{-1}	Mode	N_p	N_g	$(f_c, \phi' = 0)$ d^{-1}	$(f_c, \phi' \neq 0)$ d^{-1}	$\lvert\triangle f\rvert$ d^{-1}
1	**0.0**	f	0	0	12.9106	12.8898	.0208
		p_1	1	0	16.6759	16.6605	.0154
		p_2	2	0	20.6399	20.6211	.0188
		p_3	3	0	24.5727	24.5560	.0167

of p-modes (N_p) increases. In Table 5.5, the calculated nonradial oscillation frequencies are shown for model 2. Tables 5.6 and 5.7 display the calculated nonradial oscillation frequencies with spherical harmonic degree l=2 for model 1 and 2, respectively.

We found that, the radial frequencies that we obtained with Cowling approximation agree well with the radial frequencies that were derived with gravitational potential perturbation. Gravitational potential perturbation is more effective in p-modes than g-modes for nonradial oscillation frequencies. Finally, the effect of the gravitational potential perturbation increases with the velocity and with spherical harmonic degree l.

Table 5.3: Radial Oscillation frequencies for the rotating 1.80$M_\odot$ model (l=0).

Model	V_{rot} kms^{-1}	Mode	N_p	N_g	$(f_c, \phi' = 0)$ d^{-1}	$(f_c, \phi' \neq 0)$ d^{-1}	$\lvert\triangle f\rvert$ d^{-1}
2	**57.8**	f	0	0	12.6467	12.7128	.0661
		p_1	1	0	16.3652	16.3960	.0308
		p_2	2	0	20.3578	20.2863	.0715
		p_3	3	0	24.3825	24.1574	.2251

Table 5.4: Nonradial Oscillation frequencies for the nonrotating 1.80$M_\odot$ model (l=1).

Model	V_{rot} kms^{-1}	Mode	N_p	N_g	$(f_c, \phi' = 0)$ d^{-1}	$(f_c, \phi' \neq 0)$ d^{-1}	$\lvert\triangle f\rvert$ d^{-1}
1	**0.0**	g_4	0	4	4.489	4.458	.031
		g_3	0	3	5.671	5.608	.063
		g_2	0	2	7.584	7.437	.147
		g_1	0	1	11.026	10.676	.350
		f	0	0	13.331	13.271	.060
		p_1	1	0	17.016	17.217	.201
		p_2	2	0	20.539	21.461	.922
		p_3	3	0	23.729	25.756	2.027
		p_4	4	0	27.477	30.050	2.573

Table 5.5: Nonradial Oscillation frequencies for the rotating 1.80$M_\odot$ model (l=1).

Model	V_{rot} kms^{-1}	Mode	N_p	N_g	$(f_c, \phi' = 0)$ d^{-1}	$(f_c, \phi' \neq 0)$ d^{-1}	$\lvert\triangle f\rvert$ d^{-1}
2	**57.8**	g_4	0	4	6.892	6.881	.011
		g_3	0	3	9.126	9.007	.119
		g_2	0	2	9.990	9.427	.563
		g_1	0	1	12.953	12.952	.001
		f	1	1	16.507	16.810	.303
		p_1	2	1	19.759	20.968	1.209
		p_2	3	1	22.810	25.207	2.397
		p_3	4	1	26.579	29.485	2.906
		p_4	5	1	30.698	33.765	3.067

Table 5.6: Nonradial Oscillation frequencies for the nonrotating 1.80$M_\odot$ model (l=2).

Model	V_{rot} kms^{-1}	Mode	N_p	N_g	$(f_c, \phi' = 0)$ d^{-1}	$(f_c, \phi' \neq 0)$ d^{-1}	$\|\triangle f\|$ d^{-1}
1	**0.0**	g_4	0	4	7.441	7.607	.166
		g_3	0	3	9.173	8.636	.537
		g_2	0	2	11.506	10.799	.707
		g_1	0	1	13.369	13.275	.094
		f	1	1	15.747	14.388	1.359
		p_1	1	0	18.078	19.082	1.004
		p_2	2	0	21.098	22.107	1.009
		p_3	3	0	24.731	26.292	1.561
		p_4	4	0	28.806	30.510	1.704

Table 5.7: Nonradial Oscillation frequencies for the rotating 1.80$M_\odot$ model (l=2).

Model	V_{rot} kms^{-1}	Mode	N_p	N_g	$(f_c), \phi' = 0)$ d^{-1}	$(f_c, \phi' \neq 0)$ d^{-1}	$\|\triangle f\|$ d^{-1}
2	**57.8**	g_4	0	4	10.011	9.108	.903
		g_3	0	3	11.359	11.950	.591
		g_2	0	2	12.979	12.680	.299
		g_1	1	2	15.215	15.006	.209
		f	1	1	17.523	17.611	.088
		p_1	2	1	20.576	21.583	1.007
		p_2	3	1	24.153	25.698	1.545
		p_3	4	1	28.131	29.841	1.710
		p_4	5	1	32.262	33.961	1.699

In this part, an attempt has been made towards obtaining the observed frequencies in the variable star V1162 Ori by using the model 2 in Table 5.1. Frequencies of photometric variability have been detected by various studies. These frequencies range from 12.17 to 38.12 c/d (Hintz et al. 1998, Arentoft and Sterken 2000). Arentoft and Sterken (2000) assigned the values T_e=7400 K and M_v=1.89 to V1162 Ori. They found the Q-value as 0.029 assuming a mass of $1.80M_\odot$ and a bolometric correction of -$0^m.1$. They assumed the dominant frequency 12.708 c/d as the fundamental.

In Tables 5.8 and 5.9, the calculated radial and nonradial oscillation frequencies and the observed ones associated with the model are listed. Under the Cowling approximation, the model 2 seems to represent model of V1162 Ori best. If we consider the perturbation in the potential, the observed radial fundamental frequency agrees well with the calculated radial fundamental frequency in model 2. However, for larger l values, the differences between the observed and theoretical values get larger than that of the model with Cowling approximation (ϕ'=0).

Karino (2003) found that the Cowling approximation for the f-mode oscillations is not a good approximation in rapidly and differentially rotating stars, although rapid rotation makes this approximation better for rigidly rotating stars. This result suggests that we must be carefull when we apply the Cowling approximation to differentially rotating stars. This phenomenon can be observed in our models also. The frequency difference between the calculated radial frequency and the observed radial frequency is about 0.061 d^{-1} for the models with Cowling approximation, while the same difference is about 0.004 d^{-1} without the

Cowling approximation as displayed in Table 5.3.

For the nonradial oscillations, the model with Cowling approximation shows better fit to the almost all observed nonradial frequencies of V1162 Ori than the model with gravitational potential perturbation.

The effect of the gravitational potential perturbation on frequencies mainly depends on some physical parameters of the star model. These are the amplitude of the variations because of the density distribution of the stars and the effectiveness of He^+ ionization zones just under the stellar surface. As seen from the Figures 4.6 and 4.11, in the region near the center, the amplitudes of the variation of the g-modes are small due to the high density. So for g-modes, the effect of the gravitational potential perturbation on calculated frequencies is less. For p-modes, the gravitational potential perturbation comes to play a relatively important role because the amplitude of p-modes oscillations varies significantly in low density and under the effect of ionization and opacity in the outer region of the star. Therefore, the Cowling approximation which neglects the perturbation of the gravitational potential in basic equations yields less accurate solutions for p-modes.

Table 5.8: Oscillation frequencies for 1.80M$_\odot$, V_{rot}=57.8 kms^{-1}, $\phi' = 0$

l	m	N_p	N_g	$f_c(d^{-1})$	$f_{obs}(d^{-1})$	$\|f_c - f_{obs}\|$
0	0	0	0	12.647	12.708	0.061
1	0	0	1	12.953	12.941	0.012
1	-1	1	1	16.009	15.990	0.019
2	0	0	2	12.979	12.941	0.038
2	2	1	3	16.002	15.990	0.012
2	2	2	1	21.578	21.718	0.140

Table 5.9: Oscillation frequencies for 1.80M$_\odot$, V_{rot}=57.8 kms^{-1}, $\phi' \neq 0$

l	m	N_p	N_g	$f_c(d^{-1})$	$f_{obs}(d^{-1})$	$\|f_c - f_{obs}\|$
0	0	0	0	12.712	12.708	0.004
1	0	0	1	12.952	12.941	0.011
1	-1	1	1	16.288	15.990	0.298
1	1	2	1	21.485	21.718	0.233
2	1	0	3	12.759	12.941	0.182
2	2	1	2	15.842	15.990	0.148
2	0	2	1	21.583	21.718	0.132

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