THE ENHANCEMENT OF THE CELL-BASED GIS ANALYSES WITH FUZZY PROCESSING CAPABILITIES

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

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IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE
IN
THE DEPARTMENT OF GEODETIC AND GEOGRAPHIC
INFORMATION TECHNOLOGIES

AUGUST 2003

Approval of the Graduate School of Natural And Applied Sciences.

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ABSTRACT

THE ENHANCEMENT OF THE CELL-BASED GIS ANALYSES WITH FUZZY PROCESSING CAPABILITIES

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August 2003, 101 pages

In order to store and process natural phenomena in Geographic Information Systems (GIS) it is necessary to model the real world to form computational representation. Since classical set theory is used in conventional GIS software systems to model uncertain real world, the natural variability in the environmental phenomena can not be modeled appropriately. Because, pervasive imprecision of the real world is unavoidably reduced to artificially precise spatial entities when the conventional crisp logic is used for modeling.

An alternative approach is the fuzzy set theory, which provides a formal framework to represent and reason with uncertain information. In addition, linguistic variable concept in a fuzzy logic system is useful for communicating concepts and knowledge with human beings.

In this thesis, a system to enhance commercial GIS software, namely ArcGIS, with fuzzy set theory is designed and implemented. The proposed system allows users to (a) incorporate human knowledge and experience in the form of linguistically defined variables into GIS-based spatial analyses, (b) handle imprecision in the decision-making processes, and (c) approximate complex ill-defined problems in decision-making processes and classification.

The operation of the proposed system is presented through case studies, which demonstrate its application for classification and decision-making processes. This thesis shows how fuzzy logic approach may contribute to a better representation and reasoning with imprecise concepts, which are inherent characteristics of geographic data stored and processed in GIS.

Keywords: Fuzzy set theory, Geographic Information Systems, Uncertainty, Decision-making, Classification.

ÖZ

HÜCRE TABANLI CBS ANALİZLERİNİN BULANIK İŞLEME KAPASİTESİ İLE GELİŞTİRİLMESİ

Yanar, Tahsin Alp

Yüksek Lisans, Jeodezi ve Coğrafi Bilgi Teknolojileri Bölümü

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Ağustos 2003, 101 sayfa

Doğaya ait verilerin Coğrafi Bilgi Sistemlerinde (CBS) saklanabilmesi ve işlenebilmesi için gerçek dünyanın hesaplanabilir biçime dönüştürülmesi gerekir. Geleneksel CBS yazılımlarında belirsizlik içeren gerçek dünyanın modellenmesi için klasik küme teorisi kullanıldığından, çevredeki doğal değişkenlik uygun olarak modellenemez. Çünkü, modelleme için klasik mantık kullanıldığında gerçek dünyayı çevreleyen kesin olmayan verilerin yapay bir kesinlik içeren mekansal varlıklara dönüştürülmesi kaçınılmazdır.

Alternatif yaklaşım olarak, belirsizlik içeren bilgiyi ifade etmek ve belirsizlik içeren bilgiyi kullanarak sonuç çıkarmak için biçimsel bir çatı sunan bulanık küme teorisi kullanılabilir. Buna ek olarak, bulanık mantık sisteminde yeralan dilsel değişkenler insan düşünce ve bilgisinin aktarılmasında faydalıdır.

Bu tezde, ticari bir CBS yazılımı olan ArcGIS'i, bulanık küme teorisi ile geliştiren sistem tasarlandı ve gerçekleştirildi. Geliştirilen sistem kullanıcılara (a) insan bilgisini ve deneyimini, doğal dilde tanımlanan değişkenler aracılığı ile CBS tabanlı konumsal analizlere aktarabilme, (b) karar verme sürecinde kesin olmayan verileri işleyebilme ve (c) karar verme süreçlerinde ve sınıflandırmada yer alan kompleks, tam olarak tanımlanamayan problemleri yaklaşık olarak tanımlayabilme imkanlarını sağlar.

Önerilen sistemin işleyiş şeklinin gösterilmesi amacı ile sınıflandırma ve karar verme süreçlerinin geliştirilen sistemde uygulanması örneklendirilerek incelenmiştir. CBS'lerinde saklanan ve işlenen coğrafi verilerin kesinlik taşımaması coğrafi verilerin doğal bir karakteristiğidir. Bu tezde, kesinlik taşımayan coğrafi verilerin bulanık küme yöntemleri ile nasıl daha iyi bir şekilde işlenebileceği ve bu verileri kullanarak nasıl daha iyi sonuçlar alınabileceği gösterilmiştir.

Anahtar Kelimeler: Bulanık küme teorisi, Coğrafi Bilgi Sistemleri, Belirsizlik, Karar verme, Sınıflandırma.

To my family

ACKNOWLEDGMENTS

I would like to thank to Sevda Zuhal Akyürek for her supervision, continuous support and guidance throughout the development of this thesis. I am grateful for her friendship and encouragement for successful completion of my study. I would also like to thank to Murat İkinci for his valuable suggestions and comments.

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CHAPTER 1

INTRODUCTION

1.1 The Scope and The Objectives

Geographic Information Systems (GIS) are computer-based systems that store and process (e.g. manipulation, analysis, modeling, display, etc.) spatially referenced data at different points in time (Aronoff, 1989). Geographic data, stored and processed in a GIS, form a conceptual model of the real world (Aronoff, 1989). The abstraction of the real world to construct the conceptual model unavoidably results in differentiation between objects of the real world and their representation in GIS (i.e., computer) (Wang and Hall, 1996; Burrough, 1986). Because, classical set theory used in a conventional GIS is inadequate to express the natural variability in the environmental phenomena (Wang and Hall, 1996). Essentially, the contention is that the conventional precise quantitative techniques are intrinsically unsuited for dealing with the real world or for that matter, any system whose complexity or uncertainty degree is comparable to that of the real world. As it is stated by Heuvelink and Burrough (2002) there will often be meaningful discrepancies between reality and its representation since the reality is forced into rigid data storage formats.

The translation of geographic data from real world to conceptual (or perceptual) space is based on human cognition (Benedikt et al., 2002). Thus, an alternative approach to crisp logic is based on the key elements in human thinking (Zadeh, 1973). The most important aspects of human thinking is the ability to summarize information (i.e., approximation) (Burrough, 1986; Zadeh, 1973; Benedikt et al., 2002). The human brain may not require high degree of precision to perform most of the basic tasks. The human brain takes advantage of imprecision and encodes information into labels of fuzzy sets. Therefore, the major difference between human intelligence and machine intelligence used in most GIS software is the ability to summarize information and to manipulate fuzzy sets (Zadeh, 1973).

Since humans have the ability to summarize information (i.e., information about complex and imprecise phenomena), each word in a natural language may be viewed as a summarized description (Zadeh, 1973). Therefore, natural language has a way of deriving information and making decisions about complex and imprecise phenomena (Benedikt et al., 2002). Since language is also used as a tool for communicating knowledge in a decision-making process, the main objective of this thesis is the incorporation of human knowledge and experience in the form of linguistically defined variables into raster based GIS through the use of fuzzy set theory. To accomplish this objective, a commercial GIS software namely ArcGIS, which is a major GIS desktop system, is enhanced with fuzzy set theory. Another objective is to show how GIS technology may contribute to a better understanding of formal definitions of geographic categories.

The developed system can be viewed as a scheme for capturing experts' knowledge on a specific problem. Through the use of linguistic variables, experts' experiences in the problem domain, even though they naturally involve imprecision, are converted to fuzzy rules. Therefore, the proposed system should allow users to handle imprecision in the decision-making process by knowing only the fuzzy logic background. Easy to use graphical user interfaces (GUIs) should enable users to define fuzzy rules without necessarily knowing all the underlying concepts of the fuzzy set theory.

Fuzzy set theory has been used considerably not only in the researches but also in the industrial applications to solve a wide range of problems. Applications of fuzzy set theory in cruise control (Isuzu, Nissan, Mitsubishi), Sendai subway operation (Hitachi), automatic transmission (Honda, Nissan, Subaru), elevator scheduling (Fujitech, Mitsubishi, Hitachi), microwave oven (Hitachi, Sanyo, Sharp, Toshiba), refrigerator (Sharp), video image stabilizer (Matshushita/Panasonic), video camera autofocus (Sanyo, Canon) and washing machine (Hitachi, Matshushita, Samsung, Sanyo, Sharp, Goldstar, Daewoo) have emphasized a way for an effective use of fuzzy set theory in the complex ill-defined processes (for many additional applications see (Maiers and Sherif, 1985; Kosko, 1994; Lee, 1990a)). The proposed system should allow users to approximate complex ill-defined problems in decision-making processes and classification to obtain better results.

Fuzzy set theory has been used in many GIS researches to solve specific decision-making problems, classification processes, representing and handling uncertainty in data. The developed system should not be dedicated to a specific decision-making problem or it should not be limited with a specific classification process.

A fuzzy logic system (FLS) is a nonlinear system that maps input variables into a crisp scalar output, and is rich with the number of possible designs (Mendel, 1995). The richness of FLS was not taken into consideration while using (or integrating) fuzzy logic theory to enhance GIS operations. However, the proposed system should enable users to decide on the type of membership functions (triangular, trapezoidal, Gaussian, bell-shaped, sigmoidal, S, Π_1 type, Π_2 type), parameters of membership functions, inference methods (minimum, product), aggregation methods (maximum, sum, probabilistic-or), conjunction operators (drastic product, bounded product, Einstein product, algebraic product, Hamacher product, minimum), disjunction operators (drastic sum, bounded sum, Einstein sum, algebraic sum, Hamacher sum, maximum) and defuzzifier (center of area, bisector of area, mean of maximum, largest of maximum, smallest of maximum). To demonstrate the richness of the system, choosing among the parenthetical possibilities will lead to $2^{30} = 1073741824$ different choices. This richness of the system can be viewed as another contribution of the work within the scope of this thesis.

1.2 Organization of the Thesis

The organization of the thesis is as follows: a brief overview of fuzzy set methodologies in comparison with crisp logic and GIS interests in fuzzy set theory are explained in Chapter 2.

Chapter 3 is reserved for the design of Fuzzy Inference System. The Fuzzy Inference System has been developed on a commercial GIS software namely ArcGIS and is composed of two components, Fuzzy Inference Engine and Fuzzy Inference System Module. After giving general architecture design and workflow of the Fuzzy Inference System, design details of Fuzzy Inference Engine and Fuzzy Inference System Module are presented.

Chapter 4 begins with a brief discussion on commercial GIS software ArcGIS, which the Fuzzy Inference System has been developed on. The implementation details of Fuzzy Inference Engine and Fuzzy Inference System Module are described. In this chapter public interfaces that can be used by the client or user of functionality are presented. Chapter concludes with the description of the development environment and elements affecting performance.

In Chapter 5, causes of the employment of crisp logic in GIS and effects of employing fuzzy set theory in GIS are presented through case studies. The operation of the system is tested and results obtained from the system are examined in order to clarify differences between sharply defined rules and rules which involve imprecision.

The identification of fuzzy models, estimating the parameters of membership functions, performance of the proposed system and discussion on fuzzy rule based model types are presented in Chapter 6.

The thesis concludes with Chapter 7 in which case studies are discussed and the eventual improvements for future work are stated.

CHAPTER 2

FUZZY LOGIC

Fuzzy set theory provides a formal system for representing and reasoning with uncertain information. Linguistic variable concept in a fuzzy logic system enables to handle numerical data and linguistic knowledge simultaneously (Mendel, 1995). Even L. A. Zadeh (1965), formulated the initial statement of fuzzy set theory (Maiers and Sherif, 1985), at first never expected fuzzy sets to be used in consumer products or in geographic information systems (Perry and Zadeh, 1995).

This chapter presents the fundamental concepts in fuzzy set theory. Since fuzzy set theory generalizes classical set theory, the theoretical information is given in both classical and fuzzy sense. Chapter concludes with the discussion on applications of fuzzy set theory in Geographic Information Systems.

2.1 Classical Sets

A collection of objects of any kind form a classical set and the objects themselves are called elements or members of the set. The elements of a classical set A in a universe of discourse U can be defined by specifying a condition. One other way to identify the elements of A is by introducing characteristic function for A, denoted as $\mu_A(x)$, such that $\mu_A(x) = 1$ if $x \in A$ and $\mu_A(x) = 0$ if $x \notin A$.

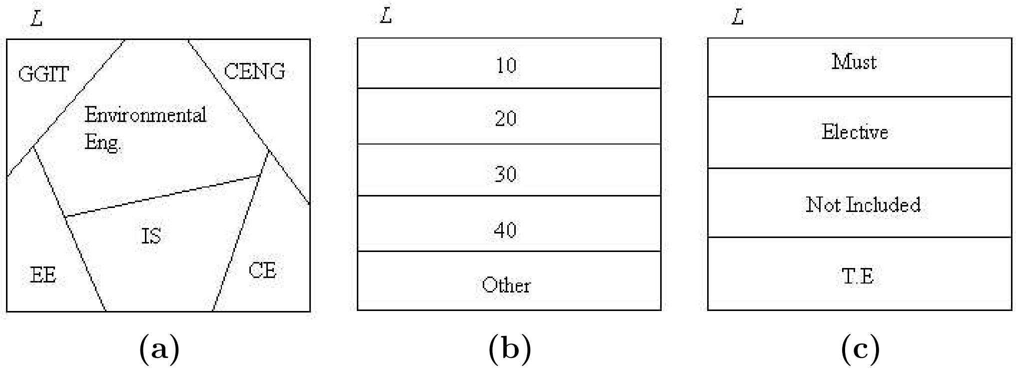


Figure 2.1: Partition of the lectures in METU into subsets by (a) departments, (b) student capacity, and (c) type.

Example 2.1 Consider the set of cities, C in Turkey whose population is greater than one million. C can be defined as $C = \{u|$ totalpopulation$(u) > 1$ Million AND country $(u) =$ Turkey$\}$. If population of a city exceeds one million, then it is completely in the set of C, hence its grade of membership is unity.

Example 2.2 Consider the set of lectures, L in METU. Many different types of subsets can be established for L. Three of them are shown in Figure 2.1. It is seen that if a course is held in GGIT, its membership function value for the subset of GGIT is unity, whereas its membership function value for the subset of CENG is zero.

2.2 Fuzzy Sets

A fuzzy set is a generalization to classical set to allow objects to take membership values between zero and unity in vague concepts (Zadeh, 1965). A fuzzy set F defined on a universe of discourse U is characterized by a membership function $\mu_F(x)$ that maps elements of universe of discourse U to their corresponding membership values which is a real number in the interval $[0, 1]$.

Example 2.3 A course can be viewed as "department specific" or not from different perspectives. One perspective is that, a course is considered to belong to a department where it is delivered. This is a very crisp definition. However,

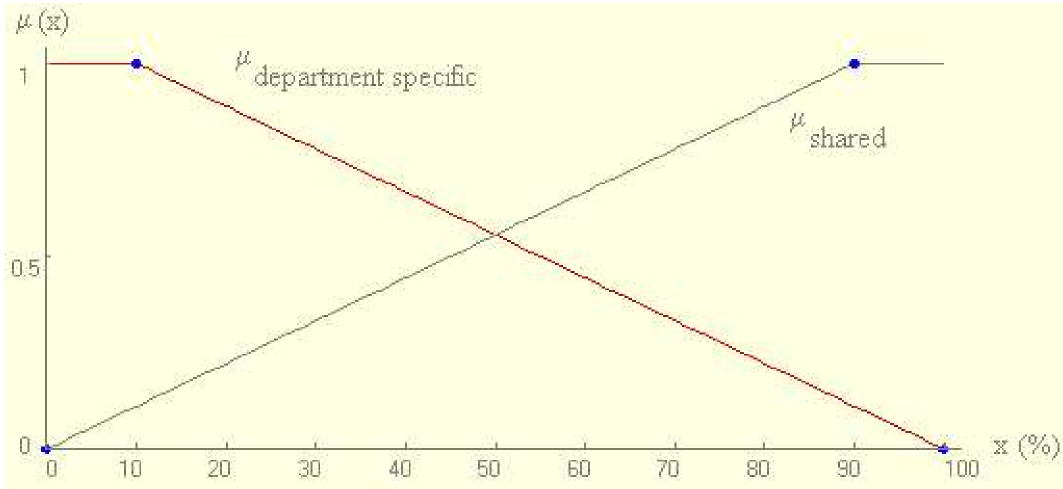


Figure 2.2: Membership functions for courses.

many students from different departments can take this course. This makes the course not as departmental as it once was. Because many of the students taking this course what is considered to be students of that department are actually not. Consequently, membership functions for courses can be defined like depicted in Figure 2.2.

It can be observed that a course exists in both subsets simultaneously - department specific and shared - but to different grades of membership. Note that it is not clear a course is department specific or shared when 50% of the students are from different departments (see (Kosko, 1994) for discussions on maximum fuzziness and paradoxes). From Example 2.3 it is obvious that an element can belong to more than one fuzzy set to different degrees. This is not allowed in classical set theory. A fuzzy set F, in a universe of discourse U can be written as (Zadeh, 1973)

$$F = \int_U \frac{\mu_F(x)}{x} \tag{2.1}$$

where integral sign stands for the union operation (details of union operation on fuzzy sets will be discussed in Section 2.6). The term $\mu_F(x)/x$ describes the degree of similarity of x in F. If U is discrete, then (2.1) may be replaced by the summation

$$F = \sum_{U} \frac{\mu_F(x)}{x} \tag{2.2}$$

in which summation sign denotes union operation.

2.3 Linguistic Variables

Developing approximate solutions, to model a system whose complexity is comparable to that of humanistic systems, i.e., human-centered, in an effective way, i.e., trade off between precision and cost, relies on the use of linguistic variables (Zadeh, 1973; Yen, 1999).

Precise quantitative techniques to model humanistic systems are not relevant to the real world because as Zadeh (1973) states, "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics". An alternative approach is to summarize information as humans. Humans do not require high degree of precision while performing basic tasks. An approximate collection of data is sufficient for human brain. The ability to summarize information is essential to characterize the complex systems. In this context, linguistic variables provide the ability to summarize information.

As pointed in the previous section fuzzy sets are characterized by membership functions. In addition to membership functions, fuzzy sets are associated linguistically meaningful terms. This association makes it easier for human experts to express their expertise using linguistic terms (Yen, 1999).

A linguistic variable is usually composed of atomic and composite labels (i.e., linguistic terms, e.g., words, phrases, and sentences) (Zadeh, 1973).

Example 2.4 Let $age(u)$ be a linguistic variable. Then its values child, young, middle-aged, old, etc., may be interpreted as set of terms linguistic variable u decomposed into: $T(age) = \{child, young, middle - aged, old\}$ where each

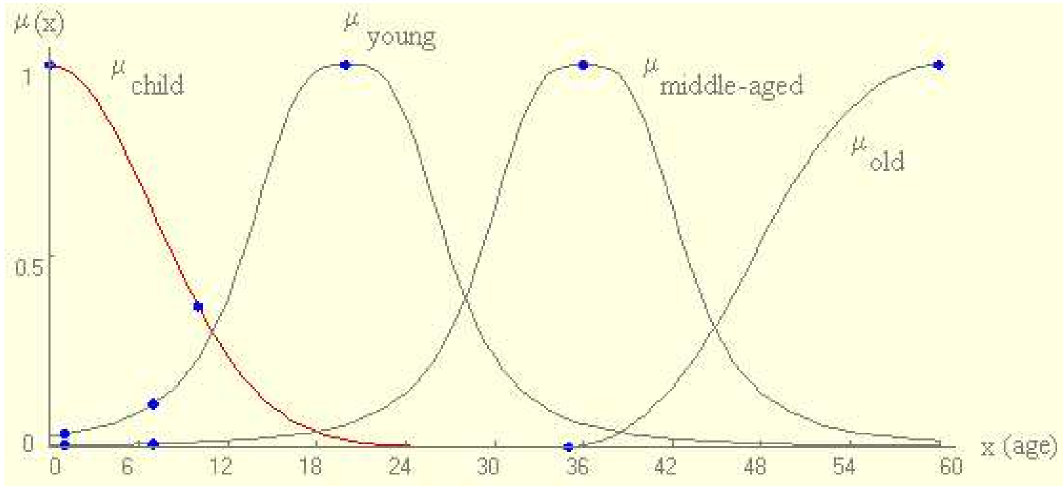


Figure 2.3: Membership functions for $T(age) = \{child,\ young,\ middle-aged,\ old\}$.

term is characterized by a fuzzy subset in the universe of discourse, $U = [0, 60]$. Membership functions of these terms are depicted in Figure 2.3.

2.4 Membership Functions

A membership function $\mu_F(x)$ maps each point in the input space to a degree of membership between zero and unity. Formally, if U is the universe of discourse and its elements are denoted by x, then a fuzzy set F in U is defined as a set of ordered pairs

$$F = \{x, \mu_F(x) | x \in X\} \tag{2.3}$$

where $\mu_F(x)$ is called the membership function of x in F. Most commonly used membership functions are in the sequel:

Triangular membership function: Triangular membership function depends on three scalar parameters a, b, and c as given by

$$triangle(x : a, b, c) = \begin{cases} 0 & x < a \\ (x-a)/(b-a) & a \leq x \leq b \\ (c-x)/(c-b) & b \leq x \leq c \\ 0 & x > c \end{cases} \tag{2.4}$$

or in more compact form, by

$$triangle(x : a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right) \tag{2.5}$$

The parameters a and c locate the "feet" of the triangle and the parameter b locates the peak. Figure 2.4(a) illustrates an example of a triangular membership function.

Trapezoidal membership function: A trapezoidal membership function depends on four scalar parameters a, b, c, and d as given by

$$trapezoidal(x : a, b, c, d) = \begin{cases} 0 & x < a \\ (x-a)/(b-a) & a \le x < b \\ 1 & b \le x < c \\ (d-x)/(d-c) & c \le x < d \\ 0 & x \ge d \end{cases} \tag{2.6}$$

or in more compact form, by

$$trapezoidal(x : a, b, c, d) = \max\left(\min\left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}\right), 0\right) \tag{2.7}$$

The parameters a and d locate the "feet" of the trapezoid and the parameters b and c locate the "shoulders". Triangular and trapezoidal membership functions have the advantage of simplicity, hence have been used extensively. Figure 2.4(b) illustrates an example of a trapezoidal membership function.

Gaussian membership function: Gaussian membership function is built on the Gaussian distribution curve and depends on two parameters m, σ

$$gaussian(x : m, \sigma) = e^{\frac{-(x-m)^2}{\sigma^2}} \tag{2.8}$$

where m and σ defines the center and width of the function. Figure 2.4(c) illustrates an example of a Gaussian membership function.

Bell-shaped membership function: A bell-shaped membership function depends on three parameters a, b, and c as given by

$$bell(x : a, b, c) = \frac{1}{1 + |\frac{x-c}{a}|^{2b}} \tag{2.9}$$

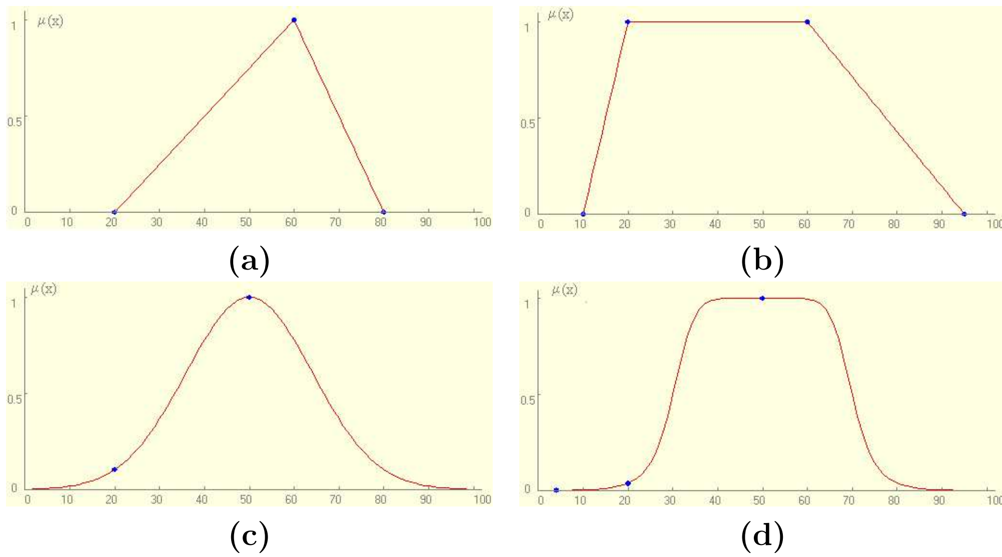


Figure 2.4: Examples of membership functions (a) $Triangle(x : 20, 60, 80)$, (b) $Trapezoidal(x : 10, 20, 60, 95)$, (c) $Gaussian(x : 50, 20)$, (d) $Bell(x : 20, 4, 50)$.

Gaussian and bell-shaped membership functions have the advantage of being smooth and non-zero at all points. Although the Gaussian membership functions and bell-shaped membership functions achieve smoothness, asymmetric membership functions, which are important in certain applications, are not specified. Figure 2.4(d) illustrates an example of a bell-shaped membership function.

Sigmoidal membership function: Sigmoidal membership function is specified by two parameters a, and c as given by

$$sigm(x : a, c) = \frac{1}{1 + e^{-a(x-c)}} \tag{2.10}$$

The sigmoidal membership function is inherently open to the right or to the left depending on the sign of the parameter a, hence is appropriate for representing concepts such as "very large" or "very negative". An example of a sigmoidal membership function is depicted in Figure 2.5(a).

S membership function: S membership function is specified by two parameters a, and b. The function is as follows:

$$s(x:a,b) = \begin{cases} 0 & x < a \\ 2\left(\frac{x-a}{b-a}\right)^2 & a \leq x \leq \frac{a+b}{2} \\ 1-2\left(\frac{x-b}{b-a}\right)^2 & \frac{a+b}{2} \leq x < b \\ 1 & x \geq b \end{cases} \tag{2.11}$$

An example of S membership function is depicted in Figure 2.5(b).

Π_1 type membership function: The first Π membership function is specified by two parameters a, and b. The function is as follows:

$$\Pi_1(x:a,b) = \frac{1}{1+\left(\frac{x-a}{b}\right)^2} \tag{2.12}$$

The shape of the function is depicted in Figure 2.5(c).

Π_2 type membership function: The second Π membership function is specified by four parameters lw, lp, rp, and rw as given by

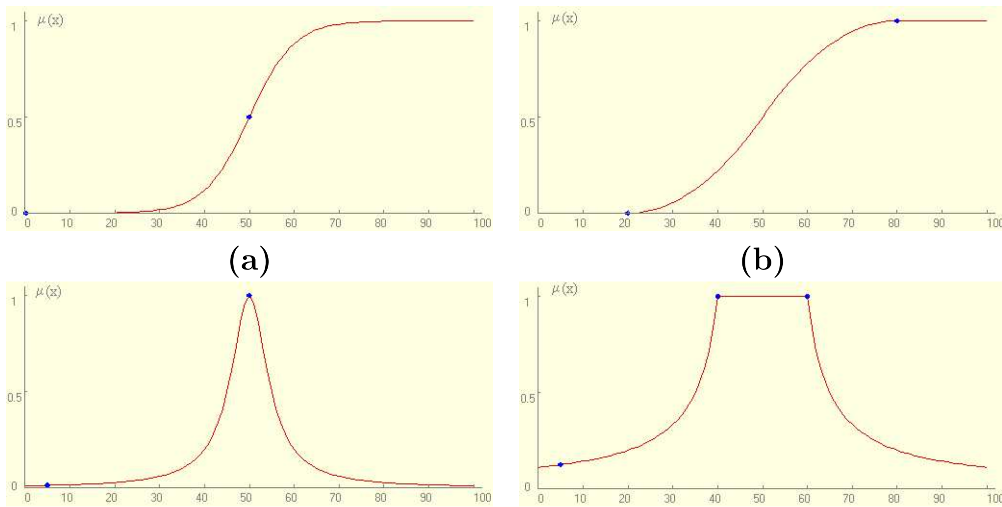
$$\Pi_2(x:lw,lp,rp,rw) = \begin{cases} \frac{lw}{lp+lw-x} & x < lp \\ 1 & lp \leq x \leq rp \\ \frac{rw}{x-rp+rw} & x > rp \end{cases} \tag{2.13}$$

The shape of the function is depicted in Figure 2.5(d).

2.5 Hedges

Meaning of a linguistic term, or more generally, of a fuzzy set can be modified through the use of linguistic hedge (Yen, 1999). For example, if *tall* is a fuzzy set, then the meaning of the fuzzy set *tall* can be modified to *very tall*, *more-or less tall*, *not tall*, and *not very tall* etc. by applying hedges very, more-or less, and not. It is noted that combinations of hedges can be applied to modify the meaning of a fuzzy set. A hedge may be regarded as an operator that acts upon a membership function of a fuzzy set.

If F is a fuzzy subset defined over the universe of discourse U and its elements are denoted by x then some of the operations on F are listed below (Yen, 1999; Mendel, 1995):



(c) (d)

Figure 2.5: Examples of membership functions (a) $Sigmoidal(x : 0.2, 50)$, (b) $S(x : 20, 80)$, (c) $\Pi_1(x : 50, 5)$, (d) $\Pi_2(x : 5, 40, 60, 5)$.

Concentration: The operation of concentration is defied by

$$\mu_{Con(F)}(x) \triangleq [\mu_F(x)]^2 \tag{2.14}$$

Concentration operation narrows the membership function as it is seen from the Figure 2.6.

Dilation: The operation of dilation is defied by

$$\mu_{Dil(F)}(x) \triangleq [\mu_F(x)]^{1/2} \tag{2.15}$$

The effect of dilation operation is the opposite of the concentration operation. Figure 2.6 depicts the effect of dilation operation.

Artificial Hedges: The artificial hedges *plus* and *minus* are defined by

$$\mu_{Plus(F)}(x) \triangleq [\mu_F(x)]^{1.25} \tag{2.16}$$

$$\mu_{Minus(F)}(x) \triangleq [\mu_F(x)]^{0.75} \tag{2.17}$$

The artificial hedges provide milder degrees of concentration and dilation than the previously defined operators (2.14) and (2.15).

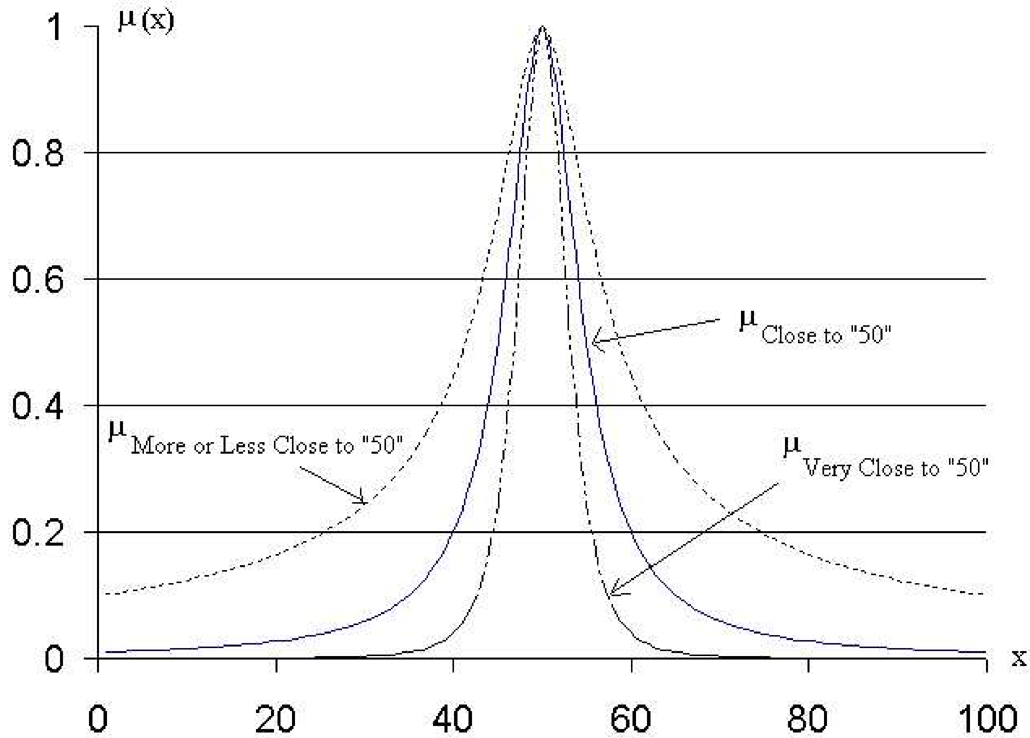


Figure 2.6: Examples of concentration and dilation.

2.6 Set Operations

After summarizing basic operations on classical sets, operations on fuzzy sets are introduced in this section.

2.6.1 Operations on Crisp Sets

It is considered that A and B are two subsets of the universe of discourse U. The union of A and B, denoted $A \cup B$, is given by $A \cup B = \{x | x \in A \text{ or } x \in B\}$ i.e., $\mu_{A\cup B}(x) = 1$ if x is contained in either A or B. The intersection of A and B, denoted $A \cap B$, is given by $A \cap B = \{x | x \in A \text{ and } x \in B\}$ i.e., $\mu_{A\cap B}(x) = 1$ if x is simultaneously in both A and B. The complement of A, denoted $\overline{A}$, is defined as $\overline{A} = \{x | x \in U \text{ and } x \notin A\}$ i.e, $\mu_{\overline{A}}(x) = 1$ if x is not contained in A. Note that these definitions can be written as follows (Yen and Langari, 1999; Mendel, 1995):

$$A \cup B \quad \Rightarrow \quad \mu_{A\cup B}(x) = \max\left(\mu_A(x), \mu_B(x)\right) \tag{2.18}$$

$$A \cap B \;\; \Rightarrow \;\; \mu_{A \cap B}(x) = \min\left(\mu_A(x), \mu_B(x)\right) \tag{2.19}$$

$$\overline{A} \;\; \Rightarrow \;\; \mu_{\overline{A}}(x) = 1 - \mu_A(x) \tag{2.20}$$

Among the others the two fundamental laws of crisp set theory are: Law of Contradiction $A \cap \overline{A} = \emptyset$ which states that an element x can either in the set A or in the complement of the set A, (i.e., $\overline{A}$). The Law of the Excluded Middle $A \cup \overline{A} = U$ asserts that a set and its complement must comprise the universe, U.

2.6.2 Operations on Fuzzy Sets

The negation not, and the set operations union and intersection are defined on fuzzy sets in terms of their membership functions. It is considered that A and B are fuzzy subsets, described by membership functions $\mu_A(x)$ and $\mu_B(x)$ and defined on a universe of discourse U. The union of fuzzy sets A and B is defined by (Zadeh, 1973)

$$A \cup B \triangleq \int_U \left(\mu_A(x) \vee \mu_B(x)\right)/x \tag{2.21}$$

The union operation corresponds to "or" connective (disjunction). The intersection of fuzzy sets A and B is defined by (Zadeh, 1973)

$$A \cap B \triangleq \int_U \left(\mu_A(x) \wedge \mu_B(x)\right)/x \tag{2.22}$$

The intersection operation corresponds to "and" connective (conjunction). The complement of fuzzy set A is defined by (Zadeh, 1973)

$$\overline{A} \triangleq \int_U \left(1 - \mu_A(x)\right)/x \tag{2.23}$$

The complement operation corresponds to negation "not". There are many choices for the fuzzy union and fuzzy intersection operators. The set of fuzzy union operators is called t-conorm (triangular conorms) or s-norm and the set of fuzzy intersection operators is called t-norm (triangular norms) operators (Yen and Langari, 1999; Yen, 1999; Mendel, 1995). Due to the principle of duality between the two operators, the choice of t-norm operator determines the choice of t-conorm operator, and vice versa. Typical t-norm, t-conorm pairs are listed below (Yen and Langari, 1999):

Drastic Product:

$$t_{Drastic}(x, y) = \begin{cases} \min\{x, y\} & \text{if } \max\{x, y\} = 1 \\ 0 & x, y < 1 \end{cases} \tag{2.24}$$

Drastic Sum:

$$s_{Drastic}(x, y) = \begin{cases} \max\{x, y\} & \text{if } \min\{x, y\} = 0 \\ 1 & x, y > 0 \end{cases} \tag{2.25}$$

Bounded Difference:

$$t_{Bounded}(x, y) = \max\{0, x + y - 1\} \tag{2.26}$$

Bounded Sum:

$$s_{Bounded}(x, y) = \min\{1, x + y\} \tag{2.27}$$

Einstein Product:

$$t_{Einstein}(x, y) = \frac{x \cdot y}{2 - [x + y - (x \cdot y)]} \tag{2.28}$$

Einstein Sum:

$$s_{Einstein}(x, y) = \frac{x + y}{[1 + x \cdot y]} \tag{2.29}$$

Algebraic Product:

$$t_{Algebraic}(x, y) = x \cdot y \tag{2.30}$$

Algebraic Sum:

$$s_{Algebraic}(x, y) = x + y - x \cdot y \tag{2.31}$$

Hamacher Product:

$$t_{Hamacher}(x, y) = \frac{x \cdot y}{x + y - (x \cdot y)} \tag{2.32}$$

Hamacher Sum:

$$s_{Hamacher}(x, y) = \frac{x + y - 2xy}{1 - (x \cdot y)} \tag{2.33}$$

Minimum:

$$t_{Minimum}(x, y) = \min\{x, y\} \tag{2.34}$$

Maximum:

$$s_{Maximum}(x, y) = \max\{x, y\} \tag{2.35}$$

Example 2.5 If the color is regarded as a linguistic variable, then its values, white, red, yellow, etc., may be interpreted as terms of fuzzy subsets of the universe. In this sense, the redness of a rose is determined by the membership function $\mu_{Red}(rose)$. Suppose a rose belongs to the set of red roses to degree 0.73, i.e., $\mu_{Red}(rose) = 0.73$. This rose also belongs to the complement of the set at a degree $\mu_{\overline{Red}}(rose) = 0.27$. Therefore, rose partially belongs to both set of red roses and its complement.

Example 2.5 shows that an element may simultaneously belong to a fuzzy set and its complement. It is seen that fuzzy sets violate the Law of Excluded Middle and the Law of Contradiction. Consequently, formula equivalents in crisp set theory are not necessarily equivalent in fuzzy set theory (Yen and Langari, 1999; Yen, 1999; Mendel, 1995). Ignoring such differences i.e., adopting wrong axioms for fuzzy sets, may incorrectly lead to rejection of fuzzy set theory (Elkan, 1994).

2.7 Relations

In this section classical relations and compositions will be used as basis for the discussion of fuzzy relations and fuzzy compositions.

2.7.1 Crisp Relations and Compositions

A crisp relation describes an association between two or more objects. Formally, if U and V are domains of the variables u and v respectively, then a relation on variables u and v is described as a set of ordered pairs in $U \times V$, such that

$U \times V = \{(x, y)|x \in U \text{ and } y \in V\}$. Note that a relation, $R(U, V)$ on variables u and v is a subset of $U \times V$. Since a relation can be viewed as a set, all basic crisp set operations can be applied to relations. A relation on variables u and v whose domains are U and V, denoted as $R(U, V)$ can be written as (Mendel, 1995):

$$\mu_R(u, v) = \begin{cases} 1 & \text{if } (u, v) \in R(U, V) \\ 0 & \text{otherwise} \end{cases} \tag{2.36}$$

It has to be also noted that compositions of relations that share a common set form crisp compositions. It is considered that relations $P(U, V)$ and $Q(V, W)$ are defined. Relation $P(U, V)$ is defined over variables u and v, whose domains are U and V and relation $Q(V, W)$ is defined on variables v and w, whose domains are V and W. The composition of relations P and Q is (Mendel, 1995):

$$R(U, W) = P(U, V) \circ Q(V, W) \tag{2.37}$$

where composition relation $R(U, W)$ is a subset of $U \times W$ and

$$(u, w) \in R \Leftrightarrow (\exists v \in V|(u, v) \in P \wedge (v, w) \in Q) \tag{2.38}$$

The composition of the relations $P(U, V)$ and $Q(V, W)$, namely $R(U, W)$ can be calculated by using the formulas (Mendel, 1995): The max-min composition

$$\mu_R(u, w) = \left\{(u, w), \max_v[\min(\mu_P(u, v), \mu_Q(u, w))]\right\} \tag{2.39}$$

The max-product composition

$$\mu_R(u, w) = \left\{(u, w), \max_v[\mu_P(u, v) \cdot \mu_Q(u, w)]\right\} \tag{2.40}$$

In the crisp case max-min and max-product compositions produce exactly the same result for $R(U, W)$.

2.7.2 Fuzzy Relations and Compositions

A fuzzy relation describes degree of association between two or more objects. In other words, a fuzzy relation is a generalization to crisp relation to allow each association between objects to take membership values between zero and unity in vague concepts (Yen and Langari, 1999; Yen, 1999; Mendel, 1995). Formally, if U and V are domains of the variables u and v respectively, then a fuzzy relation, $R(U, V)$ on variables u and v is a subset of $U \times V$ and is characterized by membership function $\mu_R(u, v)$, such that $R(U, V) = \{((u, v), \mu_R(u, v))|(u, v) \in U \times V\}$. A fuzzy relation on variables u and v whose domains are U and V, denoted as $R(U, V)$ can be expressed as (Zadeh, 1973):

$$R \triangleq \int_{U \times V} \mu_R(u, v)/(u, v) \tag{2.41}$$

Because fuzzy relations can be viewed as fuzzy sets, operations on fuzzy sets defined in Section 2.6.2 can be applied to fuzzy relations.

Example 2.6 Let R and Q are fuzzy relations defined over the same variables u and v whose domains are U and V respectively. The union and intersection of these two relations can be obtained as

$$\mu_{R \cap Q}(u, v) = \mu_R(u, v) \otimes \mu_Q(u, v) \tag{2.42}$$

$$\mu_{R \cup Q}(u, v) = \mu_R(u, v) \oplus \mu_Q(u, v) \tag{2.43}$$

where $\otimes$ and $\oplus$ are t-norm and t-conorm operators.

The compositions of fuzzy relations that share a common set are analogous to crisp compositions. Difference between the fuzzy compositions and crisp compositions is that in the fuzzy case the sets are fuzzy rather than crisp. If P is a relation from U to V and Q is a relation from V to W, then the composition of relations P and Q form a fuzzy relation R, defined by (2.38). A mathematical formula for the composition, R is (Zadeh, 1973):

$$R \triangleq \int_{U \times W} \bigvee_{v} (\mu_P(u, v) \wedge \mu_Q(v, w))/(u, w) \tag{2.44}$$

where $\bigvee$ and $\bigwedge$ denote, supremum and t-norm operator. It is noted that, fuzzy composition is not uniquely defined. Different choices of fuzzy conjunction and fuzzy disjunction operators yield different compositions. The most commonly used compositions are the sup-min and sup-product compositions.

Example 2.7 Consider the relations P and Q, where P is defined as "u is more-or less equal to v" and Q is defined as "v is smaller than w". Their relation matrices are shown below:

$$\mu_P(u,v) \quad = \quad \begin{array}{ccc} & v_1 & v_2 \\ u_1 & 0.4 & 0.9 \\ u_2 & 0.7 & 0.6 \end{array} \tag{2.45}$$

$$\mu_Q(v,w) \quad = \quad \begin{array}{ccc} & w_1 & w_2 \\ v_1 & 0.3 & 0.9 \\ v_2 & 0.8 & 0.5 \end{array} \tag{2.46}$$

The max-min composition result is:

$$\mu_{P \circ Q}(u,w) = \begin{array}{ccc} & w_1 & w_2 \\ u_1 & 0.8 & 0.5 \\ u_2 & 0.6 & 0.7 \end{array} \tag{2.47}$$

The max-product composition result is:

$$\mu_{P \circ Q}(u,w) = \begin{array}{ccc} & w_1 & w_2 \\ u_1 & 0.72 & 0.45 \\ u_2 & 0.48 & 0.63 \end{array} \tag{2.48}$$

As it is pointed in the previous section, max-min and max-product compositions, in the crisp case, produce the exactly same result for $R(U,W)$. However, Example 2.7 shows that the max-min and max-product compositions, in the fuzzy case are not the same.

2.8 Logic

Propositional logic and set theory are isomorphic. There is a one-to-one correspondence between propositional logic and set theory, which preserves the relations existing between the elements in its domain. Moreover, these two mathematical systems are also isomorphic to Boolean algebra (Yen and Langari, 1999; Mendel, 1995). Because of the connection between set theory, propositional logic and Boolean algebra, a theorem in any one of these systems has a counterpart in other two systems. For instance, if A and B are subsets of the universe of discourse, U and p and q are propositions, such that p represents the sentence "x is an element of set A" and q represents the sentence "x is an element of set B" then

$$p \text{ is true} \quad \Leftrightarrow \quad x \in A \tag{2.49}$$

$$q \text{ is true} \quad \Leftrightarrow \quad x \in B \tag{2.50}$$

$$(p \vee q) \text{ is true} \quad \Leftrightarrow \quad x \in A \cup B \tag{2.51}$$

Logic operations are not described in this section because of the isomorphism.

2.8.1 Crisp Logic

A proposition is a basic unit, i.e., a simple sentence describing statements about the world. Propositions can be combined by using conjunction (denoted $p \wedge q$), disjunction (denoted $p \vee q$), and implication (denoted $p \to q$) connectives to form complex sentences. In traditional logic, implication is used for the expressions in the form of IF A THEN B. The expression IF A THEN B where A and B are propositions is considered as a rule. A rule represents a relation between A and B and it is characterized by membership function $\mu_{A \to B}(x, y)$. A rule has two parts: a premise (i.e., antecedent, if-part of the rule) and a conclusion (i.e., consequent, then-part of the rule). Rules are also considered as a form of propositions. From truth table, depicted in Table 2.1, implication between A

Table 2.1: Truth table for implication operation

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

and B can be formulized as

$$A \to B \Leftrightarrow (\neg A) \vee B \tag{2.52}$$

and this equation can be written as

$$A \rightarrow B \Leftrightarrow \neg(A \wedge (\neg B)) \tag{2.53}$$

Membership function of an implication, $\mu_{A\rightarrow B}(x, y)$ on propositions A and B can be obtained by using equations (2.52) and (2.53) (Mendel, 1995). Using equation (2.52),

$$\mu_{A\rightarrow B}(x, y) = \mu_{\overline{A}\cup B}(x, y) \tag{2.54}$$

$$\mu_{A\rightarrow B}(x, y) = \max[1 - \mu_A(x, y), \mu_B(x, y)] \tag{2.55}$$

using equation (2.53),

$$\mu_{A\rightarrow B}(x, y) = 1 - \mu_{A\cap\overline{B}}(x, y) \tag{2.56}$$

$$\mu_{A\rightarrow B}(x, y) = 1 - \min[\mu_A(x, y), 1 - \mu_B(x, y)] \tag{2.57}$$

It has to be also noted that the implication should not conclude anything about B (conclusion) when A (premise) is false. This is due to the fact that when A (premise) is false, implication is true whether B (conclusion) is true or not (the third and the forth rows in Table 2.1). Therefore, the value of B is unknown, i.e., nothing can be inferred about B when A is false. There are two important inference schemes in traditional propositional logic:

Modus Ponens: When an implication and its premise are known to be true then modus ponens infers conclusion is true. For instance, consider an implication "IF x is A THEN y is B". It is known that the implication is true and "x is A" (i.e., premise is true), then modus ponens infers that "y is B" (i.e., conclusion is true).

Modus Tollens: When it is known that an implication is true and its conclusion is not true then modus tollens infers that its premise is not true. For instance, consider an implication "IF x is A THEN y is B". It is known that implication is true and "y is not B" (i.e., conclusion is not true), then modus tollens infers that "x is not A" (i.e., premise is not true).

It is not easy to represent uncertainty in data and reason under uncertainty in classical logic. Whereas, fuzzy logic generalizes classical logic for reasoning under uncertainty (Yen and Langari, 1999).

2.8.2 Fuzzy Logic

Fuzzy logic generalizes crisp logic to allow truth-values to take partial degrees. Since bivalent membership functions of crisp logic are replaced by fuzzy membership functions, the degree of truth-values in fuzzy logic becomes a matter of degree, which is a number between 0 and 1.

Applications of fuzzy set theory ranging from consumer products, manufacturing, robotics, control systems, finance to earthquake engineering (Maiers and Sherif, 1985; Lee, 1990a) are mostly based on the use of fuzzy if-then rules. A fuzzy if-then rule in the form of a statement such as "IF x is A THEN y is B", where $x \in X$ and $y \in Y$ has a membership function defined as $\mu_{A \to B}(x, y)$. Note that $\mu_{A \to B}(x, y)$ describes the degree of truth of the implication relation between x and y (Mendel, 1995). Examples of such membership functions are the fuzzy versions of equations (2.55) and (2.57) which were defined for crisp case.

Even though in most applications rules are connected using a t-conorm operator, there are a number of ways to connect rules (for discussions on connecting rules see (Kiszka et al., 1985a; Kiszka et al., 1985b; Lee, 1990b)).

Since linguistic variables are used in the fuzzy if-then rules to describe elastic conditions (i.e., conditions that can be partially satisfied), a fuzzy if-then rule can capture knowledge about real world that is inexact by nature and involves imprecision (Yen and Langari, 1999; Yen, 1999). Another important feature of fuzzy if-then rules is its partial matching capability (Yen and Langari, 1999; Yen, 1999; Mendel, 1995).

Example 2.8 (Mendel, 1995) Consider a rule "IF a car is big THEN its consumption is high." The premise contains a fuzzy set P, big car and the conclusion contains a fuzzy set C, high consumption. What will be the inference, (i.e., consumption of the car) if it is known that the car is a small hatchback? It is clearly seen that the given fuzzy set (a small hatchback car) and the fuzzy set in the if-part of the rule (a big car) are not the same, but they are similar. The conclusion will be the following "The consumption is moderate for this car." Again the fuzzy set moderate consumption and the fuzzy set in the then-part of the rule (high consumption) are not the same, but they are similar.

In crisp logic a rule is fired when the premise is exactly the same as the given input fuzzy set, and the result is the same as the then-part (conclusion) of the rule. However, Example 2.8 shows that in fuzzy logic, a rule is fired even the given fuzzy set only partially satisfies the if-part (premise) of the rule, and the result is a consequent which is similar to the fuzzy set in then-part (conclusion) of the rule (i.e., the consequent has a nonzero degree of similarity to the fuzzy set in the then-part of the rule).

There are two types of fuzzy if-then rules: fuzzy mapping rules and fuzzy implication rules. In the sequel these two types of fuzzy if-then rules are described.

2.8.2.1 Fuzzy Mapping Rules

A set of fuzzy mapping rules can approximate a function of interest by describing a mapping relationship between a set of input parameters to a set of output parameters. Since, a single fuzzy mapping rule approximates a small segment of a function, a set of fuzzy mapping rules are needed to approximate the whole function. A set of fuzzy mapping rules is called a fuzzy rule-based model or a fuzzy model.

To draw a conclusion from a set of fuzzy mapping rules (i.e., fuzzy model) it is based on a four-step algorithm:

Fuzzy Matching: This step involves the computation of the meaning of fuzzy conditional statements of the form IF A THEN B. The computation of the meaning of a fuzzy conditional statement, which is a combination of multiple linguistic variables connected with connectives, is primarily based on the meaning of each linguistic variable. Let T denotes a set of terms in the universe of discourse U and x represents a term in T and consider a fuzzy naming relation N between x and y where y is an object in U then the meaning of an object y is denoted by $M(x)$ and defined by (Zadeh, 1973):

$$\mu_{M(x)}(y) \triangleq \mu_N(x, y) \tag{2.58}$$

where $x \in T$ and $y \in U$. The meaning of a linguistic variable can be modified by using hedges. The computation of the meaning of a linguistic variable modified with hedge is discussed in Section 2.5. The meaning of a more complex composite term, which may involve additional terms connected with connectives, is computed by using definitions described in Section 2.6.2.

Inference: Each rule's conclusion is computed based on its matching degree. Meaning of the fuzzy conditional statement computed in the previous step is used to suppress the membership function of the conclusion (i.e., then-part of the rule). There are two types of suppression techniques:

1. The Clipping Method: The inference is made based on the minimum operator. The clipping method cuts off the values of membership function, which are higher than the matching degree.

2. The Scaling Method: The inference is made based on the product operator. The scaling method scales down the values of membership function in proportion to the matching degree.

The clipping and scaling methods are illustrated in Figure 2.7.

Combination: The value of the consequent variable for each rule is inferred through the previous two steps. Since a fuzzy model consists of multiple fuzzy if-then rules with partially overlapping conditions, a particular input may fire more than one rule. Hence, computed inferences are combined

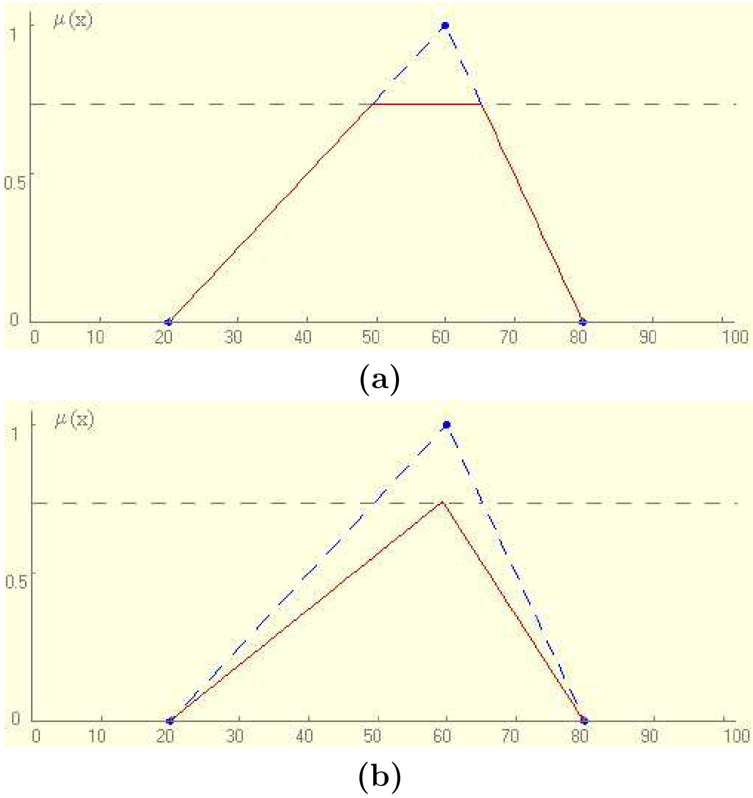


Figure 2.7: Fuzzy inference (a) clipping method, and (b) scaling method.

by applying the fuzzy disjunction operator. The combination of fuzzy conclusions is depicted in Figure 2.8.

Defuzzification: Defuzzification is an optional step which produces a crisp output for fuzzy systems whose final output needs to be in crisp. Many defuzzification methods have been proposed in the literature, some of the major defuzzification methods are: It is considered that "y is B" is a fuzzy conclusion to be defuzzified.

1. Center of Area Defuzzification (COA): Center of area defuzzification (centroid method, center of gravity method) method calculates the center of gravity of fuzzy set B. The result of center of area defuzzification is unique and is calculated by:

$$y = \frac{\int_S \mu_B(y) \times y dy}{\int_S \mu_B(y) dy} \tag{2.59}$$

where S denotes the support of $\mu_B(y)$.

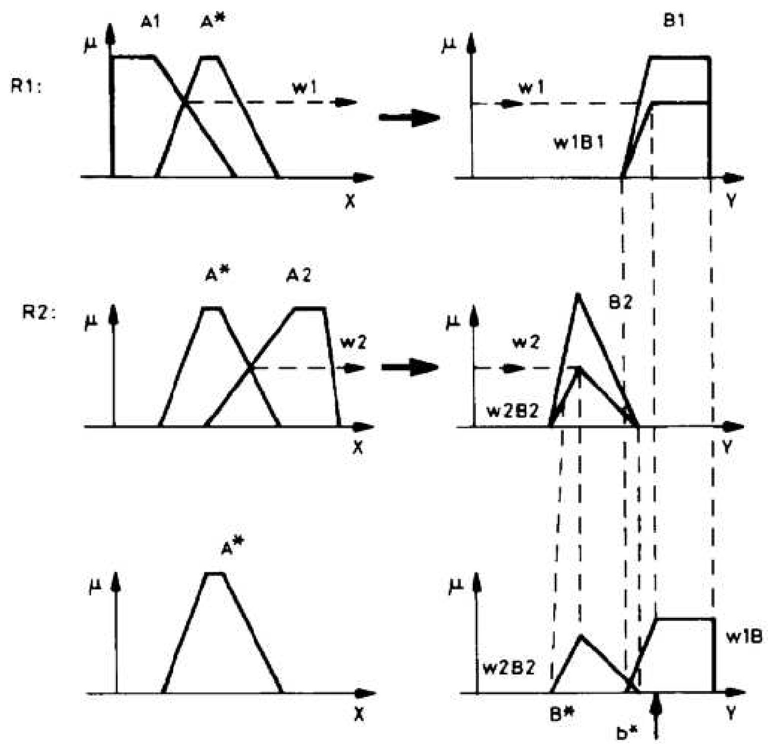


Figure 2.8: The combination of fuzzy conclusions (Koczy, 1995).

2. Bisector of Area Defuzzification: Bisector of area defuzzification (bisector) technique calculates a point which partitions the area under the membership function curve ($\mu_B(y)$) into two subregions with the same area.

$$\int_{\alpha}^{\overline{y}} \mu_B(y)dy = \int_{\overline{y}}^{\beta} \mu_B(y)dy \tag{2.60}$$

where $\overline{y}$ divides the area into two equal parts, and α and β denotes the minimum and maximum values of the support of B.

3. Mean of Maximum Defuzzification (MOM): Mean of maximum defuzzification first determines the maximum values of y for which the membership function value is a maximum. The output value is the mean of these values. Mean of maximum defuzzification can be formulized as (Yen and Langari, 1999):

$$y = \frac{\sum_{y^* \in P} y^*}{|P|} \tag{2.61}$$

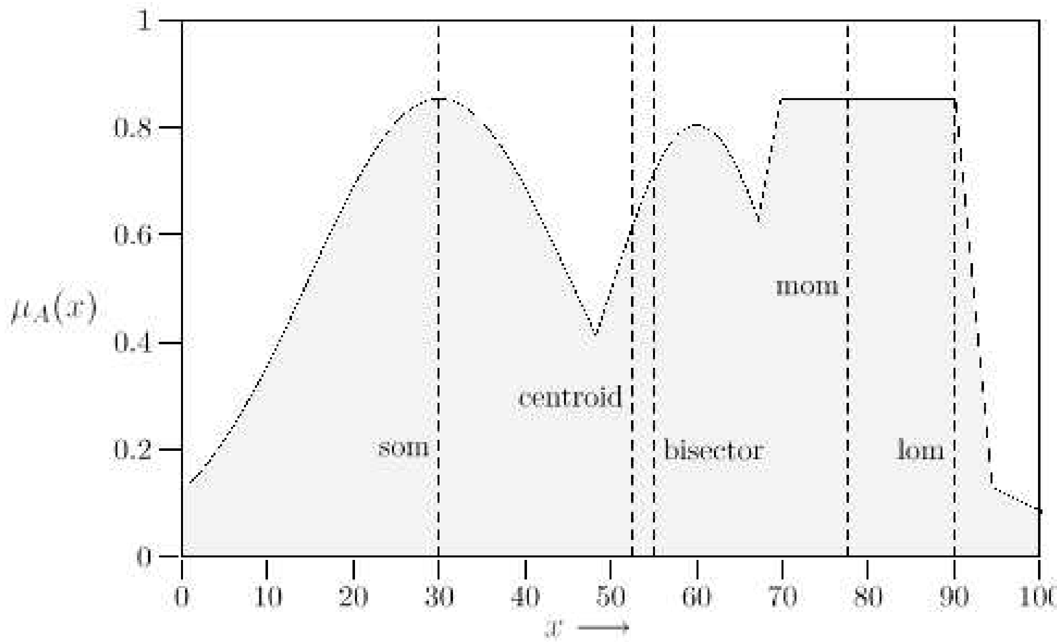


Figure 2.9: Defuzzification methods (King, 2000).

where P is a set values which has a maximum membership grade.

$$P = \{y^* | \mu_B(y^*) = sup_y \mu_B(y)\} \tag{2.62}$$

where sup stands for the supremum (i.e., the maximum value of a continuous function) over the domain y.

4. Largest of Maximum Defuzzification (LOM): Largest of maximum defuzzification method calculates the largest value (in magnitude) that maximizing the membership function $\mu_B(y)$.

5. Smallest of Maximum Defuzzification (SOM): Smallest of maximum defuzzification method calculates the smallest value (in magnitude) that maximizing the membership function $\mu_B(y)$.

Figure 2.9 illustrates discussed defuzzification methods.

2.8.2.2 Types of Fuzzy Rule Based Models

There are three types of fuzzy rule based models:

The Mamdani Model: The first fuzzy logic controller was developed by E. H. Mamdani using the model (Mamdani and Assilian, 1975). Rules of the Mamdani model are in the form of (Yen and Langari, 1999)

$$R_i : \text{IF } x_1 \text{ is } A_{i_1} \text{ AND } \dots \text{ AND } x_r \text{ is } A_{i_r} \text{ THEN } y \text{ is } C_i \tag{2.63}$$

where $i = 1, 2, \dots, N$ and N denotes the number of fuzzy rules, $x_j \in X_j(j = 1, 2, \dots, r)$ and y are linguistic variables, and A_i and C_i are fuzzy sets whose membership functions are $\mu_{A_{ij}}(x_j)$ and $\mu_{C_i}(y)$. Fuzzy model defined in (2.63) describes a mapping from $X_1 \times X_2 \times \dots \times X_r$ to Y and receives inputs in the form of "x_1 is A_1'", "x_2 is A_2'",…,"x_r is A_r'" where A_1', A_2',…,A_r' are fuzzy subsets of $X_1, X_2, \dots, X_r$. Suppose sup-min composition is used for the fuzzy inference, and maximum and minimum operations are used for all fuzzy disjunction and fuzzy conjunction operators. Then, the final output of the model (before defuzzification), C' is written as

$$\mu_{C'}(y) = \max_{i=1}^{N} \left(\mu_{C_i'}(y)\right) \tag{2.64}$$

where

$$\mu_{C_i'}(y) = (\alpha_{i_1} \wedge \alpha_{i_2} \wedge \dots \wedge \alpha_{i_N}) \wedge \mu_{C_i}(y) \tag{2.65}$$

and

$$\alpha_{ij} = sup_{x_j} \left(\mu_{A_j'}(x_j) \wedge \mu_{A_{ij}}(x_j)\right) \tag{2.66}$$

where $\wedge$ denotes the minimum operator. Since the final output (2.64) is produced by combining inference results of individual rules by superimposition, the Mamdani model is a non-additive rule model.

The Takagi-Sugeno-Kang (TSK) Model: Unlike the Mamdani model, the TSK model uses linear functions in their consequent (then-part of the rule). Therefore, the TSK model can approximate a function using fewer rules. Rules of the TSK model are in the form of (Takagi and Sugeno, 1985)

$$R_i : \text{IF } x_1 \text{ is } A_{i_1} \text{ AND } \dots \text{ AND } x_r \text{ is } A_{i_r} \text{ THEN } y = g_i(x_1, \dots, x_r) \tag{2.67}$$

where $i = 1, 2, \ldots, N$ and N denotes the number of fuzzy rules, x and y are linguistic variables and A_i are fuzzy sets and g_i denotes a function that implies the value of y.

Example 2.9 (Takagi and Sugeno, 1985)

$$R : \text{IF } x_1 \text{ is small AND } x_2 \text{ is big THEN } y = x_1 + x_2 + 2x_3 \tag{2.68}$$

According to the rule, R the value of y can be inferred by summing x_1, x_2 and $2x_3$ (x_3 is unconditioned in the premise), if the premise is satisfied (i.e., x_1 is small and x_2 is big).

The TSK model aggregates conclusions of individual rules similar to a weighted sum, the model is considered as additive model.

The Standard Additive Model (SAM): Unlike the Mamdani model, the SAM, first introduced by Kosko (1997), assumes the inputs to the system are crisp values. Moreover, the SAM uses different operators than the Mamdani model: the SAM model uses sup-product composition, product for all fuzzy conjunction operators and addition to combine conclusions of individual rules and the SAM model uses centroid defuzzification method. Rules of the SAM model are in the form of (Yen and Langari, 1999; Yen, 1999)

$$\text{IF } x \text{ is } A_i \text{ AND } y \text{ is } B_i \text{ THEN } z \text{ is } C_i \tag{2.69}$$

If inputs are $x = x_0$, $y = y_0$ then the model calculates z as

$$z = Centroid\left(\sum_i \mu_{A_i}(x_0) \times \mu_{B_i}(y_0) \times \mu_{C_i}(z)\right) \tag{2.70}$$

Like the TSK model, SAM model also uses addition to aggregate rules, thus the SAM model is an additive model.

2.8.2.3 Fuzzy Implication Rules

The semantics of fuzzy implication rules are constructed by generalizing implications in two-valued logic. Since fuzzy mapping rules are generalization to

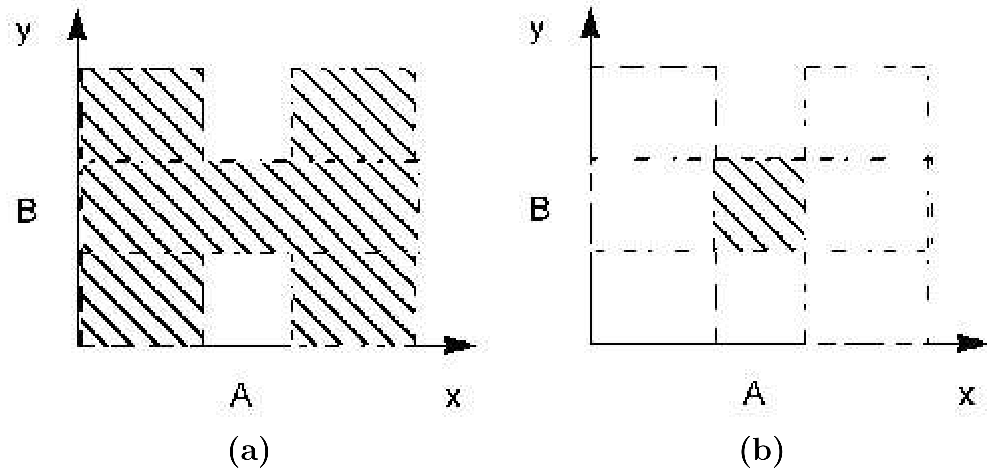


Figure 2.10: Differences between (a) implication rule, and (b) mapping rule (Yen and Langari, 1999; Yen, 1999).

set-to-set associations and fuzzy implication rules are generalization to set-to-set implications, their inference behavior are not the same. For instance, consider the rule "IF x is A THEN y is B". The differences between the implication rules and mapping rules are illustrated in Figure 2.10 (Yen and Langari, 1999). When the antecedents are satisfied both rules behave the same, however the implication rules and mapping rules behave different when their antecedents are not satisfied.

In accordance with crisp version of implication as discussed in Section 2.8.1, the expression "IF A THEN B" where A and B are fuzzy subsets of U and V, respectively is considered as a rule. A rule represents a relation between A and B and is characterized by membership function $\mu_{A \to B}(x, y)$. The membership function $\mu_{A \to B}(x, y)$ describes the truth value of the fuzzy implication rule "(x_i is A) $\to$ (y_j is B)" and written as

$$R_I(x_i, y_j) = t\left((x_i \text{ is } A) \to (y_j \text{ is } B)\right) \tag{2.71}$$

where t denotes the truth value of a proposition. Truth value of an implication in (2.71) can be defined in terms of the truth value of the antecedent (i.e., "x_i

is A") and the truth value of the consequent (i.e., "y_j is B").

$$t\left((x_i \text{ is } A) \rightarrow (y_j \text{ is } B)\right) = I(\alpha_i, \beta_j) \tag{2.72}$$

where I is called implication function, and

$$\alpha_i = t(x_i \text{ is } A) \tag{2.73}$$

$$\beta_j = t(y_j \text{ is } B) \tag{2.74}$$

2.8.2.4 Families of Fuzzy Implication Functions

There isn't a unique definition for implication function. A particular logic formulation of implication in propositional logic can be extended. These formulations are equivalent in classical logic, however because the Law of Excluded Middle and the Law of Contradiction are not the axioms of fuzzy logic, they are not equal in fuzzy logic. There are three families of fuzzy implication functions:

1. The first family is based on material implication where a material implication is defined as $A \rightarrow B = \neg A \vee B$. Then the truth value of the implication "(x_i is A)$\rightarrow$ (y_j is B)" is written as

$$t\left((x_i \text{ is } A) \rightarrow (y_j \text{ is } B)\right) = t\left(\neg(x_i \text{ is } A) \vee (y_j \text{ is } B)\right) \tag{2.75}$$

$$t\left((x_i \text{ is } A) \rightarrow (y_j \text{ is } B)\right) = (1 - \mu_A(x_i)) \oplus \mu_B(y_j) \tag{2.76}$$

2. The second family is based on the propositional calculus. The equivalence in propositional logic $A \rightarrow B = \neg A \vee (A \wedge B)$ is extended. Then the truth value of the implication "(x_i is A)$\rightarrow$ (y_j is B)" is written as

$$t\left((x_i \text{ is } A) \rightarrow (y_j \text{ is } B)\right) = t\left(\neg(x_i \text{ is } A) \vee ((x_i \text{ is } A) \wedge (y_j \text{ is } B))\right) \tag{2.77}$$

$$t\left((x_i \text{ is } A) \rightarrow (y_j \text{ is } B)\right) = (1 - \mu_A(x_i)) \oplus (\mu_A(x_i) \otimes \mu_B(y_j)) \tag{2.78}$$

3. The third family is based on the definition of standard sequence, an implication is true when the consequent is truer than the truth value of its

antecedent. Then the truth value of the implication "(x_i is A)→ (y_j is B)" is written as (Yen and Langari, 1999)

$$t\left((x_i \text{ is } A) \rightarrow (y_j \text{ is } B)\right) = sup\left\{\alpha | \alpha \in [0,1], \alpha \otimes t(x_i \text{ is } A) \leq t(y_j \text{ is } B)\right\} \tag{2.79}$$

$$t\left((x_i \text{ is } A) \rightarrow (y_j \text{ is } B)\right) = sup\left\{\alpha | \alpha \in [0,1], \alpha \otimes \mu_A(x_i) \leq \mu_B(y_j)\right\} \tag{2.80}$$

Fukami et al. (1980) have proposed a set of intuitive criteria to compare and evaluate the various definitions of the fuzzy implication functions.

2.9 Fuzzy Logic in GIS

Geographic data, stored and processed in a GIS, are captured from real world. The real world about which a GIS maintains information contains uncertainties (Roman, 1990). In a GIS operation, two different sources of uncertainties can be considered: uncertainty in data and uncertainty related with the model (Lark and Bolam, 1997; Heuvelink and Burrough, 2002).

1. Uncertainty in Data: The fundamental axioms of crisp logic limit the way of human thinking about the real world. It is necessary to be able to deal with concepts that are not necessarily "True" or "False" (i.e., concepts that are somewhere in between "True" or "False") (Burrough, 1986). For representing and handling uncertain geographic data, research began to investigate the use of fuzzy set theory. For example, same representation of boundaries is used in thematic maps for different types of changes in the real world (Burrough, 1986; Wang and Hall, 1996; Kiiveri, 1997). The representation of geographical boundaries with continuously changing properties (i.e., boundaries which are diffuse or uncertain) as sharp lines misrepresent changes in geographical properties. As pointed out by Wang and Hall (1996), fuzzy set theory can be used to improve the expressive ability of polygons.

   Fuzzy classification and identification in GIS have been widely used for many different problem domains (Lark and Bolam, 1997; Zhu et al., 1996; Ahamed et al., 2000b; Ahamed et al., 2000a; Sasikala and Petrou, 2001). The main reason for the investigation of the use of fuzzy classification is the classification error, especially, when dealing with linguistic concepts such as “flat” or “gentle” (Stefanakis et al., 1996). For example, in conventional classification of soil in a continuum landscape, an area is assigned to one soil mapping unit, and is separated from other mapping units by sharp lines. The discretization of such a continuous phenomena into distinct spatial and categorical classes results in information loss (Zhu et al., 1996).

Zhu et al. (1996) combined fuzzy logic with GIS to infer soil series from environmental conditions and stated that images produced using the proposed methodology have advantages in terms of revealing spatial patterns of soil information and detailing attribute information.

For sustainable agricultural production, crop-land suitability analysis is a prerequisite to achieve optimum utilization of the available land resources. Ahamed et al. (2000b) addressed the problems encountered when the Boolean methods are designed to assign a given area element (i.e., pixel) to a single suitability class and proposed the use of fuzzy membership approach. It is stated by Ahamed et al. (2000b) that fuzzy membership approach delineates areas of various suitability ratings to a given crop more accurately and fuzzy membership approach is found to be advantageous when determining the crops of highest suitability for a given area.

Another example of fuzzy classification is development of a method for classifying soil in soil erosion classes (Ahamed et al., 2000a). It is noted by Ahamed et al. (2000a) that various soil loss ratings are appropriately represented using fuzzy class membership approach and information in fuzzy class membership highlights the spatial variation in the severe erosion classes.

Ahlqvist et al. (2003) addressed the classification problem and introduced the idea of rough fuzzy classifications. Rough fuzzy classifications allow GIS users to reason about areas that have been classified using indiscernible concepts and for which some additional vague information is available (Ahlqvist et al., 2003). It is stated by Ahlqvist et al. (2003) that rough fuzzy classification is able to integrate uncertainty due to both vagueness and indiscernibility.

2. Uncertainty in Model: Uncertainty in the interpretation of data values within the GIS is mainly based on "early and sharp" classification (Stefanakis et al., 1996). In a GIS, a common type of operation in decision-making is a threshold model. When the underlying logic in GIS is crisp logic, then results of applying threshold values in decision-making processes are 0 or 1. This threshold model is defined as (Burrough, 1986):

$$\mu_A(x) = \begin{cases} 1 & \text{if } TH_{Low} \leq x < TH_{High} \\ 0 & \text{otherwise} \end{cases} \tag{2.81}$$

where $x \in A$ and x is an individual observation. TH_{Low} and TH_{High} represent low and high threshold values, which define the exact boundaries of set A. Such models can cause problems since they are inherently rigid. Consider the threshold value for a flat land is slope = 10%, a location with slope 9.9% is classified as flat land however a location with slope 10.1% is rejected. Stefanakis et al. (1996) addressed weakness of crisp logic in decision processes and proposed the use of fuzzy logic methodologies to overcome this problem.

Kollias and Kalivas (1998) enhanced GIS software (ARC/INFO) with fuzzy logic methodologies to allow a more precise and realistic classification and assessment of natural phenomena. New commands are added to the GIS software. The proposed system (FUZZYLAND) was developed by AML (ARC/INFO Macro Language) and operates as a set of ARC/INFO commands. Although, the developed system has important advantages when it classifies and evaluates soil data, it also has limitations in membership function types and operations on fuzzy sets. Since FUZZYLAND does not have any mechanism to capture experts' experiences, it is not easy to make decisions using linguistic variables defined by experts on a continuous landscape.

Similarly, a major desktop system ArcView is augmented to evaluate information that has vague definition, like "flat land" and "southern aspect" (Benedikt et al., 2002). The information used in the developed system (MapModels) is derived from the elevation models (e.g., slope and aspect). The developed system has a flowchart based user interface to allow easy use rather than using command like system. The proposed system can evaluate linguistic concepts to produce classified map layers. And these map layers can be combined using fuzzy map overlay operators. The fuzzy map overlay operators used in this work are based on gamma operators (Benedikt et al., 2002). Like FUZZYLAND, MapModels system can not be used for decision-making processes. Moreover, only data derived from elevation models are used in MapModels (i.e., problem specific).

Note that MapModels system uses gamma operators for map overlay instead of logical operators such as intersection (i.e., AND) and union (i.e., OR). As it is stated by Jiang and Eastman (2000), integration methods of multi-criteria need to go beyond the common approaches of union, intersection and also weighted linear combination. The aggregation approaches of fuzzy measures used in map overlay are minimum operator for intersection and maximum operator for union. The minimum operator commonly assigns suitability value to a location in terms of its worst quality (i.e., avoiding risk in decision). Whereas, the maximum operator is the opposite, it assigns suitability value to a location to the extent of its best quality (i.e., risk-taking attitude). And the averaging operator falls midway between the two extreme cases. Therefore, Jiang and Eastman (2000) propose the use of ordered weighted averaging (OWA) operator in multi-criteria evaluation for aggregation.

Martin-Clouaire et al. (2000) developed a system using possibility theory to represent and process uncertainty and imprecision present in the soil hydrological properties and showed that possibility theory can also be used for dealing with the incompleteness and vagueness pervading the soil knowledge. Since the hydrologic soil properties are computed as possibility distribution, the result can not be communicated directly by a map. Further processing is required to produce meaningful maps from possibility distributions. The proposed approach can be used for classification.

As it is stated previously geographic data contains uncertainties and these uncertain information can be captured by using fuzzy set methodologies. The visualization of uncertainty in geographic data is also an important tool for intelligent GIS. Techniques use fuzzy logic in the production of maps to visualize several types of spatially variable uncertainty in a single display (Davis and Keller, 1997). Fuzzy set theory can also be used to develop natural language interfaces for querying (Robinson, 2000; Wang, 2000).

CHAPTER 3

DESIGN OF FUZZY INFERENCE SYSTEM

This chapter presents the design issues of Fuzzy Inference System to enhance cell-based information modeling. The system has been developed on a commercial GIS software namely ArcGIS, which is a major GIS desktop system. After giving a general workflow of the Fuzzy Inference System, design details are presented.

3.1 General Architecture Design

The general architecture design and workflow of the fuzzy inference system for cell-based information modeling is shown in Figure 3.1. Commercial GIS application uses Fuzzy Inference System through public interface defined by Fuzzy Inference System Module. The interface is a collection of logically related operations that define some behavior (Kirtland, 1999). However, commercial GIS application and Fuzzy Inference System act as two separate applications.

Fuzzy Inference System is designed as an ActiveX module. Within the Fuzzy Inference System Module, precompiled libraries Fuzzy Inference Engine and ESRI ArcObjects Library are used. Since Component Object Model (COM) environment is used, applications can interact with objects only through their public interfaces. These connection points' descriptions are given in Section 4.1.1 and in Section 4.1.2.

When the Fuzzy Inference System tool is selected from the commercial GIS

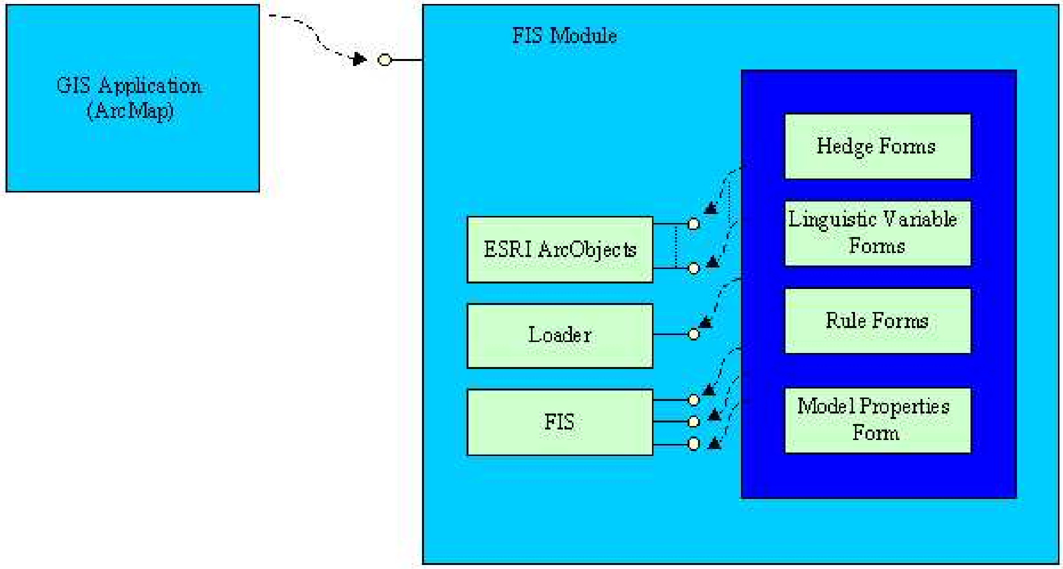


Figure 3.1: Architectural design and workflow of Fuzzy Inference System for GIS.

application, the GIS application starts execution of Fuzzy Inference System Module. The Fuzzy Inference System Module initialization phase includes creation and start of Fuzzy Inference Engine. After initialization phase, Fuzzy Inference System Module queries interfaces according to actions defined by the user.

ESRI applications are COM clients; their architecture supports the use of software components that adhere to the COM specification (ESRI, 2001). Hence, components can be built with different languages including Visual Basic and Visual C++, and these components can then be added to the applications easily. As discussed in detail in Chapter 4, Visual Basic and Visual C++ are used to create COM components to enhance the functionality of cell-based information modeling in the form of extensions. An extension is a component or a set of components that implements an interface that is expected by the application and registers itself with the application so that it may be loaded at the appropriate time. End-users can control what pieces of functionality are installed on a machine or loaded at run time. ArcGIS provides developers the key benefit of standard mechanisms for plugging extensions and other components into the

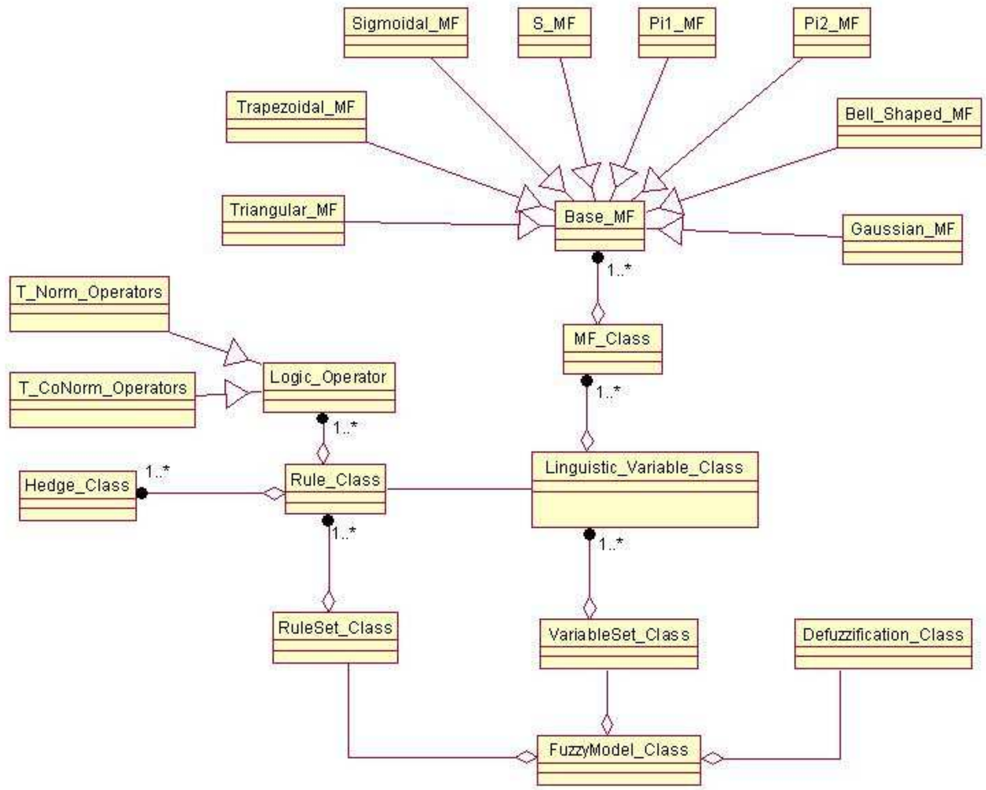


Figure 3.2: Class diagram for Fuzzy Inference Engine.

system (ESRI, 2001). In addition, previous experience in both using ArcGIS product family and ArcObjects Library lead to the selection of ArcGIS developed at ESRI for commercial GIS application.

The design of Fuzzy Inference System is mainly divided into two sections, the Fuzzy Inference Engine design and Fuzzy Inference System Module design.

3.2 Design of Fuzzy Inference Engine

Operations of fuzzy sets, linguistic variable definitions, fuzzy if-then rules and fuzzy inference compose main functionality of the Fuzzy Inference Engine. Since raster maps are major source of input for cell-based information modeling, Fuzzy Inference Engine can produce output in the form of raster map by applying fuzzy rule-based reasoning to data gathered from input raster maps.

Fuzzy Inference Engine class hierarchy is shown in Figure 3.2. Bases of the Fuzzy Inference Engine are membership function classes. Fuzzy if-then rules are constructed with the help of linguistic variables, where linguistic variables are defined by using membership functions.

3.2.1 Membership Function Classes

A fuzzy set can be defined in the Fuzzy Inference Engine by the MF_Class object. In order to form a fuzzy set, name of the fuzzy set, membership function type of the fuzzy set and membership function parameters must be known. Membership function types supported by the Fuzzy Inference Engine are:

1. Triangular membership function.
2. Trapezoidal membership function.
3. Gaussian membership function.
4. Bell-shaped membership function.
5. Sigmoidal membership function.
6. S membership function.
7. Π_1 type membership function.
8. Π_2 type membership function.

Each membership function class shown in Figure 3.2, is a subclass of Base_MF Class and computes degree of membership according to membership function parameters.

3.2.2 Operations of Fuzzy Sets

The set operations intersection and union correspond to logic operations, conjunction (and, t-norm operators) and disjunction (or, t-conorm operators) respectively. Therefore, operations on fuzzy sets in the Fuzzy Inference Engine are:

T-Norm Operators Class: A simple reusable entity class where fuzzy conjunction operators are managed. Supported t-norm operators are listed below:

1. Drastic product.
2. Bounded difference.
3. Einstein product.
4. Algebraic product.
5. Hamacher product.
6. Minimum.

T-CoNorm Operators Class: A simple reusable entity class where fuzzy disjunction operators are managed. Supported t-conorm operators are listed below:

1. Drastic sum.
2. Bounded sum.
3. Einstein sum.
4. Algebraic sum.
5. Hamacher sum.
6. Maximum.

T_Norm_Operators and T_CoNorm_Operators clasess are inherited from superclass Logic_Operator as shown in the Figure 3.2.

3.2.3 Linguistic Variables

By incorporating both linguistic term and membership function in a linguistic variable, a linguistic variable can represent human knowledge and can process numeric input data (Yen and Langari, 1999). Hence, a linguistic variable in Fuzzy Inference Engine is composed of one ore more membership functions and a linguistic term to express concepts and knowledge in human communication.

A set of problem specific linguistic variable form a variable set in the problem domain. Linguistic variables in this variable set are the basic components of fuzzy if-then rules.

3.2.4 Hedges

The meaning of a fuzzy set can be modified by hedges to create a compound fuzzy set. Mainly, a hedge is defined by a modifier name and a simple function in the form $f = \mu_A(x)^n$ for all $n \in \mathbb{R}^+$, where A represents a fuzzy set defined over the universe of discourse.

In Fuzzy Inference Engine, "NOT" keyword is considered as a hedge because it modifies the original fuzzy set to create complement of the original fuzzy set.

3.2.5 Fuzzy If-Then Rules

Fuzzy if-then rules associate an input data described using linguistic variables and fuzzy sets to a conclusion. Since input data are described using linguistic variables and fuzzy sets, fuzzy if-then rules can be viewed as a scheme for capturing imprecise knowledge.

A fuzzy if-then rule in Fuzzy Inference Engine contains one ore more linguistic variables, hedges and logic operations. Structure of a fuzzy if-then rule is as follows:

IF Linguistic Variable1 is Hedge1 Fuzzy SetA LogicOperator1
Linguistic Variable2 is Hedge2 Fuzzy SetB LogicOperator2
. . . .
. . . .
Linguistic VariableN is HedgeN Fuzzy SetM LogicOperatorN
THEN
Linguistic VariableX is Fuzzy SetY

A set of fuzzy if-then rules can represent human knowledge on a specific problem domain that is imprecise and inexact by nature. Human knowledge is captured by Fuzzy Inference Engine and is represented using fuzzy if-then rules in the

RuleSet_Class.

3.2.6 Defuzzification

Defuzzification is a step in fuzzy rule based inference where a fuzzy conclusion is converted to a crisp output. Defuzzification types implemented in Fuzzy Inference Engine are:

1. Center of area defuzzification.

2. Bisector of area defuzzification.

3. Mean of maximum defuzzification.

4. Largest of maximum defuzzification.

5. Smallest of maximum defuzzification.

3.2.7 Fuzzy Model

A set of fuzzy mapping rules form a fuzzy model (Yen and Langari, 1999). In the Fuzzy Inference Engine, fuzzy if-then rules, a set of linguistic variables and defuzzification method are used to construct the FuzzyModel_Class, where fuzzy rule based inference take place. The fuzzy rule based inference consists of four basic steps:

Fuzzy Matching: The degree to which input data match the condition of the fuzzy rules is calculated. Fuzzy matching algorithm in pseudo code is given in Figure 3.3. In the fist step of the Algorithm 1, membership grades are computed according to linguistic variables' membership function parameters. Second step involves application of hedge functions, which are defined before and represented by Hedge_Class. In the last step, selected conjunction operators (t-norm operators) and disjunction operators (t-conorm operators) are used to compute the degree of matching between input data and rule.

Algorithm 1 Fuzzy Matching Algorithm.

for each <Linguistic Variable> in the fuzzy if-then rules
    1. Compute the <Degree of Membership> to which input data belongs to the fuzzy set
    2. Apply hedge functions to computed <Degree of Membership> values
for each <Rule> in the fuzzy model
    1. Apply logic operators that connect linguistic variables to find each rule's matching degree

Figure 3.3: Fuzzy matching algorithm.

Inference: An inferred conclusion for the rule is calculated based on its matching degree. The membership function of the consequent linguistic variable is suppressed depending on the degree to which the rule is matched (Yen and Langari, 1999). There are two different methods for suppressing the membership function of the consequent:

1. The clipping method.
2. The scaling method.

Fuzzy Inference Engine supports both implication function types.

Combination: A final conclusion is generated by superimposing all fuzzy conclusions about a variable. Supported aggregation functions are:

1. The maximum operator.
2. The sum operator.
3. The probabilistic-or operator.

Defuzzification: To produce output maps, a crisp output is needed to specify the value of each pixel in the raster map. Therefore, the combined fuzzy conclusion is converted into a crisp conclusion.

Properties of the Fuzzy Inference Engine are:

1. Assumes the inputs are crisp.

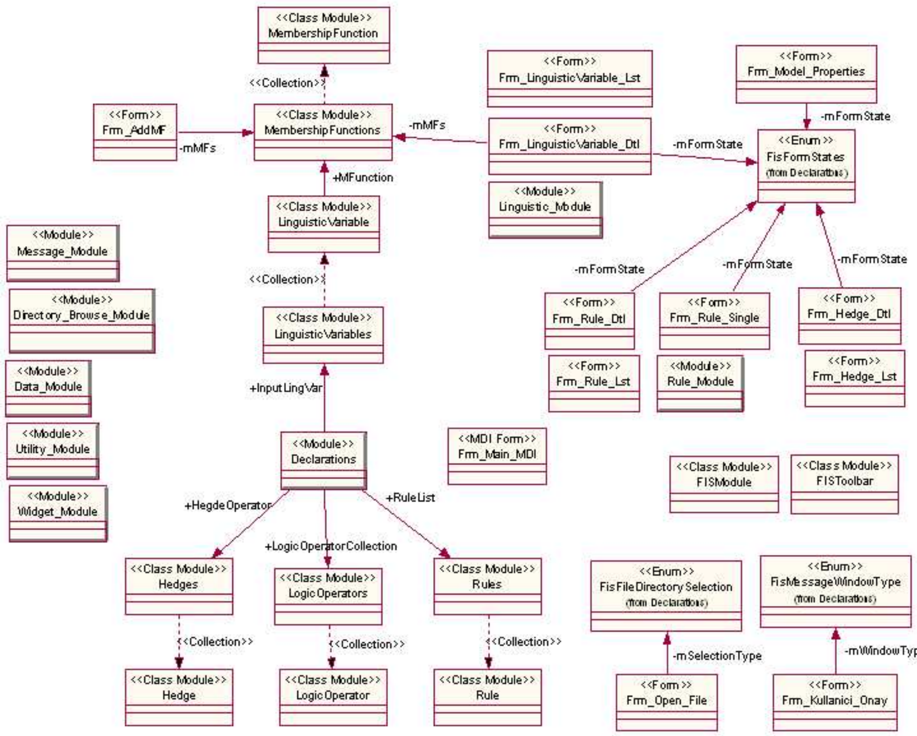


Figure 3.4: Class diagram for Fuzzy Inference System Module.

2. Can use either clipping inference or scaling inference method.
3. Supports the use of maximum, addition or probabilistic-or aggregation function to combine conclusions of fuzzy rules.
4. Does not insist on a specific defuzzification method.
5. The consequent part of the fuzzy mapping rules can be defined both as a fuzzy set and as a crisp value.

The foundations of fuzzy mapping rules and detailed discussion on the fuzzy rule based inference are given in Section 2.8.2.1.

3.3 Design of Fuzzy Inference System Module

Fuzzy Inference System Module acts as a bridge between user and the Fuzzy Inference Engine. Fuzzy Inference System Module class diagram is shown in Figure 3.4.

Fuzzy Inference System Module includes interfaces required to:

- Design membership functions.
- Define linguistic variables.
- Define hedges with their functions.
- Form fuzzy if-then rules.
- Set the fuzzy model properties.

A fuzzy rule-based, expert-like system can be designed to solve a specific problem or for a decision-making process by setting up linguistic variables, fuzzy if-then rules and model properties. The designed system can be used to approximate human knowledge in the problem domain using interfaces of the Fuzzy Inference System Module. Then, the Fuzzy Inference System Module transfers all input parameters to Fuzzy Inference Engine to produce an inferred conclusion.

CHAPTER 4

IMPLEMENTATION DETAILS

This chapter presents the implementation details of Fuzzy Inference System for ArcGIS. After giving a general information on development environment, Fuzzy Inference Engine and Fuzzy Inference System Module implementation details, and interface definitions are presented.

4.1 Implementation Overview

ESRI ArcGIS is a major desktop GIS software. ArcGIS product family includes three desktop applications: ArcMap, ArcCatalog and ArcToolbox. These applications can operate independently, however they are best thought of as three parts of an integrated desktop system.

ArcMap: ArcMap is the map-centric application for all mapping and editing tasks, as well as for map-based analysis. ArcMap provides users an environment in which to display, browse, query, link, and format geographic data.

ArcCatalog: ArcCatalog is the data-centric application that locates, browses, and manages spatial data.

ArcToolbox: ArcToolbox is a complete environment for performing the geoprocessing operations provided by ArcGIS such as data conversion, overlay processing, buffer creation, and map transformation.

As discussed in Section 3.1 ArcGIS is selected for GIS software and ArcMap is enhanced with fuzzy set methodology.

Component Object Model (COM) environment is used for developing fuzzy inference system for ArcMap, where the Component Object Model is a protocol that connects one software component, or module, with another and defining the manner by which objects interact through an exposed interface.

Since applications can interact with objects only through their public interfaces, Fuzzy Inference Engine and Fuzzy Inference System Module can be used through their interfaces. Detailed discussion on Fuzzy Inference Engine interfaces and Fuzzy Inference System Module interfaces are given in Section 4.1.1 and Section 4.1.2 respectively.

The implementation of the fuzzy inference system tool for a commercial Geographic Information System product, ArcMap is divided into two parts:

1. Fuzzy Inference Engine implementation.

2. Fuzzy Inference System Module implementation.

4.1.1 Fuzzy Inference Engine Implementation

Microsoft Visual C++ Version 6.0 is used for Fuzzy Inference Engine development. In the Fuzzy Inference Engine implementation, classes that a fuzzy system has to support are designed and implemented. Implemented classes are depicted in Figure 3.2. Every class has parametrized constructor and a destructor. Copy constructor and assignment operator is also provided in each class. Each member variable in classes has the two characteristic member functions:

SetXXX(arg ...): Set the member variable XXX with the value given in the argument.

GetXXX(): Get content of the member variable XXX.

Fuzzy Inference Engine is designed as a precompiled library. Since Component Object Model is selected as development environment, functionality provided by Fuzzy Inference Engine is used via its interfaces. As shown in Figure 4.1 Fuzzy Inference Engine has three different interfaces:

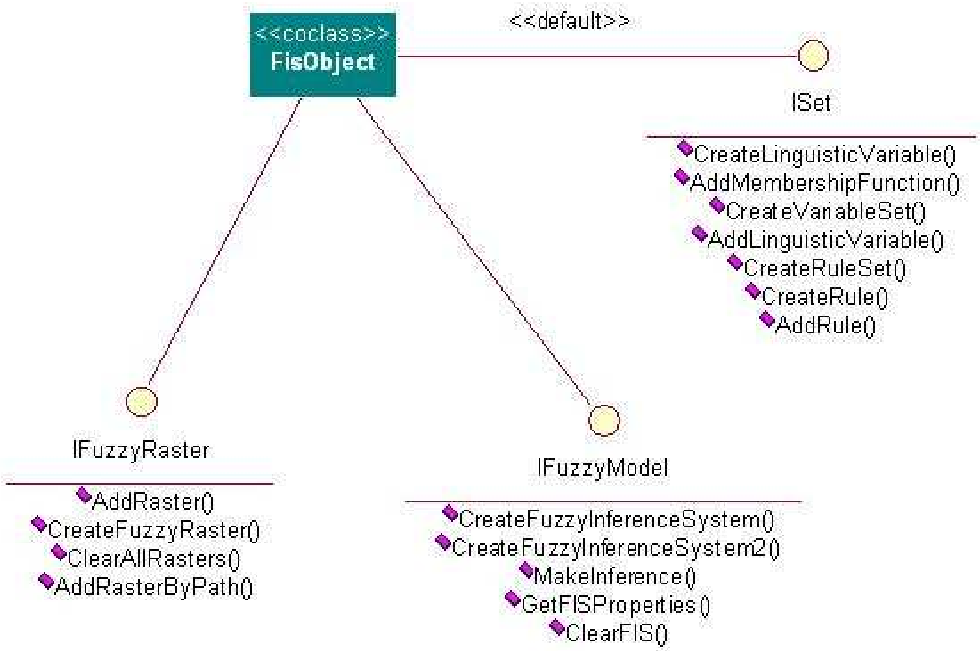


Figure 4.1: Interface diagram for Fuzzy Inference Engine.

1. ISet Interface: Provides access to members that create membership functions, linguistic variables, variable sets, rules and rule sets. Members of the interface are:

   AddLinguisticVariable: Adds a linguistic variable to variable set.

   AddMembershipFunction: Adds a membership function to linguistic variable.

   AddRule: Adds a rule to rule set.

   CreateLinguisticVariable: Creates a new linguistic variable in the domain of concern.

CreateRule: Creates a new rule that describes a mapping relationship between inputs and output.

CreateRuleSet: Creates a rule set, which will represent human knowledge about a specific problem.

CreateVariableSet: Creates a new linguistic variable set, which will define a set of linguistic variables for a specific domain.

Figure 4.2 shows the appropriate interface call sequence.

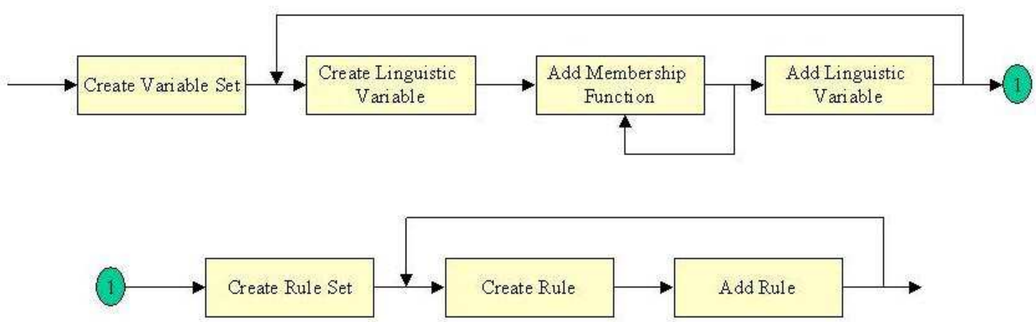


Figure 4.2: Interface call sequence diagram for Fuzzy Inference Engine ISet interface.

2. IFuzzyModel Interface: Provides access to members that create fuzzy inference system, get properties of fuzzy inference system, make inference and clear fuzzy inference system. Members of the interface are:

ClearFIS: Deletes the fuzzy inference system created by this interface.

CreateFuzzyInferenceSystem: Creates a new fuzzy inference system by setting all the parameters explicitly.

CreateFuzzyInferenceSystem2: Creates a new fuzzy inference system by using FISPropertiesType structure.

GetFISProperties: Gets the properties of the fuzzy inference system.

MakeInference: Makes inference based on defined rules and input value list.

Figure 4.3 shows the appropriate interface call sequence.

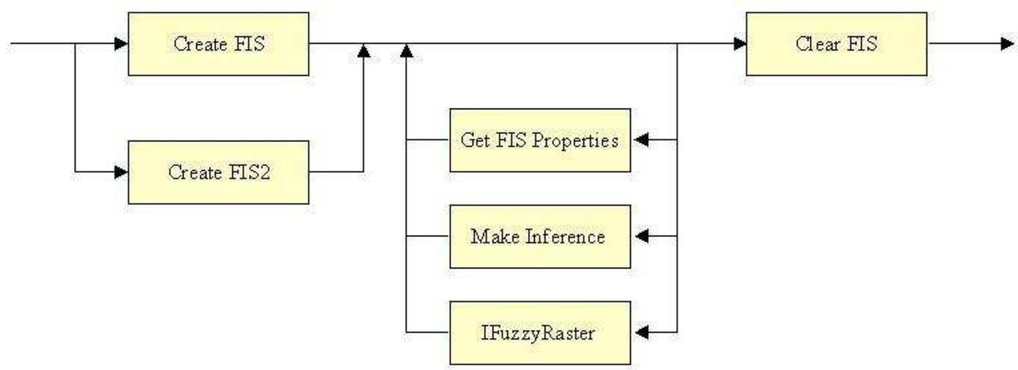


Figure 4.3: Interface call sequence diagram for Fuzzy Inference Engine IFuzzyModel interface.

3. IFuzzyRaster Interface: Provides access to members that create inferred conclusion based on defined fuzzy inference system and given input raster maps. Members of the interface are:

   AddRaster: Adds a new input raster map by reference to the raster map.

   AddRasterByPath: Adds a new input raster map by its full path in the system.

   ClearAllRasters: Deletes input raster maps from the fuzzy inference system and clears memory blocks allocated.

   CreateFuzzyRaster: Creates an inferred conclusion in the form of an output raster map. Figure 4.4 depicts algorithm for creating an inferred output raster map in pseudo code.

   Figure 4.5 shows the appropriate interface call sequence for IFuzzyRaster interface.

Figure 4.6 shows the appropriate interface call sequence to use fuzzy inference engine functionality.

Algorithm 2 Algorithm to Create Inferred Output Raster Map.

Create a new empty raster map for output
for each <Input Raster> map
  1. Create a memory block
  2. Read <Input Raster> map pixel block to allocated memory block

for each <Band> of output raster map
  1. Create a memory block to store resulting pixel block
  2. **for each** <Pixel> of output raster map
     1. Infer a conclusion by using pixel values of input raster maps of the same location
  3. Write pixel block in memory to file

Figure 4.4: Algorithm to create inferred output raster map.

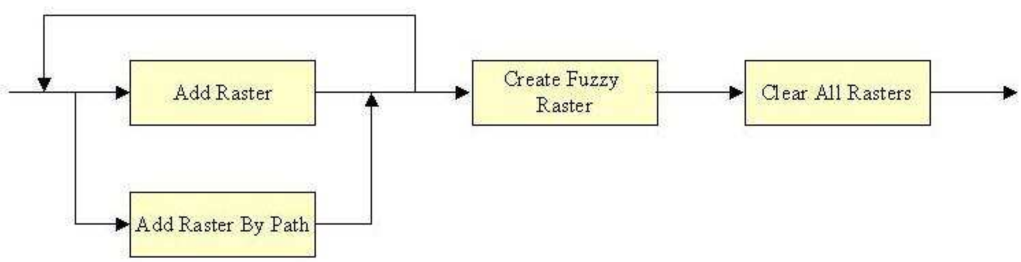


Figure 4.5: Interface call sequence diagram for Fuzzy Inference Engine IFuzzy-Raster interface.

4.1.2 Fuzzy Inference System Module Implementation

Microsoft Visual Basic Version 6.0 is used for Fuzzy Inference System Module development. In the Fuzzy Inference System Module implementation, user interfaces required to create a fuzzy inference system and interaction with ArcMap are designed and implemented. Implemented classes, modules and forms are shown in Figure 3.4.

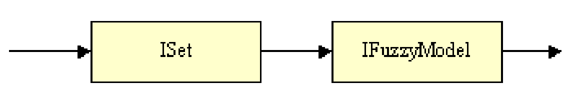


Figure 4.6: Interface call sequence diagram for Fuzzy Inference Engine.

For each form, similar design patterns are followed to standardize user interfaces and similar coding scheme is used to trace code easily. Fuzzy Inference Engine, ESRI ArcObjects precompiled libraries and COM components are used by Fuzzy Inference System Module. COM components used in the Fuzzy Inference System Module, but not mentioned here, are not related with the scope of this thesis. Interactions with other objects are shown in Figure 4.7. Fuzzy

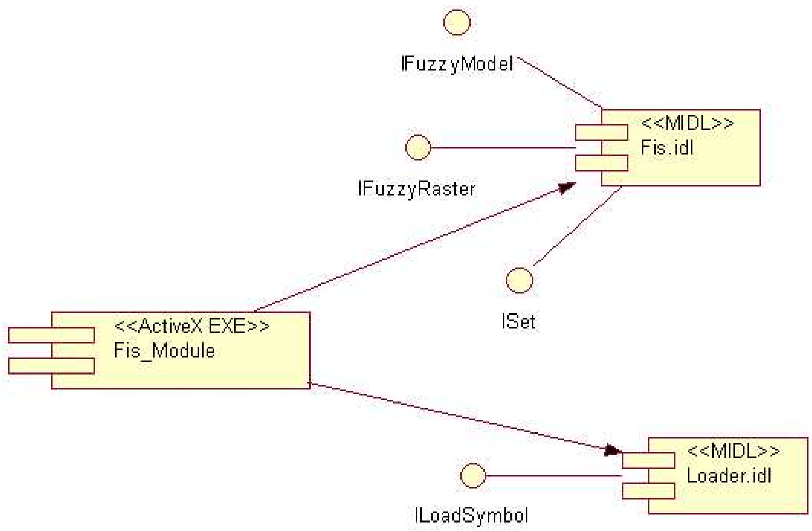


Figure 4.7: Object interactions in Fuzzy Inference System Module.

Inference System Module has a public property named “MapDocument”, and a method named “ShowMainMDI”.

Property MapDocument: Used to set reference of MxDocument of ArcMap. MxDocument represents the current ArcMap document. IMxDocument interface is a starting point for much of the other objects in ArcMap. For example, IMxDocument interface provides access to the current active view, the currently selected map, all of the maps displayed, and the style gallery (ESRI, 2001).

Method ShowMainMDI: Starts execution of Fuzzy Inference System Module.

4.2 Technical Specifications of Fuzzy Inference System

The proposed system was developed using the commercial GIS package ArcGIS 8.1, which runs at Microsoft Windows NT 4.0 Workstation (with Service Pack 6a) operating system. The developed system supports Microsoft Windows NT 4.0 Workstation, Microsoft Windows 2000 and Microsoft Windows XP operating systems. 10 MB or more free disk space is required to accommodate the system. The developed system does not insist on any hardware requirements other than the disk space. Installation notes are given in APPENDIX A.

The performance of the system is mostly affected by the size of the input raster maps (i.e., width and height in pixel), the number of input raster maps used in spatial analysis and the number of rules defined. Since operations to infer a scalar output value are mostly based on computations, the higher rates of CPU is recommended. In order to operate on input raster maps, each one of the input raster map is read to the memory, block by block. Hence, large raster maps (e.g., 50 MB or more) degrades the performance of the system. When there is a need to operate on large raster maps, it is recommended that the higher the memory and the faster the disk, it is better.

It is recommended that all input raster maps are in the same spatial reference and have exactly equal extends. Moreover, output raster map, containing inferred results in its pixel values, is created in the same spatial reference and the same extend as the first input raster map added to the Fuzzy Inference Engine.

CHAPTER 5

CASE STUDY

This chapter presents the operation of fuzzy inference system for cell-based information modeling. Operation of fuzzy inference system is exemplified through classification process and decision-making process.

5.1 Applications

To exemplify the operation of the system Digital Elevation Model (DEM) and vector maps, describing roads and town, are used for input to classification process and decision-making process. In the sequel the developed system is used for classification and taking decisions in decision-making processes.

5.1.1 Classification

Classification is defined as identification of a set of features as belonging to a group (Aronoff, 1989). In the classical sense in order to test for belonging to a group, each group is separated from other groups with sharply defined intervals. For example, in a raster-based GIS, a cell is assigned to a group if value of the cell is between the values describing that group. However, it is very difficult to work with vague concepts, which are easily comprehended by humans. The developed system can be used to classify the study area into classes, which are

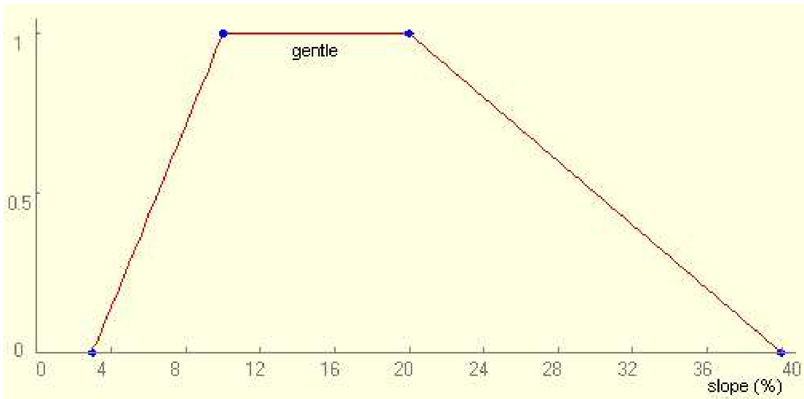


Figure 5.1: Membership function for linguistic term gentle.

defined as linguistic terms (i.e., classes do not have sharply defined intervals). In the sequel the study area is classified as linguistically defined terms “gentle”, “southern” and “close” to exemplify and test the operation of the system.

Example 5.1 To characterize a value of the slope by a natural label “gentle”, it is necessary to define the meaning of the term “gentle”. Suppose that the meaning of the term “gentle” is defined as shown in the Figure 5.1. The main purpose of the system is to assist the user to take decisions using experts' experiences in the decision-making process. Experts' knowledge are captured by fuzzy if-then rules which are in the form of IF A THEN B where A and B are terms with a fuzzy meaning. Therefore, system expects rules to be defined completely. Up to now, antecedent of the rule is constructed as “IF slope is gentle”. Since “IF slope is gentle THEN ?” is not a valid rule, what actions will be taken when the rule's antecedent is partially satisfied is not known. The consequent of the rule must also be defined. One possible consequent variable is “suitable” with meaning depicted in Figure 5.2. There are plenty of choices for consequent variable and its membership function (i.e., meaning) depending on the problem or even depending on wish. The scale of the consequent linguistic variable “suitable” is selected as $[0, 100]$. Output values will lie in the scale of the consequent variable. Depending on the scale and the meaning of the output variable (i.e., suitable) used in the example, in the output raster map pixel values

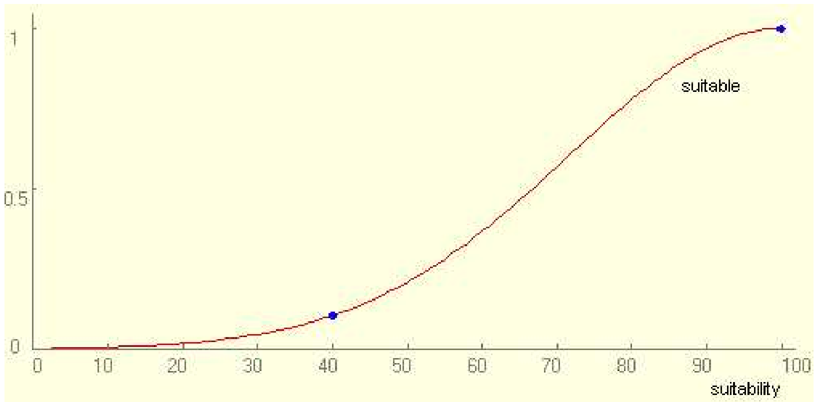


Figure 5.2: Consequent variable “suitable” and its meaning.

close to 100 means pixel definition is close to linguistic term “gentle”. Finally, to classify slope map of the study area, the rule

$$\text{IF slope is gentle THEN site is suitable.} \tag{5.1}$$

is used. Model properties are selected as shown in Table 5.1. Input slope map

Table 5.1: Model properties for classification of slope map using one rule

Model Properties	
AND operation	Minimum operator
OR operation	Maximum operator
Implication	Minimum operator
Aggregation	Maximum operator
Defuzzification process	Mean of maximum (MOM) defuzzifier

of the study area is derived from DEM and is depicted in Figure 5.3. Result of the classification based on the Rule (5.1) and the model properties listed above is depicted in Figure 5.4. In the fuzzy result map, upper parts of the region mostly has “gentle slope” with varying degrees. Higher pixel values imply pixel definition is more close to “gentle”. All pixels in the input slope map are then classified as “gentle” with varying grades. Each pixel value in the output map defines the grade of “suitability” to describe the cell as having a “gentle slope”. The exact Boolean expression of the Rule (5.1) is:

$$\text{IF cell has slope value between 10\% and 20\% THEN cell is defined as gentle.} \tag{5.2}$$

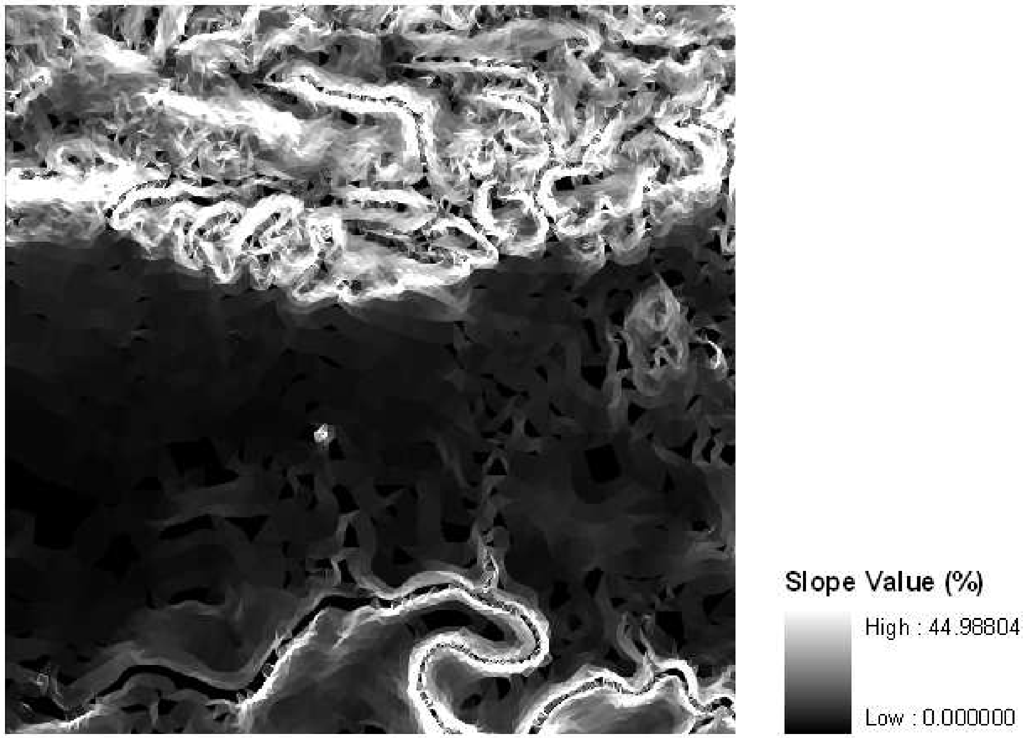


Figure 5.3: Slope map of the study area.

Result of applying Boolean expression to the input slope map is depicted in Figure 5.5. A location with slope equal to 10.1% is characterized as "gentle", while another location with slope equal to 9.9% is not. For decisions based on multiple criteria, it is usually the case an entity which satisfies majority of constraints posed by decision-maker and is marginally rejected in only one of them to be selected as valid by decision-maker. However, based on crisp logic a location with 9.9% will be rejected, even it satisfies all other constraints. Gray areas in Figure 5.4 (i.e., fuzzy result) represent locations that partially satisfy the constraint where these locations are excluded in Figure 5.5 (i.e., Boolean result). Specifically, 31.4% of the total area is represented by different tones of gray (i.e., locations that partially satisfy the constraint) in the fuzzy result map, however these locations are characterized as "not gentle" in the Boolean result map. Suppose that if Figure 5.4 (i.e., fuzzy classification) is classified as locations equal to 100 are assigned to 1 and others are assigned to 0, the

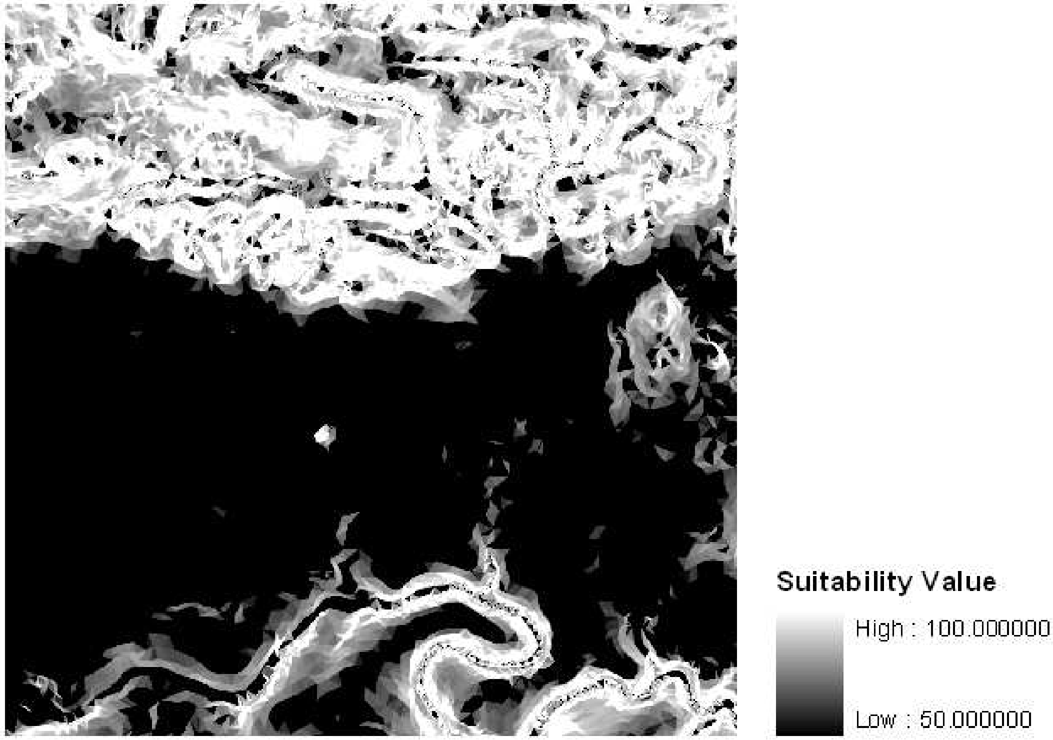


Figure 5.4: Fuzzy result map showing “gentle slope”.

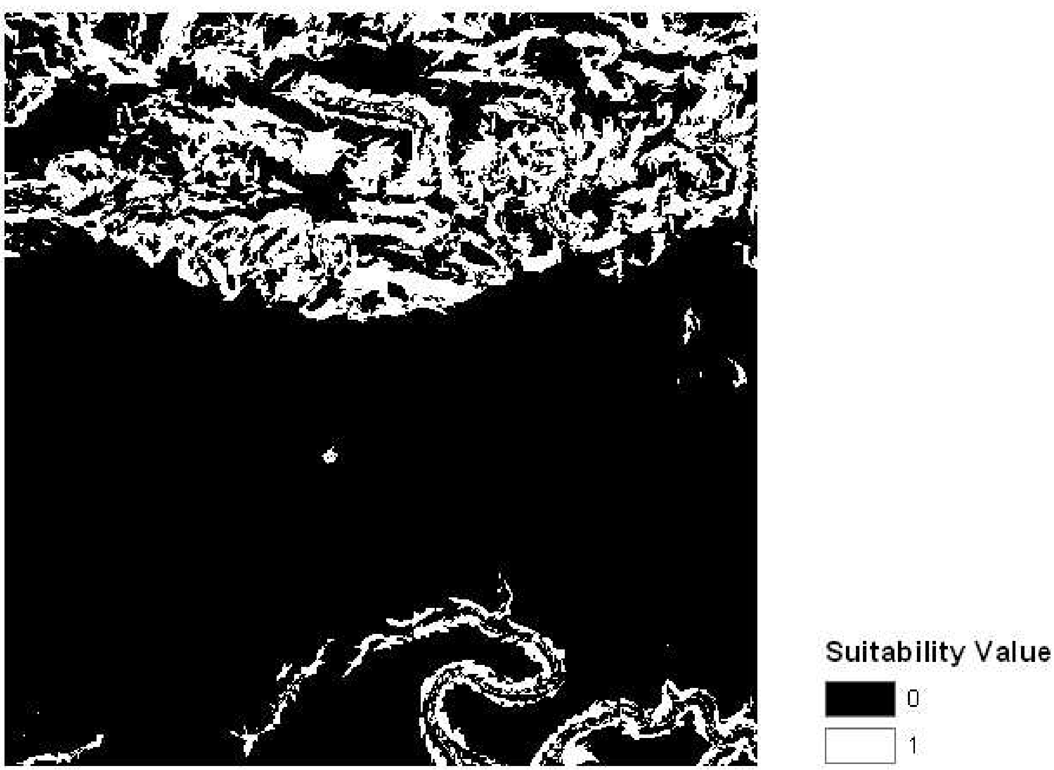


Figure 5.5: Classified locations according to Boolean expression.

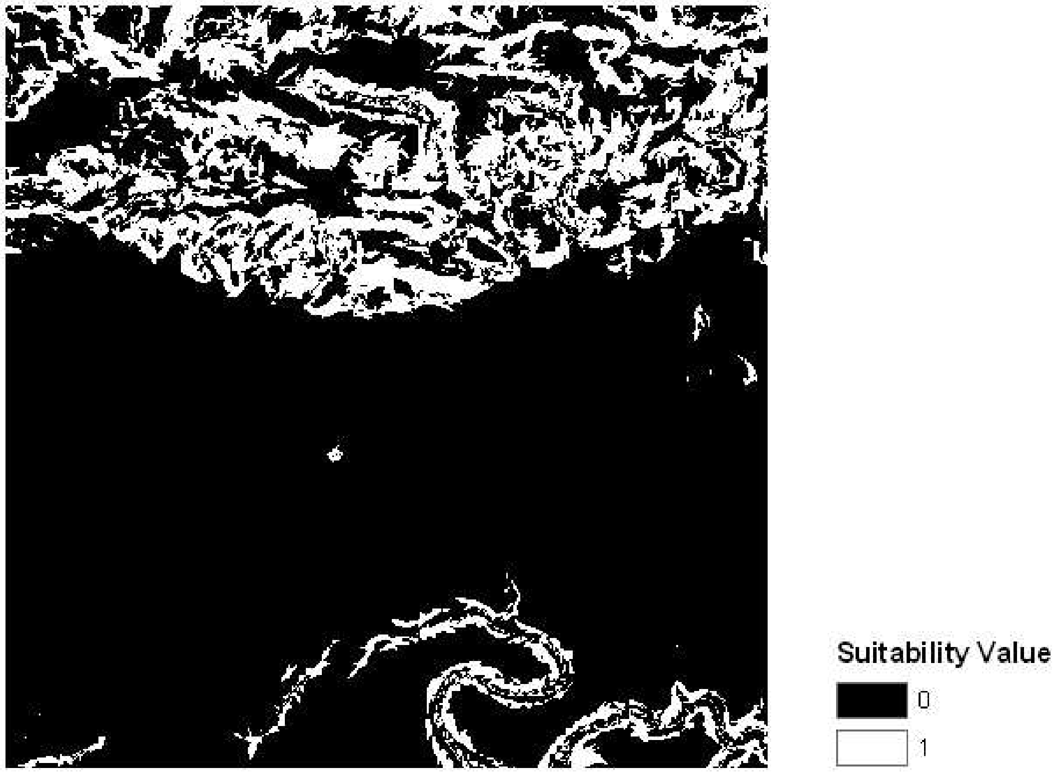


Figure 5.6: Classified fuzzy ("gentle slope") map.

result is the same as applying Boolean expression (5.2) to the input slope map. Figure 5.6 illustrates result of classification of fuzzy result. As it is easily seen that Figure 5.5 and Figure 5.6 are the same. Therefore, the result of fuzzy classification includes the result of the conventional classification (i.e., crisp logic in classification). In addition, fuzzy classification gives better results because all the pixels of input slope map contribute to the answer of the rule with a grade.

Note that scale of the consequent linguistic variable differs from the scale of the fuzzy output map shown in Figure 5.4. Because, values in the output map are in the range of $[50, 100]$. Membership functions, their parameters, model properties and defined rules affect values and their range in the produced output.

A fuzzy model describes functional mapping relationship from a set of input variables to a set of output variables using a set of fuzzy if-then rules. Fuzzy model in Example 5.1 defines a relationship from input linguistic variable "slope"

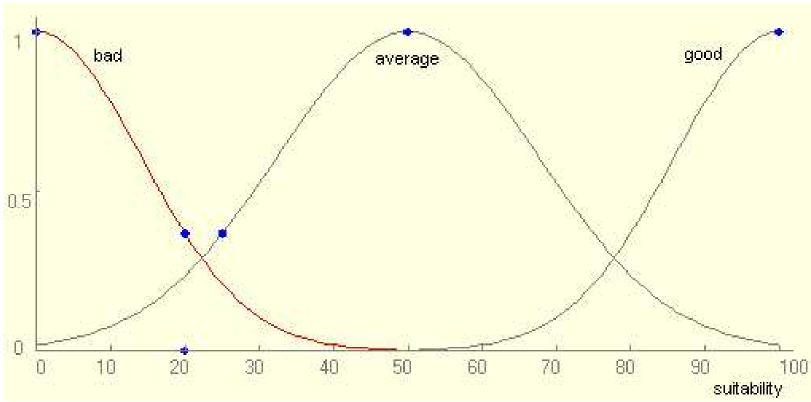


Figure 5.7: Meaning of output linguistic variable "suitable".

to output linguistic variable "site". The antecedent of a fuzzy model is defined by fuzzy partitions of input space. Generally, a fuzzy partition of an input space is a set of fuzzy subspaces whose boundaries partially overlap and whose union is the entire input space (Yen and Langari, 1999). However, input space "slope" in Example 5.1 is partitioned only in one subspace "gentle" which is defined between 3% and 40% with a core area of 10% and 20%. Therefore, other possible values for linguistic variable "slope" are not included in the model. For instance, a location with slope value equal to 2% is not mapped to any output variable using rules. These locations in Example 5.1 which are not included in the fuzzy subspace "gentle" are mapped to output value 50 (i.e., mean value in the scale of the consequent linguistic variable) based on the selected model properties.

Example 5.2 In this example, the study area is classified as suitability to build a house by using only slope as an input criterion. It is considered that the values of consequent linguistic variable suitability are "good", "average" and "bad". Meaning of output linguistic terms "good", "average" and "bad" are depicted in Figure 5.7. A fuzzy partition of the entire input space "slope" is formed by four fuzzy subregions namely "flat", "gentle", "moderate" and "steep" and their membership functions are given in Figure 5.8. To classify slope map of the study area following rules are used.

IF slope is flat THEN suitability is good. (5.3)

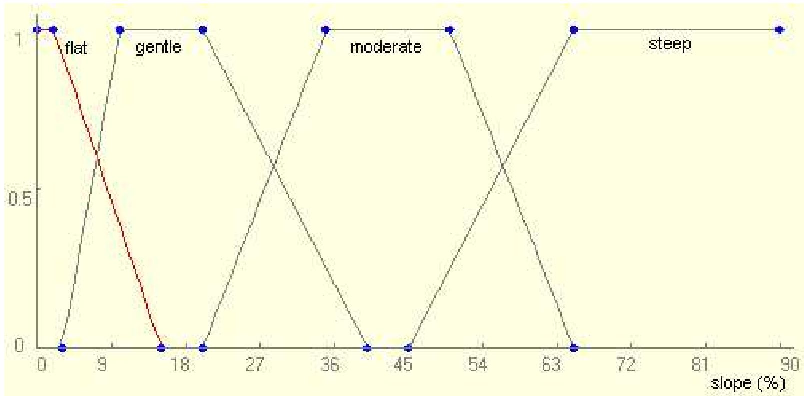


Figure 5.8: Membership functions of linguistic terms “flat”, “gentle”, “moderate”, and “steep”.

$$\text{IF} \quad \text{slope is gentle THEN suitability is average.} \tag{5.4}$$

$$\text{IF} \quad \text{slope is moderate THEN suitability is bad.} \tag{5.5}$$

$$\text{IF} \quad \text{slope is steep THEN suitability is bad.} \tag{5.6}$$

And model properties are selected as shown in Table 5.2. Result of the

Table 5.2: Model properties for classification of slope map using multiple rules

Model Properties	
AND operation	Minimum operator
OR operation	Maximum operator
Implication	Minimum operator
Aggregation	Maximum operator
Defuzzification process	Center of area (COA) defuzzifier

classification based on rules (5.3)-(5.6) and the model properties listed above is depicted in Figure 5.9. As it is easily seen from the result map, areas close to white color represent more suitable places to build a house and these areas are mostly associated with flat slope. Locations represented by dark gray and black color are not as suitable as others are because these areas can be considered as having moderate or steep slope. It is noted that qualified locations derived from the developed system can be available in orderly manner. Decisions based on multiple criteria are discussed in Section 5.1.3. As has been stated previously, membership function types and membership function parameters, model properties and defined rules affect values and their range in the produced

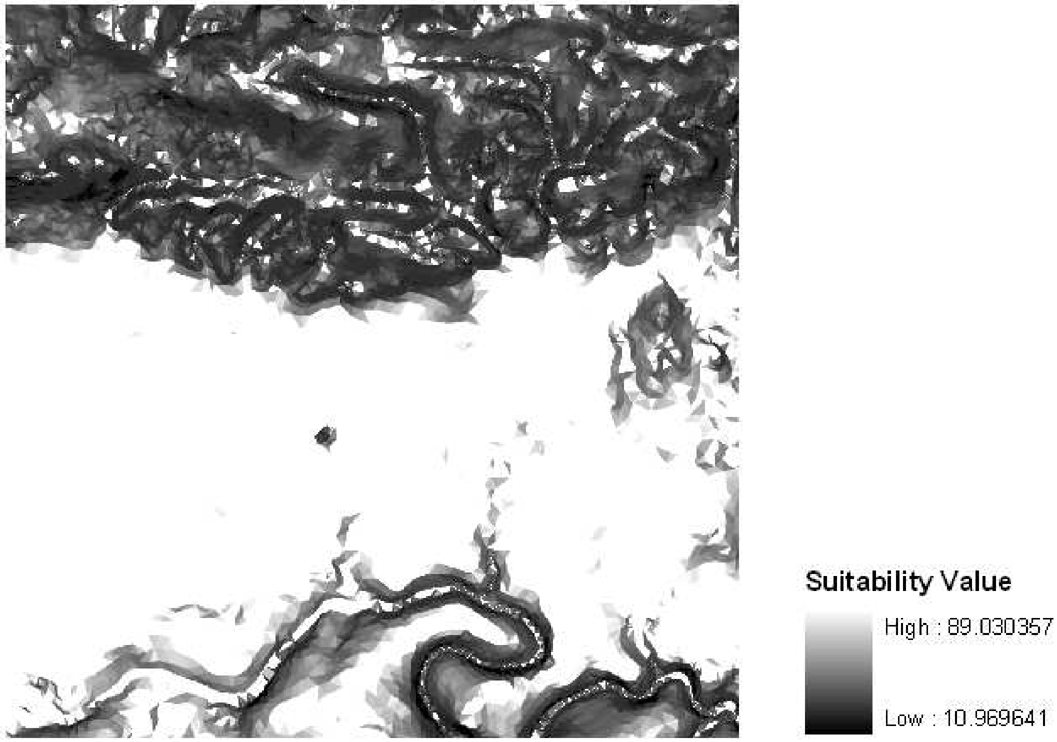


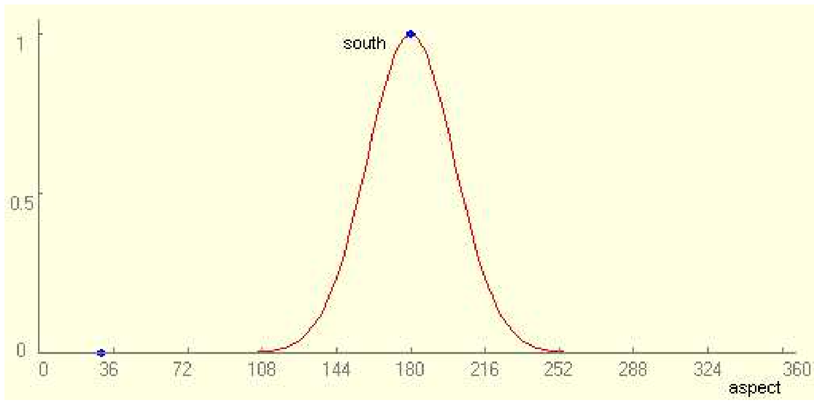
Figure 5.9: The result map showing suitability to build a house based on defined fuzzy model.

output. For instance, if mean of maximum, largest of maximum or smallest of maximum defuzzifier were selected for defuzzification the range of the output values in output suitability map would be $[0, 100]$.

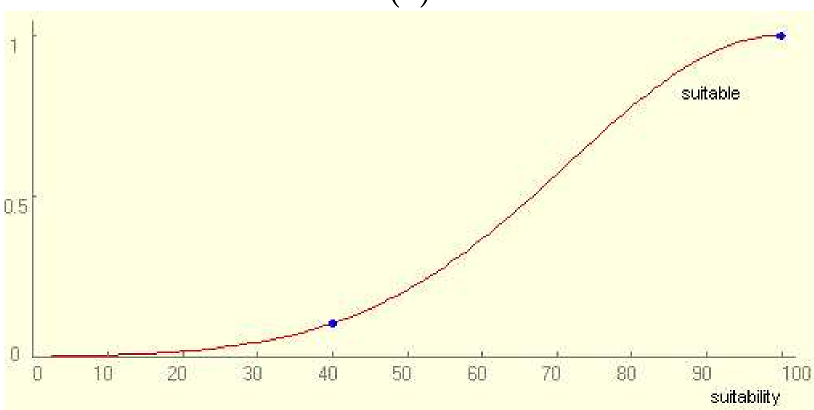
Example 5.3 This example illustrates characterizing a location by a natural label "southern". Southerly exposed locations are classified as "suitable" sites. Suppose that the meaning of the term "southern" and output linguistic term "suitable" are defined as shown in the Figure 5.10. To characterize a location by a label like "southern", aspect map of the study area is produced based on the DEM. Input aspect map of the study area is depicted in Figure 5.11. Classification is based on the rule:

$$\text{IF aspect is southern THEN site is suitable.} \tag{5.7}$$

Model properties are selected as shown in Table 5.3. Result of the classification based on the Rule (5.7) and the model properties is depicted in Figure 5.12. Note



(a)



(b)

Figure 5.10: Membership functions for linguistic term (a) "southern", and (b) "suitable".

Table 5.3: Model properties for classification of aspect map

Model Properties	
AND operation	Minimum operator
OR operation	Maximum operator
Implication	Minimum operator
Aggregation	Maximum operator
Defuzzification process	Smallest of maximum (SOM) defuzzifier

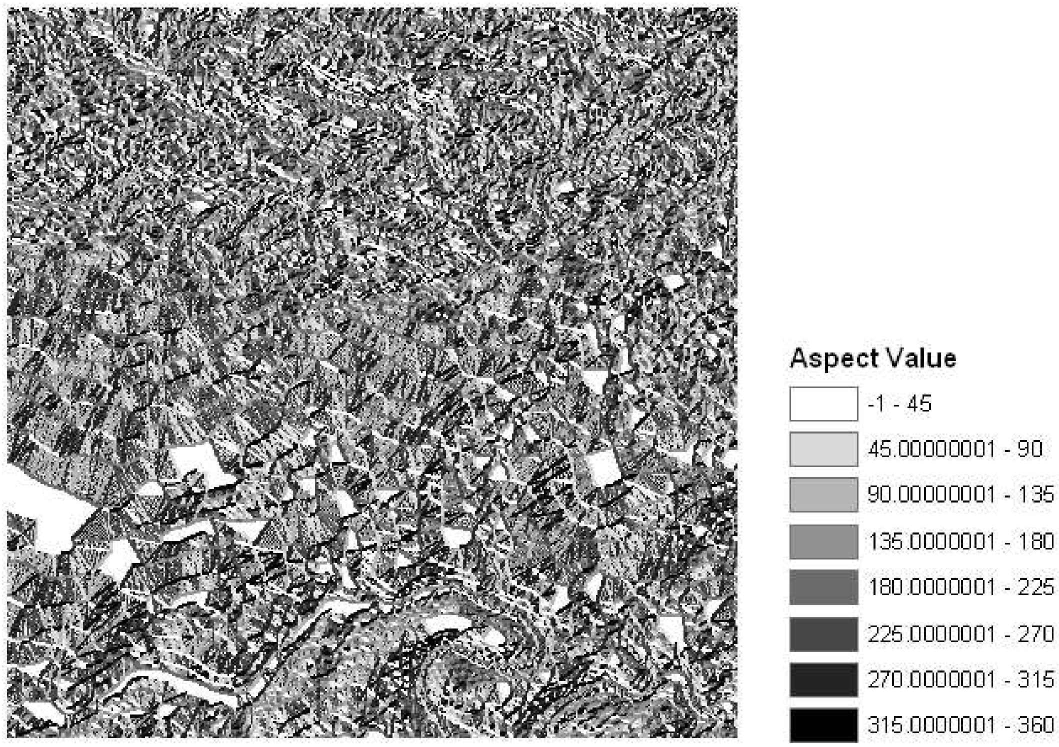


Figure 5.11: Aspect map of the study area.

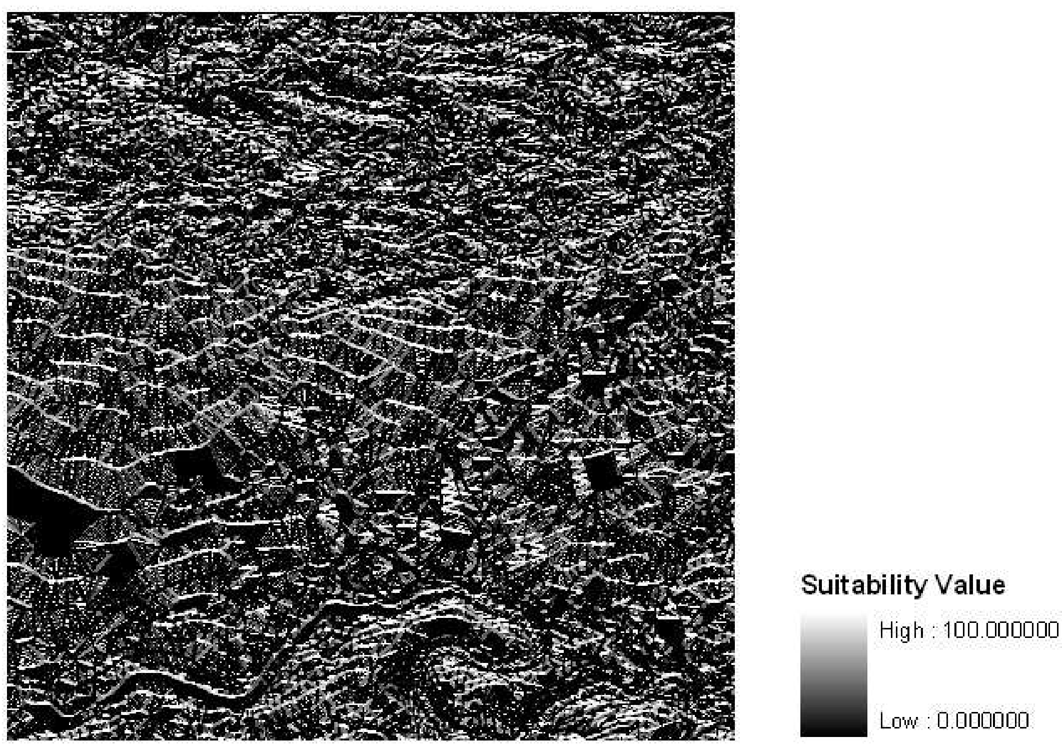


Figure 5.12: The result map showing southerly exposed locations.

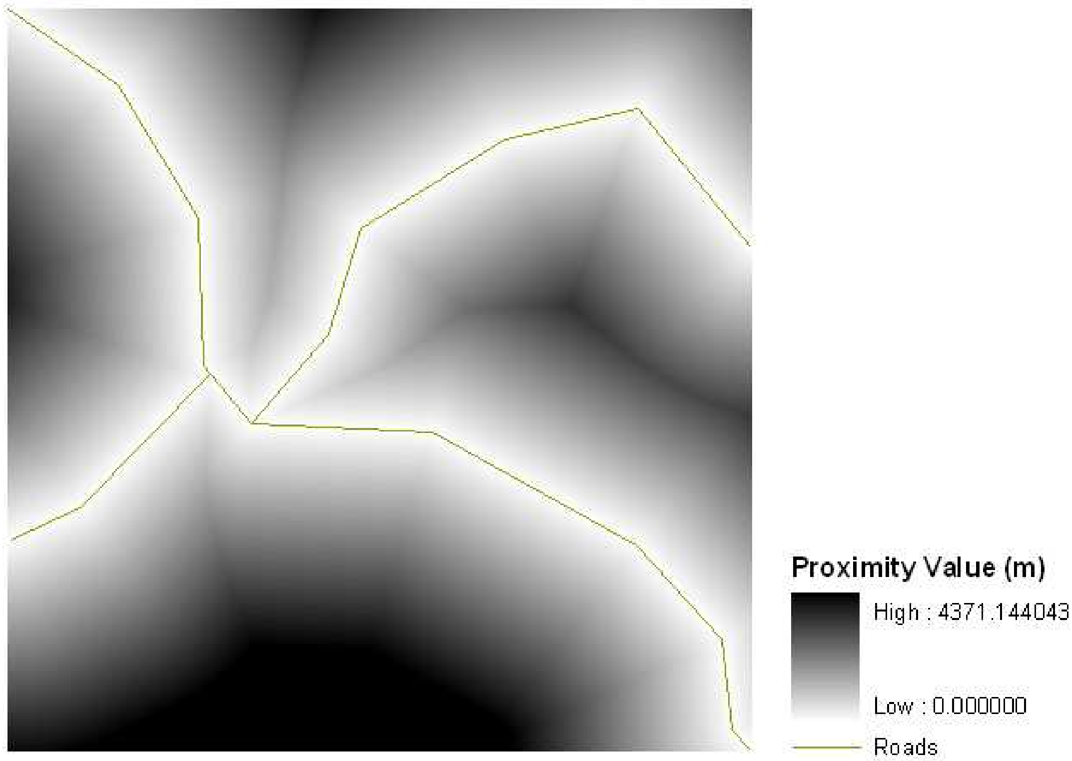


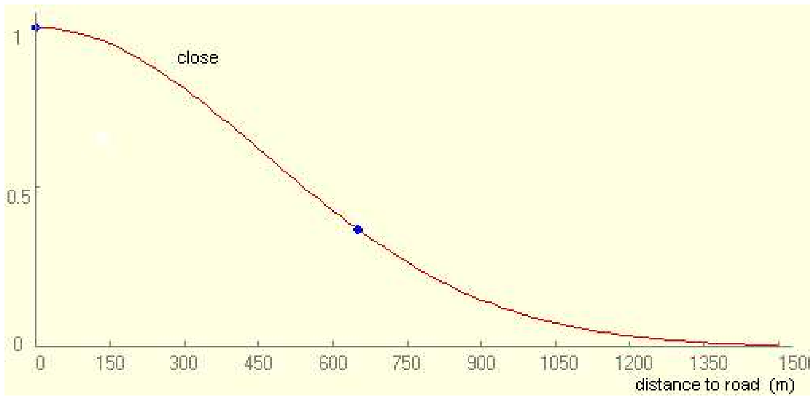
Figure 5.13: Roads and proximity to roads.

that input space is not entirely partitioned as in the Example 5.1. However, the range of the output values in the output fuzzy map is not the same as in Example 5.1. Because the model properties are different from the model properties used in Example 5.1.

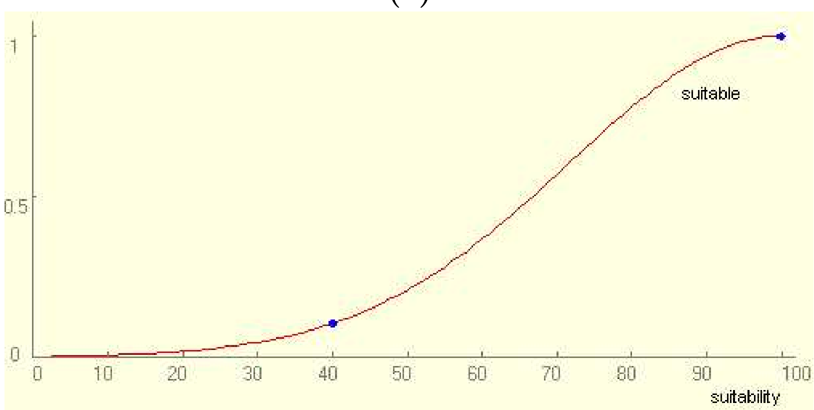
Example 5.4 It is common in most site selection problems to find sites that are close to roads. In classical approach one solution is with creating buffer zones with the defined upper limit for the term "close". This example illustrates fuzzy approach in place of using buffer zones for finding locations "close to roads". Roads in the study area and proximity to roads are depicted in Figure 5.13. Suppose that the meaning of the linguistic term "close" and the linguistic term "suitable" are defined as shown in the Figure 5.14. Model properties are selected as shown in Table 5.4. And classification is based on the rule:

$$\text{IF distance to road is close THEN site is suitable.} \tag{5.8}$$

Result of the classification based on the Rule (5.8) and the model properties



(a)



(b)

Figure 5.14: Membership functions for linguistic term (a) "close", and (b) "suitable".

Table 5.4: Model properties for classification of proximity to roads

Model Properties	
AND operation	Minimum operator
OR operation	Maximum operator
Implication	Minimum operator
Aggregation	Maximum operator
Defuzzification process	Smallest of maximum (SOM) defuzzifier

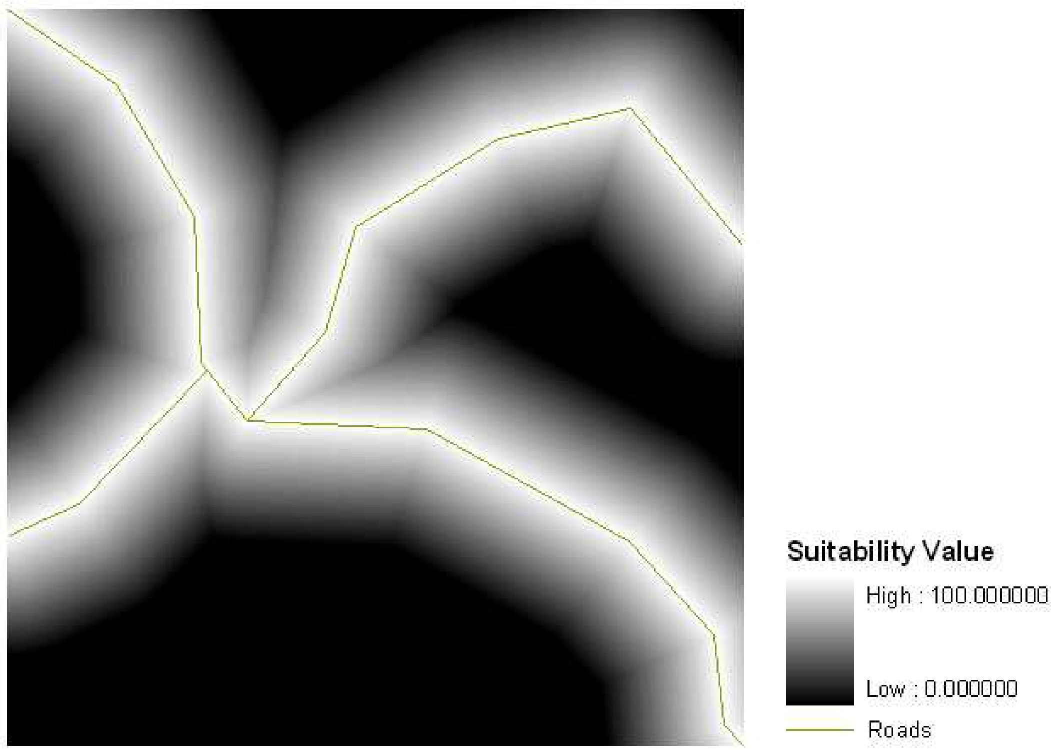


Figure 5.15: Fuzzy result map showing locations close to roads.

listed above is depicted in Figure 5.15. It is seen that the result of applying fuzzy classification resembles the buffer zones. In classical approach every location in the buffer zone has equal degree (i.e., true or 1). In the result of fuzzy classification, locations having grade equal to 100 (i.e., maximum suitability) represent these locations. In addition, fuzzy classification presents locations that partially satisfy the constraint "close to roads". These locations are represented by different tones of gray depending on suitability values. Note that suitability decreases as we moved away from the roads.

5.1.2 Hedges

Meaning of a linguistic term can be modified using linguistic hedges. The developed system provides interfaces for decision-makers to define hedges. After defining hedges, the meaning of a linguistic term in a rule can be modified using hedges.

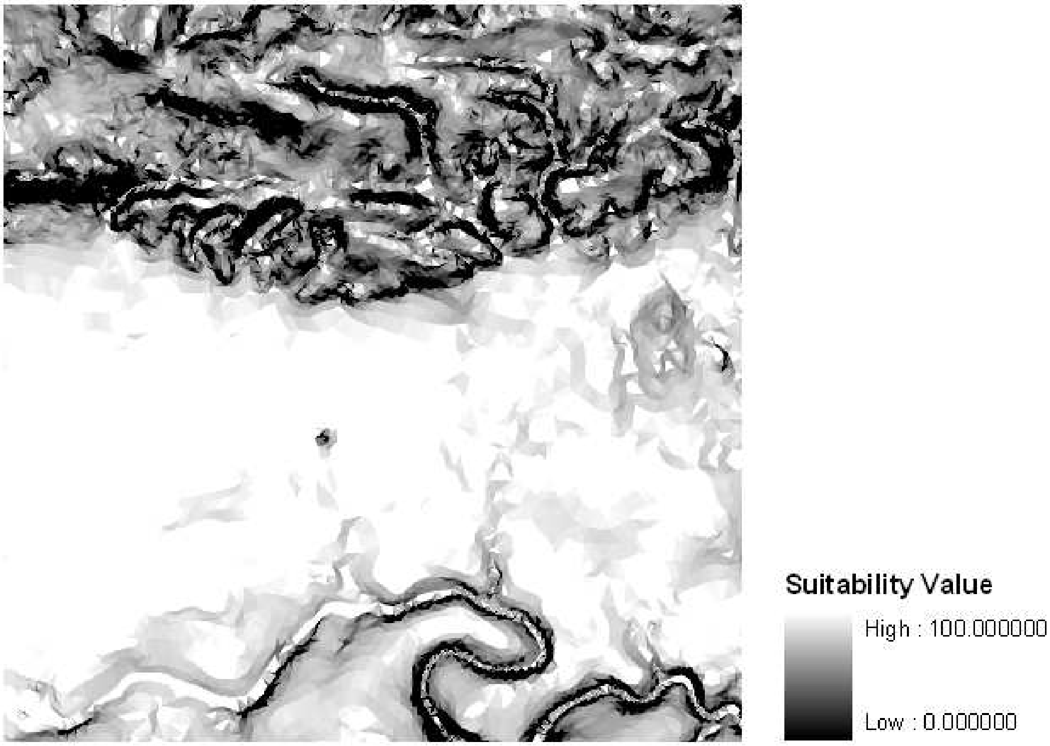


Figure 5.16: Fuzzy result map showing “flat slope”.

Example 5.5 The hedge “very” does not have a well-defined meaning in everyday use. However, in essence the hedge “very” has an intensive effect on linguistic term it operates. In general, definition of hedge “very” is given below:

$$\mu_{Very(F)}(x) = [\mu_F(x)]^2 \tag{5.9}$$

In this example the study area is classified using the Rule (5.10). Membership function of linguistic term “flat” is given in Figure 5.8.

$$\text{IF slope is flat THEN site is suitable.} \tag{5.10}$$

Figure 5.16 illustrates the result of applying Rule (5.10) to the input slope map shown in Figure 5.3. Using the definition of hedge “very” (5.9) the meaning of the term “flat” is modified to obtain a more stringent Rule (5.11).

$$\text{IF slope is very flat THEN site is suitable.} \tag{5.11}$$

Result of the classification based on the Rule (5.11) is depicted in Figure 5.17. Since the hedge “very” has an effect of narrowing the membership function,

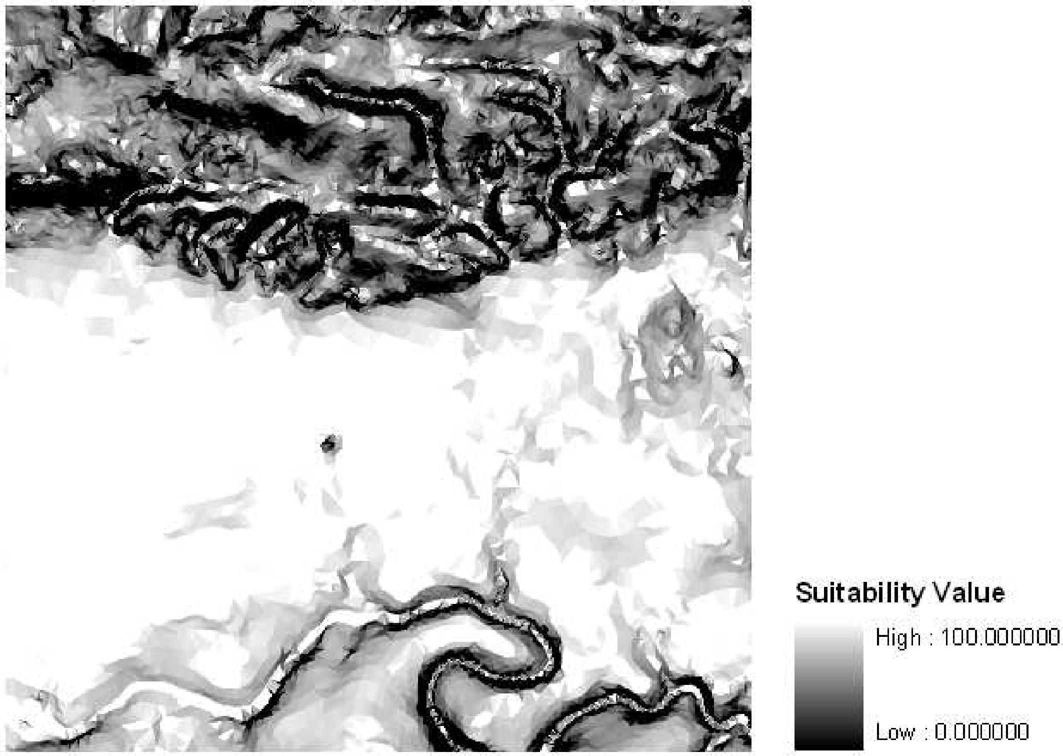


Figure 5.17: Fuzzy result map showing "very flat slope".

value of a specific location shown in Figure 5.17 is less than the value of the same location shown in Figure 5.16. As it is seen, Figure 5.17 is darker than the Figure 5.16, dark locations became darker after adding hedge "very" to the Rule (5.10). Note that it is easier to satisfy the constraint "flat slope" than the constraint "very flat slope".

Example 5.6 In Example 5.4 sites that are "close to roads" are classified as suitable according to the Rule (5.8). Like hedge "very", hedge "too" also has an effect of narrowing the membership function, while hedge "enough" widens the membership function. Because the criteria "too close to roads" should be more stringent than "close to roads", while the criteria for "enough close to roads" should be relaxed. Definitions of hedges "too" and "enough" are listed below:

$$\mu_{Too(F)}(x) = [\mu_F(x)]^2 \quad (5.12)$$

$$\mu_{Enough(F)}(x) = [\mu_F(x)]^{0.4} \quad (5.13)$$

Definitions given above can change depending on the problem, depending on context a linguistic term used, or depending on wish. Using the definitions of hedge “too” (5.12) and hedge “enough” (5.13) the meaning of the constraint “close to roads” is modified to obtain the following rules:

$$\text{IF} \quad \text{distance to road is too close THEN site is suitable.} \tag{5.14}$$

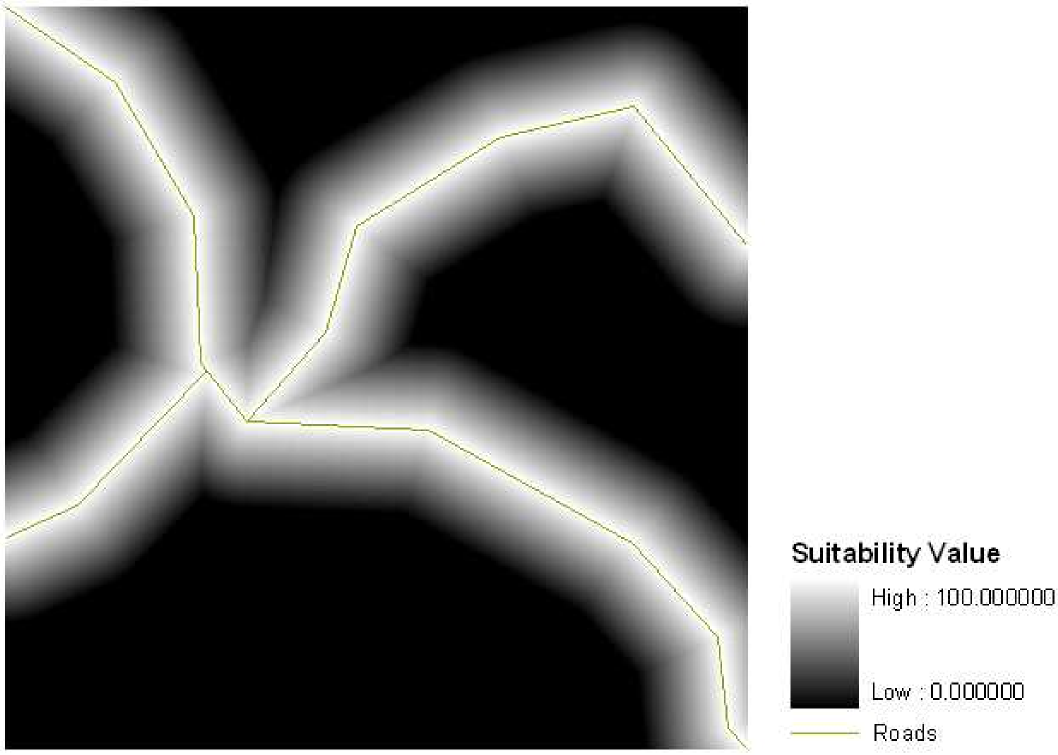
$$\text{IF} \quad \text{distance to road is enough close THEN site is suitable.} \tag{5.15}$$

Figure 5.18 depicts the results of applying rules (5.14) and (5.15) to the proximity map shown in Figure 5.13, respectively. Since it is difficult to satisfy the criteria “too close to roads” than “close to roads”, not all locations that satisfy the criteria “close to roads” satisfy the criteria “too close to roads”. In addition, the degree of satisfying the constraints “too close to roads” and “close to roads” for a specific location is not the same. The degree of satisfying the constraint “too close to roads” is less than the degree of satisfying the constraint “close to roads”. Therefore, the result of adding hedge “too” to modify the meaning of the rule results in narrowing the zone shown in the fuzzy map (Figure 5.15). On the other hand, it is easy to satisfy the criteria “enough close to roads” than “close to roads”. Therefore, more locations satisfy the constraint “enough close to roads”. Moreover, the degree of satisfying the constraint “enough close to roads” is higher than the degree of satisfying the constraint “close to roads”. Hence, hedge “enough” has an effect of widening the zone shown in the fuzzy map (Figure 5.15).

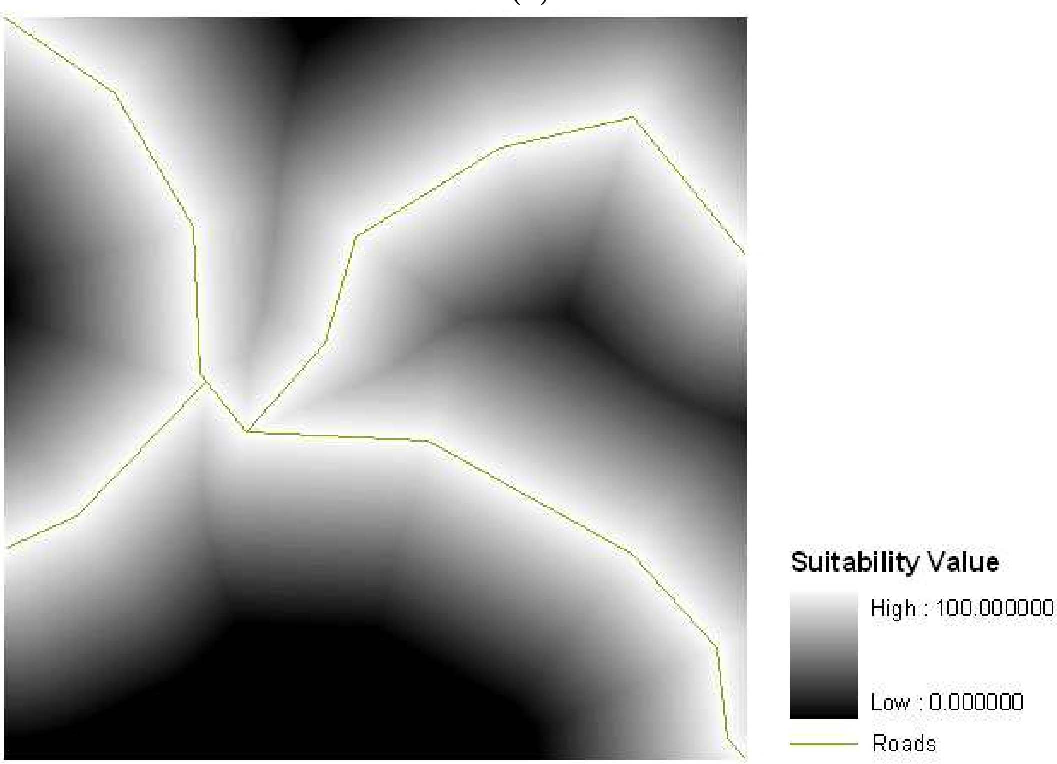
5.1.3 Decision Making

One of the main tasks in GIS is making decisions using information from different maps. The decision-making is affected by many factors and sometimes needs many criteria. In numerous situations involving a large set of feasible alternatives and multiple, conflicting and incommensurate criteria, it is difficult to state and measure these factors and criteria (Malczewski, 1996). Indeed most of the information about the real world contains uncertainties.

Threshold model is a common type of operation in decision-making. In the



(a)



(b)

Figure 5.18: The result map showing (a) "too close to roads", and (b) "enough close to roads".

threshold model, low and high threshold values limit the exact boundaries of criteria. When the underlying logic in GIS is crisp logic, then results of applying threshold values in decision-making processes are 0 or 1. Maps consisting of zero and unity values are produced for each criteria using threshold values that define the meaning of the criteria by only low and high threshold values (i.e., boundaries which are sharp or clear-cut). Then the overall result is obtained through the map overlay. Such models can cause problems since they are inherently rigid. The developed system, on the other hand, can be used to make decisions capturing uncertain information using fuzzy set methodologies. In the sequel a set of criteria is used to select suitable sites for industrial development to exemplify and test the operation of the system.

Example 5.7 To select suitable locations for industrial development humans may pose criteria such as "If site has flat or gentle slope and if site is close to roads and town then site is suitable for industrial development". It is simple for humans to comprehend and make decisions based on these vague terms. However, the conventional GIS cannot answer such vague questions. The exact Boolean criteria for the industrial development site selection is:

$$\begin{aligned}&\text{Site is suitable if (slope } <= 20\%) \text{ and (distance to road } <= 1000 \text{ m)}\\&\text{and (distance to town } <= 5000 \text{ m).}\end{aligned} \tag{5.16}$$

Associated input raster maps are: "slope map" is depicted in Figure 5.3, "proximity to roads" is depicted in Figure 5.13, and map showing "proximity to town" is depicted in Figure 5.19. Boolean answer to question (5.16) is simple. First, for each criterion (i.e., slope, distance to road and distance to town) a map containing 0s and 1s is produced. Pixel values that are less than the threshold values are assigned 1 in the output map and 0 otherwise. Second, the overall result is produced by overlaying these three maps using logical AND operation. Boolean result is depicted in Figure 5.20. The proposed system can be used to find fuzzy answer to site selection problem (see APPENDIX B). In addition, the developed system allows using vague definitions in the criteria. Hence, the

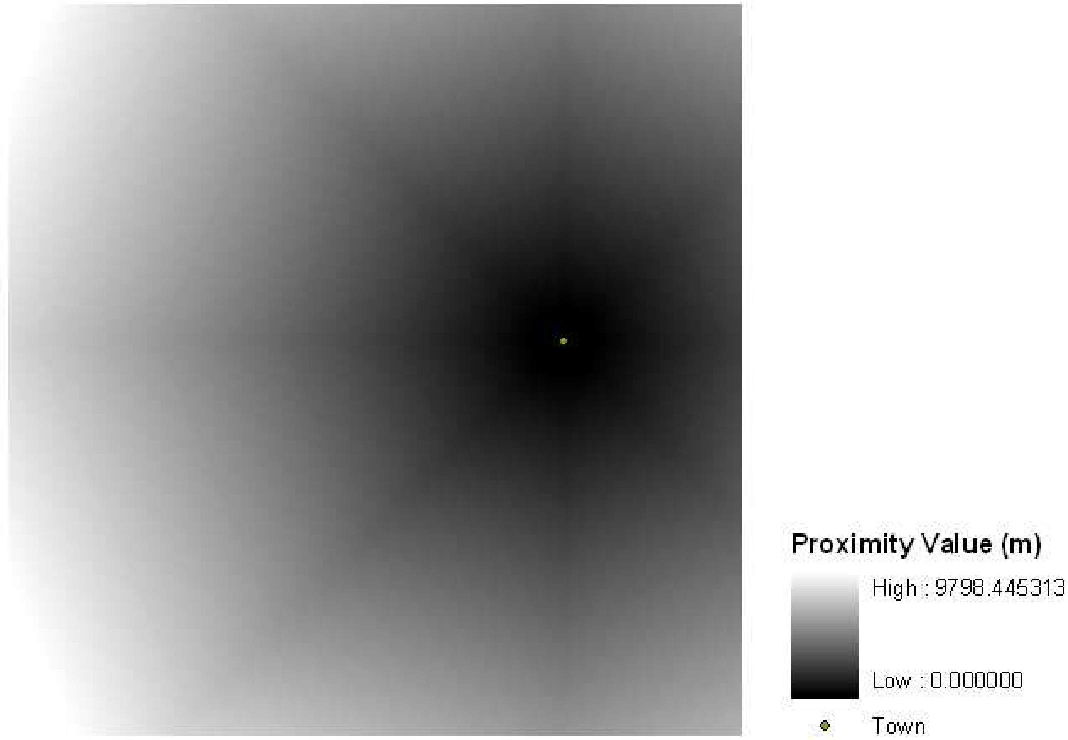


Figure 5.19: “Proximity to town” raster map for decision-making process.

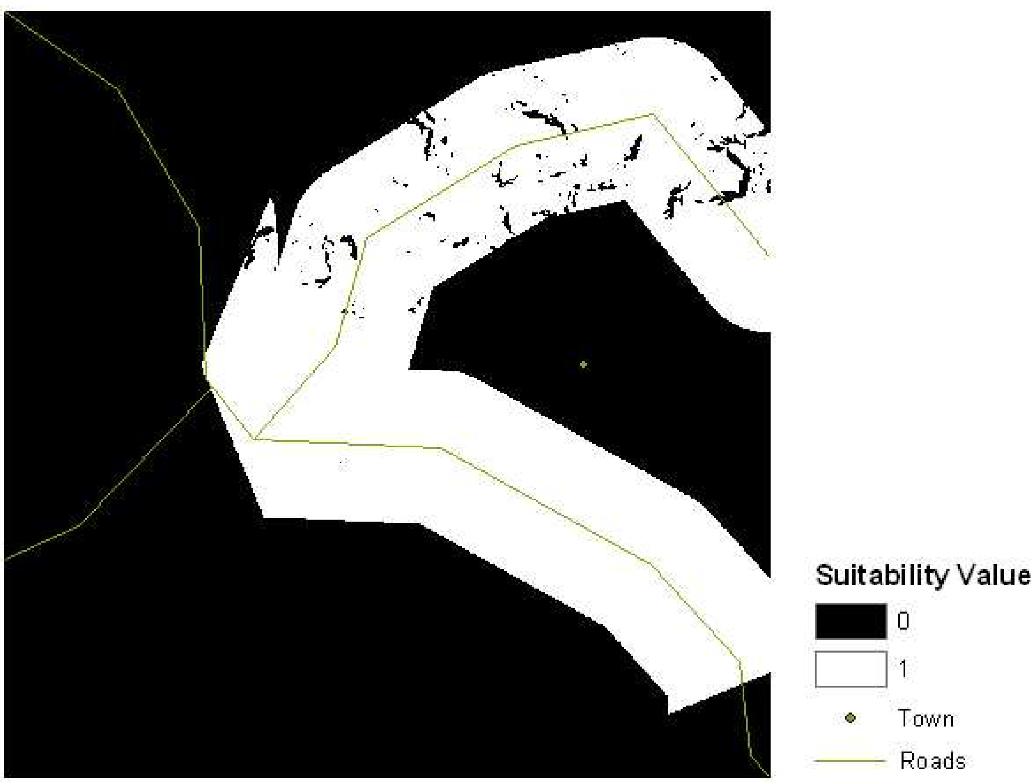


Figure 5.20: Result of Boolean analysis for suitable sites (using AND).

rules listed below are used to approximate conceptual model of the problem in expert's opinion (i.e., human cognition) and generate fuzzy answer to site selection problem:

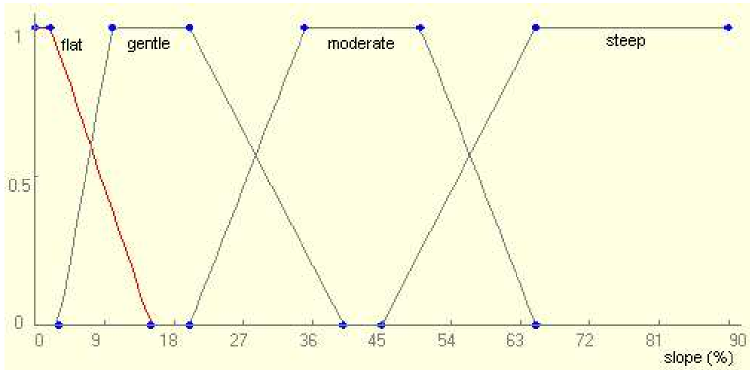
$$
\begin{array}{rl}
\text{IF} & \text{slope is flat and} \\
 & \text{distance to road is close and} \\
 & \text{distance to town is close} \\
\text{THEN} & \text{site is suitable.}
\end{array} \tag{5.17}
$$

$$
\begin{array}{rl}
\text{IF} & \text{slope is gentle and} \\
 & \text{distance to road is close and} \\
 & \text{distance to town is close} \\
\text{THEN} & \text{site is suitable.}
\end{array} \tag{5.18}
$$

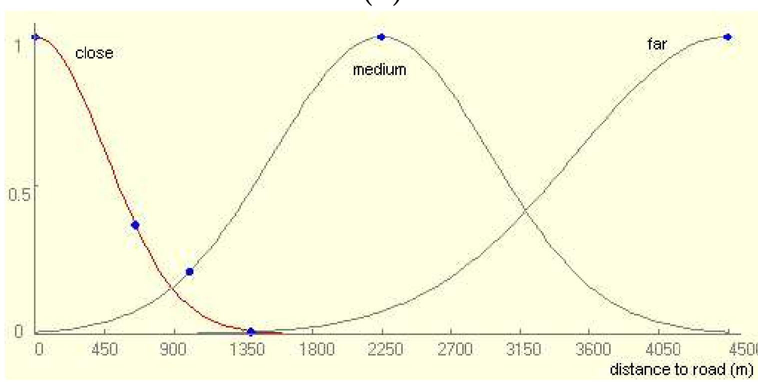
Rules (5.17) and (5.18) approximate what industrial site selection problem means to user by using linguistic terms instead of precise numerical values. These two rules can be further joined to form the rule below:

$$
\begin{array}{rl}
\text{IF} & \text{slope is flat or} \\
 & \text{slope is gentle and} \\
 & \text{distance to road is close and} \\
 & \text{distance to town is close} \\
\text{THEN} & \text{site is suitable.}
\end{array} \tag{5.19}
$$

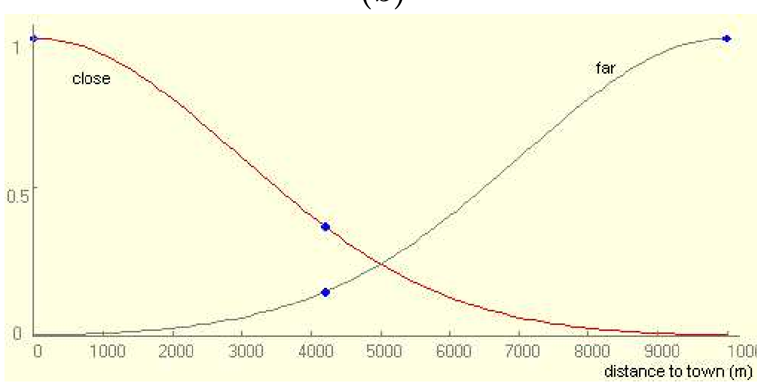
Rule (5.19) is very similar to the problem definition introduced earlier. Membership functions for linguistic terms are depicted in Figure 5.21. Membership functions can be chosen by the user arbitrarily based on user's experience, hence the membership functions for two user could be quite different depending upon their experiences and perspectives. It has to be also noted that linguistic term "close" is used twice, one stands for "close to roads" and other stands for "close to town" and two different membership functions were defined for linguistic term "close". This illustrates the fact that membership functions can be quite context



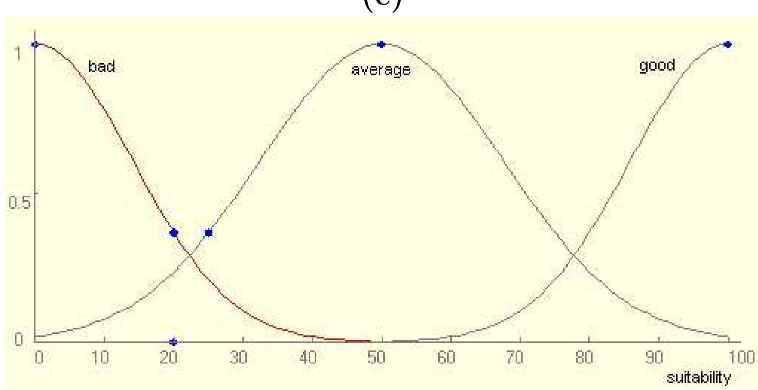
(a)



(b)



(c)



(d)

Figure 5.21: Membership functions for (a) flat and gentle slope, (b) close to roads, (c) close to town, and (d) suitability.

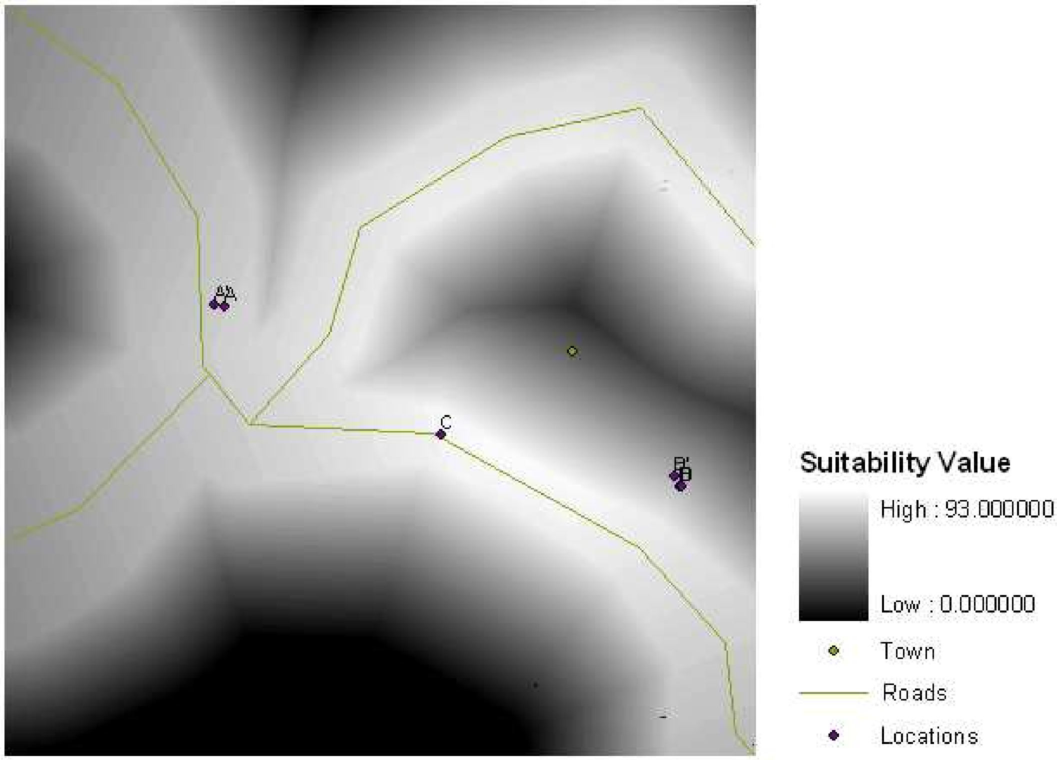


Figure 5.22: Fuzzy result to site selection problem using one rule.

dependent. Fuzzy result produced using two rules (5.17) and (5.18) is the same with the result produced using single rule (5.19). Fuzzy result is depicted in Figure 5.22. It is easily seen that fuzzy result provides a result set of locations whose attribute values partially satisfy the constraints, whereas Boolean result provides only a set of locations whose attribute values satisfy all constraints. When the underlying logic in GIS is crisp logic, locations satisfying all constraints are assigned to unity and others are assigned to 0. However, all pixels in the fuzzy output map have a suitability degree for industrial development based on satisfaction of each criterion. For instance, locations that are not close to town are not included to the Boolean result set. Hence, locations that fail to satisfy criteria "distance to town <= 5000 m" even with 1m are excluded from the result set disregarding their slope and closeness to roads. Consider location labeled A in Figure 5.22. Location A has properties as listed below:

$$\text{slope} = 3\%$$

distance to roads = 300 m

distance to town = 4953.1 m

Since, properties of location A are in the defined threshold values, location A is assigned to 1 in the Boolean result map indicating it is a suitable location. Another location near to point A is labeled as A' and has the following properties:

slope = 2.1%

distance to roads = 190 m

distance to town = 5045 m

Location A' fails to satisfy the Boolean criteria because distance to town is a little bit higher than the defined threshold value. Therefore location A' is assigned to 0 in the Boolean result map indicating it is not a suitable location. Location A' has even better slope value and more close to road but it is classified as unsuitable location based on value of distance to town, note that distance between location A and A' is less than 100 m. On the other hand, the proposed system graded suitability of location A with 77 and graded location A' with 76 (out of 100) for industrial development.

Input maps store information about real world which are continuously changing properties. Applying threshold values in Boolean analysis leads to lose information stored in the input maps. Since, the result of Boolean analysis only consists of 1s and 0s which indicate it is a suitable location or not, the user has no idea about best or worst locations satisfying all constraints. On the other hand, as has been stated previously, fuzzy site selection analysis provides locations in orderly manner; each location has a suitability degree. For example, consider points in Figure 5.22 A, A', B, B', and C. Table 5.5 gives property values and results associated with the locations. It is noted that location A, B, and C are suitable locations according to Boolean analysis. Since values of locations A, B, and C are 1 in the output Boolean map, the user has no idea about which is better for industrial development. Fuzzy result provides this information to user with no further processing requirements.

Table 5.5: Properties of locations and results of Boolean and fuzzy analysis

Locations	Slope (%)	Distance to road (m)	Distance to town (m)	Boolean result	Fuzzy result
A	3.0	300	4953.1	1	77
A'	2.1	190	5045.0	0	76
B	1.4	995.7	2352.4	1	70
B'	1.7	1051.2	2227.5	0	68
C	1.1	50	2197.3	1	90

The user may choose locations, which satisfy any of the criteria, for industrial development. Then, the rule "If site has a flat or gentle slope or site is close to road or town then the site is suitable" is used to find suitable sites. Result of applying Boolean analysis and result produced from the developed system are depicted in Figure 5.23. Locations classified as unsuitable in the Boolean analysis have lowest grade in the fuzzy result.

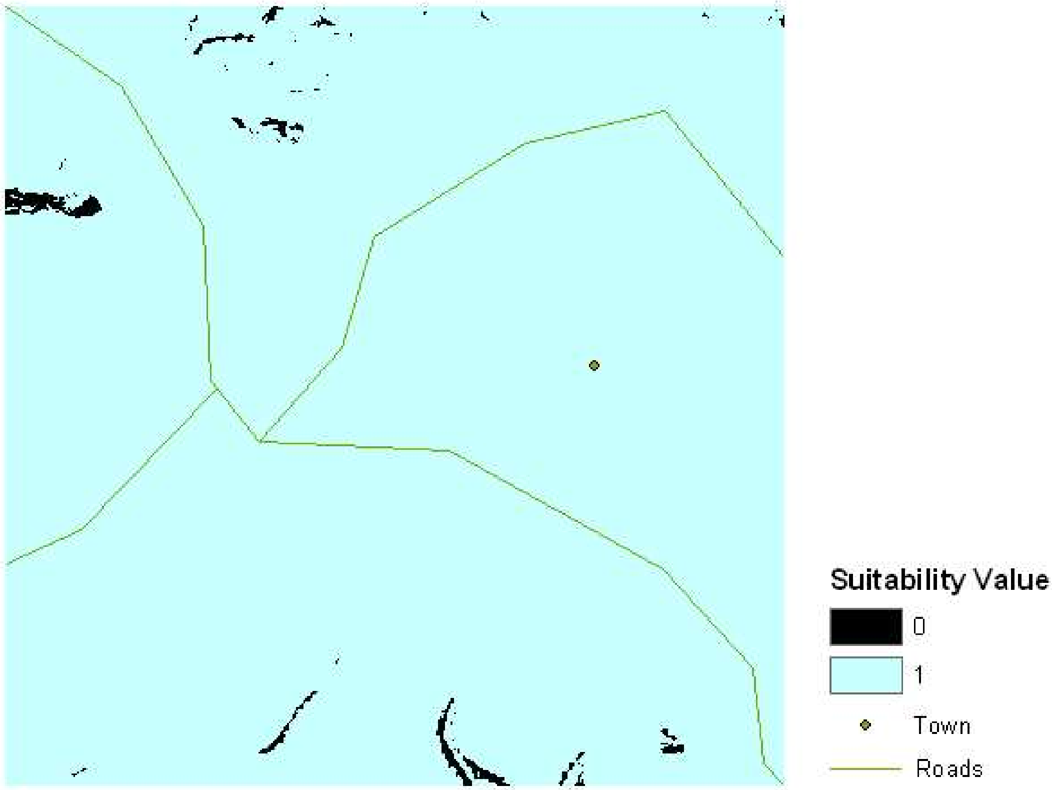
Example 5.8 To select suitable sites for industrial development three rules are defined as follows:

Rule 1. IF slope is flat and
distance to road is close and
distance to town is very close
THEN site is suitable. (5.20)

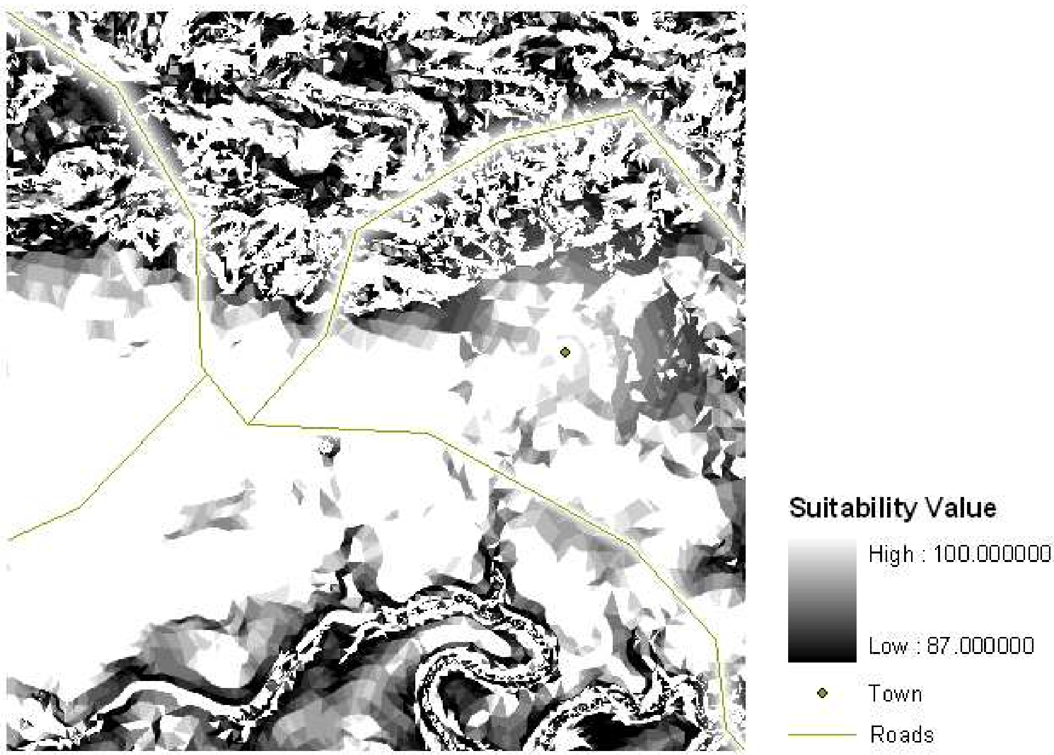
Rule 2. IF slope is flat and
distance to road is close and
distance to town is close
THEN site is average. (5.21)

Rule 3. IF slope is not flat and
distance to road is not close and
distance to town is far
THEN site is bad. (5.22)

Membership functions for linguistic terms are depicted in Figure 5.21. The



(a)



(b)

Figure 5.23: Result of (a) Boolean analysis for suitable sites (using OR), (b) fuzzy analysis.

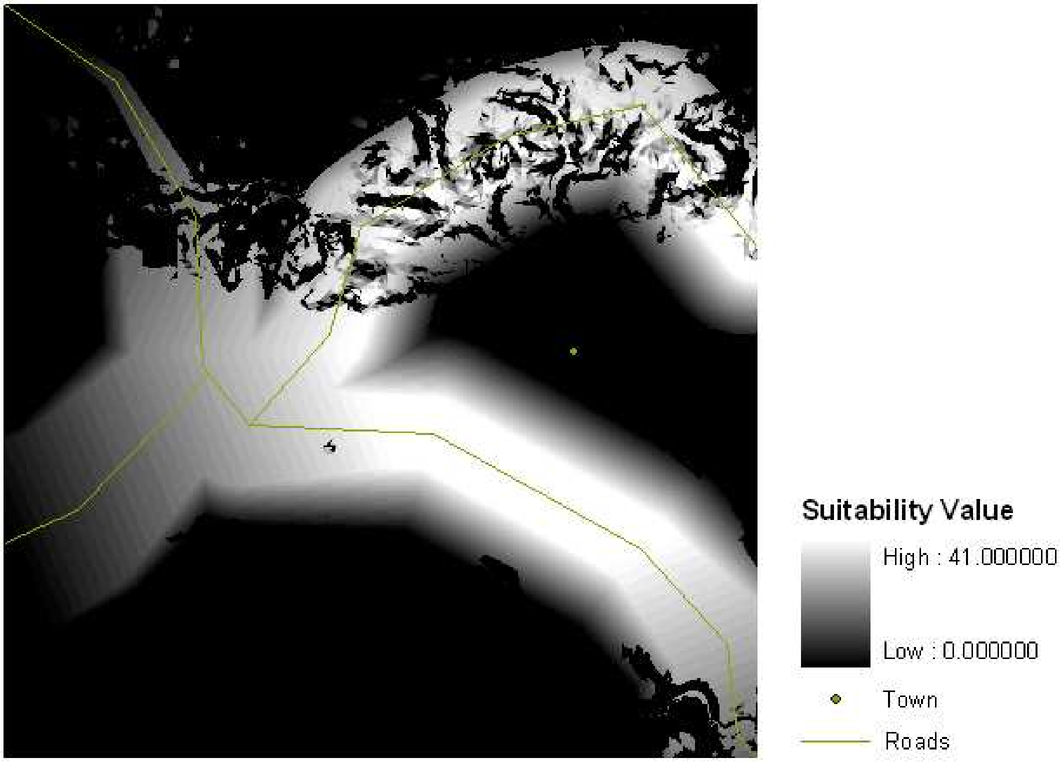


Figure 5.24: Fuzzy result to site selection problem using multiple rules.

developed system produced fuzzy result as depicted in Figure 5.24.

This example illustrates the fact that experiences of a GIS user on a specific decision-making process can be approximated easily using the developed fuzzy inference system for cell-based information modeling.

CHAPTER 6

DISCUSSION OF THE RESULTS

The choice of membership function types, membership function parameters, input and output linguistic variables, rules and fuzzy model properties depend on the problem. Hence, the identification of fuzzy models is a very important issue and consists of three main tasks (Yen, 1999):

1. Structure identification involves finding important input variables for the problem domain, partitioning of input space, specifying membership functions, and defining rules.

2. Parameter estimation involves finding unknown parameters in the model by using optimization techniques.

3. Model validation involves testing the model.

In order to estimate parameters of membership functions both linguistic information from human experts and numerical data (e.g., statistical data, data obtained from observations on the system) obtained from the actual physical system can be used (Yen, 1999).

The system has been developed on a commercial GIS software namely ArcGIS. Visual Basic and Visual C++ are used to enhance the functionality of cell-based information modeling in the form of extensions. In Visual Basic 10556 lines of code, in Visual C++ 6066 lines of code are written.

Fuzzy model approach can be used in place of classical approach in classification and decision-making processes using the proposed software. Note that classical set theory used in conventional GIS software imposes artificial precision on inherently imprecise information about real world and fails to model the way of human thinking about the real world. In addition, decision-maker has to produce maps for each criterion when using conventional GIS software based on classical set theory. However, in this thesis, it is demonstrated that using the proposed software for decision-making, decision-maker has no longer need to produce maps for each criterion. For this reason, the overall time required for the whole analysis is reduced using the developed system. Decision-making problem introduced in Example 5.7 in Section 5.1.3 contains three input maps. Each input map is in 1000-pixel height and 1000-pixel width. The developed system running on Pentium III 450MHz machine with 320MB RAM (Windows NT Workstation 4.0 platform) produced output map (i.e., 1000×1000 pixel) in 188 seconds. Note that to find Boolean answer to question first, for each criterion (i.e., slope, distance to road and distance to town) a map containing 0s and 1s is produced (i.e., three layers). Second, the overall result is produced using logical connectives. Thus, Boolean result is obtained by creating four layers.

A set of fuzzy mapping rules form a fuzzy model. There are three types of fuzzy rule based models: the Mamdani model, the TSK model and Standard Additive Model. In this thesis, the Mamdani model is used to approximate the real world or to model a decision-making process. The Mamdani model is one of the most widely used fuzzy models in practice. The consequent part of the fuzzy mapping rules can be defined both as a fuzzy set and as a crisp value in the Mamdani model. The TSK model replaces the fuzzy sets in the consequent part of the Mamdani rule with a linear equation of the input variables. The main motivation for developing TSK model is to reduce the number of rules required by the Mamdani model, especially for complex and high-dimensional problems (Yen and Langari, 1999). It is, however, usually not the case that rules approximating human knowledge and experience in classification and decision-making processes contain a linear equation of the input variables (Takagi and Sugeno, 1985). Instead, most of the rules defined in a classification and decision-making processes have linguistically defined fuzzy sets in their consequent part. The structure of rules in SAM is identical to that of the Mamdani model. The SAM uses different operators than the Mamdani model: the SAM model uses sup-product composition, product for all fuzzy conjunction operators and addition to combine conclusions of individual rules and the SAM model uses centroid defuzzification method. Parameters, rules and fuzzy model properties differ for problems. Since the Mamdani model does not insist on a specific defuzzification method as opposed to SAM, the Mamdani model provides more functionality to approximate human knowledge and experience in classification and decision-making processes. Therefore, in this thesis the Mamdani model is used to approximate a function of interest.

CHAPTER 7

CONCLUSION

Geographic data, stored and processed in a GIS, are the abstraction of the real world. Since the underlying logic in conventional GIS software systems is crisp logic, continuous nature of landscape can not be modeled appropriately. Because the real physical world is gray but crisp logic is black and white. The classical set theory used in conventional GIS software imposes artificial precision on inherently imprecise information about real world and fails to model the way of human thinking about the real world. Therefore, the abstraction of the real physical world unavoidably results in differentiation between objects of the real world and their representation in GIS.

Fuzzy logic offers a way to represent and handle uncertainty present in the continuous real world. Fuzzy logic is unique in that it provides a formal framework to process linguistic knowledge and its corresponding numerical data through membership functions. The linguistic knowledge is used to summarize information about a complex phenomenon and is used to express concepts and knowledge in human communication, whereas numerical data is used for processing.

In this thesis, fuzzy logic methodologies are used to enhance cell-based information modeling. The fuzzy inference system for commercial GIS software, namely ArcGIS, is designed and implemented. The use of fuzzy logic in GIS has become an active field in recent years (see Section 2.9). Most of the researches primarily focus on the use of fuzzy logic in classifying the continuum of the landscape. Since there is a general lack of user friendly software especially focuses on the GIS-based decision-making process, the Fuzzy Inference System has been developed to enhance a major desktop GIS software.

The main purpose of the developed system is to assist the GIS user to make decisions using experts' experiences in the decision-making process. Experts' experiences and human knowledge described in natural languages are captured by fuzzy if-then rules. In this thesis, it is tried to enable decision-makers to express their constraints through the use of natural language interfaces. The developed system enables decision-makers to express imprecise concepts that are used with geographic data. The capacity of taking linguistic information from decision-makers permits the decision-maker to more easily develop the criteria and softens the constraints and goals in order to find suitable sites.

In conventional decision-making process, a common type of operation is threshold model. For each of the criterion the study area is classified into two subregions describing whether a property value of a specific location is in the defined limit values or not. Then, maps produced for each criterion are overlaid using logical connectives (i.e., Boolean overlay). Each criterion can be weighted based on their importance to decision-maker. In this thesis, it is demonstrated that using the proposed software for decision-making, decision-maker has no longer need to produce maps for each criterion. In addition, final conclusion for multiple fuzzy rules is generated by superimposing all fuzzy conclusions (i.e., superimposing all inferred output linguistic variables) about a variable. Final scalar value is calculated by defuzzifying the superimposed fuzzy conclusions (i.e., not aggregating individual membership values in maps produced for each criterion). Therefore, operators for map overlay are not crucial for the developed system as oppose to (Benedikt et al., 2002; Jiang and Eastman, 2000). Moreover, all locations in the input space are mapped to a degree of suitability using property values of location and rules defined by the decision-maker. Therefore, values of locations in the fuzzy output map derived from fuzzy inference process can be available in orderly manner. Note that Boolean result contains only a set of 1 and 0 values. Another advantage of fuzzy inference is that fuzzy result of a decision-making process provides a set of locations whose attribute values partially satisfy the constraints posed by the user.

Rules, input and output linguistic variables, membership function types and membership function parameters and fuzzy model properties can be selected depending on the problem. Importance of a criterion can be dictated using these parameters. Variety of results can be obtained using different operators, different membership functions, different membership function parameters, different implication and aggregation operators, different defuzzifier operators and different rules. Parameters, rules and all other choices that form a fuzzy model differ for problems. Therefore, for a specific decision-making problem which properties of fuzzy model are suitable can be specified as a future work.

The developed system can be used not only to make decisions but also to classify the study area into classes, which are defined as linguistic terms (i.e., classes do not have sharply defined intervals). In this thesis, the advantages of using fuzzy logic methodologies in the classification process are demonstrated using the developed system. For the developed system, classification is similar to making decisions using rules. Using fuzzy logic methodologies in the classification avoid the high loss of information, which occurs when data are processed using conventional classification methods. Since fuzzy logic approach allows a user-defined tolerance to the class limits in the form of transition zones, intermediate conditions can be better described and gradual changes or transitions in the property values can be better expressed. Therefore, more continuous approach to classification leads to more realistic assessment of continuous landscape.

A membership function maps each point in the input space to a degree of membership between zero and unity. There exist numerous types of membership functions. In this thesis, the designed and implemented system covers the most commonly used membership functions (see Section 2.4). A fuzzy model is formed by fusing a set of fuzzy mapping rules. There are three types of fuzzy rule based models: the Mamdani model, the TSK model and Standard Additive Model. In this thesis, the Mamdani model is used to approximate a function of interest. Design and implementation of the TSK model and Standard Additive Model as an extension can be specified as a future work.

Choosing fuzzy model properties, membership function types and parameters, input-output linguistic variables, input-output relationship and operators for fuzzy inference process are crucial for developing a fuzzy model to approximate the real world or to model a decision process. Selection of appropriate parameters for specific GIS operations can be addressed as a future work.

By relying on the use of linguistic variables and fuzzy rules the incorporation of fuzzy set theory into GIS provides an approximate and yet effective means of describing the real world which is (not precise) full of uncertainties.

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APPENDIX A

FUZZY INFERENCE SYSTEM INSTALLATION

The Fuzzy Inference System for ArcMap Setup wizard automatically installs the Fuzzy Inference System executable and related files on computer. The Setup wizard decompresses files from the Fuzzy Inference System release media and copies them to a folder on hard disk. The Setup wizard will suggest a location for the folder, but user can specify a different location.

The Setup wizard logs installation progress to a file named “St6unst.log”, which is created in the installation folder. In case a problem is encountered with the installation, this file may provide information to help diagnose the problem.

ArcMap has buttons on the Standard toolbar for quickly displaying its most commonly used toolbars. The Toolbars list can be accessed by right-clicking any toolbar, the status bar, or the title bar of the table of contents in ArcMap. The Setup wizard creates a toolbar menu item named “Fuzzy Inference System for ArcMap” on the standard toolbar. Checking the mark next to the toolbar named “Fuzzy Inference System for ArcMap” will display the Fuzzy Inference System on ArcMap desktop as a floating toolbar. The Fuzzy Inference System will be executed when the associated toolbar item is selected.

The release can be removed by going to the Windows Control Panel and choosing Add/Remove Programs and choosing Fuzzy Inference System for ArcMap from the displayed list of programs.

The system requirements to install and run Fuzzy Inference System successfully are listed below:

1. The minimum disk space required to install the developed software is 10 MB.

2. The developed system requires ArcGIS software installed on the system.

3. CD-ROM drive for installation.

4. The Fuzzy Inference System supports the following operating systems:

    - Microsoft Windows NT 4.0 Workstation.
    - Microsoft Windows 2000.
    - Microsoft Windows XP.

The Fuzzy Inference System for ArcMap Setup is available from e-mail addresses tahsin_alp@yahoo.com and zakyurek@metu.edu.tr.

APPENDIX B

FUZZY INFERENCE SYSTEM SOFTWARE

The Fuzzy Inference System has interfaces required to define hedges, define input and output linguistic variables, form fuzzy if-then rules, set the fuzzy model properties and other utility functions (e.g., saving designed fuzzy model to a file, selecting output file format and output file directory etc.). In the sequel steps to design a fuzzy model for the decision-making problem introduced in Section 5.1.3 in the Example 5.7 are given. Before starting to design membership functions, associated input raster maps are added to a map in the ArcMap (Figure B.1).

To define input linguistic variable “Slope”, linguistic variable name “Slope” is written to field linguistic variable name, variable type is set to “Raster”, its status is selected as “Input” from the list, minimum and maximum values for the linguistic variable are written and associated input raster map is selected from the list. Membership functions for the linguistic variable “Slope” are added as shown in the Figure B.2.

After designing membership functions (Figure B.3), defined linguistic variable is added to linguistic variable list. Other input and output linguistic variables are defined in the same way. Membership functions for linguistic terms are depicted in Figure B.4, Figure B.5 and Figure B.6.

In the second step, rule approximating industrial site selection problem is formed by selecting linguistic variable name, hedge, linguistic term and logical

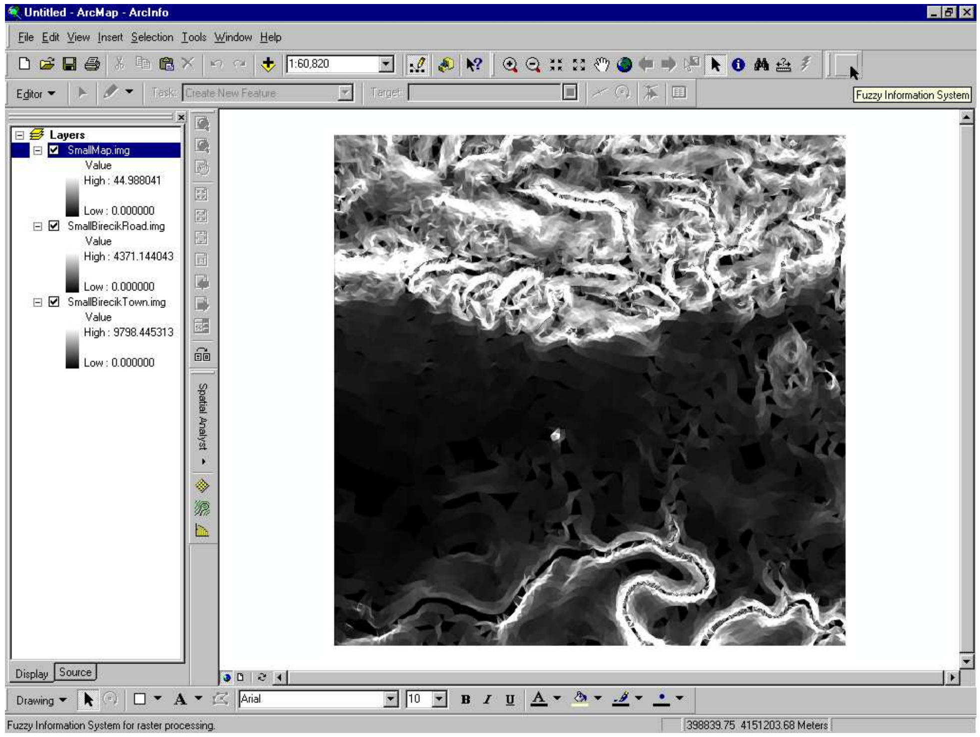

Figure B.1: Input raster maps.

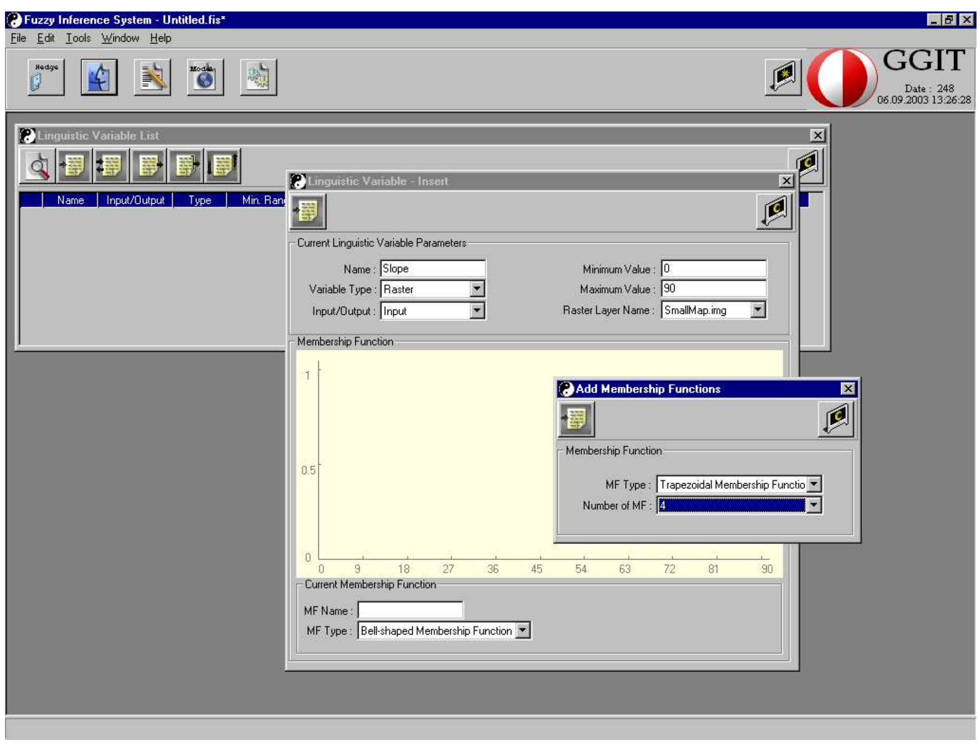

Figure B.2: Adding membership functions to linguistic variable “Slope”.

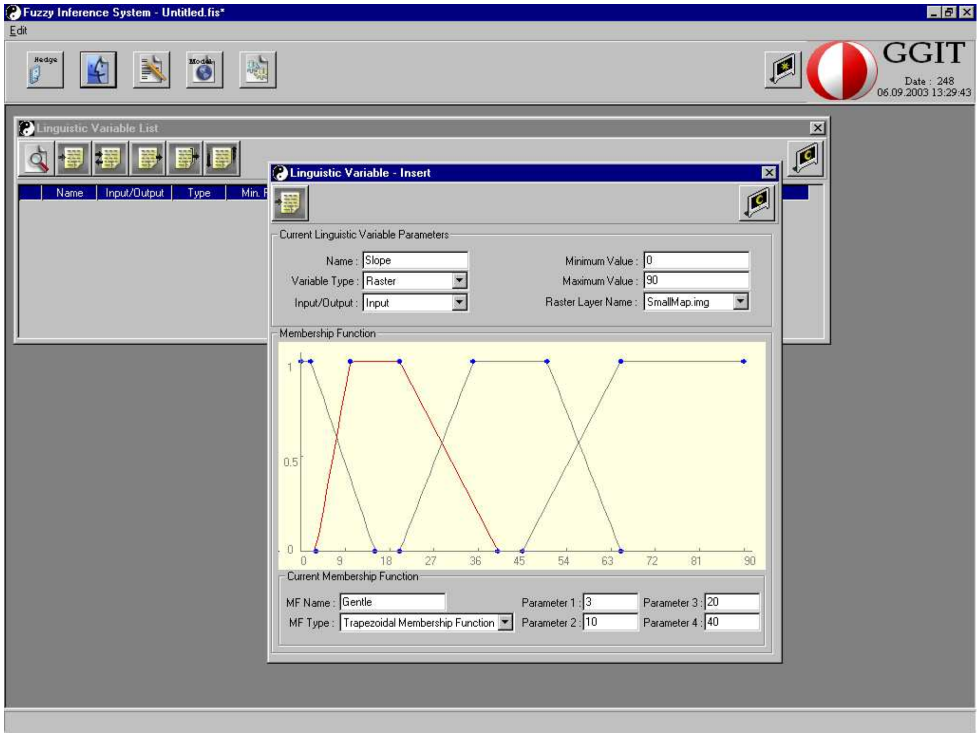

Figure B.3: Membership functions for linguistic variable “Slope”.

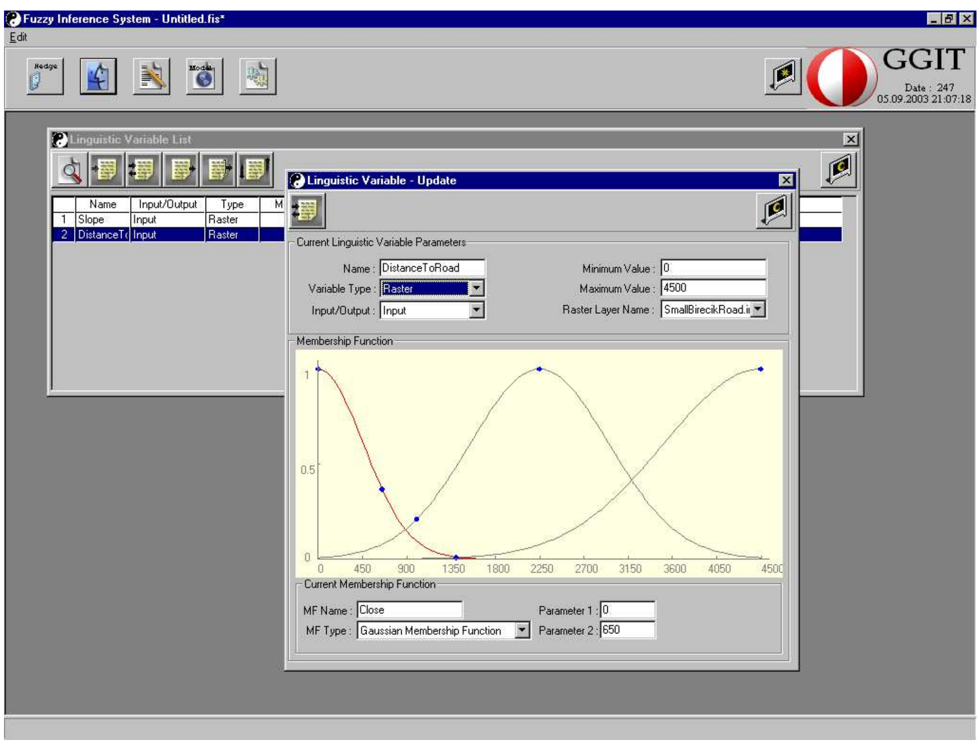

Figure B.4: Membership functions for linguistic variable “Distance to Road”.

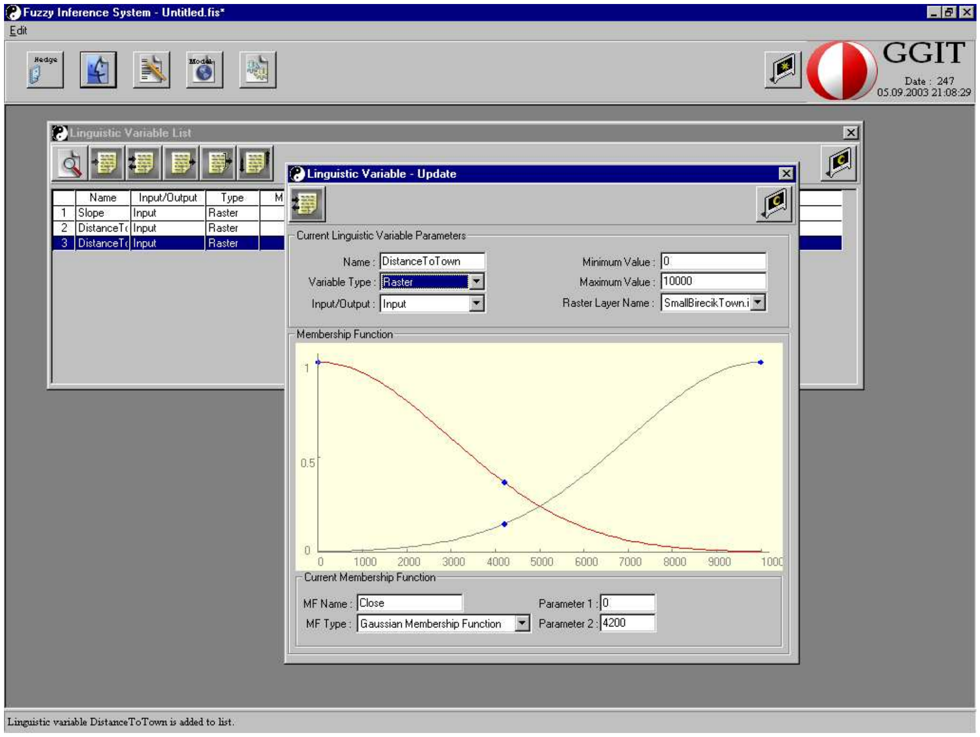

Figure B.5: Membership functions for linguistic variable “Distance to Town”.

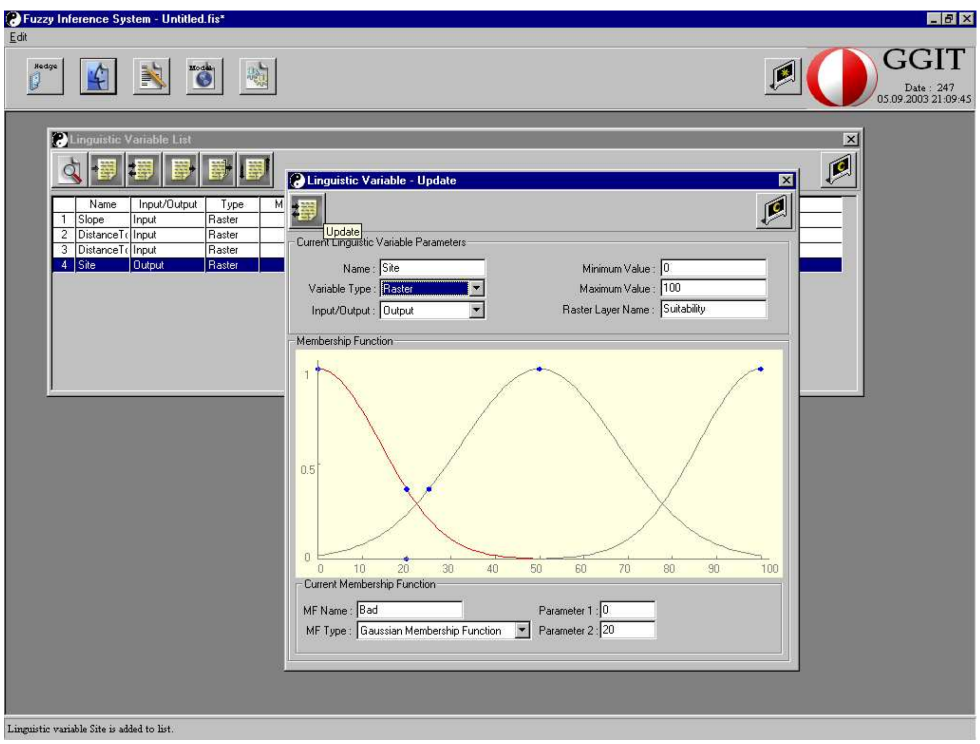

Figure B.6: Membership functions for linguistic variable “Site”.

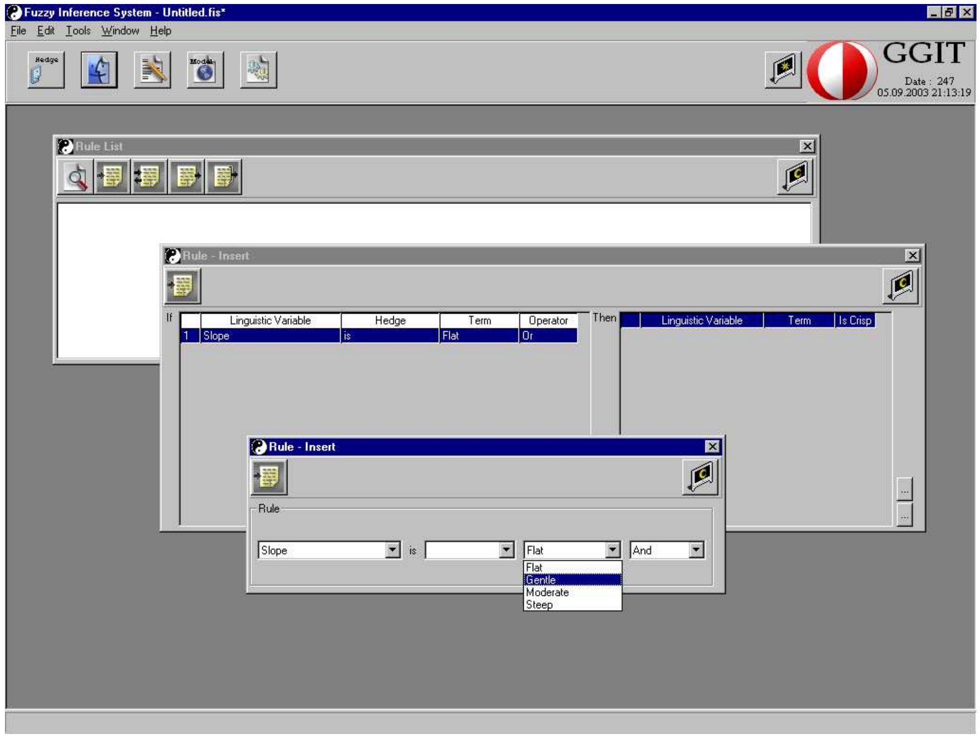

Figure B.7: Selecting linguistic terms for linguistic variable “Slope”.

connective from the associated lists. Figure B.7 and Figure B.8 depict scenes while defining if-part of the rule. Consequent part of the rule is formed as shown in Figure B.9.

Third step involves selecting implication and aggregation methods, defuzzification type and conjunction and disjunction operators from the lists (Figure B.10).

After selecting output directory for fuzzy result and output file format (Figure B.11), model is executed to generate fuzzy answer to industrial site selection problem.

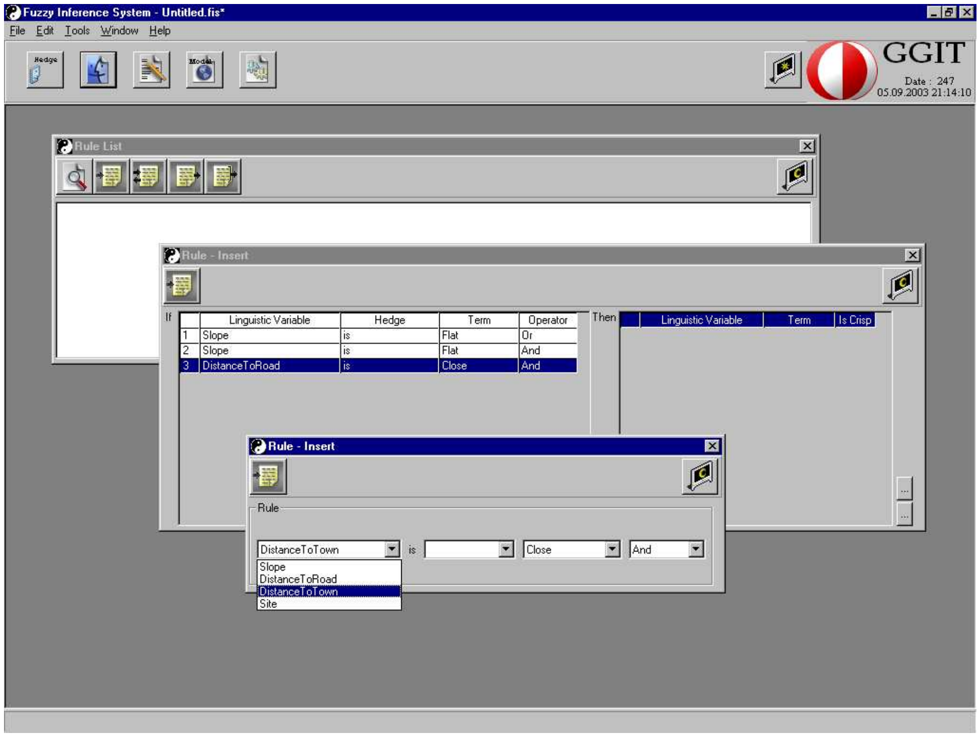

Figure B.8: Selecting linguistic terms for linguistic variable "Distance to Town".

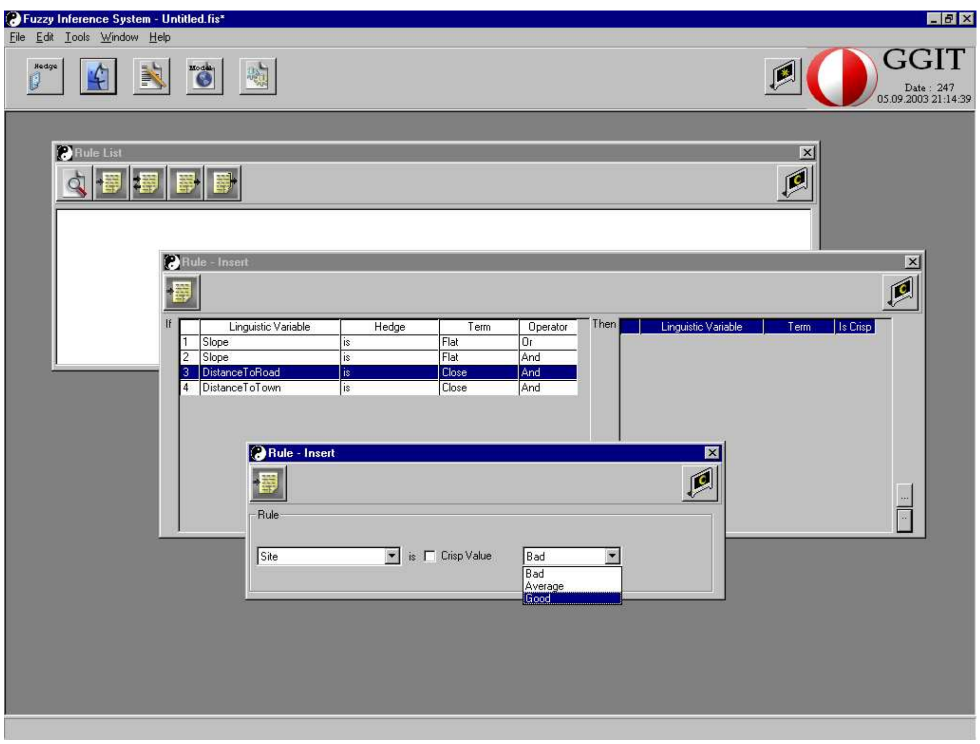

Figure B.9: Defining consequent part of the rule.

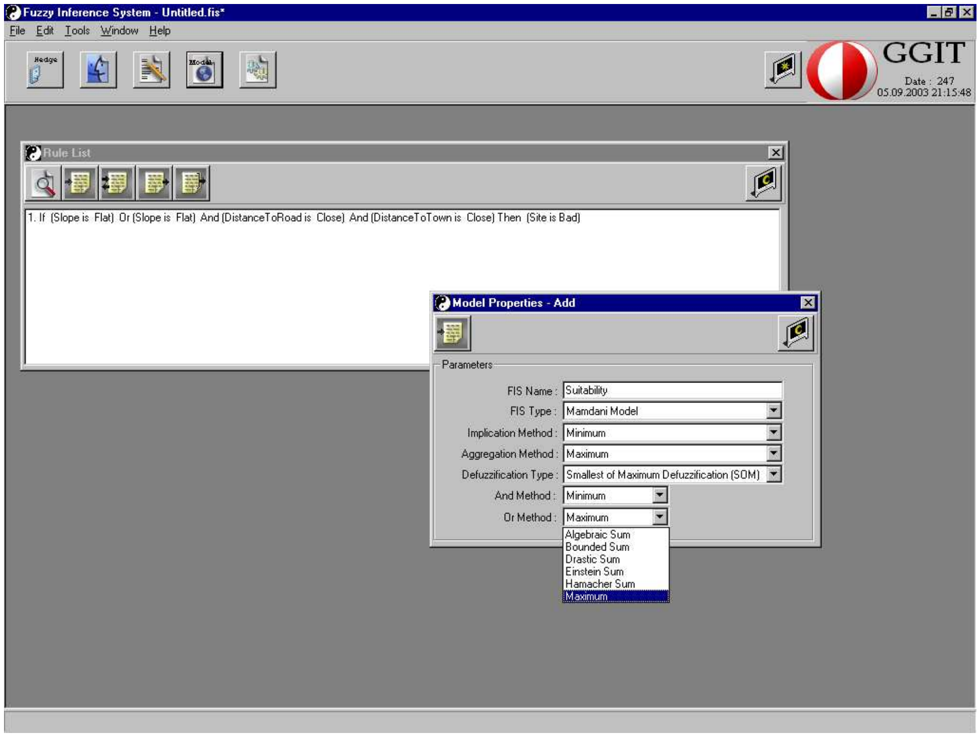

Figure B.10: Selecting model properties.

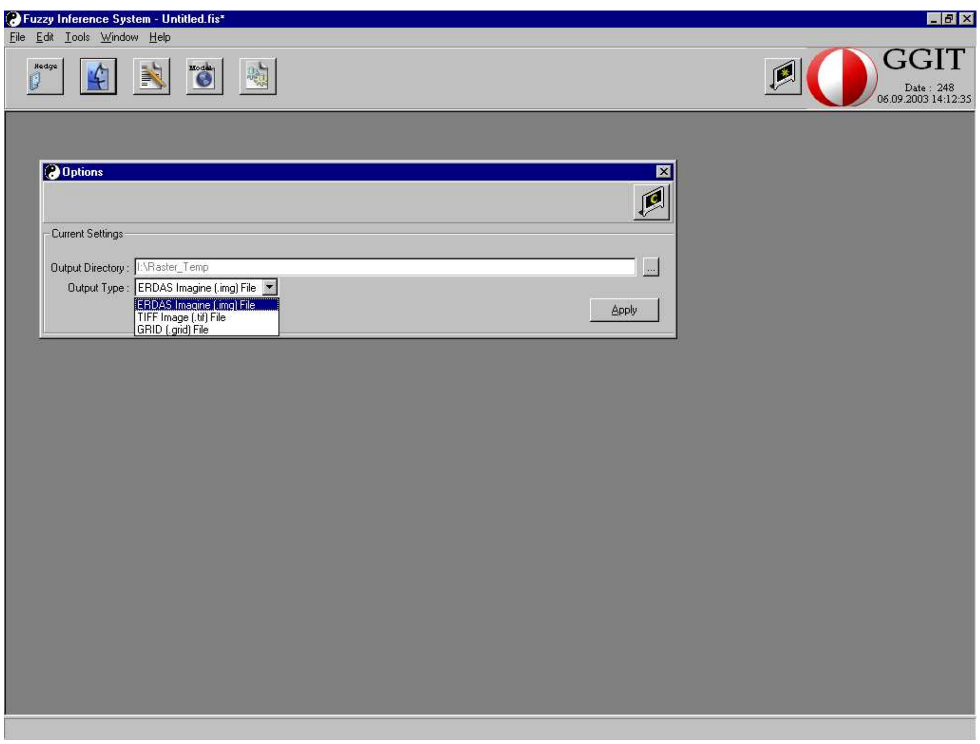

Figure B.11: Setting output directory and output file format.