A MODEL FOR CONTRACT EVALUATION: SUBCONTRACTING UNDER DYNAMIC DETERMINISTIC DEMAND ENVIRONMENT

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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This study is about contract evaluation and selection in an environment where there exist two or more parties, namely a producer and one or more outsourcing companies offering different contract options. The producer faces dynamic deterministic demand, which is known at the beginning of the planning horizon, and has to decide on the quantities of production, outsourcing, inventory carrying, and backorder. Among these decisions, the producer has the opportunity of subcontracting from a set of options offered by the subcontractors with possibly distinct contract terms. The contract options are in terms of length of the contract, fixed and variable costs associated with order placed, delivery lead-time and capacity guaranteed for use. A mathematical model is used for the evaluation of available options and for the selection of the ones in order to minimize cost incurred by the producer. The model provides desired quantities of production, inventory carrying and backorder, and also determines how different contract options will be used through the planning horizon. Extensive experimentation is performed using different factors affecting the optimal solution of the model in specific instances. These results are used in order to come up with a framework where various contracting schemes for subcontracting can be obtained. This framework can assist the producer in the decision of alternative courses of actions to be taken by him as a function of contract terms.

Keywords: Contracting, Subcontracting, Lot-Sizing

ÖZ

KONTRAT DEĞERLENDİRME MODELİ: BELLİ VE DEĞİŞKEN TALEP ORTAMINDA FASON ÜRETİM

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Bu çalışma iki ya da daha fazla grubun bulunduğu, yani bir üretici ve ona değişik kontrat opsiyonları sunan bir ya da daha fazla taşeron üretici kuruluşun bulunduğu bir ortamda kontrat değerlendirme ve seçme üzerine yoğunlaşmıştır. Planlama süresinin en başında belli, ancak değişken olabilen talep bilgisiyle karşılaşan üretici, üretim, envanter taşıma ve sonradan karşılama miktarlarına karar vermek zorundadır. Bu kararların dışında üretici taşeron firmaların sunduğu değişik kontrat opsiyonları arasından bir seçim yapma fırsatına sahiptir. Bu kontrat opsiyonları, uzunlukları, ısmarlamaya ilişkin değişken ve sabit maliyet değerleri, ısmarlamadan teslimata kadar geçen temin süreleri ve kapasite kullanım garantileri gibi özellikleri cinsinden farklılık gösterebilmektedir. Elde bulunan kontrat opsiyonlarının değerlendirilmesi ve üreticinin maliyetlerinin düşürülmesini sağlayan kontrat seçimlerini bulmak amacıyla bir matematiksel model oluşturulmuştur. Bu model istenilen üretim, envanter taşıma ve sonradan karşılama miktarlarını ve planlama süresince hangi kontrat opsiyonlarının nasıl kullanılacağına ilişkin bilgileri sağlamaktadır. Optimum çözümü etkileyen faktörleri bulmaya yönelik geniş bir deney çalışması yapılmıştır. Bu deneyin sonuçlarından faydalanılarak değişik fason üretim kontrat yapılarına ilişkin şablonlar oluşturulabileceği gösterilmiştir. Bu yapı üreticiye değişik kontrat şartlarıyla karşılaştığında nasıl hareket etmesi gerektiğine karar vermesinde yardımcı olabilecektir.

Anahtar Kelimeler: Kontrat, Fason Üretim, Parti Büyüklüğü Belirleme

To my family

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

In today's markets, competition is an important point of concern for many business operations; competition in quality, flexibility, marketing, timeliness and costs. In such a competitive environment, firms have to do their best in order to survive. One of the recent trends in scientific management is the establishment of coordination among many stakeholders of a supply chain; such as suppliers, customers, rivals, etc. Reasons for coordination can be presented as cost minimization and increasing customer satisfaction.

According to Tsay et al (1998) coordination can be achieved using different instruments, and one of these successful instruments is identified as contracting. The purposes of contracts are identified as;

- Carrying the profits near to a centrally controlled system's profits.
- Sharing risks among different parties.
- Improving system-wide performance
- Establishing long term partnerships
- Making the terms of agreement explicit with contract

The level of integration, the sharing of different responsibilities and profits are all defined using contracts. There exist a number of different contract terms including costs, commitments, bounds, length and etc. The decision of these terms is a difficult problem for the parties in need of the establishment of a contract. Different methodologies are used in this process; if one of the parties involved is much stronger, then it dictates the terms and the weaker party only has the choice of accepting it. But in situations where all parties are of equal strength, then this contracting process becomes extremely difficult.

The parties should negotiate on contract terms. In negotiation phase, contract terms are discussed together by all parties. A kind of search mechanism is utilized in order to find the terms that all parties agree on. In such situations, the contract evaluation tools may be helpful on the decision of terms and optimization of the desired objective. With such tools, the evaluation process takes a shorter time and this improves the negotiation process with increasing its iteration speed and its accuracy on the desired objectives. A framework should define the optimal behavior under different decision cases, which the decision maker may face in the negotiation process.

In real life, there may exist more complex situations where a party faces more than one contract options in an environment where there are multiple outsourcing opportunities. In this case, this party has to decide on which one(s) of the contract(s) to choose in order to optimize its objective. He may need to re-plan its own operations together with the newly arising opportunities and decide on the selection with this re-planning decision. Integrated subcontractor selection and production/inventory control tools can serve this purpose; with this kind of tools the optimal behavior can be determined. Only a framework can be sufficient to support the decision maker in this decision process.

An example for such a decision instance may be the case, where there are two subcontractors, offering contract options. The producer has limited capacity, and needs another source of product for achieving desired customer service. First subcontractor is a local firm, having smaller scale production facilities. The lead time associated with this option is shorter, but the unit cost associated with outsourcing is higher than the cost of in-house production. Second subcontractor is an overseas firm with a higher capacity. The fixed outsourcing cost of the local firm is lower relative to the overseas option. The lead time for this option is longer, but the unit cost is lower than the unit in-house production cost. The producer has to choose the option(s) to utilize in order to satisfy the demand, and plan its own operations in accordance.

Our study is about contract option selection for subcontracting some portion of the production to subcontractors with agreed terms. Tsay et al (1998) classify contracts according to their clauses. The first clause is the specification of decision rights and our model allows the control of outsourcing quantities with a lower and upper bound by both parties. For pricing, a structure allowing fixed costs and price breaks in the unit cost function is used. Minimum purchase commitments can be applied. Quantity flexibility is served using the cost structure allowing different prices for different quantities and varying lot sizes. Buyback and returns policies, allocation rules, and quality considerations are not currently modeled in our study. Lead-time is modeled in our model to allow for delayed delivery of the products after the order is placed.

In this study our aim is to develop a mathematical model in order to solve the problem of integrated subcontractor selection and production/inventory control. Our model is different then the process selection model because a contract is established in order to use a subcontracting option. This contract defines the cost structure, lead time, bounds on quantities and the length of contract, which is used to represent the time in which outsourcing option is available after the establishment of a contract. Our environment consists of a producer producing a single product and one or more subcontracting firms offering different contract options in a deterministic demand environment. The problem of the producer is to decide on;

- Which contract option(s) to use?
- How many contract(s) to establish?
- How much to utilize the subcontracting options?
- How many orders to give for subcontracting?
- How much to produce?
- How many production runs to schedule?
- How much inventory to carry?
- How much demand to backorder?
- How much to pay for a contract option?

We use a generalized model to handle different cases which can exist in real life. Our aim is to provide a tool that can answer all questions stated above. We try to come up with a framework after detailed experimentation of the model developed.

A model is constructed with a discounted cost minimization objective. All the answers for the questions previously stated are represented as our decision variables in this model. Constraints that reflect the clauses that are mentioned before are used to restrict the decisions of our model. Environmental setting is inputted using the parameters and constraints. A detailed experimental study is done in order to show the capabilities of the model and to capture the effects of different factors on system performance. A decision tool is tried to be obtained all through this study.

In the analysis part of our study we come up with general findings about the behavior of the system of our interest. In single factor analysis, we come up with the most significant factors affecting the behavior of our system. Unit outsourcing cost and fixed cost of production are identified as the most significant factors of our analysis. Their affects are investigated in the contract length versus fixed contract cost analysis. The regions where the system's behavior does not change are used for sensitivity analysis. The effect of fixed cost of production turns out to be dominant against the effect of unit cost of outsourcing. The system's choice among contracting options is investigated, where two contract options are available, under different environmental settings. Main difference of the two contract options that are used is their lengths. Effects of unit outsourcing cost and fixed contract cost are analyzed in detail. Different demand patterns are used in those analyses. Some of the observations are intuitive, but we observed some counter-intuitive results as well. The system performs well in terms of minimization of cost objective function, when the trend or seasonality of the demand is high. With an uncapacitated outsourcing option different patterns of demand may end up with decreased costs due to the use of higher lot sizes. These findings show that the behavior of systems with so many varying factors is not easily predictable. Also we observe some expected behavior. Longer contracts are almost always, more preferred against shorter ones, when the cost terms associated with both contracts are equal.

In Chapter 2, we present a literature review including the studies on supply chain modeling, contract modeling and outsourcing and make/buy decision making. The relation of our study to these studies in literature is discussed at the end of this chapter.

In Chapter 3, we present our model. Environment of this study is stated at the beginning, and then parameters, decision variables, objective function and constraints are introduced and explained in detail. Size of the model according to model parameters is discussed, and then the capabilities of the model developed are presented. The chapter concludes with a comparison of our study with the studies in the literature.

In Chapter 4, we present experimentation part of our study. First the factors of experimentation are identified and explained, and then the experimentation base used in our study is defined. Detailed experimentation is done for the case where only single contract option is available using different levels for the factors, and the results are presented. A similar study is performed with a small number of factors for the case where there are two contracts available. The results are discussed at the end of this chapter.

In Chapter 5, we present the conclusion for this study. The achievements of this study are discussed in this chapter. Further research directions for this study are presented at the end of this chapter.

CHAPTER 2

MOTIVATION AND LITERATURE

2.1 Motivation of the Study

In this study, main aim is to develop a framework that can help decision makers in simultaneously selecting a contract, negotiating contract terms and production planning decisions. With the help of a model, selection of the contract option(s), which minimize the total cost function, can be made. In this respect, this study can be classified as a contract evaluation and selection study. For contract terms negotiation, the model can be used as a tool for sensitivity analysis to achieve desired values of some performance measures and our objective function total cost. Examples to these performance measures are average inventory carried, amount of demand backordered, total number of production runs, total number of contracts, and proportion of demand satisfied from in house production or outsourcing. From this point of view, this study can be classified as a contract design study. Production planning decisions such as how much inventory to hold, when to produce, what portion of demand to backorder can be made using the results of this model. Hence, our work can also be viewed as a study on production planning. During this study, we come up with a mathematical model formulated for the cost minimization problem of a producer facing different contract options. This model and the analysis of the results gathered with extensive experimentation can be utilized as a decision aid for such producers.

2.2 Literature Review

This study is about contract evaluation and selection in an environment where there exist two or more parties; a producer and one or more outsourcing firms offering different contract options. A mathematical model is used for the evaluation of alternative options and the selection of the ones optimizing the desired objective. The results of the model are used in order to come up with general findings for some special cases. These kinds of models are considered under the main groups of supply chain modeling, contract modeling, outsourcing and make or buy decision-making literature. The related literature for these topics is discussed shortly in the following sections, and then the relation of this study to the relevant literature is provided in last section of this chapter. There exist a lot of studies related to supply chain modeling, contracting and outsourcing, but we try to select the most recent works among them, covering different areas of these topics. So our literature review is comprehensive not in the coverage of all related literature, but in coverage of distinct areas that can be found in literature.

2.2.1 Supply Chain Modeling

Supply chain modeling is the main research topic that our study can be classified as a member. This group consists of a large coverage of topics, and to state where our study is standing, first we should classify the current literature. A recent detailed literature review is taken into our survey for this purpose. Examples of different coordination mechanisms and different coordination objectives for supply chain wide efficiencies are presented. Mainly, the coordination mechanisms consist of contracts, price discounts, revenue sharing, penalties, and subsidy incentives. Different supply chain management methodologies are presented as centralized, decentralized and vendor managed structures.

According to Min and Zhou (2002), over the years, most of the firms have focused their attention on the effectiveness and efficiency of separate business functions. A growing number of firms have begun to realize the strategic importance of planning, controlling, and designing a supply chain as a whole. There exits three structures that can be used as a guideline in the analysis of supply chains;

- The type of a supply chain partnership
- Structural configuration of the supply chain network
- Characteristics of supply chain links

Key components of supply chain modeling are stated as;

- Supply chain drivers
    - Customer service initiatives
    - Monetary Value
    - Information / knowledge transactions
    - Risk elements
- Supply chain constraints
    - Capacity
    - Service compliance
    - The extent of demand
- Supply chain decision variables
    - Location
    - Allocation
    - Network structuring
    - Number of facilities and equipment
    - Number of stages (echelons)
    - Service sequence
    - Volume
    - Inventory level
    - Size of workforce
    - The extent of outsourcing

Taxonomies of supply chain modeling

- Deterministic Models
- Stochastic Models
- Hybrid Models
- IT-driven Models

Types of integrated supply chain modeling

- Supplier Selection / Inventory Control
- Production / Inventory
- Location / Inventory Control
- Location / Routing
- Inventory Control / Transportation

Mishra (2003) works on the problem of a supplier who can reduce its order costs by using an incentive system in the form of price discounts. Its aim is to change the buyers' procurement intervals to some multiples of the common replenishment epochs (CRE). They develop a generalized model that allows the selective discount policy that if profitable, can eliminate some of the buyers. The results show that this policy proves to be successful and segmenting the buyers or offering multiple CREs, can reduce the supplier's cost in many situations.

Klastorin et al (2002) investigate the situation of order coordination for a supplier, providing a product to multi retailers in a decentralized multi-echelon inventory distribution setting. Retailers face static demand and standard inventory holding costs. Supplier offers a price discount to retailers in order to coordinate the timing of their orders with his orders. They show that their policy results with a more efficient supply chain management in some conditions. They also develop a method for finding the supplier's optimal price discount in this setting.

Gerchak and Wang (2001) analyze two different types of contracts between a retailer and its supplier. First one is the vendor-managed inventory contract with revenue sharing plus surplus subsidy incentive scheme, where retailer faces uncertainty on the amounts of components demanded. They show that it is possible to come up with an all win type of result using this two-parameter contract. Second type of contract is the wholesale price driven contract with buyback option.

Khouja (2003) formulates a three-stage supply chain model in which a supplier can satisfy the demands of many customers. Three inventory coordination mechanisms are evaluated under cost minimization objective. These are equal cycle time, integer-multiplier at each stage and integer powers of two multipliers at each firm. They show that some of these inventory coordination mechanisms lead to lower cost values. They provide some insights about the cost reduction due to the complex second and third mechanisms.

Schneeweiss and Zimmer (2003) analyze operational coordination mechanisms where the local information is a valuable asset for a producer and a supplier. The coordination is achieved with the combined use of the producer's procurement policy for components and the penalty for non-correct delivery in a make to order environment. The information sharing is the main concern of the study as it is surely very important for the overall success of the supply chain. Quantitative analysis is carried out to gather a framework for the design of supply contracts.

Giannoccaro and Pontrandolfo (2003) develop a revenue sharing model for a three-echelon supply chain for achieving coordination, so that the supply chain performance can approach the centralized structure. This model shows that with the right contract parameters system may end up with a situation where supply chain efficiency is achieved and profits of all parties get higher.

Lee and Billington (1993) view the supply chain as a network of nodes where each node performs the activities of material procurement, transformation of materials to semi-finished and finished products and product distribution to markets. Centralized control of flow of materials is not always feasible or wanted, so they develop a decentralized model for the flow of materials for Hewlett-Packard Company.

Kaihara (2001) proposes supply chain management based on market-oriented programming. In this study, they use the analogy of economy computing multi agent behavior, and utilize the distributed computation as a market price system. They state the agent activities in resource acquisition based on the negotiations over tradeoffs in order to achieve multi-echelon optimization. This study proves that decision-making based on economic principles results with the optimal resource allocation all through the supply chain and supply chain management systems can be analyzed in terms of economics.

2.2.2 Contract Modeling

Contract modeling is a more specialized part of the supply chain modeling. To understand contract modeling and to present its relation to our study, we use a recent literature review as our starting point. Analysis of contracts in different environments is needed for finding out more about the effects of environments on contracting process. There exist different types of contracts such as on-time delivery, quantity flexibility, and capacity reservation. Contracting related decisions like sourcing are crucial for our study. There exist studies about the contract parameters and their effects on the behavior of different parties involved in the contracting process. Contract evaluation is an important part in contract terms negotiation phase. Several viewpoints, such as multi-party analysis is used for understanding the effects of contracts on all parties and their behaviors. Literature related to these topics indicated above, is presented and analyzed in detail.

According to Tsay (1999), supply chain management treats environments in where there exist multiple decision makers, that may be different firms or different divisions within a single firm. Management of supply chains consisting of multiple agents with possibly conflicting objectives requires consideration of the relationships among these parties. Behavior that is locally seemed to be rational can be inefficient from a global perspective, so attention turns to methods for improving system efficiencies. One of the mechanisms for making these improvements is contractual arrangement. This includes the reallocation of decision rights, rules for sharing the costs of inventory and stockout, and policies governing pricing to the end-customer or between supply chain partners.

There exist contracts that focus on how the profit is to be distributed between the two parties. This is called the *risk-sharing* objective in that it provides a mean for the buyer and supplier to share the risks arising from various sources of uncertainty, e.g. market demand, selling price, process yield, product quality, delivery time, and exchange rates. Contracts also provide a means for bringing the decentralized system profits closer to the centralized system profits, which is called the *system-wide performance improvement* objective. This is also referred to as the *channel coordination* objective. Channel coordination may be achieved by first identifying the intra-chain dynamics which cause the inefficiency, then modifying the structure of the relationship to more closely align individual incentives with global optimization. Contracts also facilitate *long-term partnerships* by defining mutual acceptance that favors the resolution of business relationship, as well as specifying penalties for non-cooperative behavior. Extending the length of the time horizon may encourage parties to engage in activities that are unfavorable in the short term but have significant payoffs over time. Another important motivation for a contract that is not typically modeled is that it *makes the terms of relationship explicit*. In fact, in the course of making expectations of each party legally definite, a contract can suggest and clearly define quantifiable performance metrics, which are prerequisite to any systematic process improvement effort.

A classification scheme for contracts that may be useful is to classify by contract clauses. These include:

a) *Specification of decision rights*. The goal is to achieve specific objectives by redistributing the control of the decision variables.
b) *Pricing*. Let Q be the amount of production quantity. Let C(Q) be a parameter of the contract between the manufacturer and retailer defining the financial terms of the supply relationship as a function of Q. Typically, $C(Q) = F + tQ$ for constants F and t. $F=0$ results in linear pricing, perhaps the most commonly assumed pricing structure. A positive F, often referred to as a franchise fee results in “two-par tariff” pricing. There exist also more complex pricing schemes as quantity discounts.
c) *Minimum purchase commitments*. Such an agreement requires the retailer to purchase a minimum quantity, either within each single transaction, or cumulatively over a specified time horizon. The manufacturer may reduce C(Q) to provide an incentive to the retailer to reach an agreement.

d) *Quantity flexibility*. In a quantity flexibility clause, the quantity the retailer finally purchases may deviate from a previous planning estimate, subject to certain constraints and/or financial consequences. To properly represent such a setting requires a stochastic demand model in which some event (such as a forecast update) occurs within the time frame of the model to motivate the exercise of flexibility.

e) *Buyback or returns policies*. A buyback clause states that the retailer may return some or all of unsold products to the manufacturer, possibly for only partial credit. Naturally, mismatches between the retailer's purchase and the market demand are only an issue when demand is assumed to be stochastic.

f) *Allocation rules*, Allocation rules specify how the manufacturer's available stock or production capacity is to be distributed among multiple retailers in a shortage scenario.

g) *Lead-time*. The lead-time for delivery of products from the manufacturer to the retailer is treated in traditional inventory models as either fixed constant, or a realization of a random variable.

h) *Quality*. Any supply relationship is premised on the quality of the delivered products. The specific notion of quality may be formalized within the contract.

Wu et al (2002) model contracting arrangements between one seller and one or more buyers where the production is done in a capital-intensive production facility, where capacity can only be expanded well in advance of output requirements. Model proposed allows both parties to negotiate bilateral contract for the goods with two-part fee structure well in advance. They show that buyer's optimal reservation level depends on seller's reservation and execution cost. The optimal behavior for the seller is to expose its cost of production while not revealing its margin. This system lets the seller to assure its ability to pay capital related costs of capacity. Provides incentives to buyers in order to take advantage of better terms on the days if alternative, cheaper options are available after the contract has been settled.

Cohen and Agrawal (1999) develop a model to determine contracting policies for a firm that is supplied by multiple suppliers. The model evaluates tradeoffs between the fixed investments, improvement possibilities and price certainty associated with long-term contracts and the flexibility offered by short-term contracts. They show that long-term contracts may not always be optimal, and discuss conditions under which short-term contracts may be justified.

Grout (1996) develops a mathematical model that can be used to decide how to structure on time delivery incentives in a contract between a supplier and a buyer where early shipments are not allowed. The buyer takes into account the supplier's cost minimizing behavior. The cost minimizing incentive that a buyer can choose is represented by a probability of on-time delivery and the relevant scheme to result in that probability. A method is developed to help buyers in their decision process; achieving exactly 100% on-time delivery is shown to be non-optimal and only feasible under specific conditions.

Serel et al (2001) investigate the sourcing decisions of a firm in the presence of a capacity reservation contract that this firm agrees with its long-term supplier in addition to the alternative spot market. This contract guarantees delivery of any desired portion of a reserved fixed capacity with a guaranteed payment by the buyer. They analyze rational action of the two parties under two distinct types of periodic review inventory control policies used by the buyer. These are two-level policy and base stock policy. The capacity commitments decrease with the inclusion of the spot market source with typical demand probability distributions.

Tsay (1999) models two parties' incentives and identifies causes of inefficiency and make suggestions for the treatment. Specific attention is given to Quantity Flexibility contracts, which combine the supplier's guarantee to deliver up to a certain percentage above the forecast with commitment of customer to purchase more than a certain percentage below the forecast. In some situations this methodology cause the self motivated supplier and customer to behave so as to achieve the systemwide optimal outcome.

Barbarosoglu (2000) proposes a decision support model that can be used by a supplier in making production and pricing decisions at contract renewal instances. First, the supplier decides on the aggregate production with a special attempt to forecast buyer commitments, and then sets the price of the item so as to satisfy his own profit and buyer cost reduction expectations together. A mathematical model is developed in order to achieve this agreement with the contract terms.

2.2.3 Outsourcing and Make/Buy Decision Making

Outsourcing or Make/Buy decisions are another specialized branch of supply chain management. Outsourcing is the process of using capacities of facilities that are not belonging to producer's system. Most important motivation for the outsourcing is the lack of capacity to meet the demands. Capacity acquisition is a substitute for outsourcing in such situations. Analysis of capacity acquisition decisions in situations, where outsourcing is available, is important. Another motivation for outsourcing may be the cost minimization objective, which can be improved with lower unit cost structure offered by subcontractors. In this situation, supplier selection and number of suppliers selected for outsourcing determine the cost structure that the producer faces. Information systems can provide a powerful aid in this selection process according to past performance or future expectations of available alternatives. Negotiation and bidding are important stages in outsourcing decisions. Outsourcing may be an option in different environments, as scheduling problems. Different methodology can be utilized in outsourcing decision process like Multiple Criteria Decision Making, and sometimes specifications are important as the objective. Literature related to these topics indicated above is presented and analyzed in detail.

Plats et al (2002) describe the results of an ongoing study to investigate the factors affecting the making of make or buy decisions and the creation of a process to guide industrial companies through making such decisions. The process is based on operationalising frameworks that address technology and manufacturing processes, supply chain and logistics, support systems, and costs. Companies following the process are taken through a structured sequence of inspection involving identification of initiatives for the make or buy decisions and the comparison of in-house and external sourcing criticized against the areas above. Multi-attribute decision-making is used to provide an overall make or buy recommendation.

According to Atamtürk and Hochbaum (2001), the fundamental question encountered in acquiring capacity to meet non-stationary demand over a multi period horizon, is how to balance the trade-offs between having insufficient capacity in some periods and excess capacity in others. In the former situation, part of the demand is subcontracted while, in the latter, capacity that has been paid for is turned out to be idle. In this paper, they investigate the trade-offs between acquiring capacity, subcontracting, production, and holding inventory to satisfy non-stationary demand over a finite horizon. They present capacity acquisition models with holding and without holding inventory and identify *forecast-robust* properties of the models that restrict the dependence of optimal capacity decisions on the demand forecasts. They develop algorithms for numerous practical cost structures involving variable and fixed charges, and prove that they all have a polynomial time complexity. For models with inventory, they solve a sequence of *constant capacity lot sizing* and *subcontracting* sub problems, which are also of independent interest.

Jolayemi and Olorunniwo (2003) develop a deterministic model for planning production and transportation quantities in multi-plant and multi-warehouse environment with extensible capacities. The model determines a product mix that maximizes total profit over a finite planning horizon. When production cannot meet demand due to lack of adequate resources, the model allows deficits to be met through subcontracting or the use of inventory. However, it does not allow subcontracting when adequate resources are available. The model gives the quantities produced at each plant, transported from each plant to each warehouse, subcontracted at each warehouse and kept in inventory in each warehouse.

Humphreys et al (2002) discuss a KBS (Knowledge Base System) designed to help companies in the make or buy decision. A model of the make or buy decision is developed conceptually from a thorough review of the literature and is supported by a series of interviews with procurement managers. The model consists of five main stages; identifying and weighting performance categories, analyzing technical capabilities, comparing internal and external capabilities, analyzing supplier organizational capabilities and total acquisition cost analysis. A KBS is developed which incorporates these five phases into the outsourcing decision.

Lee et al (2002) consider advanced planning and scheduling (APS) in which each customer order has a due date and outsourcing is available. They present a model for APS that requires an absolute due date with outsourcing in manufacturing supply chain. In practice, planning and scheduling are interdependent and should be solved together with outsourcing to guarantee that the due dates of customer orders are met. The proposed model takes into account the alternative process plans for job types, with precedence constraints for job operations. The integrated states include: selecting the best machine for each operation, deciding the sequence of operations, picking the operations to be outsourced, and minimizing the makespan for the due date of each order.

Mohebbi and Posner (1998) present sole versus dual sourcing comparison in continuous review inventory systems in the environment of a lost sales inventory system with compound Poisson demand and exponentially distributed lead times. The stationary distribution of the inventory level (stock on hand) and the corresponding cost functions with and without a service level constraint are derived using a level crossing methodology. The numerical results show that the dual sourcing performs better than sole sourcing in terms of both cost savings and service level, except for situations where one supplier is much more unreliable than other.

Kim (2003) investigates a situation in which a manufacturing company outsources its assembly operations to two contract manufacturers, taking into account the *time* (as a dynamic factor) and the *processing level* (in terms of assembling) simultaneously. Each contract manufacturer is assumed to have a different level of improvement capability of leading to supply cost reduction that in turn, benefits the manufacturing company. The decision problem faced by the manufacturing company is twofold; how much should be outsourced to each contract manufacturer (i.e. less capable or more capable); and how processed (in terms of assembling) should the semi-finished units be when returned from the contract manufacturers.

Nam et al (1995) focus on the outsourcing bidding process relevant to the selection of one contractor by a user-firm. They explore bidding situations where the vendors have different levels of expertise and cost structures. Truth-revealing mechanisms that induce the vendors to act competitively in line with their costs are utilized and MIP model is presented. The model allows the user-firm to obtain the optimal expected contract costs.

Kok (2000) considers a production facility producing a uniform product that ships a number of different package sizes to a number of stock points. The packaging capacity owned by the facility is finite and the actually used capacity must be reserved sometime before actual use. Also, the packaging capacity must be allocated among different package sizes, such that a target fill rate for each package size is achieved. They propose two different capacity reservation strategies, both derived from a periodic review order-up-to policy. One strategy assumes that excess capacity needs compared with the owned capacity cannot be filled and are postponed to the future. The other strategy assumes that excess capacity needs are outsourced. The objective of the paper is to find cost-optimal policies within each of the two classes of policies and then select the best capacity reservation policy.

Mieghem (1999) values the option of subcontracting to improve financial performance and system coordination by analyzing a competitive stochastic investment game with choice. The manufacturer and subcontractor decide separately on their capacity investment levels. Then demand uncertainty is determined and both parties have the option to subcontract when deciding on their production and sales. They analyze and present outsourcing conditions for three contract types; *price-only contracts* where a transfer price is set for each unit supplied by the subcontractor, *incomplete contracts*, where both parties negotiate over the subcontracting cost transfer, and *state-dependent* price-only and incomplete contracts for which they show corresponding result. While subcontracting with these three contract types, firm can coordinate *production* decisions in the supply systems, only state-dependent contracts can eliminate all decentralized costs and coordinate capacity *investment* decisions. They find that sometimes firms may be better off leaving some contract parameters unspecified and agreeing to negotiate. Also, a price-focused strategy for managing subcontractors can rebound because a lower transfer price may decrease the manufacturer's profit. Finally, as with financial options, the option value of subcontracting increases, as markets are more unpredictable or more negatively correlated.

Padillo and Diaby (1999) develop a multiple-criteria decision analysis methodology for the evaluation of make-or-buy strategies. The methodology represents a fundamental departure from existing frameworks by executing a dynamic, multi-attribute, and multi-objective performance evaluation of make-or-buy alternatives. The methodology contains a model that comprises four objectives: maximization of strategic competitive performance; maximization of managerial performance; minimization of sourcing risks; and maximization of financial performance. These objectives and their attributes are traded-off using composite programming, resulting in a rank ordering of the make-or-buy alternatives.

Nellore and Soderquist (2000) present analysis of outsourcing models, and propose an extension based on the role that specifications might play in outsourcing decisions. Based on how the specifications are generated and on the nature of the data it contains, it can be of significant help in outsourcing decisions. A procurement matrix is developed in which guidance for outsourcing decisions is provided in terms of specification generator, type of supplier, and contract relationship.

2.3 Relation of the Literature with our Study

According to the categorization in Min and Zhou (2002) our study can be classified as a deterministic supply chain modeling study designed for subcontractor selection, production and inventory control. Constraints in our study are the capacity and demand satisfaction constraints. Supply chain decision variables, in our model, can be presented as volume (production), inventory level and the extent of outsourcing used in the demand satisfaction.

This study is about the evaluation of supply chain contracts. Contracts are one of the major instruments used in order to achieve coordination between different levels of a supply chain. With contracts, the revenues and costs incurred by any party can be shared among all parties involved. This kind of sharing can be used in order to achieve systemwide profit optimization.

Tsay et al (1998) classify different clauses that may be involved in contract modeling in their recent review of supply chain contracting which are summarized in the previous section. Our study involves most of these clauses. The first one is the specification of decision rights, and our model allows the control of outsourcing quantities with a lower and upper bound by both parties. Pricing has an important part in our study so a structure allowing a fixed cost and price breaks in the cost function is used. Minimum purchase commitments can be applied using the lower bounds in our study. Quantity flexibility is served using the cost structure allowing different prices for different quantities, and different lot sizing in every period. Buyback and returns policies, allocation rules, and quality considerations are not currently included in our study, but they can be incorporated into our model. Lead-time is included in our model to allow delayed delivery of the products.

Mishra (2003) models the situation of a retailer that needs to restrict the replenishment intervals. In this study, price discounts are used in order to share the gains of the restricted replenishment intervals. Klastorin et al (2002) work on a multi echelon system where some incentives are used in order to coordinate the order cycles of different levels of a supply chain. We deal with the problem of a producer utilizing different options offered by subcontractors, which are similar to the suppliers in their study. We use our cost structure and bounds in order to coordinate both parties modeled in our contract evaluation problem. Gerchak and Wang (2001) analyze revenue sharing, wholesale pricing and subsidy option contracts and investigate the effects of these kinds of contracts on the level of integration. We try to implement revenue sharing using the cost structure; subsidy option is not available in our study, but can be incorporated.

Coordination is an important factor on the success of a supply chain. Therefore, there exist several studies done to understand the applications and effects of coordination better. Khouja (2003) evaluates different coordination mechanisms in a three-echelon network structure. Schneeweiss and Zimmer (2003) analyze the effects of private information on the performance of supply chains. Information sharing is not a part of our study; the demand is not available to the supplier, only the orders are available, but since all our study is about deterministic demand, this does not affect the behavior of the subcontractor. Giannoccaro and Pontrandolfo (2003) analyze the effects of revenue sharing contracts in a three-echelon network structure, and try to model the system performance according to contract parameters. Their aim is to achieve centralized efficiencies in the supply chain with revenue sharing contracts. In our study, we also tune up the contract parameters in order to get better results. In fact, we try to generate a negotiation tool, which can evaluate different parameter sets. Our aim is to define decision rules in some special environment or under special clauses defined by parameters.

In some cases, total central control is a difficult task to achieve. In these cases, the use of decentralized models can be helpful to have system-wide efficiencies. Lee and Billington (1993) analyze the usage of such models in materials management in decentralized supply chains. Their study is one of the pioneering studies in this field, inspiring many of the studies in contracting and coordination branch of supply chain management literature. Kaihara (2001) uses the economical market systems in order to achieve centralized efficiencies in decentralized environments. Our approach is based on the mathematical modeling approach, but this study is an important example for studies based on different logic bases.

There exits different types of contracts used in different environments. The parameters or the terms of the contracts are very important. Wu et al (2002) model contracts in high capital-intensive industries, where the results of investments can take effect in a long time period. Such investment decision-making is not involved in our study. Our aim is to plan the production under static capacity restrictions. Cohen and Agrawal (1999) evaluate long and short-term contracts and their advantages. In our study we evaluate different contract lengths and try to come up with a framework for the evaluation of such contracts and selection from a set of alternatives. Different parameter sets are used in order to see the effects of contract length in different settings. Grout (1996) analyzes on time delivery incentives between a supplier and a single buyer. The lead-time for delivery of orders by the subcontractor is taken as deterministic in our study, so there is no need for a penalty for late delivery. All orders are met in the given time window, but with small changes the option of not meeting the demands of producer can be incorporated into our modeling approach.

Prediction of the types of behavior in certain situations is an important tool in the design of contracts and negotiation. Serel et al (2001) analyze the principles that lie behind the behavior of different parties that meet in a long-term contract. Long-term contracts are one of the options evaluated in our study in different environmental settings. We also try to predict the behavior of parties in such environments. Tsay (1999) models incentives for the system wide optimal behavior for quantity flexibility (QF) contracts. Quantity flexibility contracts are one of the contracts evaluated in our study. Their most important property is the flexibility modeled with the minimum and maximum quantities allowed. Decision of contract terms is of vital importance; all of the parties involved in the contract may want to claim for their own rights, and an agreement may be hard to reach. Barbarosoglu (2000) develops a decision support system for suppliers, where the retailer’s concerns are also met. Our study is from a single point of view, but both parties can use the results of our work with changing efficiencies, producer can minimize its cost and retailer may want to use it in order to capture some of the profits provided by the model for the producer side.

Make or Buy decisions are one of the most frequently faced problems of the manufacturing firms. It may also be called as outsourcing, or subcontracting, but the main thing is the opportunity of satisfying the demand from a different source with a different cost structure. Platts et al (2002) try to build a guide for the make or buy decisions. Our study is similar, integrating the make or buy decisions with selection of contract options. In general, the outsourcing option can be used in cases where the capacities are not enough or there are cost reduction opportunities. Atamtürk and Hochbaum (2001) study the integrated capacity planning, subcontracting and lot-sizing problem. In their study they integrate the capacity acquisition decision using a term in the objective function but use only a single subcontracting option. We do not model capacity acquisitions integrated into our model, but we used contract option selection and complex cost structure relative to their study.

Jolayemi and Olorunniwo (2003) develop a deterministic model for planning production and transportation quantities in multi-plant and multi-warehouse environment with extendable capacities. When the in-house capacity is not sufficient to meet demand then there is the option of subcontracting. This study involves the transportation planning, that our study does not, but their study does not allow the selection of contract options and a cost structure similar to the one used in our study. Humphreys et al (2002) work on the usage of Knowledge Based Systems (KBS) in the area of strategic purchasing. Outsourcing can be an option in different levels of the production process, Lee et al (2002) model the usage of this option in a scheduling environment where the jobs have due dates. This is a good example for the make/buy decision models in different problem environments. One of the major concerns in outsourcing is the selection of the subcontractors according to a performance measure. Mohebbi and Posner (1998) evaluate the single and dual sourcing situations with the models they develop. They show that the dual sourcing has advantages over single sourcing; in our model we develop a model allowing the use of more than one contract option simultaneously.

Kim (2003) studies on the contractor selection problem, based on the current and potential capabilities of the alternatives. Expertise and opportunity of improvement are important concepts in the supplier selection process, which we do not include in to our formulation. Nam et al (1995) focus on the outsourcing bidding process where vendors have different levels of expertise and different cost structures. Capabilities type of analysis is not included in our model as a part of contract selection. This criterion may be integrated in to our approach by using Multiple Criteria Decision Making modeling. Kok (2000) models the usage of outsourcing option in scarce capacity environments where capacity reservation exits. Capacity reservation type of contract clause is not a part of our model, but the bounds can work as well.

Different methodologies can be utilized in the outsourcing decision process. Mieghem (1999) values the outsourcing options in a stochastic investment game environment. Our model is a deterministic model; some of the realizations can be modeled as random for a future extension. Minimization of cost function is the primary objective of most of the studies in this field, but there may exist different objectives as well. Padillo and Diaby (1999) use a multi-criteria process in the ranking of different contracting options. Our study uses single objective optimization, but multiple criteria evaluation may be useful in some cases. Sometimes, there exist restrictions or plans before the contract has developed. Nellore and Soderquist (2000) analyze the outsourcing decision where there exits specifications. Some sorts of specifications can be included in our study using the model parameters.

This literature survey has very much contributed to our modeling efforts and experimental design. Supply chain management literature equips us with a better understanding of the coordination, decentralization, centralization and contracting concepts. The need for the existence of these concepts, the application of these on real life problems and the results on the performances of systems enlighten us in this field. Contract modeling literature covers different types of contracts available on different environments we try to cover in our formulation. Contracting literature conveys us several potential contract clauses available in real life, so we try to investigate the effects of these clauses or terms on the behavior of the system we model. Outsourcing and make/buy part of our literature study helps us to understand distinct types of relationships that exist between different parties involved in this process. Important considerations for the determination of optimal behavior of the systems are gathered from this literature.

Our contribution to the literature may be presented as following. First of all, we present a general model, which integrates many of the contracting and make/buy concepts, to the best of our knowledge, we do not know such a model. One of the distinctions of our model from the classical literature is the cost structure involved. With our effort many practical systems can be modeled. Detailed analysis of the model capabilities is presented in the next chapter. Second contribution can be of the efforts given to analyze different settings, and the preparation of a guide for the decision process in such environments. We perform a full factorial design using various factors, and state the factors affecting the system behavior significantly. Our results can be used for guidance in similar decision instances, as they define the optimal behavior of the system

CHAPTER 3

MODEL

3.1 General Model

In this section, environment, parameters and decision variables of our problem are presented and explained. Environment of our problem is important for the model construction, all our variables and parameters highly depend on the environmental setting. Parameters are used to represent the environmental setting and different cases of problem structure. Decision variables are used to output the results of this study from the model formulated. With the analysis of these results we can come up with general principles for the behavior of such systems. Model is constructed with a cost minimization objective, and constraints are used in order to define the environment and other restrictions of our model. We also present the size and complexity of the model in terms of number of variables and parameters.

3.1.1 Environment

The environment of our study can be expressed in terms of the structure of the model and the parameters. Parameters are used in the model in order to represent the properties of the environment. The structural properties of the model formulated and the environment represented with parameters are as following:

- The environment consists of a producer and one or more subcontracting firms. The supply chain setting can be seen on Figure 3.1. The producer's aim is to satisfy the demand with a cost-minimization objective. Single item is produced by the producer having finite capacity devoted to this production process. Subcontractors are the firms producing the same product at their own facilities. A subcontractor can offer multiple contract options with different terms. Producer has the option to subcontract some of his demand to these with an agreement on cost structure and other contractual terms.

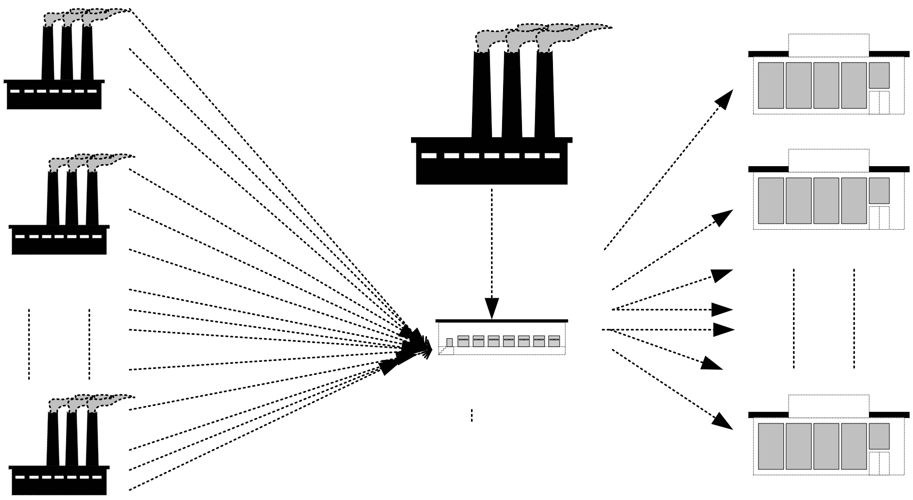

Figure 3.1: Supply chain structure of the system

- The demand for the producer is deterministic and known for a number of periods. Our aim is to create a schedule for these periods for which we have the demand information in advance.
- There exist a number of contract options that the producer can select from. These options differ in their contractual terms; cost structure, length of the contract, lead time, and bounds on the amount of products procured.

- There exist two fixed costs and a variable cost for outsourcing. Major fixed cost represents the contracting cost, which is needed in order to initiate outsourcing agreement. This cost can be thought as a sunk cost, incurred as an investment for improvement on the subcontractor side. This investment can be in different forms, it can be related to training of labor, purchasing of new machines, or providing support for any possible improvement. Minor fixed cost represents the costs incurred for the order and delivery of the outsourced units. Variable cost is charged on unit basis for every unit outsourced from a specific option.
- Lead times are important terms of the contract, which state the delay till the time of delivery of the outsourced units. With longer lead times, subcontractor can manage its production capacities in a more efficient way. With shorter lead times, he should make changes in his plans and this brings him a burden.
- Length of the contract specifies the number of periods where outsourcing option is available if a contract is established. All cost terms are very much dependent on the length of the contract. In longer contracts, generally the fixed costs increase but the variable costs decrease. On the other hand, shorter contracts are preferred for handling the temporary deviations in the demand realizations.
- A lower bound exists to limit the amount of outsourcing, and an upper bound defines the maximum allowable amount of outsourcing. These are the terms of the contract that the subcontractor can agree to reserve some of its capacity to the producer or want to have minimum purchase commitments; both these cases can be modeled using these bounds.
- Upper and lower bounds are used in a similar manner for the production amounts. For the producer upper bound is the capacity constraint limiting the production, and lower bounds are used to guarantee efficiency in production by eliminating very small batches of production.
- The variable unit costs of both outsourcing and production are non-decreasing functions of units. A constant level is defined for the unit cost within the number of units in a range. First level represents the normal working hours, and higher levels represent the overtime, so the unit cost function is a piecewise increasing function of production amount.

- There is no lead time associated with production. The units are immediately produced after the production decision.
- Backordering of the demand is allowed with a penalty per unit per period. Backorder is another source of demand satisfaction, instead of production or outsourcing, in certain cases it may be helpful to compensate the trend and seasonality of demand.
- Interest rate is used in the calculation of holding costs. Holding costs are the opportunity costs incurred for carrying inventory rather than investing the money to different alternatives, interest rate is used for representing the return of alternative investments.
- Discount factor exists for evaluating the present worth of non adjusted cost figures. Money has the time value, and its real worth diminishes with time. So, we use a discounting factor in order to evaluate the costs at a certain point in time. Using the discounted terms with this approach, we guarantee to use an equal worth basis for our evaluation.
- The items stored may need a physical storage space, which can be gathered on a unit cost basis. Some products needs special storage areas with certain temperature, humidity or concentration of some chemicals in the surrounding environment, these kinds of storage areas have a significant cost relative to inventory carrying cost.

3.1.2 Parameters

Parameters are used as input of data for the model. They can either be used in the definition of environment or setting up of scenarios. They provide the flexibility of the model to handle different cases. We use some sets in the definition of parameters and decision variables. These sets are as follows;

t:	Set of periods in the planning horizon	t=1,..,T.
k:	Set of contract option for subcontracting	k=1,..,NC.

j: Set of price breaks — j=1,..,NP for production; j=1,..,NO_k for subcontracting with contract option k

The parameters defined using these sets that are used in this model are as follows:

B_0 : Initial amount of backorders in units at the beginning of the planning horizon.

This parameter is used to represent the initial condition of delayed demand satisfaction at the beginning of the planning horizon

B_T : Desired amount of backorders in units at the end of the planning horizon

This parameter states the desired ending condition of delayed demand for later satisfaction at the end of the planning horizon.

BK_t : Unit backordering cost per unit time in period t.

This cost is incurred for every unit of demand that is not satisfied and waiting for delayed satisfaction at the end of each period.

CO_{kjt} : Unit cost value of outsourcing for contract option k for the j^{th} price break in period t.

SO_{kjt} : The amount at which unit cost for outsourcing from contract option k in period t jumps to $(j+1)^{th}$ price break. Smallest value is used to represent the lower bound of outsourcing that the subcontractor offers, and the largest value is used to represent the upper bound of outsourcing that the subcontractor offers. Setting the lower bound as zero eliminates the lower bound and setting the upper bound as infinity eliminates the upper bound.

NO_k : Total number of price breaks in the outsourcing cost structure for contract option k.

These three parameters are used together to represent the piece-wise increasing unit cost function with price breaks for outsourcing. The graphical representation of the unit cost function can be seen on Figure 3.2

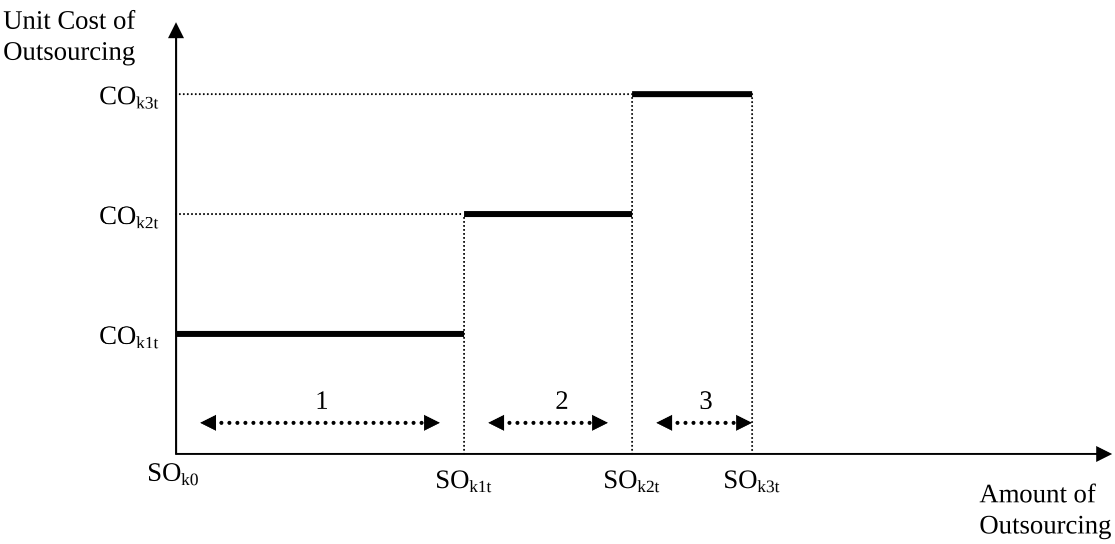

Figure 3.2: Unit cost function for outsourcing option k with NO_k=3

CP_{jt}	:	Unit cost value of production for j^{th} price break in period t.
SP_{jt}	:	The amount at which unit cost of production in period t jumps to $(j+1)^{st}$ price break.
NP	:	Total number of price breaks in the production cost structure.

These three parameters are used together to represent the piece-wise increasing unit cost function with price breaks for production. First step represents the regular time production, and further steps are used to represent overtime production. So the cost is an increasing function of the production quantity. The graphical representation of the unit cost function can be seen on Figure 3.3.

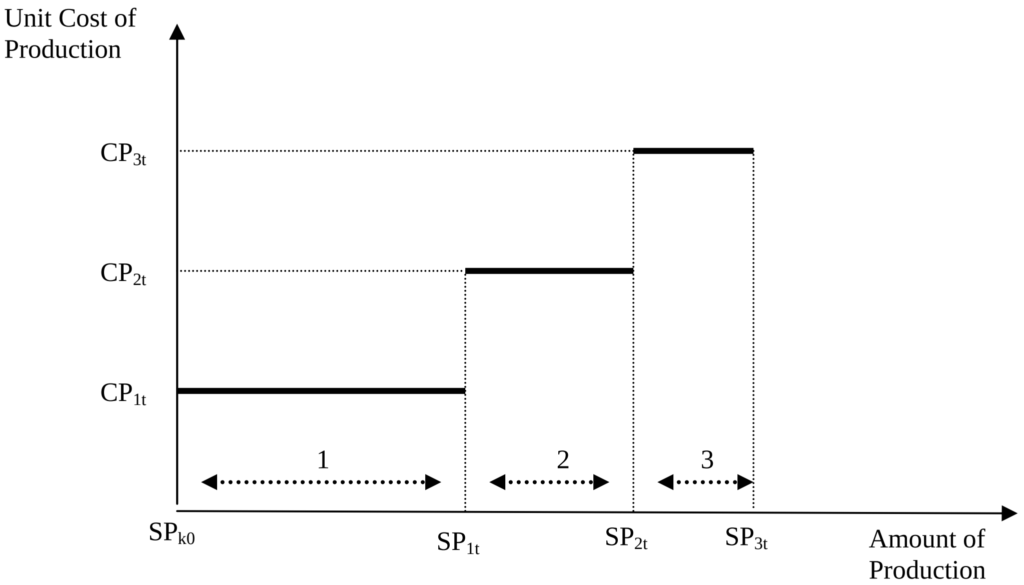

Figure 3.3: Unit cost function for production for NP=3

D_t : Demand in units for period t.

This parameter defines the amount of demand in units for every period in the planning horizon.

DIS_t : Discount factor as a multiplier per period t.

This parameter is used to find the present worth of all monetary terms. Discounting is necessary when the nominal values of monetary terms are used. If instead of nominal values, real values representing the present worth of the monetary terms are used, then there is no need for discounting. And in this case discounting factors can be set as one in order to use the real values in the calculation as themselves.

FC_{kt} : Fixed contract cost for contract option k for period t.

This cost is paid for the establishment of a contract at any period. This cost does not depend on whether the contract option is used to satisfy demand or not, once paid it is a sunk cost.

FO_{kt} : Fixed cost of outsourcing for contract option k in period t.

This cost is incurred when an order is given for outsourcing in any given period. This cost figure is related to the expenses for order giving and delivery of the products.

FP_t : Fixed cost of production in period t.

This is the cost of setup, which is needed for a production run in a given period.

I_0 : Initial amount of inventory in units at the beginning of the planning horizon.

This parameter is used to represent the initial condition of inventory on hand at the beginning of the planning horizon

I_T : Desired amount of inventory in units at the end of the planning horizon.

This parameter states the desired amount of on hand inventory at the end of the planning horizon.

INT_t : Interest rate for inventory carrying per period in period t.

The interest rate is used to calculate the cost related to inventory carrying at the end of each period. It is the opportunity cost of the money invested in inventory.

L_{kt} : Lead time for outsourcing for contract option k in period t.

Lead-time is the number of periods between the time to give an order to the subcontractor and the actual time of the delivery of the ordered units. There is no lead time associated with production since the periods length is defined in terms of production lead time. It takes less than a period to produce any lot size in production facility, so the demand can be satisfied using the new production.

LBO_{kt} : Lower bound in units for outsourcing for contract option k in period t.

UBO_{kt} : Upper bound in units for outsourcing for contract option k in period t.

These two bounds are used in order to limit the minimum and maximum values of outsourcing option utilization in any given period. Lower bound is needed to achieve efficiency of economies of scale or to apply minimum purchase commitments and an upper bound is used to control the capacity allocation of the subcontractor for the producer.

LBP_t : Lower bound in units for production in period t.

UBP_t : Upper bound in units for production in period t.

These two bounds are used in order to limit the minimum and maximum values of amount produced in any given period for capacity and efficiency purposes.

LEN_k : Length of contract in periods for contract option k.

The length of contract defines the number of periods that outsourcing can be utilized using the current running contract. There is no need for a new contract in order to outsource within the periods specified by the contract length, once the contract is signed.

NC : Total number of contract options.

This parameter defines the total number of outsourcing options offered by the subcontractors.

S_t : Unit storage cost per unit time in period t.

This cost represents the storage cost associated with the physical storage needed in order to carry inventory between periods. This cost is based on the volume or mass of the products, so it is on unit cost basis.

T : Total number of periods in the planning horizon.

This parameter sets the number of periods in our planning horizon. This parameter affects the size of the model linearly.

Some of these parameters which are related to outsourcing options are exogenous. These are the terms of contracts; unit cost structure, fixed cost of contracting, fixed cost of outsourcing, lead times, upper bounds and lower bounds. In other words a contract option C_k can be represented as follows.

$C_k=\{CO_{kjt}, SO_{kjt}, NO_k, FC_{kt}, FO_{kt}, L_{kt}, LEN_k, LBO_{kt}, UBO_{kt}\}$ such that

$j=1,..,NO_k$ $t=1,..,T$

So the contract option k can be defined using a set having $(5+2NO_k)T+2$ elements. This set definition can represent many possible contract designs, so we can call it as a generic contract definition. The producer has the opportunity to use these parameters to evaluate different options. Cost figures related to production, inventory carrying and backorder are not exogenous. These can be stated as follows; fixed cost of production, unit cost structure of production, cost of inventory carrying, storage costs, penalty cost for backordering, upper and lower bounds on production amounts, discounting factor used to carry all monetary terms to a specific point in time.

3.1.3 Decision Variables

Decision variables are used to gather the output of decisions from the model. They can also be used as performance measures or benchmarks. They define the optimal behavior of the producer in this decision process. All the production, outsourcing, inventory holding and demand backordering decisions are identified with these variables. The decision variables for our model are as follows:

B_t : Amount of demand in units backordered in period t.

This variable is used to record the amount of unsatisfied demand to be satisfied in later periods, and takes place in inventory balance equations. It is used in the penalty cost computation at the end of each period.

H_{jt} : Inventory valuation variables for j^{th} price break in period t.

In our problem setting, there exist different sources for product procurement and they all have different unit costs, even the cost of production changes with time. The average unit cost value of the products is needed for calculating the holding cost using interest rate, but in our case it is hard to record. An approach may be to hold each unit of inventory with its price, and if we record all the prices and amounts, then we can select a rule such as LIFO or FIFO and compute the holding costs according to this rule. But this approach uses too many variables and is not preferred. Instead, we develop an approximate procedure to revalue the inventory on hand at the end of each period. In our approximation, inventory is distributed to H_{jt} variables and then the holding cost is calculated using this revaluation variables. We assume that the inventory on hand is valued as if it is produced by the production plant in this period with the existing cost structure, and H_{jt} variables are used to represent different price breaks for that period, using the cost break points associated with the current period. If the inventory on hand is greater than the production capacity, then a problem arises. In order to solve this problem, capacity is taken as infinity in the inventory revaluation process. The part of the inventory exceeding the capacity is revalued from the highest price break. A numerical example is presented in the section where model capabilities are explained.

I_t : Inventory on hand in units at the end of period t.

This variable is used to record the amount of inventory kept at the end of each period and takes place in the inventory balance between periods. The storage cost is calculated using this variable at the end of each period.

O_{kjt} : Outsourcing amount in units for contract option k at the j^{th} price break in period t.

This variable is used to determine the amount ordered from contract option k with the j^{th} price break. This variable can take positive values if a contract from the k^{th} option is still working, otherwise it automatically takes zero value.

P_{jt} : Production amount in units in the j^{th} price break in period t.

The production amount in each period is represented using this variable. This variable states the amount of production done using the j^{th} price break.

ZC_{kt} : $\begin{cases} 1 & \text{if a contract is established for contract option k in period t.} \\ 0 & \text{otherwise} \end{cases}$

This variable is an indicator for the establishment of a contract from k^{th} option in period t. It is used in the calculation of fixed contract cost.

ZO_{kt} : $\begin{cases} 1 & \text{if outsourcing is exercised using contract option k in period t} \\ 0 & \text{otherwise} \end{cases}$

This variable is the indicator for the order placed from k^{th} option in period t. It is used in the calculation of fixed outsourcing cost at the end of each period.

ZP_t : $\begin{cases} 1 & \text{if a production run has occurred in period t.} \\ 0 & \text{otherwise} \end{cases}$

This variable is the indicator of a production run in period t. It is used in the calculation of fixed setup cost at the end of each period.

3.1.4 Objective Function

$$Minimize \ \sum_{t=1}^{T}\prod_{i=1}^{t}(DIS_i)[\sum_{k=1}^{NC}(\sum_{j=1}^{NO_k}(CO_{kjt}.O_{kjt}) + FO_{kt}.ZO_{kt} + FC_{kt}.ZC_{kt}) +$$

$$\sum_{j=1}^{NP}(CP_{jt}.P_{jt}) + FP_t.ZP_t + INT_t.\sum_{j=1}^{NP}(CP_{jt}.H_{jt}) + S_t.I_t + BK_t.B_t] \qquad \textbf{(3.1)}$$

Objective function (Eq. 3.1) is of minimization type with the present worth of a total cost function. Discounted period costs are summed up for the objective function calculation. A period’s cost consists of the following cost terms;

$$CO_{kjt}.O_{kjt} \qquad \textbf{(3.2)}$$

Variable outsourcing costs (3.2) calculated using the breaks in unit cost structure for each option utilized,

$$FO_{kt}.ZO_{kt} \tag{3.3}$$

Fixed outsourcing costs (3.3) cumulated for the periods in which a contract option is used.

$$FC_{kt}.ZC_{kt} \tag{3.4}$$

Fixed contract costs (3.4) summed up for the periods in which a contract is established for a contract option.

$$CP_{jt}.P_{jt} \tag{3.5}$$

Variable production costs (3.5) calculated using the breaks in unit cost structure,

$$FP_t.ZP_t \tag{3.6}$$

Fixed production costs (3.6) are summed up for periods where a production setup is done,

$$INT_t.\sum_{j=1}^{NP}(CP_{jt}.H_{jt}) \tag{3.7}$$

Inventory holding cost (3.7) calculated using the valuation variables introduced earlier and the time value of money for the producer,

$$S_t.I_t \tag{3.8}$$

Storage costs (3.8) that are related to the physical storage area are calculated using the direct inventory terms in units,

$$BK_t . B_t \tag{3.9}$$

Backorder costs (3.9) that can be thought as the loss of goodwill or some penalty due to the late satisfaction of the demand are calculated using the backorder variable.

3.1.5 Constraints

Subcontracting Constraints:

$$\sum_{i=t-LEN_k+1}^{t} ZC_{ki} \leq 1 \qquad \forall t=LEN_k,..,T \ \ \forall k=1,..,NC \tag{3.10}$$

The above constraint set is used to limit the number of working contracts from the same subcontract option to only one in a period. This constraint guarantees that a contract option cannot be used if a contract of the same option is still active. If this constraint is not set, and the same option is used more than once in any period, then the cost structure, bounds and the length of the contract will make up a new option. This is not logical in the evaluation problem.

$$ZO_{kt} \leq \sum_{i=\max(t-LEN_k+1,1)}^{t} ZC_{ki} \qquad \forall t=1,..,T \ \ \forall k=1,..,NC \tag{3.11}$$

The above constraint set guarantees that a contract has been established between the producer and a subcontractor in order to outsource some of the demand in a period.

$$O_{kjt} \leq SO_{kjt} - SO_{k,j-1,t} \qquad \forall t=1,..,T \;\; \forall j=1,..,NO_k \;\; \forall k=1,..,NC \qquad \textbf{(3.12)}$$

The above constraint set is used to limit the outsourcing amounts from a price break with the predefined interval. Intervals on the break unit cost structure are defined using this constraint. These constraints are depicted in Figure 3.4.

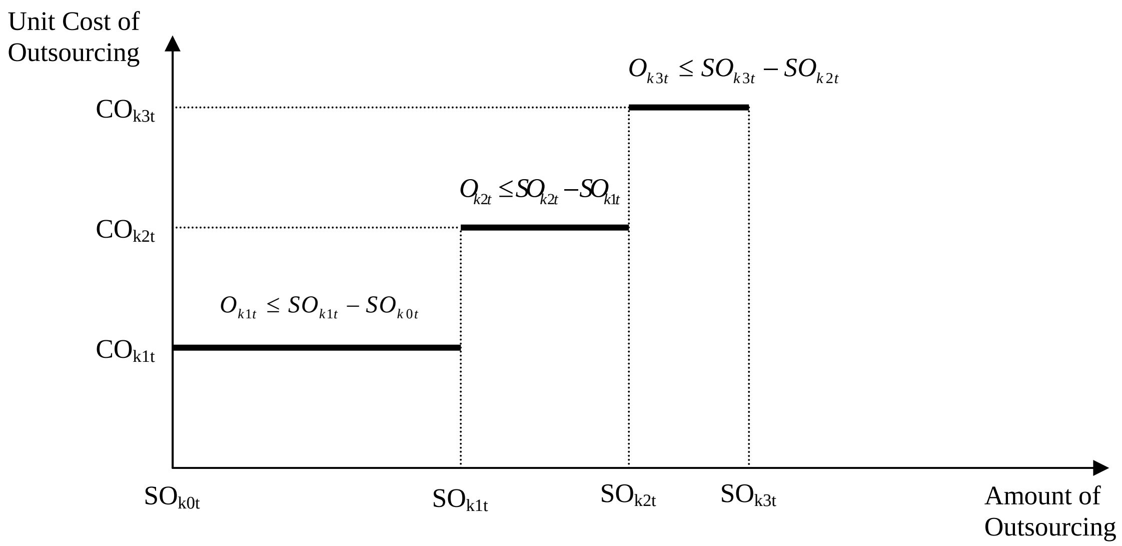

Figure 3.4: Graphical representation of the constraint set (Eq. 3.12) with NO_k=3

$$\sum_{j=1}^{NO_k} O_{kjt} \leq UBO_{kt}.ZO_{kt} \qquad \forall t=1,..,T \;\; \forall k=1,..,NC \qquad \textbf{(3.13)}$$

The above constraint set guarantees that a setup or transportation is taking place with a fixed cost in order to have outsourcing in a period. This constraint set also limits the outsourcing amounts with a predefined upper bound value. If the fixed outsourcing cost is negligible relative to other costs and there is no limitation on the outsourcing limits, then this constraint set becomes redundant.

$$LBO_{kt}.ZO_{kt} \leq \sum_{j=1}^{NO_k} O_{kjt} \qquad \forall t=1,..,T \;\; \forall k=1,..,NC \qquad \textbf{(3.14)}$$

The above constraint set is used to set a lower bound to the outsourcing quantities if an outsourcing option is utilized in a period. This lower bound can be a part of commitment set in contract, or has economic or technical feasibility meaning.

Production Constraints:

$$P_{jt} \leq SP_{jt} - SP_{j-1,t} \qquad \forall t=1,..,T \;\; \forall j=1,..,NP \qquad \textbf{(3.15)}$$

The above constraint set is used to limit the production amounts from a price break with the predefined interval. Intervals on the break unit cost structure are defined using this constraint. These constraints are depicted in Figure 3.5

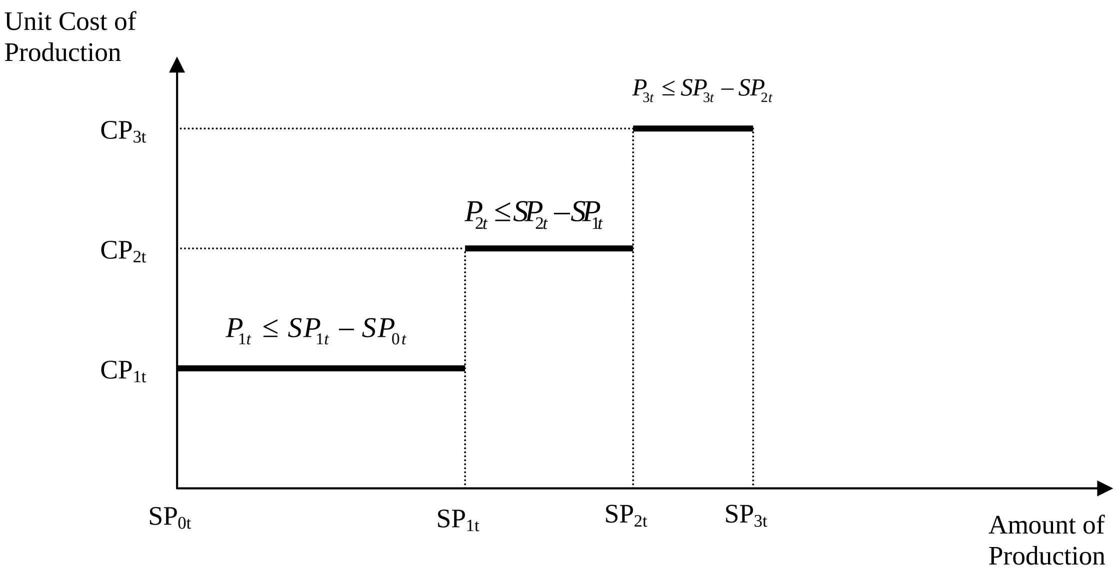

Figure 3.5: Graphical representation of the constraint set (Eq. 3.15) for NP=3

$$\sum_{j=1}^{NP} P_{jt} \leq UBP_t.ZP_t \qquad \forall t=1,..,T \qquad \textbf{(3.16)}$$

The above constraint set is the production setup constraint. In order to have production in any period, a setup must be performed before the start of the production run; also an upper bound is set to the production quantity. If the setup cost is negligible relative to the other cost figures, then this constraint can be omitted.

$$LBP_t.ZP_t \leq \sum_{j=1}^{NP} P_{jt} \qquad \forall t=1,..,T \qquad \textbf{(3.17)}$$

The above constraint set is used to set a lower bound to the production quantities if a production run is scheduled in a period. This lower bound can have economic or technical feasibility meaning.

Inventory Constraints:

$$I_{t-1} - B_{t-1} + \sum_{j=1}^{NP} P_{jt} + \sum_{k=1}^{NC} \sum_{i \in \{i:\, i+L_{ki}=t)} \sum_{j=1}^{NO_k} O_{kji} - I_t + B_t = D_t \quad \forall t=1,..,T \qquad \textbf{(3.18)}$$

The above constraint set is the well-known inventory balance constraint including the outsourced units from different contract options and backorders in a period. The demand in a period is either satisfied with a combination of the inventory, outsourced units, production in that period, or backordered for late satisfaction.

$$I_t = \sum_{j=1}^{NP} H_{jt} \qquad \forall t=1,..,T \qquad \textbf{(3.19)}$$

$$H_{jt} \le SP_{jt} - SP_{j-1,t} \qquad \forall t=1,..,T \;\; \forall j=1,..,NP\text{-}1 \qquad \textbf{(3.20)}$$

These constraint sets are used in inventory valuation process at the end of each period. The on hand inventory is re-valued using the production cost structure with infinite capacity.

Constraints due to Initial and Ending Condition:

$$I_0 = \mathrm{I}_0 \quad B_0 = \mathrm{B}_0 \qquad I_T = \mathrm{I}_\mathrm{T} \quad B_T = \mathrm{B}_\mathrm{T} \qquad \textbf{(3.21)}$$

The above constraint set is used to define the initial and ending conditions of the inventory and backorder variables. The decision variables are equated to parameter values that are introduced earlier in this chapter.

Set Constraints:

$$B_t,\, H_{jt},\, I_t,\, O_{kjt},\, P_{jt} \ge 0 \qquad ZO_{kt},\, ZP_t,\, ZC_{kt} \in \{0, 1\} \qquad \textbf{(3.22)}$$

The above constraint set is used to define the non-negativity and integrality properties of our decision variables.

The complete model is given below:

$$Minimize \;\; \sum_{t=1}^{T} \prod_{i=1}^{t} (DIS_i)[\sum_{k=1}^{NC} (\sum_{j=1}^{NO_k} (CO_{kjt}.O_{kjt}) + FO_{kt}.ZO_{kt} + FC_{kt}.ZC_{kt}) +$$

$$\sum_{j=1}^{NP} (CP_{jt}.P_{jt}) + FP_t.ZP_t + INT_t.\sum_{j=1}^{NP} (CP_{jt}.H_{jt}) + S_t.I_t + BK_t.B_t] \qquad \textbf{(3.1)}$$

Subject To

$$\sum_{i=t-LEN_k+1}^{t} ZC_{ki} \le 1 \qquad \forall t=LEN_k,..,T \;\; \forall k=1,..,NC \qquad \textbf{(3.10)}$$

$$ZO_{kt} \leq \sum_{i=\max(t-LEN_k+1,1)}^{t} ZC_{ki} \qquad \forall t=1,..,T \;\; \forall k=1,..,NC \tag{3.11}$$

$$O_{kjt} \leq SO_{kjt} - SO_{k,j-1,t} \qquad \forall t=1,..,T \;\; \forall j=1,..,NO_k \;\; \forall k=1,..,NC \tag{3.12}$$

$$\sum_{j=1}^{NO_k} O_{kjt} \leq UBO_{kt}.ZO_{kt} \qquad \forall t=1,..,T \;\; \forall k=1,..,NC \tag{3.13}$$

$$LBO_{kt}.ZO_{kt} \leq \sum_{j=1}^{NO_k} O_{k1t} \qquad \forall t=1,..,T \;\; \forall k=1,..,NC \tag{3.14}$$

$$P_{jt} \leq SP_{jt} - SP_{j-1,t} \qquad \forall t=1,..,T \;\; \forall j=1,..,NP \tag{3.15}$$

$$\sum_{j=1}^{NP} P_{jt} \leq UBP_t.ZP_t \qquad \forall t=1,..,T \tag{3.16}$$

$$LBP_t.ZP_t \leq \sum_{j=1}^{NP} P_{1t} \qquad \forall t=1,..,T \tag{3.17}$$

$$I_{t-1} - B_{t-1} + \sum_{j=1}^{NP} P_{jt} + \sum_{k=1}^{NC} \sum_{i \in \{i:\, i+L_{ki}=t\}} \sum_{j=1}^{NO_k} O_{kji} - I_t + B_t = D_t \qquad \forall t=1,..,T \tag{3.18}$$

$$I_t = \sum_{j=1}^{NP} H_{jt} \qquad \forall t=1,..,T \tag{3.19}$$

$$H_{jt} \leq SP_{jt} - SP_{j-1,t} \qquad \forall t=1,..,T \;\; \forall j=1,..,NP\text{-}1 \tag{3.20}$$

$$I_0 = \mathrm{I}_0 \quad B_0 = \mathrm{B}_0 \qquad I_T = \mathrm{I}_\mathrm{T} \quad B_T = \mathrm{B}_\mathrm{T} \tag{3.21}$$

$$B_t,\, H_{jt},\, I_t,\, O_{kjt},\, P_{jt} \geq 0 \qquad ZO_{kt},\, ZP_t,\, ZC_{kt} \in \{0, 1\} \tag{3.22}$$

3.1.6 Size of the Model

The size of our model is determined by NC+3 main parameters, which are;

NC	:	Total number of contract options.
NO_k	:	Total number of price breaks in the outsourcing cost structure for contract option k. (k=1,..,NC)
NP	:	Total number of price breaks in the production cost structure.
T	:	Number of periods in the planning horizon

There exist different measures for the size of a model. These are the total number of linear variables, binary variables, input parameters, constraint etc. All these measures capture some part of the complexity involved in a model. Number of linear variables (see Table 3.1) affects the solution time with a polynomial contribution. On the other hand binary variables (see Table 3.2) have a non-polynomial contribution to the solution time. Parameters (see Table 3.3) are important for the input process complexity, and finally constraints (see Table 3.4) build up the big matrix used for the solution in many software applications used in optimization, and affect the storage complexity of the problem.

Table 3.1: Total Number of Linear Variables

Variable Name	**Number of Linear Variables**	**Variable Name**	**Number of Linear Variables**
B_t	T	O_{kjt}	$\sum_{k=1}^{NC} NO_k .T$
H_{jt}	$NP.T$	P_{jt}	T
I_t	T	**Total Number of Linear Variables**	$(3+\sum_{k=1}^{NC} NO_k +NP).T$

Number of linear variables linearly depends on the number of periods in the planning horizon. Number of price breaks in production cost structure and outsourcing cost structure also affect the number of linear variables but not as much as the number of periods in the planning horizon.

Table 3.2: Total Number of Binary Variables

Variable Name	Number of Binary Variables	Variable Name	Number of Binary Variables
ZO_{kt}	$NC.T$	ZC_{kt}	$NC.T$
ZP_t	T	**Total Number of Binary Variables**	$(1 + 2.NC).T$

Number of binary variables linearly depends on the number of periods in the planning horizon and number of contract options.

Table 3.3: Total Number of Input Parameters

Parameter Name	Number of Parameters	Parameter Name	Number of Parameters
CO_{kjt}	$\sum_{k=1}^{NC} NO_k.T$	LBP_t	T
CP_{jt}	$NP.T$	LEN_k	NC
BK_t	T	NC	1
D_t	T	NO_k	NC
DIS_t	T	NP	1
FC_{kt}	$NC.T$	S_t	T
FO_{kt}	$NC.T$	SO_{kjt}	$\sum_{k=1}^{NC} NO_k.T$
FP_t	T	SP_{jt}	$NP.T$
INT_t	T	T	1
L_{kt}	$NC.T$	UBO_{kt}	$NC.T$
LBO_{kt}	$NC.T$	UBP_t	T
Total Number of Parameters	$2+(8+2.NP+5.NC+ 2. \sum_{k=1}^{NC} NO_k).T$		

The number of parameters linearly depends on the total number of periods in the planning horizon. Number of price breaks in production, outsourcing functions and number of contract options do not affect the number of parameter that much.

Table 3.4: Total Number of Constraints

Constraint Number	**Number of Constraints**	**Constraint Number**	**Number of Constraints**
Eq. 3.2	NC.T	Eq. 3.8	T
Eq. 3.3	NC.T	Eq. 3.9	NC.T
Eq. 3.4	T	Eq. 3.10	T
Eq. 3.5	NC.T	Eq. 3.11	T
Eq. 3.6	NP.T	Eq. 3.12	NP.T
Eq. 3.7	$\sum_{k=1}^{NC} NO_k .T$	Eq. 3.13	1
Total Number of Constraints	(4+4.NC+ $\sum_{k=1}^{NC} NO_k$ +2.NP).T+1		

The number of constraints linearly depends on the total number of periods in the planning horizon. Number of price breaks in production, outsourcing functions and number of contract options do not affect the number of constraints that much.

3.2 Capabilities of the Model

Our model has capabilities of handling different problem structures and environments. These are provided by the structure of the formulation, variables and parameters. These capabilities are explained in the next sections, and finally a compatibility type of analysis is done with the literature.

3.2.1 Capabilities over Model Structure and Variables

Capabilities can be defined using the structure of the model formulation. Variables are the main elements of the model structure, with their definition some properties are allowed or not. In our model development phase, we try to be as general as we can to allow our model to handle a large set of problems. These capabilities provided with our structure are as follows:

- **Backorder:** We use backorder as an alternative source of supplying demand. In other words, we allow the model to delay some of the demand satisfaction to later periods. A cost term is introduced for that purpose to control the amount of this quantity.
- **Outsourcing:** We use outsourcing as an alternative source of supplying demand. We allow the producer to satisfy demand not from its own production, but from subcontractors, if capacity is exceeded or it is economically efficient. A unit cost term is defined for each unit outsourced.
- **Complex cost structure:** We allow quantity dependent unit costs using price breaks in unit cost structure. A fixed cost is used to represent the cost of setup, order or delivery. At any period the cost of production or outsourcing depends on the quantities.
- **Contracting:** We allow the existence of long-term contracts; in classical literature outsourcing is done on period by period basis. But in our model we allow the establishment of long-term contracts with their own cost structure. A contract should be established in order to use the outsourcing option in any period. Length of the contract can be taken as one periods in order to model the outsourcing option into our model without the establishment of long term contracts.
- **Holding costs with price changes:** Existence of multiple sources causes a problem to occur on the holding cost calculation. An approximation is used in our model; we re-value our inventory at the end of each period as if it's produced in this period with our production facilities. A variable is

introduced to represent the different price breaks of the cost function, the last interval is assumed to be unbounded in order to handle inventories higher than our production capacity for that period. These variables are the H_{jt}. A numerical example is presented as follows:

Example:

Let $I_2=100$ $NP=2$ $CP_{12}=1$ $CP_{22}=2$ $SP_{12}=60$ $SP_{22}=75$ $INT_2=0.01$.

When our constraints work at the end of the second period, the H_{jt} variables will take the following values.

$H_{12}=60$ $H_{22}=40$

and holding cost will be calculated using the values of the variables as;

$CP_{12}.H_{12} + CP_{22}.H_{22} = 1.60 + 2.40 = 140$

and the relevant inventory holding cost is calculated as;

$INT_2.(CP_{12}.H_{12} + CP_{22}.H_{22}). = (0.01).140 = 1.4.$

- **Multiple contract options:** The model allows the use of multiple contract options. Each option is available at any time and selection of one for outsourcing, or selection of a combination is both possible. There exits only a single restriction that any contract option can be used only once at any period, this restriction is used for convenience.

3.2.2 Detailed Description of the Model Parameters and Discussion

Parameters are the main input of our model, so they shape the capabilities of our model. In this section, we try to express the flexibility provided by our definition and modeling of the parameters. The capabilities of the model, provided by the parameters are as follows:

- **Backorder Costs:** Backorder cost is a penalty used for the late satisfaction of demand. The zero value of this parameter represents the case where late satisfaction of the demand has no penalty. And if we do not want to allow backordering then we can set this penalty cost as infinity.

- **Demand:** We allow for arbitrary demand values. However, we can restrict ourselves to demand realization that can be identified by three properties. Random fluctuations, seasonality and trend. Random fluctuations model the real life realization of demand, there exits a stochastic part of the demand, which cannot be forecasted precisely. Seasonality is the change of the demand between different periods. Trend is the linear change of the demand with time. In their life cycle, many products face both positive and negative trends, also the market conditions and marketing activities can cause a trend in the demand. The constant demand represents the case where none of the properties exits for the demand of our product. Our model structure allows the existence of these two properties alone, or combined with each other through the planning horizon.
- **Discounting:** In most of the real life examples the present worth is used as a measure for the monetary terms for long planning horizons. We use a discounting factor multiplier for each period in our model, so we can come up with the present worth of the total cost and make our decision on this basis. $DIS_t=1$ represents the case where either the horizon is short, so present values practically are not affected or the parameters used already reflect the time value (i.e. they are real)
- **Dynamic parameters:** In our modeling approach we allow every parameter to be time varying (i.e. dynamic). With this approach, we increase the flexibility of our model without adding too much complexity. In practice, nearly every environmental parameter, depend on time and vary with time, has a seasonality, or trend in its value, or experience random fluctuations. These three properties are applicable to most of the parameters used in our model.
- **Fixed cost of contract:** Fixed cost of contract is one of the fundamental terms in our analysis of contract evaluation, and selection. This cost is used as the long-term bounding cost between the subcontractor and the producer. Its value can depend on other contract parameters. The zero value indicates

no cost is incurred for the establishment of the contract, or no guaranteed payments are necessary.

- **Fixed cost of outsourcing:** This cost value is related to the order and delivery costs of outsourcing from a specific option in a period. The zero value indicates that the related cost is almost negligible or these costs are incurred in the contract cost at the beginning.
- **Fixed cost of production:** Fixed cost of production is the setup related cost of a production run in a period. The zero value of this variable indicates that the setup cost is negligible relative to other cost figures.
- **Interest rate:** The model uses interest rate in the calculation of holding cost for the ending inventories of each period. The valuation of the on hand inventory is made using the break unit cost function with an infinite limit for the most expensive interval. If interest rate is taken as zero the holding cost related to time value of money will be omitted, only physical storage costs will remain.
- **Lead times:** This parameter represents the time between the delivery and the order time for an outsourcing request. This time can change over different contract options. The lead times can be zero, constant, seasonal or random fluctuating. Zero lead times means the delivery of the outsourced units are done immediately after the order takes place.
- **Length of contracts:** It defines the number of periods that a contract option can be used in outsourcing after the establishment of a contract. The length of contract can be different for each contract option. The length of only one period represents the case where there is no need to have a contract for outsourcing utilization, i.e. it corresponds to simply exercising an option to buy.
- **Lower bound on outsourcing:** There exists a lower bound on outsourcing that is used to represent the minimum efficient amount or commitments accepted by the contract, no lower amounts are allowed. The zero value indicates that there exists no efficiency limit or commitment involved in the contract.

- **Lower bound on production:** This bound is used for economic efficiency. The zero value represents the case, where the setup is costless or there is no need for a setup.
- **Multiple contract options:** The model enables the evaluation of multiple contract options. Each contract option has its own parameters; fixed costs, unit costs, price break intervals, upper and lower bounds, lead times and length. The NC parameter represents the number of contract options if this parameter is set to one, then our model can be used to evaluate a single contract.
- **Planning horizon:** The model can work with different length of planning horizons. However, to complete the description of the environment, we conceptualize planning horizons as short, medium and long.
- **Storage costs:** Storage cost is the physical storage cost incurred for storing inventory kept in a space. For many product types this kind of cost is more important than the cost related to time value of money. The zero value indicates that the physical storage costs are negligible relative to the time value of money.
- **Unit cost of outsourcing:** The model can handle price breaks in unit cost structure for outsourcing with all contract options. In unit cost function with price breaks, the cost increases as the amounts increases, the logic behind this is the need for overtime in order to produce larger quantities. For example, first break may represent the normal production capacity and the relevant costs. Second and further breaks are used to represent overtime and extra time of labor. So the intervals are much smaller than the regular ones and the costs are higher. NO_k is used to represent the number of price breaks in the unit cost structure of that contract option. If NO_k is equal to one, this case becomes equivalent to the formulation of the classical outsourcing without complex cost structure.

- **Unit cost of production:** The model can handle price breaks in unit cost structure of production. In this unit cost function with price breaks, the cost increases with the amounts, the logic behind this is the need for overtime in order to produce larger quantities. First break represents the normal production capacity and the relevant costs. Second and further breaks are used to represent overtime and extra time of labor. So the intervals are much smaller than the regular ones and the costs are higher. *NP* is used to represent the number of price breaks in the unit cost structure, and if this parameter is set to one, then our formulation corresponds to simple cost structure without price breaks.
- **Upper bound on outsourcing:** Upper bounds are about the capacity allocation of the subcontractor to the producer. The upper bounds are used over the price break structure; they can limit outsourcing amounts. If they are set to infinity, this indicates that the outsourcing facility has an infinite capacity relative to the demand of the producer.
- **Upper bound on production:** Upper bounds are used to restrict the capacity allocated to this specific product. Upper bounds are used over the price break structure; they can limit the production amount. If they are set to infinity, this indicates that there exist no capacity problems at the producer.

3.2.3 Capabilities Compared with the Lot-Sizing Literature

We incorporate a lot of issues together with the aim of satisfying supply chain considerations, not classical lot sizing. Main aim in classical lot sizing studies is to find some structural properties. However, it is important for our model to handle some of the extensions. The following studies are not mentioned before as they are related to classical lot sizing.

Wolsey (2002) classifies different problems according to their general structure. The classification includes four main types of problems; LS (Lot Sizing), WW (Wagner-Whitin), DLSI (Discrete Lot Sizing Variable Initial Stock), DLS (Discrete Lot Sizing). He uses a second classification over capacity as; C

(Capacitated), CC (Constant Capacity), U (Uncapacitated). His last classification is over extensions that can be applied to all these cases which are; B (Backlogging), SC (Start-Up Costs), ST (Start-Up Times), ST(C) (Constant Setup Times), LB (Minimum Production Levels), LB(C) (Constant Lower Bound) SL (Sales), SS (Safety Stocks). He uses these three classifications combined to classify all related problems in this field. All problems are represented using the following scheme; [LS, WW, DLSI, DLS] – [C, CC, U] - [B, SC, ST, ST(C), SL, LB, LB(C), SS], where one entry is required from each of the first two groups, and any number of entries from the last. Our model can handle most of these cases without any change, and can handle only some of them namely Start-up times, Sales, Safety stocks with additional constraints or decision variables. In order to handle start-up times, the capacity parameters for every period can be redefined using the loss of capacity due to start-up times. Sales variable can be used in the inventory balance equation, and the relevant cost reduction value can be inserted in to the objective function. Safety stock can be incorporated using a lower limit on our inventory variables.

Lee and Zipkin (1989) work on the problem of dynamic lot sizing with make-or-buy decisions with complex cost structures. They use the same structure with us for inventory balance, demand satisfaction and capacity limitation for production amounts. Their model can be converted into ours with basic cost structures. Non-linear cost structure defined in their work cannot be represented with our model.

Aksen et al (2003) work on the single item lot-sizing problem with immediate lost sales. The most important property of their model is the partial satisfaction of the demand with lost sales. Some of the demand may not be satisfied as a decision variable and a cost is associated with this lost amount. In order to represent their study with our model, we should eliminate backorder variable from both the balance equations and objective function. We should keep track of demand that is not satisfied, and insert a term for the lost sales related costs to our objective function.

Beltran and Krass (2002) model dynamic lot-sizing problem with returning items and disposals. They allow the return of items using non-positive demand figures for some periods. Some of the inventory can be disposed with a cost figure in order to minimize inventory related costs. Their model can be represented using our model with some changes; The non-negativity constraint over demand should be discarded and a disposal variable for every period should be defined and added to the inventory balance equations.

Chiu (1997) proposes a discrete time-varying demand lot-sizing model with learning and forgetting effects. Learning occurs when there exist production runs on successive periods, forgetting occurs when learning can not. In other words, if production is not scheduled for some periods, then the labor related productivity and the capacity decreases and the cost increases. This model can be represented with our model using some major changes. The cost structure should be redefined in order to handle learning and forgetting effects, also the capacity parameters should be related to the past periods production decision and productivity figures.

Sox and Gao (1999) work on the capacitated lot-sizing problem with setup carry-over. This model is different from most of others in the literature, since it allows the setup carry-over. Setup carry over means that if a production run occurs in the previous period then there is no need to do a new setup and no need to incur additional setup cost. This model can be represented in our study using a new variable for setup carry over to successive period, and this variable should be used with the classical setup variable, allowing to produce if only one of them is available either carried setup or new production setup.

Denizel et al (1997) model dynamic lot sizing with setup cost reduction. Main difference of their model from the classical literature is the setup cost reduction opportunity. This reduction is related to the amount of investment done in order to reduce setup cost. This cost figure is included in their objective function for optimization purposes. Their model can be represented using our model with using a similar investment term in our objective function and relating the setup cost value to this value.

Loparic et al (2001) work on the problem of uncapacitated lot-sizing problem with sales and safety stocks. Their model allows the sale of the unsold products with a different price value and the holding of inventory for dealing with the randomness involved in the demand realization. Their model can be represented by our model, first by setting capacity values as infinity, and then defining a new variable for sales variables in every period. These new variables should be included into our objective function with related cost values. Safety stock value can be defined using a new constraint on the minimum value of the inventory on hand for any period.

Lee et all (2001) propose a dynamic lot sizing model with demand time windows. In classical lot sizing models, demand occurs at each period, and it must be satisfied on the period of occurrence if no backordering is allowed. On the other hand if backordering is allowed, then it should be satisfied in later periods of its occurrence. In this study they propose a setting where demand comes with time windows in which no penalty is incurred for delivery. This model can be represented with our model using a new demand definition involving time windows.

Özdamar and Bozyel (2000) model the capacitated lot-sizing problem with overtime and setup times. In their model, there is an option of satisfying a portion of demand with overtime, as expected the cost of overtime is higher than regular production cost. Setup times decrease the capacity defined for every period. In order to represent their model the capacity figures should be related with the setup variables.

Shaw and Wagelmans (1998) work on the problem of single-item capacitated economic lot sizing with piecewise linear production costs and general holding costs. Their problem can be represented with our formulation if the piece-wise linear function is a piece-wise increasing function. Otherwise, we need to define new binary decision variables in order to represent the concave cost structure.

Pochet and Wolsey (1993) model the problem of lot sizing with constant batches. The capacity used in each period should be an integer multiple of a predefined amount. In order to model their problem using our structure, we should re-define the capacity values assigned for each period as decision variables which is an integer multiple of a given parameter defining the batch size.

Chyr et al (1999) propose a dynamic lot-sizing model with quantity discounts. Their model is similar to ours in inventory balance, and demand satisfaction. In order to represent their model, new binary variables should be introduced to represent the concave piecewise unit cost structure.

Martel and Gascon (1998) work on dynamic lot sizing with price changes and price-dependent holding costs. Their effort is based on the solution of the model proposed. Our model can directly represent their problem since the price changes, and price dependent holding costs are allowed in our model.

CHAPTER 4

EXPERIMENTATION AND RESULTS

In this chapter, our aim is to present the capabilities and the uses of our model in the analysis of production systems. In this analysis, we try to find out the behavior of the system under different settings. In the first section, we introduce the factors that are used to define a setting. Factors can affect the optimal behavior of the system with their varying levels. An experimentation base is created in order to analyze the effects of these factors. In the next section, Full Factorial experimental design and ANOVA are used to identify the effects of the factors, on objective function and specific performance measures that define the system behavior. Main aim for performing ANOVA is to come up with an experimentation base for the detailed analysis of two factors and their interaction. Using full factorial design, the interactions of factors among themselves can be analyzed, but there exist too much interactions and it is hard to analyze them all. A Pascal code is used to generate the models using different parameter sets, and create input model files for CPLEX. Then the created models are solved using CPLEX 6.0 and their outputs are obtained in text format in output files. Then using Pascal code the text outputs are read and the summary of the results is prepared and written on another output file. Pascal code can be provided upon request. The model is run about a hundred thousand times for detailed analysis of different levels of the factors introduced. Then, these analyses are extended to the analysis of the interaction of two specific contract parameters; Fixed Contract Cost and Length of Contract. In the third section, similar analysis is done in an environment where there are two subcontracting contract options available. In the last section, we discuss the results obtained.

4.1 Factors

Some of the parameters can have an important impact on the behavior of the system. These parameters are identified as the factors that build up different settings for our experimental analysis. There exists many other factors, but we select a proper subset of them because of the limits on total computation time. Every new factor that is introduced increases the total computation time with a multiplier of three. The importance of these factors are discussed below, also brief information is given about the values that they can get in real life. Our factors are as follows:

Factor 1: Trend of demand

Demand is one of the important parameters in our study. Demand can be identified with different parts as trend, seasonality, and randomness. We try to cover most of the possible settings within our experimental study. Demand patterns are defined using trend and seasonality and random part is omitted, since our study is about deterministic demand. Trend is selected as the first factor in our factor analysis. Trend can be either positive or negative, and can have high or low slope.

Factor 2: Seasonality of demand

Second factor used in the identification of the demand is the seasonality. Seasonality is a common property for most of the products in real world. It is defined as the change of demand between different periods independent of the trend. Different levels of seasonality can be realized in demand patterns; seasonality may be high, low or none.

Factor 3: Unit Cost of Outsourcing

Unit cost of outsourcing is an important factor in our study, because it can alone affect the amount of outsourcing used in the demand satisfaction, irrespective of the fixed costs related to outsourcing process.

Factor 4: Storage Costs

The storage cost is the factor affecting the amount of inventory carried. It is the cost of physical storage used in the inventory carrying. In our experimental setup, interest rate is taken as zero, so only factor used in objective function for inventory terms remains to be the storage cost. Inventory holding and backordering of the demand are two options that can be used to satisfy the excess demand where economies of scale are available or the capacity is not sufficient.

Factor 5: Backorder Cost

This factor affects the amount of backordered demand. Its relative value to the inventory related costs is determining the inventory and backorder amounts, which are the alternative sources of satisfying demands of customers.

Factor 6: Target Time between Two Production Runs

This factor is the indicator of the production policy that the producer uses in its operations. A small value represents that the producer prefers to produce as frequently as it is economical. Larger values show that the producer has capacity allocation restriction or other restriction preventing him from frequent production runs. We set the production setup cost in accordance with this value. If outsourcing option is not available and the producer has no capacity restrictions, then this factor's value defines the average frequency of the production runs.

Factor 7: Production Capacity

Capacity defines the amount of products that can be produced in any given period. Any restriction like machine time, labor hours or monetary investment can define this capacity. Capacity restriction can be the reason for the utilization of outsourcing option, backordering of demand or holding of inventory to balance the production amounts in different periods.

Factor 8: Length of Contract

The length of contract defines the number of periods in which the outsourcing option is available after the establishment of a contract. Its value is important, since it defines the average fixed cost of contracting with the fixed production setup cost. Very small values can cause non utilization of the outsourcing options and very high values can eliminate the utilization of production utilization given a fixed contract cost. This factor is important for negotiation purposes; its value can be decided using a bidding or auction process.

Factor 9: Fixed Contract Cost

Fixed contract cost is paid for the establishment of a contract for the usage of outsourcing option. Its value and the fixed cost of outsourcing value relative to fixed production setup costs determine the amount of outsourcing and production used in the satisfaction of demand. Fixed cost of contracting is defined in terms of the fixed production setup cost, length of contract and the desired average frequency of production runs.

Factor 10: Fixed Outsourcing Cost

Fixed outsourcing cost is paid for the order and delivery related cost of a period's outsourcing order. It determines the proportion of outsourcing and production utilized in the total demand satisfied. We relate the value of this factor to the average demand of customers.

4.2 Experimentation Base

For the use in experimentation and better understanding of the results, a base model environment is created. The base case aids us in designing further experimentation. In this environment, some of the parameters are taken as constants, some are used to relax constraints with their values, e.g. capacity equal to infinity relaxes the capacity constraint.

- Only a single contract option is allowed in the analysis at first, then two alternative contracts are evaluated together.
- Planning horizon is taken as 12 periods.
- Discounting factor is taken as 1 in order to cancel the effect of discounting on the results. Such that the real values are used instead of nominal values of all cost figures.
- Interest rate for each period is taken as zero for eliminating the holding cost and using only the physical storage cost for this purpose.
- Lead times are taken as zero for immediate delivery.
- Upper and lower bounds are not utilized for outsourcing, by defining an interval using zero value for lower bounds and cumulative demand for upper bounds.
- Single price break is used for outsourcing.
- Two prices are used for defining production unit cost. Two levels in the production process represent the regular time and overtime productions. The overtime interval is smaller as there exist legal restrictions on overtime relative to regular time. Cost value related to this interval is higher than the regular time production cost term.

- Regular time production cost is taken as 1 per unit, and the overtime cost is taken as 50% higher, as 1.5 per unit.
- Fixed cost of production is calculated using the desired order frequency (*F*) in classical deterministic inventory formulation, and balancing the tradeoffs between inventory holding and fixed setup costs. This desired frequency is one of the experimental factors used in our analysis, it is represented as the Factor 6 (*F6*).
- Unit backorder cost per unit per period is taken as constant for all periods. This variable is used as Factor 5 (*F5*) in our experimental analysis.
- Cost value of outsourcing is taken as constant in the entire planning horizon. Different values are analyzed for this variable defined as Factor 3 (*F3*) in our experiment.
- Different demand patterns are used in our experimentation. These patterns are identified with their seasonality and trend values. Seasonality of the demand is represented by Factor 2 (*F2*), and trend value is defined with Factor 1 (*F1*) in our experimental setup.
- Total demand during the entire planning horizon is taken as constant, so that different demand patters are comparable.
- Fixed contract cost value is taken as constant for all periods, and its relative value to the production setup cost is used as Factor 9 (*F9*) in our study.
- Fixed cost of outsourcing is taken as constant for all periods, different values for this variable is experimented as Factor 10 (*F10*).
- Length of the contract is taken as constant, different levels are analyzed as Factor 8 (*F8*).
- In the presence of zero interest rates for inventory holding cost computation, storage cost becomes an important element. Its value is taken as constant for all periods, and it is represented as Factor 4 (*F4*) in our experimental analysis.
- Capacity (Factor 7, *F7*) is taken as constant for all periods, to represent the constant capacity problem.

Specifically;

$T=12$

There exits 12 periods in the planning horizon.

$NC=1$

Only a single contract is available for outsourcing.

$NO=1$

Outsourcing has a single cost value independent of the outsourced amount.

$NP=2$

Production has two cost values, one for the regular time one for the overtime represented by the second, expensive smaller interval.

$CP_{1t}=1 \qquad \forall t=1,..,T$

Unit cost of production in regular time is 1 per unit-produced for all periods.

$CP_{2t}=1.5 \qquad \forall t=1,..,T$

Unit cost of production in overtime is 1.5 per unit-produced for all periods.

$DIS_t=1 \qquad \forall t=1,..,T$

Discounting factor is taken as 1, no discounting is applied to the cost terms for all periods.

$INT_t=0 \qquad \forall t=1,..,T$

Interest rate used for the cost computation of inventory carrying is taken as zero for all periods in order to eliminate this term.

$L_{1t}=0 \qquad \forall t=1,..,T$

Lead-time for outsourcing orders is taken as zero for all periods for immediate delivery of the products ordered in any given period.

$$\mathrm{UBO}_{1t}=\sum_{t=1}^{T} D_t \qquad \forall t=1,..,T$$

The capacity for the outsourcing option is taken as cumulative demand for uncapacitated analysis of the subcontractor in every period.

$LBP_{1t}=0 \qquad \forall t=1,..,T$

The lower bound is taken as zero not to restrict the producer’s usage of this option for every period.

Classical economic lot sizing formula balancing the inventory holding and fixed setup costs is as follows:

$$EOQ = \sqrt{\frac{2.K.\lambda}{h}}$$

We need to find a fixed cost (*K*) that guarantees that the time between two consecutive orders (*TO*) is equal to our desired order frequency (*F*). The average demand (λ) is found using the cumulative demand (*CD*) and the number of periods (*T*), unit holding cost (*h*) consists of only the unit storage cost (*S*).

$$TO = \frac{EOQ}{\lambda} \qquad \lambda = \frac{CD}{T} \qquad h{=}S$$

By substituting the average demand and holding cost terms and equating the time between two consecutive orders (TO) to our desired time we come up with the following equations;

$$F = \frac{EOQ}{\frac{CD}{T}} \qquad\qquad F = \frac{\sqrt{\frac{2.K.\frac{CD}{T}}{h}}}{\frac{CD}{T}}$$

When we take out the fixed cost of setup (K) needed to have the desired order frequency, then it is defined with the other parameters as follows,

$$\frac{F^2.CD.S}{2.T} = FP_t \qquad \forall t{=}1,..,T$$

We equate the fixed production cost for every period to the found fixed cost in order to have a system with the desired order frequency.

For deciding the levels of factors that are to be experimented, we use pre-experimentation runs. In these runs, we try to find the levels that guarantee that the performance measures' values that are observed cover a wide range. The levels of the factors are as following:

Levels of Factor 1: Trend of demand

Demand can have different amount of trend and this trend can be in both directions. It can be either positive or negative, and this trend can be with high slope or low slope. High trend is represented with a 25% change in the demand between periods, and low trend is represented with a 10% change in the demand between periods. Also there is a case where demand does not have any trend in it. Our five levels for this factor representing all different trend cases are as follows:

1st level (*F1=1*)	:	High negative trend (*Trend= -0.25 / period*)
2nd level (*F1=2*)	:	Low negative trend (*Trend= -0.10 / period*)
3rd level (*F1=3*)	:	No trend
4th level (*F1=4*)	:	Low positive trend (*Trend= 0.10 / period*)
5th level (*F1=5*)	:	High positive trend (*Trend= 0.25 / period*)

These factor levels are incorporated into our model by using the related parameter set; for this factor the parameter is the demand. This factor is not sufficient for defining the demand itself. So, the demand generation can be explained after the seasonality factor discussion. Plots of demand patterns with varying levels of this factor, for the case where there exist no seasonality in the demand, are presented in Figure 4.1.

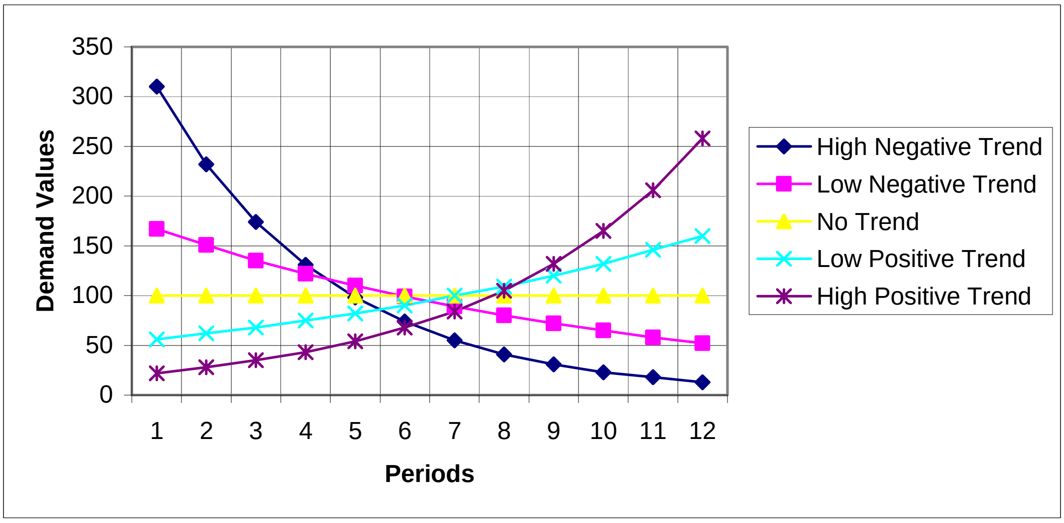

Figure 4.1: Plot of Demand patterns with varying levels of Trend Factor

Levels of Factor 2: Seasonality of demand

Different levels of seasonality can be realized in demand patterns, seasonality may be high, low or none. For high seasonality setting, the demand is assumed to change three-folds between different seasons in a cycle. In low seasonality setting, demand changes about 1.6 folds between two seasons in the planning horizon. Our three levels for this factor are as follows:

1st level (*F2=1*) : No seasonality

2nd level (*F2=2*) : Low seasonality (*Highest D. / Lowest D. = 1.66*)

3rd level (*F2=3*) : High seasonality (*Highest D. / Lowest D. = 3*)

These seasonality levels and trend levels are used together to generate the demand patterns used in our study. In general, demand is defined with three main parameters. These are;

a. *Mean demand*: This is the mean value of the demand. It is the level of demand if no trend or seasonality is involved in the demand pattern. In our study, we do not use a mean demand level, rather we prefer to use a cumulative demand level for our planning horizon. This level is selected as *100T*.

b. *Trend in demand*: This is used to add the trend effect to the mean demand levels, and generate demand levels for entire planning horizon.

c_t. *Seasonality in demand*: Seasonality is introduced using a multiplier for each period in the planning horizon. This multiplier is taken as *1* for the first three periods, where no seasonality effect exists, *1.5* for the second three periods, where the demand increases with the seasonality, *1* again for the third three periods, where no seasonality effect exists and *0.5* for the last three periods, for which the demand decreases because of the seasonality, in a cycle in the high seasonality case. The multipliers are *1*, *1.25*, *1* and *0.75* with the same order for the low seasonality case. Plots of demand patterns with varying levels of this factor, for the case where there exits no trend in the demand, are presented in Figure 4.2.

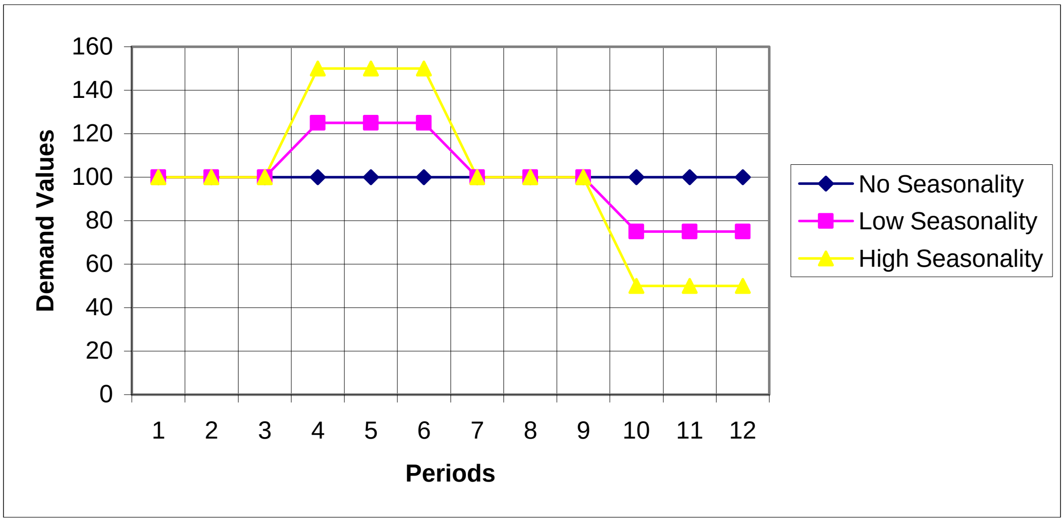

Figure 4.2: Plot of Demand patterns with varying levels of Seasonality Factor

D_t : Demand in units for period t.

In mathematical terms all this process can be summarized as;

i. b will represent the trend with values *–0.25*, *-0.1*, *0*, *+0.1*, *+0.25* with the corresponding level of the Factor 1 (*F1*).

ii. c_t will take the values mentioned above for different levels of the Factor 2 (*F2*).

iii. Demand values are generated using the following classical formula:

$D_t = (a + bt) c_t$

iv. Demand values are normalized in order to equate the cumulative demand to *100T*.

The demand patterns used in our study can be seen in Table 4.1.

Table 4.1: Demand patterns used in this study

Periods	F1=1 F2=1	F1=1 F2=2	F1=1 F2=3	F1=2 F2=1	F1=2 F2=2	F1=2 F2=3	F1=3 F2=1	F1=3 F2=2
D_1	310	295	281	167	162	157	100	100
D_2	232	221	211	151	146	141	100	100
D_3	174	166	158	135	131	127	100	100
D_4	131	155	178	122	148	172	100	125
D_5	98	117	133	110	133	155	100	125
D_6	74	87	100	99	120	139	100	125
D_7	55	52	50	89	86	84	100	100
D_8	41	39	37	80	77	75	100	100
D_9	31	30	28	72	70	68	100	100
D_{10}	23	17	10	65	47	30	100	75
D_{11}	18	12	8	58	42	27	100	75
D_{12}	13	9	6	52	38	25	100	75
TOTAL	1200	1200	1200	1200	1200	1200	1200	1200

Periods	F1=3 F2=3	F1=4 F2=1	F1=4 F2=2	F1=4 F2=3	F1=5 F2=1	F1=5 F2=2	F1=5 F2=3
D_1	100	56	58	61	22	25	27
D_2	100	62	64	67	28	31	34
D_3	100	68	71	74	35	38	43
D_4	150	75	97	121	43	60	80
D_5	150	82	107	134	54	75	100
D_6	150	90	118	147	68	93	126
D_7	100	100	104	108	84	93	105
D_8	100	109	114	119	105	117	131
D_9	100	120	125	131	132	146	164
D_{10}	50	132	103	72	165	137	102
D_{11}	50	146	114	79	206	171	128
D_{12}	50	160	125	87	258	214	160
TOTAL	1200	1200	1200	1200	1200	1200	1200

Levels of Factor 3: Unit Cost of Outsourcing

The relative value of in-house production and the outsourcing unit cost is used to define different levels for this factor. Our first level is defined as the case where outsourcing unit cost beats the in-house cost of production; second level represents the case where these two values are very close to each other. The third level represents the case where outsourcing is more expensive than the in-house production. The levels for this factor are as follows:

1st level (*F3=1*) : $CO_{11t}=0.8/unit$ $\forall t=1,..,T$

2nd level (*F3=2*) : $CO_{11t}=1/unit$ $\forall t=1,..,T$

3rd level (*F3=3*) : $CO_{11t}=1.2/unit$ $\forall t=1,..,T$

Levels of Factor 4: Storage Cost

The relative value of storage cost and backorder penalty cost determines the values of inventory holding and demand backordering. Three levels are defined for this variable. The factor levels are as follows:

1st level (*F4=1*) : $S_t=0.01/unit$ $\forall t=1,..,T$

2nd level (*F4=2)* : $S_t=0.02/unit$ $\forall t=1,..,T$

3rd level (*F4=3*) : $S_t=0.03/unit$ $\forall t=1,..,T$

Levels of Factor 5: Backorder Cost

Minimum value is taken as the average value of storage cost factors' levels. All levels are taken as two times the levels of storage costs. The levels for this factor are as follows:

1st level (*F5=1*) : $BK_t=0.02/unit$ $\forall t=1,..,T$

2nd level (*F5=2*) : $BK_t=0.04/unit$ $\forall t=1,..,T$

3rd level (*F5=3*) : $BK_t=0.06/unit$ $\forall t=1,..,T$

Levels of Factor 6: Target Time between Two Production Runs

Desired time between two production runs in number of periods is represented with *F*. Three levels are used to analyze this factor. First level represents very frequent production runs and third level represents not so frequent production runs. And second level represents the middle point of these two extremes. The parameter used and the levels used for this factor are as follows:

F: Target time between two production runs

1st level (*F6=1*) : *F=1*
2nd level (*F6=2*) : *F=3*
3rd level (*F6=3)* : *F=6*

Levels of Factor 7: Production Capacity

We use two capacity terms for our production system. First one is the regular time capacity, which is represented with the parameter R, and second is the overtime capacity which is represented with parameter O. Overtime production capacity is taken to be one-fourth of the regular time production capacity because of the legal restrictions. Also the unit cost of outsourcing is set to 50% higher than of the unit cost of regular time production. Cumulative demand value, which is set to *100T* before, is represented with CD parameter, where T represents the total number of periods in our planning horizon. For this factor, four levels are defined in order to represent all possible cases. First one is the case with no capacity restriction. Capacity is set equal to the cumulative demand. Second level allows the production of average demand for three periods, with the overtime option fully utilized. Third level represents the case where regular time production is sufficient to satisfy one period's demand, but cumulative capacity with full utilization of overtime, does not satisfy two periods' demand. In our last level for this factor, regular time capacity is not sufficient for one period demand satisfaction, so only with the outsourcing option fully utilized capacity becomes sufficient for satisfying demand of a period. The parameters mentioned above, and the levels for this factor are as follows:

R: Regular time capacity

O: Overtime capacity

CD: Cumulative demand

T: Number of periods in planning horizon

1st level (*F7=1*)	:	SP_{1t}= R= CD	$\forall t=1,..,T$
		SP_{2t}= O= CD	$\forall t=1,..,T$
2nd level (*F7=2*)	:	SP_{1t}=R=2.4 CD/T	$\forall t=1,..,T$
		SP_{2t}=O=3 CD/T	$\forall t=1,..,T$
3rd level (*F7=3*)	:	SP_{1t}=R=1.2 CD/T	$\forall t=1,..,T$
		SP_{2t}=O=1.5 CD/T	$\forall t=1,..,T$
4th level (*F7=4)*	:	SP_{1t}=R=0.8 CD/T	$\forall t=1,..,T$
		SP_{2t}=O=1 CD/T	$\forall t=1,..,T$

Levels of Factor 8: Length of Contract

Very small values of length of contract can cause the zero utilization of the outsourcing options and very high values can eliminate the production utilization with constant fixed contract costs. This factor is important for negotiation purposes and its value can be decided using a bidding or auction process. Three levels are used for inspecting its effects on the system performance. First level is that the length of contract is taken as only one period. This represents the system where no long-term contracts are needed for outsourcing. Second level is set to 3 periods to represent medium length contract, and last level is set to 6 periods for the representation of long-term contract. The levels used for this factor are as follows:

1st level (*F8=1*)	:	$LEN_1=1$
2nd level (*F8=1*)	:	$LEN_1=3$
3rd level (*F8=1)*	:	$LEN_1=6$

Levels of Factor 9: Fixed Contract Cost

The computation of the fixed production setup cost is done using the desired frequency, average storage cost and other parameters. The fixed cost of production setup, that is calculated using the EOQ formula, is represented with the parameter *K*. The production setup cost is divided into frequency in order to gather average setup cost per period. This term is used to determine the fixed cost of a contract establishment with a multiplication with its length. Three levels are used in order to inspect the affects of this factor on system performance. They are defined relative to the production setup cost. For the first level for this factor, fixed contract cost per period is set lower than the fixed production setup cost per period, for the second level, it is equal to the fixed production setup cost, and for the third and the last level, it is set higher than the fixed production setup cost. The parameters that are used, EOQ formula based equation and levels for this factor are as follows:

FP: Fixed cost of production

FC: Fixed cost of contract

FO: Fixed cost of outsourcing

K: Fixed cost of production calculated using EOQ.

$$FP_t = \frac{F^2.CD.S}{2.T} \qquad \forall t=1,..,T$$

1st level (*F9=1*) : $$FC_{1t} = \frac{LEN_1.FP_t}{F}.0.8 \quad \forall t=1,..,T$$

2nd level (*F9=2*) : $$FC_{1t} = \frac{LEN_1.FP_t}{F} \quad \forall t=1,..,T$$

3rd level (*F9=3*) : $$FC_{1t} = \frac{LEN_1.FP_t}{F}.1.2 \quad \forall t=1,..,T$$

Levels of Factor 10: Fixed Outsourcing Cost

We relate the value of this factor to the average demand of customers. Three levels are used to investigate the effects of this factor on system performance. First level is to set this value equal to zero, such that only a contract fee is paid in order to use the outsourcing option in any period irrespective of the total number of outsourcing runs. The cost term related to this is eliminated from the objective function. Second level is to charge 0.03 for every unit of average demand. This value represents the cost related with transportation and book keeping and increases with the value of the average demand. Third level is to charge 0.06, doubling the second level for every outsourcing run. The levels used for this factor in our study are as follows:

1st level (*F10=1*) : $FO_{1t} = 0$ $\forall t=1,..,T$

2nd level (*F10=2*) : $FO_{1t} = 0.03\ CD/T$ $\forall t=1,..,T$

3rd level (*F10=3*) : $FO_{1t} = 0.06\ CD/T$ $\forall t=1,..,T$

4.3 Single Contract Option: Factor Analysis

An experimental setup is prepared using the factors stated above. A full factorial design is preferred in order to capture the relations between different factors. Model is run with all possible combinations of factors in order to have a full-factorial experimental design. Some performance measures or system operation characteristic properties are recorded for the analysis of the results of the experiment. All the ANOVA results are available upon request. A summary table is prepared for the analysis of effects of each factor on the system's behavior. In each table, the mean values of the performance measures are presented for different levels of factors. Each mean is calculated using all levels of other factors so has above twenty thousand observations in it. The abbreviations used in the presentation of experiment results are as follows;

F **:** F is used to identify the factor value considered in the analysis

Obj **:** Objective function value

%O **:** Percentage of outsourcing utilization in the satisfaction of cumulative demand

%P **:** Percentage of production utilization in the satisfaction of cumulative demand

#C **:** Number of contracts established in the planning horizon.

#O : Number of outsourcing orders used in the planning horizon.

#P : Number of production runs used in the planning horizon

AvgI : Average inventory carried all over the planning horizon

AvgB : Average amount of demand backordered all over the planning horizon

Percent Change :

Percent change of the mean value of performance measures with different levels of the factor analyzed. This percentage is calculated using the percentage change over the smallest value to the greatest value. This percentage, rounded to an integer, is used as a measure of significance of the effect of the factor on the performance measures. The formula used in this percentage calculation is given in the following lines. If the denominator value of the formula takes a zero value then this percentage is not used in our tables as a measure of significance.

Min: Lowest value of the performance measure attained with any level of a given factor

Max: Highest value of the performance measure attained with any level of a given factor.

$$\text{Percent Change} = \frac{\text{Max - Min}}{\text{Min}}$$

Factor 1: Trend of demand

Trend of demand is the first factor of our analysis. Different levels changing from high negative trend to high positive trend are used in order to find out the effects of this factor on our predefined system performance measures. The effect of this factor can be expressed in two parts. These are the direction of the trend and amount of trend. The mean values of system performance measures are summarized for different levels of Factor 1 in Table 4.2. Same values are presented for the direction and amount of trend in Table 4.3 and Table 4.4.

Table 4.2: Mean values of performance measures and objective function with changing levels of Factor 1

F 1 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 H. Neg. Trend	1187.2	66.82	33.18	1.516	2.983	2.641	48.31	14.14
2 L. Neg. Trend	1189.3	63.63	36.37	1.761	3.389	3.273	53.01	14.76
3 No Trend	1189.4	62.30	37.70	1.823	3.541	3.569	50.34	15.44
4 L. Pos. Trend	1189.5	62.22	37.78	1.830	3.526	3.570	47.86	17.12
5 H. Pos. Trend	1189.5	63.33	36.67	1.773	3.417	3.279	48.17	18.70
Percent. Chg.	0%	7%	14%	21%	19%	35%	11%	32%

Table 4.3: Mean values of performance measures and objective function with changing direction of trend

Direction	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
Neg. Trend	1188,3	65,23	34,78	1,639	3,186	2,957	50,66	14,45
No Trend	1189,4	62,30	37,70	1,823	3,541	3,569	50,34	15,44
Pos. Trend	1189,5	62,78	37,23	1,802	3,472	3,425	48,02	17,91
Percent. Chg.	0%	5%	8%	11%	11%	21%	6%	24%

Table 4.4: Mean values of performance measures and objective function with changing amount of trend analysis results

Direction	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
No Trend	1189,4	62,30	37,70	1,823	3,541	3,569	50,34	15,44
Low Trend	1189,4	62,93	37,08	1,796	3,458	3,422	50,44	15,94
High Trend	1188,4	65,08	34,93	1,645	3,200	2,960	48,24	16,42
Percent. Chg.	0%	4%	8%	11%	11%	21%	5%	6%

When the trend is negative, system utilizes the outsourcing option with large batches for the first three periods where total demand is high, and utilizes the production option for the last periods where total demand is low. In no trend and positive trend cases the system utilizes outsourcing option a little less, since the high demand in last periods can be handled using inventory carrying or backordering with the production utilization. With high trend, the batch sizes for both outsourcing and production increase to compensate the sharp movements in the demand. In no trend and low trend cases outsourcing and production occur frequently and in smaller batches.

Factor 2: Seasonality of demand

Seasonality of the demand is the second factor analyzed in our study. Three levels are used to represent no seasonality, low seasonality and high seasonality cases. The mean values of system performance measures for different levels of Factor 2 are summarized in Table 4.5.

Table 4.5: Mean values of performance measures and objective function with changing levels of Factor 2

F 2 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 No Season.	1189.6	63.63	36.37	1.763	3.405	3.314	49.48	16.66
2 Low Season.	1189.1	63.54	36.46	1.746	3.379	3.274	49.57	16.07
3 High Season.	1188.2	63.81	36.19	1.713	3.329	3.212	49.57	15.35
Percent. Chg.	0%	0%	1%	3%	2%	3%	0%	9%

Only the amount of backordering is affected by seasonality. Because, the system prefers to delay some part of the demand, to handle high seasonality. The objective function does not change as expected. With capacitated production and uncapacitated outsourcing options, it can be more efficient to outsource with low prices in high seasonality case. So the objective function value remains approximately constant irrespective of the amount of seasonality.

Factor 3: Unit Cost of Outsourcing

Unit cost of outsourcing is the third factor of our analysis. Three levels are used to represent unit cost terms ranging from cheap to expensive values relative to the unit production cost. The mean values of our system performance measures for different levels of Factor 3 are summarized in Table 4.6.

Table 4.6: Mean values of performance measures and objective function with changing levels of Factor 3

F 3 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 $CO_{11t}=0.8$	1006.3	100.00	0.00	2.741	5.303	0.000	57.08	15.42
2 $CO_{11t}=1$	1240.4	72.56	27.44	1.948	3.820	2.860	44.42	12.84
3 $CO_{11t}=1.2$	1320.3	18.42	81.58	0.533	0.991	6.939	47.12	19.84
Percent. Chg.	31%	443%		414%	435%		29%	55%

This factor is the most significant factor, since about 80% of the objective function consists of the unit cost of production or outsourcing. With increasing outsourcing cost, the behavior of the system changes. The utilization of production increases, utilization of outsourcing decreases, the number of orders, contracts and production runs act in the same manner. The objective function increases as expected, but the increase does not reflect all the increase in the unit outsourcing cost, because the system changes its main source of product procurement.

Factor 4: Storage Costs

Unit storage cost per period is taken as the forth factor to be investigated in our analysis. Three levels are used to represent the values ranging from low to high for this factor. The mean values of system performance measures for changing levels of this factor are summarized in Table 4.7.

Table 4.7: Mean values of performance measures and objective function with changing levels of Factor 4

F 4 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 S_t**=0.01**	1166.1	59.94	40.06	1.660	3.094	3.649	65.86	6.89
2 S_t**=0.02**	1190.6	65.03	34.97	1.790	3.441	3.139	47.75	15.59
3 S_t**=0.03**	1210.2	66.02	33.98	1.772	3.579	3.012	35.00	25.61
Percent. Chg.	4%	10%	18%	8%	16%	21%	88%	272%

With increasing storage cost system prefers to procure smaller lots more frequently, but this increases the cost of production option, so production capacity is less utilized in this case. The amount of inventory decreases as expected and its substitute backordering increases.

Factor 5: Backorder Cost

Backorder cost is one of the other factors of our interest in this study. Three levels are used to represent the values ranging from low to high for this factor. Mean values of predefined systems performance measures for different levels of this factor are summarized in Table 4.8.

Table 4.8: Mean values of performance measures and objective function with changing levels of Factor 5

F 5 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 BK_t**=0.02**	1185.3	63.22	36.78	1.567	3.002	3.232	38.57	33.39
2 BK_t**=0.04**	1189.9	63.76	36.24	1.781	3.468	3.280	51.87	10.41
3 BK_t**=0.06**	1191.7	64.00	36.00	1.874	3.644	3.288	58.18	4.28
Percent. Chg.	1%	1%	2%	20%	21%	2%	51%	680%

Increasing the level of unit backordering cost does not change the sourcing decision as much as the unit storage cost, only system tries to decrease the lot size and to increase the frequency of outsourcing. So, the number of contracts and outsourcing orders increase. The amount of backordering decreases as expected and its substitute inventory holding increases.

Factor 6: Target Frequency of Production Runs

This factor determines the value of fixed production costs. Three levels are used to represent the cases, in which production runs range from less frequent to more frequent. The mean values of the system performance measures for different levels of Factor 6 are summarized in Table 4.9.

Table 4.9: Mean values of performance measures and objective function with changing levels of Factor 6

F 6 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 *F=1*	1148.1	46.87	53.13	1.845	3.397	6.262	21.41	8.10
2 *F=3*	1184.6	64.04	35.96	1.751	3.449	2.635	51.23	16.07
3 *F=6*	1234.1	80.07	19.93	1.625	3.268	0.903	75.98	23.93
Percent. Chg.	7%	71%	167%	14%	6%	593%	255%	195%

With increasing levels of this factor, the fixed cost of production increases, and the effects of this increase is observed as expected; the utilization of production option decreases, the number of contracts, outsourcing orders and production runs act accordingly. With the increase in fixed production cost the batch sizes increase, and increases in the amount of inventory carried and amount of backorders are observed as expected.

Factor 7: Production Capacity

Capacity is defined using the average demand per period. Four levels are defined for this factor, ranging from infinite capacity to very tight capacity. The mean values of the system performance measures for different levels of Factor 7 are summarized in Table 4.10.

Table 4.10: Mean values of performance measures and objective function with changing levels of Factor 7

F 7 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 SP_{1t}= CD SP_{2t}= CD	1166.9	52.87	47.13	1.440	2.822	2.838	59.07	17.89
2 SP1t=2.4 CD/T SP2t=3CD/T	1178.4	55.58	44.42	1.507	2.943	3.370	43.28	13.00
3 SP1t=1.2 CD/T SP2t=1.5/T	1196.2	68.09	31.91	1.826	3.524	3.499	46.58	17.15
4 SP1t=0.8 CD/T SP2t=1CD/T	1214.3	78.10	21.90	2.189	4.197	3.360	49.22	16.09
Percent. Chg.	4%	48%	115%	52%	49%	23%	36%	38%

The decrease in the production capacity causes an increase in the average cost of production with increasing the contribution of fixed cost. So the utilization of production decreases. This can also be observed on the number of contract, outsourcing orders and production runs. The cost objective increases with more restricting constraints as expected.

Factor 8: Length of Contract

The length of contract is the eight factor of our analysis. Three levels are used to represent the contracts having lengths ranging from very short to very long. The mean values of system performance measures are summarized for different level of this factor on Table 4.11.

Table 4.11: Mean values of performance measures and objective function with changing levels of Factor 8

F 8 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 $LEN_1=1$	1184.3	64.59	35.41	2.944	2.944	3.220	50.94	15.99
2 $LEN_1=3$	1189.7	63.47	36.53	1.422	3.470	3.282	48.82	15.89
3 $LEN_1=6$	1192.9	62.93	37.07	0.856	3.700	3.298	48.85	16.21
Percent. Chg.	1%	3%	5%	244%	26%	2%	4%	2%

The length of the contract only affects the outsourcing part of the system behavior. With increasing length of contract, the system prefers to establish less contracts and the number of outsourcing orders increase. Since long contracts can cover more periods.

Factor 9: Fixed Contract Cost

Fixed contract cost is one of the factors in our study. Three levels are used to represent cheaper to more expensive fixed contract cost relative to fixed production cost. The mean values of system performance measures for different levels of this factor are summarized in Table 4.12.

Table 4.12: Mean values of performance measures and objective function with changing levels of Factor 9

F 9 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 $FC_{1t} = \frac{LEN_1.FP_t}{F}.0.8$	1186.2	65.26	34.74	1.919	3.695	3.112	47.01	14.93
2 $FC_{1t} = \frac{LEN_1.FP_t}{F}.1$	1189.0	63.78	36.22	1.747	3.373	3.253	49.55	16.06
3 $FC_{1t} = \frac{LEN_1.FP_t}{F}.1.2$	1191.6	61.94	38.06	1.556	3.046	3.434	52.06	17.10
Percent. Chg.	0%	5%	10%	23%	21%	10%	11%	15%

The increase in the fixed contract cost causes a decrease in the number of contracts and number of outsourcing orders as expected. The utilization of production option increases; the number of production runs increases to cover the demand; the amount of backordering and inventory carrying increases with increasing batch sizes, and less frequent procurement.

Factor 10: Fixed Outsourcing Cost

Fixed outsourcing cost is another factor. Three levels are used to represent a range for fixed outsourcing cost. The mean values for these system performance measures for different levels of this factor are summarized in Table 4.13.

Table 4.13: Mean values of performance measures and objective function with changing levels of Factor 10

F 10 Level	Obj	%O	%P	#C	#O	#P	AvgI	AvgB
1 $FO_{1t}=0$	1179.3	69.50	30.50	2.524	5.539	2.707	30.61	11.18
2 $FO_{1t}=0.03\ CD/T$	1190.5	62.00	38.00	1.482	2.678	3.474	52.93	16.58
3 $FO_{1t}=0.06\ CD/T$	1197.1	59.48	40.52	1.216	1.897	3.620	65.08	20.33
Percent. Chg.	2%	17%	33%	108%	192%	34%	113%	82%

The effect of different levels of fixed outsourcing cost is much more significant than the effect of fixed contracting cost. With increasing levels of fixed outsourcing cost, the utilization of production option increases; average lot size of outsourcing orders increases. The number of outsourcing orders, contracts and production runs act in accordance with these changes, the inventory carrying and backorder levels increase with the increase in the lot sizes.

4.4 Single Contract Option: Contract Length and Fixed Contract Cost Analysis

Some factors should be chosen in order to continue with a detailed study of fixed contract cost negotiation in the given environments, and contract parameters. The factors that have the most significant effects on system performance measures and optimal behavior are identified as unit outsourcing cost and desired frequency of production runs. These two terms are selected according to the results of the previous single factor analysis and their effects are dominant against other factors effects on mathematical basis. All factors have significant effects on the system performance measures, but most of them only affect two system performance measures, but these two factors affect almost every system performance measure. So they are selected as the most significant factor for further analysis. The length of contract and fixed contract cost analysis is done with different levels of these factors, all other factors are taken as their medium value except demand and backordering cost. Backordering cost is taken at its highest value to eliminate the occurrence of high backorders. Three distinct demand cases are chosen in order to work on. These are cases where demand has, no trend or seasonality, low positive trend and low seasonality and no trend and high seasonality. In mathematical terms the base case for our study on contract length versus fixed contract cost analysis is as follows;

Storage Cost	$F4=2$:	$S_t=0.02$	$\forall t=1,..,T$
Backordering Cost	$F5=3$:	$B_t=0.06$	$\forall t=1,..,T$
Capacity	$F7=3$:	$SP_{1t}=R=1.2\ CD/T$	$\forall t=1,..,T$
		$SP_{2t}=O=0.3\ CD/T$	$\forall t=1,..,T$
Fixed Cost of Outsourcing	$F10=2$:	$FO_t=0.03\ CD/T$	$\forall t=1,..,T$

The model is run using these factors as constant and other factors as changing. For all contract lengths ranging from 1 period to the extreme 12 periods the model is run with different fixed contract costs; starting from zero and incrementing with 0.5 up to the point where outsourcing option is not utilized. The results are recorded using the predefined system performance measures. Detailed output of the results of study is available upon request.

Two plot representations are used in order to analyze the results. The representation used in Figures 4.3, 4.5, 4.7, 4.9 and 4.11, is the plot of percentage outsourcing as a function of fixed contract cost. The break type behavior is shown on the plot for different contract lengths. On the breakpoints system changes its outsourcing behavior. These points are the thresholds for different levels of outsourcing utilization relative to fixed contract cost. With increasing fixed contract cost, outsourcing utilization decreases with discrete breakpoints and with a very high cost term the outsourcing utilization drops to zero. This analysis can be thought as a sensitivity analysis around different values of fixed contract cost in different contract lengths. Each break represents the situations in which the changes in fixed contract cost do not change the optimal behavior of the producer. The behavior changes only with the breakpoints, the values of these breakpoints are important for such analysis.

The representation used in Figures 4.4, 4.6, 4.8, 4.10 and 4.11, is the plot of length of contract versus fixed contract cost for the values resulting with approximately the same objective function value. This objective function value is selected as the mid point of the lowest and highest objective function values observed with different contract lengths in any given setting. The reason behind this is to find an objective function value that is observed with all contract lengths. In these plots, the contract length and fixed contracting cost pairs that give the same objective function are plotted. This analysis is important because the producer will be indecisive between two alternatives when their objective function values are more or less equal within a one-percent range. In this analysis, we try to observe the change in the fixed contract cost relative to contract length in such equally treated cases. These plots show the amounts of fixed contract costs that the producer will be willing to pay for contracts with different lengths. Five different settings are analyzed in detail, these are;

Setting 1 Producer faces a demand having low positive trend and low seasonality. The outsourcing unit cost is equal to in-house production cost. This is the setting where both the producer and the subcontractor have production facilities with similar sizes. The producer desires scheduling of a production run in every three periods. In mathematical terms;

$F1=4$: Low positive trend (*Trend= 0.10 / period*)

$F2=2$: Low seasonality (*Highest D. / Lowest D. = 1.66*)

$F3=2$: $CO_t=1$ $\forall t=1,..,T$

$F6=2$: $F=3$

Two plot representations are used to analyze the results. (See Figures 4.3 and 4.4). The detailed information about the breakpoints on Figure 4.3 is presented in Table 4.14, and points on Figure 4.4 are presented in Table 4.15.

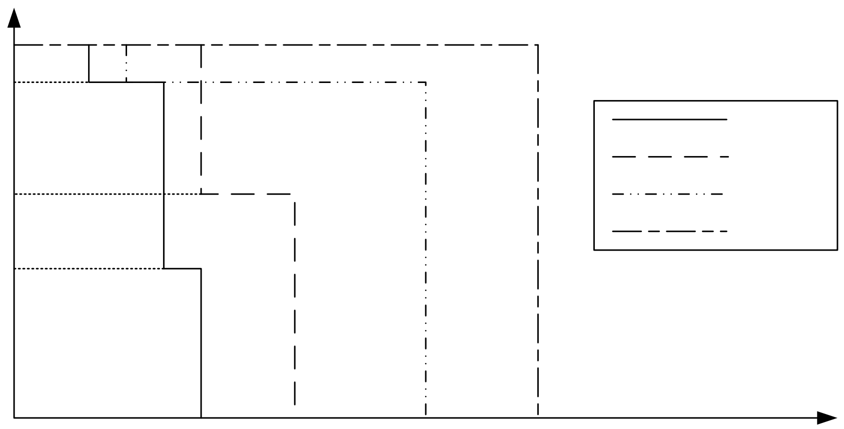

Figure 4.3: Change in Percentage Outsourced with Fixed Contract Cost and changing values of Contract Length in Setting 1

With increasing length of contract, the fixed contract costs for the breakpoints get higher. An important observation is that fixed contract cost at which the outsourcing percentage usage drops to zero is not proportional to the length of contract. With increasing length of contract, this cost term does not show a linear increase. An explanation can be made using the values in the table provided below. With increasing contract cost, system changes its primary source of products to in-house production. This behavior does not change with contract length as expected, because the cost of production is constant and outsourcing cost is increasing.

Table 4.14: Detailed data of breakpoints used in Figure 4.3

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	10	90	10	2	2	1	118,58	15,75	1274,80
1	21,5	41,58	58,42	1	1	6	60,58	6,25	1297,54
1	24,5	0	100	0	0	10	29,33	4,42	1300,22
3	24	60,08	39,92	1	2	4	75,42	4,33	1287,22
3	37,5	0	100	0	0	10	29,33	4,42	1300,22
6	15	90	10	1	4	1	81,58	6,25	1260,08
6	55,5	0	100	0	0	10	29,33	4,42	1300,22
12	70	0	100	0	0	10	29,33	4,42	1300,22

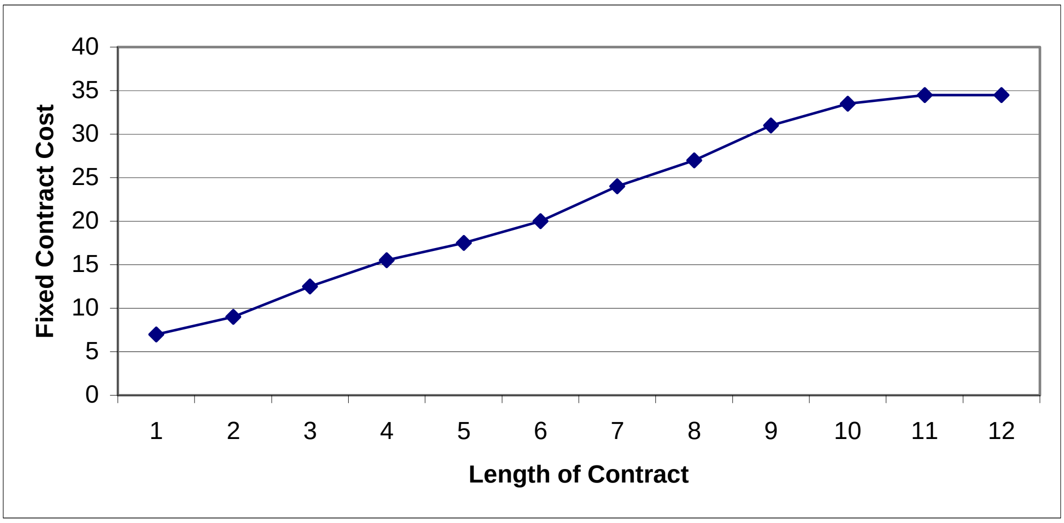

Figure 4.4: Plot of fixed contract costs providing constant objective function value under different contract lengths in Setting 1

In Figure 4.4 we observe that the fixed contract cost increases as the length of contract increases. The increase in the fixed contract cost shows a linear pattern until very high levels of contract length. Then, it converges to a value and remains there independent of the length of the contract. With shorter length contracts, the system does not change its operational decisions, but with higher levels, it cuts costs from other sources of costs and maintains the same level of objective function regardless of the length of the contract.

Table 4.15: Detailed data of points used in Figure 4.4

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	7	100	0	4	4	0	87,25	4,83	1264,42
2	9	100	0	3	6	0	80,08	0	1264,22
3	12,5	100	0	2	4	0	101,25	4,83	1264,78
4	15,5	100	0	2	6	0	52,08	4,83	1264,98
5	17,5	100	0	2	6	0	51,75	0	1265,42
6	20	90	10	1	4	1	81,58	6,25	1265,08
7	24	90	10	1	4	1	63,5	6,25	1264,74
8	27	100	0	1	5	0	82,33	4,83	1265,24
9	31	100	0	1	5	0	64,25	4,83	1264,9
10	33,5	100	0	1	6	0	42,5	4,83	1265,18
11	34,5	100	0	1	6	0	51,75	0	1264,92
12	34,5	100	0	1	6	0	51,75	0	1264,92

Setting 2 Producer faces a demand having no trend and high seasonality. The outsourcing unit cost is equal to in-house production cost. This is the setting where both the producer and the subcontractor have production facilities with similar sizes. The producer desires scheduling of a production run in every three periods. In mathematical terms;

$F1=3$: No trend

$F2=3$: High seasonality ($Highest\ D. / Lowest\ D. = 3$)

$F3=2$: $CO_t=1$ $\forall t=1,..,T$

$F6=2$: $F=3$

Two plot representations are used to analyze the results. (See Figures 4.5 and 4.6). The detailed information about the breakpoints on Figure 4.5 are presented in Table 4.16, and points on Figure 4.6 are presented in Table 4.17.

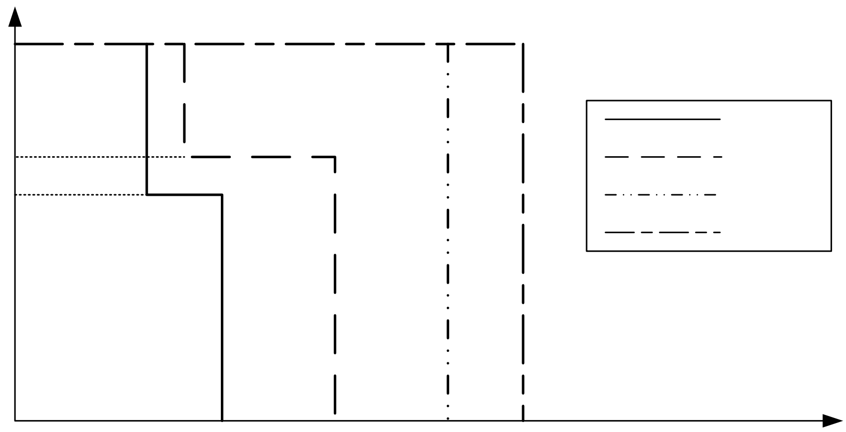

Figure 4.5: Change in Percentage Outsourced with Fixed Contract Cost and changing values of Contract Length in Setting 2

The behavior observed in Figure 4.5 is almost the same as the plot in the first setting. The fixed contract costs, eliminating the usage of outsourcing options are slightly higher than the ones in the first setting. System facing with demand having high seasonality does not leave the option of outsourcing without incurring higher contract cost, than the system facing demand with low seasonality and low trend. The reason behind this is the need to handle the large variation. So, a generalization can be made as: seasonality can be handled with outsourcing, or in highly seasonal demand environments, optimal behavior tolerates higher contract costs.

% Outsourced

100%

70%

60%

Table 4.16: Detailed data of breakpoints used in Figure 4.5

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	16	60	40	1	1	4	111,67	6,67	1286,6
1	28	0	100	0	0	10	20	5	1298,4
3	21,5	70	30	1	3	3	69,17	6,67	1278,9
3	41,5	0	100	0	0	10	20	5	1298,4
6	57,5	0	100	0	0	10	20	5	1298,4
12	68,5	0	100	0	0	10	20	5	1298,4

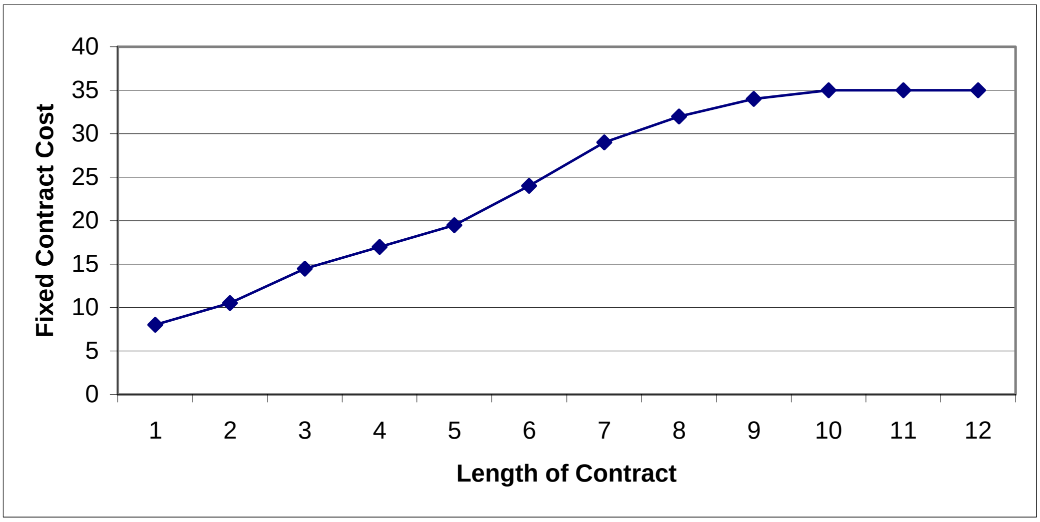

Figure 4.6: Plot of fixed contract costs providing constant objective function value under different contract lengths in Setting 2

The behavior observed in Figure 4.6 is almost the same as the plot in the first setting. The fixed contract costs, resulting with the same objective function level, are higher than the setting where demand has low seasonality and trend. In the first setting, system carries more inventory and utilize backordering to handle the trend, but in the second setting where trend does not exist, system does not carry too much inventory or backorder, so higher fixed contract cost can be tolerated.

Table 4.17: Detailed data of points used in Figure 4.6

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	8	100	0	4	4	0	87,5	0	1265
2	10,5	100	0	3	6	0	62,5	0	1264,5
3	14,5	100	0	2	4	0	100	0	1265
4	17	100	0	2	6	0	54,17	0	1265
5	19,5	90	10	1	3	1	85	10	1265,1
6	24	100	0	1	4	0	95,83	8,33	1265
7	29	100	0	1	4	0	75	8,33	1265
8	32	100	0	1	6	0	62,5	0	1265
9	34	100	0	1	6	0	54,17	0	1265
10	35	100	0	1	7	0	37,5	0	1265
11	35	100	0	1	8	0	25	0	1265
12	35	100	0	1	8	0	25	0	1265

Setting 3 Producer faces a demand having no trend and no seasonality. The outsourcing unit cost is higher than the in-house production cost. This is the setting where the producer has larger production facilities than the subcontractor. The producer desires scheduling of a production run in every six periods. In mathematical terms;

$F1=3$: No trend

$F2=1$: No Seasonality

$F3=3$: $CO_t=1.2$ $\forall t=1,..,T$

$F6=3$: $F=6$

Two plot representations are used to analyze the results. (See Figures 4.7 and 4.8). The detailed information about the breakpoints on Figure 4.7 are presented in Table 4.18, and points on Figure 4.8 are presented in Table 4.19.

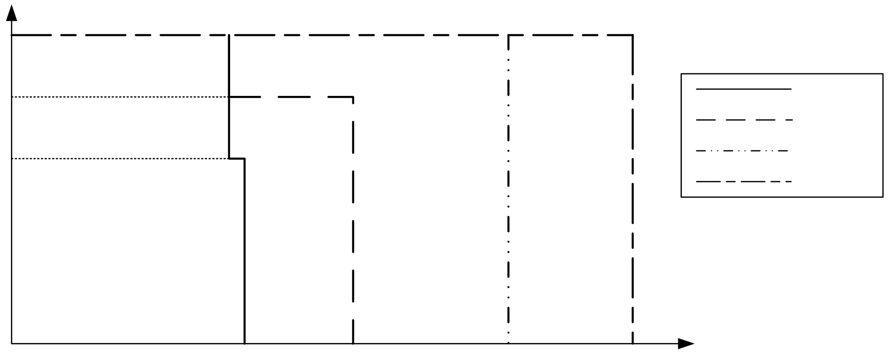

Figure 4.7: Change in Percentage Outsourced with Fixed Contract Cost and changing values of Contract Length in Setting 3

The behavior observed in Figure 4.7 is almost the same as the plots in the previous settings. The fixed contract cost terms eliminating the outsourcing option fully are much higher than the previous settings. The change in the unit outsourcing cost and frequency causes increase in the fixed contract costs. These two factors work in opposite directions; with higher unit outsourcing cost, system is willing to eliminate the outsourcing option even with smaller fixed contract cost. But the increase in the frequency of the production runs increases the cost of production more than the increase in the outsourcing, so the effect of this factor beats the effect of unit outsourcing cost. So the system tends to pay higher contract costs until it fully eliminates the outsourcing option.

Table 4.18: Detailed data of breakpoints used in Figure 4.7

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	35,5	60	40	1	1	4	126,67	16,67	1568,9
1	37,5	0	100	0	0	10	33,33	3,33	1570,4
3	36,5	80	20	1	2	2	138,33	18,33	1552,9
3	54	0	100	0	0	10	33,33	3,33	1570,4
6	78,5	0	100	0	0	10	33,33	3,33	1570,4
12	100,5	0	100	0	0	10	33,33	3,33	1570,4

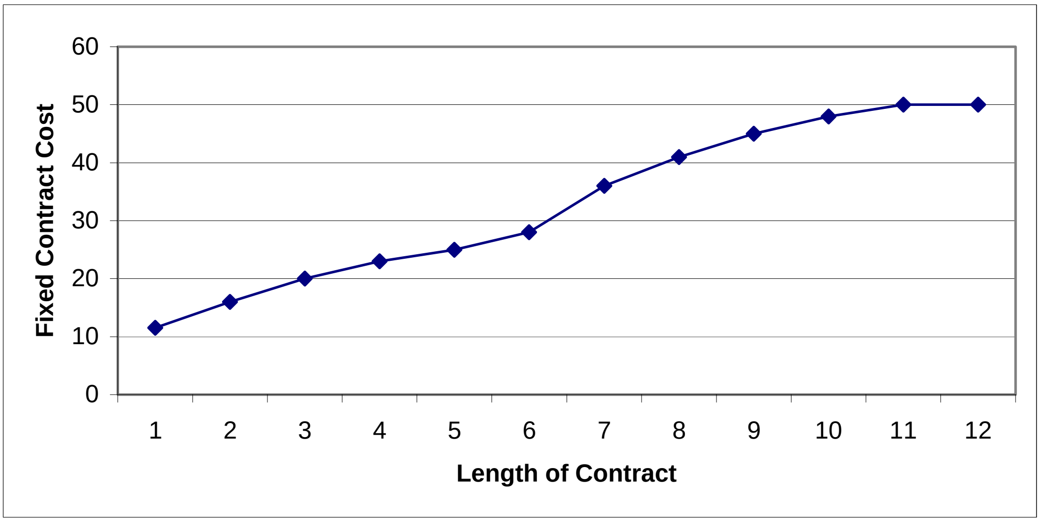

Figure 4.8: Plot of fixed contract costs providing constant objective function value under different contract lengths in Setting 3

The behavior observed in Figure 4.8 is almost the same as the plots in the previous settings. The comparison of this figure with the previous ones is not much logical since the objective value used to gather these points is not identical.

Table 4.19: Detailed data of points used in Figure 4.8

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	11,5	100	0	3	3	0	100	16,67	1519,5
2	16	100	0	2	4	0	100	16,67	1520
3	20	100	0	2	4	0	116,67	0	1520
4	23	100	0	2	6	0	66,67	0	1520
5	25	100	0	2	6	0	50	0	1520
6	28	100	0	1	4	0	141,67	8,33	1520
7	36	100	0	1	4	0	108,33	8,33	1520
8	41	100	0	1	5	0	75	8,33	1520
9	45	100	0	1	5	0	83,33	0	1520
10	48	100	0	1	6	0	58,33	0	1520
11	50	100	0	1	6	0	50	0	1520
12	50	100	0	1	6	0	50	0	1520

Setting 4 Producer faces a demand having low positive trend and low seasonality. The outsourcing unit cost is higher than the in-house production cost. This is the setting where the producer has larger production facilities than the subcontractor. The producer desires scheduling of a production run in every six periods. In mathematical terms;

$F1=4$: Low positive trend (*Trend= 0.10 / period*)

$F2=2$: Low seasonality (*Highest D. / Lowest D. = 1.66*)

$F3=3$: $CO_t=1.2$ $\forall t=1,..,T$

$F6=3$: $F=6$

Two plot representations are used to analyze the results. (See Figures 4.9 and 4.10). The detailed information about the breakpoints on Figure 4.9 are presented in Table 4.20, and points on Figure 4.10 are presented in Table 4.21.

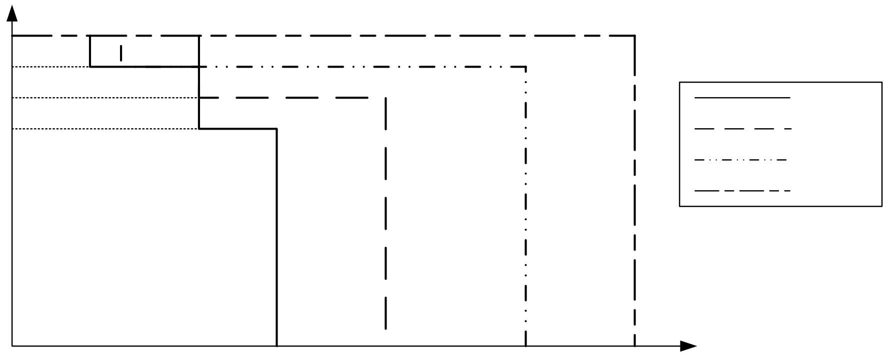

Figure 4.9: Change in Percentage Outsourced with Fixed Contract Cost and changing values of Contract Length in Setting 4

The behavior observed in Figure 4.9 is almost the same as the plots in the previous settings. The fixed contract cost values eliminating the usage of outsourcing option are slightly higher than the ones in the constant demand case in the third setting. An explanation is done previously as; the seasonality causes the need for the use outsourcing options even with higher contract costs. The fixed cost values eliminating the outsourcing option are significantly higher than the ones in our first (1) setting. The explanation for this is also provided previously as; the effect of frequency dominates the effect of unit outsourcing cost. So the fixed contract cost needed to eliminate the usage of outsourcing option increases with increasing production costs.

Table 4.20: Detailed data of breakpoints used in Figure 4.9

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	13	90	10	2	2	1	118,58	15,75	1523,8
1	30,5	70	30	1	1	3	152,33	17,42	1558,6
1	42,5	0	100	0	0	10	29,33	4,42	1570,22
3	31	80	20	1	2	2	115,5	17,42	1541,26
3	60	0	100	0	0	10	29,33	4,42	1570,22
6	18	90	10	1	4	1	81,58	6,25	1506,08
6	82,5	0	100	0	0	10	29,33	4,42	1570,22
12	100	0	100	0	0	10	29,33	4,42	1570,22

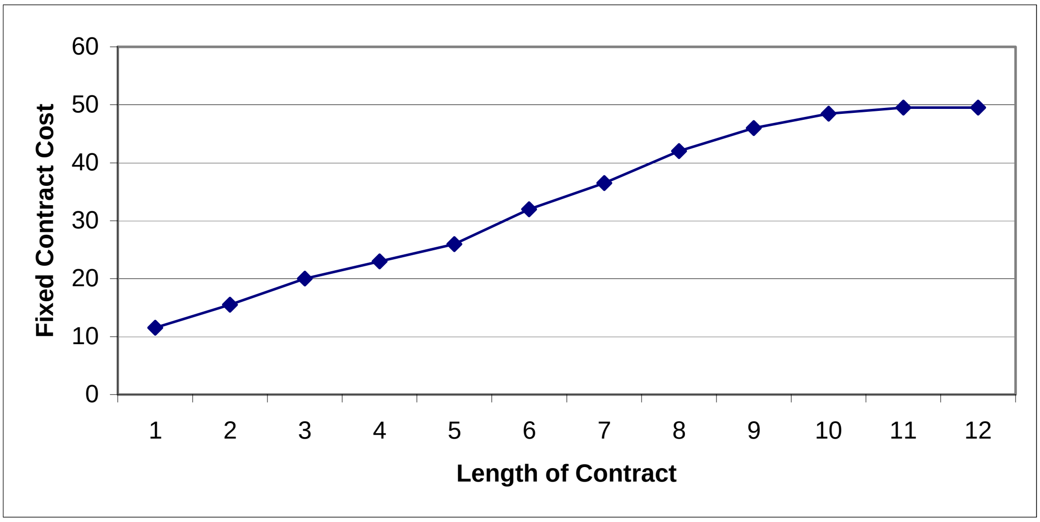

Figure 4.10: Plot of fixed contract costs providing constant objective function value under different contract lengths in Setting 4

The behavior observed in Figure 4.10 is almost the same as the plots in the previous settings. The terms are very similar to the constant demand case of third setting. Slight decreases are observed in some values. These can be explained with the trend explanation presented in the analysis of second setting (2). In our opinion trend is the main factor causing decrease in fixed contract cost values resulting with the same objective function value.

Table 4.21: Detailed data of points used in Figure 4.10

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	11,5	100	0	3	3	0	108,58	13,75	1519,46
2	15,5	100	0	2	4	0	114,25	13,5	1520,14
3	20	100	0	2	4	0	101,25	4,83	1519,78
4	23	100	0	2	6	0	52,08	4,83	1519,98
5	26	90	10	1	3	1	120,5	6,25	1520,42
6	32	90	10	1	4	1	81,58	6,25	1520,08
7	36,5	100	0	1	4	0	86,25	15	1520
8	42	100	0	1	5	0	82,33	4,83	1520,24
9	46	100	0	1	5	0	64,25	4,83	1519,9
10	48,5	100	0	1	6	0	42,5	4,83	1520,18
11	49,5	100	0	1	6	0	51,75	0	1519,92
12	49,5	100	0	1	6	0	51,75	0	1519,92

Setting 5 Producer faces a demand having no trend and high seasonality. The outsourcing unit cost is higher than the in-house production cost. This is the setting where the producer has larger production facilities than the subcontractor has. The producer desires scheduling of a production run in every six periods. In mathematical terms;

$F1=3$: No trend

$F2=3$: High seasonality (*Highest D. / Lowest D. = 3*)

$F3=3$: $CO_t=1.2$ $\forall t=1,..,T$

$F6=3$: $F=6$

Two plot representations are used to analyze the results. (See Figures 4.11 and 4.12). The detailed information about the breakpoints on Figure 4.11 are presented in Table 4.22, and points on Figure 4.12 are presented in Table 4.23.

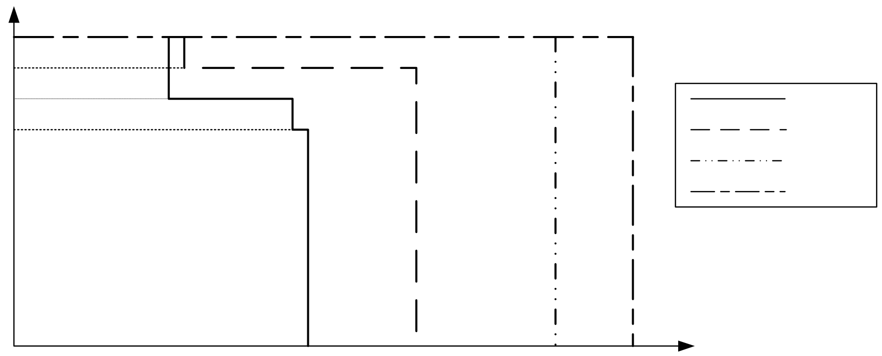

Figure 4.11: Change in Percentage Outsourced with Fixed Contract Cost and changing values of Contract Length in Setting 5

The behavior observed in Figure 4.11 is almost the same as the plots in the previous settings, but the terms are significantly higher than the third and fourth settings. The change from both of the previous settings can be explained with the existence of high seasonality causing the increase of these fixed contract cost values.

Table 4.22: Detailed data of breakpoints used in Figure 4.11

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	26	80	20	1	1	2	158,33	23,33	1547,8
1	46,5	70	30	1	1	3	108,33	23,33	1568,3
1	47	0	100	0	0	10	20	5	1568,4
3	28	90	10	1	2	1	126,67	21,67	1532
3	64,5	0	100	0	0	10	20	5	1568,4
6	87,5	0	100	0	0	10	20	5	1568,4
12	98,5	0	100	0	0	10	20	5	1568,4

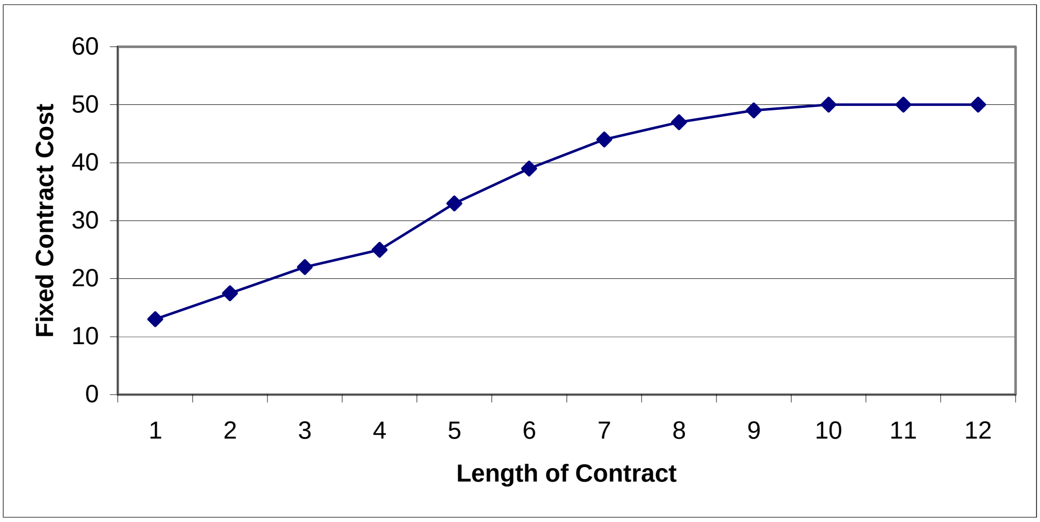

Figure 4.12: Plot of fixed contract costs providing constant objective function value under different contract lengths in Setting 5

The behavior observed in Figure 4.12 is almost the same as the plots in the previous settings. The terms are increased with respect to the third and fourth settings. In those settings system prefers to carry more inventory than no trend and high seasonality setting (5), so can pay lower costs for contract. But in the fifth setting, system carries fewer inventories and willing to pay more for the fixed contract costs.

Table 4.23: Detailed data of points used in Figure 4.12

LEN	FC	%O	%P	#Ct	#Or	#Pr	AvgI	AvgB	Obj
1	13	100	0	3	3	0	108,33	8,33	1520
2	17,5	100	0	2	4	0	112,5	8,33	1520
3	22	100	0	2	4	0	100	0	1520
4	25	90	10	1	3	1	112,5	10	1520,2
5	33	100	0	1	4	0	120,83	8,33	1520
6	39	100	0	1	5	0	83,33	8,33	1520
7	44	100	0	1	5	0	62,5	8,33	1520
8	47	100	0	1	6	0	62,5	0	1520
9	49	100	0	1	6	0	54,17	0	1520
10	50	100	0	1	7	0	37,5	0	1520
11	50	100	0	1	7	0	37,5	0	1520
12	50	100	0	1	6	0	50	0	1520

4.5 Two Contract Options: Contract Length and Fixed Contract Cost Analysis

For the detailed analysis of two alternative contract options, the base setup used in the single contract analysis is used. Desired frequency of production runs is taken as once in every three periods. Using in-house production as the first source of demand satisfaction is represented with Production (P).Two contract options are defined with different contract lengths, first one is a short contract for two periods, we name it as option 1 (O1), second is a long contract for six periods and named as option 2 (O2). Three demand cases are examined as in the previous section. Source of supply curves over fixed contract costs of both outsourcing options and source of supply curves over unit costs of the two outsourcing options are used in our analysis. In both of the source of supply curves, the regions of the plots are separated using the primary source of demand satisfaction. If all the demand is satisfied with a single source, then only the name of that source is written. In hybrid cases, one source is utilized for higher than 50% of the demand. Another source is utilized for the remaining part of the demand, the primary source is written first, and then the secondary source is written. Borders are used to identify the regions in which the changes in the combinations do not cause change in the sourcing behavior of the system. In the fixed contract cost source of supply curve the effects of the relationship between the fixed contract terms are analyzed. In the unit cost of outsourcing source of supply curve, the relationship between the terms of the two outsourcing options regarding the unit costs are analyzed. Detailed information of all system performance measures for all points used in the plots is available upon request. Our base for this part of our study can be defined as follows:

$F4=2$	:	$S_t=0.02$	$\forall t=1,..,T$	Storage Cost
$F5=3$	:	$B_t=0.06$	$\forall t=1,..,T$	Backordering Cost
$F6=2$	:	$F=3$		
$F7=3$:		$SP_{1t}=R=1.2\ CD/T$	$\forall t=1,..,T$	Capacity
		$SP_{2t}=O=0.3\ CD/T$	$\forall t=1,..,T$	
$F8$		$LEN_1=2$	$LEN_2=6$	
$F10=2$:		$FO_t=0.03\ CD/T$	$\forall t=1,..,T$	*F*ix. Cost of Outsourcing

The model is run with three different demand settings that are used in Section 4.4. The settings used in the analysis and the results gathered are as follows:

Setting 1 Constant demand setting, where demand in each period is identical: Source of supply curve for fixed contract cost can be seen in Figure 4.13. In this figure the source of demand satisfaction is identified for each fixed contract cost combination. Boundaries are used to separate the regions where the optimal selection of the main source changes from one source to another.

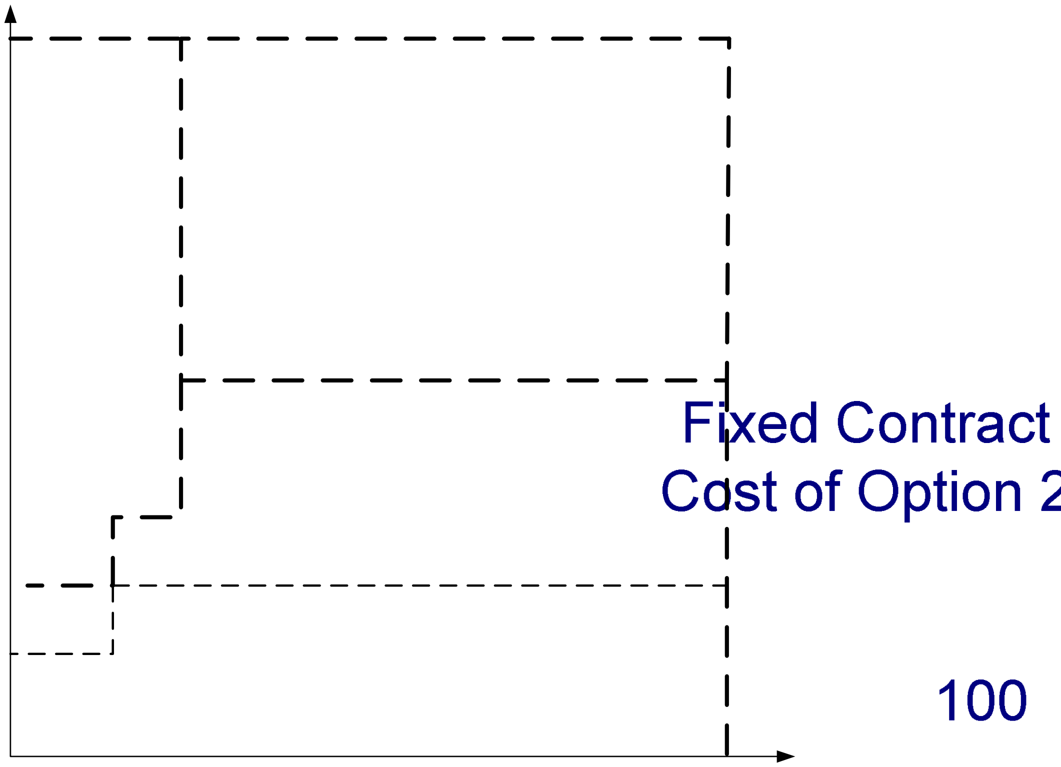

Figure 4.13: Source of supply as a function of fixed contract costs in Setting 1

When fixed contract costs of both options are small, system uses the outsourcing option fully, whereas with increasing fixed cost of outsourcing, hybrid cases occur. As expected, when fixed costs associated with both options are large, system chooses not to utilize the outsourcing option at all. For the combinations where two cost terms are equal, system always prefer to use the second option as its length is longer than the first option. First option only dominates if the fixed cost of the second option is considerably larger than the first option's fixed cost. Source of supply curve for the unit outsourcing cost is presented in Figure 4.14.

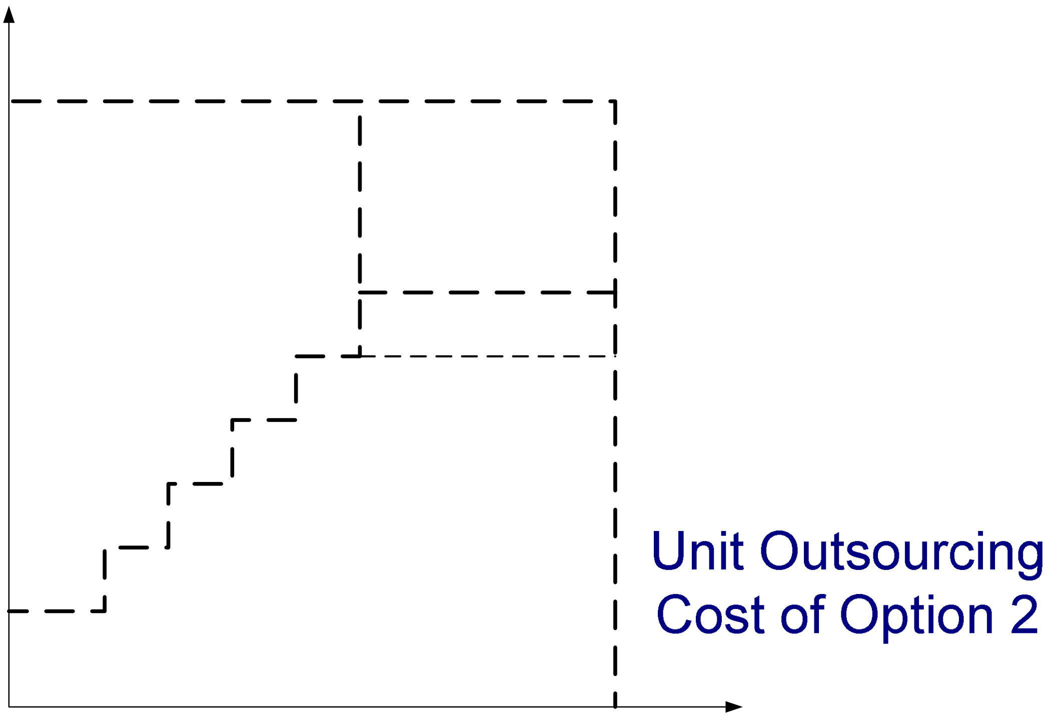

Figure 4.14: Source of supply as a function of unit outsourcing costs in Setting 1

With small unit outsourcing costs system prefers to use the outsourcing option fully. Increase in costs causes the occurrence of hybrid cases and when unit costs of both options are higher than that of the production, system does not utilize outsourcing options at all. For equal unit cost case, system always selects option two, because its length is longer. But with small deviation from the equality, the one with smaller cost is always preferred.

Setting 2 Same analysis is performed for settings where demand does not have trend but has high seasonality. Source of supply curve for fixed contract cost can be seen on Figure 4.15. On this figure the source of demand satisfaction is identified for each fixed contract cost combination.

1,08

1,06

1,04

1,02

1

0,98

0,96

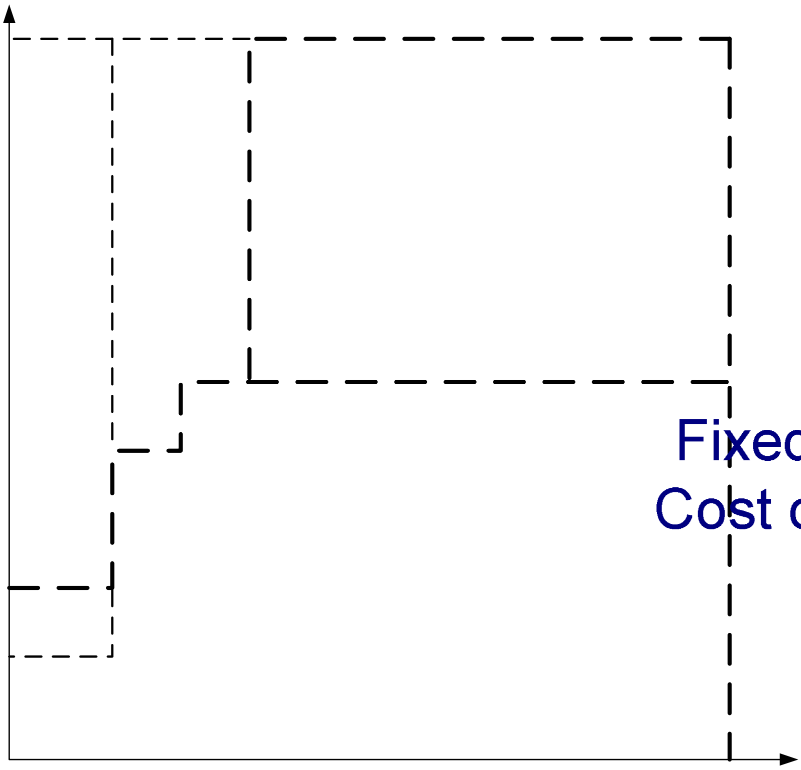

Figure 4.15: Source of supply as a function of fixed contract costs in Setting 2

For extreme cases, where both cost terms are high, or one significantly dominates the other, the system behavior is almost the same as the previous setting. Only one major difference is observed. The second and the long contract option is not used together with in-house production, but the short contract option is used in hybrid with the in-house production. Demand has its effect on these hybrid groups. High seasonality causes the use of short contracts together with production to handle variation in the demand between periods. Source of supply curve for the unit outsourcing cost is presented in Figure 4.16.

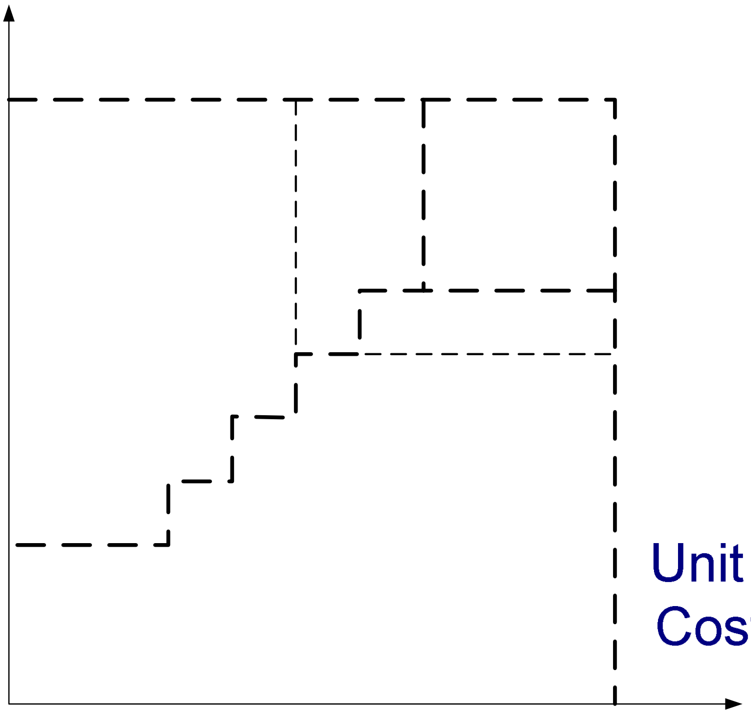

Figure 4.16: Source of supply as a function of unit outsourcing costs in Setting 2

Unit cost is less significant in the decision of outsourcing with seasonal demand. In the situations of equality and in cases where one unit cost has a slight domination to other, option 2 is still selected for its length advantage.

Setting 3 Same analysis is done for demand having low trend and low seasonality. Source of supply curve for fixed contract cost terms can be seen on Figure 4.17. On this figure the source of demand satisfaction is identified for each fixed contract cost combination.

1,08
1,06
1,04
1,02
1
0,98
0,96

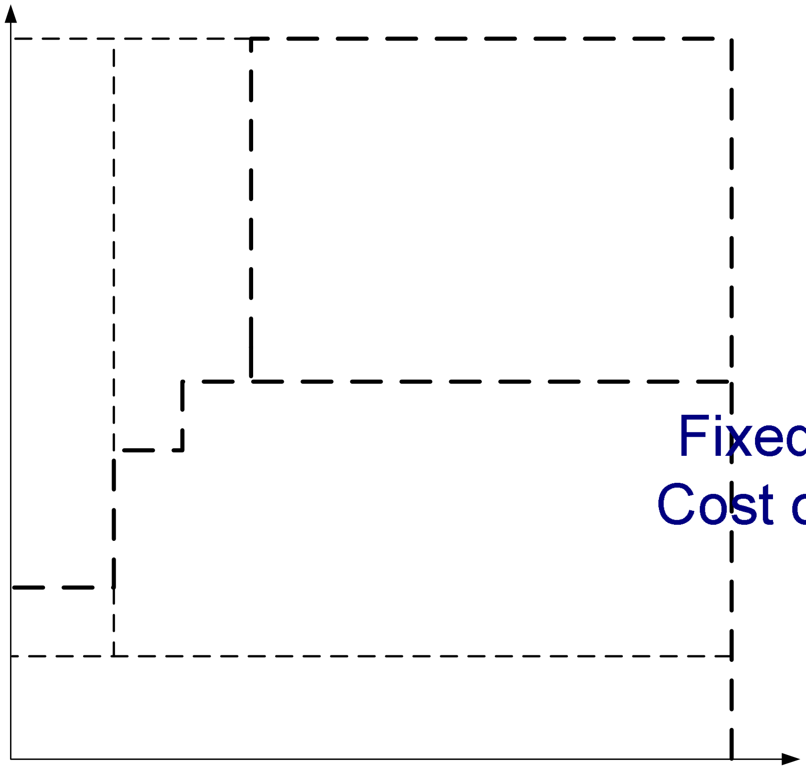

Figure 4.17: Source of supply as a function of fixed contract costs in Setting 3

Hybrid cases are used more frequently for this setting. The low trend and low seasonal demand cause the existence of hybrid cases. Long-term hybrid case is more preferred. Option 2 dominates option 1 even with higher costs, because of its long term cost minimization. Source of supply curve for the unit outsourcing cost terms is presented in Figure 4.18.

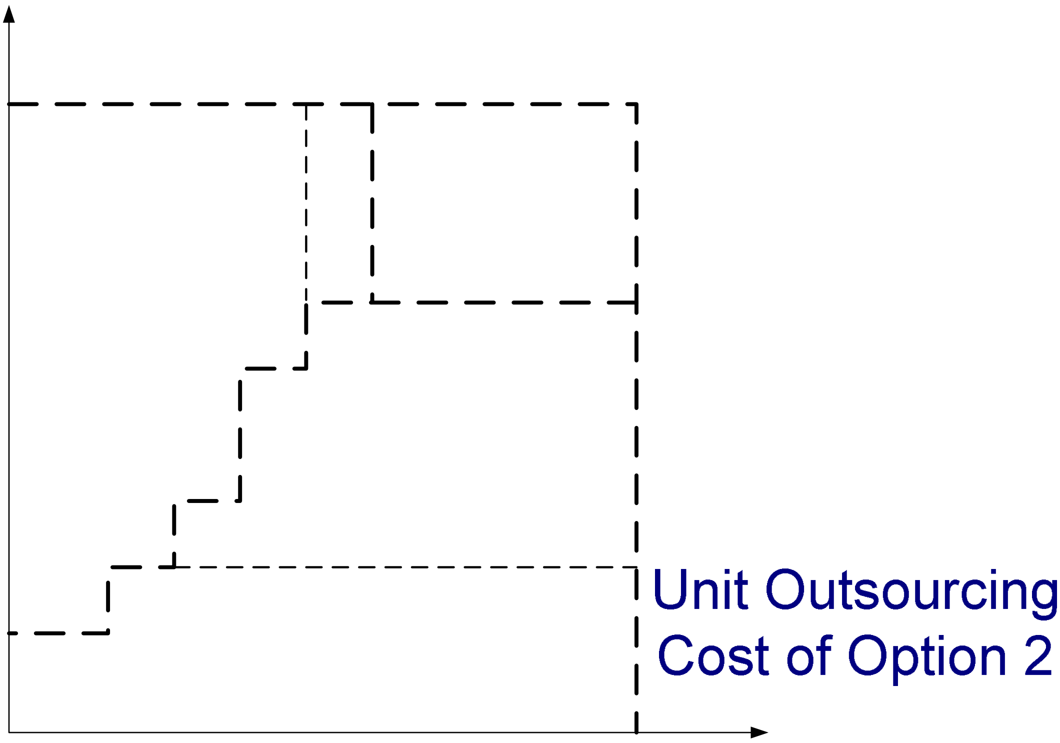

Figure 4.18: Source of supply as a function of unit outsourcing costs in Setting 3

Combinations of increasing cost diminish the amount outsourced as observed in constant and highly seasonal demand settings. Hybrid cases are frequently used as observed in the fixed contract cost source of supply curves. Variation in the demand, caused by the seasonality, is handled by the utilization of in-house production and outsourcing together. Second option dominates for the cases where cost terms are equal, but the cheaper option is selected generally for other cases.

4.6 Results and Comments

In the first section of this chapter, some parameters or environmental settings are selected as factors of our analysis. In the second section, a base case is defined for our experimental study. We expect to describe the system behavior using factors as they constitute central controllers of our analysis. In the third section, a full factorial experimental study is done in order to find the contribution of these factors to our system's performance measures. ANOVA is performed in order to find out which factors are significant and which are not, on different performance measures. In the fourth section, a more detailed setting is prepared for having a more in-depth understanding of the system behavior. Some factors are set to fixed levels in order to easily analyze the others. Fixed contract cost and length of contract are analyzed together with their relation to each other. And in the fifth section, a second contract option is incorporated for the analysis of multiple sourcing decisions. Again two contract terms are analyzed in depth to predict the optimal behavior of the system

In the single contract single factor analysis with full factorial setup, all possible combinations of the factors are input into the model and run using CPLEX 6.0. The total number of runs adds up to about a hundred thousand. Each problem is solved in a time between one second to one minute varying with their parameter sets. All the factors are analyzed independently for their effects on our eight system performance measures. The factors affecting each of our system performance measures can be identified as;

- <u>Objective Function</u>: Unit outsourcing cost
- <u>Percentage Outsourcing</u>: Unit outsourcing cost, Frequency of production, Capacity, Fixed outsourcing cost
- <u>Percentage Production</u>: Unit outsourcing cost, Frequency of production, Capacity, Fixed outsourcing cost
- <u>Number of Contracts</u>: Trend in demand, Unit outsourcing cost, Backordering cost, Capacity, Length of contract, Fixed contract cost, Fixed outsourcing cost

- <u>Number of Outsourcing Orders</u>: Trend in demand, Unit outsourcing cost, Storage cost, Backordering cost, Capacity, Length of contract, Fixed contract cost, Fixed outsourcing cost
- <u>Number of Production Runs</u>: Trend in demand, Unit outsourcing cost, Storage cost, Frequency of production, Capacity, Fixed contract cost, Fixed outsourcing cost
- <u>Average Inventory</u>: Unit outsourcing cost, Storage cost, Backordering cost, Frequency of production, Capacity, Fixed outsourcing cost
- <u>Average Backorder</u>: Trend in demand, Unit outsourcing cost, Storage cost, Backordering cost, Frequency of production, Fixed contract cost, Fixed outsourcing cost

In the single contract length and fixed contract cost analysis section, our main concern is to find out the relation of contract cost and length. First some factors are identified as significant from the analysis done in the previous section. Unit outsourcing cost is one of these factors, it affects all performance measures significantly, because it is the most important cost term in the objective function. Frequency is selected as the second factor as it nearly affects all system parameters significantly. Frequency defines the usage of production option with defining the production setup cost with its different levels. Experimentation over contract length and fixed cost is performed under different combination of the unit outsourcing cost, desired frequency of production runs and demand environments. Other factors are set to their meaningful levels for further experimental analysis. Two main analyses are done using contract length and fixed contract cost.

First is the analysis of system's choice of demand satisfaction. A figure is obtained for each environment and factor case showing the percentage of outsourcing used with a given contract length and changing the fixed contract cost. This analysis has some significant results. For longer contracts system accepts to pay higher fixed contract cost. So, the percentage outsourcing hits zero at higher levels of fixed contract cost. When both unit cost of outsourcing and desired frequency of production runs are changed together, the effect of frequency is the observed dominantly. When the demand has seasonality the system accepts to pay higher contract cost to handle the variation.

Our second analysis is to fix an objective function value approximately and find the fixed costs giving the same objective function value with different contract lengths. The results of this study show that with longer contracts higher costs can be incurred for contracting with the same level of objective function. Trend in demand pattern is a significant factor for this analysis, as we observe that with trend the system tends to carry inventories or delay demand satisfaction. To hold the same level of objective, system is willing to pay lower contract costs than the no trend and high seasonality settings, where low levels of inventory is kept or a low number of units is backordered.

In the case where two contract options are available, contract length and fixed contract cost analysis are performed. It is assumed that two contracts are available with different lengths. One of them is a contract with two periods to represent short contract option. Second contract has a length of six periods to represent the long-term contract option. A source of supply curve analysis is applied to different demand patterns for this study. System behavior is investigated using different combinations of fixed contract costs and unit outsourcing costs. System's main choice of outsourcing is identified using borders defining the regions that system's choice does not change.

For the fixed contract cost the points representing alternative combinations are separated into groups based on the sources of demand satisfaction. Single source and hybrid cases are separated and boundaries are drawn for analysis. The analysis has shown that the option with longer length always dominates the shorter one when the contract costs are equal. The fixed cost needed to change this situation in favor of the shorter option is not proportional with the lengths of the two contracts. When the fixed contract cost of the long option is about the double of the short contract option's fixed contract cost, the long contract option is eliminated. At small fixed cost combinations, system outsources all the demand, and with increases in these costs, hybrid cases occur. In cases where both fixed costs are higher, production option is utilized to satisfy all the demand. Seasonality increases the occurrence of hybrid cases of production and outsourcing together. Our opinion is that for handling the variation system uses outsourcing and production together. Trend has an opposite effect. When there is trend, the system tries to source all the demand from a single source. In our opinion, main reason is to capture the economies of scales for the single sourcing case within the capacity limits of regular time.

In the analysis of the unit outsourcing pairs a similar methodology is used. Most of the results are the same as the effects of seasonality and trend; but some minor differences are observed in these settings. When the unit costs are equal, longer contract option always dominates the shorter option but the selection changes with small differences on the unit cost structure pairs. System often chooses to source with the minimum cost. This can be the long or short outsourcing options or the production option. Long contract option is only favored when there is a very small difference in its unit cost against the shorter one. With larger differences, system chooses the short contract option for unit cost minimization.

CHAPTER 5

CONCLUSION AND FURTHER RESEARCH

In this chapter we summarize our accomplishments and present some future research directions.

5.1 Accomplishments

Our first chapter is devoted to the introduction of our study. Coordination is introduced as a success tool for improving the firms' performance for a better competition in its industry. Coordination can be achieved between different stakeholders using many instruments; contracts are highlighted as an efficient tool for coordination. The help that our study may provide for different parties involved in contracting process is discussed in detail.

Chapter 2 is the part of our study where past studies on supply chain coordination, contracting, outsourcing and make/buy decision-making are reviewed to have an insight in these topics. A recent supply chain management review by Min and Zhou (2002) is used in order to position our study in the literature and review recent trends on these topics. A recent contract modeling review by Tsay et al (1998) is used in the same manner to position our work and review recent advances in this topic. Different examples from the literature are examined to better understand these concepts; they have a valuable contribution to our modeling and experimentation efforts.

In Chapter 3, the model of our study is developed with the insight gathered from the literature review. The environment consisting of a producer and one or more subcontractor(s) providing alternative contract option(s) is explained in detail. Parameters are used to create different environmental conditions and cost structures for our model. Decision variables are used to identify the properties of the optimal operating methodology. Objective function is of cost minimization type covering all related costs of production, outsourcing, backordering penalty, and inventory carrying. Constraints are used to define restriction and the relations of the decision variables with each other. Size of the model is discussed in terms of the problem parameters; number of periods in the planning horizon, and the complexity involved in the cost structure. Capabilities of the model structure, decision variables and parameters are discussed in detail. The capabilities are the abilities to represent different settings or problem instances using the model developed in this study. The related literature is discussed with the identification of representation abilities and handicaps on previous literature.

Chapter 4 is used to show the outcomes and results that can be gathered from the use of our model. Factors that are of our interest are identified, their levels are determined for investigating the effects of these levels on system performance measures. System performance measures are statistical figures used to understand the system's behavior. Objective function only provides the amount of cost incurred by the system. But system performance measures define the main source of demand satisfaction, amount of inventory carrying, and amount of backorders and so on. A base case is defined for further experimentation purposes. Some parameters are set to fixed values in order to simplify the general structure and relax some of our constraints. The full factorial experimental study is performed for single contract option case to identify the significant effects of those factors; in the following section some of these factors are set to specific levels. A detailed analysis is used to identify the relation of length of contract and fixed contract cost combinations that make the producer indifferent on its decision. Same analysis is done for the two-contract case by using source of supply curves as the basis of our analysis. Results are discussed in detail and some comments are done on these results. Most of our results are intuitive and some of them are not that obvious but intuitive with facts. And our rare non-intuitive results are not as significant as the intuitive ones so we do not mention them much.

As a conclusion, we end up with satisfying most of our aims set at the beginning of this study; a general model is developed to provide aid in the contract terms negotiation and selection of contracts integrated with production/inventory control. In contract terms negotiation, this model may help both the producer and the subcontractor firms. Producer may use the model to evaluate different contract parameter sets, and choose the one optimizing its desired objective. Subcontractor can use the result of the model showing the contract parameter sets that are indifferent from the producer's point of view, and choose the one optimizing its desired objectives. With this two way cost minimization our coordination objective is achieved successfully. As a contract design tool our model can be used with an extended objective function including the subcontractor's cost. The gain from the centralized decision can be shared among the producer and the subcontractor according to an agreement. Experimentation over different factors is done in order to come up with a framework; many of our results can be used as a framework by the decision makers in such hard decision situations. Our results define the tendencies in the system's choices over different factors, for simple systems, the behavior of the producer can be predicted using our result as a framework. Producer firm can also use the result in the decision of production, outsourcing, inventory carrying and backordering decisions.

5.2 Future Research Directions

For the further research directions and extensions, different aspects are identified. Most important extension can be on the model structure to handle different cases that are not covered in this study. These can be identified as allowing or incorporating the following concepts into our model.

- lost sales,
- buybacks and returns,
- capacity allocation rules
- disposal of inventory,
- different effects like learning and forgetting,
- non increasing unit cost structure,
- setup cost reduction,
- quality aspects
- demand time windows,
- non linear cost structure,
- second sales opportunity,
- safety stocks,
- setup times,
- setup carry over
- multi-item extensions.

Factors other than the ones outlined in section 4.1 can be investigated so that other properties involved in the model capabilities can be observed. The additional factors that can be experimented can be presented as follows:

- discount factor,
- interest rate,
- lead time,
- upper and lower bounds for outsourcing,
- total number of periods in the planning horizon.

In this study we model all decisions as one-time decisions, in other words our model defines the optimal behavior for the producer at the beginning of the planning horizon. Our model can also be utilized on a rolling horizon basis. For the rolling horizon implementation the decisions from the previous periods should be inputted into the model as fixed and new decisions should be made for the current planning horizon.

Development of efficient solution algorithms for the model presented can be another direction of further research. In our study we do not focus on the solution algorithms or structural properties that can lead to the development of such algorithms. But in the analysis of the results obtained from our experiments, we observe some structural properties. In the contract length and fixed contract cost analysis, we observe that the fixed cost increases up to a point and then it does not have an effect on the system behavior. In the two contract option's analysis we observe that system uses hybrid production/outsourcing cases only in certain situations, and use single source in other conditions. These kinds of structural properties can be used in the development of efficient solution algorithms for the model proposed in this study.

REFERENCES

[1] Aksen, D., Altınkemer, K. and Chand, S., 2003, ‘The single-item lot-sizing problem with immediate lost sales’, European Journal of Operational Research, Vol 147, 558-566.

[2] Atamtürk, A.and Hochbaum, D. S., 2001, ’Capacity acquisition, subcontracting, and lot sizing’, Management Science, Vol 47, 1081-1100.

[3] Barbarosoğlu, G., 2000, ‘An integrated supplier-buyer model for improving supply chain coordination’, Production Planning and Control, Vol 11, 732-741.

[4] Beltran, J. L., Krass, D., 2002, ‘Dynamic lot sizing with returning items and disposals’, IIE Transactions, Vol 34, 437-448.

[5] Chiu, H. N, 1997, ‘Discrete time-varying demand lot-sizing models with learning and forgetting effects’, Production Planning and Control, Vol 8, 484-493.

[6] Chyr, F., Huang, S.-T., Lai, S. D., 1999, ‘A dynamic lot-sizing model with quantity discounts’, Production Planning and Control, Vol 10, 67-75.

[7] Cohen, M.A. and Agrawal, N., 1999, ‘An analytical comparison of long and short term contracts’, IIE Transactions, Vol 31, 783-796.

[8] Denizel, M., Erengüç, S. and Benson, H. P., 1997. ‘Dynamic lot-sizing with setup cost reduction’, European Journal of Operational Research, Vol 100, 537-549.

[9] Gerchak, Y. and Wang, Y., 2001, ‘Revenue-Sharing vs. Wholesale-Price Contracts in Assembly Systems with Random Demand’, Multi-Echelon Inventory System Conference, University of California at Berkeley, June 23-24, 2001.

[10] Giannoccaro, I. and Pontrandolfo, P., 2003, ’Supply chain coordination by revenue sharing contracts’, International Journal of Production Economics, forthcoming.

[11] Grout, J. R.,1996, ’a model of incentive contracts for just-in-time delivery’, European Journal of Operational Research, Vol 96, 139-147.

[12] Humphreys, P, McIvor R. and Huang, G., 2002, 'An expert system for evaluating the make or buy decision', Computers and Industrial Engineering, Vol 42, 567-585.

[13] Jolayemi, J. K., Olurunniwo, 2003, F. O., 'A deterministic model for planning production quantities in a multi-plant, multi warehouse environment with extensible capacities', International Journal of Production Economics, forthcoming.

[14] Kaihara, T., 2001, 'Supply chain management with market economics', International Journal of Production Economics, Vol 73, 5-14.

[15] Khouja, M., 2003, 'Optimizing inventory decisions in a multi-stage multi-customer supply chain', Transportation Research Part E, Vol 39, 193-208.

[16] Kim, B., 2003, 'Dynamic outsourcing to contract manufacturers with different capabilities of reducing the supply cost', International Journal of Production Economics, forthcoming.

[17] Klastorin, T. D., Moinzadeh, K. and Son, J., 2002, 'Coordinating orders in supply chains through price discounts', IIE Transactions, Vol 34, 679-689.

[18] Kok, G. T. de, 2000, 'Capacity allocation and outsourcing in a process industry', International Journal of Production Economics, Vol 68, 229-239.

[19] Lee, C.-Y., Çetinkaya, S., Wagelmans, A. P. M., 2001, 'A dynamic lot-sizing model with demand time windows', Management Science, Vol 47, 1384-1395.

[20] Lee, H. L. and Billington, C., 1993,'Material management in decentralized supply chains', Operations Research, Vol 41, 835-847.

[21] Lee, S.-B. and Zipkin, P. H., 1989, 'A dynamic lot-size model with make-or-buy decisions', Management Science, Vol 35, 447 – 458.

[22] Lee, Y. H., Jeong, C. S. and Moon, C., 2002, 'Advanced planning and scheduling with outsourcing in manufacturing supply chain', Computers and Industrial Engineering, Vol 43, 351-374.

[23] Loparic, M., Pochet, Y. and Wolsey, L. A., 2001, 'The uncapacitated lot-sizing problem with sales and safety stocks', Mathematical Programming Series A, Vol 89, 487-504.

[24] Martel, A. and Gascon, A., 1998, 'Dynamic lot-sizing with price changes and price-dependent holding costs', European Journal of Operational Research, Vol 111, 114-128.

[25] Mieghem, J. A., 1999, 'Coordinating investment, production, and subcontracting', Management Science, Vol 45, 954-971.

[26] Min, H. and Zhou, G., 2002, 'Supply chain modeling: past, present and future', Computer and Industrial Engineering, Vol 43, 231-249.

[27] Mishra, A. L., 2003, 'Selective discount for supplier-buyer coordination using common replenishment epochs', European Journal of Operational Research, forthcoming

[28] Mohebbi, E. and Posner, M. J. M., 1998, 'Sole versus dual sourcing in a continuous review inventory system with lost sales', Computers Industrial Engineering, Vol 34, 321-336.

[29] Nam, K., Chaudhury and A., Rao, H. R., 1995,'A mixed integer model of bidding strategies for outsourcing', European Journal of Operational Research, Vol 87, 257-273.

[30] Nellore, R. and Soderquist, K., 2000, 'Strategic outsourcing through specifications', Omega, Vol 28, 525-540.

[31] Özdamar, L and Bozyel, M. A., 2000, 'The capacitated lot sizing problem with overtime decisions and setup times', IIE Transactions, Vol 32, 1043-1057.

[32] Padillo, J. M. and Diaby, M., 1999, 'A multiple-criteria decision methodology for the make-or-buy problem', International Journal of Production Research, Vol 37, 3203-3229.

[33] Plats, K. W. and Probert, D. R., and Canez, L., 2002, 'Make vs. buy decisions: A process incorporating multi-attribute decision-making', International Journal of Production Economics, Vol 77, 247-257.

[34] Pochet, Y. and Wolsey, L. A., 1993, 'Lot-sizing with constant batches: formulation and valid inequalities', Mathematic of Operations Research, Vol 18, 767-785.

[35] Schneeweiss, C. and Zimmer, K., 2003, 'Hierarchical coordination mechanisms within the supply chain', European Journal of Operational Research, forthcoming.

[36] Serel, D. A., Dada, M. and Moskowitz, H., 2001, 'Sourcing decisions with capacity reservation contracts', European Journal of Operational Research, Vol 131, 635-648.

[37] Shaw, D. X. and Wagelmans, A. P. M., 1998, 'An algorithm for single-item capacitated economic lot sizing with piecewise linear production costs and general holding costs', Management Science, Vol 44, 831-838.

[38] Sox, C. R., Gao, Y., 1999, 'The capacitated lot sizing problem with setup carry-over', IIE Transactions, Vol 31, 173-181.

[39] Tsay A. A., Nahmias S. and Agrawal, N., 1998, 'Modeling supply chain contracts: a review', in Quantitative Models for Supply Chain Management, Tayur, S., Ganeshan, R. and Magazine, M. (eds), Kluwer Academic, Norwell. MA.

[40] Tsay, A. A., 1999, 'The quantity flexibility contract and supplier-customer incentives', Management Science, Vol 45, 1339-1358.

[41] Wolsey, L. A., 2002, 'Solving multi-item lot-sizing problems with an MIP solver using classification and reformulation', Management Science, Vol 48, 1587-1602.

[42] Wu, D., J., Kleindorfer, P. R. and Zhang, J.E.,2002, 'Optimal bidding and contracting strategies for capital-intensive goods', European Journal of Operational Research, Vol 137, 657-676.