ANALYSIS OF SAFETY STOCK FOR PRODUCTION - INVENTORY PROBLEM OF A COMPANY UNDER MULTIPLICATIVE FORM OF FORECAST EVOLUTION

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ABSTRACT

ANALYSIS OF SAFETY STOCK FOR PRODUCTION-INVENTORY PROBLEM OF A COMPANY UNDER MULTIPLICATIVE FORM OF FORECAST EVOLUTION

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In this thesis, we focus on integration issue of manufacturing and sales functions from the perspective of aggregate production planning. The manufacturing function and sales function are performed by separate affiliated companies of the same business group, which operate as an integrated supplier-buyer system. In particular, this study provides theoretical and practical insight into the use of forecast volatility measure to better match supply with demand so as to reduce the costs of inventory and stock-outs in the manufacturer-buyer relationship under described master production-scheduling environment. Nature of forecast modifications provided by the buyer lays the foundation for the study. We modify the existing aggregate production planning model to accommodate a measure of historical forecast evolution. The overall objective of the thesis is to provide management with a

forecast evolution-modeling framework to examine performance characteristics of the manufacturer-buyer interaction.

Keywords: martingale model of forecast evolution, master production scheduling, safety stock

ÖZ

ÇARPIMSAL TALEP TAHMİN EVRİMİ MODELİ ALTINDA BİR FİRMANIN ÜRETİM-ENVANTER PROBLEMİ İÇİN EMNİYET STOK ANALİZİ

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Bu çalışmada, üretim fonksiyonu ve satış fonksiyonunun toplaşık üretim planlama faaliyetleri çerçevesinde entegrasyonu üzerinde durulmuştur. Entegre bir üretici-müşteri sistemi olarak çalışan üretim ve satış fonksiyonları, aynı grubun farklı şirketleri tarafından gerçekleştirilmektedir. Özellikle, bu çalışma, incelenen üretici-müşteri ilişkisi altında arz ve talebi daha iyi dengeleyerek envanter ve stoksuzluk maliyetlerini azaltmak amacıyla, talep tahmin değişkenliği ölçütünün kullanımına teorik ve pratik bir kavrayış sunmaktadır. Müşteri tarafından sağlanan talep tahmin değişikliklerinin karakteristiği bu çalışmanın temelini oluşturmaktadır. Talep tahmin evrimi ölçütünü dahil etmek amacıyla üreticinin mevcut toplaşık üretim planlama modeli geliştirilmiştir. Bu çalışmanın genel amacı, üretici-müşteri ilişkisi

performansının incelenebilmesi için, firmaya talep tahmin evrimi modelleme çerçevesi sunmaktır.

Anahtar Kelimeler: talep tahmin evrimi modeli, toplaşık üretim planlama, emniyet stoğu

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TABLE OF CONTENTS

LIST OF TABLES

TABLE

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

In this thesis, we consider a case to show the importance and benefits of modeling sales forecast evolutions, and discuss a specific evolution modeling method that may be used in an ERP system to better integrate sales and production divisions. In particular, we consider a manufacturer-buyer interaction in a master production-scheduling environment. The manufacturer and buyer, which are affiliated companies of the same business group, operate as an integrated supplier-buyer system. Without considering end customers at first, the buyer, which is the commercial company of the group, is regarded as the first immediate customer of the manufacturer. Note that the buyer is the only customer of the manufacturer with a transfer payment scheme. With a total of 8 regional offices and more than 150 exclusive distributors scattered throughout Turkey, the buyer performs marketing and sales activities for products produced by the manufacturer.

The industry in which the group operates requires a high volume and a high variety production system with capital-intensive machinery focusing on capacity utilization, and shelf life constraints for raw materials and finished products. Producing bakery and snacking cereal products, the group operates in the fast moving consumer goods

- FMCG sector. Today, the manufacturer is faced with the challenge of producing more stock keeping units - SKUs than they have in the past and having to produce them with a shorter lead time. To accomplish this task, information visibility and interaction with the buyer, distributors and chain markets play the key role.

For the most part, how supply chain is managed lays the foundation of effectiveness, efficiency, and hence, success for the entities of the chain. If there is a need for more responsive supply chain, as in our case, supply chain decisions should be based on data that is as close to end consumers as possible. That way, in FMCG sector manufacturers ideally need to base their supply chain decisions on retailer-to-end consumer sales data. However, this practice is relatively rare in FMCG sector and few manufacturers know as much about end consumers as they would ideally like. In our case retailer-to-end consumer sales data, however, is very difficult to capture using the existing systems; and at present, buyer-to-distributors/chain markets sales data is the most reliable sales data in an individual SKU level. The group's current emphasis is to be able to act on distributor-to-retailer sales data, and accordingly, they need to capture the distributor sales data and feed into its ERP system.

Without considering manufacturer's suppliers and end consumers, the entities that comprise the supply chain under consideration are manufacturer, buyer, distributors, chain markets and retail stores. The supply chain is schematized by the following three figures representing flows of information, finished products and funds, respectively. Figure 1 represents flows of sales forecasts and demand information. Each week the buyer communicates sales forecasts in weekly level of detail to the manufacturer. The manufacturer, in turn, makes production and inventory decisions based on weekly sales forecasts. Throughout the week, the buyer processes daily orders placed by distributors and chain markets, and makes shipment decisions. Demand at distributors is observed in two different ways. First, retail stores place orders periodically at their own convenience. Second, demand can be observed while distributors visit retail stores daily, and in turn sell product from the back of the truck to satisfy the then-occurring demand. The latter can be defined as spot sales.

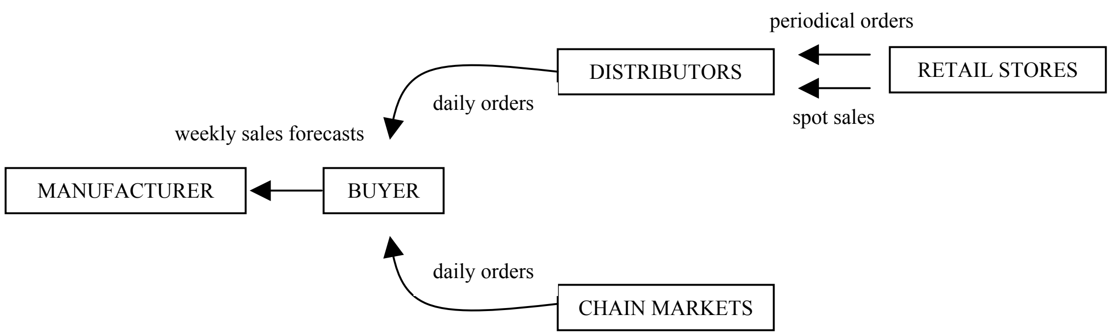

Figure 1 A Schematic Flow of Forecast and Demand Information

Figure 2 schematizes physical flow of finished products. Finished products are distributed directly from the manufacturer's plants to distributors and chain markets. The distributors visit retail stores daily and deliver customer orders. The group operates three main production facilities and has one central warehouse. Production facilities deliver their finished products to a central warehouse, which is located at one of the production facilities and managed jointly by the buyer and manufacturer. Demands are observed at the central warehouse and from here customer orders are shipped directly to distributors and chain markets.

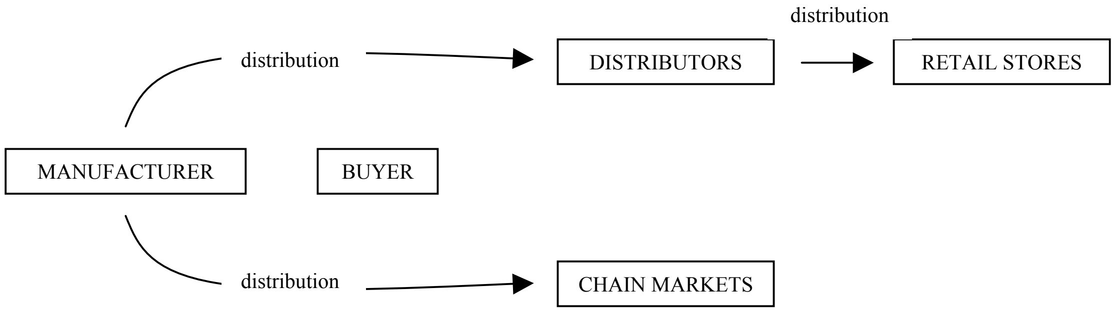

Figure 2 A Schematic Flow of Finished Products

Figure 3 represents flow of funds. Financial flows from one entity to another are bi-directional. For example, distributors and chain markets pay for the amount they demand while the buyer gives sales support in the form of incentives etc. There are four classes of sales activity throughout the supply chain under consideration: manufacturer-to-buyer, buyer-to-distributor/chain market, distributor-to-retailer, and retailer-to-end consumer, successively. Manufacturer-to-buyer sale is realized upon manufacturer-to-distributor/chain market delivery, or equivalently buyer-to-distributor/chain market sale is realized. Data on distributor-to-retailer sales and distributor inventory levels in an individual SKU level are not much reliable. At present, buyer-to-distributor/chain market sales data in an individual SKU level is used in the manufacturer's decisions.

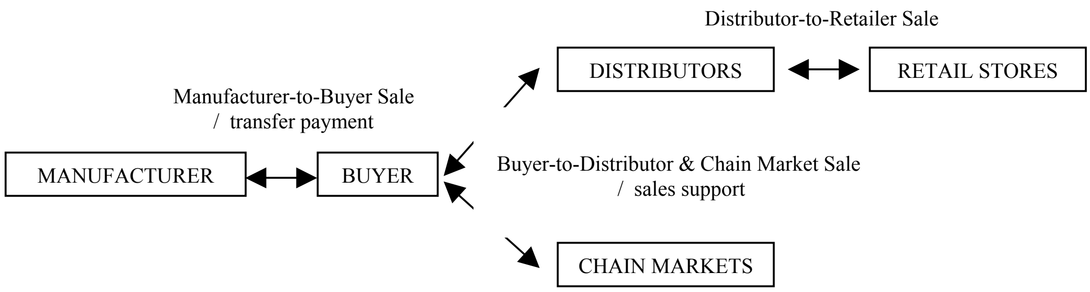

Figure 3 A Schematic Flow of Funds

Control of inventory is a problem common to all supply chains. The control of inventory directly relates to control of the above-mentioned flows throughout the supply chain. Two main types of control can be applied to manage the supply chain: centralized and decentralized control. Centralized control refers to the cases where inventory at a particular point in the supply chain is controlled while considering the inventory levels in the supply chain as a whole. As a typical example, echelon inventory policy considers the total inventory upstream. Decentralized supply chain, on the other hand, controls the inventory at an individual entity of the chain by considering only the local information at that entity. As an example, in an MRP environment, the order requirements are based on the MRP explosion by considering the forecasts as exact. Lee and Billington (1993) states that often organizational barriers between the entities exist, and information flows can be restricted such that complete centralized control of inventory in a supply chain may not be feasible or desirable. Hence, it can be said that an important requirement for implementing a centralized inventory policy is the ability to access information on inventory levels at other entities in the supply chain. In the view of this, it can be said that the supply chain under consideration has a decentralized inventory control. In fact, company structures and relationships between various business functions are among the first challenges to attain the supply chain success. Corporate structure in our case had been traditional and function-focused for almost forty years before the group's production inventory management system (called PIMS as well) was put into implementation. In this function-focused structure, managing the supply chain has been a multi-function activity in which each one of the buyer and manufacturer has considered its own local objectives rather than chain-wide emphasis within the jurisdiction of the group.

After the launchment of PIMS now, the manufacturer and buyer try to operate as an integrated supplier-buyer system. Therefore, many of the challenges facing the group's supply chain today relate to integration issues between various systems. In this environment, PIMS through its framework delivers integration tools and interfaces to the manufacturer and buyer supporting pursuits of bettering relationships between manufacturing and sales functions to respond to customer demands more efficiently. Therefore, PIMS addresses the challenge of integrating the buyer and manufacturer's activities through a set of corporate rules and software modules, supported with a PIMS database.

The work of the thesis is motivated by an in-house developed ERP project (production inventory management system – PIMS project), which deals with the development and implementation of a system for production planning and inventory management for the manufacturer-buyer setting considered. Being the main part of that project, master production scheduling - MPS problem constitutes the framework of the research in the thesis. MPS can be thought as a vehicle for coordinating the achievement of manufacturer and buyer goals. Hence the operations at MPS level should rely on two-way communications and hence information flows between the manufacturer and buyer in order to prove the success. In our framework, the main information flows between the entities of the chain are weekly sales forecasts generated by the buyer for each period of 12-week forecast horizon and daily orders placed by distributors and chain markets. Each week, the buyer reviews and updates weekly sales forecasts to more accurately predict amount of orders to be placed during a future week. And in turn, the manufacturer makes operational and tactical decisions (production decisions, determination of safety stock levels, hiring and layoff decisions, capacity investment decisions) based on these forecasts.

The research in this thesis consists of four main parts. The first part includes a description of the industry background and organizational environment for the manufacturer-buyer setting considered. An overview of forecasting activities and description of production planning and inventory management operations involved in managing the master production scheduling are presented. Corporate policies influencing activities involved in the master scheduling and some observations concerning forecasting behavior of the buyer and safety stock level determination are also discussed. In the second part of the thesis, we first give an in-depth theoretical analysis of forecast evolution modeling with a particular attention to martingale model of forecast evolution methodology. The research in the theoretical part of the thesis is motivated by the study in Heath and Jackson (1994). To have a better focus, we employ a particular set of products and environment to illustrate the application of forecast evolution modeling. By using historical forecasts and demands, we model forecast modification behavior of the buyer as a multiplicative process. To get an insight of the forecast modification behavior and to give an empirical support for the forecast evolution assumptions, we perform an empirical investigation on the data. Also, several investigations are taken to identify and solve the problems associated with the data. At the end, variance-covariance matrix of forecast updates is estimated from the historical forecast and demand data. The third part is related with the use of forecast evolution modeling and hence the estimated variance-covariance matrix. It considers the integration of forecast evolution modeling into the manufacturer's master production scheduling problem to determine target safety stock levels. We consider a variant of classical ways of determining safety stock levels (based only on forecast error variability or on demand variability). The proposed method captures the variability through the standard deviation of demand, which is calculated from the variability of forecast updates. This method is based on the fact that uncertainty in forecasts resolves in each period of the 12-week planning horizon. More precisely, using the proposed method of determining target safety stock levels we propose two other master production-scheduling models. One is just the same as the existing MPS model except the way of determining target safety stock levels. They are calculated exogenously based on demand variability that is captured by using the estimated variance-covariance matrix of forecast updates. On the other hand, the second model considers endogenous determination of target safety stock levels depending on demand variability and also correlations of forecast updates across products and time periods. In the fourth part of the thesis, we give a detailed description of the master scheduling models and conduct computational study to discuss performances of the MPS models in terms of safety inventory levels and delivery performance. As a conclusion we state that the second model, which considers demand and forecast correlations in determining target safety stock levels, provide better performance results.

The objectives of the research described in the thesis are to give the manufacturer an insight into the applicability of the forecast evolution concept and a specific evolution modeling methodology; and to integrate the forecast evolution modeling into manufacturer's master production-scheduling model, more precisely into safety stock determination method, to support pursuits of establishing better manufacturer-buyer relationships. In this thesis we make the following contributions: (1) first of all, we describe an implementation of martingale model of forecast evolution methodology in a real-life MPS problem; (2) we employ forecast evolution modeling to determine target safety stock levels. Using the fact of impact on safety stock levels of forecast volatility and correlations between forecast updates across products and time periods, we integrate forecast evolution modeling into manufacturer's MPS model to better establish target safety stock levels for a multi-product system. This method delivers a sort of allocative efficiency while allocating production time-hour capacity between products.

The thesis is structured as follows: In Chapter 2, we describe the organizational environment, buyer's forecasting activities and manufacturer's production planning operations that provide underpinning activities in managing the weekly master production planning. In addition, corporate policies influencing activities involved in master scheduling are presented. In Chapter 3, literature survey concerning the context of the research is presented. In Chapter 4 we present a methodology for forecast evolution modeling with special reference to Martingale Model of Forecast Evolution (MMFE) technique. In this chapter, empirical support for the data is also presented. Using the multiplicative form of the MMFE technique, application and integration of forecast evolution modeling into linear-programming models to establish target safety stock levels are discussed. In Chapter 5, details of the MPS models are given. Experimental runs are performed to compare the models' performances. Finally, Chapter 6 contains a summary of conclusions.

CHAPTER 2

INDUSTRY BACKGROUND

2.1. An Overview of Organizational Environment

In the thesis we consider manufacturer-buyer setting and study their interaction from the perspective of master production scheduling. The manufacturer and buyer are affiliated companies of the same business group. The manufacturer produces and inventories a portfolio of SKUs based on sales forecasts provided by the buyer, and in turn, the buyer performs marketing and sales activities for the products.

The manufacturer is one of the largest and oldest bakery and snacking cereal products producers in Turkey, holding a leading position in the industry today. The buyer deals with approximately 150 distributors scattered throughout Turkey and works directly with large chain markets. The group's assortment includes over more than hundred items of bakery and snacking products and they have several established brands. As the competition on the market for bakery products increases, the group constantly enlarges the assortment launching new products and supports the new brands with advertising and promotion. In near future the group plans to optimize the assortment and launch new products. Group's brands have the high level of consumer awareness and consumption. Most of the group's brands are positioned in the middle-price segment. The group's products are also exported to various countries.

The manufacturer is a cost-conscious company, which emphasizes cost reduction for manufacturing and inventory. The buyer, on the other hand, is sales-focused. Today, marketing expenditure and transportation costs account for significant share in the buyer's costs. The manufacturer makes expenditures on production modernization and new products development. In general terms, the industry in which the group operates is a high-volume, low-value margin industry where many of the manufacturers find it difficult to survive. Threshold level for the entry in this industry is relatively low. For success, however, it is very high, and success depends on operational efficiencies and effectiveness throughout the supply chain.

Without considering manufacturer's suppliers and end consumers, the players that comprise the supply chain under consideration are manufacturer, buyer, distributors, chain markets, and retail stores. The structures of the supply chain and distribution network are shown in Figure 4. It indicates that the distribution structure is different from the chain structure. Distribution strategy and network are established to deliver directly to distributors and chain markets from the manufacturer's production facilities. However, the first immediate customer of the manufacturer is the buyer, not distributors or chain markets. In general, regarding the nature of the supply chain, in which the manufacturer and buyer are the main players, it can be said that it has a decentralized inventory control. Inventory at the production facilities and inventory at the distributors are the main points of finished products inventory pools within the supply chain.

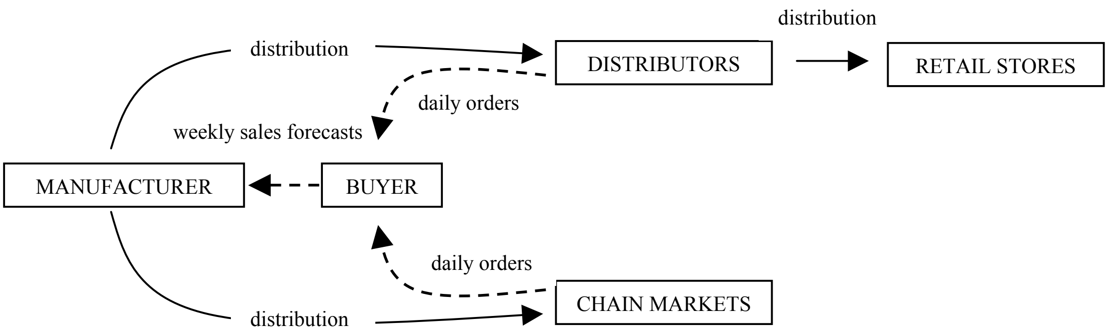

Figure 4 Distribution and Chain Structure

Amongst the largest national producers in its sector, the manufacturer produces, packages and inventories more than hundred SKUs at three group-owned production facilities. Finished products are then purchased by the buyer and shipped from the plants to over 150 distributors and a lot of chain markets throughout the country. They are further delivered from the distributors to over 190,000 retail stores. Primary distribution, referred to as manufacturer-to-distributors/chain markets delivery, is owned and governed by the buyer, so it has significant control over the distributors and their associated activities/policies regarding inventory management. The primary concern for transportation of products to distributors and chain markets is to be able to load the trucks with full truckload capacity since transportation costs account for the major part of the buyer's costs. In addition, significant logistical costs are associated with loading and delivery of multiple orders per truck.

The demarcation of sales territory results in eight regions, each of which is responsible for retail sales organization and activities of distributors and management of chain market accounts. Accounts in each region are comprised mainly of many individual exclusive distributors and chain markets. The buyer operates two major delivery networks to service end consumers. Main channel of distribution is through distributors, sales of which dominate the current sales mix (almost 82%) since the market is dominated by individual retail stores. In this channel, the buyer chooses to sell indirectly to end consumers through distributors to which products are channeled directly from the plants. The distributors are exclusive distributors in the sense that they only sell the company's products, and no other competing products. They have dedicated sales force, warehouse and vehicle fleet to service the distribution. The buyer establishes sales and distribution standards and manages sales of the distributors through its distributor supervisors. However, product distribution strategy induced by the existing performance evaluation and incentive system is not effective compared to that of the major competitor. Consequently, distributors give insufficient emphasis on distribution and merchandising, the two keys to success in retail stores, such that little compliance with service frequency and numeric distribution coverage for its territory exists.

Accounting for nearly 15% of the volume, the other major distribution channel is to sell direct to end consumers through chain markets, to which products are channeled directly from the plants. For this channel, merchandising strategies related with packaging, display, pricing, special offers etc. are becoming more important and intense. This channel will create significant volume opportunities in the future, but numeric distribution coverage is still the key to effectively service small retail stores in the distributor channel.

Distributors and chain markets use computer ordering system to communicate with order processing center in the central warehouse. The buyer's shipment representative manages the order fulfillment process. (S)he receives and monitors the product availability data contained in the database. The shipments to distributors and chain markets are based on the forecasts and target safety stock levels. Order lead time is short such that orders from distributors and chain markets are usually placed with one or two day lead time. Together with the fact of short order lead time, an increasing number of SKUs with short life cycle and scattered distributor locations underline the importance of accurate forecasts for the manufacturer since sales forecasts drive manufacturer's production planning.

The competitive strategy of the organization is that products are processed and delivered into stock according to sales forecast-driven production planning, and in turn customer orders are filled from the finished goods inventory. Production planning is based on forecasted manufacturer-to-buyer sales data provided by the buyer, whereas order fulfillment processes respond to orders from distributors and chain markets. Actual sales data of distributors and chain markets are not being pushed upstream to the manufacturer, and in turn operations at the manufacturer level do not avoid the bullwhip effect. The manufacturer is in the phase of transition from production focus to customer focus. In the past, production-focus has led the manufacturer to emphasize manufacturing-cost reduction instead of emphasis on filling periodical customer orders. Being affected by this situation, production-planning environment is structured by the organization's strategic response both to the interests of the buyer and to the actions of competitors.

After examining the organizational environment and corporate policies influencing production planning activities, forecasting and aggregate production planning activities involved in managing the master scheduling are described in the following sections.

2. 2. An Overview of Forecasting Activities

While the industry is becoming more supply chain driven and hence more competitive, efficacy of forecasting activities and integration of forecasting process with production planning represent a fundamental need for the group’s success. Therefore, an understanding of the forecasting activities and environment is essential to the first stage of the manufacturer’s production planning operations – determining master production schedule.

The buyer is the solely responsible of the forecasting activities and hence manages the forecasting system. They determine sales forecasts and communicate to the manufacturer at an individual SKU level. The forecast horizon, the length of the horizon for which nontrivial forecasts are available, comprises 3-month period. Sales forecasts are estimates of the quantity distributors and chain markets are going to order from the buyer. In fact, the structure of the forecast horizon is two-fold. The buyer considers 3-month forecast horizon and determines monthly sales forecasts accordingly when carrying out the main forecasting activities. As a consequence of the existing performance evaluation and incentive alignment system, the buyer strives to achieve monthly target of sales volume rather than weekly target. Whereas, the manufacturer considers 12-week planning horizon and they require weekly sales forecasts over the 12-week forecast horizon. The existing performance evaluation system requires the manufacturer to satisfy weekly customer orders from the finished goods inventory. Another notable characteristic of the environment is that the manufacturer bears the full cost to guarantee reliable supply to market by holding safety inventory although the buyer is the solely responsible of the quality of sales forecasts. Therefore, the way of handling safety inventory at this stage implies that a complete decentralized inventory control fails in this case. In the view of this, it can be said that the degree of centralization for inventory control at this stage is somewhere between a complete centralized control and a complete decentralized control.

This discrepancy in horizons of the buyer and manufacturer results in two-stage forecasting process. The first stage, which is the main stage, comprises monthly operations by which 3-month rolling sales forecasts are developed. Figure 5 illustrates the 3-month forecast horizon. The arrow labeled as STAGE-1 represents the decision point at which forecasting activities involved with the first stage of forecasting process are performed. At the end of month-0, monthly forecasts for the third month of the forecast horizon are generated for the first time, and monthly forecasts for the first and second months are updated on condition that percentage of update must be within the pre-specified percentage revision limits. An intermediate stage occurring between the first and second stages of the forecasting process transforms monthly sales forecasts into weekly forecasts by using an allocation scheme. The allocation scheme considers the fact that if a price change is going to happen in the forecast horizon then sales pattern before and after the change is affected since distributors and chain markets decide on how much to buy today by taking into account any future price-advantage. The intermediate stage plays the main part in trying to eliminate the effect of that discrepancy between the buyer and manufacturer's horizons. However, the allocation scheme may not always reflect the realities of demand pattern over the weeks of month. The second stage of the forecasting process includes weekly operations by which weekly forecasts are repeatedly revised and updated by the buyer. Figure 5 illustrates the 12-week forecast horizon. The arrow labeled as STAGE-2 denotes the time at which weekly forecasts are made. At the beginning of week-0, weekly forecasts for each period of the 12-week forecast horizon are generated and updated as long as percentage of update is within the pre-specified percentage revision limits. The revision limits may be violated in the case of reasonable unexpected demand conditions, which account for the tail probabilities of demand and forecast distributions.

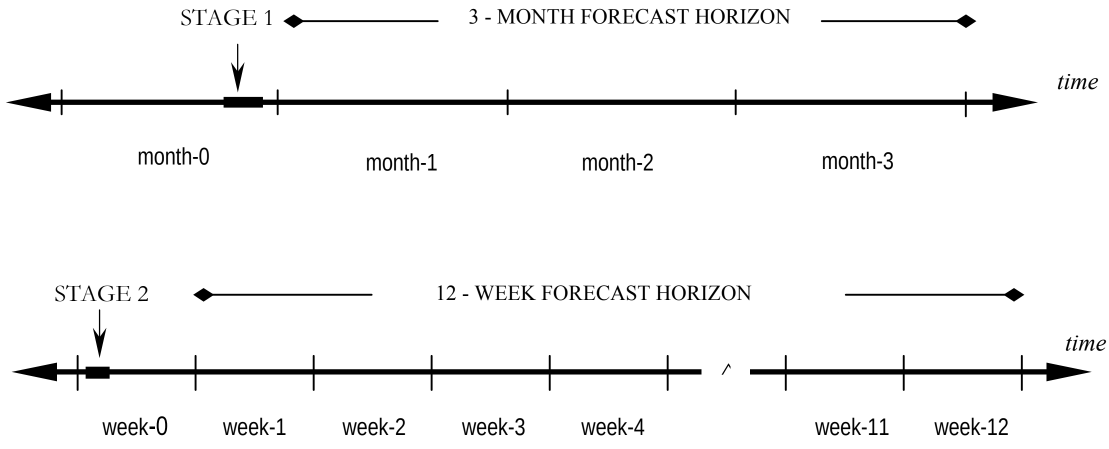

Figure 5 3-month and 12-week forecast horizons

In the first stage of the forecasting process, both analytical methods and judgment are considered in order to generate monthly sales forecasts. Statistical forecasts from analytical methods are based on historical three-year buyer-to-distributor/chain markets sales data in monthly level of detail captured in the PIMS database. The buyer generates statistical forecasts. And, judgment is incorporated during judgmental forecasting process, in which regional offices and distributor supervisors determine their judgmental sales forecasts in monthly level of detail for chain markets and distributors, respectively.

An extranet system, called distributor management information system, has been set up, which is used to process judgmental sales forecasts and to share sales activity information with regional offices and distributors. Before judgmental forecasting process, the buyer communicates sales activity information to regional offices and distributors. Sales activity information consists of planned price changes, trade promotions, product advertisements, and products to be produced over the planning horizon. Beside sales activity information, distributor supervisors consider the following factors to determine their monthly judgmental forecasts:

- Regional and seasonal factors
- Historical sales data
- Sales support activities performed by the buyer (ads, promotion, incentive etc.)
- Sales support activities performed by distributor (sales personnel, vehicles etc.)
- Judgmental estimates of sales personnel
- Inventory status
- Rival activities within its territory
- Information about new products

Judgmental forecasts generated by regional managers for chain markets and distributor supervisors for distributors are fed into distributor management information system. Regional managers monitor the judgmental forecasts of distributor supervisors to revise and approve them. That is, this practice provides regional managers with managerial intervention with the claim of improving the quality of forecasts generated and of incorporating expert opinion. At the end of the judgmental forecasting process, the buyer consolidates all judgmental forecasts generated into one judgmental forecast figure for each SKU.

Using statistical forecasts and consolidated judgmental forecast for each SKU, forecast intervals for each month of the forecast horizon are constructed. In the view of forecast intervals and budgeted sales the buyer makes three-month ahead forecasts for the first time and updates one-month ahead and two-month ahead forecasts.

The buyer allocates these monthly final forecasts to sales regions based on the pre-specified allocation percentages. Each regional office, in turn, allocates its own portion of monthly forecast to distributors operating within its territory. Allocation percentages are determined by considering historical regional sales and judgmental forecasts. Key account managers at regional offices, who manage the chain market channel within their sales territories, determine aggregate forecast amount to be allocated to the chain markets channel as a whole in their territories. Allocated monthly forecasts by region and by distributor are then treated as respective sales quotas for regional offices and distributors, on which performance evaluation and incentive alignment systems are based.

In the second stage of the forecasting process, weekly sales forecasts generated in an individual SKU level for the 12-week planning horizon are shared with the manufacturer. The manufacturer reviews the weekly sales forecasts and performs rough-cut capacity planning to determine whether they have sufficient production line capacity, man-hour capacity, raw material availability etc. to respond to these weekly forecasts. Possible changes in the forecasts are negotiated with the buyer to attain their feasibility. As a result of this interaction, actually, the nature of forecasts might change, and in turn forecasts just become respective production orders. The output of the forecasting process defines the expected weekly sales and it is used to support weekly production planning decisions of the manufacturer. The manufacturer takes weekly feasible forecasts as respective production orders for the current planning cycle, on which weekly aggregate production planning - master production scheduling is based. The master production scheduling model requires weekly sales forecasts and inventory safety level estimates for product items. The most important planning decisions are the weekly aggregate production plan, scheduling of raw materials acquisitions, planning of inventory and capacity, and hiring/layoff decisions.

The manufacturer develops an operational plan every week to determine production quantities for its products. Operational level day-to-day operations are quite critical for the continuity of daily production processes. These are operations such as inventory control, receiving and shipping, scheduling, and allocating workers to jobs. They are based on the master production schedule, a weekly production plan determining general characteristics of the production, and the other decisions made by the top level. Ideally, production planning system helps the manufacturer to design and improve the coordination and arrangement of processes, and helps devise logistics so that materials flow from suppliers, through processing plants, and to distributors and finally retail stores. Given all known data, the manufacturer tries to answer the question of what is the minimum cost solution that determines production, inventory and workforce levels to meet weekly forecast requirements.

2. 3. Production Inventory Management System – PIMS and Master Production Scheduling Problem

An increasing number of companies require some sort of enterprise resource planning - ERP system to coordinate its operations and mathematical programming models to support fact-based decision-making. Such models assist them in making better decisions about capacity and production planning, inventory management, raw material acquisition etc. Similarly, the management of the group had been concerned for some time about the lack of a formal integrated framework for production planning that facilitates coordination and collaboration among different business functions. To mitigate this problem, the group has launched an in-house developed enterprise resource planning system, called Production Inventory Management System – PIMS which provides a sort of model-based production inventory management system. The aim of PIMS, therefore, is to develop an integrated planning framework for the manufacturer-buyer setting that can be implemented with in-house developed systems and modules. In this section, we give a general description of manufacturer's planning process with particular attention to its master production scheduling (MPS) problem from the perspective of PIMS. Some details of the MPS model are given in Chapter 5.

PIMS helps the manufacturer to manage its operational activities and support some tactical decisions to satisfy orders from the inventory with the most economical commitment of all production means and capacity (production line capacity, manpower etc.). Though further development efforts continue, PIMS is tailored to solve company-specific problems for particular functional areas of business.

PIMS is basically compiled in series of modules, each covering particular functional areas of the business such as sales planning, production planning and inventory management. Modules are either stand-alone or combined with other modules to give an integrated system, and they are usually able to operate on IBM AS/400 systems. Production planning function is managed mainly through Production Planning Module - PPM and through other modules of PIMS, which are custom-tools to complement that function. PPM assists the manufacturer to make its production operation more efficient by providing tools for production control over aggregate production plans, shop floor schedules, and manpower resources. It can be said that PIMS has facilitated the transition from the purely decentralized production planning process to a more centralized process guided by PPM.

Production planning module - PPM is comprised mainly of three multi-period, large-scale mathematical programming models (master production scheduling model, shop floor scheduling model and manpower assignment model) to direct operational weekly production, inventory, raw material acquisition, and workforce decisions. PPM is employed weekly for operational planning and for dealing with tactical issues like capacity investment / reassignment, hiring - firing decisions, and overtime / undertime decisions.

Master production scheduling - MPS model of PPM is to define production requirements in weekly time buckets to meet forecasted demands over the 12-week planning horizon. Being the highest level of operational production planning in the integrated structure, the MPS model focuses on individual products and deals with weekly production, inventory levels and workforce usage under the constraints of the manufacturing operation. Once a master production schedule is developed, it becomes the master plan and ultimately drives every activity in the manufacturer -

from timely acquisition of raw materials to customer delivery. Master production scheduling model is formulated as a linear programming model and is implemented via a 12-week rolling planning horizon. The MPS problem was modeled into a GAMS code, and is solved using Cplex solver on the GAMS server.

The overall objective of MPS is to allocate all the manufacturing resources in an efficient manner while satisfying the forecasted demands over the planning horizon. The objective function of MPS model calls for minimizing combined cost of:

- underachievement of beginning finished products inventory requirements, which are needed to cover weekly forecasts provided by the buyer
- underachievement of target safety stock level requirements, which are needed to cover unexpected sales
- holding inventory per period
- and overtime man-hour usage.

Unsatisfied portion of weekly forecast and unsatisfied portion of target safety stock level for each SKU at each period of the 12-week planning horizon represent the respective shortfalls in the desired inventory levels for that SKU at that period. The objective function is the weighted sum of the above four different objectives. Weights are used to give them some sort of priority while determining the optimal solution. In the view of this, the role of the objective function, therefore, can be described as priority-setter, and hence, the optimal value of the objective function is of no concern to our research in a sense. The highest-priority objective is to minimize the costs associated with shortfalls in weekly forecast requirements and is therefore dealt with first. To minimize the shortfalls in target safety stock levels is the next highest-priority objective.

The MPS model is based on the policy of preparing, at the beginning of each period of the 12-week planning horizon, beginning finished goods inventory as much as that period's forecast provided by the buyer plus target safety stock level determined by the manufacturer for each product. This policy ensures that the manufacturer meets forecasted demand and target safety stock level for all items in its entirety as much as possible.

At the manufacturer's production facility considered for computational study, there are eight production lines. There is a set of bakery products associated with each production line, which are bounded by the technological features/capabilities of the line. All the production lines are capable of producing multiple products. There is finite production capacity and finite storage capacity for processed products. For each period of the 12-week planning horizon, the major production and inventory related decision variables associated with master production scheduling planning include:

- production quantities for each SKU at each period of the 12-week planning horizon
- projected on-hand inventory levels for each SKU at the beginning of each period of the 12-week planning horizon
- shortfall in weekly forecast requirement for each product at each period of the 12-week planning horizon

- shortfall in target safety stock level for each product at each period of the 12-week planning horizon
- and other manpower and production line related decision variables

Production time-hours usage and man-hours usage are constrained by respective resource availabilities. Production line constraint ensures that each week's production does not exceed the limit set by the total number of available production hours. Constraints on man-hours availability ensure that regular and overtime man-hours at each period do not exceed the allowable amounts.

Master scheduling model makes production decisions for each SKU at a weekly level of detail, and employs the standard production, inventory, and demand recursion. This inventory balance equation determines the beginning inventory status of each SKU for each period of the 12-week planning horizon from the beginning inventory status of the previous period plus production during the previous period minus forecasted demand for the previous period.

Figure 6 shows MPS plans made at two successive periods. The arrows denote the time at which MPS plan is made for the next 12-week planning period. At the beginning of period s, the buyer provides the manufacturer with weekly forecasts for each period of the 12-week planning horizon starting period $s + 1$. Based on these forecasts, the manufacturer employs the production planning module - PPM and hence the master production scheduling - MPS model to maintain a balance between the anticipated supply plan for products and future demands. The resulting master schedule drives the material and capacity planning, and it becomes the basis for the subsequent manufacturing operations. However, only the first period's production requirements are implemented. Lead-time for raw material acquisitions is required therefore the resulting master plan is frozen one week ahead. As the time for production, period $s + 1$, approaches, labor and material are committed. Production is made one period before the realization of demand. Therefore, amount corresponding to demand of period $s + 2$ is produced during period $s + 1$ and is made available in finished products inventory at the beginning of period $s + 2$. And in turn, orders placed by distributors and chain markets during period $s + 2$ are filled from that beginning inventory. This policy reflects realistic lead-time constraints in terms of raw material acquisitions, production and competitive factors.

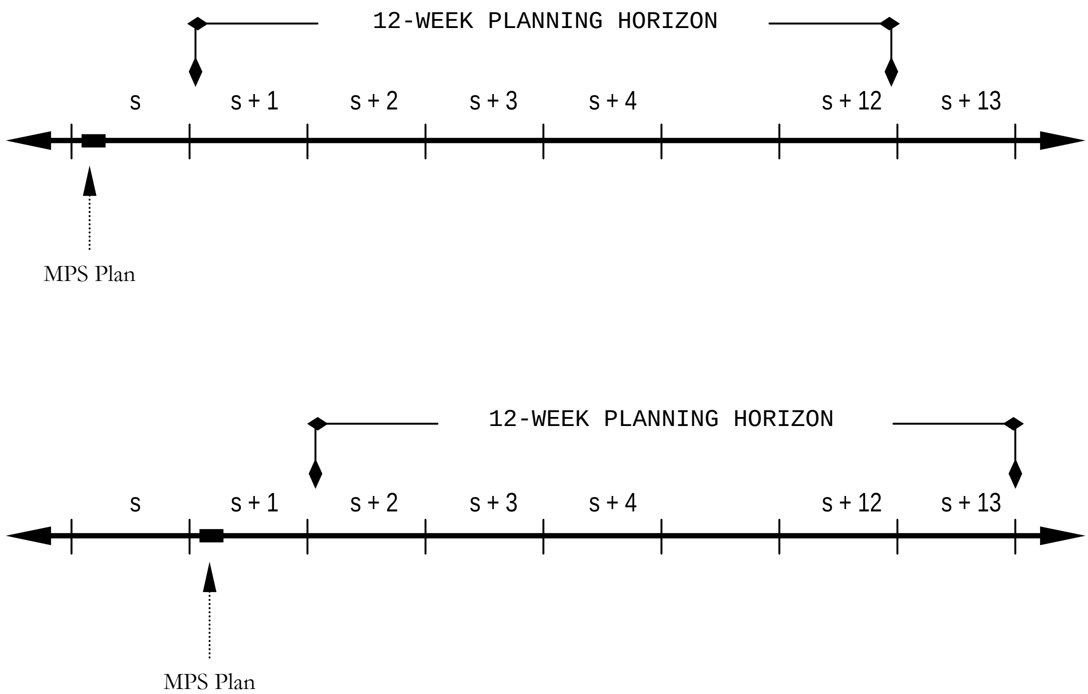

Figure 6 Master production scheduling cycle

Similarly, at the beginning of period $s + 1$, the manufacturer has a forecast vector containing forecasts for each period of the 12-week planning horizon, and in turn, master production plan for the next 12-week planning horizon covering week $s + 2$ and beyond is made. Production of period $s + 2$ is prepared for demand of week $s + 3$ and hence put into finished products inventory ready for shipment at the beginning of week $s + 3$. Orders placed by distributors and chain markets during week $s + 3$ are filled from finished products inventory.

2. 4. Observations and Discussion

In this section we identify and discuss some of current operating problems directly related with the research of the thesis, the causes of the problems and the areas that improved master scheduling can address.

The first observation is concerned with the forecasting behavior of the buyer. The buyer updates weekly forecasts for the next 12-week planning horizon at the beginning of each week. They, however, focus on updating only those weekly forecasts for the first 4-5 weeks of the 12-week planning horizon. Delayed forecast updates lead to significant forecast changes as the time for order execution approaches. This practice leads to inefficiencies and an extra burden to the production system in the sense that decisions concerning periods beyond the first 4-5 weeks of the planning horizon do rely on significantly inaccurate, misleading forecasts. Consequently, this practice cause shortsighted decisions concerning workforce resources, raw material acquisitions etc., which result in extra costs, extra hours and unsatisfied customer orders.

Another notable observation concerning the forecasting behavior is that there is a considerable influence of budgeted sales and monthly forecasts on the operational weekly forecasts, which drive the master planning process. Budgeted sales are generated to create business plan for the next financial year and to create budget plan for marketing etc. Coupled with the influence of the existing performance measurement and incentive alignment system, operational forecasts, therefore, have being adjusted to attain the respective sales quotas of the buyer and distributors. As mentioned before, the starting point of the forecasting is monthly forecasts over the 3-month forecast horizon, which are mainly based on judgment and experience, and therefore, may be manipulated to achieve performance targets. Because allocated monthly forecasts by region and by distributor are treated as sales quotas, on which performance evaluation and incentive alignment system are based. Consequently, the primary concern of distributors becomes achievement of these monthly sales quotas rather than product focus, service frequency and numeric distribution coverage. This leads to insufficient emphasis on potential market volume of some products. Distributors under this incentive system focus only on sales of some products overmuch, especially those being more likely to be sold, and are not concerned with the sales of the remaining. This results in operational inefficiencies in some cases. Integrating the evolution of operational weekly forecasts into master scheduling can mitigate the influence on production planning of this practice given the existing performance measurement system.

Production quantities requested by the MPS are based on manufacturing economics and forecasts as well as target safety stock levels. To implement the MPS in a rolling horizon context, the one-week freeze period is determined. However, forecast errors are so dramatic that it is difficult to maintain the stability of the resulting master production schedules from one period to the next. Hence amounts left over in stock or rapidly depleting stock may cause changes to the MPS. In addition, shortfalls in the beginning inventory for each product are being satisfied from the running week's production, which is prepared for the next week's customer demand. On the other hand, due to revisions made to the MPS plan, up and downs are observed for the first period of the planning horizon. This fact impacts the efficacy of the freeze period because changes have been made to master plan within the freeze period. To design aggregate plans that follow closely the up and downs of actual demands is not practical because it is usually too costly to vary output levels significantly from one period to the next**.** Clearly, this practice unbalances and overburdens the production system and also leads to shortage in the following weeks' beginning inventory or to extra working hours and hence extra costs.

Being designed to protect from demand variations when actual sales are different from predicted sales, target safety stock levels for the existing master planning model have being calculated using the historical differences between one-week ahead forecasts provided by the buyer and actual shipments to distributors and chain markets. The forecasts and orders are adjusted along the way as actual demand fluctuated from the forecast. In any week, the buyer can ship any amount of a product up to the sum of its one-week ahead forecast and target safety stock level. Therefore, the buyer can manipulate the level of target safety stock by shipping less or more than its one-week ahead forecast to achieve its monthly sales quota. Employing the proposed method of establishing target safety stock levels can mitigate this drawback. The proposed method calculates target level by considering estimated variances and covariance of historical forecast updates made by the buyer. Forecast evolution modeling represents the underlying forecasting behavior of the buyer better than the simple forecast error distribution. Furthermore, integrating the multiplicative form of martingale model of forecast evolution with safety stock calculation provides more responsive safety stock method in the sense that it adjusts the target safety stock level each time a new forecast vector is available to the manufacturer.

Note that this thesis study does not help in generating better forecasts. However, it takes into account the evolution of forecasts and uses that information. Hence, we expect to have even better results when better forecasting practices are applied. By characterizing the behavior of forecasting system and by adapting it to the existing planning activities, the manufacturer is able to react more efficiently to forecast changes and not have to rely on costly reactions to late-changing forecasts. This means that by capturing the possible evolution of forecasts, the company could obtain greater accuracy at an earlier point in time. The effort here is not to attempt improving accuracy of the forecasts. Hence, forecasts provided by the existing forecasting mechanism are considered as outcomes of the underlying stochastic process.

CHAPTER 3

LITERATURE SURVEY

Every business has integrated systems of facilities, equipment, processes, people and information to get the finished products to the customer. Challenge is to design, optimize, and redesign these systems. Lee and Billington (1992) provides a list of common pitfalls in current supply chain management practices and some corresponding opportunities. They state that as the complexity of logistics network increases organizations could more likely gain operational efficiencies by focusing on inventory. To overcome this challenge the correct use of performance measurement system for the supply chains represents a fundamental need for the organization's success.

FMCG supply chains are complex and dynamic in the sense that they involve multiple players, they are characterized by constantly evolving relationships between players, they support a huge number of stock keeping units, and they use a mixture of manufacturing techniques like make-to-stock and make-to-order to fulfill orders. The current challenge facing organizations is the ability to have effective integration and coordination across the supply chain. It is critical therefore to focus on the performance of the supply chain as an integrated whole, rather than as a collection of separate processes or companies. Hence the question of whether the existing performance evaluation system is compatible with the supply chain management initiatives should be explored. Lee and Whang (1999) studies performance measurement and incentive alignment for decentralized multi-echelon supply chains where each entity maximizing its local objectives. They define a performance measurement scheme as a set of corporate rules such as accounting methods, transfer pricing schemes, performance metrics and various operational constraints. Furthermore, they state that such a set of corporate rules is one way in order to mitigate the problem of incentive misalignment in a decentralized supply chain. In our case, PIMS is governed by a set of corporate rules, may be viewed as a contract, expressing relationships between the manufacturer and buyer.

Some studies in supply chain literature indicate that buyer-manufacturer relationships are becoming more dependent on factors like delivery performance, flexibility in contract, and commitment to work together, as opposed to traditional relationships based mainly on cost. Hausman et al (2000) investigates the manufacturing-marketing interface from a behavioral perspective and explore strategies, conflict and morale on business performance. They conclude that an increased emphasis on the importance of marketing and manufacturing strategy improves functional harmony, in turn influencing competitive position and profit performance. They illustrate that the new emphasis on supply chain management may be indicative of this phenomenon.

Lee et al. (2000) analyzes a manufacturer and a retailer supply chain from the information sharing perspective and its mitigating impact on bullwhip effects. They conclude that information sharing could provide significant inventory reduction and cost savings to the manufacturer with significantly correlated demands over time. For example, sharing capacity information permits better planning and coordination between trading partners and helps reduce the gaming that occurs in product shortage situations. Lee, Padmanabhan & Whang (1997) explores four rational factors leading to create the bullwhip effect.

Because forecasting is fundamental to business management, there are also many reading materials about forecasting outside of the supply chain literature. The concept of predictability, sales forecast modeling and conditional forecasting to predict future demands has received enormous attention in the research literature. In this study, the concept of unpredictability and its implications for sales forecasting are discussed. Related to the concept of predictability, similar studies, which have considered the problem of forecast evolution and its impact on inventory cost, have been made in the supply chain literature (for example, Heath and Jackson 1994, Graves et al. 1998, Çakanyıldırım and Roundy 1999, Kaminsky and Swaminathan 2002).

Sales forecasts drive planning activities in supply chain production-inventory systems. Analyzing how these forecasts evolve over time can lead to significant insights into the impact of forecast accuracy on supply chain costs. Note that forecasts do not necessarily become more accurate as they are updated and hence such forecast churning could cause inefficiencies if the manufacturer relies only on the wrong forecast update.

By characterizing the behavior of forecasting system and by adapting it to the existing planning activities, the manufacturer is able to react more efficiently to forecast changes and not have to rely on costly reactions to late-changing forecasts. This means that by capturing the possible evolution of forecasts, the company could obtain greater accuracy at an earlier point in time. The effort here is not to attempt improving accuracy of the forecasts. Hence, forecasts provided by the existing forecasting mechanism are considered as outcomes of the underlying stochastic process.

Several classes of forecast evolution models exist in supply chain literature. Scarf (1959) and Azoury (1985) study Bayesian models for forecast revisions in an inventory setting. Later, Bitran et al. (1986) presents Bayesian forecast updates for style goods under capacity restrictions and adapts to a stochastic mixed integer programming formulation

Fisher and Raman (1996) introduces a response-based, two period stochastic production planning problem from the fashion goods industry. After initial demand has been observed, an improved forecast drives the decision of how much additional amount of products to produce at different points during the season. In this environment, the manufacturer needs to determine production quantities in two periods, with lower production costs in the first period but improved forecast information available in the second.

Kaminsky and Swaminathan (2002) presents the multi-period demand case of the study by Kaminsky and Swaminathan (2001), which models a forecasting process getting refined over time as new information is available. In their study, forecasts are represented by a series of bands and forecast evolution process is introduced based on these bands. Subsequent forecasts have a smaller band and are contained within the band defined by previous forecasts as time elapses. They adapt the procedure to capacitated production planning environments and develop heuristics which utilize knowledge of demand forecast evolution. They also consider the impact on the system of improved forecast updates.

Hausman (1969) considers a recurring sequential decision problem and new improved forecasts before each decision stage. He treats ratios of successive forecasts as independent lognormal variates and assumes that forecast evolutions have the quasi Markovian property of no memory. To illustrate the approach, a dynamic programming formulation with the current forecast being the state variable is developed.

Hausman and Peterson (1972) considers a stochastic forecast revision process for a capacity constrained, multi-product production scheduling problem. They assume that ratios of successive forecasts of total orders for a seasonal product are mutually independent lognormal variates. They formulate the problem as a dynamic programming problem and consider alternative heuristic approaches for solving the problem.

Graves et al. (1986) considers a set of aggregate production smoothing models to analytically optimize the tradeoffs between inventory requirements and production smoothing considering demand variability and forecast uncertainty. For the case of an uncapacitated system with i.i.d. demand, they model the monthly forecast revisions of the aggregate forecast process as a single item version of the martingale model of forecast evolution (MMFE). This paper is considered to be the first MMFE paper in the supply chain literature. Hausman (1969), Graves et al. (1986, 1998) and Heath and Jackson (1994) are the main contributory papers for the development of the MMFE model and provide detailed discussion and motivation of the MMFE.

Heath and Jackson (1994) introduces an MMFE for a multi-item system. They study the impact of forecast error on cost and customer service by taking into account correlations in demands across products and time periods. They adapt the MMFE as forecast updates generation procedure to an existing LP model of production and distribution system and conduct a simulation study to analyze the relationship between safety stock levels and improvement in forecasts.

In his study that considers production inventory system from a different perspective by incorporating forecasts explicitly in production inventory systems with uncorrelated demand, Güllü (1996) models the effects of forecast evolution on system performance by considering a special case of the MMFE. He compares a single item system that employs one period ahead demand forecast with the one that does not and showed that using forecast information results in better system performance in terms of expected costs and inventory levels. Güllü (1997) considers a two-echelon allocation problem consisting of a central depot and multiple retailers with the aim of determining a depot allocation policy to minimize the system-wide costs. Finally he demonstrates the value of forecast information. Toktay and Wein (2001) considers an MMFE demand process and model the effects of forecast evolution on system performance for discrete-time make-to-stock queues under heavy traffic assumptions for a capacitated single server.

In the buyer-manufacturer setting considered, the forecast volatility problem studied in the above work leads to the question of how sufficiently accurate the forecasts provided by the buyer is in order to justify the manufacturer acting on it in master scheduling. To implement any basic MPS model in the rolling planning horizon context, demands are assumed to be known with certainty, and accordingly, the standard recursion of inventory, production and demand is employed. Uncertainty in the forecasts is typically accommodated afterwards by adding safety stocks to production requirements. However, the control and maintenance of safety stocks are a problem common to all manufacturers. According to every manufacturer's own situation, different methods of safety stock determination are applied to manage production plans. Forecast error variability and demand variability are used in many settings as better methods to determine safety stock levels. The manufacturer is one of the companies to use forecast error-based method to establish target safety stock levels. The effective and efficiency of a safety stock method depends on a clearer understanding and then proper reflection of the nature of uncertainty in forecasts. This understanding determines the relationship of the safety stocks with the uncertainty in forecasts (how safety stocks are related to the error in forecasts).

Baker and Peterson (1979) describes a general framework for analyzing rolling schedules and examines analytically a fundamental quadratic cost model for the effects of the uncertainty in forecasts, the periodicity of demand and the length of the planning interval. Enns (2002) illustrates the importance of considering forecasting, safety stock and planned lead time in evaluating the MRP performance, and studies the effects on master scheduling performance of demand uncertainty and forecast bias in a batch production environment. By using an integrated MRP planning and execution test bed, Enns evaluates the use of safety stock and inflated planned lead time to accommodate for forecast error. Enns concludes that an increase in the forecast-to-demand ratio deteriorates the MPS due date performance and increases in demand uncertainty have mixed effects on MPS due date performance.

Campbell (1995) outlines two popular safety stock determination methods (the constant cycle service level method and the constant safety stock method) and develops a third method which distributes safety stock in an optimal manner over the planning horizon and is used as a basis for evaluating the performances of the other two safety stock methods. He considers the relationships of the methods with order interval length, lead time, and forecast errors. He applies the third method to a basic master production problem for a single end product with no limits on production capacity. The problem assumes a finite planning horizon so that a rolling horizon is not considered. At the end of experimental analysis of safety stock methods, he concludes that the constant safety stock method performs better than the constant cycle service level method. And he also demonstrates that the optimal safety stock method results in measurable safety stock savings. Zinn and Marmorstein (1990) presents a simulation to compare the two alternative methods of determining safety stock levels, referred to as the demand system and the forecast system. The demand system depends upon the variability of demand whereas in the forecast system the required safety stock level depends upon the variability of demand forecast errors. They quantify the safety stock required under the two systems and discuss the managerial implications of the simulation results.

In this thesis, we focus on a set of bakery products that relate to each other in terms of package design, brand name, production line etc. The demand pattern is altered to some degree through pricing, promotion, backlogs and launching new products. In master scheduling, the manufacturer attempts to satisfy demand by manipulation of the size and combination of the variables in control. The main issues addressed in the thesis are firstly, to investigate what the methods are for integrating forecast evolution modeling into manufacturer's master production scheduling model with the aim of better inventory management; and secondly and more precisely to develop a method of establishing target safety stock levels for different products in different planning periods that accommodates the historical demand and forecast correlations captured from the estimated variance-covariance matrix of forecast updates.

Safety stock is established to accommodate the variability or exceptional supply and demand conditions. Hence correct use of safety stock in the master scheduling is important. The existing safety stock method captures the variability through the forecast errors. The proposed method, on the other hand, captures it through the standard deviation of demand, which is calculated based on the fact that uncertainty in forecasts resolves in each period of the 12-week planning horizon.

CHAPTER 4

APPLICATION OF FORECAST EVOLUTION MODELLING

4. 1. Martingale Model of Forecast Evolution

Having outlined the operating environment of the manufacturer-buyer setting, it is now clear that success in the industry depends on flexibility of the production to react to evolving business conditions on a frequent basis together with adequately accurate sales forecasts on which tactical and operational decisions rely on. Forecasting has always concerned itself with predicting future demand and then with the management of variances between forecast and actual demand. As time evolves, forecasting environment changes and additional information becomes available. Using additional information, new forecasts for products are made periodically. In order to characterize the underlying forecasting behavior, this periodical modification activity can be described as a forecast evolution model.

As mentioned before, sales forecasts of the buyer are a combination of judgment and statistical estimates. Qualitative part of forecasts is effective in the sense that it may reflect knowledge of events that have not been observed in the past but are expected in the future; knowledge of advertising, promotional plan and competitor's activities;

knowledge of recent events whose effects have not yet been observed in time series data. Hence, judgmental forecasts are useful but it is very difficult and not practical to fit a time series model on them. To characterize and model behavior of the forecasting system having qualitative estimates, Martingale Model of Forecast Evolution - MMFE technique has desirable properties and its assumptions are consistent with the realities of the most real life cases.

Incorporating forecast evolution modeling into the existing planning operations promises to assist the organization in several ways: Martingale Model of Forecast Evolution technique (1) in general provides an effective probabilistic method of distilling huge amounts of historical data on forecasting activities into information that is useful for operational activities like inventory management and production planning; (2) provides a method by which the relationship between variability in forecast updates from one period to the next and the system performance can be captured and characterized. In this section, we provide an overview of some of the issues that arise in formulating MMFE, which can then be used as the basis for examining relationships between forecasts.

For simplicity, we limit our case and focus on a set of products, which are related to each other in terms of package design, brand name, production line etc. At the manufacturer production facility, approximately 60 SKUs are being processed, packaged and inventoried on a total of eight production lines. The production lines are capable of producing multiple products. However, there are a certain number of products associated with each production line, which are bounded by the technological features/capabilities of the line. There is finite production capacity and finite storage capacity for processed products. The products on the same production line, therefore, compete for the same processing and packaging capacity. Although we will solve our MPS formulations and carry out computational study and comparison for the chosen products, our method of determining target safety stock levels will be generic so that it will supply output for the other products as well.

In order to characterize the existing forecast evolution the notation we employ is as follows:

- Consider for each product $p \in \{ 1 , \dots , P \}$ a univariate stochastic process $\{ D_t^p \}$ for a sample of $t \in \{ 1 , \dots , T \}$. The values that are tracked are described as realized demands in period t for product p.

- $F^p_{s,s+n}$ represent forecast generated at period s for the amount demanded at period $s + n$ for $n = 0, 1, 2, \dots, M$ and for product $p = 1, \dots , P$ where M (= 12) is the length of the horizon for which nontrivial forecasts are available. When $n = 0$, $F^p_{s,s}$ denotes the actual demand realized at period s. For $n = M + 1, M + 2, \dots, \infty$ $F^p_{s,s+n}$ represents the expected demand, μ^p.

At the beginning of period s, we have a random vector of forecasts generated in that period for each product p under consideration:

$$\mathbf{F}^p_s = [F^p_{s,s}, F^p_{s,s+1}, F^p_{s,s+2}, \dots, F^p_{s,s+M}, \mu^p, \mu^p, \mu^p, \dots]$$

At the start of any period s, forecast modification activity is carried out for the next M = 12 periods, but not for $n = M + 1$ and beyond. For instance, $F^p_{s-1,s+2}$ is replaced by $F^p_{s,s+2}$ in period s as the best estimate for the quantity demanded in period $s + 2$. This modification activity can be described as a finite vector of forecast updates on which the perceived forecast evolution is based. Forecast updates are defined as follows:

- $\varepsilon^p_{s,s+n}$ represent modification from period $s - 1$ to period s in the forecast generated in period $s - 1$ for the amount demanded in period $s + n$. Therefore, in any period s, we have a finite vector of forecast updates generated in that period for each product p such that random forecast update vector, $\boldsymbol{\varepsilon}^p_s$, contains the updates made from period $s - 1$ to period s in the forecast vector, $\mathbf{F}^p_{s-1}$, generated in period $s - 1$.

$$\boldsymbol{\varepsilon}^p_s = [\varepsilon^p_{s,s}, \varepsilon^p_{s,s+1}, \varepsilon^p_{s,s+2}, \ldots, \varepsilon^p_{s,s+M-1}, \varepsilon^p_{s,s+M}]$$

Given the initial vector of forecasts $\mathbf{F}^p_0 = [F^p_{0,1}, F^p_{0,2}, \ldots, F^p_{0,M}, \mu^p, \mu^p, \mu^p, \ldots]$, any prediction vector $\mathbf{F}^p_s$ generated at period s can be obtained by modifying the entries of prediction vector $\mathbf{F}^p_{s'}$ generated at period $s' < s$ by random updates. For example, the entry $F^p_{s,s+1}$ of the forecast vector of the period s equals to a function

$g\left(F^p_{s-1,s+1}, \varepsilon^p_{s,s+1}\right)$ of the entry $F^p_{s-1,s+1}$ of the forecast vector of the previous period s - 1 and some random variable $\varepsilon^p_{s,s+1}$.

$$\mathbf{F}^p_s = \begin{bmatrix} F^p_{s,s} \\ F^p_{s,s+1} \\ \vdots \\ F^p_{s,s+M-1} \\ F^p_{s,s+M} \end{bmatrix} = \begin{bmatrix} g\left(F^p_{s-1,s}, \varepsilon^p_{s,s}\right) \\ g\left(F^p_{s-1,s+1}, \varepsilon^p_{s,s+1}\right) \\ \vdots \\ g\left(F^p_{s-1,s+M-1}, \varepsilon^p_{s,s+M-1}\right) \\ g\left(\mu^p, \varepsilon^p_{s,s+M}\right) \end{bmatrix}$$

where $g\left(F^p_{s-1,s+n}, \varepsilon^p_{s,s+n}\right)$ is a function of $F^p_{s-1,s+n}$ and $\varepsilon^p_{s,s+n}$.

The MMFE is one way of modeling this forecast evolution mechanism. It is a powerful and descriptive technique, which characterizes the above random vectors and the relations between them. The MMFE defines the above function $g(*,*)$ in two different ways such that forecast evolution mechanism can be explained by additive or multiplicative operations.

Additive model describes forecast evolution mechanism by an addition operation so that random forecast update vector at period s, $\boldsymbol{\varepsilon}^p_s$ becomes:

$$\begin{bmatrix} F^p_{s,s} \\ F^p_{s,s+1} \\ \vdots \\ F^p_{s,s+M-1} \\ F^p_{s,s+M} \end{bmatrix} - \begin{bmatrix} F^p_{s-1,s} \\ F^p_{s-1,s+1} \\ \vdots \\ F^p_{s-1,s+M-1} \\ \mu^p \end{bmatrix} = \begin{bmatrix} \varepsilon^p_{s,s} \\ \varepsilon^p_{s,s+1} \\ \vdots \\ \varepsilon^p_{s,s+M-1} \\ \varepsilon^p_{s,s+M} \end{bmatrix} = \boldsymbol{\varepsilon}^p_s$$

The additive model is meaningful as long as the size of changes in predictions is unrelated to the size of the predictions.

In our case, however, forecasts can be updated directly on condition that percentage of change must be within the pre-specified weekly, monthly and cumulative percentage revision limits. Therefore, for the forecasting system under consideration it can be said that an update of 10% being independent of the forecast size is more reasonable. Putting another way, the size of difference between successive forecasts depends on the size of the forecasts. This observation suggests the use of multiplicative process to describe the underlying forecast evolution, and hence we shall focus on multiplicative process of forecast evolution hereafter. The multiplicative process has proved to often provide a good approximation to actual evolution in the data for the business environment (see Heath and Jackson 1992).

Multiplicative model describes forecast evolution mechanism by a multiplication operation so that it describes the forecast revision made in period s as the ratio of successive forecasts generated in periods s and $s-1$ for the amount demanded in period $s+n$. Hence, random forecast update vector at period s , $\mathbf{R}_s^p$ becomes:

$$\begin{bmatrix} F_{s,s}^p / F_{s-1,s}^p \\ F_{s,s+1}^p / F_{s-1,s+1}^p \\ \vdots \\ F_{s,s+M-1}^p / F_{s-1,s+M-1}^p \\ F_{s,s+M}^p / \mu^p \end{bmatrix} = \begin{bmatrix} R_{s,s}^p \\ R_{s,s+1}^p \\ \vdots \\ R_{s,s+M-1}^p \\ R_{s,s+M}^p \end{bmatrix} = \mathbf{R}_s^p$$

The idea here is that random evolution of a system is expressed as a percentage of its current size, and is independent of its size in the past.

In modeling the distributions of the size of evolution, lognormal distribution, denoted by $LN\ [\mu\ ,\ \sigma^2]$, has been particularly useful. The lognormal distribution with parameters μ and σ^2 is the distribution of a random variable whose logarithm follows a normal distribution with mean μ and variance σ^2. That is,

If $Y \sim N[\mu, \sigma^2]$ then $X = e^Y \sim LN[\mu, \sigma^2]$ with the probability density function:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}\ e^{-(\ln x - \mu)^2/2\sigma^2} \quad , \quad x > 0\ .$$

Consider $e^{Y_i} = X_i \sim LN[\mu_i, \sigma_i^2]$ and $e^{Y_j} = X_j \sim LN[\mu_j, \sigma_j^2]$. The first two moments of X_i and covariance of X_i and X_j are as follows:

$$E[X_i\ ] = e^{(\mu_i + \sigma_i^2/2)} \tag{4-1}$$

$$var(X_i) = e^{(2\mu_i + \sigma_i^2)}(e^{\sigma_i^2} - 1) \tag{4-2}$$

$$cov(X_i, X_j) = e^{\mu_i + \mu_j + (\sigma_i^2 + \sigma_j^2)/2}(e^{cov(Y_i, Y_j)} - 1) \tag{4-3}$$

Hence, random vector of forecast updates at period s can also be described as η_s^p by taking the natural logarithm of $\mathbf{R}_s^p$:

$$\ln(\mathbf{R}_s^p) = \begin{bmatrix} \ln(R_{s,s}^p) \\ \ln(R_{s,s+1}^p) \\ \vdots \\ \ln(R_{s,s+M-1}^p) \\ \ln(R_{s,s+M}^p) \end{bmatrix}$$

$$= \begin{bmatrix} \ln(F_{s,s}^p) \\ \ln(F_{s,s+1}^p) \\ \vdots \\ \ln(F_{s,s+M-1}^p) \\ \ln(F_{s,s+M}^p) \end{bmatrix} - \begin{bmatrix} \ln(F_{s-1,s}^p) \\ \ln(F_{s-1,s+1}^p) \\ \vdots \\ \ln(F_{s-1,s+M-1}^p) \\ \ln(\mu^p) \end{bmatrix} = \begin{bmatrix} \eta_{s,s}^p \\ \eta_{s,s+1}^p \\ \vdots \\ \eta_{s,s+M-1}^p \\ \eta_{s,s+M}^p \end{bmatrix} = \boldsymbol{\eta}_s^p$$

where $\eta_{s,s+n}^p$ represent the natural logarithm of ratio of successive forecasts generated in periods s and $s-1$ for the amount demanded in period $s+n$. Therefore, in any period s, we have a finite vector of logarithmic forecast updates generated in that period for each product p such that random forecast update vector, $\boldsymbol{\eta}_s^p$, contains the updates made from period $s-1$ to period s in the forecast vector, $\mathbf{F}_{s-1}^p$, generated in period $s-1$.

MMFE imposes a certain structure to the evolution of forecasts, and some of these restrictions can be tested. As in Heath and Jackson (1992) there is a set of assumptions, which are concerned with mostly conditional behavior of the system. According to Heath and Jackson (1992), the additive and multiplicative models given above are governed by the following assumptions:

Assumption 1: The information available to make predictions in period s grows as time passes. In other words, if Δ_s describe the information set at time s and Δ_{s+1} describe the information set at time $s + 1$ then $\Delta_s \subseteq \Delta_{s+1}$.

The rationale for this assumption is straightforward. Forecasting systems use available information to learn the true state space of the environment and make predictions of future demands. In fact, this type of forecasting process can be modeled using expectations conditional on information sets. Realized demands D_t^p observed in period $t < s$ constitute a part of the information set Δ_s.

Assumption 2: The forecast update vector ε_s^p is uncorrelated with the information in set Δ_{s-1} for the additive model, and hence is uncorrelated with all linear combinations of vectors ε_{s-1}^p, ε_{s-2}^p, ε_{s-3}^p, …. Also, $E(\varepsilon_s^p) = 0$.

For the multiplicative model, similarly, the vector $\mathbf{R}_s^p$ is assumed to be uncorrelated with the information in set Δ_{s-1} , and hence should be uncorrelated with all linear combinations of vectors R_{s-1}^p, R_{s-2}^p, R_{s-3}^p, …. Also we require $E(\mathbf{R}_s^p) = 1$.

By using the properties of lognormal distribution, it can be shown that if the above assumption is satisfied for the vector $\mathbf{R}_s^p$, then the vector $\ln(\mathbf{R}_s^p) = \eta_s^p$ is also uncorrelated with the information in set Δ_{s-1} , and hence with all linear combinations of η_{s-1}^p, η_{s-2}^p, η_{s-3}^p, … for all s, and $E(\eta_s^p) = 0$. To show this, Let $\mathbf{R}_s^p$ and $\mathbf{R}_{s-1}^p$ be forecast update vectors at periods s and s-1, which are lognormally distributed with parameters (μ_1, σ_1^2) and (μ_2, σ_2^2), respectively. If $\mathbf{R}_s^p$ and $\mathbf{R}_{s-1}^p$ both satisfy the above assumption then it follows that (see Law and Kelton 2000)

$$
\begin{aligned}
cov(\mathbf{R}_s^p, \mathbf{R}_{s-1}^p) &= e^{\mu_1+\mu_2+(\sigma_1^2+\sigma_2^2)/2}(e^{cov(\ln(\mathbf{R}_s^p),\ln(\mathbf{R}_{s-1}^p))}-1)=0 \\
E[\mathbf{R}_s^p] &= e^{\mu_1+\sigma_1^2/2}=1 \\
E[\mathbf{R}_{s-1}^p] &= e^{\mu_2+\sigma_2^2/2}=1
\end{aligned}
\qquad (4\text{-}4)
$$

The above expected value expressions lead to $(\mu+\sigma^2/2)=0$. And, by inserting the expression $\mu=-\sigma^2/2$ into the above covariance equation produces:

$$
cov(\mathbf{R}_s^p, \mathbf{R}_{s-1}^p) = e^{cov(\eta_s^p,\eta_{s-1}^p)}-1=0
$$

Hence, η_s^p and η_{s-1}^p are also uncorrelated.

The reason for making this assumption is to specify a model in which forecast updates are not predictable linearly or nonlinearly, by any method, using past data like observed demands. This rationale is meaningful, since if this assumption does not hold (i.e. when there exists some linear combination of the available information which is correlated with ε_s^p) then this would result in some other nontrivial linear forecast of ε_s^p, which is better than the original one. Consequently, original forecast can be improved with this new better forecast of ε_s^p.

The issue of justifying this assumption may be viewed as one of assuming that successive forecasts for the same quantity demanded form a martingale. Thus the change in forecast becomes a martingale difference. It can be shown, using properties of conditional expectations that the values of any martingale difference series must be uncorrelated, and must have zero mean.

Assumption 3: The forecast update vectors η_s^p for all s (and ε_s^p for additive process) form a stationary stochastic process.

The rationale for this assumption relates to predictability over time. If the forecast update vectors η_s^p forms a non-stationary process (if assumption 3 is violated) then the degree of predictability (from assumption 2) becomes relative to the time period considered and it becomes then possible for η_s^p and for some k that

$$E(\eta_s^p \mid \Delta_{s-1}) \neq 0 \text{ for } s = T + 1, \ldots, T + \mathrm{k}$$

$$\text{while } E(\eta_s^p \mid \Delta_{s-1}) = 0 \text{ for } s = T + \mathrm{k} + 1, \ldots, T + \mathrm{k} + \mathrm{m}$$

Therefore, if the vector process η_s^p for all s is non-stationary then the variance-covariance matrix of the η_s^p vectors may not be sufficient to capture all important forecast evolution model characteristics.

Assumption 4: The forecast update vectors η_s^p (and ε_s^p for the additive process) are multivariate normal random vectors. For the multiplicative process, ratios of successive forecasts $\mathbf{R}_s^p$ are multivariate lognormally distributed.

Under these assumptions, martingale model of forecast evolution produces an additive model in which $\boldsymbol{\varepsilon}_s^p$ vectors are independent, identically distributed multivariate normal random vectors with $E(\boldsymbol{\varepsilon}_s^p) = 0$. Similarly, for a multiplicative process, MMFE states that $\boldsymbol{\eta}_s^p$ vectors are independent, identically distributed multivariate normal random vectors with $E(\mathbf{R}_s^p) = 1$, which leads to the mean of each coordinate of vector $\boldsymbol{\eta}_s^p$ being equal to the negative of one half of its variance. Therefore, to characterize the forecasting behavior for the additive and multiplicative models, we require only the variance-covariance matrix of the forecast update vectors and the initial state of the forecasting system.

4. 2. Empirical Support and Covariance Matrix Estimation

We have, so far, discussed martingale model of forecast evolution in somewhat abstract terms. At this point, it is needed to give an empirical support for the data on hand. The main focus here is in calculating the variance-covariance matrix of the forecast updates. In this section, hence, we describe the data on hand and give some details of covariance matrix estimation.

In the empirical study part of the thesis we focus on a set of products, covering 17 different stock keeping units. The selected SKUs can be divided into two main categories: One consists of products sharing a common production line and the other comprises products having the same brand. Being related to the current status of all products produced, some drivers affecting product demand and product availability are considered in order to construct the set of products we focus on. These drivers consider factors such as the phase of product life cycle, package design, brand name, substitution, sales volume and some distributional characteristics. Also, considering these drivers, we consider nine product families. The assignment of products to these groups is not in a mutually exclusive manner such that products from different brand groups may belong to the same product group containing items being processed on the same production line.

Let $p \in \{ 1 , \ldots , P = 17 \}$ be index on different SKUs and $t \in \{ 1 , \ldots , T \}$ be index on different observation points in time. For each product under consideration three historical data sets covering approximately three years period ($T = 125$ weeks) are used to consider:

- Sales forecasts in weekly level of detail for approximately three years period. For period t we have a forecast vector generated in that period for each product p. A forecast vector comprises sales forecasts corresponding to each period of the 12-week planning horizon. Therefore, we have a total of 125x12 forecasts, 12 for each observation point.
- Quantity demanded by distributors and chain markets in weekly level of detail. For period t we have a demand observation realized in that period for each product p. All the products considered have stable demand over extended periods of time, but are affected by promotions, price changes, competitor's actions etc.

- Actual shipments to distributors and chain markets in weekly level of detail. For period *t* we have an observation on actual shipment made in that period for each product *p*.

The data sets used in the thesis are extracted from the PIMS database and analyses performed are based on disguised data. The analysis of the data indicates the presence of missing values almost in each of the series. A missing forecast value is replaced with that part of budgeted monthly sales, which is calculated by considering the working days of the week for which the missing forecast value estimates the quantity demanded. Similarly, a missing demand value and shipment value are replaced by considering the budgeted sales.

In order to estimate the actual values for the variance-covariance matrix, the following steps need to be taken:

Step 1: Clean the Data

Though a full analysis of the data set involves some product specific features there are two common motivations in data analysis: (1) to identify actual pattern of the majority of the data, and (2) to obtain more reliable information about the nature of the underlying process.

Figures given in Appendix A show samples of time series plots and box-plots for some product demands available. These products provide a general picture of the data characteristics. Weekly demand was provided covering approximately three years period (a total of 125 observations). The analysis of time series plots and box-plots shows the presence of potential outlying observations in the series.

To clean the effects of exogenous events on historical demands and sales forecasts, our first task is to detect any influencing outliers and eliminating if present. The analysis of the data indicates the presence of outliers almost in each of the series. Outliers may be attributable to skewness of the distribution of random error, some unassignable causes, or even chance. Some outliers are exceptional data; others are not (see Law and Kelton 2000). Hence the interpretation of suspected outliers and unusual data is important since they may convey some interesting information. In addition, any peculiarities in the data which can disturb the pattern and might destroy the value of estimates are identified. Observed peculiarities generally result from such factors as special one-off orders, promotions, price discounts, run-outs of stock, competitor's temporary strategy etc. For example, Figure 7 in Appendix A shows an irregular demand value at observation point 95. This demand value reflects a special one-off order situation occurred at that period. Outlier detection and peculiarity identification are essential in the sense that they clean the data so that more robust tools and analysis for inventory management and planning can be developed.

Step 2: Eliminate Systematic Forecasting Errors

Estimation biases may be induced by incorrect information or some inherent forecasting behavior. Analysis regarding forecasting bias reflects and measures the predictive performance of the forecasting system considered. We assess empirically this performance by testing whether the expected value of prediction error is zero or not.

Assuming that the η_s^p vectors are independent identically distributed multivariate normal random vectors with $E(\eta_s^p) = 0$ requires the demand process to be stationary and forecasts to be unbiased. Accordingly the issue of justifying $E(\eta_s^p) = 0$ in Assumption 2 can be viewed as one of satisfying that predictions within the forecast horizon are unbiased estimates of demand so that there is no systematic tendency to either underestimate or overestimate the true value of demand.

Let D_t^p be the demand process for product *p*; and F_{j+1}^p be *(j + 1)*-step ahead forecasts for $j \in \{0, 1, \ldots, 11\}$.

If $E(D_t^p) = E\left(F_{j+1}^p\right)$ for a particular *j* then it can be concluded that *(j + 1)*-step ahead forecasts are unbiased. Hence we can investigate the estimation biases in terms of two-sample t-tests of the individual hypotheses:

$$H_0 : E(D_t^p) - E\left(F_{j+1}^p\right) = 0 \qquad \text{for each } j$$

It can be shown by using forecast evolution mechanism that testing the above null hypothesis is equivalent to testing the hypothesis of $E(\eta_s^p) = 0$ or of $E(\mathbf{R}_s^p) = 1$.

Hence one sample t-tests for the null hypotheses $H_0 : E(\mathbf{R}_s^p) = 1$ are carried out and biased forecasts are adjusted when the null hypothesis is rejected such that adjusted forecasts result in $E(\mathbf{R}_s^p) = 1$. For example, 3-step ahead forecasts for product $p2$ appear to be generally almost 15% higher than realized demand. This upward bias, hence, is eliminated by adjusting all the historical forecast values by the scaling value of (100/115) before being used in further calculations. Analyzing the systematic errors and making adjustments to obtain unbiased forecasts is critical for the purposes of estimating a variance-covariance matrix that is based only on random fluctuations.

<u>Step 3: Obtain Forecast Update Vectors & Validate the Assumptions</u>

Modification activity at period *s*, described by forecast update vector $\mathbf{R}_s^p$ for the multiplicative process is given as:

$$\begin{bmatrix} F_{s,s}^p / F_{s-1,s}^p \\ F_{s,s+1}^p / F_{s-1,s+1}^p \\ \vdots \\ F_{s,s+M-1}^p / F_{s-1,s+M-1}^p \\ F_{s,s+M}^p / \mu^p \end{bmatrix} = \begin{bmatrix} R_{s,s}^p \\ R_{s,s+1}^p \\ \vdots \\ R_{s,s+M-1}^p \\ R_{s,s+M}^p \end{bmatrix} = \mathbf{R}_s^p \quad \text{for any } s$$

In Assumption 4, it is stated that ratios of successive forecasts are lognormal variates and hence forecast update vectors $\mathbf{R}_s^p$ are lognormal random vectors. The forecast updates described as ratios can, of course, be transformed into normal variates by taking natural logarithms. Hence, we may equivalently study the hypothesis that natural logarithms of ratios of successive forecasts are distributed normally.

Tables given in Appendix B show samples of some logarithmic forecast updates expressed as natural logarithms of ratios of successive forecasts. In the tables, η_j for $j \in \{ 0, 1, \ldots , 11 \}$ represent logarithmic forecast updates made in *(j + 1)*-period ahead forecasts to obtain *j*-period ahead forecast. In particular, column η_0 represents logarithmic forecast errors expressed as natural logarithms of ratios of one-period ahead forecasts and actual demands. From these tables we can obtain a general picture of the forecast evolution characteristics of the underlying forecasting system. An important finding obtained from the tables is that for almost every product logarithmic forecast updates having zero value are dominant in the columns corresponding to variables η_4 till η_{11}. This finding proves the observation made before on the forecasting behavior of the buyer. The buyer does not pay any attention to updating the forecasts for the last 7-8 weeks of the 12-week planning horizon. This practice causes delayed forecast updates leading to a need for significant forecast changes as the time for order execution approaches.

Another notable finding that can be obtained from the tables is that forecast updates having zero value corresponding to column η_1, representing logarithmic forecast updates made in two-period ahead forecasts to obtain one-period ahead forecasts, are also dominant. This proves the implementation of one-week freeze period in the master production scheduling.

Both graphical techniques and statistical tests are used to check the normality assumption of logarithmic forecast updates. In carrying out the hypothesis test we both perform Kolmogorov-Smirnov test with the significance level of 0.05 and construct normal probability plot. Note that K-S test is valid for any sample size (see Law and Kelton 2000). The results are given for some products in Appendix C. The normal probability plots do not have large deviations from a straight line, suggesting that the logarithmic updates are normal. And, the results of the hypothesis tests indicate that in all but two cases the data is consistent with the normality assumption. The degree of violation for those two cases is not too significant (p-values for those two cases are 0.041 and 0.043) and hence they are assumed to be normal.

Another assumption, which is even harder to satisfy than the assumption of normality, is the assumption of mutual independence for bivariate forecast updates. To test whether the forecast update vectors can be regarded as being independent, we calculate bivariate correlations based on Pearson's correlation coefficient, which requires normally distributed data (see Law and Kelton 2000). In our case, it is not reasonable to calculate bivariate correlations for the variables from η_4 till η_{11} since the samples for them are composed dominantly of logarithmic forecast updates having zero values. The results of the correlation analysis for the remaining forecast update variables are contained in Table 13 given in Appendix C, together with the value of two-tailed probabilities corresponding to the 0.05 and the 0.01 levels of significance. Correlation coefficients significant at the 0.05 level are identified with a single asterisk, and those significant at the 0.01 level are identified with two asterisks. The presence of significant correlations indicates that the independence assumption is violated. Therefore, it is needed to adjust biased forecasts so that the resulting forecast updates of adjusted forecasts satisfy the independence assumption (see Hausman 1969). After making the necessary adjustments (by scaling the biased forecasts to eliminate the bias), the results indicate that the assumption of independence is supported. These biases should be reimposed later while employing the forecasts to quantify demand variability in the subsequent sections.

One may express any observation on the time series, D_t^p, in terms of the random forecast update variates, either as,

$$D_s^p = F_{s,s}^p = \prod_{j=0}^{n-1} (R_{s-j,s}^p) * F_{s-n,s}^p$$

or as,

$$\ln(D_s^p) = \ln(F_{s,s}^p) = \sum_{j=0}^{n-1} \ln(R_{s-j,s}^p) + \ln(F_{s-n,s}^p).$$

Note that in the multiplicative model, if we begin with a forecast value $F_{s-n,s}^p$ and if every step yields an independent and identically distributed random multiplier $R_{s-j,s}^p$ from lognormal distribution, then any resulting distribution after n steps is again lognormal since it arises from the combination of random terms by a multiplicative process.

Step 4: Estimate Variance and Mean of the Forecast Update Vectors

It is easy to show that under Assumption 2 the condition $E(\mathbf{R}_s^p) = 1$ leads to $\mu_j^p = -var(\eta_j^p)/2$ for all $j \in \{0, 1, \ldots, 11\}$.

From equation (4-4), $E(R^{p}_{s,s+j}) = e^{\mu^{p}_{j} + var(\eta^{p}_{j})/2} = 1$. And after a bit of manipulation,

$$E[\eta^{p}_{j}] = \mu^{p}_{j} = -var(\eta^{p}_{j})/2 \qquad (4\text{-}5)$$

Variance of the forecast update η^{p}_{j}, $var(\eta^{p}_{j})$, can be represented by:

$$\begin{aligned} var(\eta^{p}_{j}) &= E[\ (\eta^{p}_{j} - \mu^{p}_{j})^{2}\ ] \\ &= E[(\eta^{p}_{j})^{2}] - (\mu^{p}_{j})^{2} \\ var(\eta^{p}_{j}) &= E[(\eta^{p}_{j})^{2}] - (var(\eta^{p}_{j}))^{2}/4 \qquad \text{by substituting (4-5)} \end{aligned}$$

Hence $var(\eta^{p}_{j})$ can be calculated solving the above quadratic equation. And the mean values μ^{p}_{j}'s of random variates, η^{p}_{j}'s, can be estimated by using the result $\mu^{p}_{j} = -var(\eta^{p}_{j})/2$. Table 1 below tabulates samples of variances and means of η^{p}_{j}'s for product $p13$.

Table 1 Estimated variance and mean values of η_{j} for product $p13$

	$var(\eta_{j})$	μ_{j}
j = 11	0.00628	-0.00314
j = 10	0.02065	-0.01033
j = 9	0.00588	-0.00294
j = 8	0.00385	-0.00193
j = 7	0.00268	-0.00134
j = 6	0.00526	-0.00263
j = 5	0.00468	-0.00234
j = 4	0.00976	-0.00488
j = 3	0.00823	-0.00411
j = 2	0.01201	-0.00601
j = 1	0.00723	-0.00362
j = 0	0.32209	-0.16104

Step 5 : Variance - Covariance Matrix Estimation

The variance-covariance matrix of random forecast update variates is estimated from the historical data. By using the estimated mean values μ_j^p's of the random variates η_j^p's , the issue of estimating the variance-covariance matrix from the historical data is simplified.

Let η_j^p represent the random variable that generates the updates made to *(j + 1)*-step ahead forecasts to obtain *j*-step ahead forecasts for product $p \in \{ p1, p2, \ldots, p17 \}$ and period $j \in \{ 0, 1, \ldots , 11 \}$ Hence η_0^p represents the random variable that generates forecast error between one-step ahead forecast and realized demand for product *p*.

Denote the length of an update vector with *N* and the number of products with *P* and the length of the forecast horizon with *H* such that $\mathbf{E}$ be the *N* by *P x H* data matrix consisting of such *P x H* variables as:

$\eta_0^{p1}, \eta_1^{p1}, \eta_2^{p1}, \ldots, \eta_{11}^{p1}$

$\eta_0^{p2}, \eta_1^{p2}, \eta_2^{p2}, \ldots, \eta_{11}^{p2}$

.

.

$\eta_0^{p17}, \eta_1^{p17}, \eta_2^{p17}, \ldots, \eta_{11}^{p17}$ each observed on *N* individuals.

In multivariate analysis, we seek to examine the relationships between the *P x H* variables with the aim of characterizing the underlying forecast evolution mechanism.

Let $\sum$ be the $P\ x\ H$ by $P\ x\ H$ variance-covariance matrix of the N by $P\ x\ H$ data matrix $\mathbf{E}$. Covariance terms on the off-diagonal entries and variances on diagonal entries of the matrix $\sum$ are as follows:

$$cov(\eta_i^p, \eta_j^{p'}) = \sum_t^N (\eta_{i,t}^p - \mu_i^p)(\eta_{j,t}^{p'} - \mu_j^{p'}) / N$$

$$var(\eta_j^p) = \sum_t^N (\eta_{j,t}^p - \mu_j^p)^2 / N$$

Variance $var(\eta_j^p)$ values calculated in Step 4 and in Step 5 can be compared to assure the validity of the assumptions used in these calculations.

Given the complexity of the underlying processes, note that some limitations on our ability to analyze the data fully affect the results. This is especially more important when the dimensionality of the variance-covariance matrix is large compared to the sample size of data, resulting in few degrees of freedom available. For this reason, all conclusions concerning the problem at hand must be tentative, and may be revised by looking at the same type of data at a later stage. Obviously, with more historical forecast and demand data one is expected to estimate better.

Consider the estimated variance-covariance matrix $\sum$. For any product p, using the variances $var(\eta_j^p)$ for all $j \in \{ 0, 1, \ldots , 11 \}$ the percentages of total forecast variability that is resolved as the system evolves from one period to the next period are calculated. As can be seen from Table 2, a significant proportion of total forecast variability for each product is not resolved until the period of realization for that product. In other words, there exists a considerable amount of forecast error in the system. For example, for product *p*4, a significant percentage, 70.4%, of total forecast variability is not resolved until the period of realization. This observation reflects the performance of the forecasting system (or the accuracy of the forecasts provided by the buyer) and indicates that the manufacturer cannot effectively respond to demand variability without holding significant amount of safety stock.

Table 2 Percentages of total forecast variability resolved by period

	PRODUCTS																
PERIODS / j	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16	P17
11	3.9	2.2	2.5	2.6	2.3	2.1	1.4	4.1	3.2	2.6	5.4	2.2	1.5	2.4	2.0	4.2	6.5
10	8.3	8.1	5.3	5.6	4.0	6.9	5.2	7.0	6.8	8.3	8.6	7.6	5.1	6.9	6.3	9.6	10.9
9	2.1	2.8	1.7	1.7	1.2	2.6	1.3	3.9	3.3	1.6	2.9	1.9	1.4	2.0	1.3	4.5	5.8
8	1.5	1.7	1.5	1.5	1.8	1.7	1.0	1.1	1.0	1.4	1.0	1.3	0.9	1.4	1.4	2.2	2.1
7	1.1	0.8	0.7	0.7	1.4	1.2	0.9	2.7	0.6	1.1	0.6	0.9	0.7	0.6	0.7	0.5	0.6
6	1.6	1.6	1.3	1.2	1.9	1.5	1.0	1.2	1.2	1.6	1.0	2.2	1.3	1.3	1.4	2.7	2.8
5	1.2	1.5	0.9	0.9	1.8	1.5	2.9	1.9	4.2	1.2	1.4	2.0	1.1	1.3	1.3	2.5	1.7
4	3.3	4.0	0.9	0.9	1.0	0.8	2.7	2.9	4.4	3.7	2.3	3.4	2.4	2.7	2.9	2.0	2.4
3	2.7	3.6	5.2	3.2	6.1	3.0	4.0	4.6	5.8	3.2	3.0	2.1	2.0	3.5	2.6	2.4	3.5
2	6.6	4.2	6.8	3.9	4.9	7.2	7.3	4.8	7.9	5.2	3.4	3.6	2.9	3.9	3.7	4.5	6.2
1	4.1	2.6	2.0	7.3	0.9	1.3	0.3	3.2	4.6	4.3	1.7	3.3	1.8	2.0	1.3	2.2	2.5
0	63.5	66.8	71.2	70.4	72.8	70.1	72.1	62.5	57.0	65.8	68.7	69.4	78.8	72.1	75.0	62.9	55.1
	100	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***	***100***

Step 6 : The Resultant Data

We have analyzed the historical forecast and demand data for a set of products with the aim of giving empirical support and hence validating the assumptions of martingale model of forecast evolution. First we have checked for and eliminated, if necessary, any influencing outliers and peculiarities, which may cause the misleading results in the subsequent analysis. In the setting of multiplicative processes used to describe the growth of the system, we have calculated historical forecast updates from the past forecast and demand data. We have performed two main hypothesis tests to check for the normality assumption and mutual independence assumption of forecast updates. Finally, we have estimated the variance-covariance matrix of the forecast updates from the historical data, and hence obtained the variance terms $var(\eta_j^p)$, covariance terms $cov(\eta_i^p, \eta_j^{p'})$ and expected values $E[\eta_j^p]$ for each product $p \in \{ p1, p2, \dots, p17 \}$ and for period $j \in \{ 0, 1, \dots, 11 \}$. Appendix D contains some part of the estimated variance-covariance matrix.

4. 3. Integration of Forecast Evolution Modeling with MPS

In this study, we analyze the manufacturer-buyer interaction at the aggregate planning level and empirically investigate the relationship between the buyer's forecasting behavior and in turn the manufacturer's delivery performance. By tracking forecast volatility, characterized by the size and frequency of forecast updates, we explain the buyer's forecasting behavior and in a sense measure its predictive performance. The manufacturer's delivery performance, on the other hand, is tracked by its ability to meet weekly delivery requests (weekly forecasts) provided by the buyer. Based on the empirical analysis, we attempt to show how forecast volatility could be incorporated into the production planning and inventory management activities at the MPS level to improve the manufacturer's delivery performance. Naturally, with more accurate forecasts provided delivery performance will be better.

Forecasts are periodically updated as the buyer observes the realized demands and obtains new market information. By being updated successively, however, forecasts do not necessarily become more accurate (accuracy relating to having lower standard deviation of forecast errors). Existence of much uncertainty about the market demand when the buyer makes prediction for a period for the first time gives rise to forecast volatility problem, which might cause inefficiencies if the manufacturer relies only on this relatively less accurate forecasts. Hence, the extent to which degree of accuracy of forecasts provided by the buyer determines the degree of justification for manufacturer's acting on those forecasts. The manufacturer who acts only on given less accurate forecasts will probably face significant future adjustments and costs.

Much uncertainty is associated with the data, especially with actual demands, for a long-term production-inventory model. In a long-term production-inventory model forecast unreliability becomes more important since it results in either huge inventory or low customer fill rate. Obviously, safety stocks for individual products held in inventory are critical to the execution of an effective inventory policy. Indeed, the issue at all levels of planning is how they should be computed. Practical considerations and realities of the supply chain suggest in practice that combinations of judgment and statistical analysis should be used in determining safety stock levels for products.

The manufacturer calculates safety stock levels for products by considering differences between historical actual shipment and forecast data. Hence by doing so, the manufacturer bears the full cost of forecast error and the buyer shares uncertainty of demand process with the manufacturer. They prepare, therefore, for both uncertain demands and unforeseen production problems.

Let $F^{p}_{s-1,s}$ be the forecast generated at week s-1 for the amount demanded at week s for product p (that is, it represents one-week ahead forecast generated at week s-1). Let S^{p}_{s} be the shipment amount to distributors and chain markets made at week s for product p. Each month the following steps are taken to calculate the current target safety stock level for product p:

1. From historical 40-week shipment and forecast data, deviations of actual shipments from one-week ahead forecasts are calculated as:

   $$\Delta^{p}_{s} = S^{p}_{s} - F^{p}_{s-1,s} \quad \text{for } s \in \{1, \ldots, 40\}$$

   Note that observations are numbered in time order such that more weight is given to closer observations. The largest four and the lowest four deviation values are eliminated from the set. Let the remaining set be S' and $\|S'\| = 32$.

2. Using the remaining 32 deviations, weighted mean absolute deviation-WMAD value is calculated as:

$$WMAD^p = \sum_{s \in S'}^{32} (| \Delta_s^p | * s) \ / \ \sum_{s \in S'}^{32} s$$

3. Standard deviation (σ_{Δ^p}) of the remaining 32 deviations (Δ_s^p) is calculated. And, by using a 95 percent service level, safety stock level for product *p*, SS^p is determined as:

$$SS^p = WMAD^p + 1.65 * \sigma_{\Delta^p} \tag{4-6}$$

Current safety stock levels are deterministic target inventory levels being updated each month based on historical forecast error distribution. Target safety stock levels are calculated exogenously and fed into the master production scheduling model as an input. Master scheduling model takes target safety stock levels at a constant level for each period of the 12-week planning horizon. That is, they do not vary by periods of the planning horizon.

In order to compute target safety stock level for a specific product *p*, the manufacturer determines variance of the deviations, Δ_s^p, for product *p* independently from the other products as:

$$var(\Delta^p) = \sigma_{\Delta^p}^2$$

However, due to realities of the supply chain considered and the resulting nature of demand patterns for many products, some of the products are expected to be correlated, and hence demands to be interdependent. This observation motivates us to investigate the effect of demand and forecast correlations on target safety stock levels.

The importance of demand and forecast correlations on target safety stock levels is apparent if some drivers (including factors like promotions, ads, phase of the product life cycle, brand, package design, logistical needs of products, substitute and complementary products etc.) affecting product demand and product availability are emphasized. All these factors and events have varying impacts on product demands. For example, promotions may lead to temporary demand lifts for the promoted SKUs, but they may also depress demand for other SKUs. Moreover, products in the maturity phase of the life cycle experience predictable demand than ones in the innovation phase. When any marketing instrument is applied to a product, there may be an effect on demand for related products, either substitutes or complements. In some cases, stimulations of demand for one item may result in an increase of demand for a complementary product; while in other cases, the effect is opposite for competing products. Hence, it is important that to accurately represent the outcomes in the environment under consideration interaction effects between products should be captured and modeled explicitly and be used in production planning. Estimated variance-covariance matrix of random forecast updates and the method proposed for determining target safety stock levels lay the foundation of the way of integrating the buyer’s forecast evolution with the manufacturer’s production planning model.

In the proposed integration method, calculation of target safety stock levels is based on demand variability rather than forecast error variability. In determining the target level of safety stocks based on the variability of demand, we have to calculate variance of the demand, $var(D^{p}_{s+n})$. Derivations made to quantify the variability of demand, and hence to determine target safety stock levels are given in the following subsections.

4. 3. 1 Derivations for Demand Variability

Suppose at the beginning of period s we have a finite vector of preliminary forecasts, $\mathbf{F}^{p}_{s}$, generated in that period:

$$\mathbf{F}^{p}_{s} = [F^{p}_{s,s+1}, F^{p}_{s,s+2}, F^{p}_{s,s+3}, \dots, F^{p}_{s,s+M}]$$

where M denotes the length of the planning horizon, which is equal to 12 weeks in our case.

In the setting with multiplicative process used to describe the evolving path of the system, forecasts of the demand move in discrete time steps and the predictions change accordingly.

For product p, the actual demand to be realized at period s + n, D^{p}_{s+n}, can be expressed in terms of random forecast updates and a preliminary forecast value, $F^{p}_{s,s+n}$, as follows:

$$D^p_{s+n} = F^p_{s+n,s+n} = \prod_{j=1}^{n} (R^p_{s+j,s+n}) * F^p_{s,s+n}$$
$$= \prod_{j=1}^{n} (exp(\eta^p_{s+j,s+n})) * F^p_{s,s+n} \quad (4\text{-}7)$$

where $n \in \{ 1, \ldots , M \}$ such that $F^p_{s,s+n}$ represents n-week ahead preliminary forecast available at week s. $R^p_{s+j,s+n}$ is a lognormal random forecast update observing on the ratio of j-week ahead forecasts to (j + 1)-week ahead forecasts for product p. And, $\eta^p_{s+j,s+n}$ represents normal random forecast update observing on the difference between logarithms of j-week ahead forecasts and logarithms of (j +1)-week ahead forecasts for product p such that $\ln(R^p_{s+j,s+n}) = \eta^p_{s+j,s+n}$.

We would like to emphasize, at this point, that the lognormal distribution has multiplicative reproductive properties (see Hines and Montgomery 1990). These properties are as follows:

<u>Property 1:</u> If $exp(\eta^p_{s+j,s+n})$ has a lognormal distribution with parameters μ and σ^2, and $F^p_{s,s+n}$ is a constant, then $F^p_{s,s+n} * exp(\eta^p_{s+j,s+n})$ has a lognormal distribution with parameters $(\ln(F^p_{s,s+n}) + \mu)$ and σ^2.

<u>Property 2:</u> If $exp(\eta^p_{s+j,s+n})$ and $exp(\eta^p_{s+j-1,s+n})$ are independent lognormal variables with parameters (μ_1, σ_1^2) and (μ_2, σ_2^2), respectively, then $exp(\eta^p_{s+j,s+n}) * exp(\eta^p_{s+j-1,s+n})$ has a lognormal distribution with parameters $[(\mu_1 + \mu_2), (\sigma_1^2 + \sigma_2^2)]$.

We present the variability derivations for an individual product and for a group of products separately in the following two subsections.

4. 3. 1. a. Variability Derivations for an Individual Product

In determining the target level of safety stocks based on the variability of demand, we have to calculate variance of the demand, $var(D^p_{s+n})$ using equation (4-7) as:

$$
\begin{aligned}
var(D^p_{s+n}) &= var\left(\prod_{j=1}^{n} (exp(\eta^p_{s+j,s+n})) * F^p_{s,s+n}\right) \\
&= var\left(exp(\eta^p_{s+n,s+n}) * exp(\eta^p_{s+n-1,s+n}) * \ldots * exp(\eta^p_{s+1,s+n}) * F^p_{s,s+n}\right) \\
&= var(LNV^p_n * F^p_{s,s+n}) \qquad (4\text{-}8)
\end{aligned}
$$

Where $LNV^p_n = exp(\eta^p_{s+n,s+n}) * exp(\eta^p_{s+n-1,s+n}) * \ldots * exp(\eta^p_{s+1,s+n})$ for each period $n \in \{1, \ldots, M=12\}$ is lognormal since it is the product of independent lognormal variates (from the above lognormal Property 2). Therefore, the whole term $LNV^p_n * F^p_{s,s+n}$ in the variance expression is a lognormal variable multiplied by a constant, $F^p_{s,s+n}$.

However, it is not possible to calculate directly variance of the nonlinear term, $LNV_n^p * F_{s,s+n}^p$, in equation (4-8). Therefore, it is more reasonable first to take the natural logarithm of the nonlinear term and then to calculate the variance of $\ln(LNV_n^p)$ and expected value of $\ln(LNV_n^p * F_{s,s+n}^p)$ in an effort to calculate the variance of D_{s+n}^p.

$$\ln(LNV_n^p * F_{s,s+n}^p) = \ln(LNV_n^p) + \ln(F_{s,s+n}^p) \tag{4-9}$$

where $\ln(LNV_n^p)$ in terms of random forecast updates is:

$$\begin{aligned}\ln(LNV_n^p) &= \ln(exp(\eta_{s+n,s+n}^p) * exp(\eta_{s+n-1,s+n}^p) * ... * exp(\eta_{s+1,s+n}^p)) \\ &= \eta_{s+n,s+n}^p + \eta_{s+n-1,s+n}^p + ... + \eta_{s+1,s+n}^p \\ &= \textstyle\sum_{j=1}^n (\eta_{s+j,s+n}^p)\end{aligned} \tag{4-10}$$

We calculate variance of the lognormal term, $var(\ln(LNV_n^p))$, by using equation (4-10) as:

$$\begin{aligned}&var(\ln(LNV_n^p)) = \\ &\quad = var(\textstyle\sum_{j=1}^n (\eta_{s+j,s+n}^p)) \\ &\quad = var(\eta_{s+n,s+n}^p + \eta_{s+n-1,s+n}^p + ... + \eta_{s+1,s+n}^p) \\ &\quad = var(\eta_{s+n,s+n}^p) + var(\eta_{s+n-1,s+n}^p) + ... + var(\eta_{s+1,s+n}^p)\end{aligned} \tag{4-11}$$

The last equation follows from the fact that $\eta^{p}_{s+n,s+n}$, $\eta^{p}_{s+n-1,s+n}$, …, $\eta^{p}_{s+1,s+n}$ represent the forecast updates made at different points in time (at periods $s+n$, $s+n$-1, …, $s+1$, respectively), and hence each one comes from the different independent, identically distributed random update vector. We can obtain the individual variance values $var(\eta^{p}_{s+j,s+n})$ from the estimated variance-covariance matrix $\sum$ of lognormal forecast updates, and in turn we can calculate value of the whole term, $var(\ln(LNV^{p}_{n}))$.

Similarly, we calculate expected value of the lognormal term, $E[\ln(LNV^{p}_{n} * F^{p}_{s,s+n})]$, by using equation (4-9) as:

$$E[\ln(LNV^{p}_{n} * F^{p}_{s,s+n})] =$$

$$= E[\ \textstyle\sum^{n}_{j=1} (\eta^{p}_{s+j,s+n}) + \ln(F^{p}_{s,s+n})\ ]$$

$$= E[\eta^{p}_{s+n,s+n} + \eta^{p}_{s+n-1,s+n} + \ldots + \eta^{p}_{s+1,s+n}] + E[\ln(F^{p}_{s,s+n})]$$

$$= E[\eta^{p}_{s+n,s+n}] + E[\eta^{p}_{s+n-1,s+n}] + \ldots + E[\eta^{p}_{s+1,s+n}] + \ln(F^{p}_{s,s+n}) \qquad (4\text{-}12)$$

We can calculate the individual expected values $E[\eta^{p}_{s+j,s+n}]$ by using equation (4-5), which is the result of Assumption 2 given in Section 4.1. Hence we can calculate value of the whole term, $E[\ln(LNV^{p}_{n} * F^{p}_{s,s+n})]$.

When LNV_n^p for a particular period $n \in \{ 1, \ldots , M \}$ is lognormal with parameters μ_n^p and $\sigma_{p,n}^2$, $LNV_n^p * F_{s,s+n}^p$ has a lognormal distribution with parameters $(\ln(F_{s,s+n}^p) + \mu_n^p)$ and $\sigma_{p,n}^2$ from the above lognormal Property 1. Therefore, using the relationship between lognormal distribution and normal distribution, which is stated by equation (4-2), $var(D_{s+n}^p) = var(LNV_n^p * F_{s,s+n}^p)$ is calculated as:

$$var(LNV_n^p * F_{s,s+n}^p) =$$
$$= e^{2*E[\ln(LNV_n^p * F_{s,s+n}^p)] + var(\ln(LNV_n^p))} * (e^{var(\ln(LNV_n^p))} - 1)$$

where $var(\ln(LNV_n^p))$ and $E[\ln(LNV_n^p * F_{s,s+n}^p)]$ are given by equations (4-11) and (4-12), respectively.

Therefore, using a 95 percent service level, target safety stock level of product p for period $n \in \{ 1, \ldots , M \}$ of the 12-week planning horizon, SS_n^p, based on demand variability becomes:

$$SS_n^p = z * \sqrt{var(D_{s+n}^p)} \text{ where } z = 1.645. \tag{4-13}$$

4. 3. 1. b. Variability Derivations for Product Groups

In empirically assessing the integration method, we focus on nine product families. Product families are constructed according to two main factors: brand and production line. Products belonging to the same brand category are put into the same product family. And similarly, products sharing the same production line are in the same product family. The assignment of products to these groups is not in a mutually exclusive manner such that products with distinct brands can be produced on the same production line or vice versa. Other product group compositions that follow any practical classification are also possible. For example, grouping can consider products' sales opportunities in a particular sales region. Let $G = \{G1, G2, \ldots, G9\}$ be the set of product indices that form a specific product group Gi, $i = 1, \ldots, 9$. In determining the target level of aggregate safety stock for a specific group Gi based on the variability of group's total expected demand, we have to calculate the variance of group demands, $var(\sum_{p \in Gi} D^p_{s+n})$ as:

$$var(\sum_{p \in Gi} D^p_{s+n}) = \sum_{p \in Gi} var(D^p_{s+n}) + 2 * \sum_{p, p' \in Gi} cov(D^p_{s+n}, D^{p'}_{s+n})$$

Individual variance terms, $var(D^p_{s+n})$'s, in the above expression are calculated as in the previous section. Covariance term in the above variance expression can be expressed in terms of random forecast updates and a preliminary forecast value, $F^{p}_{s,s+n}$, by using equation (4-7), as follows:

$$cov(D^{p}_{s+n}, D^{p'}_{s+n}) =$$

$$= cov\left(\prod_{j=1}^{n} (exp(\eta^{p}_{s+j,s+n}))) * (F^{p}_{s,s+n}) ,\ \prod_{j=1}^{n} (exp(\eta^{p'}_{s+j,s+n}))) * (F^{p'}_{s,s+n})\right)$$

$$= cov(exp(\eta^{p}_{s+n,s+n}) * exp(\eta^{p}_{s+n-1,s+n}) * ... * exp(\eta^{p}_{s+1,s+n}) * (F^{p}_{s,s+n}),\ exp(\eta^{p'}_{s+n,s+n}) * exp(\eta^{p'}_{s+n-1,s+n}) * ... * exp(\eta^{p'}_{s+1,s+n}) * (F^{p'}_{s,s+n}))$$

$$= cov(LNV^{p}_{n} * F^{p}_{s,s+n} ,\ LNV^{p'}_{n} * F^{p'}_{s,s+n}) \qquad (4\text{-}14)$$

Where $LNV^{p}_{n} = exp(\eta^{p}_{s+n,s+n}) * exp(\eta^{p}_{s+n-1,s+n}) * ... * exp(\eta^{p}_{s+1,s+n})$ (similarly $LNV^{p'}_{n}$) is lognormal since it is the product of independent lognormal variates (from the above lognormal Property 2). Therefore, the whole term $LNV^{p}_{n} * F^{p}_{s,s+n}$ (similarly $LNV^{p'}_{n} * F^{p'}_{s,s+n}$) in the covariance expression is a lognormal variable multiplied by a constant, $F^{p}_{s,s+n}$ (similarly $F^{p'}_{s,s+n}$).

It is not possible, however, to calculate directly covariance of the nonlinear terms, $LNV^{p}_{n} * F^{p}_{s,s+n}$ and $LNV^{p'}_{n} * F^{p'}_{s,s+n}$, in equation (4-14). Therefore, as in the previous section it is more reasonable first to take the natural logarithm of these nonlinear terms and then to calculate the covariance of $\ln(D^{p}_{s+n})$ and $\ln(D^{p'}_{s+n})$.

Therefore, by using equation (4-9), covariance of $\ln(D^{p}_{s+n})$ and $\ln(D^{p'}_{s+n})$ can be expressed as:

$$cov(\ln(D^{p}_{s+n}), \ln(D^{p'}_{s+n})) =$$

$$= cov(\ln(LNV^{p}_{n} * F^{p}_{s,s+n}) \;,\; \ln(LNV^{p'}_{n} * F^{p'}_{s,s+n}))$$

$$= cov(\Sigma^{n}_{j=1} (\eta^{p}_{s+j,s+n}) + \ln(F^{p}_{s,s+n}) \;,\; \Sigma^{n}_{j=1} (\eta^{p'}_{s+j,s+n}) + \ln(F^{p'}_{s,s+n}))$$

$$= cov(\; \eta^{p}_{s+n,s+n} + \eta^{p}_{s+n-1,s+n} + \ldots + \eta^{p}_{s+1,s+n} + \ln(F^{p}_{s,s+n}) \;,$$
$$\eta^{p'}_{s+n,s+n} + \eta^{p'}_{s+n-1,s+n} + \ldots + \eta^{p'}_{s+1,s+n} + \ln(F^{p'}_{s,s+n}) \;)$$

$$= cov(\; \eta^{p}_{s+n,s+n} + \eta^{p'}_{s+n,s+n}) + \ldots + cov(\; \eta^{p}_{s+n,s+n} + \eta^{p'}_{s+1,s+n})$$
$$+ cov(\; \eta^{p}_{s+n-1,s+n} + \eta^{p'}_{s+n,s+n}) + \ldots + cov(\; \eta^{p}_{s+n-1,s+n} + \eta^{p'}_{s+1,s+n})$$
$$\vdots$$
$$\vdots$$
$$+ cov(\; \eta^{p}_{s+1,s+n} + \eta^{p'}_{s+n,s+n}) + \ldots + cov(\; \eta^{p}_{s+1,s+n} + \eta^{p'}_{s+1,s+n}).$$

$$= cov(\ln(LNV^{p}_{n}), \ln(LNV^{p'}_{n}))$$
$$= \Sigma^{n}_{k=1} \Sigma^{n}_{j=1} cov(\; \eta^{p}_{s+k,s+n} + \eta^{p'}_{s+j,s+n}) \qquad (4\text{-}15)$$

We can obtain the individual covariance values $cov(\eta^{p}_{s+k,s+n}, \eta^{p'}_{s+j,s+n})$ from the estimated variance-covariance matrix $\sum$ of lognormal forecast updates, and hence we can calculate value of the whole term, $cov(\ln(D^{p}_{s+n}), \ln(D^{p'}_{s+n}))$.

When LNV^{p}_{n} for a particular period $n \in \{1, \ldots, M\}$ is lognormal with parameters μ^{p}_{n} and $\sigma^{2}_{p,n}$, $LNV^{p}_{n} * F^{p}_{s,s+n}$ has a lognormal distribution with parameters $(\ln(F^{p}_{s,s+n}) + \mu^{p}_{n})$ and $\sigma^{2}_{p,n}$ from the above lognormal Property 1. Therefore, using the relationship between lognormal distribution and normal distribution, which is stated by equation (4-2), $cov(D^{p}_{s+n}, D^{p'}_{s+n})$ is calculated as:

$$
\begin{aligned}
cov(D^{p}_{s+n}, D^{p'}_{s+n}) &= cov(LNV^{p}_{n} * F^{p}_{s,s+n}, LNV^{p'}_{n} * F^{p'}_{s,s+n}) \\
&= e^{E[\ln(LNV^{p}_{n} * F^{p}_{s,s+n})] + E[\ln(LNV^{p'}_{n} * F^{p'}_{s,s+n})]} \\
&\quad * e^{(var(\ln(LNV^{p}_{n})) + var(\ln(LNV^{p'}_{n})))/2} \\
&\quad * (e^{cov(\ln(LNV^{p}_{n}), \ln(LNV^{p'}_{n}))} - 1) \qquad (4.16)
\end{aligned}
$$

where $cov(\ln(LNV^{p}_{n}), \ln(LNV^{p'}_{n}))$ is given by equation (4-15), and $var(\ln(LNV^{p}_{n}))$ and $E[\ln(LNV^{p}_{n} * F^{p}_{s,s+n})]$ are given by equations (4-11) and (4-12), respectively.

Therefore, using a 95 percent service level, target level of aggregate safety stock for product group *Gi* for period $n \in \{1, \dots, M\}$ of the 12-week planning horizon, SS_n^p, based on demand variability becomes:

$$SS_n^p = z * \sqrt{var(\Sigma_{p \in Gi} D_{s+n}^p)} \quad \text{where } z = 1.645. \qquad (4\text{-}17)$$

The way to link forecast evolution with production planning should be adaptive to changes in the planning environment, which is a dynamic and constantly changing. Hence, it is important that the way of integration has the property of making use of new data and inputs to adapt to new environments. Accordingly when that happens, the MPS model adjusts and updates itself to determine target safety stock levels. Under multiplicative model, given (4-13) and (4-17) and all expressions that specify (4-13) and (4-17), forecast values by period of the planning horizon are employed to obtain the corresponding variance and covariance terms. Therefore, the period in which the MPS model runs has an impact on target safety stock levels through incorporating the forecast vector available at that period into calculation of variance and covariance terms.

CHAPTER 5

COMPUTATIONAL STUDY AND COMPARISON

The emphasis in computational study is restricted to analysis of beginning inventory levels of finished products and delivery performance of the manufacturer under three different methods of establishing target safety stock levels in the described master production scheduling environment. Each safety stock method is considered as a vehicle for expressing the relationship between forecast updating behavior of the buyer and inventory levels of the manufacturer. Hence the way of establishing safety stock is essential to adequately and efficiently accommodate the related uncertainty inherent in forecast evolution. Naturally, more accurate forecasts in the master schedule result in more stable production plans leading to low inventory levels together with high delivery performance. In practice, however, variations between forecasts and realized demands are inevitable and forecast accuracy problem may appear.

We present details of three master production scheduling - MPS models below. The first one is the original MPS model currently in use. The other two models are variants of the first model. Each one of the linear programming models described below makes production decisions for each SKU at a weekly level of detail, and employs the standard production, inventory, and demand recursion. The models are based on the policy of preparing period's forecasts at the beginning of that period. In other words, at the beginning of a period, MPS tries to prepare the finished goods inventory as much as that period's forecast and target safety stock level.

5. 1. MPS Model – 0

The first master scheduling model, which may be referred to as MODEL-0, is abstracted from the MPS model that is currently in use. For MODEL-0, target safety stock levels are determined exogenously based on the variability of historical forecast errors as stated before in equation (4-6). They are fed into the model at a constant level for each period of the 12-week planning horizon.

The following indices are used for parameter and variable definitions:

p for products such that	$p \in \{ p1, p2, \ldots, p17 \}$
w for weeks of the planning horizon such that	$w \in \{1, 2, \ldots, 12 \}$
L for production lines such that	$L \in \{L1, L2, \ldots, L8 \}$
s for labor type such that	$s \in \{s1, s2 \}$

The production and inventory related decision variables are defined as follows:

I_w^p : total beginning inventory on hand for product p at the start of period w. It is an input for week $w = 1$

Y_w^p : total production amount of product p produced during period w

$Fslack_w^p$: represent shortfall in w-step ahead forecast for product p

$SSslack_w^p$: represent shortfall in target safety stock level of period w for product p

$LHU_{L,w}$: total production time-hours usage for production line L in period w

$MHU_{s,w}$: total man-hours usage of labor type s in period w

$oMHU_{s,w}$: total overtime man-hours usage of labor type s in period w

The following production and inventory related parameters are used:

SSL^p : target safety stock level for product p determined by the current method using equation (4-6) and fixed at a constant level for each period of the 12-week planning horizon

$rMH_{s,w}$: total regular man-hours available for labor type s in period w

$oMH_{s,w}$: total overtime man-hours available for labor type s in period w

LR_L^p : total number of workers required to produce one unit of product p on production line L

$LH_{L,w}$: total production time-hours available for production line L in period w

T_L^p : average production rate per hour for product p on production line L

SR_L^p : average scrap rate for product p on production line L

BR_L^p : average machine breakdown rate in production line L when producing product p

OD_w : overtime decision at period w, which is 1 if overtime is planned at period w and zero otherwise

The objective function will be defined in terms of the following economic parameters:

PC_L^p : total production costs per unit of product p on production line L accrued until finished goods inventory

$WFslack$: objective function weight for per unit of shortfall in forecast requirements

$WSSslack$: objective function weight for per unit of shortfall in target safety stock level, which is much lower than $WFslack$

OC_s : cost per overtime man-hour for labor type s

P^p : priority of product p

h : multiplier for unit holding cost per period for products

The below linear programming model determines the production requirements of products at each period of the 12-week planning horizon with the objective of minimizing weighted combined costs of penalties per unit-violation of forecast requirements, penalties per unit-violation of target safety stock level requirements, inventory holding cost, and overtime man-hour cost.

MINIMIZE $\sum$ (Penalty for per unit of $Fslack_w^p$ + Penalty for per unit of $SSslack_w^p$ + Penalty for per unit of $SLFslack_w^p$ + Inventory Holding Costs + Overtime Costs)

SUBJECT TO

I_1^p = Initial Inventory of product p

(C1) $I_{w+1}^p = I_w^p + Y_w^p - (F_w^p - Fslack_w^p)$ $\forall$ product p and period w

(C2) $I_w^p \geq (F_w^p - Fslack_w^p) + (SSL_w^p - SSslack_w^p)$ $\forall$ product p and period w

(C3) $SSslack_w^p \leq SSL_w^p$ $\forall$ product p and period w

(C4) $I_w^p \le (\sum_{j=0}^{n} F_{w+j}^p) + SSL_w^p - SLFslack_w^p$ $\forall$ product p and period w

(C5) $MHU_{s,w} = \sum_p LR_s^p * Y_w^p \;\; / \;\; (T_L^p * (1 - SR_L^p) * (1 - BR_L^p))$

(C6) $MHU_{s1,w} \le rMH_{s1,w} + oMH_{s1,w}$ $\forall$ period w

(C7) $\sum_s MHU_{s,w} \le \sum_s (rMH_{s,w} + oMH_{s,w})$ $\forall$ period w

(C8) $oMHU_{s,w} \le oMH_{s,w} * OD_w$ $\forall$ labour type s and period w

(C9) $LHU_{L,w} = \sum_p Y_w^p \;\; / \;\; (T_L^p * (1 - SR_L^p) * (1 - BR_L^p))$ $\forall$ line L and period w

(C10) $LHU_{L,w} \le LH_{L,w}$ $\forall$ line L and period w

Constraint (C1), the inventory balance equation, relates the production, inventory, and demand variables to each other for product p and for period w. This constraint balances the beginning inventory status of each SKU at the beginning of week w+1 from the beginning inventory status of the previous week w plus production during the previous week w minus forecasted demand for the previous week w. Here the model assumes that demand of week w is known with certainty and hence forecasted demand figures are used in the recursion to represent the shipment amounts made at week w.

According to the existing performance measurement system, the manufacturer's beginning inventory I_w^p at period w must be at least the minimum inventory level specified for that item to satisfy high percentage of demand in that period from the finished goods inventory. Constraint (C2) expresses this condition, which requires that forecasted demand and target safety stock level for all items must be met in its entirety as much as possible. Production is also constrained by the shelf life decisions. Constraint (C4) limits the beginning inventory of items with short shelf life.

Constraints (C1), (C2), (C3) and (C4) include slack variables to represent amount of violation for the corresponding constraints. For illustration, by using slack variable $Fslack_w^p$ in inventory balance equation (C1), negative inventory positions are eliminated from the consideration. Constraints containing slack variables are associated with relevant objective function terms. When a slack variable takes value greater than zero it means that corresponding constraint will be violated at a cost, which is linear penalty per unit of violation. The occurrence of such a violation may indicate that a little overtime is needed, or it may signal a capacity bottleneck that cannot be avoided in the short term.

Constraint (C5) computes total man-hours usage of labor type *s* in period *w* by using production rates, labor requirements and total production amounts for all products produced at period *w*. Constraint (C6) and (C7) ensure that total man-hours usage for labor types is less than or equal to total regular man-hours plus total overtime man-hours available in period *w*. Constraint (C8) ensures that total overtime man-hours usage of labor type *s* in period *w* is less than total overtime man-hours available.

Constraints (C9) and (C10), line capacity constraints, relate the total production time-hours consumed by production line *L* to total production time-hours available for that line at period *w*. And additional constraints on man-hours availability, which relate the man-hours usage to regular and overtime man-hours available per manpower types for that period, are also considered. For simplicity, additional constraints that are included in the analysis are not reproduced here since they are not relevant directly to safety stock analysis of the thesis.

5. 2. MPS Model – 1

The second master production scheduling model considered, MODEL-1, has the same structure as the original model, MODEL-0, except that target safety stock levels are determined from demand variability as opposed to forecast error variability. For MODEL-1, target safety stock level for product *p*, SSL^p, is determined exogenously employing the estimated variance-covariance matrix of forecast updates under the assumption of independent product demands by using equation (4-13). Being a parameter, they are fed into the model at a constant level for each period of the 12-week planning horizon as in MODEL-0.

5. 3. MPS Model – 2

The third master scheduling model, which may be referred to as MODEL-2, employs the proposed method of establishing target safety stock levels. They are based on demand variability that is calculated from the variability of forecast updates by the expressions that specify equation (4-13) and (4-17). In that method we take into account correlations of forecast updates across products and time periods, which are captured from the estimated variance-covariance matrix.

Different from the existing safety stock method, the proposed safety stock method determines target safety stock levels endogenously within the as MODEL-2 and does not fix target safety stock at a constant level over the planning horizon. The premise of this practice is that target safety stock levels vary over the 12-week planning horizon as uncertainty is being resolved while forecasts evolve. This can be seen from the above $var(\ln(D_s^p))$ expression in which as the period being forecasted becomes far from the current period, the number of random-update terms increase. For illustration, for the third period of the planning horizon target safety stock level for product *p* is established by quantifying variability of demand in that period of the MPS planning horizon.

In addition to indices used in the MODEL-0 the index *Gi* for product groups such that $Gi \in \{G1, G2, \ldots, G9\}$ is used in MODEL-2.

Being a parameter in MODEL-0, target safety stock level for product *p*, SSL^p, which is determined by the current safety stock method using equation (4-6) and fixed at a constant level for each period of the 12-week planning horizon, is now replaced by the following decision variable in MODEL-2 as:

$SSvar_w^p$: target safety stock level for product *p* in period *w* determined endogenously within MODEL-2 (decision variable)

The following parameters are employed within the related constraints in order to determine the optimal value of the decision variable $SSvar_w^p$:

$SSind_w^p$: target safety stock level for product p in period w determined independently of the other products using equation (4-13)

$SSgroup_w^{Gi}$: target aggregate safety stock level for product group Gi in period w determined by considering correlations across products and time periods using equation (4-17)

The objective function coefficients and production and inventory related parameters are the same as in MODEL-0. However, in MODEL-2, decision variables $SSslack_w^p$ are handled in a different way from the other two models such that products having lower priority result in lower target safety stock levels.

The below linear programming model determines the production requirements of products at each period of the 12-week planning horizon with the objective of minimizing weighted combined costs of penalties per unit-violation of forecast requirements, penalties per unit-violation of target safety stock level requirements, inventory holding cost, and overtime man-hour cost.

MINIMIZE $\sum$ (Penalty for per unit of $Fslack_w^p$ + Penalty for per unit of $SSslack_w^p$ + Penalty for per unit of $SLFslack_w^p$ + Inventory Holding Costs + Overtime Costs)

SUBJECT TO

I_1^p = Initial Inventory of product p

(C1) $I_{w+1}^{p} = I_{w}^{p} + Y_{w}^{p} - (F_{w}^{p} - Fslack_{w}^{p})$ $\forall$ product p and period w

($\overline{\text{C2}}$) $I_{w}^{p} \geq (F_{w}^{p} - Fslack_{w}^{p}) + (SSvar_{w}^{p} - SSslack_{w}^{p})$ $\forall$ product p and period w

($\overline{\text{C3}}$) $SSslack_{w}^{p} \leq SSvar_{w}^{p}$ $\forall$ product p and period w

(C-I) $SSvar_{w}^{p} \leq SSind_{w}^{p}$ $\forall$ product p and period w

(C-II) $\sum_{p \in Gi} SSvar_{w}^{p} \geq SSjo\text{int}_{w}^{Gi}$ $\forall$ product group Gi and period w

(C-III) $SSvar_{w}^{p} \leq SSvar_{w+1}^{p}$ $\forall$ product p and period w

(C4) $I_{w}^{p} \leq (\sum_{j=0}^{n} F_{w+j}^{p}) + SSvar_{w}^{p} - SLFslack_{w}^{p}$ $\forall$ product p and period w

(C5) $MHU_{s,w} = \sum_{p} LR_{s}^{p} * Y_{w}^{p} \;\; / \;\; (T_{L}^{p} * (1 - SR_{L}^{p}) * (1 - BR_{L}^{p}))$

(C6) $MHU_{s1,w} \leq rMH_{s1,w} + oMH_{s1,w}$ $\forall$ period w

(C7) $\sum_{s} MHU_{s,w} \leq \sum_{s} (rMH_{s,w} + oMH_{s,w})$ $\forall$ period w

(C8) $oMHU_{s,w} \leq oMH_{s,w} * OD_{w}$ $\forall$ labour type s and period w

(C9) $LHU_{L,w} = \sum_{p} Y_{w}^{p} \;\; / \;\; (T_{L}^{p} * (1 - SR_{L}^{p}) * (1 - BR_{L}^{p}))$ $\forall$ line L and period w

(C10) $LHU_{L,w} \leq LH_{L,w}$ $\forall$ line L and period w

The inventory balance equation, constraint (C1), remains the same as in MODEL-0. However, constraint ($\overline{\text{C2}}$) now strives to attain that the beginning inventory, I_{w}^{p} at period w is equal to forecasted demand plus target safety stock level as much as possible. Constraint ($\overline{\text{C3}}$) is now comprised of two different decision variables and optimal value of a decision variable is constrained by the optimal value of the other decision variable.

Constraint (C-I) ensures that target safety stock level for product $p, SSvar_w^p$, is equal to at most $SSind_w^p$, which is target safety stock level determined under the assumption of independent product demands. Constraint (C-II) defines a lower limit, $SSgroup_w^{Gi}$, for the total of individual target safety stock levels of products belonging the same product group. Constraint (C-III) ensures that target safety stock level at period w is less than or equal to that of period w + 1 to remain consistent with the fact that more uncertainty is resolved in period w than in period w + 1 over the planning horizon.

A collection of individual constraints (C-I) for the set of products belonging product group Gi can be consolidated into the following expression for product group Gi as:

$$\sum_{p \in Gi} SSvar_w^p \leq \sum_{p \in Gi} SSind_w^p \qquad \forall \text{ product group } Gi \text{ and period } w \tag{5-1}$$

It can be written for product group Gi by arranging the above expression and constraint (C-II) that:

$$SSjoint_w^{Gi} \leq \sum_{p \in Gi} SSvar_w^p \leq \sum_{p \in Gi} SSind_w^p \qquad \forall \text{ product group } Gi \text{ and period } w \tag{5-2}$$

$$SSjoint_w^{Gi} \leq \sum_{p \in Gi} SSind_w^p \qquad \forall \text{ product group } Gi \text{ and period } w \tag{5-3}$$

Actually, this is the natural ordering we would like to obtain as a result of optimization. Note that if products belonging group Gi had independent demands actually, then covariance terms in the expressions that specify equations (4-13) and (4-17) would be zero and constraint (C-I) and expression (5-2) would become as follows:

$$SSvar_w^p = SSind_w^p$$
$$SSjoint_w^{Gi} = \sum_{p \in Gi} SSvar_w^p = \sum_{p \in Gi} SSind_w^p$$

Expression (5-2) follows from the fact that $-1 \le corr(D_s^p, D_s^{p'}) \le 1$, or equivalently,

$$-1 \le \frac{cov(D_s^p, D_s^{p'})}{\sqrt{var(D_s^p)}\sqrt{var(D_s^{p'})}} \le 1 \qquad \Rightarrow \qquad cov(D_s^p, D_s^{p'}) \le \sqrt{var(D_s^p)}\sqrt{var(D_s^{p'})}$$

Hence by using this result in $SSjoint_w^{Gi}$ calculation we can obtain expression (5-3) as follows:

$$\begin{aligned}
SSjoint_w^{Gi} &= \text{K} * \sqrt{var(D_s^p + D_s^{p'})} \\
&= \text{K} * \sqrt{var(D_s^p) + var(D_s^{p'}) + 2 * cov(D_s^p, D_s^{p'})} \\
&\le \text{K} * \sqrt{var(D_s^p) + var(D_s^{p'}) + 2 * \sqrt{var(D_s^p)}\sqrt{var(D_s^{p'})}} \\
&\le \text{K} * \sqrt{\left(\sqrt{var(D_s^p)} + \sqrt{var(D_s^{p'})}\right)^2} \\
&\le \text{K} * \left(\sqrt{var(D_s^p)} + \sqrt{var(D_s^{p'})} \right) \\
SSjoint_w^{Gi} &\le \sum_{p \in G} SSind_w^p
\end{aligned}$$

5. 4. Experimental Runs

Before making the experimental runs, several relevant MPS problem characteristics are worth noting:

- Planned cumulative production lead time is set at one-week. Hence MPS requirements to be produced during period w of the 12-week planning horizon are meant to be available for delivery at the beginning of period $w + 1$.
- Although corresponding shipments are actually made during period $w + 1$, we assume that total amount of shipments to be made during period $w + 1$ are made all at once at the beginning of period $w + 1$.
- It is assumed that the demand for any product that is not satisfied due to stockouts is treated as lost sales. Indeed, in the current environment backlogging is allowed, that is, amount of customer order placed by distributors and chain markets that remains unsatisfied can be met later than it occurs. Therefore the demand data made available to us reflects the backlogged amount of past demands.
- Production decisions in the running week are fixed. That is, as the models are solved at the beginning of each period in a rolling format, it recommends production levels for the second period (one week later the running week) and beyond covering 12 weeks. The main reason for this is that materials are not constrained in the models and it takes time to procure raw materials and packaging materials in place for production.

We consider three performance metrics for each product:

1. Total amount of customer orders quantified in tons that are filled from the stock
2. Total number of customer orders that are completely filled from the stock
3. Average projected inventory on hand at the beginning of week

Note that objective function values are not comparable for all models as the safety stock values are handled differently. Moreover, the weighting structure in the objective function results in values that are not numerically interpretable and comparable.

To experiment the above MPS models in parallel for comparison in the rolling planning horizon context, we roll each model for a 12-week period in succession using historical forecast and demand data and obtain 12 different, successive master production plans. At each simulation week, we execute the first week of the resulting master production plan by implementing the standard inventory, production and demand recursion. In this recursion, we treat MPS production requirements as actual production amounts and employ the historical demands of the related week to balance the beginning inventory that will be used in the next roll.

In each roll, we calculate and collect the following statistics corresponding to the performance metrics considered for each product:

- target and realized safety stock levels for each period of the 12-week planning horizon

- total inventory left over at the end of the first period of the resulting master production plan
- total amount of demand that are filled from the beginning inventory
- the number of successes (a success occur if demand is completely filled from the beginning inventory)

We present the results of computational study with these statistics and explore the benefits of integrating forecast volatility measure into master production scheduling models through the use of different methods of establishing products' target safety stock levels.

Some relevant general observations from the experimental runs are worth noting.

- Weekly forecasts provided for each period of the 12-week planning horizon are satisfied in the same way. That is, decision variable $Fslack_{w}^{p}$ for a particular product p and period w takes the same value as a result of optimization with each one of the three models.

- Using different ways of determination, target safety stock levels are established in a different way in each model. MODEL-2 establishes lower total level of target safety stock. And, employing correlations and product priorities it allocates production time-hour capacity between products differently from the other two models.

- Target safety stock requirements are satisfied in different way in each model. That is, decision variable $SSslack_w^p$ for a particular product p and period w takes the different values as a result of optimization with each one of the three models. And hence, production quantities Y_w^p recommended for a particular product p and period w by the models are different.

For each MPS model and product p, averaged over the 12-week experimentation period target safety stock and ending inventory levels are obtained and presented in Table 3. Average target safety stock levels indicate that MODEL-2 supports holding lowest of total amount of safety stock levels. Note that the values in the row labeled as TOTAL are simply the sum of the values in the respective columns. Therefore, these totals have no significance with respect to inventory value. Considering bivariate correlations of forecast updates across products and time periods and employing the product priority scheme more effectively, MODEL-2 determines more realistic target safety stock levels. For example, it determines higher levels for some products than MODEL-0 and lower levels for some others. When operating almost at full capacity, impact of product priorities on target safety stock levels becomes more apparent if MODEL-2 is used since the issue of allocating production line time-hours availability between products is dealt with more effectively by MODEL-2.

Table 3 Average target safety stock levels and average ending inventory levels

	Average target safety stock levels (tons)			Average ending inventory levels (tons)		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p1	15.8	29.4	9.7	27.6	40.2	21.1
p2	14.5	11.5	12.0	15.0	11.9	12.3
p3	33.8	40.5	34.6	42.8	50.1	43.8
p4	3.3	2.4	2.4	2.2	1.3	1.5
p5	8.7	9.0	4.0	11.5	12.0	6.9
p6	23.6	28.4	28.7	43.2	48.1	49.0
p7	8.5	13.6	2.3	23.1	28.4	18.2
p8	4.8	6.7	0	7.9	9.8	3.2
p9	4.4	6.7	5.5	8.0	10.4	9.3
p10	5.8	4.7	2.0	9.3	8.1	5.6
p11	3.9	3.6	3.6	4.5	4.2	4.1
p12	7.7	9.8	10.7	13.0	14.9	15.9
p13	10.0	15.9	9.8	14.6	20.1	14.1
p14	6.3	8.8	7.2	9.4	11.7	10.4
p15	4.5	7.5	7.8	8.3	11.1	11.4
p16	21.2	35.3	29.9	46.7	60.2	54.9
p17	26.5	32.2	18.4	36.2	42.5	28.3
TOTAL	***203.0***	***265.9***	***188.6***	***323.3***	***385.0***	***310.0***

In Table 4 below, entries of Table 3 is given as a percent of mean weekly demand for each product *p*. Note that for MODEL-2 results in lower target safety stock levels in terms of percent of mean weekly demand.

Table 4 Average target safety stock levels in percent of mean weekly demand (%)

	MODEL-0	MODEL-1	MODEL-2
p1	57.3	106.6	35.2
p2	96.7	76.7	80.0
p3	78.9	94.6	80.8
p4	150.5	109.4	109.4
p5	75.9	78.5	34.9
p6	54.6	65.8	66.5
p7	36.8	58.9	10.0
p8	60.6	84.6	0.0
p9	54.7	83.4	68.4
p10	62.2	50.4	21.5
p11	86.1	79.5	79.5
p12	59.1	75.2	82.1
p13	68.4	108.8	67.1
p14	66.7	93.1	76.2
p15	54.2	90.3	93.9
p16	45.4	75.7	64.1
p17	73.2	88.9	50.8
TOTAL	***62.9***	***82.2***	***58.3***

Under MODEL-2 for each period of the 12-week planning horizon, Table 5 reports target safety stock levels for each period of the planning horizon, $SSvar_w^p$, averaged over the 12-week experimentation period. Target safety stock levels do vary by period of the 12-week planning horizon. The results are consistent with the premise of MODEL-2, which is that target safety stock levels vary over the 12-week planning horizon as uncertainty is being resolved while forecasts evolve.

Table 5 Averages of target safety stock levels, $SSvar_w^p$, by periods of the planning horizon under MODEL-2 (in tons)

		PERIODS											
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
PRODUCTS	**p1**	9.1	9.4	9.4	9.7	9.7	9.9	9.9	9.9	9.9	9.9	9.9	9.9
	p2	11.0	11.8	11.8	11.9	12.0	12.0	12.0	12.0	12.2	12.3	12.4	12.4
	p3	31.8	32.2	32.2	34.0	34.6	35.0	35.0	35.1	35.6	35.9	36.9	36.9
	p4	2.0	2.1	2.1	2.1	2.2	2.3	2.4	2.4	2.5	2.6	2.9	3.0
	p5	2.0	2.5	2.5	2.6	2.8	3.4	3.8	4.2	4.8	5.6	7.0	7.2
	p6	21.6	23.3	24.4	26.5	28.1	28.4	29.0	29.7	30.5	31.4	35.5	36.2
	p7	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.1	2.8	2.8	3.0	3.0
	p8	0	0	0	0	0	0	0	0	0	0	0	0
	p9	3.3	3.5	3.5	3.9	5.0	5.0	5.2	6.9	7.4	7.4	7.6	7.6
	p10	1.2	1.2	1.3	1.5	2.1	2.1	2.2	2.3	2.4	2.4	2.4	2.4
	p11	3.3	3.4	3.4	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
	p12	9.3	9.9	10.1	10.5	10.7	10.8	11.0	11.0	11.1	11.2	11.6	11.7
	p13	7.0	7.1	7.1	7.9	9.0	9.5	10.2	11.1	11.7	12.1	12.7	12.8
	p14	5.4	5.9	6.2	6.6	6.9	7.1	7.2	7.5	7.6	7.8	9.0	9.2
	p15	6.9	6.9	7.0	7.2	7.6	7.7	7.7	8.0	8.0	8.1	9.0	9.0
	p16	20.7	22.0	25.0	29.5	30.9	30.9	31.4	32.0	32.3	33.1	35.0	35.6
	p17	15.3	16.5	18.1	18.2	18.3	18.3	18.3	18.5	18.8	19.5	20.4	20.8
Total		**151.8**	**159.6**	**166.2**	**177.7**	**185.6**	**188.1**	**191.0**	**196.4**	**201.3**	**205.6**	**218.8**	**221.4**

It is important to note that the performance of MODEL-2 is more seriously affected by the product priority scheme than that of the other models. This occurs because

MODEL-2 recommends higher target safety stock levels for high priority products and lower levels for low priority products. For example, it sets target safety stock level of product *p*8, which has the lowest product priority over the other products, at zero.

Another notable observation obtained from Table 5 is that there is a considerable increase in average target safety stock level of product *p*5, $SSvar_w^{p5}$, from period 1 to period 12. It goes up from 2.0 tons in period 1 to 7.2 tons in period 12. This observation is a result of what MODEL-2 intends to perform. Product *p*5 is contained in product group *G*2 together with product *p*4 and also in group *G*7 with products *p*1, *p*2, *p*3, *p*4. Actually, product *p*4 and *p*5 have the same brand with different package size and hence they can be viewed as substitute products. Investigating the covariance terms $cov(\ln(LNV_n^{p}),\ln(LNV_n^{p'}))$ for product *p*5 (calculated using equation (4-15) and specifies equation (4-16)) reveals that value of the covariance term, $cov(\ln(LNV_n^{p5}),\ln(LNV_n^{p4}))$ goes up from 0.05224 in period 1 (n = 1) to 0.36729 in period 12 (n = 12). In other words, degree of correlation between products *p*5 and *p*4 is higher in period 12 than in period 1. This results in higher $cov(D_{s+n}^{p},D_{s+n}^{p'})$ value calculated using equation (4.16), and hence higher target safety stock level for product group *G*2, $SSjoint_w^{G2}$, in period 12 determined by using equation (4.17). Table 6 contains $SSjoint_w^{G2}$'s averaged for each period of the 12-week planning horizon. Table 7 tabulates target safety stock levels for individual products $p4$ and $p5$, $SSind_w^p$'s, averaged for each period of the 12-week planning horizon. Note that $SSind_w^p$'s for a particular product p are calculated independently from the other products using equation (4-13).

Table 6 Averaged target safety stock levels for product group $G2$, $SSjoint_w^{G2}$, by periods of the planning horizon under MODEL-2 (in tons)

	PERIODS											
	1	2	3	4	5	6	7	8	9	10	11	12
G2	3.3	3.5	3.9	4.5	5.0	5.6	6.2	6.6	7.2	8.1	9.9	10.2

Table 7 Averaged target safety stock levels for individual products $p5$ and $p4$, $SSind_w^p$, by periods of the planning horizon (in tons)

	PERIODS											
	1	2	3	4	5	6	7	8	9	10	11	12
p4	2.4	2.4	2.5	2.6	2.7	2.7	2.8	2.8	2.8	2.9	3.1	3.1
p5	8.8	9.1	9.3	9.7	9.8	10.0	10.3	10.3	10.6	11.3	13.1	14.2

Therefore, constraints (C-I), (C-II) and (C-III) for MODEL-2 produce the above observation concerning $SSvar_w^{p5}$ for product $p5$. Given the increase in $SSjoint_w^{G2}$ from period 1 to period 12, constraints (C-II) ensure that the sum of $SSvar_w^{p4}$ and $SSvar_w^{p5}$ for product $p4$ and $p5$ in period 12 is higher than that of period 1. However, how an increase in this sum should be allocated to an individual $SSvar_w^p$ for each product p within a particular product group is determined by constraints (C-I) and

(C-III). Table 7 indicates that from period 1 to period 12 there is more increase in $SSind_w^p$ for product $p5$ than for product $p4$. Therefore, given product priority scheme and correlations, constraint (C-I) ensures that the increase in $SSvar_w^{p5}$ is higher than the increase in $SSvar_w^{p4}$.

We evaluate the other performance measures considered in terms of product groups. As mentioned before we construct nine product groups in terms of brand and production line. Product group $G1$ is a brand group including products with different package sizes. For products contained in product group $G1$ and for each MPS model, Table 8 reports two performance metrics, total number of demands completely filled from the beginning inventory and percentages of total amount of demand filled from the beginning inventory. It also tabulates averages of realized safety stock levels at the first period of the 12-week planning horizon. The results indicate that, overall, MODEL-2 performs better than MODEL-0 and MODEL-1. Figures corresponding to two performance metrics demonstrate that MODEL-2 results in measurable savings in safety stock levels while performing better delivery performance than MODEL-0 and nearly the same performance as MODEL-1. For example, for product $p1$ MODEL-0 and MODEL-1 determine excessive safety stock levels. Obviously, with more experimental runs and hence more resulting computational data, figures for performance measures will be more reliable with respect to service level and one is expected to compare the performances of the models better.

Table 8 Performance measures for product group *G*1

	averages of realized SS levels (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p1	6.8	13.6	3.9	9	10	9	87.4	91.8	90.2
p2	7.1	4.7	5.9	10	9	10	96.8	97.3	97.9
p3	15.6	19.8	16.1	9	10	10	89.1	95.2	93.7

Product groups *G*2, *G*3, *G*4 and *G*5 exhibit a similar behavior as product group *G*1. Product group *G*4 differs from the other brand groups in logistical demands such that products in *G*4 are channeled to end consumers through chain markets.

Each one of product groups *G*7, *G*8 and *G*9 contains products that are being processed on the same production line so that they compete for the same production line capacity. Therefore, under the low capacity setting, efficacy of MODEL-2 becomes also more apparent for these product groups.

Table 9 Performance measures for product group *G*9

	averages of realized SS level (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p12	2.7	3.6	4.1	10	11	11	92.3	95.0	96.0
p13	3.9	6.8	3.7	10	11	11	93.7	94.9	94.1
p14	1.9	3.2	2.4	9	10	10	88.0	90.8	90.2
p15	1.1	2.4	2.6	9	10	11	89.2	91.9	92.4

Table 9 shows the results corresponding to product group *G*9. Note that, for product *p*13 of that group, MODEL-2 propose lower average realized safety stock levels than the other two models while achieving better delivery performance than MODEL-0 and the same delivery performance as MODEL-1 does. This finding reflects what MODEL-2 intends to perform. Similar tables for the other product groups are given in Appendix E.

CHAPTER 6

CONCLUSION

This thesis has explored how forecast volatility is incorporated into an aggregate production planning problem with the aim of determining target safety stock levels. There can be no doubt of direct impact of more accurate forecasts provided by the buyer on customer delivery performance of the manufacturer, as well as its indirect impact on chain-wide efficiency.

We use a probabilistic model where forecast modification activity performed by the buyer over time is characterized by random variables. We bridge theory and practice in the second part of the thesis. For our application in the manufacturer's MPS problem, we provide a way of safety stock level determination that describes typical information and data flow in an abstract manner.

In the supply chain under consideration, higher delivery performance with fewer inventories represents a vital competitive dimension for the manufacturer and for the buyer as a whole. Basically, inventory management reveals the way the business is being run and indicates in a sense how well the business is performing against the competition. Through a focused study, we have addressed incorporating forecasting characteristics of the buyer into production planning activities of the manufacturer. In particular, we examined the concept of correlations captured by variance-covariance matrix of forecast modifications and their effect on target safety stock levels, and in turn, on delivery performance.

By determining safety stock levels using variability of demand as opposed to variability of forecast errors, the manufacturer can prevent the buyer from manipulating the operations to conform to its functional objectives, which is the reality of the supply chain considered since she buyer performs the dominant functions, which sales and marketing activities and in turn has more power than the manufacturer.

For the MPS environment described, we compared the original production planning model, which considers variability of historical forecast accuracy in determining safety stock levels, with the proposed models that are based on variability of demand. One of the proposed models also takes into account correlations in forecast changes and establishes target safety stock levels endogenously within the model. We have observed that under multiplicative form of martingale model of forecast evolution the latter results in lower expected target safety stock levels with better allocating production capacity between products using priority scheme more effectively.

This study provides some contributions to fill a gap in current production/inventory policies literature concerning the impact on safety stock levels of correlations in forecast modifications and of forecast volatility measured by variability of forecast accuracy.

On a broader level, the recommendation to management of the group is to maintain and promote the corporate emphasis on efforts to achieve manufacturer and buyer's working together much more closely to realize their mutual business goals. To achieve greater coordination across the supply chain, information exchange barriers, such as a lack of appropriate corporate rules, tools and interface must be removed and information transparency must be achieved. Indeed, the lack of information visibility beyond the immediate partner in the supply chain can limit the efficacy of the results. Then analysis and models such as those presented in this thesis can help assess the impact of particular actions and prioritize improvement opportunities.

This information visibility includes sharing immediate customer's demand information as well as sharing inventory status data, and even capacity information. For example, forecasts depend solely on the demand signal from immediate downstream customers results in excessive safety stock, which is a direct effect of the bullwhip effect. Manufacturer's production planning system is based on production orders provided by the buyer and adjusted to support reliable delivery of a wide variety of products.

To manage the flow of the materials, production and delivery, and to link all production activity, the manufacturer has laid great emphasis on forecasts provided by the buyer, as expressed by production orders. However it is preferable to place some emphasis on other information about customers, suppliers and production, which affect the accuracy of the schedules derived from the master production schedule, where processes make items regardless of actual end demand, i.e. forecast oriented method.

The ultimate vision of higher delivery performance with fewer inventories assumes a supply chain capable of effectively responding to customer orders with just value-added amounts of inventory between each stage. Opportunities for future work on the topics addressed by this thesis involve ways of prioritizing products more dynamically with the aim of contributing to transform this vision into reality. Priority scheme of products may be determined by taking into account inventory status of distributors or even pipeline inventory so that if pipeline inventory for a product is relatively high then that product takes lower priority. Another possible alternative is to prioritize the products according to amount of backorder for that product so that higher backorder amount during lead time reflects higher priority. Echelon stock idea to prioritize the products is the third possible alternative. Echelon inventory level for a product is constrained by a management-specified upper level. Greater difference between actual echelon inventory level and this upper level for a product implies higher priority for that product.

The goal of this thesis is to address the critical issue of forecast churning and its related impact on inventory levels. Another measure of success will be if it stimulates the company to evaluate future opportunities to apply this work to practical business cases.

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APPENDIX A

SOME TIME SERIES PLOTS AND BOX-PLOTS

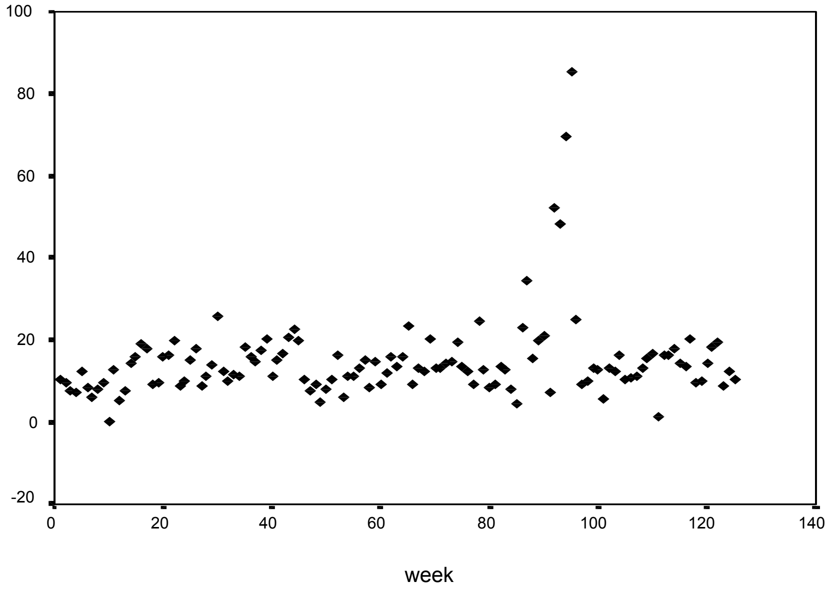


Figure 7 Time Series Plot for product $p13$

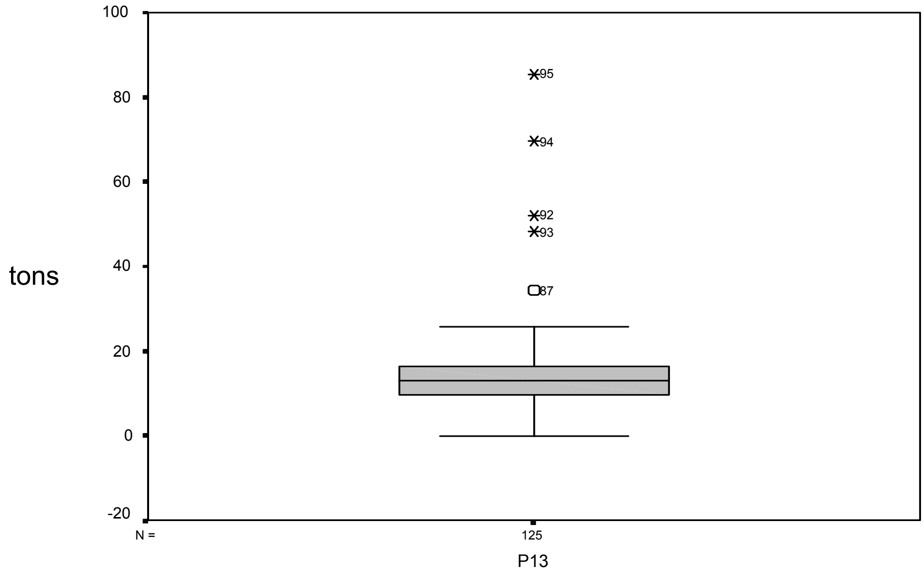


Figure 8 Box-Plot for product $p13$

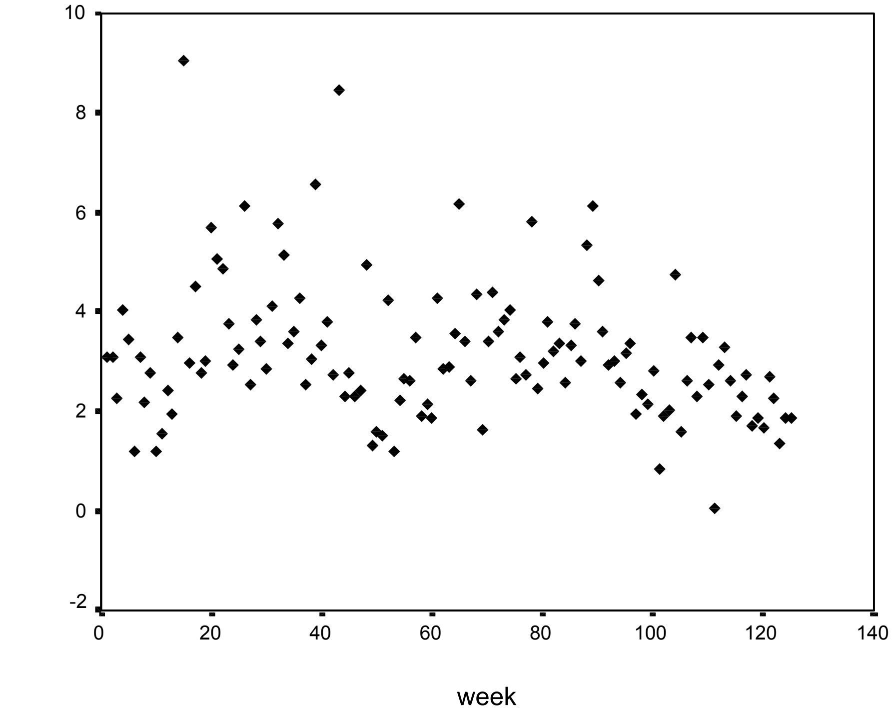


Figure 9 Time Series Plot for product $p16$

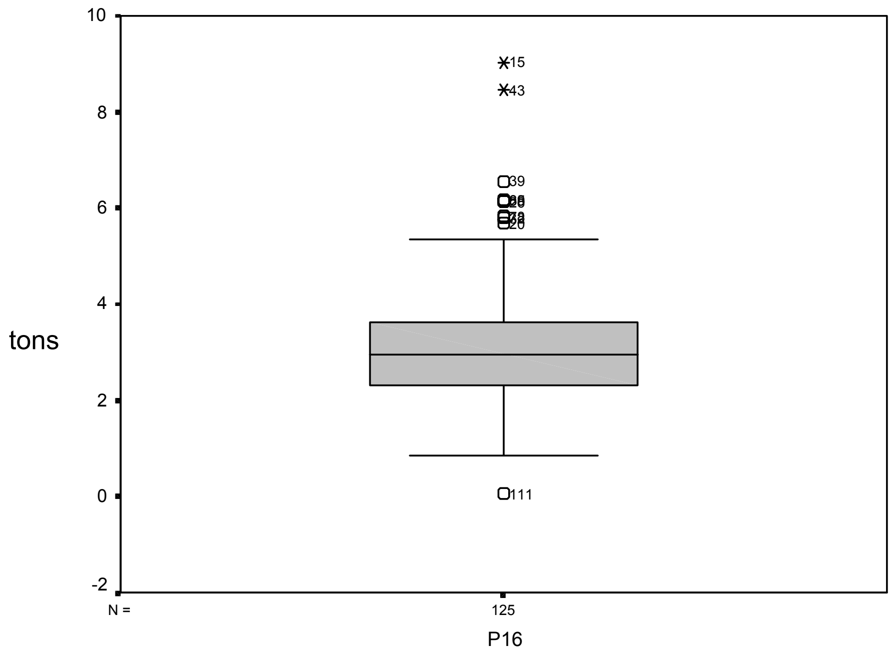


Figure 10 Box-Plot for product $p16$

APPENDIX B

SOME EXAMPLES OF FORECAST UPDATES

Table 10, 11 and 12 below contains samples of some logarithmic forecast updates expressed as natural logarithms of ratios of successive forecasts for product *p*13, *p*16 and *p*5, respectively. Where η_j for $j \in \{0, 1, \ldots, 11\}$ represent logarithmic forecast updates made in (j + 1)-period ahead forecasts to obtain j-period ahead forecast. In particular, column η_0 represents forecast errors expressed as natural logarithms of ratios of one-period ahead forecasts and actual demands.

Table 10 Some of Normal Forecast Updates for product $p13$

T	η_{11}	η_{10}	η_9	η_8	η_7	η_6	η_5	η_4	η_3	η_2	η_1	η_0
1	0	0	0	0	0	0	0	0	-0.02	-0.03	0	0.052
2	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.31
3	0	0	0	0	0	0	0	0	-0.02	-0.03	0.058	-0.65
4	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.79
5	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.15
6	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.05
7	0.373	0.368	0.059	-0.09	-0.02	0	0.1	-0.18	-0.13	0.054	0.047	-0.52
8	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.73
9	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.11
10	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.19
11	0	0	0	0	0	0	0	0	-0.02	-0.03	0	0.124
12	-0.11	-0.05	0.042	-0.01	-0.36	-0.36	-0.36	-0.35	-0.24	-0.26	-0.23	-0.78
13	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.51
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
94	0.154	0	0	0	0	0	0	0	-0.02	-0.03	-0.06	0.333
95	0	0	0	0	0	0	0	0	-0.02	-0.03	0	0.07
96	-0.06	0.021	-0.06	0	0	0	0	0	-0.02	-0.03	0	-0.27
97	0	0	0	0	0	0	0	0	0.165	-0.03	0	0.358
98	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.02
99	-0.13	0	0	0	0	0	0	0	-0.02	0.121	0	0.069
100	0	0	0	0	0	0	0	0	-0.02	-0.03	0	0.096
101	0	0	0	0	0	0	0	0	-0.02	0.149	0	-0.59
102	0	0	0	0	0	0	0	0	-0.02	-0.03	0	-0.23
103	0	0	0	0	0	0	0	0	-0.02	0.073	0	-0.24
104	-0.14	-0.06	0	0	0	0	0	0	-0.02	-0.03	0	0.5
105	0	0	0	0	0	0	0	0	-0.02	-0.03	0	0.098
106	0	0	0	0	0	0	0	0	0.064	0.146	0	-0.55
107	0	0	0	0	0	0	0	0	-0.02	0.101	0	-0.11

Note that for j = 4, 5, …, 11 variable η_j takes zero values dominantly. Variables η_0, η_3, η_2, on the other hand, take nonzero values. This indicates that there exists a forecast modification activity only for the first 3-4 periods of the 12-week planning horizon.

Table 11 Some of Normal Forecast Updates for product $p16$

T	η_{11}	η_{10}	η_9	η_8	η_7	η_6	η_5	η_4	η_3	η_2	η_1	η_0
1	0	0	0	0	0	0	0	0	0	0	0	0.664
2	0	0	0	0	0	0	0	0	0	0	0	0.392
3	0	0	0	0	0	0	0	0	0	0	0	-0.02
4	0	0	0	0	0	0	0	0	0	0	0	0.475
5	0	0	0	0	0	0	0	0	0	0	0	0.599
6	0	0	0	0	0	0	0	0	0	0	0	0.148
7	0.416	0.416	0.104	-0.09	-0.02	0	0.1	-0.17	-0.12	0.085	0.05	-0.18
8	0	0	0	0	0	0	0	0	0	0	0	-0.31
9	0	0	0	0	0	0	0	0	0	0	0	0.429
10	0	0	0	0	0	0	0	0	0	0	0	0.215
11	0	0	0	0	0	0	0	0	0	0	0	0.245
12	0.382	0.451	0.556	0.446	0	0	0	0	0	0	0	-0.11
13	0	0	0	0	0	0	0	0	0	0	0	-0.45
:	.	.	.	.	.	.	.	.	.	.	.	.
:	.	.	.	.	.	.	.	.	.	.	.	.
94	0.305	0	0	0	0	0	0	0	0	0	0	-0.69
95	0	0	0	0	0	0	0	0	0	0	0	-0.14
96	0.075	0.164	-0.08	0	0	0	0	0	0	0	0	-0.27
97	0	0	0	0	0	0	0	0	0	0	0	-0.1
98	0	0	0	0	0	0	0	0	0	0	0	-0.58
99	0.055	0	0	0	0	0	0	0	0	0	0	-0.27
100	0	0	0	0	0	0	0	0	0	0	0	-0.27
101	0	0	0	0	0	0	0	0	0	0	0	-0.71
102	0	0	0	0	0	0	0	0	0	0	0	-0.48
103	0	0	0	0	0	0	0	0	0	0.196	0	-0.74
104	9E-04	0.095	0	0	0	0	0	0	0	0	0	0.081
105	0	0	0	0	0	0	0	0	0	0	0	-0.68
106	0	0	0	0	0	0	0	0	0.182	0.278	0	-0.95
107	0	0	0	0	0	0	0	0	0	0.08	0	-0.53

Note that for j = 1, 2, …, 11 variable η_j takes zero values dominantly. Variable η_0, on the other hand, takes nonzero values, which denotes forecast errors.

Table 12 Some of Normal Forecast Updates for product $p5$

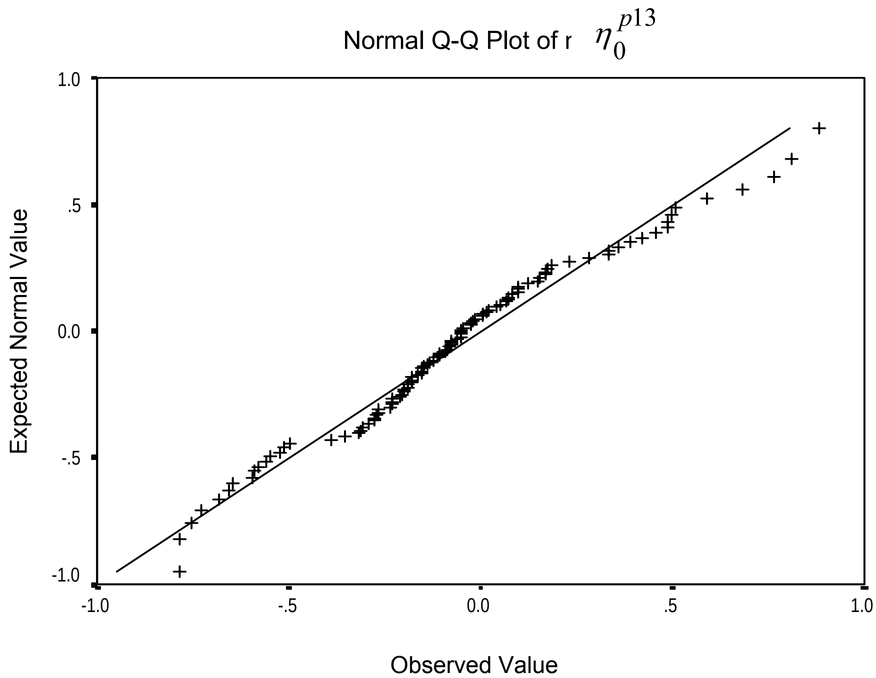
T	η_{11}	η_{10}	η_9	η_8	η_7	η_6	η_5	η_4	η_3	η_2	η_1	η_0
1	0	0	-1.47	0	0	0	0	0	0	0	0	-0.3
2	0	0	0	0	0	0	0	0	0	0	0	-0.48
3	0	0	0	0	0	0	0	0	0	0	0.056	-0.63
4	0	0	0	0	0	0	0	0	0	0	0	-0.16
5	0	0	0	0	0	0	0	0	0	0	0	-0.1
6	0	0	0	0	0	0	0	0	0	0	0	0.233
7	0.079	0.074	-0.23	-0.03	0.046	0.064	0.165	-0.04	0.07	0.26	0.219	-0.18
8	0	0	0	0	0	0	0	0	0	0	0	-0.67
9	0	0	0	0	0	0	0	0	0	0	0	0.182
10	0	0	0	0	0	0	0	0	0	0	0	0.014
11	0	0	0	0	0	0	0	0	0	0	0	0.256
12	0.004	0.057	0.158	0.363	0	0	0	0	-0.11	-0.11	-0.11	0.086
13	0	0	0	0	0	0	0	0	0	0	0	-0.47
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
94	0.251	0	0	0	0	0	0	0	0	0	0	0.116
95	0	0	0	0	0	0	0	0	0	0	0	-0.53
96	-0.08	-0	-0.08	0	0	0	0	0	0	0	0	-0.17
97	0	0	0	0	0	0	0	0	0.183	0	0	0.385
98	0	0	0	0	0	0	0	0	0	0	0	-0.51
99	-0.02	0	0	0	0	0	0	0	0	0	0	-0.16
100	0	0	0	0	0	0	0	0	0	0	0	0.324
101	0	0	0	0	0	0	0	0	0	0.351	0	0.862
102	0	0	0	0	0	0	0	0	0	0.502	0	0.076
103	0	0	0	0	0	0	0	0	0	0.198	0	-1.48
104	-0.11	-0.01	0	0	0	0	0	0	0	0	0	-0.81
105	0	0	0	0	0	0	0	0	0	0	0	-0.75
106	0	0	0	0	0	0	0	0	0.235	0.354	0	-1.5
107	0	0	0	0	0	0	0	0	0	0.28	0	-0.66

Similarly, Table 12 comprised of zero values for the majority of variables η_j. Only the variable η_0 takes nonzero values, which denotes forecast errors.

APPENDIX C

SOME RESULTS OF THE HYPOTHESIS TESTS

The results of the hypothesis of normal logarithmic forecast updates are given below for some of the variables η_j^p.

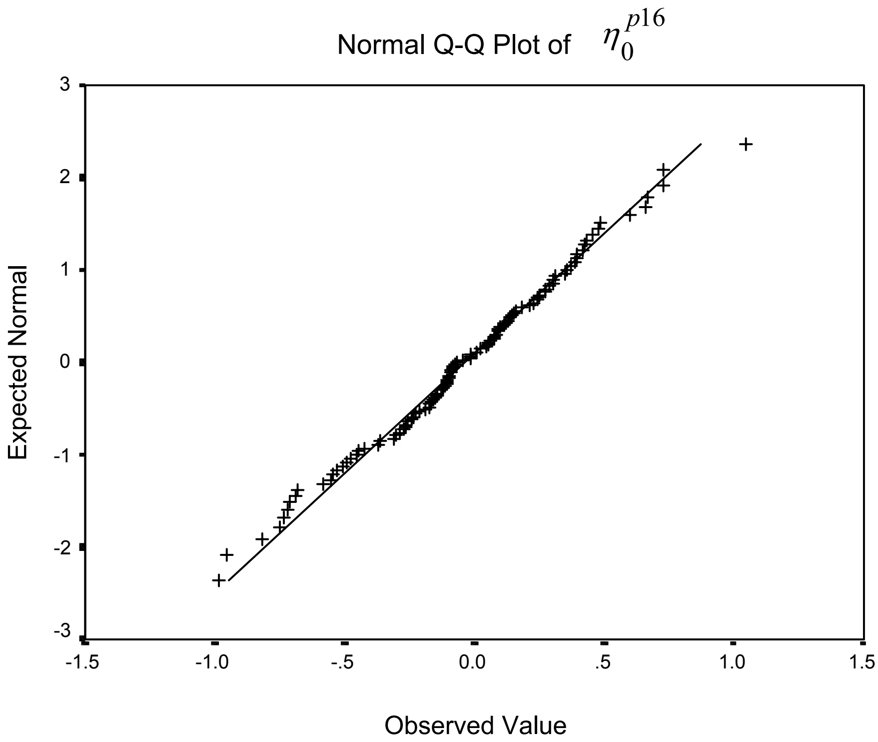


Tests of Normality

	Kolmogorov-Smirnov [a]		
	Statistic	df	Sig.
N0	.080	103	.105

a. Lilliefors Significance Correction

Figure 11 Normal Probability Plot of Forecast Update η_0 for product $p13$



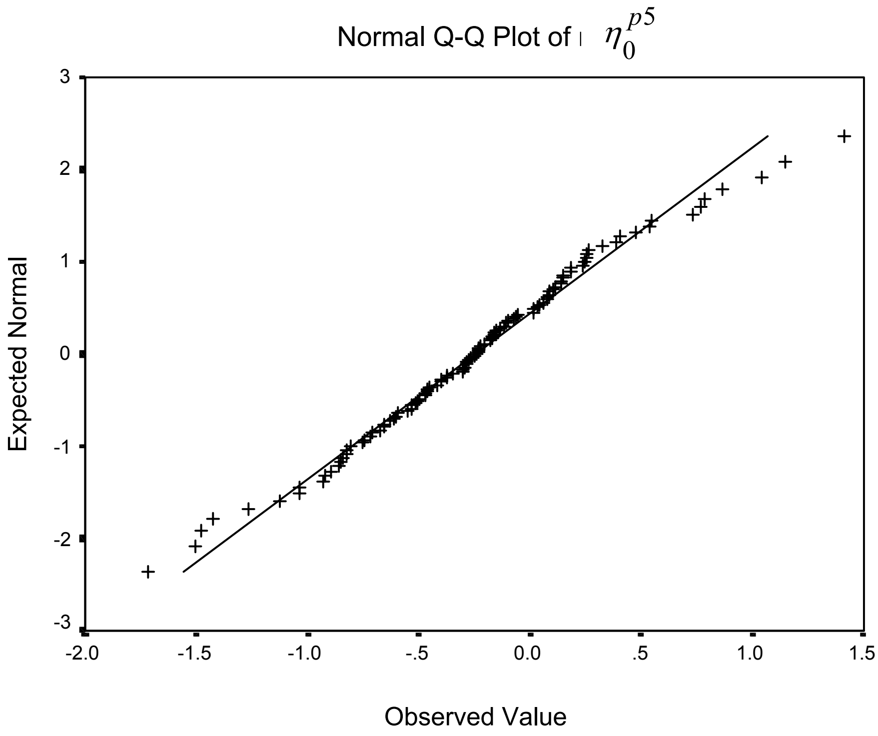
Tests of Normality

	Kolmogorov-Smirnov[a]		
	Statistic	df	Sig.
N0	.054	107	.200*

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Figure 12 Normal Probability Plot of Forecast Update η_0 for product $p16$



Tests of Normality

	Kolmogorov-Smirnov[a]		
	Statistic	df	Sig.
N0	.059	107	.200*

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Figure 13 Normal Probability Plot of Forecast Update η_0 for product $p5$

The above normal probability plots do not have large deviations from a straight line, suggesting that the logarithmic updates are normal. Normal probability plots and Kolmogorov-Smirnov tests indicate that normality assumption is not violated.

The results of the hypothesis of independent forecast updates are given in the following correlation matrix for the forecast update variables not taking zero values dominantly.

Table 13 Bivariate Correlations of Some Forecast Updates for product $p13$

Correlations

		N0	N1	N2	N3
Pearson Correlation	N0	1.000	.076	.037	.194*
	N1	.076	1.000	.052	.098
	N2	.037	.052	1.000	.653**
	N3	.194*	.098	.653**	1.000
Sig. (2-tailed)	N0	.	.436	.705	.045
	N1	.436	.	.591	.314
	N2	.705	.591	.	.000
	N3	.045	.314	.000	.
N	N0	107	107	107	107
	N1	107	107	107	107
	N2	107	107	107	107
	N3	107	107	107	107

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

The results are given together with the value of two-tailed probabilities corresponding to the 0.05 and the 0.01 levels of significance. Correlation coefficients significant at the 0.05 level are identified with a single asterisk, and those significant at the 0.01 level are identified with two asterisks. The presence of significant correlations indicates that the independence assumption is violated. Therefore, it is needed to adjust biased forecasts so that the resulting forecast updates of adjusted forecasts satisfy the independence assumption (see Hausman 1969). The results indicate that generally the assumption of independence is supported.

APPENDIX D

THE ESTIMATED VARIANCE-COVARIANCE MATRIX

Table 12 contains variance and covariance values for normal random variables, η_j^p's $p \in \{ p1, p2, \ldots, p17 \}$ and period $j \in \{ 0, 1, \ldots , 11 \}$. η_j^p represent the normal random variable that generates the updates made to *(j + 1)*-step ahead forecasts to obtain *j*-step ahead forecasts for product *p* and period *j*. Hence η_0^p represents the random variable that generates forecast error between one-step ahead forecast and realized demand for product *p*. $\sum$ is the 17*x*12 by 17*x*12 variance-covariance matrix of the 125 by 17*x*12 data matrix.

	p	p1												p2			·······	p17			
p	j	11	10	9	8	7	6	5	4	3	2	1	0	11	10	9	·······	11	10	···	0
p1	11	0.0120	0.0068	0.0027	0.0001	0.0013	0.0007	-0.0006	-0.0020	0.0000	0.0015	-0.0002	-0.0024	0.0051	0.0019	0.0006	·······	0.0012	-0.0003	···	-0.0073
	10	0.0068	0.0259	0.0049	-0.0001	0.0006	0.0003	-0.0006	-0.0022	-0.0036	-0.0012	-0.0007	0.0077	0.0002	0.0190	0.0014	·······	-0.0030	0.0199	···	0.0064
	9	0.0027	0.0049	0.0067	0.0011	0.0009	0.0017	-0.0012	0.0000	-0.0006	0.0023	-0.0006	0.0009	-0.0003	0.0023	0.0045	·······	-0.0023	0.0017	···	0.0018
	8	0.0001	-0.0001	0.0011	0.0046	0.0019	0.0028	-0.0003	0.0000	0.0011	-0.0019	0.0015	0.0009	0.0002	-0.0005	0.0009	·······	0.0033	0.0022	···	-0.0018
	7	0.0013	0.0006	0.0009	0.0019	0.0034	0.0020	0.0004	0.0030	0.0018	0.0019	0.0021	0.0009	0.0004	-0.0007	-0.0004	·······	0.0009	-0.0008	···	0.0004
	6	0.0007	0.0003	0.0017	0.0028	0.0020	0.0050	-0.0007	0.0008	0.0011	0.0005	0.0020	0.0019	0.0001	-0.0008	0.0010	·······	0.0001	-0.0015	···	-0.0033
	5	-0.0006	-0.0006	-0.0012	-0.0003	0.0004	-0.0007	0.0037	0.0013	0.0008	0.0000	0.0015	0.0007	0.0000	-0.0012	-0.0016	·······	0.0015	-0.0013	···	-0.0024
	4	-0.0020	-0.0022	0.0000	0.0000	0.0030	0.0008	0.0013	0.0104	0.0028	0.0015	0.0001	0.0038	-0.0016	-0.0021	-0.0004	·······	-0.0020	-0.0043	···	-0.0009
	3	0.0000	-0.0036	-0.0006	0.0011	0.0018	0.0011	0.0008	0.0028	0.0085	0.0045	0.0028	-0.0022	0.0000	-0.0035	-0.0004	·······	-0.0004	-0.0052	···	0.0028
	2	0.0015	-0.0012	0.0023	-0.0019	0.0019	0.0005	0.0000	0.0015	0.0045	0.0205	0.0033	0.0011	0.0005	-0.0018	0.0011	·······	-0.0028	-0.0051	···	0.0049
	1	-0.0002	-0.0007	-0.0006	0.0015	0.0021	0.0020	0.0015	0.0001	0.0028	0.0033	0.0127	-0.0021	0.0008	-0.0004	-0.0020	·······	0.0003	-0.0020	···	0.0047
	0	-0.0024	0.0077	0.0009	0.0009	0.0009	0.0019	0.0007	0.0038	-0.0022	0.0011	-0.0021	0.1978	-0.0019	0.0060	0.0017	·······	0.0055	0.0059	···	0.0380
p2	11	0.0051	0.0002	-0.0003	0.0002	0.0004	0.0001	0.0000	-0.0016	0.0000	0.0005	0.0008	-0.0019	0.0047	-0.0002	0.0000	·······	0.0058	0.0007	···	-0.0072
	10	0.0019	0.0190	0.0023	-0.0005	-0.0007	-0.0008	-0.0012	-0.0021	-0.0035	-0.0018	-0.0004	0.0060	-0.0002	0.0173	0.0014	·······	-0.0001	0.0207	···	0.0050
	9	0.0006	0.0014	0.0045	0.0009	-0.0004	0.0010	-0.0016	-0.0004	-0.0004	0.0011	-0.0020	0.0017	0.0000	0.0014	0.0059	·······	0.0004	0.0033	···	0.0000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮		⋮
p17	11	0.0012	-0.0030	-0.0023	0.0033	0.0009	0.0001	0.0015	-0.0020	-0.0004	-0.0028	0.0003	0.0055	0.0058	-0.0001	0.0004	·······	0.0265	0.0175	···	-0.0158
	10	-0.0003	0.0199	0.0017	0.0022	-0.0008	-0.0015	-0.0013	-0.0043	-0.0052	-0.0051	-0.0020	0.0059	0.0007	0.0207	0.0033	·······	0.0175	0.0446	···	-0.0001
		⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮		⋮	⋮		⋮
	2	0.0008	0.0004	0.0019	-0.0022	-0.0005	-0.0010	-0.0017	-0.0006	0.0009	0.0093	-0.0031	-0.0026	0.0000	0.0003	0.0025	·······	-0.0027	0.0021	···	0.0135
	1	-0.0008	0.0009	-0.0024	0.0007	-0.0002	0.0006	0.0006	-0.0014	0.0007	-0.0039	0.0049	-0.0061	-0.0001	0.0008	-0.0024	·······	-0.0004	0.0023	···	0.0052
	0	-0.0073	0.0064	0.0018	-0.0018	0.0004	-0.0033	-0.0024	-0.0009	0.0028	0.0049	0.0047	0.0380	-0.0072	0.0050	0.0000	·······	-0.0158	-0.0001	···	0.2250

Table 14 Some Part of the Estimated Variance–Covariance Matrix (204 x 204)

APPENDIX E

PERFORMANCE MEASURES FOR PRODUCT GROUPS

Table 15 Performance measures for product groups *G*2

	averages of realized SS levels (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p4	1.9	1.0	1.1	11	11	11	96.3	95.9	96.1
p5	4.5	4.9	2.9	10	11	11	97.2	98.4	96.6

Table 16 Performance measures for product group *G*3

	averages of realized SS levels (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p12	2.7	3.6	4.1	10	11	11	92.3	95.0	96.0
p13	3.9	6.8	3.7	10	11	11	93.7	94.9	94.1

Table 17 Performance measures for product group *G*4

	averages of realized SS levels (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p14	1.9	3.2	2.4	9	10	10	88.0	90.8	90.2
p15	1.1	2.4	2.6	9	10	11	89.2	91.9	92.4

Table 18 Performance measures for product group *G*5

	averages of realized SS levels (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p16	11.5	18.4	15.6	8	10	10	89.6	96.7	97.1
p17	14.6	17.2	10.3	10	11	11	94.9	98.3	97.6

Table 19 Performance measures for product group *G*6

	averages of realized SS levels (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p6	10.8	13.3	13.5	7	9	8	85.3	97.6	97.4
p7	4.1	5.8	1.5	8	10	10	87.6	96.5	95.9
p8	2.2	3.5	0	10	11	9	96.2	99.1	95.3
p9	2.1	3.6	2.5	10	11	11	96.7	99.2	99.3
p10	3.1	2.3	1.2	6	7	5	85.6	93.3	88.9
p11	2.1	1.9	1.7	10	11	11	98.8	99.4	99.3

Table 20 Performance measures for product group *G*7

	averages of realized SS levels (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p1	6.8	13.6	3.9	9	10	9	87.4	91.8	90.2
p2	7.1	4.7	5.9	10	9	10	96.8	97.8	98.2
p3	15.6	19.8	16.1	9	10	10	89.1	95.2	93.7
p4	1.9	1.0	1.1	11	11	11	96.3	95.9	96.1
p5	4.5	4.9	2.9	10	11	11	97.2	98.4	96.6

Table 21 Performance measures for product group *G*8

	averages of realized SS levels (in tons)			number of orders filled completely			% of total amount of orders filled		
	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2	MODEL-0	MODEL-1	MODEL-2
p6	10.8	13.3	13.5	7	9	8	85.3	97.6	97.4
p7	4.1	5.8	1.5	8	10	10	87.6	96.5	95.9
p8	2.2	3.5	0	10	11	9	96.2	99.1	95.3
p9	2.1	3.6	2.5	10	11	11	96.7	99.2	99.3