FACE RECOGNITION USING EIGENFACES AND NEURAL NETWORKS

A THESIS SUBMITTED TO

THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

OF

THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

VOLKAN AKALIN

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN THE DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

DECEMBER 2003

Approval of the Graduate School of Natural and Applied Sciences

Prof. Dr. Canan ÖZGEN
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Mübeccel DEMİREKLER
Head of Department

This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Prof. Dr. Mete SEVERCAN
Supervisor

Examining Committee Members

Prof. Dr. Gönül Turhan SAYAN
Prof. Dr. Mete SEVERCAN
Assoc. Prof. Dr. Gözde Bozdağı AKAR
Assoc. Prof. Dr. A.Aydın ALATAN
Dr. Uğur Murat LELOĞLU

ABSTRACT

FACE RECOGNITION USING EIGENFACES
AND NEURAL NETWORKS

AKALIN, Volkan
M.S., The Department of Electrical and Electronics Engineering
Supervisor: Prof. Dr. Mete SEVERCAN
December 2003, 91 Pages

A face authentication system based on principal component analysis and neural networks is developed in this thesis. The system consists of three stages; preprocessing, principal component analysis, and recognition. In preprocessing stage, normalization illumination, and head orientation were done. Principal component analysis is applied to find the aspects of face which are important for identification. Eigenvectors and eigenfaces are calculated from the initial face image set. New faces are projected onto the space expanded by eigenfaces and represented by weighted sum of the eigenfaces. These weights are used to identify the faces. Neural network is used to create the face database and recognize and authenticate the face by using these weights. In this work, a separate network was build for each person. The input face is projected onto the eigenface space first and new descriptor is obtained. The new descriptor is used as input to each person's network, trained earlier. The one with maximum output is selected and reported as

the host if it passes predefined recognition threshold. The algorithms that have been developed are tested on ORL, Yale and Feret Face Databases.

Keywords: Face recognition, Face authentication, Principal component analysis, Neural network, Eigenvector, Eigenface

ÖZ

ÖZYÜZLER VE YAPAY SİNİR AĞLARI KULLANARAK
YÜZ TANIMA

AKALIN, Volkan
Yüksek Lisans, Elektrik ve Elektronik Mühendisliği Bölümü
Tez Yöneticisi: Prof. Dr. Mete SEVERCAN
Aralık 2003, 91 Sayfa

Bu tezde, ana bileşen analizi ve yapay sinir ağlarına dayanan bir yüz tanıma sistemi geliştirilmiştir. Sistem üç aşamadan oluşmaktadır; önişlem, ana bileşen analizi, ve tanıma. Önişlem aşamasında, parlaklık dengelenmesi ve baş ayarlanması yapılmıştır. Yüz tanıma için çok önemli olan yüz görünüşlerinin bulunması için ana bileşen analizi uygulanmıştır. Başlangıç eğitim setinden özvektörler ve özyüzler hesaplanmıştır. Yüzler, özyüzler ile geliştirilmiş uzaya yansıtılmış ve özyüzlerin ağırlıklı toplamları ile ifade edilmişlerdir. Bu ağırlıklar yüzleri ayırt etmek için kullanılacaktır. Bu ağırlıkları kullanarak, yüz veritabanını oluşturmak ve yüzleri tanımak için yapay sinir ağları kullanılmıştır. Bu çalışmada, her bir kişi için ayrı bir yapay sinir ağı kullanılmıştır. Verilen yüz ilk olarak özyüz uzayına yansıtılarak yeni tanımlayıcıları elde edilir. Bu yeni tanımlayıcılar daha önce eğitilmiş ağlara giriş olarak kullanılır ve her bir kişinin ağına uygulanır. En yüksek sonucu veren ağ eğer daha önce tanımlanmış eşik değerinin üzerindeyse seçilir ve bu ağa sahip kişi

aranan kişi olarak belirtilir. Geliştirilen bu algoritmalar, ORL, Yale ve Feret yüz veritabanları üzerinde tets edilmiştir.

Anahtar Sözcükler : Yüz tanıma, Yüz doğrulama, Ana bileşen analizi, Yapay Sinir ağı, Özvektör, Özyüz

ACKNOWLEDGEMENTS

I would like to thank my supervisor, Prof Dr. Mete Severcan, for his supervision and constructive critics in the development of this thesis. I would also express my great gratitude to my family for their continuous support and great love. Special thanks to Sertan Aygün, Baykal Yıldırım, and Cem Ciğdemoğlu for their great friendships. Finally, I would like to thank to my company Aselsan and my boss Özcan Kahramangil for their support and sensitivity in this study.

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CHAPTER 1

INTRODUCTION

The face is our primary focus of attention in social intercourse, playing a major role in conveying identity and emotion. Although the ability to infer intelligence or character from facial appearance is suspect, the human ability to recognize faces is remarkable. We can recognize thousands of faces learned throughout our lifetime and identify familiar faces at a glance even after years of separation. This skill is quite robust, despite large changes in the visual stimulus due to viewing conditions, expression, aging, and distractions such as glasses, beards or changes in hair style.

Face recognition has become an important issue in many applications such as security systems, credit card verification and criminal identification. For example, the ability to model a particular face and distinguish it from a large number of stored face models would make it possible to vastly improve criminal identification. Even the ability to merely detect faces, as opposed to recognizing them, can be important. Detecting faces in photographs for automating color film development can be very useful, since the effect of many enhancement and noise reduction techniques depends on the image content.

A formal method of classifying faces was first proposed by Francis Galton in 1888 [1, 2]. During the 1980's work on face recognition remained largely dormant. Since the 1990's, the research interest in face recognition has grown significantly as a result of the following facts:

1. The increase in emphasis on civilian/commercial research projects,

2. The re-emergence of neural network classifiers with emphasis on real time computation and adaptation, ,
3. The availability of real time hardware,
4. The increasing need for surveillance related applications due to drug trafficking, terrorist activities, etc.

Although it is clear that people are good at face recognition, it is not at all obvious how faces are encoded or decoded by the human brain. Developing a computational model of face recognition is quite difficult, because faces are complex, multi-dimensional visual stimuli. Therefore, face recognition is a very high level computer vision task, in which many early vision techniques can be involved.

The first step of human face identification is to extract the relevant features from facial images. Research in the field primarily intends to generate sufficiently reasonable familiarities of human faces so that another human can correctly identify the face. The question naturally arises as to how well facial features can be quantized. If such a quantization if possible then a computer should be capable of recognizing a face given a set of features. Investigations by numerous researchers [3, 4, 5] over the past several years have indicated that certain facial characteristics are used by human beings to identify faces.

There are three major research groups which propose three different approaches to the face recognition problem. The largest group [6, 7, 8] has dealt with facial characteristics which are used by human beings in recognizing individual faces. The second group [9, 10, 11, 12, 13] performs human face identification based on feature vectors extracted from profile silhouettes. The third group [14, 15] uses feature vectors extracted from a frontal view of the face. Although there are three different approaches to the face recognition problem, there are two basic methods from which these three different approaches arise.

The first method is based on the information theory concepts, in other words, on the principal component analysis methods. In this approach, the most relevant information that best describes a face is derived from the entire face image. Based on the Karhunen-Loeve expansion in pattern recognition, M. Kirby and L. Sirovich have shown that [6, 7] any particular face could be economically represented in terms of a best coordinate system that they termed "eigenfaces". These are the eigenfunctions of the averaged covariance of the ensemble of faces. Later, M. Turk and A. Pentland have proposed a face recognition method [16] based on the eigenfaces approach.

The second method is based on extracting feature vectors from the basic parts of a face such as eyes, nose, mouth, and chin. In this method, with the help of deformable templates and extensive mathematics, key information from the basic parts of a face is gathered and then converted into a feature vector. L. Yullie and S. Cohen [17] played a great role in adapting deformable templates to contour extraction of face images.

1.1. Human Recognition

Within today’s environment of increased importance of security and organization, identification and authentication methods have developed into a key technology in various areas: entrance control in buildings; access control for computers in general or for automatic teller machines in particular; day-to-day affairs like withdrawing money from a bank account or dealing with the post office; or in the prominent field of criminal investigation. Such requirement for reliable personal identification in computerized access control has resulted in an increased interest in biometrics.

Biometric identification is the technique of automatically identifying or verifying an individual by a physical characteristic or personal trait. The term “automatically” means the biometric identification system must identify or verify a human characteristic or trait quickly with little or no intervention from the user. Biometric technology was developed for use in high-level security systems and law enforcement markets. The key element of biometric technology is its ability to identify a human being and enforce security [18].

Biometric characteristics and traits are divided into behavioral or physical categories. Behavioral biometrics encompasses such behaviors as signature and typing rhythms. Physical biometric systems use the eye, finger, hand, voice, and face, for identification.

A biometric-based system was developed by Recognition Systems Inc., Campbell, California, as reported by Sidlauskas [19]. The system was called ID3D Handkey and used the three dimensional shape of a person's hand to distinguish people. The side and top view of a hand positioned in a controlled capture box were used to generate a set of geometric features. Capturing takes less than two seconds and the data could be stored efficiently in a 9-byte feature vector. This system could store up to 20000 different hands.

Another well-known biometric measure is that of fingerprints. Various institutions around the world have carried out research in the field. Fingerprint systems are unobtrusive and relatively cheap to buy. They are used in banks and to control entrance to restricted access areas. Fowler [20] has produced a short summary of the available systems.

Fingerprints are unique to each human being. It has been observed that the iris of the eye, like fingerprints, displays patterns and textures unique to each human and that it remains stable over decades of life as detailed by Siedlarz [21]. Daugman designed a robust pattern recognition method based on 2-D Gabor transforms to classify human irises.

Speech recognition is also offers one of the most natural and less obtrusive biometric measures, where a user is identified through his or her spoken words. AT&T has produced a prototype that stores a person's voice on a memory card, details of which are described by Mandelbaum [22].

While appropriate for bank transactions and entry into secure areas, such technologies have the disadvantage that they are intrusive both physically and socially. They require the user to position their body relative to the sensor, and then pause for a second to declare himself or herself. This pause and declare interaction is unlikely to change because of the fine-grain spatial sensing required. Moreover, since people can not recognize people using this sort of data, these types of identification do not have a place in normal human interactions and social structures.

While the pause and present interaction perception are useful in high security applications, they are exactly the opposite of what is required when building a store that recognizing its best customers, or an information kiosk that remembers you, or a house that knows the people who live there.

A face recognition system would allow user to be identified by simply walking past a surveillance camera. Human beings often recognize one another by unique facial characteristics. One of the newest biometric technologies, automatic facial recognition, is based on this phenomenon. Facial recognition is the most successful form of human surveillance. Facial recognition technology, is being used to improve human efficiency when recognizing faces, is one of the fastest growing fields in the biometric industry. Interest in facial recognition is being fueled by the availability and low cost of video hardware, the ever-increasing number of video cameras being placed in the workspace, and the noninvasive aspect of facial recognition systems.

Although facial recognition is still in the research and development phase, several commercial systems are currently available and research organizations, such as Harvard University and the MIT Media Lab, are working on the development of more accurate and reliable systems.

1.2. Eigenfaces for Recognition

We have focused our research toward developing a sort of unsupervised pattern recognition scheme that does not depend on excessive geometry and computations like deformable templates. Eigenfaces approach seemed to be an adequate method to be used in face recognition due to its simplicity, speed and learning capability.

A previous work based on the eigenfaces approach was done by M. Turk and A. Pentland, in which, faces were first detected and then identified. In this thesis, a face recognition system based on the eigenfaces approach, similar to the one presented by M. Turk and A. Pentland, is proposed.

The scheme is based on an information theory approach that decomposes face images into a small set of characteristic feature images called eigenfaces, which may be thought of as the principal components of the initial training set of face images. When the eigenfaces of a database is constructed, any face in this database can be exactly represented with the combination of these eigenfaces. In combination of these eigenfaces, the multipliers of them are called the feature vectors of this face, and this face can be represented this new descriptors. Each person in database has his/her own neural network. Firstly, these neural networks are trained with these new descriptors of the training images. When an image needs to be recognized, this face is projected onto the eigenface space first and gets a new descriptor. The new descriptor is used as network input and applied to each person's network. The neural net with maximum output is selected and reported as the host if it passes predefined recognition threshold.

The eigenface approach used in this scheme has advantages over other face recognition methods in its speed, simplicity, learning capability and robustness to small changes in the face image.

1.3. Thesis Organization

This thesis is organized in the following manner: Chapter 2 deals with the basic concepts of pattern and face recognition. Two major approaches to the face recognition problem are given. Chapter 3 is based on the details of the proposed face recognition method and the actual system developed. Chapter 4 gives the results drawn from the research and finally in Chapter 5, conclusion and possible directions for future work are given.

CHAPTER 2

BASIC CONCEPTS OF FACE RECOGNITION

2.1. Introduction

The basic principals of face recognition and two major face recognition approaches are presented in this chapter.

Face recognition is a pattern recognition task performed specifically on faces. It can be described as classifying a face either "known" or "unknown", after comparing it with stored known individuals. It is also desirable to have a system that has the ability of learning to recognize unknown faces.

Computational models of face recognition must address several difficult problems. This difficulty arises from the fact that faces must be represented in a way that best utilizes the available face information to distinguish a particular face from all other faces. Faces pose a particularly difficult problem in this respect because all faces are similar to one another in that they contain the same set of features such as eyes, nose, and mouth arranged in roughly the same manner.

2.1.1. Background and Related Work

Much of the work in computer recognition of faces has focused on detecting individual features such as the eyes, nose, mouth, and head outline, and defining a face model by the position, size, and relationships among these features. Such approaches have proven difficult to extend to multiple views and have often been quite fragile, requiring a good initial guess to guide them. Research in human strategies of face recognition, moreover, has shown that individual features and their immediate relationships comprise an insufficient representation to account for the performance of adult human face identification [23]. Nonetheless, this approach to face recognition remains the most popular one in the computer vision literature.

Bledsoe [24, 25] was the first to attempt semi-automated face recognition with a hybrid human-computer system that classified faces on the basis of fiducially marks entered on photographs by hand. Parameters for the classification were normalized distances and ratios among points such as eye corners, mouth corners, nose tip, and chin point. Later work at Bell Labs developed a vector of up to 21 features, and recognized faces using standard pattern classification techniques.

Fischler and Elschlager [26], attempted to measure similar features automatically. They described a linear embedding algorithm that used local feature template matching and a global measure of fit to find and measure facial features. This template matching approach has been continued and improved by the recent work of Yuille and Cohen [27]. Their strategy is based on deformable templates, which are parameterized models of the face and its features in which the parameter values are determined by interactions with the face image.

Connectionist approaches to face identification seek to capture the configurationally nature of the task. Kohonen [28] and Kononen and Lehtio [29] describe an associative network with a simple learning algorithm that can recognize face images and recall a face image from an incomplete or noisy version input to the network. Fleming and Cottrell [30] extend these ideas using nonlinear units, training the system by back propagation.

Others have approached automated face recognition by characterizing a face by a set of geometric parameters and performing pattern recognition based on the parameters. Kanade's [31] face identification system was the first system in which all steps of the recognition process were automated, using a top-down control strategy directed by a generic model of expected feature characteristics. His system calculated a set of facial parameters from a single face image and used a pattern classification technique to match the face from a known set, a purely statistical approach depending primarily on local histogram analysis and absolute gray-scale values.

Recent work by Burt [32] uses a smart sensing approach based on multiresolution template matching. This coarse to fine strategy uses a special purpose computer built to calculate multiresolution pyramid images quickly, and has been demonstrated identifying people in near real time.

2.1.2. Outline of a Typical Face Recognition System

In Figure 2.1, the outline of a typical face recognition system is given.

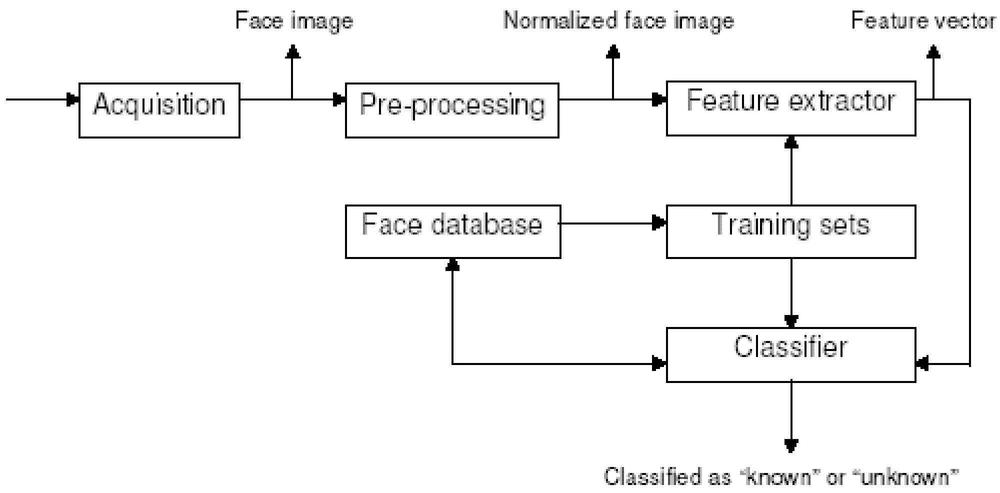


Figure 2.1. Outline of a typical face recognition system

There are six main functional blocks, whose responsibilities are given below:

2.1.2.1. The acquisition module. This is the entry point of the face recognition process. It is the module where the face image under consideration is presented to the system. In other words, the user is asked to present a face image to the face recognition system in this module. An acquisition module can request a face image from several different environments: The face image can be an image file that is located on a magnetic disk, it can be captured by a frame grabber or it can be scanned from paper with the help of a scanner.

2.1.2.2. The pre-processing module. In this module, by means of early vision techniques, face images are normalized and if desired, they are enhanced to improve the recognition performance of the system. Some or all of the following pre-processing steps may be implemented in a face recognition system:

- **Image size normalization**. It is usually done to change the acquired image size to a default image size such as 128 x 128, on which the face recognition system operates. This is mostly encountered in systems where face images are treated as a whole like the one proposed in this thesis.

- **Histogram equalization**. It is usually done on too dark or too bright images in order to enhance image quality and to improve face recognition performance. It modifies the dynamic range (contrast range) of the image and as a result, some important facial features become more apparent.

- **Median filtering.** For noisy images especially obtained from a camera or from a frame grabber, median filtering can clean the image without loosing information.

- **High-pass filtering**. Feature extractors that are based on facial outlines, may benefit the results that are obtained from an edge detection scheme. High-pass filtering emphasizes the details of an image such as contours which can dramatically improve edge detection performance.

- **Background removal**. In order to deal primarily with facial information itself, face background can be removed. This is especially important for face recognition systems where entire information contained in the image is used. It is obvious that, for background removal, the preprocessing module should be capable of determining the face outline.

- **Translational and rotational normalizations**. In some cases, it is possible to work on a face image in which the head is somehow shifted or rotated. The head plays the key role in the determination of facial features. Especially for face recognition systems that are based on the frontal views of faces, it may be desirable that the pre- processing module determines and if possible, normalizes the shifts and rotations in the head position.

- **Illumination normalization**. Face images taken under different illuminations can degrade recognition performance especially for face recognition systems based on the principal component analysis in which entire face information is used for recognition. A picture can be equivalently viewed as an array of reflectivities r(x). Thus, under a uniform illumination I, the corresponding picture is given by

$$\Phi(x) = Ir(x) \tag{2.1}$$

The normalization comes in imposing a fixed level of illumination I_0 at a reference point x_0 on a picture. The normalized picture is given by

$$\Phi(x) = \frac{Io\Phi(xo)}{I(xo)} \tag{2.2}$$

In actual practice, the average of two reference points, such as one under each eye, each consisting of 2 x 2 arrays of pixels can be used.

2.1.2.3. The feature extraction module. After performing some pre-processing (if necessary), the normalized face image is presented to the feature extraction module in order to find the key features that are going to be used for classification. In other words, this module is responsible for composing a feature vector that is well enough to represent the face image.

2.1.2.4. The classification module. In this module, with the help of a pattern classifier, extracted features of the face image is compared with the ones stored in a face library (or face database). After doing this comparison, face image is classified as either known or unknown.

2.1.2.5. Training set. Training sets are used during the "learning phase" of the face recognition process. The feature extraction and the classification modules adjust their parameters in order to achieve optimum recognition performance by making use of training sets.

2.1.2.6. Face library or face database. After being classified as "unknown", face images can be added to a library (or to a database) with their feature vectors for later comparisons. The classification module makes direct use of the face library.

2.1.3. Problems that May Occur During Face Recognition

Due to the dynamic nature of face images, a face recognition system encounters various problems during the recognition process. It is possible to classify a face recognition system as either "robust" or "weak" based on its recognition performances under these circumstances. The objectives of a robust face recognition system are given below:

2.1.3.1. Scale invariance. The same face can be presented to the system at different scales as shown in Figure 2.2-b. This may happen due to the focal distance between the face and the camera. As this distance gets closer, the face image gets bigger.

2.1.3.2. Shift invariance. The same face can be presented to the system at different perspectives and orientations as shown in Figure 2.2-c. For instance, face images of the same person could be taken from frontal and profile views. Besides, head orientation may change due to translations and rotations.

2.1.3.3. Illumination invariance. Face images of the same person can be taken under different illumination conditions such as, the position and the strength of the light source can be modified like the ones shown in Figure 2.2-d.

2.1.3.4 Emotional expression and detail invariance. Face images of the same person can differ in expressions when smiling or laughing. Also, like the ones shown in Figure 2.2-e, some details such as dark glasses, beards or moustaches can be present.

2.1.3.5. Noise invariance. A robust face recognition system should be insensitive to noise generated by frame grabbers or cameras. Also, it should function under partially occluded images. A robust face recognition system should be capable of classifying a face image as "known" under even above conditions, if it has already been stored in the face database.

Figure 2.2 (a) Original face image
(b) Scale variance
(c) Orientation variance
(d) Illumination variance
(e) Presence of details

2.1.4. Feature Based Face Recognition

It was mentioned before that, there were two basic approaches to the face recognition problem: Feature based face recognition and principal component analysis methods. Although feature based face recognition can be divided into two different categories, based on frontal views and profile silhouettes, they share some common properties and we will treat them as a whole. In this section, basic principals of feature based face recognition from frontal views [33] are presented.

2.1.4.1. Introduction

The first step of human face identification is to extract the features from facial images. In the area of feature selection, the question has been addressed in studies of cue salience in which discrete features such as the eyes, mouth, chin and nose have been found important cues for discrimination and recognition of faces.

After knowing what the effective features are for face recognition, some methods should be utilized to get contours of eyes, eyebrows, mouth, nose, and face. For different facial contours, different models should be used to extract them from the original portrait. Because the shapes of eyes and mouth are similar to some geometric figures, they can be extracted in terms of the deformable template model [27]. The other facial features such as eyebrows nose and face are so variable that they have to be extracted by the active contour model [34, 35]. These two models can be illustrated in the following:

2.2.4.1.1. Deformable template model.

The deformable templates are specified by a set of parameters which uses a priori knowledge about the expected shape of the features to guide the contour deformation process. The templates are flexible enough to change their size and other parameter values, so as to match themselves to the data. The final values of these parameters can be used to describe the features. This method works well regardless of variations in scale, tilt, and rotations of the head. Variations of the parameters should allow the template to fit any normal instance of the feature. The deformable templates interact with the image in a dynamic manner. An energy function is defined which contains terms attracting the template to salient features such as peaks and valleys in the image intensity, edges and intensity itself. The minima of the energy function correspond to the best fit with the image. The parameters of the template are then updated by steepest descent.

2.2.4.1.2. Active contour model (Snake).

The active contour or snake is an energy minimizing spline guided by external constraint forces and influenced by image forces that pull it toward features such as lines and edges. Snakes lock onto nearby edges, localizing them accurately. Because the snake is an energy minimizing spline, energy functions whose local minima comprise the set of alternative solutions to higher level processes should be designed. Selection of an answer from this set is accomplished by the addition of energy terms that push the model toward the desired solution. The result is an active model that falls into the desired solution when placed near it. In the active contour model issues such as the connectivity of the contours and the presence of corners affect the energy function and hence the detailed structure of the locally optimal contour. These issues can be resolved by very high-level computations.

2.1.4.2. Effective Feature Selection

Before mentioning the facial feature extraction procedures, we have the following two considerations:

1. The picture-taking environment must be fixed in order to get a good snapshot.
2. Effective features that can be used to identify a face efficiently should be known.

Despite the marked similarity of faces as spatial patterns we are able to differentiate and remember a potentially unlimited number of faces. With sufficient familiarity, the faces of any two persons can be discriminated. The skill depends on the ability to extract invariant structural information from the transient situation of a face, such as changing hairstyles, emotional expression, and facial motion effect.

Features are the basic elements for object recognition. Therefore, to identify a face, we need to know what features are used effectively in the face recognition process. Because the variance of each feature associated with the face recognition process is relatively large, the features are classified into three major types:

2.1.4.2.1. First-order features values.

Discrete features such as eyes, eyebrows, mouth, chin, and nose, which have been found to be important [4] in face identification and are specified without reference to other facial features, are called first-order features. Important first-order features are given in Table 2.1.

2.1.4.2.2. Second-order features values.

Another configurable set of features which characterize the spatial relationships between the positions of the first-order features and information about the shape of the face are called second-order features. Important second-order features are given in Table 2.2. Second order features that are related to nose, if nose is noticeable are given in Table 2.3.

2.1.4.2.3 Higher-order feature values.

There are also higher-level features whose values depend on a complex set of feature values. For instance, age might be a function of hair coverage, hair color, skin tension, presence of wrinkles and age spots, forehead height which changes because of receding hairline, and so on.

Variability such as emotional expression or skin tension exists in the higher-order features and the complexity, which is the function of first-order and second-order features, is very difficult to predict. Permanent information belonging to the higher-order features can not be found simply by using first and second-order features. For a robust face recognition system, features that are invariant to the changes of the picture taking environment should be used. Thus, these features may contain merely first-order and second-order ones. These effective feature values cover almost all the obtainable information from the portrait. They are sufficient for the face recognition process.

The feature values of the second-order are more important than those of the first-order and they are dominant in the feature vector. Before mentioning the facial feature extraction process, it is necessary to deal with two preprocessing steps:

> **Threshold assignment**. Brightness threshold should be known in order to discriminate the feature and other areas of the face. Generally, different thresholds are used for eyebrows, eyes, mouth, nose, and face according to the brightness of the picture.
>
> **Rough Contour Estimation Routine (RCER).** The left eyebrow is the first feature that is to be extracted. The first step is to estimate the rough contour of the left eyebrow and find the contour points. Generally, the position of the left eyebrow is about one-fourth of the facial width. Having this a priori

information, the coarse position of the left eyebrow can be found and its rough contour can be captured. Once the rough contour of the left eyebrow is established, the rough contours of other facial features such as left eye, right eyebrow, mouth or nose can be estimated by RCER [29]. After the rough contour is obtained, its precise contour will be extracted by the deformable template model or the active contour model.

Table 2.1 First-order features

Measurement	Facial Location
Area, angle	left eyebrow right eyebrow left eye right eye mouth face
Distance	Length of left eyebrow Length of right eyebrow Length of left eye Length of right eye Length of mouth Length of face Height of face

Table 2.2 Second-order features

Measurement	Facial Location				
Distance	left eyebrow	-	right eyebrow		
	left eye	-	right eyebrow		
	left eyebrow	-	left eye		
	right eyebrow	-	right eye		
	left eyebrow	-	mouth		
	right eyebrow	-	mouth		
	left eye	-	mouth		
	right eyebrow	-	mouth		
	eyebrow	-	side of face		
	eye	-	side of face		
	mouth	-	side of face		
	mouth	-	lower part of face		
Angle	left eyebrow	-	left eye	-	left eyebrow
	right eyebrow	-	right eye	-	right eyebrow
	left eye	-	left eyebrow	-	left eye
	right eye	-	right eyebrow	-	right eye
	left eyebrow	-	mouth	-	right eyebrow
	left eye	-	mouth	-	right eye
	left eyebrow	-	left eye	-	mouth
	right eyebrow	-	left eye	-	mouth

Table 2.3 Features related to nose, if nose is noticeable

Measurement	Facial Location				
Distance	left nose	-	right nose		
	left eyebrow	-	left nose		
	right eyebrow	-	right nose		
	left eye	-	left nose		
	right eye	-	right nose		
	left nose	-	mouth		
	right nose	-	mouth		
Angle	left eyebrow	-	center of nose	-	right eyebrow
	left eye	-	center of nose	-	right eye
	left nose	-	mouth	-	right nose
	left eyebrow	-	left eye	-	left nose
	right eyebrow	-	right eye	-	right nose

2.1.4.3. Feature Extraction Using the Deformable Templates

After the rough contour is obtained, the next step of face recognition is to find the physical contour of each feature. Conventional edge detectors can not find facial features such as the contours of the eye or mouth accurately from local evidence of edges, because they can not organize local information into a sensible global perception. There is a method to detect the contour of the eye by the deformable template which was originally proposed by Yullie [27]. It is possible to reduce computations at the cost of the precision of the extracted contour.

2.1.4.3.1. Eye Template

The deformable template acts on three representations of the image, as well as on the image itself. The first two representations are the peak and valleys in the image intensity and the third is the place where the image intensity changes quickly. The eye template developed by Yullie et al. consists of the following features:

- A circle of radius r, centered on a point (x_c , y_c) , corresponding to the iris. The boundaries of the iris and the whites of the eyes are attracted to edges the image intensity. The interior of the circle is attracted to valleys, or low values in the image intensity.

- A bounding contour of the eye attracted to edges. This contour is modeled by two parabolic sections representing the upper and lower parts of the boundary. It has a center (x_c , y_c), with 2w, maximum height h_1 of the boundary above the center, maximum height h_2 of the boundary below the center, and an angle of rotation.

- Two points, corresponding to the centers for the whites of the eyes, which are attracted to peaks in the image intensity.

- Regions between the bounding contour and the iris which also correspond to the whites of the eyes. These will be attracted to large intensity values.

The original eye template can be modified for the sake of simplicity where the accuracy of the extracted contour is not critical. The lack of a circle does not affect the classified results because the feature values are obtained from other information. The upper and lower parabola will be satisfactory for the recognition process. Thus, the total energy function for the eye template can be defined a combination of the energy functions of edge, white and black points.

The total energy function is defined as

$$E_{total} = E_{edge} + E_{white} + E_{black} \tag{2.3}$$

where E_{edge} , E_{white} , E_{black} are defined in the following:

- The edge potentials are given by the integral over the curves of the upper and lower parabola divided by their length:

$$\begin{aligned} E_{edge} &= -\frac{w_1}{upper\_length} \int_{upper\_bound} \Phi_{edge}(x,y)ds \\ &- \frac{w_2}{lower\_length} \int_{lower\_bound} \Phi_{edge}(x,y)ds \end{aligned} \tag{2.4}$$

where upper-bound and lower-bound represent the upper and lower parts of the eye, and Φ_{edge} represents the edge response of the point (x,y).

- The potentials of white and black points are defined as the integral over the area bounded by the upper and lower parabola divided by the area:

$$E_{w,b} = -\frac{1}{Area} \iint_{para-area} (-w_b N_{black}(x,y) + w_w N_{whıhı}(x,y))dA \tag{2.5}$$

where $N_{black}(x, y)$ and $N_{white}(x, y)$ represent the number of black and white points, and w_b , w_w are weights related with black and white points.

In order to be not affected by an improper threshold, the black and white points in Eq.(2.5) are defines as

P(x,y) is a black point if I(x,y) $\lesssim$ (threshold - tolerance),
P(x,y) is a white point if I(x,y) $\gtrsim$ (threshold + tolerance),
P(x,y) is an unambiguous point if I(x,y) is in between. (2.6)
where I(x,y) is the image intensity at point (x,y).

By the energy functions defined above, we can calculate the energy in the range of little modulations of 2w, h_1 , h_2 and ф. When the minimum energy value takes place, the precise contour is extracted.

2.1.4.3.2. Mouth Template

In the whole features of the front view of the face, the role of the mouth in relatively important. The properties of the mouth contour are heavily involved in the face recognition process. The deformable mouth template changes its own shape when it comes across the image areas of edge (which the intensity changes quickly), and white and black points. Generally, features related to middle lips, lower and upper lips are extracted. Because of the effect of brightness in the picture taking period, the middle of the lower lip may not be apparent. RCER can not find the approximate height of the lower lip. Fortunately, the length of the mouth can still be found by RCER. Usually, the height of the lower lip is between one-fourth and one-sixth of the mouth's length.

The mouth contour energy function consists of the edge term E_{edge} and the black term E_{black} . The edge term dominates at the edge area, where as the black term encloses as many black points belonging to the mouth as possible.

$$E_{total} = E_{edge} + E_{black} \quad . \tag{2.7}$$

- The edge energy function consists of three parts: middle lip (gap between lips), lower lip and upper lip separated at philtrum. The equation of the middle lip part is

$$E_{edge} = -\frac{w_{lower}}{lower}\int_{lower}\Phi_{edge}(x,y)ds - \frac{w_{left}}{left}\int_{left}\Phi_{edge}(x,y)ds - \frac{w_{right}}{right}\int_{right}\Phi_{edge}(x,y)ds$$

(2.8)

where *lower* represents the lower boundary of mouth, *left* represents the left part of upper lip, *right* represents the right part of upper lip, and Φ_{edge} (x,y) represent the edge response of point (x,y).

- The black energy function helps the edge energy to enclose black points belonging to the mouth and is defined as:

$$E_{black} = \frac{1}{Area}\int_{lbound}^{ubound}\int -w_{black}N_{black}(x,y)dA + \frac{1}{mid\_length}\int_{mid} -w_{mid}N_{black}(x,y)dS$$

(2.9)

where *Lbound* represents lower lip, *Ubound* represents upper lip, and *mid* represents number of black points. The black points are defined by Eq. (2.6) The weights w_{black} , w_{mid} , w_{lower} , w_{left} and w_{right} are experimentally determined.

2.1.4.4. Feature Extraction Using the Active Contour

The shapes of eyebrow, nostril and face, unlike eye and mouth, are even more different for different people and their contours can not be captured by using the deformable template. In this case, the active contour model or the "snake" is used. A snake is an energy minimizing spline guided by external constraint forces and influenced by image forces that pull it toward features such as lines and edges. This approach differs from traditional approaches which detect edges and then links them. In the active contour model, image and external forces together with the connectivity of the contours and the presence of corners will affect the energy function and the detailed structure of the locally optimal contour. The energy function of the active contour model [35] is defined as:

$$E_{snake} = \int_0^1 E_{snake}(v(s))dS = \int_0^1 E_{\text{int}\,ernal}(v(s))dS + \int_0^1 E_{images}(v(s))dS + \int_0^1 E_{constraions}(v(s))dS \tag{2.10}$$

where v(s) represents the position of the snake, $E_{ernal\ \text{int}}$ represents the internal energy of the contour due to bending, E_{images} gives rise to the image forces, and $E_{cons\,tra\text{int}}$ represents the external energy.

2.1.4.4.1. The Modified Active Contour Model

The original active contour model is user interactive. The advantage of its being user interactive is that the final form of the snake can be influenced by feedback from a higher level process. As the algorithm iterates the energy terms can be adjusted by higher level processes to obtain a local minima that seems most useful to that process. However, there are some problems with minimization procedure. Amini et al [37], pointed out some problems including instability and a tendency for points to bunch up on a strong portion of an edge. They proposed a dynamic programming algorithm for minimizing the energy function. Their approach had the advantage of using points on the discrete grid and is numerically stable, however the convergence is very slow.

It is possible to find a faster algorithm for the active contour [36]. Although this model still has the disadvantage of being unable to guarantee global minima, it can solve the problem of bunching up on a strong portion in the active contour. This problem occurs during the iterative process when contour points will accumulate at certain strong portions of the active contour. Besides, its computation speed is faster and thus, it is more suitable for face recognition. Active contour energy can be redefined [33] as:

$$E_{total} = \int_0^1 (\alpha(s))E_{continuity}(v(s)) + (\beta(s)E_{curvature}(v(s)) + (\delta(x)E_{images(v(s))dS} \qquad (2.11)$$

The definition of v(s) is similar to Eq (2.13) and the following approximations are used:

$$\left|\frac{dv_i}{ds}\right| \approx |v_i - v_{i-1}|^2 and \left|\frac{d^2 v_i}{ds^2}\right| \approx |v_{i-1} - 2v_i + v_{i+1}|^2 \qquad (2.12)$$

- **Continuity force.** The first derivative $|v_i - v_{i-1}|^2$ causes the curve to shrink. It is actually minimizing the distance between points. It also contributes to the problem of points bunching up on strong portions of the contour. It was decided that a term which encouraged even spacing of the points would satisfy the original goal of first order continuity without the effect of shrinking.
  Here, this term uses the difference between the average distance points, ***d***, and this distance between the two points under consideration, $|d - |v_i - v_{i-1}||$ Thus points having a distance near the average will have the minimum value. At the end of each iteration a new value of *d* is computed.

- **Curvature force.** Since the formulation of the continuity term causes the points to be relatively evenly spaced, $|v_{i-1} - 2v_i + v_{i+1}|^2$ gives a reasonable
  and quick estimate of the curvature. This term, like the continuity term, is normalized by dividing the largest value in the neighborhood, giving a number from 0 to 1.

- **Image force.** E_{images} is the image force which is defined by the following operations:

  1. We have eight image energy measurements (Mag), for eight neighbors.

  2. To normalize the image energy measurements, we select the minimum (Min), and maximum (Max) terms from those eight measurements, and then do the calculation, (Min - Max)/(Max - Min) to obtain the image force.

At the end of each iteration, the curvature is determined at each point on the new contour. If the value is larger than the threshold, β is set to 0 for the next iteration. The greedy algorithm [36] is applied for fast convergence. The energy function is computed for the current location of vi and each of its neighbors. The neighbor having the smallest value is chosen as the new position of v_i . The greedy algorithm can be applied to extract the features of eyebrow, nostril, and face. In order to prevent the snake failing at inaccurate local minima, contour estimation is done via RCER before the snake iterates. RCER uses a priori knowledge to find a rough contour as a starting (initial) contour for the snake.

2.1.4.4.2. Boundary Extraction of a Face

In order to demonstrate the use of active contour models on facial contour extraction, energy function associated to the boundary extraction of a face is presented in this section.

Unlike eyebrow extraction, the boundary extraction of a face is more time consuming because the rough contour of a face can not be estimated by RCER. However, the rough contour needs to be approximated as accurately as possible. The energy function associated with boundary extraction of a face is defined as

$$E_{face} = \sum_{i=1}^{n} (\alpha_i E_{continuity} + \beta_i E_{curvature} + \delta_i E_{images}) \qquad (2.13)$$

where $E_{continuity}$ and $E_{curvature}$ are defined in *section 2.1.4.4.1* and E_{images} is defined as

$$E_{images} = w_{lines} E_{lines} + w_{edge} E_{edge} \tag{2.14}$$

The goal of E_{lines} is to attract more white points and less dark ones inside the active contour, where as the goal of E_{edge} is to attract more edge points on the active contour boundary.

E_{lines} and E_{edge} are defined as

$$E_{lines} = \begin{cases} -Mag_{white} & , \text{ if } \beta(x,y) = 0 \text{ and } I(x,y) \geq \text{threshold+ tolerance} \\ Mag_{dark} & , \text{ if } \beta(x,y) = 0 \text{ and } I(x,y) < \text{threshold- tolerance} \\ -Mag_{white} & , \text{ if } \beta(x,y) = 1 \text{ and } I(x,y) \geq \text{threshold+ tolerance} \\ Mag_{dark} & , \text{ if } \beta(x,y) = 1 \text{ and } I(x,y) < \text{threshold- tolerance} \\ -Mag_{contour} & , \text{ if } \beta(x,y) = 255 \text{ and } I(x,y) \geq \text{threshold+ tolerance} \\ Mag_{contour} & , \text{ if } \beta(x,y) = 255 \text{ and } I(x,y) < \text{threshold- tolerance} \\ 0 & , \text{ otherwise} \end{cases}$$

$$E_{edge} = \left| \begin{aligned} & I(x,y) - \frac{1}{8}(I(x+1,y+1) + I(x+1,y) + I(x+1,y-1) + I(x,y+1) \\ & + I(x,y-1) + I(x-1,y+1) + I(x-1,y) + I(x-1,y-1)) \end{aligned} \right|$$

(2.15),

(2.16)

where *I(x,y)* represents the intensity value of point (x,y), *threshold* is the same as face threshold, *tolerance* is experimentally determined.

To extract the boundary of a face without a beard, the snake iterates and moves toward the chin because E_{face} decreases constantly. The convergent process of the snake is based on the greedy algorithm. When the iterative process stops, iterations can be re-started in order to find its next convergent place. If they are similar, the position is accurate.

2.1.5. Face Recognition Based on Principal Component Analysis

In section 2.1.4, we have reviewed a face recognition method based on feature extraction. By using extensive geometry, it is possible to find the contours of the eye, eyebrow, nose, mouth, and even the face itself.

Principal component analysis for face recognition is based on the information theory approach. Here, the relevant information in a face image is extracted and encoded as efficiently as possible. Recognition is performed on a face database that consists of models encoded similarly.

In mathematical terms, the principal components of the distribution of faces or the eigenvectors of the covariance matrix of the set of face images, treating an image as a point (vector) in a very high dimensional face space is sought. In Chapter 3, a principal component analysis method will be presented in more detail.

CHAPTER 3

FACE RECOGNITION USING EIGENFACES

The aim of this chapter is to introduce the explanation of a principal component analysis and usage of neural networks for face recognition. For principal component analysis, eigenface method will be used for extracting feature vectors.

3.1. Introduction

Much of the previous work on automated face recognition has ignored the issue of just what aspects of the face stimulus are important for face recognition. This suggests the use of an information theory approach of coding and decoding of face images, emphasizing the significant local and global features. Such features may or may not be directly related to our intuitive notion of face features such as the eyes, nose, lips, and hair.

In the language of information theory, the relevant information in a face image is extracted, encoded as efficiently as possible, and then compared with a database of models encoded similarly. A simple approach to extracting the information contained in an image of a face is to somehow capture the variation in a collection of face images, independent of any judgment of features, and use this information to encode and compare individual face images.

In mathematical terms, the principal components of the distribution of faces, or the eigenvectors of the covariance matrix of the set of face images, treating an image as point (or vector) in a very high dimensional space is sought. The eigenvectors are ordered, each one accounting for a different amount of the variation among the face images.

These eigenvectors can be thought of as a set of features that together characterize the variation between face images. Each image location contributes more or less to each eigenvector, so that it is possible to display these eigenvectors as a sort of ghostly face image which is called an "eigenface".

Sample face images, the average face of them, eigenfaces of the face images and the corresponding eigenvalues are shown in Figure 3.1, Figure 3.2, Figure 3.3, and Figure 3.4 respectively. Each eigenface deviates from uniform gray where some facial feature differs among the set of training faces. Eigenfaces can be viewed as a sort of map of the variations between faces.

Figure 3.1 Sample Faces

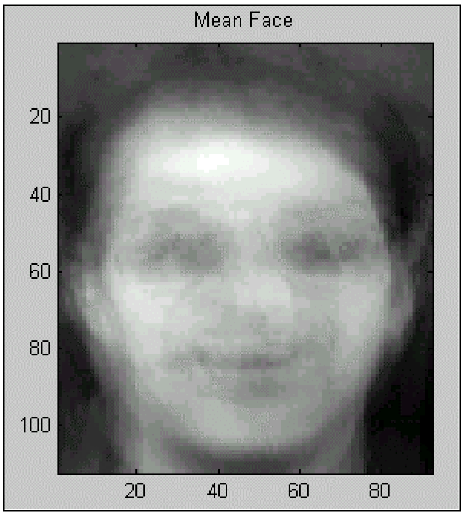


Figure 3.2 Average face of the Sample Faces

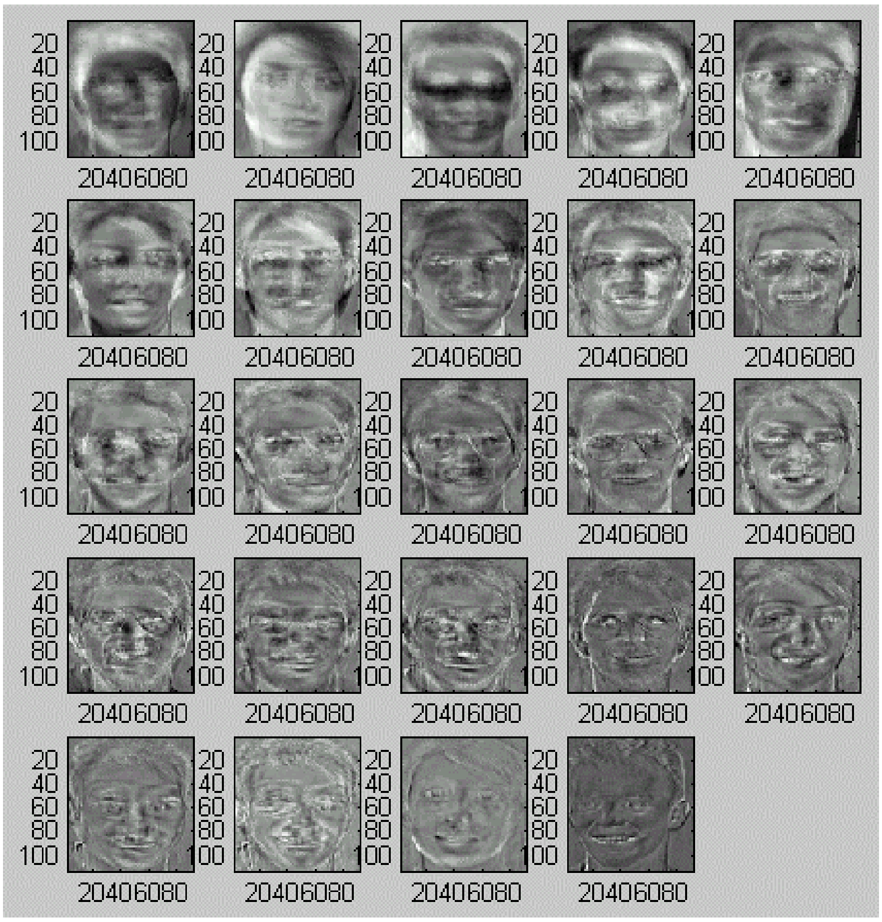


Figure 3.3 Eigen Faces of the Sample Faces

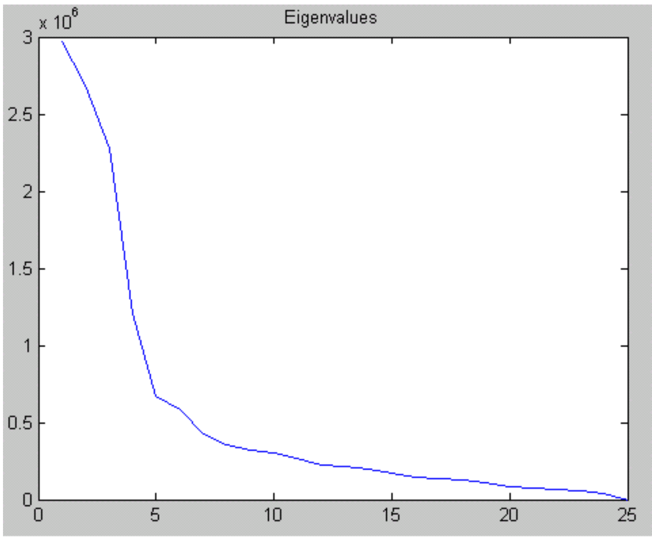


Figure 3.4 Eigenvalues corresponding to eigenfaces

Each individual face can be represented exactly in terms of a linear combination of the eigenfaces. Each face can also be approximated using only the "best" eigenfaces, those that have the largest eigenvalues, and which therefore account for the most variance within the set of face images. As seen from the Figure 3.4, the eigenvalues drops very quickly, that means one can represent the faces with relatively small number of eigenfaces. The best M eigenfaces span an M-dimensional subspace which we call the "face space" of all possible images.

Kirby and Sirovich [6, 7] developed a technique for efficiently representing pictures of faces using principal component analysis. Starting with an ensemble of original face images, they calculated a best coordinate system for image compression, where each coordinate is actually an image that they termed an "eigenpicture". They argued that, at least in principle, any collection of face images can be approximately reconstructed by storing a small collection of weights for each face, and a small set of standard pictures (the eigenpictures). The weights describing each face are found by projecting the face image onto each eigenpicture.

Turk and A. Pentland [12] argued that, if a multitude of face images can be reconstructed by weighted sum of a small collection of characteristic features or eigenpictures, perhaps an efficient way to learn and recognize faces would be to build up the characteristic features by experience over time and recognize particular faces by comparing the feature weights needed to approximately reconstruct them with the weights associated with known individuals. Therefore, each individual is characterized by a small set of feature or eigenpicture weights needed to describe and reconstruct them. This is an extremely compact representation when compared with the images themselves. The projected image of the Face1 with a number of eigenvalues is shown in Figure3.5. As seen from the figure among the 25 faces database (25 eigenfaces), 15 eigenfaces are enough for reconstruct the faces accurately. These feature or eigenpicture weights are called feature vectors and as seen in Section 3.3, they will be used as new descriptors of the face images and used fro recognition purposes.

Figure 3.5 Reconstruction of First Image with the number of Eigenfaces.

3.2. Calculation of Eigenfaces

Let a face image I(x,y) be a two-dimensional N x N array of 8-bit intensity values. An image may also be considered as a vector of dimension N^2, so that a typical image of size 256 x 256 becomes a vector of dimension 65,536, or equivalently a point in 65,536-dimensional space. An ensemble of images, then, maps to a collection of points in this huge space.

Images of faces, being similar in overall configuration, will not be randomly distributed in this huge image space and thus can be described by a relatively low dimensional subspace. The main idea of the principal component analysis (or Karhunen-Loeve expansion) is to find the vectors that best account for the distribution of face images within the entire image space.

These vectors define the subspace of face images, which we call "face space". Each vector is of length $N,^2$ describes an N x N image, and is a linear combination of the original face images. Because these vectors are the eigenvectors of the covariance matrix corresponding to the original face images, and they are face-like in appearance, we refer to them as "eigenfaces". Some examples of eigenfaces are shown in Figure 3.3.

Definitions:

An N x N matrix A is said to have an eigenvector X, and corresponding eigenvalue λ if

$$AX = \lambda X. \tag{3.1}$$

Evidently, Eq. (3.1) can hold only if

$$\det \left| A - \lambda I \right| = 0 \tag{3.2}$$

which, if expanded out, is an N^{th} degree polynomial in λ whose root are the eigenvalues. This proves that there are always N (not necessarily distinct) eigenvalues. Equal eigenvalues coming from multiple roots are called "degenerate".

A matrix is called *symmetric* if it is equal to its transpose,

$$A = A^T \text{ or } a_{ij} = a_{ji} \tag{3.3}$$

it is termed *orthogonal* if its transpose equals its inverse,

$$A^T A = AA^T = I \tag{3.4}$$

finally, a real matrix is called *normal* if it commutes with is transpose,

$$A^T A = AA^T \tag{3.5}$$

Theorem: Eigenvalues of a real symmetric matrix are all real. Contrariwise, the eigenvalues of a real nonsymmetric matrix may include real values, but may also include pairs of complex conjugate values. The eigenvalues of a normal matrix with nondegenerate eigenvalues are complete and orthogonal, spanning the N dimensional vector space.

After giving some insight on the terms that are going to be used in the evaluation of the eigenfaces, we can deal with the actual process of finding these eigenfaces.

Let the training set of face images be $\Gamma_1 \;\; \Gamma_2 \ldots\ldots\ldots\ldots \;\; \Gamma_M,$ then the average of the set is defined by

$$\Psi = \frac{1}{M} \sum_{n=1}^{M} \Gamma_n \tag{3.6}$$

Each face differs from the average by the vector

$$\Phi_i = \Gamma_i - \Psi \tag{3.7}$$

An example training set is shown in Figure 3.1, with the average face Ψ shown in Figure 3.2.

This set of very large vectors is then subject to principal component analysis, which seeks a set of M orthonormal vectors, u_n , which best describes the distribution of the data. The kth vector, u_k , is chosen such that

$$\lambda_k = \frac{1}{M}\sum_{n=1}^{M}(u_k^T\Phi_n)^2 \tag{3.8}$$

is a maximum, subject to

$$u_l\, u_k = \delta_{lk} = \begin{cases} 1, \text{ if } l=k \\ 0, \text{ otherwise} \end{cases} \tag{3.9}$$

The vectors u_k and scalars l_k are the eigenvectors and eigenvalues, respectively of the covariance matrix

$$C = \frac{1}{M}\sum_{n=1}^{M}(\Phi_n\Phi_n^T) = A\,A^T \tag{3.10}$$

where the matrix A= [Φ_1 Φ_2 …… Φ_M]. The covariance matrix C, however is N^2 x N^2 real symmetric matrix, and determining the N^2 eigenvectors and eigenvalues is an intractable task for typical image sizes. We need a computationally feasible method to find these eigenvectors.

If the number of data points in the image space is less than the dimension of the space (M < N^2), there will be only M-1, rather than N^2, meaningful eigenvectors. The remaining eigenvectors will have associated eigenvalues of zero. We can solve for the N^2 dimensional eigenvectors in this case by first solving the eigenvectors of an M x M matrix such as solving 16 x 16 matrix rather than a 16,384 x 16,384 matrix and then, taking appropriate linear combinations of the face images Φ_i .

Consider the eigenvectors v_i of $A\,A^T$ such that

$$A^T A v_i = \mu_i\, v_i \tag{3.11}$$

Premultiplying both sides by A, we have

$$AA^T Av_i = \mu_i Av_i \tag{3.12}$$

from which we see that Avi are the eigenvectors of $C= AA^T$

Following these analysis, we construct the M x M matrix $L = AA^T$, where $L_{mn} = \Phi_m^T \Phi_n$ and find the M eigenvectors, v_l , of L. These vectors determine linear combinations of the M training set face images to form the eigenfaces u_l .

$$u_l = \sum_{k=1}^{M} v_{lk} \Phi_k \tag{3.13}$$

With this analysis, the calculations are greatly reduced, from the order of the number of pixels in the images (N^2) to the order of the number of images in the training set (M). In practice, the training set of face images will be relatively small (M << N^2), and the calculations become quite manageable. The associated eigenvalues allow us to rank the eigenvectors according to their usefulness in characterizing the variation among the images.

The success of this algorithm is based on the evaluation of the eigenvalues and eigenvectors of the real symmetric matrix L that is composed from the training set of images. Root searching in the characteristic equation, Eq. (3.2) is usually a very poor computational method for finding eigenvalues. During the programming phase of the above algorithm, a more efficient method [38] was used in order to evaluate the eigenvalues and eigenvectors. At first, the real symmetric matrix is reduced to tridiagonal form with the help of the "Householder" algorithm. The Householder algorithm reduces an N x N symmetric matrix A to tridiagonal form by N - 2 orthogonal transformations. Each transformation annihilates the required part of a whole column and whole corresponding row. After that, eigenvalues and eigenvectors are obtained with the help of QR transformations. The basic idea behind the QR algorithm is that any real symmetric matrix can be decomposed in the form A = QR where Q is orthogonal and R is upper triangular. The workload in the QR algorithm is $O(N^3)$ per iteration for a general matrix, which is prohibitive. However, the workload is only O(N) per iteration for a tridiagonal matrix, which makes it extremely efficient.

3.3. Using Eigenfaces to Classify a Face Image

The eigenface images calculated from the eigenvectors of L, span a basis set with which to describe face images. Sirovich and Kirby evaluated a limited version of this framework on an ensemble of M = 115 images of Caucasian males digitized in a controlled manner, and found that 40 eigenfaces were sufficient for a very good description of face images. With M' = 40 eigenfaces, RMS pixel by pixel errors in representing cropped versions of face images were about 2%.

In practice, a smaller M' can be sufficient for identification, since accurate reconstruction of the image is not a requirement and, it was observed that, for a training set of fourteen face images, seven eigenfaces were enough for a sufficient description of the training set members. But for maximum accuracy, the number of eigenfaces should be equal to the number of images in the training set.

In this framework, identification becomes a pattern recognition task. The eigenfaces span an M' dimensional subspace of the original N^2 image space. The M' significant eigenvectors of the L matrix are chosen as those with the largest associated eigenvalues.

A new face image (Γ) is transformed into its eigenface components (projected onto "face space") by a simple operation,

$$w^k = u_k^T (\Gamma - \Psi) \qquad (3.14)$$

for k = 1,...,M'. This describes a set of point by point image multiplications and summations, operations performed at approximately frame rate on current image processing hardware, with a computational complexity of $O(N^4)$.

The weights form a feature vector,

$$\Omega^T = [w_1\ w_2 \ldots\ldots\ w_M] \tag{3.15}$$

that describes the contribution of each eigenface in representing the input face image, treating the eigenfaces as a basis set for face images. The feature vector is then used in a standard pattern recognition algorithm to find which of a number of predefined face classes, if any, best describes the face. The face classes Ω_i can be calculated by averaging the results of the eigenface representation over a small number of face images (as few as one) of each individual.

3.4. Rebuilding a Face Image with Eigenfaces

A face image can be approximately reconstructed (rebuilt) by using its feature vector and the eigenfaces as

$$\Gamma' = \Psi + \Phi f \tag{3.16}$$

where

$$\Phi_j = \sum_{j=1}^{M} w_j u_j \tag{3.17}$$

is the projected image.

Eq. (3.16) tells that the face image under consideration is rebuilt just by adding each eigenface with a contribution of w_i Eq. (3.17) to the average of the training set images. The degree of the fit or the "rebuild error ratio" can be expressed by means of the Euclidean distance between the original and the reconstructed face image as given in Eq. (3.18).

$$\text{Rebuild error ratio} = \left\| \frac{\Gamma^l - \Gamma}{\Gamma} \right\| \tag{3.18}$$

It has been observed that, rebuild error ratio increases as the training set members differ heavily from each other. This is due to the addition of the average face image. When the members differ from each other (especially in image background) the average face image becomes messier and this increases the rebuild error ratio.

3.5. Usage of Neural Networks for Recognition:

3.5.1. Introduction

Neural networks are composed of simple elements operating in parallel. The neuron model shown in Figure 3.6 is the one that widely used in artificial neural networks with some minor modifications on it.

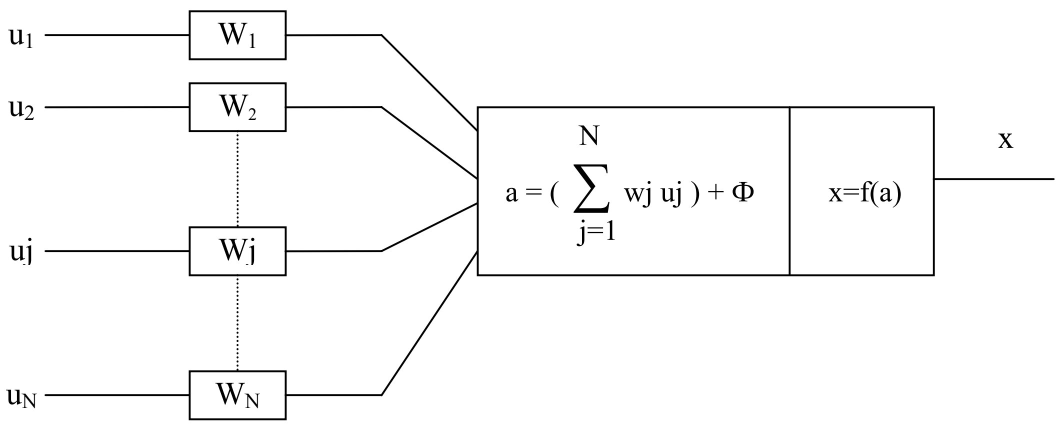


Figure 3.6 Artificial Neuron

The artificial neuron given in this figure has N input, denoted as u_1, u_2,…u_N. Each line connecting these inputs to the neuron is assigned a weight, which is denoted as w_1,w_2,….,w_N respectively. The threshold in artificial neuron is usually represented by Φ and the activation is given by the formula:

$$a = (\sum_{j=1}^{n} w_j u_j) + \Phi \tag{3.19}$$

The inputs and weight are real values. A negative value for a weight indicates an inhibitory connection while a positive value indicating excitatory one. If Φ is positive, it is usually referred as bias. For its mathematical convenience (+) sign is used in the activation formula. Sometimes, the threshold is combined for simplicity into the summation part by assuming an imaginary input $u_0 = +1$ and a connection weight $w0 = \Phi$. Hence the activation formula becomes

$$a = (\sum_{j=1}^{n} w_j u_j) \tag{3.20}$$

The vector notation

$$A = w^T u + \Phi \tag{3.21}$$

is useful for expressing the activation for a neuron.

The neuron output function f(a) can be:

Linear :

$$f(a) = K(a) \tag{3.22}$$

Threshold :

$$f(a) = \begin{cases} 0 & a \leq 0 \\ 1 & 0 < a \end{cases} \tag{3.23}$$

Ramp:

$$f(a) = \begin{cases} 0 & a \leq 0 \\ a/K & 0 < a \\ 1 & K < a \end{cases} \tag{3.24}$$

Sigmoid :

$$f(a) = 1/(1 + \exp(-Ka)) \tag{3.25}$$

Function implementations can be done by adjusting the weights and the threshold of the neuron. Furthermore, by connecting the outputs of some neurons as inputs to the others, neural network will be established, and any function can be implemented by these networks. The last layer of neurons is called the output layer and the layers between the input and output layer are called the hidden layers. The input layer is made up of special input neurons, transmitting only the applied external input to their outputs. In a network, if there is only the layer of input nodes and a single layer of neurons constituting the output layer then they are called single layer network. If there are one or more hidden layers, such networks are called multilayer networks. There are two types of network architecture: recurrent and feed forward neural network.

3.5.2. Recurrent Neural Networks:

The structures, in which connections to the neurons of the same layer or to the previous layers are allowed, are called recurrent networks and shown in Figure 3.7

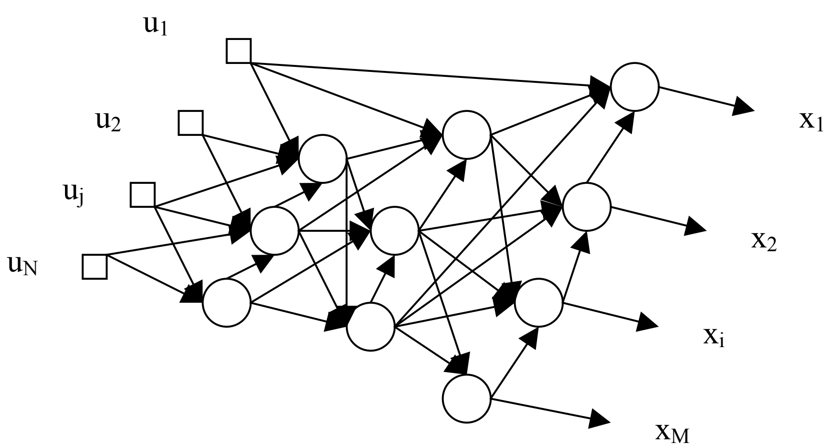


Figure 3.7 Recurrent Neural Networks

3.5.3. Feedforward Neural Networks:

In this kind of networks, the neurons are organized in the form of layers. The neurons in a layer get input from the previous layer and feed their output to the next layer. In this kind of networks connections to the neurons in the same or previous layers are not permitted. Figure 3.8 shows typical feedforward neural network.

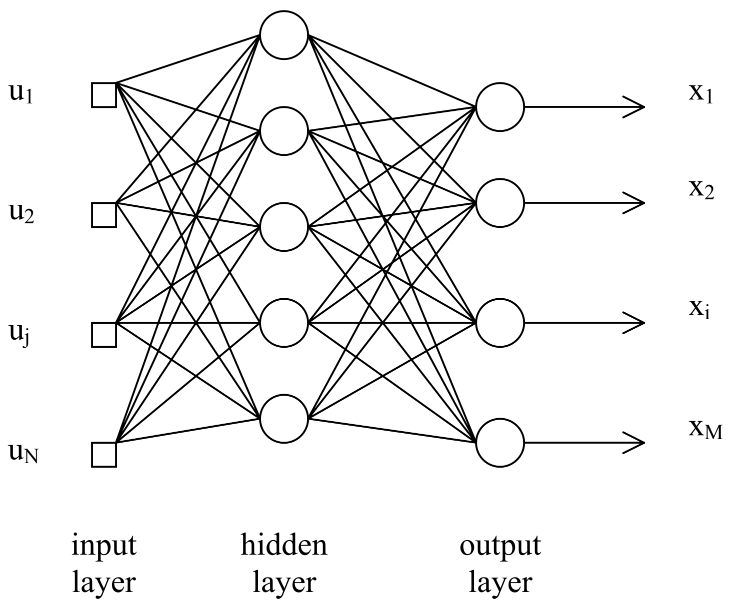


Figure 3.8. Feedforward Neural Networks

For a feedforward network always exists an assignment of indices to neurons resulting in a triangular weight matrix. Furthermore if the diagonal entries are zero this indicates that there is no self-feedback on the neurons. However in recurrent networks, due to feedback, it is not possible to obtain triangular weight matrix with any assignment of the indices.

3.5.4. Training and Simulation of Neural Networks for Recognition

Neural networks have been trained to perform complex functions in various fields of application including pattern recognition, identification, classification, speech, vision and control systems.

In this thesis, there is one neural network for each person in the database. After calculating eigenfaces, the feature vectors are calculated for the faces in the database. These feature vectors are used as inputs to train the each person's networks. In training algorithm, the faces feature vectors that belong to same person are used as positive examples for the person's network (such that network gives ”1” as output), and negative examples for the others network. (such that network gives ”0” as output), Figure 3.9 shows schematic diagram for the networks training.

When the new image is come for recognition, its feature vectors are calculated from the eigenfaces found before, and this image gets its new descriptors. These new descriptors are inputted to every network and the networks are simulated with these descriptors.. The network outputs are compared. If the maximum output exceeds the predefined threshold level, then this new face is decided to belong to person with this maximum output.

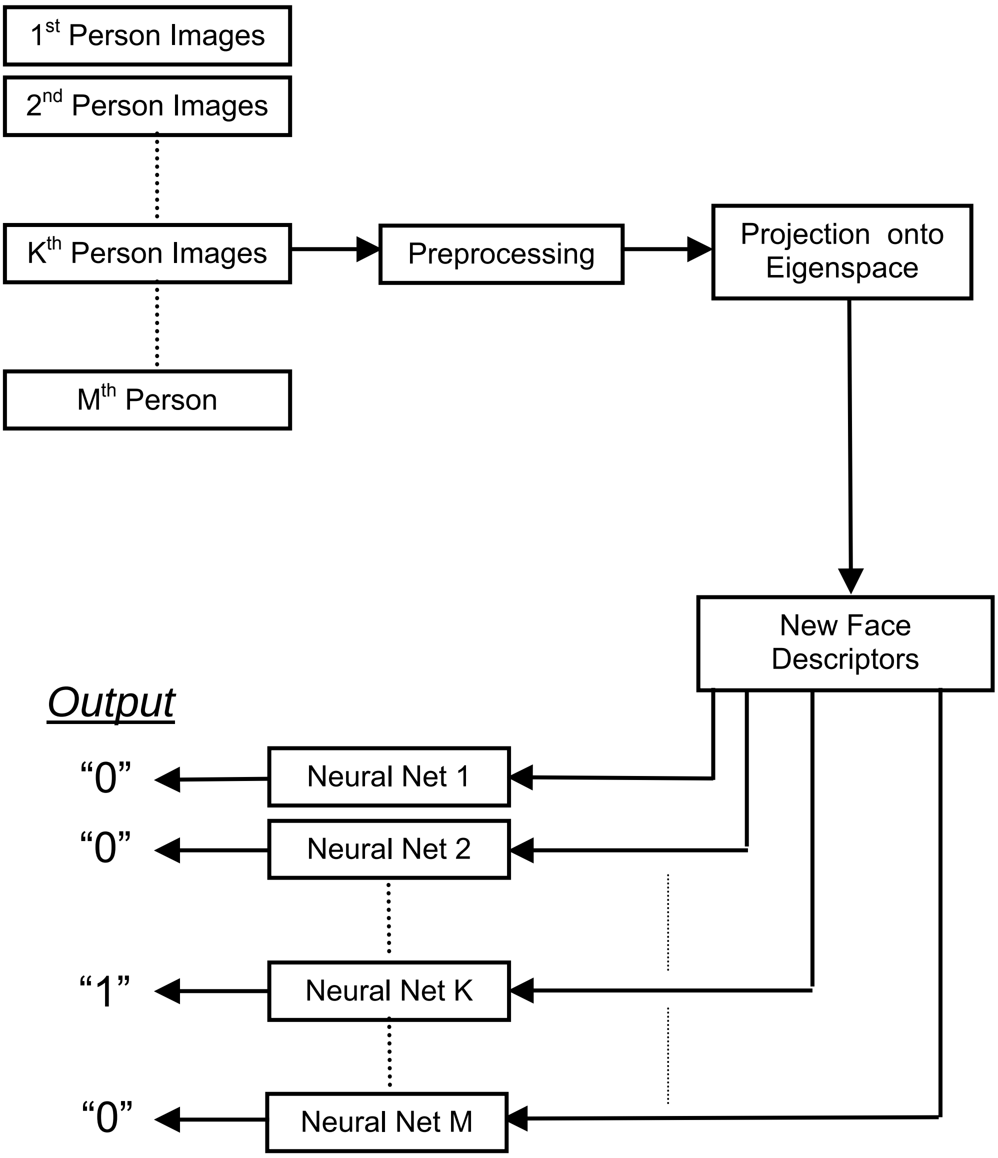


Figure 3.9 Training of Neural Networks

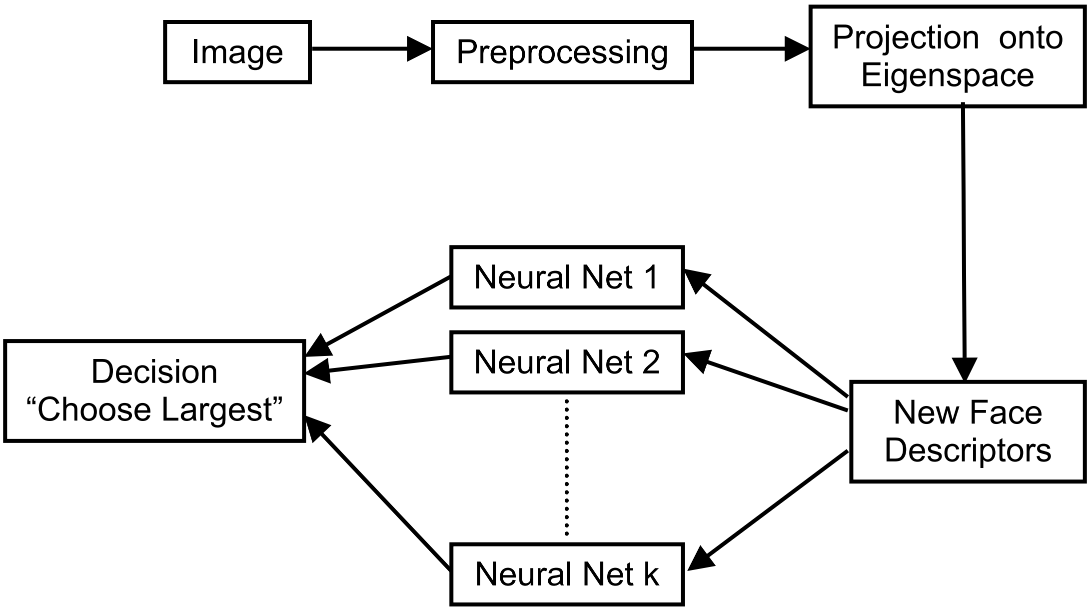


Figure 3.10 Simulation of Neural Networks for Recognition

3.6. Summary of the Eigenface Recognition Procedure

The eigenfaces approach to face recognition can be summarized in the following steps:

1. Form a face library that consists of the face images of known individuals.
2. Choose a training set that includes a number of images (M) for each person with some variation in expression and in the lighting.
3. Calculate the M x M matrix L, find its eigenvectors and eigenvalues, and choose the M' eigenvectors with the highest associated eigenvalues.
4. Combine the normalized training set of images according to Eq. (3.13) to produce M' eigenfaces. Store these eigenfaces for later use.
5. For each member in the face library, compute and store a feature vector according to Eq. (3.15).
6. Create Neural Network for each person in the database

7. Train these networks as the faces will be used as positive examples for their own networks and negative examples for all other networks
8. For each new face image to be identified, calculate its feature vector according to Eq. (3.15).
9. Use these feature vectors as network inputs and simulate all networks with these inputs
10. Select the network with the maximum output. If the output of the selected network passes a predefined threshold, it will be reported as the host of the input face. Otherwise it will be reported as unknown and adds this member to the face library with its feature vector and network.

3.7. Comparison of the Eigenfaces Approach to Feature Based Face Recognition

These two different approaches are compared based on the following aspects of face recognition:

3.7.1. Speed and simplicity: Feature based face recognition involves complex computations such as deformable templates and active contour models. The evaluation of these parameters is very time consuming even on today's computers. Eigenfaces approach is superior in its speed and reasonably simple implementation. In Eq. (3.14), it is seen that the evaluation of a feature vector involves merely additions and multiplications. On a machine that is capable of executing an addition and a multiplication in one clock cycle, this feature evaluation and comparison can be done in real time. During the experiments, although no special hardware (and special software optimizations for matrix operations) was used, the speed of the proposed face recognition system was near real time.

3.7.2. Learning capability: Feature based face recognition systems are generally trained to optimize their parameters in a supervised manner. That is, the system designer presents known individuals to the system and checks system response. In the eigenfaces approach, training is done in an unsupervised manner. User selects a training set that represents the rest of the face images. Eigenfaces are obtained from the training set members and feature vectors are formed.

3.7.3. Face background: Background of the face images is extremely important in the eigenface approach. Eigenfaces and feature vectors are evaluated by image multiplication and additions. As a result of this, entire information contained in the face image is used. If this information changes due to face background, recognition performance can significantly decrease. In order to avoid this, a "background removal" algorithm can be implemented in a preprocessing step. Feature based face recognition algorithms can be less sensitive to face background in case, they generally locate face contours in order to extract facial features.

3.7.4. Scale and orientation: The recognition performance decreases quickly as head size or orientation is misjudged. The head size and orientation in the input image must be close to that of the eigenfaces for the system to work well. In order to overcome this problem, multiscale eigenfaces can be used or the head can be removed from the rest of the image, and then scaled or rotated to meet the specifications of the eigenfaces. Again, feature based face recognition algorithms can score better in this comparison because they find facial features by using deformable templates and active contour models that are less sensitive to scale and orientation.

3.7.5. Presence of small details: Feature based face recognition algorithms can suffer when some details are present on the face image such as dark glasses or beards. For a feature based face recognition system, it is quite impossible to extract the features that are related to the eyes when dark glasses are present on the face. Also, active contour models can suffer when a beard is present on the face while locating the face contour. Eigenfaces approach excels in this aspect of face recognition. Small changes in face images such as glasses, beards or moustaches does not cause a decrease in the face recognition performance because the information that is present in the rest of the face image makes it enough to be classified correctly.

CHAPTER 4

RESULTS

The proposed method is tested on three different databases: ORL, Yale and FERET face databases. Each database has more than one face images with different conditions (expression, illumination,...etc.), of each individual.

In the following section, detailed information for these three databases and their corresponding performance results for the proposed face recognition method are given.

The number of networks used for these databases are equal to the number of people in these databases. The initial parameters of the neural networks used in these tests are given below:

- Type: Feed forward bacpropagation network
- Number of layers: 3 (input, one hidden, output layer)
    - Number of neurons in input layer : Number of eigen faces to describe the faces
    - Number of neurons in hidden layer : 10
    - Number of neurons in output layer : 1
- Transfer function of the i^{th} layer: Tansig
- Training Function: Trainlm
- Number of epochs used in training: 100
- Backprop weight/bias learning function: learngdm
- Performance function: mse

4.1. Test Results for the Olivetti and Oracle Research Laboratory (ORL) Face Database

The Olivetti and Oracle Research Laboratory (ORL) face database is used in order to test our method in the presence of headpose variations. There are 10 different images of each of 40 distinct subjects. For some subjects, the images were taken at different times, varying lighting, facial expressions (open / closed eyes, smiling / not smiling), facial details (glasses / no glasses) and head pose (tilting and rotation up to 20 degrees). All the images were taken against a dark homogeneous background. Figure 4.1 shows the whole set of 40 individuals, 10 images per person from the ORL database.

Since the number of networks is equal to the number of people in the database, forty networks, one for each person were created. Within the ten images, first 4 of them are used for tanning the neural networks, then these networks are tested and their properties are updated with the next 3 pictures for getting the minimum squared error function, and these networks will be used for later use for recognition purposes.

For testing the whole database, the faces used in training, testing and recognition are changed and the recognition performance is given for whole database.

For this database, the mean face of the whole database, the calculated top 30 (with the highest eigenvalues) eigenfaces, and their corresponding eigenvalues are shown in Figure 4.2, Figure 4.3 and Figure 4.4 respectively.

The recognition performance increases with the number of faces used to train the neural networks, so the recognition rates, with using different number of faces used in training, are calculated and given in Table 4.1.

The number of eigenfaces that is used to describe the faces, and the number of neurons in hidden layer also affects the recognition rate, so the tests are done with different number of eigenfaces and neurons, and the results are given in Table 4.2.

Histogram equalization enhances the contrast of images by transforming the values in an intensity image so that the histogram of the output image approximately matches a specified histogram. Table 4.2 also shows the recognition rates with and without histogram equalization.

Up to now, the recognition rates are calculated with neural networks only for first choice (largest output), The recognition rates with neural networks other choices are also given in Table 4.3

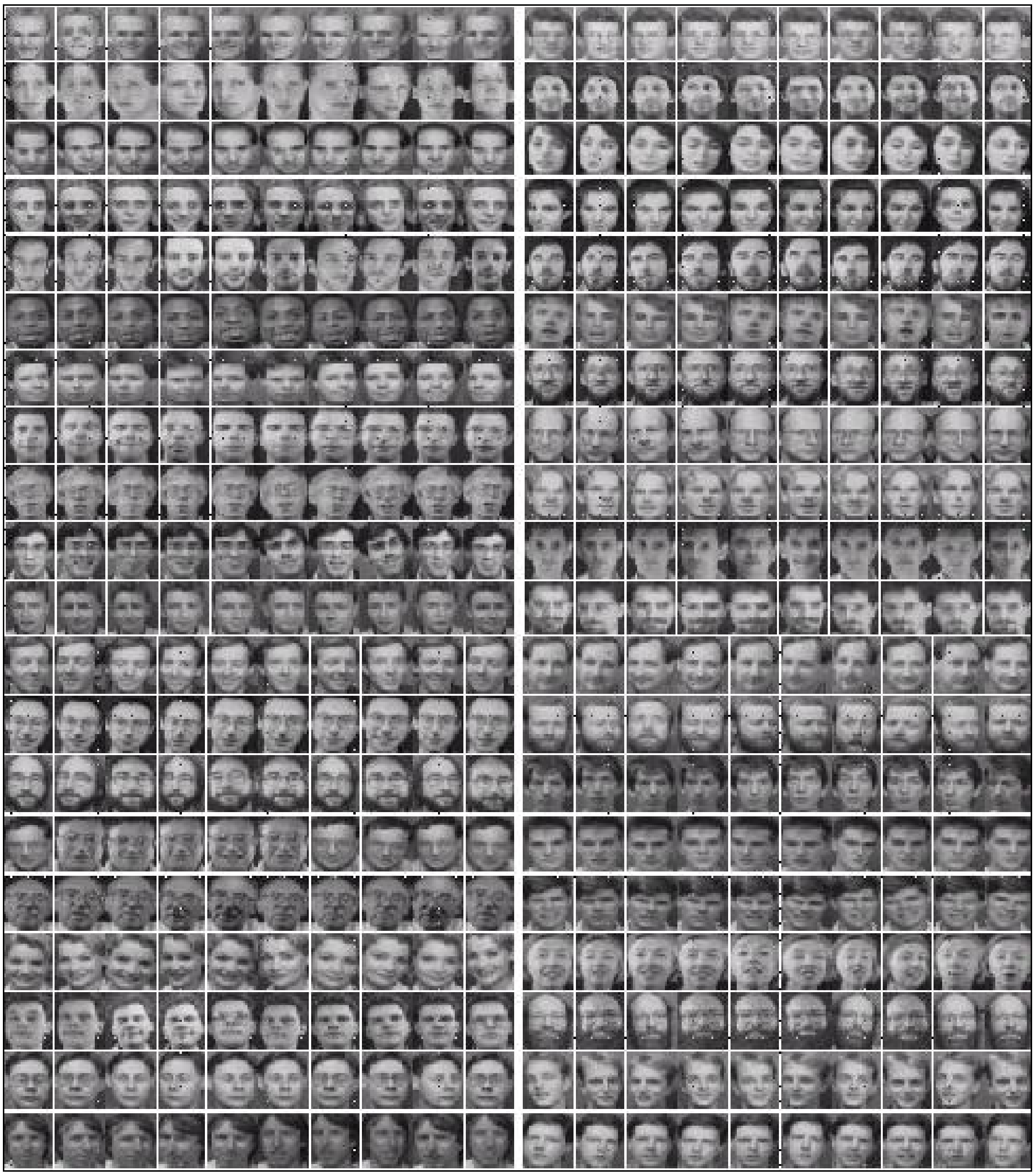


Figure 4.1 ORL Face Database

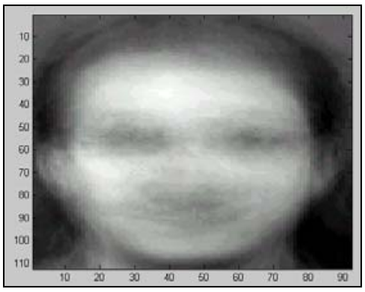


Figure 4.2 Mean face for ORL Face Database

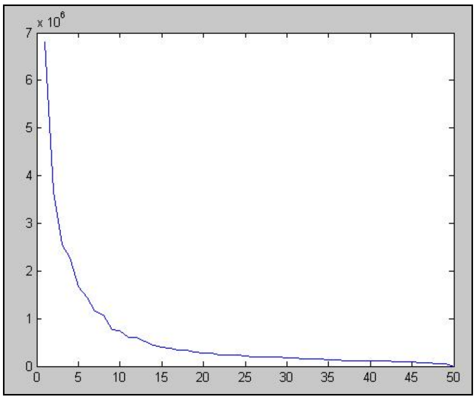


Figure 4.3 The eigen values for ORL Face Database

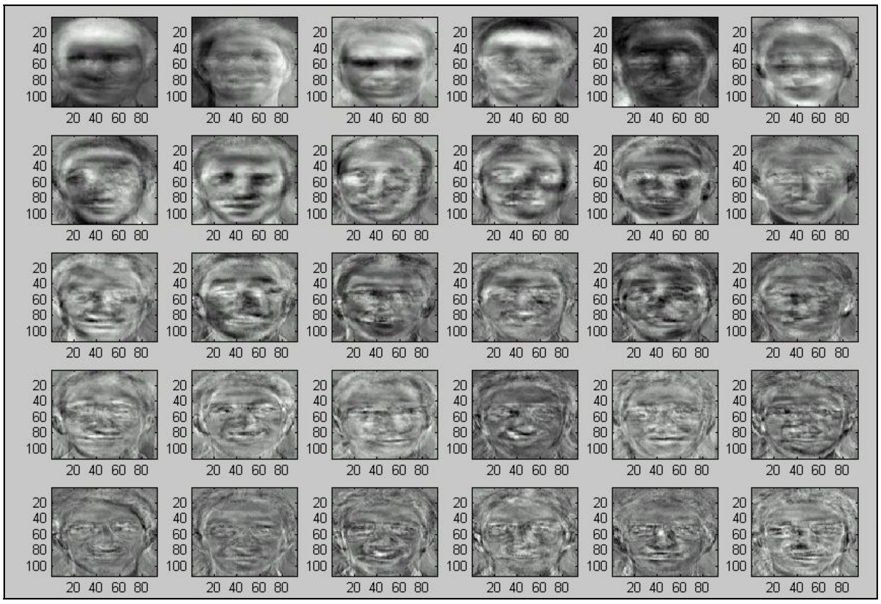


Figure 4.4 The top 30 eigen faces for the ORL Face Database

Table 4.1 Recognition Rate using different Number of Training and Test Images, and w/wo Histogram Equalization

Number of Images used in Training (per individual)	***Number of Images used in Testing (per individual)***	***Number of Eigenfaces***	***Hist. Equ.***	***Recognition Rate (%)***
1	9	10	√	8.8
1	9	10	X	9.7
2	8	20	√	25
2	8	20	X	26
3	7	25	√	51.7
3	7	25	X	51.7
4	6	30	√	75
4	6	30	X	76.2
5	5	50	√	87.5
5	5	50	X	90
6	4	60	√	90
6	4	60	X	92.5
7	3	100	√	89.1
7	3	100	X	91.6
8	2	100	√	88.8
8	2	100	X	91.2
9	1	100	√	87.5
9	1	100	X	90

Number of Neurons in Hidden Layer: 15

Table 4.2 Recognition Rate using different Number of Eigen Faces and neurons in Hidden Layer

Number of EigenFaces	***Number of Neurons in Hidden Layer***	**Recognition** ***Rate (%)***
40	5	40
	10	58.3
	15	72
	20	74.5
45	5	41
	10	73.5
	15	80.8
	20	88.5
50	5	54.8
	10	73
	15	87.5
	20	90.8
55	5	50
	10	81.8
	15	88.5
	20	91.5
60	5	48.8
	10	81.3
	15	89.5
	20	92.8
65	5	52
	10	81.8
	15	91.5
	20	94.3
70	5	50
	10	86.3
	15	92.3
	20	93.3

Number of Images used in Training : 5

Number of Images used in Testing : 5

Histogram Equalization : Done for all images

Table 4.3 Recognition Rate with Neural Networks different choices

	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5
Number of Correct Decision (Over 400 Images)	360	21	7	4	2
Recognition Rate	90%	95.2 %	97%	98%	98.5 %

Number of Testing Images : 5

Number of Eigenfaces : 50

Number of H. L. Neurons : 15

4.2. Test Results for the Olivetti and Oracle Research Laboratory (ORL) Face Database with Eye Extraction

In this section, Eye based recognition is tested on this database. Eyes are cropped from the faces and then recognition performance is tested only with eyes. Firstly a rectangular with size 15 x 62 is created (shown in Figure 4.5) and then eyes are cropped from the faces manually with this rectangle. The mean eye, and the top 30 (with highest eigenvalues) eigeneyes are shown in Figure 4.6 and Figure 4.7 respectively. The test results and the recognition ratios with different number of training images and w/wo histogram equalization, and different number of eigenfaces and neurons in hidden layer are given in Table 4.4 and Table 4.5 respectively. The recognition rates with neural networks first, second, third, forth and fifth choices are also given in Table 4.6

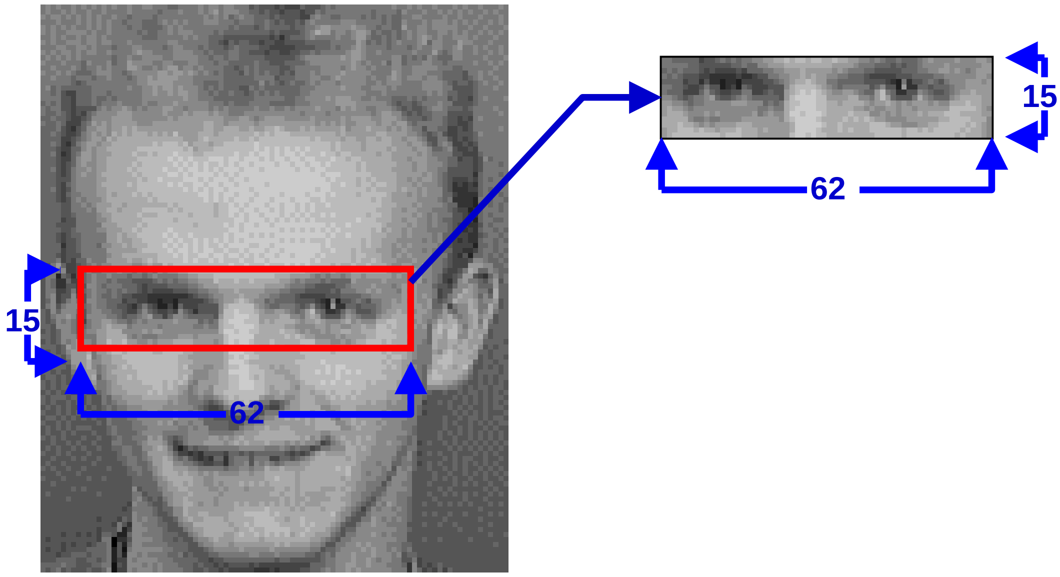


Figure 4.5: The eye extraction

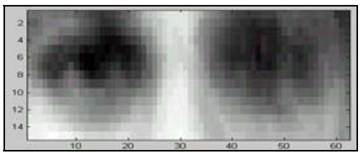


Figure 4.6: The mean eye

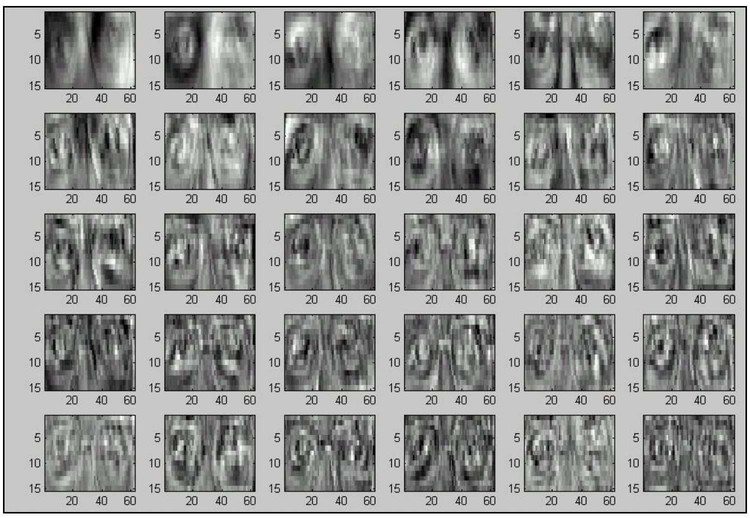


Figure 4.7: The top 30 Eigeneyes

Table 4.4 Recognition Rate using different Number of Training and Test Images

Number of Images used in Training (per individual)	*Number of Images used in Testing (per individual)*	*Number of Eigenfaces*	*Hist. Equ.*	*Recognition Rate (%)*
1	9	10	√	8.3
1	9	10	X	8.3
2	8	20	√	22.5
2	8	20	X	23.3
3	7	25	√	41
3	7	25	X	42
4	6	30	√	62.5
4	6	30	X	66.5
5	5	50	√	79
5	5	50	X	82.5
6	4	60	√	81.5
6	4	60	X	84
7	3	100	√	82.5
7	3	100	X	85.8
8	2	100	√	85
8	2	100	X	85
9	1	100	√	87.5
9	1	100	X	87.5

Number of Neurons in Hidden Layer: 15

Table 4.5 Recognition Rate using different Number of Eigen Faces and neurons in Hidden Layer

Number of EigenFaces	***Number of Neurons in Hidden Layer***	**Recognition** ***Rate (%)***
40	5	30.5
	10	45.5
	15	62
	20	66.5
45	5	37
	10	42
	15	68.5
	20	71.5
50	5	43.5
	10	50.5
	15	79
	20	79
55	5	47.5
	10	54.5
	15	79.5
	20	81
60	5	48.8
	10	63.5
	15	81
	20	84.5
65	5	44.5
	10	61
	15	81.5
	20	80.5
70	5	50
	10	60.5
	15	82
	20	82.5

Number of Images used in Training : 5

Number of Images used in Testing : 5

Histogram Equalization : Done for all images

Table 4.6 Recognition Rate with Neural Networks different choices

	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5
Number of Correct Decision (Over 400 Images)	330	17	11	5	2
Recognition Rate	82.5 %	86.7 %	89.5%	90%	91.2 %

Number of Testing Images : 5

Number of Eigenfaces : 50

Number of H. L. Neurons : 15

4.3. Test Results for Yale Face Database

The Yale Face Database is used in order to test our method in the presence of headpose variations. The Yale Face Database contains 165 grayscale images in GIF format of 15 individuals. There are 11 images per subject, one per different facial expression or configuration: center-light, w/glasses, happy, left-light, w/no glasses, normal, right-light, sad, sleepy, surprised, and wink. The whole database is shown in Figure 4.8.

Since the number of networks is equal to the number of people in the database, fifteen networks one for each person was created. Within the eleven images, first 4 of them are used for tanning the neural networks, and then these networks are tested and updated with the next 3 pictures for getting the minimum squared error function. And these values will be used for later use for recognition of the faces.

For testing the whole database, the faces used in training, testing and recognition are changed and the recognition performance is given for whole database.

For this database, the mean face of the faces, the calculated top 30 (with highest eigenvalues) eigenfaces, and their corresponding eigenvalues are shown in Figure 4.9, Figure 4.10 and Figure 4.11 respectively.

The test results and the recognition ratios with different number of training images and w/wo histogram equalization, and different number of eigenfaces and neurons in hidden layer are given in Table 4.7 and Table 4.8 respectively. The recognition rates with neural networks first, second, third, forth and fifth choices are also given in Table 4.9

Figure 4.8 YALE Face Database

Figure 4.9 Mean face for YALE Face Database

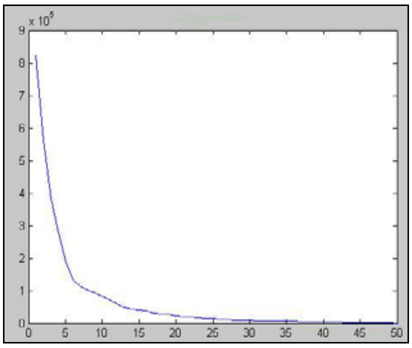


Figure 4.10 The eigen values for ORL Face Database

Figure 4.11 The top 30 eigen faces for the YALE Face Database

Table 4.7 Recognition Rate using different Number of Training and Test Images and with and without Histogram Equalization

Number of Images used in Training (per individual)	***Number of Images used in Testing (per individual)***	***Number of Eigenfaces***	***Hist. Equ.***	***Recognition Rate (%)***
1	10	10	√	16.6
1	10	10	X	12
2	9	15	√	37
2	9	15	X	33.3
3	8	30	√	54.2
3	8	30	X	45.8
4	7	30	√	75.2
4	7	30	X	71.4
5	6	30	√	94
5	6	30	X	90.6
6	5	30	√	93
6	5	30	X	88
7	4	50	√	91.6
7	4	50	X	86.7
8	3	50	√	91.1
8	3	50	X	88.8
9	2	50	√	90
9	2	50	X	90
10	1	50	√	93.3
10	1	50	X	93.3

Number of Neurons in Hidden Layer: 15

Table 4.8 Recognition Rate using different Number of Eigen Faces and neurons in Hidden Layer

Number of EigenFaces	*Number of Neurons in Hidden Layer*	*Recognition Rate (%)*
10	5	37
	10	53
	15	68.5
	20	81.2
15	5	35.1
	10	71
	15	80
	20	89
20	5	46
	10	74
	15	84.2
	20	90
25	5	51
	10	71
	15	78.8
	20	89
30	5	53.3
	10	80
	15	94
	20	95.1
35	5	46
	10	79.4
	15	95.1
	20	94.5
40	5	49
	10	80
	15	92.7
	20	94

Number of Images used in Training : 5

Number of Images used in Testing : 6

Histogram Equalization : Done for all images

Table 4.9 Recognition Rate with Neural Networks different choices

	Number of Correct Matches				
Person	**Rank 1**	**Rank 2**	**Rank 3**	**Rank 4**	**Rank 5**
1	9	1		1	
2	11				
3	10	1			
4	11				
5	11				
6	11				
7	10		1		
8	11				
9	10	1			
10	9	2			
11	11				
12	10		1		
13	10				1
14	11				
15	10	1			
Recognition Rate	94%	97.5 %	98.7 %	99.3 %	100%

Number of Testing Images : 5

Number of Eigenfaces : 30

Number of H. L. Neurons : 15

4.4. Test Results for the Yale Face Database with Eye Extraction

In this section eyes are cropped from the faces and then recognition performance is tested only with eyes. In Yale Database, all pictures have 100x100 dimensions and the left and right eye centers are placed at (30,30) and (70,30) coordinates respectively. The eye regions are cropped with the rectangle with dimension 15x68 and centered at the middle coordinates of the eyes. The eye extraction, the mean eye and the top 30 eigeneyes (with highest eigenvalues) are shown in Figure 4.12, Figure 4.13, and Figure 4.14 respectively. The test results and the recognition ratios with different number of training images, eigenfaces and number of neurons in hidden layer are given in Table 4.10 and Table 4.11 respectively. The recognition rates with neural networks first, second, third, forth and fifth choices are also given in Table 4.12

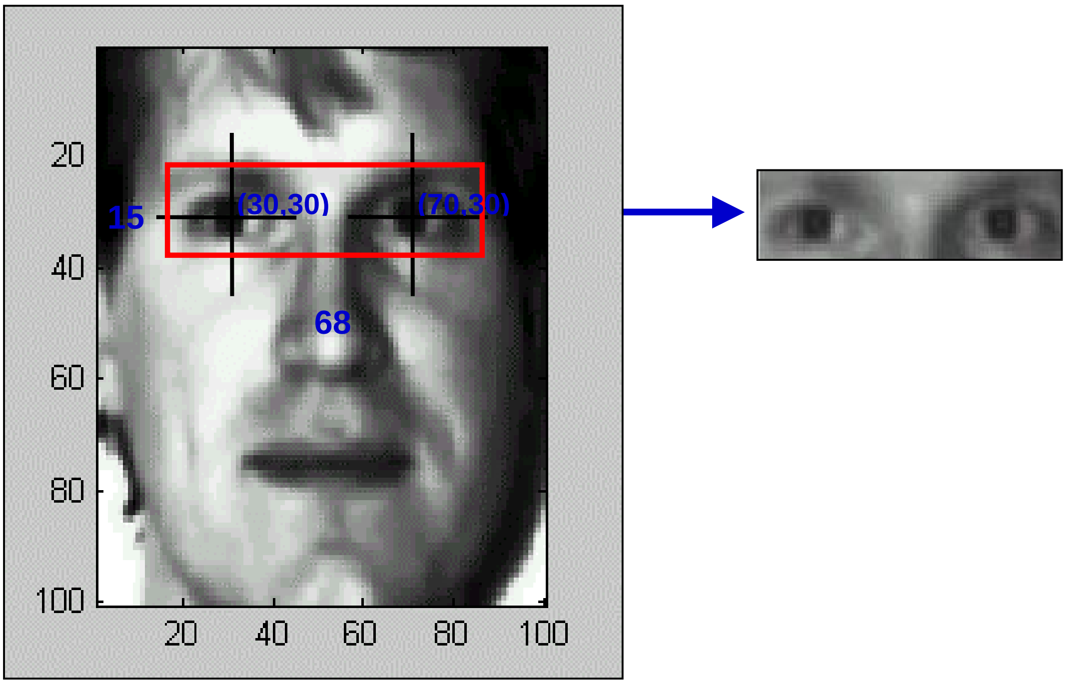


Figure 4.12 Eye Extraction

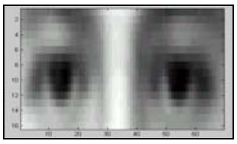


Figure 4.13 The YALE Face Database Mean Eye

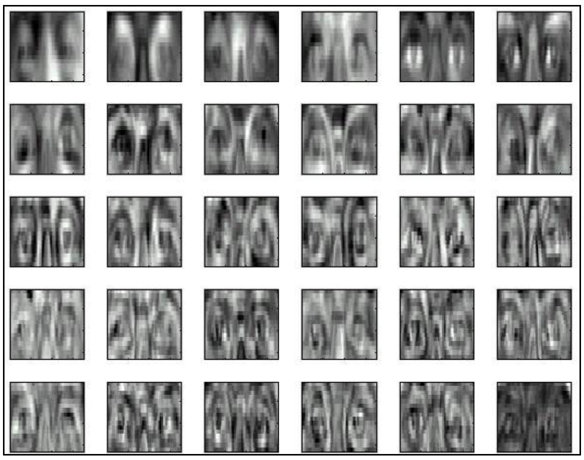


Figure 4.14 The YALE Face Database Top 30 Eigeneyes

Table 4.10 Recognition Rate using different Number of Training and Test Image

Number of Images used in Training (per individual)	***Number of Images used in Testing (per individual)***	***Number of Eigenfaces***	***Hist. Equ.***	***Recognition Rate (%)***
1	10	10	√	13.3
1	10	10	X	12
2	9	15	√	33.3
2	9	15	X	30
3	8	30	√	50
3	8	30	X	46
4	7	30	√	57.1
4	7	30	X	52.3
5	6	30	√	69.5
5	6	30	X	64
6	5	30	√	78
6	5	30	X	70.5
7	4	50	√	86.5
7	4	50	X	79
8	3	50	√	87
8	3	50	X	80.5
9	2	50	√	90
9	2	50	X	76.7
10	1	50	√	86.7
10	1	50	X	73.3

Number of Neurons in Hidden Layer: 15

Table 4.11 Recognition Rate using different Number of Eigen Faces and neurons in Hidden Layer

Number of EigenFaces	***Number of Neurons in Hidden Layer***	***Recognition Rate (%)***
10	5	35.7
	10	51
	15	62
	20	65
15	5	38
	10	49
	15	60
	20	64
20	5	41
	10	61
	15	65
	20	68
25	5	42
	10	64
	15	64
	20	69
30	5	48.5
	10	69
	15	69.5
	20	69.5
35	5	46
	10	70
	15	70
	20	73
40	5	46
	10	68
	15	70
	20	73

Number of Images used in Training : 5

Number of Images used in Testing : 6

Histogram Equalization : Done for all images

Table 4.12 Recognition Rate with Neural Networks different choices

	Number of Correct Matches			
Person	**Rank 1**	**Rank 2**	**Rank 3**	**Rank 4**
1	8	2	1	1
2	8	3		
3	7	3	1	
4	10	1		
5	6	3	2	
6	9	2		
7	8	1	2	
8	6	5		
9	8	3		
10	8	2		1
11	9	1	1	
12	7	4		
13	9	2	1	
14	6	5		
15	7	2	2	
Recognition Rate	69%	92.7 %	98.7 %	100%

Number of Testing Images : 5
Number of Eigenfaces : 30
Number of H. L. Neurons : 15

4.5. Test Results for FERET Face Database

Until recently, there was no common face recognition technology evaluation protocol which includes large databases and standard evaluation methods. The Face Recognition Technology (FERET) program sponsored by the Department of Defense's Counterdrug Technology Development Program and through the Defense Advanced Research Products Agency (DARPA) and ran from 1993 through 1997 has addressed both issues through the FERET database of facial images and the establishment of the FERET tests.

The FERET database has made it possible for researchers to develop algorithms on a common database and to report results to the literature based on this database. The results that exist in the literature do not provide a direct comparison between algorithms, since each researcher reports results using different assumptions, scoring methods, and images. The independently administered FERET test allows for a direct quantitative assessment of the relative strengths and weaknesses of different approaches.

The FERET database released in March 2001, consists of 14051 eight-bit grayscale images from 1196 individuals. The images are stored in .TIF format and as raw 8-bit data. They are 256 x 384 (width x height). Attempts were made to keep the intraocular distance (the distance between the eyes) of each subject to between 40 and 60 pixels. The images consist primarily of an individuals head, neck, and sometimes the upper part of the shoulders.

The naming convention for the FERET imagery in this distribution is of the form nnnnnxxfffq_yymmdd.ext where:

1. nnnnn is a five digit integer that uniquely identifies the subject
2. xx is a two lowercase character string that indicates the kind of imagery:

Table 4.13. Explanation of Naming Convention

Two Letter code	*Pose Angle (degrees)*	*Description*	*Number in Database*	*Number of Subjects*
Fa	0 = frontal	Regular facial expression	1762	1010
Fb	0	Alternative facial expression	1518	1009
ba	0	Frontal "b" series	200	200
bj	0	Alternative expression to ba	200	200
bk	0	Different illumination to ba	200	200
bb	+60	Subject faces to his left which is the photographer's right	200	200
bc	+40		200	200
bd	+25		200	200
be	+15		200	200
bf	-15	Subject faces to his right which is the photographer's left	200	200
bg	-25		200	200
Bh	-40		200	200
bi	-60		200	200
ql	-22.5	Quarter left and right	763	508
qr	+22.5		763	508
hl	-67.5	Half left and right	1246	904
hr	+67.5		1298	939
pl	-90	Profile left and right	1318	974
pr	+90		1342	980
Ra	+45	Random images. See note below. Positive angles indicate subject faces to photographer's right	322	264
Rb	+10		322	264
Rc	-10		613	429
Rd	-45		292	238
Re	-80		292	238

<u>*Notes:*</u>

- fa indicates a regular frontal image
- fb indicates an alternative frontal image, taken seconds after the corresponding fa
- ba is a frontal images which is entirely analogous to the fa series
- bj is an alternative frontal image, corresponding to a ba image, and analogous to the fb image
- bk is also a frontal image corresponding to ba, but taken under different lighting

- bb through bi is a series of images taken with the express intention of investigating pose angle effects (see below). Specifically, bf - bi are symmetric analogues of bb - be.
- ra through re are “random” orientations. Their precise angle is unknown. It appears that the pose angles are random but consistent. The pose angles in the table were derived by manual measurement of inter-eye distances in the image, and in their corresponding frontal image.

3. fff is a set of three binary (zero or one) single character flags. In order these denote:
    a. Indicates whether the image is releasable for publication. The flag has fallen into disuse: All images are available via this CDROM distribution, but still none may be published without the explicit written permission of the government. See the restrictions in the release agreement.
    b. Image is histogram adjusted if this flag is 1
    c. Indicates whether the image was captured using ASA 200 or 400 film, 0 implies 200.

4. q is a modifier that is not always present. When it is, the meanings are as follows:
    a. Glasses worn. Note that this flag is a sufficient condition only, images of subjects wearing glasses do not necessarily carry this flag. Some retroactive re-truthing of such images to fix this problem is warranted. See also “c” below.
    b. Duplicate with different hair length.
    c. Glasses worn and different hair length
    d. Electronically scaled (resized) and histogram adjusted.
    e. Clothing has been electronically retouched.
    f. Image brightness has been reduced by 40%

g. Image brightness has been reduced by 80%
h. Image size has been reduced by 10%, with white border replacement
i. Image size has been reduced by 20%, with white border replacement
j. Image size has been reduced by 30%, with white border replacement

Note that the modifications d through j are the result of applying various off-line operations to real images in the database; the "parent" image is that image without the "q" modifier present at all.

5. The three fields are the date that the picture was taken in year, month, and day format.
6. The filename extension is .tif. The images on the CDROMs carry an additional .bz2 suffix that indicates that the files have been losslessly compressed using the free bzip2 compressor, supplied with the database in misc/bzip2/.

The FERET tests can be conducted using the gallery (training) and probe (test) images suggested by the FERET team in the FERET CD-ROM. Table 4.9 shows the suggested gallery and probe images in the database and which probe names test which evaluation task. The gallery set is the same for all the four test setups. Such a suggestion is stated in order to standardize the test results. It is important to point out here that the gallery and probe sets use only the frontal images which are also the main concern of this thesis. Also note that the tests used a single gallery containing 1196 images. The Duplicate I probe images (dup1) were obtained anywhere between one minute and 1031 days (34 months) after their respective gallery matches. The harder Duplicate II probe images (dup2) are a strict subset of the Duplicate I images; they are those taken between 540 and 1031 days (18 months) after their gallery entries. There is usually only a few seconds between the capture of the gallery-probe pairs in facial expression evaluation probe images (fafb).

Table 4.14: The Gallery and Probe Sets used in the standard FERET test in September 1996.

Evaluation Task	**Recognized Names**	**Gallery (1196)**	**Probe Set**
Aging of subjects	Duplicate I	gallery.names	probe_dup_1_*.names (722)
Aging of subjects	Duplicate II	gallery.names	probe_dup_2_*.names (234)
Facial Expression	fafb	gallery.names	probe_fafb_*.names (1195)
Different camera and Illumination	fafc	gallery.names	probe_fafc_*.names (194)

Information about the standard FERET September 1996 gallery and probe sets is given in Table 4.9. As there is 1 image per individual in the FERET database standard gallery set, the tests are performed using this set and different gallery and probe sets constructed using some other criteria. The studied test sets [40] are as follows;

a) The standard FERET dataset: The files given in the FERET distribution CD of March 2001 are;

1. *gallery.names*: contains 1196 images from 1196 individuals.
2. *probe_fafb_expression.names*: contains 1195 images with alternative facial expressions.
3. *probe_fafc_diffcamera_diffillum.names*: contains 194 images taken with different camera and under different illumination conditions.
4. *probe_dup1_temporal_zero_to_34_months.names*: contains 722 images taken at anytime between the one minute and 34 months.
5. *probe_dup2_at_least_18_months.names*: contains 234 images taken at anytime between 18 months and 34 months. This set is a subset of the dup1 set.

b) The modified FERET dataset: This dataset shows the performance of a system trained by images with different expressions, and tested for illumination and aging changes.

1. *NewGalleryFaFb.names*: contains 1016 images from 508 individuals. First image of an individual comes from the original FERET *gallery.names* file and the second image comes from the *probe_fafb_expression.names* file
2. Same probe sets are used to test the systems, except for the *probe_fafb_expression.names* file as the images in this file are used in the training phase.

Example set of images belonging to one person is shown in Figure 4.15. As shown in Figure 4.16 the coordinates of the Left Eye, Right Eye, Nose and the Mouth are given as a separate text file for FERET Face Database. These values are used to crop the face and eye region from the entire image.

Figure 4.15 Sample Faces

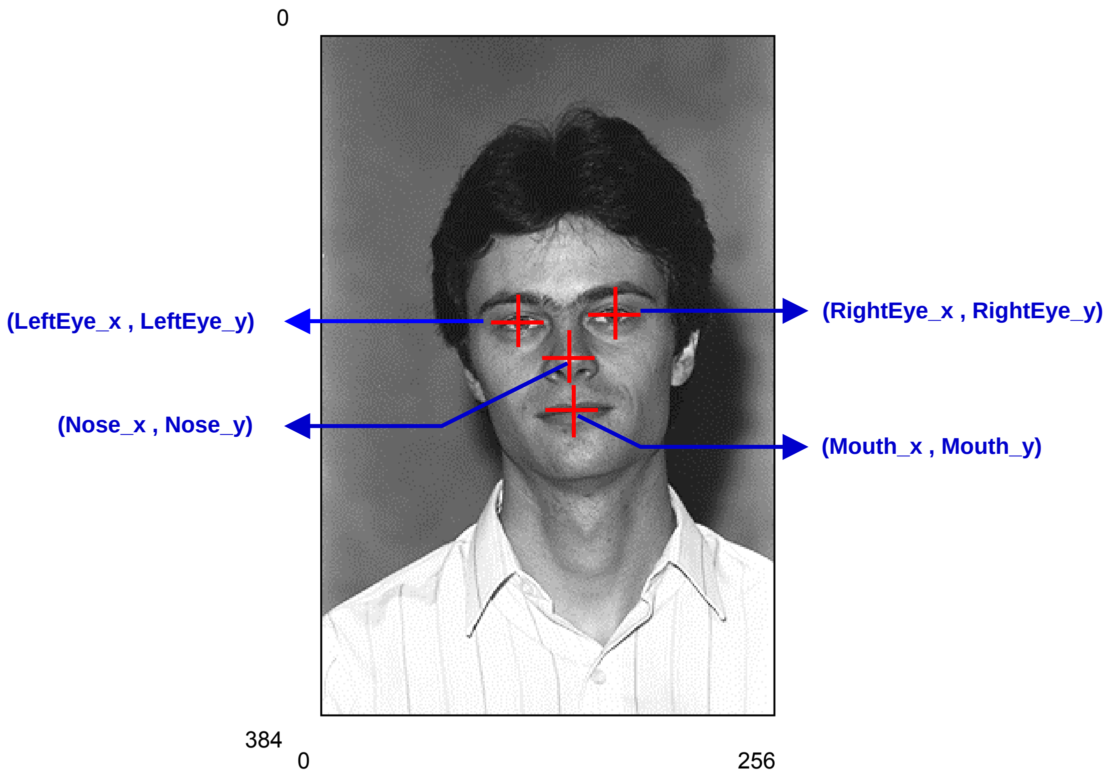


Figure 4.16 Sample Face with given coordinates

4.5.1. Preprocessing Stage

The images are preprocessed to improve the recognition performance in this stage. Firstly tilt compensation is done over the images. When the positions for the right and left eyes are known, the inverse tangent of the angle between the eye centers can be calculated. The image can be rotated using the calculated angle. then the coordinates of the eyes nose and mouth are updated according to tilt compensation

The calculation of the tilt angle is given in Figure 4.19.

The region of the image where the face is located is cut out from the image and only this area is used in the process of face recognition. The face is cropped from the entire image with using the Left Eye, Right Eye and the mouth coordinates. The cropping procedure is shown in Figure 4.20 with the width and the length of the cropping rectangle specified by means of the coordinates of the faces.

After the preprocessing stage, all the new images should have same dimension, so after cropped the face from the entire image, these new images resized to 30 x 30 pixels.

Histogram equalization enhances the contrast of images by transforming the values in an intensity image so that the histogram of the output image approximately matches a specified histogram. The histogram of an image before and after the equalization is shown in Figure 4.21.

Finally, to remove the background from the images, predefined mask [40] is applied to all faces as seen from Figure 4.22.

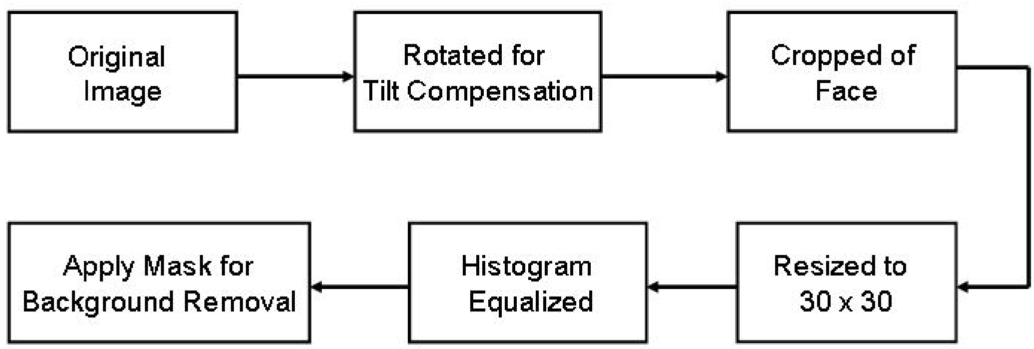


Figure 4.17 Preprocessing Stage Block Diagram

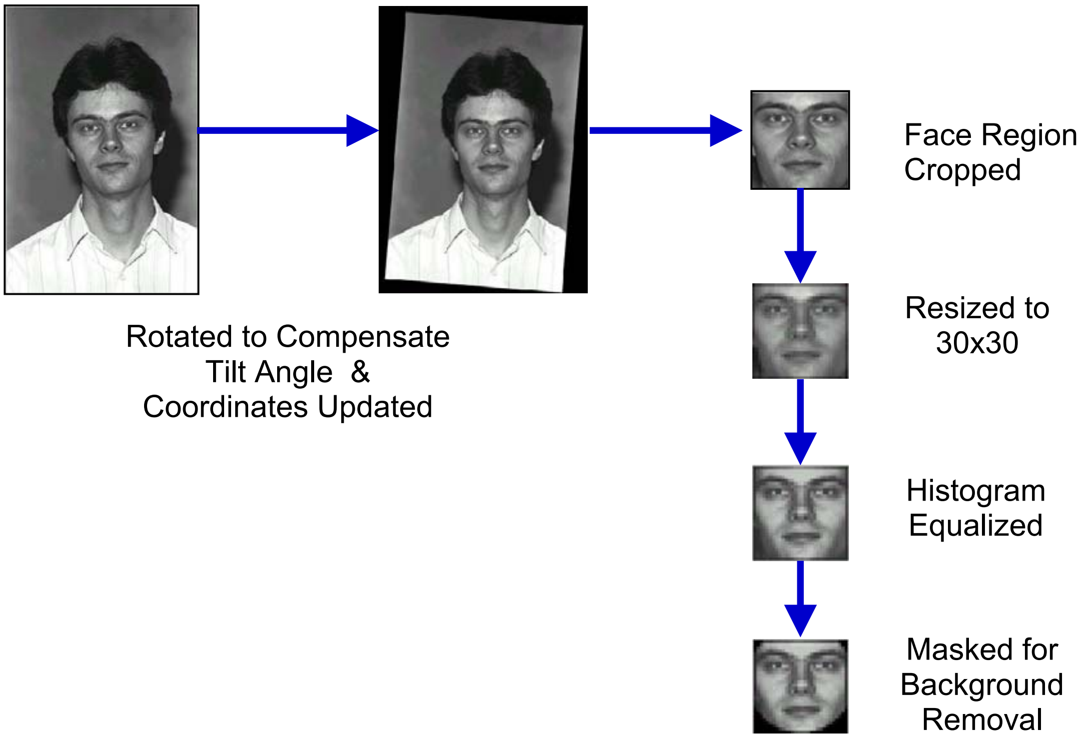
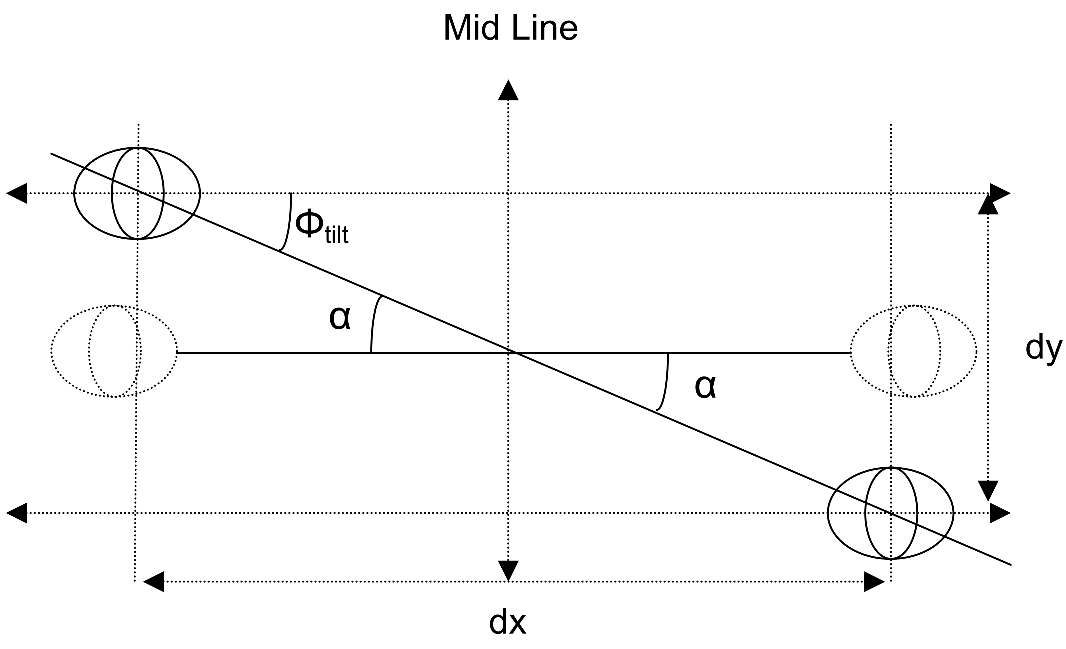


Figure 4.18 Preprocessing Stage



$$\Phi_{tilt} = \arctan\,(dy\,/dx)$$

Figure 4.19 Tilt Compensation

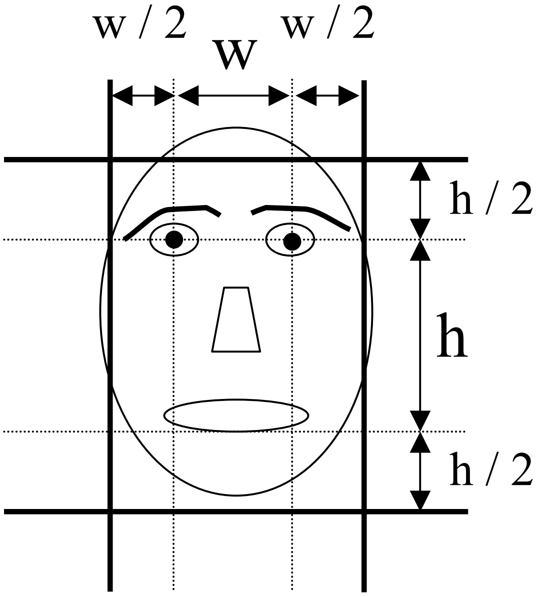


Figure 4.20 Cropping of Face Region

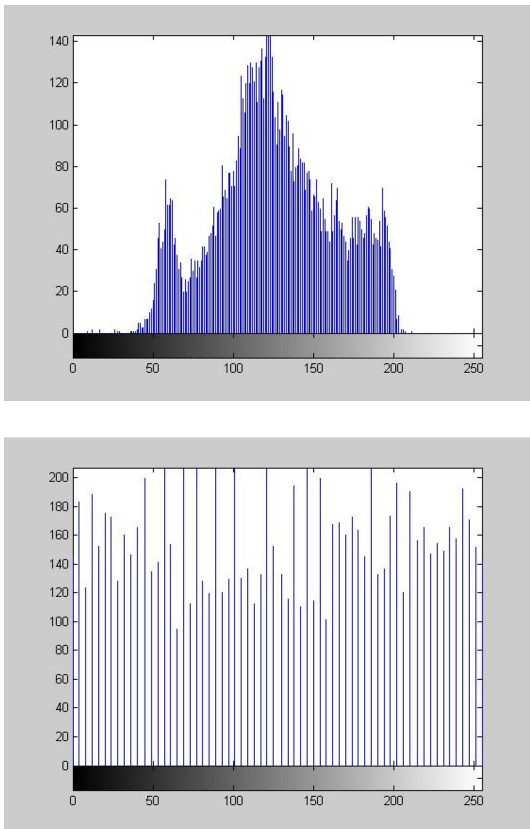


Figure 4.21. The histogram of an image before (up) and after (down) the histogram equalization.

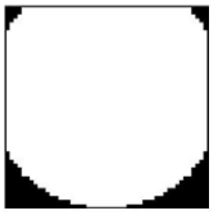


Figure 4.22 The mask used for background removal

Table 4.15 Feret Face Database Test Result

Training Images	*Testing Images*	*Number of Eigenfaces*	*Number of Neurons in H.L.*	*Hist. Eq.*	*Recog. Rate (%)*
1196 (gallery.names)	1195 (FaFb)	250	20	√	71
1196 (gallery.names)	1195 (FaFb)	250	20	X	68
1196 (gallery.names)	194 (FaFc)	250	20	√	27
1196 (gallery.names)	194 (FaFc)	250	20	X	26
1196 (gallery.names)	722 (Dup 1)	250	20	√	28
1196 (gallery.names)	722 (Dup 1)	250	20	X	25
1196 (gallery.names)	234 (Dup 2)	250	20	√	21
1196 (gallery.names)	234 (Dup 2)	250	20	X	20
1016 (FaFbMOD)	194 (FaFc)	250	20	√	24
1016 (FaFbMOD)	194 (FaFc)	250	20	X	22
1016 (FaFbMOD)	722 (Dup I)	250	20	√	36
1016 (FaFbMOD)	722 (Dup I)	250	20	X	33
1016 (FaFbMOD)	234 (Dup II)	250	20	√	13
1016 (FaFbMOD)	234 (Dup II)	250	20	X	11

4.6. Results for the FERET Face Database with Eye Extraction

The tests for Feret database done above is repeated with only the eye region of the images, not whole face, The extraction of the eye regions from the entire images and the cropping of eye region are shown in Figure 4.23 and Figure 4.24 respectively, and the related test results is given in Table 4.11.

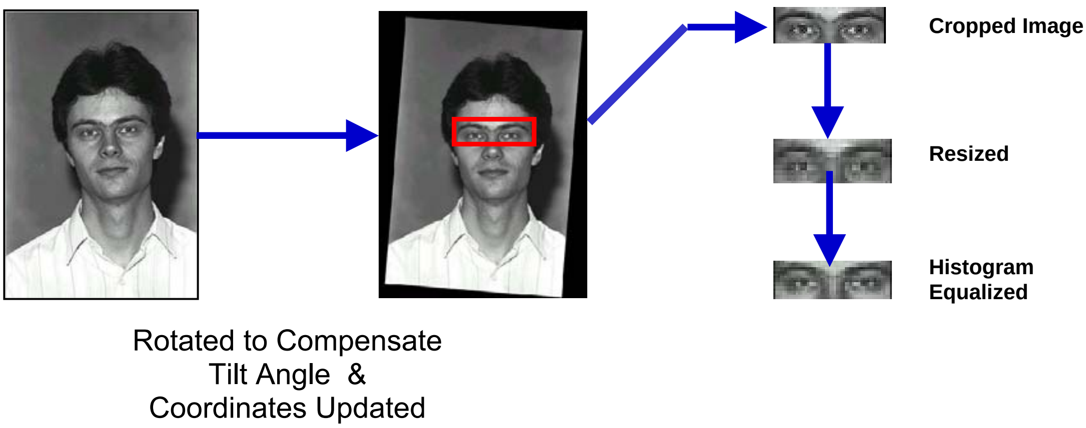


Figure 4.23 Eye Extraction

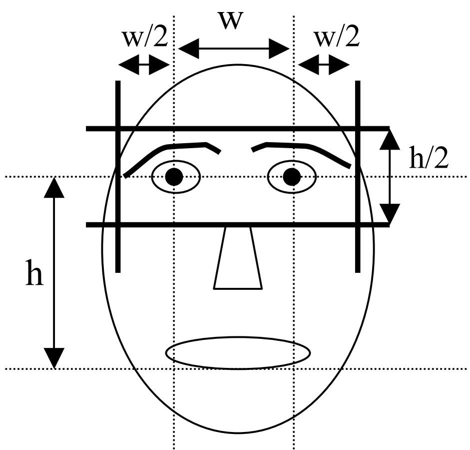


Figure 4.24 Cropping of Eye Region

Table 4.16 Feret Face Database Tets Results with Eye Extraction

Training Images	*Testing Images*	*Number of Eigenfaces*	*Number of Neurons in H.L.*	*Hist. Eq.*	*Recog. Rate (%)*
1196 (gallery.names)	1195 (FaFb)	250	20	√	54
1196 (gallery.names)	1195 (FaFb)	250	20	X	51
1196 (gallery.names)	194 (FaFc)	250	20	√	17
1196 (gallery.names)	194 (FaFc)	250	20	X	14
1196 (gallery.names)	722 (Dup 1)	250	20	√	19
1196 (gallery.names)	722 (Dup 1)	250	20	X	17
1196 (gallery.names)	234 (Dup 2)	250	20	√	20
1196 (gallery.names)	234 (Dup 2)	250	20	X	18
1016 (FaFbMOD)	194 (FaFc)	250	20	√	19
1016 (FaFbMOD)	194 (FaFc)	250	20	X	17
1016 (FaFbMOD)	722 (Dup I)	250	20	√	29
1016 (FaFbMOD)	722 (Dup I)	250	20	X	25
1016 (FaFbMOD)	234 (Dup II)	250	20	√	12
1016 (FaFbMOD)	234 (Dup II)	250	20	X	11

CHAPTER 5

CONCLUSION

In this research, two major approaches to the face recognition problem have been studied and a face recognition system based on the eigenfaces approach was proposed. Major properties of these two approaches are:

- **Feature based face recognition** makes use of the individual properties of the organs that are found on a face such as eyes, mouth and nose as well as their relationships with each other. Most common way of evaluating these features is the use of deformable templates and active contour models. Facial features are located firstly by a rough contour estimation method, and then by minimizing some energy function, exact locations are extracted. The basic characteristic of this approach is its dependency on extensive geometry.
- **Principal component analysis**, approaches to the face recognition problem by means of information theory concepts. The most relevant information that is contained in a face image is extracted. Eigenfaces method is a principal component analysis approach, where the eigenvectors of the covariance matrix of a small set of characteristic pictures are sought. These eigenvectors are called eigenfaces due to their resemblance of face images. Recognition is performed by obtaining feature vectors from the eigenvectors space and using of neural networks to compare these feature vectors.

Eigenfaces based approach excels in its speed, simplicity and learning capability.

A robust face recognition system should be insensitive to

- Changes in illumination,
- Changes in head orientation and scale,
- Presence of facial details such as glasses, beards,
- Face background.

With the principal component analysis method, face authentication, based on face and only eye region is implemented. Experimental results have shown that, the proposed face recognition method was very sensitive to face background and head orientations. Changes in the illumination did not cause a major problem to the system.

Proposed method is applied on three different databases, ORL (Olivetti Research Laboratory) Database, Yale Database and FERET (Face Recognition Technology) Database.

Firstly ORL face database is used for tests. In this database there are 10 different images of each of 40 distinct subjects. The recognition rates, with different number of eigenfaces and hidden layer neurons in neural network system are implemented. It is found that for the whole database, 5 training images per person, 50 eigenface for total database and 15 neurons in hidden layer are enough for satisfactory recognition rates (about 90 %). The eigenface method is very sensitive to head orientations, and the mismatches occur for the images with large head orientations. After these tests, the eye region are cropped from all of the images, and tests are repeated for authentication of the faces based on eye region and see that the authentication performance decreases but still remains satisfactory (about 93 %). Again the mismatches occur when there are large head orientations and when the eyes of the person are closed. The recognition results for this database are compared with the early techniques and the results are given in Table 5.1.

Table 5.1 Performance results for ORL Face Database

Method	Recognition Rate (%)
Elastic Graph Mapping	80
Line Based	97
LDA	80
Bayesian PCA	93
Proposed Method	93

Then the tests are repeated for Yale Face Database. This database is used in order to test our method in the presence of headpose variations. There are 15 different people with 11 images per subject, one per different facial expression or configuration. We get better recognition results for Yale database (about 95 %) because eigenface method is less sensitive to illumination changes and presence of small details than head orientations. Then eye regions are cropped from all images and the tests are repeated. We see that the recognition performance decreased slightly. Since the authentication is implemented only with eye region, especially images of person wearing glasses, or with closed eyes causes mismatches in these tests. Recognition results for this database with early techniques [41] are given in Table 5.2.

Table 5.2 Performance results for Yale Database

Method	Recognition Rate (%)
Euclidean Distance	60
Template Matching	85
Proposed Method	95

FERET database is the final database used during simulations. There is one image per individual in the standard FERET training set, one more modification of the standard FERET subset is also produced for the tests. The recognition rates for the FERET database is much less than for the ORL and Yale databases, because there is just one training image per person in standard, and just two training images per person in modified FERET database. Recognition results for this database with early techniques [39] are given in Table 5.3.

Arl_cor is a normalized correlation based algorithm For normalized correlation, the images were (1) translated, rotated, and scaled so that the center of the eyes were placed on specific pixels and (2) faces were masked to remove background and hair. Arl_ef is a principal components analysis (PCA) based algorithm These algorithms are developed by U.S. Army Research Laboratory and they provide a performance baseline. In the implementation of the PCA-based algorithm, all images were (1) translated, rotated, and scaled so that the center of the eyes were placed on specific pixels, (2) faces were masked to remove background and hair, and (3) the non-masked facial pixels were processed by a histogram equalization algorithm. The training set consisted of 500 faces. Faces were represented by their projection onto the first 200 eigenvectors and were identified by a nearest neighbor classifier using the L1 metric.ef_hist_dev_ang, ef_hist_dev_anm, ef_hist_dev_l1, ef_hist_dev_l2, ef_hist_dev_md, ef_hist_dev_ml1, ef_hist_dev_ml2 are seven eigenface based system from National Institute of Standards and Technology (NIST) with a common image processing front end and eigenface representation but differing in the distance metric. Algorithms are also tested from Excalibur Corp. (Carlsbad, CA), and from University of Maryland (umd_mar_97). There are two algorithms developed by MIT Media Laboratory using a version of eigenface transform; mit_mar_95 is the algorithm is the same algorithm that was tested in March 1995, algorithm retested in order to measure improvements, and mit_sep_96 is the algorithm developed since March 1995. And finally the usc_mar_97 is an elastic graph matching algorithm from Southern California University.

Table 5.3 Performance results for Feret Face Database

	Recognition Rates (%)			
	fafb	**fafc**	**DupI**	**DupII**
arl_cor	82.7	52	36.3	17.1
arl_ef	79.7	18.6	41.0	22.2
ef_hist_dev_ang	70.1	72	34.1	12.4
ef_hist_dev_anm	77.4	23.7	44.6	20.9
ef_hist_dev_11	77.2	25.8	35.0	13.2
ef_hist_dev_12	71.6	41	33.1	13.7
ef_hist_dev_md	74.1	23.2	42.2	16.7
ef_hist_dev_ml1	73.3	39.2	30.5	12.8
ef_hist_dev_ml2	77.2	30.9	34.6	12.8
excalibur	79.4	21.6	41.4	19.7
mir_mar_95	83.4	15.5	33.8	17.1
mit_sep_96	94.8	32.0	57.6	34.2
umd_mar_97	96.2	58.8	47.2	20.9
usc_mar_97	95.0	82.0	59.1	521
Proposed Method	71	28	27	21

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